Essays On Using Weather Derivatives In Dairy Production

DISSERTATION

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By

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2005
Dairy farms confront unique risks from weather conditions. Hot and humid weather induces heat stress, which brings a series of risks to dairy farm operations including reductions in milk production and pregnancy rate and increases in cull rate and death rate. Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling. Adoption of abatement equipment, however, is hindered by its high initial cost and possibly long payback period, especially for small- and medium-scale firms. Moreover, although the abatement equipment is only seasonally useful, it is fixed asset whose price rises with efficacy. Weather derivatives provide an alternative method of risk management for dairy producers. Instead of reducing production losses, weather derivatives make payments based upon observed weather conditions over a period of time so that they offer the potential to offset profit losses caused by adverse weather events. Chapter 2 tests the risk management value of weather derivatives acting as a substitute for traditional abatement technologies within a utility maximization framework. The results suggest that weather derivatives offer an opportunity to improve the efficient portfolio frontier, and simultaneously using weather derivatives and abatement equipment is more favorable than using each of them alone.

Previous research has identified the problem of basis risk in weather derivatives (Turvey, 2001; Vedenov and Miranda, 2001). Little theoretical or empirical work
has been done to examine the effect of basis risk on weather derivatives. Chapter 3 examines the effect of basis risk in weather derivatives, and whether the existence of basis risk mitigates the usefulness of weather derivatives for dairy risk management. In this research, I investigate two kinds of basis risk: geographical basis risk and reference-index basis risk. Assuming a representative dairy producer has access to both weather derivatives and traditional heat abatement equipment, a closed-form solution for his/her optimal portfolio choice problem in the presence of basis risk is derived within a mean-variance utility framework. First-, second-, and third-degree stochastic dominance criteria are used to test the risk management effectiveness with less restrictive assumptions. Also transaction costs are imposed on weather derivative prices to allow the desirability of these contracts to their issuers. The results suggest that although basis risk reduces the effectiveness of weather derivatives, it still is a significant improvement to use weather derivatives along with abatement equipment in dairy profit risk management.

Since abatement equipment can be used for many years once installed, and its maintenance costs will increase and efficacy will decrease with age, a decision that must regularly be made by a dairy farmer is when to maintain his abatement equipment and when to replace it with a new one. This decision affects both current and expected future revenues. Considering that weather derivatives can be purchased periodically, Chapter 4 tests the risk management value of weather derivatives for dairy producers and examines how weather derivatives can affect dairy producers’ abatement equipment decisions. In this chapter, I employ a dynamic programming framework to study the case that a representative dairy farmer maximizes his long-run utility using weather derivatives and abatement equipment. The finding is that
simultaneously using weather derivatives and abatement technologies will outperform using abatement technologies alone and using weather derivatives will reduce not only the investment in abatement equipment, but the frequency of asset replacement and maintenance.
Dedicated to my family
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CHAPTER 1

INTRODUCTION

From a survey conducted by the U.S. Department of Commerce in 2004, approximately $3.8 trillion of economic activity in the U.S. is exposed to some type and degree of weather risk. This accounts for about 30 percent of total gross domestic product (GDP) for that year (Finnegan, 2005).

Since the first transaction in the weather derivatives market took place in 1997, the market of weather derivatives has been growing rapidly. The number of over-the-counter and exchange-based contracts traded in the weather derivatives market grew by 43% between April 2001 and April 2002, and total notional values grew by 72% to $4.3 billion (Lyon, 2002). They include the fields of energy, electricity, agricultural plant, and other weather sensitive industries such as ski resorts.

Weather derivatives as a new financial product possess several unique properties distinguishing them from other derivatives. First, their underlying (weather) is not a tradable asset. Second, they hedge against volumetric risk instead of price risk. The indemnity is calculated based on a weather index (Cooling/Heating Degree-Days, rainfall, etc) rather than asset price. Third, they are not as liquid as traditional standard derivatives. If we assume away transaction costs, the traditional financial
derivative markets are liquid. Weather, by its nature, is location-specific. Different locations have different weather conditions whether at the same time or across time.

Weather conditions are a primary source of dairy production risk. Hot and humid weather induces heat stress, which reduces both the quantity and quality of dairy production (Barth, 1982; Thompson, 1973). Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling (Turner et al., 1992; Lin et al., 1998). Adoption of abatement equipment, however, is hindered by its high initial cost and possibly long payback period, especially for small- and medium-scale firms. Moreover, abatement equipment is only seasonally useful. Weather-based derivatives pay during undesirable weather conditions. These products cannot reduce production risk but can offset revenue losses. They can be purchased to cover only certain time periods, and may be substitutes for abatement equipment at the margin. Chapter 2, in a one-period setting, tests the risk management value of weather derivatives to reduce weather-induced profit risk and to act as substitutes to traditional abatement technologies. Chapter 3 examines the effect of basis risk in weather derivatives, and whether the existence of basis risk mitigates the usefulness of weather derivatives for dairy risk management. Two kinds of basis risk are investigated: geographical basis risk and reference-index basis risk. Also transaction costs are imposed on weather derivative prices to allow the desirability of these contracts to their issuers. Chapter 4 employs a dynamic programming framework to study the case that a representative dairy farmer maximizes his long-run utility using weather derivatives and abatement equipment.
Weather conditions are a primary source of dairy production risk. Hot and humid weather induces heat stress, which reduces both the quantity and quality of milk production. Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling. Usually, they can increase the producers’ expected profit, but cannot cover all the lost revenue from heat stress. Weather derivatives can act to reduce weather-induced profit risk and thus act as a substitute at the margin for traditional abatement technologies for risk management. We test the risk management value of weather derivatives in a utility maximization framework. The results suggest that weather derivatives offer an opportunity to improve the efficient portfolio frontier, and simultaneously using weather derivatives and abatement equipment is more favorable than using either of them alone.

2.1 Introduction

Weather conditions are a primary source of dairy production risk. Hot and humid weather induces heat stress, which reduces both the quantity and quality of dairy production. Traditional heat abatement technologies control the environment through
ventilation, misting or evaporative cooling. Adoption of abatement equipment, however, is hindered by its high initial cost and possibly long payback period, especially for small- and medium-scale firms. Moreover, abatement equipment is only seasonally useful. Weather-based derivatives are a relatively new financial product that pay during undesirable weather conditions. These products cannot reduce production risk but can offset revenue losses. They can be purchased to cover only certain time periods, and may be substitutes for abatement equipment at the margin. The objective of this study is to test the risk management value of weather derivatives to reduce weather-induced profit risk and to act as substitutes to traditional abatement technologies.

The analysis is conducted by constructing two profit models. One is for a representative producer’s profit without using weather derivatives or abatement technologies; the other is for his profit using both of these two instruments. Then the producer’s optimal portfolio choice is derived in a utility maximization framework. From the utility framework, the benefit of using weather derivatives for managing risk is measured. The assumptions implicit in this paper are that (1) the producer has Pratt’s absolute risk aversion and chooses mean-variance efficient portfolios with a one-period horizon; (2) weather conditions are the only common risk factor to all producers in summer; and (3) the market is fully efficient in that there are no transaction costs, indivisibilities, taxes, or basis risk.

In each of California, Idaho, Washington, and Wisconsin, one county is identified that produces a significant quantity of milk and for which the National Climactic Data Center has adequate weather records. Using the St-Pierre, Cobanov, and Schnitkey (SCS, 2003) biological model of heat stress in dairy cattle, the losses for representative
producers are computed for the 36 years for which data is available.\(^1\) These losses are computed for scenarios in which producers use neither abatement equipment nor derivatives, as well as for the use of each individually and simultaneously. The results indicate that although abatement is effective at reducing economic losses from heat stress, weather derivatives can significantly benefit the producer compared to using only physical abatement technologies.

### 2.2 Weather and Weather Derivatives

Weather derivatives are financial contracts in which two parties agree to exchange payments on the basis of observed weather conditions. Common weather derivatives include those on the basis of temperature during a month, the number of cooling degree days during a month, or the amount of rainfall. Weather derivatives have several unique properties. Because the payoff is calculated based on an observable weather index there is little moral hazard. Moreover, since weather information is perfectly symmetric, adverse selection is eliminated. Therefore, weather derivatives have an advantage over traditional insurance for hedging against weather-related losses, because there is no need to prove damages to receive payoffs.

Weather derivatives have been the focus of much research. Dischel (1998) argues that due to the non-tradable nature of weather, weather derivatives cannot be valued by the Black-Scholes (1973) option pricing model, and instead a stochastic Monte Carlo simulation with a weather forecast model may be more effective. Turvey (2001) examines the weather effects on crop yields and states that weather derivatives might be used as a form of agricultural insurance. Cao and Wei (2001) propose a model

\(^1\)The economic losses from heat stress can include decreased milk production, decreased reproduction, and increased mortality. But in this research, our focus is on the milk production.
for daily temperature, which can incorporate several key properties such as seasonal cycles and uneven variations throughout the year, and develop a pricing model based on Lucas’ equilibrium asset pricing model. Diebold and Campbell (2005) propose a non-structural time series model of daily average temperature, which incorporates seasonal changes of temperature levels and variations throughout the year. Most previous research only examines temperature and/or rainfall derivatives to manage weather risk for energy and field crop markets. To our knowledge, there has been no research on the potential of using weather derivatives to hedge against livestock profit risk.

Economic losses occur in the dairy industry when ambient conditions are outside dairy cows’ thermal comfort zone. According to SCS, heat stress in dairy cattle is a function of the temperature-humidity index (THI, also known informally as the ‘heat index’). Johnson reports that a THI higher than 72 degrees is likely to have adverse effects on per-cow yield. SCS assert that the lower heat tolerance of the current selection of dairy cows implies the THI threshold to trigger heat stress should be lowered to 70 degrees. Therefore 70 degrees is used as the threshold for heat stress, \( THI_{\text{threshold}} \). According to the National Oceanic and Atmospheric Administration (NOAA, 1976), the standard formula of THI is: \( THI = T - (0.55 - 0.55H)(T - 58) \), where \( T \) is temperature in degrees Fahrenheit and \( H \) is relative humidity in percent. Since \( H \) is expressed as a percentage, it is easy to see that THI is positively correlated with temperature.

THI varies over the course of the day due to changes in temperature and relative humidity. The maximum THI is in the afternoon, when the temperature is highest and relative humidity is lowest; and the minimum THI is in the night, when the
temperature is lowest and relative humidity is highest. In this paper, daily THI refers to daily maximum THI. If the maximum THI is lower than 70 degrees in a day, there is no heat stress for dairy cows.

2.3 Dairy Profit Model

Consider a dairy producer who produces without using abatement equipment or weather derivatives. His profit is \( \tilde{y} = P\tilde{Q} - C \), where \( P \) is milk price, \( \tilde{Q} \) is the stochastic yield, and \( C \) denotes total cost. For analytical simplicity, it is assumed there is no price risk, therefore price is normalized to unity\(^2\); likewise herd size is also normalized to unity. The tilde (\( \tilde{\quad} \)) denotes a random variable.

Suppose the expected profit of a producer is his historical average, \( \mu \), so the difference between \( \tilde{y} \) and \( \mu \) is his profit risk.

\[
\tilde{y} = \mu + \theta f(\tilde{x}) + \tilde{\varepsilon} \tag{2.1}
\]

The profit risk is orthogonally decomposed into two parts. One is systematic risk which comes from weather conditions; the other is nonsystematic risk which reflects the individual’s production variability not arising from weather and is assumed uncorrelated with weather conditions. The coefficient \( \theta \) quantifies the sensitivity of the producer’s individual profit to systematic risk. The final term \( \tilde{\varepsilon} \) is a nonsystematic risk component. Note that \( f(\tilde{x}) \) captures systematic risk and increases with \( \tilde{x} \). The functional form of \( f(\tilde{x}) \) is assumed to be linear, i.e. \( f(\tilde{x}) = \alpha \tilde{x} \), where \( \alpha > 0 \).

\[
\tilde{x} = E(\tilde{z}) - \tilde{z} \tag{2.2}
\]

\[
\tilde{z} = \max(\tilde{THI} - THI_{\text{threshold}}, 0) \tag{2.3}
\]

\(^2\)The ‘natural hedge’ from price changes might offset some of the losses to heat stress, especially in high production areas such as Wisconsin or California. This issue is not considered in this study.
The factor \( \tilde{z} \), which is common to all producers in a region, measures the degree of heat stress, and the factor \( \tilde{x} \) is the difference between expected and actual heat stress. If \( \tilde{z} \) is less than \( E(\tilde{z}) \), then heat stress is milder than its expectation, and \( \tilde{x} \) is positive. Under the assumptions that

\[
E(\tilde{y}) = \mu, \quad E(\tilde{\varepsilon}) = 0, \quad \text{var}(\tilde{\varepsilon}) = \sigma^2_{\tilde{\varepsilon}}, \quad \text{cov}(\tilde{z}, \tilde{\varepsilon}) = 0, \quad \text{cov}(\tilde{x}, \tilde{\varepsilon}) = 0
\]

then

\[
\theta = \frac{\text{cov}(\tilde{y}, f(\tilde{x}))}{\text{var}(f(\tilde{x}))}
\]

By substitution, equation (2.1) becomes

\[
\tilde{y} = \mu + \alpha \theta \tilde{x} + \tilde{\varepsilon} = \mu + \beta \tilde{x} + \tilde{\varepsilon}
\]

where

\[
\beta = \frac{\text{cov}(\tilde{y}, \tilde{x})}{\text{var}(\tilde{x})}.
\]

Since the risk to dairy producers is from excessively high THI, call options on THI are the only derivatives considered. The underlying index is \( \tilde{THI} \), and the strike price is \( THI_{\text{threshold}} \). Without loss of generality, the tick size is set as 1, namely, one option will bring the holder 1 dollar for each degree of THI above the strike level. Therefore, the value of the payoff from a weather call option, \( \tilde{n} \), is equal to \( \tilde{z} \).

The option premia are calculated on the basis of actuarial fairness (also called 'burn-rate' method); therefore, purchasing weather options cannot change the producer’s expected profit. The option premium, \( \pi \), equals the expected payoff, \( E(\tilde{n}) \).

Also suppose that the producer is free to choose his abatement equipment investment \( \eta \) (\( \eta \geq 0 \); where \( \eta = 0 \) means he does not install abatement equipment). By using abatement equipment, the production loss from heat stress can be reduced. The
biological functional form of the effectiveness of abatement equipment is formulated as:

\[ \tilde{m} = g(\eta, \widetilde{THI}) = (a + b \cdot \widetilde{THI}) \cdot \sqrt{\eta} \]  \hspace{1cm} (2.6)

where \( \tilde{m} \) is the profit increase resulting from the abatement investment, \( \eta \) is abatement investment, and \( a \) and \( b \) are parameters.

It is easy to see that \( \tilde{m} \) is increasing with \( \eta \) and \( \widetilde{THI} \). When \( \eta = 0 \), \( \tilde{m} \) is also equal to 0. And with fixed \( \eta \), \( \tilde{m} \) is increasing with \( \widetilde{THI} \). That is because although the profit is low when \( \widetilde{THI} \) is high, the abatement equipment will be more useful; on the other hand, when \( \widetilde{THI} \) is low (i.e. weather is good), the abatement equipment is of little use, so the benefit is small. Since the net payoff from using abatement technologies is \( (a + b \cdot \widetilde{THI}) \cdot \sqrt{\eta} - \eta \), that is to say if \( \widetilde{THI} \) is high enough, the net payoff from investing abatement equipment will be positive; otherwise, the net payoff is negative.

With weather options and abatement equipment, the producer’s net profit equals:

\[ \tilde{y}^{\text{net}} = \tilde{y} + \phi \cdot (\tilde{n} - \pi) + \tilde{m} - \eta \]  \hspace{1cm} (2.7)

where \( \phi \) is weather option purchase amount. Therefore, there are two elements that the producer is free to choose: quantity of weather options, \( \phi \), and spending on abatement, \( \eta \). It is assumed these two choices are determined simultaneously in a portfolio taking the remaining parameters as given.

The producer’s optimal portfolio choice of weather option purchase and abatement investment is derived using a utility maximization model. The producer is assumed

\[ \text{Since abatement equipment is useful for many years once installed, the installation cost is annualized at a certain rate (say 15%) for yearly analysis. Using the ‘burn-rate’ method causes the expected THI to vary little over time. The producer’s yearly optimal decision on weather option purchase amount and abatement investment will not change once determined based on current information. This is a simple one-period, one-agent model.} \]
to have a mean-variance utility function\(^4\) of

\[ U = E(\bullet) - \frac{1}{2} A \cdot \text{var}(\bullet) \]

(2.8)

where \( A \) is an index of an agent’s aversion to taking on risk. Then the representative producer’s objective is to choose his optimal option purchase \( \phi \) and abatement spending \( \eta \) to maximize his utility from using weather options and abatement equipment\(^5\):

\[ \max_{\phi, \eta} U^{\text{net}} = E(\tilde{y}^{\text{net}}) - \frac{1}{2} A \cdot \text{var}(\tilde{y}^{\text{net}}). \]

(2.9)

Specifically,

\[ U^{\text{net}} = E(\tilde{y}) + E(\tilde{m} - \eta) - \frac{1}{2} A \cdot [\text{var}(\tilde{y}) + \phi^2 \text{var}(\tilde{m}) + \text{var}(\tilde{m})] + 2\phi \text{cov}(\tilde{y}, \tilde{m}) + 2\phi \text{cov}(\tilde{m}, \tilde{m})]. \]

(2.10)

\[ = \mu + (a + b\mu_{\tilde{THI}})\sqrt{\eta} - \eta - \frac{1}{2} A \cdot [\beta^2 \sigma_z^2 + \sigma_{\tilde{z}}^2 + \phi^2 \sigma_{\tilde{z}}^2 + b^2 \eta \sigma_{\tilde{THI}}^2 - 2\beta \phi \sigma_{\tilde{z}}^2 - 2b \sqrt{\eta} \text{cov}(\tilde{THI}, \tilde{z}) + 2\phi b \sqrt{\eta} \text{cov}(\tilde{THI}, \tilde{z})]. \]

Take the first order condition with respect to \( \phi \) and \( \eta \) respectively,

\[ \phi \sigma_z^2 - \beta \sigma_{\tilde{z}}^2 + b \sqrt{\eta} \text{cov}(\tilde{THI}, \tilde{z}) = 0, \]

(2.11)

\[ (a + b\mu_{\tilde{THI}}) \cdot \frac{1}{2} \eta^{-\frac{1}{2}} - 1 - \frac{1}{2} A \left[ b^2 \sigma_{\tilde{THI}}^2 - \beta b \eta^{-\frac{1}{2}} \text{cov}(\tilde{THI}, \tilde{z}) + \phi b \eta^{-\frac{1}{2}} \text{cov}(\tilde{THI}, \tilde{z}) \right] = 0. \]

(2.12)

Then equation system of (2.11) and (2.12) can be solved simultaneously.

It follows from (2.11) that

\[ \phi^* = \beta - \frac{b \text{cov}(\tilde{THI}, \tilde{z})}{\sigma_{\tilde{z}}^2} \sqrt{\eta}. \]

(2.13)

\(^4\)This framework is equivalent to expected utility function if (net) profit is normally distributed. But either normally distributed profit or a quadratic utility function is sufficient to validate the mean-variance utility model. Meyer has shown that the mean-variance model is consistent with expected utility model under much weaker restrictions. See Pratt (1964) and Meyer (1987).

\(^5\)In principle, if \( \phi \) in the optimal portfolio is negative, that means the decision maker can benefit from selling weather call options.
Substituting (2.13) into (2.12) and rearranging, it follows
\[
\sqrt{\eta} = \frac{a + b\mu_{\widetilde{THI}}}{2 + Ab^2 \left[ \sigma^2_{\widetilde{THI}} - \frac{\text{cov}^2(\widetilde{THI},\widetilde{z})}{\sigma^2_{\widetilde{z}}} \right]}.
\] (2.14)

It follows from (2.13) that:

**Proposition 1.** The optimal weather option purchase amount is decreasing with abatement equipment investment. Thus it indicates that weather options can act as a substitute for abatement equipment.

In (2.14), it is not difficult to see that the denominator is positive, because
\[
b^2 \left[ \sigma^2_{\widetilde{THI}} - \frac{\text{cov}^2(\widetilde{THI},\widetilde{z})}{\sigma^2_{\widetilde{z}}} \right] = b^2 \sigma^2_{\widetilde{THI}} \left[ 1 - \rho^2_{\widetilde{THI},\widetilde{z}} \frac{\sigma^2_{\widetilde{THI}} - \sigma^2_{\widetilde{z}}}{\sigma^2_{\widetilde{THI}} - \sigma^2_{\widetilde{z}}} \right] = b^2 \sigma^2_{\widetilde{THI}} \left( 1 - \rho^2_{\widetilde{THI},\widetilde{z}} \right) > 0,
\]
the correlation coefficient \( \rho_{\widetilde{THI},\widetilde{z}} \in (0, 1) \).

Since \( \widetilde{m} = (a + b\widetilde{THI}) \cdot \sqrt{\eta} \), this inequality \( (a + b\mu_{\widetilde{THI}}) > 0 \) implies that abatement investment can reduce loss from heat stress. Then the numerator of (2.14) is also positive. So it follows that:

**Proposition 2.** The optimal abatement investment is positive.

And Proposition 3 also follows from (2.14):

**Proposition 3.** The optimal abatement investment is negatively related to the producer’s risk aversion degree (i.e. \( A \)). That is, the more risk-averse the producer, the less he would invest in abatement equipment.

By substituting (2.14) back into (2.13), it follows:
\[
\phi^* = \beta - b \frac{\text{cov}(\widetilde{THI},\widetilde{z})}{\sigma^2_{\widetilde{z}}} \cdot \frac{a + b\mu_{\widetilde{THI}}}{2 + Ab^2 \left[ \sigma^2_{\widetilde{THI}} - \frac{\text{cov}^2(\widetilde{THI},\widetilde{z})}{\sigma^2_{\widetilde{z}}} \right]} \cdot \rho_{\widetilde{THI},\widetilde{z}} \sigma_{\widetilde{THI}} \sigma_{\widetilde{THI}} \left( 1 - \rho^2_{\widetilde{THI},\widetilde{z}} \right) \cdot \sigma_{\widetilde{z}}.
\] (2.15)

From (2.15), it follows that:
**Proposition 4.** The optimal option purchase amount is increasing with $\beta$. That means that the more the producer’s profit is sensitive to the systematic risk, the more options he should purchase, ceteris paribus.

It also follows from (2.15):

**Proposition 5.** The optimal option purchase amount is increasing with producer’s risk aversion degree, $A$.

Propositions 3 and 5 say that high risk adverse farmers should consider buying more weather derivatives and investing less on abatement equipment. The intuition is that abatement equipment typically can increase expected return, but as to reducing risk, it does not work as well as weather options, because it cannot cover all the risk from heat stress in a very hot summer. Therefore, when balancing between the expected return and risk, the high risk-aversion farmers should buy more weather derivatives.

**Proposition 6.** The optimal option purchase is decreasing with $a$ and $b$.

By substituting (2.15) and (2.14) back into (2.10), the maximized increased utility from using weather options and abatement can be derived from:

$$
\Delta U = U^{net}(\phi^*, \eta^*) - U(0,0) \quad (2.16)
$$

$$
= (a + b\mu_{\tilde{THI}}) \cdot \sqrt{\eta} - \eta - \frac{1}{2} A [\phi^2 \sigma^2_{\tilde{z}} + b^2 \eta \sigma^2_{\tilde{THI}} - 2 \beta \phi \sigma^2_{\tilde{z}} - 2 \beta b \sqrt{\eta} \text{cov}(\tilde{THI}, \tilde{z})] + 2 \phi b \sqrt{\eta} \text{cov}(\tilde{THI}, \tilde{z})].
$$
It is also possible to compare the cases in which the producer only uses one of these two instruments. The simultaneous usage of weather options and abatement equipment will be more favorable. Therefore, weather derivatives can act as substitutes for traditional abatement technologies.

2.4 Data

To empirically evaluate the effectiveness of managing weather risk for dairy producers, equations (2.4) and (2.6) must be estimated. Three types of data are needed: weather data, profit data and abatement investment data. One county from each of California, Idaho, Washington, and Wisconsin is chosen on the basis of milk production and availability of temperature and humidity data. For the four counties selected, Kern County in California, Ada County in Idaho, Yakima County in Washington, and Dane County in Wisconsin, 36 years of data were available from the National Climactic Data Center.\(^6\) The weather data for each county include daily maximum and minimum temperatures and daily maximum and minimum relative humidities. Daily temperature and dew point\(^7\) both follow seasonal patterns during the year; therefore, the ‘burn-rate’ method works well with these data for pricing weather options. Daily maximum temperature-humidity index (THI) is derived from daily maximum temperature and minimum relative humidity.

\(^6\)The 36 years of data are actually 1949-1964 and 1984-2003. Between 1965 and 1983, there are no daily relative humidity data available from the NCDC.

\(^7\)Dew point measures how much water vapor is in the air. In many places, the air’s total vapor content varies only slightly during an entire day, and so it is the changing air temperature that causes the variation in relative humidity. Related information can be found at: http://www.usatoday.com/weather.
For each county, a representative producer’s heat stress losses and the effect on these losses of using abatement equipment are generated from the models in SCS.\(^8\) Because 97\% of heat stress occurs between May 1st and October 31st, weather options are assumed to be written only for the summer. The payoff of an option is the cumulative \(\tilde{n}\) in the summer, and the premium is the expected payoff. Equations (2.4) and (2.6) are estimated using the cumulative summer data. Table 2.1 shows the descriptive statistics of the cumulative summer weather data.

### 2.5 Estimation and Results

Following SCS, \(THI_{threshold}\) is 70 degrees. The daily milk loss during the 36 years and the corresponding daily \(\tilde{THI}\) are calculated using the weather data and the SCS milk loss model. By accumulating the milk loss and \(\tilde{z}\) for each of the 36 summers, 36 observations of cumulative losses and \(\tilde{x} = E(\tilde{z}) - \tilde{z}\) are available. From a least squares regression, \(\beta\) is estimated. Table 2.2 gives the estimation results of the four counties. For example, the beta of a representative producer in Kern County (CA) is 0.75kg of milk per cow, which means each degree of \(\tilde{z}\) beyond its expectation will induce 0.75 kg milk loss. The milk price is fixed at $0.287/kg, so the milk loss is $0.22 per degree of \(\tilde{x}\). The sensitivity of milk profits to \(\tilde{x}\) varies because of the different climates in the various counties.

\(^8\)See the Appendix A and B for detail. Note that one limitation in the SCS models is that they are daily models and therefore do not incorporate the cumulative effect of heat stress.
The daily weather data are analyzed using the abatement effect model\(^9\) to calculate the daily reduced THI corresponding to seven abatement levels. Multiplying the estimated \(\beta\) and milk price, the increased profit due to abatement investment is calculated. The daily profit effect and THI are accumulated over each summer. Thus for each county, there are 36 observations of cumulative profit changes and THI for each of the seven abatement investment levels. Parameters \(a\) and \(b\) in equation (2.6) are estimated by least squares regression. Table 2.3 shows the regression results.

With the estimates of \(\beta\), \(a\) and \(b\) can we calculate the optimal portfolio choice and investigate the risk management value of weather derivatives and abatement equipment using equations (2.14), (2.15), and (2.16). For the purpose of illustration, the representative producer’s risk aversion level, which is represented by Pratt’s Absolute Risk Aversion (PARA), is set as 0.20.\(^{10}\)

To compare the risk management effectiveness of the instruments, three scenarios are investigated:

1. Only use abatement equipment;

2. Only use weather options;

3. Use both abatement equipment and weather options.

\(^9\)In SCS there are three abatement effect models corresponding to three abatement intensity levels. The first model is for only using fans or sprinklers; the second model is for a combination of fans and sprinklers; and the third model is for a specific system, the Korral Cool system, which is used in the Southwest and other dry and hot areas, such as Arizona and Texas. For this paper, six abatement effect functions are linearly simulated using a combination of fans and sprinklers. See Appendix B.

\(^{10}\)See, for example, Pratt (1964). Note that in this paper, we make no assumption about whether the risk aversion parameter is constant, decreasing, or increasing with initial wealth levels. We are studying the case that a representative farmer faces an opportunity to buy weather options which will not change his expected wealth level and needs to decide how much money to invest on weather options. So we have an implicit assumption that changes of expectation and variance of profit due to using abatement equipment and weather options will not affect his risk aversion degree.
The results are reported in table 2.4. Using a producer in Kern County as an example, the optimal use of abatement equipment results in a $94.00 net increase in certainty equivalent utility. The use of only weather options is less effective, resulting in a $73.93 increase in certainty equivalent utility. However, in scenario (3), when a producer is free to use both abatement equipment and options, the producer’s utility increases by a certainty equivalent amount of $140.25.

And according to our data, if without using the instruments, the mean and variance of the Kern County producer’s summer revenue loss due to heat stress are $161.46 and 1132.28 respectively. Thus by the mean-variance model, the utility loss of a farmer with PARA of 0.20 is $\left(-161.46 - \frac{1}{2} \cdot 0.2 \cdot 1132.28\right) = -$274.68 in certainty equivalent. Therefore, in scenario (3), the optimal use of these two instruments can reduce utility loss by $140.25/274.68=51\%$. And the utility loss can be reduced by 34\% and 27\% in scenario (1) and scenario (2) respectively.

Note that in table 4, $\eta^*$ is physical investment on abatement equipment, while $\phi^*\pi$ is the financial investment on weather options. In principle, the price of weather options can be lowered given the payoffs based on revealed weather information are lowered in the same amount, because the usage of weather options is not to change the expected revenue but to reduce the revenue variation. This issue falls into the area of contract design, which is not the focus of this paper.

Figure 2.1 gives the increased utility corresponding to different PARAs in the three scenarios. The PARAs range from 0 to 0.35. And the increased utility is in percentage to the utility loss at the corresponding PARA level. Figure 2.1 shows that the optimal portfolio choices bring more utility than only using abatement equipment or weather options. When a producer’s PARA is relatively low, using abatement
equipment alone will bring more utility than using weather options alone; otherwise, using weather options alone will be more favorable than using abatement equipment alone. An extreme case is that a producer is risk neutral, i.e., his PARA is zero. Then using weather options will bring no benefit to him because weather options are actuarially-fairly priced. Hence the increased utility from optimal portfolio is equal to that from solely using abatement equipment.

As is apparent from figure 2.1, location also influences the relative value of weather derivatives. Of the four locations, Kern County, California benefits the most from the availability of weather derivatives—producers there would be willing to pay up to $46.25 to have weather derivatives available in addition to abatement equipment. Dane County, Wisconsin producers would be only $33.14 better off, whereas residents of Ada County, Idaho and Yakima County, Washington would be less than $10 better off. The differences in benefits to the counties are influenced by the level and variability of temperature and relative humidity in the four locations. As shown in table 2.1, Kern County, CA has the highest number of THI degree days and variability in degree days, and for this reason, the use of weather derivatives is most effective.

2.6 Cross-Validation Analysis

The robustness of these results can be investigated through cross-validation. Specifically, every 35-year subset of the 36 years of data is used to estimate the $\beta$ in equation (2.4) and $a$ and $b$ in equation (2.6). From the parameter estimates, we derive the optimal portfolio choice, i.e. $\phi^*$ and $\eta^*$. By applying the optimal portfolio to the remaining year, the out-of-sample performance of weather options and abatement equipment can be evaluated and compared.
Results shows that the estimates of $\beta$, $a$, and $b$ are quite robust.\textsuperscript{11} For instance, the mean of the 36 estimates of $\beta$ in Kern County is 0.75 kg/cow, and the standard deviation of these estimates is 0.0898. The optimal portfolio choices are robust as well. The results are shown in table 2.5. Both the means and the standard deviations of 36 out-of-sample profit reductions in the four counties are significantly decreased when using weather derivatives and abatement equipment. For instance, the mean of the 36 out-of-sample observations in Kern County is -$161.46 per cow without any abatement, and the standard deviation is 33.65; when using the two instruments, the mean and standard deviation are -$95.22 and 20.48.

According to the cross-validation analysis, the optimal portfolio is preferable to the abatement-only portfolio in most years. Moreover, the two-instrument portfolio is only less profitable when weather conditions favor milk production, namely the milk losses are relatively low. Therefore, using weather options together with abatement equipment can smooth the producer's yearly net revenue—a desirable result for a risk averse producer.

2.7 Conclusion

This is the first study to investigate the potential of weather derivatives in hedging against dairy profit risk, arising from heat stress. A representative dairy producer’s profit risk is decomposed into systematic risk from weather conditions and idiosyncratic risk which is uncorrelated with weather condition. With access to hypothetical weather derivatives and abatement equipment, the producer’s optimal portfolio choice of these two instruments is derived in a mean-variance utility maximization.

\textsuperscript{11}To save space, the detailed results of parameter estimates are not reported.
framework. The results suggest that weather derivatives can act as a substitute for abatement technologies and the simultaneous usage of them is more favorable than using either of them alone.

This paper provides a link of the burgeoning weather derivatives literature in agricultural economics to a real-world application in which an easily-quantifiable weather metric (daily THI in excess of a biological threshold) is the primary source of production risk for a major agricultural commodity. Further, unlike other possible applications of weather derivatives, dairy is unique in that weather derivatives are likely substitutable for capital investment in heat abatement equipment, such as fans or water misters.

This research also raises many questions of relevance to the economic community, such as the optimal contract design, basis risk from location difference between weather derivatives and actual production area, whether the existence of these contracts reinforces economies of scale in dairy production, what level of sophistication is required to effectively utilize these tools, and finally, what size of a dairy is required to use weather derivatives. These questions may be of interest for further research.
<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>$\mu_z$</th>
<th>$\mu_{THI}$</th>
<th>$\sigma_z$</th>
<th>$\sigma_{THI}$</th>
<th>$\rho_{THI} z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>Kern</td>
<td>1271.51</td>
<td>14086.92</td>
<td>128.31</td>
<td>146.46</td>
<td>0.98</td>
</tr>
<tr>
<td>ID</td>
<td>Ada</td>
<td>515.67</td>
<td>12838.77</td>
<td>83.57</td>
<td>165.27</td>
<td>0.78</td>
</tr>
<tr>
<td>WA</td>
<td>Yakima</td>
<td>450.97</td>
<td>12816.44</td>
<td>99.57</td>
<td>166.55</td>
<td>0.88</td>
</tr>
<tr>
<td>WI</td>
<td>Dane</td>
<td>574.64</td>
<td>12815.72</td>
<td>121.83</td>
<td>181.54</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Table 2.1: Descriptive Statistics of Cumulative Summer Weather Data
<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>$\beta$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>Kern</td>
<td>0.75 (0.089)</td>
<td>0.67</td>
</tr>
<tr>
<td>ID</td>
<td>Ada</td>
<td>0.40 (0.030)</td>
<td>0.83</td>
</tr>
<tr>
<td>WA</td>
<td>Yakima</td>
<td>0.30 (0.018)</td>
<td>0.89</td>
</tr>
<tr>
<td>WI</td>
<td>Dane</td>
<td>0.63 (0.044)</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are standard errors.

Table 2.2: Beta Coefficient Estimation

<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>$a$</th>
<th>$b$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>Kern</td>
<td>-47.85 (11.53)</td>
<td>0.0046 (0.00082)</td>
<td>0.86</td>
</tr>
<tr>
<td>ID</td>
<td>Ada</td>
<td>-31.98 (2.24)</td>
<td>0.0029 (0.00017)</td>
<td>0.91</td>
</tr>
<tr>
<td>WA</td>
<td>Yakima</td>
<td>-38.90 (1.43)</td>
<td>0.0033 (0.00011)</td>
<td>0.93</td>
</tr>
<tr>
<td>WI</td>
<td>Dane</td>
<td>-50.37 (3.88)</td>
<td>0.0046 (0.0003)</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are standard errors.

Table 2.3: Coefficient Estimation of Abatement Effectiveness
<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>$U\text{Loss}$</th>
<th>Scenario (i)</th>
<th>Scenario (ii)</th>
<th>Scenario (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\phi^*$</td>
<td>$\eta^*$</td>
<td>$\Delta U$</td>
<td>$\eta^*$</td>
</tr>
<tr>
<td>CA</td>
<td>Kern</td>
<td>274.68</td>
<td>202.71</td>
<td>66.45</td>
<td>140.32</td>
</tr>
<tr>
<td>ID</td>
<td>Ada</td>
<td>46.22</td>
<td>47.88</td>
<td>7.51</td>
<td>16.51</td>
</tr>
<tr>
<td>WA</td>
<td>Yakima</td>
<td>32.76</td>
<td>31.45</td>
<td>3.86</td>
<td>11.20</td>
</tr>
<tr>
<td>WI</td>
<td>Dane</td>
<td>113.10</td>
<td>79.54</td>
<td>19.32</td>
<td>66.48</td>
</tr>
</tbody>
</table>

Note: $U\text{Loss}$ is the utility loss due to heat stress without using the instruments; 
$\Delta U$ is the increased utility from using one or two of the instruments.

(Unit: dollar)

Table 2.4: Risk Management Effectiveness
<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>ProfitLoss</th>
<th>NetProfitLoss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std</td>
</tr>
<tr>
<td>CA</td>
<td>Kern</td>
<td>161.46</td>
<td>33.65</td>
</tr>
<tr>
<td>ID</td>
<td>Ada</td>
<td>35.13</td>
<td>10.53</td>
</tr>
<tr>
<td>WA</td>
<td>Yakima</td>
<td>24.24</td>
<td>9.23</td>
</tr>
<tr>
<td>WI</td>
<td>Dane</td>
<td>56.78</td>
<td>23.73</td>
</tr>
</tbody>
</table>

Note: *ProfitLoss* is the summer profit loss due to heat stress without using the instruments; *NetProfitLoss* is the summer net profit loss due to heat stress with using the two instruments; *Positive* is the number of positive net profit from using the two instruments in the 36 out-of-sample tests; Unit of Mean, Min, and Max is dollar.

Table 2.5: Cross-Validation Results
Figure 2.1: Increased utility with different PARAs
CHAPTER 3

WILL BASIS RISK DISABLE WEATHER DERIVATIVES?
AN EXAMINATION IN MANAGING DAIRY PROFIT RISK

This study is an extension of Chapter 2. We make the analysis more general by relaxing several assumptions in Chapter 2. Chapter 2 gives a basic idea of the advantage from combined usage of weather derivatives and abatement technologies in managing dairy profit risk. There are several assumptions in it. We assumed that there is no basis risk or transaction costs. Also we used a hypothetical weather derivative - weather options on THI. This chapter examines the effect of basis risk in weather derivatives, and whether the existence of basis risk mitigates the usefulness of weather derivatives for dairy risk management. Assuming a representative dairy producer has access to both weather derivatives and traditional heat abatement equipment, a closed-form solution for his/her optimal portfolio choice problem in the presence of basis risk is derived within a mean-variance utility framework. First-, second-, and third-degree stochastic dominance criteria are used to test the risk management effectiveness with less restrictive assumptions. Also proportional transaction costs are imposed on weather derivative prices calculated on the basis of actuarial fairness to allow the desirability of these contracts to their issuers.
3.1 Introduction

Since the first transaction in the weather derivatives market took place in 1997, the market of weather derivatives has been growing rapidly. The number of over-the-counter and exchange-based contracts traded in the weather derivatives market grew by 43% between April 2001 and April 2002, and total notional values grew by 72% to $4.3 billion. They include the fields of energy, electricity, agricultural plant, and other weather sensitive industries such as ski resorts.

Weather derivatives as a new financial product possess several unique properties distinguishing them from other derivatives. First, their underlying (weather) is not a tradable asset. Second, they hedge against volumetric risk instead of price risk. The indemnity is calculated based on a weather index (Cooling/Heating Degree-Days, rainfall, etc) rather than asset price. Third, they are not as liquid as traditional standard derivatives. If we assume away transaction costs, the traditional financial derivative markets are liquid. Weather, by its nature, is location-specific. Different locations have different weather conditions whether at the same time or across time.

Due to their properties, weather derivatives have advantages over traditional financial derivatives for hedging against weather risk. Because there is no need to prove damage to receive payment, there is little moral hazard. Furthermore, as weather information is almost perfectly symmetric, adverse selection is eliminated. At the same time, the use of weather derivatives is accompanied by basis risk caused by the fact that an end-user’s location often is not the same location as the reference location of the weather derivatives he holds.

There have been a series of studies addressed on weather derivatives. Most of them are concerned with weather derivative pricing, and the usage of weather derivatives...
on reducing risk in energy and field crop markets. In Chapter 2, we have studied the potential of weather derivatives to reduce weather-induced dairy profit risk and to act as a substitute for traditional abatement technologies in dairy production. Our results indicate that using weather derivatives and abatement equipment can significantly improve producers’ efficient portfolio frontier for risk management of dairy production.

For analytical simplicity, there are several assumptions in Chapter 2. We assume that there is no basis risk or transaction costs. Also the weather derivative we used is a hypothetical one – so far, there have not been weather derivatives on THI. In this chapter, we extend the analysis of Chapter 2 by relaxing these assumptions and testing the risk management value of weather derivatives in a more realistic setting. We add in two kinds of basis risk. One is geographical basis risk, which comes from the location difference between weather derivatives and actual production area. The other is reference-index basis risk, which comes from the reference-index difference between weather index that affects dairy production and the index of available weather derivatives. In Chapter 2, we use hypothetical weather options on heat stress degree (THI) for risk management decision making. But the currently prevailing weather derivatives include weather option/swap in heating degree days (HDD) and cooling degree days (CDD). In this study, we will assume the case that weather derivatives available are on temperature only.

Hedging against one instrument’s risk using different but related derivatives is formally called cross-hedging. This often happens when there are no derivatives for the instrument being hedged, or a suitable derivatives contract exists but the market is highly illiquid. The success of cross-hedging depends on how strongly correlated
the instrument being hedged is with the cross-hedging instrument. Thus the effect of using weather derivatives in temperature to cross-hedging dairy profit risk will depend on the degree of correlation between temperature and heat stress and the correlation between weather conditions in production area and weather derivative location.

In addition, proportional transaction costs are imposed on weather derivative prices calculated on the basis of actuarial fairness to allow the desirability of these contracts to their issuers. As to testing the risk management effectiveness, we use mean-variance utility function, first-, second-, and third-degree stochastic dominance criteria. The results show that although basis risk reduces the effectiveness of weather derivatives, it still is a significant improvement to use weather derivatives along with abatement equipment in dairy profit risk management.

3.2 Literature Review

There are ample studies in basis risk and cross-hedging. Anderson and Danthine (1981) is the first to address the issue of cross-hedging in futures markets. They provide a theoretical description of hedging including option cash and futures positions and optimal cross-hedging for individuals concerned with the mean and variance of their profit. Hayenga and DiPietre (1982) argue that even though most wholesale meat products have no futures market, established futures trading in live hogs and cattle may offer hedging opportunities for firms handling large volumes of related meat products. The basis risk in using the live hog futures market as a risk management tool for hedging several wholesale pork products is evaluated in the standard error of forecast. They find that the correspondence between pork product prices and live hog futures prices generally is quite high. Their estimation results provide a
guide to hedging pork products prices risk with live hog futures contracts. Figlewski (1984) states that cross-hedging is involved in nearly every case of application of index futures. He studies the hedging effectiveness and basis risk in stock index futures by examining the basis and different sources of basis risk on the Standard and Poor’s 500 index contract. The finding concludes cross-hedging may be more effective with a more specialized instrument such as an industry group index option or future, or an individual stock option; and hedge duration and time to expiration of the futures contract are important to cross-hedging performance. Zacharias et al. (1987) study the use of wheat futures contracts to hedge against the price risk of rough rice due to the absence of viable futures markets for rice. Their procedure is a general one in that it evaluates the risk-efficiency of cross-hedging by employing a wider range of criteria including mean-variance analysis, first-, second-, and third-degree stochastic dominance and explicitly treats the problems of futures transactions costs and lumpiness along with the timing of the cross-hedging decision. Their results indicate that farm-level cross-hedging can be considered a viable marketing alternative. Elam, Miller and Holder (1986) investigate the feasibility of forward pricing sales of rice bran via cross-hedging and compare the effectiveness of simple and multiple cross-hedging of rice bran from simulated transactions. Their results indicate that cross-hedging is feasible and simple cross-hedging using corn futures would be most effective in reducing price risks. Cecchetti, Cumby and Figlewski (1988) point out two major problems in traditional approaches to designing an optimal futures hedge: one is that focus is only on minimizing risk, so no account is taken of the impact on expected return; the other is that the basis risk from time variation in the distribution of cash and futures price changes is often neglected. They correct these problems and estimate the
optimal futures hedge by maximizing the investor’s expected utility instead of mini-
mizing risk and employing an ARCH model to capture the property of time-varying
distribution of cash and futures price changes. Also they put the technique into a real
consider the hedging and production problem of a producer facing joint risk of price,
basis and production. They derive an exact solution for futures and options under
the assumption of a constant absolute risk aversion utility function and multivariate
normal distribution of the three risks. Mahul (2002) derives and examines the first-
best hedging rule by relaxing two assumptions that are normal in most other models
so that the spot price is not necessarily linear in both the settlement price and the
basis risk, and futures contracts and options on futures at different strike prices are
available. He suggests that the role of options in the firm’s hedging strategy is ratio-
nalized by the nonlinear payoff function of the first-best hedging instrument, which
can be caused by a nonlinear relationship between the spot price and the settlement
price, and/or by a non-additive basis risk, and/or by biased financial markets.

Some researchers have come up with the problem of basis risk in weather deriva-
tives (Turvey, 2001). Little theoretical or empirical work has been done to examine
the effect of basis risk on using weather derivatives. But Dischel (2000) analyzes the
correlation of precipitation in three different but close cities in Washington and finds
that there is strong correspondence of rainfall between these cities. He suggests that
precipitation basis risk might be overstated and useful correlations among sites might
be estimated.

Another set of research studies the effect of basis risk on weather-related insurance
companies. Harrington and Niehaus (1999) study the potential hedging effectiveness
of insurance derivatives based on regional estimates of catastrophe losses. Their results indicate that state-specific Property Claims Service catastrophe derivatives would have provided effective hedges for many insurers. Thus they suggest that basis risk is not likely to be a significant problem with state-specific catastrophe derivative contracts. Zeng (2000) introduces an alternative measures of industry loss warranty (ILW) basis risk, which is the conditional probability that the policyholder does not get a payoff from ILW given that the losses occur. The upside of basis risk is also examined from the effectiveness in reducing loss volatility. The conclusion is that a carefully structured ILW can be an effective and innovative instrument for a large insurer or reinsurer to manage the severity and volatility of catastrophic losses. Cummins, Lalonde, and Phillips (2004) analyze basis risk by measuring the effectiveness of hedge portfolios, consisting of a short position each insurer’s own catastrophe losses and a long position in catastrophic-loss index call spreads, in reducing insurer loss volatility, value-at-risk, and expected losses above specified thresholds. Two types of loss indices are used – a statewide index based on insurance industry losses in Florida and four intra-state indices based on losses in four quadrants of the state. Their main finding is that firms in three largest Florida market-share quartiles can hedge almost as effectively using the intra-state index contracts as they can using contracts that settle on their own losses.

3.3 Models

The models in this section are similar to those in Chapter 2 except equation (3.8) and the optimization problem solutions, i.e. equations (3.17) and (3.18). To keep the integrity of this chapter, we do not omit the models in this chapter, which are
identical to those in Chapter 2. Readers who recall the models of Chapter 2 may only pay attention to equations (3.8), (3.17) and (3.18).

Consider a dairy producer who produces without using abatement equipment or weather derivatives. His profit is \( \tilde{y} = P \cdot \tilde{Q} - TC \), where \( P \) is milk price, \( \tilde{Q} \) is the stochastic yield, and \( TC \) denotes a total cost. For analytical simplicity, it is assumed there is no price risk; therefore price is normalized to unity. The tilde (\( \tilde{\cdot} \)) denotes a random variable.

Suppose expected profit of a producer is his historical average, \( \mu \), so the difference between \( \tilde{y} \) and \( \mu \) is his profit risk. The profit risk is orthogonally decomposed into two parts. One is systematic risk which comes from weather condition; the other is nonsystematic risk which reflects the individual’s production variability not arising from weather and is assumed uncorrelated with weather conditions.

\[
\tilde{y} = \mu + \theta \cdot f(\tilde{x}) + \tilde{\varepsilon}, \tag{3.1}
\]

where

\[
\tilde{x} = E(\tilde{z}) - \tilde{z} \quad \tag{3.2}
\]

\[
\tilde{z} = \max(\tilde{THI} - THI_{\text{threshold}}, \ 0) \quad \tag{3.3}
\]

\[
\theta = \frac{\text{cov}(\tilde{y}, f(\tilde{x}))}{\text{var}(f(\tilde{x}))} \quad \tag{3.4}
\]

\[
E(\tilde{y}) = \mu, \ E(\tilde{z}) = 0, \ \text{var}(\tilde{z}) = \sigma_{\tilde{z}}^2, \ \text{cov}(\tilde{z}, \tilde{\varepsilon}) = 0, \ \text{cov}(\tilde{x}, \tilde{\varepsilon}) = 0. \tag{3.5}
\]

The coefficient \( \theta \) quantifies the sensitivity of the producer’s individual profit to systematic risk. The factor \( \tilde{z} \), which is common to all producers in a region, measures the degree of heat stress, and the factor \( \tilde{x} \) denotes the weather condition compared to its expectation. If \( \tilde{z} \) is lower than \( E(\tilde{z}) \), that means the heat stress is milder
than its expectation. In this case, $\tilde{x}$ is positive. And $f(\tilde{x})$ captures systematic risk and increases with $\tilde{x}$. Also for analytical simplicity, the functional form of $f(\tilde{x})$ is assumed to be linear, i.e. $f(\tilde{x}) = \alpha \cdot \tilde{x}$, where $\alpha$ is a positive parameter of the linear relationship. The final term $\tilde{\varepsilon}$ is a nonsystematic risk component.

Then equation (3.1) becomes,

$$\tilde{y} = \mu + \theta \cdot \alpha \cdot \tilde{x} + \tilde{\varepsilon} = \mu + \beta \cdot \tilde{x} + \tilde{\varepsilon}$$  

(3.6)

where

$$\beta = \frac{\text{cov}(\tilde{y}, \tilde{x})}{\text{var}(\tilde{x})}.$$  

(3.7)

Since here the risk is from excessively high THI, temperature call options will be used. The payoff from a weather call option is:

$$\tilde{n} = \max(\tilde{I} - I_{\text{threshold}}, 0)$$  

(3.8)

where $\tilde{I}$ is the stochastic value of a weather index, and $I_{\text{threshold}}$ is the strike level. Equation (3.8) captures the presence of both index and geographical basis risk. If the reference index $\tilde{I}$ is temperature rather than THI, it reflects index basis risk. If the reference index $\tilde{I}$ is weather condition of a location other than the production area, then geographical basis risk exists. Note if $\tilde{I}$ is THI of the production area, there is no basis risk and $\tilde{n} = \tilde{z}$.

Transaction costs are also imposed on the option premium. So purchasing weather options can decrease the producer’s expected profit. The option premium equals the expected payoff plus proportional transaction costs:

$$\pi = (1 + \gamma)E(\tilde{n})$$  

(3.9)
where the loading rate \( \gamma > 0 \) reflects transaction costs related to administrative and implementation fees and the desirability to the issuers. If \( \gamma \) is zero, the weather options are actuarially-fairly priced.

Also suppose that the producer is free to choose his abatement equipment investment \( \eta \) ( \( \eta \geq 0 \); and \( \eta = 0 \) means he does not install abatement equipment). By using abatement equipment, the production loss from heat stress can be reduced. The biological functional form of the effectiveness of abatement equipment is formulated as:

\[
\tilde{m} = g(\eta, \overline{THI}) = (a + b \cdot \overline{THI}) \cdot \sqrt{\eta}
\]

(3.10)

where \( \tilde{m} \) is the reduced profit loss, i.e. the increased profit from using abatement, \( \eta \) is abatement investment.

With weather options and abatement equipment, the producer’s net profit equals:

\[
\tilde{y}^{net} = \tilde{y} + \phi \cdot (\tilde{n} - \pi) + \tilde{m} - \eta
\]

(3.11)

where \( \phi \) is weather option purchase amount. Therefore, there are two elements that the producer is free to choose: spending on weather options, \( \phi \), and spending on abatement, \( \eta \). It is assumed these two choices are determined simultaneously in a portfolio taking the remaining parameters as given.

### 3.3.1 Optimal Decisions

The producer’s optimal portfolio choice of weather option purchase and abatement investment is derived using a utility maximization model. The producer is assumed to have a mean-variance utility function of

\[
U = E(\bullet) - \frac{1}{2} A \cdot \text{var}(\bullet)
\]

(3.12)
where $A$ is an index of agents’ aversion to taking on risk. Then the representative producer’s objective is to choose his optimal option purchase $\phi$ and abatement spending $\eta$ to maximize his utility from using weather options and abatement equipment:

$$
\max_{\phi, \eta} U^{net} = E(\tilde{y}^{net}) - \frac{1}{2}A \cdot \text{var}(\tilde{y}^{net}).
$$

Specifically,

$$
U^{net} = E(\tilde{y}) + \phi E(\tilde{n} - \pi) + E(\tilde{m} - \eta) - \frac{1}{2}A \cdot \text{var}(\tilde{y}) + \phi^2 \text{var}(\tilde{n}) - 2\phi \text{cov}(\tilde{y}, \tilde{n}) - \frac{1}{2}A \cdot \text{var}(\tilde{m}) + 2\phi \text{cov}(\tilde{y}, \tilde{m})
$$

where $\mu = E(\tilde{y}), \mu_n = E(\tilde{n}), \mu_{THI} = E(\tilde{THI}); \sigma_n^2 = \text{var}(\tilde{z}), \sigma_n^2 = \text{var}(\tilde{n}), \sigma_{THI}^2 = \text{var}(\tilde{THI}); \sigma_{\tilde{z}, \tilde{n}} = \text{cov}(\tilde{z}, \tilde{n}), \sigma_{\tilde{THI}, \tilde{n}} = \text{cov}(\tilde{THI}, \tilde{n}).$ And all these are positive numbers.$^{12}$

Take the first order condition with respect to $\phi$ and $\eta$ respectively,

$$
\gamma \mu_n + A[\phi \sigma_n^2 - \beta \sigma_{\tilde{z}, \tilde{n}} + b \sqrt{\eta} \sigma_{\tilde{THI}, \tilde{n}} - \frac{1}{2}A \cdot \text{var}(\tilde{THI}) - \beta \sigma_{\tilde{THI}, \tilde{z}} \eta^{-\frac{1}{2}} + \phi b \sigma_{\tilde{THI}, \tilde{n}} \eta^{-\frac{1}{2}}] = 0,
$$

$$
\frac{1}{2}(a + b \mu_{THI}) \eta^{-\frac{1}{2}} - 1 = \frac{1}{2}A[2\beta \sigma_{\tilde{THI}} \eta^{-\frac{1}{2}} - \beta \sigma_{\tilde{THI}, \tilde{z}} \eta^{-\frac{1}{2}} + \phi b \sigma_{\tilde{THI}, \tilde{n}} \eta^{-\frac{1}{2}}] = 0.
$$

Then equation system of (3.15) and (3.16) can be solved simultaneously.

It follows from (3.15) that

$$
\phi^* = - \frac{\gamma \mu_n + A[\phi \sigma_n^2 - \beta \sigma_{\tilde{z}, \tilde{n}} + b \sqrt{\eta} \sigma_{\tilde{THI}, \tilde{n}} - \frac{1}{2}A \cdot \text{var}(\tilde{THI}) - \beta \sigma_{\tilde{THI}, \tilde{z}} \eta^{-\frac{1}{2}} + \phi b \sigma_{\tilde{THI}, \tilde{n}} \eta^{-\frac{1}{2}}]}{A \sigma_n^2 - \beta \sigma_{\tilde{THI}, \tilde{n}} - b \sigma_{\tilde{THI}, \tilde{n}} \sqrt{\eta}}.
$$

$^{12}$Even if there exists index and geographical basis risk, $\text{cov}(\tilde{z}, \tilde{n})$ and $\text{cov}(\tilde{THI}, \tilde{n})$ are still positive as long as weather conditions of production area and reference location covary positively.
Substituting (3.17) into (3.16) yields
\[
\sqrt{\eta} = \frac{a + b\mu \tau_{THI} + \gamma\mu \tilde{n} b \frac{\sigma_{\tau_{THI}, \tilde{n}}}{\sigma_{\tilde{n}}} + A\beta b [\sigma_{\tau_{THI}, \tilde{z}} - \frac{\sigma_{\tau_{THI}, \tilde{n}}}{\sigma_{\tilde{n}}}]}{2 + Ab^2 [\sigma_{\tau_{THI}}^2 - \frac{\sigma_{\tau_{THI}, \tilde{n}}^2}{\sigma_{\tilde{n}}^2}]}.
\] (3.18)

### 3.3.2 Comparative Static Analysis

It follows from (3.17) that:

**Proposition 7.** The optimal weather option purchase amount is decreasing with abatement equipment investment. Thus it indicates that weather options can act as a substitute for abatement equipment in dairy risk management strategies.

In (3.18), it is not difficult to see that the denominator is positive, because
\[
b^2 [\sigma_{\tau_{THI}}^2 - \frac{\text{cov}^2 (\tau_{THI}, \tilde{z})}{\sigma_{\tilde{z}}^2}] = b^2 \sigma_{\tau_{THI}}^2 [1 - \frac{\rho_{\tau_{THI}, \tilde{z}}^2 \sigma_{\tau_{THI}}^2 \sigma_{\tilde{z}}^2}{\sigma_{\tau_{THI}, \tilde{z}}^2 \sigma_{\tilde{z}}^2}] = b^2 \sigma_{\tau_{THI}}^2 (1 - \rho_{\tau_{THI}, \tilde{z}}^2) > 0,
\]

since the correlation coefficient \(\rho_{\tau_{THI}, \tilde{z}} \in (0, 1)\).

Since \(\gamma, \mu, b, \sigma_{\tau_{THI}}, \sigma_{\tilde{n}}^2\) are all positive, then
\[
d\sqrt{\eta}^* \over d\gamma = \frac{\mu \tilde{n} b \frac{\sigma_{\tau_{THI}, \tilde{n}}}{\sigma_{\tilde{n}}} \frac{-A\sigma_{\tilde{n}}^2}{A\sigma_{\tilde{n}}^2} - b \frac{\sigma_{\tau_{THI}, \tilde{n}}}{\sigma_{\tilde{n}}}}{2 + Ab^2 [\sigma_{\tau_{THI}}^2 - \frac{\sigma_{\tau_{THI}, \tilde{n}}^2}{\sigma_{\tilde{n}}^2}]} > 0.
\]

It follows that:

**Proposition 8.** The optimal abatement investment is positively related to the weather option loading rate \(\gamma\).

The intuition of Proposition 8 is that higher transaction costs make weather options less attractive, and thus abatement equipment becomes more attractive to dairy producers.

From (3.17) and Proposition 8, it can be shown that
\[
d\phi^* \over d\gamma = -\frac{-\mu \tilde{n}}{A\sigma_{\tilde{n}}^2} - b \frac{\sigma_{\tau_{THI}, \tilde{n}}}{\sigma_{\tilde{n}}^2} \frac{d\sqrt{\eta}^*}{d\gamma} < 0.
\]
Proposition 9. The optimal weather option purchase amount is negatively related to the weather option loading rate $\gamma$.

It is not straightforward to see the impact of a small change of $\beta$ on $\sqrt{\eta^*}$, and $\phi^*$. From (3.18),

$$
\frac{d\sqrt{\eta^*}}{d\beta} = \frac{A\beta b\left[\sigma_{\tilde{THI},\tilde{z}} - \frac{\sigma_{\tilde{THI},\tilde{n}}\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2}\right]}{2 + Ab^2\left[\sigma_{\tilde{THI}}^2 - \frac{\sigma_{\tilde{THI},\tilde{n}}^2}{\sigma_{\tilde{n}}^2}\right]}, \text{ thus}
$$

$$
\text{sign}\left[\frac{d\sqrt{\eta^*}}{d\beta}\right] = \text{sign}\left[\sigma_{\tilde{THI},\tilde{z}} - \frac{\sigma_{\tilde{THI},\tilde{n}}\sigma_{\tilde{z},\tilde{n}}}{\sigma_{\tilde{n}}^2}\right] = \text{sign}\left[\rho_{\tilde{THI},\tilde{z}} - \rho_{\tilde{THI},\tilde{n}}\rho_{\tilde{z},\tilde{n}}\right].
$$

By the definitions of $\tilde{z}$ and $\tilde{n}$ in (3) and (8), it is a reasonable assumption that $\rho_{\tilde{THI},\tilde{z}} > \rho_{\tilde{THI},\tilde{n}}$. Together with the fact that $\rho_{\tilde{z},\tilde{n}}$ is less than 1, we have $\frac{d\sqrt{\eta^*}}{d\beta} > 0$.

Proposition 10. The optimal abatement investment is increasing with $\beta$, under an unrestrictive assumption on the relationship of weather condition in two locations.

Proposition 11. The optimal weather option purchase amount is increasing with $\beta$, under an unrestrictive assumption on the relationship of weather conditions of production area and reference location.$^{14}$

The proof of Proposition 11 is found in the Appendix C.

3.3.3 Effect of Basis Risk

If there is no basis risk, the dairy farmer can buy weather derivatives written on THI of his production area, $\tilde{n}$ will be the same as $\tilde{z}$, and thus $\text{var}(\tilde{z}) = \text{var}(\tilde{n}) = \text{var}(\tilde{n})$.

$^{13}$This inequality holds for all of our weather data.

$^{14}$The assumption is on the correlation coefficients, specifically, $\frac{\rho_{\tilde{THI},\tilde{z}}\rho_{\tilde{THI},\tilde{n}}}{\rho_{\tilde{z},\tilde{n}} < 1}$. 

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cov(\tilde{z}, \tilde{n})$, and cov($\widehat{THI}, \tilde{z}$) = cov($\widehat{THI}, \tilde{n}$). Therefore the optional portfolio choice in (3.17) and (3.18) becomes:

$$\phi^* = -\frac{\gamma \mu_\tilde{z}}{A\sigma^2_{\tilde{z}}} + \beta - b \frac{\sigma_{\overline{THI}, \tilde{z}}}{\sigma^2_{\tilde{z}}} \sqrt{\eta}$$

(3.19)

$$\sqrt{\eta^*} = \frac{a + b \mu_{\overline{THI}} + \gamma \mu_\tilde{z} b \frac{\sigma_{\overline{THI}, \tilde{z}}}{\sigma^2_{\tilde{z}}}}{2 + A b^2 [\sigma^2_{\overline{THI}} - \frac{\sigma^2_{\overline{THI}, \tilde{z}}}{\sigma^2_{\tilde{z}}}]}. \quad (3.20)$$

Compared with (3.17) and (3.18), the existence of basis risk causes decrease in $\phi^*$ and increase in $\eta^*$, ceteris paribus. From (3.19) and (3.20), an increase in $\beta$ will not affect $\eta^*$ but only increase $\phi^*$. That is because using weather options is the most efficient way to hedge against weather risk if there is no basis risk. And an increase in risk aversion degree $A$ will reduce the optimal abatement investment and increase the weather option purchase amount. However, no analytical inference about the impact of a small change of $A$ on $\sqrt{\eta^*}$ and $\phi^*$ can be derived with the presence of basis risk.

### 3.3.4 Risk Management Effectiveness

By substituting $\phi$ and $\eta^*$ in (3.17) and (3.18) back into utility function in (3.14), the maximized increased utility in certainty equivalent from using weather options and abatement equipment can be derived from:

$$\Delta U = U^{net}(\phi^*, \eta^*) - U(0, 0)$$

$$= -\phi \gamma \mu_{\tilde{n}} + (a + b \mu_{\overline{THI}}) \sqrt{\eta} - \eta - \frac{1}{2} A \cdot [\phi^2 \sigma^2_{\tilde{n}} + b^2 \eta \sigma^2_{\overline{THI}, \tilde{z}} - 2\beta \phi \sigma_{\tilde{z}, \tilde{n}} - 2\beta b \sqrt{\eta} \sigma_{\overline{THI}, \tilde{z}} + 2 \phi b \sqrt{\eta} \sigma_{\overline{THI}, \tilde{n}}].$$

(3.21)
It is also viable to compare it with the cases in which the producer only uses one of these two instruments. The simultaneous usage of weather options and abatement equipment will be more favorable.

3.4 Data

For the empirical part of this study, we need to estimate equations (3.6) and (3.10). Three types of data are needed: weather data, profit data and abatement investment data. The 35-year (1949 to 1964 and 1984 to 2002) weather data of 14 weather stations in IL, OH, NY, and WI are from the National Climate Data Center (NCDC), a subsidiary of the National Oceanic Atmospheric Administration (NOAA).\textsuperscript{15} The weather data in each station include daily maximum and minimum temperature and daily maximum and minimum relative humidity. Daily temperature and dew point both follow routinely seasonal patterns each year. So the “burn-rate” method works well with these data for pricing weather options. Daily maximum temperature-humidity index (THI) is derived from daily maximum temperature and minimum relative humidity.

Corresponding to the daily weather data of each station, a representative producer’s daily milk loss from heat stress and reduced loss from using abatement equipment are generated by employing the results in SCS.\textsuperscript{16} Abatement investment cannot change in a relatively long period once fixed. Also weather options are assumed to be written on summer basis, i.e. the payoff is cumulative $\tilde{n}$ of a summer and premium is

\textsuperscript{15}It is a quite common phenomenon that daily relative humidity data from 1965 to 1983 are missing across weather stations in NCDC database. The selected 14 stations are the only ones which have the least amount of missing data of 1949 to 1964 and 1984 to 2002 in the four states.

\textsuperscript{16}See the Appendices for detail.
the expected payoff. Thus, equations (3.6) and (3.10) are estimated based on cumulative summer data. The summer period is set from May 1st to Oct. 31st every year, because 97% of heat stress occurs in this period.

In the 14 selected weather stations, a major city in each of the four states is selected as the weather options reference city of that state. The 10 stations that are left are treated as production areas. Table 3.1 lists the selected production areas and their corresponding weather options reference cities. Distance between each pair of production area and reference city is reported as well.

3.5 Empirical Results

3.5.1 Estimate Parameters in Equation (3.6)

Note all results of the empirical illustration in this paper is for one dairy cow, namely, the herd size is normalized into unity. Following SCS, $THI_{\text{threshold}}$ is set as 70 degrees. For each selected production area, the daily milk loss during summers of the 35 years and the corresponding daily $\tilde{\text{THI}}$ are calculated using the weather data and the SCS milk loss model. Then by accumulating the daily milk loss and $\tilde{\zeta} = \max(\tilde{\text{THI}} - THI_{\text{threshold}}, 0)$ during each summer in the 35 years, 35 observations of cumulative milk loss and $\tilde{x} = E(\tilde{\zeta}) - \tilde{\zeta}$ are obtained. From a least squares regression, $\beta$ is estimated. Table 3.2 shows the estimation results of the selected production areas. For example, the beta of a representative farmer in Summit County is 0.56 kg milk per cow, which means each degree of $\tilde{\zeta}$ beyond its mean will induce 0.56 kg milk loss. The milk price is set as $0.287/kg as in SCS, so the profit loss is $0.16 per degree of $\tilde{x}$. The average beta across the selected production areas is 0.62 kg milk per cow, and the standard deviation is 0.13 kg.
3.5.2 *Estimate Parameters in Equation (3.10)*

The daily weather data are put into the SCS abatement effect models\(^{17}\) to calculate the daily reduced THI corresponding to seven abatement levels. Multiplying the estimated \(\beta\) and milk price, we calculate reduced profit loss (in dollars) due to abatement investment (in dollars). The daily reduced profit loss and THI are accumulated for each summer. Thus in each selected production area, there are 35 observations of accumulated reduced profit loss and accumulated THI for each of the seven abatement investment levels. Parameters \(a\) and \(b\) in equation (3.10) are estimated by least squares regression. Table 3.3 shows the regression results.

3.5.3 *Risk Management Results*

With coefficient beta and parameters \(a\) and \(b\) can we calculate the optimal portfolio choice and investigate the risk management value of weather derivatives and abatement equipment using equations (3.17), (3.18) and (3.21). The loading rate \(\gamma\) in equation (3.9) is set as 5%. And the producer’s risk aversion level, which is represented by Pratt’s Absolute Risk Aversion (PARA), is set as 0.20. Farmers are assumed to be able to choose strike levels of weather options.

Table 3.4 shows the effectiveness of risk management strategies under different scenarios. First, for the sake of comparison, suppose a representative farmer does not employ either weather derivatives or abatement equipment. With \(\beta, a\) and \(b\), his annual revenue loss from heat stress can be calculated, which can then be put

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\(^{17}\) In SCS there are three abatement effect models corresponding to three abatement intensity levels. The first model is for only using fans or sprinklers; the second model is for a combination of fans and sprinklers; and the third model is for a specific system, the Korral Cool system, which is used in the Southwest and other dry and hot areas. In the research, we use the second model, and based on this model, we linearly simulate six abatement effect functions corresponding to six different fixed cost levels. See Appendix B.
into mean-variance framework to calculate his utility loss. The increased utility by using these two instruments is calculated by equation (3.21) in certainty equivalent and presented in table 3.4. No analytical results of the optimal strike level can be derived. The increased utility corresponding to a series of alternative strike levels are examined and the optimal strike level is chosen as the one with the maximum increased utility. Six different scenarios are examined:

1. no basis risk, no transaction costs;
2. only with reference-index basis risk, no transaction costs;
3. only with geographical basis risk, no transaction costs;
4. with both kinds of basis risk, no transaction costs;
5. with both kinds of basis risk and transaction costs;
6. only using abatement equipment, no weather derivatives used.

Table 3.4 displays that the increased utility is decreasing along the 6 scenarios. Take a representative farmer in Summit County of Ohio as an example. His yearly utility loss is $91 in certainty equivalent without using any instruments.\(^\text{18}\) In scenario 1, his increased utility is $52.4. So, the optimal use of weather options and abatement equipment can reduce utility loss by 58%. In scenario 2, when the only available weather options is on temperature of Summit County, his increased utility is $47.7. Then the reduced utility loss decreases from 58% to 53% because of the presence of reference-index basis risk. In scenario 3, there are no weather derivatives available on basis of Summit County’s weather indices and the farmer only has access to call options written on THI of Cleveland (around 50 miles away from Summit County). With the geographical basis risk, his increased utility is $47.21, which reduces utility

\(^{18}\)The annual net revenue from a dairy cow typically is around $330, which is calculated based on Willett.
loss by 52%. In the presence of both kinds of basis risk, his reduced utility loss is 48% and 46% with and without transaction costs. However, the reduced utility loss is 32% if only using abatement equipment, which is lower than those of other scenarios. The results of other production areas are similar. Figure 3.1 shows the average reduced utility loss across the selected production areas from above scenarios 1, 5 and 6. Therefore, although basis risk reduces the effectiveness of weather derivatives, weather derivatives can significantly improve upon the effectiveness of abatement equipment for reducing dairy profitability risk.

3.5.4 Distance Effect

It is interesting to know whether the distance in geographical basis risk plays a role in risk management effectiveness. In other words, we examine if the larger the distance between the production area and weather options reference city, the less the effectiveness in using weather derivatives. The reduced effect due to basis risk and transaction costs is measured in percentage by dividing the difference between increased utility in scenario 1 ($\Delta U$) and scenario 5 ($\Delta U^{IGT}$) by increased utility in scenario 1.

\[
\text{Reduced Effect} = \frac{\Delta U - \Delta U^{IGT}}{\Delta U} \times 100\%
\]

Figure 3.2 shows the reduced effect against different distances in the 10 production areas and there is not a clear linear relationship between distance and reduced effect.

3.5.5 Different Risk Aversion Degrees

So far, the producer’s risk aversion degree, PARA, is set as 0.2. In this subsection, a series of different PARAs are examined. Again, Summit County is taken as an
example. Corresponding to different PARAs, the percentage of increased utility in the utility loss without using the instruments is measured under three scenarios. Supposing the presence of both kinds of basis risk and transaction costs, the three scenarios are examined: (i) using abatement equipment alone; (ii) using weather options alone; (iii) using both instruments. The PARAs range from 0 to 0.55. Figure 3.3 shows the results. The optimal portfolio choices bring more utility than only using abatement equipment or weather options. If the producer’s PARA is less than 0.37, using abatement equipment alone will bring more utility compared with using weather options alone; if his PARA is higher than 0.37, using weather options alone will be more favorable than using abatement equipment alone. An extreme case is that the producer is risk neutral, i.e., his PARA is zero. Then buying weather options will be of negative benefit because weather options bear transaction costs.\(^{19}\)

### 3.5.6 Cross-Validation and Stochastic Dominance

Cross-Validation is an often-used data resampling method. The procedure is that 34 of the 35 years of data are used to estimate parameters in equations (3.6) and (3.10), and determine the optimal portfolio (\(\phi^*\) and \(\eta^*\)) and strike level, which, together with the remaining one year’s worth of data, are then put into equation (3.11) as an out-of-sample evaluation to obtain the net profit of using the portfolio. This procedure is repeated 35 times by successively omitting each of the 35 observations each time.

The cross-validation results allow the robustness of parameter estimation to be investigated. The results show that the estimates of \(\beta\), \(a\) and \(b\) are robust. For instance, the mean of the 35 estimates of \(\beta\) in Summit County is 0.564kg/cow, and the standard deviation of these estimates is 0.0342. The means and standard deviations

\(^{19}\)In this case, \(\phi^*\) is zero if selling weather options is forbidden to the producer.
of these parameter estimates are very close to the estimates and standard errors reported in table 3.2 and 3.3.\textsuperscript{20}

Then, with the cross-validation results, we can test the effectiveness of using weather derivatives and/or abatement equipment by stochastic dominance criteria, which compare the performance of any two risk management strategies requiring much less restrictive assumptions than a specific utility function. First degree stochastic dominance criterion (FSD) holds for all decision makers who prefer more to less. Second degree stochastic dominance (SSD) holds for all decision makers who have diminishing marginal utility, or equivalently a concave preference function. And third degree stochastic dominance (TSD) holds for all decision makers who are decreasingly averse to risk as they become wealthier.

The four risk management strategies to be compared are: (i) use neither of the two instruments; (ii) use abatement equipment alone; (iii) use weather options alone; (iv) use weather options and abatement equipment simultaneously. Weather options here have two kinds of basis risk as well as transaction cost, i.e. scenario 5. A comparison is made and yields similar results across the selected production areas. As an illustration, the results of Summit County are shown in table 3.5. By first and second degree stochastic dominance, using both instruments dominates using none of them and using weather options alone. By third degree stochastic dominance, using both instruments dominates all the other three strategies. This finding is consistent with that in the mean-variance utility framework. We also notice that the mean of 35 out-of-sample profit loss of Summit County is $-49.70 with a standard deviation of $20.30; the mean of 35 out-of-sample net profit loss with using the two instruments is

\textsuperscript{20}To save space, we have omitted reporting the detailed parameter estimates in cross-validation.
$34.40 with a standard deviation of $9.70. Thus, we see that using weather options and abatement equipment can significantly reduce both the mean and variance of profit loss from heat stress in summer. And we also observe that 27 of the 35 years the net profit from using the optimal portfolio is positive. The maximum is $48.9 and the minimum is -$8.6. That means in most cases, optimally using weather options and abatement equipment can increase net profit. Moreover, negative net profit from using these instruments only happens when weather conditions favor milk production, namely the milk losses are relatively low. Therefore, using weather options together with abatement equipment smooths the producer’s yearly net revenue. That is a desirable result for a risk averse producer.

3.6 Conclusion

This chapter examines the effect of basis risk in weather derivatives, which is one of the main concerns when investigating the viability and values of weather derivatives acting as a risk management tool in weather sensitive industries. This research is conducted by exploring the use of weather derivatives together with traditional abatement technologies to manage dairy profit risk from heat stress and performing a theoretical and empirical analysis of the effect of two main kinds of basis risk in weather derivatives.

A representative dairy producer’s profit risk is decomposed into systematic risk from weather conditions and idiosyncratic risk which is uncorrelated with weather conditions. With the access to weather options and abatement equipment, the producer’s optimal portfolio choice of these two instruments is derived in a mean-variance
utility maximization framework. First-, second-, and third-degree stochastic domi-
nance criteria are used to compare the different risk management strategies with
unrestrictive assumptions. The results suggest that although basis risk reduces the
effectiveness of weather derivatives, it still is a significant improvement to use weather
derivatives along with abatement equipment in dairy profit risk management.
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<tr>
<td>NY</td>
<td>Albany</td>
<td>Buffalo</td>
<td>252.1</td>
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<tr>
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<td>Broome</td>
<td>Buffalo</td>
<td>148.6</td>
</tr>
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<td>Queens</td>
<td>Buffalo</td>
<td>291.9</td>
</tr>
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<td>NY</td>
<td>Monroe</td>
<td>Buffalo</td>
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</tr>
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<td>Onondaga</td>
<td>Buffalo</td>
<td>125.6</td>
</tr>
<tr>
<td>WI</td>
<td>Dane</td>
<td>Milwaukee</td>
<td>79.6</td>
</tr>
</tbody>
</table>

Note: County is the production area; Distance is between the weather stations of the production area and of reference city.

Table 3.1: Production Areas and Options Reference Cities
<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>β</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OH</td>
<td>Summit</td>
<td>0.56</td>
<td>0.89</td>
</tr>
<tr>
<td>OH</td>
<td>Franklin</td>
<td>0.71</td>
<td>0.87</td>
</tr>
<tr>
<td>OH</td>
<td>Trumbull</td>
<td>0.52</td>
<td>0.88</td>
</tr>
<tr>
<td>IL</td>
<td>Rock Island</td>
<td>0.76</td>
<td>0.84</td>
</tr>
<tr>
<td>NY</td>
<td>Albany</td>
<td>0.61</td>
<td>0.89</td>
</tr>
<tr>
<td>NY</td>
<td>Broome</td>
<td>0.38</td>
<td>0.92</td>
</tr>
<tr>
<td>NY</td>
<td>Queens</td>
<td>0.84</td>
<td>0.93</td>
</tr>
<tr>
<td>NY</td>
<td>Monroe</td>
<td>0.60</td>
<td>0.93</td>
</tr>
<tr>
<td>NY</td>
<td>Onondaga</td>
<td>0.59</td>
<td>0.93</td>
</tr>
<tr>
<td>WI</td>
<td>Dane</td>
<td>0.63</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are standard errors.
<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>a</th>
<th>b</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OH</td>
<td>Summit</td>
<td>-57.41 (3.45)</td>
<td>0.0051 (0.00027)</td>
<td>0.91</td>
</tr>
<tr>
<td>OH</td>
<td>Franklin</td>
<td>-48.13 (5.66)</td>
<td>0.0046 (0.00043)</td>
<td>0.90</td>
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<tr>
<td>OH</td>
<td>Trumbull</td>
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<td>0.0050 (0.00024)</td>
<td>0.91</td>
</tr>
<tr>
<td>IL</td>
<td>Rock Island</td>
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<td>0.0055 (0.00049)</td>
<td>0.91</td>
</tr>
<tr>
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<td>Albany</td>
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<td>0.91</td>
</tr>
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<td>Broome</td>
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<td>0.0036 (0.00011)</td>
<td>0.90</td>
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<tr>
<td>NY</td>
<td>Queens</td>
<td>-60.14 (5.32)</td>
<td>0.0056 (0.00041)</td>
<td>0.92</td>
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<tr>
<td>NY</td>
<td>Monroe</td>
<td>-56.16 (2.80)</td>
<td>0.0050 (0.00022)</td>
<td>0.93</td>
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<tr>
<td>NY</td>
<td>Onondaga</td>
<td>-58.18 (2.67)</td>
<td>0.0052 (0.00021)</td>
<td>0.93</td>
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<td>Dane</td>
<td>-50.37 (3.97)</td>
<td>0.0046 (0.00031)</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are standard errors.

Table 3.3: Coefficient Estimation of Abatement Effectiveness
### Table 3.4: Risk Management Effectiveness

<table>
<thead>
<tr>
<th>State</th>
<th>County</th>
<th>$U_{Loss}$</th>
<th>$\Delta U$</th>
<th>$\Delta U^I$</th>
<th>$\Delta U^G$</th>
<th>$\Delta U^{IG}$</th>
<th>$\Delta U^{IGT}$</th>
<th>$\Delta U^A$</th>
</tr>
</thead>
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<td>OH</td>
<td>Summit</td>
<td>90.80</td>
<td>52.39 (71)</td>
<td>47.73 (76)</td>
<td>47.21 (74)</td>
<td>43.41 (80)</td>
<td>42.06 (81)</td>
<td>28.73</td>
</tr>
<tr>
<td>OH</td>
<td>Franklin</td>
<td>178.38</td>
<td>114.52 (71)</td>
<td>107.87 (77)</td>
<td>104.10 (70)</td>
<td>98.51 (76)</td>
<td>95.66 (77)</td>
<td>64.98</td>
</tr>
<tr>
<td>OH</td>
<td>Trumbull</td>
<td>78.98</td>
<td>44.25 (71)</td>
<td>41.04 (75)</td>
<td>39.82 (74)</td>
<td>35.13 (75)</td>
<td>33.87 (80)</td>
<td>24.02</td>
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<tr>
<td>IL</td>
<td>Rock Island</td>
<td>186.59</td>
<td>108.02 (72)</td>
<td>97.46 (76)</td>
<td>96.54 (75)</td>
<td>85.05 (76)</td>
<td>82.05 (80)</td>
<td>62.58</td>
</tr>
<tr>
<td>NY</td>
<td>Albany</td>
<td>79.31</td>
<td>44.62 (71)</td>
<td>41.11 (77)</td>
<td>40.67 (72)</td>
<td>39.09 (75)</td>
<td>37.39 (78)</td>
<td>25.16</td>
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<td>Broome</td>
<td>37.76</td>
<td>17.38 (71)</td>
<td>16.69 (73)</td>
<td>14.40 (71)</td>
<td>14.20 (75)</td>
<td>13.02 (79)</td>
<td>7.19</td>
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<td>NY</td>
<td>Queens</td>
<td>155.83</td>
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<td>91.81 (76)</td>
<td>82.69 (71)</td>
<td>79.25 (74)</td>
<td>77.05 (75)</td>
<td>62.15</td>
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<tr>
<td>NY</td>
<td>Monroe</td>
<td>82.61</td>
<td>50.96 (71)</td>
<td>47.60 (75)</td>
<td>48.65 (72)</td>
<td>45.75 (75)</td>
<td>43.70 (78)</td>
<td>26.01</td>
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<tr>
<td>NY</td>
<td>Onondaga</td>
<td>83.54</td>
<td>50.50 (71)</td>
<td>47.15 (75)</td>
<td>47.38 (72)</td>
<td>44.08 (75)</td>
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<td>27.55</td>
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<tr>
<td>WI</td>
<td>Dane</td>
<td>114.44</td>
<td>67.14 (71)</td>
<td>61.07 (75)</td>
<td>59.55 (70)</td>
<td>57.35 (68)</td>
<td>54.16 (74)</td>
<td>33.32</td>
</tr>
</tbody>
</table>

Note: $U_{Loss}$ is the utility loss in dollars without using the two instruments;

$\Delta U$ is the increased utility in dollars using the two instruments without basis risk or transaction costs;

$\Delta U^I$ is the increased utility in dollars using the two instruments with index basis risk;

$\Delta U^G$ is the increased utility in dollars using the two instruments with geographical basis risk;

$\Delta U^{IG}$ is the increased utility in dollars using the two instruments with index and geographical basis risk;

$\Delta U^{IGT}$ is the increased utility in dollars using the two instruments with index and geographical basis risk and transaction costs;

$\Delta U^A$ is the increased utility in dollars from using abatement equipment alone.

Numbers in parentheses are optimal strike levels.
<table>
<thead>
<tr>
<th>FSD</th>
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</thead>
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<td>Both ≻ None</td>
<td>Both ≻ None</td>
<td>Both ≻ None</td>
</tr>
<tr>
<td>Both ≻ Options</td>
<td>Both ≻ Options</td>
<td>Both ≻ Options</td>
</tr>
<tr>
<td>Abatement ≻ None</td>
<td>Abatement ≻ None</td>
<td>Both ≻ Abatement</td>
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<tr>
<td>Abatement ≻ Options</td>
<td>Abatement ≻ Options</td>
<td>Abatement ≻ None</td>
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<td>Options ≻ None</td>
<td>Abatement ≻ Options</td>
<td>Options ≻ None</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The sign “≻” means “stochastically dominates”; “Both” means using both instruments; “Abatement” means using abatement equipment alone; “Options” means using weather options alone; “None” means using none of the two instruments.

Table 3.5: Stochastic Dominance Comparison Results
Figure 3.1: Effectiveness comparison

Figure 3.2: Distance effect
Figure 3.3: Increased utility with different PARAs
CHAPTER 4

A DYNAMIC PROGRAMMING FRAMEWORK FOR USING WEATHER DERIVATIVES TO MANAGE DAIRY PROFIT RISK

In this chapter, we employ a dynamic programming framework to study the case of a representative dairy farmer who maximizes his long-run utility using weather derivatives and abatement equipment. The discrete action is a choice from three alternative treatments to abatement equipment – no action, replacement, or service; and the continuous action is weather derivative purchase amount.

Weather derivatives provide an alternative method for dairy farmers to manage risk. Since abatement equipment can be used for many years once installed, and its maintenance costs will increase and efficacy will decrease with age, a decision that must regularly be made by a dairy farmer is when to maintain his abatement equipment and when to replace it. The decision affects both current and expected future revenues. Considering that weather derivatives can be purchased periodically, the objective of this study is twofold: first, to test the risk management value of weather derivatives for dairy producers; second, to examine how weather derivatives can affect dairy producers’ abatement equipment decisions.
4.1 Motivation

In Chapter 2 and Chapter 3, we use a one-period model to derive a dairy farmer’s yearly management decision using weather derivatives and abatement equipment to reduce the profit risk induced by heat stress. In this chapter, we employ a dynamic programming framework to study the case that the farmer maximizes his long-run utility.

Investment in abatement equipment is assumed to be both irreversible\textsuperscript{21} and deferrable.\textsuperscript{22} So keeping the money on hand and deferring the investment is similar to keeping an American call option. These irreversible investment opportunities are typically called real options. They are valuable and the value is highly sensitive to the uncertainty of future cash flow of the investment. Once a farmer has made an investment in abatement equipment, the real option has been exercised and he gives up the possibility of waiting for new information to arrive that might affect the desirability or timing of the investment expenditure.\textsuperscript{23} The value of the real option is an opportunity cost that must be included as part of the investment cost. So whether or not to investment will not only depend on whether the Net Present Value (NPV) of a unit of capital is positive, but also whether the positive NPV is larger than the value of the real option. Pindyck (1991) shows that this kind of investment decision making problem under irreversibility and uncertainty can be solved by using two qualitatively equivalent approaches: option pricing and dynamic programming.

\textsuperscript{21}Due to, for example, prohibitively high sunk cost, the “lemons” problem or that the investment is firm-specific.

\textsuperscript{22} “There may be a cost to delay - the risk of entry by other firms, or simply foregone cash flows - but this cost must be weighed against the benefits of waiting for new information.” pp. 1111, Pindyck (1991).

\textsuperscript{23}For example, see McDonald and Siegel (1986).
The efficacy of abatement equipment is increasing with the initial investment level, so the representative dairy farmer must first make a decision on the initial installation investment on abatement equipment. Since abatement equipment can be used for many years once installed, and its maintenance costs will increase and efficacy will decrease with age, a decision that must regularly be made by the dairy farmer is when to maintain his abatement equipment and when to replace it with a new one. This decision affects both current and expected future revenues. Many similar problems have been studied by using dynamic programming models in the literature. Examples include dairy cow replacement (Smith, 1972; Miranda and Schnitkey 1995), swine production (Chavas et al., 1985), poultry production (McClelland et al., 1989), livestock feeding (Burt, 1993), and bus engine replacement (Rust, 1987).

Considering that weather derivatives can be purchased periodically, the objective of this study is twofold: first, to test the risk management value of weather derivatives for dairy producers; second, to examine how weather derivatives can affect dairy producers’ abatement equipment decisions. A dynamic programming approach is employed to determine the optimal actions to abatement equipment and optimal weather derivatives purchase. Specifically, we employ a dynamic programming framework to study the case that a representative dairy farmer maximizes his long-run utility using weather derivatives and abatement equipment. The actions that the dairy farmer need to determine at the start of each period include weather derivatives purchase amount and a choice from three alternative treatments to abatement equipment – no action, replacement, or maintenance, based upon the abatement equipment age and its maintenance history. A solution as the optimal risk management strategy for  

\(^{24}\)Abatement equipment typically has a lifespan of 7-10 years depending on the maintenances.
abatement equipment replacement and weather derivatives purchase is derived using policy iteration.

The empirical analysis is performed using 35-year daily weather data from Summit County and Cleveland, Ohio. The results suggest that simultaneously using weather derivatives and abatement technologies will outperform using abatement technologies alone, even in presence of basis risk and transaction costs. Thus weather derivatives expand the portfolio choice set, and make more desirable profit distributions available. Moreover, using weather derivatives will reduce the initial investment in abatement equipment and postpone the asset replacement, which mitigates the problem of high initial installation cost of abatement equipment for small- and medium-size firms.

4.2 One-period Setting

In a one-period setting, we develop a representative dairy farmer’s net profit model with using weather derivatives and abatement equipment to reduce the profit risk induced by heat stress. In next section, this one-period profit function will be used in dynamic programming analysis for determining optimal sequential risk management strategies. To reduce the visual complexity, we drop the time subscripts of model notations in this section.

Consider a dairy farmer who produces without using abatement equipment or weather derivatives. His profit is \( \tilde{y} = P \cdot \tilde{Q} - TC \), where \( P \) is milk price, \( \tilde{Q} \) is the stochastic yield, and \( TC \) denotes a total cost. For analytical simplicity, it is assumed there is no price risk; therefore price is normalized to unity. The tilde (\( \sim \)) denotes a random variable.
Suppose expected profit of a farmer is his historical average, \( \mu \), so the difference between \( \tilde{y} \) and \( \mu \) is his profit risk. The profit risk is orthogonally decomposed into two parts. One is systematic risk which comes from weather conditions; the other is nonsystematic risk which reflects the individual’s production variability not arising from weather and is assumed uncorrelated with weather conditions. Equations (4.1)-(4.10) give the daily models, because equation (4.3), by its nature, is for daily calculation. However, they can be easily transformed into yearly models by using yearly cumulative values of the variables.

\[
\tilde{y} = \mu + \theta \cdot f(\tilde{x}) + \tilde{\epsilon}, \quad (4.1)
\]

where

\[
\tilde{x} = E(\tilde{z}) - \tilde{z} \quad (4.2)
\]

\[
\tilde{z} = \max(\bar{THI} - THI_{threshold}, 0) \quad (4.3)
\]

\[
\theta = \frac{\text{cov}(\tilde{y}, f(\tilde{x}))/\text{var}(f(\tilde{x}))}{(4.4)}
\]

\[
E(\tilde{y}) = \mu, \quad E(\tilde{\epsilon}) = 0, \quad \text{var}(\tilde{\epsilon}) = \sigma_{\tilde{\epsilon}}^2, \quad \text{cov}(\tilde{z}, \tilde{\epsilon}) = 0, \quad \text{cov}(\tilde{x}, \tilde{\epsilon}) = 0. \quad (4.5)
\]

The coefficient \( \theta \) quantifies the sensitivity of the farmer’s individual profit to systematic risk. The factor \( \tilde{z} \), which is common to all producers in a region, measures the degree of heat stress, and the factor \( \tilde{x} \) denotes the weather condition compared to its expectation. If \( \tilde{z} \) is lower than \( E(\tilde{z}) \), it means the heat stress is milder than its expectation. In this case, \( \tilde{x} \) is positive. And \( f(\tilde{x}) \) captures systematic risk and increases with \( \tilde{x} \). The functional form of \( f(\tilde{x}) \) is assumed to be linear, i.e. \( f(\tilde{x}) = \alpha \cdot \tilde{x} \), where \( \alpha \) is a positive parameter of the linear relationship. The final term \( \tilde{\epsilon} \) is a nonsystematic risk component.
Then equation (4.1) becomes,

\[ \tilde{y} = \mu + \theta \cdot \alpha \cdot \tilde{x} + \tilde{\varepsilon} = \mu + \beta \cdot \tilde{x} + \tilde{\varepsilon} \]  

(4.6)

where

\[ \beta = \frac{\text{cov}(\tilde{y}, \tilde{x})}{\text{var}(\tilde{x})}. \]  

(4.7)

Suppose that weather derivatives are available for purchase. Since here the risk is from excessively high THI, weather derivatives that will be used are focused on weather call options. The underlying index is \( \text{THI} \), and the strike price is \( THI_{\text{threshold}} \). The payoff from a weather call option is:

\[ \tilde{n} = \max(\tilde{\text{THI}} - THI_{\text{threshold}}, 0) = \tilde{z}. \]  

(4.8)

The option premium is calculated on the basis of actuarial fairness. So purchasing weather options cannot change the farmer’s farmer’s expected profit. The option premium equals the expected payoff:

\[ \pi = E(\tilde{n}) = E(\tilde{z}). \]  

(4.9)

The timing of using weather derivatives is as follows: at the start of the period, the farmer buys weather derivatives; then at the end of the period, when all the weather information of this period has been observed, the farmer receives a payoff (if any) from any weather derivatives purchased.

Also suppose that the producer is free to choose his abatement equipment investment and maintenance strategy. By using abatement equipment, the production loss from heat stress can be reduced. The biological effectiveness of abatement equipment is a function of its initial investment, age, maintenance, and weather condition. Let \( \tilde{m} \) denote the reduced profit loss, i.e. the increased profit from using abatement
equipment; and let \( \lambda \) denote the costs of abatement equipment operating strategy. The explicit functional form will be given in the next section.

Thus with weather options and abatement equipment, the producer’s net profit equals:

\[
\tilde{y}^{\text{net}} = \tilde{y} + \phi \cdot (\bar{n} - \pi) + \tilde{m} - \lambda
\]

where \( \phi \) is weather options purchase amount. Therefore, there are two elements that the producer needs to determine: weather options purchase amount and abatement equipment operating strategy.

### 4.3 Dynamic Programming Model

This section presents a theoretical model of analyzing an asset replacement problem and examining how weather derivatives will affect the decisions. The assumptions include that first and second moments of accumulated weather conditions are constant and a representative farmer has mean-variance utility. Thus the abatement investment and operating strategy will not depend on the current weather conditions, instead it will depend on the first and second moments of weather conditions.

Suppose a representative dairy farmer is seeking an optimal risk management strategy with the aim to maximize the present value of his long-run utility. Since abatement equipment is a kind of fixed asset, its investment level cannot be changed once installed. So the farmer needs to decide the lump-sum initial installation investment. Then at the beginning of each following year, he observes the state, i.e. the status of his abatement equipment: (i) its age, and (ii) how many years since last maintenance – the state information affects the effectiveness of abatement equipment. Then the farmer needs to determine: (i) which action to take for his abatement
equipment in three alternative choices: no action, replacement, and service, and (ii) weather derivatives purchase amount.

Formally, the representative farmer’s decision problem is written as:

$$\max_{t=0}^{\infty} \sum \delta^t U(s_t, x_t)$$

(4.11)

where $t$ is time index in calendar year; $\delta \in (0,1)$ is the farmer’s discount factor reflecting interest rate and the individual’s impatience; $U(s_t, x_t)$ is utility of year $t$; $s_t$ is a vector of state variables at $t$; and $x_t$ is the farmer’s decisions at $t$. This is an infinite horizon, deterministic model.

More explicitly, the discrete time dynamic programming model consists of the following objects: a state space, an action space, transition rule, a utility function, and a discount factor $\delta$. We now turn to the detailed description of the model.

4.3.1 **State and Action Variables**

The *state vector* $s_t = (a_t, h_t)$ consists of two variables:

- $a_t = \{1, 2, \ldots, \text{lifespan}\}$: age of abatement equipment;
- $h_t = \{1, 2, \ldots, \text{lifespan}\}$: maintenance history (i.e. how many years since last maintenance).

The state variables, observed at the start of period $t$, represent the abatement equipment’s current status that affects its effectiveness and maintenance costs in period $t$.

And the *action vector* $x_t = (i_t, \phi_t)$ also consists of two variables:

$$i_t = \begin{cases} 
1, & \text{no action} \\
2, & \text{maintenance} \\
3, & \text{replace} 
\end{cases}$$
is abatement equipment operating decision at time $t$; 
\[ \phi_t \in \mathbb{R}^+ \] : weather derivatives purchase amount at time $t$.

Once the state values have been observed, the farmer’s decision problem is to choose the abatement equipment operating action, which will affect the current and future utility; and weather derivatives purchase amount, which will only affect utility of period $t$ since the payoff (if any) will be claimed at the end of that period.

The abatement status transition function is:
\[
(a_{t+1}, h_{t+1}) = \begin{cases} 
(a_t + 1, h_t + 1), & \text{if } i_t = 1 \text{ (no action)} \\
(a_t + 1, 1), & \text{if } i_t = 2 \text{ (maintenance)} \\
(1, 1), & \text{if } i_t = 3 \text{ (replace)}. 
\end{cases}
\]

### 4.3.2 Formulating Profit and Utility Functions

In the one-period setting, net profit is given by:
\[
\tilde{y}_t^{net} = \tilde{y}_t + \phi_t \cdot (\tilde{n}_t - \pi) + \tilde{m}_t - \lambda_t,
\] (4.12)

where $\tilde{y}_t = \mu + \beta \cdot \tilde{x}_t + \tilde{\varepsilon}_t$ and $\pi = E(\tilde{n}_t)$.

In (4.12), the biological functional form of the effectiveness of abatement equipment is formulated as:
\[
\tilde{m}_t = \kappa^{(h_t + a_t/n)}[(b + c \cdot \widehat{THI}_t)\sqrt{n}],
\] (4.13)

where $\tilde{m}_t$ is the reduced profit loss, i.e. the increased profit from using abatement equipment; $\widehat{THI}_t$ is cumulative THI of period $t$; $\eta$ is initial abatement equipment installation investment; $\kappa \in (0, 1)$ reflects the declining effectiveness with aging and less frequent maintenance; and $b$ and $c$ are parameters reflecting the effect of weather condition.

Suppose that the producer is free to choose his initial abatement equipment investment $\eta$ ( $\eta \geq 0$ ; and $\eta = 0$ means he does not install abatement equipment). It
is easy to see that \( \tilde{m}_t \) is increasing with \( \eta \) and \( \tilde{THI}_t \). When \( \eta = 0 \), \( \tilde{m}_t \) is also equal to 0. And with fixed \( \eta \), \( \tilde{m}_t \) is increasing with \( \tilde{THI}_t \). That is because although the profit is low when \( \tilde{THI}_t \) is high, the reduced profit loss will be high with using abatement equipment; on the other hand, when \( \tilde{THI}_t \) is low (i.e. weather is good in period \( t \)), the abatement equipment is not of much use, so the reduced loss is low. Thus the parameter \( c \) is positive.

In (4.12), the costs of abatement equipment operating strategy, \( \lambda_t \) are specified as:

\[
\lambda_t = \begin{cases} 
0, & i_t = 1 \\
(b_1 a_t + b_2 h_t) \cdot \eta, & i_t = 2 \\
\eta, & i_t = 3.
\end{cases}
\] (4.14)

Here, \( k_1 \) and \( k_2 \) are positive parameters. So maintenance costs are increasing with age \( a_t \) and maintenance history \( h_t \). Replacement costs are equal to initial abatement investment by assuming the residual value of the old equipment equals its uninstall costs.

In summary, the profit function is specified as:

\[
y_{net} = \begin{cases} 
\tilde{y}_t + \phi_t \cdot (\tilde{n}_t - \pi) + \kappa (h_t + a_t/n) [(b + c \cdot \tilde{THI}_t) \sqrt{\eta}], & i_t = 1 \\
\tilde{y}_t + \phi_t \cdot (\tilde{n}_t - \pi) + \kappa (a_t/n) [(b + c \cdot \tilde{THI}_t) \sqrt{\eta}] - (k_1 a_t + k_2 h_t), & i_t = 2 \\
\tilde{y}_t + \phi_t \cdot (\tilde{n}_t - \pi) + [(b + c \cdot \tilde{THI}_t) \sqrt{\eta}] - \eta, & i_t = 3.
\end{cases}
\] (4.15)

Note that at the beginning of a period, if the farmer chooses to replace his abatement equipment with a new one, \( a_t \) and \( h_t \) turn to zero in that period; if he chooses to have maintenance service, \( h_t \) turns to zero in that period.

The producer is assumed to have a mean-variance utility function\(^{25}\) of

\[
U = E(\bullet) - \frac{1}{2} A \cdot \text{var}(\bullet)
\] (4.16)

\(^{25}\)This framework is equivalent to expected utility maximization if (net) profit is distributed normally and producers’ utility function is exponential. But Meyer has shown that the mean-variance model is consistent with expected utility model under much weaker restrictions. See Pratt (1964) and Meyer (1987).
where $A$ is an index of agents’ aversion to taking on risk. Then the representative producer’s objective is to choose initial abatement investment $\eta$, optimal option purchase $\phi_t$, and abatement equipment operating strategy $i_t$ to maximize his long-run utility:

$$\max_{\eta, \phi_t, i_t} \sum_{t=0}^{\infty} \delta^t U_t \equiv \max_{\eta, \phi_t, i_t} \sum_{t=0}^{\infty} \delta^t \left[ E(\tilde{y}_{t}^{\text{net}}) - \frac{1}{2} A \cdot \text{var}(\tilde{y}_{t}^{\text{net}}) \right].$$  (4.17)

Thus far, the dynamic risk management problem is summarized by equations (4.15) and (4.17). In order to examine how weather derivatives will affect the decisions, we will compare the optimization solutions of the cases with and without using weather derivatives. Then the next task is to find the solution to this dynamic optimization problem.

### 4.4 Deriving Optimization Solution

We first derive the optimal weather options purchase within one period setting taking the state and action variables as given. Then a dynamic programming problem across periods will be solved numerically. And the optimal initial abatement investment is chosen by examining a series of different investment levels and picking up the one with maximum Bellman value.

#### 4.4.1 Optimization Within One Period

Because weather derivatives purchase amount in each period will only affect utility at that period and will not affect state variables, the optimal weather derivatives purchase, $\phi_t$, can be decided within one period setting.
\[
\max_{\phi_t} U_t = \max [E(\tilde{y}_{t}^{\text{net}}) - \frac{1}{2} A \cdot \text{var}(\tilde{y}_{t}^{\text{net}})] .
\] (4.18)

Specifically,
\[
U_t = E(\tilde{y}_t) + \phi_t E(\tilde{m}_t - \pi) + E(\tilde{m}_t - \lambda_t) - \frac{1}{2} A \cdot \text{var}(\tilde{y}_t) + \phi_t \text{var}(\tilde{n}_t)
\] (4.19)
\[
= \mu + \kappa^{(h_t + a_t/n)}(b + c\mu_{THI}) \sqrt{\eta} - \lambda_t - \frac{1}{2} A \cdot [\beta^2 \sigma_z^2 + \sigma_z^2 + \phi_t \sigma_m^2 - 2\beta \phi_t \sigma_{\tilde{z},\tilde{n}}
\]
\[
+ \kappa^{(2h_t + 2a_t/n)} c^2 \eta \sigma_{THI}^2 - 2\beta \kappa^{(h_t + a_t/n)} c \sqrt{\eta} \sigma_{THI,\tilde{z}} + 2\kappa^{(h_t + a_t/n)} \phi_t c \sqrt{\eta} \sigma_{THI,\tilde{n}}]
\]
where \(\mu = E(\tilde{y}_t)\), \(\mu_{THI} = E(\tilde{THI}_t)\); \(\sigma_z^2 = \text{var}(\tilde{z}_t)\), \(\sigma_n^2 = \text{var}(\tilde{n}_t)\), \(\sigma_{THI}^2 = \text{var}(\tilde{THI}_t)\); \(\sigma_{\tilde{z},\tilde{n}} = \text{cov}(\tilde{z}_t, \tilde{n}_t)\), \(\sigma_{THI,\tilde{z}} = \text{cov}(\tilde{THI}_t, \tilde{z}_t)\), \(\sigma_{THI,\tilde{n}} = \text{cov}(\tilde{THI}_t, \tilde{n}_t)\). From equation (4.9), we know that \(\tilde{n}_t = \tilde{z}_t\). Therefore \(\sigma_n^2 = \sigma_z^2 = \sigma_{\tilde{z},\tilde{n}}\) and \(\sigma_{THI,\tilde{z}} = \sigma_{THI,\tilde{n}}\). And all these are positive numbers.

Take first order condition with respect to \(\phi_t\),
\[
\phi_t \sigma_n^2 - \beta \sigma_{\tilde{z},\tilde{n}} + \kappa^{(h_t + a_t/n)} c \sqrt{\eta} \sigma_{THI,\tilde{n}} = 0 .
\] (4.20)

From (4.20), the optimal weather options purchase amount is
\[
\phi_t^* = \beta - \kappa^{(h_t + a_t/n)} c \frac{\sigma_{THI,\tilde{z}}}{\sigma_z^2} \sqrt{\eta} .
\] (4.21)

The following three propositions are derived from equation (4.21):

**Proposition 12.** The optimal option purchase amount is increasing with \(\beta\). It means that the more the producer’s profit is sensitive to weather risk, the more options he should purchase, ceteris paribus.

**Proposition 13.** The optimal weather option purchase amount is decreasing with initial abatement equipment investment \(\eta\). Thus it indicates that weather options can act as a substitute for abatement equipment.
Proposition 14. The optimal weather options purchase amount is increasing with age $a_t$ and maintenance history $h_t$.

Propositions 13 and 14 say, for the best risk management results, if the farmer’s initial abatement investment is relatively low or the abatement equipment is old and less-frequently maintained, he will buy more weather options to hedge against risk from excessive heat stress.

4.4.2 Optimization Across Periods

Except in rare and highly specialized cases, it is impossible to derive analytical solution for dynamic programming problems. Here we employ numerical methods to find the optimization solution.

The Bellman “principle of optimality” formally is expressed in the form of Bellman’s equation:

$$
V(a_t, h_t) = \max_{\phi_t \in \mathbb{R}^+; \ i_t \in \{1,2,3\}} \{ U(a_t, h_t; \phi_t, i_t) + \delta V(a_{t+1}, h_{t+1}) \} 
$$

(4.22)

where the Bellman value, $V(a_t, h_t)$, is the maximum discounted sum of current and future utility starting from period $t$.

Because the dynamic problem has an infinite horizon, the value function in equation (4.22) will not depend on time $t$.\textsuperscript{26} We may drop the time subscripts for the sake of visual clarity. Let $v$ and $x$ denote the vectors of Bellman values and optimal decisions corresponding to all possible states. And the Bellman equation is rewritten

\textsuperscript{26}Our empirical analysis shows that there is not an obvious difference in optimal decisions and Bellman values between problems of infinite horizon and of a long finite horizon (say, 40 years), because after these many years the present discounted value of future utility is negligible.
as a vector fixed-point equation:

\[ v = \max_x \{ u(x) + \delta v \}. \] (4.23)

The fixed-point equation can be solved to obtain unique and exact \( v \) and \( x \) by the \textit{policy iteration} method. Specifically, the policy iteration algorithm consists of two steps: \textit{policy evaluation} and \textit{policy improvement}. Suppose that at iteration \( j \), we have candidate decision rule \( x_j \). Then the value function \( v_j \) can be recovered via \textit{policy evaluation}, i.e.

\[ v_j = \{ u(x_j) + \delta v_j \} = (1 - \delta)^{-1} u(x_j). \] (4.24)

Next, at iteration \( j + 1 \), with \( x_j \) and \( v_j \), a new policy \( x_{j+1} \) is formed via a \textit{policy improvement} step:

\[ x_{j+1} = \arg \max \{ u(x_j) + \delta v_j \}. \] (4.25)

Then \( x_{j+1} \) is put into equation (4.24) to update the Bellman value. The loop of (4.24) and (4.25) is iterated until it converges. Therefore, the policy iteration algorithm converts the dynamic programming problem into the problem of computing a fixed point with a certain contraction mapping. The contraction property guarantees that the unique fixed point solution exists and is insensitive to rounding errors as long as the discount rate \( \delta \) is less than 1.

### 4.5 Basis Risk and Transaction Costs

Basis risk exists in incomplete real-world markets. Basis risk in this study comes from the difference between the underlying index of weather derivatives and the weather factor that affects the dairy profit (namely, THI). Two kinds of basis risk are
investigated. One is geographical basis risk, which occurs from the difference in location between the reference site of weather derivatives and the actual production area. The other is reference-index basis risk, which occurs because weather derivatives are typically based upon temperature, yet biological stresses occur as a function of THI. Accordingly, the payoff from a weather call option, equation (4.8), is changed into:

$$\tilde{n} = \max(\tilde{I} - I_{\text{threshold}}, 0)$$ \hspace{1cm} (4.26)

where $\tilde{I}$ is the stochastic value of a weather index, and $I_{\text{threshold}}$ is the strike level. Equation (4.26) captures the presence of both reference-index and geographical basis risk. If the reference index $\tilde{I}$ is temperature rather than THI, it reflects index basis risk. If the reference index $\tilde{I}$ is weather condition of a location other than the production area, then geographical basis risk exists. Note that if $\tilde{I}$ is THI of the production area, there is no basis risk.

Transaction costs in weather options are imposed by setting the option premium as the expected payoff plus proportional transaction costs. Then equation (4.9) is revised into:

$$\pi = (1 + \gamma)E(\tilde{n})$$ \hspace{1cm} (4.27)

where the loading rate $\gamma > 0$ reflects transaction costs related to administrative and implementation fees and the desirability to the issuers. If $\gamma$ is zero, the weather options are actuarially-fairly priced.

The presence of basis risk and transaction costs changes the optimal weather derivatives purchase amount, i.e. equation (4.21), into:

$$\phi^* = -\frac{\gamma \mu_{\tilde{n}}}{A \sigma_{\tilde{n}}^2} + \beta \frac{\sigma_{\tilde{z}_{\tilde{n}}}}{\sigma_{\tilde{n}}^2} - \kappa (h_t + a_t/n)C \frac{\sigma_{\tilde{THI}_{\tilde{n}}}}{\sigma_{\tilde{n}}^2} \sqrt{\eta}.$$ \hspace{1cm} (4.28)
Propositions 12–14 still hold in equation (4.28). In addition, equation (4.28) shows that the optimal weather derivative purchase amount is negatively related to the loading rate $\gamma$. The intuition is that due to transaction costs, purchasing weather derivatives lowers expected profit, and thus higher transaction costs make weather derivatives less attractive.

### 4.6 Risk Management Effectiveness

In order to derive the increased utility from using weather options and abatement equipment, we define the utility using neither of the two instruments as a benchmark.

$$U_t^0 = E(\tilde{y}_t) - \frac{1}{2} A \cdot \text{var}(\tilde{y}_t) \tag{4.29}$$

And the increased utility in certainty equivalent at $t$ is:

$$\Delta U_t = U_t - U_t^0 = [E(\tilde{y}_t^{net}) - E(\tilde{y}_t)] - \frac{1}{2} A \cdot [\text{var}(\tilde{y}_t^{net}) - \text{var}(\tilde{y}_t)]. \tag{4.30}$$

Clearly, the optimal decision rule satisfies:

$$\arg\max_{\eta, \phi, i} \{\sum_{t=0}^{\infty} \delta^t U_t\} \equiv \arg\max_{\eta, \phi, i} \{\sum_{t=0}^{\infty} \delta^t \Delta U_t\}. \tag{4.31}$$

It is also viable to study the cases in which only one of the two instruments is used by imposing the value of the other instrument to be zero. Therefore, the risk management value of weather derivatives can be examined by comparing the cases of using both two instruments and using abatement equipment alone.

### 4.7 Data

For the empirical illustration, we use 35 years (1949 to 1964 and 1984 to 2002) of weather data of Summit County and Cleveland, Ohio collected from the National
Climate Data Center (NCDC), a subsidiary of the National Oceanic Atmospheric Administration (NOAA).\textsuperscript{27} Summit County is treated as the actual production area. Cleveland, as a metropolitan city in the same state as the production area, is selected as the weather derivatives reference city for the purpose of investigating the effect of basis risk.

The weather data include daily maximum and minimum temperature and daily maximum and minimum relative humidity. Daily temperature and dew point both follow routinely seasonal patterns each year. So the "burn-rate" method works well with them for pricing weather options. Daily maximum temperature-humidity index (THI) can be derived from daily maximum temperature and minimum relative humidity. Note in the models, $\tilde{THI}$ corresponds to maximum THI. When maximum THI is lower than 70 degrees in a day, there is no heat stress for dairy cows.

Corresponding to the weather data of Summit County, a representative producer’s milk loss from heat stress and reduced loss from using abatement equipment are generated by employing the results in SCS.\textsuperscript{28} Abatement investment cannot change in a relatively long period once fixed. Also weather options are assumed to be written on summer basis, i.e. the payoff is cumulative $\tilde{n}$ of a summer. Thus, Equations (4.6) and (4.13) will be estimated based on cumulative summer data. The summer period is set from May 1\textsuperscript{st} to Oct. 31\textsuperscript{st} every year, because 97\% of heat stress occurs in this period.

\textsuperscript{27}It is a quite common phenomenon that daily relative humidity data from 1965 to 1983 are missing across weather stations in NCDC database.

\textsuperscript{28}See the Appendices for detail.
4.8 Empirical Illustration

Note that for the results of the empirical illustration, the herd size is normalized into unity.

In order to solve the dynamic programming problem, values of the parameters in the models are needed. Some of the parameters can be estimated from the data. Unfortunately, others such as the abatement maintenance cost parameters cannot be estimated due to lack of data. As a reference point for the empirical illustration, for parameter values which cannot be empirically estimated, values are established based upon discussions with experienced people and reasonable calibrations. Deviations from these base values provide insight into how changes in abatement equipment parameters and investors’ preference affect the risk management strategies and their corresponding results.

A more favorable case would be to have data of dairy farmers’ abatement maintenance and replacement records, and effectiveness of abatement equipment in different states. Assuming the farmers’ asset maintenance/replacement decisions coincide with the dynamic optimization principle, several alternative functional forms of abatement effectiveness and maintenance costs can then be estimated and tested. With the estimated parameters, the optimal dynamic decision while simultaneously using weather derivatives and abatement equipment can be given.

Table 4.1 reports the summary of the parameters, which are either estimated from data or assigned. By and large, the empirical part of this paper is to provide an illustration on how weather derivatives can be used together with abatement equipment to enhance a dairy farmer’s risk management effectiveness.
4.8.1 Parameter Estimation

Following SCS, $THI_{\text{threshold}}$ is set as 70 degrees. From the weather data of Summit County and the SCS milk loss model, we calculate the daily milk loss during summers of the 35 years and the corresponding daily $\widetilde{THI}$. Then by accumulating the milk loss and $\tilde{z} = \max(\widetilde{THI} - THI_{\text{threshold}}, 0)$ during each summer in the 35 years, we have 35 observations of cumulative profit loss and $\tilde{x} = E(\tilde{z}) - \tilde{z}$. From a least squares regression, $\beta$ is estimated, which is 0.5635 kg milk per cow with Student’s $t$-value and R-squared of 16.47 and 0.89. That is to say each degree of $\tilde{z}$ beyond its mean will induce 0.5635 kg milk loss. The milk price is set as $0.287/kg as in SCS, so the milk loss is $0.1617$ per degree of $\tilde{x}$.

We put the daily summer weather data into the SCS abatement effect model\textsuperscript{29} to calculate the daily reduced THI corresponding to seven abatement levels. Because the abatement levels in SCS are expressed as yearly investment with an annualization rate of 15%, we calibrate these levels into initial abatement investment levels $\eta$ with the assigned parameters $\kappa$ and $n$ in equation (4.13) and with the assumption that the states of equipment in SCS is $a_t = 6$ and $h_t=0$, or $a_t = 2$ and $h_t = 1$. Multiplying the estimated $\beta$ and milk price, we calculate daily reduced profit loss (in dollars) due to abatement investment (in dollars). The reduced profit loss and THI are accumulated for each summer. Thus there are 35 observations of cumulative reduced profit loss and cumulative THI for each of the seven abatement investment levels. By a least

\textsuperscript{29} In SCS there are three abatement effect models corresponding to three abatement intensity levels. The first model is for only using fans or sprinklers; the second model is for a combination of fans and sprinklers; and the third model is for a specific system, the Korral Cool system, which is used in the Southwest and other dry and hot areas. In the research, we use the second model, and based on this model, we linearly simulate another six abatement effect functions corresponding to six different fixed cost levels. See Appendix B.
squares regression, the estimates of $b$ and $c$ are -32.0920 and 0.0029 with Student’s $t$-values of 16.66 and 19.06 and R-squared of 0.91.

### 4.8.2 Dynamic Optimization Results

With the estimated and assigned parameter values can we solve the dynamic problem using *Policy Iteration* Method.\(^{30}\) For the purpose of examining how weather derivatives can affect abatement equipment decisions, we study the dynamic optimization problem under two scenarios: (i) using abatement equipment alone; and (ii) simultaneously using abatement equipment and weather derivatives. In this subsection, basis risk and transaction costs are not investigated, and therefore weather data used here are merely from Summit County, Ohio.

The optimal initial abatement investment levels, $\eta^*$, for scenarios (i) and (ii) are found by examining the dynamic optimization solutions corresponding to a series of different investment levels and picking up the levels with maximum Bellman values. The optimal levels are $105.6$ and $67.2$ for the two scenarios, respectively. Hence, using weather derivatives can mitigate the problem of high initial installation cost of abatement equipment.

Table 4.2 gives the optimal abatement equipment maintenance/replacement decisions, $i_t^*$, corresponding to all possible states. It can be seen that the decisions for the cases with and without using weather derivatives are quite similar. The only three different decisions show that scenario (i) is a bit inclined to replace the equipment earlier. However, for an equipment no more than 6-year-old, the best strategy is to have maintenance.

\(^{30}\)CompEcon Toolbox in Miranda and Fackler (2002) is used to perform policy iteration.
Figure 4.1 (a) and (b) give the optimal abatement equipment replacement rules for scenarios (i) and (ii). Without using weather derivatives, a set of new abatement equipment can be used for 9 years before replaced; while with weather derivatives, it can be used for 10 years. Therefore, using weather derivatives can postpone the replacement of abatement equipment. Figure 4.2 gives the optimal abatement maintenance policies. Under scenario (i), maintenance services should be given each year except the first and last years during the life of abatement equipment; while under scenario (ii), maintenance will not be given in the 8th year besides the first and last years. Thus, using weather derivatives reduces the frequency of maintenance.

The optimal purchase amount of weather derivatives $\phi_t^*$ under scenario (ii) is displayed in figure 4.3. As can be seen, a farmer should buy more weather derivatives as his abatement equipment turns older, especially in the 8th and 10th years when no maintenance is given. It means that as the equipment turns old, its effectiveness is declining, and therefore more weather derivatives should be used as compensation.

The risk management value of weather derivatives and abatement equipment is investigated using equations (4.29)-(4.31). First, as a benchmark, suppose this representative farmer employs neither weather derivatives nor abatement equipment. According to our data and estimated $\beta$, the mean and variance of his annual revenue loss due to heat stress are $49.69$ and $411.19$. By the mean-variance utility model, the annual utility loss of the farmer with risk aversion $\lambda$ of 0.20 is $(-49.69 - \frac{1}{2} \cdot 0.2 \cdot 411.19) = -90.81$ dollars in certainty equivalent. Thus the long-run utility loss from heat stress is $\sum_{t=0}^{\infty} \delta^t U_{loss} = \sum_{t=0}^{\infty} 0.9^t \cdot 90.81 = 908.1$ dollars. Under scenario (i), by using abatement equipment alone, the maximized increased utility in the dynamic optimization, i.e. $\max_{\eta;\Delta t} \sum_{t=0}^{\infty} \delta^t \Delta U_t$, is $248.4$ in certainty equivalent. So
the optimal use of abatement equipment can reduce the utility loss by 27.35%. And under scenario (ii), the maximized increased utility, i.e. \( \max_{\eta, \phi} \sum_{t=0}^{\infty} \delta^{t} \Delta U_t \), is $499.8 in certainty equivalent. The optimal use of these two instruments can reduce utility loss by 55.04%. Therefore, using weather derivatives together with abatement equipment can significantly enhance the risk management effectiveness over using abatement equipment alone.

4.8.3 Effect of Basis Risk and Transaction Costs

In this subsection, Cleveland serves as the weather option reference city. Therefore, weather data of Cleveland are used to calculate the payoff and premium of weather options in equations (4.26) and (4.27). The option strike level, \( I_{\text{threshold}} \) in (4.26), is assumed to be chosen by the buyers. The optimal strike levels and optimal initial abatement investment levels are determined by examining the dynamic optimization solutions corresponding to a series of combinations of different strike levels and initial investment levels and then picking up the combinations with highest Bellmans values. And transaction cost loading rate \( \gamma \) is set as 5%.

Besides the two scenarios in the last subsection, another two scenarios are analyzed here: (iii) simultaneously using abatement equipment and weather options, but in the presence of geographical basis risk and transaction costs; and (iv) simultaneously using abatement equipment and weather options, but in the presence of both kinds of basis risk and transaction costs.

Under scenario (iii), the optimal strike level is 75 degrees of THI, and the optimal initial abatement investment level is $74. The maximized increased utility is $425.4 in certainty equivalent. Under scenario (iv), the optimal strike level is 81 Fahrenheit
degrees of temperature, and the optimal initial abatement investment level is $77. The maximized increased utility is $391.3 in certainty equivalent. So although basis risk and transaction costs reduce the risk management effectiveness of weather derivatives, compared with using abatement equipment alone, it still is a significant improvement to use weather derivatives together with abatement equipment in dairy profit risk management (figure 4.4).

The optimal maintenance/replacement decisions under scenarios (iii) and (iv) are the same as those of using abatement equipment alone, i.e. scenario (i). And the optimal weather option purchase amount is displayed in figure 4.3. Presence of basis risk and transaction costs lowers the optimal purchase amount of weather options.

4.9 Conclusion and Discussion

In this chapter, we employ a dynamic programming framework to study the case that a representative dairy farmer maximizes his long-run utility using weather derivatives and abatement equipment to reduce profit loss and risk from heat stress. An exact solution as the optimal risk management strategy for weather derivatives purchase and abatement equipment replacement is derived numerically using policy iteration method. The empirical results suggest that simultaneously using weather derivatives and abatement technologies will outperform using abatement technologies alone, even in the presence of basis risk and transaction costs. And using weather derivatives reduces the investment in abatement equipment, which mitigates the problem of high initial installation cost of abatement equipment faced by small- and medium-size firms. Besides, the optimal purchase of weather derivatives depends on age and maintenance status of abatement equipment.
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<th>Symbol</th>
<th>Value</th>
<th>Interpretation</th>
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<td>$\delta$</td>
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<td>Intertemporal discount factor</td>
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<td>$A$</td>
<td>0.2</td>
<td>Pratt’s Absolute Risk Aversion</td>
<td>(16)</td>
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Note: “Equation” in the last column gives the numbers of equations where the parameters show up for the first time. The symbol *** indicates the estimates are at the 1% significance level.

Table 4.1: Summary of Parameters

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Note: Inside the decision matrix, numbers 1, 2, and 3 denote “no action”, “maintenance”, and “replacement” respectively. These numbers are the optimal decisions with using weather derivatives. Numbers in superscripts are different decisions in the case of not using weather derivatives.

Table 4.2: Optimal Maintenance/Replacement Decisions
Figure 4.1: Optimal replacement rule

Figure 4.2: Optimal maintenance rule
Figure 4.3: Optimal weather derivatives purchase

Figure 4.4: Comparison of risk management effectiveness
CHAPTER 5

SUMMARY AND CONCLUSION

This thesis provides an applicable link of the burgeoning weather derivatives literature in agricultural economics to a real-world application in which an easily-quantifiable weather metric (daily THI in excess of a biological threshold) is the primary source of production risk for a major agricultural commodity. Further, unlike other possible applications of weather derivatives, dairy is unique in that weather derivatives are likely substitutable for capital investment in heat abatement equipment, such as fans or water misters.

This study investigates the potential of weather derivatives in hedging against dairy profit risk, arising from heat stress. A representative dairy producer’s profit risk is decomposed into systematic risk from weather conditions and idiosyncratic risk which is uncorrelated with weather condition. With access to hypothetical weather derivatives and abatement equipment, the producer’s optimal portfolio choice of these two instruments is derived in a utility maximization framework. Basis risk and multi-period decisions are addressed in this study. The results suggest that weather derivatives can act as a substitute for abatement technologies and the simultaneous usage of them is more favorable than using either of them alone.

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In addition, this research raises many questions of relevance to the economic community, such as the optimal contract design, whether the existence of these contracts reinforces economies of scale in dairy production, what level of sophistication is required to effectively utilize these tools, and finally, what size of a dairy is required to use weather derivatives. These questions may be of interest for further research.
APPENDIX A

MILK LOSS FUNCTION

The milk loss model in SCS (2003) is: \( MILK_{loss} = 0.0695 \times (THI_{max} - THI_{threshold})^2 \times Duration \), where \( MILK_{loss} \) is in kilograms, and \( Duration \) is the proportion of a day in which heat stress occurs (i.e. \( THI_{max} > THI_{threshold} \)) for dairy cows maintained in a system of minimal cooling.

With the assumption that daily THI follows a perfect sine function with a period of 24 hours,\(^{31}\) the process to calculate the \( Duration \) of heat stress is:

\[
THI_{mean} = (THI_{max} + THI_{min})/2
\]

if \( THI_{max} < THI_{threshold} \)

\( Duration = 0 \)

elseif \( THI_{min} \geq THI_{threshold} \)

\( Duration = 24 \)

elseif \( THI_{mean} > THI_{threshold} \)

\( Duration = (\pi - 2 \times \arcsin(THI_{threshold}/THI_{max} - THI_{mean}))/\pi \times 12 \)

else

\(^{31}\)This assumption accounts for the extent and cumulative severity of heat stress over the course of a day. SCS state that this assumption underestimates the duration of heat stress at higher latitudes in summer, but gains in accuracy from using more complex models are overall small.
\[ Duration = (\pi + 2 \times \arcsin(\frac{THI_{\text{mean}} - THI_{\text{threshold}}}{THI_{\text{max}} - THI_{\text{mean}}})) / \pi \times 12 \]
end
APPENDIX B

ABATEMENT EFFECT FUNCTION

In SCS, for a 50 m² cow pen, which can hold 7.1759 dairy cows, when the annualized fixed costs are $310, the corresponding operating costs are $0.0685/hour of operation. The abatement effect is: \( \Delta THI = -17.6 + (0.36 \times T) + (0.04 \times H) \), where \( \Delta THI \) is the change in apparent THI, \( T \) is ambient temperature (°C), and \( H \) is ambient relative humidity in percent.

Based on the above specifications, we linearly simulate six abatement effect functions corresponding to six fixed cost levels. The six fixed cost levels are 130, 190, 250, 370, 430, and 490 dollars respectively. That is, all the parameters in a simulated model are proportional to those in the SCS model, with the proportion equal to the ratio of fixed cost levels.

We define the reduced loss by: \( \tilde{m} = \max(\min(THI_{\text{max}} - THI_{\text{threshold}}, \Delta THI), 0) \times \beta \times \text{MILK price} \).
APPENDIX C

PROOF OF PROPOSITION 11

From equations (3.17) and (3.18),

\[
\frac{d\phi^*}{d\beta} = \frac{\sigma \tilde{z}, \tilde{n}}{\sigma^2_n} - b \frac{\sigma \tilde{T_{HI}}, \tilde{n}}{\sigma^2_n} \frac{Ab[\sigma \tilde{T_{HI}}, \tilde{z} - \sigma \tilde{T_{HI}}, \tilde{n}]}{2 + Ab^2[\sigma^2 \tilde{T_{HI}} - \sigma^2_n]} \]

\[
= \frac{1}{\sigma^2_n} \left[ \sigma \tilde{z}, \tilde{n} - \frac{Ab^2\sigma \tilde{T_{HI}}, \tilde{n}}{2 + Ab^2[\sigma^2 \tilde{T_{HI}} - \sigma^2_n]} \left[ \sigma \tilde{T_{HI}} - \sigma \tilde{T_{HI}}, \tilde{n} \sigma \tilde{T_{HI}} - \sigma \tilde{T_{HI}}, \tilde{n} \right] \right] \]

\[
= \frac{1}{\sigma^2_n} \left[ \sigma \tilde{z}, \tilde{n} - \frac{Ab^2\rho \tilde{T_{HI}}, \tilde{n}\sigma \tilde{T_{HI}}[\rho \tilde{T_{HI}}, \tilde{z} - \rho \tilde{T_{HI}}, \tilde{n}] + Ab^2\sigma^2 \tilde{T_{HI}}[1 - \rho^2 \tilde{T_{HI}}, \tilde{n}]}{2 + Ab^2\sigma^2 \tilde{T_{HI}}[1 - \rho^2 \tilde{T_{HI}}, \tilde{n}]} \right] \]

If the last item of the numerator in the big bracket, i.e. \([\rho \tilde{T_{HI}}, \tilde{z} - \rho \tilde{T_{HI}}, \tilde{n}\rho \tilde{z}, \tilde{n}]\) is negative\(^{32}\), then it is easy to see that \(\frac{d\phi^*}{d\beta} > 0\).

If \([\rho \tilde{T_{HI}}, \tilde{z} - \rho \tilde{T_{HI}}, \tilde{n}\rho \tilde{z}, \tilde{n}]\) is positive, it follows from (A.1):

\(^{32}\)This normally does not happen. Refer to the derivation of Proposition 10.
\begin{equation}
\frac{d\phi^*}{d\beta} = \frac{1}{\sigma_n^2} \left[ \sigma_{\tilde{z}, \tilde{n}} - \frac{Ab^2 \sigma^2_{\text{THI}} \sigma_n \sigma_{\tilde{z}} [\rho_{\text{THI}, \tilde{z}} \rho_{\text{THI}, \tilde{n}} - \rho^2_{\text{THI}, \tilde{n}} \rho_{\tilde{z}, \tilde{n}}]}{2 + Ab^2 \sigma^2_{\text{THI}} [1 - \rho^2_{\text{THI}, \tilde{n}}]} \right]
\end{equation}

\begin{align}
&> \frac{1}{\sigma_n^2} \left[ \sigma_{\tilde{z}, \tilde{n}} - \frac{Ab^2 \sigma^2_{\text{THI}} \sigma_n \sigma_{\tilde{z}} [\rho_{\text{THI}, \tilde{z}} \rho_{\text{THI}, \tilde{n}} - \rho^2_{\text{THI}, \tilde{n}} \rho_{\tilde{z}, \tilde{n}}]}{Ab^2 \sigma^2_{\text{THI}} [1 - \rho^2_{\text{THI}, \tilde{n}}]} \right] \\
&= \frac{\sigma_{\tilde{z}, \tilde{n}}}{\sigma_n^2} \left[ 1 - \frac{\rho_{\text{THI}, \tilde{z}} \rho_{\text{THI}, \tilde{n}} - \rho^2_{\text{THI}, \tilde{n}} \rho_{\tilde{z}, \tilde{n}}}{1 - \rho^2_{\text{THI}, \tilde{n}}} \right] \\
&= \frac{\sigma_{\tilde{z}, \tilde{n}}}{\sigma_n^2} \left[ 1 - \frac{\rho_{\text{THI}, \tilde{z}} \rho_{\text{THI}, \tilde{n}} - \rho^2_{\text{THI}, \tilde{n}} \rho_{\tilde{z}, \tilde{n}}}{1 - \rho^2_{\text{THI}, \tilde{n}}} \right]
\end{align}

So if this inequality holds,

\begin{equation}
\frac{\rho_{\text{THI}, \tilde{z}} \rho_{\text{THI}, \tilde{n}}}{\rho_{\tilde{z}, \tilde{n}}} < 1,
\end{equation}

then,

\begin{equation}
\frac{d\phi^*}{d\beta} > 0.
\end{equation}

Hence, the inequality (A.3) is the sufficient condition for (A.4) to hold, i.e. Proposition 11. By the definitions of $\tilde{z}$, and $\tilde{n}$ in (3) and (8), it is a reasonable assumption that $\rho_{\tilde{z}, \tilde{n}} > \rho_{\text{THI}, \tilde{n}}$. Together with the fact that $\rho_{\text{THI}, \tilde{z}}$ is less than 1, we have the inequality (A.3). Actually this inequality holds for all of our weather data.


