ESSAYS ON SLUGGISHNESS IN MACROECONOMICS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

Takayuki Tsuruga, B.A., M.A.

* * * * *

The Ohio State University

2005

Dissertation Committee:

Paul Evans, Adviser
Bill Dupor
Masao Ogaki

Approved by

Adviser
Graduate Program in Economics
© Copyright by

Takayuki Tsuruga

2005
ABSTRACT

My dissertation studies sluggishness in macroeconomics. It consists of three essays.

The first essay (Chapter 1) develops a model which can explain the hump-shaped impulse response of inflation to a monetary shock. A standard New Keynesian (NK) model is augmented to include a dynamic externality as well as sticky wages and variable capital utilization. In simulations, I assume purely forward-looking nominal rigidities in nominal prices and wages à la Calvo. Nevertheless, we can show that inflation is hump-shaped under a reasonable range of parameters. It will be also shown that, in order for inflation to be hump-shaped, sticky wages and variable capital utilization are important as well as a dynamic externality.

The second essay (Chapter 2) studies the role of the information updating scheme in the sticky information model. I compare the predictions of the sticky information model under two different information updating schemes: Mankiw and Reis’ original updating scheme and an updating scheme where all firms do so deterministically. On the surface, both models are reasonable a priori. However, the sticky information model under our alternative scheme suffers from diminished persistence and a reduced hump-shape in its impulse response function for inflation. The attractive results in Mankiw and Reis critically depend on their information updating scheme.
Third essay (Chapter 3) extends the sticky information model by Mankiw and Reis. They apply the sticky information assumption to a model in which everything is originally static and the optimal choice is made in a static manner. In contrast, this essay considers a dynamic model in which the optimal choice is originally made dynamically due to adjustment costs. This essay applies the sticky information assumption to this type of dynamic model. A useful aggregation result with sticky information is shown. Using the neoclassical model of investment, fixed investment tends to have a hump-shaped response to a shock to profitability and to have additional persistence notwithstanding the model’s simplicity.
Dedicated to my wife, Satoe.
First and foremost, I would like to appreciate Paul Evans for his invaluable advice, encouragement and patience. He read all drafts of the chapters, suggested many ways to improve them, and pointed out many mistakes in earlier drafts of the chapters. Without his help, I could never have completed this dissertation.

I would also like to thank my dissertation committee members. Bill Dupor’s advice and comments on Chapter 1 led to great improvement of the earlier version. He and I shared much time to discuss the model in Chapter 2. I learned invaluable lessons through discussions with him. Masao Ogaki gave advice and comments from empirical viewpoints. His encouragement was also helpful.

My thanks go to Pok-sang Lam, Joseph Kobaski and Huston McCulloch in the department. Pok-sang Lam, who is my candidacy committee member, gave suggestions on Chapter 1. Joseph Kobaski has devoted his time to discussions on a draft of Chapter 1. Huston McCulloch kindly gave comments on Chapter 1 and 2. Furthermore, I would like to thank Ricardo Reis at Princeton University and Kenneth West at University of Wisconsin - Madison for their helpful comments on Chapter 2.

I also thank Ryo Kato, Takuji Kawamoto and Shinichi Nishiyama at Bank of Japan and Yoshiaki Ogura at Columbia University, Leonard Kiefer, Junhee Lee, Virgilu Midrigan, Tohkir Mirzoev and Toyoichiro Shirota for many helpful discussions and comments. Ryo Kato and Shinichi Nishiyama offered me an invaluable support.
during my 1st and 2nd year and gave me opportunities for discussions even after their graduation. Without them, my dissertation would have been totally different and much worse than this dissertation. Takuji Kawamoto, Yoshiaki Ogura and Toyooichiro Shirota gave me helpful comments on an earlier draft of the first essay. The competition and friendship with Junhee Lee, Virgiliu Midrigan and Tohir Mirzoev always offered me a strong motivation for my research. Leonard Kiefer found the time for interesting discussion as well as for proofreading Chapter 3. I am also indebted to Hankyoung Sung, who is one of my best friends in the department, for his friendship.

Finally but certainly not least, I would like to express my appreciation to my parents and family. My parents, Tsugio Tsuruga and Sayoko Tsuruga, gave me a chance of my graduate study in Ohio State University. My son, Kota, made my life in Ohio pleasant. Special thanks are given to my wife, Satoe Tsuruga, for her unconditional support in countless aspects. My graduate studies could not have reached to this stage without her support. Therefore, this dissertation is dedicated to her.
VITA

December 26, 1972 ..................... Born - Aichi, Japan

1996 .................................. B.A. Economics, Waseda University

1998 .................................. M.A. Economics, Waseda University

2002 .................................. M.A. Economics,
The Ohio State University

2002-present ........................ Graduate Teaching Associate,
The Ohio State University.

PUBLICATIONS


FIELDS OF STUDY

Major Field: Economics
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Vita</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xi</td>
</tr>
<tr>
<td>Chapters:</td>
<td></td>
</tr>
<tr>
<td>1. Hump-shaped Behavior of Inflation and Dynamic Externality</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 The model</td>
<td>5</td>
</tr>
<tr>
<td>1.2.1 Firms</td>
<td>6</td>
</tr>
<tr>
<td>1.2.2 Households</td>
<td>8</td>
</tr>
<tr>
<td>1.2.3 Closing the model</td>
<td>12</td>
</tr>
<tr>
<td>1.2.4 Model solution and parameters</td>
<td>13</td>
</tr>
<tr>
<td>1.3 Effects of a money growth rate shock</td>
<td>16</td>
</tr>
<tr>
<td>1.3.1 Stylized facts and unit labor costs</td>
<td>16</td>
</tr>
<tr>
<td>1.3.2 The role of the dynamic externality</td>
<td>18</td>
</tr>
<tr>
<td>1.3.3 The role of sticky wages and variable capital utilization</td>
<td>24</td>
</tr>
<tr>
<td>1.4 Robustness</td>
<td>26</td>
</tr>
<tr>
<td>1.4.1 Habit formation and adjustment cost of capital</td>
<td>27</td>
</tr>
<tr>
<td>1.4.2 Robustness to returns to organizational capital</td>
<td>31</td>
</tr>
<tr>
<td>1.5 Why is inflation hump-shaped in CEE?</td>
<td>32</td>
</tr>
<tr>
<td>1.6 Conclusion</td>
<td>36</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>2. Sticky Information: The Impact of Different Information Updating Assumptions</td>
<td>38</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>38</td>
</tr>
<tr>
<td>2.2 Different updating schemes</td>
<td>41</td>
</tr>
<tr>
<td>2.2.1 Random duration updating</td>
<td>41</td>
</tr>
<tr>
<td>2.2.2 Fixed duration updating</td>
<td>43</td>
</tr>
<tr>
<td>2.3 Calibration</td>
<td>44</td>
</tr>
<tr>
<td>2.3.1 The effect of a change in the growth rate of nominal expenditure</td>
<td>45</td>
</tr>
<tr>
<td>2.3.2 Anticipated disinflation</td>
<td>51</td>
</tr>
<tr>
<td>2.4 Concluding remarks</td>
<td>53</td>
</tr>
<tr>
<td>3. Sticky Information under Dynamic Models</td>
<td>55</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>55</td>
</tr>
<tr>
<td>3.2 The Model</td>
<td>60</td>
</tr>
<tr>
<td>3.2.1 Set up</td>
<td>60</td>
</tr>
<tr>
<td>3.2.2 Sticky information</td>
<td>63</td>
</tr>
<tr>
<td>3.2.3 Aggregation</td>
<td>67</td>
</tr>
<tr>
<td>3.3 Simulating the model</td>
<td>75</td>
</tr>
<tr>
<td>3.3.1 Parameter choice</td>
<td>75</td>
</tr>
<tr>
<td>3.3.2 Investment dynamics</td>
<td>76</td>
</tr>
<tr>
<td>3.4 Concluding remarks</td>
<td>83</td>
</tr>
</tbody>
</table>

Appendices:

A. The response of output and inflation under the fixed duration scheme 85
   A.1 Impulse response functions under the fixed duration scheme 85
   A.2 Disinflationary policy under the fixed duration scheme 87

B. The log-linearized solution 90

C. Proof of propositions 92
   C.1 Proof of proposition 3.2.1 92
   C.2 Proof of proposition 3.2.3 100

Bibliography 102
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Calibrated parameters in the model</td>
<td>15</td>
</tr>
<tr>
<td>1.2 Ingredients for hump-shaped inflation in CEE and the model</td>
<td>36</td>
</tr>
<tr>
<td>2.1 Autocorrelations for inflation</td>
<td>48</td>
</tr>
<tr>
<td>2.2 Autocorrelations for output</td>
<td>48</td>
</tr>
<tr>
<td>3.1 The quarter of peak response in the investment-capital ratio: different values of $c$</td>
<td>80</td>
</tr>
<tr>
<td>3.2 The quarter of peak response in the investment-capital ratio: different values of the degree of inattentiveness</td>
<td>80</td>
</tr>
<tr>
<td>3.3 The quarter of peak response in the investment-capital ratio: different values of persistence of the profitability</td>
<td>81</td>
</tr>
<tr>
<td>3.4 Autocorrelations for the investment-capital ratio</td>
<td>82</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>17</td>
</tr>
<tr>
<td>1.2</td>
<td>18</td>
</tr>
<tr>
<td>1.3</td>
<td>19</td>
</tr>
<tr>
<td>1.4</td>
<td>23</td>
</tr>
<tr>
<td>1.5</td>
<td>25</td>
</tr>
<tr>
<td>1.6</td>
<td>28</td>
</tr>
<tr>
<td>1.7</td>
<td>30</td>
</tr>
<tr>
<td>1.8</td>
<td>33</td>
</tr>
<tr>
<td>2.1</td>
<td>47</td>
</tr>
<tr>
<td>2.2</td>
<td>50</td>
</tr>
<tr>
<td>2.3</td>
<td>52</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.1 Histories of information updating and distribution of capital stocks under the random duration scheme</td>
<td>70</td>
</tr>
<tr>
<td>3.2 Histories of information updating and distribution of capital stocks under the fixed duration scheme</td>
<td>73</td>
</tr>
<tr>
<td>3.3 Impulse response functions: the investment-capital ratio to a one standard deviation shock to a measure of profitability</td>
<td>77</td>
</tr>
</tbody>
</table>
CHAPTER 1

HUMP-SHAPED BEHAVIOR OF INFLATION AND DYNAMIC EXTERNALITY

1.1 Introduction

Sticky prices are one of the most important elements in the New Keynesian model (NK model) and the policy analysis based upon it. Under nominal rigidities á la Calvo (1983) or á la Rotemberg (1982), an expression for inflation can be obtained in a very simple form called the New Keynesian Phillips Curve (NKPC). It has been one of the fundamental equations for the analysis of the monetary policy as discussed in Clarida, Gali, and Gertler (1999).

The NKPC is theoretically appealing because it can be derived from a rational expectation model with staggered price contracts and gives us intuitive descriptions of the supply side in the economy. Despite its theoretical appeal, however, the NKPC has been subject to criticism due to its counterfactual predictions. For example, Fuhrer and Moore (1995) and Fuhrer (1997) point out that NKPC predicts the expected change in inflation must decrease when the output gap is positive. Nelson (1998) concludes that standard Calvo (1983) type staggered price setting cannot generate a
hump-shaped impulse response function (IRF hereafter) that estimated VARs imply.\footnote{Delayed responses of inflation to a monetary policy shock can be seen from the VAR literature. Stock and Watson (2001) ran a simple VAR with the inflation rate, the unemployment rate and the federal funds rate, and concluded the responses of inflation to federal funds rate shock is delayed. Gali (1992) estimated a structural VAR with long-run and short-run restrictions. His IRF of inflation to an M1 shock is hump-shaped and its peak is eighth periods after the monetary policy shock.}

Mankiw and Reis (2002b) report similar results: In the sticky price model, monetary policy shocks have their maximum impact on inflation immediately.\footnote{In some exceptional cases, Taylor (1980) type nominal rigidities seem to be able to generate hump-shaped impulse response for price inflation. For example, Erceg (1997) uses Taylor type staggered wages and flexible prices to show that inflation can be hump-shaped in response to a monetary shock, although the reason hasn’t been explored clearly.}

In general, the literature has considered two ways of extending the NKPC to generate a hump-shaped IRF for inflation to monetary policy shocks. First, the inclusion of lagged inflation in the equation can yield a hump-shaped IRF for inflation. Fuhrer and Moore (1995) propose relative wage contracting, which allows inflation to be a function of lagged inflation.\footnote{Nelson (1998) reports that Fuhrer and Moore (1995)’s expression for inflation is the only one in which the inflation response could be hump-shaped.}

Roberts (1997), Roberts (2001) and Ball (2000) stress the importance of less than perfectly rational economic agents who expect the future inflation by univariate forecasting with lagged inflation. Gali and Gertler (1999) estimated a hybrid NKPC, which assumes that a fraction of the firms determines its price according to a backward looking rule of thumb. Christiano, Eichenbaum, and Evans (2005, CEE) derive a hybrid NKPC with many empirically supported frictions and succeed in accounting for the observed sluggishness in inflation and the output gap.\footnote{Although they conclude that their model performance for inflation is not substantially affected by removing backward-looking indexation, their conclusion also relies on other frictions in their model. We discuss this point in the section 1.5.}

However, these efforts to include lagged inflation are hard to defend and less than convincing, because they have no clear theory explaining why economic agents
expect future inflation by a univariate forecasting rule, why a fraction of economic agents are non-rational and why monopolistically competitive firms follow the backward indexation rule specified in CEE.

Second, learning or delayed information adjustment can generate a hump-shaped response of inflation to monetary policy shocks. Erceg and Levin (2003) focus on the imperfect information between the private sector and the central bank. They model serially correlated forecast errors with learning through Kalman filtering. Although Erceg and Levin (2003) did not show a hump-shaped IRF for inflation, Keen (2003), following the same idea as Erceg and Levin (2003), showed the inflation is hump-shaped in response to a monetary shock. Moreover, Dellas (2004) assumes that economic agents observe some of variables such as the output gap and inflation with error. Using a Kalman filter, he reports hump-shaped IRFs for the output gap and inflation under this imperfect information.

This paper explores another possible explanation for the hump-shaped response of inflation. In the paper, I assume neither a hybrid NKPC nor Kalman filtering. Hence, there is neither non-rationality nor lags in decision making. Instead, I introduce a dynamic externality into a NK model. The dynamic externality occurs through a production spillover in which the stock of organizational capital accumulates over time according to the level of aggregate output. The paper shows that a dynamic externality can be a powerful mechanism for generating a hump-shaped IRF for inflation if it is combined with sticky wages and variable capital utilization.

In the RBC literature, a number of papers have analyzed the effect of organizational capital as a propagation mechanism. For example, Cooper and Johri (1997)
focused on dynamic complementarities which are external to an individual firm. Similarly, Cooper and Johri (2002) and Chang, Gomes, and Schorfheide (2002) study the effect of learning-by-doing as a propagation mechanism in a RBC model. In a NK model, on the other hand, organizational capital may play a more important role for inflation than for output, because changes in organizational capital directly affect firms’ marginal costs via changes in the productivity. In their analysis, the learning-by-doing is assumed to be internal rather than external. However, the dynamics of organizational capital are extremely similar to our case in that organizational capital (or human capital in their context) accumulates over time according to the level of production activity.

The intuition behind a hump-shaped IRF for inflation is as follows. Expansionary monetary shocks generate two effects on marginal cost. The first effect operates through factor prices. The increased demand for goods raises the demand for inputs to increase, thereby bidding up factor prices and increasing marginal cost. The second effect operates through production spillovers, which increase productivity and reduce marginal cost. The second effect at least partially offsets the first effect and may actually reduce marginal cost in the short-run. The intermediate-run increase in marginal cost can be moderated and delayed if sticky wages and variable capital utilization slow down the increase in factor prices. Given this marginal cost behavior, forward-looking firms raise their current price only moderately in the short-run because they weight on both of the short-run decreases and intermediate-run increases in marginal cost in their determination of their current price. In the future, they will raise their price appreciably because they will no longer be putting any weight on the short-run decreases in marginal cost which lie in the past. As a result, expansionary
monetary shocks have a delayed impact on inflation under a purely forward-looking NKPC.

The rest of the paper is organized as follows. In section 1.2, I present the specific model used in the simulations. The section 1.3 shows that the IRF of inflation to a money growth shock is hump-shaped and explains the mechanism underlying that shape. The model therefore qualitatively replicates the stylized facts on the estimated IRF. The model with a dynamic externality also explains the observed IRF of marginal cost better than one without it. In the section 1.4, we consider the robustness of hump-shaped inflation to real frictions and parameters. We find that it is quite robust to these changes in the model. The section 1.5 explains the differences between our model and CEE’s, focusing on the behavior of inflation. The section 1.6 concludes the paper.

1.2 The model

In this section, we describe the model economy. We assume monopolistic competition in both the goods and labor markets as in Erceg, Henderson, and Levin (2000, EHL). The model consists of a representative goods aggregator, a representative labor aggregator and a government as well as monopolistic competitive firms and monopolistic competitive households. To include sticky prices and wages, we assume that the nominal price and wage adjustments are possible only at some constant hazard rate. This Calvo (1983)-style timing of the nominal rigidity gives us a NKPC for both price and wage inflation.
1.2.1 Firms

Following the literature, we introduce an output aggregator with constant-returns to scale technology of the Dixit-Stiglitz form and intermediate good firms under monopolistic competition. An output aggregator produces a final good $Y_t$ for household’s consumption and investment in the perfect competitive market. The final good is a transformation of a continuum of differentiated goods, each of which is produced by a single monopolistic firm. Under these assumptions, the demand function for intermediate goods takes the following form:

$$Y_t(f) = \left[ \frac{P_t(f)}{P_t} \right]^{-\epsilon_p} Y_t,$$

where $Y_t(f)$ denotes a differentiated good and $P_t(f)$ is its price. $P_t$ is the aggregate price index. $f$ is the index for intermediate good firms distributed uniformly on [0,1]. $\epsilon_p > 1$ is the elasticity of substitution between the differentiated goods.

We assume Calvo type staggered price setting so that each firm is allowed to change its price only with a probability. Instead of deriving it, we simply start with the NKPC that has been derived in the literature from that assumption. Let $\pi_t$ denote the gross inflation rate $\pi_t = P_t/P_{t-1}$ and $\hat{\pi}_t = \log(\pi_t) - \log(\pi)$, where $\pi$ is the steady state value of the gross rate of inflation. Then, the NKPC is given by

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Psi_p \hat{mc}_t,$$

where $\Psi_p$ and $\beta$ are parameters satisfying $\Psi_p > 0$ and $0 < \beta < 1$ and $\hat{mc}_t$ is the log-deviation of marginal cost from the steady state value.

The intermediate good firm faces perfectly competitive factor markets for the effective capital input (defined below) $\tilde{K}_t(f)$ and the labor input $L_t(f)$, which it
rents in competitive factor markets. For this reason, each intermediate good firm takes the rental price of effective capital, $R^k_t$, and the aggregate wage index, $W_t$, as given.

Suppose that the production function for firm $f$ is Cobb-Douglas in effective capital and labor:

$$Y_t(f) = \tilde{K}_t(f)^\alpha L_t(f)^{1-\alpha} X^\phi_t,$$

where $\alpha \in (0, 1)$ and $X_t$ is external organizational capital. The steady state value of $X$ is assumed to be one, so that production function converges to constant returns to scale in the long run.

Organizational capital accumulates through production following the law of motion:

$$\log(X_t) = \gamma \log(X_{t-1}) + \eta \log \left( \frac{Y_t}{Y} \right),$$

where $\gamma \in (0, 1)$ captures the persistence of the external effect and $\eta > 0$ captures the effect of current aggregate output on individual production. $Y$ is the steady state level of aggregate output. Thus, there is a spillover effect in the production process.

Given the production function and the assumption of perfectly competitive factor markets, the real marginal cost function $mc_t$ and the marginal rate of substitution between labor and effective capital from the static cost minimization problem take the form:

$$mc_t = (1 - \alpha)^{(1-\alpha)} \alpha^{-\alpha} w_t^{1-\alpha} (r^k_t)^\alpha X^{-\phi}.$$

$$\frac{w_t}{r^k_t} = 1 - \alpha \frac{\tilde{K}_t}{\alpha \bar{L}_t},$$

5It is not a contradiction to the assumption of monopolistic competitive households in their labor market. The households sell the labor to the labor aggregator in monopolistic competitive markets, but the labor aggregator sells its aggregate labor to the intermediate good firms in a competitive market. For this reason, we may assume the intermediate goods firm face a competitive labor market.
where $w_t$ is real wage rate (i.e. $w_t = W_t/P_t$) and $r^k_t$ is real rental cost of effective capital (i.e. $r^k_t = R^k_t/P_t$). Note that the index $f$ is dropped because all intermediate good firms face identical factor prices.

### 1.2.2 Households

Each household, indexed by $h \in (0, 1)$, is assumed to supply a differentiated labor service to firms. As in EHL, we assume a representative labor aggregator which buys households’ differentiated labor supply $L_t(h)$ to produce a single composite labor service $L_t$ which it sells to intermediate good firms. This formation is parallel to the output aggregator. Hence, we get the demand function for the differentiated labor:

$$L_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-\epsilon_w} L_t,$$

where $\epsilon_w > 1$ is the elasticity of substitution between the differentiated labor. $W_t(h)$ is the nominal wage for differentiated labor.

We set up the household’s maximization problem. We assume the following expected utility function of the money-in-utility form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t(h)^{1-\sigma_C}}{1-\sigma_C} - \frac{L_t(h)^{1+\sigma_L}}{1+\sigma_L} + \log \left( \frac{M_t(h)}{P_t} \right) \right],$$

where $C_t(h)$ is the consumption and $M_t(h)$ is the end-of-period money holding.

Next, let us consider the household’s budget constraint. It is given by

$$W_t(h)L_t(h) + R^k_t \tilde{K}_t(h) + \Gamma_t(h) + T_t(h)$$

$$= P_t \left[ C_t(h) + I_t(h) + a(U_t(h))K_t(h) \right] + B_t(h) - R_{t-1}B_{t-1}(h) + M_t(h) - M_{t-1}(h).$$

On the income side, his source of income is labor income $W_t(h)L_t(h)$, returns from effective capital service $R^k_t \tilde{K}_t(h)$, the sum of the profits from the firms in the economy
\( \Gamma_t(h) \), and a lump-sum transfer from the government to the household \( T_t(h) \). Effective capital \( \tilde{K}_t(h) \) is defined as the product of the actual capital stock \( K_t(h) \) and capital utilization \( U_t(h) \):

\[
\tilde{K}_t(h) = U_t(h)K_t(h).
\] (1.10)

On the spending side of the budget constraint, the household purchases the final goods for consumption and investment. In utilizing the actual capital \( K_t(h) \), he loses final goods in the form of the capital utilization costs given by \( a(U_t(h))K_t(h) \). The function \( a(U_t(h)) \) is assumed to be increasing and convex in \( U(h) \). (i.e. \( a'(\cdot) > 0, a''(\cdot) > 0 \).) We assume that the cost is zero when the utilization rate is equal to the steady state value of one (i.e. \( a(1)=0 \)). Finally, he spends his income for financial assets in the form of the net increase in money \( (M_t(h) - M_{t-1}(h)) \) and one period nominal bonds \( (B_t(h) - R_{t-1}B_{t-1}(h)) \). We assume a constraint \( B_t(h) > -\bar{B} \) for some large positive number \( \bar{B} \).

In the formulation of the capital accumulation equation, we follow Christiano and Fisher (1998, CF).\(^6\) The actual capital stock evolves according to

\[
K_{t+1}(h) = F((1 - \delta)K_t(h), I_t(h)) = [a_1(1 - \delta)^\nu K_t(h)^\nu + a_2I_t(h)^\nu]^{\frac{1}{\nu}},
\] (1.11)

where \( \nu \leq 1 \). As in CEE and CF, we can choose the parameter \( a_1 \) and \( a_2 \) so that \( \nu \) doesn’t affect the steady state properties of the economy. By setting \( a_1 = (1 - \delta)^{1-\nu} \) and \( a_2 = \delta^{1-\nu} \), it can be guaranteed that 1) \( I_t(h) = \delta K_t(h) \) in the steady state, 2) Under the special case of \( \nu = 1 \), the equation (1.11) corresponds to the standard law of motion of capital and 3) when \( \nu < 1 \), it implicitly assumes convex capital

\(^6\)CEE use the adjustment cost of investment rather than the adjustment cost of capital. As CEE themselves point out, it is more typical to use capital adjustment cost in the business cycle literature. We avoid using investment adjustment cost because the adjustment cost of investment adds a new state variable to the model. Thus, this new state variable not only complicates the model but also makes the effect of dynamic externality on inflation and the output gap less clear.
adjustment cost function and 4) the first derivatives of the equation with respect to 
\((1 - \delta)K_t(h)\) and \(I_t(h)\) are one in the steady state, allowing us to derive the steady state value which is unaffected by the adjustment cost parameter \(\nu\).

We assume that every household faces the same initial conditions and that the contingent markets are complete. Then, we have the symmetric equilibrium value for control variables except for \(W_t(h)\). These assumptions allow us to drop the household index \(h\) for \(C_t(h), I_t(h), U_t(h), M_t(h), B_t(h),\) and \(K_{t+1}(h)\).

In order to make a decision for these variables, the household maximizes his expected utility function (1.8) subject to (1.9), (1.10) and (1.11). The first order conditions are as follows:

\[
1 = \beta E_t \left[ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \frac{R_t}{\pi_{t+1}} \right],
\]
\[
m_t = E_t \left[ \frac{R_t}{R_t - 1} \right],
\]
\[
\lambda_t = Q_t F_l((1 - \delta)K_t, I_t),
\]
\[
r_t^k = a'(U_t),
\]
\[
Q_t = \beta E_t \left[ \lambda_{t+1} \left( r_{t+1}^k U_{t+1} - a(U_{t+1}) \right) \right] + \beta(1 - \delta) E_t \left[ Q_{t+1} F((1 - \delta)K_{t+1}, I_{t+1}) \right],
\]

where \(m_t \equiv M_t/P_t\) is the real money holding at the end of the period \(t\). \(Q_t\) is the Lagrange multiplier for the capital accumulation equation (1.11) and \(\lambda_t\) is the marginal utility for current consumption:

\[
\lambda_t = C_t^{-\sigma_C}. 
\]
In the section 1.4, we introduce habit formation into the model in order to improve performance of the output gap. In this case, the marginal utility for current consumption $\lambda_t$ takes a form different from (1.17). To keep the structure of the model constant, we retain $\lambda_t$ in it.

These first order conditions are quite standard. Equation (1.12) and (1.17) imply a consumption Euler equation that equates the marginal utility of consumption today with the discounted marginal utility of consumption tomorrow. Equation (1.13) is the money demand function: Real money holding are negatively related to the marginal utility of consumption and the opportunity cost of holding money. Equation (1.14) is the first order condition for investment. The household equates the marginal utility of investment ($Q_t F_t(\cdot, \cdot)$) with the marginal utility of consumption by allocating his resources across consumption and investment. Equation (1.15) is the marginal condition for variable capital utilization. Utilizing more of the capital stock gives the household an additional income of $r^k_t$ per unit of capital stock but it requires he pay the marginal cost of $a'(U_t)$ per unit of capital stock. Equation (1.16) determines the shadow value of capital in terms of utility.

**Wage setting**

We go to the wage setting behavior. We assume that the nominal wage contracts are analogous to the price setting behavior. In each period, the household is allowed to reoptimize its nominal wages with a probability. As in EHL, Calvo type staggered wage setting gives us the following wage NKPC to a first order approximation:

$$\hat{\pi}_w t = \beta E_t \hat{\pi}_w t+1 + \Psi_w \left[ \sigma_L \hat{L}_t - \hat{\lambda}_t - \hat{\omega}_t \right], \quad \Psi_w > 0 \quad (1.18)$$
where $\hat{\pi}^w_t$ is the log-deviation of wage inflation from the steady state value. That is, $\hat{\pi}^w_t = \log(\pi^w_t) - \log(\pi^w)$, where $\pi^w_t = W_t/W_{t-1}$ and $\pi^w$ is the gross rate of wage inflation in the steady state. Similarly, $\hat{\lambda}_t$ and $\hat{\bar{w}}_t$ are the log-deviations from the steady state of the marginal utility of consumption and the real wage, respectively. Finally, $\Psi_w$ is a parameter. The first two terms inside the brackets are the log-deviation of the marginal rate of substitution between labor and consumption from the steady state. Thus, the difference between the marginal rate of substitution and the real wage affects the wage inflation rate.

### 1.2.3 Closing the model

To close the model, we specify the government budget constraint, the monetary policy and the market clearing condition. The government budget is balanced every period (i.e. $(M_t - M_{t-1}) = \int_0^1 T_t(h)dh$, for all $t$). Its total lump-sum transfer is set equal to seignorage revenue.

We specify monetary policy in terms of the growth rate of money supply. The gross growth rate of money supply $g_t \equiv M_t/M_{t-1}$ is given by AR(1) process in logarithms:

$$
\log(g_{t+1}) = \rho_m \log(g_t) + e_{t+1}, \quad e_t \sim iid,
$$

where $0 < \rho_m < 1$.

Market clearing condition is given by

$$
Y_t = C_t + I_t + a(U_t)K_t.
$$

We also have two model identities for the real wage rate and real balances:

$$
\begin{align*}
    w_t &= \frac{\pi^w_t}{\pi_t} w_{t-1}, \\
    m_t &= \frac{g_t m_{t-1}}{\pi_t}.
\end{align*}
$$
1.2.4 Model solution and parameters

Model solution

The log-linearized model is used to analyze the solution to the model. Since some of the equations such as (1.2) and (1.18) are already log-linearized, we take log-linearizations of other Euler equations and several model identities around the steady state. There are 16 equations to be log-linearized: (1.3)-(1.6), (1.10)-(1.17), (1.19)-(1.22). As a result, we obtain 18 log-linearized equations consisting of 18 unknowns which is log-linearized around the steady state value, $\hat{Y}_t$, $\hat{\pi}_t$, $\hat{\pi}_w$, $\hat{r}_k$, $\hat{R}_t$, $\bar{m}c_t$, $\hat{L}_t$, $\hat{U}_t$, $\hat{\bar{K}}_t$, $\hat{I}_t$, $\hat{Q}_t$, $\hat{\lambda}_t$, $\hat{C}_t$, $\hat{X}_{t-1}$, $\hat{K}_t$, $\hat{\bar{w}}_{t-1}$, $\bar{m}_{t-1}$ and $\hat{g}_t$.

It should be noted that the log-linearized model does not require any specific functional form on the variable capital utilization cost function $a(U_t)$ in (1.15), (1.16) and (1.20). Instead, it requires the elasticity of the variable capital utilization cost with respect to $U_t$ evaluated at the steady state value (i.e. $a''(1)/a'(1)$). We assume that the elasticity $\mu_a \equiv a''(1)/a'(1)$ is a constant.\(^7\)

The last five variables in the list of the variables are the state variables in the model. That is, lagged organizational capital, the capital stock, the lagged real wages, lagged real balances, and the money growth rate are predetermined or exogenous. Finally, the log-linearized system of equations has a unique equilibrium at the model parameters calibrated below.

\(^7\)When (1.15) is evaluated at the steady state, we obtain $r^k = a'(1)$ where $r^k$ is the steady state value of the real rental cost of capital. Using this steady state relation, the log-linearized version of (1.15) and (1.20) is given by $\hat{r}^k_t = \mu_a U_t$ and $\hat{Y}_t = \frac{\gamma}{\delta} \hat{C}_t + \frac{\delta}{\gamma} \hat{I}_t + r^k \frac{\psi}{\gamma} \hat{U}_t$, respectively, where any variable without a time subscript is the corresponding steady state value. Finally, $U_t$ in (1.16) affects the shadow price of the capital goods only to the second order terms. Hence, we need only the elasticity of $a(U_t)$ with respect to $U_t$ alone.
Calibration

We have parameters to be specified from outside the model. Since the model is calibrated at a quarterly frequency, we assume that $\beta = 0.99$. The preference parameters in the utility function follow the literature: $\sigma_C = 1$ and $\sigma_L = 2$. The elasticities of demand functions are $\epsilon_p = \epsilon_w = 11$, which implies 10% mark-up in the steady state. The parameters $\Psi_p$ and $\Psi_w$ in two NKPCs are given in a standard way: $\Psi_p = 0.0858$ and $\Psi_w = 0.0037$.

We assume $\delta = 0.025$, which implies 10% depreciation in a year in the steady state. The capital adjustment cost parameter $\nu$ in (1.11) is assumed to be $\nu = 0.23$ from CF. These parameters imply $a_1 = (1 - \delta)^{1-\nu} = 0.955$ and $a_2 = \delta^{1-\nu} = 0.0013$. The parameter of variable capital utilization $\mu_a$ is set to 0.01, following CEE.

As for the production side, we need to assign calibrated values for $\phi$, $\gamma$, $\eta$, and $\alpha$. Cooper and Johri (2002) estimate their production functions with organizational capital, using different sets of data. Among their specifications, the most useful for our purpose is IRS-PF (Increasing Returns to Scale in the Production Function). IRS-PF assumes a constant returns to the effective capital stock and labor and that $\eta = 1 - \gamma$. Their estimates of $\phi$ then range from 0.26 to 0.35 while the estimates of $\gamma$ range from 0.50 to 0.55. Considering their estimates, we take $\gamma = \eta = 0.5$ for the dynamics of $\hat{X}_t$ and $\phi = 0.26$ for returns to organizational capital. Finally, we assume that the total cost share for effective capital is 0.36; i.e. $\alpha = 0.36$.

\footnote{In the literature of the NK model, for example Gali (2002), $\Psi_p$ is a function of $\beta$ and the probability that firms can reoptimize their nominal price. Letting $1 - \xi_p$ be the probability, the parameter $\Psi_p$ is given by $\frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}$. $\Psi_p = 0.0858$ is standard, because $\xi_p = 0.75$ gives $\Psi_p = 0.0858$. Similarly, letting $1 - \xi_w$ be the probability that households can change their nominal wage, $\Psi_w$ is given by $\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\frac{1}{(1+\epsilon_w\sigma_L)}$. The values of $\xi_w = 0.75$, $\sigma_L = 2$ and $\epsilon_w = 11$ gives $\Psi_w = 0.0037$.}
The monetary policy parameter $\rho_m$ is assumed to be 0.5, as suggested as Christiano, Eichenbaum, and Evans (1998). The value of $\rho_m$ does not affect the qualitative results for the IRF analysis.

Table 1.1 summarizes the calibrated parameters.

<table>
<thead>
<tr>
<th>Preference parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_C$</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Real rigidities</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>11</td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>11</td>
</tr>
<tr>
<td>Nominal rigidities</td>
<td></td>
</tr>
<tr>
<td>$\Psi_p$</td>
<td>0.0858</td>
</tr>
<tr>
<td>$\Psi_w$</td>
<td>0.0037</td>
</tr>
<tr>
<td>Capital accumulation technology</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.23</td>
</tr>
<tr>
<td>$\mu_a$</td>
<td>0.01</td>
</tr>
<tr>
<td>Technology in the production function</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.26</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Money supply</td>
<td></td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1.1: Calibrated parameters in the model
1.3 Effects of a money growth rate shock

1.3.1 Stylized facts and unit labor costs

To confirm the stylized facts in the VAR literature, I estimated a simple VAR similar to Walsh (2002)'s. He ran a three-variable VAR (output, inflation and the money growth rate) to generate some stylized facts about the IRFs of inflation and the output gap. Into this simple VAR, I add the logarithm of unit labor cost as a proxy for marginal cost, because the behavior of marginal cost is critically important in our analysis.\(^9\)

Fig 1.1 shows the IRFs resulting from a one standard deviation shock to the money growth rate.\(^10\) The number of lags was selected to be three on the basis of AIC. As shown in Fig. 1.1, inflation and the output gap have hump-shaped IRFs: The peak for inflation occurs in the eighth quarter while the peak for the output gap occurs in the fourth quarter.

In certain circumstances, unit labor cost proxies well for marginal cost.\(^11\) The data therefore suggests that the IRF of real marginal cost falls for two quarters after a money growth rate shock and increases only after three quarters. The peak for unit labor cost occurs in the eighth quarter.

---

\(^9\)We could have run a much larger VAR to confirm the same stylized facts as CEE did. However, a VAR with many variables usually has large standard error bands and often lacks robust IRFs. For our purposes, it will be enough just to confirm the stylized facts about inflation and the output gap and to generate stylized facts about marginal cost.

\(^10\)I HP filtered the logarithm of real GDP and the logarithm of real unit labor cost to get the output gap and the marginal cost gap. I log-differenced the CPI and M2 to obtain the inflation rate and the money growth rate. The order of the variables in the VAR is the output gap, the inflation rate, the marginal cost gap and the money growth rate.

\(^11\)In order for this relation to hold, we need to assume Cobb-Douglas technology and free mobility of all inputs.
NOTE: IRF for inflation, the output gap and real unit labor cost in response to one standard deviation shock in money growth rate shock. The sample is from 1965:1 to 2002:4. The IRFs are estimated four variable VAR with lag three.

Figure 1.1: Estimated IRFs for inflation, the output gap and real unit labor cost
1.3.2 The role of the dynamic externality

Fig 1.2 and 1.3 show several IRFs implied by our model for an increase of 1% in the money growth rate in the benchmark case.

![Simulated IRFs for inflation, the output gap and marginal cost under dynamic externality](image)

Figure 1.2: Simulated IRFs for inflation, the output gap and marginal cost under dynamic externality

Fig 1.2 shows the simulated IRF for inflation, the output gap and the marginal cost. First, inflation is strongly hump-shaped as shown in the first panel of the figure. The response of inflation peaks in the 10th quarter after a monetary shock with slightly negative initial response. This slow and gradual increase in inflation is...
Figure 1.3: Simulated IRFs for nominal interest rate, productivity, consumption, investment and money growth rate under dynamic externality
qualitatively consistent with our IRF shown in Fig 1.1 and CEE’s VAR. Second, the output gap (shown in the second panel) peaks at the period of the monetary shock. This is not consistent with the VAR analysis, but in the next section, I will show that this counterfactual prediction can be easily fixed by introducing habit formation. Finally, the marginal cost in the third panel behaves interestingly. The log-deviation of marginal cost is negative from the period zero through the eighth period after a shock, returning to positive values only after the 9th period. As a result, the marginal cost decreases for several periods and then increases.

The IRFs of other variables of interest are shown in Fig. 1.3. As in standard NK models, we encounter the lack of a liquidity effect in the first panel in Fig. 1.3. Due to the dynamic externality, productivity shows a strong procyclical response to an expansionary monetary shock. Consumption and investment show a surge in responses in their IRF, which is the main reason for unreasonable response for the output gap.

To get the intuition behind hump-shaped IRF for inflation, consider IRF of marginal cost first. The real marginal cost at each period is decomposed into effects of factor prices and productivity, as shown in (1.5).

1. The effect of the real wage: marginal cost is higher, the higher the real wage.

2. The effect of the rental cost of effective capital: marginal cost is higher, the higher the rental cost of capital.

3. The effect of productivity through the dynamic externality: the higher the externality $X_t$, the lower marginal cost.
When the money growth rate increases, increased real balances cause the real interest rate to decrease, which in turn causes marginal utility to decrease from (1.12).\textsuperscript{12} To induce the decreased marginal utility, consumption must increase (see (1.17)) and the investment must increase because the marginal utility of investment must decrease from (1.14). Thus, the increase in consumption and investment in turn raises the demand for goods. To meet the increased demand for goods, firms need to hire more labor and capital. Thus, the real wage and rental cost of capital are bid upward. The increase in factor prices gives firms the incentive to increase price. However, in our model, organizational capital is accumulated due to the increased consumption and investment. For an individual firm, the increase in the organizational capital raises firm's productivity, because this effect is external to the firm. The higher productivity gives the firm the incentive to price low. Because the effects of productivity and factor prices are offsetting, the marginal costs may increase or decrease, depending on the dynamic structure of these three elements. In the simulation, the marginal cost decreases for several periods after the shock and then increases.

Now, the reason for hump-shaped IRF for inflation is straightforward. As a first approximation, let $\beta = 1$. Then, (1.2) becomes

$$E_t \hat{\pi}_{t+1} - \hat{\pi}_t = -\Psi_p \hat{\mu}_t.$$  

(1.23)

In our simulation shown in Fig. 1.2, $\hat{\mu}_t$ is \textit{negative} until the ninth quarter after a monetary shock. Thus, inflation is expected to increase and actually increases rather than decreasing over time, even though the output gap during these periods is \textit{positive}. After these periods, $\hat{\mu}_t$ becomes \textit{positive} until it converges to its steady

\textsuperscript{12}To see this, we solve the log-linearized equation of (1.12) forward. Because the marginal utility positively depends on the sum of short-term real interest rate, the decrease in real interest rate causes the marginal utility to decrease.
state level of zero, implying that inflation decreases over time. Inflation increases as long as $\dot{mc}_t$ is negative and decreases as long as $\dot{mc}_t$ is positive.\(^{13}\) This generates the hump-shaped response of inflation.

The hump-shaped IRF for inflation can be interpreted as follows. After an unexpected monetary shock, a firm observes higher demand for its goods and needs to set the price of its goods in response to higher demand. When it is possible to reset the price, the firm will predict that the marginal cost will be high in the future but will be low for several periods before it increases. Since inflation is determined in a purely forward-looking manner, the firm will take the low marginal costs in the short-run and the high marginal costs in the intermediate-run into consideration for the determination of its price. Therefore, the firm will hesitate to price high while the marginal cost is low in the short-run. However, $\dot{mc}$ is increasing over time, as the externalities weaken and factor prices increase. When it is possible to reset the price again, the forward looking firm no longer has such a low marginal cost. At this point, the incentive to price low created by externalities has become small, leading the forward looking firm to set its price higher. Thus, inflation response is hump-shaped not because firms are backward-looking, but because they are forward-looking.

For comparison, Fig. 1.4 shows the IRFs for inflation, the output gap and the marginal cost in the absence of organizational capital ($\phi = 0$). In the figure, one can see that the maximum response of inflation occurs in the period of the monetary shock and that the marginal cost shows uniformly positive responses. The reason is simple. Without externalities, productivity does not change in response to the increase in

\(^{13}\)Note that this argument is based on the approximation $\beta \simeq 1$. It is possible that inflation increases while $\dot{mc}$ is positive, because $\beta < 1$. That is, $\beta E_t \pi_{t+1} - \pi_t < 0$ and $E_t \pi_{t+1} - \pi_t > 0$ can occur when $\dot{mc}$ is close enough to zero. In the simulation, $\dot{mc}$ at 9th quarter after the shock is positive, but inflation is increasing from 9th to 10th quarter.
production. Marginal cost increases because of higher factor prices. Therefore, \( \hat{mc} \) is uniformly positive and inflation is decreasing over time as (1.23) suggests. Note that the model excludes only the dynamic externality, continuing to maintain the assumption of sticky wages and variable capital utilization. Including sticky wages and variable capital utilization enables marginal cost to behave sluggishly as CEE point out. However, it does not generate hump-shaped behavior of inflation under the purely forward-looking NKPC. What we need for a hump shape of inflation is a short-run negative response in marginal costs.

Figure 1.4: Simulated IRFs for inflation, the output gap and marginal cost under no dynamic externality.
1.3.3 The role of sticky wages and variable capital utilization

In this subsection, we discuss the role of sticky wages and variable capital utilization. In the analysis in the previous subsection, we found that the behavior of marginal cost is important: When the log-deviation of the marginal cost takes negative values in the short-run and positive values in the intermediate-run, inflation can be hump-shaped. It will be shown that sticky wages and variable capital utilization are important for generating such behavior of marginal cost. Without both elements, we lose the short-run decrease in marginal cost and thus inflation is not hump-shaped.

To analyze the effect of sticky wages and variable capital utilization, we use (1.5) to take the log-linearization of the marginal cost around the steady state:

\[
\hat{mc}_t = \hat{P}_t^f - \phi \hat{X}_t,
\]

where \( \hat{P}_t^f \equiv (1 - \alpha)\hat{w}_t + \alpha \hat{r}_k^t \). In other words, \( \hat{P}_t^f \) is the weighted average of real wages and rental cost of effective capital. The second term in the equation \( \phi \hat{X}_t \) shows the log-deviation of productivity stemming from the dynamic externality.

Fig 1.5 makes the effect of sticky wages and capital utilization clearer. At each panel of the figure, the IRFs of factor prices \( \hat{P}_t^f \) and productivity \( \phi \hat{X}_t \) to a monetary shock are shown. The upper left panel of the figure is the benchmark case while the lower right panel of the figure is the case of flexible wages and constant capital utilization. In the off-diagonal panels of the figure are the case in which either sticky wages or variable capital utilization is missing in the simulation.

Note that only in the upper left panel does the productivity effect exceed the factor-prices effect for several periods. Thus, \( \hat{mc} \) is first negative and then positive. As shown in the previous subsection, inflation is hump-shaped due to this behavior of
NOTE: The line with (+) is the log-deviation of factor prices from the steady state value $\hat{P}_f$ and the line with (-) is productivity. The factor price minus productivity is the log-deviation of the marginal cost.

Figure 1.5: Factor prices and productivity
$m\hat{c}$. On the other hand, the factor prices in the other panels of the figure are always larger than productivity in the log-deviation response, which implies that $m\hat{c}$ always takes positive values in response to a monetary shock. Therefore, inflation is never hump-shaped.

We can see that both sticky wages and variable capital utilization are important. In the upper right panel of the figure, capital utilization is variable but real wages are flexible. In this case, the factor price effect overwhelms the productivity effect in its magnitude, because real wages is adjusted upward quickly. In the lower left panel, real wages are sticky but capital utilization is constant. As a result, the rental cost of effective capital is adjusted upward so much that factor prices exceed productivity, although the magnitude of factor prices shifts down closer to that of productivity.\(^{14}\) Finally, under flexible wages and constant capital utilization, the effect of factor prices is so strong that the productivity effect is almost negligible in comparison.

### 1.4 Robustness

In this section, we consider the robustness of the hump-shaped behavior of inflation. I will show that the mechanism and the hump-shape we obtained in the previous section are quite robust to types of real frictions such as habit formation and a quadratic adjustment cost for capital. Also, we investigate the robustness to different value of $\phi$. It will be shown that an even smaller $\phi$ could generate hump-shaped inflation.

\(^{14}\)Although real wages $\hat{w}$ show a hump shape in its response, $\hat{P}^f$ does not. This is because $\hat{P}^f$ is the weighted average of real wages and rental cost of capital and the response of $\hat{r}^k$ is much larger in its magnitude than in the case of variable capital utilization. While real wages are hump-shaped, the rental cost of capital is not only front loaded but also jumps up to a large extent. Because of the large front loaded rental cost, $\hat{P}^f$ as the weighted average doesn’t exhibit a hump shape in its IRF.
1.4.1 Habit formation and adjustment cost of capital

We extend the model by including habit formation in it. In our simulation results, the IRF for the output gap are not consistent with the stylized fact on an IRF that VARs characterize in that they don’t show their delayed response to monetary shocks. Other researchers have often found that habit formation helps to reconcile the persistence of real variables such as output and consumption in models with what is found in the data. We seek then to investigate whether our hump-shape of inflation is robust to the introduction of habit formation.

To introduce habit formation in the model, we replace (1.8) with the expected utility function with habit formation:

\[
E_0 \sum_{t=0}^\infty \beta^t \left[ \frac{(C_t(h) - bC_{t-1}(h))^{1-\sigma_C}}{1-\sigma_C} - \frac{L_t(h)^{1+\sigma_L}}{1+\sigma_L} + \log \left( \frac{M_t(h)}{P_t} \right) \right],
\]

where \( b \) is the habit parameter. Then, the marginal utility for current consumption are given by

\[
\lambda_t = (C_t - bC_{t-1})^{-\sigma_C} - \beta b E_t(C_{t+1} - bC_t)^{-\sigma_C},
\]

instead of (1.17). We set the parameter \( b \) to 0.65 in our simulation.\(^{15}\)

Fig 1.6 shows the IRF for inflation, the output gap and marginal cost with and without habit formation. The latter case merely replicates our findings from section 1.3. From the perspective of the IRF for inflation, we can see no substantial difference between the two cases. On the other hand, the output gap in the extended model now shows a clear hump shape in its response, which is a much better prediction than in the benchmark model.

\(^{15}\)This calibrated value of the habit parameter is the same as in CEE.
NOTE: IRFs for inflation, the output gap and marginal cost in response to 1% of money growth rate shock. The line with (+) is an IRF for the case without habit formation. The line with (◦) is an IRF for the case without habit formation.

Figure 1.6: Habit formation
Also, the marginal cost behaves similarly expect for responses for first two years. The short-run response of the marginal cost is smaller under habit formation than otherwise. This is because a smaller initial response of $\hat{Y}_t$ weakens response of $\hat{X}_t$, resulting in a smaller initial response of $\hat{mc}_t$. Nevertheless, the inflation response is not affected substantially, because forward-looking firms weight future marginal cost in their determination of their nominal prices and the behavior of future marginal costs is little affected by the habit formation. As a consequence, the IRF of inflation with habit formation is at least as good as with the benchmark model, and we have much better predictions for the output gap.

Next, we investigate a quadratic cost of capital adjustment. Since our accumulation equation is a little unorthodox, one may be interested in considering adjustment costs instead. We use the specification

$$K_{t+1}(h) = F((1 - \delta)K_t(h), I_t(h)) = (1 - \delta)K_t(h) + I_t(h) - \frac{\phi_k}{2} \left( \frac{I_t(h)}{K_t(h)} - \delta \right)^2 K_t(h),$$

(1.24)

instead of (1.11). We set $\phi_k = 10$ which is roughly consistent with the empirical literature on investment; e.g. Eberly (1997).

Our result on inflation is robust to this modification of the model. Assuming habit formation for consumption, Fig 1.7 shows the IRFs for inflation, the output gap and marginal cost. With the quadratic adjustment cost function, investment jumps up sharply in response to monetary shocks and thus the output gap becomes more similar to the investment response. As a result, the output gap becomes less persistent under the quadratic adjustment cost function. However, since marginal cost behavior is almost the same as the benchmark model, a hump-shaped behavior of inflation is almost unaffected.
NOTE: IRFs for inflation, the output gap and marginal cost in response to 1% of money growth rate shock. The line with (+) is an IRF for the case of non-quadratic adjustment cost given by (1.11). The line with (◦) is an IRF for the case of quadratic adjustment cost given by (1.24). The model includes habit formation.

Figure 1.7: Quadratic and non-quadratic adjustment cost of capital
1.4.2 Robustness to returns to organizational capital

It is natural to ask how robust our externality-driven hump-shaped IRF for inflation is to the extent of the externality. Assuming habit formation for consumption, Fig 1.8 shows the IRFs for inflation and the output gap for several values of $\phi$. Since Cooper and Johri (2002)'s estimates of $\phi$ in the production function estimation range between 0.26 and 0.35, I consider the values of $\phi$ of 0, 0.10, 0.20, 0.26 and 0.35 in this sensitivity analysis. Lines in a panel differ in the degree of returns to organizational capital $\phi$.

As the top panel in Fig 1.8 shows, inflation is hump-shaped even for values of $\phi$ as small as 0.10. For $\phi \geq 0.20$, a pronounced hump shape is obtained with a peak response at the eighth quarter.

For $\phi = 0.35$, a negative response appears and lasts for about one year. In other words, the price puzzle that some researchers have found in data could stem from sufficiently large externalities. The reason is straightforward: The short-run productivity effect is so great that it is optimal for firms to price low in the short-run.

The bottom panel in Fig 1.8 tells us how much the dynamic externality amplifies the effect of monetary shocks on the output gap. Although the hump-shape dynamics of the output gap largely rely on habit formation, the dynamic externality plays the role of an amplification mechanism. The larger the degree of externality, the more pronounced the hump-shape in the output gap. Thus, externalities amplify the effect of habit formation.

\footnote{In results not shown here, the dynamic externality alone actually makes a hump-shape in the output gap when $\phi$ is large. In our simulation of the model without habit formation, $\phi = 0.35$ can generate the maximum impact on the output gap with one year delay.}

31
The intuition for amplification is again simple. The increased demand for goods raises productivity. The price for goods is lower when there are externalities than otherwise due to increased productivity. Because of the lower price of goods, the increase in the demand for goods is larger. Habit formation restricts the increase in consumption so that the initial response in consumption is almost the same for all values of $\phi$. However, the effect of the monetary shock becomes stronger as $\phi$ becomes larger. As a result, a hump-shape in the output gap is more pronounced when a dynamic externality exists than otherwise.

1.5 Why is inflation hump-shaped in CEE?

In this section, we evaluate the model in CEE and our model. In our discussion, the model with sticky wages and variable capital utilization does not show hump-shaped behavior of inflation unless there is a dynamic externality (See Fig 1.2 and 1.4). In CEE, they conclude sticky wages and variable capital utilization are important for generating quantitatively plausible inflation inertia. In particular, CEE succeed in generating a hump shape in inflation without introducing a dynamic externality. Moreover, they point out that a hump-shape is actually obtained in CEE even if the assumption of backward-looking indexation is dropped. In this section, we discuss why they are able to generate the observed hump-shaped inflation without any dynamic externality.

To discuss this question, it is necessary to disentangle the assumptions CEE made in the benchmark model. In their model, there are two important assumptions for hump-shaped IRF for inflation. First, one of the most important assumptions is
NOTE: IRFs for inflation and the output gap in response to 1% of money growth rate shock based on different values of φ. The model includes habit formation.

Figure 1.8: Robustness to returns to scale
their use of a hybrid NKPC that incorporates backward-looking indexation in nominal prices and wages. This backward-looking indexation transforms inflation from a jump variable to a state variable, because inflation becomes a function of the lagged inflation. In our notations, their hybrid NKPC for prices is given by

\[ \hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} + \frac{\Psi_p}{1 + \beta} \hat{mc}_t \]

As a result, the first term in the equation gives the model inflation inertia in their benchmark model.

Second, they have the assumption of working capital channel. In other words, firms must borrow their wage bill from financial intermediaries at the beginning of each period and repay it plus interest at the end of the period. As a result, the real marginal cost is given by

\[ mc_t = (1 - \alpha)^{-(1-\alpha)} \alpha^{-\alpha} (R_t w_t)^{1-\alpha} (r^k_t)^\alpha, \]

where \( R_t \) is the gross nominal interest rate. Since marginal cost depends on the nominal interest rate, it can decrease appreciably in the short run if monetary shocks reduce the nominal interest rate substantially. That is, if a liquidity effect happens and is great enough and persistent enough to generate a sufficiently long-lived decrease in the marginal cost, inflation will be hump-shaped.

Due to these two assumptions, a model dropping only one of them can still generate hump-shaped inflation as CEE show in their analysis. When they replace backward-looking indexation with purely forward-looking indexation, they conclude that “inflation continues to be inertial.” Indeed, the peak of the IRF occurs in the fourth quarter following a monetary shock, while the peak of the IRF with indexation is in
the 10th quarter. This relatively quick but hump-shaped response of inflation is because the assumption of working capital creates short-run a decrease in the marginal cost. They also conclude “the role of the working capital assumption in our model is relatively minor” when they drop it. (The peak response occurs at the fifth quarter.) It is because their hybrid NKPC plays a crucial role for generating a hump shape in inflation.

The simulation results shown in Fig 1.4 can be interpreted as the model dropping both working capital assumption and backward-looking indexation for prices and wages. Because there is neither lagged inflation nor short-run decrease in the marginal cost, the peak response of inflation must be at the time of monetary policy shocks. Thus, sticky wages and variable capital utilization alone cannot generate the observed hump-shaped behavior of inflation.

In order for the working capital assumption alone to work for generating a hump shaped IRF for inflation, we require large and persistent liquidity effect so that the effect of the financial cost reduction overwhelms the effect of factor prices. In CEE, several special assumptions about timing and other frictions are required to generate a large and persistent liquidity effect as well as their so-called critical assumptions of sticky wages and variable capital utilization. For example, firms’ wage bills are determined prior to monetary shocks, implying that changes in real balances must be absorbed entirely by households and allowing the model to generate a large initial response of nominal interest rate to monetary shocks. As a result, these assumptions make their model complicated.

In our model, on the other hand, we rely on fewer assumptions than CEE, namely, sticky wages, variable capital utilization and a dynamic externality. Table 1.2 lists sets
of the assumptions in CEE and the model discussed in the paper. Notwithstanding this parsimony, our model accounts for the slow observed response of inflation. In particular, we have no need for the ad hoc assumption of a hybrid NKPC.

1.6 Conclusion

This paper provides a dynamic general equilibrium model which includes no non-rational economic agents and symmetric information between private agents and the central bank, but can explain the hump-shaped response for inflation to money growth shocks. The key assumption for the hump-shaped IRF for inflation is a dynamic externality. In response to expansionary monetary shocks, a dynamic externality gives firms an incentive to price low because firms observe increase in the productivity. This incentive weakens the incentive to price high due to the effect of increased factor prices.

In order for this low-pricing incentive to generate hump-shaped inflation, we require the assumption of sticky wages and variable capital utilization. As CEE point
out, these assumptions help fluctuations in marginal cost to be damped so that in the short-run the effect of productivity induced by the dynamic externality overwhelms the effect of factor prices in the short-run. Sticky wages also lead to an increase in marginal cost in the intermediate-run because the real wages are adjusted upward slowly. As a consequence, marginal cost can decrease for the first several periods but increase in the end. Given the dynamics of marginal cost, forward-looking firms raise their prices only moderately for several periods and the response of inflation to a monetary shock can be hump-shaped for a reasonable range of parameters.

We found the delayed response of inflation is quite robust to real frictions such as habit formation. It is also robust to returns to organization capital. Inflation is still hump-shaped even when we calibrate returns to organizational capital which are less than a half of Cooper and Johri (2002) estimate. When the dynamic externality is large enough, a price puzzle can even emerge. A dynamic externality could therefore be a possible explanation for the price puzzle that some researchers have found.

We compare CEE’s mechanism of hump-shaped inflation in CEE with ours. To generate plausible inflation inertia under a purely forward-looking NKPC, CEE require a financial friction that they model using a limited participation model while our model requires the dynamic externality. Note that both models predict the short-run decrease in marginal cost. Because of the absence of the empirical evaluation of these two models, Ockham’s Razor might therefore be a reasonable criterion for the choice between the two models.
CHAPTER 2

STICKY INFORMATION: THE IMPACT OF DIFFERENT
INFORMATION UPDATING ASSUMPTIONS

2.1 Introduction

Due to the many flaws of sticky price models, there is a growing literature that studies the real effect of monetary policy in terms of sticky information or sticky plans. Researchers begin with this assumption instead of sticky prices to explain non-neutrality, because of the attractive predictions of the model.\textsuperscript{17}

Mankiw and Reis (2002b, MR) develops the sticky information model to explain the stylized facts that cannot be explained by the sticky price model. They state that a firm may set its price flexibly but collect information slowly over time due to the costs of acquiring and processing information. Their model with this slow adjustment of information successfully explains: (a) a hump-shaped impulse response function for inflation; (b) disinflationary policy being able to generate a recession, which cannot be generated by the standard sticky price model.

\textsuperscript{17}Examples are Mankiw and Reis (2002b), Keen (2004), Koenig (1996) and Reis (2004b). Mankiw and Reis (2002a) and Carroll (2003) are examples of empirical studies of the sticky information model. Ball, Mankiw, and Reis (2003) study monetary policy under the sticky information model. Burstein (2005) develops a dynamic general equilibrium model of sticky plans using the state dependent model.
This paper studies the role of the information updating scheme in the sticky information model. The finding of this paper is that MR’s attractive results critically depend on the assumption that each firm has a random probability of obtaining its new information set. If, on the other hand, new information is obtained infrequently but deterministically with a fixed duration: (a) an impulse response function for inflation suffers from unrealistic behavior; (b) disinflationary policy is unable to explain a recession if it is credible and its announcement is made sufficiently far in advance; Moreover, (c) the sticky information model suffers from lack of inflation persistence; (d) under a short fixed duration, the output response suffers from lack of persistence and a weak hump-shape in its impulse response function, though it can be recovered by a longer fixed duration.

The change in inflation dynamics can be explained intuitively with an example of cars on a highway. Suppose that highway drivers determine their speed by two objectives: the legal limit and the average speed on the highway. They must obey the legal limit in the end but they also care about the average speed on the highway in the adjustment to a change in the legal limit. Two information updating schemes are compared here: the random duration scheme and the fixed duration scheme. Under the random duration scheme, drivers on the highway know the change in the legal limit randomly. For example, ten percent of all drivers are informed at the change in the first minute. Then, in the next minute, ten percent of all drivers are informed at the change in the legal minute with resampling. Under the fixed duration scheme, on the other hand, all drivers are separated into ten groups and each of the groups learns the change every minute in turn. This occurs in the staggered manner, but all drivers eventually know the change in ten minutes.
Consider a highway where all drivers run at the legal limit in the steady state. Then, suppose that the legal limit unexpectedly falls by ten miles per hour. Under the fixed duration scheme, only the first group knows the drop in the legal limit immediately and considers a plan of reductions in its speed, depending on the average speed on the road. The first group knows not only that the legal limit drops, but also that the new information will be available to half of highway drivers in five minutes and all of the drivers in ten minutes, etc. The first group chooses quickly declining speed paths. The speed chosen in ten minutes will be the new legal limit because they know that all drivers will know the new legal limit at that point of time. The future groups choose similar speed reduction plans. In ten minutes, everybody runs at the legal limit.

Under the random duration scheme, on the other hand, the decline in the average speed is slower. The first ten percent of all drivers also know the new legal limit immediately, but its speed reduction is slow. Especially, the first group’s speed is still above the legal limit even after ten minutes, because some of drivers will be driving at the old legal limit and those drivers affect the average speed on the road. Thus, the first group cannot reduce their speed quickly. This behavior of the first group prevents the average speed from declining and affects the second group’s speed reduction plans. After all, the average speed reaches at the legal limit only in the steady state and is much slower than that under the fixed duration scheme.

The paper is organized as follows. The section 2.2 shows the sticky information model under different schemes. The section 2.3 conducts simulations under the two different schemes. The simulations are twofold. The first examines the effect of a shock to the nominal expenditure growth rate. The other simulation looks at the
dynamic paths of inflation and output in response to a disinflationary policy. The section 2.4 concludes the paper.

2.2 Different updating schemes

2.2.1 Random duration updating

The first scheme we consider is the random duration scheme developed originally by MR. Every period, a firm indexed by \( i \) obtains new information with a constant probability of \( \lambda \). With the remaining probability, the firm continues to hold its old information set.

Suppose that a large number of firms produce differentiated goods under monopolistic competition. In the sticky information model, each firm is allowed to set its price \( P_t(i) \) flexibly, but it is assumed to be costly to gather the information about the economy and thus the information updating occurs only infrequently.

The optimal price plans \( \{P_{t+j}(i)\}_{j=0}^{\infty} \) are given by solving the following profit maximization problem.\(^{18}\)

\[
\max_{\{P_{t+j}(i)\}_{j=0}^{\infty}} \sum_{t=0}^{\infty} \{\beta(1 - \lambda)\}^t E_t \left[ \frac{u'(C_{t+j})}{P_{t+j}} \left( P_{t+j}(i)Y_{t+j}(i) - W_{t+j}(i)L_{t+j}(i) \right) \right],
\]

given the demand function for the good \( Y_t(i) = [P_t(i)/P_t]^{-\theta}Y_t \) and the production function for firm \( i \). Here \( Y_t(i), Y_t, W_t(i) \) and \( L_t(i) \) denote the output of its differentiated good, a composite of differentiated goods, the nominal wage and its labor input, respectively. \( P_t \) is the aggregate price index given by \( P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \). \( u'(C_t) \) is the representative household’s marginal utility of consumption, which is defined

\(^{18}\)There are two interpretations in the sticky information model. The one interpretation is that each firm solves the static profit maximization problem every period given the information set available at \( t-j \) for \( j \geq 0 \). The other interpretation is that each firm maximizes the discounted sum of current and future profits to find its price plans when the information is updated. Because they are both equivalent, we follow the second interpretation here.
later. We assume that each firm uses a specialized labor for its production. We do so because this specification allows us to derive “strategic complementarity” between firms’ pricing decisions.\textsuperscript{19} The technology is assumed to be linear, \( Y_t(i) = L_t(i) \).

The optimal price plans in this profit maximization problem must satisfy

\[
P_{t+j}(i) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t(u'(C_{t+j})Y_{t+j}(i)W_{t+j}(i)/P_{t+j})}{E_t(u'(C_{t+j})Y_{t+j}(i)/P_{t+j})}, \text{ for } j = 0, 1, 2, \ldots . \tag{2.1}
\]

Log-linearizing (2.1) yields

\[
p_{t+j}(i) = E_t(w_{t+j}(i)) \text{ for } j = 0, 1, 2, \ldots , \tag{2.2}
\]

where a lower case letter is the log-deviation of a variable from its steady state. The equation tells us that the nominal price depends on the expected nominal marginal cost for firm \( i \), which equals its expected nominal wage.

To specify the real marginal cost for firm \( i \), suppose that the representative household’s preference in period \( t \) is given by

\[
u(C_t) - \int_0^1 \nu(L_t(i))di = C_t^{1-\sigma} - \int_0^1 \frac{L_t(i)^{1+\psi}}{1+\psi}di.
\]

Because each firm uses a specialized labor to produce its output, the representative household supplies her differentiated labor to each firm \( i \). In this case, the marginal rate of substitution between labor supply and consumption is given by

\[
\frac{W_t(i)}{P_t} = C_t^\sigma L_t(i)^\psi = \left( \frac{P_t(i)}{P} \right)^{-\theta \psi} Y_t^{\sigma + \psi},
\]

\textsuperscript{19}Strategic complementarity is a key assumption in the sticky information model. Keen (2004) considers the sticky information model under strategic substitutability. He finds that the sticky information model is not successful under strategic substitutability. His results suggest that a Yeoman farmer model as in Ball and Romer (1990) or the specialized factor market model is necessary for generating plausible inflation and output dynamics, because they can create sufficient real rigidity. Woodford (2003) discusses other ways to have strategic complementarity.
since $C_t = Y_t$ in equilibrium. Log-linearization yields

$$w_t(i) - p_t = -\theta \psi(p_t(i) - p_t) + (\sigma + \psi)y_t. \quad (2.3)$$

Combining (2.2) and (2.3), we get

$$p_{t+j}(i) = E_t[p_{t+j} + \alpha y_{t+j}] \text{ for } j = 0, 1, 2, \cdots, \quad (2.4)$$

where $\alpha = (\sigma + \psi)/(1 + \theta \psi)$. 

Now, consider the price level at a point of time $t$. We aggregate a set of individual price settings according to $P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{1/\theta}$. Let $P_{t,j}$ be the price set at period $t$ based on the information set available at $t - j$. In the set-up by MR, the log-linearized price-level equation is given by

$$p_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j P_{t,j}. \quad (2.5)$$

Thus, the (log) price-level is a weighted average of all prices based on different information. The probability of information updating affects how fast the weights decline.

Combining (2.4) and (2.5) yields

$$p_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j}[p_t + \alpha y_t].$$

Thus, the price level depends on past expectation of the current optimal price. MR use this price level equation to derive the sticky-information Phillips curve.

$$\pi_t = \frac{\alpha \lambda}{1 - \lambda} y_t + \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j-1}(\pi_t + (y_t - y_{t-1})), \quad (2.6)$$

where $\pi_t = p_t - p_{t-1}$.

### 2.2.2 Fixed duration updating

The second scheme is the fixed duration scheme. Under this scheme, firms update its information about the state of the economy every $N$ periods. Moreover, it occurs
in a staggered manner as in Fischer (1977): a fraction $1/N$ of all firms update their information set each period. We call this scheme the fixed duration scheme, because the duration of holding information is deterministically determined.

In this case, the optimal price plans chosen in period $t$ solve

$$\max_{\{P_{t+j}(i)\}_{j=0}^{N-1}} \frac{1}{N} \sum_{t=0}^{N-1} \beta^j E_t \left[ \frac{u'(C_{t+j})}{P_{t+j}} \{P_{t+j}(i)Y_{t+j}(i) - W_{t+j}(i)L_{t+j}(i)\} \right].$$

The optimal price plans satisfy the condition:

$$P_{t+j}(i) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t(u'(C_{t+j})Y_{t+j}(i)W_{t+j}(i)/P_{t+j})}{E_t(u'(C_{t+j})Y_{t+j}(i)/P_{t+j})}, \text{ for } j = 0, 1, 2, \ldots, N - 1.$$ 

They are identical to (2.1) but have a truncation at $N - 1$, because the duration is fixed.

One can derive for the price level equation in the same manner as the random duration model, but aggregation over individual prices involves a weighted average with equal weights in logarithm.

$$p_t = \frac{1}{N} \sum_{j=0}^{N-1} E_{t-j}[p_t + \alpha y_t]. \quad (2.7)$$

Note that the cross-sectional distribution of prices has a truncation at $N - 1$ due to the fixed duration.

### 2.3 Calibration

We compare the dynamic properties of the model under the two different duration schemes. To conduct our comparisons, we make criteria for comparisons. One possible criterion is to match the expected duration of a newly set individual price. In other words, we equalize the average length of time between when information is updated at a given firm and when it will next be updated at the same firm. Under the random
duration scheme, it is $1/\lambda$, while under the fixed duration scheme, it is $N$. The other possible criterion is to match the cross-sectional expected age of individual prices in the economy at a point of time. Under the random duration scheme, the average age is $1/\lambda$ because of the property of a geometric distribution. Under the fixed duration scheme, the cross-sectional average age at a point of time is $(1 + 2 + \cdots + N)/N = (N + 1)/2$, because there are $N$ cohorts in the economy.

For comparisons, it is sensible to use both criteria, because both are reasonable. For the random duration scheme, we select $\lambda = 0.25$ for simulations. Therefore, $N$ should be four if one matches the expected duration of an individual price and seven if one matches the cross-sectional expected age of prices in the economy.

Finally, we follow MR by specifying the demand side of the economy by $M_t = P_t Y_t$, where $M_t$ is nominal GDP at period $t$, which is taken as exogenous in the model. Obviously, one can interpret $M_t = P_t Y_t$ as a quantity equation with a constant velocity or cash-in-advance constraint. In that case, $M_t$ would be the nominal money supply.

### 2.3.1 The effect of a change in the growth rate of nominal expenditure

We simulate dynamic paths of output and inflation by generating an unexpected increase in the growth rate of nominal expenditure $\Delta m_t$. Suppose that the growth rate of nominal expenditure follows AR(1) process.

\[
\Delta m_t = \rho \Delta m_{t-1} + \varepsilon_t,
\]

where $\rho = 0.5$ and $\varepsilon_t$ is iid with the standard deviation of 0.007, following MR. In our simulations, a one standard deviation $\varepsilon_t$ shock occurs at period zero. We select $\sigma = 1$, $\psi = 9$ and $\theta = 11$, implying $\alpha = 0.1$. 

45
Calibration I: Matching the expected duration of a newly set price

Impulse response functions of inflation and output are shown in Figure 2.1. Under the random duration scheme, both inflation and output show significant hump-shapes. Output and inflation peak at period three and seven respectively.

However, under the fixed duration scheme, the dates of the peak responses in inflation and output are dramatically reduced. Inflation peaks at period three and output at period one. If one considers $M_t$ as the nominal money supply, one can see that money is neutral from period three and inflation follows the same path as the money growth rate from period four, because all firms are informed about the new state of the economy and the adjustment of expectation completely ends by this period. Thus, the impulse response functions under the fixed duration scheme suffer from reduced hump-shapes in both output and inflation.

Table 2.1 presents the population autocorrelations implied by the two information updating schemes. The first and the fourth columns of the table are identical to columns in Table 1 by MR. The second column is the population autocorrelations for the fixed duration scheme for $N = 4$. The autocorrelation functions in the column show substantial reduction in persistence. Thus, the sticky information model under the fixed duration scheme suffers from lack of inflation persistence.

Table 2.2 also confirms that monetary non-neutrality disappears beyond periods of information adjustment durations. Moreover, while the random duration scheme is successful in generating output persistence, the persistence of output under the fixed duration scheme is remarkably low.
NOTE: Both models assume sticky information. See the appendix A for the derivation for impulse response functions under the fixed duration scheme.

Figure 2.1: Impulse response functions of the output gap and inflation to a positive one standard deviation (0.007) shock to $\Delta m_t$—Matching the expected duration. ($N = 4$)
<table>
<thead>
<tr>
<th></th>
<th>Random Duration</th>
<th>Fixed Duration</th>
<th>Actual GDP deflator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N = 4</td>
<td>N = 7</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.986</td>
<td>0.231</td>
<td>0.309</td>
</tr>
<tr>
<td>2</td>
<td>0.948</td>
<td>0.100</td>
<td>0.155</td>
</tr>
<tr>
<td>3</td>
<td>0.892</td>
<td>0.038</td>
<td>0.092</td>
</tr>
<tr>
<td>4</td>
<td>0.822</td>
<td>0.009</td>
<td>0.059</td>
</tr>
<tr>
<td>5</td>
<td>0.742</td>
<td>0.005</td>
<td>0.035</td>
</tr>
<tr>
<td>6</td>
<td>0.655</td>
<td>0.002</td>
<td>0.013</td>
</tr>
<tr>
<td>7</td>
<td>0.567</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>8</td>
<td>0.481</td>
<td>0.001</td>
<td>0.000</td>
</tr>
</tbody>
</table>

NOTE: The population autocorrelation functions calculated here are based on AR(1) growth rate in nominal expenditure. The first and the last columns are from MR. The autocorrelations of inflation under the fixed duration model are newly calculated. The model parameters are identical to MR.

Table 2.1: Autocorrelations for inflation

<table>
<thead>
<tr>
<th></th>
<th>Random Duration</th>
<th>Fixed Duration</th>
<th>GDP gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N = 4</td>
<td>N = 7</td>
<td>Quadratically detrended</td>
</tr>
<tr>
<td>1</td>
<td>0.960</td>
<td>0.685</td>
<td>0.885</td>
</tr>
<tr>
<td>2</td>
<td>0.882</td>
<td>0.283</td>
<td>0.692</td>
</tr>
<tr>
<td>3</td>
<td>0.786</td>
<td>0.475</td>
<td>0.78</td>
</tr>
<tr>
<td>4</td>
<td>0.684</td>
<td>0.267</td>
<td>0.71</td>
</tr>
<tr>
<td>5</td>
<td>0.584</td>
<td>0.010</td>
<td>0.65</td>
</tr>
<tr>
<td>6</td>
<td>0.490</td>
<td>0.000</td>
<td>0.61</td>
</tr>
<tr>
<td>7</td>
<td>0.404</td>
<td>0.000</td>
<td>0.58</td>
</tr>
<tr>
<td>8</td>
<td>0.328</td>
<td>0.000</td>
<td>0.58</td>
</tr>
</tbody>
</table>

NOTE: The population autocorrelation functions are calculated based on AR(1) growth rate in nominal expenditure. The population autocorrelations of output are newly calculated. The data for output gap is quadratically detrended and HP filtered log real GDP from 1960:1 to 2004:4. The model parameters are identical to MR.

Table 2.2: Autocorrelations for output
Calibration II: Matching the expected age of prices in the economy

The second criterion matches the cross-sectional expected age of prices in the economy. Figure 2.2 presents the impulse response functions under the random and the fixed duration.

A hump-shape in output can be recovered by increasing durations from four to seven. The third column in Table 2.2 also shows that output persistence can be recovered though autocorrelations are again zero once the order of autocorrelation function reaches to $N - 1$, the maximum duration of inattentiveness. Thus, the fixed duration scheme can now explain a hump-shape and persistence of output relatively well.

On the other hand, a dynamic response of inflation still remains unrealistic. It is delayed because its peak is at period six, but inflation decreases quickly in the next period. To see the reason for the quick reduction in inflation, suppose that the quantity equation holds. Once firms are all informed about a shock to the money growth rate, non-neutrality disappears so that inflation must be equal to the money growth rate at period seven. Because money growth rate in period seven has almost converged to its steady state of zero, inflation must decrease very quickly.

Table 2.1 confirms the lack of inflation persistence under the longer fixed duration scheme. The autocorrelations of inflation under $N = 7$ (the third column in the table) remain very low. Thus, the fixed duration model is not successful in generating inflation persistence.
Figure 2.2: Impulse response functions of the output gap and inflation to a positive one standard deviation (0.007) shock to \( \Delta m_t \) – Matching the cross-sectional expected age of prices. \((N = 7)\)
2.3.2 Anticipated disinflation

Another attractive result in MR is that their sticky information model can show a consistent contractionary disinflation. Especially, the sticky information model predicts a recession under credible disinflationary policy and the prediction is consistent with Ball (1994)'s empirical facts.

Does this result hold under the fixed duration model as well? We conduct the same experiment as in MR. Suppose that $\Delta m_t$ is 2.5% per period until period -1 and then falls to 0% in period zero. To make this policy change credible, suppose that the policy is announced $h$ periods in advance.

Not surprisingly, the attractive result does not hold under the fixed duration model when $h$ is longer than $N - 1$. In other words, when the disinflationary policy is announced more in advance than maximum length of inattentiveness, all firms have a plenty of time to become informed about the disinflationary policy. Thus, the disinflationary policy never affects output and the cost of disinflation is zero when the announcement is made far enough in advance.

What if $h$ is shorter than $N$? Figure 2.3 shows an experiment in which the disinflationary policy is announced one year ($h = 4$) in advance and the duration of information delay is seven periods ($N = 7$).

Under the random duration scheme, the disinflationary policy announced a year in advance generates a long-lasting recession. From period zero, output starts to reduce and its reduction lasts for a long time. On the other hand, under the fixed duration scheme, the length of a recession is only half a year and much shorter than the random duration model. Moreover, inflation takes an unreasonable path under the fixed duration scheme. Although inflation overshoots under the two schemes,
NOTE: The cross-sectional expected ages of prices are matched under the two schemes. When the expected durations of a newly set price are matched ($N = 4$), the cost of disinflation is zero and a dynamic path of inflation is identical to the path of $\Delta m_t$ specified in the main text. See the appendix A for the derivation of the dynamic responses of variables.

Figure 2.3: Anticipated disinflation: dynamic path of output and inflation given an announcement at date -4 of a 2.5% decrease in nominal expenditure growth at date 0 ($h = 4, N = 7$)
its overshooting under the fixed duration scheme is much larger than the random duration scheme.

2.4 Concluding remarks

This paper compares two schemes of how information set is updated in the sticky information model. One is the random duration information updating scheme originally developed by MR and the other is the fixed duration information updating scheme.

Our finding is that MR’s attractive results are not robust to an alternative type of information updating scheme. For comparison, we used two criteria that matched the expected age of a newly set individual price or the cross-sectional expected age of prices in the economy. Under both criteria, the fixed duration model fails to generate inflation persistence, given a shock to the nominal expenditure growth rate. The peaks in the impulse response function for inflation are delayed under both criteria. (After the shock, the peak is three and six, respectively), but the impulse response functions show unreasonable patterns because inflation must quickly decrease once all firms are informed.

Additionally, we conducted an experiment with an anticipated disinflation. In our analysis, we found that the cost of disinflation is zero or much smaller under the fixed duration model. When a disinflationary policy occurs after all firms have become informed by an announcement, the cost of disinflation is obviously zero and the disinflationary policy is completely successful. On the other hand, even if the
announcement of disinflation occurs before all firms are informed, the cost of disinflation is significantly reduced and the dynamic path of inflation is inplausible in that inflation unreasonably overshoots due to lack of inflation persistence.

The results shown here have an implication for model building. Equation (2.6) involves infinite lags on the right hand side. Due to the randomness of information updating and the necessity of computing each cohort’s pricing decision, the random duration model does not give us computational simplicity. An important advantage of the random duration in the sticky price model is that one does not need to keep track of the distribution of prices in the economy. While the random duration scheme in the sticky price model reduces the model’s state space, it does not do so in the sticky information model.

On the other hand, the size of the state space in the fixed duration scheme is the same under both sticky price and information model. Because the random duration scheme by MR doesn’t simplify computation, the decision between which scheme should be used for calibrations mainly depends on their empirical plausibility for the cross-sectional distribution of information. Our findings in the partial equilibrium model suggest that the decision between two schemes is not innocuous.

Which scheme is more plausible for calibration? Reis (2004b) considers this question theoretically. He provides a micro-foundation for the random duration model. However, empirical plausibility of the cross-sectional distribution would be an interesting question for a line of the future research.

---

20His finding is that aggregation over a large number of inattentive firms leads to the cross-sectional distributions under the random duration model due to a law of large numbers.
CHAPTER 3

STICKY INFORMATION UNDER DYNAMIC MODELS

3.1 Introduction

Most macroeconomists agree that many macroeconomic variables tend to be sticky or persistent to some extent. Researchers have sought to explain the sluggish behavior of macroeconomic variables. Some of them emphasize the role of limited information as a device for explaining the sluggish behavior. There are many examples in this direction. For example, the literature on adaptive learning, including Evans and Honkapohja (2001). Sims (2003) and Moscarini (2003) use the assumption of an information processing constraint to explain sluggish behavior of macroeconomic variables. Woodford (2002) and Hellwig (2002) use the concept of imperfect common knowledge to explain the dynamics of inflation and output. Reis (2004a) and Reis (2004b) give the microfoundation for the sticky information model.

The sticky information model originally developed by (Mankiw and Reis 2002b, MR) has drawn attention because of its great simplicity. In their model, firms’ choice variable is the price of their good under monopolistic competition. They consider that firms may collect information and recompute the optimal price only infrequently. For
this reason, they may not necessarily have up-to-date information for the determination of their actual price. As an approximation, they assume that economic agents have a constant probability of being informed about the state of the economy, regardless of when they last obtained information. Their sticky information model can explain stylized facts on inflation dynamics that cannot be explained by a sticky price model with rational expectations.

The goal of this paper is to explore the applicability of the sticky information model to a different context: the observed sluggish behavior of macroeconomic variables. In particular, we consider a dynamic optimization problem often seen in macroeconomic textbooks, add sticky information and seek an expression for aggregate variables.

Exploring this is an interesting extension of the sticky information model. In MR, the Blanchard and Kiyotaki (1987) model is utilized. In Blanchard and Kiyotaki (1987), the optimization problem for a firm is static and the dynamics don’t appear until the sticky information assumption is introduced. The optimal choice that a firm would make without the information restriction is only a function of the current state of the economy, because the original model is static. More generally, $X_t$, the optimal choice that an economic agent would make in period $t$ if there were no information restrictions, is modeled to be a function of $Y_t$, the state of the economy in period $t$:

$$X_t = f_s(Y_t),$$

where $f_s(\cdot)$ is the policy function of the static optimization problem. In many applications in macroeconomics, on the other hand, the optimal choice that an economic agent would make is a function not only of the current state but also the future state
of the economy. Formally,

\[ X_t = f_d(X_{t-1}, Y_t, E_t Y_{t+1}, E_t Y_{t+2}, \cdots), \]

where \( f_d(\cdot, \cdot, \cdots, \cdot) \) is the policy function from the dynamic optimization problem to a second-order approximation or typically the solution to the second order linear difference equation. One example is the permanent income model. In this case, \( X_t \) is the household’s assets in period \( t \) and \( Y_t \) is exogenous income. Another example is the neo-classical model of investment. In this case, \( X_t \) is the capital stock in period \( t \) and \( Y_t \) is the profitability for a firm. In both examples, the optimal choice has both a feedback part and a feedforward part.

I introduce MR’s sticky information assumption into this type of dynamic model. However, a problem for the extension may arise. In the static case, the optimal choice is independent of choices made in the past. In the dynamic case, by contrast, the optimal choice \( X_t \) depends on the optimal choice made in the past. In other words, the predetermined choice is important for the current optimal choice. This fact implies that the state variable \( X_t \) becomes history-dependent once the sticky information assumption is imposed. When agent’s information set is updated randomly, the predetermined choice \( X_{t-1} \) might have been determined based on a relatively new information set or a far older information set. Since the current choice \( X_t \) depends on \( X_{t-1} \), what information set was used to determine \( X_{t-1} \) matters for the current choice. This current choice affects the choices in the future through the dynamic structure of the model. As a result, the difference in information updating in the past affect a whole path of individual endogenous state variables. Moreover, if the endogenous state variables are different because of different histories of information updating, jump variables are also affected differently via endogenous state variables.
This history dependence of endogenous state variables makes aggregation difficult, especially in introducing timing of information updating that MR assumes: the newest information randomly arrives with a constant probability. When endogenous state variables are aggregated, all histories of information updating should be distinguished and tracked of. Because the number of histories increases exponentially, the extension to dynamic models is extremely difficult.

The first finding in this paper is that, notwithstanding history dependence of the individual endogenous state variable, the aggregate equation turns out to be very simple. To obtain the aggregate dynamics of variables under the sticky information assumption, all that need be done is to solve the rational expectations model first and to change the rational expectation operator into MR’s formulation of expectations. When our interest is limited to dynamic properties of aggregate variables, this slight change is sufficient.

The result obtained here implies that the sticky information assumption is easily applicable to different dynamic contexts. Because I consider only a partial equilibrium model, the paper does not investigate whether a simple aggregation equation holds in general equilibrium. For simplicity, I also rule out the case where strategic complementarities exist. Nevertheless, there are still many applications that can be considered. I discuss only the example of fixed investment in this paper, but one can apply the sticky information assumption to other partial equilibrium models such as consumption, housing investment, labor demand, and inventory investment.

The second finding in this paper is that the sticky information model can be an alternative explanation for the sluggish behavior of investment. As one might expect, our simulation results show that the impulse response of the investment to shocks are
strongly hump-shaped and also that sticky information increases persistence. This hump-shaped behavior and additional persistence give us a better understanding of investment dynamics.

The hump-shaped behavior of investment has been investigated by several authors in the literature. For example, Wen (1998) focuses on a time-to-build model to explain persistent and cyclical investment behavior in a dynamic general equilibrium model. His simulations show that investment is hump-shaped in response to a technology shock under his time-to-build assumption.\textsuperscript{21}

Another approach assumes investment adjustment costs rather than capital adjustment cost. Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Linde (2004) introduce an investment adjustment cost function to generate a hump-shaped response of investment obtained in their VAR analysis. They point out that a hump-shaped pattern of investment is not generated by the capital adjustment cost. Also, the investment response under the capital adjustment cost peaks in the first period of a shock.

The time-to-build model and investment adjustment cost model explain the sluggish behavior of investment by simply changing the firm’s investment from a jump variable to an endogenous state variable. Without this change, aggregate investment is likely to show the maximum immediate response to a shock. In contrast, the

\textsuperscript{21}His assumption is different from the standard time-to-build model in Kydland and Prescott (1982). Kydland and Prescott (1982) assume that there are pure gestation lags until investment spending is embodied in the fixed capital stock. Wen (1998) assumes that fixed investment spending is demanded at each stage of constructing the fixed capital until it is complete. By modeling this kind of the time-to-build, he can explain a hump-shaped response of the investment, which otherwise cannot be explained by Kydland and Prescott (1982)’s standard formulation.
sticky information assumption does not make such a change but nonetheless generates a hump-shape for aggregate investment. It is obtained because most firms don’t update their information to know when a shock has occurred.

This paper is organized as follows. Section 3.2 describes the model and introduces sticky information. In section 3.3, I show that investment dynamics under the sticky information are hump-shaped and persistent. Section 3.4 concludes.

3.2 The Model

3.2.1 Set up

I apply the sticky information assumption to a neo-classical investment model with capital adjustment costs. My approach to the application is as follows. First, I present a neo-classical investment model and derive the log-linearized policy for the capital stock with no information restriction. Second, I consider the expectation of the optimal choice conditional on the information set which is updated only infrequently. I refer this conditional expectation to the optimal plans. Finally, I discuss the cross-sectional distribution of information sets and individual capital stocks. I show an aggregation result of capital stocks across all firms.

Consider the following partial equilibrium model. The economy consists of a large number of firms indexed by $i$ distributed uniformly on $[0, 1]$. They use an identical technology and sell their output in competitive markets. Factor markets are also competitive. I assume individual firm $i$ chooses $K_t(i)$ and $I_t(i)$ to maximize
\[ V(K_{t-1}(i), A_t) = \max_{\{K_t(i),I_t(i)\}} A_t K_{t-1}(i)^\alpha - P^I I_t(i) - \frac{c}{2} \left( \frac{I_t(i)}{K_{t-1}(i)} - \delta \right)^2 K_{t-1}(i) + \beta E_t V(K_t(i), A_{t+1}) \]
\[ \text{s.t.} \quad K_t(i) = (1 - \delta) K_{t-1}(i) + I_t(i), \]

given \( K_{t-1}(i) \). Here \( I_t(i) \) is the firm’s investment, \( P^I \) is the relative price of new capital goods and constant for simplicity. I refer to \( A_t \) as a measure of profitability since variations in technology, factor and demand price are reflected in \( A_t \).\(^{22}\) Due to competitive factor markets, I assume that \( A_t \) is the same across all firms. When solving this problem, the firm has the up-to-date information set \( \Omega_t \). As usual, I assume that the information set contains the firm’s past decisions and the sequence of exogenous variables up to that period.

To keep the model simple and clarify the effect of the sticky information assumption, I abstract from non-convex adjustment cost and irreversibility of investment and assume a convex adjustment cost of the form \( \frac{c}{2} \left( \frac{I_t(i)}{K_{t-1}(i)} - \delta \right)^2 K_{t-1}(i) \). Under this specification of the adjustment cost, one can obtain a linear relationship between the investment-capital ratio and average \( Q \), because the adjustment cost is homogeneous of degree one in \( I_t(i) \) and \( K_{t-1}(i) \). In addition, the introduction of the capital adjustment cost makes the model dynamic.

\(^{22}\)In this formulation, I abstract from labor inputs. Implicitly, the production function may be assumed to be \( F(K_{t-1}(i), L_t(i)) = Z_t K_{t-1}(i)^\gamma L_t(i)^\nu \), where \( Z_t \) is the technology shock and \( L_t(i) \) is the labor demand. Since my interest is in the effect of shock to a measure of profitability on the investment, I abstract from the firm’s labor decision of the firm by solving out the static labor demand condition.
The first order conditions are

\[
q_t(i) = c \left[ \frac{I_t(i)}{K_{t-1}(i)} - \delta \right] + P^I \tag{3.2}
\]

\[
q_t(i) = \beta E_t V_K(K_t(i), A_{t+1}) \tag{3.3}
\]

\[
V_K(K_{t-1}(i), A_t) = \alpha A_t K_{t-1}(i)^{\alpha-1} - \frac{c}{2} \left[ \frac{I_t(i)}{K_{t-1}(i)} - \delta \right]^2 + c \left[ \frac{I_t(i)}{K_{t-1}(i)} - \delta \right] \frac{I_t(i)}{K_{t-1}(i)}
\]

\[+(1 - \delta)q_t(i), \tag{3.4}\]

where \(q_t(i)\) is the Lagrange multiplier on the capital accumulation equation. Equation (3.2) equalizes the marginal cost of investment with the marginal return \(q_t(i)\). This “marginal q” is the expected present value of the derivative of the value function as shown in (3.3). By the envelope theorem (3.4), \(q_t(i)\) is the expected present value of the marginal profit to capital.

Since the model is non-linear, I log-linearize it to obtain a linear policy function for the log-deviation of capital from the steady state, given the path of the log-deviation of profitability from the steady state. In what follows, the hatted variables are used to denote the log-deviation from the steady state. In Appendix B, the log-linearization of the first order conditions yields the optimal policy function for the capital stock:

\[
\dot{K}_t^*(i) = \zeta_1 \dot{K}_{t-1}(i) + \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_t(\dot{A}_{t+k+1}), \tag{3.5}
\]

where \(\dot{K}_t^*(i)\) denotes the solution under the rational expectations model to a first order approximation. The parameter \(\zeta_1 < 1\) and \(\zeta_2 > 1\) are roots of the characteristic equation derived from Euler equation with respect to \(\dot{K}_t(i)\) and \(h > 0\) is a function of deep parameters and the steady state value of \(A\) and \(K\). They are defined in Appendix B. Because \(h > 0\), a positive shock to the firm’s profitability causes its capital stock and investment to increase. Moreover, due to the capital adjustment
cost, the capital stock today is affected by the capital stock last period and expected future profitability.

In the rational expectations model, the firm’s information set is updated every period. The optimal (log-linearized) capital stock in the future period is written as

\[
\hat{K}_{t+j}^*(i) = \zeta_1 \hat{K}_{t+j-1}^*(i) + \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_{t+j} \hat{A}_{t+j+k+1}.
\]  

(3.6)

Note that the feedforward part of the solution (the discounted sum of future log-linearized profitability) is conditional on the information set \(\Omega_{t+j}\). In other words, the optimal capital stock \(\hat{K}_{t+j}^*(i)\) is a function of \(\Omega_{t+j}\). By analogy, \(\hat{K}_{t+j-1}^*(i)\) is a function of \(\Omega_{t+j-1}\).²³

### 3.2.2 Sticky information

In this subsection, I define the optimal plan with sticky information. Under the sticky information assumption, firms are informed only infrequently. For this restriction, the firm’s future optimal plans for the capital stock \(\{K_{t+j}(i)\}_{j=0}^{\infty}\) are all conditional not on \(\Omega_{t+j}\) but on \(\Omega_t\), if they have \(\Omega_t\) and will continue to have \(\Omega_t\) in the future. In order to make the notation as simple as possible, I still continue to assume that firm \(i\) holds the initial capital stock \(K_{t-1}(i)\) and it has \(\Omega_t\) when solving its maximization problem. My notation for the optimal plans in the sticky information model is

\[
\hat{K}_t^{(0)}(i) = \hat{K}_t^*(i)
\]

(3.7)

\[
\hat{K}_t^{(j,j-1,j-2,\ldots,1,0)}(i) = E_t \hat{K}_{t+j}^*(i) \text{ for } j = 1, 2, 3, \ldots,
\]

(3.8)

²³These future variables can be also interpreted as a complete contingency plan on the information set. If the firm solves the dynamic optimization problem by Lagrangian, it chooses the sequence of current and future capital stocks simultaneously as a complete contingency plan. \(\hat{K}_{t+j}^*(i)\) will be actually set as a value only after \(\Omega_{t+j}\) becomes available at the beginning of every period. (Sargent (1987, Chapter 14 p391-392))
where I use a superscript \((0)\) for the capital stock in period \(t\), and \((j, j-1, \cdots, 1, 0)\) for the capital stock in period \(t+j\) to describe firm \(i\)’s history of information updating. Numbers in a superscript show a length of information delay. For example, \(\hat{K}_t^{(0)}(i)\) means that the capital stock in period \(t\) is chosen by firm \(i\) with an informational delay of zero periods, because the choice of \(\hat{K}_t\) is based on \(\Omega_t\). On the other hand, \(\hat{K}_{t+j}^{(j,j-1,\cdots,1,0)}(i)\) denotes that the capital stock in period \(t+j\) is determined based on the \(j\)-period old information set. The superscript in (3.8) has \(j+1\) elements such as \(j, j-1, \cdots, 1, 0\). These elements indicate that the lagged capital stocks \((\hat{K}_t(i), \hat{K}_{t+1}(i), \cdots, \hat{K}_{t+j-1}(i))\) are determined based on \(\Omega_t\) as well as \(\hat{K}_{t+j}(i)\). If the firm is uninformed in period \(t+j\) and must choose this capital stock \(\hat{K}_{t+j}^{(j,j-1,\cdots,1,0)}(i)\), it must hold \(\Omega_t\) until period \(t+j\), suggesting that the firm must have held \(\Omega_t\) in period \(t+j-1, t+j-2, \cdots, t+1\) and \(t\) to determine its capital stock during these periods. The first element in \((j, j-1, \cdots, 1, 0)\) is the length of information delay in period \(t+j\) and the second element is that in period \(t+j-1\). Other elements in this superscript are explained in the same way. Later, I will discuss the rationale for describing a history of information updating in more detail.

By taking the expectation of (3.5) and (3.6) conditional on \(\Omega_t\), the optimal plans with sticky information are written as

\[
\hat{K}_t^{(0)}(i) = \zeta_1 \hat{K}_{t-1}(i) + \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_t(\hat{A}_{t+k+1}) \tag{3.9}
\]

and

\[
\hat{K}_{t+j}^{(j,j-1,\cdots,1,0)}(i) = \zeta_1 \hat{K}_{t+j-1}^{(j-1,j-2,\cdots,1,0)}(i) + \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_t(\hat{A}_{t+j+k+1}), \tag{3.10}
\]

where \(\hat{K}_{t+j}^{(j,j-1,\cdots,1,0)}(i)\) is a function of \(\hat{K}_{t+j-1}^{(j-1,j-2,\cdots,1,0)}(i)\) and the expected future path of profitability. Both feedback and feedforward parts are conditional on \(\Omega_t\) in (3.10).
Applying a lag operator $B^{-1}$ to the feedforward part in the solution, the optimal plans are\footnote{This lag operator should be distinguished from the usual lag operator. $B^{-1}$ is defined such that $B^{-1}E[x_{t+j} | \Omega_t] = E[x_{t+j+1} | \Omega_t]$. In words, it does not alter the information set. In contrast, the usual lag operator $L$ is defined such that $L^{-1}E[x_{t+j} | \Omega_t] = E[x_{t+j+1} | \Omega_{t+1}]$. See Sargent (1987, chapter 11 p395) for the detail.}

\begin{equation}
\hat{K}_t^{i(0)}(i) = \zeta_1 \hat{K}_{t-1}(i) + \phi(B^{-1})E_t(\hat{A}_{t+1})
\end{equation}

and

\begin{equation}
\hat{K}_t^{(j,j-1,\ldots,0)}(i) = \zeta_1 \hat{K}_{t+j-1}^{(j-1,j-2,\ldots,0)}(i) + \phi(B^{-1})E_t(\hat{A}_{t+j+1}),
\end{equation}

where $\phi(B^{-1}) = \frac{\zeta_1 h}{1-\zeta_2 B^{-1}}$.

Now, consider the information updating for the individual firm. Suppose that the firm continues to have the information set $\Omega_t$ until $t+J-1$ and updates the information set at the beginning of the period $t+J$ where $J \geq 0$. Then, the old optimal plans are ignored and new optimal plans are re-calculated based on the new information set $\Omega_{t+J}$, given the capital stock $K_{t+\mathbf{J}-1}^{(J-1,J-2,\ldots,0)}$. Given the information updating in period $t+J$, the log-linearized optimal plans are similar to (3.11) and (3.12).

\begin{equation}
\hat{K}_{t+J}^{(0,J-1,J-2,\ldots,0)}(i) = \zeta_1 \hat{K}_{t+J-1}^{(J-1,J-2,\ldots,0)}(i) + \phi(B^{-1})E_{t+J}(\hat{A}_{t+J+1})
\end{equation}

and

\begin{equation}
\hat{K}_{t+J+j}^{(j,j-1,\ldots,0,J-1,J-2,\ldots,0)}(i) = \zeta_1 \hat{K}_{t+J+j-1}^{(j-1,j-2,\ldots,0,J-1,J-2,\ldots,0)}(i) + \phi(B^{-1})E_{t+J}(\hat{A}_{t+J+j+1}),
\end{equation}

for $j = 1, 2, 3, \ldots$.

What is the rationale for describing the complete history of a firm’s information updating? In equation (3.13), firm $i$’s history of information updating in period $t+J$
is that $\Omega_t$ was held until period $t + J - 1$ and $\Omega_{t+J}$ was obtained in period $t + J$. On the other hand, imagine another firm $i'$ experiencing a slightly different history. Suppose that it holds $\Omega_t$ until $t + J - 2$ but wins a streak of updating in $t + J - 1$ and $t + J$. Its information sets in these periods are $\Omega_{t+J-1}$ in period $t + J - 1$ and $\Omega_{t+J}$ in period $t + J$. Then, its optimal plan for period $t + J$ is

$$\hat{K}_{t+J}^{(0,0,J-2,J-1,\ldots,1,0)}(i') = \zeta_1 \hat{K}_{t+J-1}^{(0,J-2,\ldots,1,0)}(i') + \phi(B^{-1})E_{t+J}A_{t+J+1},$$

(3.15)

because its capital in period $t + J - 1$ is determined based on the information set $\Omega_{t+J-1}$. Note that the feedback part in (3.15) is different from that in (3.13) while their feedforward parts are the same. Due to the feedback part, this stock of capital chosen by $i'$ might be different from the capital stock in (3.13) if the profitability changes in period $t + J - 1$. In addition, the difference affects the optimal plans for the subsequent periods. The subsequent optimal plans for the firm $i'$ are

$$\hat{K}_{t+J+j}^{(j,J-1,\ldots,0,0,J-2,\ldots,0)}(i') = \zeta_1 \hat{K}_{t+J+j-1}^{(j-1,J-2,\ldots,0,0,J-2,\ldots,0)}(i') + \phi(B^{-1})E_{t+J}A_{t+J+j+1}.$$  

Thus, subsequent plans may be also affected by choices made a long time ago. The feedback part of the solution requires us to describe the complete history of information updating in the firm’s optimal plans.

It should be noted that this complicated characteristic is peculiar to a dynamic model as I discussed in the introduction. In a static model, it is not necessary to describe the complete history of information updating, because the fact that the optimal choice is determined in a static manner implies that there is no restriction on the determination for optimal plans except for the information set and they can freely move in response to a change in the state of the economy. On the other hand, this is not true in our dynamic setting. Since the optimal choice that a firm would
make if there were no information restriction has the feedback part, the optimal plans
with sticky information also have the feedback part and thus cannot freely move in
response to a change in the state of the economy. The feedforward parts are common
across firms having the same information set, but the feedback parts are not because
the histories of information updating might be different.

Moreover, this history dependence suggests that the original maximization prob-
lem (3.1) must differ across firms. In the maximization problem (3.1), firms take
$K_{t-1}(i)$ as given. However, going back to the past, $K_{t-1}(i)$ differs across firms if
their histories of information updating differ. If the capital stock in period $t-1$ is
determined conditional on $\Omega_{t-1}$, then the capital stock is given by $K_{t-1}^{(0,\cdots)}(i)$ instead
of $K_{t-1}(i)$. If the capital stock was determined conditional on $\Omega_{t-j}$ for any positive
$j$, then the capital stock should be given by $K_{t-1}^{(j-1,j-2,\cdots,0,\cdots)}(i)$. By analogy, it is
necessary to go back to infinitely many past periods.

To aggregate individual optimal plans with sticky information, it is necessary to
trace the distribution not only of the information sets but also of the optimal plans. In
the next subsection, I show how the aggregate capital stock in the economy behaves.
Although the aggregation is complicated, the result turns out to be simple.

3.2.3 Aggregation

In the set-up of the problem, firms have the same technology and face competitive
goods and factor markets. But by introducing the sticky information assumption,
heterogeneity of the optimal plans appears because of the different information sets
of firms. Moreover, a dynamic structure of the original model - an adjustment cost
function in our setting - leads to more complicated heterogeneity.
A question arises: Is there any simple aggregate equation for the capital stock in the economy?

For this aggregation question, I consider two schemes of information updating, following Dupor and Tsuruga (2005). The first scheme is the random duration scheme originally developed by MR. Under the random duration scheme, firms’ information is updated every period but only with a constant probability $\lambda$. If information is not updated, firms must continue to use their old information set. This process occurs regardless of when they last updated their information set. Due to the property of random arrival of information, the cross-sectional distribution of their information becomes geometric. The second scheme is the fixed duration scheme. Under this scheme, information updating occurs deterministically. All firms in the economy use their information for $N$ periods and then receive up-to-date information. However, the updates are staggered as in Fischer (1977). In other words, there are $N$ cohorts in the economy and each cohort is equally distributed with a fraction $1/N$. In any period, one cohort has up-to-date information while other cohorts have information up to $N - 1$ periods old.

The propositions state that the answer to the question is asymptotically yes under the random duration scheme and yes under the fixed duration scheme. I show the propositions for the two duration scheme in turn.

**Random duration scheme**

For the random duration scheme, I make the following assumptions.
Assumption 3.2.1  1. All firms have a common capital stock $K_{t-D-1}$ in period $t-D-1$. That is, $K_{t-D-1}(i) = K_{t-D-1}$ for $\forall$ $i$, where $D$ is some non-negative number.

2. In the beginning of period $t-D$, all firms have a common information set $\Omega_{t-D}$.

Under the assumptions, every firm solves the maximization problem given the same level of the initial capital stock and has the common information set in period $t-D$. The maximization problem is

$$V(K_{t-D-1}, A_{t-D}) = \max_{\{K_{t-D(i)}, A_{t-D(i)}\}} A_{t-D}K_{t-D-1}^\alpha - \frac{c}{2} \left( \frac{I_{t-D}(i)}{K_{t-D-1}} - \delta \right)^2 K_{t-D-1}$$

$$-P^n I_{t-D}(i) + \beta E_t V(K_{t-D}(i), A_{t-D+1})$$

s.t. $K_{t-D}(i) = (1 - \delta)K_{t-D-1} + I_t(i)$, given $K_{t-D-1}$.

Firms calculate their optimal plans for the current and future capital stocks. Every period, some firms update their information and the others don’t due to the randomness of information arrival. Informed firms will recompute the maximization problem given their new information and the predetermined lagged capital stock. Whether they are informed or not, the capital stock they actually choose will be dependent on the information set they have and their history of information updating expressed by the lagged capital stock.

The tree in Fig 3.1 portrays the case of $D = 2$. In this case, every firm has the common capital stock $K_{t-3}$. When determining the capital stock in period $t-2$, each firm also has $\Omega_{t-2}$ in common. Therefore, given $K_{t-3}$ and $\Omega_{t-2}$, the capital stock in period $t-2$ is written as $K_{t-2}^{(0)}(i)$. In the next period, each firm chooses $K_{t-1}(i)$ given its information restriction. A fraction $\lambda$ of firms choose their capital stock based on
NOTE: $D$ is set to two in the figure for expository purpose. The index $i$ is suppressed here. With a probability $\lambda$, the information set held by a firm is updated regardless of when it previously updated information.

Figure 3.1: Histories of information updating and distribution of capital stocks under the random duration scheme

the new information set $\Omega_{t-2}$ and thus the capital stock in this period is $K_{t-2}^{(0)}(i)$. On the other hand, the others don’t update information and their information set remains $\Omega_{t-2}$ and thus their capital stock is written as $K_{t-1}^{(1,0)}$. The number of histories or types are 2 in period $t - 1$.

In period $t$, the number of histories becomes $2^2$. From the top node in period $t-1$, a fraction $\lambda$ of the firms on the node update information and thus their capital stock is $K_{t-1}^{(0,0,0)}(i)$. But the others choose $K_{t-1}^{(1,0,0)}(i)$. Similarly, a fraction $\lambda$ of the firms on the bottom node are informed in the beginning of period $t$ and their capital stock is $K_{t-1}^{(0,1,0)}(i)$. The others remain to have $\Omega_{t-2}$ so that their capital is $K_{t-1}^{(2,1,0)}(i)$. The others remain to have $\Omega_{t-2}$ so that their capital is $K_{t-1}^{(2,1,0)}(i)$.
In general, the number of histories is $2^D$ in period $t$ so that the superscript for $K_t(i)$ has $D + 1$ elements. In an extreme case, the superscript might be $(0, 0, \cdots, 0)$ if a firm has updated information every period (with a very low probability $\lambda^D$). In another extreme case, the superscript might be $(D, D-1, \cdots, 1, 0)$ if a firm has been uninformed since the initial period (with very low probability $(1 - \lambda)^D$).

In general, we obtain the following results under the assumption 3.2.1:

**Proposition 3.2.1** Suppose that the firms in the economy solve the maximization problem (3.16). Under Assumption 3.2.1 and the assumption that the firm’s information set is updated every period with a constant probability $\lambda$, the log-deviation of the aggregate capital stock $\hat{K}_t$ is expressed as follows:

$$\hat{K}_t = \zeta_1 \hat{K}_{t-1} + \lambda \sum_{d=0}^{D-1} (1-\lambda)^d \left[ \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_{t-d} \hat{A}_{t+k+1} \right] + (1-\lambda)^D \left[ \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_{t-D} \hat{A}_{t+k+1} \right]$$

(3.17)

for $\forall D \geq 1$.

**Proof**: See appendix C.1. □

In this proposition, the capital stock in the initial period is pinned down up to finitely many periods by the assumption 3.2.1. However, as discussed in the previous subsection, it is necessary to go back infinitely many periods. The next proposition is the case for $D \rightarrow \infty$:

**Proposition 3.2.2** As $D \rightarrow \infty$, equation (3.17) tends to be the following equation:

$$\hat{K}_t = \zeta_1 \hat{K}_{t-1} + \lambda \sum_{d=0}^{\infty} (1-\lambda)^d \left[ \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_{t-d} \hat{A}_{t+k+1} \right].$$

(3.18)

**Proof**: Trivial, because $0 < \lambda < 1$. □

The implications of Proposition 3.2.2 are twofold. First, the rational expectations and sticky information models differ only in their feedforward part in the aggregate
equation. Indeed, setting $\lambda$ to one yields the rational expectations solution to the maximization problem (3.1). In the rational expectations model, the feedforward part of the solution has only the expectation of $\{\hat{A}_{t+k+1}\}_{k=0}^{\infty}$ conditional on $\Omega$. In the sticky information model, by contrast, the feedforward part of the solution involves not only the expectation conditional on $\Omega$ but also expectations conditional on older information sets. Second, the parameters $\zeta_1$ and $\zeta_2$ in the solution can be calculated from the standard solution technique for the model with rational expectations. As a result, whenever the rational expectations model can be solved at least to a second-order approximation, the sticky information model can also be solved to a second-order approximation. It is unnecessary to re-calculate eigenvalues and eigenvectors for the sticky information model itself.

From Proposition 3.2.2, the aggregate capital stock can be obtained in the following steps: 1) Set up a representative agent model under rational expectations. 2) Solve the model under rational expectations to find roots of the characteristic equation of the model. 3) Replace the feedforward part in the rational expectations model with the expectation formulation in the sticky information model. Therefore, when only an aggregate variable is of interest, we may short-cut aggregation and complicated histories of information updating by this procedure.

**Fixed duration scheme**

Next, I consider the fixed duration scheme. While the number of histories exponentially increases under the random duration scheme, there are always only $N(<\infty)$ histories of information updating under the fixed duration scheme.

Fig 3.2 portrays the case of $N = 4$. Firm $i$ is categorized into one out of four histories, depending on its initial information set. The first line in the figure shows
Figure 3.2: Histories of information updating and distribution of capital stocks under the fixed duration scheme

a history for a cohort being informed in period $t$. The capital stock in period $t$ is conditional on $\Omega_t$ and written as $K_{t-4}^{(0,3,2,1,\cdots)}(i)$. In this history of information updating, the firm’s capital stock between $t - 4$ and $t - 1$ is all conditioned on $\Omega_{t-4}$, as implied by a superscript $K_{t-4}^{(0,3,2,1,\cdots)}(i)$. In the second line in the figure, the capital stock held by the cohort is $K_{t}^{(1,0,3,2,\cdots)}(i)$, because the information updating occurred in the previous period. For the other lines, the path of capital stock can be understood in the same way. The only difference is the timing of the information updating.

The aggregate capital stock under the fixed duration scheme is defined as the unweighted average of the individuals’ capital stock. When aggregating the capital stocks, we take the average of capital stocks vertically. Intuitively, it is straightforward...
to obtain the aggregation equation, but I make some technical assumptions to have consistency with the discussion in the previous subsection.

**Assumption 3.2.2**  
1. All firms within the \( n \)-th cohort have a common initial capital stock in period \( n - 1 \), where \( n = 1, 2, \ldots, N \).

2. In the beginning of period \( n \), firms in the \( n \)-th cohort have a common information set \( \Omega_n \).

Assumption 3.2.2 is required to make sure that all firms in the economy have their initial information set and capital stock. Under this assumption, we obtain the following aggregation result:

**Proposition 3.2.3** Under Assumption 3.2.2 and the assumption that information updating occurs every \( N \) periods and the timing of updating is staggered as in Fischer (1977), the log-deviation of the aggregate capital stock \( \hat{K}_t \) is expressed as follows:

\[
\hat{K}_t = \zeta_1 \hat{K}_{t-1} + \frac{1}{N} \sum_{d=0}^{N-1} \left[ \zeta_1 h \sum_{k=0}^{\infty} \zeta_2^{-k} E_{t-d} \hat{A}_{t+k+1} \right],
\]

(3.19)

for \( \forall \ t > N \).

*Proof*: See the appendix C.2 □

As in the random duration scheme, the proposition implies that only the feed-forward part of the solution with rational expectations should be changed into the unweighted average of expectations. In order to obtain the path of the aggregate capital stock with sticky information under the fixed duration scheme, one can derive the solution to the rational expectations model first and then replace the feedforward part with the expectation formulation in the sticky information model.
In the next section, I calibrate the investment model and show some interesting aspects of investment behavior. Both models can account for impulse responses of the investment - capital ratio and its persistence.

3.3 Simulating the model

3.3.1 Parameter choice

I calibrate some parameters to evaluate the model quantitatively. The model frequency corresponds to a quarter because the original sticky information model considers this frequency. Given my choice of a quarterly frequency, I set $\beta = 0.99$ and $\delta = 0.025$. Also, I take $c = 2$ for the adjustment cost parameter as the benchmark case from Moore and Schaller (2002) and Cooper and Ejarque (2003). The curvature of the revenue function $\alpha$ is 0.77. This value comes from an implicit assumption of the production function. When the production function is given by Cobb-Douglas production function $Z_tK_t^\gamma L_t^\nu$, the exponent on capital turns out to be $\gamma/(1 - \nu)$ after solving out the firm’s labor choice. Thomas (2002) employs $\gamma = 0.325$ and $\nu = 0.580$. By following her calibration, the exponent on capital is around 0.77.\(^{25}\)

For a calibration value of inattentiveness under the random duration scheme, I set $\lambda$ to 0.25 which is a calibration value suggested by the sticky information literature.\(^{26}\)

For the fixed duration model, the length of duration is set to $N = 4$ and 7.\(^{27}\)

\(^{25}\)Cooper and Ejarque (2003) estimates the curvature of the revenue function under convex adjustment cost function. Their estimate is 0.699. This value is not much different from my calibrated value for $\alpha$.

\(^{26}\)For example, Carroll (2003) uses household survey data on inflation expectation to estimate $\lambda$ and Mankiw and Reis (2002a) also estimates the estimate of $\lambda$ from aggregate data on inflation under the assumption that wage settings are based on sticky information model. Because these estimates are not based on the firm’s investment behavior, an appropriate calibration value of $\lambda$ in our model might be different from 0.25. For this reason, I will perform a sensitive analysis for $\lambda$ later.

\(^{27}\)See Dupor and Tsuruga (2005) for the detail of these two calibration values for $N$.  

75
To obtain a series that measures profitability $\hat{A}_t$, I construct the real capital stock and real output for the US economy. Using the calibrated value for $\alpha$, I calculated a profitability measure covering 1959:2 - 2004:1. This series is approximately a random walk. Hence, I assume $\hat{A}_t = \hat{A}_{t-1} + \varepsilon_t$. I set the standard deviation of $\varepsilon_t$ so that the investment-capital ratio in the simulation matches that in the actual data.

### 3.3.2 Investment dynamics

**Impulse response function**

I begin with an analysis of impulse response functions. The upper-left panel in Fig 3.3 plots the impulse response function of the aggregate investment-capital ratio to a shock to $\hat{A}_t$ from an estimated bivariate VAR in levels. The VAR includes a linear time trend and seven lags of the measure of profitability and investment-capital ratio, 1961:1-2004:1, where the variables are all in logarithm.

I emphasize that the impulse response function of the investment-capital ratio shows a clear hump-shaped dynamic in the data shown in the upper-left panel. In response to a shock, the investment-capital ratio moves only sluggishly and the effect of the exogenous profitability shock is delayed: The peak response occurs in period five. This hump-shaped behavior is robust to the order and lags of variables. A similar pattern of hump-shaped behavior is observed if the level of (log) investment is used in estimation instead of the (log) investment capital ratio, though the impulse response function is not reported here.

---

28The source is the Bureau of Economic Analysis. The capital stock includes nonresidential equipment and software and nonresidential structures. The real capital stock is calculated by the perpetual inventory method with the depreciation rate of 2.5%. The initial real capital stock is the annual capital stock figure for year-end 1946.
NOTE: The impulse response function in the upper-left panel is an estimated response from the data. It is estimated from a bivariate VAR consisting a linear time trend and seven lags of the profitability and investment capital ratio.

The impulse response functions in the upper-right panel is the simulated responses to a one standard deviation shock from the rational expectations model. The lower panels are the simulated responses from the two schemes of sticky information. For the fixed duration scheme, the two calibrated lengths of duration ($N=4$ and $7$) are used. In all simulated responses, the standard deviations are adjusted so that each of the standard deviations of the investment-capital ratio matches to the investment-capital variation of actual data.

Figure 3.3: Impulse response functions: the investment-capital ratio to a one standard deviation shock to a measure of profitability
Other panels in the figure show the simulated responses of the investment-capital ratio to a one standard deviation shock to the measure of profitability. Note that the impulse response function under rational expectations fails to replicate the hump-shaped pattern in the investment-capital ratio as shown in the upper-right panel. By contrast, a hump-shaped pattern is obtained with sticky information. The two lower panels show the impulse response functions under the random duration and the fixed duration sticky information schemes. Under random duration sticky information, the peak response occurs in period six. Under fixed duration, the peaks are similar to the random duration model: three under $N = 4$ and six under $N = 7$. Thus, the difference between the rational expectations and sticky information models is quite striking.

Not surprisingly, the difference from the rational expectations model comes from the information delay. When a profitability shock happens, only a fraction of firms know about the profitability shock and respond to it. Under the random duration scheme, 25% of firms increase the investment in the first period and 25% plus 18.75% ($=0.25(1-0.25)$) of firms know about the shock and respond to it in the next period. The other firms are uninformed about the shock and don’t increase investment until the shock is known to them. Due to the uninformed firms, the aggregate investment-capital ratio behaves sluggishly.

In this model, the individual investment-capital ratio per se increases quickly like the rational expectations model once a firm is informed about the profitability shock. The sluggish behavior of the aggregate investment-capital ratio appears through the aggregation. Except for the information delay, the model setting in the individual level is the same as the rational expectations model and thus there is no mechanism
to generate sluggish behavior of individual investment. This is not inconsistent with the observed lumpy investment behavior in the investment literature.\textsuperscript{29} However, the time-to-build model and the investment adjustment cost function model introduce sluggishness by setting the investment to a state variable in the model and thus they cannot generate a quick response of investment in the individual level. This would be inconsistent with the lumpy investment literature.

The parameters in the model affect the impulse response functions. The results of sensitivity analysis are shown in Table 3.1 - 3.3. First, Table 3.1 shows the sensitivity analysis on the adjustment cost parameter $c$. The estimates of the adjustment cost parameter vary widely across empirical papers. The table shows the quarters in which the peak response occurs, depending on a variety of values $c$.\textsuperscript{30} Although the parameter affects the magnitude of the response, the hump-shaped pattern in the simulation are robust: the peaks occur in period 3 - 11 under the random duration model and the peaks under the fixed duration model remain the same. Thus, for empirically plausible adjustment costs, the slow response of investment is not altered much.

Second, the degree of inattentiveness ($\lambda$ and $N$) has a strong effect on the sluggishness of the investment-capital ratio. Table 3.2 shows the quarters in which the peak response occurs, depending on the probability of information updating and the length of duration of inattentiveness. In the table, $\lambda$ ranges from 0.10 to 1.00 and $N$ ranges

\textsuperscript{29}Empirics on lumpy investment are discussed in Doms and Dunne (1998).

\textsuperscript{30}For example, in Tobin’s q approach, Hayashi (1982) estimated the adjustment cost parameter to be about 23. Eberly (1997), using individual firm data, estimated it to be around 11. In Euler equation approach, Whited (1992) found estimates in the range 0.54-2.05.

79
Table 3.1: The quarter of peak response in the investment-capital ratio: different values of $c$

<table>
<thead>
<tr>
<th>$c$</th>
<th>Random duration</th>
<th>Fixed duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>30</td>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3.2: The quarter of peak response in the investment-capital ratio: different values of the degree of inattentiveness

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Random duration</th>
<th>$N$</th>
<th>Fixed duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>12</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>0.20</td>
<td>7</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>0.25</td>
<td>6</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>0.40</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>0.50</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>0.67</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1.00</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

from 19 to 1.\textsuperscript{31} As the table shows, the more frequently firms update information, the less sluggish the investment-capital ratio is.

Finally, the sensitivity analysis on the persistence parameter of the profitability is shown in Table 3.3. Suppose that $A_t$ follows an AR(1) process instead of a \textsuperscript{31}In each row in Table 3.2, the cross-sectional average period of inattentiveness is approximately matched. Dupor and Tsuruga (2005) discuss the cross-sectional average period of inattentiveness under the two schemes as one of criteria for comparisons. While it is $1/\lambda$ under the random duration scheme, it is $(1 + 2 + \cdots + N)/N = (N + 1)/2$ under the fixed duration scheme. For example, when $\lambda = 4$, $N$ should be 7 if one use the criterion matching the cross-sectional average period of inattentiveness.
random walk process: $\hat{A}_t = \rho \hat{A}_{t-1} + \varepsilon_t$. Under both schemes, a peak response of the investment-capital ratio is immediate if the persistence of the profitability is low enough. However, for plausible values of persistence parameter, the peak responses are robust to the persistence parameter.

Although one cannot directly compare our case with the model of inflation and output, it is worth pointing out that MR obtained a strongly hump-shaped impulse response function for inflation with a relatively low persistence parameter of 0.5. But, in our case, the hump-shape is largely reduced with a low persistence parameter $\rho$.

The intuition behind this is the assumption of strategic dependence of individual decisions. In the neo-classical investment model under perfect competition, individual firms don’t need to consider the behavior of aggregate investment. In the case of the inflation model, by contrast, the desired price is determined in a strategic manner. Monopolistically competitive firms care about not only the path of nominal money supply but also the path of the price level when they determine their own price. In this setting, even informed firms set their price sluggishly because their desired
### Table 3.4: Autocorrelations for the investment-capital ratio

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Random duration</th>
<th>Fixed duration</th>
<th>Rational expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>N = 4</td>
<td>N = 7</td>
</tr>
<tr>
<td>1</td>
<td>0.895</td>
<td>0.854</td>
<td>0.899</td>
<td>0.932</td>
</tr>
<tr>
<td>2</td>
<td>0.712</td>
<td>0.656</td>
<td>0.683</td>
<td>0.783</td>
</tr>
<tr>
<td>3</td>
<td>0.478</td>
<td>0.449</td>
<td>0.416</td>
<td>0.584</td>
</tr>
<tr>
<td>4</td>
<td>0.225</td>
<td>0.253</td>
<td>0.159</td>
<td>0.360</td>
</tr>
<tr>
<td>5</td>
<td>-0.015</td>
<td>0.078</td>
<td>-0.031</td>
<td>0.135</td>
</tr>
</tbody>
</table>

NOTE: The data is BEA and is for 1959:1-2004:1. The number of observation is 180. The model’s predictions are obtained by the unweighted average of 100 simulations consisting of 180 observations. All data and model-generated time series are HP detrended series. In each set of simulations, the variance of the shocks in the profitability measure is adjusted so that the standard deviation of the investment-capital ratio is matched to that in the data.

Price is affected by the sluggish path of the price level which consists of informed and uninformed individual prices. In the model of inflation and output, strategic complementarities become an additional source of persistence. In our case, however, the competitive firms don’t have to consider their influence on the aggregate investment behavior. Therefore, all sluggishness of investment must come from the persistence of the exogenous shocks and the sluggishness of information.

**Persistence**

Next, I consider the persistence in the investment-capital ratio. Table 3.4 shows autocorrelations of the investment-capital ratio in the data and the model. The table suggests that the sticky information models perform much better than the rational expectations model.
3.4 Concluding remarks

This paper shows that the sticky information model proposed by MR can be incorporated into a model in which the economic agent’s optimal policy is determined dynamically rather than in a static manner. Using the example of fixed investment, I showed that the sticky information assumption can be used in a model which involves endogenous state variables. The aggregate path of the endogenous state variable depends on their previous value and a feedforward part that is the solution of the rational expectations model with MR’s sticky information expectation formulation.

Fixed investment under the sticky information model, in which information updating is infrequent, shows some interesting aspects as contrasted with the rational expectations model, in which information updating occurs every period. In particular, the impulse response function to a shock to the profitability is hump-shaped. The sticky information model also generates more persistence in the investment-capital ratio than the rational expectations model.

Much room is left for theoretical extensions. This paper did not investigate whether our formulation is also applicable in general equilibrium settings. To do so will be a hard task because economic agents must form the expectation of prices in good and factor markets by looking at the distribution of the optimal plans and thus the individual firms must know others’ expectation. For the same reason, the paper did not consider a case of strategic complementarities. When aggregate investment affects the individual firm’s optimal choice, we again need to have a model in which individual firms must know the distribution of individual firms. Since computation of the cross-sectional distribution is complicated in dynamic models, I considered a
simpler case in which agents form the expectation of purely exogenous state variables. Including strategic dependence would be interesting for future research.

Much room is also left for other applications. For example, one can apply the propositions to consumption, housing investment, dynamic labor demand and inventory investment. At the aggregate level, these variables are likely to show persistence and a sluggish response to exogenous shocks but may not be fully explained by the rational expectations model. Sims (1998) points out that many classic macroeconomic variables tend to show delayed response to all except for own shocks. Therefore, applications to other partial equilibrium models would be an interesting line of research.

The model can be easily defined in the context of the LQ control problem. Sims (2003) discusses his limited information model in the context of LQ control problem, too. Hence, the comparisons of Sims (2003) and MR’s sticky information model may be also an interesting research agenda.
APPENDIX A

THE RESPONSE OF OUTPUT AND INFLATION UNDER THE FIXED DURATION SCHEME

A.1 Impulse response functions under the fixed duration scheme

We derive the impulse response functions under the fixed duration scheme. See the appendix in MR for impulse response functions under the random duration scheme.

We employ the method of undetermined coefficients as in MR. From (2.8), the nominal expenditure has an MA(∞) form:

\[ m_t = \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i-k} = \sum_{i=0}^{\infty} \omega_i \varepsilon_{t-i}, \]

where \( \omega_i = \omega_{i-1} + \rho^i \) with \( \omega_0 = 1 \).

Under the fixed duration scheme, the price level is given by

\[ p_t = \frac{1}{N} \sum_{j=0}^{N-1} E_{t-j} (p_t + \alpha y_t). \]

Using \( m_t - p_t = y_t \), we have

\[ p_t = \frac{1 - \alpha}{N} \left[ \sum_{j=0}^{N-1} E_{t-j} p_t \right] + \frac{\alpha}{N} \left[ \sum_{j=0}^{N-1} E_{t-j} m_t \right]. \]
Our guess for the price level has an MA(∞) form similar to the nominal money supply:

\[ p_t = \sum_{i=0}^{\infty} \eta_i \varepsilon_{t-i} , \]

where \( \{\eta_i\}_{i=0}^{\infty} \) are coefficients to be determined. Substituting the guess and the MA form of nominal money supply, we get

\[ \sum_{i=0}^{\infty} \eta_i \varepsilon_{t-i} = \frac{1 - \alpha}{N} \sum_{j=0}^{N-1} \sum_{i=0}^{\infty} E_{t-j} \eta_i \varepsilon_{t-i} + \frac{\alpha}{N} \sum_{j=0}^{N-1} \sum_{i=0}^{\infty} E_{t-j} \omega_i \varepsilon_{t-i} . \]

Rearranging terms yields:

\[ \sum_{i=0}^{\infty} \eta_i \varepsilon_{t-i} = \frac{1 - \alpha}{N} \left[ \sum_{i=0}^{N-1} (i + 1) \eta_i \varepsilon_{t-i} + N \sum_{i=N}^{\infty} \eta_i \varepsilon_{t-i} \right] \]
\[ + \frac{\alpha}{N} \left[ \sum_{i=0}^{N-1} (i + 1) \omega_i \varepsilon_{t-i} + N \sum_{i=N}^{\infty} \omega_i \varepsilon_{t-i} \right] . \]

By matching coefficients, we get

\[ \eta_i = \frac{(i + 1)\alpha}{N - (i + 1)(1 - \alpha)} \omega_i \quad \text{for} \quad i = 0, 1, 2, \ldots, N - 1 \]
\[ \eta_i = \omega_i \quad \text{for} \quad i = N, N + 1, N + 2, \ldots . \]

The obtained sequence of \( \{\eta_i\} \) gives us the impulse response function of the price level.

We use the price response to find the inflation response. Guess \( \pi_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} . \) Then, \( p_t = \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-k-i} . \) Given the sequence of \( \{\eta_i\}_{i=0}^{\infty} , \{\gamma_i\}_{i=0}^{\infty} \) is calculated as:

\[ \gamma_0 = \gamma_0 \]
\[ \gamma_i = \eta_i - \eta_{i-1} \quad \text{for} \quad i \geq 1 . \]

Finally, the impulse response function of the output gap is directly obtained from \( m_t - p_t = y_t . \) Guess that \( y_t = \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} . \) Substitute all of the MA forms to get
\[
\sum_{i=0}^{\infty} (\omega_i - \eta_i) \varepsilon_{t-i} = \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}.
\]
The impulse response function of output is given by \(\{\phi_i\}_{i=0}^{\infty}\). Once these coefficients are calculated, the population autocorrelations are easily calculated in the standard way.

### A.2 Disinflationary policy under the fixed duration scheme

Next, we derive dynamic responses for inflation and output to an anticipated disinflationary policy. The policy experiment is a drop in \(\Delta m_t\) from 0.025 to 0 at period zero but it is announced at period \(-h\), where \(h \geq 0\). The exogenous nominal expenditure in logarithm is assumed to be \(m_t = (0.025)(1+t)\) for \(t < 0\) and \(m_t = 0\) for \(t \geq 0\). For the solution of dynamic paths of inflation and output under the random duration model, see MR again. We focus only on the fixed duration model here.

First, consider the path of \(p_t\) for \(t < 0\). The expected path of the price level must be the same among all firms regardless of informed or uninformed firms because the economy is in the steady state and the policy change has not occurred yet. \(E_{t-j}m_t = m_t = 0.025(1+t)\) for all \(j\) so \(p_t = 0.025(1+t)\).

Second, we consider the path of \(p_t\) for \(t \geq 0\). Decompose the price level in (2.7) into two parts: informed individual prices and uninformed individual prices.

\[
p_t = \frac{1}{N} \min\{N-1,h+t\} \sum_{j=0}^{\min\{N-1,h+t\}} E_{t-j}[(1-\alpha)p_t + \alpha m_t] + I_{N-1}(h+t) \frac{1}{N} \sum_{j=t+h+1}^{N-1} E_{t-j}[(1-\alpha)p_t + \alpha m_t],
\]

where we used the definition of nominal GDP \(m_t = y_t - p_t\) and \(I_{N-1}(\cdot)\) is an indicator function defined as

\[
I_{N-1}(x) = \begin{cases} 
1 & \text{if } x < N - 1 \\
0 & \text{otherwise.}
\end{cases}
\]

The decomposition allows us to consider price setting of firms who are informed and uninformed. The first term is the sum of individual prices for informed firms. The second term is the sum of prices for uninformed firms.
We work on the second term first. If \( h + t < N - 1 \), there are some uninformed firms who expect that \( m_t = 0.025(1 + t) \). These uninformed firms also expect that all other firms’ prices are \( 0.025(1 + t) \) because the uninformed have not noticed the change in \( m_t \). Therefore, the second term in the decomposed price level equation turns out to be

\[
I_{N-1}(h + t) \frac{1}{N} \sum_{j=h+t+1}^{N-1} [(1 - \alpha)E_{t-j}p_t + \alpha E_{t-j}m_t] \\
= I_{N-1}(h + t) \frac{1}{N} \sum_{j=h+t+1}^{N-1} 0.025(1 + t) \\
= I_{N-1}(h + t) \frac{N - h - 1}{N} 0.025(1 + t).
\]

The first term in the price level equation is for informed firms. The informed firms expect \( m_t = 0 \) but \( E_{t-j}p_t = p_t \), because they have no uncertainty. Therefore, the first term in the decomposed price level equation is

\[
\frac{1}{N} \min\{N-1,h+t\} \sum_{j=0}^{\min\{N-1,h+t\}} (1 - \alpha)p_t \\
= (1 - \alpha) \frac{\min\{N-1,h+t\} + 1}{N} p_t.
\]

Therefore, the price level for any \( t \geq 0 \) is given by

\[
|tl = (1 - \alpha) \frac{\min\{N,h + t + 1\}}{N} p_t + I_{N-1}(h + t) \frac{N - h - t - 1}{N} (0.025)(1 + t) \\
= I_{N-1}(h + t) \frac{(N - h - t - 1)(0.025)(1 + t)}{N - (1 - \alpha) \min\{N,h + t + 1\}},
\]

or

\[
p_t = \begin{cases} 
\frac{(N - h - t - 1)(0.025)(1 + t)}{N - (1 - \alpha)(h + t + 1)} & \text{if } h + t < N - 1 \\
0 & \text{otherwise.}
\end{cases}
\]

Hence, a large \( h \) such that \( h \geq N - 1 \) is a sufficient condition that the indicator function \( I_{N-1}(h + t) = 0 \) for any \( t \geq 0 \), implying \( p_t = m_t = 0 \) and \( y_t = 0 \) for \( t \geq 0 \).
is obvious from $m_t = y_t + p_t$). It means that there is no cost of disinflation. On the other hand, $p_t$ is non-zero for a small $h$. In fact, $p_t$ is positive for $0 \leq t < N - 1 - h$ under strategic complementarity ($0 < \alpha < 1$).
APPENDIX B

THE LOG LINEARIZED SOLUTION

We log-linearize (3.2), (3.3) and (3.4) around the steady state. The steady state values of variables are respectively

\[ K = \left( \frac{\alpha \beta A}{(1 - (1 - \delta)\beta)} \right)^{1/(1 - \alpha)}, \]

\[ q = P^I \]

and \[ I = \delta K. \] Given these steady state values, one can find the log-linearized first order conditions and the law of motion of capital. By eliminating the log-deviation of the marginal \( q \) and the investment from the steady state, we obtain the second order difference equation for \( \dot{K}_t \).

\[
c(\dot{K}_t - \dot{K}_{t-1}) = \alpha \beta K^{\alpha-1}[E_t\dot{A}_{t+1} + (\alpha - 1)\dot{K}_t] \\
+ c\delta\beta[E_t\dot{K}_{t+1} - \dot{K}_t] + (1 - \delta)\beta c[E_t\dot{K}_{t+1} - \dot{K}_t].
\]

Rearranging the equation yields

\[
\beta E_t\dot{K}_{t+1} + \psi \dot{K}_t + \dot{K}_{t-1} = -hE_t\dot{A}_{t+1},
\]

where \( h = \alpha \beta AK^{\alpha-1}c^{-1} > 0 \) and \( \psi = -(1 + \beta + (1 - \alpha)h) \). We can solve this second order difference equation in a usual manner.\(^{32}\) The parameters \( \zeta_1 \) and \( \zeta_2 \) can be obtained by solving the characteristic equation \( 1 + (\psi / \beta)z + (1 / \beta)z^2 = (1 - \zeta_1 z)(1 - \zeta_2 z) \)

\(^{32}\)For example, see Sargent (1987, Chapter 9 and 14) for the solution technique.
= 0. Without loss of generality, we obtain \( 0 < \zeta_1 < 1 < \zeta_2 \). In particular,

\[
\zeta = \frac{1}{2} \left[ -\frac{\psi}{\beta} \pm \sqrt{\left(\frac{\psi}{\beta}\right)^2 - \frac{4}{\beta}} \right].
\]

The smaller root is \( \zeta_1 \) while the larger root is \( \zeta_2 \).
C.1 Proof of proposition 3.2.1

To prove the proposition, I use the method of successive approximation. In every step, I consider the firms’ plans in period $t$. In the first step, I assume that every firm has $K_{t-1}$ as its initial stock of capital and $\Omega_t$ in period $t$ in common and show the aggregate capital stock in period $t$. In the next step, I assume that every firm has $K_{t-2}$ as its initial capital stock and $\Omega_{t-1}$ in common and consider information updating for period $t$. I show the aggregate capital stock in period $t$ under the information assumption. In the $d$-th step, the initial common capital stock is assumed to be $K_{t-d-1}$ and $\Omega_{t-d}$ and then I consider the aggregate capital stock in period $t$, according to the pattern of information updating.

It is useful to introduce the sub-aggregated capital stock $K_t^{(d)}$ for $d \geq 0$. It is defined as the sub-aggregate capital stock over firms who have $\Omega_{t-d}$, regardless of past histories for the individual firms. In a dynamic model, the history dependence of the optimal plans makes the distribution of the optimal plans different from the distribution of information sets. What we will do here is to aggregate individual capital stocks for the firms having the same information set in period $t$, and then
aggregate these sub-aggregated capital stocks based on the distribution of information sets.

**Step 1**: Under the assumption of the same initial capital stock among firms, $K_{t-1}(i) = K_{t-1}$. Therefore, its individual capital stock is

$$
\hat{K}_t^{(0)}(i) = \zeta_1 \hat{K}_{t-1} + \phi(B^{-1}) E_t \hat{A}_{t+1},
$$

(C.1)

for all $i$. Since all firms in the economy have $\Omega_t$, $\hat{K}_t = \hat{K}_t^{(0)} = \hat{K}_t^{(0)}(i)$ for all $i$. The aggregate capital stock is

$$
\hat{K}_t = \zeta_1 \hat{K}_{t-1} + \phi(B^{-1}) E_t \hat{A}_{t+1}.
$$

(C.2)

**Step 2**: Suppose that every firm has a common initial stock in period $t-2$. Also, suppose that all firms are informed in period $t-1$ and thus they have the information set $\Omega_{t-1}$ in the beginning of period $t-1$. In this case, the capital stock for all firms is $K_{t-1}(i)$. However, a firm chooses either $K_t^{(0,0)}(i)$ or $K_t^{(1,0)}(i)$, depending on whether it is informed or not in period $t$.

An informed firm recomputes optimal plans and the capital stock actually chosen in period $t$ is

$$
\hat{K}_t^{(0,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(0)}(i) + \phi(B^{-1}) E_t \hat{A}_{t+1},
$$

(C.3)

An uninformed firm continues to use the old information set. Therefore,

$$
\hat{K}_t^{(1,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(0)}(i) + \phi(B^{-1}) E_{t-1} \hat{A}_{t+1}.
$$

(C.4)

Next, we aggregate individual capital stocks according to the length of information delay. From our definition of $\hat{K}_t^{(0)}$ and $\hat{K}_t^{(1)}$, it is obvious that $\hat{K}_t^{(0)} = \hat{K}_t^{(0,0)}(i)$ and
\( \hat{K}_t^{(1)} = \hat{K}_t^{(1,0)}(i) \). Hence, (C.3) and (C.4) can be expressed as follows:

\[
\hat{K}_t^{(0)} = \zeta \hat{K}_{t-1} + \phi(B^{-1})E_t \hat{A}_{t+1} \quad \text{(C.5)}
\]
\[
\hat{K}_t^{(1)} = \zeta \hat{K}_{t-1}^{(0)} + \phi(B^{-1})E_{t-1} \hat{A}_{t+1}, \quad \text{(C.6)}
\]

where I used the fact that \( \hat{K}_{t-1} = \hat{K}_{t-1}^{(0)} = \hat{K}_{t-1}^{(0)}(i) \).

Finally, we obtain the aggregate capital stock in period \( t \) according to the distribution of information sets. Since there are only two types in the economy, the aggregation is simple. There are informed firms with a fraction \( \lambda \) and uninformed firms with a fraction \( 1 - \lambda \). Therefore,

\[
\hat{K}_t = \lambda \hat{K}_t^{(0)} + (1 - \lambda) \hat{K}_t^{(1)}
\]

\[
= \zeta \hat{K}_{t-1} + \phi(B^{-1}) \left\{ \lambda E_t \hat{A}_{t+1} + (1 - \lambda) E_{t-1} \hat{A}_{t+1} \right\}. \quad \text{(C.7)}
\]

**Step 3**: Suppose that every firm has a common initial capital \( K_{t-3} \) and all firms are informed in period \( t - 2 \). As the step 2 shows, there are two types of firms in the economy in period \( t - 1 \), but there are four types of firms in the economy in period \( t \). (See Figure 3.1.) All two types of firms in period \( t - 1 \) have an equal opportunity to update their information with a probability \( \lambda \). The capital stock for which firms can update their information are respectively \( K_t^{(0,0,0)}(i) \) and \( K_t^{(0,1,0)}(i) \). However, there are also two other uninformed firms in period \( t \). The capital stock for uninformed firms are \( K_t^{(1,0,0)}(i) \) and \( K_t^{(2,1,0)}(i) \). For each type, we have

\[
\hat{K}_t^{(0,0,0)}(i) = \zeta \hat{K}_{t-1}^{(0,0)}(i) + \phi(B^{-1})E_t \hat{A}_{t+1} \quad \text{(C.8)}
\]
\[
\hat{K}_t^{(0,1,0)}(i) = \zeta \hat{K}_{t-1}^{(0,1)}(i) + \phi(B^{-1})E_t \hat{A}_{t+1} \quad \text{(C.9)}
\]
\[
\hat{K}_t^{(1,0,0)}(i) = \zeta \hat{K}_{t-1}^{(1,0)}(i) + \phi(B^{-1})E_{t-1} \hat{A}_{t+1} \quad \text{(C.10)}
\]
\[
\hat{K}_t^{(2,1,0)}(i) = \zeta \hat{K}_{t-1}^{(1,0)}(i) + \phi(B^{-1})E_{t-2} \hat{A}_{t+1}. \quad \text{(C.11)}
\]
Next, we aggregate individual capital stock in period $t$ according to the length of information delay. Note that the capital stocks in the first two types are conditional on the same up-to-date information as we can see from the feedforward term in (C.8) and (C.9). In order to obtain $\hat{K}_t(0)$, we take a weighted average of (C.8) and (C.9) with the weights used in step 2. In other words, $\hat{K}_t(0) = \lambda \hat{K}_t^{(0,0,0)}(i) + (1 - \lambda) \hat{K}_t^{(0,1,0)}(i)$.

It is also necessary to take care of uninformed firms in (C.10) and (C.11). Since their information sets are different and so are the histories of information updating, we distinguish one from the other. The aggregate capital stock for uninformed firms on $\Omega_{t-1}$ is given by $\hat{K}_t^{(1)} = \hat{K}_t^{(1,0,0)}(i)$. The aggregate capital stock for uninformed firms but on $\Omega_{t-2}$ is given by $\hat{K}_t^{(2)} = \hat{K}_t^{(2,1,0)}(i)$.

As a result, the equations (C.8) - (C.11) are expressed as follows.

$\hat{K}_t^{(0)} = \zeta_1 \hat{K}_{t-1} + \phi(B^{-1})E_t \hat{A}_{t+1}$ \hspace{1cm} (C.12)

$\hat{K}_t^{(1)} = \zeta_1 \hat{K}_{t-1}^{(0)} + \phi(B^{-1})E_{t-1} \hat{A}_{t+1}$ \hspace{1cm} (C.13)

$\hat{K}_t^{(2)} = \zeta_1 \hat{K}_{t-1}^{(1)} + \phi(B^{-1})E_{t-2} \hat{A}_{t+1}$. \hspace{1cm} (C.14)

Here, we aggregate individual capital stocks based on the distribution of information sets. Note that the first term in the right hand side in (C.12) is the aggregate capital stock in period $t - 1$. This is because the equation (C.8) and (C.9) differ only in their lagged term and the lagged terms amount to the aggregate capital in period $t - 1$ once the weights in step 2 are used.

Finally, we obtain the aggregate capital stock in this period. The capital stock in the economy is aggregated over these three equations with the weight for each information sets in this period. Therefore, $\hat{K}_t = \lambda \hat{K}_t^{(0)} + \lambda(1 - \lambda) \hat{K}_t^{(1)} + (1 - \lambda)^2 \hat{K}_t^{(2)}$.

95
so that

\[ \hat{K}_t = \zeta_1 \hat{K}_{t-1} \quad \text{(C.15)} \]

\[ + \phi(B^{-1}) \left\{ \lambda E_t \hat{A}_{t+1} + \lambda(1 - \lambda) E_{t-1} \hat{A}_{t+1} + (1 - \lambda)^2 E_{t-2} \hat{A}_{t+1} \right\}. \]

We will use the weights for subsequent steps.

**Step 4**: Consider the case in which the initial common capital stock is \( K_{t-4} \) and all firms are informed in period \( t - 3 \). All four types of firms in \( t - 1 \) have an equal opportunity to update their information so that there are \( 2^3 \) (=8) types of firms in the economy in period \( t \). (depending on whether they informed or uninformed in this period) Among eight types, \( 2^2 \) (=4) types of firms receive the newest information, \( 2^1 \) (=2) types of firms continue to have the second newest information, and one type has the third newest information and one type has the oldest information. The equations for individual firms’ capital stock are

\[
\hat{K}_t^{(0,0,0,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(0,0,0,0)}(i) + \phi(B^{-1}) E_t \hat{A}_{t+1} \quad \text{(C.16)}
\]

\[
\hat{K}_t^{(0,0,1,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(0,1,0,0)}(i) + \phi(B^{-1}) E_t \hat{A}_{t+1} \quad \text{(C.17)}
\]

\[
\hat{K}_t^{(0,1,0,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(1,0,0,0)}(i) + \phi(B^{-1}) E_t \hat{A}_{t+1} \quad \text{(C.18)}
\]

\[
\hat{K}_t^{(0,2,1,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(2,1,0,0)}(i) + \phi(B^{-1}) E_t \hat{A}_{t+1} \quad \text{(C.19)}
\]

\[
\hat{K}_t^{(1,0,0,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(0,0,0,0)}(i) + \phi(B^{-1}) E_{t-1} \hat{A}_{t+1} \quad \text{(C.20)}
\]

\[
\hat{K}_t^{(1,0,1,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(0,1,0,0)}(i) + \phi(B^{-1}) E_{t-1} \hat{A}_{t+1} \quad \text{(C.21)}
\]

\[
\hat{K}_t^{(1,2,1,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(2,1,0,0)}(i) + \phi(B^{-1}) E_{t-2} \hat{A}_{t+1} \quad \text{(C.22)}
\]

\[
\hat{K}_t^{(2,1,0,0)}(i) = \zeta_1 \hat{K}_{t-1}^{(1,0,0,0)}(i) + \phi(B^{-1}) E_{t-3} \hat{A}_{t+1} \quad \text{(C.23)}
\]

Next, we aggregate individual capital stock according to the length of information delay. Note that, as in step 3, the second term of the right hand side in (C.16) - (C.19)
are the same, because these firms have the newest information. For this reason, all
differ in these equations are the first term. We can use step 3 to aggregate (C.16) -
(C.19) in order to derive \( \hat{K}^{(0)}_{t-1} \). Note also that the second terms in (C.20) and (C.21)
are identical and all differ are that a firm described by (C.20) updated its information
in period \( t-2 \) while a firm described by (C.21) did not. Therefore, we take a weighted
average of these in the same way as step 2 in order to derive \( \hat{K}^{(1)}_t \). For the other types
of firms in (C.22) and (C.23), we obtain \( \hat{K}^{(2)}_t = \hat{K}^{(2,1,0,0)}_t (i) \) and \( \hat{K}^{(3)}_t = \hat{K}^{(3,2,1,0)}_t (i) \),
because both types are unique in their histories of information updating.

As a result, the equations (C.16) - (C.23) are expressed as follows.

\[
\begin{align*}
\hat{K}^{(0)}_t &= \zeta_1 \hat{K}_{t-1} + \phi (B^{-1}) E_t \hat{A}_{t+1} \quad \text{(C.24)} \\
\hat{K}^{(1)}_t &= \zeta_1 \hat{K}^{(0)}_{t-1} + \phi (B^{-1}) E_{t-1} \hat{A}_{t+1} \quad \text{(C.25)} \\
\hat{K}^{(2)}_t &= \zeta_1 \hat{K}^{(1)}_{t-1} + \phi (B^{-1}) E_{t-2} \hat{A}_{t+1} \quad \text{(C.26)} \\
\hat{K}^{(3)}_t &= \zeta_1 \hat{K}^{(2)}_{t-1} + \phi (B^{-1}) E_{t-3} \hat{A}_{t+1}. \quad \text{(C.27)}
\end{align*}
\]

Finally, we obtain the aggregate capital stock in this period. The capital stock in
the economy is aggregated over these four equations (C.24) - (C.27) with new weights
for each information sets in this period. Therefore, \( \hat{K}_t = \lambda \hat{K}^{(0)}_t + \lambda (1-\lambda) \hat{K}^{(1)}_t + \lambda (1-\lambda)^2 \hat{K}^{(2)}_t + (1-\lambda)^3 \hat{K}^{(3)}_t \) so that

\[
\begin{align*}
\hat{K}_t &= \zeta_1 \hat{K}_{t-1} \\
&+ \phi (B^{-1}) \left[ \lambda E_t \hat{A}_{t+1} + \lambda (1-\lambda) E_{t-1} \hat{A}_{t+1} + \lambda (1-\lambda)^2 E_{t-2} \hat{A}_{t+1} \\
&+ (1-\lambda)^3 E_{t-3} \hat{A}_{t+1} \right]. \quad \text{(C.28)}
\end{align*}
\]

Again, we will use these weights for subsequent steps.
Step 5: In the subsequent periods, the procedure is the same. The initial common capital stock is assumed to be $\hat{K}_{t-5}$ and all firms are informed in period $t-4$ in this step. In this case, $2^3$ types of firms specified in period $t-1$ have an equal opportunity to update their information. The number of types in the economy becomes $2^4 = 16$. Among these 16 types, $2^3 (=8)$ types have the newest information set based on $\Omega_t$, $2^2 (=4)$ types have the second newest information set, $2^1 (=2)$ types have the third newest, one type has the fourth newest information set and one type has the oldest information set. To obtain $\hat{K}_t^{(0)}$, we aggregate eight types of firms in the same way as in step 4. For $\hat{K}_t^{(1)}$, we use the same way of aggregation as step 3 to aggregate four types of individual capital stock. To find $\hat{K}_t^{(2)}$, one can use the same way of aggregation as step 2. Finally, since the remaining other types of firms are only one type for each delay, $\hat{K}_t^{(3)} = \hat{K}_t^{(3,2,1,0)}(i)$ and $\hat{K}_t^{(4)} = \hat{K}_t^{(4,3,2,1,0)}(i)$. From the analogy of the equation from (C.24) to (C.27), one can obtain

\[
\hat{K}_t^{(0)} = \zeta_1 \hat{K}_{t-1} + \phi (B^{-1}) E_t \hat{A}_{t+1} \tag{C.29}
\]

\[
\hat{K}_t^{(1)} = \zeta_1 \hat{K}_{t-1}^{(0)} + \phi (B^{-1}) E_{t-1} \hat{A}_{t+1} \tag{C.30}
\]

\[
\hat{K}_t^{(2)} = \zeta_1 \hat{K}_{t-1}^{(1)} + \phi (B^{-1}) E_{t-2} \hat{A}_{t+1} \tag{C.31}
\]

\[
\hat{K}_t^{(3)} = \zeta_1 \hat{K}_{t-1}^{(2)} + \phi (B^{-1}) E_{t-3} \hat{A}_{t+1} \tag{C.32}
\]

\[
\hat{K}_t^{(4)} = \zeta_1 \hat{K}_{t-1}^{(3)} + \phi (B^{-1}) E_{t-4} \hat{A}_{t+1}, \tag{C.33}
\]

98
and the aggregate capital stock

\[
\hat{K}_t = \zeta_1 \hat{K}_{t-1} + \phi(B^{-1}) \left[ \lambda E_t \hat{A}_{t+1} + \lambda(1 - \lambda) E_{t-1} \hat{A}_{t+1} + \lambda(1 - \lambda)^2 E_{t-2} \hat{A}_{t+1} + \lambda(1 - \lambda)^3 E_{t-3} \hat{A}_{t+1} + \cdots + (1 - \lambda)^D E_{t-D} \hat{A}_{t+1} \right].
\] (C.34)

**Step D**: In step D, the sub-aggregated capital stock according to the distribution of information set becomes as follows.

\[
\hat{K}_t^{(0)} = \zeta_1 \hat{K}_{t-1} + \phi(B^{-1}) E_t \hat{A}_{t+1}
\]

\[
\hat{K}_t^{(1)} = \zeta_1 \hat{K}_{t-1}^{(0)} + \phi(B^{-1}) E_{t-1} \hat{A}_{t+1}
\]

\[
\vdots
\]

\[
\hat{K}_t^{(D-1)} = \zeta_1 \hat{K}_{t-1}^{(D-2)} + \phi(B^{-1}) E_{t-D+1} \hat{A}_{t+1}
\]

\[
\hat{K}_t^{(D)} = \zeta_1 \hat{K}_{t-1}^{(D-1)} + \phi(B^{-1}) E_{t-D} \hat{A}_{t+1},
\]

for \(D > 0\). By aggregating these capital stocks, we obtain

\[
\hat{K}_t = \zeta_1 \hat{K}_{t-1} + \phi(B^{-1}) \left[ \lambda E_t \hat{A}_{t+1} + \lambda(1 - \lambda) E_{t-1} \hat{A}_{t+1} + \lambda(1 - \lambda)^2 E_{t-2} \hat{A}_{t+1} + \cdots + (1 - \lambda)^D E_{t-D} \hat{A}_{t+1} \right],
\]

for all \(D \geq 1\).

By expanding the lag polynomial \(\phi(B^{-1})\), we obtain the equation in (3.17).
C.2 Proof of proposition 3.2.3

Under the fixed duration scheme with $N$ periods, firms are assigned to a cohort in the economy. Unlike the random duration scheme, firms which have the same information set in any period must experience the same history of information updating. This is because no firm can get out of a cohort and get into a different cohort through information updating. Moreover, by the assumptions 3.2.2, all firms within a cohort have the same initial capital stock and thus all firms will choose the same of capital stock.

Next, when $t = N$, firms in the last cohort start to operate. In the next period ($t = N + 1$), firms in the first cohort update their information set and their sub-aggregate capital stock will be $\hat{K}_t^{(0)} = \zeta_1 \hat{K}_{t-1}^{(N-1)} + \phi(B^{-1})E_t\hat{A}_{t+1}$. In general, when $t > N$, the sub-aggregated capital stocks in each cohort are

\[
\hat{K}_t^{(0)} = \zeta_1 \hat{K}_{t-1}^{(N-1)} + \phi(B^{-1})E_t\hat{A}_{t+1} \\
\hat{K}_t^{(1)} = \zeta_1 \hat{K}_{t-1}^{(0)} + \phi(B^{-1})E_{t-1}\hat{A}_{t+1} \\
\hat{K}_t^{(2)} = \zeta_1 \hat{K}_{t-1}^{(1)} + \phi(B^{-1})E_{t-2}\hat{A}_{t+1} \\
\vdots \\
\hat{K}_t^{(N-1)} = \zeta_1 \hat{K}_{t-1}^{(N-2)} + \phi(B^{-1})E_{t-N+1}\hat{A}_{t+1}.
\]

The equations hold for any $t > N + 1$, because one of the $N$ cohorts updates its information set every period.

The weight for each cohort is $1/N$. That is, the aggregate capital stock can be written as $\bar{K}_t = N^{-1} \sum_{d=0}^{N-1} \hat{K}_t^{(d)}$. By aggregating these sub-aggregate capital stock
with this equation, we obtain

\[ \hat{K}_t = \zeta_1 \hat{K}_{t-1} + \phi(B^{-1}) \frac{1}{N} \sum_{d=0}^{N-1} E_{t-d} \hat{A}_{t+1}. \]  

\hspace{1cm} (C.35)

By expanding the lag polynomial \( \phi(B^{-1}) \), the equation (3.19) is obtained.


105