DESIGN OF A LINEAR CONTROL SYSTEM OF REDUCED INPUTS FOR MULTIPLE SPACECRAFT FORMATION FLYING

A Thesis

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By

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ABSTRACT

Multiple spacecraft formation flying involves the creation and maintenance of a closely-packed grouping of spacecraft for the purpose of completing a single mission. Current research is being directed toward the development of automatic control systems that can perform formation keeping and formation changing maneuvers without the aid of human guidance. This thesis motivates and then develops such an automatic design using linear-quadratic regulator theory.

The design proposed makes use of the observation that control thrust need only be applied coplanar to the local horizon to achieve complete controllability of a two-satellite formation. Without the need for zenith-nadir thrust, simplifications and reduction of the weight of the propulsion system may be accomplished. This work focuses on the validation of this horizontal-only control system on its own merits, and in comparison to a related system which does provide thrust perpendicular to the horizontal plane.

Simulations are for point-mass, Keplerian orbits only, with no perturbations or sensor noise included. Performance measures are developed, including propellant use and maximum thrust required. Performance results based on the simulated regulation of certain trajectories to the origin of the state-space are interpreted to validate the horizontal-only controller as a practical design option for formation flying.
To my family and friends,

& in memory of heroes past.
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CHAPTER 1

INTRODUCTION

Automatic control has been used to maintain or alter the attitude of satellites in many missions to date. However, automated stationkeeping has seen somewhat less use because of the large expenditures of propellant required for most stationkeeping maneuvers, and because of the rather limited supply of propellant available to most spacecraft over their entire lifetimes. Instead of programming a computer both to decide and to execute these potentially costly maneuvers, human controllers have ordered maneuvers from the ground based on their analysis of sensor data.

When a group of several spacecraft fly in close proximity, however, automatic decision-making and execution becomes imperative. Such a close-proximity group of spacecraft is known as a formation, and multiple spacecraft formation flying (MSFF) is the application of control theory, systems theory, decision-making, artificial intelligence, and many other fields to allow such formations of spacecraft to successfully operate.

Among the first applications of MSFF were for the rendezvous of the space shuttle orbiter with other orbiting spacecraft [Gustafson and Kriegsman, 1970]. It was only much later, with the desire to minimize the costs associated with single, large
spacecraft that further benefits were projected from the development of MSFF. This is part of the reasoning given by the Space Advisory Board for its observation that MSFF would be one of the most important emerging space technologies for the next two decades [TechSat-21 factsheet, June 1998]. In general, spacecraft formations may be close groupings of planetary satellites, or they may follow heliocentric orbits, depending on the mission. In maintaining the desired spacing and geometry for a formation on a deep-space trajectory, the control designer need not be very concerned with the unforced, nearly Keplerian orbits that the individual spacecraft must follow. For such a formation, the individual orbits followed may be similar enough compared to the length scale of the orbit that the MSFF is much like the formation flying of aircraft on Earth. By contrast, a formation in orbit around the Earth consists of two or more satellites with differing orbits. Because of the proximity of the gravity well, the effort required to maintain a forced orbit, which is significantly different from the unforced orbit the spacecraft would otherwise follow, is usually prohibitive for all but the shortest missions (such as the SSO rendezvous mentioned above).

One very useful purpose to which formations of satellites may be directed is the creation of a large-baseline synthetic aperture for observation at a particular wavelength. By carefully measuring and possibly controlling the relative positions of the spacecraft which make up a formation, the separations of those spacecraft may be used to gain a great deal of resolution at the observed wavelengths. This physical example demonstrates that what is truly important in many MSFF applications is not only the actual positions of the member spacecraft, but also their positions and velocities relative to one another.
Currently, the United States Air Force is sponsoring the TechSat-21 mission, a proposed synthetic-aperture-radar space mission that will employ formation flying techniques to meet mission goals. The TechSat-21 spacecraft will maintain a formation in the shape of an orbiting disk from 100 meters to 1 kilometer in diameter. The Air Force Research Laboratory is performing research to determine and meet the requirements for this important new enterprise.

The purpose of this thesis is to aid in the development of feasible new technologies for use in the automatic control of satellite formations, including TechSat-21. The thesis provides a simple and general method for the automatic control of orbiting formations which meet certain geometrical criteria. Essentially, these criteria are that the formation must be either a line of satellites all sharing a single orbit, or a formation of satellites whose relative orbits sweep out a circle in a plane horizontal to the planet's surface. The controllers may be applicable for other formation geometries, but since many formations discussed in the literature fit the first category, and since TechSat-21 meets the conditions for the second, this thesis does not discuss any further geometries.
CHAPTER 2

MULTIPLE SPACECRAFT FORMATION FLYING

2.1: Number of Spacecraft

The formationkeeping of satellites during a mission can take several different forms, depending on the variables included. For instance, the consideration of possible collision is very nearly unnecessary for a two-satellite formation, since one spacecraft can be uncontrolled and the other can perform formationkeeping maneuvers with respect to the uncontrolled spacecraft. This pairing may be called by many different terms: e.g. beacon and member, chief and deputy, leader and follower. Throughout the remainder of this thesis, such an uncontrolled-controlled pair of satellites will be called a leader-follower arrangement. For larger numbers of spacecraft, however, all except perhaps one of the spacecraft must actively perform formationkeeping duties, and so a collision is quite possible if the satellites do not inform each other of their intentions to maneuver. Also, even if collisions are avoided, the impingement of propulsion exhaust on other spacecraft must be considered. In this paper, the leader-follower model will be employed due to its simplicity, but all results should still be applicable to larger formations, so long as collision avoidance is included.
2.2: Orbital Eccentricity

Since the eccentricity vector will have nearly the same magnitude and
direction for any satellite in the formation, it is meaningful to speak of a single
eccentricity for the entire formation. This may be the mean eccentricity of the
formation orbits, or perhaps the eccentricity of the leader satellite’s orbit. This
“formation eccentricity” (rigorously defined below) becomes another important factor
in determining the control strategies available for maintaining a given formation. If
the eccentricities of the member satellite orbits are very nearly zero (i.e. the orbits of
the satellites are nearly circular), then considerable simplifications to the dynamical
models can be made by performing the coordinate transformation described below.
One simplification is that the transformed system has only very weak coupling of the
dynamical equations, allowing the full range of linear control design techniques to be
used in formation keeping strategies. For this reason, this paper will deal only with
formations whose members follow very nearly circular orbits. However, it is worth
noting here that, as eccentricity increases, the performance of a linear, time-invariant
control law will degrade because the orbital rate varies with time for non-zero
eccentricities.

2.3: Relative Coordinate Frame for the Formation

The orbits of the satellites, when considered from an inertial frame of
reference, will have very similar geometries in the large. For this reason, it is often
useful to define a formation origin. This moving point may or may not be the actual
trajectory of a satellite; whether or not a satellite occupies the origin, the origin is said to move in a Keplerian orbit as though it were affected by gravitation. Hence, the formation orbit may be defined as the orbit which would be traveled by a massive object located at the formation origin, and moving according to Keplerian motion. The various attributes of the formation orbit can be similarly denoted as the attributes of the formation; e.g., the "formation eccentricity" is equivalent to the eccentricity of the formation orbit. Though this study does not consider the center of mass of the formation, it should be noted that the formation center of mass is not, in general, located at the formation origin as defined here. The satellites of the formation are then considered to have positions in a coordinate system specific to the formation defined in part by this formation origin. In this way, the dynamics of the entire formation may be considered without always having to refer to subtle differences (often, of order less than 0.001%) in the sets of orbital elements that correspond to the member satellites.

There may be a loss in efficiency associated with this transformation. The navigation of satellites may be achieved by calculating the appropriate orbital elements and performing maneuvers at appropriate points in the orbit to directly change these elements. Such a procedure has been suggested by Schaub et. al. [1999]. This method is currently under consideration as a baseline for the TechSat-21 MSFF control system [private communication, R. Burns, August, 00]. Nonlinear control methods must then be applied, and since the scope of this thesis includes only linear control design, such methods are not fully explored here. However, the observation that some efficiency may be gained from the application of traditional astrodynamical
techniques will prove useful in the understanding of the methods that are presented by this paper.

A set of axes for the formation must be determined once the origin has been chosen. Customarily, one of the axes lies along the position vector of the origin, from the center of the "spherical" planet to the origin itself, another is parallel to the angular momentum vector of the formation orbit, and the third is perpendicular to the first two. This is sometimes known as a velocity-normal-binormal coordinate system since, for circular orbits, this third axis will lie along the orbit of the origin, either in the same direction or the opposite direction as the velocity. The axes are often labeled x, y and z, according to a right-handed coordinate system, but there is no consistency beyond that point. Most conceivable combinations have been used by various authors, so it is necessary to simply choose one arbitrarily. For this study, in all references to the relative coordinate frame axes, the x-axis corresponds to the opposite of the direction of the instantaneous velocity vector (the "anti-velocity" direction), the y-axis to the direction pointing directly away from the center of the earth, and the z-axis in the same direction as the angular momentum vector for the formation orbit. Stated differently, motion occurring in the orbital plane is described by the x- and y-axes, with the x-axis parallel to the formation orbit and the y-axis parallel to the vector radius of the formation origin, and motion perpendicular to the orbital plane is described through the use of the z-axis. Henceforth, these axes may also be referred to by the following designations, as context requires: the x-axis denotes the "along-track" dimension; the y-axis, the "radial" dimension; and the z-axis, the "cross-track"
dimension. The leader-follower formation, with the accompanying coordinate frame just defined, may be seen in Figure 1.

The above coordinate definitions are essential to the comprehension of this thesis. Since the novelty of the design proposal made in this thesis is the elimination of any control along the radial axis in this relative frame of reference, the discussions of simulation procedure, results, and conclusions will necessarily make extensive use of, and often interchange between, the above notations.

Note that the coordinate system defined above is not an inertial frame of reference, even though the speed of the formation is constant. The coordinate system rotates about the z-axis with an angular velocity corresponding to \( \omega \), the orbital rate or frequency. So in fact, there is a difference between a constant applied thrust in the inertial frame and the same vector in the relative frame. This difference must be considered when changing from one frame of reference to the other. With that in mind, note that the simulations described in Chapter 4 do not refer at all to the inertial frame, so the difference does not present a problem.

2.4: Formation Dynamics in the Relative Frame

The derivation of the nonlinear dynamical equations that describe the motion of one spacecraft in relation to another may be found in any of several sources; Clohessy & Wiltshire [1960] and Kapila [1998] use the same relative coordinate system as defined above. Essentially, the inertial-frame position vectors of the two spacecraft are subtracted, and the time derivatives are calculated, taking into account the rotational frequency of the reference frame. In this way, the orbital frequency comes
into the equations as a constant parameter. The relative frame accelerations of a "follower" satellite with respect to a leader satellite, located at the origin of a formation with an unperturbed, Keplerian, circular orbit are given by

\[ \ddot{x} = 2\omega \dot{y} + \omega^2 x[1 - g(x, y, z, r)] + F_x \]  (2.1)

\[ \ddot{y} = -2\omega \dot{x} + \omega^2 (y + r)[1 - g(x, y, z, r)] + F_y \]  (2.2)

\[ \ddot{z} = -\omega^2 z g(x, y, z, r) + F_z \]  (2.3)

where \( r \) is the orbital radius, \( F \) denotes any specific forces, such as disturbances and control forces, and

\[ g(x, y, z, r) \equiv \left[ 1 + \frac{2y}{r} + \frac{1}{r^2} (x^2 + y^2 + z^2) \right]^{-\frac{3}{2}} \]  (2.4)

Since most formation flying occurs in a regime where \( x, y, z \ll r \), the term \( g \) is approximately unity. By applying this approximation, the above nonlinear equations become linear. The linearized dynamics for the leader-follower model of formation flying are given by the Clohessy-Wiltshire equations [Clohessy and Wiltshire, 1960]:

\[ \ddot{x} = 2\omega \dot{y} + F_x \]  (2.5)

\[ \ddot{y} = -2\omega \dot{x} + 3\omega^2 x + F_y \]  (2.6)

\[ \ddot{z} = -\omega^2 z + F_z \]  (2.7)

By examining equations (2.5)-(2.7), it can be seen that the motion in the orbital plane is decoupled from the motion perpendicular to the orbital plane in the linearized model. Therefore, it makes sense to treat the in-plane dynamics and the out-of-plane dynamics as two separate linear systems.
Note that there is no attempt to model the attitude of the spacecraft that make up the formation. The attitudes of the member spacecraft would have negligible effect on the unforced orbital dynamics, but when control forces are included, care must be taken to ensure that the thrust vector is not diverted by an offset in spacecraft attitude. In this study, the attitude is assumed to be constant in the relative frame, such that a propulsion device directed along the positive y-axis (i.e. local zenith), for example, would always be directed in this same direction.

2.5: Formation Geometry

A third important factor in formationkeeping is the geometry of the formation itself. Adding the fuel-saving restriction that the Keplerian orbits of all member spacecraft be unforced reduces the possibilities somewhat. It has been demonstrated [Yeh & Sparks, 2000] that the motion thus restricted must follow an elliptical path in the relative frame. This ellipse may be degenerate (i.e. a single point), has a center lying at any point on the along-track axis, and projects an ellipse in the orbital plane of eccentricity $\sqrt{3}/2$ whose major axis is aligned with the along-track axis. These geometric restrictions still allow many possibilities, since both the magnitude of the semi-major axis of the orbital plane projection and the maximum out-of-plane displacement may take any values, including zero.

There are four special geometries which may be chosen for each pair of satellites in the formation: in-plane, in-track, circular, and projected circular [Sabol, Burns and McLaughlin, 1999]. The in-plane formation has a pair sharing the same orbit with only a phase shift separating them. A pair in the in-track formation shares
a common ground track. The follower in a circular formation follows a path in the reference frame which is always a constant distance from the leader. Finally, the projected circular formation has the follower revolving about the leader on an elliptical path whose projection onto the local horizontal plane (i.e. the x-z plane) is a circle.

Since the TechSat-21 program has settled on using several satellites in projected circular formations about a single formation origin, the projected circular configuration will be addressed by the experiments detailed in later chapters of this thesis. Diagrams of this formation may be found in Figure 2. The in-plane formation will also be treated, since it has been of interest in formation flying literature to date [e.g. Vassar and Sherwood, 1985].
CHAPTER 3

STATE-SPACE REALIZATION AND CONTROL DESIGN

3.1: Linearization of the Relative Dynamics

The linearization of the nonlinear, relative-frame dynamics given by the Clohessy-Wiltshire equations (2.5)-(2.7) allows the use of a state-space representation of the two-satellite formation flying system. Placed in a typical formulation, the system is given by

\[ \dot{x} = Ax + Bu, \]  

where the state and control vectors are

\[ x = [x \; \dot{x} \; y \; \dot{y} \; z \; \dot{z}]^T, \]  

\[ u = [u_x \; u_y \; u_z]^T, \]

and the state and control matrices, A and B respectively, are

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2\omega & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & -2\omega & 3\omega^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0_{4x2} \\
0_{2x4} \\
\end{bmatrix},
\]  

\[
B = \begin{bmatrix}
0 & 1 \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 1 \\
-\omega^2 & 0 \\
\end{bmatrix}.
\]
\[ B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}_{4 \times 1}, \]

It was noted earlier that the in-plane dynamics are decoupled from the out-of-plane dynamics in the linearized equations. This fact is made clear by the state-space representation (3.1)-(3.5), as indicated by the dark lines that separate the decoupled subsystems in the definitions of \( A \) and \( B \).

For the remainder of this chapter, the in-plane dynamics will be treated as a system separate from the out-of-plane dynamics. The following system definitions will be employed throughout the paper when dealing with the decoupled state space representations of these subsystems:

\[ A_{xy} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 3\omega^2 & 0 \end{bmatrix}, \]

\[ B_{xy} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \]

\[ A_{z} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}. \]
When these systems are described using their decoupled state and control matrices, the state and control vectors, \( x \) and \( u \), should be construed as the appropriate portions of the full state and control vectors defined above.

### 3.2: Out-of-Plane Dynamics

The out-of-plane, or cross-track, state matrix, \( A_x \), has eigenvalues of \( \pm j\omega \), indicating that the motion is that of a simple, harmonic oscillator of frequency \( \omega \). The system is stable in the sense of Lyapunov and should therefore require only occasional corrections to perturbations and unmodelled dynamics. The pair \((A_x, B_x)\) is completely controllable. In fact, the pair \((A_x[1 0]^T)\) is also completely controllable; however, no continuous control may be applied to change the velocity directly. As such, \( B_x \) as defined above is the only appropriate control matrix for the cross-track dynamics. Observability is not an important factor in this paper, but it may be worth noting that the out-of-plane dynamics are normal for observability; i.e. the pairs \((A_x[1 0])\) and \((A_x[0 1])\) are completely observable.

In the relative frame, the cross-track component of the motion is the movement of the follower back and forth across the orbital plane of the leader satellite. This motion in the relative frame is the result of a slight difference in inclination between the orbits of the two spacecraft. The two times that \( z \) equals zero during each cycle correspond to the two points in the follower's orbit where the orbit groundtracks cross. Since the difference in inclination is the angle between the orbital planes of the
leader and the follower, trigonometry yields that the maximum value of $z$ during a cycle is equal to the orbital radius multiplied by the sine of the difference in inclination:

$$z_{\text{max}} = r \sin(\Delta i).$$  \hspace{1cm} (3.10)

This relation may be put to good use in some stationkeeping and maneuvering strategies.

The most economical method for changing the inclination of a spacecraft in orbit is to apply an impulsive change in velocity ($\Delta V$) at just the point in space where the present orbit and the target orbit cross [Fund. of Astrodynamics, pp. 169-170]. Considering the relative coordinate frame in light of this fact, the most efficient use of propulsive control applied in the cross-track direction would be to exert control forces only at or very near the points where $z = 0$. The control strategy of Schaub et al. [1999] takes advantage of this result from orbital dynamics. If the propulsion system cannot deliver a large enough $\Delta V$ for a given inclination change to be realized, then a different strategy may be need to be employed. Even in this case, though it is important to note that control effectiveness decreases approximately linearly with distance from the desired orbital plane. Note also that the inclination difference can be known to very high accuracy, based on GPS measurement over an entire orbit and the trigonometric relation between maximum cross-track displacement and inclination difference.
3.3: In-Plane Dynamics

The in-plane dynamics prove to be much more complex than the out-of-plane dynamics. The state matrix, $A_{xy}$, has eigenvalues equal to 0, 0, and $\pm j\omega$. Hence, it can be seen that the oscillatory component of the motion is a harmonic oscillation of the same frequency as the out-of-plane dynamics. However, the repeated root at the origin makes the system bounded-input/bounded-output (BIBO) unstable. Any non-zero value in the radial direction will cause the separation in the velocity direction to increase linearly without bound. Since the flight of the formation is known to occur in an environment full of unmodelled dynamics such as drag, solar radiation forces, and higher order gravitational terms ($J_2$, etc.), the formation must surely drift apart if it is not maintained through the application of control forces.

Fortunately, the pair $(A_{xy}, B_{xy})$ is completely controllable and completely observable. Also, the pair $(A_{xy}, [1 \ 0 \ 0 \ 0])$ is completely observable, allowing full knowledge of the state based only on measurements of the x-position. Note also that every pair of the form $(A_{xy}, [0 \ C_3 \ C_4 \ C_4])$, where the $C_i$ are arbitrary, is unobservable and undetectable. In some cases, this observability result may be very helpful, since a significant amount of weight may be necessary to measure multiple dimensions, and some of that weight may be eliminated.

The pair $(A_{xy}, B_i)$, where $A_{xy}$ is defined by equation (3.6) and

$$
B_i = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},
$$

(3.11)
is completely controllable; i.e., there exists a control gain matrix for each arbitrary set of eigenvalues chosen for the closed-loop system, so long as along-track accelerations are controlled. However, the pair \((A_{xy}, B_y)\), where
\[
B_y = \begin{bmatrix}
0 \\
0 \\
0 \\
1 
\end{bmatrix},
\]
is uncontrollable and unstabilizable. This result suggests that the along-track dimension of control may be more important than the radial dimension. Hence, the thesis of this paper is that a stable, linear, automatic control algorithm may be designed for MSFF maneuvers that: 1) does not require any radial (y-axis) control accelerations; and 2) performs well enough to be a feasible option in the planning of future missions. The extent to which control system performance suffers due to the loss of radial control is partially determined by the results presented in this paper.

3.4: MSFF Control Design

In designing a control system for an MSFF formation, the ability of the system to correct relative displacements caused by perturbations must be demonstrated. The method by which the effects of perturbations are mitigated may take different forms, depending primarily upon the mission objectives. For consistent accuracy in relative position and velocity (together referred to as the relative state), very short time intervals between subtle corrections would be preferable to longer intervals between larger corrections. Conversely, a mission which does not need for the state to be
extremely restricted but demands low fuel and/or power usage may require longer
time intervals between corrections of greater magnitude and complexity.

Other mission specifications may include the capability to significantly modify
an existing formation geometry. The control system would then need to be capable of
efficient maneuvers from one point to another in the moving reference frame. Again,
different missions will imply different definitions of efficiency. A formation of many
closely-packed satellites may need to execute very direct maneuvers, or have the
capacity to choose one of several maneuvers, in order to avoid collisions. A formation
of relatively light spacecraft would likely require fuel efficiency above most other
considerations, since the weight of propellant and propulsive apparatus can be a large
percentage of a small spacecraft’s weight budget. Still another mission objective with
a need for rapid repositioning will require time optimal control, and possibly a highly
efficient means of communication between spacecraft.

Before implementing any new control design, the issues of long-term
formation maintenance must be addressed by full, orbital dynamics simulations. Of
course, such simulations would model the nonlinear dynamics, and they should also
take into account such sources of perturbation as the oblateness of the Earth, sensor
and controller inaccuracies, and drag and solar radiation pressure differentials for
spacecraft pairs with different physical characteristics or orientations. Such issues are
not addressed by this thesis; rather the purpose here is to determine the feasibility of
the elimination of the radial axis of control for idealized maneuvers. It is hoped that
these results will help guide future attempts to thus simplify MSFF control systems.
3.5: Motivation for Linear-Quadratic Regulator Control Design

Linear-quadratic regulation, or LQR, is a well-established control design technique which, given a state-space pair \((A, B)\), provides a linear gain matrix, \(G\), based on a quadratic form cost function, \(J\), such that the global, asymptotic stability of the closed-loop system, \(A + BG\), is guaranteed. The typical cost function is given by

\[
J = \int_{t_0}^{t_f} \begin{bmatrix} x \end{bmatrix}^T Q \begin{bmatrix} x \end{bmatrix} + \begin{bmatrix} u \end{bmatrix}^T R \begin{bmatrix} u \end{bmatrix} \ dt ,
\]

(3.13)

where \(Q\) is the positive definite or semidefinite state weighting matrix, and \(R\) is the strictly positive definite control weighting matrix.

Since stability is asymptotic, the equilibrium point of the system is the origin of the state-space; if another constant state is desired, a new set of state variable are defined such that the desired state is the origin of the new state-space. Since stability is global, any initial deviations from equilibrium are eventually corrected. For the purposes of MSFF, LQR may be applied so long as the distance separating the follower and the leader is much less than the orbit radius of the leader; recall that this is the condition for the linearized Clohessy-Wiltshire equations to accurately model the system.

As for options other than LQR, nonlinear design is an extremely active area of MSFF control system research. Indeed, such nonlinear control methods as switching control, deadzone, and adaptive control are currently being applied to the MSFF at the Air Force Research Laboratory. Though these methods may be, by one criterion or another, the most efficient means of maintaining a given formation, the movement of MSFF control theory through the experimentation stage and into the widespread use
of control technologies expected in the next two decades will probably require the incremental adaptation of spacecraft control technologies that are already trusted and spaceworthy at the present. This need for incremental advances would come from a desire from end users to invest in "risky" new technology as little as possible. In fact, this risk-aversion in mission design strategy is a major driver for the development of MSFF technology, since one benefit of the formation is the mitigation of capital loss due to single event failure of spacecraft [presentation by Andrew Sparks at AFRL, July, 2000].

Because of such conservative approaches to space mission design and actualization, the linear control regime may be attractive. Linear controllers are in widespread use in many industrial applications, including spacecraft attitude control systems. If such a well-established spaceborne technology could be modified in minor respects to serve as the core of a formation flying control strategy, it could be an important step toward establishing the reliability of MSFF technology for potential users.

Even though some nonlinear designs may prove simple to implement, results obtained from the simulations of similar linear designs will provide a standard by which to measure the effectiveness of these novel approaches to the problem. Of the linear control design techniques available, LQR is arguably the simplest. Since LQR design does not provide fuel optimization directly, it is reasonable to expect that, if a certain level of fuel-efficiency can be attained by a linear-quadratic regulator, then further adapting the control gains to achieve the desired system performance will likely result in better fuel efficiency. The same logic might be applied to other
performance measures, such as maximum thrust required or time required to maneuver, which are not directly optimized by LQR. For these performance measures, the LQR design provides a baseline upon which further design iterations, whether using linear or nonlinear techniques, may be expected to make improvements. For these reasons, the development of a MSFF control design based strictly on linear-quadratic regulation is an important and worthwhile pursuit.

3.6: Control Structure Geometries

The term "control structure" may, in general, be construed as any of a number of control law characteristics: linearity or nonlinearity, degrees of freedom, LQR or linear-quadratic-Gaussian, etc. ad infinitum. In the context of this paper, the term "structure" is considered to indicate the means of actuation commanded by the controller. In a sense, this is a more strictly mechanical constraint on the term. In a mathematical sense, the term "control structure" as used here means simply the number of inputs indicated by the control matrix, B.

This report evaluates various linear-quadratic regulators used to implement two possible "structures" of automatic maneuver execution. One system, which is the conventional approach to linear control design in the MSFF problem, has three orthogonal axes of control thrust. Since the in-plane and cross-track dynamics are only very weakly coupled even in the nonlinear system of equations, and since the cross-track dynamics are simple harmonic motion, some authors consider only the control of the in-plane dynamics [Vassar and Sherwood, 1985; Leonard, Hollister and Bergmann, 1989; Kumar and Seywald, 1995]. In such considerations, though only two
axes of control (the x- and y-axes) are being included in the control design, the
dynamics, perturbations, and pursuant control along the third dimension are usually
implied.

The other case considered here is a structure in which control thrusts are
applied only in the in-track and cross-plane directions, omitting the capacity for direct
thrust in the radial direction (i.e. along the y-axis). The theoretical basis for this
control structure is discussed at length in Chapter 3. Some work has been done with
this structure of control; for example, Leonard, Hollister and Bergmann [1989] detail a
design which makes use of differential drag to maintain a leader-follower
configuration. In the relative coordinate frame of reference, differential drag must
necessarily provide only along-track net forces. These authors propose a nonlinear,
on-off strategy for the maintenance of the formation, but there is little reference in the
literature to the use of linear controllers in which the radial axis of control has been
eliminated.

Note that the difference between these two modes of control may be
considered in relation to the local horizon. The former, more conventional structure
uses both horizontal and vertical forces to control the relative state of the follower
spacecraft. The latter structure makes use only of horizontal control forces; thrust is
only applied parallel to the surface of the Earth. To make discussion and comparison
of these two structures less awkward, a nomenclature based on this difference will be
adopted – the system using all three orthogonal axes of control will be called the
horizontal-vertical, or “H-V” system, and the system firing only parallel to the Earth’s
surface will be referred to as the horizontal-only, or “H-only” system. This
nomenclature is particularly suitable because it applies whether or not the cross-track dynamics are being considered.

This paper will make some comparisons between the two control structures just outlined. The H-V control structure has been studied by several other researchers, so the performance of such a control system is well known. By making comparisons between the well-known H-V control structure and the H-only control structure designed here, it is hoped that those reading this document will be better able to place the results of this study in the context of the field of satellite formation flying as a whole.
CHAPTER 4

SIMULATION OF TWO-SATELLITE FORMATION FLYING MANEUVERS

4.1: The Model

The continuous-time simulations made use of a simple model based on the second-order, nonlinear dynamics (2.1)-(2.4). This model describes the continuous motion of one satellite, designated as the "follower," with respect to another satellite, known as the "leader." In this study the leader spacecraft is assumed to be in a constant, circular orbit around a spherical planet with no perturbation forces.

A low-Earth orbit (LEO) of 7000 km radius (~600 km altitude) and angular velocity 0.0011 radians/sec was chosen as the formation orbit. This orbit is comparable to that suggested for TechSat, was used for the simulation. It was desired to know the performance characteristics of the linear control laws for a suitable range of initial conditions. Such a range was chosen to include the most current sketch of the mission requirements for the TechSat-21 constellation. The goals for the TechSat mission include the ability to interchange between projected circular geometries of radii 100 m and 1 km. Accuracy of positioning should be within five percent (5%) of the distance from the formation center, and the trajectory should be such that the projections of the relative orbits about the formation center onto the Earth's surface are circular. To include these objectives in this study, a range of initial radii (from the
formation center to the outermost formation member) from 35 m to 3.5 km was studied.

In all cases simulated, the difference caused by any variation in initial radius was always less than 1%, when length dimensions were normalized by the initial radius and with all other factors being equal. For example, if the initial along-track separation for a certain simulation was 100 m, and the maximum relative velocity observed were measured to be 2 cm/s, then the maximum velocity for a simulation with initial along-track separation 50 m would be 1 cm/s, with no deviations greater than 10 mm/s; and for an initial separation of X meters, the maximum relative velocity could be predicted to be 0.02(X) cm/s. In addition, the maximum velocity would occur at the same time for the two cases, with no deviation greater than 1%. Therefore, a single initial radius of approximately 100 m was chosen and used for all further simulations.

Simulations of both the full, nonlinear dynamics and the linearized version (3.1)-(3.5) were performed. However, because there was no appreciable difference in the nonlinear and linear results, the results are provided only for the simulations that used the linearized versions of relative frame dynamics. Out of the over 1500 data points produced by the simulations for each case, the largest difference between any two data points was 1.2% for a single completion time data pair. All other differences were less than 0.5%, most were on the order of 0.001%, and many data sets were exactly the same within machine accuracy. In addition, all trends and trajectories are so geometrically similar as to be indistinguishable. Since error tolerances for the
missions being considered on the order of 5%, the linear and nonlinear versions of the simulations were considered to have the same results.

The neglect of nonlinear dynamics is certainly appropriate for the continuous-time data. However, no nonlinear, sampled-data version of the type of simulation described here was conducted, since some earlier work suggested that the linear and nonlinear results would be similarly identical. However, the absence of these results from this thesis should not be interpreted to indicate complete confidence that there are no significant differences in the behavior of the sampled-data controllers when taken from a linear to a nonlinear dynamical regime.

To help ensure the accuracy of the linear algebraic calculations, $\omega$, the orbital rate (equivalent to angular velocity) was normalized to unity. The radius of the leader's orbit was also normalized to unity, and these two unit changes resulted in the tangential velocity also being unity, since tangential velocity is equal to both the total velocity in a circular orbit and the product of orbital rate and distance from the gravitational focus ($v_1 = \omega r$). The only minor inconvenience caused by the normalization was that the quantities simulated are in unfamiliar units, so that changes in units had to be performed to provide realistic initial conditions or to interpret some data. This additional arithmetic was necessary, however, since the orbital rate in kms units caused the state matrix to be so badly conditioned that the simulation would not propagate. With this normalization applied, the state matrix becomes
\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 2 \\
0 & 0 & 0 & 1 \\
0 & -2 & 3 & 0 \\
\end{bmatrix}_{4\times 4}
\]

with eigenvalues of 0, 0, and ±j. For all simulations, the orbital rate was normalized to unity, so that the unit of simulation time was \( \omega^{-1} \), and the radius of the circular orbit was taken to be the unit of length. Once collected from the simulations, data were multiplied by appropriate constants to appear in familiar metric units. Note that simulations performed in this way would be applicable to any circular orbit so long as the proper unit transformations were performed on the output. However, in this context the data are interpreted according to a specific reference orbit as described below.

To simplify the problem somewhat, the origin of the state space (i.e. formation radius and velocity equal to zero), which is the state of the leader satellite in this model, was specified as the target of the simulated maneuvers. In reality, the maneuver objective would be to reach a new location on the x-axis for the in-plane maneuver, or to reach a target projected circular trajectory different from the origin for the projected circular maneuver. Because of the linearity of the state dynamics mentioned above, the net distance covered by the relocation of a satellite on the x-axis may be equivalenced, by scaling for length, to a maneuver to any other point on the x-axis, including the origin. It might seem at first glance that such a simple equivalence cannot be so easily equivalenced. However, the problem of moving a spacecraft in a
projected circular trajectory of radius $r_1$ to another, coplanar trajectory of radius $r_2$ is geometrically equivalent in all respects to the problem of moving a spacecraft from a projected circular trajectory of radius $r = r(r_1, r_2, \Delta \theta)$ to the origin, where $\Delta \theta$ is the difference in phase (i.e. angular location on projected circle) between the two spacecraft. The appendix to this paper derives an expression for $r = r(r_1, r_2, \Delta \theta)$ using the parametric equations for a projected circular trajectory of radius derived by [Yeh & Sparks, 2000]. And, since the performance characteristics scale with the starting radius, it does not matter what radius is tested; all coplanar projected circular trajectory transfers may be tested through the simulation of a single projected circular trajectory being transferred to the origin of its state space.

This geometric relationship may be visualized if one considers that any point in the formation may be treated as the origin. If one imagines a spacecraft residing in the position on the projected circular trajectory of radius $r_2$ and that this spacecraft now becomes the origin of the formation, then there is a path which the spacecraft at radius $r_1$ now follows in the new frame of reference. The periods of the paths must be equal, since $\omega$ does not change in a simple change of frame of reference. Visualizing the movements of the two points about a common center, one may notice that the two spacecraft are traveling in the same sense, counterclockwise or clockwise, about each other as they are traveling about their common origin. As such, one satellite may serve as a new formation origin for the other, and in this new coordinate system, the path traveled by the “follower” is exactly a projected circular path.
Were a transfer between noncoplanar projected circular trajectories desired, the geometric equivalence would fail, so a more detailed analysis would be required. However, since such noncoplanar trajectories must necessarily travel in opposite senses, it is difficult to imagine any situation where an advantage is gained from changing between noncoplanar trajectories that is sufficient to warrant the larger propellant expenditure.

4.2: Performance Criteria

The performance criteria used to compare the two control regimes were: the maximum difference between the trajectory and the desired final state (both position and velocity), the maximum control acceleration ordered for a single axis during the execution of the maneuver, the fuel or propellant used during the maneuver, and the time required to complete the maneuver.

The measure of propellant use deserves some discussion. The LQR formulation provides an optimal control law in the sense that a cost function equal to the time integral of the sum of the squares of the values of the weighted state variables and of the weighted control variables is minimized. Such a cost function, \( J \), is given by the formula

\[ J = \int_{t_0}^{t_f} \dot{x}^T Q x + u^T R u \ dt, \]  

(3.13)

where \( Q \) and \( R \) are the state and control weighting matrices, respectively. Since a higher weight on the control variables would decrease both control usage and state sensitivity, it might seem that there is a simple trade-off between fuel efficiency and
deviation of the trajectory before approaching the origin. Though this trade-off does
exist to some extent, it is important to note that fuel use is not really related to the
vector sum of the control variables, as a sum-of-squares approach might suggest.
Instead, fuel use is related to the activity of the propulsive apparatus. In fact, the fuel
used by each propulsive component is much more directly proportional to the time
integral of the absolute value, rather than of the square, of the acceleration effected by
that component. Even this relationship is an approximation whose accuracy is highly
dependent on the specific type of propulsion system used. In this study, each axis is
presumed to have thrusters dedicated only to that direction, such that all thrust in the
along-track direction is provided by different thrusters than that provided in the
radial or cross-track directions, and so on. From this assumption, the additional
simplification is made that the rate of fuel use for a given axis, \( \dot{m}_f \), will be treated as
directly proportional to the specific thrust, or acceleration, \( F \), that is provided:

\[
\dot{m}_f = \beta F. \tag{4.2}
\]

The proportionality constant, \( \beta \), is equal to the spacecraft mass \( M_{sc} = 100 \) kg divided
by the acceleration of gravity at sea level \( g = 9.807 \) m/s\(^2\) and the specific impulse of
the component \( I_{sp} = 2300 \) sec:

\[
\beta = \frac{M_{sc}}{g I_{sp}}. \tag{4.3}
\]

The values given were chosen to be suitable for a microsatellite employing electric
propulsion to actuate control commands [Space Propulsion Analysis and Design, pp. 509-
598].

30
For the H-only case, the minimization of the integrated sum of squares of the control accelerations will be similar to a minimization of the absolute values of those same accelerations. This is because only one control variable relating to the motion in the orbital plane is being minimized, and so the absolute value and the square of that variable have equivalent minima. For the H-V case, however, the sum of squares of the x- and y-axis accelerations is equal to the square of the vector sum of those accelerations, whereas the sum of the absolute values of those two variables is quite a different value. To use a geometrically equivalent example, in general, the minimization of the hypotenuse of a right triangle is not equivalent to the minimization of the sum of the legs of the triangle.

The time required to complete a maneuver was based on an error box of a size dependent upon the initial displacement of the satellite from the origin. Each maneuver was considered complete when both of two events had occurred: 1) The simulated satellite had entered and remained within a sphere centered on the origin and with radius equal to 5% of the initial displacement from the origin, and the total velocity of the satellite had reduced to and remained less than 1% of the product of initial displacement and orbital rate. For a projected circular formation, the relative velocity is of the same order of this product, and the same constraint is applied to the in-plane maneuvers somewhat arbitrarily. The position constraint does not take into account the relative importance of along-track, radial and cross-track displacements; for example, a non-zero radial displacement will create a non-zero along-track velocity, and yet the radial displacement is not held to a more stringent criterion. The
constraint is deemed adequate because it is not used in the feedback loop, but only to
define a performance measure.

4.3: Evaluation And Comparison of Controller Performance

Presumably, eliminating an entire axis of control would significantly reduce
the weight, complexity, and power requirements of the propulsion subsystem. For
gravity-gradient stabilized spacecraft with extendable booms, the problems associated
with jet-boom interactions can also be mitigated. The performance tradeoffs of this
design feature need to be better understood so that the benefits just mentioned can be
quantitatively compared with the reductions in some aspects of system performance
that may result.

Evaluations of performance will be made based on some of the criteria
suggested in the first section of this chapter. Specifically, the criteria evaluated are:
the fuel used to complete the maneuver, the time required, the maximum distance
between the trajectory and the maneuver target, the maximum velocity relative to the
target trajectory, and the maximum control acceleration required along any single axis
by the control law.

Comparisons between the H-V and H-only control structures will also be
based upon these five criteria. In order to make useful comparisons, a method of
choosing comparable LQR laws must be established. For each set of data having the
same initial conditions, there will be several LQR control laws calculated for each
control structure. These laws will all use the identity matrix as the state weighting
matrix, Q, and a scalar constant, $\rho$, multiplied by the appropriately sized identity
matrix as the control weighting matrix, $R$. The performances of all the laws in actuating the same maneuver will be noted for each of the two control structures.

It is worth noting that, though the independent variable plotted in most of the figures following the text of this paper is the control weight, $\rho$, it is often inappropriate to compare a H-only controller of a certain control weight with a H-V controller of the same control weight. This is because the control system designer may choose any control weight in LQR, or any combination of positive definite weighting matrices at all, so long as the demands on the system are met. In order to establish fair comparisons between the H-only and H-V control structures, a range of control weights is provided, and the entire range of performance available to each controller should be compared. For example, in Figure 58, the maximum accelerations commanded by various sampled-data controllers during a projected circular maneuver are presented. Even though the maximum acceleration commanded by the H-only controller at any given value of $\rho$ is less than the maximum acceleration commanded by the H-V controller of the same $\rho$-value, it is not on this basis that any comparison should be made. Instead, the important information for comparison provided by this figure is that the lowest maximum acceleration used by any H-only controller simulated is about $2.4 \times 10^3$ m/sec$^2$, and this is lower than the lowest maximum acceleration commanded by any H-V controller simulated.

4.4: Continuous-Time Simulation Method

The control laws for both the H-V and the H-only systems were calculated from the MATLAB linear-quadratic regulator routine using the command $\text{lqr}$.
linearized Clohessy-Wiltshire equations (2.5)-(2.7) served to provide the state and control matrices (3.4) and (3.5). The 6x6 identity was employed as the state weighting matrix, and a variable, $\rho$, multiplied the appropriate identity matrix (either 2x2 for H-only or 3x3 for H-V) to create the control weighting matrix for each law. For each case - H-V or H-only, continuous-time or discrete - this variable was given 19 values, from 0.1 to 9.1 in increments of 0.5, to create a selection of control laws which would allow reasonable comparisons of the performance criteria of the two control structures (H-only vs. H-V).

The continuous-time model was programmed using the MATLAB Simulink simulation package and propagated using a numerical integration scheme based on a fourth/fifth-order Runge-Kutta evolution routine, ode45. Several ODE solvers were used and the results compared; there was no appreciable difference in the trajectories calculated by any of the high order solvers. Also, the tolerances chosen and the maximum integration time step allowed were of little consequence. A simple propagator of the nonlinear equations served as the open-loop plant, and a controller implementation script, which determined from the appropriate linear control law the accelerations for any given state, was used to close the loop.

For each comparison of the continuous-time data sets, one performance criterion will be chosen such that there exists one response from the H-V set of matrix gains which is within 5% of the response from one of the H-only set of matrix gains. Then, the performance of two corresponding gains will be compared in light of the remaining criteria. This method of comparison was established before the actual data sets were produced, and works well for the in-plane maneuver data. However, the
behavior of the control systems performing projected circular maneuvers demonstrated strong dependence on the initial conditions as well as the control weight. The dependence was not a simple relationship, and so while the discussion of design feasibility in light of system performance is still quantitative, comparisons between the two systems are discussed somewhat more qualitatively.

4.5: Sampled-Data Simulation Method

For the sampled-data simulations, only the linear state-space realization of the relative frame dynamics, with state matrix given by (4.1), was used. The sampled-data system was calculated from the continuous dynamics using the MATLAB command c2d. This command discretizes a continuous system by computing the integrals of the state and control over the sampling interval, $T$, which is chosen by the user. Control laws were then calculated using the MATLAB command dlqr and the same sets of weighting matrices as for the continuous-time case. Thus, control laws based on the discretized Clohessy-Wiltshire equations commanded zero-order hold accelerations at the beginning of each discrete time interval. Examples of the sampled-data state-space pairs for each sampling period tested are given by

$$A_d = \begin{bmatrix} 1 & -2.36 & 6.54 & 2.83 \\ 0 & -4.66 & 8.50 & 1.82 \\ 0 & -2.83 & 5.25 & 0.91 \\ 0 & -1.82 & 2.73 & -0.42 \end{bmatrix}, \quad B_d = \begin{bmatrix} -0.34 & 2.18 \\ -2.36 & 2.83 \\ -2.18 & 1.42 \\ -2.83 & 0.91 \end{bmatrix}$$

(4.4)

for $T = 31$ minutes, which is equal to $2/\omega$, and
\[
A_d = \begin{bmatrix}
  1 & -15.03 & 28.54 & 3.31 \\
  0 & -5.61 & 9.92 & -1.51 \\
  0 & -3.31 & 5.96 & -0.76 \\
  0 & 1.51 & -2.27 & -0.65 
\end{bmatrix}, \quad
B_d = \begin{bmatrix}
  -17.39 & 9.51 \\
  -15.03 & 3.31 \\
  -9.51 & 1.65 \\
  -3.31 & -0.76 
\end{bmatrix}
\] (4.5)

for \( T = 62 \) minutes, which is \( 4/\omega \). Note that \( 2\pi/\omega \) equals one orbital period.

In addition to the zero-order hold matrix gains for each time interval and control weight, control gains were calculated for impulsive control actuation. In these laws, each control applied was an instantaneous change in velocity (\( \Delta V \)). This formulation makes the assumption that the propulsion system will be able to deliver the entire required impulse in an amount of time that is very small compared to the time interval. Vassar and Sherwood used the \( \Delta V \) formulation successfully in designing a linear control law for the formation flying problem [Vassar and Sherwood, 1985]. The same gain matrix design approach as was used by Vassar and Sherwood was used to produce the impulsive-thrust gain matrices for this study; the sampled-data discretization was carried out as for zero-order hold control, but the discrete control matrix, \( B_d \), was replaced by the second and fourth columns of the discrete state matrix, \( A_d \), corresponding to the velocity state variables \( x_2 = \dot{x} \) and \( x_4 = \dot{y} \). Then, a discrete LQR minimization was performed using the state matrix, \( A_d \), and the impulsive-thrust control matrix, \( B_d^{imp} \). In this way, a discrete control command takes the form of a \( \Delta V \) which is added directly to the current velocity state, as opposed to being integrated over the sampling period. The sampled-data pair for \( T = 62 \) minutes is shown below for comparison with the zero-order hold pair, (4.5).
\[
A_d = \begin{bmatrix}
1 & \text{1st col. } B \\
0 & -5.61 & 9.92 & -1.51 \\
0 & -3.31 & 5.96 & -0.76 \\
0 & 1.51 & -2.27 & -0.65
\end{bmatrix}, \quad
B_d = \begin{bmatrix}
-15.03 & 3.31 \\
-5.61 & -1.51 \\
-3.31 & -0.76 \\
1.51 & -0.65
\end{bmatrix}
\] (4.6)

The simulation of the sampled-data system was performed using a simple MATLAB script. This simulation script produced both the state and control trajectories when provided with an initial state vector, a time interval, and a length of simulation. The performance measures were calculated based on these trajectories in much the same fashion as for the continuous-time trajectories.

The methodology used for the analysis of the results of the sampled-data simulation is similar to that employed for the continuous-time case. The same performance attributes were measured and compared, but because many of the comparisons are similar to those for the continuous-time simulation results, the discussion of sampled-data results focuses on the interesting departures from continuous-time behavior.
CHAPTER 5

CONTINUOUS-TIME RESULTS

5.1: Cases Tested

As mentioned before, the maneuvers tested fell into one of two categories. One type was a maneuver in which the along-track (x-axis) displacement was reduced to zero ("in-plane"). Note that when the length scale is normalized, this type of maneuver is clearly equivalent to the transfer of the follower spacecraft from any point on the formation orbit to any other point on that same orbit, based on the observation that maneuvers that differ only in their length scale show equivalent performance when normalized to the same length scale. The other type of maneuver was a movement from a projected circular trajectory to the origin, which is equivalent to transfer between coplanar projected circular trajectories of different radii.

5.2: Maneuver for In-Plane Trajectory

5.2.1: Discussion of maneuver

In this paper, the first type of maneuver mentioned above is referred to as an "in-plane" maneuver, since the formation described by the initial condition is the in-plane formation detailed by [Sabol, Burns and McLaughlin, 1999]. In fact, this
maneuver is equivalent to making a change in the true anomaly of the follower spacecraft in the inertial frame without changing any other orbital elements in the long-term. This maneuver could conceivably be used to correct the drift caused by perturbations of the linear dynamics, since the unstable behavior of the open-loop plant manifests in the two spacecraft drifting apart along the velocity dimension at a rate proportional to the displacement along the radial dimension. It remains to be seen how well this simple maneuver can correct perturbations.

In simulations of this maneuver, the initial state of the formation was $x_0 = [105 \ 0 \ 0 \ 0 \ 0]^T$, such that the follower satellite was 105 m behind of the formation origin and had zero relative velocity. At time $t = 0$, the simulated follower satellite immediately began to apply its linear control law to reach the origin. The linear control laws were obtained in identical fashion, with only the scalar control weighting factor (see Section 4.2), $\rho$, varying from 0.1-9.1. A qualitative understanding of the controller response may be gained from an inspection of the relative-frame example trajectory plotted in Figure 3 ($\rho = 4$ for this trajectory).

5.2.2: Horizontal-only controller performance

The H-only controller performed the task of transferring its spacecraft from one point on the formation orbit to another by first accelerating to place itself in a slightly more elliptical orbit, and then decelerating to rendezvous with the origin as it approached. As a result, the distance from the origin first increased, and then decreased for all $\rho$-values tested. The maximum distance from the origin ranged from
109-116 m, or 4-10% of the initial displacement of 105 m, with 10% corresponding to \( \rho = 0.1 \) and 4% corresponding to the range \( 5.6 < \rho < 9.1 \). These results may be seen in Figure 11, and those that follow in this section may be seen in Figures 12-15. Note from the figures that the data for \( \rho < 1 \) are often much different from most of the rest of the data from the same sets. Interestingly, by increasing the importance of the state in the cost function, which is accomplished by reducing \( \rho \), the maximum displacement increased for the H-only controller.

In earlier sections, the point was made that when the length scale was the only parameter varied in a set of simulations, the data became identical within 1% when normalized for length. In this context, this geometric similarity means that, for any maneuver between in-plane trajectories, the maximum distance from the origin may be expected to be between 3-11% of the distance between the initial and final relative positions. This does not provide any simple equivalence between maneuvers of any other geometrical type.

The maximum relative velocity attained (Figure 12) was between 4.4-5.5 cm/s, with 4.4 cm/s corresponding to \( \rho = 9.1 \) and 5.5 cm/s corresponding to \( \rho = 0.1 \). The maximum control acceleration (specific thrust) ordered for any single axis by the controller (Figure 13) was between \( 7.7-85 \times 10^6 \) m/s\(^2\), again with the larger value corresponding to the smallest \( \rho \), 0.1, and the smallest value corresponding to \( \rho = 9.1 \). For a 100-kg satellite, these accelerations would translate to 0.77-8.5 mN of thrust as the largest required value. Note again and throughout this entire section that the length corresponds to an initial distance of 105 m. For an initial distance of 1 km, one
could expect that all values listed would be multiplied by a factor of approximately 10. Hence, the thrust required for a 100-kg satellite automatically moving an along-track distance of 1 km would have a range of 7.7-85 mN. This is still less than 100 mN, which puts such a control requirement within the range of proven Hall thruster capability [York, 1991]. However, there are new Hall thruster designs being considered for TechSat-21 which have maximum, constant thrusts of only 10-17 mN [private communication, R. Burns, August, 2000]. Since this thrust range is much less than the low-\( \rho \) control laws demand, only the high-\( \rho \) controllers would be compatible with these thrusters.

The time that the H-only controller required to complete the maneuver with 5% position accuracy (Figure 14) was between 1.25-1.5 orbits, the \( \rho = 0.1 \) control law taking 1.25 orbits and the \( \rho = 9.1 \) law taking 1.5. Since an orbit at 7000 km takes about 97 minutes, this range is equivalent to 120-150 minutes, or 2.0-2.4 hours. The precision of these completion times is questionable, especially since the 1% criterion for velocity is somewhat arbitrary (see Section 4.2). Also, note that the completion times do not scale with length; the controllers increase the control forces in proportion to the initial displacement such that the maneuver is completed in the same amount of time.

In order to calculate a value for the mass of propellant expended during the maneuver, some assumptions about the actuating propulsion system were required. The pertinent satellite characteristics assumed are a total, constant spacecraft mass of 100 kg and a specific impulse of 2300 seconds. This specific impulse is within the range of electric propulsion technology, but may be quite different from the propulsion system actually used by TechSat-21. The total mass of propellant thus
calculated for the maneuver (Figure 15) ranged from 0.14-0.49 milligrams, for $\rho = 9.1$ and $\rho = 0.1$, respectively. These values scale with length as well (through the acceleration term), so that a 1-km maneuver would require 1.4 - 4.9 mg of propellant. No attempt has been made to determine a feasible pulse-bit, so these propellant amounts may be much lower than a real propulsion system could possibly manage. Nonetheless, these figures represent characteristics of the space of automatically controlled maneuvers executable by a continuous, H-only controller, and so are provided as they were calculated, with no minimum value imposed.

5.2.3: Comparison with horizontal-vertical controller performance

Figures 11-15 show all the in-plane maneuver results for both the H-only and the H-V controller simulations in continuous-time. Figures 5-7 provide graphical representations of the H-only trajectory for comparison with those of the H-V controller in Figures 8-10. The maximum distances from the origin attained by the satellite controlled by the H-V laws were the same for all control weights, $0.1 < \rho < 9.1$. This result indicates that the initial distance was the maximum attained; the H-V controller was able to immediately reduce the distance from the origin, never exceeding the initial distance. The maximum velocity attained by the H-V controller fell in the range 4.2 - 6.8 cm/s (for $\rho = 9.1$ and $\rho = 0.1$, respectively), a range somewhat similar to the 4.4 - 5.5 cm/s of the H-only controller.

The maximum accelerations ordered by the H-V controllers were in the range 39-320 x $10^{-6}$ m/s$^2$, corresponding to maximum thrusts for a hypothetical 100-kg satellite of 3.9-32 mN. These values are 4-5 times the maximum thrusts ordered by
the H-only controllers. The maneuver completion time for the H-V set ranged from 1-1.4 orbits. This roughly corresponds to the range of the H-only controllers, giving times in hours of 1.6-2.3 hrs; the H-V completion times were lower by several minutes, but the determination of these completion times was not accurate to the minute.

The mass of propellant utilized during the maneuver ranged from 0.2-1.3 milligrams, the lowest value of which corresponding to \( \rho = 9.1 \). This lowest value falls well within the range of possible values for the H-only controller. Therefore, it should be possible to find one \( \rho \)-value for the H-only controllers and another for the H-V controllers such that the fuel consumption for the maneuver was equal. For \( \rho = 2.6 \), the H-only controller utilized 0.2 mg of fuel, as did the H-V controller for \( \rho = 8.6 \).

In this comparison, which is shown on Table 2, both controllers had similar completion times, the H-only controller requiring 3 minutes more than the H-V to complete the over 2-hour maneuver. The maximum displacement from the origin for the H-only controller was 111 m, 6% more than the initial displacement, which the H-V controller never exceeded. The maximum velocity relative to the origin reached was 14% higher for the H-only controller than for the H-V controller. Thus, the H-V controller requires a smaller region of space in order to maneuver successfully. In order to achieve this more confined movement, the H-V controller commanded a maximum acceleration of \( 4 \times 10^3 \text{ m/s}^2 \), nearly 2.5 times greater than the maximum acceleration commanded by the H-only controller.
5.3: Maneuver for Projected Circular Trajectory

5.3.1: Discussion of maneuver

The projected circular trajectory, or formation, is actually an elliptical path in the three-dimensional relative coordinate frame [Sabol, Burns and McLaughlin, 1999]. The ellipse is chosen such that, for an observer far above the leader and moving with the same velocity as the leader, the path of the follower appear to be a circle. This circle is the projection of the ellipse onto a locally horizontal plane. The peculiar geometry involved requires that the displacement in the cross-track direction (z-axis) always be exactly twice the magnitude of the displacement in the radial direction (y-axis). The two displacements may have either alike or opposite signs, but for a given formation, that sign relationship cannot change.

It is useful to consider the projection onto the horizontal plane of the position vector of the follower. This projected vector sweeps out an angle of arc of the projected circle in direct proportion to elapsed time. In other words, if the angle between the formation orbit and the projected position vector is defined to be the phase angle $\theta$, and if $\theta = \theta_0$ when time $t = 0$, then $\theta = \omega t + \theta_0$. The equations of motion for the projected circle formation are, as adapted from Yeh and Sparks [2000]:

\[ x(t) = r \cos \theta \]  \hspace{1cm} (5.1)

\[ y(t) = \frac{r}{2} \sin \theta \]  \hspace{1cm} (5.2)

\[ z(t) = \pm r \sin \theta \]  \hspace{1cm} (5.3)

where

\[ \theta = \omega t + \theta_0 \]  \hspace{1cm} (5.4)
and \( r \) corresponds the radius of the projected circle. From these relations, it is easy to see that during each orbit, there is a one-to-one correspondence between \( \theta \) and the states possible in the unforced projected circular configuration.

In transferring the controlled follower spacecraft from one projected circular trajectory to another in a formation, the question of when in the orbit to attempt the maneuver becomes an important factor. Because the phase angle describes the entire state of the follower spacecraft with one variable (provided the sign relationship between \( y \) and \( z \) is known), \( \theta \) is a useful variable for describing when a maneuver should take place.

If the nonlinear effects are negligible, the relative state space should show \( 180^\circ \) point symmetry about the origin. That is, if a certain initial state, \( X_0 \), yields a certain trajectory, \( x(t) \), then the negative of that state, \(-X_0\), should yield the negative of the trajectory mentioned, \(-x(t)\). This symmetry is characteristic of a linear system. In fact, in the nonlinear version of this dynamical system, this characteristic is observed with very high accuracy (\(< 0.1\% \) error in all cases), implying even further that nonlinear couplings are negligible. Therefore, only the phase angles from \( 0^\circ \) to \( 180^\circ \) need be considered; the behavior at \( \theta = 200^\circ \) was point symmetric to the behavior at \( \theta = 200 - 180 = 20^\circ \) with high precision. Note that only point symmetry can be thus guaranteed; for example, different behaviors are seen at points of equal \( y \)- and \( z \)-values because the velocities have different orientations with respect to the similar position vectors. However, the behavior at these points is often similar enough that the discussion will be limited to phase angles from \( 0^\circ \) to \( 90^\circ \). Charts showing controller performance during continuous-time maneuvers that begin at phase angles from \( 0^\circ \) to \( 90^\circ \) are
included as Figures 16-35; Figures 16-20 show performance results for maneuvers starting from \( \theta = 0^\circ \), Figures 21-25 show results for \( \theta = 30^\circ \), etc. It is no longer possible to find \( \rho \)-values for each controller such that fuel consumption is the same for all phase angles. However, it was noticed that for high \( \rho \)-values, the performance measures did not vary much with \( \rho \). Therefore, the following discussion considers the range of \( \rho \) over which controller performance varies little, so that the corresponding performance values may be compared.

5.3.2: Controller performance

From \( \theta = 0^\circ - 90^\circ \), both the H-only and the H-V controllers show maximum displacements from the origin very close to the initial value. In fact, the H-only controller exceeds the initial separation slightly at \( \theta = 90^\circ \), as can be seen in the example maneuver in Figures 6 and 9. Also, there is some increase in separation for both controllers at \( \theta = 30^\circ \) (Figure 21) and \( \theta = 60^\circ \) (Figure 26). Still, these increases are very small compared to the total distance covered during the maneuver.

The H-only needed to increase slightly the velocity to complete the maneuver; again, as with the in-plane maneuver, the lack of vertical thrust means that an increased orbital velocity must be used to increase the eccentricity and thereby allow a rendezvous. The H-V controller does not need to rely on orbital dynamics quite so much, and so was able to keep the total velocity lower.

The maximum acceleration commanded for any one direction by each controller was, in most cases, under \( 10^{-4} \text{ m/s}^2 \), or 10 mN for a 100-kg satellite. This
increase over the in-plane maneuver reflects the larger initial values for relative velocity. The performance of the H-V controller was fairly consistent over the range of θ; the H-only controller required about twice as much thrust as the H-V at θ = 0° (Figure 18), but needed the same thrust as did the H-V controller at θ = 90° (Figure 33).

Also increasing with respect to the in-plane maneuver performance was the time required to complete the maneuver. These times ranged from 1 - 3.5 orbits, but the two controllers performed with nearly identical speed (e.g. Figure 24); only low ρ-values saw a difference between the two values, but these low ρ-values will likely be avoided since they require much more propellant for very little improvement in other performance factors.

In terms of propellant use (Figures 20, 25, 30 and 35), the H-V controller again performed with great consistency over the range of phase angles from 0° to 90°, using under 0.4 mg of propellant for a specific impulse of $I_{sp} = 2300$ sec, and a spacecraft mass of $m_{s/c} = 100$ kg. The H-only controller performed as well or better than the H-V in fuel efficiency, with the exception of the low ρ-values at θ = 30°. For θ = 0°, the total propellant used for maneuvering by either controller was approximately 0.3 mg. For a 1 km radius change, this would increase to about 3 mg.

Altogether, the H-V controller performs with a great deal of consistency, allowing similar performance at any phase angle. The H-only controller can improve on this performance (greatly in the case of propellant use), but only at certain phase angles; θ = 90° saw the best overall performance for the H-only control structure (Figures 31-35).
CHAPTER SIX

SAMPLED-DATA RESULTS

6.1: Cases Tested

Among the range of propulsion options available to the satellite formation designer, the two simplest possibilities for the actuation of control forces are to use impulsive-thrust or zero-order hold. These two propulsion regimes allow for the adaptation of many well-established sampled-data control techniques. For the impulsive-thrust case, the entire control force is exerted over a time interval that is very short compared to the sampling interval. This allows the control designer to assume that the impulse results in an instantaneous change in velocity (ΔV). The zero-order hold propulsive actuator provides a constant thrust over the entirety of each sampling interval, altering its thrust only at the sampling times.

These two methods of actuating propulsive control create very different demands on the propulsion subsystem. The impulsive-thrust control regime requires the capability to produce a large maximum thrust that need only be operable for a few seconds of each sampling period, allowing any power required to be drawn from a battery which probably recharges between periods of control actuation. Chemical thrusters are well-suited to this regime, and have actuated attitude control and orbital
maneuvers for large spacecraft for several years using this method. In contrast, the zero-order hold method demands very little in the way of maximum thrust. Instead, zero-order hold means that the spacecraft must provide a constant power supply to the propulsion system over the entire period of the orbit correction or change. This technique allows the low-thrust but fuel-efficient electric propulsion technologies to operate effectively.

Since many microsatellites, such as TechSat-21, will be using some form of high-impulse, electric propulsive device for the actuation of formation maintenance and alteration, the zero-order hold regime is of immediate interest. Therefore, the majority of this section of the thesis will concern the results of simulations based on a zero-order hold method of control actuation. However, any benefits gained or detriments suffered from the elimination of the radial axis of control might be applicable to a system which used the impulsive control regime. To understand more thoroughly the behavior of the horizontal-only control structure, simulations implementing an impulsive regime of propulsive actuation were carried out. Some of those results will be discussed briefly, but this paper does not provide a full treatment on that aspect of the problem.

6.2: Effects of Sampling Period on Performance

Zero-order hold simulations were performed using sampling times of 2, 4, 6 and 8 radians of orbit, or roughly 0.5, 1, 1.5 and 2 hours for the 7000 km orbit. It is well-established that, as the sampling time of a system approaches the natural period of a periodic system, the control space becomes smaller and smaller, until the system
is uncontrollable when the sampling time equals the period [cf. Sampled-Data Control Systems, pp. 182-190]. This behavior was observed for the case currently under consideration as well. The sampling period of 93 minutes, which is very close to the orbital period of 97 minutes, produced controllers that were much less stable. Remarkably, the controllers produced with this sampling period all managed to reach their targets very quickly (in the first time step) for the in-plane maneuvers. However, since all of the projected circular maneuvers suffered severe delays in reaching the targets, and in the case of the H-only controllers, outright instability, the 93-minute sampling period is unacceptable. In the opinion of the author, the quick response for the in-plane maneuvers is due entirely to the unperturbed quality of the problem allowing the controller to predict the dynamics to unrealistic precision, and would therefore be entirely inaccurate.

By increasing the sampling period beyond the orbital period, to 124 minutes, the controller performance was once again acceptable, but not as good for any criterion or for either controller as for the half-hour or hour sampling periods. Therefore, further results will only include the data for these two suborbital periods: 31 minutes (half-hour), and 62 minutes (hour).

6.3: Maneuver for In-Plane Trajectory

6.3.1: Zero-order hold controllers

Using sampling periods of 31 minutes and 62 minutes for the in-plane maneuvers produced controllers that were stable and quickly reached the vicinity of their targets (within the first 1-4 sampling periods – see Figures 36 and 41). For both
the H-V and H-only controllers, the maximum velocities and maximum accelerations commanded decreased by factors of 2-3 when going from a half-hour sampling period to an hour sampling period (Compare the results of Figures 37 and 42 and of Figures 38 and 43). For both control structures, the maximum displacement was always the initial displacement (105 m) for all ρ-values and both sampling times.

The H-V used more propellant for maneuvering with the hour-long sampling period than it did with the half-hour period, whereas the H-only used roughly the same amounts of propellant with either sampling period (Figures 40 and 45). The time required by the H-only controller reduced by a factor of 2 by increasing the sampling period from a half-hour to an hour, whereas the time required by the H-V controller was roughly the same for both sampling periods.

Because the response time of the H-only controllers improved from an increase in sampling period, and the fuel efficiency of the H-only controllers deteriorated from the same increase, simply choosing one sampling period over the other could create a bias when making performance comparisons. However, the response time for all cases was on the order of 1 orbit (0.7 - 1.4 orbits), which is much smaller than the TechSat-21 maneuvering time allotment of 5-10 orbits [private communication, A. Sparks, June 2000]. When projected circular trajectories are considered, it will be seen that the comparisons are even more complex.

Clearly, the comparison of these data only makes sense in the context of a given mission. One must know whether or not a small gain in maneuver completion time is worth the expenditure of additional propellant. In fact, such concerns may create different requirements within the framework of a single mission. Therefore, all
pertinent data from both the half-hour and the hour sampling periods appears in Figures 36-85, and though certain figures are referred to as examples, few general comparisons, such as those made in the previous discussion of continuous-time performance, are made in this chapter. Rather, such comparisons are made in Chapter 7, and only in conjunction with recommendations in the context of specific mission objectives.

6.3.2: Impulsive-thrust controllers

The performance of the impulsive-thrust controllers was, on the whole, poorer than that of the zero-order hold controllers for the half-hour and hour sampling periods. Times for completion were comparable, but fuel usage was multiplied by a factor of 2 or more for the H-V controllers and by 9 or more for the H-only controllers. Some slight (10%) reductions in maximum velocity occurred for the impulsive-thrust H-only controller, but the maximum acceleration ordered was multiplied by a factor of 3. Conversely, there was some slight (10-15%) reduction in maximum acceleration ordered for the impulsive-thrust, high-p H-V controllers, but the maximum velocity attained increased by a factor of 2. Except for differences in the factors of reduction or increase, all of the same generalizations regarding the impulsive-thrust controllers as compared to the zero-order hold controllers held true for the projected circular maneuvers.

Since the zero-order hold controllers will be very useful for microsatellites using electric propulsion, and since they perform as well or better than the impulsive-thrust controllers for both maneuvers tested, the impulsive-thrust controllers will no
longer be considered. However, it should be noted that this cursory investigation of impulsive-thrust controllers indicated that the H-only performance may suffer much more than the H-V performance if a design shifts from the zero-order hold regime to the impulsive-thrust regime.

6.4: Maneuver for Projected Circular Trajectories

Similar to the continuous-time case, the discretization of the control system allowed both controllers to consistently keep the displacements below their initial values when maneuvering from projected circular trajectories. In addition, the performance of the H-only controllers improved in that, for all p-values tested, the relative velocities did not exceed their initial values either. There is no figure included which compares the geometry of the trajectories commanded by the sampled-data H-V and H-only controllers largely because the figures were, to the eye, identical.

Unlike in the continuous-time case, the H-only controller ordered maximum accelerations that were consistently less than those ordered by the H-V controller. Also, the H-only maximum thrusts were lower than for the continuous-time H-only case, going only as high as 3-6 mN at \( \theta = 90^\circ \) (Compare Figure 33 with Figures 64 and 83.). These highest values for the sampled-data H-only case are similar to the lowest maximum thrust values required by either type of continuous-time controller.

Maneuver completion times for the H-only controller were approximately twice those of the H-V controller for all phase angles (Figures 49, 54, 59, 64, 69, 74, 79 and 84). The completion times for the H-only controllers were approximately 2 - 2.5
orbits for all values of \( p \), while those of the H-V controllers were approximately 1 - 1.5 orbits, again for all control weights, \( p \).

The mass of propellant expended by the H-only controller to complete each maneuver with a half-hour sampling time was often similar to and sometimes more than that of the H-V controller (Figures 50, 55, 60 and 65). The sampling time appears to disrupt whatever phenomenon causes the H-only controller to be more fuel-efficient in most other settings. However, for the sampling time of one hour, the propellant use for the H-only controller was less, equaling less than 2 mg of propellant for the worst case (\( \theta = 0^\circ \); Figure 70) and just under 1 mg for the best case (\( \theta = 90^\circ \); Figure 85). In this best case, however, the H-V controller provided very nearly the same fuel-efficiency.
CHAPTER 7

CONCLUSION

7.1: Synthesis of Results

The continuous-time case studied above is strictly theoretical, and not necessarily practical for the purposes of long-term formationkeeping. The propulsion technology available to a mission designer may not be precise enough to allow for the tiny increments of thrust that would be required for such formation maintenance. Large, geometry-changing maneuvers, however, would very likely be able to use a sampling period as small as 1-2 minutes. This small sampling period resembles the continuous-time regime closely enough to make these results quite valid in a practical as well as a theoretical sense.

For an increase or decrease in the radius of a projected circular formation, the above results are valid so long as the change in radius is coplanar. The only obvious reason for which noncoplanar formation changes would be implemented would be if greater efficiency in some performance measure were gained such that the additional fuel required were justified. That possibility presents yet another question to be answered by the mission designer. To keep matters clear, the recommendations
presented here will assume coplanar formation changes, since they will probably be used in most cases.

If the ability to change the radius of the formation must be available at all points of an orbit, then the vertical control capability of the H-V controller seems to allow for more consistent performance from any angle of maneuver commencement. The H-only controller is quite capable of making the same maneuvers from any phase angle and with the same or better fuel efficiency. However, the thrust requirement doubles from 5mN at $\theta = 90^\circ$ to 10mN at $\theta = 0^\circ$ for a 100-meter radius change.

If the thrust available from the Hall thruster is assumed to be the minimum of the typical range, 0.1N, then it is barely feasible to make 1-km maneuvers from any phase angle of the projected circle using the linear controller without vertical thrust. The reduction of the thrust requirement by one half should be carefully considered in light of the pursuant savings in propulsion subsystem weight. The propulsion capability for an entire axis of control may be eliminated, and the weight of the propellant used by the remaining thrusters may be less than the additional weight of thrusters required if the third axis of thrust were retained.

7.2: Design Recommendation

Since there are only small performance differences between the horizontal-and-vertical and the horizontal-only controllers, and since the largest of those differences is the savings in propellant mass required by the H-only controller, the recommendation based on continuous-time analysis is that the H-only controller
should be seriously considered and tested for use as the control structure for any projected circular or in-plane formation.

With that recommendation must come a strong caveat: The effects of perturbations are unknown, and it is quite possible that recovering a follower spacecraft from an unstable trajectory will be much more difficult with an H-only control system than for a system with all three axes of control. The trajectories tested here are all stable in the absence of perturbations, but unstable in the presence of perturbations of any sort. Performance in long-term, perturbed environments is the next research path that the author hopes to follow.

The study of the sampled-data system seems to yield more questions than answers. The apparent dependence of propellant use on the sampling period for both H-V and H-only controllers is not a simple one. Because of this dependence, there may be a sampling time which maximizes the fuel efficiency. The discretized system has the effect of reducing or eliminating differences in H-only and H-V state trajectories. One must ask whether this is a real phenomenon, or merely a result of too many simplifications in the model. If the effect is real, the reason could lie in the orbital dynamics of the problem. Because the effectiveness of a given level of thrust in a certain direction is highly dependent on the relationship between the present and target orbits, constant attempts to correct every variable in the state vector will result in a great deal of wasted effort. The choice of a proper sampling time may relieve that deficiency of the continuous-time LQR approach.

New research directions aside, the performance analysis of the sampled-data cases brings to light only one point that could change the recommendation based on
the continuous-time results. If, from a judicious combination of control weight and commencement phase angle, very nearly the same fuel efficiency can be obtained for the H-V controller as for the H-only controller, then fuel efficiency in large maneuvers is no longer as strong an argument for the elimination of the radial axis as before. The reduction in weight and complexity offered by the elimination of the additional propulsive components is still rather significant, though. Unless further studies of the effects of perturbations show that the loss of radial axis control is severely detrimental to the mission, perhaps by greatly increasing propellant consumption in long-term formationkeeping, then the recommendation of the horizontal-only controller will stand firm.

7.3: Automatic Radius Change for a Formation of Many Satellites

The TechSat-21 program seeks to place in orbit a group of several satellite in projected circular formations about a common formation origin. In this geometry the formation members may act as a synthetic aperture for surveillance of the Earth's surface. Control over the formation geometry should allow the radius of the formation to be changed from 100m to 1km, or vice versa, whenever mission objectives require, and an autonomous, decentralized control system should maintain the formation between these large-scale formation maneuvers. Importantly, a method of preventing collisions must be found, despite the mostly independent action of the members of the formation.

The exact arrangement of the TechSat-21 formation has not been determined as of yet. For the purpose of discussion, the formation may be imagined as a set of N
concentric subgroups of member satellites, where all satellites of one subgroup are evenly spaced on a projected circular trajectory about the formation center. The subgroups may or may not have equal number of satellites, and there may or may not be a satellite located at the formation origin. One problem that must be solved if such an arrangement were considered is the problem of changing the formation radius. The linear controls laws described by this paper may be capable of performing such a radius change automatically and without the occurrence of collisions, without any command required except notification to the satellites that the formation should adopt a specific new radius.

Since the results provided in this thesis suggest that there is some performance advantage to beginning a radius change maneuver at a particular phase angle, \( \theta \) (see Chapters 5 & 6), it will be somewhat desirable to have all maneuvers begin at this same phase angle. Let this maneuver phase angle be denoted by \( \theta_m \). Then, it becomes possible to imagine the appearance a change of radius would take; as the phase angle of each satellite at a certain initial radius equals \( \theta_m \), that satellite begins an automatic maneuver to the new radius. The beginnings of the maneuvers will be separated in time by \( (\omega M)^{-1} \), where \( M \) is the number of satellites in the subgroup that shares a single projected circular trajectory. This maneuver could be accomplished with a linear controller by setting the target trajectory - radius and time-dependent phase angle - of each maneuvering satellite as the origin for that satellite.

Collisions between satellites of different initial radii could be avoided rather easily. If the formation radius is being reduced, then the group with the smallest radius should maneuver first, and then the group with the next largest radius, and so
on to the outermost group. If the formation radius were being increased, then the outermost group should move first, etc. Since the trajectories followed by spacecraft controlled by these linear controllers do not increase their initial radius significantly, and do not greatly overshoot the final radius, if at all, there would be an annular safety region during the maneuver of each subgroup into which no satellites would enter.

Due to the largely linear nature of the state-space of the formation, the trajectory for each spacecraft would be similar, since each begins and ends with the same state. Each trajectory would take the form of a spiral path from the initial radius to the final radius, and the time required for each maneuver from a given radius would be similar as well. Since all of the satellites would be following the same trajectory, but separated in time by \((\omega M)^{-1}\), the satellites would follow one another in a sort of chain. If \(M\) were small enough, and therefore the time between consecutive satellites passing through the same point in relative space large enough, collision avoidance would be automatic. The factors determining the feasible size of \(M\) – e.g. magnitude of oscillations about the final trajectory, measurement error, size of spacecraft, perturbation forces and unmodelled dynamics resulting from ellipticity of the formation orbit – would need to be included in any model seeking to validate this simple method of control.

7.4: Future Research

For designers to be able to use the information presented here, a great deal more research must be conducted. The most important use of the automatic control
system used in formation flying is arguably not for the performance of large maneuvers but for the maintenance of the formation in between maneuvers. The differential drag associated with small satellites at LEO may have rather large effects on the formations, since these spacecraft can often have very different ballistic coefficients depending on the current orientation of their solar panels. Earth-oblateness effects will cause the satellites in projected circular formations to drift apart, since the individual orbits have different inclinations in the inertial frame. So, perturbation forces should be included in further simulations of the horizontal-only control design.

The noise associated with sensing relative positions and velocities should also be included in future studies. Recent work suggests that the noise associated with GPS navigation still may be severe even after filtering out common errors [Inalhan, Busse and How, 2000]. In this case, some method of filtering the high-frequency signal variations due to noise out of the signal by taking data over a certain time interval and performing a calculation to determine a state which is very accurate. The mean orbit elements method proposed by Schaub et al. [1999] does accomplish this, but to date, that method has not been applied to a control system which leaves out the radial axis of thrust. Such a design may prove particularly fruitful, since the mean orbit elements design takes advantage of the best points in each orbit to apply forces in each relative direction.

Finally, this control design, as any other formation control algorithm, would need to be tested on orbit. Fortunately, such testing would not require an entirely different mission from tests of other MSFF control systems. Most MSFF designs to
date have had three orthogonal axes of control, so to test the H-only controller, one need only turn off one axis and direct the other two to be coplanar with the local horizontal. It is hoped that, because of the low additional cost associated with testing this system, it will be tested among the algorithms proposed for the TechSat-21 mission.
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Projected Circular Trajectory

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APPENDIX

Theorem:

Let two satellites be moving along coplanar, projected circular trajectories: \( x_1(t) \) with projected radius \( r_1 \), and \( x_2(t) \) with projected radius \( r_2 \). Let these two trajectories share a common formation origin, \( O_f \), with a common coordinate system defined by the formation orbit. If one of these two satellites is defined as a new formation origin, \( O_f^* \), then the trajectory, \( x^*(t) \), of the other satellite about the origin \( O_f^* \) is also a projected circular trajectory.

Proof:

The parametric equations, as adapted from Yeh and Sparks [2000], for the linearized trajectory of a satellite following a projected circular path relative to a leader satellite are:

\[
\begin{align*}
x(t) &= r \cos \theta \\
y(t) &= \frac{1}{2} r \sin \theta \\
z(t) &= \pm r \sin \theta
\end{align*}
\]

where the phase angle is defined by

\[
\theta = \omega t + \theta_0
\]

and \( r \) corresponds to the radius of the projected circle. Note that \( y(t) = \frac{1}{2} z(t) \) in the parametric equations above. Therefore, the underlined \( x(t) \) will henceforth denote the projected position vector, \( \underline{x}(t) = [x(t) \ z(t)]^T \), since \( y(t) \) is implicit in this vector.

The observation is made that, since the formation motion does not change significantly with a change in formation origin (this is the main effect of linearizing the relative frame dynamics). Therefore, the directions of the coordinate axes and the angular momentum of the relative frame (vector \( \omega \)) both may be kept constant with a
change in origin. If $\mathbf{x}_1(t)$ denotes the trajectory of satellite #1, and $\mathbf{x}_2(t)$ the trajectory of satellite #2, then the trajectory of #2 with respect to #1 is obtained from differencing the two: $\mathbf{x}^*(t) = \mathbf{x}_2(t) - \mathbf{x}_1(t)$. By taking the dot product of each side of this equation with itself (i.e. the norm squared), and observing that the norms of $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ are the constant radii of their respective projected circles, the following result is obtained:

$$
\|\mathbf{x}(t)\| = \|\mathbf{x}_2(t)\| + \|\mathbf{x}_1(t)\| - 2(\mathbf{x}_2(t) \cdot \mathbf{x}_1(t)).
$$

(A.5)

This relation is reminiscent of the law of cosines, since

$$
\mathbf{x}_2(t) \cdot \mathbf{x}_1(t) = \|\mathbf{x}_2(t)\| \cdot \|\mathbf{x}_1(t)\| \cos(\varphi(t)),
$$

(A.6)

where $\varphi(t)$ is the angle between the vectors. For this particular case, the angle between the vectors may be obtained by differencing the two phase angles, $\varphi(t) = \theta_2(t) - \theta_1(t)$.

Recalling the definition provided in (A.4), it is noted that

$$
\varphi(t) = \theta_2(t) - \theta_1(t) = (\omega t + \theta_{02}) - (\omega t + \theta_{01}),
$$

(A.7)

where $\theta_{02}$ and $\theta_{01}$ are the phase angles of the respective satellites at some initial time, $t_0$. Since $\theta_{02}$ and $\theta_{01}$ must be constants, therefore $\varphi$ is also a constant. Now, since the norms of (A.5) and (A.6) are also constant, then the norm of $\mathbf{x}^*(t)$ is constant. This means that the projection of this trajectory onto the local horizontal plane of satellite #1 (which is parallel with the local horizontal of the original leader) is a circle.

Furthermore, since the functions $y(t)$ are all implicit in the representations of $z(t)$, it may now be easily shown that the trajectory $\mathbf{x}^*(t)$ is a projected circular trajectory.

Through use of the law of cosines, the radius of the projected circle is given by

$$
r^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1).
$$

(A.8)

and with some geometry combined with the law of sines, the phase angle, $\theta^*$, of satellite #2 in its new projected circle may be derived as

$$
\theta^*(t) = \theta_1(t) - \sin^{-1}\left(\frac{r_2}{r_1} \sin \theta_2(t)\right).
$$

(A.9)

Thus the problem of changing the radius of a projected circular trajectory to any other radius is reduced to a regulation problem which uses exactly the same state-space representation, (A,B), as is used throughout the text of this paper.

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GLOSSARY

along-track:
Refers to the axis, dimension, or direction corresponding to the parallel to the
ground-track of an orbit. For a circular orbit, this direction is always parallel to
the velocity and the instantaneous tangent to the orbit. Perpendicular to both
the cross-track and radial directions.

control structure (definition for this paper only):
Term which indicates the geometry of the control actuation system, which is
the propulsion subsystem. Specifically, a propulsion system capable of
providing thrust along all of three orthogonal axes is said to be of a different
structure than a propulsion system where one axis of control has been
eliminated.

cross-track
Refers to the axis, dimension, or direction corresponding to the horizontal
perpendicular to a given orbit. Always parallel to the angular momentum
vector for the orbit, and perpendicular to both the along-track and radial
directions.

eccentricity vector:
One of a set of six Keplerian orbital elements, the eccentricity vector has
magnitude equal to the ratio of the length between the foci to the semi-major
axis (or unity, in the case of a parabola), and it points from the center of the
planet to the periapse of the orbit. If there is no periapse (i.e., if the orbit is
circular), then the eccentricity vector is the zero vector.
formation eccentricity:

Eccentricity of the orbital path followed by the origin of the formation relative coordinate frame. For the leader-follower arrangement, this is the eccentricity of the orbit traveled by the "leader."

formation orbit:

Orbit that a spacecraft would follow were it located at the formation origin and exhibiting Keplerian motion.

formation origin:

Point of geometrical reference moving with a formation selected to aid in MSFF. Typically, the formation origin follows an orbit similar to those followed by members of the formation, whether or not the origin is occupied by a spacecraft.

formation keeping:

Active maintenance of a given spacecraft formation over a desired duration, such as the mission lifetime.

horizontal-only (H-only):

The control structure in which it is unnecessary and virtually impossible to provide control forces along the radial dimension.

horizontal-and-vertical (H-V):

The control structure which can and does provide control forces along the radial dimension.
leader-follower arrangement:
Formation (or portion of a formation) of two spacecraft in which the trajectory of one spacecraft, designated the "follower," is maintained according to its motion relative to an uncontrolled spacecraft, designated the "leader." This is one of many terms describing the given arrangement.

multiple spacecraft formation flying (MSFF):
Refers to the tandem control of several spacecraft that are moving in close proximity to one another. Such missions may be in orbit about a planet or on an interplanetary or deep-space trajectory.

orbital rate:
Time derivative of the angle swept by an object in orbit. Equivalent to the angular or rotational frequency.

radial:
In this context, refers to the axis, dimension, or direction parallel to the instantaneous position vector from the gravitational focus outward to the orbit. Perpendicular to both the along-track and cross-track directions.

relative coordinate frame:
Non-inertial frame of reference described by a set of rotating Cartesian axes with a moving origin that follows an identifying orbit for a formation, such as that of the leader spacecraft. The relative coordinate frame used in this paper has x-axis directed along the anti-velocity vector of the leader, y-axis directed along the inertial position vector of the leader, and z-axis directed along the angular momentum vector of the leader's orbit.
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