RESPONSE MODELLING OF PAVEMENT SUBJECTED TO DYNAMIC SURFACE LOADING BASED ON STRESS-BASED MULTI-LAYERED PLATE THEORY

A Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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ABSTRACT

According to the Federal Highway Administration (FHWA Highway Statistics, 2004), almost $900 billion was spent on the maintenance and reconstruction of the U.S. highway system during the ten year period from 1995 to 2004. It is clear that improving the pavement analysis and design methods could result in annual savings in the millions and possibly billions of dollars.

The response models based on multi-layer elastic theory and displacement-based finite element methods are currently the most widely used and both are adopted as the structural response models in the recently released Mechanistic Empirical Pavement Design Guide (MEPDG). These models are capable of predicting global responses such as surface deflections but are not able to accurately predict the transverse stress distribution which is imperative to model the realistic behavior of in-service pavement systems and prevent premature failure caused by pavement layer debonding.

A stress-based model developed at Ohio State for composite laminates has shown the capability of accurately predicting the dynamic stresses at layer boundaries while retaining the ability to determine displacement behavior. In this study, the stress-based multi-layer plate theory was extended to layered pavement systems as an alternative to existing pavement response models for the analysis and design of pavements.
The proposed model was verified by comparing its solutions to existing analytical, numerical solutions, and experimental results. Good agreement was obtained in the predicted surface deflection response from existing analytical, numerical solutions and the stress-based model. It was shown that the current stress-based model can overcome the limitation of displacement-based method and predicted more accurate and realistic transverse stress at the pavement layer interfaces. Overall, a reasonably close prediction was obtained between calculated and measured responses from the two full-scale pavement experimental studies.

Moreover, a sensitivity study was carried out in order to obtain a better understanding of the different factors that affect the interface transverse stresses at the interface between surface layer and base layer. Finally, the stress-based model was used to analyze thin concrete overlay rehabilitation of rigid and flexible pavements.
DEDICATION

Dedicated to my family
ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. William E. Wolfe, for intellectual guidance, encouragement, and support throughout the course of this study, and for his patience in correcting my grammar and scientific errors. I also like to thank Dr. Tarunjit S. Butalia for sharing his invaluable thoughts on the stress-based discrete model and Dr. Fabian H. Tan for his suggestions and being on the examination committee for this dissertation.

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Finally, words cannot adequately express my gratitude to my family for their support, encouragement, and patience throughout this period of study and being my constant source of inspiration.
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<table>
<thead>
<tr>
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<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>$\pm a, \pm b, \pm \frac{h}{2}$</td>
<td>Plate dimensions</td>
</tr>
<tr>
<td>$B_{r}$</td>
<td>Bilinear mapping on $V_r \times V_r$</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker’s delta</td>
</tr>
<tr>
<td>$E_{ijkl}$</td>
<td>Elasticity Tensor</td>
</tr>
<tr>
<td>$\varepsilon_{kl}$</td>
<td>Symmetric strain tensor</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>Local Cartesian components of the unit normal to the surface</td>
</tr>
<tr>
<td>$\mathbf{f}_i$</td>
<td>The body force vector</td>
</tr>
<tr>
<td>$\mathbf{F}_i$</td>
<td>Generalized body force vector</td>
</tr>
<tr>
<td>$g_{(k)}^{(k)}$</td>
<td>Traction boundary conditions</td>
</tr>
<tr>
<td>$g_{u}^{(k)}$</td>
<td>Displacement boundary conditions</td>
</tr>
<tr>
<td>$J$</td>
<td>The total energy of a body</td>
</tr>
<tr>
<td>$M_{\alpha\beta}$</td>
<td>In-plane moment resultants</td>
</tr>
<tr>
<td>$M_{33}$</td>
<td>Transverse moment resultants</td>
</tr>
<tr>
<td>$N_{\alpha\beta}$</td>
<td>In-plane stress resultants</td>
</tr>
<tr>
<td>$N_{33}$</td>
<td>Transverse normal stress resultants</td>
</tr>
</tbody>
</table>
\( \nu_y \)  

Poisson’s ratio

\( \overline{v}_x^{(k)} \)  

Generalized in-plane displacements

\( \overline{v}_3^{(k)} \)  

Generalized transverse displacements

\( \Omega \)  

Linear functional

\( p_i \)  

Momentum density vector

\( \Pi \)  

Potential energy

\( \rho \)  

Mass density

\( R \)  

Open connected region in Euclidean space

\( \overline{R} \)  

Closure of \( R \)

\( S \)  

Boundary of \( R \)

\( S_{\sigma} \)  

Portion of a boundary where traction is prescribed

\( S_u \)  

Portion of a boundary where displacement is prescribed

\( S_{ijkl} \)  

Elastic compliance tensor

\( \sigma_{ij} \)  

Symmetric Cauchy stress tensor

\( \sigma_{\overline{ij}}^{z} \)  

Value of the stress component at \( \pm \frac{h}{2} \)

\( t \)  

Time variable

\( T \)  

Kinetic Energy

\( u_i \)  

Displacement vector
\( \tilde{u}_i \)  
Displacement resultant

\( \bar{u}_i \)  
First moment of the displacement

\( \hat{u}_i \)  
Second moment of the displacement

\( \overline{u}_i \)  
Third moment of the displacement

\( \hat{\overline{u}}_i \)  
Fourth moment of the displacement

\( v_1 \)  
In-plane displacement in the 1 direction at \(-a\)

\( v_2 \)  
In-plane displacement in the 2 direction at \(-b\)

\( \tilde{v}_a \)  
\( v_a \) resultant

\( \overline{v}_a \)  
First moment of \( v_a \)

\( V \)  
Total volume of a body

\( V_a \)  
Transverse shear stress resultants

\( W \)  
Strain energy density

\( \overline{\phi}_a^{(k)} \)  
Generalized in-plane displacement

\( \overline{\phi}_s^{(k)} \)  
Generalized transverse displacement

\( \psi \)  
Interpolation function

\( Z_i \)  
Initial conditions
CHAPTER 1

INTRODUCTION

1.1 Background and Problem Statements

The US highway system serves as the most critical transportation link in the economic development of this nation. About seventy-four percent of all the commodities delivered in the US are transported by trucks on highways (TRIP, 2006). As the nation’s highway infrastructure grows older, rehabilitation and reconstruction of the highway network has become increasingly important. According to the 2004 annual pavement condition survey by the Federal Highway Administration (FHWA), 68 percent of the major urban roads are in mediocre or poor condition (TRIP, 2006). Almost 900 billion dollars were spent on the maintenance and reconstruction of the highway system during the period of 1995 to 2004 (FHWA Highway Statistics, 2004). As the traffic volume continues to increase, a significant portion of the current highway system will need major
rehabilitation or reconstruction and the cost to maintain the highway system will increase correspondingly. Clearly, improvement of the current pavement analysis and design methods to produce long-lasting pavement could result in annual savings in the millions and possibly billions of dollars.

There are three types of pavements: flexible, rigid, and composite pavements. A flexible or asphalt pavement includes an asphalt concrete (AC) wearing course, an asphalt binder course underlain by one or more base and subbase layers which may or may not be stabilized. A rigid or Portland cement concrete (PCC) pavement consists of a Portland cement concrete slab, a base and a subbase layer. A composite pavement is typically composed of an AC layer as wearing surface and a PCC slab as a base. Due to its high cost, this type of pavement is rarely used as a new construction. Nearly all the composite pavements constructed are rehabilitated pavements using overlays (Huang, 2004).

![Figure 1.1 Typical Sections of Flexible and Rigid Pavements](image-url)
Pavement design methods can be classified into two broad categories: empirical methods and mechanistic methods. The empirical methods are developed based on the observed performance of actual test roads with known pavement materials and structures subjected to certain traffic and environmental loads. The mechanistic methods are based on fundamental laws of physics and strength of materials.

The empirical methods have been used as the primary design procedures for many decades. According to a report by the National Highway Institute (NHI, 2007), most of the states are still using the empirical AASHTO 1993 pavement design guide or an even older version as their design procedures. Recently, there has been a movement away from the traditional empirical design approach toward mechanistic design procedures because of the serious limitations of empirical methods including deficiencies when accommodating heavier vehicle loads, new material properties, climatic effects, and an inability to handle long-life pavement designs. The Mechanistic-Empirical Pavement Design Guide (MEPDG) has been recently completed (NHI, 2007) and released by the National Cooperative Highway Research Program (NCHRP) 1-37A in 2004. FHWA considers the implementation of the new design guide as one of its highest priorities.

The backbone of a mechanistic method is the pavement response model which is used to calculate the stresses, strains, and deflections in the pavement structure due to traffic loads and/or climatic factors. The thickness and material properties of each layer are then selected in such a way that the critical stresses and strains are smaller than the
allowable values to satisfy the pavement design life based on the fatigue properties of the materials as measured in the laboratory.

Numerous mechanistic pavement response models have been developed over the years, ranging from Boussinesq’s one-layer model, to multi-layer elastic theories to finite element models. The response models which are based on multi-layer elastic theory and displacement-based finite element methods are currently the most widely used and both are adopted as the structural response models in the MEPDG (NHI, 2002; NCHRP, 2004). These models are capable of predicting global responses such as surface deflections but are not able to accurately predict the transverse stress distribution which is imperative to model the realistic behavior of in-service pavement systems and prevent premature failure caused by pavement layer debonding.

A common assumption in most pavement response models is that all the pavement layers are fully bonded. However, the repeated localized transverse stresses included at pavement layer interfaces induced by traffic loads may result in a progressive debonding or even complete separation of layers. This debonding of the pavement layers can cause significant increases in the stresses and strains in the surface course and lead to premature fatigue failure. Khweir and Fordyce (2003) and Kruntcheva et al. (2005) have reported that debonding of pavement layers can reduce the life of pavement by more than 80 percent.
A problem analogous to the pavement layer debonding failure is the delamination in composite laminates. Due to their attractive properties, composite laminates have found applications in a wide variety of applications in the past several decades, and a large body of information exists in the literature on theoretical methods to determine static and dynamic characteristics of composite laminates. Some stress-based models developed for composite laminates have shown the capability of accurately predicting the dynamic stress and displacement behavior which can be a promising alternative method to be adapted to pavement applications.

1.2 Objectives

The main objective of this study is to develop and implement a pavement response model using a stress-based discrete layer theory capable of accurately predicting not only the deflections, but also the interface transverse stresses in pavements subject to dynamic loading.

The detailed listing of the tasks and schedule to accomplish this objective are:

1) Review the state of art of pavement response models for characterization of interface stresses and strains and elucidate the importance to improve the interface stress prediction to prevent premature failure caused by pavement layer debonding;

2) Adapt the stress-based discrete layer finite element model to a pavement
dynamic response model and calculate the interface stresses;

3) Verify the stress-based pavement model by comparing the prediction of displacement and stress from the stress-based model and traditional displacement-based model and examine the qualitative difference in the predictions from both models;

4) Conduct parametric studies of effects of layer thickness and material properties using the stress-based model;

5) Compare the predicted displacements and interface stresses with the measured dynamic responses from a previous full-scale accelerated pavement testing program and a on-going in-service experimental road;

6) Application of the stress-based pavement response model to analysis of bonded concrete overlays (BCO) and ultrathin whitetopping (UTW).
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Pavement structural analysis includes three main issues: material characterization, theoretical model for structural response, and environmental conditions.

Three aspects of the material behavior are typically considered for pavement analysis (Yoder and Witczak, 1975):

(1) the relationship between the stress and strain (linear or nonlinear);

(2) the time dependency of strain under a constant load (viscous or nonviscous);

(3) the degree to which the material can recover strain after stress removal (elastic or plastic).

Theoretical response models for the pavement are typically based on a continuum mechanics approach. The model can be a closed-formed analytical solution or a
Numerical approach. Various theoretical response models have been developed with different levels of sophistication from analytical solutions such as Boussinesq’s equations based on elasticity to three-dimensional dynamic finite element models.

Environmental conditions can have a great impact on pavement performance. Two of the most important environmental factors included in pavement structural analysis are temperature and moisture variation. Frost action, the combination of high moisture content and low temperature can lead to both frost heave during freezing and then loss of subgrade support during thaw significantly weakening the structural capacity of the pavement leading to structural damage and even premature failures. In addition, both the diurnal temperature cycle and moisture gradient have been shown experimentally and analytically to cause significant curling and warping stresses in the concrete slab of rigid pavements (NHI, 2002).

It is very complicated and impractical to consider all three issues in the highest degree of complexity in a pavement analysis. Appropriate levels of simplifications are made to one or more of the three issues while the important factors are recognized and kept in the model based on the engineer’s judgment. This study will focus on the second issue: the theoretical model for pavement analysis. Environmental conditions are not considered in the pavement model and the pavement materials are assumed to be linear elastic.
This chapter includes a literature review of the current pavement response models and related research regarding pavement layer interface condition and debonding failures.

2.2 Pavement Response Models

Flexible and rigid pavements respond to loads in very different ways. Consequently, different theoretical models have been developed for flexible and rigid pavements.

2.2.1 Response models for flexible pavements

2.2.1.1 Single Layer Model

Boussinesq (1885) was the first to examine the pavement's response to a load. A series of equations was proposed by Boussinesq to determine stresses, strains, and deflections in a homogeneous, isotropic, linear elastic half space with modulus \( E \) and Poisson’s ration \( \nu \) subjected to a static point load \( P \). Figure 2.1 shows the polar coordinate notation for Boussinesq’s equations, where \( z \) is the depth and \( r \) is the horizontal distance to the point of interest for the responses under the point load \( P \). This single layer model may be the simplest way to model a pavement structure.
Some of the Boussinesq’s equations in polar coordinates are given in Table 2.1, where $R = \sqrt{z^2 + r^2}$. 

Figure 2.1 Polar Coordinates for Boussinesq’s Equations
### Normal Stresses

\[
\sigma_z = \frac{3P}{2\pi R^2} \cos^3 \theta
\]

\[
\sigma_r = \frac{P}{2\pi R^2} \left[ 3 \cos \theta \sin^2 \theta - \frac{1-2\nu}{1+\cos \theta} \right]
\]

\[
\sigma_t = \frac{(1-2\nu)P}{2\pi R^2} \left[ -\cos \theta + \frac{1}{1+\cos \theta} \right]
\]

### Shear Stresses

\[
\tau_z = \frac{3P}{2\pi R^2} \cos^2 \theta \sin \theta
\]

\[\tau_r = 0\]

\[\tau_t = 0\]

### Normal Strains

\[
\varepsilon_z = \frac{(1+\nu)P}{2\pi R^2 E} \left[ 3 \cos^3 \theta - 2\nu \cos \theta \right]
\]

\[
\varepsilon_r = \frac{(1+\nu)P}{2\pi R^2 E} \left[ -3 \cos^3 \theta + (3-2\nu) \cos \theta - \frac{1-2\nu}{1+\cos \theta} \right]
\]

\[
\varepsilon_t = \frac{(1+\nu)P}{2\pi R^2 E} \left[ -\cos \theta + \frac{1-2\nu}{1+\cos \theta} \right]
\]

### Displacements

\[
d_z = \frac{(1+\nu)P}{2\pi RE} \left[ 2(1-\nu) + \cos^2 \theta \right]
\]

\[
d_r = \frac{(1+\nu)P}{2\pi RE} \left[ \cos \theta \sin \theta - \frac{(1-2\nu) \sin \theta}{1+\cos \theta} \right]
\]

\[d_t = 0\]

Table 2.1 Boussinesq’s Equations for a Point Load (Ullidtz, 1998)
As can be seen, the elastic modulus does not influence any of the stresses and the vertical normal stress $\sigma_z$ and shear stresses are independent of the elastic parameters.

Boussinesq's equations were originally developed for a static point load. Later, Boussinesq's equations were further extended by other researchers for a uniformly distributed load by integration (Newmark, 1947; Sanborn and Yoder, 1967). Although Boussinesq’s equations are seldom used today as the main design theory, his theory is still considered a useful tool for pavement analysis and it provides the basis for several methods that are being currently used.

Yoder and Witczak (1975) suggested that Boussinesq theory can be used to estimate subgrade stresses, strains, and deflections when the modulus of base and the subgrade are close. Pavement surface modulus, the equivalent “weighted mean modulus” calculated from the measured surface deflections based on Boussinesq’s equations, can be used as an overall indicator of the stiffness of pavement (Ullidtz, 1998).

The Method of Equivalent Thickness (MET) is based on work by Odemark (1949). Odemark assumed the deflections of a multi-layer pavement with layer moduli, $E_i$, and corresponding layer thickness, $h_i$, may be transformed into an equivalent single layer system with thickness, $H$, and modulus, $E_0$, if the thickness is chosen to be

$$H = \sum C_i h_i \left(\frac{E_i}{E_0}\right)^{1/3}$$
where

\[ C = \text{constant, approximately 0.8 to 0.9.} \]

After transformation, the stresses, strains, and displacements can be estimated in the one-layer half-space system using Boussinesq’s equations.

### 2.2.1.2 Burmister’s Two-layer Elastic Models

Pavement systems typically have a layered structure with stronger/stiffer materials on top instead of a homogeneous mass as assumed in Boussinesq’s theory. Therefore, a better theory is needed to analyze the behavior of pavements. Burmister (1943) was the first to develop solutions to calculate stresses, strains and displacement in two-layered flexible pavement systems (Figure 2.2).

The basic assumptions for all Burmister’s models include:

1. The pavement system consists of several layers; each layer is homogeneous, isotropic, and linearly elastic with an elastic modulus and Poisson’s ratio (Hooke’s law);
2. Each layer has a uniform thickness and infinite dimensions in all horizontal directions, resting on a semi-infinite elastic half-space;
3. Before the application of external loads, the pavement system is free of stresses and deformations;
4. All the layers are assumed to be weightless;

5. The dynamic effects are assumed to be negligible;

6. Either of the two cases of interface continuity boundary conditions given below is satisfied (Figure 2.3):

   (1): fully bonded: at the layer interfaces, the normal stresses, shear stresses, vertical displacements, and radial displacements are assumed to be the same. There is a discontinuity in the radial stresses $\sigma_r$ since they must be determined by the respective elastic moduli of the layers;

   (2): frictionless interface: the continuity of shear stress and radial displacement is replaced by zero shear stress at each side of the interface.
Figure 2.2 Burmister’s Two Layer System (Burmister, 1943)

Figure 2.3 Boundary and Continuity Conditions for Burmister’s Two Layer System

(Burmister, 1943)
Burmister derived the stress and displacement equations for two-layer pavement systems from the equations of elasticity for the three-dimensional problem solved by Love (1923) and Timeshenko (1934). To simplify the problem, Burmister assumed Poisson’s ratio to be 0.5. He found the stresses and deflections were dependent on the ratio of the moduli of subgrade to the pavement ($E_2/E_1$) and the ratio of the radius of bearing area to the thickness of the pavement layer ($r/h_1$). For design application purpose, equations for surface deflections were also proposed:

1. Flexible load bearing:

$$ W = 1.5 \frac{pr}{E_2} F_w $$

2. Rigid load bearing:

$$ W = 1.18 \frac{pr}{E_2} F_w $$

where: $W$: the surface deflection at the center of a circular uniform loading;

$p$: pressure of the circular bearing;

$E_2$: elastic modulus of the subgrade layer;

$F_w$: deflection factor.

Influence curves of deflection factor were proposed for a practical range of values of these two ratios.
2.2.1.3 Multi-layer Elastic Models

To attain a closer approximation of an actual pavement system, Burmister extended his solutions to a three-layer system (Burmister, 1945) and derived analytical expressions for the stresses and displacements. In his series of papers, Burmister did not publish any stress values. However, based on Burmister’s theory, Acum and Fox (1951) presented an extensive tabular summary of normal and radial stresses in three-layer systems at the intersection of the axis of symmetry with the interfaces. The variables considered in their work were the radius of the uniformly loaded circular area, the thickness of the two top layers, and the elastic moduli of the three layers. Jones (1962) extended Acum and Fox’s work to cover a much wider range of the same parameters. Peattie (1962) presented Jones’s table in graphical form and brought convenience in analysis and design of pavement for engineers before the modern computer was widely available.

The above cited research considered the pavement to be either a two or three layer system with a concentrated normal force or a uniformly distributed normal load. Therefore, vehicle thrust (tangential loads) and non-uniform loads were not considered. Moreover, to simplify the problem, Poisson’s ratio of 0.5 was assumed in most cases. Schiffman (1962) developed a general solution to the analysis of stresses and displacements in an N-layer elastic system. His solution provides an analytical theory for the determination of stresses and displacements of a multi-layer elastic system subjected
to non-uniform normal surface loads, tangential surface loads, rigid, semi-rigid and slightly inclined plate bearing loads. Schiffman presented the equations in an asymmetric cylindrical coordinate system (Figure 2.4). Each layer has its separate properties including elastic modulus (E<sub>i</sub>), Poisson’s ratio (ν<sub>i</sub>), and thickness (h<sub>i</sub>).

Figure 2.4 Element of Stress in a Multi-layer Elastic System (Schiffman, 1962)
Schiffman’s analytical solutions required quite extensive computations and could not be directly applied in engineering practice. However, with the advent of modern computers, the multi-layer elastic theory was used to analyze pavement systems with any number of layers using Schiffman’s solution. Many elastic layered computer programs have been developed based on the multi-layered elastic theory since then. Some of the
programs that have been widely used in pavement analysis and design are CHEVRON, BISAR, ELSYM5, KENLAYER, and WESLEA.

The CHEVRON program was developed by the Chevron research company and is based on linear elastic theory. The original program allowed up to five structural layers with one circular load area (Michelow, 1963). Revised versions now accept more than 10 layers and up to 10 wheel loads (NHI, 2002).

Bitumen Structures Analysis in Roads, or BISAR, was developed by the Royal/Shell Laboratory in 1973 (De Jong et al. 1973). It is based on linear elastic theory and allows layer bonding by using an interface friction parameter. BISAR is capable of modeling of the horizontal loading condition caused by vehicle braking.

ELSYM5 was developed by FHWA to analyze pavement structures up to five different layers under 20 multiple wheel loads (Kopperman et al., 1986).

The KENLAYER program can model the pavement layers as being either linear elastic, nonlinear elastic, or viscoelastic. It allows up to 19 layers to be modeled. However, the program is restricted to only one circular load area (Huang, 1993).

WESLEA is a multi-layer linear elastic program developed by the U.S. Army Corps of Engineers Waterways Experiment Station (Van Cauwelaert et al., 1989). The current versions have the capability of analyzing more than ten layers with more than ten loads.
2.2.1.4 Other Models

Several numerical programs have been developed to model flexible pavement systems. Raad and Figueroa (1980) developed a 2-D finite element program called ILLI-PAVE to model the flexible pavement behaviors. Nonlinear constitutive relationships were used for pavement materials and the Mohr-Coulomb theory was used as the failure criterion for subgrade soil in ILLI-PAVE.

Siddharthan et al. (2000) proposed a continuum-based finite-layer model called 3D-MOVE to evaluate the dynamic response of flexible pavements. The model can consider some important response factors such as vehicle speed, viscoelastic material properties, and complex contact stress distributions. Good agreement was found between the calculated tensile strain at the bottom of an asphalt concrete (AC) layer with that measured in several field studies.

2.2.1.5 Summary

Multi-layer elastic theory is still the most commonly used pavement response model for flexible pavements today (NCHRP, 2004; NHI, 2002). A modified version of WESLEA, referred as JULEA, has been integrated into the recently released Mechanistic
Empirical Pavement Design Guide (MEPDG) as the major response model for flexible pavements (NCHRP, 2004). Although a 2-D finite element program is included in the MEPDG, it is recommended only for research purpose, not design.

2.2.2 Response Models for Rigid Pavements

Because of concrete’s high elastic modulus, the PCC slab supplies most of the structural capacity, and tends to transfer the traffic loads to a relatively wider area than does asphalt, producing a very different stress distribution from the one generated by a flexible pavement (WSDOT, 2003). Furthermore, variable slab sizes, the presence of different types of discontinuities (e.g., longitudinal, transverse joints), a variety of load transfer mechanisms (e.g., dowel bars, aggregate interlocks), and high sensitivity to environmental conditions (e.g., temperature curling, moisture warping) make the analysis of rigid pavement a more complicated and challenging problem. The multi-layer elastic theory is generally not considered an appropriate tool for rigid pavement response analysis (Ullidtz, 1987).
2.2.2.1 Westergaard’s Analytical Solution

The early advances in rigid pavement analysis started in the 1920s. In 1926, Westergaard derived closed form analytical solutions for stresses and deflections due to thermal curling and traffic loading in jointed rigid pavements (Westergaard, 1926a; 1926b). To simplify the problem, he assumed that the subgrade can not transfer shear stresses, i.e., Winkler foundation condition. The subgrade is characterized by a single parameter, the modulus of subgrade reaction or the $k$ value. The vertical pressure of the subgrade to the concrete slab is a constant which equals to subgrade reaction ($k$) times the vertical deflection. The following assumptions were made in Westergaard’s original work (Westergaard, 1926a):

1) The concrete slab acts as a homogeneous, isotropic, elastic solid in equilibrium;

2) The classic Kirchhoff plate theory is assumed for the concrete slab, i.e., the transverse shear stresses are ignored;

3) The reaction of the subgrade is only vertical and is proportional to the deflection of the slab;

4) The concrete is resting on a set of springs with the spring constant $k$, independent of the slab deflection;

5) The thickness of the slab is uniform;

6) Three loading conditions are considered: interior loading, corner, and edge;
7) The loading pressure is assumed to be distributed uniformly over a circular or semi-circular area with radius $a$ (Figure 2.6);

8) The slab is only subjected to one load.

Based on Westergaard’s work (1926a; 1926b; 1947), the curling and load induced stresses as well as deflections under these three loading locations can be calculated.
Westergaard’s equations have been computerized (Packard, 1968; Darter, 1987) and the problems can be solved considering a wide variety of variables. Because of the highly idealized assumptions, Westergaard’s analytical solutions have too many limitations to model realistic pavement behaviors. For instance, the inability to predict responses at arbitrary locations could be a serious impediment in the response analysis of special loading conditions. For cases where span-to-depth ratio is less than 100, the Kirchhoff assumptions lead to errors in predicting edge stresses (Hammons and Metcalf, 1999). Furthermore, the inability to handle multiple wheel loads is also a serious constraint in application.

2.2.2.2 Improved Models Based on Westergaard’s Theory

Since Westergaard’s original work, some researchers have made improvements on Westergaard’s theory. Pickett and Ray (1951) developed influence charts that allow the Westergaard equations to be applied to multiple wheel loadings. Two cases were considered in their study: the Winkler (dense liquid) subgrade and the elastic solid subgrade. Pickett and Ray’s influence charts have been used by the Portland Cement Association (PCA) for rigid pavement design. The charts for interior loading were used
for the design of airport pavements (PCA, 1955) while the edge loading charts were used for the design of highway pavements (PCA, 1966).

Salsilli et al. (1993) applied the Newton-Raphson iteration procedure to convert multiple wheel loadings to an equivalent single loaded area that would produce the same bending stress and used this transformed loading in Westergaard’s equations. Three wheel load configurations were considered: dual, tandem and tridem.

### 2.2.2.3 Finite Element Models

Although closed-form analytical solutions are very desirable for practicing engineers in routine pavement analysis and design, the assumptions made to develop those solutions place too many limitations on the application. To overcome the limitations of analytical solutions, the finite element method has become a widely used tool for rigid pavement analysis since the early 1970s.

Wang and his colleagues (1972) studied the responses of rigid pavements to wheel loads using a two dimensional (2-D) linear elastic finite element model. The concrete slab was modeled with medium-thick plate elements assuming the classical plate theory based on Kirchhoff hypothesis. The foundation was treated as an infinite elastic half space, for which the stiffness matrix of the foundation was obtained by inverting the flexibility
matrix obtained Boussinesq's equation for surface deflection. Stresses and deflections computed with the finite element model were compared with those from Westergaard’s equations. The comparison showed that the analytical solutions give lower stresses and deflections.

Huang (1974) presented another 2-D elastic finite element model for rigid pavements. In his model, the foundation was modeled as an elastic continuum and the effect of load transfer from adjacent slabs and loss of contact were considered as well. The model was verified by comparing the response with analytical solutions and in-situ pavement response measured during the Arlington Road Test (Huang, 1974).

Following the early developments of 2-D elastic finite element models, Tabatabaie and Barenberg (1978, 1980) developed a more general 2-D finite element program called ILLI-SLAB. The concrete slab was modeled using medium-thick elements like earlier models but the effect of bonded or unbonded base layer could be incorporated using a second layer of plate elements below the slab. The subgrade was modeled as Winkler foundation and dowel bars at joints were modeled as discrete bar elements. The results were compared with analytical solutions and the field data from the AASHO Road Test.

The 2-D finite element program developed by Huang and Wang (1973) were modified and extended by Chou (1981) at the U.S. Army Engineer Waterways Experimental Station. Two programs were developed: WESLIQID and WESLAYER. Both programs were based on the classical medium-thick plate theory. The main
difference between the two was the subgrade modeling. In WESLIQID the subgrade was modeled as a Winkler foundation, while in WESLAYER, the subgrade was idealized as either a linear elastic solid or a linear elastic layered system. The partial or full loss of subgrade support over designated region of the slabs could be considered in the WESLAYER model.

Huang (1983), Huang and Deng (1985) also extended the earlier model KENSLABS to include the capability of modeling multiple slabs and various load transfer mechanisms in a manner similar to the ILLI-SLAB. The subgrade was characterized as an elastic half space. The loss of subgrade contact and the effects of mesh refinement were also studied.

A finite element program called RISC was developed as part of a mechanistic design procedure for rigid pavements for FHWA (Majidzadeh et al. 1984). In this program, the concrete slab was modeled using thin elastic shell element assuming Kirchhoff’s theory. In RISC, the slab foundation could be modeled as a Boussinesq elastic solid or as Burmister’s multilayer structure. The pavement materials were modeled as linearly elastic.

Tayabji and Colley (1984) developed a 2-D finite element program called JSLAB to analyze jointed reinforced concrete pavements. The slab was modeled using medium-thick plate elements. The subgrade was modeled as a Winkler foundation. The program had the capability of modeling partial contact between slab and subgrade. Non-uniformly spaced dowels were also considered.

Tia et al. (1987) developed a 2-D finite element program named FEACONS (Finite
Element Analysis of CONcrete Slabs) to analyze the response of jointed concrete pavements to load and temperature variations. Similar to other 2-D finite element program, the concrete slab was modeled using medium-thick plate elements while the subgrade was assumed as a Winkler foundation. The load transfer between slabs was modeled by means of linear joint stiffness and torsional joint stiffness. The results were verified by comparing the results with field data from test roads in Florida.

Krauthammer and Western (1988) investigated the effects of shear transfer on pavement behavior using a 2-D plane strain dynamic finite element model developed in the commercially available finite element program ADINA. Variations in shear and horizontal stresses in the concrete slab, joint efficiency, deflections adjacent to the joints, and the shear stresses in the subgrade were examined. The results were compared with the Falling Weight Deflectometer (FWD) tests.

Ioannides and Donnelly (1988) examined the effect of subgrade nonlinearity using a existing 3-D finite element program called GEOSYS. Linear 8-node brick elements were used to model the slab and subgrade, with varying degree of mesh refinement. Their vertical mesh refinement studies showed that sufficient accuracy was achieved when two layers of elements were used for the slab. Interior, edge and corner loading conditions were examined and the results were compared with analytical solution and 2-D analyses reported by Ioannides et al. (1984). Comparison showed that relatively close agreement
was achieved for slab deflections while the discrepancies in the stresses in the slab were found to be greater. The stresses in the subgrade were found to be quite significant.

Channakeshava et al. (1993) developed a 3-D, nonlinear static finite element model to study the 3-D response of plain concrete with dowelled joints. The slab were modeled with 20-node, quadratic isoparametric brick elements. The subgrade was modeled as a Winkler foundation with three discrete linear springs at each node on the base of the slab. Both wheel loads and thermal loading induced by diurnal temperature cycling were considered in their study.

Zaghloul et al. (1994) investigated the load equivalent factors by using a 3-D, nonlinear dynamic finite element model with the commercially available finite element program ABAQUS. The slab and subgrade were modeled with 3-D brick elements. The concrete was modeled as a bilinearly elastic-plastic solid. The granular base, subbase and subgrade were modeled with an elastic-plastic Druker-Prager model. The clayey subgrade was modeled using a Cam-Clay model. The predicted deflections for the static loading condition were compared with those determined from Westergaard’s analytical solution and a separate finite element program to verify the model. Different thickness and subgrade conditions were examined. The dynamic modeling capability of the program was verified by comparing the slab response with the data from in-situ monitoring.

Chatti et al. (1994) extended the existing static 2-D model ILLI-SLAB to a linear dynamic finite element program, called DYNA-SLAB, to study the effects of dynamic
loading applied by trucks on the response of rigid pavements. The concrete slab was modeled with plate elements and the foundation was treated as a Winkler foundation or a layered viscoelastic medium over a rigid or an infinite half-space. An analytical method was developed by the authors to determine the frequency dependent stiffness and damping coefficients used in the Winkler model. The model was verified by comparing the results to the analytical solution for a transient point load on a beam of finite length supported by a viscoelastic Winkler foundation, and to an approximate solution for a point load on an infinite plate by a Winkler foundation. A parametric study was carried out using the model to investigate whether a dynamic analysis is necessary for rigid pavement response prediction. The parameters included vehicle velocity, pavement roughness, pavement layer thickness, and load transfer efficiencies. The authors concluded that it is important to use expected pavement roughness to determine the peak magnitudes and locations for truck wheel loading history but the dynamic effects on the slab response are not significant. Moreover, the author pointed out that the existence of a shallow stiff layer or bedrock may increase the dynamic effects enough to require them to be considered in the analysis.

Uddin et al. (1995) reported a study of the effect of pavement discontinuities on surface deflections of a rigid pavement subjected to a standard FWD load using a 3-D elastic finite model with the general purpose finite element program ABAQUS. The concrete slab, cement-treated base, and subgrade were modeled using 3-D elastic brick
elements. Cracks in the pavement were modeled using gap elements while dowels were modeled with beam elements. Backcalculation of the pavement layer moduli was carried out based on the deflection data from static load-deflection test and FWD results for both cracked and uncracked pavement sections. The model was capable of accurately predicting the deflection response of the slab subjected to the standard FWD loading using the backcalculated moduli.

Kuo et al. (1995) developed a 3-D elastic finite element model using ABAQUS to investigate the various factors affecting rigid pavement support including base thickness and stiffness, interface bonding, slab curling and warping due to temperature and moisture gradient, load transfer at the joints and lane widths. Significant effort was made to determine the optimum mesh refinement and best element type. Both 2-D plate elements (4-noded and 8-noded) and 3-D brick elements (8-noded and 20-noded) were considered for the slab modeling. The cement-treated base was modeled using another layer of 3-D elements. The subgrade was treated as a Winkler foundation. The interface between layers was modeled using a membrane element coupled with a special interface element. The model was verified by comparing the model predictions with both Westergaard’s analytical solution and the predictions from the 2-D finite element model ILLI-SLAB. Good agreement was found for the cases when 2-D plate elements were applicable, i.e., thin slab with fairly large loading area. The predicted pavement responses from the model were also compared with full-scale field test data from the AASHO Road
Tests, PCA tests on cement-treated bases, and the Arlington Road Tests. Generally good agreement was found between the model predictions and the field test data.

Zaman and Alvappillai (1995) studied the effect of moving aircraft loads on a jointed multi-slab rigid pavement system using a 2-D dynamic finite element model. The pavement slabs were modeled using 4-noded, rectangular medium-thick plate elements. The subgrade was treated as a viscoelastic Winkler foundation. The dynamic interaction between the aircraft and pavement was modeled using a parallel spring and dashpot with an associated mass. Longitudinal joints were modeled using discrete displacement springs while transverse joints were modeled as doweled, and debonding and gaps between dowels and slabs were allowed. Significant effort was made to study the effect of dynamic vs. static loading, and the effects of dowel looseness on joint-performance. Based on the results, Zaman and Alvappillai concluded that static loading conditions are more critical for determining pavement thickness while dynamic conditions lead to larger losses in joint efficiencies due to dowel looseness.

Masad et al. (1996) developed a 3-D finite element model using ABAQUS to examine the response of rigid pavements to the thermal loading. Both the slab and subgrade were modeled with 8-noded brick elements. The slab and foundation were both assumed to be linearly elastic. Longitudinal joints, friction and loss of contact between the slab and foundation were considered in the analysis. Both linear and nonlinear temperature gradients were examined. Results from the model were compared with those
from existing 2-D finite element models including ILLI-SLAB and JSLAB. Maximum curling stresses predicted from the 3-D model and the existing 2-D models were found to be in reasonable agreement.

In another study reported by Kim et al. (1997), a 3-D elastic finite element model was developed using ABAQUS to analyze the response of a single rigid pavement slab to the heavy multiple-wheel loading applied by aircraft landing. The slab, cement-treated base, and subgrade were modeled using linear hexahedral elements. Bonded and unbonded bases were considered in the analyses. A linearly elastic and a nonlinear Drucker-Prager elastoplastic constitutive law were used for the base layer. All other layers in the pavement were modeled as linearly elastic. Different wheel configurations were examined including single, dual, and triple tandem axle loading. It was found that the maximum deflections were generally proportional to the total load regardless of wheel configuration. However, the maximum stresses in the slab were governed by the curvature of the slab, which greatly depends on the wheel spacing. In general, the smaller wheel space, the higher greater stresses due to the high interaction between the wheels.

Brill et al. (1997) developed a 3-D static finite element model for rigid pavements using the public domain finite element program NIKE3D. Unlike most 3-D finite element models, the slab was modeled with 4-noded plate elements while the subgrade was modeled using linear 8-noded hexahedral elements. Different types of joints between slabs were considered, including aggregate interlock and dowel shear transfer mechanism.
modeled with linearly elastic hexahedral elements. The predicted stresses were compared with analytical solutions. Significant differences between the two solutions were observed in most cases.

Another 3-D static finite element model, EVERFE, was developed by Davids (1998) to model the response of jointed plain concrete pavement systems to wheel loads and environmental effects. The slab, base, and subgrade were modeled using 20-noded quadratic hexahedral elements. All pavement layers were treated as linearly elastic materials. A Winkler foundation was modeled below the subgrade using an 8-noded quadratic interface element. The dowel and aggregate interlock mechanisms were modeled with specialized elements and constitutive relations. Linear, bilinear, and trilinear thermal gradients through the slab thickness were simulated. The model was verified by comparing the predicted displacements and strains with the measured values from scaled-model tests of dowelled rigid pavement systems in laboratory. Reasonable agreement was found between the predicted and measured displacements. Prediction of the strains was found less accurate.
2.2.2.4 Summary

Two main types of pavement response models have been developed for rigid pavements since the early 1920s: closed-form analytical solutions and finite element models. The analytical solutions including influence diagrams based on the work of Westergaard can be used to calculate responses under limited loading conditions without load transferring mechanism. Finite element models can describe more realistic pavement behaviors such as pavement discontinuities, multi-wheel loads with non-uniform load distribution. Numerous finite element programs are available for rigid pavement analysis. These programs can be divided into general-purpose finite element programs and finite element codes developed specifically for analysis of pavement systems. The programs from the first group are either commercial (e.g., ABAQUS) or public domain (e.g., NIKE3D) general purpose finite element programs. They are more powerful and capable of modeling 3-D nonlinear dynamic analysis. However, the application of these programs requires considerable expertise in engineering mechanics from the user and each analysis demands a large amount of computation time. The applications of these programs are usually limited to research purposes. The programs from the second group are expedient and incorporate most of the expertise needed for modeling rigid pavements and therefore are more practical. A summary of the available finite element programs specifically for rigid pavements is presented in Table 2.2. The majority of these programs use 2-D finite
elements and consider static loading conditions. They are considered to be the state-of-practice methods in rigid pavement analyses (NCHRP, 2004). An updated version of the 2-D finite element program ILLI-SLAB, called ISLAB2000, has been integrated into the new Mechanistic-Empirical Pavement Design Guide as the rigid pavement structural response model.

All the finite element models that have been developed for rigid pavement analysis are based on displacement formulations. In a displacement based finite element model, the displacement functions are assumed. The displacements at the nodes of the elements are calculated first as the primary variables. Then the stresses and strains, which are more important for design purposes, are calculated by numerically differentiating the approximate solutions. In a pavement structure, the interfaces between layers are usually locations of large stress and strain gradients because of the discontinuity in material properties. Although accurate displacement and in-plane stress distributions can be predicted, predictions of transverse stress distributions ($\sigma_{13}$, $\sigma_{23}$, $\sigma_{33}$) across the pavement thickness are generally not accurate due to the inherent limitations of displacement based approaches. The localized interface transverse stresses can contribute to pavement layer progressive debonding or even complete separation. A better response model is needed to improve the prediction of stresses distribution.
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<td>Static</td>
<td>Displacement</td>
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<td>Winkler</td>
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<td>Static</td>
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<td>Static</td>
<td>Displacement</td>
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</table>

Table 2.2 Summary of Available Finite Element Programs for Rigid Pavement Analysis
2.3 **Pavement Layer Debonding**

A pavement structure consists of several layers each made of different materials. To simplify the modeling and computation, a common assumption used in the current pavement response models is that all the layers are fully bonded to each other. Therefore, to achieve the design pavement life, good bonding between the pavement layers must be guaranteed. A typical technique used for assuring the bonding between the flexible pavement layers is spreading an asphaltic tack coat over an underlayer before paving. Unfortunately, the interface adhesion and bonding strength are unknown functions of the amount and type of the tack coat, layer materials, traffic loading, temperature, and time (Kruntcheva et al. 2006; Kennedy and Lister 1980; Peattie 1980; Pell 1980; Pös et al. 2001). The layer bonding strength may not be enough to resist the localized interface stresses and may lead to progressive debonding of pavement layers or even complete separation.

Once debonding occurs, pavement performance and service life can be seriously compromised. Tschegg et al. (1995) used an analogy between the bearing capacity of a multilayered beam to illustrate the importance of pavement layer bonding (Figure 2.7). It was demonstrated that the deflection of a beam that consists of several well-bonded layers acting as a whole was nine times lower than the deflection of a debonded beam. In
addition to the reduction in the structural bearing capacity, the debonding between the surfacing layers reduces the serviceability of the road and seriously affects its ride quality.

Debonding of the pavement layers has been recognized as a cause of premature failure of pavements for a long time (Livneh and Schlarsky, 1962). With the introduction of thin overlay rehabilitation methods (e.g., mill and overlay, thin or ultra-thin whitetopping, bonded concrete overlay), the problem of pavement surface layer debonding problem has increased. As cited by Kruntcheva et al. (2005), a number of premature debonding failures on recently overlaid pavements were reported by the Northern Ireland government agent (Shaat, 1992). Lepert et al. (1992) estimated that 5%
of the French highway network suffered severe damage as the result of pavement debonding problem in 1989. In the US, debonding has been identified as a major distress form in overlaid concrete pavements (Hutchinson, 1982). Tayebali et al. (2000) reported that pavement sections in Division 13 of the North Carolina Department of Transportation were experiencing excessive distress such as large scale shoving and pushing of the asphalt concrete mat as a result of delamination in the AC surface layer. Considerable effort has gone in to investigating the pavement debonding failures in the last three decades.

2.3.1 Effects of Layer Debonding on Pavement Performance

A series of studies on the debonding of the surface course of pavements was conducted by the Transport Research Laboratory (TRL) of the United Kingdom in the mid-1970s (Kennedy and Lister 1980). The studies were focused on the investigation of factors that might contribute to the slippage failure caused by debonding of the surface course in existing pavements and full-scale experimental roads under controlled conditions. The results showed that debonding failures were most likely to develop when surface courses were paved and compacted at elevated temperatures on cold bases laid on subgrades of inadequate stiffness. However, no significant findings regarding the
mechanism of debonding development were reached in their study (Pell, 1980) and no further research has followed by TRL.

Shahin et al. (1986) presented the results of a numerical study of layer debonding of an overlaid asphalt concrete airfield pavements. The stress and strain distribution of in a debonded and intact pavement section subjected to the aircraft loading was determined by the multi-layer elastic program BISAR. A fatigue model was used to estimate the pavement life from the calculated maximum horizontal tensile strain at the bottom of the asphalt layer. A parametric study performed to evaluate the influence of layer thickness, asphalt stiffness, pavement loading, and layer slippage on the life of the pavement found that increasing the thickness of the overlay decreased pavement fatigue life for thickness less than six inches. Furthermore, high asphalt stiffness in the overlay reduced the fatigue life of the pavement.

Van Dam et al. (1987) reported on a study of the effects of debonding on overlaid Portland cement concrete airfield pavements. The maximum tensile stress and surface deflection induced by aircraft loadings were calculated using the ILLI-SLAB while the curling stresses were determined using the Westergaard-Bradbury equation. They found a good bond between the overlay and underlying slab significantly reduced maximum tensile stress and corner deflections under wheel loads. A summary of important factors to achieve a successful bonding was also presented. It was suggested that corner deflection obtained from nondestructive testing could be used to detect debonding.
Grove et al. (1993) presented an experimental study on the bonding contribution of whitetopping (Portland cement concrete resurfacing over existing deteriorated asphalt concrete pavements). Various techniques to enhance bond strength between the PCC overlay and existing AC surface were used to resurface a 1.676 km test road in Dallas County, Iowa. The Road Rater, a type of steady state deflection equipment similar to the Dynaflect, was used to determine the structural rating of the pavement before and after the overlay construction. Cores were taken from each test section to test the shear strength of the bonding between the PCC overlay and AC layer. It was found that milling and air blasting before overlaying generally produced enhanced bonding over existing AC layer. Bond strength did not show a direct relation to the structural contribution of the existing AC pavement. A tack coat with a cationic emulsion was found to reduce the bond strength while cement grout did not show significant contribution to the bonding strength compared with other sections.

Lau et al. (1994) investigated the interface shear stresses in overlaid concrete pavements using a finite element model. The concrete slab was modeled as a thin plate resting on a Winkler foundation and the effect of transverse shear stresses were ignored. The calculated interface in-plane shear stresses were compared with laboratory prepared overlaid samples. The interface shear stresses were found to be small compared with the measured bonding strength. Based on the findings, they concluded that debonding in
overlay construction in the field is likely to result from stress concentrations caused by local defects.

Kim et al. (2003, 2004) developed 2-D and 3-D models using ABAQUS to predict the early-age behavior of bonded concrete overlays (BCO). To provide field data for comparison, a test BCO was constructed on a continuously reinforced concrete pavements (CRCP). Strains, deflections and temperature gradients were monitored by embedded instrumentation in the overlay and in the concrete slab. Calculated strains and deflections were compared with the measured response. Results showed that daily vertical movements at the edge of concrete slab and overlay were mainly caused by the diurnal temperature cycles. The longitudinal rebar in the CRCP restrained the underlying slab from curling deformation thereby increasing the debonding of the overlay from the CRCP. The calculated stresses from their 2-D and 3-D models showed good agreement with the measured values.

Khweir and Fordyce (2003) investigated the influence of pavement layer debonding on the prediction of pavement life. BISAR was used to model a typical asphalt pavement structure with different layer bonding conditions. The pavement life was estimated using fatigue formulae based on the calculated strains. Results showed that debonding of two pavement layers can reduce the pavement life by more than 80%.

Kruntcheva et al. (2005) presented a similar theoretical study to investigate the
effect of bonding conditions on flexible pavement performance. Both multi-layer elastic and static 2-D finite element models were used to model the pavement structure. The results indicated that the debonding can reduce the life of pavement structure up to 80%.

Nishiyama et al. (2005) reported a study of bonding condition in concrete overlays by laboratory testing, numerical modeling and field evaluation. Field evaluation of the influence of long-term bonding strength on the pavement surface distress was performed based on three different test roads. Nine cores were taken from the test sections and tested in the laboratory for bonding shear strength using direct shear tests. Two different 3-D finite element models were developed to simulate the debonding phenomenon between the layers at different bonding conditions. The concrete slab and overlay were modeled using eight-node solid elements and a spring element was used to quantify the bonding levels. A set of spring constant was obtained for different bonding levels and overlay thicknesses through trial and error process.

Martin et al. (2005) presented an experimental program to investigate the behavior of overlaid concrete slab under cyclic loading. Nine simply supported overlaid reinforced concrete slabs were subjected to static pure bending and repetitive loading up to 50,000 cycles. It was found that if the overlaid area was in compression, the induced stresses lead to a low risk of debonding. In the contrast, if the overlaid area was in tension, the reinforcing bars in the overlaid area had a significant influence of controlling the
development of debonding. Systematic debonding was observed in the areas with no reinforcement.

Kruntcheva et al. (2006) reported an experimental study on the properties of asphalt concrete layer interfaces. An apparatus known as the Nottingham shear box was used to apply cyclic shear loading to samples with different mix designs and binding conditions. Both horizontal and vertical displacements at the interface were monitored by LVDTs. It was found that the sample compacted with a dry and clean interface between two layers exhibited similar properties to the samples with a standard quality tack coat.

2.3.2 Bonding Strength Measurement and Bonding Assessment

In an early experimental study, Uzan et al. (1978) used a custom-built shear box to determine the interface shear strength between asphalt concrete layers. Asphalt concrete specimens with different tack coats were tested at ambient and elevated temperatures under five different normal stresses. It was found that shear resistance of the interface decreased significantly with the increasing in temperature and increased with increasing normal stresses.

Lepert et al. (1992) presented an overview of the causes of pavement layer interface
problems and some results from an experimental study to compare the ability of existing non-destructive testing (NDT) methods to detect layer debonding. Traditional FWD, lightweight vibrator, and ground penetrating radar tests were conducted on special pavement sections with different interlayer conditions. A new dynamic NDT method based on the mechanical impedance of the pavement structure was developed to detect pavement layer debonding.

Tschegg et al. (1995) proposed a wedge splitting testing method to characterize the bonding strength between asphalt concrete layers. Load-displacement curves were obtained from samples tested at different temperatures. A damage simulation concept was presented using a finite element model to simulate the crack initiation and propagation in asphalt concrete.

Hakim et al. (2000) developed a back-analysis to assess layer bonding conditions for flexible pavements from FWD test results. A two-stage database approach was applied to evaluate several newly constructed pavements with suspected debonding.

Delatte and Sehdev (2003) reported an experimental study on the bonding strength of a concrete overlay. Eight different concrete-overlay mix designs were investigated including Class C fly ash, blast-furnace slag, and synthetic polypropylene fiber reinforced concrete mixes. Three different surface roughness levels (smooth, broom, and rough) were studied to evaluate the effects of surface preparations on the bonding strength. It was found that, for most mixes, the broom base had the highest bonding strength,
followed by smooth and rough bases. The addition of fibers was found to have an adverse effect on interface bonding. All the mixes exhibited good performance in terms of freeze-thaw durability especially the fly ash concrete mix.

Kruntcheva et al. (2004) developed a NDT method to quantify the bonding condition of asphalt pavement layers. An instrumented impulse hammer provided a transient excitation and an accelerometer was used to monitor the vertical dynamic response close to the impact point. The bonding condition was assessed based on a spectral analysis of the measured dynamic response. NDT was used to evaluate the bonding condition of full-scale test pavement sections according to three interface bond levels (bonded, debonded, and partially bonded). Cores taken from the tested locations showed good agreement between predicted interface condition and interface shear strength test results.

2.3.3 Summary

It is well recognized that debonding of the pavement layers is a cause of premature failure of pavements. Studies have shown that debonding of pavement layers can reduce the life of pavement up to or more than 80%. Most of the past research focused on the consequence of debonding and the measurement of interface bonding strength. However, little has been done to characterize the interlayer stresses, which are the underlying cause
of debonding. In this regard, the utility of the state of art pavement response models, multi-layer elastic models for flexible pavements and 2-D displacement based finite element models for rigid pavements, is limited due to the inherent assumptions of the models for transverse normal and shear stress or strain components. Stress-based models developed for composite laminates that have shown the capability of accurately predicting the dynamic interlaminar stress distributions can be a promising alternative method to be adapted to study the pavement debonding.
CHAPTER 3

STRESS BASED DISCRETE LAYER PAVEMENT RESPONSE MODEL

3.1 Introduction

The state-of-art in response modeling for both flexible and rigid pavements has been summarized in Chapter Two. It has been shown that the multi-layer elastic theory and the 2-D displacement-based finite element method are currently used as the response models for the analysis and design of flexible and rigid pavements, respectively (NCHRP, 2004).

Multi-layer elastic models are extended from Burmister’s two-layer elastic theory. In these models, each pavement layer is assumed to extend infinitely in the horizontal directions, reducing the 3-D problem to an axisymmetric 2-D problem. It has been found that the deflections and strains calculated with the multi-layer elastic models are similar
to those measured within the pavement structure (NHI, 2002). However, the stress distributions predicted by these models are generally not accurate.

In the 2-D displacement-based finite element models for rigid pavements, the concrete slab and the base course are modeled using classic medium-thick plate elements based on the Kirchhoff hypothesis. The subgrade is modeled as either a Winkler foundation or an elastic half space. Based on the Kirchhoff hypothesis, both transverse normal and transverse shear strains are assumed to be zero. Since the transverse strains are neglected, the transverse stresses are not included in the formulation of the virtual work statement and thus are neglected. The deformation of the pavement layers is assumed to be entirely due to bending and in-plane stretching. These models can predict surface deflections and in-plane stresses and strains. However, they do not have the capability to provide the transverse stress distribution which is imperative to model the realistic behavior of in-service pavement systems and prevent premature failure caused by pavement layer debonding.

A stress-based discrete layer finite element model developed at Ohio State for composite laminates has shown the capability of accurately predicting the dynamic stresses at layer boundaries while retaining the ability to determine displacement behavior. The extension of this model to layered pavement systems is a promising alternative to existing approaches to pavement life estimates. This chapter presents a brief summary of the derivation and finite element implementation of this stress-based model.
3.2 Stress Based Discrete Layer Model

A problem analogous to the pavement layer debonding failure is the delamination of composite laminates. Due to their attractive properties, composite laminates have found applications in a wide variety of applications in the past several decades. For composite laminates, the problem of delamination has been a great concern for designers and researchers. A large body of information exists in the literature that focuses on the development of theoretical methods to predict accurate stresses, particularly interlaminar stresses which contribute to the delamination failures. It is well recognized that displacement based finite element models accurately predict displacements and in-plane stress but are not accurate predicting the transverse stress distributions (Van Hoa and Feng, 1998; Butalia, 1996). To overcome the disadvantages of displacement models, stress based finite element models have been developed in order to provide efficient and accurate predictions of transverse stress distribution in composite laminates.

Reissner (1947, 1950) presented one of the earliest stress based variational theories for smeared plates. The theory assumed a linear variation of in plane stress components across the plate thickness for transverse bending. Pagano (1978a, 1978b) extended Reissner’s theory by assuming a linear variation of in-plane stresses over each lamina. Continuity of both displacement and traction was ensured at the lamina interfaces. By
comparison with existing solutions of the laminate free-edge class of boundary value
problems, Pagano’s static stress based theory (1978a) showed dramatic improvements in
accuracy of predicting free-edge stress field.

Chyou (1989) rewrote Pagano’s theory in a self-adjoint form and implemented the
equations in a finite element formulation. Schoeppner (1991) extended Chyou’s work to
include inertia effects and developed a dynamic stress based discrete layer theory.
Schoeppner was able to predict both global behavior (e.g., frequencies and
displacements) and local behavior (e.g., transverse normal and shear stresses). Butalia
(1996) further extended Schoeppner’s model to account for variable mass density. For
small strains, the effect of variable mass density was included using pseudo-body force
terms. A linear stress distribution was assumed for the in-plane stresses through the
lamina thickness. The numerical model was verified by comparing the results with 3D
solutions for free and forced vibration response of isotropic, orthotropic and laminated
composite plates.

The stress based discrete layer model can be a useful tool for stress analysis of
pavement structures for predicting accurate interface stress fields, which are known to
cause debonding of pavement layers. The derivations of the stress-based model presented
here are summarized from the work by Schoeppner (1991) and Butalia (1996). They are
modified only where necessary and given here for clarity and completeness of
presentation. Details can be found in the work by Schoeppner (1991) and Butalia (1996).
3.2.1 Equation of Motion

The equations of motion for a single layer (lamina) with variable density were derived using equilibrium equations, constitutive law and kinematic relations.

3.2.1.1 Equilibrium Equations

Consider a rectangular lamina with thickness $h$, bounded by $x_1 = \pm a$, $x_2 = \pm b$, and $x_3 = \pm \frac{h}{2}$ as shown in Figure 3.1. From the principle of conservation of linear momentum, the equilibrium equations for a single lamina can be written as:

$$\sigma_{ij,j} + f_i - \dot{p}_i = 0$$

(3.1)

The in-plane and transverse equations of equilibrium can be separately written as:

$$\sigma_{a\alpha,\beta} + \sigma_{a3,3} + f_a - \dot{p}_a = 0$$

(3.2)

$$\sigma_{a3,a} + \sigma_{33,3} + f_3 - \dot{p}_3 = 0$$

(3.3)

where $\sigma_{ij}$ is the symmetric Cauchy stress tensor, $f_i$ is the body force per unit volume and $\dot{p}_i$ is the momentum density of the medium. The momentum density term can be written in terms of medium mass density, $\rho$, and the displacement, $u_i$, as:

$$\dot{p}_i = \frac{\partial (\rho u_i)}{\partial t} = \dot{\rho} u_i + \rho \ddot{u}_i$$

(3.4)
The superposed dots represent differentiation with respect to time. The $\dot{\rho}$ term represents the rate of change of mass density which can be expressed in terms of the rate of change of volumetric strains as (Hiremath, 1987):

\[
\dot{p}_i = -\rho \dot{\epsilon}_{jj} = -\rho \ddot{u}_{j,j}
\] (3.5)

Substituting Equation 3.5 into Equation 3.4 leads to

\[
\dot{p}_i = -\rho \ddot{u}_{j,j} \ddot{u}_i = \rho (\ddot{u}_i - \ddot{u}_{j,j} \ddot{u}_i)
\] (3.6)

The nonlinear term $\ddot{u}_{j,j} \ddot{u}_i$ results from the compressibility of the material.

Figure 3.1 Lamina Geometry and Coordination System
3.2.1.2 Constitutive Law

In this study, the materials are assumed to be linearly elastic, i.e. satisfying the generalized Hooke’s Law. For a monoclinic material (there is only one plane of symmetry, i.e., the plane $x_3 = 0$), the constitutive equations can be written as

$$\sigma_{\alpha\beta} = E_{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta} + E_{\alpha\beta33}\varepsilon_{33}$$

$$\sigma_{33} = \sigma_{3\alpha} = 2E_{\alpha3\beta3}\varepsilon_{\beta3}$$

$$\sigma_{33} = E_{33\gamma\delta}\varepsilon_{\gamma\delta} + E_{3333}\varepsilon_{33}$$

(3.7)

or conversely as

$$\varepsilon_{\alpha\beta} = S_{\alpha\beta\gamma\delta}\sigma_{\gamma\delta} + S_{\alpha\beta33}\sigma_{33}$$

$$\varepsilon_{33} = \varepsilon_{3\alpha} = 2S_{\alpha3\beta3}\sigma_{\beta3}$$

$$\varepsilon_{33} = S_{33\gamma\delta}\sigma_{\gamma\delta} + S_{3333}\sigma_{33}$$

(3.8)

where $E_{ijkl}$ and $S_{ijkl}$ are the components of the rate independent isothermal elasticity and compliance tensors.

3.2.1.3 Kinematics

For small displacement, the Green-Lagrange strain tensor reduces to the infinitesimal strain tensor in Cartesian component form
\( \varepsilon_{ij} = u_{(i,j)} = \frac{1}{2} (u_{i,j} + u_{j,i}) \) \tag{3.9}

The displacement components can be obtained by integrating Equation 3.9 along the lamina axes and substituting the constitutive law (Equation 3.8):

\[
\begin{align*}
  u_1(x_1, x_2, x_3, t) &= u_1(-a, x_2, x_3, t) + \int_{-a}^{x_1} \left[ S_{11a\beta} \sigma_{a\beta} + S_{1133} \sigma_{33} \right] h \eta_1 \\
  u_2(x_1, x_2, x_3, t) &= u_2(x_1, -b, x_3, t) + \int_{-b}^{x_2} \left[ S_{22a\beta} \sigma_{a\beta} + S_{2233} \sigma_{33} \right] h \eta_2 \\
  u_3(x_1, x_2, x_3, t) &= u_3 \left( x_1, x_2, -\frac{h}{2}, t \right) + \int_{-\frac{h}{2}}^{x_3} \left[ S_{33a\beta} \sigma_{a\beta} + S_{3333} \sigma_{33} \right] h \eta_3 \tag{3.10}
\end{align*}
\]

### 3.2.1.4 Generalized Equations of Motion

By integrating over the thickness of the lamina, the 3D spatial problem is reduced to 2D. Substituting Equations 3.6 and 3.10 into Equations 3.2 and 3.3 with the constitutive law (Equation 3.8) yields

\[
\begin{align*}
  \sigma_{a\beta, \beta} + \sigma_{a33} + f_a - \rho [\ddot{u}_a - \dot{u}_a S_{kk\gamma\delta} \dot{\sigma}_{\gamma\delta} - \dot{u}_a S_{kk33\sigma_{33}}] &= 0 \tag{3.11} \\
  \sigma_{a3, \alpha} + \sigma_{333} + f_3 - \rho [\ddot{u}_3 - \dot{u}_3 S_{kk\gamma\delta} \dot{\sigma}_{\gamma\delta} - \dot{u}_3 S_{kk33\sigma_{33}}] &= 0 \tag{3.12}
\end{align*}
\]

Integrating Equations 3.11 and 3.12 and the first moment of Equation 3.12 over the thickness of the lamina yields

\[
N_{a\beta, \beta} + \left( \sigma_{a3} - \sigma_{a3}^{-} \right) + F_a - \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho [\ddot{u}_a - \dot{u}_a S_{kk\gamma\delta} \dot{\sigma}_{\gamma\delta} - \dot{u}_a S_{kk33\sigma_{33}}] dx_3 = 0 \tag{3.13}
\]
\[ V_{a,\beta} + \left( \sigma_{33}^+ - \sigma_{33}^- \right) + F_3 - \int_{-h/2}^{h/2} \rho \left[ \tilde{u}_3 - \tilde{u}_3 S_{kk,0} \sigma_{\gamma\delta} - \tilde{u}_3 S_{kk,33} \sigma_{33} \right] dx_3 = 0 \]  
(3.14)

\[ M_{a\beta,\beta} + \frac{h}{2} \left( \sigma_{a3}^+ + \sigma_{a3}^- \right) - V_a - \int_{-h/2}^{h/2} \rho \left[ \tilde{u}_3 - \tilde{u}_3 S_{kk,0} \sigma_{\gamma\delta} - \tilde{u}_3 S_{kk,33} \sigma_{33} \right] x_3 dx_3 = 0 \]  
(3.15)

where \( N_{a\beta,\beta} \), the in-plane stress resultant, \( \int_{-h/2}^{h/2} \sigma_{a\beta} dx_3 \);

\( M_{a\beta,\beta} \), the in-plane moment resultant, \( \int_{-h/2}^{h/2} \sigma_{a\beta} x_3 dx_3 \);

\( V_{a,\beta} \), the transverse shear stress resultant, \( \int_{-h/2}^{h/2} \sigma_{a3} dx_3 \);

\( \rho \), the mass density, \( \frac{\rho_0}{1 + \varepsilon_{ij}} \cong \rho_0 \left( 1 - \varepsilon_{ij} \right) \) for small strains \( \varepsilon_{ij} \ll 1 \).

The “+” and “−” superscripts denote the value of the variable at top \( x_3 = \frac{h}{2} \) and bottom \( x_3 = -\frac{h}{2} \) surface of the lamina.

The body force per unit volume is assumed to be constant over the thickness of the lamina, so the generalized body force is

\[ F_i \equiv \int_{-h/2}^{h/2} f_i dx_3 = hf_i \]  
(3.16)

Defining in-plane weighted displacements as

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the generalized equations of motion for a single lamina can be written from the Equations 3.13 to 3.15 as

\[
N_{a\beta,\beta} + \left( \sigma_{a3}^+ - \sigma_{a3}^- \right) + F_\alpha + P_\alpha - \frac{\rho_0 h}{2} \ddot{u}_\alpha = 0
\]

(3.18a)

\[
V_{a,\alpha} + \left( \sigma_{33}^+ - \sigma_{33}^- \right) + F_3 + P_3 - \frac{\rho_0 h}{2} \ddot{u}_3 = 0
\]

(3.18b)

\[
M_{a\beta,\beta} + \frac{h}{2} \left( \sigma_{a3}^+ + \sigma_{a3}^- \right) - V_\alpha + R_\alpha - \frac{\rho_0 h^2}{4} \dddot{u}_\alpha = 0
\]

(3.18c)

where \( P_\alpha = \rho_0 \int_{\frac{h}{2}}^{\frac{h}{2}} \left[ \int_{-\frac{h}{2}}^{\frac{h}{2}} \epsilon_{ij} \ddot{u}_\alpha \right] dx_3 \)

\( P_3 = \rho_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( 1 - \epsilon_{ij} \right) \ddot{u}_3 \left( S_{k3j} \ddot{\sigma}_{j3} + S_{k33} \ddot{\sigma}_{33} \right) dx_3 \)

\( R_\alpha = \rho_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} \epsilon_{ij} \ddot{u}_\alpha \left( S_{k3j} \ddot{\sigma}_{j3} + S_{k33} \ddot{\sigma}_{33} \right) dx_3 \)

(3.19)

\( P_\alpha \) and \( R_\alpha \) are the nonlinear pseudo-body force terms attributed to the compressibility of the monoclinic material.
3.2.2 Consistent Stress Field

Typically, plate theories are developed by assuming the form of either the through the thickness displacement or stress field to be a linear combination of unknown functions and the thickness coordinates. In the present stress based model, the in-plane stress distribution is assumed to be linear across the thickness of each lamina as

\[
\sigma_{\alpha\beta} = \frac{N_{\alpha\beta}}{h} + \frac{12M_{\alpha\beta}}{h^3} \chi_3 \tag{3.20}
\]

Integrating Equation 3.2 through the thickness of the lamina and substituting the momentum density term (Equation 3.6) gives

\[
\sigma_{\alpha3} = \sigma_{\alpha3}^+ - \int_{\frac{h}{2}}^{\chi_3} \sigma_{\alpha3,\beta3} d\eta_3 \left[ \int_{\frac{h}{2}}^{\chi_3} f_{\alpha} d\eta_3 + \rho_0 \int_{\frac{h}{2}}^{\chi_3} \ddot{u}_{\alpha} d\eta_3 \right]
\]

\[
- \rho_0 \int_{\frac{h}{2}}^{\chi_3} \epsilon_{\beta3} \ddot{u}_{\alpha} d\eta_3 - \rho_0 \int_{\frac{h}{2}}^{\chi_3} \left(1 - \epsilon_{\beta3}\right) \ddot{e}_{\beta3} \ddot{u}_{\alpha} d\eta_3 \tag{3.21}
\]

The distribution of transverse shear stress is obtained by substituting Equation 3.20 into Equation 3.21 and use Equations 3.18a and 3.18c as

\[
\sigma_{\alpha3} = \left(\sigma_{\alpha3}^+ - \sigma_{\alpha3}^-ight) \frac{\chi_3}{h} + \left(\frac{\sigma_{\alpha3}^+ + \sigma_{\alpha3}^-}{4}\right) \left[ \frac{12x_3^2}{h^2} - 1 \right] + \frac{3}{2h} \left(1 - 4x_3^2\right) - \frac{\rho_0}{2} \left[ \ddot{u}_{\alpha} \left(\chi_3 + \frac{h}{2}\right) \right]
\]

\[
+ \ddot{u}_{\alpha} \left(\frac{3x_3^2}{h} - \frac{3h}{4}\right) + \rho_0 \left[ \ddot{u}_{\alpha} d\eta_3 - \rho_0 \int_{\frac{h}{2}}^{\chi_3} \ddot{e}_{\beta3} \ddot{u}_{\alpha} d\eta_3 - \rho_0 \int_{\frac{h}{2}}^{\chi_3} \left(1 - \epsilon_{\beta3}\right) \ddot{e}_{\beta3} \ddot{u}_{\alpha} d\eta_3 \right] + \frac{1}{h} \int_{\frac{h}{2}}^{\chi_3} \rho_0 \left[ \ddot{u}_{\alpha} \left(\chi_3 + \frac{h}{2}\right) \right]
\]

\[
+ \rho_0 \int_{\frac{h}{2}}^{\chi_3} \rho_0 \left[ \ddot{u}_{\alpha} \left(\chi_3 + \frac{h}{2}\right) \right]
\]

\[
+ \frac{12}{h^3} \int_{\frac{h}{2}}^{\chi_3} R_{\alpha3} d\eta_3 \tag{3.22}
\]
Similarly, the transverse normal stress is obtained by integrating Equation 3.3 over the thickness of the lamina and substituting momentum density term (Equation 3.6) as

$$
\sigma_{33} = \sigma_{33} - \int_{-\frac{h}{2}}^{\frac{x_3}{2}} \sigma_{a33,a}d\eta_3 - \int_{-\frac{h}{2}}^{\frac{x_3}{2}} f_3 d\eta_3 + \rho_0 \int_{-\frac{h}{2}}^{\frac{x_3}{2}} \dot{u}_3 d\eta_3 - \rho_0 \int_{-\frac{h}{2}}^{\frac{x_3}{2}} \varepsilon_{33} \dot{u}_3 d\eta_3
$$

$$
- \rho_0 \int_{-\frac{h}{2}}^{\frac{x_3}{2}} (1 - \varepsilon_{33}) \dot{\varepsilon}_{33} \dot{u}_3 d\eta_3
$$

(3.23)

The expanded form of Equations 3.22 and 3.23 can be simplified by neglecting terms with order higher than $O(h^1)$. This leads to a cubic distribution of transverse normal stress as

$$
\sigma_{33} = \frac{(\sigma_{33}^+ - \sigma_{33}^-)}{4} \left( \frac{12x_3^2}{h^2} - 1 \right) + \frac{(\sigma_{33}^+ - \sigma_{33}^-)}{4} \left( \frac{40x_3^3}{h^3} - \frac{3}{2} h \left( 1 - \frac{4x_3^3}{h^2} \right) \right)
$$

$$
+ \frac{15M_{33}}{h^2} \left( \frac{2x_3}{h} - \frac{8x_3^3}{h^3} \right)
$$

(3.24)

and quadratic distribution of transverse shear stresses as

$$
\sigma_{13} = \frac{(\sigma_{13}^+ - \sigma_{13}^-)}{h} \left( \frac{12x_3}{h^2} - 1 \right) + \frac{3V_{1}}{2h} \left( 1 - \frac{4x_3^2}{h^2} \right)
$$

$$
+ \frac{\rho_0}{h} \int_{-\frac{h}{2}}^{\frac{x_3}{2}} S_{1333} \left[ \frac{h}{4} (\sigma_{33}^+ + \sigma_{33}^-) - \frac{1}{2} \dot{N}_{33} + \frac{4x_3^3}{h^2} - x_3 \right] d\eta_3
$$

$$
+ \frac{h^2}{12} (\sigma_{33}^+ - \sigma_{33}^-) \dot{M}_{33} \left[ \frac{30x_3^4}{h^4} - \frac{9x_3^2}{h^2} + \frac{3}{8} \right] d\eta_3
$$

$$
- \frac{\rho_0}{h} \left( x_3 + \frac{h}{2} \right) \int_{-\frac{h}{2}}^{\frac{x_3}{2}} \dot{u}_3 (-a, x_2, x_3, t) dx_3
$$

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\[ -\frac{\rho_0}{h} \left( \frac{6x_3^2}{h^2} - \frac{3}{2} \right) \int_{-h/2}^{h/2} \ddot{u}_i (-a, x_2, x_3, t) x_3 dx_3 + \rho_0 \int_{-h/2}^{h/2} \ddot{u}_i (-a, x_2, \eta_3, t) d\eta_3 \]

\[ + \frac{1}{h} \int_{-h/2}^{x_3} P_1 d\eta_3 + \frac{12}{h^3} \int_{-h/2}^{x_3} R_1 \eta_3 d\eta_3 - \rho_0 \int_{-h/2}^{x_3} \varepsilon_{ij} \ddot{u}_i d\eta_3 \]

\[- \rho_0 \int_{-h/2}^{x_3} (1 - \varepsilon_{ij}) \ddot{u}_j d\eta_3 \quad (3.25) \]

and

\[ \sigma_{23} = \left( \sigma_{23}^+ - \sigma_{23}^- \right) \frac{x_3}{h} + \frac{\left( \sigma_{23}^+ + \sigma_{23}^- \right)}{4} \left( \frac{12x_3^2}{h^2} - 1 \right) + \frac{3V_2}{2h} \left( 1 - \frac{4x_3^2}{h^2} \right) \]

\[ + \frac{\rho_0}{h} \int_{-h}^{x_3} S_{233} \left[ \frac{h}{4} \left( \sigma_{33}^+ + \sigma_{33}^- \right) - \frac{1}{2} \tilde{N}_{33} \left[ \frac{4x_3^3}{h^2} - x_3 \right] \right] \]

\[ + \left[ \frac{h^2}{12} \left( \sigma_{33}^+ - \sigma_{33}^- \right) - \tilde{M}_{33} \left[ \frac{30x_3^4}{h^4} - \frac{9x_3^2}{h^2} + \frac{3}{8} \right] \right] d\eta_2 \]

\[- \frac{\rho_0}{h} \left( x_3 + \frac{h}{2} \right) \int_{-h/2}^{h/2} \ddot{u}_2 (x_2, -b, x_3, t) dx_3 \]

\[ - \frac{\rho_0}{h} \left( \frac{6x_3^2}{h^2} - \frac{3}{2} \right) \int_{-h/2}^{h/2} \ddot{u}_2 (x_1, -b, x_3, t) x_3 dx_3 + \rho_0 \int_{-h/2}^{h/2} \ddot{u}_2 (x_1, -b, \eta_3, t) d\eta_3 \]

\[ + \frac{1}{h} \int_{-h/2}^{x_3} P_2 d\eta_3 + \frac{12}{h^3} \int_{-h/2}^{x_3} R_2 \eta_3 d\eta_3 - \rho_0 \int_{-h/2}^{x_3} \varepsilon_{ij} \ddot{u}_2 d\eta_3 \]

\[- \rho_0 \int_{-h/2}^{x_3} (1 - \varepsilon_{ij}) \ddot{u}_j d\eta_3 \quad (3.26) \]
3.2.3 The Governing Field Equations

In general, the governing equations of a solid mechanics problem can be derived using either a vector approach (e.g., Newton’s Second Law of motion) or a variational approach (e.g., the principle of virtual displacements). The vector approach provides an easy and direct way to derive the governing equations but becomes cumbersome for complicated systems. In the contrast, the variational approach yields not only the governing equations but also the associated boundary conditions (Reddy, 2004; Bathe, 1996). For the current model, the generalized variational technique presented by Sandhu and Salam (1975) is used to derive the governing field equations.

3.2.3.1 Constitutive Equations

The following ten constitutive equations are obtained for the field variables $N_{a\beta}, M_{a\beta}, V_\alpha, N_{33}$, and $M_{33}$:

$$\tilde{u}_{(a,\beta)} = \frac{2}{h} \left[ S_{a\beta\gamma\delta} N_{\gamma\delta} + S_{a\beta 33} N_{33} \right] \quad (3.27)$$

$$\tilde{u}_{(a,\beta)} = \frac{4}{h^2} \left[ S_{a\beta\gamma\delta} M_{\gamma\delta} + S_{a\beta 33} M_{33} \right] \quad (3.28)$$
\[
\tilde{u}_{1,\alpha} - \tilde{u}_{3,\alpha} = \frac{4}{h} \tilde{u}_\alpha = \frac{8}{15} S_{a3\beta3} \left( \sigma^+_{\beta3} + \sigma^-_{\beta3} \right) - \frac{32}{5h} S_{a3\beta3} V_\beta \\
- \frac{\rho_0}{h} \frac{16}{35} S_{a3\beta3} \frac{\partial^2}{\partial \eta_\beta^2} \int_{-a,-b}^{x_a} \left[ \frac{h^2}{12} \left( \tilde{\sigma}^+_{33} - \tilde{\sigma}^-_{33} \right) - \tilde{M}_{33} \right] d\eta_\beta \\
+ \frac{\rho_0}{h} \frac{4h^2}{3} S_{a3\beta3} \tilde{V}_\beta - \frac{\rho_0}{h} \frac{8h^2}{5} S_{a3\beta3} \tilde{V}_\beta \\
+ \frac{\rho_0}{h} \frac{16}{3} S_{a3\beta3} \left( \frac{h^2}{2} \int_{-a,-b}^{x_a} \frac{6x_a^2}{h^2} - \frac{3}{2} \int_{-h}^{h} \tilde{V}_\beta d\eta_3 d\tau = \frac{8}{3} \psi_3 \right)
\]

(3.29)

\[
6\tilde{u}_3 = 2S_{33\alpha\beta} N_{\alpha\beta\gamma} + \frac{12}{5} S_{3333} N_{33} - \frac{h}{5} S_{3333} \left( \sigma^+_{33} + \sigma^-_{33} \right) - \frac{\rho_0}{h} \frac{h^3}{8} S_{a3\alpha3} \left( \tilde{u}_3 - \tilde{\bar{u}}_3 \right)
\]

\[
- \frac{\rho_0}{h} \frac{h^3}{175} S_{3333} \left( S_{a3\alpha3} - S_{3333} \right) \left[ \frac{h}{4} \left( \tilde{\sigma}^+_{33} + \tilde{\sigma}^-_{33} \right) - \frac{1}{2} \tilde{N}_{33} \right]
\]

\[
- \frac{\rho_0}{h} \frac{2h^2}{S_{a3\alpha3}} \int_{-a,-b}^{x_a} \left[ -\frac{1}{8} \left( \tilde{u}_3 - \tilde{\bar{u}}_3 \right) + \frac{h}{30} S_{a3\beta3} \left( \tilde{\sigma}^+_{\beta3} - \tilde{\sigma}^-_{\beta3} \right) \right] d\eta_\alpha
\]

\[
+ \frac{\rho_0}{h} \frac{h^3}{8} S_{1133} \left[ \tilde{u}_3(-a,x_2,x,t) - \tilde{\bar{u}}_3(-a,x_2,x,t) \right]
\]

\[
+ \frac{\rho_0}{h} \frac{h^3}{8} S_{2233} \left[ \tilde{u}_3(x_1,-b,x_2,t) - \tilde{\bar{u}}_3(x_1,-b,x_2,t) \right]
\]

(3.30)

\[
3\tilde{u}_3 - \tilde{\bar{u}}_3 = \frac{4}{5h} S_{33\alpha\beta} M_{\alpha\beta}\gamma + \frac{8}{7h} S_{3333} M_{33} - \frac{h}{35} S_{3333} \left( \sigma^+_{33} - \sigma^-_{33} \right)
\]

\[
- \frac{\rho_0}{h} \frac{h^3}{32} S_{a3\alpha3} \left( \tilde{u}_3 - \tilde{\bar{u}}_3 \right) - \frac{\rho_0}{h} \frac{h^3}{160} S_{a3\alpha3} \left( \tilde{u}_3 - \tilde{\bar{u}}_3 \right)
\]

\[
- \frac{\rho_0}{h} \frac{h^2}{735} S_{3333} \left( S_{a3\alpha3} - S_{3333} \right) \left[ \frac{h^2}{12} \left( \tilde{\sigma}^+_{33} - \tilde{\sigma}^-_{33} \right) - \tilde{M}_{33} \right]
\]

\[
- \frac{\rho_0}{h} \frac{h^2}{15} S_{a3\alpha3} \left[ -\frac{3}{4} \left( \tilde{u}_3 - \tilde{\bar{u}}_3 \right) + \frac{3h}{35} S_{a3\beta3} \left( \tilde{\sigma}^+_{\beta3} + \tilde{\sigma}^-_{\beta3} \right) - \frac{6}{35} S_{a3\beta3} \tilde{V}_\beta \right] d\eta_\alpha
\]

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\[ + \frac{\rho_0}{h} \frac{h^3}{80} S_{1133} \left[ \frac{5}{2} \dddot{u}_3(-a,x_2,x_3,t) - 3 \dddot{u}_1(-a,x_2,x_3,t) + \frac{1}{2} \dddot{u}_3(-a,x_2,x_3,t) \right] \]
\[ + \frac{\rho_0}{h} \frac{h^3}{80} S_{2233} \left[ \frac{5}{2} \dddot{u}_3(x_1,-b,x_3,t) - 3 \dddot{u}_1(x_1,-b,x_3,t) + \frac{1}{2} \dddot{u}_3(x_1,-b,x_3,t) \right] \quad (3.31) \]

where
\[ \psi_\alpha = - \frac{1}{h} S_{a3\beta3} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{12 x_3^2}{h^3} - \frac{3}{h} \right) \frac{h}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} P_\beta d\eta_3 d\eta_3 - \frac{12}{h^3} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{12 x_3^2}{h^3} - \frac{3}{h} \right) \frac{h}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} R_\beta \eta_3 d\eta_3 d\eta_3 \]
\[ \rho_0 S_{a3\beta3} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{12 x_3^2}{h^3} - \frac{3}{h} \right) \frac{h}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[ \varepsilon_{ji} \dddot{u}_\beta + \left( 1 - \varepsilon_{ji} \right) \dddot{u}_\beta \right] d\eta_3 d\eta_3 \quad (3.32) \]

### 3.2.3.2 Equations of Motion

For an arbitrary admissible variation of the weighted displacement quantities, nine

generalized equations of motion are obtained:
\[ N_{a\beta,\beta} + \left( \sigma_{a3}^+ - \sigma_{a3}^- \right) - \rho_0 \int_{-a,-b}^{S_{a\gamma\delta}} \left( S_{a3\gamma\delta} \dddot{N}_{\gamma\delta} + S_{a333} \dddot{N}_{333} \right) d\eta_\alpha - \rho_0 \frac{h^2}{2} \dddot{v}_a + F_a \]
\[ + \rho_0 \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[ \varepsilon_{ji} \dddot{u}_\alpha + \left( 1 - \varepsilon_{ji} \right) \dddot{u}_\alpha \right] d\eta_3 = 0 \quad (3.33) \]
\[ M_{a\beta,\beta} + \left( \sigma_{a3}^+ + \sigma_{a3}^- \right) - V_a - \rho_0 \int_{-a,-b}^{S_{a\gamma\delta}} \left( S_{a3\gamma\delta} \dddot{M}_{\gamma\delta} + S_{a333} \dddot{M}_{333} \right) d\eta_\alpha - \frac{\rho_0 h^2}{4} \dddot{v}_a \]

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\[ + \rho_0 \int \left[ \varepsilon_{j} \dot{u}_{a} + (1 - \varepsilon_{j}) \dot{e}_{j} \dot{u}_{a} \right] dx = 0 \quad (3.34) \]

\[ V_{a, a} - \frac{\hbar}{6} \left( \sigma_{a3}^+ - \sigma_{a3}^- \right) - \left( \sigma_{a3}^+ - \sigma_{a3}^- \right) + \frac{20}{h^2} M_{33} + \frac{2}{3} F_3 - \frac{2\rho_0 h}{3} \dot{u}_{3} \]

\[- \rho_0 S_{33} \left( \frac{\hbar}{3} \dot{N}_{\rho \phi} - \ddot{M}_{\rho \phi} \right) - \rho_0 S_{33} \left( \frac{\hbar}{3} \dot{N}_{\rho \phi} - \ddot{M}_{\rho \phi} \right) \]

\[ + \frac{\rho_0}{h} \left( S_{33} - S_{33} \right) \left[ \frac{h^2}{12} \left( \sigma_{33}^+ - \sigma_{33}^- \right) - \ddot{M}_{33} \right] \]

\[ + \frac{3\rho_0}{3} \int \left[ \varepsilon_{j} \dot{u}_{a} + (1 - \varepsilon_{j}) \dot{e}_{j} \dot{u}_{a} \right] dx = 0 \quad (3.35) \]

\[ N_{33} - \frac{\hbar^2}{12} \left( \sigma_{a3, 3}^+ - \sigma_{a3, 3}^- \right) - \frac{1}{2} \left( \sigma_{33}^+ + \sigma_{33}^- \right) + \frac{\rho_0 h^2}{12} S_{33} \dot{N}_{\rho \phi} + \frac{\rho_0 h^2}{12} S_{33} \dot{M}_{33} \]

\[ + \frac{\rho_0 h^2}{12} \left( S_{33} - S_{33} \right) \left[ \frac{h^2}{12} \left( \sigma_{33}^+ - \sigma_{33}^- \right) - \ddot{M}_{33} \right] = 0 \quad (3.36) \]

\[ \frac{60}{h^2} M_{33} - \frac{\hbar}{2} \left( \sigma_{a3, 3}^+ + \sigma_{a3, 3}^- \right) + V_{a, a} - 5 \left( \sigma_{33}^+ - \sigma_{33}^- \right) + \rho_0 S_{33} \dot{M}_{\rho \phi} + \rho_0 S_{33} \ddot{M}_{33} \]

\[ + \frac{3\rho_0}{2} \left( S_{aa33} - S_{33} \right) \left[ \frac{h^2}{12} \left( \sigma_{a3, 3}^+ - \sigma_{a3, 3}^- \right) - \ddot{M}_{33} \right] = 0 \quad (3.37) \]

\[ \frac{h}{4} \left( \sigma_{33}^+ + \sigma_{33}^- \right) - \frac{1}{2} \dot{N}_{33} = 0 \quad (3.38) \]

\[ \frac{h^2}{12} \left( \sigma_{33}^+ - \sigma_{33}^- \right) - \ddot{M}_{33} = 0 \quad (3.39) \]
3.2.3.3 Interface Displacement Equations

For an arbitrary variation of the transverse stress components on the top and bottom boundaries of the plate, six interface displacement equations are obtained:

\[
\begin{align*}
    u_\alpha^+ &= -h \left( \frac{3}{8} \tilde{u}_{3,\alpha} - \frac{1}{8} \tilde{u}_{3,\alpha} - \frac{3}{2h} \tilde{u}_{\alpha} \right) - \left( \frac{h}{4} \tilde{u}_{3,\alpha} - \frac{1}{2} \tilde{u}_{\alpha} \right) + 4S_{a3\beta3} \left[ \frac{(4\sigma_{\beta3}^+ - \sigma_{\beta3})h}{30} - \frac{V_\beta}{10} \right] \\
    &\quad - \frac{\rho_0}{h} \frac{2h^2}{15} S_{a3\beta3} S_{\beta\gamma3} \int_{-a,-b} h \left( \tilde{\sigma}_{33}^+ + \tilde{\sigma}_{33}^- \right) - \frac{1}{2} \tilde{N}_{33} \right] d\eta_\beta \\
    &\quad - \frac{\rho_0}{h} \frac{6h}{35} S_{a3\beta3} S_{\beta\gamma3} \int_{-a,-b} \frac{h^2}{12} \left( \tilde{\sigma}_{33}^+ - \tilde{\sigma}_{33}^- \right) - \tilde{M}_{33} \right] d\eta_\beta \\
    &\quad - \rho_0 \frac{h^3}{6} S_{a3\beta3} \tilde{\nu}_\beta - \rho_0 \frac{h^3}{10} S_{a3\beta3} \tilde{\nu}_\beta \\
    &\quad - \rho_0 \frac{h}{h} S_{a3\beta3} \int_{-a,-b} \frac{12x_3^2}{h} - 4x_3 + h \right] \tilde{v}_\beta d\eta_\beta d\eta_\beta + 2K_\alpha \\
    u_\beta^- &= h \left( \frac{3}{8} \tilde{u}_{3,\beta} - \frac{1}{8} \tilde{u}_{3,\beta} - \frac{3}{2h} \tilde{u}_{\beta} \right) - \left( \frac{h}{4} \tilde{u}_{3,\beta} - \frac{1}{2} \tilde{u}_{\beta} \right) - 4S_{a3\beta3} \left[ \frac{(4\sigma_{\beta3}^+ - \sigma_{\beta3})h}{30} - \frac{V_\beta}{10} \right] \\
    &\quad - \frac{\rho_0}{h} \frac{2h^2}{15} S_{a3\beta3} S_{\beta\gamma3} \int_{-a,-b} h \left( \tilde{\sigma}_{33}^+ + \tilde{\sigma}_{33}^- \right) - \frac{1}{2} \tilde{N}_{33} \right] d\eta_\beta \\
    &\quad - \frac{\rho_0}{h} \frac{6h}{35} S_{a3\beta3} S_{\beta\gamma3} \int_{-a,-b} \frac{h^2}{12} \left( \tilde{\sigma}_{33}^+ - \tilde{\sigma}_{33}^- \right) - \tilde{M}_{33} \right] d\eta_\beta \\
    &\quad - \rho_0 \frac{h^3}{6} S_{a3\beta3} \tilde{\nu}_\beta + \rho_0 \frac{h^3}{10} S_{a3\beta3} \tilde{\nu}_\beta
\end{align*}
\]
\[- \frac{\rho_0}{h} S_{a_3 \beta 3} \int_{-h}^{h} \left( - \frac{12 x^2}{h} - 4 x_3 - h \right) \int_{\frac{-h}{2}}^{\frac{h}{2}} \tilde{\nu}_\mu d\eta_3 dx_3 - 2 L_\alpha \]  

(3.41)

\[ u^+_3 = \frac{3}{4} (5 \tilde{u}_3 - \bar{u}_3) + \frac{3}{2} \bar{u}_3 - \frac{1}{70 h} S_{3333} \left[ (6 \sigma^+_3 + \sigma^-_3) h^2 - 7 h N^3_3 - 30 M^3_3 \right] \]

\[ + \frac{\rho_0}{h} S_{\xi \xi 33} \left[ \frac{5 h^3}{128} (\tilde{\nu}_3 - \bar{u}_3) + \frac{h^3}{32} (\bar{u}_3 - \tilde{u}_3) + \frac{h^3}{128} (\tilde{u}_3 - \bar{u}_3) \right] \]

\[ + \frac{\rho_0}{h} \frac{h^3}{700} S_{3333} (S_{\xi \xi 33} - S_{3333}) \left[ \frac{h}{4} (\tilde{\sigma}^+_3 + \bar{\sigma}^-_3) - \frac{1}{2} \tilde{N}^3_3 \right] \]

\[ + \frac{\rho_0}{h} \frac{h^2}{588} S_{3333} (S_{\xi \xi 33} - S_{3333}) \left[ \frac{h^2}{12} (\tilde{\sigma}^+_3 + \bar{\sigma}^-_3) - \bar{M}^3_3 \right] \]

\[ + \frac{\rho_0}{h} \frac{h^2}{16} S_{\alpha_2 \beta 3} \int_{-a}^{-b} \left[ (5 \tilde{\bar{\nu}}_3 - 3 \bar{u}_3 \right) + (3 \bar{u}_3 - \tilde{u}_3) \]

\[ - \frac{8 h}{105} S_{a_3 \beta 3} (5 \tilde{\sigma}^+_3 + 2 \bar{\sigma}^-_3) + \frac{8}{35} S_{a_3 \beta 3} \tilde{\nu}_\beta \right] d\eta_3 \]

\[ + \frac{\rho_0}{h} S_{1133} \left[ - \frac{5 h^3}{128} \tilde{u}_3 (-a, x_2, x_3, t) - \frac{h^3}{32} \bar{u}_3 (-a, x_2, x_3, t) + \frac{3 h^3}{64} \tilde{u}_3 (-a, x_2, x_3, t) \]

\[ + \frac{h^3}{32} \bar{u}_3 (-a, x_2, x_3, t) - \frac{h^3}{128} \tilde{u}_3 (-a, x_2, x_3, t) \]

\[ + \frac{\rho_0}{h} S_{2233} \left[ - \frac{5 h^3}{128} \tilde{u}_3 (x_1, -b, x_3, t) - \frac{h^3}{32} \bar{u}_3 (x_1, -b, x_3, t) + \frac{3 h^3}{64} \tilde{u}_3 (x_1, -b, x_3, t) \]

\[ + \frac{h^3}{32} \bar{u}_3 (x_1, -b, x_3, t) - \frac{h^3}{128} \tilde{u}_3 (x_1, -b, x_3, t) \]  

(3.42)

\[ u^-_3 = \frac{3}{4} (5 \tilde{u}_3 - \bar{u}_3) - \frac{3}{2} \bar{u}_3 - \frac{1}{70 h} S_{3333} \left[ (6 \sigma^+_3 + \sigma^-_3) h^2 - 7 h N^3_3 - 30 M^3_3 \right] \]

\[ + \frac{\rho_0}{h} S_{\xi \xi 33} \left[ \frac{5 h^3}{128} (\bar{u}_3 - \tilde{u}_3) - \frac{h^3}{32} (\bar{u}_3 - \tilde{u}_3) + \frac{h^3}{128} (\tilde{u}_3 - \bar{u}_3) \right] \]
\[-\frac{\rho_0}{h} \frac{h^3}{700} S_{3333} \left( S_{3333} - S_{3333} \right) \left\{ \frac{h}{4} (\sigma_{11}^+ + \sigma_{11}^-) - \frac{1}{2} \frac{\sigma_{11}}{\dot{N}_{33}} \right\}
\]
\[+ \frac{\rho_0}{h} \frac{h^2}{588} S_{3333} \left( S_{3333} - S_{3333} \right) \left\{ \frac{h^2}{12} (\sigma_{11}^+ - \sigma_{11}^-) - \dot{M}_{33} \right\}
\]
\[+ \frac{\rho_0}{h} \frac{h^2}{16} S_{3333} \int_{-a}^{-b} \left[ \left( 6 \ddot{u}_{a} - 3 \dddot{u}_{a} \right) + \left( 3 \dddot{u}_{a} - \dddot{u}_{a} \right) \right]
\]
\[- \frac{8h}{105} S_{3333} \left( 5 \sigma_{11}^+ - 2 \sigma_{11}^- \right) - \frac{8}{35} S_{3333} \dddot{\sigma}_{11} \right\} d\eta_a
\]
\[= \frac{h^3}{128} \dddot{u}_{3}(-a, x_2, x_3, t) + \frac{h^3}{128} \dddot{u}_{3}(-a, x_2, x_3, t)
\]
\[= \frac{h^3}{128} \dddot{u}_{3}(x_1, -b, x_3, t) + \frac{h^3}{128} \dddot{u}_{3}(x_1, -b, x_3, t)
\]
\[= \frac{h^3}{128} \dddot{u}_{3}(x_1, -b, x_3, t) + \frac{h^3}{128} \dddot{u}_{3}(x_1, -b, x_3, t)
\]

where

\[K_\alpha = -\frac{1}{h} S_{3333} \int_{-h}^{h} \left[ -6 \frac{x_3}{h} - \frac{2 x_3}{h} + \frac{1}{2} \right] P_\alpha d\eta_3 dx_3
\]
\[-\frac{12}{h^3} S_{3333} \int_{-h}^{h} \left[ -6 \frac{x_3}{h} - \frac{2 x_3}{h} + \frac{1}{2} \right] R_{,\eta_{3}} d\eta_3 dx_3
\]
\[= \rho_0 S_{3333} \int_{-h}^{h} \left[ -6 \frac{x_3}{h} - \frac{2 x_3}{h} + \frac{1}{2} \right] \left[ \ddot{\varepsilon}_{,\eta_{3}} + \left( 1 - \varepsilon_{,\eta_{3}} \right) \ddot{\varepsilon}_{,\eta_{3}} \right] d\eta_3 dx_3
\] (3.43)
\[ L_\alpha = \frac{1}{h} S_{a3}\beta_3 \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{6x_3^2}{h^2} - \frac{2x_3}{h} - \frac{1}{2} \right) \int_{-\frac{h}{2}}^{\frac{h}{2}} P_{\mu} d\eta_3 dx_3 \]

\[ - \frac{12}{h^3} S_{a3}\beta_3 \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{6x_3^2}{h^2} - \frac{2x_3}{h} - \frac{1}{2} \right) \int_{-\frac{h}{2}}^{\frac{h}{2}} R_{\mu} \eta_3 d\eta_3 dx_3 \]

\[ - \rho_0 S_{a3}\beta_3 \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{6x_3^2}{h^2} - \frac{2x_3}{h} - \frac{1}{2} \right) \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[ \varepsilon_{\mu} \ddot{u}_\mu + (1 - \varepsilon_{\mu}) \ddot{e}_{\mu} \ddot{u}_\mu \right] d\eta_3 dx_3 \] (3.45)

### 3.2.3.4 Operator Form of the Governing Equations

The governing equations (Equations 3.37 to 3.43) are derived for a single lamina. To derive the equations for a N-layer laminate, the following generalized displacements are defined:

\[
\left( g, \bar{g}, \dot{g}, \bar{g}, \ddot{g} \right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \begin{array}{c} 2x_3, \frac{2x_3^2}{h^2}, \frac{4x_3^3}{h^3}, \frac{8x_3^4}{h^4}, \frac{16x_3^4}{h^4} \end{array} \right) \frac{2}{h} dx_3
\] (3.46)

\[
\tilde{v}_\rho^{(k)} \equiv \frac{\ddot{u}_\rho^{(k)}}{2}
\] (3.47)

\[
\tilde{\phi}_\rho^{(k)} \equiv \frac{3\ddot{u}_\rho^{(k)}}{k}
\] (3.48)

\[
\tilde{v}_3^{(k)} \equiv \frac{3}{4} (\ddot{u}_3 - \ddot{u}_3)^{(k)}
\] (3.49)

\[
\tilde{\phi}_3^{(k)} \equiv (5\ddot{u}_3 - \ddot{u}_3)^{(k)}
\] (3.50)

where \( k = 1 \) and \( k = N \) represent the top and bottom laminae (Figure 3.2).
Figure 3.2 Coordinate System for a N-layer Laminate
After rewriting into self-adjoint form and eliminating the time derivatives using Gurtin’s bilinear mapping (1963, 1964), the governing equations (Equations 3.37 to 3.43) for the \( k \)-th lamina can be expressed in an operator form as:

\[
[ A ]^{(k)} \{ u \}^{(k)} + [ B ]^{(k)} \{ \sigma \}^{-(k)} + [ C ]^{(k)} \{ \sigma \}^{+(k)} + [ D_u ]^{(k)} \{ F \}^{(k)} + [ H ]^{(k)} + \{ E_u \}^{(k)} + \{ Z_u \}^{(k)} = 0
\]

(3.51)

where \([ A ]^{(k)}, [ B ]^{(k)}, [ C ]^{(k)}\) and \([ D_u ]^{(k)}\) : the field operator matrices for the \( k \)-th layer \( (k = 1, 2, \ldots N) \);

\([u]^{(k)}\) and \( \{ \sigma \}^{\pm(k)}\) : field variable vectors;

\([F]^{(k)}\) : generalized body force vector;

\([H]^{(k)}\) : pseudo body force vector;

\([E]^{(k)}\) : in-plane boundary term vector;

\([Z_u]^{(k)}\) : explicit initial conditions vector from Gurtin’s convolution bilinear mapping.

The vectors of \((20N + 3)\) field variables are defined as

\[
\{ u \}^{(k)} = \begin{bmatrix} [v]^{(k)} & [\phi]^{(k)} & [\varphi]^{(k)} & [\bar{u}]^{(k)} & [N_{\alpha\beta}]^{(k)} & [N_{33}]^{(k)} & [M_{\alpha\beta}]^{(k)} & [M_{33}]^{(k)} & [V]_{y}^{(k)} \end{bmatrix}
\]

(3.52)

\[
\{ \sigma \}^{\pm(k)} = \begin{bmatrix} [\sigma_{y3}]^{(k)} & [\sigma_{33}]^{(k)} \end{bmatrix}
\]

(3.53)

The interface continuity equations for the \( k \)-th layer can be written as

\[
[\Lambda]^{(k)} \{ \sigma \}^{-(k-1)} + [\bar{B}]^{(k)} \{ u \}^{(k)} + [\bar{C}]^{(k)} \{ \sigma \}^{+(k)} + [\bar{D}_s]^{(k)} \{ F \}^{(k)} + [\bar{D}]^{(k+1)} \{ u \}^{(k+1)} + [\bar{E}_s]^{(k)} \{ F \}^{(k+1)} + [\bar{Z}_s]^{(k)} + [\bar{Z}_u]^{(k+1)} = 0
\]

(3.54)
The governing function for the self-adjoint form of the initial boundary value problem was derived using the generalized variational formulation presented by Sandhu and Salam (1975). The number of field variables was reduced from \(20N+3\) (Equations 3.51 to 3.54) to \(10N+3\) by the following specializations:

1. Eliminating the derivatives on generalized stresses;
2. Enforcing stress boundary conditions at free edges;
3. Eliminating the self-adjoint terms of the interface continuity equations;
4. Neglecting the acceleration terms associated with interface transverse shear stress continuity conditions;
5. Neglecting the body forces (pseudo-body force accounted for);
6. No jump discontinuities in the interior of the laminate.

The reduced field variables are

\[
\begin{bmatrix}
\{u\}^{(k)} \vspace{1pt} \\
\{\phi\}^{(k)} \vspace{1pt} \\
\{\bar{v}\}^{(k)} \vspace{1pt} \\
\{\bar{\phi}\}^{(k)} \vspace{1pt} \\
\{\bar{u}\}^{(k)}
\end{bmatrix}
\] (3.55)

\[
\begin{bmatrix}
\{\sigma\}^{(k)}
\end{bmatrix}
= 
\begin{bmatrix}
\sigma^{(k)}_{\alpha 3} & \sigma^{(k)}_{33}
\end{bmatrix}
\] (3.56)
3.2.4  Finite Element Implementation

3.2.4.1  Spatial Discretization

To implement finite element method to solve the initial boundary value problem, the domain of interest $R$ is divided into a number of elements $R_e$, such that

$$
R = \lim_{m \to \infty} \sum_{e=1}^{m} R_e
$$

where $e = 1, 2, 3, \ldots, m$.

The elements are non-overlapping and connected with each other through nodal points. For each element, an unknown field variable $\bar{v}_\alpha$ can be approximated in matrix form as

$$
\bar{v}_\alpha = [\psi)_e^T \{a_e\}_e
$$

where

$[\psi]_e$ is the set of base functions;

$\{a_e\}_e$ is the column vector of unknown coefficients.

The value of any arbitrary point in the element can be approximated as

$$
\bar{v}_\alpha = [\psi]_e [T^T] \{v\}_e
$$

where

$[\psi]_e$ is the set of interpolation function, $[\psi]_e [T^T]^{-1} = [\psi]_e [\psi]^{-1}$.
For the $k$-th layer,

$$
\bar{v}_{\alpha}^{(k)} = [\psi_{\alpha}]^{T} \{ \bar{v}_{\alpha} \}^{(k)}_e
$$

(3.60)

Similarly, for the other unknown field variables in Equations 3.55 and 3.56 can be approximated as

$$
\bar{\phi}_{\alpha}^{(k)} = [\psi_{\alpha}]^{T} \{ \bar{\phi}_{\alpha} \}^{(k)}_e
$$

(3.61)

$$
\bar{v}_{3}^{(k)} = [\psi_{3}]^{T} \{ \bar{v}_{3} \}^{(k)}_e
$$

(3.62)

$$
\bar{\phi}_{3}^{(k)} = [\psi_{3}]^{T} \{ \bar{\phi}_{3} \}^{(k)}_e
$$

(3.63)

$$
\bar{u}_{3}^{(k)} = [\psi_{3}]^{T} \{ \bar{u}_{3} \}^{(k)}_e
$$

(3.64)

$$
\sigma_{r3}^{(k)} = [\psi_{r3}]^{T} \{ \sigma_{r3} \}^{(k)}_e
$$

(3.65)

$$
\sigma_{33}^{(k)} = [\psi_{33}]^{T} \{ \sigma_{33} \}^{(k)}_e
$$

(3.66)

Substituting Equations 3.60 to 3.66 into the specialized governing equation, the matrix form of the governing function for each element can be written as

$$
\Omega_e = -U_e^{T}[M_e]U_e - t \ast U_e^{T}[K_e]U_e + 2U_e^{T}([R_0]_e + t \ast [R]_e - t \ast [\bar{R}]_e)
$$

(3.67)

where $[M_e]$ is the element mass matrix;

$[K_e]$ is the element stiff matrix;

$[U_e]$ is the vector of field variables at nodal points of an element;

$[R_0]_e$ is the equivalent nodal load vector due to initial conditions at nodal points of an element;

$[R]_e$ is the applied load vector at nodal points of an element;

$[\bar{R}]_e$ is the vector of pseudo body forces at nodal points of an element;
“∗” is the convolution integral.

The governing function for the global system can be assembled from the element governing function (Equation 3.67) as

$$
\Omega_e = \sum_{e=1}^{m} \Omega_e = -\{U\}^T [M]\{U\} - t * \{U\}^T [K]\{U\} + 2\{U\}^T \{R_o\} + t * \{R\} - t * \{R\}
$$

To minimize the governing function, taking the differential of Equation 3.68 with respect to the field variables \([U]\) and setting it to zero yields:

$$
[M]\{U\} + t * [K]\{U\} = \{R_o\} + t * \{R\} - t * \{R\}
$$

where \([M]\) is the global mass matrix;

\([K]\) is the global stiff matrix;

\([U]\) is the vector of field variables at nodal points;

\([R_o]\) is the equivalent nodal load vector due to initial conditions;

\([R]\) is the applied load vector;

\([R]\) is the vector of pseudo body forces.

Differentiating Equation 3.69 with respect to time twice finally yields

$$
[M]\{U\} + [K][U] = \{R\} - \{R\}
$$

Based on the specialization of the governing equation, it has been assumed that the field variables \(\bar{v}_a^{(k)}\), \(\bar{\varphi}_a^{(k)}\), \(\bar{v}_3^{(k)}\), \(\bar{\varphi}_3^{(k)}\), \(\bar{u}_3^{(k)}\), and \(\bar{\sigma}_{33}^{(k)}\) are continuous across interelement boundaries. For rectangular elements, bilinear interpolation is needed for \(\bar{v}_a^{(k)}\), \(\bar{\varphi}_a^{(k)}\), \(\bar{v}_3^{(k)}\), \(\bar{\varphi}_3^{(k)}\), \(\bar{u}_3^{(k)}\), and \(\bar{\sigma}_{33}^{(k)}\) while the transverse shear stress \(\bar{\sigma}_{a3}^{(k)}\) requires higher order of
interpolation.

The Heterosis element first introduced by Hughes and Cohen (1974) was selected to apply the stress-based theory in the finite element model. The Heterosis element (Figure 3.3) includes a synthesis of the selectively integrated nine-node Lagrange element and eight-node serendipity element. Hughes (1987) showed that Heterosis element has the advantage over both nine-node Lagrange element and eight-node serendipity element for combining their attributes but avoiding their shortcomings. Nine-node Lagrangian interpolation was used for the in-plane generalized displacements, $v_a^{(k)}$, $\phi_a^{(k)}$, and transverse shear stresses $\sigma_{a3}^{(k)}$. The generalized transverse displacements, $\tilde{v}_3^{(k)}$, $\tilde{\phi}_3^{(k)}$, $\tilde{u}_3^{(k)}$, and the transverse normal stress, $\sigma_{33}^{(k)}$ were approximated by 8-node isoparametric quadratic interpolation scheme.

![Figure 3.3 Heterosis Element](image-url)
3.2.4.2 Temporal Discretization

After spatial discretization, the governing equations have been reduced to a system of ordinary differential equations in the time domain (Equation 3.70). To complete the solution process and solve this linear dynamic problem numerically, the equations must be integrated. The direct integration methods which are widely used in computational structural dynamics (Dokainish and Subbaraj, 1989; Subbaraj and Dokainish, 1989) can be subdivided into explicit and implicit methods. Explicit time integration methods demand less storage than implicit methods but generally require small time steps to ensure numerical stability. Implicit methods usually require considerably more computational effort per time step than the explicit methods but the time steps may be larger and many implicit methods are unconditionally stable for linear analysis.

Wilson’s $\beta - \gamma - \theta$ step-forward implicit integration method (Ghaboussi and Wilson, 1972; Hiremath et al., 1988) was used to solve the differential matrix equations for the dynamic stress-based theory.

The field variable vector and its derivative at time step $(t_n + \theta \Delta t)$, $\{U\}_{n+\theta}$ and $\{\dot{U}\}_{n+\theta}$, can be expressed in terms of $\{U\}_n$, $\{\dot{U}\}_n$, and $\{\ddot{U}\}_n$ at time step $t_n$ as

$$
\{U\}_{n+\theta} = \{U\}_\theta + \theta \Delta t \{\dot{U}\}_n + \left(1 - \beta\right) \left(\theta \Delta t\right)^2 \{\ddot{U}\}_n + \beta \left(\theta \Delta t\right)^2 \{\ddot{U}\}_{n+\theta}
$$

(3.71)
\[ \{ \bar{U} \}_{n+\theta} = \{ \bar{U} \}_0 + (1 - \gamma)(\theta \Delta t)\{ \bar{U} \}_n + (\gamma \theta \Delta t)\{ \bar{U} \}_{n+\theta} \]  

(3.72)

where \( \beta, \gamma, \) and \( \theta \) are integration constants.

Substituting Equations 3.71 and 3.72 into Equation 3.70 yields

\[ \{ K^{*} \} \{ U \}_{n+\theta} = \{ R^{*} \}_{n+\theta} - \{ R \}_{n+\theta} \]  

(3.73)

where

\[ [ K^{*} ] = [ K ] + \frac{1}{\beta(\theta \Delta t)^2} [ M ] \]  

(3.74)

\[ \{ R^{*} \}_{n+\theta} - \{ R \}_{n+\theta} = [ R ]_{n+\theta} - [ R ]_{n+\theta} + \frac{1}{\beta(\theta \Delta t)^2} [ M ] \{ a \}_{n+\theta} \]  

(3.75)

\[ \{ a \}_{n+\theta} = \{ U \}_n + (\theta \Delta t)\{ \bar{U} \}_n + \left( \frac{1}{2} - \beta \right)(\theta \Delta t)^2\{ \bar{U} \}_n \]  

(3.76)

Assuming cubic variation of nodal field variables over the time step \((t_n, t_{n+1})\) in terms of \( \{ U \}_n, \{ \bar{U} \}_n, \) and \( \{ \bar{U} \}_n, \) the values of these variables at time \((t_n + \Delta t)\) are

\[ \{ U \}_{n+1} = \frac{1}{\theta^3} \{ U \}_{n+\theta} + \left( 1 - \frac{1}{\theta^3} \right) \{ U \}_n + \left( 1 - \frac{1}{\theta^2} \right) \Delta t \{ \bar{U} \}_n + \frac{1}{2} \left( 1 - \frac{1}{\theta} \right)(\Delta t)^2 \{ \bar{U} \}_n \]  

(3.77)

\[ \{ \bar{U} \}_{n+1} = \frac{\gamma}{\beta \theta^3 \Delta t} (\{ U \}_{n+\theta} - \{ U \}_n) + \left( 1 - \frac{\gamma}{\beta \theta^2} \right) \{ \bar{U} \}_n + \left( 1 - \frac{\gamma}{2 \beta \theta} \right) \{ \bar{U} \}_n \]  

(3.78)

\[ \{ \bar{U} \}_{n+1} = \frac{1}{\beta \theta^3 \Delta t^2} (\{ U \}_{n+\theta} - \{ U \}_n) - \frac{1}{\beta \theta^2 \Delta t} \{ \bar{U} \}_n + \left( 1 - \frac{1}{2 \beta \theta} \right) \{ \bar{U} \}_n \]  

(3.79)

The equations are solved iteratively. For each time step, the field variable and its derivatives \( \{ U \}, \{ \bar{U} \}, \) and \( \{ \bar{U} \} \) are first calculated by neglecting the pseudo-body force term \( \{ R \} \). Then \( \{ R \} \) is calculated and substituted back to Equation 3.73 and the updated \( \{ U \}, \{ \bar{U} \}, \) and \( \{ \bar{U} \} \) are obtained.
3.3 Summary

This chapter presents the theoretical derivations and finite element implementation of the stress-based discrete layer model with variable density. The development of the governing equations follows the derivations presented in the work by Butalia (1996) Schoepner (1991) and Chyou (1989). The stress-based model was modified only where necessary and the derivations were given here for clarity and completeness of presentation. In the next chapters, the stress-based model will be verified by comparing with existent solutions and experimental results.
CHAPTER 4

MODEL EXTENSION TO PAVEMENT SYSTEMS AND VERIFICATION

Previous studies [Chyou (1989); Schoepner (1991); Butalia (1996)] have shown that the stress-based finite element model can overcome the disadvantages of displacement-based methods and provide not only accurate predictions of displacements but also transverse stress distributions in composite laminates. In this chapter, the stress-based model is extended to analyze layered pavement systems. The new pavement model is then verified by comparing its solutions to existing analytical and numerical solutions, and to experimental results.

4.1 Analysis of the Numerical Integration Method

Both pavements and composite laminates are alike in that both are layered systems consisting of different materials which may serve as monolithic structures or as structural
members. However, the two systems are very different in terms of the physical dimensions and material properties of the individual components (layers). To adapt the stress-based finite element developed for composite laminates to pavement applications, it is necessary to re-examine the entire methodology. Of particular concern is whether numerical stability of the model using pavement dimensions and material properties can be achieved.

Two important aspects that must be considered in the model extension are the appropriateness of the integration scheme and the identification of a practical time step $\Delta t$. Furthermore, the boundary conditions of the stress-based model are modified to accommodate the behavior of pavement systems. The new stress-based pavement model is implemented in a FORTRAN computer program by the extension of the finite element codes written by Chyou (1989), Schoeppner (1991) and Butalia (1996).

Figure 4.1 and Table 4.1 show the geometry and material properties of a rigid pavement modeled with a 10-layer stress-based finite element model using a 2 by 2 mesh.
Figure 4.1 10-Layer Model for a Rigid Pavement System
A triangular impulse loading is applied at the center of the pavement in transverse normal direction with a peak load of –1000 kPa at time $\tau = 10$ ms. The applied loading can be expressed as:

$$q(x_1,x_2,t) = q\left(\frac{a}{2}, \frac{b}{2}, t\right) = \begin{cases} q_0 \frac{2t}{\tau} & \text{if } t \leq \frac{\tau}{2} \\ 2q_0 \left(1 - \frac{t}{\tau}\right) & \text{if } \frac{\tau}{2} \leq t \leq \tau \\ 0 & \text{if } t \geq \tau \end{cases}$$

where $\tau = 20$ ms, $q_0 = -1000$ kPa.
The applied triangular impulse load is close to the low velocity impact load applied to the pavement surface during a typical Falling Weight Deflectometer testing. The boundary conditions are applied assuming the in-plane displacements, $U_1, U_2 = 0$ at the pavement edge and the through-the-thickness displacement, $U_3 = 0$ for the bottom layer.

### 4.1.1 Numerical Integration Schemes

Wilson’s $\beta - \gamma - \theta$ step-forward implicit integration method was used to solve the differential matrix equations for the dynamic stress-based theory. Different integration methods can be used by choosing appropriate integration constants (Ghaboussi and Wilson, 1972). In this study, the value of $\gamma = \frac{1}{2}$ is used since $\gamma < \frac{1}{2}$ introduces a negative damping and leads to divergence while $\gamma > \frac{1}{2}$ introduces a positive damping but can sometimes damp out desirable features of the dynamic response of the system (Ghaboussi and Wilson, 1972). Six different well-known integration schemes (Table 4.2) were studied to examine the numerical stability using appropriate values of integration constants $\beta$ and $\theta$. The time step of the analysis is selected to be $2 \times 10^{-5}$ sec, which was about 80% of the shortest time required for the transverse wave to propagate through any of the layers.
### Table 4.2 Results of Convergence Study for Different Integration Schemes for the Wilson's $\beta-\gamma-\theta$ Method

Two of the six integration schemes studied, Wilson’s method and the stiff linear acceleration method, led to convergent solutions. The results from these two integration schemes are shown in Figures 4.2 and 4.3. Figure 4.2 shows the generalized through-the-thickness displacement ($U_3$) calculated at the center of the pavement surface where the load is applied for the two methods. Figure 4.3 compares the two calculated interlayer transverse normal stress time histories between the concrete layer and the base course right below the location where the impact load is applied. As can be seen, the results from the analyses using these two convergent integration schemes are identical. The Wilson’s
integration method ($\beta = 1/6$ and $\theta = 2.0$) were selected as the numerical integration scheme for the pavement response models in the remaining analyses.

Figure 4.2 Comparison of Vertical Displacement ($U_3$) of the First Layer from Two Different Integration Schemes
Figure 4.3 Comparison of Interface Transverse Normal Stress ($\sigma_{33}$) between Concrete and Base Layers from Two Different Integration Schemes
4.1.2 Numerical Integration Time Step

The goal in the numerical integration of the equilibrium equations for the dynamic finite element models is to obtain an accurate dynamic response with the necessary computational cost. The computational cost of a dynamic finite element model using direct integration method is directly proportional to the number of required time steps. In selecting an appropriate time step $\Delta t$, the following conditions should be considered (Farhoomand, 1970):

1) must be small enough to approximate the loading function accurately.

2) must be so small that the procedure yields a sufficiently accurate result.

3) must be sufficiently small to make the procedure stable.

An integration method is called unconditionally stable if the solution for any initial conditions is convergent for any time step $\Delta t$. The method that is convergent for any initial conditions but only when the time step $\Delta t$ is smaller than a critical limit value $\Delta t_{cr}$ is called conditionally stable (Bathe, 1996).

When using unconditionally stable integration schemes (such as the Wilson’s method) the time step is still limited since it should be small enough that the response in all modes that have significant contributions to the total structural response is calculated
accurately. Comprehensive analyses of the stability of the numerical integrations have been presented by Farhoomand (1970) and Nickell (1971).

For the analyses in the Section 3.3.1.1, the time step of the analysis is selected to be $2 \times 10^{-5}$ sec, which is about 80% of the shortest time required for the transverse wave to propagate through any of the layers. To verify the accuracy of the solution, the same finite element model is solved with a smaller time step ($\Delta t = 1 \times 10^{-5}$ sec). Results from the analyses using two different time step are compared in Figures 4.4 and 4.5.

![Figure 4.4 Comparison of Vertical Displacement ($U_3$) of the First Layer using Different Time Steps](image)

Figure 4.4 Comparison of Vertical Displacement ($U_3$) of the First Layer using Different Time Steps
As shown in the figures, the two analyses with different time steps yield identical results. Therefore, the larger time step (80% of the shortest time required for the transverse wave to propagate through any of the layers) was chosen to reduce the computation time.
4.2 Comparison of Surface Deflections with Existing Solutions

To test the accuracy of the stress-based model, the results of the proposed finite element model were compared with existing analytical and numerical solutions.

Measurements of pavement surface deflection are the primary means of non-destructive evaluation of the structure integrity of pavement systems. During a typical testing, the pavement’s surface deflections are measured as the result of an applied (either static or dynamic) load. It is assumed that surface deflection reflects the overall stiffness and structural conditions of a pavement system and the measured surface deflections can be used to backcalculate the actual stiffness of the different pavement layers. As discussed in Chapter Two, a number of analytical and numerical solutions are available to predict the pavement surface deflections. The surface deflection \( U_3 \) from the stress-based model are compared with the existing solutions.

4.2.1 Westergaard’s Closed Form Analytical Solution

Based on Westergaard’s closed form analytical solutions, the maximum deflections due to interior loading conditions (see Figure 4.6) can be expressed as (Westergaard,
\[ \Delta_i = \frac{P}{8k\ell^2} \left\{ 1 + \frac{1}{2\pi} \left[ \ln\left( \frac{a}{2\ell} \right) - 0.673 \left( \frac{a}{\ell} \right)^2 \right] \right\} \]  

(4.1)

where:

\[ \Delta_i = \text{surface deflection under interior loading}; \]
\[ P = \text{applied interior load}; \]
\[ \nu = \text{Poisson’s ratio}; \]
\[ h = \text{thickness of the slab}; \]
\[ a = \text{radius of the circular loaded area}; \]
\[ b = a \text{ when } a \geq 1.724h, \]
\[ b = \sqrt{1.6a^2 + h^2} - 0.675h \text{ when } a < 1.724h; \]
\[ \ell = \text{radius of relative stiffness} = \left[ \frac{Eh^3}{12(1-\nu^2)k} \right]^{0.25} \]
\[ k = \text{the modulus of subgrade reaction}. \]
Considering the same rigid pavement system presented in the Section 4.1, the elastic modulus and Poisson’s ratio of the 200 mm thick concrete slab are 41 GPa and 0.17, respectively. The applied load is taken to be 70650 N (15880 lb) that uniformly distributed over a 150 mm radius (6 in) circular area. The composite modulus of subgrade reaction \( (k) \) is estimated to be 1085 MN/m\(^3\) (4000 pci) according to the AASHTO pavement design guide (AASHTO, 1993). The calculated maximum surface deflection using the Westergaard’s equation (Equation 4.1) is \( 4.78 \times 10^{-5} \) m. As shown in Figure 4.7,
the pavement deflection for the Westergaard solution is about 20% greater than the peak response predicted by the stress-based model.

![Comparison of Solution of Stress-based Model and the Westergaard’s Solution](image)

Figure 4.7 Comparison of Solution of Stress-based Model and the Westergaard’s Solution

### 4.2.2 Displacement-Based 3D Finite Element Model

Several researchers have verified that displacement-based finite element models can accurately predict displacements and strains [i.e., Huang, 1974; Tabatabaie and
Therefore, it is useful to verify the stress-based model by comparing predicted pavement surface deflections from the stress-based model with those obtained from displacement-based finite element models.

The 3D finite element program ABAQUS is used to model the same rigid pavement system described in the Section 3.3.1. The material properties and mesh discretization are listed in Table 4.3. Equivalent boundary conditions are applied to the pavement system as in the stress-based model: \( U_1, U_2 = 0 \) at the pavement edge and the vertical displacement, \( U_3 = 0 \), at the bottom of the pavement.

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Modulus (Pa)</th>
<th>Poisson’s Ratio</th>
<th>Density (kg/m³)</th>
<th>Discretization (( N_x \times N_y \times N_z ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>4.10E+10</td>
<td>0.17</td>
<td>2400</td>
<td>34\times34\times5</td>
</tr>
<tr>
<td>CCP Stabilized Base</td>
<td>7.60E+09</td>
<td>0.35</td>
<td>1200</td>
<td>22\times18\times2</td>
</tr>
<tr>
<td>Lime-Fly-Ash Stabilized Subgrade</td>
<td>5.50E+08</td>
<td>0.45</td>
<td>2100</td>
<td>22\times18\times5</td>
</tr>
</tbody>
</table>

Table 4.3 Material Properties and Spatial Discretization of the 3D Finite Element Model

Both static and dynamic loading are simulated in the 3D ABAQUS finite element model. For the static analysis, a uniformly distributed pressure of \( –1000 \) kPa is applied to the center of the pavement surface (see Figure 4.8). The loaded area is assumed to be a 130 mm \( \times \) 130 mm (5 in \( \times \) 5 in) square which has the equivalent area of a circle with a
radius of 150 mm (6 in). For the dynamic analysis, a triangular load is applied over the square loaded area at the center of the pavement with a peak load of −1000 kPa at time \( \tau = 0.01 \) sec. The time step for the dynamic analysis was chosen as \( 5 \times 10^{-6} \) sec. Five elements are used in the through-the-thickness direction (Figure 4.9) for the concrete layer (so to make the time step smaller than the least travel time through any element), the time step used is smaller than the one for the stress-based model.
Figure 4.8 3D Finite Element Rigid Pavement Model in ABAQUS

\[ q = 1000 \text{ kPa} \]

Concrete

Base

Subgrade
Figure 4.9 3D Finite Element Mesh for the Rigid Pavement Model in ABAQUS
Figure 4.10 compares the pavement surface deflection at the center of loading from the stress-based model and both static and dynamic analysis of 3D ABAQUS finite element model and the Westergaard solution.

Figure 4.10 Comparison of Pavement Surface Deflection under the Loading Center

It can be seen from this figure that there is a good agreement in the predicted pavement surface deflection between the stress-based model and both 3D ABAQUS models.
4.3 Comparison of Interface Transverse Stress with Existing Solutions

As summarized in Chapter Two, debonding of the pavement layers has long been recognized as a cause of premature failure of pavements. A lot of effort has been made to investigate the effect of pavement debonding to the pavement life and the measurement of interface bonding strength in the last three decades. However, little has been done to study the underlying cause of pavement layer debonding.

In the mechanics of composite laminates, it has been shown that delamination failure is governed by the localized transverse stresses in the vicinity of the lamina interface (Abrate, 1991). In Chapter Two, the literature review has shown that the existing pavement response models either do not have the capability to predict the transverse stress distribution (e.g., ISLAB2000 in the new Mechanistic-Empirical Pavement Design Guide) or they are incapable of providing accurate predictions. A summary of the available pavement finite element models was presented in Table 2.2. The only existing models that can provide the transverse stresses are 3D displacement finite element models. Therefore, the comparisons of transverse stress solution from the stress-based model are only made with the solutions from the 3D displacement-based finite element model described in Section 4.2.2.

The predicted transverse normal stresses at the interface between the concrete slab and base course are compared using ABAQUS and the stress-based model. To examine
the effect of mesh configuration on convergence and accuracy of solution, five 3D
ABAQUS models with same dimensions and material properties are solved using
different mesh configurations and time steps (See Table 4.4). Time steps are chosen
according to the number of elements used through-the-thickness.
### Table 4.4 Mesh Discretization for the 3D ABAQUS Models

<table>
<thead>
<tr>
<th>Model#</th>
<th>Concrete Slab Mesh ((N_x \times N_y \times N_z))</th>
<th>Base Course Mesh ((N_x \times N_y \times N_z))</th>
<th>Subgrade Mesh ((N_x \times N_y \times N_z))</th>
<th>Total number of Elements</th>
<th>Time Step (\Delta t) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12×10×1</td>
<td>12×10×1</td>
<td>12×10×2</td>
<td>480</td>
<td>2.0E-4</td>
</tr>
<tr>
<td>2</td>
<td>12×10×2</td>
<td>12×10×2</td>
<td>12×10×3</td>
<td>840</td>
<td>2.0E-4</td>
</tr>
<tr>
<td>3</td>
<td>26×26×3</td>
<td>22×18×2</td>
<td>22×18×5</td>
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<td>2.0E-4</td>
</tr>
<tr>
<td>4</td>
<td>26×26×4</td>
<td>22×18×2</td>
<td>22×18×5</td>
<td>5476</td>
<td>2.0E-4</td>
</tr>
<tr>
<td>5</td>
<td>30×24×3</td>
<td>22×18×2</td>
<td>22×18×5</td>
<td>8552</td>
<td>5.0E-6</td>
</tr>
</tbody>
</table>
Figure 4.11 compares the pavement surface deflection from the five 3D ABAQUS models and the stress-based model. As can be seen, the 3D ABAQUS solutions appear to converge to almost the same value when the through-the-thickness direction of the concrete slab is modeled with three or more elements. All five 3D ABAQUS models predict the pavement surface deflections close to the solution from the stress-based model with the maximum difference of less than 20% in the peak deflection.

Figure 4.11 Comparisons of Pavement Surface Deflection from 3D ABAQUS Models with Different Mesh
Figure 4.12 compares the interface transverse normal stress at the bottom of the concrete slab. It can be seen that the number of elements used through-the-thickness has a significant effect on the results. While the displacement results converge to almost the same value, the transverse normal stress continues to change when four and five elements are used for concrete slab.

![Graph showing transverse normal stress comparison](image)

Figure 4.12 Comparisons of Transverse Normal Stress at the Bottom of Concrete Slab from 3D ABAQUS Models with Different Mesh
In the displacement-based finite element model, the nodal displacements are calculated as the primary variables while stresses and strains are calculated by numerically differentiating the approximate solutions. Hence the continuity of displacement is ensured exactly at the layer interfaces but the transverse stress continuity can not be enforced due to the inherent limitations of displacement based formulation.

Figure 4.13 compares the vertical displacement, $U_3$, at the interface between concrete slab and base course from the 3D ABAQUS model with mesh #5.
Figure 4.13 Comparison of Vertical Displacement at the Interface between Concrete Slab and Base Layer

Figures 4.14 to 4.18 compare the predicted transverse normal stress from the stress-based model and the 3D ABAQUS models with different mesh configurations.
Figure 4.14 Comparison of Transverse Normal Stress at the Interface between Concrete Slab and Base Layer (One Element Through-the-Thickness of Concrete Slab)
Figure 4.15 Comparison of Transverse Normal Stress at the Interface between Concrete Slab and Base Layer (Two Elements Through-the-Thickness of Concrete Slab)
Figure 4.16 Comparison of Transverse Normal Stress at the Interface between Concrete Slab and Base Layer (Three Elements Through-the-Thickness of Concrete Slab)
Figure 4.17 Comparison of Transverse Normal Stress at the Interface between Concrete Slab and Base Layer (Four Elements Through-the-Thickness of Concrete Slab)
As can be seen, the transverse stress is discontinuous across the interface. The difference between the stress prediction from the concrete and base layers decreases when more elements are used in the through-the-thickness direction. When five elements are used in the concrete slab through-the-thickness, the predicted transverse normal stress
on top of the base layer is still about 40% less than the one from the bottom of the concrete layer. This difference may continue to decrease when more elements are used in through-the-thickness direction. However, continuity in transverse stress cannot be achieved no matter how many elements are used due to the inherent limitations of displacement based formulation.

4.4 Comparison with Experimental Results

Further verification of the stress-based finite element model can be made by comparing with experimental results. Measured surface deflection and interface stress from two full-scale pavement testing program are used to compare with calculated responses from the stress-based model.

4.4.1 Comparison with OSU Accelerated CCP Pavement Test Results

A full-scale pavement testing program was conducted to evaluate the feasibility of using Coal Combustion Products (CCPs) as pavement construction materials by the Ohio Coal Combustion Products Extension Program at the Ohio State University (Tu, 2005;
Wolfe et al., 2006). Six full-scale pavement sections (three each for rigid and flexible pavement sections) were designed, constructed, and tested under accelerated loading and controlled environmental conditions at the Ohio Accelerated Pavement Loading Facility (APLF). All six test pavement sections were instrumented with response and environmental sensors. Pavement response, performance, and standard Falling Weight Deflectometer (FWD) tests were performed at regular intervals throughout the accelerated loading period equivalent to 32-year traffic on a typical state route in Ohio. Pavement surface deflections and vertical compressive stresses at the interface of subbase and subgrade measured by single layer deflectometers and pressure cells during the initial FWD tests are used compare with the calculated responses from the stress-based model.

4.4.1.1 APLF Test Pavement Sections

Each test section was designed to be 4.57 m (15 ft) long with the standard highway lane width of 3.66 m (12 ft). Figure 4.19 shows the plan view of the test pavement sections. Figure 4.20 presents the typical instrumentation plans for flexible pavement sections.
Figure 4.19 Plan View of the APLF Test Pavement Sections
Figure 4.20 Instrumentation Plan of APLF Test for Flexible Pavement Sections
4.4.1.2 Pavement Response Comparison

The 10-layer stress-based FEM model is used to calculate the dynamic response of surface deflection and interface transverse normal stress between the subbase and subgrade. The model parameters for APLF AC control section is shown in Table 4.5. The elastic moduli are backcalculated from the initial FWD test performed on flexible pavement sections. Poisson’s ratios are assumed based on typical values of material type. Details of the backcalculation can be found in Tu (2005). The calculated response is compared with the measured response during the initial FWD test.

<table>
<thead>
<tr>
<th>Layer #</th>
<th>Material</th>
<th>Thickness (mm)</th>
<th>Elastic Modulus (MPa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AC</td>
<td>50</td>
<td>22000</td>
<td>0.35</td>
</tr>
<tr>
<td>2</td>
<td>AC</td>
<td>50</td>
<td>22000</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>Base</td>
<td>76</td>
<td>2200</td>
<td>0.35</td>
</tr>
<tr>
<td>4</td>
<td>Base</td>
<td>76</td>
<td>2200</td>
<td>0.35</td>
</tr>
<tr>
<td>5</td>
<td>Subbase</td>
<td>152</td>
<td>2200</td>
<td>0.35</td>
</tr>
<tr>
<td>6</td>
<td>Subgrade</td>
<td>106.7</td>
<td>310</td>
<td>0.45</td>
</tr>
<tr>
<td>7</td>
<td>Subgrade</td>
<td>106.7</td>
<td>310</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>Subgrade</td>
<td>213.4</td>
<td>310</td>
<td>0.45</td>
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<td>320.1</td>
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<td>0.45</td>
</tr>
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<td>10</td>
<td>Subgrade</td>
<td>320.1</td>
<td>310</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 4.5 Model Parameters for APLF AC Control Section Analyses
Figure 4.21 shows the typical impulse load applied to the pavement surface during FWD tests. The time duration (difference between \( t = A, B, \) and \( C \)) depends on the drop height. Since only the peak value is recorded in the FWD, a triangular impulse load with the measured peak value of 770 kPa (111 psi) and duration of 25 ms is assumed as the applied dynamic load in the stress-based model.

![Figure 4.21 Typical Shape of Dynamic Impulse Load during FWD Tests](image)

Figures 4.22 and 4.23 present the measured surface deflection and interface transverse normal stress between subbase and subgrade during the initial FWD test performed on the AC control section.
The four peaks in the figures correspond to the preconditioning weight dropping and three impact loads applied by the FWD machine. The visco-elastic behavior of the asphalt concrete can be clearly observed in the measured surface deflection response. The amount of visco-elastic deflection is more than 30% of the measured total deflection. Because the pavement materials are assumed to be linearly elastic, the visco-elastic
behavior is not considered in the stress-based model. Moreover, the damping of the system is not considered in the formulation (see Chapter Three). Therefore, only the dynamic portion of the response under the heaviest FWD impact load (circled in Figure 4.22) is compared with the calculated response.

Figure 4.23 Measured Transverse Normal Stress between the Subbase and Subgrade from the initial FWD Test of APLF AC Control Section
Figures 4.24 and 4.25 compare the measured and calculated surface deflection and interface transverse normal stress between subbase and subgrade from the stress-based model.

![Figure 4.24 Comparison of Calculated and Measured Surface Deflection from the initial FWD Test of APLF AC Control Section](image)

Figure 4.24 Comparison of Calculated and Measured Surface Deflection from the initial FWD Test of APLF AC Control Section
Figure 4.25 Comparison of Calculated and Measured Transverse Normal Stress from the initial FWD Test of APLF AC Control Section

It can be seen that overall the stress-based model yielded a reasonably close prediction compared with measured values. The stress-based model over-predicted the surface deflection and transverse normal stress by about 15 and 30 percent, respectively. This difference is likely a consequence of limits in the accuracy level of the embedded
response instrumentation combined with the simplifying assumptions made in the formulation of the stress-based model.

4.4.2 Comparison with OSU FDR Pavement Test Results

The stress-based model is further verified by comparing the calculated response with measured field response from the ongoing OSU Full Depth Reclamation (FDR) research project. In this study, two severely deteriorated Ohio county roads were rehabilitated using the FDR technology with different stabilization agents including fly ash, lime-klin dust, cement, and bituminous emulsion. During construction, load response and environmental sensors were installed to monitor the stresses, strains, and deflections in the pavement, and subsurface water quality. Standard FWD tests and pavement response data have been collected and monitored on a quarterly basis since the completion of construction in September 2006. Pavement surface deflections and vertical compressive stresses at the interface of subbase and subgrade measured by single layer deflectometers and pressure cells during the initial FWD tests are used to compare with the calculated responses from the stress-based model.
4.4.2.1 FDR Test Pavement Sections

The OSU FDR project involved the construction of a 4-mile-long pavement section in Delaware County and a 0.4-mile-long pavement section in Warren County, Ohio. The Delaware county test site includes nine sections constructed with six different mixes. The Warren county test site includes one lime-fly-ash section and one control section. Figures 4.26 and 4.27 show the plan view and instrumentation plan of the test pavement sections.
FDR Test Sections

- **127 mm (5") Mill and Fill**
- **2% Cement with 1.6 gals/yd^2 Emulsion**
- **5% Cement only**
- **3% LKD with 1.4 gals/yd Emulsion**
- **5% LKD with 5% Fly ash**
- **4% Lime with 6% Fly ash**

Instrumentation Sites

- ○ Strain gages and Lysimeter only
- ≻ Full instrumentation

Figure 4.26 Plan View of OSU FDR Pavement Site at Delaware County, Ohio
4.4.2.2 Pavement Response Comparison

The 10-layer stress-based FEM model is used to calculate the dynamic response of interface transverse normal stress between the subbase and subgrade. The model parameters for FDR DEL section is shown in Table 4.6. The elastic moduli are
backcalculated from the FWD test. Poisson’s ratios are assumed based on typical values of corresponding material types. Details of the backcalculation can be found in Chapman (2007). The calculated dynamic transverse normal stress is compared with the stress during response test. Response tests were conducted with the right rear tires of a tandem-axle dump truck passing over the sensor at a speed of 24 km/h (15 mph). An impulse load with a peak value of 900 kPa (130 psi) and duration of 0.6 sec is assumed as the applied dynamic load in the stress-based model.

<table>
<thead>
<tr>
<th>Layer #</th>
<th>Material</th>
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<th>Elastic Modulus (MPa)</th>
<th>Poisson’s Ratio</th>
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<tr>
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</tr>
</tbody>
</table>

Table 4.6 Model Parameters for Response Analysis of FDR DEL Section #6
As can be seen in Figure 4.28, the discrepancy between the predicted and measured response was larger than that of APLF testing. The stress-based model over-predicted the surface deflection and transverse normal stress by almost 60 percent. The larger discrepancy in FDR field pavement testing is likely because of the variations in
environmental conditions such as moisture content (ground water level) and temperature which were kept as constant values in the APLF in-door pavement testing.

4.5 Summary

In this chapter, the numerical stability of the dynamic stress-based finite element model is first examined using different integration schemes and time steps. Then the stress-based model is verified by comparing the results with existing analytical and numerical solutions. The stress-based model was found to be capable of predicting displacements consistent with the solutions obtained from 3D displacement-based finite element models and Westergaard’s equations. Moreover, the stress-based model can provide more realistic and accurate transverse stress distributions at the pavement layer interfaces than the 3D displacement-base finite element models. Overall, a reasonably close prediction was obtained between calculated and measured responses from the full-scale pavement testing projects. The stress-based model allows one to accurately estimate the stress distributions at the pavement interfaces, which is the crucial step to predict pavement layer debonding failure and damage development.
CHAPTER 5

SENSITIVITY STUDY

The stress-based model was shown in the previous chapter to provide not only accurate predictions of displacement but also more realistic and accurate distribution of interface transverse stresses than traditional displacement-based models for pavements. In this chapter a sensitivity study is carried out to investigate the effect of pavement cross section dimensions, material properties, and load levels on the transverse stresses at pavement layer interfaces between surface layer and base course.

5.1 Effect of Surface Layer Thickness

Rigid pavements are those which are surfaced with Portland cement concrete (PCC). Because of the high elastic modulus of PCC, the majority of the structural load is carried by the surface course in rigid pavement. The thickness of the surface layer of a rigid pavement typically ranges in 150 mm (6 in) to 300 mm (12 in) depending on the traffic
Flexible pavements, which are composed of a bituminous surface course, deflect more and pass more of the applied load to the underlying layers. A wider range of thickness of surface course is used for flexible pavements than for rigid pavements. The minimum thickness of Hot Mix Asphalt (HMA) surface course of 75 mm (3 in) or 100 mm (4 in) is usually applied while a surface course of HMA thicker than 400 mm (16 in) can be used for high traffic with heavy truck loads (Huang, 2004; WSDOT, 2003). In this section, two models with typical material properties of rigid or flexible pavements are analyzed to examine the effect of thickness on the transverse stress at the interface between the surface and base layer.

5.1.1 Rigid Pavements

The 10-layer pavement model employing the stress-based formulation verified in Chapter Four is analyzed with different surface layer thicknesses and load levels. A longitudinal profile and plan view of the rigid pavement are shown in Figure 5.1. The layer thickness of the stress-based FEM model for the rigid pavement analysis is shown in Table 5.1. Three load levels (peak loads of 210 kPa, 420 kPa, and 700 kPa) are analyzed for each of the six pavement configurations.
Figure 5.1 Longitudinal Profile and Plan View of the Rigid Pavement for Sensitivity Study Using 10-Layer Stress-based FEM Model
<table>
<thead>
<tr>
<th>Layer #</th>
<th>Material</th>
<th>Thickness (mm)</th>
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<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
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</table>

Table 5.1 Layer Thickness of the Stress-Based FEM Model for Rigid Pavement Analysis

Figures 5.2 to 5.4 compare the transverse normal stress at interface between the concrete slab and stabilized base layer subjected to different load levels and slab thickness from 50 mm (2 in) to 300 mm (12 in). Figures 5.5 and 5.6 show the functions of peak transverse normal stress vs. surface layer thickness and peak transverse normal stress vs. applied load levels.
Figure 5.2 Transverse Normal Stress at Interface between Concrete Slab and Base Layer under 210 kPa Pressure Loading
Figure 5.3 Transverse Normal Stress at Interface between Concrete Slab and Base Layer

under 420 kPa Pressure Loading
Figure 5.4 Transverse Normal Stress at Interface between Concrete Slab and Base Layer under 700 kPa Pressure Loading
Figure 5.5 Peak Transverse Normal Stress vs. Surface Layer Thickness under Different Load Levels for Rigid Pavement Analysis
As expected, the transverse normal stresses at the interface of slab and base layer decrease as the thickness of slab increases and decrease as the applied loading increases.

It can be seen from Figures 5.5 and 5.6 that changing the magnitude of the applied loads...
increases the calculated stresses linearly but the effect on the calculated stresses of varying the surface layer thickness is non-linear.
5.1.2 Flexible Pavements

A similar sensitivity study was carried out on a typical flexible pavement structure using the same 10-layer stress-based FEM model and following the same procedure for varying the magnitude of the applied load and the thickness of the surface layer. The longitudinal profile and plan view of the flexible pavement is shown in Figure 5.7. The layer thickness of the stress-based FEM model for the flexible pavement analysis is shown in Table 5.2.
Figure 5.7 Longitudinal Profile and Plan View of the Flexible Pavement for Sensitivity Study Using 10-Layer Stress-based FEM Model
Table 5.2 Layer Thickness of the Stress-Based FEM Model for Flexible Pavement Analysis

Figures 5.8 to 5.10 compare the transverse normal stress at interface of the Asphalt Concrete (AC) surface course and stabilized base layer under different load levels for slab thickness in the range of 75 mm to 300 mm. Figures 5.11 and 5.12 show the peak transverse normal stress vs. surface layer thickness and peak transverse normal stress vs. applied load levels.
Figure 5.8 Transverse Normal Stress at Interface between AC Surface Course and Base Layer under 210 kPa Pressure Loading
Figure 5.9 Transverse Normal Stress at Interface between AC Surface Course and Base Layer under 420 kPa Pressure Loading
Figure 5.10 Transverse Normal Stress at Interface between AC Surface Course and Base Layer under 700 kPa Pressure Loading
Figure 5.11 Peak Transverse Normal Stress vs. Surface Layer Thickness under Different Load Levels for Flexible Pavement Analysis
Figure 5.12 Peak Transverse Normal Stress vs. Applied Load for Different Surface Layer Thicknesses for Flexible Pavement Analysis

It can be seen from the figures that the interface transverse stresses for the flexible pavement examples show a similar trend to rigid pavements when subjected to different load levels with variation in surface layer thickness. Figure 5.13 compares the function of normalized transverse normal stress and surface layer thickness for flexible and rigid
pavements, where $\sigma_{33\text{max}}$ and $q_{\text{max}}$ are the peak interface transverse normal stress and peak value of the applied load, respectively.

![Graph showing comparison of normalized interface transverse normal stress of flexible and rigid pavements of different surface layer thicknesses.](image)

Figure 5.13 Comparison of Normalized Interface Transverse Normal Stress of Flexible and Rigid Pavements of Different Surface Layer Thicknesses

The flexible pavement examples show significantly higher transverse normal stresses at the interface. The magnitude of the applied loads has a linear effect on the induced stresses in both the rigid and flexible pavements because Hooke’s law was
assumed to be the appropriate constitutive law for both types of pavements. The effect of the thickness of the surface layer on the stresses at the surface layer-base interface appear to be linear for the flexible pavement but is clearly nonlinear for the rigid (i.e., high modulus) pavements.

Although the effect of HMA surface layer with thickness of 75 mm to 300 mm is studied, it should be noted that due to the compaction requirement and construction equipment limitations, the HMA surface course has to be constructed in lifts typically with maximum thickness of no more than 100 mm (4 in) (FHWA, 2003). Therefore, the critical potential debonding surface may be within the surface layer itself, depending on the bond strength between individual lifts.

### 5.2 Effect of Material Properties

In this study, it was assumed that all paving materials satisfied Hooke’s Law. The two material properties used in this model to characterize the behavior of a Hookean material were the modulus of elasticity and Poisson’s ratio. Because the Poisson’s ratio has been shown to have a relatively small effect on pavement response (Huang, 2004), it was held constant while the sensitivity of the calculated stresses to varying the elastic
modulus of the pavement layers was examined. The typical values of elastic moduli for different paving materials are listed in Table 5.3.

<table>
<thead>
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<th>Material</th>
<th>Typical Value (MPa)</th>
<th>Range of Typical Elastic Modulus (MPa)</th>
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</thead>
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<td>PCC</td>
<td>31000</td>
<td>21000 to 45000</td>
</tr>
<tr>
<td>AC</td>
<td>5000</td>
<td>4°C (40°F) 6900 to 20700 21°C (70°F) 3450 to 7600 37°C (100°F) 700 to 2200</td>
</tr>
<tr>
<td>Aggregate Base</td>
<td>2000</td>
<td>70 to 7000</td>
</tr>
<tr>
<td>Cement Stabilized Base</td>
<td>14000</td>
<td>6900 to 20700</td>
</tr>
<tr>
<td>Lime-fly ash Stabilized</td>
<td>7600</td>
<td>3500 to 17000</td>
</tr>
<tr>
<td>Soil Cement</td>
<td>6900</td>
<td>350 to 14000</td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>90</td>
<td>50 to 120</td>
</tr>
</tbody>
</table>

Table 5.3 Summary of Typical Values of Elastic Modulus for Paving Materials

(Huang, 2004)

5.2.1 Rigid Pavements

The 10-layer stress-based model for the rigid pavement shown in Figure 5.1 was analyzed. A 200 mm (8 in) thick concrete slab was used as the surface layer. The layer
thickness of the 10-layer stress based model is listed as the Case #5 in Table 5.1. The effect of the elastic modulus of an individual pavement layer was studied by varying the modulus of that layer within a range of values typical for that material while fixing the moduli of other layers. Table 5.4 summarizes the elastic moduli used in the rigid pavement model. A triangular impulse loading was applied in all cases at the center of the pavement in transverse normal direction with a peak value of –700 kPa.
Table 5.4 Sensitivity Analysis of Elastic Moduli for Rigid Pavement

<table>
<thead>
<tr>
<th>Layer</th>
<th>Fixed $E$ (MPa)</th>
<th>Case 1 $E$ (MPa)</th>
<th>Case 2 $E$ (MPa)</th>
<th>Case 3 $E$ (MPa)</th>
<th>Case 4 $E$ (MPa)</th>
<th>Case 5 $E$ (MPa)</th>
<th>Case 6 $E$ (MPa)</th>
<th>Case 7 $E$ (MPa)</th>
<th>Case 8 $E$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCC</td>
<td>31500</td>
<td>21000</td>
<td>24500</td>
<td>28000</td>
<td>31500</td>
<td>35000</td>
<td>38500</td>
<td>42000</td>
<td>45000</td>
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<tr>
<td>Base</td>
<td>7600</td>
<td>70</td>
<td>70</td>
<td>2100</td>
<td>6300</td>
<td>7600</td>
<td>9000</td>
<td>14000</td>
<td>20000</td>
</tr>
<tr>
<td>Subgrade</td>
<td>550</td>
<td>50</td>
<td>120</td>
<td>350</td>
<td>550</td>
<td>1000</td>
<td>2500</td>
<td>5000</td>
<td>14000</td>
</tr>
</tbody>
</table>
Figures 5.14 to 5.16 show the function of pavement layer moduli vs. the transverse normal stress at the interface of concrete slab and base course and pavement surface deflection. In each figure, the modulus of the layer of interest is varied in the typical range of that material while the moduli of the other two layers are fixed at the typical values shown in Table 5.4.
Figure 5.14 Effect of Modulus of Concrete on the Peak Interface Transverse Normal Stress (between Surface and Base Layer) and Surface Deflection
### Range of Typical Elastic Modulus for Base Materials

<table>
<thead>
<tr>
<th>Base Material</th>
<th>Range of Typical Elastic Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Base</td>
<td>3.0 – 3.2</td>
</tr>
<tr>
<td>Cement Stabilized Base</td>
<td>3.4 – 3.6</td>
</tr>
</tbody>
</table>

#### Figure 5.15: Effect of Modulus of Base Layer on the Peak Interface Transverse Normal Stress and Surface Deflection for Rigid Pavement

**Concrete Subgrade:**
- Stress: $E = 15.0$ GPa, $\nu = 0.17$, $\rho = 2200$ kg/m$^3$
- Deflection: $E = 31.5$ GPa, $\nu = 0.35$, $\rho = 1200$ kg/m$^3$

**Aggregate Base:**
- Stress: $E = 0.55$ GPa, $\nu = 0.45$, $\rho = 2400$ kg/m$^3$

**Cement Stabilized Base:**
- Stress: $E = 700$ kPa, $\nu = 0.20$, $\rho = 2000$ kg/m$^3$

**Surface Deflection Output:**
- $U_3 = -50$ mm
- $U_3 = -100$ mm
- $U_3 = -150$ mm
- $U_3 = -200$ mm
- $U_3 = -250$ mm

**Interface Stress Output:**
- $\sigma_{33} = -10$ kPa
- $\sigma_{33} = -15$ kPa
- $\sigma_{33} = -20$ kPa
- $\sigma_{33} = -25$ kPa
- $\sigma_{33} = -30$ kPa

---

155
Figure 5.16 Effect of Modulus of Subgrade on the Peak Interface Transverse Normal Stress (between Surface and Base Layer) and Surface Deflection for Rigid Pavement

The peak pavement surface deflections under the dynamic load decrease when the elastic modulus of any of the pavement layers increases. The peak interface transverse normal stress decreases when the elastic modulus of concrete surface layer increases. On
the other hand, an increase in the elastic modulus of either of the underlying layers causes the interface transverse normal stresses to increase. This is likely because the concrete surface layer supplies more structural capacity by bending when the base and/or subgrade layers are less stiff. Therefore, less amount of the load is transmitted onto the underlying layers. Combining three stress vs. modulus curves (solid symbols) from Figures 5.14 to 5.16, Figure 5.17 compares the sensitivity of the individual layer elastic moduli to the interface transverse normal stresses between concrete slab and the base layer.
The fixed modulus for each layer is marked by the dashed vertical lines. It can be readily seen that the interface transverse normal stress is more sensitive to the subgrade and base layer moduli than for the concrete surface layer within the range of moduli examined. Based on these results, it is clear that the presence of a very stiff subgrade
 (> 5 × 10^4 MPa) can result in an interface transverse normal stress two to three times higher than the value of interface stress when a typical natural subgrade is used.

### 5.2.2 Flexible Pavements

Similarly, the flexible pavement structure shown in Figure 5.7 was analyzed with the 10-layer stress-based model. A 100 mm (4 in) thick HMA course was used as the surface layer. The layer thickness of the model is listed as the Case #2 in Table 5.2. A dynamic load was applied in all cases at the center of the pavement in the transverse normal direction with a peak value of -700 kPa. By varying the modulus of an individual layer within the range typical of that material while fixing the moduli of other layers with typical values, the sensitivity of layer modulus of flexible pavement could be studied. Table 5.5 summarizes the elastic moduli used for the flexible pavement model.
<table>
<thead>
<tr>
<th>Layer</th>
<th>Fixed E (MPa)</th>
<th>Case 1 E (MPa)</th>
<th>Case 2 E (MPa)</th>
<th>Case 3 E (MPa)</th>
<th>Case 4 E (MPa)</th>
<th>Case 5 E (MPa)</th>
<th>Case 6 E (MPa)</th>
<th>Case 7 E (MPa)</th>
<th>Case 8 E (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>5000</td>
<td>70</td>
<td>2200</td>
<td>3450</td>
<td>5000</td>
<td>6900</td>
<td>7600</td>
<td>14000</td>
<td>20700</td>
</tr>
<tr>
<td>Base&amp;Subbase</td>
<td>2000</td>
<td>70</td>
<td>700</td>
<td>2000</td>
<td>6300</td>
<td>7600</td>
<td>9000</td>
<td>14000</td>
<td>20000</td>
</tr>
<tr>
<td>Subgrade</td>
<td>550</td>
<td>50</td>
<td>120</td>
<td>350</td>
<td>550</td>
<td>1000</td>
<td>2500</td>
<td>5000</td>
<td>14000</td>
</tr>
</tbody>
</table>

Table 5.5 Sensitivity Analysis of Elastic Moduli for Flexible Pavement
Figures 5.18 to 5.20 show the function of pavement layer moduli vs. the transverse normal stress at the interface of AC surface layer and base course and pavement surface deflection. In each figure, the modulus of the layer of interest is varied in the typical range of that material while the moduli of the other two layers are fixed at the typical values shown in Table 5.5.

![Graph showing AC Surface Layer Modulus of Elasticity vs. Transverse Normal Stress and Deflection](image)

Figure 5.18 Sensitivity of AC Elastic Modulus on the Peak Interface Transverse Normal Stress between Surface and Base Layer and Surface Deflection
Figure 5.19 Sensitivity of Base and Subbase Elastic Modulus on the Peak Interface Transverse Normal Stress between Surface and Base Layer and Surface Deflection for Flexible Pavements
Figure 5.20 Sensitivity of Subgrade Elastic Modulus on the Peak Interface Transverse Normal Stress between Surface and Base Layer and Surface Deflection for Flexible Pavements

The elastic modulus of AC is highly temperature dependent. The surface layer modulus of a flexible pavement can vary over a much wider range than that of rigid pavement. As a result of the AC layer modulus variation, the peak interface transverse normal stress between surface course and the base for a flexible pavement during the
summer can be about two to three times as much as that of winter. Overall, the interface transverse stress and surface deflection show the same qualitative trends for flexible and rigid pavements when layer modulus increases. The pavement surface deflections under the dynamic load decrease when the elastic modulus of any of the pavement layer increases. An increase in the elastic modulus of either of the underlying layers causes the interface transverse normal stress to increase. Because of the lower surface layer stiffness, the interface stresses and surface deflection of flexible pavements are more sensitive to the modulus of base, subbase, and subgrade. As can be seen in Figures 5.19 and 5.20, the stress and deflection exhibit a sharp change when the moduli are less than half of the surface layer modulus. Figure 5.21 compares the sensitivity of the individual layer elastic modulus to the interface transverse normal stress between the surface and base layer.
Figure 5.21 Comparison of Sensitivity of Pavement Layer Moduli for Flexible Pavements

5.3 Summary

In this chapter, a sensitivity study was conducted to establish the understanding of the effect of different parameters (pavement layer thickness, layer moduli, and applied load levels) on the calculated interface stresses between surface course and base layer.
The effect of each parameter was studied by varying the values of that parameter within a range of typical values while fixing other parameters.

Subjected to the same surface load, the flexible pavement examples show significantly higher transverse normal stresses at the interface than the rigid pavements. The magnitude of the applied loads has a linear effect on the induced stresses in both the rigid and flexible pavements while the effect of the thickness of the surface layer on the stresses at the surface layer-base interface appear to be linear for the flexible pavement but is clearly nonlinear for the rigid pavements.

The peak pavement surface deflections under the dynamic load decrease when the elastic modulus of any of the pavement layers increases. The calculated interface transverse normal stress decreases when the elastic modulus of concrete surface layer increases. On the other hand, an increase in the elastic modulus of either of the underlying layers causes the interface transverse normal stresses to increase. It was shown that a very stiff subgrade can result in an interface transverse normal stress two to three times higher than the value of interface stress when a typical natural subgrade is used for rigid pavements. The interface stresses and surface deflection of flexible pavements are more sensitive to the modulus of base, subbase, and subgrade than those of rigid pavements. Both interface stresses and deflections exhibit a sharp change when the moduli are less than half of the asphalt concrete modulus.
CHAPTER 6

APPLICATIONS TO BONDED THIN CONCRETE OVERLAYS

Increasing travel demand especially in heavy truck traffic coupled with the shortfall of highway program funds has put more and more stress on this nation’s aging highway infrastructure. According to the 2004 annual pavement condition survey by the Federal Highway Administration, 68 percent of all major urban roads are in mediocre or poor condition (TRIP, 2006). The study estimated that while $11.2 billion was spent in 2004 to preserve the urban roads and highways at all government levels, $15.6 billion to $19.3 billion was actually needed to maintain or improve the condition of the urban roads and highways. As a result, there is a great demand for technically sound and cost-effective pavement rehabilitation methodologies. Recently, Bonded Concrete Overlay (BCO) and UltraThin Whitetopping (UTW) have gained great popularity (e.g., Mack et al., 1998; Better Roads, 2001; Delatte and Sehdev, 2003).
The greatest concern regarding the application of BCO and UTW is the debonding and premature failure of the overlay (Lin and Wang, 2005; Trevino et al., 2006). Some of the existing design methods for concrete overlays include the Portland Cement Association (PCA), the New Jersey and the AASHTO methods. However, none of these methods includes a mechanistic response model capable of accurately predicting accurate interface stresses, the direct cause of debonding failure. Therefore, the stress-based pavement response model which has shown the capability of accurately predicting the dynamic stresses at pavement layer boundaries is a promising tool for the analysis and design of BCO and UTW.

This chapter first briefly discusses the existing design procedures and models for thin concrete overlays. A literature review of the mechanisms of overlay debonding and the current bond strength testing methods are then summarized. Finally, the stress-based pavement model is used to identify the parameters that significantly affect the interface stress between the overlay and existing pavement and to compare calculated in service stresses with published bond strength data.
6.1 Background

Although concrete overlays have been used as a pavement rehabilitation method for almost 90 years (Better Roads, 2001; Smith et al., 2004; Rasmussen and Rozycki, 2004), there has been a renewed interest in them over the last decade, primarily in response to the rapidly deteriorating highways and the dual problem of shortfall in maintenance funds and rising construction costs. Figure 6.1 shows the number of UTW projects in the United States from during the period of 1989 to 2002.

Figure 6.1 UTW Construction in the United States (Rasmussen and Rozycki, 2004)
The thickness of thin concrete overlay is typically in the range of 50 mm (2 in) to 100 mm (4 in). The overlay is called BCO when it is constructed over a rigid pavement, whereas it is termed as UTW if it is bonded to an existing flexible pavement. Overlay rehabilitations serve three major design functions: 1) strengthen the existing pavement structure against further deterioration due to fatigue cracking; 2) improve smoothness and restore riding quality; and add skid resistance. Both BCO and UTW rely on full-bond between the concrete overlay and existing pavement, therefore, the overlaid pavement can behave as a monolithic structure with enhanced structural capacity.

6.2 Existing Design Methods and Models

6.2.1 PCA Design Method

PCA developed a design method for UTW based on a displacement-based 3D finite element model (Wu et al., 1998; Rasmussen and Rozycki, 2004). The pavement response (stresses and strains) under the design load is first calculated using regression equations developed based on the 3D finite element model. Then the design life of the overlaid pavement is estimated using damage models for the fatigue cracking of concrete overlay and the existing pavement surface layer.
The PCA design procedure is empirical-mechanistic in nature which is considered an improvement over some previous design procedures that are purely empirically based. However, there are some limitations that users need to be made aware of. In the PCA design method, two modes of failure are considered: fatigue cracking of concrete overlay at the corner and fatigue cracking of existing HMA layer beneath the UTW. Overlay debonding was not considered as a failure mode because the displacement-based finite element response model can not provide accurate interface stress responses.

6.2.2 New Jersey Method

The New Jersey Department of Transportation and the New Jersey Concrete and Aggregate Association developed a similar design procedure for UTW (Gucunski, 1998). In their study, a 3D displacement-based finite element model was used to conduct an extensive parametric study on different factors on the response of UTW overlay including overlay thickness, existing AC surface layer thickness, modulus of elasticity of overlay, AC and base layer, and overlay slab size. Design equations to calculate maximum bending tensile stress due to load and temperature variation were developed based on the finite element analysis. The PCA fatigue criterion was used to determine the life traffic
volume. The New Jersey design method does not consider debonding of the overlay as a failure mechanism.

6.2.3 AASHTO Method

The newly released Mechanistic-Empirical Design Guide (MEPDG) does not include the design procedures for thin and ultra-thin concrete overlays (NHI, 2007). The AASHTO method introduced in the 1993 AASHTO Pavement Design Guide is still the most widely used concrete overlay design procedure among state agencies. It is essentially an empirical method because the design equations were developed from regression analyses based on the AASHO Road Test data. The AASHTO method assumes that the overlay satisfies a deficiency between the structural capacity required to support traffic over some future design period and the existing structural capacity. This principle is known as the “structural deficiency” or “remaining life” approach. First, the thickness of a new pavement is determined based on the traffic, materials to be used and other design parameters considered in new pavement design. Then the effective thickness of the existing pavement is calculated based on the pavement deterioration condition and the serviceability left to be further consumed. The required thickness of overlay to be bonded to the existing pavement is the “structural deficiency” or the difference in the
thickness calculated for a new pavement and the effective thickness in the existing pavement.

6.3 Debonding Mechanism

Although debonding of the pavement layers has long been recognized as a cause of premature failure of pavements (Livneh and Schlarsky, 1962), there is a lack of fundamental research identifying pavement layer debonding mechanisms and failure criteria. This is partly because of the lack of reliable pavement response models capable of accurately predicting transverse interface stresses. As discussed in Chapter Two, a great deal of effort has been made to investigate the consequence of pavement layer debonding and bonding strength measurement. However, reliable failure criteria for pavement layer debonding are not yet available, even though a few attempts were made to investigate the debonding initiation mechanism (Granju, 2001; Granju et al., 2004).

Granju (2001) reported on the results of a study investigating the debonding mechanism of thin cement-based overlays. Granju summarized a previous experimental study by Do (1989) that aimed to reproduce debonding of overlay by shear along the interface using both static and cyclic load (Figure 6.2). However, debonding was neither achieved nor initiated by either static or cyclic load. The samples actually failed by
crushing the concrete overlay. Granju hypothesized that debonding may not be directly caused by the interface shearing in the vicinity of the load because the applied load introduces a compression at the interface that is beneficial in preventing debonding. To examine his hypothesis, an experimental study was carried out on composite specimens shown in Figure 6.3. It was found out that for pavement overlays the debonding is caused by the tensile transverse normal stress introduced by bending of the structure which is contrary to the common held belief that debonding is caused by the shear at the interface in the vicinity of applied load.

![Figure 6.2 Experimental Device Used to Study Debonding of Overlay by Do (1989)](Granju, 2001)

Figure 6.2 Experimental Device Used to Study Debonding of Overlay by Do (1989)
In the study by Granju et al. (2004), the two sources of debonding considered were:

1) mechanical origin, i.e. a consequence of the flexural straining of the structure by applied wheel loads, and 2) length change origin, i.e. changes in length of the overlay and base because of shrinkage and temperature variations (shown in Figure 6.4). Based on their study, it was concluded that the debonding of concrete overlay is governed by the tensile stress perpendicular to the interface caused by the coupled effect of both sources considered.
Lemieux et al. (2005) reported on an experimental study that investigated the performance of reinforced concrete slab panels with thin bonded concrete overlays subjected to static and cyclic loading of up to 500,000 cycles. The aim was to link the delamination damage to interface location and overlay thickness. No delamination was observed under the compression zone in the vicinity of loading area while slight to severe interlayer delamination was found in tension zone. The most severe delamination damage was found in the tension zone with no reinforcement.
Figure 6.4 Two Origins of Overlay Debonding (after Granju et al., 2004)
6.4 Bond Strength Test Methods

Both laboratory and in-situ test methods have been developed to measure the bond strength between an overlay and the existing pavement surface (either PCC or AC). Depending on the expected failure mode, most of the experimental methods can be categorized into either tension or shear tests. Because the debonding of thin concrete overlay is governed by the tensile stress perpendicular to the interface, only tension tests will be reviewed here. Tensile tests can be classified into two categories: direct or indirect tension tests.

6.4.1 Direct Tension Test

In a direct tension test, a specimen is pulled apart by tensile force applied perpendicularly to the bonded interface. In direct tension tests, special gripping is required to transfer the uniaxial tensile force without introducing eccentricity or bending moment to the sample. Figures 6.5 and 6.6 show two laboratory direct tension tests to measure the bond strength between PCC and concrete overlay interface.
In the friction grip setup, two identical split pipe pieces are used to tightly hold the specimen by closing the side split parallel to the axial of the pipe. A rubber “O” ring is used as a spacer between the two split pipe pieces at the bond interface. Two universal
ball and socket connections are used to minimize the eccentricity of the applied tensile force.

Figure 6.6 Pipe Nipple Grip Setup for Laboratory Direct Tension Tests (Kuhlmann, 1990)

In the pipe nipple grips, the cylindrical base concrete sample is first fit into a steel pipe nipple and bonded to the pipe using an epoxy. After the epoxy has cured, the sample is inverted and an identical black steel pipe nipple is mounted on top of the base concrete sample and the overlay concrete is poured into the empty steel pipe. After the overlay concrete is cured, pipe caps with universal ball and socket connections are installed on
both ends of the composite specimen. This method was developed by Kuhlmann (1990) and has been adopted as ASTM Standard C1404M-98.

Either of the aforementioned two direct tension tests can be used to measure the bond strength between the concrete and concrete interface, but they are only suitable for testing on a small scale in the laboratory. The pull-off test is a direct tension test to measure the in situ bond strength. Pull-off tests have gained popularity recently because they measure the in situ bond strength and can be used in both strength monitoring during construction and quality control purpose. Pull-off test devices have been developed in several countries including the United States, the United Kingdom, Denmark, Germany, and Switzerland (McDonald and Vaysburd, 2001). However, there is not much standardization. ASTM has not yet adopted a standard test method for in situ pull-off test. There are some variations between individual pull-off devices, but the general procedures in the test are the same. In a typical pull-off test, a direct tensile load is applied to a partial-depth core which penetrates through the overlay and into the substrate AC or PCC layer (Figure 6.7).
6.4.2 Indirect Tension Test

The indirect tension or splitting test is a laboratory test that measures the tensile bond strength by splitting the composite specimen at the interface. During the splitting test, a prism with circular or square cross-section is placed under longitudinal compressive loading along the material interface. Splitting test is performed in the same way as ASTM standard test C496 for splitting tension test for homogeneous samples.
using a standard compression test machine. Tschegg et al. (1995) developed a “Wedge Splitting Test” capable of measuring the force and displacement during crack propagation until complete splitting of the specimen. Compared with direct tension test, an indirect tension test is simple to perform. However, it is not suitable for in situ bond strength measurement and the test results were found to be less consistent than those of other test methods (Delatte and Sehdev, 2003).

6.5 Bond Strength of Thin Concrete Overlay

6.5.1 Tension Bond Strength of PCC to PCC Interface

Knab and Spring (1989) reported a laboratory study that evaluated three bond strength test method for screening and selecting repair material in thin concrete overlay rehabilitation of existing PCC surface. Two uniaxial tension bond strength test methods (friction grip and pipe nipple grip method) were studied. The bond strength of three repair PCC mixtures were examined: (1) 14-day-old PCC over 80-day-old PCC, (2) 7-day-old latex modified concrete (LMC) with high air content over 94-day-old PCC, (3) 10-day-old LMC with normal air content over 129-day-old PCC. The measured tension bond strength for PCC over PCC after 14-day-curing ranged from 1.9 MPa (275 psi) to
2.9 MPa (420 psi). It was concluded that pipe nipple grip test method yielded more repeatable results based on a smaller coefficient of variation. The tension bond strength measured by pipe nipple grip test method was higher than those from the friction grip method.

A laboratory study by Kuhlmann (1990) investigated the tension bond strength of LMC over PCC. The reproducibility of the test results was studied by comparing the coefficient of variation of test results from different operators and materials. It was found that the average tension bond strength of LMC to PCC exceeded 0.48 MPa (70 psi) after 24-hour curing at room temperature and exceeded 3.1 MPa (450 psi) at 90 days.

Delatte and Sehdev (2003) reported the early-age bond strength of eight different concrete overlay mix designs using pull-off tests and splitting tests after 1, 3, 7, and 14 days of curing. Fly ash and/or blast-furnace slag was used to replace 30% of the cement in all overlay concrete mixtures. All design mixes were found to have satisfactory bond strength for overlay construction. The tension bond strength of normal-strength plain concrete mix measured in pull-off tests was about 1.4 MPa (205 psi) at 28 days. The tension bond strength from laboratory splitting tests from the specimens made of the same mix was about three to five times of the values from the pull-off tests.

McDonald and Vaysburd (2001) evaluated three pull-off test devices and test procedures for in situ bond testing for concrete overlay repairs. A total of 266 tests were conducted from 77 in situ repairs at three experimental sites with drastically different
climate conditions located in Southern Florida, Illinois, and Arizona. All tests were performed three years after construction. The results exhibited a wide range of pull-off tension bond strength for different concrete mixes, ranging from 0.4 MPa to 3.4 MPa. The average bond strength of all mixes exceeded 1.5 MPa (217 psi). Moreover, climatic conditions did not appear to have a significant impact on tension bond strength.

Momayez et al. (2005) investigated the effect of different test methods on bond strength between overlay and concrete substrate. Pull-off tests, splitting tests, and two shear bond strength tests were evaluated. All tests were performed at 28-day age. The measured tension bond strengths from pull-off and splitting tests were 1.25 MPa and 1.27 MPa, respectively.

Figure 6.8 summarizes the published tension bond strength of normal-plain concrete overlay mix in the literature. As can be seen, the bond strength measured by in situ pull-off tests presented as hollow symbols is typically higher than those from laboratory testing. Results of in situ pull-off tests show the tension bond strength varies between 1.5 MPa and 2.2 MPa. The tension bond strengths measured from laboratory splitting test appear to be the least consistent, ranging from one to five times of the values from the pull-off test (1.27 MPa to 4.5 MPa).
6.5.2 Tension Bond Strength of PCC to AC Interface

As shown in Figure 6.1, the number of UTW projects is growing very rapidly and there is undoubtedly more potential in the thin concrete overlay applications for flexible pavement rehabilitation because more than 80% of the paved roads in the US are flexible
pavements. However, the majority of the published concrete overlay bond strength data are for concrete to concrete interface (BCO) while very limited information is available regarding the tensile bond strength between PCC and AC interface (UTW).

Qi et al. (2004) evaluated the in situ bond strength of eight full-scale UTW sections constructed with various design features at the FHWA’s accelerated pavement testing facility. The bond strength between PCC and AC was found to be significantly lower than the typical bond strength between PCC and PCC interface. At 11 of the 32 tested locations, debonding occurred right after coring, therefore the pull-off tests could not be continued. The average tension bond strength was found to be approximately half of the measured interface shear strength. Figure 6.9 shows the histogram of measured tension bond strength from pull-off tests.
In the NCHRP synthesis 338 for UTW, Rasmussen and Rozycki (2004) reported that, based on the studies they reviewed, the average value of the interfacial shear strength was 0.70 MPa (102 psi) while the average value of the direct tensile pull-off test was 0.51 MPa (74 psi).

6.6 Parametric Study

As discussed in Section 6.3, for concrete pavement overlays the debonding is caused by the tensile interface transverse normal stress rather than the shear at the interface in
the vicinity of applied load. An extensive parametric study is conducted in this section to identify important parameters that significantly affect the peak tensile transverse normal interface stress between the thin concrete overlay and existing pavement surface layer. The maximum peak tensile transverse normal stress can be compared with the published tension bond strength data to determine whether debonding is likely to occur under typical dynamic wheel loading.

6.6.1 Analysis of BCO

The stress-based model verified in Chapter Four was used to analyze the dynamic response of rigid pavements with BCO rehabilitation. Figure 6.10 shows the longitudinal profile and plan view of the BCO rigid pavement studied. The parameters evaluated include: 1) overlay slab size; 2) concrete overlay thickness; 3) elastic modulus of concrete overlay; 4) thickness of substrate concrete layer; and 5) elastic modulus of substrate concrete layer. The effect of each of these five parameters is studied by varying that parameter within a range of typical values while keeping all other parameters with fixed typical values.

The dimensions and material properties of the model are summarized in Tables 6.1 and 6.2. Under the current practice, the tire loads applied on the pavement are typically
assumed to be uniform and equal to the inflation pressure. In reality, the contact pressure between tire and pavement surface is not uniform. Contact pressure up to 100% higher than the inflation pressure has been reported (De Beer et al. 1997). Fernando et al. (2006) conducted a comprehensive study on the effects of tire size and inflation pressure on tire contact stresses and pavement response. The measured peak vertical contact stress for an 11R24.5 tire inflated at a pressure of 900 kPa (130 psi) was found to be approximately 1.2 MPa (174 psi). Therefore, 1.2 MPa will be used as the peak dynamic loading in the analyses of BCO and UTW parametric study.
Figure 6.10 Longitudinal Profile and Plan View of the BCO Rigid Pavement Using Stress-based FEM Model
<table>
<thead>
<tr>
<th>Layer #</th>
<th>Material</th>
<th>Layer Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fixed Case</td>
</tr>
<tr>
<td>1</td>
<td>PCC Overlay</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>PCC Overlay</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>PCC</td>
<td>112.5</td>
</tr>
<tr>
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Table 6.1 Dimensions of the Stress-Based FEM Model for BCO Rigid Pavement Analysis
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<th>Case 1 E (MPa)</th>
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Table 6.2 Elastic Properties of the Stress-Based FEM Model for BCO Rigid Pavement Analysis
Figures 6.11 to 6.13 show the peak interface tensile transverse normal stress between overlay and substrate concrete layer at the edge of BCO slab for different slab sizes, layer thicknesses, and elastic moduli of overlay and original PCC surface layer.
Figure 6.11 Effect of Slab Size on Peak Tensile Transverse Normal Stress at Edge of the Slab of BCO

The effect of slab size is examined by varying the in plan slab dimensions with the thickness and elastic properties kept at the fixed values as listed in Tables 6.1 and 6.2. As the slab size increases, the maximum tensile transverse normal stress at the edge of the
slab decreases. The values vary from about 140 kPa for a slab of 0.91 m × 0.91 m (3 ft × 3 ft) to about 80 kPa for a slab of 2.44 m × 2.44 m (8 ft × 8 ft).

Similarly, the effect of layer thicknesses of overlay and original PCC surface layer is examined by varying the thickness of one layer with other dimensions and elastic properties.
properties kept at the fixed values. As can be seen, the thickness of overlay has slightly more effect than the thickness of existing PCC slab. As the layer thickness increases, the maximum tensile transverse normal stress at the slab edge increases. For the studied range of thickness, the transverse normal stresses at edge vary from about 90 to 130 kPa.

Figure 6.13 Effect of Elastic Modulus on Peak Tensile Transverse Normal Stress at Edge of the Slab of BCO

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The tensile transverse normal stress increases as the elastic modulus of the overlay increases or the elastic modulus of the substrate PCC decreases. The maximum transverse normal stresses at the edge vary from about 100 to 130 kPa for typical values of elastic modulus of PCC.

In all cases, the tensile transverse normal stress at the edge of slab is well below the reported tensile bond strength between PCC and PCC interface shown in Figure 6.8. The slab size and the thickness of the overlay were found to have the most significant effect on the transverse normal stress at the edge of slab. Other parameters such as the thickness of the existing PCC layer, the modulus of overlay and PCC layer were found to have minor effects on the tensile transverse normal stress at the slab edge.

6.6.2 Analyses of UTW

Figure 6.14 shows the longitudinal profile and plan view of the UTW flexible pavement. The parameters studied include: 1) overlay slab size; 2) concrete overlay thickness; 3) elastic modulus of concrete overlay; 4) thickness of substrate AC layer; and 5) elastic modulus of substrate AC layer. The effect of each of these five parameters is studied by varying that parameter within a range of typical values while keeping all other parameters with fixed typical values. The dimensions and material properties of the
model are summarized in Tables 6.3 and 6.4. A triangular load with a peak value of 1.2
MPa and duration of 0.02 sec is used as the dynamic wheel loading in the analyses of
UTW parametric study.
Figure 6.14 Longitudinal Profile and Plan View of the UTW Flexible Pavement Using Stress-based FEM Model

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<tr>
<td>Slab Size</td>
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<td>Fixed Case</td>
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<tr>
<td>(m×m)</td>
<td>1.22×1.22</td>
<td>0.91×0.91</td>
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<td>(ft×ft)</td>
<td>4×4</td>
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Table 6.3 Dimensions of the Stress-Based FEM Model for UTW Flexible Pavement Analysis
<table>
<thead>
<tr>
<th>Layer</th>
<th>Fixed E (MPa)</th>
<th>Case 1 E (MPa)</th>
<th>Case 2 E (MPa)</th>
<th>Case 3 E (MPa)</th>
<th>Case 4 E (MPa)</th>
<th>Case 5 E (MPa)</th>
<th>Case 6 E (MPa)</th>
<th>Case 7 E (MPa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCC Overlay</td>
<td>41000</td>
<td>21000</td>
<td>24500</td>
<td>28000</td>
<td>31500</td>
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<td>38500</td>
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Table 6.4 Elastic Properties of the Stress-Based FEM Model for UTW Flexible Pavement Analysis
As can be seen, the slab size does not appear to have a significant effect on the maximum transverse normal edge stress for UTW. The values vary from about 127 kPa to 137 kPa with the maximum stress obtained for a slab size of 1.83 m × 1.83 m (6 ft × 6 ft).
Figure 6.16 Effect of Layer Thickness on Peak Tensile Transverse Normal Stress at Edge of the Slab of UTW

The effect of thicknesses of overlay and thickness of the original AC surface layer is examined by varying the thickness of one layer while keeping the other layer’s dimensions and elastic properties fixed at the values shown in Tables 6.3 and 6.4. Similar to the observations made in the BCO analyses, the thickness of the existing AC surface

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layer has minor effect on the transverse normal edge stress. However, in contrast to the results obtained for BCO overlay, the transverse normal stress at the slab edge decreases when the overlay thickness increases for UTW.

Figure 6.17 Effect of Elastic Modulus on Peak Tensile Transverse Normal Stress at Edge of the Slab of UTW
The elastic modulus of the substrate AC layer has a more significant effect on the tensile transverse normal stress at the slab edge than the elastic modulus of overlay. The stress first increases then decreases as the modulus of AC layer increases. The tensile edge stress decreases only slightly (about 3 kPa) when the elastic modulus of overlay increases from 21 to 41 GPa.

The overlay thickness and elastic modulus of the existing AC surface layer were found to have the greatest effect on the tensile transverse normal edge stress. The maximum tensile stress is obtained when a thin overlay is used on a severely deteriorated flexible pavement.

The tensile transverse normal stress at the edge of slab was less than 150 kPa in all cases in the parameter study. The average tensile bond strength for PCC to AC interface is about 500 kPa (Rasmussen and Rozycki, 2004). However, in situ bond strengths lower than 140 kPa have been also reported (Qi et al., 2004).

### 6.7 Summary

BCO and UTW have gained great popularity as pavement rehabilitation tools in recent years. However, the current design methods do not have a mechanistic response model capable of accurately predicting accurate interface stresses, the direct cause of
debonding failure. In this chapter, the proposed pavement model was used to analyze both thin concrete overlay rehabilitation methods.

A number of design parameters were examined for both BCO and UTW to identify the impact of these parameters on the tensile transverse normal edge stress, the direct cause of initiation of overlay debonding. Based on the results of BCO analyses, the slab size and the thickness of the overlay have the most significant effect on the transverse normal stress at the edge of slab. For UTW overlay, the thickness of overlay and the elastic modulus of the existing AC surface layer were found to have the greatest effect on the tensile transverse normal edge stress.
CHAPTER 7

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary and Conclusions

According to the Federal Highway Administration (FHWA Highway Statistics, 2004), almost $900 billion was spent on the maintenance and reconstruction of the U.S. highway system during the ten year period from 1995 to 2004. Currently, almost all states are still using the AASHTO pavement design guide issued in 1993 or older versions as their design procedure. Increasingly, the deficiencies in these purely empirical procedures concern designers. It is clear that improving the pavement analysis and design methods could result in annual savings in the millions and possibly billions of dollars. FHWA has considered the replacement of the empirical design procedures with more rational design procedures based on mechanistic pavement response models a critical element in improving the national highway system.
A number of mechanistic pavement response models have been developed over the years. Those models which are typically based on multi-layer elastic theory and displacement-based finite element methods are currently the most widely used and both are adopted as the structural response models in the recently released MEPDG. These models are capable of predicting global responses such as surface deflections but are not able to accurately predict the transverse stress distributions which must be evaluated in order to model the realistic behavior of in-service pavement systems and prevent premature failure caused by pavement layer debonding.

A common assumption in most pavement response models is that all the pavement layers are fully bonded. However, the repeated localized transverse stress at pavement layer interfaces induced by traffic loads could result in a progressive debonding or even complete separation of layers. The debonding of the pavement layers can cause significant increases in the stresses and strains in the surface course and lead to premature fatigue failure. It has been reported that debonding of pavement layers can reduce the life of pavement by more than 80 percent.

A stress-based model developed at Ohio State for composite laminates has shown the capability of accurately predicting the dynamic stresses at layer boundaries while retaining the ability to determine displacement behavior. The main objective of this study was to extend this stress-based multi-layer plate theory to layered pavement systems as
an alternative to existing pavement response models for the analysis and design of pavements.

A comprehensive literature review was first carried out to study the existing pavement response models particularly their capability of characterizing the interface stresses and strains and elucidate importance to improve the interface stress predictions to prevent premature failure caused by pavement layer debonding. It was shown that the existing response models are not suitable for predicting interface stress distribution. The state-of-art of research related to the pavement layer debonding failures was also reviewed. Although a lot of effort has been made to investigate the consequence of pavement debonding and evaluate the pavement interface bond strength, it was found out that little has been done to characterize pavement interface stresses, which are the underlying cause of debonding.

To extend the OSU stress-based model to layered pavement analysis, the derivation of the equations and finite element implementation were briefly summarized. Then, the stress-based model was extended to analyze layered pavement system. The numerical stability of the model was examined including different numerical integration schemes and length of time steps. The model was verified by comparing its solutions to existing analytical, numerical solutions, and experimental results.

Good agreement was achieved in the predicted surface deflection response from existing analytical, numerical solutions and the stress-based model. The 3-D ABAQUS
displacement-based model yielded transverse normal stresses closer to the stress-based model when more elements were used in the through-the-thickness direction. However, it was also shown that the continuity in the transverse stress can not be achieved in displacement-based models no matter how many elements are used.

The stress-based model was further verified by comparing predicted with experimental results. Dynamic surface deflections and transverse normal stresses between the subbase and base measured in the OSU APLF testing were compared with the predicted response from the stress-based model. Overall, a reasonably close prediction was obtained between measured and predicted responses. The stress-based model over-predicted the surface deflection and transverse normal stress by about 15 and 30 percent, respectively. The difference was likely a result of limitation in the accuracy level of the embedded response instrumentation and the assumptions made in the formulation of the stress-based model.

A sensitivity study was carried out in order to obtain a better understanding of the different factors that affect the interface transverse stresses at the interface between surface layer and base layer. Rigid and flexible pavements with different layer thicknesses and elastic modulus were analyzed. When material properties were fixed at typical values, the interface transverse stress of flexible pavement showed a similar trend but significantly higher stress levels at the interface with variation in surface layer thickness due to its lower surface layer modulus. When the pavement layer thicknesses
were fixed, it was found out that the interface transverse normal stress is more sensitive to the subgrade and base layer modulus than that of the concrete surface layer in the studied modulus range. Based on the rigid pavement analysis result, a very stiff subgrade can result in the interface transverse normal stress two to three times as high as the value of interface stress when a typical natural subgrade is used.

Finally, the stress-based model was used to analyze thin concrete overlay rehabilitation of rigid (BCO) and flexible pavements (UTW). Based on a literature review, the debonding of concrete overlay is governed by the tensile stress perpendicular to the interface (transverse normal stress) at the pavement edge caused by the coupled effect of both traffic and environmental loading. A parametric study was conducted to identify the important variables that affect the tensile transverse normal stress at the interface between overlay and substrate layer at pavement edge. For BCO, the slab size and the thickness of the overlay were found to have the most significant effect on the transverse normal stress at the edge of slab. For UTW, the overlay thickness and elastic modulus of the existing AC surface layer were found to affect the tensile transverse normal edge stress the most. The worst scenario is obtained when a thin overlay is used for a severely deteriorated flexible pavement.

In all cases, the calculated critical tensile stresses at BCO interface were well below the measured bond strength between PCC and PCC interface as reported in literature. The observed BCO debonding failure may contributed to both traffic load as well as
environmental effects (e.g., curling and warping) which were not considered in the current study. The critical tensile transverse normal stress in the UTW were below the published average tensile bond strength data between PCC and AC interface but in situ bond strengths lower than the calculated critical stresses have also been reported.

7.2 Recommendations for Future Work

The stress-based model extended from the composite laminate theory has shown it can be a promising alternative to existing response models in pavement analysis and design. It has the capability of accurately predicting the dynamic transverse stresses at pavement layer boundaries while retaining the ability to determine displacement responses. However, there are a number of important future research tasks need to be continued based on the current study. These major recommendations are:

1. Integration of constitutive laws that better characterize the pavement materials.

In this study, the pavement materials were assumed to be linearly elastic. Therefore, the current model does not have the capability of modeling the aspects of real pavement behavior which deviate from this assumption. For instance, the amount of visco-elastic deflection was more than 30% of the total deflection measured in the FWD testing on AC control section in OSU CCP APLF pavement testing (Figure 4.22). The stress-based
model will be able to describe more realistic pavement behaviors if more appropriate constitutive relations are included.

2. Adding load transfer system for concrete pavement modeling.

The current model was extended from the multi-layer plate theory for composite laminates. Only a single slab can be analyzed for rigid pavements and a load transfer mechanism between adjacent slabs is not included in the model. The addition of load transfer elements (e.g., dowel bars, shear interlocks) can extend the capability of this model to analysis of multiple slabs of rigid pavements.

3. Inclusion of environmental effect.

It is well recognized that environmental loading can have significant impact on the overall pavement behavior. The inclusion of environmental effects, such as the curling of a PCC slab due to temperature gradient, can allow the model to predict more realistic pavement behaviors.


Both laboratory and field experimental studies have been conducted to investigate the bond strength between pavement layers. However, no completely suitable failure criteria are available for pavement debonding failure. When appropriate failure criteria are
developed and included, the stress-based model can be used to predict the initiation of debonding.

5. Inclusion of discontinuity terms in the finite element formulation.

The current model can only be applied to pavement systems without pre-existing debonding defects. The inclusion of discontinuity terms in the finite element formulation will allow the model to analyze deteriorated pavements with existing debonding areas and model the debonding growth under dynamic traffic loading.
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