THE EFFECTS OF CONCRETENESS ON LEARNING, TRANSFER, AND REPRESENTATION OF MATHEMATICAL CONCEPTS

DISSERTATION

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ABSTRACT

The present research investigated the effects of concrete and generic instantiations on learning and transfer of a mathematical concept. The current work distinguished two types of concreteness. Irrelevant concreteness is extraneous information that is unrelated to the to-be-learned concept and has been shown in previous research to hinder both learning and transfer. Relevant concreteness involves symbols or storyline that help communicate the to-be-learned conceptual structure. In a series of experiments, undergraduate college students learned instantiations of a mathematical group that were generic, relevantly concrete, or irrelevantly concrete. Relevant concreteness was found to promote quick learning. However, the benefit of relevant concreteness did not extend to transfer. Relevant concreteness hindered recognition and alignment of structure between the learned domain and a novel isomorph which resulted in transfer failure. With slightly protracted training, the generic instantiation was learned equally well as the relevantly concrete one. Most importantly, the generic instantiation resulted in successful structural alignment and transfer. The benefit of the generic instantiation was also demonstrated for children. In a separate experiment, after learning a relevantly concrete instantiation, participants were unable to recognize learned structure when presented with novel elements. However, participants who learned the generic or irrelevantly concrete instantiation were able to recognize structure in the context of novel elements, suggesting
that relevant concreteness obfuscates the analogy between the domains. When participants were given the correspondence of elements across learning and transfer domains, relevantly concrete instantiations did result in successful transfer. Furthermore, learning a generic instantiation was shown to have an advantage for transfer over learning multiple instantiations. Finally, relational diagrams that convey global structure resulted in transfer to isomorphs as well as modification and transfer to non-isomorphic systems of the same structure category. This research argues that relevant concreteness may give a leg-up in the learning process. However, this benefit comes at the cost of transfer. Learning a generic instantiation can provide a direct route to forming an abstract internal representation that can facilitate transfer.
Dedicated to my father, Vincent R. Kaminski
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A goal of mathematics education is conceptual understanding (Bransford, Brown, Cocking, Donovan, & Pellegrino, 2000; Kilpartick, Swafford, & Findell, 2001; NCTM, 2000). Conceptual knowledge is evidenced by the ability to recognize mathematical structure in different contexts and can provide the basis for solving new and unfamiliar problems (Kilpatrick et al, 2001; Lesh, Post, & Behr, 1987). In other words, an important goal of mathematics education is the ability to transfer conceptual knowledge from the learning situation to a novel isomorphic situation.

The purpose of the present research was to investigate learning and transfer of mathematical concepts. While the acquisition of mathematical concepts is affected by a large number of factors including prior knowledge, learner’s motivation, and social scaffolding (Bransford, et al, 2000; Kilpartick, et al, 2001) to name just a few, this research was focused on factors underlying transfer of learned knowledge to novel situations. Specifically, this research compared the effects of concrete and generic instantiations on initial learning and subsequent transfer of a mathematical concept.
Nature of Mathematical Concepts

Mathematics can be considered the formalized study of structure (Bourbaki, 1984; Corry, 1996). As a result, mathematical concepts differ from everyday concepts in two ways. First, unlike everyday concepts, such as dog, that are ill-defined (see Solomon, Medin, & Lynch, 1999), mathematical concepts, such as function or derivative, have precise definitions based on their relational structure. Mathematical ideas can be stated as relations of elements. Elements are unitary or unitized entities and relations are ways in which those elements are associated with each other. Instantiations are created by specifying elements and possibly other superficial information. For example, a group is an algebraic structure; it is a set of elements upon which a binary operation is defined such that the following properties hold: (a) an identity element exists in the set, (b) each element of the set has an inverse in the set, and (c) the operation is associative (Herstein, 1975). A group is a mathematical concept. The group of integers under addition modulo 3 is an instantiation of the concept whose elements are the three equivalent classes that contain 0, 1, and 2 respectively.

The nature of mathematical concepts allows an unlimited number of instantiations each communicating additional, nonessential perceptual or conceptual information. For example, there are an infinite number of instances of the concept derivative. Distance functions, cost functions, and any continuous function have derivatives which represents an infinitesimal rate of change. The more information that is specified, for example a particular distance function at a particular time of a particular car, creates a more concrete instantiation of the concept. Just as knowledge of velocity as rate of change in distance with respect to time is not knowledge of derivative, knowledge of any abstract concept is
not merely knowledge of one conceptual instance. Conceptual knowledge is general knowledge that multiple instances exist and an ability to recognize at least some of these instances (Skemp, 1987).

The second way mathematical concepts differ from everyday concepts is in their manner of acquisition (Skemp, 1987). Everyday concepts, such as chair and cat, are often acquired incidentally through encounters with concrete instances that typically share a great deal of perceptual similarity. On the other hand, instances of mathematical concepts can differ greatly in their superficial features likely making it difficult to recognize their common structure (see Goldstone, Medin, & Gentner, 1991). For example, without a priori knowledge of exponential decay, it is probably difficult to recognize the commonality of metabolism of medication in the body and a cooling cup of coffee. As a result, acquisition of mathematical concepts typically requires some form of supervision, often explicit instruction and deliberate learning (Skemp, 1987) which begins with an initial instantiation.

In addition, any given instantiation of a concept can occur or be expressed in different ways. To be precise, an instantiation can be communicated through different types of external representation. External representations fall into distinct categories: (1) experience-based "scripts" in which knowledge is organized around "real world" events; (2) manipulable models such as Cuisenaire rods, base ten blocks, and fraction bars; (3) pictures or diagrams such as graphs and other static figural; (4) spoken language; (5) written symbols and accompanying text or spoken language (Lesh, Post, & Behr, 1987). The choice of instantiation as well as external representation can affect what a learner can do with the acquired knowledge (e.g., Bassok & Holyoak, 1989; Goldstone & Sakamoto,
2002; Sloutsky, Kaminski, & Heckler, 2005; Zhang, 1997; Zhang and Norman, 1994). Throughout the present research, the words “instantiation” and “domain” will be used interchangeable and will refer to a specific instantiation of a concept and its associated external representation.

With regard to conceptual understanding of a mathematical concept, an effective instantiation must promote both learning and transfer. Learning is evidenced by the application of acquired knowledge within the learning domain, while transfer is the ability to apply conceptual knowledge to novel isomorphic domains. This research considered the effects on learning and transfer of different instantiations and their accompanying external representations of a given concept. These instantiations varied in their degree and type of concreteness. For the purposes of this research, concreteness will be defined as follows.

Concreteness will not imply tangible, physical objects, but rather contextualized instantiations and their external representations that many might refer to as semi-concrete. This interpretation of concreteness has been used by other researchers (e.g., Gentner, Loewenstein, & Thompson, 2003; Goldstone & Sakamoto, 2003; Goldstone & Son, 2005). Concrete versus abstract is not a dichotomy; it is a continuum where concrete instantiations provide the learner with more information than abstract instantiations. For a given concept, instantiation A is more concrete than instantiation B if A provides the learner with more information than B. Consider the increase in conveyed information as concreteness increases from a stick figure of a person to an elaborate drawing to a photograph to a real person. This conveyed information may be perceptual or conceptual in nature. For example, base ten blocks provide perceptual information regarding
magnitude of numbers. On the other hand, monetary arithmetic may involve prior knowledge and familiarity of the currency system. Both of these examples can be considered more concrete than real numbers or variables.

Sometimes the additional information of a concrete instantiation easily conveys knowledge relevant to the given concept. In this case, concreteness will be labeled *Relevantly Concrete* and may perhaps facilitate learning. However, not all concrete instantiations easily communicate relevant conceptual structure. *Irrelevantly Concrete* instantiations communicate extraneous irrelevant information such as perceptual richness. Therefore, concreteness could be considered to have two dimensions: the degree of relevant concreteness and the degree of irrelevant concreteness. Figure 1.1 illustrates different types of concreteness communicated through instantiations of proportion. Instantiations that communicate little or no concreteness, such as \( \frac{x}{y} \), are defined as *Generic* instantiations.

In the present research, concreteness was manipulated through choice of storyline and the specific symbols for elements. Perceptual richness of symbols was increased to add irrelevant concreteness. Of particular interest were instantiations that will be referred to as: (1) Generic; (2) Relevantly Concrete; (3) Perceptually Rich; and (4) Relevantly Concrete / Perceptually Rich. The Generic instantiation was intended to mimic traditional symbolic mathematics and involved simple black symbols and sentential expressions. The Relevantly Concrete instantiation involved a storyline and symbols that might help communicate the to-be-learned structure. Symbols of the Perceptually Rich and Relevantly Concrete / Perceptually Rich instantiations used colorful counterparts of the
Figure 1.1 Types of concreteness for instantiations of proportion.

Generic and Relevantly Concrete Instantiations respectively. All instantiations will be explained in more detail in Chapter 3.

**Analogical Transfer**

Transfer is the degree to which a behavior will be repeated in a new situation (Detterman, 1993). Therefore, transfer is a broad category of learning phenomena. Detterman describes different levels of transfer. *Near transfer* occurs between highly similar situations. In this case, domains may share a high degree of surface similarity such as common objects or elements. Transfer across less similar situations is *far transfer*. In addition, there is a distinction between *specific transfer* of facts or content knowledge and *general transfer* of strategies or abstract principles. Transfer of conceptual knowledge across isomorphic situations is general transfer of deep structure. Such transfer based on relational structure is analogy.
While transfer is an important goal of education, spontaneous transfer is notoriously difficult to achieve (Bassok & Holyoak, 1989; Gick & Holyoak, 1980, 1983; Reed, Dempster, & Ettinger, 1985; Reed, Ernst, & Banerji, 1974; Ross, 1987, 1989). A seminal study conducted by Gick and Holyoak (1980) investigated transfer of problem solutions. When given the solution to one problem, can or do people transfer the solution or solution strategy to a different structurally analogous problem? Adult participants were given the task of solving the radiation problem. In this problem, a malignant tumor needs to undergo radiation treatment. A radiation ray of high intensity will destroy the tumor, but unfortunately it will also destroy the healthy tissue in its path. Lower intensity beams will not destroy the healthy tissue, but are also not effective on the tumor. Participants were asked what kind of treatment design would be effective in eliminating the tumor and not the surrounding healthy tissue. The solution is to direct many low intensity rays toward the tumor at different angles. Prior to being asked the radiation problem, participants were given the solution to an analogous story. For example, an attacking army cannot approach its target by traveling over one bridge which cannot support the weight of the entire army. The army must split into small groups and approach from multiple angles.

Not all participants were able to see the relevant analogous information; only 20%, spontaneously transferred the solution. When given a suggestion that the previously learned problem and solution may be helpful, 92% of participants transferred the solution strategy. This poor pattern of spontaneous transfer has been shown in numerous studies with adults and children (Goswami, 1991; Novick, 1988; Reed, Ernst, & Banerji, 1974; Simon & Reed, 1976). For example, college students who learned solutions to algebra
word problems involving distance, mixture, or work were unable to transfer their
to solve novel isomorphic problems (Reed, Dempster & Ettinger, 1985).

Most research on transfer and analogical reasoning has focused on the following
processes that have been proposed to play a role in transfer: (a) retrieval of a prior
analogous domain (Gentner, Rattermann, & Forbus 1993; Gick & Holyoak, 1980; Ross,
1987, 1989), (b) alignment and mapping of structure between the base and novel domains
(Gentner, 1983, 1988, 1989), (c) implementation of the analogy (Novick, 1988), and (d)
schema formation (Gick & Holyoak, 1983; Reed, 1993). In order to transfer knowledge
from a previously learned domain to a novel target domain, one must first represent the
target domain, then retrieve an analogous prior domain, align the two domains according
to relevant structure, and finally implement the analogy. As a result of analogical
reasoning, abstract conceptual schema may be formed that can facilitate future analogical
transfer. Chapter 2 presents a review of this literature as well as literature documenting
development of analogical reasoning and symbol use, which can be construed as a simple
form of analogical transfer.

In addition to the above mentioned processes, the present research argues that the
nature of the initial learning domain can significantly impact the learner’s ability to
transfer. The present research focused on the manner in which the type of initial
instantiation(s) learned affects the ability to structurally align the two domains and
successfully transfer. Specifically, this research investigated the effects of concrete versus
generic instantiations on learning, transfer, alignment, and internal representation.
Effect of concreteness on transfer

The present series of studies built on previous research conducted by Sloutsky, Kaminski, and Heckler (2005) that investigated the effect of irrelevant concreteness on learning and transfer of learning across isomorphic domains. In particular, the goal of the study was to consider transfer between two domains, one generic such as mathematics and the other concrete such as a scientific isomorph. Other researchers have found more transfer from mathematics to physics than the reverse (Bassok & Holyoak, 1989), suggesting transfer is more likely to occur from a generic domain to a concrete one than from a concrete to a generic domain. However, prior knowledge, experience, and expectation about mathematics and science might have influenced transfer performance. For example, freshmen college students have generally learned more formal mathematics than formal physics. Similarly, it is possible that students may have an expectation that knowledge and strategies of mathematics can be applied to solve physics problems, but physics information may not be applicable to mathematics. To minimize such potential confounds, two artificial domains were constructed that shared structure. One domain was generic and intended to mimic mathematics, while the other domain was more concrete and was intended to resemble an isomorphic scientific situation.

Both domains were commutative mathematical groups of order 3, each being isomorphic to the integers under addition modulo 3. Concreteness of the materials was manipulated by increasing the perceptual richness of the symbols denoting the elements of each domain. The generic domain (hereafter “Math”) involved three types of symbols and was intended to mimic the symbolic, generic nature of pure mathematics. This domain was presented as a language in which symbols combine to yield a resulting
symbol. Expressions were sentential statements. The concrete domain (hereafter “Science”), was intended to resemble a scientific phenomenon. Images of 3-dimensional objects from three classes dynamically moved toward each other and merged to form a resulting object.

In a pair of experiments, undergraduate students were trained and tested isomorphically in both domains. In the first experiment, half of the participants learned Math first and then Science; and half of the participants learned Science first and then Math. Transfer was measured by comparing average test scores on a given domain as a function of prior learning of the other domain. In other words, transfer due to Math was taken to be the difference in mean Science score between the students who studied Math first and those who did not. Similarly transfer due to Science was the difference in Math scores for the two groups of students. Test questions probed students’ ability to apply learned knowledge to complex, novel problems.

Results of the study were that the mean Science score was significantly higher after learning Math, but there was no difference in mean Math score due to learning Science. In other words, the students who learned Math first were able to transfer their knowledge of the concept to help them learn Science. However, knowledge of Science did not facilitate acquisition of Math. Therefore, it seems that domains that represent elements using perceptually sparse, generic symbols facilitate transfer to more concrete, isomorphic domains.

The results of a second experiment further supported this argument. The generic symbols of the Math were replaced with very colorful and interesting images of real objects. Now the Math was more concrete than the Science because both were
perceptually rich, but the Math involved recognizable objects, a vase, a ring, and a dagger. Unlike the shapes involved in the Science, these objects have labels and likely carry affordances. The results of this experiment revealed a reversal of the transfer pattern between Math and Science. Learning Science first helped students learn the Math, while learning Math did not help students learn the Science. These results can be interpreted as transfer is more likely to occur from a less concrete domain to a more concrete domain than the reverse.

Not only was concreteness found to hinder transfer, it also hindered learning. In a third experiment, students learned Math. The perceptual richness of the symbols was varied across conditions. There were four experimental conditions in which students learned with symbols that were black, colorful and patterned, classes of colorful and patterned symbols, or classes or real objects. Test scores were significantly higher for students who learned the topic using the generic, black symbols than for the students who learned with perceptually rich symbols. Additionally there was no difference in performance among the perceptually rich groups. The students who learned with classes of objects performed no lower than those who learned with only three colorful, patterned symbols. In summary, both transfer and learning were facilitated when elements of a concept were represented using perceptually sparse, generic symbols.

Similar hindering effects of concreteness on transfer have also been demonstrated by other researchers. Goldstone and his colleagues have conducted a series of transfer studies in which college students learned instantiations of complex adaptive systems including simulated annealing and competitive specialization (Goldstone & Sakamoto, 2003; Goldstone & Son, 2005). For example, students learned strategies of competitive
specialization involving ants foraging for food and then were tested on their ability to apply learned strategies to an analogous letter-pattern recognition situation. Students learned about the ant foraging using either a concrete external representation involving images of ants and apples or an idealized representation in which ants were represented by dots and food by irregular shapes. Students freely explored computer simulations and then were tested on their ability to apply the learned strategies to the new pattern recognition domain. Poorer-performing students transferred more with idealized representations than concrete representations (Goldstone & Sakamoto, 2003). In subsequent studies, the representation of elements switched midway through the learning phase from concrete to idealized or vice versa. The best transfer occurred when originally concrete elements became idealized (Goldstone & Son, 2005). Goldstone and Son conclude that “concreteness fading” or progressive idealization of concrete elements is an effective way to promote transfer.

Therefore, there is evidence suggesting that concrete instantiations hinder transfer. However, these studies involved concreteness that either hindered learning (Sloutsky et al, 2005) or did not significantly facilitate it (Goldstone and Sakamoto, 2003; Goldstone & Son, 2005). In addition, Goldstone and his colleagues investigated effects of varying concreteness of an external representation of a fixed instantiation, not the effects of the instantiations themselves. It is possible that the hindering effects of concreteness are limited only to instantiations that do not significantly facilitate transfer. If this is the case, then concrete instantiations that significantly facilitate learning would also facilitate transfer. It is possible that some choice of elements and storyline might help communicate the to-be-learned conceptual rules. In this case, concreteness would be
relevant to the given concept, as opposed to the concreteness of the previously discussed study was irrelevant.

If the hindering effects of concreteness are due to factors other than learning, such as difficulty recognizing an isomorph or difficulty aligning common structure between domains, then relevant concreteness may also hinder transfer. If relevant concreteness does indeed hinder transfer, there are several questions that should be addressed. The present series of studies addressed the effects on learning, transfer, and internal representation of instantiations that were relevantly concrete, irrelevantly concrete, or generic.

**Research Questions**

1. What is the effect on learning of relevantly concrete, irrelevantly concrete, and generic instantiations of a simple mathematical concept?

2. What is the effect of relevantly concrete, irrelevantly concrete, and generic instantiations on transfer to a novel isomorph? Are students who learn a concrete instantiation able to structurally align it with a novel isomorph?

3. How does providing the correspondence of analogous elements across learning and transfer domains affect transfer?

4. What is the nature of the internal representations constructed during learning with relevantly concrete, irrelevantly concrete, and generic instantiations? In particular, does the constructed representation allow learned structure to be recognized when expressed with novel elements?

5. What is the effect on transfer of learning more than one instantiation?
How do relational diagrams affect learning, transfer, and schema abstraction?

Relational diagrams are external representations that perceptually symbolize aspects of the relevant relations of the system and not only the elements.

How does relevant concreteness affect children’s ability to learn and transfer?

**Theoretical Framework**

The present research was developed with a basic framework that proposes the following. Transfer of conceptual structure is analogy. Analogical reasoning from a known to a novel domain involves a process of alignment of relational structure between the two domains as proposed by Gentner’s Structure-Mapping Theory (Gentner, 1983, 1988). Evidence of structural alignment is reflected not only in transfer performance, but also formation of element correspondences and similarity ratings. The concreteness of domains affects alignablity, as evidenced in studies of the process of comparison (Markman & Gentner, 1993).

**Analogical Reasoning and Structure Mapping**

According to Gentner’s Structure-Mapping Theory (Gentner, 1983, 1988), similarities and differences of situations are determined through an alignment of structure. Representations of the two systems or scenes are matched in a manner to maximize structural correspondence. Structures are aligned so that preferred matches satisfy two constraints: (a) one-to-one correspondence by which relevant elements of one system correspond to relevant elements of the other and (b) parallel connectivity by which arguments and further related elements of matched elements also have counterparts in the other system. Although there may be many different alignments of the
two systems, the *systematicity principle* states that the mapping with the broadest and deepest relational matches is preferred. According to the theory, correspondences exist between elements, but it is the structure that is actually mapped across systems.

Successful analogical reasoning depends on recognition of common relational structure. People, especially novices and young children, tend to focus on surface features of a system or display more than structure (Ben-Zeev & Star, 2001; Gholson et al., 1997; Holyoak, Junn, & Billman, 1984; Holyoak & Koh, 1987; Novick, 1988; Schoenfeld & Herrmann, 1982; Sloutsky & Yarlas, 2000).

*The Relational Shift*

Gentner (1988) proposed that a “relational shift” occurs in development. Early in development, individuals attend to surface features or elements of a given system. Through the course of development, a shift occurs toward a focus on relational structure. Most researchers agree that a relational shift occurs from observation of similarities between elements to awareness of similarities between relations. However, there has been some dispute over the nature of the shift. In particular two causal explanations are proposed. Gentner and her colleagues (see Gentner, Ratterman, Markman, & Kotovsky, 1995) posit that the shift is knowledge-driven. When explicit knowledge of a relation is gained, the ability and tendency to perceive the relations also increases. In familiar domains even preschool children attend to relations and demonstrate successful analogical reasoning (Alexander, White, & Daugherty, 1989; Brown, 1989).

An alternative explanation for the cause of the relational shift is proposed by Halford (Halford et al., 1995; Halford et al., 1998a; Halford et al., 1998b) who argues that the relational shift is due to an increase in processing capacity. Young children lack the
cognitive capacity which would allow them to perceive relations and relational similarities.

Halford’s argument does not account for the fact that the relational shift occurs at different times in different domains. It will be mentioned in Chapter 2 that 4-year-old children are capable of responding to relations in analogies in familiar settings (Gentner, 1988; Brown & Kane, 1988), while failing to do so in other contexts (Gentner, 1988; Kotovsky & Gentner, 1996). Furthermore, across a variety of domains including chess (Chase & Simon, 1973), physics (Chi, Feltovich, & Glaser, 1981), and mathematics (Bassok, 1997; Novick, 1988; Schoenfeld & Herrmann 1982; Sloutsky & Yarlas, 2000), experts represent structural aspects of problems or situations, while novices focus on surface features. Knowledge appears to be a major driving factor behind analogical reasoning ability and not necessarily developmental cognitive capacity (Brown, 1989; Gentner et al., 1995). Metacognitive competences such as learning strategies and a disposition to reflect on thinking (Brown, 1989) are likely other contributing forces.

Therefore, the ability to perceive relational structure appears to depend on the familiarity of the domain. In addition, superficial features of the domain may also affect structural recognition.

**Structural Recognition and Alignment**

Evidence of the effect of superficial information on alignment of structure can be found in research on the process of comparison. Gentner and her colleagues (Gentner & Medina, 1998; Kotovsky & Gentner, 1996; Markman & Gentner, 1993; Goldstone, Medin, & Gentner, 1991) have argued that the process of comparison involves alignment of structure between the two compared external representations. They have investigated
the manner in which participants align structures by asking them to match an element from one visual display to an element of an analogous visual display. The displays involved relations such as monotonic increase of shape sizes or actions between people such as giving food to someone. Among the possible matches were the structurally analogous element and a perceptually similar element. The prevalent response to these one-shot mapping tasks was the perceptually similar element. Adults and children tend to overlook the relational match indicating that structural alignment did not occur.

When study participants were first encouraged to compare the two displays by asking them to rate their similarity, they were more likely to match elements to their structurally analogous counterparts than were participants who were asked to match elements without prior comparison (Markman & Gentner, 1993). Therefore, comparison leads to structural alignment through which common relations become more salient and in turn can support future analogical comparisons (Gentner & Medina, 1998; Kotovsky & Gentner, 1996; Markman & Gentner, 1993).

It is not only domain-specific knowledge and the comparison process that underlie recognition and alignment of common structure. The nature of the two compared stimuli also plays a role. In particular, Markman and Gentner (1993) found that study participants were much less likely to make relational matches across two displays when the elements involved were very perceptually rich as opposed to when the elements were perceptually sparse. Thus perceptually sparse stimuli increased the alignability of the two displays.

Goldstone, Medin, and Gentner (1991) suggest that during similarity judgments, there is competition between superficial features and abstract features. They describe the
competition as separate pools for abstract, relational similarities and for superficial similarities. As one pool gets larger, it attracts attention toward itself and away from the other pool. This competition between relational structure and surface features may not only affect judgments of similarity of two domains, but also other tasks which require structural alignment including transfer across isomorphic domains. Goldstone and Sakamoto (2003) suggest that this competition between structure and superficial features is responsible for differential transfer performance. Concrete representations can distract from structure resulting in lower transfer for concrete than idealized representations.

Thus, there are reasons to suggest that concreteness, relevant as well as irrelevant, may hinder transfer of conceptual knowledge. If relevant concreteness is shown to hinder transfer, it is important to investigate why. Do relevant and irrelevant concreteness hinder transfer in the same manner? In particular, can the learner structurally align the domains? Can the learner recognize common structure in an isomorph? Additionally, does learning more than one concrete domain have benefits for transfer? Finally, do diagrams that communicate global structure facilitate transfer and schema abstraction? These questions were addressed in the present series of experiments in which undergraduate college students learned instantiations of a mathematical group that varied in concreteness. Participants were then presented with a novel isomorphic transfer domain. They were tested in the transfer domain and then asked to match elements and rate similarity of the domains to obtain additional indicators of whether common structure was aligned.
CHAPTER 2

LITERATURE REVIEW

Effective Symbols

While concreteness has been shown to hinder transfer of complex concepts (Goldstone & Sakamoto, 2003; Goldstone & Son, 2005; Sloutsky et al, 2005), it has also been shown to affect reasoning involved in simpler transfer tasks. Symbol use can be construed as a process of transfer. For example, to effectively use a map as a symbol for a real location, one must transfer the perceived relations between entities on the map to their real-world analogs. Adults generally do not have difficulty interpreting maps or using other simple symbols. For children, this ability is not necessarily in place; it develops. Furthermore, it is affected by the concreteness of the potential symbols.

DeLoache and her colleagues (DeLoache, 1995a, 1995b, 1997, 2000) demonstrated that young children have difficulty using concrete and perceptually rich objects as symbols. DeLoache proposes that to use an object as a symbol, one must achieve dual representation. That is, one must attain “representational insight” where the object is represented both as the object itself and as a reference to its intended referent. Dual representation involving concrete objects is difficult for young children in part because of a conflict between the symbolic and non-symbolic uses of the object.
To explore the acquisition of dual representation, DeLoache investigated the ability of 2½ to 3-year-olds to use objects symbolically. The experimenter showed children a 3-dimensional scaled model of a real room and told them that a stuffed animal was hidden in the actual room. The experimenter then placed a miniature toy in the model telling the children that the location of the miniature toy in the model corresponded to the location of the actual toy in the real room. The children were then asked to retrieve the real toy. Only 16% of the children were able to make errorless retrieval of the actual toy. The children were then asked to retrieve the miniature toy. The accuracy of the miniature toy retrieval was 88% implying that the poor performance on the retrieval of the actual toy is not due to inability to remember the location, but an inability to realize that the model symbolically represented the actual room. In subsequent studies, the salience of the model was decreased by putting it behind a glass window. Under this condition, more than half of the participants accurately retrieved the toy. Similarly, when children were shown the location in a picture and not a 3-dimensional model, 80% of participants ably retrieved the real toy. In sum, decreasing the salience of the object increased the ease of its symbolic use.

By 3 years of age, most children are successful in such a task. However, when the 3-year-old study participants were encouraged to play with the model first only 44% of them successfully retrieved the toy, compared to 78% of 3-year-olds who retrieved the object with no opportunity to play. The physical interaction with the model made it more difficult for the children to treat it as a symbol. In summary, dual representation is not achieved in a stage-like, all or nothing, manner. Instead, it depends on the particular stimuli and situation. Objects with a high level of physical salience are more difficult for
young children to treat symbolically. A lower level of salience leads to better performance.

Concreteness does not only affect children’s reasoning; adults also tend to reason about pictures differently based on their degree of concreteness (Schwartz, 1996). In a study conducted by Schwartz, participants were shown drawings of a hinge with marks on the top and bottom boards. They were asked to judge whether the marks would meet if the hinge was closed. Some participants were shown detailed, realistic drawings of the hinge with a string attached to one end being held by a hand. The others were shown an abstract representation of two line segments meeting at an enlarged circular vertex point. Depending on the type of representation viewed, participants reasoned differently when drawing a conclusion. People who saw the detailed drawing tended to simulate the movement of the hinge closing, while those who saw the abstract representation were inclined to solve the problem by geometrically comparing the length of the subsegments of each side of the boards. Schwartz concluded that pictures with high fidelity to their referents lead people to think about those referents. With lower fidelity, people are more likely to think of the picture as a symbol. Therefore, concreteness affects adult reasoning as well as that of children.

In sum, concreteness of objects hinders children’s ability to use them as symbols. In the course of development children attain this ability. However, it is unclear whether older children and adults can overcome the obstacle of concreteness when attempting to transfer more complex relations such as mathematical concepts (see also Uttal, Liu, & DeLoache, 1999).
Analogical Reasoning

In effective analogical reasoning, the source domain is often familiar, well-understood, or previously learned while the target may be new. Specifically, analogies are the vehicles for advancement of thinking in the following ways: (1) acquisition of knowledge (Vosniadou, 1989), (2) conceptual change and innovative thinking (Gentner et al., 1997), and (3) problem solving (Holyoak & Koh, 1987).

The literature on analogy is extensive and ranges across age and complexity of task. Studies have considered interpretation of simple metaphors (Gentner, Ratterman, Markman, & Kotovsky, 1995) to transfer of complex systems (Goldstone & Sakamoto, 2003). The following sections will present a more detailed review of the literature than was discussed in Chapter 1. This discussion will be organized by research addressing the following subprocesses of analogical reasoning: (a) representation of the target domain, (b) retrieval of prior analogous base domain, (c) alignment and mapping of structure, (d) implementation of the analogy, and finally (e) schema formation.

Representation – Recognition of relational structure throughout development

For successful analogical reasoning, one must recognize relevant relational structure. There appears to be a relational shift over the course of development from focus on superficial features or elements of a representation to attention to relations (Gentner, 1988; Gentner et al., 1995). Gentner (1988) investigated children’s and adults’ interpretation of metaphor. When given a metaphor that could be interpreted in either an attributional or relational manner, adults tend to prefer the relational metaphor, while young children prefer the attributional interpretation. For example, given the metaphor a
plant stem is like a straw, the adult interpretation is that both carry water. A typical
response from a child is that both are thin and straight.

Gentner proposed that the relational shift from attention to superficial features to
attention to relations is a function of domain knowledge. As knowledge of domain-
specific relations increases, the ability to recognize and attend to relations also increases
(Gentner, 1988; Gentner et al., 1995). In familiar domains, young children can
demonstrate analogical reasoning. For example, when 4-year-olds were shown a picture
of a tree and asked, “If a tree had a knee, where would it be?”, they interpreted the
relational correspondence and responded as accurately as adults. Brown and Kane (1988)
conducted a study of preschool children, aged 3 to 5 years. Children learned problem-
solving strategies presented to them through example problems. The problems involved
simple biological mechanisms such as mimicry and camouflage. For example, using lady
bugs to control aphids is analogous to using purple martins to control mosquitoes. Results
indicate that young children can select appropriate solution strategies and apply them
correctly to solve analogous problems. Three-year–old children benefit from reflection,
such as discussion. Four-year-olds needed no hints to effectively transfer, especially in
the context of similar elements across domains such as mosquitoes and aphids. The
performance of five-year-olds was near ceiling.

In many situations, even adults tend to focus on elements over relations. Gentner and
Kotovsky and Gentner (1996) conducted a study of simple similarity comparison in
adults. Adults were shown two displays, \( A \) and \( B \), of three circles each arranged in a
monotonically decreasing pattern. The middle sized circle of configuration \( A \) was the
same size as the largest circle of configuration \( B \). Adults were asked to make a one-shot
mapping of this middle object in display $A$ to an object in display $B$. The experimenter pointed at the middle object in display $A$ and asked the participant to select the object in the display $B$ that best went with the indicated object. Most participants chose the circle of the same size. In other words, they matched on the basis of elements and not relations which would have matched middle circle to middle circle. Therefore, even when given a very simple task, the tendency of most adults is often to attend to elements over relations.

Ability to attend to relations and reason analogically appears to be assisted by knowledge of the domain and similarity of the corresponding elements. Furthermore, there is evidence that the process of comparison promotes recognition of abstract commonalities between representations which in turn can assist analogical reasoning. In the Kotovsky and Gentner (1996), when adults were first asked to rate the similarity of the two displays of monotonically decreasing circle and then choose the matching element in the second display, most selected the relational match. The task of rating the similarity forced them to make a comparison of the two configurations.

The benefit of comparison for highlighting relations was also demonstrated for children (Kotovsky & Gentner, 1996). Children 4 years of age were able to make relational matches across displays of geometric shapes involving symmetry, monotonic increase or decrease. Older children of age 8 were able to recognize higher-order relational matches across different dimensions. For example, they recognized the common relation between a display of increasing size and a display of increasing color saturation.

Comparison also promotes adults’ ability to transfer knowledge on complex tasks (Gentner, Loewenstein, & Thompson, 2003). When adult participants first compared and
noted similarities of two stories that involved the use of the same negotiation strategy, they were more successful transferring the strategy to a novel situation than were participants who read and summarized the stories separately.

In addition to knowledge and comparison of exemplars, relational language promotes attention to relations. By labeling the elements of a triad configuration “baby”, “mommy”, and “daddy”, preschoolers’ ability to make relational matches improved significantly (Gentner, Rattermann, Markman, & DeLoache, 1995). Both physical juxtaposition and symbolic juxtaposition invite comparison (Gentner & Loewenstein, 2002). Through physical juxtaposition, one observes exemplars. Symbolic juxtaposition occurs through the use of relational language that invites the comparison process. In either case, the resulting comparison process promotes attention to relations.

While children and adults often overlook simple common relations, they can recognize them in the context of relational language or comparison. In complex domains such as mathematics and science, the tendency to focus on surface features over relational structure is greater than for simple concepts. Seminal research conducted by Chi, Feltovich, and Glaser (1981) demonstrated that physics experts represent primarily abstract physics principles, while novices represent superficial features. This conclusion was made by examining the manner in which experts and novices sorted physics problems. The pattern of expert representation of structure and novice representation of surface features has been demonstrated in other domains including chess (Chase & Simon, 1973) and mathematics (Bassok, 1997; Novick, 1988; Schoenfeld & Herrmann 1982; Sloutsky & Yarlas, 2000).
Sloutsky and Yarlas (2000) investigated the representation of arithmetic equations by novices and experts. Undergraduate students and mathematics graduate students participated in a recognition study. Participants were asked to study 30 equations reflecting the associative and commutative properties. Shortly later they were presented with “old” and “new” equations and asked to judge whether or not they had studied them earlier. Some of the new equations shared either common relations or common elements, namely number, with previously studied equations. Both experts and novices performed accurately on the recognition task, indicating that they encoded and accessed both surface features and relational features of the equations. However, when study time was shortened, the accuracy of novices dropped. The pattern of accuracy suggested that with shortened encoding time, novices encoded surface features and not relational principles. Therefore in some situations, novices may represent elements and not relations. Experts, however, did not drop in accuracy with shorter study time, indicating that their representations reflected both elements and relations.

The findings of further studies indicated that children of age 8 and 11 have a preference for focusing on superficial aspects of arithmetic equations and not relations (Yarlas & Sloutsky, 2000). When asked to match equations, children tend to form matches based on superficial features such as the same number of terms and not presence of mathematical principles such as associativity and commutativity. Yet, when the surface features did not compete with relations, the older children were capable of noticing common relations between equations.

In sum, there is a variety of studies indicating that children as well as adults spontaneously represent superficial aspects of a domain over relational structure. Yet
when relations are familiar, people often do attend to them, particularly in the absence of salient competing element commonality or during structural alignment that occurs with comparison. These results suggest that when learning a novel concept, a generic instantiation may have benefits over a concrete representation because the generic has fewer salient superficial features.

Retrieval

When presented with a novel domain, successful transfer of conceptual knowledge requires retrieval of a previously learned isomorphic domain. As mentioned in Chapter 1, spontaneous transfer often does not occur (Gick & Holyoak, 1980). Yet when study participants were given a suggestion that a previously learned solution could be applied to the novel problem, the majority successfully applied the solution to the novel problem. Thus transfer failures are often due to failure to retrieve a previously learned analogue.

Retrieval of an appropriate learned domain is facilitated when the novel and learned domains share surface similarities. Gentner, Rattermann, and Forbus (1993) conducted a study in which participants were asked to read a large number of stories. Later they were given a probe story that resembled the original stories through surface features or relational analogy. They were then asked to write any of the original stories that came to mind. The results indicated that participants’ ability to retrieve stories was higher when the surface similarity was high. However, when participants were also asked to rate the inferential soundness of the similarity of the old and new stories, their judgments were based on relational similarity of the stories and not surface features. Therefore, high similarity judgments can suggest an awareness of common relational structure.
Surface similarity affects more than retrieval of previous knowledge. Ross (1987, 1989) examined the effects of different types of superficial similarities on both access and use of previously learned problems. Undergraduate students studied methods of solving probability problems involving combinations and permutations. Training presented formulas and solved examples. Test questions were constructed to be similar or dissimilar to training questions either through storyline or elements. To determine the specific effect of elements, some test questions presented cross-mapping of elements. In other words, elements of the training questions were given different roles in the test questions. For example, one problem was to compute the probability that three given mechanics from a group of size eight would work on the cars of three specific people from a group of size eleven. In the training example, the mechanics randomly chose the cars. The test question involved a similar storyline, but the customers randomly chose mechanics.

Comparison of average performance on test questions across conditions was used to measure the effect of different types of similarities. The effect of storyline similarity was determined by measuring the difference in performance between students who received problems with similar storylines and those who received problems with dissimilar storylines. In the same manner, the effect of element similarity was measured. Furthermore, access and use of prior examples was determined by giving or withholding appropriate formulas during testing. Access of solution strategy was indicated when a student used a formula that was not explicitly provided. When a formula was explicitly given to the students, the manner in which they used similar elements could be observed.
Storyline affected access of prior problems. Appropriate formulas were more likely to be recalled when the test questions had a storyline similar to that of the training examples. However, similar storyline did not facilitate the actual use of formulas. Many participants recalled the appropriate formulas, but did not correctly implement the solution strategy. On the other hand, similar elements did affect the use of solution formulas. Correct solutions were more often stated when elements played roles corresponding to their roles in the training examples. When elements were crossed-mapped from training to testing, performance dropped significantly. Participants were very likely to place elements in the roles they held in training questions regardless of whether or not this was appropriate for test questions. Therefore, similar superficial features can improve access of prior solution strategies. In addition similar elements tend to affect alignment of structure and implementation of solution strategies.

Spurious correlations between a base and target domain can influence what is retrieved. Even experienced students can be misled by irrelevant surface features of problems. Ben-Zeev and Star (2001) investigated what aspects of learned mathematical problems were encoded and later used to solve novel problems. Undergraduate students from Yale University who scored at least 700 on the mathematics portion of the SAT were taught different algorithms to solve mathematical problems. Elements of the problem were deliberately correlated with specific algorithms. Each study consisted of two phases. The first phase was an incidental training phase where participants were asked to rate the difficulty of quantitative comparison problems to be given to high school students. The quantities presented were rational expressions involving radicals or logarithms. Two different solution strategies were given for specific problems. The
problem solution strategies were correlated spuriously with irrelevant surface features. Specifically, multiplying one expression by \( \frac{n}{n} \) was associated with radicals in the expression; and multiplying both quantities by \( n \) was correlated with the presence of logarithms. Testing involved one of three possible decision tasks. In the first experiment, participants were shown various problem-algorithm pairs and given a yes/no recognition tasks. Participants were also asked to sort problem-algorithm pairs into meaningful groups. In a subsequent experiment, students were to suggest a solution strategy.

The results of all experiments showed a pattern of responses indicating that participants focused on the superficial features. They grouped radicals and logarithms with the correlated algorithms from the training phase; they also false-alarmed on problems-algorithm pairs with the same type of associations. Even experienced problem solvers encode spurious correlations during the process of learning. Furthermore, choices of solution strategies may be affected by the surface features of the training problems.

In sum, retrieval of an isomorph does not necessarily occur spontaneously. In the course of learning, people encode superficial information. When a novel domain shares surface aspects with a previously learned isomorph, the likelihood of retrieval increases.

The goal of the present research was to investigate the effect of instantiation on transfer. Because spontaneous transfer is notoriously difficult, participants of the present studies were explicitly told that the previously acquired knowledge can help them in the transfer domain. In addition, because degree of superficial similarity between learned and novel domains can influence transfer, the transfer domain was designed to be concrete and equally similar on the surface level to the different instantiations under consideration.
Alignment and Mapping of Structure

According to Structure- Mapping Theory of Analogy (Gentner, 1983, 1988), once a previous system has been retrieved, the two systems are aligned such that the elements of each are placed in a one-to-one correspondence and structure is mapped from the learned to the novel system. This process of alignment and mapping has also been shown to contribute to incorrect analogical reasoning. In particular, interference effects and interpretation effects underlie misalignment and incorrect mapping (Bassok, 1997, 2001).

The retention of content in the representation of the base and target domains can interfere with analogical reasoning. People have a tendency to place similar elements in the base and target domains in the same structural roles. In the previously mentioned studies conducted by Ross (1987, 1989), participants placed common elements in the same roles they held in the base domain regardless of whether or not this was appropriate. Elements that are crossed-mapped across domains can often interfere with correct alignment and mapping of structure.

This interference effect of cross-mapped elements was also demonstrated on simple comparison tasks with young children (Gentner, Rattermann, Markman, & Kotovsky, 1996). When preschoolers were presented with two displays of three objects arranged in monotonically increasing or decreasing manner and asked to choose which object in display B had the same relative size and position as an indicated object in display A, children were generally able to do so when the stimuli were perceptually sparse flower pots. However when perceptually rich stimuli, such as coffee mugs and toys, were cross-mapped, children did not make the relational choice. Instead they chose the element of the same identity.
Clearly similar or identical elements can interfere with correct alignment and mapping of structure. Analogical reasoning can also go awry because of misinterpretation of structure. Bassok (1997) explains that people often use content of base and target domains to interpret structure. In particular, better students make use of content to interpret structure. When content and structure are correlated, this is an effective approach. For example, given a problem involving students and teachers, people are more likely to consider the ratio of students per teacher than the ratio of teachers per student. However, content is not always correlated with structure. And while relevant structure is present in all isomorphic systems, irrelevant aspects of structure are not necessarily correlated in both. Students can often have difficulty distinguishing relevant from irrelevant structure. Without knowing what structure is relevant, analogical reasoning and transfer of solution strategies are bound to fail.

Bassok has proposed that misinterpretation of relevant structure is the cause of the asymmetric transfer of solution strategies between algebra and physics. Bassok and Holyoak (1989) examined transfer between algebraic knowledge and physics knowledge, namely between arithmetic-progression problems and isomorphic constant-acceleration problems. High school and college students, who were unfamiliar with both of these domains, learned one of these topics and then were given word problems involving the other topic. The measure of transfer was whether the learned method had been applied to the structurally isomorphic problems in the unstudied domain. Students who had learned arithmetic-progression first easily and spontaneously applied the learned method to correctly solve constant-acceleration problems. At the same time, the students who learned the physics topic showed essentially no transfer of method to the arithmetic-
progression problems. Bassok suggests that students misinterpreted the relevant structure of the physics problems not recognizing the analogy to the algebra problems. Specifically, students might have focused on the disconnect between continuous acceleration and discrete arithmetic progression problems.

In another study, undergraduate students were taught solutions to problems involving continuous change such as speed change or discrete change such as monetary investment (Bassok & Olseth, 1995). Then the students were asked to solve problems with analogous solutions. The target problems involved entities that either changed continuously (i.e., melting ice) or discretely (e.g., deliveries of ice to a restaurant). Students who learned discrete change problems easily transferred solutions to continuous change problems. However transfer from the continuous-change to discrete-change problems was rare. When irrelevant structural aspects are included in the representations of base or target domains, the alignment of relevant structure often fails.

In sum, alignment of two isomorphic domains is often difficult. Learners can misinterpret structural relevance. In addition, common elements often drive the alignment process. The process of recognition and alignment of common structure was of particular interest in the present research. Performance on a test in a novel isomorphic domain was taken as indirect evidence of structural alignment. In addition, participants were asked to match elements across the learned and novel domain. Ability to match analogous elements was taken as more direct evidence of structural alignment.

Implementation

Appropriate representation and successful alignment and mapping of structure are necessary but not sufficient for successful analogical reasoning. Novick and Holyoak
(1991) conducted a study in which undergraduate students were given problems and solution and asked to transfer the solution to an analogous problem. The goals of the problems were to determine set sizes; the solutions involved finding the least common multiple of subset orders involved. The possible problem storylines were number of vegetables in a garden, number of students in the band, and the number of seashells in a collection. Students were given one of two levels of hints. One group of students received a hint that the new problem can be solved in the same manner as the previous problem. This hint effectively eliminated the retrieval process for the students. Another group received this hint along with directions to align corresponding numbers in the problems. For example, failing to accommodate 2 plants in the garden problem was the same as failing to accommodate 1 student in the band problem. In effect, this more detailed hint provided students with alignment and mapping of structure. Results were that students who received hints were more likely to transfer the solution strategy than students who received no hint. Students who received the more detailed alignment and mapping hint performed better than those who received the simple suggestion hint. However, receiving the alignment and mapping hint did not guarantee transfer; only 50% correctly transferred the solution strategy.

The present research attempted to isolate the effect of instantiation type on implementation by comparing both ability to align corresponding elements and transfer test scores. If two instantiations result in correct element alignment but differential transfer test scores, then these instantiations affect analogical implementation differently.
Schema Formation

As a result of experience with multiple analogues and successful transfer, conceptual knowledge may be represented in a more abstract form, such as a schema which in turn can facilitate subsequent transfer (Reed, 1993). A schema is a cluster of interrelated knowledge representing a particular generic procedure, object, percept, event, or situation (Reed, 1993). A schema is a skeleton structure of a concept that can be instantiated by filling the placeholders with specific elements. Novick and Holyoak (1991) analyzed students’ descriptions of their solution procedures for the presence of schema formation. They concluded that schema induction is a natural consequence of analogical transfer. Furthermore, the schema was found to coexist with the problems from which it was induced.

Schemata are believed to underlie experts’ ability to categorize problems by the structure (Chi, Feltovich, & Glaser, 1981; Schoenfeld & Herrmann, 1982) as well as remember large amounts of information, in their domain, after only a brief presentation (Chase & Simon, 1973). It seems that schemata development is an important aspect in the development of expertise. Furthermore, multiple analogues and explicit instructions to compare them can enhance schema induction (Gick & Holyoak, 1983).

Finally, an abstract schema does not appear to supplant individual exemplars. The abstract and specific knowledge coexist. This agrees with the nature of abstract thought described by Medin and Ross (1989). Reasoning is often case-based, influenced by specific examples and not purely by abstract schema. Induction is conservative in the sense that considerable information from specific instances is stored in the conceptual representation.
Successful transfer and generic descriptions of learned solutions are indirect evidence of schema formation. The present research involved a more stringent test of schema existence. If participants did form a schema, then they may be more likely to modify the structure when trying to transfer to a novel nonisomorphic domain of the same mathematical structure than would participants who were using another form of reasoning such as case-based reasoning.

**Diagrammatic Reasoning**

The majority of the present discussion has focused on the effect of instantiation and concreteness on transfer of conceptual knowledge. The manner in which an instantiation is externally represented also affects transfer. In particular, the perceptual information available in the representation can affect what the learner does. Zhang (1997) argues that external representations are not merely stimulus to the mind; they guide, constrain, and even determine cognitive behavior (see also Zhang and Norman, 1994). To investigate effects of external representations on problem solving, college students learned isomorphs of Tic-Tac-Toe. The goal of the first isomorph was to choose three numbers whose sum is 15. The second isomorph involved choosing circles containing common shapes. The third involved selecting circles with common colors. Participants played one type of game against a computer. One of the significant differences between the games is that the number isomorph required more than simply perceptual observations such as shape and color comparisons. In the number game, players needed to perform addition calculations; internal representations needed to reflect the sum at different stages of the game.
Analysis of student behavior revealed that different representations of the same abstract structure caused different cognitive behaviors. Games involving shapes or colors elicited a more-is-better bias, while the number game elicited a larger-is better bias. When these biases were consistent with game goals, problem solving was easier. When bias and goals were inconstant, problem solving was more difficult. Zhang (1997) proposed “representational determinism”. When learning the abstract structure of a novel concept, the format of the representation can determine what information can be perceived, what processes can be activated, and what structures can be discovered. Representational determinism is primarily the result of two factors. First, external representations may contain invariant information that can be directly perceived without mediation. Second, external representations may directly activate prior knowledge.

One category of external representation that has been studied is diagrams. Larkin and Simon (1987) distinguished two types of external representations, sentential and diagrammatic. A sentential representation is a data structure in which elements appear in a single sequence. Expressions in sentential representations have a propositional form and correspond on a one-to-one basis to sentences in natural language. A diagrammatic representation is a data structure in which information is indexed by two-dimensional location. Diagrammatic representations can explicitly preserve the topological and geometric relations among the components of a system.

From an external representation, the learner constructs an internal representation. There is some debate about the nature of internal representations. Some researchers (Pylyshyn, 2002) have argued that internal representations are propositional, while others believe that visual images are analogues of their real-world counterparts. In other words,
visual images are pictures in the head (see Palmer, 1999). Barsalou (1999) challenges the notion that conceptual processing involves sequential processing of amodal symbols in propositional structures. Instead, Barsalou posits a perceptual symbol system that can function as a full conceptual system. The perceptual symbol system can go beyond mere recording of information to perform such functions as interpretation, categorization and inference. One implication of a perceptual symbol system is that when viewing a diagram, much information can be encoded and recognized at one time. The analogue image contains considerable information that does not necessarily need to be accessed sequentially. Furthermore, this chunk of information can function as a symbol itself. Therefore, diagrams have the potential to quickly and easily communicate information.

Larkin and Simon (1987) have analyzed different diagrammatic representations including a pulley system and a geometric construction of intersecting lines. They found that diagrams offer computational efficiency leading to advantages in the search, recognition, and inference processes. A search for information on a diagram can be restricted to a subset. Information can be recognized in both sentential and diagrammatic representations; however recognition from a diagram is often faster and easier than from a sentential representation. Typically, the same inferences can be drawn from either type of representation. However, diagrams may have the advantage of not requiring substantial computation. Because diagrams can make some information explicit, inferences can be made more easily.

Geometry is a subject with tremendous benefits from diagrams. Imagine a diagram of a line transversing two parallel lines. This representation easily conveys a lot of information. Labeling needs are minimized or eliminated. In fact, labeling would only be
required for producing a conventional proof in sentential form. Recognition of various aspects of the representation, such as the eight resulting angles, is quick and easy. Furthermore, many inferences are available at essentially no cost because they are, in a sense, perceptual (Larkin & Simon, 1987). We can “see” the congruency of opposite angles.

In summary, diagrams are computationally efficient, offering advantages over sentential representations in search, recognition, and inference processes. Effective diagrams do not only describe spatial arrangements. They can represent relational properties that offer “psychological utility” (Larkin & Simon, 1987) for problem solving. For example, graphs in economics and free body diagrams in physics are well used in these disciplines.

Not all diagrams are equally effective. Bauer and Johnson-Laird (1993) showed that diagrams that illuminated alternative possibilities improved students’ reasoning involving logical disjunction, while other diagrams did not improve reasoning. Pedone, Hummel, and Holyoak (2001) showed that diagrams that depicted multiple points of convergence in the radiation problem were used successfully to solve analogous problems. Furthermore, displays that animated the convergence were retrieved spontaneously more frequently than the static displays. The animation encouraged interpretation of the diagram as a helpful analogue and consequently enhanced transfer.

The present research considered the effect of a type of diagram referred to as a Relational Diagram. A relational diagram is similar to what Zhang has called “relational information display” (Zhang, 1996) which is a display that represents the relations between dimensions of the to-be-learned task in a manner such that the perceivable
information matches the required information of the task. This research proposes that a relational diagram is a display that represents the global structure of the system through the following: (a) perceptual symbol(s) for the relevant relation(s) that helps communicate the manner in which elements relate, (b) perceptual placeholders for elements, and (c) a minimal amount of extraneous information. Relational diagrams can provide the learner with a perceptual representation of the to-be-learned relations that may anchor behavior and possibly facilitate learning. Additionally because the diagram can symbolize the global structure of the system and not simply local rules in sentential format, learning from a relational diagram may be more likely to result in an abstract schema. The learner can possibly encode, access, and modify a single “image” of global structure. The hypothesis that a relational diagram can promote learning, transfer, and schema formation was tested in Experiment 6.
CHAPTER 3

METHODOLOGY

Basic Experimental Paradigm

The theoretical foundation of this research was that successful transfer requires recognition and alignment of structure which can be influenced by the concreteness of the initially learned instantiation. This framework is applicable to both adults and children. Experiments 1 - 6 involved undergraduate college students. However, children may be more affected by concreteness than adults as discussed in the previous chapter. Therefore, Experiment 7 investigated effects of concrete and generic instantiations on learning and transfer for fifth and sixth grade students.

Design

This research involved two basic types of experiments: Learning (Experiment 1) and Transfer (Experiments 2 - 7). Students learned the rules of an Abelian group of order 3. As in the previously conducted study (Sloutsky, Kaminski, & Heckler, 2005), the group was instantiated through artificially constructed domains. The type of instantiation learned was a between-subjects condition.

Experiment 1 investigated learning. Students were presented with the rules of a group and were then given a 24-question multiple-choice test (see Appendix A). Experiments 2 – 7 had two phases. Phase 1 was a learning phase in which students were presented with
the rules of a group and then were given a multiple-choice test. In Phase 2, participants were presented with a novel isomorphic transfer domain and asked a series of questions. The complete details will be discussed in the following chapters that present the individual experiments and will reference Appendices B – J. The basic design is illustrated in Figure 3.1.

![Figure 3.1 Phases of Experiments 2 – 7.](image)

**Material**

The specific conditions of a given experiment will be explained in greater detail in the chapters presenting the individual experiments. The possibilities are relevantly concrete, irrelevantly concrete, both relevantly and irrelevantly concrete, or generic. Relevant concreteness was manipulated through storyline and choice of symbols. For the relevantly concrete conditions, the storyline and symbols was designed to help communicate the concept. Perceptual richness added irrelevant concreteness. Colorful, patterned symbols were used in the perceptually rich, irrelevantly concrete conditions, while black symbols were used in the conditions with no irrelevant concreteness.

The structure of systems was that of a commutative group of order 3. In other words the rules were isomorphic to addition modulo 3. The idea of modular arithmetic is that only a finite number of elements (or equivalent classes) are used. Addition modulo 3 considers
only the numbers 0, 1, and 2. Zero is the identity element of the group and is added as in regular addition: \(0 + 0 = 0, \ 0 + 1 = 1, \) and \(0 + 2 = 2\). Furthermore, \(1 + 1 = 2\). However, a sum greater than or equal to 3 is never obtained. Instead, one would cycle back to 0. So, \(1 + 2 = 0, \ 2 + 2 = 1, \) etc. To understand such a system with arbitrary symbols (not integers as above) would involve learning the rules presented in the No Relevant Concreteness column.

<table>
<thead>
<tr>
<th>Elements</th>
<th>Relevant Concreteness</th>
<th>No Relevant Concreteness</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Rules of Commutative Group:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associative</td>
</tr>
<tr>
<td>For any elements (x, y, z):</td>
</tr>
<tr>
<td>(((x + y) + z) = (x + (y + z)))</td>
</tr>
<tr>
<td>Commutative</td>
</tr>
<tr>
<td>For any elements (x, y): (x + y = y + x)</td>
</tr>
<tr>
<td>Identity</td>
</tr>
<tr>
<td>There is an element, (I), such that for any element, (x): (x + I = x)</td>
</tr>
<tr>
<td>Inverses</td>
</tr>
<tr>
<td>For any element, (x), there exists another element, (y), such that (x + y = I)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specific Rules:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{} ) is the identity</td>
</tr>
<tr>
<td>(\text{} ) is the identity</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>These combine</th>
<th>Remainder</th>
<th>Operands</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{} )</td>
<td>(\text{} )</td>
<td>(\bullet \bullet )</td>
<td>(\text{} )</td>
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</tbody>
</table>

**Table 3.1** Stimuli and rules across domains.
of Table 3.1. However, a context can be created in which prior knowledge and familiarity may assist learning. In this type of situation the additional information is relevant to the concept.

To construct a condition that communicates relevant concreteness, a scenario was given for which students could draw upon their everyday knowledge to determine answers to test problems. The symbols were three images of measuring cups containing varying levels of liquid (see Table 3.1). Participants were told they need to determine a remaining amount when different measuring cups of liquid are combined. In particular, \( \text{ } \) and \( \text{ } \) will fill a container. So for example, combining \( \text{ } \) and \( \text{ } \) would have \( \text{ } \) remaining. Additionally, participants were told that they should always report a remainder. Therefore they should report that the combination of \( \text{ } \) and \( \text{ } \) would have remainder \( \text{ } \). In this domain, \( \text{ } \) behaves like 0 under addition (the group identity element). \( \text{ } \) acts like 1; and \( \text{ } \) acts like 2. For example, the combination of \( \text{ } \) and \( \text{ } \) does not fill a container and so \( \text{ } \) remains. This is analogous to \( 1 + 1 = 2 \) under addition modulo 3. Furthermore, the perceptual information communicated by the symbols themselves can act as reminders of the structural rules. In this case, the storyline and symbols may facilitate learning. Black symbols were used for the Relevantly Concrete/Perceptually Sparse condition (RC/PS) and colorful, patterned symbols for the Relevantly Concrete/Perceptually Rich condition (RC/PR).

The conditions with no relevant concreteness were presented to the participants as a symbolic language in which three types of symbols combine to yield a resulting symbol.
(see Table 3.1). Combinations were expressed as written statements. Again, the symbols were either black for the Not Relevantly Concrete / Perceptually Sparse condition (No RC/PS) or colorful and patterned for the Not Relevantly Concrete/ Perceptually Rich condition (No RC/PR).

Training and testing in all conditions were isomorphic and presented via computer. Training consisted of an introduction and explicit presentation of the rules through examples. For instance, participants in the relevantly concrete conditions were told that combining $\textcolor{black}{\text{□}}$ and $\textcolor{black}{\text{□}}$ has a remainder of $\textcolor{black}{\text{□}}$. Analogously, in the not relevantly concrete conditions where students were told that symbols combine to yield a resulting symbol the analogue to the above rule was presented as $\textcolor{red}{\text{◆}},\textcolor{red}{\text{◆}} \rightarrow \textcolor{black}{\text{●}}$. To illustrate a more complex combination, an additional example was given in which three operands combine.

After training, the participants were given a 24-question multiple choice test designed to measure the ability to apply the learned rules to novel problems. Many questions required application of multiple rules. The following are examples of test questions in the not relevantly concrete conditions. Participants in the relevantly concrete conditions saw the analogues of these questions.

(1) What can go in the blanks to make a correct statement?

$$\_\_, \textcolor{red}{\text{◆}},\_\_, \textcolor{black}{\text{●}} \rightarrow \textcolor{black}{\text{●}} \quad ?$$

(2) Find the resulting symbol:

$$\textcolor{red}{\text{◆}},\textcolor{black}{\text{●}},\textcolor{black}{\text{●}}, \textcolor{blue}{\text{□}} \rightarrow \_\_. $$
The Appendices present the complete training and testing for the different experiments and will be referenced specifically in the discussions of the specific experiments.

Procedure

For all experiments, training and testing was presented to individual participants on a computer screen using Super Lab Pro 2 software (Cedrus Corporation, 1999) in a quiet room. They proceeded through training and testing at their own pace. All participants responded by pressing buttons on the computer keyboard. Their responses were recorded.

Analysis

A quantitative analysis of results was performed for each experiment. Mean test scores and similarity ratings were compared across conditions using independent samples t-tests, analysis of variance, and analysis of covariance when appropriate. Chi-squared test were conducted to determine whether the proportion of participants who correctly matched corresponding elements were different between conditions.
CHAPTER 4

EFFECTS OF CONCRETENESS ON LEARNING AND TRANSFER

The purpose of the first experiment was to ascertain that the described “relevantly concrete” domain did indeed offer an advantage for learning over a generic or irrelevantly concrete isomorph. Students were tested after receiving only minimal training of one domain. Minimal training was given in order to potentially maximize performance differences between conditions.

EXPERIMENT 1

Method

Participants

Eighty-eight undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Twenty-two students were randomly assigned to each of four conditions that specified the type of representation they learned.

Material and Design

The experiment had a between-subjects design with two factors: Relevant Concreteness (Present, Absent) and Perceptual Richness (Present, Absent). Therefore, there were four
experimental conditions: Relevantly Concrete/Perceptually Rich (RC/PR); Relevantly Concrete/Perceptually Sparse (RC/PS); Not Relevantly Concrete/Perceptually Rich (No RC/PR); and Not Relevantly Concrete/Perceptually Sparse (No RC/PS).

As described in the general experimental paradigm, the participants were presented with one instantiation of a group (see Appendix A). Training in Experiment 1 was minimal because it consisted only of presentation of each of the rules and one additional example involving three operands. No practice problems or feedback were given. With minimal training, participants in the relevantly concrete conditions should have a greater advantage over participants in the not relevantly concrete conditions. If additional examples and practice were to be given, this advantage would be likely to decrease. After training, the multiple-choice test was given.

Procedure

All training and testing was presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.

Results and Discussion

The data are presented in Figure 4.1. Participants in all conditions were able to learn the presented set of rules and answer novel questions. Mean scores were significantly above chance score of 9, one sample t-tests, $t_s (21) > 5.94, ps < .001$. There was a significant difference in test scores across conditions. Scores were submitted to a 2-way ANOVA with presence/absence of relevant concreteness and presence/absence of perceptual richness as factors. Results revealed a significant effect of relevant concreteness, $F (1, 84) = 22.46, p < .001, \eta^2_p = .211$, and no effect of perceptual richness, $F (1, 84) = .006, p = .93, \eta^2_p = .000,$
nor any interaction, $F(1, 84) = 1.29, p = .258, \eta^2_p = .015$. These results support the notion that relevant concreteness can promote efficient learning. When minimal training was given, relevantly concrete instantiations had a clear advantage for initial learning over instantiations that do not communicate relevant concreteness. Experiment 2 addresses the question of whether this advantage also holds for transfer.

![Mean Test Score](image)

**Figure 4.1 Mean Learning Scores in Experiment 1.** Error bars represent standard error of the mean. Chance score is 9.

The purpose of Experiment 2 was to investigate the effects on transfer of learning different types of instantiations. Students learned an instantiation of a mathematical group that, as in Experiment 1, varied in concreteness. Then they were presented with examples of statements from a novel isomorphic domain and asked to answer a series of questions. Because spontaneous transfer is often poor (e.g. Gick & Holyoak, 1980, 1983), participants were explicitly told that the knowledge that they had just acquired can be used to answer the questions.
Analysis of transfer involved three measurements, performance on the test questions in the transfer domain, ability to match corresponding elements across the learning and transfer domains, and a similarity rating of the two domains.

EXPERIMENT 2

Method

Participants

One hundred undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students were randomly assigned to each of five conditions that specified the domain they learned in the first phase of the experiment.

Material and Design

The experiment included two phases: (1) training and testing in a learning domain and (2) testing of the transfer domain. The five conditions specified what domain was learned by participants in phase 1, whereas the transfer domain was the same for participants in all conditions. Four of the learning domains were the same four used in Experiment 1 and were isomorphic to the transfer domain. This experiment had a between-subjects design with two factors: Relevant Concreteness (Present, Absent) and Perceptual Richness (Present, Absent). Therefore, there were four experimental conditions: Relevantly Concrete/ Perceptually Rich (RC/ PR); Relevantly Concrete/ Perceptually Sparse (RC/ PS) (see Appendix C); Not Relevantly Concrete/ Perceptually Rich (No RC/ PR); and Not Relevantly Concrete/ Perceptually Sparse (No RC/ PS) (see Appendix B). A fifth learning domain was constructed as a baseline for spontaneous
performance in the transfer domain (see Appendix D). This domain involved unrelated arithmetic and matching questions, thus training in the base domain should not have facilitated performance in the transfer domain in this condition. Transfer was indicated if the transfer domain score in a given condition was greater than the mean transfer score of the baseline group.

In the four experimental conditions, the learning domain tests were the same 24-question tests used in Experiment 1. The transfer domain test was isomorphic to these tests. Training in the learning domain across these four conditions was isomorphic and similar to, but more detailed than that of Experiment 1. Participants were given five questions with corrective feedback and saw several complex examples.

The transfer domain was described as a children’s game involving three objects (see Table 4.1 and Appendix E). Children sequentially point to objects and a child who is “the winner” points to a final object. The correct final object is specified by the rules of the game, rules of a mathematical group. Participants in the experimental conditions were told that the game rules are like the rules of the system they just learned. They were told to figure out the rules of the game by using their prior knowledge. Participants were then asked to study a series of examples from which the rules could be deduced. Afterward, the multiple-choice test was given. Questions were presented individually on the computer screen along with four key examples at the bottom of the screen. The same four examples were shown with all test questions. Following the multiple-choice questions, participants in the four experimental conditions were asked to match corresponding elements across learning and transfer domains and then indicate a level of similarity between the two domains.
Because increasing superficial similarity between a learned domain and a transfer domain can increase the likelihood of transfer (Ross, 1987, 1989), a preliminary experiment was conducted to ensure that the transfer domain was equally similar to each of the learning domains. Forty undergraduate students were given brief descriptions of one of the learning domains, either the relevant concreteness or the no relevant concreteness, and the transfer domain. The descriptions were roughly equal in length and included the storyline, elements, and one example statement. Then the students were asked to rate the similarity between the two domains on a scale from 1, completely dissimilar, to 5, structurally identical. The mean rating for the Relevant Concreteness condition was 2.95 (SD = 1.15); and the mean rating for the No Relevant Concreteness condition was 3.20 (SD = 1.05). An independent samples t-test found no significant difference between these ratings, $t (38) = .717, p = .478$. Therefore, any resulting differential transfer performance can not be attributed to differential similarity between learning and transfer domains.

Table 4.1 Stimuli for the transfer domain.

<table>
<thead>
<tr>
<th>Elements:</th>
<th>Operands</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Procedure

As in Experiment 1, training and testing was presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.

Results and Discussion

The analysis included comparisons across conditions of learning test scores, transfer test scores, similarity ratings, and accuracy of matching corresponding elements. Six participants (two RC/PR, two No RC/PS, and two No RC/PR) were removed from the analysis because they failed to learn: their learning scores were less than 11 and therefore no different than the chance score of 9. In addition, seven participants were removed whose learning or transfer scores were more than two standard deviations from the mean of their respective conditions (three RC/PS, three RC/PR, and one No RC/PS). Participants in the four experimental conditions successfully learned the rules in the learning domain (see Figure 4.2). Test scores were significantly above chance, one sample t-tests, $t_s > 11.38$, $p_s < .001$. However, unlike Experiment 1, relevant concreteness offered no advantage over no relevant concreteness/perceptually sparse. Learning scores were submitted to a $2 \times 2$ ANOVA with presence of relevant concreteness and presence of perceptual richness as factors. There was no main effect of relevant concreteness, $F(1, 63) = 2.12, p = .150, \eta_p^2 = .033$; but there was a significant main effect of perceptual richness, $F(1, 63) = 3.86, p = .05, \eta_p^2 = .058$, with perceptually sparse instantiations eliciting somewhat better learning than perceptually rich ones. There was no interaction between relevant concreteness and perceptual richness, $F(1, 63) = .724, p = .39, \eta_p^2 = .011$. With the protracted training of Experiment 1, participants in the generic No RC/PS condition
learned equally well as those in the relevantly concrete conditions, while perceptual richness slightly hindered learning in the no relevant concreteness condition.

![Figure 4.2 Mean Test Scores in Experiment 2. Error bars represent standard error of the mean. Horizontal bar represents mean transfer score in the Baseline condition.](image)

Most importantly, participants in the Relevantly Concrete conditions performed significantly lower on the transfer test than those in the No Relevant Concreteness conditions. Transfer scores were submitted to a $2 \times 2$ ANCOVA with presence of relevant concreteness and presence of perceptual richness as factors and learning score as a covariate. When adjusted for difference in learning score, results revealed a significant main effect of relevant concreteness, $F (1, 62) = 48.8, p < .001, \eta_p^2 = .44$, indicating that relevant concreteness resulted in poorer transfer. There was also a marginally significant effect of perceptual richness, $F (1, 62) = 3.44, p = .068, \eta_p^2 = .053$, indicating that perceptually-rich stimuli resulted in somewhat less transfer than perceptually impoverished. There was no significant interaction, $F (1, 62) = .155, p = .695, \eta_p^2 = .002$. 

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Furthermore, transfer scores in each condition were compared with those of the Baseline condition. The analyses indicated that in all the experimental conditions except RC/ PR, transfer scores were above the scores of the Baseline group, one-way ANOVA post-hoc Tukey, $p < .006$, for the RC/ PR $p = .24$.

In addition to differences in transfer scores, similarity ratings were also different across conditions. The given similarity scale ranged from 1, completely dissimilar, to 5, structurally identical. Participants in the No Relevant Concreteness conditions rated the learning and transfer domains as highly similar ($M = 4.53$, $SD = .717$ for the perceptually sparse group and $M = 4.50$, $SD = .707$ for the perceptually rich group). However, participants in the Relevant Concreteness conditions did not rate the domains as highly similar ($M = 3.24$, $SD = 1.48$ for the perceptually sparse group and $M = 3.13$, $SD = 1.25$ for the perceptually rich group). Ratings were submitted to a two-way ANCOVA with Relevant Concreteness and Perceptual Richness as factors and learning score as a covariate. There was a significant effect of Relevant Concreteness, $F (1, 62) = 25.11, p < .001, \eta^2_p = .288$; no effect of perceptual richness, $F (1, 62) = .013, p = .90, \eta^2_p = .000$; no interaction, $F (1, 62) = .036, p = .84, \eta^2_p = .001$; and no effect of learning score, $F (1, 62) = .268, p = .60, \eta^2_p = .004$. These differences in ratings are remarkable given that, as mentioned above, no difference in similarity was found prior to learning.

Accuracy of matching elements followed a similar pattern of performance. Ninety-four percent of participants in both No RC conditions were able to accurately match elements of the learning domain to analogous elements of the transfer domain. However, participants in the RC conditions were not able to align the elements. Only 35% of the RC/ PS participants and 40% of the RC/ PR participants made the correct matching.
These proportions were significantly different, $\chi^2(\text{df}=3, N=67) = 29.11, p < 0.001$. Note that the expected proportion due to random guessing would be 33%. Therefore, perceptual richness hinders learning and transfer, but does not hinder the ability to perceive similarity and match elements between domains.

The results of Experiment 2 make several points. First, with slightly lengthier training, participants learned the generic no relevant concreteness/ perceptually sparse instantiation equally well as those who learned the relevantly concrete instantiations. Second, despite demonstrating high levels of knowledge in the learning domain, participants in the Relevant Concreteness conditions transferred very little, while participants in the generic Relevant Concreteness/ Perceptually Sparse condition transferred well. Finally, participants in the generic No Relevant Concreteness/ Perceptually Sparse condition transferred more knowledge than the participants in each of the other three conditions suggesting that relevant concreteness as well as perceptual richness hindered transfer when compared to a generic instantiation that communicates no relevant concreteness or extraneous perceptual richness. More specifically, relevant concreteness and perceptual richness appear to affect transfer in different ways. Participants in the RC conditions were unable to correctly match elements across domains and did not find the learning and transfer domains to be highly similar. However, participants in the No RC/ PR group correctly matched elements and rated the domains as highly similar.

The inability of participants in the Relevant Concreteness condition to match corresponding elements across domains suggests that these students were unable to align structure across domains. The hypothesis that relevant concreteness hinders alignment is
further supported by low similarity ratings given by these students, as alignable structures are considered more similar than non-alignable structures (Markman & Gentner, 1993). If students in the relevant concreteness conditions were unable to significantly transfer conceptual knowledge because they did not structurally align the two domains, then assisting students in alignment may result in successful transfer. One way to possibly assist students in aligning structure is to explicitly state the correspondence of elements. The purpose of Experiment 3 was to investigate the possibility that providing the correspondence of elements would result in successful transfer.

EXPERIMENT 3

Method

Participants

Eighty-three undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students were randomly assigned to one of five conditions that specified the domain they learned in the first phase of the experiment and whether or not they were given the correspondence of elements.

Material and Design

Material and design were nearly identical to that of Experiment 2. The experiment includes two phases: (1) training and testing in a learning domain and (2) testing of the transfer domain. Two types of learning instantiations were considered: Relevantly Concrete (Relevant Concreteness/Perceptually Sparse) and Generic (No Relevant Concreteness/Perceptually Sparse). Training and testing of these domains was identical to that used in
Experiment 2 (see Appendices B and C). Also as in Experiment 2, the same transfer domain (see Appendix F) was used for all conditions and was isomorphic to the Relevantly Concrete and Generic domains. However, half the participants who learned the relevantly concrete domain and half the participants who learned the generic domain were given the mapping of elements of the learning domain to the corresponding elements of the transfer domain. The other participants were not given this alignment. Therefore, the experiment had a 2 (Learning Domain: Relevantly Concrete vs. Generic) by 2 (Alignment: Mapping vs. No Mapping) between-subjects design. Therefore, there were four experimental conditions, Relevantly Concrete Map, Relevantly Concrete No Map, Generic Map, and Generic No Map. As in Experiment 2, a fifth unrelated learning domain was constructed as a baseline for spontaneous performance in the transfer domain.

With the exception of stating the correspondence of elements, phase 1 and phase 2 were identical to that of Experiment 2. As in the earlier experiment, participants were told that their knowledge of the first domain can help them answer the questions about the transfer domain. In addition to this suggestion to transfer, participants in the Map conditions were also given the correspondence between elements of the learning and transfer domains. For example, in the Generic Map condition, they were told that 🍵 is like 🐻, 🥤 is like 🎈, and 🍺 is like 🍺. Participants in the Relevantly Concrete Map condition were shown the analogous correspondences. In the Generic No Map and Relevantly Concrete No Map, the correspondences were not given. Then participants were asked to study a series of examples from which the rules could be deduced, afterward the multiple-choice test was given. Following the multiple-choice questions, participants in the four experimental
conditions were asked to indicate a level of similarity between the learning and transfer domains.

Procedure

All training and testing was presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.

Results and Discussion

Two participants (one Generic Map, one Generic No Map) were removed from the analysis for failing to learn; their learning test scores were less than 11 and no different than chance score of 9. In addition, two participants (one Relevantly Concrete Map, one Relevantly Concrete No Map) were removed because their learning test scores were more than two standard deviations from the mean of Relevantly Concrete learning score. One participant was removed from the Generic No Map condition who spent very little time on the task, total time was more than two and one half standard deviations below the mean time spent on the experiment.

Participants in all conditions successfully learned. Mean scores were significantly above chance score of 9, one sample t-tests, \( t > 15.49, p < .001 \). Between the Generic and Relevantly Concrete conditions, no significant difference was found in learning, independent samples t-test, \( t (60) = 1.06, p = .29 \). Mean learning test score = 20.4 (SD = 2.35) for Relevant Concreteness and mean = 19.6 (SD = 3.75) for Generic. There were clear differences in transfer across conditions (see Figure 4.3). Transfer test scores for the Generic Map, Generic No Map, and Relevantly Concrete Map groups were significantly above the baseline group, one-way ANOVA with condition as a factor, post-hoc
Figure 4.3 Mean Transfer Test Scores in Experiment 3. Error bars represent standard error of the mean. Horizontal bar represents mean transfer score for the Baseline condition.

Tukey, $p < .001$. For the Relevantly Concrete No Map condition, transfer was only moderately above that of the baseline, $p = .06$. Most importantly, when the mapping of elements was given, participants who learned the relevantly concrete domain transferred as well as those who learned the generic domain. There were no differences in transfer scores between the Generic Map, Generic No Map, and the Relevant Concreteness Map groups, post hoc Tukey, $p > .832$. At the same time, the Relevant Concreteness No Map group scored lower than the other three experimental conditions, post hoc Tukey, $p < .02$. Transfer scores were submitted to an ANCOVA with learning domain condition (Generic or Relevant Concreteness) and alignment (Map or No Map) as factors and learning score as a covariate. The results revealed significant effects of both condition, $F(1, 57) = 10.12, p < .003, \eta^2_p = .15$, and alignment, $F(1, 57) = 9.04, p < .005, \eta^2_p = .14$, and a significant interaction between the two $F(1, 57) = 15.59, p < .001, \eta^2_p = .22$. Learning score was also a contributing factor, $F(1, 57) = 30.32, p < .001, \eta^2_p = .35$. The strong interaction emphasizes the fact that giving the alignment allowed participants in
the Relevantly Concrete condition to successfully transfer, yet it did not result in any improvement for participants in the Generic condition. This suggests that alignment and mapping of structure is spontaneous when learning a generic instantiation, but is not spontaneous for learning a relevantly concrete instantiation.

Similarity ratings followed a similar pattern to that of transfer test scores. Participants in the Generic Map, Generic No Map, and Relevantly Concrete Map conditions rated the learning and transfer domains as highly similar ($M = 4.93, SD = .258$; $M = 4.67, SD = .617$; $M = 4.27, SD = .961$ respectively on a scale from 1 to 5). However, participants in the Relevantly Concrete No Map conditions did not rate the domains as highly similar ($M = 3.24, SD = 1.48$). The difference in ratings was significant. Ratings were submitted to a two-way ANCOVA with learning domain condition (Generic or Relevant Concreteness) and alignment (Map or No Map) as factors and learning score as a covariate. There were significant effects of learning domain, $F(1, 57) = 20.15, p < .001, \eta^2_p = .26$, and alignment, $F(1, 57) = 5.42, p < .03, \eta^2_p = .087$, with moderate interaction between the two, $F(1, 57) = 3.77, p = .057, \eta^2_p = .062$. Learning score was not significant, $F(1, 57) = 2.39, p = .127, \eta^2_p = .04$.

Experiment 3 demonstrated that when participants who learn the relevantly concrete instantiation were given the mapping of elements from the learning domain to the transfer domain, they were able to transfer their knowledge as well as those who learned the generic instantiation. Stating the correspondences of elements provided no additional benefit for participants who learned the generic instantiation. These results further support the hypothesis that the difficulty transferring structural knowledge from a relevantly concrete domain is due to an inability to align the two domains.
SUMMARY

The results of Experiment 1 demonstrate that some concreteness can indeed promote learning. Under conditions of minimal training, participants who learned a relevantly concrete instantiation had higher scores on a test of the learning domain than those who learned an instantiation that was not relevantly concrete. However, when protracted training was given in Experiment 2, this difference in test performance disappeared. Most importantly, the benefit of relevant concreteness did not extend to transfer. After learning the initial instantiation, participants were presented with a novel isomorph and asked to answer questions. Participants who learned a generic instantiation scored markedly higher than participants who learned a relevantly concrete or irrelevantly concrete instantiation. Therefore, it appears that learning a generic instantiation can promote transfer. At the same time, learning an instantiation that communicates any type of concreteness hinders transfer with relevant concreteness hindering transfer significantly.

Furthermore, it appears that relevant concreteness and irrelevant concreteness hinder transfer in different ways. Participants in the generic and irrelevantly concrete conditions were generally able to match elements across domains and rated the domains as highly similar. This suggests that they were able to recognize common structure and align the two domains. However, participants who learned a relevantly concrete instantiation generally did not make the correct correspondence between elements and did not find the two domains to be highly similar. Therefore it appears that they were not able to align the two domains suggesting that relevant concreteness obfuscated the analogy between the domains.
The results of Experiment 3 further support the hypothesis that relevant concreteness hinders alignment of structure and as a result inhibits transfer. In this experiment, half of the participants were given the correspondence of elements of the learning and transfer domains. The findings were that when participants in the relevantly concrete conditions were given this correspondence they transferred as well as those in the generic condition. Therefore, giving this correspondence helped the learners align the structure of the two domains and implement the analogy. For participants in the generic condition, giving the element correspondence did not improve their transfer ability.
CHAPTER 5

THE NATURE OF THE INTERNAL REPRESENTATIONS

The results of Experiments 2 and 3 suggest that transfer from a relevantly concrete domain is poor because of difficulty structurally aligning the learned and novel domains. The inability to align domains cannot be attributed to poor learning, as participants in the generic and relevantly concrete conditions learned equally well. This suggests that categorically different internal representations were constructed during learning for the generic and relevantly concrete domains where one type allowed recognition and alignment of structure and the other did not.

The purpose of Experiment 4 was to investigate the nature of the internal representation constructed during learning of different types of instantiations. An internal representation may contain structural information, superficial information, or both. Specifically, if the conceptual instantiation was successfully learned, the representation will contain the relational structure that defines the concept. In addition, the representation may contain superficial information including the elements that instantiate the concept. While an internal representation may contain both elements and relations, the attentional weights given to each may differ significantly. It is possible that a weak representation of relational structure or a representation in which elements and relations
are tightly bound together may leave the learner unable to recognize it in a novel isomorph which would lead to transfer failure.

Specifically, the goal of Experiment 4 was to test whether learned structure can be recognized when expressed with novel elements. After learning one instantiation, participants were tested on their ability to recognize the learned structure (and its violations) when presented with the learned elements or novel elements. If the representation of relational structure is weak, then students may not be able to distinguish between a true instantiation and a violation even when expressed with familiar elements. As a result they may accept as truth any expressions involving the familiar elements. If learners extract the relational structure, but bind it to specific elements, the learner should be very accurate at distinguishing truth instantiations from violations for familiar elements and highly inaccurate in doing so for novel elements. If relations are represented in a sufficiently strong manner and not bound to the learning context, then the learner should be able to accurately distinguish truths from violations in the context of both familiar and novel elements.

EXPERIMENT 4
Method
Participants

Fifty one undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students
were randomly assigned to one of three conditions that specified the type of instantiation they learned.

**Material and Design**

The experiment consisted of two phases. In phase 1, all participants learned an instantiation of a mathematical group as in the previous experiments. The type of instantiation learned was a between-subjects factor: Relevantly Concrete, Perceptually Rich, and Generic. The instantiations were the same used in Experiments 2 and 3 in the Relevant Concreteness/Perceptually Sparse (see Appendix C), No Relevant Concreteness/Perceptually Rich and the No Relevant Concreteness/Perceptually Sparse respectively (see Appendix B). In phase 1, training and testing was identical to that of Experiment 2.

In phase 2 of the experiment, participants were presented with sets of expressions involving either familiar elements from phase 1 or novel elements and were asked whether the expressions involved the same rules as those of phase 1. Phase 2 consisted of 26 test trials (see Appendix F). On each trial, participants were presented with a set of three expressions. They were told that each set is from a new system and asked whether the new system follows the same type of rules as the system they has previously learned. Three independent expressions were given per trail because violations of structure would be completely detectable. From one and possibly two expressions, the structure could not be definitely determined. Four types of trials were used. Table 5.1 shows examples of each trial type, as expressed for the Relevantly Concrete condition. For the Generic and Perceptually Rich conditions, the analogous statements were expressed with the generic black symbols or their colorful counterparts respectively. Six trials involved the same elements as the learning phase and the same relational structure (E+/R+). Six trials
involved the familiar elements, but different relational structure (E+/R-). Six trials involved novel elements and the familiar relational structure (E-/R+). Another six trials involved both novel elements and novel relations (E-/R-). In addition, two questions were posed in which familiar elements were cross-mapped to play different roles in the same relational structure. For example, in the statement , is playing the role that held in the learning domain.

Table 5.1 Examples from Phase 2 of Experiment 4.

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Results and Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td>All training and testing was presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.</td>
<td></td>
</tr>
<tr>
<td>Three participants (one Perceptually Rich, two Generic) were removed from the analysis for failing to learn; their learning test scores were less than 11 and no different</td>
<td></td>
</tr>
</tbody>
</table>
than chance score of 9. In all conditions, participants successfully learned the concept. The mean test scores of 19.6 ($SD = 4.1$) for the Relevant Concreteness group, 17.3 ($SD = 4.0$) for the Perceptually Rich group, and 18.2 ($SD = 2.8$) for the Generic group were above a chance score of 9, one sample $t$-tests, $ts > 8.54$, $ps < .001$. The differences between groups was not significant, one-way ANOVA, $F (2, 47) = 1.67$, $p = .199$, $\eta_p^2 = .066$.

While there were no differences in learning across condition, there were considerable differences in ability to discriminate familiar and novel relational structure in phase 2 depending on the presence of familiar or novel elements. Table 5.2 presents the percent of trials for which participants responded “same structure as learning phase”. In the context of familiar elements, all participants were highly accurate in discriminating structure. Across conditions, mean accuracy on all trials involving familiar elements was above chance, one sample $t$-test $ts > 9.34$, $p < .001$. To measure discriminability in the context of familiar elements, the number of “yes – same structure” responses for E+/R+ trials minus the number of erroneous “yes – same structure” responses for E+/R- trials was calculated (see Figure 5.1) and submitted to an ANCOVA with condition as a between-subjects factor and learning test score as a covariate. The results found no difference in discriminability across condition, $F (2, 42) = .289$, $p = .75$, $\eta_p^2 = .01$; and a significant effect of learning $F (1, 46) = 11.67$, $p < .001$, $\eta_p^2 = .20$. This discriminability accuracy supports the proposition that successful learning results in representations that contain both elements and relations.
Table 5.2 Portion of “Yes – Same Structure” Response in Phase 2 of Experiment 4.

<table>
<thead>
<tr>
<th>Condition</th>
<th>E+/ R+</th>
<th>E+/ R-</th>
<th>E-/ R+</th>
<th>E-/ R-</th>
<th>Cross-Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relevantly Concrete</td>
<td>0.94</td>
<td>0.04</td>
<td>0.38</td>
<td>0.11</td>
<td>0.00</td>
</tr>
<tr>
<td>Perceptually Rich</td>
<td>0.90</td>
<td>0.07</td>
<td>0.74</td>
<td>0.06</td>
<td>0.29</td>
</tr>
<tr>
<td>Generic</td>
<td>0.96</td>
<td>0.05</td>
<td>0.84</td>
<td>0.06</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 5.1 Discriminability: Percent of “Same Structure” Responses on R+ trials minus “Same Structure” Responses on R- trials. Error bars represent standard error of the mean.

However, when trials involved novel elements, there were striking differences in discriminability, where participants in the Relevantly Concrete condition were unable to recognize the familiar structure while participants in both the Generic and the Perceptually Rich conditions were able. Scores of the number of “yes – same structure”
responses for E-/R+ trials minus the number of erroneous “yes – same structure” responses for E-/R- trials were calculated (see Figure 5.1) and submitted to an ANCOVA with condition as a between-subjects factor and learning test score as a covariate. Results reveal a significant effect of condition, \(F(2, 46) = 12.22, p < .001, \eta^2_p = .35\). In addition, there was an effect of learning \(F(1, 46) = 7.62, p < .009, \eta^2_p = .14\). More specifically, all participants accurately rejected E-/R- trials, one sample t-tests \(t = 7.35, p < .005\). However, while participants in the Generic and Perceptually Rich conditions were clearly above chance in recognizing structure on E-/R+ trials, one sample t-test, \(t = 3.42, p < .004\), participants in the Relevant Concreteness condition were no better than chance, one sample t-test, \(t(16) = 1.26, p < .226\). Therefore, in the context of novel elements, the ability to recognize learned structure does not depend as much on how well the initial instantiation was learned, but rather on what type of initial instantiation was learned.

Responses to cross-mapped trials provide additional evidence that when learning a relevantly concrete instantiation, structure is tightly bound to the elements as presented during learning. These trials presented the learned structure, but switched the roles of familiar element. None of the participants in the Relevant Concreteness condition were able to recognize structure when elements were crossed mapped, while 25% of participants in the Generic condition and 29% in the Perceptually Rich condition correctly recognized familiar relational structure with cross mapped elements. Scores for these questions were submitted to an ANCOVA with condition as a factor and learning test score as a covariate. There was a main effect of condition, \(F(2, 46) = 3.68, p < .04, \eta^2_p = .14\), and no significant effect of learning scores, \(F(1, 46) = 1.49, p = .22, \eta^2_p = .03\).
Therefore, the type of instantiation from which the concept was learned significantly affected the learner’s ability to recognize the same relational structure in the context of novel elements. Learning an instantiation that communicated no relevant concreteness, whether generic or perceptually rich, allowed participants to recognize relational structure elsewhere, while learning a relevantly concrete instantiation did not. For relevant concreteness, relational structure is embedded in the learning context creating an inability to recognize structure in an isomorph that results in an obstacle for structural alignment and successful transfer.
CHAPTER 6

EFFECTS OF LEARNING MULTIPLE INSTANTIATIONS

The results of Experiments 2 and 4 indicated that learning a relevantly concrete instantiation hindered recognition of structure in a novel isomorph, thus creating an obstacle to structural alignment and successful transfer. Experiment 3 demonstrated that when learners were given the analogical correspondence of elements across domains they did successfully transfer. However, the correspondence of elements may not always be available to the learner. Real-world situations are very unlikely to provide this correspondence.

While learning one relevantly concrete instantiation did not promote transfer, it is possible that learning more than one relevantly concrete instantiation will. Multiple instantiations of a concept may improve transfer performance by encouraging the learner to form an abstract schema (Gick & Holyoak, 1983; Reed, 1993). While both infants and adults have demonstrated an ability to detect common structure from sequential exemplars in an artificial language (Gomez, 2002), during more complex problem solving, formation of a ‘high quality’ schema and successful transfer are not necessarily guaranteed. For example, Gick and Holyoak (1983) investigated the effect on transfer of learning from two analogues of the radiation problem. They found that while two
analogues increased the likelihood of successful transfer, the majority of participants did not spontaneously transfer solution strategies. When two analogues were given, only 45% of study participants transferred, while only 21% transferred when given only one instantiation. As in their earlier study (Gick & Holyoak, 1980), giving participants a suggestion to transfer greatly increased the likelihood of doing so. With a hint 80% of participants transferred from two analogues while 53% transferred from one instantiation.

Abstraction of common structure from two isomorphs may not be an automatic process. The failure to do so may lie in an inability to recognize or focus on the structure common to the two isomorphs. Abstraction may instead result from the deliberate alignment structure (Kotovsky & Gentner, 1996; Ross & Kennedy, 1990). Students may not spontaneously align two learning domains particularly if the domains are concrete, as it is more difficult to recognize relational structure common to two concrete, perceptually rich situations than between more generic situations (Gentner & Medina, 1998; Markman & Gentner, 1993). If students learn two isomorphs but do not align them according to relevant relational structure, it is possible that learning two isomorphs will show little or no advantage for transfer over learning one instantiation.

The purpose of Experiment 5 was to investigate the effect on transfer of learning more than one instantiation of a concept. To help promote alignment between the learning domains, some students were given the correspondence of elements between the two domains, while other students were not given this correspondence. The results of Experiment 3 indicated that providing correspondence of elements enabled students who learned the relevantly concrete instantiation to successfully transfer. This experiment investigated the possibility that providing students with the correspondence of elements
across two domains will facilitate subsequent transfer to a third domain. Additionally, some students learned relevantly concrete instantiations, while others learned one relevantly concrete instantiation and one generic instantiation.

EXPERIMENT 5

Method

Participants

One hundred forty undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students were randomly assigned to one of seven conditions that specified the domain or domains they learned in the first phase of the experiment and whether or not they were given the correspondence of elements between the learning domains.

Material and Design

Material and design are similar to that of Experiments 2 and 3. Students learned one or more instantiations of a mathematical group of order 3. Then they were presented with the transfer domain as in Experiments 2 and 3. Therefore, the experiment included two phases: (1) training and testing in the learning domain(s) and (2) testing of the transfer domain.

The learning phase of this experiment involved four possible instantiations of a mathematical group: Generic, Relevantly Concrete A, Relevantly Concrete B, and Relevantly Concrete C. The Generic and Relevantly Concrete A domains were the same as those of the previous experiment. Two additional relevantly concrete domains were constructed. Participants learned one, two, or three instantiations. The following seven conditions were considered: (1) Generic: learn one generic domain (see Appendix B), (2)
Relevantly Concrete A: learn one relevantly concrete domain (see Appendix C), (3)
Relevantly Concrete A & B: learn two relevantly concrete domains, (4) Relevantly
Concrete A/ Generic: learn one relevantly concrete and one generic domain, (5) Relevantly
Concrete A & B – Map: learn two relevantly concrete domains with correspondence of
elements given, (6) Relevantly Concrete A/ Generic – Map: learn one relevantly concrete
and one generic domain with correspondence of elements given, and (7) Relevantly
Concrete A, B, & C: learn three relevantly concrete domains (see Appendix G for all
multiple instantiation conditions). No baseline condition was included because the baseline
conditions of Experiments 2 and 3 indicated that without learning an isomorphic domain
participants scored no better than chance on the target domain. Therefore, a chance score
of 9/24 was used for comparison purposes as baseline transfer.

For the conditions with two instantiations, participants completed training and testing in
one domain, then proceeded to training and testing of the second. Similarly, when learning
three instantiations, participants were sequentially trained and tested on the first, then the
second, and then the third instantiation. Training in the learning phase was equated across
conditions so that the same examples, questions with feedback, summaries of rules, and 24
test questions that were used in the previous experiments were spread across the learning
domains. For example, five questions with feedback were posed in the conditions involving
one learning domain. When learning with two domains, three of these questions were asked
of the first instantiation and the remaining two were asked of the second instantiation. For
learning three instantiations, participants were asked two questions of the first instantiation,
two questions of the second, and the final question of the third.
To approximate the effect of presenting the rules in more than one domain, additional summaries of the rules were given when learning with fewer domains. In other words, for all conditions, the rules were presented along with several examples, questions with feedback, and brief summaries of the rules. In addition, when learning one instantiation, two detailed summaries of the rules were given. When learning two domains, the first domain had one detailed summary and the second did not have a detailed summary. When learning three domains, no detailed summaries were given for any of the domains.

The same 24-question multiple-choice test was used as in the previous experiments. Questions were distributed evenly over the learning domains. Participants who learned two instantiations had a twelve-question test over the first domain and the remaining twelve questions over the second domain. Participants who learned three instantiations had eight-question tests over each domain.

The Generic domain was the same as that of the previous experiments. The Relevantly Concrete A instantiation was also the same as the previous experiments. For Relevantly Concrete B, students were told a story about ordering pizzas. Every time an order is placed, a predictable portion of the order was burned. All pizzas were the same size and always had three slices. Orders were made up of individual orders of one, two, or three slices. The symbols used for this instantiation were: \( \bullet \), \( \mathbf{\bullet} \), and \( \clubsuit \) indicating the amount of an individual order. The amount burned followed the rules of the mathematical group such that \( \mathbf{\bullet} \) behave as 0, \( \bullet \) acts as 1, and \( \mathbf{\bullet} \) acts as 2 in modular arithmetic. The participant’s job was to determine the burned portion of specific orders. For example, if an order of \( \bullet \) and \( \mathbf{\bullet} \) was placed, \( \bullet \) would be burned.
The Relevantly Concrete C domain was presented as a situation in which a tennis ball company had a machine that makes tennis balls. The machine produced small batches of three balls that were placed into a container. The machine was not working properly so that it produced two, one, or zero balls per batch. The symbols of the domain indicate the number of balls produced in a batch:  and . When a series of two or more batches were produced, balls were packed into container. The participant’s job was to determine how many extra tennis balls there would be. For example, if these batches, and , were made, then would be extra.

For the conditions with multiple learning domains, participants were told that the domains are the same type of system and the rules work in the same way. In addition, for the Relevantly Concrete A & B – Map and the Relevantly Concrete A/ Generic – Map conditions, participants were told the correspondence between the elements of the two learning domains. For example, is like for the Relevantly Concrete A & B – Map condition; and is like for the Relevantly Concrete A/ Generic – Map condition.

After the learning phase, participants proceeded to the transfer phase which was identical to that of Experiment 2. They were presented with the novel domain and received the same 24-question multiple-choice test (see Appendix E). Following the multiple choice questions, participants were asked to match corresponding elements between the transfer domain and their first or only learning domain. Then they were asked to indicate a level of similarity between these two domains.

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Procedure

All training and testing was presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.

Results and Discussion

Sixteen participants (two Generic, five Relevantly Concrete/ Generic, three Relevantly Concrete/ Generic – Map, three Relevantly Concrete A & B, and three Relevantly Concrete A, B, & C) were removed from the analysis because they failed to learn as evidenced by learning score(s) not above chance. The criterion for learning was set as followings. Composite learning score (the sum of scores in all learning domains) was greater than 10. In addition, for conditions with two learning domains, learning in each individual domain was greater than 4. For the condition with three learning domains, learning in each domain was greater than 3. While there were differences in attrition rates across condition, these differences were insignificant, $\chi^2 (df = 6, N = 140) = 9.60, p = .143$.

In all conditions, participants successfully learned the material. For participants who learned more than one instantiation, learning scores in each domain were summed to create a composite learning score that was compared to learning scores in condition involving only one domain (see Table 6.1). Therefore, for all participants the maximum composite learning score was 24. These scores were significantly above a chance score of 9, one-sample t-tests, $t_s > 9.62, ps < .001$ (see Figure 6.1). In addition, learning in each individual learning domain (see Table 6.1) was above chance, one-sample t-tests, $t_s > 7.87, ps < .001$. 

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<table>
<thead>
<tr>
<th>Number of Learning Domains</th>
<th>Domain 1</th>
<th>Domain 2</th>
<th>Domain 3</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Relevantly Concrete</td>
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<td>18.20</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>2</td>
<td>9.88</td>
<td>8.94</td>
<td>2.30</td>
</tr>
<tr>
<td>Relevantly Concrete - Map</td>
<td>2</td>
<td>10.55</td>
<td>9.20</td>
<td>1.94</td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>3</td>
<td>6.41</td>
<td>6.29</td>
<td>1.10</td>
</tr>
<tr>
<td>Relevantly Concrete/Generic</td>
<td>2</td>
<td>10.60</td>
<td>9.27</td>
<td>2.34</td>
</tr>
<tr>
<td>Relevantly Concrete/Generic - Map</td>
<td>2</td>
<td>10.12</td>
<td>8.70</td>
<td>2.20</td>
</tr>
<tr>
<td>Generic</td>
<td>1</td>
<td>19.27</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.1 Learning scores in individual domains of Experiment 5.

![Figure 6.1 Mean Composite Learning Scores in Experiment 5.](image)

Figure 6.1 Mean Composite Learning Scores in Experiment 5. Error bars represent standard error of the mean.

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Effect of Multiple Relevantly Concrete Instantiations

To analyze the effect of learning more than one relevantly concrete instantiation, test scores were compared for only those participants in the Relevantly Concrete A, Relevantly Concrete A & B, and Relevantly Concrete A, B, & C conditions. Learning scores were submitted to a one-way ANOVA with number of learning instantiations as a factor. The results revealed no differences in learning, $F(2, 51) = .928, p = .402, \eta^2_p = .035$ (see Figure 6.1). Similarly, learning multiple instantiations offered no significant benefit for transfer (see Figure 6.2). Transfer scores were submitted to an ANCOVA with composite learning score as a covariate. While learning scores had a significant effect on transfer, $F(1, 50) = 9.94, p < .004, \eta^2_p = .166$, number of instantiations did not $F(2, 50) = .526, p = .594, \eta^2_p = .021$.

![Figure 6.2 Mean Transfer Scores in Experiment 5. Error bars represent standard error of the mean.](image-url)
Effect of Giving Correspondence of Elements between Relevantly Concrete Instantiations

Learning and transfer scores were compared for participants in the Concrete A & B and Concrete A & B – Map conditions. While giving participants the correspondence between elements in a learned relevantly concrete domain and an isomorphic transfer domain facilitated transfer (see Experiment 3), giving element correspondence between two relevantly concrete domains resulted in no improvement in learning of the second domain or transfer to a novel domain. Independent sample t-test revealed no difference in learning score in the second learning domain between those who were told the correspondence of elements and those who were not, \( t(35) = .371, p = .713 \) (see Table 6.1). Transfer scores were submitted to an ANCOVA with composite learning score as a covariate. Neither learning nor element correspondence contributed significantly to transfer, \( F(1, 34) = 2.32, p = .137, \eta_p^2 = .064 \), and \( F(1, 34) = .462, p = .501, \eta_p^2 = .013 \) for learning and correspondence respectively.

Effect of learning a Relevantly Concrete Instantiation and a Generic Instantiation

To analyze the effect of learning a relevantly concrete instantiation and then a generic instantiation, learning and transfer scores were compared for participants in only the Generic, Relevantly Concrete A/ Generic, and Relevantly Concrete A/ Generic – Map conditions. No differences were found in composite learning scores, one-way ANOVA \( F(2, 47) = .477, p = .624 \). In particular, providing students with the correspondence of elements offered no improvement in learning of the second (generic) domain (see Table 6.1), independent sample \( t(30) = .698, p = .491 \).

Learning one relevantly concrete and one generic domain offered no advantage for transfer over learning one generic (see Figure 6.2). An ANCOVA with condition (Generic,
Relevantly Concrete A/ Generic and Relevantly Concrete A/ Generic – Map) as a factor and composite learning scores as a covariate revealed only a small effect of condition on transfer, $F(2, 46) = 2.56, p = .088, \eta^2_p = .100$. At the same time, learning level had a significant effect on transfer, $F(1, 46) = 20.80, p < .001, \eta^2_p = .311$). More specifically, providing the correspondence of elements resulted in a slightly higher transfer score than without the correspondence, ANCOVA, $F(1, 29) = 3.69, p = .065, \eta^2_p = .113$.

**Benefit of Learning a Generic Instantiation**

Learning multiple instantiation, whether all relevantly concrete or relevantly concrete and generic, appears to offer no benefit for either learning or transfer over learning one generic instantiation. Composite learning scores were submitted to a one-way ANOVA and revealed no differences in learning level across condition, $F(6, 117) = 7.38, p = .652$.

While condition resulted in no differences in learning level, striking differences were found for transfer. Transfer scores were compared to a chance score of 9. The Generic, Relevantly Concrete A/ Generic, and Relevantly Concrete A/ Generic – Map resulted in scores that were greatly above chance (see Table 6.2). However, transfer scores for Relevantly Concrete A, B, & C were only moderately above chance and scores for Relevantly Concrete A, Relevantly Concrete A & B and Relevantly Concrete A & B – Map were no better than guessing.

Furthermore, participants in the Generic conditions scored higher on the transfer test than participants in all of the other conditions, with significant differences from conditions in which only relevantly concrete instantiations were learned, one-way ANOVA, $F(6, 117) = 9.56, p < .001, \eta^2_p = .329$, post hoc Tukey, $ps < .001$.  

82
<table>
<thead>
<tr>
<th>Number of Learning Domains</th>
<th>( t )</th>
<th>( df )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relevantly Concrete</td>
<td>1.74</td>
<td>19</td>
<td>.097</td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>1.47</td>
<td>16</td>
<td>.161</td>
</tr>
<tr>
<td>Relevantly Concrete - Map</td>
<td>.943</td>
<td>19</td>
<td>.357</td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>2.78</td>
<td>16</td>
<td>.013</td>
</tr>
<tr>
<td>Relevantly Concrete/ Generic</td>
<td>4.01</td>
<td>14</td>
<td>.001</td>
</tr>
<tr>
<td>Relevantly Concrete/ Generic – Map</td>
<td>6.06</td>
<td>16</td>
<td>.000</td>
</tr>
<tr>
<td>Generic</td>
<td>7.56</td>
<td>17</td>
<td>.000</td>
</tr>
</tbody>
</table>

**Table 6.2** Independent sample t-tests for comparison of transfer scores to chance score in Experiment 5.

*Ability to Match Elements*

After the transfer test, participants were asked to match corresponding elements between the transfer domain and their first or only learning domain. The ability to form the correct correspondence was significantly different (see Table 6.3), \( \chi^2 (df = 6, N = 124) = 37.15, p < 0.001 \). Participants in the three conditions that involved learning a generic instantiation were much more likely to correctly match elements than participants in the conditions that involved learning only relevantly concrete instantiations. Note that the expected proportion due to random guessing would be 33%.
<table>
<thead>
<tr>
<th>Learning Condition</th>
<th>Number of Learning Domains</th>
<th>Percent of Participants Making Correct Element Correspondence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relevantly Concrete</td>
<td>1</td>
<td>30.0</td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>2</td>
<td>23.5</td>
</tr>
<tr>
<td>Relevantly Concrete - Map</td>
<td>2</td>
<td>15.0</td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>3</td>
<td>41.2</td>
</tr>
<tr>
<td>Relevantly Concrete/Generic</td>
<td>2</td>
<td>86.7</td>
</tr>
<tr>
<td>Relevantly Concrete/Generic – Map</td>
<td>2</td>
<td>70.6</td>
</tr>
<tr>
<td>Generic</td>
<td>1</td>
<td>83.3</td>
</tr>
</tbody>
</table>

Table 6.3 Percentage of participants making correct element correspondence split across learning condition of Experiment 5.

**Similarity Ratings**

Similarity ratings for the first learning domain and the transfer domain were higher for participants in conditions that involved learning the generic domain \((M = 3.86, SD = 1.05)\) than for conditions that involved learning only relevantly concrete domains \((M = 2.61, SD = 1.42)\), independent sample t-test, \(t (122) = 5.32, p < .001\). There were no differences between conditions within the generic and relevantly concrete categories, one-way ANOVA \(F (2, 47) = .769, p = .469\) for the generic and \(F (2, 70) = .426, p = .735\) for the relevantly concrete (see Table 6.4).
<table>
<thead>
<tr>
<th>Learning Condition</th>
<th>Number of Learning Domains</th>
<th>Similarity Rating</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relevantly Concrete</td>
<td>1</td>
<td>2.70</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>2</td>
<td>2.76</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Relevantly Concrete - Map</td>
<td>2</td>
<td>2.30</td>
<td>.979</td>
<td></td>
</tr>
<tr>
<td>Relevantly Concrete</td>
<td>3</td>
<td>2.71</td>
<td>1.83</td>
<td></td>
</tr>
<tr>
<td>Relevantly Concrete/ Generic</td>
<td>2</td>
<td>3.73</td>
<td>.884</td>
<td></td>
</tr>
<tr>
<td>Relevantly Concrete/ Generic – Map</td>
<td>2</td>
<td>4.11</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>1</td>
<td>3.72</td>
<td>1.02</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.4  Similarity ratings between first or only learning domain and transfer domain of Experiment 5.

**SUMMARY**

Learning multiple instantiations appears to offer no benefit for transfer. In particular, learning multiple relevantly concrete instantiations resulted in little or no transfer. At the same time, learning a generic instantiation whether alone or following learning of a concrete instantiation did facilitate transfer. However, the combination of relevantly concrete and generic resulted in no better transfer than learning the generic instantiation alone.
It appears that learning a generic instantiation allowed the learner to recognize relational structure, while learning concrete instantiations did not. Not only were participants in the Generic condition able to correctly match learned generic symbols to their corresponding elements of the transfer domain, but also participants in the Relevantly Concrete/ Generic, and Relevantly Concrete/ Generic – Map conditions were able to correctly match elements of the relevantly concrete domain A to their counterparts in the transfer domain. Therefore, learning a generic instantiation facilitated recognition of common structure between domains not only in the context of the learned generic elements, but also in the context of concrete elements. In addition, participants in the conditions involving learning the generic domain found the transfer domain to be more similar to the first domain they learned than did participants in the conditions involving only relevantly concrete domains. These results further support the proposition that learning a generic domain facilitates transfer to novel isomorphs by allowing recognition and alignment of common structure.
CHAPTER 7

EFFECT OF LEARNING A RELATIONAL DIAGRAM

As demonstrated in Experiments 2 – 5 as well as previous studies (Sloutsky, Kaminski, & Heckler, 2005) generic instantiations can be effective promoters of learning and transfer. In addition, the results of Experiment 3 revealed differences in the internal representations constructed from generic and concrete instantiations. Both resulted in the ability to recognize learned relations in the context of learned elements. However generic and irrelevantly concrete learning allowed learned relations to be recognized in the context of novel elements, while relevantly concrete learning did not. What remains unanswered is whether learning a generic instantiation results in the construction of a genuine schema or simply an ability to reason analogically from one domain to another.

Novick and Holyoak (1991) investigated transfer across isomorphic mathematical word problems. They argue that schema formation is a natural consequence of analogical transfer. They rated schema formation by evaluating study participants written similarities of learning and transfer domains. While evaluating similarity descriptions can yield some indirect evidence of schema existence, a more robust test would be to measure transfer performance to a situation in which the structure itself has been modified in an abstract manner. In this situation, the learning and transfer domains would not be
isomorphic and therefore simple analogical reasoning would not be successful. Successful transfer would require recognition and modification of the relational structure.

A mathematical group is an ideal concept to test in such a manner for schema existence. The previous experiments measure transfer from a learned group of order 3 to a novel group of order 3. Participants who transferred well may have formed schema or they may have succeeded in transfer via other types of reasoning such as base-based reasoning. A more stringent test of schema formation would be to test ability to transfer to a novel mathematical group of different order. In this scenario, to successfully transfer participants would need abstract knowledge of relational structure which could be modified to accommodate not only novel elements, but a different number of elements. Such skeletal structural knowledge would be considered a schema (Reed, 1993).

Experiment 6 investigated ability to transfer knowledge of a mathematical group of order 3 to a novel group of order 3 as well as to a group of order 4. As in the previously discussed experiments, the type of instantiation learned was a between-subjects factor. Of particular interest is the effect of a relational diagram on learning and transfer of conceptual knowledge. A relational diagram is a generic instantiation that helps to communicate the global structure of a given system. In particular, a relational diagram is an external representation that has the following: (a) perceptual symbol(s) for the relevant relation(s) that helps communicate the manner in which elements relate, (b) perceptual placeholders for elements, and (c) a minimal amount of extraneous information. A hypothesis of this experiment was that learning a relational diagram would facilitate transfer to isomorphic instantiations as well as to domains of different order than that learned.
The instantiations in the previous experiments have external representations that involve symbols for elements and rules are expressed as statements such as \( \diamondsuit, \heartsuit \rightarrow \). 

\( \bullet \) or when \( \mathcal{A} \) and \( \mathcal{B} \) combine, \( \mathcal{C} \) is left-over. These representations capture individual rules and not the global system.

Research investigating the effects of diagrams on learning and transfer has shown mixed results. Gick and Holyoak (1983) found that representing the radiation problem with a diagram indicating convergence from multiple points did not guarantee spontaneous transfer. However, when the diagram was accompanied by a verbal statement of the convergence principle, transfer was more likely (Pedone, Hummel, & Holyoak, 2001). Additionally, animated convergence diagrams lead to a marked increase in transfer.

Experiment 6 compared levels of learning and transfer when learning occurs with a generic, symbolic sentential instantiation versus a generic relational diagram. In addition, a more concrete relational display was also considered to investigate whether concreteness had any effects on learning and transfer involving diagrams. As in the previous experiment, students learned an instantiation of a mathematical group of order 3 and then attempt to answer questions about a novel isomorphic group of order 3. Afterward, students were asked to answer ten multiple-choice questions involving a novel group of order 4. If relational diagrams can communicate global structure of the system and sentential representations only depict local rules, there should be a clear advantage of diagrams in transfer to a group of order 4.
EXPERIMENT 6

Method

Participants

Sixty undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students were randomly assigned to one of three conditions that specified the type of instantiation they learned.

Material and Design

The design of the experiment was similar to that of Experiment 2 except there was an additional transfer test. There were three phases: 1) training and testing in a learning domain, (2) testing of the transfer domain, and (3) testing of transfer domain 2. Three conditions specified the domain to be learned: (1) Generic (see Appendix B), (2) Relational diagram (see Appendix H), and (2) Concrete (see Appendix H). The Generic domain was the same as the Generic domain used in the previous experiments.

In the Relational diagram condition, participants were told about a code-breaking device that decodes sequences of symbols (see Figure 7.1). Sequences involved three symbols: ●, ▲, and ◆. Given a sequence of symbols, the decoder could be used to determine a resulting symbol by starting at the position on the decoder of the first symbol and moving clockwise around. If the next symbol is ◆, move one position ahead. If the next symbol is ●, move two positions. If the next symbol is ▲, then move three positions. For example given ●◆, start at position ● and move clockwise one position to stop at ▲. Then ▲ is the resulting symbol.
In the Concrete condition, participants were told a scenario involving cars driving around an oval track that has three equidistant markers (see Figure 7.2). They were told a starting point and a specific battery or batteries to use. Different batteries allowed the car to travel different distances. Their task was to determine the stopping point of the car.

The elements of this domain were: \[\text{Marker 1}, \text{Marker 2}, \text{Marker 3}\]. These represent locations on the track as well as batteries. In particular, \[\text{Battery 1}\] would allow the car to travel one position; \[\text{Battery 2}\] would allow the car to travel two positions; and \[\text{Battery 3}\] would allow the car to travel three positions. For example, if the car starts here \[\text{Starting Point}\] and uses this battery \[\text{Battery 2}\], then it will stop here \[\text{Stop Point}\].
Figure 7.2 Display used in Concrete Condition of Experiment 6.

Transfer domain 1 was the same as that used in Experiments 2, 3, and 5. Also, as in the previous experiments, participants were given the same 24-question multiple-choice tests for both the learning domain and transfer domain 1. Following the transfer test 1, participants were asked to match corresponding elements across learning and transfer domains and then indicate a level of similarity between the two domains.

In the third phase of the experiment, participants were given a paper and pencil ten-question multiple-choice test (see Appendix I). They were told that the questions are about a different system that works in the same manner as the ones they previously learned. More specifically, they were told that the rules are very similar, but instead of involving three different entities, this new system involved four: ♦, ★, ●, and □. They were shown five example statements from which the complete set of rules could be deduced. Then they were asked to answer the questions to the best of their ability.

Procedure

Training and testing of phases 1 and 2 were presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own
pace; and their responses were recorded. After completing the first two phases, each participant was given the paper and pencil test over Transfer Domain 2. Participants had no more than ten minutes to complete the Transfer Test 2.

Results and Discussion

Five participants (two Generic Sentential, two Relational Diagram, and one Concrete) were removed from the analysis because they failed to learn: their learning scores were less than 11 and therefore no different than the chance score of 9. In addition, one participant in the Concrete condition was removed because the learning score was more than 2.5 standard deviations below the mean of that group.

Participants in all conditions successfully learned the rules in the learning domain (see Figure 7.3). Test scores were significantly above chance, one sample t-tests, \( t(17) > 12.09, ps < .001 \). No differences were found in learning between the conditions. Learning scores were submitted to a one-way ANOVA with learning condition as a factor. There was no effect of condition, \( F(2, 51) = 1.91, p = .159, \eta_p^2 = .070 \).

While there were no differences in learning across conditions, there were marked differences in transfer, ANCOVA with learning score as a covariate, \( F(2, 50) = 11.10, p < .001, \eta_p^2 = .308 \). Learning score was also a contributing factor, \( F(1, 50) = 7.86, p < .008, \eta_p^2 = .136 \). Thus, while level of learning contributes to ability to transfer, the specific instantiation learned affects ability to transfer more than learning. Participants in the Concrete condition transferred significantly less than those in the other two, one-way ANOVA, \( F(2, 51) = 8.00, p < .002, \eta_p^2 = .239 \), post-hoc Tukey, \( ps < .009 \).
As in the previous experiments, similarity ratings were high for conditions with successful transfer and low for conditions with low levels of transfer. The given similarity scale ranged from 1, completely dissimilar, to 5, structurally identical. Participants in the Relational diagram condition and the Generic condition rated the learning and transfer domains as highly similar ($M = 4.22, SD = 1.17$ and $M = 4.89, SD = .323$ respectively). Ratings for the Concrete group were considerably lower ($M = 2.89, SD = 1.08$) than the other groups, one-way ANOVA, $F(2, 51) = 21.31, p < .001, \eta^2 = .455$, post-hoc Tukeys $ps < .001$.

Similarly, the majority of participants in the Generic and Relational diagram conditions accurately matched elements between the learning and transfer domains (78% and 83% respectively). However, only 33% of the participants in the Concrete condition were able to make the correct correspondence. Note that the expected proportion due to random
guessing would be 33%. The proportions were clearly different, $\chi^2 (df = 2, N=54) = 11.86, p < 0.004$.

A different pattern of performance was found on the second transfer test (see Figure 7.4). While participants in the Generic condition transferred well on the first transfer test, they failed to do so on the second. Their transfer scores ($M = 4.1, SD = 3.31$) were no better than a chance score of 3, one sample t-test, $t (17) = 1.43, p = .171$. At the same time, participants in both the Diagram and Concrete conditions scored above chance ($M = 6.78, SD = 4.11$ for the Relational Diagram and $M = 6.06, SD = 3.51$ for the Concrete), one sample t-test, $t (17)s > 3.69, ps < .003$.

![Figure 7.4](image-url)  

**Figure 7.4** Mean percentage above chance score on transfer test 1 and transfer test 2 of Experiment 6. Error bars represent standard error of the mean.

The differences in performance on the second task were even more dramatic when the distribution of scores was considered (see Table 7.1). Ten participants in the Relational Diagram condition received perfect scores, while four in the Concrete condition received only
one in the Generic condition scored perfectly. The proportion of participants in the Generic condition scoring 8 or above was lower than that of the other two conditions, chi-squared test between the proportion of participants scoring 8 – 10 and the proportion of participants scoring 0 – 7 found a clear differences across condition, $\chi^2 (df = 1, N = 54) = 5.40, p < 0.025$.

<table>
<thead>
<tr>
<th></th>
<th>Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Score: 0 - 4</td>
</tr>
<tr>
<td>Generic</td>
<td>11</td>
</tr>
<tr>
<td>Relational Diagram</td>
<td>7</td>
</tr>
<tr>
<td>Concrete</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 7.1 Distribution of scores on transfer test 2 of Experiments 6.

Because the second transfer domain was a mathematical group of order 4, not of order 3 as presented in the learning phase, these results suggest that the participants in the Diagram and Concrete conditions gleaned general knowledge of mathematical group structure. The low scores of participants in the Generic condition suggest the knowledge they acquired during learning allowed them to transfer to an isomorph, but was not knowledge that could easily be adapted to a similar type of system of a larger order. Participants in the Diagram and Concrete conditions were able to modify their knowledge and transfer it to a similar system of larger order. These findings indicate that during the course of learning these participants formed an abstract schema that was flexible enough to be modified to higher-order groups.
The Concrete condition produced interesting transfer results. Participants in this condition were not able to transfer well to the first transfer domain and yet they transferred very well to the second. This differential performance is most likely due to the different degrees of concreteness of the two transfer domains. Because transfer domain 1 was very concrete, participants in the Concrete condition faced a difficult task of transfer between two concrete domains. They generally failed to align structure and successfully transfer. This was an expected result as structural alignment between two concrete situations it has been shown to be difficult (see Experiments 2 and 3 and Markman & Gentner, 1993).

When presented with the second transfer domain, there were two factors that likely facilitated transfer. First, the second transfer domain was a very perceptually sparse external representation involving simple arbitrary symbols, thus extraction of structure would be easier than from a concrete domain. Second, the Concrete condition involved a visual display that possessed two of the three characteristics of a relational diagram. The race track with markers is a perceptual symbol for the relations with place holders for the elements. However, the third requirement for relational diagrams, minimal extraneous information, was violated. Participants in the Concrete condition learned a very concrete scenario involving a perceptually rich display. When presented with a concrete transfer domain, the concreteness of both the learning and transfer domains created an obstacle to structural alignment and transfer. However, when a novel generic domain was presented, participants were able to recognize structure and utilize the diagrammatic aspects of the learning domain.
CHAPTER 8

EFFECTS OF CONCRETENESS ON CHILDREN’S LEARNING AND TRANSFER

The results of the previous experiments demonstrated that relevantly concrete instantiations may promote quick learning, but these instantiations significantly hinder transfer. At the same time, generic instantiations can facilitate transfer. Moreover, when protracted training was given to students, the generic instantiation was learned equally well as the relevantly concrete instantiation. The participants of these experiments were adults and therefore the benefit of generic instantiations for learning and transfer can not necessarily be generalized to children. Perhaps relevant concreteness would have a different effect on children’s ability to learn and transfer mathematical concepts. The purpose of Experiment 7 was to investigate the effect of relevant concreteness on learning and transfer for children.

From one perspective, children may need a concrete instantiation to begin to grasp an abstract concept. This argument finds support in constructivist theories of development (e.g. Inhelder & Piaget, 1958) that posit that development proceeds from the concrete to the abstract and therefore learning should do the same. In addition, concrete instantiations may
be more appealing to children than traditional generic symbols; and therefore children may be more engaged in learning (Ball, 1992; Moyer, 2001).

On the other hand, if the inability experienced by learners of relevantly concrete instantiations to recognize structure in the context of novel elements is related to attentional focus, then concreteness may be at least as detrimental for children’s transfer as it is for adults’. Children are less able to control their focus of attention than adults (Dempster & Corkill, 1999; Napolitano & Sloutsky, 2004). In addition, it has been well documented that children have difficulty using concrete objects as symbols for other entities (DeLoache, 2000). If concreteness creates an obstacle to successful transfer of such simple relations, it is likely to make transfer of complex abstract relation such as that of mathematical concepts difficult. Therefore, there are reasons to believe that even with younger participants, abstract, generic instantiations could be more advantageous for transfer than concrete instantiations.

The purpose of the Experiment 7 was to investigate the effects of relevant concreteness on children’s ability to learn and transfer conceptual knowledge. Fifth and sixth grade students were chosen to participate because according to Piagetian Theory, children of this age are still in the concrete operational stage in which their thinking and problem solving is bound to the concrete (Inhelder & Piaget, 1958). At the same time, these children are familiar with learning some complex concepts and strategies and manipulating some symbolic expressions. For example, fifth and sixth graders are generally competent users of subtraction and multiplication algorithms for multi-digit numbers.
EXPERIMENT 7

Method

Participants

Twenty-one fifth grade students (mean age 11.2 years) and nineteen sixth-grade students (mean age 11.8 years) from elementary and middle schools in suburbs of Columbus, Ohio participated in the experiment. Children were randomly assigned to one of two conditions that specified the type of instantiation they learned, either generic or relevantly concrete.

Materials and Design

The design was similar to that of Experiment 2. The experiment included two phases: (1) training and testing in a learning domain and (2) testing of the transfer domain. Participants were randomly assigned to one of two conditions that specified the learning domain: Relevantly Concrete or Generic. The relevantly concrete, generic and transfer instantiations were the same as those used in the previous experiments. Training and testing was similar to that of Experiment 2, but slightly shorter in length (see Appendix J for Learning and Appendix K for Transfer). The learning and transfer tests were isomorphic and consisted of sixteen multiple-choice questions that were similar to those of Experiment 2. Following the multiple-choice questions, participants were asked to match corresponding elements across learning and transfer domains and then indicate a level of similarity between the two domains.

Procedure

Training and testing was presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their
responses were recorded. A female experimenter was present while the children completed the activity.

Results and Discussion

Three participants (two Generic condition: one fifth grader, one sixth grader and one Relevantly Concrete condition: one sixth grader) were eliminated from the analysis because their scores in the learning domain (Phase 1) were less than 8 and therefore no different than chance.

In both conditions participants successfully learned the base domain, with mean learning scores being significantly above chance score of 6, one sample t-tests, \( t_s > 10.08, ps < .001 \) (see Figure 8.1). The relevantly concrete instantiation did result in higher learning scores than did the generic instantiation. Learning scores were submitted to an ANCOVA with grade (fifth or sixth) as a covariate. There was a significant effect of condition, \( F(1, 33) = 14.2, p < .002, \eta^2_p = .30 \). There was no difference in learning across the two grade levels, \( F(1, 33) = 0.084, p = .774, \eta^2_p = .003 \). The mean learning scores were 13.5 (SD = 2.25) for fifth graders and 12.5 (SD = 1.60) for sixth graders in the relevantly concrete condition and 10.2 (SD = 2.2) for fifth graders and 10.9 (SD = 1.7) for sixth graders in the generic condition.

While relevant concreteness offered an advantage for learning, there was a clear advantage of generic instantiations for transfer. These findings were supported by an ANCOVA with learning score and grade level as covariates. There was a significant effect of condition, \( F(1, 32) = 9.1, p < .006, \eta^2_p = .222 \), and no effect of grade level, \( F(1, 32) = 1.04, p = .314, \eta^2_p = .032 \). The mean transfer scores were 7.9 (SD = 2.25) for fifth graders and 7.0 (SD = 2.5) for sixth graders in the relevantly concrete condition and 8.33 (SD =
1.9) for fifth graders and 11.1 \( (SD = 4.0) \) for sixth graders in the generic condition. Also, there was a marginal effect of learning on transfer, \( F (1, 32) = 4.47, p = .042, \eta_p^2 = .12. \) Furthermore, transfer scores in the Generic condition were significantly different from chance, one-sample t-test, \( t (17) = 4.66, p < .001. \) At the same time in the Relevant Concreteness condition, transfer scores were only marginally different from chance, one sample t-test \( t (18) = 2.06, p = .055. \)

![Bar chart showing mean test scores for Experiment 7.](image)

**Figure 8.1** Mean Test Scores for Experiment 7. Note: Error bars represent standard error of mean. Chance score is 6.

Additional analyses focused on the ability to match corresponding elements across domains, which differed markedly between the Generic condition and the Relevant Concreteness condition. Only 26% of participants in the Relevant Concreteness condition correctly matched elements. While 72% of participants in the Generic condition made the correct correspondences, \( \chi^2 (df=1, N=37) = 7.8, p < 0.006. \) Furthermore, there was a high correlation between matching ability and test score, point biserial correlation, \( r_{pb} = .69, p < \)
The mean transfer score for those who made the correct matching was 11.0 (SD = 3.20), while the mean score for those who did not make the correct matching was 6.32 (SD = 1.70). This difference was clearly significant, independent samples t-test, \( t(35) = 5.610 \) \( p < .001 \).

Similarity ratings also differed as a function of ability to match elements. Participants who correctly matched elements rated the domains as highly similar, mean of 4.2 (SD = .981) on a scale from 1 (completely different) to 5 (almost identical). At the same time, participants who did not match elements correctly gave a mean similarity rating of 3.2 (SD = 1.08). Again this was a significant difference, independent samples t-test, \( t(33) = 2.77, p < .01 \). Taken together, these findings suggest that those participants who aligned the two domains exhibited a greater ability to match elements between the domains, perceived the domains as more similar, and exhibited greater transfer. Furthermore, the likelihood of alignment was greater with generic than with relevantly concrete instantiations.

In sum, as with adult participants, generic instantiations resulted in significantly higher transfer than concrete instantiations. As in the previous experiments, high similarity ratings and ability to match elements supports the argument that a generic instantiation can promote transfer by facilitating recognition and alignment of structure. The fact that the generic condition resulted in slightly lower learning scores clearly demonstrates that while learning is necessary for successful transfer, it is not sufficient. Transfer is also a function of the type of instantiation learned.
CHAPTER 9

GENERAL DISCUSSION

The results of this research demonstrate that transfer is not simply a function of learning performance. Transfer is also a function of the type of instantiation initially learned. Concrete instantiations are packed with extraneous information that can hinder transfer. At the same time, generic instantiations can be learned equally well and can in turn promote transfer.

Summary of Findings

This research differentiated two types of concreteness that may occur through instantiations of abstract concepts such as mathematical concepts. Irrelevant concreteness is extraneous information that is unrelated to the to-be-learned concept and has been shown in previous research to hinder both learning and transfer (Sloutsky, Kaminski, & Heckler, 2005). Some concrete instantiations may involve symbols and storyline that help communicate the to-be-learned conceptual structure. Such relevant concreteness can promote quick learning (see Experiment 1). However, the ease of initial learning comes at the cost of transfer. Experiment 2 demonstrated that with slightly protracted training, generic and relevantly concrete instantiations resulted in equal levels of learning. Most importantly, participants who learned the relevantly concrete instantiation were unable to
transfer their knowledge to a novel isomorph. At the same time, participants who learned a generic instantiation were able to successfully transfer knowledge to a novel isomorph. While both relevant and irrelevant concreteness resulted in lower transfer than generic, they appear to do so for different reasons. Participants in the generic and irrelevantly concrete conditions were able to match corresponding elements across the learning and transfer domains and rated the two domains as highly similar. However, participants in the relevantly concrete condition were not able to correctly match elements and did not find the domains to be highly similar.

Low similarity ratings suggest that participants who learned the relevantly concrete instantiation were unable to recognize the analogy between the two domains, as alignable structures are perceived to be more similar than non-alignable structures (Markman & Gentner, 1993). Inability to correctly match elements is more direct evidence suggesting that these participants could not recognize and align structure. Therefore, transfer failure for relevantly concrete instantiations appears to be due to an inability to recognize and align common structure.

In Experiment 3, participants who learned the relevantly concrete instantiation and were given the correct correspondence of elements across domains successfully transferred their knowledge to the novel domain. Giving the correspondence of elements resulted in no additional benefit for participants who learned the generic instantiation. These findings support the proposition that learning a relevantly concrete instantiation makes structural recognition and alignment difficult thus resulting in transfer failure.

The results of Experiments 1 – 3 demonstrate that learning is not sufficient for transfer. Furthermore, differential transfer performance with equally high levels of
learning suggests that categorically different internal representations were constructed when learning instantiations that communicate relevantly concreteness versus those that do not. More specifically, the results of Experiment 4 were that participants who learned the generic or irrelevantly concrete instantiation were able to recognize learned structure when presented with novel elements. However, participants who learned the relevantly concrete instantiation were unable to recognize structure in the context of novel elements. Taken together, these findings suggest that learning a relevantly concrete instantiation results in an internal representation from which isomorphs cannot be recognized. Learning an irrelevantly concrete instantiation does allow recognition of analogous domains, but can negatively affect other aspects of analogy implementation.

In Experiment 5, some participants learned more than one instantiation. The findings were that learning two or three relevantly concrete instantiations did not result in significant transfer. When learning involved a generic instantiation, participants did successfully transfer. In particular, learning one generic instantiation resulted in higher transfer than when learning was divided between the relevantly concrete instantiation and then the generic instantiation. Also, giving participants the correspondence of elements across two learning instantiations led to no improvement in transfer compared to not being told the correspondence. These findings indicate that generic instantiations have a clear advantage for transfer.

Learning a generic instantiation appears to result in an internal representation which allows structural recognition, alignment, and successful transfer to novel contexts. Many might argue that a conceptual representation that is abstract enough to allow transfer to a superficially dissimilar isomorph is a schema (see Reed, 1993). However, from the
results of Experiments 2 – 5, it is not clear whether learning a generic instantiation results in the construction of an abstract schematic representation or whether successful transfer occurred because of a less abstract process such as case-based reasoning.

Experiment 6 was designed to test the degree of abstractness and flexibility of the internal representation formed from the generic instantiation by considering transfer to a non-isomorphic mathematical group of order 4. Because there is no one-to-one mapping between learning and transfer domains, successful transfer to the group of order 4 cannot be accomplished through case-based reasoning. Instead, transfer would require the knowledge of higher-order principles that define a mathematical group, such as existence of an identity element. Participants who learned the generic instantiation successfully transferred to a group of order 3, but were unable to transfer to the higher-order group. However, participants who learned the relational diagram ably transferred to novel groups of order 3 and 4. The differential transfer ability to the non-isomorphic domain is likely due to different types of information directly available from the learned visual displays. The generic instantiation presented the system in a sentential format in which an individual statement represented only partial structure. The relational diagram visually depicted the entire system and communicated global structure. These findings suggest that learning a relational diagram may result in a representation that is more schematic and would allow transfer to novel isomorphs as well as modification and transfer to non-isomorphs of the same structure category.

Finally, the results of Experiment 8 extend the findings of Experiment 2 to children. While the relevantly concrete instantiation did result in slightly higher learning, children were able to learn the generic instantiation. Most importantly, learning the generic
instantiation resulted in significant transfer while learning the concrete instantiation did not. As with adults, learning the relevantly concrete instantiation left participants unable to correctly match analogous elements across domains suggesting that transfer failed due to an inability to recognize and align structure.

**Mechanisms of Transfer**

The present research does more than demonstrate a phenomenon, it adds to the theory of analogical transfer. Much of previous research has documented successes and failures of analogical transfer (Gick & Holyoak, 1980, 1983) and demonstrated that in familiar domains where relations are known, even children reason analogically (Alexander et al., 1989; Brown & Kane, 1988; Brown, 1989; Gentner, 1988; Gentner et al., 1995). However, when learning novel relations, such as problem-solving strategies, spontaneous transfer typically does not occur (Gick & Holyoak, 1980, 1983; Novick & Holyoak, 1991). Some research has focused on the effects of common surface features promoting recall and alignment (Gentner, Rattermann, & Forbus, 1993; Ross, 1987, 1989). The general finding is that surface similarity can prompt retrieval of an analogous domain and common elements can drive alignment correctly or incorrectly depending on whether or not they are playing the same structural role across domains. The current research considered the acquisition of a novel mathematical concept and has demonstrated that in the absence of glaring similarities, extraneous information in a learned concrete instantiation can significantly hinder transfer to a novel isomorph. Concreteness that helped initial learning was found to be particularly detrimental to transfer.

These findings clearly indicate that transfer is more than a function of relational knowledge and similarity of domains. The proposed mechanism responsible for
differential transfer is attentional focus during the course of initial learning. When a specific conceptual instantiation is successfully learned, the learner constructs an internal representation that contains elements and other superficial features as well as defining relational structure. However, the attention given to elements versus relations may be vastly different, resulting in different representational weights. Concrete instantiations are packed with extraneous information that captures attention, allowing little of limited attentional resources for relational structure. For generic instantiations, bland symbols and storylines carry less information and consequently have less power to grab attention. More attentional resources are available to be given to relational structure; and as a result the representation of structure is not overwhelmed by representation of the superficial. Strong representation of structure likely results in ability to demonstrate relational knowledge in the learning domain as well as the ability to recognize structure elsewhere which is pivotal to successful transfer.

Therefore, differential attentional focus offers an explanation for poor transfer from concrete instantiations as well as lower learning levels for irrelevantly concrete instantiations in comparison to generic instantiations. Furthermore, differential attentional focus can explain differences in structural recognition and transfer ability between conditions of relevant versus irrelevant concreteness. All concreteness is likely to grab attention, but for relevant concreteness, ease of learning may require less deliberate attention to relational structure. In addition, because concrete elements grab attention and easily communicate structure, the representational position of relational structure is all but subsequent to that of elements. If relations are subsequent to elements in the representation, access to relations in the absence of elements would be virtually
impossible. Thus, students who learned the relevantly concrete instantiation were unable to recognize structure when presented with novel elements.

Learning a relevantly concrete instantiation appears to lead to encapsulated relational knowledge where relations are known but only in the learning context. Giving the correspondence of elements across the learned and novel domains helped the learner align the domains and successfully transfer. However, providing the correspondence between two concrete instantiations resulted in no benefit for transfer to a third. These findings suggest that the encapsulation of relational knowledge acquired from relevantly concrete instantiations is rather robust. Making the correspondence of elements to a novel domain allowed transfer to that domain, but does not appear to disembed relations in order to be recognized in another context.

Some research has demonstrated that learning two instantiations as opposed to one increased the likelihood of transfer (Gick & Holyoak, 1983). Yet, the present research found no transfer from two concrete instantiations and very little from three. A possible explanation for this discrepancy is that transfer is also a function of the complexity of the to-be-learned concept. Analogies involving simple concepts can often efficiently communicate conceptual information via transfer of relations from a familiar domain to a novel one. For example, the heart is like a pump. However, mathematical concepts, including that considered in the present research, are considerably more complex than concepts involved in everyday analogies as well as the Gick and Holyoak study. The solution strategy for the radiation problem can be stated in a simple sentence, “use multiple points of convergence”. The concept of a commutative mathematical group requires stating four principles. As concept complexity increases, the likelihood of
successful transfer from a concrete situation most likely decreases. Concept complexity is another factor in the process of transfer that merits future investigation.

For complex abstract concepts such as mathematical concepts, schema formation and successful transfer is not guaranteed to result from learning more than one concrete instantiation. The results of the present research suggest that a direct route to creating an abstract internal representation (that shares the skeletal characteristics of a schema) is to learn the concept via a generic instantiation such as an external representation involving traditional symbolic expressions. In this research, the highest level of transfer occurred when learning involved only the generic instantiation. Participants who learned the relevantly concrete instantiation and the generic instantiation successfully transferred, but not as well as those who learned only the generic. Therefore, learning the relevantly concrete instantiation prior to the generic provided no additional benefit for transfer. The performance of the participants in this condition makes another interesting point. These participants were able to align the elements of the relevantly concrete domain and the transfer domain. This finding suggests that learning a generic instantiation allows the learner to align learned structure not only between the generic and novel concrete domains, but also between two concrete domains. This alignment was not possible for those who only learned the relevantly concrete instantiation.

While knowledge gleaned from a generic sentential instantiation can facilitate recognition of common structure in concrete contexts and can promote successful conceptual transfer, it is not spontaneously adapted to non-isomorphic domains of the same structure category. At the same time, learning a relational diagram resulted in transfer to both the isomorphic and non-isomorphic mathematical groups. These findings
suggest that the constructed internal representations directly reflect the information in the learned instantiation, particularly the information available in the visual displays. The sentential form of the generic instantiation depicted partial structure through each individual symbolic expression; and it appears that this local structure was recognized and transferred to the isomorph. On the other hand, the relational diagram captured the global structure of the system through the visual display. Learning the relational diagram appears to produce an internal representation that depicts global structure of the system and allows for successful transfer as well as modification.

Relational diagrams have a clear advantage for maximizing transfer. In addition, if learning a relational diagram results in an internal representation that depicts global knowledge, then it is reasonable to think that such as representation may also facilitate reification of the concept and subsequent acquisition of higher-order knowledge. For example, knowledge of mathematical groups can lead to knowledge of other algebraic systems such as rings and fields, which in turn can lead to knowledge of theories such as Galois Theory. Therefore, there are reasons to believe that relational diagrams can lead to broad transfer of knowledge as well as hierarchical building of knowledge. The second prediction merits future investigation.

The advantages of relational diagrams may be tremendous, yet relational diagrams may not exist for all mathematical concepts. As Larkin and Simon (1987) demonstrated, not all diagrams are effective. Most likely, relational diagrams need to be carefully developed by experts in the field in conjunction with learning researchers, to construct visual displays that can capture relevant global relations. In the field of mathematics, relational diagrams have been developed by mathematicians and have endured
throughout history. The Cartesian Plane is a stellar example of a relational diagram that easily communicates the relationship between two variables through a perceptual representation. Experts are aware of the relevant relations of a specific concept and how they build upon other relations within a domain. Furthermore, experts are more likely to realize potential limitations of particular diagrams. For example, the circular diagram used in Experiment 6 well represented the structure of the mathematical groups considered in this research. However, this diagram is limited to cyclic groups and therefore would not be applicable to all groups. For example, there are actually two groups of order 4, the cyclic group, as considered in this study, and the non-cyclic group.

Regardless of the structural form, superficial information is likely retained in the internal representation and can potentially divert attention from relational structure. From both the concrete sentential instantiation involving containers of liquid (Experiments 2, 3, 4, 5, and 7) and from the race car instantiation that helped convey global structure (Experiment 6), transfer to the isomorph was very poor. The fact that participants who learned the race car domain did transfer to the group of order 4 makes two points. First, like the relational diagram, the race track display helped to communicate global structure. However the concreteness of the context distracted from structure and hindered transfer to the novel concrete isomorph. Second, transfer to the non-isomorphic group was easier because the transfer domain was generic involving statements of symbols. The second point suggests that transfer from a concrete instantiation to a generic is easier than transfer from a concrete to a concrete. This is a novel finding that will be further investigated in future research.
Conclusions

The general conclusion of this research is that our internal representations of abstract concepts are a direct consequence of the conceptual instantiations we learn. For mathematical concepts that are more complex than every day concepts, concrete instantiations, particularly those that may help communicate structure, can result in internal representations dominated by superficial information. As a result relational knowledge is often bound to the learning context, unable to be recognized elsewhere. While learning multiple concrete instantiations may result in the formation of an abstract schema (Gick & Holyoak, 1983; Novick & Holyoak, 1991), schema formation and successful transfer may also fail for novel complex concepts (Experiment 5). Instantiating an abstract concept in concrete contexts places the additional demand on the learner of ignoring irrelevant, salient superficial information, making the process of abstracting common structure more difficult than if a generic instantiation were considered.

Learning a generic instantiation can be a direct route to the formation of an abstract internal representation that can facilitate transfer. For some concepts, a diagram may help depict global structure, which is in turn reflected in the internal representation allowing the learner to transfer structure, modifying it when necessary.
REFERENCES


Gentner, D., Loewenstein, J., & Thompson, L. (2003). Learning and transfer: A general role for analogical encoding. *Journal of Educational Psychology, 95*, 393-408.


APPENDIX A

TRAINING AND TESTING FOR EXPERIMENT 1

The following pages present the training and testing of the Generic Domain. The training and testing for the Perceptually Rich (Irrelevantly Concrete) Condition was identical except it involved these perceptually rich symbols:
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.

Press the space bar to advance when there is no question.

Allow me to introduce myself. I am the world renowned archaeologist, Maximilian Peabody.

WHAT?? You say you have never heard of me!

Well, I have made numerous discoveries of profound importance.

Recently, I have been honored by my colleagues who presented me with the prestigious Carter Award for Archaeological Finds.

Let me tell you about the discovery for which I was bestowed with this great honor.

It was a few years back. My team and I were in the barren desert of Wadi Schmadi. Serendipitously, we discovered tablets and papers, of a sort, on which symbols were inscribed.

Here is an example of a tablet with an inscription:

After long and careful analysis, I determined that the inscriptions follow specific rules.

In fact, it seems that the inscriptions are statements in a sort of language.

The statements are made by combining symbols.
### Combining Symbols

This combination of symbols gives a predictable resulting symbol. It is written as:

$$\text{Symbol}_1 \ , \ \text{Symbol}_2 \rightarrow \text{Resulting Symbol}$$

### The Symbols

There are many symbols present in the inscriptions, but I realized that they fell into three categories.

Different symbols of the same category have the same meaning within a statement.

### So, we use

- to mean any member of the Flag category,
- to mean any member of the Circle category, and
- to mean any member of the Diamond category.

### For example, if we want a symbol from a specific category in one of the blanks below,

we can use any symbol from that category. The meaning will be the same.

$$\_\_\_\_\_\_\_ \ , \ \_\_\_\_\_\_\_ \rightarrow \_\_\_\_\_\_\_$$

### Rules of Combining Symbols:

1. The order of the two symbols on the left does not change the result.

For example

- $$\downarrow \ , \ \downarrow \rightarrow \downarrow$$

is the same thing as

- $$\downarrow \ , \ \downarrow \rightarrow \downarrow$$

Rule 2. When any symbol combines with $$\downarrow$$, the result will always be the other symbol.

For example:

- $$\downarrow \ , \ \downarrow \rightarrow \downarrow$$ and
- $$\bullet \ , \ \downarrow \rightarrow \downarrow$$

Rule 3. $$\bullet \ , \ \downarrow \rightarrow \downarrow$$

Rule 4. $$\bullet \ , \ \bullet \rightarrow \downarrow$$

Rule 5. $$\downarrow \ , \ \bullet \rightarrow \bullet$$

So, you have the idea. Statements are made by combining the symbols $$\downarrow \ , \ \bullet \ , \ \downarrow$$

Now let me teach you the rules of combination.
There is one more thing you need to know about the language... how more than two symbols combine...

<table>
<thead>
<tr>
<th>Rule 6. The result does not depend on which two symbols combine first.</th>
</tr>
</thead>
<tbody>
<tr>
<td>For example: [ \text{\ding{67}}, \text{\ding{62}}, \text{\ding{67}} \rightarrow \text{\ding{62}} ]</td>
</tr>
<tr>
<td>It does not matter if we do [ \text{\ding{67}}, \text{\ding{62}} ] first and then [ \text{\ding{67}} ] or [ \text{\ding{62}}, \text{\ding{67}} ] first and then [ \text{\ding{67}} ].</td>
</tr>
</tbody>
</table>

Let's find the resulting symbol:
\[ \text{\ding{67}}, \text{\ding{62}}, \text{\ding{67}} \rightarrow \_ \_ \_ \]

First \[ \text{\ding{67}}, \text{\ding{62}} \rightarrow \text{\ding{62}} \]
Next we have \[ \text{\ding{62}}, \text{\ding{67}} \rightarrow \text{\ding{67}} \]
So the resulting symbol is \[ \text{\ding{67}} \]

Let's summarize Key Ideas for the specific rules for combining symbols...

**Summary key ideas:**
\[ \text{\ding{62}}, \text{other symbol} \rightarrow \text{other symbol} \]
For example:
\[ \text{\ding{62}}, \text{\ding{67}} \rightarrow \text{\ding{67}} \]
\[ \text{\ding{62}}, \text{\ding{62}} \rightarrow \text{\ding{62}} \]
\[ \text{\ding{62}}, \text{\ding{62}} \rightarrow \text{\ding{62}} \]

Remember that order doesn't matter and these key ideas.
That is all you need to know!

<table>
<thead>
<tr>
<th>Summary key ideas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Here are the results of the other possible combinations:</td>
</tr>
<tr>
<td>[ \text{\ding{62}}, \text{\ding{67}} \rightarrow \text{\ding{62}} ]</td>
</tr>
<tr>
<td>[ \text{\ding{62}}, \text{\ding{62}} \rightarrow \text{\ding{62}} ]</td>
</tr>
<tr>
<td>[ \text{\ding{67}}, \text{\ding{67}} \rightarrow \text{\ding{62}} ]</td>
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</table>

<table>
<thead>
<tr>
<th>key ideas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \text{\ding{62}}, \text{other symbol} \rightarrow \text{other symbol} ]</td>
</tr>
<tr>
<td>[ \text{\ding{62}}, \text{\ding{67}} \rightarrow \text{\ding{62}} ]</td>
</tr>
<tr>
<td>[ \text{\ding{62}}, \text{\ding{62}} \rightarrow \text{\ding{62}} ]</td>
</tr>
<tr>
<td>[ \text{\ding{67}}, \text{\ding{67}} \rightarrow \text{\ding{62}} ]</td>
</tr>
</tbody>
</table>
Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

1. Find the resulting symbol
   \[ \bullet, \ast \rightarrow \_ \_ \_ \]
   1.) \[ \ast \] 2.) \[ \bullet \] 3.) \[ \bullet \]

2. Find the resulting symbol
   \[ \bullet, \flat, \flat \rightarrow \_ \_ \_ \]
   1.) \[ \ast \] 2.) \[ \bullet \] 3.) \[ \bullet \]

3. What symbols go in the blanks to make a correct statement?
   \[ \_ \_ \_ \_ \_ \rightarrow \bullet \]
   1.) \[ \ast \] and \[ \ast \] 2.) \[ \bullet \] and \[ \bullet \]
   3.) \[ \ast \] and \[ \bullet \] 4.) \[ \ast \] and \[ \ast \]

4. What goes in the blanks to make a correct statement?
   \[ \_ \_ \rightarrow \ast \]
   1.) \[ \ast \] and \[ \flat \] 2.) \[ \bullet \] and \[ \flat \]
   3.) \[ \ast \] and \[ \bullet \] 4.) \[ \ast \] and \[ \ast \]

5. What goes in the blank to make a correct statement?
   \[ \ast \_ \rightarrow \_ \_ \]
   1.) \[ \ast \] 2.) \[ \bullet \] 3.) \[ \bullet \]

6. What can go in the blank to make a correct statement?
   \[ \_ \_ \rightarrow \_ \_ \_ \_ \_ \_ \]
   1.) \[ \ast \], \[ \ast \] 2.) \[ \bullet \], \[ \bullet \]
   3.) \[ \ast \], \[ \bullet \] 4.) \[ \bullet \], \[ \bullet \]

7. What expression has the same result as the following expression?
   \[ \flat, \bullet, \ast, \_ \_ \_ \_ \rightarrow \_ \_ \_ \_ \]
   1.) \[ \ast \], \[ \bullet \] 2.) \[ \bullet \], \[ \bullet \]
   3.) \[ \bullet \], \[ \bullet \] 4.) none of the above
8. Some of my team members were discussing what symbol could be placed in the first blank below. Which of their responses do you agree with?

\[
\text{__, }, \text{ }, \text{ }, \text{ }, \text{ } \rightarrow \text{ }
\]

1.) any symbol  
2.) any symbol except ◆  
3.) any symbol except ●  
4.) any symbol except □

9. When we were analyzing tablets, I overheard two of my team members talking. They were arguing about whether these inscriptions mean the same thing (have the same result!).

What do you think?

1 - Same
2 - Different

11. Do the following give the same result?

\[
\text{__, }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ } \rightarrow \\
\text{__, }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ } \rightarrow \\
\]

1 - Yes  
2 - No

12. What goes in the blanks to make a correct statement?

\[
\text{__, }, \text{ }, \text{ }, \text{ } \rightarrow \\
\]

1.) □ and □  
2.) ◆ and ◆  
3.) ● and ●  
4.) □ and ◆

13. Which of the following symbols combine to give ◆ ?

1.) □ and ●  
2.) ◆ and ◆  
3.) ● and ●  
4.) none of the above

14. How many ●'s could combine with themselves to get □?

1.) four  
2.) five  
3.) six  
4.) seven

15. Find the resulting symbol

\[
\text{__, }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ }, \text{ } \rightarrow \\
\]

1.) □  
2.) ◆  
3.) ●

16. What symbol goes in the blank to make a correct statement?

\[
\text{●, ●, ●, ●, ●, ●, ●, ●, __ } \rightarrow \text{◆}
\]

1.) □  
2.) ◆  
3.) ●  
4.) we need more information to answer
17. When we were working, a tablet was broken. We tried to figure out what it stated.

We did not know the result, but we did know that there were two symbols on the left; one of them was ●. We were trying to figure out what the result could be. Here are some opinions of my team members. Which do you agree with?

1) the result could be any symbol
2) the result could only be ▲ or ▼
3) the result can only be ♠

18. Find the resulting symbol

▲, ▼, ♠ → ____

1) ▲ 2) ♠ 3) ●

19. Do the following statements mean the same thing?

▲, ▼, ♠, ▼, ● →

▲, ♠, ●, ▼ →

1 - Yes 2 - No

20. Do the following statements mean the same thing?

●, ●, ●, ●, ●, ● →

♠, ♠, ♠, ♠, ♠, ♠ →

1 - Yes 2 - No

21. Do the following statements mean the same thing?

●, ●, ●, ●, ●, ●, ●, ● →

♠, ♠, ♠, ♠, ♠, ♠, ♠, ♠ →

1 - Yes 2 - No

22. What goes in the blank to make a correct statement?

▲, ♠, ♠, ♠, ___ → ♠

1) ▲ 2) ♠ 3) ●

23. What is the result of the following?

▲, ●, ●, ●, ●, ●, ●, ● →

1) ▲ 2) ● 3) ♠

24. What goes in the blank to make a correct statement?

●, ●, ___ → ▲

1) ▲ 2) ● 3) ♠ 4) none of the above
You have completed this experiment.

Thank you for participating in our study.
The following pages present the training and testing of the Relevantly Concrete Domain. The training and testing for the Perceptually Rich (Relevantly and Irrelevantly Concrete) Condition was identical except it involved these perceptually rich symbols:
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.
Press the space bar to advance when there is no question.

Allow me to introduce myself. I am Melvin, chief engineer for efficiency at the Bubblinski Detergent Company.

At the Bubblinski Detergent Company we mix solutions of different kinds to fill containers of detergents.

Recently, we have been having some problems with the mixing of the solutions. The detergent mixes may not be correct.

I need your help to determine the amount of left-over solution that results in the mixing process. Then I will test the left-over amount to make sure that the detergent is being made correctly.

Let me tell you how the quantities of solutions are combined so that we can determine how much solution is left-over.

In the process of making the detergent, cups of solutions are combined.

Different kinds of solutions are used, but what is important is the quantities of solutions.

So to find the left-over amount, we can use \( \frac{1}{4} \), \( \frac{1}{2} \), and \( \frac{3}{4} \) to represent the different possible quantities of any of the solutions.
For example, if and are combined, the quantities will fill one detergent container and will be left-over.

1. The order by which two cups of solution are combined does not change the left-over result.

   For example, combining with has a left-over quantity of .

   And combining with has the left-over quantity .

Remember, we are filling containers, but we always want to have a cup of solution to test for quality when we are done.

Rule 2. and will fill a container, but we need a quantity of solution to test, so we consider as the left-over.

Rule 3. When any kind of cup of solution combines with , the result will always be the other solution cup.

   For example:

   When and combine, is left-over.

   And when and combine, is left-over.

Rule 4. A combination of and does not fill a container, so the left-over is .

Rule 5. A combination of and fills one container and has left-over.

Rule 6. Finally, you need to know that when mixing more than two cups of solution, the order of combining solutions does not matter. The left-over is the same no matter which cups are combined first.

   For example: When we combine , , and , the left-over is .

   It does not matter if we do and first and then OR and first and then .
To summarize,

Cups of solutions are mixed together to fill detergent containers. Your job is to find the cup with the remainder quantity.

We always want to have a quantity remaining, so the possible remainders are:

Let's figure out the left-over when \( \text{p}, \text{q}, \text{r} \) are mixed...

\( \text{p} \) and \( \text{q} \) mix together, with \( \text{r} \) left-over,
then \( \text{p} \) and \( \text{r} \) mix, leaving \( \text{q} \) left-over.

So \( \text{q} \) is our answer.

You can combine 2 or more cups of solution in any order; the left-over will be the same.

Next let's review the left-overs for specific cup combinations...

Here is what happens when \( \text{p} \) mixes with other cups.

Here are the left-overs from all of the other possible cup combinations:

<table>
<thead>
<tr>
<th>Mixing these 2 cups</th>
<th>results in this left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{p} )   ( \text{q} )</td>
<td>( \text{r} )</td>
</tr>
<tr>
<td>( \text{p} )   ( \text{r} )</td>
<td>( \text{q} )</td>
</tr>
<tr>
<td>( \text{q} )   ( \text{r} )</td>
<td>( \text{p} )</td>
</tr>
</tbody>
</table>

Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{p} )</td>
<td>( \text{q} )</td>
</tr>
<tr>
<td>( \text{q} )</td>
<td>( \text{r} )</td>
</tr>
<tr>
<td>( \text{r} )</td>
<td>( \text{p} )</td>
</tr>
<tr>
<td>Question</td>
<td>Examples</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1. What is left-over when () and () combine?</td>
<td>() and () and ()</td>
</tr>
<tr>
<td>2. What is left-over when the following cups of solution are combined?</td>
<td>() () ()</td>
</tr>
<tr>
<td>3. What possible cups of solution can combine with () to have a left-over of ()?</td>
<td>() () () and ()</td>
</tr>
<tr>
<td>4. Which cups of solution can combine to have () left-over?</td>
<td>() () () and ()</td>
</tr>
<tr>
<td>5. What can combine with () to have () left-over?</td>
<td>() () () and ()</td>
</tr>
<tr>
<td>6. What can mix with () to have () as a left-over?</td>
<td>() () () () ()</td>
</tr>
<tr>
<td>7. What combination of cups has the same left-over as the following?</td>
<td>() () () () ()</td>
</tr>
<tr>
<td>8. Some of the Bublinski employees were analyzing a batch of detergent. The left-over was ()</td>
<td>() () () () ()</td>
</tr>
</tbody>
</table>
9. Later I overheard two employees talking. They were arguing about whether these two mixtures of solution (below) would have the same left-over.

What do you think?

Mix 1:

Mix 2:

1 - Same  2 - Different

10. True or false...

When the cups are mixed, mix 1 and mix 2 will have the same left-over.

Mix 1:

Mix 2:

1 - True  2 - False

11. How about these mixtures, will they have the same left-overs?

Mix 1:

Mix 2:

1 - Yes  2 - No

12. What cups can combine with \( \text{P} \) and \( \text{P} \) to result in a left-over of \( \text{P} \)?

1.) \( \text{P} \) and \( \text{P} \)  2.) \( \text{P} \) and \( \text{P} \)

3.) \( \text{P} \) and \( \text{P} \)  4.) \( \text{P} \) and \( \text{P} \)

13. Which cups can combine to give a left-over of \( \text{P} \)?

1.) \( \text{P} \) and \( \text{P} \)  2.) \( \text{P} \) and \( \text{P} \)

3.) \( \text{P} \) and \( \text{P} \)  4.) none of the above

14. How many \( \text{P} \)'s could combine with themselves to get \( \text{P} \)?

1.) four  2.) five

3.) six  4.) seven

15. What is left-over when the following cups are mixed?

1.) \( \text{P} \)  2.) \( \text{P} \)  3.) \( \text{P} \)

16. What cup can mix with the following and have \( \text{P} \) left-over?

1.) \( \text{P} \)  2.) \( \text{P} \)

3.) \( \text{P} \)  4.) we need more information to answer
17. One day, a batch of detergent was spilled. We did not know the left-over quantity, but we did know that there were two cups in the mixture; one of them was \[ \text{cup} \]. We were trying to figure out what the left-over could have been. Here are some opinions of the employees. Which do you agree with?

1.) the left-over could be any cup
2.) the left-over could only be \[ \text{cup} \] or \[ \text{cup} \]
3.) the left-over could only be \[ \text{cup} \]

18. What is left-over when the following cups of solution combine?

1.) \[ \text{cup} \]
2.) \[ \text{cup} \]
3.) \[ \text{cup} \]

19. Do the following mixtures have the same left-overs?

<table>
<thead>
<tr>
<th>Mix 1</th>
<th>Mix 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \text{cup} ] [ \text{cup} ] [ \text{cup} ]</td>
<td>[ \text{cup} ] [ \text{cup} ] [ \text{cup} ]</td>
</tr>
</tbody>
</table>

1 - Yes
2 - No

20. Do the following mixtures have the same left-overs?

<table>
<thead>
<tr>
<th>Mix 1</th>
<th>Mix 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \text{cup} ] [ \text{cup} ] [ \text{cup} ] [ \text{cup} ]</td>
<td>[ \text{cup} ] [ \text{cup} ] [ \text{cup} ] [ \text{cup} ]</td>
</tr>
</tbody>
</table>

1 - Yes
2 - No

21. How about the following, do they have the same left-over?

<table>
<thead>
<tr>
<th>Mix 1</th>
<th>Mix 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \text{cup} ] [ \text{cup} ] [ \text{cup} ] [ \text{cup} ]</td>
<td>[ \text{cup} ] [ \text{cup} ] [ \text{cup} ] [ \text{cup} ]</td>
</tr>
</tbody>
</table>

1 - Yes
2 - No

22. What cup needs to mix with the following to have a left-over of \[ \text{cup} \]?

1.) \[ \text{cup} \]
2.) \[ \text{cup} \]
3.) \[ \text{cup} \]

23. What is left-over when the cups below are mixed?

[Image of mixed cups]

1.) \[ \text{cup} \]
2.) \[ \text{cup} \]
3.) \[ \text{cup} \]

24. What cups need to mix with \[ \text{cup} \] to have \[ \text{cup} \] left-over?

1.) \[ \text{cup} \]
2.) \[ \text{cup} \]
3.) \[ \text{cup} \]
4.) none of the above
You have completed this experiment.

Thank you for participating in our study.
APPENDIX B

TRAINING AND TESTING OF GENERIC DOMAIN OF PHASE 1 OF EXPERIMENTS 2, 3, 4, 5, AND 6
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.

Press the space bar to advance when there is no question.

| WHAT?? You say you have never heard of me! |
| Well, I have made numerous discoveries of profound importance. |

Recently, I have been honored by my colleagues who presented me with the prestigious Carter Award for Archaeological Finds.

Let me tell you about the discovery for which I was bestowed with this great honor.

It was a few years back. My team and I were in the barren desert of Wadi Schmadi. Serendipitously, we discovered tablets and papers, of a sort, on which symbols were inscribed.

Here is an example of a tablet with an inscription:

![Symbol Image]

After long and careful analysis, I determined that the inscriptions follow specific rules.

In fact, it seems that the inscriptions are statements in a sort of language.

The statements are made by combining symbols.

| Combining Symbols |
| This combination of symbols gives a predictable resulting symbol. It is written as: |

Symbol 1, Symbol 2 → Resulting Symbol
The Symbols

There are many symbols present in the inscriptions, but I realized that they fell into three categories.

Different symbols of the same category have the same meaning within a statement.

So, we use

- to mean any member of the Flag category,
- to mean any member of the Circle category,

and

- to mean any member of the Diamond category.

For example, if we want a symbol from a specific category in one of the blanks below,
we can use any symbol from that category.
The meaning will be the same.

________, _______  \rightarrow  _______

Rules of Combining Symbols:

1. The order of the two symbols on the left does not change the result.

For example

\[ \boxed{\text{ }, \bigtriangleup, \bigtriangleup} \rightarrow \bigtriangleup \]

is the same thing as

\[ \bigtriangleup, \boxed{\text{ }, \bigtriangleup} \rightarrow \bigtriangleup \]

Here is a question for you.

Suppose you know that \[ \boxed{\text{ }, \bigtriangleup, \bigtriangleup} \rightarrow \bigtriangleup \]
Then what symbol would go in the blank \[ \bigtriangleup, \text{ }, \bigtriangleup ? \]

Enter 1, 2, or 3:

1.) \[ \boxed{\text{ }} \]
2.) \[ \bigtriangleup \]
3.) \[ \bigtriangleup \]

\[ \boxed{\text{ is correct}} \]

Because the order of symbols on the left does not matter,

If \[ \boxed{\text{ }, \bigtriangleup, \bigtriangleup} \rightarrow \bigtriangleup \]
then \[ \bigtriangleup, \text{ }, \bigtriangleup \rightarrow \bigtriangleup \]

Let's try another question.

Suppose you know that \[ \boxed{\text{ }, \bigtriangleup, \bigtriangleup} \rightarrow \bigtriangleup \]
Then what symbol would go in the blank \[ \bigtriangleup, \text{ }, \bigtriangleup \rightarrow \bigtriangleup ? \]

Enter 1, 2, or 3:

1.) \[ \boxed{\text{ }} \]
2.) \[ \bigtriangleup \]
3.) \[ \bigtriangleup \]
<table>
<thead>
<tr>
<th>Rule 2.</th>
<th>When any symbol combines with $\text{□}$, the result will always be the other symbol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>For example:</td>
<td></td>
</tr>
<tr>
<td>$\text{□}, \text{♦} \rightarrow \text{♦}$ and</td>
<td></td>
</tr>
<tr>
<td>$\text{●}, \text{□} \rightarrow \text{●}$</td>
<td></td>
</tr>
</tbody>
</table>

So tell me what goes in this blank?

$\text{□}, \text{●} \rightarrow$ ____

1.) $\text{□}$  2.) $\text{♦}$  3.) $\text{●}$

Whenever $\text{□}$ combines with another symbol, the result is that other symbol.

So, $\text{□}, \text{●} \rightarrow \text{●}$

Let's try another...

What goes in the blank?

$\text{◆}, \text{□} \rightarrow$ ____

1.) $\text{□}$  2.) $\text{◆}$  3.) $\text{●}$

Whenever $\text{◆}$ combines with another symbol, the result is that other symbol.

So, $\text{◆}, \text{□} \rightarrow \text{◆}$

Now, let me tell you three more rules about specific combinations.

Rule 3. $\text{●}, \text{◆} \rightarrow \text{□}$

Rule 4. $\text{●}, \text{●} \rightarrow \text{◆}$

Rule 5. $\text{◆}, \text{◆} \rightarrow \text{●}$

There is one more thing you need to know about the language... how more than two symbols combine.
### Rule 6
The result does not depend on which two symbols combine first.

For example:

\[ \text{\textbullet}, \text{\textcheckmark}, \text{\textbullet} \rightarrow \text{\textcheckmark} \]

It does not matter if we do

\[ \text{\textbullet}, \text{\textcheckmark} \] first and then \[ \text{\textbullet} \] or

\[ \text{\textcheckmark}, \text{\textbullet} \] first and then \[ \text{\textbullet} \].

### Think about this question.
Remember the last rule... the order of symbols on the left does not matter.

If \( \text{\textbullet}, \text{\textcheckmark}, \text{\textbullet} \rightarrow \text{\textcheckmark} \) then \( \text{\textbullet}, \text{\textbullet}, \text{\textbullet} \rightarrow ? \)

1.) \[ \text{\textbullet} \]
2.) \[ \text{\textbullet} \]
3.) \[ \text{\textbullet} \]

---

**is correct.**

Remember the order of the symbols on the left does not matter.

If \( \text{\textbullet}, \text{\textcheckmark}, \text{\textbullet} \rightarrow \text{\textcheckmark} \) then \( \text{\textcheckmark}, \text{\textbullet}, \text{\textbullet} \rightarrow \text{\textcheckmark} \)

Because the same symbols appear in both. They just have a different order.

---

**is correct.**

The order of the symbols on the left does not matter.

If \( \text{\textbullet}, \text{\textbullet}, \text{\textbullet} \rightarrow \text{\textbullet} \) then \( \text{\textbullet}, \text{\textbullet}, \text{\textbullet} \rightarrow \text{\textbullet} \)

Because the same symbols appear in both. They just have a different order.

---

**Summary key idea:**

If the same symbols appear on the left,

Order does not matter.

The result is the same.

---

**Summary key ideas:**

1. \( \text{\textbullet}, \text{other symbol} \rightarrow \text{other symbol} \)

For example:

\[ \text{\textbullet}, \text{\textcheckmark} \rightarrow \text{\textcheckmark} \]

\[ \text{\textbullet}, \text{\textcheckmark} \rightarrow \text{\textbullet} \]

\[ \text{\textbullet}, \text{\textcheckmark} \rightarrow \text{\textbullet} \]
<table>
<thead>
<tr>
<th>Summary key ideas:</th>
<th>Summary key ideas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. The following combination gives a result of ꜭ.</td>
<td>3. We can make combinations to result in ◆ and  ● also:</td>
</tr>
<tr>
<td>◆ , ◆ → ꜭ</td>
<td>◆ , ● → ◆</td>
</tr>
<tr>
<td>◆ , ◆ → ●</td>
<td></td>
</tr>
</tbody>
</table>

Next, let's do more questions.

Rule reminders will appear, but try to remember the rules.

Later we won't have the reminders.

<table>
<thead>
<tr>
<th>What goes in the blank for the following statement?</th>
<th>What goes in the blank?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ꜭ , ● → ___</td>
<td>2. ◆ , ● → ___</td>
</tr>
<tr>
<td>Rule Reminders</td>
<td>Rule Reminders</td>
</tr>
<tr>
<td>1. Order of symbols on left doesn't matter</td>
<td>1. Order of symbols on left doesn't matter</td>
</tr>
<tr>
<td>2. If ꜭ goes with another symbol, the result is the other symbol</td>
<td>2. If ◆ goes with another symbol, the result is the other symbol</td>
</tr>
<tr>
<td>3. ◆ , ● → ꜭ</td>
<td>3. ◆ , ● → ◆</td>
</tr>
<tr>
<td>4. ◆ , ● → ●</td>
<td>4. ◆ , ● → ●</td>
</tr>
<tr>
<td>5. ◆ , ● → ●</td>
<td>5. ◆ , ● → ●</td>
</tr>
<tr>
<td>6. More than 2 symbols on left combine in any order</td>
<td>6. More than 2 symbols on left combine in any order</td>
</tr>
</tbody>
</table>

◆ is correct by rule #2.

<table>
<thead>
<tr>
<th>What goes in the blank?</th>
<th>What goes in the blank?</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. ◆ , ● → ___</td>
<td>3. ◆ , ◆ → ___</td>
</tr>
<tr>
<td>Rule Reminders</td>
<td>Rule Reminders</td>
</tr>
<tr>
<td>1. Order of symbols on left doesn't matter</td>
<td>1. Order of symbols on left doesn't matter</td>
</tr>
<tr>
<td>2. If ◆ goes with another symbol, the result is the other symbol</td>
<td>2. If ◆ goes with another symbol, the result is the other symbol</td>
</tr>
<tr>
<td>3. ◆ , ◆ → ◆</td>
<td>3. ◆ , ◆ → ◆</td>
</tr>
<tr>
<td>4. ◆ , ◆ → ◆</td>
<td>4. ◆ , ◆ → ◆</td>
</tr>
<tr>
<td>5. ◆ , ◆ → ◆</td>
<td>5. ◆ , ◆ → ◆</td>
</tr>
<tr>
<td>6. More than 2 symbols on left combine in any order</td>
<td>6. More than 2 symbols on left combine in any order</td>
</tr>
</tbody>
</table>
What goes in the blank?

3. ◆, ◆ → ____

- is correct by rule #5.

Rule Reminders:
1. Order of symbols on left doesn't matter
2. If ◆ goes with another symbol, the result is the other symbol
3. ◆ ◆ → ◆
4. ◆ ◆ → ◆
5. ◆ ◆ → ◆
6. More than 2 symbols on left combine in any order

What goes in the blank?

4. ◆, ◆ → ____

- is correct by rule #4.

Rule Reminders:
1. Order of symbols on left doesn't matter
2. If ◆ goes with another symbol, the result is the other symbol
3. ◆ ◆ → ◆
4. ◆ ◆ → ◆
5. ◆ ◆ → ◆
6. More than 2 symbols on left combine in any order

What goes in the blank?

5. ◆, ◆, ◆ → ____

- is right because:

So we have:

Now, you have been using the rules a little. Let me review the 3 Key Ideas.

Summary key ideas:

1. ◆, other symbol → other symbol

For example:

2. The following combination gives a result of ◆:

- ◆, ◆ → ◆

- ◆, ◆, ◆ → ◆

- ◆, ◆, ◆ → ◆
Summary key ideas:

3. We can make combinations to result in ♦ and ◇ also:
   
   ◇, ◇ → ♦
   ♦, ♦ → ◇

Let's summarize these key ideas.

Remember that order doesn’t matter and these 3 key ideas.

That is all you need to know!

key ideas:

1. ◇, other symbol → other symbol
2. ♦, ◇ → ◇
3. ◇, ◇ → ♦

Let me show you an example.

Determine the result of

♦, ◇, ◇, ♦, ♦, ♦, ♦, ♦ →

1st Take advantage of the ◇ key idea:

◇, other symbol → other symbol

So we get:

♦, ♦, ♦, ♦, ♦ →

Now we have

♦, ♦, ♦, ♦, ♦ →

Next, incorporate the other key ideas:

Because ♦, ♦ → ◇ we get:

◇, ♦, ♦ →

Next: ◇, ◇ →

Remember the Key Ideas:

1. ◇, other symbol → other symbol
2. ♦, ◇ → ◇
3. ◇, ◇ → ♦

Finally, ◇, ◇ → ♦

Let’s do some more questions without reminders…
<table>
<thead>
<tr>
<th>Determine the result of the following</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="symbols" /> <img src="image2" alt="symbols" /> <img src="image3" alt="symbols" /> <img src="image4" alt="symbols" /> → ____</td>
<td><img src="image5" alt="symbols" /> is correct</td>
</tr>
<tr>
<td>1.) <img src="image6" alt="symbols" /> 2.) <img src="image7" alt="symbols" /> 3.) <img src="image8" alt="symbols" /></td>
<td><img src="image9" alt="symbols" /> How did you think about this? Let me show you how I did...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><img src="image10" alt="symbols" /> <img src="image11" alt="symbols" /> <img src="image12" alt="symbols" /> <img src="image13" alt="symbols" /> → ____</th>
<th>What two symbols combine to produce <img src="image14" alt="symbols" />?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image15" alt="symbols" /> goes with other symbols, it does not affect the result. So: <img src="image16" alt="symbols" /> <img src="image17" alt="symbols" /> <img src="image18" alt="symbols" /> →</td>
<td><img src="image19" alt="symbols" /> 1.) <img src="image20" alt="symbols" /> and <img src="image21" alt="symbols" /> 2.) <img src="image22" alt="symbols" /> and <img src="image23" alt="symbols" /></td>
</tr>
<tr>
<td>We know that <img src="image24" alt="symbols" /> <img src="image25" alt="symbols" /> → <img src="image26" alt="symbols" /> So we get: <img src="image27" alt="symbols" /> <img src="image28" alt="symbols" /> → <img src="image29" alt="symbols" /></td>
<td>3.) <img src="image30" alt="symbols" /> and <img src="image31" alt="symbols" /> 4.) <img src="image32" alt="symbols" /> and <img src="image33" alt="symbols" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Either <img src="image34" alt="symbols" /> and <img src="image35" alt="symbols" /> or <img src="image36" alt="symbols" /> and <img src="image37" alt="symbols" /></th>
<th>Determine the result of the following</th>
</tr>
</thead>
<tbody>
<tr>
<td>Because: <img src="image38" alt="symbols" /> other symbol → other symbol So: <img src="image39" alt="symbols" /> <img src="image40" alt="symbols" /> → <img src="image41" alt="symbols" /></td>
<td><img src="image42" alt="symbols" /> 1.) <img src="image43" alt="symbols" /> 2.) <img src="image44" alt="symbols" /> 3.) <img src="image45" alt="symbols" /></td>
</tr>
<tr>
<td>or</td>
<td></td>
</tr>
<tr>
<td>So: <img src="image46" alt="symbols" /> <img src="image47" alt="symbols" /> → <img src="image48" alt="symbols" /></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><img src="image49" alt="symbols" /> is correct. Let me show you. We want to know <img src="image50" alt="symbols" /> <img src="image51" alt="symbols" /> <img src="image52" alt="symbols" /> <img src="image53" alt="symbols" /> → ____</th>
<th>Remember the Key Ideas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>When <img src="image54" alt="symbols" /> goes with other symbols, it does not affect the result. So: <img src="image55" alt="symbols" /> <img src="image56" alt="symbols" /> <img src="image57" alt="symbols" /> <img src="image58" alt="symbols" /> →</td>
<td>1. <img src="image59" alt="symbols" /> other symbol → other symbol</td>
</tr>
<tr>
<td>We know that <img src="image60" alt="symbols" /> <img src="image61" alt="symbols" /> → <img src="image62" alt="symbols" /> So we get: <img src="image63" alt="symbols" /> <img src="image64" alt="symbols" /> → <img src="image65" alt="symbols" /></td>
<td>2. <img src="image66" alt="symbols" /> <img src="image67" alt="symbols" /> → <img src="image68" alt="symbols" /></td>
</tr>
<tr>
<td>3. <img src="image69" alt="symbols" /> <img src="image70" alt="symbols" /> → <img src="image71" alt="symbols" /> <img src="image72" alt="symbols" /> <img src="image73" alt="symbols" /> → <img src="image74" alt="symbols" /></td>
<td></td>
</tr>
</tbody>
</table>
Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

<table>
<thead>
<tr>
<th>1. Find the resulting symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bigcirc, \bullet \rightarrow _ _ _ _ _ )</td>
</tr>
<tr>
<td>1.) ( \bigcirc ) 2.) ( \bullet ) 3.) ( \bigcirc )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. Find the resulting symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bigcirc, \bullet, \bullet \rightarrow _ _ _ _ _ )</td>
</tr>
<tr>
<td>1.) ( \bigcirc ) 2.) ( \bullet ) 3.) ( \bigcirc )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. What symbols go in the blanks to make a correct statement?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( _ _ _ _ _ \bigcirc \rightarrow _ _ _ _ _ )</td>
</tr>
<tr>
<td>1.) ( \bullet ) and ( \bigcirc ) 2.) ( \bullet ) and ( \bigcirc )</td>
</tr>
<tr>
<td>3.) ( \bigcirc ) and ( \bigcirc ) 4.) ( \bullet ) and ( \bigcirc )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. What goes in the blanks to make a correct statement?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( _ _ \rightarrow _ _ _ )</td>
</tr>
<tr>
<td>1.) ( \bigcirc ) and ( \bullet ) 2.) ( \bullet ) and ( \bullet )</td>
</tr>
<tr>
<td>3.) ( \bigcirc ) and ( \bigcirc ) 4.) ( \bullet ) and ( \bullet )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5. What goes in the blank to make a correct statement?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( _ _ \bigcirc \rightarrow _ _ _ _ _ )</td>
</tr>
<tr>
<td>1.) ( \bigcirc ) 2.) ( \bullet ) 3.) ( \bigcirc )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6. What can go in the blank to make a correct statement?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( _ _ \rightarrow _ _ _ _ _ )</td>
</tr>
<tr>
<td>1.) ( \bigcirc ), ( \bigcirc ) 2.) ( \bullet ), ( \bullet )</td>
</tr>
<tr>
<td>3.) ( \bigcirc ), ( \bigcirc ) 4.) ( \bullet ), ( \bullet )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>7. What expression has the same result as the following expression?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( _ _ _ _ _ _ _ _ \rightarrow _ _ _ _ _ _ _ _ _ _ _ _ )</td>
</tr>
<tr>
<td>1.) ( \bullet ), ( \bigcirc ) 2.) ( \bullet ), ( \bigcirc )</td>
</tr>
<tr>
<td>3.) ( \bigcirc ), ( \bigcirc ) 4.) none of the above</td>
</tr>
</tbody>
</table>
8. Some of my team members were discussing what symbol could be placed in the first blank below. Which of their responses do you agree with?

___, ●, ●, ___ \[→ \]

1.) any symbol  
2.) any symbol except ◆  
3.) any symbol except ●  
4.) any symbol except ◆

9. When we were analyzing tablets, I overheard two of my team members talking.

They were arguing about whether these inscriptions mean the same thing (have the same result).

What do you think?

10. How about the following?

Do they mean the same thing?

●, ●, ●, ●, ○ \[→ \]

●, ●, ●, ○, ◆ \[→ \]

1 - Same  
2 - Different

11. Do the following give the same result?

●, ●, ●, ○, ○, ○, ○, ○ \[→ \]

●, ●, ●, ○, ○, ○, ○, ○ \[→ \]

1 - Yes  
2 - No

12. What goes in the blanks to make a correct statement?

___, ◆, ●, ___ \[→ \]

1.) ● and ●  
2.) ◆ and ●  
3.) ● and ◆  
4.) ● and ○

13. Which of the following symbols combine to give ◆?

1.) ● and ●  
2.) ◆ and ○  
3.) ● and ◆  
4.) none of the above

14. How many ○’s could combine with themselves to get ●?

1.) four  
2.) five  
3.) six  
4.) seven

15. Find the resulting symbol

●, ●, ●, ●, ●, ●, ○, ○, ○, ○, ○, ○, ○, ○, ○, ○, ○, ○, ○, ○, ○ \[→ \]

1.) ●  
2.) ◆  
3.) ●
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>16. What symbol goes in the blank to make a correct statement?</td>
<td>![Symbols] (1) 2. ♠ 3. ○ 4. we need more information to answer</td>
</tr>
<tr>
<td>17. When we were working, a tablet was broken. We tried to figure out what it stated. We did not know the result, but we did know that there were two symbols on the left; one of them was ○. We were trying to figure out what the result could be. Here are some opinions of my team members. Which do you agree with? 1.) the result could be any symbol 2.) the result could only be ⊙ or ○ 3.) the result can only be ♠</td>
<td></td>
</tr>
<tr>
<td>18. Find the resulting symbol</td>
<td>![Symbols] (1) 2. ♠ 3. ○</td>
</tr>
<tr>
<td>19. Do the following statements mean the same thing?</td>
<td>![Symbols] (1 - Yes)  2 - No</td>
</tr>
<tr>
<td>20. Do the following statements mean the same thing?</td>
<td>![Symbols] (1 - Yes)  2 - No</td>
</tr>
<tr>
<td>21. Do the following statements mean the same thing?</td>
<td>![Symbols] (1 - Yes)  2 - No</td>
</tr>
<tr>
<td>22. What goes in the blank to make a correct statement?</td>
<td>![Symbols] (1) 2.) ♠ 3.) ○</td>
</tr>
<tr>
<td>23. What is the result of the following?</td>
<td>![Symbols] (1) 2.) ♠ 3.) ♠</td>
</tr>
</tbody>
</table>
24. What goes in the blank to make a correct statement?

![Diagram](image)

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1.)</td>
<td>2.)</td>
<td>3.)</td>
<td>4.)</td>
</tr>
<tr>
<td>♦, ♦</td>
<td>♦, ♦</td>
<td>♦, ♦</td>
<td>none of the above</td>
</tr>
</tbody>
</table>

You have completed the first portion of this experiment.

Press the space bar when you are ready to proceed to the next portion.
APPENDIX C

TRAINING AND TESTING OF RELEVANTLY CONCRETE DOMAINS OF PHASE 1 OF EXPERIMENTS 2, 3, 4, AND 5
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.

Press the space bar to advance when there is no question.

At the Bubblinski Detergent Company we mix solutions of different kinds to fill containers of detergents.

Recently, we have been having some problems with the mixing of the solutions. The detergent mixes may not be correct.

I need your help to determine the amount of left-over solution that results in the mixing process. Then I will test the left-over amount to make sure that the detergent is being made correctly.

Let me tell you how the quantities of solutions are combined so that we can determine how much solution is left-over.

In the process of making the detergent, cups of solutions are combined.

Different kinds of solutions are used, but what is important is the quantities of solutions.

So to find the left-over amount, we can use \( \text{cups} \), \( \text{cups} \) and \( \text{cups} \) to represent the different possible quantities of any of the solutions.
For example,

if and are combined, the quantities will fill one detergent container and will be left-over.

So, you have the idea. We need to find the left-over quantity when the following types of cups of solution are combined.

Now let me teach you the specific rules for finding the left-over quantities.

1. The order by which two cups of solution are combined does not change the left-over result.

For example, combining with has a left-over quantity of .

And combining with has the left-over quantity .

Here is a question for you.

Suppose you know that and combine to yield a left-over of .

Then what is left-over when and combine?

Enter 1, 2, or 3.

1.) 2.) 3.)

 is correct

Because the order of combination does not matter.

When is combined with , is left-over.

AND, when is combined with , is left-over.

Let’s try another question.

If combines with , and is left-over.

Then what is the left-over when combines with ?

1.) 2.) 3.)

 is correct

Because the order of combination does not matter.

If and combine with left-over, then is left-over when and combine.

So you know that the order of combining solutions does not matter. Now, let me tell you some specifics about combinations.
Remember, we are filling containers, but we always want to have a cup of solution to test for quality when we are done.

Rule 2. $\text{and } \text{will fill a container,}$
but we need a quantity of solution to test, so we consider $\text{as the left-over.}$

Rule 3. When any kind of cup of solution combines with $\text{, the result will always be the other solution cup.}$

For example:

When $\text{and } \text{combine, } \text{is left-over.}$

And when $\text{and } \text{combine, } \text{is left-over.}$

So tell me what is left-over when $\text{and } \text{combine?}$

1.) $\text{2.) } \text{3.) } \text{$$

$\text{is correct}$

Whenever $\text{combines with another solution cup, the result is that other cup.}$

So, $\text{is left-over when } \text{and } \text{combine.}$

Let's try another...

What is left-over when $\text{and } \text{combine?}$

1.) $\text{2.) } \text{3.) } \text{$$

$\text{is correct}$

Whenever $\text{combines with another cup of solution, the left-over is that other solution cup.}$

So, $\text{is left-over when } \text{and } \text{combine.}$

Rule 4. A combination of $\text{and } \text{does not fill a container, so the left-over is } \text{.$}$

Rule 5. A combination of $\text{and } \text{fills one container and has } \text{left-over.}$

Rule 6. Finally, you need to know that when mixing more than 2 cups of solution, the order of combining solutions does not matter. The left-over is the same no matter which cups are combined first.

For example: When we combine $\text{, } \text{and } \text{, the left-over is } \text{.$}$

It does not matter if we do $\text{and } \text{first and then } \text{OR } \text{and } \text{first and then }$
Here is a question.

If mixing \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \) has a left-over of \( \text{cup} \), then what is left-over when mixing \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \)?

1.) \( \text{cup} \)  
2.) \( \text{cup} \)  
3.) \( \text{cup} \)

\( \text{cup} \) is correct.

Remember the order of combinations does not matter, the left-over will be the same.

So, combining \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \) is the same as combining \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \).

Both have \( \text{cup} \) as a left-over.

Let's try another question.

If \( \text{cup} \) is left-over when \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \) combine, then what is left-over when \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \) combine?

1.) \( \text{cup} \)  
2.) \( \text{cup} \)  
3.) \( \text{cup} \)

\( \text{cup} \) is correct.

The order of the symbols on the left does not matter. So

If \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \) combine and have \( \text{cup} \) left-over, then \( \text{cup} \), \( \text{cup} \) and \( \text{cup} \) combine with \( \text{cup} \) left-over.

To summarize,

Cups of solutions are mixed together to fill detergent containers. Your job is to find the cup with the remainder quantity.

We always want to have a quantity remaining, so the possible remainders are:

\( \text{cup} \)  
\( \text{cup} \)  
\( \text{cup} \)

You can combine 2 or more cups of solution in any order, the left-over will be the same.

Next let's review the left-overs for specific cup combinations...

Here is what happens when \( \text{cup} \) mixes with other cups.

<table>
<thead>
<tr>
<th>Mixing these 2 cups</th>
<th>results in this left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{cup} )</td>
<td>( \text{cup} )</td>
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<tr>
<td>( \text{cup} )</td>
<td>( \text{cup} )</td>
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<tr>
<td>( \text{cup} )</td>
<td>( \text{cup} )</td>
</tr>
</tbody>
</table>

Here are the left-overs from all of the other possible cup combinations:

<table>
<thead>
<tr>
<th>Mixing these 2 cups</th>
<th>results in this left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{cup} )</td>
<td>( \text{cup} )</td>
</tr>
<tr>
<td>( \text{cup} )</td>
<td>( \text{cup} )</td>
</tr>
<tr>
<td>( \text{cup} )</td>
<td>( \text{cup} )</td>
</tr>
</tbody>
</table>
1. **What is left-over**
   - **Reminders:** You can combine 2 or more cups of solution in any order, the left over will be the same.
   - **Left-overs for specific mix combinations:**
     | Solution Cup Mix | Left-over |
     |------------------|-----------|
     |                  |           |
     |                  |           |
     |                  |           |

2. **What is left-over**
   - **Reminders:** You can combine 2 or more cups of solution in any order, the left over will be the same.
   - **Left-overs for specific mix combinations:**
     | Solution Cup Mix | Left-over |
     |------------------|-----------|
     |                  |           |
     |                  |           |
     |                  |           |

3. **What is left-over**
   - **Reminders:** You can combine 2 or more cups of solution in any order, the left over will be the same.
   - **Left-overs for specific mix combinations:**
     | Solution Cup Mix | Left-over |
     |------------------|-----------|
     |                  |           |
     |                  |           |
     |                  |           |

4. **What is left-over**
   - **Reminders:** You can combine 2 or more cups of solution in any order, the left over will be the same.
   - **Left-overs for specific mix combinations:**
     | Solution Cup Mix | Left-over |
     |------------------|-----------|
     |                  |           |
     |                  |           |
     |                  |           |
5. **What is left-over**
   
   When \( T \), \( T \), and \( T \) mix?
   
<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
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</tbody>
</table>

**Reminders**

You can combine 2 or more cups of solution in any order, the left over will be the same.

**Left-over for specific mix combinations:**

1.)
   - Left-over

2.)
   - Left-over

3.)
   - Left-over

---

Remember, you can combine 2 or more cups in any order.

Next, let's summarize the left-over cups resulting from different combinations of solution quantities...

---

Here is what happens when \( T \) mixes with other cups.

<table>
<thead>
<tr>
<th>Mixing these 2 cups</th>
<th>results in this left-over</th>
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</tbody>
</table>

Here are the left-overs from all of the other possible cup combinations:

<table>
<thead>
<tr>
<th>Mixing these 2 cups</th>
<th>results in this left-over</th>
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</thead>
<tbody>
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</tbody>
</table>

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Here is what you need to remember.

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

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Let's do an example...
Let's find the left-over quantity when the following cups of solution are mixed.

1st Let's mix the

So we get: 

Now we have 

We know and combine with left-over.

So we get: 

Then we can mix and to get left-over.

Finally, we have and which mix to leave left-over.

Remember, this is what you need to know.

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

What is the left-over when these cups are mixed?

1.) 

2.) 

3.) 

is correct. Here is how we can figure it out.

To mix these: 

First, we can mix and to get .

Then we have: 

and leave . So we get: 

Finally and combine to leave left-over.

Here is a question for you. What possible cups of solution can combine so that left-over?

1.) and 

2.) and 

3.) and 

4.) and 

Either and 

Or and 

Combine to have left-over

What is the left-over when these cups are mixed?

1.) 

2.) 

3.) 

158
is correct. Here is how we can figure it out.

To mix these:  

First, we can mix  and  to get .

Then we have:  

and leave . So we get:  

Finally  and  combine to leave  left-over.

Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

1. What is left-over when  and  combine?
   1.)  2.)  3.)

2. What is left-over when the following cups of solution are combined?

   1.)  2.)  3.)

3. What possible cups of solution can combine with  to have a left-over of ?
   1.)  and  2.)  and  
   3.)  and  4.)  and

4. Which cups of solution can combine to have  left-over?
   1.)  and  2.)  and  
   3.)  and  4.)  and

5. What can combine with  to have  left-over?
   1.)  2.)  3.)

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td></td>
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</tr>
</tbody>
</table>
6. What can mix with \(\begin{array}{c}
\end{array}\) to have \(\begin{array}{c}
\end{array}\) as a left-over?

1.) \(\begin{array}{c}
\end{array}\)  
2.) \(\begin{array}{c}
\end{array}\)  
3.) \(\begin{array}{c}
\end{array}\)  

7. What combination of cups has the same left-over as the following?

1.) \(\begin{array}{c}
\end{array}\)  
2.) \(\begin{array}{c}
\end{array}\)  
3.) \(\begin{array}{c}
\end{array}\)  
4.) none of the above

8. Some of the Babblinski employees were analyzing a batch of detergent. The left-over was \(\begin{array}{c}
\end{array}\). There were 4 cups of solution that were mixed. Two of these cups were \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\), but the other two cups were not known. The guys were discussing which other cups could possibly have been involved in the mix. Which of their responses do you agree with?

1.) any cup  
2.) any cup except \(\begin{array}{c}
\end{array}\)  
3.) any cup except \(\begin{array}{c}
\end{array}\)  

9. Later I overheard two employees talking. They were arguing about whether these two mixtures of solution (below) would have the same left-over.

What do you think?

Mix 1: \(\begin{array}{c}
\end{array}\)  
Mix 2: \(\begin{array}{c}
\end{array}\)

1 - Same  
2 - Different

10. True or false...

When the cups are mixed, mix 1 and mix 2 will have the same left-over.

Mix 1: \(\begin{array}{c}
\end{array}\)  
Mix 2: \(\begin{array}{c}
\end{array}\)

1 - True  
2 - False

11. How about these mixtures, will they have the same left-overs?

Mix 1: \(\begin{array}{c}
\end{array}\)  
Mix 2: \(\begin{array}{c}
\end{array}\)

1 - Yes  
2 - No

12. What cups can combine with \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\) to result in a left-over of \(\begin{array}{c}
\end{array}\)?

1.) \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\)  
2.) \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\)  
3.) \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\)

13. Which cups can combine to give a left-over of \(\begin{array}{c}
\end{array}\)?

1.) \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\)  
2.) \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\)  
3.) \(\begin{array}{c}
\end{array}\) and \(\begin{array}{c}
\end{array}\)  
4.) none of the above
14. How many \( \text{\ding{82}} \)’s could combine with themselves to get \( \text{\ding{82}} \)?

1.) four  
2.) five  
3.) six  
4.) seven

15. What is left-over when the following cups are mixed?

1.)  
2.)  
3.)

16. What cup can mix with the following and have \( \text{\ding{82}} \) left-over?

1.)  
2.)  
3.)  
4.) we need more information to answer

17. One day, a batch of detergent was spilled. We did not know the left-over quantity, but we did know that there were two cups in the mixture, one of them was \( \text{\ding{82}} \). We were trying to figure out what the left-over could have been. Here are some opinions of the employees. Which do you agree with?

1.) the left-over could be any cup  
2.) the left-over could only be \( \text{\ding{832}} \) or \( \text{\ding{833}} \)  
3.) the left-over could only be \( \text{\ding{82}} \)

18. What is left-over when the following cups of solution combine?

1.)  
2.)  
3.)

19. Do the following mixtures have the same left-overs?

Mix 1:  
Mix 2:  
1 - Yes  
2 - No

20. Do the following mixtures have the same left-overs?

Mix 1:  
Mix 2:  
1 - Yes  
2 - No

21. How about the following, do they have the same left-over?

Mix 1:  
Mix 2:  
1 - Yes  
2 - No
22. What cup needs to mix with the following to have a left-over of?

1) 2) 3)  

23. What is left-over when the cups below are mixed?

1) 2) 3)  

24. What cups need to mix with to have left-over?

1) 2)  

3) 4) None of the above

You have completed the first portion of this experiment.

Press the space bar when you are ready to proceed to the next portion.
APPENDIX D

PRESENTATION AND TESTING FOR BASELINE CONDITION OF PHASE 2 OF EXPERIMENTS 2 AND 3
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, or 3 to indicate your answers to the questions.

Press the space bar to advance when there is no question.

Allow me to introduce myself. I am Mr. Schmister, teacher of mathematics.

We are coding arithmetic problems that will be given to students.

We use the following symbols to represent different aspects of the problem.

- means the result is a positive number
- means the result is an even number
- means the result is either greater than 10 or less than -10

A problem can have more than one symbol apply.

For example the symbols

Apply to the problem $15 + 12 - 1 =$ __

Because $15 + 12 - 1 = 26$
- 26 is positive so ♦ applies
- 26 is even so ● applies
- 26 > 10 so ❌ applies

Also a problem can have no symbol apply.

For example consider the problem $15 - 12 - 4 =$ __

Because $15 - 12 - 4 = -1$
- -1 is negative, not positive
- -1 is odd, not even
- -1 is not > 10 and -1 is not < -10.
- So none of the symbols apply.

Also, one or two symbols can apply.

So what symbols do you think apply to the problem on the next screen?

Choose: $5 \times 3 - 32 =$ __

1.) ♦ 2.) ❌ 3.) ● ❌

Remember:
- ♦ means positive
- ● means even
- ❌ means > 10 OR < 10
The correct answer is 2.

Because $5 \times 3 - 32 = 15 - 32 = -17$

-17 is negative, not positive
-17 is odd, not even
-17 < -10

So only ☐️ applies.

<table>
<thead>
<tr>
<th></th>
<th>1. $2 + 4 = ____$</th>
<th>2. $4 \times 2 + 10 = ____$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose:</td>
<td>1.) ☐️ 2.) ☐️ 3.) ☐️ ☐️</td>
<td>1.) ☐️ 2.) ☐️ 3.) ☐️ ☐️</td>
</tr>
<tr>
<td>Remember:</td>
<td>☐️ means positive</td>
<td>☐️ means positive</td>
</tr>
<tr>
<td></td>
<td>☐️ means even</td>
<td>☐️ means even</td>
</tr>
<tr>
<td></td>
<td>☐️ means $&gt; 10$ OR $&lt; 10$</td>
<td>☐️ means $&gt; 10$ OR $&lt; 10$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>3. $25 - 26 = ____$</th>
<th>4. $-2 \times 6 = ____$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose:</td>
<td>1.) ☐️ 2.) ☐️ 3.) none</td>
<td>1.) ☐️ 2.) ☐️ 3.) ☐️ ☐️</td>
</tr>
<tr>
<td>Remember:</td>
<td>☐️ means positive</td>
<td>☐️ means positive</td>
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<td>☐️ means even</td>
<td>☐️ means even</td>
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<tr>
<td></td>
<td>☐️ means $&gt; 10$ OR $&lt; 10$</td>
<td>☐️ means $&gt; 10$ OR $&lt; 10$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>5. $100 - 50 = ____$</th>
<th>6. $2 + 4 + 9 = ____$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose:</td>
<td>1.) ☐️ ☐️ 2.) ☐️ 3.) ☐️</td>
<td>1.) ☐️ 2.) ☐️ 3.) ☐️</td>
</tr>
<tr>
<td>Remember:</td>
<td>☐️ means positive</td>
<td>☐️ means positive</td>
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<td>☐️ means even</td>
<td>☐️ means even</td>
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<td>☐️ means $&gt; 10$ OR $&lt; 10$</td>
<td>☐️ means $&gt; 10$ OR $&lt; 10$</td>
</tr>
</tbody>
</table>
7. $3 \times 4 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) none

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

8. $10 - 12 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) ✑

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

9. $2 \times 5 \times 2 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) ✐

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

10. $1 \times 2 \times 3 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) ✐

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

11. $1 + 11 + 0 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) ✐

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

12. $9 \times 9 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) none

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

13. $10 + 10 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) none

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

14. $0 - 4 = \_$
Choose:
1.) ♦ 2.) ✖ 3.) ✐

Remember:
♦ means positive
✖ means even
 ⁑ means > 10 OR < 10

166
<p>| | | | |</p>
<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>15.</td>
<td>(100 - 1 = _)</td>
<td>Choose:</td>
<td>1.) ⬤</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remember:</td>
<td>⬤ means positive</td>
<td>⬤ means even</td>
<td>⬤ means (&gt; 10) OR (&lt; 10)</td>
</tr>
<tr>
<td>16.</td>
<td>(-3 \times 4 = _)</td>
<td>Choose:</td>
<td>1.) ⬤ ⬤</td>
</tr>
<tr>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Remember:</td>
<td>⬤ means positive</td>
<td>⬤ means even</td>
<td>⬤ means (&gt; 10) OR (&lt; 10)</td>
</tr>
<tr>
<td>17.</td>
<td>(5 \times 5 - 30 = _)</td>
<td>Choose:</td>
<td>1.) ⬤</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>Remember:</td>
<td>⬤ means positive</td>
<td>⬤ means even</td>
<td>⬤ means (&gt; 10) OR (&lt; 10)</td>
</tr>
<tr>
<td>18.</td>
<td>(5 - 3 = _)</td>
<td>Choose:</td>
<td>1.) ⬤ ⬤</td>
</tr>
<tr>
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<td></td>
</tr>
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<td>⬤ means even</td>
<td>⬤ means (&gt; 10) OR (&lt; 10)</td>
</tr>
<tr>
<td>19.</td>
<td>(-5 \times 3 = _)</td>
<td>Choose:</td>
<td>1.) ⬤</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remember:</td>
<td>⬤ means positive</td>
<td>⬤ means even</td>
<td>⬤ means (&gt; 10) OR (&lt; 10)</td>
</tr>
<tr>
<td>20.</td>
<td>(-7 \times 1 = _)</td>
<td>Choose:</td>
<td>1.) ⬤ ⬤</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Remember:</td>
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<td>⬤ means even</td>
<td>⬤ means (&gt; 10) OR (&lt; 10)</td>
</tr>
</tbody>
</table>

You have completed the first portion of this experiment.

Press the space bar when you are ready to proceed to the next portion.
APPENDIX E

PRESENTATION AND TESTING OF TRANSFER DOMAIN FOR PHASE 2 OF EXPERIMENTS 2, 3, 5, AND 6
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.

Press the space bar to advance when there is no question.

Allow me to introduce myself. I am Mr. Gublini, professor of cultural studies.

I know that you just finished learning the rules of a system involving 3 entities.

You learned the rules and answered some questions.

Now I need your help figuring out something new... a children's game from another country.

What you learned before in the previous system can help you understand how this new game works.

In the game, the children point to two or more different objects, one at a time.

Then a different child who is "it" points to one final object. If this child points to the correct final object, then he or she is the winner.

There are 3 kinds of objects; and the rules of the game state what the final object should be. So the rules tell the winner what final object to point to.

The rules of the last system are like the rules of this game.

So, use what you know about the last system to help you figure out the rules of the game.

Let's get started. I will tell you what I know about the game.

There are three kinds of objects used in the game:

- Ring:
- Bug:
- Vase:
Children point to at least two objects. Then the person who is “it” tries to win by pointing to the correct final object.

Here is an example.

Some children pointed to \( \square \) and then \( \square \).

Then the child who was “it” pointed to \( \square \) and won.

There are specific rules that tell the winner which object to point to.
I need your help figuring out those rules to answer some questions.

Next, I will show you some specific examples where the “it” child was a winner. Please study these examples on the next 7 slides and then we will do some questions.

Some children pointed to \( \square \) then \( \square \).

The winner pointed to \( \square \).

Some children pointed to \( \square \) then \( \square \).

The winner pointed to \( \square \).

The children pointed to \( \square \) then \( \square \).

The winner pointed to \( \square \).

The children pointed to \( \square \) then \( \square \).

The winner pointed to \( \square \).
Some children pointed to \[ \text{ then } \] .

The winner pointed to \[ \text{ then } \] .

The children pointed to \[ \text{ then } \] .

The winner pointed to \[ \text{ then } \] .

The children pointed to \[ \text{ then } \] . The winner pointed to \[ \text{ then } \] .

The children pointed to \[ \text{ then } \] . The winner pointed to \[ \text{ then } \] .

The children pointed to \[ , \text{ then } \] . Then \[ \text{ then } \] .

The winner pointed to \[ \text{ then } \] .

Now let’s take a look at some questions.
Remember you can use what you learned in the previous system to help you answer these question about the game.

1. What object do you think the winner will point to when the other kids point to \[ \text{ then } \] ?

Choose:

1.) \[ \text{ then } \]

2.) \[ \text{ then } \]

3.) \[ \text{ then } \]

Use these examples to help you figure out the answer:

If the kids point to these:

Then the winner points to this:

2. What object does the winner point to when the other kids point to \[ \text{ then } \] ?

Choose:

1.) \[ \text{ then } \]

2.) \[ \text{ then } \]

3.) \[ \text{ then } \]

Use these examples to help you figure out the answer:

If the kids point to these:

Then the winner points to this:

3. If a group of kids wants the winner to point to \[ \text{ then } \] , and they first point to \[ \text{ need to point to } ? \]

What other objects do they 1.) \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \]

2.) \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \]

3.) \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \]

Examples:

If the kids point to these:

Then the winner points to this:

4. If the winner pointed to \[ \text{ then } \] , what objects might the other kids have pointed to?

1.) \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \]

2.) \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \]

3.) \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \] \[ \text{ and } \]

Examples:

If the kids point to these:

Then the winner points to this:
5. What object do the children need to point to along with 果汁, so that the winner points to 果汁?

1.) 果汁 2.) 3.)

Examples:
If the kids point to these:
Then the winner points to:

6. What objects can the kids point to along with 果汁, so that the winner points to 果汁?

1.) 2.) 3.) 4.)

Examples:
If the kids point to these:
Then the winner points to:

7. Suppose that a group kids points to these objects:

If other groups (in different games) point to the objects below, which group would have a winner pointing to the same object as the group above?

1.) 2.)
3.) 4.) none of the above

Examples:
If the kids point to these:
Then the winner points to:

8. A group of kids wants the winner to point to 果汁.
They have already pointed to 果汁 and 果汁, and they want to point to two more. What can their next object be?

1.) any object 2.) any object except 果汁
3.) any object except 果汁 4.) any object except 果汁

Examples:
If the kids point to these:
Then the winner points to:

9. Two groups of children were playing separate games.
One group pointed to these objects:

The other group pointed to these:

Will the winner point to the same object in each case?

1 - Yes 2 - No

Examples:
If the kids point to these:
Then the winner points to:

10. How about these objects? In separate games,
If one group points to these:
And another group points to these:

Will the winner point to the same object in each case?

1 - Yes 2 - No

Examples:
If the kids point to these:
Then the winner points to:

11. In separate games, suppose that the one group of children points to these:

And another group points to these:

Will the winner point to the same object in each case?

1 - Yes 2 - No

Examples:
If the kids point to these:
Then the winner points to:

12. A group of children want the winner to point to 果汁.
They first point to 果汁 and 果汁.

What objects should they point to next?
1.) 果汁 and 果汁 2.) 果汁 and 果汁
3.) 果汁 and 果汁 4.) 果汁 and 果汁

Examples:
If the kids point to these:
Then the winner points to:
13. What objects should the kids point to so that the winner points to?

1.)  
2.)  
3.)  
4.) none of the above

Examples:
If the kids point to these:
Then the winner points to this:

14. How many times could the kids point to so that the winner points to?

1.) four  
2.) five  
3.) six  
4.) seven

Examples:
If the kids point to these:
Then the winner points to this:

15. After the children point to those objects:

What will the winner point to?

1.)  
2.)  
3.)

Examples:
If the kids point to these:
Then the winner points to this:

16. The children point to these objects:

What additional object should they point to so that the winner points to?

1.)  
2.)  
3.)  
4.) we need more information to answer

Examples:
If the kids point to these:
Then the winner points to this:

17. Three children were playing together. One child pointed to. They were going to point to one more object, but before they did, they were trying to decide which object the winner would point to. Here are their opinions about the winning object. Which do you agree with?

1.) the winning object could be any kind
2.) the winning object could only be
3.) the winning object could only be

Examples:
If the kids point to these:
Then the winner points to this:

18. If the kids point to these objects:

What object will the winner point to?

1.)  
2.)  
3.)

Examples:
If the kids point to these:
Then the winner points to this:

19. In separate games, one group of kids pointed to these objects:

And another group pointed to these:

Will the winners of each game point to the same object?

1 - Yes  
2 - No

Examples:
If the kids point to these:
Then the winner points to this:

20. How about these objects?

In one game, the kids point to these objects:
and in another game, the kids point to these:

Will the winners of each game point to the same object?

1 - Yes  
2 - No

Examples:
If the kids point to these:
Then the winner points to this:
21. How about this?
   In one game, the kids point to these objects:
   and in another game, the kids point to these:
   Will the winners of each game point to the same object?
   1 - Yes   2 - No

Examples:
If the kids point to these:
Then the winner points to this:

22. A group of children pointed to these objects:
   What additional object do they need to point to so that the winner will point to?
   1.)   2.)   3.)

Examples:
If the kids point to these:
Then the winner points to this:

23. What will the winner point to if the children point to these?

1.)   2.)   3.)

Examples:
If the kids point to these:
Then the winner points to this:

24. If the group of children point to these objects:
   What additional objects should they point to so that the winner will point to?
   1.)   2.)   3.)   4.) none of the above

Examples:
If the kids point to these:
Then the winner points to this:
The following three slides were included in the Generic and No Relevant Concreteness/Perceptually Rich conditions and not in the Relevant Concreteness and Relevant Concreteness/Perceptually Rich conditions.
The following three slides were included in the Relevant Concreteness and Relevant Concreteness/Perceptually Rich conditions and not in the Generic and No Relevant Concreteness/Perceptually Rich conditions.

**Can you match the entities from each of the systems you learned?**

<table>
<thead>
<tr>
<th>This system</th>
<th>Previous system</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td><img src="image2.png" alt="Diagram 2" /></td>
</tr>
</tbody>
</table>

Which of the previous system acts most like

Choose: (1) ![Diagram 3](image3.png) (2) ![Diagram 4](image4.png) (3) ![Diagram 5](image5.png)

<table>
<thead>
<tr>
<th>This system</th>
<th>Previous system</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image6.png" alt="Diagram 6" /></td>
<td><img src="image7.png" alt="Diagram 7" /></td>
</tr>
</tbody>
</table>

Which of the previous system acts most like

Choose: (1) ![Diagram 8](image8.png) (2) ![Diagram 9](image9.png) (3) ![Diagram 10](image10.png)

**On a scale from 1 – 5**

How similar do you think that the two systems are?

Enter:

1 2 3 4 5

You have completed this experiment.

Thank you for participating in this study.
APPENDIX F

PRESENTATION AND TESTING OF PHASE 2 OF EXPERIMENT 4

The following pages present the training and testing for the Generic Condition. The presentation and testing for the Perceptually Rich (Irrelevantly Concrete) Condition was identical except it involved these perceptually rich symbols:
Hello. I am Professor Gublini.
I know that you just finished
learing a system and
answering questions about it.

In particular, I need your help to
determine whether these systems
follow the same type of rules as those
of the system you just learned.

So, I will show you some sets of
examples.

Your job is to decide whether the
examples follow the same type of rules
as those of the previous system.

1. Does this set follow the same type of rules as
the system you learned earlier?

Enter: 1 – yes  2 – No

![Set 1]

2. Does this set follow the same type of rules as
the system you learned earlier?

Enter: 1 – yes  2 – No

![Set 2]

3. Does this set follow the same type of rules as
the system you learned earlier?

Enter: 1 – yes  2 – No

![Set 3]

4. Does this set follow the same type of rules as
the system you learned earlier?

Enter: 1 – yes  2 – No

![Set 4]
5. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

6. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

7. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

8. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

9. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

10. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

11. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

12. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No
13. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No

14. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No

15. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No

16. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No

17. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No

18. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No

19. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No

20. Does this set follow the same type of rules as the system you learned earlier?
Enter: 1—yes  2—No
21. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

22. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

23. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

24. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

25. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

26. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

You have completed this experiment.

Thank you for participating in our study.
The following pages present the presentation and testing for the Relevantly Concrete Condition.

**Hello. I am Professor Gublini.**
I know that you just finished learning a system and answering questions about it.

**Now I need your help looking at displays from other systems.**

**In particular, I need your help to determine whether these systems follow the same type of rules as those of the system you just learned.**

**So, I will show you some sets of examples.**
Your job is to decide whether the examples follow the same type of rules as those of the previous system.

1. Does this set follow the same type of rules as the system you learned earlier?
   Enter: 1 – yes 2 – No

   ![Set 1](image1)

2. Does this set follow the same type of rules as the system you learned earlier?
   Enter: 1 – yes 2 – No

   ![Set 2](image2)
3. Does this set follow the same type of rules as
the system you learned earlier?
Enter: 1 – yes 2 – No

4. Does this set follow the same type of rules as
the system you learned earlier?
Enter: 1 – yes 2 – No

5. Does this set follow the same type of rules as
the system you learned earlier?
Enter: 1 – yes 2 – No

6. Does this set follow the same type of rules as
the system you learned earlier?
Enter: 1 – yes 2 – No

7. Does this set follow the same type of rules as
the system you learned earlier?
Enter: 1 – yes 2 – No

8. Does this set follow the same type of rules as
the system you learned earlier?
Enter: 1 – yes 2 – No
9. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

11. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

13. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

15. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

10. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

12. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

14. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No

16. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes 2 – No
17. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

18. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

19. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

20. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

21. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

22. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

23. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

24. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No
25. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

26. Does this set follow the same type of rules as the system you learned earlier?

Enter: 1 – yes  2 – No

You have completed this experiment.
Thank you for participating in our study.
APPENDIX G

TRAINING AND TESTING OF PHASE 1 OF EXPERIMENT 5 FOR MULTIPLE INSTANTIATION CONDITIONS

The following pages present the training and testing of the condition in which participants learned two Relevantly Concrete Domains.
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.
Press the space bar to advance when there is no question.

At the Bubblinski Detergent Company we mix solutions of different kinds to fill containers of detergents.

Recently, we have been having some problems with the mixing of the solutions. The detergent mixes may not be correct.

I need your help to determine the amount of left-over solution that results in the mixing process. Then I will test the left-over amount to make sure that the detergent is being made correctly.

Let me tell you how the quantities of solutions are combined so that we can determine how much solution is left-over.

In the process of making the detergent, cups of solutions are combined.

Different kinds of solutions are used, but what is important is the quantities of solutions.

So to find the left-over amount, we can use \[\text{\frac{1}{4}}, \quad \text{\frac{1}{3}} \quad \text{and} \quad \text{\frac{1}{2}}\] to represent the different possible quantities of any of the solutions.
For example,

if \( \) and \( \) are combined, the quantities will fill one detergent container and \( \) will be left-over.

1. The order by which two cups of solution are combined does not change the left-over result.

   For example, combining \( \) with \( \) has a left-over quantity of \( \).

   And combining \( \) with \( \) has the left-over quantity \( \).

Here is a question for you.

Suppose you know that \( \) and \( \) combine to yield a left-over of \( \).

Then what is left-over when \( \) and \( \) combine?

Enter 1, 2, or 3:

1.) \( \)
2.) \( \)
3.) \( \)

is correct

Because the order of combination does not matter.

When \( \) is combined with \( \), \( \) is left-over.

AND, when \( \) is combined with \( \), \( \) is left-over.

Remember, we are filling containers, but we always want to have a cup of solution to test for quality when we are done.

Rule 2. \( \) and \( \) will fill a container, but we need a quantity of solution to test, so we consider \( \) as the left-over.

Rule 3. When any kind of cup of solution combines with \( \), the result will always be the other solution cup.

For example:

When \( \) and \( \) combine, \( \) is left-over.

And when \( \) and \( \) combine, \( \) is left-over.
So tell me what is left-over when

\[ \begin{align*}
\text{and } & \text{ combine?} \\
1.) & \quad 2.) & \quad 3.)
\end{align*} \]

\[ \text{is correct} \]

Whenever \[ \text{combines with another solution cup, the result is that other cup.} \]

So, \[ \text{is left-over when } \text{and } \text{ combine.} \]

\[ \text{Rule 4. A combination of } \text{ and } \text{ does not fill a container, so the left-over is } \text{.} \]

\[ \text{Rule 5. A combination of } \text{ and } \text{ fills one container and has } \text{ left-over.} \]

\[ \text{Here is a question.} \]

If mixing \[ \text{, } \text{ and } \text{ has a left-over of } \text{, then what is left-over when mixing } \text{, } \text{ and } \text{?} \]

\[ 1.) \quad 2.) \quad 3.) \]

\[ \text{is correct.} \]

Remember the order of combinations does not matter, the left-over will be the same.

So, combining \[ \text{, } \text{ and } \text{ is the same as combining } \text{, } \text{ and } \text{.} \]

Both have \[ \text{ as a left-over.} \]

To summarize,

Cups of solutions are mixed together to fill detergent containers. Your job is to find the cup with the remainder quantity.

We always want to have a quantity remaining, so the possible remainders are:

\[ \text{, } \text{, } \text{, } \]

\[ \text{Let me ask you some questions.} \]
1. What is left-over when  and mix?  

Remember: You can combine 2 or more cups of solution in any order, the left over will be the same.

Left-overs for specific mix combinations:

<table>
<thead>
<tr>
<th>Solution Cup</th>
<th>Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When  and  are mixed together is left-over.

2. What is left-over when  and mix?  

Remember: You can combine 2 or more cups of solution in any order, the left over will be the same.

Left-overs for specific mix combinations:

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When  and  are mixed together is left-over.

3. What is left-over when  and mix?  

Remember: You can combine 2 or more cups of solution in any order, the left over will be the same.

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Because  and  mixed together with left-over, then  and mix leaving  left-over.

Remember, you can combine 2 or more cups in any order. Next let's summarize the left-over cups resulting from different combinations of solution quantities...

<table>
<thead>
<tr>
<th>Mixing these 2 cups</th>
<th>results in this left-over</th>
</tr>
</thead>
<tbody>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here is what happens when  mixes with other cups.
Here are the left-overs from all of the other possible cup combinations:  

<table>
<thead>
<tr>
<th>Mixing these 2 cups</th>
<th>results in this left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Cups Diagram" /></td>
<td><img src="image2" alt="Cups Diagram" /></td>
</tr>
<tr>
<td><img src="image3" alt="Cups Diagram" /></td>
<td><img src="image4" alt="Cups Diagram" /></td>
</tr>
</tbody>
</table>

Let's do an example...  

![Cups Diagram](image5)  

Now we have ![Cups Diagram](image6)  

We know ![Cups Diagram](image7) and ![Cups Diagram](image8) combine with ![Cups Diagram](image9) left-over.  

So we get: ![Cups Diagram](image10)  

Then we can mix ![Cups Diagram](image11) and ![Cups Diagram](image12) to get ![Cups Diagram](image13) left-over.  

Finally, we have ![Cups Diagram](image14) and ![Cups Diagram](image15) which mix to leave ![Cups Diagram](image16) left-over.

Let's find the left-over quantity when the following cups of solution are mixed.  

1st Let's mix the ![Cups Diagram](image17)  

So we get: ![Cups Diagram](image18)  

What is the left-over when these cups are mixed?  

1.) ![Cups Diagram](image19)  

2.) ![Cups Diagram](image20)  

3.) ![Cups Diagram](image21)

192  

Here is a question for you. What possible cups of solution can combine so that ![Cups Diagram](image22) left-over?  

1.) ![Cups Diagram](image23) and ![Cups Diagram](image24)  

2.) ![Cups Diagram](image25) and ![Cups Diagram](image26)  

3.) ![Cups Diagram](image27) and ![Cups Diagram](image28)  

4.) ![Cups Diagram](image29) and ![Cups Diagram](image30)  

Either ![Cups Diagram](image31) and ![Cups Diagram](image32)  

Or ![Cups Diagram](image33) and ![Cups Diagram](image34)  

Combine to have ![Cups Diagram](image35) left-over.
Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

1. What is left-over when and combine?

1.) 2.) 3.)

2. What possible cups of solution can combine with to have a left-over of ?

1.) 2.) 3.) 4.)

3. What can combine with to have left-over?

1.) 2.) 3.)

4. What combination of cups has the same left-over as the following?

1.) 2.) 3.) 4.) None of the above

5. Later I overheard two employees talking. They were arguing about whether these two mixtures of solution (below) would have the same left-over.

What do you think?

Mix 1:

Mix 2:

1 - Same 2 - Different

6. True or false...

When the cups are mixed, mix 1 and mix 2 will have the same left-over.

Mix 1:

Mix 2:

1 - True 2 - False
7. How about these mixtures, will they have the same left-overs?

Mix 1:  
Mix 2:  

1 - Yes  2 - No

8. What is left-over when the following cups are mixed?

1.)  2.)  3.)

9. One day, a batch of detergent was spilled. We did not know the left-over quantity, but we did know that there were two cups in the mixture; one of them was . We were trying to figure out what the left-over could have been. Here are some opinions of the employees. Which do you agree with?

1.) the left-over could be any cup  
2.) the left-over could only be or  
3.) the left-over could only be

10. What is left-over when the following cups of solution combine?

1.)  2.)  3.)

11. What cup needs to mix with the following to have a left-over of ?

1.)  2.)  3.)

12. What cups need to mix with to have left-over?

1.)  2.)

3.)  4.) none of the above

You have completed the first portion of this experiment.

Press the space bar when you are ready to proceed to the next portion.
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.
Press the space bar to advance when there is no question.

Now you will learn about a new system.
This system works the same way as the last system you learned.
The rules of the last systems are like the rules of this new one.

Hi. I am Antonio, a very big pizza lover. I eat pizza every day for lunch.

Every day I order pizzas for myself and my friends at work. We order from Mamma Maria’s Pizza Kitchen. It’s the best pizza in town.

Mamma Maria makes 1 size of pizza that has 3 slices.

At work, everyone orders 1, 2, or 3 slices.

If someone wants 1 slice, he gives me a card like this.
If someone wants 2 slices, he gives me a card like this.
If someone wants 3 slices, he gives me a card like this.

I place an order at the Pizza Kitchen by giving the cards to Mamma Maria. Then she gives me the number of whole pizzas and/or individual slices we need.

There is one problem though. Mamma Maria always burns part of our order. So, some people have burned slices.
Here is what is happening.

Mamma Maria collects the cards for the order.
She makes enough whole pizzas to fill the order.
All of the pizzas come out ok, except the very last portion that she cooks. It may be a whole pizza or less than a whole pizza.
Let me show you an example...

If I give her the following order

We would get 1 pizza that is ok,
but this much of our order would be burned

My friends and I are trying to figure out how much pizza is burned when different orders are placed?
Will you help us?
First, let me tell you what I know about placing the orders and how much is burned.

There is never more than 1 whole pizza burned.

So the burned amount will always be

1. What order first or second doesn’t matter. The same amount gets burned.

For example, if I order this first and then this.

then this much is given to us burned.

The same thing happens if I order this first and then this.

We get this much burned.

So, let me ask you...

If I order this then this, and this much is given to us burned, then how much would be burned if I order this and then this?

Enter 1, 2, or 3:

1)  
2)  
3)  

2. If I order this with any other single amount, the other amount is always burned.

Here are a couple of examples:

If I order and , then is burned.
If I order and , then is burned.
So, how much will be burned if I order \( \bigcirc \) and \( \bigcirc \) ?

Choose:
1) \( \bigcirc \) 2) \( \bigcirc \) 3) \( \bigcirc \)

\( \bigcirc \) is burned.

Whenever this \( \bigcirc \) is ordered, the other amount is burned.

So if I order \( \bigcirc \) and \( \bigcirc \), then \( \bigcirc \) is burned.

3. If I order \( \bigcirc \) and \( \bigcirc \), then only one pizza is made and \( \bigcirc \) is burned.

4. If I order \( \bigcirc \) and \( \bigcirc \), then one pizza is made and \( \bigcirc \) is burned.

5. If I order \( \bigcirc \) and \( \bigcirc \), then one pizza is ok, but \( \bigcirc \) is burned.

6. If I turn in more than 2 order cards, the order that I turn them in doesn't matter. The same amount ends up burned.

For example, if I turn in \( \bigcirc \) and \( \bigcirc \) and then \( \bigcirc \), then \( \bigcirc \) is burned.

The same amount is burned if I turn in \( \bigcirc \) and \( \bigcirc \) and then \( \bigcirc \).

Let me ask you a question…

If \( \bigcirc \) is burned when I turn in \( \bigcirc \), \( \bigcirc \) and then \( \bigcirc \), then how much is burned if I turn in \( \bigcirc \), \( \bigcirc \) and \( \bigcirc \) ?

Choose:
1) \( \bigcirc \) 2) \( \bigcirc \) 3) \( \bigcirc \)

\( \bigcirc \) is burned.

Remember the order that I turn the cards in doesn't matter, the same amount will be burned.

So this order \( \bigcirc \) \( \bigcirc \) \( \bigcirc \) has this \( \bigcirc \) burned;

and this order \( \bigcirc \) \( \bigcirc \) \( \bigcirc \) also has this \( \bigcirc \) burned.
Let me summarize what I know about how much pizza gets burned.

- No matter how much pizza we order, we never get more than 1 whole pizza burned.

The burned amount will always be:

- \[ \frac{1}{3} \]
- \[ \frac{2}{3} \]
- \[ \frac{3}{3} \]

- I can give 2 or more order cards in any order and it won't change the amount that is burned.

- If I order this \[ \frac{1}{3} \] then \[ \frac{2}{3} \] is burned.
- If I order this \[ \frac{1}{3} \] then \[ \frac{2}{3} \] is burned.
- If I order this \[ \frac{1}{3} \] then \[ \frac{2}{3} \] is burned.

For example,

This order \[ \frac{1}{3}, \frac{1}{3} \] will have \[ \frac{2}{3} \] burned.

Next, I want to ask you some questions...

1. How much will be burned in this order: \[ \frac{1}{3}, \frac{1}{3} \] ?

1.) \[ \frac{1}{3}, \frac{1}{3} \]

Remember:
- Any single amount ordered with \[ \frac{3}{3} \] will be burned
- Order \[ \frac{1}{3}, \frac{1}{3} \] and \[ \frac{2}{3} \] will be burned
- Order \[ \frac{1}{3}, \frac{1}{3} \] and \[ \frac{2}{3} \] will be burned
- You can place 2 or more orders at one time
- The order that you turn the cards in doesn't effect how much is burned.

\[ \frac{2}{3} \] will be burned.

When this \[ \frac{1}{3}, \frac{2}{3} \] is ordered, \[ \frac{3}{3} \] is burned.

2. How much will be burned in this order: \[ \frac{1}{3}, \frac{2}{3} \] ?

1.) \[ \frac{1}{3}, \frac{2}{3} \]

Remember:
- Any single amount ordered with \[ \frac{3}{3} \] will be burned
- Order \[ \frac{1}{3}, \frac{2}{3} \] and \[ \frac{3}{3} \] will be burned
- Order \[ \frac{1}{3}, \frac{2}{3} \] and \[ \frac{3}{3} \] will be burned
- Order \[ \frac{1}{3}, \frac{2}{3} \] and \[ \frac{3}{3} \] will be burned
- You can place 2 or more orders at one time
- The order that you turn the cards in doesn't effect how much is burned.
will be burned.

When this [icon] is ordered, [icon] is burned.

How much will be burned in this order?

1.) [icon] 2.) [icon] 3.) [icon]

is burned

To figure out how much of this order is burned, let's look at it in steps of 2 at a time.

1. If just this [icon] is ordered, then [icon] would be burned.

   So, let's just consider this [icon] and the remaining [icon].

2. This [icon] would have this [icon] burned.

3. Finally, this [icon] would have this [icon] burned.

Let me sum up the idea.

We place orders of 2 or more cards. Depending on what specific order is placed, Mamma Maria burns one of the following amounts:

You can figure out how much is burned by looking at an order 2 cards at a time and remembering what happens with 2-card orders...

This is what you need to remember.

<table>
<thead>
<tr>
<th>Order:</th>
<th>Burned:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

1. How much is burned in the following order?

1.) [icon] 2.) [icon] 3.) [icon]
2. What order could have been placed so that this much was burned?
   1.)  
   2.)  
   3.)  
   4.)  

3. What can be ordered with to have burned?
   1.)  
   2.)  
   3.)  
   4.)  

4. The other day, some coworkers were arguing about the pizza order that was placed. This much was burned. And they knew that these cards were part of the order. The guys were discussing what other cards could have been part of the order. Which of their responses do you agree with?
   1.) any card  
   2.) any card except  
   3.) any card except  
   4.) any card except  

5. What could have been ordered with to get this much burned?
   1.)  
   2.)  
   3.)  
   4.)  

6. What could be ordered to have this much burned?
   1.)  
   2.)  
   3.)  
   4.) none of the above  

7. How many of these could be ordered to have this much burned?
   1.) four  
   2.) five  
   3.) six  
   4.) seven  

8. What additional amount can be ordered with the following to get this burned?
   1.)  
   2.)  
   3.)  
   4.) we need more information to answer  

9. Do the following orders have the same burned amount?
   Order 1:  
   Order 2:  
   1 - Yes  
   2 - No  

200
10. Do the following orders have the same burned amount?

Order 1: 
Order 2: 

1 - Yes  2 - No

11. How about the following, do they have the same burned amount?

Order 1: 
Order 2: 

1 - Yes  2 - No

12. How much will be burned in this order?

1.)  2.)  3.)

You have completed this portion of the experiment.
Press the space bar when you are ready to proceed to the last portion of this experiment.
The following pages present the training and testing of the condition in which participants learned on Relevantly Concrete and One Generic Domain.
In the process of making the detergent, cups of solutions are combined. Different kinds of solutions are used, but what is important is the quantities of solutions.

So to find the left-over amount, we can use \( \text{mL} \), \( \text{mL} \) and \( \text{mL} \) to represent the different possible quantities of any of the solutions.

For example, if \( \text{mL} \) and \( \text{mL} \) are combined, the quantities will fill one detergent container and \( \text{mL} \) will be left-over.

So, you have the idea. We need to find the left-over quantity when the following types of cups of solution are combined.

Now let me teach you the specific rules for finding the left-over quantities.

<table>
<thead>
<tr>
<th>1. The order by which two cups of solution are combined does not change the left-over result.</th>
<th>Here is a question for you.</th>
</tr>
</thead>
<tbody>
<tr>
<td>For example, combining ( \text{mL} ) with ( \text{mL} ) has a left-over quantity of ( \text{mL} ). And combining ( \text{mL} ) with ( \text{mL} ) has the left-over quantity ( \text{mL} ).</td>
<td>Suppose you know that ( \text{mL} ) and ( \text{mL} ) combine to yield a left-over of ( \text{mL} ). Then what is left-over when ( \text{mL} ) and ( \text{mL} ) combine?</td>
</tr>
<tr>
<td>1.) ( \text{mL} ) 2.) ( \text{mL} ) 3.) ( \text{mL} )</td>
<td>Enter 1, 2, or 3:</td>
</tr>
</tbody>
</table>

\( \text{mL} \) is correct

Because the order of combination does not matter. When \( \text{mL} \) is combined with \( \text{mL} \), \( \text{mL} \) is left-over.

AND, when \( \text{mL} \) is combined with \( \text{mL} \), \( \text{mL} \) is left-over.

So you know that the order of combining solutions does not matter. Now, let me tell you some specifics about combinations.
Remember, we are filling containers, but we always want to have a cup of solution to test for quality when we are done.

Rule 2. \[\text{and } \text{ will fill a container, but we need a quantity of solution to test, so we consider } \text{ as the left-over.} \]

Rule 3. When any kind of cup of solution combines with \[\text{, the result will always be the other solution cup.} \]

For example:

When \[\text{and } \text{ combine, } \text{ is left-over.} \]

And when \[\text{and } \text{ combine, } \text{ is left-over.} \]

So tell me what is left-over when \[\text{ and } \text{ combine?} \]

1.) \[\text{ }\]
2.) \[\text{ }\]
3.) \[\text{ }\]

\[\text{ is correct.} \]

Whenever \[\text{ combines with another solution cup, the result is that other cup.} \]

So, \[\text{ is left-over when } \text{ and } \text{ combine.} \]

Rule 4. A combination of \[\text{ and } \text{ does not fill a container, so the left-over is } \text{.} \]

Rule 5. A combination of \[\text{ and } \text{ fills one container and has } \text{ left-over.} \]

Rule 6. Finally, you need to know that when mixing more than 2 cups of solution, the order of combining solutions does not matter. The left-over is the same no matter which cups are combined first.

For example: When we combine \[\text{ , } \text{ and } \text{, the left-over is } \text{.} \]

It does not matter if we do \[\text{ and } \text{ first and then } \text{ OR } \text{ and } \text{ first and then } \text{.} \]

\[\text{ is correct.} \]

Here is a question.

If mixing \[\text{, } \text{ and } \text{ has a left-over of } \text{, then what is left-over when mixing } \text{, } \text{ and } \text{?} \]

1.) \[\text{ }\]
2.) \[\text{ }\]
3.) \[\text{ }\]

Remember the order of combinations does not matter, the left-over will be the same.

So, combining \[\text{, } \text{ and } \text{ is the same as combining } \text{, } \text{ and } \text{.} \]

Both have \[\text{ as a left-over.} \]
To summarize,

Cups of solutions are mixed together to fill detergent containers. Your job is to find the cup with the remainder quantity.

We always want to have a quantity remaining, so the possible remainders are:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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1. What is left-over
when 🚿 and 🚿 mix?

Reminders
You can combine 2 or more cups of solution in any order, the left over will be the same.

Left-overs for specific mix combinations:

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2. What is left-over
when 🚿 and 🚿 mix?

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You can combine 2 or more cups of solution in any order, the left over will be the same.

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3. What is left-over
when 🚿, 🚿, and 🚿 mix?

Reminders
You can combine 2 or more cups of solution in any order, the left over will be the same.

Left-overs for specific mix combinations:

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Because 🚿 and 🚿 mixed together with 🚿 left-over, then 🚿 and 🚿 mix leaving 🚿 left-over.
Remember, you can combine 2 or more cups in any order. Next let's summarize the left-over cups resulting from different combinations of solution quantities...

Here is what happens when \( \text{cups} \) mixes with other cups.

<table>
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Here are the left-overs from all of the other possible cup combinations:

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</tr>
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</table>

Let's do an example...

Now we have \( \text{cups} \) and \( \text{cups} \) combine with \( \text{cups} \) left-over.

We know \( \text{cups} \) and \( \text{cups} \) combine with \( \text{cups} \) left-over.

So we get: \( \text{cups} \) left-over.

Then we can mix \( \text{cups} \) and \( \text{cups} \) to get \( \text{cups} \) left-over.

Finally, we have \( \text{cups} \) and \( \text{cups} \) which mix to leave \( \text{cups} \) left-over.

Let's find the left-over quantity when the following cups of solution are mixed.

1st Let's mix the \( \text{cups} \)

So we get: \( \text{cups} \) left-over.

What is the left-over when these cups are mixed?

1.) \( \text{cups} \)  
2.) \( \text{cups} \)  
3.) \( \text{cups} \)

To mix these: \( \text{cups} \)

First, we can mix \( \text{cups} \) and \( \text{cups} \) to get \( \text{cups} \) left-over.

Then we have: \( \text{cups} \) left-over.

So we get: \( \text{cups} \) left-over.

Finally \( \text{cups} \) and \( \text{cups} \) combine to leave \( \text{cups} \) left-over.
Here is a question for you. What possible cups of solution can combine so that left-over?

1. and  
2. and  
3. and  
4. and  

Either and  
Or and  

Combine to have left-over

Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

1. What is left-over when and combine?
   1.)  
   2.)  
   3.)  

2. What possible cups of solution can combine with to have a left-over of ?
   1.) and  
   2.) and  
   3.) and  
   4.) and  

3. What can combine with to have left-over?
   1.)  
   2.)  
   3.)  

4. What combination of cups has the same left-over as the following?
   
   1.)  
   2.)  
   3.)  
   4.) none of the above
5. Later I overheard two employees talking. They were arguing about whether these two mixtures of solution (below) would have the same left-over.

What do you think?

Mix 1:

Mix 2:

1 - Same  
2 - Different

6. True or false...

When the cups are mixed, mix 1 and mix 2 will have the same left-over.

Mix 1:

Mix 2:

1 - True  
2 - False

7. How about these mixtures, will they have the same left-overs?

Mix 1:

Mix 2:

1 - Yes  
2 - No

8. What is left-over when the following cups are mixed?

1.)  
2.)  
3.)

9. One day, a batch of detergent was spilled. We did not know the left-over quantity, but we did know that there were two cups in the mixture; one of them was . We were trying to figure out what the left-over could have been. Here are some opinions of the employees. Which do you agree with?

1.) the left-over could be any cup  
2.) the left-over could only be or  
3.) the left-over could only be

10. What is left-over when the following cups of solution combine?

1.)  
2.)  
3.)
11. What cup needs to mix with the following to have a left-over of?

1.)  
2.)  
3.)  

12. What cups need to mix with to have left-over?

1.)  
2.)  
3.)  
4.) none of the above

You have completed the first portion of this experiment.

Press the space bar when you are ready to proceed to the next portion.
The following pages present the training and testing of the condition in which participants learned three Relevantly Concrete Domains.

Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.

Press the space bar to advance when there is no question.

Allow me to introduce myself. I am Melvin, chief engineer for efficiency at the Bubblinski Detergent Company.

At the Bubblinski Detergent Company we mix solutions of different kinds to fill containers of detergents.

Recently, we have been having some problems with the mixing of the solutions. The detergent mixes may not be correct.

I need your help to determine the amount of left-over solution that results in the mixing process. Then I will test the left-over amount to make sure that the detergent is being made correctly.

Let me tell you how the quantities of solutions are combined so that we can determine how much solution is left-over.
In the process of making the detergent, cups of solutions are combined.

Different kinds of solutions are used, but what is important is the quantities of solutions.

So to find the left-over amount, we can use \( \text{, \quad \text{ and \quad \text{}} } \) to represent the different possible quantities of any of the solutions.

For example,

if \( \text{ and \quad \text{ are combined, the quantities will fill one detergent container and } \text{ will be left-over.} \)

So, you have the idea. We need to find the left-over quantity when the following types of cups of solution are combined.

Now let me teach you the specific rules for finding the left-over quantities.

1. The order by which two cups of solution are combined does not change the left-over result.

   For example, combining \( \text{ with } \) has a left-over quantity of \( \).

   And combining \( \text{ with } \) has the left-over quantity \( \).

   Here is a question for you.

   Suppose you know that \( \text{ and } \) combine to yield a left-over of \( \text{. Then what is left-over when } \text{ and } \) combine?

   Enter 1, 2, or 3:

   1.) \( \) 2.) \( \) 3.) \( \)

   \( \text{ is correct} \)

   Because the order of combination does not matter.

   When \( \) is combined with \( \), \( \) is left-over.

   AND, when \( \) is combined with \( \), \( \) is left-over.

So you know that the order of combining solutions does not matter. Now, let me tell you some specifics about combinations.
<table>
<thead>
<tr>
<th>Remember, we are filling containers, but we always want to have a cup of solution to test for quality when we are done.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 2. ( \text{and} ) will fill a container, but we need a quantity of solution to test, so we consider ( \text{as} ) the left-over.</td>
</tr>
<tr>
<td>Rule 3. When any kind of cup of solution combines with ( ), the result will always be the other solution cup.</td>
</tr>
<tr>
<td>For example:</td>
</tr>
<tr>
<td>When ( ) and ( ) combine, ( ) is left-over.</td>
</tr>
<tr>
<td>And when ( ) and ( ) combine, ( ) is left-over.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>So tell me what is left-over when ( ) and ( ) combine?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.) ( ) \hspace{1cm} 2.) ( ) \hspace{1cm} 3.) ( )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( ) is correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whenever ( ) combines with another solution cup, the result is that other cup.</td>
</tr>
<tr>
<td>So, ( ) is left-over when ( ) and ( ) combine.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rule 4. A combination of ( ) and ( ) does not fill a container, so the left-over is ( ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 5. A combination of ( ) and ( ) fills one container and has ( ) left-over.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rule 6. Finally, you need to know that when mixing more than 2 cups of solution, the order of combining solutions does not matter. The left-over is the same no matter which cups are combined first.</th>
</tr>
</thead>
<tbody>
<tr>
<td>For example: When we combine ( ), ( ) and ( ), the left-over is ( ).</td>
</tr>
<tr>
<td>It does not matter if we do ( ) and ( ) first and then ( ) OR ( ) and ( ) first and then ( ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Here is a question.</th>
</tr>
</thead>
<tbody>
<tr>
<td>If mixing ( ), ( ) and ( ) has a left-over of ( ), then what is left-over when mixing ( ), ( ) and ( )?</td>
</tr>
<tr>
<td>1.) ( ) \hspace{1cm} 2.) ( ) \hspace{1cm} 3.) ( )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( ) is correct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remember the order of combinations does not matter, the left-over will be the same.</td>
</tr>
<tr>
<td>So, combining ( ), ( ) and ( ) is the same as combining ( ), ( ) and ( ).</td>
</tr>
<tr>
<td>Both have ( ) as a left-over.</td>
</tr>
</tbody>
</table>
To summarize,

Cups of solutions are mixed together to fill detergent containers. Your job is to find the cup with the remainder quantity.

We always want to have a quantity remaining, so the possible remainders are:

![Cups with remainders]

1. What is left-over

   when 🟢 and 🟣 mix?

   **Reminders**
   You can combine 2 or more cups of solution in any order, the left over will be the same.

   **Left-overs for specific mix combinations:**

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>🟢</td>
<td>🟢</td>
</tr>
<tr>
<td>🟡</td>
<td>🟢</td>
</tr>
<tr>
<td>🟢+🟢</td>
<td>🟢</td>
</tr>
<tr>
<td>🟡+🟢</td>
<td>🟢</td>
</tr>
<tr>
<td>🟡+🟡</td>
<td>🟡</td>
</tr>
<tr>
<td>🟡+🟡+🟢</td>
<td>🟢</td>
</tr>
</tbody>
</table>

   ![Left-over combinations]

   When 🟢 and 🟢 are mixed together 🟢 is left-over.

2. What is left-over

   when 🟢, 🟡, and 🟢 mix?

   **Reminders**
   You can combine 2 or more cups of solution in any order, the left over will be the same.

   **Left-overs for specific mix combinations:**

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>🟢</td>
<td>🟢</td>
</tr>
<tr>
<td>🟡</td>
<td>🟢</td>
</tr>
<tr>
<td>🟢+🟢</td>
<td>🟢</td>
</tr>
<tr>
<td>🟡+🟢</td>
<td>🟢</td>
</tr>
<tr>
<td>🟡+_FLUSH️</td>
<td>🟢</td>
</tr>
<tr>
<td>🟡+_FLUSH️+_FLUSH️</td>
<td>🟢</td>
</tr>
</tbody>
</table>

   ![Left-over combinations]

   Because 🟢 and 🟢 mixed together with 🟢 left-over; then 🟢 and 🟢 mix leaving 🟢 left-over.

Let’s do an example...

Let’s find the left-over quantity when the following cups of solution are mixed:

![Mixed cups]

1st Let’s mix the 🟢

So we get:

![Resulting cups]
Now we have

We know and combine with left-over.

So we get.

Then we can mix and to get left-over.

Finally, we have and which mix to leave left-over.

What is the left-over when these cups are mixed?

1.)

2.)

3.)

Remember, this is what you need to know.

1. What is left-over when combine?

1.)

2.)

3.)

Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

2. What possible cups of solution can combine with to have a left-over of ?

1.) and

2.) and

3.) and

4.) and

3. What can mix with to have as a left-over?

1.)

2.)

3.)

4.)
4. Later I overheard two employees talking. They were arguing about whether these two mixtures of solution (below) would have the same left-over. What do you think?

Mix 1: 
Mix 2: 
1 - Same 2 - Different

5. True or false...
When the cups are mixed, mix 1 and mix 2 will have the same left-over.

Mix 1: 
Mix 2: 
1 - True 2 - False

6. What is left-over when the following cups are mixed?

1.) 
2.) 
3.)

7. What cup needs to mix with the following to have a left-over of ?

1.) 
2.) 
3.)

8. What cups need to mix with to have left-over?

1.) 
2.) 
3.) 
4.) none of the above

You have completed the first portion of this experiment.
Press the space bar when you are ready to proceed to the next portion.

Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.
Press the space bar to advance when there is no question.

Now you will learn about a new system.
This system works the same way as the last system you learned.
The rules of the last systems are like the rules of this new one.
Hi. I am Antonio, a very big pizza lover. I eat pizza every day for lunch.

Every day I order pizzas for myself and my friends at work. We order from Mamma Maria’s Pizza Kitchen. It’s the best pizza in town.

Mamma Maria makes 1 size of pizza that has 3 slices.

At work, everyone orders 1, 2, or 3 slices.

If someone wants 1 slice, he gives me a card like this.

If someone wants 2 slices, he gives me a card like this.

If someone wants 3 slices, he gives me a card like this.

I place an order at the Pizza Kitchen by giving the cards to Mamma Maria. Then she gives me the number of whole pizzas and/or individual slices we need.

There is one problem though. Mamma Maria always burns part of our order. So, some people have burned slices.

Here is what is happening.

Mamma Maria collects the cards for the order.

She makes enough whole pizzas to fill the order.

All of the pizzas come out ok, except the very last portion that she cooks. It may be a whole pizza or less than a whole pizza.

Let me show you an example...

If I give her the following order.

We would get 1 pizza that is ok, but this much of our order would be burned.
My friends and I are trying to figure out how much pizza is burned when different orders are placed? Will you help us? First, let me tell you what I know about placing the orders and how much is burned.

There is never more than 1 whole pizza burned.

So the burned amount will always be

\[ \begin{array}{c}
\text{1.)} \\
\text{2.)} \\
\text{3.)} \\
\end{array} \]

1. What I order first or second doesn't matter. The same amount gets burned.

For example, if I order this \( \text{1.)} \) first and then this \( \text{2.)} \), then this much \( \text{3.)} \) is given to us burned.

The same thing happens if I order this \( \text{4.)} \) first and then this \( \text{5.)} \).

We get this much \( \text{6.)} \) burned.

So, let me ask you...

If I order this \( \text{7.)} \), then this \( \text{8.)} \), and this much \( \text{9.)} \) is given to us burned, then how much would be burned if I order this \( \text{10.)} \) and then this \( \text{11.)} \)?

Enter 1, 2, or 3:

1.) \( \text{12.)} \) 2.) \( \text{13.)} \) 3.) \( \text{14.)} \)

2. If I order this \( \text{15.)} \) with any other single amount, the other amount is always burned.

Here are a couple of examples:

If I order \( \text{16.)} \) and \( \text{17.)} \), then \( \text{18.)} \) is burned.

If I order \( \text{19.)} \) and \( \text{20.)} \), then \( \text{21.)} \) is burned.

So, how much will be burned if I order \( \text{22.)} \) and \( \text{23.)} \)?

Choose:

1.) \( \text{24.)} \) 2.) \( \text{25.)} \) 3.) \( \text{26.)} \)

Whenever this \( \text{27.)} \) is ordered, the other amount is burned.

So if I order \( \text{28.)} \) and \( \text{29.)} \), then \( \text{30.)} \) is burned.
3. If I order \[ \square \] and \[ \square \], then only one pizza is made and \[ \square \] is burned.

4. If I order \[ \square \] and \[ \square \], then one pizza is made and \[ \square \] is burned.

5. If I order \[ \square \] and \[ \square \], then one pizza is ok, but \[ \square \] is burned.

6. If I turn in more than 2 order cards, the order that I turn them in doesn’t matter. The same amount ends up burned.

   For example, if I turn in \[ \square \] and \[ \square \] and then \[ \square \], then \[ \square \] is burned.

   The same amount is burned if I turn in \[ \square \] and \[ \square \] and then \[ \square \].

Let me ask you a question...

If \[ \square \] is burned when I turn in \[ \square \], \[ \square \] and then \[ \square \],

Then how much is burned if I turn in \[ \square \] \[ \square \] \[ \square \] then \[ \square \]?

Choose:

1) \[ \square \] 2) \[ \square \] 3) \[ \square \]

Remember the order that I turn the cards in doesn’t matter, the same amount will be burned.

So this order \[ \square \] \[ \square \] \[ \square \] has this \[ \square \] burned;

and this order \[ \square \] \[ \square \] \[ \square \] also has this \[ \square \] burned.

Next, I want to ask you some questions...

1. How much will be burned in this order: \[ \square \] \[ \square \] ?

   1) \[ \square \] 2) \[ \square \] 3) \[ \square \]

Remember:

* Any single amount ordered with \[ \square \] will be burned
* Order \[ \square \] and \[ \square \] will be burned
* Order \[ \square \] and \[ \square \] will be burned
* Order \[ \square \] and \[ \square \] will be burned
* You can place 2 or more orders at one time
* The order that you turn the cards in doesn’t affect how much is burned.
219

When this card is ordered, the card is burned.

2. How much will be burned in this order: card, card?

1.) 

2.)

3.) Remember:

- Any single amount ordered with another card will be burned.
- Order card, card, and card will be burned.
- Order card, card, and card will be burned.
- Order card, card, and card will be burned.

- You can place 2 or more orders at one time.
- The order that you burn the cards in doesn't effect how much is burned.

When this card is ordered, the card is burned.

What order would have this card burned?

1.) card and card

2.) card and card

3.) card and card

4.) card and card

Either card and card

Or card and card

Combine to have card left-over

Let me sum up the idea.
We place orders of 2 or more cards.
Depending on what specific order is placed, Mamma Maria burns one of the following amounts:

This is what you need to remember.

<table>
<thead>
<tr>
<th>Order:</th>
<th>Burned:</th>
</tr>
</thead>
<tbody>
<tr>
<td>card</td>
<td>card</td>
</tr>
<tr>
<td>card</td>
<td>card</td>
</tr>
<tr>
<td>card</td>
<td>card</td>
</tr>
<tr>
<td>card</td>
<td>card</td>
</tr>
</tbody>
</table>
Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

1. How much is burned in the following order?

   1.)  
   2.)  
   3.)  

2. What order could have been placed so that this much was burned?

   1.)  and  
   2.)  and  
   3.)  and  

3. What order has the same burned amount as the following order?

   1.)  
   2.)  
   3.)  
   4.)  none of the above

4. The other day, some coworkers were arguing about the pizza order that was placed. This much was burned.

   And they knew that these cards were part of the order. The guys were discussing what other cards could have been part of the order. Which of their responses do you agree with?

   1.) any card  
   2.) any card except  
   3.) any card except  
   4.) any card except

5. What additional amount can be ordered with the following to get this burned?

   1.)  
   2.)  
   3.)  
   4.) we need more information to answer

6. Do the following orders have the same burned amount?

   Order 1:  
   Order 2:  

   1 - Yes  2 - No

7. How about the following, do they have the same burned amount?

   Order 1:  
   Order 2:  

   1 - Yes  2 - No
8. How much will be burned in this order?

<table>
<thead>
<tr>
<th>1.)</th>
<th>2.)</th>
<th>3.)</th>
</tr>
</thead>
</table>

You have completed this portion of the experiment.
Press the space bar when you are ready to proceed to the last portion of this experiment.

Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.
Press the space bar to advance when there is no question.

Now you will learn about another system.
This system works the same way as the last system you learned.
The rules of the last systems are like the rules of this new one.

Hi. I am Vincent. I own the Great American Tennis Ball Factory.

At the Great American Tennis Ball Factory, we make tennis balls and pack them into containers of 3 balls.

Normally, our ball-making machine produces small batches of 3 balls that get directly put into a container. But recently we have been experiencing some problems.

The machine is not producing 3 balls per batch.

Now it produces 0, 1, or 2 balls in a batch.

No matter how many balls were produced in a batch we need to fill our containers with 3 balls.

So we make two or more batches of balls and fill up as many containers as we can.

But some times there are extra balls.
I need your help finding out how many extra balls there are when different batches are produced.

So batches will have 0, 1, or 2 balls.
Let's let this \( \bigcirc \) stand for a batch with 0 balls.
let this \( \bigcirc \) stand for a batch with 1 ball.
let this \( \bigcirc \) stand for a batch with 2 balls.

Here is an example to help you see what I mean.
Suppose we have these two batches of balls,

We could fill one container, but we would have
this much \( \bigcirc \) extra.

Next, let me tell you some specific things about figuring out how many extra balls you will have.

The extra number of balls will always be

\( \bigcirc \), \( \bigcirc \), or \( \bigcirc \).

1. The order of the batches doesn't matter. The number of extra balls will be the same.
For example, if this batch \( \bigcirc \) is made first and then this \( \bigcirc \).
then this much \( \bigcirc \) is extra.

The same thing happens if this batch \( \bigcirc \) is made first and then this \( \bigcirc \).
We will have this much \( \bigcirc \) extra.

So, let me ask you...
If this \( \bigcirc \) is made then this \( \bigcirc \), and this much \( \bigcirc \) is extra, then how much would be extra if this \( \bigcirc \) is made and then this \( \bigcirc \)?

Enter 1, 2, or 3:
1) \( \bigcirc \) 2) \( \bigcirc \) 3) \( \bigcirc \)

\( \bigcirc \) is extra.

Making \( \bigcirc \) then \( \bigcirc \) is the same as
Making \( \bigcirc \) then \( \bigcirc \). So \( \bigcirc \) is extra.
2. If this batch [●] is made with any other single batch, the other amount is always extra.

Here are a couple of examples:

If this [●] and this [●] are made, then [●] is extra.

If this [●] and this [●] are made, then [●] is extra.

So, if these batches [●] are made, how much will be extra?

Choose:

1) [●]  2) [●]  3) [●]

[●] is extra.

Whenever this batch [●] is made, the other amount is extra.

So if [●] and [●], then [●] is extra.

3. If [●] and [●] are produced, then one container can be filled and [●] is extra.

4. If [●] and [●] are produced, then we cannot fill a container. So, [●] is extra.

5. If [●] and [●] are made, then one container can be filled and [●] is extra.

6. If more than two batches are produced, the order in which they are made doesn’t matter. The extra will be the same.

For example, if [●] and [●] and then [●] are made, then [●] is extra.

The same amount is extra if [●] and [●] and then [●].

Let me ask you a question...

If [●] is extra when [●] and then [●] are made, then how much is extra if [●], [●] then [●] are made?

Choose:

1) [●]  2) [●]  3) [●]
is extra.

Remember the order that batches are made doesn’t matter, the same amount will be extra.

So when these \( \bullet \bullet \bullet \bullet \) are made, \( \bullet \bullet \) is extra;

and when these \( \bullet \bullet \bullet \bullet \) are made, this \( \bullet \) is also extra.

Next, I want to ask you a question…

Suppose these batches \( \bullet \bullet \bullet \) were made. How many extra balls would there be?

Choose:

1.) \( \bullet \bullet \)  
   Remarks:
   1. Two or more batches can be made at a time.
   2. The order that batches are made doesn’t matter.
   3. If \( \bullet \bullet \) is made with another batch, the other batch is extra.
   4. Making these \( \bullet \bullet \) has \( \bullet \bullet \) extra.
   5. Making these \( \bullet \bullet \) has \( \bullet \bullet \) extra.
   6. Making these \( \bullet \bullet \) has \( \bullet \bullet \) extra.

2.) \( \bullet \)  

3.) \( \)  

Making \( \bullet \bullet \) and \( \bullet \bullet \) will not fill a container.

So \( \bullet \bullet \) is extra.

What is extra when the following batches are made?

1.) \( \)  
2.) \( \bullet \)  
3.) \( \bullet \bullet \)  

When these batches are made \( \bullet \bullet \bullet \bullet \),
We can forget about \( \bullet \bullet \).

So now we have \( \) .

When these \( \bullet \bullet \bullet \) are made, \( \bullet \bullet \) is extra.
So now we have \( \bullet \bullet \bullet \bullet \) .

And so \( \bullet \bullet \) is extra.

Let me summarize the idea.

Two or more batches of balls are made; and the balls are put into containers of 3.

The number of extra balls will be

\( \bullet \), \( \bullet \bullet \), or \( \bullet \bullet \bullet \).

Here is what you need to remember:

<table>
<thead>
<tr>
<th>If these batches are made:</th>
<th>Then this is extra:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bullet \bullet \bullet )</td>
<td>( \bullet \bullet \bullet )</td>
</tr>
</tbody>
</table>
Now let me ask you some final questions.

Please answer them as accurately and as quickly as possible.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Two groups of batches were made. Do they have the same amount extra?</td>
<td>3. Four batches of balls were made including and . This was extra. What were the other two batches?</td>
</tr>
<tr>
<td>Group 1</td>
<td>1.) and 2.)</td>
</tr>
<tr>
<td>Group 2</td>
<td>3.) and</td>
</tr>
<tr>
<td>1 - Yes</td>
<td>2 - No</td>
</tr>
<tr>
<td>3.) and 4.)</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Which two batches below could have been made so that is extra?</td>
<td>5. How many groups of could be made so that is extra?</td>
</tr>
<tr>
<td>1.) and 2.) and</td>
<td>1.) four 2.) five</td>
</tr>
<tr>
<td>3.) and 4.) none of the above</td>
<td>3.) six 4.) seven</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6. One day two batches of balls were made. One of the batches was , but we didn't know the other. Some guys at the factory were arguing about how much would be extra. Which do you agree with?</td>
<td>7. How much is extra if these batches were produced?</td>
</tr>
<tr>
<td>1.) the extra could be any amount</td>
<td>1.)</td>
</tr>
<tr>
<td>2.) the extra could only be or</td>
<td>2.)</td>
</tr>
<tr>
<td>3.) the extra can only be</td>
<td>3.)</td>
</tr>
</tbody>
</table>

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8. Two groups of batches were made. Do they have the same amount extra?

Group 1

Group 2

1 - Yes  2 - No

You have completed this portion of the experiment.
Press the space bar when you are ready to proceed to the last portion of this experiment.
APPENDIX H

TRAINING AND TESTING OF PHASE 1 OF EXPERIMENT 6 FOR RELATIONAL DIAGRAM AND CONCRETE CONDITIONS

The following pages present the training and testing of the Relational Diagram condition.
Please read the following information and answer the questions that are presented along the way.

Enter 1, 2, 3, or 4 to indicate your answers to the questions.

Press the space bar to advance when there is no question.

Hi.
Let me introduce myself. I am Melvin Schelvin. I work as a code breaker.

I want to tell you about a device I use for figuring out sequences of symbols.

Here is the idea. We get information in the form of sequences of symbols. Based on a given sequence, we figure out a resulting symbol.

Only three kinds of symbols are used:

We figured out that if we place the symbols in specific positions around the dial and move around the dial according to specific symbols we can determine the resulting symbol.

So we placed the symbols like this. Next, I will show you what I mean about moving around the dial.
Here is an example. If I receive the sequence ◆ ⊘,
I start at the ◆ position on the decoder dial.

Here is an example. If I receive the sequence ◆ ⊘,
I start at the ◆ position on the decoder dial.

Here is an example. If I receive the sequence ◆ ⊘,
THEN I move clockwise forward 2 positions
( ◆ tells us to move forward 2 positions).

I will stop at ⦿ position.
So, the resulting symbol is ⦿.

Here is an example. If I receive the sequence ◆ ⊘,

So the idea is:
We get a sequence of 2 or more symbols.
The 1st symbol tells us where to start on the dial.
The following symbols tell us how many positions to move clockwise ahead.

In particular:
◆ means move ahead 1 position
⊘ means move ahead 2 positions
⦿ means move ahead 3 positions

Now, let me show you more specific examples.

1. No matter what the first symbol is, if the second symbol is ⦿, the resulting symbol will be the first symbol.
   (⦁ ⦿ takes you all the way around the dial.)
Here is an example...
If the sequence is \( \bullet \),
Start here \( \bullet \)
and move ahead 3 positions.

Here is an example...
If the sequence is \( \bullet \),
Start here \( \bullet \)
and move ahead 3 positions.

Here is an example...
If the sequence is \( \bullet \),
Start here \( \bullet \)
and move ahead 3 positions.

Here is an example...
If the sequence is \( \bullet \),
Start here \( \bullet \)
and move ahead 3 positions.

You stop here \( \bullet \)
So the resulting symbol is \( \bullet \).
We can write this as:
\( \bullet \) \( \bullet \) \rightarrow \( \bullet \).

Here is another example...
If the sequence is \( \bullet \),
Start here \( \bullet \)
and move ahead 3 positions.

Here is another example...
If the sequence is \( \bullet \),
Start here \( \bullet \)
and move ahead 3 positions.

Here is another example...
If the sequence is \( \bullet \),
Start here \( \bullet \)
and move ahead 3 positions.
Here is another example...
If the sequence is ✧ ✧ ✧.

You will stop here ✧.
So
✧ ✧ ✧ → ✧.

Now, let me ask you...
If you have this sequence ✧ ✧ ✧, what is the resulting symbol?

Choose:
1.) ✧
2.) ●
3.) ✧

is correct.
✧ ✧ ✧ → ✧.

Start here ✧.
Move ahead 3...

Let me ask you another question.
If you have this sequence ● ● ●, what is the resulting symbol?

Choose:
1.) ✧
2.) ●
3.) ✧

is correct.
● ● ● → ●.

Start here ●.
Move ahead 3...

You will stop here ☑.

is correct.

Start here 
Move ahead 3...

is correct.

Start here 
Move ahead 3...

is correct.

Start here .
You will stop here .

Now let me show you more examples.

2. 
Start here and 
the next means move ahead 1

2. 
So you stop here .

3. 
Next...
Start here and 
means move ahead 1
3.  

You stop here....

Next...

4.  

Start here.

● means move ahead 2

4.  

Start here.

● means move ahead 2

Here is something else you should know...

You can switch the order of the symbols in a sequence and the result will be the same.

Let me show you an example...

5.  

Watch the dot move...

1st start here ●

and move ahead 1 (for ● )

and

● ☐

Watch the dot move...

1st start here ●

and move ahead 1 (for ☐ )

You stop here....
Now start here  
and move ahead 2 (for  )

Here's a question. 
If  ,
then  ?
Choose:
1.)  
2.)  
3.)  

You stop here  

Because the order of the symbols doesn't matter. 
If  ,
then  

Let me ask you another 
If  ,
then  ?
Choose:
1.)  
2.)  
3.)  


Finally you need to know that sequences can be more than 2 symbols long.

And the order of the symbols doesn't matter, the resulting symbol will be the same.

Here is an example...
1. Sequences can have 2 or more symbols.

2. The order of the symbols doesn't matter—the result will be the same.

For example:

\[ \bullet \downarrow \downarrow \rightarrow \bullet \]

and

\[ \downarrow \bullet \rightarrow \bullet \]

Next I will tell you some specifics to remember...

3. When \( \text{☆} \) is in a sequence with any other symbol, the result will be the other symbol.

For example,
4. This symbol ◆ will move you ahead 1 position.
   For example, ◆ ◆ → ◆
   For example, ◆ ◆ → ◆

5. This symbol ⬤ will move you ahead 2 positions.
   For example, ⬤ ◆ → ◆
   For example, ⬤ ⬤ → ⬤

Next I want to ask you some questions...

1. What is the resulting symbol?
   Choose:
   1.)
   2.)
   3.)
   Remember:
   1. Sequences can have 2 or more symbols
   2. The order of symbols in sequence doesn't matter
   3. Any symbol ⬤ ⬤ -> same symbol
   4.)
   5.)
   6.)

   is correct
   Watch....
   Start here ⬤
   ◆ means move ahead 2

   is correct
   Watch....
   Start here ⬤
   ◆ means move ahead 2

2. What is the resulting symbol?
   Choose:
   1.)
   2.)
   3.)
   Remember:
   1. Sequences can have 2 or more symbols
   2. The order of symbols in sequence doesn't matter
   3. Any symbol ⬤ ⬤ -> same symbol
   4.)
   5.)
   6.)

You stop here ⬤
So the result is ⬤...
3. What is the resulting symbol?

Choose:

1. 
2. 
3. 

Remember:
1. Sequences can have 2 or more symbols.
2. The order of symbols in sequences doesn't matter.
3. Any symbol $\rightarrow$ same symbol.
4. 
5. 
6. 

4. What is the resulting symbol?

Choose:

1. 
2. 
3. 

Remember:
1. Sequences can have 2 or more symbols.
2. The order of symbols in sequences doesn’t matter.
3. Any symbol $\rightarrow$ same symbol.
4. 
5. 
6. 

Watch....
Start here

You stop here

So the result is

Watch....
Start here

You stop here

So the result is
Let me summarize the how this works...

1. Sequences can have 2 or more symbols.
2. The order of the symbols doesn't matter—the result will be the same.

For example:

Next I will tell you some specifics to remember...
3. When is in a sequence with any other symbol, the result will be the other symbol.

For example,

\[
\begin{align*}
\text{ } & \rightarrow \text{ } \\
\text{ } & \rightarrow \text{ }
\end{align*}
\]

4. This symbol \( \text{ } \) will move you ahead 1 position.

For example,

\[
\begin{align*}
\text{ } & \rightarrow \text{ } \\
\text{ } & \rightarrow \text{ }
\end{align*}
\]

5. This symbol \( \text{ } \) will move you ahead 2 positions.

For example,

\[
\begin{align*}
\text{ } & \rightarrow \text{ } \\
\text{ } & \rightarrow \text{ }
\end{align*}
\]

Let's find the resulting symbol for this sequence

\[
\begin{align*}
\text{ } & \rightarrow \text{ } \\
\text{ } & \rightarrow \text{ }
\end{align*}
\]

Let's find the resulting symbol for this sequence

\[
\begin{align*}
\text{ } & \rightarrow \text{ } \\
\text{ } & \rightarrow \text{ }
\end{align*}
\]

Let's find the resulting symbol for this sequence

\[
\begin{align*}
\text{ } & \rightarrow \text{ } \\
\text{ } & \rightarrow \text{ }
\end{align*}
\]

Let's find the resulting symbol for this sequence

\[
\begin{align*}
\text{ } & \rightarrow \text{ } \\
\text{ } & \rightarrow \text{ }
\end{align*}
\]

SUMMARY

1. Sequences can have 2 or more symbols
2. The order of symbols in sequence doesn't matter
3. Any symbol \( \text{ } \rightarrow \text{ same symbol} \)
4. \( \text{ } \rightarrow \text{ }
5. \text{ } \rightarrow \text{ }
6. \text{ } \rightarrow \text{ }

Let's start here \( \text{ } \) and take care of these \( \text{ } \) symbols first
(they take us all the way around the dial).

Let's start here \( \text{ } \) and take care of these \( \text{ } \) symbols first
(they take us all the way around the dial).

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Let's find the resulting symbol for this sequence

Let's start here and take care of these symbols first
(they take us all the way around the dial).

Let's find the resulting symbol for this sequence

Let's start here and take care of these symbols first
(they take us all the way around the dial).

Let's find the resulting symbol for this sequence

Let's start here and take care of these symbols first
(they take us all the way around the dial).

So now we have

So now we have

So let's start here and move 2 positions forward for EACH of these.

So let's start here and move 2 positions forward for EACH of these.

Let's find the resulting symbol for this sequence

Let's find the resulting symbol for this sequence

So now we have

So now we have

And we stop here.

So the resulting symbol is.
What is the resulting symbol for this sequence

Choose:
1.)
2.)
3.)

Let me show you.
Start here and
Go around 3 (for )
1 (for )
and 2 (for )

Let me show you.
Start here and
Go around 3 (for )
1 (for )
and 2 (for )

Let me show you.
Start here and
Go around 3 (for )
1 (for )
and 2 (for )

You stop here.
So the result is.
Which of the sequences below would have this resulting symbol? 

Choose:
1.)
2.)
3.)
4.)

Either
or

Watch

1st let's do

You stop here

Now let's do

Either
or

Now let's do

Either
or

We stop here

What is the resulting symbol for this sequence:

Choose:
1.)
2.)
3.)

Either
or

Let me show you.

Start here and

Go around 3 (for ) 1 (for ) and 2 (for )
Let me show you.
Start here ♦ and
Go around 3 (for ♦)
1 (for ♦)
and 2 (for ♦)

Let me show you.
Start here ♦ and
Go around 3 (for ♦)
1 (for ♦)
and 2 (for ♦)

Let me show you.
Start here ♦ and
Go around 3 (for ♦)
1 (for ♦)
and 2 (for ♦)

Let me show you.
Start here ♦ and
Go around 3 (for ♦)
1 (for ♦)
and 2 (for ♦)

You stop here ♦.
So the resulting symbol is ♦.

Let me review what you need to know.

SUMMARY
1. Sequences can have 2 or more symbols
2. The order of symbols in sequence doesn't matter
3. Any symbol ♦ → same symbol
4. ♦ ♦ → ♦
5. ♦ ♦ → ♦
6. ♦ ♦ → ♦
Now let me ask you some final questions. Please answer them as accurately and as quickly as possible.

1. What is the resulting symbol for this sequence?

   \[ \bullet \bullet \text{ } \rightarrow \text{ } \]

   Choose:
   1.) \[ \bullet \bullet \]  2.) \[ \bullet \]  3.) \[ \bullet \bullet \bullet \bullet \bullet \bullet \]

2. What is the resulting symbol for this sequence?

   \[ \bullet \text{ } \bullet \text{ } \bullet \text{ } \text{ } \rightarrow \text{ } \]

   1.) \[ \bullet \bullet \]  2.) \[ \bullet \]  3.) \[ \bullet \bullet \]

3. What two symbols can go in the blanks of the sequence below?

   \[ \text{ } \rightarrow \]  \[ \text{ } \text{ } \rightarrow \]

   1.) \[ \bullet \text{ and } \bullet \]  2.) \[ \bullet \text{ and } \bullet \]
   3.) \[ \bullet \text{ and } \bullet \]  4.) \[ \bullet \text{ and } \bullet \]

4. Which sequence has this resulting symbol?

   \[ \text{ } \rightarrow \]

   1.) \[ \bullet \bullet \]  2.) \[ \bullet \bullet \]  3.) \[ \bullet \bullet \]  4.) \[ \bullet \bullet \bullet \]

5. What symbol can go in the blank of the sequence below?

   \[ \text{ } \rightarrow \]

   1.) \[ \bullet \bullet \]  2.) \[ \bullet \]  3.) \[ \bullet \bullet \bullet \bullet \bullet \bullet \]

6. What symbols can go in the blanks of the sequence below?

   \[ \text{ } \bullet \text{ } \rightarrow \text{ } \]

   1.) \[ \bullet \bullet \]  2.) \[ \bullet \bullet \bullet \]  3.) \[ \bullet \bullet \bullet \bullet \bullet \bullet \]

7. Which sequence below would have the same resulting symbol as the following?

   \[ \text{ } \text{ } \rightarrow \]

   1.) \[ \bullet \bullet \]  2.) \[ \bullet \bullet \bullet \bullet \bullet \bullet \]
   3.) \[ \bullet \bullet \]  4.) None of the above
8. Some guys I work with were discussing what symbol could be placed in the first blank of the sequence below. Which of their responses do you agree with?

1.) any symbol  
2.) any symbol except ◇  
3.) any symbol except ●  
4.) any symbol except  

9. The other day, I overheard two other guys talking. They were arguing about whether these sequences would have the same resulting symbol.

What do you think?

1 - Same  
2 - Different

10. How about these sequences? Would the same resulting symbol?

1 - Same  
2 - Different

11. Do these sequences have the same resulting symbol?

1 - Same  
2 - Different

12. What symbols can go in the blanks of the sequence below?

1.)  and  
2.)  and  
3.)  and  
4.)  and  

13. Which sequence has this resulting symbol ◇ ?

1.)  
2.)  
3.)  
4.) none of the above

14. How many of this symbol ● would need to be in a sequence to have this resulting symbol ◇ ?

1.) four  
2.) five  
3.) six  
4.) seven

15. What is the resulting symbol for this sequence?

1.)  
2.)  
3.)  

16. What symbol can go in the blank of this sequence?
   \[
   \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \rightarrow \blacklozenge
   \]
   1.) \[
   \blacklozenge
   \]
   2.) \[
   \blacklozenge
   \]
   3.) \[
   \bullet
   \]
   4.) we need more information to answer

17. One day some guys and I were walking. We intercepted a partial sequence and we were trying to figure out what the result could be. We know there were only two symbols and one of them was \[
\bullet
\]. What do you think the resulting symbol could be?
   1.) the resulting symbol could be any symbol
   2.) the resulting symbol could only be \[
\blacklozenge
\] or \[
\bullet
\]
   3.) the resulting symbol could only be \[
\blacklozenge
\]

18. What is the resulting symbol for this sequence?
   \[
   \blacklozenge \ \bullet \ \blacklozenge \rightarrow
   \]
   1.) \[
   \blacklozenge
   \]
   2.) \[
   \blacklozenge
   \]
   3.) \[
   \bullet
   \]

19. Do the following sequences have the same resulting symbol?

20. Do the following sequences have the same resulting symbol?

21. Do the following sequences have the same resulting symbol?

22. What symbol can go in the blank of this sequence?
   \[
   \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \rightarrow \blacklozenge
   \]
   1.) \[
   \blacklozenge
   \]
   2.) \[
   \blacklozenge
   \]
   3.) \[
   \bullet
   \]

23. What is the resulting symbol for this sequence?
   \[
   \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge \bullet \rightarrow
   \]
   1.) \[
   \bullet
   \]
   2.) \[
   \bullet
   \]
   3.) \[
   \blacklozenge
   \]
24. What symbols can go in the blanks of the sequence below?

1.) and  
2.) and  
3.) and  
4.) none of the above

You have completed this portion of this experiment.

Press the space bar to proceed to the next portion.
The following pages present the training and testing of the Concrete condition.

| Please read the following information and answer the questions that are presented along the way. |
| Let me introduce myself. I am Gian Mario Presto, founder of the Presto-Electro Automobile Company. |

| Enter 1, 2, 3, or 4 to indicate your answers to the questions. |
| I would like your help. Let me explain to you how our cars work. |

| Press the space bar to advance when there is no question. |
| We have developed a new car, powered completely by electricity, and we are testing its performance. |

| The cars are powered completely by battery. A battery will enable the car to drive a specific distance. Stronger batteries will let the car drive farther than weaker batteries. |
| We test a car by driving it around an oval track. When a battery runs out we can put another one in, or just record the position where the car stopped. |
The track has three equidistant positions marked with flags. The distance between marked positions is 1 unit.

Next let me show you the batteries.

This battery will allow the car to travel 1 unit.
This battery will allow the car to travel 2 units.
This battery will allow the car to travel 3 units.

Now, let me show you an example.

If we start at the point \( \text{flag} \) and use this battery \( \text{battery} \), the car will stop at this point \( \text{flag} \).

If we start at the point \( \text{flag} \) and use this battery \( \text{battery} \), the car will stop at this point \( \text{flag} \).
So you will know a starting point and one or more batteries to use. Your job will be to determine the stopping point of the car.

Now I will show you more specific examples.

No matter where you start, using this battery will take you all the way around and you will end up at your starting point. Here is an example...

Suppose you start here and use this battery.

You will end up at your starting point of

Here is another example... Suppose you start here.

This battery will take you all the way around.

This battery takes you all the way around.

This battery takes you all the way around.
Let me ask you a question…
if you start here and use this battery, where will you stop?

Choose:
1.)  
2.)  
3.)  

Let me ask you another question…
if you start here and use this battery, where will you stop?

Choose:
1.)  
2.)  
3.)  

Now let me show you more examples.
Next...

If you start at this point and use this battery, you will end up at this point. Watch!

If you start at this point and use this battery, you will end up at this point. Watch!

Next...

If you start at this point and use this battery, you will end up at this point. Watch!

If you start at this point and use this battery, you will end up at this point. Watch!
Have you noticed that you can switch the role of the starting point and the added battery and you will get the same stopping point?

It's true. Let me show you an example.

If you start here and use then you stop here.

If you start here and use then you stop here.

If you start here and use then you stop here.

Here is a question... Suppose you know that starting here and using this battery will let you stop here.

Then where will you stop if you start here and use this battery?

Choose:

1.)  
2.)  
3.)  

is correct.

Starting here and using battery, you stop here.
Here is another question...
If you know that starting here and using this battery will let you stop here.
Then where will you stop if you start here and use this battery?  
Choose:
1.)  2.)  3.)

Finally you need to know that you can use as many batteries as you want. And the order that you use them doesn't matter.
Here is an example...

Suppose that starting here and using these batteries will let you stop here.
Then where will you stop if you start here and use these batteries?
Choose:
1.)  2.)  3.)

Let me ask you another question...
Suppose that starting here and using these batteries will let you stop here.
Then where will you stop if you start here and use these batteries?
Choose:
1.)  2.)  3.)
1. You can use one or more batteries in any order. You will stop at the same point.

2. You can switch the roles of one battery and starting point. You will stop at the same point.

For example, if you start here \( \square \) and use this battery \( \square \),
you will stop here \( \square \).

And, if you start here \( \square \) and use this battery \( \square \),
you will stop at the same point \( \square \).

Next I will tell you some specifics to remember...

3. Using this battery \( \square \) will take you all the way around the track to stop at your starting point.

For example, if you start here \( \square \) and use this battery \( \square \),
you will stop here \( \square \).

For example, if you start here \( \square \) and use this battery \( \square \),
you will stop here \( \square \).

4. If you use this battery \( \square \), you will move ahead 1 unit.

For example, starting here \( \square \) and using this battery \( \square \),
will take you to this stopping point \( \square \).

For example, starting here \( \square \) and using this battery \( \square \),
will take you to this stopping point \( \square \).

5. If you use this battery \( \square \), you will move ahead 2 units.

For example, starting here \( \square \) and using this battery \( \square \),
will take you to this stopping point \( \square \).

For example, starting here \( \square \) and using this battery \( \square \),
will take you to this stopping point \( \square \).

1. Where would you stop if you started here \( \square \) and used this battery \( \square \)?

Choose:

1) \( \square \) 2) \( \square \) 3) \( \square \)

Remember: You can:
1. Switch the roles of one battery and starting point
2. Use more than one battery in any order

- Starting Point: \( \square \) any path \( \square \)
- Stopping Point: \( \square \) any place

\( \square \) is correct

If you start here \( \square \) and use this \( \square \), you will stop here \( \square \).

Watch!

\( \square \) is correct

If you start here \( \square \) and use this \( \square \), you will stop here \( \square \).

Watch!
2. Where would you stop if you started here \( \text{P} \) and used this battery \( \text{P} \)?

Choose:
1) \( \text{P} \) 2) \( \text{P} \) 3) \( \text{P} \)

3. Where would you stop if you started here \( \text{P} \) and used this battery \( \text{P} \)?

Choose:
1) \( \text{P} \) 2) \( \text{P} \) 3) \( \text{P} \)

Remember: You can
1. use the same battery anywhere
2. use the same battery in any order

Watch!
4. Where would you stop if you started here and used this battery?

Choose:
1) 2) 3) 

Starting Point: Stopping Point:

If you start here and use this battery, you will stop here.

Watch!

5. Where would you stop if you started here and used these batteries and ?

Choose:
1) 2) 3) 

Starting Point: Stopping Point:

If you start here and use these batteries, you will stop here.

Watch!
Let me summarize how this works...

1. You can use one or more batteries in any order. You will stop at the same point.

2. You can switch the roles of one battery and starting point. You will stop at the same point.
   For example, if you start here \( P \) and use this battery \( P \), you will stop at this point \( P \).
   And, if you start here \( P \) and use this battery \( P \), you will stop at this point \( P \).

Next I will tell you some specifics to remember...

3. Using this battery \( P \) will take you all the way around the track to stop at your starting point.
   For example, if you start here \( P \) and use this battery \( P \), you will stop here \( P \).
   For example, if you start here \( P \) and use this battery \( P \), you will stop here \( P \).

4. If you use this battery \( P \), you will move ahead 1 unit.
   For example, starting here \( P \) and using this battery \( P \), will take you to this stopping point \( P \).
   For example, starting here \( P \) and using this battery \( P \), will take you to this stopping point \( P \).

5. If you use this battery \( P \), you will move ahead 2 units.
   For example, starting here \( P \) and using this battery \( P \), will take you to this stopping point \( P \).
   For example, starting here \( P \) and using this battery \( P \), will take you to this stopping point \( P \).

### SUMMARY

<table>
<thead>
<tr>
<th>Starting Point</th>
<th>Battery</th>
<th>Stopping Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>( P )</td>
<td>( P )</td>
</tr>
</tbody>
</table>

Remember:
1. You can switch the role of the starting point and the battery
2. You can use more than one battery in any order

Let's find where you would stop if you start here \( P \) and use these batteries \( P, P, P, P, P \).

First let's use these batteries \( P, P \). Watch...
Start here \[ \infty \] with these batteries \[ \infty \infty \infty \infty \].

Use \[ \infty \infty \infty \]. Watch...

So we stop here \[ \infty \].

Summary

<table>
<thead>
<tr>
<th>Starting Point</th>
<th>Battery</th>
<th>Stopping Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \infty ]</td>
<td>[ \infty ]</td>
<td>[ \infty ]</td>
</tr>
</tbody>
</table>

Remember:
1. You can switch the role of the starting point and the battery.
2. You can use more than one battery in any order.

Where would you stop if you started here \[ \infty \infty \infty \] and used these batteries: \[ \infty \infty \infty \] ?

Choose: 1.) \[ \infty \infty \infty \] 2.) \[ \infty \] 3.) \[ \infty \infty \]

If you start here \[ \infty \] and use these: \[ \infty \infty \infty \], you will stop here \[ \infty \]. Watch!

If you start here \[ \infty \] and use these: \[ \infty \infty \infty \], you will stop here \[ \infty \]. Watch!
If you start here \( P \) and use these: \( P \), \( P \), \( P \), you will stop here \( P \). Watch!

What combination of starting point and battery would let you stop at this point \( P \) ?

Choose: 1.) \( P \) and \( P \) 2.) \( P \) and \( P \) 3.) \( P \) and \( P \) 4.) \( P \) and \( P \)

Either:

- starting here \( P \) with this battery \( P \)
- starting here \( P \) with this battery \( P \)

will get you to this point \( P \).
Either:
starting here \( \square \) with this battery \( \square \)
or
starting here \( \square \) with this battery \( \square \)
will get you to this point \( \square \).

Either:
starting here \( \square \) with this battery \( \square \)
or
starting here \( \square \) with this battery \( \square \)
will get you to this point \( \square \).

Either:
starting here \( \square \) with this battery \( \square \)
or
starting here \( \square \) with this battery \( \square \)
will get you to this point \( \square \).

Either:
starting here \( \square \) with this battery \( \square \)
or
starting here \( \square \) with this battery \( \square \)
will get you to this point \( \square \).

Where would you stop if you started here \( \square \)
and used these batteries: \( \square \) ?

Choose: 1.) \( \square \) 2.) \( \square \) 3.) \( \square \)

\( \square \) is correct

If you start here \( \square \) and use these: \( \square \) \( \square \) \( \square \),
you will stop here \( \square \). Watch!

If you start here \( \square \) and use these: \( \square \) \( \square \) \( \square \),
you will stop here \( \square \). Watch!

\( \square \) is correct

If you start here \( \square \) and use these: \( \square \) \( \square \) \( \square \),
you will stop here \( \square \). Watch!

If you start here \( \square \) and use these: \( \square \) \( \square \) \( \square \),
you will stop here \( \square \). Watch!

\( \square \) is correct

\( \square \) is correct
is correct

If you start here and use these: , you will stop here. Watch!

Let me review what you need to know.

is correct

If you start here and use these: , you will stop here.

Now let me ask you some final questions. Please answer them as accurately and as quickly as possible.

SUMMARY

<table>
<thead>
<tr>
<th>Starting Point</th>
<th>Battery</th>
<th>Stopping Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>p</td>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>any point</td>
<td>same place you started</td>
<td></td>
</tr>
</tbody>
</table>

Remember:
1. You can switch the role of the starting point and the battery
2. You can use more than one battery in any order

1. If you start here and use this battery , where will you stop?

Choose: 1) 2) 3) 

2. If you start here and use these batteries , where will you stop?

Choose: 1) 2) 3) 

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3. Suppose you use 2 batteries; one of them is this. What could your starting point and other battery be so that you stop here?

Choose:
1) \[ \text{PP} \] and \[ \text{PP} \]
2) \[ \text{PP} \] and \[ \text{PP} \]
3) \[ \text{PP} \] and \[ \text{PP} \]

4. What starting point and battery will get you to this point?

Choose:
1) \[ \text{PP} \] and \[ \text{PP} \]
2) \[ \text{PP} \] and \[ \text{PP} \]
3) \[ \text{PP} \] and \[ \text{PP} \]

5. If you start here what battery could you use so that you stop at this point?

Choose: 1) \[ \text{PP} \] 2) \[ \text{PP} \] 3) \[ \text{PP} \]

6. What starting point and battery can you use along with this battery to get to this stopping point?

Choose:
1) \[ \text{PP} \] and \[ \text{PP} \]
2) \[ \text{PP} \] and \[ \text{PP} \]
3) \[ \text{PP} \] and \[ \text{PP} \]

7. Where would you stop if you start here and use these batteries?

Choose:
1) \[ \text{PP} \] and \[ \text{PP} \]
2) \[ \text{PP} \] and \[ \text{PP} \]
3) \[ \text{PP} \] and \[ \text{PP} \]

8. Some guys at the company were discussing what possible starting point could you have with these batteries AND 1 other battery so that you would stop here. Which of their responses do you agree with?

Choose:
1) any starting point
2) any starting point but \[ \text{PP} \]
3) any starting point but \[ \text{PP} \]

9. I overheard two company employees talking. They were arguing about whether these two situations would have the same stopping point.

Start here and use these batteries.

What do you think?

1 - Same
2 - Different

10. How about these situations. Will you get to the same stopping point?

Start here and use these batteries.

What do you think?

1 - Same
2 - Different
11. How about these situations. Will you get to the same stopping point?

Start here, use these.

Start here, use these.

What do you think?

1 - Same

2 - Different

12. Suppose you start here and use this battery. What other batteries do you need so you stop here?

Choose:

1) and

2) and

3) and

4) and

13. What starting point and battery will get you to this stopping point?

Choose:

1) and

2) and

3) and

4) none of the above

14. Suppose you start here, how many of these batteries could you use so that you stop here?

Choose:

1) three

2) four

3) five

4) six

15. Where will you stop if you start here and use these batteries?

Choose: 1) 2) 3) 4) we need more information to answer

16. Suppose you start here and use these batteries. What additional battery do you need so you stop here?

Choose:

1) 2) 3) 4) we need more information to answer

17. One day some employees were testing the cars. They forgot to write down the starting point, but they knew that the battery was used. They were trying to figure out where the car will stop. Here are some opinions of the employees. Which do you agree with?

1) the stopping point could be anywhere

2) the stopping point could only be or

3) the stopping point could only be

18. Where will you stop if you start here and use these batteries?

Choose: 1) 2) 3) 4)
19. Will you get to the same stopping point in these two situations?

Start here: use these batteries

Start here: use these batteries

What do you think?

1 - Same  2 - Different

20. How about these situations.
Will you get to the same stopping point?

Start here: use these batteries

Start here: use these batteries

What do you think?

1 - Same  2 - Different

21. How about these situations.
Will you get to the same stopping point?

Start here: use these batteries

Start here: use these batteries

What do you think?

1 - Same  2 - Different

22. Suppose you start here and use these batteries. What additional battery do you need so that you stop here?

Choose:  1)  2)  3)  

23. Where will you stop if you start here and use these?

Choose:  1)  2)  3)  

24. Suppose you start here and use these batteries. What additional batteries do you need so that you stop here?

Choose:

1) and  2)  and 
3) and  4) none of the above

You have completed this portion of this experiment.

Press the space bar to proceed to the next portion.
APPENDIX I

TESTING OF PHASE 3 OF EXPERIMENT 6
You just learned about a system involving 3 different entities. Then you then figured out a new system (a children’s game) that works the same way as the first one. There were 3 different objects. The rules which determined the final object in the game were the same as the rules of the first system you learned.

Now you will see a third system. This system also works like the first one. The rules are very similar, but now there are 4 different entities.

Can you try to figure out this new system? Use the paper and make any notes or drawings that might help you.

Here are some true statements:

\[ \begin{align*}
\huge \text{\textbullet} \quad \& \quad \text{\textstar} & \rightarrow \quad \text{\textstar} \\
\huge \text{\textbullet} \quad \& \quad \text{\textCircle} & \rightarrow \quad \text{\textCircle} \\
\text{\textstar} \quad \& \quad \text{\textstar} & \rightarrow \quad \text{\textCircle} \\
\text{\textCircle} \quad \& \quad \text{\textCircle} & \rightarrow \quad \text{\textbullet} \\
\text{\textCircle} \quad \& \quad \text{\textCircle} & \rightarrow \quad \text{\Box}
\end{align*} \]

Please answer these questions. Circle your answer:

1.) \( \text{\Box} \quad \& \quad \text{\textCircle} \rightarrow ? \)

2.) \( \text{\textbullet} \quad \& \quad \text{\Box} \rightarrow ? \)

3.) \( \text{\textstar} \quad \& \quad \text{\Box} \rightarrow ? \)
4.) \[ \square, \square, \star \rightarrow ? \]

5.) \[ \bullet, \star, \circ, \square \rightarrow ? \]

6.) \[ \square, \square \rightarrow ? \]

7.) \[ \square, \bullet, \star, \star \rightarrow ? \]

8.) \[ \circ, \square, \circ, \square \rightarrow ? \]

9.) \[ \bullet, \bullet, \square, \bullet, \star, \bullet \rightarrow ? \]

10.) What symbol can be placed in the blank below?

\[ \_ \_ , \square, \square \rightarrow \square \]

Circle your answer:

\[ \bullet, \star, \circ, \square \]
APPENDIX J

TRAINING AND TESTING OF PHASE 1 OF EXPERIMENT 7

The following pages present the training and testing of the Generic Domain.
Please read the information on the computer screen.

Use the computer mouse to click on the blue arrow to go to the next screen.

When there is a question, click on your answer.

Allow me to introduce myself. I am the world renowned archaeologist, Maximilian Peabody.

WHAT?? You say you have never heard of me! Well, I have made many discoveries of great importance.

Let me tell you about the discovery for which I was given this great honor.

It was a few years back. My team and I were in the barren desert of Wadi Schmadi. Luckily, we discovered stone tablets on which symbols were inscribed.

Here is an example of a tablet with an inscription:

After long and careful analysis, I determined that the inscriptions follow specific rules.
The inscriptions are statements in a sort of language.

The statements are made by combining symbols.

So statements look like this

_____ , _____ → _____

Where symbols would go in the blanks.

The symbol on the right is the result.

AND you can always figure out the resulting symbol.
You just use the rules of combination.
Now let me teach you these rules.

Rules of Combining Symbols:

1. The order of the two symbols on the left does not change the result (the symbol on the right).

For example

↓ , ◆ → ◆

is the same thing as

↓ , ◆ → ◆

Here is a question for you.
Suppose you know that ○ , ◆ → ◆ .
Then what symbol would go in the blank
◆ , ○ → ___?

Click on your answer:

↓  ◆  ●

Because the order of symbols on the left does not matter,

If ● , ◆ → ◆

then ◆ , ● → ◆
Let's try another question.
Suppose you know that \( \text{○, } \text{□} \rightarrow \text{○} \).
Then what symbol would go in the blank \( \text{□, } \text{○} \rightarrow \_ \)?

\[
\text{□} \quad \text{○} \quad \_ \\
\text{□} \quad \_ \quad \text{○}
\]

\[
\text{□} \quad \text{○} \quad \_ \\
\text{□} \quad \_ \quad \text{○}
\]

Because the order of symbols on the left does not matter,
If \( \text{○, } \text{□} \rightarrow \text{○} \)
then \( \text{□, } \text{○} \rightarrow \text{○} \)

So tell me what goes in this blank?
\( \text{□, } \text{○} \rightarrow \_ \)

\[
\text{□} \quad \text{○} \quad \_ \\
\text{□} \quad \_ \quad \text{○}
\]

Let's try another...
Whenever \( \text{□} \) combines with another symbol, the result is that other symbol.
So, \( \text{□, } \text{○} \rightarrow \text{○} \)

\[
\text{□} \quad \text{○} \quad \_ \\
\text{□} \quad \_ \quad \text{○}
\]

\[
\text{□} \quad \text{○} \quad \_ \\
\text{□} \quad \_ \quad \text{○}
\]

Now, let me tell you three more rules about specific combinations.
Rule 3. \( \text{○, } \text{□} \rightarrow \text{□} \)
Rule 4. \( \text{○, } \text{○} \rightarrow \text{○} \)
Rule 5. \( \text{□, } \text{□} \rightarrow \text{□} \)
There is one more thing you need to know about the language... how more than two symbols combine.

Think about this question. Remember the last rule... the order of symbols on the left does not matter.
If $\text{ }, \text{ }, \text{ } \rightarrow \text{ }$ then $\text{ }, \text{ }, \text{ } \rightarrow$?

Rule 6. The result does not depend on which two symbols combine first.

For example: $\text{ }, \text{ }, \text{ }, \text{ } \rightarrow \text{ }$

It does not matter if we do $\text{ }, \text{ }, \text{ }$ first and then $\text{ }$ or $\text{ }, \text{ }$, $\text{ }$ first and then $\text{ }$.

Because the **same symbols** appear in both.
They just have a different order.

Let's try another question.
If $\text{ }, \text{ }, \text{ } \rightarrow \text{ }$ then $\text{ }, \text{ }, \text{ } \rightarrow$?

Summary key idea:
If the same symbols appear on the left,
Order does not matter.
The result is the same.
Summary key ideas:

1. ☐, other symbol → other symbol
   
   For example: ☐, ☐ → ☐
   
   ☐, ☐ → ☐
   
   ☐, ☐ → ☐

Summary key ideas:

2. The following combination gives a result of ☐.
   
   ☐, ☐ → ☐

Next, let's do more questions.

Rule reminders will appear, but try to remember the rules.

Later we won't have the reminders.

What goes in the blank of the following statement?

1. ☐, ☐ → ___
   
   ☐ ☐ ☐

   Rule Reminders:
   1. Order of symbols on left doesn't matter
   2. If ☐ goes with another symbol, the result is the other symbol
   3. More than 2 symbols on left combine in any order

What goes in the blank?

2. ☐, ☐ → ___
   
   ☐ ☐ ☐

   Rule Reminders:
   1. Order of symbols on left doesn't matter
   2. If ☐ goes with another symbol, the result is the other symbol
   3. More than 2 symbols on left combine in any order

   ☐ is correct

1. ☐, ☐ → ___
   
   ☐ ☐ ☐

   ☐ is correct

2. ☐, ☐ → ___
   
   ☐ ☐ ☐
What goes in the blank?

3. ♦, ♦ → 

Rule Reminders:
1. Order of symbols can shift, doesn't matter.
2. If a ♦ goes with another symbol, the result is the other symbol.
3. More than 2 symbols on left combine in any order.

is correct

3. ♦, ♦ → ●

What goes in the blank?

4. ●, ● → 

Rule Reminders:
1. Order of symbols can shift, doesn't matter.
2. If a ♦ goes with another symbol, the result is the other symbol.
3. More than 2 symbols on left combine in any order.

is correct

4. ●, ● → ●

What goes in the blank?

5. ♦, ♦, ♦ → 

Rule Reminders:
1. Order of symbols can shift, doesn't matter.
2. If a ♦ goes with another symbol, the result is the other symbol.
3. More than 2 symbols on left combine in any order.

is right

5. ♦, ♦, ♦ → ●

because: ♦, ● → ●

So we have ●, ● → ●

Now, you have been using the rules a little. Let me review the 3 Key Ideas.

Summary key ideas:

1. ♦, other symbol → other symbol

For example: ♦, ♦ → ●

●, ● → ●

●, ● → ●
Summary key ideas:

2. The following combination gives a result of \( \text{\#} \).

\[ \bullet, \text{\#} \rightarrow \text{\#} \]

Remember that order doesn’t matter

and these 3 key ideas.

That is all you need to know!

Summary key ideas:

3. We can make combinations to result in \( \bullet \) and \( \text{\#} \) also:

\[ \bullet, \bullet \rightarrow \bullet \]
\[ \text{\#}, \text{\#} \rightarrow \bullet \]

key ideas:

1. \( \text{\#}, \text{other symbol} \rightarrow \text{other symbol} \)

2. \( \bullet, \text{\#} \rightarrow \text{\#} \)

3. \( \bullet, \bullet \rightarrow \text{\#} \)
\[ \text{\#}, \text{\#} \rightarrow \bullet \]

Determine the result of

\( \text{\#}, \bullet, \bullet, \bullet \rightarrow \)

1st. Take advantage of the \( \text{\#} \) key idea:

\( \text{\#}, \text{other symbol} \rightarrow \text{other symbol} \)

So we get:

\[ \text{\#}, \bullet, \bullet \rightarrow \]

Now we have

\( \text{\#}, \bullet, \bullet \rightarrow \)

Next, use the other key ideas:

Because \( \text{\#}, \bullet \rightarrow \text{\#} \) we get:

\[ \bullet, \bullet \rightarrow \]

Finally:

\[ \text{\#}, \bullet \rightarrow \bullet \]

Remember the Key Ideas:

1. \( \text{\#}, \text{other symbol} \rightarrow \text{other symbol} \)

2. \( \bullet, \text{\#} \rightarrow \text{\#} \)

3. \( \bullet, \bullet \rightarrow \text{\#} \)
\[ \text{\#}, \text{\#} \rightarrow \bullet \]

Let’s do some more questions without reminders...
Determine the result of the following

\[ \bullet, \blacklozenge, \blacksquare, \bullet \rightarrow \_\_\_ \]

What two symbols combine to produce \( \bullet \)?

When \( \blacksquare \) goes with other symbols, it does not affect the result.

So:

\[ \bullet, \blacklozenge, \bullet \rightarrow \_\_\_ \]

We know that \( \bullet, \blacklozenge \rightarrow \blacksquare \)

So we get:

\[ \blacksquare, \bullet \rightarrow \bullet \]

Remember the Key Ideas:

1. \( \blacksquare, \text{other symbol} \rightarrow \text{other symbol} \)
2. \( \bullet, \blacklozenge \rightarrow \blacksquare \)
3. \( \bullet, \bullet \rightarrow \_\_\_ \)

Now let me ask you some final questions.

Please answer them as best you can.
2. Find the resulting symbol
   \( \bullet, \diamond, \diamond \to \) __

3. What symbols go in the blanks to make a correct statement?
   \( \_ , \_ , \bullet \to \bullet \)

4. What goes in the blanks to make a correct statement?
   \( \_ , \_ \to \) \( \bullet \)

5. What goes in the blank to make a correct statement?
   \( \bullet , \_ \to \) \( \bullet \)

6. What can go in the blank to make a correct statement?
   \( \_ , \diamond \to \) \( \bullet \)

7. What expression has the same result as the following expression?
   \( \diamond, \bullet, \bullet, \diamond \to \) __

8. When we were analyzing tablets, I overheard two of my team members talking. They were arguing about whether these inscriptions mean the same thing (have the same result).

   \( \diamond, \bullet, \bullet, \bullet \to \) \( \bullet, \bullet, \bullet, \bullet \to \) 
   What do you think?
   
   Same
   Different

9. How about the following?
   Do they mean the same thing?

   \( \bullet, \bullet, \bullet, \diamond \to \) __

   \( \bullet, \bullet, \bullet, \diamond \to \) __

   Same
   Different
<table>
<thead>
<tr>
<th></th>
<th>10. What goes in the blanks to make a correct statement?</th>
<th>11. Which of the following symbols combine to give  ( \Box )?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( _ ), ( \Box ), ( _ ), ( \Box \to \Box )</td>
<td>( \Box ) and ( \Box )  ( \Box ) and ( \Box )  ( \Box ) and ( \Box )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>12. Find the resulting symbol</th>
<th>13. What symbol goes in the blank to make a correct statement?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Box ), ( \Box ), ( \Box ), ( \Box ), ( \Box ), ( \Box ), ( \Box ), ( \Box ), ( \Box ), ( \Box ) ( \to ) ( \Box )</td>
<td>( _ ), ( _ ), ( _ ), ( \Box \to \Box )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>14. What goes in the blank to make a correct statement?</th>
<th>15. What goes in the blank to make a correct statement?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Box ), ( _ ) ( \to ) ( \Box )</td>
<td>( \Box ), ( \Box ), ( \Box ), ( \Box \to \Box )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>16. What goes in the blank to make a correct statement?</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Box ), ( _ ) ( \to ) ( \Box )</td>
<td>You have finished the first part. Click on the blue arrow to go to the next part.</td>
</tr>
</tbody>
</table>

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The following pages present the training and testing of the Relevantly Concrete Domain.

Please read the information on the computer screen.
Use the computer mouse to click on the blue arrow to go to the next screen.

When there is a question, 
Click on your answer.

Allow me to introduce myself. I am Melvin, chief engineer for efficiency at the Bubblinski Detergent Company.

At the Bubblinski Detergent Company we mix solutions of different kinds to fill containers of detergents.

Recently, we have been having some problems with the mixing of the solutions. The detergent mixes may not be correct.

I need your help to determine the amount of left-over solution that results in the mixing process. Then I will test the left-over amount to make sure that the detergent is being made correctly.
Let me tell you how the quantities of solutions are combined so that we can determine how much solution is left-over.

In the process of making the detergent, cups of solutions are combined.

Different kinds of solutions are used, but what is important is the quantities of solutions.

So to find the left-over amount, we can use

![Images of cups]

to represent the different possible quantities of any of the solutions.

For example,

If ![Image of cups] and ![Image of cup] are combined, the quantities will fill one detergent container and ![Image of cup] will be left-over.

So, you have the idea. We need to find the left-over quantity when the following types of cups of solution are combined

![Images of cups]

Now let me teach you the specific rules for finding the left-over quantities.

1. The order by which two cups of solution are combined does not change the left-over result.

For example, combining ![Image of cups] with ![Image of cup] has a left-over quantity of ![Image of cup].

And combining ![Image of cup] with ![Image of cup] has the left-over quantity ![Image of cup].

Here is a question for you.

Suppose you know that ![Image of cups] and ![Image of cup] combine to yield a left-over of ![Image of cup].

Then what is left-over when ![Image of cup] and ![Image of cup] combine?

Click on your answer:

![Image of cups]  ![Image of cup]  ![Image of cup]
Let's try another question.

If \( \text{cup} \) combines with \( \text{cup} \); and \( \text{cup} \) is left-over.

Then what is the left-over when \( \text{cup} \) combines with \( \text{cup} \)?

\[ \text{cup} \]
\[ \text{cup} \]
\[ \text{cup} \]

\[ \text{cup} \]

So you know that the order of combining solutions does not matter.
Now, let me tell you some specifics about combinations.

Remember, we are filling containers, but we always want to have a cup of solution to test for quality when we are done.

Rule 2. \( \text{cup} \) and \( \text{cup} \) will fill a container, but we need a quantity of solution to test, so we consider \( \text{cup} \) as the left-over.

Rule 3. When any kind of cup of solution combines with \( \text{cup} \), the result will always be the other solution cup.

For example:
When \( \text{cup} \) and \( \text{cup} \) combine, \( \text{cup} \) is left-over.
And when \( \text{cup} \) and \( \text{cup} \) combine, \( \text{cup} \) is left-over.

So tell me what is left-over when \( \text{cup} \) and \( \text{cup} \) combine?

\[ \text{cup} \]
\[ \text{cup} \]
\[ \text{cup} \]

\[ \text{cup} \]

Let's try another...

Whenever \( \text{cup} \) combines with another solution cup, the result is that other cup.

So, \( \text{cup} \) is left-over when \( \text{cup} \) and \( \text{cup} \) combine.

\[ \text{cup} \]
\[ \text{cup} \]
\[ \text{cup} \]

\[ \text{cup} \]
\( \text{is correct} \)

Whenever \( \text{combines with another cup of solution, the left-over is that other solution cup.} \)

So, \( \text{is left-over when} \) \( \text{and} \) \( \text{combine.} \)

\begin{align*}
\text{Rule 4. A combination of} & \quad \text{and} \quad \text{does not fill a container, so the left-over is} \quad \text{.} \\
\text{Rule 5. A combination of} & \quad \text{and} \quad \text{fills one container and has} \quad \text{left-over.} \\
\end{align*}

\begin{align*}
\text{Rule 6. Finally, you need to know that when mixing more than 2 cups of solution, the order of combining solutions does not matter. The left-over is the same no matter which cups are combined first.} \\
\text{For example: When we combine} \quad \text{,} \quad \text{and} \quad \text{, the left-over is} \quad \text{.} \\
\text{It does not matter if we do} \quad \text{and} \quad \text{first and then} \quad \text{OR} \quad \text{and} \quad \text{first and then} \quad \\
\end{align*}

\begin{align*}
\text{Here is a question.} \\
\text{If mixing} \quad \text{,} \quad \text{and} \quad \text{has a left-over} \\
of \quad \text{, then what is left-over} \\
\text{when mixing} \quad \text{,} \quad \text{and} \quad \text{?} \\
\end{align*}

\( \text{is correct.} \)

Remember the order of combinations does not matter, the left-over will be the same.

So, combining \( \text{,} \quad \text{and} \quad \text{is the same} \)
as combining \( \text{,} \quad \text{and} \quad \text{.} \\
\text{Both have} \quad \text{as a left-over.} \\

\begin{align*}
\text{Let's try another question.} \\
\text{If} \quad \text{is left-over when} \quad \text{,} \quad \text{and} \quad \text{combine, then what is left-over when} \\
\text{,} \quad \text{and} \quad \text{combine?} \\
\end{align*}

\( \text{is correct.} \)

Remember the order of combinations does not matter, the left-over will be the same. So

If \( \text{,} \quad \text{and} \quad \text{combine and have} \\
\text{left-over, then} \\
\text{,} \quad \text{and} \quad \text{combine with} \quad \text{left-over.} \\

\begin{align*}
\text{To summarize,} \\
\text{Cups of solutions are mixed together to fill detergent containers. Your job is to find} \\
\text{the cup with the remainder quantity.} \\
\text{We always want to have a quantity remaining, so the possible remainders are:} \\
\end{align*}
You can combine 2 or more cups of solution in any order, the left-over will be the same.

Next let's review the left-overs for specific cup combinations...

Here is what happens when \[ \text{solution} \] mixes with other cups.

Mixing these 2 cups results in this left-over

1. What is left-over when \[ \text{solution} \] and \[ \text{solution} \] mix?

2. What is left-over when \[ \text{solution} \] and \[ \text{solution} \] mix?

Let me ask you some questions.
3. What is left-over when \( \text{Solution 1} \) and \( \text{Solution 2} \) mix?

Reminder:
- You can combine 2 or more cups in any order, the left over will be the same.
- Left over for specific mix combinations:

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Solution 1} )</td>
<td>( \text{Solution 2} )</td>
</tr>
<tr>
<td>( \text{Solution 2} )</td>
<td>( \text{Solution 1} )</td>
</tr>
</tbody>
</table>

\( \text{Solution 1} \) is correct

When \( \text{Solution 1} \) and \( \text{Solution 2} \) are mixed together \( \text{Solution 1} \) is left-over.

4. What is left-over when \( \text{Solution 3} \) and \( \text{Solution 4} \) mix?

Reminder:
- You can combine 2 or more cups in any order, the left over will be the same.
- Left over for specific mix combinations:

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Solution 3} )</td>
<td>( \text{Solution 4} )</td>
</tr>
<tr>
<td>( \text{Solution 4} )</td>
<td>( \text{Solution 3} )</td>
</tr>
</tbody>
</table>

\( \text{Solution 3} \) is correct

When \( \text{Solution 3} \) and \( \text{Solution 4} \) are mixed together \( \text{Solution 3} \) is left-over.

5. What is left-over when \( \text{Solution 5} \), \( \text{Solution 6} \), and \( \text{Solution 7} \) mix?

Reminder:
- You can combine 2 or more cups in any order, the left over will be the same.
- Left over for specific mix combinations:

<table>
<thead>
<tr>
<th>Solution Cup Mix</th>
<th>Left-over</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Solution 5} )</td>
<td>( \text{Solution 6}, \text{Solution 7} )</td>
</tr>
<tr>
<td>( \text{Solution 6} )</td>
<td>( \text{Solution 5}, \text{Solution 7} )</td>
</tr>
<tr>
<td>( \text{Solution 7} )</td>
<td>( \text{Solution 5}, \text{Solution 6} )</td>
</tr>
</tbody>
</table>

\( \text{Solution 5} \) is correct

Because \( \text{Solution 5} \) and \( \text{Solution 7} \) mixed together with \( \text{Solution 6} \) left-over, then \( \text{Solution 5} \) and \( \text{Solution 6} \) mix leaving \( \text{Solution 5} \) left-over.

Remember, you can combine 2 or more cups in any order. Next let's summarize the left-over cups resulting from different combinations of solution quantities...
Here are the left-overs from all of the other possible cup combinations:

Mixing these 2 cups results in this left-over

Let's do an example...

Let's find the left-over quantity when the following cups of solution are mixed.

1st Let's mix the

So we get:

Now we have

We know and combine with left-over.

So we get:

Then we can mix and to get left-over.

What is the left-over when these cups are mixed?

is correct. Here is how we can figure it out.

To mix these:

First, we can mix and to get

Then we have:

and leave . So we get:

Finally and combine to leave left-over.
Here is a question for you. What possible cups of solution can combine so that left-over?

Either and
Or and
Combine to have left-over

Now let me ask you some final questions.

Please answer them as best you can.

1. What is left-over when and combine?

2. What is left-over when the following cups of solution are combined?

3. What possible cups of solution can combine with to have a left-over of ?

4. Which cups of solution can combine to have left-over?
5. What can combine with to have left-over?  

6. What can mix with to have as a left-over?  

7. What combination of cups has the same left-over as the following?  

8. One day, I overheard two employees talking. They were arguing about whether these two mixtures of solution (below) would have the same left-over.  
   Mix 1:  
   Mix 2:  
   What do you think?  

   Same  
   Different  

9. How about these mixtures? Will mix 1 and mix 2 will have the same left-over?  
   Mix 1:  
   Mix 2:  
   Same  
   Different  

10. What cups can combine with and to result in a left-over of ?  

11. Which cups can combine to give a left-over of ?  

12. What is left-over when the following cups are mixed?
13. What cup needs to mix with the following to have a left-over of?

14. What cup needs to mix with to have a left-over of?

15. What cup needs to mix with and to have a left-over of?

16. What cup needs to mix with to have a left-over of?

You have finished the first part.

Click on the blue arrow to go to the next part.
APPENDIX K

PRESENTATION AND TESTING OF TRANSFER DOMAIN IN PHASE 2 OF EXPERIMENT 7
Allow me to introduce myself. I am Mr. Gubini, social studies teacher.

I know that you just finished learning the rules of a symbolic language. You learned the rules and then answered some questions.

Now I need your help figuring out something new... a children's game from another country.

What you learned about the symbolic language can help you understand how this new game works.

In the game, the children point to two or more different objects, one at a time.

Then a different child who is "it" points to one final object. If this child points to the correct final object, then he or she is the winner.

There are three kinds of objects used in the game:

- **Ring:**

- **Bug:**

- **Vase:**

The rules of the game tell you what the final object should be. So the rules tell the winner what final object to point to.
The rules of this game are like the rules of the language you learned.

So, use what you know about the language to help you figure out the rules of the game.

Children point to at least two objects.
Then the person who is "it" tries to win by pointing to the correct final object.

Here is an example.

Some children pointed to and then .

Then the child who was "it" pointed to and won.

There are specific rules that tell the winner which object to point to.
I need your help figuring out those rules to answer some questions.

Next, I will show you some examples where the child who was "it" was a winner.
Please study these examples on the next 7 slides and then we will do some questions.

Some children pointed to then .

The winner pointed to .

Some children pointed to then .

The winner pointed to .
The children pointed to \[ \text{then} \]

The winner pointed to \[ \text{then} \]

Some children pointed to \[ \text{then} \]

The winner pointed to \[ \text{then} \]

The children pointed to \[ \text{then} \]

The winner pointed to \[ \text{then} \]

The children pointed to \[ \text{then} \]

The winner pointed to \[ \text{then} \]

Now let's take a look at some questions.
Remember you can use what you learned about the language to help you answer these questions about the game.

1. What object do you think the winner will point to when the other kids point to \[ \text{then} \]?

2. What object does the winner point to when the other kids point to \[ \text{then} \]?
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 3. If a group of kids wants the winner to point to 
   , and they first point to 
   , what other objects do they need to point to? |
|   | Examples: |
|   | If the kids point to these: |
|   | Then the winner points to this: |
| 4. If the winner pointed to 
   , what objects might the other kids have pointed to? |
|   | Examples: |
|   | If the kids point to these: |
|   | Then the winner points to this: |
| 5. What object do the children need to point to along with 
   , so that the winner points to ? |
|   | Examples: |
|   | If the kids point to these: |
|   | Then the winner points to this: |
| 6. What objects can the kids point to along with 
   so that the winner points to ? |
|   | Examples: |
|   | If the kids point to these: |
|   | Then the winner points to this: |
| 7. Suppose that a group kids points to these objects: |
|   | Examples: |
|   | If the kids point to these: |
|   | Then the winner points to this: |
| 8. Two groups of children were playing separate games. |
|   | One group pointed to these objects: |
|   | The other group pointed to these: |
|   | Will the winner point to the same object in each case? |
|   | Yes  |
|   | No  |
| 9. How about these objects? In separate games, If one group points to these: And another group points to these: Will the winner point to the same object in each case? |
|   | Examples: |
|   | If the kids point to these: |
|   | Then the winner points to this: |
| 10. A group of children want the winner to point to . They first point to . What objects should they point to next? |
|   | Examples: |
|   | If the kids point to these: |
|   | Then the winner points to this: |
11. What objects should the kids point to so that the winner points to?

Examples:
If the kids point to these:
Then the winner points to this:

12. After the children point to these objects:

What will the winner point to?

Examples:
If the kids point to these:
Then the winner points to this:

13. The children point to these objects:

What additional object should they point to so that the winner points to?

Examples:
If the kids point to these:
Then the winner points to this:

14. If the kids point to this what other object should they point to so that the winner points to?

Examples:
If the kids point to these:
Then the winner points to this:

15. A group of children pointed to these objects:

What additional object do they need to point to so that the winner will point to?

Examples:
If the kids point to these:
Then the winner points to this:

16. If the kids point to this what other object should they point to so that the winner points to?

Examples:
If the kids point to these:
Then the winner points to this:

Let's try to match the objects and symbols from each of the systems you learned.
The following three slides were included in the Generic condition and not in the Relevant Concreteness condition.
The following three slides were included in the Relevant Concreteness condition and not in the Generic condition.