NON-MONOTONIC STRAIN HARDENING AND ITS
CONSTITUTIVE REPRESENTATION

DISSERTATION

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ABSTRACT

Modeling sheet metal forming operations requires understanding of the plastic behavior of sheet alloys along complex strain paths. In most materials, plastic deformation in one direction will affect subsequent deformation in another direction. For one-dimensional deformation, this phenomenon is known as the Bauschinger effect. The Bauschinger effect in heat-treatable aluminum alloys is heavily dependent on the presence of hardening precipitates.

A new method was developed to test sheet materials under uniaxial reversed loading to compressive strains greater than 0.20. Studies of commercial aluminum alloys 2524 and 6013, show a larger Bauschinger effect for materials that have been artificially aged past peak strength, where the precipitates are semi-coherent or incoherent. Not only is there a substantial reduction of the reverse yield stress, the period of the transient behavior following the load reversal is also lengthened. This effect is seen in over-aged materials after prestrains as small as 0.4%. Materials with less aging had shorter transient periods but showed some permanent softening after the reversal, which was a function of the prestrain.

A constitutive model was developed, based upon the nonlinear kinematic hardening model, which is capable of describing the reduction in the reverse yield stress and the
hardening transient observed after the reversal. The ability to model the permanent offset was introduced by the addition of a new term into the formulation, which can be defined as a function of plastic strain. This model has been implemented in a finite element program that successfully reproduces the main features of the experimental results for both uniaxial and more general strain paths. Simulations of draw-bead forces showed significant reductions, up to 25%, because of the Bauschinger effect. Simulations of the springback angle using the new model were within 1.5° of the experimental results for the 2524 over-aged and peak-aged tempers that show a large Bauschinger effect.
To my family for their loving support.
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CHAPTER 1

INTRODUCTION

The fundamental purpose of materials science is to uncover linkages between the properties of materials to the underlying structure. If this relationship is uncovered, the hope is that the properties of the material can be tailored for different applications by altering the structure through different processing conditions. The purpose of this project is to uncover these linkages between the precipitate structure of aluminum alloys and their mechanical properties. Specifically, this work is concerned with non-proportional strain paths, where the loading direction changes during deformation. This problem is of great importance to the continuum modeling community, because the mechanical behavior during non-proportional deformation is often different than the monotonic behavior usually assumed.

Chapter 2 describes the material aspects that determine the mechanical properties of age-hardenable aluminum alloys, and provides a background on the specific metallurgical characteristics of the two commercial alloys used in this work, 6013 and 2524. There is also a discussion on how these microstructural features can be used to describe the strength of the material.

Chapter 3 gives a brief introduction to the anisotropy of plastic flow under non-proportional deformation. For fully reversed strain paths, this phenomenon is called the
Bauschinger effect. The common explanations of the Bauschinger effect found in the literature will be introduced and discussed.

Chapter 4 is a brief introduction to plasticity theory, beginning with the methods used to model anisotropic behavior through the yield surface, and then introducing equations used to modify this yield surface to describe the hardening evolution of deforming materials. Several types will be discussed, some of which focus on the mechanics of the problem and are not as concerned with the mechanisms of deformation within the structure, while others still continuum in nature, try to take into account specific microstructural feature of the material.

Chapter 5 introduces the method used in this work to implement the constitutive equations introduced in Chapter 4 into a finite element program.

Chapter 6 investigates the techniques used in this work, including the experimental tests and some comments on the methods of data analysis and quantification.

Chapter 7 provides the main experimental data on the Bauschinger effect of two commercial aluminum alloys, 2524 and 6013, under various degrees of natural aging. The results from mechanical testing these materials are introduced and discussed to elucidate the effects of different precipitate structures.

Chapter 8 is concerned with simulating the results the mechanical testing with the finite element model, and shows comparisons between the experimental results and the output from the simulations.

Chapter 9 provides a summary of the major conclusions of this work.
After the conclusions, there is an Appendix of a paper which was submitted in similar form to the *International Journal of Plasticity*, which describes the creation and optimization of the compression-tension device used to provide the majority of the experimental data for this work.
2.1. Precipitation Hardening in Aluminum Alloys

Precipitation hardening is the primary hardening mechanism in heat treatable aluminum alloys. In precipitation hardening, the material is strengthened by a very fine dispersion of second phase particles that impede dislocation motion. When a moving dislocation encounters an obstacle, it will bow around it until the force on the dislocation is balanced by the line tension. Looking at the force balance at the blocking particle, the force acting on the particle can be related to the line tension, $T$, by,

$$F = 2T \cos(\phi/2) \tag{2.1}$$

Figure 2.1: Schematic of dislocation particle interaction.
where $\phi$ is the **breaking angle**, shown in Figure 2.1, which can be used to classify the obstacle strength. This equation shows that as the size of the precipitate increases, and $F$ becomes larger, the breaking angle must decrease, and eventually, becomes zero. At this point, the dislocation will by-pass the obstacle rather than cut through it. The radius at which a dislocation bows is related to the applied stress in the following way

$$\tau b = \frac{T}{R} \tag{2.2}$$

where $\tau$ is the shear stress on the slip plane, $b$ is the Burgers vector, $T$ is the line tension and $R$ is the radius of curvature of the bowed segment [1]. Furthermore, from the geometry in Figure 2.1, it can be seen that $2R \sin \theta = \lambda$. Substituting this into Equation 2.2 forms the basis for the Orowan equation.

$$\tau = \frac{2T \sin \theta}{b\lambda} \tag{2.3}$$

where $\sin \theta$ is related to the **breaking angle** in the following way, $\cos(\phi/2) = \sin \theta$. The line tension, $T$, can be calculated from the energy per unit length of the elastic stress field around the dislocation. As such, this value is dependent upon the nature of the dislocation (i.e. screw or edge) and requires some cut-off radius when integrating the stress field. However, for a typical dislocation density of $10^8$ per cm$^2$, the line tension, $T$, can be assumed to be on the order of $T = Gb^2/2$ [2], and the applied stress can be solved for as

$$\tau = \frac{Gb \cos(\phi/2)}{\lambda} \tag{2.4}$$
which shows that the applied stress to by-pass a rigid obstacle is inversely proportional to the particle spacing. As in the force equation, this equation has a maximum when the breaking angle goes to zero, which is the stress needed for dislocation by-pass.

In order to obtain significant hardening, a large volume fraction of precipitates is needed, which requires alloying aluminum with elements that allow significant solid solubility at high temperatures and low solubility at the service temperature. A fine distribution of refined particles is produced by a series of phase transformations. Typically a solution heat-treatment is performed by heating the material above the solvus line and holding it until the alloying elements are dissolved in the aluminum matrix. The material is quenched to room temperature and subjected to controlled aging which causes decomposition of the supersaturated solid solution into the matrix and particles. This aging process is often done at intermediate temperatures to take advantage of non-equilibrium phase transformations that produce a finer precipitate distribution. The first transformation in most systems is the formation of Guinier-Preston (GP) Zones—ordered, solute-rich clusters of atoms, which may be one or two atoms in thickness and approximately 100Å in diameter. These GP Zones retain structure of matrix and are coherent with it, which produces elastic distortions about the zones. These GP Zones may act as nucleation sites for further precipitate formation.

Intermediate phases are usually much larger than GP zones and are only partly coherent with lattice planes of the matrix. Even though the intermediate phases are not the most stable structure, they form because of kinetic restrictions on the equilibrium structure. These phases establish a specific composition and intermediate crystal
structure related to the final equilibrium precipitate. The spacing between these particles is extremely small, so that it is not possible for the dislocation lines to bend around the individual particles, so they must be cut. For small particles, the interface between the particle and matrix is coherent producing elastic stresses which must be overcome to move the dislocation. For ordered precipitates, the cutting dislocation also has to overcome the energy associated with breaking this order and increasing the area of the particle-matrix interface. As the particle grows in size, these hardening effects become more significant.

As the particle distribution coarsens to a larger scale, the distances between the particles increases, and the strength of the obstacles increases. Both of these factors lead promote the transition from dislocation cutting to dislocation by-pass, which becomes the dominate mechanism. As the dislocation bows around the particles, segments of opposite sign meet and annihilate, and the dislocation continues to glide while leaving a dislocation loop around the particle. If particle size and spacing, are sufficient for by-pass, further aging coarsens the precipitate structure leading to an increase in $\lambda$, and a corresponding lowering of the strength.

2.1.1. *Alloying and heat treatment of 2524 and 6013*

Aluminum alloys 6013 and 2524 were obtained in sheet form to study the effects of precipitation on non-proportional loading. Both these alloys are heat-treatable aluminum alloys. 2524 is a 2XXX (Al-Cu) series alloy, used in aircraft applications because of its high strength with the appropriate heat treatments. It is a close cousin of the most common 2XXX series alloy, 2024, an Al-Cu-Mg alloy. 2524 has similar
nominal values for the alloying elements as 2024, but has much tighter specifications on the alloying content and contains smaller concentrations of Fe, the major aluminum impurity element [3].

6XXX (Al-Mg-Si) series aluminum alloys, such as 6013 are not as strong as the 2XXX series alloys, but they possess good atmospheric corrosion resistance and can be extruded and welded much easier than 2XXX and 7XXX series alloys, making them prime candidates for automotive applications [4]. The nominal composition of both alloys is shown in Table 2.1. [5, 6]

In commercial aluminum alloys, many different types of second-phase intermetallic particles may be present in the aluminum matrix. The largest particles are generally referred to as constituent phases. These are intermetallic phases that contain elements with very low solubility in aluminum and do not dissolve during the solution heat treatment stage. These intermetallics often contain iron, the most common impurity element in aluminum. The size and distribution of these particles is determined by the primary processing conditions, but they are often on the order 1-30µm depending on the material cleanliness [7]. The constituent phases often identified in 2524 are Al₇Cu₂Fe,

<table>
<thead>
<tr>
<th></th>
<th>Cu</th>
<th>Mg</th>
<th>Mn</th>
<th>Si</th>
<th>Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2524</strong></td>
<td>4.0 - 4.5</td>
<td>1.2 - 1.6</td>
<td>0.45 - 0.70</td>
<td>0.12 max</td>
<td></td>
</tr>
<tr>
<td><strong>6013</strong></td>
<td>0.6 - 1.1</td>
<td>0.8 - 1.2</td>
<td>0.8 - 1.2</td>
<td>0.6 - 1.0</td>
<td>0.5 max</td>
</tr>
</tbody>
</table>

Table 2.1: Nominal compositions for aluminum alloys 2524 and 6013.
Al\textsubscript{12}(Fe,Mn)\textsubscript{3}Si, and Al\textsubscript{6}(Fe,Cu), while the most common phase in 6013 is Al\textsubscript{12}(Fe,Mn)\textsubscript{3}Si [7].

Dispersoid phases are similar, in that they are unaffected by solution heat treatment; however, these phases are purposely introduced through the addition of transition metals, such as Mn. These alloying elements have relatively slow diffusivity in aluminum and form relatively small particles less than 1\textmu m in size. Dispersoid phases act to inhibit grain boundary mobility and delay recrystallization during primary processing. In 2524, the common dispersoid phase is Al\textsubscript{20}Cu\textsubscript{2}Mn\textsubscript{3}, and in 6013, the dispersoid phase for 6013 is Al\textsubscript{12}Mn\textsubscript{3}Si. [7].

While the complex precipitate structure in commercial aluminum alloys often makes quantitative characterization difficult, the size and spacing of the dispersoid and constituent particles (on the order of 0.1 to 10\textmu m) are at least one order of magnitude larger than the precipitate phases. Because of the inverse relationship between spacing and strength predicted by the Orowan equation, it is generally assumed these particles do not affect the hardening response of materials. While the validity of this assumption can be debated, even if there are dislocation-particle interactions, they are equivalent for all tempers because of the constancy of these phases during heat-treatment.

For the composition range of 2524, the ternary Al-Cu-Mg phase diagram predicts predominately $S$ phase which dominates over the $\theta$, which is also present at equilibrium. Upon quenching, the Cu and Mg are distributed independently throughout the matrix in clusters that precede GP zone formation [7, 8]. These clusters significantly harden the material, accounting for nearly 70\% of the hardening [8]. Upon aging, these clusters
transform to GP zones, which form into two dimensional disks of diameter 3-5nm on 
{100} and {210} matrix planes [7, 8]. The structure of the S phase and its precursors are 
very similar and may be considered as the same structure with distortion associated with 
matrix coherency strains [8]. The transition phase $S'$ is initially coherent with the matrix 
and grows as laths on {210} planes along the $<100>$ directions at the expense of the GP 
zones [9]. The transition to the equilibrium $S$-phase is a continuous process as the 
precipitate grows and becomes incoherent with the matrix and the lattice constants 
change. Peak hardness occurs with the formation of the equilibrium $S$ phase [7]. Unlike 
the Al-Ge-Si system, where peak hardness occurs when the largest volume fraction of 
precipitates are present [10], there is still a considerable amount of retained Cu in the 
solution at the peak-strength condition [11, 12], and the maximum precipitate volume 
fraction actually occurs in the over-aged condition. This raises the possibility that some 
of the softening occurring during over-aging is caused by the loss of copper solute 
strengthening in addition to Orowan effects of ripening precipitates.

The presence of Si in addition to Cu and Mg creates a different precipitation 
sequence for 6013. The Si addition enables the formation of $\beta$ (Mg$_2$Si) and the 
quaternary phase, $Q$ [7, 13]. The most potent hardening precipitate during the initial 
stages of aging is the metastable $\beta''$ phase [7, 14, 15]. It is a needle shaped precipitate 
with the long axis along the $<100>$ directions of the matrix [16]. Near peak aging, some 
of the needle shaped $\beta''$ transforms to the non-shearable $\beta'$ which forms rods along the 
same $<100>$ direction. The remainder of the $\beta''$ transforms to the precursors of the $Q$-
phase, which have the same orientations along $<100>$ direction, but form as laths along
the \{150\} habit plane of the matrix. This lath phase has a hexagonal crystal structure and appears to play a strong role in the strengthening process in quaternary alloys as it gradually replaces the \(\beta'\) [14]. With increasing aging the morphology of this phase remains constant, with only a size increase [7].

In this work, both alloys were solution heat treated at 530\(^\circ\)C and then water quenched to create a super-saturated solid-solution. Some material was stored in dry ice for testing in the solution heat treated (SHT) condition. Three other tempers were produced through various amounts of artificial aging. One was near peak-hardness (PA) and the other two were at similar hardness in the under-aged (UA) and over-aged conditions (OA). The aging curves for each of the alloys at 220\(^\circ\)C are shown in Figure 2.2. The 6013 samples were heat treated for 20 minutes to produce the UA temper, 4

![Figure 2.2: Aging curves for aluminum alloys 6013 and 2524.](image-url)
hours for PA and at 250°C for 96 hours to obtain OA. The 2524 was aged at 220°C for 20 minutes for UA, 90 minutes for PA and 72 hours for OA.

Transmission electron microscopy (TEM) was performed on these heat treated samples to compare the precipitate arrangement to that reported in the literature. Figure 2.3 shows the TEM images for the three 2524 tempers. The large, dark phases on the order of hundreds of nanometers are constituent or dispersoid phases. The hardening phases are the finer, lower contrast features. As aging proceeds, the density of these phases increases from the relatively sparse situation in the under-aged sample to the more dense distribution in the longer-aged samples. These micrographs show the complexity of the particle distribution and the difficulties in determining a single, representative precipitate spacing. Even in the over-aged situation, where the precipitates are larger and well-defined, their distribution is irregular and difficult to quantify.

![Figure 2.3: TEM images of 2524 UA (a), PA (b) and OA (c).](image)

Dispersoids S’ (AlCuMg)
The 6013 micrographs, shown in Figure 2.4, also show constituent and dispersoid phases on the order of hundreds of nanometers, with the important hardening phases appearing as a finer structure. As in 2524, the under-aged sample contains very little visible precipitates. At peak hardness, the microstructure contains a very fine distribution of precipitates, which coarsens as the aging proceeds.

In a companion study to this work, precipitation strengthening of a dilute Al-Ge-Si system was found to be dominated by the precipitate volume fraction. In that system, the particles were incoherent and non-shearable even at very small sizes. Therefore, the strength of the material increased until the point at which all the solute in the matrix had precipitated. After this point, the volume fraction remained constant and the yield stress dropped because of Orowan effects related to the coarsening structure [10].

The situation in these two commercial alloy systems is different from the Al-Ge-Si system in two important aspects. First, the structure and coherency of the precipitates
change during aging. Non-coherent precipitates are easier to cut than coherent phases, because of the absence of coherency stresses. For both 2524 and 6013, the peak age condition was found to coincide with the formation of non-coherent precipitates rather than a critical Orowan spacing. Once these precipitates lose their coherency the strength drops, even though the precipitate volume fraction is still increasing. Secondly, the commercial alloys have a much higher alloy concentration than the Al-Ge-Si studied. This leads to significant concentrations of precipitates and solute present at the same time during aging. The simultaneous presence of these two mechanisms makes decoupling the effects of each mechanism difficult. For example, during over-aging of 2524 the strength drop may be attributable to precipitate evolution as well as matrix solute depletion.

2.2. Dislocation State-Variable Theories

Deformation in metallic materials is accomplished by the movement of crystal dislocations. Therefore any obstacles to dislocation motion, whether second-phase particles or other dislocations, will increase a material’s strength. As was discussed above, weak or closely spaced obstacles are cut by the dislocations. However, as the obstacles become stronger or the spacing between them grows larger, the dislocations pass between the obstacles at a stress inversely proportional to the spacing, according to the Orowan equation

\[ \sigma_{\text{by-pass}} = \frac{\eta Gb}{\lambda} \]  

(2.5)

where \( \eta \) is a constant on the order of unity based on the obstacle strength.
In dislocation hardened materials, the strength of the material is then related to the
dislocation density which can be related to the average dislocation spacing \([17, 18]\). The
increase in strength of a material with a particular dislocation density is \([17]\).

\[
\Delta \tau = \alpha G b \rho^{\frac{1}{2}} \tag{2.6}
\]

Because the dislocation density has the units of inverse area, this relationship is exactly
the Orowan equation. This relationship assumes all the hardening is due to dislocation-
dislocation interactions, rather than other strengthening mechanisms, such as
precipitation hardening.

Using this form, the work-hardening of the material can then be described through
the evolution of the state variable, \(\rho\) with plastic strain \(\varepsilon_p\), which depends on the physical
processes of dislocation accumulation and annihilation and is often assumed to take the
form \([17]\)

\[
\frac{\partial \rho}{\partial \varepsilon_p} = k_1 \rho^{\frac{1}{2}} - k_2 \rho \tag{2.7}
\]

where \(k_1\) and \(k_2\) are constants. The first term represents the athermal storage of
dislocations in the structure, which are assumed to become immobilized after traveling
the average dislocation spacing \((\rho^{1/2})\). The second term is a dynamic recovery term.
Combining these two equations, the evolution of the flow stress caused by dislocation
accumulation is
\[
\frac{\partial \hat{\sigma}}{\partial \varepsilon^p} \equiv \theta = \theta_s \left(1 - \frac{\hat{\sigma}}{\hat{\sigma}_s}\right)
\]  

(2.8)

where

\[
\theta_s = \frac{\alpha G b k_i}{2}
\]  

(2.9)

and

\[
\hat{\sigma}_s = \frac{\alpha G b k_i}{k_z}
\]  

(2.10)

where \( \theta \) is the slope of the flow curve. These equations are precisely the equations used to describe a material with Voce-type saturation behavior [19]. This fact is useful because it allows for some linkages between the features of the flow curve to the constants in the dislocation evolution laws. Equation 2.9 shows \( k \) is directly proportional to the initial work-hardening rate of the material, and taking the stress derivative of Equation 2.8,

\[
\frac{\partial \theta}{\partial \hat{\sigma}} = \frac{\theta_s}{\hat{\sigma}_s} = \frac{k_z}{2}
\]  

(2.11)

shows this derivative is directly proportional to the dynamic recovery term, \( k_z \).

For precipitate containing materials, the precipitate spacing may be less than the average dislocation spacing. Therefore, the distance the dislocation travels before encountering a pinning site may be smaller and independent of the dislocation separation.
Based on this realization, the dislocation evolution equation presented in Equation 2.7 may be rewritten,

\[ \frac{\partial \rho}{\partial \varepsilon^p} = k_{pp} - k_2 \rho \]  

(2.12)

where \( k_{pp} \) is a different material constant related to the precipitate spacing. The constant \( k_2 \) is still related to the dynamic recovery rate, however, its value may be different if precipitates change the mechanisms of dynamic recovery [20].

Using dislocation density as a state variable is useful because the constants describing the physical accumulation and recovery processes can be directly related to the flow curve. However, the basis of the theory, focusing on dislocation-obstacle interactions, is difficult to interpret when there are obstacles of varying strength and spacing, such as precipitates, forest dislocation and grain boundaries. It also neglects non-length scale strengthening mechanisms such as solute hardening.

Severe problems also arise when trying to incorporate anisotropic behavior, specifically reversed loadings. It has been shown that the Bauschinger effect causes the reverse flow curve to be quite different than the monotonic curve. Using these dislocation models, one would then obtain significantly different values for the dislocation evolution constants. While it is true that the accumulation and recovery processes described by these parameters may initially be affected by the reversal, one would expect them to converge to the values in the forward deformation for large proportional strains after the reversal. That is not the case in this situation, where the
constants derived from the reverse loading curve are remain very different from those derived the monotonic curve, even when the individual dislocation behavior is the same.
CHAPTER 3

THE BAUSCHINGER EFFECT

3.1. Main Features of the Bauschinger Effect

The Bauschinger effect is named after Johann Bauschinger, who in 1886 observed that the compressive yield strength of steel samples is reduced by prior tensile deformation [21]. Since his original observation, the term Bauschinger effect has expanded to mean not only a reduction of the yield stress upon load reversal, but encompasses any changes in plastic behavior caused by changes of the stress state. Because this definition is broad and the process of plasticity is highly material dependent, it is no surprise that a universal description or even definition of the Bauschinger effect has been elusive.

Figure 3.1 shows a reverse loading curve for aluminum alloy 6022 in which the compressive flow curve has been rotated into the first quadrant by plotting stress magnitude versus total accumulated strain. The curve for continued monotonic tension is shown for comparison. Comparing the reverse curve to that obtained by monotonic tension illustrates some of the features usually associated with the Bauschinger effect. Attributes commonly observed during reverse loading of aluminum alloys are a reduced yield stress followed by a period of increased strain-hardening and permanent softening at high strains where the two curves achieve parallelism. These features are not observed
in all materials, and when they are present, they may take different forms. For example, in some materials the two hardening curves recombine and no permanent softening is observed. In yet other cases, such as mild steel subjected to orthogonal deformation, the reloading curve is higher than the monotonic curve because of latent hardening [22].

Figure 3.2 shows a hardening curve of a material with Voce hardening[19] with two other curves offset from the original curve along the stress and strain axis. These two types of offset can be easily differentiated by plotting the work-hardening rate, $d\sigma/d\varepsilon$, as a function of the applied stress, as in Figure 3.3. When plotted in this manner the strain offset is exactly the same as the original curve, while the stress offset is shifted. The dotted line in Figure 3.3 represents the onset of tensile necking as determined by Considère’s criteria [23]. The intersection of the work-hardening curves with this line is the ultimate tensile strength (UTS). In the case of the strain offset the effect is temporary,
and both curves have the same fracture stress. In the stress case, there is a permanent offset and softening of the material.

The body of research on the sources of the Bauschinger effect can be loosely divided into three general groups. The first focuses on the residual stresses that develop
during forward deformation, and how these stresses aid the external force after load reversal. The second theory is based on the anisotropy that develops on individual slip planes. The final one is more microstructurally based and depends on the rearrangement and redistribution of dislocations after the load reversal.

3.2. Residual Stress Explanation

One of the first attempts to describe the origin of the Bauschinger effect was developed by Heyn around 1918 [24, 25] and supported by experiments by Masing [26-28]. Heyn showed that residual stresses which develop in a material during deformation can give rise to Bauschinger-like behavior. To illustrate this hypothesis, a material with a Voigt state of parallel volume elements is assumed. Each volume element is modeled as an elastic/perfectly-plastic material with a variation in yield strengths subjected to the same strain. When a load is applied, each element behaves elastically until its yield stress, after which it deforms at that same stress level, and any increase in the macroscopic stress is caused by other elements which are still elastic. The mechanism by which this material would develop a Bauschinger effect can be understood by considering the behavior around the weakest element. During forward plastic deformation, the macroscopic flow stress exceeds the yield stress of the weakest element, that is $\sigma_f > \sigma_y_{\text{elem}}$. When the load is removed, the weakest element experiences a compressive residual stress because the other elements at higher stress levels will elastically contract more than it does. During a full load reversal, the weakest element will again be the first to yield after it has deformed though twice its elastic range ($2\sigma_y_{\text{elem}}$). This means that during reverse loading, the sample will begin to yield at a stress of $\sigma_f - 2\sigma_y_{\text{elem}}$, which is smaller
in magnitude than the forward flow stress $\sigma_f$. The Heyn/Masing model is useful because it shows how features associated with the Bauschinger effect arise naturally from residual stresses, which are always present to some extent in inhomogeneous, deformed bodies.

In materials with hard second-phase particles, the plasticity mismatch between the two phases is significant and residual stresses can be important. The influence of residual stresses on a deforming matrix have been researched for a variety of materials, including metal matrix composites [29], dispersion hardened aluminum [30], dual phase steels [31], and structural aluminum alloys. Wilson [32], Abel and Ham [33], Moan and Embury [34], and Bate et al. [35] studied the Bauschinger effect as a function of precipitate structure in Al-4 wt % Cu alloys, and found the reverse yield stress was sensitive to the presence of non-coherent precipitates that lead to residual stresses in the matrix.

Figure 3.4: Continuum effects of precipitates on the Bauschinger effect [10].
Most engineering alloys contain multiple phases, which is why many current hardening models incorporating the Bauschinger effect are based on the residual stresses around non-deformable second phases. One of these approaches, the elastic inclusion model, will also be introduced in detail in Section 4.2.5. In this model, the inclusions act as non-deformable elements within the plastic matrix leading to residual stresses. While these stresses can be calculated, the way in which they influence the Bauschinger effect and evolve with plastic strain is not obvious. Continuum simulations of elastic precipitates in a plastic matrix show there is some reduction of the yield point on reversal, but the effect saturates within 0.4% for 3% precipitate concentration and within 1% for very large concentrations of 6% [10]. This period is dramatically shorter than the long terms transients observed in real materials, such as those observed in Figure 3.1.

3.3. Dislocation Resistance Explanation (Orowan Mechanism)

Orowan noted that although the residual stress explanation produces a reduced yield point and permanent softening it does not accurately describe the reverse flow curve. He argued that the residual stress models predict the reverse flow curve to be exactly the same shape as the original stress strain curve just displaced along the stress axis, which does not capture the transient behavior observed in many materials [36-38]. To further examine these problems Orowan’s group conducted a series of experiments that had been previously suggested by Zener [39]. The hypothesis was that if the Bauschinger effect is caused by residual stresses, low-temperature anneal after the forward deformation allows thermally activated by-pass of obstacles, thereby lowering the residual stress associated with dislocation pileups. If this occurs, the sample will
permanently deform during the anneal and the Bauschinger effect will be erased; however, if the sample does not deform, or still displays a Bauschinger effect, then another explanation must be made for its source.

After doing the experiments on Al, Cu, Ni, brass and decarburized steel (0.008 wt % C) there was some ambiguity in the results. During the annealing, there was deformation of about 0.2% - 0.3% strain and the degree of permanent softening decreased—both factors suggesting the relief of residual stresses. However, other features of the Bauschinger effect—reduced yield point and transient hardening—persisted until the recrystalization temperature, at which point they were erased with the cold-worked microstructure. Orowan concluded that the Bauschinger effect was affected by residual stresses but there was likely another source acting, which was related to the cold-worked microstructure of the sample.

As an example of what this other mechanism might be, he proposed a model that produces a Bauschinger effect without the need for residual stresses. This model has become generally recognized as the textbook explanation of the Bauschinger effect [2], even though it also has its limitations. A schematic of the Orowan model is shown in Figure 3.5. The small circles represent a randomly distributed obstacle profile that is assumed to remain constant in the slip plane of a moving dislocation. For sufficiently strong obstacles, the dislocation can not cut through them and will instead be extruded between them at a shear stress that is inversely proportional to the obstacle spacing. When a stress is applied, the dislocation will move across the slip plane until it encounters an array of obstacles that are sufficiently strong and closely spaced to inhibit
Figure 3.5: Schematic of the Orowan mechanism [2].

its forward motion, which occurs at position 1 in Figure 3.5. According to Orowan, “If the applied stress is removed, the dislocation may adjust itself locally, but in the absence of sufficiently strong back-stresses it will stay more or less where it was.”[38] If the load is reversed, the stress necessary to move the dislocation in the reverse direction is lower because the row of obstacles now in front of the dislocation is less dense than the row that arrested its forward motion. Therefore, the dislocation may travel a large distance before it encounters another distribution of obstacles, at position 2, that are as strong and closely distributed as those impeding its initial forward motion. This anisotropy in the obstacle distribution leads to excess strain for each stress level and manifests itself as a Bauschinger effect.

The shear strain attainable from this type of deformation can be approximated. The total dislocation density can be decoupled into the mobile and immobile parts. As shown in Section 2.2, the immobile density increases the hardness of the material. This immobile density can be divided into dislocations that locked and those that are pinned up against obstacles and can reverse their direction. The pinned dislocation lead to what
we will refer to later as isotropic hardening and the pinned dislocation can be identified with the kinematic hardening. In the most severe cases of the Bauschinger effect that will be shown in Section 7.1, the maximum amount of this kinematic hardening observed is about 108 MPa. Using the relationship between the stress increase and the dislocation density in Equation 2.6, this corresponds to a dislocation density of about $2.4 \times 10^9$ cm$^{-2}$. The shear strain caused by the motion dislocations of density $\rho$, a distance, $\lambda$ is

$$\varepsilon = b\lambda \rho$$

(3.1)

The Bergers vector for Al is $2.36 \times 10^{-10}$. Assuming the precipitates in the overaged material are non-shearable, the effective spacing, $\lambda$, can be estimated as 50.8 nm from Equation 2.4 and the yield stress of the OA material, 143 MPa. Using these values the amount of strain expected from these dislocations is 0.034%, over two orders of magnitude smaller than the transients observed in this material. Therefore the Orowan description of the Bauschinger effect does not tell the entire story.

3.4. Effect of Dislocation Evolution on the Bauschinger Effect

Studies of microstructure effects on the Bauschinger effect tend to group materials into one of two broad categories. The first is materials that contain “significant” fractions of hardening phases, such as dispersoids or precipitates, as were discussed in Section 3.2. The other is predominately single phase materials, where the hardening derives from dislocation interactions with grain boundaries, solutes and other dislocations. This distinction is somewhat arbitrary because there is a continuum of
behavior as the proportions of the phases change; however, it is often made because the two general classes do show different behavior at the extremes.

Many studies of the Bauschinger effect in the single phase systems have been motivated by TEM observations of the dislocation structure before and after a load reversal. When the loading direction is reversed, dislocation structures created during forward deformation untangle, and in some cases may fully disintegrate, before a fresh structure is created based on the new loading state [40-42]. Because of this observation, most Bauschinger studies on single-phase alloys have been concerned with the polarity of dislocation structures and the untangling of cell walls after the sign of the load is reversed [43-45].

Because the strength of the material is linked to the obstacle spacing, it is natural to search for linkages between the obstacle structure and the observed mechanical behavior. For volume defects, such as precipitates, the characterization of the size and spacing is often difficult because of non-uniform distributions. Unfortunately, the task is considerably more complex for dislocations because they are signed defects and can be either homogeneously distributed or localized into dense walls between relatively dislocation free zones. This arrangement is highly sensitive to the material, testing temperature, and even the mode of deformation, making characterization a highly complex problem.

Fortunately, recent work by Rauch [18] provides some encouragement. The dislocation structure morphology is a function of several factors—such as the amount of cross slip or climb, lattice friction, stacking fault energy, grain size, and precipitate
concentrations. Materials with these different observed structures show a corresponding
difference in mechanical behavior. Rauch hypothesized the mechanical behavior was
dependent only on the dislocation mechanics determined by these contributing factors,
and the dislocation morphology was simply a by-product of these same forces, rather than
the source of the mechanical disparity. Samples of low carbon steel was prestrained in
two temperature regimes to produce either cellular or homogeneous dislocation
structures, then reloaded at the same temperature. Despite the difference in the initial
dislocation structure, there was no difference in the strength or the work hardening rate of
these materials when they were reloaded under the same conditions, confirming the
original hypothesis.

Because the dislocation morphology seems to be a derivative of the dislocation
mechanics rather than a source, if the dislocation behavior were well understood the
Bauschinger effect could be predicted. Some of the more quantitative studies in this field
were by Hasegawa and Yakou [40]—who made predictions of the magnitude of the
Bauschinger effect caused by untangling dislocation structures—and Teodosiu [41] —
who incorporated the polarity of the dislocation structures into a continuum plasticity
formulation that will be discussed in a later section.

This approach can do a very good job of describing the Bauschinger effect, but as
mentioned before, the individual dislocation mechanics can be affected by a variety of
chemical and microstructural features, and therefore any descriptions for a particular
material may not be applicable in other systems. Despite these limitations, this approach
seems to have the best opportunity to describe the Bauschinger effect in a physical way.
The way this physical phenomenon is incorporated into mechanical analysis will be discussed in the next chapter.
CHAPTER 4

MECHANICAL MODELING OF PLASTICITY

The laws governing plasticity are constructed based on several observations of plastic deformation. First, plasticity is assumed to be volume conserving, and independent of hydrostatic pressure. Therefore, the relationships can be described based on the deviatoric stress. Secondly, plastic deformation is an irreversible process that dissipates energy as it proceeds. This dissipation has two important consequences. The first is that upon unloading, the elastic strains in the body are recovered, while the plastic strains persist. This allows the total strain to be decomposed additively into the elastic and plastic strain, \( \varepsilon = \varepsilon_p + \varepsilon_e \). The second consequence of plastic dissipation is path dependency, which means that, unlike elasticity, given the current value of stress or strain it is impossible to determine the other quantity without some model to provide a functional relationship between the current loading state and history of the body.

The essential construct of classical plasticity theory is the yield surface, which when formulated in stress space identifies the stress states necessary for plastic deformation. The general form of the yield surface, \( f \) is

\[
f = \sigma_{eq}(\sigma) - k \leq 0 \quad (4.1)
\]
where $\sigma_e$ is some scalar measure of the tensor stress state, $\sigma$. When this effective stress measure is equal to $k$, some intrinsic resistance to deformation, $f = 0$, and the material plastically deforms. Otherwise, the stress state is elastic. In associated plasticity theory, the yield surface also acts as a plastic potential, where the direction of the strain increment is normal to the yield surface.

$$d\kappa = d\lambda \frac{df}{d\sigma}$$

(4.2)

where $d\lambda$ is a scalar constant.

4.1. Initial Yield Anisotropy

Orowan description of the Bauschinger effect, presented in Section 3.3, suggests a mechanism where a material can develop anisotropic mechanical response through directional differences in dislocation resistance. However, there is another source of anisotropy that is present even in the absence of this slip plane anisotropy. In materials that possess non-random grain orientations, the distribution of active slip systems introduces anisotropy as well. If the texture is known, polycrystal models can construct the yield surface by calculating the resolved shear stress on each slip system. However, this construction requires texture measurement, which is often time consuming. In addition, this numerically created surface is often of limited use for implementation into associated plasticity formulations because the surface normal is difficult compute. Because of this situation, differentiable, analytical representations of the yield surface are desired for numerical modeling.
The Von Mises yield criteria [46] is an isotropic yield surface based on the assumption that plastic deformation occurs when the distortional energy reaches a certain critical value. In general coordinates, the Von Mises yield surface takes the following form.

\[
2\sigma^2_{eq} = (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + (\sigma_{11} - \sigma_{22})^2 + 6(\sigma_{23}^2 + \sigma_{13}^2 + \sigma_{12}^2)
\]

which when expressed in principal coordinates is

\[
2\sigma^2_{eq} = (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2
\]

The Von Mises yield function is regularly used for basic analysis because it can be analytically differentiated, which makes it very easy to implement.

An alternative yield criteria, proposed by Tresca [47], assumes the material yields when the maximum shear stress attains a critical value, making the yield equation in principal stresses:

\[
\sigma_{eq} = \max\left(\left|\sigma_3 - \sigma_2\right|,\left|\sigma_1 - \sigma_3\right|,\left|\sigma_2 - \sigma_1\right|\right)
\]

The yield surface of most materials lies in between the limits defined by the Von Mises and Tresca criteria. The main deficiency of the Tresca yield surface is the presence of vertices, where the yield surface normal is indeterminate.

Another isotropic form that does not contain vertices, and more closely matches real materials was proposed by Hosford [48]. In this formation the Von Mises quadratic form is generalized as:
\[ 2\sigma_{eq}^2 = |\sigma_2 - \sigma_3|^m + |\sigma_3 - \sigma_1|^m + |\sigma_1 - \sigma_2|^m \quad (4.6) \]

where the exponent \( m = 8 \) for FCC and 6 for BCC materials [49].

These previous yield functions are represented in principal strains, which implies isotropic behavior, however, most engineering materials are strongly anisotropic. Hill’s anisotropic, quadratic yield function (Hill’48) has been widely used because of its applicability to general stress states and its ease of implementation [50]. The equivalent stress is defined as

\[ 2\sigma_{eq}^2 = F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 \quad (4.7) \]

where the coefficients \( G, G, H, L, M \) and \( N \) are material constants. This yield function is applicable for general stress states with no restriction on the relative orientation of the principal stress axis and the anisotropy axis. If the principal stress directions are coincident with the anisotropy axis, the shear terms are zero and the coefficients \( F, G, \) and \( H \) can be determined from the strain ratios, or r-values, measured at different angles to the sheet rolling direction,

\[ r_0 = \frac{H}{G}, \quad r_{90} = \frac{H}{F}, \quad r_{45} = \frac{H}{F + G} - \frac{1}{2} \quad (4.8) \]

As in the case of the Von Mises yield function, the quadratic nature of the function does not adequately reproduce the shape of the yield surface for most materials. However, Hill’48 can be generalized to a non-quadratic form, Hill’79 [51],
\[ \sigma_{eq}^m = f|\sigma_2 - \sigma_3|^{m} + g|\sigma_3 - \sigma_1|^{m} + h|\sigma_1 - \sigma_2|^{m} + a|2\sigma_1 - \sigma_2 - \sigma_3|^{m} \\
+ b|2\sigma_2 - \sigma_2 - \sigma_3|^{m} + c|2\sigma_3 - \sigma_2 - \sigma_3|^{m} \] \hspace{1cm} (4.9)

where \( f, g, h, a, b \) and \( c \) are a new set of coefficients. However, in making this generalization the principal and anisotropy directions are assumed to be coincident.

In the same way Hill’48 modifies the Von Mises yield function by adding anisotropy coefficients, Hosford’s non-quadratic yield function can also be made anisotropic through the addition of material anisotropy coefficients \( F, G, \) and \( H \) \cite{52}.

\[ 2\sigma_{eq}^2 = F|\sigma_2 - \sigma_1|^m + G|\sigma_3 - \sigma_1|^m + H|\sigma_1 - \sigma_2|^m \] \hspace{1cm} (4.10)

This yield function is also of limited use because it again requires the anisotropy axis to be along the principal stress directions. Barlat’s YLD’91 \cite{53} overcomes this deficiency by introducing the anisotropy to the stress vector rather than the yield function itself.

Using the Hosford non-quadratic yield function,

\[ 2\sigma_{eq}^2 = |S_2 - S_1|^m + |S_3 - S_1|^m + |S_1 - S_2|^m \] \hspace{1cm} (4.11)

\( S_i \) are the eigenvalues of the transformed deviatoric stress tensor, whose components are

\[
\begin{align*}
S_{11} & = \frac{c(\sigma_{11} - \sigma_{22}) - b(\sigma_{33} - \sigma_{11})}{3} \\
S_{22} & = \frac{a(\sigma_{22} - \sigma_{33}) - c(\sigma_{11} - \sigma_{22})}{3} \\
S_{33} & = \frac{b(\sigma_{33} - \sigma_{11}) - a(\sigma_{22} - \sigma_{33})}{3} \\
S_{12} & = h\sigma_{12} \\
S_{13} & = g\sigma_{13} \\
S_{23} & = f\sigma_{23}
\end{align*}
\] \hspace{1cm} (4.12)
where $a, b, c, h, g$ and $f$ are again anisotropy coefficient. This function was shown to have some limitations, especially in fitting the behavior in shear stress states, so two improved iterations of the yield function were proposed (YLD’94 [54] and YLD’96 [55]), which added additional terms to the yield equation to account for this discrepancy in shear

$$2\sigma_{eq}^2 = \alpha_1|S_2 - S_3|^m + \alpha_2|S_3 - S_1|^m + \alpha_3|S_1 - S_2|^m$$  \hfill (4.13)

where $\alpha_1, \alpha_2$ and $\alpha_3$ differ in their derivation in YLD’94 and YLD’96. These yield equations do a better job of reproducing the yield surface, but their convexity is not assured for all cases, which is a requirement for reliable convergence of associated plasticity algorithms.

The function YLD2000-2D [56] was created to assure convexity and simplify the determination of the material parameters. This yield surface is a linear combination of two convex potentials, which assures the convexity of the resultant surface.

$$2\sigma'' = \phi' + \phi''$$  \hfill (4.14)

where $a \geq 1$ is a material parameter, but is recommended to be 8 for FCC as in Hosford’s analysis, and

$$\phi' = |X'_1 - X'_2|^a$$

$$\phi'' = |2X''_1 + X''_1|^a + |2X''_2 + X''_2|^a$$  \hfill (4.15)

The values $X'_i$ and $X''_i$ are the principal directions of the following linear transformations of the stress deviator defined as
These eight parameters can be calculated from the yield stresses and r-values of tensile tests in the directions 0°, 45° and 90° from the rolling direction and the balanced biaxial test. In addition to simplifying the material parameter calculation and assuring convexity, YLD2000 also improves the yield surface prediction over the prior Barlat yield functions [57].

4.2. Evolution of Anisotropy with Deformation

The above yield equations were proposed to account for anisotropy caused by material texture, and do a reasonable job of describing the initial yield surface of the material. As equally important as the initial shape of the yield surface, is how it evolves and changes with strain. Since, by definition, all stress states lie on or within the yield surface, if the flow stress is to increase the surface must necessarily change its size, shape or position to accommodate this new stress state. During deformation, there is a gradual rotation of the crystal orientations, producing a change in the material texture. It has been found that this change of texture can be reasonably represented by a change of the yield surface exponent in the non-quadratic yield function as a function of strain [58, 59].

However, in contrast to this gradual texture evolution, most materials show comparatively rapid changes to the yield stress with deformation as evidenced by work
hardening and the development of the Bauschinger effect. If these effects can not be explained by texture evolution, they must be caused by changes in the dislocation mobility on the individual slip systems. If all slip systems are hardened equivalently, the material is said to experience isotropic hardening. In this situation, the plastic potential retains the same form and the yield surface uniformly expands with a constant shape. In the description of the yield surface in equation 4.1, this type of hardening corresponds to an increase in the value of $k$.

One major deficiency of isotropic hardening is its inability to account for the Bauschinger effect, where the yield stress is lowered by prior deformation in the opposite direction. Experimentally, the yield surface has been measured after prestraining using small offset strains on the order of $10^{-4}$ - $10^{-6}$ [60, 61], which show a dramatic flattening of the yield surface in the direction opposite the original straining direction. Models that account for this type of distortion have been proposed for both viscoplastic [62] and irreversible thermodynamic [63] methods as well as elasto-plastic formulations. The elasto-plastic methods involve either defining the yield surface as some strain dependent parametric equation [64, 65] or by multiplying the original yield surface by some strain dependent anisotropy tensor [66, 67].

Typically, for metal forming analysis, these microplastic effects are neglected, and the Bauschinger effect is modeled by kinematic hardening. Simple linear kinematic hardening methods, propsed by Prager [68] and Ziegler [69], introduced a term, typically called the backstress, $\alpha$, which acts to translate the center of the yield surface.

$$ f = \sigma_{eq} (\sigma - \alpha) - k \leq 0 $$

(4.17)
The term “backstress” has its origins in the residual stress explanations of the Bauschinger effect, but is often used broadly without regard to the mechanisms of the mechanical behavior. In Prager’s model the translation of the yield surface occurs in the direction of the plastic strain increment

$$d\alpha = c\, d\epsilon^p$$ (4.18)

whereas Ziegler’s model assumes the translation is along the radial direction along the vector connecting the current stress to the center of the yield surface, \((\sigma - \alpha)\)

$$d\alpha = \frac{c}{\sigma_c} (\sigma - \alpha) dp$$ (4.19)

where \(dp\) is the modulus of the plastic strain rate, \(|d\epsilon^p|\) for the uniaxial case. Ziegler’s assumption prevents any inconsistencies that may exist between the yield surface normal in the 1D, 2D and 3D cases [70-72].

Most attempts to improve the description of strain hardening seek to generalize these two types of hardening to better match the behavior seen in materials. One of the first attempts was Hodge’s [72] mixed hardening, which assumes the hardening rate is a linear decomposition of isotropic and kinematic hardening. The evolution equations for \(k\) and \(\alpha\) became

$$d\alpha = mH^p d\epsilon^p$$

$$dk = (1 - m)H^p |d\epsilon^p|$$ (4.20)
where $\mathbf{d}\varepsilon^p$ is the plastic strain increment, $H^p$ is the plastic modulus of the monotonic hardening curve and $m$ is constant between 0 and 1 that determines the relative contributions of isotropic and kinematic hardening.

These linear models, while useful for providing a base, are inadequate for predicting the highly nonlinear behavior of real materials. More useful are the multi-surface models, such as that proposed by Mroz [73] or the family of nonlinear kinematic hardening models, first introduced by Armstrong and Frederick [36] and then expanded by many researchers including Chaboche [74], Geng [75], and Teodoisu [22].

4.2.1. Mroz and Two-Surface Models

The Mroz model uses Prager’s kinematic hardening model as a base, but assumes there is a field of work hardening moduli, which correspond to a series of yield surfaces each with a constant hardening modulus. During the loading between surfaces $f_i$ and $f_{i+1}$, the surface $f_i$ is active and the interior surfaces from $f_i$ to $f_1$ harden kinematically according to the active tangent modulus, $H_i$. When the surface $f_i$ (along with the other interior surfaces) comes into contact with $f_{i+1}$, $f_{i+1}$ becomes active and its hardening modulus is used. This field of hardening moduli reproduces the behavior predicted by the Masing [28] model.

The Mroz model is attractive because it can be fit using only the uniaxial flow curve, and a Bauschinger effect follows naturally. The flow curve is assumed to be composed of piecewise linear segments corresponding to each yield surface. The size of the surface is equal to the initial stress value of the associated segment, and the segment slope is the kinematic hardening coefficient.
The simplicity of the Mroz model is also its biggest disadvantage, because it is unable to describe the correct material response in some situations. Improving the Mroz model to overcome some of these deficiencies is still an active area of research [76, 77]. For practical use, the Mroz model is also computationally expensive because each surface requires the storage of a scalar describing its size, and a tensor value locating its position. Given the large number of surfaces that are required to reasonably approximate the flow curve of a typical material, the number of stored values can rapidly become unwieldy.

The two-surface models of Krieg [78] and Dafalias and Popov [79] attempt to overcome this deficiency by replacing the multiple surfaces with two—the yield surface and a bounding surface. Rather than a piecewise-linear plastic modulus, the two-surface model assumes the tangent modulus is a continuous function of the distance between the yield and bounding surfaces. This distance is calculated from the current stress state to the point on the bounding surface with the same outward normal. The yield surface moves within the bounding surface such that it moves toward the point on the bounding surface with the same outward normal. Both Krieg’s and Dafalias and Popov’s models enable the yield and bounding surfaces to experience both isotropic and kinematic transformations. One possible drawback of two-surface models is they require some search algorithm to find the common normal on the bounding surface. While this search is a straightforward operation, it is often preferable to reformulate the problem in terms of differential equations governing the evolution of the yield surface. Recasting these equations can enable the use of more efficient computational algorithms. The
relationship between the two surface models and their differential forms will be discussed in the next section.

4.2.2. **Nonlinear Kinematic Hardening**

This class of models asserts that even though the dissipative nature of plasticity destroys the coincidence between the stress and plastic strain, the stress can be uniquely determined from the proper set of internal state variables. The goal of any hardening theory then becomes to identify these state variables and determine the differential equations which govern their evolution.

These state variables may or may not be physically measurable quantities. In one basic form, the internal state variables can be assumed to be the backstress and yield surface size, leading to constitutive equations that look similar to the Prager / Ziegler / Hodge type models. The Armstrong Frederick [36] model, which has been heavily used and updated by Chaboche [74, 80, 81] assumes the backstress evolves by the following differential equation.

\[
p \gamma = \alpha d \varepsilon - \alpha dp
\]  

(4.21)

where \( c \) and \( \gamma \) are material constants. The second term on right-hand side of the equation is a recall term, whose direction depends on the current value of the backstress and provides the strain path dependence. The evolution of the backstress before and after a strain reversal produces different behavior because the direction of the first term reverses, while the second term does not. For a long proportional path, the backstress saturates to a value of \( c/\gamma \) at a rate determined by the magnitude of \( \gamma \). This saturation behavior is
equivalent to an “implied” two-surface model where the radius of the bounding surface is larger than the yield surface by the constant \( c/\gamma \).

For monotonic deformation the backstress can be integrated explicitly as

\[
\alpha(\varepsilon^p) = \frac{c}{\gamma} \left( 1 - e^{-\gamma_1 \varepsilon^p} \right) \tag{4.22}
\]

Chaboche [74] showed that this model can be generalized by introducing several backstresses, i.e.,

\[
\alpha = \sum_{i=1}^{m} a_i
\]

\[
da_i = c_i d\varepsilon^p - \gamma_i a_i dp
\]

This approach significantly improves the degrees of freedom available and makes it much easier to match the model to the material data because the total backstress is not determined by a single exponential, but by a sum of, \( i \), independent exponential functions. The addition of multiple backstresses has one other useful advantage. Adding an additional linear backstress, with the recall term omitted (\( \gamma = 0 \)), changes the “implied” two-surface model to allow kinematic hardening of the bounding surface in this case, the backstress for the monotonic situation would be.

\[
\alpha(\varepsilon^p) = \frac{c_1}{\gamma_1} \left( 1 - e^{-\gamma_1 \varepsilon^p} \right) + c_2 \varepsilon^p \tag{4.24}
\]
Equation 4.17, shows that this calculation for the backstress does not saturate. If some saturation of the linear term is desired, it can be introduced through a strain dependence of $c_2$.

The concept of multiple backstresses has been found to be of great use in modeling some of the more complex problems of plasticity. “Ratcheting” is the accumulation of strain under a cyclic load with a non-zero mean stress. Ratcheting is generally considered to be dominated by kinematic hardening, but the strain accumulation is not predicted by the simple equations above. There has been reasonable success in modeling this problem by decomposing the backstress into multiple backstresses that only operate in certain stress or strain regimes [81-85].

If ratcheting is the accumulation of strain under a cyclic load, cyclic hardening is the stress based counterpart—an increase (or decrease) of the peak load during cycles of strain. Many materials cycled between fixed strain limits tend toward stable hysteresis behavior, at which point one could assume the hardening is purely kinematic with no isotropic component. This transition from mixed hardening to purely kinematic hardening is often modeled by introducing a non-hardening surface into the evolution equation for the isotropic hardening in either strain space [86, 87] or stress space [88, 89]. In either case, this surface is analogous to the yield surface, using similar expressions for the consistency equations. Isotropic hardening only occurs if the current strain state is on the surface of the non-hardening surface, and the increment is outward pointing. This switching off and on of the hardening requires some procedure to assure a smooth stress-strain response. For example, Ohno [87] introduces an additional
kinematic hardening term that activates when the material is in a non-hardening state, but tends to zero as the non-hardening surface is approached. While these formulations end up being rather complex, the generally provide a reasonable approximation of the cyclic behavior.

4.2.3. Geng-Wagoner Hardening

The additions to hardening theory provided through ratcheting and cyclic hardening research have greatly improved the descriptions of material hardening. Unfortunately, they have also dramatically increased the complexity of the constitutive equations. For metal forming applications, where there may only be a few stress reversals, this complexity may not be necessary. More useful for these applications is a material model that is sufficiently general to efficiently describe a variety of material behavior and is computationally efficient. This was the goal of the model proposed by Geng and Wagoner [90]. Geng-Wagoner hardening has both a yield and bounding surface, centered at $\alpha$ and $\beta$ respectively, which obey separate evolution equations. Although the model is introduced in terms of two surfaces, the formulation is similar to the nonlinear kinematic hardening model, in that the relationships are represented in terms of differential equations and do not explicitly use the distance between the yield and bounding surface to determine the tangent modulus. The equation for the backstress is

$$\text{d}a = \frac{c}{\sigma_e} (\sigma - a) \text{d}p - \gamma (a - \beta) \text{d}p \quad (4.25)$$
where $\beta$ is the center of the bounding surface. This equation, based on Zeigler’s model, replaces $\alpha$ in the recall term with the vector connecting the centers of the yield and bounding surface. In the original implementation of this model, the parameters $c$ and $\gamma$ were calculated from the hardening rates in the forward and reverse directions on either side of the reversal, and as such, are strain dependent.

This model allows the bounding surface to translate, which is a necessary first requirement for modeling materials with a permanent stress offset of the yield surface. The bounding surface evolves according to mixed hardening rule

$$d\beta = \frac{mH^p}{\sigma_{\beta_0}}(\sigma_{\beta} - \beta)dp$$

$$d\sigma_{\beta_0} = (1-m)H^pd\sigma$$

where $d\sigma_{\beta_0}$ is the size of the bounding surface and $\sigma_{\beta}$ is the point on the bounding surface with the same outward normal. Notice from equation 4.19, that the size of the bounding surface is not used in the backstress calculation. Rather, the mixed hardening parameter, $m$, in equation 4.17 defines the bounding surface translation as a proportion of the plastic modulus, $H^p$. In the original work, $m$ was also a function of plastic strain in order to accurately reproduce the permanent offset. This model can reduce to the case of nonlinear kinematic hardening with an additional linear backstress, $\alpha_2$, corresponding to $\beta$.

$$d\alpha = (c_1 + c_2)de^\sigma - \gamma_1 \alpha_1 dp$$
In this representation, the translation of the bounding surface can be explicitly defined by $c_2$ rather than being defined as a proportion of the plastic modulus. Whether one uses the Geng-Wagoner formulation or the nonlinear kinematic version, the result is a flexible model which can reproduce a wide variety of material responses.

4.2.4. Teodosiu Model

Classical plasticity theory is not generally concerned with the mechanisms of deformation. While concepts such as the yield surface can be derived from crystallographic orientation and texture, plasticity is a continuum formulation and is unconcerned with features such as microstructure or dislocation behavior. Despite this fact, the material properties are derived from the structure, so there is a strong motivation to make linkages between the two regimes.

Perhaps the most thorough link between the mechanics based models and microstructure is by Teodosiu and Hu [22]. This model is based on the anisotropic behavior observed in predominately single phase materials where the hardening is dislocation-based. It seeks to explain not only the reduced yield stress for load reversals, but also the elevated yield observed for orthogonal loading caused by latent hardening.

In this model, the mechanical response is determined by three tensors, $S$, $P$ and $X$, which are all initially zero. The first tensor, $S$, describes the directional strength of the dislocation structures. It has the same value for forward and reverse directions, but it may vary in other directions. It can roughly be thought of as the density of obstacles to dislocation motion in a certain direction. The polarity of this structure is described by $P$, which is initially zero, but evolves toward the direction of the plastic strain rate tensor, $N$. 
The final tensor, $X$, is a familiar variable, the backstress. As $P$ and $N$ slowly evolve with plastic deformation, $X$ is necessary to capture the rapid changes in stress under reversal caused by dislocation pile-ups. The yield function for this model is

$$f = \sigma_x (\sigma - X) - k_o - f|S| \leq 0$$  \hspace{1cm} (4.28)

where $k_o$, is the initial yield stress, and $f$ is a scalar between 0 and 1 determining the contribution of the dislocation structure to the isotropic hardening. As mentioned above, $P$ saturates to the current straining direction

$$dP = C_{P} (N - P) dp$$  \hspace{1cm} (4.29)

The evolution equation for the backstress is similar to the previous formulations,

$$dX = C_{X} (X_{sat} N - X) dp$$  \hspace{1cm} (4.30)

however, instead of a fitting constant $X_{sat}$ is a function of the persistent dislocation structure, $S$. This dislocation structure, $S$, is decomposed into direct effects from the active slip systems and latent hardening.

$$S = S_D N \otimes N + S_L$$  \hspace{1cm} (4.31)

where $S_D$ and $S_L$ each have their own evolution equations. It is in the evolution equation for $S_D$, where the dislocation polarity, $P$ enters into the model.

Overall the model has 10 material parameters that must be determined from experiments, which is significant but manageable in some situations. The number of
parameters is reduced considerably by assumptions about the nature of the dislocation interactions, so it must be reformulated for materials with drastically different deformation mechanisms. However, in its range of applicability—mild AKDQ steel—it has shown good agreement with the experimental data.

4.2.5. Elastic Inclusion Model

Other researchers have also tried to correlate the internal state variables with microstructural features to give some microstructural motivation behind the mechanical models. One example of this is the effort to correlate the presence of precipitates to the residual stresses in aluminum alloys. Wilson [32], Abel and Ham [33], Moan and Embry [34], and Bate et al. [35] studied the Bauschinger effect as a function of precipitate structure in Al-4 wt % Cu alloys, and found the reverse yield stress was sensitive to the presence of non-coherent precipitates. It was assumed that this was caused by residual stresses in the matrix, and based on these results, the elastic inclusion model was created to correlate the backstress to the elastic stress [91]. Inspired by Eshelby’s solution of a stress field around an elliptical inclusion [92], this model assumes the backstress in the matrix, $\alpha$, is

$$\alpha = f \{\sigma^i\} = f C \{I - S\} \epsilon^p$$

(4.32)

where $\sigma^i$ is the stress caused by the $i^{th}$ precipitate, $f$ is the precipitate volume fraction, $C$ is the elastic modulus matrix of the precipitate, $I$ is the forth order identity matrix, $S$ is the Eshelby tensor and $\epsilon^p$ is the plastic strain in the matrix. In this equation, curly brackets indicate directional averaging over all the possible precipitate orientations. This model
assumes a perfect interface between the particle and matrix, so that the entire plastic strain in the matrix is accompanied by an elastic strain in the particle. This is not a realistic assumption because some of these stresses may be relieved by plastic relaxation of the matrix around the particle. Because experimental results indicate the elastic backstresses vanish at larger strains [32-34], To fit this backstress to the long-term Bauschinger transient, Barlat multiplied the above equation by a function $h(\varepsilon^p)$, which tends to zero as the plastic strain increases. For uniaxial testing, this decay term may be dependent on all aspects of the precipitate including size shape and spacing as well as dislocation creation and annihilation. For reverse loading, it may also depend on any rearrangement of the dislocations around the precipitate. There is some significant concern with the use of this relaxation parameter because it assumes the elastic stresses between the precipitate and matrix gradually are relieved over many percent of strain, but there is little experimental evidence to support this theory.
CHAPTER 5

NUMERICAL IMPLEMENTATION OF CONSTITUTIVE MODELS

Because of the nonlinear nature of most finite element analysis, an incremental solution is usually required. The total deformation is incrementally applied over several time steps, \( \Delta t \). At each of these time steps, the plasticity algorithm is responsible for defining the stiffness of the structure (i.e. the stress/strain relationship) and updating the values of the stress, strain and any internal variables to their final values at the end of the increment. For the standard isotropic/kinematic model discussed in this document, the following differential equations, in Table 5.1:, must be satisfied [93]: The complementary conditions assure that plasticity only occurs when the stress state is on the yield surface and the loading direction is pointing outward from the yield surface, assuring the material is being loaded rather than unloaded. The consistency equation, in turn, assures the stress point remains on the yield surface during the plastic increment.

Using these relations, the values of the stress, strain and state variables at the start of the increment are used to calculate the values at the end of the increment. Once these final values are known, the consistent tangential matrix, \( C_t \) is often computed. This matrix, also known as the material Jacobian, is used in the Newton-Raphson iterations of implicit algorithms to ensure a quadratic rate of convergence of the nodal displacements [94].
Elastic stress-strain relationship

\[ \dot{\sigma} = C^{el} : (\dot{\varepsilon} - \dot{\varepsilon}^p) \]

Flow Rule and Hardening Law

\[ \dot{\varepsilon}^p = \dot{\lambda} a = \dot{\lambda} \frac{\partial f}{\partial \sigma} \]
\[ \dot{a} = \dot{\lambda} g(\sigma, \alpha) \]
\[ \dot{k} = \dot{\lambda} H_{iso} \]

Yield Condition

\[ f = \sigma_v (\sigma - a) - \sigma_o - k \leq 0 \]

Kuhn-Tucker complementary conditions

\[ \lambda \geq 0, \quad f \leq 0, \quad \lambda f = 0 \]

Consistency Equation

\[ \lambda \dot{f} = 0 \quad (\text{if} \quad f(\sigma, \alpha) = 0) \]

Table 5.1: Family of differential equations for plasticity analysis.

5.1. Forward Integration

One simple way to integrate the differential equations is to calculate them using the values of the variables at the beginning of the increment. Making this assumption, the plastic multiplier is [95]:

52
\[
\Delta \lambda = \frac{a : C^{el} : \Delta \varepsilon}{a : C^{el} : a + \frac{\partial f}{\partial \alpha} \cdot g(\sigma, \alpha) + H_{iso}}
\]  

(5.1)

and the updated quantities become:

\[
\begin{align*}
\varepsilon_{n+1} &= \varepsilon_n + \Delta \varepsilon \\
\varepsilon_p^{n+1} &= \varepsilon_p^n + \Delta \lambda a_n \\
\alpha_{n+1} &= \alpha_n + \Delta \lambda g_n(\sigma_n, \alpha_n) \\
\sigma_{n+1} &= \sigma_n + C^{el} : (\Delta \varepsilon - \Delta \varepsilon^n)
\end{align*}
\]  

(5.2)

The forward integration method is generally not as effective as other methods because the yield surface is not enforced at the end of the increment, so the updated quantities may deviate from the actual solution. In order for this deviation to be corrected, some method must be made to return the values at the end of the increment back to the yield surface.

5.2. Backward Euler Method

The Backward Euler scheme for stress updating in strain-driven problems uses an elastic predictor followed by a plastic correction through closest projection. In the first step, a purely elastic strain increment is assumed and the trial stress is calculated.

\[
\sigma_{trial, n+1} = \sigma_n + C : \Delta \varepsilon
\]  

(5.3)

The trial values for the plastic strain and internal state variables are assumed to be the same as the previous increment. The first value of \(\Delta \lambda\) is found using a Taylor expansion of the yield function at the Backward Euler stress point with respect to this trial stress.

Residual tensors are then defined as the differences between the current values of the stress and state variables and the most recent Backward Euler predictions.
\[ r_\sigma^n = \sigma - \left( \sigma^{\text{trial}} - \Delta \lambda C : a \right) \]
\[ r_\alpha^n = \alpha - \left( \alpha^{\text{trial}} + \Delta \lambda g \right) \]  
(5.4)

The Taylor expansion of each of these residual quantities produces a new residual,

\[ r_\sigma^n = r_\sigma^o + \dot{\sigma} + \dot{\lambda} C a + \Delta \lambda C \frac{\partial \lambda}{\partial \sigma} (\dot{\sigma} - \dot{\alpha}) \]
\[ r_\alpha^n = r_\alpha^o + \dot{\alpha} + \dot{\lambda} g + \Delta \lambda \dot{g} \]  
(5.5)

which is set to zero. These equations can combined with the truncated Taylor series for the yield function

\[ f^{\text{new}} = f^{\text{old}} + \dot{\sigma}_{eq} - H_{iso} \dot{\lambda} \]  
(5.6)

to produce the incremental changes in the stress, backstress and plastic multiplier that occur for each iteration. This procedure is repeated until the yield function is satisfied within a given tolerance.

5.3. Implementation of Multiple Nonlinear Kinematic Hardening into ABAQUS

The commercial finite element program ABAQUS allows users to implement their own material models into the software. This user material subroutine, or UMAT, is called at each integration point within the model, so it must be as computationally efficient as possible [96]. Each time the procedure is called, ABAQUS passes to the UMAT the total strain increment, and values of the stress, strain, and user defined internal variables from the last converged step. The UMAT must then supply ABAQUS with updated values of these quantities as well as the Jacobian matrix.

The implementation of a multiple nonlinear kinematic hardening model was done by Jianfeng Wang using the Backward Euler method, and the detailed derivation can be
found in his PhD dissertation document [97]. For this work the formulation was modified to allow the linear backstress term to vary as a function of strain in order to allow for more flexibility to match the experimental results, as well as a strain dependence on the $\gamma$ terms.

The material model chosen for this work uses the following yield surface.

$$f = \sigma_e (\sigma - a) - \sigma_o - k \leq 0$$ \hspace{1cm} (5.7)

The evolution equation for the backstress is decoupled into two backstresses, one of which is linear.

$$da = \frac{C_1}{\sigma_e} (\sigma - a_1) dp - \gamma \sigma_e (\sigma - a_1)$$ \hspace{1cm} (5.8)

In this implementation, $C_1$ is a constant over the entire deformation, and $\gamma$ is a function of strain. Their values determine not only the magnitude of the Bauschinger effect, but also the nature of the reloading transient. The magnitude of $a_2$ is equal to the stress offset associated with any permanent softening, so $C_2$ as a piecewise linear function of the plastic strain can be readily calculated. For uniaxial deformation the flow stress is the sum of the backstress and the yield surface radius. The backstress can be numerically integrated as a function of strain, so the size of the yield surface can be calculated for a given set of kinematic constants by subtracting this value from the flow curve. The calculated table of yield surface size versus plastic strain was input into the UMAT for computation of the isotropic hardening modulus.
CHAPTER 6

BAUSCHINGER TESTING TECHNIQUES

6.1. Mechanical Testing Methods

Sheet metal forming is a significant component of the total metal forming industry. Sheet metal is produced from primary ingots or slabs through a series of rolling operations that produce an anisotropic grain structure and induce a characteristic crystal texture. As mentioned in Section 4.1, texture has a large influence on the mechanical behavior. Because this texture is different from bulk specimens, mechanical testing techniques for sheet material are desired. Unfortunately, these techniques are often limited by elastic buckling in the thickness direction, and specialized testing methods must be developed.

The majority of the testing performed was in-plane compression followed by tension, using a novel device developed for this work. This testing method, introduced in the Appendix, is a significant advance in sheet metal testing, enabling compressive strains on the order of 0.02 without elastic buckling. The second test used is the simple shear test which has been developed and used by several researchers [98-100]. While not capable of producing pure tension or compression (for comparison with standard tensile information), it can be conducted to larger strains, well beyond the tensile range [98, 101].
Three types of mechanical tests were performed in this work to probe the mechanical properties of sheet material under various non-proportional paths—two continuous tests and a two-stage interrupted test. All tests were conducted under displacement control to produce a nominal rate of $10^{-3}$ strain per second.

6.1.1. Uniaxial Testing

Monotonic deformation—unreversed tension or compression—is used as an input for the finite element analysis and as a basis for comparison for the reverse flow curves. The attainable strain for compression is potentially larger than that attainable in tension because the sample does not neck. This is especially important for the aged materials, which show limited tensile ductility as compared to the under-aged materials. Figure 6.1 and Figure 6.2 show both the monotonic tension and compression curves for aged

![Figure 6.1: Monotonic tension and compression curves for Al 2524 showing assumed monotonic curve for PA (a) and OA (b) tempers.](image)
Figure 6.2: Monotonic tension and compression curves for Al 6013 showing assumed monotonic curve for PA (a) and OA (b) tempers.

6013 and 2524. The figures showing the uniaxial results also show the extrapolations used for the monotonic flow curves at large strains for the finite element analysis. These extrapolations assume Voce-type saturation at large strains. As the strains increase, the monotonic compression curve deviates from the monotonic tension curve.

This is likely a result of locking caused by increasing friction in the tension-compression device as the gauge length thickens during compression. The onset of this locking is nearly impossible to detect or account for, which makes it difficult to assess large strain features. For this reason, the compression-tension test is more reliable than the tension-compression tests for Bauschinger work because it allows the reverse flow curve to be reported with greater confidence that there are no external forces clouding the interpretation.

Although there are problems with locking, the individual tension-compression and compression-tension results are remarkably similar. The comparisons are shown for UA,
PA and OA 2524 and 6016 in Figure 6.3 and Figure 6.4, respectively. Locking is most visible in the peak and overaged tempers, where the strain hardening is lower making.

Figure 6.3: Tension-compression and compression-tension curves for 2524 UA (a), PA (b) and OA (c) tempers.
differences in the load more noticeable. Most results presented in this work are compression-tension, but some tension-compression results are presented when comparison the UA and SHT tempers in Section 7.1

![Figure 6.4](image-url): Tension-compression and compression-tension curves for 6013 UA (a), PA (b) and OA (c) tempers.
6.1.2. *Shear Testing*

The shear test is shown schematically in Figure 6.5. At large strains, the sample may experience localization of the shear, and influence from the ends of the specimen. Unfortunately, as in compression locking, the onset of these effects is difficult to identify, which makes the assessment of large-strain effects somewhat difficult.

Two-stage non-reverse tests utilize large tension specimens, which are deformed and cut into sub-size specimens, as shown in Figure 6.6. Three specimens were cut from the 50 mm wide tensile gauge region to test the tensile properties orthogonal to the initial loading direction (I), and two shear orientations. One shear orientation produced tensile deformation along the initial tensile axis (II) and the other orientation induced compression along that direction (III).

![Figure 6.5: Schematic of simple shear device.](image-url)
Figure 6.6: Diagram of sample orientations for alternate strain path measurements.
6.2. Quantification of the Bauschinger Effect

When comparing and discussing the Bauschinger behavior, it is often useful to have some method of quantifying the response using a scalar measurement. Various scalar measures of the Bauschinger effect have been used in the literature [24]. The goal of these values are to highlight the Bauschinger features, while mitigating the effects of experimental error or evaluator bias. Broadly, the measures can be classified as stress based, strain based, or work or energy based. The stress based values are concerned with the reduction of the yield point, and the strain based values with the transient length. The work based values are often a hybrid of the other two classes and has the potential to give a broader measure of the response.

6.2.1. Stress Based Parameter - $\beta$

The stress based Bauschinger measures are derivatives of the difference between the forward and reverse flow stresses, $\Delta \sigma$. If $\Delta \sigma$ is normalized by the forward flow stress, the parameter $\beta$ is produced

$$\beta = \frac{\Delta \sigma}{\sigma_{\text{forward}}} = \frac{\sigma_{\text{reverse}} - \sigma_{\text{forward}}}{\sigma_{\text{forward}}} \quad (6.1)$$

$\beta$ describes the fractional reduction in the yield stress magnitude after reversal, and is zero if there is no Bauschinger effect, one if the material yields at zero applied stress and two if the material yields immediately upon reloading. The normalization inherent in the parameter $\beta$ eases the comparison of the Bauschinger effect between materials with
different flow stresses. In this calculation, the forward flow stress is simply the final flow stress before the load reversal. Several choices can be made for the reverse stress, depending on the offset used to determine the reverse yield stress. Figure 6.7 shows $\beta$ values calculated as a function of aging time for 2524, computed using various plastic strain offsets to determine the reverse yield stress. If the offset is too small, the results can be significantly clouded by very small continuum elastic effects or experimental anomalies [10]. These continuum effects can be eliminated by using a yield offset definition of 0.4%, which is double the commonly used 0.2% offset. Using this larger offset removes the scatter while preserving the trends in the data. Large strain offsets decreases the value of $\Delta\sigma$, effectively compressing the $\beta$ axis, which makes differentiating between samples with similar Bauschinger effects more difficult. For this example, the magnitude of the Bauschinger change is large enough that there is virtually no disadvantage to using offsets as large as 2%.
6.2.2. **Stress Based Parameter – $\beta'$**

An alternative Bauschinger stress measure, $\beta'$, can be defined by normalizing by the yield surface diameter rather than the forward flow stress.

$$\beta' = 2 \frac{\Delta \sigma}{\sigma_{\text{forward}} + \sigma_{\text{reverse}}} = \frac{\sigma_{\text{forward}} - \sigma_{\text{reverse}}}{\sigma_{\text{forward}} + \sigma_{\text{reverse}}}$$  \hspace{1cm} (6.2)

Figure 6.8 shows the data from Figure 6.7 plotted as $\beta'$, which qualitatively shows the same trends as $\beta$. For small Bauschinger effects, the two parameters are identical, but because of the changing denominator, $\beta'$ increases more rapidly as the severity of the Bauschinger effect increases.

Figure 6.9 shows these two parameters compared to the non-normalized value, $\Delta \sigma$, using a 0.4% offset. The values for $\Delta \sigma$ are plotted on a secondary y-axis with units of MPa. This figure shows that the general form of the response is unaffected by the

![Figure 6.8: $\beta'$ values as a function of aging time for compression-tension tests of 2524, computed using various plastic strain offsets.](image-url)
Figure 6.9: Comparison of $\beta$, $\beta'$, and $\Delta\sigma$ as a function of aging time for compression-tension tests of 2524.

normalization procedure. The underaged and overaged materials have a similar yield stress, so the forward stress used in the normalization is similar for these two situations. The largest impact of the normalization is in the peak aged condition, but because the Bauschinger effect is rapidly increasing in this region, any change in the behavior is difficult to discern.

6.2.3. Strain Based Parameter - $\xi$

In addition to the flow stress difference, it is also helpful to quantify the transient period observed after the load reversal. As mentioned in the introduction to the Bauschinger effect, the long term transient appears to be dominated by dislocation rearrangement and redistribution. For many materials, after some strain, the material returns to the same state it was during the initial forward deformation. If the long-term strength of the material after the transient is a function of the effective strain, the reverse curve will rejoin with the monotonic curve, and the characterization of the transient is
relatively straightforward. In situations where there is a permanent offset, the transient interpretation is more difficult because the steady-state value is different from the monotonic curve. Similar uncertainty arises for large strains where the equivalent strain is larger than the uniform elongation of the monotonic tension test. In this situation some assumptions about the form of the monotonic curve must be made. For the aluminum alloys used in this work, the Voce hardening law provided an acceptable fit and allowed the flow stress to be extrapolated to the higher strains.

Because of the dramatic difference in the forward and reverse work-hardening rates, one possible measurement of the Bauschinger transient length is the strain required to obtain a work-hardening rate of some comparable magnitude to the original forward rate. One possible choice would be the strain needed to obtain a work hardening rate

![Figure 6.10: Method to calculate Bauschinger parameter ξ.](image)

Figure 6.10: Method to calculate Bauschinger parameter ξ.
Figure 6.11: Tensile work-hardening rate as a function of plastic strain for 2524-OA showing hardening rate at several strains.

Figure 6.12: Tensile work-hardening of 2524 OA after compressive prestraining to the values in Figure 6.11, with the $\xi$ values labeled.
twice that experienced before the strain reversal. Figure 6.10 shows this strain, which will be referred to as $\xi$.

Figure 6.11 shows the work-hardening rate of monotonically deformed 2524-OA, with the work-hardening rate at several prestrains identified. Figure 6.12 shows the work-hardening slopes of reverse flow curves after these same prestrains. The points along these reloading curves that have work-hardening rates of approximately twice the forward rates shown in Figure 6.12, are identified with the associated $\xi$ values, which are quite different. This difference is not caused by changes in Bauschinger effect; all three curves show similar work-hardening behavior. Rather, the differences is caused by variation in the forward work-hardening rate, which determines the reverse slope used to measure $\xi$. Therefore, this value is not a reliable for quantifying the Bauschinger effect.

### 6.2.4. Strain Based Parameter - $\gamma$

A convenient way to display Bauschinger data is by plotting $\Delta \sigma$ as a function of the plastic strain after reversal. This method clearly shows the length of the transient, and because it is plotted against plastic strain, the yield stress at any offset is identified by a vertical line at that strain. Examples for 2524 OA tested at various prestrains are shown in Figure 6.13. In this case, there is no permanent offset between the forward and reverse flow curves, i.e. $\Delta \sigma$ tends to zero as the plastic strain increases. For materials exhibiting a permanent stress offset, $\Delta \sigma$ will saturate to a non-zero value.

If one assumes the isotropic hardening is a function of the equivalent plastic strain, and the material is well represented by a Chaboche backstress evolution equation, then $\Delta \sigma$ is the difference between the backstress evolution for continued forward
deformation and the evolution for reverse loading from that point. For uniaxial deformation, these evolution equations can be solved analytically as,

\[
\alpha = \nu \frac{c}{\gamma} + \left(\alpha_o - \nu \frac{c}{\gamma}\right)e^{-\gamma(\eta - \epsilon_{po})}
\]  

(6.3)

where \(\nu = \pm 1\) is the direction of flow, and \(\alpha_o\) and \(\epsilon_{po}\) are the values of the backstress and plastic strain at the point of the reversal. If the curves in Figure 6.13 are fit using this equation, \(\gamma\) is the measure of the transient length. Figure 6.14 shows \(\gamma\), which is roughly a linear function of the inverse plastic strain. This plot shows the best fit gamma is a strong function of prestrain, which invalidates the constant \(\gamma\) assumption made in the curve fitting. However, if this discrepancy is ignored, \(\gamma\) can still be a very effective scalar parameter to quantify the data.

Figure 6.13: \(\Delta\sigma\) as function of \(\epsilon^p\) for 2524 OA after the indicated prestrains.
6.2.5. **Strain Based Parameter - $\delta$**

Another parameter to describe transient lengths is the half life. In this case, the half-life can be determined as the strain at which the flow curve is half-way back to the steady state value. Again, if one assumes the isotropic hardening is a function of the equivalent plastic strain, this value can be computed by finding the intersection of the reverse flow curve with the monotonic curve that has been shifted down $\Delta \sigma/2$ from the forward flow curve. This parameter is shown schematically as $\delta$ in Figure 6.15. One problem with this approach is that, when present, a permanent stress offset can severely affect the construction line determining the transient length. For severe offsets, the flow curve may never intersect and $\delta$ would be infinite, even if the reverse curve has reached a steady-state value. Perhaps even more problematic, this method depends on the calculation of $\Delta \sigma$ to determine the position of the offset monotonic curve. As shown in
Figure 6.7, if the offset used to define the reverse yield stress is changed from 0.4% to 0.6% offset, there is a difference of nearly 30% in $\Delta \sigma$, which would then propagate to differences in $\delta$.

6.2.6. *Strain Based Parameter - $\phi$*

In Figure 6.16, a strain measure is shown that is based only on the forward flow stress. By measuring the strain needed to regain a certain percentage of the forward flow stress, the complicating factors associated with the reverse yield determination can be eliminated. Another possible advantage is that this parameter can be used with no knowledge of the monotonic flow curve. While, neglect of the isotropic hardening evolution may lead to some loss of accuracy, the determination of the parameter is
unaffected by any inaccuracies in the shape of the monotonic flow curve. It may also be used in situations where the monotonic curve is unavailable. This parameter is an
inherent part of the reversed flow curve. While this parameter can also be infinite for situations with a severe permanent stress offset, this is a less likely scenario because the reference value is not increasing with strain. If it is a problem, a smaller percentage of the flow stress can be used as a reference value with little loss in accuracy, as shown in Figure 6.17. A version of $\phi$ normalized by the prestrain has been reported in the literature [24]. This is similar to the normalization done with the stress parameters to account for materials with different yield stresses. Because $\phi$ and $\gamma$ are both linearly related to prestrain, they are of similar usefulness in quantifying the transient length.

![Diagram of stress and true strain with areas $W_1$, $W_2$, and $W_3$ highlighted](image)

Figure 6.18: Areas used to calculate the energy based Bauschinger parameters.

### 6.2.7 Work Based Parameter

The final class of quantification parameters is based on the energy or work differences in the forward and reverse directions. These parameters have the advantage
in that they are based on both the stress and strain and can potentially an average 
response of the stress and strain measures. Figure 6.18 shows the areas that represent the 
work quantities used in the analysis. In the literature, the value W_1/W_p, is proposed to be 
a measure of the proportion of the energy input into the system that is regained on the 
reversal [24]. As such, it is zero if there is no Bauschinger effect and increases as the 
Bauschinger effect becomes more severe. The advantage of this parameter is that it can 
be calculated without knowledge of the monotonic curve.

However, with the monotonic information present the parameter (W_1 + W_2) is 
proposed here as better measurement. This area is the difference in the work per unit 
volume needed to deform the material to a particular strain value in the forward and 
reverse directions. In this case, the strain chosen was the strain needed for the reverse 
curve to reach the forward flow stress, but other strain values are possible. This 
parameter can be used in the current form as the work per unit volume as in Figure 6.19 
or alternatively it can be normalized by the work input during the forward deformation to 
give a unit-less parameter describing the relative energy differences. The curve in Figure 
3.1 shows that this parameter is also a strong indicator of the Bauschinger effect, and can 
be used to assess the differences between various curves.
Figure 6.19: Work based Bauschinger parameter as a function of prestrain for 2524 OA.
CHAPTER 7

YIELD SURFACE EVOLUTION OF ALUMINUM ALLOYS 6013 AND 2524

7.1. Effect of Precipitation Heat Treatment.

Figure 7.1 and Figure 7.2 plot the tensile flow curves up to the necking point for 2524 and 6013, respectively. These results show the proposed heat treatments are effective in producing a peak-aged temper which is significantly higher than both over-aged and under-aged conditions. In alloys, the hardening curve of the under-aged sample is similar to a solution-heat-treated sample, with only a small increase in the flow stress. Looking the curve for 2524, one sees the flow curve for the over-aged condition is below even the solution heat treated case. This is largely a result of the rapid cluster formation discussed in Section 2.1.1 that occurs even when the specimens are stored in dry ice, as well as a lack of solute hardening in the over-aged material.

Figure 7.3 shows the relative increase of the yield stress and Bauschinger effect for 2024 as it is artificially aged at 230°C for a 4% prestrain. The values at each time are normalized by the initial values in the SHT condition to give a relative increase. The first quantity plotted is the yield stress, showing the evolution of the aging process. The other two parameters are stress and strain based Bauschinger parameters, $\beta$ and $\phi$, introduced in Section 6.2. As the material is artificially aged, the yield stress reaches a maximum after
several hours of aging, whereas the magnitude of the Bauschinger effect continues to increase and reaches a maximum value in the over-aged condition. For the times observed, the strain value appears to saturate at the same values, whereas the stress parameter decreases. This decrease is partially attributable to the falling yield stress, but
it is not solely an artifact of this change because the magnitude of the reverse yield stress is also increasing in this region. Both these factors contribute to the decrease in $\beta$. These results are consistent with results of Hidayetoglu, et al. who found the Bauschinger effect of 2024 increases during aging at 190°C until about 100 hours at which the effect stabilizes. They also found that for extreme overaged conditions (2 hours at 300°C) the Bauschinger effect was intermediate between the initial SHT value and the maximum observed during the continuous aging at 190°C [102].

The companion aging curve for 6013 is found in Figure 7.4. The behavior in this alloys system is similar to 2524, but the trends are not as well defined. Again, the Bauschinger parameters reach a maximum value at aging times after the peak hardness is attained. As in 2524, the Bauschinger stress reaches a peak value, but the peak is much sharper for 6013. As in 2524, this decrease in the Bauschinger parameter is caused both
by a decrease in the forward flow stress as well as an increase in the magnitude of the reverse flow stress. The strain parameter is somewhat difficult to interpret because it displays significant fluctuation in the over-aged condition.

In both these materials, especially 2524, the SHT condition is unstable and the material may naturally age before testing. Because the Bauschinger behavior develops more slowly than the yield stress, the more kinetically stable UA temper may be able to be used exclusively in place of both UA and SHT. Tension-compression curves for 2524 and 6013 are shown in Figure 7.5 and Figure 7.6. As mentioned before, these tension-compression results are susceptible to the locking problems discussed in Section 6.1.1. Qualitatively, the curves show similar magnitudes of yield point reduction, transient length and offset. When these two tempers are compared with other tension-compression data from the aged conditions, Figure 7.7, the \( \beta \) for the UA and SHT tempers are similar, suggesting analysis of both UA and SHT is unnecessary.
Figure 7.5: Tension / compression results for Al 2524 SHT (a), and UA (b) tempers.

Figure 7.6: Tension / compression results for Al 6013 SHT (a), and UA (b) tempers.
To uncover the effect of prestrain on the Bauschinger effect, compression-tension tests were performed on both alloys in the UA, PA and OA conditions. The compression-tension curves for 2524 and 6013 are plotted up to the UTS (ultimate tensile strength) in Figure 7.8 and Figure 7.9. As was mentioned in Section 6.2, a useful way of looking at the Bauschinger response is to plot $\Delta \sigma$ as a function of $\varepsilon^p$ after the reversal. The data is plotted in this way in Figure 7.10 and Figure 7.11. Note that the strain range of the UA plot is larger than PA and OA. As was seen in Figure 7.3 and Figure 7.4, the Bauschinger effect is much larger in the OA condition than the UA condition. There is a dramatic difference between the UA and OA tempers for all three important Bauschinger features—reverse yield stress, hardening transient and permanent offset. In between these two extremes, the PA condition is an intermediate transition state, and as such, it displays some features from both the OA and UA conditions.
Figure 7.8: Compression-Tension results for 2524 UA (a), PA (b) and OA (c) tempers.
Figure 7.9: Compression-Tension results for 6013 UA (a), PA (b) and OA (c) tempers.
Figure 7.10: $\Delta \sigma$ versus reverse plastic strain for 2524 UA (a), PA (b) and OA (c).
Figure 7.11: $\Delta \sigma$ versus reverse plastic strain for 6013 UA (a), PA (b) and OA (c).
Both the reverse flow curves and plots of $\Delta \sigma$ show the reverse yield point of the OA temper is reduced more severely than UA. The $\Delta \sigma$ plots also clearly show the differences in the length of the yield transient. The OA materials have a significantly longer strain transient, which is a much stronger function of the prestrain.

Looking at the OA temper, the reverse flow curves saturate to the same stress level as the monotonic flow curve, producing no permanent offset. In the UA condition, the UTS for each of the materials is a function of the prestrain. Materials with higher prestrains saturate to a lower stress and produce a larger permanent stress offset. For 2524 UA the evolution of this offset magnitude as a function of prestrain is easily seen in Figure 7.10(a). For 6013 UA at low prestrains, there initially appears to be an offset, but after a very long transient, it eventually regains its strength and breaks at a similar UTS. This is the feature of a strain offset as was described in Figure 3.3. However, even though the offset does not appear to be permanent its intermediate values also increase with prestrain.

The reverse loading curves for the largest prestrained samples of each temper are shown in Figure: 7.12. These curves clearly show the increase in the transient length associated with increasing aging, with the PA materials showing an intermediate response. In 2524, PA has a similar initial yielding behavior to UA, but then like OA shows no permanent offset. In 6013 the curve for PA lies within the bounds of the UA and OA samples. The long-term transient shown in the OA tempers is intriguing because it is much longer than the initial prestrain, and can not effectively be explained by residual elastic stresses arguments.
Figure: 7.12: $\Delta \sigma$ versus $\varepsilon^p$ for largest prestrains tested for 2524 (a) and 6013 (b).

7.2. Effect of Prestrain on the Bauschinger Effect

7.2.1. Uniaxial Testing

Because 2524 OA shows the most severe Bauschinger effect of the alloys and tempers studied in this work, a more detailed study of the effect of prestrain was performed on this condition. Figure 7.13 shows the monotonic tension curve along with the tensile reloading curves of various compression-tension tests. In this figure, the reloading curves have been shifted by their prestrain values, so that all the curves start at the origin and can be compared to one another. The most obvious observation from this figure is that as the prestrain increases, the length of the transient increases. Another interesting feature is the curves are similar up until a plastic strain of about 0.003, after which they begin to deviate and show disparate behavior. This strain magnitude is
Figure 7.13: Monotonic and reverse loading curves of 2524 OA with various prestrains.

similar to that of the elastic continuum effects caused by the elastic mismatch of the precipitates and the matrix [10]. After this period of common behavior, increasing aging leads to a drop in the reverse flow stress. This suggests the different nature of the curves is attributable to non-continuum interactions with the precipitates.

As was shown in Figure 6.17, the length of the Bauschinger transient is a linear function of the prestrain where \( \phi \), or \( \varepsilon_{\text{transient}} \), is equal to 1.8 \( \varepsilon_{\text{prestrain}} \) with a slope of about 1.8. This result is quite reasonable in light of Orowan’s explanation of the Bauschinger response. Assuming on one dimension there is a single dislocation. If the material is initially isotropic on the dislocation’s slip plane, the resistance to deformation will be the same in each direction. This situation is schematically represented in Figure 7.14, where \( \tau \) is a representation of the resistance to the dislocation’s motion as a function of distance it moves on the slip plane. Assume during the forward deformation the dislocation
Figure 7.14: Schematic of dislocation and the resistance to motion on its slip plane.

moves a distance $d$ along the slip plane. If the sign of the force is then reversed, the dislocation will be able to move a distance $2d$ before it reaches another point on the slip plane with a similar resistance. This is, admittedly, a very basic picture of the real situation, but seems to agree quite well with the value of 1.8 observed in 2524 OA. The minor difference can be partially attributed to the fact that during the forward deformation, slip activity on other slip systems likely increase the forest dislocation content, increasing the resistance in the opposite direction making motion more difficult.

This explanation suggests that the length of the Bauschinger transient depends on how severely the slip plane resistance changes with deformation. For materials where the slip plane resistance changes very little, the Bauschinger transient will be very large with a maximum of $2\varepsilon_{\text{pre}}$ as is seen in materials with stabilized hysteresis behavior. Whereas, materials where the microstructure is rapidly changing, such as the UA and SHT conditions where dislocation networks develop and dynamic precipitation is occurring, the transient length is shorter.

This explanation does a good job at describing the presence of the long-term transient response, but it does not do a good job describing rapid evolution of the
backstress with prestrain. At very small prestrains, of 0.0038 show significant yield and hardening changes when the loading direction is reversed. In this case, the length of the transient is even greater than the maximum value of $2\varepsilon^{\text{pre}}$ predicted for pure hysteresis behavior. As will be discussed later, modeling these contrasting features presents some difficulty.

### 7.2.2. Multiaxial Testing

The directionality of the yield surface contraction can be probed for each alloy and temper using the shear and orthogonal testing techniques introduced in Section 6.1. Figure 7.16 shows the results for 2524, and Figure 7.15 shows the information for 6013. Each specimen was subjected to the tensile prestrain indicated in the figure, then tested in one of the following modes: continued tension ($+\sigma_{11}$), uniaxial compression ($-\sigma_{11}$), orthogonal tension ($+\sigma_{22}$), shear where original tensile axis is compressed ($-\sigma_{12}$), and shear where original tensile axis is elongated ($+\sigma_{12}$). In order to compare the different tempers and minimize the effects of material variation, each of the stresses are normalized by the yield stress of that test at zero prestrain. These figures again show the decreased yield point in the reverse direction as compared to the forward deformation, especially in the aged conditions. The most basic observation derived from these tests is that the strength along the different directions does not rise and fall proportionally, which suggests the yield surface shape is changing.

The orthogonal test is illustrative because it can be an effective way to decouple the kinematic and isotropic hardening, because there is no backstress evolution in the
Figure 7.15: Comparison of yield strengths for various strain paths of 2524 .UA (a), PA (b), and OA (c).

orthogonal direction for uniaxial deformation. For materials with a large precipitate induced Bauschinger effect, the forward deformation is dominated by kinematic hardening. There is little hardening, and perhaps some softening in the orthogonal direction. In the UA condition, the isotropic component is a much higher proportion of the total hardening. In the under-
Figure 7.16: Comparison of yield strengths for various strain paths of 6013 .UA (a), PA (b), and OA (c).

aged material, the hardening in the orthogonal direction is similar to the original tensile direction.

The shear test is also interesting because the induced deformation is a combination of the initial loading direction and the orthogonal direction. In the case, where the original tensile axis is elongated, one would expect the strength to be some
combination of the tensile and orthogonal strength. When the shear is oriented so that the original tensile axis is under compression, the reasonable assumption is that the strength would be related to the orthogonal and compressive strengths. In general this is what is observed 2524.
CHAPTER 8

MODELING THE BAUSCHINGER EFFECT

Because 2524 shows very distinct Bauschinger behavior in the UA and OA conditions, this alloy is used to display the effectiveness of the numerical modeling routine.

8.1. Fitting the Material Data to the Hardening Model

As mentioned in Section 5.3, the non-linear kinematic hardening model with an additional linear backstress was chosen to model the material behavior. This form was chosen because it enables the modeling of a permanent stress offset and provides flexibility in the reversal transient. This evolution equation for the backstress in this model is:

\[
d\alpha_1 = C_1 \frac{\sigma - \alpha_1}{\sigma_e} dp - \gamma_1 \alpha_1 dp \\
d\alpha_2 = C_2 \frac{\sigma - \alpha_2}{\sigma_e} dp \\
d\alpha = d\alpha_1 + d\alpha_2
\]  

(8.1)

The most straightforward parameter in this equation to fit is \(C_2\), which is related to the permanent stress offset. The final offset measured at the UTS of the reverse flow curve is plotted for 2524 UA in Figure 8.1. The permanent stress offset is exactly \(\alpha_2\), so the slope
Figure 8.1: Amount of permanent stress offset measured from the compression-tension of 2524 UA.

of this curve as a function of plastic strain is $C_2$. 2524 PA and OA are assumed to have no stress offset.

The parameter $\gamma_1$ controls the saturation rate of the backstress. Finding $\gamma_1$ is the most difficult aspect of fitting process because it governs both the rate at which the backstress develops with prestrain, as well as the length of the reverse transient hardening. In the discussion of the Bauschinger parameter $\gamma$, in Section 6.2, Figure 6.14 showed that $\gamma$ can be represented as a linear function of the inverse of the prestrain. For each temper, $\gamma_1$ can be plotted as a function of prestrain and the slope and intercept values calculated using a linear-least-squares analysis to provide an expression for the strain dependency of $\gamma_1$.

Once $C_2$ and a strain dependent $\gamma_1$ are known, the final kinematic variable to determine is $C_1$. This variable can be found by matching the numerically computed backstress to that experimentally measured from the forward and reverse yield stress.
produce a reasonable fit, some method must be made to account for the disparity between
the rapid evolution of the Bauschinger effect with prestrain and the long-term transient
behavior on reversal. As discussed in Section 4.2, one way to account for this anisotropy
is through rapid changes in the yield surface shape [60, 61]. However, because this
evolution is so rapid, an alternative method, utilized here, is to assume an initial value for
the backstress, $a_o$. The biggest drawback to this method is that care must be taken to
assure this initial backstress is in the proper direction to assure the yield anisotropy is in
the correct direction. Once this assumption is made the analytical representation of the
backstress is fit to the experimental results by changing $C_1$ and $a_o$, as is shown in Figure
8.2. Table 8.1 shows the final values for the kinematic hardening variable used to model
the three tempers of 2524.

![Figure 8.2: Experimental and numerical representation of the backstress evolution for 2524.](image)
### Table 8.1: Values of kinematic hardening variable, $\gamma$, for each test condition.

| Temper | $\gamma_1$ | $C_1$ | $|a_0|$ |
|--------|------------|-------|-------|
| UA     | -0.189     | 255   | 43.0  | 0     |
| PA     | 0.021      | 25.7  | 30.8  | 52    |
| OA     | 0.438      | 16.6  | 76.1  | 54    |

8.2. **Results of the Material Model.**

These kinetic variables and the monotonic flow curve, were input into the UMAT and used in ABAQUS to model tension and compression for four shell elements. Figure 8.3 shows the comparison of the measured compression-tension tests to the results obtained from ABAQUS. In general, the material model is able to reproduce the major features of the flow curve. The largest discrepancy is OA 2524, in which the hardening is not well reproduced for the larger prestrains. The model does reproduce the increase in the transient length, as seen from the strain at which it rejoins the monotonic curve, but the experimental hardening curve is more linear than the exponential function used in Chaboche’s kinematic hardening model. The other two tempers, where the reverse hardening curve is better represented by the exponential function show very good agreement for both the reverse yield and hardening behavior.
Figure 8.3: Comparison of experimental and simulated tension-compression results for 2524 UA (a), PA (b), and OA (c).
8.3. **Comparison of Simulated and Experimental Results**

8.3.1. *Tension / Orthogonal Tension*

The tension compression experiments were used to fit the experimental parameters, so a reasonable fit between the experimental results and the simulations is expected, limited only by the number of parameters in the model, in this case the yield curve and five kinematic variables. The orthogonal tension results were not used in the fitting procedure and can provide an independent assessment of the model under a more general strain path. Figure 8.4 the evolution of the yield stress along the different strain paths, where the flow stress at each prestrain is normalized by the initial yield stress. This normalization shows the relative hardening along each strain path and facilitates comparisons for materials with different flow stresses. The model does in a qualitative way reproduce the dissimilar behavior of the under-aged material to the other two tempers. The simulated orthogonal strength of the UA larger than the other two tempers due to the more isotropic hardening. The large deviation of the PA and OA tempers is consequence of the initial backstress tensor, which does not have a component in the orthogonal, even though there is, clearly, limited hardening in this direction. In general, the experimental results for PA and OA materials, show a similar work-hardening rate as the simulations, but disagree on the starting values in the first 2% strain. This change could be corrected by replacing the assumption of a constant yield surface shape and initial backstress and replacing it with a yield surface distortion model.
8.3.2. Drawbead Simulation

To assess the effects of this UMAT on the results of real metal forming analysis, the UA, PA and OA material models were input into a model of a drawbead, and the results using the UMAT were compared to the same model assuming isotropic hardening. In this simulation, the sheet is modeled by 1000 shell elements, which are clamped and drawn through the drawbead. An image of the final deformed shape is after being pulled through the drawbead is shown in Figure 8.5. The maximum plastic strain in the part is approximately 0.22. The representative history of the force needed to pull the sample through the bead is shown in Figure 8.6. After some initial period, a steady state value for the drawing force is established. The variation is due to changing contact conditions and can be reduced by mesh refinement. The average draw force was computed for each case by taking the average of the points from 1.25 to 1.75 seconds. These forces are
Figure 8.5: ABAQUS model for the draw-bead simulation.

Figure 8.6 Simulated force vs. time for 2524 OA assuming non-linear kinematic hardening.
Table 8.2: Comparison of steady-state forces computed in ABAQUS drawbead simulation for 2524.

<table>
<thead>
<tr>
<th>Isotropic Hardening</th>
<th>UMAT Force (N)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>UA</td>
<td>1270</td>
<td>1150</td>
</tr>
<tr>
<td>PA</td>
<td>1500</td>
<td>1330</td>
</tr>
<tr>
<td>OA</td>
<td>1060</td>
<td>810</td>
</tr>
</tbody>
</table>

shown in Table 2.1. These results indicate a significant change in the drawing force predicted by the material model used in this work and that predicted assuming isotropic hardening. This improvement does come at a computational cost. The difference in the computation times for these analyses is shown in Table 8.3. These differences are significant, but are reasonable in light of the drastic difference in the simulated forces.

Table 8.3: Comparison of CPU time for ABAQUS drawbead simulation for 2524.

<table>
<thead>
<tr>
<th>Isotropic Hardening</th>
<th>UMAT CPU Time(s)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>UA</td>
<td>1567</td>
<td>2525</td>
</tr>
<tr>
<td>PA</td>
<td>1506</td>
<td>2020</td>
</tr>
<tr>
<td>OA</td>
<td>1961</td>
<td>2740</td>
</tr>
</tbody>
</table>
8.3.3. Results of Draw-Bend Testing

The draw-bend test has been proven to be a valuable tool for examining springback of formed parts because it closely mirrors the actual conditions in a forming operation, as the sheet deforms by a combination of bending and stretching. In this test, a strip of material is drawn over a tooling radius under the constant influence of a back-force, which resists the drawing motion [75, 90, 97, 103]. As shown in the schematic in Figure 8.7, the strip, which is initially 90°, springs back an angle, $\theta$, the springback angle.

![Figure 8.7: Schematic of the draw-bend test [97].](image-url)
Accurate prediction of the springback angle requires careful numerical methods [103] and accurate material representations of both the initial yield anisotropy and the hardening evolution [90]. The implementation of the model in this work was used to simulate the draw-bend test. The results of this simulation and simulations assuming isotropic hardening were then compared to experimental results.

Both the simulations and the experiments pulled the sheet 101.6 mm over a tool of radius 19.05 mm. The model used to simulate the draw-bend process consisted of 2100 shell elements, with 51 integration points through the thickness of the sheet. The simulation run time using the UMAT material model was approximately three times longer than the isotropic hardening model.

The results of the draw-bend tests and simulations are shown in Figure 8.8. The springback angle is plotted against the back force normalized by the yield force. The transition in the springback angle at intermediate back forces is caused by the development of anticlastic curvature, and is not expected to be reasonably captured with the simple Von Mises yield function assumed in this initial implementation [97]. At lower forces, this curvature does not develop, and the simulations are expected to produce reasonable results.

The 2524 UA show the unexpected result that the springback results are better explained by the isotropic hardening model. The reason for this discrepancy is as yet undetermined, but could be real or due to errors in either the simulation or experiment. The OA and PA results show better agreement between the experimental and the simulation using the new UMAT. In both these cases, the new model is able to predict
the experimentally measured angle within 1.5°, while the isotropic hardening assumption differs by over 10° at the lower back-forces.

Figure 8.8: Results of the draw-bend test for 2524 UA (a), PA (b) and OA (c).
CHAPTER 9

CONCLUSIONS

This work sought to understand the effect of changing precipitate structure on the mechanical properties of age-hardenable aluminum alloys subjected to non-proportional deformation, and incorporate these findings into a finite element program. In order to accomplish this, advances were necessary in the mechanical testing as well as numerical modeling fields.

The following conclusions were made about the measurement and analysis of the Bauschinger effect:

1. A new method has been developed for measuring continuous tension-compression curves of sheet metal using a tensile frame and some simple tooling. The criteria for specimen optimization were established, and curves and techniques for estimating the limits of the measurable strain as a function of sample material and geometry were presented. This device also allows for efficient exploration of many different types of physical phenomena associated with non-proportional loading of sheet metals, including room-temperature creep, and anelasticity.

2. While the tension-compression device does suppress buckling, the samples may experience locking during compressive deformation. This locking occurs gradually, making corrections difficult. Because the Bauschinger effect manifests itself in the reverse flow curve and tensile deformation does not experience
locking, the preferred strain path in this work is compression followed by tensile
reloading.

3. Standard scalar measurements of the Bauschinger stress change were found to
depend significantly on the offset strain used to define the reverse yield point. An
offset greater than 0.004 was determined to be necessary to avoid noise in the
measurement associated with experimental error and small elastic continuum
effects. Both the raw stress difference, $\Delta \sigma$, and the normalized parameters $\beta$ and
$\beta'$ show the same dependence of aging on the Bauschinger effect.

4. Measurements of the length of the hardening transient based upon work hardening
rates in the forward and reverse direction have been shown to be unreliable.

5. A measurement of the transient length, $\phi$, can be computed from the strain
necessary for the reverse flow stress to regain a fraction of the original forward
stress was found to be a simple and reliable way to measure the transient. For
2524 OA, this parameter varies linearly with the amount of prestrain.

6. If the monotonic curve is available and certain assumptions are made for the
kinematic hardening of the material, a parameter $\gamma$ can be calculated. This
parameter was found to be a linear function of the inverse of the prestrain, and can
be used to help fit the kinematic hardening model.

7. A Bauschinger work parameter was also suggested to which compares the energy
needed to deform the material in the forward and reverse directions up to a certain
strain.
Experimental studies of the Bauschinger effect of 2524 and 6013 revealed the following characteristics:

8. For both 2524 and 6013 the Bauschinger effect increases as the sample is artificially aged, and reaches a maximum in the over-aged state, after peak-hardness had been attained.

9. Because initially the Bauschinger effect evolves relatively slowly with aging, the SHT condition was not studied extensively because the UA state was found have sufficiently similar behavior, while being more metallurgically stable.

10. All three of the main features identified with the Bauschinger effect—reverse yield point, hardening transient and offset—are significantly different in the UA and OA tempers. The PA temper shows an intermediate response between the two.

   • The reverse yield point for the OA material is reduced more than in the UA case.

   • The UA temper contains a prestrain dependent offset, while the OA does not. This offset appears to be a permanent stress offset for 2524 UA, and a strain offset for 6013 UA.

   • The OA material also shows a much longer period of transient hardening after the stress reversal., which is a strong function of prestrain.

11. The initial plastic behavior up to 0.004 strain was very similar for 2524 OA samples with various amounts of prestrain. After this initial period, which is on
the order of the strains associated with continuum elastic effects, the Bauschinger
effect was found to be a strong function of prestrain.

12. Experimental tests show transients that persist for reverse strain over 10% strain,
suggesting explanations for the Bauschinger effect based on residual elasticity or
Orowan dislocations are insufficient.

13. The yield surface appears to change rapidly with deformation. Because the
change is so rapid, it can be approximated for modeling purposes by an initial
backstress, $a_0$; however, the directionality of this backstress does not assure the
correct behavior for multi-axial loading.

The following results were obtained from the numerical implementation of the
material model.

14. A material model was created and implemented into a user material subroutine for
the commercial finite element program ABAQUS. This material model was fit
for 2524 using the results from the compression-tension experiments and is able
to reproduce the experimentally observed behavior except for the reverse
hardening of the OA temper, which is not well fit by the exponential function
assumed in the non-linear kinematic hardening model used.

15. As an independent assessment tool, experimental results from tensile tests
orthogonal to an initial tensile prestrain were compared to the predictions from the
material model. These comparisons showed that while the general trends were
followed there is room for improvement, perhaps by better understanding the
relation of the yield surface to deformation.
16. A strip of material drawn through a drawbead was simulated in ABAQUS using the material model created in this work and an isotropic hardening assumption. Using the non-linear kinematic hardening model increases the computation time for this problem by 61% for UA, 34% for PA and 40% for OA. However, using the model significantly changed the computed steady-state drawing force by 10% in the UA case and as much as 24% for OA.

17. Simulations of the drawbead test using the new material model have been performed and require an increase the run time by a factor of three compared to isotropic hardening models. While there is some disagreement between the simulated and experimental results in 2524 UA, for the region where agreement was expected, the simulated angles for 2524 OA and PA were within 1.5° of the experimental results.
APPENDIX A

CONTINUOUS, LARGE STRAIN, TENSION-COMPRESSION TESTING OF SHEET MATERIAL

This portion of the manuscript has been published in slightly modified form in the

A.1. History of Compression Testing

The fact that a material’s mechanical properties depend on its loading path has
been well known for over a century from the work of Johann Bauschinger [104].
Bauschinger’s results showed that the yield stress of mild steel is lowered by prior strain
in a direction opposite of the testing direction. Subsequent phenomenological and
microstructural descriptions of the Bauschinger effect have been concerned not only with
the initial reverse yield point, but with the entire stress-strain response after reversal.
Review articles have appeared describing the Bauschinger effect for many materials. [24, 105, 106].

In recent years, there has been renewed emphasis on understanding mechanical
behavior under non-proportional paths as it relates to simulating forming processes.
Simple material models, such as isotropic hardening, are not sufficient to predict
springback of formed parts after removal from a die [107]. Because of the need for more
refined constitutive relations, many new continuum and physical models have been
developed to describe materials that undergo load reversals [75, 80, 89, 108, 109]. In order to fit these new constitutive equations, experimental methods are required to test materials under non-proportional loading in an accurate, reliable and reproducible manner.

Several methods have been developed to test materials along reverse loading paths. Reverse torsion [44, 50, 110] or a combination of torsion and tension [111] has been used, with strains approaching 6.0 possible with appropriate samples [44]. Sheet material can be tested using torsion methods by welding the sheet into a thin-walled tube. However, rolling and welding the sheet to form the tube can change the structure of the sheet, thus altering its macroscopic properties. Another disadvantage of torsion testing is that stress and strain are not uniform throughout the cross section of the sample, making the transformation of the torque/twist data to shear-stress/shear-strain curves indefinite. This is similar to the situation in reverse-bend tests [75], where the constitutive behavior derived is not unique if the reverse loading curve varies with prestrain.

The Bauschinger effect has been studied through reverse shear, [112] as well as combined loading with shear and some other mode, such as tension [56], with strains up to 0.5 attainable. With appropriately shaped specimens, the stress and strain distributions in the sample are relatively uniform for low strains. However, as the strains increase, shear bands may develop and end effects become problematic [100]. The strain levels at which this localization becomes a problem depends on both the material and geometry of the specimen.

These torsion and shear techniques, along with more exotic methods, such as thin-walled tubes under axial load and internal pressure [113], are valuable for understanding
plastic behavior; however, uniaxial, in-plane testing is preferred for constitutive equation
development because the deformation is uniform over the entire sampled volume and the
results are more easily interpreted. Uniaxial compression testing of bulk material has
been standardized in the industrial community [114] using cylindrical specimens with
favorable aspect ratios to prevent buckling for strains over 0.05 [115, 116].
Unfortunately, large-strain, in-plane compression is difficult to attain in sheet materials
because of buckling modes that develop.

Compression testing of sheet material generally takes one of two forms. The first
approach emulates the geometry of the bulk compression test, which has a length-to-
diameter ratio of 3 [114]. Following this approach, Bauschinger tests have been
developed using small cylindrical [105] or rectangular specimens [33, 117]. The
effectiveness of these techniques depends on the gage-length/thickness ratio of the
deforming sheet. Tests in the literature show attainable strain ranges that vary from 0.01
to 0.15 as the length/thickness ratio varies from 16 to 2 [33, 105]. For comparison, a
standard ASTM tensile bar requires a gage-length/thickness ratio greater than 2.67 and is
often larger than 20 [118]. For higher-strain compression tests, where the length-to-
thickness ratio approaches 2, the stress state in the deforming volume is unlikely to be
either uniform or uniaxial. The other major disadvantages of this type of test are that the
specimen size is often so small that the material may not be homogeneous for large-
grained microstructures, and strain measurements using traditional methods can be
difficult.

In-plane compression testing can also be accomplished using standard-sized
specimens with side loading, or constraint, to suppress buckling in the thickness
direction. Traditional techniques support the sides of the sample with a series of steel pins or rollers [119, 120], or use solid supports [121-126]. A laminate of several samples may also be used in conjunction with side constraint in what has been called the “pack method” [120]. These methods are unable to probe large strains because of buckling outside of the supported region. The largest unsupported length that can be tolerated for the above methods is approximately 1.25 mm, which typically enables compressive strains of 0.01-0.02 [120]. In addition, samples cannot be continuously deformed during a tension-compression reversal because of the gripping arrangement. Studies of the Bauschinger effect using these methods require a two-step method where the sample is prestrained, unloaded, remounted, and then compressively loaded in another fixture. This is disadvantageous, particularly for cases where reverse flow begins before the sample is completely unloaded from the initial tension. Continuous measurement is preferable for all reverse testing because it allows for consistent observation of the transition from tensile to compressive flow, and assures that little or no microstructural change or aging takes place during resting times between segments of the tests [56].

Recent work utilizing solid supports has been reported by Tan et al. [127] and Yoshida et al. [88]. Both groups used dogbone samples that could be pulled in tension and compression. To prevent buckling in the unsupported region, Tan et al. created small samples with gage-length-to-thickness ratios from 10 to 2, allowing compressive strains of 0.03 for the larger ratio and almost 0.20 for the smaller ratio. This hybrid approach, utilizing both small specimen size and side support, effectively improves the attainable strain range, but suffers from the same limitations of the small scale tests, in addition to buckling in the unsupported region. Yoshida et al. [128] used a variation of the pack
method, where 5 sheets were laminated together to provide support in addition to plates. This method was able to measure compressive strains up to 0.25 for mild steel and 0.13 for high strength steel.

Another approach to improve upon the limited strain range of supported specimens was developed by Kuwabara et al. [129], who used two pairs of comb-shaped, or fork-shaped, dies to support the sample. This design is an improvement over the solid supports because, as the sample is compressed, the male and female dies slide past each other allowing the entire length of the specimen to be supported. By eliminating the interference problem between the platens and the support fixture, strains on the order of 0.15-0.20 were attainable for single sheets of material under compressive loading [129].

Balakrishnan and Wagoner [130] extended this method to test un laminated sheet material in continuous, sequential, compression/tension tests. Special fixtures were designed for use with a standard tensile frame and a dogbone specimen. The sample was sandwiched between two sets of fork-shaped supports, similar to those of Kuwabara [129] and a supporting force was applied to the sample using a hydraulic hand pump. This device was able to achieve compressive strains of 0.08 continuously during both reversed and cyclic tests [75, 130].

The fork devices have several limitations. The specimen design is rather long and slender, and it is difficult to maintain axial alignment of the tensile axis for various sheet thicknesses. This misalignment leads to reductions in the compressive strain range attainable before buckling. The misalignment can be compensated for, to some extent, by increasing the side load, but this introduces increased error through larger friction and stress biaxiality. From a practical standpoint, repeated use at the required large force
eventually damages the forks, which are expensive and complex to machine for periodic replacement.

In the current work, a new test design was sought combining the advantages of the miniature, side-supported tests—continuous, large-strain, reversed strain paths—with the practical advantages of larger specimen sizes—homogeneity, self-alignment in a standard tensile testing machine, and simple, accurate measurement of uniform stress and strain.

A.2. New Tension-compression Approach

In order to avoid the limitations of existing designs for in-plane compression, a new approach was developed, shown in Figure A.1. Solid, flat plates are used for buckling constraint, and a special specimen design was developed to minimize buckling outside of the constrained region. The solid plates offer several advantages over fork designs including better self-alignment, much easier machining, and better durability. As will be shown, the improved alignment also allows reduced constraining force, and therefore more-nearly uniaxial loading.
Figure A.1: Schematic of the flat plate supports and final sample dimensions.

For 6022-T4 (thickness = 2.5 mm):
G = 36.8 mm, W = 15.2 mm
B = 50.8 mm, L > 3 mm

Figure A.2: Assembly of new plate method.
Replacing the forks with solid side plates reintroduces an undesirable (for buckling) unsupported region of the specimen. To prevent the sample from buckling in this region, an exaggerated dogbone specimen was designed to assure the load in the unsupported gap will be lower than the critical buckling load. By adjusting the dimensions of the sample, a large enough clearance can be sustained to allow considerable stable compressive strain.

The clamping system used to provide side support for the fork device [130] was modified slightly for the new method, Figure A.2. The same Enerpac™ P141 hand pump and RWH200 hydraulic cylinder are used to apply a restraining force to the sample through four sets of hardened steel rollers. In the previous fork device, this assembly was bolted into the fixture that held the sample. Because this fixture was eliminated for the new method, the clamping assembly currently attaches to the bottom hydraulic grip. The entire assembly is mounted on an Instron™ 1322 test frame and operated using an Interlaken™ 3200 series controller. The hydraulic clamping system is a significant improvement over the other methods discussed earlier. Control of the supporting force at a specific value allows for more robust biaxial and friction corrections than in systems where the support is provided by plates connected by bolts or springs, where the actual supporting force is unknown or uncontrolled.

The experimental conditions were optimized to achieve two competing goals: maximize the attainable compressive strains, and maximize the uniformity of strain and stress in the gage length of the specimen. Buckling and strain distribution analysis were conducted using commercial finite-element analysis software [131]. Two models were used. One was an explicit analysis using 600 linear solid elements to probe the buckling
behavior. The other analysis modeled one-quarter of the sample using an implicit model containing 7260 quadratic elements to observe the stress and strain distribution. When optimizing the part, there are essentially three buckling failure modes that need to be suppressed: buckling in the thickness direction within the supports (t-buckling), buckling in the unsupported gap (L-buckling), and buckling in the width direction (W-buckling). If these three modes are suppressed, the measurable strain is eventually limited by the non-uniformity introduced by barreling in the gage region. Examples of these four failure modes are shown in Figure A.3
Figure A.3: Examples of specimen failure though t-Buckling, L-buckling, W-buckling
A.2.1. Optimization of Sample Geometry

Regions of the sample that are prone to buckling can be optimized using standard column buckling equations. The eccentricity of a load applied to a column creates a bending moment, which must be supported in addition to the axial force. The secant formula, found in many mechanics books, [132, 133] calculates the maximum stress, $\sigma_{\text{max}}$, in a column from the combined effects of the axial force and bending moment:

$$
\sigma_{\text{max}} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \left( \frac{L_e}{2r} \sqrt{\frac{P}{EA}} \right) \right], \quad (A.1)
$$

where,

- $P$ = axial load
- $A$ = cross sectional area of the column
- $e$ = eccentricity of the load, as measured from the column neutral axis to the line of action of the force
- $c$ = distance from the neutral axis to the outer fiber where $\sigma_{\text{max}}$ occurs
- $L_e$ = effective length of the column in bending plane = $L/2$ for fixed ends
- $E$ = elastic modulus
- $r$ = radius of gyration, $r^2 = I/A$, where $I$ is the moment of inertia computed about the bending axis.

The maximum stress approaches infinity as the value within the secant approaches $\pi/2$; this point marking the stability limit for the column. It is equivalent to the value of the critical buckling load determined by the Euler method [132, 133],
Another feature of Equation A.1 is that for short, squat columns, the value of the secant approaches one and Equation A.1 reduces to,

\[ \sigma_{\text{max}} = \frac{P}{A} \left[ 1 + \frac{e c}{r^2} \right], \]  

which can be rearranged in terms of load as follows:

\[ P = \frac{A \sigma_{\text{max}}}{1 + \frac{e c}{r^2}}. \]  

For these short columns, (such as the unsupported gap region of the tension-compression specimen) failure is caused by plastic yielding of the column rather than buckling. The eccentricity of the load only serves to increase the stress by the moment it induces.

When Equation A.4 is used for design, \( \sigma_{\text{max}} \) is often set to the yield stress, predicting the maximum elastic load the column can sustain. This is a conservative criterion because the onset of buckling may not coincide with the start of plastic deformation. Substituting the sample geometry described in Figure A.1, Equation A.4 becomes

\[ P = \frac{B t \sigma_y}{1 + \frac{6 e}{t}}. \]  

In terms of the specimen geometry, the flow stress in the gage region is \( \sigma_f = P/W t \).

Solving this relation for \( P \) and substituting into Equation A.5, predicts the maximum flow
stress that can be tested before plastic deformation initiates in the unsupported gap as a function of the sample design and specimen thickness,

\[
\sigma_{f}^{\text{max before L-buckling}} = \frac{B \sigma_{y}}{W \left[ 1 + \frac{6e}{t} \right]}. \tag{A.6}
\]

If the function \( \sigma_{f} = f(\varepsilon) \) and its inverse \( \varepsilon = f^{-1}(\sigma_{f}) \) are known, then Equation A.6 can alternatively be framed in terms of a maximum strain criterion.

Because W-buckling occurs in the gage region, where yielding is necessary, an alternate method must be used to establish the limit strain. The question becomes not whether the column is yielding, but whether this deformation is stable. This is the same question asked in the Euler method, which led to Equation A.2. Because the gage region is plastically deforming during the test, the elastic modulus must be replaced with the tangent modulus and Equation A.2 becomes

\[
P = \frac{\pi^{2}E_{t}I}{L_{e}^{2}}, \tag{A.7}
\]

where \( E_{t} \) is the tangent modulus, \( d\sigma_{f}/d\varepsilon \) evaluated at \( P \). Again, substituting the test geometry and the relationship between \( P \) and the flow stress gives,

\[
\sigma_{f}^{\text{max before W-buckling}} = \frac{\pi^{2}E_{t}W^{2}}{3G^{2}}. \tag{A.8}
\]

Note this equation is only a function of the gage width and length, and unlike Equation A.6, is independent of the sample thickness. Equations A.6 and A.8, in conjunction with knowledge of a material’s stress-strain relationship, enable the calculation of the
maximum compressive strain attainable for any sample geometry before L or W-buckling.

Because the sample optimization is dependent on the mechanical properties, the final sample geometry varies for different materials. The material used in the initial optimization of the specimen geometry was aluminum alloy 6022-T4 from the same lot used by Balakrishnan and Wagoner [130] which has a thickness of 2.5 mm. The effect of material on the optimization results will be shown by comparing 6022 to aluminum-killed-drawing-quality (AKDQ) steel and Mg alloy AZ31B of the same thickness. The elastic modulus and assumed flow curves for these three materials shown in Table A.9.1. These relations are applicable fit from the 0.002 offset yield point to a strain of 0.10 for AKDQ, 0.19 for magnesium, and approximately 0.27 for 6022.

Using the relationships introduced above, the sample geometry can be optimized following the procedure summarized in Figure A.4. Each of the variables in Equations A.6 and A.8 affects the attainable compressive strain range, but there are external constraints on these values. For example, the width of the grip region, B, should be as

<table>
<thead>
<tr>
<th>Modulus (GPa)</th>
<th>Flow Curve (MPa)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 6022</td>
<td>$\sigma = 389 - 220e^{-8.44e}$</td>
<td>[130]</td>
</tr>
<tr>
<td>AKDQ Steel</td>
<td>$\sigma = 522e^{0.22}$</td>
<td>[unpublished research]</td>
</tr>
<tr>
<td>Mg AZ31B</td>
<td>$\sigma = 323 - 172e^{-13.05e}$</td>
<td>[unpublished research]</td>
</tr>
</tbody>
</table>

Table A.1: Material relations assumed for d tension-compression device development.
Figure A.4: Procedure for optimizing sample geometry through finite element simulations and experiments.
Figure A.5: Maximum compressive strain attainable as a function of sample width before W-buckling.

large as possible to discourage buckling in the unsupported region, but the size of the hydraulic grips available for the current work, 50 mm, limits this dimension. Buckling outside the gage is progressively inhibited for smaller gaps, L; but L mechanically limits the compressive strain that can be attained. Also, Equation A.8 shows that reducing G suppresses in-plane buckling tendencies. However, as mentioned in previous discussions, G must remain large enough so that the stress state in the measured gage length is uniform and uniaxial. A trial G, chosen at the beginning of the process, must be checked at the end of the optimization procedure to assure this condition is satisfied.

Once a material and trial gage length are chosen, Equation A.8 can determine the maximum flow stress before W-buckling as a function of the gage width, W. Using the material flow law, this result can be presented in terms of the maximum attainable compressive strain before W-buckling, $\varepsilon_{\text{max}}^{W-\text{buckling}}$. Figure A.5 shows this relationship for
Figure A.6: Maximum compressive strain attainable as a function of sample width for L- and W-buckling.

6022, AKDQ steel and Mg-AZ31B for a gage length of 36.8 mm. This figure indicates that the maximum attainable compressive strain is sensitive to the flow curve of the material, and is only reliable in the range where the flow stress equation is known accurately. The deviation of the steel curve from the other two materials at large widths is because the equations describing strain hardening of Mg and Al are of the saturation type [19], while hardening for steel is better described by a power-law equation [134].

In a similar manner, Equation A.6 can predict the dependence of W on the maximum strain before L-buckling, $\varepsilon_{\text{max}}^{L-\text{buckling}}$. The inputs for this prediction are the flow equations, the value of B, sheet thickness, and an estimate of the load eccentricity, which must be determined experimentally for a given machine and fixture. Figure A.6 shows the maximum attainable strain before buckling as a function of width for 6022. The intersection of these two curves indicates the optimum gage width for this particular material and thickness. For each thickness, the intersection of the L and W-buckling
Figure A.7: Optimum relationship between sample width and thickness.

curves gives a width, which is plotted as a function of thickness in Figure A.7. This curve represents the optimum width and thickness combination. Samples with dimensions lying to the left of this curve are too thin and will buckle in the gap at a strain lower than the maximum strain that was predicted in Figure A.6. Samples that lie to the right are thick enough to resist L-buckling, and will not benefit from increased thickness because of the independence of thickness on W-buckling, shown in Equation A.8. The final sample dimensions used in this work are shown by the square in Figure A.7, and were listed in Figure A.1. One can note that although this geometry was derived for 6022, it also happens to work well for AKDQ, but is not optimized for Mg AZ31B.

Figure A.6 also indicates the maximum compressive strain that the specimen can attain, which determines the necessary gap, L, to prevent mechanical interference between the supports and the grips. The minimum possible L is 3 mm, which is the smallest gap that can be consistently attained with the current fixture. After the size of
the gap is determined, it is possible to evaluate whether or not the short column assumption, which led to Equation A.6, is valid for the trial geometry. If Equation A.6 is not valid, Equation A.1 must be solved iteratively instead. For the case in Figure A.6, where the suggested L is 5.5 mm, the short column assumption results in a specimen width within 0.05% of the answer obtained using Equation A.1. For a gap of 10 mm, the error associated with the assumption increases to 0.2% and for a gap of 20 mm the two methods only differ by 1.05%, suggesting that for most cases the short column assumption is robust.

Once the trial geometry is created, it can be tested experimentally and these results can be fed back into the analysis to refine the value of the eccentricity. To determine the eccentricity, a design should be chosen that is prone to L-buckling, such as one with a small thickness or large W. In this case, 6022 with a thickness of 0.9 mm was available. In repeated tests of this material, the maximum strain attainable before L-
buckling was 0.01. The buckling map for this thinner material was created and the value of the eccentricity was adjusted until the curve describing the L-buckling had a maximum strain of 0.01 at a width of 15.2 mm. As seen in Figure A.8, this corresponds to an eccentricity of 0.3 mm.

After the trial geometry is determined, finite element simulation is used to find a gage length that provides a uniform, uniaxial stress state over the entire measurement range of a 25.4 mm extensometer. In a finite element model, each design and the ASTM tensile standard are loaded to the same stress. The strain is calculated using a “virtual extensometer” by measuring the relative displacement of two material points initially 25.4 mm apart. The gage length, G, is adjusted until the strain measured from the new geometry agrees with the results from the ASTM sample within a desired tolerance. Some iteration is required to find this value as the other dimensions of the sample will change. The final gage length chosen for 6022 was 36.8 mm, which differed from the ASTM standard by 1.2% after 0.075 strain.

A.2.2. **Optimization of the Supporting Force**

The clamping force of the supporting plates also affects the failure mode. If the clamping force is too large, frictional effects redistribute the load from the gage region onto the material that is in the unsupported gap, leading to L-buckling at the entrance of the fixture. However, if the clamping force is too small, the plates will not prevent wrinkling, or t-buckling. Using the ABAQUS/Explicit buckling model, these two extreme failure modes can be observed, with t-buckling at low side force and L-buckling at higher forces. In between these two extremes, there is a peak in the compressive strain
obtained before failure, seen in Figure A.9. The optimum side force is about 7 kN, where the side forces are sufficient to prevent t-buckling, yet the frictional effects are minimized. This is considerably lower than the forces used in the fork-device, which were 12.5 kN for 6022 and up to 19 kN for HSLA steel [130]. In practice, the range from 5-10 kN has proven to be acceptable, although higher forces are sometimes needed for thinner material, or when testing to high compressive strains, where the flow stress becomes large.

A.2.3. Strain Measurement

Because the flat side plates are in contact with the entire surface of the sample, it is impossible to attach strain gages or mount an extensometer in the conventional way. During the initial phase of the optimization program, a mechanical extensometer was mounted on the edge of the specimen. To assure the supporting plates did not come into contact with the extensometer blades, it was necessary to position the plates so as to
create a small unsupported ledge on which to mount the extensometer. This free edge caused significant wrinkling and buckling of this side of the sample face. Current tests use a non-contact, EIR™ laser extensometer, which enables the plates to cover the entire surface of the specimen. Initial results show this is an effective way to eliminate buckling along the edge.

A.2.4. *Uniaxial Data Corrections*

Because of the need to constrain the sample in the thickness direction to prevent buckling, all raw stress-strain results require corrections for frictional and biaxial effects arising from this supporting force. The addition of a constraining force, $F_2$ creates a readily calculated through-thickness stress, $\sigma_2$. Knowing this value, the Von Mises effective stress is

$$\bar{\sigma} = \sqrt{\frac{1}{2}[\sigma - \sigma_2]^2 + \sigma_2^2 + \sigma^2]} \quad \text{(A.10)}$$

where $\sigma$ is the axial testing stress. Because the thickness stress is much smaller than the stress in the testing direction, the biaxial effect is small and the choice of the constitutive equation defining the effective stress is not critical. If Equation A.10 is replaced with an effective stress based on Hill’s anisotropic yield surface with $r = 0.6$, [50] the two curves differ by 0.4%.

The friction correction is more significant and more complex because the direction of the frictional force reverses when the loading direction changes. To reduce friction, the side supports are covered with a 0.35 mm Teflon sheet, and the supporting force is transmitted
from the hydraulic pump to the supports through a series of rollers that allow the plates to move with the sample along the loading axis.

The actual force deforming the sample, $F_{\text{deform}}$, is the value measured from the load cell, $F_{\text{meas}}$, with the additional frictional force, $F_{\text{friction}}$, subtracted,

$$F_{\text{deform}} = F_{\text{meas}} - F_{\text{friction}}. \quad (A.11)$$

The frictional behavior is represented by a Coulomb friction law, [23] as follows:

$$F_{\text{friction}} = \mu F_2 \quad (A.12)$$

where $\mu$ is the friction coefficient, which is assumed to be the same in tension and compression. When the load is reversed, there is a range equal to $2F_{\text{friction}}$, where the external load, applied by the tensile frame, is changing to overcome friction in the new direction; however, the internal load on the sample is constant. The data points measured within this range are not included in the corrected results.

The magnitude of the frictional effect has been estimated by Balakrishnan [130], who found a friction coefficient in the range of 0.06-0.09 by measuring changes in the yielding force of nominally identical samples as a function of side force. This range corresponds to a difference in the flow stress of about 4% for a 5kN side force. Although this method gives satisfactory results, it can be complicated by material, contact, and geometry variation requiring several tests to gage variability. Some variability in the friction coefficient is expected, related to variations in the sample surface condition and accumulated damage to the Teflon coating. Therefore in practice, the friction coefficient is adjusted slightly so that the supported, tensile deformation matches the baseline, unsupported curve. When the friction coefficient is adjusted in this way, the values used
are never larger than the range 0.06-0.09 predicted by Balakrishnan with the fork device. In fact for the new plate method, values around 0.03-0.06 produce better agreement between the supported and unsupported flow curves. Figure A.10 shows the relative effects of each step in the data corrections. After both friction and biaxial corrections, the flow curve of the supported sample agrees well with the unsupported uniaxial tension test with a standard deviation between the two curves of under 0.1 MPa.

A.2.5. Experimental Validation of Method

The results of reverse loading experiments on AA-6022 obtained with the new approach are consistent with those obtained using the older, established, fork device, Figure A.11. It is apparent both approaches reveal the same features of the Bauschinger effect; however, tests using the flat plate supports show much smoother stress-strain behavior in compression, and larger attainable strains. Figure A.12 shows reloading curves for tension-compression and compression/tension tests using the new device. The
results are nearly symmetric with respect to tension and compression, suggesting the new method successfully stabilizes the compressive loading and leads to compressive stress-strain curves that are comparable with uniaxial tension.

Figure A.11: Comparison of current flat-plate device to previously used fork device for compression after tensile prestrain.
Figure A.12: Comparison of tension-compression and compression/tension curves.

Figure A.13: Comparison of tension-compression and compression/tension curves for the new method.

A.3. Demonstration of Capabilities of the New Method

A.3.1. Testing of the Bauschinger Effect and Material Hardening

Since the new measurement method was established, it has been used to study the Bauschinger effect. The flat plate supports were successful in allowing continuous
measurement of reverse loading in both tension and compression under displacement control for a variety of prestrains and materials. Several different aluminum, magnesium, zinc and steel samples with various thicknesses greater than 1 mm have been tested, Figure A.13 The robustness of the method can be seen in the attainable strain range and the smoothness of the stress-strain curves. (The asymmetric behavior of Mg in tension and compression is physical in nature, caused by twinning.) This capability will form the basis of the experimental results discussed in Chapter 7.

Figure A.14: Cyclic hardening curve for HSLA-50 steel for 8 cycles.
A.3.2. Cyclic Hardening and Ratcheting Tests

Being able to measure continuous tension and compression for samples of different conditions and texture orientations, allows for many other tests that can be used for constitutive equation development. Cyclic hardening tests under strain control, Figure A.14, and ratcheting tests under load control, Figure A.15, were demonstrated using high-strength, low-alloy steel (HSLA-50) with a thickness of 1.63 mm [135]. Much of the previous research on cyclic hardening and ratcheting uses cylindrical samples, [102, 136, 137] which has different properties than rolled sheet. Only recently has information from sheet material been used, such as Kang et al. [138] using data from Yoshida [128]. The ability to conduct these tests on textured sheets with a relatively simple device gives valuable baseline information on saturation behavior that is essential to development of robust material models for sheet material.
A.3.3. *Room Temperature Creep after Load Reversal*

The anelastic contribution to creep has been carefully examined through the use of stress-dip tests by researchers, including Gibeling and Nix [139]. In the stress-dip test, the load is quickly reduced while the strain transient is measured. This new technique has the ability to extend this method by allowing the stress to be completely reversed. This procedure has been used to measure the effects of room temperature creep of aluminum alloy 6022 after reverse loading.

The Instron™ tensile frame was used in load control while the strain was recorded using a National Instruments™ NI-4350 voltage measurement device capable of measuring smaller strain changes. The samples were loaded to an initial value of $\sigma_0 = -240$ MPa at 8 MPa/sec. This load roughly corresponds to a compressive strain of 0.07. After reaching this load level, the load was changed at the same rate to a new value ranging from ±75% of the original flow stress and held for one hour.

![Graph showing room temperature creep results from Al 6022.](image)

Figure A.16: Room temperature creep results from Al 6022.
Figure A.16 shows results from a series of such tests, showing the creep rates for various degrees of load reversal. Note that because the initial loading ($\sigma_0$) is compressive, the curves labeled $-0.75\sigma_0$, $-0.50\sigma_0$ and $-0.25\sigma_0$ correspond to tensile loading. At stresses near the flow stress, the creep rate in the same direction as the initial loading is slower than the rate for the same load in the reversed direction. A more refined series of tests similar to these could be used to ascertain the backstress in the material, induced by the prestrain as the stress value where the strain rate is zero. Similar techniques that probe a variety of different prestrains and/or loading rates could also be used to develop a strain-dependent room-temperature creep laws for materials that experience a load reversal. One factor that must be assessed when performing these creep tests is to assure the side force does not adversely affect the creep results because the strains are so small. For a stress reversal, the problem can be avoided entirely by making the initial deformation in the compressive direction, then releasing the side force for the reversed tensile creeping. In this way, the results are completely free from any frictional or biaxial effects.

A.3.4. Anelasticity after Strain Reversal

Like creep, anelasticity is another time dependent effect that is altered by reverse deformation. Specimens of Al-6022 and drawing-quality-silicon-killed (DQSK) steel were first compressively loaded to a true strain of -0.045 using the plate supports. The samples were then unloaded and the clamping device was removed so an extensometer could be securely attached to the wide portion of the specimen to assure the most accurate strain measurement. An additional tensile strain, $\varepsilon_{\text{ten}}$, was applied to the samples, after which, the load was quickly dropped to zero by opening the hydraulic grips. Removing the instantaneous elastic contraction, Figure A.17, shows the anelastic strain
change at zero applied stress with time for both 6022 and DQSK steel. For large strains after the reversal, \(\varepsilon_{\text{ten}} = 0.11\) the sample behaves similarly to a uniaxial test. That is, after the grips are opened and the sample is unloaded, there is some anelastic response that contracts the sample, producing a negative strain that accumulates with time. For smaller reversals, the behavior is quite different, and perhaps somewhat surprising. After the tensile load is removed, the sample again shows an anelastic response, although in the opposite direction. The elongation of the sample with time is likely attributable to the structure and dislocation arrangement from the initial compressive loading that persists during the initial stages of reversed tensile loading. The same mechanisms appear to be present in both steel and aluminum alloys as seen from the similarities in the curves for each material. Wang and Wagoner [140] have used these results to assess the possible influences of this sort of anelasticity on the time-dependent springback of aluminum alloys and concluded that it may have some effect during the first few minutes after removal from the die.
Figure A.17: Results from anelastic testing of Al 6022 and DQSK steel.
REFERENCES


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