STUDENTS’ MODELS IN SOME TOPICS OF ELECTRICITY & MAGNETISM

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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ABSTRACT

In recent years the need for model-based learning have been emphasized by many researchers. Furthermore, many theories have been put forward by researchers on how students reason. However, how the theories of reasoning are manifested within the context of electricity and magnetism and how to implement a model-based learning environment within such a context has not been the object of research. In this dissertation, we address the above two concerns.

We take mental models as a primary framework with which students reason about electric and magnetic phenomena. We also look for phenomenological primitives and ontological categories that exist within students’ reasoning. We probe students’ reasoning through a model-based diagnostic instrument. The instrument consists of a set of related multiple-choice questions. The questions are related in the sense that they can be categorized as belonging to the same conceptual domain, and that the contextual features of a set are kept to a minimum. Questions in the model-based instrument were created by visiting classrooms and studying the contexts within which concepts are introduced and within which learning takes place. Based on these observations, hypotheses were formed on the possible models students would form, which were then tested through students’ written explanations.
We find that students are mostly inconsistent with respect to the selection of choices, but that the inconsistency is only a surface feature. Students’ responses are tied to the models they have constructed or construct on the spot when faced with novel situations. We find that the concepts such as electric fields and electric potentials exist as mere “definitions” and do not contribute to forming a set of working models, and as such the need for the use of such concepts cannot be easily recognized. We also find that students function within a set of procedural rules. Whether these rules are extended directly from familiar situations through analogies or lead to constructing a set of new rules is constrained by the underlying models and the context of the questions. Models also either exist or are constructed in ways that lead students to overlook the common sense reality of physical phenomena. We also find that the way questions are perceived and interpreted are dependent on the underlying models and that different models exist without conflicting with each other.

Based on the above findings, we argue that students’ reasoning is context specific and is sensitive to the way the learning has taken place. However, all the pitfalls of a particular way of learning cannot be addressed within the time span of a course. Thus, we suggest a recontextualization process as a specific model-based learning environment to help students learn electricity and magnetism. To facilitate such a learning process we have designed most of the questions so that a student model that works correctly in one question would lead automatically to a student answering a related question incorrectly. The step-by-step guidance through a series of such related questions would then elucidate the context within which concepts are introduced, the limitations of particular representations and the ontological demands required by the subject.
To my parents
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Vita</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>Chapters:</td>
<td></td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Goals</td>
<td>3</td>
</tr>
<tr>
<td>1.2 Model-based education</td>
<td>11</td>
</tr>
<tr>
<td>1.3 A note on error</td>
<td>13</td>
</tr>
<tr>
<td>1.4 Outline</td>
<td>13</td>
</tr>
<tr>
<td>2. Cognitive frameworks in physics learning</td>
<td>15</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>15</td>
</tr>
<tr>
<td>2.2 Intuitive physics</td>
<td>16</td>
</tr>
<tr>
<td>2.3 Intuitive physics as a coherent theory</td>
<td>17</td>
</tr>
<tr>
<td>2.4 Intuitive physics as knowledge in pieces</td>
<td>20</td>
</tr>
<tr>
<td>2.5 Ontological categories</td>
<td>22</td>
</tr>
<tr>
<td>2.6 Mental models</td>
<td>23</td>
</tr>
<tr>
<td>2.7 Conclusions</td>
<td>25</td>
</tr>
</tbody>
</table>
3. Consistency of reasoning in selected topics of electricity & magnetism ........................................ 27
   3.1 Introduction ................................................. 27
   3.2 Newton’s third law in electrostatics ......................... 27
   3.3 Neutral metal rod near a charged object .................... 31
   3.4 Charges on insulators and conductors - superposition of electric fields 35
   3.5 Gaussian surfaces & electric fields .......................... 40
   3.6 Charge distribution on a conductor .......................... 44
   3.7 Electrostatic potential of a conductor ....................... 54
   3.8 Potential due to a shell of charge ........................... 63
   3.9 A grounded conductor ........................................ 68
   3.10 Charged conducting spheres - force on a charge ............... 89
   3.11 Current in an electric circuit ............................... 93
   3.12 Magnetic force - right-hand versus left-hand rule .......... 98
   3.13 Current as vectors - Ampère’s Law .......................... 104
   3.14 Current enclosed - Ampère’s Law ............................ 108
   3.15 Force on a charge in a magnetic field ....................... 110
   3.16 Magnetic induction - translational versus rotational movement ... 116
   3.17 Magnetic induction - change in field and area ............... 122
   3.18 Magnetic induction - Induced electric fields ................ 129
   3.19 Neutral spheres and neutral point particles ................. 135

4. A closer look at students’ models .................................. 138
   4.1 Introduction ................................................. 138
   4.2 Newton’s third law in electrostatics ......................... 138
   4.3 Neutral metal rod near a charged object .................... 145
   4.4 Charges on insulators and conductors - superposition of electric fields 153
   4.5 Gaussian surfaces & electric fields .......................... 162
   4.6 Electrostatic potential of a conductor ....................... 166
   4.7 Potential due to a shell of charge ........................... 171
   4.8 A grounded conductor ........................................ 174
   4.9 Charged conducting spheres - force on a charge ............... 185
   4.10 Magnetic force - right-hand versus left-hand rule .......... 190
   4.11 Force on a charge in a magnetic field ....................... 195
   4.12 Magnetic induction - movement ................................ 202
   4.13 Magnetic induction - change in field and area ............... 205
   4.14 Magnetic induction - induced electric fields ................ 209
   4.15 Neutral spheres and neutral point particles ................. 211
5. Conclusion ................................................................. 215
   5.1 Results and Discussion ........................................ 216
   5.2 Implications for teaching ...................................... 222
   5.3 Future directions ............................................... 225
   5.4 Summary ......................................................... 228

Appendices:

A. Questions ............................................................. 230

B. Summary of models & context factors ............................. 289

Bibliography .............................................................. 299
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Response distribution for Q-1 and Q-2 for algebra-based students.</td>
<td>30</td>
</tr>
<tr>
<td>3.2 Percentage transitions between Q-1 and Q-2 for algebra-based students.</td>
<td>30</td>
</tr>
<tr>
<td>3.3 Response distribution for Q-1 and Q-2 for calculus-based students.</td>
<td>30</td>
</tr>
<tr>
<td>3.4 Percentage transitions between Q-1 and Q-2 for calculus-based students.</td>
<td>31</td>
</tr>
<tr>
<td>3.5 Response distribution for Q-3 and Q-4 for algebra-based students.</td>
<td>33</td>
</tr>
<tr>
<td>3.6 Percentage transitions between Q-3 and Q-4 for algebra-based students.</td>
<td>33</td>
</tr>
<tr>
<td>3.7 Response distribution for Q-3 and Q-4 for calculus-based students.</td>
<td>34</td>
</tr>
<tr>
<td>3.8 Percentage transitions between Q-3 and Q-4 for calculus-based students.</td>
<td>34</td>
</tr>
<tr>
<td>3.9 Response distribution for Q-5 and Q-6 for algebra-based students.</td>
<td>37</td>
</tr>
<tr>
<td>3.10 Percentage transitions between Q-5 and Q-6 for algebra-based students.</td>
<td>38</td>
</tr>
<tr>
<td>3.11 Response distribution for Q-5 and Q-6 for calculus-based students.</td>
<td>39</td>
</tr>
<tr>
<td>3.12 Percentage transitions between Q-5 and Q-6 for calculus-based students.</td>
<td>39</td>
</tr>
<tr>
<td>3.13 Response distribution for Q-7 and Q-8 for calculus-based students.</td>
<td>43</td>
</tr>
<tr>
<td>3.14 Percentage transitions between Q-7 and Q-8 for calculus-based students.</td>
<td>43</td>
</tr>
<tr>
<td>3.15 Response distribution for Q-9 and Q-10 for FEH students.</td>
<td>49</td>
</tr>
</tbody>
</table>
3.16 Response distribution for Q-13, Q-14, and Q-15 for calculus-based students ........................................ 58
3.17 Categorical distribution for Q-13, Q-14, and Q-15 for calculus-based students ........................................ 59
3.18 Response distribution for Q-16 and Q-17 for calculus-based students .................................................... 62
3.19 Response distribution for Q-18, Q-19 and Q-20 for calculus-based students ........................................... 67
3.20 Response distribution for Q-21 and Q-22 for calculus-based students .................................................... 71
3.21 Percentage transitions between Q-21 and Q-22 for calculus-based students ........................................... 72
3.22 Response distribution for Q-23 and Q-24 for calculus-based students .................................................... 75
3.23 Percentage transitions between Q-23 and Q-24 for calculus-based students ........................................... 75
3.24 Contingency table for response transitions in the presence of positive and negative external charge .......... 76
3.25 Response distribution for Q-23 and Q-24 for algebra-based students ..................................................... 76
3.26 Percentage transitions between Q-23 and Q-24 for algebra-based students ........................................... 77
3.27 Response distribution for Q-25 and Q-26 for calculus-based students .................................................... 79
3.28 Percentage transitions between Q-25 and Q-26 for calculus-based students ........................................... 80
3.29 Response distribution for Q-27 and Q-28 for calculus-based students .................................................... 82
3.30 Percentage transitions between Q-27 and Q-28 for calculus-based students ........................................... 83
3.31 Response distribution for Q-29 and Q-30 for FEH students ................................................................. 87
3.32 Percentage transitions between Q-29 and Q-30 for FEH students ........................................................ 87
3.33 Response distribution for Q-31 and Q-32 for calculus-based students . 92
3.34 Percentage transitions between Q-31 and Q-32 for calculus-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 92
3.35 Response distribution for Q-33 and Q-34 for calculus-based students . 95
3.36 Percentage transitions between Q-33 and Q-34 for calculus-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 95
3.37 Response distribution for Q-34, Q-35, and Q-36 for FEH students . . 97
3.38 Percentage transitions between Q-34 and Q-35 for calculus-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 98
3.39 Percentage transitions between Q-35 and Q-36 for calculus-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 98
3.40 Percentage transitions between Q-34 and Q-36 for calculus-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 98
3.41 Response distribution for Q-37 and Q-38 for calculus-based students . 100
3.42 Percentage transitions between Q-37 and Q-38 for calculus-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 101
3.43 Response distribution for Q-39 and Q-40 for algebra-based students . 103
3.44 Percentage transitions between Q-39 and Q-40 for algebra-based students 103
3.45 Response distribution for Q-41 and Q-42 for calculus-based students . 106
3.46 Percentage transitions between Q-41 and Q-42 for calculus-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 107
3.47 Response distribution for Q-43 for calculus-based students . . . . . . 109
3.48 Response distribution for Q-44, Q-45, and Q-46 for algebra-based stu-
dents . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 113
3.49 Percentage transitions between Q-44 and Q-45 for algebra-based students

3.50 Percentage transitions between Q-45 and Q-46 for algebra-based students

3.51 Response distribution for Q-44, Q-45, and Q-46 for calculus-based students

3.52 Response distribution for Q-47 and Q-48 for calculus-based students

3.53 Percentage transitions between Q-47 and Q-48 for calculus-based students

3.54 Response distribution for Q-49, Q-50, Q-51, Q-52, and Q-53 for algebra-based students

3.55 Response distribution for Q-55, Q-56, and Q-57 for algebra-based students

3.56 Response distribution for Q-59 and Q-60 for algebra-based students
CHAPTER 1

INTRODUCTION

Physics education research is concerned with students’ learning of the concepts of physics. The need for action lies in the fact that standard instruction often does not lead to meaningful concept learning [1]. Much of the existing research in physics education can be considered as studies in students’ misconceptions of physics concepts and the development of instructional strategies to overcome misconceptions through active participation and knowledge construction through laboratory modules. Although each instructional method has its own advantages and limitations, we believe that active knowledge construction by itself is not enough, and that an awareness of the context within which that knowledge is constructed is also essential. The reason for this is that knowledge construction begins with a set of existing knowledge elements. A set of knowledge elements that functions very well within one context (see below) would not be sufficient within another context.

The principal reason for the focus on student learning is the fact that physics concepts have to be considered as an abstract representation of the physical world and that an explicit awareness of what we experience and what we abstract has to be distinguished. Also, in physics, many different representations can be used to describe a given physical situation. We should consider knowledge elements as inherently tied
to a representation of the situation at hand and its interpretation. We consider misconceptions as originating from the use or the appeal of one representation over another and not as inherently incorrect understanding. Certain representations may be more appealing to students than others in a given situation because they have provided students with “satisfactory explanations” within “similar” situations. It is not enough to pinpoint students’ incorrect understanding; students must explicitly be made aware of the particular representations they are implicitly using, the limitations of such representations, and their ontological presuppositions.

The research presented in this dissertation is important because it investigates the existence of the above notions experimentally within the context of electricity and magnetism. Conceptual change is not a one step process but requires revisiting a single concept under different contexts. The issue we address is not to list a set of “misconceptions” but rather to understand students’ reasons for using particular reasoning patterns by examining how well the ideas put forward by many theoretical researchers on human reasoning (see Chapter 2) manifest themselves in such contexts.

We assume that students construct internal representations and therefore take their mental models as the key framework for our understanding of their understanding of electric and magnetic phenomena. The identification of students’ internal representations and the identification of certain theoretical ideas of reasoning is not enough. As mentioned above, it is of great importance to make students realize the limitations of their representations by giving opportunities to reason through a single concept under varied contexts. We set out to achieve this goal by designing a set of multiple-choice questions with specific features (see below), which are to be used in class to provide
prompt feedback to students. The importance of such a set of questions is to help engage students actively in realizing what has been learned, the specific representations used in the learning, the context within which the concepts have been introduced and learned, the limitations of the representations, the ontological presuppositions, and the need to construct functional knowledge or working models in order to better understand the world of electric and magnetic phenomena.

1.1 Goals

Learning occurs within a particular context. The context involves the classroom environment, the material presented, the way it is presented and the type of problems discussed, to name a few. Students who begin study of a particular subject, say, electricity and magnetism, do not enter the class with a blank mind but have a certain intuitive understanding of the way the world works. Thus, the intuitive knowledge that students possesses can constrain new knowledge acquisition and the interpretation of the novel concepts introduced in the class [2].

By “constraining,” we do not mean that students reject the ideas put forth by an instructor or discussed in the texts. Rather, the comprehension of concepts and knowledge organization can be significantly affected by their preconceptions. Preconceptions are not the only factor that affect concept learning. For example, material learned in the first chapter of the text may influence knowledge acquisition, comprehension and perhaps the fundamental outlook on the subject as a whole.
Unfortunately, we cannot open students’ minds and find out how their knowledge is being structured. Even if we did, we might not know how someone is going to react with certainty within a particular context. Still, some generalizations can be made ([3], pg. x).

In this dissertation, we attempt to identify a set of models with which students might function in a given context. Our goals are to understand:

1. what knowledge elements are specifically manipulated by students and what models are constructed,
2. what models are used consistently (not necessarily correctly) in answering a set of questions,
3. what context features of the questions are influencing the use of particular models, and
4. what knowledge elements seem to constrain students’ reasoning.

The context is provided by a set of multiple-choice questions that are posed to students. Since we do not have any means of investigating students’ minds directly, we must rely solely on indirect observables. These observables include the answer (choice) a student would select within the context set by a particular question and the explanation provided by that same student.

On what do the above observables depend? In other words, can we attempt an explanation of how and why a particular choice is selected? Rather than attempting to provide an absolute answer in a direct fashion, we approach an explanation in a relative sense. That is, we do not analyze students’ performance based on an isolated question by itself but we use the students’ explanations on several related questions.
Before proceeding further, we must address what is meant by related questions. The relatedness of a set of questions is determined by our belief as to which mental models students should be forming and employing to arrive at a correct answer and give a valid explanation. The models can be also thought of as related in the sense that they can be categorized as belonging to the same conceptual domain. In this sense, the use of mental models is equivalent to scientific models. This need not hold at all times. To quote Vosniadou ([4], pg. 29): “Mental models are... generative and dynamic, in the sense that they can be used as the basis for constructing explanations of phenomena that were not explicit in the knowledge base...”. From the point of view of a theory of learning, mental models are important because they are the point where new information is integrated in the knowledge system and therefore represent a major source of cognitive change in existing knowledge structures.” The questions are also related in the sense that the language and pictorial representations of such questions largely remain the same.

Why a set of questions on a given conceptual topic rather than a single question? As mentioned earlier, learning takes place within a specific context. As such, the understanding of the subject (concepts) is derived within a specific context. Traditionally, concepts are exemplified through a single or several questions. However, when several questions are used (traditionally) they are not necessarily related in the sense that picture-like and/or language-like representations can differ largely among questions. Thus, comparisons between questions are difficult. This is particularly the case when different questions have different answers, not to mention the differences that may be needed in terms of mathematical tools among different questions. Such variability is kept to a minimum in our questions. A single question cannot
make students aware of all the details of a concept that is buried within a particular context. Similarly, a single question may not reveal students’ state of knowledge to an instructor for him or her to interfere constructively with the students’ learning process. Thus, the context of a question has to be expanded in such a way that the student does not feel lost in his/her attempt at the new question. We call this expansion in context - *recontextualization* to emphasize that the student must actively construct the context. This in turn has several advantages.

From the point of view of the present study it helps us identify students’ models and how consistently or inconsistently they are used. It helps us test our hypotheses on students’ reasoning or models.

From the point of view of the student, the first questions mostly refer to a familiar case while the others that follow may not. At this point the students are more likely to manipulate the knowledge elements the same way as in the previous (familiar) question, in which case they are driven to choose the incorrect solution. This choice in turn may lead to a cognitive conflict. The instruction can then guide students toward the correct solution by emphasizing features of the concept that were not apparent in the previous contexts. The reason for the need for such an emphasis is that a concept exists as a representation of the real world and that a given phenomenon can be expressed through many representations. Thus, a representation that is used in one context might not be appropriate in another.

The first question often helps us establish the knowledge base of the students. This gives us confidence that most students are able to reason within a familiar context and with a familiar set of representations. The question(s) that follow attempt to elucidate the limitations of reasoning suggested in the earlier question(s).
Giving a familiar question in the beginning might also help students access the necessary knowledge elements easily. A very unfamiliar question from the beginning can make students struggle with the retrieval of necessary knowledge elements [5]. Also, a single question cannot take into account variations in the context and the reasoning that is dependent on them.

A particular set of questions is largely motivated by informal observations of classroom instruction and informal and formal interviews with students. The data presented in the following pages, however, all result from students’ explanations gathered through web-based surveys and examinations. Although interviews, as mentioned earlier, were helpful in identifying the areas with which students have conceptual difficulties, we did not rely on them explicitly to reveal students’ models. There are two reasons for this. The first is our experience with the interview process which led us to view it as a very fluid process. That is, that students’ reasoning seems to be very sensitive to interviewers questions. As such, we wanted to have data without any presence of a questioner. We believe that this process reveal the most “natural state” of students’ reasoning. The second is the need for a large enough sample so that the data can be subjected to statistical analysis.

These observations give us an understanding of students’ knowledge base. Based on this understanding, we posit hypotheses concerning how certain knowledge elements may be combined or manipulated within a given context. This, in turn, guides us in arriving at a set of questions.
While the questions serve as a test-bed for the hypotheses we form and answering the abovementioned goals, the primary purpose is to use them in an instructional environment to help make students aware of the shortcomings of their reasoning. Two major reasons can be given as to why the answers to the goals stated above are important.

1. The demands placed by physics courses are high. Learning physics can be compared to learning a new human language. On top of such demands are the time constraints of the course. The amount of information given in a single lecture is enormous. The grasp of material in both conceptual and mathematical terms is required. Furthermore, in most cases, a shift in ontology is implicitly required by the subject matter itself. To help students achieve a scientific understanding of physics concepts under such demands is difficult unless we know how students tend to reason when faced with these new ideas.

2. Instruction, as mentioned earlier, takes place within a particular context. Certain demonstrations, representations, and the very explanations used in one context may (incorrectly) be carried over to other contexts by students. However, attempting to discuss all the pitfalls that students may make at once is fruitless from both the instructor’s and student’s viewpoint. Hence, we believe that a recontextualization in terms of small steps would lead to a better grasp of the material presented. This may be facilitated by having a set of questions for which we have an understanding of students’ models and the context factors that affect them.
The ideal goal will be to achieve a one-to-one correspondence between a model and a choice of a given question such that there will be no error in interpretation. This, unfortunately, cannot be achieved within the practical limitations of our study although varying degrees of successes have been achieved among different types of questions. (At this stage we do not know whether there is a fundamental limitation on achieving such a correspondence.) For some questions there exist more than one way (not necessarily correct) of arriving at a single choice of a given question. However, a reasoning that leads to a correct answer in one question may lead to an incorrect answer in another. This is the advantage of posing several related questions.

The difficulty of our task is that there should emerge a way in which we can identify a certain kind of reasoning (models) through students’ explanations. Since we rarely can transcend ourselves, we cannot approach a student and ask him or her to explain his or her mental model in a given question. The question it raises is how to identify a model. In this study, models are identified by a set of coherent explanations that would lead to a particular choice of a given question. Severe constraints are imposed on what particular sets of questions can be used due to the following reasons:

1. the kind of mental models we hypothesize,
2. the questions should be posed within approximately the same context (this is important for comparisons between questions and is a severe limitation),
3. the number of conceptual models needed to arrive at a solution for a given set of questions is kept to a minimum,
4. the questions do not require any involved calculations and therefore memorization of formulae is not essential. Vector properties are not needed to arrive at solutions.
Simply stated, the questions are largely conceptual. When we demand any mathematical sophistication from students, they are of very familiar, and in our view, straightforward situations.

5. questions pertinent to one group may not be conflated with those pertinent to another group (our aim is to see how individual students consider the entire set; this constrains the possible number of questions in a given set to prevent boredom; this is particularly the case since the context is largely the same among questions within a set), and

6. Even if all of the above were satisfied, not any question could be given to any type of class.

For example, in constraint 6, the types of questions that can be explored in an algebra-based electricity and magnetism class is very limited for both conceptual and mathematical reasons. Also the course’s time constraints lead to a narrow window during which a particular set of questions could be investigated.

Data presented in this dissertation are from three types of electricity and magnetism courses; algebra-based, calculus-based, and the calculus-based freshman engineering honors (FEH). For the algebra-based courses (Spring 2001, Spring 2003) and for the calculus-based courses (Spring 2002, Spring 2003), data were gathered through web surveys. Surveys were given each week (except in 2001, during which only five were carried out) on topics that corresponded to the concepts that were covered in that particular week. Two exceptions to the weekly pattern are the surveys on electrostatic phenomena (carried out over the weeks that discuss electric circuits) and on magnetic phenomena (carried out over the weeks that discuss waves and vibrations
and optics). Participation in the surveys was optional. The midterm examination for the Spring 2003 calculus-based class was also used to gather data in paper format.

The data from the FEH class (Winter 2003) was gathered through the use of a computerized voting machine system. The class was divided into groups of four students (17 groups in total). Each group responded to the questions projected on a large screen via a remote controller. The instructor posed a question to the class at a time he saw fit. Based on the responses, the instructor then decided to move to a related question or provide feedback before moving on to the next question. A maximum of three minutes was allowed for a given question after ensuring that students understood the question. Students working in groups were encouraged to discuss and respond. The texts used were Halliday, Resnick, and Walker [6] (calculus) and Giancoli [7] (algebra).

1.2 Model-based education

Several studies exist within classical mechanics, where attempts have been made to identify students’ models (e.g., [8], [9]). While models of electricity has been studied by several researchers (e.g., [10], [11], [12], [13], [14]), electrostatic phenomena has been studied in a descriptive sense ([15]) rather than in an analytical sense. In the past decade, the value of models and modeling in science education has been increasingly recognized. With this awareness, the need for model-based learning and teaching has become important ([16]).

Model-based learning can be defined as the construction of mental models of phenomena by students. Although it may be impossible to know precisely the nature and the content of a mental model in any human being, it has been suggested that
researchers can draw certain inferences about the nature of students’ mental models based on the types of reasoning a learner exhibit. Despite many attempts at instruction, students fail to reason coherently on the phenomena using the concepts they have learned. Hestenes [17] has called for a style of physics instruction in which students are taught from the beginning that in science, “modeling is the name of the game.” His arguments rest within a constructive epistemology in which meanings about the physical world are constructed and matched with experience. According to Hestenes, the main idea is to teach a system of explicit modeling principles and techniques in familiarizing the students with a basic set of physical models and to provide guidance in model building to explain and predict physical phenomena. In such a process, in addition to achieving conceptual understanding, the student follows the essential steps of scientific creation and forms an appreciation towards the methodology of science.

As discussed by Clement [18], in a model-based learning environment, efforts should be geared towards moving the student from model $M_n$ to model $M_{n+1}$. The sequence of intermediate steps from preconception to any target model form what Niedderer and Goldberg have called a learning pathway [19].

As mentioned above, the goal of our study is to attempt an understanding of students’ reasoning through use of a set of questions. These questions in turn may be used as a tool to aid in model-based learning. According to Clement [18], the instructional framework specifies the goal of a target or desired knowledge state that one wishes students to possess after instruction. There is no need for the target model to be as sophisticated as the one possessed by the expert. A model-based
diagnostic instrument could be useful in any instructional environment to identify general patterns of thinking among students and the possible causes for such patterns.

1.3 A note on error

The number of responses registered during any particular week through web surveys was not large compared to the class population. Most of the sample sizes (N) are approximately 40. This is enough to apply the large sample criterion of \( N \geq 30 \) used in standard statistical analysis. However, due to “small” sample sizes, the standard errors are comparably large. All errors are calculated at 95% confidence level and no errors were calculated for sample sizes less than 30. This is also true for the FEH class, where a single group response is counted only as a single response. Furthermore, the errors are calculated for the proportions (p) that satisfy the conditions \( Np \geq 5 \) and \( N(1-p) \geq 5 \). This does not hinder the main goals of the study discussed above and of course does not take away the validity of a student’s explanation.

We assume that students’ thinking is revealed in their written explanations. Although the absence of any explanations or explanations that are difficult to interpret does not suggest an absence of model building and thinking, our interpretations of such responses may introduce additional errors of inference.

1.4 Outline

The outline of the dissertation is as follows:
Chapter 2 provides examples of several cognitive frameworks in existence and what ways cognitive scientists have proposed in order to explain and understand student
thinking. We take mental models as a useful framework for describing students’ reasoning in electricity and magnetism.

Chapter 3 provides a discussion of the topics investigated and provides quantitative data. Our main purpose is to see to what extent students have shifted in terms of choices between related questions.

Chapter 4 looks qualitatively at the data on students’ explanations and derives possible models. The explanations also provide a window into the knowledge elements, ontological constraints, and context specific reasoning.

Appendix A provides a list of questions for quick reference while Appendix B summarizes the questions, their intended measure, identifiable models, and the related context factors.
CHAPTER 2

COGNITIVE FRAMEWORKS IN PHYSICS LEARNING

2.1 Introduction

Cognitive science can be thought of as the science of intelligence. As such, its main statement is that human mind works through representation and computation [20]. Mental representations may take the form of concepts, logical propositions and rules, images, and analogies. Mental procedures function mainly as computer data structures having among others the functions of search and retrieval. The usefulness of cognitive science theories in physics education research had been recognized [21] a decade ago, although cognitive scientists have studied physics problem solving even before that [22]. The use of such theories in physics education research has two main purposes:

1. Physics education research begins with observations of how students respond to particular physics problems. As such there is a wide range of data. Existing theories from cognitive science are used to organize data, and provide a set of fundamental explanations for such data. However, except for the attempt by Greca and Moreira [23], no such research exists within electricity and magnetism. Our research attempts to fill this gap.
2. The fundamental theoretical position one takes with respect to the cognitive theory leads to specific instructional decisions. This can take several forms, such as defining instructional goals, construction of curricula, and implementation of such instruction.

This chapter discusses several cognitive theories that we rely on to provide explanations, and furthermore, guided us in predicting certain outcomes as described in Chapter 3.

2.2 Intuitive physics

Physics is inherently an abstract science. By “abstract,” we mean that it deals with concepts and ideas that may not match with our common experience in any trivial or straightforward way. Aristotle believed that the natural state of an object is the state of rest and an action of a force is necessary in order to cause any motion. Push harder, the object would move faster. Thus, more cause leads to more effect. Aristotle concluded that

\[ F \propto \text{speed}. \]

Human beings could arrive at this conclusion naturally based on their experience with the external world - both as active participants and as observers. However, the conclusion at which Newton arrived,

\[ \vec{F} \propto \vec{a}, \]

is a highly non-trivial deduction about the behavior of the physical world. Our sensory experiences are such that we feel and detect the magnitude of the impulses in
question and not the rate of change of such impulses. However, the level of abstraction needed does not end here. The statement that the acceleration is the rate of change of speed will not take us far. In circular motion, an object may go around at a constant speed but yet have an acceleration toward the center of the circle. To overcome the putative dilemma, we need to introduce vector concepts and replace speed by velocity, which not only has a magnitude but also a direction. Thus, acceleration is a vector quantity and should properly be defined as the rate of change of velocity. The vector concepts carry an additional level of abstraction. Thus, in order to go beyond the simple observations of falling apples and rotating planets and explain the nature of such motion requires a machinery that not only is non-trivial but also may at times be counterintuitive. Not only do we receive sensory experiences through our observations and interactions with nature, we implicitly create assumptions and models based on such experiences. Such model-building and assumption-making extend to any formal learning environment as well. Such knowledge acquisition based on experience and notions built upon such experience form the set of intuitive physics.

2.3 Intuitive physics as a coherent theory

McCloskey [24] has argued that the novices (students or non-experts), although they may have the wrong model in the eyes of the expert, possess knowledge that is coherent. According to him ([24], pg. 321) “people develop on the basis of their everyday experience remarkably well-articulated naive theories of motion.” The majority of the subjects in McCloskey’s study believed that an object moves because of an impetus imparted on the object by the mover. The reason for the object coming
to rest has been argued as the dissipation of the imparted impetus. The issue that McCloskey raises is the connection that such reasoning has in common with the medieval theory of impetus. That is to say that from a similar set of observations people tend to extract similar deductions beyond historical or cultural boundaries.

Similar arguments are made by Vosniadou [25], who argues that beliefs are tied to and constrained by a set of ontological and epistemological presuppositions. As a result, beliefs do not operate “in pieces,” but form a coherent structure. As such, the main difference between the novice and the expert is not that the novice’s physical knowledge is in pieces and expert’s tied to physical laws and principles, but that the novice’s knowledge is tied to ontological and epistemological presuppositions that provide a radically different explanatory framework. Hence, misconceptions are not caused by local, isolated, false beliefs that could be corrected easily by instruction but are very hard to revise. This is because such beliefs are tied down to everyday experiences. According to Vosniadou, it is the presuppositions that are difficult to change and resist instruction and not the misconceptions.

Vosniadou developed two frameworks of reasoning called the naive framework theory and the specific theory. Naive frameworks are built early in infancy and consist of certain fundamental ontological and epistemological presuppositions. They are not available to conscious awareness and hypothesis testing and constrain the process of knowledge acquisition. An example is the presupposition that Earth is flat.
Specific theories consist of a set of propositions and beliefs that are interrelated and describe the behavior of physical objects. These theories are easily alterable. For example, children who think that there is air and water on the moon did not find it difficult to change their belief when told that astronauts had to carry air and water to the moon.

Vosniadou has studied mental models in children about the shape of Earth, giving many fascinating results. Mental models, according to her, are defined as dynamic and generative representations which can be manipulated mentally to provide causal explanations of physical phenomena and make predictions. The flat rectangular and disk shapes proposed for Earth are termed initial models since they seem to be based on everyday experience and do not show any influence from the scientific model.

The combination of initial models with culturally accepted (i.e., scientific) models give way to synthetic models. It is hypothesized that children’s initial models of Earth have their roots in a specific theory about Earth that is based on interpretations of observations and scientific information under the constraints of naive framework theory. The important identification made by Vosniadou is that of the synthetic models, in which children are able to accommodate and function using two different models to explain a single issue. The incompatibility of the two models (e.g., flat Earth and the round planet) is not realized. Such reasoning originates from the presupposition that one attaches to Earth’s “special” frame of reference.

The instructional approach that is suggested by the coherent theorists is that of cognitive conflict where novices are able to realize the shortcomings of their ideas and be able to make a transition to the expert view.
2.4 Intuitive physics as knowledge in pieces

diSessa [26] opposes the coherent knowledge theory and argues that humans gradually acquire a sense of mechanism in dealing with the world (e.g., causality). These “senses” consists of a diverse and diffuse collection of judgments and impressions that are not as precisely defined as principles. Since these “senses” are not well defined and organized, they correspond to “knowledge in pieces.” In assessing the dynamics of the pieces of knowledge elements, he defines the phenomenological primitives or p-prims.

Phenomenological primitives can be thought of as simple abstractions from common experience. They are phenomenological in the sense that they often originate in nearly superficial interpretations of the experiences of reality. They are primitive in the sense that they consists of relatively unsophisticated explanations.

diSessa identifies what he calls Ohm’s p-prim: an agent that is the locus of an impetus that acts against a resistance to produce some sort of result ([27], pg. 126). The main effect of Ohm’s p-prim is the justification of a set of proportionalities. For example, in $V = IR$, to have the same current with a larger resistance a larger voltage is required. $F = ma$ can also be included in Ohm’s p-prim if $F$ is considered as the agent, $m$ as the resistance and $a$ as the result. Another p-prim is “force as a mover.” That is, things go in the direction they are pushed.
As observed by McDermott and Shaffer [28], students’ belief that a higher number of resistors gives rise to a high equivalent resistance regardless of the configuration of the circuit can also be identified as a p-prim: more ⇒ more. All these fall into the generic Ohm’s p-prim according to diSessa in the sense: more effort (agent) ⇒ more result, more resistance ⇒ less result, etc.

The way a student would reason, according to diSessa, has its roots in direct experience with the world. P-prims consists of fragmented knowledge abstracted from such experiences. An important aspect of p-prims is that they do not need any explanation. For example, there is no need to explain why more force leads to greater speed - they are taken to be natural and events to be expected. In this regard, p-prims are not limited to relations such as more ⇒ more. Students’ belief that a table does not exert any force on a book that is on it is another example of a p-prim. We are intuitively familiar with everyday notions such as “push harder,” “pull more,” etc. Such experiences lead us to regard action of force as leading to dynamical effects. The table cannot be exerting a force on the book since the book is at rest and is passive. Why is that the book does not fall down? In our everyday lives we do not question such issues but simply take them as the “way things are,” the way the world behaves. If we demand an explanation, we are more likely to get responses such as “the table is just in the way of the book.” The nature of p-prims are such that there are no underlying substructures to them. They are to be taken as fundamental entities of reasoning requiring no deeper analysis.
P-prims are analogous in some ways to Resnick’s “ontological commitments” [29]. The preconception that motion implies force according to Resnick is derived from the presupposition of the rest condition. While p-prims are ad hoc naive explanations of physics, ontological commitments assert that a small number of principles can be identified that can act as generators for naive conceptions of physics.

2.5 Ontological categories

Chi et al., [30] argue that conceptual change occurs as a shift between categories. Why some concepts are harder to grasp and learn than the others can then be explained by whether or not a conceptual change or a transition occurs within the same categories. Hence, misconceptions arise based on whether students have placed the concepts in question under the correct or the incorrect ontological categories.

Two main categories - matter and processes - are identified by Chi et al. Accordingly, there is a fundamental reason why certain concepts are difficult to learn. They transcend the difficulty in the mathematics, mathematical and conceptual abstractness, and the technicalities of the language involved. The difficulty comes from the incompatibility between the categorical representation that the students bring to an instructional context and the ontological category to which the science concepts truly belong. For example, students categorize force as a substance under the category of matter, which an object possesses, whereas a physicist considers force as an interaction belonging to processes.
Similarly, if a student categorizes electric current as a liquid under matter, then the student is likely to reason in terms of “volume” and “space.” Chi et al. argue that this is the reason why misconceptions such as “current being used up” and “current being stored in the battery” appear. From this point of view, students’ reasoning is more matter-based whereas physicists’ reasoning is more process-based, having in addition the ability to shift between the two categories.

2.6 Mental models

The modern formulation of mental models is due to Craik [31]. He has argued that human beings translate external events and experiences into internal models and reason by manipulating the resulting internal representations. Humans, then, can translate the resulting internal representations back into actions or recognize a correspondence between them and the external events. Craik writes: “By a model we thus mean any physical or chemical system which has a similar relation-structure to that of the processes it imitates. By ‘relation-structure’ I do not mean some obscure physical entity which attends the model, but the fact that it is a physical working model which works in the same way as the processes it parallels, in the aspects under consideration at any moment...” ([31], pg.51.) Thus, for Craik, a mental model is a dynamic representation of the world where human beings are capable of translating external events and experiences into internal models and reason by manipulating the resulting internal representations [32].
Building on Craik’s arguments, Johnson-Laird [33] has extended the mental model theory attempting to provide a general explanation of human thought. Johnson-Laird’s main thesis is that human beings represent the world they are interacting with through mental models. That is, though the physical phenomena are propositionally codified through verbal statements or mathematical formulations, comprehension should involve the construction of mental models for entities or processes that they represent. As such, mental models allow us to explain and make predictions about the physical world. According to Johnson-Laird, our ability to give explanations is intrinsically tied to understanding. In order to understand any phenomenon we must have a working or an operational model of it.

A main argument of mental model theory is that all mappings have to be described as procedures. For each new piece of information, a search is made to ensure that the proposition is consistent with any earlier information encountered. If an appropriate model can be found to accommodate the proposition, the model will be cued, applied, and perhaps modified. If no appropriate model can be found, the relevant procedures will be employed to construct a mental model from scratch. The study by Greca and Moreira [23] has found that students function in terms of propositional representations regarding the concept of field.

The word mental model has taken different meanings based on what particular structures one is looking at. Johnson-Laird describes mental models as structural analogues between external phenomena and internal representations whereas Gentner and Stevens [34] consider the analogies that are carried from one internal to another internal representation. However, the underlying theme that emerges is that
people attempt to form an understanding of a newly presented phenomenon by drawing inferences from an existing mental model or constructing them on the spot. Based on Norman [35], the following properties of mental models can be listed:

1. Mental models consists of propositions, images, rules of procedure and statements. These establish the criteria of usage of mental models.
2. They may contain contradictory elements.
3. They may be incomplete.
4. People may not know how to “run” the procedures present in their mental models.
5. Elements of a mental model do not have firm boundaries.
6. Mental models tend to minimize expenditure of mental energy or effort.

2.7 Conclusions

Many theories of human thought and comprehension have been put forward by cognitive science researchers. The mental model theory is put forward as a theory of representations in the mind of real or imaginary situations as an attempt to generalize human cognition. An important feature of mental model theory is its claim of dynamic and generative representations and that people tend to solve problems by using prior information and knowledge about similar problems if a similarity exists between the structure of the models. Other theories can be considered as attempts to classify and make sense of myriad of observations within specific domains. However, this is a viewpoint of our own and not of common agreement. The authors of such theories as p-prims can disagree to varying degrees. For example, Wilson and Rutherford [36]
view mental models as the instantiation of other theories of knowledge. Within mental model arguments (Johnson-Laird, Gentner and Stevens, Norman, Vosniadou) we do not take into account the variations and subtleties among different definitions. For our purposes we consider different versions as being complimentary. We argue that a healthy coexistence of several theories is necessary, aiding us to move across domains of specific applicability. This is also the viewpoint taken by Thagard ([37] pg. 5).
3.1 Introduction

In this chapter, we present statistics collected and the inferences we make based on them. Under each question we discuss the motivation and the hypotheses we made on possible students’ models before testing the questions. The proportion of students who answered a particular question and the shift from one set of choices in one question context to another in a different, but related, context is our concern. Statistical errors are shown when possible.

3.2 Newton’s third law in electrostatics

Almost all courses in electromagnetism begin with an introduction of Coulomb’s law. The attraction between unlike charges and the repulsion between like charges is discussed in detail.

In classical mechanics, it is well known that Newton’s third law in certain context settings provides a substantial barrier to the understanding of its meaning. Students often think that a larger, heavier object exerts more force than a smaller, lighter object [38]. This can be thought of as a mental model students would form based
on their everyday experience. Larger, heavier, stronger objects do seem to do more damage on smaller, lighter, weaker objects. Such observations simply mask the notion of equal and opposite forces.

However, phenomena in electricity and magnetism provide less evidence in terms of forming mental models in the sense discussed above. In everyday situations people only deal with current, batteries, “static cling” and such, which are dissociated from notions of classical mechanics and the concepts of force. We hypothesize that Newton’s third law can be carried on to the context involving charges, in the sense that “more charge will lead to more force.” That is, if we have two charges $Q$ and $q$ such that $Q > q$, charge $Q$ will exert more force on charge $q$ than charge $q$ exert on charge $Q$. We are also interested in looking at two more aspects. One is, how a distribution of charge will be treated as opposed to a point charge; secondly, whether the physical dimensions would make students associate with everyday experiences such as larger objects exerting more force on smaller objects. The following two questions were posed with these ideas in mind.

Q-1

Two stationary point charges $+3$ C and $+1$ C are separated by some distance. What can be said about the Coulomb force between the two charges?

A. The $+3$ C charge exerts more force on the $+1$ C charge.

B. The $+1$ C charge exerts more force on the $+3$ C charge.

C. Both charges exert an equal amount of force on each other.
In the second question, the larger charged sphere has a smaller amount of charge (+1 C) assigned to it. The point charge is assigned a higher charge (+3 C). This would enable us to understand the responses, which deal with the sphere exerting a larger force on the point charge as being due to the ideas involving its physical size and could not be due to its charge (see below). The percentage distributions and the transitions among the two questions in the algebra-based class are shown in Tables 3.1 and 3.2, respectively.

Q-2
A stationary point charge of +3 C is kept at some distance from a large fixed sphere, which has a total charge of +1 C. What can be said about the Coulomb force between the point charge and the sphere?

A. The point charge exerts more force on the sphere.
B. The sphere exerts more force on the point charge.
C. Both the sphere and the point charge exert an equal amount of force on each other.
From the transition table we see that 69% consistently selected the correct model. Choice A was selected by 18% consistently. It is interesting to note that no student selected choice B for Q-1 whereas in the context of Q-2, 11% selected it. Seven percent of the students who selected the correct model in the context of Q-1 shifted to choice B or the dimension model in the context of Q-2.

The response distribution and the transitions for the same two questions among the calculus-based students are shown in Tables 3.3 and 3.4, respectively.
<table>
<thead>
<tr>
<th>Q-2→</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-1↓</td>
<td>17 ± 11%</td>
<td>9%</td>
<td>0%</td>
</tr>
<tr>
<td>A</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>B</td>
<td>4%</td>
<td>2%</td>
<td>67 ± 14%</td>
</tr>
</tbody>
</table>

Table 3.4: Percentage transitions between Q-1 and Q-2 for calculus-based students

From the transition table we see that 67% consistently selected the correct model. Choice A was selected consistently by 17%. As with the algebra-based sample, no student from the calculus-based sample selected choice B in the context of Q1.

As we can see, the percentage distributions for the two classes seem similar. The two populations seem to perform equally among the two questions.

3.3 Neutral metal rod near a charged object

The following two questions attempt to look at the consistency in students’ reasoning when an uncharged metal rod is brought closer to a charged object. The question was motivated by the fact that attraction was demonstrated by bringing only one end of the charged rod near the neutral piece of metal. The effect was not demonstrated for both ends.
Q-3
One end of an uncharged metal rod is brought close to a charged object. What will happen?
A. Metal rod is attracted by the charged object.
B. Metal rod is repelled by the charged object.
C. Nothing happens.
D. Cannot say without knowing what type of charge the object has.

Q-4
What happens if the other end of the metal rod (refer Q-3) is brought close to the same charged object?
A. Metal rod is attracted by the charged object.
B. Metal rod is repelled by the charged object.
C. Nothing happens.
D. Cannot say without knowing what type of charge the object has.

The correct answer to Q-3 and Q-4 depends on the realization that the metal rod as stated is neutral and the polarization of charges within the metal rod will involve the unlike charges being attracted to the end that is closer to the charged object, regardless of the type of charge the object may carry.

Thus, the correct model involves choice A in both cases. The idea that, if one end of the metal rod attracts, then the other end should necessarily repel is represented by choice B. This model can be given a shortened name - “fixed poles” model. Choice C can be taken as arising from thinking that the metal rod is electrically neutral and
will remain neutral. Choice D will result in arguing that the type of charge on the object must be known - that is, attempting to look for a match for the statement “like repel and unlike attract” within the problem context, although the metal rod is neutral.

The percentage distributions and the transitions for the algebra-based sample is shown in Tables 3.5 and 3.6.

<table>
<thead>
<tr>
<th></th>
<th>N = 54</th>
<th>Q-3</th>
<th>Q-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>83 ± 10%</td>
<td>44 ± 13%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>4%</td>
<td>39 ± 13%</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0%</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>13 ± 9%</td>
<td>15 ± 9%</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: Response distribution for Q-3 and Q-4 for algebra-based students

<table>
<thead>
<tr>
<th>Q-4→</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>44 ± 13%</td>
<td>35 ± 13%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>4%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>C</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>D</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>13 ± 9%</td>
</tr>
</tbody>
</table>

Table 3.6: Percentage transitions between Q-3 and Q-4 for algebra-based students
We see that 44% consistently selected the correct model for both questions. Thirteen percent of the students consistently selected the null model. Thirty-five percent of the students who selected the correct model in the context of Q-3 selected Choice B in the context of Q-4. We see that students’ performance on Q3 and Q4 are statistically different with respect to choices A and B.

The percentage distributions and the transitions for Q-3 and Q-4 for the calculus-based sample are given in Tables 3.7 and 3.8.

<table>
<thead>
<tr>
<th>$N = 46$</th>
<th>Q-3</th>
<th>Q-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>61 ± 14%</td>
<td>20 ± 11%</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>52 ± 14%</td>
</tr>
<tr>
<td>C</td>
<td>22 ± 12%</td>
<td>13 ± 10%</td>
</tr>
<tr>
<td>D</td>
<td>17 ± 11%</td>
<td>15 ± 10%</td>
</tr>
</tbody>
</table>

Table 3.7: Response distribution for Q-3 and Q-4 for calculus-based students

<table>
<thead>
<tr>
<th>Q-4→</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-3↓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>20 ± 11%</td>
<td>39 ± 14%</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>C</td>
<td>0%</td>
<td>9%</td>
<td>11 ± 9%</td>
<td>0%</td>
</tr>
<tr>
<td>D</td>
<td>0%</td>
<td>4%</td>
<td>0%</td>
<td>13 ± 10%</td>
</tr>
</tbody>
</table>

Table 3.8: Percentage transitions between Q-3 and Q-4 for calculus-based students
We see that only 20% consistently selected the correct model for both questions. Thirty-nine percent of the students who selected the correct model in the context of Q-3 selected choice B in the context of Q-4.

We see that the selection of the correct choice (A) is significantly different between the questions. The differences in the eigenvalues for the algebra- and the calculus-based classes suggest that the algebra-based students’ individual model vectors are more in agreement with each other than those among the calculus-based students. We also see that choice C is significantly present among the calculus-based students than that with the algebra-based students. These students seem to reason in terms of the neutrality of the metal rod. The questions cannot distinguish whether polarization is considered but the net neutrality of the metal rod led to selecting choice C or whether the word “neutral” is mapped directly to imply the non-existence of any dynamical effects.

3.4 Charges on insulators and conductors - superposition of electric fields

A fundamental property of the electromagnetic field is that electric and magnetic fields can be superposed. In everyday situations charges reside either on insulators or conductors. It is found that experiential and socially formed knowledge of insulators being “safe” and conductors being “dangerous” exist among students [39]. The following two questions were given to extract students’ models on the contribution of charges on insulators and conductors to the electric field at a given point.
Q-5

Two plates made up of insulator material are kept fixed at some distance apart as shown in the figure below. The charges on the plates are distributed as shown.

The electric field at point P is due to?

A. All the charges on the positive plate.
B. All the charges on the negative plate.
C. All the charges on both plates.
D. Only the charges on the positive plate, which are on the side closer to P.
E. Only the charges on the negative plate, which are on the side closer to P.
F. Charges on both plates, which are on the sides closer to P.
Q-6

What is the electric field at point P due to if the plates are made up of conducting material (refer question Q-5) and having the same charge distribution as shown in the previous question?

A. All the charges on the positive plate.
B. All the charges on the negative plate.
C. All the charges on both plates.
D. Only the charges on the positive plate, which are on the side closer to P.
E. Only the charges on the negative plate, which are on the side closer to P.
F. Charges on both plates, which are on the sides closer to P.

The percentage distribution for Q-5 and Q-6 for the algebra-based sample and the transition table among the most popular choices are given in Tables 3.9 and 3.10, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Q-5</th>
<th>Q-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>B</td>
<td>7%</td>
<td>5%</td>
</tr>
<tr>
<td>C</td>
<td>27 ± 13%</td>
<td>59 ± 15%</td>
</tr>
<tr>
<td>D</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>E</td>
<td>7%</td>
<td>0%</td>
</tr>
<tr>
<td>F</td>
<td>57 ± 15%</td>
<td>34 ± 14%</td>
</tr>
</tbody>
</table>

Table 3.9: Response distribution for Q-5 and Q-6 for algebra-based students
A note on Q-5 and Q-6 is in order. The charge distribution on the conductor is assumed to be the same as that of the insulator. This is possible since we consider a realistic case where the plates have finite dimensions and the size of the plates can be taken as small compared to the separation between them. However, these assumptions were not explicitly stated since such information could lead to the construction of models that would make Q-6 unstable with respect to what it attempts to measure. In other words, we do not know how additional information will be interpreted by students since the notions mentioned above are more likely to be absent in the students’ knowledge base.

The desired realization is that regardless of the charge distribution or exactly where charges are located, all charges contribute to the electric field at a given point. Choice F attempts to distinguish between insulators and conductors - insulators as “inhibitors of electric field” and conductors as “supporters of electric field” [40] in terms of student reasoning. This is reflected in whether or not only the charges that are closer to point P are considered. Let us select choice F as the “near charge model.” Considering student explanations (see Chapter 4), a unique one-to-one correspondence does not

<table>
<thead>
<tr>
<th>Q-6 $\rightarrow$ C Q-5</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C 18 $\pm$ 11% 7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F 30 $\pm$ 13% 25 $\pm$ 13%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.10: Percentage transitions between Q-5 and Q-6 for algebra-based students
exist between the choices and the models as has been the case for the previous topics. The percentage distribution and the transitions among choices C and F for Q-5 and Q-6 for the calculus-based sample is given in Tables 3.11 and 3.12.

<table>
<thead>
<tr>
<th></th>
<th>Q-5</th>
<th>Q-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>C</td>
<td>38 ± 13%</td>
<td>62 ± 13%</td>
</tr>
<tr>
<td>D</td>
<td>5%</td>
<td>7%</td>
</tr>
<tr>
<td>E</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>F</td>
<td>55 ± 13%</td>
<td>27 ± 12%</td>
</tr>
</tbody>
</table>

Table 3.11: Response distribution for Q-5 and Q-6 for calculus-based students

<table>
<thead>
<tr>
<th>Q-6→</th>
<th>C</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-5↓</td>
<td>29 ± 12%</td>
<td>7%</td>
</tr>
<tr>
<td>C</td>
<td>34 ± 12%</td>
<td>18 ± 10%</td>
</tr>
</tbody>
</table>

Table 3.12: Percentage transitions between Q-5 and Q-6 for calculus-based students

Choices C and F are selected by the calculus-based students in ways similar to that of the algebra-based students. However, no calculus-based student selected choices B and E - those involving the negative charges. The algebra-based students selected choice B in both questions and choice E in relation to Q-5. Furthermore, choice D is absent in both questions from the algebra-based students. The number of students who select choices C and F between the two questions seem to differ significantly.
3.5 Gaussian surfaces & electric fields

The use of Gaussian surfaces can lead to the conception that only the charges inside the Gaussian surface are responsible for the electric field at a point of interest [40]. The following two questions were given to identify the extent to which the notion of a Gaussian surface would be used to find the electric field between two uniformly charged conducting plates.
Q-7

Two parallel conducting plates are kept at some distance apart as shown below. One plate carries a uniform charge density $\sigma$ and the other a uniform charge density $-\sigma$. A cylindrical Gaussian surface is drawn as shown below. What is the magnitude of the electric field between the plates?

A. $\frac{\sigma}{2\epsilon_0}$
B. $\frac{\sigma}{\epsilon_0}$
C. Zero
D. None of the above
Q-8

Two parallel conducting plates are kept at some distance apart as shown below. One plate carries a uniform charge density $\sigma$ and the other a uniform charge density $-\sigma$. A cylindrical Gaussian surface is drawn as shown below. What is the magnitude of the electric field between the plates?

A. $\frac{\sigma}{\epsilon_0}$
B. $\frac{2\sigma}{\epsilon_0}$
C. Zero
D. None of the above

In Q-7 a cylindrical Gaussian surface is drawn including only one plate, while in Q-8 both plates are included. If the conception exists that, in determining the electric field between the plates, one has to start by identifying the net charge enclosed in the Gaussian surface, then the student will give a nonzero answer only for Q-7. That idea
would yield zero as the value of the electric field within the context of Q-8. On the other hand, if students model that taking both plates into account should double the value of the electric field between the plates, then they should select choice B in the context of Q-8 provided they selected choice B within the context of Q-7. The same reasoning is possible between Q-7 and Q-8 for choice A. The percentage distribution of student responses is given in Table 3.13.

<table>
<thead>
<tr>
<th></th>
<th>N = 49</th>
<th>Q-7</th>
<th>Q-8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>29 ± 13%</td>
<td>37 ± 13%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>57 ± 14%</td>
<td>18 ± 11%</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>8%</td>
<td>39 ± 14%</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>6%</td>
<td>6%</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.13: Response distribution for Q-7 and Q-8 for calculus-based students

It can immediately be seen that the selection of choice C increased by a large factor in going from Q-7 to Q-8. The percentage transitions is given in Table 3.14 below.

<table>
<thead>
<tr>
<th>Q-8→ Q-7</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12 ± 9%</td>
<td>6%</td>
<td>6%</td>
<td>4%</td>
</tr>
<tr>
<td>B</td>
<td>24 ± 12%</td>
<td>10 ± 8%</td>
<td>22 ± 12%</td>
<td>0%</td>
</tr>
<tr>
<td>C</td>
<td>0%</td>
<td>2%</td>
<td>6%</td>
<td>0%</td>
</tr>
<tr>
<td>D</td>
<td>0%</td>
<td>0%</td>
<td>4%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Table 3.14: Percentage transitions between Q-7 and Q-8 for calculus-based students
As can be seen from the above table, the most prominent are the B to A and B to C transitions in the order of Q-7, Q-8. B to A shifts can be thought of as occurring due to consistency in reasoning and having the correct model. Students who shift from B to C can be thought of reasoning in relation to net charge enclosed or the “flux zero” model. A to A and B to B shifts can be thought of as occurring due to the reasoning that “two plates lead to twice as much electric field” or simply the “charge double” model.

3.6 Charge distribution on a conductor

The absence of the electric field under static conditions inside a conductor is an important fact that has been discussed in calculus-based electromagnetism courses. To what extent this fact is used within the contexts of simple charge distributions on a conductor is our next topic of investigation. The questions were given in classroom to FEH students for a vote using their remote controllers. Before the questions were posed, the following facts had been discussed explicitly:

1. Charges on a conductor try to arrange themselves so that like charges attempt to get as “far away” as possible from each other.

2. The absence of the electric field inside a conductor under static conditions. This fact was repeated many times during instruction, particularly under the discussions and calculations using Gauss’s law.

3. The path independence of the electrostatic field was also discussed.
Questions Q-9 and Q-10 were motivated by the need to explore the possible effect fact 2 would have on the reasoning in terms of fact 1. In other words, we investigated what possible models students may have constructed under fact 2 and how such models influence the reasoning under fact 1. Q-9 and Q-10 are shown below.
Q-9

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. Which of the following represent the charge distribution on the inner and outer walls of the shell?
Q-10

A positive charge is kept (fixed) off-center inside a fixed spherical neutral conducting shell. Which of the following represent the charge distribution on the inner and outer walls of the shell?

1

2

3

4

5

6 None of the above
In Q-10, choice 3 attempts to take fact 1 into account. Here, the nearness of the inner positive charge towards one side of the inner wall induces more negative charges on the inner wall in that region. The inner negative charges induce positive charge on the outer wall of the conductor. But these outer positive charges are judged in relation to the inner positive charge and the outer charges move to the “far side” from the inner positive charge. Let us call this reasoning the “far side” model.

In choice 2 of Q-10, the familiar sequential reasoning of charge induction is employed: The positive inner charge induce negative charges on the inner wall and these charges in turn induce positive charges on the outer surface. Here, the charge density of the inner wall is directly mapped to the charge density of the outer wall without any spatial relations to the inner positive charge. Hence, let us call it the “density model.” Note that the induction argument may not necessarily be needed to identify the fact that the outer wall is positive. The same conclusion can be arrived at by thinking that the inner positive charge would repel the “positive charges” in the conductor making the outer wall positive. The question cannot distinguish between two such representations.

Q-9 and Q-10 were given one after the other without providing any feedback in between. Table 3.15 shows the percentage distribution for the choices.

It is seen that in the context of Q-9 all groups selected the correct choice. This question alone cannot distinguish between the correct, “far side” or the “density” models, because of the symmetry suggested in the question. However, within the context of Q-10, the models are distinguishable. Although many groups selected the correct choice, the number of groups having the alternative models is significant. Among the alternative models, the “density model” seems to be more prevalent. This
Table 3.15: Response distribution for Q-9 and Q-10 for FEH students

<table>
<thead>
<tr>
<th>$N_G = 14$</th>
<th>Q-9</th>
<th>Q-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0%</td>
<td>43%</td>
</tr>
<tr>
<td>2</td>
<td>0%</td>
<td>36%</td>
</tr>
<tr>
<td>3</td>
<td>0%</td>
<td>21%</td>
</tr>
<tr>
<td>4</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>5</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>6</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

is understandable due to the direct mapping of charges between the inner and outer walls. It is noted that the idea of the absence of an electric field inside the conductor under contexts such as Gauss’s law does not necessarily translate into an absence of influence. When charges are involved, students seems to be reasoning with respect to the direct consequences between charges.

A pattern that emerges within the context of Q-10 is that of sequential steps of reasoning. Here, the inner positive charge is taken as the “anchor” and the induction reasoning follows from that. Once the inner surface charge has been induced, the reasoning can branch out depending on whether the “density” or the “far side” model is employed. Employing the “density” model is the most straightforward since it takes only the inner surface charge into account. In contrast, as mentioned earlier, the “far side” model has to take into account the spatial relation of the outer surface charge with respect to the inner positive charge.
In both of the alternative models, the absence of the electric field inside the conductor, although explicitly discussed in the class, was not considered. This may result from a strong tendency to reason in terms of charges rather than in terms of fields, which dictate the behavior and distribution of charges. Question Q-11, was given after providing feedback on Q-10.
Q-11

A positive charge $Q$ is kept fixed at the center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. Which of the following represents the charge distributions?

1

2

3

4

5 None of the above
The correct choice (4) was selected by all groups considered groups that gave invalid responses are not included in the discussion). To see how students would respond to a similar but less straightforward question, a question from CSEM [41] was given immediately following Q-11.

Q-12

A positive charge (+q) is kept at the center inside a spherical neutral conducting shell. Outside the sphere is a charge +Q. Which of the following describes the net electric forces on each charge?

1. Both charges experience the same net force directed away from each other.
2. No net force is experienced by either charge.
3. There is no force on Q but a net force on q.
4. There is no force on q but a net force on Q.
5. Both charges experience a net force but they are different from each other.
Choices 1, 2, 4, and 5 were selected 15%, 15%, 61%, and 8%, of the time respectively. Choice 1 may be identified as treating the situation as if the conducting shell is not present [42]. The absence of the electric field inside the conductor could lead to forming the model that there is no net force on either charge (choice 2). Such a model is interesting since, in the context of Q-12, attention has been shifted to the absence of the electric fields and so students tend to ignore the presence of the outer surface charge. This could be a result of the feedback provided to students in terms of the electric field (or the absence thereof) in the conductor in questions Q-9 and Q-10.

Students who would reason in terms of a net force on charge q are very likely to reason in terms of a net force also on charge Q, since, if there is any symmetry argument to be taken into account, it exists inside the shell and not on the outside. This may explain the absence of any votes for choice 3 and the small proportion choosing 5. Students who relate to choice 4 can argue on the basis of the asymmetry of charge distribution on the outside, and the symmetry of charge distribution on the inside in relation to Q-11.

We see from the above that having answered Q-11 correctly does not necessarily imply that students would view Q-12 in a manner that would translate relationships among charge distribution, electric field, and force in a one-to-one fashion. The connections among the three facets are non-trivial and the context seems to have an influence on the transfer of knowledge and models between the questions. The differences can also be understood as differences in representations. It is easier to comprehend charge distributions on a conductor by developing spatial and density relational structures with respect to some reference charge than that to consider the
resultant electric fields (e.g., the surface charge distribution from “density” and “far side” models). However, this mode of representation has to be changed to the field representation if one needs to find the correct outer surface charge distribution. The field representation then should be extended towards any external charge to find the forces. Students’ falling back on the charge representation can lead them to ignore the field representation and hence to make incorrect deductions.

### 3.7 Electrostatic potential of a conductor

By the time instruction in a calculus-based electromagnetism class progresses to the electrostatic potential, students have acquired many elements of knowledge. The attraction and repulsion of charges, the ability of charges to move freely in a conductor until static conditions are reached, the absence of the electric field inside a conductor under such conditions, and the relevant mathematical relations such as $\frac{1}{r^2}$ dependence of the field and Gauss’s law. We developed the questions Q-13, Q-14, and Q-15 so that the reasoning required to arrive at a deduction (regardless of whether it is right or wrong) may make use of different representations.

All three questions deal with the comparison of electric potential at two given points on a conductor. The equilibrium state is suggested implicitly by providing a pictorial representation of charge distribution but not explicitly since the word “equilibrium” may lead students to the idea of a natural “equality” rather than being based on any physical reasoning.
Q-13

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The charge distribution is shown below. What can be said about the electric potential between points A & B which are on the inner surface of the shell?

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-14

A positive charge is kept (fixed) off-center inside a fixed spherical neutral conducting shell. The charge distribution is shown below. What can be said about the electric potential between points A & B which are on the inner surface of the shell?

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-15
A positive charge $Q$ is kept fixed at the center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. The charge distribution is shown below. What can be said about the electric potential between points A & B which are on the outer surface of the shell?

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.

The percentage distribution of choices for the three questions are listed in Table 3.16.

The correct answer in the context of Q-13 may be derived from the symmetry properties. These properties include either the uniform distribution of charges, the
Table 3.16: Response distribution for Q-13, Q-14, and Q-15 for calculus-based students

<table>
<thead>
<tr>
<th></th>
<th>Q-13</th>
<th>Q-14</th>
<th>Q-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0%</td>
<td>11 ± 9%</td>
<td>77 ± 12%</td>
</tr>
<tr>
<td>B</td>
<td>98 ± 4%</td>
<td>14 ± 10%</td>
<td>11 ± 9%</td>
</tr>
<tr>
<td>C</td>
<td>2%</td>
<td>75 ± 13%</td>
<td>5%</td>
</tr>
<tr>
<td>D</td>
<td>0%</td>
<td>0%</td>
<td>7%</td>
</tr>
</tbody>
</table>

equal distances to the induce charges from the center charge, or both. Although the selection of choices for Q-14 and Q-15 can be argued simply as arising due the asymmetries implied, a closer look at students’ explanations reveal two representations. One include reasoning in terms of the distance \( \frac{1}{r} \) relation and the other include the charge “density” relations, that is, more charge imply more potential.

We have categorized students explanations as falling mainly into the above two categories. The classification was carried out by looking for key words “closer,” “near,” and reasoning in terms of the proportionality arguments such as \( \frac{1}{r} \) in the distance relation category. The reference charge is the inner positive charge or the external negative charge. The key words “crowding of charge,” “more charge at...,” “excess charge,” and “uniform charge” were considered in the charge “density” relations category. Such key words were looked for occurring independently within a given question context. When both categories seem to be occurring, they were included in the “other” category. This also include the mentioning of “symmetry” (alone) since it is not straightforward whether the symmetry implies uniformity of charge or equal distances or both. Explanations that cannot be reasonably and logically understood
were also put in the “other” category. Students who did not provide any explanations were not analyzed. Table 3.17 summarizes the percentage occurrence of categories within each question.

<table>
<thead>
<tr>
<th></th>
<th>Q-13</th>
<th>Q-14</th>
<th>Q-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance relations</td>
<td>56 ± 15%</td>
<td>73 ± 14%</td>
<td>32 ± 14%</td>
</tr>
<tr>
<td>Charge ‘density’ relations</td>
<td>15 ± 11%</td>
<td>12 ± 10%</td>
<td>51 ± 15%</td>
</tr>
<tr>
<td>Other</td>
<td>29 ± 14%</td>
<td>15 ± 11%</td>
<td>17 ± 12%</td>
</tr>
</tbody>
</table>

Table 3.17: Categorical distribution for Q-13, Q-14, and Q-15 for calculus-based students

Only 22% of the students can be identified as using distance models consistently, whereas no student can be identified as consistently referring to “density” models.

Both points (A & B) in Q-13, Q-14, and Q-15 are either on the inner or the outer surface of the shell. Q-16 and Q-17 were given to take into account points on different surfaces. However, these two questions were given five weeks later. The figures were drawn in such a way as to extract any differences in reasoning involving positive and negative charges in addition to the above mentioned distance and “density” relations. In Q-17, the number of positive and the negative charges drawn as “concentrated” is the same although this is not dictated by any principle but by the closeness of the external and the inner charges to the respective surfaces.
Q-16

A positive charge $Q$ is kept fixed at the center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. The charge distribution is shown below. What can be said about the electric potential between points A & B? (Point A is on the outer surface and point B is on the inner surface.)

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-17

A positive charge $Q$ is kept fixed off-center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. The charge distribution is shown below. What can be said about the electric potential between points A & B? (Point A is on the outer surface and point B is on the inner surface.)

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.

The percentage distribution of choices for the two questions are listed in Table 3.18. Although the correct choice was selected by the majority within the context of Q-17, it is largely induced by the context rather than the correct reasoning. Only four students were consistent in selecting choice B and just three explanations clearly
<table>
<thead>
<tr>
<th></th>
<th>Q-16</th>
<th>Q-17</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>59%</td>
<td>17%</td>
</tr>
<tr>
<td>B</td>
<td>24%</td>
<td>52%</td>
</tr>
<tr>
<td>C</td>
<td>10%</td>
<td>17%</td>
</tr>
<tr>
<td>D</td>
<td>7%</td>
<td>14%</td>
</tr>
</tbody>
</table>

Table 3.18: Response distribution for Q-16 and Q-17 for calculus-based students

identify that the potential will be the same throughout the sphere. Within the context of Q-16 and Q-17, only once can we explicitly identify a case where a distance relation is been used. This happens if students ignore the external charge in both questions and consider only the relative position of the inner charge with respect to the inner surface.

It is evident that the predominant model is the charge “density” model. Even if the shift of the inner charge toward the inner surface could develop a distance relation, such a relation can be overwritten by the density model since it provides an “explanation” in the context of Q-17. For example, suppose that a student identifies that with reference to the inner charge the distance between it and the inner surface decreases in Q-17, thereby increasing the potential at point B. However, this reasoning would not answer the question “by how much” in relation to point A. If the student is confident on this fact choice D could result. But if charge imbalance has already been recognized in the context of Q-16 then the opposite notion would naturally lead to selecting choice B in Q-17. This is supported by the fact that the most common are the A → B transitions (38%).
3.8 Potential due to a shell of charge

The equation for the electrostatic potential due to a point charge provides a set of procedural rules. These include the identification of the charge responsible for the potential and the distance from the charge to the point of interest. The previous section offered an example where reasoning in terms of distance relations in one context not being present in another. This section further explores this aspect specifically in relation to the equation $V = \frac{kq}{r}$ through questions Q-18, Q-19, and Q-20.
Q-18

A conducting thin spherical shell of radius \( R \) has a total charge \( +Q \) distributed uniformly on its surface. What is the electric potential (relative to a point at infinity) at a point a distance \( r \) from the center where \( r > R \)?

A. \( \frac{Q}{4\pi\epsilon_0(R-r)^2} \)

B. \( \frac{Q}{4\pi\epsilon_0(r-R)} \)

C. \( \frac{Q}{4\pi\epsilon_0(R-r)} \)

D. \( \frac{Q}{4\pi\epsilon_0r} \)

E. \( \frac{Q}{4\pi\epsilon_0R} \)

F. None of the above
Q-19

A conducting thin spherical shell of radius $R$ has a total charge $+Q$ distributed uniformly on its surface. What is the electric potential (relative to a point at infinity) at a point on the surface?

A. $\frac{Q}{4\pi \epsilon_0 R^2}$  
B. Zero

C. Infinite  
D. $\frac{\sigma}{\epsilon_0 R}$

E. $\frac{Q}{4\pi \epsilon_0 R}$  
F. None of the above
Q-20
A conducting thin spherical shell of radius $R$ has a total charge $+Q$ distributed uniformly on its surface. What is the electric potential (relative to a point at infinity) at a point a distance $r$ from the center where $r < R$?

A. $\frac{Q}{4\pi \varepsilon_0 (R-r)^2}$

B. Zero

C. $\frac{Q}{4\pi \varepsilon_0 (R-r)}$

D. $\frac{Q}{4\pi \varepsilon_0 r}$

E. $\frac{Q}{4\pi \varepsilon_0 R}$

F. None of the above
The percentage distribution of choices for the three questions are listed in Table 3.19. The correct choice (D) in the context of Q-18 can occur through two lines of reasoning, by simply assigning $r$ in the equation or by explicitly regarding all of the charge to be concentrated at the center of the shell. When the former happens, it also ignores the fact that charge is distributed throughout.

<table>
<thead>
<tr>
<th></th>
<th>Q-18</th>
<th>Q-19</th>
<th>Q-20</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2%</td>
<td>9%</td>
<td>2%</td>
</tr>
<tr>
<td>B</td>
<td>33 ± 14%</td>
<td>12 ± 10%</td>
<td>21 ± 12%</td>
</tr>
<tr>
<td>C</td>
<td>2%</td>
<td>9%</td>
<td>35 ± 14%</td>
</tr>
<tr>
<td>D</td>
<td>51 ± 15%</td>
<td>5%</td>
<td>19 ± 12%</td>
</tr>
<tr>
<td>E</td>
<td>5%</td>
<td>65 ± 14%</td>
<td>14 ± 10%</td>
</tr>
<tr>
<td>F</td>
<td>7%</td>
<td>0%</td>
<td>9%</td>
</tr>
</tbody>
</table>

Table 3.19: Response distribution for Q-18, Q-19 and Q-20 for calculus-based students

Choice B, in Q-18 is given to take into account the reasoning that would occur if the distance relation ($r - R$) is explicitly identified in relation to charges. The same reasoning is expected with respect to choice C in the context of both Q-19 and Q-20. Thus, if the reasoning is consistent, we would expect a student who would start with choice B in Q-18 to proceed to choices C, C, respectively in Q-19 and Q-20. However, only one student did this. Eight students selected choice B in Q-18 and C in Q-20. These students in the context of Q-19 shifted to selecting A (1), B (2), C (1), and E (4), where the number of times the choice was selected is shown in brackets.

The correct selection of choice (E) within the context of Q-19, although prominent, also can occur through two lines of reasoning. These are to consider the charge to be concentrated at the center or simply to identify $R$ as the distance that should be
present in the equation. Again, the latter mode of reasoning ignores the fact that the charge is distributed. We infer that most of the correct choices within the context of Q-18 and Q-19 occur through incorrect reasoning. This is supported by the absence of the correct choice to the same extent within the context of Q-20. Only three students consistently selected the correct choices in all three questions for the correct reasons.

It is also interesting to note that if choice D is selected by a student in both Q-18 and Q-20, then choice E was also selected by the same student in the context of Q-19. Such a set of choices were selected by 14% of students. These students seem to be functioning consistently considering all of charge to be at the center of the shell for all three questions.

3.9 A grounded conductor

As discussed in the previous sections, the reasoning in terms of attraction and repulsion of charges, and more charge implying more potential are more common than reasoning involving the superposition of fields and potentials, equipotential surfaces, and absence of electric fields inside conductors. In short, the models that students generate in specific contexts are more charge-oriented than field- and/or potential-oriented. We attempt to extract such reasoning further through questions Q-21 and Q-22.
Q-21

A positive charge is brought and kept fixed in location close to a neutral conducting rod. The end further away from the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?
A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-22

A positive charge is brought and kept fixed in location close to a neutral conducting rod. The end closer to the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).

Both questions appeared in the midterm examination of the calculus-based electricity and magnetism class. In Q-21, the far side of the conducting rod is grounded. We hypothesized that the following reasoning would be used: Students would consider polarization of charges in the conductor due to the external charge. Since like charges repel, the far side will then contain charge that is similar to the external charge. The grounding then acts as a path for these charges to flow or move away from the conductor. This answer could also result from thinking (as was discussed
under Q-9, Q-10, and Q-11) that the external charge will repel the like charges farthest from itself. The process finally leaves charge that is opposite in sign to the external charge.

It is often the case that in the discussion of “charging by induction,” the attraction and repulsion of charges are used to provide a rigid representation of the process that takes place. That is, the ground wire is shown to be connected to the side which is opposite to that of the external charge. This we hypothesized could lead to the formation of a mental model in which the charge that resides around the ground wire will *always* be removed. If this mode of reasoning forms a rigid mental model, then it is to be expected that charge of same type as that of the external charge would remain when the side closer to the external charge is grounded. This is explored in Q-22 following Q-21. The choice C could result in taking the ground connection to remove all charge, the ground connection making the conductor neutral, etc. The response distribution for the two questions among the calculus-based students and the percentage transitions are shown in Tables 3.20 and 3.21, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Q-21</th>
<th>Q-22</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10 ± 6%</td>
<td>37 ± 10%</td>
</tr>
<tr>
<td>B</td>
<td>66 ± 10%</td>
<td>29 ± 9%</td>
</tr>
<tr>
<td>C</td>
<td>24 ± 9%</td>
<td>34 ± 10%</td>
</tr>
</tbody>
</table>

Table 3.20: Response distribution for Q-21 and Q-22 for calculus-based students
We see that the selection of choices A and B between the two questions are statistically different. It is seen that B to A transitions are the most frequent. Twenty-six percent and 20% of the students seem to consistently select choices B and C. It can be understood that a student who selects choice C within the context of Q-21 would select the same within the context of Q-22, since there are no changes within the question contexts that would suggest or cue a major shift from having a “neutral model” to another. The questions Q-23 and Q-24 were given to a different group of students in their midterm examination in the calculus-based electricity and magnetism course.

<table>
<thead>
<tr>
<th>Q-22→</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>4%</td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>B</td>
<td>29 ± 9%</td>
<td>26 ± 9%</td>
<td>11 ± 7%</td>
</tr>
<tr>
<td>C</td>
<td>3%</td>
<td>0%</td>
<td>20 ± 8%</td>
</tr>
</tbody>
</table>

Table 3.21: Percentage transitions between Q-21 and Q-22 for calculus-based students
Q-23

A negative charge is brought and kept fixed in location close to a neutral conducting rod. The end further away from the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-24

A negative charge is brought and kept fixed in location close to a neutral conducting rod. The end closer to the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).

Q-23 and Q-24 differ from Q-21 and Q-22 only with respect to the external charge. By highlighting this difference, we hoped to find any significant differences in reasoning involving the positive and the negative external charges. Such a comparison can be carried out since both groups of students received similar instruction from the same instructors. The responses and the transitions between answers to the two questions are shown in Tables 3.22 and 3.23, respectively.
Table 3.22: Response distribution for Q-23 and Q-24 for calculus-based students

<table>
<thead>
<tr>
<th></th>
<th>Q-23</th>
<th>Q-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>76 ± 9%</td>
<td>31 ± 10%</td>
</tr>
<tr>
<td>B</td>
<td>6 ± 5%</td>
<td>26 ± 9%</td>
</tr>
<tr>
<td>C</td>
<td>19 ± 8%</td>
<td>43 ± 10%</td>
</tr>
</tbody>
</table>

Table 3.23: Percentage transitions between Q-23 and Q-24 for calculus-based students

<table>
<thead>
<tr>
<th>Q-24→ Q-23</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>23 ± 9%</td>
<td>24 ± 9%</td>
<td>28 ± 9%</td>
</tr>
<tr>
<td>B</td>
<td>3%</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>C</td>
<td>5%</td>
<td>1%</td>
<td>13 ± 7%</td>
</tr>
</tbody>
</table>

Comparing the tables for Q-21, Q-22 and Q-23, Q-24, there seems to be a statistically significant difference in the occurrence of choice C. That is, when the positive external charge is involved, 11% shifted to choice C in Q-22 after selecting the correct choice (B) in Q-21, whereas when the negative external charge is involved, 28% of the students shifted to choice C in Q-24 after selecting the correct choice (A) in Q-23. A contingency table can be drawn for transitions with respect to the correct choice. For example, in the context of Q-21 and Q-22, 23 students were consistent in selecting choice B (call it - → -), while 26 students shifted from choice B to choice A (- → +). Ten students shifted from B to C (call it a shift to neutral or → 0). A similar identification can be made among Q-23 and Q-24 with respect to choice A. The contingency Table 3.24 is given below.
The contingency table above yields a $\chi^2(2, N = 124) = 6.23, p < 0.05$. Thus, we may conclude that the sign of the external charge is statistically significant in determining the transitions. We see from the table that the dominant factor in determining this significance is the selection of choice C. If the third column is absent in the contingency table, this yields a $\chi^2(1, N = 90) = 0.03, p > 0.05$. Thus, the transitions from the correct choice to choice C is significantly different between the two external charges.

Questions Q-23 and Q-24 were also given to algebra-based students during the sixth week of instruction (that is after the midterm examination of the calculus-based students). The responses and the transitions between the two questions are shown in Tables 3.25 and 3.26, respectively.

<table>
<thead>
<tr>
<th>$N = 47$</th>
<th>Q-23</th>
<th>Q-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40 ± 14%</td>
<td>15 ± 10%</td>
</tr>
<tr>
<td>B</td>
<td>28 ± 13%</td>
<td>34 ± 14%</td>
</tr>
<tr>
<td>C</td>
<td>32 ± 13%</td>
<td>51 ± 14%</td>
</tr>
</tbody>
</table>

Table 3.25: Response distribution for Q-23 and Q-24 for algebra-based students
A comparison of the frequency distributions for Q-23 and Q-24 for the calculus and algebra-based students show that there is a statistically significant difference in responses for Q-23 ($\chi^2(2, N = 133) = 18.97, p < 0.05$) but not for Q-24 ($\chi^2(2, N = 133) = 4.43, p > 0.05$).

Comparing Tables 3.26 and 3.23, we see that while the B $\rightarrow$ B transitions is absent among the calculus-based students, it is present among the algebra-based students.

Questions Q-25 and Q-26 are a variation on Q-21 and Q-22. The basic difference is that, instead of a conducting block, a conducting sphere is present. Since the inner and the outer surfaces are at the same potential in both questions, any positive charge on the outer surface will be removed.

<table>
<thead>
<tr>
<th>Q-24$\rightarrow$ Q-23</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9%</td>
<td>11 $\pm$ 9%</td>
<td>21 $\pm$ 12%</td>
</tr>
<tr>
<td>B</td>
<td>4%</td>
<td>19 $\pm$ 11%</td>
<td>4%</td>
</tr>
<tr>
<td>C</td>
<td>2%</td>
<td>4%</td>
<td>26 $\pm$ 12%</td>
</tr>
</tbody>
</table>

Table 3.26: Percentage transitions between Q-23 and Q-24 for algebra-based students
Q-25

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The outer surface of the sphere is then grounded. What type of charge remain on the outer surface after grounding?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-26

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The inner surface of the sphere is then grounded. What type of charge remain on the outer surface after grounding?

A. Positive charge.

B. Negative charge.

C. No charge (Neutral).

The responses and the transitions between the two questions are shown in Tables 3.27 and 3.28, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Q-25</th>
<th>Q-26</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14 ± 11%</td>
<td>42 ± 16%</td>
</tr>
<tr>
<td>B</td>
<td>25 ± 14%</td>
<td>25 ± 14%</td>
</tr>
<tr>
<td>C</td>
<td>61 ± 16%</td>
<td>33 ± 15%</td>
</tr>
</tbody>
</table>

Table 3.27: Response distribution for Q-25 and Q-26 for calculus-based students
Table 3.28: Percentage transitions between Q-25 and Q-26 for calculus-based students

<table>
<thead>
<tr>
<th></th>
<th>Q-26→</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>8%</td>
<td>3%</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>17 ± 12%</td>
<td>6%</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>17 ± 12%</td>
<td>17 ± 12%</td>
<td>28 ± 15%</td>
<td></td>
</tr>
</tbody>
</table>

While selection of choice C within the context of Q-26 drops with respect to Q-25, the selection of choice A has increased from Q-25 to Q-26. These again are reflective of the fact that the charge on the particular surface being grounded has been removed. However, more students seem to be consistent between the two questions with respect to choice C. Questions Q-27 and Q-28 further explore similar reasoning by asking about the charge on the inner surface after grounding of the outer and the inner surface, respectively.
Q-27

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The outer surface of the sphere is then grounded. What type of charge remain on the inner surface after grounding?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-28

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The inner surface of the sphere is then grounded. What type of charge remain on the inner surface after grounding?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).

The responses and the transitions between the two questions are shown in Tables 3.29 and 3.30, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Q-27</th>
<th>Q-28</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>22 ± 14%</td>
<td>14 ± 11%</td>
</tr>
<tr>
<td>B</td>
<td>56 ± 16%</td>
<td>44 ± 16%</td>
</tr>
<tr>
<td>C</td>
<td>22 ± 14%</td>
<td>42 ± 16%</td>
</tr>
</tbody>
</table>

Table 3.29: Response distribution for Q-27 and Q-28 for calculus-based students
The correct choice (B) within the context of Q-28 seems to occur more frequently than the correct choice (C) within the context of Q-26. Also, as expected, choice C within the context of Q-28 increases. Again, more students choose the correct choice consistently between the two questions.

In the FEH class, the potential on a charged shell was discussed with the active involvement of students. Students were asked to predict the shape of the $V$ versus $r$ curve for the outside and the inside of the shell. Although students seemed to be successful in their predictions for the region outside the shell, this was not so for inside the shell. The instructor then discussed explicitly the absence of electric field inside the shell, thereby implying that the potential is the same everywhere inside the shell including the surface. It was also stated that the entire shell can be treated as a point charge at the center of the shell at distances greater than the radius of the shell.
The ability demonstrated by students in predicting the shape of the curve for outside of the shell shows that the equation for the point charge potential is directly mapped onto the sphere, measuring distances from the surface. This agrees with the reasoning exhibited by students within the context of Q-18 through Q-20. Questions Q-29 and Q-30 were administered after these discussions.
Q-29

Consider two concentric conducting spherical shells of radii a and b where \( b > a \). The inner and outer shells carry charges of \(+Q\) and \( Q\) respectively distributed uniformly on its surfaces. Imagine grounding the outer shell. What is the charge on the outer shell after grounding?

1. \(+Q\)
2. Zero
3. \(-Q\)
4. None of the above
Q-30

Consider two concentric conducting spherical shells of radii a and b where \( b > a \). The inner and outer shells carry charges of \(+Q\) and \( Q\) respectively distributed uniformly on its surfaces. Imagine grounding the inner shell. What is the charge on the inner shell after grounding?

1. \(+Q\)
2. Zero
3. \(-Q\)
4. None of the above

The reason for asking the above questions is twofold. One is to see whether the existing knowledge elements can be combined to arrive at the correct solution. The other is to allow us to infer the possible student models. By combining the knowledge of the potential inside and outside of a sphere, the superposition principle, and grounding as means of making the potential zero (all elements that were provided
during instruction at one time or another), one can solve the two questions consistently. The responses and the transitions between the two questions are shown in Tables 3.31 and 3.32, respectively.

\[
\begin{array}{c|cc}
G & Q-29 & Q-30 \\
\hline
1 & 0\% & 24\% \\
2 & 31\% & 59\% \\
3 & 69\% & 12\% \\
4 & 0\% & 6\%
\end{array}
\]

Table 3.31: Response distribution for Q-29 and Q-30 for FEH students

\[
\begin{array}{c|cccc}
Q-30 & 1 & 2 & 3 & 4 \\
\hline
Q-29 & 0\% & 25\% & 0\% & 6\% \\
1 & 0\% & 0\% & 0\% & 0\% \\
2 & 0\% & 25\% & 0\% & 6\% \\
3 & 25\% & 31\% & 13\% & 0\% \\
4 & 0\% & 0\% & 0\% & 0\%
\end{array}
\]

Table 3.32: Percentage transitions between Q-29 and Q-30 for FEH students

The correct solution for Q-29 and Q-30 are choices 3 and 4, respectively. Comparison of the occurrence of these choices among the two questions show that the students did not select the correct choice within the context of Q-29 based on the knowledge elements described above. Choice 2 is basically the model that grounding removes the charge. However, it is selected more frequently within the context of Q-30 than in Q-29. This may be due to the student model that electrons (negative
charges) transfer from the ground to the shell neutralizing the charge on the shell. This is feasible only in the case of Q-30, where there is positive charge. The negative charge transfer model may also explain why choice 3 is prominent within the context of Q-29. The negative charges on the outer shell may repel the negative charges that attempt to transfer from the ground - leaving the outer shell "undisturbed."

The reasoning required to solve Q-29 is not limited to the superposition of potentials. Gauss's law can be employed to argue that, in order to make the electric field zero inside the outer shell, the amount of negative charge should not be changed. In fact, this explanation was provided to students after they answered Q-29 but before administration of Q-30. The $3 \to 1$ transition may be occurring as a direct result of specific instruction. The argument students may be employing is that, since the absence of the electric field inside the shell is a must, the charge $+Q$ on the inner shell should remain the same regardless of grounding.

The point that we make here is that higher-order knowledge elements always seems to take a back seat in relation to the most perceptible superficial features presented in a question. Reasoning in terms of "attraction," "repulsion," "transfer," and "removal" seems to be dominant, rather than abstractions such as "absence of field," and "superposition."
3.10 Charged conducting spheres - force on a charge

A course in electricity and magnetism takes students to an increased level of abstraction as the course progresses. Forces are discussed in terms of fields, and fields, in turn, are discussed in terms of potentials. The following two questions provide an opportunity for the students to reason using the different representations mentioned above. The questions were presented to the calculus-based students during spring 2002 after they had completed discussion of the electrostatic potential.
A and B are two identical metal spheres. Sphere A carries a net charge of +4 C and sphere B carries a net charge of +2 C. Initially they were kept apart from each other. The two spheres are then brought together and were made to touch each other for some time before separating them again.

Q-31
Think about the instant (a snapshot) when the two spheres make contact. What can you say about the Coulomb forces acting on a charge on sphere A?
A. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is zero.
B. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is not zero.
C. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is zero.
D. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is not zero.
E. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is zero.
F. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is not zero.
Think about an instant (a snapshot) after some time where the two spheres are still in contact. What can you say about the Coulomb forces acting on a charge on sphere A?

A. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is zero.
B. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is not zero.
C. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is zero.
D. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is not zero.
E. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is zero.
F. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is not zero.

The questions directly require students to determine the force on a charge “on sphere A.” Therefore, it is to be expected to a large degree that the knowledge associations that students would make will include “force.” How this force manifests, however, is constrained by the models they have or that they create on the spot. The responses and the transitions between the two questions are shown in Tables 3.33 and 3.34, respectively. Only transitions corresponding to popular choices are considered.
<table>
<thead>
<tr>
<th>$N = 76$</th>
<th>Q-31</th>
<th>Q-32</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>B</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>C</td>
<td>5%</td>
<td>13 ± 8%</td>
</tr>
<tr>
<td>D</td>
<td>35 ± 11%</td>
<td>14 ± 8%</td>
</tr>
<tr>
<td>E</td>
<td>4%</td>
<td>30 ± 10%</td>
</tr>
<tr>
<td>F</td>
<td>48 ± 11%</td>
<td>30 ± 10%</td>
</tr>
</tbody>
</table>

Table 3.33: Response distribution for Q-31 and Q-32 for calculus-based students

<table>
<thead>
<tr>
<th>Q-32→ Q-31↓</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>12 ± 7%</td>
<td>4%</td>
<td>11 ± 7%</td>
</tr>
<tr>
<td>F</td>
<td>1%</td>
<td>22 ± 9%</td>
<td>18 ± 9%</td>
</tr>
</tbody>
</table>

Table 3.34: Percentage transitions between Q-31 and Q-32 for calculus-based students

Choices D and E exhibit statistically significant difference between Q-31 and Q-32. A D to D transition can be expected on the basis that sphere B is considered as an external charge and reasoning purely in terms of forces between charges with direct mapping relations to Coulomb’s law. However, a D to F transition is unexpected on that basis. F to E transitions are expected on the basis of correct reasoning. F to F transitions also seem to occur in the same proportion (within the errors) as that of F to E.
3.11 Current in an electric circuit

Resistors control the current in a circuit. In instruction, we make the approximation that the connecting wires have negligible distributed resistance compared to the lumped resistance (dictated by the resistor) in the circuit. Current is then taken to be low if the resistor has high resistance and vice versa. How does such a proportionality statement translate within a single circuit with respect to current? The following two questions attempt to measure this.

Q-33
Consider the circuit below. The cross sectional area of the conducting wire is the same throughout. Rank the currents at points A & B.

A. $I_A > I_B$
B. $I_A = I_B$
C. $I_A < I_B$
Consider the circuit below. The cross sectional area of one segment of the conducting wire is less than the rest of the wire. Rank the currents at points A & B.

A. \( I_A > I_B \)
B. \( I_A = I_B \)
C. \( I_A < I_B \)

It is expected that the students would consider the cross-sectional area of segments and model them as resistors. Then localized reasoning in terms of the inverse proportionality between current and resistance should lead them to select choice A within the context of Q-34. The responses and the transitions between the two questions are shown in Tables 3.35 and 3.36, respectively.

We see clearly the shift from the correct to incorrect choice between the two questions. However, the reasoning is consistent. The idea of resistance is considered in a localized sense as expected. Such reasoning may due to the procedural knowledge...
of Ohm’s law and the specific contexts under which students learn to apply it. The other important aspect worthy of discussion is how the notion of current is modeled by students.

One student remarked that, due to the larger area at A, the “current [is] able to flow faster” in the context of Q-34. Since current is the amount of charge per unit time, it can only be meaningfully interpreted as been “high” or “low.” The use of “flow faster” implies that current itself is taken as a material (e.g., charge) and not as a process that involves a rate. Another student makes a similar comment within the context of Q-34: “...more current can pass through... .” This again implies thinking in terms of charges, although the word current is used. This suggests that, implicit within the above reasoning is the notion of “containing.” The larger area seems to “hold” or allow for more current (charge) than the smaller area. Consider the statement:
“there is less current at B than there is at A because the area of the wire is so much smaller at B.” The opposite choice (C) in the context of Q-34 basically occurs from the argument that, narrower the area, the smaller is the resistance, leading to a higher current. Students who selected the correct choice in both questions were less explicit in their reasoning, but understood that the current has to be the same throughout the wire. Only one student seemed to consider the entire wire as a single resistor.

Q-34 was also given to algebra-based students with an additional option (choice D) of “cannot say for sure,” which 15% of the students selected. Choices A, B, and C were selected by 52%, 17%, and 15%, respectively. The total number of students tested was 46. Neglecting the extra choice given, there is no significant difference between the calculus- and the algebra-based students with respect to Q-34 ($\chi^2(2, N = 75) = 0.214, p > 0.05$). However, the algebra-based students were more explicit in their reasoning, particularly with respect to selecting the correct choice; “…current is the flow of electrons, and they do not jump in and out of a circuit, the current must be the same throughout the circuit.”

Q-34 was also given with the extra choice stated above to FEH students (The choices were named 1, 2, 3, and 4 instead of A, B, C, and D in that order to allow for the voting machine). In addition, Q-35 and Q-36 were also administered to these students (see Appendix A). Table 3.37 shows the distribution of choices.

We see that majority of students selected choice 1 within the context of Q-34. This is in agreement with choices by earlier calculus- and algebra-based students. If a proportionality condition is derived for the electric field from the current, then we again expect choice 1 to be dominant within the context of Q-35. However, choice 3 is the most popular choice in Q-35. Questioning by the instructor revealed that students
Table 3.37: Response distribution for Q-34, Q-35, and Q-36 for FEH students

<table>
<thead>
<tr>
<th></th>
<th>Q-34</th>
<th>Q-35</th>
<th>Q-36</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>67%</td>
<td>33%</td>
<td>13%</td>
</tr>
<tr>
<td>2</td>
<td>13%</td>
<td>7%</td>
<td>27%</td>
</tr>
<tr>
<td>3</td>
<td>20%</td>
<td>60%</td>
<td>60%</td>
</tr>
<tr>
<td>4</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

function with respect to the idea of resistance in Q-35 as well. The higher resistance in the narrow segment leads students to identify a higher potential difference across it. This, in turn would make the electric field in the narrow segment greater than that in the wider segment. What may be noted is the handling of variables between Q-34 and Q-35. The necessity of considering currents in Q-34 has led students to identify “obstacles to the flow of current,” hence, to reason in terms of resistance. Higher resistance, then, is taken to imply less current. However, within the context of Q-35, higher resistance is taken to imply higher potential, leading to a higher electric field. In the former, the notion of potential is dropped, whereas in the latter, the notion of current is dropped.

Similarly, if proportionality conditions are derived from current to the drift speed, we expect choice 1 to be dominant in the context of Q-36. However, this is not so and seems rather to be generated from the electric field. It is possible, however, that students’ selection of choice 2 in Q-36 may be generated by the idea of equalities in the current, since feedback was provided on this fact immediately after Q-34. The transition tables (3.38, 3.39, 3.40) among the three questions are shown below. Choice 4 is not included since it was never selected.
### Table 3.38: Percentage transitions between Q-34 and Q-35 for calculus-based students

<table>
<thead>
<tr>
<th>Q-35→ Q-34</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13%</td>
<td>0%</td>
<td>53%</td>
</tr>
<tr>
<td>2</td>
<td>7%</td>
<td>0%</td>
<td>7%</td>
</tr>
<tr>
<td>3</td>
<td>13%</td>
<td>7%</td>
<td>0%</td>
</tr>
</tbody>
</table>

### Table 3.39: Percentage transitions between Q-35 and Q-36 for calculus-based students

<table>
<thead>
<tr>
<th>Q-36→ Q-35</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7%</td>
<td>13%</td>
<td>13%</td>
</tr>
<tr>
<td>2</td>
<td>0%</td>
<td>0%</td>
<td>7%</td>
</tr>
<tr>
<td>3</td>
<td>7%</td>
<td>13%</td>
<td>40%</td>
</tr>
</tbody>
</table>

### Table 3.40: Percentage transitions between Q-34 and Q-36 for calculus-based students

<table>
<thead>
<tr>
<th>Q-36→ Q-34</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0%</td>
<td>27%</td>
<td>40%</td>
</tr>
<tr>
<td>2</td>
<td>7%</td>
<td>0%</td>
<td>7%</td>
</tr>
<tr>
<td>3</td>
<td>7%</td>
<td>0%</td>
<td>13%</td>
</tr>
</tbody>
</table>

### 3.12 Magnetic force - right-hand versus left-hand rule

The right-hand rule combines the velocity of a charged particle, the direction of the magnetic field, and the direction of the magnetic force relative to a right-handed coordinate system. Given the directions of velocity and the magnetic field, the direction of the magnetic force can be determined using the right-hand rule. However,
this does not mean that employing a left-handed coordinate system would change the direction of the magnetic force on a particle given the same velocity and field direction. The following two questions were developed to see to what extent students refer to physical reality versus functioning in terms of defined rules. The questions were given only to calculus-based students.

Q-37

A positive charge moving horizontally at constant speed enters a magnetic field region from the left side. The magnetic field in the region is pointing in to the page. Which way will the charge move?

1. P direction
2. Q direction
3. R direction
Q-38

If instead of the Right Hand Rule a *Left Hand Rule* is used, which way will the charge move? (Refer Q-37)

![Diagram showing vectors P, Q, R, and B]

1. P direction
2. Q direction
3. R direction

The responses and the transitions between the two questions are shown in Tables 3.41 and 3.42, respectively.

<table>
<thead>
<tr>
<th></th>
<th>N = 44</th>
<th>Q-37</th>
<th>Q-38</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68 ± 14%</td>
<td>27 ± 13%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7%</td>
<td>7%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>25 ± 13%</td>
<td>66 ± 14%</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.41: Response distribution for Q-37 and Q-38 for calculus-based students
Table 3.42: Percentage transitions between Q-37 and Q-38 for calculus-based students

<table>
<thead>
<tr>
<th>Q-38→</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-37↓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11 ± 9%</td>
<td>0%</td>
<td>57 ± 15%</td>
</tr>
<tr>
<td>2</td>
<td>0%</td>
<td>5%</td>
<td>2%</td>
</tr>
<tr>
<td>3</td>
<td>16 ± 11%</td>
<td>2%</td>
<td>7%</td>
</tr>
</tbody>
</table>

We see that the majority of the students shifted from the correct choice to the diametrically opposite choice between the two questions. Even if a student incorrectly responded to Q-37 by considering the corresponding response to Q-38 (that is, whether it is the opposite of Q-37) we may infer the reasoning involved. We see that the next highest in Table 3.42 is the transition from 3 to 1 between the two questions. Thus, the two questions taken together can largely eliminate the errors associated with applying the right-hand rule per se and identify the way students function with a set of given rules, or in shifting from Q-37 to Q-38, how rules (e.g., a left-hand rule) are constructed.

A similar set of questions was given to the algebra-based students. Here, instead of a charged particle, the attraction or the repulsion of two parallel current-carrying wires was investigated, depending on whether the right- or a left-hand rule was applied.
Q-39
Two parallel wires carry currents in the same direction. Applying the Right Hand Rule, what can be said about the magnetic force between the two wires?

A. The wires attract each other.
B. The wires repel each other.
C. Nothing happens.

Q-40
Two parallel wires carry currents in the same direction. If instead of the Right Hand Rule a *Left Hand Rule* is used, what can be said about the magnetic force between the two wires?

A. The wires attract each other.
B. The wires repel each other.
C. Nothing happens.
The responses and the transitions between the two questions are shown in Tables 3.43 and 3.44, respectively. We see that the majority of the students consistently selected the correct choice. It is also interesting to note that no student who selected the correct choice within the context of Q-39 shifted to any other choice within the context of Q-40. Although Q-39 and Q-40 differ from Q-37 and Q-38, the reasoning required in both sets is the same. Both Q-38 and Q-40 ask for the use of a left-hand rule. However, the responses in the two contexts seems to be significantly different. We explore students’ comments in Chapter 4.

<table>
<thead>
<tr>
<th>N = 26</th>
<th>Q-39</th>
<th>Q-40</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>46%</td>
<td>58%</td>
</tr>
<tr>
<td>B</td>
<td>31%</td>
<td>19%</td>
</tr>
<tr>
<td>C</td>
<td>23%</td>
<td>23%</td>
</tr>
</tbody>
</table>

Table 3.43: Response distribution for Q-39 and Q-40 for algebra-based students

<table>
<thead>
<tr>
<th>Q-40→Q-39</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-39↓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>46%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>B</td>
<td>12%</td>
<td>15%</td>
<td>4%</td>
</tr>
<tr>
<td>C</td>
<td>0%</td>
<td>4%</td>
<td>19%</td>
</tr>
</tbody>
</table>

Table 3.44: Percentage transitions between Q-39 and Q-40 for algebra-based students
3.13 Current as vectors - Ampère’s Law

Ampère’s law states that the integral of the magnetic field around a closed path is proportional to the net current enclosed by the path (Ampèrian loop). Given the discussion in the text ([6], Section 30-3) used by the calculus-based students, it is feasible that students take an extension of the law to mean that the currents should be treated as coming out or going in straight through the loop. This may occur because the text consider only such situations. Accordingly, this may lead students to treat current as a vector quantity and not in terms of a flux (flow). The following questions investigate the existence of such models. The responses and the transitions between the two questions are shown in Tables 3.45 and 3.46, respectively.
Q-41

An Amperian loop is drawn around two current carrying wires as shown below. What is $\int \vec{B} \cdot d\vec{s}$ equal to around the loop?

A. $\mu_0(i_1 - i_2)$

B. Zero

C. $\mu_0(i_1 + i_2)$
Q-42

An Amperian loop is drawn around two current carrying wires as shown below. The angle between the two wires is $\theta$. What is $\int \vec{B} \cdot d\vec{s}$ equal to around the loop?

\[
\mathbf{A. \ \mu_0 (i_1 \cos \theta + i_2)}
\]

\[
\mathbf{B. \ \mu_0 (i_1 + i_2)}
\]

\[
\mathbf{C. \ \mu_0 (i_1 + i_2 \cos \theta)}
\]

<table>
<thead>
<tr>
<th></th>
<th>Q-41</th>
<th>Q-42</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3%</td>
<td>76%</td>
</tr>
<tr>
<td>B</td>
<td>7%</td>
<td>21%</td>
</tr>
<tr>
<td>C</td>
<td>90%</td>
<td>3%</td>
</tr>
</tbody>
</table>

Table 3.45: Response distribution for Q-41 and Q-42 for calculus-based students
We see that the majority of the students selected choice A within the context of Q-42. In Q-42, it has not been said that the current $i_2$ is perpendicular to the plane of the loop. Thus, students have taken the general direction of current $i_2$ on face value and internalized it as being perpendicular. Current $i_1$ is then mapped onto the direction of current $i_2$ by taking the cosine of the angle between the two wires. This shows the model that the currents that are “straight” through the loop has to be considered. Furthermore, the reasoning shows that the currents are treated as vectors. Explanations such as “only components perpendicular to the loop are used in the calculation” and “... need to find the vertical component of current 1 to find the answer” support this view. The text, of course, does not state anywhere that only vertical currents should be considered. However, the examples that are discussed seem to lead students to such a conclusion.
3.14 Current enclosed - Ampère’s Law

Ampère’s law is analogous to the Gauss’s law in electrostatics. The electric flux is proportional to the charge enclosed \( q_{\text{enc}} \) by the Gaussian surface, while \( \int \vec{B} \cdot d\vec{s} \) around the Ampèrian loop is proportional to the current enclosed \( i_{\text{enc}} \). As discussed in the context of Gauss’s law (see Section 3.5), the notion of “enclosed charge” can lead to students ignoring the superposition principle and even overlooking the “reality” of the electric field. We hypothesized that similar reasoning is possible in the context of Ampère’s law once the ideas of Ampèrian loop and \( i_{\text{enc}} \) are introduced. Question Q-43 was administered to identify these facets. The responses are shown in Table 3.47.
Q-43

Three current carrying wires 1, 2 & 3 are shown below. The wires 2 & 3 are enclosed by an Amperian loop. The magnetic field at point P is due to?

A. 1 only
B. 2 & 3 only
C. All wires

Table 3.47: Response distribution for Q-43 for calculus-based students

<table>
<thead>
<tr>
<th>N = 29</th>
<th>Q-43</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7%</td>
</tr>
<tr>
<td>B</td>
<td>72%</td>
</tr>
<tr>
<td>C</td>
<td>21%</td>
</tr>
</tbody>
</table>

Table 3.47: Response distribution for Q-43 for calculus-based students
The most obvious feature in Q-43 is the Ampèrian loop. A point P on the loop is specifically identified to remind students that the question is not asking for $\int \vec{B} \cdot d\vec{s}$ around the loop since that cannot be defined for a single point. Thus, it is the magnetic field at point P that is of interest in the question.

Students seem to extract $\vec{B}$ out of $\int \vec{B} \cdot d\vec{s}$ and associate the resulting field to point P. Since it is been extracted from the integral expression, the field is then interpreted to result from the current enclosed by the loop. This can be seen in the explanations such as “by Ampere’s law we know that only currents enclosed by the loop produce the magnetic field” and “because current outside the loop does not account for any magnetic field.” One student who claimed that all currents contribute phrased the explanation in terms of the superposition principle, while others simply commented that all wires produce a field at point P.

3.15 Force on a charge in a magnetic field

Determination of the magnetic force on a charged particle in a magnetic field poses a challenge to students since it employs the right-hand rule, which has to be performed in three dimensions. It is observed that students apply the right-hand rule in many different ways in a given problem. Also, it is only a procedural rule, which may not require the construction of a mental model as far as orienting one’s hand is concerned. We attempt to by pass such difficulties in interpretation by giving a set of questions that does not require the application of the right-hand rule but can be answered based on the knowledge of the behavior of charged particles in a magnetic field. In Q-44 through Q-46, the knowledge required is the same and all three lead
to a “null” result. However, we vary the context of the questions from pictorial and language to purely language. Also, the questions consider slightly different notions—direction and speed.

Q-44

A positive charge is placed at rest in a magnetic field as shown below. Which way will the charge move?

A. Left
B. Right
C. Up
D. Down
E. It will not move at all
Q-45
A positively charged particle is moving horizontally to the right in a uniform magnetic field which is pointing in the same direction. What is the direction of the magnetic force on the charge?

A. Left
B. Right
C. Up
D. Down
E. None of the above

Q-46
A positively charged particle is moving at a constant speed in some direction in a region where there are no fields present. Suppose the particle now enters a magnetic field region, which is uniform and pointing in the same direction as the particle is initially moving. What happens to the speed of the particle?

A. Stays the same
B. Decreases
C. Increases
The pictorial representation was given to look for the particular context factors that would affect student reasoning. For example, in Q-44, the image of a set of arrows representing the magnetic field may cause students to consider it as applying a force on the charge in the direction of the field despite the charge being at rest. In Q-45, the notion that the charge is moving in the same direction as the field may cue students to consider explicitly the mathematical representation of such a situation. Q-46 was given with the hypothesis that although the velocity of the charge is parallel to the field, the field would exert a force on the particle as it enters the region. Such a scenario would be thought of in terms of everyday experience where the presence of a force would lead objects to change their state of motion. Implicit within this notion is the fact that the charge enters the field region, thus introducing itself to a novel (field or force) situation. The responses for the three questions for the algebra-based students is shown in Table 3.48.

<table>
<thead>
<tr>
<th>N = 44</th>
<th>Q-44</th>
<th>Q-45</th>
<th>Q-46</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0%</td>
<td>2%</td>
<td>61 ± 14%</td>
</tr>
<tr>
<td>B</td>
<td>18 ± 11%</td>
<td>9%</td>
<td>16 ± 11%</td>
</tr>
<tr>
<td>C</td>
<td>30 ± 13%</td>
<td>23 ± 12%</td>
<td>23 ± 12%</td>
</tr>
<tr>
<td>D</td>
<td>23 ± 12%</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>30 ± 13%</td>
<td>61 ± 14%</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.48: Response distribution for Q-44, Q-45, and Q-46 for algebra-based students
We see a significant difference in the selection of the correct choice between Q-44 and Q-45. Student selection of the correct choice has doubled in moving from the former to the latter question. The correct choice seems to hold in same proportion in Q-46 as that of Q-45. The transition tables (3.49, 3.50) between Q-44, Q-45, and Q-45, Q-46 are shown below. Only the most popular choices are considered.

<table>
<thead>
<tr>
<th>Q-45→ Q-44</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>7%</td>
<td>0%</td>
<td>23 ± 12%</td>
</tr>
<tr>
<td>D</td>
<td>9%</td>
<td>2%</td>
<td>9%</td>
</tr>
<tr>
<td>E</td>
<td>5%</td>
<td>0%</td>
<td>23 ± 12%</td>
</tr>
</tbody>
</table>

Table 3.49: Percentage transitions between Q-44 and Q-45 for algebra-based students

<table>
<thead>
<tr>
<th>Q-46→ Q-45</th>
<th>A</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td>E</td>
<td>45 ± 15%</td>
<td>9%</td>
</tr>
</tbody>
</table>

Table 3.50: Percentage transitions between Q-45 and Q-46 for algebra-based students
Students’ explanations suggest that the picture context of Q-44 made them to consider it in a different manner from that of the other two. All three questions were posed to the calculus-based students. We see that most students selected the correct choice (see Table 3.51).

<table>
<thead>
<tr>
<th></th>
<th>Q-44</th>
<th>Q-45</th>
<th>Q-46</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5%</td>
<td>7%</td>
<td>86 ± 10%</td>
</tr>
<tr>
<td>B</td>
<td>23 ± 12%</td>
<td>11 ± 9%</td>
<td>2%</td>
</tr>
<tr>
<td>C</td>
<td>16 ± 11%</td>
<td>2%</td>
<td>11 ± 9%</td>
</tr>
<tr>
<td>D</td>
<td>0%</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>57 ± 15%</td>
<td>77 ± 12%</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.51: Response distribution for Q-44, Q-45, and Q-46 for calculus-based students

Although the frequency of responses differs among the algebra- and the calculus-based students, it can be noted that the proportion of the correct choice is higher (within the errors) within the context of Q-45 than in Q-44, even among the calculus-based students.
3.16 Magnetic induction - translational versus rotational movement

As identified by Allen [43], “some kind of motion” is identified by students as giving rise to induced currents in the context of magnetic induction. We hypothesize that there could be a difference in the way different types of motion (for example, rotational as opposed to translational) are perceived by students as giving rise to magnetic induction. This hypothesis is motivated by the question of why the idea of “some motion” is prominent in such contexts. Questions Q-47 and Q-48 demonstrate that there are indeed students who distinguish between the two types of motion. Our answer to the question of “why” raised above is deferred until Chapter 4.
Q-47

The following situations involve a CONDUCTING rectangular wire loop in a magnetic field. The magnetic field has CONSTANT magnitude and direction in the region shown by a cross X. The magnetic field is into the paper.

In the three pictures shown below the wire loop is moving at a constant velocity (i.e. at a constant speed in the fixed direction as shown by the arrow).

In picture I the loop is just moving into the magnetic field region.
In picture II the loop is moving in the field.
In picture III the loop is moving out of the field region.
Which situations correspond to an induced current in the loop?

A. I
B. II
C. III
D. I and II
E. II and III
F. I and III
G. All
H. None
Q-48

The following situations involve a CONDUCTING circular wire loop in a magnetic field. The magnetic field has CONSTANT magnitude and direction in the region shown by a cross X. The magnetic field is into the paper.

Consider the following three pictures:

In picture IV the loop is at REST in the magnetic field region.

In picture V the loop is MOVING AT CONSTANT VELOCITY in the direction shown.

In picture VI the loop is ROTATING clockwise AT CONSTANT ANGULAR VELOCITY about an axis through its center.
Which situations correspond to an induced current in the loop?

A. IV  
B. V  
C. VI  
D. IV and V  
E. V and VI  
F. IV and VI  
G. All  
H. None

To eliminate the perceptual difficulties associated with three-dimensional situations, the questions deal with only two-dimensional figures. Although the magnetic field is directed into the page, the need to apply the right-hand rule is eliminated by not requiring students to specify the direction of the induced magnetic field or the current, again eliminating errors associated with three-dimensional situations. Q-47 is a standard question in the context of magnetic induction that includes only translational motion. Q-48 includes both translational and rotational motions.

The responses and the transitions between the two questions are shown in Tables 3.52 and 3.53, respectively. Only the transitions between popular choices are shown.

It is seen that more students selected the choice involving only translational motion as compared to students selecting both the translational and rotational motions within the context of Q-48. Thus, students seem to distinguish between translational...
and rotational motions. The prominence of choice G in the context of Q-47 is expected based on Allen’s [43] comment cited above. We see that very few students who selected the correct choice (F) within the context of Q-47 selected choice B or E within the context of Q-48. This is the opposite of the case of students who selected the incorrect choice (G) within the context of Q-47. No student selected the correct choice (H) within the context of Q-48 after having selected choice G in Q-47. G to E transitions can be thought of as functioning consistently regardless of any motion, whereas G to B transitions can be thought of as functioning consistently only for linear motion.
3.17 Magnetic induction - change in field and area

The set of questions considered below investigates how consistently students reason with respect to changes in the magnetic field and the area of a wire loop. No application of the right-hand rule is required, merely a recognition of the direction of the induced magnetic field. Also, all questions refer to two-dimensional pictorial representations.
A wire loop is placed in a uniform external magnetic field as shown below. If the magnitude of the external magnetic field increases, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
Q-50

A wire loop is placed in a uniform external magnetic field as shown below. If the magnitude of the external magnetic field decreases, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
Q-51

A wire loop is placed in a constant uniform external magnetic field as shown below.

If the external magnetic field is suddenly switched-off what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
Q-52

A wire loop is placed in a constant uniform external magnetic field as shown below.

If the loop starts shrinking, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
Q-53

A wire loop is placed in a constant uniform external magnetic field which is confined to the circular area as shown below (i.e. the magnetic field is present only within the circular area).

If the loop starts expanding, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field

In Q-51 the word “suddenly” is used to make students think of an immediate change and not the absence of a magnetic field to begin with. We hypothesize that switching off of the field will lead students to consider an absence of cause, which in turn will lead them to consider the absence of any other effect. In Q-53, we expect
students would simply consider the area of the loop. This, we believed, would be the case through the use of examples which with they are familiar or the need to identify firm boundaries via a line drawing which would give rise to the notion of area. The responses from algebra-based students is shown in Table 3.54.

<table>
<thead>
<tr>
<th></th>
<th>Q-49</th>
<th>Q-50</th>
<th>Q-51</th>
<th>Q-52</th>
<th>Q-53</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>14 ± 10%</td>
<td>66 ± 14%</td>
<td>14 ± 10%</td>
<td>45 ± 15%</td>
<td>32 ± 14%</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>80 ± 12%</td>
<td>27 ± 13%</td>
<td>5%</td>
<td>36 ± 14%</td>
<td>52 ± 15%</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>7%</td>
<td>7%</td>
<td>82 ± 11%</td>
<td>18 ± 11%</td>
<td>16 ± 11%</td>
</tr>
</tbody>
</table>

Table 3.54: Response distribution for Q-49, Q-50, Q-51, Q-52, and Q-53 for algebra-based students

We see that most students selected the correct choice within the contexts of Q-49 and Q-50, with 61% being consistent with the correct choice. However, a drastic difference in reasoning is exhibited within the context of Q-51. There, the change of the magnetic field from having a non-zero value to zero seems to be not viewed as a decrease in the field in the same sense as that of Q-50. Although in both Q-50 and Q-52 the flux through the loop is decreasing, only 32% of the students were consistent in selecting choice A in both cases. As expected, the increase in area of the loop in Q-53 seems to be taken as an increase in flux through the loop itself, thereby requiring an opposing induced field. Only one student consistently selected the correct choice for all five questions. This student also gave excellent explanations for all five.

A question similar to that of Q-53 (see Appendix A; Q-54) was administered to the FEH students (14 groups). The group choices were 64%, 14%, and 21% in the order increase, decrease, and stays the same. The question was given after students
seemed to be comfortable in answering many forms of questions about magnetic induction. For example, out of fifteen groups, fourteen predicted the direction of the induced magnetic field correctly in a question similar to that of Q-51. However, such questions were posed after students were trained in class using the ALPS [44] kit.

3.18 Magnetic induction - Induced electric fields

As discussed in the previous section, the switching off of the magnetic field was considered as the absence of the field itself. Induction is almost exclusively discussed in terms of wire loops immersed within regions of magnetic fields. Students consider induced magnetic fields produced by induced currents. The currents are, however, moving charges. Are students able to identify the force on individual charges in the context of changing magnetic fields? Is there a conceptual barrier that exists between identifying currents as opposed to identifying the force on a single charge in such contexts? The following three questions attempt to identify the existence of any such notions.
Q-55

A wire loop is placed in an external magnetic field which is confined to the circular (gray) area as shown below (i.e. the magnetic field is present only within the gray area).

If the external magnetic field increases only within the gray area, what is the direction of the *induced* current in the loop?

A. Clockwise
B. Counterclockwise
C. There is no induced current
Q-56

A square wire loop is placed in an external magnetic field which is confined to the circular (gray) area as shown below (i.e. the magnetic field is present only within the gray area).

If the external magnetic field increases only within the gray area, what is the direction of the *induced* current in the loop?

A. Clockwise
B. Counterclockwise
C. There is no induced current
A constant external magnetic field is confined to the circular (gray) area as shown below (i.e. the magnetic field is present only within the gray area). Charge Q is placed at rest inside the magnetic field area where as charge q is placed at rest outside the magnetic field area.

If the external magnetic field now increases only within the gray area, what can be said about the two charges?

A. Only Q will start moving  
B. Only q will start moving  
C. Both charges will start moving  
D. None of the charges will move

The questions were given to algebra-students and we do not expect these students to have the physics knowledge of “changing magnetic fields produce electric fields.” It is to eliminate the (possibility) of absence of such knowledge that we pose the
questions together. In the context of Q-55 and Q-56, if a student is able to identify the presence of a current then the student should be able to identify that a force must have been exerted on both the charge $Q$ and $q$. This is due to the fact that one can recognize $q$ as belonging to the wire in Q-55 and $Q$ as belonging to the wire in Q-56. Both charges were drawn in a single diagram expecting students would not consider electric forces between the two charges. This was of concern after the questions were given but only one student considered such a scenario in his or her explanations. Thus, it did not affect our interpretation of answers to the question. The responses are shown in Table 3.55.

<table>
<thead>
<tr>
<th></th>
<th>Q-55</th>
<th>Q-56</th>
<th>Q-57</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>45 ± 15%</td>
<td>32 ± 15%</td>
<td>52 ± 15%</td>
</tr>
<tr>
<td>B</td>
<td>48 ± 15%</td>
<td>52 ± 15%</td>
<td>15 ± 11%</td>
</tr>
<tr>
<td>C</td>
<td>8%</td>
<td>15 ± 11%</td>
<td>18 ± 12%</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td>15 ± 11%</td>
</tr>
</tbody>
</table>

Table 3.55: Response distribution for Q-55, Q-56, and Q-57 for algebra-based students
Let us not concern ourselves with whether the students applied the right-hand rule correctly or not in determining the direction of the current within the context of Q-55 and Q-56 (most students seemed to answer them correctly). The fact to note is that, by identifying some direction of the current, they accept the existence of the current in both Q-55 and Q-56. However, the majority indicated that only the charge $Q$ would be moving within the context of Q-57. Thus, there seems to be a clear difference in reasoning involving closed loops of currents and single charges. Furthermore, the absence of the magnetic field in the region in which $q$ is placed seems to play a role in determining whether or not $q$ will move.

A similar set of questions was given to the calculus-based students. However, instead of drawing both charges in the same figure (Q-57), two figures were drawn within the same question (see Appendix A, Q-58). For some students, there seemed to be a confusion in the two diagrams and the magnetic field was taken to be non-zero outside and not inside. Furthermore, only 16 students participated in the survey. Among the participants, however, 62% selected the correct choice within the context of Q-55 and Q-56, while 19%, 31%, 25%, and 25% selected choice A, B, C, and D, respectively, within the context of Q-58. Also, the reasons given by the calculus-based students were hard to interpret, although the ideas such as absence of the magnetic field not influencing $q$ and charges being placed at rest, thereby not exerting any force on them, can be readily identified.
3.19 Neutral spheres and neutral point particles

During the last week of instruction, the following questions were given. The purpose of the questions was to see how students would handle the concept of “neutral” and the concept of a “point.” The reason for considering the concept of “neutral” is to see whether it is considered as a charge by itself. This is a possibility since the entire instruction dealt with the concept of “charge.” The concept of a “point” is coupled to the “neutral” concept by forming a neutral point particle.
Q-59
Two neutral metal spheres are kept fixed at a close distance. What will happen?

A. Attract
B. Repel
C. Nothing happens

Q-60
A stationary point charge is kept at a close distance from a stationary neutral point particle. Which of the following is true?

A. The point charge exerts more force on the neutral point particle.
B. The neutral point particle exerts more force on the point charge.
C. Both exerts equal amount of force on each other.
D. There is no force between the point charge and the neutral point particle.

We hoped that students would not consider the gravitational attraction between the neutral spheres in Q-59. On the other hand, we did not want to use the word “electrostatic force” in order not to confuse the students. Only one student selected the choice attraction based on gravitational forces. Thus, the gravitational force does not seem to cause problems within the present context. In Q-60, a point charge has been brought close to a neutral point particle. The responses are shown in Table 3.56.
Table 3.56: Response distribution for Q-59 and Q-60 for algebra-based students

<table>
<thead>
<tr>
<th></th>
<th>Q-59</th>
<th>Q-60</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12%</td>
<td>23%</td>
</tr>
<tr>
<td>B</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>C</td>
<td>88%</td>
<td>62%</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>15%</td>
</tr>
</tbody>
</table>

It is immediately seen that the choices are dependent on whether the consideration is between two neutral spheres or between a point charge and a point neutral particle. Clearly the presence of a charge in Q-60 brings with it a different set of conceptions to students’ minds. Most students who reasoned that neutral spheres would experience no forces between them argued that the point charge, “having charge,” would exert some force on the neutral point particle. This takes two different forms: either “having charge” would lead students to identify the point charge as having an advantage, thereby exerting more force on the neutral particle or the neutral particle is itself considered by students as a “charge,” leading them to apply Newton’s third law.
CHAPTER 4

A CLOSER LOOK AT STUDENTS’ MODELS

4.1 Introduction

In this chapter, we discuss students’ own explanations for their answers. Our reasons for providing the explanations are twofold. One is to validate the hypotheses we had formed before administering the questions to students as discussed in Chapter 3. The other is to obtain knowledge of what elements of learning students manipulate in order to construct their models. We seek to understand whether the models are applied rigidly in the questions, or whether they are applied piecemeal or are easily altered. If so, we would like to determine what context factors cue the shift in reasoning. We also look for the possible ontological constraints that may be functioning at a deeper level.

4.2 Newton’s third law in electrostatics

Q-1 and Q-2 differ from each other essentially with respect to the pictorial representation. No picture was given in the context of Q-1 while Q-2 includes a picture. Furthermore, in Q-1 we have two point charges while in Q-2 we have a point charge and a charge distribution. The sphere, however, carries a total charge less than that of the point charge. Regardless of whether students are algebra-based or calculus-based,
no student selected the choice that the +1 C point charge would exert a greater force on the +3 C point charge. The situation is different within the context of Q-2, where the sphere carried a +1 C charge. About 11% of the students in each group thought that the sphere exerted more force on the point charge.

Let us look at some individual responses:

S1:

Q-1: “A” because it has a stronger charge, therefore it will exert more force on the point with the smaller charge.

Q-2: A. The size of the object does not matter in these forces. Only the distance between the two and the amount of charge matters.

S2:

Q-1 A. The +3 exerts more force because it has a greater electric charge.

Q-2 A. Again the point has more electric charge thus exerts more force.

S3:

Q-1 A. Coulombs and force are directly related.

Q-2 A. The charge is condensed but it’s still 1 vs 3. 3 is more.
S4:

Q-1: A. Because 3 is larger than 1 and a Coulomb unit is the force which exists on a charged object.

Q-2: Because it doesn’t matter how large the object is, what does matter is the amount of charge each object has. This is why the small charged rod in lecture moved a very large piece of lumber.

It is clear that students who consistently selected choice A for both Q-1 and Q-2 selected it because they saw it as a “stronger charge.” However, it is doubtful that some students who reasoned in such manner did not know that the Coulomb force involves both charges and not just one. This is indicated by S1 who, in the context of Q-2, mentions that the force depends on the “distance between the two and the amount of charge.” However, the whole of the relationship of the Coulomb’s law involving both charges is not considered, only proportionality relating to a single charge, which is extracted to mean “unequal charges imply unequal forces.”

Another student argued in terms of $F = Eq$, again extracting $F \propto q$, not considering the presence of the term $E$. The explanation given by S3 seems to make an association to a competitive situation where “more” seems to imply an advantage. This particular student also makes a comparison of the point charge with the charge distribution arriving at the notion that the charge is condensed in the point charge. The student is considering the charge distribution and the physical dimension but settles on the notion that higher charge should impart more force.
Student S4 makes an association to their classroom demonstration. In this context, although the student claims that the physical dimension does not matter, it seems to have been taken into account through analogy. The large piece of lumber is identified with the large sphere, whereas the small charged rod is identified with the point (small) charge. However, the dimensional analogy is dropped in the light of the observation. The observation that the small rod was explicitly charged and the piece of lumber was not seems to lead the student to identify a “charge difference” (perhaps the student knows that the piece of lumber is neutral to begin with). The small charged rod carries more charge, which seems to be again matched with the point charge with the observational event attached to it. The student does not distinguish between mechanical forces and the Coulomb force. However, what is important in the present context is the structural analogy that has been made between the demonstration and the features present in Q-2. It must be noted that both S3 and S4 extract a relationship between the unit of charge—the Coulomb—and the electrostatic force. Implicit in this reasoning is the notion that charge units imply force units. Since the amount of charge is an inherent property, the above relation may naturally lead to reasoning about electrostatic force as an inherent property and not as an interaction between two or more charged objects. This can be seen in S4’s remark “...a Coulomb unit is the force which exists on a charged object.” We can hypothesize that the concept of force as originating “within” rather than through interactions has its roots from the varied commonsense observations we make about the world we live in. The superficial argument that the “stronger,” “greater” charge exerts more force can be identified as a p-prim. Let us take a look at the following comments.
S5:

Q-1: C. Because the equation for Coulomb’s law uses both charges to find the force in between them so therefore for force between them/on them is equal.

Q-2: B. Because the point charge it [is] thought of as approaching zero and has no effect on the sphere.

S6:

Q-1: C. When object A exerts a force on object B, object B exerts an equal and opposite force on object A, even if the two systems aren’t equal in mass or charge.

Q-2: B. The point charge is always very tiny (close to zero) so that it doesn’t affect the force on the sphere, however, the force of the sphere affects the point charge.

Students S5 and S6 reasoned correctly within the context of Q-1 but incorrectly within the context of Q-2. Both students do not explicitly mention the word “point” within the context of Q-1, but they use the Coulomb force law to arrive at the correct conclusion. S6 explicitly states that inequalities in mass or charge do not matter with regard to equal and opposite forces. However, the presence of the point charge in Q-2 is not viewed the same way as in Q-1. The point charge in the latter case was judged in relation to the large sphere in terms of dimension, rather than with the concept of charge. We argue that the difference in reasoning arises due to a change in representation chosen by the students. In Q-1, both students are using purely the language representation to arrive at their conclusions. In the case of Q-2, the picture-like representation is interpreted literally together with an explicit meaning (that is, of insignificance) derived for the point charge in relation to the sphere. The argument
that the above students make within the context of Q-2 could also have been applied to the context of Q-1 with regard to the point charge since, theoretically, a point charge is dimensionless. Thus, it should have been insignificant in both questions. Thus, the shift in reasoning is parallel to a shift in the representation.

Let us look at two students who answered both questions correctly.

S7:

Q-1: C. Because though they have different charges they are pulling on each other equally just like I’m pulling on the earth and the earth is pulling on me equally.

Q-2: C. Same as number one for the [same] reasoning even though they have a different mass.

S8:

Q-1: C. Because (a) Newton’s third law applies and (b) the formula (Coulomb’s Law) gives the same magnitude answer in each case, although the direction will be different.

Q-2: It should still be C, because of Newton’s third law. Determining r requires more info about the charge distribution on the sphere.

Students S7 and S8 reason correctly and consistently in both questions. S7 further makes an association with the gravitational force, which seems to be aiding him or her in understanding the present situation. This student seems to be mapping the larger charge to a larger mass (that of Earth) and a smaller mass to him/herself. The same reasoning guides S7 within the context of Q-2. S8 brings in knowledge of charge distributions and correctly identifies that the charge distribution has to
be known in order to determine r, in which case the Coulomb’s law equation can be directly applied. The argument that S8 makes in “determining r...” is a highly non-trivial step. This student essentially tries to understand the context through Coulomb’s law, the equation that applies directly only for point charges. S8 makes the reductionist attempt in “looking” for the “center of charge.” Once this step is achieved or envisioned, Q-2 is the same as Q-1. While S7 relates explicitly a specific mental model, S8 seems to function within propositional representations.

The reasoning pattern between the algebra- and the calculus-based students does not differ except for the fact that some of the calculus-based students arrived at the correct answer by considering the charge on the sphere to be concentrated at the center - “The charge on the sphere is concentrated at the center. For the same reason as in Q-1, the two charges are multiplied by each other. Therefore, they each exert an equal force on each other.”

The two questions can be used to identify the two major reasoning patterns that emerge within the contexts. The correct model-equal amounts of force–and the alternative model - that a higher value of charge will exert a larger force on a charge of smaller value do not consider the fact that both charges are involved in the electrostatic force. The model, which involves the ideas of dimension, can also be identified in the idea that the physical size of the object carrying the charge is given the advantage in terms of force.
4.3 Neutral metal rod near a charged object

Questions Q-3 and Q-4 were motivated by the classroom demonstrations where the concept of polarization in metals is demonstrated. A neutral metal rod is brought close to a pivoted charged rod, which is shown to attract. The idea of polarization in the metal rod is then discussed—with the unlike charges attracting to the end close to the charged object and the like charges repelling to the far end of the metal. What sort of models are students building within such a context? We hypothesized two major models:

1. The metal rod as a whole is neutral. Therefore the polarization occurs in the same way regardless of which end of the rod is brought close to the charged object (correct model).

2. By bringing in one end of the metal rod, the charges of same sign as the object were repelled to the far end. If the far end is now brought close, becoming the near end, then the charged object should repel the metal rod. Within this category we further hypothesized that the idea of charge being fixed at one end, or the idea of “oppositeness,” could lead to incorrect reasoning. The idea of oppositeness is simply a statement of the form “near end attracts implies far end repels.” In this case, the oppositeness of “near - far” are matched to the oppositeness of “attract - repel.” We expected the two questions to be regarded by students in an empirical and observable sense, meaning that the far end does not become the near end instantaneously beating the time it takes for polarization to occur. Of course, there is no action-at-a-distance in electricity and magnetism and, furthermore, the polarization times are of the order
of $10^{-18}$ s ([45], pg. 57). Since the demonstrations were carried out in class, we expected students to imagine the same experimental procedures for the two questions.

The following are a representative sample of student responses:

S1:

Q-3: A. Because the metal has flowing electrons that can polarize.

Q-4: A. Because the metal rod is not charged, the object is. Nothing changes as far as why the metal rod is attracted to the object.

S2:

Q-3: A. The electrons will rearrange in the uncharged metal rod depending on the charge the object has. Regardless of the charge of the object the uncharged metal rod will move the opposite charge as the charged object closer to the object, therefore, attraction occurs.

Q-4: A. Metals are conductors, the electrons are free to move around, therefore, regardless of what side of the rod is touched, the same charge that the object has will move to the opposite side of the rod, therefore, opposite charges will be closer to each other and attraction will occur.
Q-3: A. Because some electrons will move within the metal rod in response to the proximity of the charge. If the charged object is positive, some electrons will move closer to the charged object, yielding a negative charge at the close end and attraction between that end and the charged object. If the charged object is negative, some electrons in the rod will move away from the charged object, yielding a positive charge at the close end and repulsion between that end and the charged object.

Q-4: I think B [repulsion] should occur instantaneously, but then the electrons in the metal rod would move again and there would be attraction as above. Since electrons move very quickly, the only result we would observe is A. Therefore A would be the right answer.

S4:

Q-3: A. This is a case of induction, where charge is induced with no contact from the object inducing this charge. The metal rod will be attracted because the metal rod’s charge will be redistributed and an excess of the charge opposite that of the charged object is then present on the side facing the charged object. Thus, with the metal rod and charged object having opposite charges, they are attracted to each other.

Q-4: A. As soon as the metal rod is pulled away, the metal rod becomes neutrally charged again. When brought close to the same charged object, the now neutrally charged rod will have its charge redistributed, through induction. On the side closest to that of the charged object, an excess of charge opposite that of the object will be present on the rod, thus making them attracted them to one another.
All of the students quoted above use correct reasoning. S3 also thought about the instantaneous scenario but identified the observable effect. A sample of responses from students who shifted from identifying an attraction to that of repulsion is given below.

S5:

Q-3: A. Metal rod is attracted by the charged object because the charged object would displace the charge on the uncharged object. Therefore, the two objects would be attracted toward each other.

Q-4: B. Metal rod is repelled by the charged object because the two similar charges would be next to each other and would want to repel each other.

S6:

Q-3: A. Since the metal rod is uncharged, it is presumed to be neutral. Unlike charges from the neutral object are attracted by the charged object’s charges that are opposite.

Q-4: B. Since the other end of the metal rod now has all the same charges at that end as the charged object, it will be repelled by the charged object.
S7:

Q-3: Answer A because it doesn’t matter what the charge is. The charge on the object will push the same charges to the far side of the rod and attract the opposite charges in the rod. So the metal rod will be attracted.

Q-4: Answer B because all the charges that are the same as the charge on the object will be on the other end of the rod. When the rod is brought close to the object, they will have the same charges and therefore repel each other.

S8:

Q-3: A. Metal rod is attracted by the charged object. ... if it was negatively charged, the electrons in the uncharged metal rod would be repelled as far away as possible from the negatively charged object. This would leave protons on the side closest to the negatively charged object. Unlike charges attract so the metal rod would be attracted to the charged object. This would also be the case for a positively charged object except for the fact that protons would be repelled in the uncharged object leaving the electrons behind ... thus attracting to the positively charged object.

Q-4: B. Metal rod is repelled by the charged object. Because a charged was induced onto the rod, if the same charged object was brought close to the other end of the metal rod the charges would have been alike. Like charges repel so the metal rod would be repelled by the charged object.
S9:

Q-3: A, the charge of the object will attract the opposite charges within the rod and repel the like charges within the rod. Therefore, when these charges have been induced, the rod will be attracted to the charged object.

Q-4: B, the charge has been induced, meaning that the like charges have been pushed to the opposite side of the rod. Therefore, the opposite side of the rod has a charge like that of the charged object, meaning that they will repel.

S10:

Q-3: A because the charged object will attract the opposite charges in the metal rod to the end closer to the charged object and repel the like charges to the far end of the rod.

Q-4: B because the opposite end of the rod will now contain the same charges as the object and they will repel each other.

All of the answers from S5 - S10 are consistent with each other. The responses indicate that students tend to concentrate on one process and not another. All of the students above identified the process of polarization or of a distribution of charges occurring within the context of Q-3. However, the same process was not identified within the context of Q-4, and suggest implicitly the charge “fixation” model, whereas the response from S10 seems to support the fixed charge model explicitly.

Consider the following responses.
S11:

Q-3: A. The electrons in the uncharged metal object will redistribute so that unlike charges are attracted to each other.

Q-4: C. Since the initial object and the metal rod never touched no net charge was created in the metal rods, it’s electrons simply redistributed. They will not redistribute again.

S12:

Q-3: D. you cannot say without knowing what type of charge the object has. if the object is positive, the metal will probably attract since the electrons will move to the side closest to the charged object. However, if the charged object is negatively charged, the uncharged metal will most likely repel since like charges attract and electrons move better than protons do.

Q-4: D. Again you cannot say without knowing what type of charge the object has. Whatever happens to one side of the uncharged metal ... the opposite will happen on the reverse side.

The explanation of S11 suggests that, once the initial polarization has taken place, nothing else would happen. We argue that an important aspect of student reasoning emerges from the above examples, which may be explained using the idea proposed by Chi, et al [30]. It focuses on the difference involving the reasoning in terms of matter and in terms of processes. As described in Chapter 2, Chi et al. have argued
that conceptual changes are difficult to actuate between different categories. The main categories involve matter and processes, where processes essentially describe the interactions.

Within the context of Q-3, students correctly recognize the process (polarization) involved that unlike charges attract and like charges repel. To this extent, students make use of microscopic models. But once polarization has taken place, the above students seem to function in a matter mode within the context of Q-4. By this we mean that the end of the metal rod is given a “permanent” set of charges and that the next line of reasoning occurs with respect to these charges and not with respect to the polarization process.

Similar reasoning was also observed among the FEH students. Two insulator rods were charged with opposite charges and the attraction/repulsion effects on similarly-charged insulator rods were demonstrated to the students. Students were then asked to predict the effect if a metal sheet were to be brought close to one of the charged rods. Many students responded that it attracts. It was then demonstrated that the metal sheet indeed is attracted. When asked what would happen if the sheet of metal were brought near an oppositely charged rod, some students responded that it would repel. Some students argued that a different sheet of metal is needed to see any effect. This type of reasoning seems to agree with that of the student S11, who argued that no process would take place after the initial polarization. It should be mentioned that in the (honors) lecture hour during which the demonstrations were performed, the idea of polarization was not explicitly discussed. Thus the reasoning can be thought
of as arising due to students’ previous knowledge, classroom demonstrations, and
the models they may be forming on the spot. The comment of student S12 that
“whatever happens to one side of the uncharged metal ... the opposite will happen
on the reverse side” is along our starting hypothesis of “oppositeness.”

Another fundamental argument can be proposed to explain the fixed charge model.
This argument relies on the cause-and-effect relation and the idea of irreversibility,
in particular, the irreversibility of resources. In common experience, processes occur
in particular directions and in the course of the process certain resources are used.
The end effect is a fixed-end result that is not easily reversed back to the initial
state. For the fixed-charge model, the cause of the external charge leads to the effect
of charge separation. Since the process of polarization has taken place, the notion
of irreversibility of a process can now lead into the fixed charge model. The causal
explanations coupled with irreversibility of resources may also explain the “current
consumption” model often observed in the context of electric circuits containing light
bulbs ([11],[12],[13]).

4.4 Charges on insulators and conductors - superposition of electric fields

In the following discussions, algebra students will be referred to as Sa whereas
calculus students will be referred to as Sc. Consider the following explanations.
S1a:

Q-5: C. Because both sides have charges and anything that has a charge can have an electric field.

Q-6: C. It doesn’t matter if it is conducting material or not because it is an isolated system. Both sides will still have an electric field that effects point P.

S2a:

Q-5: C. No matter the material, the field is a result of all charges.

Q-6 C. Same as [Q-5].

S3c:

Q-5: C, both plates have charges which influence the electric field at point p so they both define the field for p.

Q-6: C, the fact that the plates are now a conducting material doesn’t matter since the charges on each sides of the plates were the same so the point p is still influenced the same on the electric field of point p.

S4c:

Q-5: C. Because you have to add both plates.

Q-6: C. Because all the charge must be added.
All of the above students bring in the principle of superposition implicitly and identify that all charges must contribute to the field at the given point regardless of the material. These students reason consistently between the two questions. The “all charge model,” however, can also be reasoned via models that do not necessarily include the principle of superposition. Consider the following explanations:

S5a:

Q-5: F. Both charges effect the force at point P. Only the charges closer though because the plates are insulators.

Q-6: C. All the charges on both plates affect point P because the charges are free to move around within the plates because they are conductors.

S6c:

Q-5: F. The electric field is due to the charges on both plates but because they are plates of insulator material the charges on the sides farther from P do not have an effect on the electric field.

Q-6: C. Since the plates are made of conducting material all of the charges on both plates effect the electric field at point P.

S7c:

Q-5: F. the charges on the opposite sides do not affect the point P because the electric field does not travel through insulator material.

Q-6: C. All the charges affect point P because the electric fields can travel through the conducting material.
S8c:

Q-5: F. Charge is not free to move in an insulator. So you can determine the electric field of attraction by looking at the charges on the outside of the plate closest to P.

Q-6: C. In a conducting material, charge moves freely. Therefore, charges on both sides of each plate must be accounted for determining the electric field.

S9c:

Q-5: The electric field at point is due to the charges on both plates which are on the sides closer to P. The charged particles in insulators are not free to move. So only the charges on both plates which are on the sides closer to P can produce an electric field.

Q-6: The electric field at point P is due to all charges on both plates if the plates are made up of conducting material. Conductors are materials in which charged particles are free to move. So the charges on both sides of the plates can produce an electric field.

Students S5a through S9c all moved from the “near charge model” to the “all charge model” considering differences between insulators and conductors. In selecting choice C within the context of Q-6, the above students considered conductors as having the ability to “pass the electric field through them” or having the property such that “charge could move freely on them.” In the latter case, the notion that charges could move freely on the conductors has been taken as the criterion to judge whether charges on both sides of the plates would contribute to the electric field. We argue that
such reasoning falls into the category of models for which the possibility of dynamics leads students to consider those effects in the question. The above students only seem to be considering the possibility of charges moving in the conductor. However, some students also consider an explicit charge rearrangement within the context of Q-6. A representative sample of such explanations is given below:

S10a:

Q-5: F. Charges on both plates which are on the sides closer to P. Because it is made up of insulator material, the charges are not free to move so the electric field at point P is due to only the charges on the sides closer to P. The charges on the outer side stay where they are and are insulated from affecting the field.

Q-6: C. All the charges on both plates. Because the plates are made up of conducting material, the charges are free to move to the sides closer to P so the electric field at point P is due to all the charges on both plates.

S11a:

Q-5: F. All charges on the side closer to P directly affect the electric field at P. The charges on the other side of the plate will not affect the electric field at point P because the plate is blocking any type of electric field that these charges may emit.

Q-6: C. All charges on both plates will affect P because the charges on the side further from P will now distribute to the side closer to P due to the attraction caused by the opposite charge that is on the other plate. All charge will then be on the side closer to P and will directly affect the electric field at P.
S12c:

Q-5: F, due to the insulation of each plate, it makes it harder for either plate to move its charge around or THROUGH the other side of the plate, so the field around P is due to the charge on the sides of the plates facing P.

Q-6: C, since the charge is free to move around and through the plate, the field is due to the entire charge on each plate.

S13c:

Q-5: F. Since the material is made up of insulator material, the charges on the outer sides of the plates will not have an effect on P.

Q-6: C. If the material is made up of conducting material, the charges will be free to move through the plates and have an effect on P.

Students S10a-S13c consider the movement of charge explicitly within the context of Q-6. Under the attraction of positive (or negative) plate, negative (or positive) charges move to the side facing point P. Once this happens, there are no charges on the “outer” sides of the plates and the reasoning automatically lead to the “all charge” model.
We see that under the moving charge model, students are led to a different interpretation (or manipulation) of the problem itself. All of the above models can be reversed to accommodate the reasoning within the context of Q-5 and Q-6. For example, “allowing the field through” within the context of Q-6 can be inverted, leading to a blockade of the field due to conductors. The “no field model” inside a conductor led some students to shift from “all charge model” to “near charge model” as the following explanations suggest.

S14a:

Q-5: C because all the charges, positive and negative can contribute to a field, so A, B, D, and E cannot be right. Even though the electrons on the far side of the plates cannot move to the closer side to different atoms or molecules, they can contribute attractive or repulsion forces from their locations.

Q-6: It is hard to see how that could be the charge distribution because charge will distribute itself evenly on the surface of a good conductor. However, given the assumption, the answer would be F because there would be no field within the conductor or plate, so only the charges on the close side would make a field between them.
S15c:

Q-5: C. Because the material is an insulator the electric field produced will be the result of all charges present.

Q-6: F. Because the material is a conductor, separate electrical fields will be produced on the outsides and inside of the plates. The electric field within a conductor is always zero.

The absence of a net electric field inside a conductor has led the above two students to consider the contributions from the two sides of a given plate separately. It is particularly striking that S15c within the context of Q-5 identifies correctly that the “electric field produced will be the result of all charges,” while the knowledge of the absence of a net electric field inside the conductor within the context of Q-6 leads to a dissociation of the contribution of the same charges. In this case, the absence of the field seems to have been taken as a “void” inhibiting the charges on one side, contributing to the field on the other side.

Students who consistently selected the “near charge model” seem to have selected it because of the reasoning that, regardless of the material, only charges nearest to a given point are influential at that point. This is reflected in the following explanations:
S16a:

Q-5: F. Because both sides act on the point, but only charges nearest the point have any effect.

Q-6: F. For the same reasons as [above].

S17c:

Q-5: F. Although the two plates are made up of insulator material they do have a charge, whether they are positive or negative. Since the point P is in the middle of the plates it is only affected by the charges on both plates that are near point P.

Q-6: F. Although the two plates are made up of conducting material they do have a charge, whether they are positive or negative. Since the point P is in the middle of the plates it is only affected by the charges on both plates that are near point P.

An exception to the reasoning of S16a and S17c is that of the following student:

S18c:

Q-5: F. The electric field between two plates is uniform between two plates, except for fringing along the edges. Since field lines cannot cross, the field lines on the outside of the plates do not interfere with point P.

Q-6: F. Same as above.

It is not clear from the explanation given how to interpret the word “cross.” Since the student seem to know that an electric field exist between the plates, the “crossing” can be interpreted as the field from the “outer” charges not interfering with the field of the “interior” charges. If this is the case, then the student has formed a rigid
model of field lines. However, it is also possible that the student is inconsistent within the statement itself and “crossing” is interpreted literally to mean that the field due to outer charges cannot “cross” to the interior.

4.5 Gaussian surfaces & electric fields

As discussed in Section 3.5, three prominent models can be identified within the current topic. The desired model is that once the electric field for Q-7 is known, the answer to Q-8 should follow immediately (since the field in between the plates cannot change its value based on how we choose to draw the Gaussian surface). Whether the latter Gaussian surface is useful in finding the value of the field is a different question, one we are not interested in addressing in the present context. The second model involves the notion that enclosing two plates as opposed to one leads leads students embracing it automatically to double the field. The third model takes into account the procedural knowledge that students have on Gauss’s law. This would lead them to consider the net charge enclosed by the Gaussian surface. More generally this involves a “flux zero model.” Students’ explanations will be provided below to validate the above arguments. Consider the following explanations:

S1:

Q-7: B. The flux ... through the surface is \[\pi r^2(E)\]. That is set equal to sigma times the area of the charge which is also \[\pi r^2\]. They cancel and leave you with the E field equal to B.
Q-8: C. Zero. There is no E field inside a conductor. By symmetry, the E field would be perpendicular to the surface of the plates, therefore it won’t pass through any of the Gaussian surface. The flux will be 0. So the E field will be 0 too.

S2:

Q-7: B from Gauss’ law.
Q-8: C because the total enclosed charge is zero.

S3:

Q-7: B. The electric field from a conducting plate includes only one side of the cylinder.
Q-8: C. Zero...The charges cancel each other out.

Student S1 seems to have the necessary knowledge elements concerning electric fields within the specific context. In the context of Q-8, S1 arrives at the notion of zero flux not through net charge enclosed but by reasoning purely in terms of electric fields. By realizing that there is no field inside the conductor, and that the field is perpendicular to the plates, S1 correctly concludes that there is no flux through the Gaussian surface. The important fact to realize is that the student is functioning in the “flux model,” looking for the flux before calculating the electric field. This can also be identified in the context of Q-7. If we look carefully at S1’s explanation, we see that the presence of an electric field between the plates has been already recognized - “By symmetry, the E field would be perpendicular to the surface of the plates...” Yet, the student arrives at the final conclusion that the field is zero, bypassing the above
argument. The example shows how the fixation on the “flux model” as defining a set of procedures would lead students to ignore the reality of the electric field. The same arguments apply to S2 and S3 although these students seem to be directly reasoning in terms of the net charge enclosed.

S4:

Q-7: B. The electric field goes one way through the cylinder, thus \[EA = \frac{q}{\epsilon}\]. \[q = (\text{surface charge})(A)\]. Thus substituting in E \[\Rightarrow B\].

Q-8: B. The electric field goes one way through the cylinder, thus \[EA = \frac{2q}{\epsilon}\]. \[q = (\text{surface charge})(A)\]. Thus substituting in E \[\Rightarrow B\]. Total charge contained within Gaussian surface is twice the amount of q.

S5:

Q-7 A. Only the positive side is contributing to the electric field.

Q-8: A. Both sides contribute to the electric field. There is twice as much charge. The “2” cancels with the “2” in the equation.

S6:

Q-7: A. You only take the field on one side.

Q-8: A. You would add up the charges on both sides.
The explanations from S4-S6 falls into the category of “charge double” model. Here, only the enclosed charge by the Gaussian surface has been taken as contributing to the field. Therefore, enclosing both plates has led to considering charges on both plates, implying that the field has to be doubled. The principle of superposition is explicitly absent from this model.

S8:

Q-7: B. The electric field between the plates is equal to the sum of the electric field from each individual plate. Each plate has an electrical field of $E = \frac{(\text{charge density})}{2\epsilon_0}$. Thus the net electric field is $2 \times E$. This is equal to $E = \frac{(\text{charge density})}{\epsilon_0}$.

Q-8: A. The reason for this answer is the same explanation as in [Q-7].

S9:

Q-7: B. For one plate, $E = \frac{(\text{charge density})}{2\epsilon_0}$. The two plates of opposite charge contribute the same type [or] size of electric field, so the amount doubles, cancelling the 2 in the equation.

Q-8: A. For the same reasons as the answer above, the only difference is that the gaussian surface is differently sized, but that doesn’t make the field any different.

S10:

Q-7: B. The E field due to a sheet of charge is $\frac{\sigma}{2\epsilon_0}$. By superposition, the E field due to multiple sheets of charge should be the sum of the field due to the individual sheets. So, I choose B.

Q-8: A. I might be missing something but to me this seems the same as [Q-7]. I don’t think it matters where we draw the Gaussian surface for this problem.
Students S8-S10 consistently selected the correct answer and used the superposition principle explicitly. It is important to note that S8-S10 started with the field for a single plate \( \frac{\sigma}{2\epsilon_0} \) and used the superposition principle and did not explicitly consider the Gaussian surfaces to arrive at the electric field.

Gauss’s law presents a unique challenge in the instructional process. The uniqueness is both conceptual and procedural. The superposition of electric fields is often discussed before Gauss’s law. However, from a conceptual point of view, under Gauss’s law, the principle of superposition is not explicit. From a procedural point of view, given the easiness with which electric fields can be determined under certain symmetry assumptions of the field using Gauss’s law, a procedural model is easily formed which takes the form of “flux implies electric field.” The “net charge enclosed” model is contained within such an implication.

### 4.6 Electrostatic potential of a conductor

As was discussed in Section 3.7, students’ explanations can be identified as primarily belonging to the distance relations or the charge density relations category. Among students answering Q-13 and Q-14, the use of distance relations is more often used than that of the charge density relations. The opposite occurs for the most part within the context of Q-15. Let us take a look at some of the students’ explanations.
S1:

Q-13: B. The symmetry is in the picture... .

Q-14: C. The charge is closer to point b.

Q-15: A. The charges are more crowded around a.

S2:

Q-13: B. because the charge stays the same as well as the radius is the same so the electric potential stays the same. \[ V = \frac{q}{r} \]

Q-14: C. Because the charge is the same for both points where as the distance is different. the shorter the distance the greater the potential. \[ V = \frac{q}{r} \]

Q-15: A. Because the charge is greater on the left side (A side) resulting in the greater charge on point A which contributes towards greater potential.

S3:

Q-13: B, because charges are uniformly distributed.

Q-14: C, because R is less from charge to B....so more potential.

Q-15: A, because charge is greater than [at] B.
S4:

Q-13: B. The electric potentials are equal because they are the same distance from the positive charges.

Q-14: C. B would be larger because the formula is \[ V = \frac{kq}{r} \] and the radius is smaller for B.

Q-15: A. A would be larger because it has more positive charge acting on it.

S5:

Q-13: B. They are equal because they are the same distance from the charge.

Q-14: C. The potential at B will be greater since it is closer to the charge.

Q-15: A. The potential at A will be greater because there is a greater positive charge there.

S6:

Q-13: B. They are both the same distance away from the charge.

Q-14: C. point B is closer to the charge.

Q-15: A. The charge is concentrated more around point A.

In all of the above explanations, the distinction in argument as far as the usage of words are concerned is clear between Q-14 and Q-15. We argue that the usage of distance relations explicitly within the context of Q-14 is based on that context. Comparing Q-13 and Q-14, we see that the only difference between the two is that in Q-14 the charge is moved toward the right from the center. This may cue the student to look for the relation between Q-13 and Q-14 and identify “distance” to the inside.
charge (from the center or from the inner surface) as the main parameter that has changed. This in turn would lead to making associations between the charge, the distance, and the potential. However, this is not explicit to the same extent within the context of Q-15. Judging in relation to Q-13, Q-15 differs only with respect to the presence of the external negative charge. A major factor, we argue, for not reasoning in terms of distance relations within the context of Q-15 is that the external charge is not represented as a “point” charge. Although we have not explicitly stated in the question that the internal positive charge is a point charge, drawing a ‘+’ sign inside a small circle is taken to imply that it is a point charge. This is the common representation used in the textbook and in the classroom. It is under the point charge context that the equation $V = \frac{kq}{r}$ is introduced and the superposition of potentials first discussed. The point charge property is not explicit in the external (negatively charged) object. This may in turn lead to students considering only the relative charge densities dropping any distance relations.

By the above argument we do not imply that students are taking into account the point versus charge distribution distinction explicitly, but rather, that the procedural models that students have in mind in finding the potentials make students look for relations that would manifest in the above fashion.

The inconsistency in reasoning between Q-14 and Q-15 by the above students may also suggest the fundamental belief system or a model of “imbalance.” This is in line with diSessa’s p-prims, where the basic relation structure “more implies more” is now
mapped onto charges. From this point of view a distance relation can be thought of as an accommodating argument. While such an accommodating argument is provided within the context of Q-14, it is absent within the context of Q-15, perhaps due to the representational reasons suggested above.

Only one student explicitly said that the potential must be the same at points of concern in all three questions. This student derived knowledge from the laboratory work stating that “... selected (B) because... there’s no voltage drop over the conductor. In the lab we saw that the voltage difference across a metal block was near zero. ... think that is what is going on here.” Another student arrived at the correct answer by considering the absence of the electric field inside the conductor both in the context of Q-13 and Q-14, but used a charge density relation within the context of Q-15. Even in Q-13 and Q-14 the above student’s reasoning appears to be in error since the student seems to be considering the absence of an electric field everywhere including in between the positive charge and the inner surface of the conductor. As was discussed in Section 3.7, the distance relations are not used in any explicit way within the context of Q-16 and Q-17. The charge density models seems to be dominant. Several of the students’ explanations are given below.

S7:

Q-16: A. Position A will have much more charge around it than position B.

Q-17: B. The surrounding charges on both positions are just about equal.
Q-16: A. The charge concentration is different, A has more concentration.
Q-17: B. A and B are same concentrations, so they have equal potential.

Q-16: A. Greater charge density and therefore greater potential.
Q-17 B. Equal charge densities and therefore equal potentials.

It can be seen that students are not making use of the knowledge of the absence of electric field inside a conductor, the principle of superposition or the path independence of the electrostatic field to make deductions. Such knowledge structures seem not to exist in an integrated form. Explanations seems to be occurring closest to the specific and immediate features suggested within a given context.

4.7 Potential due to a shell of charge

As discussed in Section 3.8, students were inconsistent in their reasoning among questions Q-18, Q-19, and Q-20. Consider Q-18; the most popular response was choice D (correct) followed by choice B. If the students are searching for the distance variable as part of the procedural knowledge they have in using \( V = \frac{kq}{r} \), there are two such relations offered in Q-18; namely, \( R \) and \( r \). How then, is the student to decide which distance to use?

The question Q-18 asks for the potential at a point exterior to the shell. A possible mechanism at work is suggested by the mental model theory, in which relational
structures are mapped from the shell to a point charge, for which the notion of the potential outside or exterior to the shell act as a generator. What we mean by this is the following. The equation \( V = \frac{kq}{r} \) ideally applies to a point charge, where \( r \) is the distance measured from the point charge to the point of interest. Thus, the notion of “outside of charge” is inherent within the above statement. If the students have such a notion, the distance that corresponds directly to the outside of the shell is taken as the one to be used in the equation. This procedure automatically leads to mapping the entire charge on the shell to a point at its center, since the distance is measured from the center.

Such a mechanism can also explain the relative absence of choice D within the context of Q-20 compared to Q-18. In Q-20, the point of interest is not an exterior point but an interior point. Thus, the mapping of the shell to a point does not occur to the same degree.

The second most prevalent choice, B, within the context of Q-18 occurs by considering the distance relative to the point at which the double arrow representing the distance \( r \) crosses the shell. The fact that the charge is distributed throughout the shell is not considered. A fundamental issue at work here is the recurring theme of local reasoning. The charge “nearest” to the point of interest is taken as the causal charge and not the entire distribution of charge. A similar line of reasoning is involved for choice C within the context of Q-20. We can argue that measuring the distances from the shell should evidently lead to selecting choice C within the context of Q-19 to the same extent. But this does not happen, as was discussed in Chapter 3. The notion of an infinity may not seem to refer to a realistic situation and may push students build alternative models that would accommodate the situation.
Choice B within the context of Q-20 can occur for two reasons. One involves the absence of the electric field inside the shell implying (incorrectly) the absence of potential. This is the well-known model that the absence of one entity necessarily leads to the absence of another related entity. The second is the absence of any charge in the interior of the shell.

The latter model suggested in the earlier paragraph is particularly interesting when it occurs between choices D and E in the context of Q-18 and Q-19, respectively. One student selected choices D and E within Q-18 and Q-19 by considering the charge to be concentrated at the center of the shell but shifted to choice B in Q-20. If the student is fixed on the idea of a charge at the center, consistent with the previous questions, then the natural choice should have been D under Q-20. Another student following the same sequence selected choice B for the same reason. However, this student seemed to be using the distance variables directly within the context of Q-18 and Q-19. As was discussed earlier, such usage lead to an implicit mapping of the shell to a point charge at the center. This should have led to choice D in Q-20. The inconsistency in the reasoning suggests that distance relations were not used with explicit awareness of the charges that generate the potential at a given point. The occurrence of the notion of zero charge inside the shell leading to zero potential inside may also due to the model suggested earlier, in which an “exterior” point to the charges are identified in relation to the equation for the potential of a point charge. When there are no charges within a radius ($< r$) an exterior point (“from the charges”) at a distance $r$ from the center cannot be defined--hence the absence of potential.
4.8 A grounded conductor

As shown in Section 3.9, students seem to shift with respect to their answer between the questions Q-21 and Q-22, although the answer in both cases should remain the same. The conductor is an equipotential surface due to the combined effect of polarized charges in the conductor and the external charge. Once this fact is identified, the point at which the ground connection is made is irrelevant since the charges react to the potential differences. Thus, the questions investigate the models due to a certain way of representing electric phenomena.

Students’ existing knowledge of polarization is apparent among the drawings they made during reasoning on the questions Q-21 and Q-22. Like charges repel and unlike charges attract seems to be employed. When the far side is grounded (Q-21), either the positive charges leave the conductor for the ground or the negative charges are attracted to the conductor from the ground. Several procedures for students following this reasoning can be proposed. If the students start with the external positive charge, then this external charge attempts to “push” like charges from it as far away as possible. The ground connection thus provide a path for the positive charges to leave the conductor. On the other hand, the students can start from the positive charges on the far side and imagine that negative charges being attracted from the ground to the conductor due to these (positive) charges. Both procedures will lead to the conductor being negatively charged. Within the context of Q-22, the external positive charge can attract the negative charge on the near side of the conductor and to the grounding wire, leaving the conductor positively charged. It is also possible that the negative charges at the near end will attract positive charges, thereby making
the conductor positive overall. In all of the above procedures, however, reasoning
can only occur sequentially with respect to the presence of a dominant agent. The
most straightforward model seems to be the one that would provide the rule that the
ground connection removes charge around the area where the connection is made. If
the familiar conditions provided by Q-21 is then directly mapped to Q-22, negative
charges will be removed instead of positive charges.

However, as noted in Chapter 3, there seems to be a significant difference in reason-
ing when the external charge is negative rather than positive. Consider the following
explanation by an algebra-based student.

S1a:

Q-23: A. The rod will have a positive charge, because the free electrons in the
conductor will move to the end opposite the negative charge (because like charges
repel). They will move down the wire, and when it is disconnected, the rod will
have more positive charges left (because they were initially attracted to the negative
charge).

Q-24: C. There will be a neutral charge on the rod when the connection is re-
moved, because the electrons in the rod will not go down the wire. They will be
repelled because of the negative charge near the wire. Therefore, there will still be
the same number of positive and negative charges as before.
Q-23: A. ... free electrons in the rod are repelled away toward and into the earth from the force of the negative charge. This gives the rod a net positive charge. The rod will remain positively charged if the wire is cut while the negative charge is kept close.

Q-24 C. ... it would be neutral because the grounded wire would essentially serve no purpose. The negative charge being close to the wire will prevent, or repel, the free electrons in the rod to the right side of the rod. The positive charge will be attracted toward the left side (negative charge), but it should not continue down the wire because the positive charge within the rod will want to stay near the negative charge. Therefore, when the wire is cut the rod will still be neutral, with a separation of charge.

In both of S1a and S2a, a dominant (external) charge model is apparent. If a student reasons the movement of charges in terms of electrons (which is the more natural, given that only free electrons in the metal are mobile), the presence of the negative external charge could keep the electrons not moving to ground. On the other hand, if a student reasons in terms of the positive charges the attraction of the external negative charge will hold their movement thus making the rod neutral as a whole.
However, it could be argued that the same reasoning that student S2a applies could be used to reason in the context where a positive external charge is present. Given the difference in reasoning, it is possible that most students would reason in terms of moving negative charges (or electrons) rather than moving positive charges. This fact, coupled with the dominant presence of an external negative charge provides us with an explanation for the difference in reasoning in the two contexts.

Another example of similar reasoning is provided by the following student.

S3a:

Q-23: A. Positive charge. The object is grounded, but when the negative charge is brought close, the free electrons will be repelled and move down the wire, leaving the conducting rod positive.

Q-24: C. Neutral. The negative charge would keep all the electrons from moving down the wire leaving the same amount in the rod.

Several reasoning mechanisms were suggested earlier in the context of Q-21 and Q-22. The analogous mechanisms occurring in the context of Q-23 and Q-24 are provided in the following explanations:

S4a:

Q-23: A - free electrons in the metal wire will move down the grounding wire so the metal wire will have an overall positive charge once the grounding wire is removed
Q-24: B - A negative charge will be left after a the grounded wire is removed because the positive charges are closer to the grounding wire and some of them will leave through the wire leaving the overall negative charge.

S5a:

Q-23: A. With the negative charge near the neutral conducting rod the charges within the rod orient themselves so that negative charges move away from the charge and positive charges in the rod move toward the negative charge. Once the ground is in place the electrons in the rod moving away will go into the ground, leaving only positive charges in the rod if the ground is suddenly removed a positive net charge will be left in the rod.

Q-24: B. The negative charge first causes the positive charge to come close and the negative charges to repel. When the ground is added to the end with many positive charges electrons will flow up the ground to neutralize this positive end. If the ground is suddenly cut the overall net charge will be negative.

The correct answer in both cases may be obtained not by considering the complete polarization process but rather by looking at the type of charge that would be attracted to the external positive charges. The sketches of several calculus students also support this view - see the following explanation.

S6a:

Q-23: A. ...[A] is the correct answer because once the conducting rod is not grounded the negative charge will cause the positive charge in the neutral rod to attract toward the negative charge that is brought close to the rod.
Q-24: ...the correct answer is A. The reason is because once again the negative charge brought close to the neutral ... [rod] is going to cause the neutral rod to have a positive charge because the [attraction] for ... the rod is going to feel from the negative charge.

The consistent selection of “neutral” seems to be prompted from the fact that to start with the conducting rod is neutral. Thus, although there is a separation of charge due to polarization, the net charge remain zero - hence, neutral. The other reason provided, although not common among the algebra-students, is that the charge is simply removed due to grounding, leaving no charge on the conductor. A common explanation coupled with the above reasons is that the external charge has to come in contact with the neutral rod in order to impart any charge to it. This may be due to the knowledge of charging objects without induction.

As discussed in Chapter 3, there is a significant difference in responses among the calculus- and the algebra-based students within the context of Q-23 but this is not so within the context of Q-24. We see that algebra-based students selected the “neutral” choice in the context of Q-23 more than that of the calculus-based students, for the reasons discussed in the earlier paragraph. Calculus-based students seems to employ their knowledge of polarization and grounding to a significant extent at this stage with respect to Q-23. However, it seems that both calculus- and algebra-based students are comparable with respect to selecting “positive” within the context of Q-23 and then selecting “neutral” within the context of Q-24. Thus, the extent to which the models discussed above has been used in the two classes seems to be the same with respect to the “positive” and “neutral” choices in the context of Q-24.
In the above questions, polarization with respect to the external charge plays a significant role in the sequential reasoning that may proceed afterwards. However, within the context of Q-25, Q-26, Q-27, and Q-28, this cannot be observed to the same extent. The explanations provided by students are often vague and shift incoherently between the questions. The word “incoherency” is used here not with respect to the choices selected but with respected to the explanations. We discuss below a few that we can meaningfully identify to some extent.

Consider the following set of correct responses:

S7:

Q-25: C. No charge will be put on the sphere. When it is grounded nothing will happen to it because there is no charge on the outside of the sphere.

Q-26: C. No charge will be put on the sphere. By grounding it, it will make it neutral.

Q-27: B. The conductor is neutral so there must be a negative charge on the inside of the conducting shell. The charge will not change when the sphere is grounded.

Q-28: B. The inside of the sphere will be negative and when it is grounded will have no effect on the inside of the shell at all.
In the above, the student seems to be taking the inner positive charge and combining it with a set of negative charges to accomplish the overall neutrality of the shell as stated in the question. This automatically neglects positive charge present on the outer surface of the shell before grounding. However, the explanation provided within the context of Q-26 is not compatible with the rest. Especially, within the context of Q-25, the student argues that there is no charge on the outside, whereas in Q-26 it seems that the grounding will make it neutral. This is the disparity in explanations stated above. A similar disparity occurs in the following explanation:

S8:

Q-25: C. The inner surface will gain enough negative charge to equal the positive charge of the fixed charge, the outer shell will not need a charge [because] everything in the system is balanced.

Q-26: C. The inner surface will gain a negative charge equal to the positive charge of the fixed charge, but this will not affect the charge on the outer surface.

Q-27: B. The inner surface will gain enough negative charge to equal the positive charge of the fixed charge, the outer shell will not need a charge [because] everything in the system is balanced.

Q-28 B. The inner surface will gain a negative charge equal to the positive charge...

Again, the student argues that there is no need for a charge on the outside in the context of Q-25, whereas in Q-26 there seems to be a charge that exists on the outside although it is “not affected.” The above student seems to be using the neutrality of the shell implicitly which manifests in the explanations as a “balance of charge.”
We could expect that students who reason in terms of polarization of charges in the shell followed by grounding would register the sequence C A B C in the above questions, in that order. Here, grounding simply refers to the model of charge removal. Consider the following explanation:

S9:

Q-25: C Reason: Since the outer surface is grounded, the excess charge that would collect there goes into the Earth.

Q-26: A Reason: Since the ground is connected to the inside, the inner negative charge goes to the ground while the outer positive charge remains.

Q-27: B Reason: This is just the inverse of the previous question.

Q-28: C Reason: Since the inner surface is grounded, all the excess charge that it would collect goes into the Earth.

In the above, we again see the existence of simple models based on sequential reasoning of charges. Instead of a “charge removing” model, if the ground connection is modeled as a path for the electrons to move from the ground to the shell coupled with all of the above reasoning in terms of polarization, we would expect choice B within the context of Q-28. This could be explicitly identified from one student’s comment “electrons from the ground are attracted to the positive charge.” This student consistently reasoned in terms of negative charges (electrons) moving from the ground to the shell there by providing the sequence C A B C. Here, the choice C within the context of Q-25 arise from the neutralization of the positive charges on the outer surface by the negative charges from the ground.
The questions Q-25 through Q-28 provide the context for students to reason in terms of extra charge on the outer surface of the shell. This is due to the fact that the conducting shell provide the context to associate both an inside and an outside. Such an association can cue their knowledge that an “extra charge on a conducting shell always reside on the outer surface.” However, this knowledge is incomplete by itself since it applies in the case where there are no charges inside, say, the cavity of the shell. Consider the following.

S10:
Q-25: C because the sphere is grounded.
Q-26: B because the charge goes to the outer shell and this is not grounded.
Q-27: C because there is no charge on the inner sphere.
Q-28: C because there is no charge and if there was it would be grounded.

Here the student seems to start with the process of polarization. However, the charge on the outer surface has not been mentioned in any explicit way. This is understandable, since the student is only concerned with the knowledge of the “absence of extra charge in the interior.” Thus, movement or existence of charge is judged with respect to the interior and not with respect to the exterior. In all cases, the charges induced on the interior are moved to the exterior of the shell followed by the model of grounding as charge removal. A similar reasoning is suggested in the following:
S11:

Q-25: C The electrons will run off when the grounding wire is attached.

Q-26: B There are no charges on the inner sphere so nothing will run off. The shell will stay the same.

Q-27: C There shouldn’t be any charge on the inner surface of the shell.

Q-28: C There shouldn’t be any charge on the inner surface of the shell.

Although the reasoning in terms of electrons in Q-25 may be flawed, the basic argument seems to be in agreement with that of S10 and the discussion in the preceding paragraph. The use of knowledge of the “absence of extra charge in the interior” is suggested in the usage of “shouldn’t” by S11.

Though B to A transitions in the context of Q-25 and Q-26 are suggestive of a similar reasoning mechanism in Q-21 and Q-22, no student explicitly gives evidence as such. Thus, in the context of Q-25 no student seems to be selecting choice B for the reason that the negative charges will spread across the shell after grounding the outer surface and vice versa in the context of Q-26. One transition from B to A occurred when the student considered negative charges moving from the ground to the outer surface making it negative in Q-25, and then arguing in terms of negative charge removal from the inner surface making it positive. This may give some implicit evidence to the mechanism in question. The consistent selection of choice C in Q-25 and Q-26 also occur for the simple model that the shell is neutral to begin with. However, only three students selected choice C in all four questions. Shifts from selecting C consistently in Q-25 and Q-26 to others in Q-27 and Q-28 were found difficult to interpret.
4.9 Charged conducting spheres - force on a charge

The questions Q-31 and Q-32 ask for the net force on a single charge on sphere A. However, some models function in such a way that the entire charge on a sphere is taken as a single entity and mapped onto Coulomb’s law. This may be cued by the question asking for force. The word “force” makes students associate Coulomb’s law with the question, which in turn triggers them to consider the total charges $+4$ C and $+2$ C as single entities. This procedure, then, ignores considering individual charges. Consider the following explanations:

S1:

Q-31: D, because A is only affected by sphere B.

Q-32: D, because B is still completely outside of A.

S2:

Q-31: D. It takes two charges to make a force, since A alone has no force it takes the charge form B to make a force and of course it is not zero.

Q-32: D. A experiences a force [from] B, because it take two charges to make a force. And it cannot be zero.
In the above, we see that the student identifies only complete entities - the two spheres as charges, and not the individual charges on them. Then we see that the situations are mapped to Coulomb’s law. Once the spheres are considered as entire charges in themselves, only an external sphere (charge) can exert any force on the sphere (charge) of concern. This is further seen when S1 uses the word “outside” within the context of Q-32. These arguments are in line with those made within the context of Q-18 through Q-20.

The selection of choice D within the context of Q-31 also occurs by consideration of the “instant of contact” as still being “outside.” That is, that the spheres are really not “settled in contact,” thereby using the argument given in the previous paragraph. This “settling” may also be cued by the “imbalance” of charges. However, after some time has elapsed, the two spheres are taken to be properly in contact having similar charge distributions. The “similarity” in charge distribution may cue students to consider the two spheres in contact as a whole (“both”) leading to choice F within the context of Q-32. Consider the following explanations:

S3:

Q-31: D. At the instant the two make contact and in the time before that when they were brought closer together only the charge on sphere A experienced force from the charge on B and the net force on the charge was not zero. Both spheres are conductors so the charge is found only on the surfaces. Also the net force is not zero because both were positive charges of different magnitudes.
Q-32: F. The charge on sphere A will experience forces from the charges on both spheres but the net force is not zero. This is because both the spheres are in contact with each other and their net charges will be equal but there are still two separate entities with positive charges and the charge on sphere A will still experience a force.

S4:

Q-31: D. Because charge A suppose to repel charge B but was brought to charge B. That means charge A experiences other forces from outside.

Q-32: F. During the contact, both charges balance out.

However, both S3 and S4 uses the words “charge” and “charges” interchangeably. Although both identify that the “charges” will be equal within the context of Q-32, the force is modeled through Coulomb’s law, and spheres, as discussed earlier, are mapped to individual charge entities thereby obtaining a non-zero force. Consider the following reasoning:

S5:

Q-31: F. Because there are forces acting on the particle from both directions, but there is an unequal distribution so there will be a net force in one direction.

Q-32: F. Because the two spheres will be of equal charge, but the charge of sphere A is closer to the charge on A, so it has more of an effect.

The use of the word “direction(s)” in the answer to Q-31 by S5 suggests the following model: Sphere B as a whole makes one charge entity and all the charges on sphere A except the charge of interest make up another charge entity (as a whole).
Only if this is the case can we interpret the usage of “both directions.” This can be confirmed within the context of Q-32, where the student mentions that the “charge of sphere A is closer to the charge on A.” This statement seems to imply that all the charges on sphere A except for the charge of interest are considered to be closer to A than that of the “charge” B. Implicit in this model is again Coulomb’s law. Although the student makes use of the inequality of charges by the usage of “unequal distribution” to arrive at a non-zero net force within the context of Q-31, the equality argument within the context of Q-32 seems to be redundant.

As can be expected (see the section on the role of Newton’s third law in electrostatics), a commonly held model is that the magnitude or the “balance” or “imbalance” of charge is to be mapped directly to force. However, the balance of charge translates into net force being zero in the present context, and not net force being equal.

Consider the following explanation:

S6:

Q-31: F. Magnitude of forces from charges on sphere B will be less since $|4C| > |2C|$ so more force from the one with 4C.

Q-32: E. Some charge will transfer from one to the other so they have the same charge.

In the above, S6 seems to consider the charge of concern on sphere A as not being a part of the charges that make up 4 C. However, the force is directly associated with the magnitude of the charges. Within the context of Q-31, however, the existence of a non-zero net force is assumed from the difference in magnitude of the charges, whereas within the context of Q-32, having the “same charge” leads to the absence of
a net force (choice E). Guruswamy et al.[46] have shown how students’ view of charge
transfer depends on the type of charge on the spheres. However, their study focused
solely on the final charge that the spheres acquire rather than explicitly considering
the force between the charges. Furthermore, they have not attempted to arrive at a
general explanation of their observations.

A recurring theme in the explanations of S3 through S6 within the context of Q-32
is the equality of charges. This is perhaps cued by the question, because identical
spheres are involved. A possible reasoning pathway is that equal spheres → (con-
tact) → equal charges, where the word “equality” of spheres is directly mapped onto
“equality” of charges. Only one student explicitly stated the non-equilibrium and
equilibrium conditions:

S7:

Q-31: F. ...because the charge will be repelled by the other charges on sphere A
and B as the charges move toward equilibrium between the surfaces. ...the net force
is not zero at that instant because the particles have not moved to reach equilibrium
yet.

Q-32: E. ...same as the reason above except now the charges have reached equi-
librium and have a net charge of zero. ...they no longer are moving until some other
outside charge disrupts it.

The mapping of several charges to a single entity and then considering that entity
of charge as influencing only external charges was seen by students’ model concerning
the forces within an atom. In the Spring 2001 algebra-based class we posed a question
that concerned whether the electrons in an atom would land on the nucleus of the
atom based on the Coulombic attraction between unlike charges. While no student considered it as a possibility (intuitive physics at work), one of the reasons include non-existence of forces within an atom and the Coulomb forces only to be taken between atoms. As such, conceptually, there seems to be a fundamental constraint in modeling internal forces compared to external forces. This may be due to the causal reasoning involving external agents.

4.10 Magnetic force - right-hand versus left-hand rule

As discussed in Section 3.12, Q-38 and Q-40 ask students to apply a “left-hand rule” instead of a right-hand rule. No knowledge construction is necessary to answer the two questions, only a recognition of “common sense reality” of force. In Q-38, the construction of two rules was expected. One is to map the right-hand rule in one-to-one correspondence to the left-hand and carry out the procedure of finding the magnetic force in the exact way the right hand would be used. In such a procedure, the fingers of both the right and left hand would be curved from $\vec{v}$ to $\vec{B}$. The other is to construct only a verbal rule and not actually carry out a procedure involving the left hand. This rule take the form of “opposites”; that is, “if the use of right hand lead to a particular direction, then the left hand should lead to the opposite direction.” Here, the oppositeness of left-right is mapped onto the oppositeness of directions. Both of the above rules would lead to identification of the opposite direction within the context of Q-38 to that of Q-37. Consider the following explanations.
S1:

Q-37: 1 (P direction) Reason: The velocity and magnetic field are perpendicular, so there IS a magnetic [force]. The direction of the force is given by the “right-hand rule” which would make the particle move up.

Q-38: 3 (R direction) Reason: opposite of the P direction since it is the opposite hand.

S2:

Q-37: P...the charge will move up due to the right hand rule
Q-38: R direction...since the thumb points down using the left hand

S3:

Q-37: 3. From the right hand rule the charge will move downward.
Q-38: 1. Because that would be the opposite to the right hand rule.

S4:

Q-37: 3) The force is pointing down by the right hand rule so the particle will follow along path R.
Q-38: 1) Because using your left hand your thumb [indicate that] the force will point up meaning the particle will [follow] along the P direction.

We see that S1 and S3 are more likely to be using the verbal rule of “oppositeness,” while S2 and S4 are more likely to actually be carrying out the procedure in one-to-one manner using the left hand. As suggested in Chapter 3, the two questions together
can eliminate the actual errors in applying hand rules in any single question and
determine the consistency of reasoning between two questions. This is shown by S3
and S4, who argue in reverse.

The shift to oppositeness between Q-37 and Q-38 suggests the absence of an on-
tology concerning the concept of force for the majority of the students. Examples of
students who are consistent within the two questions are given below:

S5:

Q-37: 1. It moves in direction p because of the right hand rule.

Q-38: 1. The direction would be the same because the force is always acting the
same upon the particle regardless of “which hand rule” one uses to find the force.

S6:

Q-37: 1. From the right hand rule.

Q-38: P. Whether we use right hand rule or left hand rule, the direction of the
force does not change.

As discussed earlier in Section 3.12, the algebra-based students did not shift from
correct to incorrect between Q-39 and Q-40, though both Q-37, Q-38 and Q-39, Q-
40 have the same elements; that is, both asks for the magnetic force and the right-
and the left-hand rule. If the students are to shift, the correct identification of an
attraction between the wires within the context of the right-hand rule should translate
into a repulsion in the context of the left-hand rule. However, as can be seen, this
can only happen if students are using only the verbal rule of “oppositeness.” It does
not happen in the case of one-to-one application of the procedures between the two
hands. However, within the context of Q-39 and Q-40, both of these models seems to be bypassed by the students. The main context factor seems to be that the wires are carrying currents in the “same direction.”

S7:

Q-39: A. When the current in two wires is travelling in the same direction, the wires will be attracted to each other.

Q-40: A. The wires will still attract, because their currents are flowing in the same direction.

S8:

Q-39: A. This is because according to the right hand rule, the magnetic fields from both wires are going in the opposite direction between the two wires. This leads to an attractive force between the two wires.

Q-40: A. According to the right hand rule, or a left hand rule, as long as the wires are parallel and the current is going in the same direction, the magnetic field will still be going in opposite directions and exerting an attractive force between the wires.

S9:

Q-39: A. Currents in the same direction attract each other.

Q-40: A. Currents in the same direction attract each other.
S10:

Q-39: A. When the currents run in the same direction, their magnetic fields add and the wires attract each other.

Q-40: A. The wires would still attract each other, the only thing that would change is the predicted direction of the magnetic field.

In all of the above explanations, it is clear that the direction of the current played a key role in determining the student’s choice. This shows that direction of the current is taken as the perceptible feature and the handedness of rules does not count for much. The above explanations all lead to the selection of the correct choice. However, the arguments are the same even when an incorrect choice is selected. Consider the following:

S11:

Q-39: The wires repel each other because the currents push the wires away from each other.

Q-40: The same thing. The current still travels in the same direction and causes the wires to repel each other.

The use of the “opposite” rule is suggested in the following explanation although the incorrect choice was selected in Q-39.

S12:

Q-39: B) Using both right hand rules, the forces created by the fields oppose each other, and therefore repel.

Q-40: A) It will be the opposite of the first scenario.
Ontological reasoning is suggested by one student who commented that “wires don’t care what rule you use, they’re still going to react as they did before” (the correct choice was selected in both cases).

As can be seen, the reasoning between the calculus- and algebra-based students seems to be influenced by the context factors presented within the questions. Once the identification of “attraction” is made in Q-39, the direction of the current being the same in Q-40 as that of Q-39 cues the algebra-based students to arrive at the correct solution. Thus, the direction of the currents in the two wires help algebra-based students to build a much stronger model of invariance with respect to the force. However, this is not so for the calculus-based students, since Q-37 and Q-38 do not provide such cues. For these students, two entities (e.g., two wires) cannot be identified since only a single charge is presented in the questions. For the algebra-based students, the similarity structures (e.g., currents in the same direction) exist within a single question, whereas such similarity structures are absent in the questions for the calculus-based students. The similarities for the calculus-based students exist among the questions (e.g., direction of $\vec{v}$ and the direction of $\vec{B}$). However, these similarities seems to be difficult for them to identify since they are not fundamentally tied to any ontological reasoning.

4.11 Force on a charge in a magnetic field

Questions Q-44 through Q-46 provide an example of how a pictorial context and a purely language context influence the mode of perception. This is observed to a significant extent among the algebra-based students. Consider the following explanations:
S1:

Q-44: C, the current and electric field are perpendicular to each other so the positive charge will be pushed up by the field.

Q-45: E, if the electric field and motion (velocity) are in parallel directions, no force will exist on the particle.

Q-46: A, if the field and current are parallel, the charge will receive no applied force and will continue moving at the speed it has been.

S2:

Q-44: C, based on the right hand rule, it will move up.

Q-45: E, There is no magnetic force if the particle is moving in the same direction as the magnetic field.

Q-46: A, it has no force to work with it or against it.

S3:

Q-44: C. The charge will move up because the force is into the paper, the field is to the right, so by the right hand rule the charge moves up.

Q-45: E. If the movement of the particle and the magnetic field are in the same direction, then there will be no force acting on the particle due to the magnetic field.

Q-46: A. The particle will continue going the same way, because the magnetic field and the particle are moving in the same direction. This means there is no force acting on the particle, so it will continue at the same velocity it was previously going.
S1 reasons in terms of current and electric fields. Although moving charges are currents, an electric field is not present in the questions. It can be noticed that the use of electric field is a mistake rather than a way of reasoning about the questions. What is of interest is that S1 seems to be looking for a moving entity in all of the questions. Thus, the student interprets the charge at rest in Q-44 as a current by itself and also identifies a direction, perhaps by considering the symbol ⊗, which denotes a current or a charge moving “into” with the symbol for the positive charge ⊕. If this is the case, the force should point downward rather than upward. Thus, the student has applied the right-hand rule incorrectly. However, for our discussion this error is not important. What is of importance is the existence of the correct rules within the context of Q-45 and Q-46 for S1. These correct rules are all identified in the context of some “movement.”

In both Q-45 and Q-46, the charge is moving. This cues the student to associate directly with the existing rule on how the relative directions of a moving charge (current) and the magnetic field affect the force on it. Since the student seems to be fixed on the search for movement of charges, Q-44 leads to an incorrect inference (it does not provide such a context). However, the student is able to identify a moving situation. Note that S2 and S3 also function in similar ways. For S3, the positive charge symbol is interpreted as force “into the paper.” As stated above, the inconsistency of this reasoning is not of importance—it is rather how the correct rule is identified within the other two questions’ contexts. It seems that the students are triggered by the key word “move” to associate a rule such as “if the charge is moving parallel (or in the same direction) to the field then there is no force.” Given the fact that the correct choice was not selected by this student in Q-44, it seems that the
above rule exists purely as a statement by itself without any mathematical formulas such as \( F = qvB \). This argument is further supported by the following explanations.

S4:

Q-44: D. Right hand rule.
Q-45: E. Because the field and the velocity are parallel, not perpendicular.
Q-46: A. Because a non-perpendicular magnetic field will exert no force on the particle.

S5:

Q-44: D. The positive will travel in the direction of the current which points downward.
Q-45: E. The magnetic field is parallel to the current therefore there will be no force.
Q-46: A. Because they are moving in the same direction there will be no change in its velocity.

S6:

Q-44: D. Charge will move down because of the right hand rule.
Q-45: E. There should be no force because the velocity and direction of the field are not perpendicular to each other.
Q-46: A. The velocity should stay the same because the field is not acting perpendicular on the charge; it is in the same direction.
In the explanations of S4 through S6, the right-hand rule is applied within the context of Q-44 but the correct choice is selected within the context of Q-45 and Q-46. The direct use of rules cued by the language in the latter two questions is apparent as opposed to the pictorial context provided in the former question.

Consider the following explanations:

S7:

Q-44: B. The positive charge will move to the right. It has no force or velocity to go so it will attract to the negatively charged side of the magnetic field.

Q-45: E. None of the above... Although the charge is moving, the magnetic field and the velocity must be perpendicular to each other. In this case, they are parallel so there is no force.

Q-46: C. ...it will increase. When it enters the magnetic field it enters the positive side so the pole will want to repel the positive charge thus speeding up the charge toward the opposite pole. Then it reaches a certain point where it becomes very attractive toward the negative pole and so it speeds up to get to that pole.

The student seems to consider positive and negative “magnetic” charges, in agreement with the findings of Maloney [47]. Since the student identifies the charge as at rest within the context of Q-44, it seems that the magnetic field is identified in relation to an electric field. However, we see again that within the context of Q-45, the student correctly makes use of the knowledge of rules in a similar manner to that of the earlier students. The magnetic field is again mapped onto the electric field within the context of Q-46.
In Chapter 3, it was hypothesized that the entrance of the charge into a field region could be thought of as “introducing” a force on the charge even though the field is parallel to the velocity (Q-46). The following explanation seems to support such a notion.

S8:

Q-44: E, it will not move......forces aide in charge motion and since magnetic fields will not exert a force on stationary charges, then the charge will not move.

Q-45: The force is zero if the charged particle moves parallel to the field lines.......F = qvB where B is perpendicular to the forces and current. ...(sin 0)=0 on a parallel course.

Q-46: A force must be present to increase or decrease the speed of a particle. And, forces acting in the direction of motion will increase the speed of the particle.

It seems that the student employs the formula $F = qvB$ within the context of Q-45 and this may very well be aiding in the reasoning of Q-44, unlike for the students mentioned earlier, who only used a “statement rule” within the context of Q-45.

Students who function with the mathematical rule of $F = qvB$ together with the “statement rules” seem to make the correct deductions in all cases. The mathematical rule was heavily employed by the calculus-based students as opposed to the algebra-based students. Two examples (for algebra-based students) are given below.
S9:

Q-44: E. It will not move at all because the charge has no initial velocity which means there will be no force acting upon it due to the magnetic field.

Q-45: E. There is no magnetic force on the charge because the direction of the particle is in the same direction (parallel) to the direction of the magnetic field.

Q-46: A. The speed stays the same because the particle’s velocity is parallel to the direction of the magnetic field, thus, there is no force on the particle due to the magnetic field.

S10:

Q-44: E. It won’t move at all. The equation $F=qvB$ shows that if the velocity equals zero, the force on the charge also equals zero. Therefore it won’t move.

Q-45: E. The force is zero if the charge moves parallel to the magnetic field.

Q-46: A. It stays the same because the force on the particle is zero: particle and magnetic field are parallel.

Clearly the question context plays a significant role in which elements of knowledge are activated. The algebra-based students are heavily influenced by the “statement rules,” and, when such rules are not cued by the question, seem unable to reason correctly. They “see” movement when there is none (Q-44). When the key words, “move,” “same direction” exist within the question context, the correct deductions were made with use of the existing “statement rules.” Calculus-based students relied heavily on the mathematical rules rather than just the “statement rules,” thereby selecting the correct choices regardless of the specific cues present in the questions.
4.12 Magnetic induction - movement

We attempt to explain why “movement” seems to be a popular reasoning within the context of induced currents. Why is “some form of movement” identified in place of “change of flux” through a loop? We argue that the context within which induction is introduced leads to formulation of a mental model of “movement” necessary for induction. During instruction, one of the first experiments that introduces induction is the sliding of a bar magnet in through a coil, stopping, and sliding it out. Students observe that only when the magnet is in motion that a current is produced in the coil. Our conjecture is that this demonstration leads to making causal connections between movement (cause) and that of producing currents (effect). Given the abstractness involved in reasoning in terms of flux, change of flux, and the rate of change of flux, a simple causal connection provide a “comfortable” set of relations in which the student is able to function. We argue that the above notions may make the idea of movement within the context of induction a rigid model difficult to overcome. Consider the following statements.

S1:
Q-47: G, [because] as long as there is a magnetic field and it has velocity, there is an induced current.
Q-48: B, [because] there is velocity perpendicular to the [magnetic] field.
S2:

Q-47: G. All will induce current in the loop because if there is a magnetic force acting on the loop, then it will produce current.

Q-48: B. Because the loop changes direction in a linear fashion. This will induce current.

Student S1 seems to implicitly use the idea of magnetic force in Q-48. This is because the idea of velocity being perpendicular to the field is discussed within the context of magnetic forces on currents and moving charged particles. The reasoning seems to be particularly cued by the presence of the rotating loop from which the student attempts to find a distinction. S2 directly argues in terms of force within the context of Q-47. There, it seems that the movement of the loop is directly associated with a force acting upon it. However, it seems that the concept of magnetic force on the loop is taken as the cause for the existence of current by both S1 and S2 in Q-48 and Q-47, respectively. Perhaps implicit within the idea is the force as the cause for all dynamical effects. If this is the case, then movement is taken as producing the existence of force (cause), which then will lead to the effects. This idea may be extended in light of the model that has been formed through common experience with the world - that force implies movement and vice versa. Given the fact that this model is observed to be extremely rigid and common it may also explain why “movement” simply leads students to associate it with an induced current.

There are two main reasons why translational motion is preferred as opposed to rotational motion. One is the direct association with the common linear motions involved in the context of induction through demonstrations and common textbook
examples. The second may due to a deeper notion of what constitute “change.”
Within the context of Q-48, the change may be considered as the change in position,
which would lead to identifying translations as opposed to rotations. Consider the
following statement.

S3:

Q-47: G. All because coil is moving relative to magnetic field in all cases.
Q-48 B. [Because] this [V] loop is the only one with translational relative speed.

The above statement at face value can be interpreted as simply looking for transl-
local motion, although the second reason suggested in the above paragraph may
also provide the necessary elements of reasoning. Which interpretation in particular
cannot be distinguished for S3. However, the following student seems to be reasoning
more in line with the notion of spatial change.

S4:

Q-47: G, because in all cases the loop is moving.
Q-48: B, its the only loop moving a distance.

Students who did not distinguish between translational and rotational motion seem
simply to be functioning consistently with respect to any form of motion as identified
by Allen [43]. On the other hand, students who selected the correct answer in both
cases used the notion of “change of flux” through the loop consistently.
Consider the following explanation:

S1:

Q-49: B, because the magnetic flux, or induced magnetic field always wants to oppose change.

Q-50: A, in same direction because since it [is] decreased, it wants to bring it back up in order to keep it the same.

Q-52: B, because the area decreases and it wants to oppose the change. The flux is proportional to area. It wants to oppose force [flux?]. Because flux is equivalent to BxA.

Q-53: A, the flux of the coil must decrease to compensate for the increase in area. [Because] flux is equivalent to BxA

A possible set of rules with which this student may function can be proposed. S1 states that an increase in the field must be opposed and a decrease in the field must be supported in the sense of bringing it back to the initial state. Hence, let us suppose that the student has the following rules: increases → opposite direction and decrease → same direction. Then within the context of Q-52, a decrease in area leads to a decrease in flux. But to oppose this change, the flux has to be increased hence leading to selection of choice B (opposite direction). Similarly, in Q-53 the area of the loop
increases, increasing the flux (although this is not true). But to oppose this change, the flux has to be decreased hence leading to selection of choice A (same direction). Thus, the required change is not reasoned in physical terms but rather through a set of verbal rules. Consider the following set of explanations.

S2:

Q-49: (B) This is because the induced current has to be opposite of the external magnetic field so it can compensate for the increase in external magnetic field. Therefore the induced current should decrease.

Q-50: (B) This is also in the opposite direction because when the external magnetic field is decreased then the induced magnetic field needs to compensate for this loss and thus needs to increase.

Q-52: (A) This is because the induced magnetic field wants to stay the same and when the loop size decreases the flux decreases and the induced current wants to oppose the change in flux.

Q-53: (B) The induced field is opposite because the wire loop is increasing which makes the flux increase so the induced field needs to decrease.

In both Q-49 and Q-50, it seems that the student functions with a set of verbal rules. The student seems to map the opposites suggested in the pair increase-decrease. This lead to selection choice B in both cases. (The student make use of the term “induced current” incorrectly and inconsistently between questions.) However, it is not clear within the context of Q-52 whether the use of the word “same” actually reflects S2’s thinking, which in turn influenced the selection of choice A (which also consists of the word “same” direction), or whether the student is still consistent with
notion of oppositeness but mistakenly selected choice A. Although the argument in Q-53 is correct, the selection of choice B maybe interpreted as due to the idea of opposites in the light of Q-50 rather than to a physical understanding. A similar set of reasoning is suggested by the following student:

S3:

Q-49: B - there’s an induced current to create a new B-field which will oppose the direction of increase so it’s in the opposite direction.

Q-50: A - it’s induced if there’s a change in the B-field but it’s only opposed if it increases, and it will be in the same direction if it decreases.

Q-52: A - it’ll stay the same because it’s decreasing and the outside ones cancel each other out so the same remains inside.

Q-53: B - if it’s increasing it’ll oppose the direction.

It is worth noting that S3 states that “it’s only opposed if it increases and it will be in the same direction if it decreases” within the context of Q-50. Thus, the rule: increases → opposite direction and decrease → same direction seems to be functional. Q-53 also suggests this reasoning, which does not reflect physical reasoning. Again in Q-52, the problem we encountered in the case of S2 is apparent. Perhaps the use of “stay the same” is suggestive of the “same direction” stated in choice A. However, it is possible that this sameness is identified through the word “decrease” cued either by a decrease in the area of the loop or the flux, which one is not clear in S3’s statement for Q-52 (it should be noted that S3 states that “it’s decreasing” within the context of Q-52).
The selection of choice C within the context of Q-51 seems to provide a clue for a fundamental way of thinking by students within a context where prominent rules and models are difficult to identify. The model that arises within the context of Q-51 is the need for the existence of a magnetic field at all times for magnetic induction to occur. The changes in the field have to occur when the field itself is present. This may due to the ontological understanding humans have on the nature of presence of things and the causal effects they lead to. An effect cannot arise in the absence of a “thing.” Consider the following statements which represent the majority of reasoning in Q-51:

S4:
C. There is no induce field because with no external magnetic field there is no emf, and with no emf there is no current and no induced magnetic field.

S5:
C. With no magnetic field applied, there is nothing for the loop to oppose.

S6:
C. If the one is shut off....the other would be too.

Thus, it is clear that the switching-off of the field is not considered as a change but simply the absence of it. Only four students selected the correct choice with the correct explanation, two of which are shown below:

S7:
A because it is still trying to resist change in flux, but it would only last a moment

S8:
A. The switching off of the magnetic field is the same as reducing the magnetic field to “zero.” Because of this the magnetic field through the loop will act the same
as a reduction of field, by inducing a field that is opposite the change. The field opposite the change in this case is in the direction of the original external magnetic field.

Within the context of Q-53, most students considered a change in flux as due to the change in the area of the loop. This is due to identifying the flux as the product of the field and the area, which is taken to be applied independently of the magnetic field. That is, by area is meant the area of the loop and not the area through which the magnetic field penetrates. The existence of such a notion may be directly derived from other question contexts, which did not pose any conflict in such identification.

4.14 Magnetic induction - induced electric fields

As mentioned in Section 3.18, the presence of currents does not translate directly to presence of forces on charges. Students seems to have developed procedures where the changes in magnetic fields are readily identified together with closed wire loops to produce induced currents. In the absence of such wires, the similar changes in the magnetic fields do not translate into force on charges. What emerges is a set of mechanical or procedural rules that students apply when a certain set of elements have been identified. These include the presence of a magnetic field, the changing of that field, and the presence of a closed wire. The reasoning for how it is possible for a current to be produced in the wire seems to be irrelevant. Within the context of Q-57 when only single charges are present, the force on the charges was considered in terms of whether they are placed inside or outside of the magnetic field region. In this context, the “pure” knowledge of charges placed at rest in a magnetic field is also absent for the large part. Only one student explicitly considered the charges
were at rest to begin with: “since the charge is already at rest, it will not move. The external [magnetic] field fluctuating won’t matter.” A representative sample of the popular reasoning is given below:

S1:
A. Only Q will start moving because it is the only particle inside the magnetic field. The particle must be inside the magnetic field in order for the magnetic field to act upon it.

S2:
Q will move because it is affected by a changing magnetic field. q, however is not inside the field so it stays in place.

S3:
A. Because this is the only place where the magnetic field is increased.

The most obvious feature at the places where the two charges are kept is the presence or the absence of the magnetic field. Thus, it seems that the presence of the field is simply taken as leading to effects while the absence of it lead to “nothing.” The issue also reflects the microscopic versus the macroscopic difference in reasoning [48]. The selection of the correct response within the context of Q-57 is difficult to interpret in most cases. Only one student gave a detailed explanation: “C. Both charges will start moving. If the external magnetic field starts to increase, so will the flux. To oppose that and decrease the flux, the charge q will start to move in the [opposite] direction to that of which Q is moving.” This student correctly identified the currents in Q-55 and Q-56 to be counterclockwise. However, in Q-57 the student has the notion that the charges would move in opposite directions perhaps mapping
opposite relations such as inside the field-outside the field. We had not stated the sign of the charges in the question, however. Similarly, the movement of only charge \( q \) is difficult to interpret from the explanations.

4.15 Neutral spheres and neutral point particles

As stated in the corresponding section in Chapter 3, the two neutral spheres are considered to behave differently from a point charge and a point neutral particle. The reasoning is fundamentally constrained by the point charge in some cases. It seems that in the presence of the point charge, the neutral point particle is considered as a neutral sphere with the ability to polarize. Consider the following explanation:

S1:

Q-59: C. Nothing happens. Since both spheres are neutral, they cannot induce a charge onto the other. Each sphere has an evenly distributed amount of positive and negative charge throughout.

Q-60: A. ... the point charge will exert a greater force onto the point charge [neutral particle?], by pushing away the like charge and attracting the opposite charge in the neutral particle.
By considering the explanation given by S1 within the context of Q-59, it seems that the student is functioning in terms of polarization. The presence of the charge in Q-60 provides the necessary context for the student to apply the polarization argument to the neutral point particle. The student seems to function in terms of key words such as charge, neutral, and polarization. The concepts of point and particle have no meaning. The following student also seems to function the same way although the explanation is less explicit.

S2:

Q-59: Nothing happens. the spheres are neutral and produce no force on each other.

Q-60: The point charge exerts more force on the neutral point particle because it causes it to acquire charge.

One can also treat the neutral point particle as a charge by itself. Consider the following:

S3:

Q-59: C. Nothing happens. There is no charge, so there is no attraction or repulsion between the two objects.

Q-60: A. The electric field from the charge is greater than the neutral particle.

Supposing that the student has the knowledge (as the student suggests) that electric fields are created by charges, the use of the words “greater than” suggests that the neutral point particle also creates an electric field, only that it is weaker. What is worth noting is that the same argument can be used in Q-59, identifying that similar neutral “charges” should repel. However, the student comments that “there is
no charge” in Q-59. Thus, it seems that the presence of the word “charge” in Q-60 cues the student to generate the explanation in that specific context although it is inconsistent with that of Q-59.

The concept of neutral charge is also seen in the following. However, the “neutral charge” is considered as not being able to apply any force. Therefore, only the (proper) charge is able to apply a force on the “neutral charge.”

S4:

Q-59: C. Because the two metal spheres are neutral - that is they have no attractive or repulsive forces.

Q-60: A. Because the neutral charge does not apply a force on anything.

The above explanations lead us to suggest that considering “neutral” as a type of charge ([15], pg. 48) is only a surface feature. If students believe that neutral is another type of charge then the two neutral spheres should have repelled each other. We see that such a surface feature appears in the presence of a (proper) charge by comparisons and analogies to the situations students are familiar with. Thus, such notions seems to be heavily context dependent.

As discussed in Section 3.19, the “size of the charge” still seems to matter for some students: “because the objects are the same size, they will have equal but opposite forces on each other.” The two equal sizes within the context of Q-60 are “points.” The presence of the word “stationary” in Q-60 made some students consider that there were equal forces on them, thereby leading them to select choice C. The stationarity, however, also led students to select choice D, perhaps through reasoning in terms of a net force. Choice C also results from considering the neutral point particle as a charge
and then applying Newton’s third law. However, the explanations can be understood through any mechanism (e.g., “C. Equal magnitude of charge on each other. The directions will be opposite”).

The argument that we can make in the present context is that there is a fundamental lack in the language used to present electrostatics in physics classes. Students’ consideration of the neutral point particle as a charge by itself or as capable of polarization shows that their reasoning is fundamentally constrained by the situations and notions with which they are familiar. This seems to lead them to a complete reinterpretation of the question in order for them be able to accommodate their existing knowledge, and hence to the generation of specific models on the spot.
CHAPTER 5

CONCLUSION

In this study we have looked at how students respond to two or more related questions in some selected topics of electricity and magnetism. Our goal has not been to describe students’ difficulties but rather identify the models with which they function and to examine how consistently or inconsistently they are used within a given set of questions. Rather than considering students’ explanations in an isolated sense, we have followed student explanations from question to question for a given student. This method helps us understand whether the reasoning is dependent on particular context factors or cues provided by the questions and, if so, how. We also pay attention to the associations students make to their experiential knowledge and the abstractions that are derived from the subject itself.

We have attempted to provide a specification for students’ reasoning. This, we believe, will help achieve a predictive and explanatory framework of students’ reasoning beyond the existing descriptive framework. Adhering to such a framework, questions were developed with hypotheses predicting particular reasoning outcomes. Our study largely focused on the possible reasons for student difficulties and ways to address them.
5.1 Results and Discussion

We find that two or more related questions can be perceived or internally represented in distinct ways. The way they are perceived and the way certain knowledge elements are manipulated in order to arrive at an answer is dependent on the specific context of the question. In this sense, students are mostly inconsistent with respect to the selection of choices. The specific context can cue associations that are either experiential or abstract in nature. The student who maps a large piece of lumber to the large charged sphere and the small charged rod to the point charge functions in terms of experiential knowledge, whereas the student who maps the entire charged sphere to a point charge function abstractly.

Furthermore, statements such as “more charge imply more force” have to be viewed as fundamental causal arguments. This is the only way such a result could be explained. As such, it must be considered as a concept that is very difficult to revise. This is also supported by evidence from answers to questions such as Q-59 and Q-60. Also, causal connections exist in one-to-one relations (e.g., force-charge) and not in terms of one-to-many relations (e.g., force-charges). This, we argue, is the reason that the complete expression of the Coulomb’s law is not considered within such contexts although we are confident that it exists in students’ knowledge base. Relations such as $F = Eq$ in these contexts exist purely at the propositional level dissociated from causal relations.
In electric phenomena, the basic element that is manipulated by students is the concept of charge. The basic procedure built upon this concept is the attraction or repulsion of charges when causal events are reasoned. Such reasoning seem to take a center stage as opposed to reasoning in terms of electric fields and potentials.

Reasoning in terms of electric fields and potentials is fundamentally difficult, because they do not provide for direct reasoning in terms of “push” and “pull.” In other words, electric fields and potentials do not allow students to built relational structures out of their experiential knowledge. Thus, reasoning in terms of charges, attraction, and repulsion form a rigid set of models. As such, these notions are manipulated consistently among different question contexts. The concepts of electric field and potential function purely at the propositional level, that is, as mere definitions. Thus, specific situations that implicitly demand their usage cannot be recognized. Reasoning in terms of charges, on the other hand, can easily be coupled to external representations, which are in turn easily manipulated.

We also note that it is the concepts such as “charge” that lead students to reason locally by identifying them as the basic causal elements. Since a concept such as charge seems to function within a different ontologico-categorical hierarchy than that of electric fields and potentials, relating the former to direct causal events, a notion such as the superposition principle is an inherently difficult concept to grasp and manipulate. To this extent we see that the theoretical ideas put forth by Chi et al. [30] can be carried over to electric phenomena. Notions such as a “point” have no meaning for most students. The essential word seems to be “charge,” since it is the element with which students function throughout the course and manipulate frequently as the causal element as described above. This is revealed by the question
that involved a point charge and point neutral particle, in which case the point neutral particle is mapped to a neutral sphere. A distribution of charges can also be mapped to a single charge, which then automatically will serve as a point charge. This serves as an example of both the ontological gap that exist in terms of terminology, abstraction and the accommodation of novel situations within the existing frameworks.

We do not view the differences in reasoning in terms of insulators and conductors as a set of beliefs students have or as an inherent understanding students possesses on how such materials behave with respect to electric fields. Concepts such as “electric field” have no firm place in students’ knowledge base and were dissociated from their experiential knowledge. Once a question or a set of questions are posed that involve the elements such as “insulator,” “conductor,” and “electric field,” students may develop a sense of expectation that a difference in behavior of the field is implied. The subsequent reasoning process has to be viewed as an accommodation of the electric field within the properties implied by specific materials and as construction of models on the spot. However, the models that result agree among many student populations since the meaning derived through language use is similar. Perhaps this shows that within the constraints of such questions there are only a finite number of ways that a set of propositions can be manipulated.

The above argument is supported by comparing student responses to questions Q-5, Q-6, Q-7, and Q-8. Within the context of Q-6, most students recognize the contribution of charges to the resultant electric field between the conducting plates as opposed to Q-5. But comparing Q-6 to Q-8, we have to ask what happened to the electric field in the latter case, where most students registered an absence of the electric field between the plates. The charges are still on conducting plates in
Q-8. If the causal argument ("field if mobility") provided by Rainson et al. [40] is to be considered, then the only possible conclusion is that the causal arguments are suppressed as a result of learning Gauss's law. Thus, we argue that the results are examples of context specific reasoning and model building. In the context of Q-7 and Q-8, students have raised the procedural knowledge of Gauss's law to a working model for finding the electric field, and as such, whether charges reside on the conducting plates does not matter in such contexts.

We further argue that the working model in the context of Gauss's law \( q_{\text{enc}} \Rightarrow E \) is derived through the examples provided in instruction and in textbooks, and as such, do not belong to any causal argument as suggested by Rainson et al. [40]. The fundamental issue as we see it lies with students not realizing that \( q_{\text{enc}} \) actually determines the flux of the field rather than the field itself; that is, that Gauss's law is a statement about the electric flux and not about the electric field. The specific examples to which students are introduced in studying Gauss's law have symmetries that allow students to find the electric field without integrating over the Gaussian surface. This, in turn leads to the determination of the working model mentioned above and becomes part of their understanding. Since models are also parsimonious, such relations provide easy functionality for answering many questions. The problem seems to be generic whenever an "enclosed" entity is involved, as shown by the Ampre’s law (Q-43).
While it is known that the use of propositions and procedural rules are necessary in solving problems demanding exact numbers (content problems), they are assumed to be irrelevant in conceptual problems. While specific calculations are not needed in conceptual problems, students do seem to function in terms of propositional and procedural rules even for purely conceptual problems. In this case, rather than looking for and manipulating mathematical equations, students seem to look for the existence of certain keywords and relations between them (e.g., charge, the changing magnetic field, and the presence of a wire loop). These rules involve only a few elements. Thus, students' models in electricity and magnetism are simple. That is, their reasoning is done in terms of simple representations although the actual reasoning required may be complex. Thus, conceptual change also has to be viewed as a change in the use of one set of rules to another. This is particularly the case since the abstract nature of a subject such as electricity and magnetism lies close to reasoning in terms of abstract rules rather than through experiential knowledge.

Models in novel contexts are constructed in a way that preserves certain relations among questions. However, these relations are mapped from one context to another in the most direct way. That is, students are able to build models based on analogies. Depending on which relations among questions are preserved, students may automatically arrive at the correct or the incorrect solution. Most of the questions we have developed have features that lead students to a correct solution in one context and to an incorrect solution in another context. This is particularly the case when model building is constrained by a fundamental set of elements and procedures. Some reasoning patterns can be categorized into a set of if, then rules between questions [49]. For example, if right-hand-rule yields result “up,” then the left-hand-rule yields the
result “down;” if one side attracts, then the opposite side repels; if grounding one side remove positive charges, then grounding the other side remove negative charges.

We find that procedural rules either exist or are constructed in ways that override the “reality” of physical phenomena. Two examples are provided by the Gaussian surface problems (Q-7, Q-8) and the right-hand versus left-hand rule problems (Q-37, Q-38).

Students may see what they want to see and not what is dictated by a question itself. That is the existence of models itself affect the outlook of the question and may lead to a different interpretation than that of the intended one. For example, in Q-6, students’ model of the mobility of charges on conductors lead to manipulating all the charges on a plate to the inner wall of the plate. In Q-44, the model that exist in terms of moving charges is applied to the charge at rest taking certain features of the question to provide the context of movement. The two cases are again examples of how students perceive the questions when constrained by underlying models and how “unfamiliar” situations are accommodated within existing models.

We see that two different models can exist for the same phenomenon without conflicting with each other. For example, in Q-55 and Q-56 the existence of a set of elements, such as the wire loop and the changing magnetic field, naturally lead to the identification of a current, while in Q-57 where the loop is absent, the elements do not lead to the same identification. We are confident that most students would recognize current as moving charges (electrons) if specifically asked.
All such cases point to the important role the context play within which learning takes place. Model construction takes place within specific contexts through various representations. Therefore, students’ concept learning has to be viewed as simple constructions of models and rules. As such, small scale conceptual structures should be built through processes that involve an expansion of the context.

5.2 Implications for teaching

We find that minor variations among questions lead to different solutions although such questions may have identical solutions. This suggests that students should be given opportunities to apply their knowledge within a set of questions which has low variability among question contexts before exposing them to questions that have higher variability. We suggest that knowledge hierarchies in electricity and magnetism should be built within specific contexts where the problems of representations, both language and picture-like, and the ontological demands placed by the concepts themselves are well discussed.
The mental representations formed by students depend on the picture and languagelike representations used in instruction (including classroom and textbook). In instruction we implicitly assume that a description of a concept provided in one context with a particular set of representations would make students able to generalize the concept to any other context. Our study shows that this is not the case, and moreover, that the context from one question to another does not have to differ much in its structure in order for students to answer incorrectly. Thus, an awareness of the context within which concepts are introduced as well as the specific representations (both pictorial and language-like) used to introduce the concepts are needed.

Notions such as the absence of electric field inside a metal (in electrostatic equilibrium) and equipotential surfaces exist only at a propositional level. As such, the need to reason in such terms are not easily recognizable. The implication for teaching such concepts is to devise questions that can be reasoned through using many different representations, particularly involving charge, and to make evident how alternative representations such as the field and potentials take over when representations involving charges fail. In other words, it is not enough to use the most convenient or the optimal representation within a given context; students must explicitly be made aware of the limitations of certain representations. This may help build the necessary knowledge hierarchies.

Causal arguments play a fundamental role in reasoning. Thus, the pitfalls of causal reasoning has to be emphasized. As stated above, students’ learning has to be viewed as the construction of a set of rules and models that are largely localized and reasoned about within the most obvious features present in a given problem (external) representation. Perhaps we need to explore an instructional strategy that would start
with a concept such as electrostatic potential or field and then derive Coulomb’s law rather than first introducing the notions such as Coulomb force law. Since language plays an important role in model construction, starting with a field or potential view may help students realize from the beginning that the reasoning to be used in dealing with electricity and magnetism is different from reasoning in terms of direct causal agents characteristic of Newtonian mechanics.

As a result of the work reported in this dissertation students must be exposed to proper context and the differing representations. This is because the abstract nature of a mature science such as physics has many assumptions, representations, and methodologies buried inside, few of which may be self-evident to the novice. The situation is that even the most dedicated and motivated student may not have enough time to recognize (much less deal with) such issues during the short span of the course.

The questions we have developed provide a model-based diagnostic tool to address such concerns. They are designed to be used in class with a device such as the computerized voting machine system or some other form of feedback. Such a system provides an opportunity to reveal students’ reasoning promptly to the instructor, enabling feedback and discussion leading to a recontextualization. This involves guiding students through a series of related questions with low variability in context. Within one context, students may reason with a familiar model using a particular representation. Recontextualization then suggest the exposure of students to the limitations of their models, representations and the ontological demands placed by the subject. This may help elucidate and make students realize the fundamental structure of the electromagnetic phenomena.
A version of the voting machine system implemented in the FEH class revealed that most students favored the notion of a “series of simple questions that guide them to some big idea” (from a Likert scale ranging from -2 to +2, +1 was selected by 51% while +2 was selected by 36%). This is opposed to having “a single, more complex question that does the same thing” (0 in the Likert scale was selected by 25% and +1 was selected by 32%. Only 19% selected +2). It is our hope that the use of a set of questions will help students realize the shortcomings of their reasoning through the process of recontextualization.

The above process is important since as mentioned in the previous section learning is a context and representation dependent process. Therefore, students should be exposed to situations where they are able to react upon the context within which concepts have been learned, the specific representations used in learning, the limitations of such representations and their (students’) ontological presuppositions. This we believe could lead to the construction of a set of working models that provide a useful set of “tools” in understanding the electric and magnetic phenomena.

5.3 Future directions

In this dissertation we have covered only a selected set of topics. The selection of the topics was largely motivated by the key areas in which we observed students to have conceptual difficulties and our belief about what concepts it is most important to elucidate in order to achieve the necessary ontological shifts. The question pool we have begun should be expanded in order to give instructors a wider selection. In turn, this will give the opportunity for instructors to divide a set of related questions
between the classroom and the examinations. Such a division can then lead to useful comparisons of how well the recontextualization process help achieve conceptual change and the ontological shift required in electricity and magnetism.

Some of the questions we have designed can be answered by using more than one model and also, as we have found, the way some questions are perceived are model dependent. Although we believe that these findings are important by themselves, it will be interesting to see whether we can design a set of questions that achieve a one-to-one correspondence between a choice and a model. This will minimize the ambiguity in our interpretations and will make instructional feedback much more effective. Any difficulties inherent in achieving such a goal will shed light on the cognitive processes and the limitations for instruction in terms of model-based learning.

The recontextualization process can be used as a method to investigate conceptual change, how it is achieved in terms of models and what external representations would be ideal in various contexts and what internal representations are formed in conceptual change. This may be achieved through several iterations of a set of questions until the optimum number of desired features in the questions are achieved. This in turn will also help us understand to what extent we have to carry the recontextualization process until we are confident that students can function successfully by applying the concepts to many and varied situations.

A related and an important question to be explored is to what extent the language and pictorial representations function in students’ reasoning. For example, it is important to understand how students perceive a question without an accompanying pictorial representation. This will help us understand which particular representations can be better put to use depending on what aspects of a concept we want
to emphasize. This will also help us understand the role of representation in the
interpretation and perception of a question. Also, carrying out a set of controlled
experiments of the same questions with and without pictorial representations is rela-
tively straightforward. This will also help us understand the inherent need for some
representations and the redundancy of others. All these will help reduce “noise” in
students’ reasoning and make instructional feedback well directed and efficient. Fur-
ther research is needed to explore whether an explicit awareness of the context and
the representation with which the subject matter is introduced lead to constructing
a better knowledge structure and whether it facilitates the hierarchical knowledge of
the subject as asserted by several researchers (e.g., [50]).

It is also to be seen how a model-based diagnostic instrument and the recontex-
tualization process can be implemented in areas such as quantum mechanics. Such
an implementation will be of great value since the conceptual and ontological leap
required by a subject such as quantum mechanics is immense. A fine grained step-by-
step guidance through quantum phenomena with an explicit awareness to the context
and the representation of its concepts is called for in order to help students construct
a desired functional understanding.
5.4 Summary

We have shown how sensitively the reasoning is dependent on the context. The context involve the way concepts are introduced, the particular representations used and the features of a question. However, learning has to begin within a given context with a set of knowledge elements. Within these conditions students develop a set of internal representations or a set of working models. Thus, these models are constrained by the context within which learning took place.

As we have seen, traditional instruction does not attempt to make students explicitly aware of the pitfalls of such models, yet it demands that students construct the necessary conceptual understanding. Such a demand is extremely difficult to realize, however, given the many facets that are involved in understanding physics concepts due to the nature of representations and ontological demands on the one hand and issues such as the time constraints of course on the other. Thus, an efficient method is called for to make students aware of the context within which they are introduced to concepts, the representations used and their limitations from one context to another, and the ontological demands required by the subject.

To address the above we have developed a model-based diagnostic instrument, which consists of a set of related multiple-choice questions with low variability in context for a given concept. We have verified the usefulness of the theoretical ideas put forward by many researchers on human reasoning within the context of electricity and magnetism.
We use our findings to propose a recontextualization process: this involve students being exposed first to a familiar situation where they can draw associations to their experiential knowledge or the traditional classroom discussions with ease. This help students recall and associate with knowledge elements they see as providing the necessary explanations. Such explanations can be considered as the initial working models. The recontextualization process then build on existing models by providing opportunities for the students to rethink their learning context and the representations on the one hand and providing the instructor an opportunity to understand where his/her instruction stands with respect to model building and provide prompt feedback to students on the other. This we believe will help students revise their models of the electric and magnetic phenomena with an explicit awareness to their learning process.
APPENDIX A

QUESTIONS

In the questions that follow the correct choice is *italicized*.

Q-1

Two stationary point charges +3 C and +1 C are separated by some distance. What can be said about the Coulomb force between the two charges?

A. The +3 C charge exerts more force on the + 1 C charge.
B. The +1 C charge exerts more force on the +3 C charge.
C. Both charges exert an equal amount of force on each other.
A stationary point charge of +3 C is kept at some distance from a large fixed sphere, which has a total charge of +1 C. What can be said about the Coulomb force between the point charge and the sphere?

A. The point charge exerts more force on the sphere.
B. The sphere exerts more force on the point charge.
C. Both the sphere and the point charge exert an equal amount of force on each other.
Q-3
One end of an uncharged metal rod is brought close to a charged object.
What will happen?

A. Metal rod is attracted by the charged object.
B. Metal rod is repelled by the charged object.
C. Nothing happens.
D. Cannot say without knowing what type of charge the object has.

Q-4
What happens if the other end of the metal rod (refer Q-3) is brought
close to the same charged object?

A. Metal rod is attracted by the charged object.
B. Metal rod is repelled by the charged object.
C. Nothing happens.
D. Cannot say without knowing what type of charge the object has.
Q-5

Two plates made up of insulator material are kept fixed at some distance apart as shown in the figure below. The charges on the plates are distributed as shown.

The electric field at point P is due to?
A. All the charges on the positive plate.
B. All the charges on the negative plate.
C. All the charges on both plates.
D. Only the charges on the positive plate, which are on the side closer to P.
E. Only the charges on the negative plate, which are on the side closer to P.
F. Charges on both plates, which are on the sides closer to P.
Q-6

What is the electric field at point P due to if the plates are made up of conducting material (refer question Q-5) and having the same charge distribution as shown in the previous question?

A. All the charges on the positive plate.
B. All the charges on the negative plate.
C. All the charges on both plates.
D. Only the charges on the positive plate, which are on the side closer to P.
E. Only the charges on the negative plate, which are on the side closer to P.
F. Charges on both plates, which are on the sides closer to P.
Q-7

Two parallel conducting plates are kept at some distance apart as shown below. One plate carries a uniform charge density $\sigma$ and the other a uniform charge density $-\sigma$. A cylindrical Gaussian surface is drawn as shown below. What is the magnitude of the electric field between the plates?

A. $\frac{\sigma}{2\epsilon_0}$

B. $\frac{\sigma}{\epsilon_0}$

C. Zero

D. None of the above
Q-8

Two parallel conducting plates are kept at some distance apart as shown below. One plate carries a uniform charge density \( \sigma \) and the other a uniform charge density \(-\sigma\). A cylindrical Gaussian surface is drawn as shown below. What is the magnitude of the electric field between the plates?

\[ \text{A. } \frac{\sigma}{\epsilon_0} \]
\[ \text{B. } \frac{2\sigma}{\epsilon_0} \]
\[ \text{C. Zero} \]
\[ \text{D. None of the above} \]
Q-9

positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. Which of the following represent the charge distribution on the inner and outer walls of the shell?

Correct choice = 4
Q-10

A positive charge is kept (fixed) off-center inside a fixed spherical neutral conducting shell. Which of the following represent the charge distribution on the inner and outer walls of the shell?

Correct choice = 1
Q-11

A positive charge $Q$ is kept fixed at the center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. Which of the following represents the charge distributions?

1  

2  

3  

4  

5  None of the above

Correct choice = 4
Q-12 from CSEM

A positive charge (+q) is kept at the center inside a spherical neutral conducting shell. Outside the sphere is a charge +Q. Which of the following describes the net electric forces on each charge?

1. Both charges experience the same net force directed away from each other.
2. No net force is experienced by either charge.
3. There is no force on Q but a net force on q.
4. There is no force on q but a net force on Q.
5. Both charges experience a net force but they are different from each other.
Q-13

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The charge distribution is shown below. What can be said about the electric potential between points A & B which are on the inner surface of the shell?

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-14

A positive charge is kept (fixed) off-center inside a fixed spherical neutral conducting shell. The charge distribution is shown below. What can be said about the electric potential between points A & B which are on the inner surface of the shell?

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-15

A positive charge $Q$ is kept fixed at the center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. The charge distribution is shown below. What can be said about the electric potential between points A & B which are on the outer surface of the shell?

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-16

A positive charge $Q$ is kept fixed at the center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. The charge distribution is shown below. What can be said about the electric potential between points A & B? (Point A is on the outer surface and point B is on the inner surface.)

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-17

A positive charge $Q$ is kept fixed off-center of a spherical neutral conducting shell. A negative charge $Q$ is brought near to the outside of the sphere. The charge distribution is shown below. What can be said about the electric potential between points A & B? (Point A is on the outer surface and point B is on the inner surface.)

A. $V_A > V_B$.
B. $V_A = V_B$.
C. $V_A < V_B$.
D. Cannot say for sure.
Q-18
A conducting thin spherical shell of radius $R$ has a total charge $+Q$ distributed uniformly on its surface. What is the electric potential (relative to a point at infinity) at a point a distance $r$ from the center where $r > R$?

A. $\frac{Q}{4\pi\varepsilon_0(R-r)^2}$

B. $\frac{Q}{4\pi\varepsilon_0(r-R)}$

C. $\frac{Q}{4\pi\varepsilon_0(R-r)}$

D. $\frac{Q}{4\pi\varepsilon_0r}$

E. $\frac{Q}{4\pi\varepsilon_0R}$

F. None of the above
A conducting thin spherical shell of radius $R$ has a total charge $+Q$ distributed uniformly on its surface. What is the electric potential (relative to a point at infinity) at a point on the surface?

A. $\frac{Q}{4\pi\varepsilon_0 R^2}$  
B. Zero  
C. Infinite  
D. $\frac{\sigma}{\varepsilon_0 R}$  
E. $\frac{Q}{4\pi\varepsilon_0 R}$  
F. None of the above
Q-20

A conducting thin spherical shell of radius $R$ has a total charge $+Q$ distributed uniformly on its surface. What is the electric potential (relative to a point at infinity) at a point a distance $r$ from the center where $r < R$?

A. $\frac{Q}{4\pi\varepsilon_0 (R-r)^2}$

B. Zero

C. $\frac{Q}{4\pi\varepsilon_0 (R-r)}$

D. $\frac{Q}{4\pi\varepsilon_0 r}$

E. $\frac{Q}{4\pi\varepsilon_0 R}$

F. None of the above
Q-21

A positive charge is brought and kept fixed in location close to a neutral conducting rod. The end further away from the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge.

B. Negative charge.

C. No charge (Neutral).
Q-22

A positive charge is brought and kept fixed in location close to a neutral conducting rod. The end closer to the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge.

B. Negative charge.

C. No charge (Neutral).
Q-23

A negative charge is brought and kept fixed in location close to a neutral conducting rod. The end further away from the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-24

A negative charge is brought and kept fixed in location close to a neutral conducting rod. The end closer to the charge is then connected to the ground by a conducting wire as shown below.

What is the charge on the conducting rod after the ground connection is removed?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-25

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The outer surface of the sphere is then grounded. What type of charge remain on the outer surface after grounding?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-26

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The inner surface of the sphere is then grounded. What type of charge remain on the outer surface after grounding?

A. Positive charge.
B. Negative charge.
C. No charge (Neutral).
Q-27

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The outer surface of the sphere is then grounded. What type of charge remain on the *inner* surface after grounding?

A. Positive charge.

B. Negative charge.

C. No charge (Neutral).
Q-28

A positive charge is kept (fixed) at the center inside a fixed spherical neutral conducting shell. The inner surface of the sphere is then grounded. What type of charge remain on the *inner* surface after grounding?

A. Positive charge.

B. Negative charge.

C. No charge (Neutral).
Q-29

Consider two concentric conducting spherical shells of radii \( a \) and \( b \) where \( b > a \). The inner and outer shells carry charges of \( +Q \) and \( Q \) respectively distributed uniformly on its surfaces. Imagine grounding the outer shell. What is the charge on the outer shell after grounding?

1. \( +Q \)
2. Zero
3. \( -Q \)
4. None of the above
Q-30
Consider two concentric conducting spherical shells of radii a and b where \( b > a \). The inner and outer shells carry charges of \( +Q \) and \( Q \) respectively distributed uniformly on its surfaces. Imagine grounding the inner shell. What is the charge on the inner shell after grounding?

1. \( +Q \)
2. Zero
3. \( -Q \)
4. None of the above
A and B are two identical metal spheres. Sphere A carries a net charge of +4 C and sphere B carries a net charge of +2 C. Initially they were kept apart from each other. The two spheres are then brought together and were made to touch each other for some time before separating them again.

Q-31
Think about the instant (a snapshot) when the two spheres make contact. What can you say about the Coulomb forces acting on a charge on sphere A?

A. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is zero.
B. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is not zero.
C. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is zero.
D. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is not zero.
E. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is zero.
F. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is not zero.
Q-32

Think about an instant (a snapshot) after some time where the two spheres are still in contact. What can you say about the Coulomb forces acting on a charge on sphere A?

A. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is zero.

B. A charge on sphere A experience forces only from the charges on sphere A and the net force on the charge is not zero.

C. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is zero.

D. A charge on sphere A experience forces only from the charges on sphere B and the net force on the charge is not zero.

E. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is zero.

F. A charge on sphere A experience forces from the charges on both spheres and the net force on the charge is not zero.
Q-33

Consider the circuit below. The cross sectional area of the conducting wire is the same throughout. Rank the currents at points A & B.

A. $I_A > I_B$

B. $I_A = I_B$

C. $I_A < I_B$
Q-34

Consider the circuit below. The cross sectional area of one segment of the conducting wire is less than the rest of the wire. Rank the currents at points A & B.

A. $I_A > I_B$
B. $I_A = I_B$
C. $I_A < I_B$
Q-35

A wire is connected to a battery as shown below. The cross sectional area of one segment is lesser than the rest of the wire. How does the magnitude of the electric field compare at points A & B?

1. $E_A > E_B$
2. $E_A = E_B$
3. $E_A < E_B$
4. Cannot say for sure
Q-36

A wire is connected to a battery as shown below. The cross sectional area of one segment is lesser than the rest of the wire. How does the drift speed of the electrons compare at points A & B?

1. \(v_A > v_B\)
2. \(v_A = v_B\)
3. \(v_A < v_B\)
4. Cannot say for sure
Q-37

A positive charge moving horizontally at constant speed enters a magnetic field region from the left side. The magnetic field in the region is pointing in to the page. Which way will the charge move?

1. P direction
2. Q direction
3. R direction
Q-38

If instead of the Right Hand Rule a *Left Hand Rule* is used, which way will the charge move? (Refer Q-37)

1. P direction
2. Q direction
3. R direction
Q-39
Two parallel wires carry currents in the same direction. Applying the Right Hand Rule, what can be said about the magnetic force between the two wires?

A. The wires attract each other.
B. The wires repel each other.
C. Nothing happens.

Q-40
Two parallel wires carry currents in the same direction. If instead of the Right Hand Rule a *Left Hand Rule* is used, what can be said about the magnetic force between the two wires?

A. The wires attract each other.
B. The wires repel each other.
C. Nothing happens.
Q-41

An Amperian loop is drawn around two current carrying wires as shown below. What is $\int \vec{B} \cdot d\vec{s}$ equal to around the loop?

A. $\mu_0(i_1 - i_2)$

B. Zero

C. $\mu_0(i_1 + i_2)$
Q-42

An Amperian loop is drawn around two current carrying wires as shown below. The angle between the two wires is $\theta$. What is $\int \mathbf{B} \cdot d\mathbf{s}$ equal to around the loop?

A. $\mu_0(\mathbf{i}_1 \cos \theta + \mathbf{i}_2)$ 
B. $\mu_0(\mathbf{i}_1 + \mathbf{i}_2)$ 
C. $\mu_0(\mathbf{i}_1 + \mathbf{i}_2 \cos \theta)$
Q-43

Three current carrying wires 1, 2 & 3 are shown below. The wires 2 & 3 are enclosed by an Amperian loop. The magnetic field at point P is due to?

A. 1 only
B. 2 & 3 only
C. All wires
Q-44

A positive charge is placed at rest in a magnetic field as shown below. Which way will the charge move?

A. Left
B. Right
C. Up
D. Down
E. It will not move at all
Q-45

A positively charged particle is moving horizontally to the right in a uniform magnetic field which is pointing in the same direction. What is the direction of the magnetic force on the charge?

A. Left  
B. Right  
C. Up  
D. Down  
E. None of the above
Q-46

A positively charged particle is moving at a constant speed in some direction in a region where there are no fields present. Suppose the particle now enters a magnetic field region, which is uniform and pointing in the same direction as the particle is initially moving. What happens to the speed of the particle?

A. Stays the same
B. Decreases
C. Increases
Q-47

The following situations involve a CONDUCTING rectangular wire loop in a magnetic field. The magnetic field has CONSTANT magnitude and direction in the region shown by a cross X. The magnetic field is into the paper.

In the three pictures shown below the wire loop is moving at a constant velocity (i.e. at a constant speed in the fixed direction as shown by the arrow).

In picture I the loop is just moving into the magnetic field region.

In picture II the loop is moving in the field.

In picture III the loop is moving out of the field region.
Which situations correspond to an induced current in the loop?

A. I  
B. II  
C. III  
D. I and II  
E. II and III  
F. I and III  
G. All  
H. None
Q-48

The following situations involve a CONDUCTING circular wire loop in a magnetic field. The magnetic field has CONSTANT magnitude and direction in the region shown by a cross X. The magnetic field is into the paper.

Consider the following three pictures:

In picture IV the loop is at REST in the magnetic field region.

In picture V the loop is MOVING AT CONSTANT VELOCITY in the direction shown.

In picture VI the loop is ROTATING clockwise AT CONSTANT ANGULAR VELOCITY about an axis through its center.
Which situations correspond to an induced current in the loop?

A. IV
B. V
C. VI
D. IV and V
E. V and VI
F. IV and VI
G. All

H. None
Q-49

A wire loop is placed in a uniform external magnetic field as shown below. If the magnitude of the external magnetic field increases, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field

B. Induced magnetic field is in the direction opposite to the external magnetic field

C. There is no induced magnetic field
Q-50

A wire loop is placed in a uniform external magnetic field as shown below. If the magnitude of the external magnetic field decreases, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
Q-51

A wire loop is placed in a constant uniform external magnetic field as shown below.

If the external magnetic field is suddenly switched-off what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
A wire loop is placed in a constant uniform external magnetic field as shown below.

If the loop starts shrinking, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
Q-53

A wire loop is placed in a constant uniform external magnetic field which is confined to the circular area as shown below (i.e. the magnetic field is present only within the circular area).

If the loop starts expanding, what is the direction of the induced magnetic field through the loop?

A. Induced magnetic field is in the same direction as the external magnetic field
B. Induced magnetic field is in the direction opposite to the external magnetic field
C. There is no induced magnetic field
Two equal bulbs are connected in series. A very long current carrying solenoid (S) goes through the circuit loop. The magnetic flux through the solenoid is varied at a constant rate so that there is an emf in the circuit (Fig. 1). What happens to the brightness of the bulbs as the circuit loop expands (Fig. 2)?

A. Increases
B. Decreases
C. Stays the same
D. Cannot say for sure
A wire loop is placed in an external magnetic field which is confined to the circular (gray) area as shown below (i.e. the magnetic field is present only within the gray area).

If the external magnetic field increases only within the gray area, what is the direction of the induced current in the loop?

A. Clockwise
B. Counterclockwise
C. There is no induced current
Q-56

A square wire loop is placed in an external magnetic field which is confined to the circular (gray) area as shown below (i.e. the magnetic field is present only within the gray area).

If the external magnetic field increases only within the gray area, what is the direction of the *induced* current in the loop?

A. Clockwise

B. Counterclockwise

C. There is no induced current
A constant external magnetic field is confined to the circular (gray) area as shown below (i.e. the magnetic field is present only within the gray area). Charge Q is placed at rest inside the magnetic field area where as charge q is placed at rest outside the magnetic field area.

If the external magnetic field now increases only within the gray area, what can be said about the two charges?

A. Only Q will start moving
B. Only q will start moving
C. Both charges will start moving
D. None of the charges will move
A constant external magnetic field is confined to the circular (gray) area as shown below (i.e. the magnetic field is present only within the gray area). In the two situations shown, charge Q is placed at rest inside the magnetic field area whereas charge q is placed at rest outside the magnetic field area.

If the external magnetic field now increases only within the gray area, what can be said about the two charges?

A. Only Q will start moving
B. Only q will start moving
C. Both charges will start moving
D. None of the charges will move
Q-59
Two neutral metal spheres are kept fixed at a close distance. What will happen?

A. Attract  
B. Repel  
C. Nothing happens

Q-60
A stationary point charge is kept at a close distance from a stationary neutral point particle. Which of the following is true?

A. The point charge exerts more force on the neutral point particle.  
B. The neutral point particle exerts more force on the point charge.  
C. Both exerts equal amount of force on each other.  
D. There is no force between the point charge and the neutral point particle.
**APPENDIX B**

**SUMMARY OF MODELS & CONTEXT FACTORS**

<table>
<thead>
<tr>
<th>Measure</th>
<th>Models</th>
<th>Context Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1, Q2</td>
<td>Newton’s third law in electrostatics</td>
<td>Q1 contains two point charges while Q2 contains a point charge and a charged sphere; magnitude of charge; size of the sphere</td>
</tr>
<tr>
<td></td>
<td>Equal amounts of charge ⇒ equal amounts of force.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unequal amounts of charge ⇒ unequal amounts of force</td>
<td></td>
</tr>
<tr>
<td></td>
<td>More charge ⇒ more force</td>
<td></td>
</tr>
<tr>
<td></td>
<td>‘Coulombs’ as a quantity of force</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dimensions of the sphere ⇒ more force</td>
<td></td>
</tr>
<tr>
<td></td>
<td>More ⇒ more (p-prim)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>More: charge, size of the object, e.g. sphere</td>
<td></td>
</tr>
<tr>
<td></td>
<td>“Charge” directly mapped to Coulomb’s law extracting out the charge distribution of the sphere; charged sphere = point charge</td>
<td></td>
</tr>
<tr>
<td>Q3, Q4</td>
<td>Polarization of charges in a neutral metal rod</td>
<td>Unlike charges attract, like charges repel</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Polarization process lead to a set of fixed charges</td>
<td></td>
</tr>
<tr>
<td></td>
<td>one end attract ⇒ other end repel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Irreversibility of causal processes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Neutrality of the metal sphere ⇒ nothing happens</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bringing one end of a neutral metal rod near a fixed charge and then bringing the other end</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q5, Q6</th>
<th>Superposition of electric field</th>
<th>Insulators ⇒ inhibitors of electric field</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conductors ⇒ ‘allow’ electric field</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Insulators ⇒ ‘allow’ electric field</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Absence of electric field inside conductors ⇒ inhibiting the electric field</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Charges move freely through/on conductors</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Differences of material in insulator/conductor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Differences in charge movement in insulator/conductor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unlike charges attract ⇒ rearrangement of charges on the conductor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Notions of ‘Insulator’ and ‘Conductor’</td>
<td></td>
</tr>
<tr>
<td>Q7, Q8</td>
<td>Reality of the magnitude of the electric field between the plates</td>
<td>Gaussian surface invariant model. This model gives prominence to the actual setup of the plates and the charges, ignoring the nature of the Gaussian surface</td>
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<tr>
<td></td>
<td>Reasoning in terms of $q_{enc}$ (charge enclosed)</td>
<td>Charge enclosed model. ($q_{enc} \Rightarrow \text{flux} \Rightarrow \text{field}$)</td>
</tr>
<tr>
<td></td>
<td>Superposition of the electric field</td>
<td>Flux model (flux $\Rightarrow$ field)</td>
</tr>
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<td></td>
<td></td>
<td>‘Charge double’ model - Enclosing two plates takes into account twice as much charge ($\Rightarrow$ twice the field)</td>
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<td></td>
<td></td>
<td>Gauss’s law directly determines the electric field</td>
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<td></td>
<td></td>
<td>In Q8, the two end caps of the Gaussian surface (cylinder) are inside the two plates as opposed to one in Q7</td>
</tr>
<tr>
<td>Q9, Q10, Q11</td>
<td>Absence of an electric field inside a conductor</td>
<td>Symmetry of the picture leads to symmetry of charges</td>
</tr>
<tr>
<td></td>
<td>Reasoning in terms of attraction and repulsion of charges</td>
<td>Central positive charge attract negative charges on the near inner wall, which in turn attract positive charges on the near outer wall</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Central positive charge attract negative charges on the near inner wall but repels the positive charges on the outer wall to the far side</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Position of the inner charge</td>
</tr>
<tr>
<td>Q13, Q14, Q15, Q16, Q17</td>
<td>Superposition of potentials</td>
<td>Charge density</td>
</tr>
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<td>-------------------------</td>
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<tr>
<td>Equipotential surfaces</td>
<td>Distance from the inner charge to the walls; $\frac{1}{r}$ relation</td>
<td>more $\Rightarrow$ more (charge density relations)</td>
</tr>
<tr>
<td></td>
<td>Distance relations (p-prim) closer $\Rightarrow$ more</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q18, Q19, Q20 $\frac{1}{r}$</th>
<th>Superposition reasoning in terms of $\frac{1}{r}$</th>
<th>Charges at the center</th>
<th>The point of interest is where the potential is needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charges as been concentrated at the center (for $r &gt; R$)</td>
<td>Measuring distance from the charges on the surface</td>
<td>No charge $\Rightarrow$ no potential ($r &lt; R$)</td>
<td>No field $\Rightarrow$ no potential ($r &lt; R$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mapping a point charge to the shell</td>
<td>$\infty$ as an impossibility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Potential generated at an exterior point of a point charge</td>
<td></td>
</tr>
</tbody>
</table>

292
<table>
<thead>
<tr>
<th>Q21, Q22, Q23, Q24</th>
<th>Superposition</th>
<th>Like repel unlike attract</th>
<th>Position of the ground connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipotential surfaces</td>
<td></td>
<td>Polarization of metal; Grounding removes charges that exist around the location of grounding</td>
<td>Sign of the external charges</td>
</tr>
<tr>
<td>Charges reacting to a potential difference</td>
<td></td>
<td>Negative charges (electrons) as the type of charge coming from the ground</td>
<td>Neutral conductor</td>
</tr>
<tr>
<td>Like charges repel unlike charges attract</td>
<td></td>
<td>The external charges as the dominant charge that determines the movement to and from the ground</td>
<td></td>
</tr>
<tr>
<td>Grounding as removal of charges that exist around the location of grounding</td>
<td></td>
<td>Invariance of the neutrality of the conductor</td>
<td></td>
</tr>
<tr>
<td>Sign of the external charge has any effect in reasoning</td>
<td></td>
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</tbody>
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<tr>
<th>Q25, Q26, Q27, Q28, Q29, Q30</th>
<th>Superposition</th>
<th>Grounding removes charges</th>
<th>The location of the ground connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipotential surfaces</td>
<td></td>
<td>All charges should reside on the outside of the sphere</td>
<td></td>
</tr>
<tr>
<td>Attraction and repulsion of charges</td>
<td></td>
<td>Attraction and repulsion of charges</td>
<td></td>
</tr>
</tbody>
</table>

293
| Q31, Q32 | Superposition | Entire charge on the sphere as a single entity ⇒ only external charges can have any influence  
Potential differences  
Equal spheres → (contact) → equal charges  
Imbalance of charges ⇒ net force  
Balance of charges ⇒ no net force | Term ‘Force’  
The differences in the magnitude of charges  
Equality of the spheres |
| Q33, Q34, Q35, Q36 | Local reasoning of resistance  
Ohm’s law | Less resistance more current, more resistance less current (p-prims)  
Current = charge  
Area as the amount of charges contained (current)  
More area ⇒ more current  
Less area ⇒ less current  
Less area ⇒ less resistance ⇒ more current  
Entire wire as a single resistor  
High resistance ⇒ high potential ⇒ high electric field  
High electric field ⇒ high $v$  
Current ⇒ $v$ | Narrow and wider segments of a wire |
| Q37, Q38, Q39, Q40 | Reality of dynamics of charge in a magnetic field | If right hand (left hand) rule lead to one effect than the Left hand (Right hand) rule should lead to the opposite effect | Use of the ‘left hand’ as opposed to the right hand |
| Reality of attraction/repulsion between current carrying wires | ‘Same direction’ of currents ⇒ same effect | Only one charge in a field |
| Q41, Q42 | Ampere’s law | Currents should be perpendicular to the plane of the loop | Use of the word ‘same direction’ |
| | | | Two wires to compare |
| Q43 | Ampere’s law | Currents enclosed by the Amperian loop determines the magnetic field | Currents inside and outside the Amperian loop |
| Q44, Q45, Q46 | Absence of magnetic force for a charge at rest and moving parallel to the field | If charge is moving parallel to the field then there is no force  
Positive charge at rest attracted to negative pole of the magnetic field  
Picture ⇒ field direction ⇒ right hand rule  
Field direction as the application of force in that direction when charge enters the region | Picture uses  
Language use  
Presence of field lines and the symbol for the positive charge |
| Q47, Q48 | Magnetic-Induction Movement  
Translational movement as opposed to rotational movement | Magnetic force ⇒ currents  
Velocity ⇒ currents  
Spatial change of the loop | Stationary loop  
Loops having translational motion only  
Loops having rotational motion only |
| Q49, Q50, Q51, Q52, Q53, Q54 | Magnetic induction under changes to the field and area of the loop | Increases \((\vec{B}, A) \rightarrow \text{opposite direction}\)  
Decreases \((\vec{B}, A) \rightarrow \text{same direction}\)  
The changes in the field has to occur with the field itself been present  
\(A \Rightarrow \text{Area of the loop}\) | Increasing \(\vec{B}\)  
Decreasing \(\vec{B}\)  
Vanishing of \(\vec{B}\)  
Region of \(\vec{B}\) |
| Q55, Q56, Q57, Q58 | Induced current, charge relation under changing magnetic field | Currents are induced under changing magnetic fields  
A charge is affected only when it is directly in the changing magnetic field | Wire loops inside a changing magnetic field  
Wire loop outside but enclosing changing magnetic field  
Charges placed at rest inside and outside a changing magnetic field |
| Q59, Q60 | How the notion of a neutral point particle in the presence of a point charge is reasoned | Neutral point particle = neutral sphere | Two neutral spheres

Polarization

Neutral point particle = neutral charge

Neutral point particle and a point charge |


