MODELING AND ESTIMATION FOR STEPPED AUTOMATIC TRANSMISSION WITH CLUTCH-TO-CLUTCH SHIFT TECHNOLOGY

DISSERTATION

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2004
ABSTRACT

A major problem in designing controllers for automatic transmissions in automobiles is that many of the relevant variables characterizing the performance of the power train are not measurable because of sensor cost and reliability considerations. This dissertation presents the development of model-based estimation for a stepped automatic transmission with clutch-to-clutch shift technology, to provide real-time information about some necessary but un-measurable variables. Specifically, model-based real-time estimation of clutch pressures for the clutches involved in the gear shifting process is developed and validated.

In this dissertation, mathematical models for all systems affecting the dynamic behavior of the transmission, namely, the torque converter, transmission mechanical components, shift hydraulic system, and vehicle and driveline are developed. The main focus, however, is on the development of mathematical models describing the dynamic behavior of the transmission mechanical components, viz, the dynamics of the gear sets inside the transmission and the shift hydraulic system. This dissertation presents the development of a nonlinear dynamic model for the shift hydraulic system for the transmission of interest. The result of the model development presented here includes a fully detailed model, which is complex and highly nonlinear, as well as a simplified
model which is used for real-time implementation. Resulting models are validated against experimental data.

The availability of the shift hydraulic model leads to the development of the model-based clutch pressure observer. Sliding mode observers are used due to their ability to deal with nonlinear systems, robustness against uncertainties, and ease of implementation. In this dissertation, both continuous-time and discrete-time sliding mode observers are developed and implemented. Since the turbine torque is needed by the observer but is not available, an adaptive sliding mode observer is also implemented in this research to improve the on-line estimation of the turbine torque, and hence improve the accuracy of the clutch pressure estimation. The resulting observers are validated via off-line simulation tests, and are also implemented in real-time at different sampling frequencies on a test vehicle, in order to demonstrate observer performance and establish the feasibility of using the designed observers on current transmission control units.
Dedicated to my mother
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NOMENCLATURE

\( A \)  
- cross sectional area of the solenoid air gap (m\(^2\))
- contact area between spool and sleeve (m\(^2\))

\( A, B, C \)  
- state space matrices of a linear model
- transformation matrices defined in Table 3.2

\( A_a \)  
- accumulator piston cross sectional area (m\(^2\))

\( A_c \)  
- clutch piston cross sectional area (m\(^2\))

\( A_{aod} \)  
- OD accumulator piston area (m\(^2\))

\( A_{und} \)  
- 2ND accumulator piston area (m\(^2\))

\( A_{inc} \)  
- clutch/accumulator effective inlet orifice area (m\(^2\))

\( A_{OD} \)  
- pressurized area of the OD clutch (m\(^2\))

\( A_{OD,B} \)  
- cross sectional area of the orifice at the entrance of the B-chamber (m\(^2\))

\( A_{pcv,c} \)  
- PCV cross sectional area of the land at the command chamber (m\(^2\))

\( A_{sw,OD} \)  
- area of the switch valve at the overdrive pressure side (m\(^2\))

\( A_{sw,line} \)  
- area of the switch valve at the line pressure side (m\(^2\))

\( A_{4ii} \)  
- the \( i \)th row, \( j \)th column element of the system matrix defined in equation (3.261)

\( A_{11}, A_{12}, A_{21}, A_{22} \)  
- sub-matrices of matrix A after matrix transformation

\( A_{11}', A_{12}', A_{21}', A_{22}' \)  
- transformed sub-matrices for the reduced order observer

\( A_{2ND} \)  
- pressurized area of the 2ND clutch (m\(^2\))

\( \Delta A_{pcv1}, \Delta A_{pcv2} \)  
- differences of the valve cross sectional area for PCV (m\(^2\))

\( \Delta A_{rv,B} \)  
- area difference between the spool lands around B-chamber (m\(^2\))

\( \Delta A_{rv,C} \)  
- area difference between the spool lands around C-chamber (m\(^2\))

\( \Delta A_{rv,D} \)  
- area difference between the spool lands around D-chamber (m\(^2\))

\( B_a \)  
- viscous damping coefficient for the accumulator motion (N/m/sec)

\( a_{ij} \)  
- the \( i \)th row, \( j \)th column element of A matrix defined in Table 3.2

\( B_c \)  
- viscous damping coefficient for the clutch motion (N/m/sec)

\( B_{pcv} \)  
- friction coefficient between PCV spool and sleeve (N/m/sec)

\( B_{solp} \)  
- solenoid plunger damping coefficient (N/m/sec)

\( B_{rv} \)  
- damping coefficient of the regulator spool (N/m/sec)

\( B_{3ii} \)  
- the \( i \)th row, \( j \)th column element of the system matrix defined in equation (3.226)

\( B_1, B_2 \)  
- sub-matrices of matrix B after matrix transformation
transformed sub-matrices for the reduced order observer

$\mathbf{b}_{ij}$: the $i^{th}$ row, $j^{th}$ column element of $\mathbf{B}$ matrix defined in Table 3.2

$C_d$: discharge coefficient

$C_{xOD}$: constant calculated from the OD clutch geometry

$C_{x2ND}$: constant calculated from the 2ND clutch geometry

$C_{T_{si}}$: the $i^{th}$ row element of the output shaft torque distribution vector during 2-3 up shift inertia phase calculated using equation (3.90)

$C_{T_{siS}}$: the $i^{th}$ row element of the output shaft torque distribution vector defined in equation (5.119)

$C_{T_{ti}}$: the $i^{th}$ row element of the turbine torque distribution vector during 2-3 up shift inertia phase calculated using equation (3.90)

$C_{T_{tiS}}$: the $i^{th}$ row element of the turbine torque distribution vector defined in equation (5.119)

$C_{1{T_{si}}}$: the $i^{th}$ row element of the system matrix defined in equation (3.178)

$C_{1{T_{si}}}S$: the $i^{th}$ row element of the system matrix defined in equation (3.178)

$C_{2{T_{si}}}$: the $i^{th}$ row element of the system matrix defined in equation (3.200)

$C_{2{T_{ti}}}$: the $i^{th}$ row element of the system matrix defined in equation (3.200)

$C_{3{T_{si}}}$: the $i^{th}$ row element of the system matrix defined in equation (3.226)

$C_{3{T_{ti}}}$: the $i^{th}$ row element of the system matrix defined in equation (3.226)

$C_{4{T_{si}}}$: the $i^{th}$ row element of the system matrix defined in equation (3.261)

$C_{4{T_{ti}}}$: the $i^{th}$ row element of the system matrix defined in equation (3.261)

$c_1$: rolling friction coefficient

$c_2$: aerodynamics friction coefficient

$c_{ij}$: the $i^{th}$ row, $j^{th}$ column element of $\mathbf{C}$ matrix defined in Table 3.2

$D$: disturbance/uncertainty distribution matrix for state space model transformation matrices defined in Table 3.2

$DutyCycle$: duty cycle command input (%)

$DutyCycle_{nominal}$: nominal value of the duty cycle command (%)

d_{ex1}: diameter of the regulator spool land at the port Ex1 (m)

d_{ex2}: the diameter of the regulator spool land at the port Ex2 (m)

d_{ij}: the $i^{th}$ row, $j^{th}$ column element of $\mathbf{D}$ matrix defined in Table 3.2

d_{sol,ex}: solenoid exhaust orifice diameter (m)

d_{sol,in}: solenoid inlet orifice diameter (m)

d_{sp1}: diameter of the PCV land at the supply port (m)

d_{sp2}: diameter of the PCV land at the supply port (m)
effective coefficient of the OD clutch torque for 2\textsuperscript{nd} gear and 2-3 up shift torque phase dynamics (N.m.sec\textsuperscript{-1})
ed\textsubscript{t}
 estimation error vector for unmeasured states
c\textsubscript{ij}
 the \textit{i}\textsuperscript{th} row, \textit{j}\textsuperscript{th} column element of \textit{E} matrix defined in Table 3.2
e\textsubscript{y}
 estimation error vector for measured states
\textit{F}
 arbitrary matrix
\textit{F}_{asi}
 accumulator inner return spring force (N)
\textit{F}_{aso0}
 accumulator outer return spring preload (N)
\textit{F}_{ii}
 the \textit{i}\textsuperscript{th} row, \textit{j}\textsuperscript{th} column element of the clutch torque distribution matrix during 2-3 up shift calculated using equation (3.90)
\textit{F}_{iisS}
 the \textit{i}\textsuperscript{th} row, \textit{j}\textsuperscript{th} column element of the clutch torque distribution matrix define in equation (5.119)
\textit{F}'
 the \textit{i}\textsuperscript{th} row, \textit{j}\textsuperscript{th} column element of the clutch torque distribution matrix define in equation (5.162)
\textit{F}_{mmf}
 magnetomotive force (N)
\textit{F}_{mmf,air}
 magnetomotive force generated by the working air gap (N)
\textit{F}_{rv,B}
 force created by the pressurized fluid in B-chamber (N)
\textit{F}_{rv,C}
 force created by the pressurized fluid in C-chamber (N)
\textit{F}_{rv,D}
 force created by the pressurized fluid in D-chamber (N)
\textit{F}_{s, in}
 PCV spring preload (N)
\textit{F}_{sc0}
 clutch return spring preload (N)
\textit{F}_{srv,ini}
 regulator return spring preload (N)
\textit{F}_{1ii}
 the \textit{i}\textsuperscript{th} row, \textit{j}\textsuperscript{th} column element of the system matrix defined in equation (3.178)
\textit{F}_{2ii}
 the \textit{i}\textsuperscript{th} row, \textit{j}\textsuperscript{th} column element of the system matrix defined in equation (3.200)
f(x, u, t)
 bounded uncertainty
\textit{f}_i(x), \textit{f}'_i(x)
 uncertain states-dependent function
\textit{f}_{ij}
 the \textit{i}\textsuperscript{th} row, \textit{j}\textsuperscript{th} column element of \textit{F} matrix defined in Table 3.2
\textit{\Delta f}_i(x), \textit{\Delta f}'_i(x)
 states-dependent uncertainty
\textit{G}
 Luenberger observer gain matrix
\textit{H}_{air}
 magnetic field intensity
\textit{I}_t
 lumped inertia of the torque converter turbine (N.m.sec\textsuperscript{2})
\textit{I}_{si}
 lumped inertia of the input sun gear (N.m.sec\textsuperscript{2})
\textit{I}_{rr}
 lumped inertia of the reaction ring gear – input carrier (N.m.sec\textsuperscript{2})
\textit{I}_{sr}
 lumped inertia of the reaction sun gear (N.m.sec\textsuperscript{2})
\textit{I}_{icr}
 lumped inertia of the reaction carrier – input ring gear (N.m.sec\textsuperscript{2})
\textit{I}_{cs}
 lumped inertia of the secondary carrier (N.m.sec\textsuperscript{2})
\textit{I}_{irs}
 lumped inertia of the secondary ring gear (N.m.sec\textsuperscript{2})
\textit{I}_{iss}
 lumped inertia of the secondary sun gear (N.m.sec\textsuperscript{2})
\textit{I}_v
 equivalent vehicle inertia (N.m.sec\textsuperscript{2})
\textit{I}_{23}
 lumped inertia of the gear set moving components for 2-3 up shift (N.m.sec\textsuperscript{2})
$I_{23S}$ lumped inertia of rigid shaft transmission gear set for 2-3 up shift (N.m.sec²)

$I_{23Si}$ lumped inertia of rigid shaft transmission gear set for 2-3 up shift inertia phase (N.m.sec²)

$I_3$ lumped inertia of the primary axle during the 3rd gear (N.m.sec²)

$I_{34}$ lumped inertia of the gear set moving components for 3-4 up shift (N.m.sec²)

$I_{3S}$ lumped inertia of rigid shaft transmission gear set during 3rd gear (N.m.sec²)

$I_{45}$ lumped inertia of the gear set moving components for 4-5 up shift (N.m.sec²)

$i$ solenoid current (A)

$K_{fl}$ constant factor to match the experimental data

$K_{rv}$ regulator spool return spring (N/m)

$K_{ai}$ inner return spring constant (N/m)

$K_{ao}$ accumulator outer return spring constant (N/m)

$K_{2OD}$ OD accumulator return spring stiffness (N/m)

$K_{3OD}$ 2ND accumulator return spring stiffness (N/m)

$K_c$ clutch return spring constant (N/m)

$K_f$ flow force matching factor with experimental data

$K_{pcv}$ PCV spring constant (N/m)

$K_{pOD}$ linearized coefficient of OD clutch define in equation (5.194)

$K_{pOD}'$ linearized coefficient of input pressure for OD clutch define in equation (5.195)

$K_{p2ND}$ linearized coefficient of 2ND clutch define in equation (5.210)

$K_{p2ND}'$ linearized coefficient of input pressure for 2ND clutch define in equation (5.211)

$K_t$ axle shaft stiffness (N.m.sec)

$K_{sopl}$ solenoid plunger spring constant (N/m)

$k_{1S}$ the $i^{th}$ sliding mode observer gain for 2-3 up shift torque phase of the automotive transmission with rigid shaft

$k_{1iS}$ the $i^{th}$ sliding mode observer gain for 2-3 up shift inertia phase of the automotive transmission with rigid shaft

$LR$ low-reverse clutch

$L_1, L_2$ sliding mode observer gains

$L_{1i}, L_{2i}$ discrete sliding mode observer gains matrix for 2-3 up shift inertia phase estimation

$l_{(ij)i}$ the $i^{th}$ row, $j^{th}$ column of the discrete observer gains vector for 2-3 up shift inertia phase estimation
the $i^{th}$ element of the discrete observer gains vector for 2-3 up shift
torque phase estimation

$M_a$ accumulator piston mass (kg)

$M_{rv}$ mass of the regulator spool (kg)

$m$ number of rows of the input vector

$m_{sopl}$ mass of the plunger (kg)

$N$ number of coil turns

$N_{Si}$ number of teeth of the input sun gear

$N_{Ri}$ number of teeth of the input ring gear

$N_{Sr}$ number of teeth of the reaction sun gear

$N_{Rr}$ number of teeth of the reaction ring gear

$N_{Rs}$ number of teeth of the secondary axle ring gear

$N_{ss}$ number of teeth of the secondary axle sun gear

2ND, ND second clutch

$n$ number of rows of the state vector

OD over-drive clutch

$P_c$ clutch pressure (Pa)

$P_{c,2ND}$ 2ND clutch pressure (Pa)

$\overline{P}_{c,2ND}$ nominal value of the 2ND clutch pressure (Pa)

$\tilde{P}_{c,2ND}$ 2ND clutch pressure estimation error (Pa)

$P_{c,OD}$ OD clutch pressure (Pa)

$\overline{P}_{c,OD}$ nominal value of the OD clutch pressure (Pa)

$\tilde{P}_{c,OD}$ OD clutch pressure estimation error (Pa)

$P_f$ feeding chamber pressure (Pa)

$P_{l}$ shift hydraulic line pressure (Pa)

$\overline{P}_{l}$ nominal value of the line pressure (Pa)

$P_{line}$ supply pressure (Pa)

$P_{rv,B}$ pressure inside the regulator valve B-chamber (Pa)

$P_{sol}$ solenoid controlled pressure at the output port (Pa)

$p$ number of rows of the output vector

$Q_a$ flow rate to the accumulator chamber ($m^3/sec$)

$Q_c$ flow rate to the clutch chamber ($m^3/sec$)

$Q_{Ex1}$ flow rate to the port Ex1 ($m^3/sec$)

$Q_{Ex2}$ flow rate to the port Ex2 ($m^3/sec$)

$Q_{pcv}$ flow rate from the PCV to clutch and accumulator ($m^3/sec$)

$Q_{inf}$ inlet flow rate to the feeding chamber at PCV ($m^3/sec$)

$Q_{pump}$ supply flow rate from the pump ($m^3/sec$)

$Q_{rv,B}$ flow rate into the regulator valve B-chamber ($m^3/sec$)

$Q_{rv,net}$ net flow into the regulator system ($m^3/sec$)

$Q_{sol,net}$ net flow across the solenoid valve ($m^3/sec$)

$Q_{sol,in}$ inlet flow through the solenoid valve ($m^3/sec$)

$Q_{sol,ex}$ exhaust flow through the solenoid valve ($m^3/sec$)
R  solenoid coil resistance (Ohm)
$R_{ad}$  adaptation weighting matrix
$R_f$  final drive gear ratio
$R_{si}, R_{sr}, R_{ci}, R_{cr}$  kinematic constants defined in Table 3.2
$R_{ss}, R_{rs}$  kinematic constants define in equation (3.154)
$R_{DIR}$  reaction torque of DIR clutch (N.m)
$R_{LR}$  reaction torque of LR clutch (N.m)
$R_{2ND}$  reaction torque of 2ND clutch (N.m)
$R_{RED}$  reaction torque of RED clutch (N.m)
$R_{UD}$  reaction torque of UD clutch (N.m)
$R_{OD}$  reaction torque of OD clutch (N.m)
$R_{OWCH}$  reaction torque of OWCH (N.m)
$R_{OD}$  effective radius of the OD clutch (m)
$R_{po}$  primary axle to secondary transfer gear ratio
$R_{2ND}$  effective radius of the 2ND clutch (m)
$r$  tire radius (m)
$r_1$  known scalar depending on uncertainty bound
$\Delta r$  radial clearance between spool and sleeve (m)
$S_i$  sliding surface
$s_{i,p}(k)$  the i$^{th}$ discrete sliding surface at step k for 2-3 up shift torque phase estimation
$s_{i,p}(k)$  the i$^{th}$ discrete sliding surface at step k for 2-3 up shift inertia phase estimation
$T$  transformation matrix for a linear model
$T_{C}\delta_s$  kinematic torque acting on the inertia $I_{C}\delta_s$ of the secondary axle in 5-speed transmission (N.m)
$T_{fCr}$  kinematic torque acting on the inertia $I_{Cr}$ (N.m)
$T_{fCrd}$  dynamic torque acting on the inertia $I_{Cr}$ (N.m)
$T_{fSr}$  kinematic torque acting on the inertia $I_{Sr}$ (N.m)
$T_{fSr}$  dynamic torque acting on the inertia $I_{Sr}$ (N.m)
$T_{fRr}$  kinematic torque acting on the inertia $I_{Rr}$ (N.m)
$T_{fRrd}$  dynamic torque acting on the inertia $I_{Rr}$ (N.m)
$T_L$  LR clutch torque (N.m)
$T_{load}$  vehicle load torque (N.m)
$T_{OD}$  OD clutch torque (N.m)
$T_o$  kinematic torque acting on the inertia $I_{O}$ (N.m)
net ambient torque (absolute)  
$T_p$  pump torque (N.m)  
$T_{po}$ driving torque from the primary axle to the secondary axle in 5-speed transmission (N.m)  
$T_{po}'$ load torque from the secondary axle seen by the primary axle in 5-speed transmission (N.m)  
$T_{Rs}$ kinematic torque acting on the inertia $I_{rs}$ of the secondary axle in 5-speed transmission (N.m)  
$T_{Si}$ kinematic torque acting on the inertia $I_{si}$ (N.m)  
$T_{Sid}$ dynamic torque acting on the inertia $I_{si}$ (N.m)  
$T_{Ss}$ kinematic torque acting on the inertia $I_{ss}$ of the secondary axle in 5-speed transmission (N.m)  
$T_s$ transmission output shaft torque (N.m)  
$T_s'$ estimated transmission output shaft torque (N.m)  
$T_s''$ output shaft torque estimation error (N.m)  
$T_{sg}$ kinematic torque acting on the inertia $I_{si}$ (N.m)  
$T_t$ turbine torque (N.m)  
$T_t'$ estimated turbine torque (N.m)  
$T_t''$ turbine torque estimation error (N.m)  
$T_{UD}$ UD clutch torque (N,m)  
$T_{2ND}$ 2ND clutch torque (N,m)  
TCC torque converter clutch  
t_s sliding surface reaching time (sec)  
UD under-drive clutch  
u(t) input vector  
V Lyapunov function  
$V_c$ PCV command chamber volume (m$^3$)  
$V_{c,ini}$ initial volume of the command chamber at PCV (m$^3$)  
$V_f$ PCV feeding chamber volume (m$^3$)  
$V_j(\tilde{x}(k))$ matrix contains the feedback compensation function for the discrete sliding mode observer for 2-3 up shift inertia phase  
$V_{in}$ solenoid voltage input (V)  
$V_{rv,main}$ main chamber volume (m$^3$)  
$x(t), x$ state vector  
$\hat{x}$ estimated state vector  
x_a accumulator piston displacement (m)  
x_{acc,2ND} 2ND accumulator piston displacement (m)  
x_{acc,OD} OD accumulator piston displacement (m)
\[ \begin{align*}
\mathbf{x}_{ai} & \quad \text{piston displacement when the inner return spring starts stroking (m)} \\
\mathbf{x}_c & \quad \text{clutch piston displacement (m)} \\
\mathbf{x}_{\text{Ex1}} & \quad \text{displacement of the regulator spool for the port Ex1 to open (m)} \\
\mathbf{x}_{\text{Ex2}} & \quad \text{displacement of the regulator spool for the port Ex2 to open (m)} \\
\tilde{\mathbf{x}} & \quad \text{estimation error} \\
\mathbf{x}_{\text{in}} & \quad \text{PCV spool displacement when the supply port is opened (m)} \\
\mathbf{x}_{\text{out}} & \quad \text{displacement of the PCV before the exhaust port is opened (m)} \\
\mathbf{x}_{\text{pcv}} & \quad \text{pressure control valve displacement (m)} \\
\mathbf{x}_{rv} & \quad \text{spool displacement (m)} \\
\mathbf{x}_{\text{solp, max}} & \quad \text{initial distance of the plunger to the core (m)} \\
\mathbf{x}_{\text{solp}} & \quad \text{plunger displacement measured from the initial position (m)} \\
\mathbf{x}_1, \mathbf{x}_2 & \quad \text{sub-vectors of state vector } \mathbf{x} \text{ after matrix transformation} \\
\hat{\mathbf{x}} & \quad \text{estimated unmeasured state vector} \\
\hat{\mathbf{y}} & \quad \text{estimated measured state or estimated output vector} \\
y(t) & \quad \text{output vector} \\
\mathbf{z} & \quad \text{transformed state for the reduced order observer} \\
\alpha_i & \quad \text{Luenberger observer gain} \\
\beta & \quad \text{fluid bulk modulus (Pa)} \\
\Gamma_{i,j} & \quad \text{the } i^{th} \text{ row, } j^{th} \text{ column element of the input distribution matrix define in equation (5.212)} \\
\gamma_{i,j} & \quad \text{the } i^{th} \text{ row, } j^{th} \text{ column element of the input distribution matrix for the 2-3 up shift torque phase difference equation (5.196)} \\
\Phi_{i,j} & \quad \text{the } i^{th} \text{ row, } j^{th} \text{ column element of the system matrix define in equation (5.212)} \\
\phi & \quad \text{magnetic flux} \\
\phi_{i,j} & \quad \text{the } i^{th} \text{ row, } j^{th} \text{ column element of the system matrix for the 2-3 up shift torque phase difference equation (5.196)} \\
\gamma(t, y, u) & \quad \text{scalar function which its values higher than uncertainty bounds} \\
\eta & \quad \text{some positive scalar} \\
\mu & \quad \text{transmission fluid viscosity (N.sec/m}^2) \\
\mu_0 & \quad \text{permeability of air (Henry/m)} \\
\mu(\Delta \omega) & \quad \text{slip-speed dependent friction coefficient of the clutch plates} \\
\nu & \quad \text{switching input function} \\
\nu_{eq} & \quad \text{equivalent control function} \\
\nu_i(\tilde{x}(k)) & \quad \text{discrete sliding mode observer compensation function as a function of estimation error for 2-3 up shift torque phase estimation} \\
\nu_{i,e}(\tilde{x}(k)) & \quad \text{the } i^{th} \text{ discrete sliding mode observer compensation function as a function of estimation error for 2-3 up shift torque phase estimation} \\
\Omega & \quad \text{input speed vector for the adaptive torque converter model} \\
\end{align*} \]
$\Delta \Omega$ \hspace{1cm} input speed uncertainty vector for the adaptive torque converter model

$\theta_1, \theta_2$ \hspace{1cm} A-chamber configuration (see Figure 4.10) (degree)

$\hat{\theta}$ \hspace{1cm} vector of estimated coefficient for the adaptive torque converter model

$\tilde{\theta}$ \hspace{1cm} vector of estimated coefficient error for the adaptive torque converter model ((rad/sec)$^2$)

$\rho$ \hspace{1cm} uncertainty bound

$\rho$ \hspace{1cm} transmission fluid density (kg/m$^3$)

$\omega_{ci}$ \hspace{1cm} input carrier speed (rad/sec)

$\omega_{cr}$ \hspace{1cm} reaction carrier speed (rad/sec)

$\omega_{cs}$ \hspace{1cm} secondary axle carrier speed (rad/sec)

$\omega_o$ \hspace{1cm} transmission output speed (rad/sec)

$\omega_p$ \hspace{1cm} pump speed (rad/sec)

$\omega_{ri}$ \hspace{1cm} input ring gear speed (rad/sec)

$\omega_{rs}$ \hspace{1cm} secondary axle ring gear speed (rad/sec)

$\omega_{rr}$ \hspace{1cm} reaction ring gear speed (rad/sec)

$\tilde{\omega}_{rr}$ \hspace{1cm} reaction ring gear speed estimation error (rad/sec)

$\omega_{si}$ \hspace{1cm} input sun gear speed (rad/sec)

$\omega_{ss}$ \hspace{1cm} secondary axle sun gear speed (rad/sec)

$\omega_{sr}$ \hspace{1cm} reaction sun gear speed (rad/sec)

$\omega_t$ \hspace{1cm} turbine speed (rad/sec)

$\tilde{\omega}_t$ \hspace{1cm} turbine speed estimation error (rad/sec)

$\omega_w$ \hspace{1cm} wheel speed (rad/sec)

$\tilde{\omega}_w$ \hspace{1cm} wheel speed estimation error (rad/sec)

$\xi$ \hspace{1cm} unknown but bounded uncertainty
CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

A topic of current interest in the area of controller development for automatic transmissions with a finite number of gearshifts, namely, stepped automatic transmissions, is control of shift quality for clutch-to-clutch shifts. Gearshifts in automatic transmissions involve a change in the power flow path through the transmission and involve a change in the engagement status of two clutches, the ‘off-going’ clutch and the ‘on-coming’ clutch. The term ‘clutch-to-clutch’ shift refers to a shift in which the off-going and on-coming clutches need to be actively controlled in a coordinated manner during the shift. If one of the two clutches involved in the shift is mechanically supported such that it can transmit torque in only one direction, such a support being termed an overrunning clutch (or one-way clutch or freewheeler), the need for active control of that clutch is eliminated. Advantages of automatic transmissions relying primarily on clutch-to-clutch shifts, over transmissions relying primarily on overrunning clutches, include simplicity of mechanical design and savings in transmission weight and size, which are beneficial in terms of fuel economy and production costs. As the number of speeds in automatic transmissions increases in order to enable gains in fuel economy while meeting drivability and performance goals, these
savings become more significant. However, control of clutch-to-clutch shifts to achieve shift quality comparable to those involving overrunning clutches is a challenging problem. In a clutch-to-clutch shift, smoothness of the shift requires good timing coordination between control actions involving the on-coming as well as the off-going clutches. Improper coordination results in a capacity mismatch between the two clutches, and would cause either excessive engine speed or clutch tie-up, the latter increasing torque change during the shift and resulting in a harsher shift.

Many researchers have studied clutch-to-clutch shifts and have proposed different control methods to improve shift quality (Bai et al, 2002; Yang et al, 2001; Minowa et al, 1999; Hebale and Ghoneim, 1991; Cho, 1987). A major problem in designing the controller, not only for clutch-to-clutch shifts but also for other transmission control problems, is that many of the relevant variables characterizing the performance of the power train are not measurable in production vehicles. An example of such a variable is clutch pressure in hydraulically actuated clutches. Sensed information that is usually available for control purposes in production vehicles includes speeds, namely, engine speed, transmission input and output speeds, and wheel speed. Clutch pressure sensors are usually not used in production transmissions due to sensor cost and reliability, as well as difficulty in sensor installation and maintenance. An alternative way of obtaining information about clutch pressure is to develop a mathematical model of the shift hydraulic system relating the clutch pressures to control inputs as well as measured variables, and then use model-based estimation techniques for clutch pressure estimation. However, development of a dynamic model of the shift hydraulic system for clutch
pressure estimation that is suitable for real-time application is a challenging problem, because of the complexity of the shift hydraulic circuit and its highly nonlinear nature.

Other variables needed to characterize the quality of gearshifts, but which are usually inaccessible, are transmission input and output torques, and transmission input and output accelerations. Attempts to estimate transmission output shaft torque have been reported in the literature (Masmoudi and Hedrick, 1992, Minowa, 1987, Yi et al, 2000). However, the cited approaches do not employ a model of the shift hydraulic system, and the estimation algorithms assume a-priori knowledge of clutch pressure behavior and estimation error bounds. The maximum error in the predefined clutch pressure is assumed to be known, and the output shaft torque predicted without knowing the clutch dynamic model. However, a high gain observer is required in such a case to compensate for the error bound, and results in degraded estimation accuracy in the presence of measurement noise. If a reasonably accurate shift hydraulic model is known and incorporated in the estimation algorithm, the error bound on the clutch pressure can be reduced, which in turn will allow a reduction of the observer gains and improved response in the presence of measurement noise.

Model-based closed loop estimation of clutch pressures is clearly desirable because of the expected robustness of the estimate. The challenge in such model-based estimation lies in the need to simplify the complex shift hydraulic system model to enable real-time estimation, as well as the need to use nonlinear observer techniques given the anticipated nonlinear nature of the system model.

The motivation for real-time estimation of clutch pressures is that this information is needed for coordination of the clutch pressures during shifts. It is well
known that any gearshift can be divided into two phases. The first phase is the so-called ‘torque phase’ where the load is transferred from the off-going clutch (disengaging clutch) to the on-coming clutch (engaging clutch). Proper clutch coordination is very important during this phase, and is particularly challenging since there is little variation in the measured speeds during this phase. The second phase is the so-called ‘inertia phase’. During this phase, the on-coming clutch makes the speed adjustments appropriate for the gearshift. Therefore, clutch coordination is less of an issue and only the on-coming clutch needs to be controlled during this phase. The on-coming clutch has to be controlled such that the shift is completed within an acceptable time interval while maintaining limits on torque fluctuation.

In the control of current production transmissions involving clutch-to-clutch shifts, inertia phase control is not difficult since there is only one control action required and the speed information needed for the controller to accomplish desired performance is usually available from sensors. The control of the torque phase, on the other hand, is rather difficult as real-time clutch pressure information is usually not available. In the absence of such knowledge, effective coordinated control of on-coming and off-going clutches usually depends on significant calibration work. The resulting control is open loop in nature and has the disadvantages associated with open-loop control, namely, lack of robustness to changes in system behavior with time.

1.2 Objectives of the Research

The primary objectives of this research are two-fold. The first objective is to develop a dynamic model of a typical automatic transmission that relies heavily on
clutch-to-clutch shifts. In addition to 4-speed automatic transmissions, 5-speed transmissions relying on clutch-to-clutch shifts are common in current production units. Therefore, in order to accommodate this trend, this study considers model development for both 4-speed and 5-speed cases. This study also emphasizes inclusion of a model of the shift hydraulic system in the transmission model. As mentioned earlier, transmission shift hydraulic systems are complex and consist of many components displaying linear as well as nonlinear behavior. Incorporating all such components into the model would result in a high order, complex dynamic model. This dynamic model would need to be validated against experimental data and shown to predict hydraulic system experimental response well. While such a model is suitable for off-line simulation studies, it may not be appropriate for on-line or real-time estimation and control applications. Therefore, the goal here, in addition to the development of an off-line model, is the development of a simpler on-line model that incorporates the most important components that dominate the shift hydraulic system dynamic response. Along with the model for all other subsystems also to be developed here, the overall model not only has to be able to reasonably predict the behavior of important variables under various operating conditions, viz. clutch pressure, input and output torques, and input/output speeds of the transmission, but it also has to be suitable for controller design and real-time application.

The second objective of the proposed research involves the development of model-based observers for shift-to-shift control. Observer designs will be based on measured speed signals. Since transmission dynamic models are nonlinear, nonlinear observer designs will be investigated. Such techniques have shown some promise for this application (Yang et al, 2001; Yi et al, 2000; Masmoudi and Hedrick, 1992; Cho, 1987).
The development of the observer will take advantage of the availability of the shift hydraulic system model to be developed and simplified as part of this research. Since the clutch pressure information is critical to gearshift quality control, the observer is needed to provide real-time information about clutch pressures. The complexity of the observer used for the estimation depends on the availability and complexity of the transmission dynamic model.

The main challenge in the development of such an observer is in achieving rapid convergence of the estimates, while at the same time ensuring robustness of the observer estimates in the presence of modeling uncertainties and estimation accuracy in the presence of measurement noise. Available technical literature includes description of model-based observers to estimate the output shaft torque (Masmoudi and Hedrick, 1992). However, since the dynamics of the shift hydraulic system had not been explicitly considered in the cited study, a high-gain observer was used to compensate for model uncertainties, thus degrading noise response. It is expected that with the availability and explicit inclusion of the shift hydraulic model in the estimation scheme, the uncertainty bounds can be reduced and robust observers with smaller gains can be developed. Also, estimation of the transmission input torque, or turbine torque, is important for clutch pressure estimation accuracy, since it is an unmeasured input to the transmission. Accuracy of estimation of the transmission input torque will rely in turn on the accuracy of the torque converter model used for such estimation. Some work has been reported in the literature on attempts to adjust torque converter model parameters on-line for more accurate estimation of the turbine torque (Yi et al, 2001). The effect of incorporating transmission input torque estimation on the accuracy of the clutch pressure estimation
will be explored in this research as well. In addition, the effect of uncertainty in the
torque converter model on the clutch pressure estimation accuracy will be studied as part
of robustness analysis of the estimation procedures developed here.

1.3 Organization of the Dissertation

A discussion and review of research relevant to this work is presented in Chapter
2. A review of research involving the modeling of automatic transmissions as well as
shift hydraulic components is presented first. The utilization of these models to predict
inaccessible variables in automatic transmissions, as well as a review of relevant observer
theory developments, is then presented. A survey of current research on control of clutch-
to-clutch shifts is provided, in order to better understand the resulting requirements on
on-line clutch pressure estimation.

In Chapter 3, detailed models of the transmission of interest, in both 4-speed and
5-speed versions, are presented. Simulation and validation results are presented only for
the 4-speed transmission. The simulation results show the validity and accuracy of the
developed model as compared to experimental data.

Given the importance of shift hydraulic systems in this research, a detailed model
of the shift hydraulic system for the 4-speed transmission has been developed and
presented in Chapter 4. The model has also been simplified to include the dominant
dynamics that determine clutch pressure behavior. In this chapter, simulation results for
both the detailed hydraulic model and the simplified model are shown and compared to
experimental data as a part of model validation.
In Chapter 5, the theory of sliding mode observers is reviewed first. Since the detailed theory of this observer is well documented in the literature, the mathematical conditions required to construct the observer are presented without proof. There are three sliding mode observer schemes reviewed in this chapter: i) the scheme developed by Utkin (1992), ii) the scheme developed by Slotine et al. (1987), and iii) the scheme developed by Walcot and Zak (1986, 1988). These schemes were developed with emphasis on the robustness of the observer to uncertainties. The three approaches are related closely. The theory of sliding mode observers is then applied to our system, and the design of the observer for our application is proposed. The main goal of this chapter is to present the feasibility of using sliding mode observers to successfully estimate clutch pressures and the output shaft torque in automatic transmissions.

Since the objective of this research is to develop an estimation scheme that is suitable for real-time implementation, the observer will be designed in the continuous time domain as well as the discrete time domain. Moreover, an adaptive sliding mode observer is also considered in this research in an attempt to improve the accuracy of clutch pressure estimation. Specifically, the adaptive part of the observer will be used to adjust the parameters of the torque converter model to improve the accuracy of estimation of the turbine torque, which is one of the inputs to the transmission model used by the observer. This portion of the work follows directly the work presented by Yi et al (2001). Both off-line and on-line results validating the clutch pressure estimation will be shown.

In Chapter 6, conclusions from, and contributions of, the overall research effort are presented, along with recommendations for future work.
CHAPTER 2

LITERATURE REVIEW

This chapter presents a review of the technical literature relating to the issues in
developing a model-based clutch-to-clutch controller. This problem involves three major
areas of research. The first is the development of a mathematical model describing
transmission dynamic behavior. The second area focuses on the development of on-line
estimation algorithms so that necessary variables that are important for controller
implementation, but are not measurable, can be accurately estimated. The last research
area involves the development of a controller for clutch-to-clutch shifts that ensures good
shift quality. We note that the work in this research does not include the development of a
new controller for clutch-to-clutch gearshifts, but emphasizes estimation of clutch
pressures to enable existing clutch-to-clutch controllers to achieve better shift quality.
The intent of this review is to present some approaches that have been developed to date,
and to compare them to each other. The limitations and advantages of the different
approaches are used to guide the formulation of the work described in later chapters.

2.1 Transmission Model Development

The development of a mathematical model to represent the dynamic behavior of
an automatic transmission has long been an active research subject. The availability of
such a model can simplify and reduce the cost of controller development. Specifically, the model can be used as a simulator such that any designed control strategy can be tested and tuned via simulation before the controller is installed and tested on the real vehicle or on a dynamometer. There are two approaches commonly employed in the development of the transmission model, depending on the objective of the research (Zheng, 1999). One approach is to develop a detailed engine model together with a relatively simple transmission-driveline model. This is the case when engine related research is the primary focus, gear shifts are not involved, and the load on the engine does not vary significantly. Another approach is to develop a power train model that emphasizes transmission behavior. This is the case when transmission related research is the primary focus, involving the study of shift transients, the development of a shift controller, etc. In this case, it is assumed that the engine torque is nearly unchanged if the engine control inputs are unchanged and a simplified engine model is used. Due to the fact that transmission control is of primary interest in this research, the latter approach is used here.

Clearly, the most general power train model would include detailed models of engine and transmission operation, and would be needed for the development of coordinated power train diagnostics, estimation, control strategies. While this is the ultimate goal of power train modeling and control research, for our proposed work, we restrict our goals to emphasize transmission modeling and control.

We divide the automatic transmission into four major subsystems, namely, the torque converter, the transmission mechanical subsystem, driveline and vehicle dynamics, and the shift hydraulic system.
2.1.1 Torque Converter

The first subsystem is the torque converter. The main advantage of a torque converter is its damping characteristic, which enables engine torque pulsations to be attenuated before being transmitted to the driveline. A torque converter also offers the ability to amplify the driving force when needed, namely, at low speeds.

A commonly used torque converter in automatic transmissions is the three-element torque converter. It consists of three major components, namely, a pump which is attached to and driven by the engine, a turbine which is attached to the transmission input shaft, and a stator or reactor which is attached to the housing via a one-way clutch. A simple schematic representation of the three-element torque converter is shown in Figure 2.1.

![Figure 2.1 Three-element torque converter (Tsangarides and Tobler, 1985)]
The torus or circulatory flow of the fluid inside the torque converter depends on the blade geometry of these three elements. Figure 2.2 shows the 3-D flow geometry over the turbine blade for a typical torque converter. Change of the moment of momentum of the fluid as it goes from the pump to the turbine and vice versa, along with the behavior of the one-way clutch, results in two operating modes of the torque converter at different engine speeds, viz., a torque multiplication mode and a fluid coupling mode. The torque multiplication mode occurs during vehicle launch and low speed ranges and is desired for vehicle acceleration. The fluid coupling mode, on the other hand, occurs in the high speed range, the stator is freewheeling, and the pump and turbine torques are almost equal.

Figure 2.2 3-D Flow geometry over turbine blade (By and Mahoney, 1988)
The development of a mathematical model predicting torque converter behavior has been an active research topic, and several types of torque converter models can be found in the literature (Kotwicki, 1982; Tsangarides and Tobler, 1985; Hrovat and Tobler, 1985). More recent work can be found in Duer et al. (2002), which presents a simplified linearized model based on the work of Hrovat and Tobler (1985). Based on a review of the literature, it is clear that a dynamic torque converter model that has been adequately validated and accurately describes the dynamic behavior of the torque converter over a wide range of operating conditions is still unavailable. It has been shown however that the static, nonlinear empirical input-output model developed by Kotwicki (1982) can be used for controller design purposes (Runde, 1986; Cho, 1987; Pan and Moskwa, 1995; Zheng, 1999) due to its ease of implementation and reasonable degree of validity over normal operating conditions (Cho, 1987).

2.1.2 Transmission Mechanical Subsystem

The second subsystem of the transmission is the mechanical subsystem determining the overall kinematic and dynamic relationships between the transmission input and output variables. The mechanical subsystem consists of the gear sets, the interconnections of the planetary gear sets with the torque converter and the driveline, and friction elements such as clutches and/or band brakes used to constrain relative motion.

There are several methods to represent the transmission mechanical model. The lever analogy method developed by Leising and Benford (1989) is used by Tugcu et al., (1986), Lee et al., (1997), and most recently by Mianzo (2000). This method is a simple
graphical representation which helps to simplify analysis of the kinematic and dynamic behavior of the transmission mechanical subsystem. For example, Figure 2.3a shows a simple planetary gear set which consists of a sun gear, a carrier and a ring gear, labeled by S, C, and R respectively. The lever representation for this simple planetary gear set is shown in Figure 2.3b. The length of each segment of the lever is proportional to the number of gear teeth marked by Nr, the number of teeth in the ring gear, and Ns, the number of teeth in the sun gear. K is a suitable scaling constant. Figure 2.3c shows one of the possible configurations of this planetary gear set where the speeds of each element of the gear set are also noted. The corresponding speed lever diagram for this configuration is shown in Figure 2.3d. Figure 2.3e shows the torque applied on the gear set and the corresponding torque lever diagram is shown in Figure 2.3f. While this method is simple and convenient for analysis of the steady-state behavior of the transmission, it is less so for dynamic analysis.

Figure 2.3 An example of the lever analogy
Another graphical tool that has been increasingly used in model development for complex, mixed-mode systems is the “bond graph” method (Karnopp et al., 2000). The application of this method to the development of automatic transmission models can be found in the literature in the work of Cho (1987), Hrovat and Tobler (1991), Kwon and Kim (2000), and Yang et al. (2001). Figure 2.4 shows a bond graph representation of the planetary gear set in Figure 2.3a.

![Bond Graph Representation](image)

Figure 2.4 Bond graph representation of the planetary gear set shown in Figure 2.3a

A more traditional way of developing a mathematical model, and the one that is used in this research, involves the use of appropriate conservation laws and constitutive relationships governing transmission component response. For example, Newton’s second law is used to describe the motion of mechanical components. Such an approach does yield a mathematical model of the mechanical subsystem, but does not result in graphical representations of the transmission mechanical subsystems. Examples of use of this method to develop the transmission mechanical system model can be found in Kim et al. (1994), Haj-Fraj and Pfeiffer (1999), Zheng (1999), and Jeong and Lee (1994, 2000).
2.1.3 Driveline and Vehicle Dynamics

The development of the vehicle dynamic model is a research topic in its own right. The technical literature includes models of varying complexity depending on the objectives and contexts of different research approaches. It has been shown that for the study of gearshift transients and controller development, a simple model focusing on the longitudinal vehicle dynamics including tire-road interactions (Figure 2.5) can be used effectively (Tugcu et al., 1986; Cho, 1987; Hrovat and Tobler, 1989; Masmoudi and Hedrick, 1992; Zheng, 1999; Jeong and Lee, 2000; Haj-Fraj and Pfeiffer, 2001). This type of model will be used in this work as well, and is a gross vehicle dynamics model where the driveline is modeled as a torsional spring connected to the vehicle which, in turn, is modeled as a lumped inertia subjected to tire forces or driving forces which depend on tire slip (Wong, 1978). The load torque due to wind and rolling resistance is also included in the model as a function of vehicle speed (Heywood, 1988).

Figure 2.5 Free-body diagrams for vehicle dynamics model (Cho, 1987)
2.1.4 Shift Hydraulic Subsystem

The last subsystem needed for describing transmission dynamic operation is the shift hydraulic system, which is the subsystem that we emphasize in this work. The shift hydraulic system plays a major role in the operation of automatic transmissions. In particular, one of its main functions is to generate the pressurized fluid and maintain/vary the fluid pressure to perform needed tasks satisfactorily under various operating conditions. It initiates the gear shifting process by increasing and decreasing fluid pressures in the clutches involved in the gearshift at appropriate times and to appropriate levels. Therefore, shift quality depends largely on the operation of the hydraulic system. Due to the physical complexity of the shift hydraulic circuit inside the transmission, the mathematical model that accurately describes the dynamic behavior of the system is a complex one. Figure 2.6 demonstrates the complexity of a typical shift hydraulic system for automatic transmissions.

Figure 2.6 Typical schematic for shift hydraulic system (Leising et al, 1989)
It has been shown that for a shift hydraulic system in which the clutch pressure is controlled by a linear solenoid valve, a simple linear 2\textsuperscript{nd} order model can be developed and effectively used to represent the clutch pressure dynamics (Yoon et al. 1997; Zheng, 1999). A nonlinear 2\textsuperscript{nd} order model using auto-regression with exogenous input (ARX) was also proposed by Hahn et al. (2001) and shown to be effective over a restricted range of operating conditions.

For the automatic transmission of interest here, the clutch pressure is controlled by a pulse-width-modulated (PWM) solenoid valve. The solenoid valve in this case regulates the fluid flow through the clutch chamber such that the clutch pressure is varied depending on the duty ratio of the solenoid command. The advantage of this type of solenoid valve over a linear solenoid valve is that it is less susceptible to contamination of the transmission fluid. It is also inexpensive and easy to interface to an electronic control unit (Cho et al, 2002). The mathematical model of this type of solenoid valve is usually more complex than for the linear solenoid valve. Cho et al. (1999) and Cho et al. (2002) developed a simplified linear 2\textsuperscript{nd} order model to describe the clutch pressure dynamics for this type of system, the model parameters being identified using system identification techniques. The model performed well when the duty ratio command is between 20\% - 80\%, and the frequency of the command change is less than 0.5 Hz. However, in real applications, the duty ratio command may change from one value to another instantaneously. Therefore, to accurately predict clutch pressure dynamics here, a more complicated and robust model is needed. Such a model is described in Chapter 3.
2.2 Online Estimation for Automatic Transmissions

Observers are of considerable practical interest because of their ability to provide estimates of inaccessible state variables of dynamic systems based on system dynamic models and available measurements. The original idea of the state observer was first proposed by Luenberger (1971), and has been explored extensively since then. However, the Luenberger observer is restricted to linear systems where linear mathematical models of system dynamics are appropriate. Sliding mode observers have gained much attention recently due to their applicability to nonlinear system estimation, excellent robustness properties in the presence of modeling uncertainties, and relatively simplicity of design and implementation. This method will be used in the proposed research as well. Relevant theory for the development of sliding mode observers can be found in Utkin (1992), Slotine et al. (1986), and Walcot and Zak (1986 and 1988).

Transmission input and output speed sensors are readily available in production transmissions. Variables of interest for transmission control include turbine and shaft torques, as well as transmission input and output accelerations, as they provide information more directly related to shift quality. Real-time model-based estimation of these variables based on available measurements continues to be a challenge. Preliminary results on the estimation of shaft torque using sliding-mode observers have been presented by Masmoudi and Hedrick (1992). The use of adaptive sliding-mode observers for turbine torque estimation has been described by Yi et al (2000). Though the results in the cited references are encouraging, they are preliminary in nature. Furthermore, little work has been reported on model-based clutch pressure estimation, and its use for transmission control.
Transmission input and output torques, when the transmission is in-gear, may be estimated by using an engine map in conjunction with the torque converter characteristic and the transmission gear ratio (Ibamoto et al, 1997; Ibamoto et al, 1995). Low frequency components of transmission input and output torques can be determined reasonably well thus, and can be used as feedback signals for transmission control. The estimates are less accurate during the torque phase, since the method relies on speed information which is relatively constant during the torque phase. The estimates obtained thus are further limited by the fact that available engine and torque converter models are algebraic and not dynamic, and furthermore they are nominal models which are not adapted based on measurements.

More promising methods for torque estimation use model-based observers. A model-based sliding mode observer to estimate the output shaft torque, in gear and during shifts, has been developed by Masmoudi and Hedrick (1992). Transmission output and wheel speed measurements were used as observer inputs. The clutch torques were assumed to be unknown, and robustness of the estimated shaft torque was achieved by using high sliding mode gains, presumably at the cost of increased sensitivity of the estimates to measurement noise. The study was only a simulation study, and experimental results were not reported. A similar approach has been used more recently by Yi et al (2000) to estimate the transmission input torque. An adaptive sliding mode estimator is used, the adapted parameters being the coefficients in the torque converter model. Transmission input speed, output speed, and wheel speed measurements were used as observer inputs, and estimates of shaft torque were obtained along with estimates of turbine torque.
In both of the approaches above, the shift hydraulic response was not modeled, and the clutch pressures, which are inputs to the dynamic model, were treated either as unknowns or replaced by commanded clutch pressures. The price paid for this simplicity in the estimation scheme is the need for high sliding mode gains to achieve the desired robustness of the estimates to the unknown inputs. Alternatively, the estimation problem becomes more complex, involving the simultaneous estimation of the unknown inputs as well as the unmeasured states.

2.3 Clutch-to-Clutch Controller Development

The coordinated control of engine variables such as spark advance and throttle opening, together with transmission variables such as clutch pressures, enables significant gains in shift quality control, as demonstrated by the work of Cho (1987), Yang et al (2001), and Ibamoto et al (1997). This is true whether we consider clutch-to-clutch control or overrunning clutch-to-clutch control. We restrict ourselves here to issues related to transmission control of shift quality, and to the unique problems associated with development of effective feedback control strategies and algorithms suitable for production transmissions. We discuss below the main issues involved in the development of model-based controllers for clutch-to-clutch control of shift quality.

The first obstacle to developing a good controller for clutch-to-clutch shifts is the detection of the end of the filling phase for the on-coming clutch. If the controller has no information on the filling phase, mismatch of clutch pressures easily happens (Muller, 2002). The result of clutch pressure mismatch is either engine run-away (which results if the off-going clutch is depressurized too soon) or clutch interlock (which results if the
off-going clutch is depressurized too late). While both these effects are undesirable, the latter is more so as it can damage transmissions.

Bai et al (2002) avoided the need for precise detection of the clutch fill time in their proposed control strategy as follows. The on-coming clutch pressure is used as a reducing input (“washout signal”) to the control valve pressurizing the off-going clutch. As long as the on-coming clutch is not filled, there is no reduction in the commanded pressure signal to the off-going clutch. Once the on-coming clutch is full and its pressure builds up, the commanded pressure signal to the off-going clutch is reduced, the nature of the reduction depending upon the mechanical design parameters associated with the control valve operation. While this approach avoids the need to sense clutch pressure explicitly, the reliance upon mechanical means for coordinating the controls on the two clutches comes at the expense of effectiveness over a limited range of operating conditions, and proves to be a limitation on subsequent shift control as discussed later. An approximate estimate of the fill pressure and fill time is still useful and needed in this approach, to adjust the pressure commands to the control valves for the on-coming and off-going clutches.

Another approach to detect on-coming clutch fill is based on the estimated transmission output shaft acceleration signal (Hebbale and Kao, 1995). When the oncoming clutch fills, it exerts torque on the clutch plates, causing partial load transfer from the off-going clutch and a drop in the output shaft torque and acceleration. Detection of the drop in the acceleration requires real-time estimation of the acceleration from the measured output shaft speed, which poses its own problems given the noisiness of the measured speed signal. Any model-based estimation of acceleration also has to
contend with the complexity of the transmission dynamic model, and how it may be used in the estimation scheme.

An alternative to explicit or indirect determination of the on-coming clutch fill time is operation of the off-going clutch under closed loop slip speed control with a small positive slip (Cho, 1987, Yang et al., 2001). The off-going clutch pressure is then just a little lower than that required to transmit the input torque, which makes it easier to subsequently avoid clutch tie-up at the end of the torque phase. The tradeoff here is the additional complexity associated with closed loop slip speed control, and potential problems related to control loop stability.

Setting aside the issue of detection of the end of the filling phase for the on-coming clutch for a moment, researchers have proposed several ways to control clutch-to-clutch shifts. One approach proposed by Cho (1987) and Yang et al. (2001) involves controlling the turbine speed and the transmission output speed. In this case, the controller is designed to calculate the required clutch torque for both on-coming and off-going clutches such that the turbine speed and the transmission output speeds track desired trajectories. Since it is known that during the torque phase the turbine speed does not change much, forcing it to track an inappropriate trajectory before the inertia phase occurs can cause the on-coming and off-going clutches to fight, causing the torque drop during the torque phase to become larger than the minimum theoretical value. Moreover, since the output torque is not controlled explicitly in this approach, controlling the clutch behavior to obtain a desired output speed response may not suppress the peak torque at the end of the inertia phase adequately. The solution to these problems, which was used by both Cho (1987) and Yang (2001), is to use engine torque control in addition, and to
adjust the spark timing such that engine torque can be increased or decreased as desired. The controller designs used by both Cho (1987) and Yang (2001) rely also on the assumption that both transmission input and torques are available, either by measurement or estimation. This assumption emphasizes the need for observer-based estimation of these variables.

Jeong and Lee (2000) extended the work of Cho (1987) and used the same method involving control of the turbine speed and the output speed. Based on a simple transmission model presented in their work, controlling these two speeds is equivalent to controlling the slip speed of the on-coming clutch. The work emphasized the design of the trajectory of the on-coming clutch slip speed. The performance of this control scheme depends on the assumption that the off-going clutch is fully released when the gearshift is initiated. This means that the torque phase or the load transfer is not considered in this approach. Also, even though the off-going clutch is commanded to fully release, it takes some time for the hydraulic fluid to drain and for the clutch pressure to reduce and the clutch to unlock. The advantage of this scheme is that by controlling the on-coming clutch slip, the unknown filling phase is taken care of implicitly. The desired trajectory of the slip speed in this case is calculated based on the desired inertia phase duration and the vehicle acceleration.

Minowa et al. (1999) proposed a smooth gearshift control approach using output shaft torque estimation and an H-infinity robust controller design approach. A simple table look-up is used for output shaft torque estimation in this work. The method showed improved performance and achieved a smoother gearshift control by reducing torque fluctuation during the inertia phase effectively. However, the coordination of clutches
during the torque phase is predetermined based on calibration, which may not be robust for real application in the presence of shift-to-shift variations and vehicle-to-vehicle variations. The difficulty of detecting the end of the filling phase is not considered explicitly in this work, since the commands for both the on-coming clutch and the off-going clutch are calibrated off-line as mentioned. Even with these drawbacks, this method emphasizes the need for output shaft torque estimation and shows that with output shaft torque feedback control, the inertia phase can be controlled effectively.

2.4 Conclusion

In this chapter, recent work related to transmission modeling issues, estimation of necessary variables for gearshift controller design, and the development of controllers for clutch-to-clutch shifts has been reviewed. In the area of transmission modeling, models for the torque converter, the transmission mechanical system, and the driveline and vehicle dynamics are widely discussed in the literature. Each of these subsystems is a subject in its own right. The complexity of the model for each subsystem can be varied depending on the research or development application. In the case of model development for the shift hydraulic subsystem, on the other hand, there has been much less work reported in the literature. Results from these reports show that models of complexity greater than second order are needed to facilitate gearshift controller design.

Designs of observers for transmission applications have also been reviewed here. In most such observer-based schemes, the shift hydraulic behavior is ignored, and error in the clutch pressure estimation is assumed to be bounded. This assumption can cause
problems in observer development, since observer gains have to be high to compensate for model error and cause sensitivity to measurement noise.

Some control algorithms for clutch-to-clutch shifts are also reviewed here. Work reported in the literature suggests that the torque phase and the inertia phase have to be controlled separately due to differences in control objectives and the required control actions. Reported work also shows that the estimated output shaft torque can be used as a feedback to the controller and clutch-to-clutch shifts smoothened. Since shift hydraulic dynamics are ignored in such an approach, there is a corresponding error in shaft torque estimation. Therefore, if the hydraulic system dynamics are modeled and incorporated in model-based estimation schemes, the resulting shaft torque estimation can be more accurate and can help controllers for clutch-to-clutch shifts to achieve better shift quality.
CHAPTER 3

TRANSMISSION MODEL DEVELOPMENT

The following chapter describes dynamic model development for 4-speed and 5-speed versions of an automatic transmission. Since we are studying the dynamic behavior of the transmission so that we may develop a model suitable for real time applications, a time-domain model is considered here. The overall model consists of a torque converter, the mechanical system of the transmission, and the shift hydraulic system. We note that only the torque converter model, the transmission mechanical system model and the vehicle dynamic model are presented in this chapter. The model development of the shift hydraulic system will be presented in detail in Chapter 4. We note also that a detailed engine dynamic model is not considered here for the sake of simplicity. An engine map is used to represent engine behavior throughout this work.

This chapter is organized as follows. Brief descriptions of the subsystems involved in the construction of the transmission model are given first. The development of the torque converter model used in the transmission of interest is then presented, followed by the detailed development of the models for the transmission mechanical components of both the 4-speed and 5-speed versions of the transmission. The validation of the model is then demonstrated by comparing simulation results with test data obtained
from the test vehicle. Conclusions from the work described in this chapter are given at the end of the chapter. We note here that the definition of all variables and parameters used in this chapter and throughout this thesis including their units are given in the nomenclature, if not specified.

3.1 System Components

The automatic transmissions to be described here are 4-speed and 5-speed versions of a Hyundai Motors automatic transmission, models F4A42 and F5A52 respectively. In both cases, there are four majors subsystems involved in the operation of the transmission. The first subsystem is the torque converter whose input shaft is connected to, and driven by, the engine. The torque output of the torque converter drives the second subsystem, which is the transmission mechanical subsystem. This subsystem consists of all mechanical parts of the transmission, e.g. the planetary gear sets. The transmission mechanical system is connected to the driveline exciting the vehicle dynamics. The last subsystem which determines the behavior of the transmission is the shift hydraulic system. This subsystem controls the engaging and disengaging of clutches accompanying gear shifts, as well as the powering of the main line pressure. A simple block diagram showing the interconnection of these subsystems is showed in Figure 3.1. As mentioned earlier, only the development of the torque converter model and the transmission mechanical subsystem are presented in this chapter. Due to the complexity of the shift hydraulic system, the development of its model will be discussed in Chapter 4. Figure 3.1 Subsystem connections in the simulation model of the transmission
3.2 Torque Converter Model

The development of the model describing the torque converter dynamic behavior involves some difficulty due to the three dimensional nature of the flow inside the torque converter resulting from the complexity of the blade geometry inside the torque converter. Detailed dynamic models can be found in the literature but these models have not been reported to be suitable for real-time application (Hrovat and Tobler, 1985). For the sake of simplicity, the model used in this research is a static model developed by Kotwicki (1985). This model is an empirical model and has been widely accepted for use in transmission controller design. Using the torque converter characteristic given by the manufacturer, equations describing the behavior of the torque converter used in the transmission of interest here follow.

For the torque multiplication mode, where the speed ratio between the turbine speed and the pump speed is less than 0.9, \( \frac{\omega_t}{\omega_p} < 0.9 \), the turbine torque and the pump torque are described by,
\[ T_i = 7.1315 \times 10^{-3} \omega_p^2 - 0.30135 \times 10^{-3} \omega_p \omega_t - 3.4140 \times 10^{-3} \omega_t^2 \]  \hspace{1cm} (3.1)  

\[ T_i = 7.1315 \times 10^{-3} \omega_p^2 - 0.30135 \times 10^{-3} \omega_p \omega_t - 3.4140 \times 10^{-3} \omega_t^2 \]  \hspace{1cm} (3.2)  

For the fluid coupling mode, where the speed ratio is close to one \( (\omega_t / \omega_p > 0.9) \), the turbine torque and the pump torque are equal and described by,

\[ T_i = T_p = 4.4324 \times 10^{-3} \omega_p^2 + 6.9819 \times 10^{-3} \omega_p \omega_t - 11.2612 \times 10^{-3} \omega_t^2 \]  \hspace{1cm} (3.3)  

In this case, \( T_i \) and \( \omega_t \) are the torque and speed for the turbine, respectively. Similarly, \( T_p \) and \( \omega_p \) are the torque and speed for the pump, respectively. The units for these variables are in SI-unit system.

### 3.3 4-Speed Transmission Mechanical Subsystem Model

In the 4-Speed transmission of interest, there is one compound gear set which consists of two planetary gear sets mechanically connected together. There are 5 clutches, namely, the Low-Reverse clutch (LR), the Second clutch (ND), the Under-Drive clutch (UD), the Over-Drive clutch (OD), and the Torque converter clutch (TCC). The application of clutches other than the TCC determines the operating gear ratio for the transmission. Table 3.1 shows the clutch engagement schedule and the overall transmission gear ratio for the different gears, including the final drive gear ratio of 2.84. Note that the application schedule of the TCC is not shown here, since it does not result in a change in the gear ratio of the transmission. Instead, the TCC is used to control the slip in the torque converter to improve fuel economy. Design of a controller for the TCC is a research topic in its own right, and will not be considered here.
Clutch Brake

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Table 3.1: Clutch engagement schedule for the 4-speed transmission of interest

3.3.1 Planetary Gear Train Model

The simplified schematic diagram of the transmission, the so-called “stick diagram”, showing the interconnections between the components is shown in Figure 3.2. The stick diagram, including lumped inertias as well as the corresponding torques and speeds of the planetary gears, is shown in Figure 3.3. We note that, since lumped inertias are used in the model, the model will be suitable only for studying the dynamic behavior of the transmission at low frequencies. Assuming perfect power transfer by ignoring friction and backlash at the planetary gears, the steady state speeds and torques for the elements of the planetary gear sets shown in Figure 3.3 can be written as the following.

\[ \omega_{ci} = R_{si} \omega_{si} + R_{ri} \omega_{ri} \]
\[ \omega_{cr} = R_{sr} \omega_{sr} + R_{rr} \omega_{rr} \]  

Due to the fact that two sets of planetary gears are mechanically connected together, we have the following constraints.

\[ \omega_{ci} = \omega_{rr} \]
\[ \omega_{ri} = \omega_{cr} \]  

We define,
Here, $\omega$ represents the angular velocity with its first subscript, $S$, $R$, or $C$ denoting sun-gear, ring-gear, and carrier, and its second subscript $i$ or $r$ denoting input planetary gear set and reaction gear set respectively. $N$ represents the number of teeth of the corresponding gear element which is referred to by the same subscript notation as $\omega$. Using the speed relationships and constraints from equations (3.4) and (3.5), we have,

$$\omega_{R_r} = \left( \frac{1}{1 - R_{R_i} R_{R_r}} \right) \left( R_{S_i} \omega_{S_i} + R_{R_i} R_{S_r} \omega_{S_r} \right)$$  \hspace{1cm} (3.10)

$$\omega_{C_r} = \left( \frac{1}{1 - R_{R_i} R_{R_r}} \right) \left( R_{S_r} \omega_{S_r} + R_{R_i} R_{S_i} \omega_{S_i} \right)$$  \hspace{1cm} (3.11)

From Figure 3.3, $T_{S_i}$, $T_{J_{R_r}}$, $T_{J_{fR_r}}$, and $T_o$ are the steady torques acting at the ports of the planetary gear set. The steady state torque relationship for these torques can also be derived as

$$T_o = \frac{R_{R_r} T_{J_{R_r}}}{R_{S_r}} - \frac{1}{R_{S_i}} T_{S_i}$$  \hspace{1cm} (3.12)

$$T_{J_{fR_r}} = \frac{1}{R_{S_r}} T_{J_{fR_r}} - \frac{R_{R_i}}{R_{S_i}} T_{S_i}$$  \hspace{1cm} (3.13)
\( T_{\text{Sid}} \), \( T_{\text{fSrd}} \), \( T_{\text{fRrd}} \), and \( T_{\text{od}} \) in Figure 3.3 represent the dynamic torques acting on the corresponding lumped inertias at the ports of the planetary gear sets. The relationships between these dynamic torques and their corresponding steady state values at the ports of the gear sets are the following.

\[
I_{\text{Si}} \omega_{\text{Si}} = T_{\text{Sid}} - T_{\text{Si}} \tag{3.14}
\]

\[
I_{\text{Rr}} \omega_{\text{Rr}} = T_0 - T_{\text{od}} \tag{3.15}
\]

\[
I_{\text{Cr}} \omega_{\text{Cr}} = T_{\text{fRrd}} - T_{\text{fCr}} \tag{3.16}
\]

\[
I_{\text{Sr}} \omega_{\text{Sr}} = T_{\text{fSrd}} - T_{\text{fSr}} \tag{3.17}
\]

\( I_{\text{Si}} \) is the inertia of the input sun-gear assembly, \( I_{\text{Sr}} \) is the inertia of the reaction sun-gear assembly, \( I_{\text{Rr}} \) is the inertia of the reaction ring-gear assembly, and \( I_{\text{Cr}} \) is the inertia of the reaction carrier assembly. Table 3.2 shows alternative forms of the steady state relationships for both torques and speeds at the four ports of the planetary gear train.

Figure 3.2: Stick diagram of the 4-Speed transmission of interest
Figure 3.3: Stick diagram of the planetary gear train including lumped inertias

\[
\begin{align*}
\begin{cases}
\omega_{Sr} = A \begin{pmatrix} \omega_{Si} \\ \omega_{Cr} \end{pmatrix} & \quad (\omega_{Sr}) = A \begin{pmatrix} \omega_{Si} \\ \omega_{Cr} \end{pmatrix} \\
T_{Si} = -A^T \begin{pmatrix} T_{fsr} \\ -T_o \end{pmatrix} & \quad (a)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\omega_{Si} = B \begin{pmatrix} \omega_{Rr} \\ \omega_{Cr} \end{pmatrix} & \quad (\omega_{Si}) = B \begin{pmatrix} \omega_{Rr} \\ \omega_{Cr} \end{pmatrix} \\
-T_o = -B^T \begin{pmatrix} T_{Si} \\ T_{fcr} \end{pmatrix} & \quad (b)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\omega_{Rr} = C \begin{pmatrix} \omega_{Sr} \\ \omega_{Cr} \end{pmatrix} & \quad (\omega_{Rr}) = C \begin{pmatrix} \omega_{Sr} \\ \omega_{Cr} \end{pmatrix} \\
T_{fcr} = -C^T \begin{pmatrix} -T_o \\ T_{Si} \end{pmatrix} & \quad (c)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\omega_{Cr} = D \begin{pmatrix} \omega_{Si} \\ \omega_{Sr} \end{pmatrix} & \quad (\omega_{Cr}) = D \begin{pmatrix} \omega_{Si} \\ \omega_{Sr} \end{pmatrix} \\
T_{fcr} = -D^T \begin{pmatrix} -T_o \\ T_{Si} \end{pmatrix} & \quad (d)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\omega_{Si} = E \begin{pmatrix} \omega_{Rr} \\ \omega_{Sr} \end{pmatrix} & \quad (\omega_{Si}) = E \begin{pmatrix} \omega_{Rr} \\ \omega_{Sr} \end{pmatrix} \\
-T_o = -E^T \begin{pmatrix} T_{Si} \\ T_{fcr} \end{pmatrix} & \quad (e)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\omega_{Sr} = F \begin{pmatrix} \omega_{Rr} \\ \omega_{Cr} \end{pmatrix} & \quad (\omega_{Sr}) = F \begin{pmatrix} \omega_{Rr} \\ \omega_{Cr} \end{pmatrix} \\
-T_o = -F^T \begin{pmatrix} T_{fcr} \\ T_{Si} \end{pmatrix} & \quad (f)
\end{cases}
\end{align*}
\]

\[
A = \frac{1}{R_{Sr}} \begin{bmatrix}
-R_{Sr}R_{Rr} & 1 - R_{Ri}R_{Rr} \\
R_{Sr} & -R_{Sr}R_{Ri}
\end{bmatrix} \quad B = \frac{1}{R_{Sr}R_{Sr}} \begin{bmatrix}
R_{Sr} & -R_{Ri}R_{Sr} \\
R_{Sr} & -R_{Sr}R_{Ri}\end{bmatrix}
\]

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\[
C = \frac{1}{R_i \cdot R_r} \begin{bmatrix}
-R_i R_r & R_i \\
1 - R_i R_r & -R_r
\end{bmatrix}
\]
\[
D = \frac{1}{1 - R_i R_r} \begin{bmatrix}
R_i & R_r \\
R_i R_r & R_r
\end{bmatrix}
\]
\[
E = \frac{1}{R_i} \begin{bmatrix}
1 - R_i R_r & -R_i R_r \\
R_i R_r & R_r R_i
\end{bmatrix}
\]
\[
F = \frac{1}{R_i R_r} \begin{bmatrix}
1 - R_i R_r & -R_i \\
R_i R_r & -R_r R_i
\end{bmatrix}
\]

\[
R_i = \frac{N_i}{N_i + N_{ri}}, \quad R_i = \frac{N_{ri}}{N_i + N_{ri}}, \quad R_r = \frac{N_r}{N_r + N_{rr}},
\]
\[
R_r = \frac{N_{rr}}{N_r + N_{rr}}, \quad R_i + R_{ri} = 1, \quad R_r + R_{rr} = 1
\]

Table 3.2: Steady state speed and torque relationship for F4A42 transmission

3.3.2 Models of Transmission Shift Dynamics

The main objective in developing a mathematical model for a transmission with clutch-to-clutch gearshift technology is the description of the torque phase and the inertia phase. In a conventional automatic transmission, the torque phase and the inertia phase are easily distinguishable in shifts involving a one-way clutch (OWC). Consider a gearshift involving a OWC-to-clutch load transfer. In this case, the torque phase starts when the on-coming clutch is engaged. As the torque capacity of the on-coming clutch increases, the load is transferred from the OWC, which is the off-going clutch. The inertia phase starts when the on-coming clutch takes all the load carried by the OWC. For a clutch-to-clutch gearshift, the torque phase also starts as the on-coming clutch is engaged. However, the beginning of the inertia phase cannot be determined merely by observing the load carried by the off-going clutch, since the off-going clutch is manually controlled. In other words, even if the on-coming clutch takes over the load carried by the
off-going clutch, if the off-going clutch is not released, the inertia phase does not start. In fact, the starting point of the inertia phase is defined as the moment when the off-going clutch slips, which occurs when the torque capacity of the off-going clutch is lower than the torque value for the clutch to remain in the lock-up condition. We note that, as the gearshift enters the inertia phase, since the off-going clutch is still slipping, this means that the off-going clutch still carries some load. Therefore, both off-going and on-coming clutches will “fight” and deteriorate the shift quality. The duration of the fighting as well as its severity depends on the rate of application and release of both clutches.

The mathematical model developed in this research will follow the above concept. The following sections show the model development and the resulting governing dynamic equations for the transmission undergoing a power-on up shift from first gear to the fourth gear. The derivation is done separately for the in-gear phase, the torque phase, and the inertia phase. As will be seen in following subsections, $T_i$ represents the transmission input torque or the turbine torque calculated from the torque converter model presented earlier, $I_t$ represents the turbine inertia, $\omega_t$ represents the turbine speed, $T_s$ refers to the transmission output shaft torque, $R_d$ is the final drive gear ratio, and $T_{LR}$, $T_{2ND}$, $T_{UD}$ and $T_{OD}$ are the torques from the LR, 2ND, UD, and OD clutches respectively.

As will be seen, to study the gear shift dynamics, the reaction torque at each clutch also has to be monitored. $RT_{LR}$, $RT_{2ND}$, $RT_{UD}$ and $RT_{OD}$ will represent the reaction torques at the LR, 2ND, UD, and OD clutches respectively.
3.3.2.1 First Gear and 1-2 Gear Up Shift Dynamics

(1) First Gear Dynamics

The stick diagram for the first gear including 1-2 up shift is shown in Figure 3.5. Based on Table 3.1, during the first gear, the UD clutch is fully engaged and in lock-up condition. This means that the torque capacity of the clutch is higher than the reaction torque or the torque transmitted by the clutch. Since the UD clutch connects the turbine with the input sun-gear, the speed of both the input sun-gear and the turbine are the same. The LR clutch/brake is also in lock-up condition. The functionality of the LR clutch is to hold the reaction carrier. Therefore, the reaction carrier is not moving in this gear. Moreover, since the 2ND clutch is not engaged, there is no dynamic torque entering the
planetary gear through the reaction sun-gear. We thus have the following conditions for the first gear.

\[ \omega_i = \omega_{si} \]  
\[ \omega_{Cr} = 0 . \]  
\[ RT_{2ND} = T_{2ND} = 0 \]

From Figure 3.4, we have,

\[ I_i \dot{\omega}_i = T_t - RT_{UD} \]  
\[ I_{si} \dot{\omega}_{si} = RT_{UD} - T_{si} \]  
\[ I_{re} \dot{\omega}_{re} = T_o - R_d T_s \]  
\[ I_{Cr} \dot{\omega}_{Cr} = -RT_{LR} - T_{fcr} \]  
\[ I_{Sr} \dot{\omega}_{Sr} = -RT_{2ND} - T_{fcr} \]

Using conditions (3.18) to (3.20), equations (3.21) to (3.25) become:

\[ T_t - T_{si} = (I_{si} + I_i) \dot{\omega}_{si} \]  
\[ T_o - R_d T_s = I_{re} \dot{\omega}_{re} \]  
\[ -RT_{LR} = T_{fcr} \]  
\[ -T_{fcr} = I_{Sr} \dot{\omega}_{Sr} \]

From Table 3.2:

\[ \begin{pmatrix} T_{Si} \\ T_{fcr} \end{pmatrix} = -A^{T} \begin{pmatrix} T_{fcr} \\ -T_o \end{pmatrix} \]  
\[ \begin{pmatrix} \dot{\omega}_{Sr} \\ \dot{\omega}_{re} \end{pmatrix} = A \begin{pmatrix} \dot{\omega}_{si} \\ \dot{\omega}_{Cr} \end{pmatrix} \]
Rewriting (3.27) and (3.29) in a compact form as,

\[
\begin{bmatrix}
T_{fsr} \\
-T_{ao}
\end{bmatrix} = -\begin{bmatrix}
I_{Sr} & 0 \\
0 & I_{Rr}
\end{bmatrix} \begin{bmatrix}
\dot{\omega}_{Sr} \\
\dot{\omega}_{Rr}
\end{bmatrix} + \begin{bmatrix}
0 \\
-R_dT_s
\end{bmatrix}
\]  

(3.32)

Since \(\omega_t = \omega_{si}\) and \(\omega_{cr} = 0\) as described, by substituting (3.31) and (3.32) in (3.30), we have,

\[
\begin{bmatrix}
T_{si} \\
T_{fcr}
\end{bmatrix} = \begin{bmatrix}
a_{11}^2I_{Sr} + a_{21}^2I_{Rr} \\
a_1a_{12}I_{Sr} + a_2a_{22}I_{Rr}
\end{bmatrix} \dot{\omega}_t + \begin{bmatrix}
a_{21}R_dT_s \\
a_{22}R_dT_s
\end{bmatrix}
\]  

(3.33)

Substituting \(T_{si}\) and \(T_{fcr}\) from this equation in (3.26) and (3.28), respectively, we get,

\[
(I_{si} + I_t + a_{11}^2I_{Sr} + a_{21}^2I_{Rr})\dot{\omega}_t = T_t - a_{21}R_dT_s
\]  

(3.34)

\[
RT_{LR} = -(a_{11}a_{12}I_{Sr} + a_{21}a_{22}I_{Rr})\dot{\omega}_t - a_{22}R_dT_s
\]  

(3.35)

Besides on this result, \(RT_{UD}\) can be calculated from (3.21). The speed of each gear element can be calculated from the following.

\[
\omega_{Sr} = a_{11}\omega_{si} = -1.064\omega_t
\]  

(3.36)

\[
\omega_{Rr} = a_{21}\omega_{si} = 0.38\omega_t
\]  

(3.37)

\[
\omega_o = R_d\omega_{Rr} = 0.265\omega_{Rr}
\]  

(3.38)

(2) 1-2 Up shift – Torque Phase

During the 1-2 up shift, the off-going clutch is the LR brake and the on-coming clutch is the 2ND brake. The 1-2 up shift torque phase starts as the 2ND brake is pressurized and exerts some torque on the planetary gear set. In this case, the condition in equation (3.20) is no longer valid, but the others remain the same. Therefore, the governing equations can be derived as presented in the first gear. The results are similar
to equations (3.34) and (3.35), but with an extra term representing the effect of the torque from the 2ND brake as shown below.

\[
(I_{Si} + I_t + a_{11}^2 I_{Sr} + a_{21}^2 I_{Re})\dot{\omega}_{Si} = T_t - a_{21} R_d T_s - a_{11} T_{2ND} \tag{3.39}
\]

\[
RT_{LR} = -(a_{11} a_{12} I_{Sr} + a_{21} a_{22} I_{Re})\dot{\omega}_{Si} - a_{22} R_d T_s - a_{12} T_{2ND} \tag{3.40}
\]

During this phase, the LR brake is also released. The torque capacity of the LR brake is then reduced. As the 2ND brake is pressurized, the load is transferred from the LR brake to the 2ND brake. Thus, the reaction torque calculated at the LR brake is reduced. Once the torque capacity, \(T_{LR}\), gets lower than the reaction torque calculated at the LR brake or the required torque to hold the reaction carrier, the LR brake is released from the lock up condition, and the reaction carrier starts to spin. This indicates the end of the torque phase. Therefore, we use the following conditions to determine the end of the torque phase.

\[
\left|RT_{LR}\right| > T_{LR} \tag{3.41}
\]

\[
\omega_{Cr} \neq 0 \tag{3.42}
\]

Notice that, in condition (3.41), the absolute value of the reaction torque at the LR clutch is used. This is because the LR clutch functions as a brake, which exerts torque opposed to the net torque acting on the reaction carrier. Depending on the interaction of other corresponding torques, the net torque can be either negative or positive. And since this occurs dynamically, we cannot assign a specific sign to the reaction torque, but use the absolute value instead. This will be used for all other clutches that functions as brakes.
1-2 Up shift – Inertia Phase

As the reaction carrier starts rotating, the 1-2 up shift inertia phase begins. In this case, the dynamic torque acting at the reaction carrier is the torque from the LR brake which is being reduced. At the 2ND brake, its torque capacity, $T_{2ND}$, is still being increased but is not high enough to hold the reaction sun-gear stationary. The dynamic torque acting on the reaction sun-gear is calculated from the torque capacity of the 2ND brake. The clutch UD is still fully engaged. Therefore, the speed of the input sun-gear is still the same as the turbine speed. The dynamics of this phase can be derived as follows. The equations (3.26) for the input sun-gear element and (3.27) for the reaction ring-gear still hold. For the reaction carrier and the reaction sun-gear, we can write,

$$I_{Cr} \dot{\omega}_{Cr} = -T_{LR} - T_{fcr}$$  \hspace{1cm} (3.43)

$$I_{Sr} \dot{\omega}_{Sr} = -T_{2ND} - T_{fcr}$$  \hspace{1cm} (3.44)

In this phase, we want to know the dynamics of the transmission input and output speeds, which are the input sun-gear speed, $\omega_{Si}$, and reaction ring-gear speed, $\omega_{Rr}$. From the kinematic relationships shown in Table 3.2, we use,

$$\begin{bmatrix} \omega_{Sr} \\ \omega_{Cr} \end{bmatrix} = F \begin{bmatrix} \omega_{Sr} \\ \omega_{Si} \end{bmatrix}$$  \hspace{1cm} (3.45)

$$\begin{bmatrix} -T_o \\ T_{Si} \end{bmatrix} = -F^T \begin{bmatrix} T_{fcr} \\ T_{fcr} \end{bmatrix}$$  \hspace{1cm} (3.46)

Rewriting equations (3.43) and (3.44) in the following form

$$\begin{bmatrix} T_{fcr} \\ T_{fcr} \end{bmatrix} = -\begin{bmatrix} I_{Sr} & 0 \\ 0 & I_{Cr} \end{bmatrix} \begin{bmatrix} \dot{\omega}_{Sr} \\ \dot{\omega}_{Cr} \end{bmatrix} - \begin{bmatrix} T_{2ND} \\ T_{LR} \end{bmatrix}$$  \hspace{1cm} (3.47)

And, from (3.26) and (3.27), we can write,
\[
\begin{pmatrix} -T_o \\ T_{Sl} \end{pmatrix} = \begin{pmatrix} I_{Rr} & 0 \\ 0 & I_{Sl} + I_t \end{pmatrix} \begin{pmatrix} \dot{\omega}_{Rr} \\ \dot{\omega}_{Sl} \end{pmatrix} + \begin{pmatrix} -R_d T_s \\ T_i \end{pmatrix}
\] (3.48)

Combining equations (3.45) to (3.48), we have,

\[
-\begin{pmatrix} I_{Rr} & 0 \\ 0 & I_{Sl} + I_t \end{pmatrix} \begin{pmatrix} \dot{\omega}_{Rr} \\ \dot{\omega}_{Sl} \end{pmatrix} + \begin{pmatrix} -R_d T_s \\ T_i \end{pmatrix} = F^T \begin{pmatrix} I_{Sr} & 0 \\ 0 & I_{Cr} \end{pmatrix} \begin{pmatrix} \dot{\omega}_{Rr} \\ \dot{\omega}_{Sl} \end{pmatrix} + F^T \begin{pmatrix} T_{2ND} \\ T_{LR} \end{pmatrix}
\] (3.49)

Or in a simpler form,

\[
\begin{pmatrix} F^T \begin{pmatrix} I_{Sr} & 0 \\ 0 & I_{Cr} \end{pmatrix} F + \begin{pmatrix} I_{Rr} & 0 \\ 0 & I_{Sl} + I_t \end{pmatrix} \end{pmatrix} \begin{pmatrix} \dot{\omega}_{Rr} \\ \dot{\omega}_{Sl} \end{pmatrix} = \begin{pmatrix} -R_d T_s \\ T_i \end{pmatrix} - F^T \begin{pmatrix} T_{2ND} \\ T_{LR} \end{pmatrix}
\] (3.50)

By letting:

\[
I_{12} = \begin{pmatrix} F^T \begin{pmatrix} I_{Sr} & 0 \\ 0 & I_{Cr} \end{pmatrix} F + \begin{pmatrix} I_{Rr} & 0 \\ 0 & I_{Sl} + I_t \end{pmatrix} \end{pmatrix} = \begin{pmatrix} I_{1211} & I_{1212} \\ I_{1221} & I_{1222} \end{pmatrix}
\] (3.51)

we have

\[
\begin{pmatrix} \dot{\omega}_{Rr} \\ \dot{\omega}_{Sl} \end{pmatrix} = I_{12}^{-1} \begin{pmatrix} -f_{11} T_{2ND} - f_{21} T_{LR} - R_d T_s \\ -f_{12} T_{2ND} - f_{22} T_{LR} + T_i \end{pmatrix}
\] (3.52)

In this phase, \( T_r, T_s, T_{2ND} \) and \( T_{LR} \) are inputs to the transmission, and \( T_{2ND} \) and \( T_{LR} \) can be calculated from the individual clutch characteristics. This phase ends when the torque capacity at the 2ND brake is high enough to hold the reaction sun-gear stationary. The 2ND brake goes to the lock-up condition when the clutch torque is higher than the torque required to hold the sun-gear element. In particular, this phase ends when,

\[
T_{2ND} \geq |RT_{2ND}|
\] (3.53)

\[\omega_{Sr} = 0\] (3.54)

The absolute value of the reaction torque of the 2ND brake is used here as in the case of the LR break described earlier.
3.3.2.2 Second Gear and 2-3 Gear Up Shift Dynamics

(1) Second Gear Dynamics

The stick diagram for the second gear including 2-3 up shift is shown in Figure 3.5. In this gear, the UD clutch is still fully engaged and, hence, the input sun-gear speed is equal to the turbine speed. The 2ND brake is now fully engaged so that the reaction sun-gear is not moving. The LR brake and the OD clutch are not engaged during this gear and therefore there is no dynamic torque acting on the reaction carrier. Therefore, we have the following conditions.

\[ \omega_i = \omega_{si} \quad (3.55) \]

\[ \omega_{sc} = 0 \quad (3.56) \]

\[ RT_{OD} = T_{OD} = 0 \quad (3.57) \]
From the stick diagram, we have the following equations:

\[ I_t \dot{\omega}_t = T_t - RT_{UD} - RT_{OD} \] (3.58)

\[ I_{Si} \dot{\omega}_{Si} = RT_{UD} - T_{Si} \] (3.59)

\[ I_{Rr} \dot{\omega}_{Rr} = T_o - R_d T_s \] (3.60)

\[ I_{Cr} \dot{\omega}_{Cr} = RT_{OD} - T_{fcr} \] (3.61)

\[ I_{Sr} \dot{\omega}_{Sr} = -RT_{2ND} - T_{fcr} \] (3.62)

Using conditions (3.55) to (3.57), the above equations are reduced to,

\[ T_t - T_{Si} = (I_{Si} + I_t) \dot{\omega}_{Si} \] (3.63)

\[ T_o - R_d T_s = I_{Rr} \dot{\omega}_{Rr} \] (3.64)

\[ -T_{fcr} = I_{Cr} \dot{\omega}_{Cr} \] (3.65)

\[ -RT_{2ND} = T_{fcr} \] (3.66)

Using the following kinematic relationships from Table 3.2:

\[
\begin{bmatrix}
T_{Si} \\
T_{fcr}
\end{bmatrix} = -D^T \begin{bmatrix}
-T_o \\
T_{fcr}
\end{bmatrix}
\] (3.67)

\[
\begin{bmatrix}
\omega_{Rr} \\
\omega_{Cr}
\end{bmatrix} = D \begin{bmatrix}
\omega_{Si} \\
\omega_{Sr}
\end{bmatrix}
\] (3.68)

We combine the equations and get,

\[ (I_{Si} + I_t + d^{2}_{11} I_{Rr} + d^{2}_{21} I_{Cr}) \dot{\omega}_{Si} = T_t - d^{1}_{11} R_d T_s \] (3.69)

\[ RT_{2ND} = -(d^{1}_{12} d_{12} I_{Rr} + d^{1}_{22} d_{22} I_{Cr}) \dot{\omega}_{Si} - d^{1}_{12} R_d T_s \] (3.70)

where \( RT_{UD} \) is calculated from

\[ RT_{UD} = T_t - I_t \dot{\omega}_t \] (3.71)
In this gear, we have the following speed relationships.

\[ \omega_{Cr} = d_{21} \omega_{Si} = 0.5155 \omega_t \]  \hspace{1cm} (3.72)

\[ \omega_{Rr} = d_{11} \omega_{Si} = 0.6996 \omega_t \]  \hspace{1cm} (3.73)

\[ \omega_o = R_o \omega_{Rr} = 0.265 \omega_{Rr} \]  \hspace{1cm} (3.74)

(2) 2-3 Up Shift – Torque Phase

As the OD clutch, the on-coming clutch for this gearshift, is pressurized, the torque phase starts. The condition (3.57) is void. Using the same derivation as for the second gear, the governing equations for the 2-3 up shift torque phase may be obtained and are seen to be similar to (3.69) and (3.70), but with an additional term to represent the effect of the OD clutch torque. In particular, we have,

\[ (I_{Si} + I_t + d_{11}^2 I_{Rr} + d_{21}^2 I_{Cr}) \dot{\omega}_{Si} = T_t - d_{11} R_o T_s + (d_{21} - 1) T_{OD} \]  \hspace{1cm} (3.75)

\[ RT_{2ND} = -(d_{11} d_{12} I_{Rr} + d_{21} d_{22} I_{Cr}) \dot{\omega}_{Si} - d_{12} R_o T_s + d_{22} T_{OD} \]  \hspace{1cm} (3.76)

The off-going clutch for 2-3 up shift is the 2ND brake. This clutch is released during the 2-3 up shift torque phase. The torque phase ends when the torque capacity of the 2ND brake is lower than the reaction torque calculated from equation (3.76), which is the minimum torque required to hold the reaction sun-gear. In other words, the torque phase ends as the reaction sun-gear starts rotating. Therefore, the following conditions are used to define the end of the torque phase and the beginning of the inertia phase.

\[ |RT_{2ND}| > T_{2ND} \]  \hspace{1cm} (3.77)

\[ \omega_{Sr} \neq 0 \]  \hspace{1cm} (3.78)
(3) 2-3 Up Shift – Inertia Phase

As described previously, the inertia phase for 2-3 up shift starts once the reaction sun-gear starts spinning. From this point on, the dynamic torque acting on the reaction sun-gear is clutch torque from the 2ND brake. The torque capacity of the OD clutch is being increased by commands from the controller, but initially is not high enough to stop the clutch slip or get to the lock up condition. Therefore, as for the reaction sun-gear, the dynamic torque acting on the reaction carrier is the OD clutch torque. The clutch UD is still fully engaged, while the LR brake is not. Therefore, based on the stick diagram shown in Figure 3.5, we have the following equations.

\[
I_{cr} \dot{\omega}_{cr} = T_{OD} - T_{fcr} \tag{3.79}
\]

\[
I_{sr} \dot{\omega}_{sr} = -T_{2ND} - T_{fcr} \tag{3.80}
\]

\[
(I_{si} + I_r) \dot{\omega}_{si} = T_i - T_{OD} - T_{si} \tag{3.81}
\]

\[
T_o - R_d T_s = I_{re} \dot{\omega}_{re} \tag{3.82}
\]

Similar to the derivation of the 1-2 up shift inertia phase, we want to monitor the behavior of the transmission input and output speed or the input sun-gear speed, \(\omega_{si}\), and the reaction ring-gear speed, \(\omega_{re}\). Based on the kinematic relationship in Table 3.2, we use:

\[
\begin{bmatrix}
\omega_{sr} \\
\omega_{cr}
\end{bmatrix} = F^{-1} \begin{bmatrix}
\omega_{re} \\
\omega_{si}
\end{bmatrix} \tag{3.83}
\]

\[
\begin{bmatrix}
-T_o \\
T_{si}
\end{bmatrix} = -F^T \begin{bmatrix}
T_{fcr} \\
T_{fcr}
\end{bmatrix} \tag{3.84}
\]

From (3.79) and (3.80), we have,
\[
\begin{pmatrix}
T_{fr} \\
T_{f'r}
\end{pmatrix} =
\begin{pmatrix}
I_{Sr} & 0 \\
0 & I_{Cr}
\end{pmatrix}
\begin{pmatrix}
\dot{\phi}_{Sr} \\
\dot{\phi}_{Cr}
\end{pmatrix} +
\begin{pmatrix}
-T_{2ND} \\
T_{OD}
\end{pmatrix}
\] (3.85)

Also, from (3.81) and (3.82), we have,
\[
\begin{pmatrix}
-T_o \\
T_{Si}
\end{pmatrix} =
\begin{pmatrix}
I_{Rr} & 0 \\
0 & I_{Si} + I_{I}
\end{pmatrix}
\begin{pmatrix}
\dot{\phi}_{Rr} \\
\dot{\phi}_{Si}
\end{pmatrix} +
\begin{pmatrix}
-R_d T_s \\
T_t - T_{OD}
\end{pmatrix}
\] (3.86)

Combining equations, we get,
\[
-\begin{pmatrix}
I_{Rr} & 0 \\
0 & I_{Si} + I_{I}
\end{pmatrix}
\begin{pmatrix}
\dot{\phi}_{Rr} \\
\dot{\phi}_{Si}
\end{pmatrix} +
\begin{pmatrix}
-R_d T_s \\
T_t - T_{OD}
\end{pmatrix} =
F^T\begin{pmatrix}
I_{Sr} & 0 \\
0 & I_{Cr}
\end{pmatrix}
\begin{pmatrix}
\dot{\phi}_{Rr} \\
\dot{\phi}_{Si}
\end{pmatrix} -
F^T\begin{pmatrix}
-T_{2ND} \\
T_{OD}
\end{pmatrix}
\] (3.87)

Or,
\[
\begin{bmatrix}
F^T\begin{pmatrix}
I_{Sr} & 0 \\
0 & I_{Cr}
\end{pmatrix}F +
\begin{pmatrix}
I_{Rr} & 0 \\
0 & I_{Si} + I_{I}
\end{pmatrix}
\end{bmatrix}
\begin{pmatrix}
\dot{\phi}_{Rr} \\
\dot{\phi}_{Si}
\end{pmatrix} =
\begin{pmatrix}
-R_d T_s \\
T_t - T_{OD}
\end{pmatrix} +
F^T\begin{pmatrix}
-T_{2ND} \\
T_{OD}
\end{pmatrix}
\] (3.88)

By letting:
\[
I_{23} =
\begin{bmatrix}
F^T\begin{pmatrix}
I_{Sr} & 0 \\
0 & I_{Cr}
\end{pmatrix}F +
\begin{pmatrix}
I_{Rr} & 0 \\
0 & I_{Si} + I_{I}
\end{pmatrix}
\end{bmatrix} =
\begin{bmatrix}
I_{2311} & I_{2312} \\
I_{2321} & I_{2322}
\end{bmatrix}
\] (3.89)
we have,
\[
\begin{pmatrix}
\dot{\phi}_{Rr} \\
\dot{\phi}_{Si}
\end{pmatrix} =
I_{23}^{-1}\begin{pmatrix}
-f_{11}T_{2ND} + f_{21}T_{OD} - R_d T_s \\
-f_{12}T_{2ND} + (f_{22} - 1)T_{OD} + T_t
\end{pmatrix}
\] (3.90)

In this case, \(T_r, T_s, T_{2ND}\) and \(T_{OD}\) are inputs to the transmission. Both \(T_{2ND}\) and \(T_{OD}\) can be calculated from their characteristics. This gear shift ends when the torque capacity of the OD clutch is high enough to transmit the torque from the turbine through the reaction carrier. In this case, both the input sun gear and the reaction carrier will have the same speed as the turbine. This forces the members of the planetary gear to move in unison, and the gear ratio becomes one. The constraints describing the end of the 2-3 up shift are:
\[ T_{OD} \geq RT_{OD} \quad (3.91) \]
\[ \omega_{cr} = \omega_{t} \quad (3.92) \]

3.3.2.3 Third Gear and 3-4 Up Shift Dynamics

(1) 3\textsuperscript{rd} Gear Dynamics

The stick diagram for the third gear including 3-4 up shift is shown in Figure 3.6. In this gear, both the UD clutch and the OD clutch are fully engaged. Therefore, the input sun-gear speed and the reaction carrier speed are the same as the turbine speed. The 2ND brake is not engaged. Therefore, we have the following conditions.

\[ \omega_{t} = \omega_{si} \quad (3.93) \]
\[ \omega_{cr} = \omega_{t} \quad (3.94) \]
\[ RT_{2ND} = T_{2ND} = 0 \quad (3.95) \]

Figure 3.6: Free body diagram for the third gear including 3-4 up shift
From the stick diagram, we have the following.

\[ I_t \dot{\omega}_t = T_t - RT_{UD} - RT_{OD} \]  

\[ I_{Si} \dot{\omega}_{Si} = RT_{UD} - T_{Si} \]  

\[ I_{Cr} \dot{\omega}_{Cr} = RT_{OD} - T_{fcr} \]  

\[ I_{Re} \dot{\omega}_{Re} = T_o - R_d T_S \]  

\[ I_{Sr} \dot{\omega}_{Sr} = -T_{fcr} \]  

Combining the equations (3.93) to (3.98), we get,

\[ T_t - T_{Si} - T_{fcr} = (I_{Cr} + I_{Si} + I_t) \dot{\omega}_{Si} \]  

We use the following kinematic relationships from Table 3.2:

\[ \begin{pmatrix} \dot{\omega}_r \\ \dot{\omega}_s \end{pmatrix} = A \begin{pmatrix} \dot{\omega}_c \\ \dot{\omega}_r \end{pmatrix} \]  

\[ \begin{pmatrix} T_{Si} \\ T_{fcr} \end{pmatrix} = -A^T \begin{pmatrix} T_{fcr} \\ -T_o \end{pmatrix} \]  

From (3.99) and (3.100), we can write,

\[ \begin{pmatrix} T_{fcr} \\ -T_o \end{pmatrix} = - \begin{pmatrix} I_{Sr} & 0 \\ 0 & I_{Re} \end{pmatrix} \begin{pmatrix} \dot{\omega}_r \\ \dot{\omega}_s \end{pmatrix} + \begin{pmatrix} 0 \\ -R_d T_s \end{pmatrix} \]  

Using the selected kinematic relationships, we can show that,

\[ \begin{pmatrix} T_{Si} \\ T_{fcr} \end{pmatrix} = \begin{pmatrix} a_{11} (a_{11} + a_{12}) I_{Sr} + a_{21} (a_{21} + a_{22}) I_{Re} \\ a_{12} (a_{11} + a_{12}) I_{Sr} + a_{22} (a_{21} + a_{22}) I_{Re} \end{pmatrix} \dot{\omega}_{Si} + \begin{pmatrix} a_{21} R_d T_s \\ a_{22} R_d T_s \end{pmatrix} \]  

From the fact that \((a_{11} + a_{12}) = (a_{21} + a_{22}) = 1\), we get

\[ (I_t + I_{Si} + I_{Sr} + I_{Re} + I_{Cr}) \dot{\omega}_{Si} = T_t - R_d T_s \]  

\[ RT_{UD} = (I_{Si} + a_{11} I_{Sr} + a_{21} I_{Re}) \dot{\omega}_{Si} + a_{21} R_d T_s \]
\[ RT_{OD} = (I_{Cr} + a_{12}I_{Sr} + a_{22}I_{Rr})\omega_{Si} + a_{22}R_d T_s \]  

(3.108)

In this case, we have the following speed relationships.

\[ \omega_{Rr} = \omega_{Si} = \omega_t = \omega_{Cr} \]  

(3.109)

\[ \omega_o = R_d \omega_{Rr} = 0.265\omega_{Rr} \]  

(3.110)

(2) 3-4 Up Shift – Torque Phase

The 3-4 up shift involves releasing the UD clutch and applying the 2ND brake while keeping the OD clutch fully engaged. As the 2ND brake is pressurized, the torque phase starts. Similar to the previous derivations, the following equations may be shown to govern the dynamics of the 3-4 up shift torque phase.

\[ (I_t + I_{Si} + I_{Sr} + I_{Rr} + I_{Cr})\dot{\omega}_{Si} = T_t - R_d T_s - T_{2ND} \]  

(3.111)

\[ RT_{UD} = (I_{Si} + a_{11}I_{Sr} + a_{21}I_{Rr})\dot{\omega}_{Si} + a_{21}R_d T_s + a_{11}T_{2ND} \]  

(3.112)

\[ RT_{OD} = (I_{Cr} + a_{12}I_{Sr} + a_{22}I_{Rr})\dot{\omega}_{Si} + a_{22}R_d T_s + a_{12}T_{2ND} \]  

(3.113)

As the UD clutch is being released, the torque phase ends when the UD clutch starts to slip. In this case, the speed of the input sun-gear is no longer the same as the turbine speed. Therefore, the following conditions are used to describe the end of the torque phase and the beginning of the inertia phase.

\[ RT_{UD} > T_{UD} \]  

(3.114)

\[ \omega_{Si} \neq \omega_t \]  

(3.115)
(3) 3-4 Up Shift – Inertia Phase

This phase starts when the UD clutch starts to slip. The brake 2ND is still being applied. Though the torque capacity of the 2ND brake increases, initially it’s not high enough to stop the reaction sun-gear. The clutch OD is still fully engaged and thus the speed of the reaction carrier is the same as the turbine speed. From the stick diagram, we have the following.

\[ I_{Si} \dot{\omega}_{Si} = T_{UD} - T_{Si} \]  \hspace{1cm} (3.116)

\[ I_{Rr} \dot{\omega}_{Rr} = T_o - R_d T_S \]  \hspace{1cm} (3.117)

\[ I_{Sr} \dot{\omega}_{Sr} = -T_{2ND} - T_{fcr} \]  \hspace{1cm} (3.118)

\[ T_t - T_{UD} - T_{fcr} = (I_{Cr} + I_t) \dot{\omega}_{Si} \]  \hspace{1cm} (3.119)

We are monitoring the transmission input and output speeds, which are the reaction carrier speed, \( \omega_{Cr} \), and the reaction ring-gear speed, \( \omega_{Rr} \), for this gearshift. We use the following kinematic relationships from Table 3.2.

\[
\begin{pmatrix}
\omega_{Si} \\
\omega_{Sr}
\end{pmatrix} = B
\begin{pmatrix}
\omega_{Gr} \\
\omega_{Cr}
\end{pmatrix}
\]  \hspace{1cm} (3.120)

\[
\begin{pmatrix}
-T_o \\
T_{fcr}
\end{pmatrix} = -B^T
\begin{pmatrix}
T_{Si} \\
T_{fcr}
\end{pmatrix}
\]  \hspace{1cm} (3.121)

From equations (3.116) to (3.119), we can write,

\[
\begin{pmatrix}
-T_o \\
T_{fcr}
\end{pmatrix} = -\begin{pmatrix}
I_{Rr} & 0 \\
0 & I_t + I_{Cr}
\end{pmatrix}
\begin{pmatrix}
\dot{\omega}_{Gr} \\
\dot{\omega}_{Cr}
\end{pmatrix} + \begin{pmatrix}
-R_d T_S \\
T_t - T_{UD}
\end{pmatrix}
\]  \hspace{1cm} (3.122)

\[
\begin{pmatrix}
T_{Si} \\
T_{fcr}
\end{pmatrix} = \begin{pmatrix}
I_{Si} & 0 \\
0 & I_{Sr}
\end{pmatrix}
\begin{pmatrix}
\dot{\omega}_{Si} \\
\dot{\omega}_{Sr}
\end{pmatrix} + \begin{pmatrix}
T_{UD} \\
-T_{2ND}
\end{pmatrix}
\]  \hspace{1cm} (3.123)

Combining the above equations using the selected kinematic relationships, we have,
\[
\begin{bmatrix}
I_{Rr} & 0 \\
0 & I_t + I_{Cr}
\end{bmatrix}
+ B^T \begin{bmatrix}
I_{Si} & 0 \\
0 & I_{Sr}
\end{bmatrix} B \begin{bmatrix}
\dot{\omega}_{Rr} \\
\dot{\omega}_{Cr}
\end{bmatrix}
= B^T \begin{bmatrix}
T_{UD} \\
- T_{2ND}
\end{bmatrix} + \begin{bmatrix}
-R_d T_s \\
T_r - T_{UD}
\end{bmatrix}
\] (3.124)

Letting:

\[
I_{34} = \begin{bmatrix}
I_{Rr} & 0 \\
0 & I_t + I_{Cr}
\end{bmatrix}
+ B^T \begin{bmatrix}
I_{Si} & 0 \\
0 & I_{Sr}
\end{bmatrix} B = \begin{bmatrix}
I_{3411} & I_{3412} \\
I_{3421} & I_{3422}
\end{bmatrix}
\] (3.125)

we have,

\[
\begin{bmatrix}
\dot{\omega}_{Rr} \\
\dot{\omega}_{Cr}
\end{bmatrix} = I_{34}^{-1} \begin{bmatrix}
b_{11} T_{UD} - b_{21} T_{2ND} - R_d T_s \\
(b_{21} - 1) T_{UD} - b_{22} T_{2ND} + T_r
\end{bmatrix}
\] (3.126)

In this phase, \(T_r, T_s, T_{2ND}\) and \(T_{UD}\) are inputs to the transmission. \(T_{2ND}\) and \(T_{UD}\) can be calculated from clutch characteristics. This phase ends when the torque capacity at the 2ND brake is high enough to hold the reaction sun gear, or,

\[
|T_{2ND}| \geq |RT_{2ND}|
\] (3.127)

\[
\omega_{Sr} = 0
\] (3.128)

3.3.2.4 Fourth Gear Dynamics

(1) Forth Gear Dynamics

The stick diagram for the fourth gear is shown in Figure 3.7. In this gear, the OD clutch and the 2ND brake are fully engaged. Therefore, the speed of the reaction carrier is the same as the turbine speed, and the reaction sun-gear is held still. The UD clutch is not engaged, and therefore there is no torque acting on the input sun-gear. We have the following conditions.

\[
\omega_{Cr} = \omega_t
\] (3.129)
\[ \omega_{Sr} = 0 \]  

\[ RT_{UD} = T_{UD} = 0 \]  

From the stick diagram and the above conditions, we have,

\[ I_t \dot{\omega}_t = T_t - RT_{OD} \]  

\[ I_{si} \dot{\omega}_{si} = -T_{si} \]  

\[ I_{rr} \dot{\omega}_{rr} = T_{o} - R_d T_{s} \]  

\[ I_{cr} \dot{\omega}_{cr} = RT_{OD} - T_{fcr} \]  

\[ RT_{2ND} = -T_{fcr} \]  

\[ (I_t + I_{cr}) \dot{\omega}_t = T_t - T_{fcr} \]  

Figure 3.7: Free body diagram for the forth gear

From Table 3.2, we choose,
\[
\begin{pmatrix}
\omega_{Rr} \\
\omega_{Si}
\end{pmatrix} = \mathbf{C}
\begin{pmatrix}
\omega_{Sr} \\
\omega_{Cr}
\end{pmatrix}
\] (3.138)

\[
\begin{pmatrix}
T_{fsr} \\
T_{fcr}
\end{pmatrix} = -\mathbf{C}^T
\begin{pmatrix}
-T_o \\
T_{Si}
\end{pmatrix}
\] (3.139)

Combining the equations, we get,

\[
\left( I_t + I_{Cr} + c_{12}^2 I_{Rr} + c_{22}^2 I_{Si} \right) \dot{\omega}_t = T_t - c_{21} R_d T_s
\]
(3.140)

\[
RT_{2ND} = -T_{fsr} = -(c_{11} c_{12} I_{Rr} + c_{21} c_{22} I_{Si}) \dot{\omega}_t - c_{11} R_d T_s
\]
(3.141)

\[
RT_{OD} = T_t - I_t \dot{\omega}_t
\]
(3.142)

We also have the following speed relationships.

\[
\omega_{Si} = c_{22} \omega_{Cr} = 1.938 \omega_t
\]
(3.143)

\[
\omega_{Rr} = c_{12} \omega_{Cr} = 1.357 \omega_t
\]
(3.144)

\[
\omega_o = R_d \omega_{Rr} = 0.265 \omega_{Rr}
\]
(3.145)

### 3.3.3 Clutch Torque Calculation and Clutch Friction Model

All the clutches used in both transmissions of interest are rotating clutches. The clutch plates used in the clutches are wet type clutches which are partially or fully submerged under the transmission fluid. If the clutch hydraulic pressure is known, the following simplified formula is normally used to calculate the clutch torque (Deutschman, 1975; Runde, 1986)

\[
T_C = P_C \cdot \mu (\Delta \omega_C) \cdot A_C \cdot R_C \cdot \text{sgn}(\Delta \omega_C)
\]
(3.146)

Here, \( T_C \) represents the clutch torque, \( P_C \) is the clutch hydraulic pressure, \( A_C \) is the effective pressurized area of the clutch piston, \( R_C \) is the effective radius of the clutch.
plate, and $\Delta \omega_c$ is the slip speed. $\mu(\Delta \omega_c)$ is the friction coefficient which is a nonlinear function of the slip speed. “sgn” is the sign of the clutch slip speed. All clutch torques used in this research are calculated using the above formula. All parameters are given by the manufacturer, including the friction coefficient of the clutch plate.

In this research, we initially attempted to use the following relationship between the friction coefficient and the slip speed, which has been often suggested as an appropriate model.

$$\mu C = \begin{cases} C_o & \text{if } \Delta \omega = 0 \\ C_1 \pm C_2 \cdot e^{-C_3|\Delta \omega|} & \text{if } \Delta \omega \neq 0 \end{cases} \quad (3.147)$$

The value of $C_o$ is the static friction coefficient. The numerical values of the parameters $C_i$, $i = 0, \ldots, 3$, as well as the choice of plus and minus signs depend on the clutch physical characteristics and the operating condition, i.e., the temperature of the transmission fluid, and the age of the clutch plate. Manufacturer provided experimental data, in which the clutch friction coefficient is measured versus the slip speed, does not, however, fit the model. For the sake of simplicity, an alternate nonlinear relationship between the friction coefficient and the slip speed is obtained by curve-fitting. Specifically, the following formula is used.

$$\mu C = C_1 \Delta \omega^4 + C_2 \Delta \omega^3 + C_3 \Delta \omega^2 + C_4 \Delta \omega + C_5 \quad (3.148)$$

The set of the coefficient $C_i$, $i = 1, \ldots, 5$, is different for each clutch depending on the test data. The value of $C_5$ can be considered as the static friction coefficient. Figure 3.8 shows the friction coefficient calculated from the above formula as compared to the data given by the manufacturer, for one of the clutches. It can be seen that the proposed formula gives good results as compared to the experimental data.
For a more accurate prediction of the clutch friction coefficient, the temperature of the transmission fluid needs to be considered as well. It is well-known that the friction coefficient characteristic changes as the fluid temperature changes (Haviland et al, 1993). The formulas in (3.147) and (3.148) will not able to predict the friction coefficient accurately as the temperature changes, if the parameters are considered to be constants. The normal pressure may also affect the value of the friction coefficient, which is also clearly not represented in the formulas above. Finally, friction coefficient changes due to aging are not included in the formulas.

Finding a mathematical model to incorporate all these effects is a subject in its own research. In this research, we assume that the curve-fitted model gives an accurate value for a baseline temperature and clutch condition and note that the model changes with these conditions.

Figure 3.8: Clutch friction coefficient model and experimental data
3.4 5-Speed Transmission Mechanical Subsystem Model

In the 5-Speed transmission of interest, there are 7 clutches to control the operation of the transmission, namely the Low-Reverse clutch (LR), the Second clutch (2ND), the Under-Drive clutch (UD), the Over-Drive clutch (OD), the Reduction clutch (RED), the Direct clutch (DIR) and the Torque converter clutch (TCC). As for the case of the 4-Speed transmission, the function of both LR and 2ND clutches is to stop their corresponding gear components. Therefore, they are sometime called brakes. There are also two overrunning clutches in this transmission, namely the Low-Overrunning clutch (OWCL) and the High-Overrunning clutch (OWCH). A simplified stick diagram for a particular clutch of interest is shown in Figure 3.10.

Figure 3.9: Simplified schematic diagram of the 5-speed transmission
Table 3.3 shows the clutch engagement schedule and the overall transmission gear ratios for the different gears, including the final drive gear ratio of 2.84. The application schedule of the TCC is not shown here, since it does not involve changing the gear ratio of the transmission.

<table>
<thead>
<tr>
<th>Range</th>
<th>Clutch</th>
<th>Brake</th>
<th>Overrunning Clutch</th>
<th>Gear Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UD</td>
<td>OD</td>
<td>DIR</td>
<td>REV</td>
</tr>
<tr>
<td>1st</td>
<td>X</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2nd</td>
<td>X</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3rd</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4th</td>
<td>-</td>
<td>X</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5th</td>
<td>-</td>
<td>X</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Reverse</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 3.3: Clutch engagement schedule for the 5-speed transmission of interest

As can be seen from Figure 3.9, all mechanical components inside the transmission are arranged on 2 axles. Throughout the model derivation, the main axle connecting with the engine (through the torque converter) will be called a “primary axle”. On this axle, there is a compound gear set which consists of two planetary gear sets physically connected to each other as mentioned earlier. This compound gear set will be called primary gear set, or primary set. The output of the primary axle is passed through the secondary axle, which is connected to the final drive. There is one additional planetary gear set on the secondary axle, which will be called “secondary gear set” or “secondary set”. Notice that the configuration of the primary gear set is the same as the planetary gear set in 4-Speed transmission. In the latter case, the output of the primary
The secondary gear set is directly connected to the final drive. The secondary gear set on the secondary axle reduces the gear ratio from the primary gear set and helps produce the 5th gear ratio in the 5-speed transmission.

Since the difference between the 4-Speed and 5-Speed transmissions is only in the inclusion of the additional secondary axle in the 5-Speed transmission, it will be shown that, if we take into consideration the behavior of the secondary gear set, the already developed dynamic equations describing the power-on up shifts from the first gear to fourth gear sequentially in the 4-speed transmission can be used to describe the power-on up shifts from the first gear to fourth gear in the 5-speed transmission with some minor modifications. The dynamic behavior of the secondary gear set affects the 4-5 power-on up shift. Therefore, a more detailed derivation will be shown for this shift.

The free-body diagrams of both the primary and secondary gear sets are shown in Figure 3.10 below. The descriptions for most of variables and parameters used in the 5-speed model derivation are similar to the case of the 4-speed transmission except the following. Comparing Figure 3.10 to Figure 3.3, we see that the output torque, \( T_o \), in Figure 3.3 becomes \( T_{ci} \) in Figure 3.10. Also, the load torque of the primary gear set in Figure 3.10 is the reaction torque from the secondary gear set, \( T'_{po} \), not the load from the final drive as shown in Figure 3.3, \( R_dT_r \). At the secondary gear set, \( \omega \) represents the angular velocity, with its first subscript, \( S, R, \) or \( C \) denoting sun-gear, ring-gear, and carrier, and its second subscript \( s \) representing the secondary gear set. Similarly, \( T_{Rs} \) and \( T_{Cs} \) are the static torques at the ring-gear and the carrier, respectively. \( T_o \) is the transmission output torque. The operation of the secondary gear set is controlled by the
application of DIR clutch, the RED brake, and the OWCH one-way clutch. \( T_{DIR} \) represents the torque capacity of the DIR clutch, and \( RT_{DIR} \) represents the reaction torque at the DIR clutch. \( T_{RED} \) represents the torque capacity of the RED brake, and \( RT_{RED} \) is the reaction torque calculated at the RED brake. The nomenclature is similar for the OWCH, viz, \( T_{OWCH} \) and \( RT_{OWCH} \) represent the torque capacity and the reaction torque respectively.

Figure 3.10: Free-body diagram of 5-speed transmission
3.4.1 Dynamic Behavior of the Secondary Gear Set in 1st to 4th Gears

From the clutch applications schedule in Table 3.3, it can be seen that the clutch application configuration on the secondary gear set does not change between the first gear to the fourth gear. Therefore, the dynamic response of the transmission during first gear to fourth gear operation is affected mostly by the behavior of the primary gear set. However, we do need to note down the equations describing the dynamic behavior of the secondary gear set.

From the free-body diagram in Figure 3.10, we can write the following differential equations describing the dynamics of each gear element on the secondary gear set.

\[
T_{po} - T_{Rs} = I_{Rs} \dot{\omega}_{Rs} \tag{3.149}
\]

\[
T_o - R_d T_s - T_{DIR} = I_{Cs} \dot{\omega}_{Cs} \tag{3.150}
\]

\[-T_{DIR} - T_{RED} - T_{Ss} = I_{Ss} \dot{\omega}_{Ss} \tag{3.151}\]

Based on the clutch application shown in Table 3.3, the DIR clutch is not applied which means that \( T_{DIR} = 0 \). The RED brake, on the other hand, is fully applied and in lock-up condition, which means that the clutch torque capacity is higher than the reaction torque calculated at the RED clutch. As for the case of the 4-speed transmission, we use a prefix “R” added in front of the symbol for the torque to represent the corresponding reaction torque. Therefore, in the secondary gear set between the first to the fourth gear, we have \( T_{RED} > R T_{RED} \), and the secondary sun-gear is held fixed, i.e. \( \omega_{Ss} = 0 \). Therefore, equations (3.150) and (3.151) become,

\[
T_o - R_d T_s = I_{Cs} \dot{\omega}_{Cs} \tag{3.152}
\]
From the kinematic relationship for a single planetary gear set, we have the following.

\[ \omega_{c_s} = R_{r_s} \omega_{r_s} + R_{s_s} \omega_{s_s} \implies \omega_{c_s} = R_{r_s} \omega_{r_s} \]  

(3.154)

\[ R_{r_s} = \frac{N_{s_s}}{N_{s_s} + N_{r_s}} \]

\[ R_{s_s} = \frac{N_{r_s}}{N_{s_s} + N_{r_s}} \]  

(3.155)

Combining equations (3.149) and (3.152) with the relationships in equations (3.154) and (3.155), we have the following differential equation describing the dynamics of the secondary gear set between the first gear and the fourth gear.

\[ T_{p_o} - R_{r_s} R_d T_s = \left( I_{r_s} + R_{r_s}^2 I_{c_s} \right) \dot{\omega}_{r_s} \]  

(3.156)

And from Figure 3.10, we have the following torque and the speed relationships between the output of the primary gear set and the input element of the secondary gear set.

\[ T_{p_o} = R_{p_o} T_{p_o} \]

\[ \omega_{r_s} = R_{p_o} \omega_{r_r} \]

(3.157)

\[ R_{p_o} = \frac{N'_{p_o}}{N_{p_o}} \]

Therefore, the load torque to the primary gear set, \( T_{p_o}' \), can be calculated from the equation (3.156). Specifically, we have,

\[ T_{p_o}' = R_{p_o} \left[ \left( I_{r_s} + R_{r_s}^2 I_{c_s} \right) \dot{\omega}_{r_s} + R_{r_s} R_d T_s \right] \]  

(3.158)

The reaction torque at the RED brake can also be calculated as follows. From the kinematic relationship in equation (3.155), we have,
Substituting $T_o$ from equation (3.152) into equation (3.155), we get,

$$R_{Rs} (I_{cs} \dot{\omega}_{cs} + R_s T_S) = T_{Rs}$$  \hspace{1cm} (3.160)

Combining equations (3.153), (3.159), and (3.160), the reaction torque at the RED brake can be calculated from,

$$RT_{RED} = -R_{Rs} (I_{cs} \dot{\omega}_{cs} + R_s T_S)$$  \hspace{1cm} (3.161)

We are now ready to develop the model describing the dynamics of the 5-speed transmission during the first to fourth gear by adding the equations shown in this section to those already developed for the 4-Speed transmission shown previously. To be specific, the load torque calculated by using equation (3.158) is used to replace the original load torque from the final drive in the 4-speed case. The output speed of the primary gear set is algebraically related to the output speed of the secondary gear set by the following relationship.

$$\begin{align*}
\omega_{Rs} &= R_{po} \omega_{Rr} \\
\omega_{cs} &= R_{Rs} \omega_{Rs}
\end{align*} \Rightarrow \quad \omega_{cs} = R_{Rs} R_{po} \omega_{Rr}$$  \hspace{1cm} (3.162)

3.4.1.1 First Gear and 1-2 Gear Shift Dynamics

(1) First Gear Dynamics

During the first gear, at the primary gear set, the UD clutch is fully engaged and in the lock-up condition, i.e. $RT_{UD} < T_{UD}$. As a result, the speed of the input sun-gear is equal to the turbine speed. The LR clutch/brake is also in the lock-up condition. In this
case, the speed of the reaction carrier is zero. By using the notation shown in the free-
body diagram in Figure 3.10 and applying to the dynamic equations describing the first
gear in section 3.3, we can write the dynamic equation of the primary gear set during the
first gear for the 5-speed transmission as follows.

\[
(I_i + I_t + a_{11}^2 I_{Sr} + a_{21}^2 I_{Rr}) \dot{\omega}_S = T_i - a_{21} T'_{po} \tag{3.163}
\]

\[
RT_{LR} = -(a_{11}a_{12} I_{Sr} + a_{21}a_{22} I_{Rr}) \dot{\omega}_S - a_{22} T'_{po} \tag{3.164}
\]

\[
RT_{UD} = T_i - I_t \dot{\omega}_i \tag{3.165}
\]

\[
\omega_{Rr} = a_{21} \omega_S \quad \omega_S = \omega_i \tag{3.166}
\]

Substituting \( T'_{po} \) from equation (3.158) in the equations above and using equations
(3.162) and (3.166), we can write the overall dynamic equations of the transmission for
the first gear as below.

\[
\left[ I_i + I_t + a_{11}^2 I_{Sr} + a_{21}^2 I_{Rr} + (a_{21} R_{po}^2) (I_{Rs} + R^2 I_{Cs}) \right] \dot{\omega}_S = T_i - (a_{21} R_{po} R_{Rs}) R_d T_S \tag{3.167}
\]

\[
RT_{LR} = - \left[ a_{11}a_{12} I_{Sr} + a_{21}a_{22} I_{Rr} + a_{21}a_{22} R_{po}^2 (I_{Rs} + R^2 I_{Cs}) \right] \dot{\omega}_S - (a_{22} R_{po} R_{Rs}) R_d T_S \tag{3.168}
\]

\[
RT_{RED} = -a_{21} R_{po} R_{Rs} R_{po} R_{Cs} \dot{\omega}_{Cs} - R_{Rs} R_d T_S \tag{3.169}
\]

\[
\omega_o = a_{21} R_{po} R_{Rs} R_d \omega_S \tag{3.170}
\]

(2) 1-2 Up Shift - Torque Phase

For the 1-2 up shift, the off-going clutch is the LR brake and the on-coming clutch
is the 2ND brake. When the on-coming clutch is applied, the governing equations can be
derived as in the first gear, except that the torque from the 2ND brake is not zero. In this
case, the dynamics of the primary gear set can be described as shown below.
Following the same derivation as in the case of the first gear dynamics, the overall
dynamics combining both the primary and secondary gear sets can be easily written as
follows.

\[
(I_{Si} + I_t + a_{11}^2 I_{Sr} + a_{21}^2 I_{Rr}) \dot{\omega}_{Si} = T_t - a_{21} T'_{po} - a_{11} T_{2ND}
\]  (3.171)

\[
RT_{LR} = -(a_{11} a_{12} I_{Sr} + a_{21} a_{22} I_{Rr}) \dot{\omega}_{Si} - a_{22} T'_{po} - a_{12} T_{2ND}
\]  (3.172)

The reaction torque at the RED brake and the algebraic relationships of the speeds of the
primary and the secondary gear set elements are as described in the first gear derivation.

The torque phase ends when the torque capacity of the LR brake is lower than the
reaction torque calculated from equation (3.174), i.e. \(|RT_{LR}| > |T_{LR}|\). When this occurs, the
LR brake starts to slip and the speed of the reaction carrier is no longer zero, \(\omega_{Cr} \neq 0\).

(3) 1-2 Up Shift - Inertia Phase

As the LR brake starts to slip, the reaction torque at the LR brake is now
determined by the torque capacity of the LR brake itself, i.e. \(RT_{LR} = T_{LR}\). The torque
capacity of the on-coming clutch, the 2ND brake, increases initially but is not high
enough to hold the reaction sun-gear fixed. So, the reaction torque at the 2ND brake is
determined by the torque capacity of the 2ND brake, i.e. \(RT_{ND} = T_{ND}\). Taking the results
from the 4-speed model derivation and using the notation for the 5-speed free-body diagram shown in Figure 3.10, the dynamics of the primary gear set are described by,

\[
I_{12} \begin{pmatrix}
\dot{\omega}_r \\
\dot{\omega}_s
\end{pmatrix} = \begin{pmatrix}
-f_{11} T_{ND} - f_{21} T_{LR} - T'_{po} \\
-f_{12} T_{ND} - f_{22} T_{LR} + T_i
\end{pmatrix} \tag{3.175}
\]

Substituting \( T'_{po} \) from equation (3.158), we get,

\[
I_{12} \begin{pmatrix}
\dot{\omega}_r \\
\dot{\omega}_s
\end{pmatrix} = \begin{pmatrix}
-f_{11} T_{ND} - f_{21} T_{LR} - R_{po} \left[ \left( I_{Rs} + R^2_{Rs} I_{Cs} \right) \dot{\omega}_r + R_{Rs} R_d T_S \right] \\
-f_{12} T_{ND} - f_{22} T_{LR} + T_i
\end{pmatrix} \tag{3.176}
\]

Using the known relationship between \( \omega_{Rs} \) and \( \omega_{Rr} \), and rearranging the above equation, we get,

\[
\begin{pmatrix}
I_{12} + \left[ R^2_{po} \left( I_{Rs} + R^2_{Rs} I_{Cs} \right) 0 \\
0 0
\end{pmatrix} \left( \begin{array}{c}
\dot{\omega}_r \\
\dot{\omega}_s
\end{array} \right) = \begin{pmatrix}
-f_{11} T_{ND} - f_{21} T_{LR} - R_{po} R_{Rs} R_d T_S \\
-f_{12} T_{ND} - f_{22} T_{LR} + T_i
\end{pmatrix} \tag{3.177}
\]

Letting,

\[
\begin{pmatrix}
F_{11} & F_{12} \\
F_{21} & F_{22}
\end{pmatrix} = \left( I_{12} + \left[ R^2_{po} \left( I_{Rs} + R^2_{Rs} I_{Cs} \right) 0 \\
0 0
\right] \right)^{-1} \begin{pmatrix}
-f_{11} & -f_{21} \\
-f_{12} & -f_{22}
\end{pmatrix};
\]

\[
\begin{pmatrix}
C_{1t1} \\
C_{1t2}
\end{pmatrix} = \left( I_{12} + \left[ R^2_{po} \left( I_{Rs} + R^2_{Rs} I_{Cs} \right) 0 \\
0 0
\right] \right)^{-1} \begin{pmatrix}
-R_{po} R_{Rs} R_d \\
0
\end{pmatrix};
\]

\[
\begin{pmatrix}
C_{1t1} \\
C_{1t2}
\end{pmatrix} = \left( I_{12} + \left[ R^2_{po} \left( I_{Rs} + R^2_{Rs} I_{Cs} \right) 0 \\
0 0
\right] \right)^{-1} \begin{pmatrix}
0 \\
1
\end{pmatrix}
\]\n
Then, equation (3.175) can be written as,

\[
\begin{pmatrix}
\dot{\omega}_r \\
\dot{\omega}_s
\end{pmatrix} = \begin{pmatrix}
F_{11} T_{ND} + F_{12} T_{LR} + C_{1t1} T_i + C_{1t1} T_s \\
F_{12} T_{ND} + F_{12} T_{LR} + C_{1t2} T_i + C_{1t2} T_s
\end{pmatrix} \tag{3.179}
\]

The reaction torque at the RED brake is calculated from,

\[
RT_{RED} = -R_{Rs} R_{Rs} R_{po} I_{Cs} \dot{\omega}_{Cs} - R_{Rs} R_d T_S \tag{3.180}
\]
The transmission output speed, $\omega_o$, can be calculated from,

$$\begin{align*}
\omega_{Rs} &= R_{po} \omega_{Rr} \\
\omega_{Cs} &= R_{Rs} \omega_{Rs} \\
\omega_o &= R_d \omega_{Cs}
\end{align*}$$

$$\Rightarrow \quad \omega_o = R_{po} R_{Rs} R_d \omega_{Rr} \quad (3.181)$$

The inertia phase ends when the torque capacity of the 2ND brake is high enough to stop the reaction sun-gear. And from this point on, the torque capacity of the 2ND brake will be higher than the reaction torque. Thus, we note that the inertia phase ends when,

$$|T_{2ND}| \geq |RT_{2ND}| \quad (3.182)$$

$$\omega_{Sr} = 0 \quad (3.183)$$

3.4.1.2 Second Gear and 2-3 Gear Up Shift Dynamics

(1) Second Gear Dynamics

The free body diagram of the primary gear set for the transmission in the second gear is the same as the free-body diagram of the second gear for the 4-speed transmission shown in Figure 3.3. In this gear, the UD clutch is still fully engaged which means $RT_{UD} < T_{UD}$. Therefore, as in the 1st gear, the speed of the input sun-gear is equal to the turbine speed. The 2ND brake is also fully engaged and, therefore, the reaction sun-gear is not moving. Applying the dynamic equations from the 4-speed derivation, the dynamic equations describing the 5-speed transmission during the second gear are,

$$(I_{Si} + I_t + d_{11}^2 I_{Rr} + d_{21}^2 I_{Cr}) \dot{\omega}_{Si} = T_t - d_{11}^T_{po} \quad (3.184)$$

$$RT_{2ND} = -(d_{12} d_{12} I_{Rr} + d_{22} d_{22} I_{Cr}) \dot{\omega}_{Sr} - d_{12}^T_{po} \quad (3.185)$$
\[ RT_{UD} = T_r - I_r \dot{\omega}_r \]  
\[ \omega_{Rr} = d_{i1} \omega_{Si} \quad , \quad \omega_{Si} = \omega_r \]  

We again substitute \( T_{po}' \) from equation (3.158) into equations (3.184) and (3.185), and with the use of equations (3.162) and (3.187) we get,

\[ \left[ I_{Si} + I_i + d_{i1}^2 I_{Rr} + d_{21}^2 I_{Cr} + (d_{i1} R_{po})^2 (I_{Rs} + R_{Rs}^2 I_{Cs}) \right] \dot{\omega}_{Si} = T_r - \left( d_{i1} R_{po} R_{Rs} \right) R_d T_s \]  
\[ RT_{2ND} = -\left( d_{i1} d_{22} I_{Cr} + d_{11} d_{i2} \left( I_{Rr} + R_{po}^2 \left( I_{Rs} + R_{Rs}^2 I_{Cs} \right) \right) \right) \dot{\omega}_{Si} - \left( d_{i2} R_{po} R_{Rs} \right) R_d T_s \]  

In this case, the reaction torque at the RED brake and the transmission output speed are calculated from the following.

\[ RT_{RED} = -d_{i1} R_{Rs} R_{Rs} R_{po}^2 I_{Cs} \dot{\omega}_{Cs} - R_{Rs} R_d T_s \]  
\[ \omega_o = d_{i1} R_{po} R_{Rs} R_d \omega_{Si} \]  

(2) 2-3 Up Shift - Torque Phase

The 2-3 up shift torque phase starts when the OD clutch, which is the on-coming clutch, is applied. The off-going clutch, which is the 2ND brake in this case, may be released. However, during the torque phase, the clutch capacity of the 2ND brake is still high enough to hold the reaction sun-gear of the primary gear set. As the OD clutch picks up some capacity, the dynamics of the primary gear set are described by,

\[ (I_{Si} + I_i + d_{i1}^2 I_{Rr} + d_{21}^2 I_{Cr}) \dot{\omega}_{Si} = T_i - d_{i1} T_{po}' - (d_{21} - 1) T_{OD} \]  
\[ RT_{2ND} = (d_{i1} d_{i2} I_{Rr} + d_{i2} d_{22} I_{Cr}) \dot{\omega}_{Si} + d_{i2} T_{po}' - d_{22} T_{OD} \]  

Following the same derivation of the second gear dynamics as before, the overall
dynamic equations describing the interaction between the primary and secondary gear sets during 2-3 up shift torque phase can be written as,

\[
\left[ I_{S_i} + I_t + d_{21}^2 I_{C_r} + d_{11}^2 (I_{R_r} + R_{p_0}^2 (I_{R_s} + R_{R_s}^2 I_{C_s})) \right] \dot{\omega}_{S_i} = T_t - \left( d_{11} R_{p_0} R_{R_s} \right) R_{d_s} T_S - (d_{21} - 1) T_{OD}
\]  
(3.194)

\[
R_{T_{2Nd}} = -(d_{21} d_{22} I_{C_r} + d_{11} d_{12} (I_{R_r} + R_{p_0}^2 (I_{R_s} + R_{R_s}^2 I_{C_s}))) \dot{\omega}_{S_i} - \left( d_{12} R_{p_0} R_{R_s} \right) R_{d_s} T_S - d_{22} T_{OD}
\]  
(3.195)

The reaction torque at the RED brake is the same as described in equation (3.190). The torque phase for this gear shift ends when the torque capacity of the 2ND brake is lower than the reaction torque calculated from the equation above. In this case, the 2ND brake will start to slip, which means that the reaction sun-gear is moving. In other words, the torque phase ends when the following conditions hold.

\[
|R_{T_{2Nd}}| > |T_{2Nd}|
\]  
(3.196)

\[
\omega_{S_i} \neq 0
\]  
(3.197)

(3) 2-3 Up Shift - Inertia Phase

Once the 2ND brake slips, the reaction torque is no longer determined by equation (3.195), but by the 2ND torque capacity itself, i.e. \( R_{T_{2Nd}} = T_{2Nd} \). The torque capacity of the on-coming clutch, OD clutch, continues to increase. The slip speed at the OD clutch is reduced depending on the torque capacity. In this case, the reaction torque at the OD clutch is also determined by its torque capacity, or \( R_{T_{OD}} = T_{OD} \). From the dynamic equations of the 4-speed transmission(Will you still have an Appendix A?), the dynamics of the primary gear set during the 2\textsuperscript{nd}-3\textsuperscript{rd} up shift inertia phase are,
\[
I_{23} \begin{pmatrix}
\dot{\omega}_R \\
\dot{\omega}_S
\end{pmatrix}
= \begin{pmatrix}
-f_{11}T_{ND} + f_{21}T_{OD} - T'_{po} \\
-f_{12}T_{ND} + (f_{22} - 1)T_{OD} + T_t
\end{pmatrix}
\] (3.198)

Substituting \( T'_{po} \) from equation (3.158), and \( \omega_{Rs} \) from equation (3.157), we have,

\[
\left( I_{23} + \begin{bmatrix}
R_{R_{po}}^2 (I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}\right)
\begin{pmatrix}
\dot{\omega}_R \\
\dot{\omega}_S
\end{pmatrix}
= \begin{pmatrix}
-f_{11}T_{ND} + f_{21}T_{OD} - R_{po}R_{Rs}R_dT_s \\
-f_{12}T_{ND} + (f_{22} - 1)T_{OD} + T_t
\end{pmatrix}
\] (3.199)

To simplify the above format, we let,

\[
\begin{bmatrix}
F_{211} & F_{212} \\
F_{212} & F_{222}
\end{bmatrix} = \left( I_{23} + \begin{bmatrix}
R_{R_{po}}^2 (I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}\right)^{-1} \begin{bmatrix}
f_{11} & f_{21} \\
-f_{12} (f_{22} - 1)
\end{bmatrix};
\]

\[
\begin{bmatrix}
C_{T_{r1}}^2 \\
C_{T_{r2}}^2
\end{bmatrix} = \left( I_{23} + \begin{bmatrix}
R_{R_{po}}^2 (I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}\right)^{-1} \begin{bmatrix}
-R_{po}R_{Rs}R_d \\
0
\end{bmatrix};
\]

\[
\begin{bmatrix}
C_{T_{r1}}^2 \\
C_{T_{r2}}^2
\end{bmatrix} = \left( I_{23} + \begin{bmatrix}
R_{R_{po}}^2 (I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}\right)^{-1} \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

Then, equation (3.199) becomes,

\[
\begin{pmatrix}
\dot{\omega}_R \\
\dot{\omega}_t
\end{pmatrix}
= \begin{pmatrix}
F_{211}T_{ND} + F_{212}T_{LR} + C_{T_{r1}}^2 T_t + C_{T_{r1}}^2 T_s \\
F_{212}T_{ND} + F_{222}T_{LR} + C_{T_{r2}}^2 T_t + C_{T_{r2}}^2 T_s
\end{pmatrix}
\] (3.201)

The reaction torque at the RED brake is calculated from,

\[
RT_{RED} = -R_{Rs}R_{Rs}R_{po}I_{Cs} \dot{\omega}_R - R_{Rs}R_dT_s
\] (3.202)

The transmission output speed, \( \omega_o \), can be calculated from,

\[
\begin{align*}
\omega_{Rs} &= R_{po} \dot{\omega}_R \\
\omega_{Cs} &= R_{Rs} \dot{\omega}_{Rs} \\
\omega_{o} &= R_d \dot{\omega}_{Cs}
\end{align*}
\]

\[
\Rightarrow \quad \omega_o = R_{po}R_{Rs}R_d \dot{\omega}_{Rs}
\] (3.203)

The inertia phase ends when the torque capacity of the OD clutch is high enough to transmit the turbine torque through the reaction carrier. The slip-speed at the OD
clutch is reduced to zero, meaning that the speed of the reaction carrier is equal to the
turbine speed, and the input sun-gear. Therefore, all members of the primary gear set will
move at the same speed. After this point, the torque capacity of the OD clutch will be
higher than the reaction torque calculated at the OD clutch. Therefore, the following
conditions determine the end of the inertia phase.

\[ T_{OD} \geq R_{OD} \] \hspace{1cm} (3.204)

\[ \omega_{Cr} = \omega_t \] \hspace{1cm} (3.205)

3.4.1.3 Third Gear and 3-4 Gear Up Shift Dynamics

(1) Third Gear Dynamics

In the primary gear set, and in the third gear, the UD clutch is still fully engaged.
Hence, \( R_{UD} < T_{UD} \). Therefore, as in the first and the second gear, the turbine speed is
equal to the input sun-gear. The OD clutch is also fully engaged and thus \( T_{OD} > R_{OD} \). In
this case, the speed of the reaction carrier is also equal to the turbine speed. The free-
body diagram of the primary gear set for the third gear is the same as the free-body
diagram of the third gear for the 4-speed model shown in Figure 3.6. The dynamic
equations describing the behavior of the primary gear set during the third gear are,

\[ (I_t + I_{Si} + I_{Sr} + I_{Cr})\dot{\omega}_{Si} = T_t - T'_{po} \] \hspace{1cm} (3.206)

\[ R_{T UD} = (I_{Si} + a_{11}I_{Sr} + a_{21}I_{Cr})\dot{\omega}_{Si} + a_{21}T'_{po} \] \hspace{1cm} (3.207)

\[ R_{T OD} = (I_{Cr} + a_{12}I_{Sr} + a_{22}I_{Rp})\dot{\omega}_{Si} + a_{22}T'_{po} \] \hspace{1cm} (3.208)

\[ \omega_{Cr} = \omega_{Si} = \omega_t = \omega_{Cr} \] \hspace{1cm} (3.209)
Substituting for $T'_{po}$, equations (3.206) to (3.208) become,

\[
\left( I_t + I_{si} + I_{sr} + I_{cr} + R_{po}^2 \left( I_{rs} + R_{rs}^2 I_{cs} \right) \right) \dot{\omega}_{si} = T_t - R_{po} R_{rs} R_d T_s \quad (3.210)
\]

\[
RT_{UD} = \left( I_{si} + a_{11} I_{sr} + a_{21} \left( I_{rs} + R_{po}^2 \left( I_{rs} + R_{rs}^2 I_{cs} \right) \right) \right) \dot{\omega}_{si} + a_{21} R_{po} R_{rs} R_d T_s \quad (3.211)
\]

\[
RT_{OD} = \left( I_{cr} + a_{12} I_{sr} + a_{22} \left( I_{rs} + R_{po}^2 \left( I_{rs} + R_{rs}^2 I_{cs} \right) \right) \right) \dot{\omega}_{si} + a_{22} R_{po} R_{rs} R_d T_s \quad (3.212)
\]

The reaction torque at the RED brake can be calculated from,

\[
RT_{RED} = -R_{sS} R_{po} R_{rs} R_{cs} \dot{\omega}_{rs} - R_{sS} R_d T_s \quad (3.213)
\]

And the transmission output speed is calculated from,

\[
\omega_o = R_{po} R_{rs} R_d \dot{\omega}_{si} \quad (3.214)
\]

(2) 3-4 Up Shift - Torque Phase

The 3-4 up shift involves releasing the UD clutch and applying the 2ND brake while keeping the OD clutch fully engaged. Even though the UD clutch is being released, its torque capacity is higher than the reaction torque calculated from equation (3.211). Therefore, the clutch remains in the lock-up condition. As the 2ND brake is applied, the governing equations for the primary gear set are the following.

\[
\left( I_t + I_{si} + I_{sr} + I_{cr} \right) \dot{\omega}_{si} = T_t - T'_{po} - T_{ND} \quad (3.215)
\]

\[
RT_{UD} = \left( I_{si} + a_{11} I_{sr} + a_{21} I_{rs} \right) \dot{\omega}_{si} + a_{21} T'_{po} + a_{11} T_{ND} \quad (3.216)
\]

\[
RT_{OD} = \left( I_{cr} + a_{12} I_{sr} + a_{22} I_{rs} \right) \dot{\omega}_{si} + a_{22} T'_{po} + a_{12} T_{ND} \quad (3.217)
\]

Substituting the expression for $T'_{po}$ as described earlier, the following are the overall governing equations of the transmission during the 3-4 up shift torque phase.
\[
\left(I_t + I_{Si} + I_{Sr} + I_{Cr} + R^2_{po} \left(I_{Rs} + R^2_{Rs} I_{Cs}\right) \right) \dot{\omega}_{Si} = T_t - \left(R_{po} R_{Rs}\right) R_d T_S - T_{ND} \tag{3.218}
\]

\[
RT_{UD} = \left(I_{Si} + a_{11} I_{Sr} + a_{21} \left(I_{Rs} + R^2_{po} \left(I_{Rs} + R^2_{Rs} I_{Cs}\right)\right)\right) \dot{\omega}_{Si} + \left(a_{21} R_{po} R_{Rs}\right) R_d T_S + a_{11} T_{ND} \tag{3.219}
\]

\[
RT_{OD} = \left(I_{Cr} + a_{12} I_{Sr} + a_{22} \left(I_{Rs} + R^2_{po} \left(I_{Rs} + R^2_{Rs} I_{Cs}\right)\right)\right) \dot{\omega}_{Si} + \left(a_{22} R_{po} R_{Rs}\right) R_d T_S + a_{12} T_{ND} \tag{3.220}
\]

This torque phase ends when the torque capacity of the UD clutch is lower than the reaction torque given by equation (3.219) above. The slip speed at the UD clutch is no longer zero, which means that the speed of the input sun-gear is no longer equal to the turbine speed. In other words, the condition determining the end of the torque phase for this gear shift are,

\[
RT_{UD} > T_{UD} \tag{3.221}
\]

\[
\omega_{Si} \neq \omega_t \tag{3.222}
\]

(3) **3-4 Up Shift - Inertia Phase**

As the UD clutch starts to slip, the reaction torque at the UD clutch is equal to its torque capacity, or \(RT_{UD} = T_{UD}.\) At this time, the 2ND brake is in the same condition as the UD clutch, except that it’s being applied. The reaction torque at the 2ND brake is also determined from its torque capacity or \(RT_{2ND} = T_{2ND}.\) The governing equations describing the behavior of the primary gear set during this phase are,

\[
I_{34} \left(\begin{array}{c}
\dot{\omega}_{Br} \\
\dot{\omega}_{Cr}
\end{array}\right) = \begin{pmatrix}
b_{11} T_{UD} - b_{21} T_{ND} - T'_{po} \\
(b_{21} - 1) T_{UD} - b_{22} T_{ND} + T_t
\end{pmatrix} \tag{3.223}
\]

Following the same derivation used for the previous gear, we substitute the expression for
from equation (3.158) and use the algebraic relationship between $\omega_{Rs}$ and $\omega_{Rr}$ from equation (3.157), and get,

$$
\begin{align*}
\begin{bmatrix}
I_{34} + R_{po}^2(I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_{Rr} \\
\dot{\omega}_{Cr}
\end{bmatrix}
= \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
b_{11} T_{UD} - b_{21} T_{ND} - R_{po} R_{Rs} R_d T_s \\
(b_{21} - 1) T_{UD} - b_{22} T_{ND} + T_t
\end{bmatrix}
\end{align*}
$$

(3.224)

Or

$$
\begin{align*}
\begin{bmatrix}
\dot{\omega}_{Rr} \\
\dot{\omega}_{Cr}
\end{bmatrix}
= \begin{bmatrix}
B_{311} T_{UD} + B_{321} T_{ND} + C_{3r1} T_t + C_{3r1} T_s \\
B_{312} T_{UD} + B_{322} T_{ND} + C_{3r2} T_t + C_{3r2} T_s
\end{bmatrix}
\end{align*}
$$

(3.225)

where,

$$
\begin{align*}
\begin{bmatrix}
B_{311} & B_{321} \\
B_{312} & B_{322}
\end{bmatrix}
&= \begin{bmatrix}
I_{34} + R_{po}^2(I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
b_{11} & -b_{21} \\
(b_{21} - 1) & -b_{22}
\end{bmatrix}, \\
\begin{bmatrix}
C_{3r1} \\
C_{3r2}
\end{bmatrix}
&= \begin{bmatrix}
I_{34} + R_{po}^2(I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
-R_{po} R_{Rs} R_d \\
0
\end{bmatrix}, \\
\begin{bmatrix}
C_{3r1} \\
C_{3r2}
\end{bmatrix}
&= \begin{bmatrix}
I_{34} + R_{po}^2(I_{Rs} + R_{Rs}^2 I_{Cs}) & 0 \\
0 & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\end{align*}
$$

(3.226)

The reaction torque at the RED brake and the transmission output speed are calculated from the following equations.

$$
R T_{RED} = -R_{Rs} R_{Rs} R_{po} I_{Cs} \dot{\omega}_{Rr} - R_{Rs} R_d T_S
$$

(3.227)

$$
\begin{align*}
\omega_{Rs} &= R_{po} \omega_{Rr} \\
\omega_{Cs} &= R_{Rs} \omega_{Rs} \\
\omega_o &= R_d \omega_{Cs}
\end{align*}
\Rightarrow
\begin{align*}
\omega_o &= R_{po} R_{Rs} R_d \omega_{Rr}
\end{align*}
$$

(3.228)

The inertia phase for this gearshift ends when the torque capacity of the 2ND brake is high enough to hold the reaction sun-gear at the primary gear set stationary. And from this point on, the reaction torque at the 2ND brake is lower than the torque capacity.
Therefore, the conditions we use to determine the end of the inertia phase for the 3-4 up shift are,

\[ |T_{2\text{ND}}| \geq |RT_{2\text{ND}}| \quad (3.229) \]

\[ \omega_{Sr} = 0 \quad (3.230) \]

3.4.1.4 Fourth Gear, 4-5 Up Shift and Fifth Gear Dynamics

(1) Fourth Gear Dynamics

During the fourth gear, the OD clutch is fully engaged. The input torque from the turbine is passed through the reaction carrier at the primary gear set and the speed of the reaction carrier is equal to the turbine speed, i.e. \( T_{OD} \geq RT_{OD} \) and \( \omega_{Cr} = \omega_t \). The 2ND brake is also fully engaged and holds the reaction sun-gear at the primary gear set fixed. The free-body diagram of the primary gear set of the 5-speed transmission is the same as the one used to describe the fourth gear dynamics for the 4-speed transmission shown in Figure 3.7. The following are the governing equations for the primary gear set of the 5-speed transmission for the fourth gear, as adapted from the 4-speed case.

\[
\left( I_t + I_{Cr} + c_{12}^2 I_{Rr} + c_{22}^2 I_{Si} \right) \dot{\omega}_t = T_i - c_{12} T'_{po} \quad (3.231)
\]

\[
RT_{ND} = -(c_{11} c_{12} I_{Rr} + c_{21} c_{22} I_{Si}) \dot{\omega}_t - c_{11} T'_{po} \quad (3.232)
\]

\[
RT_{OD} = T_i - I_t \dot{\omega}_t \quad (3.233)
\]

\[
\omega_{Rr} = c_{12} \omega_t \quad (3.234)
\]

Substituting \( T'_{po} \) from equation (3.158), and using equations (3.157) and (3.234) for the corresponding speed relationship, we have,
\[
\left( I_t + I_{Cr} + c_2^2 I_{Si} + c_{12}^2 \left( I_{Rr} + R_{po}^2 \left( I_{Rs} + R_{Rs}^2 I_{Cs} \right) \right) \right) \dot{\omega}_t = T_i - c_{12} R_{po} R_{Rs} R_d T_S \tag{3.235}
\]

\[
RT_{ND} = -\left( c_{21} c_{22} I_{Si} + c_{11} c_{12} \left( I_{Rr} + R_{po}^2 \left( I_{Rs} + R_{Rs}^2 I_{Cs} \right) \right) \right) \dot{\omega}_t - c_{11} R_{po} R_{Rs} R_d T_S \tag{3.236}
\]

\[
RT_{RED} = -c_{12} R_{Rs} R_{po} I_{Cs} \dot{\omega}_t - R_{Rs} R_d T_S \tag{3.237}
\]

\[
\omega_o = c_{12} R_{po} R_{Rs} R_d \omega_t \tag{3.238}
\]

(2) **4-5 Up Shift - Torque Phase**

During the 4-5 gear shift, the dynamics of the primary gear set remain the same as in the fourth gear. The ratio change occurs due to the change of the clutch application in the secondary gear set. Specifically, the 4-5 gear shift torque phase starts with the application of the DIR clutch as the on-coming clutch and the release of the RED brake as the off-going clutch. There is also one overrunning clutch involved in the 4-5 gear shift, namely, the OWCH. Based on the clutch application from the Table 3.3, both the RED brake and OWCH clutches are fully released during the 4-5 gear shift. Therefore, two cases may occur. The first one involves rapidly release of the RED clutch before the OWCH releases and allows the secondary sun-gear to move. In other words, the RED brake has no effect on the gearshift, and the transition between the torque phase and the inertia phase is determined by the OWCH. The second case refers to the case where the OWCH releases the secondary sun-gear, but the RED brake still carries some capacity. In this case, the DIR clutch and the RED brake will fight and increase the load torque on the engine. In any case, the torque phase duration is determined by the OWCH, meaning that the torque phase ends when the DIR takes over all the load from the OWCH or the load torque at the OWCH becomes zero.
Based on the free-body diagram, we can write the following.

\[ T_{po} - T_{Rs} = I_{Rs} \ddot{\omega}_{Rs} \quad (3.239) \]

\[ T_o - R_d T_s - T_{DIR} = I_{Cs} \dot{\omega}_{Cs} \quad (3.240) \]

\[ T_{DIR} - R T_{OWCH,RED} - T_{Ss} = I_{Ss} \dot{\omega}_{Ss} \quad (3.241) \]

We note that during this phase the reaction torque at the RED brake and the OWCH are the same. Therefore, the term \( R T_{OWCH,RED} \) in equation (3.241) refers to the reaction torque at either the OWCH clutch or the RED brake. The use of the OWCH ensures that the sun-gear is held fixed before the DIR clutch takes over the entire load. Therefore, equation (3.151) becomes

\[ R T_{OWCH,RED} = T_{DIR} - T_{Ss} \quad (3.242) \]

Following similar derivation used in section 3.4.1, the torque \( T'_{po} \) is described by,

\[ T'_{po} = R_{po} \left[ (I_{Rs} + R_d^2 I_{Cs}) \dot{\omega}_{Rs} + R_{Rs} T_{DIR} + R_{Rs} R_d T_s \right] \quad (3.243) \]
To find the governing equations describing the 4-5 up shift torque phase, we substitute \( T'_{po} \) from equation (3.243) into equations (3.231) and (3.232), and follow the same derivation as for the fourth gear dynamics. We then have the following,

\[
\left( I_t + I_{Cr} + c_{22}^2 I_{Si} + c_{12}^2 \left( I_{Rs} + R_{po}^2 \left( I_{Rs} + R_{Rs}^2 I_{Cs} \right) \right) \right) \dot{\omega}_t = T_t - c_{12} R_{po} R_{Rs} \left( T_{DIR} + R_d T_S \right)
\]

\[(3.244)\]

\[
RT_{ND} = -\left( c_{21} c_{22} I_{Si} + c_{11} c_{12} \left( I_{Rs} + R_{po}^2 \left( I_{Rs} + R_{Rs}^2 I_{Cs} \right) \right) \right) \dot{\omega}_t - c_{12} R_{po} R_{Rs} \left( T_{DIR} + R_d T_S \right)
\]

\[(3.245)\]

\[
RT_{OWCH,RED} = -c_{12} R_{Rs} R_{Rs}' R_{po} R_{cs} \dot{\omega}_t - R_{Rs} R_d T_S + T_{DIR}
\]

\[(3.246)\]

\[
\omega_s = c_{12} R_{po} R_{Rs} R_d \dot{\omega}_t
\]

\[(3.247)\]

As mentioned, the torque phase ends when the reaction torque held by the OWCH becomes zero. This reaction torque is calculated from equation (3.246). In this case, the secondary sun-gear will start moving. Therefore, the following conditions are used to determine the end of the torque phase.

\[
\left| RT_{OWCH,RED} \right| \leq 0
\]

\[(3.248)\]

\[
\omega_{Rs} > 0
\]

\[(3.249)\]

(3) 4-5 Up Shift - Inertia Phase

As the OWCH releases the sun-gear of the secondary gear set, the whole secondary gear set moves freely. If the RED brake is already fully released before the OWCH releases the sun-gear, then only the DIR clutch affects the behavior of the inertia phase. In particular, the torque capacity of the DIR clutch is initially increased by the commands from the controller in order to reduce the slip speed, the speed difference between the secondary sun-gear and the secondary carrier, to zero. The reaction torque at
the DIR clutch during this phase is calculated from the torque capacity of the DIR clutch, i.e. $RT_{DIR} = T_{DIR}$. However, if the RED brake is not completely released, its torque capacity affects the behavior of the inertia phase. Both the DIR clutch and RED brake will fight each other, which normally increases the load on the engine. In this case, the reaction at the RED clutch is determined by its remaining torque capacity or $RT_{RED} = T_{RED}$. The model derived in this section will be for this latter case, but the results can be used to describe the dynamic response of the former case by simply ignoring all terms with $T_{RED}$.

The free-body diagram of the secondary gear set for the inertia phase is the same as shown in Figure 3.10. However, the reaction torque at the OWCH is zero. As the secondary gear is able to move freely, the kinematic relationships for the gear elements are the following.

$$\omega_{Cs} = R_s \omega_{Rs} + R_s \omega_{Ss} \quad (3.250)$$

$$T_o = \left( \frac{1}{R_s} \right) T_{Rs} = \left( \frac{1}{R_s} \right) T_{Ss} \Rightarrow T_{Ss} = \left( \frac{R_s}{R_s} \right) T_{Rs} \quad (3.251)$$

From equation (3.241), we have,

$$T_{Ss} = T_{DIR} - T_{RED} - I_{Ss} \dot{\omega}_{Ss} \quad (3.252)$$

Using equations (3.252) and (3.251) to calculate $T_{Rs}$, and substituting the result in equation (3.239) along with the use of the speed relationship in equation (3.250), we get,

$$T_{po} = \frac{R_s}{R_{ss}} \left( T_{DIR} - T_{RED} \right) = \left[ I_{Rs} + I_{Ss} \left( \frac{R_s}{R_{ss}} \right)^2 \right] \dot{\omega}_{Rs} - \frac{R_s}{R_{ss}} I_{Ss} \dot{\omega}_{Cs} \quad (3.253)$$

By doing the same as above but replacing equation (3.239) with (3.240), we get,
\[
\left(1 - \frac{R_{Ss}}{R_{Ss}}\right)T_{DIR} - \frac{1}{R_{Ss}} T_{RED} - R_d T_S = \left(I_{Cs} + \frac{I_s}{R_{Ss}^2}\right) \dot{\omega}_{Cs} - \frac{R_{Rs}}{R_{Ss}^2} I_s \dot{\omega}_{Rs} \tag{3.254}
\]

During this gear shift, the dynamic behavior can be determined by observing the turbine speed from the primary gear set and the output speed of the secondary gear set. Therefore, we are interested in the governing equations relating \(\omega_t\) and \(\omega_{Cs}\). To obtain this, first we know that,

\[
T'_{po} = R_{po} T_{po} \tag{3.255}
\]

And the speed relationships during this gearshift can be derived as,

\[
\begin{align*}
\omega_t &= c_{22} \omega_{Cr} \\
\omega_{Rs} &= c_{12} \omega_{Cr} \\
\omega_{Re} &= R_{po} \omega_{Re}
\end{align*}
\implies \omega_{Rs} = \left(\frac{c_{12} R_{po}}{c_{22}}\right) \omega_t \tag{3.256}
\]

Using the above equations, equation (3.253) becomes,

\[
T'_{po} = \left(I_{Rs} + I_s \left(\frac{R_{Rs}}{R_{Ss}}\right)^2\right) \left(\frac{c_{12} R_{po}^2}{c_{22}}\right) \dot{\omega}_t - \frac{R_{po} R_{Rs}}{R_{Ss}^2} I_s \dot{\omega}_{Cs} + \frac{R_{po} R_{Rs}}{R_{Ss}} \left(T_{DIR} - T_{RED}\right) \tag{3.257}
\]

Similarly, substituting the speed relationship (3.256) into (3.254), we have,

\[
\left(1 - \frac{R_{Ss}}{R_{Ss}}\right)T_{DIR} - \frac{1}{R_{Ss}} T_{RED} - R_d T_S = \left(I_{Cs} + \frac{I_s}{R_{Ss}^2}\right) \dot{\omega}_{Cs} - \frac{R_{Rs}}{R_{Ss}^2} \left(\frac{c_{12} R_{po}}{c_{22}}\right) \omega_t \tag{3.258}
\]

To connect the secondary gear set to the primary gear set, we substitute \(T'_{po}\) from (3.257) into (3.231), and get,

\[
\left[I_{4s} + \frac{1}{c_{22}} \left(I_{Rs} + I_s \left(\frac{R_{Rs}}{R_{Ss}}\right)^2\right) \left(\frac{c_{12} R_{po}}{c_{22}}\right) \right] \dot{\omega}_t - \frac{c_{12} R_{po} R_{Rs}}{R_{Ss}^2} I_s \dot{\omega}_{Cs} = T_t - \frac{c_{12} R_{po} R_{Rs}}{R_{Ss}} \left(T_{DIR} - T_{RED}\right) \tag{3.259}
\]
where \( I_{45} = I_t + I_{cr} + c_{12}^2 I_{Rr} + c_{22}^2 I_{Si} \). As can be seen, both equations (3.258) and (3.259) are coupled to each other. They can be written in the following format.

\[
\begin{pmatrix}
\dot{\omega}_{c_s} \\
\dot{\omega}_t
\end{pmatrix} =
\begin{pmatrix}
A4_{11} T_{DIR} + A4_{21} T_{RED} + C4_{T11} T_t + C4_{T12} T_s \\
A4_{12} T_{DIR} + A4_{22} T_{RED} + C4_{T21} T_t + C4_{T22} T_s
\end{pmatrix}
\]

(3.260)

where,

\[
\begin{bmatrix}
A4_{11} & A4_{21} \\
A4_{12} & A4_{22}
\end{bmatrix} = (I'_{45})^{-1}
\begin{bmatrix}
1 - R_{St} \\
-\left( c_{12} R_{po} \right) R_{Rs} 
\end{bmatrix}
\begin{bmatrix}
\frac{1}{R_{St}} \\
\left( c_{12} R_{po} \right) \frac{R_{Rs}}{R_{St}}
\end{bmatrix}
\]

(3.261)

\[
\begin{bmatrix}
C4_{T11} \\
C4_{T21}
\end{bmatrix} = (I'_{45})^{-1}
\begin{bmatrix}
0 \\
-1
\end{bmatrix}
\]

\[
\begin{bmatrix}
C4_{T12} \\
C4_{T22}
\end{bmatrix} = (I'_{45})^{-1}
\begin{bmatrix}
-R_d \\
0
\end{bmatrix}
\]

(3.262)

The reaction torque at the 2ND brake, which remains fully engaged, can be calculated by first substituting \( T'_{po} \) from equation (3.257) into equation (3.232)

\[
RT_{ND} = \left[ -c_{11} c_{12} \left( I_{Rr} + I_{Rs} \left( \frac{R_{Rs}}{R_{St}} \right)^2 \right) c_{22}^2 \right] \dot{\omega}_t + c_{11} \frac{R_{po} R_{Rs}}{R_{St}^2} I_{St} \omega_{cs}
\]

(3.263)

And the transmission output speed can be calculated from
\[ \omega_s = R_d \omega_{Cs} \]  \hspace{1cm} (3.264)

The inertia phase ends when the torque capacity of the DIR clutch is higher than the reaction torque calculated at the DIR clutch, as a result of which the DIR clutch gets to the lock-up condition. The slip speed of the DIR clutch becomes zero which means that the sun-gear speed equals the carrier speed. In fact, since two elements of the planetary gear set are coupled to each other, by default, the planetary gear set will move as one. Therefore, we can write the conditions determining the end of the inertia phase as the following.

\[ T_{DIR} > R T_{DIR} \]  \hspace{1cm} (3.265)
\[ \omega_{Ss} = \omega_{Cs} = \omega_{Rs} \]  \hspace{1cm} (3.266)

(4) Fifth Gear Dynamics

In the fifth gear, at the primary gear set, the OD clutch and 2ND are fully engaged. The dynamics of the primary gear set in the fifth gear are the same as that shown in the fourth gear. At the secondary gear set, the DIR clutch is in lock-up condition and forces the whole secondary gear set unit to move as one. Therefore the gear ratio at the secondary gear set is one. In other words, the gear ratio in the fifth gear is determined from the gear ratio of the primary gear set in the fourth gear configuration, the reduction ratio between the primary and the secondary gear sets, and the final drive ratio. The kinematic relationships between torques and speeds at the secondary gear set during the fifth gear are the following.

\[ \omega_{Rs} = \omega_{Ss} = \omega_{Cs} \]  \hspace{1cm} (3.267)
\[
T_o = \left( \frac{1}{R_{Rs}} \right) T_{Rs} = \left( \frac{1}{R_{Ss}} \right) T_{Ss} \quad \Rightarrow \quad T_{Rs} = \left( \frac{R_{Rs}}{R_{Ss}} \right) T_{Ss}
\] (3.268)

And from the Figure 3.11, we can write the following.

\[
T_{po} - T_{Rs} = I_{Rs} \dot{\omega}_{Rs}
\] (3.269)

\[
T_o - R_d T_S - R T_{DIR} = I_{Cs} \dot{\omega}_{Cs}
\] (3.270)

\[-R T_{DIR} - T_{Ss} = I_{Ss} \dot{\omega}_{Ss}\] (3.271)

Combining equations (3.267) to (3.271), we get,

\[
T_{po} - R_d T_s = \left( I_{Rs} + I_{Rs} + I_{Rs} \right) \dot{\omega}_{Rs}
\] (3.272)

or, in terms of the load torque on the primary gear set

\[
T'_{po} = R_{po} \left( \left( I_{Rs} + I_{Rs} + I_{Rs} \right) \dot{\omega}_{Rs} + R_d T_S \right)
\] (3.273)

Substituting this load torque equation in the equations (3.231) and (3.232), we have the equation describing the fifth gear dynamics as shown below.

\[
\left( I_t + I_{Cr} + c_{21}^2 I_{Si} + c_{12}^2 \left( I_{Rs} + R_{po}^2 \left( I_{Rs} + I_{Rs} + I_{Rs} \right) \right) \right) \dot{\omega}_t = T_t - c_{12} R_{po} R_d T_S
\] (3.274)

The reaction torque at the 2ND brake, which is also fully engaged, can be derived in a similar way, which gives,

\[
R T'_{ND} = - \left( c_{12} \left( I_{Rs} + I_{Rs} + I_{Rs} \right) \right) \dot{\omega}_t = c_{11} R_{po} R_d T_S
\] (3.275)

To find the reaction torque at the DIR clutch, from equations (3.267) to (3.271), we can derive the following.

\[
T'_{po} = R_{po} \left( \left( R_{Rs} \frac{R_{Rs}}{R_{Ss}} \right) \dot{\omega}_{Rs} + \frac{R_{Rs}}{R_{Ss}} R T_{DIR} \right)
\] (3.276)

Therefore, it’s easy to get,
The reaction torque at the OD clutch is calculated from,
\[
RT_{OD} = T_t - I_r \dot{\omega}_t
\]  
(3.278)

and the transmission output speed is calculated from,
\[
\omega_o = c_{12} R_{po} R_{Rs} R_d \omega_t
\]  
(3.279)

### 3.5 Vehicle and Driveline Model

The model of the vehicle and driveline dynamics used here is simplified from Zheng (1999), and Cho and Hedrick (1989). The model describes the longitudinal vehicle dynamics where the final drive output shaft speed from the transmission is the input to the differential and the axle shafts. The model equations are given as follows. The output shaft dynamics are described by
\[
\dot{T}_s = K_s (\omega_o - \omega_w)
\]  
(3.280)

The driving front wheels are described by,
\[
\dot{\omega}_w = \frac{1}{I_v} (T_s - T_{load})
\]  
(3.281)

The load torque, $T_{load}$, includes the rolling friction and aerodynamic drag. Note that the wheel slip is neglected in this simplified model.
\[
T_{load} = r (c_1 + c_2 r^2 \omega_w^2)
\]  
(3.282)

For these equations, $T_s$ is the transmission output shaft torque as described before, $T_{load}$ is the load torque from the vehicle, $K_s$ is the torsional spring stiffness of the drive shaft, $R$,
c₁ and c₁ are the tire radius, the rolling friction coefficient, and the aerodynamics friction coefficient respectively, \( \omega_w \) represents the wheel speed, and \( \omega_o \) is the final drive transmission output speed, where,

\[
\omega_o = R_d \omega_{Rr} \quad \text{for the 4-speed transmission}
\]

\[
\omega_o = R_d \omega_{Cs} \quad \text{for the 5-speed transmission}
\]

Here, \( R_d \) is the final drive gear ratio, and \( \omega_{Rr} \) is the reaction ring-gear speed, which is the transmission output speed in the 4-speed transmission. \( \omega_{Cs} \) is the secondary carrier speed, which is the transmission output speed in the 5-speed transmission.

### 3.6 Transmission Mechanical Model Validation

Since we were able to obtain most of model parameters and test data for only the 4-speed transmission, only the 4-speed transmission model will be tested and validated in this research. Almost all model parameters are given by the manufacturer, except all inertia values, which were determined experimentally. The model is first tested via simulation in order to check the validity of all equations, as well as the conditions used to determine the torque phase and inertia phase as described in the model derivation. This part also shows the ability to use the model to study gear shift dynamics when the clutch-to-clutch shifting system is used. Specifically, the effect of varying the timing in applying the on-coming clutch and releasing the off-going clutch can be explored. At the end of this section, the accuracy of the model is validated by comparing the simulation results with some test data.
3.6.1 4-Speed Transmission Simulation

A Simulink® simulation model of the F4A42 transmission is built using the dynamic equations derived in the previous sections. In order to test the transmission model, the input from the engine is needed. Since we do not consider the development of the engine model in this research, we take the already developed engine model and its parameters from literature (Zheng, 1999). The over all simulation models are shown in Figure 3.12. Since we have not described the model for the hydraulic system in this chapter, clutch pressure profiles are used to represent the on-coming clutch and off-going clutch pressure inputs. They are shown in Figure 3.13. The shift logic activating the gear shifts are also arbitrarily defined. Simulation results presented here are for the power-on 1-2 and 2-3 up shifts. Since the same dynamic equations can be used for the power-on down shift as well, simulation results for the power-on 3-2 down shift are also shown.

![4-Speed Transmission Dynamics Simulator (F4A42)](image)

Figure 3.12: Powertrain simulation model with F4A42 transmission and torque converter
3.6.1.1 Power-On Up Shift

Simulation results for the power-on 1-2 and 2-3 up shifts are shown in Figure 3.14 – 3.17. In Figure 3.14, the turbine speed is shown and compared to the synchronization speed, which is the transmission output speed multiplied by the gear ratio in each gear. During the 2-3 up shift, the application and release times of the on-coming and off-going clutches are varied in order to demonstrate timing effects on the gear shift transients. The 1-2 up shift is activated at one second of the simulation time and the 2-3 shift is activated at two seconds of simulation time.

Figure 3.15 shows the output shaft torque during the gear shift and Figure 3.16 shows the enlarged portion of the output shaft torque during the transient of the 2-3 up
shift. Figure 3.17 shows the corresponding clutch torque capacity as well as the reaction torque for both on-coming clutch and off-going clutches during the 2-3 up shift.

It can be seen that, for all cases, the output torque drops because of the load transfer from the release clutch to the apply clutch. This torque drop is an inescapable characteristic of the gear up shift but can be reduced by using engine torque control (Winchell and Route, 1988). Since the drop in the output torque during the torque phase is an inescapable characteristic of an up-shift, the goal for Clutch-to-Clutch control design should be minimize this effect, as well as reducing the output torque overshoot when the inertia phase is finished.

The output torque drops until the releasing clutch slips, which is the beginning of the inertia phase. As compared to the case where both clutches are operated at the same time, if the off-going clutch is released too late, the output torque drops lower and the value at the starting point of the third gear is also low. If the on-coming clutch is applied too late, for our particular pressure profiles, it also causes a very large drop in the output torque. In this case, since the torque capacity of the on-coming clutch is too low when the inertia phase begins, engine flare occurs as can be seen from the turbine speed trace shown in Figure 3.14. As a result, the shift duration is longer than others.
Figure 3.14: Simulated turbine speed during 1-2 and 2-3 power-on up shift

Figure 3.15: Simulated output shaft torque during 1-2 and 2-3 power-on up shift
Figure 3.16: Enlarged portion of the simulated output shaft torque during 1-2 and 2-3 power-on up shift
Figure 3.17: Clutch torque capacity and reaction torque for the 2ND brake and the OD clutch during 1-2 and 2-3 power-on up shift

3.6.1.2 Power-On down shift

As mentioned earlier, the model for the power-on up shift can be used to describe transmission behavior during the power-on down shift, the simulation results for the power-on down shift 3-2 being shown in Figures 3.18- 3.22. Figure 3.18 shows the
turbine speed as compared to the synchronization speed. Figure 3.19 shows the zoom-in portion of the turbine speed during the 3-2 down shift transient. Figure 3.20 shows the output shaft torque with the transient for the 3-2 down shift also enlarged and shown in Figure 3.21. Figure 3.22 shows the corresponding clutch torque capacity and the reaction torque at the on-coming and off-going clutches during the 3-2 down shift.

As for the power-on up shift, we can study the effect on the clutch-to-clutch gear shift transient of varying the apply time of the on-coming clutch and the release time of the off-going clutch relative to each other. From Figure 3.18, it can be seen that if the on-coming and off-going clutches are applied and released at the same time, the shift is finished the fastest. But since the two clutches are fighting for some of the time, a large torque drop occurs as can be seen in Figures 3.20 and 3.21. The torque drop is even larger if we release the off-going clutch too late. The best situation for the down shift is when we apply the on-coming clutch a bit late. This forces the inertia phase to start before the load is transferred from off-going clutch to the on-coming clutch. And in this case, the decrease in the shaft torque is reduced. Normally, the quickest down shift could be achieved by rapidly releasing the off-going clutch. However, to control the output torque drop, the off-going clutch can be left partially applied and controlled in a manner so that the turbine speed reaches the target speed. During this period, the on-coming clutch can also be applied in a manner depending on the control strategy. As for the power-on up shift, the goal of the power-on down shift should also be to minimize the decrease in the output torque during the inertia phase.
Figure 3.18: Simulated turbine speed during power-on 1-2 to 2-3 up shift followed by power-on 3-2 down shift

Figure 3.19: Closed-up of the simulated turbine speed during the power-on 3-2 downshift
Figure 3.20: Simulated output shaft torque during the power-on up shift 1-2 and 2-3 followed by the power-on down shift 3-2

Figure 3.21: Closed-up of the simulated output shaft torque during the power-on down shift 3-2
Figure 3.22: Clutch torque capacity and reaction torque for the 2ND brake and the OD clutch during power-on up shift 1-2, 2-3, and power-on down shift 3-2.
3.6.2 4-Speed Transmission Model Validation

In this section, the developed model is validated against experimental data collected from the test vehicle. The closeness of the agreement between the simulation results and the experimental data will testify to the effectiveness of the developed model in predicting transmission dynamic behavior.

Since the model of the shift hydraulic system is not included here, any error in the model of the hydraulic system can be isolated and studied separately as is done in Chapter 4. In order to do this, the Hydraulic System block shown in Figure 3.12 is replaced by the clutch pressure data obtained experimentally. This allows us to concentrate only on the performance of the transmission mechanical system without bringing into the picture the performance of the hydraulic system model. Numerical values for the system parameters are either drawn from manufacturers’ specifications or measured. The measured values are mainly the inertias for the planetary gear set components.

Note that the engine model is not included in this model development. However, the effect of engine dynamics as well as the variation of load on the engine can be monitored via the engine speed. Therefore, the engine speed is used as an input to the model. The use of the static torque converter model also eliminates the effect of the fluid inertia inside the torque converter and could introduce error in the predicted turbine torque. However, it has been shown in the literature that for a low frequency analysis of a power-on up shift, which is the main interest of this research, the effect of the torque converter fluid inertia is negligible. In terms of the planetary gear model, tooth friction between the gear elements, backlash, and gear stiffness are also neglected in this model.
since only low frequency phenomena are considered here. As for the driveline dynamics, the drive axle is modeled as a torsional spring, and backlash in the differential is ignored. And for the vehicle dynamics, the load on the vehicle consists of rolling friction and aerodynamic drag and the wheel slip is neglected for simplicity. The modified Simulink® simulation model for the model validation is shown in Figure 3.23 below.

![Figure 3.23: Modified Simulink® simulation model for model validation](Image)

The model is validated only for power-on up shifts from the 1st gear to 3rd gear. Inputs to the model are the engine speed and the clutch pressure data for all clutches. The validation is done by comparing the simulated turbine speed to the measured turbine speed. An example comparison is shown in Figure 3.24, and indicates that the model is able to predict the transmission behavior quite closely. A sampling interval of 10 milliseconds is used with a fixed step-size numerical integration algorithm.
Since the non-zero initial conditions for the different operating conditions are normally unknown, one limitation of the model in its current form is that the model assumes zero initial conditions corresponding to the vehicle being at rest. Therefore, experimental data to validate the model has to be collected starting with the vehicle at rest. More experimental data is needed to validate the model over a variety of operating conditions, such additional validation being more easily done if the model can be initialized with the vehicle at different speeds and gears.

Figure 3.24: Simulated turbine speed compared to experimental data
3.3 Conclusion

This chapter presents the development and validation results for a simulation model for the automatic transmission of interest in both 4-speed and 5-speed versions. The model developed here consists of the transmission mechanical model, torque converter model, the vehicle and driveline model, and the shift hydraulic model. The model of the transmission mechanical subsystem including the torque converter and the vehicle and driveline is validated separately from the shift hydraulic system such that the error from the model of the shift hydraulic system can be isolated, and the validation result shows that the model is able to predict the transmission behavior with acceptable accuracy.
CHAPTER 4

SHIFT HYDRAULIC SYSTEM MODEL

The shift hydraulic system of an automatic transmission plays a major role in its operation. During the gear shifting operation, the hydraulic system not only initiates the shifting process but influences the quality of the gear shifting operation. For an automatic transmission which relies primarily on clutch-to-clutch shifts, quick response and precise control of clutch pressures are critical for good shift quality. Understanding the dynamic behavior of the hydraulic system not only helps us analyze transmission operation, but also guides us in controller design to improve gear shift quality. Shift hydraulic system hardware, in general, is rather complex and its behavior usually cannot be observed online using sensors or measurement devices on production transmissions due to the cost of sensors, reliability issues in service, and maintenance. Therefore, knowledge of a quantitative model for the hydraulic system dynamic response is essential because of its potential use in model-based diagnostics. The need for a quantitative model is also particularly great since the dynamic behavior of shift hydraulic systems has not received much attention as compared to research on other areas of power train system operation (Zheng and Srinivasan, 2000).
This chapter presents the development of a nonlinear, lumped-parameter, dynamic model of the hydraulic system for the automatic transmission presented in Chapter 3, which relies primarily on clutch-to-clutch shifts. A detailed model that can predict hydraulic system behavior accurately is presented. Due to the lack of information and test data for the 5-speed version of the transmission of interest, only the shift hydraulic system for the 4-speed transmission is considered in this chapter.

There are four components or systems affecting the hydraulic system dynamics. They are: the clutch and accumulator system, the pressure control valve, the solenoid valve, and the pressure regulation system. Because of interaction among these components, the resulting model is highly nonlinear and complex, and is not suitable for controller design applications. Therefore, a model simplification is also presented. The model predictions of dynamic response are validated against experimental data from a production car equipped with the automatic transmission of interest and instrumented with additional transmission sensors. The robustness of the model prediction to change in transmission operating conditions is also explored.

The outline of this chapter is as follows. The components of the hydraulic system for the transmission of interest are introduced, followed by model development for the solenoid valve, the pressure control valve, the clutch and accumulator system, and the pressure regulation system. The resulting detailed model is then simplified. Model validation against experimental data is then presented, with emphasis on the effect of adding specific component models on overall model accuracy. Conclusions are given at the end of the chapter.
4.1 Hydraulic System Components

As partially described in Chapter 3, for the 4-speed automatic transmission of interest, there are 5 clutches involved in control of the transmission of interest for all driving conditions. They are the low-reverse clutch (LR), the 2ND clutch (2ND), the underdrive clutch (UD), the overdrive clutch (OD), and the reverse clutch (REV). The clutch pressure for each clutch is individually controlled by a 3-way PWM-type solenoid valve. The clutch application schedule for each gear was already shown in the Table 3.1, Chapter 3. We note again here that we consider only forward driving conditions in this research. Therefore, reverse gear operation and the REV clutch are not considered here.

Figure 4.1 shows a simplified schematic of the hydraulic system for the transmission of interest. As shown in the figure, there are 4 elements governing the dynamics of the clutch pressure. The first is the solenoid valve, which is controlled by the duty cycle input and generates a corresponding command pressure, which is then applied to the command pressure chamber of the pressure control valve. The second subsystem is the pressure control valve. The pressure control valve modulates the line pressure to a desired value corresponding to the command pressure from the solenoid valve. The pressurized hydraulic fluid is fed to the clutch and the accumulator by the pressure control valve through the feeding chamber passing through the supply orifice. The third subsystem is the clutch and accumulator system. The last subsystem of interest is the supply line pressure regulation system.
4.2 Detailed Model for the Shift Hydraulic System

4.2.1 PWM solenoid valve model

A 3-way, normally open, PWM solenoid valve is used in this research. The cross sectional view is shown in Figure 4.2. From the figure, it is clear that when there is no current, the plunger is pushed to the left hand side due to the inner spring located inside the core. In this position, the plunger closes the orifice leading to the exhaust port. The tip
of the plunger pushes the ball to the left and away from the seat, causing the hydraulic fluid from the supply port to flow through the output port, which in turn is directly connected to the command chamber of the pressure control valve as shown in Figure 1. On the other hand, when the solenoid is energized, the magnetic force generated by the coil pulls the plunger to the right side. The ball is pushed back on the seat by its return spring, blocking the hydraulic fluid from the supply port. The opening between the plunger and the seat now connects the output port to the exhaust port. Therefore, the hydraulic fluid from the command chamber of the pressure control valve flows back to the output port, and past the exhaust orifice and the exhaust port subsequently. As a result, the controlled pressure of the hydraulic fluid in the command chamber of the pressure control valve drops to low values.

![Cross-sectional view of the PWM solenoid valve](image)

**Figure 4.2: Cross-sectional view of the PWM solenoid valve**

The mathematical model of the solenoid valve includes 3 subsystems, which are: the magnetic circuit, the plunger mechanical system, and the fluid flow system.
Magnetic circuit

We consider first the model of the magnetic circuit. In this analysis, we assume that all magnetic flux is uniform across the core and contained within the core. Also, leakage flux is negligible. From Faraday’s law, the flux inside the system can be determined from the following expression.

\[
\phi = \frac{(V_{in} - iR)}{N} \quad (4.1)
\]

Here, \(\phi\) represents the magnetic flux, \(V_{in}\) is the solenoid voltage input, \(i\) is the solenoid current, \(R\) is the coil resistance, and \(N\) is the number of coil turns. In using this expression, we also assume that there is no loading effect on the voltage input and no self heating of the coil resistance. Since most of the circuit’s reluctance is concentrated in the air gap, for the sake of simplicity, we assume that the magnetomotive force required to generate this flux consists of only the magnetomotive force from the working air gap. Therefore,

\[
F_{\text{mmf}} = F_{\text{mmf,air}} = H_{\text{air}} (x_{\text{solp,max}} - x_{\text{solp}}) \quad (4.2)
\]

\[
H_{\text{air}} = \frac{\phi}{A\mu_0} \quad (4.3)
\]

\[
i = \frac{F_{\text{mmf}}}{N} \quad (4.4)
\]

In these expressions, \(F_{\text{mmf}}\) represents the magnetomotive force, where \(F_{\text{mmf,air}}\) is the magnetomotive force generated by the working air gap. \(H_{\text{air}}\) is the magnetic field intensity, \(A\) is the cross sectional area of the air gap, and \(\mu_0\) is the permeability of air. \(x_{\text{solp,max}}\) is the initial distance of the plunger to the core before the solenoid is energized,
and $x_{solp}$ is the plunger displacement measured from the initial position. The magnetic force on the plunger can be calculated from the following equation.

$$F_{mag} = \frac{1}{2} \frac{\phi^2}{\mu_0 A}$$  \hspace{1cm} (4.5)

**Plunger mechanical system**

Plunger motion depends on the resultant of the magnetic force, the inertia force, the damping force, and the return spring force. Here, the static friction force and the force due to the hydraulic fluid acting on the plunger body are assumed to be negligible. The force from the return spring on the ball is also assumed to be negligible. The equation describing the motion of the plunger can be written as follows.

$$m_{solp} \ddot{x}_{solp} + B_{solp} \dot{x}_{solp} + K_{solp} x_{solp} = F_{mag}$$  \hspace{1cm} (4.6)

where $m_{solp}$ is the mass of the plunger, $B_{solp}$ is the damping coefficient, and $K_{solp}$ is the spring constant.

**Fluid flow system**

The last subsystem governing solenoid dynamics is the fluid flow system. As mentioned before, inlet flow occurs when the ball is pushed away from the seat by the plunger’s tip. Exhaust flow occurs when the plunger is pulled as the coil is energized, which opens the orifice connecting the output port and the exhaust port. Ignoring all leakage flow, the net flow through the solenoid valve consists of the summation of inlet and exhaust flows. We use sharp-edged orifice flow equations to represent both inlet and exhaust flows. The geometry of the orifice for both inlet and exhaust flows is shown in
Figure 4.3. Therefore, the flows across the solenoid valve’s ports can be written as follows:

\[ Q_{\text{sol,net}} = Q_{\text{sol,in}} - Q_{\text{sol,ex}} \quad (4.7) \]

\[ Q_{\text{sol,in}} = C_d A_{\text{sol,in}} \sqrt{\frac{2 |P_{\text{line}}| - P_{\text{sol}}}{\rho}} \text{sgn}(P_{\text{line}} - P_{\text{sol}}) \quad (4.8) \]

\[ Q_{\text{sol,ex}} = C_d A_{\text{sol,ex}} \sqrt{\frac{2 |P_{\text{sol}}|}{\rho}} \text{sgn}(P_{\text{sol}}) \quad (4.9) \]

where

\[ A_{\text{sol,in}} = \begin{cases} \pi d_{\text{sol,in}} (x_{\text{sol,in}} - x_{\text{solp}}), & \text{when } x_{\text{solp}} < x_{\text{tapped}} \\ 0, & \text{when } x_{\text{solp}} \geq x_{\text{tapped}} \end{cases} \quad (4.10) \]

\[ A_{\text{sol,ex}} = \pi d_{\text{sol,ex}} x_{\text{solp}} \quad (4.11) \]

\( Q_{\text{sol,net}} \) is the net flow across the solenoid valve, \( Q_{\text{sol,in}} \) and \( Q_{\text{sol,ex}} \) are the inlet and exhaust flow through the solenoid valve respectively, \( P_{\text{line}} \) is the supply pressure, and \( P_{\text{sol}} \) is the controlled pressure at the output port. \( d_{\text{sol,in}} \) and \( d_{\text{sol,ex}} \) are the inlet and exhaust orifice diameter respectively, \( \rho \) is the transmission fluid density, and \( C_d \) is the discharge coefficient, where a value of 0.61 is used for sharp-edged orifice flow.
From experimental observation of solenoid valve behavior, the controlled pressure at the output port, which is the same as the pressure inside the command chamber of the pressure control valve, cannot exceed the supply line pressure, and also cannot drop below the atmospheric pressure at the exhaust port. Therefore, equations (4.8) and (4.9) can be written as follows.

\[
Q_{\text{sol,in}} = C_d A_{\text{sol,in}} \sqrt{\frac{2(P_{\text{line}} - P_{\text{sol}})}{\rho}} \quad (4.12)
\]

\[
Q_{\text{sol,ex}} = C_d A_{\text{sol,ex}} \sqrt{\frac{2(P_{\text{sol}})}{\rho}} \quad (4.13)
\]

Most of parameters for the subsystems of the solenoid valve model can be measured physically. However, the damping coefficient due to the friction acting on the plunger cannot be so obtained. In this case, a theoretical value based on the physical dimensions is calculated first. The value is adjusted subsequently to fit model response with experimental data.
In the system hardware, the command duty cycle from the controller is applied to a 64 Hz pulse train and the solenoid circuit converts the pulse signal to solenoid current. Digital and analog integrated circuits in the driver are used to generate the pulse train. However, due to the complexity of the physical model, the observed input-output response of the solenoid electrical circuit is used instead to derive a model. The input-output response describes circuit reaction to the command input and the load. In this work, the manufacturer gives the output from the driver and the corresponding solenoid current for one duty cycle, as shown in Figure 4.4. When the pulse is turned on, the first phase is the initial pulling phase in which the driver circuit applies full voltage to achieve the fastest initial rise in current. The initial pulling phase finishes when the current reaches the level at which the magnetic force overcomes the spring force and moves the plunger. The average time of the initial pulling phase for the particular solenoid of interest is given by the manufacturer (see Figure 4.4). After the initial pulling phase, the air gap is small so that less current is needed to hold the plunger. We call this phase the holding phase, during which the driver circuit regulates the current to stay at the lower level, I\text{hold} in Figure 4.4, in order to maintain the plunger position for the remainder of the “on” portion of the duty cycle. The on/off time of the apply voltage to regulate the current is also given by the manufacturer. At the end of each pulse, the driver applies a large negative voltage, V_{off} in Figure 4.4, to achieve a fast decay of the current, which removes the magnetic field so that the plunger and the ball return to their initial positions. Ideally, the current is zero for the duration of the “off” time until the next cycle begins.
Figure 4.5 shows the simulated current of the solenoid valve model as compared to the experimental result shown in Figure 4.6. It can be seen that the behavior of the simulated solenoid current is close to the real behavior of the hardware.

Figure 4.4: The behavior of the solenoid circuit for one cycle of duty cycle command
Figure 4.5: The simulated solenoid current (50% duty cycle)

Figure 4.6: The experimental data for solenoid current (50% duty cycle)
4.2.2 Pressure Control Valve Model

The dynamics of the pressure control valve involve force balance at the spool, command chamber pressure dynamics, and feeding chamber pressure dynamics. The free body diagram of the pressure control valve is shown in Figure 4.7.

Force balance on the spool

Following Figure 4.7, the equation of motion of the valve can be derived as:

\[ B_{pcv} \dot{x}_{pcv} + K_{pcv} x_{pcv} = P_s \Delta A_{pcv1} - P_r \Delta A_{pcv2} - F_{s_in} \]  

(4.14)

where

\[ x_{pcv} = \begin{cases} \text{pressure control valve displacement} \\
0 & \text{if} \ (P_s \Delta A_{pcv1} - P_r \Delta A_{pcv2}) < F_{s_in} , \quad 0 \leq x_{pcv} \leq x_{pcv\_max} 
\end{cases} \]  

(4.15)

\[ B_{pcv} = \text{friction coefficient between spool and sleeve} \]

\[ = \frac{\mu A}{\Delta r} \]  

(4.16)

Also, \( \mu \) represents the transmission fluid viscosity, \( A \) is the contact area between spool and sleeve, \( \Delta r \) is the radial clearance between spool and sleeve, \( K_{pcv} \) is the spring constant, \( F_{s\_in} \) is the spring preload, and \( P_r \) is the feeding chamber pressure. \( \Delta A_{pcv1} \), \( \Delta A_{pcv2} \) are the differences in the valve cross sectional areas.

In equation (4.14), we assume that the inertia force is negligible. Except for the initial position and the fully compressed position, the spool always chatters at some position due to the PWM nature of the command pressure. Therefore, both static and dynamic flow forces are ignored in this analysis.
Feeding chamber continuity equation

From physical inspection, during the flow-in phase, we can assume that when the transmission fluid enters the valve from the supply port, the exhaust port is closed completely. This is also true for the exhaust phase where the supply port is closed completely when the exhaust port is opened. Thus, the analysis of the pressure variation inside the feeding chamber during the flow-in phase and the exhaust phase can be done separately. All leakage flows are ignored here. The continuity equation of the feeding chamber during the flow-in phase can be derived as:

\[ Q_{\text{inf}} - Q_{\text{inc}} = \frac{V_f}{\beta} \frac{dP_f}{dt} \]  \hspace{1cm} (4.17)
where $Q_{\text{inf}}$ and $Q_{\text{inc}}$ are the inlet and exhaust flow rates through the feeding chamber respectively, $V_f$ is the feeding chamber volume, and $\beta$ is the fluid bulk modulus. For the exhaust phase, the continuity equation becomes,

$$-Q_{\text{inc}} - Q_{\text{exf}} = \frac{V_f}{\beta} \frac{dP_f}{dt}$$  \hfill (4.18)

In this case, $Q_{\text{exf}}$ is the exhaust flow rate from the feeding chamber to the exhaust port.

**Command chamber continuity equation**

The command chamber receives pressurized hydraulic fluid from the solenoid valve. Therefore, the pressure inside the command chamber depends on the flow variation at the output port of the solenoid valve, which is described by equation (4.7). In this case,

$$Q_{\text{sol,net}} - \dot{x}_{\text{pcv}} A_{p_{\text{pcv},c}} = \frac{V_c}{\beta} \frac{dP_{\text{sol}}}{dt}$$  \hfill (4.19)

$V_c$ is the volume of the command chamber and is a function of the spool position.

$$V_c = A_{p_{\text{pcv},c}} x_{\text{pcv}} + V_{c,\text{ini}}$$  \hfill (4.20)

$A_{p_{\text{pcv},c}}$ is the cross sectional area of the land at the command chamber, and $V_{c,\text{ini}}$ is the initial volume of the command chamber.

**Flow rate through the valve inlet port**

$$Q_{\text{inf}} = C_d A_{\text{in}} (x_{\text{vin}}) \sqrt{\frac{2(P_{\text{line}} - P_f)}{\rho}}$$  \hfill (4.21)

$$A_{\text{in}} (x_{\text{vin}}) = (x_{\text{pcv}} - x_{\text{in}}) \pi d_{\text{sp1}}$$  \hfill (4.22)
In this case, $x_{in}$ is the displacement of the spool when the supply port is opened, from the spool’s initial position. $d_{sp1}$ is the diameter of the valve land at the supply port.

**Flow rate through the valve exhaust port**

$$Q_{ex} = C_d A_{ex}(x_{vex}) \sqrt{\frac{2P_t}{\rho}}$$  \hspace{1cm} (4.23)

$$A_{ex}(x_{vex}) = (x_{out} - x_{pvc})\pi d_{sp2}$$  \hspace{1cm} (4.24)

The exhaust port is initially open when the valve is at rest. As the valve moves from left to right, Figure 4.7, the exhaust port is closed. Thus, in equation (4.24), $x_{out}$ is the displacement of the valve from left to right till the exhaust is closed. Here, $d_{sp2}$ is the diameter of the valve land at the supply port.

**Flow rate output to the clutch/accumulator assembly**

As shown in Figure 4.1, we can lump all pressure drops along the line from the pressure control valve to the clutch and accumulator system, as well as any physical orifice restrictions, into one fluid restriction, the so-called “supply orifice”. Again, this is modeled as a sharp-edged orifice where its area is an effective area to represent all fluid-restrictions and pressure drops.

$$Q_{inc} = C_d A_{inc} \sqrt{\frac{2(P_t - P_c)}{\rho}}$$  \hspace{1cm} (4.25)

Here, $A_{inc}$ is the clutch/accumulator effective inlet orifice area, and $P_c$ is the clutch pressure. Note that, since $C_d$ is empirically adjusted anyway, we may use $A_{inc}$ to represent the physical orifice, without loss of generality.
4.2.3 Clutch and Accumulator Dynamics

For all plate clutches, the model is a simple piston-cylinder model as previously shown in Figure 4.1. Note that, for the particular transmission of interest, rotating clutches are used. However, due to the physical design of the clutch, which has a feature for balancing the centrifugal forces acting on the fluid in the clutch cavity (Usuki et al., 1996), our assumption of using the simple piston-cylinder model to represent the clutch behavior is valid. The free body diagram for the clutch and accumulator assembly is shown in Figure 4.8. In the figure for the clutch, the spring represents a mechanical spring, and the damper represents the damping force created by fluid viscosity. In the case of the accumulator, there are 2 return springs involved, except for the overdrive clutch. The smaller and shorter spring is placed inside the bigger and taller one. The initial preload comes from the bigger spring. There is also a damper to represent the viscous damping force exert by the hydraulic fluid. Note that both damping coefficients of the clutch and accumulator are unknown and will be parameters to be tuned during the model validation process. By neglecting the clutch plate mass, the equation of motion for the clutch piston is:

\[ B_c \ddot{x}_c + F_{sc} + F_{sc0} = P_c A_c \]

where \( A_c \) is the clutch piston cross sectional area, \( F_{sc0} \) is the clutch return spring preload, \( K_c \) is the clutch return spring constant, and \( B_c \) is the viscous damping coefficient for the clutch motion. Also,

\[ F_{sc} = K_c x_c \]  

\[ x_c = Clutch \ piston \ displacement \]

\[ = 0 \quad \text{if} \quad P_c A_c < F_{sc0} \]

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For the accumulator, except for the UD clutch, there are 2 linear springs involved in the motion of the accumulator piston. The spring with the larger diameter has a higher setting height and the one with the smaller diameter is placed inside the first one. Therefore, the motion of the accumulator piston may be considered in two parts. The first part of the stroking involves only the outer spring and the second part is where both springs are compressed. Neglecting the mass of the accumulator piston, the equation of motion of the accumulator can be derived as:

\[ B_a \ddot{x}_a + F_{aso} + F_{asi} = P_c A_a \]  

(4.29)

where \( M_a \) is the accumulator piston mass, \( A_a \) is the accumulator piston cross sectional area, \( B_a \) is the viscous damping coefficient for the accumulator motion, \( F_{aso0} \) is the outer return spring preload, and \( F_{asi} \) is the inner return spring force. Moreover,

\[
x_a = \text{Accumulator piston displacement}
\]

\[
= 0 \quad \text{if} \quad P_c A_a < F_{aso0}
\]  

(4.30)

\[
F_{aso} = F_{aso0} + K_{ao} x_a
\]  

(4.31)

\[
K_{ao} = \text{Outer return spring constant}
\]

\[
K_{ai}(x_a - x_{ai}) \quad \text{if} \quad x_{ai} < x_a < x_{a,max}
\]

(4.32)

\[
x_{ai} = \text{Piston displacement when the inner return spring starts stroking}
\]

\[
K_{ai} = \text{Inner return spring constant}
\]
Clutch/accumulator chamber continuity equation

The dynamics of clutch pressure depend on the difference between flow rates into and out of the clutch and accumulator cavity. However, before the clutch pressure starts to increase, the initial cavity has to be filled first. This filling time creates a time delay in the clutch pressure dynamic response. Unfortunately, this initial cavity volume is variable in practice, and depends on many factors such as past history of engaging and disengaging of the clutch which may relate to residual fluid in the clutch and accumulator cavity, the speed of the vehicle, fluid temperature, gear ratio, etc. This filling time is approximately between 50-180 milliseconds, (Cho, 1987). From examination of experimental data on clutch pressure response alone, the filling phase cannot be detected. Therefore, the effect of the filling time will be neglected here, and we assume that the
clutch is already fully filled when the pressure command is applied to the pressure control
valve. By ignoring the filling time, the volumetric flow rates to the clutch and
accumulator chambers are:

\[ Q_c = \dot{x}_c A_c \]  
\[ Q_a = \dot{x}_a A_a \]  

(4.33)  
(4.34)

The dynamics of the clutch pressure can be derived as:

\[ \frac{dP_c}{dt} = V_c \frac{dP_c}{dt} \]  

(4.35)

where \( Q_c \) is the flow rate to the clutch chamber, \( Q_a \) is the flow rate to the accumulator
chamber, and

\[ V_c = \text{Volume of the fluid at pressure } P_c \]
\[ = x_c A_c + x_a A_a \]  

(4.36)

4.2.4 Supply Pressure Regulation System

The simplified schematic diagram of the pressure regulation system is shown in
Figure 4.9. For the particular transmission of interest, a gerotor pump is used to feed the
hydraulic fluid through the entire system. The pump characteristic is given by the
manufacturer, and characterizes the flow rate output of the pump as a function of engine
velocity. Therefore, a table look-up is used to represent pump behavior. In Figure 4.9, the
pressure inside the main chamber of the spool is a controlled pressure which depends on
the spool position and equals the line pressure.
Figure 4.9: Simplified schematic diagram of the line pressure regulation system

To analyze the behavior of the line pressure regulation system, we consider first the dynamics inside the main chamber. The flow from the pump is fed to the supply port, I1. If the exhaust port Ex2 is opened, part of the in-flow from the pump leaks out to the A-chamber. Also, if the exhaust port Ex1 is opened, part of the fluid inside the main chamber flows out as well. During normal operation of the transmission between 1st and 4th gears, the spool is positioned such that the both Ex1 and Ex2 are open. Therefore, the flow forces due to the exhaust flow through Ex1 and Ex2 have to be considered. Since the variation of the spool position from its equilibrium is small and relatively slow, only
the static flow force is considered here. The flow force caused by the exhaust flow at Ex1
tends to close the port and can be calculated from the following equation (Merritt, 1967).

\[ F_{\text{stoi}} = 0.43\pi d_{\text{Ex1}}(x_{\text{rv}} - x_{\text{Ex1}})K_{\text{f1}}P_{\text{line}} \]  \hspace{1cm} (4.37)

where \( d_{\text{Ex1}} \) is the diameter of the spool land at the port Ex1, \( x_{\text{Ex1}} \) is the displacement of
the spool for the port Ex1 to open, \( K_{\text{f1}} \) is a constant factor adjusted to match the
experimental data, and \( x_{\text{rv}} \) is the spool displacement.

On the other hand, due to the shape of the spool body at the A-chamber, which is
built to compensate the negative flow force at the port Ex1, the flow force caused by the
exhaust flow through the port Ex2 tends to move the spool to the left and open the port
Ex2. Based on the valve dimension shown in Figure 10 below, the flow force in this case
can be calculated from the following equation.

\[ F_{\text{stoi2}} = 2C_{\text{d}}\pi d_{\text{Ex2}}(x_{\text{rv}} - x_{\text{Ex2}})P_{\text{line}}(\cos \theta_1 - \cos \theta_2) \]  \hspace{1cm} (4.38)

where \( d_{\text{Ex2}} \) is the diameter of the spool land at the port Ex2, \( x_{\text{Ex2}} \) is the displacement of
the spool for the port Ex2 to open, \( \theta_1, \theta_2 \) are obtained from the A-chamber configuration
(see Figure 4.10), and \( C_{\text{d}} \) is the discharge coefficient, which is 0.61 for a sharp-edged
orifice. Since the diameter of the spool land at both ends of the main chamber is the
same, i.e. \( d_{\text{Ex1}} = d_{\text{Ex2}} \), we assume that there is no force contributed by the pressure inside
the main chamber, \( P_{\text{line}} \). The flow through ports Ex1 and Ex2 can be calculated from

\[ Q_{\text{Ex1}} = C_{\text{d}}\pi d_{\text{Ex1}}(x_{\text{rv}} - x_{\text{Ex1}})\sqrt{\dfrac{2P_{\text{line}}}{\rho}} \]  \hspace{1cm} (4.39)

\[ Q_{\text{Ex2}} = C_{\text{d}}\pi d_{\text{Ex2}}(x_{\text{rv}} - x_{\text{Ex2}})\sqrt{\dfrac{2P_{\text{line}}}{\rho}} \]  \hspace{1cm} (4.40)
Ignoring all leakage flows inside the system, the net flow into the pressure regulator valve consists of the supply flow from the pump, the exhaust flows described by equations (4.39) and (4.40), the flow supplying the solenoid operation, and the flow supplying the pressure control valve. For the flow to the solenoid, the regulator system supplies pressurized fluid to all 4 solenoids, one for each clutch. This flow for each clutch can be calculated by using equation (4.7). Similarly, the regulator system supplies pressurized fluid to all 4 pressure control valves for the 4 clutches, and the flow for each pressure control valve can be calculated using equations (4.21). Therefore,

\[
Q_{rv,\text{net}} = Q_{\text{pump}} - Q_{E_{x1}} - Q_{E_{x2}} - \sum Q_{\text{sol,in}} - \sum Q_{\text{inf}}
\]  

(4.41)

Here, i=LR, 2ND, UD, OD. The pressure variation inside the main chamber can then be calculated.

\[
Q_{rv,\text{net}} = \frac{V_{rv,\text{main}}}{\beta} \frac{dP_{\text{line}}}{dt}
\]  

(4.42)
Here, we have $Q_{rv,net}$ to represent the net flow into the system, $Q_{Ex1}$ is the flow to the port Ex1, $Q_{Ex2}$ is the flow to the port Ex2, $Q_{pump}$ is the supply flow from the pump, and $V_{rv,main}$ is the main chamber volume.

We consider now the spool force developed at the B-chamber. From Figure 4.9, the B-chamber is connected to the overdrive clutch by the fluid passage through the switching valve. When the transmission is operated in 1st or 2nd gear, the overdrive clutch is not activated and there is no pressurized fluid flowing from the overdrive clutch to the switch valve. In this case, the switch valve is pushed to the left by the line pressure, blocking the fluid path from the overdrive clutch to the B-chamber at the pressure regulator spool. When the overdrive clutch is activated, the transmission is operated in either 3rd or 4th gear, part of the pressurized fluid going to the overdrive clutch flows to the switch valve and pushes the switch valve to the right, opening the fluid path from the overdrive clutch to the B-chamber. Therefore, pressurized fluid flows from the switch valve, passes to the fluid orifice restriction, and goes into the B-chamber. In this case, the fluid pressure in the B-chamber can be calculated as follows. At the switch valve, we assume that when the pressure from the overdrive clutch is high enough to push the switch valve to the right, the switch valve moves instantaneously.

$$Q_{rv,B} = \begin{cases} C_d A_{OD,B} \sqrt{\frac{2P_{OD} - P_{rv,B}}{\rho}} \text{sgn}(P_{OD} - P_{rv,B}), & \text{if } P_{OD} A_{sw,OD} > P_{line} A_{sw,line} \\ 0, & \text{if } P_{OD} A_{sw,OD} \leq P_{line} A_{sw,line} \end{cases}$$  \hspace{1cm} (4.43)$$

where $Q_{rv,B}$ is the flow into the B-chamber, $A_{OD,B}$ is the cross sectional area of the orifice at the entrance of the B-chamber, $P_{OD}$ is the overdrive clutch pressure, $P_{rv,B}$ is the pressure inside the B-chamber, $A_{sw,OD}$ is the cross sectional area of the switch valve at the
overdrive pressure side, and \( A_{\text{sw,line}} \) is the cross sectional area of the switch valve at the line pressure side. Therefore, the pressure inside the B-chamber and the resulting force acting on the spool can be calculated as

\[
Q_{rv,B} = \frac{V_{rv,B}}{\beta} \frac{dP_{rv,B}}{dt} \tag{4.44}
\]

\[
F_{rv,B} = P_{rv,B} \Delta A_{rv,B} \tag{4.45}
\]

For the case of the C-chamber and D-chamber, when the transmission is operated between 1st to 4th gear, transmission fluid at line pressure always fills these two chambers. Due to the differences in the spool lands around both chambers, the pressure from the transmission fluid creates a force pushing the spool to the left, and this force can be calculated as

\[
F_{rv,C} = P_{\text{line}} \Delta A_{rv,C} \tag{4.46}
\]

\[
F_{rv,D} = P_{\text{line}} \Delta A_{rv,D} \tag{4.47}
\]

In equations (4.44) to (4.47), \( F_{rv,B} \) is the force created by the pressurized fluid in B-chamber, \( F_{rv,C} \) is the force created by the pressurized fluid in C-chamber, \( F_{rv,D} \) is the force created by the pressurized fluid in D-chamber, \( \Delta A_{rv,B} \) is the area difference between the spool lands around B-chamber, \( \Delta A_{rv,C} \) is the area difference between the spool lands around C-chamber, and \( \Delta A_{rv,D} \) is the area difference between the spool lands around D-chamber.

The force developed in the E-chamber is ignored since there is no pressurized fluid entering this chamber during 1st to 4th gear. The equation of motion describing the
spool dynamics can be developed now. The spool displacement depends on the force from the pressurized fluid in each spool chamber as described by equations (4.37) and (4.38), and equations (4.45) to (4.47). Considering spool inertia as well as the damping force from friction between the spool and sleeve, the equation of motion of the spool can be written as follows.

\[
M_{rv} \ddot{x}_{rv} + B_{rv} \dot{x}_{rv} + K_{rv} x_{rv} = \sum F
\]

\[
\sum F = F_{sf1} - F_{sf2} + F_{rv,B} + F_{rv,C} + F_{rv,D} - F_{srv,ini}
\]

where \( M_{rv} \) is the mass of the spool, \( B_{rv} \) is the damping coefficient of the spool, \( K_{rv} \) is the spool return spring constant, and \( F_{srv,ini} \) is the return spring preload.

### 4.3 Hydraulic Model Simplification

The model we derived in the previous section is rather complex, highly nonlinear and high order, which is not suitable for use in controller and observer development. Therefore, the second part of the modeling work involves model simplification, which is emphasized in this section. Since the model is highly nonlinear, small signal linearization cannot be done since the system has a wide range of operating conditions, and therefore many model order reduction methods in the literature, which normally work very well for linear systems, cannot be applied here. Louca et al. (1997) introduced an energy-based model order reduction method in which the dominant dynamics of a system are identified by the energy contributions of individual elements as compared to the rest of the system. Due to its simplicity and applicability to nonlinear systems, this method is used here.
Briefly, in this method, the energy flow through each energy component of the system is measured and is called the activity of the component. Then the activity index is measured by dividing the activity of each component by the overall activity of the system.

\[
AI_i = \frac{A_i}{A_{Total}} = \frac{\int_0^t |P(t)| \cdot dt}{\sum_{i=1}^{k} \left\{ \int_0^t |P(t)| \cdot dt \right\}}, \quad i = 1, \ldots, k
\] (4.50)

Here, \( k \) is the number of components, \( P(t) \) is power through each component, and \( AI \) is the activity index. The model order is reduced by eliminating elements with low values of the activity index. This method is applied to the detailed model of the hydraulic system presented in previous sections and measures the activity index of each energy element for both cases corresponding to when the clutch is engaging as well as when it is disengaging. Energy elements with high activity index are selected by using the results from both cases. Table B.2 gives results for two selected trajectories.

<table>
<thead>
<tr>
<th>Clutch disengaging</th>
<th>Clutch engaging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic capacitance</td>
<td>37.51%</td>
</tr>
<tr>
<td>Electrical resistance</td>
<td>30.13%</td>
</tr>
<tr>
<td>Supply flow to clutch</td>
<td>15.06%</td>
</tr>
<tr>
<td>Accumulator spring</td>
<td>12.65%</td>
</tr>
<tr>
<td>Flow out from PCV to solenoid</td>
<td>1.79%</td>
</tr>
<tr>
<td>Flow in from solenoid to PCV</td>
<td>1.15%</td>
</tr>
<tr>
<td>Solenoid spring</td>
<td>0.30%</td>
</tr>
</tbody>
</table>

Table 4.1: Energy element with high activity index when the clutch is engaging and when it is disengaging
Note that inputs used for this evaluation are collected from the test vehicle under different test conditions, e.g. when clutches engage or disengage during up shift, down shift, and skipped shift, to ensure the consistency of the resulting energy index. Based on these results, the simplified model is a 3\textsuperscript{rd} order system and the mathematical derivation of the simplified model is shown below.

**Solenoid Valve**

By ignoring the spool inertia and the friction force, the spool displacement is algebraically related to the flux as follows:

\[
F_{\text{solp}} = F_{\text{mag}} \quad (4.51)
\]

where

\[
F_{\text{solp}} = K_{\text{solp}} x_{\text{solp}} \quad (4.52)
\]

\[
F_{\text{mag}} = \frac{1}{2} \mu_o A \phi^2 \quad (4.53)
\]

Substituting (4.52) and (4.53) in (4.51), we have

\[
x_{\text{sol}} = \frac{1}{2} \mu_o A K_{\text{solp}} \phi^2 \quad (4.54)
\]

The dynamics of the flux are described by

\[
\dot{\phi} = \frac{V_{\text{in}}}{N} \cdot \frac{\phi}{\mu_o A N^2} (x_{\text{solp,max}} - x_{\text{solp}}) \quad (4.55)
\]

In this equation, \( N \) is the number of turns in the coil, and \( V_{\text{in}} \) is an applied voltage. By substituting equation (4.53) in (4.54), we have
\[ \phi = \frac{V_{in}}{N} - \frac{\varphi}{\mu_o A N^2} (x_{sol, \text{max}} - \frac{1}{2} \frac{\varphi^2}{\mu_o A K_{solp}}) \]  

\( x_{sol, \text{max}} \) is the maximum displacement of the solenoid plunger. Flows from the solenoid to the command chamber at the pressure control valve, PCV, can be described by

\[ Q_{sol, \text{net}} = Q_{sol, \text{in}} - Q_{sol, \text{ex}} \]  

(4.57)

\[ Q_{sol, \text{in}} = C_d A_{sol, \text{in}} (x_{sol}) \sqrt{\frac{2(P_{line} - P_{sol})}{\rho}} \]  

(4.58)

\[ Q_{sol, \text{ex}} = C_d A_{sol, \text{ex}} (x_{sol}) \sqrt{\frac{2P_{sol}}{\rho}} \]  

(4.59)

And for the flow calculation,

\[ A_{sol, \text{in}} = \begin{cases} \pi d_{sol, \text{in}} (x_{sol, \text{max}} - x_{sol}) & \text{when } x_{sol} \leq x_{\text{ballasted}} \\ 0 & \text{when } x_{sol} \geq x_{\text{ballasted}} \end{cases} \]  

(4.60)

\[ A_{sol, \text{ex}} = \pi d_{sol, \text{ex}} x_{sol} \]  

(4.61)

**Pressure control valve, PCV**

From the energy analysis shown earlier, the spring force and the friction force acting on the spool are insignificant. Therefore, the pressure at the feeding chamber and the command chamber of the spool are algebraically related to one another by the ratio of the pressurized area of these two chambers. Also, since the compressibility effect at the feeding chamber is negligible, we assume now that there is no pressure drop at this chamber. Thus, the pressure inside this chamber will instantaneously equal the supply line pressure as the inlet port is opened. Therefore
\[ P_f = P_{line} = a P_{sol} \quad \text{where} \quad a = \frac{\Delta A_{pcv1}}{\Delta A_{pcv2}} \quad (4.62) \]

During the flow-in phase, the flow from the feeding chamber goes to the supply orifice and then to the clutch and accumulator system. By ignoring the pressure drop at the feeding chamber, the flow to the clutch can be calculated from the following.

\[ Q_{inc} = C_d A_{inc} \sqrt{\frac{2(P_{line} - P_c)}{\rho}} \quad (4.63) \]

Here \( A_{inc} = \text{equivalent supply orifice area} = \frac{A_{in} (x_{pcv}) A_{inc}}{\sqrt{A_{in} (x_{pcv})^2 + A_{inc}^2}} \quad (4.64) \)

Similarly, during the flow-out phase, the PCV opens the exhaust port and the fluid from the clutch is exhausted, the flow being calculated from

\[ Q_{exc} = -C_d A_{exc} \sqrt{\frac{2(P_c)}{\rho}} \quad (4.65) \]

Here \( A_{exc} = \text{equivalent supply orifice area} = \frac{A_{ex} (x_{pcv}) A_{inc}}{\sqrt{A_{ex} (x_{pcv})^2 + A_{inc}^2}} \quad (4.66) \)

The position of the PCV spool can be calculated based on the flow from the solenoid valve.

\[ \dot{x}_{pcv} = \frac{1}{\Delta A_{pcv1}} (Q_{sol,net}) \quad (4.67) \]
Clutch and accumulator dynamics

The dominant dynamics of the system depend on the accumulator spring. Ignoring all friction forces and the clutch dynamics, we are left with the algebraic relationship between the clutch pressure and the accumulator piston position.

\[ P_c A_a = K_a x_a \quad (4.68) \]

Based on the flow through the system, we have

\[ Q_{\text{inc}} = \dot{x}_a A_a \quad (4.69) \]

Therefore, we have the following. During the flow-in phase

\[ \dot{x}_a = \frac{1}{A_a} \left( C_d A_{\text{inc}} \sqrt{\frac{2(P_{\text{inc}} - P_c)}{\rho}} \right) \]

Or,

\[ \dot{x}_a = \frac{1}{A_a} \left( C_d A_{\text{inc}} \sqrt{\frac{2 \left[ P_{\text{line}} - \frac{K_a x_a}{A_a} \right]}{\rho}} \right) \quad (4.70) \]

And for the flow-out phase

\[ \dot{x}_a = -\frac{1}{A_a} \left( C_d A_{\text{exc}} \sqrt{\frac{2P_c}{\rho}} \right) \]

Or,

\[ \dot{x}_a = -\frac{1}{A_a} \left( C_d A_{\text{exc}} \sqrt{\frac{2 \left[ \frac{K_a x_a}{A_a} \right]}{\rho}} \right) \quad (4.71) \]
Based on the results shown here, the order of the baseline model of each clutch is reduced from 9\textsuperscript{th}-order to 3\textsuperscript{rd}-order. The same method is also applied to the pressure regulation system and the order of the model is reduced from 4\textsuperscript{th}-order to 2\textsuperscript{nd}-order. Therefore, to simulate the behavior of one clutch, the total model order is reduced from 13\textsuperscript{th}-order to 5\textsuperscript{th}-order. However, it can be noticed that even though the order of the baseline model is reduced, the simplified model is still highly nonlinear due to saturation, the orifice flow formulation, as well as the change of the model between the flow-in and flow-out phases. Note also that the clutch pressure is no longer a state variable since the fluid compressibility is negligible. Instead, clutch pressure is implicitly determined by the position of the accumulator as is shown in equation (4.67). In the physical system, the clutch piston is fully compressed shortly after the clutch is engaged. After this point, as the fluid continuously flows into the clutch chamber, the clutch pressure will increase. With the accumulator, the clutch pressure will not rise instantaneously. The behavior of the clutch pressure during this period depends on the damping characteristic of the accumulator. Therefore, the reduced order model does capture the physical behavior of the system. The validation results for the simplified model are shown in the subsequent sections.

4.4 Model Validation

The dynamic models for the subsystems developed previously are combined and validated in this section. Matlab/Simulink\textsuperscriptregistered} software is used to construct the simulation model. The experimental data used to validate the model is collected from a test car where pressure sensors are installed to measure pressures in all clutches. To validate the
model, the duty cycle command used to control the clutch in the test car is used as an input to the simulation model. Then, the simulated pressure for each clutch is compared to the data from the experiment. Note that, for the transmission of interest, the hydraulic line pressure regulation system is controlled mechanically. Therefore, there is no pressure sensor to measure the line pressure. However, the line pressure regulation system can be validated by assuming that when the clutch is fully engaged, the clutch pressure equals the line pressure. Therefore, we can compare the simulated line pressure, $P_{\text{line}}$, with the pressure data of the clutch that is fully engaged during any gear shifting process.

4.4.1 2-regime flow equation

For all models involving orifice flows here, the turbulent orifice flow model is used. However, it is well known that the simulation of the turbulent orifice flow model always causes problems with numerical integration when the pressure drop across the orifice is close to zero. As a result, the simulation is either very slow or it stops. This is because the turbulent orifice flow model has an infinite derivative when the pressure drop across the orifice is zero, which makes the system equations stiff (Ellman and Piche, 1996). Therefore, the flow model has to be changed from turbulent flow to laminar flow when the pressure difference across the orifice is close to zero. In this work, the following modified flow model is used.

$$Q = \begin{cases} C_d A \sqrt{\frac{2|\Delta P|}{\rho} \text{sgn}(\Delta P)}, & \text{when } R > R_{\text{tr}} \\ \frac{2\delta^3 D A}{\mu} (\Delta P), & \text{when } R \leq R_{\text{tr}} \end{cases} \quad (4.72)$$
Here, $C_d$ is the discharge coefficient, $A$ is the orifice cross sectional area, $D$ is the orifice diameter, $\Delta P$ is the pressure drop across the orifice, and $R$ represents Reynolds number. $R_{tr}$ is the transition Reynolds number separating the turbulent flow and the laminar flow, which in this case the value of 10 is used for a round orifice (Merritt, 1967). $\mu$ is the transmission fluid viscosity, $\rho$ is the transmission fluid density, and $\delta$ is a constant factor depending on orifice geometry, which is of 0.2 for a round orifice (Merritt, 1967). All flow calculations used in the model developed here are modified thus. Some other forms of 2-regime models, which normally give smoother transitions between flow regimes than equation (48), can be found in the literature (Ellman and Piche, 1996; Kremer, 1998).

### 4.4.2 Parameter determination

In this study, the hardware transmission is disassembled such that most of parameters used in the developed model can be measured physically. However, some parameters such as the damping coefficient of the spool valve, the damping coefficient of the clutch and accumulator, and the discharge coefficient for the flow calculation cannot be determined exactly from the physical hardware. Therefore, these unknown parameters are tuned to fit the experimental data. For the damping coefficient of the spool valve, a theoretical value can be calculated based on the tolerance between spool and sleeve, and is used as an initial guess. The damping coefficient for the clutch and the accumulator pistons cannot be calculated from the hardware and is tuned by trial-and-error. For the discharge coefficient, since all of the flow equations are developed based on the sharp-edge orifice flow model, the theoretical value, of $C_d$ of 0.61 is used. However, due to the complexity of the fluid passages inside the hydraulic system of the transmission, we
expect that the assumption of a sharp-edged orifice flow model with $C_d = 0.61$ may not hold. Therefore, we leave the discharge coefficient as a tuned parameter to fit the model with the experimental data as well. The list below shows all parameters expected to be tuned during model validation.

- $B_{pcv}$ = friction coefficient between spool and sleeve of the pressure control valve
- $B_{rv}$ = friction coefficient between spool and sleeve of the pressure regulator valve
- $B_c$ = viscous damping coefficient for the clutch motion
- $B_a$ = viscous damping coefficient for the accumulator motion
- $C_{d,inf}$ = discharge coefficient for the flow into the pressure control valve (equation (4.21))
- $C_{d,exf}$ = discharge coefficient for the exhaust flow from the pressure control valve (equation (4.23))
- $C_{d,inc}$ = discharge coefficient for the flow into the clutch and accumulator system (equation (4.25))
- $C_{Ex1}$ = discharge coefficient for the exhaust flow from the main chamber of the pressure regulator valve
- $C_{Ex2}$ = discharge coefficient for the exhaust flow from the main chamber of the pressure regulator valve
- $C_{rv,B}$ = discharge coefficient for the flow into the B-chamber of the pressure regulator valve

Note that, since the solenoid model is already tuned to fit the experimental data when we developed the model, it is not tuned again.
4.4.3 Validation without the line pressure regulation system and the solenoid valve model

As can be seen, the developed model has many tuned parameters, especially when we include all clutch models. Therefore, for the sake of simplicity, we would like to start model validation by considering only one clutch at a time. We also would like to see the effect of the solenoid dynamics on system behavior. Intuitively, solenoid response is relatively fast as compared to the rest of the system. Therefore, ignoring the solenoid model seems to be a reasonable thing to do. Therefore, in this section, we ignore solenoid valve dynamics first. However, we have already shown that the solenoid dynamics involve change of the pressure at the command chamber of the pressure control valve. To account for this relationship, we assume that when the pulse signal from the duty cycle command opens the solenoid valve, the fluid fills the command chamber of the pressure control valve instantaneously and the pressure inside the chamber equals the supply line pressure. Similarly, when the pulse signal from the duty cycle command closes the solenoid valve, the pressure inside the command chamber drops to zero instantaneously. Moreover, since the dynamics of the pressure regulation system take into account flow to all clutches, the model for the pressure regulation system cannot be used to validate the model for one clutch. To solve this problem, a table look-up is used to determine the supply line pressure for different operating conditions. Particularly, the supply line pressure for the 1\(^{\text{st}}\) and 2\(^{\text{nd}}\) gears is 150 psi, and for the 3\(^{\text{rd}}\) and 4\(^{\text{th}}\) gears is 90 psi. The following parameters will be tuned to match the model response with the experimental data.

\[ B_{pcv} = \text{friction coefficient between spool and sleeve of the pressure control valve} \]
\( B_c \) = viscous damping coefficient for the clutch motion

\( B_a \) = viscous damping coefficient for the accumulator motion

\( C_{d,\text{inf}} \) = discharge coefficient for the flow into the pressure control valve (equation (4.21))

\( C_{d,\text{exf}} \) = discharge coefficient for the exhaust flow from the pressure control valve (equation (4.23))

\( C_{d,\text{inc}} \) = discharge coefficient for the flow into the clutch and accumulator system (equation (4.25))

As an example, the model of the 2ND clutch is considered here. For this test, the experimental data we use here is for the gear up-shift from 3rd to 4th gear where the 2ND clutch is engaged, and for the gear down-shift from 4th to 3rd gear where the 2ND clutch is released. From physical measurement of the 2ND clutch components, the initial value for \( B_{pcv} = 0.015 \) N/m/sec. All discharge coefficients are assumed to be at the theoretical value, i.e. 0.61. After trial-and-error tuning of all parameters, Figures 4.11 and 4.12 show examples of the simulation response compared to the experimental data of the 2ND clutch when it is engaged and released, respectively. The duty cycle command is also shown in both figures. The parameter tuning results are shown in Table 4.2. As can be seen from both figures, the model predicts the clutch behavior with acceptable accuracy. Note that a lag time of 30 milliseconds can be seen in Figure 4.11 between the duty cycle command and the clutch pressure from the experimental data, at the beginning of the command. This lag time is due to the communication time from the transmission controller to the hydraulic system. Therefore, we shift our simulation response by 30 milliseconds to match the experimental data. As shown in Table 4.2, the numerical values
of the tuned parameters are unrealistic. Specifically, we have to use different clutch damping coefficients during clutch engaging and releasing. The value of the discharge coefficients $C_{d,\text{inc}}$ and $C_{d,\text{inf}}$ are too high, while $C_{d,\text{exf}}$ is unrealistically low. This set of parameters is also not robust to change in operating conditions. That is, the model does not give equivalent acceptable accuracy as shown in Figures 4.11 and 4.12 for different sets of operating conditions. Similar results are obtained for other clutches, but are not shown here.

The error we see from the simulation results suggests that there is an error in the modeling, and/or there are some dynamics that we have not considered. Omission of the solenoid dynamics is a likely source of modeling error and unrealistic numerical values for the tuned parameters. Even though solenoid dynamics is relatively fast, i.e. the motion of the plunger is fast, its physical dimensions, viz, the supply and exhaust orifices, affect the dynamics of the command pressure to the pressure control valve. In particular, due to the nature of the flow through the orifice, the command pressure cannot increase or decrease instantaneously as we have assumed here.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Second Clutch Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clutch parameters</td>
<td></td>
</tr>
<tr>
<td>$B_{\text{cin}} \times 10^3$ (N/m/sec)</td>
<td>8</td>
</tr>
<tr>
<td>$B_{\text{ceex}} \times 10^3$ (N/m/sec)</td>
<td>2.5</td>
</tr>
<tr>
<td>$C_{\text{d}}$</td>
<td>0.85</td>
</tr>
<tr>
<td>Accumulator parameters</td>
<td></td>
</tr>
<tr>
<td>$B_a \times 10^3$ (N/m/sec)</td>
<td>0.089</td>
</tr>
<tr>
<td>Pressure control valve parameters</td>
<td></td>
</tr>
<tr>
<td>$B_{\text{pcv}}$ (N/m/sec)</td>
<td>3.6</td>
</tr>
<tr>
<td>$C_{\text{dvin}}$</td>
<td>0.85</td>
</tr>
<tr>
<td>$C_{\text{dvex}}$</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 4.2: Numerical values for some tuned parameters for 2ND clutch
Figure 4.11: Simulation responses of the 2ND clutch compared to test car data during up shift from the 3\textsuperscript{rd} gear to the 4\textsuperscript{th} gear

Figure 4.12: Simulation responses of the 2ND clutch compared to test car data during down shift from the 4\textsuperscript{th} gear to the 3\textsuperscript{rd} gear
4.4.4 Validation of the model including solenoid dynamics

We now consider the case where the solenoid dynamics, as well as the dynamics of the command chamber of the pressure control valve, are included. All unknown parameters are tuned again to match the model response with the experimental data. The hydraulic pressure regulation system is still not considered here since we are working with only one clutch at a time. Therefore, a table look-up is still used to determine the supply line pressure for different operating conditions. To compare the result in this section with the previous simulation, we consider again the response of the 2ND clutch. Figure 4.13 shows the simulation result as compared to the experimental data for the case of 3rd to 4th gear up shift, when the pressure is rising, and for 4th to 3rd down shift, when the pressure is dropping. The numerical values for all tuned parameters are shown in Table 4.3. The simulation result shows a significant improvement of the model performance. The model predicts the clutch pressure dynamics well. Also, in Table 4.3, the tuned numerical values are more realistic. Compared with the previous test, the same value is used for the damping coefficient of the clutch when it is engaging as well as when it is disengaging. The values of the discharge coefficients are also reasonable theoretically. Moreover, the model is robust to change in the operating conditions. Figure 4.14 shows the response of the 2ND clutch for the same gear shift, but a different set of experimental data. The model still predicts clutch behavior reasonably well.

The results show that the solenoid dynamics and the dynamics of the command chamber of the pressure control valve are significant for the dynamic response of the hydraulic system. Even though the solenoid dynamics are relatively fast compared to the rest of the system, it involves a change of pressure at the command chamber of the
pressure control valve. Similarly good results are obtained for the responses of the other clutches, and model predictions match experimental responses well. The results are omitted here.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Second Clutch Parameters (New)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clutch parameters</td>
<td></td>
</tr>
<tr>
<td>( B_{cin} \times 10^3 \ (N/m/sec) )</td>
<td>8</td>
</tr>
<tr>
<td>( B_{cex} \times 10^3 \ (N/m/sec) )</td>
<td>None</td>
</tr>
<tr>
<td>( C_{din} )</td>
<td>0.7</td>
</tr>
<tr>
<td>Accumulator parameters</td>
<td></td>
</tr>
<tr>
<td>( B_{a} \times 10^3 \ (N/m/sec) )</td>
<td>0.089</td>
</tr>
<tr>
<td>Pressure control valve parameters</td>
<td></td>
</tr>
<tr>
<td>( B_{pcv} \ (N/m/sec) )</td>
<td>0.015</td>
</tr>
<tr>
<td>( C_{dvin} )</td>
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</tr>
<tr>
<td>( C_{dvex} )</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 4.3: Numerical values for some tuned parameters for 2ND clutch when solenoid dynamics are considered

Figure 4.13: Simulation responses of the 2ND clutch including solenoid dynamics compared to test car data during up shift from the 3\textsuperscript{rd} gear to the 4\textsuperscript{th} gear

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4.4.5 Validation of the Complete Model

We now combine the tuned models of all the clutches and include solenoid dynamics and the hydraulic line pressure regulation system model. Some parameters for the hydraulic pressure regulation system still need to be tuned, specifically, the damping coefficient of the regulator valve and the corresponding discharge coefficient, i.e. $B_{rv}$, $C_{E_{x1}}$, $C_{E_{x2}}$, and $C_{rv,B}$. Figures 4.15 to 4.19 show simulation results of the model compared to 2 sets of experimental data. The data used for the case shown in Figures 4.15 to 4.17 is for the case of up shift from 1st – 3rd gear. Figure 4.15 shows the simulation of the overdrive clutch and the low-reverse clutch. The low-reverse clutch decreases in pressure during 1st to 2nd gear shift. Figure 4.16 shows the simulation results for the overdrive clutch and the second clutch. Again, during the 1st to 2nd gear shift, the second clutch pressure is increased, and during the 2nd to 3rd gear shift, the second clutch pressure is
decreased. Figure 4.17 shows overdrive and underdrive clutch pressures. The overdrive clutch pressure is increased for the 2\textsuperscript{nd} to 3\textsuperscript{rd} gear shift. Figures 4.18 and 4.19 show simulation results for the shifting maneuver 4\textsuperscript{th}-3\textsuperscript{rd}-2\textsuperscript{nd}-3\textsuperscript{rd}-4\textsuperscript{th}. During this type of gear shift, there are four clutch involves (see Table 3.1). As for the case of Figures 4.15 to 4.17, we separate the plots to clearly show model performance.

From Figures 4.15-4.19, it is clear that the model has good accuracy and consistency in predicting clutch pressure dynamics. The performance of the hydraulic pressure regulation model can be seen also from Figures 4.14 to 4.16. That is, during the 1\textsuperscript{st} to 3\textsuperscript{rd} gear shift, the underdrive clutch is always fully engaged. Recalling our assumption that the clutch pressure when it is fully engaged equals the supply line pressure, Figures 4.14 to 4.16 show that the simulated clutch pressure for the underdrive clutch tracks the experimental data very well. This means that the model of the pressure regulation system can predict the behavior of the supply line pressure reasonably well too. The large error during the beginning of the simulation in every figure is the result of error in the initial state of the pressure regulation system model. It should also be noted that the measured clutch pressure when it is fully engaged is quite noisy, while the simulation response is smoother.

The hydraulic pressure regulation system is also important in its effect on the behavior of the clutch pressure dynamics as well as the overall dynamic system. The regulation mechanism maintains the supply line pressure at a desired level. Failing to maintain the pressure can cause the gear shifting process to fail if the pressure is too low, or cause a disturbing shift shock if the pressure is too high. We have shown that table look-up can be used to determine line pressure based on the hydraulic system
characteristic given by the manufacturer. However, table look-up does not account for the
dynamics of the flow variation during the gear shifting process. Therefore, a quantitative
model of the pressure regulation system is more useful and should be used in the analysis
of the transmission hydraulic system behavior.

Figure 4.15: Low-Reverse and Under-Drive clutch pressures for 1-2-3 up shift
Figure 4.16: Second and Under-Drive clutch pressures for 1-2-3 up shift

Figure 4.17: Over-Drive and Under-Drive clutch pressures for 1-2-3 up shift
Figure 4.18: Under-Drive and Over-Drive clutch pressures for 4-3-2-3-4 shift

Figure 4.19: Under-Drive and Second clutch pressures for 4-3-2-3-4 shift
4.4.6 Validation of the Simplified Model

As we did in the case of the baseline model, the simplified model is simulated and compared to the same experimental data used in the preceding section. Figures 4.20 and 4.21 show simulation results compared to experimental data for the $1^{st}$-$3^{rd}$ up shift, the experimental data being the same as in Figures 4.16 and 4.17, respectively. Similarly, Figures 4.22 and 4.23 show simulation results compared to experimental data for the $4^{th}$-$3^{rd}$-$2^{nd}$-$3^{rd}$-$4^{th}$ gear shifts, the experimental data being the same as in Figures 4.18 and 4.19, respectively. Based on the results shown, it is clear that the simplified model is less accurate than the baseline model. However, the degree of agreement with experimental data is still reasonably good, and probably adequate to justify use of the simplified model for real time application.

A step size of 1 millisecond and a fixed step-size numerical integration routine are used to simulate the simplified model. In the case of the more complex baseline model, the step size is as small as 0.1 millisecond, and numerical integration with a variable step-size has to be used. Thus, simulation time is significantly reduced when using the simplified model. Specifically, to simulate 10 seconds of system behavior, the simplified model takes 25 seconds of computation time on a 1 GHz processor Intel Pentium III computer whereas the baseline model takes more than 20 minutes of computation time on the same machine. Nevertheless, a simulation time of 25 seconds to simulate real system behavior over 10 seconds, as well as the use of a 1 millisecond step size, are still not good enough for the real-time application. The computer control unit on the current test vehicle runs with the step size as small as 16 milliseconds. Therefore, further refinement is still needed with the simplified model.
Figure 4.20: Second and Under-Drive clutch pressures for 1-2-3 up shifts

Figure 4.21: Over-Drive and Under-Drive clutch pressures for 1-2-3 up shifts
Figure 4.22: Under-Drive and Over-Drive clutch pressures for 4-3-2-3-4 shifts

Figure 4.23: Under-Drive and Second clutch pressures for 4-3-2-3-4 shifts
4.5 Conclusions

This chapter describes the detailed development of a nonlinear dynamic model for the shift hydraulic system of an automatic transmission which relies primarily on clutch-to-clutch shifts. It is shown that there are four important subsystems affecting the behavior of the hydraulic system: solenoid valve dynamics, pressure control valve dynamics, clutch and accumulator dynamics, and hydraulic pressure regulation system dynamics. The overall model is validated against experimental data. The effect of the solenoid valve dynamics on clutch pressure behavior is also evaluated. When solenoid valve dynamics are omitted, the numerical values resulting for the tuned parameters are unrealistic and the model is not robust to change in the operating conditions. By considering solenoid valve behavior, the resulting model is able to predict the hydraulic system behavior very well. The numerical values for all the tuned parameters are reasonable physically and theoretically. The model is also robust to change in the operating conditions.

Due to the complexity and nonlinearity of the fully detailed model, the model is simplified by using an energy based model order reduction method presented in this chapter. This method identifies the dominant dynamics of a system by observing the energy contributions of individual elements as compared to the rest of the system. The order of the model is reduced from 13 to 5. The validation shows that the simplified model is still able to predict the dynamic behavior of the shift hydraulic system with acceptable accuracy. The fully detailed model can be used for off-line applications such as controller validation, etc. For online applications, the simplified model can be used.
CHAPTER 5

OBSERVER DESIGN AND ONLINE IMPLEMENTATION

In this chapter, the development of an observer to estimate the variables necessary for clutch-to-clutch shift control is presented. The observer of interest is the nonlinear sliding mode observer, chosen for its applicability to nonlinear systems and accommodation of modeling uncertainties as well as ease of implementation. A review of sliding mode observer theory for state estimation is presented, including both continuous time and discrete time domain implementations. The proposed observer design for clutch pressure estimation, and turbine torque estimation is then presented, followed by evaluation on the basis of simulation and experimental results on a test vehicle.

5.1 Review of Sliding Mode Observer

In this section, a review of the approaches used in the development of the sliding mode observers potentially useful for our application is presented. As the sliding mode observer was originally developed by Utkin, we start our review of the observer schemes with the overview in Utkin (1992). We then review the observer scheme presented by Slotine et al. (1987) which emphasizes robustness considerations. The last approach that we review here is the observer developed by Walcott and Zak (1986, 1988). This
approach guarantees the stability of the estimation error and robustness in the presence of modeling uncertainty for a class of uncertain systems.

We consider first the sliding mode observer proposed by Utkin (1992), in which the objective is to force the state estimation error to zero using a discontinuous input, which is a function of an output estimation error. Consider a dynamic system with uncertainty described by

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + f(x,u,t) \\
y(t) &= Cx(t) \\
\end{align*}
\]  

where \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n} \), \( x(t) \) is a vector of state variables, \( u(t) \) is a vector of inputs, and \( y(t) \) is a vector of measured outputs. \( f(t): \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_+ \rightarrow \mathbb{R}^p \) represents bounded system uncertainty, with a known bound.

\[
\|f(x,u,t)\| \leq \rho \quad \forall x \in \mathbb{R}^n, u \in \mathbb{R}^m, t \geq 0
\]

The following are the assumptions used here:

- \( p \geq m \), the output signals are greater than or equal to the input signals, in number
- matrices \( B \) and \( C \) have full rank
- pair \( (A,C) \) is observable.

To construct the sliding mode observer, we assume for now that there is no uncertainty in the system, i.e. \( f(t) \equiv 0 \). We will relax this assumption later. Based on the assumptions above, we can assume that there exists a transformation matrix \( T \) such that the new output distribution matrix becomes

\[
CT^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix}
\]

(5.3)
The other system matrices can be written as

\[
TAT^{-1} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \quad \text{and} \quad TB = \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix}
\] (5.4)

Applying this change of coordinates, the original system can be written as

\[
\begin{align*}
\dot{x}_1(t) &= A_{11}x_1 + A_{12}y + Bu \\
\dot{y} &= A_{21}x_1 + A_{22}y + B_2u
\end{align*}
\] (5.5)

where

\[
\begin{bmatrix}
x_1 \\
y
\end{bmatrix} = Tx, \quad \text{and} \quad x_1 \in \mathbb{R}^{n-p}
\]

This coordinate transformation simply rearranges the order of the original state variables such that the measured states and the unmeasured states are separated.

The observer scheme has the following form.

\[
\begin{align*}
\dot{\hat{x}}_1(t) &= A_{11}\hat{x}_1 + A_{12}\hat{y} + B_1u + L_1L_2\text{sign}(\hat{y} - y) \\
\dot{\hat{y}} &= A_{21}\hat{x}_1 + A_{22}\hat{y} + B_2u - L_2\text{sign}(\hat{y} - y)
\end{align*}
\] (5.6)

We apply a further transformation that is normally employed in a reduced order observer. In particular, if we let \(z\) be a new state variable defined by

\[
\begin{equation}
z = x_1 + L_1y
\end{equation}
\] (5.7)

Then, the system in (4.5) becomes

\[
\begin{align*}
\dot{z} &= A'_{11}z + A'_{12}y + B'u \\
\dot{\hat{y}} &= A_{21}z + A_{22}y + B_2u
\end{align*}
\] (5.8)

where

\[
A'_{11} = A_{11} + L_1A_{21}, \quad A'_{12} = A_{12} + L_1A_{22}, \quad A'_{22} = A_{22} - A_{21}L_1, \quad \text{and} \quad B'_1 = B_1 - L_1B_2
\]

As a consequence, the observer system in (5.6) becomes,

\[
\begin{align*}
\dot{\hat{z}} &= A'_{11}\hat{z} + A'_{12}\hat{y} + B'_1u \\
\dot{\hat{y}} &= A_{21}\hat{z} + A_{22}\hat{y} + B_2u - L_2\text{sign}(\hat{y} - y)
\end{align*}
\] (5.9)
If the error vectors between the states and their estimates are defined as $e_y = \hat{y} - y$ and $e_1 = \hat{z}_1 - z_1$, then the estimation error dynamics can be written as

\[
\begin{align*}
\dot{e}_1 &= A'_1 e_1 + A'_2 e_y \\
\dot{e}_y &= A_{21} e_1 + A_{22} e_y - L_2 \text{sign}(e_y)
\end{align*}
\]  

(5.10)

For large enough $L_2$, it can be shown that a sliding motion can be induced on the error state $e_y$ in equation (5.10). The reaching condition, or the condition for sliding motion to exist for the observer (5.9), is

\[
e_y^T \dot{e}_y \leq 0
\]

(5.11)

When sliding mode is induced, after some finite time $t_s$, $e_y = 0$ and $\dot{e}_y = 0$. The error dynamics for the state $z = x_1 + L_1 y$ is then given by

\[
\dot{e}_1 = A'_1 e_1
\]

(5.12)

Since the pair $(A, C)$ is observable, the pair $(A_{11}, A_{21})$ is also observable. Therefore, $L_1$ can be chosen such that all the eigenvalues of $A'_{11} = A_{11} + L_1 A_{21}$ are negative.

From the observer development shown above, it can be seen that switching action, viz. the sign-function in equations (5.9) and (5.10), is used to make the error dynamics stable. The main difficulty with this scheme is the selection of the gain $L_2$ such that sliding motion is induced in finite time. Usually, to ensure existence of the sliding motion for arbitrary initial conditions of the states, the gain $L_2$ tends to be large. A large value of the gain $L_2$ also induces a chattering problem, which is undesirable.

Slotine et al. (1987) presented a similar approach in deriving the sliding mode observer structure. However, they proposed the use of linear gains together with switching action to extend the region where sliding motion occurs, the so-called “sliding
patch”. Details of the derivation will be omitted here, but some important concepts will be highlighted so as to compare this work to that of other researchers.

Consider a dynamic system of the following form

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1, x_2) + B_1 u(t) \\
\dot{x}_2 &= f_2(x_1, x_2) + B_2 u(t) \\
y &= x_2
\end{align*}
\]  

(5.13)

The function \( f_i \) can be used to represent a linear system as in equation (5.1), or it can be used to represent a system with nonlinearity and/or uncertainty. In either case, the function \( f_i \) is not required to be known exactly. However, the uncertainty \( |\Delta f_i| \) is assumed to be bounded by a known continuous function of its arguments.

The observer configuration in this case is in the form of

\[
\begin{align*}
\dot{\hat{x}}_1 &= \hat{f}_1(\hat{x}_1, \hat{y}) + B_1 u(t) - \alpha_y e_y - L_1 \text{sign}(e_y) \\
\dot{\hat{y}} &= \hat{f}_2(\hat{x}_1, \hat{y}) + B_2 u(t) - \alpha_y e_y - L_2 \text{sign}(e_y)
\end{align*}
\]  

(5.14)

In this case, \( e_y = \hat{y} - y \). The linear gain \( \alpha_y \) is designed based on the Luenberger observer using the linearized system corresponding to \( L_1 = 0, L_2 = 0 \). This also implies that the linearized system is observable. Based on equations (5.13) and (5.14), the error dynamics can be written as

\[
\begin{align*}
\dot{e}_1 &= \Delta f_1 - \alpha_y e_y - L_1 \text{sign}(e_y) \\
\dot{e}_y &= \Delta f_2 - \alpha_y e_y - L_2 \text{sign}(e_y)
\end{align*}
\]  

(5.15)

Here, \( \Delta f_i = \hat{f}_i - f_i \). As in the case of the scheme proposed by Utkin (1992), sliding motion exists if the following condition is satisfied.

\[
e_y^T \dot{e}_y < 0
\]  

(5.16)
\[ e^T \left( \Delta f_2 - \alpha_2 e_y - L_2 \text{sign}(e_y) \right) < 0 \]  

(5.17)

This condition is satisfied if we choose

\[ L_2 > |\Delta f_2| \]  

(5.18)

When the sliding mode is induced, after some finite time \( t_s, e_y = 0 \) and \( \dot{e}_y = 0 \).

Then, the error dynamics for \( e_1 \) become

\[ \dot{e}_1 = \Delta f_1 - L_1 \text{sign}(e_y) \]  

(5.19)

Based on the equivalent control concept proposed by Utkin (1992), system behavior after sliding motion occurs is equivalent to a system with the discontinuous term \( \text{sign}(e_y) \) being replaced by a continuous function which can be calculated from the error dynamics for \( e_y \). Specifically, assuming that \( e_y = 0 \) and \( \dot{e}_y = 0 \), from the error dynamics for \( e_y \), we have

\[ \text{sign}(e_y)_{\text{eq}} = \frac{\Delta f_2}{L_2} \]  

(5.20)

Substituting equation (5.20) in (5.19), we have

\[ \dot{e}_1 = \Delta f_1 - \frac{L_1}{L_2} \Delta f_2 \]  

(5.21)

From the result derived, the stability of the error dynamics for the unmeasured state vector \( x_1 \) is in question. Linear observer theory states that a linear system must be observable in order for any observer structure to be successful in reconstructing the states from the measurements. However, the concept of observability for a nonlinear system has not been well established. Intuitively, for the system described by equation (5.13) to be observable, \( f_2 \) must be a single valued function of \( x_1 \). It can also be seen from equation
if \( \Delta f_2 \) is a single valued function of \( e_1 \), then the term \( -(L_1/L_2)\Delta f_2 \) will have an influence on the dynamics of \( e_1 \). Even though this discussion indicates the need for restrictions on the structure of a nonlinear system for it to be observable, it also indicates that the observer can be robust to uncertainties.

Walcott and Zak (1986, 1988) presented a derivation of a sliding mode observer and sought to ensure global convergence of estimation error for a class of uncertain systems. Consider the uncertain system in the following form

\[
\dot{x}(t) = Ax(t) + Bu(t) + f(t, x, u) \\
y(t) = Cx(t)
\]

(5.22)

Again, \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n} \), \( x(t) \) is a vector of state variables, \( u(t) \) is a vector of inputs, and \( y(t) \) is a vector of measured outputs. \( f(t): \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n \) represents system modeling uncertainty. The following assumptions are used to further restrict the class of uncertain systems of interest.

- \( p \geq m \), or the output signals are greater than or equal to the input signals, in number
- matrices \( B \) and \( C \) have full rank
- pair \( (A,C) \) is observable.
- the uncertainty is matched or satisfies the “matching condition”

\[
f(t, x, u) = B\xi(t, x, u)
\]

(5.23)

where the function \( \xi: \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m \) is unknown but bounded, such that

\[
\|\xi(t, x, u)\| \leq r_1 \|u\| + \rho(t, y)
\]

(5.24)

\( r_1 \) being a known scalar and \( \rho: \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}_+ \) being a known function.
The goal is to estimate the states such that the error defined by \( e = \hat{x} - x \) is quadratically stable in the presence of the uncertainty.

The following assumptions are also used to develop the observer.

- there exists a \( G \in \mathbb{R}^{n \times p} \) such that all the eigenvalues of \( A_o = A - GC \) are stable
- there exists a Lyapunov pair \( (P,Q) \) for \( A_o \) such that the structural constraint

\[
C^TP^T = PB
\]

\[
A_o^TP + PA_o = -Q
\]

is satisfied for some \( F \in \mathbb{R}^{m \times p} \).

Based on all assumptions above, the observer is of the following form

\[
\dot{x}(t) = A\dot{x}(t) + Bu(t) - G(C\dot{x}(t) - y(t)) + P^{-1}C^TF^Tv
\]

(5.26)

where

\[
v = \begin{cases} 
-\gamma(t, y, u) & \text{if } FCe \neq 0 \\
0 & \text{otherwise} 
\end{cases}
\]

(5.27)

and the scalar function \( \gamma(.) \) is any function satisfying

\[
\gamma(t, y, u) \geq r_1 \| u \| + \rho(t, y) + \eta
\]

(5.28)

for some positive scalar \( \eta \).

Details of the proof showing the stability of the error can be found in Walcott and Zak (1988) and are omitted here. We first select a candidate Lyapunov function \( V(e) = e^T P e \), and substitute for the error \( e \) using the following error dynamics.

\[
\dot{e}(t) = (A - GC)e(t) - B\xi(t, x, u) + Bv
\]

(5.29)
The detailed derivation indicates the need for the matching condition for the uncertainty given by equation (5.23) and the structural constraint given by equation (5.25), and thus restricts the application of this scheme to a class of uncertain systems. Basically, the combination of the matching condition and the structural constraint ensures that uncertainties and/or nonlinearity terms affect only the dynamics of the measured states and hence their dynamics can be observed and compensated via the measurements. By comparison, the scheme proposed by Slotine et al. (1987) does not require the structural constraint though its ability to deal with uncertainties rests on their satisfying the same matching condition as in Walcott and Zak (1986, 1988).

Walcott and Zak’s work can be thought of as a special case of Slotine et al. (1987). To see this, consider the error dynamics equation (5.15) where, in order to satisfy the matching condition, $\Delta f_1$ is set to zero. As a result, the system has only the uncertainty $\Delta f_2$ and its effect shows only on the dynamics of the measured states. And the observer structure formed by using equations (5.25) – (5.27) will be similar to equation (5.14), but without the term $L_2 \text{sign}(e_y)$. Based on the reaching conditions (5.16) and (5.17), we are able to select the observer gain $L_2$ only to compensate for the uncertainty $\Delta f_2$. The selections of the linear gain $\alpha_i$ and the matrix $G$ for both methods are based on the same idea of placing the eigenvalues of the error dynamics for the linearized system at suitable locations on the complex plane, e.g. compare equation (5.15) to equation (5.29). The main differences between Walcott and Zak (1986) and Slotine et al. (1987) are the presence of the uncertainty $\Delta f_1$ and the switching term $L_2 \text{sign}(e_y)$ in the observer configuration of the unmeasured states. In the presence of these two terms, the error
dynamics of the unmeasured states are shown in equation (4.21). In the sliding mode control terminology, the term $\Delta f_1$ is called an “unmatched” uncertainty since its presence does not satisfy the matching condition. Note also that the use of the equivalent control method to approximate the $\text{sign}(e_y)$ function during sliding mode induces the uncertainty $\Delta f_2$ into equation (5.21). From the error equation (5.21), it can be seen that the dynamic behavior depends on the structure of both $\Delta f_1$ and $\Delta f_2$. Besides the requirement on $\Delta f_2$ that it has to be a single valued function of $e_1$, based on our assumption on the nature of the functions $f_i$, it can be assumed that $\Delta f_1$ is also a function of $e_1$. Therefore, the error dynamic equation (5.21) can be made stable, the resulting dynamics depending on the selection of the gain $L_1$. Note that, if $\Delta f_1$ is not a function of $e_1$, then the equilibrium point of $e_1$ will be offset by the upper bound of $\Delta f_1$, assuming that $\Delta f_2$ meets our requirement. In this case, we can improve the observer by obtaining a more accurate model of the system and reducing the upper bound on the uncertainty. Based on the above discussion, we consider a special case in which the nonlinear system of interest in now linearizable. We first will consider the system with only one measurement and then we will cover for the system with multiple measurements.

5.1.1 Single Measurement Case

Based on the above discussion, we consider a special case in which the nonlinear system of interest has only one measurement and is linearizable. Specifically, we
consider the linearized version of the system in equation (5.13) around some particular operating point. In this case, equation (5.13) can be written as the following.

\[
\begin{align*}
\dot{x}_1 &= \frac{\partial f_1}{\partial x_1} x_1 + \frac{\partial f_1}{\partial x_2} x_2 + \cdots + \frac{\partial f_1}{\partial x_n} x_n + \delta f_1 + B_1 u(t) \\
\dot{x}_2 &= \frac{\partial f_2}{\partial x_1} x_1 + \frac{\partial f_2}{\partial x_2} x_2 + \cdots + \frac{\partial f_2}{\partial x_n} x_n + \delta f_2 + B_2 u(t) \\
&\vdots \\
\dot{x}_n &= \frac{\partial f_n}{\partial x_1} x_1 + \frac{\partial f_n}{\partial x_2} x_2 + \cdots + \frac{\partial f_n}{\partial x_n} x_n + \delta f_n + B_n u(t) \\
y &= x_n
\end{align*}
\]

The term \( \delta f_i \), \( i = 1, \ldots, n \), represents the modeling error caused by structural deviations. The expression for \( \delta f_i \) may not be exactly known, but it is assumed to be bounded by,

\[
\| \delta f_i \| \leq \rho_i, \quad i = 1, \ldots, n
\]

By following the approach adopted by Slotine et al., (1987), the sliding mode observer for the system (5.30) is in the form of,

\[
\begin{align*}
\dot{\hat{x}}_1 &= \frac{\partial f_n}{\partial x_1} \hat{x}_1 + \frac{\partial f_n}{\partial x_2} \hat{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \hat{x}_n + B_n u(t) + k_n \text{sgn}(\hat{x}_n) \\
\dot{\hat{x}}_2 &= \frac{\partial f_n}{\partial x_1} \hat{x}_1 + \frac{\partial f_n}{\partial x_2} \hat{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \hat{x}_n + B_2 u(t) + k_2 \text{sgn}(\hat{x}_n) \\
&\vdots \\
\dot{\hat{x}}_n &= \frac{\partial f_n}{\partial x_1} \hat{x}_1 + \frac{\partial f_n}{\partial x_2} \hat{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \hat{x}_n + B_n u(t) + k_n \text{sgn}(\hat{x}_n)
\end{align*}
\]

In this case, the observer error dynamics can be derived by subtracting equation (5.32) from (5.30), and we have,
\[
\dot{x}_i = \frac{\partial f_1}{\partial x_1} \dot{x}_1 + \frac{\partial f_1}{\partial x_2} \dot{x}_2 + \cdots + \frac{\partial f_1}{\partial x_n} \dot{x}_n + \delta f_1 - k_1 \text{sgn}(\dot{x}_n) \\
\dot{x}_2 = \frac{\partial f_2}{\partial x_1} \dot{x}_1 + \frac{\partial f_2}{\partial x_2} \dot{x}_2 + \cdots + \frac{\partial f_2}{\partial x_n} \dot{x}_n + \delta f_2 - k_2 \text{sgn}(\dot{x}_n) \\
\vdots \\
\dot{x}_n = \frac{\partial f_n}{\partial x_1} \dot{x}_1 + \frac{\partial f_n}{\partial x_2} \dot{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \dot{x}_n + \delta f_n - k_n \text{sgn}(\dot{x}_n)
\]

(5.33)

Following the same design procedure shown in equations (5.16)-(5.18), sliding motion exists if the following condition is satisfied.

\[
\ddot{x}_n \dot{x}_n < 0 
\]

(5.34)

\[
\ddot{x}_n \left( \frac{\partial f_n}{\partial x_1} \ddot{x}_1 + \frac{\partial f_n}{\partial x_2} \ddot{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \ddot{x}_n + \delta f_n - k_n \text{sgn}(\ddot{x}_n) \right) < 0
\]

(5.35)

This condition is satisfied if we choose

\[
k_n > \left| \frac{\partial f_n}{\partial x_1} \ddot{x}_1 + \frac{\partial f_n}{\partial x_2} \ddot{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \ddot{x}_n + |\delta f_n| \right|
\]

\[
\geq \left| \frac{\partial f_n}{\partial x_1} \ddot{x}_1 + \frac{\partial f_n}{\partial x_2} \ddot{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \ddot{x}_n + \delta f_n \right|
\]

(5.36)

Then, after some finite time, the sliding mode is induced, \( \ddot{x}_n = 0 \) and \( \dot{x}_n = 0 \), and the error dynamics become,

\[
\dot{x}_i = \frac{\partial f_1}{\partial x_1} \ddot{x}_1 + \frac{\partial f_1}{\partial x_2} \ddot{x}_2 + \cdots + \frac{\partial f_1}{\partial x_{n-1}} \ddot{x}_{n-1} + \delta f_1 - k_1 \text{sgn}(\ddot{x}_n) \\
\dot{x}_2 = \frac{\partial f_2}{\partial x_1} \ddot{x}_1 + \frac{\partial f_2}{\partial x_2} \ddot{x}_2 + \cdots + \frac{\partial f_2}{\partial x_{n-1}} \ddot{x}_{n-1} + \delta f_2 - k_2 \text{sgn}(\ddot{x}_n) \\
\vdots \\
\dot{x}_{n-1} = \frac{\partial f_{n-1}}{\partial x_1} \ddot{x}_1 + \frac{\partial f_{n-1}}{\partial x_2} \ddot{x}_2 + \cdots + \frac{\partial f_{n-1}}{\partial x_{n-1}} \ddot{x}_{n-1} + \delta f_{n-1} - k_{n-1} \text{sgn}(\ddot{x}_n)
\]

(5.37)

Since the term \( \text{sgn}(\ddot{x}_n) \) cannot be evaluated when \( \ddot{x}_n = 0 \), applying the equivalent control method from Utkin (1992) to equation (5.35) with \( \ddot{x}_n = 0 \) gives,
\[ \text{sgn}(\tilde{x}_n) = \frac{1}{k_n} \left[ \frac{\partial f_1}{\partial x_1} \tilde{x}_1 + \frac{\partial f_n}{\partial x_n} \tilde{x}_n + \cdots + \frac{\partial f_{n-1}}{\partial x_{n-1}} \tilde{x}_{n-1} + \delta f_n \right] \] (5.38)

Substituting equation (5.38) in (5.37), we have,

\[ \begin{aligned}
\dot{\tilde{x}}_1 &= \left( \frac{\partial f_1}{\partial x_1} - \frac{k_1}{k_n} \frac{\partial f_n}{\partial x_n} \right) \tilde{x}_1 + \cdots + \left( \frac{\partial f_1}{\partial x_{n-1}} - \frac{k_1}{k_n} \frac{\partial f_n}{\partial x_{n-1}} \right) \tilde{x}_{n-1} + \left( \delta f_1 - \frac{k_1}{k_n} \delta f_n \right) \\
\dot{\tilde{x}}_2 &= \left( \frac{\partial f_2}{\partial x_1} - \frac{k_2}{k_n} \frac{\partial f_n}{\partial x_n} \right) \tilde{x}_1 + \cdots + \left( \frac{\partial f_2}{\partial x_{n-1}} - \frac{k_2}{k_n} \frac{\partial f_n}{\partial x_{n-1}} \right) \tilde{x}_{n-1} + \left( \delta f_2 - \frac{k_2}{k_n} \delta f_n \right) \\
& \vdots \\
\dot{\tilde{x}}_{n-1} &= \left( \frac{\partial f_{n-1}}{\partial x_1} - \frac{k_{n-1}}{k_n} \frac{\partial f_n}{\partial x_n} \right) \tilde{x}_1 + \cdots + \left( \frac{\partial f_{n-1}}{\partial x_{n-1}} - \frac{k_{n-1}}{k_n} \frac{\partial f_n}{\partial x_{n-1}} \right) \tilde{x}_{n-1} + \left( \delta f_{n-1} - \frac{k_{n-1}}{k_n} \delta f_n \right)
\end{aligned} \] (5.39)

Or in a matrix form as,

\[ \begin{bmatrix}
\dot{\tilde{x}}_1 \\
\vdots \\
\hat{\tilde{x}}_{n-1}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{n-1}} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_{n-1}}{\partial x_1} & \cdots & \frac{\partial f_{n-1}}{\partial x_{n-1}}
\end{bmatrix} \begin{bmatrix}
\tilde{x}_1 \\
\vdots \\
\tilde{x}_{n-1}
\end{bmatrix} + \begin{bmatrix}
\frac{k_1}{k_n} \\
\vdots \\
\frac{k_{n-1}}{k_n}
\end{bmatrix} \begin{bmatrix}
\delta f_1 \\
\vdots \\
\delta f_{n-1}
\end{bmatrix} \] (5.40)

Therefore, after the sliding mode is reached, the observer is of reduced order, viz order \( n-1 \).

1. From the theory of observer design for linear systems, we know that letting,

\[ A = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n}
\end{bmatrix} \Rightarrow \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{n-1}} \\
\frac{\partial f_2}{\partial x_1} & \cdots & \vdots \\
\vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_{n-1}}
\end{bmatrix} \] (5.41)

\[ C = x_n \]
if the pair \((A, C)\) is observable, then so is the pair \((A_{11}, A_{21})\). By using equation (5.41), equation (5.40) can be written as,

\[
\begin{bmatrix}
\dot{x}_1 \\
\vdots \\
\dot{x}_{n-1}
\end{bmatrix} = \{A_{11} - L_k A_{21}\}
\begin{bmatrix}
x_1 \\
\vdots \\
x_{n-1}
\end{bmatrix} + \begin{bmatrix}
\delta f_1 \\
\vdots \\
\delta f_{n-1}
\end{bmatrix} - L_k \delta f_n
\]

(5.42)

Therefore, the gain vector \(L_k\) can be selected such that the eigenvalues of (5.42) are at desired values in the open left-half of the complex plane. And if the structural uncertainties are all zero, i.e. \(\delta f_i, \ i=1,\ldots,n\), the estimation error dynamics in equation (5.42) can be guaranteed to be asymptotically stable. This case is similar to the observer proposed by Walcot and Zak. Once the gain vector \(L_k\) is found, the individual gains \(k_i, i=1,\ldots,n-1\), can be calculated as,

\[
[k_1 \cdots k_{n-1}]^T = k_n L_k^T
\]

(5.43)

The fact that we are able to choose the gain vector \(L_k\) for the above system validates the discussion about the structure of \(\Delta f_1\) and \(\Delta f_2\) in order to design the observer gains for unmeasured states in equation (5.21). In the case where any of \(\partial f_i\) is non-zero, care must be taken in choosing the gain vector \(L_k\) since these terms becomes a forcing function as shown in equation (5.42). To see this, we consider the equation for \(\dot{x}_1\) as an example.

From equation (5.40), we have,

\[
\dot{x}_1 = \left[\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{n-1}} \\
\frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_{n-1}}
\end{bmatrix} - \frac{k_1}{k_n} \left[\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{n-1}} \\
\frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_{n-1}}
\end{bmatrix} \right] \right] \begin{bmatrix}
x_1 \\
\vdots \\
x_{n-1}
\end{bmatrix} + \begin{bmatrix}
\delta f_1 \\
\vdots \\
\delta f_{n-1}
\end{bmatrix} - \frac{k_1}{k_n} \delta f_n
\]

(5.44)
The gain $k_n$ is chosen to enforce the sliding mode. From the theory of linear unknown input observer (UIO) design, the gain $k_1$ should be selected such that the forcing term on the right hand side of (5.44) vanishes, i.e.

$$k_1 = k_n \frac{\delta f_i}{\delta f_n}, \quad \delta f_n \neq 0$$ (5.45)

However, even for a linear system with a perfect model, this condition can usually not be satisfied without degrading the error dynamics performance. For a system with uncertainty, this condition cannot even be met.

Some special cases can be considered for this problem. First, we consider the case $\delta f_i = 0$, and $\delta f_n \neq 0$. This case refers to the system without unmatched uncertainty. Matched uncertainty is induced into the unmeasured state $x_i$ due to the use of the switching function $\text{sgn}(\dot{x}_n)$ and the equivalent control method. In this case, a preferable choice for the gain $k_1$ in (5.44) to suppress the forcing function effect is zero. However, since the gain $k_1$ is used to adjust error dynamic behavior of, it normally cannot be zero. Therefore, the choice of the gain $k_1$ will involve a trade-off between error dynamics and the level of the perturbation due to $\delta f_n$. One the other hand, if $\delta f_n = 0$, and $\delta f_i \neq 0$, then the effect of $\delta f_i$ cannot be suppressed regardless of the choice of the gain $k_1$. This case refers to the case with only unmatched uncertainty. The stability of error dynamics of unmeasured states when any of $\partial f_i$ is non-zero cannot be guaranteed. However, if we can assume that $\delta f_i$ is bounded, then one can show that the estimation error of unmeasured states is also bounded. In particular, we first assume that,
\[ A_o = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_i}{\partial x_{i-1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_{n-1}} \end{bmatrix} - \frac{k_i}{k_n} \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_{n-1}} \end{bmatrix} \] (5.46)

The solution of (5.44) is,

\[ \tilde{x}_i = e^{A_i} \tilde{x}_i(0) + \int_0^t e^{A_i(t-\tau)} (\delta f_i - \frac{k_i}{k_n} \delta f_n) d\tau \] (5.47)

If, \[ \|\tilde{x}_i(0)\| \leq a \] then,

\[ \|\tilde{x}_i\| \leq a \|e^{A_i}\| + \int_0^t \|e^{A_i(t-\tau)}\| \left(\|\delta f_i - \frac{k_i}{k_n} \delta f_n\|\right) d\tau \]

\[ \leq ae^{-\lambda_{\max}(A_i)t} + \int_0^t e^{-\lambda_{\max}(A_i)(t-\tau)} \left(\|\delta f_i - \frac{k_i}{k_n} \delta f_n\|\right) d\tau \] (5.48)

In this case, \( \lambda_{\max}(A_o) \) is the maximum eigenvalue of \( A_o \). We also have,

\[ \left\|\delta f_i - \frac{k_i}{k_n} \delta f_n\right\| \leq \|\delta f_i\| + \left\|\frac{k_i}{k_n} \delta f_n\right\| \leq \|\delta f\| (1 + \left\|\frac{k_i}{k_n}\right\|) \] (5.49)

where

\[ \|\delta f\| = \max(\|\delta f_i\|, \|\delta f_n\|) \] (5.50)

Thus, if there exists a positive number \( b \), where

\[ \|\delta f\| \leq b/(1 + \left\|\frac{k_i}{k_n}\right\|) \] (5.51)

Then we have

\[ \|\tilde{x}_i\| \leq ae^{-\lambda_{\max}(A_i)t} + e^{-\lambda_{\max}(A_i)t} \int_0^t e^{\lambda_{\max}(A_i)(t-\tau)} b d\tau \]

\[ = ae^{-\lambda_{\max}(A_o)t} + \frac{b}{\lambda_{\max}(A_o)} (1 - e^{-\lambda_{\max}(A_o)t}) \]

\[ \leq ae^{-\lambda_{\max}(A_o)t} + \frac{b}{\lambda_{\max}(A_o)} \] (5.52)

Therefore, it is proved that \( \|\tilde{x}_i\| \) is bounded.
Similar analysis can be carried out for other estimation error dynamics for 
\( \tilde{x}_i, \ i = 2, ..., n-1 \). Noting that the term \( \delta f_i \) may include uncertainty due to the input function, we let,

\[
\delta f'_i = \delta f_i + B_i u(t), \quad i = 1, ..., n
\]  

(5.53)

Thus, the same analysis and design procedure can be used by replacing \( \delta f_i \) with \( \delta f'_i \).

### 5.1.2 Multiple Measurement Case

For a system with multiple measurements, design and analysis of the sliding mode observer is approximately the same as described for the system with a single measurement. We consider the system in the following form.

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1, x_2) + B_1 u(t) \\
\dot{x}_2 &= f_2(x_1, x_2) + B_2 u(t) \\
y &= x_2
\end{align*}
\]

(5.54)

By assuming that the system is locally linearizable around some operating conditions, the system dynamic equations with \( p \) measurements can be written as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
\nabla_{x_1} f_1 & \nabla_{x_2} f_1 \\
\nabla_{x_1} f_2 & \nabla_{x_2} f_2
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
\delta f_1 \\
\delta f_2
\end{bmatrix} + \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u(t)
\]

(5.55)

Here, \( \delta f_i \) represents structural uncertainties. We assume also that the pair \((A,C)\) is observable where

\[
A = \begin{bmatrix}
\nabla_{x_1} f_1 & \nabla_{x_2} f_1 \\
\nabla_{x_1} f_2 & \nabla_{x_2} f_2
\end{bmatrix}, \quad C = x_2
\]

(5.56)
By following the approach adopted by Slotine et al. (1987), the sliding mode observer for the system (5.54) is in the form of,

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
\nabla_{x_1} f_1 & \nabla_{x_2} f_1 \\
\nabla_{x_1} f_2 & \nabla_{x_2} f_2
\end{bmatrix}\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} + \begin{bmatrix}
\delta f_1 \\
\delta f_2
\end{bmatrix} + \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} \mu(t) + \begin{bmatrix}
K_1 \\
K_2
\end{bmatrix} \text{sgn}(\tilde{x}_2) \tag{5.57}
\]

We are interested in the case where the gain matrices \(K_1\) and \(K_2\) have the following special forms.

\[
K_1 = \begin{bmatrix}
k_{11} & k_{12} & \cdots & k_{1(n-p)} \\
\vdots & \vdots & \ddots & \vdots \\
k_{(n-p)1} & k_{(n-p)2} & \cdots & k_{(n-p)(n-p)}
\end{bmatrix} \tag{5.58}
\]

\[
K_2 = \begin{bmatrix}
k_1 & 0 & \cdots & 0 \\
0 & k_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & k_p
\end{bmatrix} \tag{5.59}
\]

In this case, the observer error dynamics can be derived by subtracting equation (5.57) from (5.55), and we have,

\[
\begin{bmatrix}
\dot{\tilde{x}}_1 \\
\dot{\tilde{x}}_2
\end{bmatrix} = \begin{bmatrix}
\nabla_{x_1} f_1 & \nabla_{x_2} f_1 \\
\nabla_{x_1} f_2 & \nabla_{x_2} f_2
\end{bmatrix}\begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2
\end{bmatrix} + \begin{bmatrix}
\delta f_1 \\
\delta f_2
\end{bmatrix} - \begin{bmatrix}
K_1 \\
K_2
\end{bmatrix} \text{sgn}(\tilde{x}_2) \tag{5.60}
\]

Following the same design procedure as in the single measurement case, sliding motion exists if the following condition is satisfied.

\[
\dot{\tilde{x}}_2^T \tilde{x}_2 < 0 \tag{5.61}
\]

\[
\dot{\tilde{x}}_2^T \left[ (\nabla_{x_1} f_2) \dot{\tilde{x}}_1 + (\nabla_{x_2} f_2) \dot{\tilde{x}}_2 + \delta f_2 - K_2 \text{sgn}(\tilde{x}_2) \right] < 0 \tag{5.62}
\]

This condition is satisfied if we choose each element
Specifically, from the given form of the gain matrix $K_2$ in equation (5.59), equation (5.63) can be expanded as,

\[
\begin{align*}
K_2 & > \left[ (\nabla_{x_1} f_2) \tilde{x}_1 \right] + \left[ (\nabla_{x_2} f_2) \tilde{x}_2 \right] + |\delta f_2| \\
& \geq \left[ (\nabla_{x_1} f_2) \tilde{x}_1 + (\nabla_{x_2} f_2) \tilde{x}_2 + \delta f_2 \right]
\end{align*}
\]

With these conditions satisfied, after some finite time, sliding mode is induced where $\tilde{x}_2 = 0$ and $\dot{\tilde{x}}_2 = 0$, and the error dynamics in equation (5.60) are reduced to,

\[
\dot{\tilde{x}}_1 = (\nabla_{x_1} f_1) \tilde{x}_1 + \delta f_1 - K_1 \text{sgn}(\tilde{x}_2)
\]

Again, we use the equivalent control method to evaluate the term $\text{sgn}(\tilde{x}_2)$ at $\tilde{x}_2 = 0$.

Therefore, from (5.62), when $\tilde{x}_2 = 0$, we have

\[
\text{sgn}(\tilde{x}_2) = K_2^{-1} \left[ (\nabla_{x_1} f_2) \tilde{x}_1 + \delta f_2 \right]
\]

Or by using equation (5.64), we have,

\[
\begin{align*}
\text{sgn}(\tilde{x}_{n-p+1}) & = \frac{1}{k_1} \left[ (\nabla_{x_1} f_{n-p+1}) \tilde{x}_1 + \delta f_{n-p+1} \right] \\
\text{sgn}(\tilde{x}_{n-p+2}) & = \frac{1}{k_2} \left[ (\nabla_{x_1} f_{n-p+2}) \tilde{x}_1 + \delta f_{n-p+2} \right] \\
& \vdots \\
\text{sgn}(\tilde{x}_n) & = \frac{1}{k_n} \left[ (\nabla_{x_1} f_n) \tilde{x}_1 + \delta f_n \right]
\end{align*}
\]

Substituting equation (5.66) into (5.65), we have,
\[
\dot{x}_1 = \left[ \nabla_{x_1} f_1 - K_1 K_2^{-1} \left( \nabla_{x_2} f_2 \right) \right] \dot{x}_1 + \left( \delta f_1 - K_1 K_2^{-1} \delta f_2 \right) \tag{5.68}
\]

Equation (5.68) is basically a reduced order observer with the order of n-p. As for the single measurement case, since we assumed that the pair \((A,C)\) in equation (5.56) is observable, so is the pair \((\nabla_{x_1} f_1, \nabla_{x_2} f_2)\). Therefore, the gain matrix \(K_1 K_2^{-1}\) can be chosen to place the eigenvalues of the matrix \(\left[ \left( \nabla_{x_1} f_1 - K_1 K_2^{-1} \left( \nabla_{x_2} f_2 \right) \right) \right]\) at desired locations on the left-half of the complex plane. As we discussed in the single measurement case, when the system has structural modeling uncertainties, \(\delta f_1\), care must be taken in choosing observer gains since these terms become forcing functions as shown in equation (5.68).

The uncertainty \(\delta f_2\) appears in the error dynamics equation (5.68) via the switching feedback \(\text{sgn}(\dot{x}_2)\), the gain matrix \(K_1\), and the use of the equivalent control method. Therefore, any choice of the gain matrix \(K_1\) will directly impact the perturbation due to the uncertainty \(\delta f_2\). This illustrates the trade-off between error dynamic performance and estimation accuracy in choosing the gain \(K_1\). In some cases, this problem may be simplified by using only feedback signals that carried significant information for a particular unmeasured state. In such a case, the matrix gain \(K_1\) will not be fully filled and will reduce the number of gains to be selected. As an example, if we assume for our case that most of the information about the state \(x_l\) is carried through only the measurement \(x_u\), then we can ignore the feedback from the other measurements, and the gain matrix \(K_1\) in this case is reduced to,
The selection of the observer gains for the estimation error dynamics of the state \( x_1 \) will be greatly simplified then. Methods of checking the level of information for different states carried through different measurements have been reported in the literature under the concept of the degree of observability (Healey and Mackinnon, (1975)).

For the case of \( \delta f_2 \), this uncertainty is unmatched uncertainty and we have shown from the analysis of the single measurement case that its effect cannot be suppressed regardless of the choice of the gain \( K_1 \). Regardless of the presence of any uncertainty, we have also shown that, if each element in \( \delta f_1 \) is bounded, then the estimation error for unmeasurable states is also bounded.

### 5.1.3 Effects of Measurement Noise

Consider the case of the system with a single measurement which now contains a noise component \( v = v(t) \). From equation (5.33), we have,

\[
\dot{x}_n = \frac{\partial f_n}{\partial x_1} \dot{x}_1 + \frac{\partial f_n}{\partial x_2} \dot{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} (\dot{x}_n + v) - k_n \text{sgn}(\dot{x}_n + v)
\]  

(5.70)

For the sake of simplicity of analysis here, we ignore structural uncertainty. From the above equation, sliding mode can occur only on the surface,

\[
S = \dot{x}_n + v = 0
\]

(5.71)

By repeating the analysis for the system with a single measurement, the sliding mode is attractive if we select the gain \( k_n \) such that,
The above condition is difficult to satisfy in general, since \( \dot{v} \) is not normally known and can be large which would require an extremely large value of the gain \( k_n \). Assuming for the moment that the condition in equation (5.72) is satisfied, the sliding surface in equation (5.71) is reachable. The reduced order error dynamics from the use of the equivalent control method are given by

\[
\begin{bmatrix}
\dot{x}_1 \\
\vdots \\
\dot{x}_{n-1}
\end{bmatrix} = \begin{bmatrix}
A_{11} - L_k A_{21} & \\
\vdots & \\
L_k & \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\vdots \\
x_{n-1}
\end{bmatrix}
- L_k \dot{v}
\]  
(5.73)

\[
\bar{x}_n = v
\]  
(5.74)

Here, if the gain vector \( L_k \) for the error dynamics of unmeasured states is selected such that the maximum eigenvalue of \( A_{11} - L_k A_{21} \) is much smaller in value than dominant frequencies in \( v \), it is easy to see that,

\[
\begin{bmatrix}
\bar{x}_1 \\
\vdots \\
\bar{x}_{n-1}
\end{bmatrix} \rightarrow [0], \quad t \rightarrow \infty
\]  
(5.75)

On the other hand, if the maximum eigenvalue of \( A_{11} - L_k A_{21} \) is much higher than the dominant frequencies in \( v \), then we have,
\[
\begin{bmatrix}
\dot{x}_1 \\
\vdots \\
\dot{x}_{n-1}
\end{bmatrix} \approx -\dot{v}, \quad t \to \infty
\] (5.76)

In this case, only if the function \( v \) is a constant will the estimation error for all unmeasured states will be zero. In the case where the measurement noise and its variation are bounded by some constants \( v_1 \) and \( v_2 \) respectively, the system will remain in the vicinity of a boundary layer of width \( v_1 \) for the measurable states and \( v_2 \) for the unmeasured states. From the above discussion, it can be seen that the system cannot remain in a pure sliding mode in the presence of the arbitrary measurement noise (Misawa, 1988).

There are two approaches to reduce the noise problem (Masmoudi and Hedrick, 1992). The first one, which is normally used, is to design a low pass filter to pre-filter the high frequency content of the measurement signal, with the implicit assumption that this represents noise. Alternatively, if the noise statistics are known, the Kalman filter is normally used. The second approach is to replace the switching function with a saturation function (Slotine, 1984). This second approach is also useful in reducing the chattering problem when implementing sliding mode control. For observer design problems, this method can be used effectively only when all the states are measurable. When only some states are measurable, this method is used to reduce high frequency chattering inside a boundary layer around the sliding surface. In this case, the switching function will be used as feedback to all other unmeasured states. The same method of selecting gains used for unmeasured states will have to be applied here as well.
To see use of the saturation function to help select the gain $k_n$ in our system, consider again the single measurement case without structural uncertainties as shown in equation (5.70). In this case, we replace the switching function $k_n \text{sgn}(x_n)$ with the saturation function $k_n \text{sat}(x_n/\epsilon)$ as shown in Figure 5.1 and described by

$$
k_n \text{sat}(\frac{x_n}{\epsilon}) = \begin{cases} 
  k_n \text{sgn}(\tilde{x}_n), & \text{if } |\tilde{x}_n| > \epsilon \\
  k_n \frac{\tilde{x}_n}{\epsilon}, & \text{if } |\tilde{x}_n| \leq \epsilon
\end{cases}
$$

(5.77)

![Figure 5.1: Difference between sgn function and sat function](image)

For the observer design problem, the constant $\epsilon$ is related to the desired accuracy of estimation of the state $x_n$. The ratio $k_n/\epsilon$ has to be smaller than the noise frequency. Assuming that equation (5.77) is satisfied, within a finite time period, the estimation error $\tilde{x}_n$ will be smaller than $|\epsilon|$. From the existence condition for the sliding mode, we have
\[
\ddot{x}_n \dot{x}_n = \tilde{x}_n^T \left( \frac{\partial f_n}{\partial x_1} \tilde{x}_1 + \frac{\partial f_n}{\partial x_2} \tilde{x}_2 + \cdots + \frac{\partial f_n}{\partial x_n} \tilde{x}_n - k_n \text{sat}(\frac{\tilde{x}_n}{\varepsilon}) + \dot{\nu} \right)
\] (5.78)

And since \( \tilde{x}_n < |\varepsilon| \), the above equation becomes,

\[
\dot{x}_n = -\left[ \frac{k_n}{\varepsilon} - \frac{\partial f_n}{\partial x_n} \right] \tilde{x}_n + \left[ \frac{\partial f_n}{\partial x_1} \tilde{x}_1 + \frac{\partial f_n}{\partial x_2} \tilde{x}_2 + \cdots + \frac{\partial f_n}{\partial x_{n-1}} \tilde{x}_{n-1} + \dot{\nu} \right]
\] (5.79)

Therefore, we can select the gain \( k_n \) such that the equation (5.79) becomes a low pass filter with a cut-off frequency much lower than the frequency content of the perturbation terms inside the bracket on the right hand side of the equation. As a result, the estimation error \( \hat{x}_n \) will have the same steady-state error amplitude as in equation (5.74), but with less chattering around the sliding surface. Note that this method can also be used for selecting or checking the designed observer gains for the measured states when implementing a discrete-time observer with low sampling frequency.

### 5.1.4 Adaptive Sliding Mode Observer for Nonlinear Systems

The sliding mode observer presented here can be extended to include an adaptation mechanism to adjust uncertain parameters for a large class of linearly parametrized nonlinear systems. Theories of adaptive observers for both linear and nonlinear systems are well documented in the literature (e.g. Kudva et al., 1973; Bastin and Gevers, 1988; Teel et al., 1993; Ioannou and Sun, 1996, etc.), and are not repeated here. However, there has not been much reported in the area of adaptive sliding mode observers. The review of the adaptive sliding mode observer presented in this section is based on the work presented by Yi et al (2000). In particular, the same adaptation
mechanism as in the cited reference is used here, but with a slightly different approach in the sliding mode observer design.

Consider a nonlinear system in the following form,

\[
\begin{align*}
\dot{x}(t) &= f(x, \theta, t) + g(x, t)u(t) \\
y(t) &= Cx(t)
\end{align*}
\]  

(5.80)

where \( x \in \mathbb{R}^n \), \( y \in \mathbb{R}^m \), \( u \in \mathbb{R} \) represent the state, the output or measurement, and the input respectively. \( C \in \mathbb{R}^{nm} \) represents the output distribution matrix, \( f \) and \( g \) are vectors of time-varying nonlinear functions, and \( \theta \in \mathbb{R}^p \) represents the vector of unknown or uncertain parameters. Here, \( f \) is assumed to be linearly parametrized in unknown parameters in the following form.

\[
f(x, \theta, t) = F(x, t)\theta
\]  

(5.81)

In this case, \( F(x, t) \in \mathbb{R}^{nxp} \). Therefore, the system in equation (5.80) becomes,

\[
\begin{align*}
\dot{x}(t) &= F(x, t)\theta + g(x, t)u(t) \\
y(t) &= Cx(t)
\end{align*}
\]  

(5.82)

Following the sliding mode observer design presented previously, the observer is in the following form.

\[
\begin{align*}
\hat{x}(t) &= \hat{F}(\hat{x}, t)\hat{\theta} + \hat{g}(\hat{x}, t)u(t) - K \text{sgn}(\hat{x})
\end{align*}
\]  

(5.83)

In this case, \( \hat{x} = \hat{x} - x \), and \( K \in \mathbb{R}^{nm} \) represents the observer gain matrix. Letting,

\[
\begin{align*}
\hat{F}(\hat{x}, t)\hat{\theta} - F(x, t)\theta &= \Delta F\hat{\theta} + \hat{F}(\hat{x})\hat{\theta} \\
\Delta F &= \hat{F}(\hat{x}, t) - F(x, t) \\
\Delta g &= \hat{g}(\hat{x}, t) - g(x, t)
\end{align*}
\]  

(5.84)

(5.85)

the resulting error dynamics can be found by subtracting equation (5.82) from (5.83).
\[
\hat{x}(t) = \Delta F \hat{\theta} + \hat{F}(\hat{x})\hat{\theta} + \Delta \hat{g}(\hat{x},t)u(t) - K \text{sgn}(\hat{x}) \quad (5.86)
\]

Both \(\Delta F\) and \(\Delta g\) are assumed bounded. The sliding surface in this case is,

\[
S = [s_1, s_2, \ldots, s_m]^T \quad ; \quad s_i = \hat{y}_i - y_i = \tilde{y}_i \quad , i = 1, \ldots, m \quad (5.87)
\]

To prove the stability of the observer, the following Lyapunov function candidate is used.

\[
V = \frac{1}{2} S^T S + \frac{1}{2} \hat{\theta}^T R \hat{\theta} \quad (5.88)
\]

The derivative of \(V\) along the trajectory of the error dynamics (5.86) can be found as,

\[
\dot{V} = S^T \dot{S} + \hat{\theta}^T R \dot{\hat{\theta}} = S^T C \left( \Delta F \dot{\hat{\theta}} + \hat{F}(\hat{x})\dot{\hat{\theta}} + \Delta \hat{g}(\hat{x},t)u(t) - K \text{sgn}(\hat{x}) \right) + \hat{\theta}^T R \dot{\hat{\theta}} \quad (5.89)
\]

Assuming that \(\theta\) is slow varying, or,

\[
\dot{\hat{\theta}} = \hat{\theta} - \dot{\theta} = \ddot{\hat{\theta}} \quad , \dot{\hat{\theta}} \approx 0 \quad (5.90)
\]

Equation (5.89) can be written as,

\[
\dot{V} = S^T C \left( \Delta F \dot{\hat{\theta}} + \Delta \hat{g}(\hat{x},t)u(t) - K \text{sgn}(\hat{x}) \right) + \hat{\theta}^T \left( \hat{F}^T(\hat{x})C^T S + R \dot{\hat{\theta}} \right) \quad (5.91)
\]

From this equation, if the last term on the right hand side vanishes, the switching gain matrix \(K\) within the first parentheses on the right hand side can be selected following the methodologies of sliding mode observer design presented in previous sections. Therefore, we have the following adaptation law,

\[
\dot{\hat{\theta}} = (R^{-1}) \hat{F}^T(\hat{x})C^T S \quad (5.92)
\]

Based on the theory of the adaptive control, it should be noted that the estimated parameters are not guaranteed to converge to their true values, unless the function \(\hat{F}(\hat{x})\) is considered to belong to a class of a persistently exciting functions (Ioannou and Sun,
However, the parametric error magnitude is guaranteed to be bounded. Ioannou and Sun (1996) have shown an extensive proof for this result. More proofs can also be found in Yi et al. (2000) for the case of sliding observer design with the saturation function replacing the sign function.

5.2 Discrete Sliding Mode Observer

As for the case of the continuous time sliding mode control, discrete time sliding mode control has been extensively studied in literature (e.g. Milosavljevic, 1985; Sarpturk et. Al, 1987; Drakunov and Utkin, 1989; Furuta, 1990; Ramirez, 1991; Corradini and Orlando, 1996; Guo and Zhang, 2002; among others). It seems intuitive in real-time applications to design a controller in the discrete time domain since the controller is normally executed using a digital computer where the input to, and output of, the controller are sampled discretely in time. Nevertheless, the main motivation for using discrete sliding mode control is normally related to problems encountered during implementation of continuous time sliding mode control with a digital computer. In particular, the chattering problem is always an inherited phenomenon of continuous time sliding mode control when implemented with a slow sampling digital computer. The use of discrete sliding mode control helps in getting rid of such chattering. The main difference of discrete sliding mode control or observer as compared to their continuous time counter parts is the following. For discrete sliding mode, the equivalent control term is used to replacing the switching function. As a result, the discrete sliding mode cannot guarantee maintenance of the system on the sliding surface unless the system is perfectly known. In stead, during the sliding mode, the system is maintained in the vicinity of the
sliding manifold. The boundary of the sliding manifold depends on bounding magnitudes on modeling error and uncertainties.

The concept of discrete sliding mode control has also been extended to the discrete sliding mode observer. Early work in this subject can be found in Aitken and Schwartz (1995) and Caminhas et al (1996) etc. As for the case of discrete sliding mode control, it has been proven that the switching term in the continuous time sliding mode observer has to be replaced by a continuous function which can be calculated by using the equivalent control method, for the discrete sliding mode observer. A brief review of the discrete sliding mode observer presented here is adapted from the theory of discrete sliding mode control presented in Utkin et al. (1999).

Consider the linear system, from equation (5.5), which we recall here below.

\[
\begin{align*}
\dot{x}_1(t) &= A_{11}x_1 + A_{12}x_2 + B_1u \\
\dot{x}_2(t) &= A_{21}x_1 + A_{22}x_2 + B_2u \\
y(k) &= x_2(k)
\end{align*}
\]

Here, \( x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \mathbb{R}^n \) is an overall state vector, \( x_1 \in \mathbb{R}^{n-p} \) is a vector of unmeasurable states, and \( x_2 \in \mathbb{R}^p \) and \( y \in \mathbb{R}^p \) are a vector consisting of measurable states and an output vector respectively. The coefficient matrices have the sizes indicated: \( A_{11} \in \mathbb{R}^{(n-p)\times(n-p)} \), \( A_{12} \in \mathbb{R}^{(n-p)\times p} \), \( A_{21} \in \mathbb{R}^{p \times (n-p)} \), and \( A_{22} \in \mathbb{R}^{p \times p} \); \( B_1 \in \mathbb{R}^{(n-p)\times m} \) and \( B_2 \in \mathbb{R}^{p \times m} \). Also, \( u \in \mathbb{R}^m \) is the input vector and might include known as well as unknown components. Transforming equation (5.93) to discrete time domain with a sampling interval of \( \Delta t \), we get
\[ x_1(k+1) = \Phi_{11} x_1(k) + \Phi_{12} u_2(k) + \Gamma_1 u(k) \]
\[ x_2(k+1) = \Phi_{21} x_1(k) + \Phi_{22} x_2(k) + \Gamma_2 u(k) \]  
(5.94)

where
\[
\begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{bmatrix} = e^{A \Delta t},
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2
\end{bmatrix} = \int_0^{\Delta t} e^{A(t-\tau)} B d\tau
\]
(5.95)

The proposed observer is in the following form
\[
\hat{x}_1(k+1) = \Phi_{11} \hat{x}_1(k) + \Phi_{12} \hat{x}_2(k) + \Gamma_1 \hat{u}(k) - L_1 L_2 \nu(\hat{x}_2(k))
\]
\[
\hat{x}_2(k+1) = \Phi_{21} \hat{x}_1(k) + \Phi_{22} \hat{x}_2(k) + \Gamma_2 \hat{u}(k) - L_1 \nu(\hat{x}_2(k))
\]  
(5.96)

In this case, \( \nu(\hat{x}_2(k)) \) is a compensation function, which is a function of the error between the measurement and the estimation. \( L_i, i=1,2 \), are the observer gain matrices.

We also assume that the input function \( u \) can contain known as well as unknown components. \( \hat{u} \) represents the estimated input which may be calculated from a model for the case when the input is not measurable. Using equations (5.94) and (5.96), the error dynamic equations are,
\[
\tilde{x}_1(k+1) = \Phi_{11} \tilde{x}_1(k) + \Phi_{12} \tilde{x}_2(k) + \Gamma_1 \tilde{u}(k) - L_1 L_2 \nu(\tilde{x}_2(k))
\]
\[
\tilde{x}_2(k+1) = \Phi_{21} \tilde{x}_1(k) + \Phi_{22} \tilde{x}_2(k) + \Gamma_2 \tilde{u}(k) - L_1 \nu(\tilde{x}_2(k))
\]  
(5.97)

The sliding surfaces in this case are defined by,
\[
S(k) = [s_1(k) \quad s_2(k) \cdots s_p(k)]^T
\]  
(5.98)

\[ s_i = \tilde{x}_i - x_i, \quad i = 1, 2, \ldots, p \]  
(5.99)
Our objective is to find the function $v(\tilde{x}_2(k))$ and the observer gain $L_i$ such that the sliding surface is reached at the next time step, $k+1$, and the system motion stays on the surface from that instant on. From (5.97), considering the measurable states, we have,

$$\tilde{x}_2(k+1) = S(k+1) = \Phi_{21}\tilde{x}_1(k) + \Phi_{22}\tilde{x}_2(k) + \Gamma_2\tilde{u}(k) - L_i v(\tilde{x}_2(k)) \quad (5.100)$$

To monitor the estimation error, the above function can be written as,

$$S(k+1) = -S(k) + (\Phi_{21} + I_{(n-p)\times p})\tilde{x}_1(k) + \Phi_{22}\tilde{x}_2(k) + \Gamma_2\tilde{u}(k) - L_i v(\tilde{x}_2(k)) \quad (5.101)$$

If all terms are known, by using the equivalent control method, our objective is easy to achieve by selecting the compensation function $v(\tilde{x}_2(k))$ as,

$$S(k+1) = 0 \Rightarrow v(\tilde{x}_2(k)) = -L_i^{-1}S(k) + L_i^{-1}\left[\Phi_{21}\tilde{x}_1(k) + (\Phi_{22} + I_{p\times p})\tilde{x}_2(k) + \Gamma_2\tilde{u}(k)\right] \quad (5.102)$$

However, since $x_i(k)$ is not measurable in this case, and $u(k)$ may be uncertain as well, i.e. $\tilde{u}(k) \neq 0$, the equation (5.102) cannot be used to calculate the compensation function.

Discrete sliding mode control theory suggests that, for uncertain system, the function $v(\tilde{x}_2(k))$ can be selected based on the equation (5.102) by including only the known quantities. In particular, for our system, if only $x_2(k)$ is measurable, $v(\tilde{x}_2(k))$ becomes

$$v(\tilde{x}_2(k)) = -L_i^{-1}S(k) + L_i^{-1}\left[\Phi_{22} + I_{p\times p}\right]\tilde{x}_2(k) \quad (5.103)$$

In this case, substituting this equation back to (5.101), we get,

$$S(k+1) = \Phi_{22}\tilde{x}_2(k) + \Gamma_2\tilde{u}(k) \quad (5.104)$$

From the discretization, the matrices $\Phi_{22}$ and $\Gamma_2$ are of order $\Delta t$. Therefore, even though the system motion does not stay on the surface, it will stay in the $\Delta t$-order vicinity of the sliding manifold $S = 0$ (Utkin et al., 1999). It should be noted that for the system in the form of equation (5.93), using the compensation function shown in
equation (5.103), the gain \( L_1 \) does not affect the behavior of the error dynamics. The overall dynamics of the measurable state estimation error are determined by the choice of the function \( v(\tilde{x}_2(k)) \).

For the system we present here where the states are partially measurable, and the system is uncertain, there are not specific results in the literature discussing the proper way to design the gain \( L_2 \) so that desired error dynamic behavior may be achieved. In order to get around this problem, we use the same method as in the continuous time sliding mode observer presented earlier. Specifically, the gain \( L_2 \) can be found by first assuming that the system is able to achieve,

\[
S(k) = S(k+1) = 0 = S(k+2) = S(k+3) = \ldots \tag{5.105}
\]

Then, from equation (5.102), we have,

\[
v(\tilde{x}_2(k)) = L_1^{-1} \left[ \Phi_{21} \tilde{x}_1(k) + \Gamma_2 \tilde{u}(k) \right] \tag{5.106}
\]

Therefore, we have the following error dynamics for the unmeasured state.

\[
\tilde{x}_i(k+1) = \left( \Phi_{11} - L_2 \Phi_{21} \right) \tilde{x}_i(k) + \left( \Gamma_1 - L_2 \Gamma_2 \right) \tilde{u}(k) \tag{5.107}
\]

Provided that the pair \((\Phi_{11}, \Phi_{21})\) is observable, the observer gain \( L_2 \) can be chosen arbitrarily to meet the desired dynamics for the estimation of the unmeasured states. However, care must be taken since the value of the gain \( L_2 \) also affects the forcing function due to the uncertain input \( \tilde{u}(k) \). Moreover, the equation (5.107) is derived based on the assumption that the system stays on the sliding surface. However, as we have shown for uncertain systems, the system is not guaranteed to stay on the sliding surface. Instead, the system will stay in the vicinity of the sliding manifold. Therefore, the behavior of the error dynamics for the unmeasurable states may not be exactly as shown...
in the equation (5.107). This implies that the robustness property of the discrete sliding mode is not as strong as the continuous time case. The magnitude of the manifold is determined by the sampling interval as well as model accuracy.

Equation (5.107) shows also that, as in the case of the continuous-time sliding mode observer, once the system is in the sliding mode, the dynamics of the system correspond to the reduced-order Luenberger observer. Specifically, from the equation (5.94), the discrete reduced-order Luenberger observer can be written as the following.

\[
\dot{x}_1(k+1) = \left(\Phi_{11} - L_1 \Phi_{21}\right)\dot{x}_1(k) + \left(\Phi_{12} - L_1 \Phi_{22}\right)x_2(k) + L_1 x_2(k+1) + \left(\Gamma_1 - L_1 \Gamma_2\right)u(k)
\]  

(5.108)

The error dynamics can be easily derived, the result being the same as equation (5.107). While the equation (5.107) is not guaranteed for the discrete sliding mode observer for the reasons mentioned above, it is guaranteed for the reduced-order Luenberger observer due to the use of the direct measurement as shown in equation (5.108). Even if the system has no uncertainties, the equation (5.107) cannot be established for the discrete sliding mode observer until the sliding mode occurs. On the other hand, equation (5.107) is used to describe the error dynamics for the Luenberger observer following the start of the estimation. Therefore, the lack of sufficient robustness of the discrete sliding mode observer makes the reduced-order Luenberger the more appealing choice. However, the reduced-order observer usually has a higher bandwidth as compared to the full-order observer due to the direct transmission term from the measurement through the observer gain to the estimated states. Therefore, the reduced-order observer may be less attractive if sensor noise is significant.

More work still needs to be done in the field of discrete sliding mode observer development. It should be noted that due to the lack of more results on discrete sliding
mode observer development in the literature, for our real-time application, the method of
discrete sliding mode observer design presented in this section will be used. More
concrete developments and proofs are left for future work.

5.3 Clutch Pressure Estimation for Rigid Shaft Case

The observer design to estimate clutch pressures when the output shaft or drive
shaft is rigid, i.e. infinitely stiff, is considered in this section. Though it has been shown
by Watechagit and Srinivasan (2003(b)) that estimation of clutch pressures using a
flexible shaft model is feasible, estimation results for some states show oscillations which
are presumably caused by the switching function exciting some modes in the model. For
the model presented in Chapter 3, the highest mode is exhibited during the inertia phase.
Observation of experimental data also suggests the use of a rigid shaft model. When the
output shaft is assumed rigid, the output shaft torque will no longer be a state variable in
the system. Therefore, only the clutch pressure can be estimated in this case. In order to
estimate the output shaft torque, estimation methods reported in literature that have been
shown to be accurate can be used (Ibamoto et al., 1995; Ibamoto et al., 1997)

In the following subsections, we first develop the transmission model assuming
that the output shaft is rigid. The proposed observer algorithm is then formulated to
estimate the clutch pressures. Since the candidate shift that we use as a platform for this
research is the 2-3 power-on up shift, we concentrate here only on the estimation of the
pressure of the clutches involved during this shift. In particular, we are interested in
estimating the pressure of the OD clutch, which is the on coming clutch, and the 2ND
clutch, which is the off going clutch. The same method presented here can be applied to other shifts with minor modifications.

We note first that clutch pressures cannot be estimated in-gear since clutches involved in that particular gear are in the lock-up condition. The clutch torque in this condition corresponds to the static friction coefficient, and determination of the clutch torque is insufficient to determine uniquely the clutch capacity and hence the clutch pressure. Therefore, in the rigid shaft case, since the output shaft torque is not estimated, the observer then estimates only the measurable states, i.e. transmission input and output speeds. While estimating only the measurable states during in-gear may seem fruitless, by maintaining estimated speeds close to their measured values, i.e. by maintaining the system on the sliding surfaces, the problem of mismatched initial conditions or reaching phase when the clutch pressure observer is activated in the torque phase is minimized. The in-gear observer is also important for the adaptation scheme that we will propose to adjust the parameters of the torque converter model to improve the turbine torque estimation, and hence the clutch pressures. This will be shown in section 5.5.

Since the configuration of the observer when the transmission is in-gear can be considered as a special case of the torque phase estimation, viz., the 2-3 up shift torque phase observer can be used for the speed estimation during 2\textsuperscript{nd} gear, we will omit description of the in-gear observer construction. When the gear shift starts, due to the difference in the nature of the transmission dynamics during the torque phase and the inertia phase, the estimation algorithms for clutch pressure and transmission output shaft torque are developed separately for each phase. The robustness of the proposed observer schemes is also analyzed.
5.3.1 Model Formulation

5.3.1.1 Combined transmission mechanical system and vehicle dynamics

The state space model combining the vehicle dynamics model, the transmission mechanical model, and the shift hydraulic system is formulated using the developed model presented in Chapter 3 and Chapter 4. For the case of the second gear, we have the following.

$$\begin{bmatrix}
\dot{\omega}_t \\
\dot{\omega}_w \\
\dot{T}_s
\end{bmatrix} = \begin{bmatrix}
\frac{-d_{11}R_d T_s}{I_{23}} \\
\frac{1}{I_v} (T_s - r(c_1 + c_2 r^2 \omega_w^2)) \\
K_s R_d d_{11} \omega_t - K_s \omega_w
\end{bmatrix} + \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} T_i$$

(5.109)

where $I_{23} = (I_{si} + I_t + d_{11}^2 I_{rr} + d_{21}^2 I_{cr})$. If the output shaft is assumed rigid, the dynamics can be modified as follows. From (5.109), we have,

$$\dot{T}_s = K_s (R_d d_{11} \omega_t - \omega_w)$$

(5.110)

Assuming that $K_s \to \infty$, we have.

$$\omega_w = R_d d_{11} \omega_t$$

(5.111)

Combining equations (5.110) and (5.111), we get,

$$\dot{\omega}_w = \frac{1}{I_v} (T_s - r(c_1 + c_2 r^2 \omega_w^2))$$

(5.112)

Solving for $T_s$, we have

$$T_s = I_v \dot{\omega}_w + r(c_1 + c_2 r^2 \omega_w^2)$$

(5.113)
Using $T_s$ from (5.113), the equation (5.109) is reduced to,

$$\dot{\omega}_i = \frac{1}{I_{23S}} \left( -R_d d_{11} r(c_i + c_2(R_d d_{11} r)^2 \omega_w^2) + T_i \right) \tag{5.114}$$

where $I_{23S} = I_{23} + (R_d d_{11})^2 I_v$. In this case, equation (5.114) is the only equation describing the dynamics of the transmission during the second gear. Similarly, the combined dynamics equations for the transmission during the 2-3 up shift torque phase can be written in state space form.

$$\begin{bmatrix} \dot{\omega}_i \\ \dot{\omega}_w \\ \dot{\tau}_s \end{bmatrix} = \begin{bmatrix} \frac{1}{I_{23}} (T_s - r(c_i + c_2 r^2 \omega_w^2)) \\ \frac{1}{I_{23}} \left( T_i + \frac{-(d_{21} - 1)}{I_{23}} \right) \end{bmatrix} \begin{bmatrix} \frac{1}{I_{23}} \\ 0 \end{bmatrix} \begin{bmatrix} -R_d d_{11} r \omega_i - K_s \omega_w \end{bmatrix} \tag{5.115}$$

When the output shaft is assumed rigid, the above equation is reduced to the one equation below.

$$\dot{\omega}_i = \frac{1}{I_{23S}} \left( -R_d d_{11} r(c_i + c_2(R_d d_{11} r)^2 \omega_w^2) - (d_{21} - 1)T_{OD} + T_i \right) \tag{5.116}$$

where $I_{23S}$ is as described in the 2nd gear dynamics.

The combined dynamic equations for the 2-3 up shift inertia phase can be written in state space form as follows.

$$\begin{bmatrix} \dot{\omega}_r \\ \dot{\omega}_i \\ \dot{\omega}_w \\ \dot{\tau}_s \end{bmatrix} = \begin{bmatrix} C_{T11} T_s \\ C_{T12} T_s \\ \frac{1}{I_v} (T_s - r(c_i + c_2 r^2 \omega_w^2)) \\ K_s R_d \omega_r - K_s \omega_w \end{bmatrix} + \begin{bmatrix} F_{11} & F_{21} \\ F_{12} & F_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_{2ND} \\ T_{OD} \end{bmatrix} + \begin{bmatrix} C_{T11} \\ C_{T22} \end{bmatrix} T_i \tag{5.117}$$

From this equation, as $K_v \rightarrow \infty$, the shaft torque dynamics is reduced to,
\[
\frac{\dot{T}_s}{K_s} \approx 0 = R_d \omega_{Rr} - \omega_w \implies R_d \omega_{Rr} = \omega_w
\] (5.118)

Using the above relationship and the output shaft torque, \(T_s\), from equation (5.113), the following can be derived.

\[
\begin{bmatrix}
\dot{\omega}_{Rr} \\
\dot{\omega}_t
\end{bmatrix} = \begin{bmatrix}
C_{T,13} r (c_1 + c_2 R_d^2 \omega_{Rr}^2) \\
C_{T,23} r (c_1 + c_2 R_d^2 \omega_{Rr}^2)
\end{bmatrix} + \begin{bmatrix}
F_{11S} & F_{21S} \\
F_{12S} & F_{22S}
\end{bmatrix} \begin{bmatrix}
T_{2ND} \\
T_{\theta \theta}
\end{bmatrix} + \begin{bmatrix}
C_{T,1S} \\
C_{T,2S}
\end{bmatrix} T_t
\] (5.119)

Here,

\[
\begin{align*}
C_{T,1S} &= \frac{C_{T,1}}{I_{23S}^2} \\
C_{T,2S} &= \frac{C_{T,2}}{I_{23S}^2} \\
C_{T,1S} &= \frac{C_{T,1} I_d}{I_{23S}^2} C_{R1} + C_{T,2} \\
C_{T,2S} &= \frac{C_{T,2} I_d}{I_{23S}^2} C_{R1} + C_{T,2} \\
F_{11S} &= \frac{F_{11}}{I_{23S}^2} \\
F_{21S} &= \frac{F_{21}}{I_{23S}^2} \\
F_{12S} &= \frac{C_{T,2} I_d R_d}{I_{23S}^2} F_{11} + F_{12} \\
F_{22S} &= \frac{C_{T,2} I_d R_d}{I_{23S}^2} F_{21} + F_{22} \\
I_{23S} &= 1 - C_{T,1} I_d R_d
\end{align*}
\] (5.120)

Using a similar approach, the dynamic equation describing the transmission with rigid output shaft during the third gear is,

\[
\dot{\omega}_t = \frac{1}{I_{3S}} \left( -R_d r (c_1 + c_2 (R_d r)^2 \omega_r^2) + T_t \right)
\] (5.121)

where \(I_{3S} = I_3 + (R_d)^2 I_v\) and \(I_3 = (I_t + I_{Si} + I_{Sc} + I_{Rr} + I_{Cr})\).

5.3.1.2 Shift hydraulic system model

The shift hydraulic system has been developed and presented in Chapter 4. The fully detailed model is very complex, of high order and highly nonlinear. Therefore, the model is simplified by using the energy-based model reduction method. The simplified
model, even though of reduced order and less complexity as compared to the fully detailed model, is too complex for observer design purposes. A closed form equation relating the input and the duty cycle to the output clutch pressure is difficult to develop. We will further simplify the simplified model from Chapter 4 and use the resulting model in the observer design in the later sections.

Consider first the flows in and out, from and through, the feeding chamber to the clutch/accumulator subsystem for the OD clutch. From the simplified model developed in Chapter 4, we write the following.

\[
\dot{P}_{c,OD} = C_{s,OD} \sqrt{P_1 - P_{c,OD}} \quad \text{for flow-in to the clutch} \quad (5.122)
\]

\[
\dot{P}_{c,OD} = -C_{s,OD} \sqrt{P_{c,OD}} \quad \text{for flow-out from the clutch} \quad (5.123)
\]

where

\[
C_{s,OD} = C_d \sqrt{\frac{2}{\rho} A_{inc,E} \frac{K_{a,OD}}{A_{a,OD}}} \quad (5.124)
\]

\(K_{a,OD}\) is the OD accumulator spring stiffness, and \(A_{a,OD}\) is the accumulator pressurized area. Ignoring all delays due to the port closing and opening, i.e. all ports are opened and closed instantaneously when commanded, we write the following,

\[
\dot{P}_{c,OD} = -C_{s,OD} \sqrt{P_{c,OD}} + u C_{s,OD} \left[ \sqrt{(P_1 - P_{c,OD})} + \sqrt{P_{c,OD}} \right] \quad (5.125)
\]

where

\[
u = \begin{cases} 
1 & \text{for } t_k < t \leq t_k + \tau_i(P_{c,OD}(t))T \\
0 & \text{for } t_k + \tau_i(P_{c,OD}(t))T < t \leq t_k + T
\end{cases} \quad (5.126)
\]

Here \(\tau_i(P_{c,OD}(t))\) is the duty ratio, a 100% value meaning “on” and an open inlet port. \(T\) is the PWM period. Equation (5.125) can be written in another form as,
From PWM controlled system analysis (Sira-Ramirez, 1989; Choi and Cho, 2001), equation (5.127) can be rewritten by replacing $u$ with the actual duty ratio as,

$$
\dot{P}_{c,OD} = (u - 1)C_{s,OD}\sqrt{P_{c,OD}} + uC_{s,OD}\sqrt{(P_i - P_{c,OD})}
$$

(5.127)

In the observer design, the equation above will be linearized around some operating point. It can be shown that the linearized version of the above equation gives the same answer as the following model.

$$
\dot{P}_{c,OD} = (\tau - 1)C_{s,OD}\sqrt{P_{c,OD}} + \tau C_{s,OD}\sqrt{(P_i - P_{c,OD})}
$$

(5.128)

or

$$
\dot{P}_{c,OD} = C_{s,OD}\sqrt{(0.01 \times (DutyCycle) \times P_i - P_{c,OD})}
$$

(5.129)

where \textit{DutyCycle} is the percentage duty cycle command. Moreover, simulation results showed that both (5.129) and (5.128) give similar answers. Since the equation (5.129) is more compact, it will be used in the design of the observer for the rest of the chapter. This will be applied to other clutches involved in our design as well. We emphasize here again that the model in (5.129) is developed for the observer design purpose. The simplified model showed in Chapter 4 will be used for the observer construction during implementation.

\textbf{5.3.2 Observer Design for 2-3 Up Shift Torque Phase}

5.3.2.1 Observer formulation

Using the developed model from the previous section, the dynamic behavior of the transmission with a rigid shaft during second gear can be described by the following.
\[
\begin{bmatrix}
\dot{\omega}_t \\
\dot{P}_{c,OD}
\end{bmatrix}
= \left[ -\frac{1}{L_{23S}} R_d d_{11} r (c_1 + c_2 (R_d d_{11} r)^2 \omega_t^2) - d'_{21} P_{c,OD} \right] + \left[ \frac{1}{L_{23S}} \right] T_i 
\] (5.131)

where

\[ d'_{21} = (d_{21} - 1) \mu (\omega_t - \omega_{c,r}) R_{OD} A_{COD} \text{sgn}(\omega_t - \omega_{c,r}) \] (5.132)

We follow directly the sliding mode observer design presented previously. From equation (5.131), letting \( x_1 \) and \( x_2 \) represent \( \omega_t \) and \( P_{c,OD} \) respectively, we can write

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \begin{bmatrix}
f'_1(x) \\
f'_2(x)
\end{bmatrix} 
\] (5.133)

where

\[
x = \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}^T = \begin{bmatrix}
\omega_t \\
P_{c,OD}
\end{bmatrix}^T 
\] (5.134)

\[
f'_1(x) = -\frac{1}{L_{23S}} R_d d_{11} r (c_1 + c_2 (R_d d_{11} r)^2 \omega_t^2) - d'_{21} P_{c,OD} + \frac{T_i}{L_{23S}} 
\] (5.135)

\[
f'_2(x) = C_{sOD} \sqrt{0.01 \times \text{DutyCycle} \times P_i - P_{c,OD}} 
\] (5.136)

\[
c_t = 1 / L_{23S} 
\] (5.137)

The measured signals in this case are \( x_1 \) or \( \omega_t \). We assume the observer equations in the following form.

\[
\begin{align*}
\hat{x}_1 &= \hat{f}'_1(\hat{x}) - k_{1a} \text{sgn}(\hat{x}_1) \\
\hat{x}_2 &= \hat{f}'_2(\hat{x}) - k_{2a} \text{sgn}(\hat{x}_1)
\end{align*} 
\] (5.138)

Here

\[
\hat{f}'_1(\hat{x}) = \hat{f}_1(\hat{x}) + c_t \hat{T}_i 
\] (5.139)
where $\hat{T}_i$ is assumed to be known by estimation from measured turbine and pump speeds and the torque converter model. Also, $\hat{x}_i = \hat{x}_i - x_i$, and $k_{it}$ are switching gains. The sliding surfaces are given by

$$S_i = \hat{x}_i = \hat{x}_i - x_i = 0$$

and the sliding conditions by

$$S_i \dot{S}_i < 0$$

or

$$S_i \dot{S}_i = \hat{x}_i [\Delta f_i' - k_{is} \sgn(\tilde{x}_i)] < 0$$

Here, the term $\Delta f_i' = \hat{f}_i'(\tilde{x}) - f_i'(x)$ represents the modeling error, which is assumed to be bounded, i.e.,

$$|\Delta f_i'| < \alpha_i(x, t) > 0$$

$\Delta f_i'$ in equation (5.143) includes the uncertainty due to the turbine torque input. The switching gains, $k_{is}$ must be chosen to be large enough to satisfy (5.143). In particular, based on equation (5.142), the sliding surfaces are attractive if

$$k_i > \alpha_i(x, t) > |\Delta f_i|$$

For this particular system, when the sliding surfaces are reached, the observer becomes a reduced order observer of order 1. In particular, the estimation error dynamics computed by using equations (5.133) and (5.138) are given by

$$\dot{\hat{x}}_1 = \Delta f_i' - k_{is} \sgn(\tilde{x}_i)$$

$$\dot{\hat{x}}_2 = \Delta f_2 - k_{2is} \sgn(\tilde{x}_i)$$

Using the equivalent control concept, when the sliding surface is reached, we have
\begin{align}
\dot{x}_1 - x_1 & \approx 0 \quad \rightarrow \quad \dot{x}_1 \approx 0, \quad \ddot{x}_1 \approx 0 \\
\tag{5.146}
\end{align}

Therefore, by substituting equation (5.146) in equation (5.145), the switching terms can be approximated by
\[
\text{sgn}(\dddot{x}_1) = \frac{\Delta f'_i}{k_{1iS}}
\tag{5.147}
\]

The equivalent dynamics on the reduced order manifold are described by the error dynamics of $\dddot{x}_2$, where the switching terms are now substituted using equation (5.147). Specifically, we have
\[
\dot{x}_2 = \Delta f'_2 - k_{2iS} \frac{\Delta f'_i}{k_{1iS}}
\tag{5.148}
\]

Selection of the gains $k_{iS}$ is discussed in the next section. When the transmission is in 2nd gear, the dynamic behavior of the transmission can be described by equation (5.131) with the OD clutch pressure set equal to zero, $P_{c,OD} = 0$. The observer in this case is of same form as equation (5.138), with the dynamics of $\dot{x}_2$ being ignored. In this case, the gain $k_{1iS}$ selected in the next section is used to maintain the system on the sliding surface.

5.3.2.2 Observer gain selection

The selection of the gains $k_{1iS}$ and $k_{2iS}$ is presented in this section. The value of the gain $k_{1iS}$ depends on estimated upper bounds for the uncertainties as described by equation (5.144). Its value will guarantee the attraction of the sliding surface, $S_i$. The
equivalent dynamics on the reduced order manifold described by equation (5.148) will then be used to find the value of gains $k_{2s}$.

**Gain $k_{2s}$**

Since $f'_i(x)$ includes some nonlinear terms, we can find the $\Delta f'_i(x)$ by considering the variation of the clutch friction coefficient, the estimation error of turbine speed, the turbine torque, and the clutch pressure in $f'_i(x)$ from equation (5.135). We then have,

$$
\Delta f'_i(x) = -\frac{1}{I_{23s}}(2c_2(R_d d_1 r)^2 \partial_i \partial_i) - \frac{d'_{21} \bar{P}_{c,OD}}{I_{23s}} - \frac{\bar{P}_{c,OD} \partial d'_{21}}{\partial \mu} + c_i \bar{T}_i
$$

(5.149)

At this point, for the sake of simplicity we will ignore the uncertainty due to the friction coefficient $\mu$, which is embedded in the parameter $d'_{21}$, and the uncertainty due to the turbine torque input. The former assumption is practically valid since, based on the manufacturer’s data, the friction efficient of the clutch plate is almost constant when the slip speed is high. And this is basically the case for the on-coming clutch during the torque phase. However, the effect of both uncertainties in the turbine torque input and the friction coefficient can be considered, and are considered, in the error analysis section at the end of this chapter. Therefore, equation (5.149) becomes the following.

$$
\Delta f'_i(x) = -\frac{1}{I_{23s}}(2c_2(R_d d_1 r)^2 \partial_i \partial_i) - \frac{d'_{21} \bar{P}_{c,OD}}{I_{23s}}
$$

(5.150)

Numerical values used here follows the development presented in Wateraghit and Srinivasan (2003(b)). In particular, the maximum error bound for the OD clutch pressure is 85 psi or 0.58 MPa., and for the turbine speed is 30 rad/sec. To evaluate uncertainty bounds, we consider the combination of component uncertainties in the absence of
polarity information. Consider \( f_i \) as a functional relationship between the variables \( x_1, x_2, \ldots, x_L \),

\[
f_i = f(x_1, x_2, \ldots, x_L)
\]  

(5.151)

The uncertainty in \( f_i \) due to uncertainties in the dependent variables can be estimated reasonably as the RMS value of the contributions from the component uncertainties (Figliola and Beasley, 1991). Thus, we have

\[
|\Delta f_i| = \sqrt{\left( \frac{\partial f_i}{\partial x_1} \right|_{x_1=\bar{x}_1} \Delta x_1)^2 + \left( \frac{\partial f_i}{\partial x_2} \right|_{x_2=\bar{x}_2} \Delta x_2)^2 + \cdots + \left( \frac{\partial f_i}{\partial x_L} \right|_{x_L=\bar{x}_L} \Delta x_L)^2}
\]  

(5.152)

\( \bar{x}_i \) is the nominal value of the corresponding variable and \( \Delta x_i \) is the uncertainty in that variable. With this result, the uncertainty bound can be evaluated by applying equation (5.152) to equation (5.150), and substituting numerical values including the approximate uncertainty bounds described. We then have the following.

\[
|\Delta f_i| \approx 20.61 \text{ rad/sec}^2 \rightarrow k_i \approx 30 \text{ rad/sec}^2
\]  

(5.153)

**Gain \( k_{2S} \)**

Selection of the gain \( k_{2S} \) affects the estimation error dynamics of the clutch pressure \( P_{C,OD} \). Based on equation (5.148), the error dynamics of \( P_{C,OD} \) can be written as

\[
\dot{P}_{C,OD} = \Delta f_2 - k_{2S} \frac{\Delta f_1'}{k_{1S}}
\]  

(5.154)

In this case, \( \Delta f_2 \) represents the modeling error of the shift hydraulic system. Due to the nonlinearity of \( \Delta f_2(x) \), we consider two limiting cases in the design of the gain \( k_{2S} \). The
first case is when the clutch pressure is near zero, and the second case is when the clutch pressure is near the line pressure input.

i) $P_{c, OD} \approx 0$

This case represents the moment when the OD clutch cavity is almost full but its pressure is still low. By linearizing $f_1(x)$ around this point, and substituting $\Delta f'_1$ from equations (5.150), we get

$$
\dot{\tilde{P}}_{c, OD} = \left[ \frac{-0.5 \times C_{sod}}{\sqrt{(0.01 \times \text{DutyCycle} \times \bar{P})}} + \frac{k_{2S} d'_{21}}{k_{1S} I_{23S}} \right] \tilde{P}_{c, OD} + \frac{k_{2S}}{k_{1S} I_{23S}} \left[ \frac{1}{2 c_d (R_d d_1 \omega_2)} \tilde{\omega}_i \right]
$$

Since this equation is defined after the sliding surfaces are reached, we can assume that $\tilde{\omega}_i \approx 0$. Therefore, equation (5.155) is reduced to

$$
\dot{\tilde{P}}_{c, OD} = \left[ \frac{-0.5 \times C_{sod}}{\sqrt{(0.01 \times \text{DutyCycle} \times \bar{P})}} + \frac{k_{2S} d'_{21}}{k_{1S} I_{23S}} \right] \tilde{P}_{c, OD}
$$

Note that terms with bars represent nominal values of corresponding variables and parameters, i.e. $\bar{P}_{c, OD} = 0$, $\bar{P}_l$ = line pressure for 2nd-3rd gearshift, and $\text{DutyCycle}$ is at 100%. In can be seen that the dynamics of the estimation error for $P_{c, OD}$ are stable if the coefficient of $\tilde{P}_{c, OD}$ is less than zero. The gain $k_{2S}$ can be selected to ensure satisfactory response of $\tilde{P}_{c, OD}$. In our application, we need the estimation error to go to zero relatively fast as compared to the shift duration, especially the duration of the torque phase which is
in the range of 100-150 millisecond. Therefore, a 10 millisecond decay time for this estimation error is acceptable. As a result, we force the estimation error for $P_{e,OD}$ to go to zero within 10 milliseconds. Therefore, we place the eigenvalue of equation (5.156) at –100. After substituting numerical values, this choice of eigenvalue gives

$$\left[ \frac{-0.5 \times C_s}{\sqrt{\left(0.01 \times \text{DutyCycle} \times P_l\right)}} + \frac{k_{2S}d_{21}'}{k_{1S}I_{23S}} \right] = -100 \rightarrow k_{2S} = -8.37 \text{ Pa/sec} \quad (5.157)$$

The gain $k_{2S}$ chosen above is for the case when the clutch pressure is nearly zero as mentioned. Fortunately, as the clutch pressure gets higher, this choice of gains still seems to be usable. In fact, if one keeps on linearizing the function $f_2(x)$ as $\tilde{P}_{e,OD}$ gets higher, the coefficient of $\tilde{P}_{e,OD}$ will increase, which yields a faster decay rate of $\tilde{P}_{e,OD}$ without changing the value of $k_{2S}$. However, when the clutch pressure is close to the line pressure input, $P_{e,OD} \approx P_l$, the function $f_2(x)$ is no longer linearizable. In fact, when $P_{e,OD} \approx P_l$, the flow across the orifice becomes laminar flow and equation (5.136) is no longer valid. In this case, the validity of the gain $k_{2S}$ selected above is checked in the following case.

**ii) $P_{e,OD} \approx P_l$**

As mentioned, the flow across the orifice when $P_{e,OD} \approx P_l$ becomes laminar flow. In this case, the clutch pressure dynamics in equation (5.136) is replaced by,

$$\dot{\tilde{P}}_{e,OD} = \frac{2\delta^2 DA}{\mu} (P_l - P_{e,OD}) = f_2(x) \quad (5.158)$$

Assuming that the line pressure input is exactly known ($\tilde{P}_l = 0$), $\Delta f_2$ can be calculated as,
\[
\Delta f_2(x) = \frac{2\delta^2 DA}{\mu_{OD}} (\tilde{P}_t - \tilde{P}_{c,OD}) \quad \Rightarrow \quad -\frac{2\delta^2 DA}{\mu_{OD}} \tilde{P}_{c,OD} \quad (5.159)
\]

Therefore, equation (5.156) becomes,

\[
\dot{\tilde{P}}_{c,OD} = \left[-\frac{2\delta^2 DA}{\mu_{OD}} + \frac{k_{2S} d_{21}'}{k_{1S} I_{23S}}\right] \tilde{P}_{c,OD} \quad (5.160)
\]

By using the value for the gain \(k_{2S}\) selected previously,

\[
k_{2S} = -8.37 \text{ Pa/sec} \quad \rightarrow \quad \left[-\frac{2\delta^2 DA}{\mu_{OD}} + \frac{k_{2S} d_{21}'}{k_{1S} I_{23S}}\right] = -96.89 \quad (5.161)
\]

Surprisingly, the value of the gain \(k_{2S}\) offers error dynamics similar to the first limiting case. Therefore, it can be assumed that the value of the gain \(k_{2S}\) chosen in the previous case can be used for the entire operating range.

### 5.3.3 Observer Design for Inertia Phase

#### 5.3.3.1 Observer formulation

The differential equation governing the dynamics of the transmission during the 2-3 up shift inertia phase for the rigid shaft case is given by,

\[
\begin{bmatrix}
\dot{\omega}_{Re} \\
\dot{\omega}_t \\
\dot{P}_{c,2ND} \\
\dot{P}_{c,OD}
\end{bmatrix} =
\begin{bmatrix}
C_{T1S} \omega_{Re}\left(c_1 + c_2 \omega_{Re}^2\right) + F'_{11S} P_{c,2ND} + F'_{21S} P_{c,OD} \\
C_{T1S} \omega_{Re}\left(c_1 + c_2 \omega_{Re}^2\right) + F'_{12S} P_{c,2ND} + F'_{22S} P_{c,OD} \\
C_{x2ND} \sqrt{0.01 \times \text{DutyCycle} \times P_t - P_{c,2ND}} \\
C_{xOD} \sqrt{0.01 \times \text{DutyCycle} \times P_t - P_{c,OD}}
\end{bmatrix} =
\begin{bmatrix}
C_{T1S} \\
C_{T2S} \\
0 \\
0
\end{bmatrix} T_i \quad (5.162)
\]

where
\[ F'_{11S} = F_{11S} \left( \mu(\omega_{Sr}) R_{2ND} A_{2ND} \text{sgn}(\omega_{Sr}) \right) \]
\[ F'_{12S} = F_{12S} \left( \mu(\omega_{Sr}) R_{2ND} A_{2ND} \text{sgn}(\omega_{Sr}) \right) \]
\[ F'_{21S} = F_{21S} \left( \mu(\omega_{i} - \omega_{Cr}) R_{OD} A_{OD} \text{sgn}(\omega_{i} - \omega_{Cr}) \right) \]
\[ F'_{22S} = F_{22S} \left( \mu(\omega_{i} - \omega_{Cr}) R_{OD} A_{OD} \text{sgn}(\omega_{i} - \omega_{Cr}) \right) \]

(5.163)

All parameters are defined previously. We define
\[ \mathbf{x}^T = [x_1 \ x_2 \ x_3 \ x_4]^T = [\omega_{Br} \ \omega_i \ P_{c,2ND} \ P_{c,OD}]^T \]

(5.164)

The sliding mode observer in this case is given by
\[
\begin{bmatrix}
\dot{\omega}_{Br} \\
\dot{\omega}_i \\
\dot{P}_{c,2ND} \\
\dot{P}_{c,OD}
\end{bmatrix} = \begin{bmatrix}
\dot{f}_1'(^{\hat{\mathbf{x}}}) \\
\dot{f}_2'(^{\hat{\mathbf{x}}}) \\
\dot{f}_3(^{\hat{\mathbf{x}}}) \\
\dot{f}_4(^{\hat{\mathbf{x}}})
\end{bmatrix} - \begin{bmatrix}
k_{1IS} & 0 \\
0 & k_{2IS} \\
k_{3IS} & k_{4IS} \\
k_{5IS} & k_{6IS}
\end{bmatrix} \begin{bmatrix}
\text{sgn}(\hat{\omega}_{Br}) \\
\text{sgn}(\hat{\omega}_i)
\end{bmatrix}
\]

(5.165)

where
\[
\Delta f_1'(^{\hat{\mathbf{x}}}) = C_{T1S} c_2 r^3 R_d^2 \hat{\omega}_{Br}^2 + F_{11S}^i \hat{P}_{c,2ND} + F_{21S}^i \hat{P}_{c,OD} + C_{T1S} \hat{T}_i
\]

(5.166)

\[
\dot{f}_2'(^{\hat{\mathbf{x}}}) = C_{T2S} r(c_1 + c_2 r^2 R_d^2 \hat{\omega}_{Br}^2) + F_{12S}^i \hat{P}_{c,2ND} + F_{22S}^i \hat{P}_{c,OD} + C_{T2S} \hat{T}_i
\]

(5.167)

\[
\dot{f}_3(^{\hat{\mathbf{x}}}) = C_{s,2ND} \sqrt{0.01 \times \text{DutyCycle} \times P_i - P_{c,2ND}}
\]

(5.168)

\[
\dot{f}_4(^{\hat{\mathbf{x}}}) = C_{s,OD} \sqrt{0.01 \times \text{DutyCycle} \times P_i - P_{c,OD}}
\]

(5.169)

Here, \( k_{iIS} \) are switching gains. \( \hat{T}_i \) is assumed to be estimated from measured turbine and pump speeds and the torque converter model. The sliding surfaces are given by
\[
S_1 = \hat{\omega}_{Br} = \hat{\omega}_{Br} - \omega_{Br} = 0
\]
\[
S_2 = \hat{\omega}_i = \hat{\omega}_i - \omega_i = 0
\]

(5.170)

and the sliding conditions by
\[
S_i \hat{S}_i < 0, \quad i = 1, 2, 3
\]

(5.171)
Specifically,

\[ S_1 \dot{\tilde{\omega}}_i = \tilde{\omega}_i' [\Delta f'_i - k_{1iS} \text{sgn}(\tilde{\phi}_i)] < 0 \]
\[ S_2 \dot{\tilde{\omega}}_i = \tilde{\omega}_i' [\Delta f'_2 - k_{2iS} \text{sgn}(\tilde{\phi}_i)] < 0 \]  \tag{5.172}

Here, the term \( \Delta f'_i = \hat{f}_i'(\dot{x}) - f_i'(x) \) represents the modeling error, which is assumed to be bounded, i.e.,

\[ |\Delta f'_i| < \alpha_i(x,t) > 0 \]  \tag{5.173}

The switching gains \( k_{1iS} \) and \( k_{2iS} \) must be chosen to be large enough to satisfy (5.173). In particular, based on equation (5.172), the sliding surfaces are attractive if

\[ k_{1iS} > \alpha_{1iS}(x,t) > |\Delta f'_1| \]
\[ k_{2iS} > \alpha_{2iS}(x,t) > |\Delta f'_2| \]  \tag{5.174}

The switching gains, \( k_{3iS} \) to \( k_{6iS} \), are designed based on uncertainties in the model and their effects on estimation of \( P_{c,2ND} \) and \( P_{c,OD} \).

When the sliding surfaces are reached, the observer becomes a reduced order observer of order 2 where the equivalent dynamics on the reduced order manifold are described by

\[ \dot{\hat{P}}_{c,2ND} = \Delta f_1 - k_{3iS} \frac{\Delta f'_1}{k_{4iS}} - k_{4iS} \frac{\Delta f'_2}{k_{2iS}} \]
\[ \dot{\hat{P}}_{c,OD} = \Delta f_4 - k_{5iS} \frac{\Delta f'_1}{k_{6iS}} - k_{6iS} \frac{\Delta f'_2}{k_{2iS}} \]  \tag{5.175}

5.3.3.2 Observer gain selection

Selection of observer gains for the inertia phase can be done following procedures similar to those presented earlier. We consider first selection of the gains \( k_{1iS} \) and \( k_{2iS} \). The values of these gains depend on estimated upper bounds for uncertainties described by
equation (5.174). The equivalent dynamics on the reduced order manifold described by equation (5.175) are then used to find the values of gains \( k_{3iS} \) to \( k_{6iS} \).

**Gain \( k_{iJS} \)**

As described in the torque phase design, \( \Delta f'_i \) can be found by calculating the variation of \( f'_i(x) \) due to the state estimation error, the clutch friction coefficients, and the turbine torque input. From equation (5.162), we can write,

\[
\Delta f'_i(\tilde{x}) = (2C_{T,1S}c_2 r^3 R_j^2 \bar{\omega}_{Rr}) \bar{\omega}_{Rr} + F'_{11S} \tilde{P}_{e,2ND} + F'_{21S} \tilde{P}_{e,OD} \\
+ \tilde{P}_{e,2ND} \frac{\partial F'_{11S}}{\partial \mu_{2ND}} + \tilde{P}_{e,OD} \frac{\partial F'_{21S}}{\partial \mu_{OD}} + C_{T,1S} \tilde{T} 
\]

\( (5.176) \)

We will ignore the uncertainty due to the clutch friction coefficient and the turbine torque input for now. Therefore, equation (5.176) becomes the following.

\[
\Delta f'_i(\tilde{x}) = (2C_{T,1S}c_2 r^3 R_j^2 \bar{\omega}_{Rr}) \bar{\omega}_{Rr} + F'_{11S} \tilde{P}_{e,2ND} + F'_{21S} \tilde{P}_{e,OD} 
\]

\( (5.177) \)

The maximum estimation error for the 2ND clutch is 54 psi or 0.41 MPa, and for the OD clutch is 85 psi or 0.58 MPa, and the estimation error for \( \bar{\omega}_{Rr} \) is assumed to be 30 rad/sec. Applying equation (5.152) to equation (5.177), and substituting numerical values including the approximate uncertainty bounds described, we have the following.

\[
|\Delta f'_i| \approx 5.83 \times 10^3 \text{ rad/sec}^2 \quad \rightarrow \quad k_{iJS} \approx 6 \times 10^3 \text{ rad/sec}^2 
\]

\( (4.168) \)

**Gain \( k_{2JS} \)**

By using a similar approach and assumptions in selecting the gain \( k_{iJS} \), we can write the function \( \Delta f'_2(x) \) in the following form.
\[
\Delta f_2' (\tilde{x}) = (2C_{T\Delta S}c_2 r^3 R_2^2 \tilde{R}_{Rr} \tilde{R}_{Rr} + F'_{12S} \tilde{P}_{c,2ND} + F'_{22S} \tilde{P}_{c,OD} \\
+ \tilde{P}_{c,2ND} \frac{\partial F'_{12S}}{\partial \mu_{2ND}} + \tilde{P}_{c,OD} \frac{\partial F'_{22S}}{\partial \mu_{OD}} + C_{T\Delta S} \tilde{T})
\]

(5.178)

And by ignoring the uncertainty due to the turbine torque input, the above equation becomes

\[
\Delta f_2' (\tilde{x}) = (2C_{T\Delta S}c_2 r^3 R_2^2 \tilde{R}_{Rr} \tilde{R}_{Rr} + F'_{12S} \tilde{P}_{c,2ND} + F'_{22S} \tilde{P}_{c,OD}
\]

(5.179)

Applying equation (5.152) to equation (5.179), and substituting numerical values including the approximate uncertainty bounds described previously, we have the following.

\[
|\Delta f_2'| \approx 29.16 \text{ rad/sec}^2 \rightarrow k_{2IS} \approx 30 \text{ rad/sec}^2
\]

(5.180)

Gain \(k_{3IS}\) and \(k_{4IS}\)

Selection of the gains \(k_{3IS}\) and \(k_{4IS}\) affects the estimation error dynamics of the clutch pressure \(P_{c,2ND}\). Recalling from equation (5.175), the error dynamics of \(P_{c,2ND}\) after the sliding surfaces are reached can be written as

\[
\dot{\tilde{P}}_{c,2ND} = \Delta f_3 - k_{3IS} \frac{\Delta f_1'}{k_{1IS}} - k_{4IS} \frac{\Delta f_2'}{k_{2IS}}
\]

(5.181)

where \(\Delta f_3\) represents the modeling error of the shift hydraulic system. During the inertia phase, the 2ND clutch is the off-going clutch, and the pressure of the 2ND clutch is normally low. Therefore, we will design the observer gains \(k_{3IS}\) and \(k_{4IS}\) around the point where the clutch pressure is zero. By linearizing \(f_3(x)\) around this point, and substituting
\( \Delta f'_1 \) and \( \Delta f'_2 \) from equations (5.177) and (5.179) respectively in equation (5.181), when the system is on the sliding surface, we get

\[
\dot{\tilde{P}}_{c,2ND} = -\left[ \frac{0.5 \times C_{x,2ND}}{\sqrt{(0.01 \times DutyCycle \times \tilde{P})}} + \frac{k_{3IS}}{k_{4IS}} F'_{11S} + \frac{k_{4IS}}{k_{2IS}} F'_{12S} \right] \tilde{P}_{c,2ND} \\
- \left[ \frac{k_{3IS}}{k_{4IS}} F'_{21S} + \frac{k_{4IS}}{k_{2IS}} F'_{22S} \right] \tilde{P}_{c,OD} \tag{5.182}
\]

It can be seen that the dynamics of the estimation error for \( \tilde{P}_{c,2ND} \) are stable if the coefficient of \( \tilde{P}_{c,2ND} \) is less than zero. Thus, the gains \( k_{3IS} \) and \( k_{4IS} \) can be selected to ensure stability and satisfactory response of \( \tilde{P}_{c,2ND} \). However, the selection of the gains \( k_{3IS} \) and \( k_{4IS} \) also affects the forcing function due to the estimation error for \( P_{c,OD} \). The selection of the gain \( k_{3IS} \) and \( k_{4IS} \) should therefore be such that the estimation error for \( P_{c,OD} \) should have less effect on \( \tilde{P}_{c,2ND} \). It is desired to force the estimation error for \( P_{c,2ND} \) to go to zero within 10 milliseconds. Therefore, we place the eigenvalue of equation (5.182) at \(-100\) rad/sec. Substituting numerical values, we have

\[
\begin{align*}
\frac{0.5 \times C_{x,2ND}}{\sqrt{(0.01 \times DutyCycle \times \tilde{P})}} + \frac{k_{3IS}}{k_{4IS}} F'_{11S} + \frac{k_{4IS}}{k_{2IS}} F'_{12S} &= 100 \\
\frac{k_{3IS}}{k_{4IS}} F'_{21S} + \frac{k_{4IS}}{k_{2IS}} F'_{22S} &= 0
\end{align*}
\]

As one may expect, the choice of the gains \( k_{3IS} \) and \( k_{4IS} \) to ensure that the forcing function \( \tilde{P}_{c,OD} \) becomes zero can never be satisfied practically due to modeling uncertainties. However, we have shown earlier that as long as \( \tilde{P}_{c,OD} \) is bounded, \( \tilde{P}_{c,2ND} \) is also bounded.
Gains $k_{5iS}$ and $k_{6iS}$

The method of selecting the gains $k_{5iS}$ to $k_{6iS}$ is similar to that for the gains $k_{3iS}$ to $k_{4iS}$ based on the dynamics of the error $\hat{P}_{c,2ND}$. From equation (5.175), the error dynamics of $P_{c,OD}$ after the sliding surfaces are reached can be written as

$$\dot{\hat{P}}_{c,OD} = \Delta f_4 - k_{5iS} \frac{\Delta f'_1}{k_{1iS}} - k_{6iS} \frac{\Delta f'_2}{k_{2iS}}$$  (5.184)

Again, $\Delta f'_6$ represents the modeling error of the shift hydraulic system. The analysis we use in designing the gains $k_{5iS}$ to $k_{6iS}$ for the OD clutch here is the same as the one we used to design the gains $k_{2iS}$ for the OD clutch during the torque phase. We consider first the linearization of $f_5(x)$ around the zero clutch pressure, and substitute $\Delta f'_1$ and $\Delta f'_2$ from equations (5.177) and (5.179) respectively in equation (5.184), to get

$$\dot{\hat{P}}_{c,OD} = -\left[ \frac{0.5 \times C_{x,OD}}{\sqrt{(0.01 \times DutyCycle \times \bar{P}_i)}} \right] \frac{k_{5iS}}{k_{1iS}} f'_1 + \frac{k_{6iS}}{k_{2iS}} f'_2 - \left[ \frac{k_{5iS}}{k_{1iS}} F'_1 + \frac{k_{6iS}}{k_{2iS}} F'_2 \right] \hat{P}_{c,2ND}$$  (5.185)

As in the case of designing gains for $\tilde{P}_{c,2ND}$, we have

$$\frac{0.5 \times C_{x,OD}}{\sqrt{(0.01 \times DutyCycle \times \bar{P}_i)}} \frac{k_{5iS}}{k_{1iS}} f'_1 + \frac{k_{6iS}}{k_{2iS}} f'_2 = 100$$

$$\frac{k_{5iS}}{k_{1iS}} f'_1 + \frac{k_{6iS}}{k_{2iS}} f'_2 = 0$$

$$k_{5iS} = -7.81 \text{ Pa/sec}$$
$$k_{6iS} = -5.54 \text{ Pa/sec}$$  (5.186)

As the clutch pressure gets higher, these choices of gains are still valid. However, when $P_{c,OD} \approx P_i$, the flow across the orifice becomes laminar flow. The clutch dynamics
model used in the above derivation is not valid and must be replaced by the laminar flow model. It was shown earlier that with the laminar flow model, the gains $k_{5iS}$ and $k_{6iS}$ shown here are also still valid. Therefore, the detailed calculation is omitted here.

5.3.4 Simulation Results

The observers developed in the previous sections are now validated via simulation in this section. The validation is done using the transmission simulator which is developed and validated in Chapter 3 to produce input signals to the observer, i.e. all speed signals that can be measured on the test vehicle. To simulate the actual behavior of the transmission, engine speed and clutch pressure data obtained experimentally on an instrumented test vehicle equipped with the transmission of interest are used as inputs to the transmission model. With the use of real pressure data, the error and uncertainties from the hydraulic system model can be isolated. The estimated clutch pressure is compared with the clutch pressure input to the transmission model. Since the output shaft data is not available because it is not measured on the test car, the estimated shaft torque is compared with the shaft torque computed by the transmission simulator. For the observer, besides taking speed signals from the transmission simulator, it needs the turbine torque computed by using the torque converter model, and the clutch duty-cycle command as inputs.

Figure 5.2 shows the Simulink® simulation diagram used in the validation. We note here that the system is simulated using a fixed step-size integration routine with a step-size of 0.1 millisecond. This corresponds to a sampling frequency of 10 kHz. Since only the 2-3 up shift is emphasized here, the clutch pressures of interest are the OD clutch...
pressure and 2ND clutch pressure. The observer for the OD clutch pressure is activated starting from the torque phase and lasts until the end of the inertia phase. For the reasons mentioned in the design process, the observer for the 2ND clutch pressure is activated only during the inertia phase.

Figure 5.2: Simulink® simulation diagram for observer validation

Figures 5.3 to 5.6 show estimation results obtained using the observer designed in the previous sections. Figures 5.3 and 5.4 show the comparison of the estimated turbine speed with the turbine speed from the transmission simulator and the experimentally measured turbine speed obtained from the test vehicle. A small amount of offset can be seen between the simulated turbine speed using the transmission simulator and the measurement. The characteristic of the mismatch suggests that the turbine torque calculated from the torque converter model may be higher than the real turbine torque. The difference in terms of the initial conditions between the transmission model and the
test vehicle could cause an offset in the results shown as well. Regardless of the mismatch between the results from the transmission model and the measurement, the estimated turbine speed tracks the simulated turbine speed very well. The same results can be seen in the case of the transmission output speed estimation shown in Figure 5.4.

Figures 5.5 and 5.6 show the estimation results of the OD clutch pressure and the 2ND clutch pressure respectively, as compared to measurements. Figure 5.5 also shows the simulated OD clutch pressure obtained open loop from the OD clutch model in order to compare the benefit of using the observer over the open-loop estimation. The observer is able to estimate both clutch pressures very closely in both the torque phase and the inertia phase. One feature that can be seen also from the Figure 5.5 is the ability of the observer to estimate the clutch pressure accurately despite the need for clutch filling. Based on the experimental data used in this simulation, the delay time between the duty cycle command and the response of the OD clutch is approximately 100 milliseconds. This could be only the effect of the filling phase, which can vary from 50-150 millisecond (Cho, 1989). This delay time can also include communication delay within the control unit. In any case, the observer is still able to estimate the clutch pressure closely despite the presence of this unknown delay time.
Figure 5.3: Estimated turbine speed as compared to the simulation result and the measurement

Figure 5.4: Estimated transmission output speed as compared to the simulation result and the measurement
Figure 5.5: Estimated OD clutch pressure as compared to experimental data

Figure 5.6: Estimated 2ND clutch pressure as compared to experimental data
In order to show the performance for the set of gains selected in the previous section, especially the set of gains involved in the clutch pressure estimation, we now assume that we would like to place the poles of equations (5.157), (5.182) and (5.186) at -10 rad/sec in stead of -100 rad/sec. In this case, for the torque phase we have,

\[
\left[\frac{-0.5 \times C_s}{\sqrt{(0.01 \times \text{DutyCycle} \times \bar{P})}} + \frac{k_{21s} d_{21}}{k_{1s} I_{23s}}\right] = -100 \rightarrow k_{2is} = -8.37 \text{ Pa/sec} \tag{5.187}
\]

and for the inertia phase, we have

\[
\begin{align*}
\frac{0.5 \times C_{x,2ND}}{\sqrt{(0.01 \times \text{DutyCycle} \times \bar{P})}} + \frac{k_{3is} F'_{11s}}{k_{1is}} + \frac{k_{4is} F'_{12s}}{k_{2is}} &= 100 \\
k_{3is} F'_{21s} + \frac{k_{4is} F'_{22s}}{k_{2is}} &= 0 \\
k_{3is} F'_{11s} + \frac{k_{6is} F'_{12s}}{k_{2is}} &= 0
\end{align*}
\tag{5.188}
\]

\[
\begin{align*}
\frac{0.5 \times C_{x,OD}}{\sqrt{(0.01 \times \text{DutyCycle} \times \bar{P})}} + \frac{k_{5is} F'_{21s}}{k_{1is}} + \frac{k_{6is} F'_{22s}}{k_{2is}} &= 100 \\
k_{5is} F'_{11s} + \frac{k_{6is} F'_{12s}}{k_{2is}} &= 0
\end{align*}
\tag{5.189}
\]

and by using these gains, the clutch pressure estimation results are shown in Figure 5.7 and 5.8.

As can be seen, the above set of gains affects the clutch pressure estimation dynamics. From both figures, the estimation error converges more slowly to zero and the estimation starts to track the measurement almost at the end of the gear shift. For the case of the OD clutch, due to slow error dynamics, the error during the clutch filling phase is very obvious. Figure 5.9 shows the estimation error of the OD clutch pressure estimation.
from the simulation shown in Figure 5.5 as compared to Figure 5.7. Clearly, the set of gains used in the former case is preferable to the latter.

Figure 5.7: Estimated OD clutch pressure as compared to experimental data (pole of the error dynamics is at -10 rad/sec)
Figure 5.8: Estimated 2ND clutch pressure as compared to experimental data (pole of the error dynamics is at -10 rad/sec)

Figure 5.9: Error from OD clutch pressure estimation when the error dynamics have poles at -10 rad/sec as compared to at -100 rad/sec
5.3.5 Error Analysis

The observer designs presented in the previous sections rely heavily on assumptions that some variables and parameters are known, especially the turbine torque or the transmission input torque, and the friction coefficient of the clutch plate. These uncertainties have to be considered in the observer design process to ensure that the observer is robust to these model and input errors and gives acceptable estimation error.

In this section, the estimation error for $P_{c,OD}$ is considered. If both uncertainties in the friction coefficient and the turbine torque are considered, equation (5.150) can be written as,

$$
\Delta f'_{r}(x) = -\frac{1}{I_{23S}}(2c_{2}(R_{u}d_{1},r)\gamma)\tilde{\omega}_{r} - \frac{d_{21}}{I_{23S}} \tilde{P}_{c,OD} - \frac{P_{c,OD}}{I_{23S}} \Delta d_{21} + \frac{1}{I_{23}} \tilde{T}_{r}
$$

(5.190)

where $\Delta d_{21} = (d_{21} - 1)\Delta \mu(\Delta \omega)R_{OD}A_{COD} \text{sgn}(\Delta \omega)$

(5.191)

$
\tilde{P}_{c,OD}$ and $d_{21}'$ being nominal values of $P_{c,OD}$ and $d_{21}$ respectively. The estimation error dynamics for $P_{c,OD}$ given by equation (5.156) can be written as

$$
\dot{\tilde{P}}_{c,OD} = \left[ \begin{array}{c}
\frac{-0.5 \times C_{c,OD}}{\sqrt{0.01 \times \text{DutyCycle} \times \tilde{P}_{r}}} + \frac{k_{2s} d_{21}'}{I_{1s} I_{23S}} \\
\frac{k_{2s} d_{21}'}{I_{1s} I_{23S}} - \frac{1}{I_{23S}} \tilde{T}_{r} - \frac{P_{c,OD}}{I_{23S}} \Delta d_{21}'
\end{array} \right]
$$

(5.192)

It can be seen that only the value of the gain $k_{2s}$ now affects the forcing function on the right hand side of equation (5.192), which depends on the error in the estimated turbine torque and in the assumed friction coefficient $\mu$. Figure 5.10 shows the estimated clutch pressure assuming that the turbine torque from the torque converter model to the transmission simulator is higher by 10% than the turbine torque assumed by the observer.

Using the gain $k_{2s}$ selected earlier, the figure shows more error in the clutch pressure
estimate as compared to Figure 5.5, which corresponded to zero turbine torque error. Figure 5.11 shows the effect of the friction coefficient error on the OD clutch pressure estimation. In this case, both OD and 2ND clutches in the transmission model are assumed to have a higher value of friction coefficient than the one that is used in the observer. Figure 5.11 shows estimated that the OD clutch pressure obviously disagrees with the test data. Therefore, care must be taken in choosing the value of the gain $k_{2,5}$. However, since both uncertainties from the turbine torque estimation and the friction coefficient are considered as matched uncertainties, if both uncertainties are bounded, then the estimation error of the clutch pressure is also bounded. The choice of gains normally depends on the trade-off between the desired estimation error dynamics and the desired accuracy of estimation.

From the literature, the sliding model observer can be made robust to matched uncertainty. The robustness property of this observer simply guarantees that as long as all sliding surfaces are attractive, the estimation error can be made bounded assuming that all matched uncertainties are bounded. Section 5.1 showed analytical results confirming the above fact. The ability to deal with unmatched uncertainty depends on internal interconnections between subsystems. When the system consists of unmatched uncertainty, the selection of observer gains can emphasize lower estimation error but may degrade the error dynamics.
Figure 5.10: Estimated OD clutch pressure as compared to the experimental data with 10% turbine torque error

Figure 5.11: Estimated OD clutch pressure as compared to the experimental data with 10% friction coefficient error
5.4 Clutch Pressure Estimation using Discrete Observer

5.4.1 Discrete Observer Design for 2-3 Up Shift Torque Phase

For the discrete sliding mode observer development, we first linearize the model used in the previous section, and then discretize the linearized model to covert the continuous time model to the discrete time version. Recall from the previous section that the governing equation of the transmission with a rigid shaft during second gear is

\[
\begin{bmatrix}
\dot{\omega}_t \\
The discretization can be easily done using the specific command “c2d” in MATLAB® software package. In using this command, the sampling period and a method of discretization need to be specified. For our problem, a sampling period of 16 milliseconds is used, which is the sampling period used on the transmission control unit in the test vehicle. The “zero-order-hold” discretization method is commonly used, and is our

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choice for this research. After the discretization, equation (5.193) can be written in the following form.

\[
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
    \phi_{11} & \phi_{12} \\
    \phi_{21} & \phi_{22}
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k)
\end{bmatrix} +
\begin{bmatrix}
    \gamma_{11} & \gamma_{12} \\
    \gamma_{21} & \gamma_{22}
\end{bmatrix}
\begin{bmatrix}
    T_i(k) \\
    P_i(k)
\end{bmatrix}
\] (5.197)

Here, \(x_1\) and \(x_2\) represent \(\omega_i\) and \(P_{c,OD}\) respectively. The measured signals in this case are \(x_1\) or \(\omega_i\). We assume the observer equations to be in the following form.

\[
\begin{bmatrix}
    \hat{x}_1(k+1) \\
    \hat{x}_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
    \phi_{11} & \phi_{12} \\
    \phi_{21} & \phi_{22}
\end{bmatrix}
\begin{bmatrix}
    \hat{x}_1(k) \\
    \hat{x}_2(k)
\end{bmatrix} +
\begin{bmatrix}
    \gamma_{11} & \gamma_{12} \\
    \gamma_{21} & \gamma_{22}
\end{bmatrix}
\begin{bmatrix}
    \hat{T}_i(k) \\
    \hat{P}_i(k)
\end{bmatrix} -
\begin{bmatrix}
    l_{i_1}v_i(\hat{x}_i(k)) \\
    l_{i_2}l_{i_1}v_i(\hat{x}_i(k))
\end{bmatrix}
\] (5.198)

where \(\hat{T}_i\) is assumed to be estimated from measured turbine and pump speeds and the torque converter model. \(\hat{P}_i(k)\) is also assumed to be estimated using the hydraulic pressure regulation model. \(v_i(\hat{x}_i(k))\) is an observer compensation function which is a function of the error between the measurement and the estimation. \(\hat{x}_i(k) = \hat{x}_i(k) - x_i(k)\) and \(l_{i_1}\) are observer gains. The sliding surfaces are given by

\[
s_{i_1}(k) = \hat{x}_i(k) = \hat{x}_i(k) - x_i(k)
\] (5.199)

The error dynamics can be derived by subtracting equation (5.198) from equation (5.179), and we have the following.

\[
\begin{bmatrix}
    \hat{x}_1(k+1) \\
    \hat{x}_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
    \phi_{11} & \phi_{12} \\
    \phi_{21} & \phi_{22}
\end{bmatrix}
\begin{bmatrix}
    \hat{x}_1(k) \\
    \hat{x}_2(k)
\end{bmatrix} +
\begin{bmatrix}
    \gamma_{11} & \gamma_{12} \\
    \gamma_{21} & \gamma_{22}
\end{bmatrix}
\begin{bmatrix}
    \hat{T}_i(k) \\
    \hat{P}_i(k)
\end{bmatrix} -
\begin{bmatrix}
    L_{i_1}v_i(\hat{x}_i(k)) \\
    L_{i_2}L_{i_1}v_i(\hat{x}_i(k))
\end{bmatrix}
\] (5.200)

Using the results from section 5.3, we can write the following,

\[
s_{i_2}(k+1) = s_{i_1}(k) + (\phi_{i_11} + 1)\hat{x}_i(k) + \phi_{i_12}\hat{x}_2(k) + \gamma_{i_11}\hat{T}_i(k) + \gamma_{i_12}\hat{P}_i(k) - l_{i_1}v_i(\hat{x}_i(k))
\] (5.201)
In this case, \( \tilde{x}_2(k) \), \( \tilde{T}_t(k) \), and \( \tilde{P}_l(k) \) are unknown since the clutch pressure, the turbine torque and the supplied line pressure are not measured. Therefore, following the theory of the discrete sliding mode observer reviewed in section 5.3, we would like to find the function \( v_1(\tilde{x}_1(k)) \) such that \( s_{t_t}(k+1) \) is zero and stays at zero for future times. However, since not all terms in equation (5.201) are known, \( v(\tilde{x}_1(k)) \) is chosen as,

\[
v_1(\tilde{x}_1(k)) = \frac{1}{l_{t_t}}(s_{t_t}(k) + (\phi_{s11} + 1) \tilde{x}_1(k)) \quad (5.202)
\]

With this choice of \( v_1(\tilde{x}_1(k)) \), equation (5.201) becomes,

\[
s_{t_t}(k+1) = \phi_{s12} \tilde{x}_2(k) + \gamma_{s11} \tilde{T}_t(k) + \gamma_{s12} \tilde{P}_l(k) \quad (5.203)
\]

If \( \Delta t \) is the sampling period used for the model discretization, each term on the right hand side is of order \( \Delta t \) due to the discretization, and the system motion will be in a vicinity of order \( \Delta t \) of the sliding manifold \( s_{t_t} = 0 \) (Utkin et al., 2002). As discussed in section 5.3, it should be noted that due to the formation of the dynamic equation used in this problem, the gain \( l_{t_t} \) has no effect on the estimation dynamics for the measured state.

For the second equation in equation (5.200), the observer gain \( l_{s_t} \) is designed by assuming that the system is in the sliding mode, i.e.,

\[
0 = s_{t_t}(k) = s_{t_t}(k+1) = s_{t_t}(k+2) = .... \quad (5.204)
\]

Then, from equation (5.201), we have,

\[
l_{t_t}v_1(\tilde{x}_1(k)) = \phi_{s12} \tilde{x}_2(k) + \gamma_{s11} \tilde{T}_t(k) + \gamma_{s12} \tilde{P}_l(k) \quad (5.205)
\]

Therefore, we have the following error dynamics for the unmeasured state.

\[
\tilde{x}_2(k+1) = (\phi_{s22} - l_{t_t} l_{s_t} \phi_{s12}) \tilde{x}_2(k) + (\gamma_{s21} - l_{t_t} l_{s_t} \gamma_{s11}) \tilde{T}_t(k) + (\gamma_{s22} - l_{t_t} l_{s_t} \gamma_{s12}) \tilde{P}_l(k) \quad (5.206)
\]
Therefore, the observer gain $l_{2t}$ can be chosen arbitrarily to meet the desired dynamics for
the estimation of the unmeasured state, $P_{c,OD}$ in this case, providing that the pair
$(\phi_{22}, \phi_{12})$ is observable, which is the case for our problem. Nevertheless, care must be
taken when selecting the gain $l_{2t}$ since its value affects the forcing function terms
associated with the estimation error of the turbine torque and the supplied line pressure.
Moreover, equation (5.206) is valid only when the condition in (5.204) is satisfied.
However, we have shown already that for the system with uncertainties, this condition is
difficult to achieve for the discrete sliding mode observer. We note that this is a limitation
of the current approach.

Substituting numerical values for the model and observer presented so far, we
pick the pole location of equation (5.206) at 0.9 inside the unit circle in the z-plane which
gives,

$$l_{2t} = -2.4751 \times 10^{-2}$$ (5.207)
This corresponds to the pole at -100 rad/sec in s-domain.

5.4.2 Observer Design for Inertia Phase

Recall from section 5.3.1 that the differential equation governing the dynamics of
the transmission during the 2-3 up shift inertia phase for the rigid shaft case is given by,

$$\begin{bmatrix}
\dot{\omega}_R \\
\dot{\omega}_t \\
\dot{P}_{c,ND} \\
\dot{P}_{c,OD}
\end{bmatrix} =
\begin{bmatrix}
C_{T_{13}}(c_1 + c_2 r^2 R_d^2 \omega_R^2) + F'_{113} P_{c,ND} + F'_{213} P_{c,OD} \\
C_{T_{14}}(c_1 + c_2 r^2 R_d^2 \omega_R^2) + F'_{124} P_{c,ND} + F'_{224} P_{c,OD} \\
P_{c,ND} \sqrt{0.01 \times \text{DutyCycle} \times P - P_{c,ND}} \\
P_{c,OD} \sqrt{0.01 \times \text{DutyCycle} \times P - P_{c,OD}}
\end{bmatrix} + \begin{bmatrix}
C_{T_{15}} \\
C_{T_{25}} \\
0 \\
0
\end{bmatrix} T_t$$ (5.208)

where
The linearized model can be derived which gives

\[
\begin{bmatrix}
\dot{\omega}_r \\
\dot{\omega}_t \\
\dot{P}_{c,2\text{ND}} \\
\dot{P}_{c,\text{OD}}
\end{bmatrix} = 
\begin{bmatrix}
2C_{T1S}c_2 r^3 R_d^2 \tilde{\omega}_r \\
2C_{T2S}c_2 r^3 R_d^2 \tilde{\omega}_r \\
\end{bmatrix}
\begin{bmatrix}
\omega_r \\
\omega_t \\
\end{bmatrix} + 
\begin{bmatrix}
F'_{11S} + F'_{12S} + F'_{21S} + F'_{22S} \\
-K_{p2\text{ND}} F_{11S} + K_{p2\text{ND}} F_{21S} + K_{p2\text{ND}} F_{22S} \\
\end{bmatrix}
\begin{bmatrix}
P_{c,2\text{ND}} \\
P_{c,\text{OD}}
\end{bmatrix} + 
\begin{bmatrix}
C_{T1S} 0 \\
C_{T2S} 0 \\
K'_{p2\text{ND}} \\
0 K'_{p2\text{ND}}
\end{bmatrix} 
\begin{bmatrix}
T_t \\
P_l
\end{bmatrix}
\]  

(5.210)

where

\[
K_{p2\text{ND}} = \frac{0.5 \times C_{2\text{ND}}}{\sqrt{(0.01 \times \text{DutyCycle} \times \bar{P}_{c,2\text{ND}} - \bar{P}_{c,2\text{ND}})}}
\]  

(5.211)

\[
K'_{p2\text{ND}} = \frac{0.005 \times \text{DutyCycle} \times C_{2\text{ND}}}{\sqrt{(0.01 \times \text{DutyCycle} \times \bar{P}_{c,2\text{ND}} - \bar{P}_{c,2\text{ND}})}}
\]  

(5.212)

MATLAB’s command, c2d, is again used to discretize the linearized model above. As for
the case of the torque phase design, the sampling period, \(\Delta t\), that we use in our
application is 16 milliseconds and the zero-order-hold discretization method is selected to
transform the above continuous time model to the discrete time model. After
discretization, we can write the resulting model in a form similar to that presented for the
torque phase design. In particular, we have

\[
\begin{bmatrix}
\Phi_{i11} & \Phi_{i12} \\
\Phi_{i21} & \Phi_{i22}
\end{bmatrix}
\begin{bmatrix}
x_1(k+1) \\
x_2(k+1)
\end{bmatrix} = 
\begin{bmatrix}
\Phi_{i11} & \Phi_{i12} \\
\Phi_{i21} & \Phi_{i22}
\end{bmatrix}
\begin{bmatrix}
x_1(k) \\
x_2(k)
\end{bmatrix} + 
\begin{bmatrix}
\Gamma_{i11} & \Gamma_{i12} \\
\Gamma_{i21} & \Gamma_{i22}
\end{bmatrix}
\begin{bmatrix}
T_t(k) \\
P_l(k)
\end{bmatrix}
\]  

(5.213)

In this case

219
Thus, each \( \Phi_i \) is a 2×2 matrix, and \( \Gamma_i \) is a 2×1 column vector. All measurements are in the \( x_i \) vector. We assume that the observer equations are in the following form.

\[
\begin{bmatrix}
\hat{x}_1(k+1) \\
\hat{x}_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
\Phi_{i11} & \Phi_{i12} \\
\Phi_{i21} & \Phi_{i22}
\end{bmatrix}
\begin{bmatrix}
\hat{x}_1(k) \\
\hat{x}_2(k)
\end{bmatrix} +
\begin{bmatrix}
\Gamma_{i11} & \Gamma_{i12} \\
\Gamma_{i21} & \Gamma_{i22}
\end{bmatrix}
\begin{bmatrix}
\hat{T}_i(k) \\
\hat{P}_i(k)
\end{bmatrix} -
\begin{bmatrix}
L_{i1}V_i(\hat{x}_1(k)) \\
L_{i2}L_{ii}V_i(\hat{x}_1(k))
\end{bmatrix}
\]

(5.215)

\( \hat{x}_i(k) = \hat{x}_i(k) - x_i(k) \), \( V_i(\hat{x}_i(k)) \) is an observer compensation function for the inertia phase, where,

\[
V_i(\hat{x}_i(k)) =
\begin{bmatrix}
v_{i1}(\hat{\omega}_{Rr}(k)) \\
v_{i2}(\hat{\omega}_i(k))
\end{bmatrix}
\]

(5.216)

\( L_{ii} \) and \( L_{2i} \) are observer gains, which are assumed to have the following form.

\[
L_{ii} =
\begin{bmatrix}
l_{ii} & 0 \\
0 & l_{2i}
\end{bmatrix}
\]

(5.217)

\[
L_{2i} =
\begin{bmatrix}
l_{3i} & l_{4i} \\
0 & l_{6i}
\end{bmatrix}
\]

(5.218)

The sliding surfaces are given by,

\[
S_{ii}(k) = \hat{x}_i(k) =
\begin{bmatrix}
\hat{\omega}_{Rr}(k) - \omega_{Rr}(k) \\
\hat{\omega}_i(k) - \omega_i(k)
\end{bmatrix}
\]

(5.219)

The error dynamics can be written as

\[
\begin{bmatrix}
\hat{x}_1(k+1) \\
\hat{x}_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
\Phi_{i11} & \Phi_{i12} \\
\Phi_{i21} & \Phi_{i22}
\end{bmatrix}
\begin{bmatrix}
\hat{x}_1(k) \\
\hat{x}_2(k)
\end{bmatrix} +
\begin{bmatrix}
\Gamma_{i11} & \Gamma_{i12} \\
\Gamma_{i21} & \Gamma_{i22}
\end{bmatrix}
\begin{bmatrix}
\hat{T}_i(k) \\
\hat{P}_i(k)
\end{bmatrix} -
\begin{bmatrix}
L_{i1}V_i(\hat{x}_1(k)) \\
L_{i2}L_{ii}V_i(\hat{x}_1(k))
\end{bmatrix}
\]

(5.220)

Using the results from section 5.3, we can write the following,
\[ S_{ii}(k+1) = S_{ii}(k) + (\Phi_{i11} + I_{2,2}) \tilde{x}_1(k) + \Phi_{i12} \tilde{x}_2(k) + \Gamma_{i11} \tilde{T}_r(k) + \Gamma_{i12} \tilde{P}_i(k) - L_{i1} V_i(\tilde{x}_i(k)) \]  

(5.221)

Since the clutch pressures, the turbine torque, and the supplied line pressure are not measured, \( \tilde{x}_2(k) \), \( \tilde{T}_r(k) \), and \( \tilde{P}_i(k) \) are unknown. The function \( V_i(\tilde{x}_i(k)) \) is selected such that \( S_{ii} \), from the \((k+1)\) step on, is zero. However, since not all terms in the equation (5.221) are known, \( V_i(\tilde{x}_i(k)) \) is chosen as,

\[
V_i(\tilde{x}_i(k)) = L_{i1}^{-1} \left[ S_{ii}(k) + (\Phi_{i11} + I_{2,2}) \tilde{x}_1(k) \right]
\]

(5.222)

With this choice of \( V_i(\tilde{x}_i(k)) \), equation (5.221) becomes,

\[
S_{ii}(k+1) = \Phi_{i12} \tilde{x}_2(k) + \Gamma_{i11} \tilde{T}_r(k) + \Gamma_{i12} \tilde{P}_i(k)
\]

(5.223)

As discussed before, even though \( S_{ii}(k+1) \) cannot be forced to zero due to uncertainties, the system motion will be in \( \Delta t \)-order vicinity of the sliding manifold \( S_{ii} = 0 \). The formation of the dynamic equation for this phase eliminates the effect of the gain \( L_{i1} \) on the estimation dynamics for the measurable state. The observer gain \( L_{2i} \) is designed by assuming that the function \( V_i(\tilde{x}_i(k)) \) is able to force the system to reach the sliding mode, i.e.,

\[
0 = S_{ii}(k) = S_{ii}(k+1) = S_{ii}(k+2) = \ldots
\]

(5.224)

Then, from equation (5.221), we have,

\[
L_{i1} V_i(\tilde{x}_i(k)) = \Phi_{i12} \tilde{x}_2(k) + \Gamma_{i11} \tilde{T}_r(k) + \Gamma_{i12} \tilde{P}_i(k)
\]

(5.225)

Therefore, we have the following error dynamics for the unmeasured state.

\[
\tilde{x}_2(k+1) = (\Phi_{i22} - L_{2i} \Phi_{i12}) \tilde{x}_2(k) + (\Gamma_{i21} - L_{2i} \Gamma_{i11}) \tilde{T}_r(k) + (\Gamma_{i22} - L_{2i} \Gamma_{i12}) \tilde{P}_i(k)
\]

(5.226)
Provided that the pair \((\Phi_{i22}, \Phi_{i12})\) is observable, the observer gain \(L_{2i}\) can be chosen arbitrarily to meet the desired dynamics for the estimation of the unmeasured states, \(P_{c,OD}\) and \(P_{c,2ND}\). As was the case for the torque phase design, care must be taken when selecting the gain \(L_{2i}\) to avoid unnecessarily large perturbations from \(\tilde{T}_r(k)\) and \(\tilde{P}_f(k)\).

Substituting numerical values for the model and observer presented so far, we pick the pole location of equation (5.107) at 0.9, which corresponds to the pole at \(-100\) rad/sec in s-domain, for both unmeasured states, which gives,

\[
L_2 = \begin{bmatrix}
-2.1528 \times 10^{-5} & -2.1703 \times 10^{-5} \\
-5.2132 \times 10^{-3} & -3.0504 \times 10^{-3}
\end{bmatrix}
\]  
(5.227)

5.4.3 Simulation Results

The observer designed in the previous section is now simulated to study its performance in estimating the clutch pressure. The simulation is done in the same manner as presented for the case of the continuous time observer development. Specifically, the already developed plant model is used to represent the real transmission. All measurable signals are extracted from the plant model and fed to the observer. The use of the discrete observer allows the development of the multi-rate simulation, meaning that the sampling frequency of the observer and the plant model can be controlled separately. This technology allows us to study the performance of the observer under low sampling frequency while keeping the plant model at high sampling frequency to preserve the accuracy of the plant model, and hence the signals fed to the observer. We note here that the results shown here will not be as intensive as the continuous time observer case. Only
the clutch pressure estimation results will be shown here. The same test data set used in
the continuous time observer development is used here as well for a straightforward
comparison. All other properties such as the effect of the modeling error as well as the
gain selection are similar to the case of the continuous time observer.

Figures 5.12 and 5.13 show the estimation results for the OD clutch pressure and
the 2ND clutch pressure respectively. In this case, the plant model is executed with 10
kHz sampling frequency while the discrete observer is executed at 1 kHz sampling
frequency. The observer gains in this case are the same as we presented in the model
development sections. It can be seen that the discrete observer is able to predict the clutch
pressure as well as the continuous time observer. As described from the simulation of the
continuous time observer, the data set also contains the clutch-fill duration. It can be seen
from Figure 5.12 that the discrete observer is also able to deal with this uncertainty by
tracking the experimental data even under the presence of the clutch-fill duration.
Figure 5.12: OD clutch pressure estimation using multi-rate simulation - TDSim @ 10kHz, estimator @ 1kHz

Figure 5.13: 2ND clutch pressure estimation using multi-rate simulation: TDSim @ 10kHz, estimator @ 1kHz
Figures 5.14 and 5.15 show the results for OD clutch pressure estimation and the 2ND clutch pressure estimation respectively, when the estimator is executed with a sampling frequency of 64 Hz. This frequency is exactly what the transmission control unit on the test vehicle uses. The observer gains designed previously have to be changed according to the sampling frequency variation. The pole at 0.9, when used with the use of the sampling frequency of 64 Hz, gave poorer results with the estimated clutch pressures exhibiting some large amplitude oscillation. Therefore, in this simulation, we change the pole location to 0.45, which corresponds to a pole at –50 rad/sec in the s-domain. Then we have the following observer gains,

\[ L_2 = \begin{bmatrix} -5.0736 \times 10^{-6} & -2.1485 \times 10^{-5} \\ -1.2186 \times 10^{-3} & -3.0443 \times 10^{-3} \end{bmatrix} \]  

(5.229)

The simulation results in Figures 5.14 and 5.15 show that the observer cannot maintain the same estimation quality as shown in Figures 5.11 and 5.12. This is due to reduction of the sampling frequency. However, the performance of the observer is still within an acceptable level. Notice in Figure 5.13 that the discrete observer cannot handle the presence of the clutch-fill as well as in the previous case. The decrease in the sampling frequency delays the ability of the observer to compensate any uncertainty.
Figure 5.14: OD clutch pressure estimation using multi-rate simulation: TDSim @10 kHz, estimator @ 64 Hz - Estimator poles location: $z = 0.45$ ($s = -50$ rad/sec)

Figure 5.15: 2ND clutch pressure estimation using multi-rate simulation: TDSim @10 kHz, estimator @ 66 Hz - Estimator poles location: $z = 0.45$ ($s = -50$ rad/sec)
5.5 Adaptive Torque Converter Model for Turbine Torque Estimation

In this section, we apply the theory of the adaptive sliding mode observer reviewed in section 5.2 to improving the accuracy of the clutch pressure estimation. In the design of the clutch pressure estimation thus far, the turbine torque is assumed known from the estimation using the static torque converter model. However, it is well known that the characteristic of the torque converter is not as the same as that shown in a performance chart, over a wide range of operating conditions. And as we showed in the error analysis section, error in estimating the turbine torque induces error in the clutch pressure estimation as well. The main factor that alters the characteristic of the torque converter is fluid viscosity, which varies depending on the transmission fluid temperature. In particular, as low temperature, the fluid viscosity is high which means that the torque loss due to viscosity is high. As a result, a low turbine torque can be expected. On the other hand, as the temperature of the transmission fluid increases, the viscosity decreases, torque loss due to the viscosity is then low, and hence a high turbine torque can be produced from the torque converter.

Since the static torque converter model does not incorporate the effect of temperature change in the model, the estimated turbine torque is not accurate. Attempting to develop a better model for the torque converter is a research subject in its own right. Instead of developing a new model, we use the theory of the adaptive sliding mode observer to improve accuracy of the static model by attempting to adjust the model parameters so that the estimated turbine torque is more accurate regardless of the operating conditions.
The adaptation scheme presented here follows the work presented in Yi et al. (2000). However, while the work presented there was intended to adjust the torque converter model parameters continually, we concentrate here on adjusting the model parameters only during in-gear operation. The reason for this is two-fold. Firstly, it will be shown that the adaptation scheme is applicable for adjusting slowly varying parameters. This is in fact the case for change in torque converter characteristic due to the change of transmission fluid temperature. In other words, the transmission fluid temperature does not drastically change much in a short time period. Therefore, enabling the adaptation scheme only during in-gear operation should be adequate to accommodate the effect of temperature changes.

Secondly, use of the adaptation scheme during the gear shift may conflict with the developed clutch pressure estimation presented previously. The adaptation mechanism we present here relies on information from the error between speed signals produced by the model and the measurements as the main source for identifying error in the model. And recalling from the previous development, the clutch pressure estimation scheme also relies on the same source of information. Therefore, if both clutch pressure estimation and the torque converter model adaptation mechanism are enabled at the same time, problems in terms of uniqueness of the solution will most certainly rise. In other words, using only the error from the speed estimation is not enough to identify both the clutch pressure and the turbine torque simultaneously.
5.5.1 Adaptation Law Development

Based on the above discussion, in this work, the torque converter model adaptation scheme is activated during in-gear operation only. The adapted parameters are held constant at their last adapted values as the gear shift starts, and the clutch pressure estimation is then activated. The following derivation shows the development of the torque converter parameter adaptation mechanism for operation in the second gear. The same approach can be easily applied to other gears.

Consider the dynamic behavior of the transmission with a rigid shaft in second gear, described by the following equation.

\[
\dot{\omega}_i = -\frac{1}{I_{235}} R_d d_{11} r (c_1 + c_2 (R_d d_{11} r)^2 \hat{\omega}_t^2) + \frac{1}{I_{235}} T_t
\]  

(5.230)

With some modification from the 2-3 upshift torque phase estimation, the observer for the second gear is of the form,

\[
\dot{\hat{\omega}}_i = -\frac{1}{I_{235}} R_d d_{11} r (c_1 + c_2 (R_d d_{11} r)^2 \hat{\omega}_t^2) + \frac{1}{I_{235}} \hat{T}_t - k_{ds} \text{sgn}(\hat{\omega}_t)
\]  

(5.231)

Here, based on the torque converter model, the turbine torque can be parametrized as follows.

\[
\hat{T}_t = \hat{\theta}^T \Omega
\]  

(5.232)

where

\[
\hat{\theta}^T = \begin{cases} 
[\hat{c}_{1tc} \hat{c}_{2tc} \hat{c}_{3tc}]; & \text{if } \frac{\omega_t}{\omega_i} < 0.9 \\
[\hat{c}_{4tc} \hat{c}_{5tc} \hat{c}_{6tc}]; & \text{if } \frac{\omega_t}{\omega_i} \geq 0.9 
\end{cases}
\]  

(5.233)

\[
\Omega^T = \begin{bmatrix} \omega_p^2 & \omega_i \omega_t & \omega_t^2 \end{bmatrix}
\]  

(5.234)
The use of the adaptation scheme developed here eliminates the switching between the torque multiplication mode and the fluid coupling mode. Therefore, the equation (5.233) is reduced to,

\[
\dot{\Theta}^T = \begin{bmatrix} \hat{c}_{1a} & \hat{c}_{2a} & \hat{c}_{3a} \end{bmatrix} \text{ for all } \frac{\dot{\omega}_i}{\omega_i} \tag{5.235}
\]

Following the same derivation as the one presented for the 2-3 up shift torque phase observer development, the following error dynamics can be derived.

\[
\dot{\omega}_i = -\frac{1}{I_{23S}} (2c_2 (R_d d_{11} r)^2 \tilde{\omega}_i)\dot{\omega}_i + \frac{1}{I_{23S}} \hat{T}_i - k_{i1S} \text{sgn}(\dot{\omega}_i) \tag{5.236}
\]

The sliding surface in the case is defined by,

\[
S_i = \tilde{\omega}_i = \hat{\omega}_i - \omega_i = 0 \tag{5.237}
\]

The following Lyapunov function candidate is assumed,

\[
V = \frac{1}{2} S^T S + \frac{1}{2} \tilde{\Theta}^T R \tilde{\Theta} \tag{5.238}
\]

In this case, \( R \) is a \( 3 \times 3 \) positive definite matrix to be selected. We then have,

\[
\dot{V} = \tilde{\omega}_i \dot{\omega}_i + \tilde{\Theta}^T R \tilde{\Theta}

= \tilde{\omega}_i \left( -\frac{1}{I_{23S}} (2c_2 (R_d d_{11} r)^2 \tilde{\omega}_i)\dot{\omega}_i + \frac{1}{I_{23S}} \left( \dot{\Theta}^T \Delta \Omega + \tilde{\Theta}^T \Omega \right) - k_{i1S} \text{sgn}(\dot{\omega}_i) \right) + \tilde{\Theta}^T R \dot{\Theta}

= \tilde{\omega}_i \left( -\frac{1}{I_{23S}} (2c_2 (R_d d_{11} r)^2 \tilde{\omega}_i)\dot{\omega}_i + \frac{1}{I_{23S}} \dot{\Theta}^T \Delta \Omega - k_{i1S} \text{sgn}(\dot{\omega}_i) \right) + \tilde{\Theta}^T \left( \frac{1}{I_{23S}} \dot{\omega}_i \Omega + R \dot{\Theta} \right) \tag{5.239}
\]

Here, we use,

\[
\hat{T}_i = \dot{\Theta}^T \Delta \Omega + \tilde{\Theta}^T \Omega \tag{5.240}
\]
and also assume that the parameter $\theta$ is slowly varying, i.e. $\dot{\theta} = \hat{\dot{\theta}} = \dot{\hat{\theta}}$. The switching gain $k_{1s}$ is already selected from the 2-3 up shift torque phase estimation development and will not be repeated here. From equation (5.239), the stability requirement of the estimation scheme gives the following adaptive law.

$$\dot{\hat{\theta}} = R_a^{-1} \frac{1}{I_{23s}} \tilde{\omega} \Omega$$

(5.241)

The adaptation law above can be transformed into the discrete time domain in order to be used with the discrete estimation developed earlier. Here, we assume that the discrete observer is

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Delta t \left( R_a^{-1}(k) \frac{1}{I_{23s}} S^T(k) \Omega(k) \right)$$

(5.242)

5.5.2 Simulation Results

The adaptive torque converter model developed in the previous section is simulated here. Figure 5.16 shows the Simulink® simulation model used for this study. As for the case of the observer design simulation, the plant model, which consists of the models developed in Chapter 3 and 4, is used to represent the test vehicle. In the observer part, the sliding mode observer developed and validated in the previous sections is used. The adaptive torque converter model is added (compared to Figure 5.2) to produce the turbine input torque to the observer.
As we discussed at the onset of this section, we are concentrating on the adaptation of the torque converter model during in-gear operation. Since the observer developed in this research has been presented for only the 2-3 up shift, we concentrate on the adaptive torque converter operation during the second and the third gear only. In implementing the adaptation scheme, the gain matrix $R_a$ needs to be designed. The value of each component in $R_a$ matrix affects the speed of the adaptation, i.e. the speed of the adapted variable in reaching its true value. A high value of $R_a$ normally speeds up the convergence rate but may cause instability problems. In this work, $R_a$ is tuned by trial and error and the result of using a low value of $R_a$ and a high value of $R_a$ are presented here.
Due to the lack of test data to study the variation of the turbine torque under realistic conditions, we do the following. We assume that the torque converter of the plant (denoted by TCp in Figure 5.16), which is the original static model developed in Chapter 3, produces 20% higher torque. To do this, we simply multiply the parameters of the static model with the gain of 1.2. This torque will be called the actual torque.

Figure 5.17 shows the simulation of the turbine torque. In this figure, the actual torque is shown as compared to the torque from the adaptive torque converter model as well as the torque that would be the input to the observer if the adaptive torque converter model is not used. It can be seen that the adaptive scheme works well. The static model produces a very low torque in this case and this can cause much error in the estimated clutch pressure. The convergence speed can be adjusted by the gain matrix $R_a$. For the simulation results shown here, the following gain matrices $R_a$ are used. For the high $R_a$ case,

$$
R_a = \begin{bmatrix}
1 \times 10^5 & 0 & 0 \\
0 & 1 \times 10^5 & 0 \\
0 & 0 & 1 \times 10^5
\end{bmatrix}
$$

(5.243)

And for the low $R_a$ case,

$$
R_a = \begin{bmatrix}
1 \times 10^4 & 0 & 0 \\
0 & 1 \times 10^4 & 0 \\
0 & 0 & 1 \times 10^4
\end{bmatrix}
$$

(5.244)

From this figure, it can be seen that for a large $R_a$, the turbine torque converges to the actual torque very quickly. However, the result of a high gain is also a high pulse in the transient. Figure 5.18 shows the adaptation of each coefficient during the simulation for the case of using a high gain matrix $R_a$. Base on the gain matrix $R_a$ used here, since the
values of the components on the diagonal are equal and the off diagonal elements are all zero, the amount of adaptation for all coefficients are equally weighted as can be seen from Figure 5.18. Figure 5.19 shows the estimated OD clutch pressure resulting from using the adapted turbine torque and the torque from the static model. As can be seen, the adaptive torque converter model helps improve the accuracy of the clutch pressure estimation. We note here that the high gain matrix $R_a$ will be used in the real-time implementation presented next.

Figure 5.17: The adapted turbine torque as compared to the actual torque and the torque from the static torque converter model
Figure 5.18: Adaptation of the coefficients used to calculate the turbine torque

Figure 5.19: Estimated OD clutch pressure using the adapted turbine torque as compared to the result of using the turbine torque from the static torque converter model
5.6 Real-Time Clutch Pressure Estimation Implementation

In this section, the observers described in this chapter are implemented in real-time on the test vehicle. A simplified schematic of the test setup is shown in Figure 5.20. The test vehicle is equipped with all the standard sensors, viz. the transmission input/output speed sensors, the engine speed sensor, the transmission fluid temperature sensor, etc, as well as the pressure transducers to measure the clutch pressures. The pressure transducers are installed only for controller development and diagnostic purposes, and are not installed on production vehicles. Standard sensors available on production vehicles report measured signal values to the transmission control unit for feedback control and other purposes. Therefore, in the test setup used in this research, in order to extract information from the transmission control unit, the dSPACE® Autobox is used. The Autobox can also collect other information directly from the sensor, in this case, the pressure transducer, using installed data acquisition boards. All information collected by the Autobox is read into a personal computer using the dSPACE® program which normally runs with MATLAB/Simulink® software. In the estimation implementation, dSPACE® and MATLAB/Simulink® with the Real-Time Implementation toolbox are used to convert the Simulink® simulation program into an executable code which is loaded into the Autobox. During the implementation, the Autobox executes the code using input signals obtained from the transmission control unit and pressure transducers. dSPACE® Control Desk is used to monitor the functionality and the performance of the estimator from the Autobox.

In this work, the personal computer used here is equipped with an Intel Pentium III 700 MHz and 256 MB of RAM. As mentioned, the estimator is programmed using
Simulink®, and the model used in this test setup is shown in Figure 5.21. Recall from the simulation work during the observer design that the observer took all necessary data from the plant model. However, in this test set up, only the observer model is used, so we call this a “stand-alone observer model”, and all other necessary signals are obtained from the test car or the Autobox directly (represented by the green color blocks in Figure 5.21). Specifically, the data needed to implement the observer online are the clutch pressures, the engine speed, the transmission input speed, the transmission output speed, the clutch command duty cycle, the shift phase which indicates the torque phase and the inertia phase which is determined by the transmission control unit, and the command gear. The use of the shift phase signal together with the command gear signal is important because they help in switching the dynamic equations from torque phase to inertia phase as they are different in terms of dynamic equations as well as designed observer parameters. The current gear state and transmission fluid temperature are also collected for diagnostic purposes.
Figure 5.20: Simplified schematic for on-line clutch pressure observer implementation

Figure 5.21: The observer used for on-line implementation
The choice of the sampling frequency generally affects the performance of the sliding mode observer. Due to the use of the switching function as a compensation term, the higher is the sampling frequency, the smoother is the compensation signal. In this case, it can be assumed that the system stays on the sliding surface, and hence the estimation results of the unmeasured states would be more accurate. On the other hand, if a low sampling frequency is used for implementation, the estimation of the measured states will fluctuate about the sliding surface, which causes fluctuation in the compensation signals. As a result, the estimation of unmeasured states will not be as accurate as when high sampling frequency is used. The results shown in this section are based on implementation using two sampling frequencies. A high sampling frequency of 1 kHz, which is the highest sampling frequency allowed for real time implementation using the designated computer, is used to show the performance of the resulting observer. A low sampling frequency of 64 Hz, which is the frequency used by the transmission control unit, is also used in order to test the feasibility of implementing the resulting observer on the current production vehicle. It is noted here that all data input into the observer are captured by the transmission control unit at a sampling frequency of 128 Hz.

We note finally that the observer gains used in most of the results shown here are fine tuned during the implementation. For the case of the continuous time sliding mode observer, the switching gains, \( k_{1S} \), remain the same as designed in section 5.3. However, the switching gain for the clutch pressure error dynamics, \( k_{2S} \), had to be reduced by a factor of 0.01. As a result, the pole locations for the estimation error dynamics for the OD clutch and the 2ND clutch are reduced from -100 rad/sec to -8.2 rad/sec and -6.1 rad/sec, respectively. These values are about 2 times higher than the open loop pole locations of
the OD clutch model and the 2ND clutch model. However, the use of the original gain presented in section 5.3 resulted in high oscillation for all estimated states. This is most likely the effect of using a low sampling frequency for implementation. Recall from the simulation results shown in section 5.3 that the simulation was done at 10 kHz sampling frequency and the design gains produced good results. The estimated turbine speed was indistinguishable from the simulated one. However, the use of 1 kHz sampling frequency for implementation is not enough to produce the same quality results as shown in section 5.3. During the sliding mode, the system does not completely stay on the sliding surface. The low sampling frequency causes the estimated turbine speed to fluctuate around the measurement, and hence causes the fluctuation in the turbine speed error calculation. And since this error signal is used as a feedback to adjust the model in order to accurately predict the clutch pressure, the oscillation of the error signal causes fluctuation in the estimated error as well. To reduce the fluctuation, the switching gain $k_{2S}$ has to be reduced. The switching gain $k_{1S}$, the switching gain for the measurable state, normally cannot be decreased, in order to ensure the existence of the sliding mode. In the case of the discrete time sliding mode observer, the original observer gains designed previously in section 5.4 work well during the implementation. Therefore, there is no need for fine tuning the gains in this case.
5.6.1 Continuous Time Sliding Mode Observer Implementation

5.6.1.1 High sampling frequency tests

The results of the implementation using the observer designed in section 5.3 are shown here. Figures 5.22-5.25 show the estimation results from 2-3 power-on up shift with wide throttle acceleration, ≈85–90%. In each figure, a so-called “flag” is used to indicate the gear shift phase, viz. torque phase, inertia phase, or in-gear. Figure 5.22 shows OD clutch pressure estimation results. It can be seen that the observer is able to predict the OD clutch pressure well. The result for the estimation of the 2ND clutch pressure is shown in Figure 5.23. Due to the fluctuation of the signal caused by the limited sampling frequency, the observer seems to have difficulty capturing the 2ND clutch pressure at low levels. Therefore, the result does not seem as good as the OD clutch pressure, though it can be considered to be acceptable. Figure 5.24 shows estimation results for the turbine speed. The sliding mode occurs as the estimated turbine speed tracks the measurement well. As can be seen from the figure, due to the limited sampling frequency, the estimated turbine speed fluctuates with small amplitude around the measurement during the sliding mode. Figure 5.25 shows the transmission output speed estimation result. Even though the estimation of the output speed occurs during the inertia phase only, Figure 5.25 shows that the estimation is relatively inseparable from the measurement which means that the sliding mode is reached extremely fast.
Figure 5.22: OD clutch pressure estimation for wide throttle acceleration

Figure 5.23: 2ND clutch pressure estimation for wide throttle acceleration
Figure 5.24: Turbine speed estimation and sliding surface for wide throttle acceleration

Figure 5.25: Transmission output speed estimation and sliding surface during wide throttle acceleration

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Figures 5.26-5.29 show the clutch pressure estimation results during power-on 2-3 up shift but with low throttle acceleration, \( \approx 35-45\% \). It should be noted that, based on many tests for low throttle acceleration, the observer has to be modified to accommodate some phenomena during low-throttle acceleration. Specifically, during the power-on 2-3 up shift using low throttle acceleration for the particular transmission on the test car, engine flare always occurs (Figure 5.28), which means that the 2ND clutch or the off-going clutch slips before the OD clutch has enough clutch capacity. Theoretically and mathematically, when engine flare occurs, the nature of the transmission behavior during engine flare can be considered to be similar to that in the inertia phase. Initially the observer was designed to rely on the shift phase signal from the transmission control unit to switch the observer dynamic equations corresponding to the shift phase. Therefore, early estimation results for the low throttle acceleration showed error in every estimated state (not shown here). To get around this problem, an algorithm which checks the engine flare is added to the observer such that the inertia phase is activated whenever engine flare occurs, and activated by the shift phase signal if there is no engine flare. Figure 5.26 shows the OD clutch pressure estimation results. It can be seen that the observer is still able to estimate the clutch pressure accurately. Notice the difference where the inertia phase observer is activated as compared to the inertia phase indicated by the transmission control unit. Figure 5.27 shows the result of the 2ND clutch pressure estimation. The estimation error can be obviously seen in this case. Part of the error for the off-going clutch pressure estimation is that the initial condition for the off-going clutch is set to zero in the observer. The use of the open-loop estimation for the off-going clutch, i.e. having the model run open loop without any corrective term, will help set up
proper initial conditions and the accuracy of estimation of the off-going clutch pressure, or 2ND clutch pressure in this case, will be improved. Figures 5.28 and 5.29 show the estimation results for the turbine speed and the output speed respectively. It can be seen that, in both cases, the estimated signals track the measurements well.

Figure 5.26: OD clutch pressure estimation during low throttle acceleration
Figure 5.27: 2ND clutch pressure estimation during low-throttle acceleration

Figure 5.28: Turbine speed estimation during low throttle acceleration with engine flare
Figures 5.30 to 5.33 show the results of clutch pressure estimation during the power-on up shift with low throttle acceleration. In this case, the adaptive scheme to adjust the torque converter model parameters is also used, and the open loop off-going clutch pressure estimation is also incorporated. It is noted that the engine flare problem does not occur in this test. The OD clutch pressure estimation and the 2ND clutch pressure estimation are shown in Figures 5.30 and 5.33 respectively. In these figures, the resulting estimation using the static torque converter model without the adaptation scheme is also presented. As can be seen, the estimation of both clutch pressures are acceptably accurate. The adaptive torque converter model gives the same results as the static torque converter model. This implies that torque converter behavior during this test condition is close to the behavior of the static model. The open loop estimation helps set
up the initial condition for the off-going clutch when the inertia phase starts. However, as discussed before, due to the low pressure level of the off-going clutch pressure during the inertia phase, the observer has a hard time tracking the measurement. The turbine torque estimated from the adaptive torque converter and the static torque converter are shown in Figure 5.32. The adaptation of the coefficients for the adaptive model is shown in Figure 5.33. In this case, the value of each coefficient during the gear shift, i.e. the final value after the adaptation during the second gear, is lower than the original value used in the static model by approximately 4-5%. As can be seen from Figure 5.32, the difference between the two estimations is small, which confirms the results shown in Figures 5.30 and 5.31.

![Figure 5.30: OD clutch pressure estimation during low throttle acceleration with the adaptive torque converter model](image)

Figure 5.30: OD clutch pressure estimation during low throttle acceleration with the adaptive torque converter model
Figure 5.31: 2ND clutch pressure estimation during low throttle acceleration with adaptive torque converter model and the 2ND clutch open loop estimation

Figure 5.32: Turbine torque calculated from the adaptive torque converter model as compared to the one calculated from the static model
Figure 5.33: Adaptation of coefficients used in the adaptive torque converter model

Figures 5.34-5.37 show the estimation results from the wide throttle acceleration with the use of the adaptive torque converter model and the open-loop off-going clutch pressure estimation. The observer is still able to predict the clutch pressures well for both the OD clutch pressure shown in Figure 5.34 and the 2ND clutch pressure shown in Figure 5.35. In these figures, the resulting clutch pressure estimation using the static model shows some small error as compared to the experimental data. Figure 5.36 shows that, in this test, the turbine torque calculated from the static model is lower than the calculation using the adaptive torque converter model, where the adaptation time-history for each coefficient is shown in Figure 5.37. The adapted values of all coefficients are higher than their original values used in the static torque converter model. This basically validates the adaptation scheme presented in this research.
Figure 5.34: OD clutch pressure estimation during wide throttle acceleration using the adaptive torque converter model as compared to the use of the static model

Figure 5.35: 2ND clutch pressure estimation during wide throttle acceleration using the adaptive torque converter model as compared to the use of the static model
Figure 5.36: Turbine torque during the wide throttle acceleration calculated from the adaptive torque converter model and the static model

Figure 5.37: Adaptation of coefficients used in the adaptive torque converter model
5.6.1.2 Low sampling frequency test

This section shows the implementation results from using the continuous time sliding mode observer with a sampling frequency of 64 Hz. The observer used in this case includes the adaptive torque converter model and the open-loop off-going clutch pressure estimation. The observer gains used in this case are the same as those in the high sampling frequency test. The results from the low throttle acceleration testing are shown in Figures 5.38-5.42. Figure 5.38 shows the OD clutch pressure estimation, and it can be seen that the observer is still able to predict the clutch behavior well even under the low sampling frequency condition. The same level of accuracy can be seen from the estimation of the 2ND clutch pressure shown in Figure 5.39. Due to the low frequency sampling, the observer is not able to stay on the sliding surface as discussed. This can be obviously seen from the turbine speed estimation in Figure 5.40. The use of low sampling frequency makes the estimated turbine speed fluctuate with high amplitude. The fluctuations can be reduced by reducing the switching gain in the observer dynamic equations for the measurable states. Even if the fluctuation is high as in this case, the observer is still able to identify the clutch pressure with acceptable accuracy. Figure 5.41 shows the estimation result for the transmission output speed. Since the switching gain for the output speed dynamics is significantly lower than the one designed for the turbine speed, the estimated output speed does not show the fluctuation. Figure 5.42 shows the estimated turbine torque using the adaptive torque converter model as compared to the result from the static model. And the adaptation for each coefficient of the adaptive model is shown in Figure 5.43. The adaptation scheme seems to be working well as expected. The results from implementation for the wide throttle acceleration testing yield
the same conclusions as the case presented here. Therefore, we omit these test results here.

Figure 5.38: OD clutch pressure estimation using continuous time sliding mode observer at 64 Hz sampling frequency – low throttle acceleration

Figure 5.39: 2ND clutch pressure estimation using continuous time sliding mode observer running at 64 Hz sampling frequency – low throttle acceleration
Figure 5.40: Turbine speed estimation using the continuous time sliding mode observer running at 64 Hz sampling frequency – low throttle acceleration

Figure 5.41: Transmission output speed estimation using the continuous time sliding mode estimation running at 64 Hz sampling frequency – low throttle acceleration
Figure 5.42: Turbine torque estimation from the adaptive torque converter model and the static model – 64 Hz implementation – low throttle acceleration

Figure 5.43: Adaptation of coefficients used in the adaptive torque converter model
5.6.2 Discrete Time Sliding Mode Observer Implementation

The developed discrete sliding model observer including the adaptive torque converter model and the open loop off-going clutch pressure estimation is implemented online in this section. Again, two test conditions are of interest, which are the low throttle acceleration and wide throttle acceleration. However, only the results from the former case are shown here. The implementation will be executed using two sampling frequencies, i.e. 1 kHz and 64 Hz. We note here that, unlike the continuous time observer case, the observer gains used for the implementation are the same as the original design presented in section 5.4.

5.6.2.1 High sampling frequency test

Figures 5.44-5.48 show implementation results from the low throttle acceleration testing. The OD clutch pressure estimation result is shown in Figure 5.44. The 2ND clutch pressure estimation result is shown in Figure 5.45. From both figures, it can be seen that the accuracy of the estimated clutch pressures using the discrete sliding mode observer as compared to the experimental data is acceptable. A noticeable discrepancy can be seen for the estimation of the 2ND clutch pressure. On closer investigation of Figure 5.46, which shows the estimated turbine speed as compared to test data, it can be seen that the engine flare occurs in this particular test. However, since the observer cannot detect the occurrence of engine flare well, the inertia phase is activated at the wrong time. It is believed that if the engine flare can be detected correctly, the estimation of the 2ND clutch pressure will be improved. This suggests need for a further refinement of the engine flare detection mechanism. The estimation of both turbine speed and the
transmission output speed is accurate as can be seen from Figure 5.46 for the turbine speed and Figure 5.47 for the transmission output speed. Figure 5.48 also shows the comparison between the turbine torque calculated using the adaptive model and the one calculated from the static model. The adaptation history for each coefficient is shown in Figure 5.49. In this case, the turbine torque calculated from the static model appears to be higher than the calculation using the adaptive torque converter model. The results for the wide throttle acceleration test have the same quality as those shown here can be obtained.

Figure 5.44: OD clutch pressure estimation using the discrete sliding mode observer running at 1 kHz sampling frequency – low throttle acceleration
Figure 5.45: 2ND clutch pressure estimation using discrete time sliding mode observer running at 1 kHz sampling frequency – low throttle acceleration

Figure 5.46: Turbine speed estimation using discrete sliding mode observer running at 1 kHz sampling frequency – low throttle acceleration
Figure 5.47: Transmission output speed estimation using discrete time sliding mode observer running 1 kHz sampling frequency – low throttle acceleration

Figure 5.48: Turbine torque estimation using discrete time adaptation scheme as compared to the static model – low throttle acceleration
5.6.2.2 Low sampling frequency test

The implementation results for low throttle acceleration testing using the observer at the low sampling frequency of 64 Hz are presented here. The test results are shown in Figures 5.50 – 5.54. Figure 5.50 shows the OD clutch pressure estimation result. It can be seen that the discrete sliding mode observer is able to predict the clutch pressure well compared to the experimental data. The use of the discrete sliding model seems to reduce the fluctuation of the estimated clutch pressure but not by much as compared to the previous cases. The 2ND clutch pressure estimation result is shown in Figure 5.51. The observer again gives close agreement as compared to the experimental data. As for the other cases, the observer seems to have some difficulty in capturing the off-going clutch pressure at low magnitude. The open loop off-going clutch pressure estimation helps set up the initial condition for the off-going clutch when the inertia phase observer is
activated. Figure 5.52 shows the result of estimating the turbine speed. From the figure, it can be seen that engine flare occurs during the gearshift. The already designed engine flare detection seems to be working accurately. From the figure, the discrete sliding mode observer cannot maintain the system on the sliding surface. This result is similar to the case of the continuous time observer using a low sampling frequency. In this approach, the estimated turbine torque has less fluctuation than that shown in the previous section.

The transmission output speed is shown in Figure 5.53. A small discrepancy between the estimated output speed and the measurement can be seen from the figure as well. Figure 5.54 shows the turbine torques calculated from the adaptive torque converter model as compared to the one using the static model. As we have seen, during the low throttle acceleration, the behavior of the torque converter seems to be close to the behavior described by the static model. Therefore, from Figure 5.54, it can be seen that the turbine torques calculated from the two approaches are very close to each other. Again, the same quality of observer performance can be expected as when using high frequency sampling.
Figure 5.50: OD clutch pressure estimation using the discrete sliding mode observer running at 64 Hz sampling frequency – low throttle acceleration

Figure 5.51: 2ND clutch pressure estimation using the discrete sliding mode observer running 64 Hz sampling frequency – low throttle acceleration
Figure 5.52: Turbine speed estimation using discrete sliding mode observer running at 64 Hz sampling frequency – low throttle acceleration

Figure 5.53: Transmission output speed estimation using discrete sliding mode observer running at 64 Hz sampling frequency – low throttle acceleration
5.6.3 Discussion

This section has shown online implementation results for the developed observer. It can be seen that the limitation of the computational ability of the transmission control unit as well as the sliding mode observer imposes some difficulties in successfully and accurately identifying clutch pressures during the gear shift. Specifically, to be able to use the sliding mode observer effectively, a high sampling frequency is needed to execute the observer implementation. The use of the discrete sliding mode observer helps in reducing the chattering problem, or in this case the fluctuation of the estimated signals. However, the discretization we use here relies on the linearized model, which reduces the accuracy of the model. Moreover, sliding mode theory has shown that the robustness property of the discrete sliding mode observer is not as strong as the continuous time
case. Therefore, based on the implementation results, it seems that the use of a more powerful microprocessor on the transmission control unit will improve the accuracy of the clutch pressure estimation significantly.

The implementation results also have shown that the adaptive torque converter model can be used effectively to estimate the turbine torque under realistic conditions. It has been shown in the error analysis section that error in the estimation of the turbine torque and the friction coefficient affects the accuracy of the clutch pressure estimation. In this research, it is felt that the model of the friction used in the observer development, as well as in the transmission model, is a good model since it is developed based on experimental data. Since only speed measurement is available, it is difficult to develop an observer which can identify both the clutch pressure and the friction coefficient simultaneously. The observer scheme developed here is in fact identifying the clutch torque during the gear shift. The clutch torque is a result of the product of the clutch pressure and the clutch friction coefficient along with some other geometrical parameters. Therefore, the feasibility of identifying the clutch pressure is based on the assumption that the clutch friction coefficient is known. And this is the assumption that this research has been using. To be able to identify the friction coefficient and the clutch pressure simultaneously, more information will be needed from the transmission. This problem will be left for future research.

Even though implementation results for the 64 Hz sampling frequency are not as impressive as for the 1 kHz case, and the estimated clutch pressure in this case may not be accurate enough for controller development, the results can be used in some other ways. The obvious application is the identification of the clutch-fill. It is known that the
time needed to fill a clutch cavity can vary depending on the operation of the transmission. Since the clutch fill period represents a dead time in the response of the clutch, failing to accurately identify this dead time affects the performance of the controller. In this research, the implicit result from accurately estimating the clutch pressure is that the point where the clutch pressure rises up sharply is accurately identified. This means that the observer is able to detect the end of the clutch-fill duration accurately. And this information can be immediately useful to many of the existing controllers on production vehicles.

5.7 Conclusion

A model-based adaptive sliding mode observer for the estimation of clutch pressure and transmission input torque, or the turbine torque, for an automatic transmission is presented here. Both continuous time observers and discrete time observers are considered. The reason for working with both domains is to accommodate the limitation of the data acquisition and the computational ability of the transmission control unit. From the simulation results, the developed observer is seen to be able to estimate the turbine torque as well as clutch pressures for both off-going and on-coming clutches with reasonable accuracy.

Even though the developed sliding mode observer has shown promising preliminary results, it is robust only to matched uncertainties with known bounds. The analysis shows that matched uncertainties can affect the accuracy of the estimation of unmeasured variables by inducing bounded error. The ability to deal with unmatched
uncertainties depends on the system structure. Proper selection of observer gains can help reduce this error, but may degrade the error dynamics.

The online implementation results using the developed observers to estimate the clutch pressure and the turbine torque in real-time are also presented here. Two sampling frequencies are considered during the implementation. The high sampling frequency of 1 kHz is used to demonstrate the performance of the designed observer when there is less of a computational limitation imposed on the implementation. In this case, both continuous time and discrete time observers are able to predict both clutch pressures and the turbine torque accurately. The low sampling frequency of 64 Hz is used to study the feasibility of using the developed observer on the current production vehicle. The implementation results show that both continuous time and discrete time observers are able to predict the clutch pressures as well as the turbine torque with some degree of accuracy.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The research presented here has involved the development and validation of a mathematical model of an automatic transmission for an automobile and the design and validation of model-based observers for clutch pressures. The motivation for this work is the use of the estimated clutch pressure for control purposes, such estimation being necessary for improved control of clutch-to-clutch shifts. The stepped automatic transmission of interest here includes both 4-speed and 5-speed versions. All major components affecting the dynamics of the transmission are considered, namely, the torque converter, the transmission mechanical components, the shift hydraulic system, and the vehicle and driveline. Simplified models available in the literature are used for both the torque converter and the vehicle and driveline. However, models of the transmission mechanical components and the shift hydraulic system are developed in detail here. Since the 5-speed version involves an extension of the design of the 4-speed transmission, the mathematical model for the 5-speed transmission can be easily developed by extending the mathematical model for the 4-speed transmission. In this research, a combined overall base-line model for the 4-speed transmission is developed and validated against experimental data, and has been shown to be capable of predicting transmission dynamic behavior well.
As mentioned, one of the main focus areas of this research is the development of the model-based observer. The feasibility of the estimation methods proposed here depends on the availability of a validated hydraulic system model. However, since the physics-based model developed here is relatively high order and complex, simplified models have been developed and validated as well. One other contribution of the research here is the further simplification of the hydraulic model such that it is suitable for real-time uses in controller and estimator design, while retaining an acceptable level of accuracy. Uncertainties resulting from the model simplification will be accommodated appropriately by observer design.

The estimation of clutch pressures using a model-based observer is also presented in this research. A nonlinear adaptive sliding mode observer is used due to the ease of the design process as well as its robustness to uncertainties. An adaptive scheme is used to adjust the torque converter model parameters in order to improve the accuracy of estimation of the turbine torque, and hence of the clutch pressure, as the turbine torque is an input to the equations describing clutch response. The sliding mode observers presented in this research include both continuous time and discrete time versions. The performance of the observers was initially investigated extensively using simulation studies, where the clutch pressures measured experimentally during gear shifts were used in conjunction with the transmission model to determine the accuracy of the observer estimates of clutch pressures. These simulation studies showed that the observer is able to estimate clutch pressures and turbine torque with acceptable accuracy. In particular, the robustness of the observer estimates to model error (e.g. fill time error) in the hydraulic
system model used within the observer is confirmed. The tradeoff in selecting observer poles, between observer robustness and noise susceptibility, is also confirmed.

The developed observers are also implemented online to study their performance under realistic conditions corresponding to implementation in the test vehicle. The implementation is done using both low frequency and high frequency sampling. Low frequency sampling is used to study the feasibility of implementing the proposed observers using the current transmission control unit. High frequency sampling is used to demonstrate the improved performance of the proposed observers when the computational ability of the current transmission control unit is not a constraint. Given the rapid changes in the computational capabilities of on-board computers in automobiles, this is an indication of what production units a few years into the future would be capable of implementing. The results from the online implementation show that the proposed observers are able to estimate clutch pressures during gearshifts with acceptable accuracy. Adaptation of torque converter model parameters is also seen to work successfully in that it resulted in better estimates of clutch pressures. The continuous time sliding mode observer worked best at high sampling rates, whereas at low sampling rates it resulted in significant chattering about the sliding surface. The discrete time version worked reasonably well, though its performance was inferior to the continuous time observer.

On-line implementation reveals other demands on the estimation scheme. For example, in some of the tests involving 2-3 up shifts at low throttle, the off-going clutch would start slipping before the on-coming clutch had enough load capacity, resulting in engine flare. The occurrence of this is a function of shift controller actions on the test
vehicle. The requirement on the estimator is that it recognize this event and use the estimator equations corresponding to the inertia phase from this point on, since the normal progression of the torque phase does not occur.

6.2 Contributions

The research contributions are summarized:

1. One important contribution of this research with implications for clutch-to-clutch shift controller design is the development of a hydraulic system model. Due to the complexity of the hydraulic circuit, the hydraulic models reported in the literature are either too complicated or over simplified. In the former case, the dynamic behavior of far too many components are included, so that the resulting model has high order and is highly nonlinear. While this type of model is good for studying the behavior of the hydraulic system over widely varying conditions, it is not suitable for shift controller design and real time application, where low frequency behavior is of primary interest. In reducing the complexity and the order of the model, if the dominant dynamics of the system are not identified properly, the model ends up being over simplified and can then be used only within a limited operating range. This type of model is normally a linear model or an empirically determined model.

In this research, a base-line model of the shift hydraulic system is constructed by carefully studying the physical components of the hydraulic circuit. All necessary components are included in the model. The base-line model is then simplified in a systematic way using an energy-based model order reduction method. Both the base-line
model and the simplified model are able to predict hydraulic system behavior reasonably accurately.

2. Even though the main modeling emphasis of this research is in the development of the shift hydraulic system model, the development of models for other components involved in the power train system, viz, the torque converter, the transmission mechanical components, and the vehicle and driveline, is also a contribution of this research. The combined model can not only be used to study the dynamic behavior of the transmission under various operating conditions, but can also be used for estimator and controller development. Specifically, existing controllers can be validated and refined by using the developed model in conjunction with a simulation test bed, before they are tested on the vehicle. It should be noted the model presented here is suitable for power-on up shifts, but can be extended to other shifts with minor modifications. Similarly, the model of the vehicle dynamics here is restricted to longitudinal dynamics in straight line motion, and tire slip is neglected, but these simplifications can be modified relatively easily to accommodate tire slip effects as well as other vehicle maneuvers.

3. The development of a model-based clutch pressure observer is a major contribution of this research. This development results from the availability of the hydraulic model. It is well known that knowledge of clutch pressures during gear shifts is critical, especially for control of clutch-to-clutch shifts. Since clutch pressure sensors are usually not installed on production cars and reasonable hydraulic models are not available, clutch coordination in current production vehicles depends largely on calibration work. The result of the clutch pressure observer development proposed here gives one the ability to track clutch pressures during gear shifts without requiring clutch
pressure sensors. Therefore, clutch coordination during gear shifts can be done more accurately and effectively without using pressure sensors.

4. Another contribution of this research is the implementation of the static torque converter model involving a quadratic model relating pump and turbine torques and pump and turbine speeds with on-line adjustment of model parameters, as a way of improving turbine torque estimation. While this work is adapted from work presented in the literature, its incorporation in conjunction with clutch pressure estimation leads to differences from the reported approach in its implementation. Specifically, it has been shown that, in using the torque converter model adaptation scheme in conjunction with clutch pressure estimation, it is reasonable to adapt the torque converter parameters only during in-gear operation, and not during the gear shift as presented in the literature. This research also includes implementation of the adaptation scheme online, and the results show that the adaptation scheme helps to improve the accuracy of turbine torque estimation and hence clutch pressure estimation. The simplicity of the proposed approach, along with the results from the online implementation, suggests that this torque converter model adaptation scheme can be used immediately to assist current transmission controllers as well as engine controllers, as estimates of turbine torque and hence of shaft torque and pump torque can vastly enhance the functionality of these controllers.

5. The combination of open loop estimation of clutch pressure with observer-based closed loop estimation is another important contribution of the research. Open loop estimation of the off-going clutch pressure during the torque phase prior to clutch slip
gives information on the clutch pressure that can not be obtained from closed loop estimation, and complements the latter very nicely and simply.

6. An important contribution of this research is that the proposed clutch pressure observer can be utilized to assist and improve the performance of current production controllers immediately. The estimated clutch pressures can be used to better characterize clutch fill effects as well as assist in clutch piston stroke tracking. Specifically, the end of the clutch-fill duration can be identified as the instant when the estimated clutch pressure starts to rise sharply. Also, as the clutch pressure is directly related to the clutch piston position, the estimated clutch pressures of the off-going and on-coming clutches can be used to identify the positions of the clutch pistons during and after the gear shifts. Such clutch piston position tracking would rely upon clutch dynamic response models and would involve open loop estimation.

7. Finally, the value of the research described here is related to one of the motivating factors for the research, namely, use of the estimated clutch pressures in developing clutch-to-clutch shift control schemes. While it has been noted that our reliance upon manufacturer-provided friction data is a weakness given that clutch friction characteristics change with clutch use as well as condition of the automatic transmission fluid, it should be noted that the estimation schemes should be more properly considered as clutch torque estimation schemes. Viewed thus, and noting that it is knowledge of clutch torques as well as output shaft torque that is needed for control, we conclude that the work described here is an important prerequisite for clutch-to-clutch shift controller development.
From the above discussions, it can be seen that the development of the shift hydraulic system model and its subsequent use for on-line estimation is the central theme of this research. The existence of a simple but accurate hydraulic model enables us to develop real-time observers. Implementation of the estimation schemes in a test vehicle, together with on-line adaptation of torque converter models and open loop estimation of clutch pressures in the absence of clutch slip, validates the proposed approaches.

6.3 Recommendations for Future Research

This research has covered in detail the development of the mathematical model of the automatic transmission, and the utilization of the resulting model for off-line simulation purposes as well as observer design for on-line implementation. Many parts of this work need to be extended in order to gain the full benefit from this work in the area of off-line simulation and controller/observer design.

The first recommendation for future work is to extend the developed hydraulic model for the 4-speed transmission to the 5-speed version of the transmission. Preliminary observation of the hydraulic circuit for the 5-speed transmission shows that it has the same configuration as the hydraulic circuit for the 4-speed transmission, with two additional clutches controlling the secondary axle, and an additional supply line pressure control mechanism. Therefore, the methodology and the model presented in Chapter 4 can be easily extended to cover the hydraulic system for the 5-speed transmission.

The second recommendation is to validate the 5-speed transmission model presented in Chapter 3. This can be done without the availability of the model for the shift hydraulic system, but experimentally measured clutch pressures must be available
for such validation. Numerical values for most of the model parameters are already known or measured, except for the inertia of the transmission mechanical components. As in the case of the 4-speed transmission, the developed model for the 5-speed transmission is intended to study the dynamics of the transmission during power-on up shifts. In addition, the dynamics of power-on sequential down shifts are important as well. Therefore, experimental data must be collected from these two types of gear shifts.

The third recommendation is to extend the development of the overall transmission model, including the torque converter model, the transmission mechanical system model, the shift hydraulic system model, and the vehicle model, to accommodate other gear shift types as well as other vehicle maneuvers. As has been presented, the current model is built to represent the transmission dynamics only for power-on up shifts and power-on sequential down shifts. The power-on skip-shift can be easily developed with some minor modifications. However, for the model to represent the dynamics of power-off gear shifts, the development is more involved due to change in the direction of the power flow from an engine driving condition to an engine breaking condition. For example, the torque converter equations would be considerably different since the torus flow in the torque converter would be reversed and the turbine would be driving the pump. This extended work should be done for the 4-speed transmission as well as the 5-speed transmission, assuming that the first two recommendations above are implemented.

The fourth recommendation is to extend the clutch pressure observer development to other gear shifts as well as other vehicle maneuvers. Due to the simplicity of the observer design, this extension can be easily done as long as model components are available to describe the expanded range of gear shifts and vehicle maneuvers. The
availability of the current transmission model would limit this extension to only the power-on up shift and power-on sequential down shift. As the transmission model accommodates a greater range of gear shifts, so can the clutch pressure observer be extended to these gear shifts. Also, as a greater range of vehicle maneuvers are modeled and accommodated, other model adaptation needs would become apparent. Thus, just as it became necessary to incorporate an adaptive torque converter model with the clutch pressure estimation scheme, it may become necessary to develop other adaptation schemes to monitor changes in vehicle dynamic models, for instance, and incorporate them in the clutch pressure estimation scheme.

Finally, clutch-to-clutch shift controllers should be developed using the proposed clutch pressure observers. Due to advances in microprocessor technology, it is believed that progressively more powerful microcomputers will continue to be used in production transmission control units. We have shown that, at high sampling frequencies, the developed clutch pressure observer performs well. Knowledge of clutch pressures during gear shifts is critical for better control of clutch-to-clutch gear shifts. Therefore, improved estimation of clutch pressures should improve the performance of clutch-to-clutch shift controllers. Currently, limitations of production transmission control units in terms of computational ability may degrade the accuracy of estimated clutch pressures, and result in corresponding difficulties in new controller development. However, based on the results presented here, estimated clutch pressures obtained as shown here can be used to assist current production transmission controller units by providing other information such as detection of clutch-fill duration, or clutch piston stroke tracking, etc. We believe that the approach presented in this research does give more reliable information for
adjusting control action, when compared to the tabulated data which has been used in many current transmission controllers. So, the presented results can benefit current production transmission controllers, as well as offering opportunities for enhanced shift quality control in future versions of transmission controllers.
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