COORDINATED CONTROL OF THE TURBO ELECTRICALLY ASSISTED VARIABLE GEOMETRY TURBOCHARGED DIESEL ENGINE WITH EXHAUST GAS RECIRCULATION.

DISSERTATION
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ABSTRACT

This work investigates the improvements in NO\textsubscript{x} emissions for a variable geometry turbo (VGT) charged diesel engine with exhaust gas recirculation (EGR) when electric assist is applied to the turbo-compressor. The method relies on selecting a fixed air to fuel ratio (AFR) above the no visible smoke limit that and regulating to that AFR. Simulated NO\textsubscript{x} emission results for the electrically assisted VGT engine w/ no EGR are first compared to a conventional VGT-EGR diesel engine. Then EGR control is added to the electrically assisted system and its influence on the electric assist system is studied.

The control problem related to the complex gas exchange process in a Variable Geometry Turbocharged (VGT) diesel engine with Exhaust Gas Recirculation (EGR) has been proposed. The underlying assumptions regarding the sensor set to be used, however, are often not aligned with production intent goals for industrial applications. One such assumption is the availability of an exhaust gas pressure sensor. The exhaust gas pressure measurement is essential for the prediction of the flow rates over the EGR valve and the VGT vane. Additionally assumptions are often made regarding the availability of the air mass fraction in the intake and/or exhaust gas mixtures through a wide band oxygen sensor, commercially known as the Universal Exhaust Gas Oxygen (UEGO) sensor. While the appropriate sensors do exist, cost and reliability issues often force engine manufacturers and OEM’s to preclude one or more of these sensors from the production intent sensor set. It therefore becomes essential to find alternate means of
predicting these state variables and control inputs. In order to circumvent the assumption that all the state variables and control inputs are available, observers for the exhaust manifold pressure and the air mass fractions in the intake, and exhaust manifolds are proposed. A mean value diesel engine model is used and the performance of the observers is validated against data from a 2.4L Fiat VGT-EGR diesel engine.
Dedicated to:

My parents and the Glenn engineering legacy
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CHAPTER 1

INTRODUCTION

As Diesel engines become more accepted amongst automakers, two major emission challenges common to all Diesel engines remain at the center of research activity, Nitrogen Oxides (NO\textsubscript{x}) and Particulate Matter (PM). Advanced techniques such as high pressure common rail fuel injection combined with multiple injections per cycle are commonly used to minimize in-cylinder production of NO\textsubscript{x} and PM. However, to meet the Environmental Protection Agency (EPA) and California Air Resources Board (CARB) mandated standards it is essential that further improvements are made. Additionally if the diesel engine is to find acceptance amongst the US passenger car users it is essential that it shakes off the image of a sluggish machine by performing at the level of the Spark Ignition (SI) engine in terms of rapid acceleration response. Technically this amounts to either reducing turbo-Lag to an indiscernible level or eliminating it completely. For a diesel engine it takes roughly 20,000 Liters of air for each liter of fuel. For an SI engine it takes roughly 9,000 Liters of air for each liter of fuel used [16]. Therefore it is obvious that engine power is not just determined by displacement but also by breathing capability, which is greatly improved with the addition of a turbocharger.
The turbocharger (T/C) compressor, a rotary machine, and the internal combustion engine (ICE), a volumetric machine, are difficult to match. A compressor coupled to engine for mechanical supercharging maintains a boost pressure corresponding to the speed of the engine. Hence the speed of the compressor does not lag the speed of the engine, and the boost availability although lagging is imperceptible. Of course superchargers must be sized appropriately to best support engine performance in a narrow operating regime. The exhaust gas T/C on the other hand is coupled to the engine through the exhaust flow over the turbine. Hence, apart from T/C inertia effects, its speed and boost pressure generation depend on the engine operating condition rather than engine speed. At idle speeds only low boost pressures are available from an exhaust gas T/C spinning at low speeds. Large power demand by the engine will have to be supported by substantial airflow rates from the T/C compressor. However, before the desired airflow rate is available to the engine the T/C must first be accelerated to a higher speed by the exhaust gas, hence if the load transition is from low load (turbo spinning slow) to high load (large air demand) the air supply will lag the power demand. This is referred to as turbo-lag. Being mechanically decoupled from the engine turbo-lag implies that a T/C will accelerate or decelerate out of phase with the engine; hence turbo-lag will affect the response of the engine itself. It is obvious that the T/C inertia has a direct effect on turbo-lag; since turbo-lag is directly linked to breathing capability; several techniques have been implemented over the years to reduce this effect.

One of the simplest and most commonly used techniques to improve the response of the diesel engine for limited operating ranges only to swift increases in load demand is the use of a small capacity turbine housing. The restricted flow area increases the
pressure drop across the turbine, thus increasing the turbine torque needed for acceleration. However the smaller capacity turbine limits the engine power at higher speeds. If the maximum allowable pressure ratios are exceeded, a waste gate is employed [1], thus increasing fuel consumption over a wide range of engine operating points. This and the fact that waste-gating is an inherently inefficient process forced researchers to search for alternate means of improving T/C performance.

Improved dynamic response of the engine can also be achieved by the introduction of a variable geometry turbine (VGT) of the vane passages from the scroll to the rotor. Conventional, fixed Geometry turbochargers have a limited operating range; such turbochargers are not made, they are still being used designed for either low-end operation or high-end operation. Hence the turbine will be sized either for low load response or for high-load. The introduction of the VGT allowed a dynamic sizing of the turbine making the turbocharger compatible to a much wider engine operating range.

Variable geometry allows control over the effective flow area of the exhaust gas through external actuation. Hence a turbocharger can be appropriately sized for faster response at low engine speeds and greater power output at higher engine speeds. Two types of VGT’s are commonly used, variable nozzle and variable area. In the variable area design the flow area available to the exhaust is varied by a plate much like a conventional throttle. A variable area turbine on the other hand varies the angle of attack of the exhaust gas by varying the angle of the turbine nozzles. A third method is to divide the exhaust inlet area into two or more orifices and vary the flow volume to these orifices [1].
Additionally, to increase the boost pressure in the intake manifold during an engine transient external compressed air may supplied. Depending on the boost requirement, the external compressed air can be supplied in several ways: into the intake manifold, into the inlet of the compressor, into the exhaust manifold or into a separate turbine nozzle, or directly into the cylinder. However large quantities of compressed air are required for this method [1].

The notion of electrically assisting the T/C is not new; its benefits have long been established in theory. Developments in high-speed Permanent Magnet Brushless DC machines and the introduction of modular designs with the DC machine integrated into the T/C housing [2-8] have made electrical assist of T/C a realistic candidate for turbo-lag reduction/elimination. Taking advantage of the fact that the DC machine can in fact act both in the motor (assist) mode and in the generator (regenerative) mode, an optimized control strategy would seek to engage the electrical assist during start up and transients and switch to regenerative mode during high load and speed engine operation. In this mode the Electrical Machine (EM) is in essence functioning as an “efficient” waste-gate.

Obviously the load torque, $T_{load}$ applied by the EM is the make up torque to be supplied to compensate for the lack of power available to the compressor during the turbo-lag period. Apart from eliminating Turbo-lag and providing the possibility of improving overall engine performance (Torque and Power); electrical assist strategy also results in the following benefits:

- Improved drivability
- Possibility of lower emissions and opacity
- Improved specific fuel consumption
• Improved turbo matching
• Potential for improved cold start
• Potential of enhanced engine braking with control of boost
• Improved management of EGR flow

Electric motor technology is established enough to allow for operation at very high speeds as required by the integrated turbo electric assist systems. Still the motor technology has not been developed to the point where it is readily commercially available. The integrated electric turbo system is shown in Figure 1.1. The additional electric subsystems required for the system make it quite a difficult engineering task to produce a system that meets design and cost specifications.
Several automotive companies such as Honeywell, Inc., Caterpillar Inc., and TurboDyne Inc. are currently developing prototypes[2,3,4,5] for such systems. Figure 1.2 depicts a prototype in development by Garret Engine Boosting Systems of Honeywell Inc. The system consists of a 12 Volt DC 2 kW Permanent Magnet Synchronous Motor/Generator (PMSM) coupled to the T/C shaft. Caterpillar has developed a similar 340V, 40kW nominal, 60kW peak Permanent Magnet Synchronous Motor (PMSM) prototype depicted in figure 1.3. However little information is available in the open literature about these prototypes since they are in the development stage and are
proprietary. The PMSM exhibits superior efficiencies at the higher rotational speeds when compared to induction motors. The advantage of using an induction motor is their cost and there is no need to be concerned with the demagnetization observed in the PMSM, caused by the high operating temperatures of the motor. Therefore there is a tradeoff depending on the specifications of the overall system for which the electrically assisted turbo is being designed.

Figure 1.2 Honeywell prototype EAT GT25V
1.1 SYSTEM MODELING

Before beginning the development of any model, it is important to understand the desired system outputs. For a variety of control purposes, a state space representation of the system is typically used. The states chosen are those that can provide the insight necessary to evaluate system response to a certain control input. In the case of this work, the effects of the Exhaust Gas Recirculation (EGR) and VGT actuator settings, as well as the Turbo Electric Assist (TEA) on the engine are desired. The engine response to these
inputs is influenced by the thermodynamic characteristics and compositions of the intake and exhaust gas flows. For this reason the intake and exhaust manifolds mass, pressure, temperature, fresh air composition, and the compressor power of the turbocharger should be readily available in the model.

Traditionally, engine researchers have felt that in order to accurately represent the dynamics of a Compression-Ignition, Direct-Injection, CIDI engine complex models of the combustion, mixture, and other thermodynamic processes were required. However, it is essential that the modeling process be motivated by the specific goals of the desired control system design. An exhaustive and very accurate model is not necessarily the best because it may not be useful in attaining the intended control goals effectively. While the CIDI engine system is in fact very complicated, models needed for feedback control design of the VGT-EGR flow regulation problem need not be as equally complex.

Mean value models of diesel engines are a combination of quasi-steady state models and filling and emptying models. They have been used by many researchers for feedback control development [6-10, 12, 15, 19]. For control system design purposes the model need only characterize the dominant dynamics, while keeping the interrelationships among the measured variables consistent with the physical processes. Usually this implies a frequency range of less than 10Hz. This allows the model to be fast enough to accurately predict the overall dynamic behavior of the engine, without attempting to capture the sub-cycle events of the combustion process. The model assumes average pressure, temperature, and mass flows in the sub models. Transient performance data is then used to fit and validate the parameter constants of the sub model equations.
The fact that engine combustion processes can be modeled in a simple empirical fashion with relatively high accuracy ensures the CIDI engine is conducive to the mean value modeling approach. The system characterization is limited to using lumped parameter techniques for the dynamics of the intake, exhaust, turbo, and EGR loop coupled by empirical formulations to characterize the operation of the combustion processes in terms of torque production, temperature rise, and mean value NO$_x$ production.

As previously stated, it is important to identify the effects of the EGR and VGT actuator settings on the engine, more specifically the amount of NO$_x$ and unburned hydrocarbons produced in the exhaust. The effect of exhaust gas recirculation on the NO$_x$ is well known [16,17]. Its typical influence is presented in figure 1.3. The increase in percentage exhaust gas recirculation reduced the NO emissions for the entire brake mean effective pressure range. Its effect on increasing HC was significant near full load. Therefore there is a tradeoff between the amount of NO and HC produced as EGR is increased.
Figure 1.4 Effect of exhaust gas recirculation on the emissions of a diesel engine. (From Reference [17])

The result of the VGT setting can be seen through the effect of air to fuel ratio (AFR) on emissions. Figure 1.4 shows the effect of fuel-air ratio on NO concentration in a diesel engine [16,17]. The observed NO concentration in the exhaust increased with the increase in fuel-air ratio. It should be noted that the observed NO concentration does not represent the extent of NO formation by the combustion process because of the dilution with the excess air. The NO concentration corrected to the stoichiometric ratio shows that NO decreased with the increase of fuel-air ratio. The figure also shows the influence of fuel-air ratio on smoke, gas temperature and pressure.
It is obvious there is a tradeoff between the NO\textsubscript{x} and smoke emissions in a diesel engine in regards to the EGR and VGT vane settings. For simplicity this work will concentrate on reducing NO\textsubscript{x} in a no visible smoke AFR regime and disregard other pollutants. Therefore to alleviate the diesel NO\textsubscript{x} emissions obstacle with feedback control of the EGR and VGT vane settings, algorithms should be based on an analytical
emissions model that incorporate the transient effects of the control inputs. Thus, motivating the need for an emissions model to be used in conjunction with a mean value lumped parameter model for intake flow coordination.

Emissions models with predictive capabilities are very complex. Several modeling approaches have been implemented for predicting Diesel emissions. Engine simulations and thermo-fluid modeling of various processes have been used together with experimental techniques to develop an understanding of emission formation [21,22,23,24,25]. While thermodynamic models are dynamic in nature, the models based on regression of experimental data are static. Simple zero-dimensional thermodynamic models [21] can be used when a degree of high accuracy is not necessary. On the other hand multi-zone phenomenological models are capable of accuracies comparable with CFD based models [26, 27].

In a multi-zone model [26] the combustion process is visualized as a complex sequence of coupled events that are completed in a very rapid time. Fuel injection is modeled in extreme detail and characterized as intermittent pockets of fuel being released at each injection. Atomization, air entrainment, droplet evaporation, spray-wall impingement, and injection delay processes are all modeled [28] and lead up to combustion. Heat transfer is modeled on a zonal basis. Depending on the engine RPM and intake and exhaust valve events, these actions can take place anywhere from 1 to 50 ms [26].

A detailed emission model must be based on the kinetics of the numerous species [29,30,31]. Therefore chemical equilibrium calculations are essential in developing
detailed emission information. In addition, calculating emission species concentration requires information of the spatial and temporal dependence of pressure, temperature, and local equivalence ratio in the combustion chamber [24, 32]. This is because species such as NO\textsubscript{x} and soot are formed in a non-equilibrium manner and their concentration in the engine exhaust may not be accurately predicted based purely on the overall equivalence ratio [31]. In order to understand the local equivalence ratio, pressure, and temperature histories it is important to understand the detailed mechanism of spray burning [25, 33]. Therefore a heat release model is essential in predicting the concentrations of chemical species in discrete spray packages that are burned in consecutive crank increments. As a result it is apparent that the entire emissions model has a level of complexity and a time scale that prevent it from being used as any part of a systems level model for control development, thus motivating the need for a mean value NO\textsubscript{x} representation.

Mean value models generally do not include the formation of exhaust species such as NO\textsubscript{x}. This is because the models do not describe the combustion process. Therefore to this point control development has been confined to static emission models based on a regression analysis of experimental data.

As previously stated, a complex engine model representing combustion dynamics is on a different time scale than the flow dynamics from the VGT-EGR actuators and therefore would not be useful in regulating their respective flows. The rationale for developing a mean value NO\textsubscript{x} model is so it may be used to develop control algorithms for the VGT-EGR flow regulation problem. To develop a model it is extremely important to recognize the significant factors in NO\textsubscript{x} production such as: fueling rate, airflow rate,
engine speed, intake manifold temperature and pressure, EGR rate, air to fuel ratio and start of injection [24,25,26].

Previous attempts to statically model NOx production have been successful using neural networks [37, 38], and statistical techniques [39]. Although neural networks and statistical models have been somewhat effective in modeling and understanding NOx formation they do not physically describe the NOx formation process, thus are not ideal to use in model based feedback control algorithms regulating the intake flow in VGT-EGR diesel engines.

1.2 CONTROL DESIGN

Modern automotive Diesel engines may employ VGT based boosting to meet torque demands and copious amounts of EGR for in-cylinder NOx abatement. It has been shown that while the presence of two actuators (VGT and EGR) increases the air-loop control bandwidth, it also introduces complex and often competing interactions [10,12,15]. However, there is minimal significant work in the open literature dealing with the analysis and control of coordinating the electrical assist system to a conventional VGT-EGR system.

Control aspects related to turbo electric assist (TEA) have been investigated for conventional fixed vane turbocharged diesel engines [6-8]. Kolmanovsky et al. [6] investigated improvements in the transient response of the electrically assisted system. The method relies on formulating and numerically solving an appropriate minimum time optimal control problem. The focus of the work is on the optimization of the acceleration performance of a turbo charged diesel engine with turbo electric assist. Comparison with
a conventional turbo charged diesel engine response reveals the mechanism by which acceleration improvements are achieved while maintaining high fuel efficiency and equivalent smoke emissions levels. The study is based on an idealization where a six state mean value model of a turbo charged diesel engine is used assuming no EGR, a fixed vane position for the VGT, and the dynamics and energy addition/absorption efficiencies of the electric motor are neglected.

Others have studied the improvements of the dynamic characteristics of an automotive engine assisted by turbo electric assist [7,8]. In [8], a new concept of an asynchronous motor with a very thin rotor (low inertia) was applied to support the turbo charger during the transient engine operation. The general characteristics of the electric motor were determined separately through experiments then combined with a zero-dimensional filling and emptying model of the turbocharged diesel engine. The simulation work showed faster transient response, i.e. better load acceptance of the engine can be obtained by applying adequate electric assist to the turbocharger.

However none of the previous work has attempted to quantify the possible emissions benefits afforded by the coupled Turbo Electric Assist (TEA) system for the VGT-EGR diesel engine. The focus of this work is to investigate the possible benefits of the TEA system as it pertains to NOx emissions levels of the VGT diesel engine with and without EGR. The effect on PM emissions are not considered and visible smoke is avoided by appropriately fixing the AFR. This is based on choosing a fixed AFR that stays above the no smoke limit of the engine that reduces NOx. Specifically for the 2.4L Fiat VGT-EGR diesel engine studied in [9]. In order to include the dynamics of the
1.3 OBSERVER DESIGN

Various researchers [56,12,10,15,57,58] have assumed the presence of an exhaust gas pressure sensor to predict both the EGR flow rate and the flow rate over the VGT turbine when dealing with the control of these actuators. In [12,15,58] the desired control actions are the positions of the VGT vane and the EGR valve. The common approach used was to treat the mass flow rates, over the EGR valve and the VGT vane, as the control inputs. The desired objective being to track or regulate these flows to fulfill any desired flow profiles. The appropriate EGR valve or VGT vane position commands to the respective actuators were subsequently evaluated by inverting the standard orifice flow model. The control design challenge lay in the associated regulation/tracking problem resulting from the inherent interaction between the VGT-EGR actions and the inverse response of the system. While the exhaust pressure in the exhaust manifold can be obtained using an exhaust pressure sensor (differential or gauge pressure sensors exist for such applications), however the long time durability of such sensors in the harsh diesel exhaust environment has not been proven as yet with any degree of confidence. Hence prediction of the exhaust manifold pressure remains of practical significance. In [59], a nonlinear observer was proposed for a turbo-charged diesel engine using the passivity framework presented in [60]. However this approach requires an orifice representation of the flow through the turbine as a function of the vane angle and exhaust
manifold pressure. This introduces modeling uncertainties because the effective area of
the orifice, as will become apparent later. This approach also lacks the influence of EGR
which can introduce the same modeling errors with respect to the orifice flow equations.
In [10], a sliding mode observer for the exhaust gas pressure was proposed for the
coordinated VGT-EGR control design, but it is assumed that the flow through the turbine
is known.

An alternate problem that was tackled in [61] dealt with the regulation of the air
mass fraction in the intake manifold via appropriate EGR control with the ultimate goal
of reducing the in-cylinder NOx. The fact that the oxygen content in the cylinder air-
charge has a direct impact on the feedgas NOx generation has been well documented
[16,11]. In [61], an estimate of the oxygen concentration in the engine out exhaust (and
hence in the EGR) was used to predict the oxygen contribution of the EGR in the air
charge. This information was used to design EGR control. The Oxygen concentration in
the exhaust gas was established via a UEGO sensor placed in the post Turbo Charger
(TC) location. While traditional EGR control in Diesel engines does not rely on an
exhaust UEGO such regulation has great application in Diesel Homogeneous Charge
Compression Ignition (DHCCI) or Diesel Low Temperature Combustion (DLTC). In
these applications information on the oxygen content in the intake (hence in the air
charge) is critical for control of the injection parameters. For effective control of the air
loop specifically the three states that need to be estimated are the pressure in the exhaust
manifold, the mass fraction of air in the intake manifold, and the mass fraction of air in
the exhaust manifold. Additionally there is a need for the estimation of the EGR and
VGT flow rates, because practically all that is known for both are only the actuator positions.

Ideally, the mass fraction of air in the intake manifold may be measured with a wide band Universal Exhaust Gas O₂ (UEGO) Sensor. Yet, it is well known that the excessive moisture in the intake adversely impacts the UEGO sensor element. The dew point strategy utilized for the heater control of the UEGO sensor in normal exhaust gas environments has proven to be ineffective for diesel intake manifold applications. This justifies the measurement of the mass fraction of air in the exhaust manifold as an alternative. However, this work will demonstrate that this measurement is not necessary in either manifold in order to estimate the mass fraction of air in both manifolds.
CHAPTER 2

MODELING FOR CONTROL

2.1 INTRODUCTION

In this chapter we present the detailed mathematical development of a mean value model for a modern CIDI engine, particularly a Fiat Jet Turbo Diesel (JTD) 166 engine that features a variable geometry turbocharger with exhaust gas circulation as derived by [10] and validated by [40]. A complete list of the specifications of the engine is given in Table 2.1. The theoretical effect of turbo-electric assist is added to the model and a new empirical formulation for the average NO\textsubscript{x} emissions of the engine is also developed as part of this work.

2.2 PROBLEM STATEMENT

In order to study the transient performance of the T/C and its effect on the transient behavior of the engine, a mean value model of a turbocharged diesel engine was developed [10]. The model development is based on the familiar concept of filling and emptying where several thermodynamic volumes are assumed to be interacting with each other and with the surroundings through mass, heat and work transfer.
Table 2.1 Fiat 2.4L JTD 166 Engine Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cylinders</td>
<td>5</td>
</tr>
<tr>
<td>Arrangement</td>
<td>Inline</td>
</tr>
<tr>
<td>Bore x Stroke</td>
<td>82x90.4 mm</td>
</tr>
<tr>
<td>Compression Ratio</td>
<td>18.45:1</td>
</tr>
<tr>
<td>Displacement</td>
<td>2.387 L</td>
</tr>
<tr>
<td>Maximum Power</td>
<td>100 kW @ 4200 RPM</td>
</tr>
<tr>
<td>Maximum Torque</td>
<td>304 Nm @ 2000 RPM</td>
</tr>
<tr>
<td>Valve Train</td>
<td>2 valves per cylinder, 1 OHC</td>
</tr>
<tr>
<td>Turbocharger</td>
<td>Allied Signal VNT 25, with VGT</td>
</tr>
<tr>
<td>Intercooler</td>
<td>2.5 L air-to-air</td>
</tr>
<tr>
<td>EGR cooler</td>
<td>None</td>
</tr>
<tr>
<td>Fuel Injection Method</td>
<td>Direct Injection</td>
</tr>
<tr>
<td>Fuel Control</td>
<td>Solenoid controlled common rail injector</td>
</tr>
<tr>
<td>Intake Volume</td>
<td>2.3 L</td>
</tr>
<tr>
<td>Exhaust Volume</td>
<td>0.85 L</td>
</tr>
</tbody>
</table>

Table 2.1 Fiat 2.4L JTD 166 Engine Specifications

Modeling Assumptions:

1. Intake and Exhaust manifolds are modeled as open thermodynamic control volumes.

2. Each volume contains an ideal gas mixture of air and combustion gas. The combustion gas is assumed to consist of species resulting from the stoichiometric combustion of air and fuel.

3. Mixture properties are assumed to be uniform across the volume.

4. The difference between static and total enthalpies is negligible owing to relatively low gas velocities.
Since we are dealing with gas mixtures we will need to establish mixture properties for thermodynamic analysis of the system. As mentioned above F, the composition state represents the mass fraction of air in a given volume, hence $F_1$ is the mass fraction of air in the intake volume. These mass fractions are essential to account for both the combustion products and air re-circulated back to the engine via EGR. Due to the lean operation of Diesel engines, the EGR flow may have more than 50% air that can participate in combustion if properly controlled.

Figure 2.1: Control Volume representation of a VGT-EGR Diesel Engine with Turbo Electric Assist
\( \dot{m}_i \) : are the mass flow rates, the flow is from volume “i” to volume “j” [kg/sec]

\( \dot{m}_{i2} \) : is the reverse flow from the intake manifold to the exhaust

\( \dot{m}_f \) : is the mass flow rate of fuel injected into the engine.

\( \beta \) : is the vane angle control command for the VGT

\( \alpha \) : is the EGR valve position command

\( F_i \) : mass fraction of air in the control volume

2.3 LUMPED PARAMETER MODEL DEVELOPMENT

2.3.1 MIXTURE PROPERTIES

Using basic thermodynamic properties of mixtures [25] we can define the various mixture properties as follows:

Specific heat at constant volume \( c_v \) for the intake and exhaust volumes (kJ/kgK)

\[
c_{v1} = (1 - F_i) c_{vb} + F_i c_{vw} \tag{2.1}
\]

\[
c_{v2} = (1 - F_2) c_{vb} + F_2 c_{vw} \tag{2.2}
\]

Internal energy (kJ/kgK)
\[ u_1 = (c_{vb} T_1 + u_{av})(1 - F_1) + (c_{va} T_1 + u_{bv})F_1 \]  
\[ u_2 = (c_{vb} T_2 + u_{av})(1 - F_2) + (c_{va} T_2 + u_{bv})F_2 \]  

Gas constants

\[ R_1 = \frac{R_{mol}}{M_1}, \quad R_2 = \frac{R_{mol}}{M_2} \]  

The molecular weight of a gas mixture is defined as the sum of the mass over the total number of moles, \( M_{mix} = \frac{\sum M_i n_i}{n_{total}} \). Then molecular weights of the gas in each of the manifolds can be defined can be represented as a function of the gas composition as follows:

\[ M_1 = \frac{M_a}{F_1 + \frac{M_a}{M_b}(1 - F_1)} \]  
\[ M_2 = \frac{M_a}{F_2 + \frac{M_a}{M_b}(1 - F_2)} \]

Specific heat at constant pressure \( c_p \) for the intake and exhaust volumes (kJ/kgK)

\[ c_{p1} = c_{vl} + R_1, \quad c_{p2} = c_{v2} + R_2 \]  

Specific enthalpies for the various flow rates \( h_i \) (kJ/kg)

\[ h_i = u_i + R_i T_i \]

where:

subscripts “a” and “b” refer to air and combustion gas respectively.
subscripts “1” and “2” refer to the respective volumes.

\( c_{va} = 0.7165\) (kJ/kgK): specific heat at constant volume for air.

\( c_{vb} = 0.7833\) (kJ/kgK): specific heat at constant volume for combustion gas.

\( R_{mol} = 8.314\) (kJ/kgK): universal gas constant.

\( u_{ao} = -2.5422\) (kJ/kg): specific enthalpy of formation of air.

\( u_{bo} = -2868.8\) (kJ/kg): specific enthalpy of formation of combustion gas.

\( M_a = 28.97\) (kg/kg-Mol): Molecular weight of air.

\( M_b = 28.9074\) (kg/kg-Mol): Molecular weight of combustion gas.

### 2.3.2 MASS FLOW DYNAMICS

The intake and exhaust manifolds are dynamic control volumes with gas flows into and out of several times every engine cycle. These interacting flows are depicted in Figure 2.1. The rates of change of mass in the intake and exhaust manifold volumes are found by applying the law of conservation of mass (filling and emptying) to the intake and exhaust manifolds.

\[
\frac{dm_1}{dt} = \dot{m}_{1} + \dot{m}_{EGR} - \dot{m}_{1e} - \dot{m}_{12} \quad \text{(Intake manifold)} \quad (2.10)
\]

\[
\frac{dm_2}{dt} = \dot{m}_{12} + \dot{m}_{2} - \dot{m}_{EGR} - \dot{m}_{2t} \quad \text{(Exhaust manifold)} \quad (2.11)
\]
2.3.3 GAS MIXTURE DYNAMICS

Next we establish the equations that characterize the dynamics of the gas mixture variation in the inlet and exhaust manifolds, from the definition of $F_1$ we can write:

$$F_1 = \frac{m_{\text{air}}}{m_i}$$  \hspace{1cm} (2.12)

where $m_{\text{air}}$ is the mass of air in the intake manifold volume with a total mass $m_i$, hence:

$$F_1 m_i = m_{\text{air}}$$

⇒ (differentiating both sides w.r.t time)

$$\dot{F}_1 m_i + F_1 \dot{m}_i = \dot{m}_{\text{air}}$$

⇒

$$\dot{F}_1 = \frac{\dot{m}_{\text{air}} - F_1 \dot{m}_i}{m_i}$$

$\dot{m}_i$ is available form equation (2.10), we now need to establish a representation for $\dot{m}_{\text{air}}$, this is easily done by accounting for all the flows into and out of the intake volume as shown in Figure 2.1.

The EGR flow, $\dot{m}_{\text{EGR}}$, has a composition $F_2$ which implies that for a flow of a unit mass of EGR:

$$F_2 \dot{m}_{\text{EGR}}$$ is the flow rate of air into the intake volume via EGR.

$$(1-\text{F}_2) \dot{m}_{\text{EGR}}$$ is the flow rate of combustion gas into the intake volume via EGR.
Using the above rationale and assuming no reverse flow we can write the following representation for \( \dot{m}_{\text{air}} \)

\[
\dot{m}_{\text{air}} = \dot{m}_{c1} + F_2 \dot{m}_{\text{EGR}} - \dot{m}_{1e} F_1
\]

\[
\Rightarrow
\]

\[
m_1 \dot{F}_1 = \left( \dot{m}_{c1} + F_2 \dot{m}_{\text{EGR}} - F_1 \dot{m}_{1E} \right) - F_1 \left( \dot{m}_{c1} - \dot{m}_{1E} + \dot{m}_{\text{EGR}} \right)
\]

\[
\Rightarrow
\]

\[
\dot{F}_1 = \frac{\dot{m}_{\text{EGR}} (F_2 - F_1) + (1 - F_1) \dot{m}_{c1}}{m_1}
\]  
(2.13)

Following the procedure described above and assuming no reverse flow we can write a similar equation for \( \dot{F}_2 \):

\[
F_2 = \frac{m_{\text{air}}}{m_2}
\]  
(2.14)

where \( m_{\text{air}} \) is now the mass of air in the exhaust manifold volume with a total mass \( m_2 \), hence:

\[
\dot{F}_2 = \frac{\dot{m}_{\text{air}} - F_2 \dot{m}_2}{m_2}
\]  
(2.15)

We now establish a representation for \( \dot{m}_{\text{air}} \) by accounting for all the flows into and out of the exhaust manifold

\[
\dot{m}_{\text{air}} = F_{e2} \dot{m}_{e2} - F_2 \dot{m}_{\text{EGR}} - F_2 \dot{m}_T
\]
where $\dot{m}_{e2}$ is the mass flow rate and $F_{e2}$ is composition state of the mixed gas leaving the engine volume. It is assumed there is no dynamic associated with the engine flow and the amount of air consumed by combustion is stoichiometric. Therefore they are represented as follows:

$$\dot{m}_{e2} = \dot{m}_{1e} + \dot{m}_f$$

$$F_{e2} = \frac{F_1 \dot{m}_{1e} - AFR_s \dot{m}_f}{\dot{m}_{e2}}$$  \hspace{1cm} (2.16)

where AFR$_s$ is the stoichiometric air to fuel ratio for diesel. Substituting these equations back into the differential equation for $F_2$ yields:

$$\hat{F}_2 = \frac{\dot{m}_{e2} (F_{e2} - F_2)}{m_2}$$  \hspace{1cm} (without reverse flow)  \hspace{1cm} (2.17)

Substituting in 2.16 yields the following:

$$\hat{F}_2 = \frac{\left(\dot{m}_{1e} + \dot{m}_f\right) \left(F_1 \frac{\dot{m}_{1e} - AFR_s \dot{m}_f}{\dot{m}_{1e} + \dot{m}_f} - F_2\right)}{m_2}$$  \hspace{1cm} (2.18)

### 2.3.4 MIXTURE TEMPERATURE DYNAMICS

The equations for the variations in temperature can be derived from an energy balance on the control volume. The representation will consider the heat transfer
term. In both cases (intake and exhaust), heat is lost to the environment. However, it must be noted that for steady state operating condition the heat transfer effects may be neglected.

\[ \dot{E}_{cv1} = \dot{m}_1 h_{z1} + \dot{m}_{c1} h_{e1} - \dot{m}_{ie} h_{ie} - \dot{m}_{z2} h_{z2} - \dot{Q}_1 \] (2.19)

\[ h_{ie} = h_{z2} \]

\[ E_{cv1} = m_1 \left( c_v T_1 + u_{io} \right) \]

where

\[ c_v = (1 - F_i) c_{vb} + F_i c_{va} \]

and

\[ u_{io} = F_i u_{ao} + (1 - F_i) u_{bo} \]

is a function of \( F_i \) only, hence

\[ \dot{E}_{cv1} = \dot{m}_1 \left( c_v T_1 + u_{io} \right) + m_1 \dot{c}_v T_1 + m_1 T_1 \left( \frac{\partial c_v}{\partial F_1} \frac{dF_1}{dt} \right) + m_1 \frac{\partial u_{io}}{\partial F_1} \frac{dF_1}{dt} \] (2.20)

substituting we have:

\[ \dot{m}_1 \left( c_v T_1 + u_{io} \right) + m_1 \dot{c}_v T_1 + m_1 T_1 \left( \frac{\partial c_v}{\partial F_1} \frac{dF_1}{dt} \right) + m_1 \frac{\partial u_{io}}{\partial F_1} \frac{dF_1}{dt} \]

\[ = \dot{m}_2 h_{z1} + \dot{m}_{c1} h_{e1} - \dot{m}_{ie} h_{ie} - \dot{m}_{z2} h_{z2} - \dot{Q}_1 \]

Substituting for \( \dot{m}_1 \) and rearranging we get:
\[
\frac{dT_1}{dt} = \frac{\dot{m}_{21}(h_{21} - u_1) + \dot{m}_{c1}(h_{c1} - u_1) - (\dot{m}_{1c} + \dot{m}_{12})(h_{12} - u_1) - \left(T_1 \frac{\partial c_{el}}{\partial F_1} + \frac{\partial u_{le}}{\partial F_1}\right) m_1 \dot{F}_1}{c_{vl} m_1} \\
- \frac{\dot{Q}_1}{c_{vl} m_1}
\]

using the relationship:

\[h = u + RT\]

we can write the above equation as:

\[
\frac{dT_1}{dt} = \frac{\dot{m}_{21}(h_{21} - u_1) + \dot{m}_{c1}(h_{c1} - u_1) - (\dot{m}_{1c} + \dot{m}_{12})R_1 T_1 - (T_1 (c_{va} - c_{vb}) + u_{va} - u_{vb}) m_1 \dot{F}_1}{c_{vl} m_1} \\
- \frac{\dot{Q}_1}{c_{vl} m_1}
\]

(2.21)

similarly we can write the equation for the exhaust manifold temperature dynamics as follows:

\[
\frac{dT_2}{dt} = \frac{\dot{m}_{c2}(h_{c2} - u_2) + \dot{m}_{12}(h_{12} - u_2) - (\dot{m}_{21} + \dot{m}_{22})R_2 T_2 - (T_2 (c_{va} - c_{vb}) + u_{va} - u_{vb}) m_2 \dot{F}_2}{c_{vl} m_2} \\
- \frac{\dot{Q}_2}{c_{vl} m_2}
\]

(2.22)

It can be verified through a comparison of the values of \(c_{va}\) and \(c_{vb}\) that the differences between the thermodynamic properties of air and combustion gases are not significant hence we can assume that \(c_v\) and \(c_p\) are constant. This assumption allows us to make a simplification to the model developed so far. We can eliminate the dependence
of the mixture properties on the composition states $F_1$ and $F_2$. This allows us to eliminate the composition state altogether and the temperature dynamics reduce to the following:

$$\frac{dT_1}{dt} = \frac{m_{21}(h_{21} - u_1) + m_{c1}(h_{c1} - u_1) - (m_{1e} + m_{12})R_T T_1}{c_v m_1} - \frac{\dot{Q}_1}{c_v m_1} \quad (2.23)$$

$$\frac{dT_2}{dt} = \frac{m_{c2}(h_{c2} - u_2) + m_{12}(h_{12} - u_2) - (m_{21} + m_{2e})R_T T_2}{c_v m_2} - \frac{\dot{Q}_2}{c_v m_2} \quad (2.24)$$

In the event that a dynamic heat transfer term is needed to account for transient temperature or pressure variation, a standard conductive heat transfer term as explained in reference [43] could be used.

$$\dot{Q} = h_t A(T_{\text{gas}} - T_{\text{wall}}) \quad (2.25)$$

In this equation the entire surface area of the manifold is used in $A$ and $h_t$ is an empirical heat transfer coefficient. In reference [44], a method is proposed that allows for calculation of the heat transfer coefficient by assuming the manifold acts as a pipe with certain diameter for which Reynolds and Nusselt numbers can be found and related to solve for $h_t$. For the purpose of this model the heat transfer team for both manifolds will be ignored.
2.3.5 MIXTURE PRESSURE DYNAMICS

Equations (2.3.14) and (2.3.15) can be quite burdensome to use in a model due in large part to the need for internal energy and enthalpy constants to be defined. The need for these values can be eliminated if the dependence of the equations on composition, \( F \), can be removed. Since it is the fact that specific heat is different for air and exhaust gas that creates the need to know the composition in calculating temperature, an assumption that \( c_{va} \) and \( c_{vb} \) are constant eliminates \( F \) dependence. This is a reasonable assumption because \( c_{va} \) and \( c_{vb} \) differ only by 0.01 kJ/kgK. Now the temperature equation can be replaced with the more desirable pressure state equation derived from the ideal gas law [10, 20].

\[
p_1 = m_1 R_1 T_1 \quad \text{(2.26)}
\]

Applying the chain rule,

\[
V_1 \frac{dp_1}{dt} = m_1 R_1 \frac{dT_1}{dt} + T_1 R_1 \frac{dm_1}{dt}
\]

Substituting equations 2.10 and 2.1.23

\[
V_1 \frac{dp_1}{dt} = m_1 R_1 \left[ \frac{m_{e21} (h_{21} - u_1) + m_{e1} (h_{e1} - u_1) - (\dot{m}_{1e} + \dot{m}_{12}) R_1 T_1}{c_{v1} m_1} \right] + \\
R_1 T_1 c_{v1} \left[ \frac{\dot{m}_{e1} + \dot{m}_{EGR} - \dot{m}_{1e} - \dot{m}_{12}}{c_{v1}} \right]
\]
Applying the relations for enthalpy, $h_1 = c_p T_1$, and internal energy, $u_1 = c_v T_1$, reduces to equation 2.28.

\[
\frac{dp_1}{dt} = \frac{\gamma R_1}{V_1} (m_{c_1} T_{c_1} + \dot{m}_{v_2} T_2 - \dot{m}_{v_1} T_1) \tag{2.28}
\]

Similarly the following relation can be obtained for the exhaust manifold.

\[
\frac{dp_2}{dt} = \frac{\gamma R_2}{V_2} (m_{e_2} T_{e_2} - \dot{m}_{v_2} T_2 - \dot{m}_{v_1} T_2) \tag{2.29}
\]

2.4 EMPIRICAL FORMULATIONS FOR FIAT 2.4L JTD 166 ENGINE

2.4.1 MODELING INTAKE CHARGE

It is common to model the pumping process in four stroke engines with the speed density equation [16]; hence we can model the mass flow rate of the intake charge, $\dot{m}_{v_1}$ into the cylinder as follows:

\[
\dot{m}_{v_1} = \frac{\eta_{vol} \rho_i N V_d}{120} \tag{2.30}
\]

where

$\eta_{vol}$ : is the volumetric efficiency of the engine

33
\[ \rho_1 = \frac{m_i}{V_1} \] is the density of the gas mixture in the intake manifold

\( V_1 \) and \( V_d \) are the intake manifold volume and the engine displacement volumes respectively and \( N \) is the engine RPM.

The volumetric efficiency is a measure of the amount of intake charge the engine is able to induct per cycle. Defined as a ratio the volumetric efficiency can be expressed as: \[ \frac{m_{in,eng}}{V_d \rho_1} \], hence it is the ratio of the mass of charge in the volume \( V_d \) to the total mass of charge in the same volume at inlet conditions. Note that the charge has a density of \( \rho_1 \), and the total mass that can occupy the volume \( V_d \) is given by: \( V_d^* \rho_1 \). Volumetric efficiency will therefore take values between zero and one but under some situations intake manifolds can be tuned to allow volumetric efficiencies greater than one for a very restricted engine operating range [16]. Identification of the volumetric efficiency is critical for modeling engine behavior. For this work we adopt the following formulation of volumetric efficiency as validated by Hopka [40].

\[ \eta_{vol} = -1.21 - 0.081 \frac{p_1}{\sqrt{T_1}} + 0.090 \frac{p_1}{\sqrt{T_1}} \frac{p_2}{p_1} + 0.0046 T_1 \] (2.31)

### 2.4.2 ENGINE ROTATIONAL DYNAMICS

From a dynamics perspective, the speed of the engine is the result of the difference between the engines indicated torque and the friction and load torques as it acts on the engine inertia. When the engine is producing more torque than can be accounted for
in torque and speed, the engine will increase speed and vise versa. This effect is very easy to model using Newton’s second law.

\[ J_e \dot{\omega} = T_{\text{ind}} - T_{\text{friction}} - T_{\text{load}} \]  

(2.32)

where

\( J_e \) is the engine inertia, for the Fiat engine this is known to be 0.5 Nm/s\(^2\)

\( \dot{\omega} \) is the rotational acceleration of the crankshaft in rad/s\(^2\)

\( T_{\text{ind}} \) is the indicated torque

\( T_{\text{friction}} \) is the friction torque and other parasitic losses

\( T_{\text{load}} \) is the torque at the flywheel available to do work

### 2.4.2.1 Estimation of the Engine Torque Production

The amount of work an engine can theoretically produce is based on the energy content of the fuel and the amount of fuel that is provided for combustion. The equation for indicated power from [16] characterizes the indicated engine torque.

\[ T_{\text{ind}} = 30 \frac{P_{\text{ind}}}{\pi N} = 30 \frac{m_{\text{fuel}} Q_{\text{LHV}} \eta_{fc}}{\pi N} \]  

(2.33)

where \( Q_{\text{LHV}} \) is the lower heating value of the fuel and \( \eta_{fc} \), the fuel conversion efficiency, is the lumped combustion efficiency and diesel cycle thermal efficiency. The fuel conversion efficiency is mainly a function of AFR ratio, and generally follows the trend that as AFR increases \( \eta_{fc} \) increases except when AFR is less than
stoichiometric in which case the conversion efficiency decreases. More detailed relationships of the dependence of fuel conversion efficiency on AFR are necessary to accurately describe the torque production of the engine. Therefore equation 2.34, validated in [40], is used for the Fiat engine being simulated. This functional form is more implicit and allows for the visualization of the obvious direct relationship of fuel consumption, the influence of air, scavenging efficiency, EGR rate, temperature of the intake charge, and the reduction in torque production due to high speed.

\[ T_{\text{ind}} = 19358\dot{m}_{\text{fuel}} + 1.26 \times 10^7 \frac{\dot{m}_{\text{fuel}}}{N} + 5.69 \times 10^{-10} \dot{m}_{\text{air}} - 6 \times 10^{-5} N(p_2 - p_1) - 3 \times 10^{-6} T_1 \]  

(2.34)

2.4.2.2 ENGINE PARASITIC LOSSES

Engine losses are defined as the difference between the indicated torque, the torque that theoretically can be obtained from the amount of fuel that is being injected, and the actual, or engine brake torque. There are a variety of sources for parasitic drain on the torque the engine is producing during combustion. The most significant of these is due to friction in engine bearings, piston rings and seals. Other sources of loss include the alternator and high-pressure fuel pump. The contribution of these to the parasitic loss varies as a function of engine temperature and most importantly engine speed. By nature, engine friction is modeled as an empirical fit in terms of speed to real engine data. Heywood [16] suggests a friction fit in terms of the average speed of the piston in the cylinder; however, the mean piston speed is merely a function of the engine speed so a friction fit as a polynomial of speed will suffice. The coefficients of the polynomial are determined empirically.
In many cases, the engine parasitic drain can be defined as a second order polynomial function of speed. For others the need for a third order polynomial is most likely due to characteristics in the fuel pump and engine bearing friction having different speed dependencies. Hopka [23] proposed and validated the following in order to ensure that for zero engine speed, the friction torque was zero.

\[ T_{fr} = 21.66 + 10.53N + 7.68N^2 + 3.62N^3 \]  \hspace{1cm} (2.35)

### 2.4.3 ENGINE OUT EXHAUST TEMPERATURE

The temperature of the gas leaving the engine is of great importance to this model because it controls the temperature in the exhaust manifold. Predicting engine temperature independent of combustion processes is very difficult. In general, the rise in temperature across the engine is a function of the inlet temperature, the amount of fuel injected, the air to fuel ratio, the engine speed, the coolant temperature, the pressure of intake and exhaust gas, and flow rate of EGR. Many mean value models utilize a function for temperature rise with proportionality to the fueling rate over the exhaust flow rate as in:

\[ \Delta T_e = T_e - T_i \propto \frac{m_{\text{fuel}}}{m_{\text{fuel}} + m_{\text{air}} + m_{\text{egr}}} + F_i \]  \hspace{1cm} (2.36)

This model contains the important influence of EGR on the engine out temperature. EGR acts primarily to control the rate of heat release during combustion. EGR cannot be burned with fuel, so the combustion process is slowed because the air to
fuel ratio is reduced without reducing the amount of mass in the cylinder. While equation (2.36) is predicting the influence of EGR on temperature, this form has no dependence on speed and will have difficulty predicting the temperature for the variety of speeds that will be encountered in simulation. The form in equation (2.36) can be preserved and include the effect of speed if proportionality constants in speed are included [40].

\[
\Delta T = T_{ex_{out}} - T_i = k_1 \left( \frac{\dot{m}_f}{\dot{m}_f + \dot{m}_{c1} + \dot{m}_{c2}} \right) + k_2 F_i + 23.6SOI
\]  

(2.37)

where

\[k_1 = 5.95 + 0.868N + 113.61N^2\]

\[k_2 = 86568 + 0.448N + 23.53N^2\]

Notice that equation (2.37) also includes the influence of injection timing. This is important because as the fuel injection is retarded, more heat ends up as waste in the exhaust manifold.

### 2.4.4 EGR VALVE

A poppet style valve in the exhaust manifold controls the flow of EGR. The flow through such valves is considered adiabatic and compressible and to have only one dimension. The EGR actuator controls the lift of the poppet valve from its seat, effectively controlling the area of the orifice. The ratio of upstream to downstream
pressure across the valve also has significant influence over the flow rate across the valve. The pressure ratio also determines the flow regime experienced by the valve, either subsonic (un-choked) or sonic (choked). The pressure ratio at which the flow changes from subsonic to sonic is called the critical pressure and is defined as a function of the ratio of specific heats of the gas, $\gamma$, as follows:

$$P_{crit} = \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad (2.38)$$

In spite of the fact that the above pressure definition refers to the ratio of pressures between two reservoirs of different $\gamma$, the selection of $\gamma$ is always the upstream condition. The two flow regimes each have distinct equations defining the flow rate as a function of area, a discharge coefficient and the pressure ratio. When the ratio of downstream to upstream pressure is greater than the critical pressure, subsonic flow, i.e.:

$$\frac{p_1}{p_2} > \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad (2.39)$$

The following relation can define the flow:

$$m_{eet} = C_d A p_2 \sqrt{\frac{2\gamma}{R_2 T_2(\gamma - 1)}} \left( \frac{p_1}{p_2} \right)^{\frac{2}{\gamma}} - \left( \frac{p_1}{p_2} \right)^{\frac{\gamma+1}{\gamma}} \left( \frac{p_1}{p_2} \right) \quad (2.40)$$
When the ratio of downstream to upstream pressure is less than or equivalent to the critical pressure, sonic flow, as in:

\[
\frac{p_2}{p_1} \leq \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}
\]  

(2.41)

The flow rate becomes:

\[
\dot{m}_{egr} = C_d A p_2 \sqrt{\frac{\gamma}{R_T T_2} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}}
\]  

(2.42)

where the following conventions are used:

\( p_1 \) is the stagnation pressure of the intake manifold

\( p_2 \) is the stagnation pressure of the exhaust manifold

\( \dot{m} \) is the mass flow rate

\( v \) is the velocity of air

Then any stagnation pressure \( p_o \), and temperature, \( T_o \), can be calculated from:

\[
p_o = p \left( \frac{T_o}{T} \right)^{\frac{\gamma}{\gamma - 1}}
\]

\[
T_o = T + \frac{v^2}{2c_p}
\]

(2.43)

\[
v = \frac{\dot{m}RT}{PA}
\]
Notice in equation (2.40) and (2.42), an area term is included. This area is the actual area of the poppet valve. Clearly this area varies as a function of the EGR position setting $\alpha$. The $C_d$ term in the equations is an empirical discharge coefficient term. This term is not a constant and varies with flow rate, pressure ratio and valve position. It is common practice to lump the discharge coefficient and the area into a single term called the effective area. Other models in [45, 46] utilize an effective area that is only a function of the position of the EGR valve. This method gives a poor fit to actual engine data so other relationships must be introduced, and the effective area must be fit to each engine over a range of EGR flow cases. For the Fiat engine being simulated reference [40] indicates a particularly useful EGR effective area function to be:

\[
A_{\text{eff}} = 3.58\alpha \left( \frac{p_1}{p_2} \right)^{-48.46+0.345 N - 7 \times 10^{-9} N^2}
\]  

(2.44)

Notice that the effective area equation in (2.44) is a function of the pressure ratio to the power of a function in speed. This accounts for the influence of the engine speed on turbulence in the exhaust manifold. As the engine speed increases the speed of the flow in the exhaust manifold increases, resulting in an increase in turbulence. This turbulence has a detrimental effect on the EGR flow rate and results in the speed polynomial value that causes the effective area’s dependence on pressure ratio to decline.
2.4.5 TURBOCHARGER

The turbocharger is consists of a mechanically coupled turbine and compressor. The turbine extracts power from the exhaust gas and transfers it to the compressor, which impels air into the intake manifold. Applying Newton’s second law:

\[
J_c \dot{\omega}_c = \frac{P_t \eta_{m, tc} - P_c}{\omega_c}
\]  

(2.45)

\(P_t\) refers to turbine power, \(P_c\) refers to compressor power, and \(\eta_{m, tc}\) is the mechanical efficiency defining the fraction of mechanical energy from the turbine that can be transferred to the compressor. For the Fiat engine it is assumed the mechanical efficiency of the turbine is 90%. The transmission of power from the turbine to the compressor is modeled with a 1st order dynamic with a time constant, \(\tau\), and another mechanical efficiency as follows:

\[
\dot{P}_c = -\frac{1}{\tau} (\eta_{m, tc} P_t - P_c)
\]  

(2.6)

The time constant of the turbo best describes the lag typically associated with the turbo. This lag is never constant, but a function of the engines operating condition. For the purpose of this work a time constant of 0.25 is assumed to give a simulated turbo lag of approximately 1 second.

2.4.5.1 COMPRESSOR

A turbo compressor functions by accelerating a gas/fluid to a high speed and decelerating that fluid in a specialized collector called a volute. The power required by the compressor is related to the amount of kinetic energy the machine passes on to
the gas/fluid and the mass of the gas/fluid. Therefore, the compressor power is related to the difference in energy in the incoming and compressed outgoing gas/fluid [10].

\[ P_c = \dot{m}_{c1} c_p \left( T_{c1} - T_{atm} \right) \]  \hspace{1cm} (2.47)

The influence of the compressor on the intake air is evident from an equation with temperature as in (2.1.40) however; it is necessary to have a relationship in terms of pressure. Thus, the relationship between isentropic temperature and pressure ratios is utilized.

\[ \frac{T_{c1}}{T_{atm}} = \left( \frac{p_{c1}}{p_{atm}} \right)^{\frac{\gamma-1}{\gamma}} \]  \hspace{1cm} (2.48)

Realistically isentropic compression does not occur, so the introduction of an efficiency term is necessary. The isentropic efficiency can be used to modify (2.48) into:

\[ \frac{T_{c1}}{T_{atm}} = 1 + \frac{1}{\eta_{c, is}} \left[ \left( \frac{p_{c1}}{p_{atm}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \]  \hspace{1cm} (2.49)

Combining (2.47) with (2.48) results in a relationship between the power of the compressor and the amount the intake air pressure increases for a given flow rate. The power of the compressor can be used to solve for the mass flow rate through the compressor.
The efficiency of the compressor is typically calculated from manufacturer data. It can vary from as low as 0 at very low mass flow rates to 0.75 at moderate flow rates and high compressor rotational speed. Using a compressor map the efficiency for a given pressure ratio, mass flow rate, and blade speed, efficiency can be determined. For this model the compressor efficiency is assumed to be a constant of 0.70, and is the mean of efficiencies that occur over the operating range of interest for the Fiat engine.

2.4.5.2 TURBINE

The turbine acts as an enthalpy-extracting device in the exhaust gas. Energy is extracted as gas expands through the turbine blades at high speed. Kinetic energy from the gas is transferred to the turbine blade as the gas flows over it. Similar to the compressor, the analysis of energy availability across the turbine can be used to determine the power generated.

\[ P_t = \dot{m}_{2t} (h_2 - h_{atm}) \]  

(2.51)

\[ P_t = \dot{m}_{2t} c_p (T_2 - T_{atm}) \]  

(2.52)

where the subscript 2 refers to the upstream conditions in the exhaust manifold, and the subscript atm refers to the downstream conditions at the outlet. As was the case for the compressor, it is desirable to have relationship for power that involves
pressure ratio. Using the isentropic assumption again the pressure and temperature can be related by [42]:

\[
\frac{T_2}{T_{2,\text{is}}} = \left( \frac{p_2}{p_{\text{atm}}} \right)^{\frac{\gamma - 1}{\gamma}}
\]

(2.53)

Once again since the isentropic prediction does not give the actual temperature at the turbine outlet, an isentropic efficiency is defined as:

\[
\eta_{t,\text{is}} = \frac{T_2 - T_{\text{atm}}}{T_2 - T_{2,\text{is}}}
\]

(2.54)

The isentropic efficiency can be used in equation (2.52) to obtain:

\[
P_t = \dot{m}_{2i} c_p \eta_{t,\text{is}} T_2 \left[ 1 - \left( \frac{p_{\text{atm}}}{p_2} \right)^{\frac{\gamma - 1}{\gamma}} \right]
\]

(2.55)

Similar to the compressor, an isentropic efficiency is defined for the turbine that can be determined from manufacturer supplied maps. This efficiency is defined the same as the compressor efficiency. However, it is also a function of the VGT vane angle. Turbine efficiency ranges from about 25% to 85% over the range of turbine speeds and pressure ratios typically seen in normal engine operation. For simplicity the turbine efficiency is assumed to be a constant of 0.60, and is the mean of efficiencies that occur over the operating range of interest for the Fiat engine.

Similar to the EGR, orifice flow equations may also be used to characterize turbine flow through the VGT nozzle orifice [47]. As with EGR, the turbine flow equations utilize an effective area to lump the actual turbine area and the discharge coefficient.
Much like the EGR case, the area is dependent on the orifice setting through the vane angle of the VGT. The turbine flow becomes:

\[
\dot{m}_t = \frac{A_{eff} \, p_2}{\sqrt{T_2}} \sqrt{\left(\frac{p_{atm}}{p_2} - g + 1\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{p_{atm}}{p_2} - g + 1\right)^{\frac{\gamma+1}{\gamma}}} \tag{2.56}
\]

for cases when the ratio \(\frac{p_{atm}}{p_2} > \left(\frac{2}{1+\gamma}\right)^{\frac{\gamma}{\gamma-1}}\) and:

\[
\dot{m}_t = \frac{A_{eff} \, p_2}{\sqrt{T_2}} \sqrt{\left(\frac{\frac{2}{\gamma} \left(\frac{2}{1+\gamma} - g + 1\right)^{\frac{\gamma}{\gamma-1}} - \frac{2}{\gamma} \left(\frac{2}{1+\gamma} - g + 1\right)^{\frac{\gamma+1}{\gamma}}}{\frac{\gamma}{\gamma-1}} \tag{2.57}
\]

for all other pressure ratios.

In equations (2.56) and (2.57) there is a parameter \(g\) that has a subtractive effect from the pressure ratio. In [10,40] this is called the geometry parameter. Due to the fact that the setting of the VGT vanes influences the shape of the pressure ratio vs. mass flow curve, this parameter is necessary. The value of \(g\) is determined to be the point at which on a plot of mass flow as a function of pressure ratio, the flow line intersects the pressure ratio axis. The determination of the \(g\) parameter can be visualized in Figure 2.2. For the Fiat engine [40] the value of \(g\) is fit to the following polynomial function of the VGT setting.

\[
g = -2.057 \, \beta^4 + 6.177 \, \beta^3 - 7.11 \, \beta^2 + 3.56 \, \beta + 0.32 \tag{2.58}
\]
The effective area of the turbine is also of importance here. This area will be a function of the VGT vane setting, and there will also be some dependence on the amount of flow through the turbine as was the case for the EGR valve. For the Fiat engine being studied Hopka [40] proposed the following:

\[
A_{\text{eff}} = -0.06 \frac{P_{\text{aim}}}{P_2} + \left( \frac{P_{\text{aim}}}{P_2} \right)^{3.296 - 0.698\beta} - 0.0236
\]  

(2.59)
2.5 SYSTEM MODEL WITH ELECTRICAL ASSIST

The basic idea of a turbo-compressor system with an electric motor incorporated into the design is depicted in figure 2.3. Where: $P_T, P_c, P_{em}$ are the turbine power (delivered), compressor power (absorbed), and electrical machine power (delivered/absorbed) respectively. The other terms have the usual meaning.

Developments in high-speed Permanent Magnet Brushless DC machines and the introduction of modular designs with the DC machine integrated into the T/C housing have made electrical assist of the T/C a reality. For more information on the actual design of the motor-generator assisted variable geometry turbo charging systems for use with internal combustion engines the reader is referred to [2, 3, 4, 5, 48, 49].

![Figure 2.3: Schematic for electrical assist of T/C](image)

For the purpose of this work the power supplied by the electric machine to the T/C shaft will be treated as a third input. Work has been done controlling the Permanent Magnet Synchronous Motors (PMSM) for these high speed applications [13,14] and is
extremely complex. However to simplify this effort we will assume the requested power of the PMSM, $P_{em}$, in particular the current of the motor, will be able to be achieved by the controller. To recognize how this additional power source will influence the engine recall the original equation for compressor power:

$$\frac{dP_c}{dt} = \frac{1}{\tau}(\eta_m P_T - P_c)$$  \hspace{1cm} (2.60)

Equation (2.60) can be modified as follows to reflect both the assist mode and the regenerative mode of the electrical assist system.

$$\frac{dP_c}{dt} = \frac{1}{\tau}(\eta_m P_T - P_c + P_{em}\text{sign}\left|P_{em}\right|)$$  \hspace{1cm} (2.61)

hence

$$\begin{cases}
\text{sign}\left|P_{em}\right| > 0 \quad \text{……assist} \\
\text{sign}\left|P_{em}\right| < 0 \quad \text{……regen}
\end{cases}$$

Similarly the equation for the T/C acceleration can be modified as follows:

$$(J_{tc} + J_{EM})\omega_{nc} \frac{d\omega_{nc}}{dt} = \frac{1}{\tau}(\eta_m P_T - P_c + P_{em}\text{sign}\left|P_{em}\right|)$$  \hspace{1cm} (2.62)

Now referring back to the equation for the flow through the compressor into the intake manifold, 2.50, it is now obvious how the addition of the turbo-electric assist can influence the engine intake dynamics.

### 2.5.1 Modeling the PMSM Motor

The structure of the PMSM-based high speed drive system is shown in figure 2.4. N and S denote the magnetic north and south, respectively; n is the neutral point of the
stator windings; $u_a$, $u_b$, and $u_c$ are the potential differences between points a, b, c and the neutral point n, respectively. The dynamics of the motor can be established using the basic physical laws

\[
U = RI + \frac{d\Psi}{dt} \\
\Psi = LI + \Psi_M
\]  

(2.63)

Where $U$, $I$, and $\Psi$ are the voltage vector, the current vector and the flux vector respectively; $R$, $L$, and $\Psi_M$ are the resistance matrix, inductance matrix, and the flux vector caused by the permanent magnet, respectively.

Equation 2.63 is a general description of the electromagnetic effects and is independent of the coordinate system used. For PMSMs three reference frames are typically used to describe the dynamic behavior of the motor: the phase frame (a,b,c) coordinate frame, i.e.; the stator frame, i.e. the ($\alpha,\beta$) frame; and the field-oriented or rotor coordinate frame, i.e. the (d,q) coordinate frame.
For a symmetrical PMSM in the (a,b,c) coordinate system, the flux components generated by the permanent magnet are given by the following:

\[
\begin{align*}
\Psi_{ma} &= \lambda_0 \cos \theta_e \\
\Psi_{mb} &= \lambda_0 \cos \left( \theta_e + \frac{2\pi}{3} \right) \\
\Psi_{mc} &= \lambda_0 \cos \left( \theta_e - \frac{2\pi}{3} \right)
\end{align*}
\] (2.64)

where \(\lambda_0\) is the flux linkage of the permanent magnet and \(\theta_e\) is the electrical angular position of the motor rotor. Neglecting the reluctance effects, the electrical motion equations can be written as:

\[
\begin{align*}
\frac{di_a}{dt} &= -\frac{R}{L} i_a - \frac{1}{L} e_a + \frac{1}{L} u_a \\
\frac{di_b}{dt} &= -\frac{R}{L} i_b - \frac{1}{L} e_b + \frac{1}{L} u_b \\
\frac{di_c}{dt} &= -\frac{R}{L} i_c - \frac{1}{L} e_c + \frac{1}{L} u_c
\end{align*}
\] (2.65)

Where \(R\) is the winding resistance and \(L\) is the winding inductance; \(i_a, i_b,\) and \(i_c\) are the phase currents and \(u_a, u_b, u_c\) are the phase voltages. Also, \(e_a, e_b,\) and \(e_c\) are the induced EMF components given by the following:

\[
\begin{align*}
e_a &= \frac{d\Psi_{ma}}{dt} = -\lambda_0 \omega_e \sin \theta_e \\
e_b &= \frac{d\Psi_{mb}}{dt} = -\lambda_0 \omega_e \sin \left( \theta_e + \frac{2\pi}{3} \right) \\
e_c &= \frac{d\Psi_{mc}}{dt} = -\lambda_0 \omega_e \sin \left( \theta_e - \frac{2\pi}{3} \right)
\end{align*}
\] (2.66)

where \(\omega_e\) is the electrical angular speed of the motor rotor.
It will become apparent later in this work that it is desirable to work with the model in the (d,q) coordinate frame. Which can be obtained by transforming the motor model from the (a,b,c) coordinates resulting in the following.

\[
\frac{d}{dt} i_d = -\frac{R}{L} i_d + \omega_L i_q + \frac{1}{L} u_d 
\]

\[ (2.67) \]

\[
\frac{d}{dt} i_q = -\frac{R}{L} i_q - \omega_L i_d - \frac{1}{L} \lambda_0 \omega_c + \frac{1}{L} u_q
\]

\[ (2.68) \]

where \( i_d \) and \( i_q \) are the stator currents in the (d,q) coordinate frame; \( u_d \) and \( u_q \) are the stator voltages in the same coordinate frame. The term \( \lambda_0 \omega_c = e_q \) is the q-component of the induced EMF generated by the permanent magnet; the d-component of the EMF \( e_d \) is equal to zero. Note that in equation (2.68), if the current \( i_q \) is made to be zero then we get exactly the behavior of a constant-excited DC motor. This is the main idea of field-oriented control, to decouple the motor dynamics such that the resulting system behaves like a DC motor.

Lastly, the electrical torque and mechanical power are given by

\[
\tau_e = K_t i_q \\
\frac{P}{\omega_r} = \tau_r \omega_r
\]

\[ (2.69) \]

Where \( K_t \), the torque constant, is assumed to be equal to \((3/2)\lambda_0 N_r\), with \( N_r \) being the number of poles pairs of the motor and \( \omega_r \) is the mechanical angular speed of
the motor rotor, and is equal to the angular speed of the turbo charger, $\omega_{tc}$. Note that it is assumed there is no reluctance torque in the PMSM model. Additionally, for electrical angular position/speed and the mechanical angular position/speed the following relations are used:

\[
\begin{align*}
\omega_e &= N_r \omega_r \\
\theta_e &= N_r \theta_r
\end{align*}
\]  

(2.70)

Throughout the following sections the motor parameters used to verify the design concepts are $L=28 \, \mu H$, $R=0.0055 \, \Omega$, $J_{EM}= 0.01 \, \text{kg m}^2$, $N_r=2$, $\lambda_d=0.0252 \, \text{V s rad}^{-1}$, $K_r=0.0755 \, \text{Nm A}^{-1}$, and the supplied voltage $u_0= 340\text{V}$.

2.6 DEVELOPMENT OF NOx EMISSIONS MODEL

This model assumes the well established extended Zeldovich mechanism as the kinetics of NO formation,

\[
\begin{align*}
O + N_2 & \leftrightarrow NO + N \\
N + O_2 & \leftrightarrow NO + O \\
N + OH & \leftrightarrow NO + H
\end{align*}
\]  

(2.71 

(2.72) 

(2.73)

with forward and reverse rate constants $k_1^+, k_1^-, k_2^+, k_2^-, k_3^+, k_3^-$ respectively. The first two reactions were recognized by Zeldovich [51], and the third was incorporated by Lavoie et al. [52].

53
The law of mass action maintains that the rate at which product species are produced and the rate at which reactant species are removed are proportional to the product of the concentrations of the reactant species, with the concentration of each species raised to its stoichiometric coefficient [53] as follows:

\[
-\frac{d[O]^+}{dt} = \frac{d[NO]^+}{dt} = k_1^+ [O\text{I}N_2] \tag{2.74}
\]

\[
-\frac{d[NO]^+}{dt} = \frac{d[O]^+}{dt} = k_1^- [NO\text{I}N] \tag{2.75}
\]

\[
-\frac{d[N]^+}{dt} = \frac{d[NO]^+}{dt} = k_2^+ [N\text{I}O_2] \tag{2.76}
\]

\[
-\frac{d[NO]^+}{dt} = \frac{d[N]^+}{dt} = k_2^- [NO\text{I}O] \tag{2.77}
\]

\[
-\frac{d[N]^+}{dt} = \frac{d[NO]^+}{dt} = k_3^+ [N\text{I}OH] \tag{2.78}
\]

\[
-\frac{d[NO]^+}{dt} = \frac{d[N]^+}{dt} = k_3^- [NO\text{I}H] \tag{2.79}
\]

Ignoring the influence of the reverse the rate constants, as there effects have been shown to be relatively insignificant [52], results in the rate of NO formation given by the following:

\[
\frac{d[NO]}{dt} = k_1^+ [O\text{I}N_2] + k_2^+ [N\text{I}O_2] + k_3^+ [N\text{I}OH] \tag{2.80}
\]

An analogous relation can be written for \(d[N]/dt\):
\[
\frac{d[N]}{dt} = k_1^+ [O][N_2] - k_2^+ [N][O_2] - k_3^+ [N][OH]
\]  
(2.81)

Since \([N]\) is much less than the concentrations of other species of interest \([16]\), the steady state assumption that \(d[N]/dt\) equal to zero is made. Then equation (2.81) is set to zero and used to eliminate \([N]\) from the \([NO]\) formation rate, resulting in equation (2.82).

\[
\frac{d[NO]}{dt} = 2k_1^+ [O][N_2]
\]  
(2.82)

Thus, it is apparent that the NO formation rate can be minimized by decreasing \([N_2]\) and \([O]\). Bear in mind previous research has tried to use the \(N_2\) removing membranes for NO\(_x\) abatement and \([O]\) removal is easily achieved via EGR. Decreasing \(k_1\) will also lead to reduced NO\(_x\) and thus brings in the temperature effect on NO\(_x\) formation rate into play \([54]\).

While the Nitrogen concentration can easily be measured or estimated by using equilibrium in the burnt gas fraction, the estimation /measurement of \([O]\) is not as straightforward. Measurement is expensive and slow, notice we are discussing the measurement of \([O]\) not \([O_2]\), which is stable and easy to measure, and can be used to estimate \([O]\). An approximation of \([O]\) can be obtained thru the partial equilibrium assumption.

The equilibrium oxygen atom concentration is given by

\[
[O]_e = \frac{K_{p(0)}[O_2]^{1/2}}{\sqrt{RT}}
\]  
(2.83)

where \(K_{p(0)}\) is the equilibrium constant for the reaction.

\[
\frac{1}{2} O_2 = O
\]  
(2.84)
For simplicity, $K_p(0), R, T$ are lumped into one constant, $K_0$ where

$$K_0 = \frac{[O]_e}{[O_2]_e^{1/2}} \quad (2.85)$$

substituting the oxygen concentration after combustion, $[O_2]^{1/2}$, back into equation 2.6.12 results in

$$\frac{d[NO]}{dt} = 2k_{1}\cdot K_0 [O_2]^{1/2} [N_2]_e \quad (2.86)$$

then lumping $2k_{1}\cdot K_0$ into one constant, $\theta$, equation 16 reduces to

$$\frac{d[NO]}{dt} = \theta [N_2]_e [O_2]_e^{1/2} \quad (2.87)$$

where $\theta$ is a constant to be determined empirically from data.

The next step is to relate $[O_2]$ to $m_f$ (mass flow rate of fuel), $m_{egr}$ (EGR mass flow rate), $m_c$ (Mass flow rate through the compressor) and the states of the system [1]. In order to accomplish this consider the basic oxygen consumption reactions

$$C + O_2 = CO_2 \quad (2.88)$$

$$S + O_2 = SO_2 \quad (2.89)$$

$$2H_2 + O_2 = 2H_2O \quad (2.90)$$

Looking at these reactions we know there are 12 grams of Carbon to 32 grams of Oxygen, 32 grams of Sulfur to 32 grams of Oxygen, and 4 grams of Hydrogen has to 32
grams of Oxygen. Therefore 1 gram of Carbon equals $8/3$ grams of Oxygen, 1 gram of Sulfur equals one gram of Oxygen, and 1 gram of Hydrogen equals 8 grams of oxygen.

Now consider a fuel, $C_xH_y+2S$, assuming that the fuel composition can be defined by the fraction by weight of the constituents: Carbon, Hydrogen and Sulfur, let $C$, $H$ and $S$ represent these fractions respectively. Thus for a known mass flow rate of fuel, the oxygen consumption by the fuel based on stoichiometric combustion can be written as

$$m_{O_2,\text{Combustion}} = \left(\frac{C}{3} + H + 8S\right)m_f$$

(2.91)

Defining

$$\phi_{ST} = \frac{C}{3} + H + 8S$$

gives

$$m_{O_2,\text{Combustion}} = \phi_{ST}m_f$$

(2.92)

Assuming air is composed of 21% oxygen and 78% Nitrogen, the amount of oxygen and nitrogen flowing from the intake manifold are the following:

$$m_{O_2,\text{in}} = 0.21F_1m_{e}$$
$$m_{N_2,\text{in}} = 0.78F_1m_{e}$$

(2.93)

where $F_1$ is the mass fraction of air in the intake manifold, and $m_e$ is the total mass flow through the intake manifold. Therefore the resulting amount of oxygen in the exhaust is given by the following
Next the concentration of O$_2$, can be approximated with
\[
\left[O_2\right] = \frac{M_{EX}}{M_{OX}} \frac{0.21F_i \dot{m}_{ie} - \phi_{ST} \dot{m}_f}{\dot{m}_{ie} + \dot{m}_f}
\]  
(2.95)

and the concentration of N$_2$, can be approximated with
\[
\left[N_2\right] = \frac{0.78F_i \dot{m}_{ie}}{\dot{m}_{ie} + \dot{m}_f}
\]  
(2.96)

Then substituting these expressions into equation 87 for [O$_2$]$_e$ along with equation 25 for [N$_2$]$_e$ yields the following dynamic NO expression.

\[
\frac{d[NO]}{dt} = \theta \left[ \frac{0.78F_i \dot{m}_{ie}}{\dot{m}_{ie} + \dot{m}_f} \right] M_{EX} \left[ \frac{0.21F_i \dot{m}_{ie} - \phi_{ST} \dot{m}_f}{\dot{m}_{ie} + \dot{m}_f} \right]^\frac{3}{2}
\]  
(2.97)

Where [NO] is the molar concentration of NO in the engine out feedgas, $\theta$ is an empirical parameter determined from data, and $\phi_{ST}$ is the stoichiometric air to fuel ratio. Also, since 90% of all engine out NO$_x$ is NO we use NO for this study. It should be noted that this model ignores the NO decomposition reactions as presented in [10,11,16] and assumes a unity fuel use fraction, that is all fuel is combusted stoichiometrically.
2.6.1 COMBUSTION CHAMBER AS A REACTOR

Equation 2.6.27 defines the instantaneous rate of NO production of the engine. For evaluation of intake flow regulation control algorithms, as well as after-treatment control implementation, information on the mean value NO concentration, $C_{[NO]}$, in the feedgas is desired. This is what is sensed by the NOx sensor shown in figure 2.4. To model the mean value NO concentration in the engine out exhaust we approximate the cumulative engine combustion volume as an Isothermal Continuously Stirred Tank Reactor (ICSTR) as depicted in figure 2.5. The underlying assumptions with this approach are the reactor is a constant volume, flow rate is conserved, and the molar concentration in the reactor, $C_{[NO]}$, equals the molar concentration in the reactor.

![Figure 2.5: Typical CSTR system for single reactant.](image)

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Performing a molar balance for NO we get:

\[
V \frac{d}{dt} C_{[NO]} = FC_{[NO]in} - FC_{[NO]} + R_{NO}
\]

\[\Rightarrow \]

\[
\frac{d}{dt} C_{[NO]} = \frac{F}{V} C_{[NO]in} - \frac{F}{V} C_{[NO]} + \frac{1}{V} R_{NO}
\] (2.98)

Where, \( FC_{[NO]in} \) is the rate of NO input into the reactor, \( FC_{[NO]} \) is the rate of NO leaving the reactor, and \( R_{NO} \) is the rate of production through the combustion reaction mechanism as given in equation 2.98. Then the resulting NO model is as follows.

\[
\frac{dC_{[NO]}}{dt} = \theta \left[ \frac{0.78F\dot{m}_e}{\dot{m}_e + \dot{m}_f} \left[ \frac{M_{EX}}{M_{OX}} \frac{0.21F\dot{m}_e - \phi_{ST}\dot{m}_f}{\dot{m}_e + \dot{m}_f} \right] \right]^{\frac{1}{2}} + \frac{1}{\tau} \left( C_{[NO]in} - C_{[NO]} \right)
\] (2.99)

Where \( \tau \) is the time constant associated with the flow-reactor volume relationship. The second term on the right hand side of equation (2.99) represents flow into and out of the reactor. \( C_{[NO]in} \) is the NO concentration input to the reactor prior to the combustion process and is the contribution of NO introduced by EGR. \( C_{[NO]}(t) \bigg|_{t=0} = C_{[NO]}^0 \) is the initial NO contribution due to the residual gas fraction and is assumed negligible for this study. The solution to equation 2.6.28 yields the concentration of NO at the engine out location. Under the assumption that the exhaust gas is well mixed it safe to conclude that the concentration of NOX in the exhaust gas remains homogenous throughout the system.
2.6.2 PARAMETER ESTIMATION AND MODEL VALIDATION

The next step was to establish the value of $\theta$ and $\tau$ using a least squares curve fit. The data was fit using a least squares optimization [55] routine for seven separate step tests on a 2.4 L JTD VGT-EGR diesel engine, by giving the engine stepped fueling commands. For each case the empirically fit parameters, $\theta$, $\tau$, were found to be 1.35e-6 and 0.5 respectively. The results of the step tests are seen in figures 2.6-2.12. Each outcome is plotted against its fueling input.

Next the robustness of the model was then tested against data from the same engine for two separate drive cycles. There was no commonality between the data sets used for identification and validation. The measured mass flow rate of air, mass fraction of air in the intake manifold, and mass flow rate of fuel were input to the model. The predicted mass flow rate of NO$_x$ is compared to the actual data from the engine for the first two bags of the FTP75 drive cycles in figures 2.13 and 2.14. The drive cycle comparisons show some relatively good agreement between actual and estimated NO$_x$ flow rate. As well as some steady state error most likely due to the lumping $\theta$ of into one constant. Future improvements could possibly be made by making $\theta$ a function of temperature, humidity, start of injection [11, 34], or other system parameters.
Figure 2.6 NOx model fit for first stepped fuel test

Figure 2.7 NOx model fit for third stepped fuel test
Figure 2.8 NOx model fit for third stepped fuel test

Figure 2.9 NOx model fit for fourth stepped fuel test
Figure 2.10 NOx model fit for fifth stepped fuel test

Figure 2.11 NOx model fit for sixth stepped fuel test
Figure 2.12 NOx model fit for seventh stepped fuel test

Figure 2.13 Comparison between fit NOx model and data for the first bag of the FTP 75 drive cycle
Figure 2.14 Comparison between fit NOx model and data for the second bag of the FTP 75 drive cycle
CHAPTER 3

VGT-EGR TURBO-ELECTRIC ASSIST COORDINATED CONTROL PROBLEM

3.1 PRESENTATION OF PLANT

The model is made up of two open thermodynamic volumes representing the intake and exhaust manifolds, and two adjustable area orifices model the flow through the EGR and VGT actuators [9, 10], as represented in figure 2.1. For the sake of completeness the model derived in [10] and validated in [9] for the 2.4 liter inline Fiat engine being studied is reproduced. A first order lag approximates the dynamic power transfer between the turbine and the compressor, as well as the actuator dynamics for the adjustable orifices of the VGT and EGR. For simplicity we assume no reverse flow through the EGR valve and the rate of heat transfer from both the intake and exhaust manifolds are constant. Note the model has a singularity at $P_1 = P_{\text{atm}}$, this corresponds to compressor stall. A similar singularity exists when the turbine stalls. The model is only valid as long as the boost pressure is larger than the atmospheric. However the vector space $\Omega=\{(P_1,P_2,P_c):P_1>P_{\text{atm}}, P_2>P_{\text{atm}}, P_c>0\}$ is known to be invariant subspace, meaning all state trajectories originating $\Omega$ in stay in $\Omega \forall t>t_0$. The complete model is presented with control inputs $u_1$, $u_2$, $u_3$, and $u_4$: 

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\[
\frac{dm_1}{dt} = m_{e1} - \dot{m}_{1e} + \dot{m}_{EGR} 
\] (3.1)

\[
\frac{dm_2}{dt} = \dot{m}_{c2} - \dot{m}_{EGR} - \dot{m}_{2t} 
\] (3.2)

\[
\dot{F}_1 = \frac{(1 - F_1)m_{c1} + \dot{m}_{EGR}(F_2 - F_1)}{m_1} 
\] (3.3)

\[
\dot{F}_2 = \frac{(F_1\dot{m}_{1e} - AFR_1\dot{m}_f - (\dot{m}_{1e} + \dot{m}_f)F_2)}{m_2} 
\] (3.4)

\[
\frac{dp_1}{dt} = \frac{\gamma R_1}{v_1} \dot{m}_{e1} T_{c1} - \frac{\gamma R_1}{V_1} \dot{m}_{1e} T + \frac{\gamma R_1 T_1}{V_1} \dot{m}_{EGR} 
\] (3.5)

\[
\frac{dp_2}{dt} = \frac{\gamma R_2}{v_2} \dot{m}_{e2} T_{c2} - \frac{\gamma R_2 T_2}{V_2} \dot{m}_{EGR} - \frac{\gamma R_2 T_2}{V_2} \dot{m}_{2t} 
\] (3.6)

\[
\dot{P}_c = -\frac{1}{\tau} (\eta_m P_1 - P_c + P_m) 
\] (3.7)

\[
\frac{d\alpha}{dt} = \frac{1}{\tau_{EGR}} (-\alpha + u_1) 
\] (3.8)

\[
\frac{d\beta}{dt} = \frac{1}{\tau_{2t}} (-\beta + u_2) 
\] (3.9)

Where the states in the model are

\[m_1, m_2: \text{mass (kg) of gas in the intake and exhaust manifolds}
\]

\[F_1, F_2: \text{Burnt gas fraction in intake and exhaust manifolds}
\]

\[P_1, P_2: \text{Pressure (kPa) in intake and exhaust manifolds}
\]

\[P_c: \text{Compressor Power (kW)}
\]

\[\alpha: EGR \text{ valve position}
\]
\( \beta \): VGT vane position

**EGR Flow:**

\[
p_1 > \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad \dot{m}_{egr} = A_{eff} p_2 \sqrt{\frac{2}{R_2 T_2 (\gamma - 1)}} \left( \left( \frac{p_1}{p_2} \right)^{\frac{2}{\gamma}} - \left( \frac{p_1}{p_2} \right)^{\frac{\gamma + 1}{\gamma}} \right) \quad (3.10)
\]

\[
p_2 \leq \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad \dot{m}_{egr} = A_{eff} p_2 \sqrt{\frac{\gamma}{R_2 T_2}} \left( \frac{2}{\gamma + 1} \right)^{\gamma - 1} \quad (3.11)
\]

where \( A_{eff} = 3.58 \alpha \left( \frac{p_1}{p_2} \right)^{-48.46+0.345 N-7*10^{-4} N^2} \)

**Turbine Flow and Power**

\[
p_{atm} > \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad \dot{m}_{tr} = A_{eff} p_2 \sqrt{\frac{p_{atm}}{T_2}} \left( \left( \frac{p_{atm}}{p_2} - g + 1 \right)^{\frac{2}{\gamma}} - \left( \frac{p_{atm}}{p_2} - g + 1 \right)^{\frac{\gamma + 1}{\gamma}} \right) \quad (3.12)
\]

\[
p_{atm} \leq \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad \dot{m}_{tr} = A_{eff} p_2 \sqrt{\frac{2}{1 + \gamma}} \left( \frac{2}{\gamma - 1} - g + 1 \right)^{\frac{\gamma + 1}{\gamma}} \quad (3.13)
\]

where \( A_{eff} = -0.06 \frac{p_{atm}}{p_2} + \left( \frac{p_{atm}}{p_2} \right)^{3.296-0.698 \beta} - 0.0236 \)

and \( g = -2.057 * \beta^4 + 6.177 * \beta^3 - 7.11 * \beta^2 + 3.56 * \beta + 0.32 \)
\[ P_t = \dot{m}_2 c_p \eta_{t,ls} T_2 \left( 1 - \left( \frac{p_{atm}}{p_2} \right)^{\frac{\gamma - 1}{\gamma}} \right) \] (3.14)

Compressor Relationships

\[ \frac{T_{c,1}}{T_{atm}} = 1 + \frac{1}{\eta_{c,ls}} \left[ \left( \frac{p_{c,1}}{p_{atm}} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \] (3.15)

\[ \dot{m}_{c,1} = \frac{P_{c} \eta_{c,ls}}{T_{c,1} c_p \left( \frac{p_{c,1}}{p_{atm}} \right)^{\gamma} - 1} \] (3.16)

T/C rotational dynamics

\[ (J_{rc} + J_{EM}) \dot{\omega}_{rc} \frac{d\omega_{rc}}{dt} = \frac{1}{\tau} (\eta_m P_T - P_c + P_{em} \text{sign}[P_{em}]) \] (3.17)

Engine Rotational Dynamics

\[ J_e \dot{\omega} = T_{ind} - T_{friction} - T_{load} \] (3.18)
\[ T_{\text{ind}} = 19358 \dot{m}_{\text{fuel}} + 1.26 \times 10^5 \frac{\dot{m}_{\text{fuel}}}{N} + 5.69 \times 10^{-10} F_i \dot{m}_{ie} - 6 \times 10^{-5} N (p_2 - p_1) - 3 \times 10^{-6} T_i \]

(3.19)

\[ T_{je} = 21.66 + 10.53N + 7.68N^2 + 3.62N^3 \]

(3.20)

**NO\textsubscript{x} Emission Model**

In order to quantify the effect of TEA on the NO production of the engine being studied a mean value NO model was adopted. The model relies on the well known mechanisms that describe the NO chemical kinetics as derived in [11,16] and validated for the Fiat engine being studied in [9].

\[
\frac{dC_{\text{[NO]}}}{dt} = \theta \left[ \frac{0.78F_i \dot{m}_{ie}}{\dot{m}_{ie} + \dot{m}_f} \right] \left[ \frac{M_{EX}}{M_{OX}} \frac{0.21F_i \dot{m}_{ie} - \phi_{ST} \dot{m}_f}{\dot{m}_{ie} + \dot{m}_f} \right]^{\frac{1}{3}} - \frac{1}{\tau} C_{\text{[NO]}}
\]

(3.21)

Where \( C_{\text{[NO]}} \) is the molar concentration of NO in the exhaust feedgas, \( \theta \) is an empirical parameter determined from data, and \( \phi_{ST} \) is the stoichiometric air to fuel ratio. Since 90% of all engine out NO\textsubscript{x} is NO therefore we use NO for an emission model. It should be noted that this model ignores the NO decomposition reactions as presented in [10,16] and assumes a unity fuel use fraction, that is all fuel is combusted stoichiometrically.


**Electrical Motor model**

The d,q coordinate frame [14,15] is used to describe the dynamics of the electric motor, thus decoupling the motor dynamics such that the resulting system behaves like a DC motor.

\[
\begin{align*}
\frac{di_q}{dt} &= -\frac{R}{L} i_q - \omega \lambda_q i_d - \frac{1}{L} \lambda_0 \omega - \frac{1}{L} u_q \\
\frac{di_d}{dt} &= -\frac{R}{L} i_d + \omega \lambda_q i_q + \frac{1}{L} u_d
\end{align*}
\]

where \(i_d\) and \(i_q\) are the stator currents in the (d,q) coordinate frame; \(u_d\) and \(u_q\) are the stator voltages in the same coordinate frame. The term \(\lambda_0 \omega - e_q\) is the q-component of the induced EMF generated by the permanent magnet; the d-component of the EMF \(e_d\) is equal to zero. Note that in equation 24, if the current \(i_q\) is made to be zero then we get exactly the behavior of a constant-excited DC motor. This is the main idea of field-oriented control, to decouple the motor dynamics such that the resulting system behaves like a DC motor.

Lastly, the electrical torque and mechanical power are given by

\[
\begin{align*}
\tau_e &= K i_q \\
P &= \tau_e \omega
\end{align*}
\]
Where $K_t$, the torque constant, is assumed to be equal to $(3/2)\lambda_0 N_r$, with $N_r$ being the number of poles pairs of the motor and $\omega_r$ is the mechanical angular speed of the motor rotor, and is equal to the angular speed of the turbo charger, $\omega_{tc}$. Note that it is assumed there is no reluctance torque in the PMSM model. Additionally, for electrical angular position/speed and the mechanical angular position/speed the following relations are used:

$$\omega_e = N_r \omega_r$$
$$\theta_e = N_r \theta_r$$

(3.25)

Throughout the following sections the motor parameters used to verify the design concepts are $L=28 \ \mu H$, $R=0.0055 \ \Omega$, $J_{EM}= 0.01 \ \text{kg m}^2$, $N_r=2$, $\lambda_0=0.0252 \ \text{V s rad}^{-1}$, $K_c=0.0755 \ \text{Nm A}^{-1}$, and the supplied voltage $u_0= 340V$ similar to the system presented in [2].

### 3.2 MULTIVARIABLE ASYMPTOTIC OUTPUT TRACKING OF NONLINEAR SYSTEMS VIA OUTPUT FEEDBACK

Output tracking control via feedback linearization is a well known technique for control design of Nonlinear Systems [62]. The foundation of this method is to establish a straightforward relationship between the system input and system output. This is accomplished through differentiation of the output until the input appears explicitly in the
output dynamic equation. In this section we describe the design of a nonlinear feedback tracking controller. Consider a generic MIMO system:

\[
\dot{x} = f(x) + g(x)u \quad x \in \mathbb{R}^n \\
y = h(x) \quad y \in \mathbb{R}^m
\]  

(3.26)

differentiating the output \( y \) results in:

\[
\frac{dy_i}{dt} = \frac{\partial h_i(x)}{\partial x} \dot{x} = \frac{\partial h_i(x)}{\partial x} (f(x) + g(x)u)
\]

The number of differentiations, \( r \), required to attain an explicit input-output relationship is called the relative degree of the system. The partial relative degree, \( r_i \), is the smallest integer such that at least one of the inputs appears in \( y_i^{(r_i)} \), that is:

\[
y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^{m} L_g L_f^{r_i-j} h_j u_j \quad \text{where} \quad L_g L_f^{r_i-j} h_i \neq 0 \text{ for at least one } j \text{ in some } \Omega_i \text{ neighborhood of equilibrium } x_o.
\]

The system is said to have a vector relative degree of \((r_1, r_2, \ldots, r_m)\) at \( x_o \) and a total relative degree of \( r = \sum_{i=1}^{m} r_i \). Employing this procedure to each output yields:

\[
\begin{bmatrix}
y_i^{r_i} \\
\vdots \\
y_m^{r_m}
\end{bmatrix} = \begin{bmatrix} L_f^{r_i} h_i(x) \\
\vdots \\
L_f^{r_m} h_m(x) \end{bmatrix} + A(x)u
\]  

(3.27)
where \( A(x) = \begin{bmatrix} L_{g1} L_{j1}^{-1} h_1(x) \cdots L_{gn} L_{jn}^{-1} h_1(x) \\ L_{g1} L_{j1}^{-1} h_2(x) \cdots L_{gn} L_{jn}^{-1} h_2(x) \\ \vdots & \ddots & \vdots \\ L_{g1} L_{j1}^{-1} h_m(x) \cdots L_{gn} L_{jn}^{-1} h_m(x) \end{bmatrix} \) is an \( mxm \) nonsingular matrix at \( x = x_0 \).

If indeed the partial relative degrees are well defined we can define a finite neighborhood of \( x_0 \), \( \Omega \), as the intersection of \( \Omega_f \) and \( A(x) \) is nonsingular over \( \Omega \), then the input transformation:

\[
\begin{bmatrix} v_1 - L_j^r h_i(x) \\ \vdots \\ v_m - L_j^r h_m(x) \end{bmatrix}
\]

results in \( m \) decoupled equations of the form

\[
y_i^o = v_i
\]

where

\( v \) is chosen to meet some given control specification, e.g., to assign a specific set of eigenvalues to the tracking performance. This technique is valid only if \( A(x) \) is invertible over \( \Omega \). However if \( A(x) \) is singular alternate techniques can be applied to generate input-output linearization. Two such procedures are "Dynamic Extension" and "Output Redefinition", for more details refer to [61].

With this definition and an appropriate selection of control, \( u \), it is possible to define a dynamic equation for the error dynamics, \( y - y_d \), of the \( r \)th order. The remaining dynamics of the order \( n-r \) are referred to as the internal or zero dynamics [61] of the system. The zero dynamics are the dynamics that are rendered unobservable by the I/O linearization process. It is essential that the internal dynamics remain bounded during
tracking if the control is to be effective, in the BIBO sense. In some special cases the relative degree, \( r \), is equal to the order of the system, \( n \), and there are no internal dynamics. In this case input output linearization leads to input state linearization and both state regulation and output tracking are easily achievable.

3.2.1 OUTPUT REGULATION OF THE VGT-EGR SYSTEM

It is evident from Equation (3.21) that the NO formation rate can be minimized by decreasing \([N_2]\) and \([O]\). \(N_2\) removal with membranes has been shown to be effective, but expensive, for \(NO_x\) abatement. Formation of NO can also be reduced by decreasing the in-cylinder combustion temperature and pressure. To a large extent EGR has been proven effective in lowering the in-cylinder combustion temperature. Still further improvements are necessary. The most significant control objective for this configuration is to maintain a desired air to fuel ratio that minimizes smoke and \(NO_x\) emissions. The previously presented \(NO_x\) formulation, Equation (3.21), suggests extremely lean engine operation with high concentrations of \(O_2\) worsens \(NO_x\) [10,16]. This is certainly not the only factor contributing to elevated \(NO_x\) levels, but is the main influence associated with this control problem. Therefore the goal of the intake flow regulation will be to maintain a predetermined AFR. There are many factors that influence the desired AFR set point including:

- Drivability transients, higher AFR allows for more air supplied by electric assist in order to improve acceleration
- Torque leveling
• Combustion efficiency

• Exhaust gas temperature management. The effect of AFR on $T_{e2}$ needs to be managed for proper after treatment.

• Low Smoke/Soot Generations and Low NO$_x$.

For this problem we will focus on maintaining an AFR that achieves low smoke and low NOx. The main way to reduce NO$_x$ with the TEA system is to provide less O$_2$ to the engine. There are many transient regimes where the limited bandwidth of the conventional system provides too much O$_2$ to the engine. With the electric assist there is an opportunity to increase the bandwidth of the system and decelerate the turbo when excess air is entering the engine. Therefore it is desired to choose an AFR in the no smoke regime (AFR > 20) which also minimizes NOx. For these reasons we choose the desired set point to be $\text{AFR}^d = 25$. Similarly other AFR set points can be chosen that best optimize some engine operational criteria, such as torque demand or drivability and the control design applied similarly to achieve the desired AFR set points, established via a separate AFR optimization exercise.
The control methodology is depicted in Figure 2 as follows.

- First a RPM is chosen to meet the drive cycle vehicle velocity
- Using the desired RPM, equation 20 is used to solve for the necessary fuel mass flow rate
• Given the fuel mass flow rate the controllers for the EGR, VGT, and EM determine the necessary inputs to the system.

Once the controller determines the required supply voltage to the EM in the d,q coordinate frame it is still necessary to transform the control signal back to the a,b,c coordinate frame. For further details on this procedure the reader is referred to the following [14,15].

First the controller for the conventional VGT-EGR is presented similar to [10,11,12,13,16] and compared to the TEA plant with no EGR to show that not only does the conventional system need additional boost at times to prevent a rich AFR, but it is also needed to absorb the excess compressor energy that introduces excess air into the intake manifold leading to high O₂ concentrations. This motivates the need for an electric assist architecture, a bi-directional energy transfer turbo system. Next the controller is expanded to include EGR to investigate the effect on the load requirements on the electric assist system.

We define the following output vector, Equation (3.29), as the error between the desired and the actual F₁ and P₂ states. The motivation for this choice is clear. The F₁ state influences the AFR and can be manipulated by control input \( \dot{m}_{\text{EGR}} \). The P₂ state is chosen because it is recognized as the unstable dynamics of the system [19,12] and can be stabilized by the \( \dot{m}_{2i} \) control input. Assuming the intake charge and the fueling rate are known, the desired mass fraction of air in the intake manifold is chosen as: \( F_1^d = \frac{\text{AFR}^d \cdot \dot{m}_i}{\dot{m}_{1e}} \). The pressure in the exhaust manifold is chosen as a multiple of the
intake manifold pressure $C \cdot P_1$. This assures stability of the internal dynamics and allows $\dot{m}_{\text{EGR}}$ to achieve its desired flow rate. $C$ is also chosen such that $\frac{P_2}{P_1}$ does not increase the pumping losses to an unacceptable level, an influence seen in the volumetric efficiency eq 2.34.

\[ y_1 = F_1 - F_1^d \]
\[ y_2 = P_2 - P_2^d \]

Following the procedure outline in the previous section we solve the output tracking problem as follows:

\[ \dot{y}_1 = \dot{F}_1 - \dot{F}_1^d = \dot{F}_1 - \frac{\partial F_1^d}{\partial \dot{m}_f} \frac{d\dot{m}_f}{dt} - \frac{\partial F_1^d}{\partial \dot{m}_{\text{ie}}} \frac{d\dot{m}_{\text{ie}}}{dt} \]

assuming the fuel mass flow rate and its derivative are known and recalling the expressions for approximating intake charge and volumetric efficiency, equations 2.30 and 2.31 respectively, yields:

\[ \dot{y}_1 = \dot{F}_1 - \dot{F}_1^d = \frac{\dot{m}_{\text{EGR}} (F_2 - F_1^d) + (1 - F_1) \dot{m}_{\text{cl}}}{m_1} - \frac{AFR^d}{m_{\text{ie}}} \frac{d\dot{m}_f}{dt} + \ldots \]
\[ \ldots + \frac{AFR^d \dot{m}_f}{\dot{m}_{\text{ie}}} \frac{d\dot{m}_{\text{ie}}}{dt} \]
\[
\ddot{y}_1 = \frac{\partial \dot{y}_1}{\partial \alpha} \ddot{F}_1 + \frac{\partial \dot{y}_1}{\partial m_i} \ddot{m}_i + \frac{\partial \dot{y}_1}{\partial m_i} \frac{d m_i}{d t} + \frac{\partial \dot{y}_1}{\partial \dot{m}_i} \frac{d m_i}{d t} + \frac{\partial \dot{y}_1}{\partial \dot{m}_i} \frac{d m_i}{d t} + \ldots
\]
\[
\ldots + \frac{\partial \dot{y}_1}{\partial m_f} \frac{d m_f}{d t} + \frac{\partial \dot{y}_1}{\partial \dot{m}_f} \frac{d m_f}{d t} + \frac{\partial \dot{y}_1}{\partial P_1} \dot{P}_1 + \frac{\partial \dot{y}_1}{\partial P_2} \dot{P}_2 - \frac{\partial \dot{y}_1}{\partial \tau_{EGR}} \alpha + \frac{\partial \dot{y}_1}{\partial \tau_{EGR}} \frac{1}{u_1}
\]
(3.31)

Similarly an expression can be derived for the second output variable:

\[
\ddot{y}_2 = \dot{P}_2 - \dot{P}_2^d = \dot{P}_2 - CP_1
\]
\[
= \frac{\gamma R_2}{V_2} \left( (m_{ie} + m_f) \dot{T}_{e2} - \dot{m}_{z2} (P_{im}, P_2, T_2, \beta) \dot{T}_2 - \dot{m}_{EGR} (P_1, P_2, T_2, \alpha) \dot{T}_2 \right) + \ldots
\]
(3.32)

\[
\ldots - \frac{C \gamma R_1}{V_1} \left( \dot{m}_{ie} T_{c1} + \dot{m}_{EGR} (P_1, P_2, T_2, \alpha) \dot{T}_1 - \dot{m}_{ie} T_1 \right)
\]

\[
\ddot{y}_2 = \frac{\partial \dot{y}_2}{\partial m_ie} \frac{d m_{ie}}{d t} + \frac{\partial \dot{y}_2}{\partial m_f} \frac{d m_f}{d t} + \frac{\partial \dot{y}_2}{\partial T_{e2}} \frac{d T_{e2}}{d t} + \frac{\partial \dot{y}_2}{\partial T_{c1}} \frac{d T_{c1}}{d t} - \frac{\partial \dot{y}_2}{\partial \beta} \frac{\beta}{\tau_{2r}} + \frac{\partial \dot{y}_2}{\partial \tau_{2r}} \frac{1}{u_2} + \ldots
\]
\[
\ldots + \frac{\partial \dot{y}_2}{\partial \dot{T}_2} \frac{d T_2}{d t} + \frac{\partial \dot{y}_2}{\partial \dot{T}_1} \frac{d T_1}{d t} + \frac{\partial \dot{y}_2}{\partial \dot{T}_{c1}} \frac{d T_{c1}}{d t} + \frac{\partial \dot{y}_2}{\partial \dot{P}_1} \dot{P}_1 + \frac{\partial \dot{y}_2}{\partial \dot{P}_2} \dot{P}_2 - \frac{\partial \dot{y}_2}{\partial \tau_{EGR}} \alpha + \frac{\partial \dot{y}_2}{\partial \tau_{EGR}} \frac{1}{u_1}
\]
(3.33)

Therefore the vector relative degree is \([2\ 2]\) and the total relative degree is 4. The control design must be designed for all the flow regimes of the VGT and EGR as follows:

\[
A = \begin{bmatrix}
\frac{\partial \dot{y}_1}{\partial \alpha} & 1 & 0 \\
\frac{\partial \dot{y}_1}{\partial \beta} & 1 & 0 \\
\frac{\partial \dot{y}_2}{\partial \alpha} & 1 & 0 \\
\frac{\partial \dot{y}_2}{\partial \beta} & 1 & 0
\end{bmatrix}
\quad \text{and} \quad
A^{-1} = \begin{bmatrix}
1 \\
\frac{\partial \dot{y}_1}{\partial \alpha} \tau_{EGR} \\
\frac{\partial \dot{y}_2}{\partial \alpha} \tau_{EGR} \\
\frac{\partial \dot{y}_1}{\partial \beta} \tau_{2r} \\
\frac{\partial \dot{y}_2}{\partial \beta} \tau_{2r}
\end{bmatrix} \begin{bmatrix}
\frac{1}{\tau_{EGR}} \\
0 \\
0 \\
0
\end{bmatrix}
\]

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if \( \frac{p_1}{p_2} > \left( \frac{2}{\gamma+1} \right)^\frac{\gamma}{\gamma-1} \) then

\[
\frac{\partial y_1}{\partial \alpha} = \left( \frac{F_2 - F_1}{m_1} \right) \left[ 3.58 \left( \frac{p_1}{p_2} \right) \right] \left[ \begin{array}{c}
\frac{-48.46 + 0.345N - 7\times10^{-6}N^2}{p_2} \\
\sqrt{\frac{2\gamma}{R_2T_2(\gamma-1)}} \left( \frac{p_1}{p_2} \right)^\frac{1}{\gamma} - \left( \frac{p_1}{p_2} \right)^\frac{1}{\gamma}\end{array} \right] (3.34)
\]

and

\[
\frac{\partial y_1}{\partial \alpha} = -\gamma T_2 \left( \frac{R_2}{V_2} + \frac{CR_1}{V_1} \right) \left[ 3.58 \left( \frac{p_1}{p_2} \right) \right] \left[ \begin{array}{c}
\frac{-48.46 + 0.345N - 7\times10^{-6}N^2}{p_2} \\
\sqrt{\frac{2\gamma}{R_2T_2(\gamma-1)}} \left( \frac{p_1}{p_2} \right)^\frac{1}{\gamma} - \left( \frac{p_1}{p_2} \right)^\frac{1}{\gamma}\end{array} \right] (3.35)
\]

if \( \frac{p_2}{p_1} \leq \left( \frac{2}{\gamma+1} \right)^\frac{\gamma}{\gamma-1} \) then

\[
\frac{\partial y_1}{\partial \alpha} = \left( \frac{F_2 - F_1}{m_1} \right) \left[ 3.58 \left( \frac{p_1}{p_2} \right) \right] \left[ \begin{array}{c}
\frac{-48.46 + 0.345N - 7\times10^{-6}N^2}{p_2} \\
\sqrt{\frac{\gamma}{R_2T_2 \left( \frac{2}{\gamma+1} \right)^\frac{\gamma}{\gamma-1}}}\end{array} \right] (3.36)
\]

and

\[
\frac{\partial y_2}{\partial \alpha} = -\gamma T_2 \left( \frac{R_2}{V_2} + \frac{CR_1}{V_1} \right) \left[ 3.58 \left( \frac{p_1}{p_2} \right) \right] \left[ \begin{array}{c}
\frac{-48.46 + 0.345N - 7\times10^{-6}N^2}{p_2} \\
\sqrt{\frac{\gamma}{R_2T_2 \left( \frac{2}{\gamma+1} \right)^\frac{\gamma}{\gamma-1}}}\end{array} \right] (3.37)
\]
If
\[ \frac{p_{\text{atm}}}{p_2} > \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \]
then
\[
\hat{y}_2 = -\frac{\gamma R_2 \sqrt{T_2 p_2}}{V_2} \left( \frac{p_{\text{atm}} - g + 1}{p_2} \right)^{\frac{2-\gamma}{\gamma}} + \frac{\gamma + 1}{\gamma} \left( \frac{p_{\text{atm}} - g + 1}{p_2} \right)^{\frac{1}{\gamma}} \frac{\partial g}{\partial \beta} + \ldots
\]

\[
\partial y_2 = \frac{1}{2} \left\{ \left( \frac{p_{\text{atm}}}{p_2} - g + 1 \right) - \left( \frac{p_{\text{atm}}}{p_2} - g + 1 \right)^{\frac{\gamma + 1}{\gamma}} \right\} \frac{\partial g}{\partial \beta} + \ldots
\]

\[
\ldots \left( -0.698 \ln \left( \frac{p_{\text{atm}}}{p_2} \right)^{3.296 - 0.698 \beta} \right)
\]

(3.38)

and if
\[ \frac{p_{\text{atm}}}{p_2} \leq \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \]
where
\[
\frac{\partial g}{\partial \beta} = -8.228\beta^3 + 18.531\beta^2 - 14.22\beta + 3.56
\]

Thus yielding the following control law given in Equation (3.40), where \(k_1, k_2, k_3, \) and \(k_4\) establish the desired convergence rate. Note that there are multiple instances where the decoupling matrix, \(A\), can become singular. They are when \(F_2 = F_1\), When \(P_2 = P_1\), and when
\[
p_{\text{aim}} - g + 1 = 0.
\]
Physically this means control cannot be applied via the EGR when the mass fraction of air in the intake manifold is equal to that in the exhaust manifold, or when the exhaust manifold pressure is equal to the pressure in the intake manifold where no forward flow can be achieved. Likewise control can not be applied via the VGT when the turbine exhibits zero flow, a condition determined by the geometry parameter, \(g\).
The following figures present the performance of the proposed controller. Where design constants we chosen to be $k_1 = 2500$, $k_2 = 100$, $k_3 = 2500$, and $k_4 = 100$. 

![Image](image_url)
Figure 3.3 Exhaust to intake pressure manifold ratio for A/F ratio regulation

Figure 3.4 EGR and VGT actuator responses for A/F ratio regulation
Figure 3.5 Remaining VGT-EGR state trajectories

Figure 3.2 shows the regulation of air to fuel ratio for a fixed fueling command and fixed rpm. Figure 3.3 illustrates acceptable stabilization of the exhaust manifold pressure, meaning it allows for a physically realizable EGR flow. The EGR and VGT actuator responses are given in Figure 3.4 and the remaining states in Figure 3.5. Notice the VGT actuator becomes saturated at its fully closed position and turbine flow can not be further decreased to provide for a higher pressure ratio. It is obvious that this approach is very tedious and requires exact plant information. Thus, motivating the need for additional techniques that eliminate the need for exact plant information and that improve robustness to uncertainties and disturbances.
3.3 EXTENSION OF I/O LINEARIZATION TO SLIDING MODE FOR MIMO SYSTEMS

In this section we present the Sliding Mode method for MIMO nonlinear systems [13,62]. Considering the many empirical relationships describing the VGT-EGR system dynamics, the robustness of sliding mode control to plant uncertainties and disturbance inputs makes it an attractive technique for this application. Consider a nonlinear system of the form:

\[
\dot{x} = f(x) + \sum_{k=1}^{m} g_k(x)u_k
\]

\[
y_i = h_i(x)
\]

\[
u \in \mathbb{R}^m; \; x \in \mathbb{R}^n; \; y \in \mathbb{R}^m
\]

\[f(x), g(x), and h(x) are C^\infty vector fields on \mathbb{R}^n.\]

Now define \(S\) as a set of surfaces \(\{S_i\}\), \(i = 1\ldots m\), where \(S_i\) can be defined as a function of the \(i^{th}\) output \(y_i\) up to the \((r_i-1)^{th}\), where \(r_i\) is the equivalent relative degree of the \(i^{th}\) system output,

\[
S_i = \sum_{k=1}^{p-1} \alpha_{ik}y_i^{[k]}
\]

(3.42)
Where $\alpha_{ik}$ are design constants that are chosen to meet control specifications. For a tracking control problem $y_i^{[k]}$ is replaced with $e_i^{[k]}$ where $e_i^{[k]} = y_i^d - y_i$. To design control we examine the time derivative of equation 3.4.2.

$$
\dot{S}_i = \sum_{j=0}^{\hat{r}} \alpha_{i,j+1} y_i^{d(j)} - \sum_{j=0}^{\hat{n}} \alpha_{i,j+1} L_j h_i - \alpha_{i,r-1} \sum_{k=1}^{m} L_{ik} L_i^{k-1} h_i u_k 
$$

(3.43)

We now define the attractiveness condition for each surface such that, $S_i, \dot{S}_i < 0$. Then choose control to be

$$
u_i = M_i \text{sign}(S_i) 
$$

(3.44)

Where $M_i$ is a constant selected to insure the attractiveness condition is satisfied for $\forall \, t > t_0$. The sliding mode output regulation decomposes the dynamics of a non-linear system into 2 parts, an external (input-output) part and an internal (unobservable or zero-dynamics) part. Stability of the zero dynamics are essential for meaningful control design [13,17]. Next we illustrate this design process for the VGT-EGR system.

3.3.1 SLIDING MODE CONTROL OF THE VGT-EGR SYSTEM

The control design is first carried out for the conventional VGT-EGR system without electrical assist as is done in [10, 12, 13, 16] in order to investigate the impact on NOx. The following sliding manifolds are proposed for the EGR and VGT controllers:
\[ S_1 = (F_1 - F_1^d) + \gamma (\dot{F}_1 - \dot{F}_1^d) \]  
\[ S_2 = (P_2 - P_2^d) + \zeta (\dot{P}_2 - \dot{P}_2^d) \]

where \( S_1 \) and \( S_2 \) are used in equation 26 to control EGR and VGT actuator positions respectively, and \( \gamma \) and \( \zeta \) are design constants used to determine the rate of convergence of the error dynamics. EGR control is proposed to control AFR since when AFR is differentiated the EGR control first appears as is apparent from Equations (3.47).

\[ AFR = \frac{F_1 \cdot \dot{m}_f}{\dot{m}_{ie}} \]  

Recalling Equation (3.31) and taking the time derivative of 3.4.1 we can define the attractiveness condition to find suitable control by choosing \( M_1 \) and \( M_2 \) such that

\[ S_i \dot{S}_i < 0 \] , sliding mode is enforced, and all arguments within the sign functions are equal to zero. The time derivatives of the sliding manifolds are as follows:

\[
\dot{S}_1 = (\dot{F}_1 - \dot{F}_1^d) + \alpha (\dot{\dot{F}}_1 - \dot{\dot{F}}_1^d)
= \frac{m_{EGR}}{m_1} (F_2 - F_1) + (1 - F_1) \dot{m}_{c_1} - \frac{AFR^d}{\dot{m}_{ie}} \frac{\dot{m}_f}{dt} + \frac{AFR^d}{\dot{m}_{ie}^2} \frac{\dot{m}_f}{dt} + \ldots
\]

\[
\ldots + \alpha \left( \frac{\partial \dot{y}_1}{\partial F_1} \dot{F}_1 + \frac{\partial \dot{y}_1}{\partial \dot{m}_f} \frac{\dot{m}_f}{dt} + \frac{\partial \dot{y}_1}{\partial \dot{m}_{c_1}} \frac{\dot{m}_{c_1}}{dt} + \frac{\partial \dot{y}_1}{\partial \dot{F}_2} \dot{F}_2 + \frac{\partial \dot{y}_1}{\partial \dot{m}_{ie}} \frac{\dot{m}_{ie}}{dt} + \frac{\partial \dot{y}_1}{\partial \dot{m}_{ie}} \frac{\dot{m}_{ie}}{dt} + \ldots \right)
\]

\[
\ldots + \frac{\partial \dot{y}_1}{\partial m_f} \frac{\dot{m}_f}{dt} + \frac{\partial \dot{y}_1}{\partial \dot{m}_f} \frac{\dot{m}_f}{dt} + \frac{\partial \dot{y}_1}{\partial \dot{m}_f} \frac{\dot{m}_f}{dt} + \frac{\partial \dot{y}_1}{\partial \dot{F}_1} \dot{P}_1 + \frac{\partial \dot{y}_1}{\partial \dot{P}_2} \dot{P}_2 + \frac{\partial \dot{y}_1}{\partial \dot{T}_2} \dot{\dot{T}}_2 - \frac{\partial \dot{y}_1}{\partial \alpha} \tau_{EGR} - \alpha \tau_{EGR} \]

\[
= M_1 \text{sign}(S_1)
\]
Next the controller was simulated and evaluated for the same rpm step input as the I/O Linearization in section 3.3. The responses are presented in figures 3.6-3.9. Notice the sliding mode controller does a much better job of regulating to the desired AFR and does not allow for EGR actuator saturation.
Figure 3.6 Air to Fuel Ratio Regulation Using Sliding Mode Control

Figure 3.7 Exhaust to intake pressure manifold ratio using sliding mode
Figure 3.8 EGR and VGT actuator responses using sliding mode control

Figure 3.9 Remaining VGT-EGR State Trajectories
Next the controller was simulated for the first bag of the FTP75 drive cycle. Figure 3.10 depicts the AFR response of the controller. The pressure, actuator, and remaining system dynamics are presented in figures 3.11-3.13. Notice there are many instances where the controller can not achieve the desired AFR. A closer examination of an instance where the controller attains too rich of an AFR is given in figures 3.14 and 3.15. Notice the EGR and VGT actuators can not respond quickly enough to maintain the desired AFR. Figures 3.16 and 3.17 demonstrate an instance where the controller performance yields too lean of an AFR. Notice the EGR actuator becomes saturated. To overcome the lag and saturation of the EGR actuator the VGT actuator is next be used in conjunction to regulate AFR as opposed to maintaining a certain pressure in the exhaust manifold.

Figure 3.10 Air to fuel ratio regulation using sliding mode for FTP 75 Drive Cycle
Figure 3.11 Exhaust to intake pressure manifold ratio using sliding mode for FTP 75 Drive Cycle

Figure 3.12 EGR and VGT actuator responses using sliding mode control for FTP 75 Drive Cycle
Figure 3.13 Remaining VGT-EGR state trajectories for FTP75 drive cycle

Figure 3.14 Examination of rich AFR instance
Figure 3.15 Corresponding actuator responses for rich AFR instance

Figure 3.16 Examination of lean AFR instance
3.3.2 AFR REGULATION VIA SLIDING MODE CONTROL OF THE VGT-EGR ACTUATORS

It is known that AFR regulation cannot be achieved through EGR alone therefore the VGT actuator is used to force $S_1$ to zero as well. Therefore both VGT and EGR actuators are being used to regulate AFR and the unstable zero dynamics, the exhaust manifold pressure state [12, 18], are treated as stable and are ignored. To compensate for the EGR valve lag and saturation presented in the previous section the following sliding manifold selections are proposed.
\[ S_1 = (AFR - AFR^d) + \gamma (A\dot{FR} - A\dot{FR}^d) \]  
\[ S_2 = (S_1) + \zeta (S_1) \]  

With this selection of sliding manifolds the vane angle is used in conjunction with the EGR valve to regulate AFR. Assuming the time derivatives of fuel flow rate, RPM and volumetric efficiency are all known the sliding manifolds consist of the following:

\[ AFR = \frac{F_i \cdot \dot{m}_f}{\dot{m}_{ie}} \]

\[ A\dot{FR} = \frac{\partial AFR}{\partial F_i} \dot{F}_1 + \frac{\partial AFR}{\partial \dot{m}_f} \dot{m}_f + \frac{\partial AFR}{\partial \dot{m}_{ie}} \dot{m}_{ie} \left( \frac{\partial \dot{m}_{ie}}{\partial \dot{m}_1} \ddot{m}_1 + \frac{\partial \dot{m}_{ie}}{\partial \dot{N}} \ddot{N} + \frac{\partial \dot{m}_{ie}}{\partial \dot{\eta}_v} \ddot{\eta}_v \right) \]

\[ A\ddot{FR} = \frac{\partial A\dot{FR}}{\partial P_1} \ddot{P}_1 + \frac{\partial A\dot{FR}}{\partial P_2} \ddot{P}_2 + \frac{\partial A\dot{FR}}{\partial \ddot{m}_2} \dddot{m}_2 + \frac{\partial A\dot{FR}}{\partial \dddot{m}_1} \dddot{m}_1 + \frac{\partial A\dot{FR}}{\partial \dddot{\eta}_v} \dddot{\eta}_v + \cdots \]

Next the controller was also simulated for the first bag of an FTP75 drive cycle. Figure 3.18 depicts the AFR response of the controller. Again, there are many instances where the controller can not achieve the desired AFR. Figure 3.19 gives the pressure ratio performance for the controller. Take note that the exhaust manifold pressure does not become unstable. A closer examination of an instance where the controller attains too rich of an AFR is given in figures 3.22 and 3.23. Notice the EGR and VGT actuators still become saturated and can not maintain the desired AFR, thus additional boost is
required. Figures 3.24 and 3.25 demonstrate an instance where the controller performance yields too lean of an AFR. Again, notice the EGR and VGT actuators become saturated. This case demonstrates an instance where it is desirable to remove energy from the turbo-charger, thus motivating the need for additional actuation such as turbo electric assist.

Figure 3.18 Air to fuel ratio regulation using new sliding manifold for bag 1 of FTP 75 Drive Cycle
Figure 3.19 Exhaust to intake pressure manifold ratio using new sliding manifold for bag 1 of FTP 75 Drive Cycle

Figure 3.20 EGR and VGT actuator responses using new sliding manifold for bag 1 of FTP 75 Drive Cycle
Figure 3.21 Remaining VGT-EGR state trajectories for bag 1 of FTP75 drive cycle

Figure 3.22 Examination of rich AFR instance for new sliding manifold
Figure 3.23 Corresponding actuator responses for rich AFR instance

Figure 3.24 Examination of lean AFR instance for new sliding manifold
3.4 AFR REGULATION OF THE ELECTRICALLY ASSISTED VGT SYSTEM

The previous section motivated the need for additional turbo assist and turbo de-energizing for tighter AFR regulation. Not only does the conventional system need additional boost at times to prevent a rich AFR, but it is also needs to absorb the excess compressor energy that introduces excess air into the intake manifold and high O₂ concentrations, thus motivating the need for electric assist, a turbo architecture that is capable of supplying additional turbo energy as well as absorbing excess energy.

Next the sliding mode controller is designed for the electrically assisted VGT system with no EGR (\(\alpha = 0\)). The following sliding manifolds are proposed.
Where $S_3$ is used to regulate $i_q$ and $S_4$ regulates $i_d$. $i_d^*$ is chosen as zero such that the resulting electrical system behaves like a DC motor. $S_2$ is used to influence the power supplied or absorbed by the electric motor with the VGT and is defined as the current necessary to force $S_3 = 0$. For this case we assume the desired motor current is zero. However, it could be chosen to be any attainable value depending on the state of the energy storage system. Then chose $M_i$, $i=2$ to 4 in Equation (3.44), such that sliding mode is enforced, i.e. all arguments within the sign functions are equal to zero. Even though the electrical dynamics are included in the system, this is an idealization because it is assumed that the electric motor can achieve the desired power at all times, also any AFR sensing dynamics are ignored.

First the TEA controller was evaluated against the conventional VGT-EGR system for stepped desired AFR of 25 and a constant fuel rate of 0.0026 kg/s. The results are compared with the step response of a conventional VGT-EGR system in figures 3.26-3.33. As expected the conventional system reaches the desired AFR for this particular fuel rate must slower than the system with additional boost. Meaning the TEA response spends less time in the smoke producing regime. Notice the rapid rise in boost pressure as seen in figure 3.28 attributed to the energy added by the electric motor given in figure 3.32.
Figure 3.26 AFR step response for TEA compared with conventional VGT-EGR

Figure 3.27 EGR and VGT actuator responses for stepped AFR with TEA compared with conventional VGT-EGR
Figure 3.28 Compressor mass flow rate for TEA compared with conventional VGT-EGR

Figure 3.29 Boost Pressure for TEA compared with conventional VGT-EGR
Figure 3.30 Exhaust manifold pressure for TEA compared with conventional VGT-EGR

Figure 3.31 Intake charge for TEA compared with conventional VGT-EGR
Figure 3.32 Turbocharger speed for TEA compared with conventional VGT-EGR

Figure 3.33 Compressor, Turbine, and EM power for TEA compared with conventional VGT-EGR
Next the controller was evaluated for a stepped sequence ranging from 25 to 85 for the same fixed fueling rate. The AFR response is depicted in Figure 3.34. Notice the TEA system has no problem achieving any of the desired air to fuel ratios, while the conventional system can only maintain the first desired AFR of 25. As seen in figures 3.35-3.42, the VGT actuator becomes saturated and cannot supply sufficient air through the compressor.
Figure 3.35 AFR response for stepped AFR sequence with TEA compared with conventional VGT-EGR

Figure 3.36 EGR and VGT actuator responses for stepped AFR sequence with TEA compared with conventional VGT-EGR
Figure 3.37 Compressor mass flow rate for TEA compared with conventional VGT-EGR

Figure 3.38 Boost Pressure for TEA compared with conventional VGT-EGR
Figure 3.39 charge for TEA compared with conventional VGT-EGR

Figure 3.40 Turbocharger speed for TEA compared with conventional VGT-EGR
Figure 3.41 Compressor, Turbine, and EM power for TEA compared with conventional VGT-EGR

Figure 3.42 Intake manifold mass for TEA compared with conventional VGT-EGR
The controller was applied over Bag1 of the simulated Federal Test Procedure, FTP75 drive cycle. Figure 3.44 compares the simulated AFR responses, recalling a desired AFR of 25 is chosen for both cases in order to reduce the \([\text{O}_2]\) concentration in the engine while remaining above the smoke limit of the engine. Notice the assisted system maintains the desired AFR throughout the entire drive cycle.
Figure 3.44 Comparison of Air to fuel ratio regulation for turbo assisted and conventional system for bag 1 of FTP 75 Drive Cycle

Figure 3.45 compares the VGT and EGR actuators for the two configurations; notice the VGT actuator goes into saturation very often for this case. There are several instances in Figure 3.45 where the VGT vane is fully open and the AFR is still below the desired set point at the corresponding time as seen in Figure 3.44. Therefore additional boost is still required to maintain the desired AFR even though the VGT actuator cannot provide any further boost. Conversely, when the VGT actuator is saturated at its minimum angle, there are many instances where the AFR is far above the desired AFR. Therefore during these occurrences the excess energy of the T/C needs to be absorbed by the EM in order to prevent excess air from entering the engine.
Figure 3.45 Comparisons of EGR and VGT actuator responses for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle

The additional control authority of TEA reduces the air flow relative to the conventional system by decelerating the T/C as seen Figure 3.46. Notice the regenerative capability of TEA also provides a lower average intake manifold pressure as seen in Figure 3.47. The reduction in pressure allows for better EGR management as will later become apparent.
Figure 3.46 Comparison of compressor flow rate for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle

Figure 3.47 Comparison of boost pressure for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle
The required power of the electric motor to maintain the desired AFR is compared against the turbine and compressor powers for both systems in Figure 3.48. Observe that a majority of the time the electric motor is spent in the regenerative mode. The cumulative power of the motor for the bag of the drive cycle is -333 kJ; hence there is a significant amount of energy available from the turbo charger. The absorption of this energy has also significantly decelerated the T/C as seen in Figure 3.49. The lower RPM is good for the lifetime and durability of the T/C and the high speed bearings.

One concern for VGT-EGR control solution has been to ensure that the exhaust pressure does not grow without bound. Figure 3.50 shows the exhaust manifold pressure for both systems do not grow without bound even though no control is applied with VGT to ensure stability. This is because the limited mass flow rate into the system ensures stability.

The possibility of reduced NOx benefit due to optimal air flow can translate to reduced EGR flow rates necessary to achieve air mass fractions compatible with desired NOx production. Figure 3.51 compares the air mass fraction for the two configurations and it is apparent that even though conventional EGR can achieve air mass fractions less than 0.8 it still cannot lower the concentration of [O2] to the level of the electrically assisted system as is apparent by the prediction of substantial reduction of NOx concentration in exhaust as seen Figure 3.52.
Figure 3.48 Comparison of compressor, turbine, and EM power for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle

Figure 3.49 Comparison turbo speed for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle
Figure 3.50 Comparison of exhaust manifold pressure for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle

Figure 3.51 Comparison intake manifold air mass fraction for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle
Reduction in exhaust gas NO\textsubscript{x} concentration obviously results in lower NO\textsubscript{x} engine production. Figure 3.53 shows that there has been a significant reduction in the amount of NO produced by the engine. The conventional system produced a predicted 2.49 grams while the TEA system produced 1.41 grams.

Figure 3.52 Prediction of NO concentration in exhaust feedgas for electrical assisted and conventional system for bag 1 of FTP 75 Drive Cycle
3.5 NOX EMISSION STANDARDS

Assessment of diesel engine emission performance is typically quantified on a grams per mile basis. Using EPA Equation (3.54) it is possible to extrapolate for an estimation of the amount of grams of NO produced per mile for the entire FTP75 drive cycle. Typically the first bag of the cycle accounts for roughly 54% of the total NO produced for the entire cycle, the second bag accounts for 15%, and the third 31%. Therefore knowing the predicted production of the first bag for the TEA system is 1.41 grams and the conventional engine is 2.49 grams, using Equation (3.54) it can be
estimated that the assisted system produces 0.2 grams per mile compared to the .34 grams per mile for the conventional VGT-EGR system.

\[
\frac{\text{mass } NO_x}{\text{mile}} = 0.43\left(\frac{\text{mass } NO_x \text{ for Bag1}}{\text{dis tan ce(Bag1 + Bag2)}}\right) + 0.43\left(\frac{\text{mass } NO_x \text{ for Bag2}}{\text{dis tan ce(Bag1 + Bag2)}}\right) \\
+ 0.57\left(\frac{\text{mass } NO_x \text{ for Bag2}}{\text{dis tan ce(Bag2 + Bag3)}}\right) + 0.53\left(\frac{\text{mass } NO_x \text{ for Bag3}}{\text{dis tan ce(Bag2 + Bag3)}}\right)
\] (3.54)

Medium duty diesel engine NOx requirements are 0.05 grams per mile at the tail pipe. Assuming a modern after treatment system with 90% conversion efficiency, the engine may produce roughly .35 to .40 grams per mile prior to after treatment and therefore both systems meet the requirements. However, the idealistic simulations with the TEA system show conversion requirements on the after treatment system maybe reduced by 42%.

The previously shown comparison mainly focused on the influence of the electric motor as it pertains to NOx emissions when run as a generator and is mainly braking/decelerating the T/C. The following comparison shows the effect of the TEA when additional boost is required. Figures 3.54 demonstrates an instance during the drive cycle where the VGT actuator is fully opened, as previously seen in figure 3.45, and the conventional system cannot provide enough air to the engine and the operating condition comes close to the smoke producing regime. Observe power is added to the compressor from the motor during this transient as seen in Figure 3.55 and boost pressure is rapidly increased as seen in Figure 3.56.
Figure 3.54 Instance where conventional system cannot provide sufficient air to the engine whereas this limitation is overcome by TEA

Figure 3.55 Compressor, Turbine, and EM comparison for boost situation
3.6 AFR REGULATION OF THE ELECTRICALLY ASSISTED VGT-EGR SYSTEM

The previous section demonstrated the advantage of turbo electric assist for tighter AFR regulation. Not only is the electric motor needed for additional boost to prevent rich AFR excursions but it is also predominantly needed to brake/decelerate the T/C to curb the undesired excess air during T/C decelerations of the conventional system. In order to reduce the regenerative load requirements on the electric motor EGR is introduced. EGR control can be chosen to influence the desired current determined from Equation (3.51) just as the VGT is used to influence it in Equation (3.51) in the previous section. The following sliding manifolds are proposed:
Setting equation 3.51 equal to zero and solving for \( u_1^* \) yields \( u_1^* \). Again the desired motor current is chosen to be zero. \( S_1 \) is chosen as the integral of the difference of the desired EGR control input since with sliding mode \( u_1 \) must take discontinuous values, therefore when sliding mode is reached \( u_{1eq}=u_1^* \). Again \( S_3 \) is used to regulate \( i_q, i_d^* \) is chosen to be 0, and \( S_2 \) is also used to influence the desired motor current.

Next the controller was evaluated for the first bag of the FTP75 drive cycle and compared against the TEA system with no EGR. Again the desired AFR is maintained throughout the drive cycle as seen in figure 3.57.

\[
S_1 = \int_0^1 (u^*_1(\zeta) - u_1(\zeta)) d\zeta \tag{3.55}
\]

\[
S_2 = i_q(S_3) \tag{3.56}
\]

\[
S_3 = (AFR - AFR^d) + \gamma(A\dot{F}R - A\dot{F}R^d) + \zeta\left(\frac{(1 - 1)}{(1 - 1)}\right) + \gamma\left(A\dot{F}R - A\dot{F}R^d\right) \tag{3.57}
\]

\[
S_4 = i^*_d - i_d \tag{3.58}
\]
Figure 3.57  Air to fuel ratio regulation for electrical assisted system with EGR for bag 1 of FTP 75 Drive Cycle

Figure 3.58 shows there is a dramatic influence on the EGR on the load requirement of the EM. The introduction of EGR lowered the total regenerative requirements of the EM to -89 kJ from -333 kJ as compared to the TEA system without EGR. Reduction of the amount of energy required for the EM to absorb will also ease the requirements of the associated cooling system for the motor/generator.
Figure 3.58 Required electrical motor power with and without EGR.

The same engine out NOx concentration is predicted with EGR as for the case without as seen in Figures 3.52 and 3.53. This is because the NOx model is not based on temperature but only \([O_2]\) concentration and both systems see the same engine oxygen concentration since they are maintaining the same AFR.
Figure 3.59 Predicted NO\textsubscript{x} concentration for electrical assisted system with EGR for bag 1 of FTP 75 Drive Cycle

It is also interesting to note the effect EGR has on the T/C when it goes from a decelerated to an accelerated state as seen in Figures 31-34. These show two instances where boost is required to the compressor and less power is required from the electrical motor when EGR was used because previously in the drive cycle less energy was removed from the compressor in order to maintain the desired AFR when excess compressor flow was being introduced.
Figure 3.60 Comparison of required boost from electrical motor for system with and without EGR

Figure 3.61 Comparison of compressor under assist for system with and without EGR
Figure 3.62 Comparison of required boost from electrical motor for system with and without EGR

Figure 3.63 Comparison of compressor under assist for system with and without EGR
CHAPTER 4

OBSERVER DESIGN FOR VGT-EGR SYSTEM

As discussed earlier, not all states necessary for control implementation are readily available. For the purpose of this work it is assumed that the unavailable states are $F_1$ (intake manifold air mass fraction), $F_2$ (exhaust manifold air mass fraction, and $P_2$ (exhaust manifold pressure). Estimating exhaust manifold pressure in a turbocharged diesel engine has been successfully documented previously [10,58, 59, 60]. In this work, the observer design is extended to include the influence of EGR as well as estimate the oxygen fraction states in the intake and exhaust manifolds, $F_1$ and $F_2$ respectively. A seven state engine model as proposed in [12, 57] is used. The engine is divided into multiple subsystems and each is modeled individually, as shown in Figure 2.1. The complete model and the relevant empirical fits for a 2.4 Liter inline 5 Fiat TDI engine are presented in the Chapter 2. Observer design is based on the assumption of the availability of the following measurements: $P_1$, $T_1$, $T_2$, $T_{e2}$, $m_{i_1}$, $m_f$, and $N$. The necessary sensor set is presented in Table 1.
Table 4.1 Sensor set

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measurement</th>
<th>Sensor availability in standard set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>Intake manifold pressure</td>
<td>MAP</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Intake manifold temperature</td>
<td>IAT</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Exhaust manifold temperature</td>
<td>EGT</td>
</tr>
<tr>
<td>$T_{e2}$</td>
<td>Engine out Exhaust temperature</td>
<td>EGT/regression</td>
</tr>
<tr>
<td>$m_{c1}$</td>
<td>Compressor air flow</td>
<td>MAF</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Fuel mass flow rate</td>
<td>MFDES</td>
</tr>
<tr>
<td>$N$</td>
<td>Engine speed</td>
<td>RPM</td>
</tr>
</tbody>
</table>

Where MAP is the manifold absolute pressure sensor, IAT is the intake air temperature sensor, EGT is the exhaust gas temperature sensor, $T_{e2}$ can be obtained by either an additional EGT or estimated by regression with equation A-13, MAF is the mass air flow sensor, and MFDES is an internal PCM calculation, mass fuel desired.
4.1 SLIDING MODE OBSERVER DESIGN FOR VGT-EGR SYSTEM

The observer problem is solved in two steps. First the observers proposed in equations (1)-(4) allows us to obtain information about the empirical formulations for $\dot{m}_{\text{EGR}}$, $\dot{m}_{\text{te}}$, $\dot{m}_{2r}$, and $P_2$. In the second step, $F_1$ and $F_2$ are estimated substituting the equivalent control information into equations (4.5) and (4.6).

\[
\frac{d\hat{p}_1}{dt} = \frac{\gamma R_1}{V_1} (\dot{m}_{c_1} T_{c_1} + M_4 \text{sign} (p_1 - \hat{p}_1)) \tag{4.1}
\]

\[
\frac{d\hat{m}_1}{dt} = \dot{m}_{c_1} + M_2 \text{sign} (m_1 - \hat{m}_1) \tag{4.2}
\]

\[
\frac{d\hat{P}_c}{dt} = -\frac{1}{\tau_{ic}} \hat{P}_c + M_3 \text{sign} (P_e - \hat{P}_c) \tag{4.3}
\]

\[
\frac{d\hat{T}_2}{dt} = M_4 \text{sign} (T_2 - \hat{T}_2) \tag{4.4}
\]

\[
\hat{F}_1 = \frac{(1 - \hat{F}_1) \dot{m}_{c_1} + \hat{m}_{\text{EGR}} (\hat{P}_2 - \hat{F}_1)}{m_1} \tag{4.5}
\]

\[
\hat{F}_2 = \frac{(\hat{F}_1 \dot{m}_{c_1} - AFR \dot{m}_f - (\dot{m}_{c_1} + \dot{m}_f) \hat{F}_2)}{\hat{m}_2} \tag{4.6}
\]

\[
\dot{m}_2 = \frac{\hat{P}_2 V}{R_2 T_2} \tag{4.7}
\]
4.1.1 EXHAUST MANIFOLD PRESSURE OBSERVER

For some situations in the presence of plant disturbances and modeling uncertainties, it may be acceptable to use equivalent control methods to extract the unknown or unmodeled information [13]. For this particular nonlinear observer design, the empirical representations of the VGT, EGR, and intake charge flows are excluded. Eliminating the need for these inexact flow equations can eliminate the uncertainties in the observer design and improve their performance. Subtracting Equations 4.1-4.4 from their respective model equations given in the Appendix results in the follow mismatch dynamics:

\[
\frac{d\bar{p}_1}{dt} = \frac{\gamma R_1}{V_1} \left( \dot{m}_{1e} T_1 + \dot{m}_{EGR} T_2 - M_1 \text{sign}(\bar{p}_1) \right) \tag{4.8}
\]

\[
\frac{d\bar{m}_1}{dt} = -\dot{m}_{1e} + \dot{m}_{EGR} - M_2 \text{sign}(\bar{m}_1) \tag{4.9}
\]

\[
\frac{d\bar{P}_c}{dt} = -\frac{1}{\tau} \bar{P}_c + \frac{\eta_m}{\tau} \dot{m}_{2e} C P_{atm} \eta_{T_1,h} T_2 \left( 1 - \frac{P_{atm}}{P_2} \right)^{\frac{\gamma - 1}{\gamma}} - M_3 \text{sign}(P_c - \hat{P}_c) \tag{4.10}
\]

\[
\frac{d\bar{T}_2}{dt} = \frac{T_2^2 R}{P_2 V_2} \left( \frac{\gamma T_{c_2}^2}{T_2^2} - 1 \right) \left( \dot{m}_{1e} + \dot{m}_f \right) - (\gamma - 1) \dot{m}_{EGR} - (\gamma - 1) \dot{m}_{2e} - M_4 \text{sign}(T_2 - \hat{T}_2) \tag{4.11}
\]

Then choose $M_j, j=1$ to 4, such that sliding mode is enforced such that all arguments within the sign functions are equal to zero.
The motion in sliding mode is an idealization, realistically various imperfections make the estimated states oscillate in some vicinity of intersection between the estimates and measurements. The sign functions then switch at some finite frequency, alternately taking values of $M_j^+$ and $M_j^-$. These oscillations have high and low frequency components. The high frequency component is filtered out by the observer while its motion in sliding mode is determined by the slow component. It is reasonable to assume that the equivalent control is close to the slow component of the switching function [13] and may be derived by filtering out the high frequencies with a low-pass filter. Filtering the sliding mode terms with a low pass filter with a sufficiently small time constant to preserve the slow component but large enough to eliminate the high frequency component such as:

\[
M_j \text{sign}(\bar{e}_j)_{eq} = \frac{1}{\tau_s + 1} M_j \text{sign}(\bar{e}_j)
\]

Using these equivalent control methods allows for the necessary following information to be extracted:

\[
M_1 \text{sign}(\bar{p}_1)_{eq} = \dot{m}_{te} T_1 + \dot{m}_{EGR} T_2 \quad (4.12)
\]

\[
M_2 \text{sign}(\bar{m}_1)_{eq} = -\dot{m}_{te} + \dot{m}_{EGR} \quad (4.13)
\]
Solving equations (4.12) and (4.13) for \( \dot{m}_{EGR} \) and \( \dot{n}_{ie} \) results in the following estimates of the EGR and engine intake charges flows.

\[
\dot{m}_{EGR} = M_2 \text{sign}(\overline{m}_1)_{eq} + \frac{M_1 \text{sign}(\overline{p}_1)_{eq} - M_2 \text{sign}(\overline{m}_1)_{eq} T_2}{T_2 - T_1}
\]

Note that equations (4.16) and (4.17) become singular when \( T_2 = T_1 \), however it is extremely unlikely for this event to be physically realizable. Next it is necessary to solve for \( P_2 \) and \( \dot{m}_{2t} \) using equations (4.14) and (4.15). However an explicit solution is difficult to acquire because of the non-integer power of \( P_2 \) in equation (4.14). Therefore \( \dot{m}_{2t} \) and \( P_2 \) can be approximated with the following procedure. First solve (4.15) for \( \dot{m}_{2t} \) and substitute in equation (4.14) to yield the following.
\[
f(P_2) = M_3 \text{sign}(\bar{P})_{eq} = \\
\frac{\eta_m}{\tau} \left[-M_4 \text{sign}(T_2 - \hat{T}_2)_{eq} \frac{P_{t2}}{T_2^2 R} \left(\frac{\gamma T_2}{T_2} - 1\right) \left(\hat{m}_{in} + \hat{m}_f\right) - (\gamma - 1)\hat{m}_{EGR}\right]
\]

(4.18)

To stay consistent with the sliding mode methodology we implement the following solution search procedure.

\[
\hat{P}_2 = -M_5 \text{sign} \left(\frac{\partial f}{\partial P_2} \left(f(P_2) - f(\hat{P}_2)\right)\right)
\]

where \(f(P_2) = M_3 \text{sign}(\bar{P})_{eq}\)

(4.19)

This results in the following error dynamics:

\[
\frac{\partial (f(P_2) - f(\hat{P}_2))}{\partial t} = -M_5 \left|\frac{\partial f}{\partial P_2}\right| \text{sign}(f(P_2) - f(\hat{P}_2))
\]

(4.20)

And as \(f(P_2) \to f(\hat{P}_2)\), \(\hat{P}_2 \to P_2\) as long as \(\frac{\partial f}{\partial P_2} \neq 0\). \(\frac{\partial f}{\partial P_2} = 0\) is an impractical scenario since \(f(P_2)\) is essentially turbine power, which is monotonically increasing with \(P_2\).
4.1.2 INTAKE AND EXHAUST MANIFOLD AIR MASS FRACTION OBSERVERS

Now that the P2 observer has been designed we continue with the design of the F1 and F2 observers. Typical observers are exact model replications with the mismatch between some observed and measured state as the forcing function. For some situations where the plant is stable, it may be acceptable to use an exact plant model without the use of mismatch functions, if the rate of convergence satisfies the design criterion. For this particular system, we use an exact system model (4.5) and (4.6), or an on-line simulation of the model, and analyze the convergence of the error dynamics as follows:

\[
\dot{\hat{m}}_{AFR} = \frac{\dot{m}_{EGR} (\dot{\hat{P}}_2 F_2 - \dot{m}_{EGR} F_2 - \dot{\hat{m}}_{EGR} (\dot{\hat{P}}_2 F_2 + \dot{m}_{EGR} F_1))}{m_1}
\]

(4.21)

\[
\dot{\hat{m}}_{1e} = \left(\frac{\dot{\hat{F}}_1 m_{1e}}{\dot{\hat{m}}_{1e}} - \frac{\dot{\hat{F}}_1 m_{1e}}{m_{1e}}\right) - \left(\frac{AFR \dot{m}_{f}}{\dot{\hat{m}}_{2}} - \frac{AFR \dot{m}_{f}}{m_{2}}\right) - \left(\frac{\dot{m}_{1e} + \dot{m}_{f}}{\dot{\hat{m}}_{2}} - \frac{\dot{m}_{1e} + \dot{m}_{f}}{m_{2}}\right)
\]

(4.22)

where \(\dot{\hat{m}}_{EGR}(\dot{\hat{P}}_2)\) and \(\dot{\hat{m}}_{1e}\) are the estimated EGR and VGT flow rates given by equations (4.16) and (4.17).

Then after convergence of the P2 observer, \(m_2\) is obtained algebraically from equation (4.7) and the resulting F1 and F2 error dynamics are reduced further to:
To analyze the stability of the error dynamics equations (4.23) and (4.24) may be treated as a linear time varying system. The asymptotic stability of the error dynamics may be investigated using Lyapunov’s method.

\[ PA + A^T P = -Q \] 

(4.25)

Where Q is a the symmetric and positive definite identity matrix, I_{2x2}. Solving equation (4.23) for P, results in the following positive definite matrix:

\[ P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \]

(4.26)

where

\[ P_{11} = \frac{m_1}{2} \left( m_1 m_f + 2 m_1 m_1 + 2 m_e m_1 m_f + m_1 m_e m_c + m_2 m_e m_1 + m_1 m_1 m_c + m_1 m_1 m_c + m_1 m_1 m_c + m_1 m_1 m_c + m_1 m_1 m_c \right) \]

(4.27)
\[ P_{21} = \frac{m_1 m_2}{2} \begin{bmatrix} \dot{m}_{e_1} \dot{m}_{i_1} + 2 \dot{m}_{EGR} \dot{m}_{i_1} + \dot{m}_{f} \dot{m}_{EGR} \\ \dot{m}_{i_{e_1}} m_1 + \dot{m}_{f} m_{i_1} + \dot{m}_{EGR} m_2 + m_{i_{e_1}} m_1 \dot{m}_{e_1} + \\ 2 \dot{m}_{f} m_{i_1} \dot{m}_{EGR} + m_1 \dot{m}_{f} \dot{m}_{EGR} + m_{i_{e_1}} \dot{m}_{i_1} \\ \end{bmatrix} \]  

(4.28)

\[ P_{22} = \frac{m_2}{2} \begin{bmatrix} m_{i_{e_1}} m_1 + 2 m_{i_{e_1}} + m_{i_{e_1}} \dot{m}_{i_1} + 2 m_{i_{e_1}} \dot{m}_{EGR} \\ m_{i_{e_1}} m_1 + 2 m_{i_{e_1}} \dot{m}_{EGR} + \dot{m}_{i_{e_1}} \dot{m}_{i_1} + m_{i_{e_1}} \dot{m}_{i_1} + m_{i_{e_1}} \dot{m}_{i_1} + m_{EGR} m_2 \\ \end{bmatrix} \]  

(4.29)

\[ \text{det}(P) = \frac{1}{4} \begin{bmatrix} \frac{2 m_{i_{e_1}} \dot{m}_{i_1} + 2 m_{i_{e_1}} \dot{m}_{EGR} m_2 + 2 m_{i_{e_1}} \dot{m}_{EGR} \dot{m}_{i_1}}{m_1 m_2 + m_{i_{e_1}} m_1 + m_{i_{e_1}} m_1 + m_{EGR} m_2 \dot{m}_{i_1} + m_{i_{e_1}} \dot{m}_{i_1} + m_{i_{e_1}} \dot{m}_{i_1} + m_{EGR} m_2} \end{bmatrix} \]  

(4.30)

Since \( \dot{m}_{i_1} \), \( \dot{m}_{EGR} \), \( \dot{m}_{i_{e_1}} \), \( \dot{m}_{f} \), \( m_{i_1} \), and \( m_2 \) are all \( > 0 \), and \( \dot{m}_{EGR} \geq 0 \), \( t > 0 \), \( \text{det}(P_{11}(t)) > 0 \) and \( \text{det}(P(t)) > 0 \), then \( P(t) \) is positive definite. Consider the Lyapunov function candidate:
\[ V(t, x) = x^T P(t) x \text{ where } x^T = [F_1, F_2] \] (4.31)

The derivative \( V(t, x) \) along the time trajectories of the system is given by

\[
\dot{V}(t, x) = x^T P(t) \dot{x} + \dot{x}^T P(t) x + x^T \dot{P}(t) x \\
= x^T P(t) A(t)x + x^T A(t)^T P(t) x + x^T \dot{P}(t) x \\
= -x^T (Q - \dot{P}(t)) x 
\] (4.32)

With the assumption that \( \|\dot{P}(t)\| < \varepsilon \), or the rate of convergence for the observer is faster than the dynamics of the plant, then \( Q - \dot{P}(t) \) is positive definite, \( \dot{V}(t, x) \) is negative definite, and the error dynamics are exponentially stable. The \( F_1 \) and \( F_2 \) error dynamics are asymptotically stable and after some time \( F_1 \to 0 \) and \( F_2 \to 0 \). Since the \( \dot{P}(t) \) matrix is difficult to evaluate, it may be of interest to evaluate it for one set of initial conditions. Figure 4.1 shows the determinants of \( Q - \dot{P}(t) \) for one case. It shows that for this instance \( Q \) clearly dominates and positive definiteness is preserved.
4.2 SIMULATION RESULTS

Next, the observer was simulated to demonstrate feasibility. The equivalent control terms were obtained with $M_1=100$, $M_2=1$, $M_3=1$, $M_4=400$, $M_5=10$, and a filter constant of $\tau=0.01$. The observer was simulated over a portion of the FTP75 drive cycle to illustrate its feasibility. The performance of $P_2$ observer can be seen in Figure 4.2 and its rapid convergence is made apparent in Figure 4.3. The $F_1$ and $F_2$ observers were given initial conditions of zero, an impractical scenario, just to observe the rate of convergence. Just as the exhaust manifold pressure observer both air mass fraction observers converge within 1 second and perform well over the simulated drive cycle as can be seen in Figures 4.4-4.7. Thus suggesting no control terms are required for convergence of the air mass fraction observers.
Figure 4.2 Simulated performance of P₂ Observer over FTP75 drive cycle

Figure 4.3 Convergence of P₂ Observer
Figure 4.4 Simulated performance of F1 Observer over FTP75 drive cycle

Figure 4.5 Convergence of F1 Observer
4.3 VALIDATION RESULTS

The Fiat engine used in this research was tested on an eddy current steady state dynamometer. Temperature and pressure were measured with K-type thermocouples and
strain gage type pressure transducers respectively. Air flow was measured with a fan-type air volume flow meter.

Some of the more complicated measurements included the mixture in the intake manifold and the EGR flow rate. Intake manifold mixture can be calculated with a measurement of the intake CO₂ percentage compared to ambient levels of CO₂. EGR flow rate can be measured with a high degree of accuracy with an orifice type flow meter. Seeing widespread use as a feedback device for EGR flow control in SI engines, these flow meters use a differential pressure transducer to monitor mass flow. Such sensors are less common in Diesel engines due to higher EGR flow rates and large amounts of particulate matter that could change the flow characteristics of the sharp edge orifice. For short-term test cell measurements, such a sensor is appropriate.

To exercise the EGR and VGT independently of the specific ECU command, a dSpace-based interface was used. A separate driver circuit was interfaced to the EGR vacuum control solenoid. The VGT vane position was controlled using a modulated compressed air source in place of the air supply to the actuator from the compressor outlet. With this setup, various command input profiles were possible for both the EGR valve and VGT orifice.

Next the observer performance was evaluated against actual engine data for the 2.4L Fiat VGT-EGR diesel engine studied in [7], presented in the Appendix. The engine was given consecutive stepped VGT commands. First, from a fully close vane to 75% open, then from a fully closed vane to a fully open vane. The engine was run at a constant speed, 2050 RPM, constant fueling rate, 7.9100e-004 kg/s, and maximum EGR. Figures 4.8, 4.9, and 4.10 verify the performance of the observers against data.
Figure 4.8 Performance of P₂ Observer for stepped VGT commands and maximum EGR

Figure 4.9 Performance of F₁ Observer for stepped VGT commands and maximum EGR
Figure 4.10 Performance of $F_2$ Observer for stepped VGT commands and maximum EGR
CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 DIESEL VGT-EGR ENGINE WITH TEA MODELING AND CONTROL

The development and validation of a mean value model for a 2.4L JTD Diesel Engine was presented. The intake and exhaust manifolds were modeled as ideal gas volumes with constant specific heats. The EGR and VGT valves were modeled after the valve equations for flow through an orifice. The functions for effective areas were determined empirically [40]. It is important to note that these formulations do not incorporate and reverse flow phenomena. Volumetric efficiency, indicated torque, friction torque, and engine temperature rise are also modeled as static nonlinear functions.

An method to develop a mean value NO\textsubscript{x} model for a specific VGT-EGR diesel engine for use in real time control applications was also presented. It shows reasonably good accordance with data from 7 step tests and two separate drive cycles. Along with the previously developed mean value model of the VGT-EGR diesel engine this NO\textsubscript{x} model will provide an alternative to emission maps and allowed us to progress our research by making use of nonlinear, model based feedback control algorithms with the
specific goal of minimizing the amount of NOx emissions produced by a VGT-EGR diesel engine.

The electrically assisted turbo diesel engine was shown to produce significantly lower NOx levels in simulation when compared to a conventional VGT-EGR diesel engine. The coordination of the VGT and EGR with the turbo electric assist was also shown to reduce the load requirements on the electrical motor in both the assist and regenerative modes when compared to the electrically assisted VGT engine with no EGR. Reducing the load required of the electric motor can also reduce requirements on the subsequent cooling system necessary to prevent demagnetization of the PMSM.

Future work could include the addition of fuel flow as a control input. The introduction of another control variable such as fuel would require a much more detailed model for NOx and indicated engine torque, with respect to the fueling command, than was presented in this work.

This work also addressed some of the issues associated with measurement for real time control of a VGT-EGR diesel engine. The equivalent control approach for sliding mode observers has eliminated the need for empirical characterizations of the VGT, EGR, and intake charge flows that can introduce errors into the exhaust manifold pressure observer equations. With proper filtering, the proposed observers eliminate the need for three difficult and expensive measurements and allow for the estimation of EGR and VGT flow rates.
REFERENCES


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APPENDIX

A.1 SLIDING MODE CONTROL: AN INTRODUCTION

Sliding mode control (SMC) is one of the most widely used nonlinear control schemes due to its robustness properties and its ability to decouple systems of high dimension into a set of independent subsystems of lower dimension. Sliding mode control is employed on a set of systems called variable structure systems. Variable Structures are defined as continuous systems with appropriate switching logic. This means that they are continuous systems that consist of boundaries in which the controller switches its feedback gains. The technique of sliding mode has been extensively developed in the former Soviet Union. The genesis of the idea could be traced back to the early works of Kulebakin in the 1930’s. Sliding mode control has gained popularity in recent years as an efficient control tool with several applications enunciated by Vadim Utkin and others.

It is particularly significant in systems with discontinuous control actions like relays or “on-off” regulator systems. Though it first appeared in the context of “variable-structure” system applications, over the years the use of sliding mode control has expanded into many applications including discrete systems, feedback system design and control of infinite dimensional systems.
The phenomenon of sliding mode can be observed in any dynamic system governed by differential equations with discontinuous right hand sides. This discontinuity can be either due to discontinuous control or even due to discontinuity in the motion (e.g. due to coulombic friction). Formally "If the control as a function of the state switches at very high frequency (theoretically infinite) this motion can be classified as “Sliding Mode Control”

A.2 SLIDING MODE CONTROL OF LINEAR SYSTEMS WITH SCALAR INPUT

In order to illustrate the basic principles of sliding mode control lets investigate a simple second order time invariant system in the state space as shown in the block diagram below.

![Block Diagram of Ideal Sliding Mode Control Loop](image)

Figure A.2. Block Diagram of Ideal Sliding Mode Control Loop

We will first examine sliding mode control on a system with a scalar input. Consider a system described by the following equation
\[ \dot{x} = f(x,t,u) = Ax + Bu, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^1 \]  \hspace{1cm} (A1)

where A is the system or state matrix and B is the input vector. In sliding mode control, the control switches on the boundary of the two continuous subsystems, the so-called discontinuity surface

\[ u = \begin{cases} 
  u^+(x,t), & s(x) > 0 \\
  u^-(x,t), & s(x) < 0 
\end{cases} \]  \hspace{1cm} (A2)

where \( u^+(x,t) \) and \( u^-(x,t) \) are the continuous controls which brings the state trajectories to the discontinuity surface

The sliding mode control design procedure consists of two steps. The **first step** involves the selection of a discontinuity surface (sometimes referred to the switching surface or switching manifold) \( s(x) = 0 \) in the state space where the control undergoes discontinuities. This surface is determined so that the system behaves in a desired manner. In **step two** the continuous control functions \( u^+(x,t) \) and \( u^-(x,t) \) beyond the discontinuity surface are selected such that the control law satisfies the sufficient conditions for the existence and reachability of a sliding mode.

**A.3 WHAT IS SLIDING MODE?**

When state trajectories of discontinuous dynamic systems are directed to the discontinuity surface and then move along the surface, the motion of the state trajectories
is called sliding mode. A sliding mode for a stable system exists on a discontinuity surface if in the vicinity of this surface the distances to this surface and the velocity of its change are of opposite signs.

\[
\lim_{s \to 0} \dot{s} > 0 \\
\lim_{s \to 0} \dot{s} < 0, 
\]

(A3)

Sliding Mode Control (SMC) is highlighted in the following example:

\[
\dot{x} + a_2 \dot{x} + a_1 x = u + d(t) \\
u = -M \text{sign}(x) 
\]

(A4)

Given the system shown in equation (4) where \(d(t)\) is an unknown but bounded disturbance, and \(M, a_1,\) and \(a_2\) are constant system parameters. Design a sliding mode controller for the system.

Writing the system in the state space we get:

\[
\begin{align*}
    x_1 &= x \\
    \dot{x}_1 &= x_2 \\
    x_2 &= \dot{x} \\
    \dot{x}_2 &= u + d(t) - a_1 x_1 - a_2 x_2 
\end{align*}
\]

(A5)

*Step 1a. Check the system for controllability*
Step 1b. Select the discontinuity surface $s(x)$

We first define $s(x)$ by the following equation:

$$s(x) = \sum_{i=1}^{n} c_i x_i, \quad c_n = 1, \quad s(x) = 0$$  \hspace{1cm} (A6)

where $c_1, ..., c_{n-1}$ are defined as constant coefficients and $n$ is the order of the system. Expanding (6) yields

$$s(x) = c_1 x_1 + c_2 x_2 = c_1 x_1 + x_2 = 0 \Rightarrow x_2 = -c_1 x_1$$ \hspace{1cm} (A7)

The constant $C_2$ can be removed by dividing both sides by $C_2$. This gives a new $C_1$ value. Equation (7) can now be substituted into the first equation in (5) yielding the result:

$$\dot{x}_1 = -c_1 x_1 \Rightarrow x_1 = Ae^{-c_1 t}$$ \hspace{1cm} (A8)

Equation 8 describes the motion of the system once it reaches the discontinuity surface. This motion is called the sliding mode. The value $C_1$ can be chosen in order to speed up convergence. Notice that the dynamics of the system response (equation 8) does not contain the disturbance $d(t)$ or the system parameters ($a_1$ and $a_2$). These two properties
are called disturbance rejection and invariance with respect to plant parameters, respectively.

**Step 2. Calculate the continuous control functions** \( u^+(x,t) \) and \( u^-(x,t) \)

In sliding mode control the motion of the state trajectories can be decomposed into two parts, a relatively fast motion approaching the discontinuity surface with the control in equation (2), and a slower motion along the discontinuity surface with the control discussed in the previous step. By selecting a “sufficiently” high gain control in equation (4) the poles of the system corresponding to the fast motion can be attenuated by this high gain control. Thus we need to control only the rest of the system with the order lower than the original system by an appropriate choice of the discontinuity surface. The question now becomes what does the term “sufficient” mean? The trajectories will reach the sliding mode if \( M \) is large enough to ensure that \( s(x) \) and \( s(x) \) are of opposite signs.

\[
\dot{s} = c_i \dot{x}_1 + \dot{x}_2 = 0 = c_i x_2 + u + d(t) - a_2 x_2 - a_1 x_1 
\]

(A9)

But we defined \( x_2 = -c_i x_1 \) and \( u = -M \text{sign}(s) \) so

\[
\dot{s} = c_i \dot{x}_1 + \dot{x}_2 = 0 = (-c_i^2 + a_2 c_i - a_1) x_1 + d(t) - M \text{sign}(s)
\]

(A10)
The value $M$ must be large enough so that $s(x)$ and $\dot{s}(x)$ are of opposite signs. This will only be true if for a bounded disturbance given by $|d(t)| < d_o$, $M$ is defined by the following

$$M > \left(-c_1^2 + a_2c_1 - a_1\right)x_1 + d_o \quad (A11)$$

Notice that the dynamics of the system response (equation A8) does not contain the disturbance $d(t)$ or the system parameters ($a_1$ and $a_2$). These values are only used to get the value of $M$. In fact only the upper bound of these values are needed. The length of this domain of the sliding can also be defined by solving equation (A11) for $|x_1|$.

**A.4 SLIDING MODE CONTROL OF SYSTEMS WITH MULTIPLE INPUTS**

In the previous section we were only concerned with linear systems with a single input. We will now discuss general linear systems with multiple inputs. Consider a more general affine system given by

$$\dot{x} = f(x,t) + B(x,t)u, \ x, f \in \mathbb{R}^n, u(x) \in \mathbb{R}^m \quad (A12a)$$

$$u = \begin{cases} u_i^+(x,t), & \text{if } s_i(x) > 0 \\ u_i^-(x,t), & \text{if } s_i(x) < 0 \end{cases}, \quad s(x)^T = [s_1(x) \ldots s_m(x)] \quad (A12b)$$
with the right hand side being a linear function of control (no \( u^3, \sin(u), \ln(u), \text{etc.} \)).

Before sliding mode control of systems with multiple inputs can be considered, two concepts must be discussed first. The first is the concept of equivalent control. The second is the concept deals with an understanding of the regular form.

*Equivalent Control*

Assume there exists a sliding mode on the discontinuity surface described by equation A5.

\[
\dot{s}(x) = 0 \tag{A13}
\]

State trajectories starting outside of the discontinuity surface can be brought to the discontinuity surface by the controls in equation A2 as mentioned in section A1.1. Once the trajectories reach the discontinuity surface we want the trajectories to stay on the surface in the future. The control which makes the trajectories to stay on the discontinuity surface is called the equivalent control. Such a control can be defined as

\[
\dot{s} = \frac{\partial \hat{s}}{\partial \hat{\alpha}} = \frac{\partial \hat{s}}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial \hat{\alpha}} = Gf(x,t,u) = G\dot{x} \tag{A14}
\]

where the rows of the mxn matrix G are the gradients of the functions \( s_j(x) \).
Since the motion in the sliding mode implies $s(x) = 0$ we can assume that $\ddot{s}(x) = 0$ as well. This is because once the trajectory reaches the sliding mode we assume that it stays there for all time, thus the function ($s(x) = 0$) does not change. Equation (14) can now be written as

$$\dot{s} = G\ddot{s} = 0$$  \hspace{1cm} (A16)

Assume that at least one solution of the system of algebraic equations (equation A16) with respect to m-dimensional control does exist. Let us consider the special case in which the system becomes linear with respect to control as in equation (A12) the so-called control linear system.

Substituting equation A12 into equation (A16) yields:

$$\dot{s} = Gf + GBu = 0$$ \hspace{1cm} (A17)
If we assume that matrix \((GB)\) is nonsingular for all \(x\) and \(t\), and for a linear time invariant (LTI) system the function \(f(x,t)\) is replaced by the system matrix \(A\), then the equivalent control for the control linear system is found by the following equation

\[
\begin{align*}
\begin{align*}
 u &= u_{eq}(x,t) = -(GB)^{-1} Gf = -(GB)^{-1} GA \\
(A18)
\end{align*}
\end{align*}
\]

Substitution of equation (A18) into equation (A12) yields

\[
\begin{align*}
\begin{align*}
 \dot{x} &= f - B(GB)^{-1} Gf = Ax - B(GB)^{-1} GA \\
(A19)
\end{align*}
\end{align*}
\]

In order for \(s\) to be a stable sliding domain, a domain comprising stable sliding mode trajectories, it is necessary that the following two conditions are fulfilled. First, the equivalent control (equation 18) should have at least a single solution, and second, each component of this solution should satisfy

\[
\begin{align*}
\begin{align*}
 u_{eq,i} &\in \left[ \min(u_i^- , u_i^+) , \max(u_i^- , u_i^+) \right] \\
(A20)
\end{align*}
\end{align*}
\]

Regular Form

In terms of state space methodology the states of a given may not be unique. This is a useful premise because a systems’ dynamics may be more conveniently utilized using another set of state variables.
Note: Although their state space representations are different transformed systems have the same transfer functions!

A state vector \( P \) can be written in terms of the state variables given by axes \( x_1 \) and \( x_2 \) or it can be written in terms of the state variables given by axes \( z_1 \) and \( z_2 \). This means the same vector can be expressed in terms of different state variables. This is shown in figure 1.

In figure A1, the state variables form the axes of the state space:

![Figure A1](image)

Figure A1. Vector P given in multiple Coordinate Systems

The two-step SMC design procedure – selection of a switching manifold and then finding a control that enforces the sliding mode in this manifold is made simpler if the given system is transformed into the regular form.

\[
\dot{x} = f(x,t) + B(x,t)u, \quad x, f \in \mathbb{R}^n, u(x) \in \mathbb{R}^m
\]  

(A21)
The regular form for an affine system (equation 21) is given by

\[
\begin{align*}
\dot{z}_1 &= f_1(z_1, z_2) \\
\dot{z}_2 &= f_2(z_1, z_2) + B_2(z_1, z_2)u
\end{align*}
\] (A22a)

where equation A21 is calculated by an appropriate linear transformation \( z = Tx \). Note that equation A21 is represented in two blocks an upper block that does not depend on control and a lower block that depends on control input \( u \). If equation A21 is written in a linear form we have

\[
\dot{x} = Ax + Bu, \quad x \in \mathbb{R}^n, \quad A \in \mathbb{R}^{n \times n}, \quad u \in \mathbb{R}^n
\] (A22b)

A linear transformation \( T \) can be found such that the following is true

\[
TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad 0 \in \mathbb{R}^{(n-m) \times m}, \quad B_2 \in \mathbb{R}^{m \times m}
\] (A22c)

where the transformation is defined as

\[
z = Tx, \quad x = T^{-1}z,
\] (A22d)

Substituting (22d) into (22b) and solving for \( z \) yields

\[
\dot{z} = TAT^{-1}z + TBu
\] (A23a)

The system can now be written as
\[
\dot{z}_1 = A_{11}z_1 + A_{12}z_2 \\
\dot{z}_2 = A_{21}z_1 + A_{22}z_2 + u
\] (A23b)

where \( z_1 \in \mathbb{R}^{n-m}, \ z_2 \in \mathbb{R}^m, \ A_{11} \in \mathbb{R}^{(n-m)\times(n-m)}, \ A_{12} \in \mathbb{R}^{n-m\times m}, \ A_{21} \in \mathbb{R}^{m\times(n-m)}, \ A_{22} \in \mathbb{R}^{m\times m} \).

Now that we know how to use the transformation \((T)\), the next question is how is \( T \) calculated. Given a system in the following form

\[
\dot{x} = Ax + Bu
\] (A24)

The \( B \) matrix is partitioned such that

\[
B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}
\] (A25)

where \( B_1 \in \mathbb{R}^{(n-m)\times m} \) and \( B_2 \in \mathbb{R}^{m\times m} \) with \( \det B_2 \neq 0 \). The nonsingular coordinate transformation is given by

\[
\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T x, \quad T = \begin{bmatrix} I_{n-m} & -B_1 B_2^{-1} \\ 0 & B_2^{-1} \end{bmatrix}
\] (A26)

There are a few caveats of the regular form:

1. Calculation of the equivalent control is not necessary.
2. Sliding mode is invariant with respect to the dynamics associated with matrices $A_{21}$, and $A_{22}$. It is not invariant with respect to all plant parameters however.

3. The sliding mode equation does not depend on the gradient matrix $G$. The matrix $G$ may be needed in order to calculate the control $U$ however.

Now that the system is in regular form the two-step design process can be utilized.

Step 1. Select the discontinuity surface

In equation A23b the $m$-dimensional state vector $z_2$ is handled as an intermediate control of the first block and designed as a function of the states in $z_1$. All $(n - m)$ eigenvalues of the upper subsystem may be assigned by a proper choice of matrix $C_1$.

$$z_2 = h(z_1) = -C_1 z_1$$  \hspace{1cm} (A27)

The discontinuity surface, which provides the desired dependence between $z_1$ and $z_2$ can be chosen as

$$s = C_1 z_1 + z_2 = 0, -C_1 z_1 = z_2$$  \hspace{1cm} (A27)
The vector $z_2$ is treated as a fictitious control to the first system and the n-dimensional control problem is reduced to a (n-m)-dimensional control problem.

\[
\dot{z}_1 = A_{11}z_1 + A_{12}z_2, \quad z_2 = -C_1z_1 \Rightarrow \dot{z}_1 = (A_{11} - A_{12}C_1)z_1 \quad (A28)
\]

The matrix $C_1$ that controls and stabilizes the upper system can be found by conventional state variable feedback control methods.

**Step 2. Calculate the continuous control functions $u^+(x,t)$ and $u^-(x,t)$**

There are multiple ways to accomplish step 2. The simplest is to apply a component-wise form of the same concepts used in step 2 for single input systems. This means a separate $u_i$ is calculated for each input. The values of each $u_i$ is calculated such that $s_i$ and $\dot{s}_i$ have the opposite sign.
A.5 LINEAR ASYMPTOTIC OBSERVERS

The underlying observer design methods may be illustrated for a linear time-invariant system such as the following.

\[
\dot{x} = Ax + Bu
\]

(A29)

With output vector

\[
y = Cx \quad y \in \mathbb{R}^l
\]

(A30)

The pair \((A,C)\) is assumed to be observable. Where \(x\) and \(u\) are the \(n\) and \(m\) dimensional state and control vectors, \(A\), \(B\) and, \(C\) are constant matrices, \(\text{rank}(A)=n\), and \(\text{rank}(C)=l\).

A linear asymptotic observer is designed in the same form as the original system with an additional input depending on the mismatch between the real values and the estimated values of the output vector:

\[
\dot{x} = Ax + Bu + L(C\hat{x} - y)
\]

(A31)

Where \(\hat{x}\) is an estimate of the system state vector and \(L \in \mathbb{R}^{nxl}\) is a gain matrix.

The motion equation with respect to the mismatch equation \(\bar{x} = \hat{x} - x\) is of the form

\[
\dot{\bar{x}} = (A + LC)\bar{x}
\]

(A32)
The behavior of the mismatch governed by the homogenous equation is determined by the eigenvalues of matrix $A+LC$. For observable systems this may be assigned arbitrarily by an appropriate choice of the gain matrix $L$. It means that any desired rate of convergence of the error dynamics may be provided.

This estimator design reconstructs the entire state vector using measurements. The full-order estimator contains some redundancies and it may be desirable to reduce the order of the observer by the number of sensed outputs to reduce computational burden. Writing the equations of the voice coil being studied as an analogous linear system in the space $(x_1, x_2, y)$, where $y$ is the measured current, as:

\[
\begin{align*}
\dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + A_{13}y + B_1u \\
\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + A_{23}y + B_2u \\
y &= A_{31}x_1 + A_{32}x_2 + A_{33}y + B_3u
\end{align*}
\] (A33)

It is sufficient to design an observer only for vectors $x_1$ and $x_2$. The design of a reduced order observer is dependent on the coordinate transformation

\[
\begin{align*}
z_1 &= x_1 + L_1y \\
z_2 &= x_2 + L_2y
\end{align*}
\] (A34)
Then the system behavior is considered in the space \((z_1, z_2, y)\), and the coordinate transformation is obviously nonsingular for any \(L\). Similar to equation (10), the observer is designed in the form of a dynamic system of the \((n-l)\th\) order system and is governed by the following error dynamics.

\[
\begin{align*}
\dot{z}_1 &= (A_{11} + L_1 A_{31}) \hat{z}_1 + (A_{12} + L_1 A_{32}) \hat{z}_2 \\
\dot{z}_2 &= (A_{21} + L_2 A_{31}) \hat{z}_1 + (A_{22} + L_2 A_{32}) \hat{z}_2 \\
\end{align*}
\]  

(A35)

Where \(\hat{z} = \hat{z} - z\) is the mismatch between the estimator and the actual state value. Again if the system is observable the eigenvalues of equation (A35) may be assigned arbitrarily and \(\hat{z}_1\) and \(\hat{z}_2\) tend to zero at any desired rate. The components of the state vector \(x_1\) and \(x_2\) are then reconstructed from equation (A34).

### A.6 SLIDING MODE OBSERVERS

Let us proceed with the design of a state observer with discontinuous functions of the mismatches as follows:

\[
\begin{align*}
\dot{x}_1 &= A_{11}\dot{x}_1 + A_{12}\dot{x}_2 + A_{13}\dot{y} + B_1 u + L_1 v \\
\dot{x}_2 &= A_{21}\dot{x}_1 + A_{22}\dot{x}_2 + A_{23}\dot{y} + B_2 u + L_2 v \\
\dot{y} &= A_{31}\dot{x}_1 + A_{32}\dot{x}_2 + A_{33}\dot{y} + B_3 u - v \\
\end{align*}
\]  

(A36)
Where  \( v = M \text{sign}(\hat{y} - y) \),  \( M > 0 \)

The discontinuous vector function  \( v \in \mathbb{R}^l \) is chosen such that sliding mode is enforced in the manifold  \( \bar{y} = \hat{y} - y = 0 \) and the mismatch between the output vector  \( y \) and its estimate is reduced to zero. A matrix  \( L \) can be found such that the mismatch between  \( x \) states and their estimates decays at the desired rate. The error dynamics are as follows:

\[
\begin{align*}
\dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + A_{13}\bar{y} + L_1v \\
\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + A_{23}\bar{y} + L_2v \\
\dot{\hat{y}} &= A_{31}x_1 + A_{32}x_2 + A_{33}\bar{y} - v \\
v &= M \text{sign}(\bar{y})
\end{align*}
\]  

(A37)

It is known that sliding mode is enforced in the manifold  \( \bar{y} = 0 \) if the matrix multiplying  \( v \) in equation (A36) is negative definite and  \( M \) takes a high but finite value. This is the case for our system since  \( v \) is multiplied by a negative identity matrix. Hence for bounded initial conditions, sliding mode is enforced in the manifold  \( \bar{y} = 0 \). It follows from the equivalent control methods that the solution to  \( v_{eq} \) from  \( \dot{\bar{y}} = 0 \) should be substituted into equation (A16) with  \( \bar{y} = 0 \) to derive the sliding mode equation

\[
\begin{align*}
v_{eq} &= A_{31}\bar{x}_1 + A_{32}\bar{x}_2 \\
\dot{\bar{x}}_1 &= (A_{11} + L_1A_{31})\bar{x}_1 + (A_{12} + L_1A_{32})\bar{x}_2 \\
\dot{\bar{x}}_2 &= (A_{21} + L_2A_{31})\bar{x}_1 + (A_{22} + L_2A_{32})\bar{x}_2
\end{align*}
\]  

(A38)
that coincides exactly with equation (A35). Therefore the observer with discontinuous input functions of the mismatch in sliding mode is equivalent to the reduced order observer in (A35). However, if the plant and observed signal are affected by noise, the sliding mode observer may turn out to be preferable due to its filtering properties, which coincide with those of a Kalman Filter [13].