THREE ESSAYS ON THE TERM STRUCTURE OF INTEREST RATES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Hyoung-Seok Lim, Ph.D.

* * * * *

The Ohio State University

2004

Dissertation Committee:

Masao Ogaki, Adviser
Paul Evans
Pok-sang Lam

Approved by

Adviser

Department of Economics
© Copyright by

Hyoung-Seok Lim

2004
ABSTRACT

Three chapters focus on the term structure of interest rates. Most Central Banks have recently employed the short term interest rate as a monetary policy instrument in the form of either a Taylor rule or Inflation Targeting. Under this framework, the term structure of interest rates play an important role in determining the effectiveness of monetary policy because economic decisions are based on long-term interest rates. The first two chapters discuss the role of the term structure of interest rates in explaining the behavior of exchange rates. Chapter 1 constructs a theoretical model and Chapter 2 provides an empirical result to support this theoretical prediction. Chapter 3 directly estimates the term structure of interest rates from Korean data. The estimated yield curves are used to extract market expectations about the future interest rates path which is essential for forward-looking monetary policy.
To my parents and Jiwon, Junsung, and Seehun
ACKNOWLEDGMENTS

As always, God has gracefully guided me through all my life. In particular, God introduced Masao to me. He is not only an adviser on my research but also a teacher about my attitude to God. He has been greatly patient enough for to me follow up his instructions and also challenged me through his commitment on The Internet Christian Fellowship. A special word of thanks is due to Jiwon. She has cared about everything neglected by me with unbelievable patience. I do not hesitate to say that these papers can not be borne without them. I also thank parents, BCF of Korean Church of Columbus, and all for helps which they have kindly provided to me.
VITA

March 6, 1966 .......................... Born - Junnam, Korea

1989 ................................. B.A. International Economics, Seoul National University, Korea


2000 ................................. M.A. Economics, The Ohio State University

2003 ................................. Ph.D. Candidate, The Ohio State University

1999-present .......................... Graduate Teaching Associate, The Ohio State University.

FIELDS OF STUDY

Major Field: Economics

Studies in:

    Time Series Econometrics
    Monetary Macroeconomics
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapters</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A Theory of Exchange Rates and The Term Structure of Interest Rates</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>2</td>
</tr>
<tr>
<td>1.2 The Model</td>
<td>8</td>
</tr>
<tr>
<td>1.3 The Rational Expectation Equilibrium</td>
<td>15</td>
</tr>
<tr>
<td>1.4 Concluding Remarks</td>
<td>24</td>
</tr>
<tr>
<td>2. The Demand for Foreign Bonds and The Term Structure of Interest Rates</td>
<td>33</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>34</td>
</tr>
<tr>
<td>2.2 The Model</td>
<td>38</td>
</tr>
<tr>
<td>2.3 An Econometric Model</td>
<td>45</td>
</tr>
<tr>
<td>2.4 Empirical Results</td>
<td>50</td>
</tr>
<tr>
<td>2.4.1 Data and Unit root test</td>
<td>50</td>
</tr>
<tr>
<td>2.4.2 Canonical Cointegrating Regression Results</td>
<td>53</td>
</tr>
<tr>
<td>2.5 Concluding Remarks</td>
<td>56</td>
</tr>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iii</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>iv</td>
</tr>
<tr>
<td>Vita</td>
<td>v</td>
</tr>
<tr>
<td>List of Tables</td>
<td>viii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>ix</td>
</tr>
</tbody>
</table>
3. Estimating The Term Structure of Interest Rates from Korean Data . . . 62
   3.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 63
   3.2 Theoretical Background . . . . . . . . . . . . . . . . . . . . . . . . 68
   3.3 Empirical Results . . . . . . . . . . . . . . . . . . . . . . . . . . . 74
   3.4 Concluding Remarks and Future Research . . . . . . . . . . . . . . 80

Appendices:

A. Extracting future yield curves . . . . . . . . . . . . . . . . . . . . . . 93
   A.1 The implied forward rates as break-even rates . . . . . . . . . . . . 93
   A.2 Extracting future yield curves by using the implied forward rates . 94
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A Monte Carlo simulation for slope coefficient ( (= \beta) ) of short-term regression</td>
</tr>
<tr>
<td>1.2</td>
<td>A Monte Carlo simulation for slope coefficient ( (= \beta) ) of long-term regression</td>
</tr>
<tr>
<td>2.1</td>
<td>Tests for the Null of the difference stationary</td>
</tr>
<tr>
<td>2.2</td>
<td>Tests for No cointegration among regressors</td>
</tr>
<tr>
<td>2.3</td>
<td>CCR results for the demand for foreign bonds</td>
</tr>
<tr>
<td>3.1</td>
<td>Daily average of data</td>
</tr>
<tr>
<td>3.2</td>
<td>Estimation results, September 2003</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Estimated yield curves for September 1</td>
<td>82</td>
</tr>
<tr>
<td>3.2</td>
<td>Estimated yield curves for September 8</td>
<td>83</td>
</tr>
<tr>
<td>3.3</td>
<td>Estimated yield curves for September 15</td>
<td>84</td>
</tr>
<tr>
<td>3.4</td>
<td>Estimated yield curves for September 22</td>
<td>85</td>
</tr>
<tr>
<td>3.5</td>
<td>Estimated yield curves for September 29</td>
<td>86</td>
</tr>
<tr>
<td>3.6</td>
<td>Extracted future yield curves given Sep. 15 yield curve</td>
<td>87</td>
</tr>
<tr>
<td>3.7</td>
<td>Extracted future yield curves given Sep. 15 yield curve</td>
<td>88</td>
</tr>
<tr>
<td>3.8</td>
<td>Extracted future yield curves given Sep. 15 yield curve</td>
<td>89</td>
</tr>
<tr>
<td>3.9</td>
<td>Extracted future yield curves given Sep. 15 yield curve</td>
<td>90</td>
</tr>
<tr>
<td>3.10</td>
<td>Extracted future yield curves given Sep. 15 yield curve</td>
<td>91</td>
</tr>
<tr>
<td>3.11</td>
<td>Extracted future yield curves given Sep. 15 yield curve</td>
<td>92</td>
</tr>
</tbody>
</table>
The purpose of this chapter is to construct a model of exchange rate determination that is consistent with the stylized facts regarding Uncovered Interest Parity for short-term and long-term interest rates. This task is especially challenging because of the forward premium anomaly found for short-term interest rates and forward exchange rates. With the assumption that investors have a short investment horizon, the model is consistent with these stylized facts even when the degree of risk aversion is low. The model predicts a complicated relationship between exchange rates and the term structure of interest rates.
1.1 Introduction

The purpose of this paper is to construct a model of exchange rate determination that is consistent with the stylized facts for short-term and long-term interest rates. This task is challenging because the forward premium anomaly is found for short-term interest rates but not for long-term interest rates.

For short-term interest rates and forward exchange rates, uncovered interest parity is typically rejected (see, e.g., Hodrick (1987) and Engel (1996) for recent surveys). As Engel (1996) emphasizes, one form of the rejection found in many recent papers is that the regression of future depreciation on the current forward premium (which is equal to the short-term interest rate differential under the covered interest parity) yields negative estimates of the slope coefficient. This is called the forward premium anomaly (also see Backus, Foresi, and Telmer (2001) for a recent discussion).

For long-term interest rates, more favorable evidence for uncovered interest parity has been found. Direct evidence is given by recent papers, such as Meredith and Chinn (1998) and Alexius (2001). They find that regressions of future depreciation over a long-horizon on the current long-term interest rate differential typically yield significantly positive estimates of the slope coefficient.\textsuperscript{1} Indirect evidence has been found in the standard exchange rate models, such as Meese and Rogoff (1988), Edison and Pauls (1993) and Baxter (1994). Under the uncovered interest parity and the long-run purchasing power parity assumptions, they show that the long-term interest rate differential is more consistent with these assumptions than the short-term rate differential. Similarly, implications of standard exchange rate models hold better in long-horizon data than in short-horizon data (see, e.g., Mark (1995)).

\textsuperscript{1}Alexius (1999) finds similar results for returns on long-term bonds over short investment horizons
To the best of our knowledge, this paper is the first to build a model that is consistent with these stylized facts for both short-term and long-term interest rates. Many explanations have been provided to find an economic explanation for the forward premium anomaly for short-term interest rates. But neither the standard consumption-based asset pricing model with risk averse investors nor the dynamic term structure model can explain it (see, e.g., Mark and Wu (1998), Wu (2002)). Alvarez, Atkeson, and Kehoe (2002) construct a model of segmented asset markets which can be consistent with the forward premium anomaly. McCallum (1994) and Meredith and Chinn (1998) provide an explanation for the forward premium anomaly based on policy reactions. However, in their models, an unspecified error term is necessary for the uncovered interest parity relationship. The model in the present paper gives an alternative explanation that is neither based on transactions costs nor on the assumption of an error term associated with the uncovered interest parity relationship.

Our model is a partial equilibrium model of exchange rate determination for a small open economy. The domestic investors have a constant absolute risk aversion utility function over their wealth in the next period and the asset returns are normally distributed conditional on the available information. We assume that there are three assets in the model: a risk free asset called domestic short-term bonds and two risky assets: domestic long-term bonds and foreign bonds. The investors are also assumed to have a short investment horizon in our model. Given that many professional traders who actively trade in foreign exchange markets are likely to be assessed based on their short-horizon performances by their employers, this assumption is justifiable.

\(^2\)Our model is consistent with these stylized facts in the sense that we observe these patterns with high probability in small samples.
Our intuition for constructing an economic model that is consistent with these stylized facts is based on effects of changes in risk premiums on foreign exchange rates. Given the conditional expectations and variances of all risky assets, we can decompose the effect of a change in the domestic short-term interest rate on the demand for foreign bonds into two components. The one is the *direct risk premium effect*. It is defined as the change in demand due to changes in the risk premium for foreign bonds when the risk premium for domestic long-term bonds is kept constant. The other is the *indirect risk premium effect*. It is the change in demand due to changes in the risk premium for domestic long-term bonds when the risk premium for foreign bonds is kept constant. The change in the demand for foreign bonds is the sum of these direct and indirect risk premium effects. In the special case of risk neutral investors, the indirect risk premium effect does not play any role. However, when investors are risk averse, it is necessary to evaluate both the direct and indirect risk premium effects in order to study how the foreign exchange rate changes when the domestic short-term interest rate changes.

The direct and indirect risk premium effects are properties of the demand for foreign bonds given the distributions of the asset returns and wealth conditional on the information available to the investors. In order to examine how these effects work in equilibrium, consider the term structure of interest rates where some shocks to short term interest rates are not transmitted to long term interest rates and the risk premium of long term bonds is only affected by them.

Suppose the domestic short term interest rates rise but the long term interest rates do not change. First, the risk premium for foreign bonds falls and the direct risk premium effect lowers the demand for foreign bonds without changing in the
exchange rate. Since the supply for foreign bonds is essentially fixed in the short-run by the cumulative current balance in the model, the domestic currency appreciates now, creating expected future depreciation of the currency in order to restore an equilibrium. Second, the risk premium for domestic long-term bonds also falls as the domestic short-term interest rates rise. If the conditional covariance of the two risky assets returns is positive, the indirect risk premium effect increases the demand for foreign bonds. In order to restore an equilibrium, the domestic currency must depreciate this period, creating an expected appreciation in response to the indirect risk premium effect.

The sign and magnitude of the indirect risk premium effect depends on the conditional covariance. In the partial equilibrium model with given stochastic processes for interest rates, we endogenously derive the demand for foreign bonds by solving for the rational expectation equilibrium of the conditional expectation, variance, and covariance of the exchange rate. In equilibrium, the conditional covariance of the two risky assets returns is positive, and the direct and indirect risk premium effects have opposite signs. We show that under some reasonable parameter configurations, the indirect risk premium effect dominates the direct risk premium effect even when the degree of risk aversion is low. As a result, the domestic currency depreciates when the domestic short term interest rates rise and the long term interest rates do not rise. This feature of the model is the reason why the model is consistent with the forward premium anomaly for the short term interest rates.

On the other hand, when the domestic long term interest rates also rise with the domestic short term interest rates, then the risk premium for domestic long term bonds do not change. In this case, the indirect risk premium effect does not affect
the equilibrium exchange rate. Therefore, the domestic currency appreciates when
the short term and long term interest rates rise together. This feature of the model
makes it consistent with the stylized facts of the exchange rate and the long term
interest rates.\footnote{The intuition behind these results for direct and indirect risk premium effects can be general-
ized with Ogaki’s (1990) concepts of direct and indirect substitution effects. This generalization is
explained in an earlier version of the present paper, Ogaki (1999).}

In this paper, we show that the indirect risk premium effect is likely to be quan-
titatively important compared with the direct risk premium effect. In particular, we
show that the indirect risk premium effect can even dominate the direct risk pre-
mium effect under reasonable parameter configurations. A Monte Carlo simulation
for short-term regressions based on this parameter specification consistently shows
the forward premium anomaly. The stronger the indirect risk premium effect, the
more statistically significant the negative slope coefficient.

The result in this paper is in sharp contrast to the conventional view that short-
term capital is more internationally mobile than long-term capital. The 1960’s Op-
eration Twist, in which the Federal Reserve and the Treasury attempted to raise
the short-term rate relative to the long-term interest rate, was evidently based on
this view. However, empirical work by Fukao and Okubo (1984) suggests that inter-
national factors are important in determining the domestic long-term interest rates.
Popper (1993) presents empirical evidence that long-term capital is as internationally
mobile as short-term capital.

The model in this paper has policy implications. It implies that the effectiveness
of central bank attempts to affect exchange rates through the control of short-term
interest rates depends on the responsiveness of long-term interest rates to changes in short-term interest rates.

The rest of the paper is organized as follows. Section 2 presents the model and Section 3 derives the rational expectation equilibrium. First, given the covariance and variance assumed by agents, the rational expectation of the mean of the exchange rate is used to solve for the exchange rate’s law of motion. Then the condition for the rational expectation for the covariance is derived. Finally, the unique stable rational expectation equilibrium is found by equating the variance assumed by agents with the one implied by the demand function. Section 4 investigates the implications of the model on the relationship between the exchange rate and the term structure of interest rates. Conclusions are given in the last section.
1.2 The Model

This paper adopts a simple partial equilibrium exchange rate model following Driskill and McCafferty (1980) and Fukao (1983). We endogenously derive the demand for foreign bonds by solving for the rational expectation of the covariance, so that the covariance assumed by agents is consistent with the one implied by the demand function.\(^4\) It is technically difficult to solve for the rational expectation of the covariance in complicated asset pricing models. For this reason, we employ three asset models.

Consider a partial equilibrium model of exchange rate determination. For simplicity, the overall price level is assumed to be constant. Alternatively, all variables can be considered to be measured in real terms. Investors are assumed to live for two periods, and the same number of investors are born every period. There are 3 assets: domestic short-term bonds (=\(B_{S,t}\)), domestic long-term bonds (=\(B_{L,t}\)) and foreign bonds (=\(B_{F,t}\)). As the foreign interest rate will be assumed to be constant, the foreign short and long-term bonds are perfect substitutes and do not need to be distinguished. The domestic short and long-term bonds are discount bonds paying one unit of the domestic currency after one period and two periods, respectively. The foreign bonds behave in the same manner. At time \(t\), a representative investor allocates his initial wealth (=\(W_t\)) among the three assets and he collects the payoffs paid by the assets he holds at the beginning of time \(t + 1\).

Let \(q_t\) be the price of domestic long-term bonds at time \(t\) and \(r_t\) be the domestic short-term interest rate. Then the rate of return on holding domestic long-term bonds

\(^4\)The demand function depends on the covariance, conditional on the available information, between the exchange rates and the short-term interest rates. At the same time, the demand for foreign bonds affects the dynamics of the exchange rate and the covariance.
for one period, \( r_{L,t} \), is

\[
 r_{L,t} = \frac{1}{q_t}(\frac{1}{1 + r_{t+1}} - q_t)
\]  

(1.1)

Since \( q_t = 1/(1 + R_t)^2 \), where \( R_t \) is the domestic long-term interest rate

\[
 r_{L,t} = (1 + R_t)^2(\frac{1}{1 + r_{t+1}} - \frac{1}{(1 + R_t)^2}) \approx 2R_t - r_{t+1}
\]  

(1.2)

The risk premium for domestic long-term bonds, \( \rho_{L,t} \), is defined to be the difference between the expected rate of return on holding long-term bonds for one period and that of short-term bonds;

\[
 \rho_{L,t} = E_t(r_{L,t}) - r_t = 2[R_t - \frac{1}{2}\{r_t + E_t(r_{t+1})\}]
\]  

(1.3)

where \( E_t \) is the expectation operator conditional on the information set in period \( t \), \( \Omega_t \). We assume that \( \Omega_t \) includes the current and past values of \( r_t, R_t, r^*_t, R^*_t \), and \( s_t \), where \( r^*_t \) and \( R^*_t \) are the foreign short and long-term interest rates, respectively, and \( s_t \) is the natural log of the exchange rate expressed in terms of the domestic currency.

The rate of return on holding foreign bonds for one period in terms of the domestic currency, \( r_{F,t} \), is

\[
 r_{F,t} = r^*_t + s_{t+1} - s_t
\]  

(1.4)
Let $\rho_{F,t}$, the risk premium for foreign bonds, denote the difference between the expected rate of return on holding foreign bonds for one period and that of short-term bonds;

$$\rho_{F,t} = E_t (r_{F,t}) - r_t = r_t^* + E_t (s_{t+1}) - s_t - r_t$$ (1.5)

The model assumes that, at time $t$, a representative investor with a constant absolute risk aversion (CARA) utility function maximizes his expected utility of wealth at the beginning of the time $t+1 (= W_{t+1})$ subject to the budget constraint;

$$\max E_t \left( -e^{-kW_{t+1}} \right)$$ (1.6)

subject to

$$W_t = B^d_{S,t} + B^d_{L,t} + B^d_{F,t}$$

where $k$ is the coefficient of absolute risk aversion, and the superscript $d$ denotes demand, so domestic currency amounts invested in domestic short, long-term, and foreign bonds are $B^d_{S,t}$, $B^d_{L,t}$, and $B^d_{F,t}$, respectively. $W_t$ is the initial wealth at time $t$, and the value of investor’s assets at the beginning of time $t+1$, $W_{t+1}$, satisfies

$$W_{t+1} = B^d_{S,t}(1 + r_t) + B^d_{L,t}(1 + r_{L,t}) + B^d_{F,t}(1 + r_{F,t})$$ (1.7)
In the partial equilibrium model, the stochastic processes for the interest rates are exogenously given, and the utility function is parameterized. The equilibrium exchange rate satisfies the foreign bonds market clearing condition, $B_{dF,t}^d = B_{dF,t}^s$, where $B_{dF,t}^s$ is the supply of foreign bonds to the domestic residents. It is assumed to be equal to the cumulative current account balance and to follow the dynamic equation;

$$B_{dF,t}^s = B_{dF,t-1}^s + C_t$$  

(1.8)

$C_t$ is the current account balance in the period $t$ satisfying\footnote{Interest received by holders of foreign bonds is neglected}

$$C_t = -a + bs_t + u_t,$$  

(1.9)

where $b$ is a positive number, and $u_t$ is the trade shock which is assumed to be white noise with variance $\sigma_u^2$.

Suppose that $W_{t+1}$ is normally distributed conditional on $\Omega_t$ and that the measure of the absolute risk aversion, $k$, is a positive constant. Under these assumptions, a representative investor’s optimization problem is equivalent to maximizing

$$\max\{B_{dF,t}, B_{dL,t}\} \quad E_t(W_{t+1}) - \frac{k}{2} \text{var}_t(W_{t+1})$$  

(1.10)

where

$$E_t(W_{t+1}) = W_t(1 + r_t) + B_{dL,t}(\rho_{L,t}) + B_{dF,t}(\rho_{F,t})$$  

(1.11)
\[
\text{var}_t(W_{t+1}) = (B^d_{L,t})^2 \text{var}_t(r_{t+1}) + (B^d_{F,t})^2 \text{var}_t(s_{t+1}) \\
- 2(B^d_{L,t})(B^d_{F,t}) \text{cov}_t(r_{t+1}, s_{t+1})
\]

First order conditions with respect to \(B^d_{F,t}\) and \(B^d_{L,t}\) are, respectively

\[
\rho_{F,t} - k(B^d_{F,t})\text{var}_t(s_{t+1}) + k(B^d_{L,t})\text{cov}_t(r_{t+1}, s_{t+1}) = 0
\] (1.13)

\[
\rho_{L,t} - k(B^d_{L,t})\text{var}_t(r_{t+1}) + k(B^d_{F,t})\text{cov}_t(r_{t+1}, s_{t+1}) = 0
\] (1.14)

Solving these FOCs for \(B^d_{F,t}\) and \(B^d_{L,t}\) gives demand functions for foreign bonds and domestic long-term bonds, respectively.

\[
B^d_{F,t}[\rho_{F,t}, \rho_{L,t}] = \psi \cdot \rho_{F,t} - \psi \cdot \phi \cdot \rho_{L,t}
\] (1.15)

\[
B^d_{L,t}[\rho_{F,t}, \rho_{L,t}] = \psi \cdot \frac{\sigma^2}{\sigma^2_r} \cdot \rho_{L,t} - \psi \cdot \phi \cdot \rho_{F,t}
\] (1.16)

where

\[
\psi = 1/k\sigma^2_s(1 - \text{cor}^2)
\] (1.17)

\[
\phi = -\text{cov}/\sigma^2_r
\] (1.18)
\[
\sigma_s^2 = E_t[s_{t+1} - E_t(s_{t+1})]^2 \tag{1.19}
\]

\[
\sigma_r^2 = E_t[r_{t+1} - E_t(r_{t+1})]^2 \tag{1.20}
\]

\[
cov = E_t[\{s_{t+1} - E_t(s_{t+1})\}\{r_{t+1} - E_t(r_{t+1})\}] \tag{1.21}
\]

\[
cor = cov/\left(\sqrt{\sigma_s^2}\sqrt{\sigma_r^2}\right) \tag{1.22}
\]

The demand function for foreign bonds, Equation (1.15), depends on \(cov\), the covariance conditional on \(\Omega_t\) between the exchange rate and the short-term interest rate, and \(\sigma_s^2\), the conditional variance of the exchange rate. At the same time, the stochastic processes of the exchange rate and \(cov\) also rely on the demand function for foreign bonds. Therefore, it is required to solve for a rational expectation equilibrium in which the values of \(cov\) and \(\sigma_s^2\) are consistent with the stochastic process of the exchange rate implied by the demand function for foreign bonds. In the next section, the rational expectation equilibrium will be derived.

When the short-term interest rate rises, there exist two opposite effects on the demand for foreign bonds given the second moments of the exchange rate and the short-term interest rate. The first effect, called the direct risk premium effect, is from the first term of Equation (1.15). This effect is defined to be the change in the demand for foreign bonds when the short-term interest rates rise holding the risk premium for long-term bonds constant. This effect is equal to \(-\psi\) and is negative. The second effect, called the indirect risk premium effect, is from the second term of Equation (1.15). This effect is defined to be the change in the demand for foreign bonds when the short-term interest rate rises holding the risk premium for foreign bonds constant.
This effect is equal to $\psi\phi$. In the rational expectations equilibrium derived in the next section, $cov$ is negative, which implies that the indirect risk premium effect is positive.

An intuitive explanation of the indirect risk premium effect is as follows: If the short-term interest rate unexpectedly rises, the price of a long-term bond falls and this drop causes long-term bond holders to suffer an unexpected capital loss. When $cov$ is negative, the exchange rate tends to appreciate and it causes investors an additional unexpected loss if they hold foreign bonds. Therefore, as long as an increase in the short-term interest rate is associated with an appreciation of the domestic currency, risk averse agents will want to avoid holding both long-term bonds and foreign bonds. The greater the appreciation of the domestic currency caused by an increase in the short-term interest rate, the stronger the substitutability of domestic long term bonds and foreign bonds. In particular, when an increase in short term interest rates reduces the risk premium for long term bonds, risk averse investors want to adjust a portfolio of risky assets toward holding more foreign bonds and less long-term bonds. This indirect risk premium effect allows the demand for foreign bonds to increase when the short-term interest rate rises.

The existence of two counter forces on the demand for foreign bonds implies that the effect of a rise in the short-term interest rate on the demand for foreign bonds depends on the relative strength of these two effects. The indirect risk premium effect dominates the direct risk premium effect if and only if $\phi > 1$. Therefore, $\phi$ may be referred to as the measure of the relative magnitude of the indirect risk premium effect. In the next section, it will be shown that $\phi$ is greater than 1 under reasonable parameter configurations.
1.3 The Rational Expectation Equilibrium

In this section, the model presented in the previous section will be used to derive the rational expectation equilibrium. The stochastic processes of interest rates are assumed to be as follows:

\[ r_t = \mu + \epsilon_t + \varepsilon_t \]  
(1.23)

\[ R_t = \frac{1}{2}d + \mu + \frac{1}{2}(1 + c)\epsilon_t \]  
(1.24)

\[ r_t^* = \mu \]  
(1.25)

\[ R_t^* = \frac{1}{2}d + \mu \]  
(1.26)

where \( \epsilon_t \) and \( \varepsilon_t \) are a persistent interest rate shock and a temporary interest rate shock, respectively. It is assumed that \( \epsilon_t \) follows an AR(1) process

\[ \epsilon_t = c\epsilon_{t-1} + v_t, \quad \text{where } |c| < 1 \]  
(1.27)

and that it is independent of \( u_t \). It is also assumed that \( \varepsilon_t \), and \( v_t \) are white noise with variance \( \sigma_\varepsilon^2 \) and \( \sigma_v^2 \), respectively, and that they are independent of each other and of \( u_t \). Finally, \( d \) and \( \mu \) are positive numbers.

The conditional expectation is assumed to coincide with the best linear prediction. Since (1.24) is a fundamental representation in the sense of linear prediction theory (see, e.g., Rozanov (1967)), observing the current and past values of \( R_t \) is equivalent
to observing the current and past values of \( e_t \) under an AR(1) process. It follows that

\[
E_t(r_{t+1}) = \mu + c e_t
\]  
(1.28)

and from (1.3) and (1.28),

\[
\rho_{L,t} = d - \varepsilon_t
\]  
(1.29)

For the purpose of this paper, we need to assume that the risk premium for long-term bonds, \( \rho_{L,t} \), is nonzero. As is shown in (1.29), the assumption employed here is that only \( e_t \) is transmitted to the long-term interest rate, so that the risk premium is equal to the sum of the mean of long-term interest rate and a temporary interest rate shock.

Define \( \eta = \sigma_e^2 / \sigma_{\varepsilon}^2 \), which may be called the measure of substitution between short-term bonds and long-term bonds. If \( \eta = 0 \), then the risk premium for long-term bonds will be the mean of the long-term interest rate, implying that the short-term bond and the long-term bond will become more substitutable. The greater the magnitude of \( \eta \), the smaller the degree of the substitution.

Let \( L \) be the lag operator. Then the equilibrium condition in the period \( t \) is,

\[
E_t[A_0(L)s_t] = a + D_0
\]  
(1.30)

where

\[
A_0(L) = -\psi L^{-1} + (b + \psi)
\]  
(1.31)
\[ D_0 = -u_t - B_{F,t-1}^\phi - \psi \phi d - \psi e_t + \psi (\phi - 1) \varepsilon_t \quad (1.32) \]

The equilibrium condition for period \( t + 1 \) is, if we take expectations conditional on \( \Omega_t \) from both sides,

\[ E_t[A(L)s_{t+1}] = a + D_1 \quad (1.33) \]

where

\[ A(L) = -\psi L^{-1} + (b + 2\psi) - \psi L \quad (1.34) \]

\[ D_1 = \psi (1 - c) e_t - \psi (\phi - 1) \varepsilon_t \quad (1.35) \]

The equilibrium condition for \( t + \tau (\tau \geq 2) \) is, if we take expectations conditional on \( \Omega_t \) from both sides,

\[ E_t[A(L)s_{t+\tau}] = a + D_2 \quad (1.36) \]

where

\[ D_2 = \psi (1 - c) e_t c^{\tau - 1} \quad (1.37) \]

Solving (1.30), (1.33), and (1.36) as a difference equation system of \( E_t(s_{t+\tau}) \) with respect to \( \tau \) provides the unique saddle point solution.
\[ s_t = \bar{s} - \left(1 - \frac{1 - \lambda}{b}\right)u_t - \left(1 - \frac{1 - \lambda}{b}\right)B_{F,t-1}^\phi - \left(\frac{\lambda}{1 - \lambda e}\right)e_t + \lambda(\phi - 1)\varepsilon_t \]  

(1.38)

where \( \bar{s} = a - (1 - \frac{1 - \lambda}{b})\phi \psi d \) is the long-run equilibrium exchange rate clearing the current account, and

\[ \lambda = 1 + \frac{b}{2\psi} - \frac{b}{2\psi} \sqrt{1 + \frac{4\psi}{b}} \]  

(1.39)

It is shown that 0 < \( \lambda < 1 \), \( \partial \lambda / \partial \psi > 0 \), \( \lim_{\psi \to 0} \lambda = 0 \), and \( \lim_{\psi \to \infty} \lambda = 1 \).

Equation (1.38) shows that the investor’s expected values of \( \text{cov} \) and \( \sigma_s^2 \) affect the exchange rate dynamics through \( \lambda \) and \( \phi \). On the other hand, the exchange rate dynamics in (1.38) imply certain values of \( \text{cov} \) and \( \sigma_s^2 \), which need to be consistent with the investor’s expected values in the rational expectation equilibrium. The equilibrium is analyzed in two steps. First, we solve for the rational expectation of \( \text{cov} \). Second, we show the uniqueness and existence of the rational expectation equilibrium by solving for the rational expectation of \( \sigma_s^2 \).

Before solving for the equilibrium, note the nature of (1.38). The discrepancy between actual and long-run equilibrium exchange rates can be explained by several factors: the trade shock (the first bracket), the cumulative current account balance (the second bracket), the persistent interest rate shock (the third bracket), and the temporary interest rate shock (the fourth bracket). The trade shock, which tends to give rise to current account surplus, makes the domestic currency appreciate. As the cumulative current account balance becomes greater, the appreciation of the domestic currency takes place.
currency increases; for an investor to have incentives to hold more foreign bonds, the domestic currency must appreciate at present, so that investors will anticipate it depreciating in the future. Prolonged increases in the short-term interest rate make the domestic currency appreciate. All of these effects are consistent with the expected directions. However, the temporary interest rate shock, \( \varepsilon_t \), has a perverse effect if the relative magnitude of the indirect risk premium effect, \( \phi \), is greater than one.

The term \( \phi \) may be obtained by solving for the rational expectation of covariance. Calculating \( \text{cov} = E_t[\{(s_{t+1} - E_t(s_{t+1}))\}{\{r_{t+1} - E_t(r_{t+1})}\}] \) from (1.23) and (1.38) by taking the one period lead yields

\[
\text{cov} = -\left(\frac{\lambda}{1 - \lambda c}\right)(1 - c^2)\sigma_e^2 + \lambda(\phi - 1)\sigma_e^2 
\]  
(1.40)

Substituting the definition of \( \phi \), (1.18), into (1.41), and solving for \( \text{cov} \) gives the rational expectation equilibrium;

\[
\text{cov} = -\left[\frac{\lambda(1 - c^2) + \lambda(1 - \lambda c)}{1 - \lambda c}\right][\frac{1 - c^2 + \eta}{1 - c^2 + \eta(1 + \lambda)}]\sigma_e^2 < 0 
\]  
(1.41)

Therefore, by (1.18),

\[
\phi = \left[\frac{\lambda(1 - c^2) + \lambda(1 - \lambda c)}{1 - \lambda c}\right][\frac{1}{1 - c^2 + \eta(1 + \lambda)}] > 0 
\]  
(1.42)

In the rational expectation equilibrium, the conditional covariance between the exchange rate and the short-term interest rate, \( \text{cov} \), is negative and the measure of the
relative magnitude of the indirect risk premium effect, \( \phi \), is positive. This implies that the indirect risk premium effect is positive as is shown in the previous section.

The main issue for the purpose of this paper is whether \( \phi \) is greater or less than one. In order to determine this, we will investigate the sign of

\[
\phi - 1 = \frac{(1 - c^2)\{\lambda(1 + c) - 1\} - \eta(1 - \lambda c)}{(1 - \lambda c)(1 - c^2 + \eta(1 + \lambda))}
\]  

(1.43)

In order to examine the sign of (1.43), we need to know how \( \lambda \) depends on the underlying parameters of the model. For this purpose, the existence and the uniqueness of the rational expectation equilibrium will be shown by solving for the rational expectation of the conditional variance of the exchange rate, \( \sigma_s^2 = E_t[\{s_{t+1} - E_t(s_{t+1})\}^2] \).

By taking one period lead of (1.38), we obtain

\[
\sigma_s^2 = \left(1 - \frac{\lambda}{b}\right)^2 \sigma_u^2 + \left[\frac{\lambda^2(1 - c^2) + \lambda^2(\phi - 1)^2 \eta(1 - \lambda c)^2}{(1 - \lambda c)^2}\right] \sigma_e^2
\]  

(1.44)

By (1.41) and the definition of \( \text{cor} \), (1.22),

\[
\text{cor} = -\sqrt{\frac{(1 - c^2 + \eta)\sigma_e^2}{\sigma_s^2}} \cdot \frac{\lambda(1 - c^2) + \lambda \eta(1 - \lambda c)}{(1 - \lambda c)(1 - c^2 + \eta(1 + \lambda))}
\]  

(1.45)

By using the definition of \( \lambda \), (1.39), we obtain

\[
\psi = \frac{b \lambda}{(1 - \lambda)^2}
\]  

(1.46)
Substituting the definition of $\psi$, (1.17), into (1.46) gives

$$\frac{1}{k} = \sigma_s^2(1 - \text{cor}^2) \frac{b\lambda}{(1 - \lambda)^2}$$

(1.47)

The condition for the rational expectation equilibrium value for $\sigma_s^2$ is obtained by substituting (1.44) and (1.45) into (1.47);

$$\frac{1}{k} = g(\lambda),$$

(1.48)

where $g(\lambda) = \frac{\sigma_a^2 \lambda}{b} + \frac{b \sigma_e^2 \lambda^3 \{1 - (\phi - 1)^2\} (1 - \lambda)^2}{(1 - \lambda c)^2 (1 - \lambda)^2} - \frac{b \sigma_e^2 (1 - c^2 + \eta) \lambda (1 - \lambda^2 + \lambda (1 - \lambda c))^2}{(1 - \lambda c)^2 (1 - \lambda)^2 (1 - c^2 + \eta (1 + \lambda))^2}$

Let $\lambda^*$ be the value of $\lambda$ that satisfies (1.48). Any such $\lambda^*$ corresponds to a rational expectation equilibrium. It can be checked that $\lim_{\lambda \to 0} g(\lambda) = 0$, and $\lim_{\lambda \to 1} g(\lambda) = \infty$. In particular, under the parameter configuration employed in the following Monte Carlo simulation, it can be shown that $g'(\lambda) > 0$. Hence, there exists a unique rational expectation equilibrium. Moreover, when $k$ is smaller, $\lambda^*$ is larger. It is shown that $\lim_{k \to 0} \lambda = 1$ and $\lim_{k \to \infty} \lambda = 0$. The value of $\psi$ can be obtained by substituting $\lambda^*$ for $\lambda$ in (1.46). The value of $\psi$ is decreased by a reduction in the variances $\sigma_a^2$ and $\sigma_e^2$ and by an increase in the measure of constant risk aversion, $k$, which in turn diminishes $g(\lambda)$.

Equation (1.43) shows that $\phi$ can be either greater or less than one, depending on the parameter values. One interesting case arises when the investor is close to
being risk neutral. For a very small $k$, an approximate formula for (1.43) with $\lambda \approx 1$ is

$$
\phi - 1 = \frac{(1 + c)c - \eta}{1 - c^2 + 2\eta}
$$

(1.49)

Under $\lambda \approx 1$ condition, we investigate what condition is required to exhibit the forward premium anomaly. The forward premium regression for short-term interest rate differential is

$$
s_{t+1} - s_t = \alpha + \beta(r_t - r_t^*) + \text{error term}
$$

(1.50)

Let $\hat{\beta}$ be the estimate of $\beta$. If the estimator is consistent, it will be

$$
\text{plim} \hat{\beta} = \frac{\text{cov}(r_t - r_t^*, s_{t+1} - s_t)}{\text{var}(r_t - r_t^*)}

\Rightarrow \frac{\text{cov}(r_t, s_{t+1} - s_t)}{\text{var}(r_t)}

(1.51)

For $\hat{\beta}$ to be negative, we need

$$
\text{cov}(r_t, s_{t+1} - s_t) < 0

\Rightarrow \text{cov}(r_t, s_{t+1}) < \text{cov}(r_t, s_t)

\Rightarrow 1 < (\phi - 1)\eta

(1.52)

However, substituting equation (1.49) into equation (1.52) does not produce a positive value of $\eta$. Instead, we conduct a Monte Carlo simulation\textsuperscript{6} to investigate the forward premium anomaly.

\textsuperscript{6}We use Gauss for Windows NT/95 Version 3.2.38 to conduct the simulation
Suppose the AR(1) coefficient of the persistent interest rate shock, \( c \), in Equation (1.27) is close to one (for example, \( c = 0.9 \)), then \( \phi \) in Equation (1.49) becomes greater than one as long as \( \eta < 1.71 \). When the investor is close to being risk neutral, the degree of substitution between short and long-term bonds must be high, and consequently, \( \eta \) should be very small. Under these parameter configurations, our model presented in the previous section predicts that when the measure of the relative magnitude of the indirect risk premium effect, \( \phi \), is greater than one, the demand for foreign bonds increases as the short-term interest rate rises, resulting in the depreciation of domestic currency to cause an expected future appreciation of domestic currency. A Monte Carlo simulation based on these parameter configurations consistently generates a negative slope coefficient to show the forward premium anomaly. As Table (1.4) shows, the stronger the indirect risk premium effect, the more statistically significant the negative slope coefficient.

On the contrary, a Monte Carlo simulation for long-term interest differential still generates, as Table (1.4) shows, a positive slope coefficient under the same parameter configurations as standard exchange rate model predicts.
1.4 Concluding Remarks

In this paper, we derive the demand function for foreign bonds endogenously by solving for the rational expectation equilibrium and investigate how a rise in the short-term interest rate affects the demand for foreign bonds. It generates two opposite effects on the demand for foreign bonds. The direct risk-premium effect comes from the fact that risk averse agents with short investment time-horizons want to reduce the demand for foreign bonds to increase the amount invested in risk free assets. On the other hand, investors have another incentive, the indirect risk-premium effect, to increase the demand for foreign bonds to minimize potential capital losses resulting from holding both risky assets. We show that, under reasonable parameter configurations, the indirect risk-premium effect can even dominate the direct risk-premium effect causing demand for foreign bonds to increase. In this case, the forward premium anomaly about the short-term interest rate can be explained; the domestic currency depreciates now, creating expected future appreciation of the currency. For the long-term interest rate differential, this model still shows the same prediction on the exchange rate like standard exchange rate models. Byeon and Ogaki (1999) find such results for many of the G7 countries with cointegrating regressions of real exchange rates onto the short-term and long-term interest rate differentials. Ogaki and Santaella (2000) obtain similar results for Mexico.

If the indirect risk-premium effect is quantitatively important, then the effectiveness of central bank attempts to affect exchange rate by controlling the short-term interest rate depends on whether the long-term interest rate responds to changes in the short-term interest rate. Anecdotal evidence suggests that further empirical investigation is warranted. For example, from the middle of March 1982 to the end of
November 1982, the Bank of Japan adopted a policy to increase the domestic short-term interest rate in order to cause an appreciation of the yen (see, e.g., Komiya and Suda [1983, pp. 347-354]). The short-term interest rate in Japan increased but the yen tended to depreciate, rather than appreciate, against the U.S. dollar during this period. One remarkable fact was that the long-term interest rate did not increase when the Bank of Japan began to increase the short-term interest rate (Komiya and Suda [1983, p.349]).

The model in this paper suggests that a much more complicated relationship might exist between the term structure of interest rates and the exchange rate than is implied by exchange rate models with risk neutral agents. In addition, the model can be applied to the relationship between the exchange rate and the term structure of various short-term rates if the investment horizon is very short (e.g., 1 month or shorter). In this sense, the model could help explain Clarida and Taylor’s (1997) finding that the information given by the term structure of 1-month to 12-months forward premiums is useful in predicting the future exchange rate.

There has been little empirical work on the interaction between the exchange rate and the term structure of interest rates relative to the large volume of empirical work on the exchange rate. Further empirical investigation is warranted.
<table>
<thead>
<tr>
<th>$\phi$</th>
<th>1.6352</th>
<th>2.7848</th>
<th>3.5593</th>
<th>5.1282</th>
<th>9.0952</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\eta = 0.7)$</td>
<td>$(\eta = 0.3)$</td>
<td>$(\eta = 0.2)$</td>
<td>$(\eta = 0.1)$</td>
<td>$(\eta = 0.01)$</td>
<td></td>
</tr>
</tbody>
</table>

| mean of $\hat{\beta}$ | 0.2016 | -0.3844 | -0.7788 | -1.5776 | -3.5976 |
| negative freq. | 39.1 | 69.4 | 84.2 | 97.2 | 100.0 |
| 5 % level | 1.9 | 8.5 | 18.0 | 49.6 | 93.2 |
| (10 % level) | (3.7) | (13.8) | (28.1) | (62.5) | (96.8) |

Note: 1) percentage of negative coefficients among total iteration (=1,000)  
2) percentage of total iterations (=1,000) rejecting $H_0$ at five percent significance level.  
Numbers in parentheses are that of ten percent significance level.  
3) sample size is 102 and $c = 0.9$

Table 1.1: A Monte Carlo simulation for slope coefficient (=$\beta$) of short-term regression

\[(s_{t+1} - s_t) = \alpha + \beta(r_t - r^*_t) + \text{error term} \]

\[H_0 : \beta = 0\]
<table>
<thead>
<tr>
<th>$\phi$</th>
<th>1.6352 ($\eta = 0.7$)</th>
<th>2.7848 ($\eta = 0.3$)</th>
<th>3.5593 ($\eta = 0.2$)</th>
<th>5.1282 ($\eta = 0.1$)</th>
<th>9.0952 ($\eta = 0.01$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean of $\hat{\beta}$</td>
<td>1.0267</td>
<td>1.0274</td>
<td>1.0278</td>
<td>1.0287</td>
<td>1.0309</td>
</tr>
<tr>
<td>positive freq.</td>
<td>92.2</td>
<td>91.9</td>
<td>92.1</td>
<td>91.3</td>
<td>87.0</td>
</tr>
<tr>
<td>5 % level</td>
<td>30.9</td>
<td>30.0</td>
<td>29.3</td>
<td>28.4</td>
<td>20.8</td>
</tr>
<tr>
<td>(10 % level)</td>
<td>(43.9)</td>
<td>(43.1)</td>
<td>(42.1)</td>
<td>(39.2)</td>
<td>(30.8)</td>
</tr>
</tbody>
</table>

Note: 1) percentage of positive coefficients among total iteration(=1,000)
2) percentage of total iteration (=1,000) rejecting $H_0$ at five percent significance level. Numbers in parentheses are that of ten percent significance level.
3) sample size is 102 and $c = 0.9$

Table 1.2: A Monte Carlo simulation for slope coefficient ($= \beta$) of long-term regression

$$(s_{t+2} - s_t) = \alpha + \beta(R_t - R_t^*) + \text{error term}$$

$H_0 : \beta = 0$
REFERENCES


New York: John Wiley.


CHAPTER 2

THE DEMAND FOR FOREIGN BONDS AND THE TERM STRUCTURE OF INTEREST RATES

This chapter estimates the demand for foreign bonds by employing cointegration techniques to investigate whether data is consistent with the prediction of a partial equilibrium exchange rate model depending on the term structure of interest rates. An implication of this theoretical model is that under reasonable parameter configurations, such as a low degree of risk aversion, the demand increases as the domestic short-term interest rates rise, resulting in the expected appreciation of the domestic currency instead of the expected depreciation. This prediction, a potential explanation for the forward premium anomaly in the international financial market, is supported by empirical results. These results are in line with other empirical research, which investigate the relationship between the exchange rate and the term structure of interest rates instead of estimating the demand function for foreign bonds.
2.1 Introduction

The purpose of this paper is to investigate whether the implication of the exchange rate determination model presented by Lim and Ogaki (2003) is consistent with data. Their model is designed to explain the forward premium anomaly found in the international financial market.

The uncovered interest parity (hereafter, UIP), one of the key building block of international economics, says that a higher interest rate currency is expected to depreciate as investors demand compensation for holding a lower interest rate currency in the international market. However, UIP is typically rejected, in particular, for the short-term interest rates and forward exchange rates. The regression of the future depreciation on the current forward premium\(^7\) yields negative estimates of the slope coefficient unlike the prediction of UIP. This is called the forward premium anomaly. (see Engel (1996) for recent survey).

For long-term interest rates, more favorable evidence for UIP has been found. Direct evidence is given by recent papers, such as Meredith and Chinn (1998) and Alexius (2001). They find that regression of the future depreciation over a long-horizon on the current long-term interest rate differential typically yields significantly positive estimates of the slope coefficient.\(^8\) Indirect evidence has been found in the standard exchange rate models, such as Meese and Rogoff (1988), Edison and Pauls (1993) and Baxter (1994). Under UIP and the long-run purchasing power parity assumptions, they show that the long-term interest rate differential is more consistent with those assumptions than the short-term rate differential. Similarly, implications

\(^7\)It is equal to the short-term interest rate differential under the covered interest rate

\(^8\)Alexius (1999) finds similar results for returns on long-term bonds over short investment horizons
of standard exchange rate models hold better in long-horizon data than in short-horizon data (see, e.g., Mark (1995)).

Lim and Ogaki (2003) construct a model that is consistent with these stylized facts for both short-term and long-term interest rates. In their model, the domestic investors have the constant absolute risk aversion utility function over the wealth of the next period and the asset returns are normally distributed conditional on the available information. They assume that there are three assets in the model: a risk free asset called domestic short-term bonds and two risky assets: domestic long-term bonds and foreign bonds.

A key element for explaining these stylized facts about UIP is based on the effects of changes in risk premiums on both foreign bonds and domestic long-term bonds under a short time investment horizon. There are two effects on the demand for foreign bonds when the domestic short-term interest rate changes. The one is the conventional effect, which they define as the **direct risk premium effect**. It is the change in the demand due to the change in the risk premium for foreign bonds when the risk premium for domestic long-term bonds is kept constant. The other channel, which they define as the **indirect risk premium effect**, results from the change in the risk premium for domestic long-term bonds with the constant risk premium for foreign bonds. Therefore, the change in the demand for foreign bonds is the sum of these direct and indirect risk premium effects. In the special case of risk neutral investors, the indirect risk premium effect does not play any role. However, when investors are risk averse, it is necessary to evaluate both direct and indirect risk premium effects to

---

9Their model is consistent with these stylized facts in the sense that they observe these patterns with high probability in small samples.

10The idea behind those effects can be generalized with Ogaki’s (1990) concepts of direct and indirect substitution effects. This generalization is explained in Ogaki (1999).
study how the foreign exchange rate changes when the domestic short-term interest rates changes. In particular, these effects are investigated under the assumption that the investors have a short investment horizon. Given that many professional traders who actively trade in foreign exchange markets are likely to be assessed by their short-horizon performances by their employers, this assumption is justifiable.

The sign and magnitude of the indirect risk premium effect depend on the conditional variance of the two risky assets returns. They show that the rational expectation equilibrium of the conditional covariance of the two risky assets returns is positive and that the direct and indirect risk premium effects have the opposite signs. Further, they show that the indirect risk premium effect is likely to be quantitatively important compared with the direct risk premium effect. A Monte Carlo simulation shows that the indirect risk premium effect can dominate the direct risk premium effect under reasonable parameter configurations, such as small degree of risk aversion. A Monte Carlo simulation for short-term regression based on this parameter specification consistently shows the forward premium anomaly. The stronger the indirect risk premium effect, the more statistically significant the negative slope coefficient. This implies that the domestic currency depreciates when the domestic short-term interest rate rises and the long-term interest rate does not rise. This feature of the model is the reason why the model is consistent with the forward premium anomaly for the short-term interest rate. On the other hand, when the domestic long-term interest rate rises with the domestic short-term interest rate, then the risk premium for domestic long-term bonds does not change, and the indirect risk premium effect does not affect the equilibrium exchange rate. Therefore, the domestic currency appreciates when the short-term and long-term interest rate rise together. This feature
of the model makes the model consistent with the stylized fact of the exchange rate and the long-term interest rate.

One contribution of Lim and Ogaki’s (2003) paper is that it can explain the forward premium anomaly without employing a high degree of risk aversion, whereas the standard consumption-based asset pricing model with risk averse investors needs implausible large coefficients of relative risk aversion to explain the forward premium anomaly (see, e.g., Mark (1985), Backus, Gregory, and Telmer (1993)).

To investigate whether this theoretical predictions are consistent with the data, I directly estimate the demand for foreign bonds as a cointegrating regression under the assumption that it is stable in the long run. If indirect risk premium effect dominates the direct risk premium effect, the regression coefficient estimate for domestic short-term interest rate is expected to be positive as Lim and Ogaki (2003) predicts, i.e., a higher interest rate currency is expected to appreciate rather than depreciate. I obtain this result for several countries. These empirical results are complementary to other previous empirical outcomes. Byeon and Ogaki (1999) use a cointegration technique by regressing the real exchange rates on short-term and long-term interest rates differentials. They find that the short-term interest rates differential and the long-term interest rates differentials have the opposite effects on the exchange rate in industrialized economies. Ogaki and Santaella (2000) obtain a similar result for Mexico.

This paper is organized as follows. Section 2 briefly presents theoretical implications of Lim and Ogaki’s (2003) model. Section 3 introduces the econometric method to test these implications. Section 4 explains data and empirical results for these implications and final conclusion is given in section 5.
2.2 The Model

Consider a partial equilibrium model of exchange rate determination. For simplicity, the overall price level is assumed to be constant. Alternatively, all variables can be considered to be measured in real terms. Investors are assumed to live for two periods, and the same number of investors are to be born every period. There are 3 assets, domestic short-term bonds ($=B_{S,t}$), domestic long-term bonds ($=B_{L,t}$) and foreign bonds ($=B_{F,t}$). As the foreign interest rate will be assumed to be constant, the foreign short and long-term bonds are perfect substitutes and do not need to be distinguished. The domestic short and long-term bonds are discount bonds paying one unit of the domestic currency after one period and two periods, respectively. The foreign bonds behave in the same manner. At time $t$, a representative investor allocates his initial wealth ($=W_t$) among 3 assets and he collects the payoffs paid by the assets he holds at the beginning of time $t+1$.

Let $q_t$ be the price of a domestic long-term bond at time $t$ and $r_t$ be the domestic short-term interest rate. Then the rate of return on holding domestic long-term bonds for one period, $r_{L,t}$, is

$$r_{L,t} = \frac{1}{q_t} \left( \frac{1}{1 + r_{t+1}} - q_t \right) \quad (2.1)$$

Since $q_t = 1/(1 + R_t)^2$, where $R_t$ is the domestic long-term interest rate, it becomes

$$r_{L,t} = (1 + R_t)^2 \left( \frac{1}{1 + r_{t+1}} - \frac{1}{(1 + R_t)^2} \right) \approx 2R_t - r_{t+1} \quad (2.2)$$
Define the risk premium for the domestic long-term bond, $\rho_{L,t}$, to be the difference between the expected rate of return on holding the long-term bond for one period and that of the short-term bond:

$$\rho_{L,t} = E_t(r_{L,t}) - r_t = 2[R_t - \frac{1}{2}(r_t + E_t(r_{t+1}))]$$  \hspace{1cm} (2.3)

where $E_t$ is the expectation operator conditional on the information set in period $t$, $\Omega_t$. We assume that $\Omega_t$ includes the current and past values of $r_t$, $R_t$, $r_t^*$, $R_t^*$, and $s_t$, where $r_t^*$, and $R_t^*$ are the foreign short and long-term interest rates, respectively, and $s_t$ is the natural log of the exchange rate expressed in terms of the domestic currency.

The rate of return on holding foreign bond for one period in terms of the domestic currency, $r_{F,t}$, is

$$r_{F,t} = r_t^* + s_{t+1} - s_t$$  \hspace{1cm} (2.4)

Let $\rho_{F,t}$, the risk premium for foreign bonds, define the difference between the expected rate of return on holding foreign bonds for one period and that of short-term bonds:

$$\rho_{F,t} = E_t(r_{F,t}) - r_t = r_t^* + E_t(s_{t+1}) - s_t - r_t$$  \hspace{1cm} (2.5)

The model assumes that, at time $t$, a representative investor with constant absolute risk aversion (CARA) utility function maximizes his expected utility of wealth
at the beginning of the time $t + 1 \ (= W_{t+1})$ subject to the budget constraint,

$$
\begin{align*}
\max & \quad E_t \left( \frac{-e^{-kW_{t+1}}}{k} \right) \\
\text{s.t.} & \quad W_t = B_{S,t}^d + B_{L,t}^d + B_{F,t}^d
\end{align*}
$$  \tag{2.6}

where $k$ is the coefficient of absolute risk aversion, and the superscript $d$ denotes demand, so domestic currency amounts invested in domestic short, long-term, and foreign bonds are $B_{S,t}^d$, $B_{L,t}^d$, and $B_{F,t}^d$, respectively. $W_t$ is the initial wealth at time $t$, and the value of investor’s assets at the beginning of time $t + 1$, $W_{t+1}$, satisfies

$$
W_{t+1} = B_{S,t}^d(1 + r_t) + B_{L,t}^d(1 + r_{L,t}) + B_{F,t}^d(1 + r_{F,t})
$$  \tag{2.7}

In the partial equilibrium model, the stochastic processes for the interest rates are exogenously given, and the utility function is parameterized. The equilibrium exchange rate satisfies the foreign bond market clear condition, $B_{F,t}^s = B_{F,t}^s$, where $B_{F,t}^s$ is the supply of foreign bonds to the domestic residents. It is assumed to be equal to the cumulative current account balance and to follow the dynamic equation;

$$
B_{F,t}^s = B_{F,t-1}^s + C_t,
$$  \tag{2.8}
where \( C_t \) is the current account balance in the period \( t \) satisfying\(^{11}\)

\[
C_t = -a + bs_t + u_t,
\]

(2.9)

where \( b \) is a positive number, and \( u_t \) is the trade shock which is assumed to be white noise with variance \( \sigma_u^2 \).

Suppose that \( W_{t+1} \) is normally distributed conditional on \( \Omega_t \) and that the measure of the absolute risk aversion, \( k \), is a positive constant. Under these assumptions, a representative investor’s optimization problem is equivalent to maximizing

\[
\max \{B^d_{F,t}, B^d_{L,t}\} \quad E_t(W_{t+1}) - \frac{k}{2} \text{var}_t(W_{t+1})
\]

(2.10)

where

\[
E_t(W_{t+1}) = W_t(1 + r_t) + B^d_{L,t}(\rho_{L,t}) + B^d_{F,t}(\rho_{F,t})
\]

(2.11)

\[
\text{var}_t(W_{t+1}) = (B^d_{L,t})^2 \text{var}_t(r_{t+1}) + (B^d_{F,t})^2 \text{var}_t(s_{t+1})
\]

\[
-2(B^d_{L,t})(B^d_{F,t}) \text{cov}_t(r_{t+1}, s_{t+1})
\]

(2.12)

First order conditions with respect to \( B^d_{F,t} \) and \( B^d_{L,t} \) are respectively

\[
\rho_{F,t} - k(B^d_{F,t})\text{var}_t(s_{t+1}) + k(B^d_{L,t})\text{cov}_t(r_{t+1}, s_{t+1}) = 0
\]

(2.13)

\(^{11}\)Interest received by holders of foreign bonds is neglected.
\( \rho_{L,t} - k(B_{L,t}^d) \text{var}_t(r_{t+1}) + k(B_{F,t}^d) \text{cov}_t(r_{t+1}, s_{t+1}) = 0 \) 

(2.14)

Solving these FOCs for \( B_{F,t}^d \) and \( B_{L,t}^d \) gives demand functions for the foreign bond and the domestic long-term bond, respectively.

\[
B_{F,t}^d[\rho_{F,t}, \rho_{L,t}] = \psi \cdot \rho_{F,t} - \psi \cdot \phi \cdot \rho_{L,t} \tag{2.15}
\]

\[
B_{L,t}^d[\rho_{F,t}, \rho_{L,t}] = \psi \cdot \frac{\sigma_s^2}{\sigma_r^2} \cdot \rho_{L,t} - \psi \cdot \phi \cdot \rho_{F,t} \tag{2.16}
\]

where

\[
\psi = 1/k\sigma_s^2(1 - \text{cor}^2) \tag{2.17}
\]

\[
\phi = -\text{cov}/\sigma_r^2 \tag{2.18}
\]

\[
\sigma_s^2 = E_t[s_{t+1} - E_t(s_{t+1})]^2 \tag{2.19}
\]

\[
\sigma_r^2 = E_t[r_{t+1} - E_t(r_{t+1})]^2 \tag{2.20}
\]

\[
\text{cov} = E_t[(s_{t+1} - E_t(s_{t+1}))(r_{t+1} - E_t(r_{t+1}))] \tag{2.21}
\]

\[
\text{cor} = \text{cov}/(\sqrt{\sigma_s^2} \sqrt{\sigma_r^2}) \tag{2.22}
\]

The demand function for the foreign bond, Equation (2.15), depends on \( \text{cov} \), the covariance conditional on \( \Omega_t \) between the exchange rate and the short-term interest.
rate, and $\sigma_s^2$, the conditional variance of the exchange rate. At the same time, the stochastic processes of the exchange rate and $cov$ also rely on the demand function for the foreign bond. Therefore, it is required to solve for a rational expectation equilibrium in which the values of $cov$ and $\sigma_s^2$ are consistent with the stochastic process of the exchange rate implied by the demand function for the foreign bond.\textsuperscript{12}

When the short-term interest rate rises, there exist two opposite effects on the demand for the foreign bond given the second moments of the exchange rate and the short-term interest rate. The first effect, called the direct risk premium effect, is from the first term of Equation (2.15). This effect is defined to be the change in the demand for foreign bonds when the short-term interest rate rises when the risk premium for the long-term bonds does not change. This effect is equal to $-\psi$ and is negative. The second effect, called the indirect risk premium effect, is from the second term of Equation (2.15). This effect is defined to be the change in the demand for foreign bonds when the short-term interest rate rises when the risk premium for the foreign bonds does not change. This effect is equal to $\psi \phi$. The rational expectations equilibrium $cov$ is negative, which implies that the indirect risk premium effect is positive.

An intuitive explanation of the indirect risk premium effect is as follows: If short-term interest rate unexpectedly rises, the price of long-term bond falls and this drop causes long-term bond holders to suffer an unexpected capital loss. When $cov$ is negative, the exchange rate tends to appreciate and it causes investors an additional unexpected loss if they hold foreign bonds. Therefore, as long as an increase in short-term interest rate is associated with an appreciation of the domestic currency,

\textsuperscript{12}For detail derivation of the rational expectation equilibrium, see Lim and Ogaki (2003).
risk averse agents want to avoid holding both long-term bond and foreign bond. The greater the appreciation of the domestic currency caused by an increase in the short-term interest rate, the stronger the incentive to adjust a portfolio of risky assets toward holding more foreign bonds and less long-term bonds as the short-term interest rate rises. This indirect risk premium effect allows the demand for foreign bonds to increase when the short-term interest rate rises.

The existence of two counter forces on the demand for foreign bonds implies that the effect of a rise in the short-term interest rate on the demand for foreign bonds depends on the relative strength of these two effects. The indirect risk premium effect dominates the direct risk premium effect if and only if $\phi > 1$. Therefore, $\phi$ may be referred to as the measure of the relative magnitude of the indirect risk premium effect. Lim and Ogaki (2003) show that $\phi$ is greater than 1 under reasonable parameter configurations, such as a small degree of risk aversion.\footnotemark

Under the identification restriction that the demand function is stable in the long run, an implication of Equation (2.15) is that the demand for foreign bonds, $B_{dF,t}$, domestic short-term interest rate, $r_t$, domestic long-term interest rate, $R_t$, and returns on holding foreign bonds are cointegrated. Further, the sign of the coefficient estimate for the domestic short-term interest rate in this cointegrating regression is expected to be positive if the indirect risk premium effect dominates the direct risk premium effect. Those implications are investigated in next section by employing cointegration techniques.

\footnotetext{For this derivation and a Monte Carlo simulation result, see Lim and Ogaki (2003).}
2.3 An Econometric Model

To investigate the indirect risk premium effect, I directly estimate the demand function for foreign bonds as a cointegrating regression under the assumption that the demand for foreign bonds is stable in the long run. The cointegrating regression I employ is of the form.\(^\text{14}\)

\[
B_{F,t}^d = \beta_0 + \beta_1 t + \beta_2 r_t + \beta_3 R_t + \beta_4 [r_t^* + E_t(\Delta s_{t+1})] + \mu_t
\]

(2.23)

where \(\mu_t\) is stationary process. A negative estimate for \(\beta_3\) is expected like a standard exchange rate model. However, the sign of the coefficient estimate for \(\beta_2\) depends on the relative strength between the indirect risk premium effect and the direct risk premium effect. If the indirect risk premium effect dominates the direct risk premium effect, then a positive estimate for \(\beta_2\) is expected as Lim and Ogaki (2003) shows.

To estimate the cointegrating vector, Park’s (1992) Canonical Cointegrating Regression (hereafter CCR) is applied to (2.23). This procedure allows us to test the null hypothesis of stochastic cointegration and the deterministic cointegration restriction.\(^\text{15}\) Park’s tests are based on Wald tests for spurious deterministic trends in the CCR procedure. A Monte Carlo simulations done by Han and Ogaki (1997) shows that those tests have reasonable size and power. Other reason for employing CCR is that a Monte Carlo simulations in Park and Ogaki (1991) show that the CCR

\(^{14}\)As long as the deterministic cointegration restriction is not satisfied, I include time trend in cointegrating regression to investigate only stochastic cointegrating relationship.

\(^{15}\)This is the case where the cointegrating vector eliminates not only stochastic trend but also deterministic trend.
estimators have better small sample properties in terms of mean square error than
Johansen’s (1988) ML estimators when the sample size is small even if the Gaussian
VAR structure assumed by Johansen is true.

The basic strategy of CCR is to estimate long-run covariance parameters and
to transform the regressand and the regressors in order to remove the endogeneity
problem while maintaining the cointegration relationship by exploiting the fact of
non-uniqueness of cointegrating regression. This newly transformed regression model
is called CCR and applying least squares to this CCR yields asymptotically efficient
estimators and chi-square tests. Consider a cointegrating system\(^{16}\)

\[
y_t = h'd_t + \beta'x_{2t} + \epsilon_t
\]  

(2.24)

\[
\Delta x_{2t} = v_t
\]  

(2.25)

where \(d_t\) is deterministic term, usually constant or constant and time trend, \(x_t = (d'_t, x'_{2t})'\), \(y_t\) and \(x_{2t}\) are difference stationary, and \(\epsilon_t\) and \(v_t\) are stationary with zero
mean. Here, \(y_t\) is a scalar and \(x_{2t}\) is a \((n - 1) \times 1\) random vector. The OLS estimator
in (2.24) is super-consistent in the sense that the OLS estimator converges to \(\beta\) at the
rate of \(T\) (sample size) even when \(\Delta x_{2t}\) and \(\epsilon_t\) are correlated. The OLS estimator,
however, is not asymptotically efficient.\(^{17}\) Let

\(^{16}\)For more detail explanation, refer to Ogaki, Jang and Lim (2003), “Structural Macroeconomet-
rics”\(^{17}\)Because the error term is correlated with the first difference of regressors at leads and lags as
well as contemporarily.
\[ w_t = (\epsilon_t, v_t')' \] (2.26)

Define \( \Phi(i) = E(w_t w_{t-i}') \), \( \Sigma = \Phi(0) \), \( \Gamma = \sum_{i=0}^{\infty} \Phi(i) \), and \( \Omega = \sum_{i=-\infty}^{\infty} \Phi(i) \). Here \( \Omega \) is the long run covariance matrix of \( w_t \). Partition \( \Omega \) as

\[
\Omega = \begin{bmatrix}
\Omega_{11} & \Omega_{12} \\
\Omega_{21} & \Omega_{22}
\end{bmatrix}
\]

where \( \Omega_{11} \) is a scalar, and \( \Omega_{22} \) is a \((n - 1) \times (n - 1)\) matrix, and partition \( \Gamma \) conformably. Define

\[
\Omega_{11,2} = \Omega_{11} - \Omega_{12} \Omega_{22}^{-1} \Omega_{21}
\] (2.27)

and \( \Gamma_2 = (\Gamma_{12}', \Gamma_{22}')' \). The CCR procedure assumes that \( \Omega_{22} \) is positive definite, implying that \( x_{2,t} \) is not itself cointegrated. This assumption assures that \((1, -\beta)\) is the unique cointegrating vector (up to a scale factor).

The idea of CCR is to choose \( \Pi_y \) and \( \Pi_x \) such that the OLS estimator is asymptotically efficient when \( y_t^* \) is regressed on \( x_t^* \), where

\[
y_t^* = y_t + \Pi_y' w_t
\] (2.28)
\[ x^{*}_{2t} = x_{2t} + \Pi'_x w_t \]  

(2.29)

and \( x^*_t = (d'_t, x^*_x)' \). Because \( w_t \) is stationary, \( y^*_t \) and \( x^*_x \) are cointegrated with the same cointegrating vector \((1, -\beta)\) like \( y_t \) and \( x_{2t} \) for any \( \Pi_y \) and \( \Pi_x \). This requires

\[ \Pi_y = \Sigma^{-1}\Gamma_2 + (0, \Omega_{12}\Omega_{22}^{-1})' \]  

(2.30)

\[ \Pi_x = \Sigma^{-1}\Gamma_2 \]  

(2.31)

In practice, long-run covariance parameters in these formulas are estimated, and estimated \( \hat{\Pi}_y \) and \( \hat{\Pi}_x \) are used to transform \( y_t \) and \( x_{2t} \). As long as these parameters are estimated consistently, the result of CCR estimator is asymptotically efficient. To obtain an estimate of the long-run covariance of the disturbances in the system, \( \Omega \), I follow Park and Ogaki’s (1991) method based on Andrews and Monahan’s (1992) prewhitened HAC estimator with the QS kernel. For prewhitening, VAR of order one is used. I bound the singular values of the VAR coefficient matrix by 0.99. Andrew’s (1991) automatic bandwidth estimator is constructed from fitting in AR(1) to each disturbance.\(^{18}\)

An important property of the CCR procedure is that linear restrictions can be tested by \( \chi^2 \) tests, which are free from nuisance parameters. For this, add the spurious deterministic trends to the regression of \( y^*_t \) on \( x^*_x \);

\(^{18}\)A CCR package written by Jang and Ogaki (2001) is employed to obtain CCR results.
\[ y_t^* = \sum_{\tau=0}^{p} \theta_{\tau} t^\tau + \sum_{\tau=p+1}^{q} \theta_{\tau} t^\tau + \beta' x_{2t}^* + \epsilon_t^* \] (2.32)

\(H(p, q)\) statistic is constructed by adjusting standard Wald statistic for the null hypothesis; \(\theta_{p+1} = \theta_{p+2} = \cdots = \theta_q = 0\) by using both the estimate of the long-run variance of \(\epsilon_t^*\), and \(\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{\epsilon}_t^2\), where \(\hat{\epsilon}_t^2\) is the estimated residual in regression (2.32). \(H(p, q)\) converges in distribution to a \(\chi^2(q - p)\) random variable under the null of cointegration, while it diverges under the alternative of no cointegration because spurious trends try to mimic the stochastic trend left in residual. The null of cointegration is rejected if the computed value of \(H(p, q)\) is greater than the critical value at given significance level. In particular, \(H(0, 1)\) test can be considered as a test for the deterministic cointegration restriction because the restriction implies that the cointegrating vector eliminates not only the stochastic trends but also the linear deterministic trends. In this case, \(H(1, q)\) statistic tests the null hypothesis of stochastic cointegration.
2.4 Empirical Results

2.4.1 Data and Unit root test

U.S. Treasury International Capital\(^{19}\) provides the monthly data of U.S. transactions with foreigners in long-term\(^{20}\) U.S. and foreign securities. Further, it has surveyed the foreign holdings of U.S. long-term securities at the end of 1974, 1978, 1984, 1989, 1994, and 2001. Let U.S. be a foreign country. To obtain stock of foreign bonds data \((B_{F,t}^d)\) at the end of each month, I accumulatively add the net purchase of U.S. debt securities\(^{21}\) by each country from U.S. residents to the 1989 survey data. So, estimation period is from January 1990 through December 2003. A three-month U.S. T-bill is used as a foreign short-term interest rates \((r^*_t)\). Since a realized exchange rate depreciation is equalized to the expected depreciation on the average under rational expectation assumptions, I use a realized exchange rate depreciation defined by the difference between the natural log of the exchange rate at time \(t + 1\) and the natural log of the exchange rate at time \(t\) to get return for holding the foreign bonds \((r^*_t + E_t(\triangle s_{t+1}))\). All variables, including domestic short-term interest rates \((r_t)\) and long-term interest rates \((R_t)\), are converted to real variables by using the Consumer Price Index.

To investigate whether data contain a unit root, I use Park’s (1990) \(J(p, q)\) test and Said and Dickey’s (1984) test \((SD)\). \(J(p, q)\) test utilizes spurious regression result. The main assumption is that a variable possesses the deterministic time polynomials up to the order \(p\) (it is typically zero or one) and the additional time polynomials are

\(^{19}\)Homepage is http://www.ustreas.gov/tic

\(^{20}\)An original maturity of more than one year

\(^{21}\)It consists of marketable U.S. treasury and federal financing bank bonds and notes, bonds of U.S. gov’t corps. and federally sponsored agencies, and U.S. corporate and other bonds
spurious time trends. Under the null of difference stationarity, $J(p,q)$ has an asymptotic distribution. The null hypothesis of the difference stationarity is rejected against the alternative of trend stationarity when $J(p,q)$ is smaller because it converges to zero under the alternative hypothesis of trend stationarity. $J(p,q)$ test has several advantages over other tests in the sense that neither the order of autoregressive nor the bandwidth parameter for estimating the long-run variance needs to be chosen.\textsuperscript{22} $SD$ test is the extended Dickey-Fuller’s $t$-ratio test to the case where the order of autoregressive is unknown. Its result is very sensitive to the choice of the order of the autoregressive. Campbell and Perron (1991) recommend to start with a reasonably large value of $p$ chosen a priori and decrease $p$ until the coefficient on the last included lag is significant. When $SD$ test statistic is negative and greater than the appropriate critical value in absolute value, then the null of a unit root is rejected in favor of the alternative.

The foreign bonds ($B_{F,t}^d$) exhibits a consistent tendency to grow over time whereas other data seem to have unknown mean. To reflect these behavior, for the foreign bonds ($B_{F,t}^d$), $SD$ test includes both a constant and a linear time trend in the regression and $J(1,5)$ test is employed. For other data, $SD$ test includes a constant in the regression and $J(0,3)$ test is investigated. (Table 2.1) says that at least one of them can not reject the null hypothesis of difference stationarity. Since unit root test results appear to indicate that each variable possesses unit root, I perform cointegrating regression under the maintained assumption that all of the variables are integrated of order one.

\textsuperscript{22}Park and Choi’s (1988) Monte Carlo experiments show that J test has little size distortion compared with Phillips-Perron test and is not dominated by the Said-Dickey test or the Phillips-Perron test in terms of size adjusted power in small samples.
<table>
<thead>
<tr>
<th></th>
<th>Canada(^4)</th>
<th>Germany(^5)</th>
<th>Japan(^6)</th>
<th>Korea(^7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B^d_{F,t})</td>
<td>(J(1, 5)) 2.0952(***)</td>
<td>6.4678(***)</td>
<td>2.4989(***)</td>
<td>2.2853(***)</td>
</tr>
<tr>
<td>SD((p))</td>
<td>SD(12) (-2.4363)(***)</td>
<td>SD(1) (-1.7365)(***)</td>
<td>SD(1) (-1.9133)(***)</td>
<td>SD(15) (-1.0276)(***)</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r_t)</td>
<td>(J(0, 3)) 0.1453(***)</td>
<td>0.0166</td>
<td>0.0642</td>
<td>0.0916</td>
</tr>
<tr>
<td>SD((p))</td>
<td>SD(13) (-1.3335)(***)</td>
<td>SD(11) (-2.0016)(***)</td>
<td>SD(11) (-2.3526)(***)</td>
<td>SD(11) (-1.5948)(***)</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_t)</td>
<td>(J(0, 3)) 0.1380(***)</td>
<td>0.0124</td>
<td>0.0210</td>
<td>0.0765</td>
</tr>
<tr>
<td>SD((p))</td>
<td>SD(13) (-1.6219)(***)</td>
<td>SD(11) (-2.7492)(***)</td>
<td>SD(11) (-2.0602)(***)</td>
<td>SD(11) (-1.6338)(***)</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r_t^* + E_t(\Delta s_{t+1}))</td>
<td>(J(0, 3)) 0.0127</td>
<td>0.1135(***)</td>
<td>0.0324</td>
<td>0.0365</td>
</tr>
<tr>
<td>SD((p))</td>
<td>SD(12) (-3.3919)(***)</td>
<td>SD(12) (-1.3083)(***)</td>
<td>SD(20) (-2.5813)(***)</td>
<td>SD(1) (-7.9639)</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: 1) Significance at 10%, 5%, and 1% is denoted by *, **, and ****, respectively.
2) \(J(1, 5)\) test \([J(0, 3)\) test]: critical value for 1 % is 0.1228 [0.1118], and 5 % is 0.2950 [0.3385], and 10 % is 0.4520 [0.5773].
3) The order of AR, \(p\), in \(SD\) test is chosen following Campbell and Perron’s (1991) recommendation. Critical values for \(B^d_{F,t}\) are -3.96 (1 %), -3.41 (5 %), and -3.12 (10 %). Those for other variables are -3.43 (1 %), -2.86 (5 %), and -2.57 (10 %).
4) (1990:01 - 2003:12), \(r_t\) (Treasury Bill, 3 month), \(R_t\) (government bond, 10 year more)
5) (1996:01 - 2003:12), \(r_t\) (FIBOR 3 month), \(R_t\) (government bond, 10 year)
6) (1990:09 - 2003:12), \(r_t\) (Gensaki bond reference, 3 month), \(R_t\) (government bond, 10 year)
7) (1990:01 - 2003:12), \(r_t\) (monetary stabilization bond, 1 year), \(R_t\) (industry finance bond, 3 year)

Table 2.1: Tests for the Null of the difference stationary
2.4.2 Canonical Cointegrating Regression Results

As CCR assumes that regressors are not itself cointegrated, I employ Engle and Granger’s (1987) Augmented Dickey-Fuller (ADF) test to see whether explanatory variables are cointegrated. It is a residual based test applying SD test to the residual of an OLS cointegrating regression. It tests the null of no cointegration against the alternative of cointegration. The asymptotic distribution of ADF test generally depend on the number of the variable in the cointegrating regression.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>Germany</th>
<th>Japan</th>
<th>Korea</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF(p)3</td>
<td>ADF(11)</td>
<td>ADF(5)</td>
<td>ADF(8)</td>
<td>ADF(4)</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(-3.0948)***</td>
<td>(-2.5947)***</td>
<td>(-3.7510)***</td>
<td>(-4.2844)***</td>
</tr>
</tbody>
</table>

Note: 1) A cointegrating regression is the form of \( R_t = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^* + E_t(\triangle s_{t+1}) + \xi_t. \)
2) The number in parentheses of ADF(p) test is the order of AR, \( p \), chosen from Campbell and Perron (1991) suggestion. Critical values are -4.3078 (1 %), -3.7675 (5 %), and -3.4494 (10 %).
3) Significance at 10%, 5%, and 1% is denoted by *, **, and ****, respectively.

Table 2.2: Tests for No cointegration among regressors

Based on the time series behavior of explanatory variables, only constant is included in regression for ADF test. The null hypothesis of no cointegration is rejected against the alternative of cointegration when ADF test statistic is smaller than the appropriate critical value (Phillips and Ouliaris (1990)). Results are reported in (Table 2.2). They do not reject the null of no cointegration among regressors at a significance level of one percent.
Since there is no cointegration among regressors, I run the Canonical Cointegrating Regression in the form of (2.23) to estimate the demand for foreign bonds. The results are reported in (Table 2.3). Both $\hat{\beta}_2$ and $\hat{\beta}_3$ show theoretically correct sign and they are statistically significant at five percent significance level. A positive estimate of the short-term interest rate, which is consistent with the theoretical prediction of Lim and Ogaki (2003), implies that the indirect risk premium is quantitatively important to explain the demand for foreign bonds. Further, $H(p, q)$ tests do not reject the null hypothesis of cointegration.
\[
\begin{array}{cccccccc}
\text{constant} & \text{time} & r_t & R_t & r_t^* + \Delta s_{t+1} & H(2, 3) & H(2, 4) \\
\hline
\text{Germany}^{2)} & 0.7660 & 0.0018 & 0.0765 & -0.0910 & 0.0022 & 1.7666 & 3.2462 \\
& (0.0975) & (0.0012) & (0.0305) & (0.0328) & (0.0010) & (0.1838) & (0.1973) \\
\text{Korea}^{3)} & 0.6205 & -0.0008 & 0.1811 & -0.2409 & 0.0230 & 0.0001 & 0.5907 \\
& (0.0343) & (0.0002) & (0.0146) & (0.0169) & (0.0021) & (0.9911) & (0.7443) \\
\text{Canada}^{4)} & 0.1283 & 0.0028 & 0.0150 & -0.0138 & -0.0010 & 2.4667 & 2.5396 \\
& (0.0186) & (0.0002) & (0.0045) & (0.0045) & (0.0003) & (0.1163) & (0.2809) \\
\text{Japan}^{5)} & 0.8953 & 0.0138 & 0.0863 & -0.1014 & 0.0179 & 1.0087 & 15.4226 \\
& (0.0391) & (0.0004) & (0.0185) & (0.0191) & (0.0023) & (0.3152) & (0.0004) \\
\end{array}
\]

Note: 1) Numbers in parentheses are either standard errors for estimates or p values for \(H(p,q)\) statistics 
2) Estimation period (1996:01 - 2003:12) 
   \(r_t\) (Germany FIBOR 3 month), \(R_t\) (Germany government bond, 10 year) 
3) Estimation period (1990:01 - 2003:12) 
   \(r_t\) (Korean monetary stabilization bond, 1 year), \(R_t\) (Korea industry finance bond, 3 year) 
4) Estimation period (1990:01 - 2003:12) 
   \(r_t\) (Canada Treasury Bill, 3 month), \(R_t\) (Canada government bond, 10 year more) 
   \(r_t\) (Japan Gensaki bond reference, 3 month), \(R_t\) (Japan government bond, 10 year) 

Table 2.3: CCR results for the demand for foreign bonds
2.5 Concluding Remarks

This paper directly estimates the demand for foreign bonds by employing a cointegration technique to investigate whether data is consistent with model predictions. An implication of this theoretical model is that under reasonable parameter configurations, such as a low degree of risk aversion, the demand increases as the domestic short-term interest rates rise, resulting in the expected appreciation of the domestic currency instead of the expected depreciation. This prediction, a potential explanation for the forward premium anomaly in the international financial market, is supported by empirical results.

A positive coefficient for the short-term interest rates and a negative coefficient for long-term interest rates are estimated, respectively. These empirical results are consistent with Lim and Ogaki’s (2003) predictions. These results imply that the forward premium anomaly for short-term interest rates, one of central puzzles in international financial markets, can be explained if the indirect risk premium effect dominates the direct risk premium effect. The empirical results of this paper are in line with other empirical papers, which directly investigate the relationship between the exchange rates and the term structure of interest rates instead of estimating demand function. (Byeon and Ogaki, 1999; Ogaki and Santaella, 2000)

A theoretical implication of Ogaki’s model is that the relationship between the term structure of interest rates and exchange rates is much more complicated than the prediction of a standard exchange rate model, which is simply based on gross substitutability assumption. Empirical results for this paper suggest that the effectiveness of central bank attempts to affect exchange rate through the control of short-term interest rates depends on the responsiveness of long-term interest rates to changes in
short-term interest rates. Further it shows that long-term interest rate differential is a more important factor than short-term interest rate differential in determining the exchange rates.
REFERENCES


Ogaki, M., K. Jang, and H. Lim (2003), *Structural Macroeconometrics*, The Ohio State University.


CHAPTER 3

ESTIMATING THE TERM STRUCTURE OF INTEREST RATES FROM KOREAN DATA

This chapter estimates the term structure of interest rates from Korean data by employing a parsimonious functional form suggested by Nelson and Siegel (1987). The parameters can be estimated fairly precisely by using nonlinear least square estimation and the employed functional form accounts for a large fraction of the variation in government bonds. The market expectations about the future interest rates paths are also extracted from a given current term structure in order to provide useful information for forward looking monetary policy. This task is especially important for The Bank of Korea that has adopted an Inflation Targeting framework since 1998. This paper is expected to give a practical contribution to The Bank of Korea by providing market expectations about future interest rates.
3.1 Introduction

The term structure of interest rates is the relationship between the interest rates on bonds with different maturities. It has long been used as an important tool to price financial instruments and to analyze monetary policy. In particular, as to the latter purpose, Goodfriend (1998) points out that the long bond rate contains a premium for expected inflation and serves as an indicator of the credibility of a central bank’s commitment to low inflation.

The need to use the term structure of interest rates as an information variable in monetary policy has recently increased in Korea. One factor is the substantially increased issue of Treasury bonds to finance the fiscal deficit resulting from supporting the restructuring of financial institutions since 1997 currency crisis. The outstanding amount of Treasury bonds at the end of 1995 was only 3 trillion won but it has reached 46 trillion won at the end of June of 2001. And the trading share of government bond including Treasury bonds in secondary market has swollen to 33.8 per cent in the first half of 2001 from 6.3 per cent in 1997.23 As Treasury bonds serve as benchmarks in bond market, their prices have reflected market participants’ expectations about the future. For example, government bonds provide the implied forward interest rates, an useful information of the market’s expectations about the future path of interest rates. (see, Anderson and Sleath (1999)).

The other institutional factor in Korea is to adopt an inflation targeting framework since 1998. The revised Bank of Korea Act explicitly states that price stability is the primary goal of monetary policy in Korea. The Bank of Korea publicly announces

23The Bank of Korea (2002)
the inflation target and elaborates a monetary plan to achieve it. Kahn and Parrish (1998) emphasize that inflation targeting requires a highly forward looking monetary policy. Considering the lag of effect of monetary policy on inflation, central banks need to forecast inflation and adjust policy in response to projected deviations of inflation from target. In this sense, Amato and Laubach (2000) stress the role of forecasts in preemptive monetary policy. Extracting market expectations about the future has become more important than ever in Korea to conduct monetary policy under inflation targeting framework.


The purpose of the paper is to estimate Korean term structure of interest rates from daily available Treasury bonds data to provide market expectations about the future path of interest rates. I employ statistical techniques by fitting data to describe

24 For evaluating the earlier performance of inflation targeting framework in Korea, see Hoffmaister (1999).

25 For more explanations about equilibrium term structure models, see Anderson, Breedon, Deacon, Derry, and Murphy (1996)
the term structure of interest rates without investigating on the factors deriving it. In particular, I follow McCulloch (1971, 1975) in estimating a discount function by fitting model prices to observed bond prices. Among several approximating functional forms\(^{26}\) which have been suggested, I employ Nelson and Siegel’s (1987) parsimonious functional form. Unlike Yum \textit{et. all} (1999) who use Industry Finance Bonds, this paper focus on estimating Treasury Bonds’ term structure. Oh \textit{et. all} (2002) also estimate Treasury Bonds’ term structure, but unlike them employing equilibrium term structure models, I adopt statistical approach to estimate it.

One practical reason for choosing Nelson and Siegel’s (1987) functional form is that the number of parameters to be estimated is relatively small compared to other functional forms. It has only 4 parameters to be estimated.\(^{27}\) Nelson and Siegel (1987), however, show that their functional form is flexible enough to represent the range of shapes generally associated with the term structure of interest rates; monotonic, humped, and S shaped. This flexibility comes from the existence of three components in function; short term, medium term, and long term. With appropriate choices of weights for these components, it can generate a various shape of the term structure of interest rates.

Another advantage is that, as Juha and Viertio (1996) point out, there is a close correspondence between the three components and the finding of Litterman and Scheinkman (1991), who find using factor analytic approach that three factors,

\(^{26}\)They include polynomials (Chambers, Carleton, and Waldman (1984)), cubic splines (McCulloch (1975), Litzennberger and Rolfo (1984), Fisher, Nychka, and Zervos (1995), and Waggoner (1997)), step functions (Ronn (1987), Coleman, Fisher, and Ibbonston (1992)), piecewise linear (Fama and Bliss (1987)), and exponential forms (Nelson and Siegel (1987)).

\(^{27}\)According to Bliss (1997), the average numbers of parameters to be estimated in his empirical results are, for example, 11 for cubic spline function and 98 for piecewise function.
called as level, slope, and curvature, explain most of the observed variation in bond returns. Diebold and Li (2002) explicitly show that the three time-varying components of Nelson and Siegel’s (1987) function may be interpreted as factors corresponding to level, slope, and curvature, respectively.

There is an argument that the parsimonious functional form fits data less accurately than other models such as cubic splines. However, as Sevensson (1994) points out, somewhat less precision is acceptable to gain a glimpse of expectations, as far as monetary policy is concerned unlike financial analysis such as pricing securities. Due to this smoothness, most central banks have adopted either the Nelson and Siegel’s (1987) function or the extended version suggested by Svensson (1994), with the exceptions of US, Japan and United Kingdom which apply the smoothing splines approach. (see, BIS (1999).) Further, Bliss (1997) tests different estimation methodologies from the US Treasury bills’ pricing point of view. He defines fitted price errors both inside and outside the issues’ bid-ask range and compares them for each functional form. His results show that the performance of Nelson and Seigel’s (1987) function does not fall short of that of others.

I employ data filtering criteria based on market convention to select only bonds that are indicative of the current market yields by excluding bonds suspected to create distortions in the estimation of the yield curve. Only bonds whose traded volume is more than 10 billions won are included to avoid the liquidity problem. Further, I include bonds with more than one year remaining to maturity to capture only reliable indicator of market expectations. The empirical results show that the parameters can be estimated fairly precisely by using nonlinear least square estimation and the employed functional form accounts for a large fraction of the variations in
government bonds. I also extract the market expectations about the future interest rates paths from a given current term structure to provide useful information for forward looking monetary policy. This task is especially important for The Bank of Korea that has adopted Inflation Targeting framework since 1998. This paper is expected to give a practical contribution to The Bank of Korea by providing market expectations about future interest rates.

The rest of the paper is organized as follows. Section 2 provides a theoretical background to use Nelson and Siegel’s (1987) functional form. Section 3 explains an empirical procedures and results. Final conclusion is given in Section 4.
3.2 Theoretical Background

Estimating the term structure of interest rates is based on the Law of One Price\textsuperscript{28}, saying that investors cannot make instantaneous profits by repackaging portfolios. In complete market, the absence of arbitrage opportunities is equivalent to the existence of a positive linear pricing rule, \(d(t, m) \forall m\), such that

\[
p_t = \sum_{m=1}^{M} c_{t,m} \cdot d(t, m) \tag{3.1}
\]

where \(p_t\) is the price of bond at time \(t\) and \(c_{t,m}\) is a cash flow paid at time \(t + m\). In the term structure literature, \(d(t, m)\) is called the discount function describing the present value of one unit payable at time \(t + m\) in the future.\textsuperscript{29} However, incomplete financial market does not allow equation (3.1) to hold exactly for any reasonable \(d(t, m)\).\textsuperscript{30} This forces us to use an approximated relation such as

\[
p_t = g(c_{t,m}, d(t, m)) + \epsilon \tag{3.2}
\]

\textsuperscript{28}The Law of One Price is meant to describe a market that has already reached equilibrium. If there are any violations of it, investors will quickly exploit them for them not to survive in equilibrium. See Cochrane (2001).

\textsuperscript{29}Anderson et. all (1996) explain that \(d(t, m)\) is sometimes referred to as a zero-coupon bond price. If an instrument exists providing a single, unit cash flow \(t + m\) years into the future, its price should correspond to the value of the discount function at that point, \(d(t, m)\). Such an instrument would be a zero-coupon bond, paying no coupon payments and a unit redemption payment on the maturity date. If we let \(s(t, m)\) be the continuously compounded interest rate for a single guaranteed nominal payment due on a future year \(t + m\), it is called either ‘spot rate’ because it is the interest rate applicable today on a \(t + m\) year loan or ‘zero-coupon yield’ since it represents the yield to maturity on a zero coupon bond. Then, the relationship between them is \(s(t, m) = \frac{-\log(d(t, m))}{m-t}\). See Shiller (1990).

\textsuperscript{30}Further, if markets are incomplete there exist multiple sets of \(d(t, m)\) satisfying equation (3.1)
where $g(\cdot)$ describes how bonds are priced. After employing a specific functional form of $d(t, m)$, fitting a data by minimizing some function of $\epsilon$, which in turn should be random\(^{31}\), is a strategy to estimate the term structure of interest rates. (see, Bliss (1997).) In this framework, the discount function, $d(t, m)$, should satisfy at least 3 conditions to eliminate arbitrage profits;

$$0 < d(t, t + m) \leq 1 \quad (3.3)$$
$$d(t, t) = 1 \quad (3.4)$$
$$\frac{\partial d(t, t + m)}{\partial m} \leq 0 \quad (3.5)$$

Nelson and Siegel (1987) obtain a specific functional form of $d(t, m)^{32}$ by explicitly assuming that the instantaneous forward rate at time $t$ with settlement date $m$, denoted $f(t, m)^{33}$, is the solution to a second-order differential equation with equal roots;

$$f(t, m) = \beta_1 + \beta_2 \cdot \exp\left(-\frac{m}{\tau}\right) + \beta_3 \cdot \frac{m}{\tau} \cdot \exp\left(-\frac{m}{\tau}\right) \quad (3.6)$$

\(^{31}\)Predictable error suggest that there is additional available information that could be included in $g(\cdot)$.

\(^{32}\)As explained later, the discount function, the instantaneous forward rate, the spot rate are closely related. Once the instantaneous forward rate function is defined, others are merely transformation of it.

\(^{33}\) $f(t, m)$ is defined as $\lim_{T \to \infty} f(t, m, T)$, where $f(t, m, T)$ is the $m$ period ahead forward rates with maturity $(T - m)$ period, i.e., $f(t, m, T)$ is an interest rate determined now ($t$) for an investment beginning in the future ($m$) and ending further in the future ($T$). It is the rate of interest from period in the future to another period in the future. Therefore, $f(t, m, T) = \frac{1}{T - m} \int_{h=m}^{T} f(t, h) \, dh$. Further, $f(t, m)$ is a marginal increase in the total return from a marginal increase in the length of the investment, i.e., $f(t, m) \equiv \frac{\partial d(t, m)}{\partial m}$. For more details about these relationships, see Shiller (1990).
where $\beta_1, \beta_2, \beta_3$ and $\tau$ are parameters to be estimated.\textsuperscript{34} The impact of these parameters on the shape of the instantaneous forward rates curve can be described as follows. (see, Bolder and Streliski (1999) and Diebold and Li (2002).)

- $\beta_1$: For long maturities, $f(t, m)$ approaches asymptotically the value $\beta_1$ which must be positive. As it is a coefficient of constant that does not decay to zero in the limit, $\beta_1$ may be viewed as a long-term factor governing the level of the term structure of interest rates. Sevensson (1994) favors this characteristic since it seems reasonable to restrict forward rates for settlement very far into the future to constant. It seems unlikely that market agents have information that allows them to have different expectations for, say, 25 or 30 years into the future.

- $\beta_2$: It represents the deviation from the asymptote, $\beta_1$, as $(\beta_1 + \beta_2)$ determines the starting value of the term structure at maturity zero. Here, $(\beta_1 + \beta_2)$ must also be positive. $\beta_2$ is a coefficient of a term decaying monotonically and quickly to zero; hence it can be viewed as a short-term factor, which is closely related to the slope of the term structure. The yield curve will have a positive (negative) slope if $\beta_2$ is negative (positive).

- $\beta_3$: This determines the magnitude and direction of the shape. Whereas the sign of $\beta_3$ determines whether it is hump (positive) or U shape (negative), the absolute size of $\beta_3$ governs the magnitude of the shape. It is a coefficient of a term starting at zero, increasing, and then decaying to zero; hence $\beta_3$ may be viewed as a medium-term factor, which is closely related with the term structure’s curvature.

\textsuperscript{34}$\beta_1, \beta_2$ and $\beta_3$ are coefficients determined by initial conditions of differential equation. And $\tau$ is time constant related with differential equation.
• \( \tau \): It determines the position of either hump or U shape and it should be positive to prevent \( \beta_1 \), the level of term structure, to be infinite. Small value of \( \tau \) allows the term structure to reflect short end yield curve relatively well compared to long end yield curve.

Having specified a functional form for the instantaneous forward rate, a spot interest rates function is derived. Integrating equation (3.6) produces spot interest rate\(^{35}\), \( s(t, m) \);

\[
\begin{align*}
    s(t, m) &= \beta_1 + \beta_2 \cdot \frac{\tau}{m} \{1 - \exp(-\frac{m}{\tau})\} \\
    &+ \beta_3 \cdot \{\frac{\tau}{m} \cdot (1 - \exp(-\frac{m}{\tau})) - \exp(-\frac{m}{\tau})\}
\end{align*}
\] (3.7)

The spot rate \( s(t, m) \) is sometimes called the zero coupon yield since it represents the yield to maturity on a pure discount or zero coupon bond. A spot yield curve is a plot of interest rate \( s(t, m) \) as a function of different maturity dates \( m \), for a given trade date \( t \). By the term structure of interest rates in this paper, I mean the spot yield curve. Usual convention to represent the term structure of interest rates is to derive the yield curves showing the yield to maturity on coupon bonds for different maturities. The yield to maturity is the single discount rate on an investment that makes the sum of the present value of all cash flows equal to the current price of the investment. However, Svensson (1994) points out that this convention is inappropriate\(^{35}\),

\(^{35}\)The reason why \( s(t, m) \) is called ‘spot’ rate is that it is the interest rate that is applicable today \( (t) \) on \( m \) year investment unlike \( f(t, m) \) which is applicable \( m \) year later.
to describe the term structure of interest rates due to reinvestment risk and coupon
effect.\textsuperscript{36}

Since the spot rate at maturity \( m \) is the average of the instantaneous forward
rates\textsuperscript{37}, they are related in the same manner as marginal and average cost of pro-
duction are related such that quantity produced corresponds to time to maturity.
This relationship is a key element in using the yield curve as a indicator for market
expectation’s future interest rates path. As long as the current term structure of spot
interest rates is obtained, we can extract the implied forward rate curve describing
the marginal one period interest rates. This produces implied expected shape of yield
curve in one period’s future time given a current term structure.

However, we cannot directly infer spot rate from the prices of coupon bonds. It
should be estimated from yields on coupon bonds. The basic idea is simply treat each
future cash flows of coupon bonds as independent zero coupon bond paying the same
amount of cash flows. The Law of One Price says that the price of coupon bonds
should be the same as that of this replicating portfolio.

From this spot interest rates, the discount function employed by Nelson and Siegel
(1987) is straightforwardly obtained\textsuperscript{38};

\[
d(t, m) = \exp(-\beta_1 \cdot m - \beta_2 \cdot \tau (1 - \exp(-\frac{m}{\tau}))) \\
\quad - \beta_3 \{ \tau \cdot (1 - \exp(-\frac{m}{\tau})) + m \cdot \exp(-\frac{m}{\tau}) \}
\]

\textsuperscript{36} A par yield curve is an alternative way to represent the term structure of interest rates. A
coupon-paying bond is said to be priced at par if its current market price is its face value. The par
yield curve describes the coupon required on a coupon paying bond with time to maturity for that
bond to trade at par.

\textsuperscript{37} Note that \( s(t, m) = \frac{1}{m} \int_{h=0}^{m} f(t, t + h) \, dh \)

\textsuperscript{38} Note that \( d(t, m) = \exp(\frac{-s(t, m)}{100} \cdot (m - t)) \)
This discount function is used to determine the price of a set of bonds as the present value of a cash flow is calculated by taking the product of this cash flow and its corresponding discount factor as explained next section. It is estimated for each trade date by minimizing price errors in equation (3.2).
3.3 Empirical Results

For coupon bearing bond with maturity of $M$ years, such as Treasury Bonds in Korea, its price\textsuperscript{39}, $P^*$, can be approximated by the sum of the discounted future cash flows\textsuperscript{40};

$$
P^*(\beta, \tau) = \sum_{k=1}^{h \cdot M} c_k \cdot d(l + \frac{1}{h} \cdot (k - 1); \beta, \tau) + fv \cdot d(M; \beta, \tau),
$$

where

- $M$: number of years to maturity
- $h$: number of coupon payments a year
- $k$: sequence of coupon payments
- $c_k$: $k$th coupon payment
- $d(\cdot; \beta, \tau)$ is the discount function in equation (3.8)
- $l$: the number of years from trading date to first coupon payment
- $h \cdot M = h \cdot M$, if it is an integer.
  $$= (\text{integral part of } h \cdot M) + 1, \text{ otherwise}$$
- $fv$: face value

\textsuperscript{39}This price is a dirty price, i.e., it includes the payment of accrued interest to compensate the seller for the period since the last coupon payment during which the seller has held the bond but for which they will receive no coupon payment.

\textsuperscript{40}For simplicity, time index is dropped below.
The model prices of Treasury Bonds, $P^*(\beta, \tau)$, are compared with the observed prices of Treasury Bonds, $p$. It is assumed that the observed prices differ from the model prices by an error term;

$$p = P^*(\beta, \tau) + \epsilon \quad (3.10)$$

By minimizing price error in equation (3.10), we obtain parameters $\beta_1, \beta_2, \beta_3, \tau$. BIS (1999) points out that using bond prices in the estimation irrespective to their durations will lead to over-fitting of the long-term bond prices at the expense of the short-term prices. I weight the price error of each bond by the value related to the inverse of its duration to correct this problem. Then, the objective function to be minimized is;

$$min_{\{\beta_1, \beta_2, \beta_3, \tau\}} \sum_{j=1}^{n} \left[ \{p_j - P^*_j(\beta, \tau)\} \cdot \Phi_j \right]^2 \quad (3.11)$$

where $p_j$ is the observed price of bond $j$, $n$ is the daily total number of bonds traded in the market and $\Phi_j = \frac{1/w_j}{\sum_{j=1}^{n} 1/w_j}$, where $w_j$ is the duration of the bond $j$. Nonlinear least squares method is employed to estimate these parameters by minimizing the sum of squared weighted price errors in equation (3.11). Once the estimated parameters are obtained, the implied forward rate and the spot rate curve are computed by substituting these parameters into equation (3.6) and equation (3.7), respectively.
The data consist of daily data of September 2003, 18 trade dates altogether. They are collected from the data base of Korean Bonds Evaluating Company. The data include Treasury Bonds (TB) issued by Government and Monetary Stabilization Bonds (MSB) issued by The Bank of Korea. Issuance involves maturities across the yield curve with original terms of maturity at issuance of 3, 5, and 10 years for Treasury Bonds, and 0.25, 0.5, 0.75, 1.0, 1.6, and 2 years for Monetary Stabilization Bonds. Monetary Stabilization Bonds are included to capture short-end yield curve behavior. All Treasury Bonds and Monetary Stabilization Bonds with two year maturities are coupon bonds paying a fixed quarterly-annual interest rate. Other Monetary Stabilization Bonds are issued at a discount and mature at their par value without paying periodic interest rates. As (Table 3.1) shows, available coupon bonds and discounts bonds on each trade date, on the average, are 15 and 1, respectively. And there are 94 transactions for coupon bonds and 1 for discount bonds on the average. Most actively traded bonds in the market are on-the-run issues mainly concentrated on recently issued 3, and 5 years Treasury Bonds.

<table>
<thead>
<tr>
<th></th>
<th>the number of bonds</th>
<th>the number of transactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>coupon bonds</td>
<td>15</td>
<td>94</td>
</tr>
<tr>
<td>discount bonds</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: 1) 18 trade dates (September 2003)

Table 3.1: Daily average of data

41For Treasury Bonds issued after March 5 2003, interest rates are paid semi-annually basis.
Data filtering criteria based on market convention are employed to select only bonds that are indicative of the current market yields by excluding bonds suspected to create distortions in the estimation of the yield curve. First is the traded volume criterion. Only bonds whose traded volume is more than 10 billions won are included to avoid the liquidity problem. Second is the maturity spectrum used, i.e., how many year remaining to maturity should be considered to be reliable indicator of market expectations. I include only bonds with more than one year remaining to maturity.

The basic process of determining the optimal parameters for the discount function that best fit the bond data is outlined as follows. (see, Bolder and Streliski (1999).)

- First, a vector of starting parameters \((\beta_1, \beta_2, \beta_3, \tau)\) is selected.
- Second, the discount function derived from Nelson and Siegel (1987), Equation (3.8), is determined using these starting parameters.
- Third, this discount function is used to determine the present value of the bond cash flows and thereby to determine a vector of theoretical bond prices by using Equation (3.9).
- Fourth, weighted price errors, Equation (3.11), are calculated by taking the weighted difference between the theoretical and observed prices.
- Fifth, nonlinear least squares procedure is used to minimize the decision variable subject to certain constraints on the parameter values. To speed up the algorithm and avoid unreasonable local optima, which are not economically feasible in the Korean yield curve environment, a constrained nonlinear optimization is applied. \(\tau\) is constrained to the range of available bond maturities, \(0 < \tau < 10\).
$\beta$'s are restricted to values that provide reasonable shapes for the resulting spot rate and forward rate, \(0 < \beta_1 < 15, \quad -10 < \beta_2 < 10, \quad -15 < \beta_3 < 15\).

- Sixth, above procedures are repeated until the objective function is minimized.

<table>
<thead>
<tr>
<th></th>
<th>Sep. 1</th>
<th>Sep. 8</th>
<th>Sep. 15</th>
<th>Sep. 22</th>
<th>Sep. 29</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$ (%/yr)</td>
<td>5.5974</td>
<td>7.8765</td>
<td>4.9787</td>
<td>5.3508</td>
<td>6.9921</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\beta_2$ (%/yr)</td>
<td>-1.2653</td>
<td>0.5384</td>
<td>0.3416</td>
<td>-0.7030</td>
<td>-2.4078</td>
</tr>
<tr>
<td></td>
<td>(0.0447)</td>
<td>(0.0781)</td>
<td>(0.0334)</td>
<td>(0.0213)</td>
<td>(0.1249)</td>
</tr>
<tr>
<td>$\beta_3$ (%/yr)</td>
<td>-1.4754</td>
<td>-13.0645</td>
<td>-2.7757</td>
<td>-2.7867</td>
<td>-4.9835</td>
</tr>
<tr>
<td></td>
<td>(0.0448)</td>
<td>(0.0782)</td>
<td>(0.0335)</td>
<td>(0.0214)</td>
<td>(0.1250)</td>
</tr>
<tr>
<td>$\tau$ (yrs)</td>
<td>1.7085</td>
<td>1.1854</td>
<td>1.0184</td>
<td>1.9896</td>
<td>2.7369</td>
</tr>
<tr>
<td></td>
<td>(0.0121)</td>
<td>(0.0074)</td>
<td>(0.0135)</td>
<td>(0.0094)</td>
<td>(0.0300)</td>
</tr>
<tr>
<td>Measures of fit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RMSE(%)^2$</td>
<td>0.1026</td>
<td>0.5475</td>
<td>0.1619</td>
<td>0.1731</td>
<td>0.1285</td>
</tr>
<tr>
<td>$MAE(%)^3$</td>
<td>0.0832</td>
<td>0.3550</td>
<td>0.1298</td>
<td>0.1378</td>
<td>0.1117</td>
</tr>
<tr>
<td>$R^2$  $^4$</td>
<td>0.9981</td>
<td>0.9024</td>
<td>0.9950</td>
<td>0.9945</td>
<td>0.9964</td>
</tr>
</tbody>
</table>

Note: 1) Numbers in parentheses are heteroskedasticity-consistent standard errors for the parameters.
2) The Root Mean Squared error in percent of the principal.
3) The Mean Absolute Error in percent of the principal.
4) R-squared comes from the regression of actual price on the fitted price.

Table 3.2: Estimation results, September 2003
(Table 3.2) reports the parameter estimates and measures of fit for five Mondays on September 2003. The parameters are estimated fairly precisely. The model accounts for a large fraction of the variations in government bonds; R-squared is around 0.99 and the root mean squared error remains between 10-17 basis points (hundredths of a percentage point) except Sep. 8. Figure (3.1)-(3.5) shows estimated spot yield curves derived from these estimated parameters. Most ones show inverted yield curves sloping downward and being inverted except Sep. 1 showing a upward-sloping yield curve. The shape of the yield curve depends on market expectations, required bond risk premiums, and the convexity bias. Investigating which factor is main force to determine each shape is beyond this paper. It is a future research topic after more attentions are given to the institutional knowledge of Korean bond market.

As long as a current term structure is known, it provides a valuable information for the Central Banks to conduct monetary policy. We can extract market expectations about the future path of interest rates from the yield curve. For example, a given term structure of Sep. 15, Figure (3.6)-(3.11) shows 3, 6, and 9 months, 1, 2, and 3 years future yield curves, respectively. As the implied forward rates over 12 months rise in a given Sep. 15 yield curve, the extracted future yield curve over 12 months show a upward-sloping shape. This information can be exploited by the forward looking monetary policy.

42Sep. 8 was two days before Thanksgiving Holidays in Korea. I also estimate Sep. 9, a day before holidays, to compare two yield curves and obtain a similar one. These results suggest that seasonal factor makes noises in pricing bond.

43For more details about the relationship between these factors, see Ilmanen (1996).

44See Appendix for detail explanations about how to extract future expected yield curve from current yield curve.
3.4 Concluding Remarks and Future Research

This paper tries to estimate Korean term structure of interest rates by employing a parsimonious Nelson and Siegel’s (1987) functional form. The fact that available bond data is relatively small in Korean market is considered to select a functional form. This parsimonious function, however, is flexible enough to represent the range of shapes generally associated with the term structure of interest rates; monotonic, humped, and S shaped. This flexibility comes from the existence of three components in function; short term, medium term, and long term. With appropriate choices of weights for these components, it can generate a various shape of the term structure of interest rates.

I employ data filtering criteria based on market convention to select only bonds that are indicative of the current market yields by excluding bonds suspected to create distortions in the estimation of the yield curve. Only bonds whose traded volume is more than 10 billions won are included to avoid the liquidity problem. Further, I include bonds with more than one year remaining to maturity to capture only reliable indicator of market expectations.

The empirical results show that the parameters can be estimated fairly precisely by using nonlinear least square estimation and the employed functional form accounts for a large fraction of the variations in government bonds. I also extract the market expectations about the future interest rates paths from a given current term structure to provide useful information for forward looking monetary policy. This task is especially important for The Bank of Korea that has adopted Inflation Targeting framework since 1998. This paper is expected to give a practical contribution to The Bank of Korea by providing market expectations about future interest rates.
This paper does not investigate what factors determine the shape of yield curve. A further research will be focused on this issue to fully understand the term structure of interest rates. Further, incorporating the term structure into Inflation Targeting framework to consider optimal monetary policy rule is another extension to be investigated later.
Figure 3.1: Estimated yield curves for September 1
Figure 3.2: Estimated yield curves for September 8
Figure 3.3: Estimated yield curves for September 15
Figure 3.4: Estimated yield curves for September 22
Figure 3.5: Estimated yield curves for September 29
Figure 3.6: Extracted future yield curves given Sep. 15 yield curve
Figure 3.7: Extracted future yield curves given Sep. 15 yield curve
Figure 3.8: Extracted future yield curves given Sep. 15 yield curve
Figure 3.9: Extracted future yield curves given Sep. 15 yield curve
Figure 3.10: Extracted future yield curves given Sep. 15 yield curve
Figure 3.11: Extracted future yield curves given Sep. 15 yield curve
APPENDIX A

EXTRACTING FUTURE YIELD CURVES

A.1 The implied forward rates as break-even rates

This extraction technique is based on the relationship between the spot rates and implied forward rates. A spot rate, $s_k$, is the annually compounded discount rate of a single future cash flow such as a zero coupon bond with the time to maturity, $k$. A forward rate, $f_{k-1,k}$, is the annually compounded interest rate for a loan between $k-1$ and $k$, contracted today. Based on no-arbitrage condition, a multi-year spot rate can be decomposed into a product of one-year forward rates, i.e., it is a geometric average of one-year forward rates:

$$(1 + s_k)^k = (1 + f_{0,1})(1 + f_{1,2})(1 + f_{2,3}) \cdots (1 + f_{k-1,k})$$

where $f_{0,1} = s_1$ and $f_{k-1,k}$ is the implied one-year forward rate between maturities $k-1$ and $k$.

The implied forward rates, $f_{k-1,k}$ show how much the spot rates need to change over the next year to make all bonds earn the same holding-period return which is a sum of a bond’s initial yield and its capital gains or losses caused by yield changes. For example, if today’s spot curve is upward-sloping, longer term bonds have a yield

\footnote{This appendix comes from Ilmanen (1996).}
advantage over the one period bond. To equate holding period returns across bonds, longer bonds have to suffer capital losses that offset their initial yield advantage. The implied forward rates show exactly how much long term rates have to increase to cause such capital losses.

A.2 Extracting future yield curves by using the implied forward rates

If spot rates are known for each maturity, we can extract a specific future yield curve with a given term structure of current spot rates. Suppose the $p$ year and $q$ year spot rates are known, where $p > q$;

$$\left(1 + s_p\right)^p = \left(1 + f_{0,1}\right)\left(1 + f_{1,2}\right)\cdots\left(1 + f_{p-1,p}\right) \quad \text{(A.2)}$$

$$\left(1 + s_q\right)^q = \left(1 + f_{0,1}\right)\left(1 + f_{1,2}\right)\cdots\left(1 + f_{q-1,q}\right) \quad \text{(A.3)}$$

Dividing the equation (A.2) by the equation (A.3) produces

$$\frac{(1 + s_p)^p}{(1 + s_q)^q} = \left(1 + f_{q,q+1}\right)\left(1 + f_{q+1,q+2}\right)\cdots\left(1 + f_{p-1,p}\right) \quad \text{(A.4)}$$

According to Equation (A.1), the right hand side of Equation (A.4) can be written as $(1 + s_{q,p})^{p-q}$, where $s_{q,p}$ is the annualized $p - q$ year spot rate $q$ years forward. Therefore,

$$(1 + s_{q,p})^{p-q} = \frac{(1 + s_p)^p}{(1 + s_q)^q} \quad \text{(A.5)}$$

For example, one-year future spot yield curve is computed by fixing $q = 1$ and letting $p$ vary starting 2. The one-year future yield curve is a plot of $s(1,p)$ as a function of different maturity years $p$. These are the future spot rates that would make all
government bonds earn the same holding period return over the next year\( ^{46} \). In this case, Equation (A.5) can be rearranged as:

\[
(1 + s_1) = \frac{(1 + s_p)^p}{(1 + s_{1,p})^{p-1}}
\]  
(A.6)

The left hand side of Equation (A.6) is the riskless return of the one-year discount bond and the right hand side is the return of buying a \( p \)-year discount bond at rate \( s_p \) today and selling it a year later at rate \( s_{1,p} \). Thus \( s_{1,p} \) is the selling rate at which the \( p \) year discount bond’s holding period return equals the return of the riskless asset, i.e., it is the level of the future one-year rate that would make investors ex post indifferent between holding either of the two discount bonds.

\( ^{46} \)This same return must be the return of the one-year discount bond because it is already known today.
REFERENCES


Anderson, N., F. Breedon, M. Deacon, A. Derry, and G. Murphy (1996), Estimating and Interpreting The Yield Curve, Baffins Lane, Chichester: John Wiley and Sons Ltd.


The Bank of Korea, Financial System in Korea, Seoul, Korea.


97