PROOF OF FEASIBILITY OF A FREE-SPACE OPTICAL CROSS-CONNECT SYSTEM USING DIGITAL MEMS

DISSertation

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By

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* * * * *

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We demonstrate two different systems: a quartic configuration and a binary con-
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For the binary configuration we design and simulate three different spot displace-
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From this evaluation we conclude that the roof prism SDD has the best performance
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An underpopulated quartic cell is simulated and experimentally analyzed. The
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function is to imitate the characteristics of a proper MEMS, except that it cannot be reconfigured in real time. We are able to control the output row by sending the beam to specific spherical mirrors as predicted. Because the optical elements are uncoated we found an experimental loss of 27.54 dB per output. If we assume coated optical components the theoretical loss is only 2.46 dB for seven bounces.

Another contribution of our work is to address the problem of beam coupling at the output plane for any White cell-based OXC. Our solution is based on curved a diffraction grating that takes advantage of a free-space architecture that makes it compatible with our OXC designs.
De una palabra a otra
lo que digo se desvanece
yo solo sé que estoy vivo
entre dos paréntesis

Octavio Paz
Toda historia tiene un inicio. El de esta tesis lo tiene hace cinco años cuando bajé de un avión con un par de maletas y sin tener la menor idea de lo que estaba haciendo en esta ciudad. La intención original era la de estudiar una maestría y regresar a México donde había dejado recuerdos que a la distancia se hacían pesados y a veces dolorosos.

Sin embargo algo sucedió, que después de terminar la maestría en lugar de hacer mis maletas y tomar el primer avión que me llevara a sur del Río Bravo decidí quedarme a escribir esto que ustedes tienen en sus manos.

Una de las razones por las que decidí postergar mi regreso a México se debe a todas aquellas personas que compartieron conmigo esta estancia en el midwest americano. Comenzando por la Dr. Betty Lise Anderson quien me motivó y alentó a que confiara en mis ideas. Al Dr. Roberto Rojas cuyas observaciones han hecho que la calidad de mi trabajo aumente. A mis compañeros de trabajo: Rashmi, Niru, Carolyn, Dave, Feras y Justin por haber hecho agradable el ir al laboratorio cada día. A Ohio State, CONACYT y Fulbright Comission quienes creyeron en mí y me dieron el apoyo necesario para estudiar el postgrado.

Sería mentira si dijera que los últimos cinco años han sido dedicados únicamente a la escuela. Sobra decir que más de un día de trabajo se perdió por culpa de una noche de juerga. Fue en esas noches en Larry’s donde tratamos de resolver el mundo.
Fue en esas noches cuando dimos la bienvenida a nuevos amigos y despedimos a los viejos, preguntándonos cuándo sería nuestro turno. Para los que están y para los que se fueron (María, Nicanor, Julio, Moriana, Jose, Pedro, Ariadna, Nacho, Thania, Guillermo, Mercedes y todos aquellos que se pierden en la memoria del alcohol), sólo deseo haberles podido dar aunque sea una décima parte de todo de lo que recibí de ustedes. ¡Salud!

Una mención especial se la dedico a mis padres. A ese par de viejos que no se cansan de apoyarme y que no dejan de creer en mí aún en esos momentos cuando yo dudaba. Un abrazo a mis hermanas a quienes extraño infinitamente y a Cata de cuya ausencia nunca he podido acostumbrarme.

Ahora que escribo estas líneas que intentan reflejar cierta sabiduría y autoreflexión no puedo dejar de voltear a mi lado y ver a esta mujer que ha aceptado ser mi esposa. Su risa inunda el cuarto y sus ojos capturan los míos. Si hay un logro del que esté orgulloso en estos años es el de saber que la siguiente etapa de mi vida esté con ella.

Querida Stephany más que dedicarte una tesis te dedico el resto de mis días.

Pero todo llega a su fin y esta tesis es el final de una etapa. El camino que sigue es largo, pero al menos he dado el primer paso.
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CHAPTER 1

INTRODUCTION

1.1 Motivation

The motivation for this dissertation comes from the problem that current optical communications are facing. The bandwidth of existing networks is limited by their electronic parts that prevent the use of the transmitting media at its full capacity. The decrease in transmission cost, combined with the capacities appearing at a network node as well as the new traffic loads imposed by the internet and data transmission, have caused a rethinking of network architectures. New architectures transparent to bit-rate, modulation format, and protocol need to be investigated. The current dissertation explores one possible solution for optical switching, i.e. directing a light beam from one input port to an output port without electrical conversion.

The objective of this dissertation is to prove the feasibility of a new design for an optical cross-connection (OXC) device based on combining Micro-Electro-Mechanical Systems technology (MEMS) with an optical system called the White cell. The attractive feature of our design is that it is a 3D design using a digital MEMS. This lead to a high number of ports and uses a simple control system for the MEMS.
We will analyze two different systems: a quartic configuration and a binary configuration. We will show that the binary configuration can be modified to implement several functions in addition to its switching capabilities. As examples we will show in appendix B a dynamic beam splitter that allows the division of several beams into up to 256 beam of equal intensity on section B.1, and also we will show in the same appendix how we can add a wavelength add-drop multiplexer (WADM) in a binary OXC system, section B.2.

The White cell will be described in detail in chapter 2. Briefly it is a set of three spherical mirrors with identical radii of curvature. The beams are refocused on each bounce to spots on one of the mirrors, and several beams can bounce simultaneously inside the White cell. We replace one mirror with a digital MEMS. Each micro-mirror on the MEMS has two or three different positions. When a beam is refocused on a micro-mirror it can be directed to different elements in the White cell. These elements can be used to perform the desired functions, such as cross-connection, add-drop multiplexing, and beam splitting.

The present chapter will introduce the basic concepts of optical networks and the effect of Wavelength Division Multiplexing (WDM) on the growth of data transmission in section 1.2.1. This growth in data transmission drives the creation of new designs for optical networks; one element of these new architectures is an optical cross-connect that redirects a beam from a specific input to another output. In section 1.2.2 we will explain some of the desired characteristics that an optical cross-connect must have, and show some of the optical cross-connect technologies presently available.
This chapter also presents the optical background needed to describe our optical cross-connect. Section 1.3 is dedicated to the description of the propagation of Gaussian beam through an arbitrary optical system. Finally, in section 1.4 aberration will be introduced, and its effects in beam propagation will be analyzed.

1.2 Communications Background

Optical fiber has been the transmission medium of choice for several years for metropolitan areas as well as for long-hauls. Optical fiber networks belong to the second and third generations of digital carrier networks. The first generation is the copper-based technology implemented in the 1960s, known as T1 or E1. The second generation systems, which use optical fibers, are called SONET (Synchronous Optical Network) or SDH (Synchronous Digital Hierarchy) and were introduced in the 1980’s. The third generation is called ASON (Automatically Switched Optical Network), also based on optical fibers, but while the second generation is still dependent on several optical/electrical/optical conversions (and the first generation being exclusively electrical), the third generation systems are expected to be all optical, in that they process optical loads, and do not need to convert the information to electrical signals for processing [1, 2, 3].

The development of communications networks is closely related to the kind of information that needs to be transmitted. First generation networks (T1) were designed to support voice traffic, and this kind of network was rapidly overcome by the need for data transmission. SONET networks allowed for the transmission of both voice and data, and these have become the standard in current communication networks.
Telecommunication services have been in increasing demand for several years, not only for services as voice and data transmission, but also to new services that appeared with the popularity of internet as real-time video, high-speed data transmission and music on-demand. The increase in users that solicit such services have pushed the current networks to the limit of the amount of bandwidth they can deliver. It is expected that data traffic will increase at a rate of 80% per year, while voice traffic will only increase in 10% per year [1]. It is obvious now that the trend is that data transmission will continue to be dominant over voice traffic. As high computer power and higher data transmission become available to end users, so the need for high bandwidth (or more precisely bandwidth on-demand) will be required.

Several ways exist to increase the allowed bandwidth; one way would be to increase the amount of installed fiber, which is costly due to the physical deployment of fiber in urban and rural areas. It is desired, instead, to take advantage of the already deployed fiber and increase its allowed bandwidth. A common way to increase the allowed bandwidth in a fiber is by allowing several wavelengths (instead of only one) to be transmitted simultaneously. Several wavelengths, each transmitting from a range of 10 to 40 Gpbs, would increase the information bandwidth by a factor as large as the number of wavelengths [1][2].

1.2.1 Wavelength Division Multiplexing

The optical technology that couples several wavelengths into the same fiber is called Wavelength Division Multiplexing (WDM). An example of WDM is shown in Figure[1.1] where $n$ fibers are connected to a WDM multiplexer which combines them into one fiber, and at the other end a demultiplexer separates each wavelength and
Figure 1.1: General Wavelength Division Multiplexing link

sends each one to an appropriate output. Several problems arise with an increased number of wavelengths, such as channel spacing, distortion, and nonlinear effects due to the amount of power coupled into the fiber [1].

To prevent different signals from interleaving with each other, the International Telecommunication Union (ITU) has published recommendations on the channel spacing between each wavelength. The most common spacing is referred to as the 100 GHz (or 0.8 nm) spacing. More dense multiplexing can be achieved with a 50 GHz (0.4 nm) or a 25 GHz (0.2 nm) spacing [4].

The range of wavelengths carried in a single fiber are divided into two different bands around the 1550 nm region. The C-band includes wavelengths from 1539.766 nm up to 1570.416 nm, and the L-band operates above the C-band in the 1574.37 nm to 1608.33 nm range [4]. There exist also the S-band (around 1480 nm) and the U-band (1650 nm); together with the C-band and the L-band these will cover a range of wavelengths from 1480 nm to 1650 nm.

A consequence of employing WDM is that it requires the development of many new enabling technologies including broadband optical amplifiers of high gain, integrated waveguided filters and multiplexers, WDM laser sources, high speed modulators, and
optical cross-connects (OXC) that provide the necessary switching capabilities at these rates of transmission.

It is the need for these new OXC systems that drives the research performed in this dissertation.

1.2.2 Optical cross-connects

In SONET systems switching has been performed entirely in electronics. In every switching node, optical signals are converted to electrical form (O/E conversion), buffered electronically, and then directed to the designated port before being converted back to an optical signal again (E/O conversion). As the network capacity increases, however, electronic switching nodes are unable to keep up with the load. Besides, electronic equipment is highly dependent on transmission rates and protocols, so any system upgrades result in addition or replacement of electronic switches, or both [5, 6, 7].

Optical cross-connects (OXC) have become a critical network element for constructing the next generation of optical network, where provisioning (reconfiguration), scalability and fast restoration will be needed [7].

- **Provisioning**, also know as reconfiguration, occurs when new data routes have to be established or existing routes modified. It may be desirable to provide bandwidth on-demand, where the available bandwidth can be allocated automatically to service requests in a matter of minutes, thus increasing network flexibility [6, 8].

- **Scalability** refers to the ability of a systems to be protocol-independent and data rate-transparent. It allows the number of wavelengths per fiber to continue
to grow without involving a major upgrade in the OXC technology. The desired port counts for OXCs are expected to be in the thousands \[7, 8, 9\].

- **Restoration** takes place where physical network failure occurs. A network switch needs to reroute traffic automatically in a time interval on the order of 100 ms, thus restoring operation of the network \[6\].

The main attraction of all-optical switching is that it enables routing of optical data signals without the need for conversion to electrical signals, and therefore is independent of data rate and data protocols. The limitations of optical component technology, i.e. lack of processing at bit level, and lack of efficient buffering in the optical domain \[5, 6, 7, 8, 9\], have largely limited optical switching to facility management applications. Several solutions are currently under research. We are particularly interested in those solutions involving MEMS technology, although we acknowledge other possibilities where fiber gratings \[10\], interferometry \[11\] and liquid crystal systems are involved \[12, 13\]. Micro-electromechanical systems (MEMS) are rapidly establishing themselves as the most attractive technology for optical switching since this technology allows for low-loss, large-port-count optical switching solutions at the lowest cost per port \[14\]. We divide the solutions based in MEMS technology into two main groups: 2D Digital MEMS and 3D Analog MEMS.

**2D Digital MEMS**

Digital switches integrated in a two dimensional array of micro-mirrors on a silicon substrate are referred as 2D MEMS. In these systems light propagates parallel to the substrate as shown in figure \[15\]. Depending on the MEMS the micro-mirrors can be tilted, rotated, or lifted into and out of the optical path to change the
Figure 1.2: General layout of a two dimensional OXC, after [16].

propagation direction of the optical beam from a particular input. Two dimensional digital switches have been shown to be capable of achieving quite good optical quality, particularly with respect to cross-talk, polarization and wavelength dependence, and bit-rate transparency [15, 16, 17, 18, 19]. An 8 x 8 research prototype switch with insertion loss as low as 1.7 dB has been demonstrated [19].

A significant advantage of this technology comes from its use of a simple binary control of the micro-mirror position; this results in simple control and short switching times (i.e. $\sim 1$ ms). A disadvantage comes due to the limited area of the silicon wafer where the micro-mirrors are fabricated, and to the insertion loss arising from divergence of the optical beam propagating in free space. While path length grows linearly with the number of ports, so does loss. The 2D architectures are found to be impractical beyond 32 input and 32 output ports [20, 21].
3D Analog MEMS

The most promising technology to date for very large-port-count OXC switches, with > 1000 input and output ports, uses a 3D analog MEMS, as shown in figure 1.3 [15]. With a 3D MEMS, a connection path is established by tilting two micromirrors independently, to direct the light from an input port to a selected output port. Each port is associated with one mirror on the MEMS array. Each micromirror can be tilted in an analog fashion, meaning that the micro-mirrors can be tilted to any position within a certain range. The optical beam from an input port passes through a lens and lands on the micro-mirror associated with that particular input port. This micro-mirror adjust its angle and redirects the beam to a micro-mirror on the second MEMS, associated with a particular output port. The second micro-mirror also adjusts its angle, and redirects the optical beam to the output port.

The main disadvantage of this technology is that of controlling the micro-mirror with the required angular precision. It is necessary to track the micromirrors angle constantly with a feedback system to achieve the required angular alignment [22].
In free-space optical switches, the insertion loss is highly sensitive to angular misalignment, as discussed in [20]. The optical path in 3D analog switches is in general longer than in 2D digital crossbar switches; therefore, the tolerance for angular misalignment precision becomes even tighter. Furthermore, in 2D digital crossbar switches, the switching motion is digital, so integrated or external alignment structures, as stops, can be incorporated to help define the angle of switch mirrors. Such approaches cannot be implemented in 3D analog switches, and controlling the angular precision of the micro-mirrors becomes a challenge.

1.3 Optics Background

Next we provide some optical background that will be useful to describe and better understand our optical cross-connection device. We will begin with a description of Gaussian beams, how they can be described by using ray matrices. We will also describe the main aberrations that exist in our systems, their origins and their graphic representation.

Light beams generated by laser sources can be described by a Gaussian intensity profile. We will describe how a Gaussian beam propagates through free space and how it is affected when transmitted through an arbitrary optical system.

The transformation of the beam through free space is defined by the spot size, \( w(z) \) at a particular point \( z \), and the wavefront radius \( R(z) \) at the same point. These are defined as:
\[ w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_r} \right)^2} \] (1.1)

\[ R(z) = z \left[ 1 + \left( \frac{z_r}{z} \right)^2 \right] \] (1.2)

where \( w_0 \) is the beam waist radius and \( z_r = \frac{\pi w_0^2}{\lambda} \) is the Rayleigh range, and \( \lambda \) is the beam wavelength. At the Rayleigh range, \( z = z_r \), the beam size \( w(z_r) = \sqrt{2}w_0 \), and \( R(z) \) is minimum.

For each point through the propagation path the combination of the respective \( w(z) \) and \( R(z) \) define a unique complex parameter, \( q(z) \), which is defined as:

\[ \frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{\pi w^2(z)} \] (1.3)

When we combined the \( q \)-parameter in the expression for the complex amplitude of a Gaussian beam given by \[23\]:

\[ U(\rho, z) = A_1 \frac{w_0}{w(z)} \exp \left[ -ik \frac{\rho^2}{2w^2(z)} \right] e^{-ikz} \] (1.4)

where \( A_1 \) is an amplitude, \( \rho^2 = x^2 + y^2 \) is the radial distance from the axis of the Gaussian beam, \( k = \frac{2\pi}{\lambda} \), and \( \lambda \) is the wavelength of the beam. It is possible to divide the complex expression in its real and imaginary parts:

\[ U(\rho, z) = A_1 \frac{w_0}{w(z)} \exp \left[ -ik \frac{\rho^2}{2w^2(z)} \right] \times \]

- Amplitude factor
- Radial phase
- Longitudinal phase
The transformation of the Gaussian beam when it propagates through free space from point $z_1$ to $z_2$, is set simply by the difference of $w(z)$ and $R(z)$ at those two points. For a given value of $\lambda$, variations of beam diameter and divergence with distance $z$ are functions of a single parameter. This is often chosen to be the beam waist radius $w_0$.

It is now desired to describe the transformation of a Gaussian beam when it propagates through an arbitrary optical system formed by different optical elements (e.g. lenses, mirrors, prisms). We can describe the propagation of a Gaussian beam through an optical system by the $ABCD$ law introduced by Kogelnik [24]. The $ABCD$ law relates the $q$-parameters $q_1$ and $q_2$ of the incident and transmitted Gaussian beams at the input and output planes by [23] [25]:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad (1.6)$$

The $A, B, C$ and $D$ elements in equation (1.6) are called the ray transfer matrix elements and they represents the optical system under study. The advantage of using the ray transfer matrix is that each element in the optical system has a particular ray transfer matrix associated to it. It is possible, then, to track any beam through any optical system, simply by multiplying in the correct order the matrices involved in the beam propagation [23] [26] [27].

Therefore, we can represent the transformation from the input to the output as a linear relationship given by the matrix equation [28]:

$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} \quad (1.7)$$
where the pair \((x, \theta)\) represents the position and slope of the input and output ray. It is necessary to say that each element on \(ABCD\) matrix has a particular meaning depending on its value \([28]\).

**D=0** In this case \(\theta_2 = Cx_1\), meaning that all the rays crossing the input plane at the same point will emerge at the output plane making the same angle with the optical axis, regardless of their input angle;

**B=0** This is the imaging case: \(x_2 = Ax_1\). This means that all rays passing through the input plane at the same point will pass through the same point at the output plane. In addition \(A\) will be the magnification of the system;

**C=0** In this case \(\theta_2 = D\theta_1\). All the rays entering the system parallel to each other will also emerge parallel at the output plane;

**A=0** In this case \(x_2 = B\theta_1\). All rays entering the system at the same angle will pass through the same point at the output plane.

As an example of how the \(ABCD\) matrix is created we assume a ray that travels a distance \(d\) in an homogeneous medium of refractive index \(n\). Because the medium is homogeneous, the ray travels in straight line. The set of equations that describe this translation are:

\[
x_2 = x_1 + \frac{d}{n} \tan \theta_1 \\
\approx x_1 + \frac{d}{n} \theta_1 \tag{1.8}
\]

\[
n\theta_2 = n\theta_1 \tag{1.9}
\]
These equations relate the output coordinates of the ray with its input coordinates. This transformation can be expressed in matrix form as [27]:

\[
\begin{bmatrix}
  x_2 \\
  \theta_2
\end{bmatrix} =
\begin{bmatrix}
  1 & d/n \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  \theta_1
\end{bmatrix}
\]

(1.10)

Similar expressions for a thin lens, spherical mirrors and other elements are found in appendix A.

We can now show how a Gaussian beam propagates through an arbitrary optical system by using $ABCD$ ray matrices. As an example we will describe the propagation of a Gaussian beam through the optical system shown in Figure 1.4. We will described this system using the $ABCD$ law. We assume that the systems have a Gaussian beam at the object plane with a radius of curvature of the wavefront $R_0 = \infty$ and a beam waist $w_0$. Therefore, the $q$-parameter at the object plane becomes: $q = q_0 = -iz_r = -i\pi w_0^2/\lambda$. 

Figure 1.4: Optical system
The 1:1 imaging system shown in figure 1.4 is described by the following \((\begin{vmatrix} A & B \\ C & D \end{vmatrix})\) matrix sequence:

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_3 \\ 0 & 1 \end{bmatrix}
\]

\(1.11\)

where the matrices represent the translation from the object plane to the first lens, the first lens, the distance between the first lens and the second lens, the second lens, the distance from the second lens to the image plane. Let us assume that the focal length of the first lens and the second lens are equal and that \(d_1 = d_3 = f\). This gives the following final system matrix:

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -2d_1 + d_2 & f^2 - 1 \end{bmatrix}
\]

\(1.12\)

Substituting equation \(1.12\) into equation \(1.6\), we get:

\[
q_2 = -\frac{q_1 f^2}{-2q_1 f + q_1 d_2 - f^2}
\]

\(1.13\)

To have a 1:1 magnification, so that \(q_2 = q_1\), the distance \(d_2\) is then \(d_2 = 2f\).

1.4 Aberrations

So far we have seen how any optical system can be described by using ray matrices. When we use ray matrices we are assuming to be working in a paraxial system; that is, the rays leaving the object travel at very small angles from the optical axis (< 5°). A consequence of the paraxial approximation is that all beams will be focused at the same point, called paraxial focal point. The question is then, what happens to the
Figure 1.5: Chief and marginal ray propagation through optical system

rays that go through or near a lens edge, away from the paraxial approximation? We may say that rays that are away from the paraxial approximation will focus at a different point from the paraxial focal point. We say then that we have an aberrated system. Geometrical aberrations are caused by the inability of the optical elements to form a perfect image at the paraxial focal point [29, 30, 31].

Figure 1.5 shows the chief ray in blue, which is the ray from the object point that passes through the center of the aperture stop and the center of both entrance and exit pupils. In order to find out if a system is aberrated we trace several rays from the object point up to the paraxial focal point (we show in the figure one marginal ray in red). Each ray will travel an equal distance to that of the chief ray. The end points of the rays will form a surface, called the system wavefront (real wavefront in the figure). The difference between the system and ideal wavefronts is the aberration of the system [26, 29, 32, 33].
The aberrations depend on the object height \( h \) and the pupil coordinates \( \vec{r} = (r, \theta) \), where the ray intersects the aperture stop. A power-series expansion of the aberration function in terms of these terms for the final position for an individual ray may be written in the form \[26, 33\]:

\[
y' = A_1 r \cos \theta + A_2 h + B_1 r^3 \cos \theta + B_2 r^2 h (2 + \cos 2 \theta) + 3 (B_3 + B_4) r h^2 \cos \theta + B_5 h^3 + C_1 r^5 \cos \theta + (C_2 + C_3 \cos 2 \theta) r^4 h + (C_4 + C_6 \cos 2 \theta) r^3 h^2 \cos \theta + (C_7 + C_8 \cos 2 \theta) r^2 h^3 + C_{10} r h^4 \cos \theta + C_{12} h^5
\]

\[x' = A_1 r \sin \theta + B_3 r^3 \sin \theta + B_7 r^2 h \sin 2 \theta + (B_9 + B_{10}) r h^2 \sin \theta + C_1 r^5 \sin \theta + C_3 r^4 h \sin 2 \theta + (C_5 + C_6 \cos 2 \theta) r^3 h^2 \sin \theta + C_9 r^2 h^3 \sin 2 \theta + C_{11} r h^4 \sin \theta\]

where \( A_n, B_n \) and \( C_n \) are constants.

The \( A_n \) terms in equation (1.14) represent the paraxial imaging conditions, where \( A_2 \) is the magnification of the object and \( A_1 \) is the distance from the paraxial focus to the image place. These terms are also called first order aberration, because the exponential order of \( r \) and \( h \) is one \[32, 33\].

The \( B_n \) elements are called the Seidel\(^1\) or third order aberrations. The spherical aberrations is described by \( B_1 \), \( B_2 \) is coma, \( B_3 \) astigmatism, \( B_4 \) is the curvature of

\(^1\)Ludwing von Seidel presented a series of papers between 1852 and 1856 where he developed a method for calculating aberrations. The method consisted of adding individual contributions of the optical elements involved in the optical system \[35\].
field aberration (or Petzval), and $B_5$ is distortion. These are the basic monochromatic aberrations defined by Seidel. For our system we will concentrate on coma, astigmatism and spherical aberrations [32] [34].

Finally, the $C_n$ elements represent the fifth-order aberrations. $C_1$ is spherical aberration. $C_2$ and $C_3$ are linear coma, $C_4$, $C_6$ and $C_7$ are the oblique spherical aberration. $C_7$, $C_8$ and $C_9$ are the elliptical coma elements. Astigmatism and curvature of field are described by $C_{10}$ and $C_{11}$ and $C_{12}$ is distortion [32] [33].

Table 1.1 shows the principal aberration terms of orders 1, 3 and 5. In general we will be concerned only with the Seidel aberrations (order 3), and within these the main concerns will be in spherical, coma and astigmatism.

<table>
<thead>
<tr>
<th>order</th>
<th>Aberration name</th>
<th>Aberration term $y'$</th>
<th>Aberration term $x'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Defocus</td>
<td>$A_1 r \cos \theta$</td>
<td>$A_1 r \sin \theta$</td>
</tr>
<tr>
<td></td>
<td>Magnification</td>
<td>$A_2 h$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Spherical</td>
<td>$B_1 r^3 \cos \theta$</td>
<td>$B_1 r^3 \sin \theta$</td>
</tr>
<tr>
<td></td>
<td>Coma</td>
<td>$B_2 r^2 h(2 + \cos 2\theta)$</td>
<td>$B_2 r^2 h \sin 2\theta$</td>
</tr>
<tr>
<td></td>
<td>Astigmatism</td>
<td>$3B_3 r h^2 \cos \theta$</td>
<td>$B_3 r h^2 \sin \theta$</td>
</tr>
<tr>
<td></td>
<td>Petzval</td>
<td>$3B_4 r h^2 \cos \theta$</td>
<td>$B_4 h^3$</td>
</tr>
<tr>
<td></td>
<td>Distortion</td>
<td>$B_5 r^3 \cos \theta$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Spherical</td>
<td>$(C_2 + C_3 \cos 2\theta) r^4 h$</td>
<td>$(C_5 + C_6 \cos 2\theta) r^3 h^2 \sin \theta$</td>
</tr>
<tr>
<td></td>
<td>Linear coma</td>
<td>$(C_4 + C_6 \cos 2\theta) r^3 h^2 \cos \theta$</td>
<td>$C_9 r^2 h^3 \sin \theta$</td>
</tr>
<tr>
<td></td>
<td>Oblique spherical</td>
<td>$(C_7 + C_8 \cos 2\theta) r^2 h^3$</td>
<td>$C_{11} r h^3 \sin \theta$</td>
</tr>
<tr>
<td></td>
<td>Elliptical coma</td>
<td>$C_{10} r h^4 \cos \theta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Petzval/Astigmatism</td>
<td>$C_{12} h^5$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Classification of aberrations
1.4.1 Spherical Aberration

Spherical aberration can be defined as the variation of focus with aperture. Figure 1.6 [24] shows a sketch of a simple lens forming an “image” of an axial object point a great distance away. The spherical aberration can be noticed by comparing the focal point of the rays as their height increases from the optical axis. The rays close to the optical axis come to a focus very near the paraxial focus position. As the distance from the optical axis to the ray increases (at the left of the figure), the position of the ray intersection with the optical axis moves away from the paraxial focus. The distance from the paraxial focus to the axial intersection of the ray is called longitudinal spherical aberration. Transverse, or lateral, spherical aberration is the name given to the same aberration when it is measured in the vertical direction [32, 34]. In the image plane (paraxial focus), where the rays should meet, they are in fact spread over the plane, causing blurring of the image.

Figure 1.6: Spherical Aberration. The rays farther from the optical axis converge sooner. From [24]
For a given aperture and focal length, the amount of spherical aberration in a simple lens is a function of object position and the shape of the lens. Depending on the system characteristics it may be better to use a biconvex lens (e.g. 1:1 magnification), a plano-convex lens (e.g. collimation) or a different shape. Thus, by choosing a different shape of the lens (while maintaining the focal length) it is possible to reduce the amount of spherical aberration. Another method of reducing the spherical aberration is by splitting the optical power of a lens into two or more elements \[36\].

1.4.2 Coma

When rays are oblique to the lens, it is possible to observe a change in the magnification of the image. Rays passing close to the edge of the lens are imaged with a different magnification from those rays that pass close to the center of the lens. As a result a comatic system will form a sharp image in the center, while presenting a blurred image on the edges \[32\].

In Figure 1.7(a)\[29\], we show a collimated bundle of rays that is incident obliquely onto a bi-convex surface. Note that the bundle of rays is displaced from the normal to the surface. As can be seen the upper ray is incident onto the surface at a very wide angle. This angle of incidence decreases as we get closer to the normal axis. Thus, the rays are imaged at different heights on the image plane, producing coma. The shape of the image of a point formed by a system with coma has the shape of a comet, as shown in Figure 1.7(b), thus the name of this particular aberration \[34\].

Coma is a particularly troublesome aberration since its shape is non-symmetrical. It is hard to determine the image position since it is much more difficult to locate the center of a coma patch than for a circular blur such as that produced by spherical
aberration. Coma varies with the shape of the lens element and also with the position of any apertures or diaphragms that limit the bundle of rays forming the image.

1.4.3 Astigmatism

Astigmatism occurs when the focus for rays in the vertical plane (i.e. tangential rays) and the horizontal plane (i.e. sagittal rays) do not coincide. In the presence of astigmatism, the image of a point source is not a point, but takes the form of two separate lines as shown in Figure 1.8[29]. Between both astigmatic foci the image of a point is blurred and it takes the shape of an ellipse; half-way between the two foci is the point-of least-confusion where the blurred point takes the shape of a circle[32,36]. Astigmatism can be formed either by a lens with different radii of curvature in different planes (e.g. a cylindrical lens), or by optical elements that are tilted or off-axis.
The amount of astigmatism in a lens is a function of the power and shape of the lens and its distance from the aperture that limits the size of the bundle of rays passing through the lens \cite{32,34,36}.

### 1.4.4 Graphic Representation of Aberrations

It is common to represent the effects of aberrations in a graphical representation. There are a number of graphical techniques for representing them. One of the most common representations used is the ray-intercept curve, which represents image-space displacement as a function of object-space fractional coordinates \cite{29,34}. The shape of the ray-intercept curve not only indicates the amount of aberration present in the system, but also the different kind of aberrations that are present. Figure 1.9 shows the formation of the ray intercept curve for spherical aberration.

Each curve of Figure 1.10 represents a different kind of aberration (or a combination of different aberrations). The ordinate for each curve is the height at which
the ray intersects the (paraxial) image plane. The ray-intercept curve for third-order spherical aberration, shown in Figure 1.10(a), is simply a cubic function, flat near the origin and increasing rapidly for large positive and negative apertures. The only difference between this case and that of the higher orders is that the higher the order, the flatter is the curve near the origin.

The curve for third order coma is shown in Figure 1.10(b). This aberration curve is a parabola (“even-function” curve). The aberration curves for higher order coma are similar to those of third order coma, but the meridional curves are much flatter, as is the case for spherical aberration.

Finally because astigmatism, Figure 1.10(c), causes the tangential and sagittal images to be sharp but displaced from each other, the ray-intercept curves are straight lines, tilted differently for meridional and sagittal rays.

When there is more than one aberration involved, the curves takes characteristics of the aberrations presented, as is the case of the graphics shown in Figure 1.10(e) and (f) that correspond to spherical aberration and coma, and astigmatism and coma, respectively.
Another common graphic representation for astigmatism is the field plot, where a set of principal rays at different object heights are traced. This creates a set of image displacements. Then the differences between the paraxial image plane and the tangential and sagittal foci are calculated. This is shown in Figure 1.11, where we can see that the difference between the sagittal and tangential foci increases with increasing object point height. In general two curves are shown one representing the sagittal and tangential image position. If both curves overlap, then no astigmatism is present. The horizontal scale shows the magnitude of the displacement.
1.5 Advances since Master’s Degree

This dissertation is an extension of the research topic developed during a Master’s degree also at The Ohio State University. In the master thesis, the main objective was to explore and develop the theoretical basis of a Binary OXC. The binary OXC combines the White cell with spot displacement devices (SDDs). We had to develop several SDD systems that shifted the position of each individual beam to different outputs.

An important point of this dissertation is that even though the binary and quartic configurations had been studied before at OSU for true-time delays (TTDs), this will be the first use of either of these configurations for optical cross-connects. Some of the SDDs proposed in the earlier Master’s theses are also to be developed in this dissertation.

Another new contribution is the concept of the beam-combiner, which will be explained in detailed in chapter 3. Here we will just mention that a basic problem.
in White cell based OXCs is how to couple light back into a fiber. We present several approaches that may be implemented in our OXC, we leave the experimental implementation and more detail analysis of these beam combiners for future work.

1.6 Organization

This dissertation describes the investigation and the physical implementation of a free-space optical switch based on the White cell for four inputs and eight outputs (4x8). Different optical systems are investigated, considering particularly the influence of the aberrations. An underpopulated quartic cell is experimentally analyzed. These experiments demonstrates the feasibility of our proposed system. This dissertation is divided into four main chapters.

In chapter 2 we describe the free space optical cross-connection using the White cell. We pay special attention in the quartic and binary configurations. We describe several SDD designs in terms of alignment tolerances that are used in the binary configuration.

The description of the beam combining problem with our system and possible solutions are described in chapter 3. The system presented is proposed as topic for a future research.

In chapter 4 we describe the alignment procedure and experimental setup for an underpopulated quartic cell. Our system allows a 4x8 switching. We described the system in term of number of ports, losses and output beam quality. Also we present several simulations for a binary cell using different designs of SDD.

Chapter 5 concludes this thesis with a summary of the most important results and conclusions.
CHAPTER 2

THEORY: OPTICAL CROSS-CONNECT BASED ON THE WHITE CELL

In this chapter we investigate the conditions and limitations of an optical switching system based on the White cell. The key parameters of the study are the number of possible outputs as a function of the number of bounces, the losses and aberrations and the effects of these in the beam quality involved in the system. Several system configurations will be presented and evaluated. A complete description of the basic elements will be given.

2.1 Description of the White Cell

Our optical cross-connection device is based on the White cell. John White first proposed the White cell in 1942 [37, 38]. The White cell is commonly used in spectroscopy, where it has been used for decades to cause a light beam to pass multiple times through a gaseous sample in order to measure the absorption of low-pressure gases. It has been adapted for many applications including optical delay lines [39], and spectroscopy.

Figure 2.1 shows the basic elements of the White cell. Light is continuously refocused between three identical spherical mirrors. The center of curvature of mirror
M lies on the optical axis. Mirrors B and C are mounted across from mirror M and separated from it by a distance equal to the radius of curvature $R$ of all three mirrors. Therefore mirrors B and C image the surface of mirror M onto itself whereas mirror M images B and C onto each other.

The centers of curvature of mirror B and C (CC(B) and CC(C)) are located on mirror M at a distance $\delta$ left and right of the optical axis, respectively. Hence the centers of curvatures are separated by $2\delta$.

![Figure 2.1: Basic White cell configuration](image)

The path of a beam through the White cell can be seen in Figure 2.2. Figure 2.2a shows how light enters the White cell through an input turning mirror located adjacent to mirror M. Light is focused to a spot on the input turning mirror. Light diverging from this input spot will propagate toward mirror C and then be refocused by mirror C onto mirror M. The input spot is located at a distance $d_1$ below the mirror C’s center of curvature. The first image of the spot will, therefore, be located on mirror M at an equal distance $d_1$ above of C’s center of curvature. Then in figure
we show how light is then reflected off mirror M and propagates toward mirror B. From here, light goes back to mirror M. Since the first image was at a distance $d_2$ above of mirror B’s center of curvature, the second image will appear below it at the same distance $d_2$. From here light will propagate again towards mirror C and the cycle will start again. As long as the dimensions of mirror B and C are large enough to contain 99.99% of the input beam, light can be imaged back and forth between them with negligible diffraction losses from the mirrors’ edges. Therefore the losses in the system are caused only by the mirrors’ reflectivity.

Figure 2.2: Top view of a White cell. a) Light entering through input turning mirror images onto M via mirror C; b) light is sent to mirror B and is refocused again in M.

This multiple-reflection configuration will result in a spot pattern on the surface of mirror M. The spot pattern formed on the front of mirror M is very predictable, depending on the location of the centers of curvature of mirror B and C. Figure 2.3 shows the sequence of spots on mirror M for two particular input spots. The spots in the figure are numbered in the order in which the light “bounces” in the White
cell before finally imaging onto an output turning mirror placed adjacent on top of mirror M.

The spacing between the spots is directly related to the distance $2\delta$ between the centers of curvature of mirrors B and C. The total number of spots on mirror M is therefore dependent on the spot spacing and the overall size of mirror M. We emphasize that the spot locations depend entirely on the alignment of the two Mirrors B and C. This will become important when we replace Mirror M with a MEMS in Section 2.2, as the spots are made to land on specific pixels.

![Figure 2.3: Spot pattern for two inputs in the White cell](image)

We now will substitute a flat mirror and a field lens for mirror M (mirror M’). This mirror will be later replaced by a MEMS micromirror array. This substitution can be seen in Figure 2.4. In order to have a beam bouncing back and forth among the spherical mirrors and the flat mirror, some imaging conditions have to be fulfilled. These imaging conditions can be stated as follows:

- Mirror M’ images into itself via mirror B or mirror C.
Figure 2.4: Substitution of Spherical mirror M for a flat mirror and lens

- Mirror B images onto mirror C via the mirror M’, and viceversa.
- The beam spot size at mirror M’ is constant, with 1:1 magnification.

A detailed solution of these conditions is shown in appendix A. Here we just mention that these conditions are established by solving the ray matrix equations, and summarize briefly. The first condition is expressed as:

\[ \text{Image}_1 = T_1L_1T_2M_{b/c}T_2L_1T_1 \]  \hspace{1cm} (2.1)

where \( T_1, L_1, T_2, \) and \( M_{b/c} \) represent the ray matrices for the distance between mirror M’ and the field lens, the field lens, the distance between the field lens and spherical mirror B (or C), and the spherical mirror B (or C), respectively. Similarly, the second
imaging condition is expressed as:

\[ \text{Image}_2 = T_2 L_4 T_1 T_1 L_1 T_2 \] (2.2)

Finally the third condition is fulfilled by solving the previous conditions assuming a 1:1 magnification. A more detailed explanation is given in appendix A.

2.2 White cell OXC configurations

There are several designs for optical cross-connections based on the White cell. In this section we will review some of them, we will describe their basic performance, and compare each system based on the number of outputs as a function of the number of bounces. In general we will describe three systems: The linear White cell OXC, a quartic OXC, and a binary White cell OXC.

2.2.1 Linear White cell OXC

The simplest of these is called a “linear cell,” because the number of outputs obtainable is proportional to the number of times a given beam bounces in the White cell. The linear White cell design is shown in Figure 2.5a. In this design we substitute mirror M’ of Figure 2.4 with a MEMS, which consists of an array of tip/tilt micro-mirrors, each with two stable positions ±\(\theta\) with respect to the MEMS normal. The linear White cell employs three spherical mirrors: A, B, and C as shown in Figure 2.5a. Additionally there is a field lens in front of the MEMS. Mirror A is located along an axis at 0° with respect to a normal axis from the MEMS plane, and mirrors B and C are at \(-2\theta\) and \(+2\theta\) with respect to the same normal, respectively.

In this system, a beam is introduced into the cell and then bounces between the MEMS and spherical mirrors. Each time it strikes the MEMS, the pixel at the
corresponding spot can be tipped in either direction. If the mirrors are all at $-\theta$, the beam bounces between mirrors A and B, and the spot traces out a pattern on MEMS surface like that of the open circles in Figure 2.5b.

If, however, a pixel is switched to $+\theta$, a beam coming from A goes to Mirror C instead of Mirror B. The center of curvature of Mirror C is aligned slightly differently, above the center of curvature of mirror B, so that the spot is re-imaged to a new location, in the same column but in a different row. The pixel at this new location is

Figure 2.5: Linear White cell OXC. a) OXC configuration; b) Bounce pattern
also switched to $+\theta$, returning the spot to Mirror A. The new spot pattern is shown using filled circles, and we see that the beam emerges from the White cell in a new location, output 1 (we are assuming that the rest of the micro-mirrors are tipped to $-\theta$.)

On the lower left part of Figure 2.5a the transition diagram is shown. This diagram shows which transitions are allowed for each spherical mirror. So, light can bounce between mirror A and Mirror B, and between mirror A and mirror C, but there is no allowed transition between mirrors B and C. It is noted, however, that if the MEMS micro-mirrors are allowed to have a $0^\circ$ state (i.e. flat), then the transition between B and C is allowed.

Every time a beam is sent to mirror C it is shifted one more row. Each new row represents, then, a new output. The maximum number of outputs will depend on the number of bounces $m$ by the following relation:

$$N_{\text{outputs}} = \frac{m}{2} + 1$$ (2.3)

Such a design is not practical for a realistic interconnection, however, because the number of bounces required is larger than the number of outputs. Since each bounce will cause additional loss, we want to produce the highest possible number of outputs for the lowest number of bounces.

2.2.2 The Quartic OXC device

Let us consider what might be achieved using tip/tilt micromirror array in which each pixel has three stable states and let us take these states to be $0^\circ$ (flat), $+\theta$, and $-\theta$. 
Figure 2.6 shows how we might assemble several White cells.

Here, we place our null cell mirrors (A and B) on the axis normal to the MEMS mirror plane, but one is directly above the other. When the pixels are flat, light goes back and forth between A and B, creating an unshifted spot pattern, hence “null cell”. If a pixel is tipped to $+\theta$, light reflects off the MEMS mirror at an angle of $2\theta$, and we place two more spherical mirrors (D and C) here. Similarly, we place two mirrors (E and F) along the $-2\theta$ axis, for when the pixel is tipped to $-\theta$.

Figure 2.6 also shows the allowed transitions. In this case, light from D can go to B (pixel at $+\theta$), or to Mirror F (pixel flat). Similarly light can go from C to A or C to E. As can be seen, light always go from an upper mirror to a lower mirror and vice versa. It is also not possible for light to go directly from C to D nor from E to F.
We observe that there are two loops in the connectivity diagram, and that each loop is closed. Let us assign half of our bounces to take place in the ACEB loop, and the other half in the AFDB loop. As mentioned before, mirror A and B produce no shift, thus both are considered to be the null cell. On the other hand mirrors C, D, E and F each produce a different specific shift that will be chosen depending on the number of times a beam is allowed to bounce on each of them. We can visit either upper mirror on every other bounce, which we allow up to $m/4$ times. Thus we assign the shift of Mirror C to be one row, and the shift produced by Mirror E to be $(m/4 + 1)$ rows. Since each of these can be visited up to $m/4$ times, the maximum possible shift that can be obtained in $m$ bounces using just these two mirrors is:

\[
N_{C,E} = \frac{m}{4} \left( \frac{m}{4} + 1 \right) + \frac{m}{4} (1)
= \left( \frac{m}{4} \right)^2 + 2 \left( \frac{m}{4} \right)
\]  

(2.4)

Next, we align the next mirror, Mirror F to produce a shift of one more than the maximum produced by C and E together, or:

\[
F \implies \left[ \left( \frac{m}{4} \right)^2 + 2 \left( \frac{m}{4} \right) + 1 \right]
\]  

(2.5)

Therefore, the maximum number of shift produced using only mirrors A, B, C, E and F is:

\[
N_{ABCEF} = \left( \frac{m}{4} \right)^3 + 3 \left( \frac{m}{4} \right)^2 + 3 \left( \frac{m}{4} \right)
\]  

(2.6)

Spherical mirror D produces a shift of one column more than equation (2.6), or
\[ D \implies \left[ \left( \frac{m}{4} \right)^3 + 3 \left( \frac{m}{4} \right)^2 + 3 \left( \frac{m}{4} \right) + 1 \right] \quad (2.7) \]

The total number of shifts (possible outputs) is obtained by visiting each mirror \( m/4 \) times. We must use at least one bounce to go through the null cell to get from one loop to the other. This means we can visit one mirror one time fewer. We choose to visit Mirror C less often to allow the transition. Since C produces a shift of one, the total number of possible outputs that can be accessed in \( m \) bounces is

\[
N_{\text{quartic}} = \left( \frac{m}{4} \right) \left[ \left( \frac{m}{4} \right)^3 + 3 \left( \frac{m}{4} \right)^2 + 3 \left( \frac{m}{4} \right) + 1 \right] + \left( \frac{m}{4} \right) \left[ \left( \frac{m}{4} \right)^2 + 2 \left( \frac{m}{4} \right) + 1 \right] + \frac{m}{4} \left( \frac{m}{4} + 1 \right) + \left( \frac{m}{4} \right) \quad (1)
\]

\[
= \left( \frac{m}{4} \right)^4 + 4 \left( \frac{m}{4} \right)^3 + 6 \left( \frac{m}{4} \right)^2 + 4 \left( \frac{m}{4} \right) - 1
\]

\[
= \left[ \left( \frac{m}{4} + 1 \right)^4 - 2 \right] \quad (2.8)
\]

This number is quartic in \( m/4 \). Twelve bounces will address 254 outputs. This number is quite significant, nevertheless, we will describe still another configuration, the binary White cell OXC. The binary cell, in addition to a considerable number of outputs it also give us the ability to add more features, like add-drop wavelength or beam splitting, to our OXC.

### 2.2.3 Binary White cell OXC

We now propose a new architecture in which we can create spot shifts with using a White cell with a two-state (or three-state MEMS), using a Spot Displacement Device (SDD), without the need to include more spherical mirrors.

We adopted an architecture proposed by Collins and Anderson [39], originally for optical true time delay devices for phased array antennas. In this section we will
describe how the White cell is modified to control the output location. Figure 2.7 shows the design for the binary White cell device. Mirror M, from the original White cell of Figure 2.1 has been replaced with a MEMS micro-mirror array and a field lens, Lens 1. On either side of the MEMS are placed two flat mirrors (Auxiliary Mirrors I and II), whose functions will be described shortly. Each of the auxiliary mirrors also has a field lens to simulate a spherical mirror. There are in addition four spherical mirrors on the right. Instead of having the centers of curvature of the spherical mirrors on the MEMS, the centers of curvature are placed outside the MEMS.

We take the possible micromirror tip angles to be $\pm \theta$. Mirrors A and B are placed one above the other, along an axis at $-\theta$. Mirrors E and F are also placed one above
the other along an axis at $+3\theta$. The center of curvature of the lens associated with
the MEMS is placed on the MEMS' normal; the center of curvature (labelled $CC_{LI}$)
of the Auxiliary Mirror I and Lens 2 is placed between mirrors A and B, and similarly,
the center of curvature $CC_{LII}$ of Auxiliary Mirror II and Lens 3 is placed between
mirrors E and F, as shown in figure 2.7.

Let's assume an input beam going from the MEMS plane is sent to mirror A, for
example after bounce 1. Light coming from this spot is imaged to a new spot on
Auxiliary Mirror I, in the column labelled “2” at the far left of Aux. Mirror I in
Figure 2.7 From there the light is reflected to mirror B, which sends the light back
to the MEMS at a new location, which will land in the column labelled “3”. If the
micromirror at that spot is set to $-\theta$, then the light is sent back to mirror A again.
So, Mirrors A and B form a White cell with the MEMS, Lens 1, Auxiliary Mirror I,
and Lens 2.

Suppose the pixel that this beam strikes on bounce 3 is tipped to $-\theta$. The light
returns to Auxiliary Mirror I and makes a spot in column 4. If, on the other hand,
that MEMS pixel at bounce 3 is instead turned to $+\theta$, then light coming from mirror
B to this micromirror will be reflected from the MEMS at an angle of $+3\theta$ with
respect to the normal to the MEMS plane. Recall there are two more mirrors along
that axis, E and F. So, light coming from B will go to E.

When light goes to mirror E the light is sent to Auxiliary Mirror II, and it forms
a spot in column 4 of that mirror. From there the light is sent to the lower mirror F,
and then back to the MEMS plane. Therefore Mirror E and F form another White
cell. If the next micromirror in the MEMS is tilted to $-\theta$, the beam from F is sent
again to the AB White cell (specifically to mirror A). If the micromirror at this point
had been tilted to $+\theta$, the light coming from F would have been reflected at $+4\theta$, a direction that is not being used in this design, and the beam is lost.

Thus, according to the connectivity diagram shown in the lower left-hand corner in Figure 2.7, light can bounce continuously (and exclusively) between the MEMS and Auxiliary Mirror I via Mirror A and B, a situation that doesn’t occur while bouncing through E and F. Light going to Auxiliary Mirror II gets there via Mirror E and returns to the MEMS via mirror F. From here light must go to Auxiliary Mirror I. Therefore, light returning from Auxiliary Mirror II needs four bounces to go back to Auxiliary Mirror II.

An input beam can be sent to Mirror A from the MEMS plane every even-numbered bounce, and to Mirror E every fourth bounce (i.e. 4, 8, 12). The odd-numbered bounces always appear on the MEMS, and the even-number spots can appear either on Auxiliary Mirror I or Auxiliary Mirror II. The light can be sent to Auxiliary Mirror II on any particular even-numbered bounce, but if the light is sent there, four bounces are required before it can be sent there again.

Now, to create a cross-connect we replace Auxiliary Mirror II with a device that shifts a spot down by some number of rows. The shifting of spots will eventually result in beams leaving the device at different locations. The spots will be shifted by what we call a Spot Displacement Device (SDD). Briefly we will mention here that we will divide Auxiliary Mirror II into columns, but now we assign one SDD column for every four bounces. The number of pixels by which a beam is shifted will be different for each SDD column. That is, each SDD column will shift a beam by a distance equal to twice that of the shift produced by the previous column. One column will
produce a shift of $\Delta$, the next column a shift of $2\Delta$, the next after a shift of $4\Delta$ and so on, thus producing a binary system.

It is important to mention that we will refer as “shift” to any change in position in the SDD and MEMS plane. This shift will be in general a multiple of the micromirrors pitch $\Delta$. Therefore the SDD will cause a shift of $\Delta, 2\Delta, 3\Delta$, etc. We make this difference to make clearer to the reader the relation between the shift caused by the SDD and the MEMS pitch. In general we will say that any change caused by the SDD will cause a shift in spot position that eventually can add up to a will cause a spot displacement of $x\Delta$, where $x$ is any real integer number.

By shifting the spots, we can control at which row any given beam reaches the output turning mirror, and we associate each row with a different output. The number of possible outputs is determined by the total number of possible shifts for a given number of bounces. In the design of Figure 2.7 a shift is made every time the light goes to the SDD, but this can only happen once every four bounces. Thus the number of outputs $N$ is given by:

$$N_{\text{binary}} = 2^{m/4}$$

(2.9)

where $m$ is the number of bounces.

We present a 12-bounce system in Figure 2.8 to illustrate the operation. Here we can see three different beams incident on the input turning mirror. The MEMS, Auxiliary Mirror I, and the SDD are divided into a grid of eight rows (for eight possible output locations) and seven columns (for each bounce on the MEMS). Each region on this grid on the MEMS is a group of eight micromirrors, so that each beam lands on a different micromirror on each bounce. Each beam, therefore, can be independently directed either to the SDD or to Auxiliary Mirror I on each bounce. The number
of columns on the SDD \((m/4)\) will determine the number of possible outputs; the other columns are not used. Every four bounces allows for a shift, so 12 bounces will produce \(2^{12/4} = 8\) different outputs for each input.

Figure 2.8: Spot pattern for multiple inputs in the binary White cell-based OXC

Initially all three input beams start in row zero. Remember that according to the connectivity diagram of Figure 2.7, an input can only go to the EF White cell every fourth bounce (those would be the 4th, 8th and 12th bounces).
Let’s assume that we need to send the “white” beam to the fifth output. Therefore we will need to send the “white” beam to the SDD on the second and tenth bounces, which correspond to displacements of $4\Delta$ and $\Delta$ respectively. The “white” beam starts bouncing in the AB White cell (i.e. the micromirrors on the MEMS are tilted to $-\theta$ position), until the second bounce at which the micromirror is tilted to $+\theta$ and the beam is sent to the SDD. Then the “white” beam goes through the SDD, which for that particular column has a value of $4\Delta$, it will send the output back to the MEMS on the fourth output instead of the zeroth output. We then keep bouncing the “white” beam in the AB White cell, until the tenth bounce, when we again send the input to the SDD. Now, the beam will land in the column with the value of $\Delta$, where it goes through the SDD and is shifted by an additional distance $\Delta$. In a similar way, we can send the “gray” beam to the second output and the “black” beam to the zeroth output.

Figure 2.9 shows the number of outputs for each different configuration as a function of the number of bounces, $m$, in the White cell. From the graph we can see than twenty bounces provide for 1294 possible outputs in the quartic configuration, compared to only 32 outputs for the binary configuration and 11 for the linear configuration. In general the quartic White cell configuration provide for a larger number of possible outputs, but as shown in appendix B the binary White cell configuration can provide additional features than its switching capabilities. Nevertheless, it is important to note that the MEMS assumed in the quartic configuration has three-state, compared with the two-state MEMS assumed for the binary and linear configurations. If we assumed the same kind of MEMS for the three configurations the binary system
allows for a higher number of outputs. We refer the reader to [40, 41, 42] for better understanding.

![Figure 2.9: Outputs vs. number of bounces for Different Configurations](image)

2.3 Spot Displacement Device Designs

In a previous master’s thesis [40] we described several SDD devices. In that thesis we discussed how the shifts were produced and what imaging conditions were required in order to get the required pitch displacement on the MEMS plane. That thesis presented six different designs, which are shown in Figure 2.10. Each produces a shift on the spot pattern on the MEMS. The master’s thesis main objective was to present possible solutions for the spot displacement. Three of these results were
published later in [41]. In the present dissertation we reexamine three of these options, two of which weren’t presented in [41], and develop them further by describing how a Gaussian beam is transformed after being propagated through them. We analyze the aberrations produced in each of them and include them into a binary White cell OXC and evaluate their performance.

Figure 2.10: SDD designs previously proposed; a) Prism; b) Parallelogram Prism; c) Spherical Mirror SDD; d) Roof Prism; e) Lens Train; f) Concatenated Prism

For clarity to the reader we will mention again some of the original work develop in the mentioned thesis. At the same time we will introduce new design considerations that weren’t considered before (i.e. fabrication tolerances, Gaussian propagation). Their simulations are presented in chapter 4.

The first step to creating the binary cross-connection device is to design a spot displacement device (SDD). The basic function of the SDD is to cause a displacement.
This shifts the spot onto a new row on the MEMS plane. Depending on which bounce the beam is sent to the SDD the beam will have a different displacement.

We have two main design criteria for our SDD designs:

- Each column of the SDD has to produce double the displacement of the previous one (1 for the first one);

- Beams in a specific column on the SDD cannot overlap to any adjacent column

Imaging conditions also need to be fulfilled. The analysis developed in [41, 42] shows that we can analyze the SDD independently from the White Cell. The SDD can change the position of the beam and still meet the imaging conditions of the White Cell, as long as the beams at the input and output of the SDD are the same (spot size, angle of propagation).

### 2.3.1 Spherical Mirror SDD

A simple approach to create the spot shift is described in detailed in [41] and summarized in the present section. This SDD is shown in Figure 2.11 and is based on the combination of a lens and a tilted spherical mirror.

In the figure, \(d_{SDD}\) is the distance between the lens and the tilted spherical mirror, \(\phi\) is the tilted angle of the spherical mirror, \(f_{SDD}\) is the focal length of the spherical mirror and \(f_l\) is the focal length of the lens.

We will place the lens in the plane of auxiliary mirror II. The light travels through lens \(f_1\), translates a distance \(d_{SDD}\), is reflected by the spherical mirror of focal length \(f_{SDD}\), which is tilted \(\phi\) radians, and goes back through the same elements towards spherical mirror E in Figure 2.11. If we take \(f_l = d_{SDD}\) the resulting matrix for the described path is described as:
Figure 2.11: Lens spherical mirror combination.

\[
SDD = \begin{bmatrix}
-1 & \left(1 - \frac{d_{SDD}}{f_{SDD}}\right) d_{SDD} + d_{SDD} & 2\phi d_{SDD} \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (2.10)

For imaging, element SDD[1,2] must be zero, making \( f_{SDD} = d_{SDD}/2 \). Thus equation (2.10) simplifies to:

\[
SDD = \begin{bmatrix}
-1 & 0 & 2\phi d_{SDD} \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (2.11)

From appendix A, we recall that the spot displacement produced on the MEMS is given by element SDD[1,3], which is \( 2\phi d_{SDD} \).

From Figure 2.12, we can see that the beams are diverging from the plane of auxiliary mirror II plane towards the tilted spherical mirrors. This imposes a limiting factor in the distance \( d_{SDD} \). The value of \( d_{SDD} \) cannot be chosen arbitrarily. If \( d_{SDD} \) is too big, the light beam will diverge and overlap to an adjacent column. We will keep \( d_{SDD} \) equal in every column in order to keep the transit time equal for each column, and vary \( \phi \).
This particular SDD has a very simple geometry and its design is somewhat similar to the White cell, in the sense that it takes free-space beams and refocuses them in the desired position by tilting an spherical mirror, which makes this SDD an appealing option to choose. We will see, however, in chapter 4 that the implementation of this SDD is more problematic that it seems.

2.3.2 Roof Prism SDD

Another approach to generating a spot displacement is shown in Figure 2.13. This design was also mentioned briefly in a previous master’s thesis [40]; we now give a more detailed analysis of the SDD and explain in detail the effects of misalignments in the output beam.

In this approach a series of micro-prisms are placed close to the plane of Auxiliary mirror II in the Binary White cell OXC. In front of these micro-prisms an array of
micro-lenses is used to reduce the divergence of the beams coming from the White cell. As the beams travel through the micro-prism they are redirected to a new position where another set of micro-lenses is used to send to beams again to the White cell with the same characteristics (i.e. spot size, divergence) as they came in.

![Diagram of micro-lenses and prisms](image)

Figure 2.13: Roof Prism SDD on the input/output plane of the SDD

The micro-lenses prevent the beams from overlapping to adjacent prisms while travelling through their corresponding SDD column, by reducing the divergence of the beams. We can see in Figure 2.14 that the light coming from the White Cell is focused at the input/output plane. The micro-lens array is placed a distance $d_0$ after this plane. As the input and output micro-lenses are symmetrical we expect to have
the beam waist of the collimated beam, \( w'_1 \), at the middle point. This middle point is located at an optical path distance \( d_1 \) from the microlens. The input beam is defined by its spot size \( w_0 \), its beam divergence \( \theta \), and its wavelength \( \lambda \). Each microlens is characterized by its diameter \( \Phi \), its radius of curvature \( R_{ml} \) and its refractive index \( n \). The micro-prism is characterized by its refractive index, which we will assume to be equal to that of the micro-lenses. The optical path for each beam inside the micro-prism is then \( L = 2d_1 \).

Figure 2.14: Input/Output plane on roof prism SDD

From Figure 2.14 we can reduce our problem to finding the micro-lens that transforms a Gaussian beam with is beam waist radius \( w_0 \) at a distance \( d_0 \) in front of the lens to another with a waist \( w'_1 \) at \( d_1 \) and back to the original beam through a symmetrical optical path. This system can be described using paraxial Gaussian beam propagation method as described in equation (1.1). The relationship of the beam parameters before \((q_0)\) and after the micro-prism \((q_1)\) is given by solving equation
which can be expressed with two equations by separating real and imaginary elements:

$$0 = \frac{2d_0 - 2d_0^2 n}{R_{ml}^2} + \frac{2d_0^2}{R_{ml}^2} + \frac{4d_1 d_0}{R_{ml} n R_{ml}^2} - \frac{4d_1 d_0}{R_{ml}^2} - \frac{4d_1 d_0^2}{R_{ml}^2} + \frac{2d_1 d_0^2 n}{R_{ml}^2} +$$

$$+ \frac{2d_1 d_0^2}{R_{ml}^2 n} + \frac{2d_1}{R_{ml}^2} - \frac{2q_0 q_1 n}{R_{ml}} + \frac{2q_0 q_1}{R_{ml}^2} - \frac{4q_0 q_1 d_1}{R_{ml}^2} + \frac{2q_0 q_1 d_1 n}{R_{ml}^2} + \frac{2q_0 q_1 d_1}{R_{ml}^2 n} (2.12)$$

$$0 = q_0 - q_1 - \frac{2q_0 d_0 n}{R_{ml}} + \frac{2q_0 d_0}{R_{ml}^2} + \frac{2q_0 d_1}{R_{ml}} - \frac{2q_0 d_1}{R_{ml} n} - \frac{4q_0 d_1 d_0}{R_{ml}^2} + \frac{2q_0 d_1 d_0 n}{R_{ml}^2} +$$

$$+ \frac{2q_0 d_1 d_0}{R_{ml}^2} - \frac{2q_1 d_0 n}{R_{ml}} - \frac{2q_1 d_0}{R_{ml}^2} - \frac{2q_1 d_1}{R_{ml} n} + \frac{2q_1 d_1}{R_{ml}^2} - \frac{4q_1 d_1 d_0}{R_{ml}^2} -$$

$$- \frac{2q_1 d_1 d_0 n}{R_{ml}^2} - \frac{2q_1 d_1 d_0}{R_{ml}^2 n} (2.13)$$

At the beam waist not only are the complex parameters $q_0$ and $q_1$ purely imaginary (i.e. wavefront’s radius of curvature is $\infty$), but for 1 : 1 magnification we also require $q_1 = q_0$. Therefore the Gaussian transformation can be described by solving equations (2.12) and (2.13) for $R_{ml}$ and $d_0$:

$$R_{ml} = \left( \frac{q_0^2 n + d_0^2 n + 2d_1 d_0 - \sqrt{q_0^4 n^2 + 2q_0^2 d_0^2 n^2 + d_0^4 n^2 - 4q_0^2 d_1^2}}{2(d_1 + d_0 n)} \right) (n - 1)$$

$$d_0 = \left[ \frac{q_0^2 - 2d_1 q_0}{n} \right]^{1/2}$$

Figure 2.15 shows possible values of $d_0$ and the focal length of the microlens (defined as $f = \frac{R_{ml}}{n - 1}$) as a function of $d_1$. We are assuming that the micro-prism and micro-lenses are made of Polymethylmethacrylate (PMMA), which has a refractive index of $n = 1.4897$. We chose PMMA because is an inexpensive polymer that has been tested successfully in the fabrication of interconnection structures, as a substrate for polymer optoelectronic devices and integrated waveguides [43, 44, 45].
Figure 2.15: Focal length and distance $d_0$ vs distance $d_1$; dotted line: radius of curvature, simple line: distance $d_0$. Where $w_0 = 15 \, \mu m$, $\lambda = 0.6328 \, \mu m$, $n = 1.4897$.

We also assume that the light source has a spot size $w_0 = 15 \, \mu m$, and a wavelength of $0.6328 \, \mu m$. In the graph we can also see the point of intersection between distance $d_0$ and the focal length $f$, which represents a $4f$ imaging system. The value of $d_1$ is set at $0.6892 \, mm$, and $d_0 = 0.4627 \, mm$ for this specific case.

We will like to work with a $4f$ imaging system because of the symmetry that the system presents. Solving equation (2.12) and (2.13) in order to find the intersection point for a general system (i.e the object is located at the focal length of the microlens) we get:
\[ R = \frac{(n - 1)d_1}{n} \]  
\[ d_0 = \frac{d_1}{n} \]  

(2.16) \hspace{1cm} (2.17)

There is a limit, however, in the extension of this imaging system. The limit is imposed by the diameter of the microlens. In order to have a clear aperture we would like the microlens radius, \( \Phi/2 \), to be at least \( 2w(z) \), where \( w(z) \) is the spot size radius of the beam at the microlens position. Assuming the Gaussian beam propagation described by equation (1.1), the limit in \( d_0 \) is set as:

\[ d_0 < \frac{\pi w_0^2}{\lambda} \sqrt{\left( \frac{\Phi}{4w_0} \right)^2 - 1} \]  

(2.18)

Another question to answer is what is the effect of any change in the distance \( d_1 \) from its ideal value? The value of \( d_1 \) can change due to manufacturing processes, and we think is important to obtain a limit to this tolerance. We will now investigate the consequences of a change of the distance between the two lenses in a 4\( f \) imaging system. A larger distance \( d_1 \) between the two curved surfaces in the micro-prism produces two effects. First, the beam size on the second lens will be larger due to the diffraction of the Gaussian beam inside the micro-prism. Second, the \( q \) factor at the output beam won’t be purely imaginary.

A way to measure how the change in distance \( d_1 \) affects the output Gaussian beam is by comparing it against the input Gaussian beam. This change can be measured as the overlap integral, \( \eta \), of the two beams. If the input and output beams are equal, then \( \eta = 1 \). The overlap integral between the input Gaussian wave function \( \phi(x, y) \) and the output Gaussian function \( \psi(x, y) \) is given by:
\[ \eta = \frac{\left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, y) \psi^*(x, y) \, dx \, dy \right|^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, y) \phi^*(x, y) \, dx \, dy \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(x, y) \psi^*(x, y) \, dx \, dy} \]  

(2.19)

where

\[ \phi(x, y) = \exp \left[ -\frac{x^2 + y^2}{w_0^2} \right] \]

\[ \psi(x, y) = \exp \left[ -\frac{x^2 + y^2}{w_1^2} \right] \exp \left[ -ik\frac{x^2 + y^2}{2R_1} \right] \]  

(2.20)

We defined \( \phi(x, y) \) and \( \psi(x, y) \) using equation (1.5). We assume that \( \phi(x, y) \)'s wavefront has an infinite radius of curvature, and \( \psi(x, y) \)'s wavefront has a finite radius of curvature \( R_1 \). For both cases we neglect the longitudinal phase.

Let us consider a distance \( 2d_1 = 2f + \delta \) between the two lenses. Using ray matrices and replacing \( 2d_1 \) by \( 2f + \delta \), the ABCD matrix for the \( 4f \) system gives

\[ R_1 = -\frac{f^2}{2n\delta} \]  

(2.21)

Equation (2.19) becomes

\[ \eta = \frac{\left| \int_{0}^{\infty} \int_{0}^{\infty} \exp\left( -\frac{2}{w} x^2 - i \frac{k}{2R_1} (x^2 + y^2) \right) \, dx \, dy \right|^2}{\int_{0}^{\infty} \int_{0}^{\infty} \exp\left( -\frac{2}{w} (x^2 + y^2) \right) \, dx \, dy \int_{0}^{\infty} \int_{0}^{\infty} \exp\left[ -\frac{2}{w} (x^2 + y^2) \right] \, dx \, dy} = \frac{1}{1 + \frac{k^2 w^4}{16R_1^2}} \]  

(2.22)

Substituting equation (2.21) into equation (2.22) and using \( z_r = \frac{\pi w^2}{\lambda} \) and \( k = \frac{2\pi}{\lambda} \):

\[ \eta = \frac{1}{1 + \left( \frac{n\delta z_r}{\lambda} \right)^2} \]  

(2.23)
Figure 2.16: Coupling efficiency for a Roof Prism SDD versus lateral displacement

Figure 2.16 shows the overlap integral for different values of $\delta$ normalized to the spot size $w_0$. We can see that we have a 20% loss in the overlap integral when the longitudinal displacement, $\delta$, has a value of 4.2886 times the spot size (for a 15$\mu$m spot size this value correspond to a $\delta$ displacement of 64.433 $\mu$m). We are assuming $n = 1.4897$, $z_r = 1.117$ mm, $d_1 = 0.6892$ and $f = 0.4627$ mm, which correspond to a Gaussian beam with the characteristic mentioned for Figure 2.15. Thus, the focus is fairly forgiving of manufacturing errors. Is important to notice that any error in the distance $d_1$ will cause a change in the wavefront radius of curvature, not in the spot size. In order to have a proper imaging system, however, it would be necessary to have the correct wavefront radius of curvature at the imaging planes.
However, tighter tolerances are found when we analyze the lateral offset. This offset occurs when the input beam is not completely on the optical axis. For this case the input beam Gaussian beam is described by:

\[ \phi(x, y) = e^{\frac{-(x + \epsilon)^2 + y^2}{w_0^2}} \] (2.24)

The overlap integral using equation (2.19) shows how the coupling efficiency decreases with increasing lateral offset \( \epsilon \) between the two Gaussian beams. A 20 % overlap integral loss is caused for an offset of only 0.4724, which corresponds to only 7.086 \( \mu m \) for \( w_0 = 15 \mu m \). The effects of the lateral offset can be seen in Figure 2.17. This tolerance is important because it will indicate how far away from the optical axis the spots are allowed to be. It can be seen that the alignment of the SDD is critical.

![Effects of Transverse Offset on Overlap Integral](image)

Figure 2.17: Coupling efficiency for a Roof Prism SDD versus transverse displacement


2.3.3 Lens Trains SDD

The last design that we will discuss here is based on a modified optical waveguide built with lenses. In Figure 2.18 we show a set of identical lenses of focal length $f$ separated by a distance $d$. From [25] we know that this system is stable when the stability condition is fulfilled, for this particular case this occurs when the distance between the lenses doesn’t exceed $4f$.

![Lens Waveguide](image)

Figure 2.18: Lens waveguide

We want to take advantage of this stability, so we can propagate a beam through long distances and therefore have longer displacements on the MEMS plane. In Figure 2.19 we show a structure based on the previous lens waveguide, where in the upper plane we have placed a flat mirror, while in the lower plane is a series of spherical mirrors. Light will bounce between spherical mirrors the same way it travelled through the lenses. The optical path, $d_{lt}$ in Figure 2.19 between spherical
mirrors and the flat mirror has to be less than $R_{lt}$, where $R_{lt}$ is the radius of curvature of the spherical mirror that is defined as $f_{lt} = R_{lt}/2$, for the light to be confined.

![Diagram](image)

Figure 2.19: Two-plane optical waveguide

We can also see from Figure 2.19 that the input/output plane has a prism-shaped entrance/exit. The prism-shaped entrance deflects the incoming light to the first spherical mirror on the device, while the prism-shaped exit returns the beam to the White cell with the equal and opposite angle as it went in with (to mimic a physical mirror). This causes the light to be displaced in an orthogonal plane while being confined by the spherical mirrors. This propagation will represent, eventually, the desired displacement on the MEMS plane.

The distance $d_{lt}$ will be limited by the divergence of the beam. To calculate the maximum distance $d_{lt}$, we have to consider the spot size at the spherical mirrors on the SDD. The distance $d_{lt}$ in Figure 2.19 will be the distance in which the size of the beam at the input plane on the spherical mirror SDD doubles. This distance was established already in [41] and the shift can then be calculated as:

$$
\Delta = 2\sqrt{3\frac{\pi w_0^2}{\lambda}} \sin \theta
$$

(2.25)
where $\theta$ is the angle of the input beam with respect to the normal of the spherical mirror as shown in Figure 2.19.

In case we need a bigger shift, instead of modifying $\theta$ or $d_{lt}$ as in the design of section 2.3, it is only necessary to repeat this structure (that we call “unit cell”) as many times as needed until the required shift is reached. Figure 2.20 shows a SDD lens train device for a $2\Delta$ displacement, the vertical dotted line dividing the device into two equal structures. Similar to the SDD of section 2.3.2 we can define each structure as a 4$f$ imaging system; the problem is then reduced to analyzing consecutive 4$f$ imaging systems.

![Figure 2.20: 2∆ pitch Lens Train SDD.](image)

We will use again the overlap integral to measure the effect of the offset $\delta$ and compare the results of this section to the ones obtained in section 2.3.2. Substituting $d_{lt} = 2(f + \delta)$ in the ABCD matrix for this 4$f$ system and substituting this result into equation (1.6) we obtain:

$$q_1 = \frac{-nq_0 f_{lt}^2 + nq_0 \delta^2 - 2f_{lt}^2 \delta + \delta^3}{n(n\delta q_0 + \delta^2 - f_{lt}^2)}$$  \hspace{1cm} (2.26)
Solving for $R_1$ and $w_1^2$, which are the radius of curvature of the Gaussian beam and the spot size defined in 1.3 and using equation (2.26):

$$R_1 = \frac{q_0^2 n^2 (f^2 - 2\delta^2) - 4\delta^2 (f^2 - \delta^2)^2}{-2n^3 \delta (f^2 - 2\delta^2) q_0^2 - 2n \delta (f^2 - \delta^2)} \quad (2.27)$$

$$w_1^2 = \frac{[q_0^2 n^2 (f^2 - 2\delta^2) - 4\delta^2 (f^2 - \delta^2)^2] \lambda^2}{n^2 f^4 \pi^2 w_0^2} \quad (2.28)$$

Substituting equations (2.27) and (2.28), the overlap integral described in equation (2.19) becomes:

$$\eta = \frac{4w_0^2 w_1^2}{(w_0^2 + w_1^2)^2 + \left(\frac{k w_0^2}{2R} \right)^2} \quad (2.29)$$

where $k = \frac{2\pi}{\lambda}$. Figure 2.21 shows the effect for a lateral displacement in the coupling efficiency for three different shifts ($\Delta$, $2\Delta$, and $8\Delta$). The overlap integral varies depending of the required shift. For the particular values of $\Delta$ shown in the figure the points where we get a 20% loss in the overlap integral are: $\delta/w_0 = 5.865$, $\delta/w_0 = 2.867$ and $\delta/w_0 = 2.007$, (which correspond to a value of $\delta_1 = 0.08797$ mm, $\delta_2 = 0.042998$ mm, and $\delta_3 = 0.0300987$ mm for a $w_0 = 15\mu$m) for $\Delta$, $2\Delta$ and $8\Delta$ respectively. We can appreciate that the tolerances became tighter as we increase the number of unit cells. We can conclude that even though the lens train SDD allows for a simple way to increase the amount of possible shifts, the offset tolerances will became more important. It may be possible to have a single unit cell and instead of repeating the structure we can modify the angle $\theta$ and distance $d_{lt}$, but in that case it will be important to study the effects these changes may have on aberrations. We will analyze these possible solutions in chapter 4.
Effects of Lateral Displacement on Overlap Integral for a Lens Train SDD

Figure 2.21: Overlap Integral for different shifts ($\Delta$) vs. Lateral offset

2.4 Chapter Summary

In this chapter we have described the different systems that can be implemented as optical cross-connectional devices. We can divide the systems in two groups. In the first group we can place the linear and the quartic configurations, in this group the shifts are produced by placing the centers of curvature in the appropriate location in the MEMS plane. So the shifts are produced every time the beams visit a particular set of mirrors. In the second group we have the binary configuration which produces the shifts not by the position of the center of curvature of its spherical mirror, but by the presence of SDDs.

The SDDs can be implemented in several ways; we have presented only three. An important part of the present dissertation is the comparison of these SDDs. This
comparison will help to make the decision of which system would have best chance of success in a real system. The comparison will consist of two parts. The first part was presented in this chapter, we wanted to see how the shifts are produced and how tight the tolerances have to be for each system. The second part of the comparison will take place in chapter 4 where several simulations will be made for each system.

Finally in chapter 3 we will present how the beams land at the output plane at a different angle and position depending on the output and the initial input beam location. This characteristic will influence how the beams are placed in the output plane, and therefore it will affect how the beams can be combined and coupled back into fiber. The problem that is address in chapter 3 is unique for the White cell OXC devices (True-time delays based on the White cell don’t present this problem, because a beam never change its position once it goes inside the system), and although several solutions have been studied, the solution presented is one that has no fan-in loss.
CHAPTER 3

BEAM COMBINER

There is, however, an important characteristic with the proposed cross-connection devices. It will be noted, in Figure 2.8 that any input directed to a particular output will land in a different place within that output region. For example the white beam was sent to output four and appeared in the upper right hand corner. Had the black beam been sent to output four, its spot would appear in the lower right hand corner. Thus, once a given input has reached the correct output region, the spots must be all made to land in the same spot, for example on a detector or a fiber core. This is non-trivial in the White cell because in addition to arriving at different locations, the beams may arrive from different angles, a factor that will seriously affect coupling into a fiber.

3.1 Beams at the output of OXC

Figure 3.1 shows two beams at their final bounce arriving at different outputs. Each beam arrives from different spherical mirrors; while the “gray” beam comes from mirror B, the “white” beam comes from mirror F. The fact that each beam comes from different spherical mirrors and goes to different outputs causes the beams to arrive at the output plane with different angles. There are actually two angles
of concern here. The first has to do with which White cell a beam is arriving from when it reaches the output region. The other angle arises from the particular output location within that output region that the spot forms.

The first angle is the most severe. Figure 3.1(a) shows the last bounce for two different beams (i.e. the “white” and “gray” beams of Figure 2.8) for a 12-bounce system. The “white” beam is sent to the fifth output, meaning it was shifted on its last bounce, so it is coming from the EF White cell. On the other hand, the “gray” beam, directed to the second output, comes from the AB White cell on its last bounce.

One simple way to solve this problem (difference of angle on which mirror the beam comes from) is to add one additional bounce. Then regardless of the output selected, each beam can be sent back to the AB White cell on its last bounce. The beams will come out at the appropriate row (i.e. output), and one column over, but now all beams will arrive at their respective output regions arriving from the same general direction, that of the final spherical mirror, Figure 3.1(b).

The beams are all arriving from the same White cell now but are still directed to different outputs. Within each output region each a beam may arrive at any of several different locations (e.g. lower corner, middle). This also creates a small difference in the angle at which a beam arrives. This small angle can be important when coupling into fiber. Furthermore, the input spot array may be two-dimensional, having both columns and rows. Therefore all the rows and columns must be combined to a single spot, and this must be done taking into account the varying angles of incidence. The output should be a single spot, of the same size and shape as any individual input spot, and the output should emerge at a specific angle, independent of the arrival angle of any particular beam.
Figure 3.1: Beams arriving at a given output region will in general land in different locations and arrive from slightly different angles (a), situation that can be solved by adding an additional bounce (b).
The major concern in the design of a beam combiner is the presence of fan-in loss. To fully understand fan-in losses and how they can be avoided we will introduce Liouville’s Theorem.

### 3.2 Liouville’s Theorem

Liouville’s theorem is mainly used in statistical mechanics [46]. Even though light rays are not usually handled by statistical theories, we can apply statistical theories in a classical way so we can described the collective motion of bundles of rays. We will use Liouville’s theorem because it provides help in situations where intuition could easily lead to errors. Therefore, it can help us to prove the physical realizability of theoretical assumptions that may seem adequate but that break a physical principle in a subtle way. Liouville’s theorem is not usually treated in optics books, so a small derivation of the theorem will be presented here for the reader.

Liouville’s theorem uses the notion of phase space to describe the optical rays. Each ray can be described by its position \((x, y)\), and its momentum (or slope) \((\delta x, \delta y)\). These variables define a space of four dimensions. The physical state of a ray can be described as a point in phase space. Instead of studying individual rays we will study several simultaneous rays that propagates through the same system, this leads to the use of statistical optics [47] [48].

Figure 3.2 represents the outer limit of a bundle of rays filling a certain area in real space. Each ray of the bundle has a representation point in phase space \((x, \delta x)\) that is different from the rest of rays on the bundle. This representation of points in phase space will fill certain finite volume (area in the figure). As the ray bundle propagates through an optical system we will observe that the volume (area in the
Figure 3.2: A bundle of rays fill an area in phase space, as it propagates through the optical system the area may deform but remains constant.

The rays themselves may move far away, causing the phase space volume to deform and stretch, but the volume formed by the bundle of rays, however, remains the same.

Let’s assume a single ray starts at an input plane $z = z_i$ with coordinates $x_i, y_i, \delta x_i, \delta y_i$ within our bundle. Following the ray from the initial point to an arbitrary point $z = z_f$ along its trajectory changes, in general, all the values of the coordinates to $x_f, y_f, \delta x_f, \delta y_f$. The rest of the rays in the bundle have coordinates whose initial values and final values are somewhat different from those of our original ray. The point to be made is that, as we vary the initial coordinates of the ray at the
plane $z = z_i$, its final coordinates at $z = z_f$ vary too. In fact, the final coordinates are functions of the initial coordinates.

$$x_f = x_f(x_i, y_i, \delta x_i, \delta y_i)$$ (3.1)

$$\delta x_f = \delta x_f(x_i, y_i, \delta x_i, \delta y_i)$$ (3.2)

Similar relations hold for the other two coordinates.

Next, we need a result from the theory of volume integrals. If an integral, which is expressed in terms of a set of variables, is to be transformed to a new set of primed variables, the transformation is accomplished with the help of the Jacobian of the transformation. This transformation property of the volume integrals can be expressed by the equation

$$dV_f = \frac{\partial(x_f, y_f, \delta x_f, \delta y_f)}{\partial(x_i, y_i, \delta x_i, \delta y_i)} dV_i$$ (3.3)

Liouville’s theorem states that the initial and the final volume must be the same, therefore we have:

$$dV_f = dV_I$$ (3.4)

As a consequence the determinant of the Jacobian matrix appearing in equation must consequently have the value unity. From the definition of the Jacobian, we obtain a version for Liouville’s theorem.
Liouville’s Theorem has interesting consequences when we try to combine several beams into a single one. Let’s assume the system shown in Figure 3.3, where a beam with a solid angle $\beta_1$ and occupying an area $A_1$ is collected into a channel that can receive light from a solid angle of $\beta_2$ and an area of $A_2$.

This system can be treated as an imaging system, where the ray position of the image plane is independent of the input ray angle (e.g. $\frac{\partial x_f}{\partial \delta_x} = 0$), and also that the $x$ and $y$ coordinates are independent (e.g. $\frac{\partial y_f}{\partial \delta_x} = 0$). These conditions are expressed in equation (3.5) as:

$$\begin{vmatrix}
\frac{\partial x_f}{\partial x_i} & 0 & \frac{\partial \delta x_f}{\partial x_i} & 0 \\
0 & \frac{\partial y_f}{\partial y_i} & 0 & \frac{\partial \delta y_f}{\partial y_i} \\
0 & 0 & \frac{\partial \delta x_f}{\partial \delta x_i} & 0 \\
0 & 0 & 0 & \frac{\partial \delta y_f}{\partial \delta y_i}
\end{vmatrix} = 1$$

(3.6)

Solving for the determinant we obtain:

$$\frac{\partial x_f \partial y_f \partial \delta x_f \partial \delta y_f}{\partial x_i \partial y_i \partial \delta x_i \partial \delta y_i} = 1$$

$$\frac{\partial x_f \partial y_f}{\partial x_i \partial y_i} \begin{vmatrix}
\frac{\partial \delta x_f}{\partial \delta x_i} & \frac{\partial \delta y_f}{\partial \delta x_i} \\
\frac{\partial \delta x_f}{\partial \delta y_i} & \frac{\partial \delta y_f}{\partial \delta y_i}
\end{vmatrix} = \frac{\partial x_i \partial y_i \partial \delta x_i \partial \delta y_i}{\partial x_i \partial y_i \partial \delta x_i \partial \delta y_i}$$

(3.7)
In terms of our elements shown in Figure 3.3, where $N$ is the number of fibers present, equation (3.7) is expressed as:

$$N \cdot A_1 \beta_1 = A_2 \beta_2$$

Thus, if an attempt is made to collect the light from $N$ channels into a single channel with identical numerical apertures and areas then only $\frac{1}{N}$ of the light will be collected and the device will suffer from a $10 \log \left( \frac{1}{N} \right)$ intrinsic fan-in loss.

### 3.3 Proposed solutions

It may seem that fan-in loss may be an unavoidable characteristic of any beam combiner. There are special circumstances, however, where the fan-in loss can be avoided. From equation (3.8) we can see that if we can increase the area of the receiving detector, $A_2$, by a factor of $N$, the combined light may reach a maximum value of 100%.
In general beam combination without fan-in loss can be possible in three cases: combining beams with different polarization modes, spatial modes, and spectral modes. The first case (polarization modes) is shown in figure 3.4 where two beams with orthogonal polarizations are combined in a single output without fan-in loss.

Now let’s assume a Y-coupler to explain spatial modes. The Y-coupler consists of two single-mode waveguides, which are joined together into a single single-mode waveguide, as shown in figure 3.5. In general, if the inputs are incoherent, only 1/2 of the power will be coupled into the output waveguide [52]. If the two input are coherent, (in the case both inputs are generated by the same source, if the input sources are phase locked, or if the relative phase of each input are carefully monitored and matched), then the power from each input can be coupled into a single waveguide. This technique is called coherent beam combination. In Figure 3.5, we show two cases for a Y-branch coupler. Figure 3.5(a) shows two input beams (at the left) that are out of phase from each other. It can be seen that no power is coupled at the output, on the other hand Figure 3.5(b) shows an example of coherent beam combination.
Figure 3.5: Y-branch coupler, (a) The input beams are out of phase, while in (b) both inputs are in phase and are added coherent at the output.

where the two inputs at the Y-branch are phase matched and their power is added at the output.

The most common technique, however, that combines beams without loss is by combining beams with different wavelengths. This has been already explained when we described WDM. In WDM several wavelengths are transmitted through the same channel without any fan-in loss.

Several wavelengths can be combined by arrayed-waveguide grating (AWG) wavelength multiplexer/demultiplexers. An AWG combines and splits optical signals of different wavelengths for use in WDM systems. The AWG consists of a number of arrayed channel waveguides that act together like a diffraction grating in a spectrometer. The grating offers high wavelength resolution, thus attaining narrow wavelength channel spacings. AWGs are basically planar lightwave circuits (PLC) that provide long-term reliability, high stability, and protection against mechanical vibrations.
The fact that AWGs are PLC devices, however, makes it inconvenient to use them in White cell-based OXC (or any free-space 3D OXC). It will be desirable then to have a free-space combiner that allows us to have full use of our 3D capabilities.

In the rest of the present chapter, we will propose a beam combiner technique that may be implemented in the White cell OXC, based on combining beams at different wavelengths. We leave further development of these solutions as future work.

### 3.3.1 Concave Grating Multiplexer

We propose the use of concave grating multiplexer (CGM) as our beam combiner. We will show in the present section that CGMs can handle several beams simultaneously, which make them ideal to work with free-space OXC devices.

Figure 3.6 shows the basic layout for our proposed combiner. Figure 3.6 shows several inputs each with a different wavelength, a single output and the diffractive element. The diffraction grating has to be designed and aligned in such a way that the first diffraction order of every input falls on the output waveguide. We are assuming that the channel wavelengths are spaced equally to respect to each other, and that the pitch of the input beams is constant ($P_x$).

Concave gratings were invented by Henry Rowland in 1883. Rowland showed that if the entrance slit and the surface of the grating both lie on a circle whose diameter is the same as the radius of curvature of the grating surface, and if the grating surface is tangent to this circle, then a focused spectrum will be produced on the circle. This circle is referred to as the Rowland circle [53, 54, 55, 56].
A reflecting grating on a concave spherical surface not only disperses the light but focuses the spectrum. The great advantage of this grating lies in the fact that no separate collimating optics are needed.

Let a beam at wavelength $\lambda_0$ (the central wavelength of our spectral range) propagate from point $A$ and be diffracted into the $n$th order at point $C(y, z)$, to be imaged at point $B$. Each input will have a different wavelength $\lambda$. In order for them to be coupled into the same output, each input will also lie on the Rowland circle at an angle:

$$\beta(\lambda) = \sin^{-1} \left[ \frac{\lambda}{\lambda_0} (\sin \alpha_0 + \sin \beta_0) - \sin \alpha_0 \right] \quad (3.9)$$

where $\beta_0$ is the angle of incidence of the different inputs with respect to the normal to the diffraction grating surface, and $\alpha_0$ is the output angle with respect to the same normal.
A serious problem with concave gratings, however, is the presence of aberrations. The Rowland mounting is attractive because of its simple geometry and also because it is free of defocus and meridional coma; the main aberrations are astigmatism and sagittal coma [57, 58, 59].

It is desired that all the inputs in Figure 3.6 are pointing to the center of the diffraction grating. If they are not, each input will fill a different area of the diffraction grating and aberrations won’t be corrected equally for all the inputs unless different gratings are defined for each input.

From [56, 60, 61] we get the terms that represent the different kind of aberrations (i.e. astigmatism, coma, spherical aberrations). The most significant aberration in our case is astigmatism which is expressed as:

$$W_0 = \frac{1}{2R} \left[ \frac{\sin^2 \alpha_0}{\cos \alpha_0} + \frac{\sin^2 \beta(\lambda)}{\cos \beta(\lambda)} - \frac{\lambda}{\lambda_0} \left( \frac{\sin^2 \alpha_0}{\cos \alpha_0} + \frac{\sin^2 \beta(\lambda)}{\cos \beta(\lambda)} \right) \right]$$ (3.10)

Differentiating $W_0$ with respect to $\lambda$ and minimizing for $\lambda = \lambda_0$ we get:

$$\frac{dW_0}{d\lambda} \bigg|_{\lambda=\lambda_0} = \frac{\sin \beta_0}{2R\lambda_0} \left[ \frac{\sin \alpha_0(2 - \sin^2 \beta_0) + \sin \beta_0}{\cos^3 \beta_0} - \frac{\sin^2 \alpha_0}{\sin \beta_0 \cos \alpha_0} \right] = 0$$ (3.11)

Solving for $\sin \alpha_0$ we obtain:

$$\sin \alpha_0 = \sin \beta_0 \frac{\cos^2 \beta_0 \pm (1 + \cos^2 \beta_0)^{1/2}}{1 + \sin^2 \beta_0 \cos^2 \beta_0}$$ (3.12)

In addition we equate the linear offset at the output plane for the different inputs with the angular dispersion defined in equation (3.9) with respect to the channel spacing $\Delta \lambda$:
\[
\frac{\Delta \beta}{\Delta \lambda} = \frac{1}{\lambda_0} \sin \alpha_0 + \sin \beta_0 = \frac{P_x}{R}
\] (3.13)

Solving for \(R\), we get:
\[
R = \frac{\lambda_0 \cos \beta_0 P_x}{\Delta \lambda (\sin \alpha_0 + \sin \beta_0)}
\] (3.14)

Assuming \(\lambda_0 = 1550 \text{ nm}\), \(P_x = 250 \mu \text{m}\), a channel spacing \(\Delta \lambda = 0.8 \text{ nm}\), and setting the output angle at \(\alpha = 20^\circ\) we find,

\[
\beta_0 = 0.699127 \text{ rad} \quad (44.357^\circ)
\]
\[
\alpha_0 = 0.349066 \text{ rad} \quad (20^\circ)
\]
\[
R = 332.6398 \text{ mm}
\] (3.15)

These values correspond to a a grating period of 635.85 lines/mm.

Let us now incorporate our proposed beam combiner in a quartic White cell OXC. We will also assume that the input beams are in a linear array. After the beams have traveled through the OXC, each beam is sent to a particular output region, as shown in figure 3.7. In the figure we can see that the output region of the OXC is actually the input of the beam combiner. In the figure we can also see that a beam is landing in a particular output in each row. It is possible, however, to send each beam to any output.

Each row in Figure 3.7 corresponds to a different intended output of the OXC. In an OXC device most likely only one position of the possible output spots in each row will actually be illuminated, although our system allows for one or more beams to arrive to the same output region. Regardless of the position in the row at which the
beam arrives, it should be directed to a single detector or optical fiber, corresponding to that row. The position of this detector/fiber is shown in 3.7.

It is important to notice that in the analysis that we presented we only assumed one row of possible inputs centered with respect to the surface of the grating surface. Therefore another topic to be developed further is the effect of several rows in the proposed system.

In section 4.4 we will show OSLO® simulations for this particular beam combiner. We will analyze the results and proposed possible modifications to be studied in a future work.
Figure 3.7: Beam Combiner implemented in a general White cell OXC system
CHAPTER 4

SIMULATIONS

In the present chapter we will simulate the quartic and binary White cell OXC systems presented in chapter 2. The objective of these simulations is to compare both systems in terms of their aberrations, specifically astigmatism, assuming an eight-output system. Also in this chapter we will simulate the beam combiner presented in chapter 3 to prove the design that we proposed earlier.

Before introducing the simulations for the quartic and the binary systems we will compare, in section 4.1, the performance of different geometries of field lens for a null-cell in a White cell system. In particular we will compare the performance of a bi-convex lens, a plano-convex, and a meniscus lens. We will also compare the effect that multiple field lenses have versus the use of a single field lens.

The simulations for an underpopulated quartic cell will be shown in section 4.2. The underpopulated quartic cell is a modification of the quartic cell design described in chapter 2. An underpopulated quartic White cell is sufficient to prove that a beam can bounce among all of the available arms, while the beams’ spot positions are controlled by the MEMS tilting angle and the position of the spherical mirror’s centers of curvature. Proper description of the modifications will be presented.
The simulations of the binary White cell OXC are presented in section 4.3. In section 4.3.1 we will evaluate the performance of the different SDD designs proposed previously in section 2.3. We will evaluate the performance of each SDD design by comparing the presence of aberrations (mainly astigmatism and spherical aberrations) for two different shifts: 500 µm and 2000 µm, which represent shifts of 2∆ and 8∆ (assuming a pitch ∆ = 250 µm), respectively. This difference in shift distance will help us to evaluate whether there is any dependence of the aberrations on the shift distance, and also it will help to determine which designs are better suited for short and long shift distances.

In section 4.3.2 we will take those SDD design(s) with the best performance and simulate a binary White cell OXC using them. This simulation will help to evaluate the beam quality in terms of accumulated aberrations through the system.

In section 4.4 we show the simulation for the beam combiner presented in chapter 3. The beam combiner is still a work in progress so no attempt to integrate the beam combiner to any of the White cell OXC systems proposed is made. We identified, however, important considerations to improve the design of the beam combiner before a physical setup is attempted.

To keep consistency in the systems simulated we will assume a wavelength λ = 0.6328, and beam spot size w₀ = 15 µm.

4.1 Selection of Field Lens Geometry

The choice of the field lens is important, since this lens is a major source of aberration to the system. The characteristics of the lens will be discussed regarding
their influence on the presence of spherical and astigmatic aberration mainly. These investigations were carried out using OLSO® simulations.

We will simulate the null cell of a White cell system. The null cell is composed of two spherical mirrors a flat mirror and a field lens, as shown in Figure 4.1. The system has a 1 : 1 magnification and the distances between elements are found by solving the ray matrices equations as shown in appendix A. It is of interest to know which field lens geometry would be best for our particular system.

Based on the system shown in Figure 4.1, we will substitute field lenses of different lens geometries (i.e. biconvex, plano-convex, and meniscus). We will maintain the size of the lens the same as well as the focal length for all of the different systems.

For all three cases we will assume that the field lens is made with BK7 glass (n = 1.52 at 0.6328 µm), and that it has a 400 mm focal length and a 45 mm diameter. Figure 4.1 shows the setup of the system with a plano-convex lens, placed with its plane surface facing the MEMS plane (on the left). For all three systems we analyzed a seven-bounce system.

Figure 4.2 shows the ray intercept curves (left) and field trace curves (right) analysis for the different systems. We show these graphs to see the different aberrations presented in the system as well as their contributions with different lenses.

Before describing the ray analysis we refer the reader back to the optics background presented in section 1.4.4. On Figure 4.2 we can see that for all four graphs, (we have two for the plano-convex lens: (I) with its curved surface facing the MEMS plane, and (II) with the flat surface facing the MEMS plane), the ray intercept curves have a straight line shape with a negative slope that is slightly curved on the lower right. This particular shape indicates that the main aberration is astigmatism (the straight
Figure 4.1: White cell using plano convex lens

line), but that there is a small amount of coma present (the curved section). The main difference among the different ray intercept curve graphs is a distinct change in the vertical scale of the curves, which varies from 0.01 mm for the meniscus lens up to 0.05 mm for the plano-convex I lens. This indicates a dependence of the amount of aberration on the field lens geometry. In all cases the horizontal scale remains normalized to the lens size, which is the same for all cases. The meniscus lens show the least aberration of the fours systems compared.

It is interesting to see that regardless of the field lens geometry the shape of the ray intercept curve is similar (changing only in amplitude), which indicates that an additional source of aberrations may be found somewhere else in the system. One source is the spherical mirrors that are off-axis and tilted, which contributes to astigmatism. The fact that the beams in the White cell do not necessarily go through
the center of the field lens may explain the presence of coma and spherical aberrations. Nevertheless, the geometry of the field lens helps to reduce the contribution of the aberrations as can be seen in Figure 4.2. The only difference that can be noted from the graphs is a change in the magnitude of the aberrations. We have seen that astigmatism is the most relevant aberration, so before concluding which lens geometry is best we will analyze the results obtained from the field trace curves.

The field trace curves, which are the graphs in the right column of Figure 4.2, show us the amount of astigmatism for each lens geometry. The astigmatism is measured as the difference from the sagittal and the tangential focal point. The fact that for all four geometries we have a pair of parallel lines indicates that the astigmatism presented in the systems remains constant along the object height.

We can appreciate a distinct change in the horizontal scale of the curves (the vertical scale is normalized to the input height) for all four geometries. Figure 4.2(a), which corresponds to a plano-convex lens with its curved surface facing the MEMS plane, has an average astigmatism of 3.063 mm. The astigmatism for a plano-convex lens is greatly reduced when we place the lens with its flat surface facing the MEMS plane, Figure 4.2(b), which presents an astigmatism of 1.8107 mm. For the biconvex lens, Figure 4.2(c), the amount of astigmatism remains almost the same at a value of 1.98 mm. For the case of a meniscus lens, however, the astigmatism is reduced to a value of 0.969 mm, Figure 4.2(d).

From the previous graphs we can conclude that the plano-convex lens (with the curved surface facing the MEMS plane) has the worst performance and that the best results are obtained when using a meniscus lens. These results are summarized in table 4.1.
Figure 4.2: Ray trace curve analysis for different field lens system. Left: Ray intercept curves. Right: Field Ray curves
Table 4.1: Aberration contribution for different field lens geometry in a seven-bounce White cell null-arm

<table>
<thead>
<tr>
<th>Lens Geometry</th>
<th>Sagittal Astigmatism</th>
<th>Tangential Astigmatism</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi-convex</td>
<td>-0.989297</td>
<td>0.994107</td>
<td>1.983404</td>
</tr>
<tr>
<td>Plano-convex I</td>
<td>-1.461668</td>
<td>1.601147</td>
<td>3.062815</td>
</tr>
<tr>
<td>Plano-convex II</td>
<td>-0.847304</td>
<td>0.963437</td>
<td>1.81074</td>
</tr>
<tr>
<td>Meniscus</td>
<td>-0.485389</td>
<td>0.484141</td>
<td>0.96953</td>
</tr>
</tbody>
</table>

So far, we have assumed that a single field lens will be used for all different arms of the QWC-OXC as shown in Figure 2.6. There is the possibility, however, of using three field lenses (one for each wing). The question then arises as what configuration would present lower aberrations: a system that uses a small MEMS titling angle and uses one field lens, or a system that has a bigger MEMS titling angle but that uses one field lens for each arm? We consider this question relevant because it will help us to show that not only the geometry of the field lens is important but also that it is desirable to transmit the beam through the center of the lenses as much a possible, a situation that won’t occur if using only one field lens, and that these factors are even more important than the tilting angle of the micro-mirrors.

To make this analysis we assume a seven-bounce system in an underpopulated quartic cell. An underpopulated quartic cell, shown in Figure 4.3, is a modification of the quartic cell previously shown in section 2.2.2. As in the original quartic cell we have a null cell formed by spherical mirrors A and B. When the micromirrors on the MEMS are flat, a beam will bounce exclusively between mirror A and mirror B. Let’s assume that a beam coming from mirror A hit a micromirror on the MEMS that is tilted to $+\theta$; the beam will be then directed to mirror C. If the beam coming from A,
Figure 4.3: Underpopulated Quartic White cell OXC.

However, hits a micromirror on the MEMS that is tilt to $-\theta$, the beam will be sent to mirror F, which is missing in the underpopulated quartic cell (mirrors D and F are shown in Figure 4.3 by a dotted circle). Therefore the beam will be lost. Similarly, a beam coming from mirror B can be sent to mirror E but cannot be sent to mirror D because D is also missing in the underpopulated quartic cell. The underpopulated quartic cell is a convenient way to prove that a beam can bounce between the null cell and the lateral arms without fully implementing all the allowed transitions. A side effect, however, of the underpopulated quartic White cell is that it has a lower number of possible outputs than a fully populated one. We are assuming that mirror C causes a shift of $\Delta$, while mirror E causes a shift of $3\Delta$.

Figure 4.4 shows the single-field lens system and the three-field lens system. All field lenses are assumed to be made from BK7 glass. Figure 4.4(a) presents a meniscus lens as the single field lens (focal length = 400 mm, $\phi = 80$ mm). The tilted angle
of the micromirrors is $\pm 5^\circ$, and the spherical mirrors have a radius of curvature of 609.8 mm. This system presents an average astigmatism of 5.24107 mm after seven bounces.

On the other hand Figure 4.4(b) uses three field lenses. The central field lens has a focal length of 400 mm, while the lateral field lenses have focal lengths of 350 mm. The lateral field lenses have a different focal length to avoid them physically overlapping with the central field lens. For this case the tilting angle of the micromirrors is $\pm 10^\circ$, and the spherical mirrors have again a 609.8 mm radius of curvature. From the field plot curve we can see that the astigmatism decreases for this case, presenting an average astigmatism of 0.620908 mm after seven bounces.

The tilting angle of the micro-mirrors is important from the point of view of the mechanical setup, because the beams will separate into each arm faster with a bigger angle and therefore we will be able to place the necessary mounts closer to the MEMS plane. Also the tilt angle affect the size of the field lens. In the case of using one field lens for the three arms we have a $\pm 5^\circ$, but it is then necessary to have a 80 mm diameter field lens. If we want to have a bigger tilt angle for this case, it would be necessary to have an even bigger field lens, which makes the system physically impractical. On the other hand if in the second case we had a smaller angle, it could be possible for the sides of the field lenses to intersect the optical paths of the beams on the null cell.
4.2 Underpopulated Quartic cell Simulations

In this section we will present OSLO® simulations corresponding to an underpopulated quartic White cell. The objective is to identify whether there is any correlation between the output position of the beam and the aberrations presented. We would also like to detect any possible problem that may occur before implementing the physical setup. Finally we want to compare the distances that we got from the paraxial design procedure in appendix A with the distances from the OSLO® simulations, and the distances that we implemented in our physical setup.

We will use these simulations to compare the quartic White cell OXC against the binary White cell OXC to be presented in section 4.3. The simulations are made using OSLO®. We present a seven-bounce system with eight possible outputs. The
layout of the underpopulated quartic system was presented in Figure 4.3. Here we will present the ray trace analysis for the eight possible outputs.

For the simulations we will assume that we are using a 400 mm focal length BK7 meniscus lens for the null cell. For both lateral wings we will assume that we are using a 350 mm focal length BK7 meniscus lens. As mentioned earlier the lateral field lenses have a different focal length to avoid overlapping with the null cell field lens. All spherical mirrors are assumed to have a 609.8 mm radius of curvature. We are assuming, also, a 3-state MEMS with tilting angles of $\pm 10^\circ$ and $0^\circ$. The wavelength is $\lambda = 0.6328 \, \mu m$, the spot size at the MEMS plane is $w_0 = 15 \, \mu m$, and we are assuming a pitch $\Delta = 250 \, \mu m$.

Figure 4.5 shows the bounce pattern and ray trace analysis for the 0\textsuperscript{th} output. To direct a beam to the zeroth output means that there is no change in the position of the beam, that is, the beam bounces exclusively between mirror A and B. We show in Figure 4.5(a) the bounce pattern for this output. On Figure 4.5(b) we show the ray trace analysis for this case. Because the beam bounces exclusively between A and B the lateral field lenses are not involved in the simulation and therefore are not shown. From the ray intercept curves we can appreciate that the main aberration is astigmatism with the small presence of coma. The astigmatism is measured at 0.518163 mm after seven bounces.

Figure 4.6 on the other hand, shows the bounce pattern and ray trace analysis for the 7\textsuperscript{th} output. The bounce pattern of the system is shown in Figure 4.6(a). For clarity we have place the bounce number next to the spot beam. In the figure we can see that the beam first bounces in the null cell (bounce 1 and 2), after mirror B the beam is sent to mirror E on bounce 3, which produces a shift of $3\Delta$ placing the beam
on the row corresponding to the 3rd output. We sent the beam to mirror C on bounce 4, which produces an additional shift of $\Delta$. After mirror C we sent the beam again to mirror E on bounce 5 shifting the beam another $3\Delta$, the two remaining bounces are made on the null cell, thus directing the beam to the seventh output.

Figure 4.6(b) shows the ray trace analysis for the seventh output. We can appreciate that there is little difference with the ray trace analysis presented for the zeroth output. The comparison between both graphs show us that the beam exhibit almost the same amount of aberration regardless of the chosen output. The reason for this is that the tilted angle of the four spherical mirrors is very similar, so there is not much difference among the different optical paths for the different outputs. The astigmatism measured for the seventh output is 0.654558 mm.
Figure 4.6: Bounce pattern and ray trace curves for the 7th output for the underpopulated quartic OXC.

We will expect however that as the number of output increases, larger shifts will be required and there will be a difference on the amount of astigmatism that some optical elements may add to the system. That is, for longer position displacements on the MEMS plane a bigger tilt angle may be required for some of the spherical mirrors, thus increasing the effect of aberrations in the output beams.

In table 4.2 we summarize the amount of astigmatism shown by the system for the eight different possible outputs in the system. We remark on the fact that the difference between the lowest at the biggest astigmatism is only of 0.136425 mm.
<table>
<thead>
<tr>
<th>Output Number</th>
<th>Sagittal Astigmatism</th>
<th>Tangential Astigmatism</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.254206</td>
<td>-0.263957</td>
<td>0.518163</td>
</tr>
<tr>
<td>1</td>
<td>0.276565</td>
<td>-0.286159</td>
<td>0.562724</td>
</tr>
<tr>
<td>2</td>
<td>0.301231</td>
<td>-0.311021</td>
<td>0.611252</td>
</tr>
<tr>
<td>3</td>
<td>0.276730</td>
<td>-0.287273</td>
<td>0.564003</td>
</tr>
<tr>
<td>4</td>
<td>0.298287</td>
<td>-0.308651</td>
<td>0.606938</td>
</tr>
<tr>
<td>5</td>
<td>0.320245</td>
<td>0.334027</td>
<td>0.654272</td>
</tr>
<tr>
<td>6</td>
<td>0.29930</td>
<td>-0.310788</td>
<td>0.610088</td>
</tr>
<tr>
<td>7</td>
<td>0.321127</td>
<td>-0.333431</td>
<td>0.654588</td>
</tr>
</tbody>
</table>

Table 4.2: Accumulated astigmatism for a 8-output underpopulated quartic cell OXC system. All measurements in mm

4.3 Binary White cell OXC Cell

In this section we will simulate the system corresponding to a binary White cell OXC. Before doing the simulations we will analyze the SDD designs described previously in section 2.3. From the SDD analysis we will pick the one with the best performance to use it in the simulations of the binary White cell OXC.

The optical element characteristics used in the simulations for the binary White cell are the same that were used for the underpopulated quartic White cell, that is we use a 400 mm focal length BK7 meniscus lens as field lens for the null cell. A 350 mm focal length BK7 meniscus field lens is used for the lateral wings. All spherical mirrors have a 609.8 mm radius of curvature. For the binary white cell OXC, however we are assuming a 2-state MEMS with tilting angles of ±10°. Another with the quartic OXC system is that in order to simulate an eight output system we need twelve bounces, instead of seven. The wavelength is \( \lambda = 0.6328 \mu \text{m} \), the spot size at the MEMS plane is \( w_0 = 15 \mu \text{m} \), and we are assuming a pitch \( \Delta = 250 \mu \text{m} \).
It is necessary, however, to evaluate first the proposed SDD designs to decide which design is to be included in the binary White cell OXC simulations.

### 4.3.1 SDD Simulations

The objective of this section is to compare the different SDD designs presented in section 2.3. This comparison will be done by a series of OSLO® simulations. These simulations are done in similar conditions for each design in order to compare the amount of aberrations introduced by each SDD design. We will assume that all SDD designs are made with PMMA, have a wavelength of $\lambda = 0.6328 \, \mu \text{m}$ and a spot size of $w_0 = 15 \, \mu \text{m}$. Each SDD design will be evaluated at two arbitrary shift displacements: $500 \, \mu \text{m}$ and $2000 \, \mu \text{m}$.

**Tilted Spherical Mirror SDD**

The first design to be simulated is the tilted spherical mirror SDD shown in Figure 2.11. This particular SDD has a simple geometry that consists of a lens and a tilt spherical mirror and can be easily be implemented in a White cell OXC. Figure 4.7 shows the simulation that corresponds to the tilted spherical mirror SDD for a shift displacement of $2\Delta$ (i.e. equivalent to $500\mu \text{m}$ for $\Delta = 250 \, \mu \text{m}$). In the right lower corner of the figure we can see the layout for this particular SDD; there are two microlenses, each with a $250 \, \mu \text{m}$ diameter. Each microlens is a plano convex lens with a $1.627 \, \text{mm}$ radius of curvature and $0.1 \, \text{mm}$ thickness. The distance from the microlens to the spherical tilted mirror is $3.1746 \, \text{mm}$, and the radius of curvature of the spherical mirror is also $3.1746 \, \text{mm}$. To cause this particular shift the tilt angle of the spherical mirror is calculated by OSLO® and set at $4.4311^\circ$. From the ray intercept curves on the left of the figure we can appreciate that the main
aberration present is astigmatism, which is expected because the light goes through the center of the microlenses but is incident on the spherical mirror which is tilted. The astigmatism for this particular setup is measured as: 0.03942 mm.

![Ray trace analysis for a tilted spherical mirror SDD for a 2Δ shift.](image)

Figure 4.7: Ray trace analysis for a tilted spherical mirror SDD for a 2Δ shift.

We now present the simulations for the same SDD design but for a shift displacement of 8Δ (i.e. equivalent to 2000 µm for a pitch of Δ = 250 µm). The system is presented in Figure 4.8, the optical elements and distances are the same as the ones of Figure 4.7. As in the previous case, the main aberration presented in this case is also astigmatism, which is also expected because the system is almost identical; as a matter of fact, the only relevant change is the titled angle of the spherical mirror, which is set for this case at 15.997°. The consequence of this increase in the tilted angle is reflected in an increase in astigmatism, which for this case is calculated as:
1.3418 mm, compared to the 0.03942 mm for the 2Δ. It can also be seen that the beams are not focusing on the second microlens. The reason for this is that as we rotate the spherical mirror, the image plane (that is located 3.1746 mm away from the spherical mirror) also rotates describing a circle (with it center at the center of the spherical mirror). It would be necessary to include an additional optical element that corrects this curved image plane. The reason we didn’t detect this in the design of section 2.3.1 is that then we were working in the paraxial regime, which is no longer valid for the tilted angle values used in the current simulation.

Figure 4.8: Ray trace analysis for a tilted spherical mirror SDD for a 8Δ shift.
Roof Prism SDD

We next simulate a roof prism SDD for a 500 μm (2Δ) shift using OSLO®. The roof prism SDD was shown in Figure 2.14 as a reference to the reader. We are assuming a 15 μm radius spot size at the image plane of auxiliary mirror II, the curved surface of the micro-lens is placed 0.4283 mm from the same image plane and that the thickness of the microlens is 0.263075 mm. We are assuming that the roof prism is made of PMMA. The radius of curvature of the microlens is set at 0.20975 mm. The ray intercept curves, field plot curves and a sketch of the roof prism SDD are shown in Figure 4.9. The ray intercept curves (left column of Figure 4.9) present a straight line with a slightly “S-shape” indicating that the main aberration of the system is astigmatism, with a slight presence of spherical aberration. The presence of astigmatism can be seen in the same curves as a change in the tilt angle, especially as the object height increases. The same astigmatism can be seen more clearly in the field plot curves. The astigmatism grows as we move away from the optical axis up to the total size of the object.

The astigmatism and spherical aberrations can be reduced using an aspheric lens at the input and output of our roof prism SDD. Spherical lenses can refract light at only small angles before spherical aberration appears. Usually is necessary to divide a single lens into several lenses to reduce the spherical aberration. We can produce the same effect, however, by using an aspheric lens. Aspheric lenses are those whose curved surfaces are not necessarily defined by the sphere equation, but rather by a polynomial. Figure 4.10 shows the ray intercept curves and field plots for the same roof prism SDD, using aspheric lenses with aspheric (or conic) constant of -1.1 (the aspheric constants give the sag of the surface in terms of its departure
from a conic, not a sphere). With the aspheric lenses implemented, the astigmatism remains constant along the object height. There is also a decrease in the spherical aberration (compared to Figure 4.9). In the ray intercept curves we can see now that astigmatism has been almost eliminated (from the change in the vertical scale) and that the main remaining aberration is spherical aberration. It is important, however, to notice that the amount of spherical aberration is very small.

We now evaluate the system that causes a shift displacement of 2000 $\mu$m ($8\Delta$). The layout of the SDD and the ray trace analysis curves are shown in Figure 4.11. To have the desired shift it is necessary to change the radius of curvature and distance from the plane of Auxiliary Mirror II to the curved surface of the microlens. We use the formulas described in section 2.3.2 to make the appropriate changes. The distance from the plane of Auxiliary mirror II to the curved surface of the microlens
is now set at 0.9318 mm, and the radius of curvature of the microlens is set for this case at 0.4563 mm. As in the previous case, we use aspheric surfaces to reduce the astigmatism and spherical aberration of the system. For this case the conic constant is set at -1.0. From the ray trace curves we can appreciate that the effect of astigmatism is minimal, and that the only aberration present is a minimum amount of spherical aberration.

Even though a significant increase in the shift displacement has been made, the change in the quantity and quality of the exhibited aberrations is minimal in this SDD design. The only drawback is that different optical elements have to be designed for different shift displacements and that the position of each SDD column with respect to the plane of Auxiliary Mirror II has also to be different.
Figure 4.11: Ray lens intercept curves and field plot for a 8∆ Roof Prism SDD using aspheric lenses

**Lens Train SDD**

As in the case of the tilted spherical mirror SDD and the roof prism SDD, we simulate a lens train SDD for a 500 µm and 2000 µm shift using OSLO®. The design of the lens train SDD is shown in Figure 2.19. We are assuming a 15 µm radius spot size at the image plane of auxiliary mirror II, the curve surface of the micromirror is placed 1.599 mm from the same image plane, and the radius of curvature of each of the spherical mirrors is $-3.253032$ mm. We are assuming that the lens trains are made also of PMMA.

The ray intercept curves, field plot curves and a sketch of the lens train SDD for the 500 µm and 2000 µm shift cases are shown in Figure 4.12(a) and (b), respectively.
The line-shaped ray intercept curves (left column of Figure 4.12) indicate that the main aberration of the system is only astigmatism. In the field plot curves we can appreciate that the astigmatism remains constant along the object with an average value of 0.07764 mm and 0.311159 mm, respectively.

It is therefore fair to say that as the shift displacement caused by the lens train SDD increases, so will the aberrations produced by it. The advantage, however, with respect to the roof prism SDD, is that this design doesn’t limit the possible shift displacement produced. This advantage will become relevant when large OXC (> 64 outputs) devices have to be designed. For OXC devices with a small number of outputs it will be sufficient to work with the roof prism SDD.

4.3.2 Simulations

One conclusion we obtained from section 4.3.1 is that the roof prism SDD presented the best performance of the SDD designs analyzed in terms of astigmatism. Unfortunately, the roof prism SDD has a limit in the size of displacement it can produce. This limit is set specifically by the microlens diameter. Therefore, roof prism SDD is the best choice until the limit is reached. Beyond this limit the lens train SDD is the most obvious choice. Unfortunately the lens train SDD increases aberrations for larger displacements, so it will be desirable to improve the lens train design to correct the aberration or at least minimize them.

In the current section we will present an OSLO® simulation for a binary cell that uses roof prism SDD as the shifting device. The roof prism SDD is designed to cause a spot shift of \( \Delta, 2\Delta \) and \( 4\Delta \). The simulated binary OXC system is a 12-bounce system necessary to allow for eight possible outputs. We will present the bounce pattern and
Figure 4.12: Ray lens intercept curves and field plot for Lens Train SDD. a) $2\Delta$ shift displacement, b) $8\Delta$ shift displacement
Table 4.3: Roof prism SDD design parameters. All measurements in mm

<table>
<thead>
<tr>
<th>Shift</th>
<th>$d_0$</th>
<th>$d_1$</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>0.344726</td>
<td>0.513075</td>
<td>-0.168348</td>
</tr>
<tr>
<td>$2\Delta$</td>
<td>0.428324</td>
<td>0.638075</td>
<td>-0.209751</td>
</tr>
<tr>
<td>$4\Delta$</td>
<td>0.597784</td>
<td>0.888075</td>
<td>-0.291932</td>
</tr>
</tbody>
</table>

Ray analysis curves for the cases when a beam goes to the zeroth output and when it goes to the seventh output. We will also present the accumulated astigmatism for the seven possible outputs. This will help us to evaluate the effects that the SDD has on the beam quality in terms of aberrations.

Before we start our simulation we show in table 4.3 the values for the $d_0$, $d_1$ and $R$ for the different roof prism SDD used in the simulations. As in the previous sections we will assume $w_0 = 15 \mu m$, $\Delta = 250 \mu m$, $\lambda = 0.6328 \mu m$ and that the roof prism is made of PMMA. For all cases we are assuming that the microlenses have a thickness of 0.263075 mm, which is included in $d_1$.

We refer the reader back to Figure 2.7 for the binary White cell OXC to be simulated. As in the case for the simulation of the quartic White cell OXC, for the binary White cell OXC simulations we will also assume that the null cell is formed by a BK7 meniscus field lens with a 400 mm focal length and two 609.8 mm radius of curvature spherical mirrors. For both lateral wings we will also assume that we are using 350 mm focal length BK7 meniscus lenses and 609.8 mm radius of curvature spherical mirrors. We are assuming a 2-state MEMS with a tilting angle of $\pm 10^\circ$. The wavelength is $\lambda = 0.6328 \mu m$, the spot size at the MEMS plane is $w_0 = 15 \mu m$, and we are assuming a pitch $\Delta = 250 \mu m$. 

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We start our analysis with Figure 4.13, which presents the ray intercept curves when the beam bounces exclusively in the null cell. We expect this to be the best case of the binary system because there is no shifting of the beam by any SDD element. From the ray trace curves (left column) we can see that the main aberration is astigmatism; there is a slight curvature in the graph, however, which represents coma. The average astigmatism is measured at 0.890795 mm.

We now show in Figure 4.14 the bounce patterns and ray trace analysis for a 5∆ shift. As mentioned before, the simulated binary White cell is a twelve-bounce system, as we can see in Figure 4.14(a). To cause a 5∆ shift it is necessary to send the beam to SDD plane two times. We can see in the bounce pattern that the beam bounces in the null cell between mirrors A and B. On the second bounce the beam,
which is coming from B, is sent to mirror E, which sends the beam to the SDD, causing a shift of $4\Delta$. From the SDD the beam is sent to mirror F which sends the beam back to the MEMS. We bounce again between the null cell and on the eighth bounce we send the beam back to mirror E which will send the beam again to the SDD. This time, however, the SDD will cause a shift of only $\Delta$. The beam is directed again to mirror F which sends the beam to the MEMS plane on the fifth row. The remaining bounces are done exclusively in the null cell. Because we have in this case an even number of bounces (12) the output plane is located at the right of the MEMS, instead of to the left as in the case of the underpopulated quartic cell that has an odd number of bounces (7).

From the ray trace curves we can see that the main aberration is again astigmatism. It is interesting to see that the curves are quite similar to those presented in Figure 4.13 for the 0th output. This is a good indication that the kind of aberrations remain similar even after the introduction of several structures like the SDD.

In table 4.4 we summarize the amount of astigmatism presented in the system for the eight different possible outputs in the system. There is an important aspect to notice in the results shown in the table: Outputs 0, 1, 2, and 4 present not only a similar total astigmatism, but also the total astigmatism is lower than for the rest of the outputs (i.e outputs 3, 5, 6 and 7). The explanation of this is that outputs 1, 2 and 4 need only one SDD element to shift the beam the required distance, while outputs 3, 5, 6, 7 require to go through a SDD structure at least two times, thus increasing the aberrations.
Figure 4.14: Bounce Pattern and Ray trace curves for the 5th output of the binary White cell OXC.
### Table 4.4: Accumulated astigmatism for a 8-output binary cell OXC system. All measurements in mm

<table>
<thead>
<tr>
<th>Output Number</th>
<th>Sagittal Astigmatism</th>
<th>Tangential Astigmatism</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.492948</td>
<td>-0.397847</td>
<td>0.890795</td>
</tr>
<tr>
<td>1</td>
<td>-0.478673</td>
<td>0.489643</td>
<td>0.968316</td>
</tr>
<tr>
<td>2</td>
<td>-0.583862</td>
<td>0.420078</td>
<td>1.00394</td>
</tr>
<tr>
<td>3</td>
<td>0.72164</td>
<td>-0.500021</td>
<td>1.221661</td>
</tr>
<tr>
<td>4</td>
<td>-0.534665</td>
<td>0.424986</td>
<td>0.959651</td>
</tr>
<tr>
<td>5</td>
<td>0.73185</td>
<td>-0.51235</td>
<td>1.2442</td>
</tr>
<tr>
<td>6</td>
<td>0.75217</td>
<td>-0.531464</td>
<td>1.283634</td>
</tr>
<tr>
<td>7</td>
<td>0.92346</td>
<td>-0.662486</td>
<td>1.585946</td>
</tr>
</tbody>
</table>

#### 4.4 Beam Combiner Simulations

Now we will show the simulations corresponding to the beam combiner presented in section 3.3.1. Figure 4.15 shows the OSLO® implementation of the curved grating beam combiner. We set the radius of curvature of the diffractive element to 332.6398 mm, so that the inputs are aligned along a circle with diameter also of 332.6398 mm. All inputs are spaced 250 µm apart on the circle. The diffraction grating is assumed to consist of straight lines with a period of 635.85 lines/mm. The beam combiner accepts seven inputs with wavelengths from $\lambda = 1547.6$ nm up to $\lambda = 1552.4$ nm with a central wavelength of $\lambda_0 = 1550$ nm and a channel spacing of $\Delta \lambda = 0.8$ nm. The input representing the central wavelength is set at an input angle of $\beta_0 = 44.357^\circ$ and all inputs are directed to a common output at $\alpha_0 = 20^\circ$.

It is clear from the field plot curves in Figure 4.15 that the sagittal astigmatism is several orders of magnitude bigger than the tangential astigmatism. From the ray intercept curves, however, we can see that the beams are in focus for all wavelengths in
the tangential plane, while the sagittal plane shows the aforementioned astigmatism. The fact that the ray intercept curves are completely flat indicates the complete absence of spherical, coma, or tangential astigmatism. The sagittal astigmatism can be seen in the ray intercept curves of the orthogonal plane as a straight line.

One reason to explain the absence of tangential astigmatism and the high presence of sagittal astigmatism is that OSLO\textsuperscript{®} assumes that the grating consists of straight lines placed in a flat surface, but when the grating is placed on a curved surface the spacing of the grooves changes. In order to properly correct astigmatism it would be necessary to assume that the grooves are created by a cylindrical or aspheric wavefront impinging in the curved surface \[62\]. This can be achieved in reality by using holographic recording techniques. We leave the further development of the beam combiner for the White cell OXC for future work.

4.5 Chapter Summary

From our simulations we have found out that the best geometry for the field lens is a meniscus lens. We also found out that it is better to have separate field lenses for each arm instead of a single field lens for all the arms.

For the binary cell we have also simulated the different SDD designs presented in section 2.3 each for 2\(\Delta\) and 8\(\Delta\) shifts. The objective of simulating the SDD designs was to evaluate and compare them. The summary results of these simulations are shown in table 4.5 in terms of the astigmatism presented for each evaluation. We conclude that the roof prism SDD presents the lowest aberration of the three systems and that an increase in the distance displacement has little effect in the aberrations observed.
Figure 4.15: Concave Grating Multiplexer, OSLO® implementation.

<table>
<thead>
<tr>
<th>SDD Design</th>
<th>Astigmatism 2Δ</th>
<th>Astigmatism 8Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tilted Spherical Mirror</td>
<td>0.03942</td>
<td>1.3418</td>
</tr>
<tr>
<td>Roof Prism</td>
<td>0.0085</td>
<td>0.0093</td>
</tr>
<tr>
<td>Lens Train</td>
<td>0.07764</td>
<td>0.311159</td>
</tr>
</tbody>
</table>

Table 4.5: Summary of results for astigmatism in the SDDs design presented. All measurements in mm

From the results of table 4.5 we simulate a binary White cell OXC assuming an eight-output system using a roof prism SDD. For an eight-output system we need three different roof prism designs corresponding to ∆, 2∆ and 4∆ displacement. The characteristics for the three roof prism SDD designs are shown in table 4.3.
Finally, table 4.6 shows the summary of both the underpopulated quartic White cell and the binary White cell, for each of the eight possible outputs in terms of the astigmatism presented. The astigmatism for the underpopulated quartic White cell is lower than the astigmatism on the binary White cell. The binary White cell, however, requires of twelve bounces to have an eight-output system, while the quartic cell requires of only eight bounces.

<table>
<thead>
<tr>
<th>Output Number</th>
<th>Quartic OXC</th>
<th>Binary OXC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.518163</td>
<td>0.890795</td>
</tr>
<tr>
<td>1</td>
<td>0.562724</td>
<td>0.968316</td>
</tr>
<tr>
<td>2</td>
<td>0.611252</td>
<td>1.00394</td>
</tr>
<tr>
<td>3</td>
<td>0.564003</td>
<td>1.221661</td>
</tr>
<tr>
<td>4</td>
<td>0.606938</td>
<td>0.959651</td>
</tr>
<tr>
<td>5</td>
<td>0.654272</td>
<td>1.2442</td>
</tr>
<tr>
<td>6</td>
<td>0.610088</td>
<td>1.283634</td>
</tr>
<tr>
<td>7</td>
<td>0.654588</td>
<td>1.585946</td>
</tr>
</tbody>
</table>

Table 4.6: Summary of results for astigmatism in the OXC architectures presented. All measurements in mm

Although it may seem that the quartic cell has a better performance than the binary cell it will be necessary to perform further simulations involving a larger number of outputs (i.e. > 64) to realize the advantages and disadvantages of the proposed systems. In the present dissertation such simulations weren’t performed because the number of surfaces required to implement the required binary White cell system exceed the maximum number of allowable surfaces by OSLO® (150 surfaces).

Finally we did simulations of the beam combiner. From the simulation we can see that several beams, each with different wavelength can be directed to the same point
in space by using a curved diffraction grating. This result is encouraging; from the aberration analysis, however, we conclude that further development of this solution is necessary.
CHAPTER 5

EXPERIMENTAL RESULTS: UNDERPOPULATED QUARTIC WHITE CELL OXC

We designed and implemented a physical setup as a proof-of-concept for the underpopulated quartic cell. No physical setup for the binary White cell OXC was implemented due to the need of custom optics and budget restrictions. In the present chapter we will present the experimental results for our setup. In section 5.1 we will describe the apparatus, the optical elements used and the alignment procedure. In section 5.2 we will show the bounce patterns obtained for each different output and demonstrate the switching capabilities of the system. Finally in section 5.3 we will measure the loss of the system and identify any discrepancy with the expected value.

5.1 Description of apparatus

We will set up an underpopulated quartic cell as our experimental system. The underpopulated quartic cell was shown in Figure 4.3. As mentioned before the underpopulated quartic cell is a modification of the quartic cell where two of its spherical mirrors are missing (shown in Figure 4.3 by dotted elements). In an underpopulated quartic cell we allow for bouncing in the null cell formed by mirrors A and B when the micromirrors of the MEMS are flat. It is possible to bounce between mirror A and
C if we tilt a micromirror to $+\theta$. It is also possible to bounce between B and E when
the micromirrors are tilted to $-\theta$. It is not possible, however, to bounce between
B and D or between A and F simply because mirrors D and F are missing in the
underpopulated quartic cell. The underpopulated quartic cell is a convenient way to
prove that a beam can bounce between the null cell and the lateral arms without
implementing all the hardware. A consequence of the lower number of spherical mir-
rors with respect to the full implemented quartic cell is a decrease in the number of
possible outputs for a given number of bounces. We will set mirror C to cause a shift
of $\Delta$, while mirror E will cause a shift of $3\Delta$. We will implement a seven bounce
system that will allow us for eight possible outputs.

The design of the underpopulated quartic cell follows the paraxial procedure shown
in appendix A. We use spherical mirrors with a radius of curvature of 609.8 mm
(Edmund Optics NT43-548 enhance aluminum coated). The field lens for the null cell
has a focal length of 400 mm (Melles Griot LMP035 uncoated), while the lateral field
lenses have focal lengths of 350 mm (Melles Griot LMP031 uncoated). All optical
elements have a 50.8 mm diameter and are made from BK7 glass ($n = 1.50056
at 0.6328 \mu m$). Figure 5.1 present an optical layout of the proposed system, all
measurements are in mm.

It can be noted that the optical layout of Figure 5.1 shows a flat arrangement
of the optical elements, while Figure 4.3 shows that the optical elements are placed
below and above an optical plane. For the experimental setup we will place all optical
elements in a single plane to simplify the set up of the system in the optical table. If
we had chosen to exactly reproduce Figure 4.3 it would have been necessary to create
special mounts to place the spherical mirrors at the correct heights.
An important element of our experimental setup is the MEMS. Due to budget restrictions and because the objective of the experiment was a proof-of-concept, it wasn’t possible to get a proper MEMS. Instead of a MEMS we designed and fabricate what we call a “pseudo-MEMS” which will be described in the next section.

5.1.1 Pseudo-MEMS

For our setup we are using what we call a “pseudo-MEMS” (or static-MEMS). The main function of the psuedo-MEMS is to serve as a substitution of a regular MEMS by imitating some of its characteristics. The main differences between our
pseudo-MEMS and a proper MEMS is the size of the micromirrors and how the tilting of the micromirrors is controlled.

Our pseudo-MEMS is shown in Figure 5.2. Figure 5.2(a) shows the case of the pseudo-MEMS, we can see that the case is able to hold up to 12 columns (A to L), each column with 9 rows (0 to 8). This gives a total of 108 positions where we can place individual rectangular pieces. Position \((A,0)\) is located on the upper left corner, while position \((L,8)\) is in the lower right corner. The rectangular pieces are shown in Figure 5.2(b). Each rectangular piece has dimensions of \(1/16\)" x \(1/16\)" x \(1/2\)". Each piece is used to simulate a micromirror, so each one of these pieces has a segment of refractive material (i.e. bare silicon) glue to its top. To produce the tilting angle, some of the rectangular pieces are cut on their top at \(10^\circ\), and some others have flat tops, so that we can simulate a 2-state MEMS or a 3-state MEMS depending on the kind of rectangular pieces used. The rectangular pieces are interchangeable, so it is possible to create a specific bounce pattern by placing the rectangular pieces with the required angles in the correct positions. In section 5.2 we will explain in more detail how we can modify the bounce pattern and direct a beam to different outputs.

It can be seen in the figure that the case is not a exactly rectangular, but it has a tilted surface on the left side. The left side is cut in this way to allow the input turning mirror (ITM) to be place next to the micromirrors. Because the ITM has to be tilted in order to introduce the light into the White cell, the extra space behind it allows for a wide range of angles to tilt the ITM (up to \(45^\circ\)).

As mentioned before a piece of reflective material is place on top of the rectangular pieces’ surfaces to simulate the micromirrors’ reflectivity. The reflective material is

\(^2\)We will like to thank William Thalgott for his time and expertise to construct this device.
Figure 5.2: Pseudo-MEMS schematics. (a) Case of the pseudo-MEMS, (b) Individual rectangular pieces with a flat top, and a tilted top.

chosen to be small pieces of silicon wafer cut to the correct size (i.e. 1/16”x1/16”). Silicon is chosen because is relatively easy to cut a silicon wafer with the required precision at the required size by using a dicing saw\textsuperscript{3}. To place the silicon on top of the rectangular pieces we used a double-sided tape. The manual procedure is to place the double-sided tape on top of the rectangular piece, and on top of them we place the silicon fragment. We cut the double side tape with a knife blade and then set the silicon fragment in place using tweezers. The critical aspects of this procedure are to avoid any contact of the double-sided tape with the silicon reflective side, to avoid any scratch of the silicon with the tweezers and to make perfect contact among the silicon fragment, the double-side tape and the top of the rectangular piece. Any error

\textsuperscript{3}Special thanks to Feras Abou Galala for taking time from his research to cut the silicon wafers.
will cause a loss in the reflectivity of the silicon, and alteration on the final angle. Figure 5.3 shows a photograph of our pseudo-MEMS.

![Figure 5.3: Pseudo-MEMS photograph](image)

5.1.2 Alignment procedure

We start the alignment by setting up the null cell. We placed a flat mirror on the MEMS plane as a temporally substitute for the pseudo-MEMS. Once the null cell is aligned will replace the flat mirror with our pseudo-MEMS. Figure 5.4 shows how the mirror is mounted. We mount the flat mirror over a rotational stage, in such a way that the mirror center is over the rotation center. We also use a translation stage that allows us to change the position of the mirror with respect to the optical axis.

As seen in Figure 5.1 we are using three meniscus lenses as field lenses. The meniscus field lens for the null cell is about 138.603 mm away from the flat mirror (as
set from the OSLO simulation). The meniscus lens is mounted in a Thorlabs LMR2 lens mount. This field lens doesn’t have any translational capability. It is set as a reference point. The flat mirror (that will be substituted for the pseudo-MEMS) and the spherical mirrors of the null cell will be set around the position of the field lens.

The spherical mirrors are mounted in CVI gimbal mounts, model 166-20. Each gimbal mount is then mount on a translation and a rotation stage, as shown in figure 5.5.

To set up the null cell we will first establish our optical axis. For this we use a HeNe laser beam ($\lambda = 0.6328 \mu m$). The laser beam is then aligned using two apertures to guarantee that the laser beam is at the right height and is travelling parallel to the optical table, Figure 5.6(a).

This beam is then set as our optical axis. Once we establish our optical axis we center the field lens on the optical axis. To center the field lens we leave the apertures in the same position that they were when we set the optical axis. We place the field
lens between both apertures. If the field lens is not centered the transmitted laser beam will miss the second aperture and the back reflection will miss the first aperture. We then modified the position of the lens until the laser beam goes again through both apertures, Figure 5.6 (b).

After the field lens is centered we placed the flat mirror at 138.5 mm from the field lens. We measured the distance with the help of a ruler on the optical table and adjust the distance using the translation stage. We set the flat mirror perpendicular with respect to the optical axis; we call this angle $0^\circ$. We check that the flat mirror is perpendicular by making sure that the reflected beam from the flat mirror goes back through the apertures, Figure 5.6 (c).

To place spherical mirror A we first measure 387.0 mm along the optical axis from the field lens. From this point we place spherical mirror A at 30 mm to the left (assuming the observer is behind the field lens and is looking towards the spherical

Figure 5.5: Setup of spherical mirrors on gimbal mounts, rotation, and translation stages
mirror plane), Figure 5.6(d). We move the input beam from its previous positions and send the light of the beam laser to an Input Turning Mirror (ITM) next to the flat mirror. The input turning mirror is set in such a way that the beam is sent to spherical mirror A. Light reflected from A is sent to the flat mirror (which is still perpendicular with respect to the optical axis), and from there light is sent to spherical mirror B approximately 30 mm on the opposite side of the optical axis.

We now have to ensure that the distances set for the spherical mirrors and the field lens are correct. To do this we have to check that the imaging conditions are fulfilled. The first imaging condition that spherical mirror B should image onto spherical mirror A (and vice versa), and the second imaging condition imaging the pseudo-MEMS plane onto itself.

Imaging spherical mirror B onto spherical mirror A will help us to set the correct distance from the flat mirror in the pseudo-MEMS plane to the meniscus field lens. To check this imaging condition we place a sharp object (i.e. tweezers) where we expect spherical mirror B will be. We illuminate the object with a diffuse laser light, so the light will be reflected from the flat mirror to where spherical mirror A is supposed to be. Because we have a -1 magnification we will have a negative image of the object. Figure 5.7 shows the object and image plane where the sharp object is being imaged at the position of mirror A. In case both images are not the right size it would be necessary to change the distance between the field lens and the flat mirror until it is. The distance between the field lens and the flat mirror is set at 139.0 mm.

We next check the second imaging condition (i.e. imaging of the flat mirror onto itself) by looking at the bounce pattern on the flat mirror by using a CCD camera. This will allow us to determine whether the beams are being focused or not. If they
Figure 5.6: Align procedure for the null cell. Top view. a) establishing the optical axis; b) placing the field lens; c) placing of flat mirror; d) placing of spherical mirrors in null cell

aren’t, we will see an increase of the beams size at each bounce. To check the bounce pattern we replicate the layout shown in Figure 5.8. The camera in mounted on a post along the optical axis and above the plane of the spherical mirrors. We use the
Figure 5.7: Spherical mirror B imaging onto spherical mirror A. A sharp object is being imaged with -1 magnification.

Lens of the CCD camera to focus on the flat mirror. We introduce the laser beam into the White cell via the input turning mirror.

The input turning mirror directs the laser beam to spherical mirror A. We change the position of the center of curvature of mirror A in such a way that the beam reflected from A is focused on the upper left corner of the flat mirror (opposite from the input turning mirror). The beam goes then to mirror B and the center of curvature of mirror B is set in such a way that the beam is then refocused next to the input turning mirror. The centers of curvature of mirrors A and B are now set so the spot pattern describes two parallel rows as shown in Figure 2.3. Once we have the parallel bounce pattern we check if there is any magnification of the beam through the different bounces. If there is any change in the spot size of the beam it will be necessary to modify the distance of the spherical mirrors until the proper magnification is achieved. The final distance for spherical mirrors A and B is set...
Figure 5.8: Layout of the null-cell to check the bounce pattern on the MEMS plane at 389.0 mm from the field lens. Figure 5.9 show the spot pattern obtained after correcting the distances of spherical mirrors A and B.

Once we have checked and made the necessary corrections for the aforementioned distances, we proceed to place spherical mirrors C and E. Because the reflective surface on the pseudo-MEMS were placed by hand it was possible that tilt angle was not exactly ±10°. To place the remaining spherical mirrors in the correct position it is necessary to place them with respect to our actual pseudo-MEMS angle. To do so we substitute the flat mirror by our pseudo-MEMS.

We use the input turning mirror to direct the light of the laser to spherical mirror A. Figure 5.10(a) shows that the laser beam is incident in position “z” of the input turning mirror (marked with a black spot). Once the ITM has been set at the correct angle to send the beam to mirror A, we change the center of curvature of mirror A
Figure 5.9: MEMS plane imaging onto itself after correcting the distances of spherical mirrors A and B. We check the bounce pattern on the flat mirror to identify any magnification of the beam.

in such a way the reflected beam is sent to position \((I, 0)\). The rectangular piece that is placed in position \((I, 0)\) has a tilted top set at +10°, indicated by blue color. The light reflected from the pseudo-MEMS plane describes the optical axis for the arm where spherical mirror C is placed. We use a pair of apertures to mark this optical path. We then place the field lens in this new arm at approximately 148.0 mm from the pseudo-MEMS plane, and the spherical mirror C at 343.0 mm from the field lens, Figure 5.10(b).

We remove the apertures and set the center of curvature of spherical mirror C in such a way that the reflected beam lands on position \((D, 8)\). In this case the rectangular piece in \((D, 8)\) has a tilted top set at +10°, indicated again in blue color. This will send the beam back to mirror A. The beam reflected from mirror A will go this time to position \((E, 0)\). In order to send the beam to mirror B the rectangular piece has to have a flat top, indicated in green color. Light reflected from B will be sent to position \((H, 8)\). The rectangular piece of position \((H, 8)\) is tilted at −10°, indicated in red. The light reflected from \((H, 8)\) in the pseudo-MEMS marks the optical axis for the arm where spherical mirror E is placed. The field lens is set at 148.0 mm
from the pseudo-MEMS plane and spherical mirror E is set at 343.0 mm from the field lens, Figure 5.10(c).

To place the field lens of spherical mirror C at the correct position we image spherical mirror C onto spherical mirror A. Similarly, we image spherical mirror B onto spherical mirror E. The procedure is the same as the one used to image spherical mirror A onto spherical mirror B using tweezers and a diffuse laser light. The field lenses are then set at 147.5 mm from the pseudo-MEMS plane along their respective optical axis.

To check whether the location of mirror C is correct, we bounce several times between mirror A and C. We check if there is any increase in the spot size and correct by adjusting the location of mirror C. We repeat the process between mirror E and B and correct the location of mirror B. Spherical mirror C and E are located at 344.5 mm from their respective field lenses. The final setup can be seen in Figure 5.11.

Once the optical elements are in the correct place we can set the center of curvature of mirror C and E are set so the appropriate shift displacement is taken place. We will described how we set their centers of curvature in section 5.2.

5.2 Bounce Patterns

In this section we will describe how different bounce patterns can be achieved to send a beam to different outputs. For all bounce patterns the beam is introduced into the White cell via the input turning mirror that is placed on the lower left corner of the case, as was shown in Figure 5.10.

Figure 5.12 shows the bounce pattern for a beam that is directed to the zeroth output. The beam is introduced into the White cell by reflecting on position “z”
Figure 5.10: Alignment for spherical mirrors C and E. (a) Pseudo-MEMS plane indicating where the beam lands. Blue color indicates a tilted surface at $+10^\circ$, while red color indicated a tilted surface at $-10^\circ$, green color indicates a flat surface. (b) Procedure to place spherical mirror C and its field lens. (c) Procedure to place spherical mirror E and its field lens.
of the ITM. From the ITM the beam is sent to spherical mirror A, the center of curvature of mirror A is already set in such a way that the reflected beam is sent to position \((I, 0)\) on the pseudo-MEMS in the first bounce. The rectangular piece at position \((I, 0)\) now has a flat top, indicated by the green color, so the beam is sent to spherical mirror B. Light reflected from B is sent to position \((D, 8)\) on the second bounce. The rectangular piece in this position has also a flat top, so the beam is sent again to mirror A. The beam land on position \((E, 0)\) on the third bounce, on \((H, 8)\) on the fourth bounce, \((A, 0)\) on the fifth bounce, \((L, 8)\) on the sixth bounce and finally the beam goes to position \((w, 0)\) outside the pseudo-MEMS on the seventh bounce. Because all the rectangular pieces involved have a flat top the bouncing is done exclusively between spherical mirrors A and B. The bounce pattern describes two parallel lines as expected.
In order to send a beam to the first output we will set the center of curvature of spherical mirror C in such a way that every time we go to C we get a $\Delta$ displacement. Figure 5.13 shows a bounce pattern to send a beam to the first output. This time the beam is introduced into the White cell by reflecting a beam in position “y” on the input turning mirror. The beam then is reflected toward spherical mirror A. The center of curvature of mirror A is the same as the one used for the zeroth output. In this case, the beam from spherical mirror A is sent to position $(J,0)$ (instead of position $(I,0)$ where it went when the input beam bounces on position “z” on the ITM). The rectangular piece of position $(J,0)$ has a flat top, so the beam is sent to spherical mirror B (bounce 1). The beam reflected from B is sent to position $(C,8)$ which has also a flat top. From $(C,8)$ the beam is sent to spherical mirror A (bounce
and from A the beam is sent to position \((F, 0)\). In this case the rectangular piece on \((F, 0)\) has a \(+10^\circ\) tilted top, indicated by blue color, sending the beam the spherical mirror C (bounce 3). Spherical mirror C has its center of curvature displaced in such a way that every time a beam is sent to C it will be displaced by \(\Delta\). From Figure 5.13(a) we can see that the center of curvature of C, \(CC(C)\), is placed above the center of curvature of mirror B, \(CC(B)\). Once the beam has been sent to C the beam is displaced by one row, so this time the beam lands on the pseudo-MEMS on position \((G, 7)\). On this position the rectangular piece has also a tilted top to \(+10^\circ\), thus the beam will be reflected towards spherical mirror A (bounce 4). The remaining bounces are done exclusively between spherical mirror A and B, so the rectangular pieces of positions \((B, 1)\) and \((K, 7)\) have a flat top. Finally, the beam goes out the pseudo-MEMS on position \((x, 1)\).

We must note that even though we send the beam to spherical mirror C on bounce 3, we could have sent it there in bounce 1 or bounce 5 and still reached the same output.

To send a beam to the second output it is necessary to send the beam two times to spherical mirror C. This case is shown in Figure 5.14. For this case the beam goes into the White cell after bouncing from position “x” on the ITM. The first two bounces are done between spherical mirrors A and B, sending the beam to positions \((K, 0)\) and \((B, 8)\), respectively. Both positions have a flat top rectangular piece. On the third bounce, in position \((G, 0)\), the rectangular piece has a \(+10^\circ\) tilted top sending the beam to mirror C, thus producing a \(\Delta\) shift in the beam position. The beam reflected from C lands on position \((E, 7)\) which has also a \(+10^\circ\) tilted top sending the beam to spherical mirror A, bounce 4. We need to send the beam one more time to
spherical mirror C. This is done in bounce 5, where the beam lands on position \((C, 1)\)
which has a \(+10^\circ\) tilted top rectangular piece, thus sending the beam to spherical
mirror C. The beam reflected from C has an additional \(\Delta\) shift and the beam is sent
this time to position \((I, 6)\). From \((I, 6)\) the beam is sent to A again (bounce 6), and
the beam is sent outside the pseudo-MEMS on position \((y, 2)\).

In order to send a beam to the third output we will send the beam one time to
spherical mirror E, so mirror E will produce a \(3\Delta\) shift in the beam position. Figure
5.15 shows a possible configuration to reach this output. The beam is introduced to
the White cell by bouncing in position “\(w\)” on the ITM. The first two bounces are
done in the null cell, positions \((L, 0)\) and \((A, 8)\), while \((L, 0)\) has a flat top rectangular
piece, \((A, 8)\) has a \(-10^\circ\) tilted top rectangular piece, shown in red. From \((A, 8)\), the
Figure 5.14: (a) Bounce pattern on pseudo-MEMS plane for third output on the underpopulated quartic White cell OXC. (b) Photograph of pseudo-MEMS

beam is sent to spherical mirror E (bounce 2), which shifts the beam position by $3\Delta$. The beam from spherical mirror E is sent to position $(H, 3)$ on the pseudo-MEMS. From $(H, 3)$ the beam is sent to spherical mirror A (bounce 3) by placing a $-10^\circ$ tilted top rectangular piece. The remaining bounces are done exclusively in the null cell. The beam gets out of the pseudo-MEMS on position $(z, 3)$.

We have so far prove that is possible to send different beam to the outputs 0 to 3. We will know send another set of inputs to the remaining outputs. To do so we arrange the rectangular pieces to create a new set of bounce patterns.

To send a beam to the fourth output is necessary to send the beam one time to mirror C, $\Delta$, and one time to mirror E, $3\Delta$. This case is shown in Figure 5.16. The beam bounces in position “$z$” on the ITM (bounce 0) and its send to $(I, 0)$ via
Figure 5.15: Bounce pattern on pseudo-MEMS plane for third output on the under-populated quartic White cell OXC.

spherical mirror A. Position \((I, 0)\) has a \(0^\circ\) tilted angle (i.e. flat top rectangular piece), so the beam is sent to position \((D, 8)\) via spherical mirror B (bounce 1). Position \((D, 8)\) has a \(+10^\circ\) angle which send the beam to \((E, 3)\) via mirror E (bounce 2). Now we send the beam to mirror C by placing a \(0^\circ\) tilted angle (bounce 3). This sends the beam to spherical mirror C, which shifts the beam by \(\Delta\). From mirror C the beam lands on position \((H, 4)\), which has a \(-10^\circ\) rectangular piece. The beam coming from C lands on position \((A, 4)\). Position \((A, 4)\) has a flat tilting rectangular piece, so the beam is sent to mirror B on bounce 5. The beam is sent to mirror A in bounce 6, and the beam gets out of the pseudo-MEMS on position \((w, 4)\).

Similarly, to send a beam to output 5 we need to send the beam one time to mirror E and two times to mirror C. To send a beam to output 6 we need to send the beam
Figure 5.16: Bounce pattern on pseudo-MEMS plane for fourth output on the under-populated quartic White cell OXC.

two times to mirror E and to send a beam to mirror 7 we need to send the beam two
times to mirror E and one time to mirror C. These cases are shown in Figures 5.17,
5.18 and 5.19, respectively.
Figure 5.17: Bounce pattern on pseudo-MEMS plane for fifth output on the under-populated quartic White cell OXC.

Figure 5.18: Bounce pattern on pseudo-MEMS plane for sixth output on the under-populated quartic White cell OXC.
Figure 5.19: Bounce pattern on pseudo-MEMS plane for seventh output on the underpopulated quartic White cell OXC.
5.3 Loss

In this section we will present the experimental loss. Before introducing the loss of the system we will show the individual loss of each of the different optical elements used in the setup.

The loss measurements were done as shown in Figure 5.20. Using our laser and a pair of apertures we set an optical axis. On one extreme of this optical axis we placed a Si-photodetector (Newport 818-SL), as shown in Figure 5.20(a). We record the readings of the photodetector; this will be our lossless measurement for our setup. Later we will introduce our field lens between the photodetector and the laser. It is important to mention that the lenses are uncoated, so the transmittance is set by the Fresnel reflections at a value of 4.2% for BK7 glass at 632.8 nm for each surfaces. The transmissivity due to Fresnel reflections alone is therefore expected to be 91.8%. The transmissivity was measured at 91.51% for the field lenses.

To measure the reflectivity of the spherical mirrors and of the psuedo-MEMS’s silicon reflective surface, we used the setup shown in Figure 5.20(b). As we can see the laser beam is reflected from the test surface and the reflection is sent to the photodetector.

The spherical mirrors are coated with enhanced aluminum, which is specified by the manufacturer to have a reflectivity > 95% for a wavelength between 0.45 - 0.65 µm [63]. We measured a 94.58% reflectivity for the spherical mirrors.

The reflectivity of the silicon fragments used in the psuedo-MEMS was expected to be 54.75%, but the measured reflectivity was only of 51.49% ±0.67. One reason for having such a low reflectivity for the silicon is that it was exposed to the environment for several weeks, and the surfaces would oxidize. Also, when the silicon was glued
Figure 5.20: Loss measurement of the individual optical elements. (a) Transmissive elements; (b) Reflective elements.

to the rectangular pieces of the pseudo-MEMS it may have been possible that we touched the surfaces and contaminated the samples.

Table 5.1 shows the theoretical loss values and the experimental loss values of our optical elements. We can see that there is a fair agreement between our expected and experimental values, except for the pseudo-MEMS silicon surface whose losses are lower by more that 3%.

<table>
<thead>
<tr>
<th>Optical Element</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field lens (transmissivity)</td>
<td>91.8%</td>
<td>91.51%</td>
</tr>
<tr>
<td>Spherical mirror (reflectivity)</td>
<td>&gt;95%</td>
<td>94.58%</td>
</tr>
<tr>
<td>Pseudo-MEMS’s silicon surface (reflectivity)</td>
<td>54.75%</td>
<td>51.49%</td>
</tr>
</tbody>
</table>

Table 5.1: Theoretical versus experimental transmissivity/reflectivity for the different optical elements used in the setup
We can see that the major contribution of loss will come from the pseudo-MEMS’s silicon surface. The fact that we are using uncoated elements also increases the loss of the system. The expected loss for a seven-bounce system using the theoretical values of table 5.1 was of 24.94 dB. If, on the other hand, we assume a reflectivity of 97% on the MEMS micromirrors and of 99% in the spherical mirrors (in case they are gold coated) and also a 98% transmissivity in the lenses, the losses for the system are calculated as low as 2.46 dB for a seven-bounce system. In Figure 5.22 we show the loss for a seven-bounce system using uncoated elements as well as coated elements, plotted as a function of bounce number.

In Figure 5.22 we show also the experimentally measured loss in our system. The loss was measured for each one of the eight different outputs in each one of its bounces on the psuedo-MEMS plane. The measurements were done using the setup shown in Figure 5.21. As we can see in the figure we used a beam splitter between the field lens and the psuedo-MEMS to send a fraction of the light (50%) to a Si-photodetector (Newport 818-SL). We used a small segment of a beam splitter, which was small enough to interrupt only one beam at a time. We took seven measurements (one per bounce) for each one of the eight possible outputs. It was necessary, therefore, to move our beamsplitter fragment in front of the correct silicon fragment on the pseudo-MEMS at each bounce. The results of these measurement are shown in Figure 5.21.

We found an experimental loss of 27.54 dB per output. One of the reasons of having such a large difference between the experimental and theoretical results is that the reflectivity of the bare silicon used in the pseudo-MEMS as micromirrors was lower than what we expected.
Figure 5.21: Loss measurement at the pseudo-MEMS plane for each one of the possible outputs (spherical mirrors not shown)
Figure 5.22: Loss presented in the system versus ideal value loss.
CHAPTER 6

SUMMARY AND CONCLUSIONS

In the present dissertation we have proposed new solutions to perform optical cross-connection (OXC). Our solutions are based in a combination of Micro-Electro-Mechanical systems (MEMS) with an optical White cell. The solutions presented have the attractive feature that even though they are free-space systems, thus allowing a high number of possible inputs and outputs, they use digital MEMS, which require a simpler control system than those usually involved in traditional free-space OXCs.

In chapter 2 we presented the original configuration of the White cell, and how this can be modified to create an OXC system. We showed that is possible to design two different systems: a quartic White cell OXC system, and a binary White cell OXC system. We did theoretical simulations on the quartic cell and the binary cell, including several spot displacement devices (SDD) designs. Experimental work was done only on the quartic cell because of its lower cost and lack of custom optics.

A contribution of this dissertation is that even though the binary and quartic configurations had been studied before at OSU for True-Time delays (TTDs), this is the first attempt to use these configurations for optical cross-connects. The SDDs designs and their performance evaluation is also another contribution of the present work.
Another important contribution is the concept of the beam-combiner, which was be explained in detailed in chapter 3. Here we will just mention that a basic problem in White cell based OXCs is how to couple light back into a fiber. We presented a White cell beam combiner approach compatible with the OXCs architecture that doesn’t present the fan-in loss problem.

6.1 Conclusions for the Spot Displacement Designs

As part of the description of the binary White cell OXC device on chapter 2 we analyzed three different designs for a spot displacement device (SDD) needed to shift the beams to the required outputs. The SDDs described in section 2.3 were: tilted spherical mirror SDD, roof prism SDD and the lens train SDD. We derived general equations for these devices in order to perform any desired beam displacement.

In order to better describe the performance of our SDD design we completed several simulations in chapter 4 for each SDD design assuming a displacement of 2Δ and 8Δ. We found out that the primary aberration was astigmatism in all cases arising from the oblique angle at which the beams are introduced into each SDD. Table 6.1 shows the main results obtained. All systems are assumed to have $\lambda = 0.6328 \mu m$, $\Delta = 250 \mu m$, and $w_0 = 15 \mu m$.

<table>
<thead>
<tr>
<th>SDD Design</th>
<th>Astigmatism 2Δ</th>
<th>Astigmatism 8Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tilted Spherical Mirror</td>
<td>0.03942</td>
<td>1.3418</td>
</tr>
<tr>
<td>Roof Prism</td>
<td>0.0085</td>
<td>0.0093</td>
</tr>
<tr>
<td>Lens Train</td>
<td>0.07764</td>
<td>0.1554</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of results for astigmatism in the SDDs design presented. All measurements in mm

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For the tilted spherical mirror SDD we found out that even though it has a simple geometry, it is suitable only for very small displacements (1∆). We found that the roof prism SDD has the best performance for a 2∆ shift and a 8∆ shift. We have to consider, however, that the size of the shift that a roof prism SDD can produce is limited by the diameter of its microlenses. For the system described the roof prism SDD will be adequate for up to 64 outputs. If longer displacements are needed it will be necessary to use a lens train SDD. Also, the roof prism SDD has higher tolerance for manufacturing errors, while the lens train SDD’s manufacturing tolerance decreases as we increase the amount of displacement. We also found that our devices are sensitive to misalignment.

6.2 Quartic System versus Binary System

In chapter 4 we did OSLO® simulations for the quartic cell and the binary cell. For both cases we simulated an eight-output system. The results of those simulations are reproduced in table 6.2. It shows the astigmatism present in a eight-output system using the underpopulated quartic White cell OXC and the binary White cell OXC with the roof prism SDD. It is necessary to remark that the quartic cell is a seven-bounce system using a three-state MEMS (±10°, 0°), while the binary OXC is a twelve-bounce system using a two-state MEMS (+10°, 0°). In both cases we are assuming \( \lambda = 0.6328 \, \mu m \), \( \Delta = 250 \, \mu m \) and \( w_o = 15 \, \mu m \).

From the table we may conclude that the quartic cell has a better performance that the binary cell. It will be necessary, however, to simulate systems with a higher number of possible outputs (> 64) to fully compare both systems and other SDD designs. Systems with such a high number of outputs weren’t simulated in the present
<table>
<thead>
<tr>
<th>Output Number</th>
<th>Quartic OXC</th>
<th>Binary OXC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.518163</td>
<td>0.890795</td>
</tr>
<tr>
<td>1</td>
<td>0.562724</td>
<td>0.968316</td>
</tr>
<tr>
<td>2</td>
<td>0.611252</td>
<td>1.00394</td>
</tr>
<tr>
<td>3</td>
<td>0.564003</td>
<td>1.221661</td>
</tr>
<tr>
<td>4</td>
<td>0.606938</td>
<td>0.959651</td>
</tr>
<tr>
<td>5</td>
<td>0.654272</td>
<td>1.2442</td>
</tr>
<tr>
<td>6</td>
<td>0.610088</td>
<td>1.283634</td>
</tr>
<tr>
<td>7</td>
<td>0.654588</td>
<td>1.585946</td>
</tr>
</tbody>
</table>

Table 6.2: Summary of results for astigmatism in the OXC architectures presented. All measurements in mm.

Dissertation because they require a higher number of surfaces than the maximum allowed by OSLO® (using sequential simulations).

The simulations of the binary cell showed that it is possible to introduce complex optical elements, like the SDD, without a significant increase in the aberrations of the system. This is an encouraging result for other systems based on the binary system, such as those shown in appendix B.

6.3 Experimental setup

In chapter 5 we realized an experimental setup of our underpopulated quartic White cell. This is the first attempt at The Ohio State University to demonstrate a quartic White cell OXC. The main objective of the experiments was to prove whether it was possible to control the final output of a particular beam by using the tilting mirrors of a MEMS.

Due to the lack of a MEMS, we designed and made a “pseudo-MEMS”. The pseudo-MEMS is a structure whose main function is to imitate the characteristics
of a proper MEMS. The main differences between our pseudo-MEMS and a proper MEMS is the size of the device and how the tilting of the micromirrors is controlled.

The alignment procedure is described in chapter 5. It is of interest to check the distances that were calculated by the imaging procedure of appendix A, with the distances obtained on the simulations using OSLO®, and the final distances obtained in the experimental setup. Table 6.3 shows the distances for our design. Null cell $d_0$ and null cell $d_1$ correspond to the distance between the MEMS plane and the field lens and the distance between the field lens and the spherical mirror on the null cell, respectively. Similarly, lateral wing $d_0$ and lateral wing $d_1$ correspond to the distances between the MEMS plane and the field lens, and the field lens and the spherical mirror on the lateral arms, respectively. We present in the first column the distances calculated by MAPLE using the paraxial procedure described in appendix A while the second column gives to the values obtained from the OSLO® simulations. The difference in these two measurements is mainly because in the Maple simulations, we are assuming that the beams are normal to the lenses and going through the center of all optical elements. From the simulations we know that the beams not necessarily go through the center of the lenses. The third column corresponds to the distances measured in the experimental setup. Again we see a discrepancy with respect to the theoretical values, nevertheless, we found a good agreement between our experimental setup distances and the OSLO® simulations.

We presented the bounce pattern for a beam sent to eight different outputs. The important aspect is that we are able to control the output row by sending the beam to specific spherical mirrors as we predicted. There is, however, a high amount of loss in the system. The expected loss for this experimental system was 24.9422 dB because...
of the uncoated optical elements. We found, however, an experimental loss of 27.54 dB (average). One of the reasons for having a discrepancy on the experimental results is that the reflectivity of the bare silicon used in the pseudo-MEMS as micromirrors was lower than what we expected. We were expecting a reflectivity of 0.5475, but the measured reflectivity was only of 0.5149 (average). One reason for the low reflectivity for our silicon is that it was exposed to the environment for several weeks, and also possible surface contamination. It is important to mention, however, that if we assume coated optical components (MEMS included) the theoretical loss is only 2.46 dB for seven bounces.

### 6.4 Beam Combiner

An important contribution is to address the problem of beam coupling at the output plane for any White cell-based OXC. This problem arises from the fact that different input beams arrive at different angles and positions when directed to a particular output. This will affect the coupling of the beam to a fiber or a detector. Our solution is based on curved a diffraction grating that takes advantage of a free-space architecture that makes it compatible with our OXC designs. Even though we have designed such a beam combiner, the simulation shows that the aberrations are

<table>
<thead>
<tr>
<th>Distance</th>
<th>Paraxial</th>
<th>OSLO</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null cell $d_0$</td>
<td>141.3062</td>
<td>138.603</td>
<td>139.0</td>
</tr>
<tr>
<td>Null cell $d_1$</td>
<td>390.87094</td>
<td>388.483</td>
<td>389.0</td>
</tr>
<tr>
<td>Lateral arms $d_0$</td>
<td>151.2364</td>
<td>148.3841</td>
<td>147.5</td>
</tr>
<tr>
<td>Lateral arms $d_1$</td>
<td>345.877</td>
<td>343.272</td>
<td>344.5</td>
</tr>
</tbody>
</table>

Table 6.3: C. All measurements in mm
high. Rather than discard this solution we believed that is necessary to redesign the
grating surface in order to correct the presented aberrations. To improve the design it
maybe necessary to assume that the grooves are created by a cylindrical or aspheric
wavefront impinging in the curved surface creating thus a user-defined diffraction
grating surface instead of the linear grating used in the present simulation.

6.5 Future work

With these results we have proven that it is possible to have a free-space OXC
and use digital MEMS thus combining the advantages of having large port counts
with simple control electronics. We based this system in the optical White cell.

As future work it will be desired to extent our simulations for a larger number
of possible outputs (≥ 64) for the quartic OXC system and the binary OXC. With
such high number of outputs it will be possible to have a better understanding of the
aberrations in each system, and have a better comparision between these two system.
Another advantage of having a high number of outputs is that it will require to have
another SDD design in the binary OXC.

It will also be important to implement a quartic OXC with a real MEMS and a
binary OXC with custom-made SDD to prove the validity of our simulations.

Another important area to be work in the future is the beam combiner, which
requires a better design, it would also be of interest to have an experimental setup
either in the quartic OXC or in a binary system.
APPENDIX A

IMAGING CONDITIONS

We will use ray 3x3 ray matrices to describe the imaging conditions of the White cell. These imaging conditions will also be analyzed for the case where an SDD has to be introduced in the beam path.

A.1 Imaging conditions for the White cell

Figure A.1 shows the basic setup for the White cell.

The ray matrix for a given optical element operates on an input vector.

\[
\begin{bmatrix}
y_0 \\
p_0 \\
1
\end{bmatrix}
\]

Where \( y_0 \) is the displacement of a ray incident on the element with respect to the optical axis, and \( p_0 \) is the slope of the incident ray. We are using 3x3 matrices because these are a more general solution than the usual 2x2 ray matrices. 3x3 ray matrices will help us to include tilted surfaces. The matrix for the element will then produce a new vector reflecting the effect of the optical element on the ray’s position and slope. The basic matrices used in this appendix are as follows:
Figure A.1: Basic setup for a White cell

\[
T_1 = \begin{bmatrix}
1 & d_1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (A.1)

is the translation matrix between the MEMS and the field Lens, where \( d_1 \) is the distance between these two elements;

\[
T_2 = \begin{bmatrix}
1 & d_2 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (A.2)

is the translation matrix between the field lens and spherical mirror A (B or C), where \( d_2 \) is the distance between the lens and any of the spherical mirrors;

\[
M = \begin{bmatrix}
1 & 0 & 0 \\
-\frac{2}{R} & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (A.3)
is the matrix for any spherical mirror of Figure A.1, where $R$ is its radius of curvature;

$$\text{Lens} = \begin{bmatrix}
1 & 0 & 0 \\
-\frac{1}{f} & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}$$ \hspace{1cm} (A.4)

is the matrix for the field lens, where $f$ is its the focal length, and

$$\text{SDD} = \begin{bmatrix}
o & p & u \\
q & r & v \\
s & t & w \\
\end{bmatrix}$$ \hspace{1cm} (A.5)

is the general matrix for our SDD (not shown in Figure A.1).

In order to have a beam bouncing back and forth among the spherical mirrors and the flat mirror some imaging conditions have to be fulfilled. These imaging conditions can be stated as follows:

- Mirror A images onto mirror B (or C) via the mirror M'.
- Mirror M' images into itself via mirror A (B or C).
- The beam spot size at mirror M' is constant, 1:1 magnification.

The first condition is expressed as:

$$\text{Image}_1 = T_2L_1T_1L_1T_2$$ \hspace{1cm} (A.6)

where $T_1, L_1, T_2$, and $M_{b/c}$ were described previously. Similarly, the second condition is expressed in equation (A.7).

$$\text{Image}_2 = T_1L_1T_2M_{b/c}T_2L_1T_1$$ \hspace{1cm} (A.7)
Solving for equation (A.6) we get:

\[
C_1 = \begin{bmatrix}
1 - \frac{2d_1 + d_2}{f} + \frac{2d_1d_2}{f^2}
& 2(d_1 + d_2) - \frac{2d_2 + 2d_1d_2}{f} + \frac{2d_1d_2}{f^2}
& 0

\frac{-2 + 2d_1}{f}
& 1 - \frac{2d_1 + d_2}{f} + \frac{2d_1d_2}{f^2}
& 0

0
& 0
& 1
\end{bmatrix}
\]  

(A.8)

In order to have imaging for this condition element \(C_1[1, 2]\) has to be zero. Solving equation (A.8) for \(f_1\), we obtain two solutions:

\[
f_1 = \frac{d_2}{d_1}
\]

These solutions represent symmetric and antisymmetric ray patterns. We choose the solution that puts any of the spherical mirrors on the focal plane of the field lens.

Now we have to consider the second condition that images the MEMS plane back into itself. From equation (A.7) we obtain:

\[
C_2 = \begin{bmatrix}
-1
& 2(d_2 - d_1) - 2d_2^2
& 0

0
& -1
& 0

0
& 0
& 1
\end{bmatrix}
\]  

(A.10)

To obtain an imaging condition element \(C_2[1, 2]\) has to be set equal to zero. Solving for \(R\):

\[
R = \frac{d_2^2}{d_2 - d_1}
\]  

(A.11)

Because of the symmetry of the White cell, the radius of curvature of the different spherical mirrors is the same.
A.2 Imaging condition for SDD

The previous matrix analysis is valid for the quartic and linear OXC; for the binary White cell it is also valid, but additional conditions are also necessary. These conditions are basically:

- Mirror E image into mirror F going through the SDD.
- The MEMS has to image back onto itself going through the SDD.

These conditions are expressed as follows:

\[ Image_3 = T_2L_1T_1SDDT_1L_1T_2 \]  \hspace{1cm} (A.12)

and

\[ Image_4 = T_1L_1T_2M_{b/c}T_2L_1T_1SDDT_1L_1T_2M_{b/c}T_2L_1T_1 \]  \hspace{1cm} (A.13)

where SDD is the general matrix defined in equation (A.5). Solving equation (A.12) gives:

\[
Image_3 = \begin{bmatrix}
\frac{q(d_2 - d_1) - r}{q(d_2^2 - d_2^2) - 2q(d_2 + 2p_1 + r) + s(d_2 + r + o) + p} & \frac{qd_2^2}{sd_2} & \frac{vd_2}{w} \\
\frac{-s(d_2^2 - d_2^2) + t}{d_2} & -o + q(d_2 - d_1) & d_2
\end{bmatrix}
\]  \hspace{1cm} (A.14)

In order to have imaging, we again have to set element \( Image_3[1,2] \) to zero. Because \( d_2 \) has already been defined previously in equation (A.9), element \( q \) on the SDD matrix has to be zero. Also, element [1,3] represents a displacement in the image. In the case of imaging E onto F, such an offset may cause the light to be displaced enough that it misses Mirror F. So we would require element [1,3] to be
zero, setting thus \( v \) to zero. Also element \([3,2]\) has to be equal to zero and element \([3,3]\) equal to 1, so element \( s \) and \( w \) are set equal to 0 and 1 respectively. Matrix \( \text{Image}_3 \) is then reduced to:

\[
\text{Image}_3 = \begin{bmatrix}
-\frac{r}{d_2} & 0 & 0 \\
\frac{d_2(o+r)+d_1(r+o)+p}{d^2} & -o & -\frac{u}{d_2} \\
-\frac{1}{d_2} & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (A.15)

Further simplification of equation (A.15) is possible before analyzing the fourth condition. Because we need magnification 1:1 we need to set elements \( r \) and \( o \) to 1, also element \( t \) will be zero. Assuming these condition \( \text{Image}_3 \) simplifies to:

\[
\text{Image}_3 = \begin{bmatrix}
-1 & 0 & 0 \\
\frac{2(d_2+d_1)+p}{d^2} & -1 & -\frac{u}{d_2} \\
0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (A.16)

Now, to reduce SDD matrix, we substitute equation (A.16) in equation (A.13):

\[
\text{Image}_3 = \begin{bmatrix}
-1 & p & -u \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (A.17)

To have imaging we set \( p \) equal to zero. The displacement caused by the SDD on the MEMS plane is then caused exclusively by element \( u \). The final matrix, therefore, that we desire for our SDD is as follows:

\[
\text{Image}_3 = \begin{bmatrix}
-1 & 0 & -u \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (A.18)
APPENDIX B

ADDITIONAL FEATURES ON THE BINARY WHITE SYSTEM BY SDD SUBSTITUTION

The optical White cell has been proven to be a very dynamic system that can be easily modified to be implemented as a true-time delay device \[^{[64]}\], and an optical cross-connection \[^{[30], [42]}\]. In the present section we will substitute other optical devices for the SDD columns to allow for even more functionality. The objective of the present appendix is to show how versatile the binary White cell systems can be.

In the first part of this appendix we will present a new design based in the binary White cell that allows for each element of a input beam array to be split into up to eight beams; thus, it is a programmable beamsplitter. The system allows one to control each beam independently.

In the second part we will introduce an optical circulator into one of the SDD columns. To include an optical circulator will provide our binary OXC system with the ability to add or drop specific wavelengths.
B.1 Beam Splitter White cell Design

Basically what we need to do is to design a system that allows us to divide a beam a determined number of times. It is also possible that a particular input won’t need to be divided at all.

The problem is then set as a dynamically controlled beam splitter, where a particular beam can pass through a beam splitter more than once. This system is based in the White cell binary configuration \[39 \ 41\].

B.1.1 Binary Cell

A detailed explanation of the basic principles of the White cell are given in \[37 \ 38\]. Here we just say that the White cell allows many input beams to bounce independently between a set of three spherical mirrors, two facing the other, with identical radii of curvature, and separated by a distance equal to their radii of curvature. The beams are refocused to spots on one mirror on each bounce. We replace that mirror with a micro-electromechanical system (MEMS) mirror array. Each micro-mirror on the MEMS has two different tilted positions, which can be controlled. When a beam is refocused on a micro-mirror it can be directed to what will be referred to as a Beam Splitter Device (BSD), or to a flat mirror (auxiliary mirror I).

The White cell by its nature produces patterns of spots in rows (or columns). When the beam is directed to the BSD, the beam will be divided in two beams of equal intensity. Every time a beam is send to a BSD, the beam will be divided in the same manner. Depending on the number of times a beam goes to the BSD we can divide it into a specific number of beams of equal intensity.
Figure B.1 shows the White cell binary system with non-specific BSD. The Figure shows four spherical mirrors, a two-state MEMS, a flat auxiliary mirror, and a BSD. According to the connectivity diagram in the lower right part of the Figure, beams can bounce back and forth between mirror A and B. When we need to divide a beam a micromirror in the MEMS changes its position sending a beam to the BSD where the beam is divided in a 50/50 fashion. Thus, two beams return from the BSD for each beam sent there.

A detailed explanation of the bounce pattern can be found in [41]. Here we just say that the beams can be directed to the BSD by tilting a micro-mirror in the MEMS to the right position.
B.1.2 Beam Splitter Device

In this section we will discuss how the beams are divided. The first step to creating the beam splitter White cell system is to design a Beam Splitter Device (BSD). Each time a beam lands on the BSD it will divide that beam into two beams, each with the same intensity (half of the original beam).

An important aspect of the BSD design is that the optical path length of every beam has to be the same. As long as the beams are bouncing between the MEMs and auxiliary mirror I the optical paths are the same. We need to guarantee that the optical path lengths of the beams when they go inside the BSD will also be the same for each beam.

Imaging conditions also need to be fulfilled. The BSD can perform several changes on the beam and still meet the imaging conditions of the White Cell, as long as the beams at the input and output of the BSD are the same (spot size, angle of propagation).

BSD Design

A simple approach to generating a Beam Splitter Design is by using a compound optical system composed of a beam splitter (BS), a penta prism (PP) and a 180 retro reflector prisms (RP). We will take advantage of total internal reflection (TIR) caused by the PP and the RP to minimize loss. All elements can be made of the same material (e.g. BK7) so that there are no Fresnel reflections from the interfaces between the elements.

In Figure B.2 we show the general design for the BSD using the elements previously mentioned. Light coming from the White cell will be split by the beam splitter into
two beams with equal intensity. One beam will propagate inside the penta prism. In general a beam coming inside a penta prism will come out at a $90^\circ$ angle with respect to the input beam. As it can be seen in Figure B.2 the beam coming out of the penta prism will go back to the White cell. On the other segment the respective beam will travel through the retro reflector prism, changing the trajectory of the beam by $180^\circ$, sending the beam back to the White cell.

Each beam will have to travel through the same length to avoid any time delay between them. So is necessary to describe the optical paths in each prism segment and make them equal.

**Optical Paths**

Figure B.3a shows a retro reflector prism with the beam splitter. The beam splitter has length $l_1$. The retro reflector is a simple right-angle prism. By simple geometry we can calculate that the optical path for the retro reflector segment is only $3l_1$. 

Figure B.2: BSD Design conformed by a beam splitter (BS), penta prism (PP) and retroflector prism (RP)
It may be fair to assume at this point that the optical paths of both segments of the BSD will be different, and that it may be necessary to compensate for this difference in the optical paths. We will do this by adding a small segment of length $l_2$. The addition of this segment can be seen in Figure B.3. The optical path of the retro reflector prism is then:

$$OP_{rp} = 3l_1 + l_2$$  \hspace{1cm} (B.1)

![Figure B.3: Retro reflector segment of the BSD](image)

The optical path of the penta prism segment is a little bit more complicated than for the retro reflector segment, but it can be also calculated by simple geometry. To do so, we divide the penta prism in simple geometrical figures as shown in Figure B.4. The penta prism is then divided into a square of side $l_1$, two triangles with one of their legs equal to $0.5l_1$, and an irregular polygon that won’t be taken into consideration. It is important to notice that the penta prism has an angle of $67.5^\circ$ as shown in the
This angle comes from the fact that after two bounces the beam is reflected by $90^\circ$.

After the original beam is divided by the beam splitter, one beam goes through the square section, in the penta prism, travelling a distance $l_1$, then it goes through the short leg of the triangle (of length $0.5l_1 \tan \theta$), bounces by internal total reflection, travels a distance $l_s$, bounces again at the second surface and goes again through a distance $l_1 + 0.5l_1 \tan \theta$. The optical path of the penta prism can then be calculated as:

$$OP_{pp} = 2 \left( l_1 + \frac{l_1}{2} \tan \theta \right) + l_s \tag{B.2}$$

where $l_s$ can also be defined as function of $l_1$ as:

$$l_s = l_1 \sqrt{2} - l_1 \tan \theta \tag{B.3}$$

where $\theta = 90 - 67.5 = 22.5^\circ$

Therefore, substituting \[B.3\] into \[B.2\] we get:
\[ \text{OP}_{pp} = 2 \left( l_1 + \frac{l_1}{2} \tan \theta \right) + l_1 \sqrt{2} - l_1 \tan \theta \]
\[ = 2l_1 + l_1 \sqrt{2} \quad (B.4) \]

To make both optical paths equal we solve for \( l_2 \):

\[ \text{OP}_{pp} = \text{OP}_{rp} \]
\[ 2l_1 + l_1 \sqrt{2} = 3l_1 + l_2 \]
\[ l_2 = l_1 (\sqrt{2} - 1) \quad (B.5) \]

The final BSD design is shown in Figure B.5, where the distance between the output beams is \( 2l_1 \).

![Final BSD design](image)

**Figure B.5:** Final BSD design
Family Tree

So far, we have seen how is possible to divide a particular beam into two beams. We have seen also that the optical path length for each beam is equal so they won’t have any time delay with respect to each other. Now, we need to know how the BSDs will be placed in the White cell.

From figure [B.5] we know that the output beams from the BSD will be displaced from the input’s position, and that this displacement will be symmetrical to the input axis of the original beam.

Figure [B.6] shows a “Family Tree” with dashed lines. It is a symmetric tree formed by our BSD. The tree represents the amount of displacement that is caused by the BSD in each beam pair. This displacement is important, because it will define the required size of the MEMS and optical elements.

Figure B.6: Family Tree caused by the BSD.
Also in figure B.6 we can see that the tree is drawn inside a mesh. Let’s assume that each square in the mesh is a micro-mirror, and that their pitch is 250 $\mu$m. From the figure we can see that the distance between the divided beams after the first time we go to the BSD are separated by nine times the pixel pitch, or 2.250 mm. After the second pass the two outermost beams are separated by 13 times the pixel pitch (3.250 mm) and after the third pass on the BSD the two outermost beams are separated by 15 times the pixel pitch (3.750 mm).

### B.1.3 Physical Implementation

Let’s assume a 256 fiber array input, with a 250 $\mu$m pitch. If we considered that the fibers are arranged in a linear array, this array will be 64 mm wide. If all beams are to be divided eight times we will need a MEMS at least of 256 x 15 micromirrors (64mm x 3.750 mm).

We would like the lenses and mirror used to be around two inches wide, or 50.8 mm. Two-inch optical elements are easier to handle and the needed mounts are also cheaper than custom elements. It is then convenient to arrange our 256-element array input in a different way. Let’s assume that we arrange our 256 beam array in a 64 x 4 array, as shown in Figure B.7. In the figure we can see four rows of 64 elements each. Each element represents an input spot (shown in red). The distance between rows is $(16)(250\mu m) = 4.00 mm$ which will allow for each individual input beam to be divided up to eight times.

With this input array the final size of the MEMS plane will be 4 rows of 64 elements with 250 $\mu$m pitch. Each row is separated by 4.00 mm. This give us a final size of 16 mm x 16 mm. With these dimensions for the MEMS plane we can calculate
the optical elements needed to provide imaging conditions in the White cell. We use the procedure described in appendix A to calculate this optical elements. Here we present only the results in Table B.1

Is important to notice from the data of table B.1 that the diameter of the field lens is set at 50.8 mm, and that the distance to the spherical mirror is 506.09 mm. The f# for this system is then $\approx 10$. For a paraxial approximation the f# should be around 8 and 10. So our system is well within that range.
### Table B.1: Summary for Beam Splitter White cell System

<table>
<thead>
<tr>
<th>Beam characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>0.25 mm</td>
</tr>
<tr>
<td>Wavelength</td>
<td>1550 nm</td>
</tr>
<tr>
<td>Array</td>
<td>64 x 4</td>
</tr>
<tr>
<td>Spot size</td>
<td>25 μm</td>
</tr>
<tr>
<td>Refractive index</td>
<td>1.5006 (BK 7 at 1550)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Field Lens: Meniscus Lens (Melles Griot)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length</td>
<td>500 mm @ 546 nm</td>
</tr>
<tr>
<td>Diameter</td>
<td>50.8 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spherical Mirror (CVI Laser)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of curvature</td>
<td>600 mm</td>
</tr>
<tr>
<td>Diameter</td>
<td>50.8 mm (2&quot;)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distances and Dimensions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance between MEMS and field lens</td>
<td>78.171 mm</td>
</tr>
<tr>
<td>Distance between field lens and Sph.</td>
<td></td>
</tr>
<tr>
<td>Mirror</td>
<td>506.09 mm</td>
</tr>
<tr>
<td>Divergence</td>
<td>0.019735 rad</td>
</tr>
<tr>
<td>Rayleigh Range</td>
<td>1.2667 mm</td>
</tr>
</tbody>
</table>
Loss

Another important consideration for the beamsplitter device is the amount of loss that each beam will experience after several bounces. We have designed assuming a beam spot size around a fourth of the micro-mirror size \( w_o = 25 \mu m \); the spherical mirror and lenses are big enough to eliminate any substantial diffraction loss. So the main source of loss will be reflection/transmission of the beam through the different optical elements.

We can assume that the BSD and the MEMS are coated with a high-reflectivity coating tuned at 1550 nm. The maximum loss will then be defined for that beam that goes to the BSD in every single bounce. A v-coating from Melles Griot can be as good as 0.1% reflection loss.

A beam needs to go through 72 interfaces to be divided into 8 beams. So the losses for this systems are:

\[
\text{Loss} = (1 - 0.001)^{72} \\
= 0.9305 \\
\Rightarrow -7.2 dB
\]  

(B.6)

B.2 Binary configuration with Wavelength Add-Drop Multiplexer

In this section we will substitute for the SDD column with an optical circulator. The introduction of the optical circulator will provide our OXC system with...
the ability to add or drop wavelengths. This is necessary in order to reuse the wavelengths somewhere else in the network, or just to prevent different signals with same wavelength to be transmitted through the same channel.

Wavelength add-drop multiplexers (WADMs) are able to insert or remove WDM channels at intermediate points in the network. Thus far, switchable WADM’s have been made using planar waveguide switches [65], micro-machined tilting or on/off mirrors [66, 67], and liquid-crystal switches [68]. It is the objective of this section to show how WADM functions can be implemented in our Binary White cell configuration.

Figure B.8 shows the basic functions performed by an WADM. In the figure we can see how three beams at different wavelengths (indicated by the colors green, red and blue) are introduced into our WADM via the input port. The wavelength represented by the red color is dropped by our WADM and two additional wavelengths are introduced via the “add” port with different wavelengths represented by colors yellow and turquoise, respectively. The green and blue wavelengths go through the WADM without any modification. As a consequence, at the output port we have four different wavelengths green, yellow, turquoise and blue.

![Figure B.8: Wavelength add-drop multiplexer basic functions.](image-url)
The problem of our WADM can be reduced to the design of a free-space optical circulator. An optical circulator is defined as a multi-port (at least three) non-reciprocal passive component. Its function is to transmit a beam from one port to the next sequential port, and to block any transmission from one port to the previous port. Optical circulators can be categorized as: (I) full circulators, in which light passes through all ports in a full loop; and (II) quasi-circulators in which light can from the last port cannot be transmitted back to the first port. For example in a quasi-circulator light passes through port 1 to port 2 and from port 2 to port 3, but any light from port 3 cannot go back to port 1 (in a full circulator light from port 3 goes back to port 1). In our case we need one input/output port, a drop port and an add port. Light from the input port is sent to the drop port, while a new wavelength is sent back to the output port from the add drop. It will be enough to implement a free-space quasi-three-port optical circulator for our system.

In Figure 13.9(a) a schematic for a four-port quasi-circulator is shown. The design is based on a Mach-Zehnder interferometer where we have included a half-waveplate and a Faraday rotator. When a beam goes through port 1 the beam splitters divides it into two orthogonal polarizations. The two beams are passed through a half wave plate and a Faraday rotator. The optical axis of the half-wave plate is rotated 22.5° with respect to the x-axis so that the vertically polarized light is rotated +45°. The Faraday rotator is chosen so that it provides a −45° when light propagates along the z-axis. As a consequence the Faraday rotator cancels the rotation caused by the half-wave plate, therefore the polarization of both beams are unchanged. The two beams are recombined by the second beam splitter and coupled into port 2.
Figure B.9: Optical quasi-circulator. (a) Free-space implementation; (b) schematics.

When a beam is sent through port 2, it is again split into two orthogonal polarizations by the second beam splitter. Now, due to the non-reciprocal nature of the Faraday rotator, when light propagates in the z-direction, the total effect after going through both elements results in a total rotation of 90°. Therefore the two beams are combined by the first beam splitter in a direction orthogonal to port 1 and coupled into port 3. Therefore, our free-space WADM can be implemented by using port two as our input/ouput port and port 1 and 3 as the add - drop port respectively, as shown in Figure B.9(b). In this particular configuration port 4 is not used.

Figure B.10 shows a binary White cell OXC where a series of SDD columns are used to switch the beam to different positions. In order to implement WADM capabilities into our OXC system we will substitute for one column of the SDD with the optical circulator we showed in Figure B.9.
Figure 3.10(b) shows in detail how an optical circulator can be introduced as a substitution for an SDD column. In the figure we show how a beam once is introduced on the optical circulator via port 2 will be “drop” on port 1, at the same time another beam with a different wavelength (shown in green in the figure) is “added” via port 3 and sent into the White cell via port 2. When a particular wavelength has to be added or dropped, the MEMS will direct the beam to the WADM where its wavelength will be changed. This add-drop function has the penalty that by eliminating one SDD column we reduce the number of possible outputs for our OXC system. This drawback can be solved by increasing the number of bounces to replace the lost SDD column.

It is interesting to understand that we can substitute any SDD column by any other optical device, like a WADM in this case. The only condition that we have to do this substitution is to guarantee that the input and output beams have the correct angle of propagation and spot size.
Figure B.10: Binary White cell OXC with WADM capabilities.
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