ESSAYS IN MONETARY AND INTERNATIONAL ECONOMICS

DISSERTATION

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This dissertation is comprised of three essays in monetary and international macroeconomics. The first essay, titled “A Dynamic Model of Exogenous Exchange Rate Pass-Through”, examines a two-country open economy model with sticky prices where exporters’ choice of invoicing currency is endogenous. Besides generating incomplete pass-through, the model yields four main results. First, firms’ invoicing strategy is generally time-varying. Second, instant pass-through into import prices is greater than into export prices when depreciation is caused by a domestic monetary expansion. Thirdly, average pass-through is asymmetric in times of persistent depreciation and appreciation. It is higher under depreciation when the destination market is more competitive. Finally, cross-country differences in money supply variability produce an origin-based asymmetry: different average pass-through rates into import and export prices.

The second essay, titled “Limited Commitment, Inaction and Optimal Monetary Policy”, examines the optimal frequency of monetary policy meetings when their schedule is pre-announced. The contribution of this paper is twofold. First, we show that in the standard New Keynesian framework infrequent but periodic revision of monetary policy may be desirable even when there are no explicit costs of policy adjustment. Adjustment of policy on a pre-announced schedule de facto acts as a commitment not to adjust in intermediate periods. We find that at short horizons
gains from such commitment outweigh welfare costs of central bank’s inaction. Second, we solve for the optimal frequency of policy adjustment and characterize its determinants. When applied to the U.S. economy, our analysis suggests that the Federal Open Market Committee should revise the federal funds target rate no more than twice a year.

Finally, the third essay, titled “Does the Federal Reserve Do What It Says It Expects to Do?”, studies the behavior of the Federal Open Market Committee in setting the federal funds target rate and making a bias announcement. The bias announcement states the likely direction of policy at the next meeting. The current bias concerning the next interest rate decision should be the optimal forecast based on the committee’s interest rate policy rule. Therefore, the interest rate implied by the estimated policy should be consistent not only with the observed rate, but also with the observed bias announcement. We jointly estimate interest rate and bias announcement decision rules and find strong consistency between the two decisions in their response to inflation. However, the response to measures of economic activity is found inconsistent.
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CHAPTER 1

A Dynamic Model of Endogenous Exchange Rate Pass-Through

1.1 Introduction

The new open economy macroeconomics (NOEM) literature has recently attracted great attention of researchers as a convenient optimization-based framework for analyzing old issues in international macroeconomics. One of the central questions in the NOEM is the elasticity of import prices with respect to exchange rate shocks, often called the “exchange rate pass-through”. The issue of pass-through is important for these models because welfare implications of monetary policy shocks and the optimal monetary policy itself have been shown to depend critically on pass-through elasticity.\(^1\)

An emerging consensus in empirical research is that short-run pass-through of exchange rate shocks into import and export prices is neither 0 nor 1. Campa and Goldberg (2002) find average pass-through for 25 OECD countries to be roughly 57%. Campa and González (2002) and Faruqee (2004) estimate pass-through elasticities for the Euro area. Most of their estimates lie strictly between 0 and 100%. Olivei (2002) and Coughlin and Pollard (2000) find incomplete, but positive pass-through into U.S.

\(^1\)See, respectively, Obstfeld and Rogoff (1995) vs. Betts and Devereux (2000), and Engel (2002).

Moreover, some evidence points to a number of asymmetries in pass-through. First, Dwyer et al. (1993) report asymmetric pass-through elasticities for Australian import and export prices between 1974 and 1992. Second, in a study of disaggregated US import prices between 1982 and 1990, Coughlin and Pollard (2000) find asymmetric pass-through rates into U.S. import prices with the direction of asymmetry varying across industries. Wickermasinghe and Silvapulle (2003) also find statistically significant higher response of Japanese manufactured import prices in times of appreciations than in times of depreciations. Theoretical literature is mostly devoted to the size of pass-through and seems to have overlooked the issue of its asymmetry.

Limited aggregate pass-through can occur for many reasons. Two recent papers, Bachetta and van Wincoop (2002) and Devereux, Engel and Storgaard (2002), examine optimal choice of invoicing currency in the context of NOEM.\(^3\) Both papers provide a useful analysis of firms’ decisions. Their models assume one period in advance pricing, so that all prices are assumed predetermined in every period. Thus, when firms are identical the only equilibrium is a corner solution: pass-through is either 0 or 1. Limited pass-through can only be generated by intratemporal heterogeneity of invoicing decisions. However, the latter often implies multiple equilibria, which complicates a dynamic analysis.

In this paper we study another scenario. We assume endogenous invoicing and


\(^3\) Menon (1995) provides a helpful review of earlier theoretical developments.
symmetric firms, but with staggered price adjustment. The model generates a pass-through that is always incomplete due to firms’ intertemporal heterogeneity and a unique equilibrium in this setting is easier to find. Another advantage is that the model is more dynamic and sheds some light on potential causes of asymmetries in pass-through.

We point out several key model ingredients. First is Taylor-type two-period price stickiness. The model assumes two cohorts of firms in every sector, each updating prices in turn and at different dates. Each date’s new prices are valid during both the current and the next period. When re-optimizing, exporting firms also choose the currency in which to specify their prices.

The size of every period’s aggregate pass-through thus depends on both past invoicing decisions of the firms whose prices don’t change as well as the size of price adjustment by re-optimizing firms. This also ensures a time-varying pass-through rate. Although in every period the pass-through into the unadjusted prices is either zero or one, pass-through into aggregate import price index is always positive and incomplete. We emphasize aggregate pass-through as opposed to pass-through into individual prices, because the available evidence is mostly based on industry-level data and tells us little about behavior of individual firms.

Another feature of Taylor pricing is that firms maximize the sum of current and future expected profits. This raises the importance of conditional means of variables. Our main finding is that optimal invoicing strategy is generally different depending on whether the currency is under- or over-valued relative to the long-run equilibrium.

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4One could think of this as being the decision of whether to index the price to the exchange rate or not. Another example would be an international catalogue seller. Before printing the catalogue, the seller could choose which currency to specify the prices in.
This makes possible asymmetric pass-through rates in times of persistent depreciation or appreciation. Finally, we assume incomplete markets, so that unexpected shocks create deviations from the interest parity and risk sharing conditions. As a result, one country’s monetary expansion will have asymmetric effects on consumptions and real wages across countries. Therefore, pass-through into import prices depends critically on the origin of the shock. This gives a simple explanation of the second type of asymmetry. In our benchmark model, domestic monetary expansions affect import prices by more than they affect the foreign currency value of export prices. As a consequence, in the long-run a country with higher money supply variability experiences higher pass-through into its import prices.⁵

To summarize, the setting we consider predicts: 1) incomplete and time-varying short-run exchange rate pass-through; 2) asymmetric average pass-through in times of persistent depreciation or appreciation; and 3) a possibility of asymmetric average pass-through into import and export prices. ⁶

The rest of this essay is organized as follows. Section 1.2 describes the model and equilibrium conditions. In section 1.3 we briefly describe how the model can be solved using the collocation method. The remaining sections discuss our findings and the intuition behind them.

⁵This result is similar to the finding in Devereux et. al. (2002), although their motivation is somewhat different.

⁶A caveat should be made before interpreting our results. In this paper we do not attempt to match the data quantitatively. A detailed ”moment-matching” exercise would require a more elaborate model (e.g. with lengthier price stickiness, distribution costs, variable markups, etc.). Instead, we concentrate on the most basic NOEM model to show that endogeneity of invoicing decisions along with staggered time-dependent price adjustment produces results that are qualitatively similar to those found in the data.
1.2 The Model

Most features of our model are standard in the NOEM literature\(^{7}\). The world is assumed to have two countries, Home and Foreign. Total population is spread over a unit interval with a fraction \(n\) living at Home. Firms in both economies are of two types - firms producing and selling domestically (nontraded goods firms) and exporters producing in the domestic economy but selling abroad. The mass of each type of firms is normalized to equal that of consumers, and each firm produces a differentiated product, acting as a constrained monopolist.

1.2.1 Consumers, Asset Markets and the Government

Each country in the model has its own currency. For simplicity we will call the Home currency \textit{dollars}, and Foreign currency \textit{euros}. Hereafter, variables expressed in euros are marked with an asterisk. A superscript \((F\) or \(H)\) indicates the origin of the good/firm and a lower script \((f\) or \(h)\) indicates the location of the consumer/government in question. As usual, lower script \(t\) indicates time. In addition, variables pertaining to exporters will have a superscript \(e\). For example, \(C^e_{Ffht}\) indicates consumption of foreign export goods by domestic agents at time \(t\); on the other hand, \(P^{eHs}_{ft}(i)\) indicates the euro price of the domestic export good \(i\) sold to foreign agents.

Consumer goods are collected into baskets \textit{à la} Dixit and Stiglitz (1977). Home country agents purchase the home goods basket \(C^H_{ht}\) from domestic firms and the

\(^{7}\)Lane(2001) and Sarno(2001) provide useful surveys.
foreign goods basket $C^{eF}_{ht}$ from foreign exporters. These aggregates are given by:

$$C^H_{ht} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\lambda}} \int_0^n C^H_{ht}(j)^{\frac{1}{\lambda}} \, dj \right]^{\frac{1}{\lambda}};$$

$$C^{eF}_{ht} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\lambda}} \int_n^1 C^{eF}_{ht}(j)^{\frac{1}{\lambda}} \, dj \right]^{\frac{1}{\lambda}}. \tag{1.1}$$

Home consumers’ aggregate consumption basket $C_{ht}$ combines the two country goods baskets in a CES fashion:

$$C_{ht} = \left\{ \frac{1}{n} C^H_{ht} \left( \frac{1}{n} \right)^{\frac{1}{\lambda}} + (1-n) \left( \frac{1}{1-n} \right)^{\frac{1}{\lambda}} C^{eF}_{ht} \right\}^{\frac{1}{1-\lambda}}. \tag{1.2}$$

Price indices associated with each consumption aggregate in (1.1) are defined as the minimum nominal cost of purchasing one unit of the relevant basket. They are given by:

$$P^H_{ht} = \left[ \frac{1}{n} \int_0^n P^H_{ht}(j)^{1-\lambda} \, dj \right]^{\frac{1}{1-\lambda}};$$

$$P^{eF}_{ht} = \left[ \frac{1}{1-n} \int_n^1 P^{eF}_{ht}(j)^{1-\lambda} \, dj \right]^{\frac{1}{1-\lambda}}. \tag{1.3}$$

The domestic consumer price index (CPI) is the minimum cost of one unit of the aggregate consumption basket (1.2). It is given by:

$$P_{ht} = \left\{ n P^H_{ht}^{(1-\omega)} + (1-n) P^{eF}_{ht}^{(1-\omega)} \right\}^{\frac{1}{1-\omega}}. \tag{1.4}$$

The baskets/indexes in (1.1)-(1.4) imply the following demands for individual goods and country baskets:

$$C^H_{ht}(j) = \frac{1}{n} \left[ \frac{P^H_{ht}(j)}{P^H_{ht}} \right]^{-\lambda} C^H_{ht}, \quad \text{and} \quad C^H_{ht} = n \left[ \frac{P^H_{ht}}{P_{ht}} \right]^{-\omega} C_{ht} \tag{1.5}$$

$$C^{eF}_{ht}(j) = \frac{1}{1-n} \left[ \frac{P^{eF}_{ht}(j)}{P^{eF}_{ht}} \right]^{-\lambda} C^{eF}_{ht}, \quad \text{and} \quad C^{eF}_{ht} = (1-n) \left[ \frac{P^{eF}_{ht}}{P_{ht}} \right]^{-\omega} C_{ht} \tag{1.6}$$
Consumption baskets and price indices relevant to the foreign consumers are defined analogously. All consumers in the model are assumed to be identical. They supply labor, eat, and hold equal ownership in all domestic firms (including exporters). In addition, agents hold money balances, from which they derive utility, and bonds.

Asset markets are incomplete. There are two types of nominal non-state-contingent bonds in this economy: a Home bond ($B_t^H$) denominated in the Home currency and sold only domestically and a Foreign bond ($B_t^*$) denominated in the foreign currency and traded internationally. Bonds acquired at date $t$ pay nominal interest rates of $i_t$ and $i_t^*$, respectively, at date $t + 1$. The assumed structure of asset markets allows ex-post deviations from the uncovered interest parity and risk sharing conditions.

Firms’ profits are distributed as dividends, which consumers take as given. Home consumers enter each period with the previous period’s bonds ($B_{t-1}^H$) and money balances ($M_{t-1}$), and choose consumption $C_{ht}$, new bond and money holding $B_t^H, B_t^*$, $M_{ht}$, and labor supply $L_{ht}$ in order to maximize expected life-time utility:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{ht}^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\varepsilon} \left( \frac{M_{ht}}{P_{ht}} \right)^{1-\varepsilon} - \kappa \frac{L_{ht}}{\psi} \right)$$

Subject to a budget constraint:

$$C_{ht}P_{ht} \leq -B_t^H - e_tB_t^* + (1 + i_{t-1})B_{t-1}^H + e_t(1 + i_t^*)B_{t-1}^* +$$

$$+ W_{ht}L_{ht} - (M_{ht} - M_{ht-1}) + T_{ht} + \frac{1}{n} \int_0^n \pi^H_{ht}(j) dj + \frac{1}{n} \int_0^n \pi^e_{ht}(j) dj; \tag{1.7}$$

where $e_t$ is the nominal exchange rate, $\pi^H_{ht}$ and $\pi^e_{ht}$ are respectively profits of non-traded and export goods producers and $T_{ht}$ is the lump-sum transfer from the government. Since all domestic consumers are identical, in equilibrium $B_t^H = 0, \forall t$.

The government injects money in the form of transfers $T_{ht}$. Its budget constraint is given by:
\[ T_{ht} = M_{ht} - M_{ht-1} \]  

Home consumers’ first order conditions (F.O.C.) are:

\[ \frac{C_{ht} - \sigma}{(1 + i_t)P_{ht}} = E_t \left( \frac{\beta C_{ht+1} - \sigma}{P_{ht+1}} \right) \]  

\[ \frac{C_{ht} - \sigma e_t}{(1 + i_t^*)P_{ht}} = E_t \left( \frac{\beta C_{ht+1}^* e_{t+1}}{P_{ht+1}} \right) \]  

\[ \frac{\chi}{C_{ht} - \sigma} \left( \frac{M_{ht}}{P_{ht}} \right)^{-\varepsilon} = \frac{i_t}{1 + i_t} \]  

\[ \frac{\kappa L_{ht}^{-\psi-1}}{C_{ht} - \sigma} = \frac{W_{ht}}{P_{ht}} \]  

Foreign country consumers face a similar problem, except they only have access to international foreign-currency denominated bonds. Their F.O.C.’s are:

\[ \frac{C_{ft} - \sigma}{(1 + i_t^*)P_{ft}^*} = E_t \left( \frac{\beta C_{ft+1} - \sigma}{P_{ft+1}^*} \right) \]  

\[ \frac{\chi}{C_{ft} - \sigma} \left( \frac{M_{ft}}{P_{ft}^*} \right)^{-\varepsilon} = \frac{i_t^*}{1 + i_t^*} \]  

\[ \frac{\kappa L_{ft}^{-\psi-1}}{C_{ft} - \sigma} = \frac{W_{ft}^*}{P_{ft}^*} \]  

**1.2.2 Nontraded Goods Firms**

Producers of nontraded goods use homogenous labor via a linear technology \((y_t = L_t)\), so that nominal marginal cost equals the nominal wage: \(MC_t = W_t\). Prices are adjusted à la Taylor with two cohorts. In each period only a half of the firms change their prices. The new prices are then fixed for two periods. The re-set price for domestic firm \(i\) is obtained by maximizing expected discounted profits:

\[ \max_{P_{ht}^H(i)} \sum_{j=0}^{1} Q_{ht+j} \left( \frac{P_{ht}^H(i) - MC_{ht+j}}{P_{ht+j}} \right) \left( \frac{P_{ht}^H(i)}{P_{ht+j}} \right)^{-\lambda} \left( \frac{P_{ht+j}^H}{P_{ht+j}} \right)^{-\omega} nC_{ht+j} \]
where $Q_{t,t+j}$ is the firms’ discount factor between dates $t$ and $t + j$. The optimal nominal re-set price is equal to a markup over a weighted average of the current and expected future real marginal costs:

$$P_{ht}^{H}(i) = \frac{\lambda}{\lambda - 1} \frac{E_t}{E_t} \left[ \sum_{j=0}^{1} Q_{t,t+j} y_{ht+j}^{(d)H}(i) mc_{ht+j} \right]$$  \hspace{1cm} (1.16)$$

where $y_{ht}^{(d)H}(i)$ is real demand for the domestic firm $i$’s output and $mc_{ht}$ is the real marginal cost defined as $\frac{MC_{ht}}{P_{ht}}$. Since firms are owned by the consumers, profits are discounted at the consumers’ marginal rate of intertemporal substitution. Real demand $y_{ht}^{(d)H}(i)$ is obtained by multiplying per capita demand in (1.5) by the mass of the Home consumers:

$$y_{ht}^{(d)H}(i) = n \left( P_{ht}^{H}(i) \right)^{-\lambda} \left( P_{ht}^{H} \right)^{\lambda - \omega} P_{ht}^{C} C_{ht}$$  \hspace{1cm} (1.17)$$

We focus on a a symmetric equilibrium, where firms within each cohort are identical. Let $P_{ht-1}^{H}(j)$ denote the common price of those firms that do not change prices at time $t$. Then, the Home goods price index (1.3) can be expressed as:

$$\left( P_{ht}^{H} \right)^{1 - \lambda} = \frac{1}{2} \left[ \left( P_{ht-1}^{H}(j) \right)^{1 - \lambda} + \left( P_{ht}^{H}(i) \right)^{1 - \lambda} \right]$$  \hspace{1cm} (1.18)$$

In equilibrium output of each firm must equal the demand (1.17). Aggregation of (1.17) together with (1.18) yields total labor demand by domestic firms.

$$L_{ht}^{(d)H} = \int_{0}^{n} y_{ht}^{(d)H}(i) dj = \left( P_{ht}^{H} \right)^{\lambda - \omega} P_{ht}^{C} C_{ht} n^{2} \left\{ \left( P_{ht-1}(j) \right)^{-\lambda} + \left( P_{ht}^{H}(i) \right)^{-\lambda} \right\}$$  \hspace{1cm} (1.19)$$

From (1.3) and (1.5) total nominal sales of the domestic nontraded goods can be expressed as:

$$\int_{0}^{n} P_{ht}^{H}(i) y_{ht}^{(d)H}(i) di = P_{ht}^{H} \left[ \frac{P_{ht}^{H} \left( P_{ht}^{H} \right)^{-\omega} n^{2} C_{ht} \right]$$
Thus, total nominal profits can be expressed as:

$$\int_{t_0}^{n} \pi^H_{ht}(i) \, di = P^H_{ht} \left[ \frac{P^H_{ht}}{P^H_{ht}} \right]^{-\omega} n^2 C_{ht} - W_{ht} L^{(d)H}_{ht}$$  \hspace{1cm} (1.20)

The problem of the producers of nontraded goods in the Foreign country is analogous.

### 1.2.3 Exporters

Exporters also set prices à la Taylor. In addition to setting the price, they have to decide which currency to specify their price in. Maximizing expected discounted profits under PCP and LCP, we obtain optimal prices under the two scenarios (see Appendix A). Under PCP the optimal reset price (in dollars) of firm \(i\) that re-optimizes at time \(t\) is:

$$P^{eH,pcp}_{ft}(i) = \frac{\lambda}{\lambda-1} \left[ \frac{E_t \sum_{j=0}^{1} Q_{t,t+j} \left[ y^{(d)eH,pcp}_{ft+j}(i) \right]}{m c_{ht+j}} \right] \left[ \frac{P^{eH*}_{ft}}{P^{eH*}_{ft}} \right]^{-\omega}$$

where \(y^{(d)eH,pcp}_{ft}(i)\) denotes foreign consumers’ real demand for the domestically produced export good \(i\) when its producer chooses PCP. It is given by:

$$y^{(d)eH,pcp}_{ft}(i) = \left( \frac{P^{eH,pcp}_{ft}(i)}{e_t P^{eH*}_{ft}} \right)^{-\lambda} \left( \frac{P^{eH*}_{ft}}{P^{eH*}_{ft}} \right)^{-\omega} (1-n)C_{ft}$$

Combining the last two equations and dividing through by \(P^{eH,pcp}_{ht}(i)\), we obtain an alternative expression for the optimal reset price under PCP:

$$P^{eH,pcp}_{ft}(i) = \frac{\lambda}{\lambda-1} \left[ \frac{E_t \sum_{j=0}^{1} Q_{t,t+j} \left[ y^{(d)eH,pcp}_{ft+j}(i) \right]}{m c_{ht+j}} \right] \left[ \frac{P^{eH*}_{ft}}{P^{eH*}_{ft}} \right]^{-\omega} \frac{P^{eH*}_{ft+j} (1-n)C_{ft+j} m c_{ht+j}}{P^{eH*}_{ft+j} P^{eH*}_{ft+j} (1-n)C_{ft+j}}$$  \hspace{1cm} (1.21)

Similarly, under local currency pricing the optimal price (in euros) is:

$$P^{eH*,lcp}_{ft}(i) = \frac{\lambda}{\lambda-1} \left[ \frac{E_t \sum_{j=0}^{1} Q_{t,t+j} \left[ y^{(d)eH,lcp}_{ft+j}(i) \right]}{m c_{ht+j}} \right] \left[ \frac{P^{eH*}_{ft}}{P^{eH*}_{ft}} \right]^{-\omega} \frac{P^{eH*}_{ft+j} (1-n)C_{ft+j} m c_{ht+j}}{P^{eH*}_{ft+j} P^{eH*}_{ft+j} (1-n)C_{ft+j}}$$  \hspace{1cm} (1.22)
Firm $i$’s invoicing decision amounts to comparing expected discounted profits under the two alternatives. These are given by:

$$
\pi^e_{ht,pcp}(i) = E_t \sum_{j=0}^{1} Q_{t,t+j}^{(d)} \left( \frac{P_{eH,pcp}(i) - MC_{ht+j}}{P_{eH,pcp}(i)} \right) y_{ft+j}^{(d)}
$$

$$
\pi^e_{ht,lcp}(i) = E_t \sum_{j=0}^{1} Q_{t,t+j}^{(d)} \left( \frac{P_{eH,lcp}(i) - MC_{ht+j}}{P_{eH,lcp}(i)} \right) y_{ft+j}^{(d)}
$$

(1.23)

At the time of re-optimization firm $i$ evaluates profits in (1.23) and chooses the invoicing strategy:

$$
a_t(i) = \begin{cases} 
1 \text{ (PCP)} & \text{if } \pi^e_{ht,pcp}(i) \geq \pi^e_{ht,lcp}(i) \\
0 \text{ (LCP)} & \text{if otherwise}
\end{cases}
$$

(1.24)

The invoicing decision is equivalent to the decision of whether to index the price to the exchange rate. If PCP is chosen, then the firm fixes the dollar value of the price and the next period’s euro value changes together with the exchange rate. On the other hand, under LCP the euro value of the price is fixed and does not respond to exchange rate depreciation. Let $j$ be the index of the ‘old’ firms, who do not re-optimize their prices at time $t$, and let $i$ be the index of the ‘young’ firms, who revise prices and invoicing decisions. Then, given the symmetry of the firms within each cohort, for arbitrary $i$ and $j$ the Home export price index (in euros) evolves as follows:

$$
(P_{eH^*})^{1-\lambda} = \frac{1}{2} \left[ a_{t-1}(j) \left( \frac{P_{eH^*}(j)}{P_{eH^*}(j)} \right) + (1 - a_{t-1}(j)) P_{eH^*}(j) \right]^{1-\lambda} + \\
+ \frac{1}{2} \left[ a_t(i) \left( \frac{P_{eH^*}(i)}{P_{eH^*}(i)} \right) + (1 - a_t(i)) P_{eH^*,lcp}(i) \right]^{1-\lambda}
$$

(1.25)

where $P_{eH^*}(j)$ is the predetermined component of the old firms’ price: $P_{eH^*}(j) = P_{eH^*,pcp}$, if $a_{t-1} = 1$, and $P_{eH^*}(j) = P_{eH^*,lcp}(j)$, if $a_{t-1} = 0$. 

11
Total labor demand by exporters, obtained in the same way as (1.19), can be expressed as:

\[
L^{(d)eH}_{ht} = (P^{eH*}_{ft} - \omega) C_{ft} \frac{(1-n)n}{2} \left\{ P^{eH}_{ft} - (1 - a_{t-1}(j))P^{eH}_{ft} \right\}^{-\lambda} + \\
+ \left[ a_t(i) \left( \frac{P^{eH,pcp(i)}_{ft}}{e_t} \right) + (1 - a_t(i)) \left( P^{eH*,lcp(i)}_{ft} \right) \right]^{-\lambda}
\]

(1.26)

Finally, aggregate current period nominal profits of the exporters, are given by:

\[
\int_0^n \pi^{eH}_ht(i)di = e_t P^{eH*}_{ft} \left( \frac{P^{eH*}_{ft}}{P^{eH*}_{ft}} \right)^{-\omega} n(1-n)C_{ft} - W_{ht}L^{(d)eH}_{ht},
\]

(1.27)

Foreign exporters also make invoicing decisions \((a^*_t)\) and solve an analogous problem.

1.2.4 Uncertainty, Market Clearing and Equilibrium

Uncertainty in the economy is due to unexpected money supply shocks. We assume that money supplies evolve as stochastic AR(1) processes:

\[
M_{ht} - 1 = \rho_h (M_{ht-1} - 1) + \varepsilon_t \\
M^{*}_{ht} - 1 = \rho_f (M^{*}_{ht-1} - 1) + \varepsilon_t^*
\]

(1.28)

where \(\varepsilon_t, \varepsilon_t^*\) are iid \(\mathcal{N}(0, \sigma^2)\) and \(\mathcal{N}(0, \sigma^2_{e^*})\) respectively.

Stationarity of the money supplies \emph{de facto} assumes stationarity of the nominal exchange rate. While empirical studies suggest near random walk behavior of major exchange rates, we view mean reversion as a useful assumption in the context of our model. The motivation lies in the fact that under the random walk assumption expected depreciation is always zero. However, financial markets and the media often report expectations of nonzero depreciations.\(^8\) Moreover, terms like “overvalued

\(^8\)The Financial Forecast Center (www.forecasts.org), for example, provides forecasts of major exchange rates for up to 18 months. Other forecasts are also available. It is easy to verify that they are not based on a random walk assumption.
currency” are often used by economists. Such use implies the existence of an estimate of some equilibrium exchange rate relative to which the currency is overvalued. Such estimates of equilibrium exchange rates are often derived from market sentiment indicators, news on countries’ current account performances and monetary and public policies. To the extend that firms pay attention to economic analysis and/or financial market reports, their invoicing decisions will reflect nonzero expected exchange rate changes. By assuming mean reversion we would like to capture the effects of such expectations on firms’ invoicing decisions.

Finally, we impose labor market clearing conditions:

\[ nL_{ht} = L^{(d)H}_{ht} + L^{(d)eH}_{ht} \]
\[ (1 - n)L_{ft} = L^{(d)F}_{ft} + L^{(d)eF}_{ft} \]

Equilibrium in this economy is characterized by a set of prices and allocations, such that all parties’ optimality conditions are satisfied, and all markets clear. Since the model is stationary, prices and allocations are determined through time-invariant policy functions, which respond to the state of nature. Below we describe how the policy functions were solved for.

1.3 Solution Method

Solving this model is complicated by the discrete nature of the invoicing decision, which precludes loglinearization and the use of standard methods of undetermined coefficients. We solve the model in its nonlinear form, using the collocation method\(^9\). As a first step, we group variables into actions \( x_t \), states \( s_t \), and expectational variables \( z_t \). Actions include current period non-predetermined variables,

\(^{9}\)Miranda and Fackler (2002) provide an extensive discussion of how collocation works.
such as prices, output, consumption, etc. States are represented by current period predetermined variables: money holdings, interest income from the previous period’s bond holdings, and previously set prices by firms at Home and abroad, i.e. 
\[ s_t = [M_t, M_t^*, (1 + i_{t-1}^*)B_t, P_{t-1}^{eH}, P_{t-1}^{eF}, P_{t-1}^H, P_{t-1}^{Fx}]' \]
Expectational variables are functions of the next period’s states and actions, which appear under the expectation sign; e.g., the right-hand sides of equations (1.9) and (1.10). Note also, that there are additional discrete actions and states. The discrete part of the action space incorporates invoicing decisions \( a_t \) and \( a_t^* \), while discreet states are the previous period’s invoicing decisions \( a_{t-1} \) and \( a_{t-1}^* \). Discrete variables are not included in \( x_t \) and \( s_t \); rather, they are treated separately.

The system of equilibrium conditions can be expressed as:
\[ F(x_t, s_t, E_tz_{t+1}) = 0 \]
where states evolve according to the law of motion in state, actions, and shocks
\[ \epsilon_{t+1} = [\epsilon_{t+1}, \epsilon_{t+1}^*]' \]
\[ s_{t+1} = g(s_t, x_t, \epsilon_{t+1}) \]
Ultimately, we seek to solve this system for decision rules of the type: \( x_t = x(s_t) \).
However, it is easier to first solve for agents’ expectations \( E_tz_{t+1} \), and obtain \( x'z \) by solving a nonstochastic nonlinear equation system. Since \( z_t \) is a function of \( s_t \) and \( x(s_t) \), we can approximate the vector \( z_t \) as a linear combination of polynomials in \( s_t \) (we use Chebyshev polynomials for the reasons discussed in Miranda and Fackler(2002)):
\[ z_t = \sum_{i=1}^{n} c_i \phi(s_t) \]
where coefficients $c_i$ are unknown. Next, we approximate the true pdf of money shocks by a discretised probability distribution: $\epsilon = \epsilon_1, \ldots, \epsilon_m$; $p_j = \text{prob}(\epsilon = \epsilon_j)$. Then, $E_t z_{t+1}$ is given by:

$$E_t z_{t+1} = \sum_{j=1}^{m} \sum_{i=1}^{n} p_j c_i \phi(g(s_t, x_t, \epsilon_{t+1}))$$

If the $c_i$'s were known, then given any state $s_t$, actions could be obtained from:

$$F \left( x_t, s_t, \sum_{j=1}^{m} \sum_{i=1}^{n} p_j c_i \phi(g(s_t, x_t, \epsilon_{t+1})) \right) = 0 \quad (1.30)$$

To solve for $c_i$'s, we approximate the continuous part of the state space $S$ by a finite number of nodes $s_1, \ldots, s_n$. We will require equation (1.30) to hold at all nodes, instead of everywhere on $S$. In our case, we first solve for the deterministic steady state and then compute the steady state value of $s$. The approximation nodes are chosen in the small neighborhood around the steady state, where we expect our economy to evolve under shocks with a small variance.

Next, consider the following iteration strategy. First, come up with a guess for the $c_i$'s and solve equation (1.30) at all $n$ nodes to obtain $x_1, \ldots, x_n$. Given $s_1, \ldots, s_n$ and $x_1, \ldots, x_n$ we can solve for current period values of $z_1, \ldots, z_n$. Then, update $c_i$’s by solving a linear system of $n$ equations $z_j = \sum_{i=1}^{n} c_i \phi(s_j)$, $j = 1, \ldots, n$. Using the new $c_i$’s, return to step 1, and continue until the change in $c_i$’s is negligible.

Finally, recall that there are two discrete states and actions $a_{t-1}, a^*_{t-1}, a_t, a^*_t$, which take values zero or one, giving a total of sixteen possible scenarios. If one is prepared to deal with the kink in the $F(\cdot)$ function and the firms’ value functions, the $[a_t; a^*_t]$ vector can be incorporated into $F(\cdot)$ and the system can be solved for four sets of coefficients in $c$ for each possible value of $[a_{t-1}; a^*_{t-1}]$. Otherwise one can solve the

10The residuals (approximation error off the nodes) in our calculations were in order of $10^{-6}$. 15
Parameter $\beta$ $\chi$ $\sigma$ $\varepsilon$ $\psi$ $\kappa$ $\lambda$ $\omega$ $n$ $\rho$ $\sigma^2 = \sigma^*_2$  

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<th>Value</th>
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</tr>
<tr>
<td>$\sigma^2 = \sigma^*_2$</td>
<td>0.000001</td>
</tr>
</tbody>
</table>

Table 1.1: Model Calibration

model for all possible scenarios to obtain sixteen sets of coefficients in $c$. We find that the two methods can be made to perform equally well in terms of accuracy, although computational speed is somewhat higher in the second case.

1.4 Choice of Parameter Values

There is little agreement on the appropriate choice parameters in NOEM models. Following Bergin (2002) we assume labor supply elasticity of unity; i.e. $\frac{1}{\psi-1} = 1$ or $\psi = 2$. Coefficient on real money balances in the utility is set to be small ($\chi = 0.05$) to reduce average money holding. For simplicity countries are assumed to be of equal size, so $n = 0.5$. The elasticity of substitution between goods ($\lambda$) is set to 6 in the benchmark model, consistent with an estimate of a 20% markup in the U.S. by Rotemberg and Woodford (1992). Estimates of the elasticity of substitution between Home and Foreign goods ($\omega$) also vary considerably from as low as 1.107 in Bergin (2002) to 12 (the upper bound in Harrigan (1993)). In the benchmark model we use $\omega = 8$. Parameter values are summarized in Table 1.

\footnote{Ghironi (2000) is one of the first attempts to calibrate the model in a structural way, but he uses a small open economy version. Bergin (2002) estimates a two-country model based on the data for the U.S. vs. rest of G-7 countries between 1973 and 2000. We borrow some of his estimates.}
Figure 1.1: Impulse Responses of Selected Variables to a Positive Home Money Shock (Benchmark model, $\rho=0.5, a_{t-1} = a_{t-1}^* = 1$).
1.5 Exchange Rate Pass-Through and Exporters Pricing Decisions

1.5.1 Depreciation due to Home Monetary Expansion

Effects on the Home country

Exchange rate pass-through in the model is determined by a joint response of preset and new prices of exporters. First, suppose the old prices were fixed in producer currency (dollars). When Home money supply increases, they decline in proportion to depreciation as the old cohort gains competitiveness. Since Home real wages also rise, the young exporters are under pressure to set higher re-set prices. However, the increase in new prices is constrained by competitiveness concerns. The net effect on the export price index is therefore negative, but is small due to the opposite movements of the two cohorts’ prices.

If predetermined export prices were fixed in euros, they would not respond to a Home money shock. The new prices would still rise, although by a smaller fraction because the absence of expenditure switching contains the increase in Home real wages. Nevertheless, the resulting prediction is surprising: Home export prices should rise when dollar depreciates.

Thus, past invoicing decisions are critical not only in determining the size, but also the sign of the instantaneous export prices’ response to Home-generated depreciations.

Effects on the Foreign Country

The offsetting effect of the re-set prices that occurs when old prices were set in producer currency does not occur in the Foreign country. In contrast, we find that Foreign export prices respond by more to Home-generated depreciations. This is
because 1) foreign exporters face higher demand from Home consumers; and 2) Home money shocks have limited effect on Foreign real wages. As an example, consider the case when old Home exporters follow PCP. Figure 1.1 documents impulse responses of various variables in both countries to a Home money shock. An imperfect asset market generates deviations from uncovered interest parity and risk sharing at the time of the shock. Unexpected expansion of the Home money supply lowers the Home nominal interest rate with little impact on the foreign interest rate. Thus, the liquidity effect only occurs in the Home country where aggregate demand rises. Foreign exporters’ labor demand rises due to Home consumers’ increased appetite, but foreign nontraded goods producers lose sales due to expenditure switching towards Home’s old exporters. These effects partially offset in equilibrium, and the total demand for foreign labor slightly falls. Real wages in the foreign country therefore fall, but only marginally. With virtually unchanged marginal costs, young exporters in the foreign country are willing to raise their price to meet higher demand at Home. The effect on the Foreign export price index is larger if old foreign export prices were fixed in euros. In both cases, however, the magnitude of the foreign export price response is substantially greater than the response of Home export prices and is always positive. Figure 1.2 plots relative responses of Home and Foreign export prices to a Home money shock. To summarize, relative responses of import and export prices to any given exchange rate depreciation are generally different depending on the origin of the depreciation. In response to Home-generated depreciations, Home import prices react by more than Home export prices. Moreover, past invoicing decisions are important for the size of instantaneous pass-through, as well as the sign of pass-through into export prices. Next, we discuss firms’ invoicing decisions.
1.5.2 Is There a Globally Optimal Invoicing Strategy?

Most theoretical studies of individual firms’ invoicing decisions examine a globally optimal invoicing strategy, i.e. when firms always price in one currency. However, in general equilibrium such strategy may not exist, i.e. firms may not always want to follow PCP or LCP but may change their behavior depending on the state of the world. In Appendix A we show that firms profits may be written as:

\[
\pi_t^{pcp} = B_t \left[ 1 + E_t \left( \frac{e_{t+1}}{e_t} \right)^\lambda \left( \frac{W_{t+1}}{W_t} \Phi_{t+1} \right) + 2 \text{cov}_t \left( \frac{e_{t+1}}{e_t} \lambda, \frac{W_{t+1}}{W_t} \Phi_{t+1} \right) \right]^{1-\lambda} \\
\left[ 1 + E_t \left( \frac{e_{t+1}}{e_t} \right)^\lambda \left( \frac{W_{t+1}}{W_t} \Phi_{t+1} \right) + 2 \text{cov}_t \left( \frac{e_{t+1}}{e_t} \lambda, \Phi_{t+1} \right) \right]^{-\lambda}
\]

(1.31)
\[
\pi_t^{lp} = B_t \left[ \frac{1 + E_t \left( \frac{W_{t+1}}{W_t} \Phi_{t+1} \right)}{1 + E_t \left( \frac{e_{t+1}}{e_t} \right) E_t \Phi_{t+1} + 2 \text{cov}_t \left( \frac{e_{t+1}}{e_t}, \Phi_{t+1} \right)} \right]^{\lambda - 1} \tag{1.32}
\]

where: \( B_t = \frac{\lambda - \lambda}{(\lambda - 1) - \lambda} W_t^{1-\lambda} e_t^\lambda \Omega_t \), \( \Phi_{t+1} = Q_{t+1} \frac{\Omega_{t+1}}{\Omega_t} \), and
\[
\Omega_{t+j} = (P_{t+j} H^*)^{\lambda - \omega} (P_{t+j}^*)^\omega P_{t+j}^{-1} (1 - n) C_{t+j}^*.
\]

Next, imagine that firms have perfect foresight. Let \( d_{t+1} = \left( \frac{e_{t+1}}{e_t} \right) \). Assume also that all covariance terms are zero and that no change is expected in other variables: \( B_t = \Phi_{t+1} = \left( \frac{W_{t+1}}{W_t} \Phi_{t+1} \right) = 1 \). The profits reduce to:

\[
\pi^{pcp} = 1 + d_t^{\lambda}
\]
\[
\pi^{lcp} = 2^{1-\lambda} (1 + d_{t+1})^{\lambda}
\]

The left-side diagram in Figure 1.3 plots the two functions. Both are convex in expected depreciation, but profits under PCP display greater curvature (since \( \lambda > 1 \)). In this case, producer currency pricing is the globally optimal strategy.

---

Figure 1.3: Price-Setting Firms’ Profits
Now consider a small alteration by assuming that nominal wages are expected to increase by 4 percent: \( \left( \frac{W_{t+1}}{W_t} \right) = 1.04 \). The right-hand side diagram in Figure 1.3 plots the resulting profit functions. The graph reveals that the existence of a globally optimal strategy disappears - firms would prefer LCP when they expect (small levels of\(^{12}\)) depreciation and PCP otherwise. Note that the last example is plausible, because nominal wages are mostly driven by Home money supply, while the exchange rate is driven by relative money supplies. Therefore, depending on the expected paths of the two countries’ money supplies either expected depreciations or appreciations are possible when Home nominal wages are expected to rise. One can construct similar examples for various values of covariances. Overall, while the full general equilibrium solution requires a numerical approach, the examples above illustrate that invoicing strategies in general equilibrium are likely to be time-varying.

1.5.3 Profits and Invoicing Under Uncertainty

Next we analyze profits in a fully stochastic and dynamic environment. Consider first the benchmark model for the parameters in Table 1.1. Figure 1.4 plots simulated observations of the domestic young exporters’ expected discounted profits under PCP and LCP. The average profit functions, obtained through a polynomial fit, have similar shapes under both strategies. They are concave around the long-run equilibrium and display some signs of convexity in times of large depreciations and appreciations.

Under LCP the profit function is flatter and with greater curvature. The polynomial fits are displayed in the left diagram of Figure 1.5. Firms’ invoicing strategies are asymmetric: on average, Home exporters tend to choose PCP in times when dollar is depreciated relative to the long-run equilibrium. The opposite is found for LCP\(^{12}\). The PCP profit line crosses the LCP profits once again under a large enough depreciation.
Figure 1.4: Average Profit Functions

(see the right-hand diagram in Figure 1.5). In simulations we find that the direction of asymmetry in firms’ invoicing decisions varies with model parameters, and most importantly with $\lambda$. For example, when $\lambda$ is low (e.g. 3, as opposed to 6 in the benchmark model), the model produces asymmetry of the opposite direction.

When simulating the model, we observe time-varying and state-dependent rate of exchange rate pass-through. To provide a quantitative assessment relative to the empirical evidence, we take an econometrician’s position. We estimate average pass-through by running a conventional regression of simulated changes in log prices on changes in logs of nominal exchange rates, marginal costs and aggregate consumption demands in the destination markets. For the Home import prices, for example, the estimated equation is:

$$\Delta \log(P_{t}^{F}) = \alpha_0 + \alpha_1 \Delta \log(e_{t}) + \alpha_2 \Delta \log(MC^{*}_{t}) + \alpha_3 \Delta \log(C_{t}^{F})$$

In the the benchmark specification with symmetric countries average pass-through into both countries’ import prices is incomplete, as expected. It is roughly equal
to 60%.\textsuperscript{13} Figure 1.6 plots the simulated nominal exchange rate and Home import prices. Next, we discuss the two-types of asymmetries documented in the data.

\subsection*{1.5.4 Directional Asymmetry: Depreciations vs. Appreciations}

Coughlin and Pollard (2000) document different responses of U.S. import prices depending on the direction of exchange rate movements with the direction of asymmetry varying across industries. Wickermasinghe and Silvapulle (2003) also find statistically significant asymmetric response of Japanese manufacture import prices: import prices in Japan were found to respond by more when yen appreciates. We now discuss how these directional asymmetries could arise in the context of our model.

The asymmetric invoicing decisions, discussed above, are the main reason for asymmetric pass-through in the model. When the dollar appreciates for several consecutive periods and Home firms price in local currency, aggregate pass-through is

\textsuperscript{13}Since the estimated relationship is very close to the truth in the model, we obtain small standard errors and double- and triple-digit t-values on all coefficients. This is why we do not report them.
entirely due to the changes in new prices. The latter are mostly driven by changes in marginal costs and foreign demand. On the other hand, when dollar continuously depreciates, Home exporters choose PCP and experience expenditure switching towards their goods. Pass-through elasticity is determined by the difference between the rate of depreciation and percentage change in new prices. Re-set prices in this cases are affected by two opposing factors: higher marginal marginal costs and competitiveness loss to the "old" exporters. In times of persistent depreciations marginal costs (Home wages) rise by more because expenditure switching raises demand for Home labor. On the other hand, loss of competitiveness contains the increase in new prices. Thus, whether pass-through is higher or lower in times of depreciations depends critically on the strength of expenditure switching towards "old" firms. The latter, in turn,
Table 1.2: Directional Asymmetry of Pass-Through into Import Prices

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<th>(λ)</th>
<th>Depreciations</th>
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<td>2</td>
<td>0.5122</td>
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<td>3</td>
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</tr>
<tr>
<td>10</td>
<td>0.6374</td>
<td>0.4914</td>
</tr>
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</table>

is mostly determined by the elasticity of substitution, λ. When markets are close to being competitive (low monopoly power, high λ) market share concerns are important, and new export prices rise by less in times of depreciations. Put differently, when λ is high, we should expect import prices to respond by more in times of depreciations.

Table 1.2 presents model-implied pass-through rates in times of persistent depreciations and appreciations \(^{14}\) for several values of λ. In the benchmark model with λ = 6 pass-through into the Home import prices is higher in times of dollar depreciations (≈ 68%) than in times of appreciations (≈ 48%). On the other hand, when we set λ = 3, asymmetry changes: pass-through into Home import prices is higher in times of appreciation, consistent with Wickermasinghe and Silvapulle (2003). Overall, the model predicts that countries/industries with less monopolistic markets should observe higher pass-through into import prices in times of depreciation and lower pass-through in times of appreciation.

\(^{14}\)Since we assume two-period price stickiness, we define depreciations as persistent when they occur in at least two consecutive quarters
1.5.5 Origin-Based Asymmetry: Export vs. Import Prices

Second type of asymmetry, reported in Dwyer et. al. (1993) is the difference in pass-through rates into export vs. import prices. Since our model has only two countries, this asymmetry can also be stated as different pass-through rates into import prices across countries. The intuition of how such asymmetry can arise in the model has been discussed in the context of the effects of Home country shocks on Foreign export prices (see section 1.5.1). There, it was found that depreciations caused by domestic monetary expansions produce a larger response by import prices.

Since the exchange rates in the model are determined by relative money supply shocks, a country with a more variable money supply should observe higher average

Figure 1.7: Model Simulation: Exchange Rate and Prices ($\rho = 0.9, \frac{\sigma_{\epsilon}^2}{\sigma_{\epsilon^*}^2} = 10$).
Relative Money Shocks

<table>
<thead>
<tr>
<th>Variability ( \frac{\sigma_{\epsilon_H}}{\sigma_{\epsilon_F}} )</th>
<th>Home Export Prices</th>
<th>Home Import Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.6041</td>
<td>0.6041</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.6617</td>
<td>0.4897</td>
</tr>
<tr>
<td>2</td>
<td>-0.6941</td>
<td>0.4670</td>
</tr>
<tr>
<td>6</td>
<td>-0.8382</td>
<td>0.2222</td>
</tr>
<tr>
<td>10</td>
<td>-0.8993</td>
<td>0.1367</td>
</tr>
<tr>
<td>15</td>
<td>-0.9394</td>
<td>0.0423</td>
</tr>
</tbody>
</table>

Table 1.3: Origin-Based Asymmetry: Pass-Through into Import and Export Prices

pass-through rate into its import prices. This conclusion is similar to that of Devereux, Engel and Storgaard (2002), although our motivation is somewhat different. In their model low variance of Home money shocks stabilizes marginal costs relative to the variability of the exchange rate, making PCP a preferred strategy for Home exporters and raising relative pass-through in the Foreign economy. In our setting, although low variance of domestic money supply also lowers correlation between exchange rates with marginal costs, it also raises the correlation of the aggregate demand in the Foreign country with the nominal exchange rate. In addition, it does not necessarily imply PCP as a more preferred strategy.

To quantify this asymmetry, in Table 1.3 we report estimated pass-through rates under different relative standard deviations of money supply shocks. In one extreme case, when the standard deviation of Foreign money supply shocks is 10 times larger that of the Home country, pass-through into export prices rises to almost 90%, while for import prices it drops to less than 14%. Figure 1.7 displays the simulated export and import price series for this extreme case.
1.6 Conclusion

In this paper we have examined a two-country new open economy macroeconomics model with two-period price stickiness à la Taylor, where exporters’ choice of currency in which to fix prices is endogenous. The model predicts an incomplete pass-through of exchange rate shocks into import and export prices. In addition, we have arrived at four main findings.

First, invoicing strategy is generally different in times of persistent depreciation and appreciation. In the benchmark model, exporters choose producer currency pricing when exchange rate is depreciated relative to its long-run equilibrium level and producer currency pricing when it is appreciated.

Second, when exchange rate depreciation is caused by a domestic monetary expansion, the response of Home country export price index is muted, even when the pre-set prices are fixed in producer currencies. This result is due to the offsetting movements of the pre-set and re-optimized prices.

Third, instant pass-through of exchange rate shocks into import and export prices is asymmetric. Depreciations caused by the Home country monetary expansions produce larger responses of import prices than export prices.

Fourth, average pass-through displays two types of asymmetries, also found in the data. First is the directional asymmetry: in times of depreciation, a country with a more competitive market should observe higher pass-through into import prices than in times of appreciation. Second is the origin-based asymmetry: a country with a more variable money supply should observe higher pass-through into its import prices.
The model and findings presented in this paper suggest several directions for future research, both empirical and theoretical. On the theoretical front, the proposed setting is well suited for analyzing optimal monetary policy in an international setting. The endogenous nature of exchange rate pass-through in a fully dynamic setting is an advantage over models that take the pass-through rate as given. As Devereux et. al. (2002) point out, it is inconsistent to analyze monetary policy for a given level of pass-through, when pass-through itself depends on monetary policy. Furthermore, the model could be improved in several dimensions. We do not model a distribution sector, which many argue is important for reconciling low pass-through into consumer prices and higher pass-through into export supply prices, which is also observed in the data\textsuperscript{15}. A more elaborate analysis should also include more determinants of nominal exchange rates, such as productivity shocks. Another interesting direction is to incorporate capital into the model to examine the business cycle properties of the model\textsuperscript{16}.

Finally, it would be useful to perform more empirical research to shed light on the virtually unexplored issue of asymmetric pass-through and firms’ invoicing decisions.

\textsuperscript{15}For example, see the discussion in Obstfeld (2002))

\textsuperscript{16}Kollman (2003) constructs an international business cycle model in the new open economy style, but assumes local currency pricing. The role of endogenous exchange rate pass-through on the business cycle properties of NOEM models has not been explored by researchers.
CHAPTER 2

Limited Commitment, Inaction and Optimal Monetary Policy

2.1 Introduction

As Clarida et. al. (1999) point out, no major central bank has announced a life-time commitment to a specific monetary policy rule. Thus, theoretical research has devoted a great deal of attention to designing policies that could in one way or another mimic long-term commitment. In this paper we consider a simple policy that to some extent is already in place: the practice of holding infrequent and periodic monetary policy meetings.

Our motivation comes from two observations. First, central banks around the world make monetary policy decisions at discrete times and with differing frequency. The Bank of England’s recent survey of over ninety central banks found that seven central banks held policy making meetings less than monthly, about thirty six had monthly meetings, while the rest made policy decisions more frequently, some even on a daily basis\textsuperscript{17}.

\textsuperscript{17}See Mahadeva and Sterne (2000), chart 7.5. \textit{How often do policy-makers meet to decide on the setting of policy instruments?} In the rest of the paper by policy adjustment we shall assume changes of the main policy instrument/target at the policy makers’ level such as the revision of federal funds target rate at FOMC meetings.
Second, in the absence of major shocks to the economy most major central banks hold policy meetings regularly. For example, the Bank of Japan’s monetary policy meetings take place twice a month, the Governing Council of the European Central Bank meets monthly, while the Federal Open Markets Committee in the U.S. revises the federal funds target rate eight times a year. Moreover, most monetary authorities in developed countries announce the schedule of policy meetings in advance.

A natural question is whether there are benefits of infrequent policy adjustment in the absence of explicit commitment to a particular policy rule. By analogy with the sticky price literature, it is tempting to justify infrequent policy meetings by appealing to administrative difficulties, or other policy adjustment costs. It would then follow that the optimal frequency of policy meetings should depend on the tradeoff between central banks’ internal cost of adjustment and social losses arising from policy makers’ inaction. However, the adjustment cost analogy is unlikely to provide a complete story. For example, it is not useful in explaining the fact that some major central banks (e.g. the Bank of Japan, the ECB and the Bank of England) have policy meetings more often than some smaller ones (e.g. the Bank of Canada or the Riksbank). More importantly, the analogy does not exploit the external effects of central banks’ actions. Note that when policy is adjusted on a pre-announced schedule, then following each policy meeting central bankers not only announce a new target, but also *de facto* promise to keep it fixed until the next meeting. Such implicit promises can be viewed as sequential short-term commitments. Therefore, the appropriate tradeoff in choosing the frequency of policy meetings is between the volatility of inflation and output arising from inaction and the benefits of short-term sequential commitments.
The contribution of this paper is twofold. First, we show that in the standard New Keynesian model and for most plausible parameter values infrequent but periodic policy adjustment is preferable to pure discretion even without any adjustment costs. Second, we solve for the optimal frequency of monetary policy meetings. Applied to the U.S. economy, our analysis suggests that the FOMC should meet no more than twice a year.

The rest of the paper is organized as follows. First, we consider a central bank in the Clarida et. al. (1999) world that is not able (or not willing) to make a life-time commitment to a policy rule (i.e. operates under pure discretion). As they show, even in the absence of the Barro and Gordon (1983) problem of inflationary bias, commitment is welfare improving because the impact of policy decisions on private sector forecasts improves the inflation-output variability tradeoff. To this we add that a welfare improving commitment need not be life-time: central banks that are unwilling to make long-term promises could instead offer short-term sequential commitments and also improve welfare relative to pure discretion. Furthermore, we solve for the optimal monetary policy under limited time commitment and find significant diminishing marginal returns from lengthier commitment. In the benchmark model announcing a new policy rule every year allows the central bank to realize about 90 percent of the total possible gains from life-time commitment. This is discussed in section 2.2.

Next, in section 2.3 we characterize a simple policy of infrequent adjustment where the central bank vows to revise the interest rate only every other period. One can think of this scenario as a policy of sequential short-term commitments to a degenerate non-state contingent rule - a commitment not adjust the policy. We find that
infrequent adjustment is preferred to pure discretion for most plausible parameter values. The intuition behind this finding is straightforward. A discretionary policy allows a timely response to exogenous disturbances, but features higher output costs of reducing inflation. On the other hand, under infrequent adjustment, the central bank acts as if under commitment every second period, but leaves some shocks in non-meeting months unanswered. While the latter contributes to the volatility of target variables, the effects of commitment are the opposite. In particular, commitment reduces the cost disinflation at the time of adjustment and prompts a more aggressive response to inflationary pressures. Quick disinflations make the effects of exogenous shocks on inflation die out faster under periodic adjustment than they do under period-by-period adjustment. This effect contains inflationary expectations and inflation itself in non-meeting months, producing lower volatility of inflation in all periods. We find that for many plausible parameter values these benefits from commitment dominate the destabilizing effects of inaction. Section 2.4 discusses the optimal frequency of policy meetings, its determinants and the application of our analysis to the U.S. case. Section 2.5 concludes.

2.2 Short-term Sequential Commitments

2.2.1 The Model

We consider a central bank in the world of Clarida et. al. (1999), hereafter CGG99. It seeks to minimize the expected present value of quadratic losses of the form:

$$E_0(L) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\alpha x_t^2 + \pi_t^2)$$ (2.1)
The economy is described by the usual New Keynesian IS and Phillips curves:

\[ x_t = -\varphi(i_t - E_t \pi_{t+1}) + E_t x_{t+1} + g_t \]  
\[ \pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t \]  

where \( i_t \) is the nominal interest rate, \( \pi_t \) is the rate of inflation, \( x_t \) captures log deviations of real output from its natural level (output gap) and the two exogenous state variables \( u_t \) (cost-push shifts) and \( g_t \) (demand shifts) evolve according to:

\[ g_t = \mu g_{t-1} + \hat{g}_t \]  
\[ u_t = \rho u_{t-1} + \hat{u}_t \]

The exact interpretation of the cost-push and demand shifts depends on the level of generality in the underlying nonlinear model. With government spending, variable markups and technology shocks each shift can be a combination of various types of disturbances: \( g_t \) usually incorporates government spending (as in Clarida et al. (1999)) and shocks to the growth rate of natural output (as in Woodford (1999)). Similarly, in the general setup \( u_t \) captures exogenous variations in deviations between the marginal disutility of labor and the marginal product of labor.\(^{18}\)

Note that the formulation of equation (2.1) assumes the absence of the inflationary bias, that occurs when the central bank targets a level of output above its natural level.\(^ {19}\) This is done for two reasons. First, inflationary bias results in higher average inflation. The recent experience of the U.S. and most European countries does not

\(^{18}\)We do not argue that the model in (1)-(5) is a good representation of reality. This framework was chosen due to its popularity and in order to better relate to existing studies. In section 2.4.3. we briefly discuss some alternative model assumptions that could be of interest for future research.

\(^{19}\)See Kydland and Prescott (1977) and Barro and Gordon (1983).
suggest the presence of such a bias. Secondly, inflationary bias is likely to lower welfare regardless of how often the policy makers meet and would not affect the rankings of the policies we consider. The primary focus of this paper is the role of commitment in removing the bias toward over-stabilization of output under discretionary policies. Unlike the inflationary bias, this stabilization bias results in excessive volatility of inflation. Volatility of inflation is likely to be a more serious problem than its average level.

In the exercises below we base our choice of parameter values on the micro-foundations behind the model. The IS equation is derived from the consumption Euler equation, where $\varphi$ is the inter-temporal elasticity of substitution. The plausible range of $\varphi$ suggested in many studies lies between 0.5 and 1 (log utility). In the benchmark model we use an intermediate value: $\varphi = 0.67$. We choose $\beta = 0.997$ since the focus is on the monthly frequency. The Phillips curve is commonly derived from Calvo pricing equation, in which $\lambda$ takes the form:

$$\lambda = \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \left( \frac{1}{\varphi} + \frac{1 - \eta + \vartheta}{\eta} \right)$$

where $\theta$ is the probability that a firm will not change its price in any given period, $\eta$ is the weight of labor in the Cobb-Douglas production function, $\vartheta$ is the inverse of the Frisch elasticity of the labor supply. Common assumptions in the literature suggest $\theta = \frac{11}{12}$ and $\eta = \frac{2}{3}$. There is little agreement about the appropriate value for the labor supply elasticity. We use a high value of elasticity ($\frac{1}{\vartheta} = 5$, or $\vartheta = 0.2$) as prescribed by Prescott (2003) and used in Rotemberg and Woodford (1992, 1997) and Gali et. al. (2003). The optimal weight of output in the loss function under mild regularity

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\(^{20}\)Blinder (1998), from his own experience as Vice Chairman of the Fed, argued that the U.S. monetary policy makers do not systematically try to push unemployment below its natural level.
conditions can be expressed as follows: \( \alpha = \frac{\lambda}{q} \) (2.7)

where \( q \) is the demand elasticity. We set the latter at 6, which implies a steady state markup over marginal costs of 20%.

Innovations to cost and demand shifts are assumed uncorrelated with each other with standard deviations of: \( \sigma_{\tilde{u}} = \sigma_{\tilde{g}} = 0.001 \). Finally, the persistence parameters are: \( \rho = \mu = 0.8 \). In section 2.4 we consider an alternative calibration of the stochastic processes based on the U.S. data.

### 2.2.2 Discretion vs. Life-Time Commitment

We begin by discussing the optimal policy under discretion and under life-time commitment. While the optimality of the latter is well known, here we re-state the main argument in order to motivate further discussion of alternative policies.

In the absence of commitment, which we will refer to as pure discretion, the central bank does not make any promises as to how it will adjust the interest rates in the future. Thus every period it minimizes contemporaneous losses while taking all forecasts as given. To obtain the solution it is convenient to first choose inflation and output that maximize the objective function subject to (2.3) and then obtain the interest rate from (2.2). The first order condition (F.O.C.) is given by:

\[
\pi_{nc}^t = -\frac{\alpha}{\lambda} x_{nc}^t
\]

Using this in (2.3), we obtain:

\[
x_t = \frac{\alpha \beta}{\alpha + \lambda^2} E_t x_{t+1} - \frac{\lambda}{\alpha + \lambda^2} u_t
\]

\(^{21}\)See Woodford (2003), chapter 6, proposition 6.2.
Recursive substitution together with (5) yields a solution:

\[
\begin{align*}
\pi_t^{nc} &= \lambda \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)} u_t; \\
x_t^{nc} &= \pi_t^{nc} u_t.
\end{align*}
\] (2.8)

Note that the resulting interest rate policy always neutralizes demand shocks in the sense that it makes inflation and output react to supply shocks only.

Next, consider the case of commitment. For simplicity, as in CGG99 we begin by examining policies that that have the same functional form as under pure discretion, i.e. \( \pi = \omega_1 \cdot u_t \) and \( x_t = \omega_2 u_t \). Then future target variables can be expressed as multiples of their current period values: \( \pi_{t+j} = \pi_t u_{t+j} \) and \( x_{t+j} = x_t u_{t+j} \). The objective function (2.1) can therefore be re-written as:

\[
L_t = \frac{1}{2} \left( \alpha x_t^2 + \pi_t^2 \right) E_t \sum_{i=0}^{T} \beta^i \left( \frac{u_{t+i}}{u_t} \right)^2
\] (2.9)

where \( T \) is the policy horizon (\( \infty \) in the standard case). Note that since the summation term in the last equation is exogenous, minimizing (2.9) is equivalent to minimizing current period losses \( (\alpha x_t^2 + \pi_t^2) \). The first order condition, obtained by minimizing the objective function w.r.t. \( x_t \) and subject to the Phillips curve (2.3), is:

\[
\alpha x_t + \frac{\partial \pi_t}{\partial x_t} \pi_t = 0
\] (2.10)

The term \( \frac{\partial \pi_t}{\partial x_t} \) is crucial in determining the tradeoff between inflation and output. Under discretion the central bank is unable to affect forecasts of the future. Hence from (2.3) we have \( \frac{\partial \pi_t}{\partial x_t} = \lambda \), i.e. the output cost of reducing inflation by one unit is \( \frac{1}{\lambda} \). On the other hand, under life-time commitment the central bank can count on the private sector to expect \( E_t \pi_{t+1} = \rho \pi_t \). This ability to credibly influence private

\[\text{interest rate that implements the solution can be recovered from (2.2). Using the notation of Clarida et. al (1999), it can be represented as: } i_t = \gamma_x E_t \pi_{t+1} + \frac{1}{\rho} y_t, \text{ where } \gamma_x = 1 + \frac{(1-\rho)\lambda}{\rho \sigma}.\]
sector forecasts modifies the Phillips curve equation to be:

$$\pi_t = \frac{\lambda}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t$$  \hspace{1cm} (2.11)

Hence, under commitment we have: $\frac{\partial \pi}{\partial x_t} = \frac{\lambda}{1-\beta\rho}$, so that only $\frac{1-\beta\rho}{\lambda}$ units of output need to be sacrificed in order to bring inflation down by one unit. The multiplicative constant $(1-\beta\rho)$ captures ‘savings’ in the form of lower output costs of reducing inflation that arise from the central bank’s ability to affect private sector forecasts.

A complete solution is obtained by combining (2.10) and (2.11):

$$\begin{align*}
\pi_t^c &= -\frac{ak}{\lambda} x_t^c \\
x_t^c &= -\frac{\lambda}{\lambda^2+ak(1-\beta\rho)} u_t \\
\pi_t^c &= \frac{ak}{\lambda^2+ak(1-\beta\rho)} u_t
\end{align*}$$  \hspace{1cm} (2.12)

where $k = 1-\beta\rho$. Note that output (inflation) under discretion is less (more) volatile than under commitment. This is the stabilization bias, which is primarily due to high costs of reducing inflation under discretion. Under commitment, lower costs of reducing inflation prompt the central to ‘buy’ more inflation reduction at the cost of some extra output volatility ($k < 1$) thus alleviating the problem of stabilization bias.

To see the welfare gains from commitment note that from (2.12) the unconditional expectation of the loss function can be expressed as: $E(L) = \frac{\alpha\lambda^2+\alpha^2}{\lambda^2+ak(1-\beta\rho)} \frac{1}{1-\beta} E(u^2)$. It is easy to show that the loss is minimized when $k = (1-\beta\rho)$ and that any policy taking the form of (2.12) with $(1-\beta\rho) < k < 1$ is preferred to the case of pure discretion ($k = 1$).

### 2.2.3 Discretion vs. Short-Term Commitment

The commitment described above was life-time. Such arrangement may be both unrealistic (e.g. because chairmen of central banks have limited terms in office) and

---

23The interest rate rule implied by the solution is given by $i_t = \gamma_\pi E_t \pi_{t+1} + \frac{1}{\phi} g_t$, where $\gamma_\pi = 1 + \frac{(1-\rho)\lambda}{\rho^2\alpha(1-\beta\rho)}$. 

39
undesirable (e.g. because of model uncertainty or because of extraordinary circumstances requiring deviations from the announced rule). As an alternative, central banks could use short-term commitments, i.e. announce policy rules that are valid for a pre-determined period of time. Such short-term promises allow the monetary authority to credibly influence private sector forecasts in the periods when the commitment is valid, but not in the long-run. Intuitively, we should expect such policy to be suboptimal relative to life-time commitment, but perform better than pure discretion.

As a simple verification, consider a monetary authority that announces a new commitment every other period\(^{24}\). Thus, its powers are restricted to affecting only one period ahead forecasts. It can be shown that under a class of linear interest rate rules considered above, the equilibrium with one-period commitments is also described by (2.12)\(^{25}\) but with \(k = \left(1 + \frac{\beta \rho}{1 + \varphi \lambda + \rho}\right)^{-1}\). Note that in this case \((1 - \beta \rho) < k < 1\), implying that even though life-time commitment is still the first best, central banks that are unwilling to commit forever can achieve a better outcome than pure discretion by offering short-term commitments.

2.2.4 Short-Term vs. Life-Time Commitment Under Unconstrained Optimum

Next, we evaluate relative welfare gains under life-time and short-term commitments when the policy under each scenario is globally optimal. Under life–time commitment the central bank chooses inflation and output to maximize the following

\(^{24}\)This is similar to the analysis of partial commitment in fiscal policy in Klein and Ríos-Rull (2003).

\(^{25}\)A complete solution is provided in Appendix B.
Lagrangean:

\[ \mathcal{L}_1 = -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \alpha x_{t+i}^2 + \pi_{t+i}^2 + \gamma_{t+i} (\pi_{t+i} - \lambda x_{t+i} - \beta E_t \pi_{t+1+i} - u_{t+i}) \right] \right\} \]  
(2.13)

The optimal policy sets inflation in proportion to the change output (see CGG99):

\[ x_{t+i} - x_{t+i-1} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 1, 2, 3, \ldots. \]  
and \[ x_{t+i} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 0 \]  
(2.14)

As an alternative, suppose that every \( T + 1 \) periods the central bank announces a new commitment that is valid for \( T \) periods in the future. It can be shown that the optimal policy announced at date \( t \) and valid until \( t + T \) (the ’commitment cycle’) closely resembles (2.14) 26:

\[ x_{t+i} - x_{t+i-1} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 1, 2, 3, \ldots T \]  
and \[ x_{t+i} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 0 \]  
(2.15)

The only difference between (2.14) and (2.15) is that the short-term commitment expires at \( t + T \) and at \( t + T + 1 \) a new announcement is made. Model stationarity implies that at the beginning of each commitment cycle the central bank will always announce the same policy. To evaluate the relative performance of the two arrangements, we solve the model above under various durations of commitment (\( T \)) and measure welfare gains implied by each policy\(^{27}\). Figure 2.1 plots welfare gains under sequential commitments of various lengths as a fraction of the total gains obtained.

\(^{26}\)Appendix B provides a complete solution. Note that this model resembles those of Schaumburg and Tambalotti (2002) and Kara (2003). Both of these studies consider a similar setting. Their models examine ‘Calvo type’ central bankers who offer life-time commitments, but each period face a constant probability of being replaced. The central bankers in our model are more of the ‘Taylor type’ - they are being replaced after serving fixed terms in office. We consider the latter to be a more plausible scenario for developed countries. In any case, they also find that most of the gains from commitment occur at short horizons.

\(^{27}\)Welfare gains were measured by the difference of the unconditional expectation of the loss function (2.1) under each case of commitment and under pure discretion.
under life-time commitment. A striking feature in Figure 2.1 is the presence of strong diminishing returns from lengthier commitment. For example, in the benchmark specification sequential 12-month commitments allow the central bank to realize about 90% of the total gains obtained under life-time commitment. When the persistence of the cost-push shocks is reduced to 0.5, then the same fraction of the gains can be obtained by committing to a new policy every six months. Intuitively, under commitment the optimal policy seeks not only to eliminate the contemporaneous effects of current exogenous shocks, but also to neutralize the predetermined component of future expected effects coming from persistence \( E_t(u_{t+1}) = \rho u_t \). When persistence is large, then current innovations affect the forecasts of inflation and output.
far into the future making long-term commitment more important. Another reason for substantial gains from short-term commitments lies in their sequential nature. In any announcement period we expect the optimal policy to influence short-term private sector forecasts in a way that allows greater stabilization of inflation within the 'commitment' cycle. Moreover, in equilibrium agents expect the same policy to be re-announced after the expiration of the current commitment. Hence, the stabilization properties of the current policy are also embedded into the agents’ long-term forecasts of future policy decisions. More stable long-term forecasts of inflation, in turn, further contain current inflation, thus multiplying the stabilizing effects of the announced commitment. This extra effect produces large overall welfare gains even when the commitment is set to expire soon.

To this end we have shown that the ability of short-term sequential commitments to affect private sector forecasts both in the short-run and in the long-run, makes them preferable to pure discretion and provides a useful alternative to life-time commitment when the latter is unfeasible. The prescription of this section is useful for central banks that possess at least a short-term commitment technology. Next, we consider a case when commitment to state-contingent rules in not feasible and examine a simple alternative - infrequent policy adjustment.

2.3 Infrequent Monetary Policy Adjustment

Central banks’ unwillingness (or inability) to commit to explicit policy rules prompts researchers to look for implementable arrangements that somehow resemble commitment. Here we consider a simple policy of infrequent but periodic monetary

\[ \text{Note, that when } \rho = 0 \text{ equation (2.12) reduces to (2.8), i.e. under the simple policy of section 2.2.1. gains from commitment disappear when shocks are completely unpredictable.} \]
policy adjustment. An attractive feature of such policy is its implementability: to some extent it is already in place since central banks around the world make monetary policy decisions at discrete points in time and with a stable frequency. For example, the Bank of Japan’s monetary policy meetings take place twice a month, the Governing Council of the European Central Bank meets monthly, while the Federal Open Markets Committee in the U.S. revises the target federal funds rate eight times a year. In addition, monetary authorities in developed countries typically announce the schedule of policy meetings well in advance.

When monetary policy is adjusted infrequently and on a pre-announced schedule, then after each adjustment the central bank not only declares a new target, but also *de facto* promises to leave it unchanged until the next policy meeting. This closely resembles short-term sequential commitments of the previous section. The difference is that by fixing the target for several periods, the central bank commits to a non-state-contingent rule (or, alternatively, it commits not to adjust). This implies a trade-off between gains from commitment and losses arising from central bank’s inaction. Moreover, the trade-off implicitly defines the optimal frequency of policy meetings - an important issue in monetary policy design.

In this section we consider a central bank that is unwilling or unable to commit to an explicit policy rule but holds policy meetings at pre-announced dates. We will assume that the economic agents believe that the monetary authority will adhere to the announced schedule of the meetings. In other words, we abstract from the possibility of unscheduled meetings. This is a reasonable assumption for major economies. For example, in the last decade only in 2001 has the FOMC had more than eight meetings a year. As long as unscheduled meetings represent true emergencies and have a small unconditional probability, their existence should not affect our main results.
of this policy relative to period-by-period adjustment under pure discretion. Then we solve for the optimal frequency of policy meetings and characterize its determinants. Finally, we examine the implications of our analysis to the case of the Federal Open Markets Committee.

2.3.1 Central Bank’s Problem Under Infrequent Adjustment

To keep things tractable, we begin with a central bank that commits to holding policy meetings every other period. In every meeting period (denote $t$) it sets a new interest rate and promises to keep it fixed for two periods. If the promise is credible, the private sector forecast of the next period’s output is given by:

$$E_t x_{t+1} = -\varphi i_t + \varphi E_t \pi_{t+2} + E_t x_{t+2} + \mu g_t$$  \hspace{1cm} (2.16)

Similarly, expected next period inflation can be written as:

$$E_t \pi_{t+1} = -\lambda \varphi i_t + \lambda \mu g_t + (\lambda \varphi + \beta) E_t \pi_{t+2} + \lambda E_t x_{t+2} + \rho u_t$$  \hspace{1cm} (2.17)

Using equations (2.16) and (2.17) in (2.2) and (2.3) allows to express the IS and Phillips curve equations as follows:

$$x_t = -i_t (2\varphi + \varphi^2 \lambda) + \varphi (\lambda \varphi + \beta + 1) E_t \pi_{t+2} + (\lambda \varphi + 1) E_t x_{t+2} + \varphi \rho u_t + (\varphi \lambda \mu + \mu + 1) g_t$$  \hspace{1cm} (2.18)

$$\pi_t = \lambda x_t - \beta \lambda \varphi i_t + (\lambda \beta \varphi + \beta^2) E_t \pi_{t+2} + \beta \lambda E_t x_{t+2} + (\beta \rho + 1) u_t + \beta \lambda \mu g_t$$  \hspace{1cm} (2.19)

In contrast to the case of pure discretion, short-term commitments expand the policy horizon to the duration of the interest rate fixity. At the time of interest rate adjustment the central bank’s problem is:

$$\max_{i_t} \frac{1}{2} \left[ (\alpha x_t^2 + \pi_t^2) + \beta E_t (\alpha x_{t+1}^2 + \pi_{t+1}^2) + F_t \right]$$  \hspace{1cm} (2.20)
where $F_t$ represents expected losses beyond $t + 1$, which are taken as given. The constraints to the problem include the modified IS and Phillips curves at dates $t$ (equations (2.18) and (2.19)) and $t + 1$:

$$x_{t+1} = -\varphi (i_t - E_{t+1} \pi_{t+2}) + E_{t+1} x_{t+2} + g_{t+1}$$

$$\pi_{t+1} = \lambda x_{t+1} + \beta E_{t+1} \pi_{t+2} + u_{t+1}$$

(2.21)

(2.22)

where forecasts of $\pi_{t+2}$ and $x_{t+2}$ cannot be manipulated by the central bank and are taken as given. Inserting the constraints into the objective function and maximizing w.r.t. $i_t$ yields the following first order condition:

$$\alpha x_t \frac{\partial x_t}{\partial i_t} + \pi_t \left( \frac{\partial \pi_t}{\partial i_t} + \frac{\partial \pi_t}{\partial \pi_t} \right) + \beta E_t \left( \alpha x_{t+1} \frac{\partial x_{t+1}}{\partial i_{t+1}} + \pi_{t+1} \frac{\partial \pi_{t+1}}{\partial i_{t+1}} \right) = 0.$$ 

Or:

$$\alpha (2 + \varphi \lambda) x_t + \lambda (2 + \varphi \lambda + \beta) \pi_t = -\beta (\alpha E_t x_{t+1} + \lambda E_t \pi_{t+1})$$

(2.23)

Since the central bank is unable to respond to shocks in the next period, it sets the instrument at the level that minimizes average expected losses between policy meetings. This is similar to what one obtains in the Taylor model of price-setting where each firm adjusts its price periodically. However, this analogy is not as close as it seems. Firms’ infrequent revision of prices is typically justified by the existence of price adjustment costs. In the absence of such costs firms would always do better by revising prices every period. On the other hand, central bank’s actions have important external effects that define a different trade-off. Although changing the interest rate on a period-by-period basis allows a timely response to exogenous shocks, it features high output costs of reducing inflation. On the other hand, infrequent

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30 Note also that in the absence of next period considerations ($\beta = 0$), the optimality condition reduces to the standard solution without commitment.
policy adjustment, while forcing the central bank to put up with extra volatility arising from inaction, creates commitment gains from the ability to affect short-term forecasts. As long as the gains from commitment exceed losses from inaction, the central bank would choose infrequent policy meetings even when there are no explicit costs of interest rate adjustment. Next, we define the equilibrium and evaluate welfare under the two alternatives.

### 2.3.2 Equilibrium

The presence of an endogenous state variable $i_{t-1}$ in periods of central bank’s inaction implies that the minimum state vector is different across periods. In particular, in periods of adjustment the relevant state is summarized by contemporaneous shocks $e_t = \{g_t, u_t\}$, whereas in periods of inaction the relevant states can be summarized as $s_t = \{e_{t-1}, e_t\}$. We use the following notion of equilibrium:

**Definition 1** A rational expectations infrequent policy adjustment equilibrium is described by a set of policy functions $\{i^a(e_t), x^a(e_t), \pi^a(e_t)\}$ and $\{i^n(s_t), x^n(s_t), \pi^n(s_t)\}$ such that:

1. **In periods of adjustment** the policy is described by $\{i^a(\cdot), x^a(\cdot), \pi^a(\cdot)\}$ which satisfy (2.18), (2.19) and (2.23).

2. **In periods of central bank’s inaction** the policy is described by $\{i^n(\cdot), x^n(\cdot), \pi^n(\cdot)\}$, where: $i^n(s_t) = i^a_{t-1}(e_{t-1})$ and $x^n(\cdot)$ and $\pi^n(\cdot)$ satisfy (2.2)-(2.3).

3. **Private sector forecasts are consistent with the policy.** That is:

   - **In periods of inaction**, forecasts in (2.2) and (2.3) are given by
     
     $$ E_t x_{t+1} = E_t x^a(e_{t+1}) \quad E_t \pi_{t+1} = E_t \pi^a(e_{t+1}) $$
In periods of adjustment forecasts in (2.23) are given by (2.16) and (2.17), while the forecasts in (2.16) and (2.17) are given by:

\[ E_t x_{t+2} = E_t x^a(e_{t+2}) \quad E_t \pi_{t+2} = E_t \pi^a(e_{t+2}) \]

Here we'll describe the solution in general terms\(^{31}\). First, start with periods of adjustment. The equilibrium is described by a system of three expectational equations: (2.18), (2.19), and (2.23). To obtain the solution, we can first substitute the interest rate out using one of the equations. Second, since the system is linear and because all state variables in adjustment periods are exogenous, we seek a solution of the form:

\[ y^a_t = De_t \]  

(2.24)

where \( y_t = [x_t, \pi_t]' \), \( e_t = [g_t, u_t]' \) and superscript \(^a\) indicates periods of adjustment. Next, note that since at \( t + 2 \) the central bank faces exactly the same problem as at time \( t \), rational expectations imply \( E_t y_{t+2} = DP^2 e_t \), where \( P \) is a diagonal matrix with persistence parameters (\( \mu \) and \( \rho \)). Using this forecasting rule leaves us with a deterministic system of 4 linear equations in the unknown coefficients of \( D \), which is straightforward to solve\(^{32}\). Given the solution (2.24), we can back out the interest rate rule, which is also linear in exogenous states:

\[ i^a_t = \Psi e_t \]  

(2.25)

In periods when the central bank rests, equilibrium is described by (2.2) and (2.3) where \( i^a_t = i^a_{t-1} = \Psi e_{t-1} \) and \( E_t \pi_{t+1} \) and \( E_t x_{t+1} \) must be consistent with the

\(^{31}\)A complete solution is provided in Appendix D.

\(^{32}\)Note that unlike in the case of pure discretion, the matrix \( D \) does not generally have zeros in the first column. This is because with the interest rate being fixed for two periods, it is no longer optimal to neutralize demand shocks at the time of adjustment. Instead, a policy of minimizing the average effects of demand and supply shocks over two periods is preferred.
adjustment policy, i.e. $E_t y_{t+1} = DPe_t$. The solution of this system takes the form:

$$y^n_t = D_1 e_{t-1} + D_2 e_t$$ (2.26)

where the superscript $^n$ indicates no-adjustment periods.

### 2.3.3 Welfare Measures

We use the unconditional expectation of the loss function (2.1) to measure social loss. In the case of infrequent adjustment welfare costs can be expressed as:

$$E(L^d) = 0.5 \frac{1}{1-\beta} \left( 0.5 E(L^n) + 0.5 E(L^n) \right)$$

where $E(L^n)$ and $E(L^n)$ represent unconditional expectations of losses in times of adjustment and inaction, respectively and are given by:

$$E(L^n) = D_1 \Omega D'_1 + D_2 \Omega D'_2 + 2D_1 \Sigma D'_2$$

where $\Omega$ and $\Sigma$ are, respectively, covariance and autocovariance matrices of $e_t$ ($\Omega = E(e_t e'_t)$ and $\Sigma = (e_{t-1} e'_t)$).

### 2.3.4 Discreteness vs. Discretion

As was mentioned above, the desirability of a policy of infrequent interventions depends on the size of the gains from short-term commitment relative to losses arising from the inability to respond to exogenous shocks in a timely fashion. In the benchmark model welfare loss under the discrete adjustment policy (0.0045) is smaller than under period-by-period adjustment with discretion (0.0053)\(^{33}\). To provide a better

\(^{33}\)As a reference point, the loss under life-time commitment is 0.0019.
intuition behind this result we examine exact numerical solutions. In the benchmark model the equilibrium in periods of intervention is given by (see eq. (2.24)):

\[
\begin{pmatrix}
  x_{t}^{da} \\
  \pi_{t}^{da}
\end{pmatrix} =
\begin{pmatrix}
  0.0445 & -23.0357 \\
  -0.0009 & 2.8620
\end{pmatrix}
\begin{pmatrix}
  g_t \\
  u_t
\end{pmatrix}
\] (2.28)

where the superscript \(^{da}\) indicates periods of adjustment under discrete policy.

Under pure discretion (superscript \(^{pd}\)) the variables in all periods evolve according to:

\[
\begin{pmatrix}
  x_{t}^{pd} \\
  \pi_{t}^{pd}
\end{pmatrix} =
\begin{pmatrix}
  0.0000 & -19.3301 \\
  0.0000 & 3.2217
\end{pmatrix}
\begin{pmatrix}
  g_t \\
  u_t
\end{pmatrix}
\] (2.29)

The expressions reveal that even though the optimal policy under infrequent adjustment allows inflation and output to react to demand shocks, their contemporaneous effect is small, and more so in the case of inflation. Thus, both inflation and output are mostly driven by the supply shocks. The response of inflation to supply shocks is smaller and the response of output is larger under infrequent adjustment. This is a 'substitution effect' of lower output costs of reducing inflation: the central bank 'buys' more inflation reduction at the expense of output.

Next, consider central bank holidays. To characterize the equilibrium, note that the interest rate adjustment policy implied by (2.28) takes the form\(^{34}\):

\[
i_{t}^{da} =
\begin{pmatrix}
  1.3368 \\
  8.2772
\end{pmatrix}
\begin{pmatrix}
  g_t \\
  u_t
\end{pmatrix}
\] (2.30)

Inflation and output are described by (2.21) and (2.22) where next period expectations must be consistent with (2.28). Combining these equations yields equilibrium in

\(^{34}\)The interest rate rule can be obtain from IS equation (2.18) where \(x_t\) and \(\pi_t\) are given by (2.28) and \(E_t x_{t+2} = \mu^2D_{11}g_t + \rho^2D_{12}u_t\) and \(E_t \pi_{t+2} = \mu^2D_{21}g_t + \rho^2D_{22}u_t\).
periods of inaction (superscript $d^n$):

$$
\begin{pmatrix}
\hat{x}^{d^n}_t \\
\pi^{d^n}_t
\end{pmatrix} = \begin{pmatrix}
-0.8912 & -5.5181 \\
-0.0160 & -0.0993
\end{pmatrix} \begin{pmatrix}
g_{t-1} \\
u_{t-1}
\end{pmatrix} + \begin{pmatrix}
1.0351 & -16.9022 \\
0.0179 & 2.9785
\end{pmatrix} \begin{pmatrix}
g_t \\
u_t
\end{pmatrix}
$$

Although target variables in periods of inaction are functions of both current and past shocks, in most cases past shocks enter with the opposing sign thus reducing the impact of current shocks. The magnitude of this offsetting effect is large when shocks are persistent. Intuitively, when persistence is high future shocks have a large predetermined component. Since the central bank at the time of adjustment seeks to balance current and future expected targets, the interest rate reacts to both contemporaneous shocks and to the predetermined component of the next period’s shocks. Thus, the larger the predetermined component, the more effective is the policy makers’ ability to neutralize shocks in two periods. An interesting finding implied by the solution is that despite infrequent adjustment, inflation is less responsive to supply shocks (and less volatile overall) in all periods, not only in periods of intervention. This is due to the ‘spillover’ effect of short-term commitments: since agents expect a more aggressive response to inflationary pressures at the next policy meeting, the effects of exogenous shocks on inflation are not expected to persist for too long. Lower inflationary expectations, in turn, contain inflation itself.

The solution further reveals that the impact of demand shocks on inflation continues to be small in periods of inaction: although the coefficient on $g_t$ is larger than in (2.28), a negative coefficient on $g_{t-1}$ together with high persistence in $g_t$ imply a smaller overall effect. The effect on output is also small, although larger than in the case of inflation. On the other hand, average impact of supply shocks on output for the most part is expected to be larger than under pure discretion (because
current and past shocks enter with the same sign). Overall we conclude that the infrequent adjustment policy generates less volatile inflation and more volatile output in all periods, thus reducing the stabilization bias of the discretionary policy. Table 2.1 summarizes standard deviations of inflation and output across periods and between policies in the baseline specification. Finally, Figure 2.2 illustrates gains in inflation-output variability tradeoff by plotting the efficient policy frontier for the two alternatives. The frontier is constructed by measuring unconditional variances of inflation and output for various values of $\alpha^{35}$. The figure shows that under the simple policy of infrequent interventions the central bank faces a more favorable choice between inflation and output variability.

<table>
<thead>
<tr>
<th>Policy (Periods)</th>
<th>St.Dev. ($x$)</th>
<th>St.Dev. ($\pi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infrequent adjustment (work)</td>
<td>0.0383</td>
<td>0.00477</td>
</tr>
<tr>
<td>Infrequent adjustment (holiday)</td>
<td>0.0359</td>
<td>0.00483</td>
</tr>
<tr>
<td>Infrequent adjustment, average</td>
<td>0.0371</td>
<td>0.00480</td>
</tr>
<tr>
<td>Pure Discretion</td>
<td>0.0323</td>
<td>0.00537</td>
</tr>
</tbody>
</table>

Table 2.1: Unconditional Volatility of Target Variables

To this end we have established that in the benchmark model the policy of infrequent adjustments is preferable to pure discretion. Below we show that this conclusion holds for a wide range of plausible parameter values.

$^{35}$The output weight ($\alpha$) was changed by picking different values of demand elasticity ($q$) in equation (2.27), so as to keep $\lambda$ unchanged. In the case of infrequent adjustment policy the depicted frontier corresponds to average variances across periods of action and inaction.
Volatility of Exogenous Shocks

Figure 2.3 plots social losses for various standard deviations of demand and cost-push innovations ($\sigma_g$ and $\sigma_u$). The left panel suggests that infrequent adjustment is more preferable when cost-push innovations are more volatile. This is because more volatile supply shocks call for greater attention to inflation stabilization. The latter (in light of the previous discussion) is easier to achieve when the central has periodic holidays. The right panel tells us that the volatility of demand shocks works against the case for more holidays. However, since the impact of demand shocks on target variables is small, in the benchmark model $\sigma_g$ must be 50 times larger than $\sigma_u$ for the central bank to choose pure discretion over periodic holidays.
As is evident from Figure 2.4, the desirability of infrequent interventions is increasing in the persistence of the cost-push shocks\(^{36}\). Intuitively, the more persistent they are, the greater is the pre-determined component of the conditional short-term forecasts of inflation and output, which, in turn, raises the importance of the central bank’s ability to affect those forecasts. A straightforward way to see this is to examine the simple commitment rules of section 2.2. Note that \( k \) under commitment (see eq. (2.12)) reduces to unity when \( \rho = 0 \), and is larger when it is large. Hence, gains from commitment are increasing in \( \rho \). In the benchmark model with infrequent interventions, when \( \rho \) is roughly less than 0.17, benefits from commitment are not sufficient to outweigh the effects of demand shocks. However, this threshold is far below what is empirically plausible.

\(^{36}\)Persistence of \( g_t \) was not found to affect the ranking of the policies.
Figure 2.4: Comparison of Policies: Role of Persistence in Supply Shocks

Length of Price Stickiness

Duration of average price stickiness in the Calvo model is determined by the firms’ probability of not adjusting prices \( \text{duration} = \frac{1}{1-\theta} \). Thus, longer price stickiness decreases \( \lambda \) (see eq. 2.6), resulting in i) smaller sensitivity of inflation to output fluctuations (hence a smaller effect of demand shocks on inflation); and ii) less weight of output in the social loss function (see eq. 2.7). In light of the discussion above, both effects work to increase the desirability of the discrete adjustment policy (see Figure 2.5).
Figure 2.5: Comparison of Policies: Role of Price Stickiness

Labor Supply Elasticity

Perhaps the most controversial parameter in macroeconomics is the elasticity of labor supply, \( \frac{1}{\theta} \). Most empirical estimates based on micro level data suggest values of elasticity in the range of near zero to 0.5 (See, for example, Altonji (1986) or Domeij and Floden (2002)). On the other hand, most of the macro studies suggest quite the opposite (e.g. Woodford and Rotemberg (1992)). Prescott (2003) argues that a highly elastic labor supply is more plausible to account for cross-country variations in labor effort\(^{37}\). In the context of our model, when labor is inelastic (high \( \theta \)), firms’ marginal cost schedules are very steep, raising the impact of exogenous shocks on inflation and making output stability a priority. The opposite is true when the labor

\(^{37}\)In a more general setting other phenomena, such as sticky or efficiency wages, can also affect output sensitivity w.r.t. inflation. A low \( \theta \) may be partly a metaphor for them.
supply is highly elastic. In our baseline model infrequent adjustment is preferred when the elasticity exceeds roughly $0.03$ (see Figure 2.6), which is plausible from both camps’ perspectives.

### 2.4 Optimal Frequency of Policy Meetings

Next, we seek to characterize the optimal frequency of adjustment. This is done in two steps. First we develop a solution to a more general model where the central bank revises the interest rate every $T + 1$ periods. Then we choose the optimal $T$ that minimizes expected social loss. A complete solution to this model is presented in Appendix D.
2.4.1 Baseline Model

Figure 2.7 plots values of the social loss function for various $T$ in the benchmark model. The optimal frequency of policy meetings is once every six months ($T = 5$) or twice a year. With the optimal frequency of adjustment the central bank can realize over 50 percent of the gains obtainable under life-time commitment. To further illustrate the gains from moving to the optimal frequency of adjustment, Figure 2.8 presents simulated series under the optimal frequency of policy meetings. It shows that choosing the frequency optimally can achieve sizable gains in stabilizing inflation. Another observation is that the interest rate is more stable. This is interesting in light of the recent research on interest rate smoothing. Interest rate stability in this model
stems in part from lengthier duration of interest rate fixity and in part from the central bank’s ability to affect longer-term forecasts. The latter is related to the discussion in Woodford (1999): when monetary authority can affect longer term forecasts, it takes a smaller change in the interest rate to achieve the desired effect on output and inflation. Note, however, that smoothing in this model is different from what is implied by Woodford’s analysis. He examines the case of life-time commitment and therefore calls for frequent, but very small adjustment of the interest rate. The main message in Figure 2.8 is that when explicit long-term commitment is not feasible, smoothing occurs as a result of infrequent but somewhat larger (although not too large) changes in the interest rate.

Finally, Figure 2.9 explores how the optimal time between policy meetings varies with model parameters. Its interpretation is directly related to the discussion in section 2.3. Factors that decrease $\lambda$ (see eq. 2.6) lower the importance of demand shocks and output stabilization. Thus they raise the benefits of infrequent adjustment and imply longer optimal duration of inaction. More persistent supply shocks raise the importance of the predetermined component of long-term forecasts and also increase the optimal duration between the meetings. Interestingly, with higher persistence the optimal length of inaction appears to rise exponentially. Finally, higher volatility of demand shocks relative to supply shocks raises expected losses from inaction and works to increase the desirable frequency of monetary policy meetings.
Figure 2.8: Model Simulation Under Optimal Frequency of Adjustment
Figure 2.9: Optimal Frequency of Adjustment and Parameter Values
### 2.4.2 More Holidays for the FOMC?

To tailor our analysis to the case of the U.S. we consider an alternative calibration of some model parameters, the most important of which are those describing stochastic processes of exogenous shocks. We interpret cost-push shocks as exogenous markup variations arising from labor market imperfections. As in Clarida, Gali and Gertler (2001) we define the wage markup as the wedge between between the consumers’ marginal disutility of labor and their marginal return from labor\(^{38}\). The generalized wage markup can be expressed as:

\[
\mu^w_t = - \frac{U_{C_t}}{U_{L_t}} w_t 
\]

(2.32)

where \(- \frac{U_{C_t}}{U_{L_t}}\) is the inverse of the marginal rate of substitution between consumption and labor and \(w_t\) is the real wage rate. Assuming a period utility function of the form:

\[
U(C_t, L_t) = C_t^{1-\frac{1}{\varphi}} - L_t^{1+\vartheta}
\]

we have:

\[
\ln(\mu^w_t) = - \frac{1}{\varphi} \ln(C_t) - \vartheta \ln(L_t) + \ln(w_t)
\]

(2.33)

We construct the \(\ln(\mu^w_t)\) series from the previous equation using benchmark parameter values and quarterly U.S. data covering 1947:1-2004:2\(^{39}\). The cost push shock \(u_t\) is taken to be the Hodrick-Prescott (HP) filtered wage markup series. The AR(1) fit of the resulting \(u_t\) is:

\[
u_t = -0.00003 + 0.78 u_{t-1} + \hat{u}_t
\]

(2.34)

\(^{38}\)See also Gali et. al. (2001)

\(^{39}\)The data series are: \(C_t\) - real personal consumption expenditures (from BEA), \(L_t\) - hours in the nonfarm business sector (from BLS), \(w_t\) - real compensation per hour in the nonfarm business sector (BLS). Consumption and labor series were transformed into per capita levels using a measure of population obtained from GDP and GDP per capita series (BEA).
where the numbers in parenthesis are standard errors, and the standard deviation of the innovation ($\sigma_{\hat{u}}$) is 0.0125. Ignoring the intercept term, and translating to monthly frequency, the persistence parameter and the standard deviation of innovations are approximately 0.92 and 0.0072 respectively.

The demand shocks $g_t$ were constructed using the deviations of the share of government spending in GDP\footnote{The exact measure uses deviations of $g_t = -\ln \left(1 - \frac{G_t}{Y_t}\right)$ from the HP trend. Here $G$ - government current expenditures, and $Y$ - nominal GDP (both series taken from the BEA).} from the HP trend. The fitted process at the quarterly frequency is:

$$
g_t = -0.0004 (0.0016) + 0.74 g_{t-1} + \hat{g}_t (0.044) + 0.542 (0.0072)
$$

with $\sigma_{\hat{g}} = 0.0243$. Similarly, approximate values of shock persistence and standard
deviation of innovations are 0.9 and 0.0140 at the monthly frequency. Finally, we consider the possibility of lower duration of average price stickiness. At the lower end we take the average estimate in Gali et. al. (2001) of 2.35 quarters, or 7 months, and at the high end we take the commonly used value of 12 months. First, consider the lower-end value. Figure 2.10 plots loss functions under the four policies. It reveals that under alternative calibration the optimal choice of the frequency of policy meetings (once every six months) the central bank can gain about 75 percent of the total gains available under life-time commitment.

As an alternative, we also quantify these gains using the "inflation equivalent" measure: a permanent deviation of inflation from its target that generates the same welfare loss as a move from commitment to discretion. Dennis and Söderström (2002) use this measure to quantify gains from commitment in New Keynesian models. As they point out, the correspondence between "inflation equivalent" and the percentage reduction in the value of the loss function is not one-to-one, making this a useful alternative. The measure can be expressed as follows:

\[ \pi = \sqrt{(1 - \beta) \left( L_{\text{discretion}} - L_{\text{alternative}} \right)} \]

Under our calibration welfare gains from life-time commitment are equivalent to a permanent reduction in the deviation of inflation from its target by 2.96% \(^{41}\). On the other hand, a move from pure discretion to the optimal frequency of policy meetings is equivalent to a permanent reduction by 2.52%, generating 85% of the total possible gains measured using the "inflation equivalent".

\(^{41}\)This is consistent with Dennis and Söderström (2002). In their models/calibrations life-time commitment reduces inflation by 0.05% to 3.6%
Finally, the optimal duration between FOMC meetings under the alternative calibration and for various durations of price stickiness is presented in Figure 2.11. The result suggests that the FOMC should allow at least five months of no adjustment or, put differently, it should meet no more than twice a year.

2.4.3 Directions for Future Research

Considerations of clarity and simplicity led us to conduct the analysis above within a very simple and a purely forward looking model. This leaves a number of interesting extensions for future research. First, in light of Fuhrer (1997), Mankiw and Reis (2002) and many others it would be interesting to examine the optimal frequency of policy decisions in an environment where some agents are either backward-looking
or do not update their information set. Second, an analysis of the effects of model uncertainty on the optimal frequency of policy meetings would certainly increase our understanding of the proper policy design. Thirdly, central banks around the world use targets at different horizons. The exact horizon of the target is likely to affect the optimal frequency of policy meetings. Fourth, our analysis could be extended by explicitly introducing emergency/unscheduled meetings. Their presence is likely to lower inflationary expectations, requiring fewer scheduled meetings. Fifth, many central banks, and the Federal Reserve in particular, often use additional tools in affecting private sector expectations, such as FOMC bias announcements. Private sector and the media perceive bias announcements as indications of future policy changes. The latter clearly gives the Fed an extra leverage in influencing private sector forecasts.

The analysis of the optimal frequency of policy decisions from the standpoint of commitment could also be applied in other areas of macroeconomics, most importantly the fiscal policy. Klein and Rios-Rull (2003) show that partial commitment has important implications for the optimal fiscal policy. Our analysis could be applied to their problem to study the optimal length of commitment. More generally, examining the optimal frequency of adjusting various income, trade and other taxes could be of great interest to macroeconomists.

2.5 Conclusion

In this paper we have examined the issue of optimal frequency of monetary policy meetings. Viewing infrequent adjustment of monetary policy as simple short-term sequential commitments, we showed that it is preferred to period by period adjustment under discretion. Crucial in our argument is the finding that benefits from commitment spread to periods of central bank’s inaction. This happens because expectations of aggressive inflation stabilization at the time of policy adjustment contain inflation and mute effects of exogenous shocks in times when the central bank is on holiday. In addition we have provided a solution for the optimal frequency of policy meetings. Under a sensible calibration describing the U.S. economy the model prescribes holding FOMC meetings twice a year or less.
CHAPTER 3

Does the Federal Reserve Do What It Says ItEXPECTS to Do? \(^{43}\)

3.1 Introduction

A vast amount of resources is tasked to predicting future U.S. monetary policy. Speculators in financial markets forecast monetary policy to help place bets in interest rate futures markets. Businesses making investment decisions and homebuyers forecast monetary policy to inform the timing and maturity choices of their borrowing decisions. U.S. Treasury officials forecast monetary policy to construct projections of the cost of financing the Federal Public Debt.

It is less well recognized that the Federal Reserve forecasts its own future decisions. Changes in either the Fed policy rule or the indicators that appear in the rule make the Federal Reserve forecasting decision non-trivial. One reason that the U.S. central bank makes this forecast is to convey information to the public through the FOMC bias announcements.

In September 1983 the Federal Open Markets Committee (FOMC) began making a bias announcement at the same meeting where it chose the current federal funds target rate. The bias announcement states whether tightening, no change or easing

\(^{43}\)This chapter is based on our joint work with Bill Dupor and Tim Conley

68
is more likely in the inter-meeting period. The bias is a useful predictor of near term policy; it correctly predicts 85 of the subsequent 160 rate change decisions between September 1983 and January 2003. Payne (2001), Lapp and Pearce (2000) and Pakko (2003) also find that the bias announcement contains significant information for predicting the adjustment decision even after conditioning for typical macro variables (e.g. inflation and output). Table 3.1 tabulates the frequency of bias announcement with different magnitudes of the following meeting federal funds target rate adjustment. The most frequent bias announcement is neutral (43.7%) and the most frequent adjustment is zero (53.8%). The neutral bias correctly predicts no adjustment at the next meeting in 57.1% of the observations. Since inaction is the most common adjustment decision (86 out of 160), simply predicting no change would do as well as the bias announcement in terms of the number of correct predictions. Predicting no change, however, cannot forecast interest rate adjustments. The final column of Table 3.1 contains the mean of the next meeting adjustment conditional on the current bias announcement.

If bias reflects the Fed’s forecast of its own future actions, then it should contain useful information about the underlying policy rule. For concreteness, suppose the rule relates the federal funds target rate to the measures of inflation and economic activity. If bias indicates the direction of the expected next meeting policy action, then

---

44 In January 2000, the Fed formally changed the bias announcement language from the likely direction of future monetary policy into a risk statement about future inflation and economic activity. Financial market participants, however, continue to perceive the bias announcement as an indication of future monetary policy. In any case, our sample ends before this change was made.

45 Note that an easing bias is about as likely to be followed by a large rate cut-twenty five basis points or more-as by a small cut. Most of these large reductions occur early in the sample as the Federal Reserve completes its period of gradual disinflation. Inaction by the Federal Reserve has become more important of late. During the last ten years, 62% of FOMC meetings resulted in no rate change as opposed to 45% in the previous decade.
<table>
<thead>
<tr>
<th>Bias Announc.</th>
<th>Magnitude of Next Meeting Adjustment (in basis points)</th>
<th>Next Meeting Adjustment (Average, in basis points)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Less than -25</td>
<td>Between -25 &amp; 0</td>
</tr>
<tr>
<td>Easing</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>Neutral</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Tightening</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>25</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 3.1: Current Meeting Bias Announcement and Next Meeting Target Adjustment

It should be based on the Fed’s forecast of the future interest rate adjustment. Put differently, bias should reflect the Fed’s forecasts of the same measures of inflation and economic activity, modified by a one period lead. Moreover, if the Fed expects to follow the same interest rate rule at the next meeting, then bias announcements should be based on the same policy rule. A natural approach to testing whether the Fed does what it says it’s expecting to do (based on bias announcements) is to compare policy parameters implied by its interest rate adjustments and bias announcements. This is the theme of this paper: we jointly estimate policy rules implied by the FOMC’s twin announcements and examine the internal consistency between them.

The rest of the paper is organized as follows. In section 3.2 we describe discrete choice models of interest rate adjustment and bias formation which are estimated separately using the standard ordered probit procedure. In section 3.3 we present a structural bivariate discrete choice model with simultaneity of the two decisions. Section 3.4 discusses our findings. Section 3.5 concludes.
3.2 Standard Discrete Choice Estimation

We begin with a standard univariate ordered probit estimation of the interest rate adjustment and bias formation models. We assume that the FOMC’s rate adjustment decisions are based on its desired target which follows a Taylor-type rule$^{46}$:

$$R_t^* = \gamma_1 E_t (x_{t+j}) + \eta_t$$  \hfill (3.1)

where $R_t^*$ is the (latent) Fed’s desired rate, $x_t$ includes inflation and measures of economic activity and $\eta_t$ is a normally distributed (i.i.d.) shock. The interest rate adjustment decision $A_t$ takes on three values:$^{47}$

$$A_t = \begin{cases} 
1 & \text{if } R_t^* \geq \delta_2 \\
0 & \text{if } R_t^* \in (\delta_1, \delta_2) \\
-1 & \text{if } R_t^* \leq \delta_1 
\end{cases} \quad \text{rate increase no change rate decrease}  \hfill (3.2)$$

The bias announcement, on the other hand is based on the Fed’s expectation of the next period’s interest rate, which is given by :

$$E_t (R_{t+1}^*) = \gamma_2 E_t (x_{t+j+1}) + \zeta_t$$  \hfill (3.3)

The bias decision $B_t$ is formed as follows:

$$B_t = \begin{cases} 
1 & \text{if } E_t (R_{t+1}^*) \geq \alpha_2 \\
0 & \text{if } E_t (R_{t+1}^*) \in (\alpha_1, \alpha_2) \\
-1 & \text{if } E_t (R_{t+1}^*) \leq \alpha_1 
\end{cases} \quad \text{tightening bias neutral bias easing bias}  \hfill (3.4)$$

If the Fed bases both bias and adjustment decisions on the same policy rule, then $\gamma_2 = \gamma_1$. This hypothesis is the topic of this paper.

As a first step, we separately estimate models (3.1)–(3.2) and (3.3)–(3.4) in a standard univariate ordered probit framework. The explanatory variables in (3.1)

$^{46}$Throughout the paper we use superscripts $^*$ to indicate unobservable (latent) variables.

$^{47}$Dueker (1999) and Hu and Phillips (2002) estimate similar univariate models of interest rate adjustment. Dueker uses five thresholds corresponding to changes of various sizes. We limit our attention to three choices due to a small number of observations.
include lagged federal funds target rate and three-quarter ahead forecasts of inflation, output and (in a separate experiment) unemployment. The explanatory variables in (3.3) are the same, but taken with a one-period lead (i.e. four-quarter forecasts and current fed funds rate). The Fed’s actual forecasts are taken from the Greenbook reports. The latter makes the frequency of the data correspond to the FOMC meeting dates. Detailed description of the data is provided in section 3.3.

The estimates for the period between February 1986 and July 1998 are presented in Table 3.2. Note that the standard ordered probit procedure does not identify the scale and location of the latent variable. Hence, the signs and standard errors of the estimates in Table 3.2 are interpretable, but not their numerical values. First, consider the model without unemployment. Columns (i) and (iii) of Table 3.2 report parameter estimates for the interest rate adjustment and bias formation models respectively. In the adjustment model every coefficient is of expected sign. Greater inflation and output increase the probability of a rate increase, while a larger lagged target rate reduces the probability of a rate increase. However, the coefficient on inflation is not statistically different from zero. Moreover, for the bias decision, in column (iii), the inflation and interest rate coefficients are not of the anticipated sign. Both are statistically insignificant at a 5% level. This suggests an inconsistency between the bias and rate adjustment decisions. Expected three period ahead inflation increases the likelihood of a current interest rate hike, while expected four period ahead inflation decreases (or has no effect) on the likelihood of a forecast of an interest rate hike at the next meeting. A similar difference arises for the response to the interest rate.

In the second specification, we add expected unemployment to the right-hand side. The estimates are presented in columns (ii) and (iv). With the addition of
unemployment, the value of the loglikelihood function improves appreciably and every coefficient in both models is of the expected sign. This is supportive of consistency across the two decisions; however, the response in the bias equation to output is still statistically insignificant.

These findings will be expanded upon in the rest of the paper. Since the bias and adjustment decisions are made jointly, univariate estimation is inappropriate. Formal testing of cross equation coefficient restrictions should be based on joint estimation of the parameters. In addition, our theoretical hypothesis of consistency implies not only identical signs and significance of parameters across models, but also their numerical equality. We would also like to develop an economic interpretation of the elasticities implied by the statistical model.

### 3.3 Structural Discrete Choice Estimation

In this section, we structure our discrete choice model in a way to provide interpretable parameter estimates, that we can compare to standard interest rate rule estimation. The common approach is based on three assumptions. First, the central bank has a desired interest rate $R^*_t$, that depends on its forecasts of inflation and measures of economic activity:

$$R^*_t = \beta_1 E_t (x_{t+j}) + e_t$$  \hspace{1cm} (3.5)

where $e_t$ is an $i.i.d.$ policy shock. Second, at each policy meeting the FOMC computes the gap between the desired rate and the actual interest rate $R_{t-1}$ set at the previous meeting:

$$h^*_t = R^*_t - R_{t-1}$$  \hspace{1cm} (3.6)

Hereafter, we refer to $h_t^*$ as the gap. Finally, the new adjustment decision moves the interest rate closer to the desired rate:

$$R_t = R_{t-1} + (1 - \rho) h_t^*$$

(3.7)

where $\rho \in [0, 1]$ is a 'smoothing parameter,' which determines the fraction of the gap that the central bank closes. Most existing studies estimate (3.5)-(3.7) using quarterly or monthly data.

### 3.3.1 A Discrete Interest Rate Rule

In our analysis, we take into account the discreteness of the FOMC’s interest rate adjustment and bias announcement decisions. The central bank decides whether to leave unchanged, raise or lower the interest rate based on the current gap $h_t^*$. If $h_t^*$ is close to zero, the central bank decides that the current desired rate is insufficiently far from the last period interest rate to adjust. Hence it leaves the rate unchanged. If $h_t^*$ is sufficiently large (or small), the central bank raises (or lowers) the interest rate.

As before, the adjustment decision $\mathcal{A}_t$ takes on three values:

$$\mathcal{A}_t = \begin{cases} 
1 & \text{if } h_t^* \geq \delta_2 \\
0 & \text{if } h_t^* \in (\delta_1, \delta_2) \\
-1 & \text{if } h_t^* \leq \delta_1
\end{cases}$$

(3.8)

Note that $(\delta_1, \delta_2)$ is the inaction region for $h_t^*$ where the interest rate is not changed.

The bias announcement, on the other hand, states the likely direction of policy at the next meeting. Therefore, a tightening (easing) bias should be announced when the central bank expects to raise (lower) the interest rate at the next meeting. A typical estimates of $\rho$ are in the neighborhood of 0.8 at a quarterly rate (see e.g. Clarida et. al. (1999)).
neutral bias signals expected inaction at the next meeting. The bias equation \( B_t \) is:

\[
B_t = \begin{cases} 
1 & \text{if } k_t^* \geq \alpha_2 \\
0 & \text{if } k_t^* \in (\alpha_1, \alpha_2) \\
-1 & \text{if } k_t^* \leq \alpha_1
\end{cases}
\] (3.9)

where \( k_t^* \) represents the central bank’s forecast of the future gap:

\[
k_t^* = E_t R_{t+1}^* - R_t = \beta_2 E_t (x_{t+j+1}) - R_t + v_t
\] (3.10)

and \( v_t \) is an iid shock. The key difference between (3.8) and (3.9) is that the latter is based on the central bank’s expectation of the future gap \( k_t^* \). Consistency between bias and adjustment decisions requires \( k_t^* \) to be the optimal forecast of the future gap \( h_t^{*+1} \), which, in turn, requires the policy parameters in (3.5) and (3.10) to be equal. Thus, a null hypothesis that the bias decisions are consistent with interest rate adjustment decisions can be formulated as:

\[
H_0 : \beta_1 = \beta_2
\] (3.11)

### 3.3.2 Identification

Our underlying statistical model is the bivariate ordered probit. The error terms in the decision rules, \([e_t, v_t]\), are assumed to be jointly normally distributed with a correlation coefficient \( \theta \). Standard ordered probit estimation only identifies the marginal effects of the right-hand side variables on probabilities. It does not identify the scale and location of \( h_t^* \) and \( k_t^* \), which preclude an economic interpretation of the parameter estimates. We use our model-implied restrictions to arrive at interpretable coefficients.

First, consider the scale issue. Note that the specification of (3.6) and (3.10) imposes \((-1)\) coefficients on \( R_{t-1} \) and \( R_t \), respectively. This pins down the scale of \( h^* \) and \( k^* \). Intuitively, by modeling the FOMC decisions as based on differences between
the actual and the desired target rate we require the desired rate \( R^* \) to have the same scale as the actual rate \( R \). Second, given the scale, the ordered probit identifies only two of the three location parameters in each model (the constant term and the two inaction boundaries). In estimations we normalize the upper boundary of each inaction interval \( (\delta_2, \alpha_2) \) to zero, so that the estimates of the lower boundaries \( (\delta_1, \alpha_1) \) can be interpreted as magnitudes of inaction intervals.

### 3.3.3 The Data

Our dependent variables are the adjustment and bias decisions \( (A_t, B_t) \). The adjustment decision is constructed from the federal funds target rate \( R_t \). We use the FOMC’s bias announcements constructed by Payne (2001), which we updated until 2003. Hereafter, we refer to the bias announcement data as the bias.

We estimate a forward-looking rule with \( j = 3 \) using real time forecasts constructed by the Board of Governors of the Federal Reserve, known as the Greenbook forecasts. In the early 1980s, Board staff began preparing macro variable forecasts for use in setting the federal funds target rate. Approximately one week before each FOMC meeting, these classified forecasts were distributed to members of the FOMC. The set of forecasted variables include: real and nominal GDP and its components, CPI inflation, the unemployment rate, housing starts and the non-farm payroll employment.\(^{50}\) These variables are forecasted up to 10 quarters into the future and up to 12 quarters in the past.\(^{51}\) The forecast horizon, however, varies over the sample. We use observations from February 1986 when the CPI less food and energy was first included in the reports. Greenbook data becomes publicly available five years after its release.

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\(^{50}\)Variables included in each Greenbook vary in the sample.

\(^{51}\)Backwards forecasting is not always trivial because of data collection lags.
constructions. Thus, our sample ends in July 1998. Within the sample we have 100 observations with two lags and four period ahead forecasts. This data is constructed by the Federal Reserve Bank of Philadelphia and available at their website.

Our vector of explanatory variables includes the core CPI inflation rate $\pi_t$, the percentage deviation of real GDP from an HP trend $y_t$ and the unemployment rate $u_t$.\textsuperscript{52} We add unemployment rate for the reasons described in section 2.\textsuperscript{53}

### 3.4 Empirical Results and Discussion

Table 3.3 presents our estimation results. The first column contains nonlinear least squares estimates of (3.5)-(3.7). The second column presents restricted bivariate ordered probit estimates of our model when we impose the consistency restriction that $\beta_1 = \beta_2$. Finally, columns (iii) and (iv) report unrestricted bivariate ordered probit estimates of (3.8) and (3.9). Several observations stand out.

First, compare the least squares estimates (column (i)) with the restricted bivariate ordered probit estimates (column (ii)). All of the coefficients have the predicted signs and have reasonable values. The inflation coefficients are extremely close - 1.41 and 1.38 respectively. The output and unemployment variables are also similar, particularly after accounting for the size of the standard errors. Both estimation procedures deliver economically plausible response to the exogenous variables. In both cases the FOMC has a greater than one for one response to inflation (i.e. policy is active) and it increases the target rate in response to higher unemployment and to lower cyclical output. Moreover, the inflation and output coefficients are similar to

\textsuperscript{52} The use of a one-sided filter to de-trend output did not alter the results qualitatively.

\textsuperscript{53} Inclusion of other variables, such as unit labor costs, changes in nonfarm business inventories, industrial production index, etc. did not improve the model fit.
those proposed by John Taylor, 1.5 and 0.5 respectively. Next compare the parameter estimates implied by the adjustment decisions (column (iii)) with those implied by the bias announcements (column (iv)). We observe strong consistency in the policy response to inflation. We cannot reject the hypothesis that with respect to inflation, the Federal Reserve does what it says it’s going to do (p-value of 0.7425). The consistency of the bias and adjustment response to inflation is the most stable finding in all of our experiments.

Output coefficients, on the other hand, are quite different, but their standard errors are large. Despite the large difference in point estimates the likelihood ratio test could not reject the hypothesis that output coefficients are equal (p-value of 0.1187).

The biggest source of inconsistency is in the decisions’ response to unemployment. The implied latent target rate from the bias model responds much more strongly to changes in unemployment than the latent rate implied by the adjustment model. Here we can easily reject the hypothesis that the two coefficients are equal (p-value of 0.0006). Overall, the joint hypothesis (3.11) is also rejected (p-value of 0.0002).

Next, consider the estimates of the FOMCs inaction regions for the adjustment and bias decisions. These numbers are strikingly large. In column (ii), the bias in-action region is 0.023, or 230 basis points. Historically, the FOMC typically makes adjustments in twenty-five or fifty basis points.

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54 All hypotheses were tested using the likelihood ratio test.
55 This is true even after accounting for the Okun law relationship between output and unemployment.
56 Without the inclusion of unemployment in the bias and adjustment policies, the hypothesis that the rules are consistent is also easily rejected.
<table>
<thead>
<tr>
<th></th>
<th>Adjustment Equation (i)</th>
<th>Adjustment Equation (ii)</th>
<th>Bias Equation (iii)</th>
<th>Bias Equation (iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation, $E_t(\pi_{t+3})$</td>
<td>0.140 (0.108)</td>
<td>0.527 (0.144)</td>
<td>-0.222 (0.136)</td>
<td>0.439 (0.161)</td>
</tr>
<tr>
<td>Output, $E_t(y_{t+3})$</td>
<td>0.294 (0.090)</td>
<td>0.251 (0.084)</td>
<td>0.159 (0.113)</td>
<td>0.010 (0.096)</td>
</tr>
<tr>
<td>Unempl., $E_t(u_{t+3})$</td>
<td>- (0.135)</td>
<td>-0.532 (0.135)</td>
<td>- (0.181)</td>
<td>-0.970 (0.181)</td>
</tr>
<tr>
<td>$R_{t-1}$</td>
<td>-0.140 (0.064)</td>
<td>-0.380 (0.088)</td>
<td>0.126 (0.081)</td>
<td>-0.327 (0.104)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.003 (0.003)</td>
<td>-0.029 (0.008)</td>
<td>0.002 (0.003)</td>
<td>-0.058 (0.012)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.007 (0.003)</td>
<td>-0.039 (0.009)</td>
<td>-0.008 (0.003)</td>
<td>-0.068 (0.012)</td>
</tr>
<tr>
<td>Loglikelihood</td>
<td>-90.104</td>
<td>-81.739</td>
<td>-100.956</td>
<td>-84.511</td>
</tr>
</tbody>
</table>

Notes:
1. standard errors are reported in parentheses;
2. sample size is 100 observations;
3. Explanatory variables (except for $R$) are 3-quarter (4-quarter) forecasts in the adjustment (bias) model;
4. Variance of the normal distribution was chosen so as to restrict the size of the inaction intervals ($|\delta_2 - \delta_1|$ and $|\alpha_2 - \alpha_1|$) in each model to 100 basis points.

Table 3.2: Standard Ordered Probit Estimates
<table>
<thead>
<tr>
<th></th>
<th>LS (i)</th>
<th>Bivariate Restricted (ii)</th>
<th>Bivariate Unrestricted (iii) Adjustment (Bias) (iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.410</td>
<td>1.375</td>
<td>1.372                                         1.322</td>
</tr>
<tr>
<td></td>
<td>(0.138)</td>
<td>(0.127)</td>
<td>(0.140)                                       (0.139)</td>
</tr>
<tr>
<td>Output</td>
<td>0.337</td>
<td>0.439</td>
<td>0.545                                         0.205</td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(0.187)</td>
<td>(0.205)                                       (0.192)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>-1.377</td>
<td>-1.662</td>
<td>-1.349                                        2.142</td>
</tr>
<tr>
<td></td>
<td>(0.198)</td>
<td>(0.229)</td>
<td>(0.197)                                       (0.300)</td>
</tr>
<tr>
<td>$R$</td>
<td>0.785</td>
<td>-</td>
<td>-                                             -</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.040)</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>\delta_1</td>
<td>$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$</td>
<td>\alpha_1</td>
<td>$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.006)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>-</td>
<td>0.012</td>
<td>0.011                                         -</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>-</td>
<td>0.016</td>
<td>-                                             0.012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Corr(e, v)</td>
<td>-</td>
<td>0.421</td>
<td>0.562                                         -</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.125)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.977</td>
<td>-</td>
<td>-                                             -</td>
</tr>
<tr>
<td>Loglikelihood</td>
<td>-167.162</td>
<td>-</td>
<td>-155.559</td>
</tr>
</tbody>
</table>

Notes:
1. Standard errors are reported in parentheses;
2. Sample size is 100 observations;
3. Explanatory variables (except for R) are 3-quarter (4-quarter) forecasts in the adjustment (bias) equation. R stands for the lagged (current) federal funds target rate in the adjustment (bias) equation.
4. Constant estimated, but not reported.

Table 3.3: Monetary Policy Estimates
Table 3.4: Bivariate Model Predictions

<table>
<thead>
<tr>
<th>Adjustment Decisions</th>
<th>Bias Announcements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut</td>
<td>No action</td>
</tr>
<tr>
<td>No. of Actual Actions</td>
<td>24</td>
</tr>
<tr>
<td>No. of Correct Pred.</td>
<td>9</td>
</tr>
<tr>
<td>% of Correct Pred.</td>
<td>38</td>
</tr>
</tbody>
</table>

Note: Number of cases when both actions were predicted correctly is 29 out of 100.

This would suggest inaction regions of the same order. Our larger inaction region stems from our identifying assumption about the desired interest rate and the gap. Recall that \( h_t^* = R_t^* - R_{t-1} = \beta_1 E_t(x_{t+j}) - R_{t-1} + e_t \). Therefore, the lagged interest rate enters the desired gap only because the Fed is assumed to base its decisions on the difference between the latent desired rate and the actual rate. Suppose, on the other hand, that the desired rate itself depended on the lagged interest rate. For example, \( R_t^* = (1 - \rho) \left( \beta_1 E_t(x_{t+j}) \right) + \rho R_{t-1} + e_t \). In this case the coefficient on \( R_{t-1} \) in (6) would be \( (\rho - 1) \) instead of -1. If \( \rho = 0.785 \), the least squares estimate of smoothing from column (i), then the adjustment inaction region for the bivariate model would be \( (1 - 0.785) \cdot 0.023 \), or 49 basis points. Note that the estimates of \( \beta_1 \) and \( \beta_2 \) would be unaffected by this alternative identification assumption.

The inconsistency of the bias and adjustment responses to unemployment is difficult to explain in the context of a rational expectations model. Here, we discuss one potential explanation. First, note from Figure 3.1 that between 1986 and 1994, the federal funds rate exhibited strong response to unemployment. However, between 1994 and 1998, despite a continuous decline in unemployment, the Federal Reserve did not respond as strongly as in the previous years. If it used the natural rate hypothesis as a working model, then declines in unemployment were likely to forecast...
increases in inflation. As a way to combat this anticipated inflation and to convince financial market participants of a strong anti-inflation stance, the FOMC could have often announced a tightening bias in response to strong employment reports (see the upper panel of Figure 3.3). However, strong employment may have never given rise to markets’ inflation expectations (perhaps as a direct consequence of the tightening bias announcement) since average inflation continued to decline throughout the nineties. Thus the FOMC did not follow through with actual target rate increases (see the upper panel of Figure 3.3).
This explanation suggests that bias announcements may play an important role in monetary policy. If announcements of a tightening bias are capable of reducing private sector expectations of inflation (and hence overall inflation), then the Fed can remain conservative with respect to raising the interest rate but stand ready to announce a bias in the presence of inflationary pressures. This may explain why in Table 3.1 the tightening bias announcements (48 cases) are more prevalent than actual interest rate increases (28 cases). Our estimation results also corroborate the story. Table 3.4 shows that interest rate increases are more difficult to predict than tightening bias announcements.
Figure 3.2 plots the actual federal funds target rate $R_t$, the latent rate implied by the adjustment model estimates $\hat{R}_t^*$ and our estimate of the Fed’s forecast of the latent rate $\hat{E}_{t-1}R_t^*$ implied by the bias formation model. Despite the inconsistencies in parameter estimates the implied series for the interest rates are remarkably close. The standard deviation of the estimated Fed’s forecast error $(\hat{R}_t^* - \hat{E}_{t-1}R_t^*)$ is 64 basis points. Note, that the largest divergence between $\widehat{R}_t^*$ and $\hat{E}_{t-1}R_t^*$ occurs in the latest part of the sample, which is consistent with the post-1994 weak response to unemployment in Figure 3.1.

Finally, Figure 3.3 presents the actual and predicted FOMC decisions. Note that in the later part of the sample the expected gap implied by the bias model (see the lower panel in Figure 3.3) was well outside the inaction region, thus prompting the FOMC to repeatedly announce a tightening bias. On the other hand, the contemporaneous gap implied by the adjustment model is close to, but still within, the boundary of the inaction interval. Hence the Fed refrained from actual interest rate increases.

### 3.5 Conclusion

Summarizing our results: i) there is strong evidence that the interest rate and bias announcement are consistent with each other in response to inflation; ii) the evidence with respect to response to output is mixed; iii) the evidence with respect to unemployment points to considerable inconsistency; iv) the joint hypothesis that the adjustment and bias decisions between 1986 and 1998 were based on the same interest rate rule is rejected. The results (iii) and (iv) can be explained by a weak
Figure 3.3: Actual and Predicted FOMC Decisions
response of the federal funds rate to strong employment in the second half of the 1990s. The FOMC repeatedly announced a tightening bias in response to positive innovations in expected employment and output; at the same time, it repeatedly did not follow through with interest rate hikes.

It is worth noting that, for two reasons, our methodology would be difficult to extend to more recent data. First, we exploit the fact that each FOMC bias announcement was one of three outcomes. However, since 2001 the FOMC has often issued more nuanced statements after its meetings. For example, at its January 1996 meeting, the bias statement was explicit regarding expected policy actions: “Moderating economic expansion in recent months has reduced potential inflationary pressures going forward. With price and cost trends already subdued, a slight easing of monetary policy is consistent with contained inflation and sustainable growth.”

On the other hand, in the May 2003 FOMC press release, the paragraph concerning the policy bias states: “Although the timing and extent of that improvement remain uncertain, the Committee perceives that over the next few quarters the upside and downside risks to the attainment of sustainable growth are roughly equal. In contrast, over the same period, the probability of an unwelcome substantial fall in inflation, though minor, exceeds that of a pickup in inflation from its already low level. The Committee believes that, taken together, the balance of risks to achieving its goals is weighted toward weakness over the foreseeable future.” The FOMC had notably: (i) moved from setting an inter-meeting bias to a bias for the ‘next few quarters’ or ‘forecastable future’; (ii) separately discussed the likely outcomes for inflation and output in terms of both magnitudes and probabilities.
Furthermore, the FOMC statements have become less informative about future policy actions. In March 2003, for example, the FOMC statement includes no bias announcement: "In light of the unusually large uncertainties clouding the geopolitical situation in the short run and their apparent effects on economic decision-making, the Committee does not believe it can usefully characterize the current balance of risks with respect to the prospects for its long-run goals of price stability and sustainable economic growth."

These changes in the bias present a new research question: how does a more sophisticated announcement, that also details a larger set of policy determinants, affect monetary policy? Greater nuance may increase transparency and give insight into the FOMC decision-making process. Greater nuance may also increase confusion among financial market participants and the public. Of course, the two are not mutually exclusive.
APPENDIX A

Exporters Profits

Here we show how to equations (1.31)-(1.32) were derived.

A.1 PCP Profits

The price-setting problem is given by:

\[
\max_{p_{pt}} \pi_{t \rightarrow t+j}^{pcp} = E_t \sum_{j=0}^{1} Q_{t \rightarrow t+j} \left( \frac{P_{pt}^e H - W_{t \rightarrow t+j}}{P_{t \rightarrow t+j}} \right) \left( \frac{P_{pt}^e H}{e_{t \rightarrow t+j} P_{t \rightarrow t+j}^e H^*} \right)^{-\lambda} \left( \frac{P_{t \rightarrow t+j}^e H^*}{P_{t \rightarrow t+j}^*} \right)^{-\omega} (1 - \gamma) C_{t \rightarrow t+j}^* = \\
= (P_{pt}^e H)^{1-\lambda} \left( E_t \sum_{j=0}^{1} Q_{t \rightarrow t+j} e_{t \rightarrow t+j}^\lambda \Omega_{t \rightarrow t+j} \right) - (P_{pt}^e H)^{-\lambda} \left( E_t \sum_{j=0}^{1} Q_{t \rightarrow t+j} W_{t \rightarrow t+j} e_{t \rightarrow t+j}^\lambda \Omega_{t \rightarrow t+j} \right)
\]

where \( \Omega_{t \rightarrow t+j} = (P_{t \rightarrow t+j}^e H^*)^{\lambda-\omega} (P_{t \rightarrow t+j}^*)^\omega P_{t \rightarrow t+j}^{-1}(1 - \gamma) C_{t \rightarrow t+j}^* \). The re-set price is given by:

\[
P_{pt}^e H = \frac{\lambda}{\lambda - 1} \left( E_t \sum_{j=0}^{1} Q_{t \rightarrow t+j} W_{t \rightarrow t+j} e_{t \rightarrow t+j}^\lambda \Omega_{t \rightarrow t+j} \right)
\]

Plugging the optimal price into the profit function and re-arranging yields:

\[
\pi_{t \rightarrow t+j}^{pcp} = \frac{\lambda^{-\lambda}}{(\lambda - 1)^{1-\lambda}} \left[ E_t \sum_{j=0}^{1} Q_{t \rightarrow t+j} W_{t \rightarrow t+j} e_{t \rightarrow t+j}^\lambda \Omega_{t \rightarrow t+j} \right]^{1-\lambda}
\]
Taking the current period values outside of each bracket, we obtain:

$$\pi^\text{pcp}_t = \frac{\lambda^\lambda}{(\lambda - 1)^{1-\lambda}} \left( W_t e_t \lambda \Omega_t \right)^{1-\lambda} \left[ 1 + E_t \left( Q_{t,t+1} \left( \frac{W_{t+1}}{W_t} \left( \frac{e_{t+1}}{e_t} \right) \right)^\lambda \Omega_{t+1} \right) \right]^{1-\lambda}$$

To save space, denote: $B_t = \frac{\lambda^\lambda}{(\lambda - 1)^{1-\lambda}} W_t e_t \lambda \Omega_t$ and $\Phi_{t+1} = Q_{t,t+1} \Omega_{t+1}$. Expanding the expectational terms, we obtain:

$$\pi^\text{pcp}_t = B_t \left[ 1 + E_t \left( \left( \frac{e_{t+1}}{e_t} \right)^\lambda \right) E_t \left( \frac{W_{t+1}}{W_t} \Phi_{t+1} \right) + \text{cov} \left( \left( \frac{e_{t+1}}{e_t} \right)^\lambda , \Phi_{t+1} \right) \right]^{1-\lambda}$$

### A.2 LCP Profits

Next, re-do the same exercise for the case of LCP. The price-setting problem is given by:

$$\max_{P^e LCP^*} \pi^\text{lcp}_t = E_t \sum_{j=0}^1 Q_{t,t+j} \left( \frac{P^e LCP^*_{t+j} \Omega_{t+j}}{P_{t+j}} \right) \left( \frac{P^e LCP^*}{P^e LCP^*_{t+j}} \right)^{-\omega} \left( 1 - n \right) C_{t+j}^* =$$

$$= \left( P^e LCP^* \right)^{1-\lambda} \left( E_t \sum_{j=0}^1 Q_{t,t+j} e_{t+j} \Omega_{t+j} \right) - \left( P^e LCP^* \right)^{-\lambda} \left( E_t \sum_{j=0}^1 Q_{t,t+j} \right) \left( W_{t+j} \Omega_{t+j} \right)$$

Optimal price:

$$P^e LCP^* = \frac{\lambda}{\lambda - 1} \frac{E_t \sum_{j=0}^1 Q_{t,t+j} \Omega_{t+j}}{E_t \sum_{j=0}^1 Q_{t,t+j} e_{t+j} \Omega_{t+j}}$$
Plug optimal price into the profits:

\[
\pi_{lcp}^t = \frac{\lambda^{-\lambda}}{(\lambda - 1)^{1-\lambda}} \left[ \frac{E_t \sum_{j=0}^{1} Q_{t,t+j} W_{t+j} \Omega_{t+j}}{E_t \sum_{j=0}^{1} Q_{t,t+j} e_{t+j} \Omega_{t+j}} \right]^{1-\lambda} = \lambda^{-\lambda} \left( W_t \Omega_t \right)^{1-\lambda} \left( e_t \Omega_t \right)^{-\lambda} \left[ 1 + E_t Q_{t,t+1} \frac{W_{t+1} \Omega_{t+1}}{W_t \Omega_t} \right]^{1-\lambda}.
\]

Finally, expanding the expectational term:

\[
\pi_{lcp}^t = B_t \left[ 1 + E_t \left( \frac{w_{t+1}}{w_t} \Phi_{t+1} \right) \right]^{1-\lambda} \left[ 1 + E_t \left( \frac{e_{t+1}}{e_t} \right) E_t \Phi_{t+1} + cov \left( \frac{e_{t+1}}{e_t}, \Phi_{t+1} \right) \right]^{-\lambda}.
\]
APPENDIX B

Equilibrium in a Simple Model of One-Period Sequential Commitments

Here we describe a solution to a model where the central bank announces commitment to a new policy every other period. Assuming that the interest rate policy also neutralizes demand shocks \((i_t = \gamma u_t + \frac{1}{\phi} g_t)\) implies that the IS equation at the time of announcement is:

\[
x_t = -\varphi(\gamma u_t - E_t \pi_{t+1}) + E_t x_{t+1}
\]  

(B.1)

Since the rule is valid in the next period, rational expectation of the next period’s output is:

\[
E_t x_{t+1} = -\varphi \gamma \rho u_t + \varphi E_t \pi_{t+2} + E_t x_{t+2} \equiv -\varphi \gamma \rho u_t + f_{2t}
\]

where the two period ahead forecasts (summarized in \(f_{2t}\)) are taken by the CB as given. Similarly, next period’s inflation forecast is:

\[
E_t \pi_{t+1} = \lambda E_t x_{t+1} + \beta E_t \pi_{t+2} + \rho u_t \equiv -\lambda \varphi \gamma \rho u_t + f_{3t}
\]

where \(f_{3t} = \beta E_t \pi_{t+2} + \lambda f_{2t} + \rho u_t\). The last two expressions can be combined with the IS curve to obtain output as a function of exogenous shocks and long-term forecasts.
Using the expression for output to modify the Phillips curve yields:

\[ \pi_t = \lambda \left( 1 + \frac{\beta \rho}{1 + \varphi \lambda \rho + \rho} \right) x_t + f_{4t} + u_t \] (B.2)

where \( f_{4t} \) summarizes two period ahead forecasts. The extra output term appearing in (B.2) represents the effect of short-term commitment on next period’s inflation forecast. The output cost of reducing inflation falls by a factor of

\[ k_2 = \left( 1 + \frac{\beta \rho}{1 + \varphi \lambda \rho + \rho} \right)^{-1} \]

relative to pure discretion.

At the time of announcement, the central bank’s problem is to minimize (2.9) with \( T = 1 \) subject to (B.2). The first order condition can be expressed as:

\[ \pi_{1t}^c = -\frac{\alpha k_2}{\lambda} x_{1t}^c \] (B.3)

Note that \( k_1 < k_2 < 1 \), i.e. under short-term commitment output cost of reducing inflation is higher than under long-term commitments but lower than under pure discretion, as hypothesized earlier.

The minimum aggregate state vector in this model is the vector of exogenous shocks \( e_t = \{ g_t, u_t \} \). The restriction that the interest rate policy must neutralize the demand shocks implies that inflation and output must be functions of supply shocks only. With these in mind, we use the following definition of a rational expectations equilibrium.

**Definition 2** A rational expectations equilibrium is described by policy functions \( i_t(e_t), x_t(u_t), \) and \( \pi_t(u_t) \) such that:

1. Equations (2.2) and (2.3) are satisfied in all periods.
2. In periods of policy announcement, policies \( x \) and \( \pi \) must also satisfy (B.3).
3. In periods when no announcement is made, inflation and output are given by\(^{57}\):

\[ x_t(u_t) = x_{t-1}(u_t), \quad \pi_t(u_t) = \pi_{t-1}(u_t) \quad i_t(e_t) = i_{t-1}(e_t). \]

4. Private sector forecasts in (2.2) and (2.3) are consistent with the central bank’s policy:

\[ E_t x_{t+1} = E_t x_{t+1}(u_{t+1}), \quad E_t \pi_{t+1} = E_t \pi_{t+1}(u_{t+1}). \]

Given the linear-quadratic structure of the problem, the last condition requires that in announcement periods expected inflation and output are given by:

\[ E_t \pi_{t+1} = \rho \pi_t \quad \text{and} \quad E_t x_{t+1} = \rho x_t. \quad (B.4) \]

Moreover, since the central bank solves the same problem every announcement period it will always announce the same rule. Hence in equilibrium (B.2) and (B.3) must hold in all periods and policy rules \( i_t(\cdot) \), \( x_t(\cdot) \), and \( \pi_t(\cdot) \) are time-invariant. The solution is obtained by combining (B.2), (B.3) and (2.3):

\[
\begin{align*}
x_t^{1c} &= -\frac{\lambda}{ak_2(1-\beta \rho)+\lambda \rho} u_t \\
\pi_t^{1c} &= \frac{ak_2}{ak_2(1-\beta \rho)+\lambda \rho} u_t
\end{align*}
\]

\(^{57}\)This is equivalent to saying that the pre-announced policy is valid in non-announcement periods.
APPENDIX C

Unconstrained Optimum Under Limited Commitment

Here we describe the unconstrained optimum under long- and short-term commitment. The case of life-time commitment has been examined in CGG99. Recall that in that case the central bank chooses inflation and output to maximize the following Lagrangean:

\[
\mathcal{L}_1 = -\frac{1}{2}E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \alpha x_{t+i}^2 + \pi_{t+i}^2 + \frac{\lambda}{\alpha} \pi_{t+i} \right] \right\}
\]

The optimal policy (see CGG99) is described by:

\[
x_{t+i} - x_{t+i-1} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 1, 2, 3, \ldots
\]

and

\[
x_{t+i} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 0
\]

Next we turn to the case of partial commitment.

C.1 Partial (Short-Term) Commitment

Suppose that at time \( t \) the central bank announces a rule that is valid until \( t + T \). Before analyzing this case, note that the Phillips curve (2.3) can be re-written as:

\[
\pi_t = E_t \sum_{i=0}^{\infty} \beta^i (\lambda x_{t+i} + u_{t+i})
\]
Since the commitment is set to expire at a finite future date, the central can only manipulate private sector forecasts until the expiration date of the announced policy rule. Hence, the sequence of constraints from the date of the announcement until $t + T$ can be presented as:

$$\pi_t = E_t \sum_{i=0}^{T} \beta^i (\lambda x_{t+i} + u_{t+i}) + \beta^{T+1} E_t \pi_{t+T+1}$$

...............................

$$\pi_{t+j} = E_{t+j} \sum_{i=j}^{T} \beta^{i-j} (\lambda x_{t+i} + u_{t+i}) + \beta^{T+1-j} E_{t+j} \pi_{t+T+1}$$

...............................

$$\pi_{t+T} = \lambda x_{t+T} + u_{t+T} + \beta E_{t+T} \pi_{t+T+1}$$

where all forecasts of $\pi_{t+T+1}$ are taken by the central bank as given. The policy for inflation and output is chosen by maximizing the following Lagrangean:

$$\mathcal{S}_2 = -\frac{1}{2} E_t \left\{ \sum_{i=0}^{T} \beta^i \left[ \alpha x^2_{t+i} + \pi^2_{t+i} \right] + F_{1t} \right\} + \gamma_t \left( -\pi_t + \sum_{i=0}^{T} \beta^i (\lambda x_{t+i} + u_{t+i}) + \beta^{T+1} F_{2t} \right) + +.................................+$$

$$+ \gamma_{t+j} \beta^{j} \left( -\pi_{t+j} + \sum_{i=j}^{T} \beta^{i-j} (\lambda x_{t+i} + u_{t+i}) + \beta^{T+1-j} F_{2t} \right) +$$

.................................+

$$+ \gamma_{t+T} \beta^{T} \left( -\pi_{t+T} + \lambda x_{t+T} + u_{t+T} + \beta F_{2t} \right) \right\}.$$  \hfill (C.3)

where $F_{1t}$ summarizes expected losses beyond $t + T$ and $F_{2t} = E_t \pi_{t+T+1}$. The first order conditions are given by:

for $i = 0$:

$$\alpha x_{t+i} + \lambda \gamma_{t+i} = 0$$

$$\pi_{t+i} - \gamma_{t+i} = 0$$  \hfill (C.4)
for $T \geq i > 0$:
\[
\alpha x_{t+i} + \lambda (\gamma_t + \gamma_{t+1} + ... + \gamma_{t+i}) = 0
\]
\[
\pi_{t+i} - \gamma_{t+i} = 0
\] (C.5)

Re-arranging, yields the solution of the same form as under full commitment:
\[
x_{t+i} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 0
\] (C.6)
\[
x_{t+i} - x_{t+i-1} = -\frac{\lambda}{\alpha} \pi_{t+i}, \quad i = 1, 2, 3, ..., T
\] (C.7)

The only difference is that the commitment expires at time $t+T$, at which point the central bank re-optimizes and announces a new commitment. Since the structure of the problem faced by the "new" central bank at time $t+T+1$ is the same as at time $t$, it will announce the same policy. We use the following notion of equilibrium.

**Definition 3** A stationary limited commitment economic equilibrium is described by a set of policy rules \(\{i_j(s^j_t), x_j(s^j_t), \pi_j(s^j_t)\}_{j=0}^T\), where $j$ indicates position of period $t$ within the commitment cycle (0—time of announcement, $T$ — commitment expiration date) and $s^j_t$ summarizes the vector of states at time $t$ relevant at $j$’th stage of the commitment cycle\(^{58}\), such that:

1. Equations (2.2) and (2.3) always hold.

2. In periods of announcement the policy also satisfies (C.6).

3. In intermediate periods the policy also satisfies (C.7).

4. Private sector forecasts are consistent with the policy. That is:

   - In periods of announcement, forecasts in (2.2) and (2.3) are given by
   \[
   E_t x_{t+1} = E_t x_1(s^1_{t+1}) \quad E_t \pi_{t+1} = E_t \pi_1(s^1_{t+1})
   \]

\(^{58}\)As will become clear below for $j = 0$, $s^j_t = u_t$, while for $j > 1$, $s^j_t = \{s^{j-1}_t, u_t\}$. 

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In intermediate periods for $j = 1, \ldots, T - 1$, the forecasts in (2.2) and (2.3) are given by:

$$E_{t+j}x_{t+j+1} = E_{t+j}x_{j+1}(s_{t+j+1}^j) \quad E_{t+j}\pi_{t+j+1} = E_{t+j}\pi_{j+1}(s_{t+j+1}^j)$$

In the period of policy expiration, the forecasts in (2.2) and (2.3) are given by:

$$E_{t}x_{t+1} = E_{t}x_{0}(s_{t+1}^0) \quad E_{t}\pi_{t+1} = E_{t}\pi_{0}(s_{t+1}^0)$$

The cyclical nature of commitments implies different processes describing output and inflation in three types of periods: periods of policy announcement, periods of policy expiration and intermediate periods. In times of policy announcement, the following 3 equations describe the evolution of inflation, contemporaneous policy and the expected policy respectively:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t; \quad x_t = -\frac{\lambda}{\alpha} \pi_t; \quad E_t \pi_{t+1} = -\frac{\alpha}{\lambda} x_{t+1} + \frac{\alpha}{\lambda} x_t$$

Combining the three, we obtain the following difference equation in output:

$$x_t = \beta \frac{\alpha}{\alpha (1 + \beta) + \lambda^2 E_t x_{t+1}} - \frac{\lambda}{\alpha (1 + \beta) + \lambda^2} u_t \quad (C.8)$$

Similarly, in intermediate periods, the following 3 equations are valid:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t; \quad x_t = x_{t-1} - \frac{\lambda}{\alpha} \pi_t; \quad E_t \pi_{t+1} = -\frac{\alpha}{\lambda} x_{t+1} + \frac{\alpha}{\lambda} x_t$$

Combining them yields:

$$x_t = \frac{\alpha}{\alpha (1 + \beta) + \lambda^2 x_{t-1}} + \beta \frac{\alpha}{\alpha (1 + \beta) + \lambda^2} E_t x_{t+1} - \frac{\lambda}{\alpha (1 + \beta) + \lambda^2} u_t \quad (C.9)$$

Finally, in the last period, when the commitment expires, we have:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t; \quad x_t = x_{t-1} - \frac{\lambda}{\alpha} \pi_t; \quad E_t \pi_{t+1} = -\frac{\alpha}{\lambda} x_{t+1}$$
Hence:

\[ x_t = \frac{\alpha}{\alpha + \lambda^2} x_{t-1} + \beta \frac{\alpha}{\alpha + \lambda^2} E_t x_{t+1} - \frac{\lambda}{\alpha + \lambda^2} u_t \]  

(C.10)

To summarize, if we start with period \( t \) when a new commitment is announced, then output between periods \( t \) and \( t + T \) evolves according to:

\[
\begin{align*}
    x_t &= \beta \delta_1 E_t x_{t+1} - \frac{\lambda}{\alpha} \delta_1 u_t \\
    x_{t+1} &= \delta_1 x_t + \beta \delta_1 E_{t+1} x_{t+2} - \frac{\lambda}{\alpha} \delta_1 u_{t+1} \\
    x_{t+T} &= \delta_2 x_{t+T-1} + \beta \delta_2 E_{t+T} x_{t+T+1} - \frac{\lambda}{\alpha} \delta_2 u_{t+T}
\end{align*}
\]

(C.11)

where \( \delta_1 = \frac{\alpha}{\alpha(1+\beta)+\lambda^2} \) and \( \delta_2 = \frac{\alpha}{\alpha+\lambda^2} \). The system of equations above fully characterizes the evolution of output within each commitment 'cycle'.

The solution presented below proceeds as follows. First, note that under the optimal policy output does not respond to demand shocks (i.e. the interest rate policy neutralizes demand shocks as it does under pure discretion). Given the linear-quadratic nature of the problem, we can guess that at time \( t \) (beginning of the commitment cycle) the policy takes the following form: \( x_t = \omega_1 u_t \). Model stationarity implies that the policy at the beginning of the next cycle will have the same form: \( x_{t+T+1} = \omega_1 u_{t+T+1} \). Hence, at time \( t \) rational agents expect \( E_t x_{t+T+1} = \omega_1 E_t u_{t+T+1} = \omega_1 \rho^{T+1} u_t \). To solve the model, we can start with \( t + T \) and through backward substitution of the equations in (C.11) express each \( x_{t+j}, j = 0, 1...T \) as a function of \( E_t x_{t+T+1} \), past outputs and shocks. Note that the resulting equation for \( x_t \) will not depend on past output and therefore can be solved for \( \omega_1 \), which gives us the equilibrium output policy in periods of announcement. Output response in all other periods can be obtained recursively. Finally, inflation is obtained from the central bank’s F.O.C.’s.
C.2 Equilibrium in Announcement Periods

Starting with period $t + T$ in (C.11), we can recursively substitute out all forecasts between $t$ and $t + T$:

at $t + T$:

$$x_{t+T} = \delta_2 x_{t+T-1} + \beta \delta_2 E_{t+T} x_{t+T+1} - \frac{\lambda}{\alpha} \delta_2 u_{t+T}$$

which implies that $E_{t+T-1} x_{t+T} = \delta_2 x_{t+T-1} + \beta \delta_2 E_{t+T-1} x_{t+T+1} - \frac{\lambda}{\alpha} \delta_2 \rho u_{t+T-1}$. Using this in the equation for $t + T - 1$ yields:

$$x_{t+T-1} = \frac{1}{1 - \beta \delta_1 \delta_2} \left( \delta_1 x_{t+T-2} + \left( \beta \delta_1 \right) \left( \beta \delta_2 \right) E_{t+T-1} x_{t+T+1} - \frac{\lambda}{\alpha} \delta_1 \left( 1 + \beta \rho \delta_1 \right) \delta_2 u_{t+T-1} \right)$$

Continuing in this fashion, it is easy to see, that output throughout the commitment cycle can be represented as follows:

For $j \in [1, T]$:

$$x_{t+j} = A_j \left[ \delta_1 x_{t+j-1} + C_j E_{t+j} x_{t+T+1} - \frac{\lambda}{\alpha} \delta_1 B_j u_{t+j} \right] \quad \text{(C.12)}$$

and in the initial period ($j = 0$):

$$x_t = A_0 \left[ C_0 E_t x_{t+T+1} - \frac{\lambda}{\alpha} \delta_1 B_0 u_t \right] \quad \text{(C.13)}$$

where the coefficients can be computed recursively as follows:

$$A_j = \frac{1}{1 - \beta \delta_1 \delta_2 A_{j+1}}; \quad C_j = (\beta \delta_1) C_{j+1} A_{j+1}; \quad B_j = 1 + \beta \rho \delta_1 A_{j+1} B_{j+1} \quad \text{(C.14)}$$

with the terminal values of:

$$A_T = \frac{\delta_2}{\delta_1}; \quad C_T = \beta \delta_1; \quad B_T = 1$$
Having solved for the coefficients, we can obtain the equilibrium in periods of announcement. Using $x_t = \omega_1 u_t$ and $E_t x_{t+T+1} = \omega_1 E_t u_{t+T+1} = \omega_1 \rho^{T+1} u_t$ in (C.13) for $j = 0$ yields:

$$\omega_1 = \left(1 - A_0 C_0 \rho^{T+1}\right)^{-1} \left(-\frac{\lambda}{\alpha} \delta_1 A_0 B_0\right)$$

(C.15)

This gives a solution for output in periods of announcement. Inflation is obtained from the central bank’s first order condition. Since both inflation and output both depend on contemporaneous shocks only, their unconditional variances and expected social losses in periods of announcement are straightforward to compute.

### C.3 Equilibrium in Non-Announcement Periods

To obtain equilibrium in other periods we exploit the functional form of the solution in periods of announcement. Note that for any $j$, $E_{t+j} x_{t+T+1} = \rho^{T+1-j} \omega_1 u_{t+j}$. Use this to iterate (C.12) forward and express output in each period as a function of exogenous shocks only.

At time $t + 1$:

$$x_{t+1} = A_1 \delta_1 \omega_1 u_t + A_1 \left[C_1 \rho^T \omega_1 - \frac{\lambda}{\alpha} \delta_1 B_1\right] u_{t+1}$$

at $t + 2$:

$$x_{t+2} = (A_2 \delta_1) (A_1 \delta_1) \omega_1 u_t + (A_2 \delta_1) A_1 \left[C_1 \rho^T \omega_1 - \frac{\lambda}{\alpha} \delta_1 B_1\right] u_{t+1} + A_2 \left[C_2 \rho^{T-1} \omega_1 - \frac{\lambda}{\alpha} \delta_1 B_2\right] u_{t+2}$$

Continuing further we obtain the following representation for output for $j > 0$:

$$x_{t+j} = \delta_1^j \left(\prod_{i=1}^j A_i\right) \omega_1 u_t + \delta_1^{j-1} \left(\prod_{i=1}^j A_i\right) \left[C_1 \rho^T \omega_1 - \frac{\lambda}{\alpha} \delta_1 B_1\right] u_{t+1} + \delta_1^{j-2} \left(\prod_{i=2}^j A_i\right) \left[C_2 \rho^{T-1} \omega_1 - \frac{\lambda}{\alpha} \delta_1 B_2\right] u_{t+2} + \cdots + A_j \left[C_j \rho^{T-j+1} \omega_1 - \frac{\lambda}{\alpha} \delta_1 B_j\right] u_{t+j}$$

(C.16)
or:

\[ x_{t+j} = a_0^{(j)} u_t + a_1^{(j)} u_{t+1} + \ldots + a_j^{(j)} u_{t+j} = \sum_{k=0}^{j} a_k^{(j)} u_{t+k} \]  

(C.17)

where the superscript \( j \) indicates the coefficients’ dependence on \( j \). Using the central bank’s first order condition, we can express inflation as:

\[ \pi_{t+j} = -\alpha \lambda (x_{t+j} - x_{t+j-1}) = -\alpha \sum_{k=0}^{j-1} (a_k^{(j)} - a_k^{(j-1)}) u_{t+k} - \alpha a_j^{(j)} u_{t+j} = \sum_{k=0}^{j} c_k^{(j)} u_{t+k} \]  

(C.18)

Using these representations of inflation and output, computing unconditional variances of inflation and output in each intermediate period is straightforward. They are given by:

\[ Var(x_{t+j}) = \left( \sum_{k=0}^{j} \left( a_k^{(j)} \right)^2 \right) \gamma_0 + 2 \left[ \sum_{i=1}^{j} \left( \sum_{k=i}^{j} a_k^{(j)} a_{k-i}^{(j)} \right) \gamma_i \right] \]  

(C.19)

\[ Var(\pi_{t+j}) = \left( \sum_{k=0}^{j} \left( c_k^{(j)} \right)^2 \right) \gamma_0 + 2 \left[ \sum_{i=1}^{j} \left( \sum_{k=i}^{j} c_k^{(j)} c_{k-i}^{(j)} \right) \gamma_i \right] \]  

(C.20)

where \( \gamma_0 = E(u^2) \) and \( \gamma_i \)'s are \( i \)-th order autocovariances of \( u_t \). Finally, in each period within the commitment cycle, expected value of the loss function is given by:

\[ E(L_j) = \frac{1}{2} \left[ \alpha Var(x_{t+j}) + Var(\pi_{t+j}) \right] \]  

(C.21)

And total unconditional expectation of social losses is a simple average:

\[ E(L) = \frac{1}{1-\beta} \left( \frac{1}{T+1} \sum_{j=0}^{T} E(L_j) \right) \]  

(C.22)
APPENDIX D

Solution to a Model of Infrequent Policy Adjustment

Here we assume that the Central Bank fixes the interest rate at \( t \) until some future date \( t + T \). The logic of the solution is the same as before. First we would like to eliminate all endogenous forecasts until date \( t + T \). Then, we solve for the optimal policy at date \( t \) and back up the equilibrium in periods of inaction. Note, that by taking all forecasts beyond \( t + T \) as given, the central bank essentially minimizes:

\[
E_t(L) = \frac{1}{2} E_t \sum_{j=0}^{T} \beta^j (\alpha x_{t+j}^2 + \pi_{t+j}^2)
\]

subject to a sequence of \( T + 1 \) Phillips curve and IS constraints. To obtain the first order condition, we first obtain the impact of the current interest rate on each future forecasts of inflation and output. Since the forecasts beyond date \( t + T \) when the commitment expires are taken by the CB as given, we start from date \( t + T \) where we have:

\[
\frac{\partial E_t x_{t+T}}{\partial i_t} = -\varphi; \quad \text{and} \quad \frac{\partial E_t \pi_{t+T}}{\partial i_t} = \lambda \frac{\partial E_t x_{t+T}}{\partial i_t} = -\lambda \varphi
\]
At date $T - 1$ the impact is:

$$\frac{\partial E_t x_{t+T-1}}{\partial t_i} = -\varphi + \frac{\partial E_t x_{t+T}}{\partial t_i} + \varphi \frac{\partial E_t \pi_{t+T}}{\partial t_i} = -\varphi + \frac{\partial E_t x_{t+T}}{\partial t_i} (1 + \varphi \lambda);$$

and:

$$\frac{\partial E_t \pi_{t+T-1}}{\partial t_i} = \lambda \frac{\partial E_t x_{t+T-1}}{\partial t_i} + \lambda \beta \frac{\partial E_t x_{t+T}}{\partial t_i}$$

Continuing in this fashion and noting that $\frac{\partial E_t \pi_{t+j}}{\partial t_i} = \frac{\partial \pi_{t+j}}{\partial t_i} \equiv B_j \pi$ and $\frac{\partial E_t x_{t+j}}{\partial t_i} \equiv B_j x$, $T \geq j \geq 0$ we can see that the impact can be expressed in a recursive form:

$$B_j^x = -\varphi + B_{j+1}^x + \lambda \varphi \sum_{k=1}^{T-j} \beta^{k-1} B_{j+k}^x$$

$$B_j^\pi = \lambda \sum_{k=0}^{T-j} \beta^k B_{j+k}^x = \lambda B_j^x + \beta B_{j+1}^x$$

for any $j \in [0, T)$ and for $j = T$, the impact is

$$B_{t+T}^x = -\varphi; \quad \text{and} \quad B_{t+T}^\pi = -\lambda \varphi$$

Then the Central Bank’s first order condition can be expressed as:

$$E_t \sum_{j=0}^{T} \beta^j \left( \alpha x_{t+j} B_{t+j}^x + \pi_{t+j} B_{t+j}^\pi \right) = 0$$

(D.3)

**D.1 Modified Structural Equations**

To solve for the equilibrium we perform repeated substitutions to express each $E_t x_{t+j}$ and $E_t \pi_{t+j}$, $j \in [0, T]$ above as a function of the current period interest rate and the variables which the Central Bank takes as given: exogenous states $u_t$ and $g_t$ and time $t + T + 1$ forecasts of inflation and output. Each forecast in the equation above can be represented as:

$$E_t x_{t+j} = B_j^x i_t + A_j^x E_t x_{t+T+1} + A_j^\pi E_t \pi_{t+T+1} + A_j^{tg} g_t + A_j^{wu} u_t$$

(D.4)

$$E_t \pi_{t+j} = B_j^\pi i_t + A_j^x E_t x_{t+T+1} + A_j^\pi E_t \pi_{t+T+1} + A_j^{tg} g_t + A_j^{wu} u_t$$

(D.5)
where coefficients $B$ and $A$ are constants. In the same way as we derived the coefficients on $i$, it is possible to show that other coefficients can be computed recursively as follows:

Expected Inflation:

\[ A_{j}^{\pi} = A_{j+1}^{\pi} + \lambda \phi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{\pi} + \phi \beta^{T-j} \]

\[ A_{j}^{\pi} = \lambda \sum_{k=0}^{T-j} \beta^{k} A_{j+k}^{\pi} + \beta^{T-j+1} = \lambda A_{j}^{\pi} + \lambda \beta \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{\pi} + \beta^{T-j+1} \]  \hspace{1cm} (D.6)

Expected output:

\[ A_{j}^{x} = A_{j+1}^{x} + \lambda \phi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{x} \]  \hspace{1cm} (D.7)

Demand Shocks:

\[ A_{j}^{xg} = \mu^{j} + A_{j+1}^{xg} + \lambda \phi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{xg} \]  \hspace{1cm} (D.8)

Cost-Push Shocks:

\[ A_{j}^{xu} = A_{j+1}^{xu} + \varphi A_{j+1}^{xu} \]  \hspace{1cm} (D.9)

for any $j \in [0, T)$ and for $j = T$ the initial values are:

\[ A_{T+T}^{\pi} = \varphi; \quad \text{and} \quad A_{T+T}^{\pi} = \beta + \lambda \varphi \]

\[ A_{T+T}^{x} = 1; \quad \text{and} \quad A_{T+T}^{x} = \lambda \]

\[ A_{T+T}^{xg} = \mu^{T}; \quad \text{and} \quad A_{T+T}^{xg} = \lambda \mu^{T} \]  \hspace{1cm} (D.10)

\[ A_{T+T}^{xu} = 0; \quad \text{and} \quad A_{T+T}^{xu} = \rho^{T} \]  

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### D.2 Equilibrium in Period of Adjustment

The first order condition can be written as:

\[
\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} \begin{pmatrix} E_t x_{t+j} \\ E_t \pi_{t+j} \end{pmatrix} = 0 \tag{D.11}
\]

As before, we guess that at time \( t \) the response of inflation and output takes the form:

\[
\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = D \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{D.12}
\]

So that:

\[
\begin{pmatrix} E_t x_{t+T+1} \\ E_t \pi_{t+T+1} \end{pmatrix} = D P^{T+1} \begin{pmatrix} g_t \\ u_t \end{pmatrix}
\]

It follows that:

\[
\begin{pmatrix} E_t x_{t+j} \\ E_t \pi_{t+j} \end{pmatrix} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix} i_t + A_j \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{D.13}
\]

where:

\[
A_j = \begin{pmatrix} A_{jx}^{xx} & A_{jx}^{x\pi} \\ A_{j\pi x}^{xx} & A_{j\pi x}^{x\pi} \end{pmatrix} D P^{T+1} + \begin{pmatrix} A_{jx}^{xu} & A_{j\pi u}^{xu} \\ A_{j\pi x}^{xu} & A_{j\pi x}^{x\pi} \end{pmatrix} \tag{D.14}
\]

The first order condition becomes:

\[
\left[ \sum_{j=0}^{T} \beta^j \left( \alpha (B_j^x)^2 + (B_j^\pi)^2 \right) \right] i_t + \left[ \sum_{j=0}^{T} \beta^j \left( \alpha B_j^x B_j^\pi \right) A_j \right] \begin{pmatrix} g_t \\ u_t \end{pmatrix} = 0
\]

We can express the optimal interest rate as:

\[
i_t = - \left[ \sum_{j=0}^{T} \beta^j \left( \alpha (B_j^x)^2 + (B_j^\pi)^2 \right) \right]^{-1} \cdot \left[ \sum_{j=0}^{T} \beta^j \left( \alpha B_j^x B_j^\pi \right) A_j \right] \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{D.15}
\]

\[
i_t = C_1 \cdot (A_3 D P^{T+1} + A_4) \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{D.16}
\]
where

\[
A_3 = \left[ \sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B^x_j & B^\pi_j \end{pmatrix} A_{1j} \right]
\]

\[
A_4 = \sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B^x_j & B^\pi_j \end{pmatrix} A_{2j}
\]

To solve for equilibrium in periods when the interest rate is adjusted, we combine the previous equation with forecast equation when \(j = 0\):

\[
D \begin{pmatrix} g_t \\ u_t \end{pmatrix} = \begin{pmatrix} B^x_t \\ B^\pi_t \end{pmatrix} i_t + A_0 \begin{pmatrix} g_t \\ u_t \end{pmatrix}
\]

Hence, \(D\) must satisfy:

\[
D = B_0^x C_1 \cdot (A_3 DP^{T+1} + A_4) + (A_{10} DP^{T+1} + A_{20})
\]

or:

\[
D - \underbrace{(B_0^x C_1 A_3 + A_{10}) DP^{T+1}}_{=C_2} = \underbrace{B_0^x C_1 A_4 + A_{20}}_{=C_3}
\]

This is a system of 4 linear equations in coefficients of \(D\) which are straightforward to solve.

**D.3 Equilibrium in Periods of Non-Adjustment**

In periods when the interest rate is fixed \((0 < j \leq T)\), equilibrium is described by:

\[
x_{t+j} = -\phi i_t + E_{t+j} x_{t+j+1} + \varphi E_{t+j} \pi_{t+j+1} + g_{t+j} \quad \text{(D.17)}
\]

\[
\pi_{t+j} = \lambda x_{t+j} + \beta E_{t+j} \pi_{t+j+1} + u_{t+j} \quad \text{(D.18)}
\]

\[A_0 = A_{10} DP^{T+1} + A_{20}\]

59. \[A_0 = A_{10} DP^{T+1} + A_{20}\]
with \( T \geq j > 0 \). These equations can be also expressed as:

\[
x_{t+j} = B_j^x i_t + A_j^{xx} E_{t+j} x_{t+T+1} + A_j^{xu} E_{t+j} u_{t+T+1} + A_j^{xT} u_{t+j} + A_j^{xT+1} \mu^j
\]

and

\[
\pi_{t+j} = B_j^\pi i_t + A_j^{\pi x} E_{t+j} x_{t+T+1} + A_j^{\pi u} E_{t+j} u_{t+T+1} + A_j^{\pi T} u_{t+j} + A_j^{\pi T+1} \mu^j
\]

where the forecasts of \((t + T + 1)\) variables must be consistent with interest rate adjustment policy:

\[
\begin{pmatrix}
E_{t+j} x_{t+T+1} \\
E_{t+j} \pi_{t+T+1}
\end{pmatrix} = DP^{T+1-j} \begin{pmatrix}
g_{t+j} \\
u_{t+j}
\end{pmatrix}
\]

Solution for the interest rate can be expressed as

\[
i_t = \Psi e_t,
\]

where

\[
e_t = \begin{pmatrix} g_{t+j} \\ u_{t+j} \end{pmatrix}.
\]

Collecting terms, we obtain:

\[
\begin{pmatrix}
x_{t+j} \\
\pi_{t+j}
\end{pmatrix} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix} e_t + \begin{pmatrix} A_{1j} D P^{T+1-j} + A_{2j} (P^j)^{-1} \end{pmatrix} e_{t+j}
\]

Let:

\[
C_{5j} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix}
\]

\[
C_{6j} = A_{1j} D P^{T+1-j} + A_{2j} (P^j)^{-1}
\]

Hence:

\[
\begin{pmatrix}
x_{t+j} \\
\pi_{t+j}
\end{pmatrix} = C_{5j} e_t + C_{6j} e_{t+j}
\]

Unconditional variances can be expressed as:

\[
E x_{t+j}^2 = (c_{5j1} \Psi) \Omega (c_{5j1} \Psi)^\prime + (c_{6j1}) \Omega (c_{6j1})^\prime + 2 (c_{5j1} \Psi) \Sigma_j (c_{6j1})^\prime
\]

and

\[
E \pi_{t+j}^2 = (c_{5j2} \Psi) \Omega (c_{5j2} \Psi)^\prime + (c_{6j2}) \Omega (c_{6j2})^\prime + 2 (c_{5j2} \Psi) \Sigma_j (c_{6j2})^\prime
\]

where \(c_{5j1}\) and \(c_{5j2}\) are first and second elements of \(C_{5j}\), \(c_{6j1}\) and \(c_{6j2}\) are rows of \(C_{6j}\), \(\Omega\) is unconditional covariance matrix of \(e_t\) and \(\Sigma_j\) is the unconditional correlation matrix \(E(e_t e_{t+j})\).

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BIBLIOGRAPHY


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[68] Schenk, Andrea, 2003, Exchange Rate Pass-Through into Canadian Import Prices, mimeo, Queen’s University.


[71] Woodford, Michael, 1999, Optimal Monetary Policy Inertia, NBER working paper no. 7261