Essays on Applied Spatial Econometrics and Housing Economics

DISSERTATION

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By

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* * * * *

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ABSTRACT

Housing purchase represents one of a household’s most significant economic decisions. The ancient joke in real estate is that the three most important criteria for selecting a house are location, location, and location. This explains the great emphasis of a household on residential location choice when he/she is buying a home. Driven by households’ demand on location, it should also play an important role in determining house prices. As a key determinant in household consumption behavior, locational context or neighborhood effects is worth investigation. This dissertation examines locational/neighborhood effects in the housing market using spatial econometric methods.

The first essay studies the importance of social interactions in a household’s location decision. I argue that individuals prefer interacting with others who have similar socioeconomic backgrounds. The hypothesis that a household desires to find a good community match is tested through the application of a discrete residential location choice model. An unwritten rule in real estate is that one should buy the cheapest house in an expensive neighborhood, which is formally the Tiebout hypothesis that households search for a community where their benefits from local public goods will exceed their local tax costs. The community matching hypothesis and the Tiebout hypothesis have different implications regarding a household’s residential location choice. Community matching implies households will prefer similarity,
while the Tiebout hypothesis implies households will prefer neighborhoods with richer neighbors. I use a nested logit (NL) regression to analyze a household’s residential community decision within Franklin County, OH. As an important input of the NL regression, a set of housing price indices are created through a spatial error model. The regression results support the hypothesis that a household prefers neighbors with like socioeconomic characteristics in almost all of the similarity dimensions and only prefers an affluent neighborhood to a moderate degree.

The second essay employs a spatial autoregressive model (SAR) to estimate housing asset prices. Applying the rational expectations hypothesis, this essay models the current value of a housing unit as the conditional expectation of the discounted stream of housing services accruing to the owner of the house. Based on the importance of location, the value of housing services is determined by neighborhood effects as well as the physical attributes of the property itself. In the existing hedonic literature, the neighborhood effects are only ascribed to prior transactions in the neighborhood. After employing the generalized method of moments (GMM) in estimating the spatial asset pricing model, I find that both expected future transactions and prior transactions in the neighborhood are significant in explaining a house’s price, and the explanation power of future neighborhood transactions is statistically equivalent to that of past neighborhood transactions. The inclusion of expected future transaction prices in the neighborhood takes into account the influence of expected changes in the community and factors these potential changes into the house prices. This is consistent with the forward-looking behavior of households. The forward-looking model generates superior out-of-sample prediction performance relative to the conventional hedonic model.
Dedicated to my husband for his unbounded love and help in many ways.
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I must confess in all humility and sincerity that only I am responsible for the shortcomings of this thesis.
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CHAPTER 1

Introduction

Neighborhood effects provide one channel through which the housing market is spatially interdependent. From a household’s perspective, neighborhood effects involve the extent of social contact with other neighbors. The benefits of social interactions prompt households to search for a neighborhood that maximizes potential social interactions. As houses are attached to their locations, spatial interdependence of house prices in a neighborhood must govern the house price determination process. This dissertation presents two essays, thematically linked, but utilizing different methodologies that attempt to explore the effects of social interactions.

Chapter 2 studies the influence of beneficial social interactions on a household’s location decision. Sociologists and psychologists have pointed out the benefits of social interaction. They have also reached an agreement that people will have better social interaction when they are around with others who share same values and have similar attitudes. Applying cost-benefit analysis, the economic interpretation of these findings is provided as: the benefit of social interaction is based on the similarity of the two individuals involved in the interaction; and the cost of it is the effort and expense that must be spent on the social interaction, which can be measured by propinquity (proximity in physical space). Because of the important facilitative roles
played by propinquity and similarity in the utility function, a household will tend to searches for potential neighbors with similar socioeconomic backgrounds in order to maximize its utility, which leads to one of the foci of chapter 2: the test of households’ searching for similarity.

An unwritten rule in real estate is that one should buy the cheapest house in an expensive neighborhood, which is formally Tiebout’s hypothesis (1956) that households search for a community where their benefits from local public goods will exceed their local tax costs. The hypothesis of searching for similarity and Tiebout’s hypothesis of searching for fiscal surplus have different implications regarding a household’s residential location choice. Similarity search implies households will prefer neighbors with similar income level, while Tiebout’s hypothesis implies households will prefer richer neighbors, which leads to the second focus of chapter 2: the test of households’ searching for fiscal surplus.

A nested logit (NL) regression is employed to analyze the household’s residential location decision within Franklin County, OH. The application of the discrete choice models enables the measurement of the impact of similar and affluent neighbors on the household’s community choice, as well as public services and community attributes on location choices. The NL calculates the probability of a household choosing a certain community; conditional on the set of attributes that determines utility and the budget constraint. Through the budget constraint, the community’s “entry price” becomes a necessary input into the household’s indirect utility function. The community entry price is defined as the premium paid by households in order to locate within this community. It is represented by a set of housing price indices which is constructed through the application of a spatial error geostatistical model. Given the presence
of omitted variables and measurement errors, the spatial error model is effective to control for the correlations among the disturbances and yield more precise estimates of the price indices.

Chapter 3 employs a spatial autoregressive model (SAR) to estimate housing asset prices. The inseparable relationship of a house and its fixed physical location fundamentally distinguishes it from other assets. The dividends derived from housing assets are therefore determined by their physical locations. The locational impacts on the dividends provide a channel that allows the communication of house prices within a common neighborhood. Following Can (1997), the spatial interdependence of residential prices have received rising attention in the hedonic literature. To address the spatial correlation of housing prices, two types of spatial models have been applied in hedonic analysis: the lattice model, which is also called the econometric approach; and the geostatistical model, which is also called as the statistical approach. The essential distinction between the two approaches is that the lattice model explicitly expresses the neighborhood influence in the functional form using predetermined spatial weighting matrices, while the geostatistical model allows the spatial interdependence to reside in the residuals and it estimates the parameterized variance-covariance matrix together with the other coefficients. I adopts the lattice modeling in the second essay.

Under the rational expectations hypothesis, the current value of a housing unit is modeled as the conditional expectation of the discounted stream of housing services accruing to the owner of the house. Because houses do not exist in isolation, neighborhood effects should serve as a determinant in measuring the value of housing services received by the owner. In the existing hedonic literature, the neighborhood effects are
only ascribed to prior transactions in the neighborhood. With the inclusion of prior sales in the neighborhood, only the concurrent neighborhood effects are taken into account, but not the expected future neighborhood effects. If the locational context in a neighborhood is anticipated to change, these effects should be capitalized into the current price of a house. Ignoring the expected neighborhood effects will therefore lead to a biased estimation. For instance, assume that an airport is going to add a runway. Flights using this runway will require a flight path that will pass over a neighborhood previously unaffected by the airport. Even though the new runway has not been added, we should expect to see an immediate impact of this news on the house prices in the neighborhood. In determining a house value, a forward looking household will not ignore anticipated changes in the neighborhood, and neither should the economic model.

The concurrent and expected future neighborhood effects take the form of the weighted current values and expected future values of nearby houses in the rational expectations asset pricing model. The weights are predetermined and based on distances between houses along time dimension and geographic space dimension. This rational expectations price model is then solved for the reduced form which turns into a spatial autoregressive model (SAR) with two spatial lags. The econometric model explicitly accounts for the expectations of future neighborhood effects as well as the concurrent neighborhood effects. I estimate the SAR model using a data set that consists of single-family housing transactions for Franklin County (OH) between 1985 and 1998. Because of the presence of endogenous variables in the SAR model, the best general method of moments (GMM) estimator is used for the estimation.
CHAPTER 2

Residential Location Choice: The Role of a Taste for Similarity

2.1 Introduction

For many households, purchasing a home is one of the most significant economic decisions that they will make. The choice of an optimal level of housing consumption requires that a household gather information regarding the features of potential residences. The characteristics of the housing structure itself will be an important determinant of a household’s choice of residence (Quigley, 1976), but other factors also will influence this decision, such as households’ individual characteristics (Gabriel and Rosenthal, 1989). Recent research has included neighborhood quality as one of the determinants of a household’s residential choice. For example, Friedman (1980) estimated the role of local public services on residential location choice in a multinomial logit framework. The finding of this study suggests local public services and other community characteristics play only minor roles in determining the residential location choices, while the quantity of housing services that the household can obtain in a community plays a major role. Quigley (1985) estimated recently moved renters’ choices in three stages by a nested multinomial logit model. He found that school and public expenditures have small negative effects on the probability of a renter in
choosing a community. In the metropolitan Pittsburgh context, he interpreted these results as households’ preferences toward residences outside the central city. Nechyba and Strauss (1997) included a large set of community quality indicators, such as public expenditures, tax, crime rate, commercial activity, etc., in their logit community choice model. Their results show that an individual households’ location decision is significantly affected by local public services and community entry prices.

In this study, neighborhood quality variables are grouped into two subsets. The first captures the nature of the community by focusing on factors, which describe the natural and sociopolitical environment, such as local tax rates, the crime rate, and parks. The second set describes the inhabitants of the community, which includes such things as the median education level, median income, median education, and median age, capturing the features of the socioeconomic environment.

Neighborhood variables in the second category, such as median income and education measure the quality of a community; however, they do not fully capture all of the features that might attract a potential incoming household. This attraction is based on two factors, the intrinsic features and the compatibility of the community with the household’s own interests. Therefore, the value of the household’s neighbors’ attributes relative to the household matter as well as the absolute values of their characteristics for the household’s location choice. The absolute values represent certain intrinsic features of the community that are directly related to positive externality flows. The relative values may affect the household’s social interactions.

A great deal of research in sociology and psychology points to the benefits of social interaction, as it promotes emotional and physical health (Diose and Mugny, 1984), affects learning and work-related skills (Rogoff, 1990), intensifies social cohesion to
achieve collective efficacy, and generates collective efficacy for children (Sampson, Raudenbush, and Earls (1997). The extent of social contact between two individuals is determined by the perceived cost and expected reward derived from that interaction. The cost is the effort and expense that must be spent on the social interaction, which can be measured by propinquity (proximity in physical space). The reward is based on the similarity of attitudes and values between the two persons in some instances (Sampson, Morenoff, and Earls (1999); Clark (1986); Hagestad and Uhlenberg, 2005) and it is based on complementary needs in other cases (Winch, 1952).¹

In economic terms, the benefits and costs from social interactions may affect an individual’s utility function (Manski, 2000). Because of the important facilitative roles played by propinquity and similarity in the utility function, a household will tend to searches for similarities between him/herself and potential neighbors in order to maximize its utility, which leads to one of the foci of this paper: the test of households’ searching for similarity.² According to the author’s knowledge, there are no economic studies that have explored the possibility that of households’ voluntarily search for similarity (in some economically relevant dimension) for the purpose of beneficial social interactions. This paper explores the impact of similarities and differences between a household and the community’s other residents along the dimensions of

¹The rewards of complementary needs are not the focus of this paper.

²The similarity search hypothesis is also associated with the relative deprivation theory in sociology. Relative deprivation refers to the discontent people feel when they compare their positions to those of other similarly situated and find out that they have less than they think they deserve. This theory implies that households tend to feel less happy if they do not do well as their neighbors, which is consistent with the idea of searching for similarities.
race, income, education and family size (i.e. whether or not the household has children)\(^3\) upon a discrete choice econometric model. The degree of similarity between a household and his/her neighbors take the value of the absolute difference between a household’s characteristics and its potential neighbors’ characteristics.

The second focus of this study is to test Tiebout’s hypothesis (1956); a households search for fiscal surplus. The conventional perspective suggested by Tiebout’s hypothesis that households vote with their feet, implies that households have preferences toward public services, local taxes, and community amenities that influence their choice of housing location. Hamilton (1975) extended Tiebout’s model by allowing a property tax instead of the community-specific head tax in Tiebout’s paper. In Hamilton’s heterogeneous community model, high income households and low income households could co-exist only if the community had very strong zoning rules.\(^4\) Accordingly, the fiscal deficit and the fiscal surplus will be capitalized in house prices. The studies of Tiebout and Hamilton reveal that both high income households and low income households prefer to live with high income households in order to avoid a fiscal deficit and to obtain fiscal surplus.\(^5\) Many previous studies have confirmed the search for affluence through a hedonic capitalization process (Yinger, 1982). The

\(^3\)In the sociology literature, the characteristics that determine households’ similarities are measured or proxied by age, race, sex, income, occupation, education, and family size (for the potential reciprocated exchange of childcare). See Priest and Sawyer (1967), Stutz (1973), Sampson, Morenoff, and Earls (1999), Clark (1986, 1991), Hagestad and Uhlenberg (2005), and Coleman (1990).

\(^4\)Zoning rules are the classification of allowable land use by a government, such as the classification for single-family residential district, the classification for multi-family residential district, the classification for business district, etc.

\(^5\)Fiscal surplus is defined as the excess of local service benefits over local property tax payments. (Hamilton, 1976) Fiscal deficit is defined as the shortage of local service benefits over local property tax payments.
confirmation of the capitalization process only weakly corroborates Tiebout’s hypothesis, while this paper takes a more direct approach. Households’ preference for living in a high income neighborhood is tested by finding a migrant household’s responses to the difference between their likely house value and its potential neighbors’ house values.

A nested logit (NL) regression is employed to analyze the household’s residential location decision within Franklin County, OH. The application of the discrete choice models enables the measurement of the impact of similarity and affluent neighbors on the household’s community choice, as well as public services and community attributes on location choices. The NL calculates the probability of a household choosing a certain community; conditional on the set of attributes that determines utility and the budget constraint. Through the budget constraint, the community’s “entry price” becomes a necessary input into the household’s indirect utility function. The community entry price is defined as the premium paid by households in order to locate within this community. It is represented by a set of housing price indices which is constructed through a hedonic price function (Rosen, 1974). To circumvent the difficulties of omitted variables and measurement errors, a spatial geostatistical model is used to control the correlations among the disturbances and yield more precise estimates of the price indices.

This paper is organized as follows. Section 2 outlines the basic model specifications. In Section 3, the data is described and a statistical summary is provided. The empirical results are presented in Section 4. Section 5 provides detailed interpretations of the empirical results. Finally, the last section offers some conclusions from the analysis.
2.2 Model Specification

This section describes the nested logit model that is employed to analyze a housing market in which households sort themselves among the set of available residential locations on the basis of utility derived from given alternatives (McFadden, 1974) and their budget constraints.

2.2.1 Nested Logit model

The nested logit model is an extension of the conditional logit model or multinomial logit model. It groups the substitutable alternatives into subgroups that allow the variance to differ across the subgroups while maintaining the independence of irrelevant alternatives (IIA) assumption within each group. Suppose, then, the $J$ alternatives can be divided into $L$ subgroups such that the choice set can be written $[C_1, \cdots, C_J] = (C_{1|1}, \cdots, C_{J|1}), \cdots, (C_{1|L}, \cdots, C_{J|L})$, with a choice of subgroups indexed $l = 1, 2, \cdots L$, and alternatives $j = 1, 2, \cdots J_l$ in subgroup $l$. Logically, the choice process can be thought as that of choosing among the $L$ choice sets and then making the specific choice within the chosen set. One way to express the utility that a household derives from choosing a location $j$ in subgroup $l$ is to use the random utility model (RUM). It is defined as:

$$U_{jl} = V_{jl} + \varepsilon_{jl} \quad (2.1)$$

6 A conditional logit (CL) model has been applied before the application of the NL model in this paper. Because the data failed to pass Hausman-McFadden Test (1984), the results from the CL are not presented.

7 This IIA property can be stated as follows: The ratio of the probabilities of any two alternatives is independent of the choice set. The IIA derives from the assumption that the stochastic disturbance terms are independent and identically distributed. In the location choice model, the IIA assumption is likely to be violated if households perceive the destination alternatives as close substitutes. When the IIA property fails to hold, the nested logit (NL) model is an appropriate method of estimation.
where

\[ \varepsilon_{jl} = \text{a random element due to unobserved attributes,} \]

\[ V_{jl} = \text{the deterministic component of the utility of a household for location } j \text{ in subgroup } l, \]

\[ U_{jl} = \text{a household’s conditional utility function (conditional on the decision) in choosing location } j \text{ in subgroup } l. \]

If the household is rational, the location he/she chose must have maximized his/her utility. Therefore, \( U_{jl} \) becomes the unconditional utility function:

\[ U_{jl} = \max(U_k) \tag{2.2} \]

Where, \( k = 1, \ldots, Jl \).

Consequently, the probability of an individual choosing community \( j \) among the total available alternatives \( Jl \) in choice set \( l \) is,

\[ \Pr_{jl|l} = \Pr(U_{jl} - \max(U_k)) = \Pr(U_{jl} \geq U_k, \forall k \neq jl) \tag{2.3} \]

Equation (2.3) means that the conditional probability of alternative \( j \) being chosen given choice set \( l \) is equal to the probability that the utility obtained by the individual from option \( j \) exceeds the utility obtained from any of the other alternatives in the given choice set. Substitution of (2.1) into (2.3) yields:

\[ \Pr_{jl|l} = \Pr(\varepsilon_{jl} + V_{jl} \geq V_k + \varepsilon_k, \forall k \neq jl) = \Pr(\varepsilon_k - \varepsilon_{jl} \leq V_{jl} - V_k, \forall k \neq jl) \tag{2.4} \]
The nested logit model can be derived from the assumption that the residuals \[ ((\varepsilon_{11}, \cdots, \varepsilon_{J1}), \cdots, (\varepsilon_{1l}, \cdots, \varepsilon_{Jl}), \cdots, (\varepsilon_{1L}, \cdots, \varepsilon_{JL})) \] have a generalized extreme-value distribution (McFadden, 1974). Then the conditional probability of choosing community \( j \) given the \( l \) subgroup, \( Pr_{j|l} \), can then be written as,

\[
Pr_{j|l} = \frac{\exp(V_{jl})}{\sum_{k=1}^{J_l} \exp(V_k)} \tag{2.5}
\]

If we further assume that the deterministic component of the total utility, \( V_{jl} \), has a linear form as,

\[
V_{jl} = \beta' x_{jl} + \alpha' Z_l \tag{2.6}
\]

where,

- \( x_{jl} = \) the vector of observed attributes of choice \( C_{j|l} \),
- \( Z_l = \) the vector of observed attributes of the choice set \( l \),
- \( \alpha \) and \( \beta \) = the vector of the unknown parameters.

Substitution of (2.6) into (2.5) yields,

\[
Pr_{j|l} = \frac{\exp(\beta' x_{jl} + \alpha' Z_l)}{\sum_{k=1}^{J_l} \exp(\beta' x_{lk} + \alpha' Z_l)} = \frac{\exp(\beta' x_{jl})}{\sum_{k=1}^{J_l} \exp(\beta' x_{lk})} \tag{2.7}
\]

where, \( I_l = \log(\sum_{k=1}^{J_l} \exp(\beta' x_{lk})) \) is defined as an inclusive value for nest \( l \), which is the expected utility for the choice of alternatives within nest \( l \). The probability of choosing nest \( l \) is derived as,

\[A \text{ generalized extreme-value distribution allows the correlation of error terms within a subgroup, which makes it partially relax the IIA assumption.}\]
Pr_l = \frac{\exp(\alpha'Z_l + (1 - \sigma_l)I_l)}{\sum_{m=1}^{L} \exp(\alpha'Z_m + (1 - \sigma_l)I_m)} \quad (2.8)

where, $1 - \sigma_l$ is a measure of the correlation in unobserved factors within nest $l$.\(^9\)

By the law of probability, the unconditional probability of the observed choice made by a household is,

$$Pr_{jl} = Pr_{j|i} Pr_l = \frac{\exp(\beta'x_{jl}) \cdot \exp(\alpha'Z_l + (1 - \sigma_l)I_l)}{\sum_{k=1}^{L} \sum_{m=1}^{L} \exp(\alpha'Z_m + (1 - \sigma_l)I_m)} \quad (2.9)$$

$$= \frac{\exp(\beta'x_{jl} + \alpha'Z_l - \sigma_lI_l)}{\sum_{m=1}^{L} \exp(\alpha'Z_m + (1 - \sigma_l)I_m)}$$

### 2.2.2 Model Parameterization

In order to parameterize the logit model, further assumptions about the form of the underlying indirect utility function are required. Specifically, a household allocates its income between non-housing consumption and housing consumption, as well as deciding the community to achieve the maximum level of the utility. The direct utility the household receives by choosing to live in a particular community can be defined as a function of housing consumption, non-housing consumption, and his/her tastes. Formally, the indirect utility of household $i$ in choosing community $jl$\(^10\) is defined as:\(^11\)

\(^9\)When $1 - \sigma_l$ equals 1 for all $l = 1, 2, \ldots, L$ (no correlation within nests), the nested logit model becomes a simple conditional logit model.

\(^10\)Community $jl$ is the community $j$ in subgroup $l$.

\(^11\)The price of the non-housing goods is normalized to 1.
\[ U_{i,jl} = U_{i,jl}(h_{i,jl}, c_{i,jl}, z_i) \]
\[
s.t. y_i = c_{i,jl} + P_{i,jl} \]

where,

\[ z_i = \text{household } i's \text{ demographic characteristics which determine the individual's preferences,} \]

\[ y_i = \text{individual } i's \text{ income,} \]

\[ c_{i,jl} = \text{the non-housing consumption of household } i \text{ in community } j,l,^{12} \]

\[ h_{i,jl} = \text{the housing services consumed by household } i \text{ in community } j,l,^{12} \]

\[ P_{i,jl} = \text{household } i's \text{ housing expenditure in community } j,l.^{13} \]

Because the housing expenditure in community \( j,l \) can be artificially decomposed into two categories including the spending on housing structural attributes, \( p_s s_i \) (where \( p_s \) is the composite price of housing structure bundle, and \( s_i \) is the vector of housing structural variables), and the spending on community characteristics, \( p_z Z_{jl} \) (where \( p_z \) is the composite price of community feature bundle, and \( Z_{jl} \) is the vector of community variables, which includes both the specific attributes relevant to community \( j \), and the common attributes relevant to subgroup \( l \)).^{14} 

Therefore, the utility function can be written as,

---

\(^{12}\)Because of the different community entry prices, household \( i's \) non-housing consumption may differ across communities. The same reason holds for the variable \( h_{i,jl} \). Both the non-housing consumption and housing consumption are indexed by the household and the community.

\(^{13}\)The local taxes are not in the budget constraint, but appear as an attribute of housing and are capitalized in house price, \( P_{i,jl} \).

\(^{14}\)The community variables, \( Z_{jl} \), represents the combination of \( x_{jl} \) and \( z_l \) in section 2.1.
\[ U_{i,jl} = U_{i,jl}(c_{i,jl}, s_i, Z_{jl}, z_i) \]  
\[ s.t. \ y_i = c_{i,jl} + p_s s_i + p_z Z_{jl} \]  

where,

\( s_i \) = household i’s consumption bundle of the housing structural attributes,

\( Z_{jl} \) = community jl’s attributes, such as local public service, socio-economic characteristics composition, median income and so on,

\( p_s \) = the composite price of the housing structure bundle,

\( p_z \) = the composite price of the community attribute bundle.

To maximize its utility, household i can choose the quantity of \( c_{i,jl} \), the value of \( s_i \), and the value of the product of \( p_z \) and \( Z_{jl} \), but not the quantity of \( Z_{jl} \). This is because the value of \( Z_{jl} \) is constant for all households who choose community jl in which to reside. Therefore household i’s budget constraint is rewritten as,

\[ U_{i,jl} = U_{i,jl}(c_{i,jl}, s_i, Z_{jl}, z_i) \]  
\[ s.t. \ y_i - p_z Z_{jl} = c_{i,jl} + p_s s_i \]  

The available income that household i can allocate between non-housing consumption and the housing structure bundle is the difference of real income and the expenditure on community attributes of location j.
After maximizing direct utility subject to the budget constraint, and substituting the derived demand functions into the direct utility function, the indirect utility function can be expressed as,\(^\text{15}\)

\[
U_{i,jl} = V_{i,jl}(y_i, p_z Z_{jl}, p_s, Z_{jl}, z_i) + \varepsilon_{i,jl}
\]  

(2.13)

where,

\[
p_z Z_{jl} = \text{the product of } p_z \text{ and } Z_{jl} \text{(expenditure on the community attributes)},
\]

\[
\varepsilon_{i,jl} = \text{the random component of the total utility},
\]

\[
V_{i,jl} = \text{the deterministic component of the total utility, which has a linear function form according to the assumption for equation (2.6)}.
\]

The community variables, \(Z_{jl}\), entering the indirect utility function through the product of \(Z_{jl}\) and \(p_z\), include both community amenities and socio-economic composition of the neighborhood.

The economic interpretation of the relevance of the neighbors’ socio-economic characteristics comes from the independent preferences suggested by Pollak (1976)\(^\text{16}\). This is the so called exogenous or contextual effect (Manski, 1993) in the neighborhood effects literature, and was used to estimate households’ demand in some previous studies.

While the inclusion of \(Z_{jl}\) in the indirect utility function reveals the role that is played by the potential neighbors’ characteristics on the household’s total utility, it

\(^{15}\)In the indirect utility function, the prices, \(p_s\) and 1, substitute for the quantities, \(s_i\) and \(c_{i,jl}\). Because the quantities of \(s_i\) and \(c_{i,jl}\) are under control of individual \(i\), but not the quantity of \(Z_{jl}\), the price of \(Z_{jl}\) does not substitute for the quantity in the indirect utility function.

\(^{16}\)Pollak (1976) stated that one’ preference may be affected by others’ consumption. Therefore, from a household’s point of view, the characteristics of neighbors may be taken as signals of their consumption.
does not describe household’s preference for homogeneous neighbors. If the household values a homogenous community, the homogeneity of the community can be thought as a special good that should also be priced. Consequently, the price of homogeneity should also enter the household’s utility function. However, this paper takes a different approach than pricing homogeneity as a special good; rather, it tests a household’s preference for a homogeneous community directly through the inclusion of dissimilarity in the household’s utility function.\footnote{Pricing homogeneity is different than the match of a particular household with a community. The former is the price of a measure of the similarity or diversity of a community that would permit a match. The latter measures the similarity or diversity between a particular household and a given community. To develop a price for homogeneity or diversity would require fully capturing the aspects of homogeneity or diversity that are included in the market price. But these aspects are not very clear. For example, a perfectly homogeneous community would be attractive to a limited group of similar people. Thus the market price could be low. Alternatively, a heterogeneous community would provide matches to a lot of people, but perhaps not good matches. Thus the market price could be low. In either case, it would be difficult to measure and difficult to develop this price. Therefore, this paper does not take the former approach. The omission of pricing of homogeneity is likely a minor omission.} Specially, the amount of dissimilarity is included in the utility function to indicate the disutility of a household that results from living with heterogeneous neighbors. The quantity of dissimilarity is measured by the absolute difference between the household’s and the neighbors’ amount of a particular characteristics. The utility function (2.13) is extended as following,

\[
U_{i,jl} = V_{i,jl}(p_z Z_{jl}, p_s, Z_{jl}, z_i, y_i, |y_i - y_{jl}|, |z_i - z_{jl}|, |w_i - w_{jl}|, (w_i - w_{jl})) + \varepsilon_{i,jl} \tag{2.14}
\]

where,

\[
\begin{align*}
    z_{jl} & \text{ = community } jl \text{'s demographic characteristics,} \\
    y_{jl} & \text{ = community } jl \text{'s median income level,} \\
    w_i & \text{ = household } i \text{'s house value,} \\
    w_{jl} & \text{ = community } jl \text{'s median house value,}
\end{align*}
\]
\[ |z_i - z_{jl}| = \text{the absolute value of the difference of household } i's \text{ demographic characteristics and the median characteristics of the neighbors in community } jl, \]

\[ |y_i - y_{jl}| = \text{the absolute value of the difference of household } i's \text{ income and the median income of the neighbors in community } jl, \]

\[ |w_i - w_{jl}| = \text{the absolute value of the difference of household } i's \text{ house value and the median value of the houses in community } jl, \]

\[(w_i - w_{jl}) = \text{the difference of household } i's \text{ house value and the median value of the houses in community } jl. \]

Three absolute difference terms are created to test a household’s preferences for similarity. The use of them captures the desires of a household to match the neighborhood in multiple dimensions. The absolute values of the differences are constructed for the purpose of avoiding the confusion arising from the signs of the estimated coefficients. This study focuses on households’ preferences over similarity, therefore; only the absolute value of the difference of a household and its neighbors is relevant for this paper.

Tiebout’s hypothesis is tested by including \((w_i - w_{jl})\), the difference of household \(i's \) house value and the median house value in a community \(j_l\). The smaller the value of \((w_i - w_{jl})\) is, the more likely that household \(i\) has a relatively smaller house than most of the houses in community \(j_l\). If the estimated coefficient of \((w_i - w_{jl})\) has a negative sign, Tiebout’s hypothesis is confirmed. In other words, households prefer to live in an affluent community.\(^{18}\)

\(^{18}\)The negative sign of \((w_i - w_{jl})\) indicates that affluent community is in general attractive to potential households, while Tiebout hypothesized that poor households prefer to live in a rich community in order to capture the fiscal surplus, and rich households also like to live in a rich community to avoid a fiscal deficit.
The probability of choosing a community \( jl \) can be computed for the NL model by substituting the linear function of \( V_{i,jl} \), which is indicated by equation (2.14), into equation (2.9). Because \( z_i, y_i, \) and \( p_s \) are constant across alternative communities, they will no longer remain in the probability function. Therefore, the vector of the explanatory attributes in the probability function is composed of \( p_z Z_{jl}, Z_{jl}, |y_i - y_{jl}|, |z_i - z_{jl}|, |w_i - w_{jl}|, \) and \( (w_i - w_{jl}) \).

The community entry price, \( p_z Z_{jl} \), in the indirect utility function needs to be constructed for each community in the choice set. The hedonic house price estimation method is employed to handle this issue (Rosen, 1974).

### 2.2.3 Hedonic House Price Index

#### Hedonic Price Function

According to Rosen, the conceptual form of the hedonic function is a relationship between the bidding function from consumers and the offering function from the suppliers in the housing market. In other words, consumers bid for structural components of housing units and packages of neighborhood amenities in order to maximize their utility; suppliers maximize their profits by offering different housing unit packages to the market. Considering the market process described above, the variation in transaction prices can be explained as a function of property characteristics. In this paper, the structural characteristics of housing units, neighborhood socioeconomic attributes, community features and public services enter the hedonic price function (the subscript \( l \) indicating the subgroup is left out for convenience):\(^{19}\)

\(^{19}\)In the approach of equation (2.15), \( school_j \) and \( Z_{j(i)} \) are employed to indicate the community quality that is capitalized into the house price. The author also tried another approach that leaves out \( Z_{j(i)} \), leaving only the dummy variables, \( school_j \), to capture the differences of non-structural attributes of a house across communities. The results show that the first approach fits better. This
\[ w_i = w(school_j, s_i, Z_j(i)) + v_i \]

where,

\[ school_j = \text{dummy variable indicating school district } j, \]

\[ w_i = \text{the transaction price}^{20} \text{ of household } i's \text{ house}, \]

\[ Z_j(i) = \text{the community variables}^{21}, \]

\[ s_i = \text{the vector of the structural attributes of household } i's \text{ house}, \]

\[ v_i = \text{the independent and identically distributed (IID) random error term.}^{22} \]

Due to the durability and the fixed position of a house, it has been broadly accepted in the housing literature that the geographic location of a house has nontrivial impact on its economic value. The inclusion of \( Z_j(i) \) and \( school_j \) is to test for capitalization of the effects of the geographic locations on house values. According to Can (1992),\(^{23}\) the elements of \( Z_j(i) \) that denote the neighbors’ socio-economic characteristics test for the capitalization of the adjacency effects. Neighbors’ socio-economic characteristics can be interpreted as the signals of maintenance decisions that may result from the utilization of more complete information in the first approach. The result of the second approach is not presented in this paper, but can be provided upon request.

\(^{20}\)All the houses in the data set used here are owned by households, therefore, rents are not considered.

\(^{21}\)In the hedonic regression, the neighbors’ socio-economic characteristics and community amenities are not distinguished from each other. They are all expressed by \( Z_j(i) \). Due to the merger of three data sets, some variables of \( Z_j(i) \) are on the census tract level, varying across census tracts within a school district; while some other variables of \( Z_j(i) \) are on the school district level that is constant within a school district. Therefore, \( Z \) is indexed by both the school district and household.

\(^{22}\)The IID assumption is relaxed in the section of Spatially Autocorrelated Error Terms in the Hedonic Regression.

\(^{23}\)Can (1992) separated location effects into two categories: The first one is called neighborhood effects, which is the effects from the absolute geographic location of a house, such as the public services and accessibilities, and can be capitalized into the house value as a premium or through varying implicit prices of attributes. The other one is adjacency effects, which are associated with the externalities from the surroundings of the housing unit.
affect the market value of a given house. Other elements of $Z_{j(i)}$ and variable $\text{school}_j$, capture the neighborhood effects of a house’s geographic location.

To parameterize the hedonic function, a linear form is assumed as,

$$w_i = \alpha_0 + \sum_{j=1}^{J-1} \alpha_{0j} \text{school}_j + \alpha_1 Z_{j(i)} + \beta s_i + v_i \quad (2.16)$$

where, $\alpha_0$, $\alpha_{0j}$, $\alpha_1$, and $\beta$ are the coefficients to be estimated, while $\hat{\alpha}_0$, $\hat{\alpha}_{0j}$, $\hat{\alpha}_1$, and $\hat{\beta}$ indicate the estimated coefficients.

The hedonic regression reveals the marginal price of each attribute given the linear hedonic form, but tells nothing about the demand function or the supply function; therefore, it is only a market clearing function determined by the equilibrium market-clearing price.

**Hedonic Price Index**

Upon the selection of the functional form and the appropriate determinants of housing values, the hedonic function is estimated, and the estimated coefficients are applied to construct the house price index for each school district. The price index is,

$$\hat{w}_j = \sum_{j=1}^{J-1} \hat{\alpha}_{0j} \text{school}_j + \hat{\alpha}_1 \hat{Z}_j \quad (2.17)$$

Where,

24For the easy interpretation of the parameters, a linear function is used. A semi log form has also been tested by the author. The results are highly consistent with the results of the linear regression. Furthermore, both the price indexes created from the semi log form or the linear form are applied in the NL model, and similar estimates are generated. The results of the semi log form are not presented in this paper, but it is available from the author under request.
\( \hat{w}_j = \text{the price index of school district } j \),

\( \hat{\alpha}_{0j} = \text{the estimated parameter of } \alpha_{0j} \),

\( \hat{\alpha}_1 = \text{the estimated parameter of } \alpha_1 \),

\( \overline{Z}_j = \text{neighborhood’s average characteristics in community } j \).

In this formulation, the price index, \( \hat{w}_j \), reflects spending on community attributes, which is one part of the entire expenditure on the house. This corresponds to the community premium denoted before. The price index defined in this paper is different from the conventional ones, because it excludes the spending on the constant-quality housing structures. The reason for making this change is to be consistent with the input element in the indirect utility function. Because the excluded part of the conventional price index is a constant number, using \( \hat{w}_j \) instead of the standard measure in the NL model does not make any difference to the results.

**Spatially Autocorrelated Error Terms in the Hedonic Regression**

The precision of the price indices can be affected by a number of factors, such as the functional form of the hedonic regression and the set of influential explanatory variables excluded from the function. This section addresses the issue of the spatially dependent error terms in order to achieve more efficient coefficient estimates and unbiased estimates of the standard errors. The assumption of IID of the stochastic error

\[ \hat{W}_j = \hat{\alpha}_0 + \sum_{j=1}^{J-1} \hat{\alpha}_{0j} \text{school}_j + \hat{\alpha}_1 \overline{Z}_j + \hat{\beta} \pi. \]

The price index, \( \hat{w}_j \), is the value of \( \hat{W}_j \) deducting a constant term, \((\hat{\alpha}_0 + \hat{\beta} \pi)\), which represents the spending on a standard housing bundle of structural attributes. According to the indirect utility function, the price index, \( \hat{w}_j \), is the entry price of community \( j \), and will be used in the logit models.
term in the hedonic function does not hold because of the possibility of measurement errors and omitted variables.

Among the determinants of house values, housing structure variables are usually measured with little error, but it is likely that many of them are omitted. The location variables may not be fully observed, and even they can be observed, some measurement errors are likely to occur, which will leave the residuals produced by the hedonic equations spatially correlated. It is reasonable to expect that the correlation between residuals are determined by the proximity of observations, given that nearby house units share the same neighborhood, which tend to create similar errors in measuring the attributes of the neighborhood. The strength of the relationship diminishes as the distance between the observations increases. To address this problem, this study models the spatial correlation of the error terms explicitly through the use of a geostatistical model (Dubin, 1988). The covariance matrix of the error terms is defined as,

\[
E\{vv'\} = \sigma^2 K = \Omega 
\]

(2.18)

Where,

\[ K \] = the correlation matrix, \\
\[ \Omega \] = the covariance matrix with nonzero off-diagonal elements.

To estimate the covariance matrix, \( \Omega \), a semivariogram model can be employed, which expresses the variance of the difference between the values of the regionalized variables as a function of separation distance. The process is,
\[ \gamma(L_i - L_j) = 0.5 \text{var}\{v(L_i) - v(L_j)\} \]
\[ = C(0) - C(L_i - L_j) \]  \hspace{1cm} (2.19)
\[ = \sigma^2(1 - K_{ij}) \]

where,

\[ L_i = (x_i, y_i) \] denotes the location of property \( i \) (\( x_i \) indicates the latitude, and \( y_i \) indicates the longitude of property \( i \)),

\[ C(L_i - L_j) = \text{cov}\{v(L_i), v(L_j)\} \] denotes the covariance of residual \( v_i \) and residual \( v_j \),

\( C(0) \) denotes the variance.

To explicitly model the semivariogram process, the functional form of the semivariogram needs to be specified. This paper applies the spherical model, which is,

\[ \gamma_{ij} = \begin{cases} 
    c_0 \left( \frac{3d_{ij}}{2a_0} - \frac{d_{ij}^3}{a_0^3} \right) & \text{if } i \neq j \text{ and } 0 < d_{ij} < a_0 \\
    0 & \text{if } i = j \\
    c_0 = (\sigma^2) & \text{if } d_{ij} > a_0 
\end{cases} \]  \hspace{1cm} (2.20)

where,

\( c_0 \) and \( a_0 \) = parameters that need to be estimated in the spherical function

\( d_{ij} \) = the distance between property \( i \) and property \( j \).

The parameters in the semivariogram function and the coefficients of the hedonic regression are estimated simultaneously through a maximum likelihood method. The price indices can then be constructed using the more efficient estimates.
2.3 Data Description

There are several sources for the data used in this essay. The Center for Urban and Regional Analysis (CURA, 1995 and 2001\textsuperscript{26} ) provides two surveys, which matches housing transaction data and households’ characteristic variables. National Center for Education Statistics (NCES) provides neighborhood socio-economic characteristics at the school district level, and census tract level variables come from the U.S. Census. The Ohio Department of Education (ODE) provides community school quality data. The Bureau of Justice Statistics (BJS) provides the crime index. The Franklin County Treasurer provides property tax data. All monetary values are converted to a base period value (2001 quarter 1) for the control of inflation. The combinations of the data sets are used to estimate the residential location choice model and the hedonic price model. The two surveys are the 2001 Housing Survey in Franklin County (pre-September 11), and the 2001 Housing Survey in Franklin County (post-September 11). There are three reasons for combining these two data sets. First, these two surveys were collected in very close time periods. Second, there is no overlap of the sample objects. Third, the questionnaires of these two surveys are same, except for some additional questions about the opinions of households’ attitudes towards terrorists that are not the concern of this paper. The conjunction of these two data sets increases the number of total observations substantially.\textsuperscript{27}

\textsuperscript{26} The data from CURA contains both households’ socio-economic information and the housing unit structural characteristics. The former is for the estimation of the NL model; the latter is for the estimation of price indices.

\textsuperscript{27} Originally, there are 1258 observations in the pre-September 11 survey, and 803 observations in the post-September 11 survey. The combination of them gives 2,061 observations. (These are the raw numbers before the data cleaning process.)
The observations in the CURA data set were recorded in 2001, but most transactions happened before that time, which means that the variables describing housing structures and households' characteristics may not be the same as when the houses were bought. In other words, the residents in the data set are not necessarily recent movers.\textsuperscript{28} This suggests that a household's past choices of a community may not be the same as what his/her current choice would be if they relocated, but they have not relocated due to transaction costs. To circumvent this incompatibility, a subset of the recent movers is employed to retest the hypothesis. The date that is used for the nested logit model is described as following.\textsuperscript{29}

Although Franklin County contains 16 major school districts,\textsuperscript{30} only 11 of them have sufficient observations to be used for the estimation of the residential location choices. They are: the Bexley city school district, the Columbus city school district, the Gahanna Jefferson city school district, the Groveport Madison local school district, the Reynoldsburg city school district, the Hilliard city school district, the Southwestern city school district, the Upper Arlington city school district, the Dublin city school district, the Westerville city school district, and the Worthington city school district. Consistent estimates are still available according to McFadden (1978).\textsuperscript{31}

\textsuperscript{28}This feature of our data has both pros and cons. As a random sample of population, sample selection bias is eliminated. On the other hand, being observations of new movers, the observed characteristics of households may not be consistent with their choices that were made before.

\textsuperscript{29}Data for the hedonic price function is reported in Appendix A1.

\textsuperscript{30}There are 22 school districts listed under Franklin County. Because some school districts are primarily located outside Franklin County, only 16 school districts are considered when the price indices are created. Because of insufficient observations, 5 school districts are left out in the NL model. They are the Canal Winchester local school district, Grandview heights school district, Hamilton local school district, Plain local school district, and Whitehall city school district.

\textsuperscript{31}McFadden (1978) suggested that the discrete choice model may be estimated by estimating a sub-sample of the alternatives from the full choice set. As long as an alternative has the logical possibility of being an observed choice, given it is included in the assigned set, consistency holds.
The data set that is used to estimate the NL model contains 1,467 observations in total. The definitions and statistical descriptions of the variables in the estimation of the location choice model are provided in Table (2.1) and Table (2.2). Table (2.1) contains some community variables as well as the set of price indices constructed from ML estimates (GEOPIND). These estimates were constructed from the hedonic regression, which is explained in detail in Appendix A2. Table (2.2) displays the key variables of this study, representing the dissimilarity of a household and its neighbors. A household’s preference over a homogenous community is studied through five categories: the difference in education background, the difference in number of children, the difference in race, the difference in income, and the difference in house value.

The indicators of dissimilarity are used to measure how different a household is from its neighbors. If households prefer to group themselves into homogenous neighborhoods, the signs of those coefficients on these indicators are expected to be negative. The coefficient on DFMASTER measures how the location decision of a household with a Masters degree is affected by the percentage of households whose highest level of education is not a Masters degree. The variable DFCHILD is included to test whether a household with children is more attracted to a community where

\[32\] The discrepancy of the observations numbers for the hedonic and the logit models come from missing variables in the CURA survey. Among the 2,061 observations in the CURA survey, only 962 of them have complete information about house structural variables needed for the hedonic estimation.; however, 1,467 observations have complete information of the household’s socio-economic characteristics, these needed in the estimation of location choice.

\[33\] The household head’s socio-economic characteristics are used to compute the difference. To use the household head’s value instead of the family average is a very common approach in the empirical studies of the housing market. This occurs primarily due to data limitations.

\[34\] The construction of the difference in house value requires every observation to have a transaction price variable. Therefore, due to the missing variables, some observations need to be deleted whenever a transaction price variable is included in the estimation, which gives us 731 total observations. The statistical description of the data with the variable of the housing value difference and the data without it are presented separately, as well as the regression results.
Panel A: Definition of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOPIND</td>
<td>Price index of geostatistical model (not standardized)</td>
</tr>
<tr>
<td>TAX</td>
<td>Property tax rate, in percentage, in the township</td>
</tr>
<tr>
<td>PUPILEXP</td>
<td>Education expenditure per pupil in 10,000 dollars in school district</td>
</tr>
<tr>
<td>CRIME</td>
<td>Crime index divided by population in the police jurisdiction</td>
</tr>
<tr>
<td>CBD</td>
<td>Distance of centroids of school districts to the CBD in miles</td>
</tr>
<tr>
<td>AVGSC9</td>
<td>Average of math, reading, and writing of the Ohio 9th-grade proficiency test</td>
</tr>
<tr>
<td>MHVALUE</td>
<td>Median house value in 10,000 dollars (from census)</td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Value</th>
<th>Std.Deviation</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOPIND</td>
<td>15.33</td>
<td>3.04</td>
<td>12.33</td>
<td>23.83</td>
</tr>
<tr>
<td>TAX</td>
<td>1.75</td>
<td>0.23</td>
<td>1.48</td>
<td>2.44</td>
</tr>
<tr>
<td>PUPILEXP</td>
<td>0.62</td>
<td>0.10</td>
<td>0.49</td>
<td>0.85</td>
</tr>
<tr>
<td>CRIME</td>
<td>5.33</td>
<td>1.54</td>
<td>1.34</td>
<td>11.81</td>
</tr>
<tr>
<td>CBD</td>
<td>8.38</td>
<td>2.78</td>
<td>0.90</td>
<td>14.41</td>
</tr>
<tr>
<td>AVGSC9</td>
<td>87.87</td>
<td>8.95</td>
<td>64.1</td>
<td>98.1</td>
</tr>
<tr>
<td>MHVALUE</td>
<td>6.77</td>
<td>1.72</td>
<td>4.54</td>
<td>10.11</td>
</tr>
</tbody>
</table>

Regarding variable, GEOPIND, Table (2.9) reports the standardized price indices. The summary report of GEOPIND in this table, which are the entry prices in the NL model, are not the standardized price index, but the original fitted values of the hedonic model. The reason of doing this is to avoid the errors caused by the standardization procedure.

Table 2.1: Descriptive Statistics for Community Variables in NL
### Difference Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sample size: 1,467</th>
<th>Mean Value</th>
<th>Std.Deviation.</th>
<th>Sample size: 731</th>
<th>Mean Value</th>
<th>Std.Deviation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFLESSHS</td>
<td></td>
<td>0.79</td>
<td>0.26</td>
<td>0.08</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>DFHS</td>
<td></td>
<td>0.12</td>
<td>0.28</td>
<td>0.11</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>DFCOLLEGE</td>
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<td>0.32</td>
<td>0.20</td>
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<tr>
<td>DFBACHELOR</td>
<td></td>
<td>0.22</td>
<td>0.34</td>
<td>0.23</td>
<td>0.35</td>
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<tr>
<td>DFMASTER</td>
<td></td>
<td>0.16</td>
<td>0.33</td>
<td>0.15</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>DFCHILD</td>
<td></td>
<td>0.26</td>
<td>0.32</td>
<td>0.33</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>DFWHITE</td>
<td></td>
<td>0.12</td>
<td>0.09</td>
<td>0.12</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>DFBBLACK</td>
<td></td>
<td>0.06</td>
<td>0.23</td>
<td>0.07</td>
<td>0.24</td>
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<tr>
<td>DFINC</td>
<td></td>
<td>3.74</td>
<td>3.59</td>
<td>3.67</td>
<td>3.61</td>
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</tr>
<tr>
<td>DFHVAL</td>
<td></td>
<td>-</td>
<td>-</td>
<td>3.13</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>DFHVALTIE</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-0.07</td>
<td>4.85</td>
<td></td>
</tr>
</tbody>
</table>

*a* A household’s highest education achievement is categorized into less than high school, high school, some college degree, bachelor’s degree, and master’s degree. For whatever level of education the household achieves, we subtract the percentage of the population with that level from 1 and assign it to the household. All others are assigned zero. For instance, if a household has attained some college, and 45% of the population has this level, the DFCOLLEGE will be assigned 0.55, while DFLESSHS, DFHS, DFBACHELOR and DFMASTER will be assigned zero. By construction, these variables are always between zero and one.

*b* For a household that has children, DFCHILD is the absolute difference of one and the percentage of households that have children in the school district. For a household that has no children, DFCHILD is zero.

*c* For a white household, DFWHITE is the difference between 1 and the percentage population that is white in the neighborhood and DFBBLACK is zero; for a black household, DFBBLACK is the difference between 1 and the percentage of population who is black and DFWHITE is zero.

*d* DFINC is the absolute value of the income difference between the household’s and the median neighbor’s (scaled by $10,000), and is always positive.

*e* DFHVAL is always positive, but DFHVALTIE can be negative or positive.

---

Table 2.2: Descriptive Statistics for Characteristic Difference between Household and Neighbors in NL
most of his/her neighbors also have children. The variable DFINC gives a measure of the dissimilarity of household and its neighbors in regards to income. In the same vein, the coefficients on DFWHITE and on DFBBLACK capture the disutility caused by moving into a community where the composition of ethnicities is different from a household’s own race.

In order to distinguish the hypothesis of households’ preferences for similarity from Tiebout’s hypothesis that households prefer to live in a community with large houses, a pair of variables are created: DFHVAL and DFHVALTIE. The former is the absolute difference of a household’s house value and its median neighbor’s, while the latter is the simple difference of house values. A negative sign of DFHVAL is expected if households have a preference for similarity. According to Tiebout (1956) and Hamilton (1975), households prefer to buy a small house in a community with many relatively large houses, for the purpose of capturing a fiscal surplus. The variable DFHVALTIE is expected to have a negative sign because positive values of DFHVALTIE imply the household would be subsidizing others in the neighborhood through property tax payments. Because of the high correlation of DFINC and DFHVAL (rich households intend to purchase more expensive houses, while poor households can only afford cheap houses), we expect it be difficult to separate the effects of these two variables.

There are a few competing hypotheses that might explain why households tend to group with similar neighbors. One of these competing hypotheses is based on the effect of zoning and limitations to the supply of housing. If a uniform zoning rule is imposed in a school district for residential lots, a homogenous community should be expected. Because there are multiple types of zoning rules governing residential housing in each
As a result, the composition of households within each community is not homogenous. This can be seen from the census data. Figure (2.1), Figure (2.2), and Figure (2.3) provide some evidence of the diversity in several sample school districts. While income levels tend to be greater in the Worthington and Southwestern school districts than in Columbus, there is a wide distribution in all areas. A similar results is observed for house values and family sizes.

Another competing hypothesis is that homogeneity is due to income segregation, which is predicted by urban economic models. This hypothesis predicts that households with similar incomes tend to group together, but it predicts nothing about

\[35\] A zoning map of Franklin County can be provided by the author upon request.
Figure 2.2: Household Income Distribution

Figure 2.3: Household Size Distribution
their preferences along the other socio-economic dimensions. If the hypothesis of similarity search is corroborated in more than just income dimension, then the income segregation hypothesis does not completely describe the results.

If black households select to live with other black households, it either suggests the pure preferences of blacks for black culture, or it might be caused by a constrained choice set faced by black households, due to racial discrimination. For example, Yinger (1998) documents that real estate agents engage in racial steering. This study can not separate discrimination from preferences as the cause of racial segregation.

2.4 Estimation: Results

In this model, the eleven possible community choice alternatives are divided into two subgroups that comprise the central city nest, and the suburbs. Given the geography of Franklin County, the nest of the central city consists of the Columbus City School District, while the nest of the suburbs consists of the rest of the school districts.\(^\text{36}\) The results of the NL regression are reported in Table (2.3).\(^\text{37}\)

Column I shows the results of the restricted model on a large sample, while column II displays the results of the full model with the house value difference variables. Because of the small sample size of the full model, column II in Table (2.3) excludes some insignificant variables to enable the convergence of the iterations. Because those insignificant variables include most of the difference variables, our discussion is mainly

\(^{36}\) Other nesting structures have been tested besides the one that is reported in this paper. Comparing the estimated parameters of the inclusive values, \(1 - \sigma_i\), only the nesting structure reported provides the most appropriate value, which is between 0 and 1 (Greene 1997).

\(^{37}\) Results of the hedonic price function are reported in Appendix A2. The set of price indices (GEOPIND) constructed using the estimates of the hedonic function enter the NL regression as one of the explanatory variables.
based on the restricted model. The major purpose of reporting the full model results is to discover the effects of variable DFHVAL and variable DFHVALTIE.\textsuperscript{38}

\textsuperscript{38}Because variable DFHVALTIE, \((w_i - w_{jt})\), is not available in the restricted model, we include MHVALE, which measures \(w_{jt}\), to explore a household’s preference for communities with large houses.
<table>
<thead>
<tr>
<th>Specification Variable</th>
<th>I Restricted Model (obs:1467)</th>
<th>II Full Model (obs: 731)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Dissimilarity Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFLESSHS</td>
<td>4.43** (1.81)</td>
<td>-</td>
</tr>
<tr>
<td>DFHS</td>
<td>2.57** (1.07)</td>
<td>-</td>
</tr>
<tr>
<td>DFCOLLEGE</td>
<td>6.89** (2.94)</td>
<td>-</td>
</tr>
<tr>
<td>DFBACHELOR</td>
<td>-7.38*** (0.91)</td>
<td>-4.35*** (1.03)</td>
</tr>
<tr>
<td>DFMASTER</td>
<td>-9.95*** (1.17)</td>
<td>-1.77 (1.50)</td>
</tr>
<tr>
<td>DIFCHILD</td>
<td>-12.05*** (1.41)</td>
<td>-1.85 (1.36)</td>
</tr>
<tr>
<td>DFINC</td>
<td>-0.12*** (0.03)</td>
<td>-0.04 (0.05)</td>
</tr>
<tr>
<td>DFWHITE</td>
<td>-1.40 (0.95)</td>
<td>-1.72 (1.15)</td>
</tr>
<tr>
<td>DFBBLACK</td>
<td>-3.15* (1.71)</td>
<td>-1.36 (1.88)</td>
</tr>
<tr>
<td>DFHVAL</td>
<td>-</td>
<td>-0.15*** (0.01)</td>
</tr>
<tr>
<td>DFVALTIE</td>
<td>-</td>
<td>-0.46*** (0.04)</td>
</tr>
<tr>
<td>Panel B: Community Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEOPIND</td>
<td>-1.84*** (0.09)</td>
<td>-0.37*** (0.10)</td>
</tr>
<tr>
<td>PUPILEXP</td>
<td>47.38*** (2.13)</td>
<td>6.46** (2.56)</td>
</tr>
<tr>
<td>CBD</td>
<td>-0.012 (0.03)</td>
<td>0.05 (0.04)</td>
</tr>
<tr>
<td>PTAX</td>
<td>-17.59*** (0.59)</td>
<td>-8.55*** (0.56)</td>
</tr>
<tr>
<td>PCRIME</td>
<td>-0.25*** (0.03)</td>
<td>-0.13*** (0.03)</td>
</tr>
<tr>
<td>AVGSC9</td>
<td>0.14*** (0.02)</td>
<td>-0.08*** (0.02)</td>
</tr>
<tr>
<td>MHVALUE</td>
<td>1.28*** (0.08)</td>
<td>-</td>
</tr>
<tr>
<td>Panel C: IV Parameters (1 – σl)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central City</td>
<td>1 (Fixed Parameter)</td>
<td>1 (Fixed Parameter)</td>
</tr>
<tr>
<td>Suburbs</td>
<td>0.94*** (0.01)</td>
<td>1.03*** (0.01)</td>
</tr>
</tbody>
</table>

Numbers in parentheses correspond to standard errors. * indicates significance at 10% level; ** indicates significance at 5% level; *** indicates significance at 1% level or less.

Table 2.3: Estimates of the Coefficient in the NL Model
Recall that when the IV parameter is 1 a nested logit model becomes a conditional logit model. The estimated IV parameters of the NL model found in Table (2.3) are very close to 1, which explains why Hausman-McFadden Test is only partially passed for the CL model.\textsuperscript{39}

In column I of Table (2.3), more than half of the dissimilarity variables perform well with the expected negative sign, which confirms the hypotheses in this paper and suggests that households are more attracted to a similar neighborhood. The community variables also perform well. Other estimation results are consistent with previous studies.

1. Preference on the similarity of education background

Among the five categories of education levels, the dissimilarity measures of the two highest education level, DFMASTER and DFBACHELOR have the expected negative coefficients, but the other three have significant positive estimates. The negative signs of DFMASTER and DFBACHELOR suggest that a household with a master’s degree or a bachelor’s degree tends to group with others who have a similar educational background, while the positive signs of DFLESSHS, DFHS, and DFCOLLEGE indicate that a household without a bachelor’s degree would be discouraged in living with someone else who is equally educated as he/she is. The former finding is consistent with the hypothesis in this paper. But the latter one seems to contradict the similarity hypotheses. To test whether the households in the three lower level educational categories would prefer an educationally heterogeneous neighborhood or

\textsuperscript{39}Actually, the point estimates of all other coefficients as well as their standard errors in both the CL model and the NL model are nearly indistinguishable. The estimates of the CL model are available from the author upon request.
would be more interested in choosing better educated neighbors because of the positive externality of education, two other variables that indicate the neighborhood education level (variable SDBA is the percentage of the population in the school district that has a Bachelor’s Degree and the variable SDMA is the percentage of the population in the school district that has a Master’s Degree) are added in the NL model. After the inclusion of these two new variables, DFLESSHS, DFHS, and DFCOLLEGE are no longer significant, while SDBA, and SDMA are significantly positive, and DFMASTER and DFBACHELOR keep their negative signs. These new results suggest that a household is in general more attracted by a better educated neighborhood, but discouraged by a poorly educated one, no matter whether he/she is well educated or not. Additionally, the unchanged negative coefficients on DFMASTER and DFBACHELOR suggest that the general preference of a household over a better educated neighborhood does not dominate the preference of a highly educated household over a residency with homogenously educated residents. In other words, a household with a bachelor’s degree or a master’s degree is not only attracted by a well educated neighborhood area, but also interested in living with others who are equally educated as he/she is, while a household with little education would prefer to choose better educated neighbors.

For those well educated households, similar high education background implies more common interests for those residents, which in turn creates a more interactive community. These households enjoy a higher utility in a highly interactive community in contrast to a segmented one. In contrast, relatively poorly
educated households, appear to prefer trading a homogenous neighborhood for a positive externality flow of education.

2. Preference for households with children

The negative sign of the coefficient on DFCHILD indicates that a household that has children is less likely to choose a community with few children per household.\textsuperscript{40} The needs of reciprocated exchange of childcare could be one of the motivations of households with children for similar neighbors. When a family chooses a residence, the parents come up with a question naturally: Will there be other children nearby for my children to play with? This consideration should lead to the preference of the with-children households for a neighborhood with a lot of other with-children residents. Residing in this kind of community, parents can exchange advice and information about childbearing and help each other with the task of controlling children, while children can socialize with each other in their social network.

3. Preference for the similarity of income

The negative sign of the coefficient on DFINC is consistent with the hypothesis that households prefer to group themselves within a neighborhood of similar income households. Because income works as a signal, indicating hobbies and interests of households, seeking similar income neighbors is another way to approximate an interactive community and therefore create greater gains from the social interaction in the neighborhood.

\textsuperscript{40}The opposite also holds. The author replaced DFCHILD by the variable, DFSINGLE, whose coefficient measures the preference of single families, and the results provided the similar conclusion: a single prefers a community with more singles.
4. Preference for the similarity of races

The coefficient on DFBBLACK is negative and significant, which implies that a black household is more likely to live with blacks. The coefficient on DFWHITE is negative but not significant, which implies that white households do not have a particular preference regarding their neighbors’ races. The insignificance of DFWHITE might result from the approximation of the percentages of white households across the sample school districts. The negative coefficient on DFBBLACK may suggest another explanation for the racial segregation phenomenon. Given results of previous studies that suggested black households’ preference over racially integrated neighborhoods, the findings in this paper point to another possible explanation. The segregation of black households and white households might be not caused by racial discrimination, but households’ own preferences over the neighbors’ ethnicities. This self-selection might result from comfort with the culture, which implies that a racially segregated community is more interactive than an integrated one for the members in it. Of course, black households’ choice might not necessarily suggest their preference, but their inability to locate in a white neighborhood, which is caused by racial discrimination. Both of the explanations are consistent with the findings in this paper, thus further study will be required.

5. Preference on the community attributes

The percentage of white households ranges between 72.8 percent and 94.7 percent for the 11 sample school districts. Meanwhile, the percentage of black households ranges between 2.1 percent and 21.2 percent. Obviously, the variation of the latter is much higher than the former. This might explain the statistical significance of DFBBLACK, and the statistical insignificance of DFWHITE.
All of the community variables have the expected significant signs. The positive sign of the coefficient on PUPILEXP suggests the greater educational expenditures per pupil; the more attractive is a community. The positive sign of AVGSC9 implies that the better the scholarly performance of students, the more desirable is a community. For parents who have children, greater school spending and a high proficiency score signal better student performance. For households without school age children, greater school spending and high proficiency score imply a better investment value of houses in the school district.\(^{42}\) The negative coefficients on TAX, CRIME and GEOPIND indicate that high tax rate, high crime rate, or high entry price of a school district make a community less attractive. The positive sign of MHVALUE suggests that households are attracted to a community with a relatively high median house value, which is consistent with Tiebout's hypothesis. The coefficient on CBD is not significant, implying the distance between the property and the CBD does not affect a household's choice.

Column II of Table (2.3) provides the results of complete model that includes the house value differences variables, most of which are consistent with the results in column I of Table (2.3), except some variables that are no longer significant. This is partially because of the reduced sample size. Another reason is that the variable DFHVAL picks up the effects of DFINC and DFMASTER. Because households' income differences and education differences are highly correlated to their house value differences, it is not surprising to see that the coefficients of DFINC and DFMASTER

\(^{42}\)The households with no children can easily extract the benefits of the greater school spending through selling the house to some family that have school age children.
are no more significant after the inclusion of DFHVAL. The negative coefficient on DFHVAL confirms the similarity hypothesis, while the negative sign of DFHVALTIE confirms Tiebout’s hypothesis. If household \(i\)'s house value, \(w_i\), is greater than the average house value in the community \(jl\), \(w_{jl}\), $10,000 increase of \(w_i\) will decrease \(i\)'s total utility by 0.61.\(^{43}\) If household \(i\)'s house value, \(w_i\), is less than the average house value in the community \(jl\), \(w_{jl}\), $10,000 increase of \(w_i\) will decrease \(i\)'s total utility by 0.31.\(^{44}\) The dominant effect of DFHVALTIE suggests that a household would always like to buy a cheaper house than his/her neighbors’ house values. This effect adds to the effect of a household’s preference on similarities, which creates an asymmetry of preferences for downward deviations over upward deviations from the average house value in the neighborhood. This finding extracts the influence of DFHVAL from the prominent influence of DFHVALTIE.

\[2.5 \quad \text{Estimation: Discussion}\]

A case study of the Columbus city school district provides more insight about the estimated results. This case study is based on the NL results in column I of Table (2.3).

To aid in interpretation of the findings, Table (2.4) contains the estimated marginal effects on the probabilities with respect to the explanatory variables in the Columbus

\(^{43}\)Household \(i\)'s total utility is affected by both DFHVALTIE \((w_i - w_{jl})\), and DFVAL \(|w_i - w_{jl}|\), while the coefficient of \((w_i - w_{jl})\) is -0.46, and the coefficient of \(|w_i - w_{jl}|\) is -0.15. Therefore, the total effect is -0.61.

\(^{44}\)Given the estimated coefficients, the marginal effect of DFHVAL and the marginal effect of DFHVALTIE have been computed. For example, the direct marginal effect of DFHVAL in Columbus school district is -2.01%, and the direct marginal effect of DFHVALTIE in Columbus school is -6.58%. These values suggest that if household \(i\)'s house value is less than the average house value in the community \(jl\), $10,000 increase of \(w_i\) will decrease the probability of \(i\) choosing Columbus school district by 4.57 percentage points.
city school district, while Table (2.5) reports the elasticity effects on the probabilities with respect to the explanatory variables in the Columbus city school district. All marginal effects and elasticity effects are evaluated for each individual household separately, and then averaged across all households of the appropriate subgroup.

45The marginal effect can be interpreted as the percentage points change of the probability of moving into school district \(j\) (where \(j\) can be any school district of the 11 school districts in the choice set) with response to a unit change in the attributes of the Columbus City School District. The elasticity effect can be interpreted as the percentage change of the probability of moving into school district \(j\) (where \(j\) can be any school district of the 11 school districts in the choice set) with response to one percentage change in the attributes of the Columbus City School District.
### Table 2.4: Variable Marginal Effects, the Columbus City School District

<table>
<thead>
<tr>
<th>School District</th>
<th>DFBAC</th>
<th>DFMA</th>
<th>DFC</th>
<th>DFINC</th>
<th>DFBL</th>
<th>GEO</th>
<th>PTAX</th>
<th>PCR</th>
<th>AVG</th>
<th>MHVA</th>
<th>PUPIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gahanna</td>
<td>6.61</td>
<td>8.91</td>
<td>10.79</td>
<td>0.10</td>
<td>2.83</td>
<td>1.65</td>
<td>15.75</td>
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<td>-0.12</td>
<td>-1.15</td>
<td>-42.43</td>
</tr>
<tr>
<td>Bexley</td>
<td>1.00</td>
<td>1.35</td>
<td>1.64</td>
<td>0.02</td>
<td>0.43</td>
<td>0.25</td>
<td>2.39</td>
<td>0.03</td>
<td>-0.02</td>
<td>-0.17</td>
<td>-6.44</td>
</tr>
<tr>
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<td>13.85</td>
<td>16.78</td>
<td>0.16</td>
<td>4.39</td>
<td>2.56</td>
<td>24.47</td>
<td>0.35</td>
<td>-0.19</td>
<td>-1.78</td>
<td>-65.93</td>
</tr>
<tr>
<td>Hilliard</td>
<td>2.66</td>
<td>3.58</td>
<td>4.34</td>
<td>0.04</td>
<td>1.14</td>
<td>0.66</td>
<td>6.33</td>
<td>0.09</td>
<td>-0.05</td>
<td>-0.46</td>
<td>-17.06</td>
</tr>
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<td>1.84</td>
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<td>0.02</td>
<td>0.58</td>
<td>0.34</td>
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<td>0.05</td>
<td>-0.03</td>
<td>-0.24</td>
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<td>Grove City</td>
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<td>9.77</td>
<td>0.09</td>
<td>2.56</td>
<td>1.49</td>
<td>14.26</td>
<td>0.20</td>
<td>-0.11</td>
<td>-1.04</td>
<td>-38.42</td>
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<td>Reynoldsburg</td>
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<td>9.89</td>
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<td>2.59</td>
<td>1.51</td>
<td>14.42</td>
<td>0.20</td>
<td>-0.11</td>
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<td>2.51</td>
<td>1.46</td>
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<td>-0.11</td>
<td>-1.02</td>
<td>-37.68</td>
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<td>-13.22</td>
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<tr>
<td>*Columbus</td>
<td>-45.78</td>
<td>-61.69</td>
<td>-74.73</td>
<td>-0.72</td>
<td>-19.56</td>
<td>-11.39</td>
<td>-109.04</td>
<td>-1.54</td>
<td>0.85</td>
<td>7.95</td>
<td>293.77</td>
</tr>
</tbody>
</table>

*Columbus indicates direct marginal effect of the attribute.
### Table 2.5: Variable Elasticity Effects, the Columbus City School District

<table>
<thead>
<tr>
<th>School District</th>
<th>DFBAC HELOR</th>
<th>DFMA STER</th>
<th>DFC HILD</th>
<th>DFINC ACK</th>
<th>DFBL PIND</th>
<th>GEO</th>
<th>PTAX</th>
<th>PCR</th>
<th>AVG</th>
<th>MHVA</th>
<th>PUPIL EXP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gahanna</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
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<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Bexley</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Upper Arlington</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Hilliard South</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Grove City</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Reynolds-burg</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Dublin</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>13.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Wester-ville</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>Worth-ington</td>
<td>0.169</td>
<td>0.133</td>
<td>0.277</td>
<td>0.039</td>
<td>0.005</td>
<td>2.605</td>
<td>3.234</td>
<td>0.098</td>
<td>-1.125</td>
<td>-0.848</td>
<td>-2.917</td>
</tr>
<tr>
<td>*Columbus</td>
<td>-1.291</td>
<td>-1.342</td>
<td>-2.805</td>
<td>-0.366</td>
<td>-0.193</td>
<td>-27</td>
<td>-34.359</td>
<td>-1.148</td>
<td>11.665</td>
<td>8.793</td>
<td>30.238</td>
</tr>
</tbody>
</table>

*Columbus indicates direct elasticity effect of the attribute
The own marginal effect of locating in the Columbus city school district for the attribute DFCHILD is negative and has the largest absolute value among all the dissimilarity variables, which implies that households are more responsive to the change of DFCHILD than the other difference variables. This is also confirmed by the elasticity of DFCHILD. For a household with children, a 1 percentage point of decrease on the number of households who also have children in the Columbus city school district will decrease the probability of choosing the Columbus city school district by 0.75 percentage point, at the same time increase the probability of the Upper Arlington city school district being chosen by 0.17 percentage points, and increase the probability of the Worthington city school district being chosen by 0.03 percentage points. The high values of the own marginal effect and the estimated elasticity of PUPILEXP in Columbus show how significant the expenditures on schools are in influencing households’ location decisions.\(^{46}\) A $1,000 increase of per pupil expenditure in Columbus leads to a 29.38 percentage points increase in the probability of moving into Columbus.\(^{47}\) A 1 percent increase of per pupil expenditure in Columbus causes a 30.24 percent increase in the likelihood of choosing Columbus. Because per pupil expenditure can be used to measure school quality, the large value of the own marginal effect of PUPILEXP implies households’ concern for school quality is a major determinant of location choice when they migrate.

\(^{46}\) A large value of PUPILEXP signals not only a school district with good public schools, but also the availability of a group of high quality children. For those families with children, this kind of school districts are particularly attractive because of the potential beneficial peer effects on their children.

\(^{47}\) Because the variable PUPILEXP measures per pupil expenditure in $10,000, the big value of the marginal effect of PUPILEXP (293.77\%) can be interpreted as 29.38 percent increase on a $1,000 change in per pupil expenditure.
2.6 Conclusions

The focus of this paper has been to test the effects of dissimilarity on households’ residential location choice. This was tested by a two step procedure. First a location choice model requires inclusion of a house price index and thus the construction of an estimate of that index. Next, with an estimate of the house price index in hand, we estimate a nested logit regression model. Several data sets containing both households’ characteristics and community attributes are utilized. The location decision of a household is modeled as a searching process for a matching community along the dimensions of households’ socio-economic characteristics. The dissimilarity variables have been grouped into five categories: educational background, with school age children or not, race, income, and house value.

The findings reveal that a household prefers neighbors who are like herself/himself with the exception that poorly educated households prefer better educated neighbors. It is hypothesized that the reason for these preferences over similarity comes from the desirability of social interaction, while the preferences of households with little education over better educated neighbors suggests the positive externality flow of education. The revealed preference of white households in our sample suggests that they are indifferent about their neighbors’ races, which might be explained by the small variations on the percentages of white households across the sample school districts.

In addition, Tiebout’s hypothesis has been tested and has been corroborated by the data in this paper. Households would like to buy a somewhat cheaper house as compared with the values of the neighbors’ houses in order to gain a fiscal surplus.
Meanwhile, the house values difference of a household and his/her neighbors’ should be small enough for the household to stay in a relatively homogeneous community.
Appendix A1. Data for the Hedonic Price Function

The recorded transaction dates (yy/mm/dd) in the 2001 surveys make it possible to adjust for inflation across time. The transaction prices are converted to a base period (1st quarter, 2001) dollar value using a quarterly house price index for Columbus Metropolitan Statistics Area (MSA) from Freddie Mac.

The variables included in the estimation of hedonic prices are categorized into two subgroups corresponding to the hedonic function (2.16). Table (2.6) provides the definition and summary statistics of the housing structural characteristics. Each house is assigned values of the selected community’s attributes and neighbors’ socio-economic characteristics, which are elements of $Z_{j(i)}$. The definition and descriptive statistics of the community variables are provided in Table (2.7). The merger of these two data sets leaves 962 observations available.
Panel A: Definition of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE</td>
<td>Transaction price in $2001(Q1) and divided by 10,000</td>
</tr>
<tr>
<td>ROOMS</td>
<td>Number of total rooms</td>
</tr>
<tr>
<td>ROOMS2</td>
<td>Number of total rooms squared</td>
</tr>
<tr>
<td>HOUSEAGE</td>
<td>Age of the structure</td>
</tr>
<tr>
<td>HOUSEAGE2</td>
<td>Squared age of the structure</td>
</tr>
<tr>
<td>BATHTOT</td>
<td>Number of full bathrooms+0.5×number of half bathrooms</td>
</tr>
<tr>
<td>LOTS</td>
<td>Lot size in thousands of square feet</td>
</tr>
<tr>
<td>LOTS2</td>
<td>Lot size squared</td>
</tr>
<tr>
<td>LOTS3</td>
<td>Lot size cubed</td>
</tr>
<tr>
<td>FIREPLACE</td>
<td>Dummy variable that indicates the existence of a fireplace</td>
</tr>
<tr>
<td>NOSTORIES</td>
<td>Number of stories and times 10</td>
</tr>
<tr>
<td>CONDITION</td>
<td>Dummy variable indicating the condition of a house</td>
</tr>
<tr>
<td>SQFT</td>
<td>Building area in thousands of square feet</td>
</tr>
<tr>
<td>CROSS1</td>
<td>Cross term of SQFT and NOSTORIES</td>
</tr>
<tr>
<td>CROSS2</td>
<td>Cross term of SQFT2 and NOSTORIES</td>
</tr>
<tr>
<td>CROSS3</td>
<td>Cross term of SQFT3 and NOSTORIES</td>
</tr>
<tr>
<td>X_COORD</td>
<td>X coordinate of the observation in square feet</td>
</tr>
<tr>
<td>Y_COORD</td>
<td>Y coordinate of the observation in square feet</td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE</td>
<td>14.65</td>
<td>9.55</td>
<td>1.01</td>
<td>152.07</td>
</tr>
<tr>
<td>ROOMS</td>
<td>6.50</td>
<td>1.38</td>
<td>3.00</td>
<td>16.00</td>
</tr>
<tr>
<td>ROOMS2</td>
<td>44.18</td>
<td>20.05</td>
<td>9.00</td>
<td>256.00</td>
</tr>
<tr>
<td>HOUSEAGE</td>
<td>39.26</td>
<td>24.37</td>
<td>3.00</td>
<td>162.00</td>
</tr>
<tr>
<td>HOUSEAGE2</td>
<td>2134.68</td>
<td>2552.31</td>
<td>9.00</td>
<td>26244.00</td>
</tr>
<tr>
<td>BATHTOT</td>
<td>1.85</td>
<td>0.69</td>
<td>1.00</td>
<td>5.50</td>
</tr>
<tr>
<td>LOTS</td>
<td>12.63</td>
<td>23.59</td>
<td>0.87</td>
<td>306.66</td>
</tr>
<tr>
<td>LOTS2</td>
<td>715.30</td>
<td>5582.33</td>
<td>0.76</td>
<td>94042.58</td>
</tr>
<tr>
<td>LOTS3</td>
<td>133044.31</td>
<td>1434433.35</td>
<td>0.66</td>
<td>28839438.10</td>
</tr>
<tr>
<td>FIREPLACE</td>
<td>0.57</td>
<td>0.50</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>NOSTORIES</td>
<td>14.64</td>
<td>4.93</td>
<td>10.00</td>
<td>25.00</td>
</tr>
<tr>
<td>CONDITION</td>
<td>2.27</td>
<td>0.58</td>
<td>1.00</td>
<td>4.00</td>
</tr>
<tr>
<td>CROSS1</td>
<td>26.30</td>
<td>16.08</td>
<td>6.72</td>
<td>128.54</td>
</tr>
<tr>
<td>CROSS2</td>
<td>53.71</td>
<td>61.53</td>
<td>4.52</td>
<td>826.13</td>
</tr>
<tr>
<td>CROSS3</td>
<td>127.58</td>
<td>293.81</td>
<td>3.03</td>
<td>5309.52</td>
</tr>
<tr>
<td>X_COORD</td>
<td>1827731.25</td>
<td>27982.74</td>
<td>1760808.20</td>
<td>1893377.37</td>
</tr>
<tr>
<td>Y_COORD</td>
<td>731159.24</td>
<td>27193.99</td>
<td>661905.00</td>
<td>779187.00</td>
</tr>
</tbody>
</table>

Table 2.6: Descriptive Statistics on the Housing Structural Variables
Panel A: Definition of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEDHHINC</td>
<td>Median household income in 10,000 dollars</td>
</tr>
<tr>
<td>PROCOL_25</td>
<td>Proportion of population (age≥25) with at least some college</td>
</tr>
<tr>
<td>PROWHITE</td>
<td>Proportion of Caucasians in census track</td>
</tr>
<tr>
<td>MEDAGE</td>
<td>Median age of households in census track</td>
</tr>
<tr>
<td>PUPILEXP</td>
<td>Education expenditure per pupil in $10,000 in school district</td>
</tr>
<tr>
<td>CRIME</td>
<td>Crime index divided by population in the police jurisdiction</td>
</tr>
<tr>
<td>TAX</td>
<td>Property tax rate, in percentage, in the township</td>
</tr>
<tr>
<td>CBD</td>
<td>Distance of property to the central business district in miles</td>
</tr>
<tr>
<td>AVGSC9</td>
<td>Average of math, reading, and writing of the Ohio 9th-grade proficiency test</td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Value</th>
<th>Std.Deviation</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEDHHINC</td>
<td>5.39</td>
<td>1.88</td>
<td>1.24</td>
<td>11.6</td>
</tr>
<tr>
<td>PROCOL_25</td>
<td>0.64</td>
<td>0.20</td>
<td>0.13</td>
<td>0.97</td>
</tr>
<tr>
<td>PROWHITE</td>
<td>0.83</td>
<td>0.18</td>
<td>0.03</td>
<td>0.98</td>
</tr>
<tr>
<td>MEDAGE</td>
<td>35.59</td>
<td>5.63</td>
<td>22.30</td>
<td>60.30</td>
</tr>
<tr>
<td>PUPILEXP</td>
<td>0.61</td>
<td>0.08</td>
<td>0.49</td>
<td>0.85</td>
</tr>
<tr>
<td>CRIME</td>
<td>6.18</td>
<td>3.8</td>
<td>1.34</td>
<td>11.81</td>
</tr>
<tr>
<td>TAX</td>
<td>1.75</td>
<td>0.26</td>
<td>1.48</td>
<td>2.44</td>
</tr>
<tr>
<td>CBD</td>
<td>7.39</td>
<td>3.02</td>
<td>0.89</td>
<td>15.27</td>
</tr>
<tr>
<td>AVGSC9</td>
<td>78.00</td>
<td>12.89</td>
<td>64.10</td>
<td>98.10</td>
</tr>
</tbody>
</table>

The crime index is provided by the Bureau of Justice Statistics (BJS). It is the total number of murders, non-negligent manslaughters, forcible rapes, robberies, aggravated assaults, burglaries, larceny-thefts, motor vehicle thefts, and arsons. The property tax rate is provided by the Franklin County Treasurer. The expenditure/pupil and efficiency score are provided by the Ohio Education Department. MEDHHINC, PROCOL_25, PROWHITE, and MEDAGE are from census data. AVGSC9 is provided by the Ohio Department of Education.

Table 2.7: Descriptive Statistics on the Community Variables in the Hedonic Model
Appendix A2. The Hedonic Price Indices

Given the selected variables from Table (2.6) and Table (2.7) in the previous section, equation (2.16) can be written as,

\[
\text{Price} = \alpha_0 + \sum_{j=1}^{J-1} \alpha_{0j} \text{school}_j + \alpha_1 \text{MEDAGE} + \alpha_2 \text{PROWHITE} \\
+ \alpha_3 \text{PROCOL}_25 + \alpha_4 \text{MEDHHINC} + \alpha_5 \text{EXPUP} \\
+ \alpha_6 \text{CRIME} + \alpha_7 \text{TAX} + \alpha_8 \text{CBD} + \alpha_9 \text{AVGSC}9 \\
+ \beta_1 \text{ROOMS} + \beta_2 \text{ROOMS}2 + \beta_3 \text{HOUSEAGE} \\
+ \beta_4 \text{HOUSEAGE}2 + \beta_5 \text{BATHTOT} + \beta_6 \text{ACRE} \\
+ \beta_7 \text{ACRE}2 + \beta_8 \text{ACRE}3 + \beta_9 \text{FI} + \beta_{10} \text{NOSTORIES} \\
+ \beta_{11} \text{CONDITION} + \beta_{12} \text{SQFT} \ast \text{NOSTORIES} \\
+ \beta_{13} \text{SQFT}2 \ast \text{NOSTORIES} + \beta_{14} \text{SQFT}3 \ast \text{NOSTORIES} + v_i
\]  

Specification I in Table (2.8) reports the OLS coefficients estimates along with their standard errors. Because the maximum likelihood estimates of the price indices are more precise when the errors are correlated, the spherical semivariogram function is employed, and the maximum likelihood estimates are generated for the hedonic regression.

\footnote{Some estimates of the community variables are statistically insignificant, but they are still left in the hedonic function when constructing the price indices.}
Before the ML method is applied, the residuals from an OLS regression are tested by fitting those residuals into a spherical semivariogram function, which is called the method of moment estimate of the semivariogram (Matheron, 1963). The results confirm the existence of auto-correlated disturbances, which recommends the use of the ML method to improve the precision of the predictions. The values of the estimated parameters of the spherical semivariogram function ($c_0$ and $a_0$) through the method of moment estimation are very close to the values of the estimates through the ML method. This fact proves the robustness of the ML estimates. The results of the ML estimates are provided in column II of Table (2.8). The chi-square value of 18.58 in Table (2.8) indicates that the geostatistical model is superior to the OLS model.\textsuperscript{49} Because the indices created from OLS are so close to the indices created from ML, we do not expect that the results of the discrete choice model will be significantly different using OLS indices or ML indices. Only the ML price indices\textsuperscript{50} are reported in Table (2.9).

\textsuperscript{49}The "Chi-Square" value is -2 times the log likelihood from the null model minus -2 times the log likelihood from the fitted model, where the null model is the one with the same explanatory variables and iid distributed errors. Because the ML estimates of a regression with iid disturbances equal the OLS estimates of such regression, the likelihood ratio test can be interpreted as the test for the MLE of geostatistical model and the OLSE of the hedonic regression. In addition, the out-sample prediction power of geostatistical model relative to OLS has been widely confirmed by the literature. Because of the small sample size, this paper skips the prediction test.

\textsuperscript{50}OLS price indices are tested in the NL framework by the author and the results are indeed not different from ML estimates. Therefore, they are not reported here.
<table>
<thead>
<tr>
<th>Specification</th>
<th>OLS</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-20.40*** (-5.63)</td>
<td>-21.98*** (5.81)</td>
</tr>
<tr>
<td>SCHOOL1</td>
<td>3.19* (1.83)</td>
<td>3.20 (2.09)</td>
</tr>
<tr>
<td>SCHOOL12</td>
<td>6.21*** (1.66)</td>
<td>6.49*** (1.8)</td>
</tr>
<tr>
<td>ROOMS</td>
<td>3.85*** (0.92)</td>
<td>4.21*** (0.89)</td>
</tr>
<tr>
<td>ROOMS2</td>
<td>-0.31*** (0.06)</td>
<td>-0.33*** (0.06)</td>
</tr>
<tr>
<td>HOUSEAGE</td>
<td>0.05 (0.03)</td>
<td>-0.06 (0.04)</td>
</tr>
<tr>
<td>HOUSEAGE2</td>
<td>4.09E-04 (3.03E-04)</td>
<td>5.29E-04* (3.04E-04)</td>
</tr>
<tr>
<td>BATHTOT</td>
<td>1.81*** (0.48)</td>
<td>1.87*** (0.47)</td>
</tr>
<tr>
<td>LOTS</td>
<td>0.10** (0.05)</td>
<td>0.11** (0.05)</td>
</tr>
<tr>
<td>LOTS2</td>
<td>-1.39E-03*** (4.97E-04)</td>
<td>-1.46E-03*** (5.11E-04)</td>
</tr>
<tr>
<td>LOTS3</td>
<td>3.59E-06*** (1.35E-06)</td>
<td>3.66E-06*** (1.40E-06)</td>
</tr>
<tr>
<td>FIREPLACE</td>
<td>0.80 (0.49)</td>
<td>0.79 (0.49)</td>
</tr>
<tr>
<td>NOSTORIES</td>
<td>0.64*** (0.23)</td>
<td>0.65*** (0.23)</td>
</tr>
<tr>
<td>CONDITION</td>
<td>1.57*** (0.42)</td>
<td>1.28*** (0.41)</td>
</tr>
<tr>
<td>CROSS1</td>
<td>-1.08*** (0.24)</td>
<td>-1.04*** (0.24)</td>
</tr>
<tr>
<td>CROSS2</td>
<td>0.53*** (0.08)</td>
<td>0.50*** (0.08)</td>
</tr>
<tr>
<td>CROSS3</td>
<td>-0.05*** (0.01)</td>
<td>-0.04*** (0.01)</td>
</tr>
<tr>
<td>MEDHHINC</td>
<td>0.47** (0.22)</td>
<td>0.53** (0.24)</td>
</tr>
<tr>
<td>PERCOL_25</td>
<td>2.41 (1.87)</td>
<td>2.68 (2.02)</td>
</tr>
<tr>
<td>PERWHITE</td>
<td>2.87** (1.44)</td>
<td>2.50 (1.54)</td>
</tr>
<tr>
<td>MEDAGE</td>
<td>0.13** (0.05)</td>
<td>0.14*** (0.05)</td>
</tr>
<tr>
<td>PUPILEXP</td>
<td>4.76 (5.05)</td>
<td>4.50 (5.49)</td>
</tr>
<tr>
<td>CRIME</td>
<td>0.05 (0.08)</td>
<td>0.05 (0.08)</td>
</tr>
<tr>
<td>TAX</td>
<td>-2.72 (2.14)</td>
<td>-2.58 (2.32)</td>
</tr>
<tr>
<td>CBD</td>
<td>0.12 (0.12)</td>
<td>0.13 (0.13)</td>
</tr>
<tr>
<td>AVGSC9</td>
<td>0.04 (0.04)</td>
<td>0.04 (0.05)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.64</td>
<td>-</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.63</td>
<td>-</td>
</tr>
<tr>
<td>$a_0$ (spherical semivariogram)</td>
<td>-</td>
<td>1146.87</td>
</tr>
<tr>
<td>$c_0$ (spherical semivariogram)</td>
<td>-</td>
<td>39.49</td>
</tr>
</tbody>
</table>

Observations: 1467 1467
Likelihood Ratio ($\chi^2$): 18.58

Numbers in parentheses correspond to standard errors. * indicates significance at 10% level; ** indicates significance at 5% level; *** indicates significance at 1% level or less.

Table 2.8: Coefficient Estimates of the Hedonic House Price Model
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<th>GEOPIND</th>
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Table 2.9: Price Indices from the ML Estimates
3.1 Introduction

The fixed geographic position of housing fundamentally distinguishes it from other assets. Unlike assets such as stocks, houses are attached to their spatial locations. Therefore, the dividends derived from housing assets are inseparable from their physical locations. The locational impacts on the dividends provide a channel that allows the communication of house prices within a common neighborhood. The interdependence of house prices in a neighborhood governs the house price determination process.

In the hedonic literature, there has been a marked increase in studies highlighting concerns about the spatial interdependence of residential prices. The spatial correlation has been introduced in hedonic analysis through two approaches: the econometric approach that employs a lattice model, and the statistical perspective that employs a geostatistical model.

Following the first type of approach, Can (1992) appears to be the first to systematically analyze the spatial dependence of house prices in the context of the hedonic approach. She verified the existence of spatial correlation using Lagrange multiplier
tests and demonstrated the superiority of the spatial model to the mainstream hedonic model. Can (1997) extended the spatial hedonic model in her previous study, and explicitly estimated the impact from prior sales on the transaction price of a house with a spatial lag functional form. Following the same path, Pace and Gilley (1997), and Pace, Barry, and Clapp (1998) adopted the lattice modeling methodology but with a slightly different approach by ascribing the spatial correlation to the residuals of a hedonic function.


The essential distinction between the two perspectives is that the lattice model explicitly expresses the neighborhood influence in the functional form using a predetermined spatial weighting matrix, while the geostatistical model allows the spatial interdependence to reside in the residuals and it estimates the parameterized variance-covariance matrix to improve the efficiency of the estimates.

This study is related to lattice modeling but takes an alternative approach. Rather than adopting the hedonic framework, it applies the rational expectations hypothesis, considering housing units as assets that generate dividends across time periods. The dividend of a housing asset at a given time is the value of housing services that accrue to the owner of the house through that time period. Because houses do not exist in isolation, neighborhood effects should play an important role in determining the value of the housing services received by the owner. Even though the neighborhood effects are not directly observable by econometricians, they are capitalized into house
values and reflected by the market prices of houses in the neighborhood. Therefore, the market prices of neighboring houses influence the value of a property through the spatially interdependent housing dividends. Furthermore, as an asset, the value of a unit is not only determined by the current dividend, but also expected future dividends. This implies that the current price of a house is governed by the expected future values of nearby houses as well as the contemporaneous values of nearby houses.

Although the concurrent neighborhood effects are well-recognized and capitalized into house prices through the inclusion of prior sales in the neighborhood, the expected future neighborhood effects have not been inspected in the existing literature. This study explores the spatial dependence of house prices by extending the investigation of neighborhood effects to incorporate expectations of the future. If the locational context in a neighborhood is anticipated to change, these effects should be capitalized into the current price of a house. For instance, assume that an airport is going to add a runway. Flights using this runway will require a flight path that will pass over a neighborhood previously unaffected by the airport. Even though the new runway has not been added, we should expect to see an immediate impact of this news on the house prices in the neighborhood. In determining a house value, a forward looking household will not ignore anticipated changes in the neighborhood, and neither should the economic model.

In particular, we initiate our investigation with a rational expectations asset pricing model\textsuperscript{51} that interprets the current price of a housing unit as a function of the

\textsuperscript{51}The assumption of rational expectations implies that households make optimal forecasts given the information and they know the true structure of economy and true behavioral relationships, i.e., they know how economy works. They make forecasts as statisticians do. This assumption provides the rules to households in forming forecasts and enabling us to solve the structural model to a reduced form for the estimation purpose.
current and the expected future values of neighboring residences in addition to its own structural characteristic attributes. The information of the current values and the expected future values of houses nearby is weighted according to the distances between houses along the time dimension and space dimension. The reduced form of this rational expectations price model is derived under the rational expectations hypothesis, and it has the form of a spatial autoregressive model (SAR) with two spatial lags. Our economic model explicitly accounts for the expectations of future neighborhood effects. We solve the expectations model for the reduced form and estimate it using a data set that consists of single-family housing transactions for Franklin County (OH) between 1985 and 1998. Because of the presence of endogenous variables in the SAR model, the best general method of moments (GMM) estimator is used for the estimation.

This paper is organized as follows. Section 2 outlines the basic model specifications and the generating process for the time-space weighting matrix. Section 3 provides the details of the GMM method that is applied in this study. Section 4 describes the data. The empirical results and the interpretations of the results are presented in Section 5. The last section offers some conclusions from the analysis.

3.2 Model Specification

This section introduces spatial interdependence into an asset pricing model through the channel of housing dividends. The integration of spatial techniques and rational expectations is through the application of two weighting matrices along both time and space dimensions.
3.2.1 Asset Pricing Model

We consider houses as an investment good, which produces a stream of dividends over its lifetime and can be traded at any time. The housing dividend is interpreted as the value of housing services that accrue to the owner of the house, which is also defined as imputed rent\(^{52}\) in housing literature. If agents are risk neutral, then the arbitrage between holding housing assets and the riskless assets should be such that the expected return on the property—given by the expected rate of capital gain plus the dividend/price ratio—should equal the riskless interest rate:

\[
\frac{E_t y_{i,t+1} - y_{it}}{y_{it}} + \frac{d_{it}}{y_{it}} = r, \tag{3.1}
\]

or equivalently

\[
y_{it} = \delta E_t y_{i,t+1} + \delta d_{it}. \tag{3.2}
\]

where \(r\) is the riskless interest rate; \(\delta\) is the discount factor, defined as \(\delta \equiv \frac{1}{1+r} < 1\); \(y_{it}\) is the market price of a house at location \(i \in (1, L)\) and at time \(t \in (1, \infty)\); and \(d_{it}\) is the dividend extracted from living in this property at time period \(t\).

\(^{52}\)Imputed rent is an imputation for the net rental income of owner-occupied housing. It is based on the assumption that owner-occupants are in the rental business and that they are renting the houses in which they live to themselves: As tenants, they pay rent to the landlords (that is, to themselves); as landlords, they collect rent from their tenants (that is, from themselves), they incur expenses, and they may have a profit or a loss from the rental business.
Because $|\delta| < 1$, the solution should be forward looking. The forward substitution proceeds by iterating forward on the system, making use of the law of iterated expectations. Continuing the process, we get

$$y_{it} = \sum_{q=1}^{\infty} \delta^{q+1} E_t(d_{i,t+q}) + \delta d_{it} + \lim_{q \to \infty} \delta^{q+1} E_t(y_{i,t+1+q}),$$

(3.3)

where $E_t$ denotes conditional expectation given information available to the economic agent at period $t$. Imposing the transversality condition, $\lim_{q \to \infty} \delta^{q+1} E_t(y_{i,t+1+q}) = 0$, we get

$$y_{it} = \sum_{q=1}^{\infty} \delta^{q+1} E_t(d_{i,t+q}) + \delta d_{it}.$$  

(3.4)

The functional form indicates that the current value of a housing unit is the discounted present value of expected dividends over its lifetime. The dividend $d_{i,t+q}$ is the value of housing services that accrue to the owner of the house $i$ at time $t + q (\forall q \geq 0)$. This study ascribes the total housing services that the owner can derive from living in house $i$ to two sources: the utility derived from the physical attributes of this housing unit, generated by the quality of house $i$; and the utility stemming from neighborhood effects, reflected by the market values of neighboring houses. The market values of houses are represented by their realized transaction prices.

---

53 O. Blanchard (1979): If the elasticity of the current price with respect to next period’s expected price is less than unity, imposing stationarity leads to the choice of a unique solution-the forward solution.

54 The law of iterated expectations states that $E_t[E_{t+q}(p_{i,t+1+q})] = E_t(p_{i,t+1+q})$.

55 The transversality condition rules out rational speculative bubbles and generate a unique solution.

56 In this study, the term "market values" are interchangeable with the term "transaction prices".
The first source of housing services has been well recognized in the hedonic literature. In addition to the structural characteristics of a house, neighborhood effects have been paid increasing attention in recent hedonic studies through the inclusion of variables describing community characteristics such as local tax rate, crime index, and median income, etc. This is primarily because the important role played by location in determining house prices. The importance of location introduces neighborhood effects as the second source of housing services.

The locational effects that were defined by Can (1992) have two levels: (1) absolute neighborhood effects,\textsuperscript{57} which represent the impacts of the absolute location of the house that relates to the array of community characteristics (public services provision, accessibility, and neighbors' characteristics) and (2) adjacency effects or neighborhood spillover effects, which are externalities spatially associated with the behavior of neighbors, such as maintenance/repair decisions made by neighbors, on a nearby household's total utility. Suppose that there are two houses with exactly the same structure located on two side-by-side streets. Along one of these two streets, all the houses are well-maintained by the owners, while the houses on the other street are poorly maintained. We should expect that a household inhabiting the house on the first street (a nicer neighborhood) would have higher utility than the household living in the house on the second street (a run-down neighborhood). Therefore, the neighborhood effects are defined as the sum of the absolute neighborhood effects and the spillover effects. A nice neighborhood that can attribute high housing services to

\textsuperscript{57}Can (1992) uses "neighborhood effects" for type (1) effects. This paper uses "neighborhood effects" as a more general term, which is interchangeable with "locational effects" including both "absolute neighborhood effects" (type (1) effects) and "neighborhood spillover effects" (type (2) effects).
the residents should be a neighborhood that has both appealing community characteristics and neat neighbors.

To represent the neighborhood effects in the dividend function, the transaction prices of neighborhood houses are used as a proxy for both the absolute neighborhood effects and the neighborhood spillover effects. Obviously, the results of maintenance/repair behavior of a neighbor is represented by the appearance of his/her house and capitalized into the house value, which justifies the approximation of spillover effects using neighborhood house values. The capitalization of community attributes into house prices back the use of neighborhood house value as the proxy of absolute locational effects. Considering the consequences of omitted variables and misspecifications using arbitrarily selected community variables, using the values of houses in the same vicinity that share the same locational factors avoids the potential problems without the loss of information.

The following expression provides the functional form of the dividend as

\[ d_{i,t+q} = x_{i,t+q}^* + y_{i,t+q}^*, \forall q \geq 0, \] (3.5)

where subscript \(-i\) denotes the set of \(i\)'s neighboring houses, the latent variable, \(x_{i,t+q}^*\), measures the value of housing services attributed from housing structures as a function of housing characteristics, and \(y_{i,t+q}^*\) quantifies the housing services arising from neighborhood effects as a function of neighborhood house values. Inserting the dividend equation into the asset price function, house \(i\)'s value at time \(t\) can be written as
\[ y_{it} = \sum_{q=1}^{\infty} \delta^{q+1} E_t(y_{-i,t+q}^* + x_{i,t+q}^*) + \delta y_{-it}^* + \delta x_{it}^* , \quad (3.6) \]

which equals

\[ y_{it} = \sum_{q=1}^{\infty} \delta^{q+1} E_t(y_{-i,t+q}^*) + \delta (y_{-i,t}^*) + \frac{\delta}{1 - \delta} (x_{i}^*), \quad (3.7) \]

where we assume housing characteristics are exogenously predetermined and invariant across time, \( x_{i,t+q}^* = x_{it}^* = x_i^* \forall q \geq 0 \), so \( E_t \) is dropped for \( x_i^* \).

To derive the values of \( y_{i,t+q}^*(q \geq 1) \) and \( y_{-i,t}^* \), we utilize the information of the market values of the neighborhood houses at the given time period. The extent of the influences of distinct housing units in the same vicinity are weighted in a prior fashion in accordance with general spatial interaction theories as reviewed by Fortheringham and O'Kelly (1989). It is assumed that the values of neighboring houses are used as proxies of neighborhood effects. Naturally, the influence diminishes as the distance between any two houses increases. Following the literature (Upton and Fingleton, 1985), the inverse distances between houses are measured and used as the weights. We denote

\[ y_{-i,t+q}^* \equiv \lambda^f \sum_{j \in -i} (s_{ij} y_{j,t+q}^*), \forall q > 0, \]

where \( y_{j,t+q}^* \) is \( i \)'s neighbor \( j \)'s market value at time \( t + q (\forall q > 0) \), the parameter \( \lambda^f \) measures the neighborhood effect and is typically referred to as the spatial autoregressive parameter, and the weight \( s_{ij} \) is the inverse of the physical distance between \( i \) and \( j \). This study names \( s_{ij} \) as the "spatial discount factor" in contrast to the time discount factor. The summation symbol, \( \sum_{j \in -i} \) denotes the summation over all the
neighborhood house values in the given time period. The first term on the right hand side of equation (3.7) is therefore

$$\sum_{q=1}^{\infty} \delta^{q+1} E_t(y_{-i,t+q}^*) = \lambda^f \sum_{q=1}^{\infty} \delta^{q+1} E_t \sum_{j \in -i} (s_{ij} y_{j,t+q}),$$  \hspace{1cm} (3.8)

which is the temporally and spatially discounted sum over the expected future (relative to time $t$) values of neighboring houses. The spatial discount factor $s_{ij}$ discounts the neighborhood effects along the dimension of spatial distance, while the time discount factor $\delta^{q+1}$ discounts the neighborhood effects along the dimension of time. Both of these discount factors assign weights to neighbors based on the "closeness" between houses, and the spatial distance and time distance determine the weight jointly.

The transactions are ordered sequentially according to the transaction dates and we assume that each transaction takes place at a distinct point in time,\textsuperscript{58} which means there is not any transaction price simultaneously determined with other transaction prices in a given neighborhood. Therefore, $y_{-i,t}$ that represents the average of neighborhood house values at time $t$ is meaningless in this context. A suitable substitute of the neighborhood effects in time $t$ will be the observed neighborhood effects from prior to time $t$. We define the second term on the right hand side of equation (3.7) as

$$\delta y_{-i,t}^* \equiv \sum_{q=1}^{t-1} \delta^{q+1} \lambda^p \sum_{j \in -i} (s_{ij} y_{j,t-q}),$$  \hspace{1cm} (3.9)

where $y_{j,t-q}$ is $i$’s neighbor $j$’s market value at time $t-q (\forall q \in [1, t-1])$, $\lambda^p$ is the spatial autoregressive parameter, and the spatial discount factor, $s_{ij}$, has the same meaning.

\textsuperscript{58} Usually, the recorded transaction dates accurate to days. Because it is rare to see many transactions happen on the same day within a given neighborhood, our assumption is fairly justified.
as for equation (3.8). To be consistent with equation (3.8), the time discount factor in equation (3.9) is symmetrically set to be $\delta^{q+1}$. Substituting equation (3.8) and equation (3.9) into equation (3.7), we get

$$y_{it} = \lambda^f \sum_{q=1}^{\infty} \delta^{q+1} E_t \sum_{j \in -i} (s_{ij}y_{j,t+q}) + \lambda^p \sum_{q=1}^{t-1} \delta^{q+1} \sum_{j \in -i} (s_{ij}y_{j,t-q}) + \frac{\delta}{1-\delta} (x^*_i). \quad (3.10)$$

The spatial autoregressive parameter $\lambda^p$ is not restricted to be the same as the spatial autoregressive parameter $\lambda^f$, and the statistic test for the equality of them will be carried out in the estimation section. The null hypothesis, $H_0: (\lambda^f = \lambda^p)$, implies that current neighborhood effects have the same influence as the discounted stream of future neighborhood effects. We have reason to believe that the estimates of $\lambda^p$ and $\lambda^f$ would be the same. The arbitrage equation (3.4) implies that the discounted future dividends will affect $y_{it}$, the current price of a given house, in the same way as the contemporaneous dividend. Meanwhile, one component of the future dividend, which is the future neighborhood effect, $y^*_{i,t+q}$, and the counterpart in the current dividend, $y^*_{i,t}$, are both constructed from neighborhood prices, which are weighted spatially in the exactly same way. Therefore, it is reasonable to expect that the impact of the expected future neighborhood effect on $y_{it}$ after being discounted appropriately should equal that of the current neighborhood effect.

Given the exogeneity of $s_{ij}$, the above equation can be rewritten as,

---

59 Equation (3.8) specifies the time discount factor for the neighborhood effects as $\delta^{q+1}$, which means that a house value in the distant past is discounted more heavily than the extend to which a house value in the recent past is discounted.

60 In a given neighborhood, the location of each house is fixed and known, so the weights $s_{ij}$ based on the distances between house $i$ and house $j$ is constant.
\[
y_{it} = \lambda^f \sum_{q=1}^{\infty} \delta^{q+1} \sum_{j \in -i} s_{ij} E_t y_{jt+q} + \lambda^p \sum_{q=1}^{t-1} \delta^{q+1} \sum_{j \in -i} s_{ij} y_{j,t-q} + \frac{\delta}{1-\delta} x^*_i,
\]

or equivalently

\[
y_{it} = \lambda^f \sum_{k=t+1}^{\infty} \sum_{j \in -i} \delta^{k-t+1} s_{ij} E_t y_{jk} + \lambda^p \sum_{k'=1}^{t-1} \sum_{j \in -i} \delta^{t-k'+1} s_{ij} y_{j,k'} + \beta^* x^*_i; \tag{3.11}
\]

where \( k \equiv t + q, \ k' = t - q \) and \( \beta^* \equiv \frac{\delta}{1-\delta} = \frac{1}{r} \). In the right-hand side of the above equation, the first summation term sums over the expected values of all \( js (\forall \ j \in [1, L] \) and \( j \neq i \)), and across all \( ks (\forall \ k > t) \); the second summation term sums over the values of all \( js (\forall \ j \in [1, L] \) and \( j \neq i \)), and across all \( k's (\forall \ k' \in [1, t-1]) \).

We define the time discount factor for the periods between time \( t \) and \( k \) as \( \delta_{tk} \equiv \delta^{k-t+1} \). The above equation becomes

\[
y_{it} = \lambda^f \sum_{k=t+1}^{\infty} \sum_{j \in -i} \delta_{tk} s_{ij} E_t (y_{jk}) + \lambda^p \sum_{k'=1}^{t-1} \sum_{j \in -i} \delta_{tk'} s_{ij} y_{j,k'} + \beta^* x^*_i.
\]

Let \( \omega_{it,jk} = \delta_{tk} s_{ij} \), then the new weight assigned to \( i \) and \( t \)th observation’s neighbor \( jk \) that is \( \omega_{it,jk} \), aggregates the discount information along both the time dimension and the space dimension.\(^{61}\) We have

\(^{61}\)In this paper, the spatial-temporal weights are created from the integration of the spatial and temporal information (element by element multiplication of the spatial weighting matrix and the temporal weighting matrix). This in turn generates a composite weighting matrix along both time dimension and space dimension. Pace, Barry, Gilley and Sirmans (2000) took another approach to the spatial-temporal weighting matrix. They constructed a spatial weighting matrix as \( S \) and a temporal weighting matrix as \( T \) respectively, and then defined the spatial-temporal weighting matrix as a function of \( S \) and \( T \) as \( W = \phi_s S + \phi_T T + \phi_{ST} ST + \phi_{TS} TS \). In their case, there will be four spatial autoregressive coefficients (\( \phi_s, \phi_T, \phi_{ST}, \)and \( \phi_{TS} \)) to be estimated. Because Pace et al. (2000) restricted the spatial and temporal relations to be among previous observations only, \( S, T, ST, \) and \( TS \) are all lower triangular matrices, the simultaneity problem did not occur in their specification, and therefore the estimation using the OLS method was not computational demanding. Further comparison of our model and the model in Pace et al. (2000) is provided in the last section of this paper.
\[ y_{it} = \lambda_f \sum_{k=t+1}^{\infty} \sum_{j \in -i} \omega_{it,jk} E_t(y_{jk}) + \lambda_p \sum_{k'=1}^{t-1} \sum_{j \in -i} \omega_{it,jk'} y_{jk'} + \beta^* x^*_i, \] (3.12)

Because the market value of a property is associated with the time when it is evaluated and the position where the property is located, we can construct an index that comprises the time and the location as \((j, K)\), where \(j \in [1, L]\) and \(K = (k \cup k') \in [1, \infty)\), and denote a set for all the house values in this neighborhood across time periods as \(\Lambda = \{(j, K) | j \in [1, L], K \in [1, \infty)\}\). Therefore,

\[ \sum_{(j,K) \in \Lambda} \omega^f_{it,jK} = \sum_{k=t+1}^{\infty} \sum_{j \in -i} \omega_{it,jk} \] (3.13)

where \( \sum_{(j,K) \in \Lambda} \) denotes the summation over housing units in the vicinity and across all time periods. With the superscript \(f\), \(\omega^f_{it,jK}\) conveys more information than \(\omega_{it,jk}\), and the set formed by \(\omega_{it,jk}s\) is a subset of the set formed by \(\omega^f_{it,jK}s\). Put another way, \(\omega_{it,jk}s\) are the non-zeros elements of \(\omega^f_{it,jK}s\). This can be formulated as \(\omega^f_{it,jK} \neq 0 \) if \(k > t\); \(\omega^f_{it,jK} = 0\) otherwise. Similarly,

\[ \sum_{(j,K) \in \Lambda} \omega^p_{it,jK} = \sum_{k'=1}^{t-1} \sum_{j \in -i} \omega_{it,jk'}, \] (3.14)

where \(\omega^p_{it,jK} \neq 0 \) if \(K < t\); \(\omega^p_{it,jK} = 0\) otherwise. Given this new notation, we have

\[ y_{it} = \lambda_f \sum_{(j,K) \in \Lambda} \omega^f_{it,jK} E_t(y_{jK}) + \lambda_p \sum_{(j,K) \in \Lambda} \omega^p_{it,jK} y_{jK} + \beta^* x^*_i, \] (3.15)

Provided function (3.15), it is more convenient to illustrate the price relationship in a vector form. We use vector \(Y\) to represent all the house values that are sorted regarding the time periods when the houses are evaluated, starting with the earliest

\(^{62}\)The set \(\Lambda\) includes all the transactions for \(L\) units of properties across the infinite time horizon.
time period. Because the total number of housing units is \( L \), there are \( L \) values for each period. This price vector \( Y \) has the form of

\[
Y = \begin{pmatrix} y_{11} & \cdots & y_{L1} & \cdots & y_{1t} & \cdots & y_{Lt} & \cdots \end{pmatrix}',
\]

where \( y_{11}, \cdots, y_{L1} \) are the housing market values in period 1, \( y_{1t}, \cdots, y_{Lt} \) are the house market values in period \( t \), etc., and \( y_{it} \in Y \) with \( i \in [1, L] \) and \( t \in [1, \infty] \).

Define row vector \( \omega^f_{it} \) as

\[
\omega^f_{it} = \begin{pmatrix} \omega^f_{it,11} & \cdots & \omega^f_{it,L1} & \cdots & \omega^f_{it,1t} & \cdots & \omega^f_{it,jk} & \cdots & \omega^f_{it,Lt} & \cdots \end{pmatrix},
\]

and a row vector \( \omega^p_{it} \) as

\[
\omega^p_{it} = \begin{pmatrix} \omega^p_{it,11} & \cdots & \omega^p_{it,L1} & \cdots & \omega^p_{it,1t} & \cdots & \omega^p_{it,jk} & \cdots & \omega^p_{it,Lt} & \cdots \end{pmatrix},
\]

where \( \omega^f_{it,jK} \in \omega^f_{it} \), and \( \omega^p_{it,jK} \in \omega^p_{it} \). Then the summation terms in function (3.15) can be replaced by the vector multiplication terms as,

\[
y_{it} = \lambda^f \omega^f_{it} E_t(Y) + \lambda^p \omega^p_{it} Y + \beta^* x^*_i.
\]

Recall \( \beta^* = \frac{1}{r} \) and \( r \) is the interest rate.

Considering \( \omega^f_{it} \) as the \( it \)th row of a weighting matrix \( W^f \), \( \omega^p_{it} \) as the \( it \)th row of weighting matrix \( W^p \), and \( x^*_i \) is the \( it \)th element of \( X^* \), the price equation system can be written in matrix form as,

\[
Y = \lambda^f W^f EY + \lambda^p W^p Y + \beta^* X^*,
\]

(3.17)
where $W^f$ is an upper triangular matrix with zeros on the diagonal, $W^p$ is a lower triangular matrix with zeros on the diagonal, and $X^*$ is a vector with exogenous and predetermined entries.

### 3.2.2 The Reduced Functional Form

The expectation equation (3.17) is not directly estimable because of the unknown expectation term $E(Y)$, so it has to be solved for the estimation purpose. Under rational expectation regime, this section derives the reduced form equation\(^63\) employing the method of undetermined coefficients. An educated guess for the problem at hand would be,

\[ Y = \Pi X^* \alpha, \quad (3.18) \]

where both matrix $\Pi$, and vector $\alpha$ are undetermined coefficients. Plugging this guess in equation (3.17), we get

\[ Y = \lambda^f W^f E \Pi X^* \alpha + \lambda^p W^p \Pi X^* \alpha + \beta^* X^*, \quad (3.19) \]

or equivalently

\[ Y = \lambda^f W^f \Pi X^* \alpha + \lambda^p W^p \Pi X^* \alpha + \beta^* X^*, \quad (3.20) \]

because all the elements of $X^*$ are exogenously predetermined and invariant throughout all the time periods. We proceed with identification using equation (3.18) and equation (3.20),

\(^63\)Equation(3.17) is also called a structural equation, which represents the endogenous variables as a function of both endogenous and exogenous variables. In comparison with structural equations, a reduced form equation expresses endogenous variables solely in terms of exogenous variables.
\[ \lambda^I W^f \Pi X^* \alpha + \lambda^p W^p \Pi X^* \alpha + \beta^* X^* = \Pi X^* \alpha, \]

which is in turn

\[ \Pi X^* \alpha = (I - \lambda^I W^f - \lambda^p W^p)^{-1} \beta^* X^*, \]

Comparing the above equation with our original guess, the identified solution is,

\[ Y = (I - \lambda^I W^f - \lambda^p W^p)^{-1} \beta^* X^*. \]

Recall that \( X^* \) measures the part of dividend that is derived from the physical condition of the house itself. To replace the latent variable \( X^* \) with the observed variables, \( X^* \) is described as a function of housing structural attributes as \( X^* = X \tilde{\beta} + \tilde{\varepsilon} \), where \( X \) is a vector of observed housing characteristics. The final reduced form of the price equation is,

\[ Y = (I - \lambda^I W^f - \lambda^p W^p)^{-1}(X \beta + \varepsilon), \tag{3.21} \]

or

\[ Y = \lambda^I W^f Y + \lambda^p W^p Y + X \beta + \varepsilon, \tag{3.22} \]

where \( \beta = \tilde{\beta} \beta^* = \frac{\tilde{\beta}}{\tau} \) and \( \varepsilon = \tilde{\varepsilon} \beta^* = \frac{\tilde{\varepsilon}}{\tau} \) is the stochastic error vector. We will assume the elements of \( \varepsilon \) are iid distributed with variance \( \sigma^2 I \) for the estimation purpose.

### 3.2.3 Weighting the Market Values of Nearby Houses

This section explains how we construct the weighting matrix \( W^f \) and \( W^p \). Recall that the elements of both \( W^f \) and \( W^p \) are the products of the time discount factor,
(δ_{tk}, \delta_{tk'})$, and the spatial distance function, \(s_{ij}\). For the estimation of the current market value of a given house, the difference between \(W^{p}\) and \(W^{f}\) is that \(W^{p}\) counts the past transactions of the neighboring houses, while \(W^{f}\) counts the future transactions of the neighboring houses. These features determine the upper triangular property of \(W^{f}\), and the lower triangular property of \(W^{p}\).

Suppose there are \(n\) transactions up to time \(n\). After we sort all the transactions, the price vector \(Y\) has the following form,

\[
Y = \begin{pmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & \cdots \end{pmatrix}^t,
\]

where \(y_1\) is the transaction observed at time 1, and \(y_2\) is the transaction observed at time 2, etc..

**Constructing the Weighting Matrix \(W^{f}\)**

To construct \(W^{f}\), we need to know the future transactions for each \(y\). For instance, \(y_1\)'s future starts from period 2. The set of \(y_1\)'s expected future neighborhood house values would be the expectations of \(\begin{pmatrix} y_2 & y_3 & y_4 & y_5 & \cdots \end{pmatrix}\). The first row of \(W^{f}\), which consists of the weights assigned to \(y_1\)'s expected future neighborhood house values, is then

\[
\omega_{1}^{f} = \begin{pmatrix} 0 & \delta^2 s_{12} & \delta^3 s_{13} & \delta^4 s_{14} & \delta^5 s_{15} & \cdots \end{pmatrix},
\]

where \(s_{ij}\) is the spatial discount factor for the impact of transaction \(i\) on transaction \(j\), and \(\delta\) is the time discount factor. For transaction \(y_2\), its set of future neighborhood values is constructed similarly.

\(^{64}\)As we assumed that the time is continuous, and there is only one transaction at any given time period. Therefore, \(n\) transactions imply \(n\) time periods.
house values would be \((y_3, y_4, y_5, \cdots)\). The second row of \(W^f\), which is consist of the weights assigned to \(y_2\)’s expected future neighborhood house values, is then

\[
\omega_2^f = \begin{pmatrix}
0 & 0 & \delta^3 s_{23} & \delta^4 s_{24} & \delta^5 s_{25} & \cdots
\end{pmatrix},
\]

In the same fashion, we can compute the weights for \(y_3\)’s future transactions, the weights for \(y_4\)’s future transactions, etc.. We have

\[
W^f = \begin{pmatrix}
0 & \delta^2 s_{12} & \delta^3 s_{13} & \delta^4 s_{14} & \delta^5 s_{15} & \cdots \\
0 & 0 & \delta^3 s_{23} & \delta^4 s_{24} & \delta^5 s_{25} & \cdots \\
0 & 0 & 0 & \delta^4 s_{34} & \delta^5 s_{35} & \cdots \\
0 & 0 & 0 & 0 & \delta^5 s_{45} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix},
\]

which has non-zero elements on the upper triangular, and zeros otherwise.

**Constructing the Weighting Matrix \(W^p\)**

To construct \(W^p\), we need to know the past of the given \(y\). For instance, \(y_2\)’s past neighborhood value is only \(y_1\), and \(y_3\)’s past neighborhood value are \(y_1\) and \(y_2\), \(y_4\)’s past neighborhood values are \(y_1, y_2\) and \(y_3\), etc.. Notice that for \(y_1\), there are no past neighborhood values exist since \(y_1\) itself is in the initial time period. Therefore the first row of \(W^p\), which consists of the weights assigned to \(y_1\)’s past neighborhood house values, has all zero entries. The second row of \(W^p\) has the following structure as

\[
\omega_2^p = \begin{pmatrix}
\delta^2 s_{21} & 0 & 0 & 0 & 0 & \cdots
\end{pmatrix}.
\]

In the same fashion, the third row of \(W^p\), which contains the weights for \(y_3\)’s past neighboring transactions is then
\[ \omega_3^p = \begin{pmatrix} \delta s_{31}^3 & \delta s_{32}^2 & 0 & 0 & 0 & \cdots \end{pmatrix} . \]

We can construct \( W^p \) as

\[
W^p = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & \cdots \\
\delta s_{21}^2 & 0 & 0 & 0 & 0 & \cdots \\
\delta s_{31}^3 & \delta s_{32}^2 & 0 & 0 & 0 & \cdots \\
\delta s_{41}^4 & \delta s_{42}^3 & \delta s_{43}^2 & 0 & 0 & \cdots \\
\delta s_{51}^5 & \delta s_{52}^4 & \delta s_{53}^3 & \delta s_{54}^2 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots 
\end{pmatrix},
\]

which has non-zero elements on the lower triangular, and zeros otherwise.

### 3.3 GMM Estimation Method

The presence of the spatially lagged dependent variables precludes the application of OLS. Because the stochastic error terms are no longer independent of explanatory variables, OLS will lead to biased and inconsistent coefficient estimates for spatial autoregressive (SAR) models. Thus, there is a need for other estimation methods. Both the maximum likelihood (ML) estimator and the general method of moments (GMM) estimator are potential candidates. We choose GMM here because it is less demanding in computation and proved to attain the same limiting distribution as the ML estimator (under normal disturbances). This section and the appendix provide the details of the GMM estimation procedure, including the construction of the feasible best instrumental variables and the moment equations.

#### 3.3.1 Moment Functions

The reduced function is a SAR model with two spatial lags, which is defined as
\[ Y_n = \lambda^f W_n^f Y_n + \lambda^p W_n^p Y_n + X_n\beta + \varepsilon_n, \]  

\[ \text{(3.23)} \]

where \( n \) is the total number of spatial units, \( X_n \) is an \( n \times k \) dimensional matrix of non-stochastic exogenous variables, \( W_n^f \) is a zero diagonal upper triangular weighting matrix, \( W_n^p \) is a zero diagonal lower triangular weighting matrix, and \( \varepsilon_{n1}, \ldots, \varepsilon_{nn} \) of the elements of vector \( \varepsilon_n \) are i.i.d. \( (0, \sigma^2) \).

Let \( \theta = (\lambda^f \quad \lambda^p \quad \beta^f) \) and \( \lambda = (\lambda^f \quad \lambda^p) \). Denote \( S_n(\lambda) = (I_n - \lambda^f W_n^f - \lambda^p W_n^p) \), and \( \varepsilon_n(\theta) = S_n(\lambda)Y_n - X_n\beta \) for any possible value of \( \theta \). In order to distinguish the true parameters from other possible values in the parameter space, we denote \( \lambda_0 \), \( \theta_0 \), and \( \sigma_0^2 \) as the true parameters that generate the sample. It follows that

\[ W_n^f Y_n = W_n^f S_n^{-1}(X_n\beta_0 + \varepsilon_n) = G_{1n}(X_n\beta_0 + \varepsilon_n), \]  

\[ \text{(3.24)} \]

and

\[ W_n^p Y_n = W_n^p S_n^{-1}(X_n\beta_0 + \varepsilon_n) = G_{2n}(X_n\beta_0 + \varepsilon_n), \]  

\[ \text{(3.25)} \]

where \( G_{1n} = W_n^f S_n^{-1} \), and \( G_{2n} = W_n^p S_n^{-1} \) with \( S_n = (I_n - \lambda_0^f W_n^f - \lambda_0^p W_n^p) = S_n(\lambda_0) \) for simplicity. To rule out the unit root case, \( S_n \) has to be nonsingular. If \( \lambda^f \) and \( \lambda^p \) are both less than one in absolute value, the row normalization of the weighting matrices of \( W_n^f \) and \( W_n^p \) is sufficient to guarantee a nonsingular \( S_n \).\(^{65}\) Because \( E((W_n^f Y_n)'\varepsilon_n) = E((G_{1n}\varepsilon_n)'\varepsilon_n) \neq 0 \) and \( E((W_n^p Y_n)'\varepsilon_n) = E((G_{2n}\varepsilon_n)'\varepsilon_n) \neq 0 \), the OLS estimator will be biased and inconsistent for the SAR model.

\(^{65}\)When \( W^f \) and \( W^p \) are row normalized, \( \|W^f\| \leq 1 \) and \( \|W^p\| \leq 1 \). For \( S_n(\lambda) = (I_n - \lambda^f W_n^f - \lambda^p W_n^p) \) to be nonsingular, the sufficient condition is that \( |\lambda^f| < 1 \) and \( |\lambda^p| < 1 \). But of course, the estimated values of \( \lambda^f \) and \( \lambda^p \) do not have to satisfy the sufficient condition.
Lee (2006) suggests the use of two types of moment conditions to construct the GMM framework for the estimation of $\theta$. One type of moment condition is based on the orthogonality of $X_n$ and $\varepsilon_n$. Let $Q_n$ be an $n \times k_x$ matrix of IVs constructed as functions of $X_n$, and $W_n$s, then the $k_x$ moment functions correspond to the orthogonality conditions of $X_n$ and $\varepsilon_n$ are $Q_n' \varepsilon_n(\theta)$. The second type of moment conditions is based on the correlations across the spatial units. For any constant $n \times n$ matrix $P_{jn}(j = 1, \cdots, m)$ that is uniformly bounded in both row and column with $tr(P_n) = 0$, $P_{jn} \varepsilon_n$ is uncorrelated with $\varepsilon_n$. This provides the theoretical ground for the second type of moments that is described as $\varepsilon_n(\theta)'P_{jn} \varepsilon_n(\theta)$. With the selected IV matrix $Q_n$ and zero diagonal matrices $P_{jn}s$, the set of moment functions forms a vector as

$$g_n(\theta) = (P_{1n} \varepsilon_n(\theta))', \cdots, P_{mn} \varepsilon_n(\theta), Q_n')' \varepsilon_n(\theta) = \begin{pmatrix}
\varepsilon_n(\theta)'P_{1n} \varepsilon_n(\theta) \\
\vdots \\
\varepsilon_n(\theta)'P_{mn} \varepsilon_n(\theta) \\
Q_n' \varepsilon_n(\theta)
\end{pmatrix}, \quad (3.26)$$

in the estimation within the GMM framework.$^{66}$

As the moment function of $Q_n' \varepsilon_n(\theta)$ is based on the joint significance of the elements in $X_n$ ($\beta_0 \neq 0$), an invalid instrument variable $Q_n$ can be generated if all regressors in $X_n$ are irrelevant. The proposed GMM estimator with the inclusion of both types of these two moment conditions are applicable even if $X_n$ is not jointly significant, and has the same limiting distribution as the ML estimator with normal disturbances.

$^{66}$At $\theta_0$, $g_n(\theta) = (\varepsilon_n'P_{1n} \varepsilon_n, \cdots, \varepsilon_n'P_{mn} \varepsilon_n, \varepsilon_n'Q_n)'$, which has a zero mean because $E(Q_n' \varepsilon_n) = Q_n' E(\varepsilon_n) = 0$ and $E(\varepsilon_n'P_{jn} \varepsilon_n) = \sigma_0^2 tr(P_{jn}) = 0$ for $j = 1, \cdots, m$. 

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3.3.2 Feasible Best IVs \((P_n \text{ and } Q_n)\)

The best selections of \(Q_n\) and \(P_{jn}\) \((j = 1, \ldots, m)\) for a SAR model with \(p\)-order spatial lags are proposed in Lee (2006).\(^67\) Based this general form, we can derive the best IVs matrices for our SAR model with 2-order spatial lags in equation (3.23) as

\[
Q_{o,n} = (G_{1n}X_n\beta_0, G_{2n}X_n\beta_0, X_n),
\]

and

\[
P_{o,n} = \begin{pmatrix} P_{o,1n} \\ P_{o,2n} \end{pmatrix} = \begin{pmatrix} (G_{1n} - \frac{\text{tr}(G_{1n}) I_n}{n}) \\ (G_{2n} - \frac{\text{tr}(G_{2n}) I_n}{n}) \end{pmatrix},
\]

where \(G_{1n}, G_{2n},\) and \(S_n\) are defined as in equation (3.24) and (3.25). Because \(Q_{o,n}\) and \(P_{o,n}\) are functions of \(\theta_0\), the best IVs can not be used directly in the GMM estimation, and the feasible ones need to be derived. With some initial consistent estimates \(\hat{\theta}\), the feasible best IV function for the linear moments is

\[
\hat{Q}_{o,n} = (\hat{G}_{1n}X_n\hat{\beta}, \hat{G}_{2n}X_n\hat{\beta}, X_n),
\]

and the feasible best IV function for the quadratic moments is

\[
\hat{P}_{o,n} = \begin{pmatrix} \hat{P}_{1n} \\ \hat{P}_{2n} \end{pmatrix} = \begin{pmatrix} (\hat{G}_{1n} - \frac{\text{tr}(\hat{G}_{1n}) I_n}{n}) \\ (\hat{G}_{2n} - \frac{\text{tr}(\hat{G}_{2n}) I_n}{n}) \end{pmatrix},
\]

\(^67\)For a SAR model with \(p\)-order spatial lags as

\[
Y_n = \sum_{j=1}^{p} \lambda_j W_{jn} Y_n + X_n\beta + \varepsilon_n,
\]

where elements \(\varepsilon_{ni}, i = 1, \ldots, n,\) of \(\varepsilon_n\) are iid\(\left(0, \sigma^2_0\right)\), and \(W_{jn}\)s are \(p\) distinct spatial matrices. Denote \(\lambda = (\lambda_1, \ldots, \lambda_p)'\), \(\theta = (\lambda', \beta')'\), \(S_n(\lambda) = (I_n - \sum_{j=1}^{p} \lambda_j W_{jn}), S_n = S_n(\lambda_0),\) and \(G_{jn} = W_{jn} S_n^{-1}\) for \(j = 1, \ldots, p\). Furthermore, let \(\varepsilon_n(\theta) = S_n(\lambda)Y_n - X_n\beta\) for any possible value of \(\theta\). Lee (2006)[25] proves the best selection of \(Q_n\) is \(Q_{o,n} = (G_{1n}X_n\beta_0, \ldots, G_{pn}X_n\beta_0, X_n)\) and the best IV function of \(P_{jn}\)s is \(P_{o,n} = (G_{jn} - \frac{\text{tr}(G_{jn}) I_n}{n})\), for \(j = 1, \ldots, p\).
where $\hat{G}_{1n} = W_n^f(I_n - \hat{\lambda}_n^f W_n^f - \hat{\lambda}_n^p W_n^p)^{-1}$, $\hat{G}_{2n} = W_n^p(I_n - \hat{\lambda}_n^f W_n^f - \hat{\lambda}_n^p W_n^p)^{-1}$, $\hat{\lambda}_n$ and $\hat{\beta}$ are the initial consistent estimates. Kelejian and Prucha (1998) suggested the use of $W_n X_n$, $W_n^2 X_n$, etc., together with $X_n$ as IVs in a 2SLS framework for estimating $\theta$. We extend this application to our spatial model with 2-order spatial lags using the following instrument matrix as$^{68}$

$$Q_{2sls,n} = \begin{pmatrix} W_n^f X_n & W_n^p X_n & W_n^f W_n^p X_n & W_n^p W_n^f X_n & X_n \end{pmatrix}.$$ 

This instrument matrix is employed to form an initial 2SLS estimator, which can then be used in the construction of $\hat{Q}_{o,n}$ and $\hat{P}_{o,n}$ for the estimation of $\theta$ within a GMM scheme. The 2SLS estimate $\hat{\theta}_{2sls,n}$ has the form of

$$\begin{align*}
\hat{\theta}_{2sls,n} &= (\hat{\lambda}_n) \\
&= [Z_n' Q_{2sls,n} (Q_{2sls,n} Q_{2sls,n})^{-1} Q_{2sls,n} Z_n]^{-1} Z_n' Q_{2sls,n} (Q_{2sls,n} Q_{2sls,n})^{-1} Q_{2sls,n} Y_n,
\end{align*}$$

where $Z_n = \begin{pmatrix} W_n^f Y_n & W_n^p Y_n & X_n \end{pmatrix}$.

### 3.3.3 Feasible Optimal GMM Estimator

The moment function for the house price equation (3.23) is

$$g_n(\theta) = \begin{pmatrix} \varepsilon_n'(\theta) P_{1n} \varepsilon_n(\theta) \\ \varepsilon_n'(\theta) P_{2n} \varepsilon_n(\theta) \\ Q_n' \varepsilon_n(\theta) \end{pmatrix}.$$ 

The derivative vector of $g_n(\theta)$ with respect to $\theta$ is

$$\begin{align*}
\frac{\partial g_n(\theta)}{\partial \theta'} &= -\begin{pmatrix}
(W_n^f Y_n)' P_{1n} \varepsilon_n(\theta) \\
(W_n^f Y_n)' P_{2n} \varepsilon_n(\theta) \\
Q_n' W_n^f Y_n
\end{pmatrix}
\begin{pmatrix}
(W_n^p Y_n)' P_{1n} \varepsilon_n(\theta) \\
(W_n^p Y_n)' P_{2n} \varepsilon_n(\theta) \\
Q_n' W_n^p Y_n
\end{pmatrix}^{-1}
\begin{pmatrix}
\varepsilon_n'(\theta) P_{1n} X_n \\
\varepsilon_n'(\theta) P_{2n} X_n \\
Q_n' X_n
\end{pmatrix}.
\end{align*}$$

$^{68}$Because $W_n^f$ and $W_n^p$ are row standarlized, if $X_n$ contains an intercept column, the products of $W_n^f$ or $W_n^p$ or both with $X_n$ will generate a constant column. These constant columns together with the constant column in $X_n$ results in a singular $Q_{2sls,n}$. To avoid this, the intercept column in $X_n$ needs to be removed before it is multiplied with the weighting matrices.
where \( P_n^s = P_n + P_n' \). It follows that,\(^{69}\)

\[
D_n = \frac{\partial E(g_n(\theta_0))}{\partial \theta'} = - \begin{pmatrix} \sigma_0^2 tr(P_n^s G_{1n}) & \sigma_0^2 tr(P_n^s G_{2n}) & 0 \\ \sigma_0^2 tr(P_n' G_{1n}) & \sigma_0^2 tr(P_n' G_{2n}) & 0 \\ Q_n' G_{1n}X_n \beta_0 & Q_n' G_{2n}X_n \beta_0 & Q_n' X_n \end{pmatrix}
\]

Provided the moment function \( g_n(\theta) \), the optimal GMM estimator is

\[
\hat{\theta}_{o,n} \equiv \arg\min_\theta g_n(\theta)' \Omega_n^{-1} g_n(\theta),
\]

where \( \Omega_n \) is the variance-covariance matrix of \( g_n(\theta_0) \) as,

\[
\Omega_n = \begin{pmatrix} (\mu_4 - 3\sigma_0^4) w_n^t w_n & \mu_3 w_n^t Q_n \\ \mu_3 Q_n w_n & 0 \end{pmatrix} + \sigma_0^4 \begin{pmatrix} \Delta_n & 0 \\ 0 & \frac{1}{\sigma_0^2} Q_n' Q_n \end{pmatrix},
\]

with \( \Delta_n = [vec(P_n'), vec(P_n')] [vec(P_n^s), vec(P_n^s)], w_n = [vec_D(P_n), vec_D(P_n)],[60] \)

\( \mu_3 = E(\varepsilon_{ni}^3), \) and \( \mu_4 = E(\varepsilon_{ni}^4). \)

For the feasible best GMM estimation with \( \hat{Q}_n \) and \( \hat{P}_n \), the corresponding \( \Omega_n \) is

\[
\hat{\Omega}_n = \begin{pmatrix} (\hat{\mu}_4 - 3\hat{\sigma}^4) \hat{w}_n^t \hat{w}_n & \hat{\mu}_3 \hat{w}_n^t \hat{Q}_n \\ \hat{\mu}_3 \hat{Q}_n \hat{w}_n & 0 \end{pmatrix} + \hat{\sigma}^4 \begin{pmatrix} \hat{\Delta}_n & 0 \\ 0 & \frac{1}{\hat{\sigma}^2} \hat{Q}_n' \hat{Q}_n \end{pmatrix},
\]

where \( \hat{\Delta}_n = [vec(\hat{P}_n'), vec(\hat{P}_n')] [vec(\hat{P}_n^s), vec(\hat{P}_n^s)], \hat{w}_n = [vec_D(\hat{P}_n), vec_D(\hat{P}_n)],[71] \)

and the vector of moment function is

\[
g_{o,n}(\theta) = \begin{pmatrix} \varepsilon_{n}'(\theta) (\hat{G}_{1n} - \frac{tr(\hat{G}_{1n})}{n} I_n) \varepsilon_n(\theta) \\ \varepsilon_{n}'(\theta) (\hat{G}_{2n} - \frac{tr(\hat{G}_{2n})}{n} I_n) \varepsilon_n(\theta) \\ \varepsilon_{n}'(\theta) (\hat{G}_{1n} X_n \beta, \hat{G}_{2n} X_n \beta, X_n)' \varepsilon_n(\theta) \end{pmatrix}.
\]

\(^{69}\) \(E(\varepsilon_{n}'(\theta_0) P_n^s W_n Y_n)\)

\( = E(\varepsilon_{n}'(\theta_0) P_n^s G_n (X_n \beta_n + \varepsilon_n(\theta_0)))\)

\( = E(\varepsilon_{n}'(\theta_0) P_n^s G_n X_n \beta_n + E(\varepsilon_{n}'(\theta_0) P_n^s G_n \varepsilon_n(\theta_0))\)

\( = 0 + E(\varepsilon_{n}'(\theta_0) P_n^s G_n \varepsilon_n(\theta_0))\)

\( = E[tr(\varepsilon_{n}'(\theta_0) P_n^s G_n \varepsilon_n(\theta_0))]\)

\( = \sigma_0^2 tr(P_n^s G_n)\)

\(^{70}\) \(vec(P_n')\) denotes the column vector that is formed from stacking all the columns of \( P_n. \) \( vec_D(P_n)\)

\( \) denotes the column vector formed with diagonal elements of \( P_n. \)

\(^{71}\) \( \hat{P}_n, \hat{Q}_n, \hat{\mu}_3, \) and \( \hat{\sigma} \) are computed using the initial 2SLS estimates.
The feasible best GMM estimator will be derived from \( \min_{\theta} g_{b,n}(\theta)\hat{\Omega}_n^{-1} g_{b,n}(\theta) \).

The first order derivative of the objective function with respect to \( \theta \) is

\[
2 \left( \frac{\partial g_{b,n}(\theta)}{\partial \theta} \right)^\prime \hat{\Omega}_n^{-1} g_{b,n}(\theta).
\]

### 3.3.4 Wald Test

The Wald tests for spatial dependence can be used to discriminate between alternative specifications. This subsection provides the technical details in constructing the Wald test statistics for the spatial dependence and the equality of the neighborhood effects.

#### Spatial Dependence Test

The Wald test for the existence of the spatial dependence of house prices in a neighborhood, tests for the null hypothesis, \( H_0 : \begin{pmatrix} \lambda^f = 0 \\ \lambda^p = 0 \end{pmatrix} \), which suggests a conventional hedonic pricing model that is specified as,

\[
P_n = X_n\beta + \varepsilon_n. \tag{3.27}
\]

The Wald statistic is then constructed as,

\[
Wald_1 = \left( \begin{array}{c} \hat{\lambda}^f \\ \hat{\lambda}^p \end{array} \right)^\prime S(\hat{\lambda}^f, \hat{\lambda}^p)^{-1} \left( \begin{array}{c} \hat{\lambda}^f \\ \hat{\lambda}^p \end{array} \right),
\]

where \( \hat{\lambda}^f \) and \( \hat{\lambda}^p \) are the estimated \( \lambda \)'s in our spatial asset model (unrestricted model), and \( S(\hat{\lambda}^f, \hat{\lambda}^p) \) is the estimated variance covariance matrix of \( \begin{pmatrix} \hat{\lambda}^f \\ \hat{\lambda}^p \end{pmatrix} \) that is the block of \( Var(\hat{\theta}) \) partitioned conformably with \( \theta (\theta = (\lambda^f \ \lambda^p \ \beta')^\prime) \). The estimate of \( Var(\hat{\theta}) \) can be derived from the negative Hessian of the GMM objective function.
Note that test statistics are asymptotically distributed as $\chi^2$ with two degrees of freedom.

**Spatial Equality Test**

The Wald statistic for the equality of the neighborhood effects flowing from the future and the past neighborhood transactions, tests for the null hypothesis, $H_0 : (\lambda^f = \lambda^p)$, with the spatial model of equation (3.23) as the alternative model. The Wald statistic is written as,

$$Wald_2 = \left( \hat{\lambda}^f - \hat{\lambda}^p \right)^2 \left( \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right) S(\hat{\lambda}^f, \hat{\lambda}^p) \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)^{-1},$$

where $\hat{\lambda}^f, \hat{\lambda}^p$ and $S(\hat{\lambda}^f, \hat{\lambda}^p)$ are the same as those in the previous Wald statistic. The test statistics are asymptotically distributed as $\chi^2$ with one degree of freedom.

**3.4 Data Description**

The data set used in this paper consists of 37,681 original deed transfers in the suburban school districts of Franklin County OH between 1985 and 1999. It is obtained from the Center for Urban and Regional Analysis (CURA) and indexed by school district code. In view of the variations in the implicit marginal prices of housing structural attributes, the estimation of the spatial asset pricing model is carried out for each school district respectively. There are 15 suburban school districts in Franklin County. They are: the Bexley city school district, the Canal Winchester local school district, the Grandview heights school district, the Hamilton local school district, the Gahanna Jefferson city school district, the Groveport Madison local school district, the Plain local school district, the Reynoldsburg city school district, the Hilliard city school district, the Southwestern city school district, the Upper Arlington city school district.
district, the Dublin city school district, the Westerville city school district, the Westerville city school district, and the Worthington city school district.\textsuperscript{72} The estimation results are reported for all of these suburban school districts except for the Plain local school district, for which the data set does not contain sufficient observations.

To test the model performance in out-of-sample predictions, it is desirable to carry out a cross validation using a different data set than that used for estimation. Therefore, we split each school district data set into 2 subsets:

1. A model-building set, which consists of the transactions during 1985 to 1996 (\(m\) observations),

2. A validation set, which consists of the transactions during 1997 to 1999 (\(n - m\) observations).

The model-building set is used for estimating the parameters, while the validation set is used for testing the predictive power of the model. Suppose there are \(n\) observations in total for a school district, we use \(m\) to denote the number of observation in the model-building set, and \(n - m\) to denote the number of observation in the validation set.\textsuperscript{73}

The recorded transaction dates (yy/mm/dd) make it convenient to adjust for inflation across time. The transaction prices are converted to a base period (1st quarter, 1983).

\textsuperscript{72}The only urban school district in Franklin County is the Columbus city school district.

\textsuperscript{73}Ideally, the validation set should include only the transaction that happens right after the last transaction in the model-building set. This is analogous with the leave-one-out cross validation test applied in cross-sectional data. This test should be repeated as time proceeds, and the average of the prediction errors should be used to measure the performance of the model. For example, if the validation set includes the transaction happens on May 30, 1983, the model-building set should include all the transactions happens before May 30, 1983. If the transaction on May 31, 1983 is the validation set, the previous model-building set should be adjusted to include the transaction on May 30, 1983. Given the structure of our spatial model, the two spatial-time weighting matrices have to be constructed whenever the model-building set and the validation set is adjusted. The cost of the computational complexity incurred by this is prohibitive.
2001) dollar value using a quarterly house price index for Columbus Metropolitan Statistics Area (MSA) from Freddie Mac.

There are seven structural attributes available in the database: the number of total rooms, the number of full bathrooms, the number of half bathrooms, the building area, the lots size, the age of the house, and the number of fireplaces. We have included six variables, namely, lots size (LOTSSQFT), building area (BLDGSQFT), number of full baths (BATH), number of half baths (HBATH), housing age (HAGE), and fireplace (FIREPLACE). The definition and descriptive statistics on variables are provided in Table (3.1).

In the specification of the weighting matrices, we consider houses within 2-mile radius as neighbors—houses that are outside the radius receive zeros weights, and limit the time lag between two transactions to be less than half a year—two transactions that happened more than six months apart are not considered to have influences on each other. The reason for selecting these screening criteria is to make our estimation results comparable with Can’s work in 1997, in which she specifies a spatial hedonic model with weighted prior sales as the spatial lag (equivalent to $W_n^p Y_n$ in our model).

Because the number of total rooms is highly correlated with the other housing structural variables, it is excluded from the estimation.

Given the average quarterly real return on riskless security—a treasury bond as $r = 0.01$. The daily time discount factor $\delta = \frac{1}{(1+r)^{1/252}} = 0.99989$.

We also tried other screening criteria. For example, we extended the geographic distance restriction from 2 miles to 3 miles, or contracted from 2 miles to 1 mile. In the same manner, we varied the restrictions on the time span. The estimation results are quite consistent for different specifications of the weighting matrices. We also find that the more neighbors included within a given geographic area (same distance restriction, but longer time span), the larger the estimates of the spatial autoregressive coefficients ($\lambda^f$ and $\lambda^p$). The average neighborhood transaction becomes more representative when more neighborhood transactions are included within a radius of a given value. Meanwhile, given the same geographic area, the average transaction in a longer time span and the average transaction in a shorter time span are equally important in the spatial context. It is not surprising to see the slight upward changes of both spatial coefficients when the time span is extended and the radius of distance is held fixed for the neighbor screening criterion. Sample
Furthermore, both of these weighting matrices are row normalized for the estimation purpose. The summary statistics of the number of neighboring transactions is also reported in Table (3.1).

estimates with the screening rules that use neighbors within 2 miles and less than 2 years apart are reported in 3.6.
### Table 3.1: Definition of Variables and Summary Statistics

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE</td>
<td>Transaction price in $2001(Q1) and divided by 1,000</td>
</tr>
<tr>
<td>LOTSQFT</td>
<td>Lot size in thousands of square feet</td>
</tr>
<tr>
<td>BLDGSQFT</td>
<td>Building area in thousands of square feet</td>
</tr>
<tr>
<td>BATH</td>
<td>Total number of bathrooms</td>
</tr>
<tr>
<td>HBATH</td>
<td>Total number of half bathrooms</td>
</tr>
<tr>
<td>HAGE</td>
<td>Age of house</td>
</tr>
<tr>
<td>FIREPLACE</td>
<td>Dummy variable that indicates the existence of a fireplace (1=fireplace, 0=no fireplace)</td>
</tr>
<tr>
<td>$\lambda^{f}$</td>
<td>Spillover coefficient measuring the spatial influence of the expected future values of neighboring houses.</td>
</tr>
<tr>
<td>$\lambda^{p}$</td>
<td>Spillover coefficient measuring the spatial influence of the current values of neighboring houses.</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>Variance of the errors</td>
</tr>
<tr>
<td>School ID</td>
<td>School District</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Franklin County</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE</td>
<td>148.02</td>
<td>56.12</td>
<td>30.05</td>
<td>299.94</td>
</tr>
<tr>
<td>LOTSQFT</td>
<td>9.51</td>
<td>3.88</td>
<td>1.31</td>
<td>32.67</td>
</tr>
<tr>
<td>BLDGSQFT</td>
<td>1.62</td>
<td>0.50</td>
<td>0.50</td>
<td>7.20</td>
</tr>
<tr>
<td>BATH</td>
<td>1.52</td>
<td>0.53</td>
<td>1.00</td>
<td>4.00</td>
</tr>
<tr>
<td>HBATH</td>
<td>0.58</td>
<td>0.51</td>
<td>0.00</td>
<td>3.00</td>
</tr>
<tr>
<td>HAGE</td>
<td>31.96</td>
<td>19.24</td>
<td>3.00</td>
<td>197.00</td>
</tr>
<tr>
<td>FIREPLACE</td>
<td>0.63</td>
<td>0.57</td>
<td>0.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>
3.5 Results

3.5.1 Parameter Estimates

Parameter estimates for the spatial asset pricing model (equation 3.23) are presented in Table (3.2). Table (3.3) reports the Wald test results on the coefficients of spatially lagged dependent variables \((Wald1\) and \(Wald2\)). The Wald test statistics corroborate the spatial asset pricing model, and therefore offer a more significant assessment of the coefficient estimates. The coefficient estimates of the hedonic model specified by equation (3.27) are provided in Table (3.4) as a reference. The estimates of the housing structural variables in Table (3.2) are quite consistent with the estimation results in Table (3.4). This fact gives credence to the coefficient estimates of the spatial asset pricing model. The following discussion is based on the results of Table (3.2) and Table (3.3).
<table>
<thead>
<tr>
<th>Schools</th>
<th>$\lambda^l$</th>
<th>$\lambda^p$</th>
<th>CONS -TANT</th>
<th>LOTS -SQFT</th>
<th>BLDG -SQFT</th>
<th>BATH</th>
<th>HBATH</th>
<th>HAGE</th>
<th>FIRE -PLACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bexley</td>
<td>0.71</td>
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<td>68.78</td>
<td>1.24*</td>
<td>43.04***</td>
<td>10.62***</td>
<td>23.61***</td>
<td>0.49***</td>
<td>10.55***</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
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<td>(0.58)</td>
<td>(1.72)</td>
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<td>(3.36)</td>
<td>(5.68)</td>
<td>(3.13)</td>
<td>(3.29)</td>
</tr>
<tr>
<td>Canal Winchester</td>
<td>0.16**</td>
<td>0.35***</td>
<td>-6.61</td>
<td>1.11***</td>
<td>33.40***</td>
<td>7.89**</td>
<td>6.82**</td>
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<td>3.62</td>
</tr>
<tr>
<td></td>
<td>(2.52)</td>
<td>(5.6)</td>
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<td>(3.9)</td>
<td>(9.72)</td>
<td>(2.46)</td>
<td>(2.41)</td>
<td>(-4.88)</td>
<td>(1.57)</td>
</tr>
<tr>
<td>Grandview Heights</td>
<td>0.46***</td>
<td>0.33***</td>
<td>-52.07***</td>
<td>2.75***</td>
<td>41.49***</td>
<td>-2.32</td>
<td>6.60**</td>
<td>0.01</td>
<td>7.31***</td>
</tr>
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<td>(5.09)</td>
<td>(-5.47)</td>
<td>(5.35)</td>
<td>(9.94)</td>
<td>(-0.61)</td>
<td>(2.27)</td>
<td>(0.12)</td>
<td>(3.17)</td>
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<tr>
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<td>43.66***</td>
<td>0.27**</td>
<td>15.52***</td>
<td>11.81***</td>
<td>7.00***</td>
<td>-0.54***</td>
<td>6.98***</td>
</tr>
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<td>(5.06)</td>
<td>(8.28)</td>
<td>(2.05)</td>
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<td>(7.39)</td>
<td>(4.98)</td>
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<td>(5.13)</td>
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<td>Gahanna Jefferson</td>
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<td>0.83***</td>
<td>26.84***</td>
<td>13.17***</td>
<td>7.16***</td>
<td>-0.69***</td>
<td>8.09***</td>
</tr>
<tr>
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<td>(11.64)</td>
<td>(7.61938)</td>
<td>(4.57)</td>
<td>(-11.92)</td>
<td>(5.8)</td>
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<td>0.24***</td>
<td>7.25***</td>
<td>1.27***</td>
<td>26.56***</td>
<td>11.84***</td>
<td>8.84***</td>
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</tr>
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<td>(12.14)</td>
<td>(4.82)</td>
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<td>(9.52)</td>
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<tr>
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<td>0.43***</td>
<td>-27.75***</td>
<td>0.62***</td>
<td>40.29***</td>
<td>5.10***</td>
<td>0.59</td>
<td>-0.54***</td>
<td>6.87***</td>
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<td>(10.31)</td>
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<td>(4.41)</td>
<td>(25.78)</td>
<td>(5.13)</td>
<td>(0.56)</td>
<td>(-14.63)</td>
<td>(7.88)</td>
</tr>
<tr>
<td>Hilliard</td>
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<td>17.22***</td>
<td>0.3</td>
<td>48.68***</td>
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<td>(3.47)</td>
<td>(1.44)</td>
<td>(26.23)</td>
<td>(0.83)</td>
<td>(2.58)</td>
<td>(-8.19)</td>
<td>(9.1)</td>
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</table>

To be continued

Table 3.2: Coefficient Estimates of the Spatial Price Model (GMM)- Two Mile and Six Months
Table 3.2: (Continued)

<table>
<thead>
<tr>
<th>Schools</th>
<th>$\lambda^f$</th>
<th>$\lambda^p$</th>
<th>CONS</th>
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<th>BLDG</th>
<th>BATH</th>
<th>HBATH</th>
<th>HAGE</th>
<th>FIRE</th>
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<td>6.93***</td>
<td>-0.40***</td>
<td>10.89***</td>
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<td>(3.45)</td>
<td>(6.45)</td>
<td>(5.41)</td>
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<td>(5.5)</td>
<td>(-11.46)</td>
<td>(10.29)</td>
</tr>
<tr>
<td>Upper Arlington</td>
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<td>0.40**</td>
<td>-4.52</td>
<td>0.51***</td>
<td>25.88***</td>
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<td>(-1.95)</td>
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<td>(11.34)</td>
<td>(12.9)</td>
<td>(9.4)</td>
</tr>
<tr>
<td>Dublin</td>
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<td>0.35***</td>
<td>-29.85***</td>
<td>0.33*</td>
<td>42.68***</td>
<td>7.21***</td>
<td>6.20***</td>
<td>-0.18**</td>
<td>8.67***</td>
</tr>
<tr>
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<td>(5.08)</td>
<td>(7.56)</td>
<td>(-6.84)</td>
<td>(1.8)</td>
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<td>(4.08)</td>
<td>(3.55)</td>
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<td>(5.27)</td>
</tr>
<tr>
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<td>0.36***</td>
<td>-45.27***</td>
<td>1.08***</td>
<td>37.89***</td>
<td>5.44***</td>
<td>3.26***</td>
<td>-0.46***</td>
<td>10.41***</td>
</tr>
<tr>
<td></td>
<td>(9.68)</td>
<td>(7.61)</td>
<td>(-11.16)</td>
<td>(5.57)</td>
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<td>(4.12)</td>
<td>(2.59)</td>
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<td>(9.17)</td>
</tr>
<tr>
<td>Whitehall</td>
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<td>26.57***</td>
<td>11.84***</td>
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</tr>
<tr>
<td>Worthington</td>
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<td>0.19</td>
<td>44.37***</td>
<td>11.42***</td>
<td>5.72***</td>
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</tr>
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<td>(4.62)</td>
<td>(-10.09)</td>
<td>(0.93)</td>
<td>(19.27)</td>
<td>(5.47)</td>
<td>(3.04)</td>
<td>(1.65)</td>
<td>(4.95)</td>
</tr>
</tbody>
</table>

76 The numbers in the parentheses are the t-statistics; neighbors are within 2-mile radius and within six months.

* indicates significance at 10% level ($p=1.65$); ** indicates significance at 5% level ($p=1.96$); *** indicates significance at 1% level or less ($p=12.58$).
<table>
<thead>
<tr>
<th>Schools</th>
<th>$\sigma^2$</th>
<th>WALD1</th>
<th>WALD2</th>
<th>OBS_E</th>
<th>OBS_P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bexley</td>
<td>2291.7</td>
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<td>0.55</td>
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<td>1757</td>
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<tr>
<td>Canal Winchester</td>
<td>678.28</td>
<td>0.61</td>
<td>2.53</td>
<td>539</td>
<td>646</td>
</tr>
<tr>
<td>Grandview Heights</td>
<td>1033.1</td>
<td>350.09***</td>
<td>1.14</td>
<td>709</td>
<td>885</td>
</tr>
<tr>
<td>Hamilton</td>
<td>270.85</td>
<td>53.27***</td>
<td>8.08***</td>
<td>930</td>
<td>1077</td>
</tr>
<tr>
<td>Gahanna Jefferson</td>
<td>1039.88</td>
<td>532.73***</td>
<td>0.15</td>
<td>3058</td>
<td>3785</td>
</tr>
<tr>
<td>Groveport Madison</td>
<td>314.47</td>
<td>132.20***</td>
<td>1.85</td>
<td>3208</td>
<td>3979</td>
</tr>
<tr>
<td>Reynoldsburg</td>
<td>378.91</td>
<td>490.90***</td>
<td>2.25</td>
<td>2575</td>
<td>3224</td>
</tr>
<tr>
<td>Hilliard</td>
<td>398.91</td>
<td>108.91***</td>
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<td>2144</td>
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</tr>
<tr>
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<td>2591</td>
</tr>
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<td>5.48**</td>
<td>3241</td>
<td>4184</td>
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<tr>
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<td>722.18***</td>
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<td>353.45***</td>
<td>3.81*</td>
<td>2210</td>
<td>2879</td>
</tr>
</tbody>
</table>

Table 3.3: Wald Tests of the Spatial Price Model (GMM)- Two Miles and Six Months

Because WALD1 follows a distribution, and WALD2 follows a distribution, the critical values of the chi-square distributions are as follows:

$\chi^2(2)=4.61$ (significance level=0.10), $\chi^2(2)=5.99$ (significance level=0.05), $\chi^2(2)=9.21$ (significance level=0.01), $\chi^2=13.82$ (significance level=0.001), $\chi^2(1)=2.71$ (significance level=0.10), $\chi^2(1)=3.84$ (significance level=0.05), $\chi^2(1)=6.64$ (significance level=0.01), $\chi^2=10.83$ (significance level=0.001)

OBS_E indicates the number of observations that are used for estimation; OBS_P indicated the number of observations that are used for predication.
<table>
<thead>
<tr>
<th>School District</th>
<th>CONSTANT</th>
<th>LOTSSQFT</th>
<th>BLDGSQFT</th>
<th>BATH</th>
<th>HBATH</th>
<th>HAGE</th>
<th>FIREPLACE</th>
<th>Adj $R^2$</th>
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</thead>
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<tr>
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<td>27.03***</td>
<td>1.00**</td>
<td>41.27***</td>
<td>11.55***</td>
<td>19.45***</td>
<td>0.39***</td>
<td>9.69***</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(4.39)</td>
<td>(2.31)</td>
<td>(14.69)</td>
<td>(5.22)</td>
<td>(9.56)</td>
<td>(4.89)</td>
<td>(5.04)</td>
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</tr>
<tr>
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<td>52.85***</td>
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<td>-0.41***</td>
<td>4.75**</td>
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<tr>
<td></td>
<td>(7.74)</td>
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<td>(7.99)</td>
<td>(12.62)</td>
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<td>(3.1)</td>
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<tr>
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To be continued

Table 3.4: Coefficient Estimates of the Conventional Hedonic Price Model (OLS)
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<th>HAGE</th>
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<td>(6.07)</td>
<td>(2.83)</td>
<td>(3.58)</td>
<td>(6.42)</td>
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</tr>
</tbody>
</table>

*indicates significance at 10% level;
** indicates significance at 5% level;
*** indicates significance at 1% level or less.
For the housing structural variables, the signs and the magnitudes of those estimates are reasonable, providing sound interpretations on the marginal prices of housing attributes. For instance, the coefficient estimates for building area range between 16 and 49. An increase of 1000 square feet of building area will lead to an increase in house price of between $16,000 and $49,000. The coefficient estimates for house age range between -0.4 and -0.7. An increase of 1 year of house age will drive down house price by around $550. The coefficient estimates for full bathroom number range between 5 and 27. Adding one full bath will lead an increase in house price of between $5,000 and $27,000. The coefficient estimates for fireplace range between 7 and 13. Adding one fireplace will lead an increase in house price of between $7,000 and $13,000.

It is observed that spatial interdependence plays an important role in determining house prices. The Wald test statistics of \( Wald1 \) verifies the joint significance of the two spatial autoregressive coefficients of \( \lambda_f \) and \( \lambda_p \). The t statistics for \( \lambda_p \) are in the rejection region (at 5% significance level) for almost all the school district with the exception of the Bexley district. The only insignificant \( \lambda_f \)s are found to be in the Bexley city school district, the Hamilton local school district and the Upper Arlington city school district. The values of those significant \( \lambda_p \)s range between 0.18 and 0.43, and the values of those significant \( \lambda_f \)s range between 0.10 and 0.48. For instance, the estimated \( \lambda_f \) and \( \lambda_p \) of the Canal Winchester local school district have the values of 0.16 and 0.35 respectively. The value of \( \lambda_p \) translates into a $350 increase in the

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Fireplace} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{Building Area} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Full Bath} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{House Age} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Fireplace} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{Building Area} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Full Bath} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{House Age} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Fireplace} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{Building Area} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Full Bath} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{House Age} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Fireplace} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{Building Area} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Full Bath} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{House Age} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Fireplace} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{Building Area} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Full Bath} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_f \times \text{House Age} \]

\[ \text{Price} = \text{Price}_{\text{base}} + \lambda_p \times \text{Fireplace} \]
transaction price of a house from a $1,000 increase in the weighted average price of the prior sales in the neighborhood. The value of $\lambda^f$ suggests that if an increase in the weighted average price of the neighboring houses is expected, it will lead to a $160 increase of the current market price of a given house. For the Grandview heights school district, an anticipated $1,000 increase of nearby house values will leads to an increase of $458 in house value; while an observed $1,000 increase of nearby houses will increase house value by $326.

For Bexley, Hamilton and Upper Arlington districts, we suspect that the non-significance of $\lambda^f$ results from the unpredictable changes of house prices over time. After testing the time-varying nature of house prices during the sample time period, we found that the house prices in these three districts do not present any specific time pattern, while the transaction prices in the other school districts follow a time trend. The more random the prices are, the less predictable the future transactions are. Thus, the expected future neighborhood prices are not informative in predicting the current market value of a house.

The joint significance of $\lambda^f$ and $\lambda^p$ confirms the contribution of the spatial lags in the asset pricing model, particularly, the significance of $\lambda^f$ in most of the school districts suggests an improvement in model performance in comparison with hedonic models that only incorporate prior transactions in the neighborhood. This also is consistent with households’ forward-looking behavior. Note that the inclusion of the expected future spatial lag in the price model takes into account of the anticipated changes in the neighborhood, regarding both neighborhood spillover effects.

---

79 We test the time-varying pattern of house prices using a separate regression, which contains a deterministic time trend as well as the housing structural attributes. The coefficient estimates of the time trend are significant for the other school districts, but not for these two districts.
and absolute neighborhood effects. Because our spatial asset pricing model contains a spatial lag on the expected future transactions in addition to the spatial lag on the past transaction, it is able to translate these expected changes into house prices. It adds a bonus to the model performance compared with spatial models with only a spatial lag on prior transactions.

With only two exceptions, the wald statistics, $Wald_2$, are inside the acceptance region (at 5% significance level), which means that the two spatial autoregressive coefficients of $\lambda_f$ and $\lambda_p$ are not statistically different from each other for the twelve school districts. This suggests that the current neighborhood effects influence the price of a house to the same degree as how the discounted stream of future neighborhood effects impact a house’s price. A $1,000 increase in the current average price of nearby houses contributes to the price change of a house by the same amount as the price change caused by an expected $1,000 increase (in present value) in the future average price of neighboring houses. The fact again supports the forward-looking behavior of households, and lends credits to the accuracy of the proxy used for the current neighborhood effects.$^{80}$

To compare our analysis with previous studies, an alternative spatial model specification as in Can (1997)$^{81}$ is considered. In this alternative model, only the prior neighborhood transactions are referred to as spillovers by households, while their forward-looking behavior is suppressed. This means that the spatial coefficient $\lambda_f$ is

---

$80$ Recall that we use the weighted average of the past transactions to approximate the current neighborhood effects.

$81$ The spatial price model in Can (1997) includes only the prior sales in controlling for the neighborhood effects. She uses the prior transactions that occurred with a 3Km (about 2 mile) radius of a current transaction within the prior six months to construct the spatial weighting matrix. Her model can be viewed as a restricted version of ours (the spatial coefficient $\lambda_f$ is restricted to be zero).
restricted to be zero and only $\lambda^p$ is estimated together with the other housing characteristic coefficients. Base upon the results that are reported in Table (3.5), we find upward bias of the estimated spatial dependence on neighborhood effects. The estimated $\lambda^p$s in most of school districts (13 out of 14) are biased upward, and the bias is more than 50 percent for 11 school districts. It is worth noting that the value of the estimated $\lambda^p$ in Table (3.5) is close to the sum of the estimated $\lambda^p$ and $\lambda^f$ in Table (3.2) for 9 out of 14 cases (within 10 percent deviation). This discovery suggests that the value of $\lambda^p$ in the restricted model approximates the sum of the current neighborhood effects and the expected future neighborhood effects.
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Table 3.5: Coefficient Estimates of the Spatial Price Model (GMM) with Only Past Neighborhood Transactions
Table 3.5: (Continued)

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The numbers in the parentheses are the t-statistics; neighbors are within 2-mile radius and within six months.

* indicates significance at 10% level (p=1.65); ** indicates significance at 5% level (p=1.96); *** indicates significance at 1% level or less (p=12.58).

OBS_E indicates the number of observations that are used for estimation; OBS_P indicated the number of observations that are used for predication.
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To be continued

Table 3.6: Coefficient Estimates of the Spatial Price Model (GMM)- Two Miles and Two Years
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<td>(4.99)</td>
<td>(3.07)</td>
<td>(0.86)</td>
<td>(4.78)</td>
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</table>

81 The numbers in the parentheses are the t-statistics; neighbors are within 2-mile radius and within two years.
* indicates significance at 10% level (p=1.65); ** indicates significance at 5% level (p=1.96); *** indicates significance at 1% level or less (p=12.58).
Schools  \( \sigma^2 \)  WALD1  WALD2  OBS_E  OBS_P  
Bexley  1610.38  19.23***  1.15  1335  1757  
Canal Winchester  650.65  188.38***  2.60  539  646  
Grandview Heights  1028.38  8343.55***  0.89  709  885  
Hamilton  268.56  61.60***  10.06**  930  1077  
Gahanna Jefferson  1046.25  510.27***  0.05  3058  3785  
Groveport Madison  228.8441  1347.13***  0.46  3208  3979  
Reynoldsburg  360.21  281.67***  1.23  2575  3224  
Hilliard  410.97  42.14***  2.31  2144  2703  
Southwestern  508.35  121.85***  1.28  2086  2591  
Upper Arlington  2112  3.75  0.15  3241  4184  
Dublin  1034.76  780.05***  0.39  3265  4102  
Westerville  736.24  757.79***  5.10*  3182  3980  
Whitehall  315.54  493.82***  3.13  1453  1889  
Worthington  1175.12  88.19***  0.04  2210  2879  

Table 3.7: Wald Tests of the Spatial Price Model (GMM) - Two Miles and Two Years

### 3.5.2 Predictive Performance

To demonstrate the out-of-sample prediction power of our asset pricing model utilizing the spatial information in the neighborhood, the SAR model here is compared with a basic hedonic house price model as in equation (3.27).

We compute the expected market values of houses in the validation sample set employing the coefficients estimated from using the model-building sample set. Instead of using equation (3.21), we use the following functional form to predict house

---

81 Because WALD1 follows a distribution, and WALD2 follows a distribution, the critical values of the chi-square distributions are as follows:
\( \chi^2(2)=4.61 \) (significance level=0.10), \( \chi^2(2)=5.99 \) (significance level=0.05), \( \chi^2(2)=9.21 \) (significance level=0.01), \( \chi^2=13.82 \) (significance level=0.001), \( \chi^2(1)=2.71 \) (significance level=0.10), \( \chi^2(1)=3.84 \) (significance level=0.05), \( \chi^2(1)=6.64 \) (significance level=0.01), \( \chi^2=10.83 \) (significance level=0.001).

OBS_E indicates the number of observations that are used for estimation; OBS_P indicated the number of observations that are used for predication.

99
values,

\[ \hat{Y}_n = (I_n - \hat{\lambda}^f W_n^f)^{-1}\hat{\lambda}^p W_n^p Y_n + X_n\hat{\beta}, \]

where \( \hat{\lambda}^f, \hat{\lambda}^c \), and \( \hat{\beta} \) are the estimated coefficients using the model-building data set; \( W_n^f, W_n^c \) and \( X_n \) are generated using the entire sample; and \( \hat{Y}_n \) is the vector of predicted values for the entire sample, of which only the last \( n - m \) entries (the first \( m \) elements are in the model-building set, and the last \( n - m \) elements are in the validation) set are used to compute the prediction statistics.

To complete the estimation and prediction processes, the weighting matrices have to be constructed twice: the first set of \( W \) s are \( m \times m \) dimensions, which only use the model-building sample set with \( m \) observations; the second set of \( W \) s are \( n \times n \) dimensions, which use both the entire sample. After computing vector \( \hat{Y}_n \), the elements of it that corresponding to the validation sample set are used to construct the measures for gauging the prediction performance of the price model. We employ three statistic measures: The first one is the average of the residuals, which is defined as,

\[ e_0 = \frac{1}{n - m} \sum_{i=m+1}^{n} e_i, \tag{3.28} \]

where \( e_i = y_i - \hat{y}_i \) is the residual in the validation data set, and \( \hat{y}_i \) is the predicted price, which is among the last \( n - m \) elements of \( \hat{Y}_n \). The second measure is the estimate of the variance of the residuals, which is defined as,

\[ \hat{\sigma}_e^2 = \frac{1}{n - m} \sum_{i=m+1}^{n} (e_i - e_0)^2. \tag{3.29} \]

\(^{82}\)The reason why \( W_n^p Y_n \) is viewed as known and put in the right hand side of the predition equation is the lower triangular property of \( W_n^p \). Based on this, \( W_n^p Y_n \) is becomes the "prior transaction prices", which are predetermined.
The last measure is the mean squared errors (MSE), which is defined as,

$$MSE = \frac{1}{n-m} \sum_{i=m+1}^{n} e_i^2 = e_0^2 + \sigma^2_e. \quad (3.30)$$

The prediction performance of the SAR model are compared with that of the conventional hedonic model of equation (3.27). The statistics of these three measures are reported in Table (3.8).
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<tr>
<th>School</th>
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<th>BIAS II</th>
<th>VARIANCE I</th>
<th>VARIANCE II</th>
<th>MSE I</th>
<th>MSE II</th>
<th>MSE%</th>
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BIAS is the average bias of the residuals.
VARIANCE is the variance of the residuals.
MSE is the mean squared error.
MSE% is the percentage improvement on MSE of the spatial asset model over the hedonic model.
OBS is the number of observations in the validation sample set.
Model I is the spatial asset price model; Model II is the hedonic model.

Table 3.8: Out-of-Sample Prediction Performance (Spatial Asset Model v.s. Hedonic Model)
Among the fourteen school districts, the spatial model outperforms the hedonic pricing model in twelve school districts in terms of the MSE statistics. Compared with the hedonic model, the spatial asset pricing model improves the out-of-sample prediction performance by 16.41% on average. Except for Grandview and Hamilton districts, the spatial model generate smaller $e_0$. For the values of $\sigma_e^2$, the hedonic model is better than the spatial model only in the Grandview, Bexley and Worthington school districts.

### 3.6 Conclusion and Extensions

Although neighborhood effects have been recognized for playing an important role in determining house prices, the existing spatial hedonic house price literature ignores the influence of expected future neighborhood effects. This study focus on both current neighborhood effects and anticipated future neighborhood effects through the inclusion of two spatial lags in a house price model.

The current value of a house has been modeled as the discounted present value of expected housing services accruing to the owner throughout its lifetime. In the framework of a rational expectations asset pricing model, the spatial dependence of house prices enters the price determination process through economic agents’ behavior in exploring housing services. The reduced form of the rational expectations model falls in the category of spatial autoregressive (SAR) regression. The SAR regression contains two spatial lags, measuring the contemporaneous spatial effects and expected future spatial effects on the price of a house.
Both the future and current neighborhood effects are found to be significant in the house price determination process. These results corroborate forward-looking behavior of households, and validate our hypothesis that the expected future values of nearby houses are relevant determinants of the market value of a given house. We also confirm that the discounted future neighborhood price movements will affect the price of a given house in the same way as the contemporaneous neighborhood price movements. Furthermore, the equality of those two spatial autoregressive coefficient estimates justifies the proxies for neighborhood effects in this paper. More impressively, the superior out-of-sample predictions of the spatial asset pricing model confirm the potential merits of applying the spatial econometric techniques to the asset pricing model.

Given the existence of neighborhood effects in the house price determination process, the omission of them in the specification might upward bias the estimates of the housing structural variables of LOTSSQFT and BLDGSQFT. These biases might come from the positive correlation of neighborhood effects and lots size/building size. It is sensible that a nice neighborhood usually has a lot big houses, and the distances between houses are far in general.

As a possible extension to this study, the spatial-temporal weighting matrix could be decomposed into a function of a spatial matrix and a temporal matrix respectively as in Pace et al. (2000). As result of the statistical equality of $\lambda^f$ and $\lambda^p$, the spatial price equation can be written as $Y_n = \lambda W_n Y_n + X_n \beta + \varepsilon_n$, where $\lambda = \lambda^f = \lambda^p$, and $W_n = W_n^f + W_n^p$. The spatial-temporal weighting matrix, $W_n$, is then specified as

$^{83}$Comparing Table (3.4) and Table (3.2), we find that with the omission of neighborhood effects, the coefficient of LOTSSQFT and the coefficient of BLDGSQFT are overestimated in 10 school districts, and underestimated in only 1 school district (unchanged in 2 school districts). There are no specific trends for the estimates of the other housing structural variables.
\[ W_n = \phi_S S_n + \phi_T T_n + \phi_{ST} S_n T_n + \phi_{TS} T_n S_n. \] 

This specification allows the individual estimations of \( \phi \)'s, which provide the details of how neighborhood effects spill across space, time, and the aggregate of space and time. Note, because the correlation for house prices is not restricted to be among previous transactions, neither \( S_n \) nor \( T_n \) is a lower triangular matrix. To estimate this model, the OLS estimator will be biased, and computational complications might be involved.

\[ ^{84} \text{In this study, the spatial-temporal weighting matrices, } W_n^f \text{ and } W_n^p, \text{ can be interpreted as the lower triangular matrix and the upper triangular matrix of } W_n \text{ (} W_n = W_n^f + W_n^p). \text{ And } W_n \text{ is constructed from the element by element multiplication of the spatial weighting matrix, } S_n, \text{ and the temporal weighting matrix, } T_n, \text{ as } W_n = S_n * T_n, \text{ where } * \text{ indicates the multiplication of element by element. Therefore, this study differs from Pace et al. in the specification of } W_n. \]
Appendix B1. The Best IV Function for GMM Estimation

The equation \( Y_n = \lambda^f W_n^f Y_n + \lambda^c W_n^c Y_n + X_n \beta + \varepsilon_n \) can be rewritten as \( Y_n = Z_n \theta + \varepsilon_n \), where \( Z_n = (W_n^f Y_n, W_n^c Y_n, X_n) \), and \( \theta = \begin{pmatrix} \lambda^f \\ \lambda^c \\ \beta \end{pmatrix} \), the IV estimator based on \( Q_n \) is,

\[
\hat{\theta}_{IV,n} = [Z_n'(Q_n^{-1}Q_n'Z_n)^{-1}Q_n'Q_n^{-1}Q_n'Y_n].
\]

It can be shown that the asymptotic distribution of \( \hat{\theta}_{IV,n} \) follows from,

\[
\sqrt{n}(\hat{\theta}_{IV,n} - \theta_0) \rightarrow^D N(0, \Sigma_{IV}),
\]

where

\[
\Sigma_{IV} = \lim_{n \rightarrow \infty} \frac{1}{n} [(G_{1n}X_n\beta_0, G_{2n}X_n\beta_0, X_n)'Q_n(Q_n^{-1}Q_n'Q_n)^{-1}Q_n'(G_{1n}X_n\beta_0, G_{2n}X_n\beta_0, X_n)]^{-1}.
\]

By the Generalized Schwartz inequality, the optimal IV matrix \( Q_{o,n} \) has the following form, \(^85\)

\[
Q_{o,n} = (G_{1n}X_n\beta_0, G_{2n}X_n\beta_0, X_n).
\]

\(^85\)Let \( Z^* = (G_nX_n\beta, X_n) \), then Schwartz inequality suggests that \( Z^*Z^* \geq Z^*Q_n(Q_n'^{-1}Q_n'^{-1})^{-1}Q_n'Z^* \).
Appendix B2. The Optimal GMM Estimator

Let $a_n$ be a sequence of constants with a full row rank greater than or equal to the number of unknown parameter, The GMM estimator of $\theta$, derived from

$$\hat{\theta}_n \equiv \arg\min_{\theta} g_n(\theta)'a_n' a_n g_n(\theta),$$

follows the distribution as

$$\sqrt{n}(\hat{\theta}_n - \theta_0) \rightarrow^D N(0, \Sigma_\theta),$$

where

$$\Sigma_\theta = \lim_{n \to \infty} \left( \frac{1}{n} D_n \right)^{-1} \left( \frac{1}{n} D_n^\prime a_n' a_n \frac{1}{n} \Omega_n a_n' a_n \frac{1}{n} D_n \right) \left( \frac{1}{n} D_n \right)^{-1}.
$$

If we set $\Omega_n^{-1} = a_n' a_n$

$$\sqrt{n}(\hat{\theta}_{a,n} - \theta_0) \rightarrow^D N(0, \Sigma_\Omega),$$

where

$$\Sigma_\Omega = \lim_{n \to \infty} \left( \frac{1}{n} D_n^\prime \Omega_n^{-1} D_n \right)^{-1}.
$$

The Generalized Schwartz inequality states the following
\[ AA' \geq AX(X'X)^{-1}X'A' \]

or

\[ AVA' \geq AX(X'X)^{-1}X'A' \]

for any matrix \(X\), where \(V\) is positive definite.

If we define \(P^{-1}X = X^*\), then

\[ AX(X'P^{-1}P^{-1}X)^{-1}X'A' = APP^{-1}X(X^*X^*)^{-1}X'P'P'^{-1}A' \]

or equivalently

\[ AX(X'P'^{-1}P^{-1}X)^{-1}X'A' = (AP)X^*(X^*X^*)^{-1}X'^*(AP)' \]

Provided the Generalized Schwartz inequality, we get

\[ (AP)(AP)' \geq (AP)X^*(X^*X^*)^{-1}X'^*(AP)' \]

or

\[ AVA' \geq AX(X'V^{-1}X)^{-1}X'A' \]

Therefore, \(\frac{1}{n}D_n'\Omega^{-1}D_n \geq (\frac{1}{n}D_n')a'_n\Sigma_{\theta,n}(\frac{1}{n}D_n)[\frac{1}{n}D_n']a'_n\Sigma_{\theta,n}(\frac{1}{n}D_n)\), \(\Sigma_{\Omega} < \Sigma_{\theta,n}\) is proved to be the optimal GMM estimator.
CHAPTER 4

Concluding Remarks

In the analysis of both household location choice and house price determination, neighborhood effects are significant and worth great emphasis. A major issue that is involved with neighborhood effects refers to correlated observations. The spatial econometric models are proved to be an effective tool for addressing these correlations that widely exist in a spatially interdependent housing market.

Given different contexts, neighborhood effects take various forms. In Chapter 2, neighborhood effects are described as social interactions among households who reside in the same community. To achieve a high degree of beneficial social interactions, this chapter hypothesizes that households tend to search for potential neighbors with similar social-economic backgrounds in their residential location decisions. This hypothesis of search for similarities is tested together with Tiebout’s hypothesis of search for fiscal surplus through the application of a nested logit model. To proceed with this nested logit model, a set of community entry prices, as the necessary input of the NL model, are constructed using a spatial error econometric model. The spatial error model is used to control for the spatial correlations of the disturbances that are caused by omitted variables and measurement errors.
The results show that a household prefers neighbors who are like herself/himself in the dimensions of education background (with the exception that poorly educated households prefer better educated neighbors), family size, race, and income level. Furthermore, Tiebout’s hypothesis has been corroborated with certain reservation. Households would like to buy a somewhat cheaper house as compared with the values of the neighbors’ houses in order to gain a fiscal surplus. Meanwhile, the house values difference of a household and his/her neighbors’ should be small enough for the household to stay in a relatively homogeneous community.

In chapter 3, neighborhood effects are represented by the spillovers from nearby house values. As a special type of asset, housing asset distinguishes from other assets for its fixed geographic location and inseparable relationship to its surroundings. Given the locational impacts from nearby houses, the dividends derived from a house are determined not only by the structural characteristics of the house itself, but also the neighborhood spillovers. Under the rational expectations hypothesis, the current value of a housing unit is modeled as the conditional expectation of the discounted stream of housing services accruing to the owner of the house. In the rational expectations model, the neighborhood effects enter the house price determination process. The reduced form of this rational expectations economic model has the form of a spatial autoregressive model (SAR) with two spatial lags, measuring the contemporaneous neighborhood effects and the expected future neighborhood effects.

The results reveal the significance of both the future and current neighborhood effects in the house price determination process, corroborating forward-looking behavior of households. Furthermore, the findings confirm that the discounted future neighborhood price movements will affect the price of a given house in the same way
as the contemporaneous neighborhood price movements. In addition, the superior out-of-sample predictions of the spatial asset pricing model show the power of the SAR model in the forecast of housing asset prices.

The application of the spatial error model and the spatial autoregressive model in this dissertation well address neighborhood effects that commonly exist in the housing market. Provided the explicit functional form of a SAR model, the extent of neighborhood effects can be directly estimated. This desirable advantage of a SAR model establishes its important assistance in the analysis of a wide range of housing issues.
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