THE EFFECT OF MANUFACTURING ERRORS ON PREDICTED DYNAMIC FACTORS OF SPUR GEAR

A Thesis

Presented in Partial Fulfillment of the Requirement for 
the Degree Master of Science in the 
Graduate School of The Ohio State University

by

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****

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THESIS ABSTRACT

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TITLE OF THESIS: THE EFFECT OF MANUFACTURING ERRORS ON PREDICTED DYNAMIC FACTORS OF SPUR GEARS

This thesis studies the effect of manufacturing errors on predicted dynamic factors of spur gears. Three types of dynamic factors are defined and studied: dynamic load factors, dynamic tooth force factors, and dynamic bending moment factors. Three different computer programs for predicting dynamic factors are introduced. These programs are a MATLAB forced vibration analysis using a six degree of freedom model, a multi-degree of freedom Dynamic Transmission Error Program (DYTEM) that uses a six degree of freedom model, and the Geared Rotor Dynamics Program (GRD) that uses a finite element method. After comparing the three program’s results with experimental data provided by NASA (National Aeronautics and Space Administration, the DYTEM program is used for dynamic factors prediction. The effects of different profile tolerances for AGMA quality 10, 12, and 14 gears are presented.

Advisor’s Signature
To My Dearest Parents,

Brother, and Sisters.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td></td>
<td>DEDICATION</td>
<td>iii</td>
</tr>
<tr>
<td></td>
<td>ACKNOWLEDGMENTS</td>
<td>iv</td>
</tr>
<tr>
<td></td>
<td>VITA</td>
<td>v</td>
</tr>
<tr>
<td></td>
<td>TABLE OF CONTENTS</td>
<td>vi</td>
</tr>
<tr>
<td></td>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td></td>
<td>LIST OF TABLES</td>
<td>xiv</td>
</tr>
<tr>
<td></td>
<td>NOMENCLATURE</td>
<td>xv</td>
</tr>
<tr>
<td></td>
<td>CHAPTER I INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>Overview</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Research Objectives</td>
<td>1</td>
</tr>
<tr>
<td>1.3</td>
<td>Literature Review</td>
<td>2</td>
</tr>
<tr>
<td>1.4</td>
<td>Outline of Thesis</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>CHAPTER II PROBLEM FORMULATION</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>Dynamic Factors</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Types of Dynamic Factor</td>
<td>6</td>
</tr>
<tr>
<td>2.2.2</td>
<td>AGMA Dynamic Factor</td>
<td>11</td>
</tr>
<tr>
<td>2.3</td>
<td>Model Discussion</td>
<td>15</td>
</tr>
<tr>
<td>2.3.1</td>
<td>MATLAB Program</td>
<td>20</td>
</tr>
<tr>
<td>2.3.2</td>
<td>DYTEM Program</td>
<td>24</td>
</tr>
<tr>
<td>2.3.3</td>
<td>HGRD/GRD Program</td>
<td>29</td>
</tr>
<tr>
<td>2.4</td>
<td>Assumptions</td>
<td>33</td>
</tr>
</tbody>
</table>
CHAPTER III  EXPERIMENTAL AND COMPUTER PROGRAMS COMPARISONS
3.1 Introduction ................................................................................. 35
3.2 Comparison to NASA Experimental and Theoretical Data ............ 36
3.2.1 NASA Gear Specification ....................................................... 38
3.2.2 Natural Frequencies Comparison ........................................... 41
3.2.3 Dynamic Tooth Load Comparison ........................................... 45
3.3 Dynamic Analysis at Different Input Torque Level ....................... 49
3.4 Computer (DYTEM, GRD, and MATLAB Programs)
Comparedisons .............................................................................. 53
3.4.1 Spur Gear Specification ........................................................ 54
3.4.2 Comparison of Dynamic Load Factors using the Three
Computer Programs ....................................................................... 57
3.4.3 Perfect and Optimized Involute Spur Gears Comparison ........... 65
3.5 Summary ..................................................................................... 65

CHAPTER IV  THE EFFECTS OF PROFILE MANUFACTURING TOLERANCES ON DYNAMIC FACTORS
4.1 Introduction ................................................................................. 66
4.2 Selection of Tooth Profiles as a Reference in Comparison .............. 67
4.3 Quality 14 Gears ....................................................................... 70
4.3.1 K-Chart of Quality 14 Gears .................................................. 70
4.3.2 Dynamic Factors Comparisons of Quality 14 Gears ................. 72
4.4 Quality 12 Gears ....................................................................... 76
4.4.1 K-Chart of Quality 12 Gears .................................................. 76
4.4.2 Dynamic Factors Comparisons of Quality 12 Gears ................. 77
4.5 Quality 10 Gears ....................................................................... 82
4.5.1 K-Chart of Quality 10 Gears .................................................. 82
4.5.2 Dynamic Factors Comparisons of Quality 10 Gears ................. 84
4.6 Dynamic Analysis Consideration ................................................ 88
4.7 Summary ..................................................................................... 93

CHAPTER V  CONCLUSIONS AND RECOMMENDATIONS
5.1 Conclusions .............................................................................. 95
5.2 Recommendations for Future Research ...................................... 96

REFERENCES .................................................................................. 98
| APPENDIX A | SAMPLE RUN OF MATLAB PROGRAM | 101 |
| A.1 | Sample Input Data | 102 |
| A.2 | Sample Output Data | 103 |
| A.3 | Graphical Results | 104 |
| A.4 | MATLAB Program | 107 |

| APPENDIX B | SAMPLE RUN OF DYTEM PROGRAM | 111 |
| B.1 | Sample Input Data from LDP | 112 |
| B.2 | Sample Input Data for DYTEM | 114 |
| B.3 | Graphical Results | 116 |
| B.4 | Runge-Kutta Fifth-Fourth Order | 118 |

| APPENDIX C | SAMPLE RUN OF GRD PROGRAM | 122 |
| C.1 | Sample Input Data for GRD | 123 |
| C.2 | Output File from GRD | 124 |
| C.3 | Graphical Results | 130 |

| APPENDIX D | Various Study of Dynamic Analysis | 131 |
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Number</th>
<th>Figure Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Schematic of Time Varying Torque</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>Schematic of Time Varying Force</td>
<td>8</td>
</tr>
<tr>
<td>2.3</td>
<td>Schematic of Time Varying Moment</td>
<td>9</td>
</tr>
<tr>
<td>2.4</td>
<td>Dynamic Factors, Cv and Kv used by American Gear Manufacturers Association (AGMA) [3]</td>
<td>12</td>
</tr>
<tr>
<td>2.5</td>
<td>The Inverted Dynamic Factors used by AGMA</td>
<td>14</td>
</tr>
<tr>
<td>2.6</td>
<td>Schematic of the Simplest Gear Dynamic System (Model A)</td>
<td>16</td>
</tr>
<tr>
<td>2.7</td>
<td>Schematic Gear Pair for Model B</td>
<td>17</td>
</tr>
<tr>
<td>2.8</td>
<td>Schematic Gear Pair for Model C</td>
<td>18</td>
</tr>
<tr>
<td>2.9</td>
<td>Schematic Gear Pair for Model D</td>
<td>19</td>
</tr>
<tr>
<td>2.10</td>
<td>Six Degree of Freedom Model used in MATLAB and DYTEM Program</td>
<td>22</td>
</tr>
<tr>
<td>2.11</td>
<td>Schematic of Geared Rotor System</td>
<td>30</td>
</tr>
<tr>
<td>2.12</td>
<td>Schematic of Discretized Geared Rotor System</td>
<td>30</td>
</tr>
<tr>
<td>3.1</td>
<td>Layout of NASA Gear Noise Rig</td>
<td>36</td>
</tr>
<tr>
<td>3.2</td>
<td>Detail of Gearbox of NASA Gear Noise Rig</td>
<td>37</td>
</tr>
<tr>
<td>3.3</td>
<td>Schematic of NASA Test Spur Gears</td>
<td>38</td>
</tr>
<tr>
<td>3.4</td>
<td>Tooth Strain Gage Data at Several Different Operating Speeds using 196.93 lb-in Torque (From Rebbechi, et. al. [7])</td>
<td>41</td>
</tr>
<tr>
<td>3.5</td>
<td>Dynamic Bending Moment at Different Operating Speed using 196.93 lb-in Torque</td>
<td>45</td>
</tr>
<tr>
<td>3.6</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 2000 rpm and at 47 % Full Torque (298.57 lb-in), ζ=0.2</td>
<td>46</td>
</tr>
</tbody>
</table>
3.7 Dynamic Tooth Load Comparisons at Operating Speed of 4000 rpm and at 31% Full Torque (196.93 lb-in), \( \zeta = 0.2 \) ........................................ 46

3.8 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm and at 31% Full Torque (196.93 lb-in), \( \zeta = 0.6 \) ........................................ 47

3.9 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm and at 47% Full Torque (298.57 lb-in), \( \zeta = 0.3 \) ........................................ 47

3.10 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm and at 94% Full Torque (597.14 lb-in), \( \zeta = 0.5 \) ........................................ 48

3.11 Dynamic Mesh Force at Different Input Torque Level, \( \zeta = 0.2 \), Speed = 4000 rpm ................................................................. 50

3.12 Dynamic Tooth Force at Different Input Torque Level, \( \zeta = 0.2 \), Speed = 4000 rpm ................................................................. 50

3.13 Dynamic Bending Moment (Pinion) at Different Input Torque Level, \( \zeta = 0.2 \), Speed = 4000 rpm ................................................................. 51

3.14 Dynamic Bending Moment (Gear) at Different Input Torque Level, \( \zeta = 0.2 \), Speed = 4000 rpm ................................................................. 51

3.15 Dynamic Factor (Pinion) at Different Input Torque Level, \( \zeta = 0.2 \), Speed = 4000 rpm ................................................................. 52

3.16 Dynamic Factor (Gear) at Different Input Torque Level, \( \zeta = 0.2 \), Speed = 4000 rpm ................................................................. 52

3.17 The Peak to Peak Static Transmission Error (PPTE) of Spur Gear at several Operating Torques using MULTILD Program ........................................ 55

3.18 Static Transmission Error of the Spur Gear at Different Operating Torques using MULTILD Programs ........................................ 56

3.19 Comparisons of the Dynamic Loads Predicted by the Three Different Dynamic Models ................................................................. 57

3.20 K-Chart of Spur Gear for Optimum Profile Modification .......... 60

3.21 Comparison of Dynamic Load Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear ........................................ 62

3.22 Comparison of Dynamic Tooth Force Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear ........................................ 63

3.23 Comparison of Dynamic Bending Moment Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear (Pinion Tooth) ................................................................. 64

3.24 Comparison of Dynamic Bending Moment Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear (Gear Tooth) ................................................................. 64

x
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Dynamic Load Factor of the Perfect Involute Spur Gear</td>
<td>67</td>
</tr>
<tr>
<td>4.2</td>
<td>K-Chart of Spur Gear used as the Reference Tooth Profile</td>
<td>68</td>
</tr>
<tr>
<td>4.3</td>
<td>K-Chart of Spur Gear for AGMA Class 14</td>
<td>71</td>
</tr>
<tr>
<td>4.4</td>
<td>Comparison of Dynamic Load Factors for the Class 14 Profiles</td>
<td>72</td>
</tr>
<tr>
<td>4.5</td>
<td>Comparison of Dynamic Tooth Force Factors for the Class 14 Profiles</td>
<td>73</td>
</tr>
<tr>
<td>4.6</td>
<td>Comparison of Dynamic Bending Moment Factors for the Class 14 Profiles (Pinion)</td>
<td>74</td>
</tr>
<tr>
<td>4.7</td>
<td>Comparison of Dynamic Bending Moment Factors for the Class 14 Profiles (Gear)</td>
<td>75</td>
</tr>
<tr>
<td>4.8</td>
<td>K-Chart of Spur Gear for AGMA Class 12</td>
<td>76</td>
</tr>
<tr>
<td>4.9</td>
<td>Comparison of Dynamic Load Factors for the Class 12 Profiles</td>
<td>78</td>
</tr>
<tr>
<td>4.10</td>
<td>Comparison of Dynamic Tooth Force Factors for the Class 12 Profiles</td>
<td>80</td>
</tr>
<tr>
<td>4.11</td>
<td>Comparison of Dynamic Bending Moment Factors for the Class 12 Profiles (Pinion)</td>
<td>81</td>
</tr>
<tr>
<td>4.12</td>
<td>Comparison of Dynamic Bending Moment Factors for the Class 12 Profiles (Gear)</td>
<td>81</td>
</tr>
<tr>
<td>4.13</td>
<td>K-Chart of Spur Gear for AGMA Class 10</td>
<td>83</td>
</tr>
<tr>
<td>4.14</td>
<td>Comparison of Dynamic Load Factors for the Class 10 Profiles</td>
<td>84</td>
</tr>
<tr>
<td>4.15</td>
<td>Comparison of Dynamic Tooth Force Factors for the Class 10 Profiles</td>
<td>85</td>
</tr>
<tr>
<td>4.16</td>
<td>Comparison of Dynamic Bending Moment Factors for the Class 10 Profiles (Pinion)</td>
<td>86</td>
</tr>
<tr>
<td>4.17</td>
<td>Comparison of Dynamic Bending Moment Factors for the Class 10 Profiles (Gear)</td>
<td>87</td>
</tr>
<tr>
<td>4.18</td>
<td>Dynamic Mesh and Tooth Forces Trace of the Perfect Involute Spur Gear at Pinion Speed of 3214 RPM</td>
<td>88</td>
</tr>
<tr>
<td>4.19</td>
<td>Static and Dynamic Moment Trace of the Perfect Involute Spur Gear at Pinion Speed of 3214 RPM</td>
<td>89</td>
</tr>
<tr>
<td>4.20</td>
<td>Dynamic Mesh and Tooth Forces Trace of the Case 4 (T1004) of AGMA Class 10 Spur Gear at Pinion Speed of 3214 RPM</td>
<td>89</td>
</tr>
<tr>
<td>4.21</td>
<td>Static and Dynamic Moment Trace of the Case 4 (T1004) of AGMA Class 10 Spur Gear at Pinion Speed of 3214 RPM</td>
<td>90</td>
</tr>
<tr>
<td>4.22</td>
<td>Dynamic Mesh and Tooth Forces Trace of the Optimum Modified Spur Gear at Pinion Speed of 3214 RPM</td>
<td>90</td>
</tr>
<tr>
<td>4.23</td>
<td>Static and Dynamic Moment Trace of the Optimum Modified Spur Gear at Pinion Speed of 3214 RPM</td>
<td>91</td>
</tr>
<tr>
<td>D.9</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 3</td>
<td>140</td>
</tr>
<tr>
<td>D.10</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 4</td>
<td>141</td>
</tr>
<tr>
<td>D.11</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 5</td>
<td>142</td>
</tr>
<tr>
<td>D.12</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 6</td>
<td>143</td>
</tr>
<tr>
<td>D.13</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 7</td>
<td>144</td>
</tr>
<tr>
<td>D.14</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 47% Full Torque (298.57 lb-in), Case 1</td>
<td>145</td>
</tr>
<tr>
<td>D.15</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 47% Full Torque (298.57 lb-in), Case 2</td>
<td>146</td>
</tr>
<tr>
<td>D.16</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 47% Full Torque (298.57 lb-in), Case 3</td>
<td>147</td>
</tr>
<tr>
<td>D.17</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 47% Full Torque (298.57 lb-in), Case 4</td>
<td>148</td>
</tr>
<tr>
<td>D.18</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94% Full Torque (597.14 lb-in), Case 1</td>
<td>149</td>
</tr>
<tr>
<td>D.19</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94% Full Torque (597.14 lb-in), Case 2</td>
<td>150</td>
</tr>
<tr>
<td>D.20</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94% Full Torque (597.14 lb-in), Case 3</td>
<td>151</td>
</tr>
<tr>
<td>D.21</td>
<td>Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94% Full Torque (597.14 lb-in), Case 4</td>
<td>152</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Number</th>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>NASA Spur Gear Parameters</td>
<td>39</td>
</tr>
<tr>
<td>3.2</td>
<td>Natural Frequencies and Dynamic Load Factors Predicted by the Three Different Modeling Methods with NASA Gear Specifications at 196.93 lb-in Torque</td>
<td>43</td>
</tr>
<tr>
<td>3.3</td>
<td>Mesh Damping Ratio used in DYTEM Program for Calculating the Dynamic Tooth Loads</td>
<td>49</td>
</tr>
<tr>
<td>3.4</td>
<td>Spur Gears Parameters</td>
<td>54</td>
</tr>
<tr>
<td>3.5</td>
<td>The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Optimum Modified Spur Gears</td>
<td>58</td>
</tr>
<tr>
<td>3.6</td>
<td>The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Optimum Modified Spur Gears</td>
<td>61</td>
</tr>
<tr>
<td>4.1</td>
<td>The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Optimum Modified Spur Gears and the Case 4 (T1204) Quality 12 Gear</td>
<td>79</td>
</tr>
<tr>
<td>4.2</td>
<td>The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Case 4 (T1004) Quality 10 Gear</td>
<td>92</td>
</tr>
<tr>
<td>A.1</td>
<td>Input Data for MATLAB Program</td>
<td>102</td>
</tr>
</tbody>
</table>
NOMENCLATURE

ψ  Helix angle
Θ₁  Angular displacement of pinion
Θ₂  Angular displacement of gear
Θ₀  Angular displacement of driver
Θₜ  Angular displacement of load
Cᵢ  Pinion bearing viscous damping value
C₂  Gear bearing viscous damping value
Cₘ  Mesh damping ratio
Cᵣ  Torsional pinion shaft damping
C₁  Torsional gear shaft damping
Cᵥ  Dynamic factor defined by AGMA
Dₚ  Dynamic Factor
D₀  Dynamic Increment
e(t)  Transmission error
F  Force
I₁  Pinion inertia
I₂  Gear inertia
k₁  Pinion bearing stiffness
k₂  Gear bearing stiffness
kₘ  Mesh stiffness
kᵣ  Torsional pinion shaft stiffness
\( k_{12} \)  Torsional gear shaft stiffness
\( K_v \)  Dynamic factor defined by AGMA
\( M \)  Moment
\( M_D \)  Dynamic Moment
\( M_S \)  Static Moment
\( N_P \)  Number of teeth
\( P \)  Normal Diametral pitch
\( Q_v \)  Gear quality number
\( Q_V \)  Gear quality number
\( S_{CD} \)  Dynamic Contact Stress
\( S_{CS} \)  Static Contact Stress
\( S_F \)  Static Tooth Force
\( S_L \)  Static Load
\( S_{TD} \)  Dynamic Root Stress
\( S_{TS} \)  Static Root Stress
\( V_{phit} \)  Profile Tolerance
\( v_{max} \)  Pitch line velocity maximum at the operating pitch diameter
\( W_d \)  Increment dynamic tooth load
\( W_i \)  Static tooth load
\( x \)  Displacement
\( Y_1 \)  Translation of pinion
\( Y_2 \)  Translation of gear
CHAPTER I

INTRODUCTION

1.1 Overview

Dynamic analysis of gears has become an important area of study in gear manufacturing. One of the areas of interest is the prediction of dynamic factors. Being able to determine a gear pair's dynamic factor, one can better predict the gear pair's life and reliability.

1.2 Research Objectives

The dynamic factor used in gear design has been the subject of many studies, and has resulted in many formulations. The existing factors used by AGMA (American Gear Manufacturers Association) and ISO (International Standard Organization) are empirical in nature. Although accuracy is a consideration, it has not been studied to any great
extent. The advent of many relatively simple models and available analysis procedures allows for dynamic factors to be predicted by a variety of methods. This thesis uses three different modeling procedures that predict dynamic factors and then uses one of the procedures to compare the effects of profile accuracy with the values used in present AGMA standards.

1.3 Literature Review

The development of the dynamic factor, using either experimental and theoretical methods, can be traced back more than one century. Walker introduced a factor which was reformulated by Barth as the ratio of static load to dynamic load [1]. Buckingham has performed systematic studies in the prediction of dynamic tooth loads for designing gears [2]. The formulation of dynamic factor has changed as more factors, such as operational speeds, tooth precision, and gear geometry, are considered. The AGMA 2001 standard provides some standard equations for the computation of the dynamic factor based on the gear quality numbers of the gears [3]. The AGMA Standard recognizes that it does not consider system resonances and is empirical in nature.

Thus, more intensive investigations in the analytical prediction of dynamic factors have been performed by many investigators. Kubo [4] presented a method whose results compared favorably with his experimental data. Kubo's measured results are presented by
Dudley [5] in his fairly detailed discussion of dynamic factors. Lin, et al. [6] have conducted an investigation in predicting the effects of both linear and parabolic tooth profile modifications on the dynamic response using a four degree of freedom torsional model. Rebbechi, et. al. [7] have performed some experimental studies to predict the gear dynamic behavior. These experimental results were compared with a computer program developed at NASA (National Aeronautics and Space Administration) for a four degree of freedom model [8]. Wang [9] has also applied time domain torsional dynamic modeling for the prediction of dynamic factors. Although not applied directly to dynamic factor modeling, Kahraman and Singh [10] and Padmanabhan and Singh [11] have studied dynamic modeling procedures for gears that have non-linear effects near system resonances.

Ozguven and Houser [12] have developed a computer program, DYTE (Dynamic Transmission Error Program) that uses either measured or computed static transmission errors to excite a single degree of freedom non-linear time domain model. This program determines the dynamic factors, dynamic tooth forces, and dynamic transmission errors. This computer program has been extended to a six degree of freedom non-linear model called DYTEM (Dynamic Transmission Error Program for Multi-degree of freedom model that provides more complete results [13]. In addition to the dynamic factors, dynamic tooth forces, and dynamic transmission errors predicted by DYTE, dynamic bearing forces, bearing displacement, and torsional shaft motions are also predicted with this time domain program. A computer program called GRD (Gear Rotor Dynamics
Program) has been developed by Kahraman, et. al. [14]. This procedure uses finite elements to analyze the dynamics of a system consisting of two shafts supported on bearings and coupled by a spur or helical gear mesh. This program uses a frequency domain approach with harmonic components of transmission error as the excitation to predict dynamic tooth loads.

1.4 Outline of Thesis

In this thesis, Chapter 2 presents the problem formulation of the dynamic factors. Three types of computer programs that predict the dynamic factors are discussed. Chapter 3 deals with the comparison between the experimental results and the computer program predictions. Chapter 4 presents the effect of profile manufacturing tolerances on dynamic factors. Several profile modifications of spur gears based on AGMA gear qualities are discussed in terms of dynamic factors. Chapter 5 provides several conclusions and recommendations for future research.
CHAPTER II

PROBLEM FORMULATION

2.1 Introduction

This chapter presents the problem formulation of dynamic factors. First of all, several types of dynamic factors are briefly explained. Dynamic Load Factor, which has been used by American Gear Manufactures Association (AGMA) is being formulated based on its empirical data. Next, four physical models are discussed in brief, which are the first step in selecting the appropriate model to determine the dynamic factors. Then, several computer programs that generate the dynamic factors of gears in mesh for specified parameters are introduced. Finally, this chapter ends with the assumptions, that are taken into account throughout the computer simulation, which are used in the experimental comparison and calculating the dynamic factors in this thesis.
2.2 Dynamic Factor

There are five different definitions that can be derived from the terminology of dynamic factors that result in the different calculation procedures. These are the Dynamic Load Factor, Dynamic Tooth Force Factor, Dynamic Bending Moment Factor, Dynamic Contact Stress Factor, and Dynamic Root Stress Factor.

2.2.1. Types of Dynamic Factor

In this section, all dynamic factors will be considered as derating factors. This means that they are typically numbers less than one that are used as multiplies in the gear rating. However, in later sections, these dynamic factors will be inverted so that as stress multipliers. This description is much easier to interpret, but contrary to the scheme used in AGMA dynamic factor definitions.

The Dynamic Load Factor is the ratio of the static load carried by a gear pair to the peak dynamic load, as shown in Equation 2.1. Figure 2.1 shows the schematic of time varying torque. This dynamic load usually does not consider load sharing between teeth so it is possible to have the highest dynamic load where two pairs of teeth are in contact, hence providing a conservative value. This is the factor used by AGMA in its standards, which will be explained more detailed in section 2.3.
\[ D_F = \frac{S_l}{S_L + D_I} \]  
(Eq. 2.1)

where,

- \( D_F \) = Dynamic Factor
- \( S_L \) = Static Load
- \( D_I \) = Dynamic Increment

![Torque Diagram](image)

**Figure 2.1** Schematic of Time Varying Torque.

The *Dynamic Tooth Force Factor* method considers each tooth pair as a separate entity and then computes the highest ratio of static force carried by one tooth relative to the highest dynamic force carried by the same tooth. This factor is close to being correct for surface durability since it is indicative of Hertzian contact stress amplitude. However, since it is not location dependent, it may or may not be indicative of the peak dynamic contact stress on a gear tooth. Equation 2.2 shows the governing equation of the dynamic tooth force factor. The schematic of the time varying force is illustrated in Figure 2.2.
\[ D_F = \frac{S_F}{S_F + D_I} \]  
\[ \text{(Eq. 2.2)} \]

where,
- \( D_F \) = Dynamic Factor
- \( S_F \) = Static Tooth Force
- \( D_I \) = Dynamic Increment
- \( F \) = Force

**Figure 2.2** Schematic of Time Varying Force.

The *Dynamic Bending Moment Factor* method takes the dynamic tooth force used in the previous dynamic factor and multiplies it by the moment arm to the location of the critical root stress as obtained in the geometry factor calculation. This factor does not directly indicate root stress, but allows for the computation of a root stress multiplier that is essentially the same as computing the ratio of static root stress to dynamic root stress. Equation 2.3 shows the governing equation of the dynamic bending moment factor. The schematic of the time varying moment is illustrated in Figure 2.3.
\[ D_F = \frac{M_S}{M_D} \]  
(Eq. 2.3)

where,

- \( D_F \) = Dynamic Factor
- \( M_S \) = Static Moment
- \( M_D \) = Dynamic Moment
- \( F \) = Tooth Force
- \( x \) = Displacement
- \( M \) = Moment

\[ M = F \cdot x \]

**Figure 2.3**  Schematic of Time Varying Moment.

The *Dynamic Contact Stress Factor* is the most sensible dynamic factor for surface durability. It is computed by taking the ratio of the highest static contact stress on a gear tooth to the highest dynamic contact stress. In most instances this factor will differ only slightly from dynamic tooth force factor, but where the peak dynamic loads occur at a different location than the peak static loads, there will be differences between the two methods. Equation 2.4 shows the governing equation of the dynamic contact stress factor.
\[ D_F = \frac{S_{CS}}{S_{CD}} \]  
(Eq. 2.4)

where,
- \( D_F \) = Dynamic Factor
- \( S_{CS} \) = Static Contact Stress
- \( S_{CD} \) = Dynamic Contact Stress

The Dynamic Root Stress Factor method calculates dynamic factor based on the root stresses of each element of a gear pair. Since the peak stress location at the root changes slightly with load location, this factor computes more precise results than the dynamic bending moment factor, but is more difficult to analytically implement. Equation 2.5 shows the governing equation of the dynamic contact stress factor.

\[ D_F = \frac{S_{TS}}{S_{TD}} \]  
(Eq. 2.5)

where,
- \( D_F \) = Dynamic Factor
- \( S_{TS} \) = Static Root Stress
- \( S_{TD} \) = Dynamic Root Stress

Three of the five dynamic factors explained above will be used in this research. They are the Dynamic Load Factor, the Dynamic Tooth Force Factor, and the Dynamic Bending Moment Factor. Recommendations will be made on methods for computing those three dynamic factors.
2.2.2. **AGMA Dynamic Factor**

AGMA has published one type of dynamic factor, referred to here as *Dynamic Load Factor* in order to distinguish it from the other previously defined dynamic factors. In general, AGMA expresses this dynamic factor as $C_v$ or $K_v$. The Dynamic Load Factor usually relates the total tooth load including internal dynamic effects to the transmitted tangential tooth load [3], as shown below:

$$C_v = K_v = \frac{W_t}{W_d + W_t} \quad (\text{Eq. 2.6})$$

where

- $C_v$ or $K_v$ = Dynamic Factor.
- $W_d$ = Increment dynamic tooth load.
- $W_t$ = Static tooth load.

AGMA extends the above dynamic factor definition to several approximate dynamic factor equations based on empirical data and the gear quality numbers ($Q_v$). Figure 2.4 shows the AGMA's Dynamic Factor formulation at several gear quality numbers at certain range of pitch line velocity. AGMA standard shows that the smaller the dynamic factor value, $C_v$ and $K_v$, the higher the stresses on the component, as stated above in Equation 2.6.

Quality 12 and above ($Q_v \geq 12$ limits) gears are considered as very accurate gearing, and have $C_v$ and $K_v$ values between 0.90 and 0.98, as shown in shaded area in Figure 2.4.
Gears with Quality between 6 and 11 (6 \leq Q_v \leq 11) have dynamic factor formulation as shown below. The plot of each dynamic factor for each quality number is provided in Figure 2.4.

\[ C_v = K_v = \left( \frac{A}{A + \sqrt{V_i}} \right)^{\varepsilon} \]  
\hspace{1cm} (Eq. 2.7)

where

\[ A = 50 + 56 (1.0 - B) \]  
\hspace{1cm} (Eq. 2.8)

\[ B = \frac{(12 - Q_v)^{0.667}}{4} \]  
\hspace{1cm} (Eq. 2.9)
\[ v_{\text{t}_{\text{max}}} = \left[A + (Q_v - 3)\right]^2 \]  \hspace{1cm} \text{(Eq. 2.10)}

\begin{align*}
v_{\text{t}} & \quad \text{Pitch line velocity} \\
Q_v & \quad \text{Gear quality number} \\
v_{\text{t}_{\text{max}}} & \quad \text{pitch line velocity maximum at operating pitch diameter}
\end{align*}

AGMA states that the dynamic factor curves for Gear Quality between 6 and 11 could be extrapolated beyond the end point shown on Figure 2.4, but each task has to be based on experience and careful consideration of factors influencing the dynamic load. Equation 2.10 is then used as a guideline for computer calculation to determine where the maximum pitch line velocity for each gear quality ends.

Finally, the dynamic factor curve for gears with Quality of 5, \( Q_v = 5 \), can be seen also in Figure 2.4. The dynamic factor approximation this quality of 5 (\( Q_v < 6 \)) has equation as shown below:

\[ C_v = K_v = \frac{50}{50 + \sqrt{v_t}} \]  \hspace{1cm} \text{(Eq. 2.11)}

where

\[ v_t \text{ does not exceed 2,500 ft/min} \]

In this thesis, the dynamic factor will always be considered as a dynamic stress multiplier in the fashion of Kubo [4] and later presented in Dudley [5]. The expression used can always be converted to an AGMA type factor by inverting the number that is computed.
This approach implies that higher dynamic factors indicate higher stresses on a component. An inverted AGMA dynamic factors plot shown in Figure 2.5. The shaded area is the region of dynamic factors for very accurate gearing \((Q_v \geq 12)\). The value of the dynamic factor gets larger as the pitch line velocity increases or as the gear quality decreases from 11 to 5.

\[ \text{Figure 2.5 } \text{The Inverted Dynamic Factors used by AGMA.} \]
2.3 Model Discussion

There are many simulation procedures for computing the various dynamic factors on a gear pair. First, one must select an appropriate physical model and then use one of several equation solving techniques to evaluate the physical model.

Many of the different physical models that have been used for gear dynamics modeling have been discussed by Ozguven, et. al. [12 - 14]. Later papers that extend this literature review include works by Blankenship and Singh [15], and by Vinayak and Singh [16]. Some of the physical models that have been used are presented below.

A) The simplest gear dynamics model considers the gear pair as two rotary inertias coupled by a spring and a damper, as shown in Figure 2.6. $I_1$ and $I_2$ are the pinion and gear inertias, respectively. The spring and damper are usually considered as the mesh stiffness ($k_m$) and the mesh damping ratio ($c_m$) of the gear pair, respectively. The mesh stiffness is time varying, but can be averaged to give a linear time invariant model. The mesh damping ratio is an unknown value since it is an extremely difficult tasks to calculate an accurate value of damping ratio for any system. The rotation on the pinion side will generate the angular displacement of the pinion, $\Theta_1$, and the rotation on the gear side will generate the angular displacement of the gear, $\Theta_2$. It is possible to predict the following quantities using such a model: (i) dynamic transmission errors, (ii) the effects of others dynamic on that gear pairs, and (iii) a single torsional natural
frequency. Furthermore, depending upon the implementation, this model may or may not include the nonlinear effects due to loss of contact.

![Diagram of Simplest Gear Dynamic System](image)

**Figure 2.6** Schematic of the Simplest Gear Dynamic System (Model A).

B) The next simplest model that is often used includes the translational motions of the gears by adding the stiffness of the supporting shafts, bearings, and housing. This model is shown in Figure 2.7. The configuration of the pinion and gear can be seen as explained in part A. The pinion and gear bearing stiffnesses are noted as $k_1$ and $k_2$, respectively. Meanwhile, $c_1$ and $c_2$ are bearing damping values on pinion and gear sides, respectively. These stiffness and damping values take into account not only the bearings, but also the effects of shafts and housing. Hence, it may be necessary to
greatly simplify the actual system in order to came up with realistic values for the stiffnesses, k's, and dampings, c's. In its simplest form, this model has three natural frequencies, each of which has various degrees of torsional and lateral coupling. Gregory, et. al. [17] was the first to identify the necessity to use this model as the absolute minimum for practical gears. Since all gears tend to have some degree of lateral/torsional coupling and except for very symmetrical cases, each of the vibration modes is excited to some extent by the dynamic excitations (transmission errors) of the mesh. The use of model "A" has been successful in modeling some test stands because efforts were made to decouple the lateral motions by using inordinately large gear wheel inertias or by having extremely compliant shafts and bearings.

![Diagram](image)

Figure 2.7  Schematic Gear Pair for Model B.
C) By adding additional shafts, driver, and load inertias to model above, one can have additional natural frequencies. One model can be seen in Figure 2.8. $I_D$ and $I_L$ are the driver and load inertias, respectively. The pinion and gear may have a configuration, as shown in Figure 2.4. For this model C, one may compute six natural frequencies. However, these added masses seldom affect the mesh forces in the frequency range studied in this research.

![Figure 2.8 Schematic Gear Pair for Model C.](image)

D) A more complex model considers the shafts and bearings to be continuous and includes shaft bending separate from bearing stiffness. This approach is the most complex of the models discussed and usually requires the application of a finite
element approach. A schematic of this system can be seen in Figure 2.9. The pinion shaft and gear shaft are modeled as several rotor elements. This method is most appropriate when there are multiple masses on the same shaft and when pinions have diameters of the same order as its attached shaft.

![Figure 2.9 Schematic Gear Pair for Model D.](image)

These physical models (Models A - D) still require an excitation. In the time varying mesh stiffness models, one may use only the time varying stiffness as the excitation. However, this method may only be applied for “perfect involute” (no profile modification) gears and even then may not account for “off-line-of-action” contact that might occur. The effects of significant displacement excitations due to tooth deflections and profile modifications or due to errors in manufacture are usually not adequately considered in these models.
Ozguven and Houser [12] developed a means of separately incorporating each of these excitations through the computation of the loaded transmission error, \( e(t) \), of a gear pair. The transmission error that is used is computed at the mean operating load, so there are some potential errors due to the time varying loads that occur in operating gears.

In this research, three different models of varying complexity are compared to see how they differed in their prediction of dynamic factors. Three computer programs will be introduced in brief for calculating dynamic factors. These are the MATLAB program, DYTEM program, and GRD program.

2.3.1 MATLAB Program

A matrix analysis program to calculate the forced vibration analysis using MATLAB [18] has been developed in Gear Dynamics and Gear Noise Research Laboratory at The Ohio State University. The program is written using English Unit System, and solves for the eigenvalues and eigenvectors of a six degree of freedom undamped torsional system. A schematic of this six degree of freedom system is shown in Figure 2.10.

\( I_D \) and \( I_L \) are the drive and load torsional inertias, respectively. \( I_1 \) and \( I_2 \) are the respective inertias of the pinion and gear. There is a connecting shaft between the pinion and driver, called the pinion shaft, which has torsional stiffness, \( k_{11} \), and shaft damping, \( c_{11} \). Likewise, the connecting shaft between the gear and load, which is called the gear shaft, has shaft
stiffness, \( k_{a2} \), and shaft damping, \( c_{a2} \). The load, driver, pinion and gear inertias, and torsional stiffnesses of pinion and gear shafts are calculated based on their input geometry, such as diameter, length, and face width.

The combined effects of bearing compliance and shaft compliance, as measured along the line of action, are indicated by the stiffnesses, \( k_2 \) and \( k_3 \), for the pinion and gear, respectively. The associated damping is modeled with discrete dampers, \( c_2 \) and \( c_3 \), but may also be included as modal damping after the individual vibration modes are decoupled in the MATLAB analysis.

The pinion and gear can be modeled as two inertias coupled by a spring and damper. The spring is assumed to have an averaged mesh stiffness value, \( k_m \), while the damper is defined in term of mesh damping ratio. This average mesh stiffness value, \( k_m \), is inverted from the compliance mesh stiffness value generated by the Load Distribution Program (LDP) [20]. Another input to this program is the transmission error value, \( e(t) \), which is considered to be the excitation to the system. The transmission error, \( e(t) \), is usually the amplitude value of the first fourier series harmonic of mesh frequency calculated from LDP, and can be obtained using the Gear Graphic (GGR) program [22].
Figure 2.10  Six Degree of Freedom Model used in MATLAB and DYTEM Program.
The MATLAB model is solved using matrix methods that assume proportional damping and then modal decomposition and modal summation methods are used to solve for the response of any of the variables as a function of the sinusoidal transmission error excitation, e(t). Six natural frequencies and their respective modes shapes are solved in the analysis. In addition, the program calculates the natural frequency of a single degree of freedom system that considers only the pinion and gear geometry and the value of the average mesh stiffness.

The computer simulation results from the forced vibration analysis using this MATLAB program can be seen in twelve graphical frequency responses outputs presented over range of selected frequencies. The twelve graphical outputs are

- Angular displacements of the driver, load, pinion, and gear,
- Translational displacement of the pinion and driver (multiplying this term by bearing stiffness will produce bearing forces),
- Dynamic mesh force,
- Dynamic transmission error,
- Mode shapes at each predicted natural frequency.

Only Dynamic Load Factors may be computed with this method since there is no means of computing the load sharing between individual tooth pairs. The Dynamic Load Factor can be calculated using the dynamic mesh force and average mesh force, as shown in equation below,
Dynamic Load Factor = 1 + \frac{Dynamic Mesh Force}{Average Mesh Force} \tag{Eq. 2.12}

The forced vibration analysis program using MATLAB, a sample input, the respective simulation output, and the graphical results are included in Appendix A.

2.3.2 *DYTEM Program*

Multi-degree of Freedom Dynamic Transmission Error Program (DYTEM) is a time domain simulation, and is developed by Ozguven, et. al. [12-13, 19]. This program is an extended model of the original program that use only a single degree of freedom program, called DYTE program [23].

DYTEM is a six degree of freedom model, shown in Figure 2.10. It has four angular rotations and two translations along the line of action. The four angular rotations, \( \Theta_D \), \( \Theta_L \), \( \Theta_1 \), and \( \Theta_2 \), are for driver, load, pinion, and gear, respectively. The two translations are the translation of pinion and gear, which are symbolized as \( Y_1 \) and \( Y_2 \), respectively. The DYTEM version 2.2 program, run only in PC, uses a fourth order Runge-Kutta technique for solving the differential equations. Recently, a program using the Runge-Kutta Fifth-Fourth Order, which is developed by Padmanabhan was implemented into the program, as shown in Appendix B. This new DYTEM version 3.0, can be used for better computer simulation solving, and for running at low speed. Furthermore, it has been extended to run in a UNIX environment. A new plotting routine, called PLOT3, has also been added.
separately. This allows the user to view the graph of DYTEM results after the DYTEM being executed without re-run from the program again.

DTYM program in solving the dynamic analysis of the six degree of freedom model considers many factors:

1. Inertia of the driver, load, pinion and gears.
2. Constant or time varying drive and load torques.
3. Time varying mesh stiffness and mesh damping.
4. Separating of teeth in mesh.
5. Backlash.
6. Back side collision that may follow tooth separation.
7. Torsional compliances of pinion and gear shafts.
8. Material damping of pinion and gear shafts.
9. Transverse compliances of pinion and gear shafts.
11. Gear errors, such as tip relief, spacing errors, tooth lead errors, and profile modifications that are considered in the LDP program.
12. Shear deformation, bending, base rotation, and Hertzian deflection in the LDP program.

There are two main inputs that are required to run the DYTEM program. One main input is from the LDP run, usually stored in FILENAME.GRD. The second main input is the program control, geometry data, dynamic data, and gear error data. This second input can
be supplied from a user assigned file or run interactively. Both of these main inputs use English Units.

The first main input from the LDP run, which is stored in FILENAME.GRD. This can be selected using the “Create DYTE File” in Program Control in the LDP input (Option E). Selecting this input option, the user will be asked to supply the value of the reference radius of the pinion and gear. The FILENAME.GRD contains the dynamic analysis information, as follows:

- **Number of positions**, which is the POSCON numbers in LDP. The number of points have to be equally spaced in every mesh. Thus, the combination of beginning and ending position constant values with the number of positions has to match out that will give an equal space division in every mesh. For example, with beginning position constant of 0.000; ending position constant of 0.950 has to be chosen to give number of positions of 20; and likewise, ending position constant of 0.960 needs to be selected to give number of positions of 25.

- **Mesh stiffness** at each position (lb/in).

- **Static transmission error** at each position (micro inches).

- **Average compliance** of the mesh (micro in/lb).

- **Maximum number of teeth in contact**.

- **Load sharing values, Load arm of pinion, and load arm of gear**.

- **Angular position of pinion**.
The second main input to DYTEM, which can be entered using a user specified data or run interactively has several different input categories, as followed:

- Program control and gear geometry data.
- Inertias of driver, load, pinion, and gear, and mass of pinion and gear.
- Torsional stiffnesses of shafts and bearing stiffnesses.
- Mesh damping ratio, shafts damping ratio, and bearings damping.
- Driver and load torques.
- Kinematics data that consists of selection of one speed run or multi speed runs.
- Backlash.
- Selection in output files information.

Two methods can be chosen in the DYTEM, one is the *Approximate Method*, and another one is the *Static Transmission Error (STE) Method*. These methods can be selected in the program control and gear geometry data. The Approximate Method uses the value of the average mesh stiffness resulted from the inverted value of the average compliance of the mesh stiffness generated by LDP. The Static Transmission Error uses the value of mesh stiffness data at each LDP points. Since the Static Transmission Error uses the varying mesh stiffness at each position, its output results are more accurate than the Approximate Method.

The mesh damping ratio, damping ratio for shafts, and bearing dampings are difficult values to evaluate since there is no accurate calculations or theories for choosing them.
The output results of DYTEM which can be seen in the form of graphical outputs and also stored in files, are as follows:

1. Dynamic Mesh and Tooth Forces (stored in FILE16).
2. Static and Dynamic Moments at Tooth Root (stored in FILE24).
3. Dynamic Factors for Pinion and Gear (stored in FILE14).
4. Static and Dynamic Transmission Errors (stored in FILE15).
5. Dynamic Bearing Forces for Pinion and Gear (stored in FILE22).
6. Dynamic Bearing Displacement for Pinion and Gear (stored in FILE23).
7. Torsional Shaft for Pinion and Gear (stored in FILE21).

Two additional output files that resulted in running the DYTEM are the FILE25 and FILE29. FILE25 consists of several parameters to be read when the PLOT3 program is being executed. FILE29 consists of the time at each position of the mesh, which is used as the x-axis values in plotting the output results of DYTEM (FILE14, FILE15, FILE16, FILE21, FILE22, FILE23, and FILE24).

For multi speed run which can be selected in the Kinematics data, two additional output files will be resulted, FILE18 and FILE19. FILE18 consists of the values of Dynamic Load Factor and the Dynamic Tooth Force Factor. FILE19 consists of the values of the Dynamic Bending Moment Factor for the Pinion and for the Gear. Both of these files are plotted after the DYTEM analysis is completed.
The *Dynamic Load Factors* are calculated by taking the ratio of maximum dynamic mesh force to the static transmitted load. The *Tooth Force Factors* on each tooth are calculated by taking the ratio of the maximum dynamic mesh force times the load sharing at each tooth to the static transmitted load. Finally, the *Dynamic Bending Moment Factors* are computed by multiplying the instantaneous tooth force by the moment arm to a critical radius at the tooth root.

A sample input from LDP (FILENAME.GRD), user defined input for DYTEM, and graphical results, are included in Appendix B.

2.3.3 **HGRD/GRD Program**

The Geared Rotor Dynamics Program (GRD or HGRD) developed by Kahraman, et. al. [14, 21] is a finite element method to analyze the dynamics of a system consisting of two shafts supported on bearings and coupled by a spur or helical gear mesh, as shown in Figure 2.11. The “H” on HGRD stands for ‘Helical’ since this HGRD program, which is the extended version of GRD, can be used for calculating the dynamic of a helical gear mesh.

In order to perform the finite element method, the gear rotor system, shown in Figure 2.11, need to be discretized to small rotor elements, like the one shown in Figure 2.12. As shown, the pinion and gear shafts are discretized into several small rotor elements.
However, the higher the amount of discretization, the higher memory is required in the program, thus the longer the time needed to analyze the gear mesh.

**Figure 2.11** Schematic of Geared Rotor System.

**Figure 2.12** Schematic of Discretized Geared Rotor System.
There are two main inputs that are needed to run GRD or HGRD program. One main input is from the LDP run stored in FILENAME.GRD. The name of this file has to be changed to DYNAM.OUT before running the GRD or HGRD program. The second main input is from the HGRDFILE run, usually under a user assigned filename. The first input uses the English Units, while the second input uses Metric Units.

The first main input is from LDP run, and is stored in FILENAME.GRD. As mentioned in the previous section, this can be selected using the "Create DYTE File" in Program Control in the LDP input (Option E). Supplying the value of reference radius of pinion and gear, the LDP program will generate a FILENAME.GRD that consists the dynamic analysis information, as mentioned in DYTEM program section. The HGRD or GRD program does not required all the information supplied on FILENAME.GRD. The following dynamic information are required in the HGRD or GRD program:

- **Number of positions**, which is the POSCON numbers in LDP. This number must be 25 since it is the default used by the HGRD or GRD program.
- **Static transmission error** at each position (micro inches).
- **Average compliance of the mesh** (micro in/1b).

The HGRD or GRD program will convert the English Units for the above values to the Metric units, automatically when the HGRD loads this data into the program.
The second main input to HGRD or GRD program is supplied by running the program HGRDFILE. The unit that is being used in this second input must be in Matrix Units. There are several categories in the data entry processing to the HGRDFILE, as follow:

- **General Data:** It consists of the number of rotor elements in all shafts used.

- **Material Properties:** It consists of the density of the shaft material, viscous damping coefficient, hysteretic damping coefficient, modulus of elasticity, and shear modulus of elasticity.

- **Element Properties:** Disk element consists of diameter, width, and material density dimension. Bearing element consists of stiffness and damping values. Rotor element consists of length, outer diameter, inner diameter, and axial load values. Gear element consists of pitch circle, face width, and gear material density values.

- **Gear Mesh Properties:** It consists of the gear mesh stiffness and damping, base circle of pinion and gear, and the helix angle.

- **Forced Response Data:** The option that can be chosen are (1) whirling orbit at a specified node, (2) deflections at specified node, (3) dynamic load to static load ratio at meshing point, and (4) acceleration at a specified node.

For the purpose this thesis, the option 3 (dynamic load to static load ratio at meshing point) has been chosen in the Forced Response Data. The additional data that is required by selecting this option are starting and upper limit of rotational speed of the first shaft, and the increment for rotational speed.
This program also uses a transmission error input computed by LDP, but treats the equations as linear time invariant and uses an average value for the mesh stiffness. Bearing stiffnesses are placed at the actual bearing locations, and shaft deflections that could only previously be incorporated into the bearing stiffness, are now independently included.

After the HGRD or GRD program is executed, an output result can be seen in a file whose name is specified by the user. This file consists of natural frequencies of the system and their correspondence mode shapes, and then the value of the dynamic load factor at each speed specified in the Forced Response Data. This dynamic load factor data can be plotted by several available graphical packages.

The sample of input files from LDP and HGRDFILE, the output file, and some graphical outputs are included in Appendix C.

### 2.4 Assumptions

An unknown of each of these three models is the amount of damping to be included. Models 1 and 3 only allow modal damping, while model 2 allows discrete dampers to be placed at the same locations as the springs. In all cases, damping values were selected to give good comparisons with experimental data taken by Rebbechi, et. al. [7].
In this thesis, a study of three of the dynamic factor computation methods is conducted. They are the *Dynamic Load Factor* as computed by AGMA, the *Dynamic Tooth Force Factor*, and the *Dynamic Bending Moment Factor*. The models used have the capability of carrying the computations through the final step to calculate contact or root stresses, but this option has not yet been implemented.

In fact, in this thesis, the intent is to see the effects of profile errors on the dynamic factors and these effects show up equally well with each of the factors that is chosen. Spacing errors, which are another significant input to dynamic factor are not included in this analysis. In this paper, the dynamic factor will always be considered as a dynamic stress multiplier in the fashion of Kubo [4] and later presented in Dudley [5]. The expression used can always be converted to an AGMA type factor by inverting the number that is computed. This approach implies that higher dynamic factors indicate higher stresses on a component.
CHAPTER III

EXPERIMENTAL AND COMPUTER PROGRAMS COMPARISONS

3.1 Introduction

This chapter presents the experimental results of spur gear pairs from NASA (National Aeronautic and Space Administration) gear noise rig obtained by Rebecchi, et. al. [7]. The experimental results are compared in terms of natural frequencies to the theoretical results taken from the computer program simulations, mainly, using DYTEM, GRD, and MATLAB programs. Next, the DYTEM program is extensively exercised for several comparisons of dynamic tooth load to the experimental tooth load data provided by NASA. Then, an optimum spur gear pair is selected that will give the minimum dynamic factor prediction. The three dynamic factors that will be explored in great detail in this section are the Dynamic Load Factor, Dynamic Tooth Force Factor, and Dynamic Bending Moment Factor on both the Pinion and Gear Teeth.
3.2. **Comparison to NASA Experimental and Theoretical Data**

NASA Lewis gear noise rig was built to enable fundamental experimental studies of the dynamic gear behavior, such as dynamic tooth loads. This gear noise rig can also provide support to gear noise reduction programs [7]. The layout and the details of the gearbox are shown in Figure 3.1 and Figure 3.2, respectively. The gearbox is extensively instrumented for strain and vibration measurement. The experimental strain gage data obtained in each run were converted to dynamic tooth data using calibration data measured under static conditions.

![Diagram of NASA Gear Noise Rig](image)

**Figure 3.1** Layout of NASA Gear Noise Rig.
Figure 3.2  Detail of Gearbox of NASA Gear Noise Rig.

Besides experimental approach, NASA also developed several computer software packages, which were used to predict dynamic loads. Two of them are the DANST and ARL_DYN. The DANST is the Dynamic Analysis of Spur Gear Transmission program [8] that has four torsional degrees of freedom consisting of the motor, pinion, gear, and load. This program predicts the effect of dynamic loads and tooth bending stress of spur gears due to the operating speed, stiffness, damping, inertia, and tooth profile. The DSTO Aeronautical Research Laboratory (ARL) developed another software for dynamic analysis called ARL_DYN. This program has the capability to calculate the dynamic load and tooth bending stress of the spur gears in more complex circumstances. It allows the detailed contact conditions of each tooth pairs to separately designated. Moreover, the shaft deflection is considered in this program.
The computer simulation predictions using ARL_DYN program are used in this chapter, as a comparison to the NASA experimental results and to the theoretical prediction obtained using computer software packages - DYTEM, GRD, and MATLAB programs.

3.2.1 *NASA Gear Specification*

Rebbechi, et. al. [7] used gears of AGMA class 13 quality in the NASA gear noise test rig. The gear pair has a unity ratio, each gear has 28 teeth and a diametral pitch of 8 in⁻¹. The schematic of these gears is illustrated in Figure 3.3. Table 3.1 shows the specification of the spur gears.

![Figure 3.3 Schematic of NASA Test Spur Gears.](image-url)
Table 3.1 NASA Spur Gear Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth pinion / gear</td>
<td>28</td>
</tr>
<tr>
<td>Normal diametral pitch</td>
<td>8 (1/in)</td>
</tr>
<tr>
<td>Center distance</td>
<td>3.5 in</td>
</tr>
<tr>
<td>Face width</td>
<td>0.25 in</td>
</tr>
<tr>
<td>Helix angle</td>
<td>0 degree</td>
</tr>
<tr>
<td>Pressure angle</td>
<td>20 degree</td>
</tr>
<tr>
<td>Theoretical contact ratio</td>
<td>1.64</td>
</tr>
<tr>
<td>Outside diameter pinion / gear</td>
<td>3.75 in</td>
</tr>
<tr>
<td>Root diameter pinion / gear</td>
<td>3.1875 in</td>
</tr>
<tr>
<td>Base diameter pinion / gear</td>
<td>3.2889 in</td>
</tr>
<tr>
<td>Drive inertia</td>
<td>2.1 in-lb-s^2</td>
</tr>
<tr>
<td>Load inertia</td>
<td>7.8 in-lb-s^2</td>
</tr>
<tr>
<td>Torsional stiffness of pinion shaft</td>
<td>3000 in-lb/rad</td>
</tr>
<tr>
<td>Torsional stiffness of gear shaft</td>
<td>110000 in-lb/rad</td>
</tr>
<tr>
<td>Shaft damping ratio</td>
<td>0.06</td>
</tr>
<tr>
<td>Average mesh stiffness</td>
<td>7.1E5 lb/in</td>
</tr>
<tr>
<td>Pinion/gear inertia</td>
<td>1.2189E-3 in-lb-s^2</td>
</tr>
<tr>
<td>Pinion/gear mass</td>
<td>9.363E-3 lb-s^2/in</td>
</tr>
<tr>
<td>Pinion/Gear bearing stiffness</td>
<td>1.5E4 lb/in</td>
</tr>
<tr>
<td>Mesh damping</td>
<td>vary</td>
</tr>
<tr>
<td>Backlash</td>
<td>0.01 in</td>
</tr>
<tr>
<td>Full Torque (Rebbechi, et. al. [7])</td>
<td>635.25 lb-in</td>
</tr>
<tr>
<td>Pinion modification amount</td>
<td>0.0009 in</td>
</tr>
<tr>
<td>Gear modification amount</td>
<td>0.0010 in</td>
</tr>
<tr>
<td>Pinion roll angle</td>
<td>24 degree</td>
</tr>
<tr>
<td>Gear roll angle</td>
<td>24 degree</td>
</tr>
</tbody>
</table>
The pinion and gear mass are $3.1212 \times 10^{-3}$ lb-s$^2$-in$^{-1}$ if they are calculated directly based on the gear geometry, which is shown in Figure 3.3. Likewise, the pinion and gear inertia are $4.063 \times 10^{-3}$ lb-s$^2$-in with the same gear geometry values. However, in simulating this gear pair system and comparing the dynamic results with the experimental data given by Rebbechi, et. al. [7], these numbers have been increased three times. The new values are $9.363 \times 10^{-3}$ lb-s$^2$-in$^{-1}$ for the pinion and gear masses, and $12.189 \times 10^{-3}$ lb-s$^2$-in for the pinion and gear inertia.

The mass and inertia numbers has been increased (three time for this comparison) so that the dynamic results can compare reasonably well with the experimental data. One reason to increase these numbers is that in modeling the gear system in the test rig, neither the allowance for gear shaft nor the spacers which rotate together with the gear are considered. Moreover, the gear hubs and gear couplings in the test rig are not being considered in the theoretical calculation. Rebbechi, et. al. [7] has shown that increasing the mass and inertia of the gear pair affects the dynamic results of the system. Thus, in this comparison, the mass and inertia values tabulated in Table 3.1 are selected. Later, in predicting the dynamic factors and other dynamic responses in section 3.4 and Chapter 4, the original value of mass and inertia of pinion and gear will be used.

In Appendix D, DYTEM program is used to calculate several dynamic responses for the gear pair with different masses, inertia, speed, and damping. From all these runs, four most favorably results are selected and included in the following section.
3.2.2 *Natural Frequencies Comparison*

Figure 3.4 presents Rebbechi’s tooth root strain gage data for two consecutive spur gears runs using the spur gear parameters tabulated in Table 3.1 and using 1 torque of 96.93 lb-in (31% of Full Torque). Four operating speeds, which are 800, 2000, 4000, and 6000 rpm, are selected. For each speed, the time axis is normalized to position along the line of action. In this case, the test rig has extremely compliant shafts so only a single torsional natural frequency predominates. The modes of vibration that have significant shaft bending are at low frequencies that do not show up in the strain data.

![Graph of tooth strain gage data at several different operating speeds using 196.93 lb-in Torque.](image)

**Figure 3.4** Tooth Strain Gage Data at Several Different Operating Speeds using 196.93 lb-in Torque (From Rebbechi, et. al. [7]).
There are three regions of interest to be noted from the experimental curves shown in Figure 3.4, as followings:

1. The region where the entire load is carried by two teeth in contact (the contact ratio for this system is 1.64). The strain of each operating speed increases rapidly as the load is taken by the tooth of interest. At this region, especially until point "A" being reached, any operating speed has little effects on the dynamic tooth strain gage of the system.

2. The region where the entire load is carried by a single tooth in contact. The dynamics on the system have a greater affect in this region that cause the oscillation on the system, depending on the operating speed. The lower the operating speed, the higher numbers of oscillation on the system.

3. The region where the entire load is carried back by two teeth in contact, in which a new tooth coming in contact. Again, at this region, the dynamics of the system does not affect the tooth strain gage in a great matter.

From this experimental data, one also can observe that at higher operating speeds, there are overshoots in the responses of the strain gage data. By inverting the periods of the oscillations observed in the strain gage data, for the three lower speeds (800, 2000, and 4000 rpm), one finds that the primary natural frequency of the system is roughly 3400 Hz. A single natural frequency is difficult to discern from 6000 rpm data because very few oscillations occur. Also, it is noted that the damping, as qualitatively assessed from the
decay rate of the oscillations, is lowest at 800 rpm and then increases as the speed is increased.

Next, DYTEM, GRD, and MATLAB programs are used for the natural frequencies comparison with the experimental data shown in Figure 3.4. The model used in DYTEM and MATLAB programs can be seen in previous chapter, which is shown in Figure 2.7. Two specific models are considered when GRD program is used. One model can be seen in Figure 2.9 in previous Chapter. This model consists of external driver, shafts, bearings, pinion, gear, and external load. For simplification in classification in this thesis, it is called GRD Model B. Another model is called GRD Model A, where the external driver and load in GRD Model B are not included in the model specification. The parameters used in all models are tabulated in Table 3.1.

<table>
<thead>
<tr>
<th></th>
<th>Natural Frequency (Hz)</th>
<th>Dynamic Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASA Experiment</td>
<td>3400</td>
<td>N/A</td>
</tr>
<tr>
<td>MATLAB Model</td>
<td>3445</td>
<td>2.4</td>
</tr>
<tr>
<td>DYTEM Model</td>
<td>(\approx 3200)</td>
<td>2.8</td>
</tr>
<tr>
<td>GRD Model A</td>
<td>(\approx 3750)</td>
<td>2.6</td>
</tr>
<tr>
<td>GRD Model B</td>
<td>(\approx 3750)</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Table 3.2 Natural Frequencies and Dynamic Load Factors Predicted by the Three Different Modeling Methods with NASA Gear Specifications at 196.93 lb-in Torque.
Table 3.2 shows that the primary natural frequencies predicted by each of the models compare quite favorably with one another. It is shown also that they compare quite favorably with the NASA experimental results. The maximum dynamic load factors are 2.4, 2.8, 2.6, and 2.1 using MATLAB, DYTEM, GRD model A, and GRD model B, respectively.

Adding the external load and driver into the GRD model A, the natural mesh frequency predicted is not affected. The natural frequencies for both models are around 3750 Hz. However, the model with external load and driver (GRD model B) has a lower dynamic factor at the major resonance but at all lower speed is nearly identical to the GRD model A. At 3750 Hz resonance, the dynamic factors are 2.6 and 2.1 for GRD model A and GRD model B, respectively. For general information, the next lower natural frequencies that are predicted by the MATLAB model are at 151 and 400 Hz., respectively. At these frequencies, the vibration mode shapes are dominated by translational motions of the inertias.

Figure 3.5 shows the dynamic bending moment results of DYTEM simulations for the four operating speeds (800, 2000, 4000, and 6000 rpm). In order to obtain reasonable agreement, the damping ratio of the primary vibration mode at 3400 Hz needed to be adjusted for each speed. The damping ratios used increased with speed and were 0.06 for 800 rpm, 0.06 for 2000 rpm, 0.2 for 4000 rpm and 0.3 for 6000 rpm. Although the trends are not perfect, the simple lumped parameter model provides a good qualitative agreement with the experimental data.
Since damping is a significant value and a “wild card” in the dynamic responses studies, the numbers above are selected from several runs. The effects of dampings can be seen in great details in Appendix D.

![Graph showing dynamic bending moment at different operating speeds](image)

**Figure 3.5** Dynamic Bending Moment at Different Operating Speed using 196.93 lb-in Torque.

3.2.3 *Dynamic Tooth Load Comparison*

Rebbechi, et. al. [7] also provided some experimental dynamic tooth loads data using NASA test rig at several operating speeds and torques. They also included their theoretical prediction using the ARL_DYN computer program. Both these NASA data are compared with the dynamic tooth load obtained using the DYTEM program at the respective operating speed and torques, as shown in Figures 3.6 to 3.10. Again, the mesh damping ratio of the system is a significant value at each test condition.
Figure 3.6 Dynamic Tooth Load Comparisons at Operating Speed of 2000 rpm and at 47% Full Torque (298.57 lb-in), $\zeta = 0.2$.

Figure 3.7 Dynamic Tooth Load Comparisons at Operating Speed of 4000 rpm and at 31% Full Torque (196.93 lb-in), $\zeta = 0.2$. 
Figure 3.8  Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm and at 31% Full Torque (196.93 lb-in), \( \zeta = 0.6 \).

Figure 3.9  Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm and at 47% Full Torque (298.57 lb-in), \( \zeta = 0.3 \).
Figures 3.6 - 3.10 show the agreement between the dynamic tooth loads predicted by NASA (both experimental and theoretical) and the dynamic tooth load computed using DYTEM program. The selection of mesh damping ratio in DYTEM program in each run are tabulated in Table 3.3. From these mesh damping ratio values, one can not directly draw a conclusion that there are some trends that relates the mesh damping ratio values to the speeds or torques. However, one can conclude that the mesh damping ratio is one of the most important factors in the dynamic analysis of gears in mesh, specially computer simulation at different operating speeds and torques.
Table 3.3 Mesh Damping Ratio used in DYTEM Program for Calculating the Dynamic Tooth Loads.

<table>
<thead>
<tr>
<th>Operating Speed</th>
<th>Mesh Damping Ratio at 31% Torque</th>
<th>Mesh Damping Ratio at 47% Torque</th>
<th>Mesh Damping Ratio at 94% Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 rpm</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000 rpm</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6000 rpm</td>
<td>0.6</td>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

3.3 Dynamic Analysis at Different Input Torque Level

From previous section, one can observed that different input torque values give a different dynamic responses. In order to examine the effect of torque variation in the gear pair system, DYTEM program is used with the data tabulated in Table 3.1. The operating speed is selected to be 4000 rpm and the torque levels are varied from 31% to 110% of Rebbechi's full torque (635.25 lb-in). Figure 3.11 shows the dynamic mesh at different torque level. One can conclude that higher torque level will yield to higher dynamic mesh force values. The same case occurs for the tooth force, which can be seen in Figure 3.12. Lower torque level will yield to lower dynamic tooth force values. Moreover, at lower torque, there is higher bouncing on the tooth force amplitude.
Figure 3.11 Dynamic Mesh Force at Different Input Torque Level, $\zeta = 0.2$, Speed = 4000 rpm.

Figure 3.12 Dynamic Tooth Force at Different Input Torque Level, $\zeta = 0.2$, Speed = 4000 rpm.
Figure 3.13 Dynamic Bending Moment (Pinion) at Different Input Torque Level, $\zeta = 0.2$, Speed = 4000 rpm.

Figure 3.14 Dynamic Bending Moment (Gear) at Different Input Torque Level, $\zeta = 0.2$, Speed = 4000 rpm.
Figure 3.15 Dynamic Factor (Pinion) at Different Input Torque Level, 
\( \zeta = 0.2, \) Speed = 4000 rpm.

Figure 3.16 Dynamic Factor (Gear) at Different Input Torque Level, 
\( \zeta = 0.2, \) Speed = 4000 rpm.
Figures 3.12 and 3.13 show the dynamic bending moment for pinion and gear, respectively, at different torque level. Like the dynamic mesh force and dynamic tooth forces mentioned earlier, the same trends occur in the dynamic bending values. The higher torque level will yield to higher dynamic moment. Figures 3.14 and 3.15 show the dynamic factor for pinion and gear, respectively, at different torque level. At lower torque level, the value of the dynamic factors for both pinion and gear are quite high. They start to reduce as the torque level is increased.

3.4 Computer (DYTEM, GRD, and MATLAB Programs) Comparison

In this session, firstly, the three computer programs (DYTEM, GRD, and MATLAB programs) are compared using a set of spur gears. The perfect involute spur gears (without any tooth profile modification) are considered, in order to validate the agreement among the three computer packages in term of the dynamic load factors. Secondly, the optimum tooth profiles spur gears that give minimum dynamic factor values are selected, and compared to the dynamic factor of perfect involute spur gears. The comparison is accomplished only by considering the DYTEM programs in the three dynamic factors which have been described in great details in Chapter 2. They are the dynamic load factor, dynamic tooth force factor, and dynamic bending moment factor for the pinion and gear.
3.4.1 Spur Gears Specification

Table 3.4 shows the specification of the spur gear pairs that are used in DYTEM, GRD, and MATLAB programs. The schematic of the spur gear can be seen in Figure 3.3. The mass and inertia for the pinion and gear are calculated directly from the gear geometry, as shown in the Table 3.4.

<table>
<thead>
<tr>
<th>Table 3.4  Spur Gears Parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth pinion / gear</td>
</tr>
<tr>
<td>Normal diametral pitch</td>
</tr>
<tr>
<td>Center distance</td>
</tr>
<tr>
<td>Face width</td>
</tr>
<tr>
<td>Helix angle</td>
</tr>
<tr>
<td>Pressure angle</td>
</tr>
<tr>
<td>Theoretical contact ratio</td>
</tr>
<tr>
<td>Outside diameter pinion / gear</td>
</tr>
<tr>
<td>Root diameter pinion / gear</td>
</tr>
<tr>
<td>Base diameter pinion / gear</td>
</tr>
<tr>
<td>Drive inertia</td>
</tr>
<tr>
<td>Load inertia</td>
</tr>
<tr>
<td>Torsional stiffness of pinion shaft</td>
</tr>
<tr>
<td>Torsional stiffness of gear shaft</td>
</tr>
<tr>
<td>Shaft damping ratio</td>
</tr>
<tr>
<td>Average mesh stiffness</td>
</tr>
<tr>
<td>Pinion/gear inertia</td>
</tr>
<tr>
<td>Pinion/gear mass</td>
</tr>
<tr>
<td>Pinion/Gear bearing stiffness</td>
</tr>
<tr>
<td>Mesh damping</td>
</tr>
<tr>
<td>Backlash</td>
</tr>
<tr>
<td>AGMA torque rating</td>
</tr>
</tbody>
</table>
A torque of 1128 lb-in has been selected in the computer program comparisons. This value is obtained by using the FAIRFIELD Gear Design Program [24] which is developed by FAIRFIELD Manufacturing Company. Running the MULTILDP program [25] for the same spur gear specifications used in FAIRFIELD program, one can find out the torque that will give a minimum static transmission error. Figure 3.17 shows the peak to peak static transmission error (PPTE) at different operating torques obtained by running MULTILDP. It shows that as the operating torques increase from 200 lb-in to 1140 lb-in, the peak to peak static transmission error decrease from 350 μ in to 100 μ in. Then, increasing for torques above 1140 lb-in, the peak to peak static transmission errors start to increase again.

**Figure 3.17** The Peak to Peak Static Transmission Error (PPTE) of Spur Gear at several Operating Torques using MULTILDP Program.
The maximum torque that gives the minimum peak to peak static transmission error lies between 1020 and 1140 lb-in. Thus, 1128 lb-in torque is selected as the operating torque for the computer programs comparison. Figures 3.18 shows variation of the static transmission errors at several operating torques.

![Graph showing static transmission error at different torques.](image)

**Figure 3.18** Static Transmission Error of the Spur Gear at Different Operating Torques using MULTILDP Programs.
3.4.2 *Comparison of Dynamic Load Factors using the Three Computer Programs*

Figure 3.19 presents a comparison of the three dynamic models when predicting the *dynamic load factor*. The excitation is the transmission error for a perfect involute (no profile modifications) spur gear pair with the specifications given in Table 3.4. Table 3.5 gives the transmission error amplitudes of the first six Fourier series harmonics of mesh frequency that are predicted by the LDP program. The predicted time domain transmission errors data were used in the analysis for the GRD and DYTEM models. However, only the first harmonic of mesh frequency was used in the MATLAB model.

![Comparison of Dynamic Loads Predicted by the Three Different Dynamic Models](image.png)
Table 3.5  The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Perfect Involute Spur Gears.

<table>
<thead>
<tr>
<th>Harmonic #</th>
<th>Amplitude (μ in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>265.410</td>
</tr>
<tr>
<td>2</td>
<td>58.948</td>
</tr>
<tr>
<td>3</td>
<td>72.898</td>
</tr>
<tr>
<td>4</td>
<td>62.154</td>
</tr>
<tr>
<td>5</td>
<td>17.917</td>
</tr>
<tr>
<td>6</td>
<td>52.369</td>
</tr>
<tr>
<td>PPTE</td>
<td>475.540</td>
</tr>
</tbody>
</table>

Two cases for the GRD model are analyzed, model A is for a gear set without external inertias (load and driver) and model B is for a gear set with external inertias (Figure 2.9 in Chapter 2 shows both models). Each of the four models predicts approximately the same “primary” mesh excited resonance which lies between 5800 and 6700 Hz. These frequencies are much higher than those of the previous example because the gear wheel inertias and masses were reduced to solid disks and the shaft stiffness was increased to a more realistic value that more closely approximates the short shafts that would be used in practice.
Differences between each model exist because each model, as described earlier, has its own set of assumptions. In this case, the DYTEM model, which is nonlinear and has time varying coefficients, shows a slight tendency towards a softening nonlinearity (Blankenship, [15]). The large number of peaks at the lower frequencies in the DYTEM and GRD models are due to the higher mesh frequency harmonics of transmission error exciting the natural frequencies. The excitation of the MATLAB model is sinusoidal, so the natural frequencies are only excited once as opposed to multiple times in the other models. In any case, each of the models provides similar trends with increasing speed.

Because the DYTEM model allows for nonlinearities and provides time domain information on forces and stresses, it will be used for all subsequent predictions. Modal damping ratios of 0.06 were used in each of these examples. However, where only dynamic load is desired and mesh stiffness is approximately constant, the other two models can work acceptably as long as one realizes that in each of the models, damping is a “wild card” that must be assumed.

3.4.3 Perfect and Optimized Involute Spur Gears Comparison

The appropriate tip and root relief to provide for minimum transmission error are obtained by iteration of the profile modifications used in the LDP analysis. The profile chart for the optimized case is given in Figure 3.20. Only linear modifications to the tip and root of the drive gear were tried and the pinion is assumed to be perfect. Of course, one could have
provided equivalent modifications on the pinion and obtained similar results. It is interesting to note that the respective starts of the modification are at roll angles closer to the highest (HPSTC) and lowest (LPSTC) points of single tooth contact than to the pitch point.

![K-Chart of Spur Gear for Optimum Profile Modification](image)

**Figure 3.20** K-Chart of Spur Gear for Optimum Profile Modification.

Table 3.6 shows the Fourier Series of Transmission Errors and the Peak to Peak Transmission Error for the Optimum Modified Spur Gear. From Table 3.5 and Table 3.6, it can be seen that both peak-to-peak transmission error and the amplitudes of the transmission error’s first three mesh frequency harmonics are all significantly reduced for the optimum profile modification.
Table 3.6  The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Optimum Modified Spur Gears.

<table>
<thead>
<tr>
<th>Harmonic #</th>
<th>Amplitude (μ in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.281</td>
</tr>
<tr>
<td>2</td>
<td>14.632</td>
</tr>
<tr>
<td>3</td>
<td>12.072</td>
</tr>
<tr>
<td>4</td>
<td>12.447</td>
</tr>
<tr>
<td>5</td>
<td>3.134</td>
</tr>
<tr>
<td>6</td>
<td>10.325</td>
</tr>
<tr>
<td>PPTE</td>
<td>85.516</td>
</tr>
</tbody>
</table>

The corresponding reduction in predicted dynamic load factor is shown in Figure 3.21. This figure points out an important fact about dynamic load factors, namely, that the same gear pair's factor can vary significantly depending upon the shapes of the individual tooth profiles. In the case of the perfect involute, the dynamic load factor exceeds 2.0 around resonance and the dynamic factor is between 1.2 and 1.6 when operating beneath the resonance. It is this lower frequency region that AGMA standards designate for the dynamic factor in [3]. For all speeds, the gear pair with the optimized profiles has a dynamic factor beneath 1.10.
Figure 3.21 Comparison of Dynamic Load Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear.

Figure 3.22 shows a similar comparison for the dynamic tooth force factor. The shapes are somewhat similar to dynamic load factors, but the values are lower, and at some speeds the dynamic tooth factor amplitudes actually drop below 1.0. This is possible because it is often the case that dynamic load factor is a maximum when two tooth pairs of teeth are in contact and this does not correspond to the same instant of time when the peak force on an individual tooth is at its maximum.
**Figure 3.22** Comparison of Dynamic Tooth Force Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear.

Figures 3.23 and 3.24 shows the *dynamic bending moment factors* for the pinion and gear teeth, respectively. The shapes of these curves look very similar to those of the dynamic tooth force factors. The dynamic bending moment factors for the gear teeth are somewhat less than those of the pinion. It is again interesting to note that the optimized dynamic bending moment factors are often beneath unity.
Figure 3.23 Comparison of Dynamic Bending Moment Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear (Pinion Tooth).

Figure 3.24 Comparison of Dynamic Bending Moment Factors of the Perfect Involute Spur Gear and Optimum Modified Spur Gear (Gear Tooth).
3.5 **Summary**

Chapter 3 has covered several comparisons between the experimental data and the theoretical computer prediction. The experimental strain gage data that obtained from NASA shows the agreement to the theoretical results that are obtained using DYTEM, GRD, and MATLAB programs. These programs compare quite favorably with the theoretical data simulated by ARL_DYN program which is developed by NASA. Mesh damping ratio is one of signification factors to determine the dynamic behavior of the gear system. However, it is a "wild card" that must be assumed.

DYTEM, GRD, and MATLAB programs compare quite good to each other in term of dynamic load factor and natural frequencies. The DYTEM program, which is able to carried out the dynamic tooth load factor and dynamic bending moment factor, is used extensively.

An optimum involute tooth profiles for a set of spur gear has been selected and compared to the perfect involute spur gears in term of dynamic factors. One can conclude that the tooth profiles of a gear has a great affect on the prediction of dynamic factors. This optimum involute tooth profiles gear will be used a base-line profile modification that will be discussed more details in Chapter 4.
CHAPTER IV

THE EFFECTS OF PROFILE MANUFACTURING TOLERANCES ON DYNAMIC FACTORS

4.1 Introduction

This chapter presents the effects of tooth profile manufacturing tolerances on the prediction of the dynamic factors. The manufacturing tolerances are considered based on the AGMA gear accuracy tolerance, which is known in term of the gear quality number. First, a set of gear pairs with specific tooth profiles is selected as a reference, so that comparison can be accomplished according to the gear quality number. Next, a K-Chart is established for a Quality 14 gear. Several profile extremes are chosen within the maximum allowable profile tolerances for the specified gear quality. Then, the predicted dynamic factors are plotted as a function of mesh frequency or operating speed. Once this analysis is completed, Quality 12 gear and Quality 10 gear are then being considered with the same procedures as taken in the Quality 14 gear. Finally, several other dynamic analysis are included for further study of dynamic behavior of the gear pairs in mesh.
4.2 Selection of Tooth Profiles as a Reference in Comparison

One quandary that was faced in initiating the effects of profile manufacturing tolerances on dynamic factor was to consider what profile should be used as a reference for which to add manufacturing errors. As has been shown in the earlier figures in Chapter 3, the shape of the tooth profile can have a drastic effect on the predicted dynamic factor. Figure 4.1 shows the dynamic load factor for the perfect involute spur gear. If one started with a perfect involute (no profile modification), the dynamic factor would be largely independent of the amount of manufacturing tolerance that is used because from a dynamics viewpoint, a perfect involute is far from ideal.

![Dynamic Load Factor of the Perfect Involute Spur Gear](image)
In fact, for speeds under 6000 rpm, a perfect involute gear would have a dynamic factor approaches 1.6, which is shown in Figure 4.1. Therefore, it was concluded that one should start with an optimized tooth profile that minimizes transmission error and manufacturing deviations would be applied as deviations from this tooth profile. Thus, the tooth profile is iteratively selected using LDP and MULTILDP programs that will give a minimum transmission error.

![PINION TOOTH PROFILE](image1)

![GEAR TOOTH PROFILE](image2)

![PINION PLUS GEAR MODIFICATION](image3)

**Figure 4.2** K-Chart of Spur Gear used as the Reference Tooth Profile.
In this thesis, only a linear tooth profile is considered. A schematic of the pinion and gear tooth profile (K-Chart) for optimum modified gear can be seen in Figure 4.2. The amount of pinion tip modifications and its starting roll angle are 0.00095 inches and 23.18 degrees, respectively. The same amount of tip modification and its starting roll angle are chosen for the gear. Certainly, one can have tip and root modifications on either pinion or gear only, but still obtain similar results, as mentioned in Chapter 3. Throughout the discussion in this chapter, the K-Chart will be shown only in the total modification of the pinion and the gear. In other words, even though both pinion and gear tip modifications at their respectively roll angles are entered into the LDP program, only the total modification results will be shown in the K-Chart.

Having the reference tooth profile of the spur gear, comparisons can be made according to the gear quality. The AGMA gear accuracy tooth profile tolerances will be used in each gear quality. The AGMA profile tooth tolerances have the equation that consists of number of teeth, normal diametral pitch, helix angle, and gear quality number, as shown in the following equation:

\[
V_{\text{phit}} = \frac{21.5}{10000} \left( \frac{N_p}{\cos(\psi)} \right)^{0.154} \left( 1.4 \left( 8 \cdot Q_v \right) \right) \left( P^{-0.589} \right) \quad \text{(Eq. 4.1)}
\]

where,

- \( V_{\text{phit}} \) = Profile tolerance
- \( N_p \) = Number of teeth
- \( Q_v \) = Gear quality number
- \( P \) = Normal diametral pitch
- \( \psi \) = Helix angle
A linear region equal to the AGMA tolerance for the prescribed quality level will be used on either side of the optimum profile modification. For each quality level studied, four profile cases were run. Their dynamic factor results are compared with the dynamic factor resulted by the optimum profile modification gear. The dynamic factors from the perfect involute (no profile modification) are also included in each figures. Since the AGMA dynamic factors are far off resonant conditions, all of the output graphs will show dynamic factors up to a running speed equivalent to a mesh frequency of 2500 Hz (the operating speed of 5400 rpm).

4.3 Quality 14 Gears

A Quality 14 gear pair is considered by AGMA to be a very accurate gearing mentioned before in Chapter 2 (see Figure 2.2). The AGMA dynamic factor values lie between 1.00 and 1.11. Using the AGMA profile tolerance formulation (Equation 4.1) for this quality, one can calculate that the amount of tooth profile tolerances are ± 0.00014 inches.

4.3.1 K-Chart of Quality 14 Gears

Figure 4.3 shows the K-Chart of the AGMA Quality 14 gear. Case 1 (T1401) uses a profile whose pinion and gear tip reliefs are 0.00014 inches greater than those of the optimal modification and have starting roll angles that begin closer to the center of the
tooth than the specified values. Case 2 (T1402) uses a profile whose pinion and gear tip reliefs are 0.00014 inches less than perfect and have starting roll angles that are farther from the tooth center than those of the optimal modification. Cases 3 (T1403) and 4 (T1404) use combinations of the extreme amplitudes and starting roll angles in order to provide two additional scenarios. It is obvious that there are an infinite number of possible profiles, but it is felt that these extreme cases provide a good envelope of the worst case possibilities.

Figure 4.3  K-Chart of Spur Gear for AGMA Class 14.
4.3.2 Dynamic Factors Comparison of Quality 14 Gears

Figure 4.4 shows the dynamic load factor prediction for the profiles given in the K-Chart of Figure 4.3. For the conditions of the simulation, the unmodified profile modification result (perfect involute) is included as is the current AGMA dynamic factor. As expected, each of the gear pairs with profile errors has a dynamic load factor that is greater than the optimum modification, but each is lower than the unmodified gear pair's dynamic load factors.

![Graph showing dynamic factor comparison](image)

**Figure 4.4** Comparison of Dynamic Load Factors for the Class 14 Profiles.
The gears with the lower magnitude profile modification (Cases T1402 and T1403) seem to yield higher dynamic factors than the gears whose profiles have larger amplitude modifications (Cases T1401 and T1404). One conclusion of these results might be that if one is to err in the relief amplitudes, it is safer to over-modify than to under-modify the profiles. Over most of the operating speed range, the predicted dynamic load factors exceed the recommended AGMA dynamic load factor line for a Quality 14 gear. The only exception to this is the optimized profile modification case.

![Graph showing dynamic factor vs mesh frequency and pinion speed for various cases.]

**Figure 4.5** Comparison of Dynamic Tooth Force Factors for the Class 14 Profiles.
Figure 4.5 shows the dynamic tooth force factor for Quality 14 gear pairs. The dynamic tooth force factor is quite close to the dynamic load factors, except between 1680 and 2100 Hz (3600 and 4500 rpm). In these operating speeds, the dynamic tooth force factor for each case is lower than the respective dynamic load factors. Again, the gears with the lower magnitude profile modification (Cases T1402 and T1403) seem to yield higher dynamic factors than the gears whose profiles have larger amplitude modifications (Cases T1401 and T1404).

Figures 4.6 and 4.7 show the dynamic bending moment factors for pinion and gear, respectively for Quality 14 gears.

![Graph showing dynamic bending moment factors for pinion and gear with different cases labeled as O, Optimal, U, Unmodified, A, AGMA, 1. T1401, 2. T1402, 3. T1403, 4. T1404.](image)

**Figure 4.6** Comparison of Dynamic Bending Moment Factors for the Class 14 Profiles (Pinion).
Figure 4.7  Comparison of Dynamic Bending Moment Factors for the Class 14 Profiles (Gear).

At Quality 14 gear, the dynamic bending moment factors for the pinion are consistently beneath the AGMA dynamic factor values for all cases run. However, it is not the case for the dynamic bending moment factors for the gear. The gear pair with higher amplitude profile modifications (Cases T1401 and T1404) produce dynamic bending moment factors that are lower than the AGMA dynamic factor values. Again, one conclusion of these results might be that if one is to err in the relief amplitudes, it is safer to over-modify than to under-modify the tooth profiles.
4.4 Quality 12 Gears

A Quality 12 gear pair is also considered by AGMA to be a very accurate gearing mentioned before in Chapter 2 (see Figure 2.2). The AGMA dynamic factor values lie between 1.00 and 1.11. Using the AGMA profile tolerance formulation (Equation 4.1) for this quality, one can calculate that the amount of profile tolerances are ± 0.000275 inches.

4.4.1 K-Chart of Quality 12 Gears

![Graph showing K-Chart of Spur Gear for AGMA Class 12.](image)

Figure 4.8 K-Chart of Spur Gear for AGMA Class 12.
Figure 4.8 shows the K-Chart of the AGMA Quality 12 gear. The AGMA tolerance for Quality 12 gears of the geometry studied here is ± 0.000275 inches. Case 1 (T1201) uses tooth profiles whose pinion and gear tip reliefs are 0.000275 inches greater than those of the optimal modification and have starting roll angles that begin at pitch point of the tooth. Case 2 (T1202) uses a profile whose pinion and gear tip reliefs are 0.000275 inches less than optimum modification and have starting roll angles that are farther from the tooth center than those of the optimal modification.

Cases 3 (T1203) and 4 (T1204) use combinations of the extreme amplitudes and starting roll angles in order to provide two additional scenarios. Case 3 (T1203) has starting roll angle at the pitch point of the tooth and the amount of modification is 0.000275 less than the optimal modification. Case 4 (T1204) is the vice versa of the case 3 (T1203), where the starting roll angle is now farther than the pitch point of the gear and the amount of modification is 0.000275 greater than the optimal modification. It is obvious that there are an infinite number of possible profiles, but it is felt that these extreme cases provide a good envelope of the worst case possibilities.

4.4.2 Dynamic Factors Comparison of Quality 12 Gears

Figure 4.9 shows the predicted dynamic load factors for the Quality 12 gears, which the K-Chart has been shown in Figure 4.8. The predicted dynamic load factors look similar
in shape to those of the Quality 14 gears except in most instances the dynamic factors for
the extreme cases are significantly higher than those of the Quality 14 gears.

A notable exception is case 4 (T1204), in which the dynamic load factors are often less
than the optimum profile modification case and are usually considerably lower than the
AGMA dynamic factor. In this case, the profile modification starts farther from the pitch
point, but has a larger amplitude of modification than the optimal modification. A study of
the Fourier series harmonic amplitudes of transmission error for case 4 (T1204) indicates
that most of the harmonics of transmission error are about the same or less than the
optimum profile modification case. Table 4.1 shows the Fourier series harmonic
amplitudes and the peak to peak transmission errors for the case 4 (T1204) Quality 12
gear and for the optimum modified spur gears.

![Graph](image)

**Figure 4.9** Comparison of Dynamic Load Factors for the Class 12 Profiles.
Table 4.1  The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Optimum Modified Spur Gears and the Case 4 (T1204) Quality 12 Gear.

<table>
<thead>
<tr>
<th>Harmonic #</th>
<th>Optimum Modified Spur Gear</th>
<th>CASE 4: T1204 Quality 12 Gear</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude (μ in)</td>
<td>Amplitude (μ in)</td>
</tr>
<tr>
<td>1</td>
<td>10.281</td>
<td>59.843</td>
</tr>
<tr>
<td>2</td>
<td>14.632</td>
<td>7.104</td>
</tr>
<tr>
<td>3</td>
<td>12.072</td>
<td>28.664</td>
</tr>
<tr>
<td>4</td>
<td>12.447</td>
<td>8.697</td>
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<tr>
<td>6</td>
<td>10.325</td>
<td>6.008</td>
</tr>
<tr>
<td>PPTE</td>
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<td>153.53</td>
</tr>
</tbody>
</table>

Figure 4.10 shows the dynamic tooth force factor for Quality 12 gear pairs. The dynamic tooth force factor is quite close to the dynamic load factor, except between 1680 and 2100 Hz. (3600 and 4500 rpm). At these operating speeds, the dynamic tooth force factor for each case is lower than the respective dynamic load factor. The gears with the higher magnitude profile modification (Cases T1201 and T1204) seem to yield to lower dynamic tooth force factor than the gears whose tooth profiles have smaller amplitude modifications. Thus, as has been concluded in previous section for Quality 14 gear, if one to err in the relief amplitudes, it is safer to over-modify than to under-modified the tooth profiles.
**Figure 4.10** Comparison of Dynamic Tooth Force Factors for the Class 12 Profiles.

Figures 4.11 and 4.12 show the dynamic bending moment factors for pinion and gear for Quality 12 gears, respectively. As with the Quality 14 gears, the dynamic bending moment factor for the pinion for Quality 12 gear are consistently beneath the AGMA value for all cases run, except for speeds between 4600 and 5000 rpm (Cases T1202 and T1203). However, it is not the case in the dynamic bending moment factor for the gear. The gears with higher amplitude profile modifications (Cases T1201 and T1204) produce dynamic bending moment factors that are lower than the AGMA dynamic factor values.
Figure 4.11  Comparison of Dynamic Bending Moment Factors for the Class 12 Profiles (Pinion).

Figure 4.12  Comparison of Dynamic Bending Moment Factors for the Class 12 Profiles (Gear).
4.5 **Quality 10 Gears**

The AGMA dynamic factor for Quality 10 gears lie between 1.00 and 1.2 for operating speeds up to 5360 rpm. The dynamic factor curve and its governing equation are shown in Figure 2.2 and Equation 2.7, respectively in Chapter 2. Using the AGMA profile tolerance formulation, Equation 4.1, with the Quality 10 gear pair, one can find that the amount of profile tolerances are $\pm 0.000538$ inches. This range of tooth profile modification is about twice larger than that used in Quality 12 gears, and is four times larger than that used in Quality 14 gears.

4.5.1 **K-Chart of Quality 10 Gears**

Figure 4.13 shows the K-Chart of the AGMA Quality 10 gear. The AGMA tolerance for Quality 10 gears of the geometry studied here is $\pm 0.000538$ inches. Case 1 (T1001) uses a profile whose pinion and gear tip reliefs are 0.000538 inches greater than those of the optimal modification and have starting roll angles that begin at pitch point of the tooth. Case 2 (T1002) uses a profile whose pinion and gear tip reliefs are 0.000538 inches less than optimum modification and have starting roll angles that are farther from the tooth center than those of the optimal modification.

Cases 3 (T1003) and 4 (T1004) use combinations of the extreme amplitudes and starting roll angles in order to provide two additional scenarios. Case 3 (T1003) has starting roll
angle at the highest point of single tooth in contact (23.18 degrees), as used in optimal modification gears, and the amount of modification is 0.000538 less than the optimal modification gear. Case 4 (T1004) has starting roll angle as used in case 2 (T10102), and the amount of modification is 0.000538 greater than used in the optimal modification gear. Again, it is obvious that there are an infinite number of possible profiles, but it is felt that these extreme cases provide a good envelope of the worst case possibilities.

Figure 4.13 K-Chart of Spur Gear for AGMA Class 10.
4.5.2 **Dynamic Factors Comparison of Quality 10 Gears**

Figure 4.14 shows the predicted dynamic load factor for the Quality 10 gear, for which the K-Chart has been shown in Figure 4.13. The dynamic load factor results exhibit similar trends to the earlier data and the dynamic load factors do seem to be higher for the yet larger manufacturing deviations from the optimal profile modification case. One interesting operating speed is at 3200 rpm, where the case 4 (T1004), dynamic load factor gets very large. This will be discussed in more detail in section 4.6.

![Graph showing dynamic factors comparison](image)

**Figure 4.14** Comparison of Dynamic Load Factors for the Class 10 Profiles.
Figure 4.15 shows the dynamic tooth force factors for Quality 10 gear set. A quite similar trends have been observed on this dynamic factor as what has been mentioned in previous section. The dynamic tooth force factor are quite close to the dynamic load factors, except between the 1680 and 2100 Hz. (3600 and 4500 rpm). In these operating speeds, the dynamic tooth force factors for each case are lower that the respective dynamic load factors. The gears with the higher magnitude profile modification (Cases T1001 and T1004) seem to yield to lower dynamic tooth force factors than the gears which have smaller amplitude modifications.

![Figure 4.15 Comparison of Dynamic Tooth Force Factors for the Class 10 Profiles.](image)
One interesting to be noted is the dynamic tooth force factor at operating speed of 3200 rpm. Most of the cases at this speed yield very high dynamic tooth force factors. There are 1.24, 1.27, 1.30, and 1.48 for cases T1001, T1002, T1003, and T1004, respectively. Meanwhile, the dynamic tooth force factor of the optimal modification, AGMA standard, and perfect involute gears are 1.11, 1.18, and 1.43, respectively. Case T1004, which has a tooth modification of 0.000578 larger than the optimum, and a starting roll angle at the pitch point of the gear, generate a dynamic tooth factor that even exceeds the dynamic tooth factor predicted by perfect involute gear. This case will be studied more extensive in the following section.

Figure 4.16 Comparison of Dynamic Bending Moment Factors for the Class 10 Profiles (Pinion).
Figures 4.16 and 4.17 show the dynamic bending moment factors for pinion and gear, respectively, for the Quality 10 gear pair. The dynamic bending moment factors for the pinion for most cases are consistently beneath the AGMA value. However, it is not the case for the dynamic bending moment factors for the gear. The gears with higher amplitude profile modifications (Cases T1201 and T1204) produce dynamic bending moment factors that are lower than the AGMA value, except at speeds between 3000 - 3300 rpm.
4.6 Other Dynamic Analysis Comparison

In order to investigate the higher value of dynamic factors predicted in the Quality 10 gear pair for case T1004 in more detail, a case study of the model's time domain results is presented in this section. Dynamic mesh loads, dynamic tooth forces, static bending moments, and dynamic bending moments are presented versus time, as shown in Figures 4.18 - 4.23. The comparison will be made with the dynamic analysis predicted by optimum profile modification gear, perfect involute gear, and case T1004 gears. Case T1004 has a starting roll angle at the pitch point of the gear and a tooth modification of 0.00149 inches on tip reliefs of the pinion and gear.

![Graph](image)

**Figure 4.18** Dynamic Mesh and Tooth Forces Trace of the Perfect Involute Spur Gear at Pinion Speed of 3214 RPM.
Figure 4.19  Static and Dynamic Moment Trace of the Perfect Involute Spur Gear at Pinion Speed of 3214 RPM.

Figure 4.20  Dynamic Mesh and Tooth Forces Trace of the Case 4 (T1004) of AGMA Class 10 Spur Gear at Pinion Speed of 3214 RPM.
Figure 4.21 Static and Dynamic Moment Trace of the Case 4 (T1004) of AGMA Class 10 Spur Gear at Pinion Speed of 3214 RPM.

Figure 4.22 Dynamic Mesh and Tooth Forces Trace of the Optimum Modified Spur Gear at Pinion Speed of 3214 RPM.
The perfect involute bending moment results of Figure 4.19 are similar to many of the perfect involute strain gage measurement results reported in the literature. The dynamic load and dynamic tooth force traces of Figure 4.18 clearly show the regions of single and double tooth pair contact. The dynamic load factor uses the ratio of peak load to the average load ratio whereas the dynamic tooth force factor uses the peak dynamic force divided by the peak static force (not shown). The ratio of the peak bending moment to the peak static bending moment shown in Figure 4.19 is used to compute dynamic bending moment factors.
The dynamic loads shown in Figure 4.21 for the erroneously modified gear (Case T1004) show a clear oscillation at the system natural frequency of 3000 Hz. In this case, the second harmonic of mesh frequency (see Table 4.2) is exciting the system natural frequency around 3000 Hz. The second harmonic of transmission error is particularly large in this instance and therefore yields the excessively high dynamic factors that are exhibited.

Figures 22 and 23 indicate how the optimum profiles significantly reduce the dynamic responses of the terms used in computing each type of dynamic factor.

<table>
<thead>
<tr>
<th>Table 4.2</th>
<th>The Fourier Series of Transmission Errors and the Peak to Peak Transmission Errors for the Case 4 (T1004) Quality 10 Gear.</th>
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<td>CASE 4: T1004</td>
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<td>Spur Gear</td>
</tr>
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</tr>
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</tr>
<tr>
<td>2</td>
<td>101.490</td>
</tr>
<tr>
<td>3</td>
<td>6.465</td>
</tr>
<tr>
<td>4</td>
<td>1.613</td>
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</table>
4.7 Summary

Chapter 4 has covered the effects of profile manufacturing tolerances on predicting the three dynamic factors - dynamic load factor, dynamic tooth force factor, and dynamic bending moment factor. Profile tolerances for Quality 14 gear, Quality 12 gear, and Quality 10 gear have been used, and compared with the dynamic factor predicted using the optimum modified, perfect involute gears. The AGMA dynamic factor values are also included.

The dynamic factor predicted by the optimum modified gear is always below the AGMA dynamic factor value. The perfect involute gear usually has the highest dynamic factor values among all case run. For most cases, the predicted dynamic factors of each run according to the Quality gear, lies between the optimum modified and perfect involute gear pair’s dynamic factors.

If one is to err in the relief amplitudes, for most cases, it is better to over-modify the tooth profile modification than to under-modify the profiles, especially in the very accurate gearing (Quality 12 and Quality 14 gears). The over-modify tooth profile gear usually lower the dynamic factor.
The computer packages, DYTEM, GRD, and MATLAB programs, have been studied in great details in this thesis. Those computer software packages are developed at the Gear Dynamic and Gear Noise Research Laboratory. A fifth order Runge-Kutta technique has been implemented to the DYTEM program. Thus, the program can be used for better simulation solving and used for running at low operating speed. Experimental data, which was provided by NASA, has been compared to the theoretical prediction. Several dynamic analysis were carried out to study the dynamic behavior of gear pairs in mesh. The effects of tooth profile tolerances on predicting dynamic factors have been established. The dynamic factors are compared based on the AGMA Quality number, for the different cases of tooth profile. Based in this work, several conclusions and recommendations can be made.
5.1 Conclusions

This exercise in dynamic analysis using computer packages has pointed out several conclusions, the following:

1. Different definitions of dynamic factors will give different values of dynamic factors, and all these types should be related to the stress value that is computed from each definition.

2. Damping factors can significantly affect the predicted dynamic analysis of the gear pairs in mesh.

3. Experimental strain gage data shows the agreement with the theoretical prediction simulated from DYTEM, GRD, and MATLAB programs. These three programs compare quite well with one another in predicting the natural frequency and dynamic load factor. Also, the DYTEM program compares quite favorably with the theoretical data provided by NASA.

4. Deviations from optimal profile modifications yield dynamic factors that often exceed the AGMA recommendations for non-optimal modification gear.
5. In order to achieve low dynamic factors, it is necessary to provide profile modifications that minimize gear transmission error.

6. If one is to err in choosing profile modifications, it is better to provide excessive modification rather than too little modification. For several cases, too little modification will produce higher dynamic factors.

5.2 Recommendations

Throughout this research, several interesting ideas were developed. However, due to the time limitations, not all of them can be implemented. Thus, several recommendations for future research are listed as following:

1. Extend the ability of DYTEM program, so that this computer program that can be used to solve the dynamic root stress factor and dynamic contact stress factor.

2. Extend the ability of DYTEM program, so that it can be used for solving dynamic analysis of helical gear pairs.
3. A more detailed experimental study for the factors that effect the various damping of the gear pairs system. Thus, the theoretical predictions of dynamic analysis, which are simulated in Chapter 3 and Appendix D, can be compared.

4. Performed an extended study on the dynamic factor predictions, based on different types of gear modifications, such as spacing errors, lead modification, and parabolic profile modification.

5. Since the HGRD/GRD program use two different units in the input files (one is Metric unit and another one is English unit), it is suggested to modify the input file that uses Metric unit to an input that uses English unit.
REFERENCES


22. GGRAPH Manual, A program to Plot the Results of the Load Distribution Program, Gear Dynamic and Gear Noise Research Laboratory, The Ohio State University, Columbus, Ohio.


24. MULTILDP Version 8.22, Computer Program, Gear Dynamics and Gear Noise Research Laboratory, The Ohio State University, Columbus, Ohio, March 1993.

APPENDIX A

SAMPLE RUN OF MATLAB PROGRAM
A.1 Sample Input Data

Table A.1 Input Data for MATLAB Program.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<td>Transmission error</td>
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<td>in</td>
</tr>
<tr>
<td>Pinion diameter</td>
<td>3.289</td>
<td>in</td>
</tr>
<tr>
<td>Pinion face width</td>
<td>0.25</td>
<td>in</td>
</tr>
<tr>
<td>Gear diameter</td>
<td>3.289</td>
<td>in</td>
</tr>
<tr>
<td>Gear face width</td>
<td>0.25</td>
<td>in</td>
</tr>
<tr>
<td>Mesh stiffness</td>
<td>707118.6</td>
<td>lb/in</td>
</tr>
<tr>
<td>Pinion bearing stiffness</td>
<td>15000</td>
<td>lb/in</td>
</tr>
<tr>
<td>Gear bearing stiffness</td>
<td>15000</td>
<td>lb/in</td>
</tr>
<tr>
<td>Pinion torsional shaft stiffness</td>
<td>3000</td>
<td>in-lb/rad</td>
</tr>
<tr>
<td>Gear torsional shaft stiffness</td>
<td>110000</td>
<td>in-lb/rad</td>
</tr>
<tr>
<td>Driver inertia</td>
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<td>in-lb-s^2</td>
</tr>
<tr>
<td>Pinion inertia</td>
<td>12.189 E-3</td>
<td>in-lb-s^2</td>
</tr>
<tr>
<td>Gear inertia</td>
<td>12.189 E-3</td>
<td>in-lb-s^2</td>
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<tr>
<td>Load inertia</td>
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<tr>
<td>Damping ratio</td>
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<td></td>
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</table>
### A.2 Sample Output Data

\[ M = \begin{align*}
2.1000e+00 & & 0 & & 0 & & 0 & & 0 & & 0 \\
0 & & 1.2189e-02 & & 0 & & 0 & & 0 & & 0 \\
0 & & 0 & & 1.2189e-02 & & 0 & & 0 & & 0 \\
0 & & 0 & & 0 & & 7.8000e+00 & & 0 & & 0 \\
0 & & 0 & & 0 & & 0 & & 9.3636e-03 & & 0 \\
0 & & 0 & & 0 & & 0 & & 0 & & 9.3636e-03
\end{align*} \]

\[ K = \begin{align*}
3.0000e+03 & & -3.0000e+03 & & 0 & & 0 & & 0 & & 0 & & 0 \\
-3.0000e+03 & & 1.8153e+06 & & -1.9123e+06 & & 0 & & 1.1629e+06 & & -1.1629e+06 \\
0 & & -1.9123e+06 & & 2.0223e+06 & & -1.1000e+05 & & -1.1629e+06 & & 1.1629e+06 \\
0 & & 0 & & -1.1000e+05 & & 1.1000e+05 & & 0 & & 0 & & 0 \\
0 & & 1.1629e+06 & & -1.1629e+06 & & 0 & & 7.2212e+05 & & -7.0712e+05 \\
0 & & -1.1629e+06 & & 1.1629e+06 & & 0 & & -7.0712e+05 & & 7.2212e+05
\end{align*} \]

\[ P1 = 2.8192e+03 \]

\[ P6 = \]
- Columns 1 through 3
  - 3.4449e+03
  - 0
  - 0
  - 0
- Columns 4 through 6
  - 0
  - 0
  - 0
  - 0
  - 6.2461e+00
  - 0
  - 0
  - 0
  - 0
  - 0
  - 0
  - 0
  - 0

\[ v = \begin{align*}
\end{align*} \]

\[ v x = \begin{align*}
-2.7608e-06 & & 1.3251e-04 & & -1.7725e-03 & & -1.5203e+00 & & 8.2223e-01 & & -4.3331e-21 \\
\end{align*} \]

\[ mesh = \begin{align*}
-2.7922e+00 & & 2.2522e-02 & & -4.7343e-03 & & 2.5702e-03 & & -4.1815e-15 & & 0
\end{align*} \]
A.3 Graphical Results

Fig. A.1 Angular Displacement of Driver

Fig. A.2 Angular Displacement of Load

Fig. A.3 Angular Displacement of Pinion

Fig. A.4 Angular Displacement of Gear
Figures A.9 Mode Shapes
A.4 MATLAB Program

This macro solves for the eigenvalues and eigenvectors of a 6 dof undamped torsional system. Inputs are such that radii and lengths of gears and disks are entered. The first 4 terms are torsional inertias, the second four are translation of the gear elements. Shafts with diameters and lengths are defined between disks 1 and 2 and between 3 and 4, respectively. Units are English.

Modification has made on October 1993, only on format of the output graphs and adding the input comments, JH
Next mod: March 1993 for the input only to follow the dytem, JH

clear

%INPUT to the system:
e=0.00123; % Transmission Error (in)
d1 = 7.35; % Driver Diameter (in)
l1 = 10; % Driver Length (in)
d2 = 3.289; % Pinion Diameter (in)
l2 = 0.25; % Pinion Face Width (in)
d3 = 3.289; % Gear Diameter (in)
l3 = 0.25; % Gear Face Width (in)
d4 = 9.05; % Load Diameter (in)
l4 = 16.5; % Load Length (in)
ds1 = 1.125; % Pinion Shaft Diameter (in)
l1s1=10; % Pinion Shaft Length (in)
ds2 = 1.1875; % Gear Shaft Diameter (in)
l1s2=10; % Gear Shaft Length (in)

km=707118.6 % Mesh Stiffness (lbs/in)
k2=150000; % Pinion Bearing Stiffness (lbs/in)
k3=150000; % Gear Bearing Stiffness (lbs/in)

%Calculation for torsional stiffness:
kt1=(pi*d1^4)*12e6/(32*l1s1); % Pinion Torsional Shaft Stiffness (in-lb/rad)
kt2=(pi*d2^4)*12e6/(32*l1s2); % Gear Torsional Shaft Stiffness (in-lb/rad)
kt1=3000;
kt2=110000;

%Inertia of Driver, Pinion, Gear, and Load:
j1=.00115*(d1/2)^4*11; % Driver Inertia (in-lb-s^2)
j2=.00115*(d2/2)^4*12; % Pinion Inertia (in-lb-s^2)
j3=.00115*(d3/2)^4*13; % Gear Inertia (in-lb-s^2)
j4=.00115*(d4/2)^4*14; % Load Inertia (in-lb-s^2)
j1=2.1;
j2=12.189e-3;
j3=12.189e-3;
j4=7.8;

%dd=j2*(0.00115* % Radius of the driver, pinion, gear, and load
r1=d1/2; % Driver Radius (in)
\[ r_2 = \frac{d_2}{2}; \quad \text{Pinion Radius (in)} \]
\[ r_3 = \frac{d_3}{2}; \quad \text{Gear Radius (in)} \]
\[ r_4 = \frac{d_4}{2}; \quad \text{Load Radius (in)} \]

\[ f_{t1} = \left( k_m \left( r_2^2 r_2 + r_3^2 r_2 + r_3^2 + j_2 j_3 \right) \right)^{-0.5}; \]

\[ \hat{m}_2 = 0.0023 r_2^2 12; \]
\[ \hat{m}_3 = 0.0023 r_3^2 13; \]
\[ m_2 = 9.3636e-3 \quad \% 3.1212e-3; \]
\[ m_3 = 9.3636e-3 \quad \% 3.1212e-3; \]

\[ M = [j_1 0 0 0 0 0; j_2 0 0 0 0 0; j_3 0 0 0 0 0; j_4 0 0 0 0 0; m_2 0 0 0 0 0; m_3 0 0 0 0 0] \]

\[ k_{11} = k_{t1}; \]
\[ k_{12} = -k_{t1}; \]
\[ k_{22} = k_m r_2^2 + k_{t1}; \]
\[ k_{23} = -k_m r_2 r_3; \]
\[ k_{24} = 0; \]
\[ k_{25} = k_m r_2; \]
\[ k_{26} = -k_m r_3; \]
\[ k_{33} = k_m r_3^2 + k_{t2}; \]
\[ k_{34} = -k_{t2}; \]
\[ k_{35} = -k_m r_3; \]
\[ k_{36} = k_m r_3; \]
\[ k_{44} = k_{t2}; \]
\[ k_{55} = k_2 + k_m; \]
\[ k_{56} = -k_m; \]
\[ k_{66} = k_m + k_3; \]

\[ K = [k_{11} k_{12} 0 0 0 0; k_{12} k_{22} k_{23} k_{24} k_{25} k_{26}; 0 k_{23} k_{33} k_{34} k_{35} k_{36}; 0 0 k_{34} k_{44} 0 0; 0 k_{25} k_{35} 0 k_{55} k_{56}; 0 k_{26} k_{36} 0 k_{56} k_{66}] \]

\[ \text{mik} = \text{inv}(M) * K; \]

\[ [v, d] = \text{eig}(\text{mik}); \]

\[ k_{t1}; \]
\[ k_{t2}; \]

\[ F_1 = \left( \left( k_m \left( j_2 r_3^2 + j_3 r_2^2 \right) \right)^0.5 \right) / (2 \pi); \]

\[ w = d^0.5; \]
\[ F_6 = (d^0.5) / (2 \pi) \]

\[ v \]
\[ v(1,:) = v(1,:) * r_2; \]
\[ v(2,:) = v(2,:) * r_2; \]
\[ v(3,:) = v(3,:) * r_3; \]
\[ v(4,:) = v(4,:) * r_3; \]
\[ v(5,:) = v(5,:); \]
\[ v(6,:) = v(6,:); \]

\[ v \]

\[ \text{mesh} = v(3,:) + v(6,:) - v(2,:) - v(5,:); \]

\[ \text{mi} = v^* M v; \]
\[ k = v^* K v; \]
\[ f_c 2 = k_m r_2^2 e; \]
\[ f_c 3 = -k_m r_3^2 e; \]
\[ f_c 5 = k_m e; \]
\[ f_c 6 = -k_m e; \]
\[ f_c = [0 f_c 2 f_c 3 0 f_c 5 f_c 6]'; \]

\[ v_{tf} = v'^* f_c; \]
\[ z_1 = .06; \]
\[ z_2 = .06; \]
\[ z_3 = .06; \]
\[ z_4 = .06; \]
z5=.06;
z6=.06;
ql=zeros(1,100);
q2=zeros(1,100);
q3=zeros(1,100);
q4=zeros(1,100);
q5=zeros(1,100);
q6=zeros(1,100);
phil=zeros(1,100);
phi2=zeros(1,100);
phi3=zeros(1,100);
phi4=zeros(1,100);
phi5=zeros(1,100);
phi6=zeros(1,100);
nf = 200;
fmin = 50.0;
fmax = 50000;
delflog = log(fmax-fmin)/nf;
figure(1)
cla

text(0.1,0.5,'PLEASE WAIT CALCULATION IN  PROGRESS');pause(3)

for i = 1:nf+1;
  f(i,1) = fmin+exp((i-1)*delflog);
  Rf1 = f(i,1)/abs(F6(1,1));
  Rf2 = f(i,1)/abs(F6(2,2));
  Rf3 = f(i,1)/abs(F6(3,3));
  Rf4 = f(i,1)/abs(F6(4,4));
  Rf5 = f(i,1)/abs(F6(5,5));
  Rf6 = f(i,1)/abs(F6(6,6));
  q1(i)=abs(vtf(1))/ki(1,1)/((1-Rf1^2)^2+(2*z1*Rf1)^2)^.5;
  phi1(i)=-atan2(2*z1*Rf1*vtf(1), (1-Rf1^2)*vtf(1));
  q2(i)=abs(vtf(2))/ki(2,2)/((1-Rf2^2)^2+(2*z2*Rf2)^2)^.5;
  phi2(i)=-atan2(2*z2*Rf2*vtf(2), (1-Rf2^2)*vtf(2));
  q3(i)=abs(vtf(3))/ki(3,3)/((1-Rf3^2)^2+(2*z3*Rf3)^2)^.5;
  phi3(i)=-atan2(2*z3*Rf3*vtf(3), (1-Rf3^2)*vtf(3));
  q4(i)=abs(vtf(4))/ki(4,4)/((1-Rf4^2)^2+(2*z4*Rf4)^2)^.5;
  phi4(i)=-atan2(2*z4*Rf4*vtf(4), (1-Rf4^2)*vtf(4));
  q5(i)=abs(vtf(5))/ki(5,5)/((1-Rf5^2)^2+(2*z5*Rf5)^2)^.5;
  phi5(i)=-atan2(2*z5*Rf5*vtf(5), (1-Rf5^2)*vtf(5));
  q6(i)=abs(vtf(6))/ki(6,6)/((1-Rf6^2)^2+(2*z6*Rf6)^2)^.5;
  phi6(i)=-atan2(2*z6*Rf6*vtf(6), (1-Rf6^2)*vtf(6));
  q1c(i)=q1(i)*exp(j*phi1(i));
  q2c(i)=q2(i)*exp(j*phi2(i));
  q3c(i)=q3(i)*exp(j*phi3(i));
  q4c(i)=q4(i)*exp(j*phi4(i));
  q5c(i)=q5(i)*exp(j*phi5(i));
  q6c(i)=q6(i)*exp(j*phi6(i));
  qt(i,1)=q1c(i);
  qt(i,2)=q2c(i);
  qt(i,3)=q3c(i);
  qt(i,4)=q4c(i);
  qt(i,5)=q5c(i);
  qt(i,6)=q6c(i);
end

tx=(v*qt')

fm=km*(r3*xt(:,3)+xt(:,6)-z2*xt(:,2)-xt(:,5)+e);
dte=r3*xt(:,3)-r2*xt(:,2);
ffmag=abs(fm);
dtemag=abs(dte);
xmag = abs(xt);
xang = angle(xt);
xreal=real(xt);
ximag=imag(xt);

% Plotting the Results:
figure(2)
subplot(221)
    loglog(f,xmag(:,1))
    title('Fig. A.1 Angular Displacement of Driver')
    ylabel('Theta (rad)');xlabel('Frequency, Hz')
subplot(223)
    loglog(f,xmag(:,4))
    title('Fig. A.2 Angular Displacement of Load')
    ylabel('Theta (rad)');xlabel('Frequency, Hz')
subplot(222)
    loglog(f,xmag(:,2))
    title('Fig. A.3 Angular Displacement of Pinion')
    ylabel('Theta (rad)');xlabel('Frequency, Hz')
subplot(224)
    loglog(f,xmag(:,3))
    title('Fig. A.4 Angular Displacement of Gear')
    ylabel('Theta (rad)');xlabel('Frequency, Hz')
orient landscape
print h1

figure(3)
subplot(221)
    loglog(f,xmag(:,5))
    title('Fig. A.4 Translational Displacement of Pinion')
    ylabel('x (in)');xlabel('Frequency, Hz')
subplot(222)
    loglog(f,xmag(:,6))
    title('Fig. A.5 Translational Displacement of Gear')
    ylabel('x (in)');xlabel('Frequency, Hz')
subplot(223)
    loglog(f,ffmag)
    title('Fig. A.6 Mesh Force')
    ylabel('Mesh Force (lbf)');xlabel('Frequency, Hz')
subplot(224)
    loglog(f,dtemag)
    title('Fig. A.7 Dynamics Transmission Error')
    ylabel('Dynamics T.E. (in)');xlabel('Frequency, Hz')
orient landscape
print h2

figure(4)
subplot(231):plot(vx(:,1));title('Fig. A.9.1 Mode Shape, f6')
subplot(232):plot(vx(:,2));title('Fig. A.9.2 Mode Shape, f5')
subplot(233):plot(vx(:,3));title('Fig. A.9.3 Mode Shape, f4')
subplot(234):plot(vx(:,4));title('Fig. A.9.4 Mode Shape, f3')
subplot(235):plot(vx(:,5));title('Fig. A.9.5 Mode Shape, f1')
subplot(236):plot(vx(:,6));title('Fig. A.9.6 Mode Shape, f2')
orient landscape
print h3
APPENDIX B

SAMPLE RUN OF DYTEM PROGRAM
### B.1 Sample Input Data from LDP

<table>
<thead>
<tr>
<th>DYNAMIC ANALYSIS PROGRAM INFORMATION</th>
<th>MAXIMUM NUMBER OF TEETH IN CONTACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUMBER OF POSITIONS</td>
<td>LOAD SHARING</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>MESH STIFFNESS (lb/in)</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.544242E+06</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.55066E+06</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.55511E+06</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.56043E+06</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.56047E+06</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.55899E+06</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.55511E+06</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.55094E+06</td>
<td>0.1305</td>
</tr>
<tr>
<td>0.54460E+06</td>
<td>0.1773</td>
</tr>
<tr>
<td>0.85141E+06</td>
<td>0.2269</td>
</tr>
<tr>
<td>0.85930E+06</td>
<td>0.2787</td>
</tr>
<tr>
<td>0.86503E+06</td>
<td>0.3232</td>
</tr>
<tr>
<td>0.86544E+06</td>
<td>0.3871</td>
</tr>
<tr>
<td>0.87198E+06</td>
<td>0.4426</td>
</tr>
<tr>
<td>0.87331E+06</td>
<td>0.4986</td>
</tr>
<tr>
<td>0.87387E+06</td>
<td>0.5546</td>
</tr>
<tr>
<td>0.87402E+06</td>
<td>0.6102</td>
</tr>
<tr>
<td>0.87389E+06</td>
<td>0.6650</td>
</tr>
<tr>
<td>0.87335E+06</td>
<td>0.7187</td>
</tr>
<tr>
<td>0.87208E+06</td>
<td>0.7706</td>
</tr>
<tr>
<td>0.86961E+06</td>
<td>0.8203</td>
</tr>
<tr>
<td>0.86555E+06</td>
<td>0.8673</td>
</tr>
<tr>
<td>0.85965E+06</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.85185E+06</td>
<td>1.0000</td>
</tr>
<tr>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STATIC TRANSMISSION ERROR (in micro-inches)</th>
<th>LOAD ARM FOR PINION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1260.349</td>
<td>0.000000</td>
</tr>
<tr>
<td>1245.669</td>
<td>0.000000</td>
</tr>
<tr>
<td>1234.797</td>
<td>0.000000</td>
</tr>
<tr>
<td>1227.583</td>
<td>0.000000</td>
</tr>
<tr>
<td>1223.949</td>
<td>0.000000</td>
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<tr>
<td>1223.861</td>
<td>0.000000</td>
</tr>
<tr>
<td>1227.318</td>
<td>0.000000</td>
</tr>
<tr>
<td>1234.351</td>
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</tr>
<tr>
<td>1245.038</td>
<td>0.000000</td>
</tr>
<tr>
<td>1259.524</td>
<td>0.000000</td>
</tr>
<tr>
<td>1174.833</td>
<td>0.000000</td>
</tr>
<tr>
<td>1195.354</td>
<td>0.000000</td>
</tr>
<tr>
<td>1213.511</td>
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</tr>
<tr>
<td>1229.141</td>
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</tr>
<tr>
<td>1241.974</td>
<td>0.000000</td>
</tr>
<tr>
<td>1251.626</td>
<td>0.000000</td>
</tr>
<tr>
<td>1257.681</td>
<td>0.000000</td>
</tr>
<tr>
<td>1259.809</td>
<td>0.000000</td>
</tr>
<tr>
<td>1257.883</td>
<td>0.000000</td>
</tr>
<tr>
<td>1252.017</td>
<td>0.000000</td>
</tr>
<tr>
<td>1242.535</td>
<td>0.000000</td>
</tr>
<tr>
<td>1229.851</td>
<td>0.000000</td>
</tr>
<tr>
<td>1214.352</td>
<td>0.000000</td>
</tr>
<tr>
<td>1196.316</td>
<td>0.000000</td>
</tr>
<tr>
<td>1178.911</td>
<td>0.000000</td>
</tr>
<tr>
<td>AVERAGE COMPLIANCE (in micro IN/Lb)</td>
<td>LOAD ARM FOR PINION</td>
</tr>
<tr>
<td>1.414189</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
0.000000 0.209183
0.000000 0.203182
0.000000 0.197283
0.000000 0.191486
0.079738 0.185793
0.082536 0.180205
0.085463 0.174721
0.098513 0.169345
0.051686 0.164076
0.094982 0.158915
0.098401 0.153864
0.101941 0.148923
0.105603 0.144094
0.109384 0.139377
0.113284 0.134773
0.117303 0.130283
0.121439 0.125908
0.125693 0.121649
0.130062 0.117507
0.134546 0.113482
0.139144 0.109575
0.143856 0.105788
0.148680 0.102121
0.153615 0.098575
0.158661 0.095150
0.163816 0.091848
0.169080 0.088669
0.174451 0.085613
0.179929 0.082681
0.185512 0.079875
0.191200
0.196991
0.202886
0.208881
0.214978
0.221174
0.227468
0.233860
0.240349
0.246933
0.253612
0.260383
0.267248
0.274204

LOAD ARM FOR GEAR
0.000000
0.000000
0.000000
0.000000
0.000000
0.000000
0.000000
0.000000
0.000000
0.274555
0.267593
0.260724
0.253947
0.247264
0.240675
0.234182
0.227785
0.221485
0.215285

ANGULAR POSITION OF PINION
POSITION THEtap
1. 0.000
2. 0.009
3. 0.018
4. 0.027
5. 0.036
6. 0.045
7. 0.054
8. 0.063
9. 0.072
10. 0.081
11. 0.090
12. 0.099
13. 0.108
14. 0.117
15. 0.126
16. 0.135
17. 0.144
18. 0.153
19. 0.162
20. 0.171
21. 0.180
22. 0.188
23. 0.197
24. 0.206
25. 0.215

θ CENTROID RADIUS AND LOADS FOR STRESS CALCULATIONS
2.53046, 1.95553, 30.86970,
10.83822, 91.05739,
2.16145, 2.32455, 18.01255,
23.69535, 594.88110,
B.2 Sample Input Data for DYTEM

**DYTEM - PROGRAM CONTROL AND GEOMETRY DATA**

A. CHANGE ALL DATA   
B. NUMBER OF TEETH ON PINION  28   
C. BASE RADIUS OF PINION (IN)  0.16445E+01  
D. BASE RADIUS OF GEAR (IN)  0.16445E+01  
E. NUMBER OF INTERPOLATION POINTS IN EACH DIVISION  3   
F. NUMBER OF MESH CYCLES OF ANALYSIS  1  
G. MESH NUMBER FOR WHICH DYNAMIC ANALYSIS WILL BE MADE  90   
H. TOLERANCE FOR INTEGRATION ROUTINE  0.10E+00

I. DYNAMIC ANALYSIS RESULTS FOR ONLY 90-TH MESH WILL BE GIVEN IN OUTPUT

J. APPROXIMATE METHOD WILL BE USED

K. STEADY STATE RESPONSE WILL BE CALCULATED

IF YOU WANT TO MAKE CHANGES, SELECT (A/B/C/D/E/F/G/H/I/J/K)  
ELSE, ENTER (N, DEF=N)

**DYTEM - DYNAMIC DATA: MASS AND INERTIA**

A. CHANGE ALL DATA  
B. DRIVE INERTIA (IN-LB-S**2)  0.21000E+01  
C. PINION INERTIA (IN-LB-S**2)  0.40630E-02  
D. GEAR INERTIA (IN-LB-S**2)  0.40630E-02  
E. LOAD INERTIA (IN-LB-S**2)  0.78000E+01  
F. PINION MASS (LB-S**2/IN)  0.31212E-02  
G. GEAR MASS (LB-S**2/IN)  0.31212E-02

IF YOU WANT TO MAKE CHANGES, SELECT (A/B/C/D/E/F/G)  
ELSE, ENTER (N, DEF=N)

**DYTEM - DYNAMIC DATA: STIFFNESS**

A. CHANGE ALL DATA  
B. TORSIONAL STIFFNESS OF PINION SHAFT (IN-LB/RAD)  0.30000E+04  
C. TORSIONAL STIFFNESS OF GEAR SHAFT (IN-LB/RAD)  0.11000E+06  
D. PINION BEARING STIFFNESS (LB/IN)  0.20000E+07  
E. GEAR BEARING STIFFNESS (LB/IN)  0.20000E+07

IF YOU WANT TO MAKE CHANGES, SELECT (A/B/C/D/E)  
ELSE, ENTER (N, DEF=N)

**DYTEM - DYNAMIC DATA: DAMPING**

A. CHANGE ALL DATA  
B. MESH DAMPING RATIO  0.0600  
C. DAMPING RATIO FOR SHAFTS  0.0600  
D. PINION BEARING DAMPING (LB-S/IN)  0.40000E+01  
E. GEAR BEARING DAMPING (LB-S/IN)  0.40000E+01
IF YOU WANT TO MAKE CHANGES, SELECT (A/B/C/D/E)  
ELSE, ENTER (N, DEF=N)

DYTEM - DYNAMIC DATA - LOADING

A. CHANGE ALL DATA
B. DRIVE TORQUE (IN-LB)  0.11280E+04
C. LOAD TORQUE (IN-LB)  0.11280E+04

IF YOU WANT TO MAKE CHANGES, SELECT (A/B/C)  
ELSE, ENTER (N, DEF=N)

DYTEM - KINEMATIC DATA

A. CHANGE ALL DATA
B. FIRST VALUE OF PINION SPEED FOR ANALYSIS (RPM)  4000.0
C. NUMBER OF SPEEDS OF ANALYSIS  1
D. INCREMENT IN SPEED (RPM)  1.0

IF YOU WANT TO MAKE CHANGES, SELECT (A/B/C/D)  
ELSE, ENTER (N, DEF=N)

DYTEM - GEAR ERROR DATA

A. CHANGE ALL DATA
B. BACKLASH (INCH)  0.00600
C. THERE ARE NO GEAR ERRORS WHICH CAUSE  
DIFFERENT STE IN EACH MESH CYCLE

IF YOU WANT TO MAKE CHANGES, SELECT (A/B/C)  
ELSE, ENTER (N, DEF=N)

DO YOU WANT TO STORE BEARING FORCES FOR ALL  
MESH CYCLES ANALYZED? (Y/N, DEF=N):

DO YOU WANT TO SAVE THIS DATA? (Y/N, DEF=N):

WOULD YOU LIKE TO SAVE THE DYNAMIC ANALYSIS (Y/N, DEF=N)

STAND BY - Working...  
It may take some time...

iend= 9200  
COMPUTATIONS ARE COMPLETED FOR SPEED= 4000.0 RPM
B.3 Graphical Results
B.4 Runge-Kutta Fifth-Fourth Order

```
subroutine dopri5 (n, fcn, x, y, xend, eps, hmax, h)

numerical solution of a system of first order ordinary
differential equations y' = f(x,y). this is an embedded
runge-kutta method of order 5(4) due to dormand and prince (1980)

input parameters

n  dimension of the system (i.e. 51)
fcn name (external) of the subroutine computing the first
derivative f(x,y):
  subroutine fcn(n,x,y,f)
  real*8 x,y(n),f(n)
  f(1)=...........

x  initial x-value
y(n) initial values for y
xend final x-value (xend-x could be + or -)
eps local tolerance
hmax maximum stepsize
h  initial stepsize guess

output parameters

y(n) output at xend

external subroutine (to be supplied by the user)

solous this subroutine is called after every successfully
computed step (and the initial value):
  subroutine solous(nr,x,y,n)
  real*8 x,y(n)

furnishes the solution y at the nr-th grid point x
(initial value is considered the first grid point). supply a dummy routine, if the solution is not desired
at the intermediated points

implicit real*8 (a - h, o - z)

parameter(np=50)
dimension y(np)
real*8 k1 (np), k2 (np), k3 (np), k4 (np), k5 (np), y1 (np),
& g6 (np), k6 (np)
```
logical reject
external fcn
common /stat/ nfcn, nstep, naccpt, nreject, alip
common /track/ndiv

common stat can be used for statistics
nfcn number of function evaluations
nstep number of computed steps
naccpt number of accepted steps
nreject number of rejected steps
data nmax/5000/, ound/5. d - 16/

nmax maximum number of steps
Rnd round smallest number satisfying 1. + round > 1
posneg = dsign (1.d0, xend - x)

hmax = dabs (hmax)
h = dmin1 (dmax1 (1.d - 4, dabs (h)), hmax)
h = dsign (h, posneg)
eps = dmax1 (eps, 7. * round)
reject = .false.
naccpt = 0
nreject = 0
nfcn = 1
nstep = 0
call solous (naccpt + 1, x, y, n)
call fcn (n, x, y, k1)

basic integration step
10 if ( (nstep .gt. nmax) .or. ( (x + 0.1 * h) .eq. x) ) go to 130
if ( (x - xend) * posneg + round) .gt. 0.) return
if ( ( (x - xend) + h) * posneg).gt. 0.) h = xend - x
nstep = nstep + 1

first six stages

do i = 1, n
   yl (i) = y (i) + h*0.2d0*k1 (i)
enddo
call fcn (n, x + 0.2d0*h, yl, k2)

doi = 1, n
   yl (i) = y(i) + h* ( (3.d0/40.d0) *k1 (i) + (9.d0/40.d0)
   & * k2 (i))
enddo
call fcn (n, x + 0.3d0*h, yl, k3)
doi = 1, n
   yl (i) = y (i) + h* ( (44.d0/45.d0) *k1 (i) - (56.d0/15.d0) *k2
   & (i) + (32.d0/9.d0) *k3 (i))
enddo
call fcn (n, x + 0.8d0*h, yl, k4)

do  i = 1, n
   yl (i) = y (i) + h* ( (1.9372d4/6.561d3) *k1 (i) - \\
   (2.5360d4/2.187d3) *k2 (i) + (6.4448d4/6.561d3) *k3 (i) - \\
   (2.12d2/7.29d2) *k4 (i) )
endo

call fcn (n, x + (8.d0/9.d0) *h, yl, k5)

do  i = 1, n
   yl (i) = y (i) + h* ((9.017d3/3.168d3) *k1 (i) - (3.55d2/3.3d1) \\
   *k2 (i) + (4.6732d4/5.247d3) *k3 (i) + (4.9d1/1.76d2) *k4 (i) \\
   - (5.103d3/1.8656d4) *k5 (i) )
endo

g6 (i) = yl (i)
xph = x + h
call fcn (n, xph, yl, k2)

do  i = 1, n
   yl (i) = y (i) + h* ( (3.5d1/3.84d2) *k1 (i) + (5.00d2/1.113d3) \\
   *k3 (i) + (1.25d2/1.92d2) *k4 (i) - (2.187d3/6.784d3) *k5 (i) \\
   + (1.1d1/8.4d1) *k2 (i) )
endo

k6 (i) = k2 (i)

calculate first step of y1-y1^*

do  i = 1, n
   k2 (i) = (7.1d1/5.7600d4) *k1 (i) - (7.1d1/1.6695d4) *k3 (i) \\
   + (7.1d1/1.920d3) *k4 (i) - (1.7253d4/3.3920d5) *k5 (i) \\
   + (2.2d1/5.25d2) *k2 (i)
endo

call fcn (n, xph, yl, k3)

do  i = 1, n
   k4 (i) = (k2 (i) - (1.0d0/40.d0) *k3 (i) ) *h
endo

nfcn = nfcn + 6

error estimation

err = 0.0d0

do  i = 1, n
   denom = dmax1 (1.d - 05, dabs (yl (i) ), dabs (y (i) ), \\
   2.d0*uround/eps)
   err = err + (k4 (i) /denom)**2
endo

err = dsqrt (err/float (n) )

calculate hnew (we require 0.2<= hnew/h <= 10.)
fac = dmax1 (0.1d0, dmin1 (5.0d0, ( (err/eps) ** (1..5) ) / 0.9 ) )
hnew = h/fac

if (err .le. eps) then
  step accepted
  naccpt = naccpt + 1
  do i = 1, n
    k1 (i) = k3 (i)
    y (i) = y1 (i)
  enddo
  x = xph
  call solous (naccpt + 1, x, y, n)
  if (abs (hnew) .gt. hmax) hnew = posneg*hmax
  if (reject) hnew = posneg*dmin1 (abs (hnew), abs (h))
  reject = .false.
else
  step rejected
  reject = .true.
  if (naccpt .ge. 1) nrejct = nrejct + 1
  end if
  h = hnew
  go to 10
fail exit
130 write (6, 140) x
return
140 format(1x,'exit of dopri5 at x=',e11.4)
end

subroutine solous (nnpnts, x, y, n)
  implicit real*8 (a-h, o-z)
  parameter(np=50)
  real*8 y (np)
  return
end
APPENDIX C

SAMPLE RUN OF GRD PROGRAM
C.2 Output File from GRD

******************************************************************************
GEARED ROTOR DYNAMICS PROGRAM
GRD
GEAR DYNAMICS AND GEAR NOISE RESEARCH LABORATORY
THE OHIO STATE UNIVERSITY
COPYRIGHT 1989, 1993
******************************************************************************

THE FIRST SHAFT:

-----------------------------

MATERIAL PROPERTIES OF THE SHAFT:
DENSITY OF THE MATERIAL...... = 0.78000E+04 KG/M**3
ELASTIC MODULUS.............. = 0.20700E+12 N/M**2
SHEAR MODULUS................. = 0.79500E+11 N/M**2
VISCOUS DAMPING COEFFICIENT... = 0.00000E+00 S
HYSTERETIC LOSS FACTOR...... = 0.00000E+00

AT X= 0.0000 M THERE EXISTS A RIGID DISK WITH THE FOLLOWING
SPECIFICATIONS:
OUTER DIAMETER....... = 0.25400E+00 M
WIDTH.................. = 0.15200E+00 M
MATERIAL DENSITY....... = 0.78000E+04 KG/M**3

AT X= 0.0000 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY= 0.35030E+13 N/M
KZY= 0.00000E+00 N/M
KXY= 0.35030E+13 N/M
KZZ= 0.00000E+00 N/M
CYY= 0.00000E+00 N-S/M
CZY= 0.00000E+00 N-S/M
CXZ= 0.00000E+00 N-S/M

AT X= 0.0000 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE
FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT...... = 0.84667E-01 M
OUTER DIAMETER............... = 0.25400E-01 M
INNER DIAMETER................ = 0.00000E+00 M
AXIAL LOAD.................... = 0.00000E+00 N

AT X= 0.0847 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY= 0.35030E+13 N/M
KZY= 0.00000E+00 N/M
KXY= 0.35030E+13 N/M
KZZ= 0.00000E+00 N/M
CYY= 0.00000E+00 N-S/M
CZY= 0.00000E+00 N-S/M
CXZ= 0.00000E+00 N-S/M

AT X= 0.0847 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE
FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT...... = 0.84667E-01 M
OUTER DIAMETER............... = 0.25400E-01 M
INNER DIAMETER................ = 0.00000E+00 M
AXIAL LOAD.................... = 0.00000E+00 N

AT X= 0.1693 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY= 0.35030E+13 N/M
KZY= 0.00000E+00 N/M
KXY= 0.35030E+13 N/M

\[
\begin{align*}
KXX &= 0.35030E+13 \text{ N/M} \\
CYY &= 0.00000E+00 \text{ N-S/M} \\
CZZ &= 0.00000E+00 \text{ N-S/M} \\
CXX &= 0.00000E+00 \text{ N-S/M}
\end{align*}
\]

AT X = 0.1693 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT........... = 0.84667E-01 M
OUTER DIAMETER................... = 0.25400E-01 M
INNER DIAMETER................... = 0.00000E+00 M
AXIAL LOAD....................... = 0.00000E+00 N

AT X = 0.2540 M THERE EXISTS A BEARING WITH THE FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY = 0.17510E+09 N/M \\
KZZ = 0.00000E+00 N/M \\
CYY = 0.00000E+00 N-S/M \\
CZZ = 0.00000E+00 N-S/M \\
CXX = 0.00000E+00 N-S/M

AT X = 0.2540 M THERE EXISTS A RIGID DISK WITH THE FOLLOWING SPECIFICATIONS:
OUTER DIAMETER.............. = 0.89000E-01 M
WIDTH....................... = 0.10700E-01 M
MATERIAL DENSITY......... = 0.78000E+04 KG/M**3

AT X = 0.2540 M THERE EXISTS A BEARING WITH THE FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY = 0.17510E+09 N/M \\
KZZ = 0.00000E+00 N/M \\
CYY = 0.00000E+00 N-S/M \\
CZZ = 0.00000E+00 N-S/M \\
CXX = 0.00000E+00 N-S/M

AT X = 0.2540 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT........... = 0.84667E-01 M
OUTER DIAMETER................... = 0.25400E-01 M
INNER DIAMETER................... = 0.00000E+00 M
AXIAL LOAD....................... = 0.00000E+00 N

AT X = 0.3387 M THERE EXISTS A BEARING WITH THE FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY = 0.35030E+13 N/M \\
KZZ = 0.00000E+00 N/M \\
CYY = 0.00000E+00 N-S/M \\
CZZ = 0.00000E+00 N-S/M \\
CXX = 0.00000E+00 N-S/M

THE SECOND SHAFT:

MATERIAL PROPERTIES OF THE SHAFT:
DENSITY OF THE MATERIAL....... = 0.78000E+04 KG/M**3
ELASTIC MODULUS.............. = 0.20700E+12 N/M**2
SHEAR MODULUS................ = 0.79500E+11 N/M**2
VISCOS DAMPING COEFFICIENT = 0.00000E+00 S
HYSTERETIC LOSS FACTOR...... = 0.00000E+00

AT X = 0.0000 M THERE EXISTS A BEARING WITH THE FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY = 0.35030E+13 N/M \\
KZZ = 0.00000E+00 N/M
KZ=  0.00000E+00 N/M  
KX=  0.35030E+13 N/M  
CYY=  0.00000E+00 N-S/M  
CZ=  0.00000E+00 N-S/M  
CX=  0.00000E+00 N-S/M

AT X=  0.0000 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE
FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT.......=  0.84667E-01 M
OUTER DIAMETER................=  0.25400E-01 M
INNER DIAMETER................=  0.00000E+00 M
AXIAL LOAD.....................=  0.00000E+00 N

AT X=  0.0847 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY=  0.17510E+09 N/M  
KYY=  0.00000E+00 N/M  
KX=  0.17510E+09 N/M  
CY=  0.00000E+00 N-S/M  
CY=  0.00000E+00 N-S/M  
CX=  0.00000E+00 N-S/M

AT X=  0.0847 M THERE EXISTS A RIGID DISK WITH THE FOLLOWING
SPECIFICATIONS:
OUTER DIAMETER.......=  0.88300E-01 M
WIDTH .....................=  0.10700E-01 M
MATERIAL DENSITY .......=  0.78000E+04 KG/M*M

AT X=  0.0847 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY=  0.17510E+09 N/M  
KYY=  0.00000E+00 N/M  
KX=  0.17510E+09 N/M  
CY=  0.00000E+00 N-S/M  
CY=  0.00000E+00 N-S/M  
CX=  0.00000E+00 N-S/M

AT X=  0.0847 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE
FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT.......=  0.84667E-01 M
OUTER DIAMETER................=  0.25400E-01 M
INNER DIAMETER................=  0.00000E+00 M
AXIAL LOAD.....................=  0.00000E+00 N

AT X=  0.1693 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY=  0.35030E+13 N/M  
KYY=  0.00000E+00 N/M  
KX=  0.35030E+13 N/M  
CY=  0.00000E+00 N-S/M  
CY=  0.00000E+00 N-S/M  
CX=  0.00000E+00 N-S/M

AT X=  0.1693 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE
FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT.......=  0.84667E-01 M
OUTER DIAMETER................=  0.25400E-01 M
INNER DIAMETER................=  0.00000E+00 M
AXIAL LOAD.....................=  0.00000E+00 N

AT X=  0.2540 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KYY=  0.35030E+13 N/M  
KYY=  0.00000E+00 N/M  
KX=  0.35030E+13 N/M  
CY=  0.00000E+00 N-S/M  
CY=  0.00000E+00 N-S/M
AT X = 0.2540 M THERE EXISTS A FINITE ROTOR ELEMENT WITH THE
FOLLOWING SPECIFICATIONS:
LENGTH OF THE ELEMENT ......... = 0.84667E-01 M
OUTER DIAMETER .............. = 0.2540E0-01 M
INNER DIAMETER .............. = 0.0000E0+00 M
AXIAL LOAD ............... = 0.0000E0+00 N

AT X = 0.3387 M THERE EXISTS A BEARING WITH THE
FOLLOWING STIFFNESS AND DAMPING COEFFICIENTS:
KXY = 0.3503E0+13 N/M
KZY = 0.0000E0+00 N/M
KXX = 0.3503E0+13 N/M
KYY = 0.0000E0+00 N/S/M
KZZ = 0.3503E0+13 N/M
CXY = 0.0000E0+00 N/S/M
CYZ = 0.0000E0+00 N/S/M
CXX = 0.0000E0+00 N/S/M
CYZ = 0.0000E0+00 N/S/M
CZZ = 0.0000E0+00 N/S/M

AT X = 0.3387 M THERE EXISTS A RIGID DISK WITH THE FOLLOWING
SPECIFICATIONS:
OUTER DIAMETER ............ = 0.3050E0+00 M
WIDTH ..................... = 0.7620E0-01 M
MATERIAL DENSITY .......... = 0.7800E0+04 KG/M**3

GEAR MESH PROPERTIES:
BASE CIRCLE DIAMETER OF GEAR 1.... = 0.0953 M
BASE CIRCLE DIAMETER OF GEAR 2.... = 0.0953 M
AVERAGE MESH STIFFNESS ........... = 0.1240E0+09 N/K
AVERAGE MESH DAMPING ............. = 0.9870E0+03 N-S/M
HELIX ANGLE ................ = 0.0000E0+00 Degrees

******************************************************************************
A. FREE VIBRATION ANALYSIS:
******************************************************************************
FOLLOWING NATURAL FREQUENCIES AND MODESHAPES ARE
CALCULATED FOR THE ABOVE SPECIFIED SYSTEM:

MODE 1

NATURAL FREQUENCY = 0.4758 (HZ.)

CORRESPONDING MODESHAPES:

<table>
<thead>
<tr>
<th>DISP. IN Y</th>
<th>DISP. IN Z</th>
<th>DISP. IN X</th>
<th>ROT. ABOUT Y</th>
<th>ROT. ABOUT Z</th>
<th>ROT. ABOUT X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.455E-09</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.125E-04</td>
<td>0.102E+01</td>
</tr>
<tr>
<td>0.539E-09</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.553E-04</td>
<td>0.102E+01</td>
</tr>
<tr>
<td>-1.64E-09</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.329E-04</td>
<td>0.101E+01</td>
</tr>
<tr>
<td>-1.30E-04</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>-0.963E-04</td>
<td>0.100E+01</td>
</tr>
<tr>
<td>-2.45E-09</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.320E-03</td>
<td>0.100E+01</td>
</tr>
<tr>
<td>0.443E-10</td>
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<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.258E-03</td>
<td>0.100E+01</td>
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<tr>
<td>0.131E-04</td>
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<td>0.000E+00</td>
<td>0.000E+00</td>
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<td>0.100E+01</td>
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<tr>
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<td>0.000E+00</td>
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<td>0.000E+00</td>
<td>0.421E-04</td>
<td>0.986E+00</td>
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******************************************************************************
MODE 2

******************************************************************************
NATURAL FREQUENCY = 25.1397 (HZ.)
CORRESPONDING MODESHAPES:

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<thead>
<tr>
<th>DISP. IN Y</th>
<th>DISP. IN Z</th>
<th>DISP. IN X</th>
<th>ROT. ABOUT Y</th>
<th>ROT. ABOUT Z</th>
<th>ROT. ABOUT X</th>
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</thead>
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<td>0.000E+00</td>
<td>-2.67E-05</td>
<td>-1.01E+01</td>
<td></td>
</tr>
<tr>
<td>-5.61E-08</td>
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<td>0.000E+00</td>
<td>-1.35E-02</td>
<td>-6.84E+00</td>
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<tr>
<td>0.124E-07</td>
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<td>0.000E+00</td>
<td>0.544E-02</td>
<td>-3.61E+00</td>
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</tr>
<tr>
<td>0.622E-03</td>
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<td>-3.75E+01</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.000E+00</td>
<td>0.157E-02</td>
<td>0.328E-01</td>
<td></td>
</tr>
<tr>
<td>-1.16E-07</td>
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<td>0.000E+00</td>
<td>0.549E-02</td>
<td>0.356E+00</td>
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</tr>
<tr>
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<td>0.000E+00</td>
<td>-1.40E-02</td>
<td>0.680E+00</td>
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<td>0.000E+00</td>
<td>0.000E+00</td>
<td>0.155E-05</td>
<td>0.100E+01</td>
<td></td>
</tr>
</tbody>
</table>

B. FORCED VIBRATION ANALYSIS:

DYNAMIC TO STATIC LOAD RATIO AT THE SPECIFIED RANGE

<table>
<thead>
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C.3 Graphical Results

Figure C.1 Dynamic Factor using GRD Program.
APPENDIX D

VARIOUS STUDY OF DYNAMIC ANALYSIS
Figure D.1 Dynamic Tooth Load Comparisons at Operating Speed of 2000 rpm, 47% Full Torque (298.57 lb-in), Case 1.
Figure D.2  Dynamic Tooth Load Comparisons at Operating Speed of 2000 rpm, 47% Full Torque (298.57 lb-in), Case 2.
Figure D.3  Dynamic Tooth Load Comparisons at Operating Speed of 4000 rpm, 31% Full Torque (196.93 lb-in), Case 1.
Figure D.4  Dynamic Tooth Load Comparisons at Operating Speed of 4000 rpm, 31% Full Torque (196.93 lb-in), Case 2.
Figure D.5  Dynamic Tooth Load Comparisons at Operating Speed of 4000 rpm, 31% Full Torque (196.93 lb-in), Case 3.
Figure D.6  Dynamic Tooth Load Comparisons at Operating Speed of 4000 rpm, 31% Full Torque (196.93 lb-in), Case 4.
Figure D.7  Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 1.
Figure D.8  Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 2.
Figure D.9  Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 3.
Figure D.10 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 4.
Figure D.11 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 5.
Figure D.12 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 6.
Figure D.13 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 31% Full Torque (196.93 lb-in), Case 7.
Figure D.14 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 47% Full Torque (298.57 lb-in), Case 1.
Figure D.15 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 
47% Full Torque (298.57 lb-in), Case 2.
**Figure D.16** Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 47% Full Torque (298.57 lb-in), Case 3.
Figure D.17 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 47% Full Torque (298.57 lb-in), Case 4.
Figure D.18 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94 % Full Torque (597.14 lb-in), Case 1.
Figure D.19 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94 % Full Torque (597.14 lb-in). Case 2.
Figure D.20 Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94% Full Torque (597.14 lb-in), Case 3.
Figure D.21  Dynamic Tooth Load Comparisons at Operating Speed of 6000 rpm, 94% Full Torque (597.14 lb-in), Case 4.