ESSAYS ON EXCHANGE RATE MODELS UNDER A TAYLOR RULE TYPE MONETARY POLICY

DISSERTATION

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ABSTRACT

This dissertation develops three exchange rate models that explicitly incorporate a Taylor Rule type monetary policy in order to study its implications on exchange rate dynamics. Since the seminal work of Taylor (1993), the Taylor Rule has become a new standard in the literatures on exchange rates as well as monetary policy. The results reported in this dissertation imply that the Taylor Rule may be very useful in understanding exchange rate dynamics better.

My first two essays attempt to improve on the performance of the existing techniques for estimating the half-life of Purchasing Power Parity (PPP) deviations. The first essay, "Half-Life Estimation under the Taylor Rule," addresses two perennial problems in the current PPP literature, namely, unreasonably long half-life estimates of PPP deviations (PPP Puzzle; Rogoff, 1996) and extremely wide confidence intervals for half-life point estimates (Murray and Papell, 2002). Using a model that incorporates a forward looking version Taylor Rule in a dynamic system of exchange rates and inflation, I obtain significantly tighter confidence intervals along with reasonably short half-lives for PPP deviations, which is roughly consistent with micro evidence. My model also indicates that real exchange rate dynamics may differ greatly, depending on the pattern of systematic central bank responses to the inflation rate.

In the second essay, "Half-Life Estimation under the Taylor Rule: Two Goods Model," I estimate and compare half-lives of PPP deviations for PPI- and CPI-based
real exchange rates. As an extension of my first essay (single good model), we employ a
gMM system method in a two-goods model, where the central bank attempts to keep
general inflation (e.g., GDP deflator inflation) in check. In a similar framework that
employs a money demand function instead of the Taylor Rule, Kim (2004) reported
much shorter half-life point estimates for the non-service consumption deflators than
those for service consumption deflators, though with quite wide confidence intervals.
In contrast, I find that half-life estimates were about the same irrespective of the
choice of aggregate price indexes, which is consistent with many other studies that
reported only moderate or no difference. Most importantly, I obtain much smaller
standard errors that enables us to make statistically meaningful comparisons between
the sizes of half-life estimates. Our model also shows that rationally expected future
Taylor Rule fundamentals help understand real exchange rate dynamics only when
the central bank responds to general inflation aggressively enough.

Finally, the third essay, "Local-Currency Pricing, Technology Diffusion, and the
Optimal Interest Rate Rule," studies optimal monetary policy and its implications on
exchange rate regimes in a dynamic stochastic general equilibrium model that features
sticky-price, local-currency pricing, and technology diffusion. The main findings of
this essay are twofold. First, in response to a favorable real shock, central banks
may raise nominal interest rates despite price-stickiness and local-currency pricing,
which seems to be consistent with empirical evidence of rising nominal interest rates
during economic boom. This outcome is exactly opposite to the Rogoff’s (2004)
prediction as well as those of many others. Second, central banks respond identically
to technology shocks that occur in tradables sectors so that optimal monetary policies
do not require any exchange rate change. However, central banks respond oppositely
to real shocks in nontradables sectors, and the resulting interest rate differential calls for exchange rate changes. Therefore, compared with Duarte and Obstfeld’s (2004) results, this outcome implies that benefits of flexible exchange rates could be quite limited if technology shocks in nontradables sectors occur infrequently. This essay also finds a case that central banks optimally do not respond to any technology shock in tradables sectors, which may be related to a dirty-float system of exchange rates.
To my parents and my family
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CHAPTER 1

INTRODUCTION

In this dissertation, I develop three exchange rate models that explicitly incorporate a Taylor Rule type monetary policy in order to investigate its implications on exchange rate dynamics. This approach contrasts with the conventional exchange rate models mainly in the following aspect. That is, central banks are assumed to target interest rates instead of money growth rates in implementing monetary policy, so that the money supply adjusts endogenously rather than exogenously. This dissertation reports many interesting results that imply the Taylor Rule may be very useful, both theoretically and empirically, in understanding exchange rate dynamics better.

This dissertation is organized as follows. In chapters 2 and 3, I develop two exchange rate models to improve upon the performance of the currently available techniques for estimating the half-life of Purchasing Power Parity (PPP) deviations. PPP has been one of the most useful building blocks for influential exchange rate models. In presence of nominal rigidities, financial factors such as monetary shocks may cause temporary deviations of the real exchange rate away from its equilibrium.
PPP level. If PPP holds in the long-run, however, such deviations will die out eventually, since nominal shocks are neutral in the long-run. The half-life refers to the time required for such a deviation to adjust halfway to its long-run equilibrium level.

In chapter 2, I attempt to provide a solution to two major problems in recent PPP literature, namely, unreasonably long half-life estimates of PPP deviations (PPP Puzzle; Rogoff, 1996) and extremely wide confidence intervals for half-life point estimates (Murray and Papell, 2002). Recognizing that these problems mostly arise when we use a single equation approach, I construct a system of stochastic difference equations that combines Mussa’s (1982) error correction model of real exchange rates with a forward looking version Taylor Rule along with uncovered interest parity condition. Using the saddle-path solution for this system, I implement efficient GMM estimations for the half-life of PPP deviations. In so doing, I obtain significantly tighter confidence intervals as well as reasonably short half-lives for PPP deviations, which is roughly consistent with micro evidence. My model also indicates that real exchange rate dynamics may differ greatly, depending on the pattern of systematic central bank responses to the inflation rate.

In chapter 3, I estimate and compare half-lives of PPP deviations for PPI- and CPI-based real exchange rates. As an extension of my first essay, which is a single good model, I employ a GMM system method in a two-goods model, where the central bank attempts to keep general inflation (e.g., GDP deflator inflation) in check. In a similar framework but employing a conventional money demand function instead of the Taylor Rule, Kim (2004) reported much shorter half-life point estimates for the non-service consumption deflators than those for service consumption deflators, though with quite wide confidence intervals. In contrast, I find that half-life estimates
were about the same irrespective of the choice of aggregate price indexes, which is consistent with many other studies that reported only moderate or no difference (e.g., Wu, 1996, Murray and Papell, 2002). More importantly, I obtain much smaller standard errors than those of Kim (2004) so that it is possible to make statistically meaningful comparisons between the sizes of half-life point estimates. Our model also shows that rationally expected future Taylor Rule fundamentals help understand real exchange rate dynamics only when the central bank responds to general inflation aggressively enough.

In chapter 4, I study the optimal interest rate rule and its implications on exchange rate regimes in a dynamic stochastic general equilibrium model that features sticky-price, local-currency pricing, and technology diffusion. In this chapter, I raise an issue with regard to a common specification of country-specific shocks. Many researches, which include works of Devereux and Engel (2006), Obstfeld (2004), and Duarte and Obstfeld (2005), specify a country-specific shock as an economy-wide shock. Put it differently, they assume that technology shocks in tradables and nontradables sectors in a country are perfectly correlated. Instead, I assume that a country-specific technology shock in the home country can diffuse to the same sector of the foreign country with a one-period lag, and vice versa, which seems to be more reasonable.

With this alternative specification, I obtain the following interesting results. First, in response to a favorable technology shock, central banks may wish to raise nominal interest rates despite price-stickiness and local-currency pricing, which seems to be consistent with empirical evidence of rising nominal interest rates during economic boom. It should be noted that this outcome is exactly opposite to the Rogoff’s (2004) prediction as well as those of many others.
Second, central banks respond identically to technology shocks that occur in tradables sectors so that optimal monetary policies do not require any exchange rate change. However, central banks respond oppositely to real shocks in nontradables sectors, and the resulting interest rate differential calls for exchange rate changes. Therefore, this result demonstrates that benefits of flexible exchange rates could be quite limited if technology shocks in nontradables sectors occur rather infrequently, which contrasts with the Duarte and Obstfeld’s (2005) strong support for the flexible exchange rate system. I also construct a case that central banks optimally do not respond at all to any technology shock in tradables sectors, which may be related to a dirty-float system of exchange rates.

Lastly, chapter 5 concludes.
CHAPTER 2

HALF-LIFE ESTIMATION UNDER THE TAYLOR RULE

2.1 Introduction

Since its revival by Cassel (1921), Purchasing Power Parity (PPP) has been one of the most useful building blocks in monetary models (e.g., Frenkel, 1976) and neoclassical models of exchange rates (e.g., Lucas, 1982), as well as in the more recent Redux model (Obstfeld and Rogoff, 1995). This paper attempts to provide a solution to two major problems that have frequently arisen in recent empirical PPP literature. These problems may be summarized as follows\(^1\).

Rogoff (1996) has described the “PPP puzzle” as the question of how one might reconcile highly volatile short-run movements of real exchange rates with an extremely slow convergence rate to PPP, and alluded to the "remarkable consensus" of three- to five-year half-life estimates for PPP deviations from long-horizon data\(^2\). In other words, though financial factors, such as monetary shocks, may successfully account for the short-run volatility of real exchange rates in the presence of nominal rigidities,

\(^1\)Due to the frequent failure of the law of one price in microeconomics data (e.g. Isard 1977), very few economists consider PPP as a short-run proposition. The empirical evidence on the validity of PPP in the long-run is mixed (see Rogoff 1996 for a survey). This paper accepts PPP as a long-run proposition without further discussing this issue.

\(^2\)More recently, Taylor (2002) report shorter half-life estimates than this consensus half-life from over 200-year long data for 20 countries. However, Lopez, Murray, and Papell (2004) claim his results are not robust to lag selection problems.
it is hard to rationalize why such PPP deviations should attenuate that slowly, if such shocks truly are largely neutral in the medium-run.\(^3\)

Another issue, raised by Murray and Papell (2002), concerns the possibility of conducting a meaningful statistical inference of the length of half-lives of real exchange rates. Their half-life estimates for many real exchange rates turned out to have extremely wide confidence intervals, even though their point estimates were consistent with Rogoff’s remarkable consensus.

It should be noted that most of the researchers named above acquired their half-life estimates using a univariate process model of real exchange rates\(^4\). It would seem, therefore, that such a single-equation approach does not facilitate a resolution of these issues. Interestingly, recently formulated panel approaches have tended to provide relatively shorter half-lives for current float data (e.g., 2.5-year half-lives, Papell, 1997, and Wu, 1996). Moreover, Murray and Papell (2005) obtained reasonably compact confidence intervals for their half-life panel estimates. Their estimates, however, confirmed Rogoff’s consensus half-life. Therefore, panel approaches have been insufficient as a solution to the PPP puzzle, even though they have provided some efficiency gains. Furthermore, the main assumption of the panel approach, that the convergence rates to PPP were the same across countries, is hard to justify. Recently,

\(^3\)Real shocks such as technology shocks may account for such a slow convergence rate, since those shocks may not be neutral even in the long-run. However, real shocks may not be able to successfully explain the short-run volatility of real exchange rates.

\(^4\)If PPP holds in the long-run, the real exchange rate \(s_t (e_t + p_t^* - p_t)\) may be represented as the following simple AR(1) process, \(s_t = \mu + \alpha s_{t-1} + \varepsilon_t\), where \(\alpha < 1\). Then, the corresponding half-life can be calculated as \(\ln(\frac{1}{2})/\ln\alpha\).
Imbs et al. (2005) showed that ignoring heterogeneous dynamics may result in quite inaccurate half-life estimates\(^5\).

One potential answer to these problems can be found in the work of Kim, Ogaki, and Yang (2003). Instead of single-equation approaches, they suggested using a system method that combines economic theories with the single-equation model of real exchange rates. Imposing theory-driven restrictions on their model of exchange rates and inflation, they estimated half-lives that were much shorter\(^6\) than the current consensus of three- to five-year half-lives for the post Bretton Woods CPI-based real exchange rates. It should be noted that one advantage of using such a system method is that it provides estimates that are more efficient than those from a single-equation method, as long as the imposed restrictions are valid. Indeed, these authors obtained limited efficiency gains, which may have allowed more suitable statistical inferences to be made from their point estimates.

The research presented in this paper constitutes a modification of the work of Kim, Ogaki, and Yang (2003), who derived restrictions from the conventional money-market equilibrium condition, which they identified using money demand functions. However, the money demand function can be quite unstable, especially in the short-run, so their use of the money-market equilibrium condition may not have been ideal. In contrast, we incorporated a forward-looking version of the Taylor Rule into a dynamic system of the exchange rate and inflation. In so doing, we attempted to determine whether Taylor Rule-based restrictions could resolve the problems noted above. In fact, we obtained a substantial efficiency improvement over the results of

\(^5\)Chen and Engel (2005) claim that such aggregation bias may not have significant impacts on panel approaches.

\(^6\)Their half-life estimates range from 0.12 to 2.22 years. They don’t report the median half-life, but it was 0.35 year from full sample estimations by our calculation.
Kim, Ogaki, and Yang (2003), and our point estimates were reasonably shorter (1.49- and 1.37-year median half-lives for GDP deflator- and CPI-based real exchange rates, respectively) than the current consensus of three- to five-year half-lives. Interestingly, our estimates are roughly consistent with those of Crucini and Shintani (2004), who reported 1.1-, 1.0-, and 1.6-year baseline half-life estimates for OECD micro data on all, traded, and non-traded good prices, respectively\(^7\).

Since the seminal work of Taylor (1993), the Taylor Rule has been one of the most popular models used in the monetary policy literature. The core implication of the Taylor Rule is that the price level would be indeterminate unless the central bank responds to inflation aggressively enough to raise the real interest rate.

One particularly interesting point has been made by Clarida, Galí and Gertler (2000), who provided strong empirical evidence of a structural break in the Fed’s reaction function\(^8\). Putting it differently, they found that the estimate of the coefficient on rationally expected near-future inflation became strictly greater than one for the period of the Volker-Greenspan era, whereas the corresponding estimate for the pre-Volker era turned out to be strictly less than one; these results are consistent with the implications of the Taylor Rule and observed inflation dynamics\(^9\). Similar findings have been provided by Taylor (1999a) and Judd and Rudebusch (1998). Clarida, Galí and Gertler (1998) also found similar international evidence for Germany and Japan.

This paper shows that consideration of such a structural break may be important in understanding real exchange rate dynamics. It turns out that the dynamics of

\(^7\)Parsley and Wei (2004) also provide similar micro-evidence.

\(^8\)Unlike Taylor (1993), they used a forward looking version of the Taylor rule. In their model, the Fed is assumed to respond to future inflation forecast rather than current inflation.

\(^9\)One can observe rapidly rising inflation rates in the pre-Volker era and declining inflation rates in the Volker-Greenspan era.
real exchange rates can differ greatly, depending on the pattern of systematic central bank responses to inflation. In what follows, we show that exchange rate dynamics can be greatly affected by the present value of rationally expected future fundamental variables only when the inflation coefficient is strictly greater than one. Interestingly, when the inflation coefficient is less than one, real exchange rate dynamics can be explained only by past economic variables and any martingale difference sequences; moreover, future fundamental variables play no role.

This paper is not the first to emphasize the importance of expected future (Taylor Rule) fundamentals in the context of real exchange rate dynamics. Engel and West (2002) drew similar conclusions using data for the US and Germany. However, they did not attempt to estimate the half-lives of PPP deviations. In addition, Mark (2005) presented similar evidence that the dynamics of the real US$-Deutschemark exchange rate could be better understood by means of Taylor Rule fundamentals in a learning framework\textsuperscript{10}.

The rest of the paper is organized as follows. In 2.2, we construct a system of stochastic difference equations for the exchange rate and inflation, explicitly incorporating a forward-looking version of the Taylor Rule into the system. Then, we describe three different estimation strategies. These include a GMM system method, which combines the single-equation method with the model-driven restriction from the Taylor Rule. In 2.3, a description of the data and the estimation results are provided. 2.4 concludes.

\textsuperscript{10}Obstfeld (2004) discusses implications of optimal interest rate rules on exchange rate regimes in a dynamic stochastic general equilibrium model. For a similar discussion, see Devereux and Engel (2005).
2.2 Model Specification

2.2.1 Gradual Adjustment Equation

We start with a simple univariate stochastic process of real exchange rates. Let $p_t$ be the log domestic price level, $p_t^*$ be the log foreign price level, and $e_t$ be the log nominal exchange rate as the price of one unit of the foreign currency in terms of the home currency. And we denote $s_t$ as the log of the real exchange rate, $p_t^* + e_t - p_t$.

Rather than econometrically testing it\textsuperscript{11}, we simply assume that PPP holds in the long-run\textsuperscript{12}. Putting it differently, we assume that there exists a cointegrating vector $[1 - 1 - 1]'$ for a vector $[p_t \ p_t^* \ e_t]'$, where $p_t$, $p_t^*$, and $e_t$ are difference stationary processes. Under this assumption, real exchange rates can be represented as the following stationary univariate autoregressive process of degree one.

$$s_{t+1} = d + \alpha s_t + \varepsilon_{t+1}, \quad (2.1)$$

where $\alpha$ is a positive persistence parameter that is less than one\textsuperscript{13}.

Note that $\alpha$ can be consistently estimated by the conventional least squares method under the maintained cointegrating relation assumption\textsuperscript{14} as long as there’s

\textsuperscript{11}The empirical evidence on the validity of PPP in the long-run is mixed. It should be noted that even when we have some evidence against PPP, such results might be due to lack of power of existing unit root tests in small samples, and are subject to the observational equivalence problem.

\textsuperscript{12}We may assume that PPP applies only to the tradable price-based real exchange rates by distinguishing tradables from non-tradables. See Kim and Ogaki (2004) or Kim (2005a) for details.

\textsuperscript{13}Note that this is a so-called Dickey-Fuller estimation model. One may estimate half-lives by an Augmented Dickey-Fuller estimation model in order to avoid possible serial correlation problems. However, as shown in Murray and Papell (2002), half-life estimates from both models were roughly similar. So it seems that AR(1) specification is not a bad approximation.

\textsuperscript{14}It is well-known that the standard errors obtained from the least squares or parametric bootstrap methods may not be valid asymptotically in presence of serially correlated errors. It would be necessary to use either Newey-West or QS kernel covariance estimators, or to use the moving block bootstrap method.
no measurement error. Once we obtain the point estimate of $\alpha$, the half-life of the real exchange rate can be obtained by $\frac{\ln(0.5)}{\ln\alpha}$.

Interestingly, Kim, Ogaki, and Yang (2003) pointed out that the equation (2.1) could be implied by the following error correction model of real exchange rates by Mussa (1982) with a known cointegrating relation described earlier.

$$\Delta p_{t+1} = b [\mu - (p_t - p_t^* - e_t)] + E_t \Delta p_{t+1}^* + E_t \Delta e_{t+1}, \quad (2.2)$$

where $\mu = E(p_t - p_t^* - e_t)$, $b = 1 - \alpha$, $d = -(1 - \alpha)\mu$, $\varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^* - \Delta p_{t+1}^*)$, and $E_t \varepsilon_{t+1} = 0$. $E(\cdot)$ denotes the unconditional expectation operator, and $E_t(\cdot)$ is the conditional expectation operator on $I_t$, the economic agent’s information set at time $t$.

One interpretation of this equation is that the domestic price level adjusts instantaneously to the expected change in its PPP, while it adjusts to its unconditional PPP level, $E(p_t^* + e_t)$ only slowly with the constant convergence rate $b$ ($= 1 - \alpha$), which is a positive constant less than unity by construction.

2.2.2 Taylor Rule Model

We assume that the uncovered interest parity holds. That is,

$$E_t \Delta e_{t+1} = i_t - i_t^*, \quad (2.3)$$

where $i_t$ and $i_t^*$ are domestic and foreign interest rates, respectively.

15If there is a measurement error problem, $\alpha$ may not be even consistent in this framework, since the aforementioned cointegrating relation assumption may not hold. We can deal with this problem by a two stage cointegration method that directly estimates the cointegrating vector rather than assuming it. See Kim, Ogaki, and Yang (2003) for details.

16Standard errors can also be obtained using the delta method.

17Recall that $b$ (convergence rate parameter) equals to $1 - \alpha$, where $\alpha$ (persistence parameter) is a positive constant less than unity.
The central bank in the home country is assumed to continuously set its optimal target interest rate \((i_t^T)\) by the following forward looking Taylor Rule\(^\text{18}\).  

\[
i_t^T = \iota + \gamma_\pi E_t \Delta p_{t+1} + \gamma_x x_t,
\]

where \(\iota\) is a constant that includes a certain long-run equilibrium real interest rate along with a target inflation rate\(^\text{19}\), and \(\gamma_\pi\) and \(\gamma_x\) are the long-run Taylor Rule coefficients on expected future inflation \((E_t \Delta p_{t+1})\) and current output deviations \((x_t)\)^\text{20}, respectively. We also assume that the central bank attempts to smooth the interest rate by the following rule.

\[
i_t = (1 - \rho)i_t^T + \rho i_{t-1},
\]

that is, the current actual interest rate is a weighted average of the target interest rate and the previous period’s interest rate, where \(\rho\) is the smoothing parameter. Then, we can derive the forward looking version Taylor Rule equation with interest rate smoothing policy as follows\(^\text{21}\).

\[
i_t = \iota + (1 - \rho)\gamma_\pi E_t \Delta p_{t+1} + (1 - \rho)\gamma_x x_t + \rho i_{t-1}
\]

\[(2.4)\]

Combining (2.3) and (2.4), we obtain the following.

\[
E_t \Delta e_{t+1} = \iota + (1 - \rho)\gamma_\pi E_t \Delta p_{t+1} + (1 - \rho)\gamma_x x_t + \rho i_{t-1} - i_t^* \tag{2.5}
\]

\(^{18}\)We do not impose any restriction on foreign central bank’s behavior, since foreign variables are pure forcing variables in our model as will be illustrated later.


\(^{20}\)If we assume that the central bank responds to expected future output deviations rather than current deviations, we can simply modify the model by replacing \(x_t\) with \(E_t x_{t+1}\). However, this does not make any significant difference to our results.

\(^{21}\)It is also straightforward to modify our model where the home central bank employs the exchange rate targeting. See Kim (2005b) for reference.
where $\gamma^s_\pi = (1 - \rho)\gamma_\pi$ and $\gamma^s_x = (1 - \rho)\gamma_x$ are *short-run* Taylor Rule coefficients.

Now, let’s rewrite (2.2) as the following equation in level variables.

$$p_{t+1} = b\mu + E_t e_{t+1} + (1 - b)p_t - (1 - b)e_t + E_t p^*_{t+1} - (1 - b)p^*_t \quad (2-2')$$

Taking differences and rearranging it, (2-2’) can be rewritten as follows.

$$\Delta p_{t+1} = E_t \Delta e_{t+1} + \alpha \Delta p_t - \alpha \Delta e_t + [E_t \Delta p^*_{t+1} - \alpha \Delta p^*_t + \eta_t], \quad (2.6)$$

where $\alpha = 1 - b$ and $\eta_t = \eta_{1,t} + \eta_{2,t} = (e_t - E_{t-1}e_t) + (p^*_t - E_{t-1}p^*_t)$.

From (2.4), (2.5), and (2.6), we construct the following system of stochastic difference equations.

$$\begin{bmatrix} 1 & -1 & 0 \\ -\gamma^s_\pi & 1 & 0 \\ -\gamma^s_x & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_{t+1} \\ E_t \Delta e_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha & 0 \\ 0 & 0 & \rho \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} E_t \Delta p^*_{t+1} - \alpha \Delta p^*_t + \eta_t \\ \eta^s_x x_t - i^*_t \\ \eta^s_x x_t \end{bmatrix} \quad (2.7)$$

For notational simplicity, let’s rewrite (2.7) in matrix form as follows.

$$\mathbf{A}E_t \mathbf{y}_{t+1} = \mathbf{B} \mathbf{y}_t + \mathbf{x}_t, \quad (2-7')$$

and thus\(^{22}\),

$$E_t \mathbf{y}_{t+1} = \mathbf{A}^{-1} \mathbf{B} \mathbf{y}_t + \mathbf{A}^{-1} \mathbf{x}_t \quad (2.8)$$

$$= \mathbf{D} \mathbf{y}_t + \mathbf{c}_t,$$

where $\mathbf{D} = \mathbf{A}^{-1} \mathbf{B}$ and $\mathbf{c}_t = \mathbf{A}^{-1} \mathbf{x}_t$. By eigenvalue decomposition, (2.8) can be rewritten as follows.

$$E_t \mathbf{y}_{t+1} = \mathbf{V} \Lambda \mathbf{V}^{-1} \mathbf{y}_t + \mathbf{c}_t, \quad (2.9)$$

\(^{22}\)It is straightforward to show that $\mathbf{A}$ is nonsingular, and thus has a well-defined inverse.
where $D = V\Lambda V^{-1}$ and

$$V = \begin{bmatrix}
\frac{\alpha\gamma^s}{\alpha - \rho} & 1 & 1 \\
\frac{\alpha\gamma^s}{\alpha - \rho} & 1 & 1 \\
\frac{\alpha\gamma^s}{\alpha - \rho} & 1 & 0
\end{bmatrix}, \quad \Lambda = \begin{bmatrix}
\alpha & 0 & 0 \\
0 & \frac{\rho}{1 - \gamma^s} & 0 \\
0 & 0 & 0
\end{bmatrix}$$

Premultiplying (2.9) by $V^{-1}$ and redefining variables,

$$E_t z_{t+1} = \Lambda z_t + h_t, \quad (2.10)$$

where $z_t = V^{-1}y_t$ and $h_t = V^{-1}c_t$.

Note that, among non-zero eigenvalues in $\Lambda$, $\alpha$ is between 0 and 1 by definition, while $\frac{\rho}{1 - \gamma^s} (= \frac{\rho}{1 - (1 - \rho)\gamma^s})$ is greater than unity as long as $1 < \gamma^s < \frac{1}{1 - \rho}$. Therefore, if the long-run inflation coefficient $\gamma^s$ is strictly greater than one$^{23}$, the system of stochastic difference equations (2.7) has a saddle path equilibrium, where rationally expected future fundamental variables enter in the exchange rate and inflation dynamics. On the contrary, if $\gamma^s$ is strictly less than unity, which might be true in the pre-Volker era in the US, the system would have a purely backward looking solution, where the solution would be determined by past fundamental variables and any martingale difference sequences.

Assuming $\gamma^s$ is strictly greater than one, we can show that the solution to (2.7) satisfies the following relation (see Appendix for the derivation).

$$\Delta e_{t+1} = \hat{i} + \frac{\alpha\gamma^s}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha\gamma^s}{\alpha - \rho} \Delta p^*_t + \frac{\alpha\gamma^s - (\alpha - \rho)}{\alpha - \rho} i^*_t \quad (2.11)$$

$$+ \frac{\gamma^s}{\alpha - \rho} \left( \frac{\alpha\gamma^s - (\alpha - \rho)}{(\alpha - \rho)\rho} \right) \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1},$$

where,

$$\hat{i} = \frac{\alpha\gamma^s - (\alpha - \rho)}{(\alpha - \rho)(\gamma^s - (1 - \rho))} i_t.$$
\[ E_t f_{t+j} = - \left[ E_t i_{t+j}^* - E_t \Delta p_{t+1}^* \right] + \frac{\gamma^s}{\gamma^s_{\pi}} E_t x_{t+j} \]
\[ = -E_t r_{t+j}^* + \frac{\gamma^s}{\gamma^s_{\pi}} E_t x_{t+j}, \]

\[ \omega_{t+1} = \frac{\gamma^s}{(\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s_{\pi}}{\rho} \right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1}) \]
\[ + \frac{\gamma^s}{\alpha - \rho} \eta_{t+1} - \frac{\alpha \gamma^s_{\pi} - (\alpha - \rho)}{\alpha - \rho} \nu_{t+1}, \]

and,

\[ E_t \omega_{t+1} = 0 \]

Or, (2.11) can be rewritten with full parameter specification as follows.

\[ \Delta e_{t+1} = \dot{i} + \frac{\alpha \gamma^s_{\pi} (1 - \rho)}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma^s_{\pi} (1 - \rho)}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha \gamma^s_{\pi} (1 - \rho) - (\alpha - \rho)}{\alpha - \rho} i_t^{\star} \]
\[ + \frac{\gamma^s_{\pi} (1 - \rho)}{(\alpha - \rho) \rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma^s_{\pi} (1 - \rho)}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1} \]  

(2-11')

Here, \( f_t \) is a proxy variable that summarizes the fundamental variables such as foreign real interest rates \( (r_t^*) \) and domestic output deviations.

Note that if \( \gamma^s_{\pi} \) is strictly less than unity, the restriction in (2.11) may not be valid, since the system would have a backward looking equilibrium rather than a saddle path equilibrium\(^{24}\). Put it differently, exchange rate dynamics critically depends on the size of \( \gamma^s_{\pi} \). As mentioned in the introduction, however, we have some supporting empirical evidence for such a requirement for the existence of a saddle path equilibrium, at least

\(^{24}\) If the system has a purely backward looking solution, the conventional Vector Autoregressive (VAR) estimation method may apply.
for the sample period we consider. So we believe that our specification would remain valid for our purpose in this paper.

2.2.3 GMM Estimation

We discuss three estimation strategies here: single equation estimation; GMM estimation by Hansen and Sargent (1980, 1982); GMM system estimation by Kim, Ogaki, and Yang (2003).

Single Equation Estimation

We start with the conventional single equation approach. For this, let’s rewrite (2.2) as follows.

\[
\Delta p_{t+1} = b [\mu - (p_t - p_t^* - \varepsilon_t)] + \Delta p_{t+1}^* + \Delta \varepsilon_{t+1} + \varepsilon_{t+1},
\]  

(2-2”)

where \( \varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^* - \Delta p_{t+1}^*) \) and \( E_t \varepsilon_{t+1} = 0 \). It is easy to see that (2-2”) implies (2.1). Then one can estimate convergence parameter \( b \) (or persistence parameter \( \alpha \)) by the conventional least square method.

GMM Estimation by Hansen and Sargent (1980, 1982)

Our second estimation strategy deals with the equation (2.11) or (2-11’). The estimation of the equation (2.11) is a challenging task, since it has an infinite sum of rationally expected discounted future fundamental variables. Following Hansen and Sargent (1980, 1982), we will linearly project \( E_t(\cdot) \) onto \( \Omega_t \), the econometrician’s

\[25\text{In contrast to Clarida, Gali, and Gertler (1998, 2000), Taylor (1999a), and Judd and Rudebusch (1998), Orphanides (2001) found the estimates of the inflation coefficient } \gamma_\pi \text{ to be consistently greater than one in both the pre- and post-Volker regimes. In any case, our model specification is still valid.}

\[26\text{However, in empirical perspectives, it is highly likely that the error terms are serially correlated due to aggregation bias especially when we use low frequency data. This problem can be fixed by correcting standard errors using either the Newey-West covariance estimator or QS kernel estimator.} \]

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information set at time \( t \), which is a subset of \( I_t \). Denoting \( \hat{E}_t(\cdot) \) as such a linear projection operator onto \( \Omega_t \), we can rewrite (2.11) as follows.

\[
\Delta e_{t+1} = \hat{\iota} + \frac{\alpha \gamma_s}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma_s}{\alpha - \rho} \Delta p^*_t + \frac{\alpha \gamma_s - (\alpha - \rho)}{\alpha - \rho} \iota_t^*
\]

(2.12)

where

\[
\xi_{t+1} = \omega_{t+1} + \frac{\gamma_s (\alpha \gamma_s - (\alpha - \rho))}{(\alpha - \rho) \rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_s}{\rho} \right)^j \left( \hat{E}_t f_{t+j+1} + \xi_{t+1} \right),
\]

and

\[
\hat{E}_t \xi_{t+1} = 0,
\]

by the law of iterated projections.

Rather than choosing appropriate instrumental variables that are in \( \Omega_t \), we simply assume \( \Omega_t = \{ f_t, f_{t-1}, f_{t-2}, \cdots \} \). This assumption would be an innocent one under the stationarity assumption of the fundamental variable, \( f_t \), and it can greatly lessen the burden in our GMM estimation by significantly reducing the number of coefficients to be estimated.

Let’s assume, for now, that \( f_t \) be a zero mean covariance stationary, linearly indeterministic stochastic process so that it has the following Wold representation.

\[
f_t = c(L) \nu_t,
\]

(2.13)

where \( \nu_t = f_t - \hat{E}_{t-1} f_t \) and \( c(L) \) is square summable. Assuming that \( c(L) = 1 + c_1 L + c_2 L^2 + \cdots \) is invertible, (2.13) can be rewritten as the following autoregressive representation.

\[
b(L) f_t = \nu_t,
\]

(2.14)

Kim, Ogaki, and Yang (2003) use the foreign inflation rate \( (\Delta p^*_t) \) as a scalar instrument variable.
where \( b(L) = c^{-1}(L) = 1 - b_1 L - b_2 L^2 - \cdots \). Linearly projecting \( \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_s^*}{\alpha - \rho} \right)^j E_t f_{t+j+1} \) onto \( \Omega_t \), Hansen and Sargent (1980) show that the following relation holds.

\[
\sum_{j=0}^{\infty} \delta^j \hat{E}_t f_{t+j+1} = \psi(L)f_t = \left[ \frac{1 - \left( \delta^{-1} b(\delta) \right)^{-1} b(L)L^{-1}}{1 - (\delta^{-1} L)^{-1}} \right] f_t, \tag{2.15}
\]

where \( \delta = \frac{1 - \gamma_s^*}{\rho} \).

For actual estimation, we assume that \( f_t \) can be represented by a finite order AR\( (r) \) process, that is, \( b(L) = 1 - \sum_{j=1}^{r} b_j L^j \), where \( r < \infty \). Then, it can be shown that the coefficients of \( \psi(L) \) can be computed recursively (see Sargent 1987) as follows.

\[
\psi_0 = (1 - \delta b_1 - \cdots - \delta^r b_r)^{-1}
\]

\[
\psi_r = 0
\]

\[
\psi_{j-1} = \delta \psi_j + \delta \psi_0 b_j,
\]

where \( j = 1, 2, \ldots, r \). Then, the GMM estimation of (2.11) can be implemented by simultaneously estimating following two equations.

\[
\Delta e_{t+1} = \hat{i} + \frac{\alpha \gamma_s^*}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma_s^*}{\alpha - \rho} \Delta p^*_t + \frac{\alpha \gamma_s^* - (\alpha - \rho) i^*_t}{\alpha - \rho} i_t \tag{2.16}
\]

\[
+ \frac{\gamma_s^* (\alpha \gamma_s^* - (\alpha - \rho))}{(\alpha - \rho) \rho} (\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1}) + \xi_{t+1},
\]

\[
f_{t+1} = k + b_1 f_t + b_2 f_{t-1} + \cdots + b_r f_{t-r+1} + \nu_{t+1}, \tag{2.17}
\]

where \( k \) is a constant scalar, and \( \hat{E}_t \nu_{t+1} = 0 \).

\[28\] We can use conventional Akaike Information criteria or Bayesian Information criteria in order to choose the degree of such autoregressive processes.

\[29\] Recall that Hansen and Sargent (1980) assume a zero-mean covariance stationary process. If the variable of interest has a non-zero unconditional mean, we can either demean it prior to the estimation or include a constant but leave its coefficient unconstrained. West (1989) showed that the further efficiency gain can be obtained by imposing additional restrictions on the deterministic term. However, the imposition of such an additional restriction is quite burdensome, so we simply add a constant here.

\[30\] In actual estimations, we normalized (2.16) by multiplying \((\alpha - \rho)\) to each side in order to reduce nonlinearity.

The system method (GMM) estimation combines aforementioned two estimation strategies. That is, we implement a GMM estimation for the three equations (2-2”), (2.16), and (2.17) simultaneously. Rather than leaving $\alpha$ in (2.16) unrestricted as in the Hansen-Sargent method, we restrict it to satisfy (2-2”) or (2.1) at the same time. In so doing, we may be able to acquire further efficiency gains by using more information.

A GMM estimation, then, can be implemented by the following $2(p + 2)$ orthogonality conditions.

\[
\hat{E} x_{1,t} (s_{t+1} - d - \alpha s_t) = 0 \tag{2.18}
\]

\[
\hat{E} x_{2,t-\tau} \left( \Delta e_{t+1} - \hat{i} - \frac{\alpha \gamma_s}{\alpha - \rho} \Delta p_{t+1} + \frac{\alpha \gamma_s}{\alpha - \rho} \Delta p_{t+1}^* - \frac{\alpha \gamma_s - (\alpha - \rho)}{\alpha - \rho} i_t^* \left( \psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_r f_{t-r+1} \right) \right) = 0 \tag{2.19}
\]

\[
\hat{E} x_{2,t-\tau} (f_{t+1} - k - b_1 f_t - b_2 f_{t-1} - \cdots - b_r f_{t-r+1}) = 0, \tag{2.20}
\]

where $x_{1,t} = (1, s_t)'$, $x_{2,t} = (1, f_t)'$, and $\tau = 0, 1, \cdots, p^{31^{32}}$.

2.3 Empirical Results

This reports estimates of the persistence parameter $\alpha$ (or convergence rate parameter $b$) and their implied half-lives from aforementioned three estimation strategies.

We use CPIs and GDP deflators in order to construct real exchange rates with the US$ as a base currency. I consider 19 industrialized countries$^{33}$ that provide 18 exchange rates. For interest rates, I use quarterly money market interest rates that $^{31}$ does not necessarily coincide with $r$.

$^{32}$ In actual estimations, we use the aforementioned normalization again.

$^{33}$ Among 23 industrialized countries classified by IMF, I dropped Greece, Iceland, and Ireland due to lack of reasonable number of observations. Luxembourg was not included because it has a currency union with Belgium.
are short-term interbank call rates rather than conventional short-term treasury bill rates, since we incorporate the Taylor Rule in the model where a central bank sets its target short-term market rate. For output deviations, we consider two different measures of output gaps, quadratically detrended real GDP gap (see Clarida, Galí, and Gertler 1998)\(^{34}\) and unemployment rate gaps (see Boivin 2005)\(^{35}\). The data frequency is quarterly and from the IFS CD-ROM and DataStream\(^{36}\). For GDP deflator-based real exchange rates, the observations span from 1979:III to 1998:IV for Eurozone countries, and from 1979:III to 2003:IV for Non-Eurozone EU countries and Non-EU countries with some exceptions due to lack of observations (see the note on Table 2.4 for complete description)\(^{37}\). For CPI-based real exchange rates, the sample period is from 1979:III to 1998:IV for Eurozone countries, and from 1979:III to 2003:IV for the rest of the countries except Sweden (see the note on Table 2.7 for complete description).

The reason that our sample period starts from 1979:III is based on empirical evidence on the US Taylor Rule. As discussed in 2.2, the inflation and exchange rate dynamics may greatly depend on the size of the central bank’s reaction coefficient to future inflation. We showed that the rationally expected future fundamental variables

\(^{34}\)I also tried same analysis with the cyclical components of real GDP series from the HP-filter with 1600 of smoothing parameter. The results were quantitatively similar.

\(^{35}\)The unemployment gap is defined as a 5 year backward moving average subtracted by the current unemployment rate. This specification makes its sign consistent with that of the conventional output gap.

\(^{36}\)I obtained Danish and Japanese GDP deflator data from DataStream. IFS CD-ROM doesn’t provide reasonable number of observations for Danish deflator data. There was a seasonality problem in Japanese GDP deflator data in IFS CD-ROM until late 1979. All other data is from IFS CD-ROM.

\(^{37}\)In this paper, relevant Eurozone countries are Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, and Spain. Non-Eurozone EU countries refer to Sweden, Denmark, and the UK. And Non-EU countries include Australia, Canada, Japan, New Zealand, Norway, Switzerland, and the US.
appear in the exchange rate and inflation dynamics only when the long-run inflation coefficient $\gamma_\pi$ is strictly greater than unity. Clarida, Galí, and Gertler (1998, 2000) provide important empirical evidence for the existence of a structural break in the US Taylor Rule. Put it differently, they show that $\gamma_\pi$ was strictly less than one during the pre-Volker era, while it became strictly greater than unity in the Volker-Greenspan era.

We implement similar GMM estimations for (2.4) as in Clarida, Galí, and Gertler (2000)\textsuperscript{38}\textsuperscript{39} with longer sample period and report the results in Table 2.1 (see the note on Table 2.1 for detailed explanation). We use combinations of two different inflation measures and two output gap measures for three different sub-samples. Most coefficients were highly significant and specification tests by $J$-test were not rejected. More importantly, our requirement for the existence of a saddle path equilibrium met only for the Volker-Greenspan era. Therefore, we may conclude that this provides some empirical justification for the choice of our sample period.

Half-life estimates by the single equation, the Hansen-Sargent method, and finally the system method are reported in Tables 2.2-2.4 for GDP deflator-based real exchange rates and in Tables 2.5-2.7 for CPI-based real exchange rates. We implemented estimations using both gap measures, but report the full estimates with unemployment gaps along with the median half-life with quadratically detrended real GDP gaps (Median$_{x_\pi}$) in order to save space\textsuperscript{40}. We chose the degrees of autoregressive

\textsuperscript{38}They used GDP deflator inflation along with the CBO output gaps (and HP detrended gaps).

\textsuperscript{39}Unlike them, we assume that the Fed targets current output gap rather than future deviations. However, this doesn’t make any significant changes to our results. And we include one lag of interest rate rather than two lags for simplicity.

\textsuperscript{40}The results with quadratically detrended real GDP gaps were quantitatively similar.
process of each fundamental term in (2.17) by the conventional BIC\textsuperscript{41}. Standard errors were adjusted using the QS kernel estimator with automatic bandwidth selection in order to deal with unknown serial correlation problems\textsuperscript{42}.

One interesting finding is that both the Hansen-Sargent method and the system method provide much shorter half-life estimates compared with ones from the single equation method (see Tables 2.2-2.7). The median half-life estimates from the single equation method were 3.06 and 2.59 years for GDP deflator-based and CPI-based real exchange rates, respectively. However, we obtained 1.49 and 1.37 years from the system method, and less than 1 year median half-life estimates from the Hansen-Sargent method\textsuperscript{43}. Interestingly, these estimates are roughly consistent with the average half-life estimates from the micro-data evidence by Crucini and Shintani (2004). For the OECD countries, their baseline half-life estimates for traded good prices were 1.0 years, while 1.1 and 1.6 years for all and non-traded good prices.

Compared with Kim, Ogaki, and Yang (2003), our half-life estimates are longer than theirs. Using the system method with the restriction that was derived from the money demand function rather than the Taylor Rule, they implemented half-life estimations for CPI-based real exchange rates. Their half-life estimates range from 0.12 to 2.22 years, and their median half-life estimates for the full sample period was

\textsuperscript{41}Chosen rs range from 2 to 4. We didn’t consider the r greater than 4 in order to avoid the potential ”many instrument” problem.

\textsuperscript{42}We impose the corresponding Taylor Rule coefficient GMM estimates (first stage estimation) throughout the whole estimation since joint estimation can be quite burdensome. We need to correct the standard errors by redefining the second step weighting matrix. However, since we acquired significant efficiency improvement over the single equation method, we believe such correction may not change the results much.

\textsuperscript{43}The results were quantitatively similar when the quadratically detrended real GDP series were used. See Median\textsubscript{x} values for details.
about 0.35 year\textsuperscript{44}, which is much shorter than our 1.37 year median half-life. However, our median half-life estimates are still shorter than the 3 to 5 year consensus half-life, and we believe that ours are reasonably short considering micro-data evidence\textsuperscript{45}. Furthermore, our results from the Hansen-Sargent method were more reliable than theirs as they obtained unreasonable estimates often.

Regarding efficiency, we obtained substantial efficiency gains from both the Hansen-Sargent method and the system method over the single equation method. As pointed out by Murray and Papell (2002), half-life estimates from single equation methods may provide virtually no useful information due to wide confidence intervals. Our half-life estimates from the single equation method were consistent with such a view. For most GDP deflator based real exchange rates with the exceptions of Australia, Denmark, and New Zealand and all CPI-based real exchange rates with the exception of Australia, standard errors for half-life estimates were very big. However, when we implement estimations by the Hansen-Sargent method and the system method, the standard errors were reduced significantly. Our results can be also considered as great improvement over Kim, Ogaki, and Yang (2003) who acquired only limited success in efficiency gains. We got even more dramatic improvement in efficiency for estimates by the Hansen-Sargent method, since their estimates by the Hansen-Sargent method produced extremely wide confidence intervals often.

Finally, we report likelihood ratio type tests and corresponding $p$-values. In the estimations by the system method, we impose a restriction that $\alpha$ in orthogonality conditions (2.18) and (2.19) are same. We can construct a likelihood ratio type test

\textsuperscript{44}They don’t report the median estimate. Their median half-life estimate was obtained by our calculation.

\textsuperscript{45}For the survey on other micro-data evidence, see Crucini and Shintani (2004).
statistic from $J$-statistics for the unrestricted estimation and the estimation with such a restriction, which obeys $\chi^2$ distribution asymptotically with degree of freedom 1. In most cases, our specification were not rejected.

Based on our results shown here, we believe that the Hansen-Sargent method and the system method that incorporate a forward looking version Taylor Rule may greatly help resolving two aforementioned issues (from single equation approaches) in the PPP literature. Furthermore, it seems that deriving restrictions from the Taylor Rule rather than the money demand function was an appropriate choice based on our estimation results from each approach.

2.4 Conclusion

This paper has addressed two perennial issues of PPP literature, namely, unreasonably long half-life estimates of PPP deviations (Rogoff, 1996) and extremely wide confidence intervals of half-life point estimates (Murray and Papell, 2002). As a means of resolving these issues, Kim, Ogaki, and Yang (2003) have suggested using a system method utilizing economic theories or models in implementing estimations. Using post Bretton Woods CPI-based real exchange rates, they estimated much shorter half-lives (0.12 to 2.22 years) than the three- to five- year consensus half-life, but with only limited efficiency gains.

In contrast, we incorporated a forward-looking version of the Taylor Rule in the model, rather than the money demand function, in order to derive restrictions for half-life estimation. We obtained reasonably short median half-life estimates for both GDP deflator- and CPI-based real exchange rates for 19 developed countries, and these half-life estimates were shorter than the current consensus half-life estimates of
three to five years. Interestingly, our median half-life estimates are roughly consistent with the micro-data evidence of Crucini and Shintani (2004).

With regard to efficiency, our half-life estimation using the Hansen-Sargent method and the system method of Kim, Ogaki, and Yang (2003) greatly outperformed the single-equation method. In these cases, we found much smaller standard errors for virtually all of our half-life estimates, as compared to those obtained using the single-equation method. Our use of the Taylor Rule, in place of the money demand function used by Kim, Ogaki, and Yang (2003), seems to have been an appropriate choice, given that we obtained substantial efficiency gains, even compared with their results.

Finally, our model has potentially important implications for exchange rate and inflation dynamics. In particular, we showed that rationally expected future fundamental variables enter these dynamics only when the long-run Taylor Rule inflation coefficient is greater than one. When it is less than one, exchange rate and inflation dynamics may be explained only by past fundamental variables and any martingale difference sequences.
Table 2.1: GMM Estimation of the US Taylor Rule

<table>
<thead>
<tr>
<th>Inflation Deviation</th>
<th>Sample</th>
<th>$\gamma_\pi$ (s.e.)</th>
<th>$\gamma_\chi$ (s.e.)</th>
<th>$\rho$ (s.e.)</th>
<th>$J$ (p&lt;sub&gt;v&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP Deflator</td>
<td>60:1-79:2</td>
<td>0.998 (0.033)</td>
<td>0.670 (0.023)</td>
<td>0.417 (0.014)</td>
<td>14.418 (0.851)</td>
</tr>
<tr>
<td></td>
<td>79:3-98:4</td>
<td>2.529 (0.068)</td>
<td>0.234 (0.086)</td>
<td>0.632 (0.017)</td>
<td>16.292 (0.753)</td>
</tr>
<tr>
<td></td>
<td>79:3-03:4</td>
<td>2.900 (0.128)</td>
<td>0.216 (0.096)</td>
<td>0.686 (0.025)</td>
<td>22.191 (0.386)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>60:1-79:2</td>
<td>0.920 (0.015)</td>
<td>0.191 (0.008)</td>
<td>0.213 (0.024)</td>
<td>12.647 (0.921)</td>
</tr>
<tr>
<td></td>
<td>79:3-98:4</td>
<td>3.311 (0.240)</td>
<td>0.222 (0.055)</td>
<td>0.744 (0.034)</td>
<td>14.998 (0.823)</td>
</tr>
<tr>
<td></td>
<td>79:3-03:4</td>
<td>3.532 (0.304)</td>
<td>0.255 (0.055)</td>
<td>0.773 (0.036)</td>
<td>16.904 (0.717)</td>
</tr>
<tr>
<td>Real GDP</td>
<td>60:1-79:2</td>
<td>0.573 (0.058)</td>
<td>1.529 (0.232)</td>
<td>0.723 (0.037)</td>
<td>16.604 (0.735)</td>
</tr>
<tr>
<td></td>
<td>79:3-98:4</td>
<td>2.903 (0.166)</td>
<td>0.084 (0.058)</td>
<td>0.742 (0.021)</td>
<td>15.949 (0.773)</td>
</tr>
<tr>
<td></td>
<td>79:3-03:4</td>
<td>2.935 (0.237)</td>
<td>0.144 (0.112)</td>
<td>0.751 (0.027)</td>
<td>19.144 (0.576)</td>
</tr>
<tr>
<td>CPI</td>
<td>60:1-79:2</td>
<td>0.944 (0.061)</td>
<td>0.255 (0.041)</td>
<td>0.594 (0.045)</td>
<td>17.284 (0.694)</td>
</tr>
<tr>
<td></td>
<td>79:3-98:4</td>
<td>2.941 (0.153)</td>
<td>0.125 (0.038)</td>
<td>0.806 (0.012)</td>
<td>16.334 (0.751)</td>
</tr>
<tr>
<td></td>
<td>79:3-03:4</td>
<td>2.539 (0.143)</td>
<td>0.105 (0.069)</td>
<td>0.793 (0.027)</td>
<td>18.414 (0.623)</td>
</tr>
</tbody>
</table>

Notes: i) Inflations are quarterly changes in log price level ($\ln p_t - \ln p_{t-1}$). ii) Quadratically detrended real GDP series are used for real GDP output deviations. iii) Unemployment gaps are 5 year backward moving average unemployment rates minus current unemployment rates. iv) The set of instruments includes four lags of federal funds rate, inflation, output deviation, long-short interest rate spread, commodity price inflation, and M2 growth rate. v) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vi) $J$ in last column refers to $J$-test statistics, and corresponding $p$-values are in parentheses.
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>2.603 (1.251)</td>
<td>0.936 (0.030)</td>
</tr>
<tr>
<td>Austria</td>
<td>3.909 (3.515)</td>
<td>0.957 (0.038)</td>
</tr>
<tr>
<td>Belgium</td>
<td>3.332 (2.786)</td>
<td>0.949 (0.041)</td>
</tr>
<tr>
<td>Canada</td>
<td>4.833 (4.223)</td>
<td>0.965 (0.030)</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.076 (0.398)</td>
<td>0.851 (0.051)</td>
</tr>
<tr>
<td>Finland</td>
<td>3.393 (2.650)</td>
<td>0.950 (0.038)</td>
</tr>
<tr>
<td>France</td>
<td>2.788 (2.171)</td>
<td>0.940 (0.045)</td>
</tr>
<tr>
<td>Germany</td>
<td>3.589 (3.335)</td>
<td>0.953 (0.043)</td>
</tr>
<tr>
<td>Italy</td>
<td>4.217 (4.300)</td>
<td>0.960 (0.040)</td>
</tr>
<tr>
<td>Japan</td>
<td>3.897 (2.889)</td>
<td>0.957 (0.032)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.876 (1.238)</td>
<td>0.912 (0.056)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>2.032 (0.948)</td>
<td>0.918 (0.037)</td>
</tr>
<tr>
<td>Norway</td>
<td>1.213 (0.630)</td>
<td>0.867 (0.064)</td>
</tr>
<tr>
<td>Portugal</td>
<td>13.47 (28.94)</td>
<td>0.987 (0.027)</td>
</tr>
<tr>
<td>Spain</td>
<td>3.851 (2.983)</td>
<td>0.956 (0.033)</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.890 (1.498)</td>
<td>0.912 (0.066)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.795 (2.011)</td>
<td>0.940 (0.042)</td>
</tr>
<tr>
<td>UK</td>
<td>2.524 (1.967)</td>
<td>0.934 (0.050)</td>
</tr>
<tr>
<td>Median (Avg)</td>
<td>3.064 (3.516)</td>
<td>0.945</td>
</tr>
</tbody>
</table>

Table 2.2: Deflator-Based Real Exchange Rates: Single Equation Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Hansen-Sargent Half Life (s.e.)</th>
<th>Hansen-Sargent α (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.842 (0.658)</td>
<td>0.814 (0.131)</td>
</tr>
<tr>
<td>Austria</td>
<td>0.808 (0.185)</td>
<td>0.807 (0.040)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.524 (0.098)</td>
<td>0.718 (0.044)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.976 (0.385)</td>
<td>0.837 (0.059)</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.762 (0.077)</td>
<td>0.797 (0.018)</td>
</tr>
<tr>
<td>Finland</td>
<td>0.956 (0.155)</td>
<td>0.834 (0.024)</td>
</tr>
<tr>
<td>France</td>
<td>0.515 (0.075)</td>
<td>0.714 (0.035)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.698 (0.100)</td>
<td>0.780 (0.028)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.694 (0.055)</td>
<td>0.779 (0.015)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.649 (0.038)</td>
<td>0.766 (0.012)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.902 (0.561)</td>
<td>0.825 (0.099)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.638 (0.045)</td>
<td>0.762 (0.015)</td>
</tr>
<tr>
<td>Norway</td>
<td>0.813 (0.142)</td>
<td>0.808 (0.030)</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.795 (0.062)</td>
<td>0.804 (0.014)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.566 (0.036)</td>
<td>0.736 (0.014)</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.435 (0.064)</td>
<td>0.671 (0.039)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.617 (0.049)</td>
<td>0.755 (0.017)</td>
</tr>
<tr>
<td>UK</td>
<td>0.677 (0.066)</td>
<td>0.774 (0.019)</td>
</tr>
<tr>
<td>Median (Avg)</td>
<td>0.696 (0.715)</td>
<td>0.780</td>
</tr>
<tr>
<td>Median_x (Avg)</td>
<td>0.711 (0.805)</td>
<td>0.784</td>
</tr>
</tbody>
</table>

Table 2.3: Deflator-Based Real Exchange Rate: Hansen-Sargent Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
<th>LR ($pv$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.735 (0.521)</td>
<td>0.905 (0.027)</td>
<td>1.735 (0.188)</td>
</tr>
<tr>
<td>Austria</td>
<td>2.839 (1.494)</td>
<td>0.941 (0.030)</td>
<td>1.931 (0.165)</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.703 (0.732)</td>
<td>0.903 (0.040)</td>
<td>1.345 (0.246)</td>
</tr>
<tr>
<td>Canada</td>
<td>3.209 (1.633)</td>
<td>0.947 (0.026)</td>
<td>1.264 (0.261)</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.761 (0.089)</td>
<td>0.796 (0.021)</td>
<td>0.397 (0.529)</td>
</tr>
<tr>
<td>Finland</td>
<td>1.368 (0.257)</td>
<td>0.881 (0.021)</td>
<td>0.346 (0.556)</td>
</tr>
<tr>
<td>France</td>
<td>1.875 (1.166)</td>
<td>0.912 (0.052)</td>
<td>0.338 (0.561)</td>
</tr>
<tr>
<td>Germany</td>
<td>2.249 (1.354)</td>
<td>0.926 (0.043)</td>
<td>1.945 (0.163)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.719 (0.060)</td>
<td>0.786 (0.016)</td>
<td>0.968 (0.325)</td>
</tr>
<tr>
<td>Japan</td>
<td>1.614 (0.494)</td>
<td>0.898 (0.030)</td>
<td>7.996 (0.005)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.664 (1.056)</td>
<td>0.901 (0.060)</td>
<td>0.097 (0.755)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.786 (0.110)</td>
<td>0.802 (0.025)</td>
<td>4.030 (0.045)</td>
</tr>
<tr>
<td>Norway</td>
<td>0.837 (0.142)</td>
<td>0.813 (0.028)</td>
<td>0.156 (0.693)</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.802 (0.023)</td>
<td>0.806 (0.005)</td>
<td>1.546 (0.214)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.533 (0.021)</td>
<td>0.722 (0.009)</td>
<td>0.182 (0.670)</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.525 (0.061)</td>
<td>0.719 (0.028)</td>
<td>2.008 (0.156)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.705 (0.064)</td>
<td>0.782 (0.018)</td>
<td>2.686 (0.101)</td>
</tr>
<tr>
<td>UK</td>
<td>1.626 (0.673)</td>
<td>0.899 (0.040)</td>
<td>3.197 (0.074)</td>
</tr>
<tr>
<td>Median (Avg)</td>
<td>1.491 (1.419)</td>
<td>0.890</td>
<td>-</td>
</tr>
<tr>
<td>Median$_x$ (Avg)</td>
<td>1.726 (1.865)</td>
<td>0.905</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: i) The US$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Median$_x$ is the median when quadratically detrended output gaps are used. iv) Chosen parameters are $(\gamma_\pi, \gamma_x, \rho) = (3.311, 0.222, 0.744)$ for countries where observations end in 1998.IV, while $(3.532, 0.255, 0.773)$ for countries where observations end in 2003.IV. v) Sample periods are 1979.II $\sim$1998.IV for most Eurozone countries (Austria, Finland, France, Germany, Italy, Netherlands, Portugal, Spain), and 1979.III$\sim$2003.IV for one Non-Eurozone EU countries (UK) and most Non-EU countries (Australia, Canada, Japan, Norway, Switzerland, US). Among Eurozone countries, Belgium’s sample spans from 1980.I to 1998.IV. Among Non-Eurozone EU countries, Denmark’s observations span from 1984.IV to 2003.IV, and Sweden’s sample period is from 1980.I to 2001.I. Among Non-EU countries, New Zealand’s sample period starts from 1982.II until 2003.IV. vi) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vii) LR in last column refers to the likelihood ratio type test statistics and corresponding $p$-values are in parentheses.

Table 2.4: Deflator-Based Real Exchange Rate: System Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>2.572 (1.188)</td>
<td>0.935 (0.029)</td>
</tr>
<tr>
<td>Austria</td>
<td>2.608 (1.798)</td>
<td>0.936 (0.043)</td>
</tr>
<tr>
<td>Belgium</td>
<td>2.038 (1.096)</td>
<td>0.918 (0.042)</td>
</tr>
<tr>
<td>Canada</td>
<td>5.970 (5.448)</td>
<td>0.971 (0.026)</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.351 (1.272)</td>
<td>0.929 (0.037)</td>
</tr>
<tr>
<td>Finland</td>
<td>3.051 (2.235)</td>
<td>0.945 (0.039)</td>
</tr>
<tr>
<td>France</td>
<td>2.015 (1.241)</td>
<td>0.918 (0.049)</td>
</tr>
<tr>
<td>Germany</td>
<td>1.841 (1.136)</td>
<td>0.910 (0.053)</td>
</tr>
<tr>
<td>Italy</td>
<td>2.607 (2.011)</td>
<td>0.936 (0.048)</td>
</tr>
<tr>
<td>Japan</td>
<td>3.188 (2.182)</td>
<td>0.947 (0.035)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.688 (1.003)</td>
<td>0.902 (0.055)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>3.142 (0.984)</td>
<td>0.946 (0.016)</td>
</tr>
<tr>
<td>Norway</td>
<td>2.142 (1.268)</td>
<td>0.922 (0.044)</td>
</tr>
<tr>
<td>Portugal</td>
<td>5.538 (5.930)</td>
<td>0.969 (0.032)</td>
</tr>
<tr>
<td>Spain</td>
<td>3.704 (3.109)</td>
<td>0.954 (0.037)</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.773 (7.641)</td>
<td>0.970 (0.039)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.976 (1.108)</td>
<td>0.916 (0.045)</td>
</tr>
<tr>
<td>UK</td>
<td>1.796 (1.170)</td>
<td>0.908 (0.057)</td>
</tr>
<tr>
<td>Median (Avg)</td>
<td>2.590 (3.000)</td>
<td>0.936</td>
</tr>
</tbody>
</table>

Table 2.5: CPI-Based Real Exchange Rate: Single Equation Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Hansen-Sargent Half Life (s.e.)</th>
<th>Hansen-Sargent $\alpha$ (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.707 (0.063)</td>
<td>0.783 (0.017)</td>
</tr>
<tr>
<td>Austria</td>
<td>0.624 (0.057)</td>
<td>0.758 (0.019)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.604 (0.070)</td>
<td>0.750 (0.025)</td>
</tr>
<tr>
<td>Canada</td>
<td>1.797 (2.778)</td>
<td>0.908 (0.135)</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.648 (0.203)</td>
<td>0.765 (0.064)</td>
</tr>
<tr>
<td>Finland</td>
<td>1.284 (0.771)</td>
<td>0.874 (0.071)</td>
</tr>
<tr>
<td>France</td>
<td>0.484 (0.127)</td>
<td>0.699 (0.066)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.717 (0.072)</td>
<td>0.874 (0.071)</td>
</tr>
<tr>
<td>Italy</td>
<td>1.156 (0.483)</td>
<td>0.861 (0.054)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.526 (0.073)</td>
<td>0.719 (0.033)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.735 (0.125)</td>
<td>0.790 (0.032)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.613 (0.078)</td>
<td>0.754 (0.027)</td>
</tr>
<tr>
<td>Norway</td>
<td>0.336 (0.117)</td>
<td>0.597 (0.107)</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.525 (0.047)</td>
<td>0.719 (0.021)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.561 (0.301)</td>
<td>0.734 (0.122)</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.931 (0.163)</td>
<td>0.830 (0.027)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.480 (0.098)</td>
<td>0.697 (0.051)</td>
</tr>
<tr>
<td>UK</td>
<td>0.428 (0.131)</td>
<td>0.667 (0.083)</td>
</tr>
<tr>
<td>Median (Avg)</td>
<td>0.619 (0.731)</td>
<td>0.756</td>
</tr>
<tr>
<td>Median $\times$ (Avg)</td>
<td>0.504 (0.558)</td>
<td>0.712</td>
</tr>
</tbody>
</table>

Table 2.6: CPI-Based Real Exchange Rate: Hansen-Sargent Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
<th>LR (pv)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Australia</strong></td>
<td>1.422 (0.282)</td>
<td>0.885 (0.021)</td>
<td>2.015 (0.156)</td>
</tr>
<tr>
<td><strong>Austria</strong></td>
<td>0.732 (0.073)</td>
<td>0.789 (0.019)</td>
<td>4.265 (0.039)</td>
</tr>
<tr>
<td><strong>Belgium</strong></td>
<td>0.679 (0.066)</td>
<td>0.775 (0.019)</td>
<td>3.501 (0.061)</td>
</tr>
<tr>
<td><strong>Canada</strong></td>
<td>8.406 (7.994)</td>
<td>0.980 (0.019)</td>
<td>0.055 (0.815)</td>
</tr>
<tr>
<td><strong>Denmark</strong></td>
<td>1.929 (0.838)</td>
<td>0.914 (0.036)</td>
<td>2.131 (0.144)</td>
</tr>
<tr>
<td><strong>Finland</strong></td>
<td>4.264 (4.322)</td>
<td>0.960 (0.040)</td>
<td>0.736 (0.391)</td>
</tr>
<tr>
<td><strong>France</strong></td>
<td>1.323 (0.537)</td>
<td>0.877 (0.047)</td>
<td>0.304 (0.581)</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td>0.844 (0.124)</td>
<td>0.814 (0.025)</td>
<td>0.400 (0.527)</td>
</tr>
<tr>
<td><strong>Italy</strong></td>
<td>1.414 (0.481)</td>
<td>0.885 (0.037)</td>
<td>0.499 (0.527)</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td>0.575 (0.056)</td>
<td>0.740 (0.022)</td>
<td>5.946 (0.015)</td>
</tr>
<tr>
<td><strong>Netherlands</strong></td>
<td>0.754 (0.124)</td>
<td>0.795 (0.030)</td>
<td>1.656 (0.198)</td>
</tr>
<tr>
<td><strong>New Zealand</strong></td>
<td>2.240 (0.598)</td>
<td>0.926 (0.019)</td>
<td>0.743 (0.389)</td>
</tr>
<tr>
<td><strong>Norway</strong></td>
<td>1.636 (0.676)</td>
<td>0.899 (0.039)</td>
<td>4.329 (0.037)</td>
</tr>
<tr>
<td><strong>Portugal</strong></td>
<td>0.932 (0.078)</td>
<td>0.830 (0.013)</td>
<td>1.410 (0.235)</td>
</tr>
<tr>
<td><strong>Spain</strong></td>
<td>4.218 (4.236)</td>
<td>0.960 (0.040)</td>
<td>1.996 (0.158)</td>
</tr>
<tr>
<td><strong>Sweden</strong></td>
<td>4.949 (6.007)</td>
<td>0.966 (0.045)</td>
<td>0.904 (0.342)</td>
</tr>
<tr>
<td><strong>Switzerland</strong></td>
<td>1.169 (0.404)</td>
<td>0.862 (0.044)</td>
<td>0.160 (0.689)</td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td>0.579 (0.108)</td>
<td>0.741 (0.041)</td>
<td>8.139 (0.004)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Median (Avg)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.369 (2.115)</td>
<td>0.881</td>
<td>-</td>
</tr>
</tbody>
</table>

|          | Median$_x$ (Avg) | 0.859 | -     |

Notes: i) The US$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Median$_x$ is the median when quadratically detrended output gaps are used. iv) Chosen parameters are $(\gamma_\pi, \gamma_x, \rho) = (2.941, 0.125, 0.866)$ for countries where observations end in 1998.IV, while $(2.539, 0.105, 0.793)$ for countries where observations end in 2003.IV. v) Sample periods are 1979.III ~1998.IV for Eurozone countries (Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, Spain), and 1979.III~2003.IV for Non-Eurozone EU countries (Denmark, UK) except Sweden, and Non-EU countries (Australia, Canada, Japan, New Zealand, Norway, Switzerland, US). Sweden’s observations span from 1979.III to 2001.IV. vi) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vii) LR in last column refers to the likelihood ratio type test statistics and corresponding p-values are in parentheses.

Table 2.7: CPI-Based Real Exchange Rate: System Method
CHAPTER 3

HALF-LIFE ESTIMATION UNDER THE TAYLOR RULE: TWO-GOODS MODEL

3.1 Introduction

Purchasing Power Parity (PPP, henceforth) has been one of the most useful building blocks for many influential exchange rate models (e.g., Frenkel 1976, Lucas 1982, Obstfeld and Rogoff 1995). The main logic of PPP is that if goods market arbitrage can ensure broad parity over a sufficiently wide range of individual products, then the overall price indices should also exhibit similar parity across countries.

In presence of nominal rigidities, financial factors such as monetary shocks may cause temporal deviations of the real exchange rate away from its equilibrium PPP level. If PPP holds in the long-run, however, such deviations will die out eventually, since nominal shocks would be neutral in the long-run.

Accepting PPP as a valid long-run proposition\(^{46}\), Kim (2004) estimated half-lives of PPP deviations, the time required for a deviation to halfway adjust to its long-run equilibrium level, in a system method framework developed by Kim \textit{et. al.} (2003), which combines economic theories with the univariate model of real exchange rates. He obtained much shorter half-life point estimates for the non-service goods

\(^{46}\)Empirical evidence on the validity of long-run PPP is mixed. See Rogoff (1996) for a survey on this issue.
consumption deflators than those for the service goods consumption deflators\textsuperscript{47,48}, though with quite wide confidence intervals. Using the same framework, Kim and Ogaki (2004) obtained similar evidence for producer price index (PPI)- and consumer price index (CPI)-based real exchange rates\textsuperscript{49}.

However, such results are in contrast to many other researches that found only moderate or no significant difference in half-life point estimates. Wu (1996), for example, obtained around 2.1- and 2.5-year half-life estimates for wholesale price index (WPI)- and CPI-based real exchange rates, respectively. Murray and Papell (2002) also found that their half-life point estimates were consistent with Rogoff (1996)’s three- to five-year consensus half life whichever price indices are used to construct real exchange rates\textsuperscript{50}. Interestingly, Wei and Parsley (1995) reported four- to five-year half-lives even when they used 12 traded good sector deflators.

This paper constitutes a modification of the work of Kim (2004), who combined the univariate model of real exchange rates with the conventional money market equilibrium condition, which he identified using a money demand function. However, the money demand function can be quite unstable especially in the short-run. So the use of the money demand function may not have been ideal. Instead, this paper constructs a system of the exchange rate and inflation that incorporates a forward looking version of the Taylor Rule monetary policy, where the central bank sets its

\textsuperscript{47}His median half-life point estimates were 0.67 and 3.36 years for traded and non-traded good prices, respectively. The medians were calculated from his Tables 2 and 4.

\textsuperscript{48}Following Stockman and Tesar (1995), he used the non-service consumption deflator as the traded good price, while the non-traded good price was measured by the service consumption deflator. See Kim (2004) for details.

\textsuperscript{49}Both Kim (2004) and Kim and Ogaki (2004) also estimated the half-lives for the general price index using a single good approach described in Kim, Ogaki, and Yang (2003).

\textsuperscript{50}They used the current float CPI-based real exchange rates and the long-horizon WPI-based real exchange rates, which extends Lee (1976)’s data.
target interest rate based on the expected future changes in the general price index such as the GDP deflator\textsuperscript{51}.

Since the seminal work of Taylor (1993), the Taylor Rule has been one of the most popular models in the monetary policy literature. One particularly interesting point has been made by Clarida, Galí, and Gertler (1998, 2000), who found strong empirical evidence of a structural break in the Fed’s reaction function to inflation. Our model shows that consideration of such a structural break may be important in understanding real exchange rate dynamics. In what follows, we show that the rationally expected future Taylor Rule fundamentals enter the exchange rate dynamics only when the central bank responds to inflation aggressively enough\textsuperscript{52}. If not, the exchange rate dynamics can be explained only by past economic variables and any martingale difference sequences.

For half-life estimations, we consider 18 industrialized countries that provide 17 current float real exchange rates based on the PPIs and CPIs. We found that the half-life estimates for both real exchange rates were about the same (1.43- and 1.87-year average and 1.61- and 1.60-median half-life estimates for PPI- and CPI-based real exchange rates, respectively). And our half-life estimates were reasonably shorter than Rogoff (1996)’s three- to five-year consensus half-life, which alluded to the ”PPP puzzle”\textsuperscript{53}.” Interestingly, our estimates are roughly consistent with micro evidence by

\textsuperscript{51}For a model where the central bank targets the CPI inflation, see Kim (2005).

\textsuperscript{52}Engel and West (2002) also emphasized the importance of expected future Taylor Rule fundamentals in understanding real exchange rate dynamics. Mark (2005) also presented similar evidence in a learning framework.

\textsuperscript{53}The ”PPP puzzle” is the question of how one might reconcile highly volatile short-run movements of real exchange rates with unreasonably long half-life estimates or extremely slow convergence rate to PPP.
Crucini and Shintani (2004) who reported 1.1-, 1.0-, and 1.6-year half-lives for all, traded, and non-traded good prices from their baseline estimates.

More importantly, we obtained a substantial improvement in efficiency over the results of Kim (2004) as well as conventional univariate estimation approaches\textsuperscript{54}, which enables us to make statistically meaningful comparisons between the size of half-life estimates.

The rest of this paper is organized as follows. In 3.2, we construct a system of stochastic difference equations for the exchange rate and inflation explicitly incorporating a forward looking version of the Taylor Rule with interest rate smoothing policy. Then, we discuss three estimation strategies for estimable equations that are derived from the system. In 3.3, a description of the data and the estimation results are provided. 3.4 concludes.

3.2 Model Specification and Estimation Strategy

In this, we assume that there are two different baskets of products in each country. Let $p_1^t$ and $p_2^t$ be the logs of the price indices of those baskets in the home country. We further assume that the log of the general price index ($p_G^t$) is a weighted average of $p_1^t$ and $p_2^t$. That is,

$$p_G^t = \delta p_1^t + (1 - \delta)p_2^t,$$

where $\delta$ is a strictly positive weight parameter. Foreign price indices are similarly defined. One natural interpretation of the equation (3.1) is that the GDP deflator is

\textsuperscript{54}Murray and Papell (2002) argue that univariate methods provide virtually no information regarding the size of the half-lives due to extremely wide confidence intervals.
a geometric weighted average of the PPI and CPI, where the GDP basket includes the CPI and PPI baskets\textsuperscript{55}.

### 3.2.1 Gradual Adjustment Equation

Our model starts with a simple univariate stochastic process of real exchange rates. Let $p_t$ be the log of the domestic price level ($p_t^1$ or $p_t^2$), $p_t^*$ be the log of the foreign price level ($p_t^{1*}$ or $p_t^{2*}$), and $e_t$ be the log of the nominal exchange rate as the unit price of the foreign currency in terms of the domestic currency. And the log of the real exchange rate, $s_t$, is defined as $p_t^* + e_t - p_t$.

Here, we simply assume that PPP holds in the long-run without any formal econometric test\textsuperscript{56}. Put it differently, we assume that there exists a cointegrating vector $[1 - 1 - 1]'$ for a vector $[p_t, p_t^*, e_t]'$, where $p_t$, $p_t^*$, and $e_t$ are all difference stationary processes. Under this assumption, real exchange rates can be represented as the following stationary univariate autoregressive process of degree one.

$$s_{t+1} = d + \alpha s_t + \varepsilon_{t+1}, \quad (3.2)$$

where $\alpha$ is a strictly positive persistence parameter that is less than one\textsuperscript{57}.

Interestingly, Kim \textit{et. al.} (2003) showed that the equation (3.2) could be implied by the following error correction model of real exchange rates by Mussa (1982) with

\textsuperscript{55}Engel (1999) and Kakkar and Ogaki (1999) also employ this assumption.

\textsuperscript{56}The empirical evidence on the validity of PPP in the long-run is mixed. It should be noted, however, that even when we have some evidence against PPP, such outcomes may have resulted from lack of power of existing unit root tests in small samples, and are subject to observational equivalence problems.

\textsuperscript{57}This is the so-called Dickey-Fuller estimation model. It is also possible to estimate half-lives by the augmented Dickey-Fuller estimation model in order to avoid a serial correlation problem in $\varepsilon_{t+1}$. However, as shown in Murray and Papell (2002), these two methodologies provide roughly similar half-life estimates. So it seems that our AR(1) specification is not a bad approximation.
a known cointegrating relation described earlier.

\[ \Delta p_{t+1} = b [\mu - (p_t - p^*_t - e_t)] + E_t \Delta p^*_{t+1} + E_t \Delta e_{t+1}, \quad (3.3) \]

where \( \mu = E(p_t - p^*_t - e_t), \ b = 1 - \alpha, \ d = -(1 - \alpha)\mu, \ \varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p^*_{t+1} - \Delta p^*_t), \) and \( E_t \varepsilon_{t+1} = 0. \) \( E(\cdot) \) denotes the unconditional expectation operator, and \( E_t(\cdot) \) is the conditional expectation operator on \( I_t, \) the economic agent’s information set at time \( t. \)

One interpretation of (3.3) is that \( p_t \) adjusts instantaneously to the expected change in its PPP, while it adjusts to its unconditional PPP level, \( E(p^*_t + e_t) \) only slowly with the constant convergence rate \( b \ (= 1 - \alpha), \) which is a strictly positive constant less than one by construction.

### 3.2.2 Taylor Rule Model

We assume that the central bank in the home country continuously sets its optimal target interest rate (\( i^T \)) by the following forward looking version of the Taylor Rule.

\[ \hat{i}^T_t = \hat{i} + \gamma_\pi E_t \Delta p^G_{t+1} + \gamma_x x_t, \quad (3.4) \]

where \( \hat{i} \) is a constant that includes a certain long-run equilibrium real interest rate along with a target inflation rate\(^{58}\), and \( \gamma_\pi \) and \( \gamma_x \) are the long-run Taylor Rule coefficients on expected future inflation \( (E_t \Delta p^G_{t+1}) \) and current output deviations \( (x_t) \)\(^{59}\), respectively. Note that we are implicitly assuming that the central bank is targeting general inflation, which is defined as changes in a general price index such as the GDP deflator.


\(^{59}\)If we assume that the central bank responds to expected future output deviations rather than current deviations, we can simply modify the model by replacing \( x_t \) with \( E_t x_{t+1}. \) However, this does not make any significant difference to our results.
We further assume that the central bank attempts to smooth the current actual interest rate \( i_t \) by the following rule.

\[
i_t = (1 - \rho)i_t^T + \rho i_{t-1}
\]  

That is, the current actual interest rate is a weighted average of the target interest rate and the previous period’s interest rate, where \( \rho \) is the interest rate smoothing parameter. Plugging (3.4) into (3.5), we derive the following forward looking version of the Taylor Rule equation with the interest rate smoothing policy.

\[
i_t = (1 - \rho)i_t^* + (1 - \rho)\gamma_x E_t \Delta p_{t+1}^G + (1 - \rho)\gamma_x x_t + \rho i_{t-1}
\]  

Next, we assume that uncovered interest parity holds. That is,

\[
E_t \Delta e_{t+1} = i_t - i_t^*,
\]

where \( i_t^* \) is the foreign interest rate.

Without loss of generality, we assume that equations (3.2) or (3.3) hold for \( p_t^1 \) and \( p_t^{1*} \) without imposing any restriction on \( p_t^2 \) and \( p_t^{2*} \). That is, PPP is assumed to hold in the long-run for the prices of first baskets of goods\(^{60}\). With this assumption, we rewrite (3.3) as the following equation in level variables.

\[
p_{t+1}^1 = b \mu + E_t e_{t+1} + (1 - b)p_t^1 - (1 - b)e_t + E_t p_{t+1}^{1*} - (1 - b)p_t^{1*}
\]

Taking differences and rearranging it, (3.8) can be rewritten as follows.

\[
\Delta p_{t+1}^1 = E_t \Delta e_{t+1} + \alpha \Delta p_t^1 - \alpha \Delta e_t + [E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t],
\]

where \( \alpha = 1 - b \) and \( \eta_t = \eta_{1,t} + \eta_{2,t} = (e_t - E_{t-1}e_t) + (p_t^{T*} - E_{t-1}p_t^{T*}) \).

\(^{60}\)This does not mean that PPP doesn’t hold for the second baskets of goods.
Since we are interested in the dynamic behavior of $\Delta p^1_{t+1}$ rather than $\Delta p^G_{t+1}$, we rewrite (3.6) in terms of $\Delta p^1_{t+1}$ using (3.1). That is,

$$i_t = \iota + \delta \gamma^s \pi E_t \Delta p^1_{t+1} + \gamma^s x_t + \rho \iota_{t-1} + (1 - \delta) \gamma^s \pi E_t \Delta p^2_{t+1}, \quad (3.10)$$

where $\iota = (1 - \rho) \tilde{i}$, and $\gamma^s = (1 - \rho) \gamma^x$ and $\gamma^s = (1 - \rho) \gamma^x$ are short-run Taylor Rule coefficients. Combining (3.7) and (3.10), we obtain the following.

$$E_t \Delta e_{t+1} = \iota + \delta \gamma^s \pi E_t \Delta p^1_{t+1} + \gamma^s x_t + \rho i_{t-1} + (1 - \delta) \gamma^s \pi E_t \Delta p^2_{t+1} - i^*_t \quad (3.11)$$

Denoting $\gamma^1_{\pi} = \delta \gamma^s$ and $\gamma^2_{\pi} = (1 - \delta) \gamma^s$, we construct the following system of stochastic difference equations from (3.9), (3.10) and (3.11).

$$\begin{bmatrix} 1 & -1 & 0 \\ -\gamma^1_{\pi} & 1 & 0 \\ -\gamma^2_{\pi} & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t \Delta p^1_{t+1} \\ E_t \Delta e_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha & 0 \\ 0 & 0 & \rho \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \Delta p^1_t \\ \Delta e_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} E_t \Delta p^1_{t+1} - \alpha \Delta p^1_{t+1} + \eta_t \\ \iota + \gamma^2_{\pi} E_t \Delta p^2_{t+1} + \gamma^s x_t - i^*_t \\ \iota + \gamma^2_{\pi} E_t \Delta p^2_{t+1} + \gamma^s x_t \end{bmatrix} \quad (3.12)$$

For notational simplicity, let’s rewrite (3.12) as the following equation in matrix.

$$\mathbf{A} E_t \mathbf{y}_{t+1} = \mathbf{B} \mathbf{y}_t + \mathbf{x}_t, \quad (3.13)$$

Note that $\mathbf{A}$ is nonsingular, and thus has a well-defined inverse. Thus, (3.13) can be rewritten as follows.

$$E_t \mathbf{y}_{t+1} = \mathbf{A}^{-1} \mathbf{B} \mathbf{y}_t + \mathbf{A}^{-1} \mathbf{x}_t \quad (3.14)$$

$$= \mathbf{D} \mathbf{y}_t + \mathbf{c}_t,$$

where $\mathbf{D} = \mathbf{A}^{-1} \mathbf{B}$, $\mathbf{c}_t = \mathbf{A}^{-1} \mathbf{x}_t$. By the eigenvalue decomposition, (3.14) can be expressed as

$$E_t \mathbf{y}_{t+1} = \mathbf{V} \Lambda \mathbf{V}^{-1} \mathbf{y}_t + \mathbf{c}_t, \quad (3.15)$$
where \( D = V \Lambda V^{-1} \) and
\[
V = \begin{bmatrix}
1 & 1 \\
\frac{\alpha \gamma s^1}{\alpha - \rho} & 1 \\
\frac{\alpha \gamma s^1}{\alpha - \rho} & 1
\end{bmatrix}, \quad \Lambda = \begin{bmatrix}
\alpha & 0 & 0 \\
0 & \frac{\rho}{1 - \gamma s^1} & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Premultiplying (3.15) by \( V^{-1} \),
\[
E_t z_{t+1} = \Lambda z_t + h_t,
\] (3.16)

where \( z_t = V^{-1}y_t \) and \( h_t = V^{-1}c_t \).

Note that, among non-zero eigenvalues in \( \Lambda \), \( \alpha \) is between 0 and 1 by definition, while \( \frac{\rho}{1 - \gamma s^1} \) is greater than unity as long as \( 1 < \delta \gamma \pi < \frac{1}{1 - \rho} \). Therefore, if the domestic central bank’s systematic response to inflation is aggressive enough \( (\delta \gamma \pi > 1)^{61} \), the system of stochastic difference equations (3.15) will have a saddle path equilibrium, in which rationally expected future fundamental variables appear in the exchange rate and inflation dynamics. Otherwise, the system would follow a purely backward looking solution path to be completely determined only by past fundamental variables and any martingale difference sequences.

Under the assumption of \( \delta \gamma \pi > 0 \), it turns out that the solution to (3.15) satisfies the following relation (see Appendix for its derivation).
\[
\Delta e_{t+1} = \bar{\iota} + \frac{\alpha \gamma s^1}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma s^1}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha \gamma s^1}{\alpha - \rho} (\frac{\alpha - \rho}{\rho} \bar{\iota}^* + \gamma s^1 (\alpha - \rho) \sum_{j=0}^{\infty} \left( \frac{1 - \gamma s^1}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1},
\] (3.17)

where,
\[
\bar{\iota} = \frac{\alpha \gamma s^1 - (\alpha - \rho)}{(\alpha - \rho)(\gamma s^1 - (1 - \rho))^t}.
\]

\(^{61}\)The condition \( \delta \gamma \pi < \frac{1}{1 - \rho} \) is easily met for all sample periods we consider in this paper.
\[ E_t f_{t+j} = -(E_t r_{t+j} - E_t \Delta p_{t+j+1}^1) + \frac{1 - \delta}{\delta} E_t \Delta p_{t+j+1}^2 + \frac{\gamma_x}{\delta \gamma_{\pi}} E_t x_{t+j} \]

\[ = -E_t r_{t+j} + \frac{1 - \delta}{\delta} E_t \Delta p_{t+j+1}^2 + \frac{\gamma_x}{\delta \gamma_{\pi}} E_t x_{t+j}, \]

\[ \omega_{t+1} = \frac{\gamma^{s1}_x (\alpha \gamma^{s1}_\pi - (\alpha - \rho))}{\rho (\alpha - \rho)} \sum_{j=0}^{\infty} \left( 1 - \frac{\gamma^{s1}_\pi}{\rho} \right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1}) \]

\[ + \frac{\gamma^{s1}_x}{\alpha - \rho} \eta_{t+1} - \frac{\alpha \gamma^{s1}_\pi - (\alpha - \rho)}{\alpha - \rho} v_{t+1}, \]

and,

\[ E_t \omega_{t+1} = 0 \]

Or, (3.17) can be rewritten with full parameter specification as follows.

\[ \Delta e_{t+1} = \bar{\iota} + \frac{\alpha \delta (1 - \rho) \gamma_{\pi}}{\alpha - \rho} \Delta p_{t+1}^1 - \frac{\alpha \delta (1 - \rho) \gamma_{\pi}}{\alpha - \rho} \Delta p_{t+1}^1 \]

\[ + \frac{\alpha \delta (1 - \rho) \gamma_{\pi} - (\alpha - \rho)}{\alpha - \rho} \eta_{t+1} \]

\[ + \frac{\delta (1 - \rho) \gamma_{\pi} (\alpha \delta (1 - \rho) \gamma_{\pi} - (\alpha - \rho))}{\rho (\alpha - \rho)} \sum_{j=0}^{\infty} \left( 1 - \frac{\delta (1 - \rho) \gamma_{\pi}}{\rho} \right)^j E_t f_{t+j+1} \]

\[ + \omega_{t+1}, \]

Note that one of the key variables \( f_t \) is a proxy that summarizes the rationally expected future fundamental variables such as foreign real interest rates \( (r_t^*) \) and domestic output deviations.

It is important to realize that if \( \delta \gamma_{\pi} \) is strictly less than unity, the restriction implied by (3.18) may not be valid, since the system would have a purely backward looking equilibrium rather than a saddle path equilibrium\(^{62}\). However, if we believe that the central bank has been reacting to inflation aggressively enough (e.g., post-Volker era), rationally expected future Taylor Rule fundamentals should be explicitly

\(^{62}\)In that case, the conventional vector autoregressive (VAR) estimation method may apply.
considered in half-life estimation procedures. In a nutshell, real exchange rate dynamics critically depends on the size of $\gamma_\pi$ given $\delta$. As will be shown later, we will provide some supporting evidence on such a requirement for the existence of a saddle path equilibrium for the sample period we consider$^{63}$.

### 3.2.3 Estimation Strategy

We discuss three estimation strategies here: univariate equation method; GMM estimation by Hansen and Sargent (1980, 1982); and GMM system estimation by Kim et al. (2003).

#### Univariate Equation Method

First approach is the conventional univariate equation approach that is based on the equations (3.2) or (3.3). Assuming that PPP holds for $p_t^1$ and $p_t^{1\ast}$, the equation (3.3) can be transformed to the following estimable equation.

$$
\Delta p_{t+1}^1 = b [\mu - (p_t^1 - p_t^{1\ast} - e_t)] + \Delta p_{t+1}^{1\ast} + \Delta e_{t+1} + \varepsilon_{t+1},
$$

(3.19)

where $\varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (E_t \Delta e_{t+1} - \Delta e_{t+1}) + (E_t \Delta p_{t+1}^{1\ast} - \Delta p_{t+1}^{1\ast})$ and $E_t \varepsilon_{t+1} = 0$.

It is straightforward to show that (3.19) implies (3.2).

Note that the convergence parameter $b$ or the persistence parameter $\alpha$ ($= 1 - b$) can be consistently estimated by the conventional least squares method under the

$^{63}$In contrast to Clarida, Gali, and Gertler (1998, 2000), Taylor (1999a), and Judd and Rudebusch (1998), Orphanides (2001) found the estimates of the inflation coefficient $\gamma_\pi$ to be consistently greater than one in both the pre- and post-Volker regimes. In any case, our model specification is still valid.
maintained cointegration assumption\(^{64}\) as long as there’s no measurement error\(^{65}\).

Once we get the point estimate of \(\alpha\), the half-life of the real exchange rate can be obtained by \(\frac{\ln(2)}{\ln(\alpha)}\), and standard errors can be calculated by the delta method.

**GMM Estimation by Hansen and Sargent (1980, 1982)**

Our second estimation strategy deals with the equations (3.17) or (3.18). It should be noted that (3.17) has an infinite sum of rationally expected discounted future fundamental variables, which complicates our estimation procedure. Following Hansen and Sargent (1980, 1982), we linearly project \(E_t(\cdot)\) onto \(\Omega_t\), the econometrician’s information set at time \(t\), which is a subset of \(I_t\). Denoting \(\hat{E}_t(\cdot)\) as such a linear projection operator onto \(\Omega_t\), we rewrite (3.17) as follows.

\[
\Delta e_{t+1} = \bar{\iota} + \frac{\alpha \gamma_{s1}^1}{\alpha - \rho} \Delta p_{t+1}^1 - \frac{\alpha \gamma_{s1}^1}{\alpha - \rho} \Delta p_{t+1}^{1*} + \frac{\alpha \gamma_{s1}^1 - (\alpha - \rho)}{\alpha - \rho} i_t^* + \frac{\gamma_{s1}^1 (\alpha \gamma_{s1}^1 - (\alpha - \rho))}{\rho (\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_{s1}^1}{\rho} \right)^j \hat{E}_t f_{t+j+1} + \xi_{t+1},
\]

where

\[
\xi_{t+1} = \omega_{t+1} + \frac{\gamma_{s1}^1 (\alpha \gamma_{s1}^1 - (\alpha - \rho))}{\rho (\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_{s1}^1}{\rho} \right)^j \left( E_t f_{t+j+1} - \hat{E}_t f_{t+j+1} \right),
\]

and

\[
\hat{E}_t \xi_{t+1} = 0
\]

by the law of iterated projections.

\(^{64}\)It is well-known that the standard errors obtained from the least squares or parametric bootstrap methods may not be valid asymptotically in presence of serially correlated errors. It would be necessary to use either Newey-West or QS kernel covariance estimators, or to use the moving block bootstrap method.

\(^{65}\)If there is a measurement error problem, \(\alpha\) may not be even consistent, since the aforementioned cointegrating relation assumption may not hold. We can deal with this problem by a two stage cointegration method that directly estimates the cointegrating vector as a first stage estimation. See Kim et al (2003) for details.
Rather than choosing appropriate instrumental variables that are in $\Omega_t$, we simply assume $\Omega_t = \{f_t, f_{t-1}, f_{t-2}, \cdots\}$. This assumption would be an innocent one under the stationarity assumption for $f_t$, and it can greatly lessen the burden of our GMM estimations by significantly reducing the number of coefficients to be estimated.

For this purpose, we assume that $f_t$ be a zero mean covariance stationary (for now), linearly indeterministic stochastic process so that it has the following Wold representation.

\[ f_t = c(L)\nu_t, \quad (3.21) \]

where $\nu_t = f_t - \hat{E}_t f_t$ and $c(L) = 1 + c_1 L + c_2 L^2 + \cdots$ is square summable. Assuming the invertibility of $c(L)$, (3.21) can be rewritten as the following autoregressive representation.

\[ d(L) f_t = \nu_t, \quad (3.22) \]

where $d(L) = c^{-1}(L) = 1 - d_1 L - d_2 L^2 - \cdots$. It can be shown that the following relation should hold when we linearly project $\sum_{j=0}^{\infty} \left(\frac{1-\rho^j}{1-\rho}\right) \hat{E}_t f_{t+j+1}$ onto $\Omega_t$.\(^66\)

\[ \sum_{j=0}^{\infty} \delta^j \hat{E}_t f_{t+j+1} = \psi(L) f_t = \left[ \frac{1 - \left(\delta^{-1}d(\delta)\right)^{-1} d(L)L^{-1}}{1 - (\delta^{-1}L)^{-1}} \right] f_t, \quad (3.23) \]

where $\delta = \frac{1-\rho^2}{\rho}$.

For actual estimation, we assume that $f_t$ can be represented by a finite order AR($r$) process\(^67\), that is, $d(L) = 1 - \sum_{j=1}^{r} d_j L^j$, where $r < \infty$. Then, it can be shown that the coefficients of $\psi(L)$ can be computed recursively as follows\(^68\).

\[ \psi_0 = (1 - \delta d_1 - \cdots - \delta^r d_r)^{-1} \]

\(^66\)See Hansen and Sargent (1980) for details.

\(^67\)We can use conventional Akaike Information criteria (AIC) or Bayesian Information criteria (BIC) in order to choose the degree of such autoregressive processes.

\(^68\)See Sargent (1987) for details.
\[\psi_r = 0\]

\[\psi_{j-1} = \delta\psi_j + \delta\psi_0d_j,\]

where \(j = 1, 2, \cdots, r\). Then, the GMM estimation of (3.17) can be implemented by simultaneously estimating next two equations\(^{69}\).

\[
\Delta e_{t+1} = \bar{i} + \frac{\alpha\gamma_{s1}}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha\gamma_{s1}}{\alpha - \rho} \Delta p_{t+1} + \frac{\alpha\gamma_{s1} - (\alpha - \rho)}{\alpha - \rho} i_t^* \\
+ \frac{\gamma_{s1}(\alpha\gamma_{s1} - (\alpha - \rho))}{\rho(\alpha - \rho)} (\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1}) + \xi_{t+1},
\]

and

\[
f_{t+1} = k + d_1 f_t + d_2 f_{t-1} + \cdots + d_r f_{t-r+1} + \nu_{t+1},
\]

where \(k\) is a constant, and \(\hat{E}_t \nu_{t+1} = 0\).

Recall that we assume a zero-mean covariance stationary process for \(f_t\) following Hansen and Sargent (1980). If the variable of interest has a non-zero unconditional mean, we can either demean it prior to the estimation or include a constant but leave its coefficient unconstrained. West (1989) showed that the further efficiency gain can be obtained by imposing additional restrictions on the deterministic term. However, the imposition of such an additional restriction is quite burdensome, so we simply add a constant here.

**GMM System Estimation by Kim et. al. (2003)**

Our last estimation strategy is a system method suggested by Kim et. al. (2003), which combines previous two estimation strategies. That is, we implement a GMM estimation for the three equations (3.2), (3.24), and (3.25) simultaneously. Note that the Hansen and Sargent method in previous leaves \(\alpha\) in (3.24) unrestricted. However,

\(^{69}\)In actual estimations, we normalized (3.24) by multiplying \((\alpha - \rho)\) to each side in order to reduce nonlinearity.
in the system method, we restrict it to satisfy (3.2) at the same time. In so doing, we may be able to acquire further efficiency gains by using more information.

A GMM estimation, then, can be implemented by the following $2(p + 2)$ orthogonality conditions.

\[
\hat{E}x_{2,t-\tau}(\text{st}_1 - d - \alpha \text{st}_t) = 0
\]

\[
(3.26)
\]

\[
\hat{E}x_{2,t-\tau}(\text{st}_1 - d - \alpha \text{st}_t) = 0
\]

\[
\hat{E}x_{2,t-\tau}(\text{st}_1 - d - \alpha \text{st}_t) = 0
\]

\[
(3.27)
\]

\[
(3.28)
\]

where $x_{1,t} = (1 \text{st}_t)'$, $x_{2,t} = (1 \text{ft}_t)'$, and $\tau = 0, 1, \ldots, p^{7071}$.

### 3.3 Empirical Results

In this section, we report estimates of the persistence parameter $\alpha$ (or convergence rate parameter $b$) and their implied half-lives from aforementioned three estimation strategies.

We use PPIs and CPIs in order to construct real exchange rates with the US dollar as a base currency. We consider 18 industrialized countries\(^\text{72}\) that provide 17 real exchange rates. For interest rates, we use quarterly money market interest rates that are short-term interbank call rates rather than using the conventional short-term treasury bill rates. This is because we incorporate the Taylor Rule in the model where the central bank sets its target short-term market rate based on changes in the GDP deflator and output deviations. For output deviations, we consider two

\(^{70}\)Note that $p$ does not necessarily coincide with $r$.

\(^{71}\)In actual estimations, we use the aforementioned normalization again.

\(^{72}\)Among 23 industrialized countries classified by IMF, I dropped Greece, Iceland, Ireland, and Portugal due to lack of reasonable number of observations. Luxembourg was not included because it has a currency union with Belgium.
different measures of output gap, quadratically detrended real GDPs (Clarida, Galí, and Gertler 1998)\textsuperscript{73} and unemployment rate gaps (Boivin 2005, Mankiw 2001)\textsuperscript{74}. The data frequency is quarterly and from IFS CD-ROM. The observations span from 1979:III to 1998:IV for most Eurozone countries, and from 1979:III to 2003:IV for Non-Eurozone EU countries and Non-EU countries with some exceptions due to lack of observations (see the notes on Tables 3.3 and 3.6 for complete description)\textsuperscript{75}.

The main reason that our sample period starts from 1979:III is based on empirical evidence on the US Taylor Rule. As discussed in 3.2, the dynamic system of the exchange rate and inflation may differ greatly, depending on the systematic response pattern of the central bank to future inflation. That is, we showed that the rationally expected future Taylor Rule fundamentals appear in the real exchange rate dynamics only when the long-run inflation coefficient is big enough ($\delta \gamma_\pi > 1$). Otherwise, the orthogonality conditions (3.27) and (3.28) would become invalid and other estimation strategies such as the VAR method should be employed.

Clarida, Galí, and Gertler (1998, 2000) provide important empirical evidence for the existence of a structural break in the US Taylor Rule. They show that the US central bank has reacted to the future inflation more aggressively during the post-Volker era (1979:III ~) than the pre-Volker era. Following Clarida, Galí, and Gertler

\textsuperscript{73}We also tried same analysis with the cyclical components of real GDP series from the HP-filter with 1600 of smoothing parameter as well as the Congressional Budget Office (CBO) output gaps. The results were quantitatively similar.

\textsuperscript{74}The unemployment gap is defined as a 5 year backward moving average subtracted by the current unemployment rate. This specification makes its sign consistent with that of the conventional output gap.

\textsuperscript{75}In this paper, relevant Eurozone countries are Austria, Belgium, Finland, France, Germany, Italy, Netherlands, and Spain. Non-Eurozone EU countries refer to Sweden, Denmark, and the UK. And Non-EU countries include Australia, Canada, Japan, New Zealand, Norway, Switzerland, and the US.
(2000)\textsuperscript{76,77}, we implemented a similar GMM estimation for the US Taylor Rule using longer sample period. The results are reported in Table 2.1 (see the note on Table 2.1 for detailed explanation). We use the quarterly changes in the GDP deflator as a general inflation measure, and considered two output gap measures (quadratically detrended real GDPs and unemployment gaps) for three sub-samples. All coefficients were highly significant and the $J$-test didn’t reject our model specification. More importantly, our requirement for the existence of a saddle path equilibrium met only for the Volker-Greenspan era\textsuperscript{78}. Therefore, we may conclude that this provides some empirical justification for the choice of our sample period.

Our half-life estimates by the univariate equation method, the GMM estimation by Hansen-Sargent method, and the GMM system method by Kim et al. (2003) are summarized in Tables 3.1-3.6 for PPI-based real exchange rates\textsuperscript{79} and CPI-based rates\textsuperscript{80}. We implemented these estimations using both output gap measures, but report the full estimates that were obtained when we used unemployment gaps. In order to save space\textsuperscript{81}, we provide only the median half-life for the cases we used

\textsuperscript{76}They used GDP deflator inflation along with the CBO output gaps (and HP detrended gaps).

\textsuperscript{77}Unlike them, we assume that the Fed targets current output gap rather than future deviations. However, this doesn’t make any significant changes to our results. And we include one lag of interest rate rather than two lags for simplicity.

\textsuperscript{78}There is no obvious way of how to measure $\delta$, the weight parameter on the first basket of goods. Since we treat the first basket as a so-called tradable goods where PPI holds without imposing any restriction on the second basket of goods, I measured $\delta$ as the ratio of the sum of durable and non-durable consumption expenditures to the total expenditures, treating service expenditure as a proxy for non-tradables. From the long-horizon US consumption data (1959:I–2003:IV), I got 0.43.

\textsuperscript{79}In this case, we consider the PPI and CPI as the prices of the first and second baskets of goods, respectively, whereas the GDP deflator is the general price index that the US central bank targets.

\textsuperscript{80}Similarly, we consider the CPI and PPI as the prices of the first and second baskets of goods in this case.

\textsuperscript{81}The results with quadratically detrended real GDP gaps were quantitatively similar.
the quadratically detrended real GDP gaps \( \text{Median}_x \)\(^82\). We chose the degree of autoregressive process for each fundamental term \( f_t \) in (3.19) by the conventional BIC\(^83\). Standard errors were adjusted using the QS kernel estimator with automatic bandwidth selection in order to deal with unknown serial correlation problems\(^84\).

One interesting finding is that both the Hansen-Sargent method and the system method provide reasonably short half-life estimates compared with ones from the univariate equation method (see Tables 3.1-3.6). The median half-life estimates from the univariate equation method were 2.64 and 2.57 years for PPI- and CPI-based real exchange rates, respectively. However, we obtained 1.61- and 1.60-year median half-lives from the system method, and less than 1 year from the Hansen-Sargent method\(^85\). Average half-life estimates were 1.43 and 1.87 years for PPI- and CPI-based rates, respectively, from the system method. Note that our half-life estimates are reasonably shorter than Rogoff (1996)’s three- to five-year consensus half-life estimate, which lead to the so-called ”PPP puzzle.” Therefore, our estimation results may provide some answers to this puzzle. It is also very interesting that our estimates are roughly consistent with the micro-data evidence by Crucini and Shintani (2004) who reported 1.0-, 1.1-, and 1.6-year baseline half-life estimates for all, traded, and non-traded good prices for the OECD countries, respectively.

\(^82\)Full estimation results are available from the author by request.

\(^83\)Chosen \( rs \) range from 2 to 4. We didn’t consider the \( r \) greater than 4 in order to avoid the potential ”many instrument” problem.

\(^84\)We impose the GMM estimates of the US Taylor Rule coefficient to the estimation system as a first stage estimation, since joint estimation can be quite burdensome. We need to correct the standard errors by redefining the second step weighting matrix as shown by Ogaki (1993). However, since we already acquired significant efficiency improvement over the univariate equation method, we believe such correction may not change the results much.

\(^85\)The results were quantitatively similar when the quadratically detrended real GDP series were used. See Median\(_x\) values for details.
Since the estimation techniques used in this paper are quite similar as those of Kim (2004) and Kim and Ogaki (2004), it would be interesting to compare our results with theirs. Kim (2004) reported, for the non-service goods consumption deflators, half-life estimates that range from 0.13 to 2.71 years, and the median half-life was 0.67 year. For the service goods consumption deflators, his estimates range from 0.30 to 15.52 years, and the median was 3.36. Kim and Ogaki (2004) also reported similar results for PPI- and CPI-based real exchange rates. Therefore, their results imply that the convergence rates might differ greatly depending on what price indices are used to construct real exchange rates, which is in contrast to other studies such as Wu (1996) and Murray and Papell (2002) who found only moderate or no significant difference.

In order to interpret such discrepancies, we focus on the efficiency issue in Kim (2004). One potentially important problem in his results is that his point estimates are associated with quite big standard errors. Especially for the non-traded good prices, it wasn’t possible to make a statistically meaningful reasoning for the size of the half-life due to extremely wide confidence intervals. Even though he obtained relatively better estimates (in terms of efficiency) for the traded goods, the standard errors were still quite big for many estimates. Kim and Ogaki (2004) also reported similar problems.

However, we obtained a substantial improvement in efficiency for both PPI- and CPI-based rates. It should be noted that our results greatly outperform the univariate equation approach too. Murray and Papell (2002) point out that the univariate

---

86 We obtained his median half-life estimates by inspecting the Table 2 in Kim (2004), since he didn’t report it.

87 Again, this value was obtained from Table 4 in Kim (2004).
equation approach provides virtually no useful information regarding the size of the half-lives due to extremely wide confidence intervals of the point estimates. Interestingly, our system method provides much tighter confidence intervals compared with the ones by the univariate equation method.

Therefore, we may conclude that even though Kim (2004) and Kim and Ogaki (2002) found significant difference in half-lives, one may not fully accept the results as statistically meaningful ones. Our results, however, provide more efficient half-life estimates for both real exchange rates and our finding that the half-lives are about the same irrespective of the choice of aggregate price indices should be considered to be statistically reliable\(^8\).

Then, the natural question is that why this paper produces completely different results from those of Kim (2004) and Kim and Ogaki (2004) even though the estimation strategies are virtually same. It should be noted that one of the advantages of using the system method is that it may provide more efficient estimates as long as the imposed restrictions are valid. Rather than following them in using a money demand function, which may be quite unstable especially in the short-run, this paper incorporates a forward looking version of the Taylor Rule. Therefore, it seems that deriving restrictions from the Taylor Rule was an appropriate choice.

Finally, in order to see if the specification in our model can be statistically justified, we implemented likelihood ratio type tests \((LR)\) and calculated corresponding \(p\)-values \((pv)\). In our system estimations, we impose a restriction that \(\alpha\) in orthogonality conditions (3.26) and (3.27) are same. We can construct a likelihood ratio type test statistic utilizing \(J\)-statistics for the unrestricted estimation and the estimation with

\(^8\)Our Hansen-Sargent estimations also outperform Kim (2004)'s, since he was not able to obtain reasonable estimates for most real exchange rates he considered.
such a restriction, which obeys a $\chi^2$ distribution asymptotically with degree of freedom $1^{89}$. In most cases, our specification were not rejected.

3.4 Conclusion

In this paper, we employed a system method framework that explicitly incorporates a forward looking version of the Taylor Rule to estimate and compare half-lives of PPP deviations for PPI- and CPI-based real exchange rates for 18 industrialized countries. In a similar framework, Kim (2004) obtained much shorter half-life point estimates for the non-service goods prices than those for the service goods prices, which is in contrast to other studies that reported moderate or no difference (e.g., Wu 1996, Murray and Papell 2002).

However, his point estimates were associated with quite wide confidence intervals, and thus provided statistically less useful information regarding the size of half-life point estimates. Kim and Ogaki (2004) also reported similar problems for PPI- and CPI-based real exchange rates.

In contrast, we obtained a substantial improvement in efficiency over the work of Kim (2004). Our half-life estimations produced much tighter confidence intervals for both point estimates, enabling statistically meaningful comparisons of the length of half-life point estimates. Thus, our conclusion that the half-lives are about the same irrespective of the choice of aggregate prices may be statistically more reliable one$^{90}$. And we believe that the use of the Taylor Rule, rather than using a money demand function as Kim (2004) did, contributed to such efficiency gains.

$^{89}$See Ogaki (1993) for details.

$^{90}$It should be noted that our comparisons are limited to the half-lives based on aggregate price indices such as the PPI and CPI, which are imperfect measures of tradable or non-tradable good prices.
It is also interesting that our estimates are reasonably shorter than Rogoff (1996)’s three- to five-year consensus half-life, which alluded to the "PPP puzzle." Our estimates, however, are roughly consistent with micro evidence by Crucini and Shintani (2004), and thus may provide some answers to the puzzle.

Finally, our model renders potentially important implications on real exchange rate dynamics. Particularly, we showed that rationally expected future Taylor Rule fundamentals enter the dynamics only when the central bank responds to the change in future inflation aggressively enough. Otherwise, exchange rate and inflation dynamics may be explained only by past fundamentals and any martingale difference sequences.
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>α (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.908 (0.781)</td>
<td>0.913 (0.034)</td>
</tr>
<tr>
<td>Austria</td>
<td>2.038 (1.205)</td>
<td>0.919 (0.046)</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.707 (0.900)</td>
<td>0.903 (0.048)</td>
</tr>
<tr>
<td>Canada</td>
<td>1.932 (0.815)</td>
<td>0.914 (0.035)</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.976 (1.911)</td>
<td>0.943 (0.035)</td>
</tr>
<tr>
<td>Finland</td>
<td>2.367 (1.493)</td>
<td>0.929 (0.043)</td>
</tr>
<tr>
<td>France</td>
<td>2.194 (1.455)</td>
<td>0.924 (0.048)</td>
</tr>
<tr>
<td>Germany</td>
<td>2.639 (1.965)</td>
<td>0.936 (0.046)</td>
</tr>
<tr>
<td>Italy</td>
<td>2.979 (2.512)</td>
<td>0.943 (0.046)</td>
</tr>
<tr>
<td>Japan</td>
<td>2.704 (1.660)</td>
<td>0.938 (0.037)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2.881 (2.173)</td>
<td>0.942 (0.043)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>3.228 (1.119)</td>
<td>0.948 (0.018)</td>
</tr>
<tr>
<td>Norway</td>
<td>1.407 (0.745)</td>
<td>0.884 (0.058)</td>
</tr>
<tr>
<td>Spain</td>
<td>2.879 (1.976)</td>
<td>0.942 (0.039)</td>
</tr>
<tr>
<td>Sweden</td>
<td>2.857 (2.129)</td>
<td>0.941 (0.043)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.084 (1.160)</td>
<td>0.920 (0.043)</td>
</tr>
<tr>
<td>UK</td>
<td>2.675 (1.911)</td>
<td>0.937 (0.043)</td>
</tr>
</tbody>
</table>

|         | Median   | 2.639 | 0.936 |
|         | Average  | 2.439 | 0.928 |

Table 3.1: PPI-Based Real Exchange Rate: Univariate Equation Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Hansen-Sargent Half Life (s.e.)</th>
<th>Hansen-Sargent α (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.156 (0.448)</td>
<td>0.861 (0.050)</td>
</tr>
<tr>
<td>Austria</td>
<td>0.320 (0.070)</td>
<td>0.581 (0.069)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.358 (0.097)</td>
<td>0.616 (0.081)</td>
</tr>
<tr>
<td>Canada</td>
<td>1.005 (0.809)</td>
<td>0.842 (0.117)</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.490 (0.144)</td>
<td>0.702 (0.073)</td>
</tr>
<tr>
<td>Finland</td>
<td>1.290 (1.024)</td>
<td>0.874 (0.093)</td>
</tr>
<tr>
<td>France</td>
<td>0.972 (0.271)</td>
<td>0.837 (0.042)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.217 (0.136)</td>
<td>0.450 (0.224)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.602 (0.044)</td>
<td>0.750 (0.016)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.462 (0.095)</td>
<td>0.687 (0.053)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.112 (0.152)</td>
<td>0.213 (0.448)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1.094 (0.270)</td>
<td>0.853 (0.033)</td>
</tr>
<tr>
<td>Norway</td>
<td>0.702 (0.076)</td>
<td>0.781 (0.021)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.095 (0.202)</td>
<td>0.161 (0.627)</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.082 (0.470)</td>
<td>0.852 (0.099)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.798 (0.381)</td>
<td>0.805 (0.083)</td>
</tr>
<tr>
<td>UK</td>
<td>0.249 (0.203)</td>
<td>0.499 (0.283)</td>
</tr>
<tr>
<td>Median</td>
<td>0.602</td>
<td>0.750</td>
</tr>
<tr>
<td>Average</td>
<td>0.647</td>
<td>0.668</td>
</tr>
<tr>
<td>Median</td>
<td>0.401</td>
<td>0.678</td>
</tr>
</tbody>
</table>

Table 3.2: PPI-Based Real Exchange Rate: Hansen-Sargent Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
<th>LR (pv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.678 (0.502)</td>
<td>0.902 (0.028)</td>
<td>1.018 (0.313)</td>
</tr>
<tr>
<td>Austria</td>
<td>0.351 (0.042)</td>
<td>0.610 (0.036)</td>
<td>0.578 (0.447)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.380 (0.065)</td>
<td>0.634 (0.049)</td>
<td>1.233 (0.267)</td>
</tr>
<tr>
<td>Canada</td>
<td>1.990 (0.902)</td>
<td>0.917 (0.036)</td>
<td>0.411 (0.521)</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.670 (0.046)</td>
<td>0.772 (0.014)</td>
<td>1.869 (0.172)</td>
</tr>
<tr>
<td>Finland</td>
<td>2.822 (1.674)</td>
<td>0.940 (0.034)</td>
<td>0.424 (0.515)</td>
</tr>
<tr>
<td>France</td>
<td>1.498 (0.674)</td>
<td>0.891 (0.046)</td>
<td>1.569 (0.210)</td>
</tr>
<tr>
<td>Germany</td>
<td>2.066 (1.317)</td>
<td>0.920 (0.049)</td>
<td>2.156 (0.142)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.619 (0.050)</td>
<td>0.756 (0.017)</td>
<td>0.640 (0.424)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.524 (0.082)</td>
<td>0.719 (0.037)</td>
<td>1.676 (0.195)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>3.723 (3.396)</td>
<td>0.955 (0.041)</td>
<td>3.002 (0.083)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1.668 (0.347)</td>
<td>0.901 (0.020)</td>
<td>0.766 (0.381)</td>
</tr>
<tr>
<td>Norway</td>
<td>0.699 (0.109)</td>
<td>0.780 (0.030)</td>
<td>1.866 (0.172)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.528 (0.233)</td>
<td>0.720 (0.104)</td>
<td>1.040 (0.308)</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.612 (0.791)</td>
<td>0.898 (0.047)</td>
<td>1.101 (0.294)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.687 (0.715)</td>
<td>0.902 (0.039)</td>
<td>0.556 (0.456)</td>
</tr>
<tr>
<td>UK</td>
<td>1.839 (0.902)</td>
<td>0.910 (0.042)</td>
<td>1.558 (0.208)</td>
</tr>
</tbody>
</table>

Median: 1.612  0.898  -  
Average: 1.433  0.831  -  
Median $x$: 1.717  0.904  -  

Notes: i) The US$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Median $x$ is the median when quadratically detrended output gaps are used. iv) Chosen parameters are $(\gamma_\pi, \gamma_{x}, \rho, \delta) = (3.311, 0.222, 0.744, 0.447)$ for countries where observations end in 1998.IV, while $(3.532, 0.255, 0.773, 0.447)$ for countries where observations end in 2003.IV. v) Sample periods are 1979.III $\sim$1998.IV for most Eurozone countries (Austria, Finland, Germany, Netherlands, Spain), and 1979.III$\sim$2003.IV for Non-Eurozone EU countries (Denmark, UK) and all Non-EU countries (Australia, Canada, Japan, New Zealand, Norway, Switzerland, US). Among Eurozone countries, Belgium and France’s sample span from 1980.I to 1998.IV, while Italy’s observations span from 1981.I to 1998.IV. Among Non-Eurozone EU countries, Sweden’s observations span from 1980.I to 2001.I. vi) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vii) LR in last column refers to the likelihood ratio type test statistics and corresponding $p$-values are in parentheses.
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>2.572 (1.188)</td>
<td>0.935 (0.029)</td>
</tr>
<tr>
<td>Austria</td>
<td>2.608 (1.798)</td>
<td>0.936 (0.043)</td>
</tr>
<tr>
<td>Belgium</td>
<td>2.310 (1.546)</td>
<td>0.928 (0.047)</td>
</tr>
<tr>
<td>Canada</td>
<td>5.970 (5.448)</td>
<td>0.971 (0.026)</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.351 (1.272)</td>
<td>0.929 (0.037)</td>
</tr>
<tr>
<td>Finland</td>
<td>3.051 (2.235)</td>
<td>0.945 (0.039)</td>
</tr>
<tr>
<td>France</td>
<td>2.187 (1.577)</td>
<td>0.924 (0.053)</td>
</tr>
<tr>
<td>Germany</td>
<td>1.841 (1.136)</td>
<td>0.910 (0.053)</td>
</tr>
<tr>
<td>Italy</td>
<td>2.887 (2.504)</td>
<td>0.942 (0.049)</td>
</tr>
<tr>
<td>Japan</td>
<td>3.188 (2.182)</td>
<td>0.947 (0.035)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.688 (1.003)</td>
<td>0.902 (0.055)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>3.142 (0.984)</td>
<td>0.946 (0.016)</td>
</tr>
<tr>
<td>Norway</td>
<td>2.142 (1.268)</td>
<td>0.922 (0.044)</td>
</tr>
<tr>
<td>Spain</td>
<td>3.704 (3.109)</td>
<td>0.954 (0.037)</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.733 (7.641)</td>
<td>0.970 (0.039)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.976 (1.108)</td>
<td>0.916 (0.045)</td>
</tr>
<tr>
<td>UK</td>
<td>1.796 (1.170)</td>
<td>0.908 (0.057)</td>
</tr>
<tr>
<td>Median</td>
<td>2.572</td>
<td>0.935</td>
</tr>
<tr>
<td>Average</td>
<td>2.893</td>
<td>0.934</td>
</tr>
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</table>

Table 3.4: CPI-Based Real Exchange Rate: Univariate Equation Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.921 (2.101)</td>
<td>0.914 (0.090)</td>
</tr>
<tr>
<td>Austria</td>
<td>0.291 (0.055)</td>
<td>0.552 (0.062)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.227 (0.081)</td>
<td>0.466 (0.127)</td>
</tr>
<tr>
<td>Canada</td>
<td>n.a.</td>
<td>1.031 (0.081)</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.932 (0.328)</td>
<td>0.830 (0.054)</td>
</tr>
<tr>
<td>Finland</td>
<td>0.523 (0.074)</td>
<td>0.718 (0.034)</td>
</tr>
<tr>
<td>France</td>
<td>0.266 (0.091)</td>
<td>0.521 (0.116)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.275 (0.067)</td>
<td>0.533 (0.082)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.736 (0.101)</td>
<td>0.790 (0.025)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.854 (0.225)</td>
<td>0.816 (0.044)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.220 (0.072)</td>
<td>0.454 (0.117)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>3.984 (3.996)</td>
<td>0.957 (0.042)</td>
</tr>
<tr>
<td>Norway</td>
<td>0.746 (0.178)</td>
<td>0.793 (0.044)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.099 (0.198)</td>
<td>0.174 (0.607)</td>
</tr>
<tr>
<td>Sweden</td>
<td>2.056 (2.726)</td>
<td>0.919 (0.103)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.230 (0.132)</td>
<td>0.471 (0.203)</td>
</tr>
<tr>
<td>UK</td>
<td>1.252 (0.778)</td>
<td>0.871 (0.075)</td>
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<tr>
<td>Median</td>
<td>0.630</td>
<td>0.790</td>
</tr>
<tr>
<td>Average</td>
<td>0.913</td>
<td>0.695</td>
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<tr>
<td>Median$_x$</td>
<td>0.531</td>
<td>0.722</td>
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Table 3.5: CPI-Based Real Exchange Rate: Hansen-Sargent Method
<table>
<thead>
<tr>
<th>Country</th>
<th>Half Life (s.e.)</th>
<th>$\alpha$ (s.e.)</th>
<th>LR (pv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.635 (0.499)</td>
<td>0.899 (0.029)</td>
<td>1.846 (0.174)</td>
</tr>
<tr>
<td>Austria</td>
<td>0.370 (0.016)</td>
<td>0.626 (0.013)</td>
<td>0.515 (0.473)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.486 (0.140)</td>
<td>0.700 (0.072)</td>
<td>0.494 (0.482)</td>
</tr>
<tr>
<td>Canada</td>
<td>4.432 (2.992)</td>
<td>0.962 (0.025)</td>
<td>1.596 (0.206)</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.276 (1.587)</td>
<td>0.927 (0.049)</td>
<td>0.970 (0.325)</td>
</tr>
<tr>
<td>Finland</td>
<td>1.419 (0.481)</td>
<td>0.885 (0.037)</td>
<td>1.073 (0.300)</td>
</tr>
<tr>
<td>France</td>
<td>0.555 (0.030)</td>
<td>0.732 (0.012)</td>
<td>0.051 (0.822)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.788 (0.193)</td>
<td>0.803 (0.043)</td>
<td>1.135 (0.287)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.699 (0.066)</td>
<td>0.781 (0.018)</td>
<td>2.815 (0.093)</td>
</tr>
<tr>
<td>Japan</td>
<td>1.546 (0.585)</td>
<td>0.894 (0.038)</td>
<td>5.264 (0.022)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.749 (0.249)</td>
<td>0.794 (0.061)</td>
<td>1.540 (0.215)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1.986 (0.485)</td>
<td>0.916 (0.020)</td>
<td>2.471 (0.116)</td>
</tr>
<tr>
<td>Norway</td>
<td>1.602 (0.742)</td>
<td>0.897 (0.045)</td>
<td>3.137 (0.077)</td>
</tr>
<tr>
<td>Spain</td>
<td>4.570 (2.278)</td>
<td>0.963 (0.018)</td>
<td>0.058 (0.810)</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.375 (7.431)</td>
<td>0.968 (0.043)</td>
<td>0.356 (0.551)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.602 (0.762)</td>
<td>0.897 (0.046)</td>
<td>6.302 (0.012)</td>
</tr>
<tr>
<td>UK</td>
<td>1.662 (0.858)</td>
<td>0.901 (0.048)</td>
<td>1.434 (0.231)</td>
</tr>
<tr>
<td>Median</td>
<td>1.602</td>
<td>0.897</td>
<td>-</td>
</tr>
<tr>
<td>Average</td>
<td>1.868</td>
<td>0.856</td>
<td>-</td>
</tr>
<tr>
<td>Median$_x$</td>
<td>1.736</td>
<td>0.905</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: i) The US$ is the base currency. ii) Unemployment gaps are used for output deviations. iii) Median$_x$ is the median when quadratically detrended output gaps are used. iv) Chosen parameters are $(\gamma_\pi, \gamma_{x1}, \rho, \delta) = (2.941, 0.125, 0.806, 0.447)$ for countries where observations end in 1998.IV, while $(2.539, 0.105, 0.793, 0.447)$ for countries where observations end in 2003.IV. v) Sample periods are 1979.III~1998.IV for most Eurozone countries (Austria, Finland, Germany, Netherlands, Spain), and 1979.III~2003.IV for Non-Eurozone EU countries (Denmark, UK) and all Non-EU countries (Australia, Canada, Japan, New Zealand, Norway, Switzerland, US). Among Eurozone countries, Belgium and France’s samples span from 1980.I to 1998.IV, while Italy’s observations start from 1981.I to 1998.IV. Among Non-Eurozone EU countries, Sweden’s observations span from 1980.I to 2001.I. vi) Standard errors were adjusted using the QS kernel with automatic bandwidth selection, and are reported in parentheses. vii) LR in last column refers to the likelihood ratio type test statistics and corresponding p-values are in parentheses.

Table 3.6: CPI-Based Real Exchange Rate: System Method

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CHAPTER 4

LOCAL-CURRENCY PRICING, TECHNOLOGY DIFFUSION, AND THE OPTIMAL INTEREST RATE RULE

4.1 Introduction

In his early work, Friedman (1953) argues that, since prices tend to adjust sluggishly, exchange rates should be flexible enough to change the relative price of home to foreign goods when a country-specific productivity shock occurs. Devereux and Engel (2003), however, question the validity of this classical case for flexible exchange rates. They point out that when prices are pre-set in local currency (local-currency pricing or LCP), exchange rate changes do not alter the relative price, and thus there’s no expenditure-switching role for the exchange rates. Furthermore, they conclude that, under such circumstances, optimal monetary policies should be associated with fixed exchange rates even in the presence of country-specific real shocks\(^\text{91}\).

This rather surprising prescription of fixed exchange rates, however, is upset by Obstfeld (2004) and Duarte and Obstfeld (2004) who introduce nontradables sectors in Devereux and Engel’s (2003) model. They demonstrate that such a fixed exchange rate prescription is primarily due to a model structure where international

\(^{91}\)In contrast, when prices are pre-set in the producer’s currency (producer-currency pricing or PCP), they show that optimal monetary policy entails flexible exchange rates.
consumptions are perfectly synchronized under flexible prices. With nontraded goods, consumptions cannot be perfectly synchronized even with perfectly integrated financial markets, since nontradables cannot be shipped abroad. Put it different, consumptions respond disproportionately to a country-specific real shock, which requires asymmetric monetary policy responses. Therefore, the case for flexible exchange rates is restored even in the absence of expenditure-switching effects of exchange rates.

In this paper, I raise an issue with respect to the specification of country-specific shocks. Obstfeld (2004) and Duarte and Obstfeld (2004) implicitly assume that technology shocks in tradables and nontradables sectors of either country are perfectly correlated\textsuperscript{92}. Such a specification can be also found in their more recent work by Devereux and Engel (2006). Instead, I assume that a technology shock in one country will diffuse to the same sector of the other country with a one-period lag. In other words, technology shocks in this paper are specified as being both country- and industry-specific, which seems to be more realistic. Other than this, I adopt the same basic model by Obstfeld (2004) that features sticky price, local-currency pricing, and the interest rate rule.

With this modification, this paper produces the following interesting results. First of all, two different types of distortion may emerge when a favorable country-specific shock occurs in either country. One source of distortion is originated from price stickiness, which requires expansionary monetary interventions. The other distortion is so-called news effect due to technology diffusion to the other country. It turns out that welfare-maximizing central banks wish to neutralize it by implementing a contractionary monetary policy. As will be shown later, when the news effect

\textsuperscript{92}For example, a technology innovation in the domestic car manufacturing industry will have an identical and immediate impact on the productivity of the home housing industry.
dominates the sticky price effect under a certain condition, central banks may raise nominal interest rates in response to a positive country-specific real shock. This result is in sharp contrast with those of other aforementioned studies. And I believe that this result is consistent with actual data, since it is common to observe rising nominal interest rates during economic boom.

Secondly, this paper demonstrates that the implications of optimal monetary policy on exchange rates can differ greatly, depending on the source of a country-specific shock. Under the current setup, central banks identically respond to a real shock that occurs in tradables sectors in either country, and the exchange rate stays the same. However, they respond oppositely to a shock in nontradables sectors, and the resulting interest rate differential calls for exchange rate changes. It can be also shown that central banks optimally don’t respond to any productivity shock in traded goods sectors with a proper interest rate rule. This finding may be consistent with the so-called dirty floating exchange rate system.

The rest of the paper is organized as follows. In 4.2, I present the basic setup of the model. 4.3 discusses the properties of a benchmark solution under flexible prices. In 4.4, I report the analytic solution of the model under price stickiness and LCP. 4.5 describes the optimal interest rate rule that is derived from the central bank’s welfare maximization problem. 4.6 concludes.

4.2 The Baseline Model

4.2.1 Preferences

There are two countries. Each country is populated by a continuum of identical monopolistically competitive producers/consumers indexed by \( h \in [0, 1] \) in the home
country and by $f \in [0, 1]$ in the foreign country. Each agent produces one traded good and one nontraded good, and provides labor.

The representative home consumer/producer $h$ maximizes,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\rho}(h)}{1- \rho} - \kappa L_t(h) \right] \right\}, \; \rho, \kappa > 0, \; \beta \in (0, 1),$$

(4.1)

where $C_t$ is an aggregate consumption index, $L_t$ is labor supply$^{93}$, and $\kappa$ is a labor disutility parameter. Following Obstfeld (2004), I don’t explicitly model the demand for money$^{94}$. The representative agent faces the following budget constraint.

$$P_t C_t(h) = P_{H,t}(h) Y_{H,t}(h) + S_t P_{H,t}^*(h) Y_{H,t}^*(h) + P_{N,t}(h) Y_{N,t}(h)$$

(4.2)

$$- WL_{t+1}(h) + (1 + R_{t+1}) WL_t(h),$$

where $P_t$ is the nominal price index of $C_t$, $S_t$ is the nominal exchange rate as the unit price of the foreign currency in terms of the home currency, $WL_t$ is the marketable nominal wealth that includes domestic and foreign bond-holdings, and $R_{t+1}$ is the nominal ex post return on $WL_t$. The representative home producer $h$ supplies a traded good to the home country ($Y_{H,t}$) and to the foreign country ($Y_{H,t}^*$), and provides a nontraded good ($Y_{N,t}$). $P_{H,t}, P_{H,t}^*, P_{N,t}$ are the corresponding prices in local currencies, respectively. Note that the agent employs price-discrimination by choosing a separate price for the traded good sold in the foreign country$^{95}$.

The overall consumption index is defined as,

$$C = \frac{C_{t}^{\gamma} C_{N}^{1-\gamma}}{\gamma^\gamma (1- \gamma)^{1-\gamma}}.$$  

(4.3)

$^{93}$We assume that the labor market in each country is perfectly competitive.

$^{94}$Since I assume that central banks adopt interest rate rules and that the money supply adjusts endogenously, the money demand doesn’t have to appear in the utility function (see Woodford, 2003). The money demand is assumed to have a negligible impact on utility.

$^{95}$Under PCP, the agent chooses a single price for the traded good sold in both markets. That is, once the domestic price $P_{H,t}(h)$ is determined, the foreign price $P_{H,t}^*(h)$ will be automatically set by $P_{H,t}(h)/S_t$. 

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where $C_T$ and $C_N$ are the consumption sub-indexes for tradables and non-tradables, respectively. The tradables sub-index $C_T$ is a function of the consumptions of home traded goods ($C_H$) and foreign traded goods ($C_F$). That is,

$$C_T = \frac{C_H^\xi C_F^{1-\xi}}{\xi^\xi (1-\xi)^{1-\xi}}$$

For simplicity, we assume that $\xi = 1/2$, which implies that there is no home bias. Then, $C_T$ can be rewritten as follows.

$$C_T = 2C_H^{1/2}C_F^{1/2} \quad (4.4)$$

Each consumption index is defined as a general CES function over consumption of all the varieties of each good as follows.

$$C_j = \begin{cases} \left[ \int_0^1 C_j(h)^{\theta-1} dh \right]^{1/\theta}, & j = H, N \\ \left[ \int_0^1 C_j(f)^{\theta-1} df \right]^{1/\theta}, & j = F \end{cases} \quad (4.5)$$

where $\theta > 1$ is the common constant elasticity of substitution across goods in each sector. Note that, under the current setup, the representative consumer spends constant shares of her total income for home traded goods ($\gamma_2$), foreign traded goods ($\gamma_2$), and nontraded goods (1-$\gamma$).

It can be shown that corresponding price indexes are (see Appendix),

$$P = P_T^{\gamma_2}P_N^{1-\gamma_2} \quad (4.6)$$

and

$$P_T = P_H^{1/2}P_F^{1/2}, \quad (4.7)$$

where

$$P_j = \begin{cases} \left[ \int_0^1 P_j(h)^{1-\theta} dh \right]^{1/\theta}, & j = H, N \\ \left[ \int_0^1 P_j(f)^{1-\theta} df \right]^{1/\theta}, & j = F \end{cases} \quad (4.8)$$
Then, the demand functions for each good are (see Appendix),

\[ C_H(h) = \frac{\gamma}{2} \left( \frac{P_H(h)}{P_H} \right)^{-\theta} \left( \frac{P}{P_H} \right) C \tag{4.9} \]

\[ C_N(h) = (1 - \gamma) \left( \frac{P_N(h)}{P_N} \right)^{-\theta} \left( \frac{P}{P_N} \right) C \tag{4.10} \]

\[ C_H^*(h) = \frac{\gamma}{2} \left( \frac{P_H^*(h)}{P_H^*} \right)^{-\theta} \left( \frac{P^*}{P_H^*} \right) C^* \tag{4.11} \]

The foreign representative producer/consumer also solves a similar maximization problem. And foreign prices and demand functions can be analogously defined and denoted by a superscript asterisk (*).

### 4.2.2 Asset Market

We assume a perfectly integrated financial market. Consumers in each country can access to a complete set of state-contingent nominal bonds\(^{96}\) originated from the home and the foreign country. Since they have to buy these bonds before the realization of the state of the world, they will equalize the marginal consumption value of one unit of the nominal bonds across countries for all states of nature. Then, as in Backus and Smith (1993), the resulting \textit{ex post} allocation implies the following risk-sharing condition.

\[ \frac{C_t^-}{P_t} = \frac{C_t^*-\rho}{S_t P_t^*} \tag{4.12} \]

for all dates and states. Note that since Purchasing Power Parity (PPP, \( P_t = S_t P_t^* \)) does not hold \textit{ex post} in this model\(^{97}\), consumptions would not be equalized across countries in general.

\(^{96}\)That is, each bond-holder receives her payoff in currencies rather than goods.

\(^{97}\)However, PPP will be still valid \textit{ex ante} for tradable goods.
4.2.3 Production Technologies

The production functions for each variety of goods are given by,

\[ Y_{H,t}(h) = A_t L_{H,t}(h), \quad Y^*_{H,t}(h) = A_t^* L^*_{H,t}(h), \quad Y_{N,t}(h) = B_t L_{N,t}(h) \] (4.13)

\[ Y^*_{F,t}(f) = A_t^* L^*_{F,t}(f), \quad Y_{F,t}(f) = A_t L_{F,t}(f), \quad Y^*_{N,t}(f) = B_t^* L^*_{N,t}(f), \]

where \( L_{j,t}(i) \) denotes employment of firm \( i \) in sector \( j \) at time \( t \), and \( A_t \) and \( B_t \) are country/industry-specific technology levels at time \( t \). Therefore, productivity shocks in this paper are specified as being both country- and industry-specific, which is in contrast with the economy-wide productivity specification\(^{98}\) by Obstfeld (2004), Duarte and Obstfeld (2004), and Devereux and Engel (2006).

Letting lower-case letters (excluding interest rates) denote natural logarithms, the technologies obey the following stochastic processes.

\[ a_t = \lambda a_{t-1} + u_t + u^*_{t-1}, \quad b_t = \lambda b_{t-1} + v_t + v^*_{t-1} \] (4.14)

\[ a^*_t = \lambda a^*_{t-1} + u^*_t + u^*_{t-1}, \quad b^*_t = \lambda b^*_{t-1} + v^*_t + v^*_{t-1}, \]

where

\[ \lambda \in [0, 1], \quad u \sim N(0, \sigma_u^2), \quad v \sim N(0, \sigma_v^2) \]

This also constitute a new feature of this paper. Note that a productivity shock in the home country will diffuse to the same sector of the foreign country with a one-period lag, and vice versa. For example, when a real shock occurs in the home tradables sector \((u_t)\) at time \( t \), it will increase the home tradables sector productivity immediately, but will have a muted impacts on the home tradables sector at time \( t+1 \). At the same time, it will be fully incorporated to the foreign tradables sector.

\(^{98}\)Under their specification, \( A_t = B_t \) and \( A^*_t = B^*_t \) for all \( t \).
productivity at time $t+1$. It should be also noted that $u_t$ is an unanticipated shock to the home country at time $t$, while all agents can anticipate its impact on the foreign country at time $t+1$ with no uncertainty. All other shocks are similarly defined.

Another interesting feature of this paper is that the productivity shocks in the home nontradables sector can diffuse to the foreign nontradables sector, and vice versa. Consequently, when there occurs a productivity shock in the foreign (home) nontradables sector, it will be shown that the home (foreign) central bank has to respond to it even before it is incorporated to the home (foreign) technology\footnote{That is, even though nontraded goods cannot be shipped abroad, its technology is mobile with a time lag.}.

### 4.2.4 Interest Rate Rules

Following Obstfeld (2004), I assume that the central bank in each country employs the nominal interest rate as the monetary instrument. That is, central banks adopt the following interest rate rules,

$$i_t = \nu + \psi p_t - \alpha_1 u_t - \alpha_2 v_t - \alpha_3 u^*_t - \alpha_4 v^*_t,$$

(4.15)

for the home country, and

$$i^*_t = \nu + \psi p^*_t - \alpha_1 u^*_t - \alpha_2 v^*_t - \alpha_3 u^*_t - \alpha_4 v^*_t,$$

(4.16)

for the foreign country. $i_t$ and $i^*_t$ denote the nominal interest rates in the home and the foreign country, respectively\footnote{It will be shown later that the price level is determinate as long as $\psi$ is strictly positive.}.

### 4.3 Benchmark Solution: Flexible Prices Equilibrium

As will be shown later, the model economy in 4.4 contains three sources of distortion: monopolistic competition, local-currency pricing, and price stickiness. It should
be noted that, under the current setup, optimal monetary policy cannot do anything about the distortion originated from the first two sources\footnote{This statement by no means implies that the distinction between LCP and PCP is not important with respect to optimal monetary policy. With PCP, optimal monetary policy can utilize exchange rate changes in order to change the relative price of home goods to foreign goods, while there is no benefit of exchange rate changes under LCP.}. However, it is still possible for central banks to alleviate distortion due to price stickiness by changing their policy stances in response to a country-specific real shock. Therefore, it would be useful to analyze the solution under flexible prices as a benchmark case.

With fully flexible prices, firms set prices by constant markup \((\frac{\theta}{1-\theta})\) pricing over nominal marginal cost in each period. We assume that labor markets are perfectly competitive so that nominal marginal costs are \(\frac{W_t}{A_t}\) and \(\frac{W_t}{B_t}\) for the home country, and \(\frac{W_t^*}{A_t^*}\) and \(\frac{W_t^*}{B_t^*}\) for the foreign country.

From the optimal labor-consumption trade-off condition,

\[
\frac{W_t}{P_t} C_t^{-\rho} = \kappa = \frac{W_t^*}{P_t^*} C_t^{*-\rho}
\]

Combining (4.6), (4.7), markup pricing rules, and (4.17), we can show that equilibrium consumptions are (see Appendix),

\[
C_t = \left[ \left( \frac{\theta - 1}{\theta \kappa} \right) A_t^2 B_t^{1-\gamma} A_t^{*2} \right]^{\frac{1}{\rho}}
\]

\[
C_t^* = \left[ \left( \frac{\theta - 1}{\theta \kappa} \right) A_t^{*2} B_t^{*1-\gamma} A_t^{*2} \right]^{\frac{1}{\rho}}
\]

Taking logs to (4.18) and (4.19), and denoting lower cases as log variables, consumption innovations in the flexible price equilibrium are,

\[
c_t - E_{t-1} c_t = \frac{\gamma}{2\rho} (u_t + u_t^*) + \frac{1 - \gamma}{\rho} v_t
\]

\[
c_t^* - E_{t-1} c_t^* = \frac{\gamma}{2\rho} (u_t^* + u_t^*) + \frac{1 - \gamma}{\rho} v_t^*
\]
That is, consumptions respond identically to all tradables sector shocks, while each
country responds only to its own nontradables sector shocks. It should be also noted
that, under flexible prices, technology diffusion has no impact on consumption in
either country.

4.4 Sticky-Price, Local-Currency Pricing Model

We assume that producers set their nominal prices for their goods in local currency
one period in advance. For example, the representative home producer $h$ sets the
prices $P_{H,t}(h), P^*_H(h), \text{ and } P_{N,t}(h)$ at time $t - 1$ using all available information, and
maintains them for one period.

Taking all aggregate prices and quantities as given, the home agent $h$ solves,

$$\max_{P_{H,t}(h), P^*_H(h), P_{N,t}(h)} E_{t-1} \left\{ \frac{C_t(h)^{1-\rho}}{1-\rho} - \kappa L_t(h) \right\}$$ (4.22)

subject to the budget constraint (4.2), the demand equations (4.9), (4.10), and (4.11),
and the labor demand function,

$$L_t(h) = \frac{Y_{H,t}(h) + Y^*_{H,t}(h)}{A_t} + \frac{Y_{N,t}(h)}{B_t}$$ (4.23)

Then, the first order condition with respect to $P_{H,t}(h)$ implies the following pricing
equation for the home tradable good in the home country (see Appendix).

$$P_{H,t}(h) = \frac{\theta \kappa}{\theta - 1} \frac{P_tE_{t-1}[C_t/A_t]}{E_{t-1}[C_t^{1-\rho}]}$$ (4.24)

Assuming a symmetric equilibrium ($C_t(h) = C_t$),

$$P_{H,t} = \frac{\theta \kappa}{\theta - 1} \frac{P_tE_{t-1}[C_t/A_t]}{E_{t-1}[C_t^{1-\rho}]}$$ (4.24)
Similarly,

\[ P_{N,t} = \frac{\theta \kappa}{\theta - 1} \frac{P_t E_{t-1} [C_t / B_t]}{E_{t-1} [C_t^{1-\rho}]} \]  

(4.25)

\[ P_{H,t}^* = \frac{\theta \kappa}{\theta - 1} \frac{P_t E_{t-1} [C_t^* / A_t]}{E_{t-1} [S_t C_t^{1-\rho} C_t^*]} \]  

(4.26)

\[ = \frac{\theta \kappa}{\theta - 1} \frac{P_t E_{t-1} [C_t^* / A_t]}{E_{t-1} [C_t^{1-\rho}]} \]  

The last equality is due to (4.12)\textsuperscript{102}.

Using (4.6) and (4.7), (4.24) can be rewritten as follows.

\[ \frac{P_{H,t}^{1-\gamma}}{P_{F,t}^2} = \frac{\theta \kappa}{\theta - 1} \frac{P_{N,t}^{1-\gamma} E_{t-1} [C_t / A_t]}{E_{t-1} [C_t^{1-\rho}]} \]  

(4.27)

From (4.24) and (4.25),

\[ \frac{P_{H,t}}{P_{N,t}} = \frac{E_{t-1} [C_t / A_t]}{E_{t-1} [C_t / B_t]} \]  

(4.28)

Note that, unlike Obstfeld (2004) and others, the relative price of the home tradables to nontradables is not one in general. Using (4.28), (4.27) can be rewritten as,

\[ \frac{P_{H,t}}{P_{F,t}} = \left( \frac{\theta \kappa}{\theta - 1} \right)^{\frac{\gamma}{2}} \frac{(E_{t-1} [C_t / A_t])^2 (E_{t-1} [C_t / B_t])^{2(1-\gamma)}}{(E_{t-1} [C_t^{1-\rho}])^{\frac{\gamma}{2}}} \]  

(4.29)

(4.26) implies the following home price of foreign traded goods.

\[ P_{F,t} = \frac{\theta \kappa}{\theta - 1} \frac{P_t E_{t-1} [C_t / A_t]}{E_{t-1} [C_t^{1-\rho}]} \]  

(4.30)

Using (4.6), (4.7), and (4.28) again,

\[ \frac{P_{F,t}}{P_{H,t}} = \left( \frac{\theta \kappa}{\theta - 1} \right)^{\frac{2}{\gamma}} \frac{(E_{t-1} [C_t / A_t])^{\frac{2}{\gamma}} (E_{t-1} [C_t / B_t])^{\frac{2(1-\gamma)}{\gamma}}}{(E_{t-1} [C_t^{1-\rho}])^{\frac{2}{\gamma}}} \]  

(4.31)

Multiplying (4.29) to (4.31),

\[ 1 = \left( \frac{\theta \kappa}{\theta - 1} \right)^{\frac{4}{\gamma (2-\gamma)}} \frac{(E_{t-1} [C_t / A_t])^{\frac{2}{\gamma}} (E_{t-1} [C_t / B_t])^{\frac{4(1-\gamma)}{\gamma}}}{(E_{t-1} [C_t^{1-\rho}])^{\frac{4}{\gamma (2-\gamma)}}} \]  

(4.32)

\textsuperscript{102}This is possible because \(P_t\) and \(P_t^*\) are known at time \(t - 1\).
Then, log normality implies the following.

\[
E_{t-1}c_t = \frac{1}{\rho} \ln \left( \frac{\theta - 1}{\theta \kappa} \right) + \frac{\gamma}{2\rho} \left( E_{t-1}a_t + E_{t-1}a^*_t + \sigma_{cu} + \sigma_{cu^*} - \frac{1}{2} \sigma_u^2 - \frac{1}{2} \sigma_{u^*}^2 \right) + \frac{1 - \gamma}{\rho} \left( E_{t-1}b_t + \sigma_{cu} - \frac{1}{2} \sigma_v^2 \right) - \frac{2 - \rho}{2} \sigma_c^2
\]

(4.33)

From the first order condition for nominal bond holdings,

\[
\frac{C_{t-1}^{-\rho}}{P_t} = (1 + i_t) \beta E_t \left( \frac{C_{t+1}^{-\rho}}{P_{t+1}} \right)
\]

(4.34)

Recognizing that \(P_{t+1}\) is known at time \(t\), log normality implies,

\[
c_t = E_t c_{t+1} - \frac{1}{\rho} \left[ \log \beta + i_t - (p_{t+1} - p_t) + \frac{\rho^2}{2} \sigma_{c^2} \right]
\]

(4.35)

Plug the interest rate rule (4.15) into (4.35), and take expectations at time \(t-1\),

\[
p_t = \frac{1}{1 + \psi} E_{t-1} p_{t+1} + \frac{1}{1 + \psi} \left[ \rho (E_{t-1} c_{t+1} - E_{t-1} c_t) - \left( \log \beta + i_t + \frac{\rho^2}{2} \sigma_{c^2} \right) \right]
\]

(4.36)

Solving (4.36) forward, we get the following.

\[
p_t = \rho \sum_{j=0}^{\infty} \left( \frac{1}{1 + \psi} \right)^{j+1} E_{t-1} (c_{t+j+1} - c_{t+j}) - \frac{1}{\psi} \left( \log \beta + i_t + \frac{\rho^2}{2} \sigma_{c^2} \right).
\]

(4.37)

with a proper transversality condition imposed.

From (4.33),

\[
E_{t-1}(c_{t+j+1} - c_{t+j}) = \frac{\gamma}{2\rho} E_{t-1}(a_{t+j+1} - a_{t+j} + a^*_{t+j+1} - a^*_{t+j}) + \frac{1 - \gamma}{\rho} E_{t-1}(b_{t+j+1} - b_{t+j})
\]

(4.38)

The stochastic processes (4.14) imply the following conditional expectations at time \(t-1\).

\[
E_{t-1}a_{t+j} = \lambda^{j+1} a_{t-1} + \lambda^j u^*_{t-1}, \quad E_{t-1}a^*_{t+j} = \lambda^{j+1} a^*_{t-1} + \lambda^j u_{t-1},
\]

\[
E_{t-1}b_{t+j} = \lambda^{j+1} b_{t-1} + \lambda^j v^*_{t-1}
\]

(4.39)
Plug (4.39) into (4.38), and use the resulting equation in (4.37), we get the following.

\[ p_t = \frac{\gamma}{2} \sum_{j=0}^{\infty} \left( \frac{\lambda}{1+\psi} \right)^{j+1} \left( \lambda - 1 \right) (a_{t-1} + a^*_{t-1}) + (1 - \gamma) \sum_{j=0}^{\infty} \left( \frac{\lambda}{1+\psi} \right)^{j+1} \left( \lambda - 1 \right) b_{t-1} \]

\[ + \frac{\gamma}{2} \sum_{j=0}^{\infty} \left( \frac{\lambda}{1+\psi} \right)^{j+1} \left( \frac{\lambda - 1}{\lambda} \right) (u_{t-1} + u^*_{t-1}) \]

\[ + (1 - \gamma) \sum_{j=0}^{\infty} \left( \frac{\lambda}{1+\psi} \right)^{j+1} \left( \frac{\lambda - 1}{\lambda} \right) v^*_{t-1} - \frac{1}{\psi} \left( \log \beta + \mu + \frac{\rho^2}{2\sigma^2_c} \right), \]

which can be solved as,

\[ p_t = -\frac{\gamma \lambda (1 - \lambda)}{2(1 + \psi - \lambda)} (a_{t-1} + a^*_{t-1}) - \frac{(1 - \gamma) \lambda (1 - \lambda)}{1 + \psi - \lambda} b_{t-1} \]

\[ - \frac{\gamma (1 - \lambda)}{2(1 + \psi - \lambda)} (u_{t-1} + u^*_{t-1}) - \frac{(1 - \gamma) (1 - \lambda)}{1 + \psi - \lambda} v^*_{t-1} \]

\[ - \frac{1}{\psi} \left( \log \beta + \mu + \frac{\rho^2}{2\sigma^2_c} \right) \] (4.40)

Note that the home price index \( p_t \) will be lowered by a positive shock \( u_{t-1} \) at time \( t - 1 \) via two channels, \( p_{H,t} \) and \( p_{F,t} \). First of all, \( p_{H,t} \) will be set at a lower level because the home tradables sector productivity will increase by \( \lambda u_{t-1} \) at time \( t \). At the same time, \( p_{F,t} \) will decline because the foreign tradables sector productivity will fully incorporate \( u_{t-1} \) at time \( t \) due to technology diffusion.

Next, we solve for realized consumption \( c_t \). Plug the interest rate rule (4.15) into (4.35), and take expectations at time \( t \),

\[ c_t = E_t c_{t+1} = \frac{1}{\rho} \ln \beta + \mu - p_{t+1} + (1 + \psi) p_t - \alpha_1 u_t - \alpha_2 v_t - \alpha_3 u^*_t - \alpha_4 v^*_t + \frac{\rho^2}{2\sigma^2_c} \] (4.41)

Updating (4.33) once,

\[ E_t c_{t+1} = \frac{1}{\rho} \ln \left( \frac{\theta - 1}{\theta K} \right) + \frac{\gamma}{2\rho} \left( E_t a_{t+1} + E_t a^*_{t+1} + \sigma_{cu} + \sigma_{cu^*} - \frac{1}{2} \sigma^2_u - \frac{1}{2} \sigma^2_u^* \right) \]

\[ + \frac{1 - \gamma}{\rho} \left( E_t b_{t+1} + \sigma_{cv} - \frac{1}{2} \sigma^2_v \right) - \frac{2 - \rho}{2} \sigma^2_c \] (4.42)
Plugging (4.42) into (4.41) and rearranging it,
\[
c_t = \frac{\gamma}{2\rho}(E_t a_{t+1} + E_t a_{t+1}^*)& + \frac{1 - \gamma}{\rho} E_t b_{t+1} + \frac{1}{\rho} p_{t+1} - \frac{1 + \psi}{\rho} p_t \\
+ \frac{1}{\rho} (\alpha_1 u_t + \alpha_2 v_t + \alpha_3 u^*_t + \alpha_4 v^*_t) + \nabla,
\]
where \(\nabla\) is a function of parameters and unconditional moments. From (4.14),
\[
E_t a_{t+1} = \lambda a_t + u^*_t, \quad E_t a^*_{t+1} = \lambda a_t^* + u_t
\]
\[
E_t b_{t+1} = \lambda b_t + v^*_t
\]
Plug (4.44) and \(p_{t+1}\) obtained from updating (4.40) once into (4.43), we get the following function of realized consumption in terms of contemporaneous shocks.
\[
c_t = \frac{\gamma \lambda \psi}{2\rho(1 + \psi - \lambda)}(a_t + a^*_t) + \frac{(1 - \gamma) \lambda \psi}{\rho(1 + \psi - \lambda)} b_t \\
+ \frac{\gamma \psi}{2\rho(1 + \psi - \lambda)}(u_t + u^*_t) + \frac{(1 - \gamma) \psi}{\rho(1 + \psi - \lambda)} v^*_t \\
+ \frac{1}{\rho} (\alpha_1 u_t + \alpha_2 v_t + \alpha_3 u^*_t + \alpha_4 v^*_t) + \tilde{\nabla},
\]
where \(\tilde{\nabla}\) denotes a function of parameters, unconditional moments, and variables dated \(t - 1\). Foreign consumption can be similarly obtained as,
\[
c^*_t = \frac{\gamma \lambda \psi}{2\rho(1 + \psi - \lambda)}(a_t + a^*_t) + \frac{(1 - \gamma) \lambda \psi}{\rho(1 + \psi - \lambda)} b^*_t \\
+ \frac{\gamma \psi}{2\rho(1 + \psi - \lambda)}(u_t + u^*_t) + \frac{(1 - \gamma) \psi}{\rho(1 + \psi - \lambda)} v_t \\
+ \frac{1}{\rho} (\alpha_1^* u^*_t + \alpha_2^* v^*_t + \alpha_3^* u_t + \alpha_4^* v_t) + \tilde{\nabla}^*
\]
Let’s assume that policy parameters \(\alpha\)’s are all zero as in the flexible prices equilibrium. Then, (4.45) implies the following.
\[
c_t - E_{t-1} c_t = \left[ \frac{\gamma}{2\rho} \left( \frac{\psi \lambda}{1 + \psi - \lambda} \right) + \frac{\gamma}{2\rho} \left( \frac{\psi}{1 + \psi - \lambda} \right) \right] (u_t + u^*_t) \\
+ \frac{1 - \gamma}{\rho} \left( \frac{\lambda \psi}{1 + \psi - \lambda} \right) v_t + \frac{1 - \gamma}{\rho} \left( \frac{\psi}{1 + \psi - \lambda} \right) v^*_t
\]
Similarly,

\[ c_t^* - E_{t-1}c_t^* = \left[ \frac{\gamma}{2\rho} \left( \frac{\psi \lambda}{1 + \psi - \lambda} \right) + \frac{\gamma}{2\rho} \left( \frac{\psi}{1 + \psi - \lambda} \right) \right] (u_t + u_t^*) \tag{4.48} \]

\[ + \frac{1 - \gamma}{\rho} \left( \frac{\lambda \psi}{1 + \psi - \lambda} \right) v_t^* + \frac{1 - \gamma}{\rho} \left( \frac{\psi}{1 + \psi - \lambda} \right) v_t \]

Compared with (4.20) and (4.21), equations (4.47) and (4.48) demonstrate the following interesting results. First, under the current setup, tradables sector productivity shocks create two types of distortions, the sticky price effect \( \left( \frac{\gamma}{2\rho} \left( \frac{\psi \lambda}{1 + \psi - \lambda} \right) \right) \) and the news effect \( \left( \frac{\gamma}{2\rho} \left( \frac{\psi}{1 + \psi - \lambda} \right) \right) \). Regarding the sticky price effect, neither country cannot generate a full response \( \left( \frac{\gamma}{2\rho} \right) \) of consumption to \( u_t \), since \( p_t \) was already pre-determined at time \( t - 1 \). Therefore, consumption responses will be muted by \( \frac{\psi \lambda}{1 + \psi - \lambda} (\leq 1) \).

At the same time, \( u_t \) generates the news effect at time \( t \). When \( u_t \) is realized, agents in both countries know it will diffuse to the foreign country, and thus \( p_F \) will be lowered at time \( t + 1 \), and so will \( p_{t+1} \). Anticipating higher \( c_{t+1} \), people will increase \( c_t \) right away (consumption smoothing), which is distortionary since \( A_t^* \) hasn’t changed yet. Note that \( v_t^* \) will generate a pure news effect in the home country, while it creates only the sticky price effect in the foreign country. Similar reasoning can be done with respect to \( u_t^* \) and \( v_t \).

It is very interesting to see that productivity shocks \( u_t \) or \( u_t^* \) may lead to a market over-reaction. Note that the combined impacts of \( u_t \) or \( u_t^* \) on either country’s consumption is,

\[ \frac{\gamma}{2\rho} \left( \frac{\psi(1 + \lambda)}{1 + \psi - \lambda} \right), \]
which is greater than the consumption response \( \frac{\gamma}{2\rho} \) under the flexible prices equilibrium\(^{103} \) as long as \( \psi > \frac{1-\lambda}{\lambda} \). Since \( \lambda \) is a persistent parameter of productivity that is often deemed to be fairly big, this condition may not be too restrictive\(^{104} \). If \( \psi \) happens to be same as \( \frac{1-\lambda}{\lambda} \), consumption responses will be identical to those of flexible prices economy.

It should be also noted that international consumptions identically respond to \( u_t \) and \( u_t^* \), while they disproportionately respond to \( v_t \) and \( v_t^* \).

Following Obstfeld (2004), I derive the endogenous covariances in this model for a welfare analysis in next . For algebraic simplicity, I assume all shocks are uncorrelated each other. From (4.45), we get the following.

\[
\begin{align*}
\sigma_c^2 &= A_1^2 \sigma_u^2 + A_2^2 \sigma_v^2 + A_3^2 \sigma_u^* + A_4^2 \sigma_v^* \\
\sigma_{cu} &= A_1 \sigma_u^2, \sigma_{cv} = A_2 \sigma_v^2, \sigma_{cu*} = A_3 \sigma_u^2, \sigma_{cv*} = A_4 \sigma_v^2,
\end{align*}
\]

where

\[
\begin{align*}
A_1 &= \frac{\gamma}{2\rho} \frac{\psi(1+\lambda)}{1+\psi-\lambda} + \frac{\alpha_1}{\rho}, \quad A_2 = \frac{1-\gamma}{\rho} \frac{\lambda \psi}{1+\psi-\lambda} + \frac{\alpha_2}{\rho} \\
A_3 &= \frac{\gamma}{2\rho} \frac{\psi(1+\lambda)}{1+\psi-\lambda} + \frac{\alpha_3}{\rho}, \quad A_4 = \frac{1-\gamma}{\rho} \frac{\psi}{1+\psi-\lambda} + \frac{\alpha_4}{\rho}
\end{align*}
\]

The foreign covariances can be analogously defined.

\(^{103}\)See equations (4.20) or (4.21).

\(^{104}\)However, there is no reason to require \( \psi \) to be greater than \( \frac{1-\lambda}{\lambda} \) under the current setup. Any strictly positive \( \psi \) works in this paper. If \( \psi \) is negative, \( p_t \) may become indeterminate.
4.5 Optimal Interest Rate Rule and Welfare Analysis

Home labor supply must be consistent with the following condition.

\[
E_t L_{t+1} = \gamma \left( \frac{P_{t+1}}{P_{H,t+1}} \right) E_t \left( \frac{C_{t+1}}{A_{t+1}} \right) + \gamma \left( \frac{P^*_t}{P^*_{H,t+1}} \right) E_t \left( \frac{C^*_t}{A_{t+1}} \right) + (1 - \gamma) \left( \frac{P_{t+1}}{P_{N,t+1}} \right) E_t \left( \frac{C_{t+1}}{B_{t+1}} \right)
\]

(4.50)

Plug (4.24), (4.25), (4.26) into (4.50), and we obtain the following.

\[
E_t L_{t+1} = \left( \frac{\theta - 1}{\theta \kappa} \right) \left\{ \left( 1 - \frac{\gamma}{2} \right) E_t C^{1-\rho}_{t+1} + \frac{\gamma}{2} E_t C^*_{t+1} \right\}
\]

(4.51)

Plug (4.51) into the home expected utility at time \( t \), and rearrange it. Then, we get,

\[
E_t \left[ \frac{C^{1-\rho}_{t+1}}{1 - \rho} - \kappa L_{t+1} \right] = E_t \left[ \left( \theta \gamma + 2 \gamma - \frac{\theta}{2} \right) E_t C^{1-\rho}_{t+1} - \frac{\gamma(\theta - 1)}{2\theta} E_t C^*_{t+1} \right]
\]

(4.52)

As we can see in (4.45), the foreign monetary intervention doesn’t affect home consumption. Thus, it is sufficient to maximize the following.

\[
E_t C^{-\rho}_{t+1} = \exp \left[ (1 - \rho) E_t c_{t+1} + \frac{(1 - \rho)^2}{2} \sigma_c^2 \right],
\]

where the last equality holds due to the lognormality assumption. Or, more simply,

\[
\max_{\alpha} \left\{ E_t c_{t+1} + \frac{1 - \rho}{2} \sigma_c^2 \right\}
\]

Using (4.42),

\[
E_t c_{t+1} + \frac{1 - \rho}{2} \sigma_c^2 = \gamma \left( \sigma_{cu} + \sigma_{cu*} \right) + \frac{1 - \gamma}{\rho} \sigma_{cw} - \frac{1}{2} \sigma_c^2 + NP,
\]

(4.53)

where NP denotes a function of non-policy variables. Using (4.49), we express (4.53) as a function of policy coefficients and unconditional moments of shocks. Then, the
maximization problem collapses down to the following.

\[
\begin{align*}
\max_{\alpha} & \left\{ A_1 \left( \frac{\gamma}{2 \rho} - \frac{1}{2} A_1 \right) \sigma_u^2 + A_2 \left( \frac{1-\gamma}{\rho} - \frac{1}{2} A_2 \right) \sigma_v^2 \\
& \quad + A_3 \left( \frac{\gamma}{2 \rho} - \frac{1}{2} A_3 \right) \sigma_{u*}^2 - \frac{1}{2} A_4^2 \sigma_{v*}^2 \right\} \\
\end{align*}
\] (4.54)

Straightforward maximization with respect to the policy parameters yield the following.

\[
\begin{align*}
\alpha_1 &= \frac{\gamma}{2} \left( 1 - \frac{\psi(1 + \lambda)}{1 + \psi - \lambda} \right) = \alpha_3 \\
\alpha_2 &= (1 - \gamma) \left( 1 - \frac{\psi \lambda}{1 + \psi - \lambda} \right) \\
\alpha_4 &= -(1 - \gamma) \frac{\psi}{1 + \psi - \lambda}
\end{align*}
\] (4.55)

Analogously, the foreign optimal policy coefficients are,

\[
\begin{align*}
\alpha_1^* &= \frac{\gamma}{2} \left( 1 - \frac{\psi(1 + \lambda)}{1 + \psi - \lambda} \right) = \alpha_3^* \\
\alpha_2^* &= (1 - \gamma) \left( 1 - \frac{\psi \lambda}{1 + \psi - \lambda} \right) \\
\alpha_4^* &= -(1 - \gamma) \frac{\psi}{1 + \psi - \lambda}
\end{align*}
\] (4.56)

Therefore,

\[
\alpha_1 = \alpha_1^* = \alpha_3 = \alpha_3^* = \alpha_4 = \alpha_4^* = \alpha_4 = \alpha_4^* (4.57)
\]

A careful inspection of (4.47) and (4.48) shows that these optimal interest rate rules recover the consumption innovations under the flexible prices equilibrium (4.20) and (4.21). Furthermore, it is easy to show that the endogenous variances under the flexible prices solution are recovered with these optimal rules. That is,

\[
\begin{align*}
\sigma_c^2 &= \left( \frac{\gamma}{2 \rho} \right)^2 \left( \sigma_u^2 + \sigma_{u*}^2 \right) + \left( \frac{1-\gamma}{\rho} \right)^2 \sigma_v^2 \\
\sigma_{cu} &= \left( \frac{\gamma}{2 \rho} \right) \sigma_u^2, \quad \sigma_{cv} = \left( \frac{1-\gamma}{\rho} \right) \sigma_u^2, \quad \sigma_{cu*} = \left( \frac{\gamma}{2 \rho} \right) \sigma_{u*}^2, \quad \sigma_{cv*} = 0
\end{align*}
\] (4.58)
Note that these variances are minima under the current setup.

One interesting feature of these results is the following. Unlike Obstfeld (2004) and others, central banks may raise nominal interest rate when a positive productivity shock occurs. This may occur when the news effect is big enough. Remember that a current shock $u_t (u_t^*)$ will lower $p_{F,t+1}$ ($p_{H,t+1}$) and thus lower $p_{t+1}$ owing to technology diffusion with a one-period lag. Assuming that central banks can commit these optimal policy responses, a strictly positive value of $\psi$ implies lower nominal interest rates at time $t + 1$. Furthermore, lower $p_{t+1}$ will lead to higher expected inflation at time $t + 1$. Hence, real interest rates at time $t + 1$ will be lower in both countries. Therefore, expecting a positive consumption growth owing to lower $p_{t+1}$ and lower real interest rates, agents will intend to increase $c_t$ at time $t$. However, such a consumption innovation at time $t$ (due to the news effect) is distortionary, and central banks wish to eliminate it. Note that such a news effect will be greater the bigger $\psi$ is, since bigger $\psi$ implies that real interest rates will decline more at time $t + 1$.

In a nutshell, if central banks commit fairly aggressive responses to prices with $\psi > \frac{1-\lambda}{\lambda}$, consumers may over-react, and the central banks may wish to respond by implementing contractionary monetary policies. This result is new, and I believe it is consistent with empirical findings. It is common to see rising productivity during economic boom, and nominal interest rates normally rise during economic boom.

Another interesting feature of this paper is the following. Unlike the models by Obstfeld (2004) and Duarte and Obstfeld (2004), optimal monetary policy may not

\footnote{Because technology diffusion will occur at time $t + 1$.}

\footnote{Under LCP, models by Devereux and Engel (2003), Obstfeld (2004), Duarte and Obstfeld (2004) predict that central banks implement expansionary monetary policies. It is easy to see that the sticky price effect alone cannot generate market over-reaction.}

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require flexible exchange rates as long as country-specific technology shocks occur in tradables sectors. This is because, under the current setup, central banks identically respond to \( u_t \) or \( u_t^* \) (see equation 4.57). However, policy responses to \( v_t \) or \( v_t^* \) will be asymmetric, and the resulting interest rate differential calls for flexible exchange rates.

This feature may also have the following implication. With \( \psi = \frac{1-\lambda}{\lambda} \), optimal responses will become,

\[
\begin{align*}
\alpha_1 &= \alpha_1^* = \alpha_3 = \alpha_3^* = 0 \\
\alpha_2 &= \alpha_2^* = \delta \\
\alpha_4 &= \alpha_4^* = -\delta,
\end{align*}
\]

where \( \delta = \frac{1-\gamma}{1+\lambda} \). That is, central banks may not react to technology shocks in tradables sectors. But they oppositely respond to real shocks in nontradables sector. If technology innovations occur less frequently in nontraded goods sectors, this feature may be related to the so-called dirty-float in the sense that monetary authorities affect exchange rates movements only infrequently.

### 4.6 Conclusion

In this paper, I develop a sticky-price, local currency pricing model with a more realistic specification of a country-specific shock. Rather than assuming that technology shocks in tradables and nontradables sectors are perfectly correlated in a country, I introduce country- and sector-specific productivity shocks, which can diffuse to the same sector of the other country with a one-period time lag.

This modification produces quite interesting results. When a central bank commits a fairly aggressive response to price changes, a favorable country-specific shock
may generate market over-reaction, which requires the central bank to raise nominal interest rate rather than lowering it as Obstfeld (2004) and others predict. This seems to be consistent with some empirical findings of rising nominal interest rates during economic boom.

This paper also shows that central banks may still identically respond to a country-specific shock (in tradables sectors) even in the presence of non-traded goods. However, real shocks originated from nontradables sectors induce opposite responses of central banks, and the resulting interest rate differential requires flexible exchange rates. This paper also may explain infrequent exchange rate interventions of central banks that may be consistent with dirty float system.
CHAPTER 5

CONCLUDING REMARKS

In this dissertation, I develop three exchange rate models that incorporate a Taylor Rule type monetary policy in attempt to investigate its empirical and theoretical implications on exchange rate dynamics.

In chapters 2, I address two perennial issues of PPP literature, unreasonably long half-life estimates of PPP deviations and extremely wide confidence intervals, which may be encountered when we use a single equation approach. Alternatively, I construct a dynamic system of exchange rates and inflation rate by combining Mussa’s (1982) error correction model of exchange rate with a forward looking version Taylor Rule along with uncovered interest parity condition. In so doing, I derive estimable equations for half-lives of PPP deviations, which produce reasonably short half-life estimates with significantly tighter confidence intervals.

In chapter 3, I extend the model in chapter 2 to a two-goods model in order to estimate and compare half-lives of PPP deviations for PPI- and CPI-based real exchange rates. In a similar framework but using a conventional money demand equation rather than using the Taylor Rule, Kim (2004) obtained significantly shorter half-life point estimates for the non-service goods consumption deflators than those for the service goods consumption deflators, though with quite wide confidence intervals.
Using a forward looking version Taylor Rule, however, I find that half-life estimates are about the same irrespective of the choice of aggregate price indexes, which is consistent with many other studies that reported only moderate or no difference (Wu, 1996, Murray and Papell, 2002). More importantly, I obtain much tighter confidence intervals so that statistically meaningful comparisons between the size of half-life point estimates.

In chapter 4, I develop a dynamic stochastic general equilibrium model of exchange rates that features price-stickiness, local-currency pricing, and technology diffusion in attempt to investigate the optimal interest rate rule and its implications on exchange rate regimes. Unlike other researches (Obstfeld, 2004, Duarte and Obstfeld, 2005, Devereux and Engel, 2006) that assume technology shocks in tradables and nontradables sectors are perfectly correlated in a country, I introduce country- and sector-specific productivity shocks, which can diffuse to the same sector of the other country with a one-period time lag.

With this modification, I obtain the following quite interesting results. First, it is shown that when a central bank commits a fairly aggressive response to price changes, a favorable country-specific shock may generate market over-reaction, which requires the central bank to raise nominal interest rate rather than lowering it as Obstfeld (2004) and others predict. I believe that my prediction is consistent with empirical evidence of rising nominal interest rates during economic boom. Second, my model also shows that central banks may still identically respond to a country-specific shock that occurs in tradables sectors, even in the presence of non-traded goods. In such a circumstance, therefore, no exchange rate change will be required in implementing
optimal monetary policies. However, real shocks that are originated from nontradables sectors induce opposite responses of central banks, and the resulting interest rate differential calls for exchange rate changes. Therefore, benefits of maintaining flexible exchange rates could be quite limited if technology shocks in nontradables sectors occur infrequently. I also construct a case that may explain infrequent exchange rate interventions of central banks, which might be consistent with dirty float system.
APPENDIX A

DERIVATION OF (2.11)

Since $\Lambda$ in (2.10) is diagonal, assuming $0 < \alpha < 1$ and $1 < \gamma_\pi < \frac{1}{1-\rho}$, we can solve the system as follows.

\[
\begin{align*}
z_{1,t} &= \sum_{j=0}^{\infty} \alpha^j h_{1,t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \\
z_{2,t} &= -\sum_{j=0}^{\infty} \left(\frac{1 - \gamma_\pi^s}{\rho}\right)^{j+1} E_t h_{2,t+j} \\
z_{3,t} &= h_{3,t-1} + v_t,
\end{align*}
\]

where $u_t$ and $v_t$ are any martingale difference sequences.

Since $y_t = Vz_t$,

\[
\begin{bmatrix}
\Delta p_t \\
\Delta e_t \\
i_{t-1}
\end{bmatrix} =
\begin{bmatrix}
\alpha & 1 & 1 \\
\alpha & 1 & 1 \\
\alpha & 1 & 0
\end{bmatrix}
\begin{bmatrix}
z_{1,t} \\
z_{2,t} \\
z_{3,t}
\end{bmatrix}
\]

(a4)

From first and second rows of (a4), we get the following.

\[
\Delta e_t = \frac{\alpha \gamma_\pi^s}{\alpha - \rho} \Delta p_t - \frac{\alpha \gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho} z_{2,t} - \frac{\alpha \gamma_\pi^s - (\alpha - \rho)}{\alpha - \rho} z_{3,t}
\]

(a5)

Now, we find the analytic solutions for $z_t$. Since $h_t = V^{-1}c_t$,

\[
h_t = \frac{1}{1 - \gamma_\pi^s}
\begin{bmatrix}
\alpha & \alpha & \alpha \\
\alpha & \alpha & \alpha \\
\alpha & \alpha & \alpha
\end{bmatrix}
\begin{bmatrix}
\Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t + \gamma_\pi^s x_t - i_t^s \\
\gamma_\pi^s (\Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t) + \gamma_\pi^s x_t - i_t^s \\
\gamma_\pi^s (\Delta p_{t+1}^* - \alpha \Delta p_t^* + \eta_t) + \gamma_\pi^s x_t - \gamma_\pi^s i_t^s
\end{bmatrix}
\]

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and thus,

\[ h_{1,t} = -\frac{\alpha - \rho}{\alpha \gamma^s_\pi - (\alpha - \rho)} (E_t \Delta p^*_{t+1} - \alpha \Delta p_t^* + \eta_t) \]  

\[ h_{2,t} = \frac{1}{1 - \gamma^s_\pi} \left[ \frac{\rho \gamma^s_\pi, t}{\alpha \gamma^s_\pi - (\alpha - \rho)} (E_t \Delta p^*_{t+1} - \alpha \Delta p_t^* + \eta_t) + t + \gamma^s \pi x_t - \gamma^s \pi t_t \right] \]  

\[ h_{3,t} = -i^*_t \]  

Plugging (a6) into (a1),

\[ z_{1,t} = -\frac{\alpha - \rho}{\alpha \gamma^s_\pi - (\alpha - \rho)} \sum_{j=0}^\infty \alpha^j \left( \Delta p^*_{t-j} - \alpha \Delta p^*_{t-j-1} + \eta_{t-j-1} \right) + \sum_{j=0}^\infty \alpha^j u_{t-j} \]  

\[ = -\frac{\alpha - \rho}{\alpha \gamma^s_\pi - (\alpha - \rho)} \Delta p^*_t + \sum_{j=0}^\infty \alpha^j u_{t-j} - \frac{\alpha - \rho}{\alpha \gamma^s_\pi - (\alpha - \rho)} \sum_{j=0}^\infty \alpha^j \eta_{t-j-1} \]  

Plugging (a7) into (a2)\(^\text{107}\),

\[ z_{2,t} = -\frac{\gamma^s_\pi}{\alpha \gamma^s_\pi - (\alpha - \rho)} \sum_{j=0}^\infty \left( \frac{1 - \gamma^s_\pi}{\rho} \right)^j (E_t \Delta p^*_{t+j+1} - \alpha E_t \Delta p^*_{t+j} + E_t \eta_{t+j}) \]  

\[ - \frac{1}{\rho} \sum_{j=0}^\infty \left( \frac{1 - \gamma^s_\pi}{\rho} \right)^j \left( t + \gamma^s \pi E_t x_{t+j} - \gamma^s \pi E_t i^*_t \right) \]  

\[ = \frac{\alpha \gamma^s_\pi}{\alpha \gamma^s_\pi - (\alpha - \rho)} \Delta p^*_t - \frac{\gamma^s_\pi}{\alpha \gamma^s_\pi - (\alpha - \rho)} \eta_t - \frac{t}{\gamma^s_\pi - (1 - \rho)} \]  

\[ - \frac{\gamma^s_\pi}{\rho} \sum_{j=0}^\infty \left( \frac{1 - \gamma^s_\pi}{\rho} \right)^j E_t \Delta p^*_{t+j+1} - \frac{\gamma^s_\pi}{\rho} \sum_{j=0}^\infty \left( \frac{1 - \gamma^s_\pi}{\rho} \right)^j \left( \frac{\gamma^s_\pi}{\rho} E_t x_{t+j} - E_t i^*_t \right) \]  

Then, denoting \( E_t f_{t+j} \) as \(- (E_t i^*_{t+j} - E_t \Delta p^*_{t+j+1}) + \frac{\gamma^s_\pi}{\rho} E_t x_{t+j} = -E_t r^*_{t+j} + \gamma^s_\pi E_t x_{t+j},\)

\[ z_{2,t} = \frac{\alpha \gamma^s_\pi}{\alpha \gamma^s_\pi - (\alpha - \rho)} \Delta p^*_t - \frac{\gamma^s_\pi}{\alpha \gamma^s_\pi - (\alpha - \rho)} \eta_t - \frac{t}{\gamma^s_\pi - (1 - \rho)} \]  

\[- \frac{\gamma^s_\pi}{\rho} \sum_{j=0}^\infty \left( \frac{1 - \gamma^s_\pi}{\rho} \right)^j E_t f_{t+j} \]  

Finally, plugging (a8) into (a3),

\[ z_{3,t} = -i^*_{t-1} + v_t \]  

\(^{107}\)We use the fact \( E_t \eta_{t+j} = 0, \ j = 1, 2, \ldots \).
Now, plugging (A.1) and (a11) into (a5),
\[
\Delta e_t = \frac{\alpha \gamma_s^\pi}{\alpha - \rho} \Delta p_t - \frac{\alpha \gamma_s^\pi}{\alpha - \rho} \Delta p^*_t + \frac{\gamma_s^\pi}{\alpha - \rho} \eta_t + \frac{\alpha \gamma_s^\pi - (\alpha - \rho)}{(\alpha - \rho)(\gamma_s^\pi - (1 - \rho))} t
\]
\[
+ \frac{\gamma_s^\pi(\alpha \gamma_s^\pi - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_s^\pi}{\rho} \right)^j E_t f_{t+j} + \frac{\alpha \gamma_s^\pi - (\alpha - \rho)}{\alpha - \rho} \iota_{t-1}
\]
\[
- \frac{\alpha \gamma_s^\pi - (\alpha - \rho)}{\alpha - \rho} \eta_t
\]

Updating (a12) once and applying law of iterated expectations,
\[
\Delta e_{t+1} = \hat{i} + \frac{\alpha \gamma_s^\pi}{\alpha - \rho} \Delta p_{t+1} - \frac{\alpha \gamma_s^\pi}{\alpha - \rho} \Delta p^*_t + \frac{\alpha \gamma_s^\pi - (\alpha - \rho)}{\alpha - \rho} i^*_t
\]
\[
+ \frac{\gamma_s^\pi(\alpha \gamma_s^\pi - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_s^\pi}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1},
\]
where
\[
\hat{i} = \frac{\alpha \gamma_s^\pi - (\alpha - \rho)}{(\alpha - \rho)(\gamma_s^\pi - (1 - \rho))} t,
\]
\[
\omega_{t+1} = \frac{\gamma_s^\pi(\alpha \gamma_s^\pi - (\alpha - \rho))}{(\alpha - \rho)\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_s^\pi}{\rho} \right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1})
\]
\[
+ \frac{\gamma_s^\pi}{\alpha - \rho} \eta_{t+1} - \frac{\alpha \gamma_s^\pi - (\alpha - \rho)}{\alpha - \rho} \eta_{t+1},
\]
and,
\[
E_t \omega_{t+1} = 0
\]
APPENDIX B

DERIVATION OF (3.17)

Note that $\Lambda$ in (3.16) is a diagonal matrix. Therefore, assuming $0 < \alpha < 1$ and $1 < \delta \gamma_\pi < \frac{1}{1-\rho}$, we can solve the system of stochastic difference equation (3.16) as the following three equations.

\begin{align*}
  z_{1,t} &= \sum_{j=0}^{\infty} \alpha^j h_{1,t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \\
  z_{2,t} &= -\sum_{j=0}^{\infty} \left(1 - \frac{\gamma s_1^2}{\rho}\right)^{j+1} E_t h_{2,t+j} \\
  z_{3,t} &= h_{3,t-1} + v_t,
\end{align*}

where $u_t$ and $v_t$ are any martingale difference sequences.

Since $\mathbf{y}_t = \mathbf{Vz}_t$,

\[
\begin{bmatrix}
  \Delta p_{1t}^1 \\
  \Delta e_t \\
  i_{t-1}
\end{bmatrix}
= \begin{bmatrix}
  1 & 1 & 1 \\
  \frac{\alpha \gamma s_1^2}{\alpha - \rho} & 1 & 1 \\
  \frac{\alpha \gamma s_1^2}{\alpha - \rho} & 1 & 0
\end{bmatrix}
\begin{bmatrix}
  z_{1,t} \\
  z_{2,t} \\
  z_{3,t}
\end{bmatrix}
\]

Multiplying the first row of (b4) by $\frac{\alpha \gamma s_1^2}{\alpha - \rho}$ and subtract it from the second row, we get the following.

\[
\Delta e_t = \frac{\alpha \gamma s_1^2}{\alpha - \rho} \Delta p_{1t}^1 - \frac{\alpha \gamma s_1^2}{\alpha - \rho} z_{2,t} - \frac{\alpha \gamma s_1^2 - (\alpha - \rho)}{\alpha - \rho} z_{3,t}
\]
Now, we need to find the analytic solutions for $z_t$. Since $h_t = V^{-1}c_t$,

$$h_t = \frac{1}{1 - \gamma_t^{s1}} \begin{bmatrix} -\frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} & \frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} & 0 \\ \frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} & -\frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t + t + \gamma_t^{s2} E_t \Delta p_{t+1}^{2*} + \gamma_x x_t - i_t^{s1} \\ \gamma_t^{s1} (E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t) + t + \gamma_t^{s2} E_t \Delta p_{t+1}^{2*} + \gamma_x x_t - i_t^{s1} \\ \gamma_t^{s1} (E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t) + t + \gamma_t^{s2} E_t \Delta p_{t+1}^{2*} + \gamma_x x_t - i_t^{s1} \end{bmatrix},$$

and thus,

$$h_{1,t} = -\frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} (E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t) \quad (b6)$$

$$h_{2,t} = \frac{1}{1 - \gamma_t^{s1}} \begin{bmatrix} \frac{\rho \gamma_t^{s1}}{\alpha \gamma_t^{s1} - (\alpha - \rho)} (E_t \Delta p_{t+1}^{1*} - \alpha \Delta p_t^{1*} + \eta_t) \\ + t + \gamma_t^{s2} E_t \Delta p_{t+1}^{2*} + \gamma_x x_t - \gamma_t^{s1} i_t^{s1} \end{bmatrix} \quad (b7)$$

$$h_{3,t} = -i_t^{s1} \quad (b8)$$

Plugging (b6) into (b1), we get,

$$z_{1,t} = -\frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j \left( \Delta p_{t-j}^{1*} - \alpha \Delta p_{t-j-1}^{1*} + \eta_{t-j-1} \right) + \sum_{j=0}^{\infty} \alpha^j u_{t-j} \quad (b9)$$

$$= -\frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} \Delta p_t^{1*} - \frac{\alpha - \rho}{\alpha \gamma_t^{s1} - (\alpha - \rho)} \sum_{j=0}^{\infty} \alpha^j \eta_{t-j-1} + \sum_{j=0}^{\infty} \alpha^j u_{t-j}$$

Plugging (b7) into (b2),

$$z_{2,t} = -\frac{\gamma_t^{s1}}{\alpha \gamma_t^{s1} - (\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_t^{s1}}{\rho} \right)^j (E_t \Delta p_{t+j+1}^{1*} - \alpha E_t \Delta p_{t+j}^{1*} + E_t \eta_{t+j}) \quad (b10)$$

$$- \frac{1}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_t^{s1}}{\rho} \right)^j (t + \gamma_t^{s2} E_t \Delta p_{t+j+1}^{2*} + \gamma_x E_t x_{t+j} - \gamma_t^{s1} i_t^{s1} E_t i_{t+j})$$

$$= \frac{\alpha \gamma_t^{s1}}{\alpha \gamma_t^{s1} - (\alpha - \rho)} \Delta p_t^{1*} - \frac{\gamma_t^{s1}}{\alpha \gamma_t^{s1} - (\alpha - \rho)} \eta_t - \frac{t}{\gamma_t^{s1} - (1 - \rho)}$$

$$- \frac{\gamma_t^{s1}}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_t^{s1}}{\rho} \right)^j E_t \Delta p_{t+j+1}^{1*}$$

$$- \frac{\gamma_t^{s1}}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_t^{s1}}{\rho} \right)^j \left( \gamma_t^{s2} E_t \Delta p_{t+j+1}^{2*} + \gamma_x E_t x_{t+j} - \gamma_t^{s1} E_t i_{t+j} \right),$$

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where we use the fact $E_t \eta_{t+j} = 0$, $j = 1, 2, \ldots$. Denoting $\hat{i} = -\frac{i}{1 - (1 - \rho)} = \frac{i}{(1 - \rho) (\gamma_{\pi} - 1)}$ and $E_t f_{t+j} = -(E_t i_{t+j}^* - E_t \Delta p_{t+j+1}^*) + \frac{\gamma_{\pi}^1}{\gamma_{\pi}} E_t \Delta p_{t+j+1}^* + \frac{\gamma_{\pi}^1}{\gamma_{\pi}} E_t x_{t+j} = -E_t i_{t+j}^* + \frac{1-\delta}{\rho} E_t \Delta p_{t+j+1}^* + \frac{\gamma_{\pi}^1}{\gamma_{\pi}} E_t x_{t+j}$, (B.1) can be rewritten as follows.

$z_{2,t} = \hat{i} + \frac{\alpha \gamma_{\pi}^1}{\alpha \gamma_{\pi}^1 - (\alpha - \rho)} \Delta p_t^* - \frac{\gamma_{\pi}^1}{\alpha \gamma_{\pi}^1 - (\alpha - \rho)} \eta_t - \frac{\gamma_{\pi}^1}{\rho} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_{\pi}^1}{\rho} \right)^j E_t f_{t+j}$, (b11)

Finally, plugging (b8) into (b3),

$z_{3,t} = -i_{t-1}^* + v_t$ (b12)

Plug (b11) and (b12) into (b5) to get,

$$
\Delta e_t = \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} \Delta p_t^* - \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} \Delta p_t^* - \frac{\gamma_{\pi}^1}{\alpha - \rho} \eta_t - \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} \hat{i}
$$

$$
+ \frac{\gamma_{\pi}^1}{\rho (\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_{\pi}^1}{\rho} \right)^j E_t f_{t+j}
$$

$$
+ \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} i_{t-1}^* - \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} v_t
$$

Updating (B.1) once and applying the law of iterated expectations,

$$
\Delta e_{t+1} = \hat{i} + \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} \Delta p_{t+1}^* - \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} \Delta p_{t+1}^* + \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} \hat{i}^*
$$

$$
+ \frac{\gamma_{\pi}^1}{\rho (\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_{\pi}^1}{\rho} \right)^j E_t f_{t+j+1} + \omega_{t+1},
$$

where,

$$
\bar{i} = \frac{\alpha \gamma_{\pi}^1}{(\alpha - \rho) (\gamma_{\pi}^1 - (1 - \rho))} i^*,
$$

$$
\omega_{t+1} = \frac{\gamma_{\pi}^1}{\rho (\alpha - \rho)} \sum_{j=0}^{\infty} \left( \frac{1 - \gamma_{\pi}^1}{\rho} \right)^j (E_{t+1} f_{t+j+1} - E_t f_{t+j+1})
$$

$$
+ \frac{\gamma_{\pi}^1}{\alpha - \rho} \eta_{t+1} - \frac{\alpha \gamma_{\pi}^1}{\alpha - \rho} v_{t+1},
$$

and,

$$
E_t \omega_{t+1} = 0
$$

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APPENDIX C

DERIVATIONS FOR THE CHAPTER 4

C.1 The Baseline Model

Given a consumption index, the consumption-based price index for the composite goods can be derived from the following minimization problem. For the home traded goods,

\[
\min_{c(h)} \int_0^1 P_H(z)C'_H(z)dz \\
\text{s.t.} \\
\left[ \int_0^1 C_H(z)^{\frac{\theta - 1}{\theta}} dz \right]^{\frac{1}{\theta - 1}} = 1
\]

From the first order condition with respect to \( C_H(h) \),

\[
P_H(h) = \lambda \frac{\theta}{\theta - 1} \left[ \int_0^1 C_H(z)^{\frac{\theta - 1}{\theta}} dz \right]^{\frac{1}{\theta - 1}} \frac{\theta - 1}{\theta} C_H(h)^{-\frac{1}{\theta}} \\
= \lambda \left[ \int_0^1 C_H(z)^{\frac{\theta - 1}{\theta}} dz \right]^{\frac{1}{\theta - 1}} C_H(h)^{-\frac{1}{\theta}}
\]

From this,

\[
\int_0^1 P_H(h)^{1-\theta} dh = \lambda^{1-\theta} \left[ \int_0^1 C_H(z)^{\frac{\theta - 1}{\theta}} dz \right]^{-1} \int_0^1 C_H(h)^{\frac{\theta - 1}{\theta}} dh \\
= \lambda^{1-\theta}
\]
Therefore,
\[
\left( \int_{0}^{1} P_H(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}} = \lambda
\]

Note that \( \lambda \) is a shadow price of one unit of the composite goods \( C_H \). Hence, the price index for the composite home goods consumption is,
\[
P_H = \left( \int_{0}^{1} P_H(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}}
\]

Similarly,
\[
P_N = \left( \int_{0}^{1} P_N(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}}, \quad P_F = \left( \int_{0}^{1} P_F(f)^{1-\theta} df \right)^{\frac{1}{1-\theta}}
\]

Now, consider the following minimum expenditure problem for the traded goods.
\[
\min_{c_H(h), c_F(f)} \int_{0}^{1} P_H(z)C_H(z)dz + \int_{0}^{1} P_F(z)C_F(z)dz
\]
\[
\text{s.t.} \quad 2 \left[ \int_{0}^{1} C_H(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{2(\theta-1)}} \left[ \int_{0}^{1} C_F(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{2(\theta-1)}} = 1
\]

From the first order condition with respect to \( C_H(h) \),
\[
P_H(h) = 2\lambda \frac{\theta}{2(\theta-1)} \left[ \int_{0}^{1} C_H(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta+2}{2(\theta-1)}} \left[ \int_{0}^{1} C_F(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{2(\theta-1)}} \frac{\theta-1}{\theta} C_H(h)^{-\frac{1}{\theta}}
\]
\[
= \lambda \left[ \int_{0}^{1} C_H(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta+2}{2(\theta-1)}} \left[ \int_{0}^{1} C_F(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{2(\theta-1)}} C_H(h)^{-\frac{1}{\theta}},
\]

where \( \lambda \) is now the shadow price of the composite consumption of traded goods. From this,
\[
\int_{0}^{1} P_H(h)^{1-\theta} dh = \lambda^{1-\theta} \left[ \int_{0}^{1} C_H(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{2}} \left[ \int_{0}^{1} C_F(z)^{\frac{\theta-1}{\theta}} dz \right]^{-\frac{\theta}{2}}
\]

Similarly from the first order condition with respect to \( C_F(f) \),
\[
\int_{0}^{1} P_F(f)^{1-\theta} df = \lambda^{1-\theta} \left[ \int_{0}^{1} C_H(z)^{\frac{\theta-1}{\theta}} dz \right]^{-\frac{\theta}{2}} \left[ \int_{0}^{1} C_F(z)^{\frac{\theta-1}{\theta}} dz \right]^\frac{\theta}{2}
\]
Multiplying these two equations, we obtain the following.

\[ \lambda^{2(1-\theta)} = \left( \int_0^1 P_H(h)^{1-\theta} \, dh \right) \left( \int_0^1 P_F(f)^{1-\theta} \, df \right) \]

Therefore,

\[ P_T = \lambda = \left( \int_0^1 P_H(h)^{1-\theta} \, dh \right)^{\frac{1}{\eta(1-\theta)}} \left( \int_0^1 P_F(f)^{1-\theta} \, df \right)^{\frac{1}{\eta(1-\theta)}} \]

\[ = P_H^{1/2} P_F^{1/2} \]

The price of home consumption goods can be obtained from solving a similar minimum expenditure problem, which is,

\[ P = P_T^{\gamma} P_N^{1-\gamma} \]

Facing market prices as given, a representative home consumer solves the following problem for the home tradable goods consumption.

\[ \text{Max}_{C_H(h)} \left( \int_0^1 C_H(z)^{\frac{\theta-1}{\sigma}} \, dz \right)^{\frac{\sigma}{\theta-1}} \]

s.t.

\[ \int_0^1 C_H(z) P_H(z) \, dz = Z_H, \]

where \( Z_H \) is any fixed total nominal expenditure for home tradables. The first order condition with respect to \( C_H(h) \) is,

\[ \frac{\theta}{\theta - 1} \left( \int_0^1 C_H(z)^{\frac{\theta-1}{\sigma}} \, dz \right)^{\frac{1}{\sigma}} \frac{1}{\theta - 1} C_H(h)^{-\frac{1}{\theta}} = \lambda P_H(h) \]

Therefore,

\[ \lambda P_H(h) = \left( \int_0^1 C_H(z)^{\frac{\theta-1}{\sigma}} \, dz \right)^{\frac{1}{\sigma}} C_H(h)^{-\frac{1}{\theta}} \]

Combining this with another first order condition with respect to \( C_H(h') \),

\[ \frac{P_H(h)}{P_H(h')} = \left( \frac{C_H(h')}{C_H(h)} \right)^{\frac{1}{\sigma}} \Rightarrow \left( \frac{P_H(h)}{P_H(h')} \right)^{\theta - 1} = \left( \frac{C_H(h')}{C_H(h)} \right)^{\frac{\theta - 1}{\sigma}} \]
Rearranging this,

\[ C_H(h')^{\theta-1} P_H(h)^{1-\theta} = C_H(h)^{\theta-1} P_H(h')^{1-\theta} \]

Taking integration over \( h' \),

\[ P_H(h)^{1-\theta} \int_0^1 C_H(h')^{\theta-1} dh' = C_H(h)^{\theta-1} \int_0^1 P_H(h')^{1-\theta} dh' \]

\[ \Rightarrow P_H(h)^{-\theta} \left( \int_0^1 C_H(h')^{\theta-1} dh' \right)^{\frac{\theta}{\theta-1}} = C_H(h) \left( \int_0^1 P_H(h')^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} \]

\[ \Rightarrow P_H(h)^{-\theta} C_H = C_H(h) P_H^{-\theta} \]

Therefore,

\[ C_H(h) = \left( \frac{P_H(h)}{P_H} \right)^{-\theta} C_H \]

Likewise,

\[ C_N(h) = \left( \frac{P_N(h)}{P_N} \right)^{-\theta} C_N \]

and

\[ C_F(f) = \left( \frac{P_F(f)}{P_F} \right)^{-\theta} C_F \]

Note that both \( C_T \) and \( C \) are Armington forms so that

\[ C_H = \frac{1}{2} \frac{P_T}{P_H} C_T, \quad C_F = \frac{1}{2} \frac{P_T}{P_F} C_T \]

and

\[ C_T = \gamma \frac{P}{P_T} C, \quad C_N = (1 - \gamma) \frac{P}{P_N} C \]

Combining these equations, we get

\[ C_H(h) = \frac{\gamma}{2} \left( \frac{P_H(h)}{P_H} \right)^{-\theta} \left( \frac{P}{P_H} \right) C \]

\[ C_N(h) = (1 - \gamma) \left( \frac{P_N(h)}{P_N} \right)^{-\theta} \left( \frac{P}{P_N} \right) C \]
and

\[ C_F(f) = \frac{\gamma}{2} \left( \frac{P_F(f)}{P_F} \right)^{-\theta} \left( \frac{P}{P_F} \right) C \]

Similarly,

\[ C_H^*(h) = \frac{\gamma}{2} \left( \frac{P_H^*(h)}{P_H^*} \right)^{-\theta} \left( \frac{P^*}{P_H^*} \right) C^* \]

### C.2 Flexible Prices Equilibrium

Using the definitions of price indexes, the first equality of the optimal labor-consumption trade-off condition can be written as,

\[
C_t^0 = \frac{W_t}{\kappa P_t} = \frac{W_t}{\kappa P_H^t P_F^1 P_N^1} = \frac{W_t}{\kappa \left( \frac{\theta - 1}{\rho - 1} \right)^{\frac{\gamma}{2}} \left( \frac{\theta - 1}{\rho - 1} \frac{S_t}{W_t} \right)^{\frac{\gamma}{2}} \left( \frac{\theta - 1}{\rho - 1} \frac{W_t}{\gamma} \right)^{1-\gamma}}
\]

The last equality holds due to markup pricing rules. Rearranging it, we get,

\[
C_t^0 = \frac{\theta - 1}{\theta \kappa} \left( \frac{W_t}{S_t W_t^*} \right)^{\frac{\gamma}{2}} A_t^\gamma B_t^{1-\gamma} A_t^* A_t^\gamma
\]

It is straightforward to show \( \frac{W_t}{S_t W_t^*} = 1 \) by combining the labor-consumption condition with the risk-sharing condition. Therefore,

\[
C_t = \left[ \left( \frac{\theta - 1}{\theta \kappa} \right) A_t^\gamma B_t^{1-\gamma} A_t^* A_t^\gamma \right]^{\frac{1}{2}}
\]

Similarly,

\[
C_t^* = \left[ \left( \frac{\theta - 1}{\theta \kappa} \right) A_t^\gamma B_t^{1-\gamma} A_t^* A_t^\gamma \right]^{\frac{1}{2}}
\]

### C.3 Sticky-Price, Local-Currency Pricing Rule

\[
E_{t-1} \left\{ \frac{C_t(h)^{1-\rho}}{1-\rho} - \kappa L_t(h) \right\}
\]
subject to,

\[ P_tC_t(h) = P_{H,t}(h)Y_{H,t}(h) + S_tP_{H,t}^*(h)Y_{H,t}^*(h) + P_{N,t}(h)Y_{N,t}(h) \]

\[ - WL_{t+1}(h) + (1 + R_{t+1})WL_t(h) \]

\[ L_t(h) = \frac{Y_{H,t}(h) + Y_{H,t}^*(h)}{A_t} + \frac{Y_{N,t}(h)}{B_t}, \]

and the demand equations

\[ Y_{H,t}(h) = \gamma \frac{\left(P_{H,t}(h)\right)}{\left(P_{H,t}\right)} C_t \]

\[ Y_{H,t}^*(h) = \gamma \frac{\left(P_{H,t}^*(h)\right)}{\left(P_{H,t}^*\right)} C_t^* \]

\[ Y_{N,t}(h) = (1 - \gamma) \frac{\left(P_{N,t}(h)\right)}{\left(P_{N,t}\right)} C_t \]

From the flow budget constraint,

\[ C_t(h) = \frac{P_{H,t}(h)Y_{H,t}(h)}{P_t} + \frac{S_tP_{H,t}^*(h)Y_{H,t}^*(h)}{P_t} + \frac{P_{N,t}(h)Y_{N,t}(h)}{P_t} - \frac{WL_{t+1}(h)}{P_t} \]

\[ + \frac{(1 + R_{t+1})WL_t(h)}{P_t} \]

\[ = \gamma \frac{P_{H,t}(h)}{P_t} \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_t}{P_{H,t}}\right) C_t + \gamma \frac{S_tP_{H,t}(h)}{2} P_t \left(\frac{P_{H,t}^*(h)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_t^*}{P_{H,t}}\right) C_t^* \]

\[ + (1 - \gamma) \frac{P_{N,t}(h)}{P_t} \left(\frac{P_{N,t}(h)}{P_{N,t}}\right)^{-\theta} \left(\frac{P_t}{P_{N,t}}\right) C_t - \frac{WL_{t+1}(h)}{P_t} + \frac{(1 + R_{t+1})WL_t(h)}{P_t} \]

\[ = \gamma \frac{C_t}{2} \frac{P_{H,t}(h)}{P_t}^{1-\theta} + \gamma \frac{S_tP_{H,t}^*}{2} P_t \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{1-\theta} + (1 - \gamma) \frac{C_t}{P_{N,t}^{1-\theta}} P_{N,t}(h)^{1-\theta} \]

\[ - \frac{WL_{t+1}(h)}{P_t} + \frac{(1 + R_{t+1})WL_t(h)}{P_t} \]
From the labor demand function,

\[
L_t(h) = \frac{1}{2} A_t \left( \frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_t}{P_{H,t}} \right) C_t + \frac{1}{2} A_t \left( \frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_t^*}{P_{H,t}} \right) C_t^*
\]

\[
+ (1 - \gamma) \left( \frac{P_{N,t}(h)}{P_{N,t}} \right)^{-\theta} \left( \frac{P_t}{P_{N,t}} \right) C_t
\]

\[
= \frac{1}{2} \frac{P_t C_t}{A_t} P_{H,t}^1 \left( \gamma \frac{1}{A_t} \left( \frac{P_{N,t}(h)}{P_{N,t}} \right)^{-\theta} + \gamma \frac{1}{A_t} \frac{P_t^* C_t^*}{P_{H,t}} P_{H,t}^1 \right) + (1 - \gamma) \frac{1}{B_t} \frac{P_t C_t}{A_t} P_{N,t}^1 \left( \gamma \frac{1}{A_t} \left( \frac{P_{N,t}(h)}{P_{N,t}} \right)^{-\theta} \right)
\]

Plug these two into the objective function, and set derivatives to zeros.

\[
P_{H,t}(h) : \frac{\gamma(1 - \theta)}{2 P_{H,t}^{1-\theta}} E_{t-1} \left[ C_t(h)^{-\rho} C_t \right] P_{H,t}(h)^{-\theta}
\]

\[
= -\frac{\theta \gamma P_t}{2 P_{H,t}^{1-\theta}} E_{t-1} \left[ C_t/A_t \right] P_{H,t}(h)^{-\theta-1}
\]

\[
P_{H,t}^*(h) : \frac{\gamma(1 - \theta) P_t^*}{2 P_{H,t}^{1-\theta} P_{H,t}^*} E_{t-1} \left[ S_t C_t(h)^{-\rho} C_t^* \right] P_{H,t}^*(h)^{-\theta}
\]

\[
= -\frac{\theta \gamma P_t^*}{2 P_{H,t}^{1-\theta}} E_{t-1} \left[ C_t^*/A_t \right] P_{H,t}^*(h)^{-\theta-1}
\]

\[
P_{N,t}(h) : \frac{(1 - \gamma)(1 - \theta)}{P_{N,t}^{1-\theta}} E_{t-1} \left[ C_t(h)^{-\rho} C_t \right] P_{N,t}(h)^{-\theta}
\]

\[
= -\frac{(1 - \gamma) \theta \gamma P_t}{P_{N,t}^{1-\theta}} E_{t-1} \left[ C_t/B_t \right] P_{N,t}(h)^{-\theta-1}
\]

Finally,

\[
P_{H,t}(h) = \frac{\theta \gamma}{\theta - 1} \frac{P_t E_{t-1} \left[ C_t/A_t \right]}{E_{t-1} \left[ C_t(h)^{-\rho} C_t \right]}
\]

\[
P_{H,t}^*(h) = \frac{\theta \gamma}{\theta - 1} \frac{P_t E_{t-1} \left[ C_t^*/A_t \right]}{E_{t-1} \left[ S_t C_t(h)^{-\rho} C_t^* \right]}
\]

\[
P_{N,t}(h) = \frac{\theta \gamma}{\theta - 1} \frac{P_t E_{t-1} \left[ C_t/B_t \right]}{E_{t-1} \left[ C_t(h)^{-\rho} C_t \right]}
\]

Assuming a symmetric equilibrium,

\[
P_{H,t} = \frac{\theta \gamma}{\theta - 1} \frac{P_t E_{t-1} \left[ C_t/A_t \right]}{E_{t-1} \left[ C_t^{1-\rho} \right]}
\]

\[
P_{H,t}^* = \frac{\theta \gamma}{\theta - 1} \frac{P_t E_{t-1} \left[ C_t^*/A_t \right]}{E_{t-1} \left[ S_t C_t^{1-\rho} C_t^* \right]}
\]

\[
P_{N,t} = \frac{\theta \gamma}{\theta - 1} \frac{P_t E_{t-1} \left[ C_t/B_t \right]}{E_{t-1} \left[ C_t^{1-\rho} \right]}
\]
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