LOW FREQUENCY FEEDFORWARD AND PREDISTORTION LINEARIZATION OF RF POWER AMPLIFIERS

DISSERTATION

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By

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* * * * *

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ABSTRACT

Linearity is an area of great concern in the development of communication systems. As the bandwidth of modulated signals becomes wider, RF power amplifiers (PA) have become a serious bottleneck due to their nonlinear response. This thesis gives a review of existing linearization systems commonly used in cellular phone base stations and which could be potentially applied to mobiles. Special focus is placed on predistortion linearization which is becoming prevalent. An alternative implementation of postdistortion often referred as low frequency feedforward linearization is also investigated. The existing issues associated with both low frequency feedforward and predistortion linearization are analyzed in details. Memory effects in PAs are identified as the origin of the degradation of the performance of linearization systems as the bandwidth is increased. Our research is concerned among other things with the development of improved RF predistortion algorithm accounting for such memory effects. In this thesis, we present theoretical work where the Volterra representation is used to analyze the performance of both predistortion and low frequency feedforward linearization systems in the presence of memory effects. A new real time algorithm is then developed for predistortion linearization which accounts for memory effects. This real time digital algorithm is then implemented in a FPGA testbed and verified using multisines and WCDMA modulated RF signals. This thesis concludes with a
discussion of possible extensions of this work combining the various novel linearization schemes proposed.
This is dedicated to Jesus and Yoonjoo Kim
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CHAPTER 1

INTRODUCTION

One of the most challenging issue in the design of radio frequency (RF) power amplifiers for base stations, is the linearity requirement. Various governing bodies such as FCC and ETSI, limit spurious emissions outside the allocated bandwidth. In addition, limitations are placed on interferences to adjacent channels in the allocated frequency bands. Nonlinearities within the components of wireless system introduce distortions in the transmitted signal and result in the generation of signals outside of the intended frequency channel or band. These unwanted distortions are potential interfering sources for other radio users and must be reduced to a sufficient low level such that all systems can operate satisfactorily.

Class A power amplifiers which are highly linear have traditionally been used to meet the strict linearity requirements of wireless standards. Class A amplifiers are however inefficient in converting DC power to RF power. This is further aggravated by the fact that power back off is used to achieve the targeted linearity. Power back off permits one to meet the linearity requirement by using a high power amplifiers at lower power levels. Clearly achieving high linearity in an amplifier with power back off calls for the use of expensive high-power transistors which degrade the power efficiency.
Among the most crucial issues in the recent development of communication systems is the improvement of the power efficiency of power amplifiers in transmitter systems. To realize highly efficient power amplifiers, alternative amplifier classes such as class B, C, D and E must be used rather than the power hungry class A. However higher power-added efficiency (PAE) power amplifiers usually exhibit poor linearity. One needs then to rely on high performance linearization techniques to improve the linearity of these power amplifiers. Various linearization techniques have been developed to linearize power amplifiers. Table 1.1 compare the typical performance of various linearization techniques [2].

<table>
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<tr>
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Table 1.1: Comparison of linearization techniques

The linearization systems used in 2G basestations were principally based on feed-forward. Feedforward linearization systems provide an excellent linearization for wide bandwidths without stability issues. However it requires expensive RF components, and its efficiency is poor since a 2nd power amplifier is required. As the performance of adaptive predistortion linearization systems is now approaching that of feed-forward systems, modern basestations are now relying mostly on predistortion linearization. However for 3G W-CDMA applications which require more bandwidth the present linearization schemes have been found to be much less effective. The source of this
problem is the memory effects associated with non-linearities. Non-adaptive predistortion algorithms do not usually account for memory effects. That is the amplitude and phase correction of the RF envelope is only a function of the instantaneous magnitude of the RF envelope. Memory effects can be classified into slow and fast memory effects. Slow memory effects such as slow thermal drift can be theoretically handled with an adaptive linearization scheme. Fast memory effects require the development of more advanced linearization algorithm applied in real-time.

To understand non-linearities with memory in amplifiers, it is necessary to study the behavioral modeling of amplifiers. The Volterra representation provides a mathematical tool for rigorously describing nonlinear effects with memory in nonlinear devices such as mixers and amplifiers. Note that the Volterra approach is not usually favored by RF designers due to convergence problems and the difficulty in extracting the Volterra kernels (VKs) [22]. Also the use of the Volterra representation for systems of higher order than order three become impractical. Nonetheless it is still the most rigorous approach for describing nonlinear systems with moderate non-linearities. Norbert Wiener is credited for casting in 1958, the theory of Volterra representation of nonlinear systems in its modern framework. This method can be used to predict the behavior of communication receivers for small power levels [23]. For example by applying small AC-signals superposed on a DC bias, bias-dependent admittances can be extracted and used to analyze electron devices. Good agreement for harmonics up to the 3rd order was demonstrated in 1991 [21]. The VKs was extracted by measuring the input and output signals including higher order interferences. Only scalar systems (one input and one output) have been rigorously studied so far for evaluating the range of accuracy [4] and the condition of the existence of Volterra
series (VS) [5]. In 1974 Julian J. Bussgang introduced a vector Volterra formalism to handle multi ports. The matrix representation of Volterra systems was then developed and allowed to conveniently treat multiple-input, multiple-output (MIMO) systems [19]. Approximate extension of the Volterra representation to strongly linear systems submitted to broadband perturbations can be realized by using a sliding kernel [7]. This has been shown to yield accurate broadband behavioral models [8] capable of accounting for memory effects in strongly non-linear systems.

As discussed above, to realize RF power amplifiers which are both linear and power-efficient and meet the requirement of broadband communication systems, improved linearization schemes accounting for memory effects are needed. A new linearization scheme called bilateral baseband modulation (BBM) linearization which is related to both low-frequency feedforward (LFFF) and predistortion will be studied. We will present preliminary work where the Volterra representation is used to analyze the capability of the BBM linearization scheme to account for memory effects. It will be verified using simulations in MATLAB that under certain operating conditions both the upper and lower IMD3s can be theoretically reduced by more than 100dB using BBM.

The organization of the thesis is as follow. Chapter 2 reviews the low frequency feedforward linearization recently investigated by a MISES student [16] and discusses the existing issues associated with both low frequency feedforward and predistortion linearization. Chapter 3 describes linearization results obtained in MATLAB without and with MEs. Chapter 4 addresses imbalance issues associated with balancing IQ modulators. Chapter 5 discusses the characterization of non-linear systems with memory effects using the large signal network analyzer. Chapter 6 and 7
present then new predistortion algorithms and their implementation into a real-time
digital system. Chapter 8 concludes this thesis with a summary of the achievements
contributed and with a discussion on possible extensions of this work.
CHAPTER 2

LOW FREQUENCY FEEDFORWARD AND RF PREDISTORTION LINEARIZATION

In this section we shall discuss the limitations of both low frequency feedforward (LFFF) linearization and digital predistortion. As mentioned in the previous section the degradation in performance of the linearization techniques with increasing signal bandwidth is due to memory effects (ME) taking place in the RF amplifier devices.

The importance of ME in digital predistortion is a well documented problem [10] [12] [13] [33] [42]. For LFFF a first order theory developed by a previous MISES student led to the derivation of a new phase constraint that the device characteristics must satisfy for LFFF to work for both intermodulation side-bands. We will show however that the device must be quasi-memory-less for this phase constraint to work and will present supportive ADS simulation results. Note that the discussion of LFFF linearization presented in this section will directly benefit the new IBM, OBM and BBM linearization schemes to be discussed in the next chapter.
2.1 Low Frequency Feedforward Linearization

Low frequency feedforward linearization[16][17] relies on the injection of a low frequency signal at the output of the amplifier as shown in Figure 2.1 to cancel the intermodulations.

\[ f_c = f_b - f_a \]

Figure 2.1: Low-frequency feedforward linearization

To understand the principle of operation of LFFF let first consider a scalar model as was done in Ref.12. When memory effects are neglected the nonlinear AC response of a transistor to small AC signals can be described with the help a Taylor series expansion:

\[
i_{ds}(t) = g_{m1}v_{gs} + g_{d1}v_{ds} + g_{m2}v_{gs}^2 + g_{m1d}v_{gs}v_{ds} + g_{d2}v_{ds}^2 + g_{m3}v_{gs}^3
+ g_{m2d}v_{gs}^2v_{ds} + g_{m1d2}v_{gs}v_{ds}^2 + g_{d3}v_{ds}^3 + g_{m4}v_{gs}^4 + g_{m3d}v_{gs}v_{ds}
+ g_{m2d2}v_{gs}^2v_{ds} + g_{m1d3}v_{gs}v_{ds}^3 + g_{d4}v_{ds}^4 + \cdots, \tag{2.1}
\]

7
where $i_{ds} = \text{drain-source current}$, $v_{gs} = \text{gate-source voltage}$, and $v_{ds} = \text{drain-source voltage}$ where $v_{gs}(t)$ and $v_{ds}(t)$ are small AC signals. In the quasi-memoryless approximation the $g$ coefficients are replaced for the frequency domain analysis by complex coefficients which are weakly frequency dependent. This expression implies that the small signal drain current is not only function of input signal $v_{gs}$ but also the output signal $v_{ds}$. Furthermore in two-tone test the high order terms generate the cross modulation signals, which could be used to decrease the intermodulation powers.

The low frequency signal must then be fed to the drain side through a amplifier with gain $G$ and phase shift $\phi$. For a perfect cancellation Shuang et al. demonstrated that the following conditions should be theoretically satisfied:

$$|G| = \frac{3}{2} \left| \frac{y_{m3}}{y_{md}} \right|$$

(2.2)

$$\phi = (2m + n + 1)\pi$$

(2.3)

$$\theta_d - \theta_m = n\pi$$

(2.4)

The new phase constraint $\theta_d - \theta_m = n\pi$ is required if both the lower and upper side are to be canceled. If this condition is not verified, only one side-band a time can be suppressed for a given pair of $G$ and $\phi$ as is illustrated in Figure 2.2. These results has recently been reproduced experimentally in our laboratory by Mr. Xian Cui (see Appendix A.1 for experimental results).

Real devices do not naturally verify the condition $\theta_d - \theta_m = n\pi$. It is however possible in some devices to find a bias condition $(V_{GS}, V_{DS})$ for which this LFFF conditions is approximately verified. But such an ”optimal” bias condition for LFFF might not correspond to an ”optimal” bias condition for the gain, P1dB, or PAE. In other devices it might not be possible to find a bias condition where both the side-bands $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ are simultaneously canceled. As we shall see in
Figure 2.2: (a) IMD3 contour as a function of phase $\phi$ and gain $G$, (b) IMD3 plotted versus phase $\phi$ for different gains $G$

In the next section, this is due to the fact that the results given above are only valid for quasi-memoryless amplifiers: that is amplifiers must have $y_{m3}$ and $y_{md}$ which remain constant within the bandwidth of operation.

**Extraction of $y_{m3}$ and $y_{md}$**

To address the issue of memory in power amplifiers let us see how the terms $y_{m3}$ and $y_{md}$ are extracted in practice. Fig. 2.3(a) shows the circuit configuration used for extracting $y_{m3}$. Consider the example of an AMPS cellular frequency band. A two-tone signals with frequency $\omega_1 = 877$MHz and $\omega_2 = 878$MHz is generated. The term $y_{m3}$ in equation (2.5) is responsible for the generation of intermodulation terms $I_{ds}(2\omega_1 - \omega_2)$ and $I_{ds}(2\omega_2 - \omega_1)$ at frequencies of $875$MHz and $881$MHz respectively. The formula for calculating $y_{m3}$ is:

$$y_{m3}(-) = \frac{4}{3} \frac{I_{ds}(2\omega_1 - \omega_2)}{V_{gs}^2(\omega_1)V_{gs}^2(\omega_2)}$$

$$y_{m3}(+) = \frac{4}{3} \frac{I_{ds}(2\omega_2 - \omega_1)}{V_{gs}^2(\omega_2)V_{gs}^2(\omega_1)}$$

(2.5)
Figure 2.3: Circuit for extracting (a) $y_{m3}$ and (b) and $y_{md}$ in ADS

Extraction of $y_{md}$ is done by applying a single-tone signal ($\omega_1 = 877\text{MHz}$) at the input side and a low frequency signal ($\omega_3 = 1\text{MHz}$) at the drain side as shown in Fig. 2.3(b). The term $y_{md}$ generates side-band intermodulation products at $\omega_1 - \omega_3 = 875\text{MHz}$ and $\omega_2 + \omega_3 = 879\text{MHz}$. The formula for calculating $y_{md}$ is:

$$y_{md}(-) = 2\frac{I_{ds}(2\omega_1 - \omega_2)}{V_{gs}(\omega_1)V_{ds}^*(-\omega_3)}$$  \quad  

$$y_{md}(+) = 2\frac{I_{ds}(2\omega_2 - \omega_1)}{V_{gs}(\omega_2)V_{ds}(\omega_3)}$$

Clearly it is seen that a unique $y_{m3}$ and $y_{md}$ is only obtained if these coefficients are frequency independent: $y_{m3}(-) = y_{m3}(+)$ and $y_{md}(-) = y_{md}(+)$. Under such conditions the amplifier is referred as quasi-memoryless. However real amplifiers exhibits memory effects. The larger the bandwidth, the more pronounced these memory effects. As a result it is not possible to cancel simultaneously the lower and upper side band using LFFF. The theory developed by Yu et al. [17] is not capable of accounting for this memory effect. We will see in Chapter 3, that a more rigorous analysis can be obtained using the Volterra formalism. New approaches for simultaneously canceling the lower and upper side bands will also be demonstrated.
2.2 RF and digital baseband adaptive predistortions

Figures 2.4 and 2.5 show the schematics an (a) RF and (b) baseband adaptive predistortion system respectively. Typically these systems are quasi-memoryless. Predistortion requires the insertion of a non-linear module before the RF power amplifier. The nonlinear module, called a predistorter, has effectively the inverse characteristics of the PA so that the overall response of the PA is linear. A predistorter can be implemented with the help of an IQ modulator. Consider an incoming signal

\[ x = A \cos(\omega t + \phi) = I \cos(\omega t) - Q \sin(\omega t) \]

with \( I = A \cos \phi \) and \( Q = A \sin \phi \). The action of the quasi-memoryless predistorter is to modify the phase and amplitude of the input signal to compensate for the in-band AM-AM and AM-PM distortion introduced by the PA. The output of the predistorter is then

\[ y = A \ B \cos(\omega t + \phi + \psi) = I' \cos(\omega t) - Q' \sin(\omega t) \]

where one easily verify that \( I' \) and \( Q' \) are given by:

\[
\begin{bmatrix}
I' \\
Q'
\end{bmatrix} = \begin{bmatrix}
B \cos \psi & -B \sin \psi \\
B \sin \psi & B \cos \psi
\end{bmatrix} \begin{bmatrix}
I \\
Q
\end{bmatrix} = \begin{bmatrix}
\alpha & -\beta \\
\beta & \alpha
\end{bmatrix} \begin{bmatrix}
I \\
Q
\end{bmatrix}
\]

A typical quasi-memoryless algorithm uses the envelope \( A = \sqrt{I^2 + Q^2} \) to evaluate \( \alpha \) and \( \beta \)

\[
\alpha(A) = 1 + \alpha_3 A^2 + \alpha_5 A^4 \\
\beta(A) = 1 + \beta_3 A^2 + \beta_5 A^4
\]

(2.6) (2.7)

This non-linear expansions is used to cancel the intermodulation products generated by quasi-memoryless 3rd order and 5th order nonlinearities. This will be demonstrated in the next chapter for LFFF and OBM which will be seen to effectively
operate like postdistortion. In practice the coefficients $\alpha_3$, $\alpha_5$, $\beta_3$ and $\beta_5$ minimizing the ACPR are obtained after an optimization search for either two-tone or multisine (CDMA like) excitations.

There are two major limitations with digital predistortion linearization: (1) the adaptation of the linearization coefficients ($\alpha$ and $\beta$) cannot be accomplished in real time and this limits its performance and (2) conventional predistortion linearization algorithms do not include memory effects which affect the output of nonlinear devices. Improved algorithms are therefore required to account for memory effects and will be proposed in the next chapter.

Figure 2.4: RF predistortion (quasi-memoryless algorithm).
Figure 2.5: Adaptive baseband digital predistortion (quasi-memoryless algorithm).
CHAPTER 3

BASEBAND LINEARIZATION OF PAS WITHOUT AND WITH MEMORY EFFECTS

In this chapter we will first study three new proposed baseband linearization schemes: IBM, OBM and BBM (input/output/bilateral baseband modulation) linearizations which are various baseband implementation of the RF LFFF linearization studied in the previous chapter. We then conclude the chapter by discussing an improved predistortion algorithm. The schematic of OBM shown in Fig. 3.1 is similar to that of LFFF (see Fig. 2.1) except that the square law operation is now performed directly on the baseband and not the RF signal. This eliminates the need for the RF coupler, square law detector and low pass filter required by the LFFF topology.

The low-frequency signal can also be injected at the input or at both the input and output of the amplifier. This first option will be referred in this thesis as IBM (input baseband modulation) and the second option as BBM (bilateral baseband modulation).

We will first demonstrate (both theoretically and using MATLAB simulations) how OBM/LFFF can completely suppress 3rd and 5th order non-linearities in memoryless systems for multiple tones excitations. This digital baseband linearization
combined with digital predistortion could then provide further improved linearization capability. However again MEs must be included to implement a linearization effectively working for wideband signals. In the presence of memory effects we will see that both IBM and OBM do not provide enough degrees of freedom to eliminate both the LSB and USB IMD in two-tone excitations. However BBM which combines both IBM and OBM, can provide in some systems the required degree of freedom to cancel both the LSB and USB as this will be demonstrated for a 3rd order system under a two-tone excitation.

3.1 Perfect IM3 and IM5 cancellation using \( g_{md} \) for memoryless systems

We consider in this section a system which does not include MEs: that is all the transconductances are real numbers. The system considered is a 5th order memoryless system with multiple inputs \( (v_{in} \) and \( v_{LFF} \)) of the form:
\[ i_{\text{out}} = g_{m3}v_{\text{in}}^3 + g_{m5}v_{\text{in}}^5 + g_{md}v_{\text{in}}v_{LFF}\]  

where the input RF signal is given by \( v_{\text{in}} = v_{BB}(t)\cos(\omega t) \) and where \( v_{LFF} \) is the low-frequency baseband signal injected at the output to linearize the amplifier. We can now use the 2\(^{nd}\) order nonlinear term \( g_{md} \) to cancel the 3\(^{rd}\) and 5\(^{th}\) order intermodulations:

\[ v_{LFF}(t) = -\frac{3}{4} \frac{g_{m3}}{g_{md}} v_{BB}^2(t) - \frac{5}{8} \frac{g_{m5}}{g_{md}} v_{BB}^4(t) \]

Fig. (a) 3.2 shows that both the 3\(^{rd}\) and the 5\(^{th}\) order intermodulation are indeed canceled.

It is demonstrated in Appendix A.2 that this method works also with an IQ modulator. If the RF signal is given by

\[ v_{in}(t) = I(t)\cos(\omega t) + Q(t)\sin(\omega t) \]

the correction for a 5\(^{th}\) order system with \( g_{m3} \) and \( g_{m5} \) terms is

\[ v_{LFF}(t) = -\frac{3}{4} \frac{g_{m3}}{g_{md}} [I^2(t) + Q^2(t)] - \frac{5}{8} \frac{g_{m5}}{g_{md}} [I^2(t) + Q^2(t)]^2 \]

Note that more complex 3\(^{rd}\) or 5\(^{th}\) order systems will involve additional 3\(^{rd}\) and 5\(^{th}\) correction terms. Consider the following 3\(^{rd}\) order system:

\[ i_{\text{out}} = g_{m3}v_{\text{in}}^3 + g_{md}v_{\text{in}}v_{LFF} + g_{md2}v_{\text{in}}^2v_{LFF}. \]  

We need now to take into account the nonlinear intermodulation terms contributed by \( g_{md2} \). In this simple case this \( g_{m3} \) effects can completely be removed by solving for the required linearization signal \( v_{LFF} \) using the following equation:

\[ g_{md} v_{LFF} + g_{md2} v_{LFF}^2 = -\frac{3}{4} g_{m3}v_{BB}^2, \]  

16
Figure 3.2: (a) IMD3, (b) IMD3 and IMD5 linearization for four tones $I$ and $Q$ excitations (MATLAB results)

where $v_{BB}$ is the base-band signal for double sideband upconversion. Two solutions are possible and are found to work equally well. Fig.(b) 3.2 shows that the 3$^{rd}$ order intermodulation is indeed canceled perfectly.

In LFFF or OBM, the term $g_{md}$ acts as like the modulator in the predistortion linearization except that the linearization is performed at the output. This explains why we are finding that like in predistortion, the correction signal in OBL is also proportional to $v_{in}^2$ and $v_{in}^4$ for 3$^{rd}$ and 5$^{th}$ order non-linearities respectively. Such a linearization techniques is referred as postdistortion [18]. Similarly IBM (input baseband modulation) linearization where the low-frequency baseband signal is applied at the input is a form of predistortion. In IBM the term $g_{m2}$ acts as like the modulator implementing the predistortion linearization.
It should be possible to extend the results derived in this section to quasi-memoryless systems where the group delay or phase is constant throughout the channel bandwidth. It is however even more important to extend these results to systems with memory. This expansion will be the object of the next three sections where we will consider two three schemes referred as BBM (bilateral baseband modulation), VPD (vectorial predistortion) and QOBM (quadratic baseband modulation). BBM the first linearization scheme considered is a combination of IBM and OBM whereas QOBM is a quadraric modulation implementation of OBM.

3.2 Volterra Analysis of BBM for Systems with Memory

In the previous section we have demonstrated the capability of low-frequency feedforward to linearize an amplifier for multitone excitations. The systems considered were multi-input single-output (MISO) memoryless systems. We shall return now to the issue of the memory effects discussed in the previous chapter.

In the previous chapter the theory of low-frequency feed-forward techniques for \textit{quasi-m}emoryless system was reviewed for two-tone excitations. That theory introduced a new phase constraint which must be verified for both the LSB and USB to be canceled. It was however pointed out that for systems with memory the cancellation of both the LSB and USB intermodulation products was not usually possible and the combination of IBM and OBM into BBM was proposed to achieve that goal. In this section the theory of BBM for two-tone excitations is now developed for a system with memory.

The most rigorous theory for including memory effects is the Volterra formalism. In that formalism the system is described by Volterra kernels of various orders. For
Figure 3.3: Example of input baseband linearization for (a) single input and single output (SISO) with memory and (b) double input and single output (DISO) with memory.

example a SISO system with a 3\textsuperscript{rd} order kernel is of the form:

\[ y(t) = \iiint_{-\infty}^{\infty} h(\tau_1, \tau_2, \tau_3) x(t - \tau_1) x(t - \tau_2) x(t - \tau_3) d\tau_1 d\tau_2 d\tau_3 \]

For a BBM system (see Fig. 3.4) with ME we need to use the vector Volterra formalism developed for MIMO [19] since we have two inputs \( v_{in} \) and \( v_{LFF} \) and one output \( i_{out} \). An example of IBM and OBM systems with memory is shown in Figure 3.3(a) and (b). We next consider the frequency response for a two-tone excitation. The intermodulation terms of interest are the in-band terms which are:

\[
\begin{align*}
i_{out}(2\omega_1 - \omega_2) &= A_0 a_1^2 b_1^* + A_1 a_1 c_1^* + A_2 a_1 c_2^* + A_3 b_1 c_1 s^2 + A_4 b_1 c_2 s^2 + A_5 b_1 c_3 s^2 \\
i_{out}(2\omega_2 - \omega_1) &= B_0 a_1^* b_1^2 + B_1 b_1 c_1 + B_2 b_1 c_2 + B_3 a_1 c_1^2 + B_4 a_1 c_2^2 + B_5 a_1 c_1 c_2 (3.5)
\end{align*}
\]

where \( A_0 \sim A_5 \) are the coefficients for the LSB IMD3, and where \( B_0 \sim B_5 \) are the coefficients for the LSB IMD3. There coefficients can be expressed in terms of the VKs of the fundamental signals (see results in Appendix A.4 presenting the results of our derivation). The two desired tones at the output of the PA are themselves given
by:

\[ i_{out}(\omega_1) = D_0a_1 + D_1b_1c_1^* + D_2b_1c_2^* + D_3a_1^2a_1^* + D_4a_1b_1^* + D_5c_1c_1^*a_1 (3.6) \]
\[ + D_6c_1^*c_2a_1 + D_7c_1c_2^*a_1 + D_8c_2c_2^*a_1 \]
\[ i_{out}(\omega_2) = E_0b_1 + E_1a_1c_1 + E_2a_1c_2 + E_3b_1^2b_1^* + E_4b_1a_1a_1^* + E_5c_1c_1^*b_1 (3.8) \]
\[ + E_6c_1^*c_2b_1 + E_7c_1c_2^*b_1 + E_8c_2c_2^*b_1 \]

where \( D_0 \sim D_8 \) and \( E_0 \sim E_8 \) are coefficients expressed in terms of the VKs of the fundamental signals (see results in Appendix A.4 presenting the results of our derivation).

The linearization goal is to set the amplitude of the LSB and USB to zero:

\[ i_{out}(2\omega_1 - \omega_2, c_1, c_2) = 0 \quad \text{and} \quad i_{out}(2\omega_2 - \omega_1, c_1, c_2) = 0 \]

The optimal BBM signals \((c_1, c_2)\) required are the solution of a bi-quadratic system.

Let us consider first the case of OBM first \((c_1 = 0)\) with 2 applied RF tones \(a_1\) and \(b_1\). The derivation is carried in Appendix A.5. It demonstrated that for OBM with \( A_4 = B_4 = 0 \) a joint simultaneous cancellation of the USB and LSB IMD3 we must have \( A_0 = B_0 = y_{m3} \) and \( A_2 = B_2 = y_{md} \) verifying \( y_{m3} - y_{md} = n\pi \). This is exactly the condition previously derived by Yu et al [16]. For the general case where \( A_4 \) and \( B_4 \) are non-zero we must further have \( A_4 = B_4 = y_{md2} \) verifying \( y_{m3} - y_{md2} = n\pi \) and \(|a_1| = |b_1|\). It results that the required conditions for both the LSB and USB to be canceled simultaneously are unlikely to be verified in OBM for general systems and arbitrary signals and therefore only an imperfect linearization is possible with OBM/LFF in nonlinear systems with memory. A reduction of 20 to 30 dB of the
two-tone IMD3 was nonetheless obtained by Yu et al [16] with the tuning of the PA transistor operating point.

To cancel both the LSB and USB one can rely the additional degree of freedom offered by BBM. To analyze its feasibility, a LDMOSFET PA was designed in ADS using load-pull techniques (see Appendix A.3). The LDMOS PA was then modeled in MATLAB with a 3rd order Volterra model. A reasonable agreement in terms of IMD0 and IMD3 is obtained between the Volterra model and the ADS simulations as is shown in Fig 3.5(a). The bi-quadratic system of equations 3.2 was then solved to implement BBM. The results obtained from MATLAB simulations are shown in Fig. 3.5(b). It is seen that under certain operating conditions both the upper and lower IMD3s can be theoretically reduced by 100dB and 200 dB respectively. Note however that, depending on the Volterra Kernel, the biquadratic system does not always admit a solution for certain power levels.
Figure 3.5: (a) Volterra model versus ADS simulation, (b) IMD3 cancellation with BBM

3.3 Volterra Analysis of QOBM for Systems with Memory

Using predistortion as a guideline we can infer that the quadratic OBM (QOBM) topology shown in Fig 3.6 will permit us to account for memory effects. This is demonstrated in Appendix A.6. Like for predistortion we define $c_I = \alpha_3 \frac{b_3 a_I^3}{2}$ and $c_Q = \beta_3 \frac{b_3 a_I^3}{2}$. It is demonstrated in App. A.6 that for a quasi-memoryless system the $\alpha_3$ and $\beta_3$ coefficients are given by:

$$
\alpha_3 = -2\text{Re} \left[ \frac{(1+j)y_{m3}}{y_{md}} \right] \quad \text{and} \quad \beta_3 = -2\text{Im} \left[ \frac{(1+j)y_{m3}}{y_{md}} \right]
$$

However as is demonstrated in Appendix for amplifier with memory the coefficients $\alpha_3$ and $\beta_3$ will be complex numbers. This implies that the baseband signals sent to the port I and Q need to appropriately phase shifted. The topology of QOBM which doubles the degrees of freedom over OBM, brings the capability to remove non-linearities with memory.
Figure 3.6: Topology of quadratic OBM
CHAPTER 4

LARGE SIGNAL NETWORK ANALYZER WITH TRIGGER FOR BASEBAND & RF SYSTEM CHARACTERIZATION WITH APPLICATION TO K-MODELING & IQ MODULATOR BALANCING

The modeling and balancing of IQ modulators can benefit from the measurement capabilities of the large signal network analyzer. Both input and output triggered LSNA can permit us to acquire the IQ baseband and RF signals in an IQ modulator for K-modeling. By combining the use of the K inverse matrix and error amplitude & phase, the best IQ balancing of the modulator will be obtained.

4.1 Introduction

K-models provide a behavioral representation of both the RF and baseband signals in communication systems using their I and Q signal components in the band of interest. Non-linear K-models can be used in transmitter systems to model the non-linearities from the digital signal process (DSP) circuits to the RF antenna. K-models of PAs and IQ modulators have been reported for several simulation tools (Cadence and CientoTM AltaTM SPW [29]) to predict the adjacent channel power ratio (ACPR) at the output of RF front ends. K-models are usually developed using
the input and output signals obtained in circuit simulators. However there has not been up to now any reported attempt to extract K-models directly from experimental data. This chapter presents a simple measurement technique for acquiring the necessary data.

First in Section 4.2 a triggered LSNA configuration and associated digital testbed is presented. The immediate result from the use of a trigger is the stabilization of the RF envelope displayed by the LSNA. In Section 4.2 a novel measurement technique for correlating the I and Q signals at RF and baseband is then presented. Unlike the previously reported 3-port LSNA [30] our system is not presently configured to extract calibrated 3-port signals. However such data are not typically required for these linear and non-linear MIMO (multiple input multiple output) system applications where the baseband signals are typically injected in an IQ modulator with buffered input ports with 50 ohm impedance.

The proposed measurement technique which correlates the I and Q signals at RF and baseband is applied in Section 4.3 to an IQ modulator. A simple theory is presented to extract its linear K-model and model the IQ imbalance of the IQ modulator. The validity of this basic K-model is then tested for its ability to actually cancel the IQ imbalance of the modeled modulator.

4.2 Digital Testbed With 2-Port LSNA

We describe in this section the triggered LSNA configuration and the digital testbed developed for using it.
4.2.1 10MHz Synchronization between LSNA and digital testbed

In our work a FPGA digital testbed is used to generate the desired I and Q modulation waveforms. An arbitrary waveform generator could have been used instead. The digital testbed used is based on an Altera Stratix FPGA/DSP S25 development board. The components of the complete digital testbed are described in more details in chapter 6. We focus here on the critical features which are used by the triggered LSNA. The 10MHz TTL signal generated by the LSNA to synchronize its own 25 MHz ADC is used in our digital testbed to synchronize the 80 MHz system clock of the FPGA. This is done using one of the PLL available in the Altera Stratix FPGA. This synchronization insures the FPGA and the LSNA use the same time base.

4.2.2 Triggered LSNA and digital testbed

The LSNA in its normal mode of operation is triggered by an internal software trigger. The ADCs of our LSNA were configured with a special trigger option [31]. The LSNA software was then augmented (work performed at NMDG) with the required modification to allow it to either generate a trigger (trigger-out configuration) when it initiates its measurement or to start its measurement when it receives an external trigger (trigger-in configuration). The trigger provides a time reference which together with the 10 MHz synchronization, permits us to synchronize the LSNA measurements with the generation and/or measurement of the baseband modulation signals. In this work both the trigger-out and trigger-in configurations will be used and compared. In the trigger-out case the trigger signal originating from the LSNA is used to start the generation of the baseband signals. In the trigger-in case, the FPGA starts the baseband waveform and then sends a trigger to the LSNA to start
Figure 4.1: The addition of an output measurement trigger to an LSNA introduces a time reference for the modulation. As a result the display of the modulation RF signals (see Fig. 4.2) is stabilized.

the measurement. In both configurations the measurement is performed automatically under Labwindows control. Although both configurations will be found to work, the trigger-in configuration can be expected to be superior because (1) the triggers generated by the 80 MHz FPGA have a 2 ns rise time instead of 5 ns for the LSNA ADC trigger signal and (2) because there is a finite delay (several clocks) for the baseband signals to emerge at the DAC outputs. As we shall see below this unwanted
delay in the trigger-out configuration induces a noise in the measurements performed by the LSNA. In the trigger-in we can adjust the delay in the FPGA such that the measurement and modulation waveforms start at the same clock beat.

![Graphs showing modulated RF signals](image)

(a) Measurement 1  
(b) Measurement 2

(a) Measurement 3  
(b) Measurement 4

Figure 4.2: Envelop of the modulated RF signals displayed by LSNA

An immediate result from the use of a trigger is the stabilization of the RF envelope displayed by the LSNA. This is demonstrated in Fig. 4.2 where the RF signals displayed in time domain for two-tone (894 and 896 MHz) RF signals at the input of the DUT are found to have the same stable envelope for different LSNA measurements.
using the experimental setup of Fig. 4.1. Without the trigger the envelope displayed would have the shape but would keep sliding to different random positions. Note that the phase of the RF signal measured at the input port of the DUT keeps varying despite the use of a trigger to stabilize the envelope. The 2-5 ns trigger permits us to stabilize the modulation envelope but not the RF phase. The experimental set up shown in Fig. 4.1 needs therefore to be modified as described in the next section if one desires to correlate the baseband and RF phases. The stability of the RF envelope is, however, useful in itself if the PA is to be tuned in real time to analyze the impact of the harmonic impedance termination on the RF envelope. The triggered LSNA facilitates also the acquisition of the baseband signals (currents and voltages) at port 1 and 2 with a scope. This is of importance as bias tees have a strong impact on the non-linear response of PAs.

4.3 Digital Testbed with 2-Port Triggered LSNA

The triggered LSNA can be also used to characterize IQ modulators. As shown in Fig. 4.3, this requires a modification of the setup so that we can acquire the phase of the RF signals at port 1 (LO input) and port 2 (modulator output). This setup permits ones to set or acquire the phase and amplitude of both the baseband and RF I & Q signals.

4.3.1 K-Modeling of IQ Modulator

The simplest model we can use to represent the IQ imbalance of an IQ modulator is the linear K-model:
\[
\begin{pmatrix}
I_{RF}(t) \\
Q_{RF}(t)
\end{pmatrix}
= 
\begin{pmatrix}
 k_{11}(\omega_m) & k_{12}(\omega_m) \\
k_{21}(\omega_m) & k_{22}(\omega_m)
\end{pmatrix}
\begin{pmatrix}
 I(t) \\
 Q(t)
\end{pmatrix}
\]

(4.1)

We can use simple I & Q excitations to evaluate \(k_{11}, k_{12}, k_{21}\) and \(k_{22}\). Consider the experiment shown in Fig. 4.4 where we inject \(I = \cos(\omega_{mt})\) and \(Q = \sin(\omega_{mt})\) in the modulator to generate the upper side-band of amplitude \(M2\) and phase \(\phi_2\) at the frequency \(\omega_2 = \omega_0 + \omega_m\). Due to the modulator imbalance a parasitic lower side-band tone of amplitude \(M1\) and phase \(\phi_1\) at the frequency \(\omega_1 = \omega_0 - \omega_m\) is also generated.

![Diagram](image)

Figure 4.3: Setup for the characterization of an IQ modulator for a given modulation frequency \(\omega_m\)

The K-matrix of the modulator can then be extracted from the phases and amplitudes \(\phi_0, M1, \phi_1, M2\) and \(\phi_2\) measured by the LSNA using the following expression:

\[
K_{MOD} = 
\begin{pmatrix}
k_{11}(\omega_m) & k_{12}(\omega_m) \\
k_{21}(\omega_m) & k_{22}(\omega_m)
\end{pmatrix}
= 
M_1 \begin{pmatrix}
\cos(\phi_1 - \phi_0) & \sin(\phi_1 - \phi_0) \\
\sin(\phi_1 - \phi_0) & -\cos(\phi_1 - \phi_0)
\end{pmatrix}
+ 
M_2 \begin{pmatrix}
\cos(\phi_1 - \phi_0) & \sin(\phi_1 - \phi_0) \\
\sin(\phi_1 - \phi_0) & -\cos(\phi_1 - \phi_0)
\end{pmatrix}
= M_1 P_1 + M_2 P_2
\]

(4.2)
Figure 4.4: The I and Q signals are selected to generate the upper side band. The $M_1$ and $\phi_1$ terms of the lower side-band arised from the impedance of the modulator.

### 4.3.2 Compensation of IQ Imbalance

The IQ imbalance can then be compensated for in the FPGA testbed by using the following IQ conversion:

$$K_{FPGA} = M_2P_2(K_{MOD})^{-1}$$

Note that this IQ conversion minimizes the change in phase by restoring the original group phase shift of the IQ modulator while removing its imbalance. Indeed implementing a zero phase shift is unnecessary and most likely undesirable as is discussed below. The K matrix conversion is implemented in the FPGA with 13 bits unsigned. As the number of bits is increased, the small off-diagonal elements are resolved better compared to the diagonal terms. Note however that the final number of bits is truncated to 14 bits in the DAC. Around 36 dBc suppressions in both upper side band (USB) and lower side band (LSB) were achieved. Note that
this was obtained after some small tuning to compensate for the 13 bits truncation of the K matrix (Fig. 4.5).

![Figure 4.5: Implementing the IQ correction $K_{FPGA2}$ in the FPGA and with some tuning reduces the imbalance of the IQ modulator at the baseband & LO frequencies $\omega_m$ and $\omega_0$](image)

As mentioned above the reduction of the K-matrix to a unity matrix is not only unnecessary but also potentially undesirable. Indeed the linear K-model used is a simplification of the IQ modulator response and it is best to use it only for small relative amplitude and phase corrections. In this spirit it is conceivable that the IQ balancing would benefit from a preliminary relative correction. To perform this IQ conversion we apply the simple K-model presented in [31] where the IQ imbalance is associated with the gain and phase error arising in the in-phase and quadrature paths of the local oscillator signal:
\[ K_{FPGA1} = \begin{pmatrix} k_{11}(\omega_m) & k_{12}(\omega_m) \\ k_{21}(\omega_m) & k_{22}(\omega_m) \end{pmatrix} = \begin{pmatrix} (1 + \frac{\epsilon}{2})\cos\frac{\theta}{2} & -(1 + \frac{\epsilon}{2})\sin\frac{\theta}{2} \\ -(1 - \frac{\epsilon}{2})\sin\frac{\theta}{2} & (1 - \frac{\epsilon}{2})\cos\frac{\theta}{2} \end{pmatrix} \] (4.4)

The gain and phase corrections \( \epsilon \) and \( \theta \), were implemented in the FPGA with 10 and 8 unsigned bits respectively to provide a preliminary compensation for the intrinsic IQ imbalance of the modulator. An IQ imbalance of 43 dBC and 46 dBC suppressions was achieved by cascading the two IQ imbalance correction techniques as is shown in Fig. 4.6

![Figure 4.6: Cascading the IQ conversion \( K_{FPGA1} \) and \( K_{FPGA2} \) in the FPGA yields the following correction for the IQ imbalance at the baseband & LO frequencies \( \omega \)](image)

**4.3.3 Error Analysis**

As was discussed in Section 4.2 the triggered LSNA comes in two flavors: input and output trigger configurations. It is of interest to compare the stability and accuracy of the phases measured with both configurations. Figures 4.7 and 4.8 show the
phases obtained after 10 successive measurements. Note that different phases and are obtained due the difference in settings of the analog circuits used to drive the IQ modulator which are manually tuned (see [10] for details). In Fig. 4.7 the circled phases in measurement 2 and 3 were actually off by 180 degree and this was manually corrected. The standard deviations obtained for the trigger-out configuration are much larger ($\sigma_{\phi_1}=12.15$ and $\sigma_{\phi_2}=11.2$) compared to those obtained for the trigger-in configuration ($\sigma_{\phi_1}=0.1$ and $\sigma_{\phi_2}=0.03$). Clearly the performance of the trigger-in configuration is much superior. This originates in part from the fact that when using the input-trigger configuration the trigger rise time (less than 2 ns) is sharper than that of the Spectrum ADC board of the LSNA. It results that the phase error of the baseband signal is reduced for shorter rise times. For 1 MHz modulation frequency and 2 ns rise time the error bound is of 0.36 degrees. The degradation of the error for the phase $\phi_1$ and $\phi_2$ for the trigger-out configuration is however much larger than the factor of 2 or 3 degradation in rise-time. Most certainly this large error and also the intermittent phase instability observed originate from the delay of the modulation signal in reaching the DAC outputs when using the trigger-out configuration. The resulting missing data induce an effective noise in the LSNA measurement which is not present when using the trigger-in configuration.

4.4 Conclusion

In this chapter we have presented a triggered LSNA measurement setup which provides the means to correlate together the amplitude and phase of the baseband and RF signals in multi-input communication systems. These measured data can be used to develop K models. As an example, a linear K-model was extracted for
Figure 4.7: Variation of $\Phi_1 = \phi_1 - \phi_0$ with measurements when using the trigger-out configuration.

modeling the IQ imbalance of an IQ modulator. Using this linear K-model the IQ imbalance could be compensated to provide 43 and 46 dBc isolation for the LSB and USB respectively.
Figure 4.8: Variation of $\Phi_1 = \phi_1 - \phi_0$ with measurements when using the trigger-in configuration.
CHAPTER 5

VOLTERRA CHARACTERIZATION OF NON-LINEAR PA WITH LSNA

The linearization of RF power amplifiers (PA) can benefit from the availability of the generalized Volterra coefficients characterizing its non-linear response. In this work a large-signal network analyzer is used to acquire the amplitude and phase of the 3rd intermodulation terms $Y_{m3-}$ and $Y_{m3+}$ of an LDMOSFET PA. The frequency dependence and difference between $Y_{m3-}$ and $Y_{m3+}$ reveals the memory effects of the RF amplifier.

5.1 Introduction

One of the most challenging issue in the design of radio frequency (RF) power amplifiers for base stations, is the linearity requirement. FCC and ETSI limit spurious emissions outside the selected bandwidth. These unwanted distortions referred to as spectral regrowth are potential interfering sources for other radio users and must be reduced to a sufficient low level such that all systems can operate satisfactorily. Various linearization techniques such as feedforward and recently predistortion have been developed to reduce the spectral regrowth [25][14]. The performance of these
linearization schemes is usually measured using in-band and out-band figures of merit such as error vector magnitude (EVM) and adjacent channel power ratio (ACPR).

It is found in practice that the performance of linearization decreases as the signal bandwidth increases. This performance degradation has been identified to be linked with the frequency dependence of the non-linearities in PAs [48]. Indeed various physical processes with memory contribute to the PA characteristics. The design of linear and power-efficient power amplifiers for wideband applications such as multi-carrier
WCDMA requires therefore the development of linearization schemes accounting for memory effects. Predistorter algorithms accounting for memory effects such as memory polynomials have been proposed and demonstrated to provide a robust solution for improving the ACPR of RF PAs in MATLAB simulations [44].

As the bandwidth of the RF envelope increases, electrical memory effects in RF PAs should be taken into an account to improve the linearization performance. Indeed for broadband RF applications, it can be verified that fast electrical memory effects usually dominate over slow thermal electrical memory effects for modulation bandwidth above 1MHz [6]. Unlike slow thermal memory effects which can be handled using adaptive techniques, electrical memory effects are best dealt with using real time algorithms due to bandwidth limitations.

In this chapter we will experimentally characterize the memory effects taking place in the PA to be linearized. For this purpose a large signal network analyzer (LSNA) will be used to measure the power-dependent generalized 3rd order Volterra coefficients of a Class AB amplifier. The accuracy of these LSNA measurements will also be investigated. The strong asymmetry between the lower and upper side bands generated by the non-linearities will provide the motivation for investigating in chapter 6 & 7 a novel predistortion algorithm which can independently linearize the lower and upper side bands.

5.2 Nonlinear PA Characterization with LSNA

In this work the LSNA is used to characterize the 3rd order intermodulation response of a Class AB LDMOSFET PA at 895 MHz for a two-tone excitation. The data acquisition is performed under LabWindows control. The LSNA is used to measure
both the amplitude and phase of the two-tone RF excitation $a_1(\omega)$ and $a_1(\omega + \omega_m)$ incident on port 1 as well as the amplitude and phase of the transmitted intermodulation RF signals $b_2(\omega - \omega_m)$ and $b_2(\omega + \omega_m)$ transmitted to port 2 as shown on Fig. 5.3. Using these intermodulation signals we calculated next the generalized Volterra coefficients $Y_{m3-}$ and $Y_{m3+}$ defined as:

\[
Y_{m3-} = \frac{b_2(\omega - \omega_m)}{a_1^2(\omega)a_1^*(\omega + \omega_m)} \quad (5.1)
\]
\[
Y_{m3+} = \frac{b_2(\omega + 2\omega_m)}{a_1^*a_1^2(\omega + \omega_m)} \quad (5.2)
\]

The amplitude and phase of these coefficients are plotted in Figs. 2(a) and 2(b) respectively as a function of tone spacing $\omega_m$ and input power $|a_1(\omega)|^2 = |a_1(\omega + \omega_m)|^2$ from -4 to 6 dBm. As shown in Fig. 5.2 the variation of $Y_{m3-}$ and $Y_{m3+}$ as a function of tone spacing $\omega_m$ reveals the presence of memory effects (frequency dependence of the non-linearity) in the PA. However up to 0.5 MHz the amplitude and phase of $Y_{m3-}$ and $Y_{m3+}$ are approximately the same. This is confirmed by Fig. 5.2(c) which plots the phase and amplitude of the difference of $Y_{m3-}$ and $Y_{m3+}$. Above 0.5 MHz the difference in phase and amplitude increases rapidly with tone spacing. This result suggests that a memory-less linearization scheme which attempts to remove the lower and upper sidebands using the same non-linear coefficients will become less efficient for this PA as the bandwidth increases above 0.5 MHz.

5.3 Accuracy of Resolution of LSNA

The 2004 version of the LSNA software when operated with modulated RF signals calculates a modified modulation frequency $\omega_m$ (for a given user-specified resolution frequency $\Delta \omega$ which approximates the one requested by the user. It is then up to
the user to ensure that not only the proper RF carrier frequency but also proper modulation frequency are provided by the RF source. The LSNA is indeed intended to work with any periodically modulated (presently limited by its software and calibration to 10 MHz bandwidth) RF sources as long as these are phase locked to the 10 MHz reference signal. The question might arise for a new LSNA user to the degree of accuracy \( \frac{\Delta \omega}{\omega_m} \) one should actually program the modulation frequency \( \omega_m \) in this external RF source given the non-integer value requested by the LSNA software.

To investigate this issue we plot in Fig. 5.4 the amplitude and phase of \( Y_{m3-} \) as a function of the accuracy \( \Delta \omega_m \) (related to the number of digit programmed) of the modulation frequency used for various input power levels (-4 dBm to 10 dBm). The resolution \( \Delta \omega \) is 190 Hz, the targeted \( \frac{\omega_m}{2} \) is 500kHz, the value \( \frac{\omega_m}{2} \) requested by the LSNA is 499.916.07666 Hz. Note that the vectorial source generator (ESG 4438C) used has 0.1 Hz resolution for the RF carrier and 1Hz modulation resolution and the data below 0.1 Hz are most likely redundant. As shown in Fig. 5.4 the data begin to degrade above 100 Hz. The amplitude \( Y_{m3-} \) requires about \( \frac{\Delta \omega}{10} \) resolution to stabilize whereas the phase required about \( \delta \omega_m \). This analysis indicates that the required accuracy \( \Delta \omega_m \) for the modulation frequency \( \omega_m \) is on the order of a fraction of the resolution frequency \( \Delta \omega \) selected by the user or the LSNA.

Note also that to further ascertain the reproducibility of these measurements, we have superposed on Fig. 5.4 the data obtained from ten successive measurements performed ith large time intervals. Clearly the data measured are highly reproducible and the phase error is usually at max a fraction of a degree; larger error being naturally observed at low input power levels due to the noise floor and dynamic range limits of the LSNA.
Figure 5.2: Comparison of the magnitude and phase of $A_0$ and $B_0$ versus tone spacing $f_b - f_a$. 
Figure 5.3: Signals measured with the LSNA

Figure 5.4: Amplitude and phase of $Y_{m3+}$ ad $Y_{m3-}$ versus the accuracy $\Delta \omega_m$ of $\omega_m$
CHAPTER 6

PREDISTORTION LINEARIZATION OF MULTI-CARRIER POWER AMPLIFIERS

In this chapter a new vectorial digital predistortion linearization with $3^{rd}$ and $5^{th}$ order corrections is implemented to account for the difference in memory effects in the lower and upper side bands. The two band predistortion linearization can linearize independently each band of a 2-carrier WCDMA in a LDMOSFET RF amplifier providing up to 40 dBc ACPR.

6.1 FPGA Algorithm for $3^{rd}$ & $5^{th}$ Order Predistortion with Memory Effects

Having evidenced in the previous chapter the strong asymmetry between the lower and upper side bands generated by the non-linearities of the PA above 0.5 MHz, we will investigate now a novel predistortion algorithm which can independently linearize the lower and upper sidebands. We describe in this section the digital testbed and the linearization algorithm before presenting the preliminary results obtained.

6.1.1 Digital testbed (FPGA) used for RF predistortion

The digital testbed is based on an Altera Stratix FPGA-DSP development board S25. This development board includes two in-board A/D and D/A converters. The 4
A/D converters are all 12 bits and the 4 D/A converters are all 14 bits. The system clock (80 MHz) is distributed via synchronized and delayed clocks to the external A/D and D/A converters. Two two-stage analog boards were developed for the adaptation of the signals between the FPGA board and the IQ modulator. The first stage increases the voltage level of the signals coming from D/A converters and provides DC offset controls to manually adjust the LO leakage in the IQ modulator. The second stage analog board which includes additional controls for the gain and differential DC offset, changes the mode of the IQ signals from single to complementary outputs.
to drive the differential IQ modulator. This original testbed was developed by Dr. Chaillot while a visiting scientist in 2003 - 2004.

6.1.2 Linearization Algorithm

The new 2-band linearization algorithm implemented in the FPGA is shown in Fig. 6.2. This linearization algorithm calculates the 3rd and 5th order power of the I and Q signals. Four different Hilbert transforms implemented using 64 taps are used. By combining (adding or subtracting) the 3rd and 5th order power terms with their Hilbert transforms with the proper weights one can scale and phase shift these initially memoryless terms. Differential memory is implemented by using different phase shifts and amplitudes for the lower side-band terms (generated by the subtraction) and the upper side-band terms (generated by the addition). 16 coefficients corresponding to the 8 phases and amplitudes of $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \iota$ and $\theta$ are therefore required to control independently the lower and upper side bands for correcting both the 3rd and 5th non-linearities. The overall approach provides a frequency selective IMD cancellation.

6.1.3 Balanced Offset of I and Q

In addition to the manual LO offset controls available in the analog boards, several software blocks were implemented to control directly from the LabWindows environment, the DC offset and IQ imbalance of the IQ modulator. The DC offsets permit us to control the LO leakage in the IQ modulator where as the IQ correction described in the previous chapter provides the means to correct for the memory effects of the IQ modulator.
Figure 6.2: FPGA algorithm developed by Dr. Chaillot for 3rd & 5th order predistortion with memory effects

6.2 RF Predistortion Linearization Results

6.2.1 Linearization Results for 2-Carrier WCDMA signals

The performance of the linearization algorithm was investigated using wideband signals (multitones and WCDMA signals) either generated using a vectorial source generator (ESG4438C) or a look-up table (LUT) in the FPGA. Fig. 6.3 shows the linearization of a class AB LDMOSFET PA for a single carrier WCDMA signal using a memory-less predistortion linearization algorithm. As is seen in Fig. 6.3, the spectral regrowth on the lower and upper side bands are reduced simultaneously (for about 40dBc ACPR) but cannot be reduced separately. Fig. 6.4 shows now a two-carrier WCDMA signal. The vertical scale is 10 dB per division and the horizontal scale is
6MHz per division. Each WCDMA band has a 5 MHz bandwidth. The center of both band is separated by 15 MHz. As indicated in Fig. 6.4(a), each band is immediately flanked on both sides by spectral regrowth bands of about 5 MHz bandwidth each originating from the interaction of each band with itself \([a_1(\omega)\mid a_1(\omega)]^2\) and the adjacent band \([a_1(\omega)\mid a_1(\omega_m)]^2\). In addition both bands interfere together and generate two intermodulation bands at -22.5 MHz \([a_1(\omega)^2a_1(\omega + \omega_m)^2]\) and +22.5 MHz \([a_1(\omega)^2a_1(\omega + \omega_m)^2]\) relatively to the LO (center tone) which was not fully cancelled. The ability of the linearization algorithm to reduce the spectral regrowth on the lower side band only or the upper sideband only is demonstrated in Figs. 8.2 and 8.3. Note that not only the intermodulation bands at -22.5 or +22.5 MHz are reduced but also the self-spectral regrowth surrounding the original band themselves. An improved algorithm will be developed in the next chapter to differentially address these two types of spectral regrowth. Finally Fig. 6.4(d) shows the combined reduction of spectral regrowth on both sides of the LO. These preliminary results demonstrate the capability of this algorithm to linearize a PA exhibiting differential memory effects (above 1MHz). Note that the non-linearities which extend over 50 MHz bandwidth are reduced by using real time processing and without using a feedback loop.

### 6.2.2 Relationship between FPGA and Volterra Coefficients

The memory effects which are the object of studies in this chapter can also originate from the IQ modulator and the Hilbert transforms used in the linearization algorithm beside the power amplifier to be linearized. It is therefore necessary in practice to compensate as well for these effects in the linearization algorithm implemented in the FPGA.
Figure 6.3: Linearization without frequency selective correction (WCDMA signal)

6.3 Conclusion

In previous chapter, 3rd order non-linear system coefficients characterizing an LDMOSFET PA were extracted directly from LSNA measurements. These measurements revealed the presence of strong differential memory effects between the lower and upper sidebands above 0.5 MHz. In this chapter we presented a novel predistortion algorithm developed by Dr. Chaillot which accounts for the asymmetry between the lower and upper side bands generated by the non-linearities. That algorithm relies on 16 parameters to independently address the 3rd and 5th order distortion while accounting for the differential memory effects of the RF PA. Independent control of the lower and upper side band spectral regrowth was then demonstrated for 2-carrier WCDMA signals for an overall 40 dBc of ACPR. In the next chapter an improved algorithm capable of linearizing separately the in-band and out-of-band (IMD) will be investigated.
Figure 6.4: Linearization without frequency selective corrections (2 multi-carrier WCDMA signal).
CHAPTER 7

FREQUENCY SELECTIVE PREDISTORTION LINEARIZATION OF RF POWER AMPLIFIERS

This chapter presents a more advanced frequency selective RF vectorial predistortion linearization system for RF power amplifiers (PAs) with memory. The new vectorial digital predistortion algorithm presented enables the linearization of the lower and upper sidebands to be addressed separately.

The validity of the frequency-selective predistortion linearization theory is first demonstrated using MATLAB simulations. Volterra series are used for modeling RF PAs with memory effects. The excitation used for this theoretical validation in MATLAB is a multitone signal (multisine).

The algorithm is then implemented in a digital predistortion testbed using an EP2S60 digital signal processing (DSP) board. The performance of the predistortion algorithm for a class AB RF amplifier (LDMOS) are then examined for both multitone and two carrier-WCDMA signals. As shall be seen an adjacent channel leakage ratio (ACLR) of up to 45dBc is achieved experimentally for both the in-band and interband intermodulation distortions. A 5th order (six bands) algorithm in addition to the
original 3\textsuperscript{rd} order (four bands) algorithm is also investigated to further reduce the out-of-band and in-band intermodulation distortions.

7.1 Introduction

The most rigorous theory for including memory effects is the Volterra formalism [19] [22] [23]. In that formalism the system is described by Volterra kernels of various orders. For example, a single input and single output (SISO) system has a 3\textsuperscript{rd} order kernel of the form:

\[
y_{\text{out}}(t) = \int_{-\infty}^{\infty} h(\tau_1, \tau_2, \tau_3) x(t - \tau_1) x(t - \tau_2) x(t - \tau_3) d\tau_1 d\tau_2 d\tau_3
\]

A simpler picture emerges for Volterra series when we switch to the frequency domain. The non-linearities of the amplifier will be assumed initially to be well represented by a 3\textsuperscript{rd} order Volterra system. When a two tones excitation \(a_1(\omega_1)\) and \(a_1(\omega_2)\) of frequency \(\omega_1\) and \(\omega_2\) respectively is applied at port 1, the output \(b_2(\omega)\) at port 2 of the amplifier can be verified to be of the following form (see Fig. 7.1):

\[
b_{2,PA}(2\omega_1 - \omega_2) = Y_{M3m} a_1^2(\omega_1)a_1^*(\omega_2)
\]

\[
b_{2,PA}(\omega_1) = Y_{Mm} a_1(\omega_1) + Y_{M3mm} a_1(\omega_1)|a_1(\omega_1)|^2 + Y_{M3mp} a_1(\omega_1)|a_1(\omega_2)|^2
\]

\[
b_{2,PA}(\omega_2) = Y_{Mp} a_1(\omega_2) + Y_{M3pm} a_1(\omega_2)|a_1(\omega_1)|^2 + Y_{M3pp} a_1(\omega_2)|a_1(\omega_2)|^2
\]

\[
b_{2,PA}(2\omega_2 - \omega_1) = Y_{M3p} a_1^2(\omega_2)a_1^*(\omega_1)
\]

The output features the two desired tones plus two intermodulation tones at \(2\omega_1 - \omega_2\) and \(2\omega_2 - \omega_1\). The complex coefficients \(Y_{Mm}, Y_{Mp}, Y_{M3m}, Y_{M3p}, Y_{M3mm}, Y_{M3mp}, Y_{M3pm}, Y_{M3pp}\) which are calculated from Volterra series can be measured using a large signal network analyzer (LSNA). For example the \(Y_{M3m}\) and \(Y_{M3p}\) intermodulation
coefficients can be calculated from the incident and reflected waves acquired by the LSNA using:

\[
Y_{M3m} = \frac{b_2(\omega_1 - \omega_m)}{a_1^2(\omega_1) a_1^*(\omega_2)}
\]

\[
Y_{M3p} = \frac{b_2(\omega_2 + \omega_m)}{a_1^*(\omega_1) a_2^2(\omega_2)}
\]

with \( \omega_m = \omega_2 - \omega_1 \) Experimental results on the variation of these coefficients versus tone spacing were presented in Chapter 5. For convenience the difference between the amplitude and phase of the \( Y_{M3p} \) and \( Y_{M3m} \) coefficients are plotted again in Fig. 7.2a and b as a function of tone spacing \( \omega_m = \omega_2 - \omega_1 \) and for various input power levels \( |a_1(\omega_1)|^2 = |a_1(\omega_2)|^2 \) from -4 to 6 dBm. As is shown in Fig. 7.2, the difference in amplitude and phase increases rapidly with tone spacing above 0.5 MHz. We refer to this effect as differential memory. It results that quasi-memoryless (scalar) linearization techniques which implicitly assume that \( Y_{M3p} = Y_{M3m} \) will not be able to cancel both the lower and upper sides (here above 0.5 MHz [10]) when applied to
such a nonlinear PA exhibiting differential memory effects. We shall develop in the next section a new vectorial predistortion theory which can address differentially the linearization of the upper and low side bands.

Figure 7.2: The difference between $Y_{M3m}$ and $Y_{M3p}$ reveals a strong differential memory effects above 1MHz.

7.2 Vectorial Predistortion

Consider the vectorial predistortion system shown in Figure 7.3.

The ideal IQ modulator used in this predistortion system is shown in Fig. 7.4. The IQ modulator is assumed to have the following coefficients submitted at the I and Q baseband inputs:

$$
\alpha(t) = \alpha_0 + \mathcal{F}^{-1} \left\{ \alpha_3(\omega_m) \mathcal{F} \left\{ E^2(t) \right\} \right\}
$$

$$
\beta(t) = \beta_0 + \mathcal{F}^{-1} \left\{ \beta_3(\omega_m) \mathcal{F} \left\{ E^2(t) \right\} \right\}
$$
Figure 7.3: Ideal vectorial predistortion linearization

with $\alpha_0 = \beta_0 = 1$ and where $E_{\text{inter}}^2$ is the AC component of the envelope square given by

$$E_{\text{inter}}^2(t) = \text{Re}[a_E(\omega_m) \exp(j\omega_m t)]$$

$$a_E(\omega_m) = \frac{a_1(\omega_2)a_1^*(\omega_1)}{2}$$

with $\omega_m = \omega_2 - \omega_1$. Note that in Figure 7.3 we are using a DC block to temporarily remove the DC component of the envelope whose contribution will be discussed in the next section.

The in-band signal IMD0 at the output of the IQ modulator is then:

$$b_{2,M}(\omega_1) = a_0 a_{1,f}(\omega_1) - \beta_0 a_{1,Q}(\omega_1) = a_1(\omega_1) + ja_1(\omega_1) = a_1(\omega_1)(1 + j)$$

$$b_{2,M}(\omega_2) = a_0 a_{1,f}(\omega_2) - \beta_0 a_{1,Q}(\omega_2) = a_1(\omega_2) + ja_1(\omega_2) = a_1(\omega_2)(1 + j)$$

where we used $a_{1,f}(\omega) = a_1(\omega)$, $a_{1,Q}(\omega) = -ja_1(\omega)$.

Similarly the IMD3 at the output of the IQ modulator is:

$$b_{2,M}(2\omega_1 - \omega_2) = a_{1,f}(\omega_1)a_{3_f}(\omega_m) - a_{1,Q}(\omega_1)a_{3_Q}(\omega_m)$$
Figure 7.4: Ideal IQ modulator used in RF predistortion

\[
\begin{align*}
\alpha(t) x(t) - \beta(t) \hat{x}(t) & = a_1(\omega_1) \left[ a_{3, I}^*(\omega_m) + j a_{3, Q}^*(\omega_m) \right] \\
& = \left[ \alpha_3^*(\omega_m) + j \beta_3^*(\omega_m) \right] \frac{1}{2} a_1^2(\omega_1) a_1^*(\omega_2) \\
(7.1) \\
\end{align*}
\]

where we used: \( a_{3, I}(\omega_m) = \alpha_3(\omega_m) a_E(\omega_m) \) and \( a_{3, Q}(\omega_m) = \beta_3(\omega_m) a_E(\omega_m) \)

Next the IMD3 correction signals \( b_{2,M}(2\omega_1 - \omega_2) \) and \( b_{2,M}(2\omega_2 - \omega_1) \) generated by the IQ modulator and amplified by the amplifier are superpositioned at the output of the PA with the IMD3 terms generated by the PA so that they are hopefully cancelled:

\[
\begin{align*}
b_{2,PA}(2\omega_1 - \omega_2) & = Y_{M3m} b_{2, M}(\omega_1) b_{2, M}(\omega_2) + Y_{Mm} b_{2, M}(2\omega_1 - \omega_2) = 0 \quad (7.3) \\
b_{2,PA}(2\omega_2 - \omega_1) & = Y_{M3p} b_{2, M}(\omega_1) b_{2, M}(\omega_2) + Y_{Mp} b_{2, M}(2\omega_2 - \omega_1) = 0 \quad (7.4)
\end{align*}
\]
The following products need to be evaluated:

\[
b_{2,M}(\omega_1)b_{2,M}^*(\omega_2) = (1+j)^2 (1-j) a_1^2(\omega_1)a_1^*(\omega_2) = 2(1+j) a_1^2(\omega_1)a_1^*(\omega_2) \quad (7.5)
\]

\[
b_{2,M}^*(\omega_1)b_{2,M}^2(\omega_2) = (1+j)^2 (1-j) a_1^2(\omega_2)a_1^*(\omega_1) = 2(1+j) a_1^2(\omega_2)a_1^*(\omega_1) \quad (7.6)
\]

Substituting Equations 7.5, 7.6, 7.1 and 7.2 into Equations 7.3 and 7.4 we obtain the following systems of equations:

\[
\alpha_3 - j\beta_3 = -4(1-j)\frac{Y_{M3m}^*}{Y_{Mm}} = Z_1
\]

\[
\alpha_3 + j\beta_3 = -4(1+j)\frac{Y_{M3p}}{Y_{Mp}} = Z_2.
\]

This linear system is easily solved for \(\alpha_3\) and \(\beta_3\):

\[
\alpha_3 = \frac{1}{2}(Z_1 + Z_2) \quad \text{and} \quad \beta_3 = \frac{1}{2j}(Z_2 - Z_1)
\]

In general, the coefficients \(\alpha_3\) and \(\beta_3\) are complex numbers. This implies that the baseband envelope signal sent to the port 3I and 3Q of the IQ modulator must be first phase shifted. This is referred in this paper as vectorial predistortion (VPD) linearization.

For a quasi-memoryless PA the following identity holds:

\[
\frac{Y_{M3m}}{Y_{Mm}} = \frac{Y_{M3p}}{Y_{Mp}} = \frac{Y_{M3}}{Y_M} = \frac{3}{4} \frac{y_{M3}}{y_M}.
\]

since we have for a quasi-memoryless PA

\[
Y_{M3m} = Y_{M3p} = Y_{M3} = \frac{3}{4} y_{M3}
\]

\[
Y_{Mm} = Y_{Mp} = Y_M = y_M
\]

It results that we have \(Z_1 = Z_2^* = Z\) and the \(\alpha_3\) and \(\beta_3\) are simply given by:

\[
\alpha_3 = -3\text{Re} \left[ (1+j) \frac{y_{M3}}{y_M} \right]
\]

\[
\beta_3 = -3\text{Im} \left[ (1+j) \frac{y_{M3}}{y_M} \right]
\]
Note that $\alpha_3$ and $\beta_3$ are now both real. No phase shift is required and the conventional scalar predistortion (SPD) linearization scheme is sufficient for such quasi-memoryless PAs.

The derivation above for PA with differential memory assumed an ideal modulator. The theory can readily extended to also account for the differential memory contribution of the IQ modulator. The scheme presented is therefore not limited to ideal IQ modulator.

7.3 Real Time Implementation for BroadBand signals

The derivation given above was conducted for simplicity assuming a two-tone excitation. In practice the RF are modulated. Consider the case of a two-carrier system where $\omega_1$ and $\omega_2$ are the center frequency. Consider the case were the modulatated carriers can be represented by each using two tones shown in Figure 7.5. 4 tones (plain lines) are therefore injected at the input of the PA and 12 tones are observed at the output. This 4 tone excitation permits us to clearly distinguish the the interband and inband intermodulation distortions generated by the various $Y_{M_1}$, $Y_{M_p}$, $Y_{M3m}$, $Y_{M3p}$, $Y_{M3mm}$, $Y_{M3mp}$, $Y_{M3pm}$, and $Y_{M3pp}$ coefficients. In the previous derivation we focused on the $Y_{M3p}$ and $Y_{M3m}$ inter-band contributions. However clearly we need also to account for the interband intermodulation distortion. For this purpose we shall see that we need to have access to the $I(T)$ and $Q(t)$ signal representation instead of only the envelope $E^2 = I^2(t) + Q^2(t)$.

Further since our goal is also to implement this linearization scheme using digital predistortion, it is desirable to recast the vectorial predistortion theory in terms of $I(T)$ and $Q(t)$ signals.
Digital predistortion is implemented by directly predistorting the I and Q signals. For example in the digital implementation the IQ modulator is effectively replaced by a matrix multiplication of the form:

\[
\begin{bmatrix}
I' \\
Q
\end{bmatrix} = \begin{bmatrix}
\alpha & -\beta \\
\beta & \alpha
\end{bmatrix} \begin{bmatrix}
I \\
Q
\end{bmatrix}
\]

In the previous section we introduced a DC block in the calculation of the enveloped as VPD calls for the use phase of shifter. However the DC components contains important data needed for the inband intermodulation linearization. In this work we focus on differential memory effect and will effectively separate the incoming frequency spectrum in terms of the lower side band \((f < f_c)\) and the upper side band \((f > f_c)\).
Figure 7.6: Digital implementation of vectorial predistortion

To perform this two band filtering the easiest is to partition the input $I$ and $Q$ in terms of even and odd excitations as follow:

$$ I_{in}(t) = I_e(t) + I_o(t) $$

$$ Q_{in}(t) = Q_e(t) + Q_o(t) $$

$$ Q_e = \hat{I}_e $$

$$ Q_o = -\hat{I}_o $$

Note that we use the notations $\hat{I}(t) = \mathcal{H}(t)$ to denote the Hilbert of $I(t)$. One can readily verify that $I_e$, $I_o$, $\hat{I}_e$ and $\hat{I}_o$ is obtained and $I$ and $Q$ and their Hilbert using:

$$ I_o = \frac{I + \hat{Q}}{2}, \quad \hat{I}_o = \frac{\hat{I} - Q}{2}, \quad I_e = \frac{I - \hat{Q}}{2}, \quad \text{and} \quad \hat{I}_e = \frac{\hat{I} + Q}{2} $$

The pair $I_e$ and $Q_e$ generates the upper side band ($f > f_c$) and the pair $I_o$ and $Q_o$ generates the lower side band ($f < f_c$). We can evaluate the envelope in terms of this two-band IQ representation:

$$ E^2(t) = I_{in}^2(t) + Q_{in}^2(t) $$
\[ E_{\text{inter}}^2(t) + E_{\text{inband}}^2(t) \]

where \( E_{\text{inter}}^2 \) and \( E_{\text{inband}}^2 \) are interband and inband envelope components defined as:

\[ E_{\text{inter}}^2 = 2I_e I_o - 2\tilde{I}_e \tilde{I}_o \]
\[ E_{\text{inband}}^2 = E_e^2 + E_o^2 \]
\[ E_e^2 = I_e^2 + \tilde{I}_e^2 \]
\[ E_o^2 = I_o^2 + \tilde{I}_o^2 \]

For a two-tone excitation one can verify that \( E_{\text{inband}}^2 \) is the DC component and \( E_{\text{inter}}^2 \) the AC component of the envelope. When dealing with general multitone signals they define respectively the time-dependent envelopes needed to correct for the inband and interband intermodulation distortions as shown in Fig. 7.5.

The \( E_{\text{inband}}^2 \) term is used to first linearize \( I_e, Q_e \) and \( I_o \) and \( Q_o \). First we calculate the \( \alpha_A(t), \beta_A(t), \alpha_B(t) \) and \( \beta_B(t) \) coefficients.

\[ \alpha_A(t) = 1 - 2 \text{Re} \left[ \frac{Y_{M3mm}^{Y_{Mm}}}{Y_{Mm}} \right] E_o^2 - 2 \text{Im} \left[ \frac{Y_{M3mp}^{Y_{Mm}}}{Y_{Mm}} \right] E_e^2 \]
\[ \beta_A(t) = 0 - 2 \text{Re} \left[ \frac{Y_{M3mm}^{Y_{Mm}}}{Y_{Mm}} \right] E_o^2 - 2 \text{Im} \left[ \frac{Y_{M3mp}^{Y_{Mm}}}{Y_{Mm}} \right] E_e^2 \]
\[ \alpha_B(t) = 1 - 2 \text{Re} \left[ \frac{Y_{M3mm}^{Y_{Mp}}}{Y_{Mp}} \right] E_o^2 - 2 \text{Im} \left[ \frac{Y_{M3mp}^{Y_{Mp}}}{Y_{Mp}} \right] E_e^2 \]
\[ \beta_B(t) = 0 - 2 \text{Re} \left[ \frac{Y_{M3mm}^{Y_{Mp}}}{Y_{Mp}} \right] E_o^2 - 2 \text{Im} \left[ \frac{Y_{M3mp}^{Y_{Mp}}}{Y_{Mp}} \right] E_e^2 \]

and calculate the resulting \( I'_o, Q'_o, I'_e \) and \( Q'_e \) using the digital IQ modulator:

\[ I'_o = \alpha_A I_o - \beta_A Q_o \]
\[ Q'_o = \beta_A I_o + \alpha_A Q_o = -\tilde{I}'_o \]
\[ I'_e = \alpha_B I_e - \beta_B Q_e \]
\[ Q'_e = \beta_B I_e + \alpha_B Q_e = \tilde{I}'_e \]
The output of the inband predistortion linearization is then obtained by reconstituting $I$ and $Q$:

$$I_{in}^{'} = I_c^{'} + I_o^{'}$$
$$Q_{in}^{'} = Q_c^{'} + Q_o^{'}$$

This inband algorithm is schematically represented in Fig. 7.6.

Next we need to calculate the interband intermodulation correction. In our Volterra theory this correction is to be added to the inband intermodulation correction we have just calculated. However we shall demonstrate below in simulations that it is preferable to proceed with the interband intermodulation correction sequentially using $I_{in}^{'}$ and $Q_{in}^{'}$ instead of $I_{in}$ and $Q_{in}$ since they have already been updated by the inband correction.

The modulator is fed the cross product terms $E_{inter}^2$ of the signal envelope.

$$\alpha_{inter}(t) = 1 + \text{Re}(\alpha_3)E_{inter}^2 - \text{Im}(\alpha_3)\tilde{E}_{inter}^2$$
$$\beta_{inter}(t) = 1 + \text{Re}(\beta_3)E_{inter}^2 - \text{Im}(\beta_3)\tilde{E}_{inter}^2$$

where according to the dicussion above we define:

$$E_{inter}^2(t) = 2\hat{I}_c\hat{I}_o - 2\hat{I}_c\hat{I}_o$$

The IQ output is then obtained from the digital modulator:

$$I_{out} = \alpha_{inter}I_{in}^{'} - \beta_{inter}Q_{in}^{'}$$
$$Q_{out} = \alpha_{inter}Q_{in}^{'} + \beta_{inter}I_{in}^{'}$$

The calculation of $\alpha_{inter}$ and $\beta_{inter}$ requires the availability of the Hilbert of $E_{inter}^2$ to perform the phase shift associated with $\alpha_3$ and $\beta_3$ (see previous section). One can
verify that the following algebraic formula provides an exact evaluation of the Hilbert for multitone baseband excitations:

\[
\hat{E}_{\text{inter}}^2 = \mathcal{H}(E_{\text{inter}}^2) = 2I_e\hat{I}_o + 2\hat{I}_cI_o
\]

Note that the calculation of Hilbert transforms is costly and the proposed 3\(^{\text{rd}}\) order algorithm presented above and 5\(^{\text{th}}\) order extension presented below only requires the Hilbert transforms of \(I_{\text{in}}\) and \(Q_{\text{in}}\) since the remaining Hilbert transforms needed are all calculated from simple algebraic expressions.

![Figure 7.7](image_url)

Figure 7.7: Comparison of vectorial predistortion linearization in RF power amplifier with Memory Effects.
To verify our vectorial predistortion algorithm in the time domain, a Volterra model of RF PAs was implemented in MATLAB. The upper sideband intermodulation of the RF power amplifier model was phase-shifted in the Volterra Kernels to implement differential memory effects. In Fig. 7.8, the results represented by the symbol ◦ symbol correspond to the case where interband corrections ($Y_{M3m}$ and $Y_{M3p}$) are applied sequentially (serial connection) after the inband corrections ($Y_{M3mm}$, $Y_{M3mp}$, $Y_{M3pm}$, $Y_{M3pp}$). The results represented by the × symbol correspond to the case where the inband and interband linearizations are performed in parallel. The results represented by the + symbol correspond to in-band linearization only and ▽ symbol to interband linearization only. In this approach the serial connection of the inband linearization and the interband linearization is more effective to linearize the Volterra model of a RF PA with differential memory effect. Note that in Fig. 7.7, interband intermodulation was linearized up to 130 dBC and inband distortion up to 70 dBC ACPR.

### 7.4 Architecture for Frequency Selective Predistortion Algorithm in FPGA Board

The new 2-band linearization algorithm implemented in the FPGA is shown in Fig. 7.8. This linearization algorithm calculates the 3rd and 5th order power of the I and Q signals. I Q signal are modulated linearly by IQ modulator.

The digital testbed is based on an Altera DSP board (EP2S60). This development board includes two in-board A/D and D/A converters. The 2 A/D converters are all 12 bits and the 2 D/A converters are all 14 bits. The system clock (100 MHz) is distributed via synchronized and delayed clocks to the A/D and D/A converters in DSP board. The two-stage analog boards were developed for the adaptation of
the signals between the DSP board and the IQ modulator. The first stage increases
the voltage level of the signals coming from D/A converters and provides DC offset
controls to manually cancel the LO leakage in the IQ modulator. The second stage
analog-board which includes additional controls for the gain and differential DC offset,
provide for the IQ signals (1) single-ended outputs to drive the IQ modulator of
the ESG4438C RF source and (2) complementary outputs to drive a differential IQ
modulator (Analog Device). The ESG4438C IQ modulator was found to have better
performance than the complementary IQ modulator in terms of the balancing and
LO leakage. The linearization results presented in this chapter were obtained using
the ESG 4438C modulator. When using the IQ modulator of the digitally modulated
RF source, the maximal voltage is limited to 1 Vpp at the inputs of the IQ modulator.
To protect the IQ modulator of the RF source a voltage limiter was used in the I and
Q channels.
Figure 7.9: Class AB RF power amplifier at frequency 895 MHz

Before linearizing the RF PA, it necessary to balance the IQ modulator. This is performed by searching with the Labwindows control panel (PC) for the optimal coefficients compensating for imbalance in gain and phase in the inphase and quadrature LO paths. Those coefficients are dependent on the modulation frequency. In our work 3 bands are used which correspond to the 1\textsuperscript{st}, 3\textsuperscript{rd} and 5\textsuperscript{th} linear IMD corrections considered below. Due to the frequency dependence of the IQ modulator used (ESG4438C), the imbalance was analyzed using multitone signals in these 3 frequency bands. A 5 tone signals were generated from an LUT with 5MHz bandwidth. Three frequency bands corresponding to 5 MHz, 10 MHz and 15 MHz offsets
from the LO were selected. The optimal balancing obtained in each band is 45 dBc, 
35 dBc and 20 dBc respectively.

Differential memory effect is implemented by using phase shift of $E^2_c$, $E^2_o$ for inband 
linearization, and of $E^2_{inter}$ and $\mathcal{H}(E^2_{inter})$ for interband linearization as shown in 
Eqs. (7.7) ~ (7.8).

$$E^2_i = E^2_{inband,i} + E^2_{inter,i}$$

$$E_{inband,i} = E_{e,i} + E_{o,i} = I^2_{e,i} + \hat{I}^2_{e,i} + I^2_{o,i} + \hat{I}^2_{o,i}$$

$$E_{inter,i} = I_{e,i}I_{o,i} + Q_{e,i}Q_{o,i} = I_{e,i}\hat{I}_{e,i} - I_{o,i}\hat{I}_{o,i}$$

$$E_{inter,i+1} = I_{e,i+1}I_{o,i+1} + Q_{e,i+1}Q_{o,i+1}$$

$$\hat{E}_{inter,i+1} = -I_{e,i+1}Q_{o,i+1} + Q_{e,i+1}I_{o,i+1} = I_{e,i+1}\hat{I}_{o,i+1} + I_{e,i+1}I_{o,i+1}$$

$$E_{inband,i+1} = E_{e,i+1} + E_{o,i+1}$$

$$I_{e,i+1} = I_{e,i} + \alpha_B \cdot I_{e,i} - \beta_B \cdot \hat{I}_e$$

$$\hat{I}_{e,i+1} = \hat{I}_{e,i} + \beta_B \cdot I_{e,i} + \alpha_B \cdot \hat{I}_e$$

$$I_{o,i+1} = I_{o,i} + \alpha_A \cdot I_{o,i} + \beta_A \cdot \hat{I}_o$$

$$\hat{I}_{o,i+1} = \hat{I}_{o,i} - \beta_A \cdot I_{o,i} + \alpha_A \cdot \hat{I}_o$$

$$E_{inter,i+1} = I_{e,i+1}I_{o,i+1} + Q_{e,i+1}Q_{o,i+1}$$

$$(I_{e,i} + \Delta I_{e,i})(I_{o,i} + \Delta I_{o,i}) + (Q_{e,i} + \Delta Q_{e,i})(Q_{o,i} + \Delta Q_{o,i})$$

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\[
E_{\text{inter},i+1} = \mathcal{H}(I_{\text{e},i}I_{\text{o},i}) + I_{\text{e},i}I_{\text{o},i} + \Delta I_{\text{e},i}I_{\text{o},i} + \mathcal{H}(\Delta I_{\text{e},i}\Delta I_{\text{o},i}) \\
- \mathcal{H}(\hat{I}_{\text{e},i}\hat{I}_{\text{o},i}) + \hat{I}_{\text{e},i}\hat{I}_{\text{o},i} + \Delta I_{\text{e},i}\hat{I}_{\text{o},i} - \mathcal{H}(\Delta I_{\text{e},i}\hat{I}_{\text{o},i})
\]

The I and Q baseband signals are divided into two paths such as even and odd regions using a phase shift which are implemented in FPGA with the help of a Hilbert transform. Then the target regions are addressed individually to be linearized with Volterra corrections (inband and interband 3rd and 5th order linearization coefficients). The phase shifting \( E_e \) and \( E_o \) with inband linearization coefficients enables the inband distortion (3rd and 5th) to be linearized. The phase shift of \( E_{\text{inter}} \) and \( \mathcal{H}(E_{\text{inter}}) \), which are 2nd itself, can address 3rd intermodulations in both upper and lower side band separately. The envelope signal for interband, \( E_{\text{inter}} \) and \( \mathcal{H}(E_{\text{inter}}) \) do not have DC terms so that those can be phase shifted in the fixed IQ domain (complex domain). RF terms (1st order) are summed and subtracted signals (I-Q and I+Q) in order to indicate the even and odd mode regions in the frequency domain. Applying \( E_o \) and \( E_e \) to itself for inband higher order and \( E_{\text{inter}} \) and \( \mathcal{H}(E_{\text{inter}}) \) to itself iteratively for interband enables higher order nonlinearities (5th and 7th) of RF PAs to be linearized as shown in Fig. 7.10. But this approach is limited by the RF accuracy of the D/A (14bits) converters and the digital dynamic range of the DSP board.

### 7.5 Analysis of Hilbert Transformation in Frequency Domain and DC Distortion in DSP Board

The two-band linearization algorithm uses two Hilbert transformations to implement the phase shifting of the I and Q signals. The Hilbert transformation, in its
Figure 7.10: FPGA algorithm for inband and interband 3rd & 5th order predistortion with memory effects

Figure 7.11: Separate addressing of even and odd mode (inband and interband) in the frequency domain

digital implementation, features a number of limitation in its amplitude performance as is shown in the frequency domain in Fig. 7.12. The amplitude errors, resulting from the digital Hilbert transformation are frequency dependent and depend on the number of taps (16 ∼ 256). The ripple in the amplitude response can be reduced by using an Hamming window. There is no phase errors in Hilbert transformation. Fig. 7.12 shows that of 64 and 128 taps Hilbert transformation can be implemented up to 10 MHz for a 80 MHz clock. The algorithm installed in FPGA was implemented
with 64 taps and with an Hamming window. Note that the EP1S25 DSP board we originally used had a strong DC/low-frequency distortion problem and the nonideal Hilbert transformation degraded it even further. The distortions originating from the EP1S25 were found to be considerably reduced when we switched to the EP2S60 FPGA board.

![Graph showing frequency dependent amplitude deviation of Hilbert Transformation](image)

**Figure 7.12: Frequency dependent amplitude deviation of Hilbert Transformation**

### 7.6 RF Predistortion Linearization Results

The performance of the linearization algorithm was investigated using wideband signals (multitones and WCDMA signals) either generated using a vectorial source generator (ESG4438C) or directly generated from a look-up table (LUT) in the FPGA. Fig. 7.13 shows that 6 band digital linearization is working with 5 tones signal, generated by a LUT in the FPGA. Each sideband has inband and interband up to 5th order. Before linearization, sufficient accurate IQ modulator balancing was
obtained to perform 5th order interband linearization. The IQ balancing accomplished were 50dBc, 45dBc and 35dBc at the fundamental, 3rd and 5th interband frequencies respectively. 3rd linearization corrections induce some 5th order distortion while linearizing 3rd interband. So 5th order linearization are needed to reduce the 5th order interband distortion.

Fig. 7.14 and 7.15 shows next results for a two-carrier WCDMA signal. The vertical scale is 10 dB per division and the horizontal scale is 6MHz per division. Each WCDMA band has a 5MHz bandwidth. The center of both band is separated by 15MHz. As indicated in Fig. 7.15 (1) each band is immediately flanked on both sides by spectral regrowth bands of about 5 MHz bandwidth each originating from the interaction of each band with itself \([a_1(\omega)] \, a_1(\omega) |^2\) and the adjacent band \([a_1(\omega)] \, a_1(\omega + \omega_m) |^2\). In addition both bands interfere together and generate two intermodulation bands at \(-22.5MHz \, [a_1(\omega)2a_1(\omega + \omega_m)]\) and \(+22.5MHz \, [a_1(\omega)^*a_1(\omega + \omega_m)^2]\) relatively to the LO (center tone) which was not fully cancelled.

The ability of the frequency selective predistortion linearization algorithm (four band regions) to reduce the spectral regrowth on the lower sideband only or the upper side band only is demonstrated in Figs. 7.15(2)(3). Note that not only the intermodulation bands at \(-22.5\) or \(+22.5MHz\) are reduced but also the self-spectral regrowth surrounding the original band themselves. Our improved algorithm differentially address these two types (inband and interband) of spectral regrowth. All the linearization coefficients are complex number to address the different amplitude and phase of intermodulation resulting from broadband memory effects.

Finally Fig. 7.15(4) shows the combined reduction of the spectral regrowth on both sides of the LO. These results demonstrate the capability of this algorithm to
linearize a PA exhibiting differential memory effects (above 1 MHz). Note that the non-linearities which extend over 50 MHz bandwidth are reduced using only real time processing and without using a feedback or adaptive loop.

Figure 7.13: 6 band linearization (in and interband) with 5 tone signal

7.7 Conclusion

In this chapter, 3rd order non-linear system coefficients characterizing an LDMOS-FET PA were extracted directly from LSNA measurements. These measurements revealed the presence of strong differential memory effects between the lower and
Figure 7.14: 4 band linearization (in and interband) with 2 carrier WCDMA signal

upper sidebands above 0.5 MHz. We proposed a novel predistortion algorithm which accounts for the asymmetry between the lower and upper side bands generated by the non-linearities. That algorithm relies on 6 parameters to independently address the 3rd order distortion while accounting for the differential memory effects of the RF PA. Independent control of the lower and upper side band spectral regrowth was then demonstrated for 2-carrier WCDMA signals for an overall 45 dBC of ACPR.

The extension from 2-carrier to multi-carrier power amplifiers could then proceed by further dividing the bandwidth in additional bands. The PA system identification
Figure 7.15: Linearization with frequency selective corrections (2 carrier WCDMA signal)

with the large-signal network analyzer (LSNA) measurement should facilitate the development of multi-carrier linearization by providing the needed multi-tone Volterra coefficients. An extension of the LSNA modulation bandwidth would be, however, greatly desirable.
CHAPTER 8

CONCLUSION AND FUTURE WORK

8.1 Conclusion

In this thesis we have focused on the development of predistortion linearization to reduce interferences in multicarrier PAs. Contributions in several areas were made in the process and are reviewed below.

A first group of contributions was concerned with low-frequency feedforward. We observed theoretically and experimentally the presence of memory effects in input (IBM) and output (OBM) baseband modulation linearization. We then proposed new bilateral (BBL) and quadratic (QBL) baseband linearization schemes which account for memory effects in the PA.

Our second group of contributions was related to the modeling and balancing of the IQ modulator. First we presented a triggered LSNA measurement setup providing the means to correlate together the amplitude and phase of the baseband and RF signals in multi-port communication systems. Next using this triggered LSNA setup we extracted a linear K-model for modeling the IQ imbalance of the IQ modulator used in our predistortion testbed. Finally we used this linear K-model to reduce the IQ imbalance to 43 and 46 dBC isolation for the LSB and USB respectively.
Our third group of contributions was related to the characterization of memory effects in the LDMOSFET PA under test. First, we measured the generalized Volterra coefficients for this PA. We observed a strong differential memory effect between the lower and upper sidebands above 0.3 MHz. We also established the measurement conditions for obtaining reliable and reproducible vectorial IMD3 measurements with the LSNA.

Our fourth group contribution was concerned with the linearization of PAs with memory using predistortion linearization. We developed a new vectorial predistortion linearization theory. An FPGA testbed was developed to test this theory. We then demonstrated experimentally the independent cancellation of the lower and upper side-band in-band and interband spectral regrowths for a 2-carrier WCDMA signal.

8.2 Combination of digital predistortion and bilateral baseband linearization

As was discussed in the introduction, current digital adaptive memoryless predistortion schemes need to be upgraded to efficiently linearize the wider bandwidth signals of 3G standards (WCDMA). One such approach was presented in this work for two-carrier WCDMA and could readily be extended for multi-carrier WCDMA larger than 2.

Another option is to augment predistortion with a digital implementation of low frequency feedforward (LFFF) linearization: BBM. Fig. 8.1 shows a possible joint implementation of BBM and baseband digital predistortion scheme for a class AB amplifier in a mobile. Four DACs are required in the full implementation. An adaptive branch can be also implemented for tracking slow thermal memory effects. Alternative approaches based are explored in the next section.
8.3 Development of Linearization Algorithms with Memory

As studied in the previous chapters by using of an IQ linearization topology (VPD, QOBL or QIBL) and by controlling independently the phase and amplitude on the I and Q signal themselves, it is possible to linearize a two-tone excitation signal in a 3rd order system exhibiting memory effect if $y_{md2}$ is negligible.

The nonlinear filter shown in Figure 8.2 is proposed as a possible scheme for introducing frequency-dependent memory effects in the linearization algorithm for each inband and interband intermodulation components. The time averaging elements are used to implement different linearization corrections depending on the time scale of the variation of the envelope $E(t)$. Note that the non-linear functions $\alpha(A) = \alpha_3 E + \alpha_5 E^2$ and $\beta(B) = \beta_3 E + \beta_5 E^2$ could be initially obtained from those established for a memoryless linearization scheme. This would greatly reduce the complexity of the extraction procedure.

Two approaches are possible. The function $\alpha(A)$ and $\beta(A)$ and the weights $s_i$ and $r_i$ could be either obtained by modeling or by empirically means.
The modeling approach requires the development of a behavioral model for the trajectory of I and Q at the PA output and the subsequent inversion of this model. Such practice is common for extracting a model using AM-PM data.

An alternative approach would be to search for the optimal coefficients empirically. A genetic algorithm could be for example used to efficiently find the optimal set of coefficients required. The LSNA could be used to record both the EVM and ACPR for multitone excitations.

![Diagram of vectorial IQ linearization](image)

Figure 8.2: Proposed vectorial IQ linearization for systems with memory.
APPENDIX A

RF PREDISTORTION AND DIGITAL BASEBAND PREDISTORTION LINEARIZATION

A.1 Experimental Results on LFFF

The lower side band (LSB) and upper side band (USB) IMD3 measured experimentally on a LDMOSFET amplifier with the low frequency signal injected at the input is shown in Figure A.1. These measured data were acquired by Mr. Cui using the analog testbed shown in Appendix A.2. As predicted by the theory both LSB and USB cannot be cancelled for the same LFFF phase and gain.
Figure A.1: LSB and USB IMD3 for IBL in a LDMOSFET amplifier as a function of the LFFF phase for various LFFF gain.
Figure A.2: Analog Testbed (left) designed by Mr. Cui with Power Amplifier (right) designed by Mr. Dai.
A.2 OBM with IQ modulator

We can easily verify that the OBM method works also with an IQ modulator. Let us consider first a 3rd order system. The RF signal is now given by

\[ v_{in}(t) = I(t) \cos(\omega t) + Q(t) \sin(\omega t). \]

The 3rd order intermodulation terms generated by \( g_{3m} \) is given by, keeping only in the inband terms:

\[
\begin{align*}
g_{3m}v_{in}^3 &= g_{3m} (I \cos(\omega t) + Q \sin(\omega t))^3 \\
&= g_{3m} \left( \frac{3}{4} I^3 + \frac{3}{4} I^2Q + \frac{3}{4} IQ^2 + \frac{3}{4} Q^3 \right) + \cdots \quad (A.1) \\
&= g_{3m} \left( \frac{3}{4} I^3 + \frac{3}{4} Q^3 \right) + \cdots \\
&= g_{3m} \frac{3}{4} I^3 + \cdots \quad (A.2) \\
&= g_{3m} \frac{3}{4} v_{in} [I^2 + Q^2] + \cdots \quad (A.3)
\end{align*}
\]

The linearization voltage required to cancel the inband 3rd order intermodulation terms generated by \( g_{3m} \) is then given by

\[
g_{md} v_{in} v_{LFF} = -\frac{3}{4} g_{m3} v_{in} (I^2 + Q^2) \quad (A.5)
\]

or simply:

\[
v_{LFF} = -\frac{3}{4} \frac{g_{m3}}{g_{md}} v_{in} (I^2 + Q^2)
\]

The validity of these equations has been verified using MATLAB simulations.
A.3 Class AB amplifier with loadpull design

Figure A.3: LoadPull Amplifier Design
Figure A.4: (a) Gain, (b) Power-Added Efficiency

Figure A.5: (a) P1dB, (b) P1 dB and Gain
A.4 Volterra kernels for BBM (IMD3 and In-band)

We present here the Volterra Kernels derived for BBM (Bilateral Baseband Linearization).

Two-tone Signal terms

\[
E_0 = P_{21}^{(1)}(f_b)
\]
\[
E_1 = \frac{1}{4} P_{21}^{(2)}(f_a, f_c) + P_{21}^{(2)}(f_c, f_a)
\]
\[
E_2 = \frac{1}{4} P_{21}^{(2)}(f_a, f_c) + P_{22}^{(2)}(f_c, f_a)
\]
\[
E_3 = \frac{1}{8} P_{211}^{(3)}(f_b, f_b, -f_b) + P_{211}^{(3)}(f_b, -f_b, f_b) + P_{211}^{(3)}(-f_b, f_b, f_b)
\]
\[
E_4 = \frac{1}{8} P_{211}^{(3)}(f_b, f_a, -f_a) + P_{211}^{(3)}(f_b, -f_a, f_a) + P_{211}^{(3)}(-f_a, f_a, f_b)
+ P_{211}^{(3)}(-f_a, f_b, f_a) + P_{211}^{(3)}(f_a, f_a, -f_b) + P_{211}^{(3)}(f_b, -f_b, f_b)
\]
\[
E_5 = \frac{1}{8} P_{211}^{(3)}(f_b, f_c, f_c) + P_{211}^{(3)}(f_b, f_c, -f_c) + P_{211}^{(3)}(-f_c, f_b, f_c)
+ P_{211}^{(3)}(f_c, -f_c, f_b) + P_{211}^{(3)}(-f_c, f_c, f_b)
\]
\[
E_6 = \frac{1}{8} P_{212}^{(3)}(f_b, f_c, f_c) + P_{212}^{(3)}(-f_c, f_b, f_c) + P_{212}^{(3)}(f_b, f_c, -f_c)
+ P_{212}^{(3)}(-f_c, f_b, f_c) + P_{212}^{(3)}(f_b, f_c, f_c)
\]
\[
E_7 = \frac{1}{8} P_{212}^{(3)}(f_b, f_c, f_c) + P_{212}^{(3)}(f_b, f_c, f_c) + P_{212}^{(3)}(-f_c, f_b, f_c)
+ P_{212}^{(3)}(f_b, -f_c, f_c) + P_{212}^{(3)}(-f_c, f_b, f_c)
\]
\[
E_8 = \frac{1}{8} P_{212}^{(3)}(f_b, f_c, f_c) + P_{212}^{(3)}(f_b, f_c, f_c) + P_{212}^{(3)}(-f_c, f_b, f_c)
+ P_{212}^{(3)}(f_b, -f_c, f_c) + P_{212}^{(3)}(-f_c, f_b, f_c)
\]

\[
D_0 = P_{21}^{(1)}(f_a)
\]
\[
D_1 = \frac{1}{4} P_{21}^{(2)}(-f_c, f_b) + P_{21}^{(2)}(-f_c, f_b)
\]
\[
D_2 = \frac{1}{4} P_{21}^{(2)}(f_b, -f_c) + P_{22}^{(2)}(-f_c, f_b)
\]
\[
D_3 = \frac{1}{8} P_{211}^{(3)}(f_a, f_a, -f_a) + P_{211}^{(3)}(f_a, -f_a, f_a) + P_{211}^{(3)}(-f_a, f_a, f_a)
\]
\[
D_4 = \frac{1}{8} P_{211}^{(3)}(f_b, f_b, -f_b) + P_{211}^{(3)}(f_b, -f_b, f_b) + P_{211}^{(3)}(-f_b, f_b, f_b)
+ P_{211}^{(3)}(-f_b, f_b, f_b) + P_{211}^{(3)}(f_b, -f_b, f_b)
\]
\[
D_5 = \frac{1}{8} P_{211}^{(3)}(f_a, f_c, f_c) + P_{211}^{(3)}(f_a, f_c, -f_c) + P_{211}^{(3)}(-f_c, f_a, f_c)
+ P_{211}^{(3)}(-f_c, f_a, f_c) + P_{211}^{(3)}(f_a, f_c, f_c)
\]
\[
D_6 = \frac{1}{8} P_{212}^{(3)}(f_a, -f_c, f_c) + P_{212}^{(3)}(f_a, f_c, -f_c) + P_{212}^{(3)}(f_a, f_c, f_c)
+ P_{212}^{(3)}(-f_c, f_a, f_c) + P_{212}^{(3)}(f_a, f_c, -f_c)
\]
\[
D_7 = \frac{1}{8} P_{212}^{(3)}(f_a, -f_c, f_c) + P_{212}^{(3)}(f_a, f_c, -f_c) + P_{212}^{(3)}(f_a, f_c, f_c)
+ P_{212}^{(3)}(-f_c, f_a, f_c) + P_{212}^{(3)}(f_a, f_c, -f_c)
\]
\[
D_8 = \frac{1}{8} P_{212}^{(3)}(f_a, -f_c, f_c) + P_{212}^{(3)}(f_a, f_c, -f_c) + P_{212}^{(3)}(f_a, f_c, f_c)
+ P_{212}^{(3)}(-f_c, f_a, f_c) + P_{212}^{(3)}(f_a, f_c, -f_c)
\]

(A.6)
IMD3 terms

\[ A_0 = \frac{1}{8} [P^{(3)}_{2111}(f_a, f_a, -f_b) + P^{(3)}_{2111}(f_b, -f_b, f_a) + P^{(3)}_{2111}(-f_b, f_a, f_a)] \]
\[ A_1 = \frac{1}{4} [P^{(2)}_{211}(f_a, -f_c) + P^{(2)}_{2111}(-f_c, f_a)] \]
\[ A_2 = \frac{1}{4} [P^{(2)}_{222}(f_a, -f_c) + P^{(2)}_{222}(f_c, f_a)] \]
\[ A_3 = \frac{1}{8} [P^{(3)}_{2111}(f_b, -f_c, -f_c) + P^{(3)}_{2111}(-f_c, f_b, -f_c) + P^{(3)}_{2111}(-f_c, -f_c, f_b)] \]
\[ A_4 = \frac{1}{8} [P^{(5)}_{2111}(f_b, -f_c, -f_c) + P^{(3)}_{2111}(-f_c, f_b, -f_c) + P^{(3)}_{2211}(-f_c, -f_c, f_b)] \]
\[ A_5 = \frac{1}{8} [P^{(3)}_{2111}(f_b, -f_c, -f_c) + P^{(3)}_{2111}(-f_c, f_b, -f_c) + P^{(3)}_{2112}(f_b, -f_c, -f_c) + P^{(3)}_{2111}(-f_c, -f_c, f_b)] \]

(A.7)

A.5 Derivation of the linearization conditions for OBM

In Chapter 3 we have introduced a system of two biquadratic equations for BBM.

This system of equation applies also to the case of OBM if we set \( c_1 = 0 \). For the LSB we obtain the following system:

\[ A_0 a_1 b_1^* + A_2 a_1 c_2^* + A_4 b_1 c_2^* = 0 \]  

(A.8)

which admits the following solution:

\[ c_2 = -\frac{A_2}{A_1} a_1^* b_1 \text{ for } A_4 = 0 \]

(A.9)

\[ c_2 = \left[ \frac{A_2}{2 A_1^*} \pm \left( \frac{1}{4} \left( \frac{A_2}{A_1^*} \right)^2 - \frac{A_0}{A_4} |b_1|^2 \right)^{1/2} \right] \frac{a_1^*}{b_1^*} \text{ for } A_4 \neq 0 \]  

(A.10)

For the USB we obtain the following system:

\[ B_0 a_1 b_1^* + B_2 b_1 c_2 + B_4 a_1 c_2^* = 0 \]  

(A.11)

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which admits the following solution:

\[
c_2 = -\frac{B_2}{B_1} a_1^* b_1 \quad \text{for } B_4 = 0 \tag{A.12}
\]

\[
c_2 = \frac{B_2}{2B_4} \left[ \left( \frac{1}{4} \left( \frac{B_2}{B_4} \right)^2 - \frac{B_0}{B_4} |a_1|^2 \right)^{1/2} \right] \frac{b_1}{a_1} \quad \text{for } B_4 \neq 0 \tag{A.13}
\]

We see that for OBM a joint simultaneous cancellation of the USB and LSB IMD3 requires that we have:

\[
\frac{A_2}{A_0} = \frac{B_2}{B_0} \quad \text{for } A_4 = B_4 = 0 \tag{A.14}
\]

\[|a_1| = |b_1| \quad \text{and} \quad \frac{A_2}{A_4^*} = \frac{B_2}{B_4} \quad \text{and} \quad \frac{A_0^*}{A_4} = \frac{B_0}{B_4} \quad \text{for } A_4 \neq 0 \quad \text{and} \quad B_4 \neq 0 (A.15)
\]

In the first case \( A_4 = B_4 = y_{md2} = 0 \) this is indeed satisfied if we have (quasimemoryless assumption) \( A_0 = B_0 = y_{m3} \) and \( A_2 = B_2 = y_{md} \) verifying \( y_{m3} - y_{md} = n\pi \). This is exactly the condition which was previously derived by Yu et al [16] (see Chapter 2). In the second case where \( A_4 \) and \( B_4 \) are non-zero we further need to have \( A_4 = B_4 = y_{md2} \) verifying \( y_{m3} - y_{md2} = n\pi \). Also the linearization is only possible if \( a_1 \) and \( b_1 \) have the same amplitude which is an unrealistic requirement.

A.6 Volterra Analysis of QOBM for Systems with Memory

The analysis of the quadratic OBM (QOBM) topology proceeds along the same line as that of predistortion but is slightly simpler as the postdistortion takes place in the amplifier itself. The topology of QOBM is shown in Fig. 3.6. The IMD3 terms at the output of the amplifier are given by (we use a 3rd Volterra system where we neglect \( y_{md2} \)):

\[
y_A(2\omega_a - \omega_b) = A_0 a_T^2 b_T^* + A_2 a_T c_T^* \tag{A.16}
\]

\[+ A_0 a_Q^2 b_Q^* + A_2 a_Q c_Q^* \tag{A.17}
\]

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\[ y_A(2\omega_b - \omega_a) = B_0 a_I^* b_I^2 + B_2 b_I c_I \]  
\[ + B_0 a_Q^* b_Q^2 + B_2 b_Q c_Q \]  
\[(A.18)\]

Using \( a_I = a_1 \), \( a_Q = j a_1 \), \( b_I = b_1 \) and \( b_Q = j b_1 \) this system of equations can be rewritten:

\[ y_A(2\omega_a - \omega_b) = A_0 a_1^2 b_1^*(1 + j) + A_2 a_1 (c_I^* + j c_Q^*) = 0 \]  
\[(A.20)\]

\[ y_A(2\omega_b - \omega_a) = B_0 a_1^* b_1^2 (1 + j) + B_2 b_1 (c_I + j c_Q) = 0 \]  
\[(A.21)\]

It results that the solution is

\[ c_I - j c_Q = -(1 - j) \frac{A_0^*}{A_2} a_1^* b_1 \]  
\[(A.22)\]

\[ c_I + j c_Q = -(1 + j) \frac{B_0}{B_2} a_1^* b_1 \]  
\[(A.23)\]

Like for predistortion let us define the coefficients \( \alpha_3 \) and \( \beta_3 \)

\[ c_I = \alpha_3 \frac{b_1 a_1^*}{2} \quad \text{and} \quad c_Q = \beta_3 \frac{b_1 a_1^*}{2} \]

It results that we can rewrite Equations A.22 and A.23 as:

\[ \alpha_3 - j \beta_3 = -2(1 - j) \frac{A_0^*}{A_2} = Z_1 \]  
\[(A.24)\]

\[ \alpha_3 + j \beta_3 = -2(1 + j) \frac{B_0}{B_2} = Z_2. \]  
\[(A.25)\]

This linear system is easily solved for \( \alpha_3 \) and \( \beta_3 \):

\[ \alpha_3 = \frac{1}{2} (Z_1 + Z_2) \quad \text{and} \quad \beta_3 = \frac{1}{2j} (Z_2 - Z_1) \]

In general the coefficients \( \alpha_3 \) and \( \beta_3 \) are complex numbers. This implies that the baseband signal sent to the port I and Q of the IQ modulator must have been first phase shifted.
For a quasi-memoryless PA the following identity holds:

\[
\frac{A_0}{A_2} = \frac{B_0}{B_2} = \frac{y_{m3}}{y_{md}}.
\]

It results that we have \(Z_1 = Z_2^* = Z\) and the \(\alpha_3\) and \(\beta_3\) are simply given by:

\[
\alpha_3 = -2\text{Re} \left[ (1 + j) \frac{y_{m3}}{y_{md}} \right] \quad (A.26)
\]

\[
\beta_3 = -2\text{Im} \left[ (1 + j) \frac{y_{m3}}{y_{md}} \right] \quad (A.27)
\]

Note that \(\alpha_3\) real and \(\beta_3\) are now real. No phase shift is required and a memoryless predistortion linearization is sufficient.
A.7 Implementation of Higher Order Predistortion Linearization

\[
E_i = E_{i_{\text{inband},i}} + E_{i_{\text{imd},i}}
\]

\[
E_{i_{\text{inband},i}} = E_{e,i} + E_{o,i}
\]

\[
E_{i_{\text{imd},i}} = I_{\text{e},i}^2 + I_{\text{o},i}^2 + \Delta I_{\text{e},i}^2 + \Delta I_{\text{o},i}^2 + H I_{\text{e},i}^2 + H I_{\text{o},i}^2
\]

\[
E_{i_{\text{imd},i+1}} = I_{\text{e},i+1} I_{\text{o},i+1} + Q_{e,i+1} Q_{o,i+1}
\]

\[
H E_{i_{\text{imd},i+1}} = -I_{\text{e},i+1} I_{\text{o},i+1} + Q_{e,i+1} I_{\text{o},i+1}
\]

\[
E_{i_{\text{imd},i+1}} = I_{\text{e},i+1} I_{\text{o},i+1} + Q_{e,i+1} Q_{o,i+1}
\]

\[
E_{i_{\text{imd},i+1}} = (I_{\text{e},i} + \Delta I_{\text{e},i})(I_{\text{o},i} + \Delta I_{\text{o},i}) + (Q_{e,i} + \Delta Q_{e,i})(Q_{o,i} + \Delta Q_{o,i})
\]

\[
= I_{\text{e},i} I_{\text{o},i} + I_{\text{e},i} \Delta I_{\text{o},i} + I_{\text{o},i} \Delta I_{\text{e},i} + \Delta I_{\text{e},i} \Delta I_{\text{o},i} - H I_{\text{e},i} H I_{\text{o},i} - H \Delta I_{\text{e},i} H I_{\text{o},i}
\]

\[
H E_{i_{\text{imd},i+1}} = H(I_{\text{e},i} I_{\text{o},i}) + I_{\text{e},i} H \Delta I_{\text{o},i} + H \Delta I_{\text{e},i} I_{\text{o},i} + H(\Delta I_{\text{e},i} \Delta I_{\text{e},i}) - H(\Delta I_{\text{e},i} H I_{\text{e},i}) - H(\Delta I_{\text{e},i} H I_{\text{o},i})
\]

\[
E_{i_{\text{imd},i+1}} = E_{\text{e},i} + I_{\text{o},i}
\]

\[
I_{\text{e},i+1} = I_{\text{e},i} + \alpha_B \cdot I_{\text{e},i} - \beta_B \cdot H I_{\text{e},i}
\]

\[
H I_{\text{e},i+1} = H I_{\text{e},i} + \Delta H I_{\text{e},i} + Q_{\text{e},i} + \Delta Q_{\text{e},i} - \Delta \alpha_B \cdot I_{\text{e},i}
\]

\[
I_{\text{o},i+1} = I_{\text{o},i} + \alpha_A \cdot I_{\text{o},i} + \beta_A \cdot H I_{\text{o},i}
\]

\[
H I_{\text{o},i+1} = H I_{\text{o},i} + \alpha_A \cdot H I_{\text{o},i} - \Delta \beta_A \cdot I_{\text{o},i}
\]
\[ E_{c,i+1} = I_{c,i+1}^2 + HI_{c,i+1}^2 \]
\[ = (I_{c,i} + I_{ep\text{minus}})^2 + (HI_{c,i} + Q_{ep\text{minus}})^2 \]
\[ = I_{c,i}^2 + 2 \cdot I_{c,i} \cdot I_{ep\text{minus}} + I_{c, ep\text{minus}}^2 \]
\[ + HI_{c,i}^2 + 2 \cdot HI_{c,i} \cdot Q_{ep\text{minus}} + Q_{ep\text{minus}}^2 \]
\[ = I_{c,i}^2 + HI_{c,i}^2 + I_{c, ep\text{minus}}^2 + Q_{ep\text{minus}}^2 \]
\[ + 2 \cdot I_{c,i} \cdot (\gamma_B \cdot I_{c,i} - \beta_B HI_{c,i}) \]
\[ + 2 \cdot HI_{c,i} \cdot (\beta_B \cdot I_{c,i} - \alpha_B HI_{c,i}) \]

\[ E_{o,i+1} = I_{o,i+1}^2 + HI_{o,i+1}^2 \]
\[ = (I_{o,i} + I_{q\text{minus}})^2 + (HI_{o,i} + Q_{q\text{minus}})^2 \]
\[ = I_{o,i}^2 + 2 \cdot I_{o,i} \cdot I_{q\text{minus}} + I_{q\text{minus}}^2 \]
\[ + HI_{o,i}^2 + 2 \cdot HI_{o,i} \cdot Q_{q\text{minus}} + Q_{q\text{minus}}^2 \]
\[ = I_{o,i}^2 + HI_{o,i}^2 + I_{c, q\text{minus}}^2 + Q_{q\text{minus}}^2 \]
\[ + 2 \cdot I_{o,i} \cdot (\alpha_A \cdot I_{o,i} + \beta_A HI_{o,i}) \]
\[ - 2 \cdot I_{o,i} \cdot (\beta_A \cdot I_{o,i} - \alpha_A HI_{o,i}) \]
\begin{align*}
I_{c,i+1}I_{o,i+1} &= (I_{\text{out},i} - Q_{\text{out},i})(I_{\text{out},i} + Q_{\text{out},i}) \\
&= (I_{\text{diff},i} + I_i \cdot \alpha_m - Q_i \cdot \beta_m - H_{\text{sum},i} - I_i \cdot \beta_p - Q_i \cdot \alpha_p) \\
&= (I_{\text{diff},i} + I_i \cdot \alpha_m - Q_i \cdot \beta_m + H_{\text{sum},i} + I_i \cdot \beta_p + Q_i \cdot \alpha_p) \\
&= (I_{\text{diff},i} - H_{\text{sum},i} + I_i \cdot (\alpha_m - \beta_p) - Q_i \cdot (\alpha_p + \beta_m)) \\
&= (I_{\text{diff},i} + H_{\text{sum},i} + I_i \cdot (\alpha_m + \beta_p) + Q \cdot (\alpha_p - \beta_m)) \\
&= I_{\text{diff},i}^2 - H_{\text{sum},i}^2 + (I_{\text{diff},i} - H_{\text{sum},i})(I_i \cdot (\alpha_m + \beta_p) - Q_i \cdot (\alpha_p - \beta_m)) \\
&= (I_{\text{diff},i} + H_{\text{sum},i})(I_i \cdot (\alpha_m - \beta_p) - Q_i \cdot (\alpha_p + \beta_m)) \\
&= -I_{c,i+1}Q_{o,i+1} \\
&= -(I_{\text{out},i} - H_{\text{out},i})(-H_{\text{out},i} + Q_{\text{out},i}) \\
&= (H_{\text{diff},i} + I_i \cdot \beta_p - Q_i \cdot \beta_m + I_{\text{sum},i} + I_i \beta_m + Q_i \alpha_m) \\
&= (H_{\text{diff},i} + I_{\text{sum},i} + I_i \cdot (\alpha_p + \beta_m) + Q_i \cdot (\alpha_m - \beta_p)) \\
&= -H_{\text{diff},i}^2 + I_{\text{sum},i}^2 + (H_{\text{diff},i} + I_{\text{sum},i})(-I_i (\alpha_p - \beta_m) + Q_i (\alpha_m - \beta_p)) \\
&= (I_{\text{sum},i} + I_i \cdot \beta_p + Q_i \cdot \alpha_p - I_{\text{diff},i} - I_i \cdot \alpha_m + Q_i \cdot \beta_m) \\
&= (H_{\text{diff},i} + I_{\text{sum},i} - I_i \cdot (\alpha_m - \beta_p) + Q_i \cdot (\alpha_p + \beta_m)) \\
&= (H_{\text{diff},i} + I_{\text{sum},i} - I_i \cdot (\alpha_p - \beta_m) + Q_i \cdot (\alpha_m + \beta_p)) \\
&= -H_{\text{diff},i}^2 - H_{\text{sum},i}^2 + (H_{\text{diff},i} + I_{\text{sum},i})(-I_i (\alpha_p - \beta_m) + Q_i (\alpha_m - \beta_p)) \\
&= (I_{\text{diff},i} + I_{\text{sum},i} + I_i \cdot (\alpha_m + \beta_p) + Q_i \cdot (\alpha_m - \beta_p)) \\
&= (I_{\text{diff},i} + I_{\text{sum},i} + I_i \cdot (\alpha_p + \beta_m) + Q_i \cdot (\alpha_p - \beta_m)) \\
&= (H_{\text{diff},i} + I_{\text{sum},i})(I_{\text{diff},i} + H_{\text{sum},i}) \\
&= (H_{\text{diff},i} + I_{\text{sum},i})(I_{\text{diff},i} + H_{\text{sum},i})(I_i (\alpha_p + \beta_m) + Q_i (\alpha_m - \beta_p)) \\
&= (I_i (\alpha_p + \beta_m) + Q_i (\alpha_m - \beta_p)) \\
&= (A.32)
\end{align*}
\[ I_{\text{out},i} = I_{\text{diff},i} + I_i \cdot \alpha_{\text{minus}} - Q \cdot \beta_{\text{minus}} \]
\[ = I_i - Q_i + I_i \cdot (\alpha_a - \alpha_b) - Q_i \cdot (\beta_a - \beta_b) \]
\[ = I_i - Q_i + I_i \cdot (E_{\text{ind},i} \cdot \Re(\alpha) - H E_{\text{ind},i} \cdot \Im(\alpha)) \]
\[ - Q_i \cdot (E_{\text{ind},i} \cdot \Re(\beta) - H E_{\text{ind},i} \cdot \Im(\beta)) \]

\[ Q_{\text{out},i} = I_{\text{sum},i} + I_i \cdot \beta_{\text{minus}} + Q_i \cdot \alpha_{\text{minus}} \]
\[ = I_i + Q_i + I_i \cdot (\beta_a - \beta_b) + Q_i \cdot (\alpha_a - \alpha_b) \]
\[ = I_i + Q_i + I_i \cdot (E_{\text{ind},i} \cdot \Re(\beta) - H E_{\text{ind},i} \cdot \Im(\beta)) \]
\[ + Q_i \cdot (E_{\text{ind},i} \cdot \Re(\alpha) - H E_{\text{ind},i} \cdot \Im(\alpha)) \]

\[ H I_{\text{out},i} = H I_{\text{diff},i} + H(I_i \cdot \alpha_{\text{minus}} - Q_i \cdot \beta_{\text{minus}}) \]
\[ = H I_i - H Q_i + I_i \cdot \alpha_{\text{minus}} - Q_i \cdot \beta_{\text{minus}} \]
\[ = H I_i - H Q_i + I_i \cdot (H E_{\text{ind},i} \cdot \Re(\alpha) + E_{\text{ind},i} \cdot \Im(\alpha)) \]
\[ - Q_i \cdot (H E_{\text{ind},i} \cdot \Re(\beta) + E_{\text{ind},i} \cdot \Im(\beta)) \]
\[ = H I_i - H Q_i + I_i \cdot \alpha_{\text{plus}} - Q_i \cdot \beta_{\text{plus}} \]

\[ H Q_{\text{out},i} = H I_{\text{sum},i} + H(I_i \cdot \beta_{\text{minus}} + Q_i \cdot \alpha_{\text{minus}}) \]
\[ = H I_i + H Q_i + I_i \cdot \beta_{\text{plus}} + Q_i \cdot \alpha_{\text{plus}} \]
\[ = H I_i + H Q_i + I_i \cdot (\beta_a + \beta_b) + Q_i \cdot (\alpha_a + \alpha_b) \]
\[ = H I_i + H Q_i + I_i \cdot \beta_{\text{plus}} + Q_i \cdot \alpha_{\text{plus}} \]

### A.8 Acronym

FCC: Federal Communications Commission

ETSI: European Telecommunications Standards Institute

FPGA: Field Programmable Gate Array

VK: Volterra Kernel

ME: Memory Effects

RF: Radio Frequency

PA: Power Amplifier

ADS: Advanced Design System

LSNA: Large Signal Network Analyzer
DSP: Digital Signal Process
SISO: Single Input and Single Output
MIMO: Multi input and Multi Output
ADC: Analog to Digital Converter
DAC: Digital to Analog Converter
I&Q: In and Quadrature
ACPR: Adjacent Channel Power Ratio
ACLR: Adjacent Channel Leakage Ration
EVM: Error Vector Magnitude
WCDMA: Wideband Code Division Multiple Access
USB: Upper Side Band
LSB: Lower Side Band
IP3: 3\textsuperscript{rd} Intercept Point
IMD3: 3\textsuperscript{rd} Order Intermodulation
IMD5: 5\textsuperscript{th} Order Intermodulation
IBM: Input Baseband Modulation
OBM: Output Baseband Modulation
QOBM: Quadrature Output Baseband Modulation
BBM: Bilateral Baseband Modulation
VPD: Vectorial Predistortion
LUT: Look-UP Table
LFFF: Low Frequency Feedforward
AM-PM: Amlitude Modulation-Phase Modulation
LO: Local Oscillator
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