DEVELOPMENT OF A GENERALIZED MECHANICAL EFFICIENCY PREDICTION METHODOLOGY FOR GEAR PAIRS

DISSERTATION

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By

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ABSTRACT

In this study, a general methodology is proposed for the prediction of friction-related mechanical efficiency losses of gear pairs. This methodology incorporates a gear contact analysis model and a friction coefficient model with a mechanical efficiency formulation to predict the gear mechanical efficiency under typical operating conditions. The friction coefficient model uses a new friction coefficient formula based on a non-Newtonian thermal elastohydrodynamic lubrication (EHL) model. This formula is obtained by performing a multiple linear regression analysis to the massive EHL predictions under various contact conditions. The new EHL-based friction coefficient formula is shown to agree well with measured traction data. Additional friction coefficient formulae are obtained for special contact conditions such as lubricant additives and coatings by applying the same regression technique to the actual traction data. These coefficient of friction formulae are combined with a contact analysis model and the mechanical efficiency formulation to compute instantaneous torque/power losses and the mechanical efficiency of a gear pair at a given position. This efficiency prediction methodology is applied to both parallel axis (spur and helical) and cross-axis (spiral bevel and hypoid) gears. In the case of hypoid gears, both face-milling and face-
hobbing processes are considered, and closed-form expressions for the geometric and kinematic parameters required by the efficiency model are derived.

The efficiency prediction model is validated by comparing the model predictions to a set of high-speed spur gear efficiency measurements covering several gear design and surface treatment variations. The differences between the predicted efficiency values and the measured ones are consistently within 0.1 percent. Influence of basic gear design parameters, tooth modifications, operating conditions, surface finish and treatments, lubricant properties, and manufacturing and assembly errors on mechanical efficiency of both parallel-axis and cross-axis gears are investigated. At the end, a set of guidelines is provided on how to improve mechanical efficiency of gear pairs through design, surface engineering and lubricant solutions.
Dedicated to my parents
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Vita</td>
<td>vi</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>viii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xiv</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>xx</td>
</tr>
<tr>
<td>Chapters:</td>
<td></td>
</tr>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Research Background and Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Literature Review</td>
<td>4</td>
</tr>
<tr>
<td>1.3 Scope and Objectives</td>
<td>10</td>
</tr>
<tr>
<td>1.4 Dissertation Organization</td>
<td>12</td>
</tr>
<tr>
<td>2 Overall Methodology</td>
<td>15</td>
</tr>
<tr>
<td>2.1 Technical Approach</td>
<td>15</td>
</tr>
<tr>
<td>2.2 Gear Contact Analysis Models</td>
<td>17</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>2.1</td>
<td>Empirical friction coefficient formulae considered in this study</td>
</tr>
<tr>
<td>3.1</td>
<td>Test matrix for the friction traction measurements</td>
</tr>
<tr>
<td>3.2</td>
<td>Surface treatment combinations used in friction measurements</td>
</tr>
<tr>
<td>3.3</td>
<td>Regression coefficients for 75W90 gear oil</td>
</tr>
<tr>
<td>3.4</td>
<td>Regression coefficients for Lubricant A</td>
</tr>
<tr>
<td>3.5</td>
<td>Regression coefficients for Lubricant B</td>
</tr>
<tr>
<td>3.6</td>
<td>Matrix of parameters used in the massive EHL model predictions</td>
</tr>
<tr>
<td>3.7</td>
<td>Coefficients for the EHL based formula</td>
</tr>
<tr>
<td>4.1</td>
<td>Example helical gear pair parameters</td>
</tr>
<tr>
<td>4.2</td>
<td>Lubricant (ATF) parameters</td>
</tr>
<tr>
<td>5.1</td>
<td>Gear pair blank data for the example hypoid gear set</td>
</tr>
<tr>
<td>5.2</td>
<td>Machine settings and cutter parameters for the example hypoid pinion</td>
</tr>
<tr>
<td>5.3</td>
<td>Machine settings and cutter parameters for the example hypoid ring gear</td>
</tr>
<tr>
<td>5.4</td>
<td>Lubricant parameters for 75W90</td>
</tr>
<tr>
<td>6.1</td>
<td>Test gear pair parameters [70]</td>
</tr>
<tr>
<td>6.2</td>
<td>Test lubricant properties for 75W90</td>
</tr>
</tbody>
</table>
7.1 Parameters of the example helical gear pair used in the parametric study …… 189
7.2 Lubricant viscosity constants used in the parametric study ……………………. 201
7.3 Surface treatment combinations considered in the parametric study ……… 202
7.4 Hypoid gear set basic parameters for design comparison ……………………. 217
8.1 Influence of an increase in different parameter values on $\eta$ of a parallel axis gear pair …………………………………………………………………………… 223
8.2 Influence of an increase in different parameter values on $\bar{\eta}$ of a hypoid gear pair …………………………………………………………………………… 224
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Flowchart for the efficiency prediction methodology</td>
<td>16</td>
</tr>
<tr>
<td>2.2</td>
<td>(a) A schematic illustration of the gear load distribution, and (b) the equivalent cylindrical contact representing contact segment $q$</td>
<td>19</td>
</tr>
<tr>
<td>3.1</td>
<td>Comparison of friction coefficient models for $R = 5 \text{ mm}$, $V_e = 5 \text{ m/s}$, $P_h = 2 \text{ GPa}$, $\nu_0 = 10 \text{ cP}$, $\nu_k = 13 \text{ cSt}$, and $S_{rms} = 0.07 \text{ m}$</td>
<td>43</td>
</tr>
<tr>
<td>3.2</td>
<td>WAM 6 universal tribology test machine</td>
<td>45</td>
</tr>
<tr>
<td>3.3</td>
<td>Measured $\mu$ as a function of $SR$. Uncoated-uncoated surfaces with 75W90 oil. (a) $P_h = 1 \text{ GPa}$, (b) $P_h = 1.5 \text{ GPa}$, and (c) $P_h = 2.0 \text{ GPa}$</td>
<td>50</td>
</tr>
<tr>
<td>3.4</td>
<td>Comparison of fitted and measured coefficient of friction, uncoated surface versus uncoated surface with 75W90, $SR &lt; 0$. (a) $P_h = 1 \text{ GPa}$, (b) $P_h = 1.5 \text{ GPa}$, and (c) $P_h = 2.0 \text{ GPa}$</td>
<td>59</td>
</tr>
<tr>
<td>3.5</td>
<td>Residual plots from the regression analysis of traction data for uncoated surface versus uncoated surface and $SR &lt; 0$; (a) residual versus fitted values; and (b) the histogram of the standardized residuals</td>
<td>62</td>
</tr>
<tr>
<td>3.6</td>
<td>Comparison of fitted and measured coefficient of friction, uncoated surface versus uncoated surface with 75W90, $SR &gt; 0$. (a) $P_h = 1 \text{ GPa}$, (b) $P_h = 1.5 \text{ GPa}$, and (c) $P_h = 2.0 \text{ GPa}$</td>
<td>64</td>
</tr>
</tbody>
</table>
3.7 Comparison of fitted and measured coefficient of friction for two coated surfaces with 75W90, DLC-B versus DLC-B. \( SR < 0 \). (a) \( P_h = 1 \) GPa, (b) \( P_h = 1.5 \) GPa, (c) \( P_h = 2.0 \) GPa, and (d) \( P_h = 2.5 \) GPa …………………………… 67

3.8 Comparison of fitted and measured coefficient of friction for two coated surfaces with 75W90, DLC-B versus DLC-B. \( SR > 0 \). (a) \( P_h = 1 \) GPa, (b) \( P_h = 1.5 \) GPa, (c) \( P_h = 2.0 \) GPa, and (d) \( P_h = 2.5 \) GPa …………………………… 71

3.9 Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant A, \( SR < 0 \). (a) \( P_h = 1 \) GPa, and (b) \( P_h = 2.0 \) GPa ……… 78

3.10 Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant A, \( SR > 0 \). (a) \( P_h = 1 \) GPa, and (b) \( P_h = 2.0 \) GPa ……… 80

3.11 Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant B, \( SR < 0 \). (a) \( P_h = 1 \) GPa, and (b) \( P_h = 2.0 \) GPa ……… 83

3.12 Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant B, \( SR > 0 \). (a) \( P_h = 1 \) GPa, and (b) \( P_h = 2.0 \) GPa ……… 85

3.13 The \( \mu \) values for (a) \( P_h = 1 \) GPa and various \( V_e \) values, and (b) \( V_e = 5 \) m/s and various \( P_h \) values. Both surfaces are uncoated and the lubricant is 75W90 … 88

3.14 The \( \mu \) values for (a) \( P_h = 1 \) GPa and various \( V_e \) values, and (b) \( V_e = 5 \) m/s and various \( P_h \) values. Both surfaces are coated with DLC-B and the lubricant is 75W90 ……………………………………………………………………… 89

3.15 Comparison of the fitted \( \mu \) values for the five cases in Table 3.2 with 75W90 oil at \( P_h = 1.5 \) GPa and four values of \( V_e \) ……………………………………… 90

3.16 Comparison of the fitted \( \mu \) values for the five cases in Table 3.2 with Shell 75W90 oil at \( V_e = 5 \) m/s and four values of \( P_h \) ……………………………………… 91

3.17 Comparison of measured \( \mu \) values for three different lubricants at \( P_h = 1.0 \) GPa ……………………………………………………………………… 93
3.18 Comparison of measured $\mu$ values for three different lubricants at $P_h = 1.5$ GPa ................................................................. 94

3.19 Comparison of measured $\mu$ values for three different lubricants at $P_h = 2.0$ GPa ................................................................. 95

3.20 Comparison between published empirical formulae and the measured traction data ............................................................................. 97

3.21 Comparison of EHL model predictions and the measured data at $P_h = 1$ GPa and various $V_e$ values. Uncoated surfaces, 75W90 oil .................. 99

3.22 Comparison of EHL model predictions and the measured data at $P_h = 1.5$ GPa and various $V_e$ values. Uncoated surfaces, 75W90 oil .................. 100

3.23 Comparison of EHL model predictions and the measured data at $P_h = 2.0$ GPa and various $V_e$ values. Uncoated surfaces, 75W90 oil .................. 101

3.24 A typical $\mu$ versus $SR$ curve ................................................................. 103

3.25 Regression of the massive predictions by the thermal EHL model, (a-g) comparison of actual and fitted $\mu$ values for various parameters, (h) residual plot of all the data. $P_h = 1.5$ GPa, $V_e = 15$ m/s, $R = 0.04$ m, $T = 100$ degC, $S = 0$ $\mu$m, unless specified ................................................................. 109

4.1 Spur gear geometry used for calculation of curvatures and surface velocities ......................................................................................... 118

4.2 Helical gear geometry used for calculation of curvatures and surface velocities ......................................................................................... 119

4.3 (a) Illustration of contact lines at mesh position $\phi = \phi$, and (b) predicted $\mu(z, \theta, \phi)$ at every discrete contact point. $S = 0.1$ $\mu$m, $L_{in} = 500$ Nm, $N_p = 4,000$ rpm and $T_{oil} = 100^\circ$C ................................................................. 124

4.4 A measured surface roughness profile of $S = 0.1$ $\mu$m RMS .................. 125

xvi
4.5 Predicted $\mu(\phi)$ and $\eta(\phi)$. $S = 0.1 \mu m$, $L_{in} = 500 Nm$, $N_p = 4,000 rpm$ and $T_{oil} = 100^\circ C$ ................................................................. 123

5.1 Kinematic principles of (a) face-milling (single indexing), and (b) face-hobbing (continuous indexing) processes [81] ................................................................. 132

5.2 Hypoid gear teeth with moving grid cells attached to capture any contact zones ................................................................. 134

5.3 (a) Set-up of the moving grid, and (b) definition of the grid on the tangent plane for $\mu$ calculation ................................................................. 135

5.4 Principal directions and surface velocities .................................................. 142

5.5 Predicted contact pressure histograms at rotational position $\phi = \phi_1$ ........... 153

5.6 Predicted contact pressure distributions at (a) $\phi = \phi_1$, (b) $\phi = \phi_2$, (c) $\phi = \phi_10$ .. 154

5.7 Distribution of (a) contact pressure, (b) combined radii of curvature, (c) sliding velocity, and (d) sum of rolling velocities along the normal of the instant line of contact at angular position $\phi_1$. $S = 0.1 \mu m$, $L_{in} = 1,600 Nm$, $N_p = 1,000 rpm$, and $T_{oil} = 60^\circ C$ ................................................................. 156

5.8 Distribution of $\mu(z, \theta, \phi)$ at every mesh grid of principal contact points predicted by using (a) the experimental $\mu$ formula, (b) the EHL-based $\mu$ formula, and (c) the actual EHL analysis. $S = 0.1 \mu m$, $L_{in} = 1,600 Nm$, $N_p = 1,000 rpm$, and $T_{oil} = 60^\circ C$ ................................................................. 158

5.9 A typical measured surface roughness profile used in the EHL analysis. $S = 0.1 \mu m$, $L_{in} = 1,600 Nm$, $N_p = 1,000 rpm$, and $T_{oil} = 60^\circ C$ ................. 159

5.10 $\bar{\mu}(\phi)$ and $\bar{\eta}(\phi)$ predicted by using the friction coefficient formulae developed in this study and the EHL model. $S = 0.1 \mu m$, $L_{in} = 1,600 Nm$, $N_p = 1,000 rpm$, and $T_{oil} = 60^\circ C$ ................................................................. 160

6.1 High-speed gear efficiency test machine [70] ........................................... 165

6.2 A schematic showing the layout of the gear efficiency test machine ....... 166
Examples of (a) 23 and 40-tooth test gears, and (b) test gears having different face widths [70] ................................................................. 170

Typical measured surface roughness profiles for the two test gears, (a) an uncoated test gear with $S_{rms} = 0.3 \mu m$, and (b) a coated test gear with $S_{rms} = 0.1 \mu m$ ................................................................. 176

Comparison of predicted and measured $\eta$ values of 23-tooth and 40-tooth gears sets at 6000 rpm and a range of input torque; (a) wide face width gears (26.7 mm), (b) medium face width gears (19.5 mm), and (c) narrow face width gears (14.2 mm) ................................................................. 177

Comparison of predicted and measured $\eta$ values of 23-tooth and 40-tooth gears sets at 406 N-m and a range of rotational speed; (a) wide face width gears (26.7 mm), (b) medium face width gears (19.5 mm), and (c) narrow face width gears (14.2 mm) ................................................................. 182

Comparison predicted and measured $\eta$ values of a 40-tooth gear pair coated with DLC-B at (a) 6000 rpm and various input torque values, and (b) 406 Nm and various rotational speed values .............................................. 185

Influence of $L_{in}$ and pinion speed $N_p$ on $\eta$; $S = 0.4 \mu m$ and $T_{oil} = 100^\circ C$ .. 191

Influence of $S$ and $T_{oil}$ on $\eta$; $L_{in} = 200 Nm$ and $N_p = 2,000 rpm$ ............. 193

Influence of $\beta_n$ and $\psi_n$ on $\eta$; $S = 0.4 \mu m$, $L_{in} = 200 Nm$, $N_p = 2,000 rpm$, and $T_{oil} = 100^\circ C$ ................................................................. 194

Influence of $\varepsilon_\beta$ and $\varepsilon_\alpha$ on $\eta$; $S = 0.4 \mu m$, $L_{in} = 200 Nm$, $N_p = 2,000 rpm$, and $T_{oil} = 100^\circ C$ ................................................................. 196

Influence of $\xi_\beta$ and $\xi_\alpha$ on $\eta$; $S = 0.4 \mu m$, $L_{in} = 200 Nm$, $N_p = 2,000 rpm$, and $T_{oil} = 100^\circ C$ ................................................................. 197

Influence of $\chi$ and $\delta$ on $\eta$; $S = 0.4 \mu m$, $L_{in} = 200 Nm$, $N_p = 2,000 rpm$, and $T_{oil} = 100^\circ C$ ................................................................. 199
7.7 Influence of $\Delta \lambda$ and $\Delta \psi_n$ on $\eta$; $S = 0.4 \mu m$, $L_{in} = 200 \, Nm$, $N_p = 2,000 \, rpm$, and $T_{oil} = 100^\circ C$ ................................................................. 200

7.8 Comparison of frictional power losses of the 23-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 \, Nm$, $N_p = 6,000 \, rpm$, and $T_{oil} = 100^\circ C$ ................................................................. 204

7.9 Comparison of frictional power losses of the 40-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 \, Nm$, $N_p = 6,000 \, rpm$, and $T_{oil} = 100^\circ C$ ................................................................. 205

7.10 Comparison of $\bar{\eta}$ of the 23-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 \, Nm$, $N_p = 6,000 \, rpm$, and $T_{oil} = 100^\circ C$ .... 206

7.11 Comparison of $\bar{\eta}$ of the 40-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 \, Nm$, $N_p = 6,000 \, rpm$, and $T_{oil} = 100^\circ C$ .... 207

7.12 Influence of $L_{in}$ and pinion speed $N_p$ on $\bar{\eta}$; $S = 0.4 \mu m$ and $T_{oil} = 60^\circ C$ ... 210

7.13 Influence of $S$ and $T_{oil}$ on $\bar{\eta}$; $L_{in} = 1,600 \, Nm$ and $N_p = 1,000 \, rpm$ ............ 211

7.14 Example hypoid gear pair geometry and illustration of errors, $\Delta V$: pinion movement along offset, $\Delta H$: pinion axial movement, $\Delta R$: pinion movement along gear axis, $\Delta \beta$: shaft angle error .................................................. 212

7.15 Influence of errors (a) $\Delta H$, (b) $\Delta V$, (c) $\Delta R$, and (d) $\Delta \beta$ on $\bar{\eta}$. $S = 0.4 \mu m$, $L_{in} = 1,600 \, Nm$, $N_p = 1,000 \, rpm$ and $T_{oil} = 60^\circ C$ .................................................. 213

7.16 Comparison of $\bar{\eta}$ values of four gear sets listed in Table 7.4. $L_{in} = 800 \, Nm$, $N_p = 1,000 \, rpm$, $S = 0.4 \mu m$, and $T_{oil} = 50^\circ C$ .................................................. 218

7.17 Comparison of $\bar{\eta}$ of the example hypoid gear pair for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 1,000 \, Nm$, $N_p = 1,000 \, rpm$, and $T_{oil} = 100^\circ C$ .................................................. 219
NOMENCLATURE

\[ A_g \] Arrangement factor in Eq. (2.22)
\[ b \] Half width of Hertzian contact
\[ B \] Face width
\[ c_0, c_1, c_2, c_3 \] Coefficients in Eq. (2.4)
\[ C_i \ (i = 1 \text{ to } 6) \] Functions of thermal parameters in Eq. (2.6)
\[ d \] Elastic deformation
\[ d_{bore} \] Bearing bore diameter
\[ D_m \] Bearing mean diameter
\[ D_o \] Element outside diameter
\[ D_q^{(1)}, D_q^{(2)} \] Deflections of point \( q \) on the pinion and the gear, respectively
\[ D_r \] Root diameter
\[ E' \] Effective modulus of elasticity
\[ e_f, e_h \] Unit vectors in the principal directions of surface 1
\[ e_s, e_q \] Unit vectors in the principal directions of surface 2
\[ f_L \] A factor depending on bearing type and method of lubrication in Eq. (2.24)
\[ F_f \] Sliding friction force
\[ F_g \] Dip factor in Eq. (2.22)
\[ F_q \] Discrete forces at contact point \( q \)
\[ F_r \] Rolling friction force
\[ F_r' \] Rolling friction per unit width
\[ F_s' \] Sliding friction per unit width
\[ FH \] Face hobbing
FM  Face milling
\( \tilde{G} \)  Dimensionless material parameter
\( h \)  Film thickness
\( h_0 \)  Reference film thickness in Eq. (2.3)
\( \tilde{H} \)  Dimensionless film thickness
\( \kappa_f, \kappa_h \)  Principal curvatures of surface 1
\( \kappa_s, \kappa_q \)  Principal curvatures of surface 2
\( \kappa_{nc} \)  Normal curvature in the direction of a contact line
\( \kappa_{nn} \)  Normal curvatures in the direction that is normal to the contact line
\( \kappa_{1,II}^{(i)} \)  Principal curvatures of surface \( \Sigma_i \), \( i = 1, 2 \).
\( K_f \)  Lubricant thermal conductivity
\( M_b \)  Bearing friction moment
\( n \)  Surface unit normal
\( n_{(i)} \)  Velocity of the tip of the surface unit normal over the surface \( \Sigma_i \)
\( L_{in}, L_{out} \)  Input torque and output torque, respectively
\( N \)  Rotation speed
\( N_p \)  Pinion speed
\( p \)  Fluid pressure
\( \tilde{P} \)  Dimensionless fluid pressure
\( P_b \)  Bearing frictional power loss
\( P_c \)  Asperity contact pressure
\( P_{ch} \)  Churning loss
\( P_g \)  Gear frictional power loss
\( P_h, P_{\text{max}} \)  Maximum Hertzian pressure
\( P_s \)  Spin power loss
\( P_{\text{seal}} \)  Seal power loss
\( P_{ta}, P_{th}, P_{tr} \)  Transitional pressures in Eq. (2.4)
\( P_w \)  Windage power loss
\( Q \)  Total number of contact points
\( \tilde{Q} \)  Dimensionless load parameter
\( q_1, q_2 \)  Angles defined in Figure 5.4
$r_1, r_2$  Radii of curvature of body 1 and body 2, respectively  
$r_i$, $r_j$  Position vectors for surface 1 and surface 2, respectively  
$R$  Effective radius of curvature  
$R_b$  Base radius  
$R_f$  Roughness factor in Eq. (2.22)  
$R_{ie}$  Radii of curvature for surface $i$ in the directions of lubricant entraining vector  
$R_{is}$  Radii of curvature for surface $i$ in the direction that is normal to the lubricant entraining vector  
$R_{ix}$  Radius of curvature for surface $i$ in the minor axis ($x$) direction  
$R_{iy}$  Radii of curvature for surface $i$ in the major axis ($y$) direction  
$R_p$  Pitch radius  
$R_q$  Distance from the point $q$ to the gear axis  
$S$  Surface roughness  
$s_0$  Fluid parameter in Eq. (2.2)  
$S_{cla}$  Surface roughness in CLA  
$SR$  Slide-to-roll ratio  
$S_{rms}$  Surface roughness in RMS  
$t$  Dimensionless time  
$t_0$  Lubricant temperature at inlet  
$t_{26}$, $t_{p26}$  Directions defined in Figure 5.3b  
$\bar{T}$  Dimensionless film temperature  
$T_{oil}$  Oil temperature at inlet  
$\bar{U}$  Dimensionless fluid velocity  
$\bar{\bar{U}}$  Dimensionless speed parameter  
$u_1, u_2$  Surface velocities of body 1 and body 2, respectively  
$U_{1x}, U_{2x}$  Surface velocities of body 1 and 2 in $x$ direction (normal to the contact line), respectively  
$\psi_{e}^{(t)}$  Relative velocity of the contact point respect to the surface $\Sigma_i$  
$V_e$  Entraining velocity  
$V_s$  Sliding velocity
\( V_r \) \hspace{1cm} \text{Sum of rolling velocity} \\
\( W' \) \hspace{1cm} \text{Unit load} \\
\( \bar{W} \) \hspace{1cm} \text{Dimensionless unit load} \\
\( W_b \) \hspace{1cm} \text{Bearing load} \\
\( X_{in}, X_{out} \) \hspace{1cm} \text{Inlet and outlet of computational domain for the contact zone} \\
\( \alpha \) \hspace{1cm} \text{Pressure viscosity coefficient} \\
\( \alpha_1, \alpha_2 \) \hspace{1cm} \text{Pressure exponential parameters in Eq. (2.4)} \\
\( \bar{\beta} \) \hspace{1cm} \text{Dimensionless coefficient of thermal expansion} \\
\( \beta_b, \beta_{op} \) \hspace{1cm} \text{Helix angles at base radius and operating pitch radius, respectively} \\
\( \beta_n \) \hspace{1cm} \text{Normal helix angle} \\
\( \chi \) \hspace{1cm} \text{Shaft misalignment} \\
\( \delta \) \hspace{1cm} \text{Center distance shift} \\
\( \delta_q \) \hspace{1cm} \text{Initial separation at contact point} \ q \\
\( \Delta \) \hspace{1cm} \text{Contact line inclination angle} \\
\( \Delta H \) \hspace{1cm} \text{Assembly error defined in Figure 7.14} \\
\( \Delta V \) \hspace{1cm} \text{Assembly error defined in Figure 7.14} \\
\( \Delta R \) \hspace{1cm} \text{Assembly error defined in Figure 7.14} \\
\( \Delta \beta \) \hspace{1cm} \text{Assembly error defined in Figure 7.14} \\
\( \Delta \lambda \) \hspace{1cm} \text{Lead angle error} \\
\( \Delta \psi_n \) \hspace{1cm} \text{Pressure angle error} \\
\( \varepsilon_\alpha \) \hspace{1cm} \text{Involute contact ratio} \\
\( \varepsilon_\beta \) \hspace{1cm} \text{Face contact ratio} \\
\( \phi_1, ..., \phi_m \) \hspace{1cm} \text{Gear mesh position (angle)} \\
\( \Phi \) \hspace{1cm} \text{Oil mixture function in Eq. (2.20)} \\
\( \bar{\gamma} \) \hspace{1cm} \text{Dimensionless coefficient of thermal expansion} \\
\( \eta \) \hspace{1cm} \text{Instantaneous efficiency} \\
\( \bar{\eta} \) \hspace{1cm} \text{Average efficiency in terms of the gear pair sliding and rolling frictional losses and the load dependent bearing loss (in Chapter 6)} \\
\( \bar{\bar{\eta}} \) \hspace{1cm} \text{Average efficiency in terms of the gear pair sliding and rolling frictional losses (in all chapters except Chapter 6)} \\
\( \varphi_T \) \hspace{1cm} \text{Thermal reduction factor}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>( \lambda )</td>
<td>Gearbox space function in Eq. (2.20)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Instantaneous coefficient of friction</td>
</tr>
<tr>
<td>( \bar{\mu} )</td>
<td>Average coefficient of friction</td>
</tr>
<tr>
<td>( \mu_b )</td>
<td>Bearing friction coefficient</td>
</tr>
<tr>
<td>( \mu_v )</td>
<td>Friction coefficient for asperity contact</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>( \bar{\nu} )</td>
<td>Dimensionless dynamic viscosity</td>
</tr>
<tr>
<td>( \nu_0 )</td>
<td>Dynamic viscosity at oil inlet under ambient pressure</td>
</tr>
<tr>
<td>( \nu_k )</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>( \theta_1, \theta_2 )</td>
<td>Roll angles of pinion and gear, respectively</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>Angular displacement of the pinion</td>
</tr>
<tr>
<td>( \bar{\rho} )</td>
<td>Dimensionless lubricant density</td>
</tr>
<tr>
<td>( \Sigma_1, \Sigma_2 )</td>
<td>Surface 1 and surface 2, respectively</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Shear stress</td>
</tr>
<tr>
<td>( \bar{\tau}_1 )</td>
<td>Dimensionless shear stress of surface 1</td>
</tr>
<tr>
<td>( \bar{\tau}_L )</td>
<td>Dimensionless limiting shear stress</td>
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<td>( \omega_1, \omega_2 )</td>
<td>Angular velocities of body 1 and body 2, respectively</td>
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<td>( \omega_b )</td>
<td>Bearing angular velocity</td>
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<tr>
<td>( \omega_{\text{in}}, \omega_{\text{out}} )</td>
<td>Input and output rotational speeds, respectively</td>
</tr>
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<td>( \xi_\alpha )</td>
<td>Tip relief</td>
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<tr>
<td>( \xi_\beta )</td>
<td>Lead crowning</td>
</tr>
<tr>
<td>( \psi_n )</td>
<td>Normal pressure angle</td>
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CHAPTER 1

INTRODUCTION

1.1 Research Background and Motivation

Gears are widely used in many applications including automotive, rotorcraft and off-highway vehicle drive trains, machine tools, and industrial gearboxes. Gears transmit power and rotational motion from one shaft to another. In this process, some of the power is inevitably lost due to friction in the system and the drag caused by the atmosphere around the gears.

Internal combustion engines and other forms of prime movers were the main focus of efficiency improvement efforts in the past. As most of the potential efficiency improvements from the engines have been realized, the focus has started to shift towards the efficiency of the remainder of the drive train including the transmission and the rear axle. In the view of the high fuel prices, the fuel economy of a vehicle became, perhaps the first time in recent history, a factor influencing the decision of the costumer. Independent of this, environmental pressures and government regulations are becoming more stringent in terms of allowable emissions in the form of hazardous gases and
particulates released to the environment. One percent improvement in drive train efficiency (from the output of the engine to the wheels) would reduce the fuel consumption and air pollutants by the same percentage. These form the motivation for gear or drive efficiency studies.

There are other benefits to improving gear efficiency as well. Since the efficiency losses amount to additional heat generation within the gearbox, several gear failure modes including scoring and contact fatigue failures are directly impacted by the efficiency of the gear pair. A more efficient gear pair generates less heat, and therefore, it is likely to perform better in terms of such failures. In the process, demands on the capacity and the size of the lubrication system and the amount and quality of the gearbox lubricant are also eased with improved efficiency. This also reduces the overall weight of the unit contributing to further efficiency improvements.

The total efficiency loss of the gearbox is attributable to sliding and rolling frictional losses between the gear teeth, windage losses due to complex interactions with the air surrounding the gears, and oil splashing and churning losses inside the gearbox, as well as the losses associated with the bearing and seals. While churning and windage losses are mostly geometry and speed related, friction losses are mainly associated with sliding velocities and load. Friction losses of hypoid gears are of primary interest here since they are a major source of losses in a drive train. As a common automotive application, the drive train of a rear-wheel-drive (or all-wheel-drive) car or truck contains
one (or two, one rear and one front) axle gearbox that is formed by a hypoid final drive
reduction unit and a differential. Unlike parallel axis gears that may have a mechanical
efficiency well over 99 percent, the efficiency of hypoid gears usually falls into the range
of 86 to 97 percent depending on the amount of relative sliding induced by the gear
geometry. This is mainly because sliding velocities in a hypoid gear contact are
significantly larger than parallel-axis gears. As a result of this, an axle unit that is formed
by a single hypoid gear set has the same levels of (and in many applications more) power
loss as a manual or automatic transmission that contains many spur or helical gears. This
presents the hypoid gear drive as a prime candidate for any efficiency improvement
efforts.

While friction power losses of an individual spur or helical gear pair are quite
small, efficiency of gear trains that use multiple gear pairs as in transmissions might be
significant. The mechanical efficiency of a gear train formed by in-series gear pairs is the
product of efficiencies of individual gear meshes. Therefore, any incremental efficiency
improvement that can be achieved from each gear pair often amounts to sizable
improvements when the entire gear train or transmission considered. Therefore,
efficiency of spur and helical gears cannot be overlooked as well.

Friction can be stated as the resistance to motion between two surfaces in relative
sliding and rolling under dry or lubricated contact conditions. Lubricant applied to a
contact significantly alters the contact conditions, and hence friction. Applied load,
speed, parameters related to contact geometry, surface roughness and lubricant all help define the lubrication conditions [1]. Gears are usually operated under mixed Elastohydrodynamic Lubrication (EHL) conditions where the lubricant film thickness is comparable to the surface asperity heights such that actual metal-to-metal contacts are possible. Therefore, the friction between the gear teeth can be considered as a hybrid of dry and fluid friction. This effective friction at each contact point of a gear plays an important role in defining friction-dependent efficiency losses as well as influencing scoring limits, contact fatigue lives and the dynamical behavior including vibration and noise [2,3].

Friction at a gear mesh has two components: sliding friction and rolling friction. Sliding friction is a direct result of the relative sliding between the two contacting surfaces while rolling friction exists due to the resistance to the rolling motion that takes place when one surface is rolled over another. Coefficient of friction $\mu$ usually refers to the coefficient of sliding friction, as a coefficient of rolling friction has no physical meaning.

### 1.2 Literature Review

Efficiency of gear systems has been of interest to gear researchers for more than a century [4]. A large number of studies have been published, especially within the last forty years, on friction and efficiency of gear trains as reviewed in references [5-7]. In
terms of the methodologies used for the study of gear efficiency, three main approaches are observed: (i) measurement of gear efficiency directly by using actual gears or representative hardware, (ii) semi-analytical prediction of efficiency using a pre-known constant or empirical $\mu$ information, and (iii) physics-based analytical prediction of gear efficiency by computing $\mu$ using an EHL model.

One group of experimental studies focused on measurement of power losses of gear pairs [8-17]. Several others adapted a simpler contact interface, most commonly a pair of cylindrical disks (twin-disks), to measure $\mu$ under conditions simulating a gear pair, so that this measured friction coefficient can be used to predict the efficiency of a gear pair [18-26, 38-39]. Some of these studies [18-25] resulted in well-known and widely used empirical formulae for $\mu$. These empirical formulae state that $\mu$ is a function of a number of parameters such as sliding and rolling velocities, radii of curvature of the surfaces in contact, load related parameters (unit load or contact pressure), amount of surface roughness, and the lubricant viscosity. The sliding velocities included in these empirical formulae are normal to the line of contact and are uniform along the contact line due to the geometry of the roller specimens and the twin-disk test configuration. There is no sliding in the direction of the contact line. Consequently, the formulae from these twin-disk experiments represent a contact line of a spur gear pair and cannot be used directly for other types of gears including helical, worm, and hypoid gears. For these types of gears, the radii of curvature vary along the
contact line. In addition, there is also a component of sliding velocity in the direction of the contact line.

Influence of the sliding in the direction of a contact line on the value of $\mu$ or the lubrication conditions is not well understood. A group of experimental investigations [27-30] attempted to bridge this gap between the contact conditions defined by a pair of cylindrical disks and the contact conditions of an actual gear pair. Hirano et al [27] studied the effect of angle between directions of sliding and line of contact on friction and wear of a roller to conclude that the sliding in the direction of the contact line results in the largest $\mu$ while the opposite is true for the direction normal to the contact line. Tan [28] and Hohn [29] stated in two independent studies for worm gears that increasing sliding in the direction of the contact line, while keeping the sliding and rolling in the direction of normal to the contact line constant, should increase the value of $\mu$.

Semi-analytical efficiency models can be reviewed in two groups. The first group of studies [31-33] investigated the efficiency of a spur gear pair by assuming a uniform (constant) $\mu$ along the entire contact surface and at every rotational position. A tangential friction force along the sliding direction was computed by using this user-defined constant friction coefficient $\mu$ and basic geometric and kinematic parameters of a spur gear pair. The amount of reduction of torque transmitted to the driven gear was used to calculate the mechanical efficiency $\eta$ of the gear pair. These models, while being very useful in bringing a qualitative understanding to the role of spur gear
geometry on efficiency, had some major shortcomings. The first shortcoming stems from the definition of $\mu$. It must be constant for every contact point of a gear pair, and it must be known beforehand. Data from the published twin-disk tests indicate that $\mu$ of a combined rolling/sliding contact is not constant. It is influenced by many contact parameters [18-25]. The second shortcoming is that these studies were limited to spur gears and many complicating effects of the tooth bending and contact deformations, tooth profile modifications and manufacturing errors were not included in the efficiency models.

The second group of semi-analytical models [34-40] can be considered as an improvement over the constant $\mu$ type models. These models relied on published experimental $\mu$ formulae such as those of references [18-21] to predict the efficiency of spur [35-38, 40] and helical [34] gear pairs. While they are potentially more accurate than the models of the first group, these models are still limited in the sense that their accuracy is dependent largely on the accuracy of the empirical $\mu$ formula used. These empirical $\mu$ formulae are not general and often represent certain types lubricants, operating temperatures, speed and load ranges, and surface roughness conditions of roller specimens that might differ from those of the actual gear pair of interest.

There are a number of gear efficiency models that use different forms of EHL models for predicting $\mu$ [42-54]. Among them, Dowson and Higginson [47] and Martin [48] assumed smooth contact surfaces for a spur gear pair and computed the
instantaneous friction coefficient caused by surface shear stress distribution caused by the fluid film at the contact from a smooth surface EHL model. Adkins and Radzimovsky [49] developed an efficiency model for lightly loaded spur gears under hydrodynamic lubrication conditions, assuming that the gear teeth are rigid. Simon [50] provided an enhancement by using a point contact EHL model for heavily crowned spur gears with smooth surfaces. He included the elastic deformations of the surfaces due to the fluid pressure distribution. Larsson [51] conducted a transient EHL analysis for an involute spur gear with smooth surfaces. In this analysis, an isothermal full film thickness was assumed with a non-Newtonian model. Gear teeth were assumed to be rigid. Tooth-to-tooth load distribution at the gear mesh was modeled as a step function. Wang, et al [52] analyzed involute spur gear lubrication by using a transient thermal EHL model for a Newtonian fluid. In this model, gear teeth were again assumed rigid and smooth. It was assumed that the load is carried by either one tooth pair or two tooth pairs with the transition from one to other modeled as a trapezoidal variation of the load.

Wu and Cheng [53] developed a spur gear friction model based on mixed EHL contact analysis. The surface roughness was modeled such that all the asperities have the same radius of curvature whose heights have a Gaussian distribution. This study used an empirical rolling friction formula that was developed by Goksem and Hargreaves [53] for isothermal contacts. In order to account for the effect of the temperature rise at high speeds, a thermal reduction factor was used to modify this formula. Michalidis et al [54] included the influence of the asperity contacts as well in calculating $\mu$. This study used a
numerically generated roughness profile as an input. Gear materials were assumed to behave according to the elastic-perfectly plastic model. Thermal effects and non-Newtonian lubricant behaviors were considered.

The EHL based models [47-54] were successful in eliminating (to a certain extent) the need for prior knowledge of $\mu$. However, they were not practical since they required significant CPU time to run. In addition, they might be more accurate in EHL aspects of the problem, their modeling of gears was limited to simple spur gears with ideal load distributions and no tooth deformations.

The literature on efficiency of helical gears is quite limited. Papers by Akin [55] and Wellauer and Holloway [56] presented formulations on how to calculate EHL film thickness along the pitch line of a helical gear contact by using the film thickness formula of Dowson and Higginson. Chittenden, et al [57] presented a generalized analysis of a smooth surface EHL problem under isothermal condition having a fluid entrainment at some intermediate angle from the minor axis of the contact ellipse, as is the case for helical gears. Simon [58] extended his EHL analysis to helical gears using a smooth surface, point contact thermal EHL model. Haizuka, et al [86] studied the influence of helix angle on frictional power loss of helical gears experimentally and found the power loss increased with the increasing of the helix angle.
In terms of efficiency of hypoid gears, Buckingham [59] proposed an approximate formula for the power loss of hypoid gears, which is the sum of the losses of corresponding spiral bevel and worm gears. Naruse et al [8,10] conducted several tests on scoring and frictional losses of hypoid gears of the Klingelnberg type. Coleman [60] used a simple $\mu$ formula with a very limited number of parameters to calculate bevel and hypoid gear efficiency. In a later study [61], he proposed a scoring formula for bevel and hypoid gears with a $\mu$ formula that is a function of only the surface velocity and surface roughness. Smooth surface thermal EHL formulations were also applied to a modified hypoid gear pair by Simon [62] using a point contact model. Jia et al [63] analyzed the elastohydrodynamically lubricated hypoid gears by the multi-level techniques to study the film thickness and pressure distributions under isothermal condition.

1.3 Scope and Objective

The models reviewed in the previous section were quite limited in terms of their representation of gears. Almost all of them assumed rigid gear teeth (no bending deflections, base rotations or gear blank deformations) and relied on theoretical idealized load distributions along the contact line. These models were not capable of including modified profiles and any type of geometric deviations resulting from the manufacturing processes, heat treatment distortions, assembly errors and deflections of support structures. This study will aim at eliminating these shortcomings by using gear contact models that are capable of including such effects. It will also aim at developing and
validating general physics-based efficiency models of gear pairs that are less demanding computationally.

Most of these previous efficiency models were not validated. Therefore, their accuracy and effectiveness in representing real-life gear pairs are not known. Consequently, this study will emphasize the validations of not only the friction coefficient models but also the overall gear pair efficiency models.

The developed efficiency methodology will combine a gear contact analysis model, an EHL-based coefficient of friction model, and an efficiency computation formulation. This methodology will be applied to spur/helical and hypoid gear pairs. Since the focus of this study is the prediction of mechanical efficiency in terms of frictional power losses, the prediction of other losses related to oil churning, gear windage, and bearings will rely on published work of others [32,36,64-69].

Specific objectives of this study are as follows:

(i) Develop a model for the prediction of mechanical efficiency of gear pairs including all key gear, lubricant, surface finish, and operational parameters.

(ii) Assess the accuracy of the published empirical $\mu$ formulae by comparing them to the EHL-based $\mu$ values predicted by an EHL model and experimental traction data.
(iii) Validate the friction coefficient predictions of the EHL model by comparing them to the measured traction data at various load, speed and sliding conditions as well as roughness and lubricant properties.

(iv) Derive new empirical and EHL-based $\mu$ formulae to minimize computational effort associated with EHL-based model so that the efficiency methodology can be used as a design and/or optimization tool.

(v) Apply the developed efficiency methodology to parallel-axis (spur and helical) and right-angle (spiral bevel and hypoid) gear pairs.

(vi) Validate the efficiency prediction methodology by comparing the predictions to actual spur gear efficiency measurements [70].

(vii) Perform parametric studies to quantify the influence of basic gear parameters, tooth modifications, operating load, speed and temperature conditions, lubricant properties, manufacturing and assembly errors, and surface finish and treatments on mechanical efficiency of gears.

The ultimate goal in this study is to identify the key parameters influencing mechanical efficiency of a gear pair and define design guidelines for improved gear efficiency.

1.4 Dissertation Outline

Chapter 2 outlines the overall technical methodology proposed in this study for the prediction of the mechanical efficiency of a gear pair and describes the main components of this methodology. In Chapter 3, differences between friction coefficient
models are identified and explained by comparing them with the measured traction data. Based on the results of this comparison, Multiple Linear Regression Analysis is employed to derive a new friction coefficient formula based on the EHL model predictions. A set of friction coefficient formulae is also developed with the measured traction data.

Chapter 4 is devoted to the application of the mechanical efficiency model to the parallel-axis gears. The load distribution model and formulations of surface velocities and radii of curvature are presented. Efficiency prediction results are presented for an example helical gear pair. The application of the developed mechanical efficiency methodology to cross-axis gears is described in Chapter 5. This chapter gives an introduction to the contact mechanics model used, provides the relevant details for the calculation of the hypoid gear tooth surface curvatures and velocities that are inputs to friction coefficient models. Variations of the surface sliding and rolling velocities, radii of curvature, load and pressure distributions as well as predicted friction coefficients are presented along the lines of contact at a given mesh angle. A set of efficiency predictions is presented for an example hypoid gear pair.

The gear pair mechanical efficiency methodology is validated in Chapter 6 by using an experimental database established for high-speed spur gears [70]. This extensive validation effort includes different speed-load conditions, different gear design variations as well as different lubricant and surface conditions.
Chapter 7 presents results of a number of parametric studies aimed at arriving at
design guidelines for efficiency of both parallel-axis and cross-axis gear pairs. Variables
in these parametric studies include the gear geometry, tooth modifications, surface finish
and treatments, lubricant parameters, manufacturing and assemble errors as well as
operating conditions. Finally, Chapter 8 presents the main conclusions and contributions
of this study and provides a list of recommendations for future work.
CHAPTER 2

OVERALL METHODOLOGY

2.1 Technical Approach

Figure 2.1 illustrates a flowchart of the efficiency computation methodology used in this study. It consists of three main components: (i) a gear contact analysis model, (ii) a friction coefficient model, and (iii) a gear pair mechanical efficiency computation formulation.

The gear contact analysis model uses the gear design parameters, machine setups and cutter parameters, operating conditions and errors associated with assembly, mounting and manufacturing of the tooth profile to predict load and contact pressure distributions at every contact point at each incremental mesh position.

Predicted load and contact pressure distributions together with geometry, kinematic parameters, surface finish, and lubricant parameters are input to a friction
Figure 2.1. Flowchart for the efficiency prediction methodology
coefficient model to determine the instantaneous friction coefficient $\mu(z, \theta, \phi_m)$ of every contact point $(z, \theta)$ on the gear tooth surfaces, where $z$ and $\theta$ are the gear surface parameters and $\phi$ is the gear rotation angle. $\mu(z, \theta, \phi_m)$ is then used by the mechanical efficiency computation module to determine the instantaneous efficiency $\eta(\phi_m)$ of the gear pair at the $m$-th incremental rotational position defined by angle $\phi_m$. This procedure is repeated for an $M$ number of discrete positions $(m = 1, 2, \ldots, M)$ spaced at an increment $\Delta\phi$ ($\phi_m = m \Delta\phi$) to cover an entire mesh cycle. These instantaneous mechanical efficiency values $\eta(\phi_m)$ are then averaged over a complete mesh cycle to obtain the average mechanical efficiency of the gear pair. In the following sections, the main components of this methodology as shown in Figure 2.1 are described briefly.

### 2.2 Gear Contact Analysis Models

For computation of the contact loads of parallel-axis gears, a load distribution model that was initially proposed by Conry and Seireg [71] and later further developed by Houser [72] is used in this study. This model is designed to compute elastic deformations at any point of the gear surface given the tooth compliance, applied torque, and the initial tooth separations under no load. It assumes that the elastic deformations are small, and hence, the macro-level gear geometry remains unchanged so that the tooth contact is contained in the near vicinity of the contact line, even after the deformations. The formulation of this model is discussed in detail in Chapter 4.
For the contact analysis of face-milled and face-hobbed hypoid gears, a commercially available finite element (FE) based gear analysis package CALYX [73] is used. This model combines FE method and a surface integral approach [74]. The contact analysis model used in this study has a special setup for the finite element grids inside the instantaneous contact zone. A set of very fine contact grid is defined automatically on hypoid gear teeth to capture the entire contact zone. These grid cells are much finer than the regular sizes of finite element meshes at elsewhere on the tooth surfaces and they are attached to the contact zones that result in more accurate contact analysis. This model also allows plug-ins developed by users so that the mechanical efficiency formulation can be linked to the gear contact analysis model. Details on this contact analysis model and derivations of related geometry and kinematics parameters for hypoid gears are presented in Chapter 5.

2.3 Friction Coefficient Models

When gears are in contact, load distribution along the contact lines on consecutive gear teeth can be obtained by the gear contact model as illustrated schematically in Figure 2.2(a). Here, the load distribution along the lines of contact is discretized. As most of the relevant parameters along the lines of contact vary, including the rolling and sliding velocities, radii of curvature, contact pressure (or load per unit contact length), and surface roughness, the friction coefficient should vary along the contact lines as well. Since it is not practical to analyze every possible contact point for its $\mu$ value, a
Figure 2.2. (a) A schematic illustration of the gear load distribution, and (b) the equivalent cylindrical contact representing contact segment $q$. 

$$R = \frac{n_1 n_2}{n_1 + n_2}$$

$$E' = \frac{2}{\frac{1 - n_1^2}{E_1} + \frac{1 - n_2^2}{E_2}}$$
discretization such as the one shown in Figure 2.2(a) is required. A more refined load distribution (more discrete segments) would result in more accurate predictions at the expense of more computational time.

At every instant in mesh, there can be one or several teeth in contact depending on the total gear contact ratio. Each contact line consists of several segments, and each segment is represented by the contact characteristics of a point located in the middle of the segment. The gear contact at each one of these discrete segments can be equated to the contact of two equivalent cylinders with radii of curvature \( r_1 \) and \( r_2 \), normal load \( w \), and surface velocities \( u_1 \) and \( u_2 \) as shown in Figure 2.2(b). This is equivalent to a cylinder with a sliding velocity \( u_1 - u_2 \) and radius of curvature \( R = r_1 r_2 / (r_1 + r_2) \) in contact with a flat surface. The sliding velocity is required to be perpendicular to the contact segment and the two contacting surfaces are usually separated by a certain amount of lubricant film. As a result, a fluid pressure and viscous shear are generated to cause friction traction along the contacting surfaces. This friction traction can be either estimated by using published empirical friction coefficient formulae at each contact segment or analyzed by using a line contact EHL formulation. In this study, both approaches will be explored.
2.3.1 Published empirical friction coefficient formulae

A large number of empirical formulae for coefficient of friction can be found in the literature. Most of these formulae were obtained by curve fitting measured data collected from twin-disk type tests. They have the following general form:

\[
\mu = f \left( v_k, v, V_s, V_r, R, W', P_{\text{max}}, S, \ldots \right).
\]  

(2.1)

Here, \( v_k \) and \( v \) are the respective kinematic and dynamic viscosities of the lubricant, both of which are measured at the oil inlet, and are functions of inlet oil temperature at ambient pressure. Parameters \( V_s, V_r, \) and \( R \) denote relative surface sliding velocity, sum of the rolling velocities and the combined radius of curvature, respectively. The load parameters are the unit normal load \( W' \) or the contact pressure \( P_{\text{max}} \). \( S \) is a surface finish parameter, representing the initial composite surface roughness of the two contacting surfaces.

A representative set of commonly cited friction coefficient formulae are listed in Table 2.1 and implemented in this study. Several other empirical formulae are also available in the literature [5-7]. An examination of Table 2.1 indicates that these formulae are quite different from each other in terms of the parameters that are included and the parameter ranges represented. For instance, formulae by Drozdov and Gavrikov [21] and Misharin [18] do not include any surface roughness parameter, and hence, influence of varying surface roughness cannot be studied using these formulae. The remaining three formulae
<table>
<thead>
<tr>
<th>Formulae and Authors</th>
<th>Applicable ranges</th>
<th>Specific units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drozdov and Gavrikov [21]:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = \left[ 0.8 \sqrt{v_k V_s + V_r \phi + 13.4} \right]^{-1}$</td>
<td>$v_k \in [4, 500]$</td>
<td>$V_s, V_r: \text{m/s}$</td>
</tr>
<tr>
<td>$\phi = 0.47 - 0.13 (10)^{-4} P_{\text{max}} - 0.4 (10)^{-3} v_k$</td>
<td>$V_s \leq 15, V_r \in [3, 20]$</td>
<td>$P_{\text{max}}: \text{kg/cm}^2$</td>
</tr>
<tr>
<td></td>
<td>$P_{\text{max}} \in [4000, 20000]$</td>
<td></td>
</tr>
<tr>
<td>O’donoghue and Cameron [20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.6 \left[ \frac{(S + 22)}{35} \right] \left[ V_s^{1/8} V_s^{1/3} V_r^{1/6} R^{1/2} \right]^{-1}$</td>
<td></td>
<td>$S: \text{\mu in, CLA}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_s, V_r: \text{in/s}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R: \text{in}$</td>
</tr>
<tr>
<td>Misharin [18]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.325 \left[ V_s V_r v_k \right]^{0.25}$</td>
<td>$V_s / V_r \in [0.4, 1.3]$</td>
<td>$V_s, V_r: \text{m/s}$</td>
</tr>
<tr>
<td></td>
<td>$P \geq 2,500 \text{ kg/cm}^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mu \in [0.02, 0.08]$</td>
<td></td>
</tr>
<tr>
<td>ISO TC60 [41]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.12 \left[ W' S / (R V_s v) \right]^{0.25}$</td>
<td></td>
<td>$V_r: \text{m/s}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R: \text{mm}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$S: \text{\mu m, RMS}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$W': \text{N/mm}$</td>
</tr>
<tr>
<td>Benedict and Kelley [19]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.0127 \left[ \frac{50}{50 - S} \right] \log_{10} \left[ \frac{3.17 (10)^8 W'}{v V_s V_r^2} \right]$</td>
<td>$\frac{50}{50 - S} \leq 3$</td>
<td>$S: \text{\mu in, RMS}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$W': \text{lbf/in}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_s, V_r: \text{in/s}$</td>
</tr>
</tbody>
</table>

**Table 2.1.** Empirical friction coefficient formulae considered in this study ($v_k$ and $\nu$ are given in centistokes and centipoises, respectively).
in Table 2.1 consider a surface parameter $S$ that is either the root mean square (RMS) or the centerline average (CLA) of the roughness profiles. Similarly, formulae by Drozdov and Gavrikov and Misharin also exclude the radius of curvature $R$. Meanwhile, ISO TC60 formula does not include $V_s$. Similarly, the formulae of O’Donoghue and Cameron [20] and Misharin are not a function of the normal load ($W'$ or $P$) and hence they cannot account for any load effects on $\mu$. In addition, each formula listed in Table 2.1 is valid within certain ranges of key system parameters. For instance, the formula proposed by Drozdov and Gavrikov is valid for $\nu_k \in [4, 500]$ centistokes, $V_s \leq 15$ m/s, $V_r \in [3, 20]$ m/s and $P_{\max} \in [4000, 20000]$ kg/cm$^2$, and would be suitable for a gear application only if these gear contact parameters stay within these ranges. These formulae will be used in this study with special attention given to their parameter sets and ranges of applications.

2.3.2 Calculation of friction coefficient using an EHL model

Distribution of the film pressure and thickness across an elastic lubricated contact can be obtained by solving the transient Reynolds equation simultaneously with a number of other equations including the film thickness equation, viscosity-pressure-temperature relationship, density-pressure-temperature relationship, the energy balance equation and the load equation [75]. In this study, a deterministic EHL model proposed by Cioc et al [75] will be used.
Assuming that a helical gear pair exhibits line contact conditions, a non-dimensional transient Reynolds equation is written as:

\[
\frac{\partial}{\partial X} \left[ s_0 \frac{P_0 b}{12 \nu_0 u} \left( \frac{\bar{\rho} \bar{H}^3}{\bar{\nu}} \right) \frac{\partial \bar{P}}{\partial X} \right] = \frac{\partial}{\partial X} \left( \bar{\rho} \bar{H} \right) + \frac{\partial}{\partial \tilde{t}} \left( \bar{\rho} \bar{H} \right) \quad (2.2)
\]

with the boundary conditions of \( P = 0 \) at inlet, and \( P = 0 \) and \( \partial P/\partial X = 0 \) at outlet. In Eq. (2.2), \( b \) is the half width of Hertzian contact zone, and \( X \) and \( H \) are the dimensionless coordinate and the dimensionless film thickness, respectively, both normalized by \( b \), \( P_h \) is the maximum Hertzian pressure, \( \bar{\nu} \) is the dimensionless dynamic viscosity normalized by \( \nu_0 \), which is the dynamic viscosity at oil inlet, and \( \bar{\rho} \) is the dimensionless lubricant density normalized by the lubricant density at inlet. \( u \) is the rolling velocity defined as the mean surface velocity of the two contacting surfaces as \( u = (U_1 + U_2)/2 \) and \( \tilde{t} \) is the dimensionless time defined as \( \tilde{t} = ut/b \). For a Newtonian fluid \( s_0 = 1 \) while \( s_0 \) is a function depending on the limiting shear stress, film thickness, fluid pressure and surface velocity that is similar to the model presented by Larsson [51] for a non-Newtonian fluid [75].

The film thickness equation has the form

\[
\bar{H} = \frac{h_0}{b} \left( \frac{b}{2R} \right) \tilde{X}^2 + \frac{d(X, \tilde{t})}{b} + S(X, \tilde{t}) \quad (2.3)
\]
where, \( h_0 \) is the reference film thickness, \( d(X, \bar{t}) \) is the total elastic deformation, \( S(X, \bar{t}) \) is the composite surface profile irregularities, and \( R \) is the combined radius of curvature of the contact.

A two-slope viscosity-pressure-temperature model [75] is used as

\[
v = \begin{cases} 
\alpha_1 P_h & P < P_{ta}, \\
e^{G_1 P - \bar{T}(T-1)} & P_{ta} \leq P \leq P_{tb}, \\
e^{G_2 (P - P_{tr}) - \bar{T}(T-1)} & P_{tb} \leq P.
\end{cases}
\]  

(2.4)

Here, \( G_1 = \alpha_1 P_h \) and \( G_2 = \alpha_2 P_h \), \( \bar{T} \) is the dimensionless coefficient of thermal expansion, and \( \bar{T} \) is the dimensionless film temperature normalized by the oil temperature at inlet. \( P_{tr} \) is the transitional pressure, a threshold value beyond which an increase in viscosity changes its slope from \( \alpha_1 \) to \( \alpha_2 \). Transition pressure values of \( P_{ta} = 0.7 P_{tr} \) and \( P_{tb} = 1.4 P_{tr} \) with \( P_{tr} = 380 \) MPa were suggested by Allen et al [75] for mineral oils. The coefficients \( c_0 \) to \( c_3 \) are determined such that that transition between the two slopes is smooth.

In addition, the following density-pressure-temperature relationship is considered

\[
\bar{\rho} = \left( 1 + \frac{C_a P}{1 + C_b P} \right) \left[ 1 - \bar{\rho}(\bar{T} - 1) \right] 
\]  

(2.5)
where $C_a = 0.6 \times 10^{-9} P_h$, $C_b = 1.7 \times 10^{-9} P_h$, and $\bar{\beta}$ is the dimensionless coefficient of thermal expansion [75].

Since a thermal EHL model is sought, energy equations for Newtonian and non-Newtonian lubricants are given, respectively, as

\[
\frac{\partial^2 T}{\partial Z^2} = C_2 \bar{U} \frac{\partial T}{\partial X} - C_3 \bar{T} \bar{U} \frac{\partial P}{\partial X} - C_4 \bar{P} \left( \frac{\partial \bar{U}}{\partial X} \right)^2 + C_6 \bar{P} \frac{\partial T}{\partial t} \tag{2.6a}
\]

\[
\frac{\partial^2 T}{\partial Z^2} = C_2 \bar{U} \frac{\partial T}{\partial X} - C_3 \bar{T} \bar{U} \frac{\partial P}{\partial X} - C_5 \frac{\bar{\tau}_1 \bar{\tau}_L}{\sqrt{\bar{\tau}_L^2 - (\bar{\tau}_1)^2}} + C_6 \bar{P} \frac{\partial T}{\partial t} \tag{2.6b}
\]

In Eq. (2.6), coefficients $C_i$ ($i = 1$ to 6) are functions of thermal parameters, $Z$ is the dimensionless coordinate normalized by $b$, $\bar{U}$ is the dimensionless fluid velocity, $\bar{\tau}_1$ is the dimensionless shear stress of surface 1, and $\bar{\tau}_L$ is the dimensionless limiting shear stress.

Finally, the load distribution predicted across the contact segment should be such that the normal load applied at the contact is balanced, i.e.

\[
\bar{W} = \int_{X_{in}}^{X_{out}} [P(X, \bar{t}) + P_c(X, \bar{t})] dX \tag{2.7}
\]
where $\bar{W}$ is the dimensionless unit load applied, $X_{in}$ and $X_{out}$ are the inlet and outlet of computational domain for the contact zone, $P_c$ is the asperity contact pressure for where asperity contact occurs. This load balance equation is checked periodically and the reference film thickness $h_0$ in Eq. (2.3) is adjusted, if necessary.

In reference [75], Eqs. (2.2) to (2.7) were solved numerically by using the finite difference method. Using the same methodology, the pressure $p(x)$, viscosity $\nu(x)$, and fluid film thickness $h(x)$ distributions across the lubricated contact zone are obtained. In the case of no asperity contacts, the surface shear stress distribution across the Hertzian contact width $-b \leq x \leq b$ is given by

$$\tau(x) = -\frac{h(x)}{2} \frac{dp}{dx} + \nu(x) \frac{U_{1x} - U_{2x}}{h(x)}.$$  

Typical ground or shaved gear tooth surfaces cannot be considered smooth unless they are chemically or mechanically polished. The actual surface roughness amplitudes are typically comparable to the fluid film thickness. As a result, the portion of the normal load might be carried by actual asperity contacts resulting in mixed-EHL conditions. In this case, Eq. (2.8) is still valid at the $x$ locations where a fluid film is maintained between the two surfaces. At the $x$ locations where actual asperity contacts take place

$$\tau(x) = \mu_x p(x)$$  

(2.9)
where $\mu_s$ is the friction coefficient between the contacting asperities. Consequently, the friction traction force per unit width of the contact can be written as

$$F_t = \int \tau(x) dx = F_r' + F_s'$$

(2.10)

where $F_r'$ is the rolling friction per unit width of the contact that comes from the viscous flow in the formation of the film thickness and squeeze motion during pure rolling,

$$F_r' = -\int h(x) \frac{\partial p}{\partial x} dx$$

(2.11a)

and $F_s'$ is the sliding friction per unit width of the contact that is caused by fluid shear and the asperity contact, respectively [76]

$$F_s' = \int v(x) \frac{U_1x - U_2x}{h(x)} dx + \int \mu_s p(x) dx.$$  

(2.11b)

Finally, the coefficient of sliding friction of the entire section of contact is given by

$$\mu = \frac{F_s'}{W'}$$

(2.12)

where $W'$ is the applied normal load per unit width of contact.
With the exception of $\mu_s$ in Eq. (2.11b) that must be defined in cases when asperity contact occurs, the approach outlined above to compute $\mu$ is entirely physics-based. One disadvantage of this approach is that it requires significant computational effort since a large number of individual EHL analyses must be carried out for each discretized segment along the lines of contact at each rotational gear position considered.

2.4 Gear Efficiency Computation

Once the coefficient of friction $\mu(z, \theta, \phi_m)$ at each contact point $(z, \theta)$ at each rotational angle $\phi_m$ ($m = 1, 2, ..., M$) are known, the sliding friction force at each contact position can be calculated by

$$F_f(z, \theta, \phi_m) = \mu(z, \theta, \phi_m)W(z, \theta, \phi_m)$$

(2.13)

where $W(z, \theta, \phi_m)$ is the normal load at the contact point of interest. The rolling friction force $F_r$ can be obtained by multiplying $F'_r$ in Eq. (2.11a) with the width of the contact. When the empirical formulae like the ones listed in Table 2.1 are used, $F_r$ can be estimated from an isothermal EHL contact [53] as

$$F_{ro} = 4.318(GU)^{0.658} Q^{0.0126} R/\alpha$$

(2.14)
where $\tilde{G} = \alpha E'$ is the dimensionless material parameter, $\tilde{U} = \frac{V_0(u_1 + u_2)}{E'R}$ is the dimensionless speed parameter, $\tilde{Q} = \frac{W'}{(E'R)}$ is the dimensionless load parameter, $R$ is the effective radius of curvature, $\alpha$ is the pressure viscosity coefficient. In order to account for the effect of temperature rise at high speed conditions, a thermal reduction factor $\varphi_T$ can be used to modify this isothermal formula such that

$$F_r = \varphi_T F_{ro}$$

(2.15)

where $\varphi_T$ is defined in reference [53] as

$$\varphi_T = \frac{1 - 13.2(P_h/E')(L^*)^{0.42}}{1 + 0.213(1 + 2.23SR^{0.83})(L^*)^{0.64}}$$

(2.16a)

where

$$L^* = \frac{\partial V}{\partial t_0} \frac{(V_c)^2}{K_f} .$$

(2.16b)

Here, $\nu$ is the absolute viscosity in cPs, $t_0$ is the lubricant temperature at inlet in degrees C, $K_f$ is the lubricant thermal conductivity in $W/m°C$, $SR$ is the slide-to-roll ratio that is defined as $SR = 2(u_1 - u_2)/(u_1 + u_2)$, and $V_c = (u_1 + u_2)/2$. 

30
The instantaneous efficiency of a gear pair is defined as the ratio of the instantaneous output power to the input power

$$\eta(m) = \frac{L_{out}(m)\omega_{out}}{L_{in}\omega_{in}}$$  \hspace{1cm} (2.17)$$

where $L_{out}(m)$ and $L_{in}$ are the values of torque acting on the output and input gears and $\omega_{out}$ and $\omega_{in}$ are the output and input rotational speeds, respectively. When only frictional losses are considered, the instantaneous output power can be written as the difference between the input power and the frictional power losses and hence the efficiency can be expressed as

$$\eta(m) = 1 - \frac{1}{L_{in}\omega_{in}} \sum_{q=1}^{Q} \left[ |F_f(u_1 - u_2)| + |F_r(u_1 + u_2)| \right]_q$$  \hspace{1cm} (2.18)$$

where $\sum_{q=1}^{Q} \left[ |F_f(u_1 - u_2)| + |F_r(u_1 + u_2)| \right]_q$ is the sum of the sliding frictional and rolling frictional power losses at position $m$ and $Q$ is the total number of load segments at the same position.
2.5 Other Sources of Power Losses

2.5.1 Windage and oil churning losses

The windage losses are difficult to predict. They are dependent on many parameters such as the width and diameter of the gear blank, speed, gear blank/rim geometry, the shape of the housing, type of the lubrication method (dip or jet lubrication), and the operating temperature and viscosity of the oil [77]. The following formula can be used to estimate the windage loss \( P_w \) of a gear having an outside diameter \( D_o \) and face width \( B \) as

\[
P_w = 10^{-17} N^3 D_o^5 B^{0.7}
\]  

(2.19)

where, \( P_w \) is the horsepower loss due to windage, \( N \) is the speed in rpm, \( D_o \) and \( B \) is the gear outside diameter and face width, respectively. Townsend [32] proposed another formula in the form

\[
P_w = 10^{-20} \Phi N^{2.9} (0.16 D_r^{3.9} + D_r^{2.9} B^{0.75} m^{1.15})
\]  

(2.20)

where \( \Phi \) is an oil mixture function that indicates the state or type of the atmosphere within the gear unit (\( \Phi = 1 \) for an oil free atmosphere), \( \lambda \) is a gearbox space function (\( \lambda = 1 \) for free space; \( \lambda = 0.6 \) to 0.7 for large enclosure and \( \lambda = 0.5 \) for a fitting
enclosure), $D_r$ is the root diameter and $m$ is the module, both in mm. Another similar empirical formula for windage losses was proposed by Anderson and Loewenthal [36] as

$$P_w = 2.82 \times 10^{-7} (1 + 2.3 \frac{B}{R_p}) N^{2.8} R_p^{4.6} (0.028 \nu + 0.019)^{0.2}$$

where $B$ and $R_p$ are face width and pitch radius in meter, $N$ is the rotational speed in RPM; $\nu$ is the dynamic viscosity in centipoises. These formulae are valid for spur gears. For helical gears, Dawson [65] provided a correction factor that describes the relationship between helix angles and the power loss as percentage of equivalent spur gears.

Churning losses result from gear blanks revolving in the oil sump and generating splash for lubricating the gear teeth, bearings, and seals. These losses are a function of speed and oil level, oil viscosity and temperature, tooth geometry and the submerged depth of the gears [32]. Several empirical formulae [67] were proposed to estimate the oil churning losses. The British Standard BS ISO/TR 14179 [32] provides a formula for oil churning losses for a smooth outside diameter, smooth side faces (both faces), and for the tooth surfaces as

$$P_{ch} = \frac{7.37 F_k \nu_k N^3 D_o^{4.7} L}{A_g 10^{26}},$$

(2.22a)
respectively. In Eq. (2.22), \( F_g \) is the dip factor (\( F_g = 0 \) if the gear is not dipped in oil, and \( F_g = 1 \) if a gear is fully dipped into oil), \( v_k \) is the kinematic viscosity at operating temperature, \( R_f \) is a roughness factor, \( \beta_n \) is the normal helix angle and \( A_g \) is the arrangement factor. In Eq. (2.22c), \( i = 1,2 \) for pinion and gear respectively.

2.5.2 Bearing friction losses

Rolling element bearing losses originate from various sources [78]: (i) rolling friction due to elastic hysteresis and deformation at raceway contacts, (ii) sliding friction from unequal curvatures in contact areas, sliding contact of cage with rolling elements and guiding surfaces, sliding between the ends of the rollers and ring flanges, and seal friction, (iii) lubricant friction due to viscous shearing on rolling element, cage and raceway surfaces, and (iv) churning and working of lubricant dispersed within the bearing cavity.
The most widely accepted bearing efficiency formulation given by Harris [78] is based on the original formulations of Palmgren. Bearing friction moment of a rolling element bearing without a seal can be written as

\[ M_b = M_P + M_L \]  

(2.23)

where \( M_P \) is the load dependent moment resulting from rolling and sliding friction in loaded rolling contacts

\[ M_P = 0.5 \mu_b W_b d_{bore} . \]  

(2.24a)

Here, \( M_P \) is calculated at the bearing bore radius of \( 0.5 d_{bore} \), where \( d_{bore} \) is the bearing bore diameter in mm, \( \mu_b \) is the coefficient of friction for the bearing. And \( W_b \) is the bearing load in N. The second component in Eq. (2.23), \( M_L \) (unit: N-mm), accounts for viscous friction and related cage friction in rolling element pockets, and at cage guiding surfaces given as [78].

\[ M_L = \begin{cases}  10^{-7} f_L (\nu_k N)^{2/3} D_m^3, & \nu_k N > 2000 \\ 1.60 \times 10^{-5} f_L D_m^3, & \nu_k N < 2000 \end{cases} \]  

(2.24b)

Here, \( f_L \) is a factor depending on bearing type and method of lubrication, \( N \) is the bearing speed in rpm and \( D_m \) is the mean diameter of bearing in mm. Finally, the total
bearing power losses can be obtained by multiplying the total frictional moment in Eq. (2.23) with the bearing angular velocity as $P_b = M_b \omega_b$. 
CHAPTER 3

VALIDATION OF FRICTION COEFFICIENT MODELS

3.1 Introduction

As stated earlier, power loss due to gear mesh friction is one of the major contributors to gear energy losses, and hence, is the primary focus of this study. For the rolling friction losses, the published model described in Chapter 2 is adapted here. For frictional losses due to sliding, both published friction coefficient $\mu$ formulae and a thermal EHL model are used. Since the load and contact pressure distributions can be predicted accurately by the available gear contact analysis models, the accuracy of the friction coefficient models should determine the accuracy of the gear mechanical efficiency predictions. Therefore, a validation of the $\mu$ models is essential before the gear mechanical efficiency predictions can be accepted with confidence. This chapter aims at achieving this goal. The following sequence of analyses will be performed for this purpose:
(i) First of all, the \( \mu \) predictions from the published empirical formulae and the thermal EHL model will be compared to determine whether they are in agreement both quantitatively and qualitatively within the typical parameter ranges of a gear contact.

(ii) Sets of measured traction coefficient data collected through simulated gear contacts will be analyzed using statistical methods to obtain a set of new \( \mu \) formulae representing gear contact in terms of materials, surface roughnesses, rolling and sliding velocities, contact pressures and lubricant parameters.

(iii) The new experimental \( \mu \) formulae will be compared to published \( \mu \) formulae and the thermal EHL model to assess their accuracy.

(iv) With the presumption that the EHL model predicts more accurate \( \mu \) values since it is physics-based, the same statistical methods will be applied to the EHL simulations to obtain a new EHL-based \( \mu \) formula. If this is possible, then the need to run the EHL model at every contact segment in real time would be eliminated, thus significantly reducing the computational effort required for gear efficiency analysis.
3.2 Comparison of Empirical and the EHL-Based μ Models

It is noted in Table 2.1 that different forms and parameter sets have been used in the published empirical formulae to represent the effect of load, speeds, the lubricant and surface condition. For instance, Drozdov and Gavrikov’s formula uses the maximum Hertzian pressure as the load parameter, while the widely used formula of Benedict and Kelley considers the load per unit length as the load parameter. Before any comparison can be made between the empirical formulae listed in Table 2.1, they must be reformulated so that each formula uses the same parameter definitions. For this purpose, two transformations are performed without influencing the accuracy of the original formulae.

(i) A new parameter, slide-to-roll-ratio, is introduced that is defined as \( SR = \frac{V_s}{V_r} \), where \( V_s = u_1 - u_2 \) and \( V_r = u_1 + u_2 \).

(ii) Maximum Hertzian pressure \( P_h \) is used to replace the load per length \( W' \). For a line contact problem, \( P_h = P_{\text{max}} = \frac{W' E'}{2 \pi R} \). Here \( R = \frac{\eta r_2}{\eta + r_2} \) is the equivalent radius of curvature where \( \eta_1 \) and \( r_2 \) are the radii of the curvature of the two contacting surfaces, and \( E' \) is the effective modulus of elasticity defined by the elasticity and Poisson’s ratio values of the mating surfaces as \( E' = 2 \left[ \frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right]^{-1} \). Thus, the load per length can be written as \( W' = \frac{2 \pi R P_h^2}{E'} \).
Applying these, the $\mu$ formulae listed in Table 2.1 can be rewritten, starting with the formula by Drozdov and Gavrikov [21]

$$f = \frac{1}{0.8\sqrt{v_k V_s + V_r \phi(P_{\text{max}}; v_k)} + 13.4} = \frac{1/V_r}{0.4\sqrt{v_k SR + \phi(P_h; v_k)} + 13.4/V_r}$$

(3.1)

where $\phi(P_h; v_k) = 0.47 - 0.13 \times 10^{-4} P_h - 0.4 \times 10^{-3} v_k$. The formula by O’donoghue and Cameron [20] takes the form

$$f = \left(\frac{S_{\text{cla}} + 22}{35}\right) \frac{0.6}{V_0^{1/8} v_s^{1/3} v_r^{1/6} R^{1/2}} = \left(\frac{S_{\text{cla}} + 22}{35}\right) \frac{0.756}{V_0^{1/8} Sr^{1/3} v_r^{1/2} R^{1/2}}.$$

(3.2)

Similarly, formulae by Misharin [18] and ISO TC 60 [41] are given respectively as

$$f = \frac{0.325}{(V_s V_r v_k)^{1/4}} = \frac{0.3865}{(SR v_r^2 v_k)^{1/4}},$$

(3.3)

$$f = 0.12 \left(\frac{W S_{\text{rms}}}{R V_r v_0}\right)^{0.25} = 0.19 \left(\frac{P_h^2 S_{\text{rms}}}{E^* V_r v_0}\right)^{0.25}.$$

(3.4)

Finally, the formula by Benedict and Kelley becomes
\[ f = 0.0127 \left( \frac{50}{50 - S_{rms}} \right) \log_{10} \left( \frac{3.17 \times 10^8 W'}{v_0 V_r V_r^2} \right) \]

\[ = 0.0127 \left( \frac{50}{50 - S_{rms}} \right) \log_{10} \left( \frac{3.98 \times 10^9 R P_h^2}{v_0 SRV_r^3 E'} \right) \]  \hspace{1cm} (3.5)

The units of these formulae and their applicable ranges remain unchanged. In these formulae, the viscosity is evaluated at either the mean surface temperature or at the oil inlet temperature under ambient pressure. The surface roughness parameter in O’donoghue and Cameron’s formula is the centerline average (CLA) value while ISO TC 60 and Benedict and Kelley’s formulae use the root-mean-square (RMS) values. Under the assumption that the surface roughness profile follows a Gaussian distribution, these two roughness values are related as \( S_{rms} = 1.25 S_{cla} \).

In order to facilitate numerical comparisons among these formulae and the EHL model predictions, an example line contact problem with the following parameters is considered:

\[ R = 5 \text{ mm}, \quad V_e = 5 \text{ m/s}, \quad P_h = 2 \text{ GPa} \]

\[ v_0 = 10 \text{ cPs}, \quad v_k = 13 \text{ cSt}, \quad S_{rms} = 0.07 \text{ \mu m}. \]

With these parameters, the calculated \( \mu \) values from published empirical formulae, Eqs. (3.1-3.5), and from the thermal EHL model described in Chapter 2 are plotted as a
function of slide-to-roll ratio (SR) in Figure 3.1. Here, three main discrepancies between the published empirical formulae and the thermal EHL-based μ are evident.

(i) When \( SR = 0 \), EHL predicts nearly zero friction (only a very small amount of rolling friction), while published empirical formulae all predict their largest \( \mu \) values. This is because \( V_s \) is in the denominator of most of these formulae. This implies that the friction coefficient is largest at the pitch point where there is no relative sliding in total conflict with the physical intuition and the EHL model prediction. Some of these investigators saw this shortcoming of these formulae and set limits for the applicable range of \( SR \). For instance, Misharin stated that his formula is valid only for \( SR \in [0.4, 1.3] \).

(ii) The second discrepancy is that when the \( SR \) is increased, the value of \( \mu \) decreases according to all these formulae except that of ISO TC 60 formula that does not consider \( V_s \) as a parameter. Meanwhile the EHL model predicts that \( \mu \) increases linearly first for small \( SR \) values, and then reaches a maximum \( \mu \) value, after which it decreases gradually with the increasing of \( SR \). The overall qualitative shape of the \( \mu \) versus \( SR \) curve from the EHL model is very different from the other \( \mu \) formulae.

(iii) The third discrepancy is that the thermal EHL model predicts much lower \( \mu \) values than the empirical formulae, regardless of the value of \( SR \).
Figure 3.1. Comparison of friction coefficient models for $R = 5 \, \text{mm}$,
$V_e = 5 \, \text{m/s}$, $P_h = 2 \, \text{GPa}$, $\nu_0 = 10 \, \text{cPs}$, $\nu_k = 13 \, \text{cSt}$, and $S_{rms} = 0.07 \, \mu\text{m}$. 
As these differences amongst the individual $\mu$ formulae as well as between the EHL model and these formulae are so great that neither can be used in the efficiency model with confidence since the resultant efficiency predictions will differ greatly. Only way to describe these discrepancies is to compare the predicted coefficients of friction to the measured ones as it will be done in the next sections using traction data collected from contact experiments simulating gear contacts.

### 3.3 Friction Coefficient Measurements

Friction coefficient was measured by the sponsor of this project by using a tribology test machine (WAM-6) as shown in Figure 3.2. The configuration of this test machine allows it to measure traction, wear, and scuffing as functions of contact stress, lubricant entrainment velocity, slide-to-roll ratio and material properties. This machine uses a ball-on-disk type arrangement. It is capable of changing speed values of the disk within $\pm 12,000$ rpm and the ball within $\pm 16,000$ rpm to achieve the desired SR ratios. The maximum normal load of the machine is nearly 1,600 N that results in $P_h$ values up to 3.3 GPa using a standard ball of 20.638 mm diameter and a standard disk of 101.6 mm diameter. The change in SR is achieved by changing the speeds of the ball and the disk as well as changing the skew and ball axis tilt angles.

The test matrix for this measurement is described in Table 3.1. The tests measured the friction traction at the sphere-disk interface under EHL conditions. The value of $P_h$ is calculated based on the ball on disk contact situation. The entrainment
Figure 3.2. WAM-6 universal tribology test machine.
### Table 3.1 Test matrix for the friction traction measurements.

| Lubricants | 1. 75W90 gear oil  
|            | 2. Alternate oil A  
|            | 3. Alternate oil B  |
| Lubricant inlet temperature ($T_{oil}$), deg C | 100 |
| Test surfaces | 1. Uncoated SAE 52100 steel  
|            | 2. DLC coating type A  
|            | 3. DLC coating type B  |
| Entrainment velocities ($V_e$), m/s | 5, 10, 15, 20 m/s |
| Slide-to-roll ratio (SR) | -1 to +1 |
| Maximum Hertzian pressure ($P_h$), GPa | 1.0, 1.5, 2.0, 2.5 |
velocity $V_e$ is defined as the mean of the sphere surface velocity $u_1$ and the disk surface velocity $u_2$. $SR$ is again defined as the ratio of the sliding velocity to the entrainment velocity.

More than 150 tests were performed with different combinations of lubricant, test surfaces, $V_e$, and $P_h$ values. In each test, $SR$ was varied continuously between the design limits of $-1.0$ and $1.0$, unless it is beyond the machine capability. The test machine was unable to reach $SR = 1.0$ for $V_e > 10$ m/s. Maximum contact pressure values are such that it covers most gear applications.

The traction tests included three different lubricants with 75W90, a common rear axle and manual transmission fluid, being the baseline fluid. The other two variations, called Lubricants A and B here, use the same base stock as 75W90 with special “low-friction” additives. Since effects of additives are not possible to capture through the EHL model, an experimentally defined friction coefficient is needed for predicting the efficiency of components running with such special lubricants.

The test matrix also included a number of surface treatments in the form of thin-film coatings, namely Physical Vapor Deposition (PVD) coatings, from the Diamond-Like-Carbon (DLC) family. Uncoated baseline traction tests were performed with chemically polished ($S \leq 0.1$ µm) balls and disks, both made of SAE 52100 material. Two variations of DLC coatings, called DLC-A and DLC-B here, were also considered in
the test matrix as shown in Table 3.2. Here DLC coated balls were tested against both DLC coated and uncoated disks, bringing the surface treatment combinations to five.

An example set of measured coefficient of friction data is shown within $-1.0 < SR < 1.0$ in Figure 3.3 for the uncoated-uncoated specimens with 75W90 gear oil at a temperature of 100 deg C for $P_h = 1.0, 1.5$ and $2.0$ GPa. Due to speed limitations of the test machine, no data was collected for $SR > 0.33$ when $V_e = 15$ m/s and for $SR > -0.33$ for $V_e = 20$ m/s. The measured data for $P_h = 1.5$ GPa, $V_e = 15$ m/s and $SR < 0$ was not used here due to the erratic behavior of the data observed.

It is observed from Figure 3.3 that measured $\mu$ values increase with increasing $P_h$ and decrease with increasing $V_e$. $SR$ has a significant influence on the coefficient of friction as well. Standard tools of Multiple Linear Regression Analysis are sought in order to describe these relationships quantitatively in a single formula. Such a formula would allow the use of this data conveniently and accurately without relying on interpolations or look-up tables. Data with negative and positive $SR$ are treated separately in this regression analysis. In the following section, details of the regression analysis are given.
<table>
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</tr>
</thead>
<tbody>
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<td>Uncoated</td>
</tr>
<tr>
<td>2</td>
<td>DLC type B</td>
<td>DLC type B</td>
</tr>
<tr>
<td>3</td>
<td>DLC type A</td>
<td>DLC type A</td>
</tr>
<tr>
<td>4</td>
<td>DLC type A</td>
<td>Uncoated</td>
</tr>
<tr>
<td>5</td>
<td>DLC type B</td>
<td>Uncoated</td>
</tr>
</tbody>
</table>

Table 3.2 Surface treatment combinations used in friction measurements.
Figure 3.3. Measured $\mu$ as a function of $SR$. Uncoated-uncoated surfaces with 75W90 oil. (a) $P_h = 1$ GPa, (b) $P_h = 1.5$ GPa, and (c) $P_h = 2.0$ GPa.
Figure 3.3 continued.

\[ Ph = 1.5 \text{ GPa}, \ Ve = 15 \text{ m/s} \]

\[ Ph = 1.5 \text{ GPa}, \ Ve = 5 \text{ m/s} \]

\[ Ph = 1.5 \text{ GPa}, \ Ve = 10 \text{ m/s} \]
Figure 3.3 continued.

(a) $P_h = 2 \text{ GPa, } V_e = 5 \text{ m/s}$

(b) $P_h = 2 \text{ GPa, } V_e = 10 \text{ m/s}$

(c) $P_h = 2 \text{ GPa, } V_e = 16 \text{ m/s}$

(d) $P_h = 2 \text{ GPa, } V_e = 20 \text{ m/s}$
3.4 Development of New \( \mu \) Formulae from Measured Traction Data

3.4.1 Basics of multiple linear regression

In the Multiple Linear Regression model, the dependent variable is assumed to be a linear function of multiple independent variables in addition to an error \( \epsilon \) introduced to account for all other factors as

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon
\]

(3.6)

where \( y \) is the dependent variable that is the friction coefficient \( \mu \) (or a function of \( \mu \) if a transformation of \( \mu \) is performed) in this case. For each one of the \( n \) independent observations, Eq. (3.6) can be written as

\[
y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \epsilon_i, \quad i = 1, 2, \ldots, n
\]

(3.7)

where \( x_{ij} \) are the settings of the \( j \)-th independent variable for the \( i \)-th observation.

Multiple linear regression can also be written in matrix form as [79]

\[
Y = X\beta + \epsilon
\]

(3.8a)

where
Here, \( Y \) is a vector of dependent (response) variable, \( X \) is a matrix of known constants, \( \beta \) is a vector of regression coefficients, and \( \varepsilon \) is the error (or residual) vector. In multiple linear regression, two basic assumptions are usually made [80]: (i) \( \varepsilon_i \) follows normal distribution as \( N(0, \sigma^2) \), and (ii) all elements of \( Y \) vector are independent of each other.

Based on these assumptions, the expected value of the dependent variable is

\[
E(Y) = X\beta
\]  

(3.9)

and the normal distribution of \( Y \) values corresponding to any particular value of \( X \) has the same standard deviation \( \sigma \).

In Eq. (3.9), elements of \( X \) and \( Y \) are all known and the coefficients \( \beta \) are estimated by the method of least squares. Let \( \hat{\beta} \) be the estimator of \( \beta \), then the estimator of \( E(Y) \) can be written as
\( \hat{Y} = X\hat{\beta} \) \hspace{2cm} (3.10)

where, \( \hat{\beta} = [\hat{\beta}_0 \ \hat{\beta}_1 \ \ldots \ \hat{\beta}_k]^T \). Then the deviation of the observed value of \( Y \) from the estimated value \( \hat{Y} \) is \( Y - \hat{Y} \) that is often called the residual. The goal of least squares is to minimize the sum of the squares of error \( SSE \) defined as

\[
SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2.
\]  \hspace{1cm} (3.11)

The minimization of Eq. (3.11) can be done through solving the following system of linear equations

\[
\frac{\partial SSE}{\partial \hat{\beta}_j} = 0, \quad j = 0,1,\ldots,k
\]  \hspace{1cm} (3.12)

In reality the variances of the observations (dependent variable) are usually not equal, requiring a weighted least squares estimation. The basic idea is to transform the observations \( Y \) to other variables \( Z \) that satisfy the assumptions and then apply the unweighted analysis to the variables obtained [80]. In the case of weighted least squares, Eq. (3.11) becomes

\[
SSE = \sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2
\]  \hspace{1cm} (3.13)
where the weights are given as \( w_i = 1/\sigma_i^2 \). In practice, \( \sigma_i^2 \) and the weights \( w_i \) are unknown and must be estimated.

The coefficient of determination \( R^2 \), which is also known as the squared multiple correction coefficient, is the proportion of the variability in the dependent variable that is explained by the regression model. \( R^2 \) is defined as the ratio of the regression sum of squares \( SSR \) to the total sum of squares \( SST \) as

\[
R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}, \quad 0 \leq R^2 \leq 1. \tag{3.14a}
\]

Here, \( SST = SSR + SSE \) or

\[
\sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n} (y_i - \hat{y}_i)^2, \tag{3.14b}
\]

where, \( \bar{y} \) is the sample mean

\[
\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \quad \text{and} \quad \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \cdots + \hat{\beta}_k x_{ik}. \tag{3.14c}
\]

A disadvantage of using \( R^2 \) as a measure of goodness of fit is that if an additional independent variable were introduced into the model, which is not strongly related to the dependent variable, \( R^2 \) would increase, implying a better fit. In order to avoid this
adverse effect, an adjusted $\hat{R}^2$, denoted by $R^2_{\text{adj}}$, is introduced. When new variables are added to the model, $R^2_{\text{adj}}$ does not increase unless the new variables have additional predictive capability. $R^2_{\text{adj}}$ is defined as

$$R^2_{\text{adj}} = 1 - \frac{\text{SSE}/(n-k-1)}{\text{SST}/(n-1)}$$

(3.15)

3.4.2. Multiple linear regression analysis of traction data

In the ball-on-disk experiments, the original dependent variable is the coefficient of friction $\mu$, the original independent variables are $P_h$, $SR$, and $V_e$. In this section, a detailed example on the regression analysis for the traction data for contact of two uncoated surfaces for $SR < 0$ is presented.

A regression equation in the following form is considered to describe the measured data shown in Figure 3.3

$$\log \mu = a_1 + a_2 \log P_h + a_3 \log |SR| + a_4 |SR| \cdot (P_h)^2 + a_5 e^{-|SR|(P_h)^2} + a_6 \log V_e$$

(3.16)

where $\mu$ is the measured coefficient of total friction, which includes both sliding and rolling friction effects, and $a_i$, $i = 1, 2, \ldots, 6$ are the unknown regression coefficients. Equation (3.16) can be converted back to the standard format given in Eq. (3.6) by letting
\[ y = \log[\mu], \quad x_1 = \log P_h, \quad x_2 = \log[SR], \]
\[ x_3 = |SR| P_h^2, \quad x_4 = e^{-|SR| P_h^2}, \quad x_5 = \log V_e. \]

Statistical software R was used in this analysis. The ordinary least square method was applied first. However, the residual plot (not shown here) showed a clear pattern that, the residual decreases as \( P_h \) increases, which violates the equal variance assumption. After examining the scatter plot of the original coefficient of friction versus the \( SR \) shown in Figure 3.3, it is obvious that the measured data have a much bigger variance at \( P_h = 1 \) GPa than at higher values of \( P_h \). Thus the weighted least square method is sought next. The weights are selected as \( w_i = \frac{1}{\hat{\sigma}_i^2} \), where \( \hat{\sigma}_i^2 \sim 1/(\log P_h)_i \).

This regression analysis yielded \( R^2_{adj} = 0.985 \), a very large coefficient of determination, which implies that 98.5 percent of the variability in the dependent variable can be explained by this regression model. The coefficients of friction from the fitted equation (3.16) and the original friction data are compared in Figure 3.4. It can be concluded from these figures that the fitted values agree very well with the measured data. Figure 3.5(a) shows the weighted residuals \( w_i^{0.5} \hat{\epsilon}_i \) versus fitted values \( w_i^{0.5} \hat{y}_i \), which reveals no evidence of violation of the equal variance assumption. In addition, Figure 3.5(b) shows that the assumption of normal distribution of the residuals is satisfied.
Figure 3.4. Comparison of fitted and measured coefficient of friction, uncoated surface versus uncoated surface with 75W90, SR < 0. (a) $P_h = 1 \text{ GPa}$, (b) $P_h = 1.5 \text{ GPa}$, and (c) $P_h = 2.0 \text{ GPa}$.
Figure 3.4 continued.

(b) 

Ph = 1.5 GPa, Ve = 5 m/s

Ph = 1.5 GPa, Ve = 10 m/s

Continued
Figure 3.4 continued.

(c) $P_h = 2 \text{ GPa}, \ V_e = 15 \text{ m/s}$

(d) $P_h = 2 \text{ GPa}, \ V_e = 20 \text{ m/s}$
Figure 3.5. Residual plots from the regression analysis of traction data for uncoated surface versus uncoated surface and $SR < 0$; (a) residual versus fitted values; and (b) the histogram of the standardized residuals.
For \( SR > 0 \), the same regression equation as Eq. (3.16) and the same analysis procedure were applied to the corresponding sets of measured data. The plots of coefficient of friction from the fitted equation together with the measured friction are shown in Figure 3.6, which again shows a very good fit.

Similarly, for case 2 of two coated surfaces (DLC-B versus DLC-B), the regression equation is selected as

\[
\log [\mu] = a_1 + a_2 \log P_h + a_3 \log |SR| + a_4 \left[ |SR| \cdot \left( P_h \right)^2 + |SR| \cdot P_h \right] \\
\quad + a_5 e^{-\left[ |SR| \cdot \left( P_h \right)^2 + |SR| \cdot P_h \right]} + a_6 \log V_e
\]

(3.17)

The plots of coefficient of friction from fitted equation together with the measured friction are shown in Figures 3.7 and 3.8 for \( SR < 0 \) and \( SR > 0 \), respectively. They also show a reasonably good fit.

By applying a similar regression analysis, the other three cases listed in Table 3.2 are also analyzed. The obtained expressions for all 5 cases, all lubricated with 75W90 gear oil, are summarized below.

\[
\mu = e^{f(SR,P_h)} \cdot P_h^{a_2} \cdot |SR|^{a_3} \cdot V_e^{a_6}
\]

(3.18a)
Figure 3.6. Comparison of fitted and measured coefficient of friction, uncoated surface versus uncoated surface with 75W90, $SR > 0$. (a) $P_h = 1$ GPa, (b) $P_h = 1.5$ GPa, and (c) $P_h = 2.0$ GPa.
**Figure 3.6** continued.

*Ph = 1.5 GPa, Ve = 5 m/s*

*Ph = 1.5 GPa, Ve = 10 m/s*

*Ph = 1.5 GPa, Ve = 15 m/s*
Figure 3.6 continued.

Ph = 2 GPa, Ve = 5 m/s

Ph = 2 GPa, Ve = 10 m/s

(c) Ph = 2 GPa, Ve = 15 m/s
Figure 3.7. Comparison of fitted and measured coefficient of friction for two coated surfaces with 75W90, DLC-B versus DLC-B. $SR < 0$. (a) $P_h = 1 \text{ GPa}$, (b) $P_h = 1.5 \text{ GPa}$, (c) $P_h = 2.0 \text{ GPa}$, and (d) $P_h = 2.5 \text{ GPa}$. 

Continued
Figure 3.7 continued.

(b)
Figure 3.7 continued.

Ph = 2 GPa, Ve = 5 m/s

Ph = 2 GPa, Ve = 10 m/s

Ph = 2 GPa, Ve = 15 m/s

Ph = 2 GPa, Ve = 20 m/s

Continued
Figure 3.7 continued.

- Ph = 2.5 GPa, Ve = 5 m/s
- Ph = 2.5 GPa, Ve = 10 m/s
- Ph = 2.5 GPa, Ve = 15 m/s
- Ph = 2.5 GPa, Ve = 20 m/s
Figure 3.8. Comparison of fitted and measured coefficient of friction for two coated surfaces with 75W90, DLC-B versus DLC-B. $SR > 0$. (a) $P_h = 1$ GPa, (b) $P_h = 1.5$ GPa, (c) $P_h = 2.0$ GPa, and (d) $P_h = 2.5$ GPa.
Figure 3.8 continued.

(a) \( P_h = 1.5 \text{ GPa}, \, V_e = 5 \text{ m/s} \)

(b) \( P_h = 1.5 \text{ GPa}, \, V_e = 10 \text{ m/s} \)

(c) \( P_h = 1.5 \text{ GPa}, \, V_e = 15 \text{ m/s} \)
Figure 3.8 continued.

**Ph = 2 GPa, Ve = 5 m/s**

![Graph showing Coefficient of friction vs SR for Ph = 2 GPa, Ve = 5 m/s.]

**Ph = 2 GPa, Ve = 10 m/s**

![Graph showing Coefficient of friction vs SR for Ph = 2 GPa, Ve = 10 m/s.]

**Ph = 2 GPa, Ve = 15 m/s**

![Graph showing Coefficient of friction vs SR for Ph = 2 GPa, Ve = 15 m/s.]

Continued
Figure 3.8 continued.

Ph = 2.5 GPa, Ve = 5 m/s

Ph = 2.5 GPa, Ve = 10 m/s

(d) Ph = 2.5 GPa, Ve = 15 m/s
\[ \mu = e^{f(SR, P_h)} P_h^{a_2} |SR|^{a_3} V_c^{a_6} \quad \text{where} \quad f(SR, P_h) = \begin{cases} a_1 + a_4 |SR| P_h^2 + a_5 e^{-|SR| P_h^3} & \text{case 1, 3, 5} \\ a_1 + a_4 |SR| P_h^{1.5} + a_5 e^{-|SR| |P_h|^{1.5}} & \text{case 4} \end{cases} \]

<table>
<thead>
<tr>
<th>Case #</th>
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<th>5</th>
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<td>DLC/A – DLC/A</td>
<td>DLC/A - Uncoated</td>
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</tr>
<tr>
<td>(a_6)</td>
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<td>-0.444290</td>
<td>-0.571847</td>
<td>-0.49046</td>
</tr>
</tbody>
</table>

**Table 3.3** Regression coefficients for 75W90 gear oil.
where

\[
f(SR, P_h) = \begin{cases} 
  a_1 + a_4 |SR| P_h^2 + a_5 e^{-|SR| P_h^2} & \text{case 1, 3, 5} \\
  a_1 + a_4(|SR| P_h^2 + |SR| P_h) + a_5 e^{-(|SR| P_h^2 + |SR| P_h)} & \text{case 2} \\
  a_1 + a_4 |SR| P_h^{1.5} + a_5 e^{-|SR| P_h^{1.5}} & \text{case 4}
\end{cases}
\]

Coefficients \( a_i \) \((i = 1, 2, ..., 6)\) in Eq. (3.18) for all these five cases are listed in Table 3.3.

The five cases analyzed above are all with lubricant 75W90. The measured traction data with lubricants A and B were processed with multiple linear regression as well. In the measurements with these two lubricants, only two levels of \( P_h \) were considered, which are at 1.0 and 2.0 GPa. Values of \( V_e \) and \( SR \) are similar to those of tests with 75W90 oil.

In the regression of the traction data for uncoated ball against uncoated disk with lubricant A, the regression equation is selected as

\[
\log |\mu| = a_1 + a_2 \log P_h + a_3 \log |SR| + a_4 |SR| \cdot (P_h)^{1.5} + a_5 e^{-|SR| (P_h)^{1.5}} + a_6 \log V_e
\]

for both negative and positive \( SR \) values. The fitted and measured \( \mu \) values for the contact of two uncoated surfaces with lubricant A for \( SR < 0 \) and \( SR > 0 \) are compared.
in Figures 3.9 and 3.10, respectively. By applying the similar regression analyses, other four cases listed in Table 3.2 are also analyzed, and the results are listed in Table 3.4.

The same regression equation of (3.16) was used for an uncoated ball against uncoated disk with lubricant B as well. For lubricant B, the fitted $\mu$ values are compared to the actual measured values in Figures 3.11 and 3.12 for $SR < 0$ and $SR > 0$, respectively. By applying the similar regression analysis, other four cases listed in Table 3.2 are also analyzed, and the results are listed in Table 3.5.

With these formulae based on traction data, several comparisons can be made. In Figures 3.13(a) and 3.13(b), the $\mu$ values for a contact of two uncoated surfaces are plotted as a function of $SR$ for various values of speed and load, respectively. In Figure 3.13(a), an increase of $V_c$ reduces $\mu$. An increase of $P_h$ mostly results in an increase of $\mu$ as illustrated in Figure 3.13(b). A similar trend is also illustrated in Figure 3.14 for case 2 with both surfaces coated with DLC-B. It is also observed in Figures 3.13 and 3.14 that the $\mu$ values tend to be lower for $SR < 0$ than those of $SR > 0$, which are more significant in Figure 3.14 for coated surfaces. This asymmetry about $SR = 0$ most likely results from the test machine.

The $\mu$ formulae defined by Eq. (3.18) with the constant coefficients given in Table 3.3 for 75W90 gear oil are used to compare all five cases listed in Table 3.2 in
Figure 3.9. Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant A, $SR < 0$. (a) $P_h = 1$ GPa, and (b) $P_h = 2.0$ GPa.
Figure 3.9 continued.

Ph = 2 GPa, Ve = 5 m/s

Ph = 2 GPa, Ve = 10 m/s

Ph = 2 GPa, Ve = 15 m/s

Ph = 2 GPa, Ve = 20 m/s
Figure 3.10. Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant A, $SR > 0$.
(a) $P_h = 1$ GPa, and (b) $P_h = 2.0$ GPa.
Figure 3.10 continued.
\[ \mu = e^{f(SR, P_h) P_h^{a_2} |SR|^{a_3} v e^{a_6}} \]  

where \[ f(SR, P_h) = \begin{cases} 
  a_1 + a_4 |SR| P_h^{a_2} + a_6 e^{-|SR| P_h^{a_2}} & \text{case 2,3,4} \\
  a_1 + a_4 |SR| P_h^{1.5} + a_6 e^{-|SR| P_h^{1.5}} & \text{case 1,5}
\end{cases} \]

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**Table 3.4** Regression coefficients for Lubricant A.
Figure 3.11. Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant B, SR < 0.
(a) $P_h = 1$ GPa, and (b) $P_h = 2.0$ GPa.
Figure 3.11 continued.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3_11}
\caption{Coefficient of friction vs. SR for different velocities and pressures.}
\end{figure}
Figure 3.12. Comparison of fitted and measured coefficient of friction for two uncoated surfaces with lubricant B, $SR > 0$. (a) $P_h = 1$ GPa, and (b) $P_h = 2.0$ GPa.
Figure 3.12 continued.
\[
\mu = e^{f(SR, P_h)} P_h^{a_2} |SR|^{a_3} V_e^{a_6}, \quad \text{where} \quad f(SR, P_h) = \begin{cases} 
  a_1 + a_4 |SR|^2 P_h^2 + a_5 e^{-|SR|^2 P_h^2} & \text{case 1, 2, 3, 5} \\
  a_1 + a_4 |SR|^{1.5} P_h^{1.5} + a_5 e^{-|SR|^0.5 P_h^{1.5}} & \text{case 4}
\end{cases}
\]

<table>
<thead>
<tr>
<th>Case #</th>
<th>Uncoated - Uncoated</th>
<th>DLC/B – DLC/B</th>
<th>DLC/A – DLC/A</th>
<th>DLC/A - Uncoated</th>
<th>DLC/B - Uncoated</th>
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</thead>
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<td>(SR&gt;0)</td>
<td>(SR&lt;0)</td>
<td>(SR&gt;0)</td>
<td>(SR&lt;0)</td>
</tr>
<tr>
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<td>1.260231</td>
<td>1.491279</td>
<td>1.505326</td>
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<tr>
<td>(a_3)</td>
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<td>0.439244</td>
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<tr>
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<td>(a_6)</td>
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<td>-0.431949</td>
<td>-0.60172</td>
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<td>-0.54265</td>
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**Table 3.5** Regression coefficients for Lubricant B.
Figure 3.13. The $\mu$ values for (a) $P_h = 1$ GPa and various $V_e$ values, and (b) $V_e = 5$ m/s and various $P_h$ values. Both surfaces are uncoated and the lubricant is 75W90.
Figure 3.14. The $\mu$ values for (a) $P_h = 1$ GPa and various $V_e$ values, and (b) $V_e = 5$ m/s and various $P_h$ values. Both surfaces are coated with DLC-B and the lubricant is 75W90.
Figure 3.15 Comparison of the fitted $\mu$ values for the five cases in Table 3.2 with 75W90 oil at $P_h = 1.5$ GPa and four values of $V_e$. 
Figure 3.16 Comparison of the fitted $\mu$ values for the five cases in Table 3.2 with 75W90 oil at $V_e = 5$ m/s and four values of $P_h$. 
terms of their friction performance. The $\mu$ values compared at various $V_e$ under a constant contact pressure $P_h=1.5\text{GPa}$ are shown in Figure 3.15. Similarly, Figure 3.16 compares the $\mu$ values at $V_e=5\text{m/s}$ for four values of $P_h$. It is observed from these figures that the cases 2 and 3 which used two coated surfaces (DLC-B versus DLC-B and DLC-A versus DLC-A, respectively) results in significantly lower $\mu$ values than the other three cases with no coating or only one coated surface. Case 2 (DLC-B versus DLC-B) resulted in the lowest values for all parameter ranges considered. This suggests that the engineered surface treatments in the form of coatings should be included in any efficiency improvement effort.

Effects of different lubricants on $\mu$ values are demonstrated in Figures 3.17 to 3.19 for various $V_e$ and $P_h$ values using Case 1 (two uncoated surfaces) as the platform for this comparison. Again, a sizable difference between 75W90 and the two alternative “low friction” oils is evident, suggesting that lubricant additives might play an important role on efficiency of a gear pair. In this particular case, lubricant A resulted in the lowest $\mu$ values compared to the other two lubricants.

### 3.5 Validation of the Friction Coefficient Models

Given this set of $\mu$ formulae obtained from the regression analysis of the actual traction data, the previous published formulae and the EHL model introduced in Chapter
Figure 3.17. Comparison of measured $\mu$ values for three different lubricants at $P_h = 1.0$ GPa.
Figure 3.18. Comparison of measured $\mu$ values for three different lubricants at $P_h = 1.5$ GPa.
Figure 3.19. Comparison of measured $\mu$ values for three different lubricants at $P_h = 2.0$ GPa.
2 can be assessed for their accuracy. For this purpose, first the traction test contact conditions will be simulated by using the formulae listed in Section 3.2 and then by using the EHL model.

### 3.5.1 Comparison of empirical formulae and the measured traction data

A comparison between the published empirical $\mu$ formulae and the measured traction data is shown in Figure 3.20. The discrepancies between these published formulae and the fitted measured data are quite similar to those between the same formulae and the EHL model predictions, discussed earlier in Section 3.2. In Figure 3.20 for $V_e = 10$ m/s and $P_h = 1.0$ GPa, the measured and predicted $\mu$ values differ in both magnitudes and qualitative shape. The published empirical formulae of others give larger $\mu$ values regardless the value of $SR$. The predicted $\mu$ values become very large as $SR$ approaches zero while the measured $\mu$ value is almost zero at $SR = 0$. Typical spur or helical gears have $SR$ values between –1 and 1. The same differences were observed at other load and speed conditions as well. The efficiency values predicted by using these published $\mu$ formulae would be much lower than the actual mechanical efficiency values. Therefore, it can be concluded here that these formulae should not be used for gear efficiency studies and the results of previous efficiency models that employed these formulae [34-37, 40] should be taken cautiously.
Figure 3.20. Comparison between published empirical formulae and the measured traction data.
3.5.2 Comparison of the EHL model predictions and the measured traction data

Given the poor comparison between the widely-used friction coefficient formulae and the measured ones under conditions representing a gear contact, the EHL-based approach for prediction of $\mu$ is employed next. The $\mu$ values predicted by the EHL model described earlier and the measured values represented by the fitted formulae are compared in Figures 3.21 to 3.23.

As discussed in Chapter 2, the EHL model calculates contributions of both rolling and sliding to friction. Therefore, Figures 3.21 to 3.23 have two curves for the EHL model, one for sliding only and one for the total (sliding plus rolling) in addition to a curve representing the measured data that represents the total friction coefficient. Figure 3.21 shows predicted and measured $\mu$ values at $P_h = 1.0$ GPa and $V_e = 5$, 10, 15 and 20 m/s. The EHL model predictions agree well with the measured data at these conditions. The overall amplitudes match as well as the shape of the curves plotted as a function of $SR$. The rolling component of the friction coefficient is about 10 percent of the total and hence the sliding component dominates the EHL model predictions.

Figures 3.22 and 3.23 show similar comparisons for $P_h = 1.5$ and 2.0, respectively. The agreement between the EHL model and the measurements are again reasonably good in these figures, while the prediction magnitudes are slightly lower at higher load values. Yet, the overall shapes are very similar. Therefore, it can be stated
Figure 3.21. Comparison of EHL model predictions and the measured data at $P_h = 1$ GPa and various $V_e$ values. Uncoated surfaces, 75W90 oil.
Figure 3.22. Comparison of EHL model predictions and the measured data at $P_h = 1.5$ GPa and various $V_e$ values. Uncoated surfaces, 75W90 oil.
Figure 3.23. Comparison of EHL model predictions and the measured data at $P_h = 2.0$ GPa and various $V_e$ values. Uncoated surfaces, 75W90 oil.
that the EHL based approach is valid for typical ranges of $SR$ values of typical gear contact conditions.

As reviewed by Martin [5], three different regions can be defined on a $\mu$ versus $SR$ (or $\mu$ versus $V_s$ at constant $V_r$) curve shown schematically in Figure 3.24. When the sliding velocity is zero, there is no sliding friction, and only very little rolling friction. Thus, the $\mu$ values approach to almost zero as $SR \rightarrow 0$. When the $SR$ is increased from zero, $\mu$ values first increase linearly with small values of $SR$. This region is defined as the linear or isothermal region. When $SR$ is increased a little more, $\mu$ reaches a maximum value and decreases as $SR$ values are increased beyond that point. This region is referred to as non-linear or non-Newtonian region. As $SR$ is increased further, the friction decreases in an almost linear fashion, which is called thermal region as the thermal effects can explain these variations. Most of the published empirical formulae seem to be obtained by measurements that are run within the thermal region only [5]. Therefore, they should be expected to be valid only in that region. Yet, gears operate in the other two regions as well and the thermal non-Newtonian EHL model used in this study for the predictions is valid in all three regions of frictions.
Figure 3.24. A typical $\mu$ versus $SR$ curve.
Based on above comparisons, the EHL based model can be considered to be validated. This is a positive outcome since the EHL model is physics-based, requiring no empirical parameters. However, it presents a major difficulty especially when used in a gear efficiency model. Each EHL analysis takes several minutes of computations (more than 2 minutes of CPU time on a 3.0 GHz PC) and a complete gear efficiency analysis might require a few hundred EHL analyses. Therefore, this approach is not practical for real-life engineering applications requiring many parameter studies. This becomes a major handicap since the goal here is to be able to use the efficiency model as a design tool.

In order to overcome this disadvantage, a matrix for massive EHL analyses is proposed here. The idea is to use the EHL model as if it is an experimental tool. Results from the massive EHL analyses would then be processed by the regression analysis, the same way it was done for the measured traction data. The goal here is to obtain a new $\mu$ formula based on the EHL approach to be used in the gear efficiency models, eliminating the difficulties of this approach in terms of the computational demand.

### 3.6.1 Design of the massive EHL model predictions

The matrix used for the massive EHL analyses is listed in Table 3.6. Around 10,000 runs of the EHL model were performed. In this “experiment”, three additional
parameters are included, namely the surface roughness amplitude $S$, oil viscosity at the inlet that is a function of the inlet oil temperature, and combined radius of curvature of two contacting surfaces. These three parameters were not included in the traction measurements presented in Section 3.3. However, they all influence the friction coefficient value to a certain extent and should be included in this “experiment”. Based on the observations from the measured traction data, reasonably small increments are used for $SR$ in order to capture drastic changes in the nonlinear region. Entrainment velocity $V_e$ is varied within the range from 1 to 20 m/s that covers most gear applications. Five different levels of temperatures from 50 to 110 deg C are modeled to cover the typical operating temperature conditions. Equivalent radius of curvature is varied from 0.005 to 0.08 m. Surface roughness is introduced as another variable to include individual surface roughness values up to 0.4 $\mu$m (RMS). A typical Hertzian pressure range of 0.5 to 2.5 GPa is considered. Gear oil 75W90 is used as the lubricant.

3.6.2 Multiple linear regression of the massive EHL model predictions

In this regression analysis, only the coefficient of sliding friction is analyzed, as the coefficient of rolling friction is meaningless. The regression equation is selected as

$$
\log \mu = b_1 + b_2 \log P_h + b_3 \log |SR| + b_4 \log |SR| \cdot (P_h)^2 \cdot \log_{10}(v_0) + b_5 e^{SR \cdot (P_h)^2 \cdot \log_{10}(v_0)} + b_6 \log V_e + b_7 \log v_0 + b_8 \log R + b_9 e^S
$$

(3.20)
where $\nu_0$ is the absolute viscosity at oil inlet temperature in cPs, $S$ is the RMS surface roughness in $\mu$m, $R$ is the effective radius of curvature in meters, $V_e$ is the oil entraining velocity in m/s and $b_i, i=1,2,...,9$ are the regression coefficients to be determined.

This regression analysis resulted in $R_{adj}^2 = 0.94$. The determined regression coefficients for Eq. (3.20) are listed in Table 3.7. Figure 3.25 compares fitted and actual values of $\mu$ for various parameter values together with the residual for all of the data. This figure demonstrates that the equal variance assumption is met. Normal probability plot of the residual is also checked and no strong evidence of violation of the normal distribution assumption is found. With this, the coefficient of sliding friction formula can then be written as

$$\mu = e^{f(SR,P_h,\nu_0,S)}P_h^{b_2}SR^{b_3}V_e^{b_4}\nu_0^{b_5}R^{b_6}$$

(3.21a)

where

$$f(SR,P_h,\nu_0,S) = b_1 + b_4|SR|P_h \log_{10}(\nu_0) + b_5e^{-|SR|P_h \log_{10}(\nu_0)} + b_6e^S.$$  

(3.21b)

This EHL-based $\mu$ formula obtained by using multiple linear regression analysis includes all the key features of a gear contact, namely $SR$, $V_e$, $P_h$, $S$, $R$, and $\nu_0$. It can be directly used to calculate $\mu$ instead of running extremely time-consuming EHL
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<th>Parameter</th>
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<td>Lubricants</td>
<td>75W90 gear oil</td>
</tr>
<tr>
<td>Inlet temperature, deg C</td>
<td>50, 60, 80, 100, 110</td>
</tr>
<tr>
<td>Radius of curvature ($R$), m</td>
<td>0.005, 0.01, 0.02, 0.03, 0.04, 0.08</td>
</tr>
<tr>
<td>Entraining velocity ($V_e$), m/s</td>
<td>1, 5, 10, 15, 20</td>
</tr>
<tr>
<td>Slide-to-roll ratio ($SR$)</td>
<td>0.005, 0.01, 0.025, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5, 0.6, 0.8, 1.0</td>
</tr>
<tr>
<td>Surface roughness ($S$), µm</td>
<td>0, 0.05, 0.1, 0.2, 0.3, 0.4</td>
</tr>
<tr>
<td>Maximum Hertzian pressure ($P_h$), GPa</td>
<td>0.5, 1.0, 1.5, 2.0, 2.5</td>
</tr>
</tbody>
</table>

Table 3.6 Matrix of parameters used in the massive EHL model predictions.
$$\mu = e^{f(SR, P_h, v_0, S)} P_h^{b_2} |SR|^{b_3} V_e^{b_6} V_0^{b_7} R^{b_8}$$

where  

$$f(SR, P_h, v_0, S) = b_1 + b_4 |SR| P_h \log_{10}(v_0) + b_5 e^{|SR| P_h \log_{10}(v_0)} + b_9 e^S$$

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<tr>
<td>$b_8$</td>
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<tr>
<td>$b_9$</td>
<td>0.620305</td>
</tr>
</tbody>
</table>

Table 3.7 Coefficients for the EHL based formula.
Figure 3.25. Regression of the massive predictions by the thermal EHL model, (a-g) comparison of actual and fitted $\mu$ values for various parameters, (h) residual plot of all the data. $P_h = 1.5$ GPa, $V_e = 15$ m/s, $R = 0.04$ m, $T = 100$ degC, $S = 0$ um, unless specified.
Figure 3.25 continued.

Ph = 2 GPa, S = 0 um

Ph = 1.5 GPa, S = 0.1 um

Ph = 0.5 GPa, S = 0.2 um

Residual v.s. Fitted

(e)  

(f)  

(g)  

(h)
analyses in real time, while still maintaining acceptable accuracy. It also avoids numerical issues associated with any EHL model.

3.7 Concluding Remarks

In this chapter, the friction coefficient values obtained by using a set of published formulae and a thermal, non-Newtonian EHL model were compared to extensive measured traction data. It was shown that the EHL model compares with the measured data well while the empirical formulae of others fail to do so.

Linear regression analysis technique was used to reduce the measured data obtained by using various surface treatments (coatings) and lubricants to a set of experimental $\mu$ formulae. A matrix that includes all relevant parameters was designed and executed by using the EHL model with a standard gear oil, 75W90. The results of the massive EHL model predictions were reduced to a single $\mu$ formula that is capable of predicting $\mu$ values for typical gear contact problems. This formula will be used in Chapters 4 to 7 for predicting the mechanical efficiency of parallel axis and cross axis gears.

There are still some limitations to the $\mu$ computation based on an EHL analysis since certain aspects of lubricants such as additives cannot be readily modeled. Therefore, one would still require experimental traction data to determine $\mu$ for such
conditions. This was the case for lubricants A and B. The same is true for surface treatments such as thin-film coatings. A coated surface and an uncoated one might have the same roughness values, but very different $\mu$ values, that was the case with DLC coatings considered in traction tests. In such cases, again the $\mu$ values need to come from experimental data. Linear regression analysis presented here can be used to reduce such data to simple formulae that will be suitable for the efficiency models.
CHAPTER 4

APPLICATION TO PARALLEL-AXIS GEAR PAIRS

4.1 Introduction

Parallel-axis spur and helical gears are known to be highly efficient in transmitting power, especially when compared to their cross-axis counterparts such as bevel and hypoid gears. While a spur or helical gear pair can have mechanical efficiencies above 99 percent, a drive train might have several gear meshes connected in series so the overall efficiency losses of the entire gear train are no longer negligibly small. A significant amount of attention has been given recently to spur and helical gear efficiency because of the constant pressure to improve the fuel economy of passenger vehicles and rotorcrafts. In this chapter, the mechanical efficiency prediction methodology proposed in Chapter 2 will be applied to spur and helical gear pairs. First, a load distribution program will be introduced. Next, curvatures and surface velocities for spur and helical gears will be determined in the form required by the friction coefficient models. Finally, the mechanical efficiency associated with the frictional losses will be computed.
4.2 The Load Distribution Model

For the computation of the contact loads of spur and helical gears, a load distribution model (LDP) that was initially proposed by Conry and Seireg [71] and later further developed by Houser et al [72] and several of his students is used in this study. This model is designed to compute elastic deformations at any point of the gear surface given the tooth compliance, applied torque, and the initial tooth separations under no load. For the solution of the gear contact problem, conditions of compatibility and equilibrium are considered.

The condition of compatibility is concerned with the condition for which certain points may come into contact under load. For any point \( q \) within the contact zone, the sum of elastic deformations of two bodies and the initial separation must be greater than or equal to the rigid body displacement. This condition can be written as

\[
D_q^{(1)} + D_q^{(2)} + \delta_q \geq R_b \Theta 
\]  

(4.1)

where \( D_q^{(1)} \) and \( D_q^{(2)} \) are the components of the deflection of point \( q \) on the pinion and the gear, respectively, and \( \delta_q \) is the initial separation at contact point \( q \) between the pinion and gear including any tooth surface errors. Due to the assumption of small deflections, the approach of the rigid body as a result of rigid body rotation is taken as the
arc length traversed along the base circle of pinion (the driver) as \( R_b \Theta \) where \( R_b \) is the base radius and \( \Theta \) is the angular displacement of the pinion.

The condition of equilibrium implies that the sum of the moments applied on a gear body must be equal to zero. By replacing the continuous but non-uniform pressure distribution which is expected along a potential line of contact by a set of discrete forces \( F_q \), this condition can then be expressed as

\[
\sum_{q=1}^{Q} F_q R_q + L_{in} = 0
\]  

(4.2)

where \( F_q \) is the discrete force acting at the contact point \( q \), \( R_q \) is the distance from the point \( q \) to the gear axis, \( Q \) is the total number of contact points considered, and \( L_{in} \) is the input torque. Equation (4.1) can be written as equality constraint by introducing a slack variable \( Y_q \) as

\[
D_q^{(1)} + D_q^{(2)} + \delta_q - R_b \Theta - Y_q = 0
\]  

(4.3)

where \( Y_q \geq 0 \). It can be seen from this equation that gears are in contact at point \( q \) if \( Y_q = 0 \), and otherwise, \( q \) does not represent a contact point. The load distribution problem can be solved iteratively by using a modified Simplex algorithm for the values
of the force vector $\mathbf{F}$, the slack variable vector $\mathbf{Y}$, and $\Theta$ for the following set of objective functions:

$$
\begin{align*}
-\mathbf{A} \mathbf{F} + R_b \Theta \mathbf{i} + \mathbf{Y} - \delta &= \mathbf{0} \\
\mathbf{F}^T \mathbf{R} + L_i &= 0 \\
F_q &= 0 \quad \text{or} \quad Y_q = 0
\end{align*}
$$

(4.4)

where $\mathbf{A}$ is the compliance matrix that includes the compliances of pinion and gear teeth and the Hertzian contact, $\mathbf{R}$ is a vector of distances from the contact points to the gear axis, $\mathbf{i}$ is an identity vector and $\delta$ is a vector of initial separations. In Eq. (4.4), all the vectors and matrices are of dimension $Q$, where $Q$ is the total number of contact points that are used to discretize the contact lines at a given mesh position and the subscript $q$ represents any contact point $q$ at that mesh position.

For each angular position $m$ ($m = 1, 2, \ldots, M$) of gears in mesh, the load distribution model computes the load distributions $W'$ (unit load for each contact segment that represents a contact point) along the lines of contact and calculates the maximum Hertzian pressure as $P_h = \sqrt{\frac{W'E'}{2\pi R}}$ [77] based on an equivalent cylindrical contact, where $E'$ is the effective Young’s modulus, and $R$ is the equivalent radius of curvature of the contacting surfaces of the pinion and gear.
4.3 Surface Velocities and Curvature

For an involute spur gear pair shown in Figure 4.1, surface velocities at a contact point are given as 
\[ u_i = \omega_i r_i, \]
where \( i = 1, 2 \) for pinion and gear surfaces, respectively. The radius of curvature \( r_i \) is a function of the roll angle of the corresponding point in contact and base radius, \( r_i = R_{bi}\theta_i \). The roll angle \( \theta_i \) can be defined as \( \theta_i = \tan(\alpha_{ix}) \), where \( \alpha_{ix} \) is the profile angle at the point of contact. With perfect involute tooth profile and no manufacturing errors and shaft misalignment, the contact lines on a spur gear tooth are parallel to the pitch line and the directions of the sliding velocities are normal to the contact lines.

For helical gears, the tooth profiles in the transverse plane are defined by the involute geometry. Therefore, the curvatures and surface velocities of a contact point in the transverse plane can be calculated by using the involute properties of the profiles, as in spur gears. However, unlike spur gear, the contact lines are not parallel to the pitch line. Due to the helix angle, an inclination angle \( \Delta \) exists between the contact line and the tooth trace in the tangent plane as shown in Figure 4.2. The angle \( \Delta \) is defined by the base helix angle \( \beta_b \) and the helix angle at the operating pitch cylinder \( \beta_{op} \) as [87]

\[ \cos \Delta = \frac{\cos \beta_{op}}{\cos \beta_b} \]  

(4.5)
Figure 4.1. Spur gear geometry used for calculation of curvatures and surface velocities.
Figure 4.2. Helical gear geometry used for calculation of curvatures and surface velocities.
In this study, when the published empirical formulae or the EHL model are used to predict $\mu$, curvatures and surface velocities are calculated in the direction that is normal to the contact line as shown in Figure 4.2 based on the considerations that empirical formulae obtained from twin-disk tests simulate a pair of spur gears in contact at one mesh position where surface velocities are perpendicular to the line of contact. In addition, a line contact EHL model is considered in the local coordinate system whose origin moves with instantaneous contact point and surface velocities are normal to the line of contact, assuming negligible side leakage in the direction of contact line. For a pair of spur gears in mesh, one contact line is equivalent to the contact between two virtual cylinders in contact. For a pair of helical gears, due to the inclination angle, modifications must be made to obtain this equivalence. Consequently, the contact line is discretized into many small segments and each segment is viewed as a contact line that is equivalent to a spur gear with an equivalent angular velocity $\dot{\omega} = \omega \cos \Delta$. For helical gears, principal directions are $x$ and $y$, both defined in the common tangent plane as shown in Figure 4.2. At any contact point $q$, the radii of curvature in these principal directions, $R_{ix}$ and $R_{iy}$, and the radii of curvature in the lubricant entrainment direction $e$ and its normal, $R_{ie}$ and $R_{is}$, can be related through Euler’s equation [84] as

$$R_{ie} = \left[ \frac{\cos^2 \Delta}{R_{ix}} + \frac{\sin^2 \Delta}{R_{iy}} \right]^{-1}$$

(4.6a)

and

120
\[ R_{is} = \left[ \frac{\sin^2 \Delta}{R_{ix}} + \frac{\cos^2 \Delta}{R_{iy}} \right]^{-1}. \] (4.6b)

Since the contact line on a helical gear is a straight line, \( R_{iy} = 0 \). Thus, the radii of curvatures and surface velocities in the direction \( x \) that is normal to the contact line are

\[ R_{ix} = R_{ie} \cos^2 \Delta \] (4.7)

and

\[ U_{ix} = \dot{\omega}_i R_{ix} = \omega_i R_{ie} \cos^3 \Delta \] (4.8)

where \( i = 1, 2 \) for the pinion and gear surfaces, respectively.

### 4.4 Mechanical Efficiency Prediction for an Example Helical Gear Pair

The mechanical efficiency model is applied to an example unity-ratio helical gear pair. Basic design parameters of this gear pair are given in Table 4.1. It is operated at a center distance of 0.15 m. An automatic transmission fluid having the main properties listed in Table 4.2 is considered as the gear lubricant.

As discussed earlier, contact lines at every rotational position within a mesh cycle are discretized into many small constant-load segments and \( \mu \) is computed for each
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</tr>
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<tbody>
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<tr>
<td>Helix angle (degree)</td>
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<tr>
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</tr>
<tr>
<td>Young’s modulus (Pa)</td>
<td>2.07x10¹¹</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Table 4.1** Example helical gear pair parameters.

| Inlet temperature (K)                          | 373.15      |
| Viscosity (Pa s)                               | 0.0065      |
| Density (kg/m³)                                | 813         |
| Pressure viscosity coefficient (1/Pa)          | 1.2773x10⁻⁸ |
| Temperature viscosity coefficient (1/K)        | 0.0217      |
| Thermal conductivity (W/m/K)                   | 0.1176      |
| Coefficient of thermal expansion (1/K)         | 6.5x10⁻⁴    |
| Specific heat (J/kg/K)                         | 2000        |

**Table 4.2** Properties of the Automatic Transmission Fluid.
segment by using the published empirical formulae, the thermal EHL model, and the new friction models developed in Chapter 3. Using a finer discretization of the load distribution results in more segments and more accurate results while the computational time is also increased accordingly. The contact parameters at the mid-point of load segment including sliding and rolling velocities, load and radii of curvature are calculated before $\mu$ can be computed.

Figure 4.3(a) illustrates two consecutive teeth of the driving gear in contact at a given mesh angle $\phi = \phi_1$. These two contact lines are discretized using 24 segments, 16 segments for the longer contact line of tooth 1 and 8 segments for the contact line of tooth 2. The instantaneous coefficients of friction at each of these segments $\mu(z, \theta, \phi_1)$ calculated by using (i) the published $\mu$ formulae listed in Table 2.1, (ii) actual EHL analyses, (iii) the $\mu$ formula, Eq. (3.21), obtained from the regression analysis of the massive EHL model predictions, and (iv) the new $\mu$ formula, Eq. (3.18), obtained from the regression analysis of the measured traction data (uncoated-uncoated) are shown in Figure 4.3(b) for the operating conditions $S = 0.1 \mu m$, $L_{in} = 500 Nm$, $N_p = 4,000 rpm$ and $T_{oil} = 100^\circ C$. In the actual EHL analysis, a measured surface roughness profile shown in Figure 4.4 is used for every contact point.

As shown in Figure 4.3(b), all of the published $\mu$ formulae give values that are significantly larger than the actual EHL model and the new formulae proposed in Chapter 3, further confirming the shortcomings of these formulae. The published $\mu$ formulae with
Figure 4.3. (a) Illustration of contact lines at mesh position $\phi = \phi_1$, and (b) predicted $\mu(z, \theta, \phi)$ at every discrete contact point. $S = 0.1 \mu m$, $L_{in} = 500 Nm$, $N_p = 4,000 rpm$, and $T_{oil} = 100^\circ C$. 
Figure 4.4. A measured surface roughness profile of $S = 0.1 \mu m$ RMS.
the exception of the ISO formula that does not include $V_s$ appear to give the maximum $\mu$

at contact point 12. This is the contact point that is on the pitch line at this particular position
and has no sliding. As described in Chapter 3, these formulae have $V_s$ in their denominator, and therefore near pitch line, they predict their largest $\mu$ values. The distribution of $\mu$ values from the actual thermal EHL model, the $\mu$ formulae from the regression of the traction data and the regression of the EHL model are in good agreement. They are all significantly lower than those of the other formulae and have their minimum (almost zero) value at the pitch point, again in agreement with Chapter 3.

Figure 4.5 shows the variation of the average value of the instantaneous coefficient of friction at every mesh angle, $\bar{\mu}(\phi_m) = Average[\mu(z,0,\phi_m)]$, and the corresponding instantaneous mechanical efficiency $\eta(\phi_m)$ of the gear pair as a function of gear mesh position for $S = 0.1 \mu m$, $L_{in} = 500 Nm$, $N_p = 4,000 rpm$ and $T_{oil} = 100^\circ C$. While the variation of $\bar{\mu}$ and $\eta$ with $\phi_m$ is minimal, the average value of $\eta$ over an entire mesh cycle is quite different for the friction coefficient models considered. Again, the results from the thermal EHL model, the EHL regression, and the measured traction data based formulae agree well with each other and yield $\eta$ values around 99.85 percent. Meanwhile four of the other formulae from the literature, with the exception of Misharin’s formula, yield $\eta$ values around 99.65 percent. Considering that most helical gears will have mechanical efficiency loss of less than 1 percent ($\eta > 99$ percent), this difference between the other formulae and the new ones proposed here is very significant.
Figure 4.5. Predicted $\bar{\mu}(\phi)$ and $\eta(\phi)$. $S = 0.1 \mu m$, $L_{in} = 500 Nm$, $N_p = 4,000 rpm$, and $T_{oil} = 100^\circ C$. 
While the efficiency model based on EHL analysis should be more accurate than the one based on the empirical formulae or fitted EHL formula, it has the disadvantage of requiring significantly more computational effort. The results shown in Figure 4.5 required more than 300 different EHL analyses and over 8 hours of CPU time on a PC having a 3.2GHz processor. Therefore, the EHL-based $\mu$ formula should be preferred to actual EHL analyses for practical engineering purposes.

### 4.5 Concluding Remarks

In this Chapter, the mechanical efficiency model described in Chapter 2 was applied to parallel-axis gears. Geometric and kinematic parameters required by the analysis were derived for both spur and helical gears. An example helical gear pair lubricated with an automatic transmission fluid is used as a numerical example for the application of the efficiency prediction model. Predicted friction coefficient values and efficiency values obtained by using the different friction models were compared. It was demonstrated that the published $\mu$ formulae listed in Table 2.1 are not accurate while using the EHL-based and experimental $\mu$ formulae result in very similar efficiency values.

The efficiency model for the parallel-axis gears will be validated later in Chapter 6 through comparisons to actual spur gear efficiency measurements. The validated model will then be used in Chapter 7 to describe the influence of gear design parameters, profile
modifications, lubricant parameters and temperature, surface roughness parameters and surface treatments such as coatings, and gear manufacturing and assembly errors on mechanical efficiency of gears.
CHAPTER 5

APPLICATION TO CROSS-AXIS GEAR PAIRS

5.1 Introduction

Unlike parallel-axis gear pairs, cross-axis gear pairs, such as spiral bevel and hypoid gears, transmit power between two intersecting or nonintersecting shafts often having a right angle between them. Hypoid gears are perhaps the most common type of cross-axis gears as they are used in many high-volume applications such as passenger car and truck axle gearboxes. If there is zero offset between the two shaft axes, i.e. the axes intersect each other at a right angle, then the hypoid gear pair reduces to a spiral bevel gear pair. Similarly setting the shaft angle and the longitudinal tooth curvature to zero, a spiral bevel gear pair can be reduced to a parallel-axis helical gear pair. Therefore, a hypoid gear pair can be considered as the most general gear type while the other common gear types define the limiting cases of it.

Hypoid and spiral bevel gears have been widely used in automotive and aerospace industries. They are manufactured by cutting machines designed by the Gleason Works
(USA) and Oerlikon (Switzerland). In terms of the manufacturing methods, there are basically two types of spiral bevel and hypoid gears, *face-milled* (FM) and *face-hobbed* (FH). Face milling is a single indexing process that only cuts one slot at a time as illustrated in Figure 5.1(a). In the face milling process, a circular face-mill type cutter is used. For the non-generated member, only the cutter rotates and is fed into the gear blank up to the full depth. The cutter withdraws when the full depth is reached and then the work piece is indexed to the next tooth to repeat the process. The lengthwise tooth form is a circular arc with a curvature equal to the curvature of the cutter. For a generated member, the flank lead function is a circular arc that is wound around a conical surface [81]. Hypoid gears generated by the face milling process usually result in tapered depth teeth and grinding may be used as the finishing process [82].

Face hobbing is a continuous cutting process that is formed by continuous rotations combined with a feed motion as illustrated in Figure 5.1(b). The generating motion that rolls out the tooth surface is superimposed on the indexing motion. When one blade group, as shown in Figure 5.1(b), is moving through one slot, the work piece rotates in the opposite direction and then the next blade group enters the next slot. This way, cutting for all the teeth are finished nearly simultaneously. This method employs a circular face-hob type cutter. The resulting lengthwise tooth curve is an epicycloid that is formed kinematically [81]. Hypoid gears generated by the face hobbing process usually result in uniform depth teeth and lapping is required as the finishing process [82].
Figure 5.1. Kinematic principles of (a) face-milling (single indexing), and (b) face-hobbing (continuous indexing) processes [81].
In this chapter, the mechanical efficiency prediction model developed earlier is applied to cross-axis gears. A face hobbed hypoid gear pair cut by Gleason machines is used as an example as it represents the highest volume applications of such gears. However the model can be used for both FM and FH type hypoid gears. The mechanical efficiency model for hypoid gears is incorporated into a finite element based gear contact analysis model. Curvatures and surface velocity calculations for hypoid gears are presented. The mechanical efficiency for hypoid gears in terms of frictional losses is computed.

5.2 Gear Contact Analysis Model

A commercially available finite element (FE) based gear analysis package CALYX [73] is used as the contact analysis tool for hypoid gears. This model combines the FE method and a surface integral approach [74]. The gear teeth are modeled by using quasi-prismatic elements. The finite element model is used to obtain large deformations of the gear away from the contact zone, while an elastic half-space model is used to obtain relative deformations within the contact zone. This requires a special set-up for the finite element grids inside the instantaneous contact zone. As shown in Figure 5.2, a very fine contact grid is automatically defined to capture the entire contact zone. These grid cells are much finer than the regular sizes of finite element meshes elsewhere on the tooth surfaces and they are attached to the contact zones that result in a more accurate contact analysis than using a pure FE method. A schematic view of these grids is shown in Figure 5.3(a). Along the entire face width direction, there are $2n+1$ slices, and within
Figure 5.2. Hypoid gear teeth with moving grid cells attached to capture any contact zones.
Figure 5.3 (a) Set-up of the moving grid, and (b) definition of the grid on the tangent plane for $\mu$ calculation.
Figure 5.3 continued.
each slice, there is a principal contact point (denoted by a dot) if contact exists. In the profile direction, there are $2m+1$ grids within each slice, some of which would be in contact due to tooth deflections and local surface deformations. Both values of $m$ and $n$ and the dimension of the grid cells in the profile direction $\Delta s$ are user defined. Here $s$ is the curve length parameter measured along the profile and $\Delta s$ is the width of the grid cell in the profile direction. Choosing an appropriate value for $\Delta s$ is crucial in obtaining correct contact pressures. As the contact analysis model adapted in this study assumes constant contact pressure over each individual grid cell, using a large value for $\Delta s$ for a fixed $n$ can result in poor resolution, because only the grid cells at the center will end up carrying the entire load. If the value of $\Delta s$ is very small, then the grid is too narrow and the entire contact zone might not be captured by the defined contact grid. Therefore, an accurate contact pressure distribution must be obtained first, before any efficiency analysis can be performed.

The $\mu$ calculation is carried out in the grid of principal contact point in the tangent plane as shown in Figure 5.3(b), which is a zoomed view of a grid cell containing the principal contact point $q$. The surface formed by dotted lines, defined by points $1'$ to $8'$ along the edges, is the grid on the actual tooth surface and the plane formed by solid lines defined by points 1 to 8 along the edges, is the grid in the tangent plane for this particular contact point $q$. Points $2' (2'), 4' (4'), 6' (6')$ and $8' (8')$ are the mid-points defined at each side of the grid. Vector $t_{26}$ that connects point 2 and 6 is approximated as the instant line of contact and vector $t_{p26}$ is normal to $t_{26}$ in the tangent plane. $n$ is
the surface normal vector at the contact point $q$. Here it will be assumed that the variation of $\mu$ within each face width slice is negligible. The $\mu$ value obtained for the principal contact point will be assigned to all other potential contact points within the same face width slice.

5.3 Calculation of Surface Velocities and Radii of Curvature

While the contact pressure values at each grid cell are provided by the contact analysis model, surface velocities and radii of curvature must be calculated from the hypoid geometry. Before this can be done the principal contact points must be determined, and principal curvatures and principal directions must be defined.

5.3.1 Principal contact points

Assume pinion surface and gear surface are defined by $r_1(s_1,t_1)$ and $r_2(s_2,t_2)$ respectively, where $s_1, t_1, s_2, t_2$ are the curvilinear surface parameters. The principal contact point is determined at the location where $r_1$ and $r_2$ become the closest to each other [74]. Surface $r_1(s_1,t_1)$ is discretized into a set of grid of points $r_{1ij} = r_1(s_{il}, t_{1j})$, and for each of these grid points, $r_{2ij} = r_2(s_{2i}, t_{2j})$ is located such that $\left\| r_1(s_1,t_1) - r_2(s_2,t_2) \right\|$ is minimized with respect to the variable $s_2$ and $t_2$. This extremization is equivalent to solving the following system of nonlinear equations

138
\[
\begin{align*}
\left[ r_{ij} - r_2(s_{2i}, f_{2j}) \right] \frac{\partial r_2(s_{2i}, f_{2j})}{\partial s_{2i}} &= 0 \\
\left[ r_{ij} - r_2(s_{2i}, f_{2j}) \right] \frac{\partial r_2(s_{2i}, f_{2j})}{\partial s_{2j}} &= 0
\end{align*}
\]

Initially, solutions from Eq. (5.1) for each of the grid point \( r_{ij} \) are obtained by the Newton-Raphson method. Then a new grid that is finer than the original grid is set up around the point \( r_{ij} \) for which the separation \( \| r_{ij} - r_{2ij} \| \) was the smallest. This search process is repeated several times with progressively smaller grids to locate the principal contact point [74]. Once the principal contact points within each face width slice on the tooth surfaces of the pinion and the gear are determined, sets of grid cells are laid out above and below these principal contact points. These added grid cells act as the candidate contact points that will likely be in contact due to elastic deformations of the two contacting surfaces.

### 5.3.2 Principal curvatures and principal directions

According to differential geometry [83], the principal curvatures and principal directions at a common contact point between two surfaces can be determined from the coefficients of the first and second fundamental form of the surface. Let \( \kappa^{(1)} \) and \( \kappa^{(2)} \) be the two eigenvalues of the eigen problem
\[ N \lambda = \kappa M \lambda \]  

(5.2)

where \( \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \),

\[
M = \begin{bmatrix}
\frac{\partial \mathbf{r}}{\partial s} & \frac{\partial \mathbf{r}}{\partial s} & \frac{\partial \mathbf{r}}{\partial t} & \frac{\partial \mathbf{r}}{\partial t} \\
\frac{\partial \mathbf{r}}{\partial s} & \frac{\partial \mathbf{r}}{\partial s} & \frac{\partial \mathbf{r}}{\partial t} & \frac{\partial \mathbf{r}}{\partial t}
\end{bmatrix}, \quad N = \begin{bmatrix}
\mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial s^2} & \mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial s \partial t} \\
\mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial t \partial s} & \mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial t^2}
\end{bmatrix},
\]

\[
n = \frac{\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t}}{\left| \frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right|}.
\]

Here, \( \mathbf{n} \) is the unit normal vector, and \( s \) and \( t \) are the surface curvilinear parameters at the common contact point. Matrices \( M \) and \( N \) contain the coefficients of the first and second fundamental form of the surface, respectively, and \( \kappa^{(1)} \) and \( \kappa^{(2)} \) represent the principal normal curvatures. If the two corresponding eigenvectors \( \lambda^{(1)} \) and \( \lambda^{(2)} \) are normalized such that \( \{ \lambda^{(i)} \}^T M \lambda^{(i)} = 1, \ i = 1, 2 \), then the two unit vectors in the principal directions corresponding to the principal curvatures are defined as [74]

\[
e^{(i)} = \{ \lambda^{(i)} \}^T \begin{bmatrix}
\frac{\partial \mathbf{r}}{\partial s} \\
\frac{\partial \mathbf{r}}{\partial t}
\end{bmatrix} \quad \text{for} \ i = 1, 2.
\]

(5.3a)
\[ e^{(2)} = \left[ \lambda^{(2)} \right]_T \begin{bmatrix} \frac{\partial r}{\partial s} \\ \frac{\partial r}{\partial t} \end{bmatrix}. \] 

(5.3b)

### 5.3.3 Orientation of the contact ellipse

Assume \( q \) is the contact point of two surfaces \( \Sigma_1 \) and \( \Sigma_2 \). Let \( e_f \) and \( e_h \) be unit vectors of the principal directions, and \( \kappa_f \) and \( \kappa_h \) be principal curvatures of surface \( \Sigma_1 \) at point \( q \). Similarly, define \( e_s \) and \( e_q \) as the unit vectors of the principal directions, and \( \kappa_s \) and \( \kappa_q \) as the principal curvatures of surface \( \Sigma_2 \) at same contact point \( q \). These values of principal directions and curvatures can be solved by Eq. (5.2). The angle \( \sigma \), formed by \( e_f \) and \( e_s \), and the angle \( \alpha \) that determines the orientation of the coordinate axes \( x \) and \( y \) with respect to \( e_f \) as shown in Figure 5.4 can be obtained as [74]

\[ \sigma = \tan^{-1} \left( \frac{e_s \cdot e_h}{e_s \cdot e_f} \right) \] 

(5.4a)

and

\[ \alpha = \frac{1}{2} \tan^{-1} \left( \frac{g_2 \sin 2\sigma}{g_1 - g_2 \cos 2\sigma} \right) \] 

(5.4b)
Figure 5.4 Principal directions and surface velocities.
where \( g_1 = \kappa_f - \kappa_h \), and \( g_2 = \kappa_s - \kappa_q \). Then, the two directions \( x \) and \( y \) can be represented as

\[
x = e_f \cos \alpha + e_h \sin \alpha \\
y = -e_f \sin \alpha + e_h \cos \alpha .
\] (5.5a)

(5.5b)

### 5.3.4 Surface velocities and radii of curvature in desired directions

Assume that the two contacting surfaces, \( \Sigma_1 \) and \( \Sigma_2 \), are in continuous tangency at the point of contact so that the position vectors and unit normals of surfaces \( \Sigma_1 \) and \( \Sigma_2 \) are equal. Based on this consideration, the following two relations can be obtained [84]

\[
u_r^{(2)} = \nu_r^{(1)} + \nu^{(12)}
\] (5.6a)

\[
n_r^{(2)} = n_r^{(1)} + \omega^{(12)} \times n
\] (5.6b)

where \( \nu_r^{(i)} \) is the relative velocity of the contact point respect to the surface as it moves over the surface \( \Sigma_i \), \( n_r^{(i)} \) is the velocity of the tip of the surface unit normal in its motion over the surface \( \Sigma_i \), and \( \omega^{(12)} = \omega^{(1)} - \omega^{(2)} \). Equations (5.6) are used for the derivation of
curvature relations of mating surfaces. As shown in Figure 5.4, the velocity vectors of point $q$ on surface $\Sigma_1$ can be represented in coordinate system $S_a(e_f, e_h)$ as

$$\mathbf{u}_r^{(1)} = \begin{bmatrix} u_f^{(1)} \\ u_h^{(1)} \end{bmatrix}^T, \quad (5.7a)$$

$$\mathbf{n}_r^{(1)} = \begin{bmatrix} n_f^{(1)} \\ n_h^{(1)} \end{bmatrix}^T. \quad (5.7b)$$

Similarly, the velocity vectors of point $q$ on surface $\Sigma_2$ can be represented in coordinate system $S_b(e_s, e_q)$ as

$$\mathbf{u}_r^{(2)} = \begin{bmatrix} u_s^{(2)} \\ u_q^{(2)} \end{bmatrix}^T \quad (5.8a)$$

$$\mathbf{n}_r^{(2)} = \begin{bmatrix} n_s^{(2)} \\ n_q^{(2)} \end{bmatrix}^T. \quad (5.8b)$$

According to the Rodrigues’ formula, vectors $\mathbf{u}_r^{(i)}$ and $\mathbf{n}_r^{(i)}$ are collinear for the principal directions, and they are related to the principal directions as $\mathbf{n}_r^{(i)} = -\kappa_r^{(i)} u_r^{(i)}$, where $\kappa_r^{(i)}$ are the principal curvatures of surface $\Sigma_i$, $i = 1, 2$. These principal curvatures are $\kappa_f$ and $\kappa_h$ for surface $\Sigma_1$ and $\kappa_s$ and $\kappa_q$ for surface $\Sigma_2$. Using the Rodrigues’ formula, Eqs.
(5.7) and (5.8) are rearranged together with appropriate coordinate transformations to obtain a system of linear equations as [84]

\[
\begin{bmatrix}
    b_{11} & b_{12} & 0 & 0 \\
    b_{21} & b_{22} & 0 & 0 \\
    0 & 0 & b_{33} & b_{34} \\
    0 & 0 & b_{43} & b_{44}
\end{bmatrix}
\begin{bmatrix}
    v_h^{(2)} \\
    v_f^{(2)} \\
    v_s^{(1)} \\
    v_q^{(1)}
\end{bmatrix}^T
= \begin{bmatrix}
    b_{15} \\
    b_{25} \\
    b_{35} \\
    b_{45}
\end{bmatrix}
\tag{5.9}
\]

where

\[
    b_{11} = -\kappa_f + \kappa_s \cos^2 \sigma + \kappa_q \sin^2 \sigma,
\]
\[
    b_{12} = b_{21} = \kappa_s \sin \sigma \cos \sigma - \kappa_q \sin \sigma \cos \sigma,
\]
\[
    b_{22} = -\kappa_h + \kappa_s \sin^2 \sigma + \kappa_q \cos^2 \sigma,
\]
\[
    b_{33} = \kappa_s - \kappa_f \cos^2 \sigma - \kappa_h \sin^2 \sigma,
\]
\[
    b_{34} = b_{43} = \kappa_f \sin \sigma \cos \sigma - \kappa_h \sin \sigma \cos \sigma,
\]
\[
    b_{44} = \kappa_q - \kappa_f \sin^2 \sigma - \kappa_h \cos^2 \sigma,
\]
\[
    b_{15} = -(\omega^{(12)} \cdot e_h) - \kappa_f (v^{(12)} \cdot e_f),
\]
\[
    b_{25} = (\omega^{(12)} \cdot e_f) - \kappa_h (v^{(12)} \cdot e_h),
\]
\[
    b_{35} = -(\omega^{(12)} \cdot e_q) - \kappa_s (v^{(12)} \cdot e_s),
\]
\[
    b_{45} = (\omega^{(12)} \cdot e_s) - \kappa_q (v^{(12)} \cdot e_q).
\]

145
Equation (5.9) can then be solved to obtain the surface velocities in the principal directions.

The surface velocities in the direction of contact line and in the direction normal to the contact line for surface $\Sigma_1$ and surface $\Sigma_2$, as shown in Figure 5.4, can be obtained by coordinate transformations as follows

\[
\begin{bmatrix}
V_p^{(1)} \\
V_t^{(1)}
\end{bmatrix} = \begin{bmatrix}
\cos q_2 & \sin q_2 \\
-\sin q_2 & \cos q_2
\end{bmatrix} \begin{bmatrix}
V_s^{(1)} \\
V_q^{(1)}
\end{bmatrix},
\]

(5.10a)

\[
\begin{bmatrix}
V_p^{(2)} \\
V_t^{(2)}
\end{bmatrix} = \begin{bmatrix}
\cos q_1 & \sin q_1 \\
-\sin q_1 & \cos q_1
\end{bmatrix} \begin{bmatrix}
V_f^{(2)} \\
V_h^{(2)}
\end{bmatrix}.
\]

(5.10b)

Sliding and rolling velocities in the direction of the contact line are defined as $V_{st} = V_t^{(1)} - V_t^{(2)}$ and $V_{rt} = V_t^{(1)} + V_t^{(2)}$, respectively. Sliding and rolling velocities in the direction that is normal to the contact line are $V_{sp} = V_p^{(1)} - V_p^{(2)}$ and $V_{rp} = V_p^{(1)} + V_p^{(2)}$, respectively. The resultant sliding and rolling velocities are $V_{s\text{total}} = \sqrt{V_{st}^2 + V_{sp}^2}$ and $V_{r\text{total}} = \sqrt{V_{rt}^2 + V_{rp}^2}$, respectively. Normal curvatures in the direction of the contact line, $\kappa_{mc}^{(i)}$, and in the direction that is normal to the contact line, $\kappa_{nn}^{(i)}$, are obtained as
\[
\kappa_{nc}^{(i)} = \kappa_f^{(i)} \cos^2 (\pi / 2 + q_i) + \kappa_{II}^{(i)} \sin^2 (\pi / 2 + q_i), \tag{5.11a}
\]

\[
\kappa_{nn}^{(i)} = \kappa_f^{(i)} \cos^2 q_i + \kappa_{II}^{(i)} \sin^2 q_i \tag{5.11b}
\]

where, \( q_i \) is the angle that is formed by the direction of interest and \( e_f \) and \( e_s \) (\( i = 1, 2 \)). The corresponding radii of curvature are simply the inverse of the curvature values.

The velocities and curvatures in Eqs. (5.10) and (5.11) in the direction that is normal to the contact line together with contact pressures predicted by the contact analysis model, can then be used in friction models to calculate \( \mu \). In this study, the sliding in the direction of the contact line is neglected in predicting the friction coefficient, while the resultant sliding and rolling velocities are used for the computation of the frictional power losses.

### 5.4 An Example Hypoid Gear Efficiency Analysis

With all necessary parameters at each contact point calculated by using the formulations and the contact model presented in previous sections, instantaneous efficiency of a hypoid gear pair can be calculated by applying the methodology defined in Figure 2.1. A face-hobbed hypoid gear borrowed from an automotive rear axle application will be used here as an example system to demonstrate the hypoid gear efficiency methodology. The main gear blank dimensions of this example system are
listed in Table 5.1. The main machine set-up and cutter parameters used for manufacturing the pinion member and the ring gear are listed in Tables 5.2 and 5.3, respectively. The pinion member is generated and the ring gear is non-generated. A typical gear oil, 75W90, is used as the lubricant whose properties are listed in Table 5.4.

As mentioned earlier, the correct contact pressure distribution must be obtained by adjusting the grid parameters, i.e., grid cell width $\Delta s$, the number of grids in the face width direction $2n+1$ and the number of grids in the profile direction $2m+1$ as well as the separation tolerance. In this example, these parameters are selected as $\Delta s = 0.2 \text{ mm}$, $m = 8$, $n = 16$, and the separation tolerance is 0.008 mm. A typical contact pressure histogram is shown in Figure 5.5 at the mesh angle $\phi = \phi_1$, which indicates total grid area is large enough to contain all contact points, yet is not too large so that multiple grids in each face width segment define the contact pressure distribution quite accurately. The contact pressure at the principal contact point, which usually lies in the middle of grid cells along the profile direction, can be approximated as the maximum Hertzian pressure. This figure maps surfaces of five gear teeth with three teeth are in contact at this particular position. For each tooth, the vertical direction refers to the face width direction, while the horizontal direction corresponds to the profile direction.

Figure 5.6 illustrates the contact pressures distribution at all the contact zones on the hypoid ring gear in contact at three different mesh positions $\phi = \phi_1$, $\phi = \phi_5$, and $\phi = \phi_0$. As discussed earlier, contact pressures, radii of curvatures, sliding velocities, and sum of rolling velocities are four of the key geometry-related parameters used in the
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Pinion</th>
<th>Ring Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teeth number</td>
<td>12</td>
<td>41</td>
</tr>
<tr>
<td>Face width</td>
<td>41.9652</td>
<td>34.2392</td>
</tr>
<tr>
<td>Pinion offset</td>
<td></td>
<td>44.45</td>
</tr>
<tr>
<td>Shaft angle (deg)</td>
<td></td>
<td>90</td>
</tr>
<tr>
<td>Hand of spiral</td>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>Mean spiral angle (deg)</td>
<td>49.9789</td>
<td>24.0784</td>
</tr>
<tr>
<td>Outer cone distance (mm)</td>
<td>103.562</td>
<td>127.787</td>
</tr>
<tr>
<td>Face angle (deg)</td>
<td>28.642</td>
<td>58.7358</td>
</tr>
<tr>
<td>Pitch angle (deg)</td>
<td>28.642</td>
<td>58.7358</td>
</tr>
<tr>
<td>Base angle (deg)</td>
<td>28.642</td>
<td>58.7358</td>
</tr>
<tr>
<td>Front angle (deg)</td>
<td>28.642</td>
<td>58.7358</td>
</tr>
<tr>
<td>Back angle (deg)</td>
<td>28.642</td>
<td>58.7358</td>
</tr>
<tr>
<td>Pitch apex beyond crossing point (mm)</td>
<td>-15.152</td>
<td>18.8428</td>
</tr>
<tr>
<td>Face apex beyond crossing point (mm)</td>
<td>-0.7962</td>
<td>20.4917</td>
</tr>
<tr>
<td>Root apex beyond crossing point (mm)</td>
<td>-20.7927</td>
<td>9.2783</td>
</tr>
<tr>
<td>Base apex beyond crossing point (mm)</td>
<td>-40.7892</td>
<td>-1.9351</td>
</tr>
<tr>
<td>Normal chordal addendum (mm)</td>
<td>6.985</td>
<td>1.4424</td>
</tr>
<tr>
<td>Normal chordal tooth thickness (mm)</td>
<td>8.9916</td>
<td>3.8608</td>
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<tr>
<td>Young’s modulus (Pa)</td>
<td></td>
<td>2.07x10^9</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td></td>
<td>0.3</td>
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*Table 5.1* Gear pair blank data for the example hypoid gear set.
<table>
<thead>
<tr>
<th>Hypoid member</th>
<th>Pinion</th>
</tr>
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<tbody>
<tr>
<td>Tooth side</td>
<td>Concave</td>
</tr>
<tr>
<td>Radial setting (mm)</td>
<td>128.538</td>
</tr>
<tr>
<td>Tilt angle (deg)</td>
<td>33.161</td>
</tr>
<tr>
<td>Swivel angle (deg)</td>
<td>-35.102</td>
</tr>
<tr>
<td>Blank offset (mm)</td>
<td>43.4963</td>
</tr>
<tr>
<td>Root angle (deg)</td>
<td>-1.1295</td>
</tr>
<tr>
<td>Machine center to back (mm)</td>
<td>-0.0314</td>
</tr>
<tr>
<td>Sliding base (mm)</td>
<td>32.0707</td>
</tr>
<tr>
<td>Cradle angle (deg)</td>
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</tr>
<tr>
<td>Ratio of roll</td>
<td>3.3520</td>
</tr>
<tr>
<td>Cutter point radius (mm)</td>
<td>88.5863</td>
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<tr>
<td>Blade eccentric angle (deg)</td>
<td>19.6959</td>
</tr>
<tr>
<td>Cutter blade angle (deg)</td>
<td>19.8601</td>
</tr>
<tr>
<td>Cutter edge radius (mm)</td>
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<tr>
<td>HF</td>
<td>5.4198</td>
</tr>
<tr>
<td>DZ</td>
<td>0</td>
</tr>
<tr>
<td>Number of cutter blade groups</td>
<td>17</td>
</tr>
<tr>
<td>Blade rake angle (deg)</td>
<td>12</td>
</tr>
<tr>
<td>Blade hook angle (deg)</td>
<td>4.42</td>
</tr>
<tr>
<td>Type of cutter</td>
<td>TOPREM</td>
</tr>
<tr>
<td>Cutter modification angle (deg)</td>
<td>2.4</td>
</tr>
<tr>
<td>Cutter modification length (mm)</td>
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**Table 5.2** Machine settings and cutter parameters for the example hypoid pinion.
<table>
<thead>
<tr>
<th>Hypoid member</th>
<th>Ring Gear</th>
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<td>Tooth side</td>
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<tr>
<td>Vertical setting (mm)</td>
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<tr>
<td>Horizontal setting (mm)</td>
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<tr>
<td>Machine center to back (mm)</td>
<td>15.6421</td>
</tr>
<tr>
<td>Root angle (deg)</td>
<td>58.7358</td>
</tr>
<tr>
<td>Cutter point radius (mm)</td>
<td>88.3364</td>
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<tr>
<td>Blade eccentric angle (deg)</td>
<td>19.7962</td>
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<tr>
<td>Cutter blade angle (deg)</td>
<td>23.0992</td>
</tr>
<tr>
<td>Cutter edge radius (mm)</td>
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</tr>
<tr>
<td>HF</td>
<td>5.4397</td>
</tr>
<tr>
<td>DZ</td>
<td>0</td>
</tr>
<tr>
<td>Number of cutter blade groups</td>
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<tr>
<td>Blade rake angle (deg)</td>
<td>12</td>
</tr>
<tr>
<td>Blade hook angle (deg)</td>
<td>4.42</td>
</tr>
<tr>
<td>Type of cutter</td>
<td>CURVED</td>
</tr>
<tr>
<td>Blade spherical radius (mm)</td>
<td>1905</td>
</tr>
</tbody>
</table>

Table 5.3 Machine settings and cutter parameters for the example hypoid ring gear.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet temperature ($K$)</td>
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</tr>
<tr>
<td>Viscosity ($Pa \cdot s$)</td>
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</tr>
<tr>
<td>Density ($kg/m^3$)</td>
<td>815</td>
</tr>
<tr>
<td>Pressure viscosity coefficient ($1/Pa$)</td>
<td>$1.344 \times 10^{-8}$</td>
</tr>
<tr>
<td>Temperature viscosity coefficient ($1/K$)</td>
<td>0.0217</td>
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<tr>
<td>Thermal conductivity ($W/m/K$)</td>
<td>0.132</td>
</tr>
<tr>
<td>Coefficient of thermal expansion ($1/K$)</td>
<td>$6.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>Specific heat ($J/kg/K$)</td>
<td>2000</td>
</tr>
</tbody>
</table>

**Table 5.4** Lubricant parameters for 75W90.
Figure 5.5. Predicted contact pressure histograms at rotational position $\phi = \phi_1$. 
Figure 5.6. Predicted contact pressure distributions at (a) $\phi = \phi_1$, (b) $\phi = \phi_5$, (c) $\phi = \phi_{10}$. 
friction model. The distributions of these parameters along the contact lines at the same mesh angle \( \phi = \phi_1 \) are shown in Figure 5.7. It is clear from this figure that all four key parameters vary significantly along the contact lines.

The values of \( \mu(z,0,\phi_1) \) at each of the principal contact points along the instantaneous lines of contact are calculated by using the new formulae developed in Chapter 3. Specifically, the \( \mu \) formula, Eq. (3.18), obtained by the multiple regression analysis of the measured traction coefficients and the \( \mu \) formula, Eq. (3.21), obtained by processing the massive EHL model predictions are used here, as well as by using the actual EHL model are shown in Figure 5.8 for \( S = 0.1 \mu m, L_m = 1600 Nm, N_p = 1000 \text{ rpm}, \) and \( T_{oil} = 60^\circ C \). For the EHL model prediction, a measured rough surface profile with \( RMS = 0.1 \mu m \) like the one shown in Figure 5.9 is used. As shown in Figure 5.8, all of the predicted \( \mu \) values show certain amount of variations along the instantaneous lines of contact. The \( \mu \) values get especially large when the contact occurs near the edge where contact pressures are large as shown in Figure 5.7. The variations of \( \mu \) predicted at each contact point, together with the variations of the contact pressures and the sliding and rolling velocities, result in the variations of the power losses at each of these locations.

Figure 5.10 shows the variation of the average value of the instantaneous coefficient of friction at every mesh angle, \( \bar{\mu}(\phi_m) = Average[\mu(z,0,\phi_m)] \), and the
\( P_{\text{max}} = 4.4 \ \text{GPa}, \ P_{\text{min}} = 0.036 \ \text{GPa} \)

(a) Contact pressure

\( R_{\text{max}} = 76 \ \text{mm}, \ R_{\text{min}} = 2 \ \text{mm} \)

(b) Combined radius of curvature along the normal of the instant line of contact

**Figure 5.7.** Distribution of (a) contact pressure, (b) combined radii of curvature, (c) sliding velocity, and (d) sum of rolling velocities along the normal of the instant line of contact at angular position \( \phi_1 \). \( S = 0.1 \ \mu m, \ L_{\text{in}} = 1,600 \ \text{Nm}, \ N_p = 1,000 \ \text{rpm}, \) and \( T_{\text{oil}} = 60^\circ C \).
Figure 5.7 continued.

\[ V_{s,\text{max}} = 1.8 \text{ m/s}, \; V_{s,\text{min}} = 0.02 \text{ m/s} \]

(c) Sliding velocity along the normal of the instant line of contact

\[ V_{r,\text{max}} = 8.0 \text{ m/s}, \; V_{r,\text{min}} = 0.6 \text{ m/s} \]

(d) Sum of rolling velocity along the normal of the instant line of contact
Experimental $\mu$ formula, $\mu_{\text{max}} = 0.0932$, $\mu_{\text{min}} = 0.0004$

(a)

EHL-based $\mu$ formula, $\mu_{\text{max}} = 0.0857$, $\mu_{\text{min}} = 0.0063$

(b)

Actual EHL analysis, $\mu_{\text{max}} = 0.1023$, $\mu_{\text{min}} = 0.0156$

(c)

Figure 5.8. Distribution of $\mu(z, \theta, \phi)$ at every mesh grid of principal contact points predicted by using (a) the experimental $\mu$ formula, (b) the EHL-based $\mu$ formula, and (c) the actual EHL analysis. $S = 0.1 \, \mu m$, $L_{in} = 1,600 \, Nm$, $N_p = 1,000 \, rpm$, and $T_{oil} = 60^\circ C$. 

158
Figure 5.9. A typical measured surface roughness profile used in the EHL analysis. $S = 0.1 \, \mu m$, $L_{in} = 1,600 \, Nm$, $N_p = 1,000 \, rpm$, and $T_{oil} = 60^\circ C$. 
\( \eta(\phi) \) predicted by using the friction coefficient formulae developed in this study and the EHL model. \( S = 0.1 \, \mu m \), \( L_{in} = 1,600 \, Nm \), \( N_p = 1,000 \, rpm \), and \( T_{oil} = 60^\circ C \).
corresponding instantaneous mechanical efficiency $\eta(\phi_m)$ of the gear pair. It is evident that the two EHL related models are in reasonably good agreement and give an average efficiency of about $\eta = 98$ percent for $S = 0.1 \text{ \mu m}$, $L_{in} = 1600 \text{ Nm}$, $N_p = 1,000 \text{ rpm}$ and $T_{oil} = 60^\circ \text{C}$. Meanwhile an average difference around 2 percent in predicted $\eta(\phi_m)$ values is observed between the $\eta(\phi_m)$ values calculated by using the experimental $\mu$ formula and by using the other two EHL related models. This difference can be explained by the fact that the experimental $\mu$ formula was obtained based on the measured traction data that were collected under the constant temperature 100 deg C and with a constant radius of curvature. Therefore, the influence of the temperature and the radii of curvature cannot be taken into account in the friction prediction by using the experimental $\mu$ formula. This is the major shortcoming of Eq. (3.18). On the other hand, as discussed in Chapter 3, the EHL model and the EHL-based $\mu$ formula tend to underestimate the friction slightly under heavily loaded conditions, as is the case in this example hypoid gear application.

5.5 Concluding Remarks

The mechanical efficiency model described in the previous chapters was applied to the cross-axis gears with a face-hobbed hypoid gear pair as a representative example. Formulations for the required surface velocities and radii of curvature were presented. An example face-hobbed hypoid gear pair lubricated with 75W90 gear oil was used as a
numerical example for the application of the efficiency prediction model. Variations of the contact pressures, sliding and rolling velocities, and effective radii of curvatures along the instant lines of contact were presented. The $\mu$ and $\eta$ values predicted by using the two new $\mu$ formulae proposed earlier were compared to each other. Mechanical power losses from a hypoid gear pair were observed to be significantly more than that of a typical helical gear pair since hypoid gears have significantly more sliding due to their unique geometry and the shaft offset.
6.1 Introduction

In this chapter, the gear pair efficiency model will be validated by comparing the model predictions to a set of efficiency data collected in a companion study by Chase [70] using a high-speed spur gear efficiency test machine. A coefficient of friction model based on the EHL model was validated in Chapter 3 by using a set of traction data from ball-on-disk tests. The comparison provided in this chapter will focus on the validation of the overall gear pair mechanical efficiency prediction methodology outlined by the flow chart of Figure 2.1. A significant body of spur gear pair measurements [70] is already available. These spur gear experiments are also compatible with the traction data presented in Chapter 3. The same system will be used here as the example system for this validation effort, providing a direct comparison between the parallel-axis gear efficiency model and the gear efficiency measurements. Since experimental efficiency data is not available for the hypoid gears, a direct validation of the hypoid version of the model will not be attempted in this study. However, it should be noted that the
methodologies used for parallel-axis and cross-axis gear efficiency are essentially the same except the gear geometry and the contact model employed. Therefore, validation of the parallel-axis gear efficiency model as well as the validation of the friction coefficient model should provide confidence in terms of the accuracy of the hypoid gear pair efficiency model as well.

6.2 High Speed Spur Gear Efficiency Tests

Chase [70] designed and procured a high-speed spur gear efficiency test machine shown in Figure 6.1. In Figure 6.2, a schematic layout is provided to define key components of the test machine. Two identical gearboxes are positioned in a mirror image configuration and connected to each other through flexible shafts to form a “four-square” arrangement. Gears are loaded mechanically through a split coupling mounted on one of the flexible shafts connecting the two gearboxes. A high-speed drive unit is connected to one of the gearboxes. It provides sufficient power to the closed power circulation loop to overcome efficiency losses. A precision torque-meter located between the gearbox and the drive unit is used to measure torque provided to the gearboxes. Since the two gearboxes are identical in terms of their content (bearings, shafts and gears), operating conditions (speed and torque transmitted), as well as their lubrication (lubricant type, flow rate and temperature), the measured torque loss can be divided into two to determine the total torque loss of a single gearbox. Details of the test machine and the
Figure 6.1. High-speed gear efficiency test machine [70].
Figure 6.2. A schematic showing the layout of the gear efficiency test machine
instrumentation can be found in reference [70]. The same reference also demonstrates the accuracy and the repeatability of the efficiency tests.

The test machine was designed to operate at various levels of load as well as no load conditions. The torque loss measurements under no load (called spin tests in [70]) represent load-independent losses in the form of windage and oil churning, in addition to load independent bearing and seal losses. Under loaded conditions, the friction-related losses at the gear mesh and the bearings are added to the spin losses. Therefore, by conducting the efficiency tests at a given speed under both unloaded and loaded conditions, one can separate the load related frictional losses of gears and bearings from the load-independent losses. Finally, by computing the load dependent bearing losses using the manufacture’s specifications and published bearing efficiency models [78] described in Chapter 2, the friction losses at the gear meshes can be quantified. These measured gear mesh friction losses will be used for the validation of the model here.

6.2.1 Gearboxes

The test gear specimens are shrink-fit mounted on relatively rigid shafts in addition to a key at each shaft-gear interface. The axial position of each gear is controlled tightly using precision spacers mounted at both sides between the gear and its bearings. This ensures the test-to-test repeatability of the gear axial positions and the repeatability of the active gear face width. There are four identical high precision cylindrical roller bearings (SKF NJ 406) in each gearbox, which have a critical speed
rating of 11,000 rpm. The bearings are not preloaded, and therefore they do not cause sizable friction losses under unloaded conditions.

6.2.2 Test specimens and test matrix

Chase [70] considered two different gear designs, a 23-tooth gear pair and a 40-tooth gear pair, with the basic design parameters listed in Table 6.1. For each basic design, there were three different gear pair samples with different face width values of 26.7 mm (wide FW), 19.5 mm (medium FW) and 14.2 mm (narrow FW). Examples of test gears having different modules and face widths are shown in Figure 6.3. In addition to these six variations that were used to investigate the influence of gear module and face width on gear efficiency, other variations associated with surface conditions were also introduced, including different roughness values and “low-friction” thin-film (PVD) coatings.

For each gear pair, Chase [70] performed tests under various test conditions. In order to quantify the influence of the load, a set of test were performed at 6,000 rpm and the torque values between 0 and 677 N-m (500 ft-lbf) with an increment of around 135 N-m (100 ft-lbf). Similarly, at 406 N-m (300 ft-lbf), tests were run at speeds between 2,000 and 10,000 rpm with an increment of 2,000 rpm to quantify the effect of the speed on gear efficiency. Additional tests were performed at all incremental torque and speed combinations within 6,000 to 10,000 rpm and 406 to 677 N-m that represent the actual operating conditions of the intended sponsor applications. In order to investigate the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design 1</th>
<th>Design 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
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<td>40</td>
</tr>
<tr>
<td>Module, <strong>mm</strong></td>
<td>3.95</td>
<td>2.32</td>
</tr>
<tr>
<td>Pressure angle, <strong>deg</strong></td>
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<td>28</td>
</tr>
<tr>
<td>Pitch diameter, <strong>mm</strong></td>
<td>90.86</td>
<td>92.74</td>
</tr>
<tr>
<td>Base diameters, <strong>mm</strong></td>
<td>82.34</td>
<td>81.89</td>
</tr>
<tr>
<td>Outside diameter, <strong>mm</strong></td>
<td>100.34</td>
<td>95.95</td>
</tr>
<tr>
<td>Root diameter, <strong>mm</strong></td>
<td>81.30</td>
<td>85.80</td>
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<tr>
<td>Start of active profile, <strong>mm</strong></td>
<td>85.38</td>
<td>87.73</td>
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<tr>
<td>Circular tooth thickness, <strong>mm</strong></td>
<td>6.435</td>
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</tr>
<tr>
<td>Root fillet radius, <strong>mm</strong></td>
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<td>0.83</td>
</tr>
<tr>
<td>Contact ratio</td>
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<td>1.02</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>1:1</td>
<td>1:1</td>
</tr>
</tbody>
</table>

**Table 6.1** Test gear pair parameters [70]
Figure 6.3. Examples of (a) 23 and 40-tooth test gears, and (b) test gears having different face widths [70].
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Measured at deg C</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, kg/l</td>
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<td>0.78</td>
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<td>Kinematic viscosity, cSt</td>
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<td>97.5</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>14.31</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>15.6</td>
<td>0.86</td>
</tr>
<tr>
<td>Specific heat, kJ/kg/C</td>
<td>100</td>
<td>2.29</td>
</tr>
<tr>
<td>Flash point, deg C</td>
<td>N/A</td>
<td>207</td>
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</table>

**Table 6.2** Test lubricant properties for 75W90.
contributions of the spin losses, a set of tests was performed at no load conditions and at all speed increments up to 10,000 rpm. All these tests were performed by using 75W90 gear oil with properties listed in Table 6.2.

6.3 Modeling of the Spur Gear Efficiency Tests

At any given speed under unloaded conditions, the spin power loss $P_s$ is defined by the gear windage losses $P_w$, oil churning losses $P_{ch}$, load independent bearing losses $P_{b,no\ load}$, and seal losses $P_{seal}$, which can be written as

$$P_s = P_w + P_{ch} + P_{b,no\ load} + P_{seal}. \quad (6.1)$$

When gears are loaded, the total power loss of the gearbox $P_t$ can be written as

$$P_t = P_s + P_g + P_{b,load} \quad (6.2)$$

where $P_g$ and $P_{b,load}$ are load dependent gear mesh and bearing power losses, respectively. $P_g$ can be written in terms of its sliding friction and rolling friction components. The differences between measured $P_t$ and $P_s$ (difference of the power losses from loaded and unloaded tests at the same speed) equals the sum of load-
dependent bearing and gear mesh losses that will be called the mechanical losses in this chapter. It is given as

\[ P_{\text{mech}} = P_t - P_s = P_{g, \text{slide}} + P_{g, \text{roll}} + P_{b, \text{load}}. \] (6.3)

Here, \( P_{g, \text{slide}} \) can be calculated for every contact point at every instant within a mesh cycle as

\[ P_{g, \text{slide}}(z, \theta, \phi_m) = \mu(z, \theta, \phi_m)W(z, \theta, \phi_m)\left[u_1(z, \theta, \phi_m) - u_2(z, \theta, \phi_m)\right] \] (6.4)

where \( \mu(z, \theta, \phi_m) \) is the instantaneous coefficient of friction, \( W(z, \theta, \phi_m) \) is the normal load carried by the teeth in contact at the specific contact point, \( u_1(z, \theta, \phi_m) \) and \( u_2(z, \theta, \phi_m) \) are the surface velocities of the pinion and gear at the contact point, respectively. The power losses due to rolling friction \( P_{g, \text{roll}} \) are obtained as

\[ P_{g, \text{roll}}(z, \theta, \phi_m) = F_r(z, \theta, \phi_m)\left[u_1(z, \theta, \phi_m) + u_2(z, \theta, \phi_m)\right] \] (6.5)

where \( F_r(z, \theta, \phi_m) \) is the rolling friction force between the gear teeth in contact at every contact point calculated by using the formula available in the literature [53] as discussed in Chapter 2 (Eq. 2.14). The load dependent losses of a single bearing can be computed as [78]
where $\mu_b$ is the coefficient of friction for the bearing ($\mu_b = 0.0011$ from the SKF catalog for a light series roller bearing), $W_b$ is the radial load applied to the bearings, and $d_{bore}$ is the bore diameter of the bearing ($d_{bore} = 30$ mm for these bearings).

As there are four identical bearings and one gear mesh in the gearbox, the mechanical efficiency of the gearbox can be defined as

$$\eta = \frac{P_{in} - P_{mech}}{P_{in}} = \frac{P_{in} - 4P_{b, load} - P_{g, slide} - P_{g, roll}}{P_{in}}. \quad (6.7)$$

Here, $P_{in} = L_{in}\omega_{in}$, where $L_{in}$ is the input torque applied to the closed loop through the split coupling in Nm and $\omega_{in}$ is the rotational speed of the input gear in rad/s.

### 6.4 Comparison of Measured and Predicted Mechanical Efficiency Values

As discussed in previous section, in the efficiency measurements, for both test gears, the load was varied from 135 Nm to 677 Nm, and the speed was varied from 2,000 to 10,000 rpm. Same conditions are used in the simulation as well. For the 40-tooth gear pair, these operating conditions resulted a maximum Hertzian pressure ranging from 0.47 GPa (at 135Nm for wide face width) to 2.0 GPa (at 677 Nm for narrow face width), slide-
to-roll ratio (SR) ranging from –0.3 to 0.3, and entraining velocity ranging from 4.7 m/s (at 2000 rpm) to 22.8 m/s (at 10,000 rpm). For the 23-tooth gear pair, these test conditions resulted a maximum Hertzian pressure ranging from 0.51 GPa (at 135 Nm for wide face width) to 2.1 GPa (at 677 Nm for narrow face width), slide-to-roll ratio (SR) ranging from -0.97 to 0.99, and entraining velocity ranging from 4.02 m/s (at 2,000 rpm) to 20.1 m/s (at 10,000 rpm). These conditions were basically covered in the matrix of the EHL predictions as discussed in Chapter 3. In addition, the lubricant used in the spur gear tests (75W90) is the same as the one used in the EHL regression. Therefore, the EHL based \( \mu \) formula given by Eq. (3.21) is suitable for all these test conditions.

The test gears have measured surface roughness values in the range from 0.2 to 0.5 \( \mu \)m (rms). An example roughness profile measured by using a surface profiler is shown in Figure 6.4(a). In the simulation, an average rms surface roughness value of 0.3 \( \mu \)m is considered.

The predicted and measured mechanical efficiency values for both the 40-tooth and 23-tooth gears are compared in Figure 6.5 at 6,000 rpm and various \( L_{in} \) values. In Figure 6.5(a) for wide face width gears \( (FW = 26.7 \text{ mm}) \), several observations can be made:

(i) The predicted and measured \( \bar{\eta} \) values agree very well for both 23- and 40-tooth gear pair regardless of the load. The maximum difference between the
Figure 6.4. Typical measured surface roughness profiles for the two test gears, (a) an uncoated test gear with $S_{rms} = 0.3 \, \mu m$, and (b) a coated test gear with $S_{rms} = 0.1 \, \mu m$. 
Figure 6.5. Comparison of predicted and measured $\bar{\eta}$ values of 23-tooth and 40-tooth gears sets at 6000 rpm and a range of input torque; (a) wide face width gears (26.7 mm), (b) medium face width gears (19.5 mm), and (c) narrow face width gears (14.2 mm).
Figure 6.5 continued.
Figure 6.5 continued.

![Graph depicting the comparison of measured and predicted efficiencies for different input torques (40T and 23T). The graph shows a nearly horizontal trend with slight decreases as the input torque increases. The measured and predicted values are indicated by different markers, with measured efficiencies represented by filled hexagons and triangles, and predicted efficiencies by unfilled circles and squares. The x-axis represents the input torque (Nm), while the y-axis represents the efficiency (η) in percent.]
predicted and measured mechanical efficiency values is less than 0.1 percent for all tests represented in this figure. This indicates that not only the gear mechanical efficiency model proposed in Chapter 3 but also the published bearing efficiency formulae are quite accurate.

(ii) There is a sizable difference in mechanical efficiency values of 23-tooth and 40-tooth gear systems as substantiated by both the predictions and measurements. For this speed value of 6,000 rpm, the $\eta$ values of the 23-tooth gearbox are around 99.5 percent while it is nearly 99.75 percent for the 40-tooth design. While this 0.25 percent difference might look rather small, it is indeed quite significant. For instance, in a gear train that uses four gear meshes like these, one would obtain overall gear train mechanical efficiency values of $(0.995)^4 = 0.98$ and $(0.9975)^4 = 0.99$ by using coarse and fine-pitch designs, respectively. This one percent difference in efficiency is obviously very significant.

(iii) The load seems to have very little influence on the mechanical efficiency of the gearbox. A very slight increase in $\overline{\eta}$ is observed with increased load.

These three observation made for wide face width gears are also valid for the other two face widths as shown in Figures 6.5(b) and 6.5(c) for $FW = 19.5$ and 14.2 mm,
respectively. The predicted $\bar{\eta}$ values are mostly within 0.1 percent deviation from the measured ones.

The same level of agreement between the measured and predicted $\bar{\eta}$ are demonstrated in Figures 6.6 as well at $L_{in} = 406$ N-m and various rotational speed conditions. Here, Figures 6.6(a), 6.6(b) and 6.6(c) show the comparisons for wide, medium and narrow face width gears, respectively. Again, the 40-tooth design is consistently more efficient than the 23-tooth design, and the model and experiments agree well. The influence of speed on $\bar{\eta}$ is slightly more obvious compared to the influence of load. An increase in speed increases $\bar{\eta}$ slightly.

One last comparison is provided here between measurements and predictions using 40-tooth gear pairs that are coated with DLC-B as described in Chapter 3 as part of the traction tests. Since the EHL model in hand cannot simulate the contact between two coated surfaces, the EHL based $\mu$ formula (Eq. 3.21) is not applicable here. Therefore, in Figure 6.7, the measured mechanical efficiency values are compared with the $\bar{\eta}$ values predicted by using the experimental $\mu$ formula that was obtained in Chapter 3 based on the measured traction data using the same coating (Eq. 3.18 case 2, DLC-B running against DLC-B). The measured surface roughness values of the coated gears were around 0.1 $\mu$m (rms) as illustrated in Figure 6.4(b), in agreement with the roughness values of traction specimens. It is evident in Figure 6.7 that the differences between the
Figure 6.6. Comparison of predicted and measured $\eta$ values of 23-tooth and 40-tooth gears sets at 406 N-m and a range of rotational speed; (a) wide face width gears (26.7 mm), (b) medium face width gears (19.5 mm), and (c) narrow face width gears (14.2 mm).
Figure 6.6 continued.
Figure 6.6 continued.
Figure 6.7. Comparison predicted and measured $\bar{\eta}$ values of a 40-tooth gear pair coated with DLC-B at (a) 6000 rpm and various input torque values, and (b) 406 Nm and various rotational speed values.
predicted and measured $\bar{\eta}$ values are mostly within 0.05 percent for all speed and load conditions considered, further suggesting that the efficiency model is indeed accurate.

6.5 Conclusion

The mechanical efficiency model proposed in this study was applied to simulate the high-speed spur gear efficiency measurements. The $\bar{\eta}$ values predicted by the efficiency model that uses the $\mu$ formula based on the EHL model were shown to be within 0.1 percent difference from the measured $\bar{\eta}$ values. This indicates that the efficiency model proposed is sufficiently accurate for most engineering applications. Similarly the model is capable of predicting efficiency of the gears having engineered surface treatments such as coatings, provided that measured $\mu$ formulae are available to represent the contact conditions defined by such treatments.
CHAPTER 7
PARAMETRIC STUDIES

7.1 Introduction

In previous chapters, the mechanical efficiency prediction methodology was proposed and applied to both parallel-axis and cross-axis gear pairs. It was shown in Chapter 3 that the design parameters, operating conditions, lubricant properties, as well as surface finish characteristics are all influential factors that determine the friction coefficient at a given lubricated sliding contact interface. The same parameters are therefore expected to influence the mechanical efficiency of a gear pair as well. Such influences can be studied both qualitatively and quantitatively through carefully designed parametric studies.

As mentioned in previous chapters, running the actual thermal EHL model requires a significant computational effort, thus not practical to perform parametric study with it. The efficiency model that uses the EHL-based friction coefficient formula (Eq. (3.21)) is capable of accounting for all of the key parameters and has been validated.
through comparisons to spur gear efficiency measurements in Chapter 6. Therefore, the parametric studies presented in this chapter will employ gear pair efficiency models that use Eq. (3.21) as the $\mu$ predictor.

For the parametric studies on different types of surface treatments and lubricants, Eq. (3.21) is not applicable since it is valid only for uncoated surfaces with 75W90 oil. In such cases, the experimental $\mu$ formulae listed in Tables 3.3 to 3.5 will be used in the efficiency model in place of Eq. (3.21). For the power losses associated with rolling friction, published empirical formula as in Eqs. (2.14) to (2.16) will be employed. Power losses due to gear windage, oil churning, and bearing frictions will not be included in this chapter. Therefore the conclusions reached as a result of the parametric studies are concerned only with friction related mechanical efficiency losses.

7.2 A Parametric Study on the Mechanical Efficiency of Parallel-Axis Gears

For the parametric study on the mechanical efficiency of the parallel-axis gears, an example helical gear pair whose basic parameters listed in Table 7.1 will be considered. This gear pair is the final drive reduction of a 6-speed front-wheel-drive manual transmission. The following parameters will be varied:
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>22</td>
<td>83</td>
</tr>
<tr>
<td>Normal module (mm)</td>
<td></td>
<td>2.34</td>
</tr>
<tr>
<td>Normal pressure angle (degree)</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>Normal helix angle (degree)</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Outside diameter (mm)</td>
<td>67.02</td>
<td>228.11</td>
</tr>
<tr>
<td>Root diameter (mm)</td>
<td>52.79</td>
<td>214.09</td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>34.8</td>
<td>34.8</td>
</tr>
<tr>
<td>Operating center distance (mm)</td>
<td></td>
<td>141.5</td>
</tr>
</tbody>
</table>

Table 7.1 Parameters of the example helical gear pair used in the parametric study.
Operating conditions: input (pinion) speed, $N_p$
input torque, $L_{in}$

Design parameters: normal helix angle, $\beta_n$
generating normal pressure angle, $\psi_n$
face contact ratio, $\varepsilon_\beta$
involute contact ratio, $\varepsilon_\alpha$

Tooth modification parameters: linear tip relief, $\xi_\alpha$
parabolic lead crown, $\xi_\beta$

Lubricant conditions: lubricant type
temperature, $T_{oil}$

Contact surface conditions: RMS surface roughness amplitude, $S$
thin-film coatings

Manufacturing and assemble errors: shaft misalignment, $\chi$
center distance shift, $\delta$
lead angle error, $\Delta\lambda$
pressure angle error, $\Delta\psi_n$

Influence of these variations on the average efficiency $\bar{\eta} = \frac{1}{M} \sum_{m=1}^{M} \eta(\phi_m)$ for the example helical gear pair are studied in this section. Figure 7.1 shows the variation of $\bar{\eta}$ as a function of $L_{in}$ for various values of $N_p$, given $S = 0.4 \mu m$ and $T_{oil} = 100^\circ C$. As observed here, a very slight reduction in $\bar{\eta}$ is observed with increased $L_{in}$. Likewise an
Figure 7.1. Influence of $L_{in}$ and pinion speed $N_p$ on $\bar{\eta}$; $S = 0.4 \ \mu m$ and $T_{oil} = 100^\circ C$. 

\[ pN = \eta \]
increase in $N_p$ increases $\bar{\eta}$ slightly. At $L_{in} = 400$ Nm, the difference between the $\bar{\eta}$ values corresponding to 1000 and 4000 rpm is nearly 0.05 percent, which happens to be the spread for all operating conditions considered.

The combined influence of the surface roughness parameter $S$ and the oil inlet temperature $T_{oil}$ is illustrated in Figure 7.2 for $L_{in} = 200$ Nm and $N_p = 2000$ rpm. Both $T_{oil}$ and $S$ have a significant influence on $\bar{\eta}$. For instance at $T_{oil} = 40^\circ C$, $\bar{\eta} = 99.2$ percent for a smooth surface while it is 98.3 percent for $S = 0.6 \ \mu m$. This is expected since the ratio of the film thickness to surface roughness (lambda ratio) is influenced by $S$. At the same roughness value of $S = 0.6 \ \mu m$, elevating lubricant temperature to $100^\circ C$ improves the efficiency from 98.3 percent to nearly 99.4 percent, partly due to the reduction in viscosity due to temperature increment [70,85].

Combined influence of two gear design parameters, normal helix angle $\beta_n$ and generating normal pressure angle $\psi_n$, on $\bar{\eta}$ is shown in Figures 7.3. Here, $\beta_n$ is varied between 30 to 33 degrees and $\psi_n$ is varied between 18 and 19.5 degrees while all other parameters remain unchanged. While $\bar{\eta}$ increases with an increase of $\psi_n$, it decreases rather significantly with an increase of $\beta_n$, which agrees with the helical gear efficiency measurements conducted by Haizuka et al [86]. The difference between the $\bar{\eta}$ values for $\beta_n = 30$ and 33 degrees is nearly 0.2 percent for $\psi_n = 18$ degrees. The combined influence of the other two design parameters considered, involute contact ratio $\varepsilon_{Ga}$ and
Figure 7.2. Influence of $S$ and $T_{oil}$ on $\bar{\eta}$; $L_{in} = 200 \text{ Nm}$ and $N_p = 2,000 \text{ rpm}$.
Figure 7.3. Influence of $\beta_n$ and $\psi_n$ on $\bar{\eta}$; $S = 0.4 \mu m$, $L_{in} = 200 Nm$, $N_p = 2,000 rpm$, and $T_{oil} = 100^\circ C$. 

$\beta_n$ [deg] 

$\psi_n = 19.5^\circ$ 

$18^\circ$ 

$\bar{\eta} \text{ [percent]}$
face contact ratio $\varepsilon_\beta$, on $\overline{\eta}$ are illustrated in Figure 7.4. Here the value of $\varepsilon_\alpha$ is varied
from 1.25 to 2 by changing the outside diameter of the pinion while the gear remained unchanged. Similarly, $\varepsilon_\beta$ is varied from 1 to 2.5 by changing the face width of both the
pinion and the gear. In Figure 7.4, the influence $\varepsilon_\beta$ is minimal perhaps since the load
applied is kept the same at $L_{in} = 200$ Nm. This agrees with the experimental data
presented in Chapter 6. Meanwhile, when the pinion addendum is enlarged to achieve a
higher $\varepsilon_\alpha$, larger sliding velocities are introduced with more significant efficiency
losses. The difference in $\overline{\eta}$ of gear pairs having $\varepsilon_\alpha = 1.25$ and 2.0 is nearly 0.1 percent,
the latter one being less efficient. An increase in $\varepsilon_\alpha$ is known to reduce vibration
amplitudes and noise of the gear set. Figure 7.4 suggests that any noise improvements
through increased $\varepsilon_\alpha$ could reduce the efficiency of the gear pair.

The influence of two key tooth surface modification parameters on $\overline{\eta}$ is shown in
Figure 7.5. A linear profile modification (tip relief) of magnitude $\xi_\alpha$ (starting at the
operating pitch point) is applied to the tips of both the pinion and the gear while a
parabolic lead crown $\xi_\beta$ is applied to the pinion only. In Figure 7.5, $\xi_\beta$ appears to have
a limited effect on $\overline{\eta}$. On the other hand, increasing $\xi_\alpha$ reduces the effective involute
contact ratio by removing material from the areas of the larger sliding velocities, and
hence, causes $\overline{\eta}$ to increase quite significantly. A gear pair having a tip relief of $\xi_\alpha = 15$
$\mu$m is nearly 0.2 percent more efficient than its unmodified counterpart.
Figure 7.4. Influence of $\varepsilon_\beta$ and $\varepsilon_\alpha$ on $\bar{\eta}$; $S = 0.4 \mu m$, $L_{in} = 200 Nm$, $N_p = 2,000$ rpm, and $T_{oil} = 100^\circ C$. 

\[
\eta = \beta \varepsilon_\alpha
\]
Figure 7.5. Influence of $\xi_\beta$ and $\xi_\alpha$ on $\bar{\eta}$; $S = 0.4 \, \mu m$, $L_{in} = 200 \, Nm$, $N_p = 2,000 \, rpm$, and $T_{oil} = 100^\circ C$. 
Figures 7.6 and 7.7 illustrate the influence of various manufacturing errors on \( \eta \) given \( L_{in} = 200 \) Nm, \( N_p = 2000 \) rpm, \( T_{oil} = 40^\circ C \), and \( S = 0.4 \) \( \mu \)m. In Figure 7.6, various amounts of pinion shaft misalignment error \( \chi \) and center distance shift error \( \delta \) are introduced. A positive \( \delta \) corresponds to a larger center distance and causes \( \eta \) to increase in \( \eta \) since the effective involute contact ratio is reduced. A misalignment slope is used to define the pinion shaft misalignment \( \chi \), which is the ratio of the misalignment magnitude to the pinion face width. The gear shaft is assumed to be perfectly aligned. With a pinion shaft misalignment, the size and location of the contact zone tends to reduce and shift toward one side of the flank along the lead direction, causing the contact pressure to increase to certain amount depending on the amount of misalignment. In Figure 7.6, it seems that \( \eta \) is minimum at perfectly aligned condition, while \( \eta \) increases more or less with the increasing of the amount of misalignment.

Influences of pressure angle and lead angle errors, \( \Delta \psi_n \) and \( \Delta \lambda \), are shown in Figure 7.7. Here neither one of these two errors appears to have any tangible influence on \( \eta \).

Three lubricants with properties listed in Table 7.2 are used to illustrate the influences of lubricant type on gear efficiency. Surface treatment combinations modeled for the gear pair listed in Table 7.3 include two uncoated surfaces as well as other combinations of coated (DLC-A and DLC-B) and uncoated surfaces. Two spur gear
Figure 7.6. Influence of $\chi$ and $\delta$ on $\bar{\eta}$; $S = 0.4 \mu m$, $L_{in} = 200 \ Nm$, $N_p = 2,000 \ rpm$, and $T_{oil} = 100^\circ C$. 
Figure 7.7. Influence of $\Delta \lambda$ and $\Delta \psi_n$ on $\bar{\eta}$; $S = 0.4 \, \mu m$, $L_{in} = 200 \, Nm$, $N_p = 2,000 \, rpm$, and $T_{oil} = 100^\circ C$. 
Density, $g/cc$

$\rho = a + b \times (T_{inlet} - 40)$

Kinematic viscosity, $cSt$

$\nu_k = Z - \exp\left(-0.7487 - 3.295 \times Z + 0.6119 \times Z^2 - 0.3193 \times Z^3\right)$

$Z = \xi - 0.7$

$log_{10}\left[log_{10}(\xi)\right] = A - B \times \log_{10}(T_{inlet} + 273.15)$

Absolute viscosity, $cPs$

$\nu = \rho \times \nu_k$

Pressure-viscosity coefficient, $1/GPa$

$\alpha = \frac{E}{\left(\frac{G}{(T_{inlet} + 273.15)} - 1\right) \times (1 + D \times T_{inlet})^2}$

where

<table>
<thead>
<tr>
<th>Lubricant</th>
<th>Density</th>
<th>Kinematic viscosity</th>
<th>Pressure-viscosity coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
<td>$A$</td>
</tr>
<tr>
<td>75W90</td>
<td>0.8353</td>
<td>-0.0006</td>
<td>7.9737</td>
</tr>
<tr>
<td>Lubricant A</td>
<td>0.8253</td>
<td>-0.0006</td>
<td>7.7572</td>
</tr>
<tr>
<td>Lubricant B</td>
<td>0.8453</td>
<td>-0.0006</td>
<td>9.0178</td>
</tr>
</tbody>
</table>

Table 7.2 Lubricant viscosity constants used in the parametric study.
<table>
<thead>
<tr>
<th>Case #</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Uncoated</td>
<td>Uncoated</td>
</tr>
<tr>
<td>2</td>
<td>DLC type B</td>
<td>DLC type B</td>
</tr>
<tr>
<td>3</td>
<td>DLC type A</td>
<td>DLC type A</td>
</tr>
<tr>
<td>4</td>
<td>DLC type A</td>
<td>Uncoated</td>
</tr>
<tr>
<td>5</td>
<td>DLC type B</td>
<td>Uncoated</td>
</tr>
</tbody>
</table>

**Table 7.3** Surface treatment combinations considered in the parametric study.
pairs used in Chapter 6 for validation of the efficiency model, 23-tooth and 40-tooth spur gear pair with the same face width of 26.67 mm, as listed in Table 6.1 are analyzed with these surface treatment combinations. In Figures 7.8 and 7.9, the predicted power loss values of the 23-tooth and 40-tooth gears for the cases listed in Table 7.3 are compared for 75W90 oils as well as the other two alternative lubricants (called lubricant A and Lubricant B in Chapter 3). Overall trend from these figures is that the uncoated-uncoated gear pairs as well as coated-uncoated ones have the largest fictional power losses. Pairs formed by two coated gears yield lower power loss values. Overall, gears coated with DLC-B type coating have the lowest friction losses regardless of the type of lubricant they use. This is in direct agreement with the traction data presented in Chapter 3. It is also evident from Figures 7.8 and 7.9 that lubricant A is the best lubricant among the three in terms of friction power losses and the 40-tooth gear pair has significantly less power losses in agreement with the results presented in Chapter 6. The corresponding $\eta$ values for the same 23-tooth and 40-tooth gear pairs are shown in Figures 7.10 and 7.11, respectively. Here, the trends in these figures are opposite of those present in Figures 7.8 and 7.9 since higher power losses amount to lower $\eta$ values. Here, an uncoated 23-tooth gear pair running in 75W90 oil has an efficiency of $\eta = 99.64$ percent. This gear pair can be replaced by a DLC-B coated 23-tooth gear pair to increase the efficiency to 99.77 percent. Further changing the oil to lubricant A improves the mechanical efficiency of the same DLC-B coated gear pair to $\eta = 99.84$ percent. In addition, if the gear design is changed to a 40-tooth one with the same DLC-B coatings and lubricant A, the efficiency improves to 99.92 percent that amounts to a nearly 0.3 percent efficiency improvement.
Figure 7.8. Comparison of frictional power losses of the 23-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 \, Nm$, $N_p = 6,000 \, rpm$, and $T_{oil} = 100^\circ C$. 
Figure 7.9. Comparison of frictional power losses of the 40-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 \, Nm$, $N_p = 6,000 \, rpm$, and $T_{oil} = 100^\circ C$. 
Figure 7.10. Comparison of $\bar{\eta}$ of the 23-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 \ Nm$, $N_p = 6,000 \ rpm$, and $T_{oil} = 100^\circ C$. 
Figure 7.11. Comparison of $\bar{\eta}$ of the 40-tooth gears for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 500 Nm$, $N_p = 6,000 rpm$, and $T_{oil} = 100^\circ C$. 
over the uncoated 23-tooth gear pair. This is a good illustration of the combined influence of lubricant, surface treatments and gear design parameters on $\eta$.

### 7.3 A Parametric Study on the Mechanical Efficiency of Cross-Axis Gears

For the parametric study on the mechanical efficiency of the cross-axis gears, an example hypoid gear pair whose basic blank dimensions, cutter and machine setup parameters listed in Tables 5.1-5.3 will be considered. Unlike parallel axis gears, the geometry related parameters of hypoid gears cannot be varied directly as they are determined by the cutter and machine setup parameters. Therefore, the parametric studies using this gear pair will be limited to the following parameters.

- **Operating conditions:** input (pinion) speed, $N_p$
  - input torque, $L_{in}$

- **Lubricant conditions:** lubricant type
  - temperature, $T_{oil}$

- **Contact surface conditions:** RMS surface roughness amplitude, $S$
  - thin-film coatings

- **Assemble errors:** displacement in pinion axial direction, $\Delta H$
  - pinion offset displacement, $\Delta V$
  - pinion displacement along gear axis, $\Delta R$
  - shaft angle error, $\Delta \beta$
Figure 7.12 shows the variation of $\bar{\eta}$ as a function of $N_p$ for $L_{in} = 500$ and 1000 Nm, given $S = 0.4 \ \mu m$ and $T_{oil} = 60^\circ C$. $L_{in}$ has a negligible influence on $\bar{\eta}$ within this load range and curves for $L_{in} = 500$ and 1000 Nm are almost identical. However, $\bar{\eta}$ increases quite significantly with an increase in $N_p$. The mechanical efficiency values at $N_p = 500$ and 2000 rpm differ by nearly 2 percent, which is due to very small rolling velocities at low speed that cause the friction coefficient increase significantly.

The combined influence of the surface roughness parameter $S$ and the oil inlet temperature $T_{oil}$ is illustrated in Figure 7.13 for $L_{in} = 1600$ Nm and $N_p = 1000$ rpm. Similar to the findings for parallel axis gears in Section 7.2, both $T_{oil}$ and $S$ have a significant influence on $\bar{\eta}$. For instance at $T_{oil} = 60^\circ C$, $\bar{\eta} = 98.3$ percent for a smooth surface while it is 96.3 percent for $S = 0.6 \ \mu m$. At the same roughness value of $S = 0.6 \ \mu m$, elevating lubricant temperature to $100^\circ C$ improves the efficiency from 96.3 percent to nearly 97.5 percent, partly due to the reduction in viscosity due to temperature increment [70,85].

Influence of hypoid gear assembly tolerances or errors, $\Delta H$, $\Delta V$, $\Delta R$, and $\Delta \beta$ that are defined in Figure 7.14 on $\bar{\eta}$ are shown in Figure 7.15. In Figure 7.14, $\Delta H$, $\Delta V$, and $\Delta R$ are all varied from $-0.5 \ mm$ to $+0.5 \ mm$ and $\Delta \beta$ is varied from $-0.1$ degree to
Figure 7.12. Influence of $L_{in}$ and pinion speed $N_p$ on $\bar{\eta}$; $S = 0.4 \mu m$ and $T_{oil} = 60^\circ C$. 
Figure 7.13. Influence of $S$ and $T_{oil}$ on $\bar{\eta}$; $L_{in} = 1,600 Nm$ and $N_p = 1,000 rpm$. 
Figure 7.14. Example hypoid gear pair geometry and illustration of errors, $\Delta V$: pinion movement along offset, $\Delta H$: pinion axial movement, $\Delta R$: pinion movement along gear axis, $\Delta \beta$: Shaft angle error.
Figure 7.15 Influence of errors (a) $\Delta H$, (b) $\Delta V$, (c) $\Delta R$, and (d) $\Delta \beta$ on $\bar{\eta}$. $S = 0.4 \mu m$, $L_{in} = 1,600 Nm$, $N_p = 1,000 \text{ rpm}$ and $T_{oil} = 60^\circ C$. 
Figure 7.15 continued.
+0.1 degrees. From Figure 7.15, it is found that shaft angle error $\Delta \beta$ has almost no influence on $\eta$, while influence of other three errors on $\eta$ is very limited.

In Figure 7.16, $\eta$ is evaluated for four different hypoid gear pairs whose basic parameters are outlined in Table 7.4. Gear set #1 is face milled while the all other three gear sets are face hobbed. Gear set #3 has a gear ratio of 3.42 while all other three gear sets have a 3.73 ratio. Gear set #1 has the smallest pinion shaft offset, while the gear set #4 has the largest offset. From Figure 7.16, the highest efficiency values are predicted for the gear set #1 for $S = 0.4 \, \mu m$, $L_{in} = 800 \, Nm$, $N_{p} = 1000 \, rpm$, and $T_{oil} = 50^\circ C$. The $\eta$ value for gear set #1 is about 1.5 percent higher than that of gear set #4, which has the largest offset. Similarly, $\eta$ value of gear set #1 is nearly one percent higher than those of gear sets #2 and #3, which have a medium offset. It is also noted that gear sets #2 and #3 have the same amount of pinion offset and very similar gear ratios, which yield very close $\eta$ values. As all these designs have similar blank dimensions, gear ratios, and spiral angles, the difference in efficiency is most likely due to the differences in shaft offset. As the offset increases, the efficiency decreases due to increases in sliding between the gear teeth in mesh.

Combined influence of surface treatments and lubricant type on hypoid gear efficiency is shown in Figure 7.17. Similar to parallel-axis gears, both coatings and lubricant type tend to influence the hypoid gear efficiency significantly. For example, the example uncoated hypoid gear pair has a mechanical efficiency of 95.8 with lubricant B,
which can be increased to 97.4 percent if lubricant A is used instead. When both gears are coated with DLC-B with lubricant A, the efficiency of this gear pair improves further to 97.7. This is a 1.9 percent improvement over the same gear pair operating in lubricant B with no coatings.
<table>
<thead>
<tr>
<th>Gear Set</th>
<th>Manufacturing Process</th>
<th>Gear Ratio</th>
<th>Offset (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FM</td>
<td>11x41</td>
<td>19.05</td>
</tr>
<tr>
<td>2</td>
<td>FH</td>
<td>11x41</td>
<td>44.45</td>
</tr>
<tr>
<td>3</td>
<td>FH</td>
<td>12x41</td>
<td>44.45</td>
</tr>
<tr>
<td>4</td>
<td>FH</td>
<td>11x41</td>
<td>56.99</td>
</tr>
</tbody>
</table>

**Table 7.4** Hypoid gear set basic parameters for the design comparison.
Figure 7.16. Comparison of $\bar{\eta}$ values of four gear sets listed in Table 7.4. $L_{in} = 800 Nm$, $N_p = 1,000 \ rpm$, $S = 0.4 \ \mu m$, and $T_{oil} = 50^\circ C$.
Figure 7.17. Comparison of $\bar{\eta}$ of the example hypoid gear pair for different lubricants and surface treatments; $S = 0.1 \mu m$, $L_{in} = 1000 Nm$, $N_p = 1000 \text{ rpm}$, and $T_{oil} = 100^\circ C$. 

219
CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

In this study, a general-purpose model for the prediction of mechanical efficiency of gear pairs has been developed. This model has been applied to both parallel-axis and cross-axis gear pairs. As the gear efficiency methodology proposed here is comprehensive and general, it has the potential to form a foundation for future modeling of efficiency of other types of gear drives.

The accuracy of the published friction coefficient formulae has been assessed by comparing them to the EHL-based $\mu$ values and the measured traction data. While $\mu$ values from the EHL analysis agreed well with the measured traction data, the published formulae failed to do so. This provides clear insight that the $\mu$ formulae developed by others using lubricants, geometries, surface characteristics and operating conditions reflecting an application other than gears might not provide accurate friction information to a gear efficiency model. This is an important point because some of these $\mu$ formulae,
especially the one by Benedict and Kelley, has been used extensively for efficiency models without any caution.

The regression analysis developed here to obtain new $\mu$ formulae based on actual traction measurements and EHL runs has proven to be very effective. These fitted formulae compared very well to the original data or simulations in more than 30 regression analyses. This is also significant since it allows reduction of experimental data and EHL analyses to simple closed-form $\mu$ formulae. This way, efficiency analyses of gears can be performed effectively in a relatively short period of time. This makes the gear pair efficiency models suitable as common design tools rather than being tedious and difficult-to-use analysis tools. This gear efficiency model should allow gear efficiency to be used as a design criterion in addition to other noise and durability related criteria when gears are being designed. Currently, gear efficiency is treated as a side effect that must be dealt with after the design is finalized.

The effectiveness of the efficiency models in describing the real-life gear applications is also commendable. The spur gear efficiency prediction using the EHL-based $\mu$ formula consistently predicted values within 0.1 percent of the actual measured values for more than 50 cases covering different gear designs and surface parameters.

Influence of basic gear geometric parameters, tooth modifications, operating conditions, surface finish and treatments, lubricant properties, manufacturing and
assembly errors on mechanical efficiency of parallel-axis gears have been studied in this project to identify key parameters influencing gear efficiency. Table 8.1 provides a qualitative account of how most of the relevant parameters influence the mechanical efficiency of a parallel-axis gear pair. Here, these parameters are categorized in three groups, \textit{influences significantly}, \textit{influences slightly}, and \textit{no influence}. An upward arrow means that an increase of this particular parameter causes an increase in mechanical efficiency of the gear pair while the opposite is true for a downward arrow. Based on the parametric studies performed in this study, a variation in surface roughness, lubricant viscosity, temperature, helix angle, pressure angle, module, involute contact ratio, tip relief, coatings, as well as center distance shift is observed to influence the gear mechanical efficiency significantly. It is worth to mention that an effort to lower gear noise through increasing the involute contact ratio was found to reduce the gear mechanical efficiency significantly. Speed, and shaft misalignment only influence the gear mechanical efficiency slightly, while load, manufacturing errors, lead crown, face width, and face contact ratio have very limited influence or no influence at all.

Influence of operating conditions, surface finish and treatments, lubricant properties, assembly errors, and design variations on the efficiency of hypoid gears have also been quantified. While hypoid gear pair is defined by a very large number of parameters, extensive generic analyses were not possible to obtain information about the influence of each parameter on the mechanical efficiency of the hypoid gear pair, still several consistent trend were observed, some of which are listed in Table 8.2 in a similar
<table>
<thead>
<tr>
<th>Parameters</th>
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<th>Influences slightly</th>
<th>No influence</th>
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<tr>
<td>Lubricant viscosity</td>
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<tr>
<td>Helix angle</td>
<td>$\beta_n$</td>
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<td>Module</td>
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<td>Face contact ratio</td>
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Table 8.1 Influence of an increase in different parameter values on $\eta$ of a parallel-axis gear pair.

223
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Influences significantly</th>
<th>Influences slightly</th>
<th>No influence</th>
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**Table 8.2** Influence of an increase in different parameter values on $\eta$ of a hypoid gear pair.
fashion as Table 8.1. This table states that the rotational speed, the shaft offset, surface roughness amplitude, lubricant viscosity and temperature, and coatings are all very influential on the mechanical efficiency of a hypoid gear pair. Meanwhile, the parameters such as load and assemble errors have a limited or no influence on hypoid efficiency.

8.2 Recommendations for Future Work

The following can be listed as related further research areas.

(i) While this study provided a validated mechanical efficiency model for parallel-axis gear pairs, no direct validation of hypoid gear pairs was attempted. An experimental test study is needed for validation of the hypoid efficiency model predictions.

(ii) This study focused mostly on the sliding and rolling friction related efficiency losses. Load independent losses due to windage and oil churning were not included in this study. Yet, a lightly loaded, high-speed gear pair might experience load independent losses as high as the friction related losses. A theoretical and experimental investigation of gear windage and oil churning/squeezing losses is needed. Such a study, combined with the frictional efficiency model proposed in this study would have the capability of predicting the total efficiency of gearboxes and axle units.
(iii) In order to be able to predict the efficiency losses of automatic transmissions, the efficiency methodology presented here should be extended to handle internal gear meshes, full-complement needle bearings and washer as well as entire planetary gear trains under complex power flow and duty cycle conditions.

(iv) The EHL model used in this study was two-dimensional allowing only line contact conditions. This EHL model must be modified to a three-dimensional one to handle point contact situations. The EHL model must also be modified to handle more excessive asperity-contact conditions that are quite common in passenger car and agricultural vehicle transmissions.

(v) The traction coefficient database must be expanded to include other common lubricants, surface roughness profiles and coatings. EHL-based regression analyses of the same conditions would provide new $\mu$ formulae allowing a better quantification of these effects.

(vi) As needed, this efficiency methodology can be applied to other types of gears such as worm gears, cross-axis helical gears, face gears, and straight bevel gears.


[41] ISO TC 60, DTR 13989


