DEVELOPING A MODEL OF COMMUNICATION FOR PRE-SERVICE ELEMENTARY TEACHERS’ WRITTEN MATHEMATICAL EXPLANATIONS

DISSERTATION

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By

Drew K. Ishii, M.A., M.S.

* * * * *

The Ohio State University
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Dissertation Committee:
Professor Patricia Brosnan, Adviser
Professor Susan Kline
Professor Stephen Pape

Approved by

____________________________

Adviser
College of Education
ABSTRACT

Written explanations to mathematics problems were analyzed from undergraduates in a *mathematics for elementary teachers* course in which two different instructional approaches were used: a traditional lecture/recitation approach and a nontraditional inquiry problem-solving approach. Twenty-four students in the nontraditional approach and 121 in the traditional approach participated with a total of 145 university juniors and seniors. All students were assessed on a common final examination. The final assessment required students to solve problems and provide written explanations for their work. The purpose of this study was to develop a communication model of the written explanations provided by the undergraduate students in an effort to better understand written mathematical explanations and communication. A grounded qualitative analysis of the data was performed using the knowledge base from sociocultural theory, constructivist communication theory, and prior similar research in mathematics education. Based upon this as a theoretical framework, a hierarchical model of explanations was developed from the written explanations. Three main categories describe the data from lowest to highest sophistication: algorithmic explanations, structural explanations, and transformative explanations. The categories in the model were developed without taking into consideration the correctness of the
explanation, rather the completeness of the explanation and the job that the explanations do in the responses. *Algorithmic* explanations are procedural in nature and may include simple translations from symbolic mathematical work into words. *Structural* explanations provide details and features beyond those that are procedural. *Transformative* explanations are characterized by showing logical progressions and connections and may include the student’s thought process in addition to elements from the previous explanatory categories. Across both instructional approaches and across all questions the most used type of explanation was the algorithmic explanation. A comparison of the types of explanations used between one section of the traditional approach and the nontraditional section showed that the explanation type use was very similar in both approaches. The frequencies were all within two instances or 8.3% of each other between the sections. For both approaches, there were very low associations between the type of explanation used and mathematical success on the question.
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VITA

November 17, 1974……………………… Born California

1996……………………………………….. B.A. Communication Studies and Mathematics,
University of San Diego

1998……………………………………….. M.S. Mathematics,
Western Kentucky University

2003……………………………………….. M.A. Mathematics Education,
The Ohio State University

PUBLICATIONS

1. Ishii, D. K. (2003). First-time teacher-researchers use writing in middle school

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CHAPTER 1
INTRODUCTION

Statement of Problem

Within the last 15 years, communication in mathematics has become increasingly important to mathematics education researchers and teachers. It is difficult to discuss teaching without mentioning communication, since it is usually the vehicle for teaching. Communication in the mathematics classroom may occur in several forms between all members of the class, not just from the teacher to the students. We see mathematical communication occurring in both oral and written forms. The current mathematical reform efforts by the National Council of Teachers of Mathematics (1989, 2000) have placed a priority on communication in the mathematics curriculum at all levels. In conjunction, the mathematics education community has moved from a view of mathematics teaching and learning as the transmission of facts and concepts, to a more situated view of learning where students are actively engaged in and responsible for their own learning (Cobb, Yackel, & Wood, 1992).

This in turn changes the role of the teacher from the disseminator of knowledge to facilitator of the learning process. If this is to be the new expectation of the teaching and learning of mathematics, then we, the mathematics education community, must better understand how to communicate with our students and be able to model effective
communication practices in order to have a standard of effective mathematical
communication. If we adopt the view of mathematical learning as embedded within
social practice, then this presents the possibility of studying the social mathematical
activities. In order to do this, it may be helpful to seek assistance from the field of
communication studies. Communication experts may help us to better understand human
interactions and social activity in general because all social interactions exist because of
the ability to communicate with others.

Applying communication theory to the study of mathematics education, as will be
seen in this study, provides us with the tools necessary to further understand the teaching
and learning of mathematics. When communication is paired with the concept of
understanding, intersubjectivity comes into play. For example, any social setting
involving communication activities requires the participants to understand what the other
is saying or meaning in order for the participants to achieve their goals (Delia, O’Keefe,
& O’Keefe, 1982). In a sociocultural sense, this involves the alignment of voices into one
voice or one perspective (Wertsch, 1991). From a communication perspective, this
involves the coordination of actions, the use of common ground or commonalities, and
the use of cooperative practices in order to gain an understanding of one another when we
communicate (Clark, 1996).

Although it is possible to communicate mathematical ideas in various ways
including graphical and symbolic representations, this research focuses on
communication through language, namely writing. This is not to say that written
communication and explanations cannot include graphical and symbolic representations,
but for the purposes of this research, constructing explanations through language will usually accompany some other type of representation.

I propose the notion that written mathematical explanations may be viewed as an external representation (Goldin & Shteingold, 2001) of students’ mathematical sense making and mathematical thinking in that it is possible to discuss the representations’ meanings and nuances, however this may depend upon the explanatory task. The process of writing mathematical explanations for someone else to understand forces one to clearly articulate a concept or procedure so that another person can understand one’s thinking. There is a distinction, however, between providing a written explanation so that another person may understand the explainer’s thinking, and providing a written explanation so that another person may understand the concept being explained. Either way the critical component lies in the perceived audience of the message. Studying mathematical communication may help teachers of students so that they may build upon current knowledge or help to dispel misconceptions. This, I believe, is at the heart of the NCTM Communication Standard, yet has not been addressed in the current research.

An increasing awareness about classroom communication has come from the National Council of Teachers of Mathematics in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) that emphasizes the role that communication should play in the mathematics classroom. The Communication Standard says that students should be able to organize, communicate, analyze, and evaluate thoughts to each other and to their teacher using the language of mathematics. Since communication can be either oral or written, students should be proficient in both modes of communication.
The current research that addresses communication in mathematics education primarily focuses on the social negotiations of meanings through classroom discussions (e.g., Cobb, Wood, & Yackel, 1993; Yackel & Cobb, 1996). This focus on classroom discussion and social interaction was motivated by a paradigm shift in what constitutes mathematical teaching and learning. Cobb, Yackel, and Wood (1992) established a view of mathematical learning as a process of enculturation, where the teacher acts as an acculturated representative of the mathematics community. Consequently, this view of learning shifted the concentration from what the teacher says to what everyone in the classroom says during classroom discussions. This broadened the conception of learning to include not only constructivist perspectives, but also sociocultural and interactionist perspectives (Sierpinska, 1998). Through all of this research, the focus on oral communication took precedence.

Writing in mathematics instruction gained recognition in the late 1980s with work from many of the authors who contributed to *Writing to Learn Mathematics and Science* (Connolly & Vilardi, 1989). This volume gives accounts of writing activities to use throughout the levels of the mathematics curriculum. Writing in mathematics usually takes the form of either journal writing or expository writing. The difference between these two types of writing is subtle in that the difference lies in the intent of the activity. Journal writing is meant to be a reflective activity where students write about their learning experiences. The writing can also serve as a thinking device when it records prior experience and is revisited later, thus providing for a reflection upon learning. Expository writing activities provide students with the opportunity to express or explain their thinking to another person, thus providing an account or demonstration of
mathematical competence. The use of writing in mathematics learning has the potential to benefit both students and teachers including improvements in problem solving, and in procedural and conceptual learning (Borasi & Rose, 1989). Writing by definition increases the communication in the classroom, and benefits teachers by opening channels of feedback and helping to tailor teaching practices to the needs of student progress (Miller, 1992).

Given the plethora of suggestions and research by practitioners and researchers, there is inconclusive evidence that writing can truly affect learning in mathematics. There is agreement that writing has the potential to influence many aspects of learning that are consistent with mathematics reform (Borasi & Rose, 1989; Drake & Amspaugh, 1994; Jurdak & Zein, 1998). Many studies and articles give anecdotes of the use of writing in mathematics, but very few studies (Pearce & Davison, 1988; Shield & Galbraith, 1998; Silver, Leung, & Cai, 1995; Stonewater, 2002) have attempted to categorize the elements and nuances of writing. None of the studies offers a way of looking at writing in terms of effective communicative practices. As a field, we do have ways of classifying student writing in terms of its structure, but not in terms of communicative competency. This aspect of NCTM’s Principles and Standards for School Mathematics (2000) has not been addressed in the literature and deserves attention.

**Focus Questions**

Studying mathematical communication by identifying effective communication activities and strategies beyond that which has been described earlier is important at all levels of mathematics teaching. The NCTM Principles and Standards for School Mathematics (2000) refers to the expectations and competencies that students should
exhibit, however in order for students to meet the Communication Standard their teachers must know how to model and teach so that their students may communicate effectively. Although some research suggests that elementary teachers lack sufficient depth of mathematical knowledge (Ball, 1990), it is important for them to be able to communicate the knowledge they do possess. Complicating the issue further, some middle school and elementary school teachers choose to teach at those levels because of their previously negative experiences with mathematics (Watson, 1995). While some research suggests that depth of mathematical knowledge and depth of pedagogical content knowledge accompany each other (Ebert, 1993), other research suggests that strong content knowledge is not sufficient and may actually impede the ability to explain material to students (Thompson, P. W. & Thompson, A. G., 1994). Regardless, elementary teachers introduce children to the beginnings of mathematics whether they have deep mathematical understanding or not. Since elementary students in this country consistently under-perform mathematically compared to students in other countries (Ma, 1999), studying pre-service elementary teachers’ communication is a logical place at which to begin in order to bring about improvements in both elementary teaching and learning of mathematics.

The purpose of this study was to develop a communication model that blends the two research disciplines of communication studies and mathematics education. Communication has become a major emphasis in mathematics education, as evidenced by NCTM (2000), and it is important to study the field of communication studies in an effort to learn what the communication researchers can teach mathematics educators about communication. Since this descriptive study connected the fields of mathematics
education and communication studies, there were a multitude of directions in which this research could have proceeded. Given the setting of an elementary mathematics course, and the scope of this research, I was interested in pursuing answers to the following questions:

- What communicative elements and processes exist in the mathematical explanations of pre-service elementary teachers?
- What relationship exists (if any) between explanations and mathematical success?
- What differences exist between success and the explanations offered by students from two different course settings?

Theoretical Framework

The framework for this study consists of elements from both education/learning theory and communication theory. The two specific theories that guided this work are sociocultural theory and constructivist communication theory. Sociocultural theory provides the overarching framework for the social aspects of learning. From there, social interactions are dissected further using constructivist communication theory, which offers insight into human communication by helping to explain the communication processes at work within social interactions.

Sociocultural Theory.

The main tenets of Vygotskian sociocultural theory that are relevant to this study are: (a) human action is mediated by tools, signs, and most importantly, language; (b) learning is inherently a social process; and (c) the zone of proximal development. For Vygotsky, language is the ultimate tool, which mediates thought and learning. Language
(or speech) is the primary means of communication in society, without which many activities would be difficult to accomplish. Because humans communicate and interact with each other, learning is closely related to the use of language. Vygotsky (1978) posits that language has an interpersonal function first, and an intrapersonal function second. In his view, language serves to mediate thought because for children action and speech are “one and the same complex psychological function” (p. 25). The result of this is egocentric speech, which is the transition between external and internal speech. For example, Vygotsky found that children solve problems with help of their speech by verbalizing their thoughts. Only later in the child’s development does it begin to internalize the socialized speech into egocentric speech, which allows it to solve problems without verbalizing thoughts, thereby illustrating the relationship between the thoughts and language of the learner.

The role of language in shaping knowledge and learning from the sociocultural perspective is integral to the study of classroom communication. Language plays a role in individual learning, in collective learning, and in shaping the culture of the classroom. Since language is embedded in culture, language and culture function similarly to shape beliefs, views, and knowledge. Both language and culture impose a structure on the situation in which they reside. They serve the same purpose of constructing or at least assisting in construction of knowledge. Since language is finite and limited in scope, it constrains reality, which shifts focus away from the conception that an objective reality exists (Kvale, 1996). In this way there is no language that can describe the objective reality, but only the structures of language that speak through the person. In a way, we come to know the meanings of objects by social exchanges and negotiations through
language, which then become internalized. Meaning, then, exists in the social contexts in which it was constituted. Wertsch (1991) describes meaning as a *renting* metaphor, borrowed from Bakhtin, where we simply rent meanings because we are a second level removed from it. We can only indirectly mean what we say because we are constrained by that language and more so by the culture.

The notion of sociocultural theory that learning takes place socially follows from the notion that language mediates human action (Vygotsky, 1962). Sociocultural theory places great emphasis on the social activities of the learner during the learning process. In describing the way in which children internalize higher psychological functions, Vygotsky (1978) explains that children take cues from adults who react to their behavior. An activity that was once external to the child begins to occur internally, and that interpersonal process becomes an intrapersonal process through a series of developmental events. Extending this notion to people in general, the cultural activities occur socially and independent of the person, then internalized so that it becomes part of the person’s regular functions. In relation to learning that occurs in the classroom, this point seems obvious. In a classroom situation, the activity of exchanging information and ideas (i.e. communication) between teacher and student or between students constitutes learning in a social context. This idea is especially important in the study of classroom communication. Since classrooms create norms and conventions that are specific to that situation, this in turn affects the learning that takes place. These norms and conventions can be thought of as the creation of culture. All cultures have origins and thus have a history, which is another aspect of Vygotsky’s sociocultural theory. Wertsch (1991)
refers to this as genetic analysis. This idea emphasizes that both origins and transitions are important in understanding the development of learning.

Wertsch’s work, largely informed by Vygotsky and Bakhtin, uses terms that are embedded in a sociocultural frame of mind such as Bakhtin’s terminology, utterance, voice, addressivity, and dialogicality. Utterance is “the real unit of speech communication” (Wertsch, 1991, p. 50). Speech is always cast in the form of an utterance belonging to a particular speaking person. Voice is the point of view from which the utterance is spoken. Addressivity refers to the ability of turning to someone else, or directing the utterance, which cannot exist without addressivity. Dialogicality is the most basic construct since addressivity involves at least two voices. This refers to the quality of speech that always involves more than one person or voice. For example, in the classroom where a teacher presents mathematical concepts to the students, the teacher’s utterances represent the voice of the mathematics culture or community. When this voice differs from that of the students’ voices, the goal of the learning process is to have students adopt the voice (Wertsch, 1991) that is consistent with the mathematics community at large.

This view of Bakhtin/Wertsch assumes a sociocultural view of the world. They align with Vygotsky’s principles that rely on history, social negotiation, and mediation. Even the meanings of words have history, in which the definition has been socially negotiated (Volosinov, 1973), thus mediating or bridging thought and action in the world. All of these Bakhtinian terms constitute the language that is the tool that helps us interpret, act on, and embrace the world.
Vygotsky’s concept of the zone of proximal development is another aspect of sociocultural theory that is useful in educational settings involving social interactions between teachers and students. The zone of proximal development is characterized as the difference between a person’s actual ability when performing a task and the person’s ability when assistance is given by a more knowing other (Vygotsky, 1962). Vygotsky used this concept when referring to children engaged in problem-solving situations, but it can be (and has been) extrapolated to any educational setting where two people of differing ability engage in an activity. He says that good learning occurs ahead of the person’s ability, namely within the zone of proximal development. An educational implication associated with the zone of proximal development is the notion of scaffolding where adults structure and manage problem-solving situations for children by participating with children in supported situations (Rogoff, 1990). Some features of scaffolding include: (a) gaining the child’s interest, (b) reducing the number of steps to solve problems, (c) focusing on the goal, (d) noting critical features of the process, and (e) demonstrating an ideal version of the task. In an educational setting, a teacher that scaffolds a problem-solving activity for a student or students is deeply engaged with meaningful communication taking place between teacher and learner. Therefore it is the communication that makes scaffolding possible.

Sociocultural theory, as conceived by Vygotsky, provides a perspective on learning that is much different than the traditional transmission of information from the teacher to the student. Sociocultural theory embeds knowledge within the interactions of teachers and students. Since learning occurs on an interpersonal plane first (Vygotsky, 1978), the social activities of the learner are extremely important to development. When
this is translated to the mathematical learning situation, the process is no different. Since
the mathematics education community is currently moving towards a view of
mathematical learning as situated within social contexts, the exchanges between the
participants provide the structure of the learning situations. From the sociocultural
perspective the teacher guides students in the activities of learning and engages in their
zones of proximal development (Rogoff, 1990). Since social interactions are so important
to sociocultural theory, a natural progression towards the study of the communication that
occurs in these situations seems appropriate. The following theory from communication
studies is a likely fit into the sociocultural frame of reference.

*Constructivist Communication.*

The constructivist approach to communication, as opposed to constructivism that
is usually associated with Piaget, was developed by Delia and his colleagues (Delia, B.
occurs in a social setting with some kind of purpose, the constructivist view posits that
communication is an interaction in which people are engaged in coordinating and seeking
to achieve their goals. This process is done through the coordination of their respective
lines of action. People know how to do this through their previous experiences and
interactions. From these experiences they organize mental schemes on which to base
future interactions (O’Keefe & Delia, 1985). People act and react according to the
meaningful ways that they have grouped into larger categories either similar or different
experiences. Basically, people use these categories or constructs\(^1\) to make sense of the
world as well as operate and communicate within it based on previous similar or different

\(^1\) George Kelly (1955) developed a theory of personal constructs, upon which constructivism is partially
based.
experiences. Therefore, communicating can be viewed as an interpretive action that is
deciphered through a person’s perceptions. Delia, O’Keefe, and O’Keefe (1982) call
these constructs “interpretive schemes” (p. 152), which are general cognitive
organizational schemes. They serve as resources for people when they interact with
others and help them fit experiences into a frame of reference where they can jointly
coordinate their actions.

Constructivist communication places people and their interpretive schemes within
a larger context, namely culture and community. Since people develop their personal
interpretive schemes from experiencing the world and then organizing the cognitive
structure accordingly, the culture or sociocultural community from which the person
operates is just as important as the individual. Because of this, interpretive schemes have
history, and “culture and community are historical processes in which forms of social
organization and interpretation are maintained and elaborated in and through processes of
social life” (Delia, O’Keefe, & O’Keefe, 1982, p. 154). Constructivism places emphasis
on both the social aspects of each situation as well as the individual participants’ social-
cognitive and behavioral aspects of communication (O’Keefe & Delia, 1985).

This facet of constructivism likens it to Vygotskian sociocultural theory in that it
both acknowledges the culture and history of the individual and considers the larger
context that both influences and constrains experiences and ways of knowing the world.
Vygotsky’s observations suggest that, “perception consists of categorized rather than
isolated perceptions” (p. 33). Both theories recognize that in order to understand the
individual, it is necessary to understand the culture, community, and history from which
the individual comes because the actions, behaviors, and knowledge emerge from the
interpretations of past experiences. “Every act collapses past, present, and future; and thus, every act emerges from a new past into a new future” (Delia et al., 1982, p. 156).

The constructivist approach sees communication as a process of alignment or coordination because individuals communicate their intentions to their communication partners, and reciprocally interpret the intentions they receive from their partners (Delia et al., 1982). This reciprocity also affects the strategies people use to communicate their intentions and interpret the communication of others. In the entire scheme of communicating and understanding each other, or intersubjectivity, coordinating actions is based upon the attention that participants pay to the practical ways of behaving and interacting with each other. Another related aspect of this requires a cognitive awareness of communication behaviors, which affects individual communicative action based upon the listener or the audience. Communication that reflects the ability to adapt messages to the listener’s characteristics and intentions shows a cognitive capacity to engage in cooperative behaviors. Relying on commonalities such as common languages and codes reflects the ability to engage others in ways that aids understanding each other’s communication.

When constructivist communication is partnered with the sociocultural view of learning, they provide a perspective into the educational setting that emphasizes the social context as well as the cognitive abilities of the participants in each situation. Taking this view into mathematics teaching and learning accentuates the culture of the mathematics community and the ways in which students and teachers interact and communicate within that culture (Cobb, Yackel, & Wood, 1992). Individual differences within the participants in the mathematics culture are honored when participants
communicate using practices that reflect their cognitive abilities in communicating such as taking another’s perspective and constructing messages that are person centered. When applied to a mathematical learning situation, there is no single definitive way to communicate mathematical ideas, and teachers as well as students must be proficient in communicating mathematics. Contrary to the belief of many neophyte mathematics students, there are reasons for every process and concept in mathematics. Teachers can provide explanations to students for this type of information while increased communication can serve to monitor a student’s grasp of the given material (NCTM, 2000). In this way, the study of explanations and the various types and functions of which supply another layer of knowledge to enhance the analysis of communicating mathematics. Further studies in the constructivist approach to communication are found in the literature review chapter.

These domains of literature provide the framework for studying classroom communication in the current study. Vygotsky’s sociocultural theory elucidates the learning theory that situates all learning in a social as well as an historical setting. Constructivist communication theory is an appropriate counterpart to sociocultural theory because it too recognizes the role that community, culture, and history play in shaping reality and the ways in which people act and communicate within it. As will be seen in the next chapter, the work in explanations is important because it provides the umbrella under which all explanations fall. That work, along with others that will be discussed in the following chapter, allows for the exploration of explanations in the mathematics classroom.
In the following chapters, the details of this study are discussed further. In the next chapter, I review the current and relevant literature that informed this study. Literature from the fields of mathematics education, general learning, and communication studies combine to form the knowledge base upon which this study is built. Within mathematics education, the literature on mathematical communication and the use of writing in the mathematics classroom is relevant to this study. The third chapter is devoted to the research methodology that guided the study. A discussion of the participants, the context, and procedures are discussed in that chapter. In the fourth chapter, I discuss the process of data analysis along with the results of that process. Since this study utilized qualitative research methods in the development of the communication model, the process of that development is given in that chapter in order for the reader to see how the model was inductively created. In the final chapter, the results, conclusions, and other meaningful outcomes of the study are discussed. The limitations, suggestions, and future research directions are included in that discussion as well.
CHAPTER 2
REVIEW OF LITERATURE

Introduction

The purpose of this literature review is to survey the theory and research germane to communication in mathematics, in addition to exposing the gap in the literature that necessitates the impetus for the current study. Although there are many facets of communication in the mathematics classroom, I will concentrate this literature review around four main areas: (a) general learning theory and its application to mathematics education; (b) literature on writing in mathematics; (c) communication theory; and (d) mathematical explanations.

The general research in mathematics education about communication focuses on two levels of communication, oral and written. Within oral communication, research concentrates on the discourse in the classroom between students, and whole class discussions between teacher and students. Written communication in mathematics is used generally in two ways, journal writing and expository writing. Theory from Communication Studies has several useful frameworks that can be valuable to the study of mathematical communication. The theories discussed include constructivist theory, message design, and various approaches to explanatory discourses in communication. The last section on mathematical explanations consists of research from mathematics
education that focuses on discursive and written explanations. Within the literature review, I articulate the theories mentioned from the theoretical framework as they apply to the main areas of concern.

A major influence in recent years on communication in mathematics has been the *Principles and Standards for School Mathematics* (National Council for Teachers of Mathematics [NCTM], 2000) document, which includes communication as one of its standards for the teaching and learning of mathematics. NCTM calls for students of all grade levels to be able to do the following with regard to communication in the mathematics classroom:

Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely (p. 60).

This Communication Standard emphasizes and almost mandates that the practice of mathematics shift from being an isolated activity to a shared activity involving peers and teachers. Mathematics reform efforts call attention to the important role of the social environment in education. Taking the Communication Standard as a point of departure for this literature review, one can see the many ways in which NCTM’s suggestions can be interpreted and implemented into both theory and practice. Although the *Principles and Standards* do not include college-level mathematics, the expectations are certainly relevant.
In this literature review chapter, I first survey sociocultural theory in order to situate this study with that particular view of learning. This perspective on learning and reliance on sign systems and language lays the foundation upon which I will discuss the constructivist theory of communication that takes a sociocultural perspective on communication activities. I survey the main principles of constructivist communication as well as communication research from that perspective including positions on the different ways of constructing messages. Following the communication theories, I return to the focus of communication and language in mathematics teaching and learning. From there, the focus of communication in the mathematics classroom shifts to writing, its uses in teaching and learning mathematics, and the relevant research. Finally, the survey of literature culminates with studies that focus on mathematical explanations and the ways in which the field is able to talk about and identify them. This next section begins the discussion of the perspective on learning upon which this study is based.

**Learning**

Learning rarely takes place in complete isolation. Some kind of social interaction occurs in the learning situation whether it is between a teacher and a student, between students, or an individual trying to make sense of something she/he has read, which was written by someone else, thus making it an interaction between two people. In a classroom setting the different types of social interactions play an important part of the learning process. Sociocultural/sociohistorical theory as developed by Vygotsky posits that mental functions occur on two planes, first on the interpsychological or social plane, and then on the intrapsychological or individual plane (Vygotsky, 1978; see also Wertsch, 1991). Here, an individual’s knowledge can be thought of as the internalization
of external knowledge derived from social living. Sociocultural theory views language as
the ultimate tool that mediates thought and action. Vygotsky places strong emphasis on
language and sign systems and their role in development. Sign systems such as language
are important for communication because he believes that “communication requires
meaning (i.e. generalizations) as much as signs” (Vygotsky, 1962, p. 6). Since learning is
seen as inherently social and that mental functions being outside of the person and later
internalized to the individual’s repertoire of functions, the communication and interaction
between those involved serves as a dynamic worth more than just simple exchanges of
ideas. This is an opportunity for intersubjectivity. Wertsch’s conception of
intersubjectivity is the degree to which people come to align their voices, or come to an
understanding of each other. Intersubjectivity is essentially univocal, meaning that one
perspective is agreed upon, as opposed to two divergent perspectives that are in conflict
with one another.

Rogoff (1990) defines intersubjectivity as “shared understanding based on
common focus of attention and some shared presuppositions that form the ground for
communication” (p. 71). Without intersubjectivity communication would not be possible,
for there would be no sense of a common goal or direction for the activity of
communication. Rogoff utilizes a Vygotskian construct called the zone of proximal
development to help to explain the interactions of a parent and a child. Vygotsky (1978)
describes the zone of proximal development as the hypothetical distance between what a
child can do alone and what the child can do with assistance or guidance from a more
capable person such as a parent or teacher. Rogoff’s description of this interaction
between a child and a caregiver as guided participation where the caregiver engages with
the child at an appropriate ability level that is within the child’s zone of proximal development. Guided participation accounts for the individual, interpersonal, and cultural practices that comprise whole learning events as a system of involvement (Rogoff, Mistry, Goncu, & Mosier, 1993). This concept focuses on the interactions between children and their caregivers in sociocultural activities in which children learn by participation and guidance, and the culturally acceptable nuances of participation and behavior in various learning situations are then passed-on to the child. Although Rogoff’s work does not extend the concept of guided participation to more than dyadic interactions, the metaphor of guided participation can be applied to situations that involve teachers and students when their activity has to do with mutually understanding each other.

In educational settings, the interactions between teachers and students that can be described as guided participation are instances of classroom activity that have the potential to be examined for the types of communicative practices at play. From the sociocultural perspective, this type of engagement between student and teacher allows the student to participate in activities slightly beyond his/her ability, and from the help of the teacher, the student learns the skills of the culture (Rogoff, 1990). From guided participation, students are cognitively engaged on the social level first, and then those skills are internally processed on the student’s individual level. Thus this is Vygotsky’s main premise in action in an educational setting. On the part of a caregiver or teachers, guided participation most always involves connecting known concepts with those that are new. It is a way of establishing commonalities in order to have a shared perspective on the activity. Organizing activities to fit within the child’s or learner’s grasp is another
practice of guided participation that varies across cultures and communities (Rogoff et al., 1993). This affects the amounts of responsibility placed on the child or the learner and the degree to which they are allowed to participate in a given activity. These sociocultural ideas may be applied to any learning situation including mathematical learning.

*Social Turn in Mathematics*

From the various perspectives on mathematical learning, and from Vygotskian-inspired perspectives of Wertsch, Rogoff and others, the increased awareness of the social aspects of teaching and learning is seen in recent research in mathematics education. This awareness encourages thinking of the learner as embedded within a broader setting or culture, instead of thinking of the learner in isolation. This shift in thinking is evidenced by Cobb, Yackel, and Wood (1992), where they propose that knowing mathematics is not the ability to re-present the mathematics that one knows, but as being able to participate in the culture or the community of mathematics (an interactionist view). Based mainly upon Constructivism associated with von Glaserfeld and Blumer’s Symbolic Interactionism; Cobb, Yackel, and Wood (1992) forward a position of mathematical learning that involves individual constructive activity as well as communal social practice. In their view, mathematics can be seen as representing a cultural body of knowledge, and through participation in the culture, people come to know the mathematics. “To know is to be able to participate in a social practice,” (p. 27) implies that to know the culture of mathematics is to know and understand the conventions and taken-as-shared meanings, which happens through mathematical acculturation. The implications of this view of mathematics for research focuses on the meaning-making situations and the interactions between students and the teacher, as the
acculturated representative of the mathematics community. The substance of the interactions that can be examined, then, is the discourse. From their perspective, the focus on mathematical discourse and the interactions in which it is embedded is studying mathematical meaning making. Adopting this view of mathematics moves the emphasis away from being able to perform mathematical procedures and towards the individual mathematical communities of learners that are constructed in each different classroom.

Cobb and his colleagues established this view of learning mathematics that incorporated elements from constructivism (von Glaserfeld, 1984), symbolic interactionism (Blumer, 1969), and ethnomethodology (Mehan & Wood, 1975) with an overall sense of meaning making taking place both individually and socially. Students participate in culturally/mathematically significant activities that are directed by the teacher who represents the culturally accepted views of the mathematics community at large. Through this participation with the teacher and with each other, students become mathematically acculturated where they learn to participate in the mathematical discourse. Cobb, Wood, and Yackel (1993) observed that as students participate in mathematical discourse, they not only talk about mathematics, but they “talk about talking about mathematics” (p. 102). The difference lies in the object of conversation. When talking about mathematics, the object of talk is the mathematical object itself. When talking about talking about mathematics, the object of talk includes the explanations and justifications being offered by the class. They conducted a teaching experiment over the course of a year in a second-grade classroom to develop instructional activities that aligned with constructivist learning. This experiment was different from traditional mathematics instruction in that it focused on social interactions and
communication and not on individual seatwork. During the course of their study, they realized that the mathematical activities and the development of what truly counted as a problem and a solution followed normative processes. Through multiple interactions and discussions, students and teacher together established certain expectations and conventions in the ways of speaking about mathematical topics and problems. The features of the inquiry mathematics of explaining, justifying, and collaborating became topics of conversation, and through that reflection on practice, students and teacher were able to constitute the norms of the classroom. When the teacher chose not to authoritatively control conversations with the correct answers and move the discussion on, the participants in the classroom were expected to give answers and justifications for their thinking. In addition, making cooperation and understanding others a topic of discussion, helped students to realize the expectations and conventions for participation.

The researchers found that both the social norms developed by the classroom as a community and the mathematical practices involved influenced the students’ mathematical learning.

The norms that Cobb, Wood, and Yackel (1993) described in the previous study are referred to as sociomathematical norms. Yackel and Cobb (1996) collaborated with second- and third-grade teachers to help them change their teaching practices, in addition to developing ways to account for students’ development of beliefs and values in the mathematics setting. The teachers’ instruction consisted of whole-class teacher-led discussions of posing problems, small-group work, and follow-up whole-class discussions of the solutions that were developed during the group work. They describe the process of establishing three sociomathematical norms (a) mathematical difference,
(b) mathematical sophistication, and (c) acceptable mathematical explanation and justification. They note that mathematical difference and sophistication are established by the taken-as-shared sense of appropriateness of the discussion. These are developed through their interactions with the teacher and negotiated through discussion. For example, when a teacher prompted students for “different ways” of performing a mathematical procedure, the students implicitly learned about mathematical difference when the various ways were presented to the class. Mathematical sophistication was subtly established in that the teacher did not explicitly evaluate explanations as being better than others; rather students inferred their value based upon the teacher’s reactions to them. This occurred when students paid attention to the language that accompanied the teacher’s reactions and responses. Although the teacher refrained from directly evaluating the students’ responses, the students became adept at evaluating the teacher’s reactions and learned variations in mathematical sophistication. The norm of acceptable mathematical explanations was established by participation in the process of contributing to the activity. Yackel and Cobb consider the notion of sociomathematical norms to be important because it establishes a way of analyzing mathematical activity in the individual microcultures that are established in individual classrooms.

Since the development of social norms in the classroom is context dependent and specific, it is difficult to study the phenomenon on a scale any larger than a few classrooms or the classes of a few teachers. Given this, many studies that examine the role of discourse and the development of normative behaviors in the mathematical context tend to be case studies, vignettes, and thick descriptions of the phenomenon in action (Cobb, 1995; Folkson, 1996; Knuth, 2001; Seeger, 1998; Silver & Smith, 1996;
Voigt, 1995; Williams & Baxter, 1996; Yackel, 1995). Among the many studies, that examined discourse and norms, Cobb, Boufi, McClain, and Whitenack (1997) studied reflective discourse, which is characterized by repeated shifts in the discourse so that their actions become the object of discussion. Instead of the discourse that is built around talking about mathematics, reflective discourse is “talking about talking about mathematics.” From their study, an example problem required the children to discuss the ways in which they could see five monkeys playing in two trees. After the students came up with various ways, the conversation shifted, with input from the teacher, to verifying that the possibilities they considered were the only ones. The results of the previous activity then became the object of the next activity. The researchers saw the teacher’s role in this type of discourse as critical so that the activity did not become a guessing game, and that the conversation did make the shift to operating on what was previously done. They are cautious in suggesting that participating in reflective discourse does not determine mathematical development, rather it enables it and constrains it.

Consequent research from the researchers that studied and developed the notion of sociomathematical norms focused on the further development and analysis of sociomathematical norms at various grade levels (McClain & Cobb, 2001; Yackel, Rasmussen, & King, 2001), and extended the study of discourse to include argumentation. Wood (1999) explored the establishment of the context that facilitates argumentation. In a second-grade classroom, she considered the context to be both a macro and micro-concept, for the social setting and for the argument itself, respectively. The analysis included the organization of the teacher’s lessons, the patterns of interactions, and further turn-by-turn analysis of the communication that occurred in
particular lessons. Wood found that the teacher established her expectations of participation in discussions from the first day of the school year. The students were expected to participate in examining, critiquing, and validating their knowledge through the discourse. She proposes that the creation of a context for argumentation is done when the teacher understands the relationship between the social processes and conceptual development. This study among others about collective argumentation (Krummheuer, 1995, 1998; Yackel, 2002) highlight the interwoven nature of the social processes of the classroom, the teacher’s role in establishing norms, and the conceptual understanding of mathematics.

The research done by Cobb and his colleagues and that done by others that is consistent with interactionist perspectives has bridged micro- and macro-analyses into an interpretive framework by which classroom activity and learning can be examined. Cobb (2000) explains that their position relies upon the reflexive relationship between the psychological and social perspectives in that both perspectives coexist and constitute each other.

…normative activities of the classroom community (social perspective) emerge and are continually regenerated by the teacher and student as they interpret and respond to each other’s actions (psychological perspective). Conversely, the teachers and students’ interpretations and actions in the classroom (psychological perspective) do not exist except as acts of participation in communal classroom practices (Cobb, 2000, p. 64).

This view established by Cobb and his colleagues shows the importance of taking into account both the individual/psychological and the communal/social perspectives because
both simultaneously comprise the classroom activities. This notion of reflexivity is taken from ethnomethodology (Mehan & Wood, 1975) and used in a way that describes how the social and psychological processes are derived from the context of the situation, but also help to create that situation.

While not a study in mathematics education, van Zee and Minstrell (1997) have studied the notion of reflective discourse in science education, namely physics classes. They define reflective discourse as vigorous interactions between student and teacher, where the teacher avoids evaluating the correctness of the student responses and focuses on student thinking. Both conceptions of reflective discourse (Cobb, Boufi, McClain, & Whitenack, 1997; van Zee & Minstrell, 1997) involve students making their thinking explicit and the object of conversation. As opposed to thinking of reflective discourse as a communication activity, van Zee and Minstrell (1997) conceived of it as a general way of fostering communication in the classroom. The main characteristics of reflective discourse are: (a) invoking metaphors for teaching and learning, (b) following students’ lead in thinking, (c) and structuring discussions to foster and monitor changes in student conceptions. Invoking metaphors for teaching and learning is done by constructing identities for students as people who are knowledgeable, and expanding contexts of understanding to a shared understanding. Following students’ lead in thinking is done by: (a) responding to student responses with restatements in neutral ways, (b) using reflective questioning that give responsibility for thinking onto the students, (c) acknowledging and encouraging students as conversational partners, and (d) invoking silence to foster student thinking. Structuring discussions to foster and monitor changes in student conceptions is done by: (a) soliciting students’ initial conceptions, (b) guiding the discussion, and (c)
engaging students in monitoring changes in their conceptions. These are communication activities that help to establish the importance of communicating and understanding the thinking and conceptions of both teacher and students.

This discussion of the focus in mathematics education on communication and discourse provides a glimpse of the value the field places on how teachers and students talk (communicate) in the classroom. It is not difficult to see that each mathematics classroom develops in its own way where norms, expectations, and conventions are culturally nurtured with the direction of the mathematics teacher. This research gives us ways to talk about learning in terms of the communication and other “cultural” activities that occur in the classroom. What is needed to supplement this research is to further investigate the communicative elements of the discourse and to discover the cultural and accepted communicative aspects of mathematics teaching and learning. Some of this needed research is discussed later in this chapter, while some of it has yet to be conducted.

The next section in this literature review focuses on communication theories that are relevant to this study. The communication perspective for this study is based in the following communication theories by which the data analysis was informed.

Communication Studies

The body of knowledge from communication studies is helpful in that it provides a perspective on mathematical communication different from that of discourse in a community. Communication theory offers several viewpoints from which to address communication, and specifically the production of messages/explanations to others in order to convince or justify thinking. This section contains theories from Communication
Studies as well as various approaches to explanations. I discuss constructivist communication theory and related research, as it provides a framework for communication and social interaction. Stemming from constructivism, message design logics give insight into the various ways in which people approach the same communication task. After constructivism and design logics have supplied a backdrop for this discussion, I discuss explanatory discourse, and the different types of approaches to explanations.

*Constructivist Communication Theory*

A theory of communication that aligns with sociocultural contexts is constructivist communication theory as forwarded by Delia, B. O’Keefe, and D. O’Keefe (1982). From the constructivist perspective, people are interpretive beings that act according to what they perceive of a situation. Communication is seen as coordinated action, which is organized through strategies through reciprocal recognition of communicative intent. Therefore, when people enter into communication activities, the act of communicating is forwarding each other's intentions, and then aligning them to achieve a communicative goal. In the constructivist view, through people's experiences they develop cognitive constructs that they use to group events and experiences. This action is guided by interpretive schemes that are context-relevant intentions and beliefs, and actualized through strategies, which are the ways in which people choose to act based on their intentions.

Delia et al. (1982) make a distinction between two types of interpretive schemes. The first is general or abstract interpretive schemes that are relevant to people throughout an instance of interaction. These are basic social rules that people use to connect their
present activity with what they already know. The second type, organizing scheme, is more specific and is relevant for very specific situations. This is social knowledge of particular acts such as, being in a meeting, question and answer situations, and standard procedures for daily rituals. Since communication is embedded within action, human interaction is seen as a coordination of lines of action, or of interpretive schemes because the schemes provide people with the alternatives for action. Interpretive and organizing schemes that people share with each other serve as resources for the coordination of activities (Delia, et al., 1982).

The constructivist perspective holds that effective and adaptable communication occurs not only when one possesses behavioral and linguistic skill, but also the social-cognitive skills, including interpersonal constructs, to be able to formulate different messages that are adapted to different listeners (Applegate & Delia, 1980). People develop increasingly complex and organized interpersonal constructs through social experience over time. Yet there is not a standard by which all people organize their cognitive constructs because of the variation in people’s social experiences. People differ in their organization and complexity of constructs; some have highly differentiated, abstract, and integrated systems, while others have sparse, concrete, and globally organized construct systems (Applegate, Burke, Burleson, Delia, & Kline, 1985). This in turn affects the ways in which people perceive and understand the communication and action of others. Within this view of communication, it is important to realize that effective communication is not achieved solely on the basis of taking the audience into consideration when creating messages. Interpersonal constructs play a role in determining
communication-relevant differences of the audience, acquiring a collection of message strategies, and goal integration (O’Keefe & Delia, 1988).

Coordinating action or co-ordination (Clark, 1996) is done through co-ordination devices, which are assumed commonality of thought, agreements, or precedents. In any given situation there can be numerous co-ordination devices available for people who wish to coordinate their actions. Therefore, these people will, according to the Principle of Joint Salience, co-ordinate based upon the most salient co-ordination device available to them with respect to their common ground (Clark, 1996). The mutual beliefs and knowledge about a situation serve as common ground for people. Usually members of communities use conventions, which are partly arbitrary regularities in behavior, as co-ordination devices in regularly occurring coordination problems.

From the constructivist view of communication, one aspect of social perception that is thought to influence effective communication is perspective-taking. Adapting communication to the listener’s role, action, character, intention, emotional state, or knowledge shows the ability of the speaker to take another’s perspective (O’Keefe & Delia, 1985). It should not be assumed that if a person takes another’s perspective, then the person’s perceptions affect the adaptation of messages to others. The relationship between perception and behavior is more complex, but perspective-taking is one process that can influence communication. This ability is socially demonstrated in the construction of different uses of language or codes. Elaborated codes or speech is that which expresses everything necessary for the listener to understand without having background knowledge, whereas restricted codes or speech is based upon common assumptions and expectations which eliminates the need to verbalize meanings.
(Applegate & Delia, 1980). Similar to elaborated codes, person-centered codes take into account the listener’s feelings and beliefs, and focuses on the individual’s unique qualities. Related to restricted codes, position-centered codes appeal to the situational obligations, assumptions, and rights of the participants.

Constructivist communication theory is considered to be a social-cognition approach to analyzing communication. Since the theory brings to light the social practices, behaviors, goals, intents, and strategies that people use in order to communicate and interpret the communication of others. From this perspective, messages do not exist in a vacuum where they are created for one sole purpose. They are created by the messenger, who takes into consideration the perspectives and cognitive abilities of the listener in order to tailor receiver-focused messages that result in outcomes that are desired by the messenger.

Since constructivist communication theory focuses on the individual making sense of the world within the confines of social practice, the work of Philipsen (1997) is relevant to this discussion of communication in that he takes a culture-specific view of communication and the ways in which people communicate given their place in their community and society. Philipsen describes culture as a code, which consists of a system of elements such as symbols, meanings, premises, and rules. Culture is not necessarily related to ethnicity; as evidenced in two of the social groups that he studied that do not represent ethnic cultures, rather micro-cultures, clusters, or neighborhoods that consist of people with certain commonalities, goals, and lives. The communication that occurs within the culture can be thought of as speech code, which he defines as a socially constructed system governing communicative conduct. This work is largely influenced
by the work of Hymes (1962) who suggested that communication is inherently culturally specific, and to study communication is to discover and describe the communication practices in the particular settings, thus establishing an ethnography of speaking.

Based upon his findings from observing specific groups of people/cultures and the communication that occurs in those groups, Philipsen (1997) proposes several characteristics of speech codes that describe the communication in specific cultures or groups of people. First, speech codes are *distinctive*, meaning that where a distinct culture exists there is a distinct speech code. Communication can be done in different ways depending on the culture, and different cultures have distinct conceptions of communication. Second, the *substance* of speech codes consists of the psychology, sociology, and rhetoric of the distinct culture. Implicit in a group’s communication are the rules and norms that make the group exist and distinguish it from others. Third, speech codes contain the *meaning* of speaking, which refers to the ways in which cultures view communicative acts. This is how members of the culture interpret the significance of speaking through vocabulary use, rituals, and meta-communicative labels. Fourth, there is a *site* of speech codes, which refers to the rules, terms, and premises that are evident in communicative behavior. These constitute the patterns, rituals, myths, and social dramas that exist in speaking. Finally, speech codes have *discursive force*, which is to say that using the speech codes provides the ability to predict, explain, and control discourse about intelligibility, prudence, and morality of communicative behaviors (Philipsen, 1997). Discursive force refers to the ability to communicate with skill, effectiveness, and cultural control. *Code systematicity* is a feature of discursive force that refers to interrelated nature of the culture, symbols, and properties of language use. Codes
are *socially legitimated*, which means that codes are learned in interactions with others and sustained within those interactions. *Code elements* refer to the artfully skilled ways in which members use codes to influence or control others through communication behaviors.

**Message Variation**

Based upon the constructivist approach to communication, much research has been done on message production and analysis. This section briefly summarizes some of the studies that have been done within the constructivist framework. From this perspective, a coding system by Clark and Delia (1976) shows the general types of messages that children produce when making requests of someone who is disinclined to make the request. They define four message strategies: (a) simple request; (b) elaborated requests, where the needs of the persuader are stressed; (c) counterarguing, where the objections of the persuadee are anticipated and refuted; and (d) advantage to other, where the advantages of compliance are stressed. They, among other constructivist researchers (Delia, Kline, & Burleson, 1979; Applegate, 1982), posit that in both children and adults the four strategies form a hierarchy of development where the progression from simple request to advantage to other shows an increase in sophistication in the ability to resolve the needs of both participants, to being able to refute the validity of the persuadee’s needs.

Important in communication is the negotiation and achievement of goals in particular situations, where people design and use messages as a means of accomplishing these goals (O’Keefe & Delia, 1985). Research can focus on the contextual meanings of messages, as well as the *job* the message is doing. Many constructivist researchers
examine the latter. A hierarchic message classification system in Applegate and Delia (1980) and further detailed in O’Keefe and Delia (1985) accounts for the message variation in trying to achieve multiple goals in regulative communication situations in which mothers were asked to respond to hypothetical examples in which they were required to modify their child’s behavior. Nine individual categories reflect the differences in communication with regard to the goals that are meant to be accomplished. Messages focused on modifying the child’s behavior consist of physical punishment, commands, and rule giving. Messages that attempt to gain the child’s compliance consist of offering reasons for rules, discussing consequences of noncompliance, and discussing general principles behind appropriate behaviors. The most complex messages were those in which the mother attempts the multiple aims of modifying behavior, offering reasons for compliance, and encouraging empathy in the child’s social conduct. These messages consist of describing feelings, helping the child to make an empathic response through analogy, and leading the child to reason through the situation (O’Keefe & Delia, 1985).

In the same study, the mothers responded to interpersonal situations in which a separate hierarchy of strategies is reflected. These messages reveal the degrees to which the mothers dealt with interpersonal conflicts such as hurt feelings their reasons and consequences, and the child’s ability to understand and empathize in conflict situations.

The constructivist approach to communication, as alignment of action, places emphasis on the social cognitive aspect of communication. The implicit and subconscious nature of employing interpretive schemes lends itself to the idea that together with a person’s cognitive complexity of schemes lies an implicit theory of communication in general. How one goes about the actual process of communication, production of
messages, is addressed by message design logics (O’Keefe, 1988). This theory was developed through the analysis of written communication in response to a communication task. People were asked to construct messages aimed at a subordinate who had not completed work that was assigned. The messages were categorized by goal and by reasoning in their design. Accordingly, O’Keefe generated three categories that progress in communication sophistication within which people design messages expressive, conventional, and rhetorical.

Several features characterize the expressive design logic. The expressive communicator does not construct messages that are sophisticated and thoughtful. They are mostly reactive, honest, and truthful statements that should be taken as such. They believe there is no hidden meaning; there is simply self-expression. The expressive communicator does not take into consideration the position, feelings or emotions of the others involved. The use of language is primarily to construe the thoughts and feelings of the expressive communicator without giving attention to the context. With the expressive communicator, conversations are usually structured as reactions to immediately prior events.

The next level in the design logics is the conventional design logic. The conventional communicator sees communication as a game played by certain socially accepted rules to secure a desired response. In this logic, the focus moves away from the speaker and towards a conception of what is deemed appropriate according to societal rules or norms. The conventional communicator sees the context of the situation and communicates within that context according to the role- and communication-based rights and obligations of the participants. Since there is such an appeal to the rules and norms of
a situation, effective communication can only occur when all communicators involved follow and adhere to those rules.

The highest level in the design logics is the rhetorical design logic. With the rhetorical design logic there is a belief that, “communication is the creation and negotiation of social selves and situations” (O’Keefe, 1988, p. 87). The function of communication is to negotiate social consensus. The rhetorical communicator is able to see that the communication process creates context, and is very flexible in communicating. Since the rhetorical communicator is flexible and adept in communicating, he/she can specifically tailor communication to the situation and to the others involved so that they can make sense of the utterance. The rhetorical communicator can anticipate interactional problems and avoid them by using communication to redefine the situation.

The constructivist perspective and approach to communication provides an avenue with which communication processes can supplement the current ways of thinking and talking about the communication that occurs in the mathematics classroom. This is something that has yet to be done in the research on mathematical communication with this or any other theory of communication. This view of communication places emphasis on individuals trying to make sense of the world using their interpretive cognitive structures that were constructed from social interactions. But more important than taking audience and communication task into account is the way in which the communicator reasons about communication (O’Keefe & Delia, 1988).
Language and Communication in Mathematics

When one thinks of mathematical activity, what usually comes to mind is the learner doing mathematics in isolation absent from any kind of social activity. This view of mathematical learning associated with Piagetian constructivism places the responsibility of learning on the individual making sense of the world by solving problems. Even in the mathematics classroom, which is inherently a social setting, emphasis is still placed on the individual learner. Here the role of the teacher is to listen, construct models of students’ ways of thinking, and to build activities based upon those models. To Piaget, the teacher guides students to help them form their own ideas about mathematical relations and properties (Sierpinska, 1998). From this perspective communication is seen as a means of transmitting one’s thoughts mediated by language, but not knowledge, since knowledge cannot be verbally transmitted to others. The point of communication is for the teacher to glimpse into the thinking of the learner provided that the learner is mature enough to communicate such thinking.

Seeing the important nature of social interaction in education as well as the applicability of Vygotsky’s sociocultural theory, one can focus on both the conception of the individual learner and the social and cultural aspects that influence the learner. From the sociocultural perspective, learning mathematics is a process of enculturation or apprenticeship (Cobb, Jaworski, & Presmeg, 1996) where the learner engages the teacher and others in meaningful social practices that expose the learner to the ways of thinking about and doing mathematics that are consistent with those from the mathematics community at large. This may transpire with the teacher, as the representative of the mathematics community, presenting the learners with a definition of a concept and
having the learners explore relationships, structures, applications, examples, and non-examples (Sierpinska, 1998). In this view, the written symbols and spoken language of mathematics are tools in the enculturation process and expose the learner to the ways in which mathematics historically evolved into the present-day mathematics.

Taking both conceptions of the individual learner and the sociocultural learner to somewhere in the middle, the interactionist perspective emphasizes the specific context in which learning is taking place. This view emphasizes the interaction between learner and culture, where language is seen as social practice (Sierpinska, 1998). Communication is seen to help students understand the language and practice of mathematics. The mathematical meanings are imbedded in the language use of that particular setting where knowledge emerges from the shared experiences of the teacher and learners that develop within the particular classroom culture (Sierpinska, 1998). In the interactionist view, the teacher does not possess the knowledge to be learned, thus the role of the teacher is to provide the mathematical discourse in which the learners participate and use to develop their meanings.

Regardless of the different perspectives of mathematical learning, the language of mathematics exists in various forms. Pirie (1998) classified six means by which students and teachers communicate mathematics to each other. Ordinary language consists of people’s everyday vocabulary, which can at times impede understanding when the everyday meanings do not fully capture the essence of the mathematical concept. The move from ordinary to mathematical verbal language can be seen in the concept of subtraction where “take away” is a common everyday understanding of it, but does not fully capture subtraction of negative numbers for example. Within mathematics the
symbolic language is another type that is not merely the direct translation of the words into symbols, although the symbols do represent the meanings of the verbal. Pirie offers the example of “subtract two from three” where if directly translated into symbols would be “– 2 3.” Of course that is written incorrectly and we see this represented as “3 – 2” (Pirie, 1998, p. 12). Visual representation is not a language, but it is a way of communicating mathematics to others. Unspoken but shared assumptions also do not constitute a language, but do act as a way of communicating. This is evident in situations where students have participated in social interactions in which they have developed shared meanings and understandings of concepts, and where someone outside of the particular culture would not be able to make sense of the communication. Finally, quasi-mathematical language is used when there is no available mathematical language to the student, but the language that is used has mathematical significance. This categorization of the language or lexicon of mathematics gives an overview to the types of language considerations with which students and teachers of mathematics encounter.

In a sense, the line between a means of communication and a mathematical representation become blurry when exploring the functions of communication and representations. Seeger (1998) brings the notions of practicing both communication and representation together in describing three types of learning with regard to language and symbol mimetic, discursive, and theoretic. Mimetic learning is based upon imitation of the process and syntax of language and symbol. Discursive learning is based upon speech and narrative. Theoretic learning is based upon externalization of knowledge. The progression of learning can be seen from the simple imitation of others in communicating and representing, to the ability to externally represent mathematical knowledge either
through communicating or representing. Seeger does not present these as stages, but they show different levels of engagement with representation and communication with students. The types of mathematical learning forwarded by Seeger, the various perspectives on learning from Sierpinska and Cobb et al., and the categories of mathematical communication developed by Pirie illustrate a picture of mathematical learning in which language and communication play an important role in the mathematics classroom.

The focus on classroom discourse has become increasingly common in the research as evidenced by the studies done by many researchers (e.g. Cobb et al., etc.). While this has addressed the NCTM communication standard, the current research has yet to deal with the issues of student communicative competency not to mention competency in communicating through writing (see next section). Much of the social norms and discourse-themed research explores the ways in which classroom communication enhances or enables mathematical learning. This research describes specific classroom discussions and interactions where the communication has played a role in the normative activities of the classroom. The research on sociomathematical norms and reflective discourse make progress towards the discovery of effective mathematical communication. These norms become facets of the classroom communication and may occur without the students completely aware of it taking place. But this type of communication addresses the fact that there is a modicum of quality control when it comes to mathematical communication that is acceptable or not to the class as a whole (i.e. what counts as an acceptable explanation or justification). In this normative practice, the students and teacher collectively decide on the appropriateness of
the mathematical communication. Reflective discourse in which students engage in conversations where their previous activity is the subject of the conversation is an activity that monitors classroom communication. Research that addresses these concerns provides the mathematics education community with guidelines and suggestions for teaching that is consistent with the reform mathematics efforts and making classroom communication an important aspect of class activity. An even further examination must be done that explores the elements of the communication that occurs in these types of discussions and exchanges.

Though the current research has explored student mathematical communication and how that social activity promotes learning, which does address the NCTM Communication Standard, there is little to no attention being paid to written communication. There is a substantial body of research in writing, which will be discussed later, that surveys the types of activities, classroom practices, and the benefits of using writing; yet there are very few studies that examine the elements of student writing for the purpose of communicating knowledge or competence to a reader. The focus has been on how writing is used to enable or assist the learning of mathematics through the various types of writing assignments and activities. To use the notion of quality control again, the current research does not address the elements of effective quality writing in mathematics.

Writing in Mathematics Instruction

Why Writing?

Writing in mathematics is implicitly addressed in the NCTM Communication Standard because writing is a form of communication that sometimes gets overlooked in
favor of oral communication; however, the *Writing to Learn* movement in the ‘80s brought prominence to writing in the mathematics curriculum. Writing is important to all school subjects, but is rarely seen in traditional mathematics classrooms. Throughout the school curriculum writing and composition classes appear at every level. Obviously writing is a very important skill for all students in the American school system to have with a certain amount of proficiency. Yet in mathematics, which has its own lexicon, structure, and ways of performance, attention to writing is disregarded in favor of procedural performance. Even Vygotsky saw the value of writing in the learning process. He describes written speech compared to oral speech as more complex, deliberate, and based upon an arbitrary sign system (Sierpinska, 1998). Vygotsky (1962, p. 99) says that writing requires “deliberate analytical action” in which much more conscious analysis and thought must go into translating inner speech into writing so that it is understandable, as opposed to the translation from inner speech to oral speech which is a subconscious process.

Writing in mathematics learning, as will be shown later, can be used in a multitude of ways. The most important aspect of writing is that it is a reflective process, regardless of the mode of implementation of writing. The entire structure of writing is based upon synthesizing ideas and reproducing them using the symbol system that we know as language. By using writing activities, students use one more step in their thinking, even if it is for a tiny moment of time. The mere act of writing forces a person to think before acting, as opposed to freely expressing thoughts. This reflection can allow people to engage in their own learning and develop deeper understanding of salient relationships and concepts.
Writing in mathematics classes is mainly used in two capacities, journal writing and expository writing. Bell and Bell (1985) use expository writing in conjunction with writing that accompanies problem solving, whereas Miller and England (1989) use the terminology in the context of responding to writing prompts. The distinction between journal writing and expository writing is not always made explicitly in the literature; rather it is a separation that I make that accounts for the ways in which each is used in terms of its communicative goal. In journal writing students usually reflect on some activity or respond to a prompt given by the teacher in order to solidify their thinking on some topic or concept. Usually the goal of journal writing is to reflect upon one’s own thoughts and not to communicate understanding to another person. With expository writing, students use writing as an active part of the learning process with teacher-provided in-class writing activities or prompts. The goal in expository writing is to communicate understanding to another person, whether it is the teacher or another student. Writing activities serve students as a representation other than the usual symbolic and graphical representations in their mathematical learning. Writing is communicating and students communicate their understanding to their teachers and to others through various writing activities.

In a study that compared student problem-solving success through the verbalization and written expression of the solving process Pugalee (2004) explored the value of writing activities in mathematical problem solving. The verbal and written problem-solving processes of 20 ninth grade algebra students were analyzed in this study. Each student solved six open-ended problems of varying degrees of difficulty (low, medium, high), both verbally and written. The verbal solvers were videotaped and their
responses transcribed in order to compare them with the written solvers’ work. The emergent categories from the qualitative analysis corresponded to an established metacognitive framework (Garofalo & Lester, 1985) that describes the problem-solving process in four activities, orientation, organization, execution, and verification (Pugalee, 2004). The analysis also determined student strategies and error patterns. The strategy of guess and check was the most used by the students, whereas working backwards was used the least. There were no significant differences in strategy use by the two groups. For error types, the majority was procedural (66.2%), followed by computational (23%), and algebraic (10.8%). The students that wrote their solving process had significantly less procedural errors compared to the verbalizing students, in addition to significantly producing a greater number of correct answers. Students that wrote used significantly more orientation and execution statements than the verbalizing students. The researcher concluded that the results support the notion that writing can support a metacognitive framework and that the process of writing is more effective than the verbalization of thinking processes.

Using Writing in the Classroom

It is possible for expository writing assignments to be interchanged with journal writing assignments, as the difference between the two can be quite subtle. As mentioned earlier, I claim that the main difference between expository writing and journal writing is the way in which the writing task is used for communication purposes. Journal writing is reflective in nature, where the student reflects upon an event, activity, or concept in attempt to solidify thinking. Expository writing is used more in an explanatory capacity to convey thinking or understanding to another person (teacher or student). The goal in an
expository writing assignment is to express knowledge or thinking whether it is to demonstrate knowledge to another person or to make thinking clear. In practice, journal-writing assignments could be read by another student or teacher, which would not necessarily qualify the assignment to be expository. A journal-writing assignment for students has them write as if someone else may not read it in that they write for themselves, and not to convey thoughts to others as an expository assignment would.

In the literature what is usually referred to as transactional writing (Birken, 1989; Powell & Lopez, 1989; Rose, 1989) can be thought of as expository writing assignments in which the writing is meant to be read by someone other than the writer, usually a teacher. Expository writing allows students to catch mistakes in addition to remembering and understanding problems better (Cai, Jakabcsin & Lane, 1996; Drake & Amspaugh, 1994). When students write they provide themselves with a concrete record of their thinking and mathematical processes which gives them a chance to check their work. Writing encourages students to think about their thinking, which adds to their understanding by supplying them with a means to reevaluate their processes. Powell and Lopez (1989) describe learning as a dynamic process where students move between experiences and reflections. Somewhere between the experiences and the reflections, students critically reflect. Writing is an activity that promotes this circular idea of learning. The writing process itself is a constant automatic reflective process on experiences where writers check their thinking in order to communicate it on paper, requiring a detachment from the actual situation (Vygotsky, 1962, 1977).

This section reviews some of the common uses of writing in the mathematics classroom taken from various research journals, teaching journals, and books. From a
middle school practitioner’s perspective, McGehe (1991) suggests using word webs, which are basically concept maps where students or the teacher brainstorm over a key word or concept. She suggests activities such as writing problems that would have a given answer, and writing a story problem to a given picture. Guided-response sheets are templates used in problem solving that outline the process for students such as: (a) understanding the problem, (b) devise a plan, (c) carry out the plan, and (d) look back. Similarly, Zupancic and Ishii (2002) used a heuristic devise with middle school students for problem solving called ODEAR, where students Organize, Define, Explore, Answer, and Reflect when solving problems. More advice from a practitioner, Burns (1995) suggests using writing journals or logs, writing solutions to problems, writing essays, and writing about learning. She also gives strategies on using writing that include talking with students about the purpose of writing, establishing the teacher as the audience, using student writing in instruction, and revise and editing activities. Along with these activities, House (1996) offers other creative activities that incorporate writing including, writing newspapers or newsletters, advertisement campaigns, poems, limericks, parodies, and songs using mathematical topics and concepts.

A structured approach to writing that combines it with verbal communication is suggested by Huinker and Laughlin (1996) with their think-talk-write strategy. In this instructional approach, the teacher initiates the activity with a concept or thought after which the students silently ponder in a reflective dialogue with themselves. Next, students discuss the concept with other members of their group for a set amount of time. Then finally, the students write about the topic based upon their own thoughts and their group discussion. Similar in structure but on an individual basis, Masingila and Prus-
Wisniowska (1996) utilize performance tasks where students both demonstrate and communicate their understanding. A performance task is structured so that it may require students to perform a task or problem, and then ask questions about their results and sub-goals along the way.

From the seminal text *Writing to Learn Mathematics and Science* (Connolly & Vilardi, 1989), several authors give anecdotes and suggestions for using writing activities in mathematics instruction. In a freshman mathematics course for the non-science major, Berlinghoff (1989) offers students problem-solving strategies such as arguing by analogy and looking for patterns when students are confronted with problems to solve. He has students write about a famous mathematician, mathematical event, or historical period. Kenyon (1989) offers writing as problem solving where he structures long-term writing techniques around stages consisting of reflection, clarification, first draft, peer inquiry, and revision. Some of his short-term writing techniques consist of explain how or why, comparing two concepts, outlining chapters, replacing two-column proofs with prose, and writing test questions.

Birken (1989) uses four types of writing assignments with her college students. In-class informal assignments have students write a brief paragraph after solving a problem. Homework problems that involve students interpreting or analyzing a concept are reflective activities. Essay questions can be put on tests and finals. Formal technical reports require students to be precise and logical in writing about a problem or concept. Gopen and Smith (1989) also suggest having students write reports that force students to think about mechanical elements of mathematics to which they would not normally pay
attention. They use them in computer-based calculus classes where students work on labs relating to the calculus concepts and write-up lab reports.

Rose (1989) offers several writing activities that fall within the categories of transactional or expressive writing. Transactional assignments require students to write in order for it to be read. These activities can be writing summaries, questions, explanations, definitions, reports, word problems, term papers, projects, essays, books, notetaking, and dialogues. Expressive writing activities require students to personally explore their thinking processes and make them explicit. These activities can take the form of free writing, journal writing, letters, admit slips, and autobiographies. Some of these activities are suggested by Frankenstein and Powell (1989), in addition to learning logs and multiple-entry logs that allow students to reflect on their learning over a period of time. Brandau (1990) has students write mathematics autobiographies in order to discover and examine their views of mathematics. Among the multitude of ways writing can be used, Keith (1989) recommends some of the previously mentioned activities in addition to: (a) formal definition or theorem writing, (b) generalizing a concept, (c) translating a visual image into words, (d) writing of an algorithm, (e) group collaborations, and (f) inventing problems.

Given all of the different types of writing activities and writing prompts, Miller and England (1989) classify writing prompts into four general categories. Contextual prompts are affective and subject-oriented. Instructional prompts are either teacher or student focused that ask students to write about their mathematical instruction. Reflective prompts promote either analysis or clarification by referencing activities or concepts familiar to the students. Miscellaneous prompts are free writing and just for fun prompts.
All of the preceding writing activities show that there are many ways in which teachers may use writing in mathematics teaching. The activities range from short informal activities that require very little writing to more involved projects and papers.

**Research on Journal Writing**

Journal writing in mathematics classes is mainly used in a reflective manner to process events or learning that has already taken place. The use of journals has many potential benefits in the classroom to both teacher and student. Borasi and Rose (1989) analyzed the benefits of journal writing and mathematics instruction in a college-level algebra course. The journals were used for three reasons: (a) for students to express and reflect upon their learning, (b) to provide the teacher with input, and (c) to establish a dialogue between the student and teacher. This study structured the algebra course in such a way that writing was an integral part of the course, but retained the usual goals and content of the course without interference to the teaching style of the instructor. The students kept journals or personal log notebooks in which they reflected on material in the class, recorded thoughts and feelings, or responded to open-ended assignments. They identified three categories of benefits of journal writing: benefits as the students write their journals, as the teacher reads the students’ journals, and as students and teacher dialogue in the journals.

Several benefits were discovered as students wrote journals. There was a therapeutic effect on the feelings and attitudes of the students, as the journals served as an outlet for the students to express themselves. There was an increased learning of mathematical content, as the students had to create meaning in order to write. There was an improvement in learning and problem-solving skills, as students’ writing helped
clarify both the process and content of problems. Finally, journal writing provides a potential for students to achieve a more appropriate view of mathematics by reflecting on their beliefs of mathematics as a discipline (Borasi & Rose, 1989). They found three benefits as the teacher reads the journals. First, the teacher can better evaluate individuals by being aware of individual needs and by having a record of individual development and progress. Second, the teacher can respond to feedback on the course and make improvements accordingly. And third, there is potential for long-term instructional improvements in that teachers are made more aware of common problems in the course and can personalize instruction in each class. Two benefits as students and teachers dialogue in the journals were discovered. One is that more individualized teaching can be achieved when the teacher directly responds to issues raised in the journals by the students. A more supportive classroom atmosphere can be created when both students and teacher build mutual respect through individual dialogue.

A twist on the traditional journal writing activities is found in multiple-entry logs. These logs blend journal writing and reflection with expository writing prompts. Powell and Ramnauth (1992) describe multiple-entry logs as a prompt for students to reflect on, and a medium for them to record multiple and layered versions of their reflections and images. Physically, multiple-entry logs are pieces of paper that have been divided into three columns where the first column contains the text in which the student responds. The text can be a writing prompt, a mathematics problem, or definition. Powell and Ramnauth used multiple-entry logs in a personal tutoring/office hours situation in which the teacher responded to the student’s work and provided comments to the student. This reflective process can be beneficial in that they give the student an opportunity to express thoughts
and feelings, and to make connections across the text, classroom material, and individual help from the teacher.

Another study on the use of journal writing done with middle and high school students suggests that there may be a progression over time of students’ writing with respect to their mathematical understanding. In this study Clarke, Waywood, and Stephens (1993) explored the implications of regular completion of student journals in mathematics. They wanted to identify indicators of any developmental process associated with student journal writing and document student beliefs regarding mathematics, mathematical activity, and the role of journals in mathematical learning. A questionnaire was administered to all students in the seventh through the twelfth grade and a random sample of 150, 25 at each grade level, was collected for statistical analysis. Teacher interviews as well as journal samples were collected as data. Three categories of journal entry types were identified: recount, summary, and dialogue. The categories served as a description of students’ perceptions of their learning of mathematics and as a progression in student mathematical activity. They suggested that the three categories represent a hierarchical process that students’ journal writing goes through from recount being the simplest to dialogue being the most complex. They note that the progression through the various types of writing also reflects the students’ perceptions of their mathematical learning where they first see knowledge as something to describe (Recount), as a collection of discrete ideas (Summary), and as creating their own knowledge (Dialogue).

Moving away from a codification of journal entry types and toward examining various effects of journal writing on procedural knowledge, conceptual understanding, problem solving, mathematics school achievement, mathematical communication, and
attitudes toward mathematics. Jurdak and Zein (1998) systematically identified the differential effect of gender, language of instruction, level of writing, and level of mathematics achievement, and examined the students’ evaluation of their own writing experiences. They used a mathematics evaluation test to measure conceptual understanding, procedural understanding, problem solving, and mathematical communication. Four sixth-grade classes (two taught in English, two in French) of 26 students each (42 girls, 62 boys) participated. The journal-writing group was given a specific question or prompt for the last 10 minutes of class while the non-journal writing group was given textbook exercises. Attitude was measured by a Likert questionnaire. Five open-ended journal questions were used to assess the journal writing experience of the treatment group. A multivariate analysis of covariance (MANCOVA) was conducted on gender, language program (English or French), level of writing achievement, and level of mathematics achievement which showed no effect on the relationship between journal writing and students’ attitudes towards the importance of mathematics and problem solving. The journal-writing group showed significantly higher levels of conceptual understanding, procedural knowledge and mathematical communication. Problem solving, attitudes toward mathematics, and school mathematics achievement were not significantly different between the groups. They concluded that students in the journal-writing group had a positive response to the journal-writing process.

Research on Expository Writing

Where journal writing assignments usually require students to reflect on concepts and practices, expository writing activities are done actively and usually as part of instruction. Problem-solving activities may require students to make use of expository
writing assignments, and Bell and Bell (1985) made the connection between the processes of writing and problem solving. They noticed that the writing process and the mathematical problem-solving process are basically the same. The writing process consists of discovering a topic, deciding what one needs to say about a topic, organizing and structuring content, writing the draft, and revising and editing. The problem-solving process mirrors the writing process in that it consists of defining the unknown, determining what information one already knows, designing a strategy or plan for solving the problem, reaching a conclusion and checking the results. They studied two ninth-grade general math classes separated into a control and an experimental group. Both classes were given the same assignments, examples, quizzes and tests for four weeks. The experimental group had a different method of teaching, which was based on teaching problem-solving skills by combining traditional mathematics techniques with a structured expository writing component. Their results support the claim that expository writing could be of value in mathematics class. In addition, they found two benefits of using this approach, namely, providing communication between student and teacher, and providing an opportunity for the teacher to give individual feedback to students to clear up incorrect processes. In general they found that communication in the classroom between teacher and student improved.

The improvement in communication between teacher and students is of a great benefit not only to the students, but also to the teacher. Miller (1992) studied the use of writing in mathematics classes from the teacher’s perspective. She examined teacher benefits from using impromptu writing prompts in high school algebra classes. Miller wanted to ascertain two things, what teachers could learn about their students’
understanding of school mathematics from reading their responses to in-class, impromptu writing prompts; and if instructional practices would be influenced as a result of reading students’ responses to in-class, impromptu writing prompts. The data consisting of both student and teacher writing samples suggested that in-class, impromptu writing prompts could produce a resource for informing teachers about students’ understanding by providing communication between teacher and student that would not normally exist. She found that by using in-class writing prompts, the teachers could assess their students’ understanding of concepts, procedures and, “memorizing facts without understanding the mathematical underpinnings” (Miller, 1992, p. 334). She also concluded that when teachers read students’ writings, their instructional practices were influenced in the following ways: (a) re-teaching immediately, (b) delaying exams due to lack of understanding, (c) providing review sessions, (d) initiating private discussions, and (e) using writing prompts at differing times during class instead of only using them at the beginning of class.

Writing can also be used in cooperative learning settings, as Johanning (2000) examined expository writing with seventh and eighth graders’ outcomes of writing and postwriting group collaboration in pre-algebra. She sought to address three things, the ways in which students approach a problem when they have to write and explain a solution rather than work computationally, the ways in which writing helps students understand and solve a problem, and the ways in which the process of reviewing and discussing each other’s work affect learning. Two classes of gifted seventh graders and advanced eighth graders were prompted to write in order to solve a problem. The students solved the problem and justified their solutions by writing. Then they discussed their
solutions in collaborative groups on the next day. While this was going on, the teacher visited each group to share with them approaches from other groups. Of the 48 students, seven participated in data collection through interviews. The data were collected in four ways: (a) participant observations, (b) interviews, (c) audiotapes of group collaboration, and (d) written solutions. The students used a variety of strategies to solve the problem such as drawing pictures and diagrams, making tables or graphs, and setting up equations. The most common strategy was guess and check. They showed a strong awareness of their reasons for particular steps. They were able to demonstrate and explain their reasoning when presenting solutions, and in interviews, students indicated that writing helped them understand a problem better as well as in different ways. The students indicated three main benefits from writing: finding mistakes, remembering the problem, and understanding the problem better. By seeing various approaches to the same problem, the group collaboration experience broadened their scope of mathematical knowledge. She also found that students were more willing to share their ideas because they had written them down first. Writing before group work allows students to create their own understanding before they collaborate with other students as well as the teacher. When students share their written solutions, they enhance their understanding and develop a pool of information and experiences to use in the future.

Porter and Masingila (2000) applied Writing To Learn Mathematics (WTLM) activities to a freshman college-level calculus class to compare with a non-writing class. They sought to answer two questions, “Does writing to learn mathematics improve students’ conceptual understanding, and does writing to learn mathematics affect students’ ability to perform routine skills and procedures” (p. 167). They determined that
the two classes in question were comparable by administering the MAA Calculus Readiness Test. The instruction for the writing class consisted of writing activities both in-class and outside of class. The same types of activities were given to the non-writing group, but did not include writing as a key element. Rather the activities required the students to be able to discuss the concepts and procedures. The data consisted of the students’ three exams and final exam. These were analyzed by error analysis and the errors were categorized as procedural or conceptual. A MANOVA comparison of the groups showed no statistically significant difference between the groups except on the second exam. They concluded that the writing activities per se may not be beneficial to the students, but that the benefits lie within the fact that these activities engage students and force them to think enough about the mathematics to be able to communicate it to others.

*Summarizing Writing*

As we have seen with the first section on writing that surveyed the various activities that teachers have at their disposal, writing in mathematics classroom instruction looks different to different people. There are distinctions in the types of writing prompts that exist (Miller & England, 1989; Rose 1989; Masingla & Prus-Wisniowska, 1996), distinctions in problem-solving methods that use writing (Berlinghoff, 1989; McGhe, 1991; Zupancic & Ishii, 2002), and distinctions in individual versus group activities (Keith, 1989; House, 1996; Huinker & Laughlin, 1996). Yet, two themes are present throughout the various articles and activities. First, students benefit from writing activities because it gives them a space to reflect upon their learning whether it is through reflective prompts, or by gathering and solidifying their thoughts in
order to explain it to others. Second, teachers benefit from using writing activities because they are given the opportunities to engage with students in their thinking and it opens channels of communication that would not normally be present.

From the research studies on both journal writing and expository writing, we see the very same benefits discussed by the researchers (Borasi & Rose, 1989; Miller, 1992; Meel, 1999; Porter & Masingila, 2000; Powell & Lopez, 1989). Journal writing, learning logs, and multiple-entry logs promote a reflective nature to the learning process that encourages students to be agents for their own learning. Giving students a physical record of their thoughts is a powerful tool in learning. Expository writing activities focus students’ attention on producing writing that is coherent and meaningful so that another person can understand their thinking. Again the act of writing out their thinking or explaining in words to another person introduces an added element/space in the learning process that is usually not present, yet provides the context for the solidifying of students understanding.

From the learning perspective, most of the studies that examine the role of writing in mathematics conclude that the writing activities did assist in students’ conceptual development in one way or another. Only Jurdak and Zein (1999) and Porter and Masingila (2000) offer inconclusive evidence that writing positively affects achievement, although they do mention other benefits that are seen as valuable from a teaching perspective, such as better attitudes towards mathematics and better conceptual understanding. The literature offers evidence of increased classroom communication as well as increased ability in problem-solving situations, but it is difficult to say with certainty that writing can increase anything more than discourse, attitudes, and skills.
Shepard (1993) concurs by saying that researching writing is difficult because of, “…the ubiquitous nature of its proposed effects on learning” (p. 287). It seems as though there is convergence in the literature that writing can positively influence many things in the context of classroom learning, and at the very least it does not hurt the process.

It is possible that many of the studies do not systematically approach writing because it is difficult to control in an experimental setting. Different writing activities require students to construct different types of messages and it is not clear from the literature if specific types of writing activities have the potential to influence learning or not. Since the use of writing strategies is consistent with reform mathematics ideas, any teacher or researcher that utilizes these is obviously familiar with other reform teaching techniques. This combination of teaching strategies is difficult to isolate in any teaching situation. For instance, a teacher who uses writing activities is concerned to some degree with the mathematical communication in the classroom, and it is unlikely that a teacher would use writing activities in isolation without using some other kind of communication enhancing teaching strategies, or having concern for building problem-solving skills, using technology, or any other nontraditional teaching techniques.

Explanations

Before the formal discussion of mathematical explanations, which is the point of this section, a discussion of explanations in general sets the tone to better understand explanations in mathematical communication. Within the many purposes of communication, argument and explanation seem to blur into the same function. They may even be thought of in terms of each other, in which giving an argument is providing an explanation, and vice versa. At some point a distinction must be made as to which is
which. Antaki (1994) says that, superficial explanations have nothing to do with leading from one statement to another, whereas arguments establish a set of statements that lead to a conclusion. Arguments contain a certain momentum towards a conclusion that explanations do not; however, Scriven distinguishes between the two by saying that, “an explanation can never give conclusive support to a statement” (as cited in Antaki, 1994, p. 141). Explanations can serve many purposes outside of drawing conclusiveness. Explanations can define, resolve puzzlement, elucidate, paraphrase, make clear, fill in detail, supply stages, reclassify, and interpret. Antaki makes the claim that those who treat argument and explanation as claim-backing activities realize that they are distinct, but not easily separable.

If we accept Antaki’s proposition to treat argument and explanation synonymously, then we open ourselves to the many functions that explanations can serve outside of the conclusion-drawing purpose of arguments. Littlejohn (2002) posits that there are two types of explanations, causal and practical. Causal explanations preserve the “argument” idea in explanations where they connect events as causal relationships toward an outcome. Practical explanations elucidate actions as goal related in which action is designed to achieve a future state. Causal explanations are provided in relation to antecedent and consequential events, and practical explanations are provided to show action as controllable and chosen.

Skill in providing explanations is important in educational situations where explanation as teaching is necessary. Rowan (1999) offers three types of explanations that promote understanding of a subject matter. The first type, elucidating explanations, help in understanding the meaning and use of a term and clarify distinctions in meaning.
Good elucidating explanations may come in the form of a definition and provide a range of varied examples and non-examples. The second type of explanation is the quasi-scientific explanation, which helps to envision complex structures or processes, usually scientific or scholarly representations. Good quasi-scientific explanations highlight main points, key structures, or critical connections by using signaling devices, figurative language, or graphic aids. The third type of explanation is the transformative explanation, which helps to understand difficult or implausible ideas. Good transformative explanations, “discuss the limitations of the implicit theory, while proposing the more accepted one, and demonstrating its greater adequacy” (p. 326).

When these concepts and theories from communication studies are brought into the realm of mathematics teaching and learning, they have the potential to open new avenues of inquiry by allowing us to examine the communication side of mathematical communication instead of the mathematical side that has previously been done. Several concepts from communication research such as coordination, common ground, types of explanations, receiver-adapted messages, and many others have yet to be incorporated into the processes of teaching, learning, and communicating mathematics. In turn, this is an avenue that is worth exploration by the mathematics education community because in the teaching and learning of mathematics, one message does not fit all. If this were the case, there would be no need for teachers and students to read all they wanted about mathematics from textbooks. As research has shown, people do not learn mathematics that way. Therefore teachers of mathematics must be increasingly aware of the means, functions, and types of practices of communication that are available to them. In addition to awareness of communication potential in the classroom, there is an expectation that
students and teachers alike are able to effectively communicate mathematical concepts to each other. By current conceptions of communication, this involves much more than producing a message and having the receiver decipher it; as expressed earlier understanding communication or achieving intersubjectivity requires work beyond that of deciphering code.

Written Mathematical Explanations

In this section, I will review a few studies from the last relevant area of research that is taken from studies of written communication in the classroom. These studies are closely related to the present study as they collected student expository writing samples and classified and organized the elements of the writing in order to categorize mathematical writing. The studies reviewed here can be thought of as points of departure for the current study.

An early study that classified mathematical writing in order to determine the elements of the writing itself was done by Pearce and Davison (1988), in which they determined the amount, kinds, and uses of writing that teachers use in junior high school mathematics classes. By looking at student samples and teacher interviews, they classified five types of writing activities used by the teachers as: (a) direct use of language, which is copying and transcribing information; (b) linguistic translation, which is translation of mathematical symbols into words; (c) summarizing/interpreting, which is summarizing, paraphrasing, and making personal notations about material from texts or other sources; (d) applied use of language, which is a situation where a mathematical idea was applied to a problem context; and finally (e) creative use of language, which is using
written language to explore and convey mathematically related language. They found that the direct use of language activities were the ones that teachers used most frequently.

Similar to Pearce and Davison’s classification efforts, Silver, Leung, and Cai (1995) compared Japanese and American fourth-grade students’ written explanations. The students wrote out their explanations of a task in which they had to derive multiple solutions. They examined the responses for correctness, evidence of strategy use, and mode of explanation. For correctness, they coded data as either at-least-partially correct or completely correct. For solution strategy to the particular problem, they identified enumeration, grouping, or restructuring. Their categories for modes of explanations were, visual, verbal/symbolic, mixed, neither, or inconsistent. They found that there was practically no difference between the students from both countries, which they note that in other comparative studies, Japanese students usually outperform American students.

In a more complex effort of generating a model of student writing, Shield and Galbraith (1998) analyzed eighth grade students’ writing and developed a coding scheme of the content of the writing. In addition to developing the coding scheme, they compared the writing samples with the type of writing that occurred in the students’ textbook. They identified six elements or features of the content of the students’ writing: (a) exemplar, (b) goal statement, (c) kernel, (d) justification, (e) link to prior knowledge, and (f) practice exercises. The most common of these was exemplar in which students gave verbal descriptions of specific examples, diagrams, conventions, and graphs. In comparison with the students’ textbook, they found that the writing samples heavily reflected the same type of writing style, which was a focus on procedures and algorithms with little elaborations and with an authoritative tone.
This last study examined two groups of college calculus student’s expository writing. Stonewater (2002) set out to determine criteria for successful and unsuccessful mathematics writers, which he calls “The mathematics writer’s checklist.” The writing task was for the students to write an essay from a prompt that addressed the topic of limits. They were to address five concepts in their essay, limit, continuity, derivative, area, and behavior. They were graded for correctness of definition and overall quality of explanation. The analysis indicated six areas that distinguish between successful and unsuccessful writers. Those six emerging themes were: (a) using appropriate mathematical language, (b) building a context for the essay, (c) including examples for elaboration, (d) utilizing multiple modes of representation in essay elaboration, (e) using mathematical notation, and (f) covering essay topics.

These studies give us some ways of classifying and explaining students’ writing in mathematics classes. The researchers recognized that the type of writing that occurs in mathematics classes is different than that of other disciplines and thus needed to be examined further to assess the elements of student’s mathematical writing. In effect, they began to discover how students communicate their knowledge to the teacher through the different types of writing activities. These studies did not, however, focus on the functions of the writing, or what job the explanations do in the writing. That issue is a communicative task that focuses on the role of the communication.

Closely related to the current study in intent and method, Kline and Ishii (2002) sought to develop a framework for thinking about explanatory communication skills in written mathematical explanations, and to examine its viability empirically with 95 remedial college-level mathematics students’ written explanations. This study utilized
theories from communication studies and mathematics education in order to build the model of explanations in that study. With college-level remedial mathematics students, the researchers collected writing samples that required the students to write a letter to a friend that would help the friend to better understand how to graph the equation of a line in two variables. The communication task was to give explanations to a peer who did not understand how, but needed to graph the equation of a line in two variables. Using theories from Communication Studies including Constructivist Communication Theory, they inductively developed a communication model that comprehensively described six communication activities evident in the written explanations. The first of the communication activities was orienteering, which is guiding the reader’s attention and stating the communication goals. The second activity was articulating relevant concepts with interpretive explanations, which includes giving definitions, templates, and outlines. The third activity was legitimizing concepts through reason-giving explanations, which focuses on using exemplars and the mathematical work in conjunction with justifications and reasoning with the exemplars. The fourth activity was guiding procedural activities with descriptive explanations, which explains the relevant process or structure with elucidating the pattern, and supplying steps of the process. The fifth communication activity was solidifying reader understanding, in which connections were made to solidify new knowledge and create connections. The final activity was facilitating linguistic control of mathematical terms, which was displaying competency with the mathematical terminology, and linking with their symbolic forms. These six communication activities were not present in every single writing sample; rather these activities represented the spectrum of use. The writing samples were coded using the model, as well as being coded
for the communication activities that were not present, called gaps. These were the same
six communication activities, but coded for the elements that were missing or that should
have been present given the particular example and style.

Consequently the Kline and Ishii (2002) study, in a sense, was a precursor to the
current study. As will be seen in the next chapters, there are similarities between the
model developed for the current study and the one in Kline and Ishii (2002). Since these
two studies use similar methodologies and the same theoretical framework from
Communication Studies, including Constructivist Communication, the models developed
in similar ways.

Summary

In this literature review, I surveyed the relevant literature in learning theories,
mathematics education, and communication studies. Sociocultural theory developed by
Vygotsky and further applied to teaching and learning by the works of Rogoff and
Wertsch provided the main perspective in general learning for this study with the
emphasis on learning in social settings and the reliance on language. Since the release of
Curriculum and Evaluation Standards (NCTM, 1989) and subsequent Principles and
Standards for School Mathematics (NCTM, 2000), communication in the mathematics
classroom has been seen as important not only to implement, but also to study. This move
on the part of NCTM, motivated by social constructivist theories, has contributed to the
philosophy of studying mathematics as personal constructions within a social context,
rather than as a concrete set of facts to be transmitted from teacher to student. From this
perspective mathematics is seen as being presented, re-presented, mediated, and enabled
through communication.
The practice of communication in the mathematics classroom is realized through both oral and written modes. In oral mathematical communication, there are those who study discourse in the context of classroom as a community of learners by which the communication aids the establishment of sociomathematical norms that enable learning to take place (Cobb, Wood, & Yackel, 1993; Yackel & Cobb, 1996). From this perspective, mathematical learning is viewed as a process of enculturation in which the metaphor of apprenticeship reflects the positions of the teacher and students in learning the ways of mathematics (Cobb, Jaworski, & Presmeg, 1996). The positions on mathematical learning forwarded by Pirie (1998), Sierpinska (1998), and Seeger (1998) focus on the role and importance of communication; the means by which teachers and students communicate mathematics, and the practice of communication and representation in learning, respectively.

In written communication, journal writing and expository writing are practices that allow for increased communication between teacher and student, and between students. In journal writing, both teachers and students can benefit by engaging in prolonged dialogue with the material, as well giving students an outlet in which to vent frustrations and confusions (Borasi & Rose, 1989; Powell, 1997). Expository writing activities are useful in problem-solving situations because it was discovered that the processes of writing and problem solving mirror each other (Bell & Bell, 1985). Some of the benefits of using expository writing include, increase in levels of understanding (Cai, Jakabcsin, & Lane, 1996), increase in student participation (Johanning, 2000), and providing feedback to the teacher that can influence instruction (Miller, 1992).
From the field of communication studies, constructivist communication theory (Delia et al., 1982) provides a framework for understanding communication situated in activities of alignment and coordination of intents and goals. The main tenet of constructivism is that people behave in situations based upon previous experiences and the way in which they interpret their current situation. A theory of message design logics (O’Keefe, 1988) facilitates the understanding of the differences in the types of messages that people create. The three design logics (expressive, conventional, and rhetorical) show a progression in sophistication of message production based upon the speaker’s implicit beliefs about the purpose of communication and how they perceive their role in it.

Explanations in communication may be seen to serve the same purposes of arguments, although arguments are usually seen as leading to a conclusion. It is easier to think of both arguments and explanations as claim-backing activities, yet the two are not easily distinguishable (Antaki, 1994). Explanations can be seen as either causal or practical (Littlejohn, 2002). In educational settings, explanations may be classified as elucidating, quasi-scientific, and transformative (Rowan, 1999).

As explained, many aspects of communication in the mathematics classroom can be studied with various approaches on how to study it. Approaches to understanding written explanations (Pearce & Davison, 1988; Shield & Galbraith, 1998; Silver, Leung, & Cai, 1995) have resulted in the production of coding systems and categories with which to identify and classify certain elements of student explanations. Most of the research on written explanations has not been done with college-level students, and has not integrated elements from communication studies with that of mathematics education.
This is the next step for studying written explanations in mathematics. Since general explanations have been studied in the field of Communication Studies, there is much to be gained by bringing that perspective to mathematics education. The beginnings of that work are evident in the model developed by Kline and Ishii (2002), which brought together elements from communication studies and mathematics education. That study provided the groundwork for the current study, which also brings together elements from the various fields. My hope is that this study will bridge the two fields, add to the literature on mathematical communication, and motivate further study from this perspective.
CHAPTER 3

METHODOLOGY

In this exploratory and descriptive study, I focused on the examination of written mathematical explanations of college students enrolled in a mathematics course for pre-service elementary teachers. There were two different approaches to the mathematics for elementary teachers course: a traditional lecture and recitation approach and a nontraditional inquiry approach. Mathematics content final exams constituted the corpus of written mathematical explanations examined in this study. In the search for effective communication practices in mathematics, the purpose of this study was twofold. First, I sought to develop a model of written communication from the students in the Math 105 course to better understand the explanations that they used in their writing. Second, I wished to use the model to make comparisons of the explanations from the different approaches of the course.

Participants

The participants in this study consisted of 145 undergraduate pre-service elementary school teachers enrolled in the first mathematics content course for the elementary education major. Of these students, 93% (n = 134) were female and 7% (n = 11) of them were male, mostly of Caucasian decent. A majority of the students, 75% (n =
were either junior- or senior-level students. There were 24 students in the
nontraditional/inquiry section, and 121 in the traditional lecture/recitation sections. The
students in the nontraditional/inquiry section were matched with one recitation section
from the traditional lecture/recitation course that met at the same time period in the
afternoon. The two sections were matched based upon previous mathematics background,
previous mathematics courses, standardized test scores, and by gender. The students in
the remaining traditional lecture/recitation sections were not randomly assigned to their
sections, since they selected the meeting time that best matched their individual
schedules.

Setting

This study took place at a large urban Midwestern university with undergraduate
students enrolled in the first of a three-course sequence that students preparing to be
elementary school teachers take to fulfill their program requirements. This course
sequence is generally known as the mathematics content courses for students who intend
to teach elementary school levels. This course is usually taught as a typical
lecture/recitation format that meets for lecture on Mondays, Wednesdays, and Fridays
with recitation meeting on Tuesdays, and Thursdays. The 10-week course was taught in
two different ways. One hundred twenty-one students were taught in what I am calling
the traditional format, which is the lecture and recitation approach. Another 24 students
were taught in a nontraditional format, which consisted of a single section that met
Monday through Friday with the same instructor using an inquiry approach to learning
the mathematical concepts. All class meetings for both types of instruction met for 48
minutes per day. In this study, I compared one section with 24 students (nontraditional) to
another section (traditional lecture/recitation) with 121 students because I wanted to compare two different approaches to teaching the same course and the traditional section was only offered in large groups.

In the fall quarter, two mathematics professors taught the two different types of the Math 105 course. One professor taught in the traditional lecture/recitation format, and the other in a small-section nontraditional/inquiry approach.

**Description of the Course**

The mathematics course, Math 105 is usually taught as a lecture and recitation format. The instructors require the students to not only be able to perform necessary calculations and procedures for the material that they may one day teach, but also be able to explain what they did. This is generally required of the students in writing on homework assignments and on exam questions. This iteration of the Math 105 course was no exception in that both sections were taught with a focus on providing explanations so that someone else could understand their work and reasoning both orally and in writing.

**Traditional – Lecture/Recitation.**

For the traditional-instruction group, there was one mathematics lecturer who coordinated the course assignments and labs, gave the lectures, and designed the exams. In addition, there were three recitation instructors (Graduate Teaching Associates) who taught two recitation sections each. All three of the recitation instructors were mathematics educators. Two of them were doctoral students in mathematics education, and the other instructor had already received his doctorate in mathematics education. The recitation instructors facilitated discussions about the homework assignments and labs that the lecturer designed to supplement the course content. The labs were mostly hands-
on activities using manipulative materials or physical objects that aided in the learning process. The students generally worked in groups of 1 to 4 people on both labs and homework assignments done in recitation classes. The course grades depended upon attendance and participation to recitation/lab sections, homework assignments, midterms, and a final exam.

The lecture format for this course had very limited interactions between the lecturer and the students in the sense that the lecturer asked very few questions and had limited engagement only with the students who sat in the rows near the front and center. During lectures, the lecturer mainly asked questions that he answered himself. In some instances the professor would ask the students questions and wait for them to answer, however, only a few students actually participated and answered his questions. These were usually the students that sat very near the front of the lecture hall. Towards the end of the lecture period when students would talk to each other and get loud or pack-up their things to go, the lecturer would talk over the noise as to not be bothered by the disturbances. In general there were very few instances where the students would actually talk and participate during the lectures. They mainly listened and quickly took notes from the overhead projector. The lecturer did allow students to ask questions for clarification, but mainly solicited questions at the beginning of the lecture period.

The purpose of the recitation meetings was mainly to give students an opportunity to work in small groups during both lab activities and homework assignments. The lecturer for the course designed the lab activities for each recitation meeting. During most weeks the students were given a lab activity each time they met. The exception to this was during exam weeks, where the recitation instructors would review the exam material
to prepare the students for the upcoming exam. The lab activities were designed to take up about half of their meeting time, approximately 20–25 minutes. The other half of the meeting time was devoted to their weekly homework assignments, which the students could work on in small groups or individually. The labs were activities that modeled mathematical relationships, algorithms, and procedures sometimes with manipulative materials. During the lab activities, the recitation instructors would circulate around the room between the groups to answer questions about the lab. Very rarely would the recitation instructors have the attention of the entire class, but in the event that they did, it was usually to address a common error or misconception, or to give advice about how to teach and explain the material to children. The recitation instructors also circulated around the room to answer questions about the homework assignments.

During the course, the students were frequently asked to provide explanations to their work and express their thinking. On many of the homework assignments from the textbook, the students were asked to explain how they would describe concepts to children or to give an example of how they would go about trying to alleviate a misconception about a particular mathematics concept. Some common homework, quiz, and exam questions were phrased as, “Explain your reasoning”, “Justify your conclusion”, “Explain in sentences…”, and “Briefly describe your procedures.” For these types of problems, students would write in sentences their reasoning, justifications, and explanation. Even in the lab activities the students were asked to provide justifications and explanations. The articulation of the students’ reasoning and explanatory ability in writing were definitely a focus. Although this approach does not appear to be traditional in nature, this was the norm for the Math 105 course because of the focus on teaching.
Therefore the students would not only need to be able to know the mathematical concepts, but also be able to teach and explain them to elementary students. Asking the students to write out their explanations was a very normal part of their instruction.

*Nontraditional – Inquiry.*

The nontraditional group that consisted of 24 students was taught by a sole mathematics instructor in an inquiry/problem solving approach with a focus on group activities. This instructor coordinated the course materials and assignments as well. Generally, Friday class meetings were taught by one of the same recitation instructors (the same one each week) from the traditional group, but done so to keep consistency with the inquiry approach for this section. Sometimes the main instructor of this class would lecture, but the general daily operations of the class consisted of worksheets or activities that the instructor developed to introduce mathematical topics and ideas without a formal declaration of the current topic and lecture on that topic. The students in the class were assigned to small groups in which they worked throughout the quarter. The groups were assigned at the beginning of the term and reshuffled at the middle of the term. These small groups consisted of four people each, and all class activities and discussions were done within those groups. The course grades depended upon class attendance and participation, homework assignments, one quiz, one midterm, and a final exam.

On days when the students would work on activities with their groups, this activity would take about 30 minutes of class time. During this time, the instructor circulated around to each group listening to their conversations, inquiring about their progress, and answering questions. The professor rarely answered questions with direct
answers; rather he replied with another question in order to direct the students’ attention to their own thinking. The remaining class time would usually consist of a whole-class discussion of the answers, the material in the problems, or the concepts at work in the activities. During these discussions the instructor would actively solicit the students’ opinions. He asked probing and open-ended questions that required explanation on the part of the students. Throughout the discussions he would ask students to improve upon each other’s statements and explanations so that the students had the practice of listening and evaluating another person’s thought process, which they will undoubtedly need to do when teaching children. The discussions concluded with the students coming to consensus on particular reasoning and explanations for the given topics, which was orchestrated by the instructor. The students in this section were always on-task, from what I observed. During the group activities, the group members monitored each other’s understanding and made sure that they did not proceed without everyone’s understanding. When they had whole-class discussions, the students eagerly offered explanations and were not forced to give answers. It seemed that they developed an understanding that their participation was vital to the class activities whether they be in groups or whole class though their willingness to participate and contribute.

This approach of the Math 105 course differed in many ways and especially in the assessment of the final grades. Compared to the traditional sections, the inquiry section had only one midterm and one quiz. The bulk of the assessment was on their weekly assignments from their course packet that was developed by the instructor. Every concept that was covered in the class was introduced with a story, game, or activity worksheet from their course packet. These scenarios modeled in some way or introduced a
mathematical concept that usually was not obvious. The scenarios required the students to think about the relationships and mathematical ideas at play. The students worked on these problems in their small groups. Each activity had accompanying open-ended questions, about the material, which served as the students’ homework assignments. The students were required to turn in the responses for all of the questions in each section. The write-ups were referred to in the class as reports, and they were collected about once a week. The questions that followed some of the activities include: “How did your problem solving strategies change from problem to problem? Why did they change? How does this inform how you will teach fractions?”, “Find a rule to express a terminating decimal as a fraction”, “Find and explain the relation between…”, and “Can you make any generalizations about…” Early in the quarter the instructor established a set of expectations for the written reports. These were explicated in the course packet as a supplement to the course material and were reinforced with discussions throughout the course.

The supplement to the course packet materials included many hints and guidelines for the students that established the expectations, philosophy, and perspective of the instructor. The supplement addressed many of the issues that made this course different from other mathematics content classes such as algebra or calculus. Among the information in the supplement, the students were given helpful checklists and suggestions for problem solving, as well as tips on how to write up mathematical solutions that include symbolic mathematical work and English sentences. This guide for writing in mathematics classes included several suggestions and guidelines including (a) clearly state the problem to be solved, (b) state the answer in a complete sentence that stands on
its own, (c) clearly state the assumptions which underlie the formulas, (d) provide a paragraph which explains how the problem will be approached, (e) label diagrams and other visual representations, (f) define variables, and (g) explain how each formula is derived (Pemantle, 2002). Also included in the supplement were excerpts from both the National Council of Teachers of Mathematics’ (NCTM) *Principles and Standards for School Mathematics* (2000) and the recommendations on the mathematical preparation of teachers by the Mathematical Association of America (MAA, 2001). The excerpts taken from these documents addressed the issues of teacher preparation expectations for the teaching and learning of school-aged students’ mathematical experiences.

**Data Collection**

The corpus of data consisted of mathematics final exams from the Math 105 course from 145 students from two different approaches of the course, observation field notes, and personal researcher’s log entries. For data sources, the final exams provided the explanations for the mathematics problems. The students were asked to provide explanations for their mathematical work and problem solutions. The observation field notes provided the information necessary to develop descriptions of the different approaches of the course. The researcher’s log served as a place for reflection upon the methods, process, and data analysis for the study itself. The methods employed in this study included elements from both qualitative and quantitative research methods.

On the first day of classes, each instructor distributed a syllabus that outlined the research that was going to be conducted throughout the quarter, as well as the content and procedures of the course. In order to characterize the differences between the two types of instruction, I conducted sporadic observations of each of the types of courses on non-
test days in order to observe the general workings of class meetings. The purpose of these observations was to document instructional practices and behaviors of the instructors and students. I made five observations for each of the nontraditional instruction section and the traditional lecture, and at least one observation of each recitation section. Observing the individual recitation sections from the traditional group was not as important to observe as the actual lecture meetings because instruction occurred in the lecture, whereas the recitation sections were strictly structured by the lecturer to maintain consistency between the sections. During these observations, I took field notes in addition to making video recordings, audio recordings, or both. These data helped in understanding how the courses were conducted and in creating the description of the two types of approaches to the course.

For the final exam, both instructors agreed to include the same six mathematics content questions on a separate page that counted towards the point total of the final exam for all students. I made the assumption that this procedure would provide bona fide final exam data because it actually counted towards the students’ grades. Since most students are concerned about their grades, I assumed that the students gave genuine effort to completing the final exam questions for this study. Only 3 of the 6 questions could be used because they specifically required the students to write explanations for their mathematical work.

A mathematics professor and a physics professor from the university developed the set of six questions. The mathematics professor was a veteran instructor for the Math 105 course sequence, but was not the lecturer for this study. He was quite familiar with the topics and expectations for the course. The content of the test consisted of questions
that addressed the following mathematical subjects: sets and patterns, percent increase, digits and place value, proportional reasoning, conceptions of subtraction, and fraction comparison (see Appendix A). Only 3 of the 6 final exam questions were used in this study because there were only three questions that clearly required that the students write out explanations to their work, thus being eligible to have the written explanations analyzed for the purposes of this study. The three questions were as follows:

**Question 1**

“How many numbers are in the set \{3, 7, 11, 15, 19, \ldots, 207\}? Show your work and provide an explanation.”

**Question 2**

“Last year you paid $100 for a plane ticket to Chicago. Since then the price of plane tickets has increased 120%. How much will you pay this year? Show your work and provide an explanation for your response.”

**Question 3**

“A recipe that serves 6 persons requires one-and-a-half quarts of milk. How many one-quart containers of milk will you have to buy if you want to make enough for 22 persons? Show your work and provide an explanation for your response.”

The first question addressed sets of numbers and cardinality of sets. The second question dealt with finding percentages and percent increase. Finally, the third question addressed the application of proportional reasoning. These questions were chosen because they asked to show the mathematical work as well as to provide explanations for the work, thereby giving the students a communication task of explaining themselves to their instructor or any reader who would grade the test. For the purposes of data analysis,
the written explanations were examined separate from the symbolic mathematical work in order to establish a relationship between mathematical success (correct or incorrect) and type of explanation used. Although it is possible to consider the symbolic mathematical work to be part of the explanation in the overall sense of mathematical communication, the goal of this study was to examine only the written responses that followed the symbolic work. The details of the coding of mathematical success and the type of explanation are discussed in the next section.

Method of Data Analysis

The process of data analysis was largely informed by the perspectives of sociocultural theory and constructivist communication (see discussions from previous chapters). The data for this study consisted of written explanations that accompanied mathematical work from students’ post-tests. I viewed this production of explanations as a cultural practice that is common in most mathematical learning settings. The students and instructors created communities of learning that can be thought of as individual cultural environments where all participants had a shared understanding of the course goals and of the appropriate ways of interacting and participating in mathematical learning. The course instructors, as representatives of the mathematics community, established the expectation for participation and communication that took place in these courses/communities. Thus the explanations provided by the students should reflect the appropriate ways of communicating within their individual environments, as well as within the broader mathematics community. From this perspective, I conducted an analysis of the data in search of the elements that reflect the duality of individual and broad mathematics communities. In a sense, I assumed that the mathematical
explanations reflected the conventional ways of communicating within the mathematics community, in addition to the conventions that were individually established within each course.

**Qualitative Analysis**

**What communicative elements are present in the mathematical explanations of pre-service elementary teachers?**

Ascertaining the communicative elements in the students’ written explanations was a main purpose of this study. In order to better understand mathematical communication a model was inductively derived from the data using principles of grounded theory (Charmaz, 2000; Glaser & Strauss, 1967) and based upon theories from both mathematics education and communication studies (as discussed in the previous chapter). Using methods of grounded theory and qualitative inquiry forces the theory generation into a reflexive process that is “contextually situated” (Charmaz, 2000, p. 523). In using the inductive procedures of grounded theory, I believe that the process of the theory development is just as useful and important as the theory itself. Therefore in the next chapter as part of the data analysis section, I include the process of generating, developing, and refining the communication model.

A subset of the data was analyzed in order to begin the inductive process of creating initial codes and descriptors. The codes and categories were derived from the data set and were compared with the rest of the data set. This generative process was conducted under the guidelines of grounded theory while being informed by concepts in the literature from sociocultural theory, mathematics education, and communication theory. Once a seemingly comprehensive coding system was established, it underwent
several iterations with the entire data set while constantly being refined to provide a clear representation of the data set.

**Validity Concerns in Qualitative Data Analysis.**

The portion of the design of this study that is qualitative in nature was in the development of the coding categories/framework according to the principles of grounded theory. As mentioned before, grounded theory is based upon strategies that generate theory rather than limit it (Charmaz, 2000). This generative process is a comparative method that constantly examines and reexamines itself with data collected from different people, individuals at different points in time, situation-to-situation, and category-to-category (Charmaz, 2000). This general practice informed my study in the way I developed the codes and theory upon which the main analysis is based.

Lather (1997) uses the same word “validity” as in quantitative methods, analogously to Lincoln and Guba’s trustworthiness, but uses it in order to “mobiliz[e] all the baggage that it carries and, us[e] it to rupture validity as a ‘regime of truth,’ to displace its historical inscription toward policing the borders between science and not-science” (p. 241). Her use of the same traditionally positivist word “validity” makes a statement that there is nothing “invalid” about qualitative inquiry while maintaining a scientific lexicon. Lather extends the idea of validity to include both methodological and contextual factors saying that “credibility, politics, and ethics equals validity” (Lather, 2002). The communication model that was established from the data in this study represents just one perspective on the analysis of the data. Other models derived in the same manner would carry the same validity/authenticity as this one does, as long as the credibility, perspective, and perspective of the researcher were clearly made.
This study like all qualitative inquiry should be examined for its trustworthiness. This term is analogous to validity and reliability in the positivist tradition. Lincoln and Guba (1985) characterize trustworthiness as credibility, transferability, dependability, and confirmability. To establish credibility, researchers usually engage in practices of prolonged engagement, persistent observation, disconfirming data, and crystallization/triangulation\(^2\) among others. Transferability refers to the applicability to other situations. This can be achieved by thick descriptions of the data. To ensure dependability, it is important that researchers make their decision-making process transparent and well documented. This is usually referred to as an audit trail. Finally, confirmability is achieved through the audit trail as well as through the constant reflexive practices of the researcher including the use of a reflective researcher journal. Applying the notion of trustworthiness to this study, it was established in ways that are consistent with the criteria by Lincoln and Guba. During my observations of the classes, I took careful field notes of the processes of the instructors and students. I visited each class several times in order to be able to provide a full description of what occurred in the classes. I crystallized (Janesick, 2000) data from various sources including observations, documents, and researcher journal/log entries. After each observation, I reflected on each experience in my researcher journal. I look to the descriptions I have provided of the courses for the reader to determine the transferability of the communication model to other similar situations. The confirmability of this study resides in the methods that were used to develop the codes of the communication model. The initial coding of the data and subsequent refining iterations followed procedures consistent with those outlined by

\(^2\) The notion of crystallization is analogous to the notion of triangulation.
Crabtree & Miller (1992) in conducting text analyses in which the text motivates and alters the codes while making further abstractions. This technique is similar to a constant comparative method (Charmaz, 2000), in which codes are compared, tested, and refined with examples and non-examples during the data analysis process in order to move from the macro-levels of the data to the micro-levels (Feagin, 1999). Although I utilized the methods of grounded theory, I did not use it as a constructivist grounded theory in the sense that there was not an interactive cycle between data collection and data analysis informing and influencing each other (Charmaz, 2000). Rather, I utilized objectivist grounded theory in which “the ‘discovered’ reality arises” (Charmaz, 2000, p. 524) as the researcher attempts the position as an outside viewer. This objectivist view of grounded theory has its roots in Glaser and Strauss’ (1967) positivist conception of grounded theory.

An important issue in educational research is the notion of confidentiality of the participants. As per the consent form that each participant signed, I guaranteed complete confidentiality of my participants. In fact, the database from which I worked did not contain students’ names; rather it identified the exams by the last six digits of the students’ identification numbers. Christians (2000) articulates how confidentiality is “the single most likely source of harm in social science inquiry” (p. 139). Lather’s definition of validity as being equal to credibility, politics, and ethics is apropos in circumstances of participant confidentiality. In this study, confidentiality was not an issue. Information gathered directly from the participants was never divulged to their instructors so that the students’ standings in the class or final grades could not have been jeopardized. Due to the nature of the research conducted in this study, none of the data or data analysis had
been shared with instructors during the quarter in which they taught the courses. Since the type of data collected was not what I would deem “highly sensitive,” confidentiality concerns were easily assuaged.

Researcher Perspective.

In this study, as in all qualitative inquiry, the perspective of the researcher is not seen as a bias that taints the data collection and data analysis process, for it is embraced and viewed as a strength that factors into the credibility of the study. I see myself as a mathematician (in the sense of someone who does math) with a background in communication studies who has a vested interest in bringing these two disciplines together to help inform each other. I see myself as an educational researcher who functions from an interpretivist framework and views reality as that which is socially constructed by those involved and influenced by culture. I see myself as a mathematics teacher who knows that all students do not learn mathematics by simply being told the concepts and that many students have a genuine fear of mathematics. This perspective is mine alone and is evident in the way in which I approached this research project.

With this background, I feel that the analysis and interpretations in this study represent only a single viewpoint from which the data could be analyzed. From a sociocultural/sociohistorical perspective, my background and previous experiences shape and influence the interpretations of my current experiences. My background and previous discourse analytic experience in Kline and Ishii (2002) and other research projects give this study strength and authenticity in that I am not an outside observer who is unchanged by the research process. Rather, I am an insider who shapes and is shaped by the data collection and analysis process. For example, when I conducted observations of the
mathematics course for this study, I could not separate myself as observer from the previous experiences I had as a Graduate Teaching Associate for the same course only a year prior. Although my purpose in this research was to “capture what is,” which is a subjective task in itself, I approached the subjective in an informed yet objective manner by objectifying that which is to be interpreted (Schwandt, 2000). My observations and subsequent descriptions were influenced by who I am as a mathematics teacher, researcher, and communicator. These personal elements made me sensitive to occurrences and phenomena that others may not have seen. Likewise, these attributes invariably led me to omit other occurrences and phenomena that others might have seen. Even by employing an objective view of the subjective, it is important to understand that the observations were conducted through my human experience, which is undoubtedly a subjective process.

Although I approached the discourse analysis of the written mathematical explanations using objectivist grounded theory methods, I do feel that my background influenced the interpretations made from the data. The goal of the data analysis was to build a model or in a sense a “big picture” of written mathematical explanations for this situation, however it was difficult as a mathematics teacher to analyze the explanations without making judgments based upon correctness. Since correctness is perceived by teachers and students to be such an integral part of mathematics, suspending judgments on correctness and focusing on the substance was a constant concern at the beginning of the analysis. Since writing samples are somewhat devoid of the person who created them, keeping an objective eye on a subjective process was easier to maintain than observing human experiences and making subsequent interpretations, inferences, and descriptions.
My point in explicitly making my assumptions and perspective known is for the reader to understand that this research was not conducted in a vacuum, in which the data collection and analysis were “pure” procedures. Rather they were interpretations, which were as objective as they could be, of an inherently subjective world.

**Quantitative Analysis**

Once the model was established using qualitative research methods, the descriptive statistics were calculated in order to provide evidence to support answers for the next two focus questions that were best supported by quantitative methods.

*What relationship (if any) exists between explanations and mathematical success?*

Since achievement (success) is important to educators, determining whether or not there was a relationship between the types of explanations the students used and their final exam success was important. Success was determined by coding each question on the final exam as either correct or not correct. Although there are varying degrees of accuracy in mathematics, in this study it was not necessary to code the data as such. The communication model consists of three main categories, as will be discussed in the next chapter. Since the data consisted of questions that were coded as either correct or not, as well as being coded by the type of explanation from the communication model, it was necessary to use a statistical test that would be able to distinguish a relationship between the two nominal (categorical) variables of success and type of explanation. The test that does this is the chi-square test for independence, which determines differences in frequencies of independent groups (Smith, 1988). In this case, the independent groups were the students who answered the question successfully, and those who did not answer it successfully. These responses were analyzed to see if there was a relationship between
success and the type of explanation used from the communication model. In order to use
the chi-square test, a contingency table of frequencies was made for each of the three
exam questions with response accuracy (success) as columns and explanation type as
rows. The chi-square test first assumes that there is no difference between those students
who were successful on the question and those who were not successful. In other words,
success on the question was unrelated to the type of explanation the student used on that
question. In addition to establishing relationships between success and explanations,
these frequencies were examined further to better understand the statistical results.

What differences exist between success and the explanations offered by each
approach to the course?

Since there were two different approaches to the Math 105 course, it was
necessary to determine the differences and similarities in success on the questions and the
types of explanations used by the students. Since there was one section of 24 students in
the nontraditional section and 121 students in the lecture/recitation, only one section of
the lecture/recitation was compared with the single nontraditional section. The same chi-
square test was performed with each of the two approach sections for all three questions.
Similar frequency contingency tables were constructed in order to conduct the chi-square
test and to make comparisons between the two sections with respect to success and
explanation used for each question. Since each section had so few students, the
frequencies for each category were low and consequent correction tests were considered
but not applied due to the nature of the contingency tables. The correction tests for chi-
square require 2 by 2 contingency tables, whereas this study has 2 by 4 contingency
tables. For this reason, comparisons between the two groups culminated in descriptions of the frequencies of the mathematical explanations that each section utilized.
CHAPTER 4
ANALYSIS OF DATA

Introduction

The process of data analysis for this study was completed in two distinct phases, a qualitative phase and a quantitative phase, in order to fully answer the focus questions that motivated this study. The first phase, which I refer to as the qualitative phase, consisted of the generation and refinement of the communication model for the students’ explanations on their final exam. This was a grounded analysis from which the model was developed. The model was developed directly and inductively from the data itself. The discovery and generation of the model from this data was very much a part of the data analysis since no existing framework was imposed on the data. Therefore, the development process of the model is explicated further in this data analysis chapter.

The second phase, which I refer to as the quantitative phase, of data analysis consisted of the various statistical calculations done with the responses coded using the developed model of communication. The use of quantitative methods in this phase was done to examine the relationship between success and type of explanations used. Frequencies and chi-square analyses were conducted in order to give further details into the communication model. Before going into the formal development and analysis of the
communication model, I include a section based on the observations that were conducted throughout the quarter.

Observation Data

This section is meant to serve more as a description than an analysis of the observation data that were collected for this study. The descriptions provide examples and vignettes of classroom situations of both the lecture/recitation sections and the nontraditional section. This section sets the stage for the discourse analysis later in this chapter. The situations described in this section were chosen because they represented typical situations in the various sections, as well as examples that were not similar to the everyday class periods. The descriptions of the observations give the reader the opportunity to see the similarities and differences of the approaches of the course.

In a lecture period of the LR (lecture/recitation) section, the lecturer was presenting the topic of Pascal’s triangle and showing how the chart can be used to solve problems such as finding the number of routes from home to school, as well as other counting problems. During this lecture period, the type of interaction was quite typical of a lecture hall experience. The students took notes while the lecturer taught the material and wrote his own notes on the overhead projector. He would periodically ask questions and pause for participation, but the interactions occurred infrequently. During the lecture some students would explain things to each other, but mostly there was very little student-to-student interaction. In this lecture period, the lecturer made many references to teaching children the concepts they were covering and how it is important to teach them that everything in mathematics has a reason. He explained why you should not go directly to the clever way, the trick, or the algorithm directly without giving the reasons
and rationale behind the algorithm. He offered advice by saying, “If I was a fourth grade teacher, I’d have a poster of Pascal’s triangle on the wall,” in order to give children access and to show its usefulness. As far as the interaction goes in this instance, the lecturer would answer his own questions during the lecture and only a few students in front were thoroughly engaged and would correct mistakes or would participate in the lecture.

In another lecture period, the lecturer was transitioning the material from fraction arithmetic to decimals. Here, he transitioned by showing that fractions and decimals are not all that different. The interactions were mainly focused on the lecturer asking questions of the first few rows where some of the students would participate and answer his questions. During this lecture period, I overheard a student next to me say, “I wish I would have learned to change decimals before today.” She was referring to changing decimals into fractions. I thought that this was an interesting comment for her to make considering that one day she will teach this material to children and that she had supposedly just learned how to convert decimals that day. It is possible that she was referring to “actually learning” meaning that she finally learned the reasoning behind the process. I noticed that students in the back rows were not as attentive as the students in the front rows, which is to be expected in a large lecture hall with over 145 people in it. Similar to other lectures, in this particular period the students seemed distracted by others that were talking in the back. But when the lecturer said something about putting a particular question on the test, all of the students paid close attention and took notes.

In the recitation sections for the LR approach, the students usually worked on lab activities. In one instance, the students worked together in groups of 2 to 4 each and 1
student actually worked alone. The recitation instructor circulated around the groups answering questions while the students were working on lab activities that pertained to the lecture material. These were worksheets that involved multiplying fractions and the students were using graph paper to represent an area model for multiplying fractions. The example they had was $\frac{1}{3} \times \frac{5}{7}$. They translated that problem into $\frac{1}{3}$ of $\frac{5}{7}$ in order to use the area model. The student-teacher interactions consisted of the instructor walking around and answering questions while the students worked. The instructor frequently made sure that the students were keeping on task. For this activity, two groups of students were constantly asking questions of the instructor. When it seemed like most of the students were finished with their activity, the instructor gained the attention of the entire class and explained the area representations of the fractions on the board in order to ensure that the students answered the lab questions correctly. When the lab activity was finished, the students moved on to working the homework problems that the lecturer assigned. During this recitation period, the instructor gained the attention of the class during the homework time to give advice on how to teach children to multiply fractions and to anticipate that the children would have questions and difficulty when they would get results where both the numerator and the denominator would become larger, yet the resulting fraction would be smaller than either of the factors. However, most of the groups did not seem to be paying attention to the instructor and they continued working on their homework.

In a separate recitation section, on the same day with the same topic, students were in groups of 2 to 4 students each in which they were working on the lab activity. In each group, it appeared that the students worked independently first, then if they
encountered problems they would consult other group members. At that point, if they could not answer the questions, they would ask the instructor. Similarly in this recitation section, the instructor would circulate between the groups while answering questions. A common topic for the instructor was clearing up confusion as to why when multiplying fractions the resulting product is actually smaller than either of the two factors being multiplied. Similar to previous sections, when the students finished with the lab activity, they would move on to homework problems and the instructor would help explain some topics on the homework. Generally, the types of interactions that occurred in the recitation sections between instructor and students were focused on answering questions, clearing up confusion and misconceptions, and giving information. Interactions between students were probably more explanatory in nature, while they would help each other answer questions and give each other advice or hints.

In the NT (nontraditional) section, the class was run quite differently than the LR sections. Since the students always met with the same instructor every day, they worked on the same activity over several class periods. On one class session, the topic was truth tables in which the instructor had the attention of the entire class only at the very beginning of the class period. However, the instructor was taking cues from the students and following their thinking in the discussions. He would say, “Whatever you think is best,” to students and would try not to offer his own opinions and information. Rather he would follow the students’ lines of thought and reword their explanations for the entire class to comprehend. The teacher was also involved in making connections between the logical arguments made by the students. He made an effort to solicit student opinions and not give his own immediately. After a few minutes, the group work began and the
students worked in pre-assigned groups where they worked on a question that was on divisibility and conditions of divisibility. Here the instructor would circulate around to the groups, and he would say to them, “Tell me about an answer that you had, and then changed and why.” He did this so that the students could give more information to him than just answers. They would go through explanations involving their confusion and why they answered something one way, then after talking it over with group members they would end up with a different conclusion or answer. Then after about 15 minutes of work, the instructor gained the attention of the entire class for discussion, and he wanted to make public the questions and answers that they had previously changed. Students would offer explanations and the instructor would prompt the students for more. The instructor served as a moderator for the discussion while directing the flow, but the students were the ones that offered most of the explanations and did most of the talking. The instructor made certain that he asked someone from each group to give an answer. He would also say things such as, “How can you prove this?” Finally, at the very end of class, the students asked for the instructor’s explanation to the problem.

In another observation of this class, the class worked just about the entire time on their worksheets and group activities and only right in the beginning and the end of class did the instructor have the attention of everyone. During this class the instructor circulated around to all the groups and spent lots of time with each individual group. He made sure that everybody in the group could explain the topic, which was working in bases other than base 10.

In another class period that was very atypical of the nontraditional section, the instructor started the class as “pick on somebody day.” In all fairness, the students were
warned about this the day before and it was not intended to be a mean-spirited move by
the instructor. This class period proceeded entirely as a discussion between the students
as a whole and the instructor. The students never worked in groups that day, as they had
worked on a group activity before, which was on the associative and commutative
properties of addition and multiplication. This class period began with the instructor
randomly picking someone to explain their answers using the different properties on the
example of $53+43$. A student went to the board, wrote her answers and stayed there while
the instructor talked through the example and the work with the class, but only using her
example. After working with that example, the instructor wanted to choose another
student to take that example and improve upon it. He let that idea rest with the class
before choosing another student. This next student gave an answer and the instructor
followed her thinking and explanations while rewording her explanations for the class. It
should be noted that during this class period all of the students appeared to be engaged in
the discussion. They all seemed to be thinking and trying to figure out different ways of
explaining and different ways of writing the work. Towards the end of the class, the
instructor asked if the students could come to a consensus on what they were trying to
answer. As a collective they were to come up with a definitive answer that would be
acceptable to everyone. At this point, the instructor asked if someone would go to the
board who did the problem a different way. A student volunteered, went to the board, and
put up her work to explain it. Her work differed from the first because she used a
horizontal approach to the addition rather than a vertical approach such as the previous
student’s work. It seemed like a completely different way for some students and it caused
unrest among other students who wanted to know if that algorithm worked with a
different example. The instructor then used a different example of $59+92$ to see if the algorithm would work in this instance. It is interesting to see in this example that the ones digit added to more than 9, which would carry over into the tens place. This troubled the horizontal algorithm process that the student offered. The instructor asked about getting a consensus for an explanation to the new problem and the students talked and offered explanations while the instructor wrote the accepted explanations and reasons line by line with the mathematics work. During this he would ask to, “Agree or disagree,” so that he would only write up on the board what was acceptable to the entire class. The instructor used words such as, “example, explanation, proof, justification,” and wanted to categorize the work on the board using those words. Towards the end of the class, they came up with an acceptable example and it was clear that the discussion was not over and needed to be carried over to the next class period.

The Analysis of Explanations

The data consisted of student responses to three questions each from 145 students’ final exams from the first quarter of a three-course sequence of the mathematics for elementary teachers course. Of these 145 students, 24 were in an inquiry section and the remaining 121 were in traditional lecture-recitation sections. The three questions addressed the mathematical topics of sets, percent increase, and proportional reasoning. The questions asked students to solve the problem and to provide an explanation for their work (see Appendix A).
Coding of the Data

Mathematical Success

This level of coding was based on the student’s success for each problem. If the student’s mathematical work was correct, it was scored as a “1” or success. If the student’s work was flawed or not completely correct, it was scored as a “0” or not-success. The purpose of this system was to code the questions based purely on their mathematical work and their mathematical thinking without the consideration of the explanation that they offered, if any. Some students provided only symbolic mathematical work, and these were scored accordingly, however, other students only provided written statements that contained symbolic work and an answer to the question. In these instances, the correctness of the response was considered success or not-success, and did not take into consideration how the answer was communicated in the written explanation. This coding was done to determine the number of students who answered each question correctly or incorrectly. The frequencies of success are discussed in a later section in conjunction with the statistical calculations that were conducted with this information.

The first question reads, “How many numbers are in the set \{3, 7, 11, 15, 19, \ldots, 207\}? Show your work and provide an explanation.” The correct solution to this problem is 52. By adding 1 to each number in the set and dividing each by 4, the set consists of the numbers 1 through 52, with 52 being the cardinality of that particular set.

The second question reads, “Last year you paid $100 for a plane ticket to Chicago. Since then the price of plane tickets has increased 120%. How much will you pay this year?” The correct solution to this problem is $220. An increase of 120% is
$120, which is then added to the previous price of $100. This results in a new price of $220.

The third question reads, “A recipe that serves 6 persons requires one-and-a-half quarts of milk. How many one-quart containers of milk will you have to buy if you want to make enough for 22 persons?” The correct solution to this problem is 6 quarts. This problem can be solved several different ways. One strategy is to use a proportion. The proportion is 6 persons: 1.5 quarts = 22 persons: x quarts. By solving the proportion for x, the result is 5.5 quarts of milk, however the question asks for the number of one-quart containers. Thus the solution is 6 quarts.

Qualitative Phase: Development of the Communication Codes

My approach to the grounded analysis of the data was to derive a model of communication activities from the data itself without imposing any previously established scheme or model onto the data. With a grounded analysis (Charmaz, 2000), the model would be completely representative and specific to this particular set of data. Without an a priori set of conditions and categories with which to analyze the data, I was free to let the data “speak for itself,” without imposing any restrictions upon the data. However, my particular grounded analysis was not devoid of a perspective, which I bring to the analysis. The data were filtered through my perceptions and the lens with which I examined it. This perspective was comprised of the theories from communication studies and mathematics education that were discussed in previous chapters. Those theories (sociocultural theory, explanatory discourse, etc.) can be thought of as my perspective in the grounded analysis of these data. The theories can be thought of as the knowledge base with which I approached the data.
The Process/Iteration

The first step in the process of data analysis and formation of a model of communication was to examine each question separately. I took the first question, read through each student’s answer, and took notes on their written responses. My notes consisted primarily of a list of communication elements and features of the written responses that I thought were being exhibited by each student along with whatever came to mind that I thought was occurring in the data. At first I took note of each sentence that comprised the total response, however many of the responses were short in nature with no more than three sentences long. In many cases, there were not complete or proper sentences to delineate, so I changed my approach to taking more of a holistic approach to the responses. I looked for the main idea or focus that was “trying to be construed” (i.e. the utterance). At times the utterances were one sentence long and at other times, they were several sentences long. I continued to take notes in this way for each of the three questions, generating a different list for each question. I did notice some similarities across the three questions, yet continued to focus on narrowing and refining the list for each individual question.

From the first iteration I noticed that across examples of the same question, student explanations tended to follow a pattern that matched the strategy they used to solve the problem. In other words, the students who chose to solve the problem a certain way had explanations that were very similar to other students who solved the problem the same way. For instance, the students who solved the proportional reasoning question (Question 3) by using a proportion and then solving mostly had similar explanations. An
example was, “…set up quarts/people = quarts/people” (707642, NT section). Students who proceeded with this method explained their work similarly. On the other hand, students who tried to solve the same problem by reasoning through the proportion and scaling had very similar explanations. An example of this was, “…used repeated addition. In the recipe you use 1.5 quarts of milk for every 6 people, so for every 6 people I added 1.5” (762630, LR section). This seemed obvious to me that similar ways of thinking about a problem would be followed by similar explanations. There were exceptions to this, but for the most part similar solution strategies were followed by similar explanations. In any case, it was an occurrence that I noted several times in that the pattern continued across all three questions.

Another facet of the analysis that struck me early in the process was that I was noticing similar types of explanations to those that we discovered in the Kline and Ishii (2002) study. Although those explanations/ narratives were longer, some of the same elements were showing up in the responses from this study such as: (a) providing reason-giving explanations, (b) guiding procedural activity with a descriptive explanation, (c) displaying symbolic exemplars, (d) verbal description, and (e) solidifying conceptual understanding by using involving phrasing (see literature review for details of Kline & Ishii (2002)). A comparison of the results from this study and those from Kline and Ishii (2002) are discussed in the next chapter. Continuing to write out what I considered were the communication elements that I was seeing, I maintained different lists for each of the three questions.

3 Denotes the data identification number and whether that student was in the lecture/recitation (LR) section or the nontraditional (NT) section.
Once the list for each question was completed, I checked the codes for redundancies. Inevitably there were redundancies and they were either examined for differences then rephrased, or they were collapsed or eliminated completely. Some codes did not represent communication codes, but were descriptive of the solution process or the thought process. Examples of these codes were the following, “reasoning with %”, “explaining nuances of the answer”, “reasoning with proportions”, and “explaining what % increase means.” Instead of reflecting communication strategies or activities, those codes reflected solution strategy, and consequently were eliminated. At this point, the list of codes for each question was manageable enough to assign letters (a through u) to each one and to proceed with the coding of each response. Therefore, each student response for each of the three questions was coded according to the list developed for each individual question.

For this round of coding, most student responses received a single code or two, while others received no more than four. I proceeded to code the entire set of the responses for the first question, and after completing the coding for that question, I found that many of the same combinations of codes were appearing. I looked back through the set of responses for the first question, and my intuitions were correct. There were in fact several combinations that occurred frequently. Some of the combinations were:

“explain meaning of symbols” and “give directives”
“provide background features, definitions” and “supply conditions for use”
“describe steps of process with reasons” and “provide rationale”
“explain meaning of symbols” and “provide background features, definitions”
“provide background features, definitions” and “describe steps of process with reasons”
“explain connections, or logical progression” and “list facts, details”.

I believed that the combinations were occurring for a couple of reasons. Either, the combination codes were coding the same thing, or they were coding something that was closely related. It was at this point that I began to think that there might possibly be a few overarching themes or broad categories to which the codes could belong. With this in mind, I proceeded to code the remaining two questions the same way I had coded the first question, by coding and identifying everything that I saw. Still, most responses were coded with no more than four codes with most of them receiving one or two.

With the notion of broader themes or categories in mind, I continued through another round of coding, but this time I tried to think of the responses as holistically as possible, while trying to think about “what the explanation is doing for this answer,” or “what is being accomplished by this explanation?” However, some cases could not be represented with only one code and required more. I called the multi-coded responses “combos”, on which this round of coding focused. For this iteration I focused on the responses that were combo coded in order to better understand the features that made them different than the other responses. From there I made another list that consisted of the combo codes. I examined the responses each case at a time to better understand the different combinations that appeared in the list. In many of the combo cases, the multiple codes were related or very similar, resulting in future collapsing and combining of the codes. After doing this for all three questions, most of the combo codes were collapsed or were similar enough to be related in some way. The process of further examining and collapsing the combo codes led to the realization that the codes bunched together in a few small groups. This occurred in all three questions, and I made comparisons across the
questions to find the same to be true. I had determined that there were three broad explanatory categories to which all of the codes could belong. I felt that these three broad categories with their respective codes that further detailed the aspects of categories was the model of communication for the written mathematical explanations for this particular set of data. From the procedures and perspectives outlined earlier, I felt that this was the model that would describe the written explanations. This analysis was just one interpretation of the data, however, depending upon the perspective and research goals, any number of different models could have been developed from these data.

The Codes and Categories

Upon realizing that there were three broad categories within the coding scheme, I decided to name them. The first category consisted of codes that related to the direct translation of the symbolic mathematical work or that seemed to be very algorithmic in nature. In these instances, the responses contained little detail, yet provided the necessary steps of the process and explained what certain symbols meant as well as answering the question. This category was named Algorithmic Explanation. The following shows the detailed codes for the category of Algorithmic Explanations, along with examples of each.

Description of symbols or work.

“…I counted by fours starting at 3 and made a tally mark for each number in the set and ended with 207. Then I counted the tallys (sic).” (983996, LR section)

Explain the meaning of symbols or formulas.

“$100 (last years price)+$120 (increased price) = 220 (this years price)” (783372, LR section)
Provide steps of the process.

“If 6 ppl requires 1 ½ then using the repeated addition method we add 1 ½ + 1 ½ +1 ½ +1 ½ which gives a total of 6 qts milk” (868879, LR section)

Give directives.

“With the set provided you have to keep adding 4 to each number until you get to 207 so for instance…and so on until you get to 207” (982508, LR section)

Provide general or nonspecific descriptions.

“We can set the problem up as a ratio of 120% of 100 is ? We then use means/extremes to solve.” (024806, LR section)

Alert reader to the conclusion/answer.

“So you would pay $220 this year” (720320, LR section)

The second overall category consisted of codes that provided additional detail for the mathematical work that gave more insight and interpretations into the concepts at work behind the solution strategy. These responses provided extra information about the process of finding a solution, as well as giving kernels of information (key information) that was necessary for solving this particular problem or ones that are similar to it. This category was named Structural Explanation. The following shows the detailed codes for Structural Explanations, along with examples of each.

Provide background knowledge, definitions, details, facets, or facts.

“For every 6 people you need 1½ quarts.” (866008, LR section)

Supply conditions for use.

“When you write all the elements of the set and line them up in a 1-1 correspondence with the counting chant you end on 51.” (884968, LR section)
Description of kernel ideas.

“If a price is increased by x%, you multiply the old price by the percentage increase and then add this number to the old price. This gives you the new price…” (685628, LR section)

Provide current knowledge, definitions, details, facets, or facts.

“…there are 4 #s before 19 in the set…there are 3#s before 15 in the set…there are 51#s before 207 in the set.” (37604, LR section)

Responses that offered explicit insight into the thought process belonged to the third overall category of Transformative Explanations. These responses provided glimpses into the logic and reasons behind the decisions and solution steps of the process. Explanations in this category give a logical chain of steps together with the reasons for action, or give a series of statements and conclusions that logically follow each other. The following shows the detailed codes for Transformative Explanations, along with examples of each.

Description of the thought process and steps of action.

“1½ quarts milk serves a six people recipe. If you divided the recipe to make it serve 2 people, you only need ¾ quart milk, but you want to serve 22 people so you take 2 and divide it into 22, getting 11. You need 11x2 person serving, which is 11x3/4 giving you 8¼ quart. Buy 9.” (946525, LR section)

Description of process with reasons given for action.

“First, I will multiply $100 by 1.20 to see what 120% of $100 is. Next I will add that answer onto the original $100 to give me the total price I will pay this year. I am adding it on because the price has increased.” (803128, LR section)
“Before each pattern repeats, there are 5 #s in each pattern. The pattern occurs 10 times and there are two number left over.” (889657, LR section)

Once these codes were established as belonging to a broader explanatory category, I put the data through another iteration of coding which was done mainly as a check to the communication model. I approached this iteration as if the communication model had existed all along and was attempting to impose that system on these data. The necessary checks and corrections were made and at that point the qualitative analysis for the study was complete. Recall, that the purpose of the qualitative analysis was to establish a communication model that would fairly and accurately describe the explanations given by the students for this course final exam. Entering the coded data into the computer provided the transition from the qualitative analysis to the quantitative analysis so that the necessary statistical tests could be conducted.

**Summary of the Communication Model**

The communication model developed from the qualitative analysis consisted of three overarching explanatory categories. Each category consisted of detailed codes that help to describe the variations within the broader explanatory categories. Table 4.1 shows the details of the three main explanatory categories together with the codes that describe each category. The specific codes belong to the broader categories of Algorithmic, Structural, and Transformative explanations. Each written explanation from the students’ final exams belonged to one of the three major communication categories. In many cases students used more than one type of explanation or communication activity and were coded with as many as the student used. In cases where the multiple codes belonged to
the same explanatory category, they were coded with that single category. In cases where there were codes belonging to two different categories, then the question was coded for the highest level that appeared. Thus this coding was individually evaluated and coded for the highest level of explanation that occurred with *algorithmic* being the lowest level and *transformative* being the highest.

<table>
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<tr>
<th>Explanatory Category</th>
<th>Description of Communication Activity</th>
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| Algorithmic          | Describe or directly translating work into words  
|                      | Explain meaning of mathematical symbols or formulas  
|                      | Provide steps of solution process  
|                      | Give directions or prescriptive action  
|                      | Provide general or nonspecific descriptions of solution  
|                      | Alert reader to conclusion or answer |
| Structural           | Provide background knowledge, definitions, details, and features  
|                      | Supply the conditions for use; providing kernel ideas  
|                      | Provide current knowledge, definitions, details, and features |
| Transformative       | Provide steps of process along with thinking and reasoning  
|                      | Explain thinking through the pattern or situation |

Table 4.1: Summary of Explanatory Categories with Corresponding Communication Activity

The *Algorithmic* category of explanations contains communication activities that generally describe the process of solving the given problem or providing an algorithmic
account of the solution. The communication activities that fall under this category include: (a) describing the work, (b) directly translating the work into words, (c) explaining the meaning of the mathematical symbols or formulas, (d) providing the steps of the solution process, (e) giving directions or prescriptive action, (f) providing general or nonspecific descriptions of the solution, and (g) alerting the reader to the conclusion. These activities contain explanations that are descriptive in nature and tend to focus on directly translating and describing the solution process, without much if any interpreting or rationale given to the mathematical actions being described. These explanations are efficient and provide the succinct descriptions of the solution process. Algorithmic explanations are similar to what Ennis (1969) offers as descriptive explanations where facts, steps, purpose, and relationships are explicated for informational purposes. The following are more examples of algorithmic explanations:

“Since the numbers increase by the number four you would use the information by making all the numbers increase by four to receive your answer. So, the numbers with the last digit of 1, 3, 7, 5, 9 will repeat. So you will have 102 numbers.” (846581, NT section)

“207-3 = 204 subtract 3 because it doesn’t start at zero, 204/4 = 51 numbers. Divide by 4 because every number increases by 4” (849512, NT section)

“120% is 120 per 100. So I set it up like that and used the p:% = n:m. 100 dollars is m the price that it was originally there. We don’t know the new price after the increase. The price is $120.” (904350, LR section)

“I used a proportion because it was easier to see what I was solving for, then with cross multiplication I set up an equation that was a missing addend and solved for the missing addend.” (049448, LR section)
“This is a ratio of proportions.” (482142, NT section)

“120% of 100 is $120 so, add that to the $100 to find ticket price.” (685312, LR section)

The *Structural* category of explanations contains communication activities that provide the reader with more insight into the solution process than a simple description provides. Within this category the explanations provide some interpretation and meaning into some of the facets of the mathematics used in the solution process. The communication activities that fall under this category include: (a) providing background knowledge whether that is definitions, details, and features; (b) supplying the conditions for use; (c) providing kernel ideas; and (d) providing current knowledge as definitions, details, and features. The aspect of these explanations that distinguishes them from the first type is that they provide insight and structure with details beyond that of simply describing the solution process. By supplying the reader with additional information, these explanations go beyond the surface features and give the reader criteria for action, clarifications, and distinctions, which is similar to interpretive explanations (Ennis, 1969). The following are more examples of *structural* explanations:

“If counting from 1 to 207, there are 104 odd numbers. Since the set is of every other odd number, you would divide 104 by 2.” (767300, NT section)

“The are 68 numbers in the set because the set starts at 3 and goes to 207, contains only odd numbers.” (806627, LR section)

“Since there was an increase of 120 percent you are paying what you paid last year plus 100 percent (or $100) plus 20 percent of last year ($20) therefore you pay $220.” (922458, LR section)
“If the price increases to 120% multiply the $100 by the percent increase for the total amount.” (888005, LR section)

“Since I’d need 5 ½ quarts of milk, I have to buy 6 quarts and have ½ a quart left over.” (861513, LR section)

“For this we need to find the ratio of people to quarts of milk. We know that for 6 people we need 1.5 qt. So we need to figure out an equivalent ratio for 22 people.” (849208, LR section)

The Transformative category of explanations contains communication activities that explicitly reflect the thought process. The argument can be made that any written explanation reflects the thought process because it shows the product of the thought process, however the distinction here is that these explanations explicitly show the logical progression and the reasoning behind the solution process. There is a logical connection between statements and decisions that is evident in a train of thought. The communication activities that fall under this category include: (a) providing the steps of the process along with thinking and reasoning, and (b) explaining thinking through a pattern or situation. These types of explanations may contain elements of the previous two types of explanations, however this type clearly reflects the reasoning, thinking, and decision-making behind the solution process. The information that is conveyed through transformative explanations is not necessarily required in order to provide a sufficient explanation, however it does reflect a deeper or more sophisticated view of this explanatory situation. Examples of transformative explanations follow.
“There are 51 numbers in the set because you find how many total numbers in between 3 and 207 by subtracting them. Since each number goes up by 4, you can then divide the total numbers in 3 and 207, which is 204, by 4, to get 51.” (889616, LR section)

“From 3 to 19 there are 5 #s. 19 is close to 20; 20x10 = 200. There are 10 groups of 5 numbers in the set. We can assume the 50th # is 199 b/c each # is increased by 4, 199+4 = 203; 203+4 = 207. So we add 2+50 = 52. There are 52 numbers in the set.” (827884, NT section)

“Since 120% of 100 is 120 and it has been increased 120%, it will be 120 more than last year’s ticket, so you add 120+100 = 220.” (806159, LR section)

“Increased 100% so increased the full amount of 100 dollars. So now the plane ticket is $200, 20% of 100 is 20. So add 20 dollars more onto $200 to get a total of $220 dollars for this year’s plane ticket.” (888274, LR section)

“For every 6 people you need 1 ½ quarts. So for the first 18 you can look at it as 3 groups of six people. so you need 3 groups of 1 ½ or 4 ½ quarts for the last 4 people you will have to make the whole recipe so you need another 1 ½ quarts totaling 6 quarts.” (682972, LR section)

“We know how many people we have and how much milk is required for them, so we have to find how much is required per person. Using the formula written above, we find that .25 quarts of milk is required per person. We now will have 22 people so again using the formula, we find that 5.5 quarts of milk are required for 22 people. Since we have to buy one-quart containers, we need to buy 6 one-quart containers.” (809874, NT section)

The three categories are somewhat hierarchical in nature in that they increase with sophistication, from *algorithmic* being the simplest and *transformative* being the most
complex. *Algorithmic* explanations simply give the process, the answer, or a translation of the mathematical symbols. This type of explanation efficiently answers the question with an adequate amount of information given to the reader. The *structural* explanation provides the reader with more insight by defining terminology and supplying extra inferences that aid in the solution process that may not have been apparent and obvious. *Transformative* explanations offer the reader insight into thought process along with the solution process. There is evidence of a logical chain of events that guides the reader through the entire solution. All three types of explanations are equally acceptable as explanations and provide a satisfactory answer to the problem. None is better than the other in terms of correctness, but the differences between the categories reflect a distinction in the reasoning ability and the subsequent expression of that ability. The initial directions for the final exam questions did not ask the students to provide as much detail as possible, nor did it ask for the reasoning process. The directions simply asked for an explanation, and the way in which the student interpreted those instructions is reflected in the variations of explanation offered through the three different types.

As I have described earlier, this communication model has some similarities and differences to the study by Kline and Ishii (2002). In Figure 4.1, the influencing fields of communication studies and mathematics education are shown in the process of development for this model.
Theorists

Communication Studies
Delia, Ennis, O’Keefe, Philipsen, Rowan

Mathematical Education
Cobb et al., Pearce & Davison, Shield & Galbraith

Integration of Communication Studies and Mathematics Education

Kline & Ishii:
Communication activities in written explanations

Current Study
Multiple iterations: raw codes, collapsing codes into 3 broad categories

Final hierarchical model:
Algorithmic explanations
Structural explanations
Transformative explanations

Figure 4.1 Lineage of the Communication Model
The process began with the influence of constructivist communication theorists and others whose work informs explanatory discourse such as Delia et al. (1982), Ennis (1967), O'Keefe (1988), Philipsen (1996), and Rowan (1999). The general frameworks of the constructivist approach to communication (Delia et al., 1982) and the general types of explanations in teaching (Ennis, 1967) provided a perspective with which to approach written explanations in Kline and Ishii (2002). Although the explanation model in that study was inductively derived from the data, the codes and categories were influenced by other communication ideas including the way in which a person/student perceives the communication situation and constructs messages accordingly (O'Keefe, 1988), speech codes and certain ways of speaking that appear in various cultural groups and subgroups (Philipsen, 1996), and the complexities of communicating scientific and “difficult” concepts to others (Rowan, 1999). These communication concepts and ideas together with those from mathematics education including: the notion that knowing mathematics is to be able to participate in mathematical activities and discussions (Cobb et al., 1992), the various writing activities used in the mathematics classroom (Pearce & Davison, 1988), and the coding scheme of written mathematical responses into its elements and features (Shield & Galbraith, 1998) further shaped the model in Kline and Ishii (2002). The communication model in the current study was inductively derived from the data similarly to Kline and Ishii (2002), yet resulted in some of the same codes because of the shared theoretical groundings of both studies. As described in earlier sections of this chapter, the codes that resulted in the current model were refined several times until an encompassing categorical system was developed. The final version of the model
produced a hierarchical model (Algorithmic, Structural, and Transformative explanations) that reflects the variation in complexity of the written explanations.

Quantitative Phase: Statistical Analysis

The second phase of this study focused on the statistical analysis of the data. Descriptive statistics were calculated in addition to chi-square analyses to determine the extent of relationships between student success and type of explanation used. The focus questions addressed two issues: (a) if there was a relationship between type of explanation and successfully answering the question, and (b) if there were any differences in the explanations between the two approaches. The following discussion describes the results from the statistical analysis in the quantitative phase for this study.

Mathematical Success and Type of Explanation

For the first question, How many numbers are in the set \{3, 7, 11, 15, 19, ..., 207\}? Show your work and provide an explanation, only slightly more than one-quarter (n = 39, 27%) of the 145 students answered correctly. The majority of the students (n = 106, 73%) answered that question incorrectly. Table 4.2 shows the percentage of students that answered the first question correctly in relation to the type of explanation that was used. One-fifth (n = 29, 20%) of the students did not offer any type of explanation; rather they answered the question completely symbolically. The algorithmic explanations were provided by the most by the students (n = 56, 39%). Structural explanations were provided less frequently, about one-quarter of the students (n = 34, 23%). The transformative explanations were used the least (n = 26, 18%), but not that much behind the second category of explanations.
| Explanation Type | Question 1 | | | Question 2 | | | Question 3 | | |
|-----------------|------------|---|---|------------|---|---|------------|---|
|                 | Incorrect  | Correct | Total | Incorrect  | Correct | Total | Incorrect  | Correct | Total |
| None            | 27         | 2     | 29   | 11         | 4       | 15    | 4          | 0       | 4     |
|                 | (20)       |       |       | (10)       |         |       | (3)        |         |       |
| Algorithmic     | 41         | 15    | 56   | 59         | 49      | 78    | 55         | 35      | 90    |
|                 | (39)       |       |       | (54)       |         |       | (62)       |         |       |
| Structural      | 27         | 7     | 34   | 8          | 14      | 22    | 14         | 20      | 35    |
|                 | (23)       |       |       | (15)       |         |       | (24)       |         |       |
| Transformative  | 11         | 15    | 26   | 5          | 25      | 29    | 4          | 12      | 16    |
|                 | (18)       |       |       | (21)       |         |       | (11)       |         |       |
| Total           | 106        | 39    | 145  | 53         | 92      | 145   | 78         | 67      | 145   |
|                 | (73)       | (27)  | (145)| (37)       | (63)    | (145) | (54)       | (46)    | (145) |

*Note.* Percentages appear in parentheses.

Table 4.2

Number of Intern Success on Final Exam Questions by Type of Explanation Used on Each Question
For the second question, *Last year you paid $100 for a plane ticket to Chicago. Since then the price of plane tickets has increased 120%. How much will you pay this year? Show your work and provide an explanation for your response*, more than half \( (n = 92, 63\%) \) of students answered the question correctly with less than half \( (n = 53, 37\%) \) that answered incorrectly. One-tenth \( (n = 15, 10\%) \) of the students did not use any type of explanation. About half \( (n = 78, 54\%) \) of the students used *algorithmic* explanations. A small percentage \( (n = 22, 15\%) \) used *structural* explanations in their writing. The *transformative* explanations were used by about one-fifth \( (n = 30, 21\%) \) of the students.

In the third question, *A recipe that serves 6 persons requires one-and-a-half quarts of milk. How many one-quart containers of milk will you have to but if you want to make enough for 22 persons? Show your work and provide an explanation for your response*, less than half \( (n = 67, 46\%) \) of the students answered correctly, whereas slightly more than half \( (n = 78, 54\%) \) the students answered incorrectly. A very small number \( (n = 4, 3\%) \) of students did not offer any type of explanation. The majority \( (n = 90, 62\%) \) of students used algorithmic explanations. *Structural* explanations were used by about one-quarter \( (n = 35, 24\%) \) of the students. Roughly one-tenth \( (n = 16, 11\%) \) used *transformative* explanations.

In order to determine if a relationship existed between mathematical success and the type of explanation used, a chi-square test for independence for each of the three questions was conducted. Since both variables (success and type of explanation) were nominal or categorical variables, the chi-square test for independence was the statistical test that determines whether the variables are related or independent (Hopkins, Hopkins, & Glass, 1996). The statistic that was used to determine relationships was Cramer’s \( V \).
because success was a dichotomous nominal variable and type of explanation was a mulitchotomous nominal variable (Hays, 1994). Cramer’s $V$ is a statistic that uses the chi-square statistic and reflects the magnitude of the relationship between the variables.

For the first question, *How many numbers are in the set \{3, 7, 11, 15, 19, ..., 207\}? Show your work and provide an explanation*, there was a low association (Cramer’s $V = 0.363, p < .000$) between answering the question correctly and the type of explanation that was used by the students. Most students answered the question incorrectly and did so using all types of explanations, however, of those that answered the question using *transformative* explanations, more students tended to answer the question correctly. For the second question, *Last year you paid $100 for a plane ticket to Chicago. Since then the price of plane tickets has increased 120%. How much will you pay this year? Show your work and provide an explanation*, there was a low association (Cramer’s $V = 0.309, p < .005$) between answering the question correctly and the type of explanation. Students who answered the question correctly tended to use *algorithmic* explanations the most, followed by *transformative* explanations, and then by *structural* explanations. For the third question, *A recipe that serves 6 persons requires one-and-a-half quarts of milk. How many one-quart containers of milk will you have to but if you want to make enough for 22 persons? Show your work and provide an explanation*, there was a very low association (Cramer’s $V = 0.292, p < .01$) between answering the question correctly and the type of explanation. Students who used *algorithmic* explanation tended to answer incorrectly. Those who used *structural* explanations tended to answer correctly. Students who used *transformative* explanations tended to answer the question correctly.
Difference-Similarities Between the Approaches

In the first question, *How many numbers are in the set \{3, 7, 11, 15, 19, ..., 207\}?

*Show your work and provide an explanation*, the lecture/recitation section (LR) had a 16% \((n = 4)\) success rate, which was comparable to the nontraditional section (NT) that had a success rate of 24% \((n = 6)\). Table 4.3 shows the percentage of usage of the explanations with success for both the LR section and NT section. In the LR section 37% \((n = 9)\) of the students used *algorithmic* explanations, whereas the NT section 37% \((n = 9)\) of the students used *algorithmic* explanations. The LR section had 21% \((n = 5)\) of the students use *structural* explanations. The NT section had 29% \((n = 7)\) of the students use *structural* explanations. Finally, the LR section had 21% \((n = 5)\) of the students use *transformative* explanations, where the NT section had 17% \((n = 4)\) use *transformative* explanations. These results show that both sections performed about the same on the first question with regard to the percentages of success and the types of explanations used.
<table>
<thead>
<tr>
<th>Explanation Type</th>
<th>Question 1 LR</th>
<th>Question 1 NT</th>
<th>Question 2 LR</th>
<th>Question 2 NT</th>
<th>Question 3 LR</th>
<th>Question 3 NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>5 (21)</td>
<td>4 (17)</td>
<td>3 (13)</td>
<td>1 (4)</td>
<td>0 (0)</td>
<td>0 (0)</td>
</tr>
<tr>
<td>Algorithmic</td>
<td>9 (37)</td>
<td>9 (37)</td>
<td>13 (54)</td>
<td>15 (62)</td>
<td>16 (66)</td>
<td>15 (62)</td>
</tr>
<tr>
<td>Structural</td>
<td>5 (21)</td>
<td>7 (29)</td>
<td>2 (8)</td>
<td>4 (17)</td>
<td>4 (17)</td>
<td>3 (13)</td>
</tr>
<tr>
<td>Transformative</td>
<td>5 (21)</td>
<td>4 (17)</td>
<td>6 (25)</td>
<td>4 (17)</td>
<td>4 (17)</td>
<td>6 (25)</td>
</tr>
<tr>
<td>Success</td>
<td>4 (16)</td>
<td>6 (24)</td>
<td>17 (71)</td>
<td>18 (75)</td>
<td>12 (50)</td>
<td>9 (37.5)</td>
</tr>
</tbody>
</table>

*Note.* Column percentages in parentheses. For each section, LR (n = 24) and NT (n = 24)

Table 4.3: Number of Student Success by Type of Explanation Used for Different Approaches

For the second question, *Last year you paid $100 for a plane ticket to Chicago.*

*Since then the price of plane tickets has increased 120%. How much will you pay this year? Show your work and provide an explanation,* the LR section had a 71% (n = 75) success rate, which was comparable to the NT section that had a success rate of 75% (n = 18). In the LR section 54% (n = 13) of the students used _algorithmic_ explanations, where in the NT section 62% (n = 15) of the students used _algorithmic_ explanations. The LR section had 8% (n = 2) of the students use _structural_ explanations, while the NT section had 17% (n = 4) of the students use _structural_ explanations. Finally, the LR section had
25% \((n = 6)\) of the students use *transformative* explanations, where the NT section had 17% \((n = 4)\) use *transformative* explanations. In this question, the LR section had a fairly higher success rate than the NT section. The use of *algorithmic* explanations was about the same for both sections. The *structural* explanations were used more by the NT section than in the LR section. Almost twice the percentage of students in the LR section used the *transformative* explanations as in the NT section.

In the third question, a *recipe that serves 6 persons requires one-and-a-half quarts of milk. How many one-quart containers of milk will you have to but if you want to make enough for 22 persons? Show your work and provide an explanation*, the LR section had a 50% \((n = 12)\) success rate, which was comparable to the NT section that had a success rate of 37.5% \((n = 9)\). In the LR section 66% \((n = 16)\) of the students used *algorithmic* explanations, where in the NT section 63% \((n = 15)\) of the students used *algorithmic* explanations. The LR section had 17% \((n = 4)\) of the students use *structural* explanations, while the NT section had 13% \((n = 3)\) of the students use *structural* explanations. Finally, the LR section had 17% \((n = 4)\) of the students use *transformative* explanations, where the NT section had 25% \((n = 6)\) use *transformative* explanations. Again for this question, the LR section had a better success rate than the NT section. Both sections’ use of *algorithmic* explanations was about the same. For *structural* explanations, both sections were near the same. In *transformative* explanations, the sections were about the same.

A similar chi-square test using the Cramer \(V\) statistic was attempted but resulted in correlations with very high \(p\)-values that were unacceptable to interpret and report. The reason for the high \(p\)-values was most likely due to the low cell count in many of the
categories for both sections of the course. Although there are correction tests such as Fisher’s exact or Yate’s Correction that can be done for situations that result in low cell counts, these are only available for contingency tables that are two-by-two. In this study, the contingency tables are two-by-four, and thus do not qualify to be able to use any of the correction tests.

In the next chapter, I further discuss the elements of the communication model and show how they tie back to the theories and research that influenced their derivation. I discuss the results of the statistics that were calculated in an effort to compare sections from the two approaches to the course. In addition, I explicate the limitations of the study and make interpretations and suggestions for future research.
CHAPTER 5
DISCUSSION

Introduction

This chapter serves as a point of discussion for the findings of the research study, its implications, and possible future directions. I discuss the results of the study and highlight some interesting findings that stand out from the rest, in addition to some unanticipated results. What this study means in the larger context of the field that also places this study amongst the current related literature in mathematics education are discussed in addition to the limitations of the study that may or may not have influenced the results. I discuss the implications of this study and make suggestions for teachers and researchers, as well suggestions for future directions in this line of research.

The process of developing a model of communication for these data has given me a way to identify and describe the variability in student written explanations. The model I developed is an interpretation of the data that accounts for differing levels of explanatory sophistication, and not necessarily ability. The algorithmic explanations are those that simply describe the process of finding the problem solution by directly translating the mathematical symbols into words and/or giving the steps without any justifications or reasoning. The structural explanations go a step further in providing some interpretation and insight into the details of the problem. They do so by providing more information
through conditions, definitions, and facts that are not obvious from the problem itself.

The transformative explanations are the most sophisticated type of explanation that provides the most insight into the solution process. These explanations show the logical progression of the solution by providing connections between ideas, and/or the student’s thought process while solving the problem. Transformative explanations can contain elements from the other two categories, but the distinguishing feature of these explanations is the inclusion of reasons and evidence of the student’s thought process.

Discussion of Results

The three levels in the study show that there are varying degrees of providing an explanation in writing, and perhaps in the future could be translated to a verbal setting, however, this model does not represent the varying degrees of correctness, rather it shows the different levels of sophistication and complexity with which students explain the solution of a problem. In this model, each type of explanation whether it is algorithmic, structural, or transformative, is equally correct and adequate as an explanation; and the explanations most likely reflect the complexity with which the student reasoned about the problem. What I have done in this study is given more depth into explaining explanations and the work they do in the responses. Although I argue that the three categories in this model are hierarchical in nature, there is no evidence yet as to the type of explanation that is preferred by other mathematics educators, or which type would be ideal. Since this study was not designed to have the explanations evaluated by other mathematics teachers or experts in terms of the quality or completeness, it is not appropriate to infer which type of explanation is preferable. At some point in the future it would be beneficial to determine if an algorithmic explanation is considered to be adequate for certain purposes,
or the *structural* explanation, or if the *transformative* explanation is the ultimate goal for the types of explanations that we, mathematics educators, want our students to use.

In this study, the first focus question, *What communicative elements and processes exist in the mathematical explanations of pre-service elementary teachers?*, addressed the identification of the communication elements that are present in the written explanations from the participants. The communication elements are those codes that make up the details in the three main explanatory categories of the model: *algorithmic*, *structural*, and *transformative* explanations. Recall that Table 4.1 describes the codes and categories that make up the communication model.

*Algorithmic* explanations are described as those that delineate the solution process by giving the steps without reasoning. The students who used this kind of explanation exhibited responses that could be considered just satisfactory enough to provide some type of explanation. These explanations do not show any of the reasons or rationale behind the solving of the problem. *Algorithmic* explanations may be seen as the least sophisticated of all three types of explanations, meaning that the communication task was accomplished with the least amount of effort, yet it was accomplished. These responses are reminiscent of O’Keefe’s (1988) expressive design logic. Recall (from the literature review) that the expressive design logic is characterized by being reactive, self-expressive, and honest without considering the context or the feelings of others. Similarly, the use of *algorithmic* explanations has the potential to be thought of in the same way. These explanations do not show a consideration for the reader/audience with any attempts at making interpretations or supplying any additional details. In the constructivist (communication) sense, the explanations or codes do not reflect a
perspective-taking approach to communication, rather they appear to be position-centered, where the person offers only as much as the situation deems necessary.

*Structural* explanations are identified as those that supply additional details and features of the process beyond the actual steps of the process. These explanations provide more information than the *algorithmic* explanations, however they may not necessarily be longer in length. In this case, it is the substance of the explanation that makes it a *structural* explanation. In these responses, the key details other than the obvious ones are expressed and are thought to be important facets by the author of the explanation. This category is similar to Rowan’s (1999) quasi-scientific explanations that consist of highlighting main points and providing important components in order to help envision complex structures and processes. *Structural* explanations tend to be person-centered codes in the sense that they are taking another’s perspective into consideration by providing more interpretation in realizing the entire solution process. These explanations give structure to the solution by affording important features of the problem solution, however a distinguishing factor of these explanations is that they could be more complete. They are not wrong explanations, just not entirely complete, as the structure of a building cannot encapsulate the essence of the building itself.

*Transformative* explanations may not only provide elements of the process along with additional details, but also show the logic and thought process behind finding the solution. With this type of explanation there is an attempt at providing more than just enough details of the steps of the process and beyond sufficient definitions and facts. The addition of the reasons and rationale behind the steps of the process gives the impression of taking the reader/audience into consideration by making logical connections and
inferences. This notion of seeing the explanatory situation as more than an opportunity to merely answer a question, but to demonstrate knowledge and to inform, hints at O’Keefe’s (1988) rhetorical design logic. A person who communicates at this level of design logics sees communication as the negotiation of situations and uses communication to redefine situations. It is possible that the use of the transformative explanation shows a parallel adeptness in communication situations. Rowan (1999) also has “transformative” explanations in her taxonomy of explanations in science teaching that are characterized as helping in understanding difficult ideas and shifting thinking from one theory to another. Since the transformative explanations in this study contain the details and information that show thought process and rationale, they are similar to Rowan’s because they have the ability to help in fully understanding a topic or process.

The second focus question, What relationship exists (if any) between explanations and mathematical success?, addressed the relationship between answering the problem successfully and the type of explanation. The results show that there were very low but statistically significant correlations (relationships) between what explanations students used and whether they get the question right or not. For all three questions, the Cramer’s V statistics was near 0.3 with all p-values < .01. I had anticipated that a certain explanation would be used more with people who answered the question correctly, or those who answered the question wrong would use a certain type of explanation. This was certainly not the case. In all three problems, the highest percentage of students used algorithmic explanations in their writing, 39%, 54%, and 62% respectively. This was probably due to the fact that this type of explanation was the easiest to provide in that they are efficient and get the point across with the least amount of effort. They describe
the steps and that is about all. It is possible that many students saw the directions of, “show your work and provide an explanation,” as the opportunity to say just enough to get the point across. Because of the minimal instructions, there was no indication to the students that they needed to provide a thorough explanation and reasoning, however some students did provide explanations that detailed their thinking and reasoning. From a communication perspective, the task of providing an explanation for their work must be thought of in terms of the audience. In this case, the audience or the reader is the person or instructor grading their work, who happens to be very knowledgeable with these concepts. Therefore the burden is not on the student to provide an explanation that is completely sound and flawless. The algorithmic explanations are not providing the reader with detailed knowledge, rather are conveying enough information to convince the reader that the student knows what s/he is doing mathematically. This might be construed as a flaw in the wording of the question since it does not elicit enough motivation or communicative burden from the student to provide detailed explanations. The type of wording found in these questions, “show your work and provide a response,” is very typical in mathematics classrooms where communication is encouraged. Unfortunately, many mathematics instructors are not knowledgeable about communication concepts and therefore think that any kind of communication is better than none, which might be the case, but we do not really know that yet.

For the first problem, How many numbers are in the set {3, 7, 11, 15, 19, …, 207}? Show your work and provide an explanation, the students were expected to find the total amount of numbers in a given set. This question was unlike the other two questions because there is not a limited number of ways to go about solving the problem.
Also, this exact type of question was probably not directly taught to the students during the course, whereas the other two questions were typical questions that the students had practice solving. Thus, this question had the lowest success rate, 27% \((n = 39)\), and the highest percentage of students who did not write any explanation at all \((20\%, n = 29)\). Over twice as many students answered the question incorrectly than answered it correctly. It is possible that this question took students by surprise and they did not know how to proceed because it was not a procedural question. Algorithmic explanations were used by 39% of the students, where structural and transformative explanations were used by 23% and 18% respectively. Some students were able to provide details and some were able to provide their thought processes, but more of the students simply explained their work in words with the algorithmic explanations. Again this is probably due to the nature of the problem that was not a typical problem for which they were prepared.

The second problem, Last year you paid $100 for a plane ticket to Chicago. Since then the price of plane tickets has increased 120%. How much will you pay this year? Show your work and provide an explanation for your response, was a typical question where the students were expected to find the price of a ticket that had increased by a certain percent from the previous year. In this case, the student success rate was 63% \((n = 92)\). That question was fairly procedural and had 54% \((n = 78)\) of the students use algorithmic explanations. This was the only question of the three that had the highest percentage \((21\%, n = 30)\) of students use transformative explanations. Since the question was fairly typical in that the students had probably answered many questions similar to it during the course, they were prepared for a question such as this one. For this type of question, there are several ways of solving the problem, but all of them involve the
calculation of the percent increase and adding that to the previous year’s price. Regardless of the thought process, those two things must be done in order to solve the problem correctly. Thus, this question was probably easier to solve and easier to explain than the first question. Therefore more students were able to express their thinking and thought processes better than on the other questions. Another possible explanation for the higher frequency of transformative explanations in this problem is that it is easier for students to communicate their thinking in more depth when they have a better understanding of a topic.

The third problem, A recipe that serves 6 persons requires one-and-a-half quarts of milk. How many one-quart containers of milk will you have to buy if you want to make enough for 22 persons? Show your work and provide an explanation for your response, was similar to the second problem in that it was a routine problem that required the students to proportionally increase a recipe to accommodate more servings. This question had a success rate of 46% (n = 67), however this question was worded so that the student had to think a little deeper to get the question fully correct because it asked for the number of quarts of milk they would have to buy. If the student attempted to solve the problem by setting up equal proportions and solving for the unknown variable, they would find an answer that was not a whole number. To answer the question correctly, however, they would have to round up to the nearest whole number. Many students who solved the problem in that way did not round up and therefore answered the question incorrectly. So there were more people who answered the question correctly in terms of the mathematics, but for the purposes of this study, they answered the question incorrectly, which accounts for the low success rate of 46%. Of the three questions, this
one had the highest percentage, 62% ($n = 90$) of students who used algorithmic explanations. Again this question was a routine problem in that the students were probably prepared to answer similar questions without taking into consideration the context of the question. Therefore this contributed to the high percentage (62%, $n = 90$) of students who used the algorithmic explanations, and subsequently the low percentage (11%, $n = 16$) of students who used transformative explanations.

While the results from the usage of explanations are not surprising because they seem to spread out across the categories, what is interesting is that the success rates for the three questions seems quite low. Recall that the success rates for the questions are 27%, 63%, and 46% respectively. This is surprising given that these questions came from their final exam. In questions one and three, more students answered the questions incorrectly than answered correctly. While more students probably did the mathematics correctly on question three, yet did not answer the question correctly. In any case, the success rates seem as though they should be higher.

The third focus question, *What differences exist between the explanations offered by each approach of the course?*, addressed the differences (and similarities) between the two approaches of the course. In both approaches of the course, the students were accustomed to explaining themselves on their homework assignments, quizzes, and tests. Although there were 121 students in the LR sections and only 24 students in the NT section, I compared the one LR ($n = 24$) section with one NT ($n = 24$) section. From my observations of the course, the students in the nontraditional (NT) section seemed to be more focused on providing explanations with the expectation of expressing their thinking clearly to their classmates and the instructor. Surprisingly the nontraditional (NT) section
did not do much better than the lecture/recitation (LR) section. For the first problem, the NT section had a higher success rate of 24% \( (n = 6) \), compared to the LR section that had a success rate of 16% \( (n = 4) \). Actually there is a difference in success of two instances between the two sections. For the second problem, a greater percentage \( (75\%, \ n = 18) \) in the NT section answered correctly compared to the LR section \( (71\%, \ n = 17) \). Again, the difference between the sections is minimal, namely a difference of one instance. With the third problem, the success rate for the LR section \( (50\%, \ n = 12) \) was better than the NT section \( (37.5\%, \ n = 9) \). But this difference is the greatest between the sections out of the three problems. Since the total numbers in each section are so low (less than 30), it was difficult to get any meaningful correlations in order to compare the two sections. Therefore the best conclusion that can be made at this point between the sections is that as far as the success rates, they seem to have done similarly, even though the percentages marginally differ. It is interesting to see the LR section had better success only on the third problem, which was a fairly standard problem. The NT section did better on the nonstandard first problem, as well as on the second problem, which was a standard problem. Recall that the differences in success in the first two problems are merely a difference of at most two instances.

As far as comparing the use of explanations, the trend across both sections is that most students used \textit{algorithmic} explanations across all three problems. In the second and third problems, the percentages were greater than the first problem. This was probably due to the fact that those problems were fairly typical and prescriptive in nature. There were obvious ways to go about finding answers to those problems, and there were fairly algorithmic ways of solving the problems. On all three problems across both sections, the
explanation use of all types differed by no more than two students. But with

*transformative* explanations, the LR section used more on the first and second problems. On the third problem, the NT section had 25% \((n = 6)\) use *transformative* explanations compared to the LR section that had 17% \((n = 4)\), yet I would have thought that the NT section would have used more *transformative* explanations in their writing across all problems because they had many assignments that required them to provide explanations throughout the course. It could be the case that the type of writing that they were encouraged to do was similar to the *algorithmic* explanations. It would be interesting to go back and examine the students’ writing in both sections from assignments that they had during the course in order to compare them with the types of explanations they gave on their final exams. This is a limitation of the study that was not part of the design that could have shown some interesting results. But for my purposes, it was something that was not foreseeable, yet can be done in future research.

The fact that the percentages of the type of explanations used seem to be close between the sections reinforces the notion that this model describes the variation in the types of explanations that the students gave regardless of the approach. It may also be the case that the frequencies are so low that it is difficult to conclude much from this situation.

An interesting outcome from the data analysis that was briefly reported because it was somewhat of an anomaly was that some of the students used what are called kernels (Kline & Ishii, 2002; Shield & Galbraith, 1998) or kernel (template) ideas in their writing. This idea is characterized as expressing the main idea or the main concept that must be known in order to proceed and solve the problem. These can also be the
expression of a metaphorical template that prescribes the actions and is applicable to any
other similar situation (Kline & Ishii, 2002). In this study, however, out of the 145
students only 3 actually used this communication idea. I noted this in my earlier
conceptions of the model, but collapsed it with another code within the structural
category. I did not want to ignore this element because it reflects a different perspective
than his/her own that the person uses. This is an example of a student using a kernel:

“If a price is increased by x%, you multiply the old price by the percentage
increase and then add this number to the old price. This gives you the new price…”
(685628, LR section)

It might be the case that these people used kernels because they see the writing situation
as something more than just expressing their reasons and they see it as an opportunity to
teach or something other than just demonstrating knowledge. I collapsed this idea into the
structural explanations because they do not offer a complete rationale; rather they
provide a key component or (kernel) idea that is necessary for completing the task
without actually giving the reasons.

Another interesting result is the appearance of what I call “inconsistent”
explanations. These are explanations that a few of the students used when they explained
their work differently than what their actual symbolic work shows. It was not a case of
incorrect information or academic dishonesty, yet their reasoning did not match their
actual symbolic work. An example of this phenomenon follows. This is the symbolic
mathematical work.

“100 + (100 * 1.20) = 100 + 120 = 220”. This is the response that followed that work.

“This year you will pay $220 for a plane ticket. 100% of the original ticket price would
be $100.00 and 20% of the original ticket price would be $20.00. These added together is $120.00, and if you add this to the original price (increase) you will get $220.00.”

It is peculiar that a student would do the symbolic mathematical work, find an answer, and then give a written explanation to a different way of thinking about and solving that problem. In this example the student’s symbolic mathematical work indicates thinking about the problem as first finding the amount the ticket increased and then adding that to the original price. The explanation that follows does not match that strategy. The explanation would seem to follow symbolic mathematical work that shows the addition of “100 + 100 + 20 = 220” or at least “100 + 20 = 120” to show the increase in price.

Both the symbolic work and the explanation are correct ways of solving the problem; they just represent two distinct ways of doing it. I cannot begin to speculate why a student would do this because it does not follow the directions of, “Show your work and provide an explanation.” This incident makes no sense, yet it is interesting. Had post-final exam interviews been built into the design of the study, this would have been one that I would have pursued.

Links to Previous Research

The current study has many similarities, including a similar theoretical framework, with a study by Kline and Ishii (2002) in which written explanations were examined where students were required to write an explanation to another student in their class. For that study, the communication task was to explain to a peer so that s/he could fully understand a mathematical concept. That task could be interpreted as an informative or a teaching opportunity on the part of the student. That communication task was
different from the current study’s communication task in that the audience or reader is
different, as well as a different communicative burden. In this study the students were
required to justify and explain enough reasoning so that they could get the answer
correct. It was a demonstration of knowledge or a communicating knowledge task. In
Kline and Ishii (2002) the writing samples were structurally different in that there was
more depth and variability because the mathematics question asked the students to
explain how to graph the equation of a line in two variables. In this study the writing
samples were much shorter and more succinct especially on the two routine problems
from the final exam. In both studies, the resulting models that emerged from the data are
similar and contain similar components. This was mainly due to the fact they utilized the
same knowledge base from communication studies and joined that with the current work
in mathematics education.

Some elements from the three broad categories in the model from this study
appear in the model from Kline and Ishii (2002). In the current study, the main structure
of the model reflects ordered categories contrary to the other model in which they were
mainly features of a framework. For this study the model attempts to explain in a
hierarchical fashion the sophistication in student writing while accounting for the various
features of the explanations. The model in the other study served as a general framework
for understanding written explanations. Both studies were exploratory in nature in that
they were both grounded analyses where the model emerged from the data itself and no
previously developed system was applied to the data. These studies connected the fields
of communication studies and mathematics education in order to better make sense of
written explanations from students.
In Kline and Ishii (2002), the analysis of the explanations yielded a coding system or model that is organized by the elements and subtasks comprising written explanations. In that framework, there are six main categories that are used to describe explanations: (a) orienteering, (b) articulating relevant concepts with interpretive explanations, (c) legitimizing concepts through reason-giving explanations, (d) guiding procedural activities with descriptive explanations, (e) solidifying reader understanding, and finally (f) facilitating linguistic control of mathematical terms. Using this framework, a student’s written explanation can be coded using some or all of the categories depending upon the elements that appear. Since the communicative task in that study required the students to write explanations that would help another person understand the mathematical concepts, the exemplars tended to be long expositions or narratives; thus necessitating a coding system that could effectively describe the student writing. In contrast to that study, the students’ writing samples were much shorter in length, since they were written explanations that accompanied symbolic mathematical work on their exam questions. The coding system or model in this study captures the overall essence and spirit of the explanation and codes according to one of the three levels of explanation: algorithmic, structural, and transformative.

In the current study the algorithmic explanations are very similar to what was called “guiding procedural activity with descriptive explanations,” in Kline and Ishii (2002). For that category the main activity of the explanation was to supply meaning and explain how to do the procedure by giving the reader the details and steps necessary for the procedure. This sentiment is mirrored in the algorithmic explanation in that it is both descriptive and procedural in nature by provides steps, directions, and direct translations
of the symbolic mathematics into words. Although these categories from both studies are similar, a distinguishing factor is found in the character and use of the descriptive explanations that Ennis (1969) uses. Kline and Ishii (2002) use Ennis’ notion of descriptive explanations for this category in that these explain the process, details, and conditions for use; whereas in this study descriptions refer to the details of the process, and supplying additional definitions and meanings are activities that are characterized by the next level of explanations.

The category of structural explanations in this study focuses on supplying more information in addition to or instead of the procedural descriptions. This category is seen as a higher level of explanation because at the very least a student could answer the question by giving the steps of the procedure. Providing additional information such as definitions and background information goes beyond the notion of the algorithmic explanation. This category has a hint of Ennis’ notion of the descriptive explanation as was used in the other study, but in this study there is a separation between descriptions of the process, and descriptions that further enlighten the concept/situation. The category relating to “solidifying reader understanding” can be linked to the current study’s structural explanations in that they both provide an additional link between new and prior knowledge of the situation and concept. In the other study this category encompasses more communication activities that make knowledgeable connections in order to get the reader to understand, such as using visual formats, involving language, and supportive or personalized moves.

Another feature from Kline and Ishii (2002) that appears in this one is the notion of articulating kernel ideas, which are key information for the reader, or the “big idea” for
the particular concept. In that study, the articulation of kernel ideas is seen as an interpretive explanatory activity, in which the student can use key definitions, and/or provide templates that outline the process so that they may be used in other situations. For this model, the articulation of kernel ideas and definitions is seen as a structural explanation in that providing this information goes beyond simply solving the problem. The definitions and key information (kernels) give more structure to the work as they do provide some interpretive explanations that are more than procedural in nature. In contrast, the template part of the category for the other study is used in this study as a procedural abstraction of the process in which a reader could take that information and use it in another similar situation because the given explanation is general and unspecific to the particular problem.

The final category in this study, transformative explanations, is similar to the category in the Kline and Ishii (2002) study, “Legitimizing concepts through reason-giving explanations.” In the current study, transformative is the highest level of the three categories because it reflects the students’ thinking and logical processes in solving the problem. These explanations may contain elements from both algorithmic and structural explanations, but they are distinguished by the fact that they show logical connections and the reasoning behind the procedures and given actions. In the other study, the notion of the reason-giving explanation is classified in terms of its use with the symbolic mathematics or exemplars by showing the connections between the actions/solution and the reasoning. For this study, providing the reasoning in the explanation is seen as the highest level of explanation because in one respect, it reflects a level of sophistication that is not present in other types of explanations. The transformative explanations do
more than just describe the process or provide additional details. They show a logical progression through a situation in which a reader can follow and arrive at the same solution.

From a sociocultural perspective, a well-crafted transformative explanation has the potential to provide a type of scaffolding (Rogoff, 1990) for the reader of the explanation. Since transformative explanations express the thinking and thought process for the mathematics task, the person constructing the explanation sees the explanatory situation as an opportunity to provide special features and decision making in the process. Within scaffolding, the more-knowing person (teacher) is in a position to focus on goals, provide ideal versions of the task, and note critical features of the process. The apprenticeship metaphor (Rogoff, 1990) for teaching and learning can be applied to effective explanations crafted in order to attain understanding on the part of a less-knowing person (audience). In an explanatory situation, the use of transformative explanations can create intersubjectivity (Wertsch, 1991) between the message producer and the reader. Since the situation may not necessarily be face-to-face because it involves written explanations, the usual sense of “communicating” to attain intersubjectivity is challenged, yet a transformative explanation has the potential to create a full picture of the concept and process, which can result in understanding the topic.

Intersubjectivity in the sociocultural sense supports the notion of coordination (Clark, 1996) in that both ideas invoke the metaphor of alignment. For intersubjectivity, it is the alignment of voice or perspective, whereas in coordination it is the alignment of goals to attain understanding a concept. These concepts are evident in transformative explanations when the message producer provides the reasoning behind the mathematical
process. The transformative explanations reflect or have the potential to reflect other elements in constructivist communication (Delia et al., 1982) such as person-centered messages and elaborated codes. An explanation that is person-centered takes into account the qualities of the listener. A transformative explanation has the potential to do this if the written task gives an indication of the audience, however a distinguishing feature of transformative explanations is the depth of elaboration and insight to the process. This notion aligns with elaborated codes (Applegate & Delia, 1980) in that the transformative explanations express information that the listener needs without having the background knowledge.

A topic consistent with constructivist communication is the speech code (Philipsen, 1997). There have not been explicit investigations into mathematical speech codes; however adopting the view of mathematical learning as participating in cultural practice (Cobb et al., 1992) shows that mathematics embodies cultural ways of communicating and acting. This is directly related to speech codes within particular communities and cultures. Transformative explanations reflecting the most sophisticated ways of explaining a concept can potentially contain speech codes and demonstrate their discursive force (Philipsen, 1997). Discursive force is the ability to communicate with skill, effectiveness, and cultural control, all of which would be mathematically demonstrated in an effective transformative explanation.

Although for this study transformative explanations are written classroom communication, they have parallels to research in verbal communication in the mathematics classroom. Since this type of explanation involves expressing the student thought process the notion of reflective discourse is an appropriate comparison. The van
Zee (2000; van Zee & Minstrell, 1997) notion of reflective discourse involves structuring discussions to foster and monitor conceptions. The *transformation* explanations have the potential to do this by providing troubleshooting details that help the reader to grasp the concept and follow the mathematical process. The Cobb et al (1997) notion of reflective discourse involves making mathematical action the object of discussion. Although this does not exactly occur in *transformation* explanations, if a class were to revisit their written explanations and discuss differences and improvements, then reflective discourse would be enacted.

In this study I did not examine explanations in terms of their correctness, for I explored the relationship between mathematical success and type of explanation. Stonewater (2002) offers successful writing tips for mathematical essays, some of which correspond with elements of the explanations in the current study. His ideas about using appropriate mathematical language mirror the idea of using definitions and terminology in the *structural* explanations. Building a context for the essay matches with the *transformation* explanations in that they both provide interpretive moves so that the audience is able to follow the reasoning in order to understand the mathematical process.

Placing this study in the broader context of the literature has it continuing with other studies that examined elements of student writing. This study goes further than studies done by Pearce and Davison (1988), Silver, Leung, and Cai (1995), Shield and Galbraith (1998), and Stonewater (2002) in that I examined writing holistically as opposed to examining the specific elements of student writing, although that was as aspect of the iterations in the current analysis. In these previous studies, the researchers codified and categorized the features of the explanations such as: using symbolic
notation, using visual representations, degrees of correctness, summarizing, and many more. In Kline and Ishii (2002) and in the current study, student writing was explored thoroughly, while noting features of the explanations, but placing them in a meaningful way that helps to see what the explanations do in the writing as far as helping the reader connect concepts through the process, and gauging the sophistication of their reasoning and communicative abilities. This study adds to the literature in that it provides a method of analyzing writing in a fashion that accounts for competency and complexity, rather than the strict codifying of writing and the cosmetic features of it.

Although the model developed in this study retains similar elements from Kline and Ishii (2002), the main distinction between the studies is that the current model offers a hierarchical interpretation of mathematical explanations, whereas the other model is arranged by communication subtasks present in the explanations. The communication research that serves as the knowledge base for this study including Clark and Delia (1976), Applegate and Delia (1980), O’Keefe (1988), and Rowan (1999) also reflect hierarchical models and coding systems that attempt to capture the variations in sophistication that exist in communication. Similar to these studies and other constructivist communication coding systems, the model in this study was developed in the spirit of describing the features of communication in order to identify “the situation-specific character of the jobs messages are meant to do.” (O’Keefe & Delia, 1985, p. 56).

Connecting back to the NCTM Principles and Standards, which provided the motivation for this research, has this study and potentially others like it consistent with the message that NCTM is conveying through the Communication Standard. While many other researchers have focused on the role of mathematical communication in the
classroom while providing excellent exemplars of mathematical discourse, there is no definitive research on the message of the Communication Standard. It says that students should be able to communicate their thinking coherently and clearly to others, but we have yet to establish guidelines on what that is or how to do it. We have plenty of research that explains the features of mathematical communication, but we do not have anything that addresses the quality of effective communication. The standards do not specify the mode of mathematical communication whether it is written or verbal, so the next logical step for research in the same line as this study would be to take this model and apply it to an oral communication situation either with students or with teachers or both. Then we can make comparisons and explore these same ideas to gain a clearer picture of all mathematical communication.

**Limitations**

This study laid the groundwork upon which further research can be built. Since the participants in this study were undergraduate college students who plan to be elementary school teachers, the mathematical concepts that they covered on the final exam were all very basic. The concepts were topics that the students would some day have to teach to children, and this situation was not exploited on the final exam in terms of the type and depth of explanation that was asked of them. In order to make the written explanations even more communicatively interesting with more details, the students could have been asked to give a response that they would offer to a student, instead of showing their reasoning to their instructors. That is to say, if the communication task required students to explain the concept to a student or someone without much knowledge, the explanations would probably have more detail. The design of the final
exam was out of my control, but that does not diminish the communication task that they were actually given. Analyzing teachers’ explanations to students has yet to be done and starting with typical writing tasks such as those in this study offers a starting point. The wording of the question is very important in trying to elicit well-crafted responses from students. For this study, the wording of the questions was vague, which enabled students to interpret their communicative burden as they saw fit. They were not explicitly required to provide transformative explanations and it is unknown whether or not the instructors of the two approaches ever required the students to give written tasks that resulted in transformative explanations. This resulted in the variations that were captured with the communication model.

One of the limitations of this study from a quantitative methodological standpoint was that there was only one person coding the data. In other studies there would be more than one person coding data in order to establish inter-rater reliabilities that would lead to more consistent coding of the data. Since I was the only person coding data in this study, it is possible that there are inconsistencies or anomalies in the coding that would be caught had there been more than one person coding. Since I went through several iterations with the data I am confident in the coding, however, it is still possible that there are inconsistencies. These would not have affected the resulting communication model. Since the main purpose of the study was to develop the communication model, this study did not rely heavily on quantitative techniques in which inter-rater reliabilities would be important.

Another limitation of this study was that there were too few numbers to compare between the sole nontraditional (NT) section and one of the lecture/recitation (LR)
sections. Both sections only had 24 students each to compare. These low numbers together with the fact that there was only one NT section made it difficult to run the statistics I had intended to calculate. For the chi-square analyses there were not enough frequencies in each of the category cells in order to run any meaningful calculations. The only statistics that I could truly compare were the frequencies, which are interesting in themselves, but cannot show any relationships. Coding the problems as either right or wrong (success or not success) as I did, limited the statistical tests that I could run because that coding scheme does not account for the variability in the students’ success. The goals of the study were not concerned so much with the correctness of the mathematics as much as the type of explanations. Even though the popular belief in mathematics is that there are only correct or incorrect answers without any gray area in between, my personal belief is that there are varying degrees of being correct or not. Take the third problem as an example where the students could do the mathematics correctly and still get the question incorrect if they did not read the question thoroughly enough or understood the question to see that it required them to do more than find the mathematical answer; they had to think about the question in terms of the context. That aspect of the study was probably not as strong as it could have been and had the questions been graded out of five points, there would have been greater variability and I could have made other comparisons using the means from each section. But again the numbers were so small between the 24 in the NT section compared to the 121 in the complete LR sections, that I could only compare one section of LR with the one section of NT. It would not have been statistically appropriate to compare the 121 students in the LR with the 24 in the NT. This problem with having a single NT section, however, was out of my
control and unavoidable. Had there been an equal split between the two sections, it would have been easier to run statistical tests that would have provided more insight to the data.

Given all of the limitations and the possibilities that could have been, there were elements of this study that went well and should be recognized. The communication model could have been complex and convoluted, but it is simple enough to be accessible to others in the various fields that would be interested in the results of the study. The communities in mathematics education teachers and researchers, along with the practitioners at various levels have the potential to be interested in this research, especially since it is beginning to get to the heart of what the NCTM (2000) Standards are imploring of our community. This is the type of research that we as the mathematics education community can conduct in order to bridge the theory and practice void in the hopes of influencing the teaching and the learning of mathematics at all levels. As it stands now, the communication model can be accessible to those who do not have a deep understanding of communication theories and principles. Obviously, more exploration and research needs to be done in order to fully develop the model into something that can be translated into a form that can be helpful to teachers and students in all mathematical situations. The important next steps are to see how to refine and improve upon the model so that suggestions can be made from here that would benefit teachers and students to help them improve upon the communication and discourse in their classrooms that is needed and required for successful teaching and learning of mathematics.

In an ideal study, the writing tasks would be better crafted with directed emphasis on providing explanations in order for the audience to achieve understanding. This aspect would be present in the homework assignments, group work, and lab exercises. It would
not be necessary for every single assignment to be crafted so that the students were
providing complete explanations in order to achieve understanding of a lesser-knowing
other, rather having an awareness between communication/writing task and explanations
would supply a better context for the course. In terms of data collection and analysis, a
content analysis of the writing assignments and prompts along with examining the
accompanying written explanations throughout the course would allow for a better
understanding of the context.

Implications

Naturally with educational research, the question of implications for teaching and
learning arises. In the case of this study, I am left with the possibility of asking even more
questions about this research. Some of these relate to further research goals, which will
be discussed in the following section, yet other questions relate to the communication
model. Since the model represents a way of examining and codifying explanations along
a continuum with each category representing varying degrees of detail and reasoning, it
can be used to give both teachers and students a goal to reach depending on the
expectations of the class. Earlier in the discussion, I made the claim that the three
categories of explanations in this model, algorithmic, structural, and transformative, are
equally correct in terms of the actual providing of an explanation. None of the three is
any more correct or incorrect as a type of explanation. Where the hierarchical notion
comes in is in the sophistication and detail that the student provides. With this in mind,
the model has the potential to be used in any mathematics classroom to serve as an
exemplar for students and teachers. This can enlighten the members of the classroom in
that it can show them the variation that exists in the simple act of answering a
mathematics question. In actuality, they can see that providing an explanation may not be all that simple of a process.

As an exemplar for students and teachers, this model can be the impetus that helps to guide the discourse in a classroom. This can provide the avenue with which teachers may help students “talk about talking about mathematics” (Cobb, Wood, & Yackel, 1993). This is the notion where teachers and students make the communication and discourse in the classroom the object of discussion. One way to use this in an activity would be to give students similar problems to the types in this study that are appropriate for their level of understanding while giving them the same instructions. Then the students can have the opportunity to share their explanations with each other while generating lists of similarities and differences. This can then be compared to the model from this study, or the model can serve as a point of departure. Both the sharing and the comparing of explanations allows students to “talk about talking about mathematics” without trying since the model is already established and would give students ideas on how to go about discussing their explanations. In conjunction with this activity, the teacher may intervene or guide students to the type of explanation s/he expects from the class in the future, which may even take them beyond the model. Either way, this activity would give students structure on how to have discussions in their writing. Activities such as this one coincide with how NCTM and other researchers such as Cobb and his colleagues envision the role of communication in the mathematics classroom.

Along similar lines of the previous activity, the generation of this model implies that teachers should be aware that there are differences in the way their students can express themselves. They may be aware that it is certainly possible for students to
express themselves differently, but actually being able to recognize and codify it themselves, might be a different issue. Having the general mathematics education community made aware of the qualitative differences between student explanations is an activity that this model can help in facilitating. In fact, with further iterations and trials of the model, we might be able to develop something that can be given to teachers and students that would serve as guiding principles in communicating about mathematics. This gets to the central motivation for this study which is, if NCTM is going to encourage communication in the classroom or that there needs to be communication between students and between teacher and student, then we need to give teachers enough background and/or knowledge on what mathematical communication is. I think the average mathematics teacher and mathematics educator translates this reform idea into the notion that students need to talk to each other, or at the very least they need to talk more and the teacher needs to listen more. There is, of course, the work by Cobb and his colleagues that focuses on creating classroom cultures where students and teacher collectively decide on what constitutes acceptable explanation or not, and other sociomathematical norms; but in general, there should be some guidelines to help mathematics educators fully achieve the ideal of what NCTM recommends. I think that this study is getting to that. I do not think it fully addresses this issue, but it is a start.

Future Research Directions

After conducting this research study and developing the communication model, it seems as though there is a wealth of further questions and directions in which this research could proceed. An obvious place to start within this line of research would be to test the model or develop similar models with students at various levels with level-
appropriate questions. This would help in the refinement of the model to be able to compare other levels of students or even to compare with other models. Thus we could attempt to build a comprehensive representation of similar types of tasks and the accompanying explanations.

It seems as though a study that examines the relationship between understanding and communicating that understanding would be worth conducting. Does knowing a concept well influence a person’s ability to communicate that concept? While this question should have an answer, I am not certain that we are able to answer it yet. That study would need to use some kind of mode or system similar to the one in this study or similar to O’Keefe (1988) to define what “communicating the concept well” means and identify exemplars of that. It may be possible that knowing and understanding a concept have little to do with how a person communicates because we do not know if there is a relationship between effective general communication and effective mathematical communication.

Using this study as a point of departure for future research studies, a study that would be worth investigating would be to conduct another discourse analysis with the course materials and assignments from the two approaches of this study. The expectations that were established in each of the courses have the potential to influence the ways in which the students proceeded with the writing they offered on the final exams. Finding similarities and commonalities would provide further insight into the variations in the communication model. In this study, the instructor for the NT section created his own course materials that served as the text for the course. A discourse analysis on this material could provide insight not only to the explanations, but also the
types of problems and questions to which the students were exposed. Since the NT section in this study had a lower percentage of success on the routine problems, but not on the non-routine problem, the discourse analysis could help to explain that phenomenon. A similar study would involve selecting writing samples from the students at different points throughout the course. Instead of analyzing the materials, the point of the study would be to analyze writing samples to see if the types of explanations the students use stay the same or change in some way. It would be interesting to see if the students would start the quarter using *algorithmic* explanations, move to using *structural* explanations later, and complete the course final exam using *transformative* explanations. This might say something about the instructors and the types of explanations they use or the type that they expect.

A likely direction this research could take would be to use this model with the explanations of teachers. A study might involve asking teachers the same types of questions to see what type of explanations they use. Of course, they would not have the same communicative burden of trying to write an explanation so that they received credit for the question, as was the case for the students in the current study. For that study the directions would have to express to the teachers that they must write as if they were the students, or to ask them to provide an explanation that they would expect to be completely correct with the fullness of details and reasoning. This type of study would allow us to see where the teachers’ thinking is relative to the types of explanations they would expect of their students.

During my grounded analysis, I made several notes where I thought that there might have been some kind of connection between the solution strategy and the written
explanation. A further analysis of this relationship could show the distinctions in reasoning and the subsequent expression of that reasoning. The potential relationship might be more evident in the routine problems as opposed to the non-routine problems that have more ways of solving. Another aspect worth further investigation that I noted during the grounded analysis would be to make a deeper comparison, if possible, with the message design logics developed by O’Keefe (1988). Her design logics are hierarchical in nature and also involve the analysis of written responses to a prompt. Because the different categories in this model form different levels of sophistication, O’Keefe’s message design logics might be useful.

More Questions and Suggestions.

There are some questions that have the potential to motivate further inquiry and study. Since this study was done with written explanations, similar research should be done with verbal explanations of both teachers and students. It is possible, but not yet known, that this model can translate to a verbal situation. Once the research gets to that point many other avenues for research will open. If the assumption is made that this model may be a way to codify explanations, what type(s) of explanations should teachers model in the classrooms? Which type of explanation should students and teachers use that would coincide with the NCTM Standards in the different situations of students explaining to their peers, students explaining to teachers, and teachers explaining to students? As a community, is there a type of explanation that we recommend our students to use? This is to say, would any of these explanations be present if specific communication standards were created? What would be a way that we could teach mathematical communication and explanations to students, either through teacher
modeling or in a text format? The mathematics education community, as well as all mathematics teachers, should be aware of the differences, and we should develop ways of incorporating communicative aspects of teaching and explaining into teacher education. We should have people conducting communication-centered research such as this study at the various mathematics levels of elementary school, middle school, high school, and college.

If I were to make some immediate recommendations directly from this study, it would be to examine the model and the role of the transformative explanation in the verbal teaching and learning of mathematics as it is now. My guess is that the transformative explanation is the direction in which we want teachers to use when they teach, and consequently the type of explanation they would want students to use. Although the model developed in this study emerged from student writing, the more important transition should be to verbal explanations because not every mathematics teacher utilizes writing in their instruction, but every mathematics classroom involves verbal communication even if it is one-way communication (from teacher to students).

A final aspect of this study that is unknown and would be worth investigating in the future is whether transformative explanations might be linked to or reflect greater understanding of the material. It seems plausible that the deeper someone understands a concept, the better the person could communicate the concept. The answer to this question has the potential to positively impact the teaching, learning, and assessment of mathematics, but I am not yet convinced that better understanding impacts better communication. Take for example the anonymous college mathematics professor who students perceive as not being able to explain well. Clearly the professor has a deep
understanding of the material, but may not be able to communicate the concepts to the specific student audience. In this instance, it is a case of a person’s communicative ability and not knowledge of the material. Of course for students communicating a mathematics concept to another person, the case is different. The potential exists for a student who has a better understanding of a concept to communicate it better. I think that research in this area must take into consideration general communicative ability and mathematical communicative ability, however this distinction has yet to be made or examined.
APPENDIX A

MATHEMATICS CONTENT TEST

1. How many numbers are in the set \{3, 7, 11, 15, 19, \ldots, 207\}? Show your work and provide an explanation.

2. Last year you paid $100 for a plane ticket to Chicago. Since then the price of plane tickets has increased 120%. How much will you pay this year? Show your work and provide an explanation for your response.

3. A class was asked to add three 6’s together. This was followed up by a class discussion of the different ways that the individual pupils did it. Several pupils reported that they added 6 +6 to get 12. Then there was one more 6 to add in. They did this by taking the digit 1 from the 12 and putting it next to the remaining 6, obtaining 16. Then the digit 2 that remained from the 12 was added to the 16 to get 18. Explain why this gives the correct result.

4. A recipe that serves 6 persons requires one-and-a-half quarts of milk. How many one-quart containers of milk will you have to buy if you want to make enough for 22 persons? Show your work and provide an explanation for your response.

5. Give two examples of “word problems” which illustrate the subtraction problem; “17-12”. The examples must illustrate two fundamentally different ways of thinking about subtraction.

6. Which number is larger, \(\frac{4}{7}\) or \(\frac{5}{8}\)? Show two different ways of proving your answer (You may not use a calculator).
LIST OF REFERENCES


Lather, P. (2002). Personal communication.


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