DEPOSIT FACILITIES AND CONSUMPTION SMOOTHING:
A DYNAMIC STOCHASTIC MODEL OF PRECAUTIONARY WEALTH
CHOICES FOR A CREDIT-CONSTRAINED RURAL HOUSEHOLD

DISSERTATION

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This dissertation examines two issues. One is does more convenient access to safer deposit facilities affect the level and composition of the precautionary wealth portfolio of credit-constrained rural households. The other one is the extent to which the resulting changes in precautionary wealth may be linked to smoother consumption patterns. This is accomplished by solving a dynamic stochastic model on wealth portfolio choices. In the model, households make three decisions: how much to consume, how much to save in a bank, and how much livestock to keep in the barn. Portfolio decisions depend on the rates of return for each asset, transaction costs, and the impact of systemic shocks on labor income, the price of livestock, and the expected return on deposits—given the probability of bank bankruptcy.

Simulation results reveal how much increased access to deposit facilities, measured in terms of a reduction in transaction costs, improves the ability of credit-constrained households to insulate consumption from income variations due to systemic shocks. This expansion of the outreach of deposit facilities will be more important for household’s welfare the riskier is the environment where they operate. The simulation results also show that effective financial regulation and supervision and the promotion of robust financial institutions, which are less vulnerable to systemic shocks, create an environment conducive to the holding of a greater share of precautionary wealth in the
form of deposits. This reduces the costs for households of risk management and consumption smoothing. It also improves the economy’s allocation of resources through increased financial intermediation. The research sheds light on policy debates about strategies for rural deposit mobilization.
Dedicated to my family,

my lovely ladies: Alicia † (mom), Sandra (wife), Natalia and Carla (daughters); and

my respected gentlemen: Vitalio (dad), Kublai and Erick (brothers).
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CHAPTER 1

INTRODUCTION: RISK, PRECAUTIONARY WEALTH, AND DEPOSITS

This dissertation explores the effects of increased access to financial deposit services on the precautionary wealth held by poor rural households in developing countries, as a strategy to cope with risk, and on their ability to buffer consumption. To accomplish this, it solves a dynamic stochastic model of precautionary wealth portfolio choices for a rural household vulnerable to a local systemic shock. Farmers are highly exposed to the adverse income shocks that result from the vagaries of weather and other exogenous events. In order to protect their consumption from income variations, rural households must adopt creative strategies to deal with the incidence of these risks. In the absence of efficient instruments to deal with these events, such as formal insurance, households have no other choice than to rely on costly income and consumption smoothing strategies.

Greater access to deposit facilities in a financial institution, measured in terms of reductions in the transaction costs associated with deposit services, as well as lower risks of insolvency or bankruptcy of the deposit-taking institution reduce the costs and risks and widen the array of options for holding precautionary wealth. Indeed, this more accessible and affordable means to hold assets in a safer and more liquid form, even if the
explicit returns from the interest earned are low, changes the composition of the portfolio of precautionary wealth at the same time that it allows the household a less frequent use of other, more costly, strategies to cope with shocks.

Furthermore, reductions in the cost of consumption protection, if protection behaves like a normal good, allow households to acquire more of it, resulting in a smoother consumption pattern. Such changes, by allowing the household to rely less on costlier strategies and by increasing protection, so consumption becomes smoother, are welfare improving at the household level. Moreover, at the aggregate level of the rural economy, additional deposits mobilized by financial intermediaries improve the efficiency of the overall allocation of resources, in comparison to a regime where (idle) precautionary wealth is held in the form of less socially productive assets.

At least four important policy implications can be derived from this research, related to the ability of policymakers to influence the degree of consumption protection attained by the rural poor. First, policies that achieve reductions in transaction costs for both depositors and deposit-taking institutions increase the outreach of deposit facilities to the rural poor and improve their welfare. Second, a system of prudential regulation and supervision that reduces the likelihood of bankruptcy of financial institutions is more conducive to the accumulation of precautionary wealth in the form of financial assets. The expanded outreach of deposit facilities increases the level of financial intermediation and reduces the social costs of holdings of unproductive assets as precautionary wealth. Third, measures that lower credit portfolio risks, either through reducing the riskiness of the income flows of borrowers or by improving the risk-managing technologies of
financial institutions, protect the precautionary wealth of depositors. Innovations in insurance (indexed contracts) and in finance (microfinance technologies) can have substantial impacts on the welfare of the rural poor. Fourth, if significant externalities emerge in the experimentation and development of new ways of providing these important deposit services in isolated regions, appropriate policy interventions may be required to promote an optimum level of financial innovation and outreach.

1.1 Motivation

As a financial examiner at the Superintendence of Banks and Financial Institutions in Bolivia, I had the opportunity to observe first hand the potential impact of access to financial services on poor rural households. I was impressed by how a sequence of loans with initial amounts as low as US$ 5 can be life-changing, when productive opportunities exist. Efficiently provided financial services, in general, and deposit facilities, in particular, can improve welfare. Such services are especially critical in the rural areas of developing countries, where 70 percent of the world’s poor live (The World Bank, 2006).

Financial services matter. At the macroeconomic level, theoretical and empirical evidence shows that deepening of the financial system promotes economic growth (King and Levine, 1993; Levine, 1997; Acemoglu and Zilibotti, 1997; Rajan and Zingales, 1998). At a microeconomic level, several studies reveal a positive impact of financial services on poverty, inequality, and household welfare (Beck, Demirguc-Kunt and Levine, 2004; Ahlin and Jian, 2005).
Among a number of valuable financial services, however, researchers and practitioners have centered their attention mostly on credit, even though deposit facilities can be as significant or even more for the poor. The reasons for the strong bias towards credit over deposit mobilization, in both theory and practice, are numerous.

In theory, a long tradition of research has focused on credit, an inclination that was renewed with the advent of new developments in asymmetric information and contract theory as a framework for examining the difficulties of lending. In practice, some of the possible reasons that may have contributed to the disproportionate attention given to credit may include, among others: (1) easy access to cheap funds from donors and governments, which would have lowered or eliminated incentives for deposit-mobilization among financial institutions; (2) a framework of prudential regulation that, if in place, usually prohibited deposit taking by non-regulated institutions and, if non-existent, contributed to the lack of trust among savers; (3) beliefs that poor rural households do not save and that deposit mobilization would be fruitless and too costly, and recently (4) the interest of some governments in using microcredit as a political instrument, a patronage that cannot be easily linked to the generosity of politicians in the case of deposit facilities.

Reflecting views from The Ohio State University, Vogel (1984) had claimed that savings mobilization, referring to the expansion of deposit services, was “the forgotten half of rural finance”. Over 20 years later, this is still true. This author attributed the neglect of deposits to the “myth” that poor rural households do not have the capacity and/or the interest to save. Thus, the poor are seen exclusively as potential borrowers and
not as potential depositors. Nonetheless, it is likely that deposit services are even more important for the rural poor than credit, among other things because the long production cycle in agriculture requires efficient means for liquidity management and because poor households need liquid reserves in order to meet emergencies.

In effect, if possible, rural households may be frequently more willing to use their accumulated financial wealth in case of an emergency rather than increasing the financial burden on their already “weak” backs with a loan. Even worse, it is often the case that credit is just not a choice for them. In many cases, however, access to deposit facilities may be prohibitively costly or may be accompanied by the risk of loss of the depositor’s financial wealth due to bankruptcy of the deposit-taking institution. In these cases, the rural poor are deprived of the welfare-enhancing opportunity to reduce the costs and risks of their consumption smoothing efforts.

1.2 Definitions

The strategies that households use as risk-coping mechanisms are classified in the literature in various ways. A commonly used classification divides them into \textit{ex ante} and \textit{ex post} strategies. While the former are implemented before the shock has occurred, the latter are implemented afterwards. As Fafchamps (2003) points out, this classification may be misleading, since for an \textit{ex post} risk-coping mechanism to be effective, \textit{ex ante} income smoothing actions may be needed. This author prefers to classify strategies into preventive and curative. In turn, Morduch (1995) classifies them into income smoothing
and consumption smoothing strategies, while Alderman and Paxson (1994) separate them into risk management and risk coping strategies.

This dissertation adopts Morduch’s classification. Income smoothing attempts to reduce the volatility of income flows. These strategies include, for instance, crop diversification and plot fragmentation, the choice of a low-return, low-risk production technology (e.g., the choice of a crop variety that is resistant to drought, pests and other risk factors, when it is cultivated in a risky environment, despite its lower rate of return), strategic migration, and labor market participation on a steady basis. Consumption smoothing strategies are those that are adopted after the shock has occurred, in order to sustain consumption. They include, for instance, temporary changes in labor market participation, sales of productive assets, emergency loans, transfers, and the use of precautionary wealth.

In the literature, there is not a clear distinction between precautionary wealth and precautionary savings. These two concepts are used interchangeably. This dissertation uses the term “precautionary wealth” to refer to the stock of assets that are held in order to smooth consumption from income shocks. It is the type of wealth that households accumulate due to uncertainty and just use it in the event of sharp drops in income or emergency spending needs (Romer, 2001). A clear distinction must be made between this precautionary wealth and precautionary savings, since wealth is a stock variable and savings are a flow variable. Thus, even if a poor rural household cannot save large amounts as a flow, the share of its precautionary wealth with respect to its total wealth or its size relative to income may be significant. The type of behavior that underlies the
accumulation of precautionary wealth, consisting of saving up to a target wealth-income ratio and dissaving to absorb income shocks, is described as a buffer stock saving behavior (Deaton, 1991).

Assets held as precautionary wealth should have some desired characteristics, such as liquidity, safety, and divisibility. Liquidity is understood as the ease to convert assets into cash, without having to incur excessively high liquidation costs. Safety is the characteristic of an asset of preserving its value over time. Divisibility is an attribute of an asset by which it can be easily broken into parts, without affecting its total value. These three characteristics matter the most when the precautionary motive prevails, while the implicit rate of return on the asset is evaluated in terms of the costs and benefits of attaining consumption protection.

The line that separates precautionary asset holdings and productive assets is sometimes blurred; e.g., chicken that lay eggs may be a productive asset and also a device to store wealth for precautionary purposes. To make a clearer distinction between these two, we focus on the purpose for which the asset was acquired. Although productive assets may be sold as part of a consumption smoothing strategy, this would happen only in extreme cases. Otherwise, it is very unlikely that households will cash in their productive physical capital. Such productive asset liquidation will make them even poorer in the future, as this decision will affect their potential earnings.

This dissertation focuses on assets that mainly serve to store wealth for precaution and not for productive purposes. Specifically, the dynamic model developed in chapter 4 focuses on precautionary assets, in the form of either deposits or livestock. One of the
most traditional ways to store wealth in developing countries is the holding of small livestock (pigs, goats, and chicken). The question is how effective these assets are in dealing with income shocks, compared to holdings of financial deposits. Transaction costs and risk are relevant for answering this question.

Given the high degree of fragmentation that characterizes developing economies, access to facilities for depositing financial wealth is differential across households, in reflection of the transaction costs involved, especially for rural households living in remote locations. The more fragmented the economy, the higher the transaction costs and, therefore, the less access to financial deposits the rural poor will have.

Increased access to deposit facilities is, in part, about reducing the costs that a household has to incur when engaging in a deposit transaction: mostly transportation costs, the opportunity cost of time, entry costs, and discrimination costs. Long distances and an absent road infrastructure may make physical access to the branch of a financial institution prohibitively costly. Entry costs refer to all the costs that the household incurs to open and maintain a savings/checking account (e.g., minimum balances, fees, cost of checks and the like). These costs can in many cases be quite substantial. Beck, Demirguc-Kunt and Martinez Peria (2006) report that, in some poor countries, the minimum amount to be able to open a checking account is higher than the GDP per capita, while in other countries fees to maintain a checking account exceed 25 percent of GDP per capita.

Because of discrimination costs, peasants who feel that they are being discriminated against have to “pay” in order to use a formal deposit service. In this
regard, Becker (1971) argues that “if an individual has a ‘taste for discrimination’ he must act as if he were willing to pay something (directly or in the form of a reduced income) to be associated with some persons instead of others” (p.8). Thus, discrimination is often associated with a loss of utility, something painful, and therefore a cost.

Increased access to deposit facilities is also about reducing the risks that a household faces when keeping its precautionary wealth with a particular deposit-taking institution. These risks may emerge from unexpected or uncompensated fluctuations in the real rate of return (e.g., due to inflation), from the threat of confiscation (e.g., due to macroeconomic mismanagement or political unrest), or from the inability to recover the deposit, due to bankruptcy of the financial institution.

1.3 Rural households in developing countries

Life in the rural areas of developing countries is not easy. The vast majority of rural households have to deal not only with low but also highly volatile incomes. While low incomes result mainly from low labor productivity, income volatility is a consequence of a risky environment and of limited access to efficient instruments to deal with risk.

In effect, the environment in poor rural areas is plagued with risks, such as those resulting from shocks typically caused by local yield and price fluctuations (systemic shocks) or by individual-specific shocks, such as illness, death or asset losses (idiosyncratic shocks). In addition, households are forced to adopt costly strategies to
cope, particularly because they seldom have access to formal insurance and credit markets, if these exist at all.

The perverse combination of low and volatile incomes in a risky environment and limited access to efficient risk-coping strategies makes these households extremely vulnerable to adverse income shocks. As a consequence, rural households must adopt costly strategies to deal with risk. Moreover, the availability only of too costly risk-coping instruments induces a choice of levels of protection of consumption from income risk below some optimum. In extreme cases, a reduction in welfare results from the incidence of severe hunger and, in a larger scale, from cases of famine. Furthermore, in the long run, this setting becomes a perfect scenario for breeding poverty.

The poorest are most vulnerable. Their vulnerability grows with their isolation in highly fragmented markets. Furthermore, the poorer the household is, the more limited is (and costly) its menu of risk-coping strategies. Indeed, the poorest of the poor often do not have access to credit at all (i.e., they are fully credit constrained), so they have to rely on self-insurance or risk-sharing strategies to cope with risk.

The accumulation of precautionary wealth is a self-insurance strategy motivated by prudent behavior. A vast literature claims that households build up wealth so they can manage it as a buffer stock, in order to smooth consumption after income shocks (Deaton, 1991; Paxson, 1992; Rosenzweig and Wolpin, 1993; Behrman, Foster, and Rosenzweig 1997; Gollier, 2003). In the literature, we find references to precautionary wealth either in the form of a single interest-earning riskless asset (Deaton, 1991) or as a group of assets, which typically do not include deposits in a financial institution as a component of
the portfolio (Rosenzweig and Wolpin, 1993; Udry, 1995; Fafchamps, Udry and Czukas, 1998; Jalan and Ravallion, 2001). The assets that have been commonly studied as components of precautionary wealth portfolios are: livestock, inventories of grain, and cash.

Scholars have often ruled out deposit facilities as a means for holding wealth by showing that households maintain a low share of wealth in the form of financial assets. For instance, Rosenzweig and Wolpin (1993) and Kochar (1995) reveal that, in rural India, the share of wealth that lies in financial assets is “trivial”, of less than 5 percent. We argue that this small share held as financial assets is due to limited access to deposit facilities, not to lack of demand for them. This does not necessarily have to be the case. The microfinance revolution has been expanding the set of choices of financial services for poor peasants (Robinson, 2001; Armendariz de Aghion and Morduch, 2006). Financial services—including savings deposit facilities—are becoming more accessible, so rural households can find financial assets more within reach.

It is likely, though, that the poorest will still face severe borrowing constraints, given that they are limited in their ability to show their creditworthiness or simply because microfinance institutions or their competitors have not yet developed a cost-effective lending technology to reach these rural households (Gonzalez-Vega et al., 2006). Thus, it is reasonable to think that, in the future, poor rural households will have greater access than now to facilities for depositing part of their wealth, but that they will still be limited in their ability to borrow or will be subject to some form of credit rationing, particularly for emergency loans (Boucher, 2000). The availability of these
deposit services will make it possible for rural households to avoid or further reduce the use of other risk-coping strategies that may be comparatively more costly.

1.4 The process of choosing a set of risk-coping strategies

Generally, households will choose not just a single risk-coping action but a combination of strategies, depending on factors such as the availability and costs of each option, the characteristics of the shocks that the households are more exposed to, and the features of each household itself.

While the choice of the most accessible and least costly will be favored, several strategies will be combined when there are covariances or increasing marginal costs for each strategy. For instance, in small villages, child labor may be a self-insurance consumption smoothing strategy that is commonly used, because fluctuations in school attendance do not result in substantial losses of human capital (Jacoby and Skoufias, 1997). Other authors have considered, however, the high costs of permanently keeping children out of school in a risky environment (Maldonado and Gonzalez-Vega, 2005). Besides the apparently lower cost, nevertheless, child labor is an “easy access” strategy, as parents have control over their children.

The characteristics of the expected shocks, such as their frequency, intensity, duration, and predictability will also influence the choice of strategies. Some strategies may be eventually exhausted if the shocks are frequent and intense. For instance, a farmer who decides to work extra hours to overcome a given shock may not be able to use this strategy further, as the extra time this agent can devote to work has natural limits.
Another issue is whether especially liquid assets will be worth holding if shocks are not frequent. If the motivation is to be protected against large but rare shocks, it is likely that households will prefer to hold illiquid assets (Carroll and Samwick, 1998).

Household characteristics, such as age and gender composition, initial level of wealth, distance to markets, and skills and education (among others), will also influence the choice of risk-coping strategies. For example, as a person becomes older, he/she becomes more risk averse (has less tolerance for risk), so we expect him/her to adopt a more conservative position in terms of the strategies chosen (Mandal, 2007).

From the discussion so far, it is clear that the choice of a set of risk-coping strategies is a complex process, because it involves various factors and because these factors change over time and with the environment. Thus, the optimum combination of strategies will change over time, as the set of available strategies and their costs, the characteristics of the shocks and of households, and the environment change.

An additional complication in the strategy-selection process is that, in some cases, the choice of a particular strategy implies sequential decisions. This is, for example, the case of precautionary wealth, which implies a further decision on the composition of the asset portfolio that the household wants to hold. This composition affects, in turn, the attractiveness of the portfolio, compared to the set of other risk-coping strategies that can be potentially adopted. The opportunity to hold a safer and more liquid portfolio of precautionary assets may, for example, reduce the degree of production diversification needed (either in crops or activities). Consequently, this would allow the household to exploit better the gains from specialization. The types of assets held as precautionary
wealth may also influence decisions about the use of other strategies, such as temporary labor market participation or the demand for emergency loans.

The profile of each asset in terms of liquidity, safety, availability, profitability and other key features will influence the decision about portfolio composition. It is expected that this decision will privilege a combination of assets that allows households to absorb income shocks in the best possible way, given the limited available options they have. Moreover, although not all assets are accumulated as precautionary wealth, eventually all assets may play the role of reserves against negative shocks. We focus here on those assets that are acquired mostly with the precautionary purpose in mind.

1.5 Precautionary wealth and access to deposit services

The use of assets as precautionary wealth has been extensively investigated. Rosenzweig and Wolpin (1993) examine the use of bullocks as buffer stock in the semi-arid tropics in India. They conclude that there is a relationship between the probability of buying/selling bullocks and positive/negative income shocks, which suggests the idea of using livestock for consumption smoothing.

As Fafchamps, Udry and Czukas (1998) claim, however, if markets are poorly integrated, local systemic income shocks will affect the relative price of the assets used as precautionary wealth. The resulting capital loss will prevent farmers from using livestock as a buffer stock or will make it unattractive. By using household panel data from the West African semi-arid tropics, they show that households adversely affected by drought sell more livestock than otherwise, but that the effect is very small. The less
frequent use of livestock as a buffer stock suggests that other assets may perform this function better, such as inventories of grain, cash, valuables, and the stock of human and farm capital.

Other authors have researched the use of inventories of grain and cash as a buffer stock. Jalan and Ravallion (2001) are motivated by the idea that credit-constrained poor rural households with a greater probability of idiosyncratic shocks may hold a greater proportion of unproductive liquid wealth with precautionary purposes, compared to households with access to credit. This behavior may perpetuate poverty. By using panel data from China, they find that higher income uncertainty results in a higher share of wealth held in unproductive liquid forms, such as grain or cash. However, the portion is very small. In contrast, Udry (1995) argues that, in Nigeria, asset drawdown is large when a negative income shock occurs.

These results about holdings of “unproductive” assets (equivalent to Keynesian idle cash hoarding) are influenced, moreover, by the assumption that only grain or cash under the mattress are available as stores of wealth. The great social advantage of using deposit facilities, instead, is that the funds deposited as precautionary wealth can then be transformed into “productive” loans, through the process of financial intermediation. In the aggregate, therefore, the apparently wasteful use of precautionary reserves disappears in the case of deposits, due to financial intermediation (Gonzalez-Vega, 2003). This, in turn, increases the productivity of resources in the rural areas.

Recently, the hypothesis of attaining protection for smoothing consumption has been challenged by new findings. For instance, by using a dynamic programming model,
Zimmerman and Carter (2003) show that the poor reduce their consumption in order to protect their assets. This asset smoothing behavior responds to their uncertainty about the future and their desire not to liquidate productive assets. These assets will be used only if the household’s income and precautionary savings are not enough to finance the subsistence threshold of consumption.

Along the same lines, Kazianga and Udry (2006) find evidence of little consumption smoothing in rural Burkina Faso between 1981 and 1985, a period of severe drought. Their main finding is that income shocks are transmitted largely to consumption. Clearly, this is not a pleasant outcome. This result would not be necessarily the case, however, if poor households had access to a more convenient, less costly and less risky means way of holding precautionary wealth: a safe and rewarding financial deposit.

It is conceivable that reducing consumption is, to some extent, one of the first strategies that is used when a shock hits; however, for the extremely poor, this may not even be a choice, because they are already at the edge of subsistence. Moreover, it is likely that the large observed (painful) impact of income shocks on consumption is precisely an indication of the inefficacy or extremely high cost of the various risk-coping mechanisms available (both for income and consumption smoothing). One of the purposes of financial deepening in the rural areas of developing countries should then be to reduce these costs.

To sum up, uninsured risk provokes changes in the household’s consumption, savings, labor, and production decisions, in a way that is detrimental for the households’ welfare, due to the costs they need to incur in attaining consumption (smoothing)
The absence of well-functioning credit, insurance and labor markets reinforces insecurity and exerts pressures on households to choose costly risk-coping strategies. One possible consumption smoothing strategy is the use of buffer stock savings or precautionary wealth. The literature is not clear which asset, if any, is used as a buffer stock. The choice of asset has important implications, however. So far, researchers have explored the use of livestock, grain or cash, with mixed results. However, researchers seem to agree on something: assets are accumulated and used as a consumption smoothing strategy.

1.6 The problem: Lack of access to deposit services

In the harsh environment described above, rural households seek efficient instruments to deal with risk. Better access to financial services, in general, and deposit facilities, in particular, becomes one of these instruments. In effect, the availability of deposit facilities makes it possible for rural households to avoid or reduce the use of other risk-coping strategies that may be relatively more costly.

This dissertation examines the effects of increased access to financial deposit facilities on the precautionary wealth that poor rural households in developing countries accumulate and use as a strategy to cope with adverse shocks. The research question has two parts:

1) Does increased access to deposit facilities—measured in terms of a reduction in the transaction costs of depositing and withdrawing at a financial institution and in terms of a reduction in the risks of depositing— influence, if at all, the
level and composition of the precautionary wealth portfolio of credit-constrained poor rural households in developing countries?

2) Does increased access to facilities to deposit financial wealth improve the capacity of credit-constrained poor rural households to buffer consumption from income fluctuations?

Let us hypothesize about the first question. On the one hand, increased access to deposit facilities for financial wealth may have an impact on the composition of the precautionary wealth portfolio of poor rural households. This may occur by augmenting the proportion of financial assets held and reducing the holdings of other types of assets used to store wealth, such as inventories of grain and livestock (portfolio composition effect). This outcome may result if financial assets are a superior instrument in terms of liquidity, profitability, and/or safety. The result may also occur if the value of the financial asset is not correlated with the price of the other assets or with the potential income shock.

On the other hand, the holding of precautionary reserves has an opportunity cost, in terms of the foregone returns from profitable projects that could have been undertaken with the same purchasing power. The more reserves are held, the greater the opportunity cost is. At the same time, larger precautionary reserves allow a greater capacity to buffer consumption. The availability of a better instrument to store wealth, such as a savings deposit in a financial institution (bank or non-bank intermediary), may produce two opposite outcomes on the level (in contrast to the composition) of the precautionary wealth portfolio:
i) Greater access to deposit facilities may increase the level of precautionary wealth, because the household may decide to rely more on this type of risk-coping mechanism—in contrast to using other strategies—as its relative attractiveness improves. The total cost of holding precautionary reserves may increase (or it may decrease, if the improvement is sufficiently strong) but, at the same time, the new arrangement may more than compensate for the potential disadvantages of otherwise engaging in more costly risk-coping strategies (strategy substitution effect).

ii) Greater access to deposit facilities may reduce the level of precautionary wealth, because the holdings of safer and more liquid reserves reduce the need for a large stock of reserves. Indeed, a portion of the value of illiquid assets is lost at the time of liquidation (melting effect). For households that have a target level for their ability to cope with risk, a level of reserves larger than what will be used is needed, in order to compensate for this melting. The holding of a more liquid asset reduces this loss (reduces the rate of melting) and results in smaller target reserves.

Whether total precautionary wealth increases or decreases, when a household has better access to deposit services, will depend on the net outcome of these two effects (strategy substitution and reduced melting effects). In either case, there is a gain in efficiency—a reduction in the total costs of risk coping—and this represents a welfare improvement.
Now, let us address the second question. Increased access to deposit facilities may help in achieving a smoother consumption pattern, because it reduces the cost of protecting consumption. In effect, either because of the smaller melting effect or the smaller cost of the set of strategies (given the substitution effect), this cost has been reduced. If the cost of smoothing consumption is lower than the cost of reducing consumption, when a shock strikes, then we can expect a smoother consumption pattern.

1.7 Research objectives

The purpose of this dissertation is to provide a conceptual framework for the analysis of deposit facilities as a mechanism for risk management and consumption smoothing. In implementing this task, the dissertation pursues some general and specific objectives.

1.7.1 General objectives

- To develop a dynamic stochastic wealth portfolio model that portrays how increased access to facilities to deposit precautionary wealth in a financial institution affects the poor rural household’s decisions on the level and composition of precautionary wealth.
- To determine whether there is a relationship between the proportion of asset holdings in the form of financial wealth (the household’s degree of deposit deepening) and its capacity to protect consumption from income shocks.
1.7.2 Specific objectives

- To identify the optimal policy rule that drives the proportion of precautionary wealth that is held in the form of financial assets.

- To explain the determinants of holding financial assets as precautionary wealth instead of holding other types of assets.

- To evaluate the effects of the risk of insolvency or bankruptcy of the financial institution on the farmer’s precautionary wealth.

- To identify policies and interventions that may be needed to achieve a socially optimum supply of deposit facilities.

1.8 Significance

Since borrowing constraints were the most critical issue under financial repression—a regime based on government interventions that did not allow an optimum level of financial intermediation and that created the conditions for the emergence of a strong credit rationing process— not much attention has been paid in the academic literature and in policymaking to deposit constraints. As a result, the potential effects of relaxing deposit constraints have been largely ignored.

This dissertation fills this gap in the development finance literature, by addressing the changes in precautionary wealth and asset portfolio composition that result from providing greater access to deposit facilities for financial wealth in the rural areas of developing countries. The combination of three elements differentiates this dissertation: 1) financial deposit facilities characterized by differential outreach influence the choices
that rural households face in accumulating precautionary wealth, 2) different levels of returns and risk for each precautionary asset require optimum portfolio decisions, and 3) the analysis is undertaken by using a stochastic dynamic wealth portfolio choice model that is constructed within a precautionary motive theoretical framework. In fact, Browning and Lusardi (1996) acknowledge that no much has been studied on “savings” portfolio decisions.

A deeper understanding of poor rural household behavior towards risk and the mechanisms available to deal with income shocks is also relevant in terms of policy and financial regulation. Moreover, De Janvry and Sadoulet (2002) point out that a better understanding about which assets people use to save and the incentives that guide these choices may shed light on other markets or institutions that deserve attention.

Finally, I also revisit some misconceptions that are shared among practitioners, regulators and policymakers. For example, they claim that poor people do not save and that, therefore, they do not accumulate wealth, because they cannot afford to do so. They also claim that poor farmers do not hold deposits because they are “financially illiterate”. I agree with Fafchamps (2003) when he claims that "…precious little is known on how the rural poor use money and other financial instruments” (p.203). He urges for more work on this topic, so adequate deposit instruments can be provided. This research aims at assisting in the design of deposit instruments and financial regulation in developing countries.

I first review the literature about the strategies used by poor rural households to deal with risk and the current debate about the types of assets that are used as
precautionary wealth. Chapter three develops a simple analytical model to illustrate the importance of transaction costs in wealth portfolio decisions. A more elaborated dynamic stochastic model of precautionary wealth portfolio decisions is presented and solved in chapter four. The results of several simulations are reported in chapter five and conclusions follow in chapter six.
CHAPTER 2

THE LITERATURE ON RISK AND PRECAUTIONARY WEALTH

2.1 Introduction

In this chapter, I explore several areas of the literature that are intimately related to the research problem. First, I review the literature that connects poverty and vulnerability, as a framework for understanding the role that risk plays in explaining poverty. Second, I examine the various strategies that poor rural households in developing countries adopt in order to deal with adverse income shocks. In particular, I focus on the use of financial and other assets as precautionary wealth. Third, I review the most relevant literature on consumption/savings and portfolio decisions, as a conceptual framework for developing a stochastic dynamic model of precautionary wealth choices.

2.2 Poverty, risk and vulnerability

Poverty is one of the most challenging problems that humankind has been trying to address, with limited success so far. In 2005, around 41 percent of the world’s population (2.6 billion people) lived with less than two dollars a day. Extreme poverty, experienced by people who lived with less than one dollar a year, accounted for one fifth of the world’s population, around 1.2 billion people. Poverty is more acute in the rural
areas of developing countries (LDCs), where 45 percent of the world’s total population lives (The World Bank, 2005).

The first of the Millennium Development Goals (MDGs), an agreement signed by 189 countries in 2000, calls for a reduction of extreme poverty by half between 1990 and 2015. Between 1990 and 2002, the incidence of extreme poverty in the world fell from 28 to 22 percent. The trends also indicate that, even though living standards have improved, wide disparities persist across regions. The World Bank projects that the MDGs may not be achieved in some parts of the developing world, particularly where the incidence of extreme poverty is among the highest, Sub-Saharan Africa (46 percent) and Latin America and the Caribbean (24 percent).

Although the poor are decreasing as a share of the total population, poverty still remains unabated. In a global economy, poverty today is not a country-level problem. Sachs (2005) and others have suggested that the best way to defeat terrorism is by combating poverty. It is key, therefore, to gain a good understanding of the nature and extent of the poverty problem, so appropriate policies can be adopted.

While there is ample agreement on the urgency of reducing poverty, there is little agreement on its definition and measurement. The World Bank defines a person as poor if his or her consumption or income levels fall below some minimum necessary to meet basic needs. This minimum is usually called a "poverty line". According to Bourguignon and Chakravarty (2003), the multidimensionality of poverty (e.g., low level of education, undernourishment, social exclusion, and the like) should be captured through various
poverty lines. A person would be considered poor if she/he falls below at least one poverty line.

In contrast to the definition of poverty, some agreement has been reached on its classification. Poverty is often classified as being either transient (transitory) or chronic (Murdoch, 1994; Dercon, 2005). The former is associated with transitory income fluctuations, while the latter is a permanent condition, regardless of any exogenous adverse income shocks. Operationally, Murdoch (1994) suggests that the transient and chronic types of poverty should be sample specific; i.e., a household that is poor in every period in the sample is chronically poor and it is transiently poor otherwise. Jalan and Ravallion (1999) propose a more comprehensive classification. They classify households as “very vulnerable” or “persistently poor” when their income is always below the poverty line; as “vulnerable” or “chronically poor” when their income is on average below the poverty line but sometimes above it; and as “not very vulnerable” or “transiently poor” when their income is on average above the poverty line but sometimes below the line.

These classifications suggest that there is a subset of the poor –the transient poor (vulnerable and not very vulnerable)– that, if effectively provided with some type of insurance, they may be able to pull out of poverty. Thus, this type of poverty may be mainly attributed to risk and to the inefficacy of instruments to deal with it. As Dercon (2005) states, looking at poverty from a risk perspective contrasts with the standard view that assesses poverty in a world of certainty, where there is no room for risk and
vulnerability. The standard view neglects the role that consumption protection mechanisms can play in poverty alleviation.

The role that risk plays in explaining poverty varies depending on the context. In developed countries, poverty seems to be generated mostly by the structural characteristics of poor households, such as low education and single parenting, and not much by risk (Murdoch, 1994). In fact, Bane and Elwood (1986) conclude that more than half of the poverty spells in the United States are due to family structure and life-cycle events, such as divorce, births and death. In LDCs, in contrast, poverty is caused, in addition to household structural characteristics, by riskiness and by the lack of instruments to deal efficiently with shocks that results from poorly developed financial institutions and weak social insurance institutions (Murdoch, 1994).

Risk, the probability of experiencing losses from an anticipated or unanticipated exogenous shock, has consequences on poverty in many ways. First, before the shock materializes (ex ante), households must adopt costly income smoothing strategies (for instance, the choice of a low-variability, low-return crop). Second, when risk materializes, there are the negative effects of the shock itself (for instance, the loss of assets and crops) as well as the adoption of costly consumption smoothing strategies, such as the sale of productive assets. Both ex ante and ex post consequences of risk lower productivity and technical efficiency (Alpizar, 2007).

The concept of risk as a cause of poverty is associated with the degree of protection and/or preparedness that households achieve; i.e., a risky scenario with a low level of protection will cause an increase in poverty or make it more persistent. The level
of protection that a household attains for some level of risk determines how vulnerable the household is. Dercon (2005) defines vulnerability as the existence and extent of a threat of poverty and destitution. The riskier the environment and the less ability households have to obtain efficient protection, the greater the threat becomes.

In LDCs, the threat for rural households comes mainly from working in the traditional agricultural sector (where a large share of labor is deployed, due to its low productivity). In this environment, the vagaries of weather and price shocks are the main sources of risk and, therefore, essential in explaining transient poverty and how severe chronic poverty can be. In other words, the poorer the economy, the more it has to rely on traditional agriculture and the more its exposure to risk arises from weather and price shocks.

Vulnerability also results from missing and/or incomplete financial and insurance markets, which limits the ability of households to attain appropriate protection and exacerbates the consequences associated with negative income shocks. Consequently, rural households must adopt second-best strategies to reduce the impact of adverse shocks on consumption. The several imaginative strategies these households use to deal with risk have received a lot of attention. Next, I review the pertinent literature.

2.3 Risk coping strategies and the costs of smoothing consumption

Extensive research has been conducted on the strategies adopted by rural households for risk mitigation (Kochar, 1995; Murdoch, 1995; Townsend, 1995; Dercon, 2005, among others). Most authors conclude that households attain some level of
protection, so negative income shocks are not fully transmitted into consumption. However, this protection far from amounts to full insurance (Townsend, 1995).

Among the class of income smoothing strategies, households may choose to plant crops with lower returns but that generate more stable income flows (Morduch, 1995). The hypothesis about planting low return–low variability crops is questioned by Kochar (1995), who states that this might be true only for excessively risk averse households. She argues that farmers make their cropping choices after the weather conditions are known and, therefore, the choice they make, even if it is of a low return–low variability crop, is a profit maximizing decision. In support of Morduch’s hypothesis, it is plausible that poor rural households in LDCs would be excessively risk averse, since they have small room for making mistakes. Moreover, their conservative choices respond to uncertainty about key future events, including unknown states of nature.

Diversification, another strategy to cope with risk, can be achieved in various ways. It can be attained through the choice of activities (e.g., farm and off-farm work, bot in agricultural and in non-agricultural occupations); spatially, across regions (e.g., village and out of village), and intertemporally, across seasons (e.g., planting and harvesting employment). The extensive use of diversification has been documented by Reardon (1997), who claims that in Sub-Saharan Africa close to half of household income comes from off-farm activities. In turn, the efficacy of this strategy has been studied by Kurosaki (2001), who finds that, in Pakistan, less diversified households are more vulnerable to risk.
Migration is another type of diversification of the sources of income. The importance of this strategy is growing, as the number of migrants has been rapidly increasing around the world. In effect, the United Nations reported that in 2002 the number of people residing out of their own country was at an all-time high of about 175 million, more than double the number in 1975. It is estimated that almost one of every ten persons living in the more developed regions is a migrant (United Nations). In turn, in some cases, remittances account for more than 10 percent of GDP.

By analyzing panel data from El Salvador, Pleitez-Chavez (2004) concludes that international migration-cum-remittances is a powerful insurance mechanism for households, since it provides protection against systemic shocks, whereas national migration protects households mainly from idiosyncratic shocks.

The adoption of risk-coping strategies implies costs. The costs of using a low return–low variability of income strategy can be measured in terms of opportunity costs. The costs associated with inefficient diversification are reflected in a lower productivity of labor, which results from not taking advantage of the gains from specialization and, consequently, not exploiting comparative advantages fully. High transaction costs are also associated with the cultivation of multiple plots.

The costs of relying on migration, which in itself is a costly and risky enterprise, can be enormous for the household, including—for example—family welfare losses from having to single-parent children, an increase of the burden of household chores on children (Chen, 2006), and the termination of marriage. The dissolution of the family could worsen the precarious situation of non-migrant members. The cost of relying on
child labor can be measured in terms of the loss in human capital formation (Maldonado and Gonzalez-Vega, 2005).

After the shock has taken place and either because the income smoothing strategies have not been sufficient for stabilizing income or because available strategies are not completely exhausted—as households may have the expectation of a continued sequence of shocks—consumption smoothing strategies are adopted. In some cases, households intentionally may decide to be ex ante under-protected and look for ex post protection. This behavior may be a reflection of the high costs of attaining ex ante protection and of people’s expectations about shock characteristics (e.g., frequency and impact).

One of the most commonly used strategies to smooth consumption in the presence of a shock is temporary employment for wages. Kochar (1993) claims that the majority of Indian households (70 percent) reported the use of such a strategy in efforts to smooth consumption after idiosyncratic crop shocks. This behavior has three implications. First, the demographic composition of the household is critical for attaining protection. Second, large shocks or shocks that directly affect the work capacity of household members (illness or death) cannot be overcome by increasing the labor supply. Third, the strategy works only if shocks are idiosyncratic; otherwise, the local labor market will also suffer. The main implication in Kochar’s argument is that, for this strategy to be effective, a well functioning labor market is a necessary condition. Takasaki, Barham and Coomes (2004) show that households in the Amazonian tropical forest cope not only with idiosyncratic
but also systemic shocks (e.g., floods) through labor supply responses, in the form of upland cropping and resource extraction.

Credit can be used as an income smoothing or consumption smoothing strategy, depending on the moment when the debt is contracted. As an income smoothing strategy, a loan may increase the household’s production capacity such that, even if income fluctuates, a higher income allows greater protection. Credit can also work as a consumption smoothing strategy, as in the case of an emergency loan. When households have access to credit, not using all that is available (i.e., keeping a credit reserve) as still another strategy.

Some consumption smoothing strategies may be used temporarily, as is the case of seasonal migration or pulling children out of school for a while (Jacoby and Skoufias, 1997). Help from extended families and friends, constitutes an implicit or explicit group-based risk sharing mechanism (safety nets). When the adoption of temporary strategies is not feasible or is not enough for mitigating the impact of income shocks, households must adopt more costly strategies, such as the sale of productive assets, with the corresponding capital losses and decline in productive capacity. In order to avoid making such costly decisions, households may choose to accumulate precautionary wealth.

To sum up, risk provokes changes in the household’s consumption, savings, labor, and production decisions in a way that is detrimental for the household’s welfare, due to the costs of attaining protection. The absence of well-functioning credit, insurance and labor markets reinforces these costly outcomes.
Nevertheless, the hypothesis of attaining protection for smoothing consumption has been challenged by new findings. Kazianga and Udry (2006) find evidence of little consumption smoothing in rural Burkina Faso during a severe drought. These authors show that income shocks are transmitted largely to consumption, an outcome with adverse consequences on welfare, which, in turn, reflects the paucity of protection mechanisms. Although it is conceivable that reducing consumption may be one of the first risk-coping strategies that is used, for the extremely poor this is not a choice, given that they are at the edge of subsistence.

2.4 The role of assets as a buffer stock

The literature amply discusses how assets are used to buffer consumption, with the theoretical contributions of Deaton (1991) and Carroll (1997), among others, and also with empirical evidence. This review focuses on the empirical contributions on this subject.

Deaton (1991) establishes the benefits of precautionary saving when credit markets are imperfect. The agent’s intertemporal utility is maximized under a set of conditions: (i) a consumer who faces borrowing constraints is prudent (a precautionary demand for saving is present) and impatient, (ii) assets offer a safe return, and (iii) income is stationary and uncertain. The solution of the infinite-time horizon problem is that households will accumulate a stock of assets in good years and exhaust them in bad years. Since it is assumed that the rate of time preference is greater than the rate of return on assets, these impatient consumers will not accumulate a large stock of precautionary
wealth. This model fits fairly well the consumption patterns observed in developing countries, which are characterized by occasional low levels of consumption and by low levels of asset holdings as well as by high variability of these holdings (Dercon, 2000).

Borrowing constraints attributed to an imperfect credit market in part explain the prudent behavior of households. The assumption of a safe return on assets seems to contradict, however, the initial assumption about the incompleteness of financial markets. On the one hand, in the rural areas of LDCs, financial shallowness may imply the lack of access to safe deposit facilities. On the other hand, in these rural areas, non-financial assets are unattractive as precautionary wealth, not so much because they have high storage or high liquidation costs but because their value is highly correlated with income fluctuations. That is, a bad harvest not only implies lower farm labor incomes but also a reduction in the value of assets sold in order to smooth consumption. Thus, the challenge for the farmer is more complex than in Deaton’s specification. In turn, financial assets themselves are not immune to systemic shocks.

Among the variety of assets that may be held for precautionary purposes, the literature has focused mainly on livestock (such as bullocks, goats, and so forth) and inventories of grain. Rosenzweig and Wolpin (1993) investigate the use of bullocks as a buffer stock in the semi-arid tropics in India. The nature of this particular productive asset requires the incorporation in the model of production decisions that are linked to consumption. The authors build a finite-horizon structural dynamic model of agricultural investment, with the following characteristics: farmers have a subsistence level of consumption, they have preference for consumption smoothing but face liquidity
constraints, and their investments in bullocks, land and irrigation equipment contribute to output and income (which are stochastic). The results, using panel data, are:

- From a maximum-likelihood ordered Probit model, they conclude that there is a relationship between the probability of buying/selling bullocks and positive/negative income shocks, which suggests the idea of consumption smoothing. Kochar (1995) argues that the increase in the probability of selling bullocks when income is low is not related to any exogenous measure of the shock, so reverse causation is possible.

- The current level of bullock stocks influences the probability of a subsequent purchase, which suggests the existence of a precautionary wealth target stock.

- Underinvestment in bullocks, compared to optimum holdings, is found to be due to the farmer’s aversion to risk and to borrowing constraints. Rosenzweig and Wolpin argue that, under complete markets, a farmer will always have two bullocks (assuming a fixed plot of land). However, they also state that this is not feasible, because weather shocks affect income. Since farmers are credit constrained and have low incomes, they will be induced to sell livestock to meet their consumption goals. Although the authors do not point this out, this situation reduces their productivity and lowers their incomes even more.

In this model, consumption smoothing above subsistence levels is achieved by bullock turnover. Bullocks are more attractive than other assets, such as land, because farmers can avoid the income-asset value correlation through selling bullocks in other villages, not affected by the local systemic shock. They emphasize that this is what makes
this type of asset particularly interesting as a means of holding precautionary wealth. However, for this to be true, some level of market integration for the particular asset, beyond the village, is needed, which is not necessarily the case in a poor economy.

As Fafchamps, Udry and Czukas (1998) claim, if markets are poorly integrated, systemic income shocks will affect the relative price of the assets used as precautionary wealth. They particularly focus on livestock. This correlation precludes farmers from costlessly using livestock as a buffer stock. The question they address is whether sales of livestock increase when a household is subject to systemic adverse rainfall shocks. This is accomplished by estimating a regression equation of the net number of animals sold, regressed on three estimated measures of income shocks while controlling for household characteristics. Panel data from the West African semi-arid tropics, with information at the household level, are used for this purpose. The results show that households adversely affected by a drought sell more livestock than otherwise, but that the effect is very small. Indeed, the authors assert that “livestock sales compensate at most 30 percent and probably closer to 15 percent of the income losses resulting from village-level rainfall shocks” (p.274). This result contradicts, to some extent, the outcome in Rosenzweig and Wolpin (1993).

Fafchamps, Udry and Czukas (1998) remark that a less frequent use of livestock as a buffer stock than is usually perceived may reflect that there are other assets that perform this function better, such as grain, cash, valuables, and the stock of human and farm capital. Moreover, these authors argue that livestock is not sold in times of drought because the returns to livestock production increase once the drought is over. Therefore,
farmers prefer to sacrifice present consumption, with the expectation of better returns in the future.

Livestock in the form of large cattle is an “ugly” asset not only because of the positive correlation between its value and income but also due to the indivisibilities that characterize this type of asset. In effect, Fafchamps, Udry and Czukas (1998) estimate that households subject to a reduction in 80,000 FCFA (monetary units) need to sell one third of a cow, which obviously is not feasible.

Jalan and Ravallion (2001) are motivated by the idea that credit-constrained poor rural households with a greater probability of experiencing idiosyncratic shocks may hold a greater proportion of unproductive liquid wealth with precautionary purposes compared to households with access to credit. By using panel data from China, these authors find that higher income uncertainty results in a higher share of wealth that is held in unproductive liquid forms, such as grain or cash. However, the portion is very small. Indeed, these authors claim that if all income risk is eliminated, the share of wealth held in liquid unproductive assets (cash or grain) would decline from 26.5 percent to 25.8 percent of the total. They do not explain, however, why people would want to hold unproductive assets when there is no risk.

Moreover, Jalan and Ravallion (2001) maintain that there is a Kuznets curve (U-inverted relationship) between precautionary wealth and permanent income, as neither the poorest nor the richest quintile seem to hold liquid wealth motivated by income risk. These authors claim that “the rich do not need it (precautionary liquid wealth), while the poor can not afford to do so” (p.46). In a similar argument, Rodriguez-Meza, Southgate
and Gonzalez-Vega (2004), using household panel data from El Salvador, find that there is a precautionary demand for land, which jointly with income per capita takes the shape of another Kuznets curve. The poor would want to hold land for precautionary purposes but cannot afford it, while the rich no longer demand land for these purposes.

Udry (1995) argues that, in the northern part of Nigeria, asset drawdown is large when a negative income shock occurs. In particular, transitory income shocks are mitigated by drawing down stocks of grain and cash savings, but livestock holdings are unaffected.

From reviewing the literature, it is not clear which asset, if any, is used as a buffer stock. So far, researchers have explored the use of livestock, grain, and cash, with mixed results. However, the literature seems to agree on something: rural households accumulate precautionary wealth as a strategy to mitigate the effects that adverse income shocks may have over consumption. In order to understand how rural households build their precautionary wealth, it is important to study the underlying consumption/saving and portfolio decisions.

2.5 Consumption / savings theory

Consumption is one area in economic theory that has experienced enormous progress, especially during the second half of the past century. Two theories/hypotheses have been the most influential: the Life Cycle Hypothesis (Modigliani, Brumberg and Ando, 1954) and the Permanent Income Hypothesis (Friedman, 1957). In the former, current consumption is determined by the value of lifetime resources while, in the latter,
consumption is determined by permanent income. In essence, the same idea lies behind both hypotheses: consumption is an intertemporal choice that is guided by preferences about consuming today or in the future, while savings are seen as a means for future consumption. Browning and Lusardi (1996) warn us, in a well written review of household savings, that the theory of savings is a theory of consumption, as savings are studied as a mere residual between income and current consumption.

The Life Cycle Hypothesis (LCH) states that assets are accumulated until workers retire, and then their consumption is financed by dissavings. This behavior generates a hump-shaped savings curve, which depends on the stage of life. In the Permanent Income Hypothesis (PIH), consumption is a function of permanent income, defined as the annuity value of current financial and human wealth (Deaton, 1992). The difference between current and permanent income is transitory income. Since permanent income is more stable than current income, the PIH can explain why consumption is smoother than current income, as the empirical evidence shows. The greater the gap between current income and permanent income, the higher is the flow of savings. Likewise, savings are negative when current income is lower than permanent income. Therefore, under the PIH, savings depend only on transitory income.

As Romer (2001) points out, the main idea behind the LCH and PIH is that individuals use saving and borrowing to achieve consumption smoothing. It is important to keep in mind that both theories assume the existence of a well functioning financial market, which is not necessarily the case in developing countries and even less likely in their rural regions.
As highlighted by Deaton (1992), one of the major drawbacks of these theories is
the neglect of the precautionary motive for saving: “a preference for constant
consumption irrespective of future uncertainty is not compatible with a prudent behavior,
and it is in this respect that the permanent income model seriously emasculates the basic
theory of intertemporal allocation.” (p.48).

The question is why these models do not include one of the most fundamental
motives for saving, the precautionary motive. In fact, this motive was the first one of
eight motives that Keynes listed in 1936.1 The answer to this puzzle comes from the
assumption of a behavior under certainty-equivalence that underlies the PIH. This
behavior implies that an individual consumes as much as she/he would if her/his income
were certain and equal to its mean. In other words, an individual will equalize the
marginal propensity to consume out of current and expected future income regardless of
the risks associated with his/her income. Therefore, uncertainty does not have any impact
on consumption. This theoretical result has been seriously misleading in interpreting
empirical facts, as will be explained later in this chapter.

In the permanent income models, behavior –under certainty equivalence– arises
from assuming a quadratic utility function, which generates a linear marginal utility. This
assumption is made for tractability rather than for gaining substance on the subject. New
developments in dynamic stochastic models have made precautionary savings significant
as a way to look beyond the PIH. In effect, Browning and Lusardi (1996) confirm that

1 The motives he lists for savings are precaution, life-cycle, intertemporal substitution, improvement,
inependence, enterprise, bequest and avarice.
the main innovation in standard consumption theory has been the inclusion of a precautionary motive for savings.\textsuperscript{2}

2.5.1 Precautionary savings

Carroll (1997) argues that most people save for emergencies rather than for being prepared for retirement. To recognize the existence of a precautionary savings motive, the marginal utility of consumption must be a convex function, not the linear relationship that results from the assumption of a quadratic utility function in the PIH.\textsuperscript{3} If marginal utility is convex then, at lower levels of consumption additional consumption will generate a higher marginal utility than at higher levels of consumption. More importantly, the rate at which marginal utility rises with shortfalls in consumption is greater when consumption is low than when it is high (Deaton, 1992).

Since uncertainty increases the number of states of nature in which consumption could be low and, therefore, the marginal utility of consumption would be high, strong incentives for precautionary savings emerge. Therefore, in the presence of uncertainty, future consumption becomes more attractive than under uncertainty and so do savings.

Precautionary savings are driven by the degree of “prudence” and not by the consumer’s risk aversion (Kimball, 1990). In contrast to risk aversion, measured by the degree of concavity of the utility function, prudence is measured by the degree of

\textsuperscript{2} They use the term “standard models” to refer to those that assume optimization. They avoid using the terms PIH and LCH because they think there is no an agreement on the terminology used.

\textsuperscript{3} A quadratic utility function has other inconveniences, such as the existence of a bliss point, beyond which the marginal utility of consumption becomes negative.
convexity of marginal utility. Although it seems logical to think that risk aversion is correlated with prudence, the relationship between these two will depend on the type of utility function that is used.

As figure 1 shows, greater uncertainty, reflected by a wider gap between \( C_A \) and \( C_B \), combined with a convex marginal utility function, causes an increase in the expected marginal utility of consumption. Under these conditions, individuals have a motivation for precautionary savings.

![Graph showing the relationship between expected marginal utility and consumption]

**Figure 2.1:** Precautionary wealth motive for saving. Romer (2001)
Deaton (1991) shows that prudent consumers who are impatient, face liquidity constraints, and have labor income that is identically and independently distributed over time will accumulate assets and use them as a buffer stock in protecting consumption against bad draws of income.

In the context of developing countries, where poor rural households have to deal with low and highly volatile incomes, precautionary savings are an essential tool for survival. This precautionary demand for savings is more important when individuals are partially credit constrained and even more when they are fully credit constrained.

### 2.5.2 Credit constraints

Consumers limited in their ability to borrow will follow a different consumption pattern, compared to those who are not credit constrained, for the same levels of income and wealth. Even if borrowing restrictions were not binding today, the possibility that they could become binding in the future affects today’s consumption decisions (Zeldes, 1989).

Credit constraints emerge in different ways. Maldonado (2004) shows a comprehensive classification of the different categories in which an individual can be credit constrained. Here, the most restrictive case is considered, that of an agent that is fully credit constrained and, therefore, cannot have negative assets at any time. A poor rural household may be fully credit constrained due to the following reasons:

- Credit constraints may emerge from asymmetric information and, more specifically, from adverse selection and moral hazard (Stiglitz and Weiss,
Credit markets do not clear through increases in price (interest rate) because of the potential for adverse selection, as only the riskier borrowers would be willing to pay the higher interest rates. To avoid default risk, lenders engage in non-price credit rationing. Increased moral hazard from higher interest rates may have the same effect (Keeton, 1979).

- High screening, monitoring, and contract enforcement costs make it unprofitable for financial institutions to grant loans to small farmers (Gonzalez-Vega, 2003).
- Subsidized credit programs and associated credit rationing benefit mostly large borrowers and not the vast majority of poor farmers (Gonzalez-Vega, 1976). As in the case of asymmetric information, a pooling equilibrium emerges that leads to non-price allocations of loans.
- Financial institutions may impose prohibitively high discrimination costs that farmers are not willing to “pay” (Becker, 1971).
- The farmers’ prudent behavior may prevent them from borrowing, when their activities are risky (Boucher, 2000).

Fully credit-constrained poor rural households have stronger incentives to accumulate precautionary wealth, even when they do not have access to efficient mechanisms to accomplish this goal. Assets will be run down when needed and replenished when possible. This is the dynamics of a buffer-stock savings behavior.
2.5.3 Buffer-stock savings: A new framework

The standard version of the LCH/PIH, a certainty equivalence model that assumes a quadratic intertemporal additive utility function, has been predominantly used as the theoretical framework to explain consumption behavior (Carroll, 1997). However, this model has encountered difficulties in explaining some stylized facts. I devote special attention to two of them:

- Consumption appears to track quite closely current income, which is often referred in the literature as the consumption-income parallel (Carroll and Samwick, 1998).

- Wealth holdings are low and volatile (Aiyagari, 1994).

The buffer stock savings models have made important contributions in attempts to solve these puzzles. Several scholars have emphasized the importance of precautionary savings and their use as a buffer stock (Zeldes, 1989; Kimball, 1990; Deaton, 1991; Carroll, 1997). The essential elements of these models are: (i) an important income uncertainty component, (ii) “prudence”, in the sense that agents have a precautionary demand for savings, and (iii) “impatience”, in the general sense that agents prefer to consume today than in the future. Each one of these models has its own peculiarities; for instance, Deaton (1991) explicitly introduces a liquidity constraint condition, while Carroll (1997) sets up a target wealth-to-permanent income ratio. However, they share a common idea: assets are not desired for their own sake but to buffer income fluctuations.

---

4 In the Carroll’s sense, patience implies that if consumers had certainty about the future they would prefer to consume more than their current income today.
When a buffer-stock savings behavior is taken into account, the anomalies identified earlier can be explained. In the first case, impatient consumers who are prudent will be unwilling to borrow, which explains why consumption tracks current income. Along the same lines, impatience explains the low levels of wealth observed, and the volatility of the stock of wealth is a natural consequence of the role that it plays in buffering income fluctuations in a risky environment.

It is critical to understand how agents choose to allocate their wealth, and portfolio theory offers some useful insights on this.

### 2.6 Portfolio theory

In a traditional portfolio problem, the agent (investor) must decide how to invest his/her wealth among a set of possible assets that earn random returns. The decision made on the portfolio by the investor will depend on determinants such as risk tolerance, the stage in which the person is in his/her life cycle, the time horizon he/she wishes for the investment, and others. The elements that seem to weight the most in the investment decision are the risk-return trade offs (Bodie and Merton, 2000).

Markowitz (1952) pioneered the idea that the optimal portfolio choice has to do with the trade off between earning a higher expected return and taking a greater risk. An individual whose decision is guided just by risk and return is said to have mean-variance preferences. A powerful result from portfolio theory is the prediction that households with this type of preferences can get a portfolio that is less risky than each asset alone by
combining two risky assets, as long as their yields are not perfectly positively correlated. This is the principle of diversification.

According to Markowitz, rational investors will only look at efficient portfolios, which offer the highest possible return for a certain level of risk or the lowest possible level of risk for a given return. The set of efficient portfolios defines the efficiency frontier. The investor chooses one portfolio based on his preference for risk. In the absence of a riskless asset, the portfolio chosen will lie on the efficiency frontier but, when a riskless asset exists, every efficient portfolio results from the combination of the multiple risky assets on the frontier and the riskless asset. Therefore, when a household seeks to reduce the risk level of the precautionary wealth it holds, some level of portfolio diversification is expected.

This dissertation focuses on a portfolio of assets that serve mainly to store precautionary wealth. They are unproductive or semi-productive assets. Specifically, I focus on financial deposits and small livestock. The approach has a two-fold purpose. First, in contrast to a priori expectations, some researchers have found that livestock plays an unimportant role in consumption smoothing in some parts of the world. However, as Fafchamps (2003) points out that more research is needed to establish the role that livestock plays as a precautionary asset.

Second, the relevant question is whether the welfare of rural households can be improved upon. Expanded deposit services seem to be a potential vehicle to achieve this goal. Under some circumstances, financial assets can offer less costly and less risky alternatives to the assets traditionally held as precautionary wealth in the rural areas of
developing countries. More importantly, wealth held in a deposit with a financial intermediary may be uncorrelated with the returns from local assets, whose value may be covariant with the household’s income shocks. This will offer an opportunity for effective diversification that most likely is less costly than migration, the other alternative for breaking the covariance (Pleitez-Chavez, 2004).

Furthermore, in contrast to other precautionary assets that are unproductive (Jalan and Ravallion, 2001), deposits contribute to financial intermediation and, thereby, transform the command over resources held by the depositor into loanable funds that, in turn, allow the exploitation of attractive productive opportunities by other producers who might otherwise be credit-constrained (Gonzalez-Vega, 2003). This mitigates the social costs of precautionary wealth holdings.

How attractive financial deposits will be as precautionary wealth will depend, in turn, on the transaction costs faced by depositors and on the likelihood of bankruptcy of the deposit-taking institution. This likelihood depends, in turn, on how correlated the financial intermediary’s assets are with the systemic shock.

I am ignoring the potential erosion of the value of deposits from inflation and devaluation, particularly under interest rate controls. I am also ignoring the risk of expropriation (freeze on deposits) due to incompetent macroeconomic policies or political instability. Interestingly enough, the deposits of the poor tend to be more stable during periods of political and financial instability, as shown by the experience of Bolivia (Gomez-Soto and Gonzalez-Vega, 2007). Clearly, the opportunities of poor rural households to protect and smooth their consumption by using financial deposits would be
severely constrained under financial repression (Shaw, 1973). In this case, the role of the authorities will be to dismantle financial repression and improve macroeconomic management (Gonzalez-Vega, 2003). The loss of access to a less costly tool for consumption smoothing is one of the ways macroeconomic instability contributes to poverty traps.
CHAPTER 3

PORTFOLIO MODEL WITH TRANSACTION COSTS

3.1 Introduction

A theoretical model of wealth portfolio is developed and solved in this chapter. The exercise is built upon a simple version of a two-period stochastic model on asset portfolio decisions, originally developed by Samuelson (1969). In this model, the economic agent decides how much to consume and how much to invest in either a risky asset (which can be seen as livestock) or a risk-free asset (which can be seen as deposits). The solution to the problem yields an optimal saving-consumption decision and optimal portfolio selection.

I modify Samuelson’s model by introducing transaction costs. In the modified version, the agent has to incur a transaction cost whenever she invests in the risk-free low-return asset (deposit). The exercise consists in computing the critical value for the transaction cost that makes the individual indifferent between investing all her wealth in the risky asset or incur a transaction cost in order to diversify her portfolio across the risky and the risk-free asset. The solution to this model offers interesting insights on the role that transaction costs play in explaining the composition of precautionary wealth and the ability of the agent to achieve a level or risk compatible with her objectives. In
particular, it reveals the cost that the agent is willing to incur in order to enjoy the benefits from diversification.

3.2 The importance of transaction costs

Financial transactions are not frictionless; they imply costs that influence decisions. For instance, opening a savings account or depositing/withdrawing money from a financial institution implies that the household has to incur various types of transaction costs (Hess, 1995). Among these costs is the time required to search for a deposit-taking institution that seems to be sound and reliable. The time spent in understanding the different characteristics of the financial deposit products offered also has an opportunity cost. Financial institutions charge fees on transactions and on outstanding balances. Taxes must be paid on interest earnings, and there are penalties for early withdrawals. There are also costs that result from restrictions imposed by the financial institution that limit the size and the number of withdrawals. Depositors are also required to keep a minimum balance. In the case of financial cooperatives, there is an additional implicit cost of holding deposits, which results from the requirement that depositors freeze some sum of money as equity. A similar cost of foregone interest emerges from the forced savings found in village banking and other microfinance programs, such as the Grameen Bank.

In the rural areas of developing countries, depositor transaction costs emerge mostly from long distances to the nearest branch of the financial institution (Gonzalez-Vega and Gonzalez-Garita, 1987). These distances, jointly with an underdeveloped
infrastructure, imply high transportation, lodging, communications, and waiting costs. Branches may be opened for service at inconvenient times and days. Procedures may require identification documents and literacy, thus excluding large segments of the population. Crime and other threats make the carrying of cash over long distances very risky. In many cases, these costs are so high that they make the holding of small deposits prohibitive (Gonzalez-Vega, 2003). Because many of these costs are independent of the size of the transaction, they particularly exclude the poor.

Transaction costs reduce the net return on deposits and push their level below the optimum. Moreover, transaction costs are inversely related to the interest rate elasticity of the supply of deposits as, the higher the transaction costs are, the less responsive households will be to changes in the interest rate paid on deposits. This feature undermines the power of using slight increases in the interest rate as the only instrument to promote savings mobilization. For poor households in the rural areas, any strategy for deposit deepening must focus, instead, on transaction costs.

Using information from 193 banks in 58 countries, Beck, Demirguc-Kunt and Martinez-Peria (2006) analyze indicators of physical access, affordability, and eligibility barriers to deposit services. These authors show that in many countries these barriers can potentially exclude a significant share of the population from using financial services. The availability of physical infrastructure is a robust predictor of barriers. Financial exclusion can retard economic growth and increase poverty and inequality (Galor and Zeira, 1993).
The importance of transaction costs for depositors is studied by Guerrero (1988). This author analyzes the determinants of deposit mobilization in the rural areas of the Dominican Republic. Guerrero adds variants to the original deposit demand model formulated by Wai (1972) and proposes a deposit demand that is a function of income, wealth, interest rates, loan expectations, education level, other financial intermediaries (competition), and transaction costs. After running an OLS regression, the author explains why Banco Agricola was successful in mobilizing deposits from a multitude of relatively small, rural and semi-urban households, when following the advice of the Rural Finance Program at the Ohio State University. The author highlights two factors as the main reasons for this outcome. First, there was a major reduction in transaction costs for depositors and, second, there was an increase in the net returns on deposits.

The influence of transaction costs on precautionary wealth portfolio choices is illustrated in the two-period stochastic model developed next as a variation of the Samuelson (1969) portfolio model.

### 3.3 Insights from a two-period stochastic model on asset portfolio decisions

In the two-period deterministic portfolio model developed by Samuelson (1969), the economic agent (household) decides how much to consume and what proportion of savings to invest between a risk-free and a risky asset. There are two states of nature in the economy (good and bad), with equal probability. The returns on the risky asset are stochastic, whereas labor income is deterministic. The agent cannot borrow.
I modify Samuelson’s model by including transaction costs. In order to acquire the riskless asset and diversify its portfolio, the household must incur a transaction cost $\tau$. This transaction cost takes the form of a fixed entry cost, which for the case of deposits can be thought of as the cost of the trip when depositing or withdrawing money from the bank. This cost is independent of the amount of the transaction. Other, more complex, specifications for $\tau$ can be explored. Here, however, the relevant feature is that transaction costs influence portfolio choices. A transaction cost is attached to investing a share of the household’s wealth in the risk-free asset, whereas investment in the risky asset is assumed to be free of transaction costs.

The household maximizes intertemporal utility as a function of consumption $U(c_t)$, over two periods. The current time is $t_0$, and after period $t_1$ there is no economic life, so the household has no bequest motive. Preferences are time-separable and period utility is logarithmic. The discount factor is $\delta > 0$. There is a risk-free asset, such as an insured deposit, which pays a fixed interest rate $r$, where $r > 0$. There is a risky asset, which pays a gross return of $\lambda$ in the good state and of $1/\lambda$ in the bad state, with equal probability. The risky asset generates a higher expected return than the risk-free asset. More formally, $\lambda > \frac{1}{2} + \frac{1}{2\lambda} > 1 + r > \frac{1}{\lambda}$.

The household’s initial wealth is a stock of assets $A_0$. It receives labor income at the beginning of each period, which is exogenously given and non-stochastic. Given its

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5 A logarithmic utility function is a Hyperbolic Absolute Risk Aversion (HARA) utility function and it carries the precautionary motive for saving, as defined by Kimball (1990).
current income \( Y_0 \), the household chooses consumption and a portfolio mix so as to maximize expected lifetime utility. The proportion of wealth that is invested in the risky asset is symbolized by \( \omega_0 \).

The general version of the optimization problem for the household is:

\[
\text{Max}_{c_t, \omega_t, \omega_0} U(C_t) + E \delta U(C_{t+1})
\]

subject to

\[
A_t = (A_0 + Y_0 - \tau - C_0)[(1-\omega_0)(1+r) + \omega_0 Z_t]
\]

where \( Z_t = \begin{cases} 
\lambda & \text{with probability } \kappa \\
\frac{1}{\lambda} & \text{with probability } 1-\kappa
\end{cases} \)

where \( \kappa = 1/2 \)

\( A_0 \) is given

\[
\delta = \frac{1}{1+\rho}, \quad \text{where } \rho \text{ is the discount rate.}
\]

I study two cases. In the first case, the household can accumulate wealth (invest) only in the form of the risky asset (\( \omega_0 = 1 \)). In the second case, the household diversifies its wealth portfolio between two assets: the risky and the risk-free asset, in some proportion (\( 0 < \omega_0 < 1 \)).

The corresponding value functions are (see appendix A for the derivation):

55
Case I: Only the risky asset

\[ V^I_0 (A_0) = \log C_0^* + \delta E \log \left( (A_0 + Y_0 - C_0^*)(Z_1 + Y_1) \right) \]  \hspace{1cm} (3.3)

Case II: Diversification between the risky and the risk-free asset

\[ V^{II}_0 (A_0) = \log C_0^* + \delta E \log \left( (A_0 + Y_0 - \tau + \frac{Y_1}{1+r} - C_0^*) \left[ (1 - \hat{a}_0^*)(1 + r) + \hat{a}_0^* Z_1 \right] \right) \]  \hspace{1cm} (3.4)

where:

\[ \hat{a}_0^* = \frac{(1 + r) \left[ 2(1 + r) - \left( \frac{1}{\lambda} + \lambda \right) \right]}{2(\lambda - (1 + r)) \left( \frac{1}{\lambda} - (1 + r) \right)} \]  \hspace{1cm} (3.5)

The solutions are found by solving backwards. For both cases, I obtain the optimal consumption decision and, for the second case, the optimal portfolio choice, which is characterized by the following equality:

\[ \frac{\lambda - (1 + r)}{(1 + r)(1 - \hat{a}_0^*) + \hat{a}_0^* \lambda} = \frac{(1 + r) - (1/\lambda)}{(1 + r)(1 - \hat{a}_0^*) + \hat{a}_0^* (1/\lambda)} \]  \hspace{1cm} (3.6)

On the left-hand-side, the numerator shows the increase in the total return the household gets in the good state, if the share of the risky asset \( \hat{a}_0^* \) is increased by one unit, while the denominator is the marginal utility of the return in the good state. On the right-hand-side, the numerator shows the reduction in the total return in the bad state, if the share of the risky asset \( \hat{a}_0^* \) is increased by one unit, while the denominator is the marginal utility of the return in the bad state. By equating these two fractions, the household is equating the increase in utility that results in the good state from increasing
the share of the risky asset $\tilde{\omega}_0$ with the reduction in utility in the bad state that an increase in $\tilde{\omega}_0$ and a reduction in the share of the risk-free asset cause. Further increases in the share of the risky asset would lead to a higher expected reduction of utility if the bad state occurs compared to the expected increase in utility if the good state happens. The optimum share $\tilde{\omega}_0$ thus depends on how different the returns in the good and the bad states ($\lambda$ and $1/\lambda$) are as well as the interest rate $r$ and the transaction cost on deposits $\tau$. The greater the difference in returns in the two states is, the lower will be $\tilde{\omega}_0$. The higher the interest rate is, the lower $\tilde{\omega}_0$ will be. The lower the transaction cost on deposits, the lower $\tilde{\omega}_0$ will be.

From solving the second case we can see that, in this simple formulation of the model, the portfolio choice problem is completely separable from the consumption-saving choice problem. In particular, the optimal portfolio choice does not affect the optimal consumption choice and vice versa. The optimal portfolio choice determines the portfolio’s expected return, but this return has no relationship with the optimal consumption-saving rule.

In the dynamic model to be developed in the next chapter, the links between consumption and portfolio decisions are more explicit, through the concept of precautionary wealth. The main motivation for a poor rural household to accumulate precautionary wealth does not emerge from the returns it can earn on its assets, per se,

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6 The solution of the optimization problem for case II relies on the handouts distributed by Dr. Pok-sang Lam in his course Macroeconomics II at The Ohio State University.
but from the possibility of smoothing consumption in the future, when a shock strikes. Thus, the simpler revised version of Samuelson is—in essence—different from the dynamic model I will next formulate. While the former focuses mainly on differences in risk and returns, the latter is about consumption smoothing.

The simpler two-period model allows us, nevertheless, to identify the importance that transaction costs play in portfolio diversification choices. To capture this effect, I equate the two value functions, $V^I_0$ and $V^{II}_0$, that result from the optimization problems (cases I and II). I then calculate a critical value for $\tau$ (the transaction cost for the depositor), which would make the household indifferent between these two choices. That is, this critical $\tau$ is the magnitude of the transaction cost that makes the household indifferent between investing all its wealth in the risky asset only or diversifying its portfolio by adding the riskless asset. At a sufficiently high transaction cost, no diversification will occur, and by holding the risky asset only, the household will be worse off.

The exercise is undertaken by assuming reasonable values for the parameters, such as a high discount factor (0.95), a low level of initial precautionary assets, equivalent to 5 percent of income at time $t_0$, a gross return to the risky asset in good times equal to 1.5 (the reciprocal, of 0.66, is the return in bad times), and a rate of return on deposits equal to 3 percent per period. The assumption of different income levels can further reflect the potential consequences of a negative labor income shock that takes place in the second period. For instance, I assume a reduction in income of 20 percent with respect to income at $t_0$ (table 3.1).
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor (δ)</td>
<td>0.95</td>
</tr>
<tr>
<td>Discount rate (ρ)</td>
<td>0.05</td>
</tr>
<tr>
<td>Initial assets (Ao)</td>
<td>50</td>
</tr>
<tr>
<td>Initial income (Yo)</td>
<td>1,000</td>
</tr>
<tr>
<td>Bad return on the risky asset (1/λ)</td>
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</tr>
<tr>
<td>Good return on the risky asset (λ)</td>
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</tr>
<tr>
<td>Income in period one after shock (Y1)</td>
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<tr>
<td>Interest rate (r)</td>
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</tr>
<tr>
<td>Probability of each state of nature (κ)</td>
<td>0.50</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Case I (Only a risky asset)</th>
<th>Case II (A risky and a risk-free asset)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky asset share (ω*)</td>
<td>1.00</td>
</tr>
<tr>
<td>Value function (Vo) - No deposit transaction costs</td>
<td>5.796</td>
</tr>
<tr>
<td>Value function (Vo) to obtain the critical τ</td>
<td>5.796</td>
</tr>
<tr>
<td>Critical value for transaction cost (τ)</td>
<td>276</td>
</tr>
</tbody>
</table>

**Table 3.1:** Computation of the value for the transaction cost that makes the household indifferent between diversifying and not diversifying its precautionary wealth portfolio.
The results of these exercises reveal that:

- As the principle of diversification predicts, the value function, which reflects the household’s lifetime utility, is higher when the portfolio of assets is diversified by adding the riskless asset, such as a risk-free deposit, despite its low rate of return, compared to the situation when the portfolio is concentrated in the risky asset only (table 3.1).

- The value of the critical $\tau$, the magnitude of the depositor transaction cost that makes the household indifferent between diversifying the asset portfolio or just holding the risky asset (i.e., the $\tau$ that equates the value functions for case I and II), is as high as 15 percent of the household’s lifetime wealth.

This result reflects the tremendous willingness to pay that the household has for having the choice of portfolio diversification through the addition of a riskless asset. The implication is that, in the rural areas of developing countries, where risks are high and cost-effective risk-coping tools are not available, households should be willing to incur high transaction costs in order to add bank deposits to their portfolios of wealth that frequently they do not. Therefore, may be an indication of the extraordinarily high transaction costs they face. This preference for financial assets will be strong, however, as long as these deposits are risk-free. As will be shown in the next chapter, a preference for deposits will still be present, even if deposits are risky, as long as the risks associated with the two types of assets are held not positively correlated.

The covariance of the returns on livestock holdings and on deposits will erode the power of diversification in containing risk and increasing welfare. As long as the
correlation is not perfect, however, diversification will be optimal. If the high willingness to pay for deposits is a reflection of the threats on consumption smoothing from the holdings of the risky assets, in the absence of deposit facilities the protection of consumption will be very costly. By reducing these costs and risks, the introduction of safe and convenient deposit facilities can have powerful welfare-enhancing effects.

That bank deposits are not frequently observed in many developing countries, therefore, could be mostly a reflection of prohibitively high transaction costs for depositors. This absence may also reflect the lack of trust of depositors in the financial institution, when the financial asset is not perceived as sufficiently risk-free. It may also result from the implicit costs of discrimination. The increase in household welfare from a reduction of these transaction costs and from the development of safer and sounder rural financial institutions could, therefore, be quite significant.

Other results include the following:

- The more patient the household is, the higher the value function (in both cases) will be. A greater accumulation of wealth in the first period increases consumption in the second period, which is valued more highly by a patient consumer—as shown by the lower discount rate. In developing countries, patience is inversely related to poverty. Thus, the welfare improvement from reducing the costs of holding precautionary wealth, by expanding the outreach of deposit facilities, will be more significant for the impatient poor. The promotion of deposit facilities (and not just credit) should then be a tool in poverty alleviation and food security programs.
- The greater the initial wealth, the higher the value function will be. Not only do wealthier households consume and invest more, but they also enjoy less costly opportunities to smooth their consumption over time.

- The greater the expected return on the risky asset, the greater its share in the portfolio of wealth becomes. However, given the assumptions of the exercise, this share does not go above 50 percent. Under these assumptions, the household always holds at least half of its portfolio in the risk-free asset, regardless of the level of return on the risky asset. The reasons are that, in this specification, as the return of the risky asset in good times $\lambda$ increases, the return in bad times $1/\lambda$ decreases. The riskier environment is characterized by these diverging outcomes. Moreover, in this exercise, both states of nature have the same probability of occurring. If the probability of a bad outcome declines, the holdings of the risky asset as a share of the total portfolio will increase. Thus, the riskier the environment in which the household operates, the larger the share of its wealth that it would like to hold in a risk-free asset. For this reason, access to safe and convenient deposit facilities is more critical in riskier environments.

- In contrast, if the return on the risk-free asset goes above 8.5 percent, then the household prefers to hold its entire wealth in the risk-free asset. This can represent the situation of a surplus unit in the economy which, by definition, does not have productive opportunities with better rates of return than a bank deposit. Moreover, under the realistic assumptions of the exercise in a risky
environment, the rate of return on a risk-free deposit does not need to be too high to attract depositors.

The simpler model thus sheds light on the important role that transaction costs play in explaining precautionary wealth portfolio decisions in risky and poor developing areas. With this insight, I now switch to a dynamic stochastic model that fits the situation of a poor rural household better: stochastic labor income, a portfolio of risky assets, covariance between income flows and the price of livestock, and the potential impact of a systemic shock on the solvency of the deposit-taking financial institution. In this case, in addition to transaction costs, the authorities have to be concerned about the safety and soundness of deposit-taking financial institutions.
CHAPTER 4

DYNAMIC STOCHASTIC MODEL OF PRECAUTIONARY WEALTH

4.1 Introduction

This chapter develops a dynamic, stochastic, infinite horizon model of precautionary wealth choices. In the model, the household chooses how much to consume and how much to add or to subtract from its stock of precautionary wealth, which is held in the form of either deposits or livestock. This approach offers richer results than the modified version of Samuelson’s original buffer stock model, presented in the previous chapter, in several ways.

First, the two assets held by the household are risky. That is, there is no safe asset, as in the original model. On the one hand, livestock holdings are threatened by changes in its price and the potential associated capital losses. On the other hand, if the deposit-taking financial institution becomes insolvent or goes bankrupt, the household may not be able to recover the whole amount of its deposits.

Second, the model captures the positive correlations that may exist between labor income, the price of livestock, and the net return on deposits, adjusted for the risk of insolvency. These correlations emerge as a consequence of a local systemic shock. In contrast to the earlier model, here labor income is stochastic. Deposits (backed by a
credit portfolio with some degree of diversification) are still less risky than livestock, except under some bankruptcy outcomes that prevent the deposit-taking institution from returning the whole amount of the deposit. A systemic shock on labor income may lead to sales of livestock, in order to protect consumption, inducing a drop in its price. Lower incomes and lower livestock prices may, in turn, be associated with a deterioration of the quality of the loan portfolio of the deposit-taking institution, thereby reducing the expected return on deposits.

Third, under a reasonable set of assumptions, the infinite horizon model delivers a certainty-equivalent steady state. This property allows the computation of a numerical solution for the problem and, from this, the exploration of the sensitivity of key results to parameter changes, through a number of simulations. These simulations highlight the role of depositor transaction costs and of the safety of deposits, two areas where the authorities can intervene. The simulations also show how differences in the riskiness of the environment explain the diverse composition of portfolios of precautionary wealth and the success of rural deposit mobilization in various types of countries.

4.2 Numerical approximation

The dynamic, stochastic, infinite horizon nature of the model makes obtaining an analytical solution particularly difficult. It is often the case that nonlinearities in the model prevent the researcher from achieving a closed-form solution. If this is the case, numerical techniques have proven to be among the most accurate ways to approximate a solution to the model.
Solving the model numerically implies the following steps. First, it involves the computation of certainty-equivalent steady-state values for the action and state variables as well as for the shadow prices. Whenever a system of equations is nonlinear, numerical techniques such as the Broyden method can be used. This root-finding method is a multivariate generalization of the univariate secant method that uses the inverse update as algorithm. The CompEcon Toolbox developed by Miranda and Fackler (2002) contains a \textit{broyden} routine that greatly facilitates the use of this method.

Second, since the infinite horizon Bellman equation can be expressed as a vector fixed-point problem, the optimal value and policy function can be computed by using standard function iteration methods, such as Newton or function iteration. The complexity of this problem arises from the fact that the unknown is not a value but a functional form.

The collocation method is used for solving the Bellman equation. This method consists in approximating the unknown Bellman equation (function) by a combination of \( n \) basis functions, whose unknown coefficients are to be determined. The basis functions will be fixed by requiring the approximant to satisfy the Bellman equation at the \( n \) collocation nodes previously defined (Miranda and Fackler, 2002). The degree of accuracy of the numerical solution is measured in terms of the orders of magnitude of the difference between the approximant and the exact solution, which is called the residual. The CompEcon Toolbox also contains a routine called \textit{dpsolve}, which efficiently handles the numerical problem.
Third, numerical methods are useful in implementing postoptimality analysis, which is useful in performing simulations, in order to examine how the system responds to changes in the parameters. Particularly, this facilitates the dynamic path analysis, which describes how the system evolves over time starting from a given initial condition.

In contrast to deterministic models, the stochastic models do not generate a single dynamic path but multiple ones. Each one of these paths emerges from the realization of random shocks. Therefore, the steady state is not a point anymore but a distribution. Monte Carlo simulation allows the generation of a sequence of random shocks, from which a representative path is generated. The CompEcon Toolbox provides the `dpsimul` routine to perform the dynamic path analysis.

4.3 Dynamic stochastic model of precautionary wealth choices

The model frames the potential effects of increased access to more convenient and safer deposit facilities for financial wealth on the portfolio composition and level of the household’s precautionary wealth. The model illustrates how a fully credit-constrained poor rural household makes two types of decisions: (i) consumption decisions, which implicitly respond to the question of how much additional precautionary wealth will be accumulated or decumulated in each period, and (ii) portfolio decisions, which answer the question of what types of assets will be used to hold precautionary wealth.

The timing of the model is the following. At the beginning of period $t$, the rural household finds itself holding a stock of deposits in a financial institution $D$ and a stock of small livestock $L$, for instance, pigs and goats. Knowing this information, the
household decides the direction (increase or decrease) and the magnitude of the change in deposits $\Delta D$ and of the change in livestock holdings $\Delta L$.

At the beginning of period $t$, this representative household observes a discrete systemic shock $z$, which affects the agricultural household’s income $Y(z)$, the price of livestock $P(z)$, and the expected rate of return on financial deposits $r(z)$. The shock is systemic in that it simultaneously affects all local households.

The model has two central characteristics. First, the local price of livestock is a function of the systemic shock. This function captures the negative effect on livestock prices of a systemic shock, which results in an excess supply of livestock, as the villagers try to sell their animals in order to overcome the impact of the shock on their consumption. The reduction in price is exogenous to the households, as it results from the collective behavior in the village. Livestock prices drop and sales of livestock become less attractive for the household, but they may still be needed, since this may be, in the model, the only way in which the household can deal with the consequences of the shock on consumption.

Second, the systemic shock may also have an exogenous impact on the rate of return on financial deposits. Some households will struggle to repay their loans when incomes and livestock prices are low and, thus, the probability of insolvency or bankruptcy of the financial institution arises in the event of an adverse systemic shock. As a result, by the next period the depositor may not recover all of its financial wealth. In practice, this effect will depend on the portfolio diversification achieved by the financial intermediary and on other risk-managing policies that determine its safety and soundness.
For instance, if a systemic shock strikes, a non-regulated, small, local financial institution with very little portfolio diversification is more likely to go into bankruptcy, compared to a regulated, large, and more diversified institution with operations over a broader space, for instance in a national scale.

The household’s objective is to maximize the sum of current and discounted expected future utility subject to: (i) a budget constraint and (ii) credit constraints. The dynamic nature of the problem creates incentives for households to accumulate precautionary wealth in good times, either in the form of deposits or livestock, in order to guarantee consumption during bad times in the future. Moreover, households are assumed to be fully credit constrained, which increases their incentives for holding precautionary wealth. Emergency loans are not available to smooth consumption after an adverse shock.

The composition of the precautionary wealth portfolio between financial deposits and livestock will depend, in this model, on the household’s relative risk aversion, impatience (discount rate), initial wealth, and income as well as on the price of livestock, the effective interest rate earned on deposits, as a measure of the net return on deposits adjusted for risk, the transaction costs incurred by the depositor, which capture transportation costs (physical distance) and/or discrimination costs (cultural distance), the risk of insolvency or bankruptcy of the financial institution, and relevant covariances between the prices of livestock and income and of each of these two with the probability of bankruptcy, as all three are correlated with the systemic shock.
The infinite horizon, stochastic, dynamic model with mixed state and continuous choice is formalized below.

4.3.1 The model

In order to formulate the Bellman equation, I define the state and action variables, the corresponding transition functions (law of motions) for each state variable, and the reward function as well as the constraints for the maximization problem.

4.3.1.1 State Variables

The model has three state variables:

\( D_t \) = Outstanding deposits, beginning of period

\( L_t \) = Livestock holdings (physical units), beginning of period

\( z_t \) = Systemic shock

The agent’s deposits and livestock holdings at the beginning of the period are continuous variables that may attain any nonnegative value. The systemic shock denoted by \( z \) is i.i.d. and discrete and it determines the level of three variables:

\( r(z) \) = The per-period rate of return on deposits, adjusted for the likelihood of loss of some of the principal due to bankruptcy of the financial institution

\( P(z) \) = Unit price of livestock

\( Y(z) \) = Labor income at the farm, independent of returns from livestock holdings
4.3.1.2 Action variables

The model has two action variables:

\[ \Delta D_t = \text{Net additions to deposits} \]

\[ \Delta L_t = \text{Net additions to livestock} \]

These action variables are constrained from below, because the household is fully credit constrained. More specifically, the lower bounds entail that the household cannot borrow at any time, neither from the financial institution (in cash) nor from informal sources (in cash or in livestock). Thus,

\[ \Delta D_t \geq -D_t \]

\[ \Delta L_t \geq -L_t \]

4.3.1.3 Transition functions

The equations that govern the evolution of the state variables over time, as a function of the dynamic choices that the household makes, are given by:

\[ D_{t+1} = (D_t + \Delta D_t) \cdot [1 + r(z')] \quad (4.1) \]

\[ L_{t+1} = g(L_t + \Delta L_t) \quad (4.2) \]

The stock of deposits in the next period is a function of the stock of deposits at the beginning of this period, the change in deposits as a result of the household’s choice, and the effective earnings from the interest rate that the deposit-taking institution pays on the new stock of deposits. In turn, the interest rate is modeled as a function of the systemic
shock, which takes place after the change in deposits has occurred and influences the amount of deposits available the following period.

The growth of livestock is a function that captures the reproduction of the initial stock of livestock and of the net additions to livestock during the period. This function will show diminishing marginal returns to herd size.

4.3.1.4 Reward function

The household’s reward function reflects the utility that it derives from consuming: \( u(c) \) is the utility function, where \( c \) is consumption.

Consumption is subject to a budget constraint, which shows that the household can consume from its labor income, from withdrawals from its initial stock of deposits, and from sales from its initial livestock holdings, at a price determined by the state of nature. To the extent to which the household decides to accumulate more precautionary wealth, it will have to reduce current consumption. Consumption is also reduced by the presence of the transaction costs that the household incurs each period in its dealings with the financial institution, if any. These costs are a function \( \tau(\Delta D) \) of the amounts deposited or withdrawn during the period.

More specifically,

\[
c_t = Y_t(z) - \Delta D_t - \Delta L_t \cdot P_t(z) - \tau_t(\Delta D_t)
\]

(4.3)

Next, the Bellman equation is formulated taking into account all of these elements.
4.3.2 Bellman equation

The Bellman equation, denoted by $V(D, L, z)$, is the maximum attainable sum of current and expected future utility over an infinite horizon, discounted at a per-period rate of $\rho > 0$. The household seeks a policy that prescribes the action that should be taken, contingent on the state, in order to maximize current and expected future rewards. The infinite horizon characteristic of the model makes the optimum solution to the dynamic optimization problem to be identical for every period and, thus, we can drop the time subscripts in the value function $V(.)$. Thus, the Bellman equation can be written as follows:

$$V(D, L, z) = \max_{\Delta D, \Delta L} \{u(c) + \delta E_z V[(D + \Delta D)(1 + r(z')), g(L + \Delta L), z']\} \quad (4.4)$$

where the maximization is carried out subject to:

$$\Delta D \geq -D \quad (4.5)$$
$$\Delta L \geq -L \quad (4.6)$$
$$c = Y(z) - \Delta D - \Delta L \cdot P(z) - \tau(\Delta D) \quad (4.7)$$

Here, $u$ denotes the agent’s utility function, $c$ denotes the agent’s consumption, $\tau$ denotes the agent’s deposit transaction costs, $g$ denotes the livestock growth (reproduction) function, $z'$ denotes next period’s systemic shock, namely the shock that occurs after this period’s decisions have been made, and $\delta = (1 + \rho)^{-1}$ denotes the per-period discount factor.

It is important to notice that net additions to deposits $\Delta D$ and to livestock holdings $\Delta L$ can be negative such that, in some periods, consumption can be greater than
labor income. This behavior would be consistent with the consumption smoothing objective.

I assume that the utility function \( u \) is continuously differentiable on \((0, \infty)\) with marginal utility nonnegative \( u' \geq 0 \) and \( u'(0) = \infty \); the livestock growth function \( g \) is continuously differentiable on \([0, \infty)\) with \( g \geq 0 \), \( g' \geq 0 \), and \( g(0) = 0 \); the transaction cost function \( \tau \) is continuously differentiable on \((-\infty, \infty)\) with \( \tau \geq 0 \), \( \tau' > -1 \), and \( \tau(0) = \tau'(0) = 0 \). In this model, transaction costs are proportional to transaction size rather than fixed, for convenience.

### 4.3.3 Euler conditions

The Euler conditions characterize equilibrium. They provide an intertemporal arbitrage interpretation so the researcher can understand the essential characteristics of the dynamic process.

The Euler conditions are obtained by applying the Karush-Kuhn Tucker conditions (4.8 and 4.9) and the Envelope Theorem (4.10 and 4.11) to the optimization problem that is contained in the Bellman equation.

\[
\mu_{\Delta D} = -u'(c) \cdot [1 + \tau'(\Delta D)] + \delta E_z \left[ \lambda_{D} (D', L') \cdot (1 + r(z')) \right] \tag{4.8}
\]

\[
\mu_{\Delta L} = -u'(c) \cdot P(z) + \delta E_z \left[ \lambda_{L} (D', L') \cdot g'(L + \Delta L) \right] \tag{4.9}
\]

\[
\lambda_{D} = \delta E_z \left[ \lambda_{D} (D', L') \cdot (1 + r(z')) \right] - \min(\mu_{\Delta D}, 0) \tag{4.10}
\]

\[
\lambda_{L} = \delta E_z \left[ \lambda_{L} (D', L') \cdot g'(L + \Delta L) \right] - \min(\mu_{\Delta L}, 0) \tag{4.11}
\]
where $\mu_{D\Delta}$ and $\mu_{L\Delta}$ capture the changes in the value function from net additions of deposits and livestock holdings, respectively. In turn, $\lambda_{D}$ and $\lambda_{L}$ are the shadow prices to the household of having one more dollar of deposits in the financial institution or holding one more unit of livestock in the barn, respectively.

If we take into account the possibility that the constraints would be binding, then a set of complementarity conditions needs to be defined, namely:

\[
\Delta D \geq -D \perp \delta E_{z} \left[ \lambda_{D}(D', L') \cdot (1 + r(z')) \right] \leq \lambda_{D}(D, L) \quad (4.12)
\]

\[
\Delta L \geq -L \perp \delta E_{z} \left[ \lambda_{L}(D', L') \cdot g'(L + \Delta L) \right] \leq \lambda_{L}(D, L) \quad (4.13)
\]

In the first case, the implication is that, either:

\[
\Delta D > -D \Rightarrow \delta E_{z} \left[ \lambda_{D}(D', L') \cdot (1 + r(z')) \right] = \lambda_{D}(D, L) \text{ since } \mu_{D\Delta} = 0, \text{ or} \quad (4.14)
\]

\[
\Delta D = -D \Rightarrow \delta E_{z} \left[ \lambda_{D}(D', L') \cdot (1 + r(z')) \right] < \lambda_{D}(D, L) \text{ since } \mu_{D\Delta} < 0 \quad (4.15)
\]

By symmetry, the same logic applies to equation (4.13). Thus, the Euler conditions simplify to:

\[
\Delta D \geq -D \perp \delta E_{z} \left[ \lambda_{D} \left[ (D + \Delta D)(1 + r(z')) \right] \cdot g(L + \Delta L); z' \right] \cdot (1 + r(z')) \leq \lambda_{D} \quad (4.16)
\]

\[
\Delta L \geq -L \perp \delta E_{z} \left[ \lambda_{L} \left[ (D + \Delta D)(1 + r(z')) \right] \cdot g(L + \Delta L); z' \right] \cdot g'(L + \Delta L) \leq \lambda_{L} \quad (4.17)
\]

\[
\lambda_{D} = u'(c) \cdot [1 + \tau'(\Delta D)] \quad (4.18)
\]

\[
\lambda_{L} = u'(c) \cdot P(z) \quad (4.19)
\]

\[
c = Y(z) - \Delta D - \Delta L \cdot P(z) - \tau(\Delta D) \quad (4.20)
\]
Equation 4.20 is the budget constraint. A nice interpretation of the equilibrium conditions along the optimal path can be derived from this set of equations (4.16 to 4.20), as follows.

4.3.4 Evolution along an optimal path

Along an optimal path, the following conditions should be met:

\[ \Delta D_t \geq -D_t \perp \delta E_t[\lambda_{D,t+1} \cdot (1 + r_{t+1})] \leq \lambda_{D,t} \]  (4.21)

\[ \Delta L_t \geq -L_t \perp \delta E_t[\lambda_{L,t+1} \cdot g_{t+1}^{'\prime}] \leq \lambda_{L,t} \]  (4.22)

\[ \lambda_{D,t} = u_{t}^{'\prime} \cdot (1 + \tau_{t}^{'\prime}) \]  (4.23)

\[ \lambda_{L,t} = u_{t}^{'\prime} \cdot P_t \]  (4.24)

where

\[ u_{t}^{'\prime} = u^{'}(Y_t - \Delta D_t - \Delta L_t \cdot P_t - \tau_t) \]  (4.25)

\[ \tau_t = \tau(\Delta D_t) \]  (4.26)

\[ \tau_{t}^{'\prime} = \tau(\Delta D_t) \]  (4.27)

\[ g_{t}^{'\prime} = g^{'}(L_t + \Delta L_t) \]  (4.28)

From equations (4.23) and (4.24), standard economic theory results for a static problem are obtained. The household will equalize its marginal utility per dollar spent in consumption to the marginal reward of a dollar either deposited in the financial institution, adjusted for a transaction cost, or used to purchase livestock.

By manipulating algebraically the set of equations (4.21 – 4.24), it follows that, along the optimal path:
The complementarity conditions imbedded in equations (4.29) and (4.30) can be interpreted as arbitrage conditions. Equation (4.29) implies that, for the household to deposit an additional dollar in the financial institution, the marginal valuation of that dollar in terms of consumption possibilities in the future (consumption protection) should be greater than the foregone marginal utility of shifting that dollar from consumption expenditure to deposits today. Equation (4.30) shows that, for the household to buy an extra unit of livestock, the marginal valuation of that unit in the future should be greater than the foregone marginal utility of shifting $P$ dollars from consumption expenditures to livestock today.

4.3.5 Certainty equivalent steady state

The steady state characterizes the long-run behavior of the controlled process. Stochastic models do not generate, however, a single dynamic path but numerous paths, each one resulting from the realization of a random shock. As a result, the steady state is a distribution. To facilitate the numerical solution of the model, however, it is convenient to start the approximation from values that emerge in the absence of shocks. Indeed, the certainty-equivalent steady state fixes the shock at its mean values, such that $P$, $Y$ and $r$ can be treated as constants. Although, for a stochastic model, the state and action variables will usually not converge to their certainty-equivalent steady state values, it is
key to compute these values in order to use them as initial guesses for the numerical approximation of the Bellman equation.

Assume $P > 0$, $Y > 0$, and $r > -1$. For this model, in the certainty equivalent steady-state, $D$, $L$, $\lambda_D$, and $\lambda_L$ are constant and satisfy:

\[
\Delta D \geq -D \downarrow \delta \lambda_D (1 + r) \leq \lambda_D \tag{4.31}
\]

\[
\Delta L \geq -L \downarrow \delta \lambda_L \cdot g'(L + \Delta L) \leq \lambda_L \tag{4.32}
\]

\[
\lambda_D = u'(c) \cdot [1 + r'(\Delta D)] \tag{4.33}
\]

\[
\lambda_L = u'(c) \cdot P \tag{4.34}
\]

\[
g(L + \Delta L) = L \tag{4.35}
\]

\[
D = (D + \Delta D) \cdot (1 + r) \tag{4.36}
\]

\[
c = Y(z) - \Delta D - \Delta L \cdot P(z) - \tau(\Delta D) \tag{4.37}
\]

Equations (4.31) to (4.34) are the Euler conditions, as previously derived. Equations (4.35) and (4.36) are the transition functions (laws of motion) for the state variables, $D$ and $L$, and equation (4.37) is the budget constraint. Since $P > 0$, $u' > 0$ and $\tau > -1$, it follows that $\lambda_D > 0$ and $\lambda_L > 0$, so the certainty-equivalent steady state conditions simplify to:

\[
\Delta D \geq -D \downarrow r \leq \rho \tag{4.38}
\]

\[
\Delta L \geq -L \downarrow g'(L + \Delta L) \leq 1 + \rho \tag{4.39}
\]

\[
g(L + \Delta L) = L \tag{4.40}
\]

\[
D = (D + \Delta D) \cdot (1 + r) \tag{4.41}
\]
\[ c = Y(z) - \Delta D - \Delta L \cdot P(z) - \tau(\Delta D) \] (4.42)

From equation (4.38), it is clear that a steady state obtains only if \( r \leq \rho \). Let’s ignore the hairline case and assume that \( r < \rho \). Thus, a corner solution is observed for deposits. Then, in the certainty-equivalent steady-state,

\[ D = \Delta D = 0 \] (4.43)

\[ \Delta L \geq -L \perp g'(L + \Delta L) \leq 1 + \rho \] (4.44)

\[ g(L + \Delta L) = L \] (4.45)

\[ c = Y - \Delta L \cdot P \] (4.46)

To guarantee a reasonable steady-state \( L \), it is sufficient to assume that there is an \( L' \) such that \( g^{-1}(L') > L' > 0 \) and \( g'(L') = 1 + \rho \), in which case the steady state \( L \) is \( g^{-1}(L') \).

Let’s summarize the assumptions that must be made to ensure that the certainty equivalent steady-state is well defined: (i) \( u \) is continuously differentiable on \((0, \infty)\) with \( u' \geq 0 \) and \( u'(0) = \infty \); (ii) \( g \) is continuously differentiable on \([0, \infty)\) with \( g \geq 0 \), \( g' \geq 0 \), and \( g(0) = 0 \); (iii) \( \tau \geq 0 \) is continuously differentiable on \((-\infty, \infty)\) with \( \tau \geq 0 \), \( \tau' < 0 \), and \( \tau(0) = \tau'(0) = 0 \); (iv) \( P > 0 \), \( Y > 0 \), and \( \rho > r > -1 \); and (v) there exists \( L' \) so that

\( g^{-1}(L') > L' > 0 \) and \( g'(L') = 1 + \rho \). If these assumptions hold, the steady state deposit and livestock holdings are \( D' = 0 \) and \( L' = g^{-1}(L') \).
4.3.6 Functional forms

The household has a constant relative risk aversion utility function (CRRA), which depends on the level of consumption $c$ and the relative risk aversion coefficient $\gamma$. The CRRA utility function exhibits convex marginal utility, which indicates the existence of a precautionary savings motive (figure 4.1 at the end of this chapter). The constant relative risk aversion characteristic of this type of utility functions implies that, as wealth increases, the household’s aversion to risk remains constant and is equal to $\gamma$. The coefficient of relative prudence is also constant and is equal to $\gamma+1$.

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma \neq 1$$  \hfill (4.47)

For tractability, the transaction cost function is assumed to be continuous and differentiable. Transaction costs increase with the size of the transaction (either a deposit or a withdrawal of funds) in response to considerations such as safety. In this case, the greater the amount of the deposit/withdrawal, the greater the risk associated with the transaction and, therefore, the higher the implicit premium for risk (or security cost) as a component of transaction costs, in addition to a fixed cost that is assumed here to be zero, such that only the variable component of the more complex function is considered. This assumption is made for convenience. Transaction costs may take other relationships with respect to the size of the deposit and they will still have a similar impact on portfolio choices. The transaction cost function represented in figure 4.2 is given by:

$$\tau = \alpha \cdot (\Delta D)^2, \quad \alpha > 0$$  \hfill (4.48)
The livestock growth (reproduction) function exhibits diminishing marginal returns, which result from the assumption of a fixed area of land and amount of labor for taking care of the herd (figure 4.3). The adopted functional form shows that livestock grows at a decreasing rate, within the range $L + \Delta L < 1$. It also shows that, for values of livestock $L + \Delta L > 1$, the herd starts to decline as mortality increases, due to the limited resources the household has in terms of food, labor and space for taking care of the animals. In other words, the rate of growth of livestock is positive for values of livestock that are less than one and negative otherwise.

$$g = (L + \Delta L)^\beta \quad 0 < \beta < 1$$  \hspace{1cm} (4.49)

Once the certainty-equivalent steady state is characterized and the functional forms are defined, the certainty equivalent steady state values can be easily computed.

4.3.7 Certainty-equivalent steady-state values

The system of equations has an analytical solution for the certainty-equivalent steady state that is characterized by the following values

$$D^* = \Delta D^* = 0$$  \hspace{1cm} (4.50)

$$L^* = \left(1 + \frac{\rho}{\beta}\right)^{\beta}$$  \hspace{1cm} (4.51)

$$\Delta L^* = \left(1 + \frac{\rho}{\beta}\right)^{-\beta} \cdot \left[1 - \left(\frac{1 + \rho}{\beta}\right)^{1 - \beta}\right]$$  \hspace{1cm} (4.52)

$$c^* = Y^* - \Delta L^* \cdot P$$  \hspace{1cm} (4.53)
From equation (4.50) it is clear that, in the certainty-equivalent steady state, the household does not hold any deposits in the financial institution. This is because, in the absence of risk, the household’s intertemporal consumption/wealth accumulation decisions do not respond to risk-return considerations. If there is no risk, the motivation for diversifying the portfolio of wealth is absent. In these circumstances, there is no precautionary motive and the household accumulates wealth only in order to optimize its intertemporal consumption. In the absence of risk, the household will only keep that asset that yields the highest rate of return, since this strategy will increase its consumption possibilities in the future more.

This wealth accumulation strategy holds as long as the rate of return on the asset is greater than the subjective discount factor. Therefore, portfolio decisions are solely driven by the rates of return and, by assumption, the rate of return on deposits is set lower than the rate of return on livestock (for low levels of holdings of the livestock). This result is compatible with portfolio theory. The household accumulates wealth only in the form of livestock, since this is, in the model, the asset that yields the greatest return. Neither the variability nor the correlation of returns are taken into account.

As a result, the holdings of livestock in the steady state are positive, and they are a function of the discount factor $\rho$ and of the livestock growth function parameter $\beta$.  

\[
\lambda^*_d = \frac{1}{(Y - \Delta L \cdot P)^\gamma} \quad (4.54)
\]

\[
\lambda^*_c = \frac{P}{(Y - \Delta L \cdot P)^\gamma} \quad (4.55)
\]
Livestock holdings and the discount factor are inversely related, \( \frac{\partial L}{\partial \rho} < 0 \). Thus, the more impatient the household is, the lower its livestock holdings will be. The relationship between livestock holdings \( L \) and the growth parameter \( \beta \), is ambiguous, as higher rates of return will make it attractive to hold more livestock (substitution effect) and, at the same time, a more rapid accumulation of the stock (wealth effect) will discourage further accumulation.

In the steady state, net additions to livestock holdings (4.52) are always negative. This result has to do with the nature of steady state, where the system is at rest because economic agents do not have incentives to change their behavior over time. The household consumes all the returns to its livestock, so livestock holdings remain constant over time. All labor income is also spent in consumption (4.53).

The shadow prices of deposits (4.54) and of livestock (4.55) are equal to the marginal utility the household derives per dollar spent in consumption.

The solution to the dynamic model is presented next.

### 4.4 Solution to the model

The solutions to the deterministic and stochastic versions of the dynamic model are computed by using numerical approximation techniques.\(^7\) More specifically, the collocation method is used to solve for the Bellman equation. The code that solves the

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\(^7\) A complete explanation of the approximation techniques that were used to solve the model can be found in the book *Applied Computational Economics and Finance* by Miranda and Fackler (2002).
model was run in MatLab 6.1 and it is attached as Appendix B.

The solution for each model is characterized by two sets of results. The first set includes the optimal choices (i.e., the optimal net additions to deposits and to livestock holdings), the value function, and the shadow price functions for deposits and livestock. Regarding the first of these outcomes, it is worthwhile to keep in mind that the optimal choices are contingent on the state values. In turn, the second set of results offers a dynamic path analysis, which shows how the variables of the system evolve over time — starting from a given initial condition.

**4.4.1 Solution of the deterministic version of the model**

The solution of the deterministic version of the model is computed after assuming the following set of parameter values, which reflect either magnitudes established in the literature or stylized facts about developing countries: a relative risk aversion coefficient \( \gamma = 0.5 \), a transaction cost function parameter \( \alpha = 1.0 \), a livestock growth function parameter \( \beta = 0.8 \), a per period interest rate on deposits \( r = 0.03 \), a price of livestock \( p = 0.65 \), labor income \( y = 1.0 \), and a discount rate of \( \rho = 0.05 \).

The large difference between the rate of reproduction of the livestock (implicit in the corresponding coefficient) and the rate of interest earned on deposits describes an environment of fragmentation where, in the presence of severe credit constraints, one asset generates high rates of return and the other one generates low rates of return, even in the absence of risk. In this fragmented environment, the arbitrage that would equate these rates of return is not possible.
Moreover, in the deterministic model, the two types of assets are held only in response to their relative returns and as a tool for the intertemporal optimization of consumption. A precautionary motive for holding wealth does not exist in this environment. Next, I describe and explain the first set of outcomes, for this deterministic version of the model.

The choice related to the household’s optimal net additions to deposits indicates that, ceteris paribus, the larger the stock of deposits that the household has in the financial institution, the more it will rely on withdrawals of these funds in order to increase current consumption (figure 4.4). This result stems from the fact that the shadow price of deposits decreases with the amount of deposit holdings; that is, the household’s marginal valuation of holding an additional dollar of deposits diminishes as the stock of deposits increases and as current consumption looks increasingly more valuable than the future consumption that these extra deposits would allow. Thus, the larger the stock of deposits, the less costly it becomes to increase current consumption by withdrawing money from the financial institution.

In contrast, if livestock holdings are high compared to deposits, the optimal choice is to rely less on deposit withdrawals for consumption or even to add to deposits. The interesting case shows up when the household holds fairly high levels of livestock but not much in deposits. In this case, despite the absence of risk, the household’s optimal choice is to increase the stock of deposits in the financial institution.

There are two reasons that explain this outcome. The first one is the presence of decreasing marginal returns on livestock, which may even become negative (due to
mortality). This reduces the incentives for holding additional animals, as livestock holdings get larger. Then, the optimal policy becomes to increase the stock of deposits instead, as their rate of return, however low, would look comparatively more attractive in these circumstances. Second, the shadow price of livestock declines with the amount of livestock holdings; that is, the marginal valuation of having one more unit of livestock decreases as livestock increases, as was also the case for wealth held as deposits.

Actually, under the values of the parameters assumed for the exercise, the choice of optimal net additions to livestock holdings results in that, regardless of the level of deposits, livestock is sold most of the time (figure 4.5). Additions to livestock holdings only take place when this stock reaches comparatively low levels.

The interpretation of the value function is straightforward; the household’s welfare is higher, the more deposits and livestock holdings there are (figure 4.6). The household derives its utility from consumption, and the accumulation of wealth increases consumption possibilities into the future. Under the parameter values used for the exercise, the rewards for the household increase more rapidly as deposits increase, compared to increases in livestock holdings. This reflects the fact that, while deposits offer a constant return under certainty, livestock exhibits diminishing marginal returns.

The shadow price of deposits (measured as the increase in the current and discounted expected future rewards the household derives from consumption when holding an additional monetary unit of deposits), decreases as the stock of deposits increases (figure 4.7). This reflects the declining marginal utility of consumption. Indeed, the shadow price of deposits is high when the amount of wealth that is held in the
form of both, livestock and deposits, is low. This result reveals that the accumulation of wealth, for whatever motive, is comparatively more “costly” for a poorer household, because it has to give up consumption that carries a higher marginal utility.

The shadow price of livestock follows the same logic: the smaller the livestock holdings, the higher their shadow price (figure 4.8). There are two reasons for this. In addition to the declining marginal utility of consumption, as wealth increases, the decreasing marginal returns associated with larger holdings of livestock make them less and less attractive for the household as an asset.

The numerical approximation technique used to solve the model requires a check of the accuracy of the approximants, which is given by the residuals. In this particular case, the residuals are in the order of $10^{-4}$ around zero, which means that the approximation is quite accurate (figure 4.9). Moreover, one of the most important features of this solution of the model is that I can liberally expand the range of the solution around the steady state. This suggests that the model is quite robust.

Once the model is solved numerically, a dynamic path analysis can be performed by using the \textit{dpsimul} routine contained in the CompEcon Toolbox. This analysis produces expected choices and state paths over time. The simulation was conducted over a 20-year horizon.

The simulations implemented with the deterministic model provide the following results. The expected deposits path (figure 4.10) and the net additions to deposits path (figure 4.11) remain at their steady state value, which is zero, for 20 years. In the absence of risk, the household chooses to hold the asset that offers the highest rate of
As the amount of livestock in the steady state is equal to 0.34, the household prefers to hold livestock, because the per period returns that it can obtain from these holdings (0.24) are higher than the returns from deposits (0.03). Once risk is introduced, as in the next section, a different expected deposits path should be expected, as in this other case the household would have incentives for accumulating precautionary wealth in the form of deposits, even if its returns are lower, in order to diversify its portfolio.

The expected net additions to livestock holdings depicted in figure 4.13 show that the household rapidly reaches the steady state value for these purchases, which is negative. On the one hand, the household sells some livestock each period; on the other hand, the herd expands even more rapidly, building the herd up to its steady state value. In steady state, these two effects exactly offset each other, and the holdings of livestock remain constant.

Under the values assumed for the exercise, the expected livestock holdings achieve their steady-state value in a four year time spam (figure 4.12). The assumption of diminishing marginal returns in the expansion of the herd reflects the typical limitations of a poor rural household. Indeed, it is easier to take care of a smaller herd, given fixed land and labor inputs, and more food would be available for each animal in this case. Thus, a faster rate of growth of livestock can be expected when the herd is smaller.

One of the most interesting advantages of numerical methods is that they provide the elements to see how the variables change when uncertainty is introduced in the model. The solution to the stochastic version comes next.
4.4.2 Solution to the stochastic version of the model

In the solution to the stochastic model, risk is introduced as a negative shock that affects both labor income and the price of livestock, while financial deposits are kept as a risk-free asset. The riskiness of deposits will be introduced later on, for the simulations. This way of introducing the shock has two purposes. First, this approach makes it easier to understand why some magnitudes change when risk is present. Second, the characteristics of this model are similar to those of the modified version of Samuelson’s portfolio analysis presented in chapter 3, where there is one safe asset. These assumptions facilitate the comparison of the results about the importance of transaction costs in the household’s precautionary portfolio decisions. This comparison and some relevant simulations are performed in chapter five.

Risk emerges from the existence of two states of nature, which lead to a good and a bad outcome, with some probabilities. The solution to the stochastic version of the dynamic model is thus computed assuming the following set of parameter values: relative risk aversion coefficient $\gamma = 0.5$, transaction cost function parameter $\alpha = 1.0$, livestock growth function parameter, $\beta = 0.8$, interest rate on deposits $r = 0.03$, price of livestock in the good state $p_G = 0.65$, price of livestock in the bad state $p_B = 0.40$, labor income in the good state $y_G = 1.0$, income labor in the bad state $y_B = 0.5$, and a discount rate of $\rho = 0.05$. The two states of nature have the same probabilities of occurring, $\kappa_G = \kappa_B = 0.5$. Thus, all parameters are the same as in the deterministic version of the model, except that now there are values for labor income and for the price of livestock that depend on the state of nature.
Under these values, the negative shock reduces income by 50 percent, a reduction that is then transmitted, through the collective choices to sell animals in the village, into a reduction of the price of livestock of 38 percent. On the one hand, the income shock is not fully transmitted into the price of livestock, as in actual situations there may be other strategies adopted to protect consumption, so the local households do not have to fully rely on livestock sales to compensate for the income drop.

On the other hand, the change in price will also depend on the elasticity of the local demand of livestock. In an extreme case, in a fully integrated market, the households in the village would not influence the national price of livestock. In this case, they would be price takers, and the shock would not cause a price drop. In a fragmented economy, however, the local households do not have access to livestock markets beyond their own village, which has been hit by the systemic shock. In this case, the impact on price of the comparatively massive local sales of animals will be substantial. In either case, nevertheless, the price of livestock will not be affected as much as the reduction in income. The assumptions of the model reflect this stylized fact of the rural areas of developing countries.

The income-cum-livestock price shock induces several changes in the model. The possibility of comparing them with the benchmark case (the deterministic model) greatly facilitates the interpretation of these results. Indeed, the presence of risk produces the following changes, compared to the deterministic model.

First, the choice related to optimal additions to deposits shows that, if the household holds low levels of livestock and deposits, its optimal policy will be to
withdraw less money from the financial institution than if there is no risk (figure 4.14). The reason for this outcome is that the marginal valuation of holding financial deposits (as part of its precautionary wealth) is high when there is risk and when both deposits and livestock holdings are low. At low levels of wealth, the household prefers to sacrifice some current consumption rather than to further deplete a stock of riskless wealth that is critical for the protection of future consumption.

The most relevant change the stochastic model delivers is on the expected deposits path (figure 4.15). Under a deterministic model, the holdings of deposits evolve towards their zero steady-state value. In the stochastic model, the simulation of this path for a 40-year horizon shows that expected deposits evolve over time to a positive level and remain relatively constant over time after the first ten years. The stock of deposits is stabilized around 10 percent of the expected per period labor income (figure 4.16).

Another interesting result is that the path for expected livestock holdings depicted in figure 4.17 shows an increase of livestock holdings from 33 percent of per period labor income (which in the deterministic model is fixed) to 46 percent of expected labor income, as a result of the presence of risk.

These changes in the dynamic paths illustrate two relevant issues. First, when risk is present, as portfolio theory predicts and as it was found in the modified Samuelson’s portfolio problem (solved in chapter 3), the household diversifies its wealth between the risky and the risk-free asset. Moreover, the total level of precautionary wealth held increases when uncertainty is introduced.
Indeed, in the presence of risk, precautionary wealth serves the additional purpose of smoothing consumption under different states of nature. For this reason, the value to the household of its precautionary wealth holdings increases and the desired stock becomes larger. Moreover, as the price of livestock is not immune to the systemic shock, the household has to keep even larger holdings of precautionary wealth (compared to a situation where the covariance is absent), as it expects to suffer a capital loss in the case of a bad state of nature. This is equivalent to the presence of a melting effect when the shock strikes.

In addition to this level effect, there is a composition effect, as the precautionary wealth portfolio is diversified between the two assets and away from the livestock only solution of the deterministic model. The diversification strategy favors increases in deposits (the risk-free asset) in a greater proportion than the increases in livestock holdings (the risky asset). This composition effect is further induced by the correlation between the labor income shock and the price of livestock and by the absence of any covariance between these two variables and the rate of return on deposits in the model.

In the next chapter, I perform a number of simulations to explore how the results of the dynamic stochastic model change with changes in the values of the parameters, including the influence of the shock on the net return on deposits adjusted for the risk of bankruptcy of the financial institution.
Figure 4.1 Constant relative risk aversion (CRRA) utility function for different values of the risk aversion coefficient (gamma).
Figure 4.2 Livestock growth transition function for different values of the exponent parameter (beta).
Figure 4.3 Transaction cost function for different values of the parameter alpha.
Figure 4.4 Optimal net additions to deposit choices (deterministic).
Figure 4.5 Optimal net additions to livestock choices (deterministic).

Model parameters
- \( \gamma = 0.5; \) Relative risk aversion coefficient
- \( \alpha = 1.0; \) Transaction cost function parameter
- \( \beta = 0.8; \) Livestock growth function parameter
- \( r = 0.03; \) Interest rate on deposits
- \( p = 0.65; \) Livestock price
- \( y = 1.0; \) Income
- \( \rho = 0.05; \) Discount rate
Figure 4.6 Value of the household – welfare (deterministic).
Figure 4.7 Shadow price of deposits (deterministic).
Figure 4.8 Shadow price of livestock (deterministic).
Figure 4.9 Approximation residuals (deterministic).
Figure 4.10 Expected deposits path (deterministic).
Figure 4.11 Expected net additions to deposits path (deterministic).
Figure 4.12 Expected livestock holdings path (deterministic).
Figure 4.13 Expected net additions to livestock path (deterministic).
STOCHASTIC VERSION OF THE MODEL:
INCOME AND PRICE RISK, AND RISK-FREE DEPOSITS

Figure 4.14 Optimal net additions to deposit choices (stochastic).

Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>gamma</td>
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<td>Relative risk aversion coefficient</td>
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<tr>
<td>alpha</td>
<td>1.0</td>
<td>Transaction cost function parameter</td>
</tr>
<tr>
<td>beta</td>
<td>0.8</td>
<td>Livestock growth function parameter</td>
</tr>
<tr>
<td>r</td>
<td>[0.03 0.03]</td>
<td>Interest rate on deposits for each state of nature</td>
</tr>
<tr>
<td>p</td>
<td>[0.4 0.65]</td>
<td>Livestock price for each state of nature</td>
</tr>
<tr>
<td>y</td>
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<td>Income for each state of nature</td>
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<tr>
<td>w</td>
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<td>Shock probabilities</td>
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<tr>
<td>rho</td>
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<td>Discount rate</td>
</tr>
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</table>
Figure 4.15 Shadow price of deposits (stochastic).
Figure 4.16 Expected deposits path (stochastic).
Figure 4.17 Expected livestock holdings path (stochastic).
Figure 4.18 Expected net additions to deposits path (stochastic).
Figure 4.19 Expected net additions to livestock holdings path (stochastic).
5.1 Introduction

In this chapter, I study how the dynamic stochastic model reacts to changes in key parameter values. I focus on changes in the optimal choices, the shadow prices of state variables, the value function, and the intertemporal actions and states paths corresponding to different parameter values.

The simulation analysis is performed by building on a baseline case, which rests on a choice of a set of parameters that are, on the one hand, consistent with the existing literature and that, on the other hand, take values that make the simulations possible within the bounds of the solution. The set of parameter values chosen is: a relative risk aversion coefficient $\gamma = 0.5$, a transaction cost function parameter $\alpha = 1.0$, a livestock growth function parameter, $\beta = 0.8$, a per period interest rate on deposits $r = 0.04$ (this is the interest rate paid on deposits in the rural areas of Bolivia), price of livestock in the good state $p_G = 0.8$, price of livestock in the bad state $p_B = 0.5$, labor income in the good state $y_G = 1.8$, labor income in the bad state $y_B = 0.9$, and a discount rate of $\rho = 0.07$. The two states of nature have different probabilities of occurring, $\kappa_G = 0.9$.
and $\kappa_b = 0.1$. This is a conservative scenario, given the high vulnerability to local systemic shocks observed in the rural areas of developing countries (e.g., floods in Bangladesh every three years). The simulation is implemented over a 40-year span of time.

The analysis is undertaken by using the *ceteris paribus* criterion – according to this approach, changes in parameters are explored one at a time. This makes the correct identification of the source of variation in each case possible. The following simulations are performed:

- Changes in the costs of transacting with the financial institution, $\tau$.
- Variations in the effective interest rate, $r$, where negative values reflect the potential loss of some of the principal of the deposit, under bankruptcy of the financial institution.
- Changes in the degree of riskiness of the environment, which are captured by the probability of each state of nature, $\alpha$.
- Changes in the degree of risk aversion (prudence) of the household, $\gamma$.
- Variations in the discount rate, to reflect changes in impatience, $\rho$.

Special attention is paid to the first simulation, as changes in precautionary wealth choices produced by a reduction in transaction costs, which can be influenced by policy, are one of the central concerns of this dissertation. Moreover, the simulation exercises allow me to draw an expected consumption path, which is critical for analyzing the influence of increased access to deposit facilities on consumption smoothing.
As an experiment, and after all the simulations are completed, I create an artificial scenario, in which different parameters are simultaneously changed in a way that encourages the household to maintain precautionary wealth in the form of deposits. The goal of this experiment is to observe links between the holding of a greater share of precautionary wealth as deposits and the corresponding consumption smoothing outcome.

The resulting greater ability to smooth consumption, at a lower cost, represents an improvement in the household’s private welfare. The share of precautionary wealth that is held as deposits also matters from the perspective of social welfare, given the consequences of financial intermediation on the overall allocation of resources. While this positive impact of financial deepening on the productivity of resources is a critical motivation for this dissertation, it is a topic that is beyond its scope.

5.2 Simulation 1: Changes in transaction costs

A first simulation consists in evaluating how the results of the dynamic model change when transaction costs vary. Three scenarios are studied, for a lower transaction cost $\tau_L = 0.5$, a higher cost $\tau_H = 1.5$, and one where the transaction cost is equal to the value for the baseline case, $\tau_B = 1$.

In order to analyze the impacts of a change in the cost of financial transactions, it is worthwhile to take a step back and revisit some fundamental features of this version of the model. The household faces uncertainty, in that there is a systemic adverse shock with low probability of occurrence (there is a 90 percent probability of the good state, in
In order to cope with risk, the household chooses a combination of two different assets. One is livestock, which is risky—because of the transmission of the negative income shock on its price—but which has no transaction costs associated to its liquidation in the local market. The other one is a deposit in a financial institution, which for now is a risk-free asset but for which there is a cost of every transaction of depositing/withdrawing money, in each period. The assumption of risk-free deposits is relaxed when the second simulation is performed.

The probability of occurrence of the adverse shock determines the portfolio structure of the household. The higher the chance that the adverse shock will take place, the more efficient it will be to use the risk-free asset as a consumption smoothing strategy, even in the presence of any transaction costs that may be associated to it. The lower the chance that the adverse shock will take place, the less efficient it will be to accumulate the riskless asset and the more relevant any transaction costs associated to that asset will become.

Therefore, given the low probability of occurrence of the adverse shock—considered in this version of the model—it follows that the household will end up with a precautionary wealth portfolio that has a higher share accumulated in the form of the risky but transaction cost-free asset (livestock) and a lower share on the riskless and costly asset (deposits).

These shares are the benchmark for understanding the impacts (if any) of a reduction in the transaction cost of deposits on the structure of the precautionary wealth
portfolio and on the degree of consumption smoothing that may be achieved by the household.

The reduction of the transaction cost of deposits in the financial institution makes them comparatively more attractive than otherwise. This induces a substitution effect (the household will accumulate a larger share of its wealth in the asset that is becoming relatively more attractive) and a wealth effect (the increased wealth that results from the reduction in the cost of holding this asset allows the household to accumulate more of both assets). By the substitution effect, the household will sell livestock and deposit the proceedings in the bank. By the wealth effect, the household will increase both its deposits and its livestock holdings. Thus, there is no doubt that deposit holdings will increase as the transaction cost shrinks, as shown by Figure 5.1 at the end of this chapter. In the case of deposits, both effects have the same sign. Moreover, the figure shows larger increases in deposit holdings when transaction costs decline toward the lower level (from 1 to 0.5, from the baseline) than when these costs are still high after the decline towards the baseline (from 1.5 to 1).

What happens to the investment in livestock depends, however, on the balance between the substitution and the wealth effects. Given the numerical assumptions for this case, though, the initial relatively small share of deposits in the portfolio implies that any changes in the price of deposits will have a moderate impact on the household’s total wealth and, therefore, that the wealth effect can be expected to be relatively modest. In fact, this is confirmed by Figure 5.2, where a 34 percent reduction in transaction costs (i.e., from 1.5 to 1) produces almost no change in the long-run level of livestock holdings.
(i.e., the wealth effect merely cancels out the substitution effect). It takes a more considerable reduction in the transaction cost of deposits (67 percent of the higher value of 1.5, namely, from 1.5 to 0.5) for the wealth effect to start dominating the substitution effect, thus inducing an increase in the long-run investment in livestock.

Figure 5.3 shows an interesting result. The optimal additions to deposits are positive and higher during the first 6 years and then negative and lower for the next 34 years (both with respect to the path observed for the base line and the higher values of the transaction cost), particularly after a 67 percent reduction in this transaction cost from its higher level. This behavior is consistent with the optimal deposits path depicted in Figure 5.1, since the household tends to accumulate most of its deposit holdings precisely during the first six years. Thus, if the accumulation pattern is higher during those first six years, it follows that the withdrawal path will also be more pronounced for the following 34 years. That is, in the presence of a lower transaction cost, the adjustment of the household’s share of deposits in precautionary wealth takes place more rapidly.

The last element to consider is that, since in this version of the model the asset whose price is declining has a minor role (defined by the smaller investment share) in the consumption smoothing strategy of the household, one would expect that this reduction in cost should not imply an important improvement in the household’s consumption smoothing performance. Actually, by looking at Figure 5.4, it is possible to confirm that the 67 percent reduction of the transaction cost parameter (from 1.5 to 0.5) has a positive impact in reducing the range of variation of consumption along the 40-year expected path. This is shown by the positive relationship that exists between the transaction cost
and the variance of consumption. For the lower transaction cost parameter $\alpha = 0.5$, the variance of consumption is equal to $1.039 \times 10^{-4}$, whereas the variance for the higher $\alpha = 1.5$ is $1.726 \times 10^{-4}$, while the variance for the benchmark case $\alpha = 1.0$ is $1.574 \times 10^{-4}$ (table 5.1).

Therefore, a clear link can be established between reductions in depositor transaction costs, increases in the level of precautionary wealth, a greater share of deposits in this wealth, and better consumption protection. In general, the impact of transaction costs on consumption smoothing could be expected to be larger, the higher the probability of occurrence of the adverse shock is.

5.3 Simulation 2: Changes on the risk of bankruptcy of the financial institution

A second simulation explores how changes in the effective interest rate, adjusted for the risk of bankruptcy, affect the optimal deposit choice and consumption path. The outcomes under the risk-free deposit interest rate $r_B = 0.04$ are compared to the case where the systemic shock has an impact on the interest rate, which reduces this expected return to $r_R = -0.1$. This new interest rate anticipates a loss of 10 percent of the principal of the deposit when the negative shock realizes.

The goal of this simulation is to capture the effects that a systemic shock may have on consumption smoothing, through the financial system channel. The premise is that, if a financial institution is able to break the covariance among labor income, the price of livestock, and the net returns on deposits, adjusted for risk, access to deposit facilities may have a positive effect in helping the household to buffer consumption from
income shocks. On the other hand, the operation of fragile financial institutions, vulnerable to local systemic shocks, may not have this power.

The covariance between labor income and the adjusted returns on the deposit arises from the fact that a negative income shock undermines the ability of those local households that have loans (different from the fully credit-constrained household considered in the model) to repay their debt. This, in turn, undermines the soundness and solvency of the financial institution, depending on how diversified its loan portfolio is and on what level of provisions for bad loans it has accumulated. A large deposit-taking institution, with a diversified loan portfolio over a space that goes beyond the local circumstances, will be less vulnerable to local systemic shocks, compared to a local financial institution, like a cooperative or a village bank. By the same token, a well-regulated financial institution will be required to hold bad-loan reserves commensurate with the level of risk in its portfolio and it will have access to a lender of last resort, to deal with liquidity shocks. Thus, the structure of the rural financial market, in terms of the types of institutions found, has consequences on consumption smoothing through this systemic shock impact on the probability of bankruptcy.

The possibility of bankruptcy of the financial institution implies that, ceteris paribus, attaining consumption protection has become more difficult for the household. Figure 5.5 shows that, when the deposit-taking financial institution is affected by the shock (10 percent reduction in the principal of its deposits), the household prefers to hold a smaller amount of deposits. The combination of transaction costs and risk has reduced, in this case, the relative attractiveness of deposits.
An interesting result is that, in the presence of a moderate interest rate (bankruptcy) risk, when a 10 percent of the deposits are lost when a negative shock strikes, the household still diversifies its precautionary wealth portfolio, by holding a share of 1.4 percent of its precautionary wealth in the form of deposits.

For the two scenarios discussed above, the expected consumption paths were generated and are depicted in Figure 5.6. The computation of the variance for each consumption path (table 5.2), used here as a rudimentary measure of the degree of consumption smoothing, reveals that consumption is smoother when deposits are risk-free and less smooth when the impact of bankruptcy risk exists. The corresponding variances are $1.560 \times 10^{-4}$, and $2.343 \times 10^{-4}$, respectively. The share of deposits in the precautionary wealth portfolio is almost negligible (1.4 percent), so deposits almost do not have any impact on consumption smoothing.

Two important implications can be derived from these results. First, households seem to be very sensitive to the risk of bankruptcy of the financial institution, which can be observed in a substantial reduction in deposits for $r_L = -0.1$ (around 86 percent). Second, local financial institutions, which are by nature more vulnerable to systemic shocks, may offer less attractive options for the households’ efforts to smooth their consumption. This has important implications for the design of rural financial systems.

5.4 Simulation 3: Changes in the degree of riskiness in the environment

The third simulation is about the consequences of the relative riskiness of the environment. This is accomplished by changing the frequency of the systemic shocks, by
altering the probability of occurrence \( \kappa \). Two scenarios are analyzed: a low risk scenario, in which the probability of an adverse event is 10 percent (benchmark), and a high risk scenario, in which this probability is 50 percent.

The results of the dynamic model show that, when a household lives in a risky environment, it will accumulate more precautionary wealth compared to a household that lives in a place that is less exposed to systemic shocks. Thus, a level effect is observed (figures 5.7 and 5.8). The composition of the portfolio will be different as well (table 5.3). In a riskier environment, households will have a greater share of precautionary wealth in the form of deposits, as this type of asset provides greater protection (risk-free asset). In other words, the riskier the environment where rural households live, the more critical will deposit facilities be in their consumption smoothing strategies.

Unfortunately, however, the riskier environments of poorer developing countries tend to be accompanied by much lower levels of financial deepening and, particularly, by a strong neglect of an appropriate supply of deposit facilities.

Not only the expected consumption path is more stable under the low risk scenario, but the possibilities of consumption are also greater than in the risky scenario (figure 5.9). Thus, the volatility of consumption, measured as its variance, for the low risk scenario is \( 1.3715 \times 10^{-4} \) while for the risky scenario it is equal to \( 2.1498 \times 10^{-4} \). The challenge of consumption protection does indeed vary much across different environments. This heterogeneity also influences the different value (cost-benefits) that the development of rural financial facilities will have.
Second, under the low-risk scenario, the household has to sacrifice less current consumption in order to keep a precautionary wealth that protects it from bad states of nature. Thus, the opportunity cost of holding this wealth is less where there is less risk. The household’s welfare will increase both from this higher consumption flows and from its greater ability to avoid sharp reductions in consumption when the bad states prevail.

5.5 Simulation 4: Changes on risk aversion

The relative risk aversion of the household is an important determinant of the effect that precautionary wealth has on expected consumption paths. Thus, an increase in risk aversion increases the motivation for a household to accumulate precautionary wealth and this, in turn, increases expected consumption growth by decreasing current consumption and increasing savings (Romer, 2001).

The stochastic dynamic model developed in this dissertation uses a relative risk aversion coefficient $\gamma$ equal to 0.5, which is consistent with the value that Gourinchas and Parker (2002) estimated using US household-level data from the Consumer Expenditure Survey (0.51). Most likely, this coefficient would be higher for the poorer and more vulnerable rural populations of developing countries.

It can be expected that, as risk aversion increases, the level of precautionary wealth will increase, which can only be accomplished at the expense of current consumption. Thus, on the one hand, the accumulation of more precautionary wealth to respond to the increased risk aversion implies a higher opportunity cost in terms of foregone current consumption while, on the other hand, more precautionary wealth
implies greater protection against negative income shocks in the future and, therefore, smoother consumption.

The simulation is conducted for two different levels of risk aversion, $\gamma = 0.50$ and $\gamma = 0.99$. Although the increase in the risk aversion coefficient is not as much as it could have been (the literature indicates a maximum value equal to 4.0), the simulation provides some interesting insights in this regard.

First, as economic theory predicts, the more risk averse the household is, the greater the accumulation of precautionary wealth will be. In this model, the higher risk aversion coefficient produces an increase of the stock of precautionary wealth from 20.5 percent of expected per period labor income to 23.8 percent of this income. This increase is mostly a result of an increase in deposits, in reflection of their assumed risk-free nature in this exercise.

A rapid increase in deposits (figure 5.10) in the early periods allows the household to withdraw more money in the subsequent periods (figure 5.11) and, therefore, to protect consumption better (figure 5.12). Although the consumption path simulated for the higher risk aversion scenario exhibits a higher variance, $2.301 \times 10^{-4}$ compared to $1.436 \times 10^{-4}$ (table 5.4), the calculation of the coefficient of variation reveals the opposite behavior. In effect, when the variability of consumption is measured as a ratio of the mean value, the coefficient of variation of consumption for the more risk averse household is lower (0.0706) compared to the less risk averse household (0.0793). This is due to the differences that exist in average consumption for each case.
5.6 Simulation 5: Changes in impatience

Gourinchas and Parker (2002) estimated a subjective discount rate for US households equal to 3.4 percent. I use a discount rate of 7 percent, which would reflect better the case of a poor, more impatient, household in a developing country. In this exercise, the simulation attempts to replicate an environment where poverty is extreme, so households are extremely impatient, in the sense that they discount future consumption heavily and value current consumption much.

The simulation exercise shows that impatient households keep a lower level of precautionary wealth, which results from the large opportunity cost from holding this wealth. Deposits are a negligible part of the portfolio (figure 5.13), and the household almost specializes completely in keeping livestock (figure 5.14). As expected, a lower level of precautionary wealth implies lower protection and, therefore, a higher volatility of consumption (figure 5.15 and table 5.5).

The relevant implication of this result is that the provision of deposit facilities in extremely poor places may not improve the situation of the villagers, as they are highly impatient. This particular insightful may be interesting for some policymakers that believe that deposit facilities are equally useful everywhere. For extremely poor people, deposit facilities may be a very expensive tool, but in return they will have to absorb most of the shocks by reductions in consumption. Other safety nets may be required in this case.

Carroll and Samwick (1997) estimated a discount rate in the vicinity of 10-15 percent for the same group of people.
5.7 Simulation 6: A scenario that is more conducive to holding deposits

For the last simulation, I create a scenario that is more conducive to holding deposits, by simultaneously changing several parameters. I assume a risk-free deposit (perhaps covered by deposit insurance) that pays a higher interest rate (6 percent) and requires a lower transaction cost ($\alpha = 0.5$ compared to $\alpha = 1.0$). The simulation is run for both scenarios, the more and the less conducive environment for deposit-taking, in an scenario that is a little riskier than the benchmark case, as the bad outcome has a probability of occurring equal to 0.15.

The results are impressive. Precautionary wealth increases from 22.2 percent to 30.5 percent of the per period labor income, and the share of deposits with respect to total precautionary wealth exhibits almost a three-fold increase (from 14.3 percent to 36.6 percent). Since deposits have become more attractive, the household chooses, in this simulation, to attain more protection by accumulating precautionary wealth (figures 5.16 and 5.17). This, in turn, allows the household to reduce the variability of consumption, measured by its variance, from $1.361 \times 10^{-4}$ to $1.325 \times 10^{-4}$ (figure 5.18 and table 5.6).

Thus, improving the relative attractiveness of deposits can have an important influence on the household’s welfare. The next chapter presents the conclusions of this research and proposes the adoption of policies to achieve this desirable outcome.
SIMULATION 1: CHANGES IN TRANSACTION COSTS

Parameter values

\begin{itemize}
  \item \text{gamma} = 0.5; \quad \text{Relative risk aversion coefficient}
  \item \text{alpha} = [1.5 \ 1.0 \ 0.5] \quad \text{Transaction cost function parameter}
  \item \text{beta} = 0.8 \quad \text{Livestock growth function parameter}
  \item \text{r} = [0.04 \ 0.04] \quad \text{Interest rate on deposits for each state of nature}
  \item \text{p} = [0.5 \ 0.8] \quad \text{Livestock price for each state of nature}
  \item \text{y} = [0.9 \ 1.8] \quad \text{Income for each state of nature}
  \item \text{w} = [0.1 \ 0.9] \quad \text{Shock probabilities}
  \item \text{rho} = 0.07 \quad \text{Discount rate}
\end{itemize}

\textbf{Figure 5.1} Expected deposits path for different transaction cost parameters.
Figure 5.2 Expected livestock path for different transaction cost parameters.
Figure 5.3  Expected net additions to deposits path for different transaction cost parameters.
Figure 5.4 Expected consumption path for different transaction cost parameters.
### Table 5.1
Simulation 1. Effects of reducing depositor transaction costs on precautionary wealth.

<table>
<thead>
<tr>
<th></th>
<th>BENCHMARK</th>
<th>SIMULATION 1: TC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α = 1.0</td>
<td>α = 0.5</td>
</tr>
<tr>
<td>Deposits (D)</td>
<td>0.035</td>
<td>0.067</td>
</tr>
<tr>
<td>Livestock (L)</td>
<td>0.315</td>
<td>0.316</td>
</tr>
<tr>
<td>Precautionary wealth (PW)</td>
<td>0.351</td>
<td>0.383</td>
</tr>
<tr>
<td>Expected Income (Y)</td>
<td>1.710</td>
<td>1.710</td>
</tr>
<tr>
<td><strong>Ratios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW / Y</td>
<td>20.5%</td>
<td>22.4%</td>
</tr>
<tr>
<td>D / PW</td>
<td>10.1%</td>
<td>17.5%</td>
</tr>
<tr>
<td>L / PW</td>
<td>89.9%</td>
<td>82.5%</td>
</tr>
<tr>
<td><strong>Consumption variability</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>$1.5740 \times 10^{-4}$</td>
<td>$1.0394 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
SIMULATION 2: CHANGES IN BANKRUPTCY RISK

Figure 5.5 Expected deposits path for different levels of bankruptcy risk.

Parameter values
- \( \gamma = 0.5 \) Relative risk aversion coefficient
- \( \alpha = 1.0 \) Transaction cost function parameter
- \( \beta = 0.8 \) Livestock growth function parameter
- \( r_{\text{no risk}} = [0.04 \ 0.04] \) Interest rate on deposits, no risk
- \( r_{\text{low risk}} = [-0.1 \ 0.04] \) Interest rate on deposits, bankruptcy risk
- \( p = [0.5 \ 0.8] \) Livestock price for each state of nature
- \( y = [0.9 \ 1.8] \) Income for each state of nature
- \( w = [0.1 \ 0.9] \) Shock probabilities
- \( \rho = 0.07 \) Discount rate
Figure 5.6 Expected consumption path for different levels of bankruptcy risk.
<table>
<thead>
<tr>
<th></th>
<th>BENCHMARK</th>
<th>SIMULATION 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r = [0.04 0.04]</td>
<td>r = [-0.10 0.04]</td>
</tr>
<tr>
<td>Deposits (D)</td>
<td>0.035</td>
<td>0.005</td>
</tr>
<tr>
<td>Livestock (L)</td>
<td>0.316</td>
<td>0.315</td>
</tr>
<tr>
<td>Precautionary wealth (PW)</td>
<td>0.351</td>
<td>0.319</td>
</tr>
<tr>
<td>Expected Income (Y)</td>
<td>1.710</td>
<td>1.710</td>
</tr>
<tr>
<td><strong>Ratios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW / Y</td>
<td>20.5%</td>
<td>18.7%</td>
</tr>
<tr>
<td>D / PW</td>
<td>10.0%</td>
<td>1.4%</td>
</tr>
<tr>
<td>L / PW</td>
<td>90.0%</td>
<td>98.6%</td>
</tr>
<tr>
<td><strong>Consumption variability</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>1.5600x10^-4</td>
<td>2.3436x10^-4</td>
</tr>
</tbody>
</table>

**Table 5.2** Simulation 2: Effects of a change in bankruptcy risk on precautionary wealth.
SIMULATION 3: CHANGES IN THE RISKINESS OF THE ENVIRONMENT

Figure 5.7 Expected deposits path for scenarios with different levels of risk.

Parameter values

- $\gamma = 0.5$ Relative risk aversion coefficient
- $\alpha = 1.0$ Transaction cost function parameter
- $\beta = 0.8$ Livestock growth function parameter
- $r = [0.04 0.04]$ Interest rate on deposits for each state of nature
- $p = [0.5 0.8]$ Livestock price for each state of nature
- $y = [0.9 1.8]$ Income for each state of nature
- $w = [0.1 0.9]$ Shock probabilities: Low risk
- $w = [0.5 0.5]$ Shock probabilities: High risk
- $\rho = 0.07$ Discount rate
Figure 5.8 Expected livestock path for scenarios with different levels of risk.
Figure 5.9 Expected consumption path for scenarios with different levels of risk.
<table>
<thead>
<tr>
<th></th>
<th>BENCHMARK</th>
<th>SIMULATION 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\kappa = [0.1 \ 0.9]$</td>
<td>$\kappa = [0.5 \ 0.5]$</td>
</tr>
<tr>
<td>Deposits (D)</td>
<td>0.035</td>
<td>0.0683</td>
</tr>
<tr>
<td>Livestock (L)</td>
<td>0.316</td>
<td>0.3235</td>
</tr>
<tr>
<td>Precautionary wealth (PW)</td>
<td>0.351</td>
<td>0.392</td>
</tr>
<tr>
<td>Expected Income (Y)</td>
<td>1.71</td>
<td>1.71</td>
</tr>
<tr>
<td><strong>Ratios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW / Y</td>
<td>20.5%</td>
<td>22.9%</td>
</tr>
<tr>
<td>D / PW</td>
<td>10.0%</td>
<td>17.4%</td>
</tr>
<tr>
<td>L / PW</td>
<td>90.0%</td>
<td>82.6%</td>
</tr>
<tr>
<td><strong>Consumption variability</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>1.3715x10^{-4}</td>
<td>2.1498x10^{-4}</td>
</tr>
</tbody>
</table>

**Table 5.3** Simulation 3. Effects of a change in the riskiness of the environment on precautionary wealth.
SIMULATION 4: CHANGES IN RISK AVERSION

Figure 5.10 Expected deposits path for different levels of risk aversion.

Parameter values

- gamma = [0.5 0.99]   Relative risk aversion coefficient
- alpha = 1.0          Transaction cost function parameter
- beta = 0.8           Livestock growth function parameter
- r = [0.04 0.04]      Interest rate on deposits for each state of nature
- p = [0.5 0.8]        Livestock price for each state of nature
- y = [0.9 1.8]        Income for each state of nature
- w = [0.1 0.9]        Shock probabilities
- rho = 0.07           Discount rate
Figure 5.11 Expected net additions to deposits path for different levels of risk aversion.
Figure 5.12 Expected consumption path for different levels of risk aversion.
Table 5.4 Simulation 4. Effects of a change in the household’s risk aversion on precautionary wealth.
SIMULATION 5: CHANGES IN IMPATIENCE

Figure 5.13 Expected deposits path for different discount rates.

Parameter values

- $\gamma = 0.5$  Relative risk aversion coefficient
- $\alpha = 1.0$  Transaction cost function parameter
- $\beta = 0.8$  Livestock growth function parameter
- $r = [0.04, 0.04]$  Interest rate on deposits for each state of nature
- $p = [0.5, 0.8]$  Livestock price for each state of nature
- $y = [0.9, 1.8]$  Income for each state of nature
- $w = [0.1, 0.9]$  Shock probabilities
- $\rho = [0.07, 0.10]$  Discount rate
Figure 5.14 Expected livestock path for different discount rates.
Figure 5.15 Expected consumption path for different discount rates.
### Table 5.5
Simulation 5. Effects of a change in the household’s impatience on precautionary wealth.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Simulation 5</th>
<th>Ratios</th>
<th>Consumption variability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho = 0.07$</td>
<td>$\rho = 0.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deposits (D)</td>
<td>0.036</td>
<td>0.000</td>
<td>10.1%</td>
<td>1.4751x10^-4</td>
</tr>
<tr>
<td>Livestock (L)</td>
<td>0.315</td>
<td>0.282</td>
<td>99.9%</td>
<td>1.6322x10^-4</td>
</tr>
<tr>
<td>Precautionary wealth (PW)</td>
<td>0.350</td>
<td>0.282</td>
<td>89.9%</td>
<td></td>
</tr>
<tr>
<td>Expected Income (Y)</td>
<td>1.710</td>
<td>1.710</td>
<td>20.5%</td>
<td></td>
</tr>
<tr>
<td>Ratios</td>
<td></td>
<td></td>
<td>16.5%</td>
<td></td>
</tr>
<tr>
<td>$PW / Y$</td>
<td>20.5%</td>
<td>16.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D / PW$</td>
<td>10.1%</td>
<td>0.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L / PW$</td>
<td>89.9%</td>
<td>99.9%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SIMULATION 6:
A MORE CONDUCIVE SCENARIO FOR HOLDING DEPOSITS

Figure 5.16 Expected deposits path for scenarios that are more/less conducive to deposit mobilization.

Parameters values
- \( \gamma = 0.5 \) Relative risk aversion coefficient
- \( \alpha = 0.5 \) Transaction cost function parameter
- \( \beta = 0.8 \) Livestock growth function parameter
- \( r = [0.06, 0.06] \) Interest rate on deposits for each state of nature
- \( p = [0.5, 0.8] \) Livestock price for each state of nature
- \( y = [0.9, 1.8] \) Income for each state of nature
- \( w = [0.15, 0.85] \) Shock probabilities
- \( \rho = 0.07 \) Discount rate
Figure 5.17 Expected livestock path for scenarios that are more/less conducive to deposit mobilization.
Figure 5.18 Expected consumption path for scenarios that are more/less conducive to deposit mobilization.
Table 5.6  Simulation 6. Effects of a change in the scenario, such that it becomes more conducive to holding deposits, on precautionary wealth.

<table>
<thead>
<tr>
<th></th>
<th>LESS CONDUCTIVE</th>
<th>MORE CONDUCTIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r = \begin{bmatrix} 0.04 &amp; 0.04 \end{bmatrix}, \alpha=1.0, \kappa=\begin{bmatrix} 0.15 &amp; 0.85 \end{bmatrix} )</td>
<td>( r = \begin{bmatrix} 0.06 &amp; 0.06 \end{bmatrix}, \alpha=0.5, \kappa=\begin{bmatrix} 0.15 &amp; 0.85 \end{bmatrix} )</td>
</tr>
<tr>
<td>Deposits (D)</td>
<td>0.053</td>
<td>0.186</td>
</tr>
<tr>
<td>Livestock (L)</td>
<td>0.317</td>
<td>0.322</td>
</tr>
<tr>
<td>Precautionary wealth (PW)</td>
<td>0.370</td>
<td>0.507</td>
</tr>
<tr>
<td>Expected Income (Y)</td>
<td>1.665</td>
<td>1.665</td>
</tr>
<tr>
<td><strong>Ratios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW / Y</td>
<td>22.2%</td>
<td>30.5%</td>
</tr>
<tr>
<td>D / PW</td>
<td>14.3%</td>
<td>36.6%</td>
</tr>
<tr>
<td>L / PW</td>
<td>85.7%</td>
<td>63.4%</td>
</tr>
<tr>
<td><strong>Consumption variability</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>1.3601x10^{-4}</td>
<td>1.3258x10^{-4}</td>
</tr>
</tbody>
</table>
CHAPTER 6

CONCLUSIONS AND POLICY RECOMMENDATIONS

Rural households in developing countries are highly vulnerable to adverse income shocks. In the absence of formal instruments to attain protection, such as insurance, they have no choice other than to rely on costly strategies to smooth their consumption. Costly protection reduces their welfare one way or the other. In some cases, with the adoption of a costly strategy, consumption smoothing is achieved, but only at a high cost. In other cases, consumption smoothing is simply not achieved, which may be even worse. Therefore, reductions in the cost of protecting consumption will be welfare-enhancing.

This dissertation examines the potential impacts of more convenient access to safer deposit facilities on the household’s precautionary wealth and on its success in consumption smoothing. More convenient access is measured as a reduction in transaction costs. Safer deposits result from a reduction in the probability of bankruptcy of the financial institution in the case of a local systemic shock.

Two models are developed for the analysis. First, Samuelson’s (1969) model is adapted to build a two-period stochastic model on asset portfolio decisions. The modified version incorporates transaction costs as a variable relevant in choosing a
portfolio. Second, a dynamic, stochastic, infinite horizon model of precautionary wealth choices for a credit-constrained rural household is used to explore these issues.

The first model is solved for two scenarios. In one scenario, the household has access to a risky asset (livestock) only. In the other one, the household incurs a transaction cost if it wants to accumulate wealth in the form of a risk-free asset (deposits), in addition to livestock. A critical value for the transaction cost—the value that makes the household indifferent between investing all of its wealth in the risky asset or incurring a transaction cost in order to diversify its portfolio between the risky and the risk-free asset—is computed. Beyond the differences in expected asset returns, the magnitude of this critical transaction cost reveals the household’s willingness to pay for an instrument that facilitates the management of risk.

The second exercise develops a dynamic stochastic model that fits the situation of a poor rural household better. In contrast with the first one, in this model labor income is stochastic, precautionary wealth can be diversified between two assets, both assets are risky, and there exists a covariance between labor income and the price of livestock and between these two and the potential impact of a local systemic shock on the solvency of the deposit-taking financial institution.

In the model, the household makes three decisions: how much to consume, how much to increase/decrease the amount of financial deposits, and how much livestock to buy/sell. The dynamic nature of the model captures the desire of the household to balance the current and expected discounted future utility derived from consumption. These intertemporal decisions are made with a precautionary purpose in mind, as the
utility function is characterized by constant relative risk aversion and, thereby, it carries the precautionary motive for saving. The household is fully credit-constrained.

Numerical methods are used to approximate a solution for the Bellman equation that emerges from the optimization problem. Using the collocation method, the approximation is nearly optimal: the residuals are in the order of $10^{-4}$ for the deterministic version of the model and $10^{-3}$ for the stochastic one.

The solution allows the simulation of several scenarios, by changing parameter values. Six simulations are performed for different values for the transaction cost parameter, different levels of bankruptcy risk (through changes in the effective interest rate paid on deposits), different levels of riskiness in the environment (through changes in the probabilities for each state of nature), different degrees of risk aversion (prudence) of the household, and variations in the discount rate, to reflect changes in impatience. In addition, an experiment simultaneously changes several parameters, to create a scenario that encourages holding assets in the form of deposits. The goal is to analyze the sensitivity of the level and composition of precautionary wealth and of consumption patterns to these changes.

The results of the modified version of Samuelson’s model reveal that, under realistic parameter values, poor rural households have a tremendous willingness to pay a transaction cost to have access to a risk-free asset (deposits) and diversify their portfolio. This model also shows that, the riskier the environment is, the greater the motivation for holding a larger share of the risk-free asset will be. Thus, in small villages in developing countries, the provision of safe and low transaction cost deposit accounts will be greatly
valued. Moreover, the model illustrates how the rate of return of the risk-free asset (deposits) does not have to be too high to attract depositors, since considerations such as safety and convenience seem to be more relevant to the households.

The solution of the dynamic model and the various simulations show several interesting results. First, as expected, the demand for deposit facilities emerges with the presence of risk. In the absence of risk, the household prefers to hold all of its wealth in the asset that yields the highest return (livestock). In the certainty-equivalent steady state, livestock holdings (all of the wealth) amount to 34 percent of the household’s per-period income. In this scenario, the only motivation for holding assets is to improve the household’s ability to consume in the future.

This situation changes, however, as soon as risk is introduced, by allowing a negative labor income shock that also affects the price of livestock. In this scenario, two effects (level and composition) influence precautionary wealth. On the one hand, the level of precautionary wealth increases (to 57 percent of the expected per-period income), as the household accumulates extra assets to protect its consumption in this risky scenario. On the other hand, deposits are added to precautionary wealth holdings (while none were held before). The composition of wealth now includes deposits for 10 percent and livestock for 47 of the expected per-period income. In the risky environment, livestock holdings increase, despite their riskiness, but in a lower proportion than deposits.

Second, a reduction in the transaction costs of deposits has two effects as well. On the one hand, it increases the level of precautionary wealth (as the cost of protecting
consumption with this strategy declines) and, on the other hand, it changes its composition, by increasing the share of deposit holdings. The resulting changes in precautionary wealth in turn allow the household to improve its ability to smooth consumption (reduce its variance). In general, the positive impact of a reduction in transaction costs on consumption smoothing is expected to be larger, the higher the probability of the adverse shock is.

How depositor transaction costs affect precautionary wealth holdings is influenced by their magnitude. At very high levels, transaction costs would make it (almost) prohibitively expensive to hold deposits. Reductions in transaction costs that still leave them at extremely high levels will have insignificant effects on the share of deposit holdings in precautionary wealth. However, as the simulations suggest, there is a threshold beyond which further reductions in transaction costs will rapidly increase the demand for deposit facilities.

Third, the risk of bankruptcy of the financial institution greatly discourages the holdings of deposits as precautionary wealth. The representative household is very sensitive to this bankruptcy risk. There are both precautionary wealth level and composition effects. On the one hand, a 10-percent probability of experiencing a bad outcome –in which the household losses its deposits– reduces precautionary wealth from 20.5 percent to 18.7 percent of expected per-period income. On the other hand, the share of deposits falls from 10 percent to 1.4 percent of total precautionary wealth. In this scenario, attaining consumption protection is more difficult, and this is reflected in an increase in the variability of the expected consumption path.
Fourth, risk aversion and impatience—both associated with poverty—influence deposit holdings in opposite directions. The more risk averse the household is, the more it will demand deposit facilities. The more impatient the household is, the less it will demand deposit facilities. Both types of behavior highlight the importance of providing safe deposit instruments in order to increase household welfare.

On the one hand, more risk-averse households are willing to incur higher costs in order to protect their future consumption and will hold more precautionary wealth. The share of deposits in their precautionary wealth will be higher than for less risk-averse households. The supply of more convenient and safer deposit facilities will lower the costs of holding precautionary wealth for these prudent households and will improve their welfare.

On the other hand, a more impatient household will value current consumption more than a more patient one. Thus, it will hold less precautionary wealth and a smaller share of deposits in this wealth. For an impatient household, the opportunity cost of protecting future consumption is higher. The supply of more convenient and safer deposit facilities will lower the costs of holding precautionary wealth for these impatient households and improve their welfare. Poor households in the rural areas of developing countries are both impatient and risk averse. The increased outreach of deposit facilities will particularly matter for them.

Fifth, the riskier the scenario is, the greater the incentive to hold precautionary wealth for attaining consumption protection will be. In a riskier, environment, deposit facilities will be demanded even if transaction costs are high, as households have a high
willingness to pay to attain protection. This is the case, however, because in the model the household does not have access to other risk-coping strategies. Otherwise, the household would prefer to adopt other less costly strategies and not hold the costly deposit. In contrast, in a low-risk scenario, there will be less incentive for holding precautionary wealth. Therefore, households will become more sensitive to high transaction costs in their demand for deposit facilities.

Sixth, a more conducive scenario for holding deposits—lower transaction costs to access safe and more rewarding deposit instruments—greatly encourages the holdings of deposits as precautionary wealth. In effect, a increase in the interest rate on deposits from 4 to 6 percent, with a simultaneous reduction in the transaction cost parameter from 1.0 to 0.5, produce almost a three-fold increase in deposits (and the share grows from 14.3 to 36.6 percent of precautionary wealth). These favorable conditions produce a level effect, and precautionary wealth increases from 22.2 to 30.5 percent of per-period income. These changes improve the ability of the household to smooth consumption and increase its welfare.

Summing up, increased access to deposit facilities matters both at a micro and a macro level. At the household level, the importance of deposits as a consumption smoothing strategy depends on: i) the riskiness of the environment (e.g., frequency and intensity of the local systemic shocks) and ii) the level of fragmentation in the economy, as the latter determines how covariant the household’s income and the price of livestock will be. In more fragmented local markets, the transmission of the income shock into the price of livestock will be greater. Access to broader national markets lowers the
covariance. Thus, the riskier the environment and the more fragmented the economy, the greater the importance of the provision of convenient and safe deposit instruments will be.

Poor rural households, which are assumed to be more risk averse and prudent, have great incentives for holding precautionary assets. The marginal utility of a loss in consumption is high for poor households. Moreover, because they are close to subsistence, they greatly fear reductions in consumption below a critical level. Poor households are more impatient, however, and this discourages the accumulation of wealth. The supply of more convenient instruments to protect consumption, in particular deposit facilities, reduces the opportunity cost of accumulating precautionary wealth for these households.

The provision of deposit facilities matters at a macro level as well. First, the associated costs and benefits should be considered in including an expansion of the outreach of deposit facilities as a poverty alleviation tool. Poverty, after all, is intimately linked with vulnerability. Deposit mobilization is seldom considered, however, among the ingredients of a rural development strategy.

Second, deposit mobilization is critical for financial intermediation, as a mechanism for improving the allocation of resources in the rural economy. Indeed, the supply of deposit facilities is a powerful way to reduce the fragmentation that characterizes poor developing economies. Through intermediation, precautionary wealth that would otherwise be held in unproductive forms can be transformed into loanable
funds that allow deficit units in the economy to take advantage of opportunities with higher rates of return.

This dissertation builds a theoretical framework that offers a strong basis for empirical research. One limitation has been that some of the parameters in the model were not estimated applying econometric methods on actual data. Another limitation is that the models assume only two assets, which induces a “natural” substitution effect. A more comprehensive analysis would include more than two assets, to explore both substitution and complementarity.

Three policy implications emerge. First, since reductions in the transaction costs associated with deposit facilities affect the composition of precautionary wealth in a way that reduces the overall risk of the household’s wealth portfolio—for a reasonably low probability of bankruptcy of the financial institution—the supply of affordable financial deposit services matters.

Financial institutions as well as governments can direct their efforts at reducing transaction costs. In the case of financial institutions, more cost-effective technologies should be developed, to allow them to mobilize small deposits from very disperse households in areas with low density of population. This is quite a challenge, and it requires a large amount of investment. Moreover, a key role for reducing transaction costs may be on the side of the government, as the creation and expansion of the physical and institutional infrastructure will greatly facilitate the achievement of this outcome.

Second, a more efficient system of regulation and supervision of the financial institutions that operate in rural areas will reduce the risk of bankruptcy and will enhance
the households’ trust in the financial system. Supervision is costly, though. In the early stages of financial development, reducing both the risk of bankruptcy and transaction costs may be easily achieved. However, there is a threshold beyond which improving the safety of the financial institution can only be accomplished at the expense of having to increase transaction costs.

Third, the design of a financial architecture that promotes financial institutions that are less vulnerable to local systemic risk creates an environment that is more conducive for deposit mobilization and more successful consumption smoothing.

In summary, in the rural areas of developing countries, a reduction in transaction costs may come from improving the physical infrastructure, so clients can be reached and they can reach the financial institution in a more cost-effective manner. Reductions in transaction costs can also come from the design of a more conducive prudential supervision and regulation system for deposit mobilization and from encouraging robust financial institutions to serve rural clienteles. This leaves ample room for the state to intervene in providing the necessary public goods and in encouraging innovation in financial technologies, given the externalities that typically characterize financial deepening and the generation of knowledge.
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APPENDIX A

ANALYTICAL SOLUTION FOR A PORTFOLIO MODEL WITH TRANSACTION COSTS
ANALYTICAL SOLUTION FOR A PORTFOLIO MODEL WITH TRANSACTION COSTS

General Case

\[ \max_{c_0, c_1, \omega_0, \omega_1} U(C_0) + E\delta U(C_1) \]

s.t. \[ A_i = (\omega_0 + Y_0 - C_0) [(1 - \omega_0) (1 + r) + \omega_0 Z_1] \]

where \( Z_i = \begin{cases} \lambda & \text{with probability } \frac{1}{2} \\ \frac{1}{\lambda} & \text{with probability } \frac{1}{2} \end{cases} \)

\( Z_0 \) given

\[ \delta = \frac{1}{1 + \rho} \]

\( U(C_i) = \log U(C_i) \)

Case I. Only one risky asset: livestock with stochastic returns (\( \omega_0 = 1 \))

\[ \max_{c_0, c_1} U(C_0) + E\delta U(C_1) \]

s.t. \[ A_i = (\omega_0 + Y_0 - C_0) Z_i \]

where \( Z_i = \begin{cases} \lambda & \text{with probability } \frac{1}{2} \\ \frac{1}{\lambda} & \text{with probability } \frac{1}{2} \end{cases} \)

\( A_0 \) given

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\[ \delta = \frac{1}{1 + \rho} \]

\[ U(C_t) = \log U(C_t) \]

Solving backwards

**Period 1:** No bequest motive

The solution is:

\[ C_t = A_t + Y_t, \]

\[ \omega_t = \text{Indeterminate} \]

**Period 0:**

\[
\text{Max}_{C_0} U(C_0) + E\delta U(C_1)
\]

After plugging the solution for \( t=1 \), the problem becomes:

\[
\text{Max}_{C_0} U(C_0) + E\delta U[(A_0 + Y_0 - C_0)Z_1 + Y_1]
\]

**F.O.C.**

\[
\frac{1}{C_0}\delta E \frac{Z_1}{(A_0 + Y_0 - C_0)Z_1 + Y_1} = 0
\]

\[
\frac{1}{\delta C'_0} = \frac{1}{2} \left[ \frac{\lambda}{(A_0 + Y_0 - C_0') + Y_1} + \frac{1/\lambda}{(A_0 + Y_0 - C_0')(1/\lambda) + Y_1} \right]
\]

Solving for \( C_0 \), we get the following implicit quadratic function:

\[
(1 + \delta)C_0^2 - \left[ (A_0 + Y_0)(2 + \delta) + Y_1(\lambda + \frac{1}{\lambda})(1 + \frac{\delta}{2}) \right]C_0 +
(A_0 + Y_0) \left[ (A_0 + Y_0) + Y_1(\lambda + \frac{1}{\lambda}) \right] + Y_1^2 = 0
\]
The value function is:

\[ V_0'(A_0) = \log C_0^* + E\delta \log \left[(A_0 + Y_0 - C_0^*)Z_1 + Y_1 \right] \]

Case II. Two assets: one risky asset (livestock with stochastic returns) and a risk-free asset (deposits)

\[
\max_{C_0,C_1,\omega_0,\omega_1} U(C_0) + E\delta U(C_1)
\]

s.t. \( A_i = (A_0 + Y_0 - C_0 - \tau)(1 - \omega_i)(1 + r) + \omega_i Z_1 \)

where \( Z_1 = \begin{cases} 
\lambda & \text{with probability } \frac{1}{2} \\
\frac{1}{\lambda} & \text{with probability } \frac{1}{2}
\end{cases} \)

\( A_0 \) given

\[ \delta = \frac{1}{1 + \rho} \]

Solving backwards

**Period 1:** No bequest motive

The solution is:

\[ C_i = A_i + Y_i - \tau \]

**Period 0:**

\[ V_0'' = \max_{C_0,\omega_0} U(C_0) + E\delta V_1(A_1) \]

After plugging the optimal consumption for \( t=1 \), the problem becomes:
After some algebra and rearranging the terms, the value function becomes:

$$\equiv \log C_0 + \delta E \log \left\{ \left( A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1 + r} - C_0 \right)^* \right\}$$

$$\equiv \log C_0 + \delta E \log \left\{ \left( 1 + r \right) \left[ 1 - \frac{(A_0 + Y_0 - \tau - C_0)}{A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1 + r} - C_0} \omega_0 \right] + \left( \frac{(A_0 + Y_0 - \tau - C_0)}{A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1 + r} - C_0} \omega_0 \right) \right\}$$

Substituting $\tilde{\omega}_0 = \frac{(A_0 + Y_0 - \tau - C_0)}{A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1 + r} - C_0}$, the value function reduces to

$$\equiv \log C_0 + \delta E \log \left\{ \left( A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1 + r} - C_0 \right) \left[ (1 + r)(1 - \tilde{\omega}_0) + \tilde{\omega}_0 Z_1 \right] \right\}$$

Then the problem is identical to:

$$\text{Max}_{c_t} \log(C_0) + \delta \log \left( A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1 + r} - C_0 \right) + \delta \text{Max}_{\tilde{\omega}_0} E \log \left( (1 + r)(1 - \tilde{\omega}_0) + \tilde{\omega}_0 Z_1 \right)$$

As it is shown, the portfolio choice problem is completely separable from the consumption-saving choice problem.

Taking the first order conditions (F.O.C.s)

$$\frac{1}{C^*_0} = \delta \frac{1}{A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1 + r} - C^*_0} \quad (1)$$

and,

$$E \frac{Z_i - (1 + r)}{(1 + r)(1 - \tilde{\omega}_0) + \tilde{\omega}_0 Z_1} = 0 \quad (2)$$
From 1 we have \( C_0^* \):

\[
C_0^* = \frac{1}{1+\delta} \left( A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1+r} \right)
\]

and from 2, we get \( \bar{\alpha}_0^* \)

\[
\frac{1}{2} \left( \frac{\lambda - (1+r)}{1+\alpha} + \bar{\alpha}_0^* \lambda \right) + \frac{(1/\lambda) - (1+r)}{(1+r)(1-\bar{\alpha}_0^*) + \bar{\alpha}_0^* (1/\lambda)} = 0
\]

Solving for \( \bar{\alpha}_0^* \),

\[
\bar{\alpha}_0^* = \frac{(1+r) \left[ 2(1+r) - \left( \frac{1}{\lambda} + \lambda \right) \right]}{2 \left( \lambda - (1+r) \right) \left[ \frac{1}{\lambda} - (1+r) \right]}
\]

The value function is

\[
V_0^{\prime\prime}(A_0) = \log C_0^* + \delta E \log \left\{ \left( A_0 + Y_0 - \tau + \frac{Y_i - \tau}{1+r} - C_0^* \right) \left[ (1-\bar{\alpha}_0^*)(1+r) + \bar{\alpha}_0^* Z_t \right] \right\}
\]

By equating the two value functions, \( V_0^{\prime} \) and \( V_0^{\prime\prime} \), I obtain the critical value for \( \tau \), which is the transaction cost that makes the household indifferent between holding the risky asset only and diversifying its portfolio.
APPENDIX B

MATLAB CODE FOR SOLVING THE DYNAMIC STOCHASTIC MODEL
MATLAB CODE FOR SOLVING THE DYNAMIC STOCHASTIC MODEL

% PRECAUTIONARY WEALTH PORTFOLIO MODEL
fprintf('STOCHASTIC PRECAUTIONARY WEALTH PORTFOLIO MODEL
')
disp('Prepared by Franz Gomez-Soto')
disp('-----------------------------------------------------------------------------------')
disp(' ')
close all
clear all

% ENTER MODEL PARAMETERS
gamma = 0.5;                            % relative risk aversion coefficient
alpha = 1.0;                               % transaction cost function parameter
beta  = 0.8;                                % livestock growth function parameter
r     = [0.04 0.04]';                      % interest rate on deposits for each state of nature
p     = [0.5 0.8]';                         % livestock price for each state of nature
y     = [0.9 1.8]';                         % income for each state of nature
w     = [0.1 0.9]';                        % shock probabilities
rho   = 0.07;                                % discount rate
rstar = r(1)*w(1)+r(2)*w(2);      % interest rate at steady state
pstar = p(1)*w(1)+p(2)*w(2);    % prices at steady state
ystar = y(1)*w(1)+y(2)*w(2);     % income at steady state
m = length(r);

% ERROR CHECKS
if rstar>rho|rho<-1|any(r<-1),
    disp('interest rate error')
end
if norm(sum(w)-1)>1.e-8,
    disp('check probabilities')
end

% PACK MODEL STRUCTURE
clear model
model.func = 'fpseudo';                                      % model functions
model.discount = 1/(1+rho);                              % discount factor
model.e = (1:m)';                                               % shock values
model.w = w;                                                     % shock probabilities
model.params = {gamma,alpha,beta,r,p,y};      % other parameters
model.discretestates = 3;                                    % third state variable is discrete
% COMPUTE CERTAINTY-EQUIVALENT STEADY-STATE
Dstar = 0; % deposits
LstarplusdLstar = ((1+rho)/beta)^(1/(beta-1)); % livestock holdings plus acquisitions
Lstar = LstarplusdLstar*beta; % livestock holdings
dLstar = LstarplusdLstar-Lstar; % livestock acqisitions
cstar = ystar-pstar*dLstar; % consumption
IDstar = cstar^(-gamma); % deposit shadow price
ILstar = cstar^(-gamma)*pstar; % livestock shadow price
vstar = (1/rho+1)*(cstar^(1-gamma))/(1-gamma); % value
sstar = [Dstar Lstar 1];
xstar = [0 dLstar];

% CHECK MODEL DERIVATIVES AT CE STEADY STATE
dpcheck(model,sstar,xstar,1);

% DEFINE APPROXIMATION SPACE
n = [20 20]; % number of state collocation coordinates
ns = m*prod(n); % number of collocation nodes
smin = [0.0 0.3*Lstar]; % minimum values for states
smax = [3.0 3.0]; % maximum values for states
fspace = fundefn('spli',n,smin,smax,[],(1:m)'); % function space
scoord = funnode(fspace); % state collocation grid coordinates
snodes = gridmake(scoord); % state collocation grid nodes

% SOLVE BELLMAN EQUATION USING FUNCTION ITERATION
optset('dpsolve','algorithm','funcit')
v = vstar + IDstar*(snodes(:,1)-Dstar) + ILstar*(snodes(:,2)-Lstar);
x = [zeros(ns,1) dLstar*ones(ns,1)];
[c,s,v,x] = dpsolve1(model,fspace,snodes,v,x);

% SOLVE BELLMAN EQUATION USING NEWTON METHOD
optset('dpsolve','algorithm','newton')
v = reshape(v,ns,1);
x = reshape(x,ns,2);
[c,s,v,x,resid] = dpsolve1(model,fspace,snodes,v,x);

% COMPUTE SHADOW PRICES
n = [length(s{1}) length(s{2}) length(s{3})];
p1 = funeval(c,fspace,s,[1 0 0]);
p2 = funeval(c,fspace,s,[0 1 0]);
p1 = reshape(p1,n);
p2 = reshape(p2,n);
% PLOT OPTIMAL POLICY (Surface)
figure(1)
hh=surf(s{1},s{2},x(:,:,1,1)');
title('Optimal Net Additions to Deposits Choice');
xlabel('Deposits'); ylabel('Livestock Holdings');
zlabel('Net Additions to Deposits');
set(hh,'FaceColor','interp','EdgeColor','interp')

% PLOT OPTIMAL POLICY (Surface)
figure(2)
hh=surf(s{1},s{2},x(:,:,1,2)');
title('Optimal Net Additions to Livestock Choice');
xlabel('Deposits'); ylabel('Livestock Holdings');
zlabel('Net Additions to Livestock Holdings');
set(hh,'FaceColor','interp','EdgeColor','interp')

% PLOT VALUE FUNCTION (Surface)
figure(3)
hh=surf(s{1},s{2},v(:,:,1)');
title('The Value of the Household');
xlabel('Deposits'); ylabel('Livestock Holdings');
zlabel('Value');
set(hh,'FaceColor','interp','EdgeColor','interp')

% PLOT SHADOW PRICE FUNCTION 1 (Surface)
figure(4)
hh=surf(s{1},s{2},p1(:,:,1)');
title('Shadow Price of Deposits');
xlabel('Deposits'); ylabel('Livestock Holdings');
zlabel('Price');
set(hh,'FaceColor','interp','EdgeColor','interp')

% PLOT SHADOW PRICE FUNCTION 2 (Surface)
figure(5)
hh=surf(s{1},s{2},p2(:,:,2)');
title('Shadow Price of Livestock');
xlabel('Deposits'); ylabel('Livestock Holdings');
zlabel('Price');
set(hh,'FaceColor','interp','EdgeColor','interp')

% PLOT RESIDUAL
figure(6)
hh=surf(s{1},s{2},resid(:,:,1)');
title('Approximation Residual');
% COMPUTE EXPECTED STATE AND POLICY PATH
nyrs = 40;
nrep = 1000;
sinit = [Dstar*ones(nrep,1) sstar(2)*ones(nrep,1) ones(nrep,1)];
[spath,xpath] = dpsimul(model,sinit,nyrs,s,x);
s1path = mean(squeeze(spath(:,1,:)));
s2path = mean(squeeze(spath(:,2,:)));
x1path = mean(squeeze(xpath(:,1,:)));
x2path = mean(squeeze(xpath(:,2,:)));

% PLOT EXPECTED STATE PATH
figure(7);
plot(0:nyrs,s1path);
title('Expected Deposits Path');
xlabel('Year');
ylabel('Deposits');

% PLOT EXPECTED STATE PATH
figure(8);
plot(0:nyrs,s2path);
title('Expected Livestock Holdings Path');
xlabel('Year');
ylabel('Livestock Holdings');

% PLOT EXPECTED POLICY PATH
figure(9);
plot(0:nyrs,x1path);
title('Expected Net Additions to Deposits Path');
xlabel('Year');
ylabel('Net Additions to Deposits');

% PLOT EXPECTED POLICY PATH
figure(10);
plot(0:nyrs,x2path);
title('Expected Net Additions to Livestock Holdings Path');
xlabel('Year');
ylabel('Net Additions to Livestock Holdings');
FUNCTION FILE

function [out1, out2, out3] = fpseudo(flag, s, x, j, gamma, alpha, beta, r, p, y, rstar, pstar, ystar, istar);

[ns, ds] = size(s);
[ns, dx] = size(x);
D = s(:, 1);
L = s(:, 2);
i = s(:, 3);
dD = x(:, 1);
dL = x(:, 2);

switch flag
  case 'b'
    out1 = [-D -L ones(ns, 1)];            % lower limit on the actions
    out2 = [inf*ones(ns, 2) 4*ones(ns, 1)];   % upper limit on the actions
  case 'f'
    c = y(i) - dD - p(i).*dL - alpha*dD.^2;
    u = (c.^(1-gamma))/(1-gamma);
    u1 = c.^(-gamma);
    u2 = -gamma*c.^(-gamma-1);
    dcdD = -1-2*alpha*dD;
    dcdL = -p(i);
    dcdDdD = -2*alpha;
    out2 = zeros(ns, dx);
    out3 = zeros(ns, dx, dx);
    out1 = u;
    out2(:, 1) = u1.*dcdD;
    out2(:, 2) = u1.*dcdL;
    out3(:, 1, 1) = u1.*dcdDdD + u2.*dcdD.*dcdD;
    out3(:, 1, 2) = u2.*dcdD.*dcdL;
    out3(:, 2, 1) = u2.*dcdL.*dcdD;
    out3(:, 2, 2) = u2.*dcdL.*dcdL;
  case 'g'
    out1 = [1+r(i).*j.*(D+dD) (L+dL).^beta j*ones(ns, 1)];
    out2 = zeros(ns, ds, dx);
    out3 = zeros(ns, ds, dx, dx);
    out2(:, 1, 1) = 1+r(i).*j;
    out2(:, 2, 1) = beta*(L+dL).^(-beta-1);
    out3(:, 2, 2) = beta*(beta-1)*(L+dL).^(-beta-2);
end