ANALYSIS OF CRACK PROPAGATION
IN ASPHALT CONCRETE
USING A COHESIVE CRACK MODEL

A Thesis

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by

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* * * * *

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<tr>
<td>A</td>
<td>cross sectional area</td>
</tr>
<tr>
<td>a</td>
<td>crack depth and notch depth (or initial notch)</td>
</tr>
<tr>
<td>b</td>
<td>beam depth</td>
</tr>
<tr>
<td>CMOD</td>
<td>crack mouth opening displacement</td>
</tr>
<tr>
<td>COD</td>
<td>crack opening displacement</td>
</tr>
<tr>
<td>d</td>
<td>plastic zone length</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus</td>
</tr>
<tr>
<td>(f_f)</td>
<td>flexural tensile strength</td>
</tr>
<tr>
<td>(f_{net})</td>
<td>net flexural tensile strength</td>
</tr>
<tr>
<td>(f_t)</td>
<td>tensile strength</td>
</tr>
<tr>
<td>(F(a/b))</td>
<td>geometry correction factor</td>
</tr>
<tr>
<td>G</td>
<td>strain energy release rate</td>
</tr>
<tr>
<td>(G_e)</td>
<td>critical strain energy release rate</td>
</tr>
<tr>
<td>(G_F)</td>
<td>fracture energy</td>
</tr>
<tr>
<td>(G(t))</td>
<td>intensity of cohesive force acting near the crack edge</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity</td>
</tr>
<tr>
<td>(J_{lc})</td>
<td>critical value of J-integral</td>
</tr>
<tr>
<td>(K_I)</td>
<td>mode I stress intensity factor</td>
</tr>
<tr>
<td>(K_{lc})</td>
<td>fracture toughness or critical stress intensity factor</td>
</tr>
<tr>
<td>(K_0)</td>
<td>modulus of cohesion</td>
</tr>
<tr>
<td>(L_{ch})</td>
<td>characteristic length</td>
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MOR  
modulus of rupture

M  
mass of beam

P  
load

s  
initial slope of load-deflection curve

t  
temperature

$U_T$  
$\text{total}$ energy to failure

W  
$\text{beam}$ width

w  
width of cohesive crack

$w_c$  
critical value of cohesive crack opening

$\sigma$  
stress

$\sigma_c$  
fracture resistance

$\sigma_{ij}$  
$\text{stress in front of a}$ crack tip

$\sigma_0$  
far field $\text{stress}$ or crack driving force

$\sigma_y$  
yield strength

$\sigma_{yy}$  
tensile stress around the hole

$\sigma^m_i$  
$\text{stress of node } i \text{ at } m^{th}$ step

$\dot{\sigma}$  
loading rate

$\varepsilon$  
strain

$\delta$  
load-line displacement

$\rho$  
radius of curvature at the crack tip

$\gamma$  
surface energy per unit $\text{area}$

$\gamma_p$  
plastic energy per unit of crack extension per crack tip

$\delta U$  
decrease in potential energy

$\delta U_{SE}$  
increase in surface energy

$\delta U_{PL}$  
plastic energy dissipated

v  
Poisson's ratio
\( \alpha_s \) shear shape factor

\( V_2(a/b) \) a function depends on the value of \( a/b \)
1.1 General

A rational pavement design method must lead to the prediction of the pavement performance during its service life as well as the prevention of catastrophic failure. Generally, due to the destructive action of the traffic load and the environment, the structural integrity of a pavement is damaged by several distress mechanisms developed in the structure. These distress mechanisms will propagate either independently or interact with each other and finally produce disintegration, distortion and fracture of the pavement.

The structural damage in flexible pavements appears in two main forms: cracking and deformation, both of which are due to load repetitions or adverse environmental conditions [1]. Damage associated with permanent deformation is believed to be due to shear distortion or volumetric changes. On the other hand, cracking occurs primarily as a result of the applied load and possible volumetric change due to environmental conditions. To illustrate possible factors that may affect crack propagation in asphalt concrete, the development of reflection cracks in an asphalt overlay is used as an example.
Reflection cracking refers to cracking in a new pavement surface that appears as a result of the presence of the crack or joint in the old pavement. This type of cracking can be caused by low temperature thermal cracks and the deflection of the underlying concrete. The movements of the cracks or joints in the old pavement cause a physical tearing of the overlay material at the discontinuity produced by the joints or cracks.

There are several causes which produce movements at the discontinuities. These causes are: (a) low temperature contraction, (b) daily temperature cycles, and (c) traffic loads [2].

As shown in Fig. 1.1 [2], low temperatures cause a horizontal opening due to the contraction in the old surface when the temperature drops (e.g., from summer to winter). This opening mode of fracture (mode I) produces a high tensile stress in the overlay directly over the cracks or joints in the original pavements. Due to the temperature drops in the overlay, an additional tensile thermal stress is produced in the overlay as illustrated in Fig. 1.2 [2]. The additional thermal stress is equal in every section of the overlay. The sum of the stresses everywhere in the overlay will be most severe over the joint or the crack where the additional stresses produced by the opening are found.

Daily temperature cycles also result in the thermal tensile stresses in the overlay. In the concrete slab, the temperature cycling produces temperature gradients that cause a curling at the joints or
Note:

1. $S_L$ is not necessarily equal to $S_R$ if joint or crack spacing varies from one slab to another.

2. $S_{\text{top}}$ may not be equal to $S_{\text{bottom}}$ which will also produce curling.

3. The horizontal opening at the top of the slab will vary daily with short term temperature cycling as well as seasonally due to long term temperature cycling.

Fig. 1.1 Horizontal movement due to temperature change in the old pavement.

Note:

The thermal crack will start from the top. With a reflection crack starting at the bottom, the stress distribution will actually propagate a crack from both directions.

Fig. 1.2 Thermal stress in the overlay above crack or joint.
cracks. Figure 1.3 [2] shows the curling action in the slab. The upward curl is produced when the top of the slab is cooler than the bottom. This curling action causes an opening of the crack or joint which occurs more often than the low temperature opening. However, since the temperature gradient in a slab beneath an overlay is relatively small compared to that of low temperature, curling action causes less damage than the temperature opening.

Another important cause of stresses and crack growth is traffic loads. Traffic loads produce bending (point B in Fig. 1.4 indicates the maximum bending stress) and shearing stresses (as shown at points A and C in Fig. 1.4) in the old pavement and in the overlay above it. Each time a load passes over a crack or joint in the old pavement, three pulses of high stress concentration occur at the tip of the crack as it grows upward through the overlay. With each pulse of high stress concentration, the crack grows a little bit more. Because there is a void beneath the old pavement, the maximum shearing stress at point C is higher than at point A.

The rate of growth of the crack depends upon the amount of load transfer that exists across the crack or joint in the old pavement. Thus, it is important to determine how much load transfer the old pavement has across its cracks or joints when designing an overlay to resist reflection cracking.

From the above discussion, it is obvious that environmental conditions (e.g., temperature) and traffic loads are important factors
Fig. 1.3 Stress caused by thermal curling of the pavement slab due to temperature differential through the pavement slab.
that affect the crack propagation of the overlay. Each type of load cycle causes the crack to propagate a different amount. To correctly predict the development of a crack in asphalt concrete pavement, one needs to know the basic properties that govern the crack resistance of asphalt concrete. However, so far, there is no rational model that can be used to properly predict the crack resistance of asphalt concrete. It is the attempt of the present study to develop a theoretical model to simulate the progressive cracking in asphalt concrete.

1.2 Research Objectives

The objectives of this study can be summarized as follows:

1. To develop a numerical model to analyze the progressive crack propagation in asphalt concrete.

2. To experimentally measure the material properties associated with the proposed analytical model for asphalt concrete.

3. To perform a parametric study on the response of asphalt concrete subjected to different environmental and loading conditions.

1.3 Scope of the study

The present study consists of the following parts:

1. A review of fracture mechanics is given in Chapter 11. This chapter includes two main topics: (a) linear elastic fracture mechanics, in which energy criterion and stress intensity factor
criterion are discussed; and (b) elastic-plastic fracture mechanics, in which discussion of the Dugdale model and the Barenblatt model is given.

2. The theoretical framework of the cohesive crack model for asphalt concrete is discussed in Chapter III. In this chapter the concept of the cohesive crack model is derived. A numerical program based on the cohesive crack model is developed and detailed.

3. An experimental program which was designed to measure the basic material properties of asphalt concrete is presented in Chapter IV.

4. Chapter V includes the following sections. First, a numerical simulation is performed to curve fit the experimental results in order to determine the stress-separation relationship for asphalt concrete. Second, the development of the process zone in asphalt concrete is analyzed and reported. Third, the effects of the specimen, notch length, and temperature on the behavior of asphalt concrete are discussed based on the results obtained from the parametric study. Finally, the temperature effect on fracture toughness ($K_{1c}$) and critical J-integral ($J_{1c}$) are investigated using the proposed model and compared with the available experimental results.

5. Chapter VI concludes the findings obtained from the present study. Recommendations are also suggested for future research.
CHAPTER II
A REVIEW OF FRACTURE MECHANICS

2.1 Introduction

Selection of failure criterion which can be used to estimate the failure strength of a structure, is an important aspect of structural design. The traditional design method often does not take into account the existence of flaws and inherent defects. However, in the microscopic aspect, materials contain defects and imperfections within the crystalline structure, and other forms of flaws resulting from manufacturing and handling processes. The existence of flaws such as joints and cracks causes a redistribution of stress and stress concentration around the discontinuous parts of the materials. As a result, the failure strength predicted using conventional strength criterion may not be reliable and may overestimate the actual strength of the structure. To account for the existence of flaws in the material, fracture mechanics approaches have been used to provide a rational analysis for measuring the reduction of the strength.
2.2 Stresses in the presence of cracks

Figure 2.1 shows an infinitely large plate of linear elastic material with an elliptical hole. The plate is subjected to a uniform tension, \( \sigma_0 \). The stress distribution around the hole was solved by Inglis in 1913 [3] and is given as:

\[
\sigma_{yy} = \sigma_0 \left( 1 + \frac{2a}{c} \right) = \sigma_0 \left( 1 + 2 \sqrt{\frac{a}{\rho}} \right)
\]  

(2.1)

in which

\( \sigma_{yy} \) = tensile stress around the hole (along the y-direction),

\( \sigma_0 \) = far field stress,

\( a \) = major axis of the ellipse,

\( c \) = minor axis of the ellipse,

\( \rho \) = radius of curvature at the crack tip.

From Equation 2.1, it can be seen that \( \sigma_{yy} \) depends on \( a \) (the half length of the hole), and the radius of curvature (\( \rho \)) at the point of interest. Note that for a circular hole with \( a = \rho \) (i.e., \( a = c = \rho \)), the tensile stress (\( \sigma_{yy} \)) around the hole is equal to \( 3\sigma_0 \). This indicates that the hole will reduce the strength of a material by three times if a strength criterion is used. For a sharp crack, where \( \rho = 0 \), an infinite stress at the ends of the crack is expected. This result suggests that a material with a crack can not sustain any applied load if one assumes the strength criterion. In reality, it has been observed
that a material with flaws (or cracks) still has the ability to resist a certain amount of applied load. This indicates that the ordinary stress criteria are not appropriate to this case and the fracture mechanics concepts, which will be discussed in the following sections, have to be applied.

![Stress Distribution Diagram](image)

Fig. 2.1 The stress distribution around an elliptical hole in an infinitely large plate subjected to a uniform tension $\sigma_0$

### 2.3 Linear Elastic Fracture Mechanics (LEFM)

#### 2.3.1 Energy criterion

The first of the fracture mechanics approaches was proposed by Griffith in 1921 [4]. He proposed that a brittle body fails because of the presence of numerous internal cracks or flaws which produce local stress concentration. He stated that the elastic body under stress must transfer from an undamaged to a damaged state by a
process during which a decrease of the potential energy takes place. He also stated that fracture instability is reached when the increase in surface energy, which is caused by the extension of the crack, is balanced by the release of elastic-strain energy in the volume surrounding the crack. Figure 2.2 shows an infinitely large plate with an initial crack length of 2a which is subjected to a uniform tension, $\sigma_0$. Based on the assumption of a constant surface energy, Griffith energy criterion for crack propagation can be represented mathematically as:

$$\delta U \geq \delta U_{SE}$$

(2.2)

in which

$$SU = \text{the decrease in potential energy due to increased crack surface},$$

$$\delta U_{SE} = \text{the increase in surface energy due to increased crack surface}.$$

The expression for the critical stress ($\sigma_c$) at which a crack will propagate based on the Griffith energy criterion can be written as:

$$\sigma_0 = \sigma_c \quad \text{(crack driving force = fracture resistance)}$$

(2.3)

and

$$\sigma_c = \sqrt{\frac{2\gamma E}{\pi a}}$$

(2.4)

in which
\[ \gamma = \text{surface energy per unit area}, \]
\[ E = \text{Young's modulus}. \]

The Griffith energy criterion is based on the assumption that there is no (or negligible) plastic deformation in the material. However, for most materials (e.g., metals), plastic deformation always takes place.

Thus, the Griffith criterion has to be modified for materials with significant plastic deformation. Orowan [5, 6] and Irwin [7, 8, 9] came to the conclusion that even a slight plastic flow which occurs in the brittle fracture case will absorb a great amount of additional energy required to create new surfaces.
Irwin's criterion for crack propagation can be expressed as follows:

\[ \delta U \geq \delta U_{SE} + \delta U_{PL} \] (2.5)

in which

\[ \delta U_{PL} = \text{the plastic energy dissipated due to increased crack surface.} \]

Similarly, the critical stress \((\sigma_c)\) at which a crack will propagate can be expressed as:

\[ \sigma_c = \sqrt{\frac{(2\gamma + \gamma_p)E}{\pi a}} \] (2.6)

in which

\[ \gamma_p = \text{the plastic energy per unit of crack extension per crack tip.} \]

Irwin recognized that the plastic energy dissipated in material is much higher than the surface energy dissipated. Therefore, he proposed that the surface energy can be neglected when compared with plastic energy.

### 2.3.2 Stress Intensity Factor Criterion

The distribution of the stresses in front of a crack tip \((\sigma_{ij})\) as shown in Figure 2.3 can be expressed by the following equation \([8]\) :

\[ \sigma_{ij} = \frac{K_1}{\sqrt{2\pi x}} + \text{higher order terms} \] (2.7)
where $x$ is the distance from the crack tip, $K_i$ is the mode I stress intensity factor, and $\sigma_{ij}$ is the near tip stress. Generally, $K_i$ depends on the applied load, the crack length, and the shape of the specimen. The mode I stress intensity factor ($K_i$) is usually expressed in the following form [10]:

$$K_i = \sigma \sqrt{\pi a} F(\frac{a}{b})$$  \hspace{1cm} (2.8)

in which $\sigma$ is the nominal stress, $a$ is the length of an edge crack or half the length of an interior crack, and $F(a/b)$ is a geometry correction factor which depends on the type of load, the crack length and the shape of the specimen.

![Stress distribution in front of a crack tip](image)

**Fig. 2.3 Stress distribution in front of a crack tip according to the theory of elasticity.**

It can be noted from Eq. 2.7 that the stresses near the crack tip are square-root singular. Therefore, stress criterion is not applicable as discussed earlier.
According to Irwin [8], the following relation can be derived between the mode I stress intensity factor \( (K_1) \) and the energy release rate \( (G) \):

\[
K_1 = \sqrt{EG} \quad \text{for plane stress} \quad (2.9)
\]

\[
K_1 = \sqrt{\frac{EG}{1 - \nu^2}} \quad \text{for plane strain} \quad (2.10)
\]

For asphalt concrete with \( \nu=0.25 \), the difference in \( K_1 \) between plane stress and plane strain cases will be less than 3.5 percent if the following equations are used:

\[
K_1 = \sqrt{EG} \quad (2.9)
\]

or

\[
K_{IC} = \sqrt{EG_c} \quad (2.11)
\]

in which \( G_c \) is the critical strain energy release rate. Basically, the use of the critical stress intensity factor as a crack propagation criterion is equivalent to the use of the critical strain energy release rate.

### 2.4 Elastic-Plastic Fracture Mechanics

#### 2.4.1 The Dugdale Model

Without modification, LEFM can not be used to describe the behavior of a real material if the fracture process zone is not negligible compared to the size (or crack) of the structure. For a real material there is always a fracture process zone of finite length in
front of a crack. This fracture process zone can result from yielding of the materials (e.g., metal) or formation of micro-cracks and stable crack growth for heterogeneous brittle materials (e.g., concrete). To simulate the process zone observed in a thin metal plate, Dugdale [11] proposed a model which assumed the length of the plastic zone to be much larger than the thickness of the sheet and assumed the plastic zone as a yielded strip ahead of the crack tip. The material is assumed to be elastic perfectly plastic so that the stress within the yielded strip equals the yield strength ($\sigma_y$). Dugdale also postulated that the effect of yielding is to increase the crack length by the extent of the plastic zone as shown in Figure 2.4 for a finite length crack in an infinite medium subjected to a uniform remote stress, $\sigma_0$. Within the yielded strip, $a \leq |x| \leq c$, the opening of the crack faces is restrained by the closing pressure, $\sigma_y$. The length $d$ of this strip can be determined from the condition that the stress field is nonsingular.

Fig. 2.4 The Dugdale model.
By using superposition of the solutions for the uncracked sheet loaded by remote tension $\sigma_0$, and for the cracked sheet with remote loading and with pressure, $p_2(x) = \sigma_0$ for $|x| \leq a$ and $p_2(x) = \sigma_0 - \sigma_y$ for $a \leq |x| \leq c$, on the crack surface (Figure 2.5), Dugdale got an expression for the length of the plastic zone:

$$d = 2a \sin \left( \frac{\pi \sigma_0}{4 \sigma_y} \right) \approx \frac{\pi}{8} \left( \frac{K_1}{\sigma_y} \right)^2$$

(2.12)

He found very good agreement between the measured lengths of the plastic zones in steels from experimental results and the predictions based on equation 2.12 for $\sigma_0$ as large as $0.9 \sigma_y$.

![Fig. 2.5 Superposition method used in the Dugdale model.](image)

### 2.4.2 The Barenblatt Model

Similar to the Dugdale model, Barenblatt [12] proposed that a cohesive force is acting across a plastic zone ahead of the real crack tip. However, the cohesive force in the Barenblatt model is from molecular cohesion unlike that in the Dugdale model.

In the model, Barenblatt proposed that the cohesive stresses at the crack tip will increase as the applied load increases. But, there is
a limit to the material's ability to restrain the crack faces from opening. Then, a dynamic crack starts to develop, Barenblatt found an expression for the modulus of cohesion ($K_0$):

$$K_0 = \int_0^d \frac{G(t) \sqrt{t}}{d}$$  \hspace{1cm} (2.13)

in which $G(t)$ is the intensity of the cohesive force acting near the crack edge, $t$ is the distance along the crack surface taken along the normal to the crack edge, and $d$ is the width of the region subjected to cohesive forces. From the integral in equation (2.13), Barenblatt was able to show that

$$K_0 = \sqrt{\pi E \gamma}$$  \hspace{1cm} (2.14)

and finally, the fracture criterion for crack propagation can be derived as:

$$K_f = \frac{K_0}{\sqrt{\pi}}$$  \hspace{1cm} (2.15)

Basically the Dugdale model and the Barenblatt model can be categorized as cohesive crack models since for both models there exists a cohesive closing force acting along the fracture process zone (or plastic zone). These two models will be extended to predict the fracture resistance of asphalt concrete and will be discussed in the next chapter.
CHAPTER III

A COHESIVE CRACK MODEL FOR ASPHALT CONCRETE

3.1 Introduction

Asphalt concrete is composed of brittle; inclusions (aggregates) and viscous matrix (asphalt cement). Due to the existence of the viscous matrix, asphalt concrete behaves like a visco-elastic material. As a result, the stress-strain characteristics is dependent on the loading rate and the environmental temperature. A good basic understanding of the response of asphalt concrete to the applied stress can be qualitatively obtained by the use of rheological models. The simplest model is the Maxwell model which consists of a spring (providing the elastic response) and a dash pot (providing the viscous response) connected in series. A more realistic representation of actual behavior can be modeled by the Burger model in Fig. 3.1 [13].

In general, the strain ($\varepsilon$) of a visco-elastic material such as asphalt concrete can be expressed as a function of time ($t$), temperature ($T$), and loading rate ($\dot{\varepsilon}$). That is

$$
\varepsilon = \varepsilon(t, T, \dot{\varepsilon})
$$

(3.1)
Fig. 3.1 The Burger rheological model for asphalt concrete.

However, it should be noted that Eq. 3.1 is only valid for the undamaged viscous material. To model crack propagation in asphalt concrete, it is necessary to include a separate criterion other than Eq. 3.1, which will be discussed later.

3.2 The Cohesive Crack Model

To properly model the crack propagation in asphalt concrete, a cohesive crack model, which is similar to the Dugdale-Barenblatt model, is proposed. Some fundamental concepts and basic assumptions regarding the proposed cohesive crack model are described in the following sections.
3.2.1 Fundamental Concept of the Cohesive Crack Model

The fundamental concept of the proposed cohesive crack model can be demonstrated by means of a tensile test as shown in Fig. 3.2. This test is controlled by a constant change of deformation to ensure a stable state so that the complete stress-deformation curve can be obtained [17].

In the direct tensile test (Fig. 3.2(a)), the deformation is measured by two equal gauge lengths A and B with the results presented in Fig. 3.2. The original length of the two gauges is equal to $L_0$. According to the assumption that the specimen is homogeneous and has a constant area, the paths A and B of the stress-deformation curve coincide until the tensile strength ($f_v$) is reached. As the deformation precedes, a process zone forms in the specimen. According to the conclusion of Petersson [17], the process zone is located in a narrow band across the specimen for concrete-like material. When the process zone develops, the force will decrease due to the formation of microcracks and the corresponding weakening of the material. The decreasing load leads to a decrease in deformation outside the process zone. Similar to the unloading in the stress-strain curve, the decreasing load results in a decrease in deformation everywhere outside the process zone (see curve B in Fig. 3.2(b)). In Fig. 3.2(a), if the whole process zone is assumed to fall within gauge length A, the deformation within gauge length B can be described by a $\sigma - \varepsilon$ curve, including an elastic unloading part. This phenomenon is presented in Fig. 3.2(c). It should be noted that the
stress-strain \((\sigma - \varepsilon)\) curve indicated in Fig. 3.2(c) can be modeled using the rheological model described earlier and will be dependent on the applied loading history and environmental temperature conditions.

The deformation within gauge length \(A\) includes the deformation of process zone and the deformation at the non-critical section. Thus, the additional deformation, \(w\), due to the process zone, is the difference between curve \(A\) and curve \(B\) (see Fig. 3.2(b) and 3.2(d)), i.e.

\[
w = \Delta l_A - \Delta l_B \quad (3.2)
\]

Now, it is possible to divide the behavior of the tested material into two parts:

1. Outside the process zone, the \(\sigma - \varepsilon\) curve can be applied. In this zone the deformation is given by

\[
\Delta l_B = \varepsilon l_o \quad (3.3)
\]

2. Within the process zone, the \(\sigma - w\) curve must be used and the deformation is given by

\[
\Delta l_A = \varepsilon l_o + w \quad (3.4)
\]

The width of the process zone can be assumed to be zero before the load is applied. According to this assumption, the process zone may be described as a tied crack with width \(w\), that is, a crack which
Fig. 3.2. The principles for division of the deformation properties into a $\sigma - \varepsilon$ curve and a $\sigma - w$ curve, where $w$ is the additional deformation due to the fracture process zone.
is able to transfer stress according to the stress-separation \((\sigma - w)\) curve (Fig. 3.2(d)).

Assuming the results obtained from the tensile test are applicable to predict the stress distribution in front of a notch or crack tip, one can then analyze the progressive growth of a crack under different mode I loading conditions. Figure 3.3 shows the stress distribution in front of a crack tip in a beam under the action of growing deformation. A process zone has developed and is described as a cohesive crack. Within the process zone, the properties are governed by the stress-separation \((\sigma - w)\) curve. A more general representation of the closing pressure along this process zone can be modeled as shown in Fig. 3.4. Similar to the strain behavior, the stress-separation relationship is also a function of the loading rate and temperature. Outside the cohesive crack zone the stress-strain \((\sigma - \varepsilon)\) curve, which can be modeled using visco-elasticity, is assumed to be valid for the material.

As the applied load increases, the stresses in front of the cohesive crack tip increase. Since no stress could be higher than the tensile strength \((f_t)\), an increase in deformation will cause the extension of a cohesive crack at that point. Thus, the stress at the cohesive crack tip of the process zone is always equal to \(f_t\) as long as the process zone extends.
Fig. 3.3. Stress distribution in front of a crack tip before and after growth of the real crack.

Fig. 3.4 The cohesive crack modeled by the Burger model.
3.2.2 Basic Assumptions of The Cohesive Crack Model

In order to describe the real fracture process for the asphalt concrete, the following assumptions were made:

1. The process zone starts developing at one point when the first principal stress reaches the tensile strength $f_t$.

2. The process zone develops perpendicular to the direction of first principal stress.

3. The material in the process zone is still able to transfer stress. The stress transferring capability of the process zone depends on its opening according to the stress-separation ($\sigma - w$) curve as shown in Fig. 3.2(d).

4. The width of the process zone in the stress direction equals to the opening of the zone.

5. The properties of the materials outside the process zone are given by stress-strain ($\sigma - \varepsilon$) curve as shown in Fig. 3.2(c).

3.2.3 Material Properties for the Cohesive Crack Model

When the numerical method is used in the cohesive crack model, one has to know the stress-strain relationship and the stress-separation relationship as described in Fig. 3.2(c) and 3.2(d). For asphalt concrete, the $\sigma - \varepsilon$ curve is assumed to be linear up to the maximum stress (Fig. 3.5) for a fixed loading rate and temperature. The position of the line is defined by the tensile strength and the
Young's modulus (E), which are dependent on the applied loading rate and environmental temperature.

To obtain the stress-separation (\(\sigma - w\)) curve for asphalt concrete, a trial and error method (or curve fitting method), which will be discussed in Chapter V, is used. Based on the results of the curve fitting method, a stress-separation (\(\sigma - w\)) curve as shown in Fig. 3.6 for asphalt concrete is suggested.

As Hillerborg suggested in 1978 [15], the area under the \(\sigma - w\) curve is considered to be equal to the amount of energy necessary to create one unit area of crack surface and is defined as fracture energy (\(G_F\)). Based on this definition, the fracture energy (\(G_F\)) can be calculated as:

\[
G_F = \frac{\int_0^{w_c} \sigma(w) dw}{A} \quad (3.5)
\]

in which \(A\) = cross sectional area; and \(w_c\) = critical value of the width of the cohesive crack zone (\(w\) value for \(\sigma = 0\)).

A direct determination of fracture energy (\(G_F\)) for concrete was suggested by Petersson in 1980 [16]. The method is extended to asphalt concrete to determine the fracture energy of asphalt concrete. By using the three-point bending test on a notched asphalt concrete beam (Fig. 3.7(b)) one can get a stable load-deflection curve (see Fig. 3.7(a)) using displacement control. The area under the \(P - \delta\) curve represents the amount of energy consumed when the crack propagates through the beam. When the area of the cross section is
known, $G_r$ can be calculated by using the total energy divided by the uncracked sectional area and can be calculated by the following equation [16]:

$$G_F = \left[ \int_0^{\delta_{\text{max}}} P(\delta)d\delta + \frac{1}{2}Mg\delta_{\text{max}} \right] / [(b-a)W] \quad (3.6)$$
where \( b = \) beam depth; \( W = \) beam width; \( a = \) notch depth; \( M = \) mass of the beam (between the supports); \( g = \) acceleration of gravity (32.2 \( \text{ft sec}^{-2} \)), and \( \delta_{\text{max}} = \) maximum deflection at which the applied load is equal to zero.

![Diagram](image)

**Fig. 3.7 Typical \( P - \delta \) and \( P - \text{CMOD} \) curves for a stable three-point bend testing.**

### 3.3 Numerical Analysis

To obtain the theoretical result using the **proposed** cohesive crack model, a numerical method has to be used except for a few special cases (e.g., a center crack in an infinite plate, see Section 2.4). In the following section, the numerical formulation of this model will be presented.

#### 3.3.1 Numerical Formulation

Consider a notched beam subjected to a load \( P \) in the midspan as shown in Fig. 3.8. It is assumed that the process zone will develop along a straight crack plane, which is reasonable for mode I crack propagation. When the beam is loaded, by introducing closing stresses over the cohesive crack and replacing the closing stresses by
equivalent nodal forces, it is possible to analyze the progressive crack propagation in the beam.

As shown in Fig. 3.8, assuming the material is linear elastic and if the deformations are small, by using method of superposition the widening of the crack at each node and the deflection at the loading point can be calculated by the following equations:

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & \cdots & a_{1(n-1)} & c_1 \\
a_{21} & a_{22} & a_{23} & \cdots & a_{2(n-1)} & c_2 \\
a_{31} & a_{32} & a_{33} & \cdots & a_{3(n-1)} & c_3 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
a_{(n-1)1} & a_{(n-1)2} & a_{(n-1)3} & \cdots & a_{(n-1)(n-1)} & c_{n-1} \\
b_1 & b_2 & b_3 & \cdots & b_{n-1} & d_p
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\vdots \\
\sigma_{n-1} \\
d_1
\end{bmatrix} =
\begin{bmatrix}
w_1 \\
w_2 \\
w_3 \\
\vdots \\
w_{n-1} \\
d_1
\end{bmatrix}
\]  

or

\[
w_i = \sum_{j=1}^{n-1} a_{ij} \sigma_j + c_i P \]  

or

Fig. 3.8 A notched beam subjected to three-point bending.
\[ \delta = \sum_{j=1}^{n-1} b_j \sigma_j + d_P P \]  

(3.9)

in which

\( a_{ij} = \) the widening of the crack at node \( i \) when an equivalent closing force (unit stress \( \times \) element length and element length=\( x/(n-1) \)) is acting at node \( j \) (in/psi),

\( a_a = \) the widening of the crack at node \( i \) when an equivalent closing force (unit stress \( \times \) half element length and element length=\( x/(n-1) \)) is acting at node \( 1 \) (in/psi),

\( c_i = \) the widening of the crack at node \( i \) when a unit load \( P \) is applied (in/lb),

\( b_j = \) the displacement at the loading point when an equivalent closing force (unit stress \( \times \) element length) is acting on node \( j \) (in/psi),

\( d_P = \) the displacement at the loading point when a unit load \( P \) is applied (in/lb),

\( \sigma_j = \) the closing pressure at node \( j \) (psi),

\( w_i = \) the width of the crack opening at node \( i \) (in), and

\( \delta = \) the load-line displacement (in).

Equation 3.7 can be rewritten as:

\[ \{C\} \{F\} = \{\Delta\} \]  

(3.10)
in which $[C]$ is a compliance matrix. This matrix can be numerically formulated using finite element methods or using a method derived from linear elastic fracture mechanics. In the present study, the second method is used and the methodology is described in Appendix A.

![Diagram](image)

Fig. 3.9 A schematic illustration of the first step.

### 3.3.2 Calculation of the First Point of the $P-\delta$ and $P$-CMOD Curves

In the calculation method, the stresses acting across the cohesive crack were replaced by equivalent nodal forces. These forces can be determined according to the stress-separation ($\sigma - w$) curve when the width at the cohesive crack zone is known. As indicated in Fig. 3.9, when the first node reaches its tensile strength, the opening at first node is still equal to zero, i.e. $\sigma_1 = f_t$, $w_1 = w_2 = \ldots = w_{n-1} = 0$. From this, one can determine the first
point of $P-\delta$ and $P$-CMOD curves. The calculation procedures for this step are described as follows:

1. Initial boundary conditions:
   \[ \sigma_i = f_i, w_1 = w_2 = \ldots = w_{n-1} = 0 \]

2. The $n$ unknowns:
   \[ \sigma_2, \sigma_3, \sigma_4, \ldots, \sigma_{n-1}, P, \delta \]

3. The $n$ equations:
   \[
   w_i = \sum_{j=1}^{n-1} a_{ij} \sigma_j + c_i P \quad (i=1,2,\ldots,n-1) \tag{3.8}
   \]
   \[
   \delta = \sum_{j=1}^{n-1} b_{j} \sigma_{j} + d_{p} P \tag{3.9}
   \]

By assuming that $\delta=1$ and solving the $n$ equations, the stresses ($\sigma_j$) ($j=1,2,\ldots,n-1$) and the load ($P$) can be obtained. However, $\sigma_i$ must satisfy the initial boundary condition, i.e., $\sigma_1 = f_i$. If $\sigma_i$ does not equal to $f_i$, say $\sigma_1 = \bar{C}$, then $\sigma_2, \sigma_3, \ldots, \sigma_{n-1}, P$, and $\delta$ must be multiplied by a correction factor ($f_i/\bar{C}$) so that $\sigma_i$ will satisfy the prescribed initial boundary Conditions. Thus, the first point of $P-\delta$ and $P$-CMOD curves is generated.

3.3.3 Calculation of the Second Point of the $P-\delta$ and $P$-CMOD Curves

When the crack starts to propagate as shown in Fig. 3.10, the first node is opened and the second node is assumed to reach the tensile strength. At this point the boundary conditions can be expressed as: $\sigma_2 = f_1$, $w_2 = w_3 = \ldots = w_{n-1} = 0$, $w_1 \neq 0$, $\sigma_i = \sigma(w_i)$. The
solution of this step can be obtained by using a superposition method. The procedures of this method are described as follows:

1. Construct the basic solution for the first step as shown in Fig. 3.11(a). By assuming \( \sigma_1 = 1, \) \( w_1 = w_2 = \ldots = w_{n-1} = 0 \) and solving the unknowns for the first step, the basic solution is obtained and stored in memory.

2. Construct the basic solution for the second step as shown in Fig. 3.11(b). By assuming \( \sigma_1 = 0, \sigma_2 = 1, w_1 \neq 0, w_2 = \ldots = w_{n-1} = 0 \) and solving this system, the basic solution for the second step is obtained and stored also.

3. By using the principle of superposition one knows that

\[
\sigma_2 = f_t = \sigma_1^1 + \sigma_2^2
\]  
(3.11)
in which $\sigma_i^m$ denotes the stress of node $i$ at the $m^{th}$ step. From Equation 3.11 one gets

$$\sigma_2^2 = f_i - \sigma_2^1$$ \hspace{1cm} (3.12)

Since $\sigma_2^1$ has been solved at the first step, $\sigma_2^2$ can be determined. Meanwhile $w_1$ can be determined. At this time $\sigma_1$ is assumed to be equal to the value obtained from the last step, i.e. in the second step the assumption is $\sigma_1 = \sigma_1^1 = f_i$. From the value $w_1$ and the given $\sigma-w$ curve, one can check the difference between $\sigma_1$ and $\sigma(w_1)$. If the difference is not tolerable, assume $\sigma_1 = \sigma(w_1)$ and go back to Eq. 3.11 to calculate $\sigma_2^1$ again. Then $(\sigma_2^2)_{\text{new}}$ and $(w_1)_{\text{new}}$ can be determined using the same procedures. These procedures have to be repeated until the tolerance is acceptable.

### 3.3.4 Calculation of Further Crack Propagation

When the crack propagates to the third node, the first two nodes are opened. This is illustrated in Fig. 3.12. The boundary conditions associated with this step are: $\sigma_1 = \sigma(w_1), \sigma_2 = \sigma(w_2), \sigma_3 = f_i, w_1, w_2 \neq 0, w_3 = w_4 = \ldots = w_{n-1} = 0$. By superposing the basic solutions obtained from steps 1, 2, 3 of Fig. 3.11, one can get the following equation:

$$\sigma_3 = f_i = \sigma_1^1 + \sigma_2^2 + \sigma_3^3$$ \hspace{1cm} (3.13)

In this step assume $\sigma_1$ and $\sigma_2$ are equal to the values obtained from the previous (second) step. From $\sigma_1$ one can get $\sigma_3^1$ and from
Fig. 3.11 Stress distribution for each step.
Fig. 3.12 A schematic illustration of the third step.

\[ \sigma_3 = f_t - \sigma_1 - \sigma_2 \]  

(3.14)

Once \( \sigma_3 \) is solved, \( w_1 \) and \( w_2 \) can be determined. Then follow the same numerical iteration procedures as the second step and the conditions \( \bar{\sigma}_1 = \sigma(w_1) \) and \( \bar{\sigma}_2 = \sigma(w_2) \) should be satisfied.

Following the same principle, the process of the crack propagation can be analyzed and all of the \( P-\delta \) and P-CMOD curves can be obtained. The flow chart of the numerical program of the cohesive crack model is listed in Fig. 3.13.
3.4 Verification of the Numerical Results

In order to check the validity of the proposed numerical method, the numerical results obtained using the aforementioned method were compared with the results reported by Petersson [17] and Roelfstra and Wittmann [18].

The numerical comparisons for the four models (Dugdale model (D), straight line approximation (SL), concrete (C), and fiber-reinforced concrete (FRC)) are presented in Fig 3.16. The material properties and beam dimensions used in this numerical example are listed as follows:

Material properties (from Petersson):

\[ G_F = 100 \text{ N/m} \quad \text{(for D, SL, and C)} \]
\[ G_F = 1430 \text{ N/m} \quad \text{(for FRC)} \]
\[ f_t = 4 \text{ MPa} \]
\[ E = 40,000 \text{ MPa} \]
\[ v = 0.2 \]

\( \sigma-w \) curves are shown in Fig 3.14.
1. Construct \( \mathbf{d}_p \) by CRCKP subroutine.
2. Construct \( \mathbf{a}_{ij} \) by CODS subroutine.
3. Construct \( \mathbf{c}_i \) by CODP subroutine.
4. Construct \( \mathbf{b}_i \) by CODP subroutine.

(see Appendix A)

\[
[C]\{F\} = \{\delta\}
\]

First Step

B.C.: \( \sigma_1 = f, w_1 = \ldots = w_{n-1} = 0 \)
Solve the n unknowns by a Equations \( \mathbf{a}, \mathbf{a}_\sigma, \ldots, \mathbf{a}_{n-1}, \mathbf{P}, \mathbf{6} \)

Get first point of the \( P-\delta \) and \( P-\text{CMOD} \) curves.

Second step

B.C.: \( \sigma_1 = \sigma(w), \sigma_2 = f - \sigma_1 w_1 \neq 0, w_2 = \ldots = w_{n-1} = 0 \)
Use superposition \( \mathbf{\sigma_2} = f - \mathbf{\sigma_1} + \mathbf{\sigma_2} \)
Assume \( \sigma_1 = f \) get \( \sigma^2 - \sigma^1 \) and \( w_1 \)

Assume \( \sigma^1 = a(w) \)

\( \left| \sigma^1 - \sigma(w) \right| < 0.01 \)

Yes

Get second point of the \( P-\delta \) and \( P-\text{CMOD} \) curves

No

Fig. 3.13 Flow chart of the numerical program of the cohesive crack model.
Third step

B.C.: $\sigma_1 = \sigma(w_1), w_1 \neq 0$, $\sigma_2 = \sigma(w_2), w_2 \neq 0$

$\sigma_3 = f_t$, $w_3 = w_4 = \ldots = w_{n-1} = 0$

Use superposition

$\sigma = f_t = \sigma_1^1 + \sigma_2^2 + \sigma_3^3$

Assume $T_{2} = \text{The values from 2nd step}$

Get $\sigma_1$ from the assumptive value $\bar{\sigma}_1$

Get $\sigma_2$ from the assumptive value $\bar{\sigma}_2$

Get $\sigma_3 = f_t - \sigma_1^1 - \sigma_2^2$, and $w_1, w_2$

Repeat the same numerical iteration procedures to get until the $(n-1)$th point of $P-\delta$ and $P$-CMOD curves.

Fig. 3.13 (continued).
(a) A $\sigma-w$ curve according to Dugdale model (D).

(b) A two-line approximation of the $a-w$ curve (C).

(c) The $\sigma-w$ curve approximated using a single, straight line (SL).

(d) An example of $\sigma-w$ curve representing a fiber-reinforced material (FRC).

Fig. 3.14 $\sigma-w$ curves for four models.

The beam dimensions analyzed by Petersson [17] and the present study are given in Fig. 3.15.

Fig. 3.15 Beam dimensions used by Petersson and the present study.
Fig. 3.16 Numerical comparisons between this study and Petersson's results.

(a) Dugdale model (D). (b) Straight line (SL) approximation.
(c) Two line (C) approximation. (d) Fiber-reinforced material.
(e) Load-deflection curves from Petersson. (f) Load-deflection curves from this study.
Fig. 3.16 (continued).

A: Petersson's result
B: Results from this research
Results from Petersson.

Results from this study.

Fig. 3.16 (continued).
It is observed that the differences in peak load between Petersson's results [17] using FEM and this study are about 6% to 8% (see Table 3.1). It can be noted that these numerical results are judged to be satisfactory when compared with the numerical results obtained by Petersson. The numerical results of P-CMOD curves, which were not reported by Petersson, are given in Fig. 3.17.

The material properties and beam dimensions of the other numerical example which was reported by Roelfstra and Wittmann, [18] are listed as follows:

Material properties (from Roelstra and Wittmann):

\[
G_f = 105 \text{ N/m} \\
\sigma_t = 3.5 \text{ N/mm}^2
\]
$E = 30,000 \text{ N/mm}^2$

$\nu = 0.2$

$\sigma - w$ curve is shown in Fig 3.18.

![Graph showing stress versus crack width in mm]

Fig. 3.18 $\sigma - w$ curve (from Roelfstra and Wittmann).

The beam dimensions for this example are given in Fig. 3.19.

![Diagram of beam dimensions]

Fig. 3.19 Beam dimensions used by Roelfstra & Wittmann and the present study.
Fig. 3.20 Comparisons of the load-deflection and the P-CMOD curves.
Again, the numerical results obtained using the present method and the FEM [18] are about the same (see Fig. 3.20). It should be noted that the peak load predicted using the proposed method is about 6% lower than that of FEM. This may be due to a relatively higher stiffness obtained using the finite element methods.

Table 3.1 The differences in peak load between this study and other researchers.

<table>
<thead>
<tr>
<th>Model</th>
<th>$P_{\text{max}}$ Present Study (lbs)</th>
<th>$P_{\text{max}}$ Petersson [17] (lbs)</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dugdale model (D)</td>
<td>15494</td>
<td>16595</td>
<td>6.6%</td>
</tr>
<tr>
<td>Straight Line (SL)</td>
<td>12624</td>
<td>13700</td>
<td>7.9%</td>
</tr>
<tr>
<td>Concrete (C)</td>
<td>11047</td>
<td>11930</td>
<td>7.4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>$P_{\text{max}}$ Present Study (lbs)</th>
<th>$P_{\text{max}}$ and Wittmann [18]</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>2325.1</td>
<td>2473</td>
<td>6.0%</td>
</tr>
</tbody>
</table>
CHAPTER IV

EXPERIMENTAL PROGRAM FOR ASPHALT CONCRETE

4.1 Material Characteristics

In this chapter, the experimental program used to evaluate the material properties associated with the proposed model will be presented. The asphalt concrete mix, testing methods and testing conditions used in the experimental program are detailed in the following sections.

4.1.1 Asphalt Concrete Mix

The asphalt used in this study is AC-20 grade, which was purchased from the Koch company. The aggregates used in this study are natural gravel and natural sand and were provided from a local aggregate company (Columbus Builders Supply, Inc.). The gradation of the aggregate is given in Table 4.1 along with the gradation requirements of 404 mix specified by Ohio Department of Transportation [19]. The mixing temperature is $300^\circ F$ and the compaction temperature for the hot mix is $280^\circ F$. Medium traffic conditions are assumed, thus for the asphalt tablets the number of compaction blows in each end of the specimen is 50. The optimum
asphalt content according to Marshall mix design has been determined as 5.15%. This optimum asphalt content was used to produce the asphalt tablets and the beam specimens for evaluation of Marshall stability, indirect tensile strength, flexural strength, and fracture energy.

Table 4.1 Aggregate Gradation Specification for Ohio Department of Transportation 404 Mix

<table>
<thead>
<tr>
<th>Sieve</th>
<th>Specification (% passing)</th>
<th>Mix Gradation (% passing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2 inch</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3/8 inch</td>
<td>90-100</td>
<td>93</td>
</tr>
<tr>
<td>No. 4</td>
<td>45-75</td>
<td>50</td>
</tr>
<tr>
<td>No. 16</td>
<td>15-45</td>
<td>20</td>
</tr>
<tr>
<td>No. 50</td>
<td>3-22</td>
<td>6</td>
</tr>
<tr>
<td>No. 200</td>
<td>0-8</td>
<td>3</td>
</tr>
</tbody>
</table>

4.1.2 Specimen Dimensions

Two types of specimens were prepared in the present study. One is the tablet specimen and the other one is the beam specimen. The asphalt tablets were used for Marshall stability tests and indirect tensile tests. The beams were used for the three-point bend tests. Some of the beams were saw cut with notch-depth ratios of 0.2, 0.4, and 0.6. The dimensions of the beam specimen are 15 inches in length, 2.9 inches in height, and 3.0 inches in width. The asphalt
tablets have the dimensions of 4 inches in diameter and 3 inches in height.

4.2 Preparation of the Specimens

The tablets were prepared based on the Mix Design Method for Asphalt Concrete (MS-2) recommended by the Asphalt Institute for Marshall stability tests [20]. The procedures of preparing the Marshall tablets are described as follows:

(a) Preparation of aggregates---The aggregates were oven dried at 300°F to 320°F for one day and then 1150 grams of aggregates were measured out, blended in the desired proportions.

(b) Preparation of asphalt concrete mix---The asphalt cement was heated to 300°F and 5.15% of asphalt cement (by weight) was added into the aggregates. The asphalt cement was mixed with the aggregates, and the asphalt concrete was then thoroughly mixed by hand.

(c) Compaction of asphalt concrete---Before compaction, the hammer and the Marshall molds were heated up to 200°F to 300°F. A piece of filter paper was placed in the mold to prevent the asphalt concrete from sticking to the mold. Then, the asphalt concrete mixture was placed in a heated Marshall mold with a base and 50 blows (for medium traffic) of impact forces were applied on each side of the specimens. Fig. 4.1 shows the compaction hammers used in this study.
The same mixing procedures were used to prepare the beam specimens. Fig. 4.2 shows the mold and the concrete mix to produce the beam specimens. The beam specimens were compacted statistically by using a Forney testing machine applying 10 cycles of 100 Kips static force to the surface of the beams (Fig. 4.3). This procedure was used to ensure that the density of the beam is similar to that of the asphalt tablets.

Fig. 4.1 Compaction hammer for Marshall Tablets preparation.
Fig. 4.2 Steel mold for beam specimens.

Fig. 4.3 Compression machine for producing beam specimens.
4.3 Testing Apparatus and Experimental Procedures

4.3.1 Marshall Stability Tests

Fig. 4.4 shows the Marshall stability testing equipment. From this test one can determine the maximum load (or called Marshall stability) and the associated displacement at the peak load (or termed flow). To simulate the adverse environment that the asphalt concrete may experience, the Marshall tablets were conditioned in a water bath of 140 °F for 30 to 40 minutes before testing. The experimental results obtained from the Marshall stability tests and the determination of the optimum asphalt content are given in Fig. 4.5. It was found that the optimum asphalt content for the aggregate gradation and asphalt cement used in this research is equal to 5.15%.

Fig. 4.4 Marshall stability test set-up.
Fig. 4.5 Results of Marshall Stability Test.
A. C. = $\frac{5.27\% + 4.86\% + 5.32\%}{3} = 5.15\%$
4.3.2 Indirect Tensile Tests

To measure the tensile strength of asphalt concrete, indirect tensile tests were performed instead of direct tensile tests because the indirect tensile test has the following advantages [21]:

1. It is relatively simple;
2. The type of specimen and equipment are the same as that used for compression testing;
3. Failure is not seriously affected by surface conditions;
4. Failure is initiated in a region of relatively uniform tensile stress and;
5. The coefficient of variation of the test results is low.

The indirect tensile tests were carried out at five different temperatures (183, 36°F, 75°F, 104°F, and 140°F). For 18°F and 36°F cases, the tablets were put in the refrigerator for one day to reach the required temperature before testing. In the cases of 104°F and 140°F, the tablets were wrapped with a plastic sheet and put in a plastic bag and then were conditioned in the water bath for six hours to gain the required temperature. An MTS testing system was used in this test. Displacement control was used in order to get a complete load versus load-line deflection curve. The loading rate in the indirect tensile tests was fixed at 0.03 inches per minute. Following this loading rate the indirect tensile test for each asphalt tablet was finished within ten minutes. Thus, the temperature change during the testing process was assumed to be negligible. The
applied load and load-line deflection were monitored and recorded by an X-Y recorder. The testing set-up for the indirect tensile test is shown in Fig. 4.6. The typical load versus load-line deflection curves obtained from the indirect tensile tests are attached in Appendix C. Figure 4.7 shows a typical load versus load-line deflection curve obtained from indirect tensile test (T=75°F).

The indirect tensile test involves loading a cylindrical specimen with compressive loads distributed along two opposite generators. This condition results in a relatively uniform tensile stress perpendicular to and along the diametral plane containing the applied load. The failure usually occurs by splitting along this loaded plane.

Fig. 4.6 MTS Testing system for Indirect Tensile Tests.
Fig. 4.7 A typical load versus load-line deflection curve of the indirect tensile tests ($T=75^\circ F$).

Based on the measured peak load, the indirect tensile strength can be measured and expressed as [21]:

$$f_t = \frac{2P_{\text{max}}}{\pi DH} \quad (4.1)$$

in which

$P_{\text{max}}$ is the measured peak load; $H$ is the thickness of the tablet; $D$ is the diameter of the tablet which is equal to 4 inches; and $f_t$ is the indirect tensile strength.

The fracture energy $G_F$, which is a measure of crack resistance, can be calculated from the complete load versus load-line deflection curve and expressed as:
\[ G_p = \left[ \int_0^{\delta_{\text{max}}} P(\delta) d\delta \right] / A = \text{area under the } P - \delta \text{ curve} / A \]  \hspace{1cm} (4.2)

in which \( A \) is the sectional area of the cylinder. The area under the \( P - \delta \) curve can be calculated by a planimeter as shown in Fig. 4.8; the cross sectional area \( A \) in this case is assumed to be equal to the product of height \( H \) and diameter \( D \) of the tablet. Thus, Eq. 4.2 can be rewritten as:

\[ G_F = \left[ \int_0^{\delta_{\text{max}}} P(\delta) d\delta \right] / (HD) \]  \hspace{1cm} (4.3)

Fig. 4.8 A Compensating Polar Planimeter for calculating the area under the curve.

### 4.3.3 Three-point Bend Test

Figure 4.9 shows the three-point bend testing set-up. Before the three-point bend testing is performed, one must check the
stiffness of the testing machine in order to obtain a stable load-deflection curve. It has been checked that the testing system is stiff enough for an asphalt concrete specimen (in this study the maximum load is no more than 200 lbs). The applied load and the load-line deflection were also recorded by an X-Y recorder. The loading rate for this test was 0.125 inch per minute and the notch-depth ratios were 0.2, 0.4, and 0.6. The dimensions for the testing beams are 12.6 inches in length, 3.0 inches in width, and 2.9 inches in height. The typical load versus load-line deflection curves for the tests are listed in Appendix D. Figure 4.10 shows a typical load versus load-line deflection curve from the three-point bend test. The fracture energy \( (G_F) \), can be expressed by Eq. 4.2. However, the sectional area \( (A) \) used here is the initial uncracked ligament area. It is equal to \( (b-a)W \). Thus, the fracture energy \( (G_F) \) can be calculated as:

\[
G_F = \left[ \int_0^{\delta_{\text{max}}} P(\delta)d\delta + \frac{1}{2}Mg\delta_{\text{max}} \right] / [(b-a)W] 
\]  

Fig. 4.9 Three-point bend testing set-up.
4.4 Test Results

The test results obtained from the above tests are summarized as follows:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Marshall Stability</td>
<td>1223.3 lbs</td>
</tr>
<tr>
<td>Average Flow (0.01 inch)</td>
<td>10.53</td>
</tr>
<tr>
<td>Average Unit Weight</td>
<td>141.27 lb/ft³</td>
</tr>
</tbody>
</table>

(b) Indirect Tensile Test.
Table 4.3 Results of indirect tensile test.

<table>
<thead>
<tr>
<th>Temp (°F)</th>
<th>Spec. No.</th>
<th>Indirect tensile strength ($f_t$) (psi)</th>
<th>Fracture Energy ($G_p$) (lb/in)</th>
<th>Average ($f_t$) (psi)</th>
<th>Average ($G_p$) (lb/in)</th>
<th>P - δ curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>18°F</td>
<td>0-37</td>
<td>293.81</td>
<td>18.41</td>
<td>305.44</td>
<td>19.98</td>
<td>Attached in Appendix C</td>
</tr>
<tr>
<td></td>
<td>0-38</td>
<td>309.56</td>
<td>20.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-39</td>
<td>324.45</td>
<td>20.33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-40</td>
<td>293.94</td>
<td>20.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36°F</td>
<td>0-28</td>
<td>183.15</td>
<td>17.63</td>
<td>189.16</td>
<td>18.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-29</td>
<td>156.17</td>
<td>18.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-41</td>
<td>213.26</td>
<td>18.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-42</td>
<td>204.06</td>
<td>20.74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75°F</td>
<td>0-20</td>
<td>70.40</td>
<td>9.53</td>
<td>63.39</td>
<td>8.71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-21</td>
<td>63.21</td>
<td>9.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-22</td>
<td>64.59</td>
<td>9.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-23</td>
<td>57.19</td>
<td>8.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-24</td>
<td>61.58</td>
<td>6.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>104°F</td>
<td>0-33</td>
<td>14.07</td>
<td>1.65</td>
<td>13.70</td>
<td>1.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-34</td>
<td>13.23</td>
<td>1.39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-35</td>
<td>13.80</td>
<td>1.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140°F</td>
<td>0-30</td>
<td>3.70</td>
<td>0.295</td>
<td>3.62</td>
<td>0.276</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-31</td>
<td>3.53</td>
<td>0.259</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-32</td>
<td>3.63</td>
<td>0.274</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The tensile strengths at different temperatures obtained from this study were compared with the results reported by Kennedy and Hudson in 1968 [21] and are listed in Fig. 4.11. From the results of Kennedy and Hudson, the loading rate had a significant effect on the indirect tensile strengths. A higher loading rate led to a larger indirect
tensile strength. Thus, the experimental results obtained from the present study (loading rate=0.03 in/min) were judged to be reasonable.

![Graph showing effect of testing temperature and loading rate on indirect tensile strength of asphaltic concrete.](image)

**Fig. 4.11** Effect of testing temperature and loading rate on indirect tensile strength of asphaltic concrete.

(c) Three-Point Bend Test.

Table 4.4 shows the results from the three-point bend test. In this Table the fracture energy ($G_F$), critical stress intensity factor ($K_{lc}$) and net flexural tensile strength ($f_{f_{net}}$) for different notch-depth ratios ($a/b=0.2, 0.4, \text{ and } 0.6$) from the three-point bend test were calculated. From Table 4.4 it can be seen that the parameters ($G_F, K_{lc}, f_{f_{net}}$) obtained from different notch-depth ratios are not quite the same. This may be due to the effect of beam weight. For a higher notch-depth ratio such as $a/b=0.6$, the beam weight becomes an...
important effect on the failure mechanism. Thus, neglecting the beam weight may not be a suitable assumption to make for asphalt concrete.

Table 4.4 Results of the three-point bend test \(T=75^\circ F\).

<table>
<thead>
<tr>
<th>a/b</th>
<th>spec. No.</th>
<th>(P_{\text{max}}) (lbs)</th>
<th>(f_f) (psi)</th>
<th>(K_I) psi - (\sqrt{\text{in}})</th>
<th>Fracture Energy ((G_F)) (lb/in)</th>
<th>(\text{due to beam weight})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>B-0-10</td>
<td>177.2</td>
<td>203.34</td>
<td>173.99</td>
<td>2.94</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td>B-0-9</td>
<td>151.2</td>
<td>183.28</td>
<td>154.68</td>
<td>2.78</td>
<td>0.297</td>
</tr>
<tr>
<td>0.4</td>
<td>B-0-8</td>
<td>77.2</td>
<td>159.65</td>
<td>129.17</td>
<td>1.90</td>
<td>0.300</td>
</tr>
<tr>
<td></td>
<td>B-0-7</td>
<td>64.8</td>
<td>128.69</td>
<td>105.18</td>
<td>2.00</td>
<td>0.385</td>
</tr>
<tr>
<td>0.6</td>
<td>B-0-6</td>
<td>21.2</td>
<td>98.64</td>
<td>67.51</td>
<td>1.33</td>
<td>0.456</td>
</tr>
<tr>
<td></td>
<td>B-0-5</td>
<td>29.8</td>
<td>144.49</td>
<td>97.87</td>
<td>1.42</td>
<td>0.400</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>153.02</td>
<td>121.40</td>
<td>2.06</td>
<td>0.355</td>
</tr>
</tbody>
</table>

\(P - \delta\) curves: Attached in Appendix D

* Fracture energy without considering beam weight.

** Fracture energy due to beam weight.

*** If beam is considered, an extral 5.45 lbs (half weight of the beam) must be added to \(P_{\text{max}}\).

4.5 Basic Material Properties of the Cohesive Crack Model

(a) Tensile strength \(f_t\) ---The tensile strengths obtained from the indirect tensile tests at different temperatures are used as the inputs for tensile strength.

(b) Fracture energy \(G_F\) ---Due to the end compressive effect of the asphalt tablets, multiple cracks were produced in the indirect
tensile test. Therefore, the fracture energy obtained from the indirect tensile test is higher than that obtained from the three-point bend test. It is proper to select the fracture energy obtained from the three-point bend tests instead of that obtained from indirect tensile test. However, since only the $T = 75^\circ F$ case was performed for three-point bend tests, an alternative method is used to normalize the fracture energy-temperature relationship. The results of average fracture energy at different temperatures from indirect tensile tests are listed in Table 4.3. By using the regression method, the temperature versus fracture energy relationship obtained using indirect tensile tests can be expressed as:

$$G_F(T)_{\text{indirect tensile}} = 10^{(1.3226 + 0.0018 T - 1.118 \times 10^{-4} T^2)}$$ \hspace{1cm} (4.4)

Assuming that the fracture energy ($G_F$) versus temperature relationship obtained from the indirect tensile tests is the same as that of the three-point bend test, then, one can calculate the fracture energy of three-point bend tests based on the $G_F$ value at $T = 75^\circ F$ ($G_F\text{indirect tensile} = 8.71 \text{ lb/in}$) as:

$$G_F(T)_{3\text{-pt.}} = G_F(75^\circ F)_{3\text{-pt.}} \times \frac{G_F(T)_{\text{indirect tensile}}}{8.71}$$ \hspace{1cm} (4.5)

This way, the $G_F$ values at different temperatures can be obtained.

(c) Young's modulus ($E$) and Poisson's ratio ($\nu$) --- The Young's modulus at different temperatures can be estimated from the initial slope of $P-\delta$ curves obtained from the indirect tensile tests as well
as three-point bend tests. Since the Young's modulus is proportional to the initial slope, the relationship between the Young's modulus and the slope can be expressed as:

$$E \propto F(s)$$  \hspace{1cm} (4.6)

in which $s$ is the initial slope of $P - \delta$ curve obtained from the indirect tensile test. From the initial slopes of $P - \delta$ curves at different temperatures, the following relationship between the initial slopes ($s$) and temperatures can be obtained using the regression method.

$$s(T) = 10^{(5.5418 - 0.0142 \cdot T)}$$  \hspace{1cm} (4.7)

An estimated value of Young's (E) at $T = 75^\circ F$ from the three-point bend testing results was calculated. The Young's modulus (E) was calculated using the following formulas [10]:

$$\Delta = \Delta_{\text{crack}} + \Delta_{\text{no crack}} = \frac{1}{E} \left[ \frac{PL^3}{48EI} + \frac{\alpha_LPL(1 + \nu)}{2A} \right] + \frac{\sigma}{E} L V \left( \frac{a}{b} \right)$$  \hspace{1cm} (4.8)

in which

$$\Delta = \text{total load-line deflection},$$

$$\Delta_{\text{crack}} = \text{load-line deflection due to crack},$$

$$\Delta_{\text{no crack}} = \text{load-line deflection calculated from structural mechanics (i.e., elastic deformation without considering the existence of crack)},$$
\[ V_2 \left( \frac{a}{b} \right) = \text{a function depends on notch-depth ratio } (a/b). \]

\[
V_2 \left( \frac{a}{b} \right) = \left( \frac{a}{b} \right)^2 \left\{ 5.58 - 19.57 \left( \frac{a}{b} \right) + 36.82 \left( \frac{a}{b} \right)^2 - 34.94 \left( \frac{a}{b} \right)^3 + 12.77 \left( \frac{a}{b} \right)^4 \right\}
\]

(4.9)

\[
a_s = \frac{12 + 11v}{10(1 + v)} \quad \text{(shear shape factor)}
\]

(4.10)

\[ v: \text{ Poisson's ratio} \]

\[
\sigma = \frac{M y}{I} = \frac{3PL}{2Wb^2}
\]

(4.11)

Knowing the value of Young's modulus (E) at \( T = 75^\circ F \) (\( E_{75^\circ F} = 75,000 \text{ psi} \)) and Eq. 4.7, one can estimate the values of E for different temperatures based on the following equation:

\[
E(T) = 10^{(5.93906 - 0.0142 T)}
\]

(4.10)

The calculated values of Young's modulus are listed in Table 4.5.

From the suggestion of Yoder [22], the effect of asphalt concrete temperature on Poisson's ratios is listed in Table 4.5.

Table 4.5 summarizes the material properties along with the characteristic length \( L_{ch} = \frac{G_F \times E}{f'_I^2} \) that will be used in the parametric study using the proposed cohesive crack model.
A trial and error (or curve fitting) method was used to back calculate the stress-separation \((a - w)\) curve, which will be detailed in Chapter V.

<table>
<thead>
<tr>
<th>Temp. Parameters</th>
<th>18°F</th>
<th>36°F</th>
<th>75%</th>
<th>104°F</th>
<th>140%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_t) (psi)</td>
<td>305.4</td>
<td>189.16</td>
<td>63.39</td>
<td>13.70</td>
<td>3.62</td>
</tr>
<tr>
<td>(G_{in-lt} F in^2)</td>
<td>4.726</td>
<td>4.413</td>
<td>2.06</td>
<td>0.364</td>
<td>0.065</td>
</tr>
<tr>
<td>E (psi)</td>
<td>483,000</td>
<td>268,000</td>
<td>75,000</td>
<td>29,000</td>
<td>9,000</td>
</tr>
<tr>
<td>v</td>
<td>0.13</td>
<td>0.25</td>
<td>0.35</td>
<td>0.46</td>
<td>0.49</td>
</tr>
<tr>
<td>(L_{ch}) (inch)</td>
<td>24.45</td>
<td>33.03</td>
<td>38.35</td>
<td>56.22</td>
<td>44.31</td>
</tr>
</tbody>
</table>
CHAPTER V

COMPARISONS AND PARAMETRIC STUDIES

5.1 Introduction

In the present chapter, determination of the stress-separation \((\sigma - w)\) curve for asphalt concrete will be discussed. This stress-separation curve will be used to perform a parametric study on the effect of temperature and size of specimen on the structural response of asphalt concrete. Comparisons between the theoretical predictions and the available experimental results reported by other researchers will be presented.

5.2 Determination of Stress-Separation Curve \((a - w)\) for Asphalt Concrete

Figure 5.1 shows a typical experimental result obtained from a three-point bend test for notch-depth ratio equal to 0.4. By using a numerical simulation method [23], it is possible to back calculate the \(\sigma - w\) curve that is suitable for asphalt concrete. In the present study, a bilinear descending \(\sigma - w\) curve is used to simulate the flexural behavior of pre-cracked specimens (see Fig. 5.2).
Specimen number: B-0-8
Beam Length: 12.6 inch
Beam Height: 2.9 inch
Beam Width: 3.0 inch
Notch-Depth Ratio (a/b): 0.4

Fig. 5.1 The experimental $P - \delta$ curve from three-point bending (a/b = 0.4)

stress $\sigma$

$\frac{f_t}{a \cdot f_t}$

knee

$c^* \quad w_c$

$E = 75,000 \text{ psi}$

$\nu = 0.35$

Fig. 5.2 A bilinear stress-separation curve for numerical simulation.
By trying different combinations of $a^*$ and $c^*$, twenty-five bilinear $\sigma-w$ curves with the same fracture energy ($G_F$) are generated. The twenty-five different combinations are:

$$a^* = 0.2, 0.4, 0.6, 0.8, \text{ and } 1.0$$

and

$$c^* = 0.1, 0.3, 0.5, 0.7, \text{ and } 0.9$$

Fig. 5.3 presents the numerical result of load versus load-line deflection curves associated with the different combinations of $a^*$ and $c^*$. It can be observed from Fig. 5.3 that the predicted load versus load-line deflection response appears to be very sensitive to the shape of the stress-separation curve ($\sigma-w$). It was found that only the group of $a^*=1.0$ can cover the experimental result. By trying $a^*=1.0$ and $c^*=0.48$, it was found that the experimental result can be reproduced with the accuracy of peak load up to 98.6% (Fig. 5.4). This stress-separation ($\sigma-w$) curve for asphalt concrete at $75^\circ F$ is represented by Eq. 5.1 and is shown in Fig. 5.5.

$$\sigma(w) = f_t, \quad \text{for } 0 \leq w \leq 0.48 \cdot w_c$$

$$\sigma(w) = \frac{1}{0.52} (1 - \frac{w}{w_c}) f_t, \quad \text{for } 0.48 \cdot w_c \leq w \leq w_c$$

(5.1)

in which

$$w_c = \frac{2 \cdot G_F}{(a^* + c^*) \cdot f_t} = \text{the critical value of the crack opening}$$
Fig. 5.3 Numerical simulation of the load versus load-line displacement response by changing the values of $a^*$ and $c^*$. 

- A: Experimental result
- B: $a^*=0.2$, $c^*=0.9$
- C: $a^*=0.2$, $c^*=0.7$
- D: $a^*=0.2$, $c^*=0.5$
- E: $a^*=0.2$, $c^*=0.3$
- F: $a^*=0.2$, $c^*=0.1$
**Fig. 5.3** (continued).
Fig. 5.3 (continued).

Fig. 5.4 Numerical simulation of the load versus load-line displacement for \( a^* = 1.0, \ c^* = 0.48 \).
5.3 Accuracy of the Numerical Simulation Method

In Section 5.2, \( a/b = 0.4 \) was selected as the experimental result for the curve fitting method. However, it is interesting to check the accuracy of the numerical simulation if other notch-depth ratios (e.g., \( a/b = 0.2 \) and \( a/b = 0.6 \)) are used. Figure 5.6 shows the comparison between the numerical simulation and experimental data. It can be noted from the figures that not all of the experimental results can be reproduced satisfactorily. This may be due to the experimental measurement, heterogeneous properties of the material and the inappropriate assumption of neglecting the beam weight. As one can see from Table 4.4, for a high notch-depth ratio with \( a/b = 0.6 \), the beam weight (about 11 lbs) becomes significant compared to its peak load (\( P_{\text{max}} \) less than 30 lbs). Thus, here the notch-depth ratio \( a/b = 0.4 \) (specimen No. is B-0-8) was used in curve fitting for reducing the effect of beam weight. Another reason for choosing
Fig. 5.6 Comparison between numerical simulation and experimental data.
a/b=0.4 was due to the fracture energy from this notch-depth ratio is about the same as the average fracture energy (see Table 4.4). Compared to the experimental dispersion, the numerical results are judged to be reasonable.

5.4 The Development of the Process Zone

By using the cohesive crack model, the development of the process zone can be analyzed. The stress distribution of the process zone at different stages on the load-deflection curve in an asphalt concrete beam subjected to three-point bending at 75°F and 18°F is shown in Fig. 5.7. The notch-depth ratio for both cases is 0.2.

The process zone starts to develop when the load is applied. It can be observed that at stage 1 indicated in Fig. 5.7, a small process
zone has developed. Note that no stress exceeds the maximum tensile strength \((f_0)\) along the process zone. In the second stage of the load-deflection curve, the cohesive crack propagates and the process zone is extended. When the peak load is reached, the stress distribution is quite different compared to the linear elastic one. It can be observed that the material at the notch tip is able to transfer stress even after the peak load and the traction free crack will not propagate until the fourth stage is reached. In the cohesive crack model, the stress distribution within the process zone is dependent on the stress-separation curve. As one can see from Fig. 5.7, the stress distribution is not linear within the process zone, which is different from the behavior of linear elastic materials. The fracture process zones for each step at two different temperatures are presented in Table 5.1. From Table 5.1 it can be seen that at the higher temperature \((75\,^\circ F)\) asphalt concrete has a larger process zone. This is in agreement with general observation, i.e., at higher temperatures the asphalt concrete is more ductile than that at lower temperatures. The crack opening displacements \((COD)\) along the crack plane at each step are shown in Fig. 5.8.
Table 5.1 The process zone length at 75°F and 18°F.

<table>
<thead>
<tr>
<th>Temp</th>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>18°F</td>
<td>Load (P) (lbs)</td>
<td>324.3</td>
<td>517.84</td>
<td>612.2</td>
<td>423.94</td>
</tr>
<tr>
<td></td>
<td>Process zone (in)</td>
<td>0.3618</td>
<td>1.0852</td>
<td>1.4469</td>
<td>1.6639</td>
</tr>
<tr>
<td>75°F</td>
<td>Load (lbs)</td>
<td>67.24</td>
<td>107.72</td>
<td>136.17</td>
<td>108.32</td>
</tr>
<tr>
<td></td>
<td>Process zone (in)</td>
<td>0.3642</td>
<td>1.0926</td>
<td>1.6025</td>
<td>1.8939</td>
</tr>
</tbody>
</table>

![Diagram](image-url)
Fig. 5.7 The process zone and stress distribution in front of the notch tip at different stages on the load-deflection curve. Three-point bend test ($a/b=0.2$).
Fig. 5.7 (continued).
Fig. 5.7 (continued).
Fig. 5.7 (continued).
Fig. 5.6 Crack opening displacement (COD) along the crack plane for each stage.

Fig. 5.8 (continued).
5.5 Parametric Study

5.5.1 Size Effect on Load-Deflection ($P-\delta$) Curves

From the results of the stress-separation curve ($\sigma-w$) obtained in Section 5.2 and other material properties, i.e., tensile strength ($f_t$), fracture energy ($G_F$), Young's modulus ($E$) and Poisson's ratio ($\nu$), it is possible to predict the size effect on the load versus load-line deflection curves using the proposed model. Six different specimen sizes with the same notch-depth ratio ($a/b=0.2$) and the same span-height ratio ($L/b=4.35$) were investigated. The dimensions of the six specimens are listed in Table 5.2.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Span (L) (inch)</th>
<th>Height (b) (inch)</th>
<th>Width (W) (inch)</th>
<th>Initial Notch (a) (inch)</th>
<th>L/b</th>
<th>a/b</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>125.98</td>
<td>28.94</td>
<td>3.0</td>
<td>5.79</td>
<td>4.35</td>
<td>0.2</td>
</tr>
<tr>
<td>B</td>
<td>75.59</td>
<td>17.36</td>
<td>3.0</td>
<td>3.47</td>
<td>4.35</td>
<td>0.2</td>
</tr>
<tr>
<td>C</td>
<td>25.20</td>
<td>5.79</td>
<td>3.0</td>
<td>1.16</td>
<td>4.35</td>
<td>0.2</td>
</tr>
<tr>
<td>D</td>
<td>12.60</td>
<td>2.89</td>
<td>3.0</td>
<td>0.58</td>
<td>4.35</td>
<td>0.2</td>
</tr>
<tr>
<td>E</td>
<td>7.56</td>
<td>1.74</td>
<td>3.0</td>
<td>0.35</td>
<td>4.35</td>
<td>0.2</td>
</tr>
<tr>
<td>F</td>
<td>2.52</td>
<td>0.58</td>
<td>3.0</td>
<td>0.12</td>
<td>4.35</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Note: Specimen D is considered to be the normal size. Specimens A, B, C are enlarged to 10, 6, 2 times respectively of the specimen D with the same ratios of span/height and notch/depth. Similarly, specimens E and F are reduced to 0.6 and 0.2 respectly of the normal size.
Fig. 5.9 shows the load versus load-line deflection curves obtained from the proposed model for six different sizes. It can be seen that the peak load increases as the specimen size increases (Fig. 5.9). In order to compare the ductility for different sizes, dimensionless load-deflection diagrams were used. By normalizing the load $P$ by $f_t \cdot b^2$ and the load-line deflection $\delta$ by $b$, one can obtain the dimensionless load-deflection diagrams as shown in Fig. 5.10. Based on the dimensionless load-deflection diagrams, one can find that for a given temperature, the ductility and the normalized peak load for asphalt concrete decrease with the increase of specimen size.

![Fig. 5.9 Load versus load-line deflection curves for venous size.](image-url)
Fig. 5.10 Dimensionless load-deflection diagrams by varying beam dimensions.
5.5.2 Notch Sensitivity and Size Effect

A structure is said to be notch sensitive if \( f^\text{net}_f \) (net flexural tensile strength) at failure decreases for non-zero initial notch length. The notch sensitivity is defined by the ratio of \( f^\text{net}_f / f_f \) (net flexural tensile strength/flexural tensile strength). \( f^\text{net}_f \) for a three-point bend notched beam can be expressed:

\[
f^\text{net}_f = \frac{3L}{2W} \frac{1}{(b - a)^2} p_{\text{max}}
\]

(5.3)

in which L is the span; W is the beam width; b is the beam depth; a is the notch depth and \( P_{\text{max}} \) is the peak load. \( f_f \) which is also termed modulus of rupture (MOR), is equal to \( f^\text{net}_f \) when the initial notch length (a) equals zero.

Table 5.3 Parameters for notch sensitivity.

<table>
<thead>
<tr>
<th>a/b</th>
<th>P_{\text{max}} (lb)</th>
<th>( f^\text{net}_f ) (psi)</th>
<th>( f^\text{net}_f / f_f )</th>
<th>a/b</th>
<th>P_{\text{max}} (lb)</th>
<th>( f^\text{net}_f ) (psi)</th>
<th>( f^\text{net}_f / f_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>185.39</td>
<td>138.08</td>
<td>1</td>
<td>0</td>
<td>348.79</td>
<td>129.89</td>
<td>1</td>
</tr>
<tr>
<td>0.2</td>
<td>114.83</td>
<td>133.57</td>
<td>0.968</td>
<td>0.2</td>
<td>226.94</td>
<td>132.06</td>
<td>1.017</td>
</tr>
<tr>
<td>0.4</td>
<td>78.29</td>
<td>161.89</td>
<td>1.173</td>
<td>0.4</td>
<td>141.41</td>
<td>146.29</td>
<td>1.126</td>
</tr>
</tbody>
</table>

\[ L = \frac{(G_F - B/\ell)^2}{63.39^2} = 38.36 \text{ inch} \]

(5.3)

\[ \text{G12.60 in}, b = 28.9 \text{ in}, W = 3.0 \text{ in} \]

\[ L = \frac{(G_F - B/\ell)^2}{63.39^2} = 38.36 \text{ inch} \]

(5.3)

\[ \text{L = 12.60 in}, b = 30.79 \text{ in}, W = 3.0 \text{ in} \]

\[ L = \frac{(G_F - B/\ell)^2}{63.39^2} = 38.36 \text{ inch} \]

(5.3)

\[ \text{L = 0.754 in}, b = 28.94 \text{ in}, W = 3.0 \text{ in} \]

\[ L = \frac{(G_F - B/\ell)^2}{63.39^2} = 38.36 \text{ inch} \]

(5.3)

\[ \text{L = 0.754 in}, b = 28.94 \text{ in}, W = 3.0 \text{ in} \]
Table 5.3 shows the results obtained for the notch sensitivity and size effect study. In this Table the net flexural tensile strengths for different notch-depth ratios and different specimen sizes were calculated using Eq. 5.3. The results from Table 5.3 are shown in Fig. 5.11. It can be noted from Figure 5.11 that the asphalt concrete beam is most sensitive to the existence of notch when the relative notch-depth ratio is close to 0.2. Furthermore, it indicates that notch sensitivity is not a pure material property since it is also dependent on the size of the beams. The notch sensitivity will increase with increasing beam dimensions. In the case of \( T = 75^\circ F \), the characteristic length \( \left( L_{ch} - \frac{G_r \cdot E}{f_t^2} \right) \) for asphalt concrete is about 38.36 inches and consequently a 5.79 inch (i.e. \( b / L_{ch} = 0.1509 \)) deep beam was found to be relatively notch insensitive. Although notch sensitivity is a reflection of the material property and structural behavior, it can be used as a relative indicator when different materials are compared using the same size of specimens. Figure 5.12 shows the relationship between modulus of rupture (MOR) and beam depth. It can be observed from this Figure that modulus of rupture decreases with increasing beam depth. The variation of modulus of rupture becomes smaller as the beam depth increases (for example, from \( b = 28.94 \) inch to \( b = 43.4 \) inch the variation of the modulus of rupture is very limited). For large specimens, because the process zone is small compared to the whole section, the net flexural tensile strength for a very large specimen will tend to its modulus of
rupture (MOR). The effect of notch-depth ratio on the load-deflection and load-CMOD curves predicted from the cohesive crack model are also shown in Fig. 5.13.

Fig. 5.11 Notch sensitivity as a function of relative notch depth and b/Lch calculated by means of cohesive crack model.

Fig. 5.12 Theoretical Prediction of Size-Effect on Modulus of Rupture (MOR) of Three-Point Bending Asphalt Specimens.
Fig. 5.13 Notch effect on load-deflection and load-CMOD curves.
5.5.3 Temperature Effect on $P-\delta$ and $P-CMOD$ Curves

Since asphalt concrete is a visco-elastic material, the mechanical behavior is sensitive to temperature. The temperature effect on $P-\delta$ and $P-CMOD$ curves are predicted using the cohesive crack model and are presented in Fig. 5.14. Note that the stress-separation ($\sigma-w$) curves at different temperatures are assumed to be the same as the diagram indicated in Fig. 5.5 or Eq. 5.1. As can be seen in Fig. 5.14, the behavior of asphalt concrete becomes much more brittle when the temperature decreases. Since asphalt concrete pavements become brittle as the temperature decreases (i.e., from summer to winter), low temperature cracking in the asphalt concrete pavements will become an important problem during the winter season.

5.5.4 Variation of Fracture Toughness with Temperature

As mentioned in section 2.3.2, fracture toughness, $K_{lc}$, (or critical stress intensity factor) can be used as a material property. In the linear elastic fracture mechanics, when a crack has reached a critical size, the stress field in the vicinity of crack tip is represented by a factor $K_{lc}$. In this investigation the $K_{lc}$ value is computed using the following formula derived by Tada et al [10]:

$$K_{lc} = \sigma_{max} \sqrt{\pi a} F\left(\frac{a}{b}\right) = \frac{3P_{max}}{2\pi b} \sqrt{\pi a} F\left(\frac{a}{b}\right)$$  \hspace{20pt} (5.4)

in which
Fig. 5.14 Temperature effect on load-deflection and load-CMOD curves (a/b=0.40).
Fig. 5.14 (continued a/b = 0).
\[
F\left(\frac{a}{b}\right) = \frac{1.99 - \frac{a}{b}(1 - \frac{a}{b})}{\sqrt{\pi}} \left[ 2.15 - 3.93 \frac{a}{b} + 2.7 \left(\frac{a}{b}\right)^2 \right] \frac{1}{(1 + 2 \frac{a}{b})(1 - \frac{a}{b})^2} \tag{5.5}
\]

In order to study the temperature effect on the fracture toughness \(K_{lc}\), numerical results of the three-point bend tests obtained using the cohesive crack model are summarized in Table 5.4. In this Table the fracture toughness \(K_{lc}\) for different notch-depth ratios at different temperatures are calculated.

Figure 5.15 shows the theoretical results of \(K_{lc}\) versus temperature from Table 5.4. It can be observed that \(K_{lc}\) is a function of temperature. The phenomenon was also reported by Karakouzian [24], i.e., the fracture toughness is sensitive to temperature (Fig. 5.16). In addition, he also found that \(K_{lc}\) is a function of crack size (except a particular dimension) and loading rate.

5.5.5 Variation of Critical \(J\)-integral \((J_{le})\) with Temperature

Among the various parameters used to characterize elastic-plastic fracture, the J-integral proposed by Rice in 1968 has been widely accepted [25]. The J-integral is defined as a path independent contour integral representing a non-linear elastic energy release rate. It can be shown that an approximate value of critical strain energy release rate \((J_{le})\) can be obtained by calculating the area under the load-deflection curve (i.e. the total energy to failure, \(U_T\)) from three-point bending for different transverse crack lengths [26]. The
Table 5.4 The average values of fracture toughness for various temperature.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>-20°F</th>
<th>-5°F</th>
<th>18°F</th>
<th>6°F</th>
<th>60°F</th>
<th>75°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (psi)</td>
<td>405.3</td>
<td>494.1</td>
<td>405.9</td>
<td>311.8</td>
<td>251.2</td>
<td>226.0</td>
</tr>
<tr>
<td>G (in-lb/in²)</td>
<td>4.13</td>
<td>4.84</td>
<td>4.73</td>
<td>4.41</td>
<td>2.92</td>
<td>2.06</td>
</tr>
<tr>
<td>E (psi)</td>
<td>6700700</td>
<td>4103170</td>
<td>1934290</td>
<td>073800</td>
<td>489910</td>
<td>300000</td>
</tr>
<tr>
<td>v</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.25</td>
<td>0.34</td>
<td>0.35</td>
</tr>
</tbody>
</table>

\[ \sigma - \omega \]

As shown in Figure 5.5

<table>
<thead>
<tr>
<th>/b=0.2</th>
<th>P_{max} (lb)</th>
<th>K_{lc}</th>
<th>/b=0.4</th>
<th>P_{max} (lb)</th>
<th>K_{lc}</th>
<th>/b=0.6</th>
<th>P_{max} (lb)</th>
<th>K_{lc}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1039.3</td>
<td>939.96</td>
<td></td>
<td>593.38</td>
<td>905.26</td>
<td></td>
<td>268.61</td>
<td>779.88</td>
</tr>
<tr>
<td></td>
<td>1179.0</td>
<td>1066.3</td>
<td></td>
<td>684.36</td>
<td>1044.06</td>
<td></td>
<td>313.91</td>
<td>911.40</td>
</tr>
<tr>
<td></td>
<td>948.84</td>
<td>858.15</td>
<td></td>
<td>549.63</td>
<td>838.52</td>
<td></td>
<td>253.46</td>
<td>735.89</td>
</tr>
<tr>
<td></td>
<td>714.34</td>
<td>646.06</td>
<td></td>
<td>413.93</td>
<td>631.49</td>
<td></td>
<td>191.20</td>
<td>555.13</td>
</tr>
<tr>
<td></td>
<td>507.40</td>
<td>458.90</td>
<td></td>
<td>294.27</td>
<td>448.94</td>
<td></td>
<td>137.90</td>
<td>400.38</td>
</tr>
<tr>
<td></td>
<td>395.79</td>
<td>357.96</td>
<td></td>
<td>232.90</td>
<td>355.31</td>
<td></td>
<td>110.87</td>
<td>321.90</td>
</tr>
</tbody>
</table>

Average: \( K_{lc} \) (psi-\( \sqrt{\text{in}} \))

|        | 875.03       | 10G7.26        |        | 810.85       | 610.89        |        | 436.07       | 345.06         |

Note: 1. \( v \) for -20°F and -5°F are assumed the same as that of 18°F since Poisson's ratio is not sensitive to the results.

2. \( E \) for various temperatures are predicted based on the normal value for \( E \) at 75°F = 300,000 psi and the relation between temperature and Young's modulus constructed in chapter IV (experimental program).

3. Tensile strength (\( f_t \)) values are estimated from T.

Van Dam and Carpenter, S. H. for AC-20 grade [14].

4. \( L=12 \) inch, \( b=3 \) inch, \( W=3 \) inch.
Fig. 5.15 Theoretical prediction of temperature effect on $K_{tc}$.

Fig. 5.16 Experimental results of temperature effect on $K_{tc}$ (from Karakouzian).
following formulas were employed to calculate $J_{lc}$ from the value of $U_T$ and the corresponding crack length,

$$J_{lc} = -\left( \frac{1}{W} \frac{dU_T}{da} \right)$$  \hspace{1cm} (5.6)

$$U_T = \int_0^{\delta(P_{max})} P(\delta)d\delta$$  \hspace{1cm} (5.7)

in which

$U_T$ = total strain energy.

In the present study, the cohesive crack model is used to simulate the three-point bending. By using the load versus load-line deflection curve obtained from the numerical analysis, one can get the $P-\delta$ curve up to the peak load as shown in Fig. 5.17. As a result, the $U_T$ can be calculated from the load-deflection curve and thus, the $J_{lc}$ can be determined as follows:

1. Total energy, $U_T$, under the load-deflection curve was calculated by integrating the area of the $P-\delta$ curve up to the peak load using numerical methods. For example, in the case of Fig. 5.17 ($T = 75^\circ F$, $a/b = 0.6$),

$$P_{max} = 110.87 \text{ lbs}$$

$$\delta(P_{max}) = 0.039852 \text{ inch}$$  \hspace{1cm} (5.7)

$$P(\delta) = 6545 \cdot 3555 \delta - 1.527 \times 10^5 \times \delta^2 + 2.083 \times 10^6 \times \delta^3 - 1.544 \times 10^7 \times \delta^4$$

The total strain energy, $U_T$, can be calculated as:
\[ P = 6545.3555x - 1.527e+5x^2 + 2.083e+6x^3 - 1.544e+7x^4 \quad R = 1.00 \]

Fig. 5.17 Load-deflection diagram for calculating total strain energy (UT).

\[ \frac{U}{W} = 5.371 - 1.7367a \]
\[ \frac{U}{W} = 5.348 - 1.934\% \]
\[ \frac{U}{W} = 3.149 - 0.9383a \]
\[ \frac{U}{W} = 24877 - 0.8442a \]
\[ \frac{U}{W} = 4.802 - 1.4808a \]
\[ \frac{U}{W} = 3.8723 - 1.0158a \]

Fig. 5.18 Energy per unit thickness versus notch (crack) length.
\[ U_T = \int_{0}^{0.039852} (6545.3555 \ 6 - 1.527 \times 10^5 \times \delta^2 + 2.083 \times 10^6 \times \delta^3 - 1.544 \times 10^7 \times \delta^4) \, d\delta \]
\[ = 2.979 \text{ in.-lb.} \]

2. The total energy per unit thickness, \( \frac{U_T}{W} \), was then plotted against the notch as shown in Fig. 5.18. The slope \( \frac{d\left(\frac{U_T}{W}\right)}{da} \) was obtained through regression. This procedure was repeated for each temperature.

3. The critical J-integral, \( J_{lc} \), was then determined for each temperature from the slope obtained from Fig. 5.18 and the following equation:

\[ J_{lc} = -\left( \frac{1}{W} \frac{dU_T}{da} \right) \]

(5.5)

The results of the \( U_T \), \( \frac{U_T}{W} \) and \( J_{lc} \) at different temperatures calculated based on the above procedures are summarized in Table 5.5.

The theoretical \( J_{lc} \) versus temperature diagram is shown in Fig. 5.19. From this Figure it is obvious that \( J_{lc} \) derived from elastic-plastic fracture mechanics is sensitive to temperature. It can be seen that the \( J_{lc} \) values increase when the temperature increases for temperatures below 40°F. However, the \( J_{lc} \) values were found to decrease at temperatures higher than 40°F. The experimental results of J-integral (\( J_{lc} \)) versus temperature relationship reported by
Table 5.5 Results of $J_{lc}$ for various temperature.

<table>
<thead>
<tr>
<th>Temperature Parameters</th>
<th>-2WF</th>
<th>-5°F</th>
<th>18°F</th>
<th>36°F</th>
<th>60°F</th>
<th>7°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_T$ (in-lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a/b=0.2</td>
<td>9.233</td>
<td>11.692</td>
<td>12.990</td>
<td>12.799</td>
<td>7.793</td>
<td>6.018</td>
</tr>
<tr>
<td>a/b=0.4</td>
<td>9.070</td>
<td>9.173</td>
<td>9.855</td>
<td>8.603</td>
<td>5.997</td>
<td>4.274</td>
</tr>
<tr>
<td></td>
<td>5.577</td>
<td>6.359</td>
<td>6.738</td>
<td>5.838</td>
<td>4.417</td>
<td>2.979</td>
</tr>
<tr>
<td>$U_T/W$ (in-lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a/b=0.2</td>
<td>3.078</td>
<td>3.897</td>
<td>4.330</td>
<td>4.267</td>
<td>2.598</td>
<td>2.006</td>
</tr>
<tr>
<td>a/b=0.4</td>
<td>3.023</td>
<td>3.058</td>
<td>3.285</td>
<td>2.868</td>
<td>1.999</td>
<td>1.425</td>
</tr>
<tr>
<td></td>
<td>1.859</td>
<td>2.120</td>
<td>2.246</td>
<td>1.946</td>
<td>1.472</td>
<td>0.993</td>
</tr>
<tr>
<td></td>
<td>1.015</td>
<td>1.4808</td>
<td>1.7367</td>
<td>1.9342</td>
<td>0.9383</td>
<td>0.8421</td>
</tr>
</tbody>
</table>

W=3.0 inch, L=12 inch, b=3 inch.

Dongre et al. [26] are shown in Fig. 5.20. Despite the large dispersion of the experimental data reported by Dongre et al, it can be concluded that $J_{lc}$ increases when the temperature increases for low temperatures (i.e., lower than 40°F). However, it is difficult to conclude the results for the temperatures higher than 40°F due to large dispersion observed in the experimental data. More experimental results are needed to verify the theoretical prediction at a higher temperature range.
Fig. 5.19 Theoretical prediction of temperature effect on Jlc.

Fig. 5.20 Experimental critical J-integral (Jlc) versus temperature plot (from Dongre, Sharma, and Anderson).
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE STUDY

6.1 Conclusions

A cohesive crack model, which is similar to the Dugdale-Barenblatt model, was proposed in this study to simulate the progressive crack development in asphalt concrete. It was found that the crack propagation process can be analyzed using the proposed model. The magnitude of the closing pressures was found to depend on the stress-separation ($\sigma - w$) relationship. To determine the stress-separation ($\sigma - w$) relationship for asphalt concrete, an experimental program was designed. Indirect tensile tests and three-point bend tests were conducted using an MTS testing system. From the experimental results, the material properties at various temperatures such as Young's modulus ($E$), fracture energy ($G_f$), and indirect tensile strength ($f_t$) were determined. A numerical simulation (or curve fitting method) was used to determine the stress-separation ($\sigma - w$) relationship for the proposed cohesive crack model.

Using the material properties obtained from the experimental program, size effect, notch sensitivity, and temperature effect on the
fracture parameters (i.e., critical stress intensity factor and critical J-integral) were studied. The theoretical predictions were found to be in good agreement with available experimental results in terms of effects of temperature and specimen size on crack propagation of asphalt concrete.

Based on the findings of this research, the following conclusions were drawn:

1. By using the cohesive crack model, the fracture zone for asphalt concrete can be computed. For asphalt concrete, a higher temperature has a larger fracture process zone since asphalt concrete tends to be more ductile at higher temperatures than at lower temperatures.

2. By studying the size effect, it was found that for a given temperature, ductility for asphalt concrete decreases with increasing size.

3. Notch sensitivity increases with increasing beam dimensions. A decrease in characteristic length \( L_{ch} \) corresponds to an increase in notch sensitivity.

4. No conclusion can be made on the critical J-integral from the available experimental results. However, the effect of temperature on the fracture toughness \( K_{ic} \) can be correctly predicted using the proposed cohesive crack model.
6.2 Recommendations for Future Study

Based on the present study, the following suggestions are recommended for future study:

1. In this study, the stress-separation ($\sigma - w$) relationships for various temperatures were assumed to be the same. To correctly evaluate the effect of temperature on the stress-separation curve, a direct tensile test performed under different temperatures is recommended. It is also recommended that three-point bending tests at different temperatures should be performed in order to get more reliable fracture energy values.

2. Since asphalt concrete pavements are subjected to a repeated dynamic loading instead of a static loading, a fatigue test is recommended to be done for further study.

3. The visco-elastic response can be incorporated in the theoretical analysis if the overall behavior of asphalt concrete subjected to different service conditions (i.e., loading rate, temperature, etc.) is to be properly simulated.
References


Appendix  A

CONSTRUCTION OF THE COMPLIANCE MATRIX
A.1. A Method for Computing Certain Displacement Relevant to Crack Problems

From linear elastic fracture mechanics (LEFM) the strain energy release rate $G_i$ can be expressed as:

$$G = \frac{\partial U_T}{\partial a} \bigg|_{P = \text{const}},$$ \hfill (A.1)

in which $U_T$ is the total strain energy; $a$ is the crack area (for unit width, area=$1 \times a$).

![Figure A.1](image)

Fig. A.1 A body with initial crack subjected to load.

Now consider a body loaded by a force, $P$, which in addition has virtual forces, $F$, applied, as shown in Fig. A.1. The total strain energy can be summed for the mode I, II, and III, i.e.

$$G = G_I + G_{II} + G_{III}$$ \hfill (A.2)
The resulting stress intensity factors for each mode due to loading force $P$ and virtual force $F$ may be added. That is to say

\[
K_I = K_{IP} + K_{IF} \quad (A.3.a)
\]

\[
K_{II} = K_{IIIP} + K_{IIF} \quad (A.3.b)
\]

\[
K_{III} = K_{IIIIP} + K_{IIIIF} \quad (A.3.c)
\]

From Eq. 2.9 obtained by Irwin [8], the mode I stress intensity factor can be expressed as

\[
K_I = \sqrt{EG} \quad (2.10)
\]

If only the opening mode is considered, the following equation can be obtained from Eqs. 2.10, A.2, and A.3.b

\[
E'G = E'G_I = K_I^2 = (K_{IP} + K_{IF})^2
\]

or

\[
E'G_1 = (K_{IP} + K_{IF})^2 \quad (A.4)
\]

Introducing Castigliano's theorem, it is possible to compute certain displacements. Castigliano's theorem states that displacement of any load $Q$ (in its own direction) may be computed by

\[
\Delta_0 = \frac{\partial U_1}{\partial Q} \quad (A.5)
\]
The total strain energy may be regarded as that due to applying loading forces with no crack present plus that due to crack. That is

\[ U_T = U_{no\ crack} + \int_0^s \frac{\partial U_T}{\partial a} da \]  \hspace{1cm} (A.6)

Introducing Eqs A.1 and A.5, \( \Delta Q \) can be computed.

\[ \Delta Q = \frac{\partial U_T}{\partial Q} = \frac{\partial}{\partial Q} \left( U_{no\ crack} + \int_0^s \frac{\partial U_T}{\partial a} da \right) \]

\[ = \frac{\partial U_{no\ crack}}{\partial Q} + \frac{\partial}{\partial Q} \int_0^s G da \]  \hspace{1cm} (A.7)

in which \( \frac{\partial U_{no\ crack}}{\partial Q} \) can be identified with \( \Delta Q_{no\ crack} \). That is to say

\[ \frac{\partial U_{no\ crack}}{\partial Q} = \Delta Q_{no\ crack} \]  \hspace{1cm} (A.8)

Substituting Eq. A.4 into Eq. A.7 and letting the virtual force \( F \) equal zero gives the following equation.

\[ \Delta p = \Delta p_{no\ crack} + \frac{2}{1} \int_0^s K_{ip} \frac{\partial K_{ip}}{\partial p} da \]  \hspace{1cm} (A.9a)

or

\[ \Delta F = \Delta F_{no\ crack} + \frac{2}{1} \int_0^s K_{ip} \frac{\partial K_{ip}}{\partial F} da \]  \hspace{1cm} (A.9b)

Thus, displacements \( \Delta p \) or \( \Delta F \) may be computed from displacements with no crack present (this part can be calculated by...
traditional structural mechanics) and a knowledge of stress intensity factor formulas by using Eq. A.9.

A.2. Formulation of Load Point Displacement and Crack Opening $w$ (or COD) for a Three-Point Bending

A.2.P Formulation of Load Point Displacement ($\delta$ due to $P$)

Consider a notch beam subjected to a point load $P$ in the midspan as shown in Fig. A.2. From equation A.9.a

$\Delta_P = \Delta_{P \, \text{no crack}} + \Delta_{P \, \text{crack}} = \Delta_{P \, \text{no crack}} + \frac{2}{E} \int_0^{a_c} K_{ip} \frac{\partial K_{ip}}{\partial P} \text{d}a$ (A.10)

in which $\Delta_{P \, \text{no crack}}$ can be obtained from traditional structural mechanics,

$\Delta_{P \, \text{no crack}} = \frac{PL^3}{48EI} + \frac{\alpha_s PL}{4GA}$ (A.11)
and $E$ is Young's modulus; $I$ is the moment of inertia; $L$ is span; $G$ is shear modulus $G = \frac{E}{2(1 + \nu)}$; $A$ is the cross sectional area; $\alpha_\ast$ is shear shape factor $\alpha_\ast = \frac{10(1 + \nu)}{1 + 2\nu}$; $\nu$ is Poisson ratio.

Thus the displacement due to crack is equal to

$$\Delta_{\text{p crack}} = \frac{2}{E} \int_0^a K_{\text{ip}} \frac{\partial K_{\text{ip}}}{\partial P} \, da \quad \text{(A.12)}$$

For three-point bending condition:

$$\sigma = \frac{6M}{b^2} = \frac{6PL}{b^4} \cdot \frac{3PL}{2b^2} \quad \text{(A.13)}$$

In the case of $L=4b$,

$$\sigma = \frac{3PL}{2b^2} = \frac{12Pb}{2b^2} = \frac{6P}{b} \quad \text{(A.14)}$$

According to Eq. 2.8

$$K_{\text{ip}} = \sigma \sqrt{\pi a} \, F(a \div b) \quad \text{(A.15)}$$

in which

$$F(a \div b) = \frac{1.99 - \frac{a}{b} \left( \frac{2.15 - 3.93 \frac{a}{b} + 2.7 \left( \frac{a}{b} \right)^2}{1 + 2 \frac{a}{b} \left( 1 - \frac{a}{b} \right)^2} \right)}{\sqrt{\pi}} \quad \text{(A.16)}$$

Substituting Eq. A.14 into Eq. A.15 one gets

$$K_{\text{ip}} = \frac{6P}{b} \sqrt{\pi a} \, F(a \div b) \quad \text{(A.17)}$$

Substituting Eq. A.15 into Eq. A.12 yields
Thus

\[ \Delta_{p, \text{crack}} = \frac{2}{E} \int_{0}^{a_c} 6P b \sqrt{\pi a} F(a / b) \left[ \frac{6}{b} \sqrt{\pi a} F(a / b) \right] da \]

Now introducing Gauss-Chebyshev Quadrature (see Appendix B) one can estimate the above integral.

\[ \Delta_{p, \text{crack}} = \frac{72P \pi}{b^2 E'} \int_{-1}^{1} \frac{(a_c)Z + (a_c)}{2} \left[ F(a / b) \right]^2 \frac{a_c}{2} dZ \]

\[ = \frac{72P \pi}{b^2 E'} \int_{-1}^{1} \frac{1}{\sqrt{1 - Z^2}} \left[ \sqrt{1 - Z^2} \frac{(a_c)Z + (a_c)}{2} \left[ F(a / b) \right]^2 \frac{a_c}{2} \right] dZ \]

\[ = \frac{72P \pi a_c}{b^2 E'} \int_{0}^{1} \frac{1}{\sqrt{1 - Z^2}} \left[ \sqrt{1 - Z^2} \frac{(a_c)Z + (a_c)}{2} \left[ F(a / b) \right]^2 \frac{a_c}{2} \right] dZ \]

\[ = \frac{72P \pi a_c}{b^2 E'} \left[ \frac{\pi}{n + 1} \sum_{i=0}^{n} F(Z_i) \right] \tag{A.19} \]

in which

\[ F(Z_i) = \sqrt{1 - Z_i^2} \frac{(a_c)Z_i + (a_c)}{2} \left[ F(a / b) \right]^2 \]

\[ a' = \frac{a_c Z + a_c}{2} \]

\[ F(a' / b) = \frac{1.99 - \frac{a'}{b}(1 - \frac{a'}{b})}{\sqrt{\pi}} \left[ (2.15 - 3.93 \frac{a'}{b} + 2.7 \left( \frac{a'}{b} \right)^2 \right]
\]

\[ (1 + 2 \frac{a'}{b})(1 - \frac{a'}{b})^{\frac{1}{2}} \]

Thus Eq. A.19 may be rewritten as
The above procedures have been programmed as the CRCKP subroutine.

**A.2.1.1 Check the Formulation of CRCKP Using Available Formulas**

Calculate load-line deflection by available formula [12].

\[ A_{\text{crack}} + \Delta_{\text{no crack}} = \frac{\sigma}{E} LV_2\left(\frac{a}{b}\right) + \frac{16P}{Et} \left(1 + \frac{\alpha t E}{16G}\right) \]

\[ = \frac{24P}{Et} + \frac{16P}{Et} \left(1 + \frac{\alpha t E}{16G}\right) \quad (A.21) \]

Table A.5 shows the comparison of the numerical method with the results from Eq. A.21.

<table>
<thead>
<tr>
<th>a/b</th>
<th>Load-line deflection (from numerical method) (inch)</th>
<th>Load-line deflection (from Eq. A.21) (inch)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.3494E-03</td>
<td>3.3417E-03</td>
<td>0.23</td>
</tr>
<tr>
<td>0.5</td>
<td>9.3303E-03</td>
<td>8.8725E-03</td>
<td>5.16</td>
</tr>
<tr>
<td>0.8</td>
<td>6.1186E-02</td>
<td>5.6277E-03</td>
<td>8.72</td>
</tr>
</tbody>
</table>
A.2.2 Formulation of Crack Opening \( w \) due to Applied Load \( P \)

Consider a beam subjected to a three-point bending load \( P \) as shown in Fig. A.3. In order to calculate the crack opening at point D the virtual force is applied at that point.

![Diagram](image)

Fig. A.3 A notched beam subjected to three-point bending with a virtual force \( F \).

From Eq. A.9.b

\[
\Delta_F = \Delta_{F_{\text{no crack}}} + \frac{2}{E} \int_{a_F}^{a_c} \frac{\partial K_{\psi}}{\partial F} da
\]

(A.22)

or

\[
\text{COD} = \text{COD}_{F_{\text{no crack}}} + \frac{2}{E} \int_{a}^{a_c} \frac{\partial K_{\psi}}{\partial F} da
\]

(A.23)

in which

\[
\text{COD}_{F_{\text{no crack}}} = 0 \quad \text{and}
\]
\[ K_{ip} = \frac{6P}{b} \sqrt{\pi a} F\left(\frac{a}{b}\right) \quad (A.17) \]

\[ F\left(\frac{a}{b}\right) = \frac{1}{\sqrt{\pi}} \left(1 - \frac{a}{b}\right)^{\frac{3}{2}} \left(2.15 - 3.93 \frac{a}{b} + 2.7 \left(\frac{a}{b}\right)^2\right) \quad (A.16) \]

\[ K_{if} = \frac{2F}{\sqrt{\pi a}} F\left(\frac{a}{a}, \frac{a}{b}\right) \quad (A.24) \]

\[ F\left(\frac{a}{a}, \frac{a}{b}\right) = \frac{3.52(1 - \frac{a}{a})}{\left(1 - \frac{a}{b}\right)^{\frac{3}{2}}} - 4.35 - 5.28 \frac{a}{a} \quad (A.25) \]

Thus Eq. A.23 yields

\[ \text{COD} = \frac{2}{E} \int_a^b \left[ \frac{6P}{b} F\left(\frac{a}{b}\right) F\left(\frac{a}{a}, \frac{a}{b}\right) \right] \, da \]

\[ = \frac{24P}{bE} \int_a^b F\left(\frac{a}{b}\right)^2 F\left(\frac{a}{a}, \frac{a}{b}\right) \, da \quad (A.26) \]

Using Gauss-Chebyshev Quadrature to calculate the integration of the Eq. A.26 one gets

\[ \text{COD} = \frac{24P}{bE} \int_a^b \frac{1}{\sqrt{1 - Z^2}} \left\{ \sqrt{1 - Z^2} \left[ F\left(\frac{a}{b}\right)^2 \right] \left[ F\left(\frac{a}{a}, \frac{a}{b}\right) \right] \right\} dZ \]

\[ = \frac{24P}{bE} \frac{a - a_F}{2} \int_a^b \frac{1}{\sqrt{1 - Z^2}} \left\{ \sqrt{1 - Z^2} \left[ F\left(\frac{a}{b}\right)^2 \right] \left[ F\left(\frac{a}{a}, \frac{a}{b}\right) \right] \right\} dZ \]
in which

\[
F(Z_i) = \sqrt{1 - Z_i^2} \left[ F(a'/b) \right] \left[ F\left(\frac{a_P}{a}, \frac{a'}{b}\right) \right] 
\]

\[
a' = \frac{(a_e - a_P) Z + (a_e + a_P)}{2}
\]

\[
F(a'/b) = \frac{1.99 - \frac{a'}{b}(1 - \frac{a'}{b}) \left[ (2.15 - 3.93\frac{a'}{b} + 2.7\frac{a'}{b}) \right]}{\sqrt{\pi}} (1 + 2\frac{a'}{b}) \frac{a'}{b}
\]

\[
F\left(\frac{a_P}{a}, \frac{a'}{b}\right) = \frac{3.52(1 - \frac{a_P}{a}) - 4.35 - 5.28\frac{a_P}{a}}{(1 - \frac{a_P}{b})} \left(1 - \frac{a'}{b}\right)
\]

\[
+ \left\{ \frac{1.30 - 0.30 \left(\frac{a_P}{a}\right)}{1 - \left(\frac{a_P}{a}\right)^2} + 0.83 - 1.76\frac{a_P}{a} \right\} \left\{1 - \left(\frac{a_P}{a}\right)\frac{a'}{b}\right\}
\]

Eq. A.27 is the final result of the crack opening at node D due to apply load P. These procedures have been programed as the CODP subroutine.

**A.2.2 Check the Formulation of CODP by Available Formulas**

Calculate the crack opening displacement \(\text{(COD)}\) by available formula [28]:

\[
\text{COD}(x) = \frac{24P\alpha}{bE} v_i(\frac{\alpha}{b}) \left\{ (1 - \frac{x}{\alpha})^2 + (-1.149\frac{\alpha}{b} + 1.081)\left[ \frac{x}{\alpha} - \left(\frac{x}{\alpha}\right)^2 \right] \right\}^\frac{1}{2} 
\]
Fig. A.4 shows the differences between available formula and numerical result.

A.2.3 Formulation of Crack Opening w Due to the Load P Acting Perpendicular to the Crack Surface
Consider a notched beam with a pair of loads at node H and a pair of virtual forces F at node E (see Fig. A.5). From the Eq. A.9.b the crack opening at node E may be expressed as

\[
\Delta_F = \Delta_{F \text{ no crack}} + \frac{2}{E} \int_{a}^{c} K_{IP} \frac{\partial K_{IP}}{\partial F} da
\]  

(A.22)
or

\[
\text{COD} = \Delta_{F \text{ no crack}} + \frac{2}{E} \int_{a}^{c} K_{IP} \frac{\partial K_{IP}}{\partial F} da
\]

(A.30)

in which

\[
\Delta_{F \text{ no crack}} = 0
\]

(A.31.a)

\[
K_{IP} = \frac{2P}{\sqrt{\pi a}} F\left(\frac{a}{a}, \frac{a}{b}\right)
\]

(A.31.b)

\[
\frac{\partial K_{IP}}{\partial F} = \frac{2}{\sqrt{\pi a}} F\left(\frac{a}{a}, \frac{a}{b}\right)
\]

(A.31.c)

\[
F\left(\frac{a}{a}, \frac{a}{b}\right) = \frac{3.52 \left(1 - \frac{a}{a}\right)}{\left(1 - \frac{a}{b}\right)^{\frac{4}{2}}} - \frac{4.35 - 5.28, \frac{a}{b}}{\left(1 - \frac{a}{b}\right)^{\frac{1}{2}}}
\]

\[
+ \left\{ \frac{1.30 - 0.30 \left(\frac{a}{a}\right)^{\frac{2}{2}}}{\sqrt{1 - \left(\frac{a}{a}\right)^{2}}} + 0.83 - 1.76 \frac{a}{a} \right\} \left\{ 1 - \left(\frac{a}{a}\right) \frac{a}{b} \right\}
\]

(A.31.d)
Substituting Eqs. A.31.a, A.31.b, and A.31.c into Eq. A.30 yields

\[
\text{COD} = \frac{2}{E} \left[ \int_{a_1}^{a_f} \frac{2P}{\sqrt{\pi a}} F(\frac{a}{b}, \frac{a}{b}) \right] \left[ \frac{2}{\sqrt{\pi a}} F(\frac{a_F}{a}, \frac{a}{b}) \right] da
\]

\[
= \frac{8P}{E} \left[ \int_{a_1}^{a_f} \frac{1}{\sqrt{\pi a}} F(\frac{a}{b}, \frac{a}{b}) \right] F(\frac{a_F}{a}, \frac{a}{b}) da
\]

Again applying Gauss-Chebyshev Quadrature to estimate the integration of Eq. A.32 yields

\[
\text{COD} = \frac{24P}{bE} \int_{-1}^{1} \frac{1}{\pi} \frac{1}{(a_c - a_F)Z + (a_c + a_F)} \left[ F(\frac{a}{b}, \frac{a}{b}) \right] \left[ F(\frac{a_F}{a}, \frac{a}{b}) \right] \frac{a_c - a_F}{2} dZ
\]

\[
= \frac{8P}{E} \frac{a_c - a_F}{2} \int_{-1}^{1} \frac{1}{\sqrt{1 - Z^2}} \left[ \sqrt{1 - Z^2} \frac{1}{\pi} \frac{1}{(a_c - a_F)Z + (a_c + a_F)} \left[ F(\frac{a}{b}, \frac{a}{b}) \right] \left[ F(\frac{a_F}{a}, \frac{a}{b}) \right] \right] dZ
\]

\[
= \frac{8P}{E} \frac{a_c - a_F}{2} \frac{\pi}{n + 1} \sum_{i=1}^{n} F(Z_i)
\]

in which

\[
F(Z_i) = \sqrt{1 - Z_i^2} \frac{1}{\pi} \frac{1}{(a_c - a_F)Z + (a_c + a_F)} \left[ F(\frac{a}{b}, \frac{a}{b}) \right] \left[ F(\frac{a_F}{a}, \frac{a}{b}) \right]
\]

\[
a' = \frac{(a_c - a_F)Z + (a_c + a_F)}{2}
\]
Eq. A.33 is the final result of the crack opening at node E due to the load $P$ acting at node H. These procedures also have been programed as the CODS subroutine.

There is no close form solution to calculate this crack opening. The numerical solution from the above formulation (i.e., subroutine CODS) is shown in Fig. A.6.
Fig. A.6 Numerical results from subroutine CODS (a/b=0.5).

- a/b=0.5
- P=1000 psi
- E=6E06 psi
- t=1 inch
- Poisson's ratio=0.2
Appendix B

GAUSS-CWEBYSWEV QUADRATURE
Consider the integral of a function $Q(x)$ which is integrated from $x=a$ to $x=b$. That is

$$I = \int_a^b Q(x)\,dx$$  \hspace{1cm} (B.1)

In order to estimate the integration of Eq. (B.1) by the numerical method, Gauss-Chebshev Quadrature is introduced [27]. The Gauss-Chebshev Quadrature formula are given by

$$\int_{-1}^{1} \frac{1}{\sqrt{1-Z^2}} F(Z)\,dZ = \sum_{i=0}^{n} w_i F(Z_i)$$  \hspace{1cm} (B.2)

in which

$$Z_i = \cos \frac{(2i+1)\pi}{2(n+2)} \quad i=0,1,2,\ldots,n$$  \hspace{1cm} (B.3)

and

$$w_i = \frac{\pi}{n+1}$$  \hspace{1cm} (B.4)

Then Eq. (B.2) can be simplified to

$$\int_{-1}^{1} \frac{1}{\sqrt{1-Z^2}} F(Z)\,dZ = \left(\frac{\pi}{n+1}\right) \sum_{i=0}^{n} F(Z_i)$$  \hspace{1cm} (B.5)

Any integral with the form of Eq. (B.1) can be expressed in the form of Eq. (B.2) by using a suitable transformation, that is to change the integration limits from the interval $[a,b]$ to the interval $[-1,1]$. In order to achieve this, the following transformation should be made

$$x = \frac{(b-a)Z + (b+a)}{2}$$  \hspace{1cm} (B.6)
\[ dx = \frac{b - a}{2} dZ \quad (B.7) \]

Substituting Eqs. (B.6) and (B.7) into Eq. (B.1) yields

\[ \int_a^b Q(x) dx = \int_{-1}^1 Q(Z) \frac{b - a}{2} dZ \]

\[ = \int_{-1}^1 \frac{1}{\sqrt{1 - Z^2}} (\sqrt{1 - Z^2} \frac{b - a}{2}) Q(Z) dZ \]

\[ = \int_{-1}^1 \frac{1}{\sqrt{1 - Z^2}} F(Z) dZ \]

\[ = \frac{\pi}{n + 1} \sum_{i=0}^{n} F(Z_i) \quad (B.9) \]

in which

\[ F(Z_i) = \frac{b - a}{2} \sqrt{1 - Z_i^2} Q(Z_i) \quad (B.10) \]
Appendix  C

EXPERIMENTAL RESULTS OF LOAD VERSUS LOAD-LINE DEFLECTION CURVES FOR ASPHALT TABLETS AT DIFFERENT TEMPERATURES: INDIRECT TENSILE TEST
Appendix D

EXPERIMENTAL RESULTS OF LOAD VERSUS LOAD-LINE DEFLECTION CURVES FOR NOTCHED BEAMS: THREE-POINT BEND TESTS
Beam specimen

\[ F_{b1} = 1.0 \]
\[ a/b_1 = 0.2 \]
\[ c/b_1 = 2.91 \text{ inch} \]

\[ F_{b2} = 0.4 \]
\[ a/b_2 = 0.2 \]
\[ c/b_2 = 2.84 \text{ inch} \]

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*10 Millimetre to the Centimetre*
Beam specimen

Load (lbs):

- \( B = 0.7 \)
- \( a/b = 0.4 \)
- \( b = 2.95 \text{ inch} \)

Deformation (inch):

- \( B = 0.6 \)
- \( a/b = 0.6 \)
- \( b = 2.89 \text{ inch} \)

- \( B = 0.5 \)
- \( a/b = 0.6 \)
- \( b = 2.84 \text{ inch} \)
END OF THESIS