MODELING AND CONTROL OF SWITCHED RELUCTANCE MACHINES FOR ELECTROMECHANICAL BRAKE SYSTEMS

DISSERATION

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ABSTRACT

Electro-mechanical brake (EMB) systems have been proposed to replace the conventional hydraulic brake systems. Due to the advantages such as fault tolerant operation, robust performance, high efficiency, and reliable position sensorless control, switched reluctance machine (SRM) has been chosen as the servomotor of the EMB systems. This research is focused on the modeling and control of switched reluctance machines for EMB systems. The overall goal is to design a robust clamping force controller without position sensors for the SRM.

An accurate model and precisely estimated parameters are critical to the successful implementation of the control system. An inductance based model for switched reluctance machine is proposed for this research. Maximum likelihood estimation techniques are developed to identify the SRM parameters from standstill test and online operating data, which can overcome the effect of noise inherent in the data.

Four-quadrant operation of the SRM is necessary for the EMB system. Based on the inductance model of SRM, algorithms for four-quadrant torque control and torque-ripple minimization are developed and implemented.
The control objective of the electro-mechanical brake system is to provide desired clamping force response at the brake pads and disk. A robust clamping force controller is designed using backstepping. The backstepping design proceeds by considering lower-dimensional subsystems and designing virtual control inputs. The virtual control inputs in the first and second steps are rotor speed and torque, respectively. In the third step, the actual control inputs, phase voltages, appear and can be designed. Simulation results demonstrate the performance and robustness of the controller.

Position sensorless control of SRM is desired to reduce system weight and cost, and increase reliability. A sliding mode observer based sensorless controller is developed. Algorithms for sensorless control at near zero speeds and sensorless startup are also proposed and simulated, with satisfactory results.

Experimental testbed for the electro-mechanical brake system has been setup in the laboratory. DSP based control system is used for SRM control. The algorithms developed in simulation have been implemented on the testbed, with corresponding results given. Future work is suggested to finalize the implementation of the electro-mechanical brake system.
Dedicated to my wife and my parents
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NOMENCLATURE

\( R, L \) : Phase resistance and inductance

\( L_\theta \) : Phase inductance at rotor position \( \theta \)

\( L_a, L_u, L_m \) : Phase inductance at aligned position, unaligned position, and a midway between the two

\( R_d, L_d \) : Resistance and inductance of damper winding

\( \theta, \omega \) : Rotor position and speed

\( \hat{\theta}, \hat{\omega} \) : Estimates of rotor position and speed

\( e_\theta, e_\omega \) : Estimation error of rotor position and speed

\( V, i \) : Phase voltage and current

\( \theta_{on}, \theta_{off} \) : Turn-on and turn-off angles

\( \lambda \) : Flux linkage

\( N_r \) : Number of rotor poles

\( T \) : Electromagnetic torque

\( T_l \) : Load torque

\( T_{ref} \) : Torque command

\( F \) : Clamping force
\( F_{\text{ref}} \) : Force command

\( \hat{\cdot} \) : Estimate of \( \cdot \)

\( [ \cdot ]^T \) : Transpose of \( [ \cdot ] \)

\( E[\cdot] \) : The operation of taking the expected value of \( [\cdot] \)

\( w(\cdot) \) : Process noise sequence

\( v(\cdot) \) : Measurement noise sequence

\( X(\cdot) \) : State vector

\( Y(\cdot) \) : Measured output vector in the presence of noise

\( Q \) : Covariance of the process noise sequence

\( R_0 \) : Covariance of the measurement noise sequence

\( R(\cdot) \) : Covariance of the state vector

\( e(\cdot) \) : Estimation error, \( e(k) = Y(k) - \hat{Y}(k) \)

\( \exp \) : The exponential operator

\( \det \) : Determinant

\( \hat{Y}(k | k-1) \) : The estimated value of \( Y(k) \) at time instant \( k \) given the data up to \( k-1 \)

\( U(\cdot) \) : Input vector

\( \theta(\cdot) \) : Parameter vector
CHAPTER 1

INTRODUCTION

1.1 Research Background

Modern vehicles have become more and more “electrical”. The implementation of by-wire systems - replacing a vehicle’s hydraulic systems with wires, microcontrollers, and electric machines - promises better safety and handling, as well as lower manufacturing costs and weight.

By-wire systems began to be installed well over a decade ago, first in military and then in commercial aircraft. In these systems the control commands are not transferred in a hydraulic/mechanical way but through electrical wires (by-wire). In this area, the advantages against classical hydraulic/mechanical systems have proved to be so substantial that the technique is expected to be used in other areas as well [1].

With the fast development of electric or hybrid electric vehicles (HEV), more and more by-wire systems have been designed for vehicle components, such as throttle-by-wire, steer-by-wire, and brake-by-wire. This dissertation is based on a brake-by-wire project sponsored by Delphi Automotive®.
1.1.1 Brake-By-Wire

The braking system is one of the most important systems in vehicles. In most of the vehicles nowadays, hydraulic actuated brake systems are used. The hydraulic brake system is composed of the following basic components (Figure 1.1): the master cylinder which is located under the hood, and is directly connected to the brake pedal, converts mechanical pressure from driver’s foot into hydraulic pressure. Steel brake lines and flexible brake hoses connect the master cylinder to the slave cylinders located at each wheel. Brake fluid, specially designed to work in extreme conditions, fills the system. Shoes and pads in the calipers are pushed by the slave cylinders to contact the drums and/or rotor disks thus causing drag, which slows the car.

![Figure 1.1 Conventional hydraulic brake systems](image)

The hydraulic brake systems have proved to be very successful in automotive industry. However, to reduce the inherent high cost and necessity of regular maintenance
of the hydraulic systems, people are motivated to find cheap and reliable substitute of the brake systems, that is, electromechanically actuated brake systems, or Brake-By-Wire.

In recent years, the automotive industry and many of their suppliers have started to develop Brake-By-Wire systems [2-4]. An electromechanical brake-by-wire system looks deceptively simple. Wires convey the driver’s pressure from a sensor on the brake pedal to electronic control unit that relays the signal to electromechanical brake actuators at each wheel. In turn, the modular actuators squeeze the brake pads against the brake disk to slow and stop the car [2].

Generally a brake-by-wire system contains the following components: four wheel brake modules (electro-mechanical brake actuators), an electronic control unit (ECU), and an electronic pedal module with pedal feel simulator, as shown in Figure 1.2.

![Figure 1.2 Brake-by-wire systems](image-url)
Brake-by-wire does everything [2]: the antilock and traction control functions of today’s antilock braking systems (ABS) plus brake power assist, vehicle stability enhancement control, parking brake control, and tunable pedal feel, all in a single, modular system.

The brake-by-wire technology is expected to offer increased safety and vehicle stability to consumers and it will provide benefits to automotive vehicle manufacturers who will be able to combine vehicle components into modular assemblies using cost effective manufacturing processes.

1.1.2 Electro-Mechanical Brake Actuators

Among the components of brake-by-wire systems, the wheel brake module (or electro-mechanical brake actuator, EMB) is the most important one. It receives the electronic commands from control unit and generates the desired braking force, by means of electric motors and corresponding mechanical systems.

A prototype of EMB developed by ITT Automotive [3] is shown in Figure 1.3. A spindle is used to actuate the inner brake pad. A bolt at the back of the brake pad, which fits in a recess of the pad support, prevents the spindle from rotating. The nut of the planetary roller gear is driven by the rotor of a brushless torque motor. By integrating the coil of the servo motor directly into the brake housing and by supporting the gear and spindle unit with one central bearing only, the compact design is made possible. A resolver measures the position of the rotor for electronic commutation.
To embed such functionality as ABS (Anti-lock brake system), TCS (Traction control system) etc. in conventional hydraulic brake systems, a large number of electro-hydraulic components are required. An electromechanically actuated brake system, however, provides an ideal basis to convert electric quantities into clamping forces at the brakes [3]. Standard and advanced braking functions can be realized on uniform hardware. The software modules of the control unit and the sensor equipment determine the functionality of the Brake-By-Wire system. The reduction of vehicle hardware and entire system weight are not the only motivational factors contributing to the development of a Brake-By-Wire system. The electro-mechanical brakes have the advantages in many other aspects:
- Environmentally friendly due to the lack of brake fluid
- Improved crash worthiness - its decoupled brake pedal can be mounted crash compatible for the passenger compartment
- Space saving, using less parts
- More comfort and safety due to adjustable pedals
- No pedal vibration even in ABS mode
- Simple assembly
- Can be easily networked with future traffic management systems
- Additional functions such as an electric parking brake can be integrated easily
- Reduced production and logistics costs due to the ‘plug and play’ concept with a minimized number of parts.

1.1.3 Switched Reluctance Machines

Electric motors are used in the electro-mechanical brake systems (EMB) to drive the brake pads. Different types of electric motors have been tried in EMB, such as DC motors, brushless DC motors, or induction motors [2,4,5]. Some prototypes of brake-by-wire system based on these motors have already been developed. However, due to the importance of the brake system and the harsh working environment, a more efficient, reliable, and fault-tolerant motor drive over wide speed range is preferred for this application. In this research, a switched reluctance motor (SRM), is proposed to be the servo-motor in the electro-mechanic brake system, which has the following advantages over the other types of electric machines:
Fault-tolerance

The brake system requires that the motor drive system continue operation in case of errors in phase windings, drive circuits, or sensors. The switched reluctance motors can operate in a reduced power rating when losing one or more phases, while the other motors may completely fail in such cases.

Fast acceleration and high speed operation

Emergency braking requires fast acceleration and high-speed operation of the motor drive. From mechanical point of view, the phase winding or permanent magnet mounted on the rotor prevents the motor from accelerating fast and running at super high speed. This problem exists in all the types of motors mentioned above except switched reluctance motors, which have neither winding nor magnet on the rotor.

Robust performance at high temperature

During braking, a lot of heat will be generated in the caliper house that may cause high temperature. The absence of magnets or windings on the rotor of SRM suggests a robust performance in the presence of high temperature.

Reliable position sensorless control

Position sensor in motor drives mean additional cost, space, and weight, as well as a potential source of failure. Integration of a reliable position sensorless control turns into a
necessary step towards development of high-speed motor drives. Due to the limited available computational time at high speeds, the sensorless method should be robust and timely efficient. This has led us to develop a novel geometry based technique that requires minimum magnetic data and computation. These, in turn, would favor SRM drive as a superior choice for this application.

**High efficiency**

The inherent simplicity of the SRM geometry and control offers high efficiency and a very long constant power speed ratio [6].

In this research, an 8/6, 42V, 50A switched reluctance motor is used in the electro-mechanical brake system.

### 1.2 Research Objectives

A block diagram of the electro-mechanical brake developed in this research is shown in Figure 1.4. The brake assembly consists of a bracket which is rigidly mounted to the vehicle chassis and the floating caliper which is typically held on two sliding pins that are in turn attached to the bracket.

The actuator inside the caliper housing constitutes the actual electromechanical energy conversion device. In this design configuration, the actuator consists of a ball screw assembly that is driven by a dual-stage planetary gear. The input gear of the first
planetary gear stage is connected to a 4-quadrant switched reluctance motor. This allows for conversion of the rotary motion (torque $T_a$) of the servomotor into linear ball screw action in order to create the required clamping force $F_{CL}$ at the wheel brake rotor.

The servomotor interfaces electrically to a 4-quadrant servo-controller that controls current and voltage (controller output $u$) to the motor. An encoder delivers rotor position information to the controller for correct commutation of the motor phases.

A clamping force sensor located in the force path between the ball screw and the caliper housing measures the clamping force $F_{CL}$ of the ball screw. The output signal of this force sensor is used to close the loop on the clamping force command $F_d$ which is generated by the higher-level brake system controller.

The overall goal of this research is to develop and implement a low-cost drive system consisting of a sensorless switched reluctance motor with power converter and controller. This drive system shall be operated as part of an electromechanical clamping device for
the purpose of converting a clamping force command into a physical clamping force in a closed-loop configuration under observation of specific dynamic requirements.

The research objectives have the following main components:

**Modeling and parameter estimation of switched reluctance machines**

Different model structures for switched reluctance machines can be found in literature. An appropriate model for the electro-mechanical brake application is to be decided. Then the parameters for the proposed model need to be estimated from test or operating data. And the identified model and its parameters must be validated by simulation and experiments. Noise effect on the parameter estimation is to be studied too.

**Four-quadrant operation of switched reluctance machines**

In electro-mechanical brake systems, the electric motor needs to be operated in all four quadrants on the torque-speed plane to realize desired clamping force response. For switched reluctance machines, this means to set correct turn-on/turn-off angles for each phase and determine the proper exciting sequence. This will be combined with the torque control described in the following part.

**Torque control and torque ripple minimization of switched reluctance machines**

Based on the structure and torque production principle of switched reluctance machines, the torque generated in each phase is a highly nonlinear function of phase
current and rotor position. And during operation, each phase will be conducting for only a fraction of the electric cycle. So the torque in any individual phase is discontinuous. Since the SRM needs to be operated in four quadrants, it is even more difficult to control the torque and minimize the torque-ripple. Based on the proposed SRM model, new torque control algorithms will be suggested and implemented.

**Robust clamping force control of electro-mechanical brake**

The control objective in electro-mechanical brake system is the clamping force between the brake pads and the brake disk. Generally the force is a nonlinear function of the distance between the pads and the disk which corresponds to the angular movement of the SRM rotor, and the torque seen by the motor is a nonlinear function of the force. For SRM, the control inputs are the phase voltages. It’s hard to define a direct relationship between the control input and the objective. A robust controller needs to be designed to meet different requirements on the clamping force. Back-stepping technology is to be used in this research to design the force controller.

**Rotor position sensorless control of switched reluctance machines**

Rotor position sensing is an integral part of switched reluctance motor control system due to the torque-production principle of SRM. Conventionally, a shaft position sensor is employed to detect rotor position. But this means additional cost, more space requirement and an inherent source of unreliability. A sensorless (without direct position or speed sensors) control system, which extracts rotor position information indirectly from
electrical or other signals, is expected. The sensorless algorithm needs to be stable in full speed range. Sensorless control for motor startup (when regular electrical signals are not available) also needs to be designed.

**Implementation of control algorithms in experimental testbed**

As the first step of the electro-mechanical brake system development, an offline simulation testbed is to be setup in Matlab/Simulink®, with the parameters identified from real motor and brake caliper. All control algorithms will be tested on the simulation testbed first. In later steps, the experimental testbed that contains the switched reluctance motor, power converter, and DSP will be built up. And the control algorithms will then be implemented and tested on the experimental testbed.

Details on how these research objectives have been achieved are described in the following chapters.

### 1.3 Dissertation Organizations

This dissertation is organized as follows. The research background and objectives are introduced in Chapter 1. Literature review of related work is summarized in Chapter 2. Chapter 3 presents the model identification and parameter estimation of switched reluctance machines from standstill test data and operating data. In Chapter 4 the four-quadrant torque control and torque-ripple minimization of SRM are presented. A robust clamping force controller for the electro-mechanical brake is described in Chapter 5. In
Chapter 6 a sliding mode observer based sensorless control algorithm for switched reluctance machines is developed and analyzed. Sensorless control at startup and low speed is also given. And In Chapter 7, the experimental setup for the electro-mechanical brake is introduced and related experimental results are shown. Overall conclusions and future work are presented in Chapter 8.
CHAPTER 2

LITERATURE REVIEW

The idea of brake-by-wire is kind of new. But the related technologies have been studied and applied in other areas for many years. According to the research goal and objectives of the electro-mechanical brake systems, the related researches are focused on electric machines (switched reluctance machines, or SRM, for this research), modeling and parameter identification, torque control and torque-ripple minimization of SRM, back-stepping controller design, and position sensorless control of electric machines. Research activities in these fields can be easily found in publications. The literature review will be based on these topics.

2.1 Modeling of Switched Reluctance Machines

To ensure the high efficiency and successful development of a complicated control system, offline simulation is often performed first. At this stage of development, a model of the system is designed and simulated offline. Then the parameters of the real system will be identified and implemented into the model. An accurate model and precisely
estimated system parameters are critical to the successful implementation of the final control system.

In this research, the model of brake caliper is available from cooperative company. So the major task is to model the switched reluctance machines and power converters.

The nonlinear nature of SRM and high saturation of phase winding during operation makes the modeling of SRM a challenging work. The flux linkage and phase inductance of SRM vary with both the phase current and the rotor position. Therefore the nonlinear model of SRM must be identified as a function of the phase current $i$ and rotor position $\theta$.

Two main types of models for SRM have been suggested in the literature – the flux linkage based model [7-10], and the inductance based model [11-12].

### 2.1.1 Flux Linkage Based SRM Model

The flux linkage based model assumes the nonlinear relationship between the phase flux linkage and phase current and rotor position. A typical flux model of SRM is as follows [10],

$$\lambda = \lambda_s (1 - e^{-f(\theta)i}), \quad (2.1)$$

where $\lambda$ and $i$ are the phase flux linkage and phase current, respectively, $\theta$ is the phase angle, $\lambda_s$ is a constant, and

$$f(\theta) = a + b \cos \theta + c \cos 2\theta + d \sin \theta + e \sin 2\theta. \quad (2.2)$$
A 3-D plot in Figure 2.1 shows the $\lambda$-$\theta$-$i$ relationship obtained from an 8/6 switched reluctance motor.

![3-D plot](image)

Figure 2.1 Nonlinear function $\lambda(\theta, i)$ of an SR motor

In an SRM control system, the phase winding is modeled as an inductance and a resistance connected in series. The resistance $R$ is assumed known and phase voltage $V$ and current $i$ can be measured. Therefore, the flux linkage $\lambda$ can be computed by

$$\lambda = \int (V - Ri) dt.$$  \hspace{1cm} (2.3)

From the relationship $\lambda$-$\theta$-$i$, rotor position angle $\theta$ can be obtained since $\lambda$ and $i$ are known. This is the basic operating principle of the flux linkage based model in rotor position estimation.
2.1.2 Inductance Based SRM Model

In the inductance-based model, the position dependency of the phase inductance is represented by a limited number of Fourier series terms and the nonlinear variation of the inductance with current is expressed by means of polynomial functions [11]:

\[ L(\theta, i) = \sum_{n=0}^{\infty} L_n(i) \cos(nN_r \theta + \varphi_n), \]  

(2.4)

where \( N_r \) is the number of rotor poles, \( L_n \) are coefficients to be decided. In practical use, only the first few terms of the Fourier series are used. The higher-order-term can be ignored without significant error.

A 3-D plot in Figure 2.2 shows the \( L-\theta-i \) relationship obtained from an 8/6 SRM.

![3-D plot showing the \( L(\theta, i) \) relationship for an 8/6 SRM.](image)

Figure 2.2 Nonlinear function \( L(\theta, i) \) of an SR motor
In electro-mechanical brake system, the switched reluctance motor will be running at very low speed (near zero) at steady state. Only one or two phases will be continuous conducting at such situation. Since the flux linkage based model utilizes the integration in Eq. (2.3) to estimate the flux linkage. The errors in parameter \( R \) and measurements \( V \) and \( i \) in the active phase may accumulate very soon, without chance to be reset (reset can only occur when the phase is not conducting). Our simulation shows that flux linkage model based rotor position estimation fails in long time zero speed case, which is the steady state of an electrical brake. It reduces the output clamping force up to 25% and creates oscillation. Same problems do not exist in inductance-based model. This makes the inductance-based model a better choice for electro-mechanical brake application.

The inductance-based model suggested by Fahimi etc. [11] can represent the SRM at standstill or low load condition very well. But for highly saturated condition under high load, the model needs to be improved to include saturation effect and core losses. Also, models with different number of terms in the Fourier series will be compared to select the best model. This will be detailed in Chapter 3.

2.2 Parameter Identification of Electric Machines

Once a model of an electric machine is selected, how to identify the parameters in the model becomes an important issue. Finite element analysis can provide model parameters that will be subjected to substantial variation after the machine is constructed with
manufacturing tolerances. Therefore, the model parameters need to be identified from test and/or operating data.

### 2.2.1 Parameter Estimation in Frequency Domain and Time Domain

Generally, the parameter estimation from test data can be done in frequency domain or time domain. Since noise is an inherent part of the test data, the noise effect on parameter estimation must be taken into consideration. In [13], Keyhani *et al.* studied the effect of noise on parameter estimation of synchronous machines in frequency-domain, and got the conclusion that: noise has significant impact on the synchronous machine parameters estimated from SSFR (steady state frequency response) test data using curve-fitting techniques. The estimated values of machine parameters are very sensitive to the value of armature resistance used in data analysis. Even a 0.5% error in the value of armature resistance could result in unrealistic estimation of machine parameters. Hence a technique should be developed which provides a unique physically realizable machine model even when the test data is noise-corrupted.

### 2.2.2 Neural Network Based Modeling

In [25], Karayaka *et al.* developed an Artificial Neural Network based modeling technique for the rotor body parameters of a large utility generator. Disturbance operating data collected on-line at different levels of excitation and loading conditions are utilized for estimation. Rotor body ANN models are developed by mapping field current \( i_{fd} \) and
power angle $\delta$ to the parameter estimates. Validation studies show that ANN models can correctly interpolate between patterns not used in training.

Nonlinear neural network modeling of other types of electric machines have also been reported [18]. However, for ANN based model, rich data set collected at different loading and exciting levels are needed for training the ANN to improve the performance of the model. This sometimes restricts the application of ANN based models.

2.2.3 Maximum Likelihood Estimation

A time-domain identification technique, which can overcome the multiple solution sets problem encountered in the frequency response technique, is used to estimate machine parameters. The new technique is maximum likelihood estimation, or MLE.

The MLE identification method has been applied to the parameter estimation of many engineering problems. It has been established that the MLE algorithm has the advantage of computing consistent parameter estimates from noise-corrupted data. This means that the estimate will converge to the true parameter values as the number of observations goes to infinity [22,23]. This is not the case for the least-square estimators which are commonly used in power system applications.

Maximum likelihood estimation techniques have been successfully applied to identify the parameters of synchronous [14-16] and induction machines [17] from noise-corrupted data. In this research, MLE techniques will be used to identify the parameters of switched
reluctance machines from standstill test and operating data. The details and the results are given in Chapter 3.

2.3 Torque Control and Torque-ripple Minimization of SRM

Due to the nature of torque production in SRM, the torque generated in any individual phase is discontinuous. The output torque is a summation of the torque generated in all active phases. Ripple-free torque control strategies for SRM have been studied extensively. The most popular approach for ripple minimization has been to store the torque-angle-current characteristics in a tabular form so that optimum phase current can be determined from position measurements and torque requirement.

The method described in [27] is based on the estimation of the instantaneous electromagnetic torque and rotor position from the phases terminal voltages and currents. The flux linkage for the active phase is computed from the voltage, current and stator resistance. Both the rotor position and torque are obtained from the third order polynomial evaluations which coefficients are pre-computed and stored in memory locations of the DSP used to implement the control. These coefficients are computed from the flux linkage versus current and rotor position characteristics curve data measured experimentally; bi-cubic spline interpolation technique is used to generate these coefficients. The estimated torque is compared with a constant reference value and the result of this comparison drives a current regulator to control the motor phase currents. Simulation results have shown that the torque ripple can be reduced from a value of about
100% for the motor operating in open loop to about 10% when the torque ripple minimization controller is utilized.

The method of ripple reduction by optimizing current overlapping during commutation at all torque levels was studied in [28]. The algorithm is based on minimize both the average and peak current hence improves the dynamic performance of the motor and inverter. It is shown that the proposed current profiling algorithm results in the highest possible torque/current inverter rating and an extended operating speed range under constant torque operation.

The research by Husain etc. [29] suggests a new strategy of PWM current control for smooth operation of the SRM drive. In this method, a current contour for constant torque production is defined and the phase current is controlled to follow this contour. The scheme is capable of taking into account the effects of saturation, although in some cases more accurate modeling of the motor inductance may be required.

In [30] Le Chenadec etc. present methods for computing simple reference currents for a current-tracking control to minimize torque ripple. In [31], nominal currents that result in constant torque are computed for reduced current peaks and slopes, under the constraint that at critical rotor positions each of the phases contributes half of the total torque. In [32], the control goal, motivated by energy considerations, is to minimize the peak phase current while requiring linear torque change in the angular range where the two phases overlap.
In [33] Russa *et al.* propose a new commutation strategy along with a PI controller to minimize torque ripple, where an easily invertible flux function is used in calculating reference phase currents. Fuzzy logic and neural network based methods are proposed in [33-34]. In [36], the algorithm proposed combines the use of a simplified model with adaptation. Explicitly, it includes dynamic estimation of low harmonics of the combined unknown load torque and the ripple in the produced torque (due to model simplification), and adds appropriate terms to the commanded current to cancel these harmonics.

In this research, an inductance based model for switched reluctance machine is used. The torque generated by the SRM can be computed directly from the phase currents and rotor position. This provides a convenient way to control the output torque. The main problem will be how to determine the torque generated from each phase during overlapping to make a ripple free resultant torque according to the rotor position. This will be detailed in Chapter 4.

### 2.4 Back-Stepping Controller Design

Over the past ten years, the focus in the area of control theory and engineering has shifted from linear to nonlinear systems, providing control algorithms for systems that are both more general and more realistic. Nonlinear control dominates control conferences and has strong presence in academic curricula and in industry.
Backstepping is a design approach whose significance for nonlinear control can be compared to root locus or Nyquist's method for linear systems. Its roots are in the theory of feedback linearization of the 1980's. With its added flexibility to handle modeling nonlinearities and some classes of systems that are not feedback linearizable, backstepping is the most important ingredient in the nonlinear control advances of the last decade.

Consider a nonlinear system [37]:

\[ \dot{\eta} = f(\eta) + g(\eta)\xi \]
\[ \dot{\xi} = u \]

(2.5) \hspace{1cm} (2.6)

where \([\eta^T, \xi^T]^T \in \mathbb{R}^{n+1}\) is the state and \(u \in \mathbb{R}\) is the control input. The functions \(f\) and \(g\) are smooth in a domain \(D \subset \mathbb{R}^n\) that contains \(\eta = 0\) and \(f(0) = 0\). A state feedback control law is to be designed to stabilize the origin \((\eta = 0, \xi = 0)\).

This system can be viewed as a cascade connection of two components; the first component is (2.5), with \(\xi\) as “virtual” input and \(\eta\) as state, and the second component is the integrator (2.6), with \(u\) as input and \(\xi\) as state.

Suppose the component (2.5) can be stabilized by a smooth state feedback control law \(\xi = \phi(\eta)\), with \(\phi(0) = 0\). Suppose further that a Lyapunov function \(V(\eta)\) that satisfies the inequality
\[
\frac{\partial V}{\partial \eta} [f(\eta) + g(\eta)\xi(\eta)] \leq -W(\eta),
\]  

(2.7)

where \(W(\eta)\) is positive definite. Integrator backstepping theory tells that the state feedback control law

\[
u = \frac{\partial \phi}{\partial \eta} [f(\eta) + g(\eta)\xi] - \frac{\partial V}{\partial \eta} g(\eta) - k[\xi - \phi(\eta)]
\]  

(2.8)

stabilizes the origin of nonlinear system (2.5-2.6), with a Lyapunov function of \(V(\eta) + [\xi - \phi(\eta)]^2 / 2\).

In this research, the electro-mechanical brake is a typical nonlinear system. There’s no direct relationship between the control objective and the input. Iterative backstepping control techniques will be used to design the clamping force controller, which is detailed in Chapter 5.

2.5 Position Sensorless Control of SRM

As is mentioned in Chapter 1, rotor position sensing is an integral part of switched reluctance machine control but a rotor position or speed sensor is not desired. This requires position sensorless control of SRM.

A large amount of sensorless control techniques have been published in the last decade. All these techniques come from the same main idea, that is, to recover the encoded position information stored in the form of flux linkage, inductance, back-EMF,
etc. by solving the voltage equation in an active or idle phase. However, according to the different signals and devices used, the sensorless techniques differ greatly in cost, accuracy, and range of application. Generally the various techniques can be divided into three categories according to the signals they use.

The first category uses external injected signal. Mehrdad Ehsani etc. propose such an indirect SRM rotor position-sensing scheme by applying a high frequency external carrier voltage [42]. Two modulation techniques – amplitude modulation (AM) and phase modulation (PM), which are commonly used in communication systems, are adopted to encode winding current that is dependent on phase inductance. Because the phase inductance is a periodic function of rotor position, the encoded signal can be decoded to obtain rotor position.

This scheme applies an external sinusoidal voltage to the unexcited phase winding to generate encoding signal, so it can keep excellent track of the rotor angle continuously and is extremely robust to switching noises. However, the fairly expensive high frequency sinusoidal wave generator will highly increase the cost of the drive system. And the encoding and decoding circuits will also increase the complexity, hence unreliability, of the drive.

Instead of using external signal, the second category utilizes internal generated signal. Iqbal Husain etc. suggest a sensorless scheme by measuring mutually induced voltage in an inactive phase winding when adjacent phases are excited by power converter [43]. Since this voltage varies significantly as the rotor moves from its unaligned position to
aligned position, a simple electronic circuit can be built to capture the variation and send it to a microprocessor to compute the rotor position.

Compared to the previous described sensing scheme, the new method directly measures an internal signal that is available without the injection of any diagnostic pulses. This reduces the space requirement, complexity and cost of the system. However, this method is not suitable for high-speed applications because the speed dependent terms of the mutually induced voltage cannot be neglected.

Despite the suitability, both these methods need to measure extra signals that are unnecessary for operation purpose. Another method, which uses ONLY operating data, is described by Gabriel Gallegos-Lopez and his colleagues [44]. The new method, called CGSM (Current Gradient Sensorless Method), uses the change of the derivative of the phase current to detect the position where a rotor pole and a stator pole starts to overlap. It can give one position update per energy conversion without \textit{a priori} knowledge of motor parameters which is an indispensable basis of the above two methods. This makes it applicable to most SRM topologies in a wide torque and speed range and for several inverter topologies. Since all calculation can be done on a cheap but powerful DSP (digital signal processor), this method has great advantages over other similar methods in terms of low-cost, easy implementation, and robust functionality. But on the other hand, it has the disadvantages such as needing a startup procedure, not being suitable for low speed, and not allowing large load torque transients.
Ideally, it is desirable to have a sensorless scheme, which uses only operating data measured from motor terminals while maintaining a reliable operation over the entire speed and torque range with high resolution and accuracy. In this research, a sliding mode observed based sensorless control algorithm is suggested and tested. Details are given in Chapter 6.

2.6 Summary

The above literature review covers most aspects of the proposed research.

From the literature, we got the basic ideas for modeling of switched reluctance machines. We will improve the inductance-based model for electro-mechanical brake application. Maximum likelihood estimation based techniques, which have been successfully utilized to other types of electric machines, will be applied to parameter estimation of SRM from standstill and online test data.

Since the model used in this research differs from the models proposed by others, the relative algorithms on torque control, torque-ripple minimization, and sensorless control will be different too. This will be described in the following chapters.
CHAPTER 3

MODELING AND PARAMETER IDENTIFICATION OF SWITCHED RELUCTANCE MACHINES

Modeling the dynamic properties of a system is an important step in analysis and design of control systems. Modeling often results in a parametric model of the system that contains several unknown parameters. Experimental data are needed to estimate the unknown parameters.

Generally, the parameter estimation from test data can be done in frequency-domain or time-domain. Since noise is an inherent part of the test data, which may cause problems to parameters estimation, the effects of noise on different parameter estimation techniques must be investigated. Studies on identification of synchronous machine parameters from noise-corrupted measurements show that noise has significant effect on frequency-domain based techniques. Sometimes unrealistic parameters are obtained from noise-corrupted data. A time-domain technique - maximum likelihood estimation (MLE) - can be used to remove the effect of noise from estimated parameters. The models and the procedures to identify the parameters of switched reluctance machines using
experimental data will be presented in this chapter, after a brief introduction to switched reluctance machines and their control.

3.1 Introduction to Switched Reluctance Machines

The name “switched reluctance” has now become the popular term for this class of electric machine. A switched reluctance motor (SRM) is a rotating electric machine where both stator and rotor have salient poles, as shown in Figure 3.1. SR motors differ in the number of phases wound on the stator. Each of them has a certain number of suitable combinations of stator and rotor poles (for example, 6/4, 8/6, …).

![Figure 3.1 Double salient structure of switched reluctance machines](image)

Phase windings are mounted around diametrically opposite stator poles (A-A, B-B, …). There is neither phase winding nor magnet on the rotor of SRM. The motor
operates on the well-known minimum reluctance law - excitation of a phase will lead to
the rotor moving into alignment with that stator pole, so as to minimize the reluctance of
the magnetic path. The motor is excited by a sequence of voltage/current pulses applied
at each phase. The exciting sequence (instead of the current direction) determines the
rotating direction of the SRM. For example, an exciting sequence of A-B-C-D for the
motor shown in Figure 3.1 will result in counterclockwise rotation; while an exciting
sequence of C-B-A-D will result in clockwise rotation of the motor.

Also note that the voltage/current pulses need to be applied to the respective phase at
the exact rotor position relative to the excited phase, so rotor position sensing is an
integral part of SRM control.

A typical drive circuit for SRM is shown in Figure 3.2.

![Figure 3.2 Typical drive circuit for a 4-phase SRM](image)

According to this configuration, two types of voltage chopping are available, as
shown in Figure 3.3:
- Soft chopping, as shown in Figure 3.3 (a), only one switch (S2) is switching during conducting period (with S1 being kept on). A positive voltage is applied to conducting phase when S2 is on, and a zero voltage is applied when S2 is off.

- Hard chopping, as shown in Figure 3.3 (b), both switches (S1 and S2) are switching during conducting period. So a positive voltage is applied to conducting phase when S1 and S2 are on, and a negative voltage is applied when S1 and S2 are off (before the current drops to zero).

In this research, soft-chopping with 20kHz switching frequency is used.

SR machines have a significant torque ripple, especially when operated in single-pulse voltage control mode. This is the price to pay for high efficiency. Algorithms must
be developed to control the voltage pulses applied to the phase windings to minimize the torque ripple. This will be described in Chapter 4.

3.2 Inductance Based Model of SRM At Standstill

There are two main types of SRM model: flux linkage based model and inductance-based model. Both models have been designed and simulated with the electro-mechanical brake system. After detailed study and comparison of the two models, the inductance-based model has been chosen for our system, which completely fulfills the requirements of the brake system and provides a flexible method for indirect rotor position sensing.

For a switched reluctance motor, the dynamic system refers to a phase winding which can be represented by an inductor in series with a resistor, as shown in Figure 3.4. The corresponding resistance and the inductance are the system parameters.

![Inductance model of SRM at standstill](Figure 3.4 Inductance model of SRM at standstill)
Since the phase inductance changes periodically with the rotor position $\theta$, it can be expressed as a Fourier series with respect to $\theta$. To simplify the inductance expression, we select the origin of the rotor position $\theta$ to be at the aligned position, as shown in Figure 3.5. Under this definition, the phase inductance $L$ reaches its maximum value $L_a$ at $\theta = 0$ (aligned position) and its minimum value $L_u$ at $\theta = \pi / N_r$ (unaligned position).

So the phase inductance can be expressed as

$$L(\theta, i) = \sum_{k=0}^{m} C_k(i) \cos k N_r \theta,$$

(3.1)

where $N_r$ is the number of rotor poles, $i$ is phase current, $\theta$ is rotor position, and $m$ is the number of terms included in the Fourier series.
To determine the coefficients $C_k(i)$ in the Fourier series, the inductances at several specific positions need to be known. Use $L_{\theta}(i)$ to represent the inductance at position $\theta$, which is a function of phase current $i$ and can be approximated by a polynomial:

$$L_{\theta}(i) = \sum_{n=0}^{p} a_{\theta,n} i^n,$$

(3.2)

where $p$ is the order of the polynomial and $a_{\theta,n}$ are the coefficients of polynomial. In our research, $p = 5$ is chosen after we compare the fitting results of different $p$ values ($p = 3, 4, 5,$ and $6$ have been tried and compared.)

For an 8/6 SR machine, we have $N_r = 6$. When $\theta = 0^\circ$ is chosen at the aligned position of phase A, then $\theta = 30^\circ$ is the unaligned position of phase A. Usually the inductance at unaligned position can be treated as a constant [11]:

$$L_{30^\circ} = \text{const}.$$

(3.3)

In [11], The authors suggest using the first three terms of the Fourier series, but more terms can be added to meet accuracy requirements.
3.2.1 Three-Term Inductance Model

If three terms \((m = 2)\) are used in the Fourier series, then we can compute the three coefficients \(C_0\), \(C_1\), and \(C_2\) from inductances at three positions: \(L_{0^\circ}\) (aligned position), \(L_{30^\circ}\) (unaligned position), and \(L_{15^\circ}\) (a midway between the above two positions). Since

\[
\begin{bmatrix}
L_{0^\circ} \\
L_{15^\circ} \\
L_{30^\circ}
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & \cos(6 \times 15^\circ) & \cos(12 \times 15^\circ) \\
1 & \cos(6 \times 30^\circ) & \cos(12 \times 30^\circ)
\end{bmatrix}
\begin{bmatrix}
C_0 \\
C_1 \\
C_2
\end{bmatrix},
\]

(3.4)

we have

\[
\begin{bmatrix}
C_0 \\
C_1 \\
C_2
\end{bmatrix} =
\begin{bmatrix}
1/4 & 1/2 & 1/4 \\
1/2 & 0 & -1/2 \\
1/4 & -1/2 & 1/4
\end{bmatrix}
\begin{bmatrix}
L_{0^\circ} \\
L_{15^\circ} \\
L_{30^\circ}
\end{bmatrix},
\]

(3.5)

Or in separate form, we have

\[
C_0 = \frac{1}{2} \left[ \frac{1}{2} (L_{0^\circ} + L_{30^\circ}) + L_{15^\circ} \right],
\]

\[
C_1 = \frac{1}{2} (L_{0^\circ} - L_{30^\circ}),
\]

(3.6)

\[
C_2 = \frac{1}{2} \left[ \frac{1}{2} (L_{0^\circ} + L_{30^\circ}) - L_{15^\circ} \right].
\]

Note that \(L_{0^\circ}\) and \(L_{15^\circ}\) are curve-fitted as a polynomial of phase current \(i\).
3.2.2 Four-Term Inductance Model

If four terms are used in the Fourier series, then we can compute the four coefficients $C_0$, $C_1$, $C_2$, and $C_3$ from phase inductance at four different positions: $L_0^\circ$ (aligned position), $L_{10^\circ}$, $L_{20^\circ}$, and $L_{30^\circ}$ (unaligned position). Since

$$
\begin{bmatrix}
L_0^\circ \\
L_{10^\circ} \\
L_{20^\circ} \\
L_{30^\circ}
\end{bmatrix}
= 
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & \cos(60^\circ) & \cos(120^\circ) & \cos(180^\circ) \\
1 & \cos(120^\circ) & \cos(240^\circ) & \cos(300^\circ) \\
1 & \cos(180^\circ) & \cos(300^\circ) & \cos(540^\circ)
\end{bmatrix}
\begin{bmatrix}
C_0 \\
C_1 \\
C_2 \\
C_3
\end{bmatrix},
$$

(3.7)

we have

$$
\begin{bmatrix}
C_0 \\
C_1 \\
C_2 \\
C_3
\end{bmatrix}
= 
\begin{bmatrix}
1/6 & 1/3 & 1/3 & 1/6 \\
1/3 & 1/3 & -1/3 & -1/3 \\
1/3 & -1/3 & -1/3 & 1/3 \\
1/6 & -1/3 & 1/3 & -1/6
\end{bmatrix}
\begin{bmatrix}
L_0^\circ \\
L_{10^\circ} \\
L_{20^\circ} \\
L_{30^\circ}
\end{bmatrix}.
$$

(3.8)

Or in separate form, we have

$$
C_0 = \frac{1}{3} \left[ \frac{1}{2} (L_0^\circ + L_{30^\circ}) + (L_{10^\circ} + L_{20^\circ}) \right],
$$

$$
C_1 = \frac{1}{3} (L_0^\circ + L_{10^\circ} - L_{20^\circ} - L_{30^\circ}),
$$

(3.9)

$$
C_2 = \frac{1}{3} (L_0^\circ - L_{10^\circ} - L_{20^\circ} + L_{30^\circ})
$$

$$
C_3 = \frac{1}{3} \left[ \frac{1}{2} (L_0^\circ - L_{30^\circ}) - (L_{10^\circ} - L_{20^\circ}) \right].
$$
3.2.3 Voltage Equation and Torque Computation

Based on the inductance model described above, the phase voltage equations can be formed and the electromagnetic torque can be computed from the partial derivative of magnetic co-energy with respect to rotor angle $\theta$. They are listed here:

\[
V = R \cdot i + \frac{d\lambda}{dt} = R \cdot i + \frac{d(Li)}{dt} = R \cdot i + L \frac{di}{dt} + i(\frac{\partial L}{\partial \theta} \omega + \frac{\partial L}{\partial i} \omega) ,
\]

where

\[
\omega = \frac{d\theta}{dt} ,
\]

\[
\frac{\partial L}{\partial i} = \sum_{k=0}^{m} \frac{\partial C_k(i)}{\partial i} \cos kN, \theta ,
\]

\[
\frac{\partial L}{\partial \theta} = -\sum_{k=1}^{m} C_k(i)kN, \sin kN, \theta .
\]

And

\[
T = \frac{\partial W_c(\theta,i)}{\partial \theta} = \frac{\partial}{\partial \theta} \{ \int [L(\theta,i)di] \}
\]

\[
= -\sum_{k=1}^{m} \{ kN, \sin(kN, \theta) \int [C_k(i)di] \}.
\]
Eq. (3.14) represents the torque generated in one phase of the switched reluctance machine. The total electromagnetic torque is a summation of the torque generated in all active phases. Since the phase inductance $L$ has an explicit expression with respect to $\theta$ and $i$, the above equations can be computed analytically.

3.3 Standstill Test

There are two parameters ($R$ and $L$) in the model that need to be estimated. Test data can be obtained from standstill tests.

The basic idea of standstill test is to apply a short voltage pulse to the phase winding with the rotor blocked at specific positions, record the current generated in the winding, and then use maximum likelihood estimation to estimate the resistances and inductances of the winding. By performing this test at different current level, the relationship between inductance and current can be curve-fitted with polynomials.

The experimental setup is shown in Figure 3.6.
Before testing, the rotor of SRM is blocked at a specific position (with the phase to be tested at aligned, unaligned, or other positions). A DSP system (dSPACE DS1103 controller board) is used to generate the gating signal to a power converter to apply appropriate voltage pulses to that winding. The voltage and current at the winding is measured and recorded. Later on, the test data is used to identify the winding parameters.

### 3.4 Maximum Likelihood Estimation

To minimize the effects of noise caused by the converter harmonics and the measurement, maximum likelihood estimation (MLE) technique is applied to estimate the parameters.
3.4.1 Basic Principle of MLE

Suppose the system dynamic response is represented by

\[
\begin{cases}
    x(k+1) = A(\theta)x(k) + B(\theta)u(k) + w(k) \\
    y(k) = C(\theta)x(k) + v(k)
\end{cases}
\]

(3.15)

where \( \theta \) represents the system parameters,

\( x(k) \) represents system states,

\( y(k) \) represents the system output,

\( u(k) \) is the system input,

\( w(k) \) is the process noise, and

\( v(k) \) is the measurement noise.

To apply the maximum likelihood estimation method, the first step is to specify the likelihood function [19-23]. The likelihood function \( L(\theta) \), is defined as

\[
L(\theta) = \prod_{k=1}^{N} \left[ \frac{1}{\sqrt{(2\pi)^{m} \det(R(k))}} \exp \left( -\frac{1}{2} e(k)^T R(k)^{-1} e(k) \right) \right],
\]

(3.16)

where \( e(\cdot) \), \( R(\cdot) \), \( N \) and \( m \) denotes the estimation error, the covariance of the estimation error, the number of data points, and the dimension of \( y \), respectively.

Maximizing \( L(\theta) \) is equivalent to minimizing its negative log function, which is defined as:

\[
V(\theta) = -\log L(\theta)
\]

\[
V(\theta) = \frac{1}{2} \sum_{k=1}^{N} [e(k)^T R(k)^{-1} e(k)] + \frac{1}{2} \sum_{k=1}^{N} \log \det(R(k)) + \frac{1}{2} mN \log(2\pi).
\]

(3.17)
The parameter vector $\theta$ can be computed iteratively using Newton’s approach [20,24]:

$$ H\Delta\theta + G = 0 $$
$$ \theta_{\text{new}} = \theta_{\text{old}} + \Delta\theta $$

(3.18)

where $H$ and $G$ are the Hessian matrix and the gradient vector of $V(\theta)$. They are defined by:

$$ H = \frac{\partial^2 V(\theta)}{\partial \theta^2} \quad G = \frac{\partial V(\theta)}{\partial \theta}. $$

(3.19)

The $H$ and $G$ matrices are calculated using the numerical finite difference method as described in [17,23].

To start iterative approximation of $\theta$, the covariance of estimation error $R(k)$ is obtained using the Kalman filter theory [18-22]. The steps are as follows:

1) Initial conditions: The initial value of the state is set equal to zero. The initial covariance state matrix $P_0$ is assumed to be a diagonal matrix with large positive numbers. Furthermore, assume an initial set of parameter vector $\theta$.

2) Using the initial values of the parameters vector $\theta$, compute the matrices $A$, $B$, and $C$.

3) Compute estimate $\hat{y}(k \mid k-1)$ from $\hat{x}(k \mid k-1)$:

$$ \hat{y}(k \mid k-1) = C\hat{x}(k \mid k-1). $$

(3.20)

4) Compute the estimation error of $Y(k)$:
\[ e(k) = y(k) - \hat{y}(k \mid k-1). \]  

(3.21)

5) Compute the estimation error covariance matrix \( R(k) \):

\[ R(k) = R_0 + C \cdot P(k \mid k-1) \cdot C^T. \]  

(3.22)

6) Compute the Kalman gain matrix:

\[ K(k) = P(k \mid k-1) \cdot C^T \cdot R(k)^{-1}. \]  

(3.23)

7) Compute the state estimation covariance matrix at instant \( k \) and \( k+1 \):

\[ P(k \mid k) = P(k \mid k-1) - K(k) \cdot C \cdot P(k \mid k-1) \]
\[ P(k+1 \mid k) = A(\theta) \cdot P(k \mid k) \cdot A^T(\theta) + Q \]  

(3.24)

8) Compute the state at instant \( k \) and \( k+1 \):

\[ \hat{x}(k \mid k) = \hat{x}(k \mid k-1) + K(k) \cdot e(k) \]
\[ \hat{x}(k+1 \mid k) = A(\theta) \cdot \hat{x}(k \mid k) + B(\theta) \cdot u(k) \]  

(3.25)

9) Solve Eq. (3.18) for \( \Delta \theta \) and compute the new \( \theta \) such that

\[ \theta_{\min} \leq \theta_{\text{new}} \leq \theta_{\max}. \]  

(3.26)

10) Repeat steps (2) through (9) until \( V(\theta) \) is minimized.

The above mechanism for maximum likelihood estimation is illustrated in Figure 3.7.
A model of the system is excited with the same input as the real system. The error between the estimated output and the measured output is used to adjust the model parameters to minimize the cost function $V(\theta)$. This process is repeated till the cost function is minimized.

### 3.4.2 Performance of MLE with Noise-Corrupted Data

To test the performance of the above algorithm, some simulations have been done in Matlab. The circuit shown in Figure 3.4 is used for this simulation. A step input voltage is applied to the circuits, different noise is added to the output currents to get noise “corrupted” data with different signal-to-noise ratio (S/N). Then MLE techniques are
used to estimate the parameters \( R \) and \( L \) with different initial guesses. The estimated results are shown in the following tables.

The true value of \( R \) is 0.157 \( \Omega \) and that of \( L \) is 0.290 \( mH \).

In Table 3.1, the initial guesses are within 90% of true values. The relative estimation errors are also shown in the table.

In Table 3.2, the initial guesses are within 10% of true values.

Some noise-corrupted signals with different signal-to-noise ratio are shown in Figure 3.8 below.

<table>
<thead>
<tr>
<th>S/N</th>
<th>( R_{\text{ini}} ) (( \Omega ))</th>
<th>( R_{\text{est}} ) (( \Omega ))</th>
<th>( e_R ) (%)</th>
<th>( L_{\text{ini}} ) (( mH ))</th>
<th>( L_{\text{est}} ) (( mH ))</th>
<th>( e_L ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \infty )</td>
<td>0.9( R )</td>
<td>0.157000</td>
<td>0.0000</td>
<td>0.9( L )</td>
<td>0.290000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2000:1</td>
<td>0.9( R )</td>
<td>0.157000</td>
<td>-0.0003</td>
<td>0.9( L )</td>
<td>0.290000</td>
<td>0.0023</td>
</tr>
<tr>
<td>1000:1</td>
<td>0.9( R )</td>
<td>0.156998</td>
<td>-0.0013</td>
<td>0.9( L )</td>
<td>0.289993</td>
<td>0.0115</td>
</tr>
<tr>
<td>500:1</td>
<td>0.9( R )</td>
<td>0.156999</td>
<td>0.0000</td>
<td>0.9( L )</td>
<td>0.289990</td>
<td>0.0068</td>
</tr>
<tr>
<td>200:1</td>
<td>0.9( R )</td>
<td>0.156998</td>
<td>-0.0077</td>
<td>0.9( L )</td>
<td>0.290143</td>
<td>0.0492</td>
</tr>
<tr>
<td>100:1</td>
<td>0.9( R )</td>
<td>0.157014</td>
<td>0.0086</td>
<td>0.9( L )</td>
<td>0.289838</td>
<td>-0.0558</td>
</tr>
<tr>
<td>50:1</td>
<td>0.9( R )</td>
<td>0.156935</td>
<td>-0.0416</td>
<td>0.9( L )</td>
<td>0.290753</td>
<td>0.2595</td>
</tr>
<tr>
<td>20:1</td>
<td>0.9( R )</td>
<td>0.156984</td>
<td>-0.0101</td>
<td>0.9( L )</td>
<td>0.290242</td>
<td>0.0833</td>
</tr>
<tr>
<td>10:1</td>
<td>0.9( R )</td>
<td>0.156836</td>
<td>-0.1046</td>
<td>0.9( L )</td>
<td>0.293724</td>
<td>1.2842</td>
</tr>
<tr>
<td>5:1</td>
<td>0.9( R )</td>
<td>0.156972</td>
<td>-0.0175</td>
<td>0.9( L )</td>
<td>0.291014</td>
<td>0.3496</td>
</tr>
</tbody>
</table>

Table 3.1 Estimation results with initial guesses within 90% of true values
<table>
<thead>
<tr>
<th>S/N</th>
<th>$R_{\text{init}}$ (Ω)</th>
<th>$R_{\text{est}}$ (Ω)</th>
<th>$e_R$ (%)</th>
<th>$L_{\text{init}}$ (mH)</th>
<th>$L_{\text{est}}$ (mH)</th>
<th>$e_L$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>0.1$R$</td>
<td>0.157000</td>
<td>0.0000</td>
<td>0.1$L$</td>
<td>0.289995</td>
<td>-0.0015</td>
</tr>
<tr>
<td>2000:1</td>
<td>0.1$R$</td>
<td>0.157002</td>
<td>0.0015</td>
<td>0.1$L$</td>
<td>0.289956</td>
<td>-0.0151</td>
</tr>
<tr>
<td>1000:1</td>
<td>0.1$R$</td>
<td>0.156998</td>
<td>-0.0014</td>
<td>0.1$L$</td>
<td>0.290034</td>
<td>0.0118</td>
</tr>
<tr>
<td>500:1</td>
<td>0.1$R$</td>
<td>0.156997</td>
<td>-0.0016</td>
<td>0.1$L$</td>
<td>0.290059</td>
<td>0.0203</td>
</tr>
<tr>
<td>200:1</td>
<td>0.1$R$</td>
<td>0.156993</td>
<td>-0.0046</td>
<td>0.1$L$</td>
<td>0.290221</td>
<td>0.0763</td>
</tr>
<tr>
<td>100:1</td>
<td>0.1$R$</td>
<td>0.157049</td>
<td>0.0314</td>
<td>0.1$L$</td>
<td>0.289092</td>
<td>-0.3129</td>
</tr>
<tr>
<td>50:1</td>
<td>0.1$R$</td>
<td>0.156984</td>
<td>-0.0101</td>
<td>0.1$L$</td>
<td>0.290154</td>
<td>0.0531</td>
</tr>
<tr>
<td>20:1</td>
<td>0.1$R$</td>
<td>0.156971</td>
<td>-0.0185</td>
<td>0.1$L$</td>
<td>0.289974</td>
<td>-0.0090</td>
</tr>
<tr>
<td>10:1</td>
<td>0.1$R$</td>
<td>0.157013</td>
<td>0.0084</td>
<td>0.1$L$</td>
<td>0.289203</td>
<td>-0.2748</td>
</tr>
<tr>
<td>5:1</td>
<td>0.1$R$</td>
<td>0.157110</td>
<td>0.0701</td>
<td>0.1$L$</td>
<td>0.286158</td>
<td>-1.3248</td>
</tr>
</tbody>
</table>

Table 3.2 Estimation results with initial guesses within 10% of true values

From the simulation results we can see that, the estimation algorithm can accurately identify the system parameters from very ‘noisy’ data (S/N > 20:1), even with poor initial guesses.
Normally the signal-to-noise ratio of voltage and current measurements is above 100:1~200:1. So MLE can assure the correct estimation of the system parameters.

### 3.5 Parameter Estimation Results From Standstill Tests

The motor used in this research is a 42V/50A 8/6 SRM. Tests are performed at several specific positions for current between 0~50 ampere. Maximum likelihood estimation is used to get the parameters of the model shown in Figure 3.4.

In Figure 3.9, the voltage and current waveforms used in standstill tests are shown. The blue dotted curve shows the current calculated from the estimated parameters ($R$ and $L$). It matches the measurement very well.
The inductance estimation and curve-fitting results at aligned, midway, and unaligned position are shown in Figure 3.10 - Figure 3.12.

The results show that the inductance at unaligned position doesn’t change much with the phase current and can be treated as a constant. The inductances at midway and aligned position decrease when current increases due to saturation.

Figure 3.10  Standstill test results for inductance at $\theta = 0^\circ$
Figure 3.11  Standstill test results for inductance at $\theta=15^\circ$

Figure 3.12  Standstill test results for inductance at $\theta=30^\circ$
A 3-D plot of inductance shown in Figure 3.13 depicts the profile of inductance versus rotor position and phase current. At theta = 0 and 60 degrees, phase A is at its aligned positions and has the highest value of inductance. It decreases when the phase current increases. At theta = 30 degrees, phase A is at its unaligned position and has lowest value of inductance. The inductance here keeps nearly constant when the phase current changes.

![3-D plot of inductance](image)

Figure 3.13  Standstill test result: nonlinear phase inductance

In Figure 3.14, the flux linkage versus rotor position and phase current based on the estimated inductance model is shown. The saturation of phase winding at high currents is clearly represented. At aligned position, the winding is highly saturated at rated current.
3.6 Verification of Standstill Test Results

The results obtained from standstill test can be verified by comparing them with the inductances calculated from the physical dimension of the switched reluctance machine, or from finite element analysis results.

A cross-section of the 8/6 SRM used in this research is shown in Figure 3.15. Phase A is now at its aligned position, and phase C is at its unaligned position.

When phase A at aligned position is excited, the flux generated by the two phase-windings will mainly cross the two airgaps between the two pairs of stator poles and rotor poles, and the stator and rotor iron (as shown in Figure 3.16).
Ignoring the contribution from the machine iron and the fringing in the airgap, the two identical inductances can be computed as

\[ L_1 = L_2 = \frac{N^2 \mu_0 A_c}{g} \]  

(3.27)

where
\[ N = 10 \] is the number of turns of the phase winding,

\[ \mu_0 = 4\pi \times 10^{-7} \] is the permeability of air,

\[ A_c = 6.68 \times 45 \times 10^{-6} \] is cross-sectional area, and

\[ g = (34.95-34.925)/2 \times 10^{-3} \] is the airgap length.

The phase inductance \( L_a \) is the equivalent of \( L_1 \) and \( L_2 \) in series. So

\[
L_a = L_1 + L_2 = 2 \times \frac{10^2 \times 4\pi \times 10^{-7} \times 6.68 \times 45 \times 10^{-6}}{0.0125 \times 10^{-3}} = 6.04 \times 10^{-3} \text{H}
\] (3.28)

According to the standstill test result, the inductance at aligned position at low current (non-saturated) is about \( 5.942 \times 10^{-3} \text{H} \), which is very close to the calculation result.

When phase C at unaligned position is excited, the flux generated by the two phase-windings will NOT cross the two major airgap between the stator poles and rotor yoke, because of the high reluctance of the airgap. Instead, the flux will cross the shorter airgap between the stator pole and two adjacent rotor poles, as shown in Figure 3.17.

Under such cases, it is very hard to estimate the equivalent airgap length. Finite element analysis (FEA) is needed to get a good estimate of the inductance here. The inductance at unaligned position got from FEA is about \( 4.2 \times 10^{-4} \text{H} \), which is close to the standstill test result \( 4.9 \times 10^{-4} \text{H} \), as shown in Figure 3.12.
3.7 SRM Model for Online Operation

For online operation case, especially under high load, the losses become significant. There are no windings on the rotor of SRM. But similar as synchronous machines, there will be circulating currents flowing in the rotor body and makes it work as a damper winding. Considering this, the model structure may be modified as shown in Figure 3.18, with $R_d$ and $L_d$ added to represent the losses on the rotor.

![Model structure of SRM under saturation](image)

Figure 3.18 Model structure of SRM under saturation
The phase voltage equations can be written as:

\[
\begin{bmatrix}
L & -L_d \\
0 & L_d \\
\end{bmatrix}
\begin{bmatrix}
\dot{i}_1 \\
\dot{i}_2 \\
\end{bmatrix}
=
\begin{bmatrix}
0 & R_d \\
-R & -R - R_d \\
\end{bmatrix}
\begin{bmatrix}
\dot{i}_1 \\
\dot{i}_2 \\
\end{bmatrix}
+
\begin{bmatrix}
0 \\
1 \\
\end{bmatrix}V, \\
\tag{3.29}
\]

where \(i_1\) and \(i_2\) are the magnetizing current and damper current.

It can be re-written in state space form as:

\[
\dot{x} = Ax + Bu \\
y = Cx + Du, \\
\tag{3.30}
\]

where

\[
x = [i_1 \quad i_2], \quad y = [i_1 + i_2], \quad u = [V], \\
A = \begin{bmatrix}
L & -L_d \\
0 & L_d \\
\end{bmatrix}^{-1}
\begin{bmatrix}
0 & R_d \\
-R & -R - R_d \\
\end{bmatrix}, \quad B = \begin{bmatrix}
L & -L_d \\
0 & L_d \\
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
1 \\
\end{bmatrix}, \\
C = [1 \quad 1], \quad \text{and} \\
D = 0.
\]

The torque can be computed as follows (notice that \(L\) is the magnetizing winding):

\[
T = \frac{\partial W_r(\theta, i_1)}{\partial \theta} = \frac{\partial}{\partial \theta}\left\{\int[L(\theta, i_1)i_1]di_1\right\}
= \sum_{k=0}^{m} \frac{\partial}{\partial \theta}\left\{C_k(i_1) \cos(kN_r\theta)i_1\right\}di_1
= -\sum_{k=1}^{m} \{kN_r \sin(kN_r\theta)\left\{C_k(i_1)i_1\right\}di_1\}.
\tag{3.31}
\]
During on-site operation, we can easily measure phase voltage $V$ and phase current $i = i_1 + i_2$. But the magnetizing current ($i_1$) and the damper winding current ($i_2$) are not measurable. Let’s assume that the phase parameters $R$ and $L$ obtained from standstill test data are accurate enough for light load case. And we want to attribute all the errors at high load case to damper parameters. If we can estimate the exciting current $i_1$ during online operation, then it will be very easy to estimate the damper parameters. A two-layer recurrent neural network is formed and trained here for such purpose.

3.8 Neural Network Based Damper Winding Parameter Estimation

During online operation, there will be motional back EMF in the phase winding. So the exciting current $i_1$ will be affected by:

- Phase voltage $V$,
- Phase current $i$,
- Rotor position $\theta$, and
- Rotor speed $\omega$.

To map the relation ship between $i_1$ and $V, i, \theta, \omega$, different neural network structures (feed forward or recurrent), with different number of layers, different number of neurons in each layer, and different transfer functions for each neuron, are tried. Finally the one shown in Figure 3.19 is used. It is a two-layer recurrent neural network. The feeding-back of the output $i$ to input makes it better in fitting and faster in convergence.
The first layer is the input layer. The inputs of the network are $V$, $i$, $\theta$, $\omega$ (with possible delays). One of the outputs, the current $i$, is also fed back to the input layer to form a recurrent neural network.

The second layer is the output layer. The outputs are phase current $i$ (used as training objective) and magnetizing current $i_i$.

![Figure 3.19 Recurrent neural network structure for estimation of exciting current](image)

A hyperbolic tangent sigmoid transfer function – “tansig()” is chosen to be the activation function of the input layer, which gives the following relationship between its inputs and outputs:
\[ n_1 = \sum_{i=1}^{d} IW_{1,i} \cdot p_i + LW_{1,2} \cdot y_1 + b_1 , \]

\[ a_i = \text{tansig}(n_i) = \frac{2}{1 + e^{-2n_i}} - 1 . \]

A pure linear function is chosen to be the activation of the output layers, which gives:

\[ n_2 = LW_{2,1} \cdot a_1 + b_2 , \]

\[ y_1 = a_2 = \text{purelin}(n_2) = n_2 ; \]

\[ n_3 = LW_{3,1} \cdot a_1 + b_3 , \]

\[ y_2 = a_3 = \text{purelin}(n_3) = n_3 . \]

After the neural network is trained with simulation data (using parameters obtained from standstill test). It can be used to estimate exciting current during on-line operation. When \( i_1 \) is estimated, the damper current can be computed as

\[ i_2 = i - i_1 , \]

and the damper voltage can be computed as

\[ V_2 = V - i \cdot R . \]

The damper resistance \( R_d \) and inductance \( L_d \) can then be identified using maximum likelihood estimation.
The data used for training is generated from simulation of SRM model obtained from standstill test. The model is simulated at different DC voltages, different reference currents, and different speed. The total size of the sample data is 13,351,800 data points. The training procedure is detailed as follows:

First, from standstill test result, we can estimate the winding parameters \((R \text{ and } L)\) and guess initial damper parameters \((R_d \text{ and } L_d)\). The guesses of \(R_d\) and \(L_d\) may not be accurate enough for online model. It will be improved later through iteration.

Second, build an SRM model with above parameters and simulate the motor with hysteresis current control and speed control. The operating data under different reference currents and different rotor speeds are collected and sent to neural network for training.

Third, when training is done, use the trained ANN model to estimate the magnetizing current \((i_1)\) from online operating data. Compute damper voltage and current according to equations (3.38) and (3.39). And then estimate \(R_d\) and \(L_d\) from the computed \(V_2\) and \(i_2\) using output error estimation. This \(R_d\) and \(L_d\) can be treated as improved values of standstill test results.

Repeat above procedures until \(R_d\) and \(L_d\) are accurate enough to represent online operation (it means that the simulation data matches the measurements well).
In our research, the neural network can map the exciting current from and $V, i, \theta, \omega$ very well after training of 200 epochs.

### 3.9 Model Validation

To test the validity of the parameters obtained from above test, a simple on-line test has been performed. In this test, the motor is accelerated with a fixed reference current of 20 ampere. All the operating data such as phase voltages, currents, rotor position, and rotor speed are measured. Then the phase voltages are fed to an SRM model running in Simulink, which has the same rotor position and speed as the real motor. All the phase currents are estimated from the Simulink mode and compared with the measured currents. The results are shown in Figure 3.20 and Figure 3.21.

Figure 3.20 Validation of model with on-line operating data
In Figure 3.20, the phase current responses are shown. The dashed curve is the voltage applied to phase winding; the solid curve is the measured current; and the dotted curve is the estimated current. An enlarged view of the curves for phase A is shown in Figure 3.21. It is clear that the estimation approximates to the measurement quite well.

![Current Response in Phase A](image)

**Figure 3.21** Validation of model with on-line operating data (Phase A)

To compare online model with standstill one, we compute the covariance of the errors between the estimated phase currents and the measured currents. The average covariance for standstill model is 0.9127, while that for online model is 0.6885. It means that the online model gives much better estimation of operating phase currents.
3.10 Modeling and Parameter Identification Conclusions

An inductance-based model has been selected for the switched reluctance motor used in this research. To represent the losses at heavy load condition, a damper winding is added into the model structure. Model validation results prove that the addition of the damper winding can represent the switched reluctance machine at on-line operation better than the simple $R-L$ model.

Noise, which is inherently presented in the field test data, has significant impact on the machine parameters estimated from frequency-domain based techniques. Maximum likelihood estimation (MLE) based techniques can identify the model parameters from very noisy test data. They are proposed in this chapter to identify the SRM parameters from standstill test and operating data.

For the improved SRM model, the voltage and current for the damper winding are not available from measurements. A two-layer recurrent neural network is proposed here to estimate the damper winding current, so the parameters of the damper winding can be identified.

Model validation by comparing the actual and estimated phase current under the same voltage pulses proves that the proposed model and the identified parameters can represent the dynamics of the SRM quite well.
CHAPTER 4
FOUR QUADRANT TORQUE CONTROL AND TORQUE-RIPPLE MINIMIZATION

To get desired clamping force response, the servomotor in electro-mechanical brake system needs to be operated in all four quadrants of the torque-speed plane. At the same time, the ripple in the torque generated by the motor must be minimized to reduce acoustic noise and mechanical vibration.

4.1 Four Quadrant Operation of Switched Reluctance Machines

In this context the four quadrants mean the four sections on the torque-speed plane. As shown in Figure 4.1, the torque ($T$) – speed ($\omega$) plane is divided into four quadrants. In the 1st quadrant, both torque and speed are positive. This corresponds to the forward motoring region of the motor. In the 2nd quadrant, speed is positive while torque is negative. This corresponds to the forward braking (or, generating) region. Accordingly, the 3rd quadrant corresponds to the backward motoring region; and the 4th quadrant corresponds to the backward braking (generating) region of the motor.
For switched reluctance motor (SRM) used in this research, the rotating direction (or the sign of the speed $\omega$) is determined by the exciting sequence. For example, using the phase definition in Figure 4.2, if the exciting sequence is phase A-B-C-D-A-…, then the rotor will rotate in counter clockwise (CCW) direction; and if the exciting sequence is phase C-B-A-D-C-…, the rotor will rotate in clockwise (CW) direction.
The working mode of the SRM (motoring or generating, or the sign of the torque $T$) is determined by the conduction angle for each phase. For the 8/6 SRM used in this application, the phase inductance has a profile shown in Figure 4.3. According to the assumed forward direction, the phase inductance increases in region $[-30^\circ, 0^\circ]$, which corresponds to \textit{forward motoring} (quadrant I) operation; and the phase inductance decreases in region $[0^\circ, 30^\circ]$, which corresponds to \textit{forward generating/braking} (quadrant II) operation. Similarly, the \textit{backward motoring} (quadrant III) operation happens in region $[30^\circ, 0^\circ]$, and \textit{backward generating/braking} (quadrant IV) operation happens in region $[0^\circ, -30^\circ]$.

![Figure 4.3 Phase inductance profile and conduction angles for 4-quadrant operation](image_url)

So for four-quadrant operation of the SRM, we just need to set the corresponding exciting sequence and conduction angles for each phase according to the desired
operating quadrant. There’s no need to consider the polarity of the voltage applied to each phase (or in other words, the direction of the current in each phase winding).

### 4.2 Torque Control and Torque Ripple Minimization

At any time, the resultant output torque of SRM is the summation of the torque in all phases:

\[
T = \sum_{j=1}^{N} T_j,
\]

where \( N \) is the number of phases, and \( T_j \) is the torque generated in phase \( j \), which is a nonlinear function of phase current \( i \) and rotor position \( \theta \). If the phase current \( i \) is fixed, the torque (of an 8/6 SRM) will have a profile as shown in Figure 4.4.

![Figure 4.4 Phase torque profile under fixed current](image)

Figure 4.4 Phase torque profile under fixed current
It is clear that high torque is not available near aligned/unaligned position even when high phase current is presented. To generate a ripple-free output torque, there must be overlapping between phases.

During phase overlapping, the current in one phase is decreasing, and that in the other phase is increasing. To obtain a constant torque, the summation of the torque generated by these currents must equal to the torque generated in non-overlapping period. To determine the desired torque produced by each phase, torque-factors are introduced here, which are defined as

\[
T = \sum_{j=1}^{N} T_j = \sum_{j=1}^{N} f_j(\theta)T_{ref},
\]

where \( f_j(\theta) \) is the torque factor for phase \( j \) at rotor position \( \theta \), and \( T_{ref} \) is the expected output torque.

The motor used in this research is an 8/6switched reluctance motor. To generate desired torque \( (T = T_{ref}) \), the torque factor must meet the following requirements \( (N = 4) \):

\[
\cdot \sum_{j=1}^{4} f_j(\theta) = 1, \quad (4.3)
\]

\[
\cdot f_j(\theta) = f_j(\theta + \pi / 3), \quad (4.4)
\]

\[
\cdot f_j(\theta) = f_k(\theta - (j - k) \times \pi / 12). \quad (4.5)
\]
There are many different choices that can meet the above requirements. Appropriately chosen torque factors will facilitate the current control and produce satisfactory torque output. In our simulation, the following torque factors are adopted.

For 8/6 SRM, the inductance increasing/decreasing period for each phase is $\pi/6$. We choose a conduction angle of $\pi/8$. This means,

$$\theta_{\text{off}} - \theta_{\text{on}} = \pi / 8.$$  \hfill (4.6)

So the phase overlapping for each two adjacent phases is $\pi/8 - \pi/12 = \pi/24$. During phase overlapping, the total torque is distributed in the two phases according to a sine function of the rotor position. The torque factor for phase $j$ is expressed as follows,

$$f_j(\theta) = \begin{cases} 0.5 - 0.5 \cos 24(\theta - \theta_{\text{on},j}) & \theta_{\text{on},j} \leq \theta < \theta_{\text{on},j} + \pi / 24 \\ 1 & \theta_{\text{on},j} + \pi / 24 \leq \theta < \theta_{\text{off},j} - \pi / 24 \\ 0.5 + 0.5 \cos 24(\theta - \theta_{\text{off},j} - \pi / 24) & \theta_{\text{off},j} - \pi / 24 \leq \theta < \theta_{\text{on},j} \\ 0 & \text{other} \end{cases}.$$  \hfill (4.7)

The turn-on/turn-off angles ($\theta_{\text{on},j}$ and $\theta_{\text{off},j}$) for phase A in different operating quadrants are chosen as follows,

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>Quadrant I</th>
<th>Quadrant II</th>
<th>Quadrant III</th>
<th>Quadrant IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\text{on},A}$</td>
<td>-30°</td>
<td>5°</td>
<td>7.5°</td>
<td>-27.5°</td>
</tr>
<tr>
<td>$\theta_{\text{off},A}$</td>
<td>-7.5°</td>
<td>27.5°</td>
<td>30°</td>
<td>-5°</td>
</tr>
</tbody>
</table>

Table 4.1  Turn-on/turn-off angles for phase A
The turn-on/turn-off angles for the other phases can be obtained by phase shifting of a multiple of $\pi/12$.

The torque factors for all four phases in forward motoring operation are shown in Figure 4.5 below.

![Figure 4.5 Torque factors for forward motoring operation](image)

In the above figure, the thick dotted line is the summation of all the four torque factors. It's obvious that it is 1 at any rotor position.
Once the torque factors are chosen, the reference current for different phases at specified rotor position can be determined through the following steps.

1) Compute the torque factors for all phases according to current rotor position and turn-on/turn-off angles (Eq. 4.7);

2) Compute the desired torque for each phase by multiplying the $T_{ref}$ with the corresponding torque factor;

3) Assume $i_{ji} = 0$, $i_{j2} = i_{\text{max}}$ (maximum phase current allowed), $j=1,2,3,4$;

4) Compute $i_j = (i_{ji} + i_{j2})/2$;

5) Calculate $T_j$ generated by $i_j$ according to Eq. (3.14);

6) Compare $T_j$ with the desired torque for phase $j$ calculated in step 2).

   If the difference between the computed $T_j$ and the designed torque is small enough, let $i_{\text{ref},j} = i_j$ and stop.

   Else if $T_j$ is less than desired torque, let $i_{ji} = i_j$, and go to step 4.

   Else if $T_j$ is greater than desired torque, let $i_{j2} = i_j$, and go to step 4.

   Iterate the steps until satisfactory results are obtained.

In DSP implementation of this algorithm, table-lookup and interpolation can be used to avoid the massive computation needed.
4.3 Hysteresis Current Control

From previous section, the reference current for each reference $i_{ref,j}$ is determined by the desired torque and current rotor position. Then hysteresis current control technique will be applied to maintain the phase current within an acceptable range around the reference current. The principle of hysteresis current control is shown in Figure 4.6.

As shown in Figure 4.6, when the reference current $i_{ref,j}$ is given, a lower limit and an upper limit for the phase current can be obtained, according to the control requirements and the switching frequency of the power converter. Then the power switches are turned on and off by comparing the actual phase current with the two limits.
4.4 Simulation Results

Based on the above torque control technique, a Simulink model is built to test the performance of the algorithm. Here we use a torque ripple coefficient to measure the ripple in the torque. It is defined as follows,

\[
TR = \frac{\sqrt{\frac{1}{\tau} \int_{t_0}^{t_0 + \tau} [T(t) - T_{ave}]^2 \, dt}}{T_{ave}} \times 100\%,
\]  

(4.8)

where

\[
T_{ave} = \frac{1}{\tau} \int_{t_0}^{t_0 + \tau} T(t) \, dt,
\]  

(4.9)

and \( t_0 \) is the time when a phase starts to conduct, \( t_0 + \tau \) is the time when the next phase starts to conduct. For the 8/6 SRM used in this research, we have \( \theta(t_0 + \tau) - \theta(t_0) = \pi / 12 \).

The simulation results for four-quadrant torque control and torque ripple minimization are shown in Figure 4.7. The upper curves show the torque command (solid red) and actual torque generated by the switched reluctance motor (dashed blue). The lower curve shows the rotor speed. It’s easy to find that the motor operated in all four quadrants. The torque ripple in all operating region is less than 4%, which is very satisfactory.
Figure 4.7  Four-quadrant torque control and torque-ripple minimization

4.5 Torque Control Conclusions

Switched reluctance machines have a significant torque ripple, especially when operated in single-pulse voltage control mode. This is not desired for electro-mechanical brake systems. To minimize the torque ripple, phase overlapping is necessary and the determination of torque generation in overlapping phases is very important in torque ripple minimization. To get a ripple-free torque, the summation of the torque generated in all active phases must always be the same. In this research, a torque factor is defined to determine the torque required in each phase according to the rotor position and the specified turn-on/turn-off angles. Then the torque command is converted into current command according to the inductance based SRM model. Finally voltage-chopping
technique is used to generate voltage pulses for each phase to maintain the phase current around the desired value.

Since four-quadrant operation of the SRM is necessary for EMB system, the above torque control algorithms need to be implemented in all four quadrants, with different turn-on/turn-off angles and exciting sequence for different quadrants. Simulations show satisfactory results of the proposed algorithms.
CHAPTER 5

DESIGN AND IMPLEMENTATION OF A ROBUST CLAMPING FORCE CONTROLLER

The control objective of the electro-mechanical brake (EMB) system is to provide desired clamping force response at the brake pads and disk. It is realized by controlling the torque, speed, and rotor position of the servomotor in the EMB system. In this research, an 8/6 switched reluctance motor (SRM) is used as the servomotor.

In this chapter, a robust clamping force controller will be designed and implemented.

5.1 Clamping Force Control System

The electro-mechanical brake system developed in this research contains the following components:

- An 8/6 switched reluctance motor, with corresponding power converter and sensors
- A dual-stage planetary gear and a ball screw, which allows for conversion of the rotary motion of the SR motor into linear ball screw action in order to create the required clamping force on the brake disk
- A clamping force sensor located in the force path between the ball screw and the caliper housing measures the clamping force
- A controller, which receives brake commands and clamping force feedback, and generates control output – the switching signal for power converter to provide voltage pulses to the SRM

A block diagram of the clamping force control system is shown in Figure 5.1.

![Block diagram of clamping force control system](image)

Figure 5.1  Block diagram of clamping force control system

Generally, the clamping force \( F \) applied at the brake disk is a nonlinear function of rotor position \( \theta \), and the load torque \( T_L \) seen by the SR motor is a nonlinear function of the clamping force \( F \). The dependence of the load torque on the clamping force and the
dependence of the clamping force on the rotor position are governed by the mechanical
coupling between the motor and the brake disk. In general, $F$ and $T_L$ are given by

$$F = \mu_F(\theta),$$  \hspace{1cm} (5.1)

$$T_L = \mu_L(\theta),$$  \hspace{1cm} (5.2)

with $\mu_F$ and $\mu_L$ being nonlinear functions.

Physically, it is meaningful to assume that $\mu_F$ is monotonic and that the slope of the
function $\mu_F$ is positive and bounded, i.e.

$$\sigma_{F,min} \leq \frac{\partial \mu_F}{\partial \theta} \leq \sigma_{F,max}.$$

Furthermore, typically, $T_L$ is proportional to $F$ with the proportionality constant being
essentially the gear ratio of the coupling between the motor shaft and the disk pads. For
generality, however, we only require that

$$|T_L| \leq \sigma_L |F| + \tau_L$$  \hspace{1cm} (5.4)

with $\sigma_L$ and $\tau_L$ being nonnegative constants.

The control objective is to make the clamping force $F$ track a given reference
trajectory $F_{ref}$. Practically, it is reasonable to consider $F_{ref}$ and its first derivative to be
bounded almost everywhere.
5.2 System Dynamics

The mechanical dynamics of the SR motor are given by

\[ J \ddot{\theta} = T - D \dot{\omega} - T_L, \quad (5.5) \]

where

- \( J \) is the rotational inertia of the motor,
- \( D \) is the viscous friction coefficient,
- \( \theta \) is the rotor position,
- \( \omega \) is the rotor speed, \( \dot{\theta} = \omega \),
- \( T_L \) is the load torque, and
- \( T \) is the electromagnetic torque generated by the SRM.

The total electromagnetic torque \( T \) is given by

\[ T = \sum_{j=1}^{N} T_j(\theta, i_j), \quad (5.6) \]

where \( N \) is the number of phases, \( T_j \) is the torque contribution of the phase \( j \), and \( i_j \) is the torque contribution of the phase \( j \). The torque \( T_j \) can be computed as

\[ T_j(\theta, i_j) = \frac{\partial W_{e,j}}{\partial \theta} = \frac{\partial}{\partial \theta} \left( \int_{0}^{\theta} L(\theta, i) \, di \right), \quad (5.7) \]

where \( L(\theta, i_j) \) is the inductance associated with phase \( j \) at rotor position \( \theta \) under phase current \( i_j \). The detailed inductance model of SRM is given in Chapter 3.

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For design simplicity, we use the three-term Fourier series model here:

\[
L(\theta, i_j) = C_0(i_j) + C_1(i_j) \cos \left( N_r (\theta - (j - 1) \frac{2\pi}{NN_r} ) \right),
\]

\[
+ C_2(i_j) \cos \left( 2N_r (\theta - (j - 1) \frac{2\pi}{NN_r} ) \right),
\]

(5.8)

where \( N_r \) is the number of rotor poles.

To determine the three coefficients \( C_0, C_1, \) and \( C_2 \), we use the inductances at three positions: \( L_a(i_j) \) (aligned position), \( L_u \) (unaligned position), and \( L_m(i_j) \) (a midway between the above two positions). Note that \( L_u \) can be treated as a constant but \( L_a(i_j) \) and \( L_m(i_j) \) are functions of the phase current \( i_j \) and can be approximated by the polynomials

\[
L_a(i_j) = \sum_{n=0}^{p} a_n i_j^n,
\]

\[
L_m(i_j) = \sum_{n=0}^{p} b_n i_j^n,
\]

(5.9)

where \( p \) is the order of the polynomials and \( a_n, b_n \) are the coefficients.

Using Eq. (3-6), the three coefficients for the Fourier series can be computed as

\[
C_0(i_j) = \frac{1}{2} \left[ \frac{1}{2} (L_a(i_j) + L_u) + L_m(i_j) \right],
\]

\[
C_1(i_j) = \frac{1}{2} \left[ L_a(i_j) - L_u \right],
\]

(5.10)

\[
C_2(i_j) = \frac{1}{2} \left[ \frac{1}{2} (L_a(i_j) + L_u) - L_m(i_j) \right].
\]
With the above definition, the torque expression given in Eq. (5.7) reduces to

\[
T_j(\theta, i_j) = -\frac{N}{4} i_j^2 [(L_a^{**}(i_j) - L_a) \sin(N_r (\theta - (j - 1) \frac{2\pi}{NN_r}))
+ (L_a^{**}(i_j) + L_a - 2L_m^{**}(i_j)) \sin(2N_r (\theta - (j - 1) \frac{2\pi}{NN_r}))]
\]

(5.11)

where

\[
L_a^{**}(i_j) = \sum_{n=0}^{p} \frac{2}{n+2} a_n i_j^n
\]

(5.12)

\[
L_m^{**}(i_j) = \sum_{n=0}^{p} \frac{2}{n+2} b_n i_j^n
\]

The electrical dynamics of the switched reluctance motor are given by

\[
V_j = R \cdot i_j + \frac{d\lambda_j}{dt} = R \cdot i_j + \frac{d(L(\theta, i_j)i_j)}{dt}
= R \cdot i_j + L(\theta, i_j) \frac{di_j}{dt} + i_j \left( \frac{\partial L(\theta, i_j)}{\partial \theta} \omega + \frac{\partial L(\theta, i_j)}{\partial i_j} \frac{di_j}{dt} \right)
\]

(5.13)

where \(V_j\) is the voltage applied on phase \(j\), and \(R\) is the resistance of phase winding.

Using “standard” from, Eq. (5.13) can be written as

\[
\frac{di_j}{dt} = \frac{1}{L(\theta, i_j) + i_j \frac{\partial L(\theta, i_j)}{\partial i_j}} \left[ V_j - R \cdot i_j - i_j \frac{\partial L(\theta, i_j)}{\partial \theta} \omega \right].
\]

(5.14)

From physical considerations, it is meaningful to assume that the denominator in Eq. (5.14) is positive within the range of operation of the SR motor, i.e.,
\[ L(\theta, i_j) + i_j \frac{\partial L(\theta, i_j)}{\partial i_j} > \varepsilon > 0. \] (5.15)

Using the inductance model in Eq. (5.8), we have

\[
\frac{\partial L(\theta, i_j)}{\partial \theta} = -\frac{N_r}{2} \left[ (L_a(i_j) - L_u) \sin(N_r(\theta - (j - 1) \frac{2\pi}{NN_r})) \\
+ (L_u(i_j) + L_u - 2L_m(i_j)) \sin(2N_r(\theta - (j - 1) \frac{2\pi}{NN_r})) \right],
\] (5.16)

and

\[
L(\theta, i_j) + i_j \frac{\partial L(\theta, i_j)}{\partial i_j} = \frac{1}{2} \left[ \frac{1}{2} (L_a^*(i_j) + L_u) + L_m^*(i_j) \right] \\
+ \frac{1}{2} \left[ L_a^*(i_j) - L_u \right] \cos(N_r(\theta - (j - 1) \frac{2\pi}{NN_r})) \\
+ \frac{1}{2} \left( L_m^*(i_j) - L_m^*(i_j) \right) \cos(2N_r(\theta - (j - 1) \frac{2\pi}{NN_r}))
\] (5.17)

where

\[ L_a^*(i_j) = \sum_{n=0}^{p} (n+1)a_n i_j^n \]
\[ L_m^*(i_j) = \sum_{n=0}^{p} (n+1)b_n i_j^n \] (5.18)

Equations (5.5)-(5.18) summarize the dynamics of the SRM based clamping force control system. A robust force controller will be designed according to these equations.
5.3 Controller Design

In this section, the clamping force controller for the switched reluctance motor based brake system is designed using robust backstepping. The backstepping design proceeds by considering lower-dimensional subsystems and designing virtual control inputs (or equivalently, state transformations).

5.3.1 Backstepping Controller Design – Basic Idea

The virtual control inputs in the first and second steps are rotor speed $\omega$ and torque $T$, respectively. In the third step, the actual control inputs $V_j, j = 1, \ldots, N$, appear and can be designed.

In the first step of backstepping, the one-dimensional system

$$\dot{F} = \frac{\partial \mu_F}{\partial \theta} \omega$$

is considered and $\omega$ is regarded as the virtual control input. The objective is to make the clamping force $F$ track the designed force response $F_{ref}$.

However, since $\omega$ is not the actual control input, the error between $\omega$ and the desired $\omega$ is formulated as

$$z_2 = \omega - \omega^*.$$
In the second step of backstepping, the torque $T$ will be regarded as the virtual control input and will be *designed* to make $z_2$ small, i.e., $\omega$ converges to the virtual control law $\omega^*$ designed for $\omega$, and hence, $F$ converges to $F_{\text{ref}}$.

Since $T$ is also not the actual control input, the process is repeated by introducing an error $z_3$ which is the difference between $T$ and the desired $T$

$$z_3 = T - T^*.$$  \hfill (5.21)

At the third step of backstepping, the control inputs $V_j, j = 1, \ldots, N$, appear and the control law is designed at that step can actually be implemented.

### 5.3.2 Backstepping Controller Design - Step 1

For the first step, the force tracking error is defined as

$$z_1 = F - F_{\text{ref}}.$$  \hfill (5.22)

A Lyapunov function

$$V_1 = \frac{1}{2} z_1^2$$  \hfill (5.23)

is used here.

Differentiating Eq. (5.23), we obtain
\[
\dot{V}_1 = z_1 \dot{z}_1 = z_1 (\hat{F} - \hat{F}_{ref}) \\
= z_1 \left( \frac{\partial \mu}{\partial \theta} \omega - \hat{F}_{ref} \right) \\
= -k_1 \frac{\partial \mu}{\partial \theta} z_1^2 + z_1 z_2 \frac{\partial \mu}{\partial \theta} - z_1 \hat{F}_{ref}
\]

where

\[
z_2 = \omega - \omega^*,
\]

\[
\omega^* = -k_1 z_1,
\]

and \( k_1 > 0 \) is a design freedom.

Choosing \( \omega = -k_1 z_1 = -k_1 (F - F_{ref}) \) would result in the practical tracking of clamping force \( F \) in the one-dimensional system described in Eq. (5.19).

Using Eq. (5.3), Eq. (5.24) reduces to

\[
\dot{V}_1 \leq -k_1 \sigma_{F,\text{min}} z_1^2 + \sigma_{F,\text{max}} |z_1| |z_2| + |z_1| |\hat{F}_{ref}|
\]

5.3.3 Backstepping Controller Design - Step 2

A new Lyapunov function is defined as

\[
V_2 = V_1 + \frac{1}{2c_2} z_2^2
\]

where \( c_2 > 0 \) is a design freedom. Differentiating Eq. (5.27),
\[
\dot{V}_2 \leq -k_1 \sigma_{F, \text{min}} z_1^2 + \sigma_{F, \text{max}} \| z_1 \| \| \dot{z}_1 \| + \| \dot{F}_{\text{ref}} \|
\]
\[
+ \frac{1}{c_2} z_2 \left[ \frac{1}{J} (T - D \omega - T_L) + k_1 \frac{\partial \mu_F}{\partial \theta} \omega - k_1 \dot{F}_{\text{ref}} \right]
\]
\[
\leq -k_1 \sigma_{F, \text{min}} z_1^2 + \sigma_{F, \text{max}} \| z_1 \| \| \dot{z}_1 \| + \| \dot{F}_{\text{ref}} \|
\]
\[
- \frac{k_2}{Jc_2} z_2^2 + \frac{1}{Jc_2} z_2 z_3 - \frac{D}{Jc_2} z_2 \omega - \frac{1}{Jc_2} z_2 T_L
\]
\[
+ \frac{k_1}{c_2} \frac{\partial \mu_F}{\partial \theta} z_2 \omega - \frac{k_1}{c_2} z_2 \dot{F}_{\text{ref}}
\]
\[
\leq -k_1 \sigma_{F, \text{min}} z_1^2 + \sigma_{F, \text{max}} \| z_1 \| \| \dot{z}_1 \| + \| \dot{F}_{\text{ref}} \|
\]
\[
- \frac{k_2}{Jc_2} z_2^2 + \frac{1}{Jc_2} \| z_2 \| \| z_3 \| + \frac{D}{Jc_2} \| z_2 \| \| z_1 \| + \frac{k_1}{c_2} \| z_1 \| \| \dot{z}_1 \| + \frac{k_1}{c_2} \| z_2 \| \| \dot{F}_{\text{ref}} \|
\]
\]
\[
(5.28)
\]

where
\[
z_3 = T - T^*,
\]
\[
T^* = -k_2 z_2 = -k_2 [\omega + k_1 (F - F_{\text{ref}})],
\]
\]
\[
(5.29)
\]

with \( k_2 > 0 \) being a controller gain to be picked by the designer.

### 5.3.4 Backstepping Controller Design - Step 3

For the third and final step of the backstepping, we use the Lyapunov function
\[ V_3 = V_2 + \frac{1}{2c_3} z_3^2 \] 

(5.30)

where \( c_3 > 0 \) is a design freedom.

To differentiate Eq. (5.30), the dynamics of the torque will be required. The torque dynamics can be derived from Eq. (5.6) and (5.11) as

\[ \dot{T} = \sum_{j=1}^{N} \dot{T}_j(\theta, i_j), \] 

(5.31)

\[ \dot{T}_j(\theta, i_j) = \frac{\partial T_j}{\partial \theta} \omega + \frac{\partial T_j}{\partial i_j} \frac{di_j}{dt} + \frac{\partial T_j}{\partial \omega} \frac{d\omega}{dt} d\theta \]

\[ = \frac{\partial T_j}{\partial \theta} \omega + \frac{\partial T_j}{\partial i_j} \left[ V_j - R_i i_j - \frac{\partial L(\theta, i_j)}{\partial \theta} \omega \right] + \frac{\partial T_j}{\partial \omega} \frac{d\omega}{dt} \]

(5.32)

where

\[ \frac{\partial T_j}{\partial \theta} = -\frac{N_r}{4} i_j^2 [(L_a^*(i_j) - L_u) \cos(N_r(\theta - (j-1)\frac{2\pi}{N_r}))]

+ 2(L_a^*(i_j) + L_u - 2L_m^*(i_j)) \cos(2N_r(\theta - (j-1)\frac{2\pi}{N_r}))], \] \( \frac{\partial T_j}{\partial i_j} \]

\[ = -\frac{N_r}{2} i_j^2 [(L_a^*(i_j) - L_u) \sin(N_r(\theta - (j-1)\frac{2\pi}{N_r})))

+ (L_a^*(i_j) + L_u - 2L_m^*(i_j)) \sin(2N_r(\theta - (j-1)\frac{2\pi}{N_r})))]

\[ + \frac{\partial L_a^*(i_j)}{\partial i_j} - \frac{\partial L_m^*(i_j)}{\partial i_j} \sin(2N_r(\theta - (j-1)\frac{2\pi}{N_r})))

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Differentiating Eq. (5.30) and using Eq. (5.28), we have

\[
\dot{V}_3 \leq -k_1 \sigma_{F,\min} z_1^2 + \sigma_{F,\max} \|z_1\| z_2 + |z_1| F_{\text{ref}}^r + \frac{k_2}{Jc_2} z_2^2 + \frac{1}{Jc_2} \|z_2\| z_3 + \frac{D}{Jc_2} \left[ \|z_2\| + k_1 |z_1| \right] + \frac{1}{Jc_2} \|z_2\| \left[ \sigma_L \left( |z_1| + |F_{\text{ref}}^r| \right) + \tau_L \right] + \frac{k_1}{c_2} \sigma_{F,\max} \|z_2\| \left[ \|z_2\| + k_1 |z_1| \right] + \frac{k_1}{c_2} \|z_2\| F_{\text{ref}}^r + \frac{1}{c_3} z_3 \left[ \hat{T} + k_2 \left( \frac{1}{T} (T - D\omega - T_L) \right) + k_1 \frac{\partial \mu_F}{\partial \theta} \omega - k_1 \dot{F}_{\text{ref}} \right]
\]

(5.35)

where, for notational convenience, we have introduced \( \hat{T} = \dot{T} \). The control law will first be designed in terms of \( \hat{T} \). The control laws in terms of the input voltages \( V_j \) will then be obtained through a commutation scheme.

Designing

\[
\hat{T} = -k_3 z_3 = -k_3 \left[ T + k_2 \omega + k_2 k_1 (F - F_{\text{ref}}) \right]
\]

(5.36)

with \( k_3 > 0 \) being a design freedom, Eq. (5.35) reduces to

\[
\dot{V}_3 \leq -k_1 \sigma_{F,\min} z_1^2 + \sigma_{F,\max} \|z_1\| z_2 + |z_1| F_{\text{ref}}^r + \frac{k_2}{Jc_2} z_2^2 + \frac{1}{Jc_2} \|z_2\| z_3 + \frac{D}{Jc_2} \left[ \|z_2\| + k_1 |z_1| \right] + \frac{1}{Jc_2} \|z_2\| \left[ \sigma_L \left( |z_1| + |F_{\text{ref}}^r| \right) + \tau_L \right] + \frac{k_1}{c_2} \sigma_{F,\max} \|z_2\| \left[ \|z_2\| + k_1 |z_1| \right] + \frac{k_1}{c_2} \|z_2\| F_{\text{ref}}^r + \frac{k_1}{c_3} z_3 \left[ \hat{T} + k_2 \left( \frac{1}{T} (T - D\omega - T_L) \right) + k_1 \frac{\partial \mu_F}{\partial \theta} \omega - k_1 \dot{F}_{\text{ref}} \right]
\]
It can be rearranged as

\[
\dot{V}_3 \leq -k_1 \sigma_{F,min} z_1^2 - \frac{k_2}{Jc_2} z_2^2 - \frac{k_3}{c_3} z_3^2 \\
+ \left| z_1 \right| z_2 \left[ \sigma_{F,max} + \frac{k_1 D}{Jc_2} + \frac{\sigma_L}{Jc_2} + \frac{k_1 \sigma_{F,max}}{c_2} \right] \\
+ \left| z_1 \right| z_3 \left[ \frac{k_1 k_2 D}{Jc_3} + \frac{k_2 \sigma_L}{Jc_3} + \frac{k_1 k_2 \sigma_{F,max}}{c_3} \right] \\
+ \left| z_2 \right| z_3 \left[ \frac{1}{c_2} + \frac{k_2^2}{Jc_2} + \frac{k_3 D}{Jc_3} + \frac{k_1 k_2 \sigma_{F,max}}{c_3} \right] \\
+ z_2^2 \left[ \frac{D}{Jc_2} + \frac{k_1 \sigma_{F,max}}{c_2} \right] + z_3^2 \left[ \frac{k_2}{Jc_3} \right] \\
+ \left| z_1 \right| \| \dot{F} \|_{\text{ref}} + \frac{\sigma_L}{Jc_2} z_2 \| F \|_{\text{ref}} + \frac{\sigma_L}{Jc_2} z_2 + \frac{k_1}{c_2} z_2 \| \dot{F} \|_{\text{ref}} \\
+ \frac{k_1 \sigma_L}{Jc_2} z_3 \| F \|_{\text{ref}} + \frac{k_3 \sigma_L}{Jc_3} z_3 + \frac{k_1 k_2}{c_3} z_3 \| \dot{F} \|_{\text{ref}}
\]

(5.38)

From Eq. (5.38), it is seen that by picking \( k_1, k_2, k_3, c_2, \) and \( c_3 \) appropriately, \( V_3 \) satisfies

\[
\dot{V}_3 \leq -\gamma V_3 + \chi \left[ \| F \|_{\text{ref}}^2 + \| \dot{F} \|_{\text{ref}}^2 + \| \tau \|_{\text{L}}^2 \right]
\]

(5.39)
where

\[
\gamma = \min \left( k, \sigma_{F,\text{max}}, \frac{k_2}{J}, k_3 \right) \tag{5.40}
\]

and \( \chi \) is a constant independent of the controller gains \( k_1 \), \( k_2 \), and \( k_3 \). The proof of Eq. (5.39) follows by quadratic over-bounding which is standard in the backstepping literature [50].

From Eq. (5.39), it follows that practical tracking is achieved, i.e., the force tracking error \( z_1 = F - F_{\text{ref}} \) can be regulated to an arbitrarily small compact set by picking \( k_1 \), \( k_2 \), and \( k_3 \) large enough.

To obtain control laws in terms of the input voltages \( V_j, j = 1, \ldots, N \), two alternative methods can be used.

**Method one: torque control and torque-ripple minimization**

Since the desired torque \( T^* \) can be obtained from Eq. (5.36), the torque control algorithm described in Chapter 4 can be applied here to determine the reference current \( i_{\text{ref},j} \) for each phase. Then hysteresis current control will be used to maintain the phase current around the reference. This is the most commonly used technique in controller implementation.
Method two: voltage commutation and PWM

The following commutation scheme is used to get the control laws in terms of phase voltages:

\[ V_j = \left( L(\theta, i_j) + i_j \frac{\partial L(\theta, i_j)}{\partial i_j} \right) \bar{V}_j + R \cdot i_j + i_j \frac{\partial L(\theta, i_j)}{\partial \theta} \omega, \]  \hspace{1cm} (5.41)

\[ \bar{V}_j = \frac{\partial T_j}{\partial i_j} \left[ \hat{T} - \sum_{j=1}^{N} \frac{\partial T_j}{\partial \theta} \omega \right] \sum_{j=1}^{N} \left( \frac{\partial T_j}{\partial i_j} \right)^2. \]  \hspace{1cm} (5.42)

From Eq. (5.42), we have

\[ \sum_{j=1}^{N} \bar{V}_j \frac{\partial T_j}{\partial i_j} = \hat{T} - \sum_{j=1}^{N} \frac{\partial T_j}{\partial \theta} \omega, \]  \hspace{1cm} (5.43)

which satisfies Eq. (5.32), and \( \bar{V}_j \) approximates \( \frac{di_j}{dt} \).

The input voltages obtained from Eq. (5.41-5.42) are continuous voltages. In SRM control, the phase voltage can only be switched among \( \pm V_{dc} \) and 0. So PWM techniques need to be applied here to approximate the desired phase voltage waveforms.

In controller implementation, method one will be used, which provides desired torque with minimal torque ripple.
5.3.5 Backstepping Controller Design – Generalized Control Law

The overall controller is given by

\[ z_1 = F - F_{ref}, \]  
(5.22)

\[ \omega^* = -k_1 z_1, \]  
(5.25)

\[ z_2 = \omega - \omega^*, \]  
(5.20)

\[ T^* = -k_2 z_2 = -k_2 [\omega + k_1 (F - F_{ref})], \]  
(5.29)

\[ z_3 = T - T^*, \]  
(5.21)

\[ \hat{T} = -k_3 z_3 = -k_3 [T + k_2 \omega + k_2 k_1 (F - F_{ref})], \]  
(5.36)

and

\[ V_j = \left( L(\theta, i_j) + i_j \frac{\partial L(\theta, i_j)}{\partial i_j} \right) \tilde{V}_j + R \cdot i_j + i_j \frac{\partial L(\theta, i_j)}{\partial \theta} \omega, \]  
(5.41)

\[ \tilde{V}_j = \frac{\partial T_j}{\partial i_j} \left[ \hat{T} - \sum_{j=1}^{N} \frac{\partial T_j}{\partial \theta} \omega \right] \frac{\sum_{j=1}^{N} \left( \frac{\partial T_j}{\partial i_j} \right)^2}{\sum_{j=1}^{N} \left( \frac{\partial T_j}{\partial i_j} \right)} \]  
(5.42)

In implementation, Eq. (5.41) and (5.42) can be replaced by torque control algorithms given in Chapter 4.

In above control laws, the controller design freedoms are \( k_1, k_2, k_3, c_2, \) and \( c_3 \).

Noting the structure of the controller, the control law can be slightly generalized to
provide additional damping in the electrical dynamics and to provide more design freedom in shaping the force dynamics. The generalized control law is given by

\[
\hat{T} = -K_p (F - F_{ref}) - K_d (\dot{F} - \dot{F}_{ref}) - K_i \int_0^t (F(t) - F_{ref}(t)) \, dt,
\]

\[
- K_T T - K_w \omega
\]

\[
\vec{V}_j = \frac{\partial T_j}{\partial i_j} \left[ \hat{T} - \sum_{j=1}^N \frac{\partial T_j}{\partial \theta} \omega \right] - \sum_{j=1}^N \left( \frac{\partial T_j}{\partial i_j} \right)^2 + \varepsilon_T
\]

\[
V_j = \left( L(\theta, i_j) + i_j \frac{\partial L(\theta, i_j)}{\partial i_j} \right) \vec{V}_j + i_j \frac{\partial L(\theta, i_j)}{\partial \theta} \omega - K_{cur} \cdot i_j
\]

with \( K_p, K_d, K_i, K_T, K_w, \) and \( K_{cur} \) being controller gains free to be picked by the designer. The term \( \varepsilon_T \) is added to avoid the singularity of the commutation scheme when all currents are zero. Eq. (5.45) and (5.46) can be replaced by the torque control algorithm given in Chapter 4.

Note that the controller design does not require the knowledge of the mechanical parameters \( D \) and \( J \) of the motor. Also, the functional form of \( F = \mu_F(\theta) \) and \( T_L = \mu_L(\theta) \) are not required to be known. Magnitude bounds on \( D, J, \mu_F, \) and \( \mu_L \) are sufficient for picking the controller gains. While the controller does require knowledge of the electrical parameters of the motor, the design does provide considerable robustness to uncertainty in the electrical parameters also. Simulation results are given in the following section.
5.4 Simulation Results

The performance of the controller is verified through simulation studies in Matlab/Simulink® environment. The Simulink model of the electro-mechanical brake system is shown in Figure 5.2. The parameters of the switched reluctance motor and the brake system are given in Appendix A.

![Figure 5.2 Simulink model of electro-mechanical brake system](image)

The force reference trajectory is specified to be 2500 $N$ till the actual clamping force reaches 2000 $N$ within 0.1 $s$, and 1600 $N$ after that. The phase currents are constrained to be maintained in the Range $[0, 50]$ $A$. The parameters of the controller were chosen to be

\[ K_p = 20, \quad K_d = 0.002, \]

\[ K_i = 2, \quad K_T = 3500, \]

\[ K_{\theta} = 85, \quad K_{cur} = 1. \]  \hspace{1cm} (5.47)
As mentioned in previous section, there are two methods to get the control law in terms of voltages after we get the torque reference. Both methods are simulated and the results are shown in the following sections.

### 5.4.1 Controller with Voltage Commutation Scheme and PWM Implementation

Using the voltage commutation scheme described in Eq. (5.45-5.46), the results are shown in Figure 5.3 through Figure 5.6.

In Figure 5.3, the reference clamping force (dotted red curve) and the actual clamping force response (solid blue curve) are shown. The force reference is set to 2500 N initially. At 0.06 second, the actual clamping force reaches 2000 N and the force reference drops to 1600 N. At 0.1 second, the clamping force reaches steady state and stays at 1600 N.

![Clamping Force Response](image)

Figure 5.3 Clamping force response
The phase current waveforms are shown in Figure 5.4. At steady state, the speed of the switched reluctance motor is nearly zero. Only phase A and D are conducting and they share the desired load torque. The currents in phase B and C are zero.

![Phase Currents](image)

Figure 5.4 Phase current waveforms

In Figure 5.5, the rotor position, speed, and torque during the control process are shown. At first, the switched reluctance motor is accelerated to 1500 RPM in a very short time. When the force reference drops at 0.06 second, the SRM is decelerated and reserved rotation direction very quickly to meet the decreased clamping force requirements. Four-quadrant operation of the SRM is necessary here to provide good clamping force response. The four-quadrant torque speed curve is shown in Figure 5.6.
Figure 5.5  Rotor position, speed and torque

Figure 5.6  Four-quadrant torque speed curve
5.4.2 Controller with Torque Control and Torque-Ripple Minimization

Using the torque control and torque-ripple minimization algorithm developed in Chapter 4, the results are shown in Figure 5.7 and Figure 5.8.

In Figure 5.7, the clamping force response under the new controller is shown. Comparing it with Figure 5.3, we can see that almost identical clamping force responses are obtained.

![Clamping Force Response](image)

Figure 5.7 Clamping force response

In Figure 5.8, the four-quadrant torque speed curve with new controller is shown. It is obvious that the torque-ripple in this case is much smaller than that in the previous controller implementation.
5.4.3 Controller Robustness Test

To demonstrate the robustness of the controller, we consider the following case:

- Only the constant terms in the inductance expression are available for use in the controller
- Errors in the parameters of resistance $R$ and unaligned inductance $L_u$ up to 50%
- Furthermore, the load is changed to include a cascaded first-order linear block with gain 1.1 and time constant 2 ms.

Figure 5.8 Four-quadrant torque speed curve
The simulation results are shown in Figure 5.9 through Figure 5.11.

**Figure 5.9** Clamping force response

**Figure 5.10** Phase current waveforms
It is seen that the performance of the force controller is retained in spite of the errors in electrical parameters and the change of load. Though the switched reluctance has a different trajectory in torque speed plane (as shown in Figure 5.11), the clamping force response is almost identical to previous cases.

### 5.5 Controller Design Conclusions

A robust nonlinear clamping force controller for an SRM based electro-mechanical brake system has been designed and simulated in this chapter. The controller is designed via a backstepping procedure. It does not require knowledge of the mechanical parameters of the motor and the functional forms of the relationships among the motor position,
clamping force, and the motor load torque. Moreover, the controller provides significant robustness to uncertainty in the SRM parameters. The proposed controller design can also be extended to electro-mechanical brake systems using other type of electric machines.
CHAPTER 6

POSITION SENSORLESS CONTROL OF SWITCHED RELUCTANCE MACHINES

Sensorless (without direct position or speed sensors) control system, which extracts rotor position information indirectly from electric signals, is important for automotive application due to the need for minimum package size, high reliability, and low cost for electric motor driven actuators.

Ideally, it is desirable to have a sensorless scheme, which uses only operating data measured from motor terminals while maintaining a reliable operation over the entire speed and torque range with high resolution and accuracy.

Various methods of sensorless control for switched reluctance machines have been investigated and reported in literature, each with its advantages and disadvantages. With the rapid progress in digital signal processor (DSP), observer-based sensorless algorithms have attracted more and more attention. Generally only operating signals are needed for this kind of sensorless algorithms. And a model of the SRM will be running in the
memory (of DSP) which has the same input as the real motor. The difference between the estimated output signals and the measured ones is used to extract the position information.

Sliding mode observer, with its advantages of inherent robustness of parameters uncertainty, computational simplicity, and high stability, provides a powerful approach to implement sensorless schemes.

However, sliding mode observers often fail at very low (near zero) speed or ultra high speed. Also for SRM at standstill, no operating signals are available for the observer to estimate the rotor position. So methods for sensorless startup and low-speed operation of the SRM must be provided.

6.1 Sliding Mode Observers

To define a sliding mode observer for SRM drive system, we first need to setup a model of the system and build the system differential equations. The sliding mode observer will be defined based on these equations.

6.1.1 System Differential Equations

The SRM drive system differential equations include the electromagnetic equations, the electromechanical equations, and the mechanical equations, which are given in Chapter 5. They are listed here again for reference:
The phase voltage equations can be expressed as

\[ V_j = R \cdot i_j + \frac{d\lambda_j}{dt} = R \cdot i_j + L_j \frac{di_j}{dt} + i_j \left( \frac{\partial L_j}{\partial \theta} \omega + \frac{\partial L_j}{\partial i_j} \frac{di_j}{dt} \right), \tag{6.1} \]

where \( L_j = L(\theta, i_j) \).

By converting to “standard” format of differential equations, Eq. (6.1) can be represented as

\[ i_j = \frac{1}{L_j + i_j \frac{\partial L_j}{\partial i_j}} \left[ V_j - R \cdot i_j - \frac{\partial L_j}{\partial \theta} \omega \cdot i_j \right]. \tag{6.2} \]

The voltage equations (6.2 with \( j = 1 \ldots N \), where \( N \) is the number of phases) are nonlinear differential equations.

The torque-speed equation can be expressed as

\[ \dot{\omega} = \frac{1}{J} \sum_{j=1}^{N} T_j - T_L, \tag{6.3} \]

where \( J \) is the moment of inertia of the rotor, \( T_L \) is the load torque, and \( T_j \) is the electromagnetic torque generated in phase \( j \). \( T_j \) can be computed as follows,

\[ T_j = \frac{\partial W_{e,j}}{\partial \theta} = \frac{\partial}{\partial \theta} \int L_j (\theta, i_j) i_j di_j. \tag{6.4} \]
The mechanical equation can be expressed as

\[ \dot{\theta} = \omega, \quad (6.5) \]

where \( \omega \) is the rotor speed.

Equations (6.2), (6.3), and (6.5) form the system differential equations of the switched reluctance machine and brake system, which are used to define the sliding mode observer.

### 6.1.2 Definition of Sliding Mode Observer

According to the system differential equations derived from the inductance model of SRM, a second-order sliding mode observer for rotor position and speed estimation can be defined as follows,

\[ \dot{\hat{\theta}} = \dot{\omega} + k_\omega \text{sgn}(e_j), \quad (6.6) \]

\[ \dot{\hat{\omega}} = \frac{1}{J} \left[ \sum_{j=1}^{N} \hat{T}_j - \hat{T}_L \right] + k_\omega \text{sgn}(e_j), \quad (6.7) \]

where

\( \hat{\theta}, \hat{i}, \hat{\omega} \) are the estimations of \( \theta, i, \omega \),

\( \hat{T}_j \) and \( \hat{T}_L \) are phase torque and load torque estimated from \( \hat{\theta}, \hat{i}, \hat{\omega} \), and
\( e_f \) is an error function based on measured and estimated variables. The details about \( e_f \) will be given later in this section.

Note that if the estimate of load torque \( \hat{T}_l \) is not available, Eq. (6.7) can be simplified as

\[
\dot{\omega} = k_\omega \text{sgn}(e_f).
\]  (6.8)

Generally the gain \( k_\omega \) will be chosen large enough; the first term in Eq. (6.7) can be neglected without any problem.

### 6.1.3 Estimation Error Dynamics of Sliding Mode Observer

To describe the observer error dynamics, the following estimation errors are defined:

\[
e_{\theta} = \theta - \hat{\theta},
\]  (6.9)

\[
e_{\omega} = \omega - \dot{\omega}.
\]  (6.10)

Differentiating Eq. (6.8) yields

\[
\dot{e}_{\theta} = \theta - \dot{\theta} = \omega - \left(\dot{\omega} + k_\omega \text{sgn}(e_f)\right) = e_{\omega} - k_\omega \text{sgn}(e_f)
\]  (6.11)
Differentiating Eq. (6.8) yields

\[ \dot{\varepsilon}_\omega = \dot{\omega} - \hat{\omega} \]  \hspace{1cm} (6.12)

Substituting Eq. (6.3) and (6.8) into (6.12), we have

\[ \dot{\varepsilon}_\omega = \frac{1}{J} \left[ \sum_{j=1}^{\infty} T_j - T_L \right] - k_\omega \text{sgn}(e_f) . \]  \hspace{1cm} (6.13)

Since the gain \( k_\omega \) may be selected to be large enough such that the first term in Eq. (6.13) may be neglected, the speed error dynamics become

\[ \dot{\varepsilon}_\omega = -k_\omega \text{sgn}(e_f) . \]  \hspace{1cm} (6.14)

Eq. (6.11) and (6.14) represent the estimation errors dynamics of the sliding mode observer.

From Eq. (6.11) we can find that if the error function \( e_f \) is chosen to carry the same sign as \( e_\theta \), and the observer gain \( k_\theta \) is chosen to satisfy

\[ k_\theta > |e_\omega| , \]  \hspace{1cm} (6.15)

then \( \dot{e}_\theta \) will always have different sign than \( e_\theta \). The sliding surface \( e_\theta = 0 \) will be reached in finite time.

Once the sliding surface is reached, the estimation error dynamics become

\[ \dot{e}_\theta = 0 , \]  \hspace{1cm} (6.16)
\[
\dot{e}_\omega = - \frac{k_{\omega}}{k_\theta} e_\omega. \tag{6.17}
\]

So the speed error \( e_\omega \) will decrease to zero exponentially.

### 6.1.4 Definition of Error Function

From above analysis we get the conclusion that the error function \( e_f \), which compares measured electrical variables with their corresponding estimated values, need to carry the same sign as \( e_\theta \).

There are many possible error functions that can meet this requirement, hence stabilize the error dynamics. In this research, the following error function is used:

\[
e_f = \sum_{j=1}^{N} \sin \left[ N_r \left( \hat{\theta} - (j - 1) \frac{2\pi}{NN_r} \right) \right] (i_j - \hat{i}_j). \tag{6.18}
\]

Now let’s analyze why \( e_f \) has the same sign as \( e_\theta \). The motor has two operating modes: motoring and generating. The analysis will be done in both modes.

**Motoring operation mode**

As shown in Figure 6.1, when the rotor position \( \theta \) lies between \([ -\pi / N_r, 0] \), the phase inductance increases with \( \theta \) and the SR motor works in motoring mode. If the estimated rotor position \( \hat{\theta}_1 \) is greater than the actual rotor position \( \theta_1 \), then we have
\[ e_\theta = \theta_1 - \hat{\theta}_1 < 0. \]  

(6.19)

Since the phase inductance \( L(\hat{\theta}_1, i_j) > L(\theta_1, i_j) \), the estimated current \( \hat{i}_j \) must be less than the actual current \( i_j \) with the same phase voltage. So

\[ i_j - \hat{i}_j > 0. \]  

(6.20)

During the motoring region \([-\pi / N_r, 0]\), we always have

\[
\sin \left[ N_r \left( \hat{\theta} - (j - 1) \frac{2\pi}{NN_r} \right) \right] < 0.
\]  

(6.21)

So we have the conclusion that \( e_f < 0 \), or in other words, \( e_f \) has the same sign as \( e_\theta \).

Same conclusion can be drawn if the estimated rotor position \( \hat{\theta}_1 \) is less than the actual rotor position \( \theta_1 \).

Figure 6.1 Phase inductance profile
Generating operation mode

As shown in Figure 6.1, when the rotor position \( \theta \) lies between \([0, \pi / N_r]\), the phase inductance decreases with \( \theta \), and the SR motor works in generating mode. If the estimated rotor position \( \hat{\theta}_2 \) is greater than the actual rotor position \( \theta_2 \), then we have

\[
e_\theta = \theta_2 - \hat{\theta}_2 < 0.
\]  

(6.22)

Since the phase inductance \( L(\hat{\theta}_2, i_j) < L(\theta_2, i_j) \), the estimated current \( \hat{i}_j \) must be greater than the actual current \( i_j \) with the same phase voltage. So

\[
i_j - \hat{i}_j < 0.
\]  

(6.23)

During the motoring region \([0, \pi / N_r]\), we always have

\[
\sin\left[ N_r \left( \hat{\theta} - (j-1) \frac{2\pi}{NN_r} \right) \right] > 0.
\]  

(6.24)

So we have the conclusion that \( e_f < 0 \), or in other words, \( e_f \) has the same sign as \( e_\theta \). Same conclusion can be drawn if the estimated rotor position \( \hat{\theta}_2 \) is less than the actual rotor position \( \theta_2 \).

In summary, the error function defined in Eq. (6.18) always carries the same sign as that of the rotor position error \( e_\theta \). It can stabilize the error dynamics of the sliding mode observer defined in Eq. (6.6) and (6.8). When the sliding surface \( e_\theta = 0 \) is reached, the
rotor position estimate \( \hat{\theta} \) equals to the actual rotor position \( \theta \), and the speed estimate \( \hat{\omega} \) converges exponentially to the actual speed \( \omega \).

### 6.1.5 Simulation Results

The sliding mode observer defined above has been implemented in Simulink model. In actual system, the voltage and current measurements are often noise-corrupted. To test the validity of the sliding mode observer under noise effect, noise signals have been added to the voltage and current signals in Simulink, as shown in Figure 6.2.

![Simulink model for sliding mode observer](image)

**Figure 6.2** Simulink model for sliding mode observer

The simulation results are shown in Figure 6.3 and Figure 6.4. In Figure 6.3, the SR motor starts to run at 500 RPM, and the speed jumps to 1000 RPM after 1 second. In Figure 6.4, the SR motor starts to run at 90 RPM, and the speed drops to 40 RPM after 1
second. In both figures, the solid red curves represent the actual rotor position and speed, and the dotted blue curves represent the estimated rotor position and speed from the sliding mode observer.

It is demonstrated that the sliding mode observer can get satisfactory estimates of rotor position and speed in despite of the noise in measurements. It uses only operating signals, so no additional sensors or external circuitry are needed. The computational simplicity and robust stability properties of sliding mode observers make it a good choice for sensorless control of SRM.

Figure 6.3  Simulation results at high speed
However, the sliding mode observer may fail at very low speed and ultra high speed. Also it cannot be used for sensorless startup because of the absence of electric signals at standstill.

In this research, the switched reluctance motor needs to be run at near zero speed at steady state. So sensorless control algorithms for such cases must be studied and developed.
6.2 Sensorless Control At Near Zero Speeds

When the rotor speed is low (near zero speeds), the sliding mode observer based controller cannot give satisfactory results. Other methods must be used for this speed region.

6.2.1 Turn-On/Turn-Off Position Detection

Re-organize the terms in phase voltage equation (6.1), we have

\[
\omega \frac{\partial}{\partial \theta} + \frac{\partial L_j}{\partial i_j} di_j + i_j \frac{\partial L_j}{\partial \theta} \omega. 
\]  

(6.25)

The last item in Eq. (6.25) represents the motional back-EMF, which can be ignored at near zero speeds. So we have

\[
L_j + i_j \frac{\partial L_j}{\partial i_j} \approx (V_j - R \cdot i_j) / \frac{di_j}{dt}. 
\]

(6.26)

Let’s observe Eq. (6.26). The right side of Eq. (6.26) can be computed directly from terminal measurements \( V_j \) and \( i_j \); and the left side is determined by rotor position and phase current. At different rotor position \( \theta \), the left side of Eq. (6.26) will have a different value (with \( i_j \) known). So if at any moment, the value of the right side matches the value of the left side at a given position \( \theta \), we can know that the rotor reaches the specified position. Based on this observation, we can easily find a way to determine the turn-on/turn-off moments for the SRM.
Let’s define

\[ F(\theta, i_j) = L_j + i_j \frac{\partial L_j}{\partial i_j}, \quad (6.27) \]

and

\[ G(V_j, i_j) = (V_j - R \cdot i_j) \frac{\Delta I}{\Delta i_j}. \quad (6.28) \]

Then when phase \( j \) is conducting and if

\[ F(\theta_{off,j}, i_j) = G(V_j, i_j), \quad (6.29) \]

then the turn-off position of phase \( j \) is arrived and phase \( j \) should be turned off.

Similarly, when phase \( j \) is conducting and if

\[ F(\theta_{on,j+1}, i_j) = G(V_j, i_j), \quad (6.30) \]

then the turn-on position of phase \( j+1 \) is arrived and phase \( j+1 \) should be turned on.

By the way, \( F(\theta, i_j) \) can be computed from the inductance model as

\[ F(\theta, i) = \frac{1}{2} \left[ \frac{1}{2} (L_a^* + L_u) + L_m^* \right] \]

\[ + \frac{1}{2} \left( L_a^* - L_u \right) \cos N, \theta \]

\[ + \frac{1}{2} \left[ \frac{1}{2} (L_a^* + L_u) + L_m^* \right] \cos 2N, \theta \]

\[ (6.31) \]
where

\[ L^*(i) = \sum_{n=0}^{p} (n+1)\alpha_n i^n, \quad (6.32) \]

and

\[ L^*_m(i) = \sum_{n=0}^{p} (n+1)\beta_n i^n, \quad (6.33) \]

### 6.2.2 Simulation Results

The above algorithm is simulated in Simulink. The desired turn-on/turn-off angles and the actual turn-on/turn-off angles (obtained from above algorithm) are listed in Table 6.1. The position estimation errors are also listed in the table.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Desired (°)</th>
<th>Actual (°)</th>
<th>Error (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_{\text{on},A})</td>
<td>-30</td>
<td>-29.49</td>
<td>0.51</td>
</tr>
<tr>
<td>(\theta_{\text{off},A})</td>
<td>-7</td>
<td>-6.14</td>
<td>0.86</td>
</tr>
<tr>
<td>(\theta_{\text{on},B})</td>
<td>-15</td>
<td>-14.49</td>
<td>0.51</td>
</tr>
<tr>
<td>(\theta_{\text{off},B})</td>
<td>8</td>
<td>8.86</td>
<td>0.86</td>
</tr>
<tr>
<td>(\theta_{\text{on},C})</td>
<td>0</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>(\theta_{\text{off},C})</td>
<td>23</td>
<td>23.86</td>
<td>0.86</td>
</tr>
<tr>
<td>(\theta_{\text{on},D})</td>
<td>15</td>
<td>15.51</td>
<td>0.51</td>
</tr>
<tr>
<td>(\theta_{\text{off},D})</td>
<td>38</td>
<td>38.86</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 6.1 Simulation results at near zero speeds
It is clear that the errors are negligible. Satisfactory results at near zero speeds (less than 20 RPM in our case) can be obtained.

### 6.3 Sensorless Startup

When the rotor is at standstill, there are no operating voltage and current signals available for position detection with the above algorithms. So some detecting voltage pulses need to be injected into phase windings for position sensing. The voltage pulses must be short enough so the torque generated in the phases can be neglected. The detecting voltage signals can be injected into only one or multiple phase windings. The easiest way is to inject an identical voltage pulse into each phase one by one, and compare the peaks of the currents generated in them. This is valid because at any rotor position, different phases have different airgap between the stator and the rotor pole (hence different inductances), as shown in Figure 6.5.

![Figure 6.5 Cross section of an 8/6 switched reluctance machine](image)

Figure 6.5  Cross section of an 8/6 switched reluctance machine
At the rotor position shown in Figure 6.5, the phase inductances have the following relationship

\[ L_D > L_C > L_A > L_B. \]  

(6.34)

If an identical voltage pulse is injected, the peaks of the currents generated in these phases will have the relationship

\[ i_{\text{peak},D} < i_{\text{peak},C} < i_{\text{peak},A} < i_{\text{peak},B}. \]  

(6.35)

At this moment, if the motor is to rotate clockwise, then phase C should be excited; and if the motor is to rotate counterclockwise, then phases D and A should be excited.

For rotor at different positions, the peaks of the currents have different but definite relationships. So the controller can decide which phases to be turned on according to this information.

The relationship among the peaks of currents for rotor at different positions is shown in Figure 6.6. The phase conduction cycle can be divided into 8 regions. In each region, the peaks of the currents have a unique relationship, and the phases to be excited can be determined by this relationship. This is summarized in Table 6.2.

Once the SR motor is started, operating voltage and current signals will be available. The sensorless algorithm proposed above can be applied, according to the range of the speed.
Figure 6.6  Peak currents at different rotor positions

<table>
<thead>
<tr>
<th>Peak currents</th>
<th>Rotor position</th>
<th>Phase to be excited</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_c &gt; I_d &gt; I_b &gt; I_a$</td>
<td>$[0^\circ, 7.5^\circ]$</td>
<td>B, C</td>
</tr>
<tr>
<td>$I_d &gt; I_c &gt; I_a &gt; I_b$</td>
<td>$[7.5^\circ, 15^\circ]$</td>
<td>C</td>
</tr>
<tr>
<td>$I_d &gt; I_a &gt; I_c &gt; I_b$</td>
<td>$[15^\circ, 22.5^\circ]$</td>
<td>C, D</td>
</tr>
<tr>
<td>$I_a &gt; I_d &gt; I_b &gt; I_c$</td>
<td>$[22.5^\circ, 30^\circ]$</td>
<td>D</td>
</tr>
<tr>
<td>$I_a &gt; I_b &gt; I_d &gt; I_c$</td>
<td>$[30^\circ, 37.5^\circ]$</td>
<td>D, A</td>
</tr>
<tr>
<td>$I_b &gt; I_a &gt; I_c &gt; I_d$</td>
<td>$[37.5^\circ, 45^\circ]$</td>
<td>A</td>
</tr>
<tr>
<td>$I_b &gt; I_c &gt; I_a &gt; I_d$</td>
<td>$[45^\circ, 52.5^\circ]$</td>
<td>A, B</td>
</tr>
<tr>
<td>$I_c &gt; I_b &gt; I_d &gt; I_a$</td>
<td>$[52.5^\circ, 60^\circ]$</td>
<td>B</td>
</tr>
</tbody>
</table>

Table 6.2  Determining phases to be excited from the relationship among the peaks of the currents in all phases
6.4 Sensorless Control Conclusions

In this chapter, a sliding-mode observer based sensorless control algorithm has been designed and simulated. Satisfactory results are obtained for medium and high speed operation of the switched reluctance machine. For very low speed operation, which is the steady state of the electro-mechanical brake system, a method by matching a position-related variable with its calculated value from voltage and current measurements is suggested. The turn-off position of the conducting phase and the turn-on position of the next phase can be easily determined through this algorithm, which is enough for control of the SRM. Also, voltage probing at standstill is suggested to start the SRM without position sensors. It is realized through comparing the relationship of the peaks of the currents generated in all phases when probed by the same short voltage pulse.

By combining the above algorithms, a sensorless scheme that yields satisfactory results over full speed range can be obtained.
CHAPTER 7

EXPERIMENTAL TESTBED AND RESULTS

In previous chapters, a model of the electro-mechanical brake system has been setup in Simulink. The clamping force controller and position sensorless control algorithms for switched reluctance machines have been designed and implemented in simulation. The next step is to setup an experimental testbed, and test and implement the controllers in the real system.

7.1 Experimental Testbed

A DSP based experimental testbed for the electro-mechanical brake system has been setup in Mechatronics laboratory of the Ohio State University. A block diagram of the testbed is shown in Figure 7.1. The testbed contained two sets of electric motor drive: one is for a switched reluctance motor, which is the servomotor of the EMB system; the other is for an induction machine, which serves as the load to the SRM. Since the caliper is not available, the induction machine will be used to simulate the brake load. A torque sensor
is shaft coupled in between the two electric machines. Both motor drives are DSP based, with its power converter, position encoder, and voltage and current sensors.

![Block diagram of experimental testbed](image)

**Figure 7.1** Block diagram of experimental testbed

The details of the components in the experimental testbed are shown in Figure 7.2. The DSP system used for SRM drive is dSPACE DS1103 controller board system. It contains dual CPU – Motorola PowerPC 604e and TI TMS320F240 DSP. DS1103 provides interfaces in Matlab/Simulink to access the hardware and software system of controller board, which is very convenient tool for control development. The DSP system used for induction motor drive is dSPACE DS1102 control system.

A picture of the testbed is shown in Figure 7.3.
Figure 7.2 Components of experimental testbed

Figure 7.3 Photo of experimental testbed
7.2 Experimental Results of Four-Quadrant Speed Control and Torque Ripple Minimization

In this experiment, a PID speed controller for the switched reluctance motor is implemented. The controller takes the speed command and the actual speed as input, and output the torque command. Torque control and torque ripple minimization algorithm mention in Chapter 4 is then applied to convert the torque command into phase current command. Finally hysteresis current control is used to decide the appropriate voltage pulses for each active phase. A block diagram of the controller is shown in Figure 7.4.

Figure 7.4  Block diagram of four-quadrant speed controller

To test the speed controller, the speed command is first set to be 500 RPM, then switched to −500 RPM after 2 seconds, and finally reset to zero. During this process, the motor is operating in all four quadrants of the torque speed plane. The experimental results are shown in Figure 7.5 through Figure 7.7.
In Figure 7.5, the torque and speed curves during the whole process are shown. The SR motor is first accelerated in forward direction, then slowed down and reversed. Finally it is stabilized at steady state. From the torque waveform we can see that the torque ripple is quite small. This proves the performance of the ripple minimization algorithm.

![Figure 7.5 Torque and speed responses](image1)

The four-quadrant torque-speed curve is shown in Figure 7.6. The motor starts from the 1st quadrant, then passes the 2nd, 3rd, and 4th quadrant, and finally returns to the origin.

In Figure 7.7, the voltage and current waveforms for phase A and B are shown. It is seen that phase overlapping exists to minimize torque ripple.
Figure 7.6 Four-quadrant torque-speed curve

Figure 7.7 Voltage and current waveforms
The experimental results verify the effectiveness of the torque control and torque ripple minimization algorithms developed in Chapter 4.

### 7.3 Experimental Results of Clamping Force Control

In this experiment, the clamping force controller developed in Chapter 5 is implemented and tested.

Since a brake caliper and a clamping force sensor are not currently available in the experimental testbed, an induction motor is used to simulate the caliper load and a “virtual” force sensor is implemented in DSP to simulate the force sensor. The virtual force sensor takes the angular movement of the rotor as input and computes the clamping force according to the following equations:

$$F = 2.5\{[(1.19 \times 10^{16}) \tilde{\theta} - 4.235 \times 10^{13}) \tilde{\theta} + 5.904 \times 10^{10}] \tilde{\theta} + 1.43 \times 10^{6}\} \tilde{\theta}, \quad (7.1)$$

$$\tilde{\theta} = \frac{\theta}{28} \left(\frac{0.00125}{\pi}\right), \quad (7.2)$$

where $\theta$ is the angular movement of the motor, $\tilde{\theta}$ is the linear movement of the brake pad, and $F$ is the clamping force between the brake pad and brake disk.

Four-quadrant torque control and torque-ripple minimization algorithms have been implemented in the DSP to convert the torque command obtained from force controller to phase current command.
A block diagram of the controller is shown in Figure 7.8.

The clamping force command is set as follows: the initial force command is 2500\(N\); when the actual clamping force reaches 2000\(N\), the force command drops to and stays at 1600\(N\).

The experiment results are shown in Figure 7.9 through Figure 7.12.

In Figure 7.9, the clamping force response is shown. Comparing with the simulation result in Figure 5.7, the experimental response is not as good but still quite satisfactory. Considering that the induction motor cannot fully represent the characteristics of the brake load, this result is quite acceptable.

The SR motor torque and speed responses are shown in Figure 7.10, with very small torque ripple. The four-quadrant torque-speed curve is shown in Figure 7.11, and the phase voltages/currents waveforms are shown in Figure 7.12.
Figure 7.9  Clamping force response

Figure 7.10  Torque and speed waveforms
Figure 7.11 Four-quadrant torque-speed curve

Figure 7.12 Phase voltage and current waveforms
7.4 Experimental Results Conclusions

In Chapters 4 and 5, algorithms for four-quadrant torque control, torque-ripple minimization, and clamping force control are developed and simulated. The validity of these algorithms is to be verified by experimental test results.

In this chapter, the experimental testbed is introduced and tests for four-quadrant speed control and clamping force control are performed. The test results prove that the performance of the controllers developed in previous chapters meet the requirements of electro-mechanical brake system.

The sensorless control algorithms proposed in Chapter 6 haven’t been tested in this research. It will be included in future work.
CHAPTER 8

CONCLUSIONS AND FUTURE WORK

8.1 Conclusions

The field of vehicle braking is on the verge of a revolution. The impact of new Brake-By-Wire technology is heralding a rate of change in brake systems technology not seen since the disc brake came into common use. This change will be driven forward by systems development and hold advantages for the vehicle manufacturer in terms of vehicle dynamics and design packaging.

In Brake-By-Wire systems, the conventional hydraulically actuated brakes are replaced by electro-mechanical brakes (EMB). Electric motors are used in the EMB to drive the brake pads onto the brake disks. An efficient, reliable, and fault-tolerant operation of the electric motor in the EMB system is highly desired. Switched reluctance motors, which operate on the principle of minimum reluctance, have the inherent advantages of fault-tolerance and ability to work with high speed and high temperature.
due to the lack of phase windings or magnets on the rotor. These characteristics make
SRM a good choice for electro-mechanical brake systems.

Modeling the dynamical properties of a system is an important step in analysis and
design of the control system. Generally, the parameter estimation from test data can be
done in frequency-domain or time-domain. Since noise is an inherent part of test data and
it has great impact on frequency-domain based methods, time-domain based methods,
such as maximum likelihood estimation (MLE), are proposed to identify the parameters
of the switched reluctance machines. An inductance-based model for the SRM is detailed
in this dissertation, and the MLE techniques are successfully applied to identify the
parameters of the SRM model from standstill test and onsite operating data.

Based on the inductance model of SRM, phase voltage equations and torque
equations can be formed, which provides a way to compute the electromagnetic torque
generated by the SRM from input phase voltages. On the other hand, when a desired
torque is given, the corresponding phase currents/voltages needed to generate this torque
can be determined if the motor speed is known and the turn-on/turn-off angles for each
phase is specified. This is the basis of the four-quadrant torque control and torque-ripple
minimization algorithms proposed in Chapter 4.

The control objective of the EMB system is to generate desired clamping force at the
brake pads and disk. It is realized by controlling the torque, speed, and rotor position of
the SRM in the EMB system. However, there’s no direct relationship between the control
objective and the control inputs, which are the phase voltages for the switched reluctance motor. In this dissertation, the clamping force controller is designed using robust backstepping, which proceeds by considering lower-dimensional subsystems and designing *virtual* control inputs. The virtual control inputs in the first and second steps are rotor speed $\omega$ and torque $T$, respectively. In the third step, the actual control inputs – phase voltages, appear and can be designed. The proposed clamping force controller does not require knowledge of the mechanical parameters of the motor and the functional forms of the relationships among the motor position, clamping force, and the motor load torque. Moreover, the controller provides significant robustness to uncertainty in the SRM parameters. Simulation results have proved this.

According to the torque generation principle of the switched reluctance machines, rotor position sensing is an integral part of SRM control. At the same time, the position sensor/encoder is a source of additional space, extra cost, and potential cause of failure. Rotor position sensorless control of SRM is desired for the electro-mechanical brake systems. Sliding mode observer (SMO), with its advantages of inherent robustness of parameters uncertainty, computational simplicity, and high stability, provides a powerful approach to implement sensorless schemes. A SMO-based sensorless control algorithm is developed and simulated in this research, with excellent performance in wide speed range. However, SMO based controller doesn’t work well at near-zero speeds, which is the steady state of the electro-mechanical brake system. A method by matching a position-related variable with its calculated value from voltage and current measurements is suggested to detect the turn-off position of the conducting phase and the turn-on position
of the next phase, which is enough for control of the SRM at low speeds. Also, by probing the phase windings of SRM at standstill and comparing the relationship of the peaks of the currents generated in all phases, sensorless startup of the SRM is realized.

To test the algorithms developed above, an experimental testbed has been setup in the lab. Experiments on four-quadrant speed control and clamping force control are performed. The test results prove the validity of the SRM model and its parameters, as well as the desired performance of the torque and clamping force controllers.

8.2 Future Work

This dissertation covers most of the issues related to the implementation of a switched reluctance motor based electro-mechanical brake system. Within the scope of this work, there still remain a few aspects to be addressed to improve:

- The sensorless control algorithms developed in Chapter 6 need to be implemented and tested on experimental testbed. Sensorless control in full speed range of the SRM in EMB needs to be verified.
- The integration of the sensorless algorithms with the torque and clamping force control algorithms needs to be completed. Currently the torque and force controllers use the rotor position signal sensed by an encoder. Later when sensorless control is tested and implemented, the estimated rotor position will be used instead. The effect of the errors in position estimation on the torque and
force control needs to be investigated. A robust force controller is to be implemented.

- The induction motor on the testbed needs to be replaced by brake caliper and all tests performed above need to be repeated and modified for the “real” brake system. A clamping force sensor is necessary in the first stage, but methods to eliminate the force sensor need to be investigated. The final goal is to realize position-sensorless and force-sensorless control of the EMB system.

- Currently all control functions are implemented on dSPACE DS1103 system which is perfect for rapid control prototyping. When all functions are verified on the development system, a low-cost implementation system needs to be recommended for mass production of manufacturer.

With the successful accomplishments of the above work, the SRM based EMB can finally be implemented in future hybrid/electric vehicles, which will hopefully appear in market around year 2010.
BIBLIOGRAPHY


APPENDIX A

PARAMETERS OF SWITCHED RELUCTANCE MOTOR AND BRAKE CALIPER SYSTEM

A.1 Parameters of Switched Reluctance Motor

- Number of phases: \( N = 4 \)
- Number of stator poles: \( N_s = 8 \)
- Number of rotor poles: \( N_r = 6 \)
- Rated Voltage: \( V = 42 \text{ V DC} \)
- Rated Current: \( I = 50 \text{ A} \)
- Phase winding resistance: \( R = 0.2996 \Omega \)
- Phase winding inductance: \( L_u = 0.50304 \text{ mH} \)
Phase winding inductance at other rotor positions: \( L_\phi(i) = \sum_{n=0}^{p} a_{\phi,n}i^n \), where \( p \) is the order of the polynomial and \( a_{\phi,n} \) are the coefficients of polynomial. They are listed in Table A.1.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( p )</th>
<th>( a_{\phi,0} \rightarrow a_{\phi,p} )</th>
</tr>
</thead>
</table>
| 0° | 5 | 5.1892e-003  
6.4641e-005  
-7.7645e-005  
6.5907e-006  
-2.1974e-007  
2.6214e-009 |
| 10° | 5 | 4.2396e-003  
-1.9346e-004  
-2.8209e-006  
4.9546e-007  
-1.3845e-008  
1.1391e-010 |
| 15° | 5 | 2.6605e-003  
-4.7806e-005  
-2.9271e-006  
4.3566e-008  
4.2784e-009  
-9.6186e-011 |
| 20° | 5 | 1.1875e-003  
-1.9716e-005  
3.8457e-006  
-2.3203e-007  
4.3266e-009  
-1.9516e-011 |

Table A.1 Phase inductance coefficients at different rotor positions

- Damper winding resistance: \( R_d = 37.9659 - 5.4105 \times i + 0.2613 \times i^2 - 0.0041 \times i^3 \Omega \)
- Damper winding inductance: \( L_d = 0.5678 \) mH
A.2 Parameters of Brake Caliper

\[ F = 2.5\{[1.19 \times 10^{16} \tilde{\theta} - 4.235 \times 10^{13}]\tilde{\theta} + 5.904 \times 10^{10}]\tilde{\theta} + 1.43 \times 10^6\} \tilde{\theta} \]

\[ \tilde{\theta} = \frac{\theta}{28} \left( \frac{0.00125}{\pi} \right) \]

\[ T_i = \frac{F}{2.5} \frac{1}{28} \left( \frac{0.00125}{\pi} \right) \]

where \( \theta \) is the angular movement of the motor, \( \tilde{\theta} \) is the linear movement of the brake pad, \( F \) is the clamping force between the brake pad and brake disk, and \( T_i \) is the load torque connected to the SR motor.
APPENDIX B

SIMULATION TESTBED OF SWITCHED RELUCTANCE MOTOR BASED ELECTRO-MECHANICAL BRAKE SYSTEM

The simulation testbed for switched reluctance motor based electro-mechanical brake system is implemented in Matlab/Simulink®. As shown in Figure B.1, it has four modules:

- Force Command (Figure B.2)
- Force Controller (Figure B.3)
- Switched Reluctance Motor and Power Converter (Figure B.4 through Figure B.7)
- Brake Caliper (Figure B.8)
Figure B.1 Simulink testbed of electro-mechanical brake system
Figure B.2  Force command module

Figure B.3  Force controller module
Figure B.4  SRM and power converter module
Figure B.5  SRM Hysteresis module (four modules for four phases)
Figure B.5  SRM back EMF module (four modules for four phases)
Figure B.6  SRM torque module (four modules for four phases)

Figure B.7  SRM mechanic part module (four modules for four phases)

Figure B.8  Brake caliper module