ESSAYS ON INFLATION AND GROWTH

DISSERTATION

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By

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* * * * *

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ABSTRACT

This dissertation investigates the relationship between inflation and economic growth emphasizing the financial sector and differences across countries.

The first essay uses a cash-in-advance model with credit as an alternative means of payment and simulates several steady-state relationships. Exogenous financial innovations lead to upward trending velocity as has been observed in the last half century or more. Differences in financial innovation across countries, or within a country and across time, affects how inflation affects output growth. Lower costs of intermediation imply that inflation affects output less than in countries with higher costs and similar effects hold within the same country when intermediation costs have changed over time. Either case presents difficulties for empirical tests.

The second essays examines the robustness of empirical inflation-growth results. Bayesian Model Averaging is used to explore the relationship between inflation and growth. Cross-sectional data over long periods of time provide no evidence of an inflation-growth relationship. Panel data, using 5-year averages, produces starkly different results, in particular, inflation is one of the more robust variables. Threshold affects are discussed but do not alter the main conclusions. Parameter heterogeneity across countries is investigated with the finding that coefficients vary widely with even small changes in the sample. The negative relationship of inflation and growth is still strong, but the importance of the point estimates is questionable.
The third essay utilizes standard time series methods. Structural inflation shocks are identified by imposing long-run restrictions on the model, and the effects of inflation on separate sectors of the economy are examined. This approach is applied to seven industrialized countries, with the results that inflation affects sectoral output differently, with little similarity across countries. Sectors such as manufacturing, agriculture and finance seem to be affected in quite different manners. The agriculture sector suggests that inflation may be harmful to growth, but the evidence is weak. Inflation is positively related to manufacturing growth and this is significant in most countries. Inflation is positively related to the finance sector in three countries and has no long run effect in four countries.
To Marda Rushing, Richard Hineline, Ginger Hineline and my children, Paige, Sydney and Grant.
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CHAPTER 1

INTRODUCTION

The effects that inflation has on growth has long been questioned, both theoretically and empirically. Unlike many monetary issues, where the question is often whether there is a specific effect or no effect, there are coherent models that predict inflation can have a positive effect, negative effect or no effect at all on output growth. Thus it seems an ideal situation to test the alternative theories against the data. Unfortunately the data often do not tell a convincing theory. Even stepping away from the difficulties of causation, the general correlation of inflation and output growth is often unclear. This is surprising, since policy-makers often have strong beliefs regarding the effects of inflation and the proper economic polices. This is more dramatic when the policy makers holding such prior beliefs are making decisions that will have repercussions on other countries, who’s own policy-makers may not share these beliefs, such as when the IMF insists on certain austerity measures.

The issue of inflation and growth has enjoyed somewhat of a revival since the early 1990’s, with the work of Barro (1996, 1997) and Fischer (1993) followed by many others. Much of the current research has used cross-country data, whether pooled in growth regressions or analyzed separately in time series analysis, yet one of the facts that has emerged is that there are great differences across countries. The
essays in this thesis focus on two common issues. First trying to better understand the direction of the effect of inflation on growth if there is such an effect, and second, how much and why do these effects vary across countries. These issues have important implications for which theoretical models are considered relevant as well as what the role of policy should be towards inflation. The conclusions are mixed. The evidence on the effect of inflation on growth is negative when using panel data and growth regressions but positive when using time series data and autoregressions which identify inflation shocks using long-run restrictions. On the other hand, the evidence is strong that the effects of inflation vary widely across countries, and studies that include a wide variety of countries may not provide good inference for making policy decisions in general.

The first paper sets up a neoclassical growth model with a cash-in-advance constraint with the additional twist that agents can use costly financial intermediation rather than money. This follows Ireland (1994), but the cost of finance is proportional to purchases rather than a fixed cost, and the financing constraint is extended to investment goods. The argument is that this will lead to a negative relationship between inflation and output, as is suggested in empirical growth regressions as well as most policy-makers beliefs. The analysis is then to examine what the effects of the additional finance sector imply. The first results are that trends in the velocity of money move in the directions that have been observed. Second, financial innovation in transactions services can lead to stronger growth, which complements findings that finance is positively related to growth. It is also shown that further development of finance will decrease the effects of inflation on growth. This has implications for the optimal inflation rate in more detailed models with additional imperfections. Thus
countries that find it more desirable to use the inflation tax for government revenue, i.e. those with less developed tax systems, are also those that are affected more by inflation because of having less developed financial services. The empirical implication is that countries with lower costs of finance should be affected less by inflation. If it is assumed that the OECD countries have less costly finance than less developed countries, this result is not supported in the results of the second essay.

The second essay investigates the direction of the effect as well as the implications of including diverse countries, but emphasizes the robustness of the results in growth regressions. Although growth regressions popularized much of the interest in the topic, these results are notoriously non-robust, in the sense that small changes to the regression equation or the omission of high inflation countries cause the results to become statistically insignificant. The robustness question is addressed by Levine and Renelt (1992) and the fact that inflation was found to be highly non-robust is seldom omitted from critiques of inflation and growth papers. The approach in this second essay is to utilize a recent method, Bayesian Model Averaging, and to focus on aspects of inflation and growth. The primary findings are that using averages over long periods in a cross country sample, as many growth regressions have done, the effects of inflation on growth are not significant, even if non-linearities are allowed for, such as only high inflation countries influencing growth. On the other hand, using panel data with seven periods of 5-year averages per country, inflation is a significant negative determinant of growth. Exploring the variability across countries, by running numerous regressions with alternate samples of countries, shows that the point estimates vary widely depending on even small changes in the sample.
The third essay takes a different econometric approach to the inflation growth question, using a structural vector autoregression. This follows papers by Bullard and Keating (1995), King and Watson (1997) and Rapach (2003), who find a positive relationship using a long-run restriction in the spirit of Blanchard and Quah (1989) to identify the VAR. Bullard and Keating and Rapach consider several countries and this evidence of a positive relationship is mixed, with some countries exhibiting a positive effect while others exhibit a negative effect or no long-run effect at all. The essay departs from previous work by looking at disaggregate data rather than aggregate GDP, with the hope of gaining a better understanding of the effects of inflation throughout the economy. The conclusion is that there are few results that are consistent across sectors and countries. Evidence that inflation permanently harms output seems possible in the case of agriculture, but even here the evidence is weak. On the other hand manufacturing seems to benefit from inflation and this effect is somewhat uniform across the sample of countries considered. Lastly, the financial sector does not respond in a uniform manner to inflation. It is positively effected in some countries and not effected at all in others, although there is no evidence of a significant negative effect.

Although the three essays are separate studies in themselves, they focus on the main theme that not only is the effect of inflation on growth open to question, but that statistical analyses are sensitive to the countries sampled and that sharp policy conclusions should not be drawn from such work. On the other hand, the results suggest that inflation does matter and that there is ample future work to be done, in contrast to the recent revolution in macroeconomics that argues that monetary matters are relatively unimportant. Although there are few concrete conclusions to
support any of the many theories that are currently popular, hopefully the results will be found valuable to those who believe that such matters are important and lead to continued work on the many details that are not well understood.
CHAPTER 2

INFLATION, FINANCE AND GROWTH IN A CASH-IN-ADVANCE ECONOMY WITH COSTLY INTERMEDIATION

2.1 Introduction

There have been numerous empirical and theoretical papers exploring the relationship between growth and output and the topic continues to receive ample interest. There are at least two good reasons why this research area remains active. On the empirical side, the effects of inflation on growth and the mechanisms through which it works is unclear, supporting a variety of possibilities, including that of no relationship. With these many possibilities comes many competing theories. On the practical side, there are policy implications and vested interest groups willing to use the newest research to support their agenda. I will begin this paper with a short survey of the empirical evidence and a very brief review of some theoretical models that produce typical results. Then I will describe how this paper’s model can explain some empirical puzzles and how it compares to similar models. The model is not drastically different than most cash in advance models, but allows the choice of cash and credit goods to be endogenous. This allows for the study of an explicit financial services sector, that is otherwise unavailable. It also allows for a much richer study
of velocity. The model predicts that inflation will have non-linear effects on output which will depend on the development of credit markets. Thus using panel data and assuming constant slope coefficients is almost sure to provide non-robust estimates.

The model is based on the insights of Ireland’s (1994) paper where there exists multiple methods of payments, in particular cash or credit. He analyzes the effects of inflation on growth and reversing the question considers the effects of growth on the use of money and credit. He concludes that money is not important to growth but that the growth of the economy has significant effects on the use of money. This is not surprising given that he assumes cash-in-advance on consumption goods only and an AK technology. This paper also will not have long-run growth effects, since a neoclassical setup with exogenous technological change is assumed. But there will be permanent steady-state consequences. The second problem is that Ireland finds a Tobin-like effect which clearly contradicts the majority of the evidence, as will be seen in the next section. The quantitative amount of this affect is small so it is not so much a problem. The main objection is that it in principle could be significant. While Ireland considers the moderate cases of 10% and 20% inflation, the growth affect would be higher for 100% and 200% inflation as well, implying that very high inflation may be good for growth, which is surely not the case. The setup below will produce the more common negative effect of inflation on growth.

Ireland (1994) introduces a method such that the allocation between cash and credit goods in the economy is endogenous. By assuming a fixed cost in spatially segmented markets, consumers choose to use credit in some markets and cash in others. Thus he produces a CIA model where consumption velocity is different from one, and the allocation between cash and credit goods is endogenous. In particular,
due to the cost of credit being fixed, as the economy grows the measure of markets where cash is used is approaching zero and M1 velocity relative to M2 is increasing exponentially. Ireland cites data over the last 100 years showing such an increase in velocity.

This paper attempts to keep the essence of Ireland’s choice between money and credit, and use it in a standard CIA model. While I propose it is more accurate with its conclusions with respect to growth than Ireland’s, the results are not very surprising considering the existing work. On the other hand it provides nice results with respect to inflation, velocity and the financial services sector. In general I get steady-state results that seem consistent with observed phenomenon: velocity is increasing with respect to inflation and improvements in credit technologies and the financial services also have important implications for the economy. I lose the result than M1 velocity is increasing indefinitely over time as the economy grows, however I do allow for exogenous changes in financial innovation that would increase velocity. Thus my model would predict structural breaks in velocity in response to a discrete number of financial innovations. With a fairly brief period of history being governed by significant growth and fiat money it is difficult to argue which is more realistic. Ireland’s results, though, do fit nicely with the recent interest in analyzing economies where money is absent. Thus an obvious next step would be allow the technologies in the credit sector to improve continuously and/or endogenously.

As mentioned, the next subsection surveys some key empirical results and the following some theoretical contributions. Section II sets out a model with costly intermediation in the spirit of Ireland, where consumption goods are subject to CIA. Section III extends the model to CIA on investment goods as in Stockman (1981).
Section IV covers the implications for empirical tests regarding inflation and growth, section V provides discussion on welfare issues and section VI concludes.

2.1.1 Empirical Results

Much of the recent empirical evidence has been motivated by the development of endogenous growth models and the vast empirical results that have come about since their inception. Early papers that emphasized the possible negative relationship between inflation and long run growth were Kormendi and Meguire (1985), Grier and Tullock (1989), and Fischer (1993) while more recent work by Barro (1996, 1997) finds the same. These papers run regressions of the long run rate of growth, usually an average over several years, on several variables that may be expected to affect growth, including the average rate of inflation. The coefficients on inflation are negative and statistically significant in most cases. For example, Barro (1997) uses 10 year medians of inflation, for the 1960’s, 70’s and 80’s, for approximately 120 countries in addition to measures of human capital, rule of law, etc. and regresses them on GDP per capita growth over 10 year periods. He finds a negative and statistically significant effect of inflation on growth. An increase in inflation of 10 percentage points reduces annual GDP growth by 0.3 percentage points. However these results are driven by high inflation countries. When omitting countries with inflation above 20% he cannot reject a zero coefficient. This presents a problem that few theoretical models of inflation and growth can explain.

Fischer (1993) regresses inflation and other variables (budget surplus, terms of trade and black market exchange premium) on GDP growth and also finds a negative
and significant coefficient. He uses a sample of 110 countries averaged over 1960-1989. In addition he uses these same variables in regressions for capital accumulation and also a Solow-residual and again finds significant and negative coefficients. The effect on capital accumulation contrasts that of several authors, including Barro, who add investment to a growth equation and conclude that inflation does not work through this channel because the coefficient is relatively unchanged. The effect on productivity is more typical, and it is often suggested that if inflation affects long run growth negatively, then it is through productivity rather than investment rates.

Another possibility is that it is inflation uncertainty that effects growth. Since average inflation and its variability are highly correlated it is difficult to use both in the same regression. Fischer runs his regressions with the standard deviation of inflation around its seven-year mean in place of average inflation and comes up with similar negative and significant results. Barro uses both in the same equation and finds the standard deviation is not significant and concludes that inflation’s effect is through average rates. However, as just mentioned, when both are included at the same time it is not clear how to attribute the individual coefficients due to the high collinearity. One problem with testing inflation uncertainty is that measures such as standard deviations do not necessarily reflect uncertainty. Grier and Grier (2003) find negative effects from inflation uncertainty using a simultaneous Garch-M model.

There is also the possibility of serious non-linearity. As mentioned above, Barro fails to find a significant effect when only moderate inflation countries are included1.

1Although, he then continues: “Thus, the clear evidence for the negative relation between growth and inflation comes from the middle and upper intervals. However, since the three estimated coefficients do not differ significantly from each other, the data are consistent with a linear relationship.”
Sarel (1996) uses a structural break test and finds that he cannot reject a zero coefficient on inflation rates below 8%. Above this he finds a significant and negative correlation. Bruno and Easterly (1998) use different subgroups divided by inflation rates and find significant differences at the 40% level. They argue two other interesting points: First, the negative growth effect is primarily present in higher-frequency data such as 5 or 10 year periods rather than 30 year periods and second, they “establish a robust finding that growth falls sharply during high inflation crises, then recovers rapidly and strongly after inflation falls (p. 3).” In fact, they find that some countries experience growth rates above their average after the recovery stage. Fischer (1993) finds different results in regards to non-linearity using a spline method, where the biggest negative effect is for countries with inflation between 0-15%, the next largest effect for those in the 15-40% range, and the smallest for those above 40%.

A interesting paper that approaches things from a completely different angle is Bullard and Keating (1995) who, rather than using a simple growth equation, use a structural vector autoregression. They decompose inflation and output into permanent and transitory components and attempt to approach the problem from a structural view, a criticism often leveled against those taking the growth equation approach. They find that some countries seem to be characterized by inflation that has a permanent component while others are not. Concentrating on those countries where they cannot reject a unit root they observe in general no effect on long run output from a permanent inflation shock. However they find that the effect may be positive for a country that starts at low inflation.
The results on inflation and growth, however, are not robust as shown by Levine and Renelt (1992). It turns out that inflation is one of the less robust variables. They use an extreme bounds analysis and introduce a small number of core variables they argue are almost always significant in growth equations. With these additional variables, inflation becomes statistically insignificant in many cases.

There are two main issues that arise from the empirical evidence: that inflation may have zero or positive growth effects at low rates and the fragility, or non-robustness, of inflation in growth regressions. Although there are some theoretical models that can produce such a non-linear relationship, the empirical evidence is not that clear cut as yet. On the other hand, this paper attempts to explain why typical regressions are fragile if we believe that inflation does in fact matter. There are also two policy issues, that of economic development and that of optimal monetary policy. I would argue that the empirical results above support Fischer’s (1993) claim that “a stable macroeconomic framework is necessary though not sufficient for sustainable growth”, whereas questions concerning the optimal rate of inflation, and whether it is indeed zero, are less clear.

2.1.2 Theoretical Results

With the above empirical results it should be no surprise that various theoretical models have been devised that predict different relationships. In fact, early work on monetary theory came ahead of good empirical work, and it now seems like the theory is trying to catch-up. A survey of the money and growth literature up to 1990 is found in Orphanides and Solow (1990). More recent surveys that cover both empirical and theoretical issues are Briault (1995) and Temple (2000). Much of
the debate on inflation and growth can be interpreted as the hypothesis presented by Tobin (1965) that inflation may be good for growth, followed by reactions and analyses of whether this in fact could be the case, first by Sidrauski (1967) and then others. Tobin’s argument was simply that households could hold money or capital and that if inflation reduced money holdings then capital accumulation would increase. Tobin’s model was criticized for taking a partial equilibrium approach, by taking individuals saving behavior as fixed. Sidrauski put money in the utility function and, through an optimal growth model, showed that inflation would have no long-run effect. It is interesting that much of the current literature addresses the possibilities of a Tobin-effect even though it has proved difficult to find an equilibrium model supporting this conjecture.

Following Sidrauski’s stark superneutrality result, models were quick in coming that allowed for inflation to have a negative impact on capital accumulation. For example, if money was instead placed in the production function, inflation would decrease capital accumulation. In general, these results have been repeated under varying arguments, that money needs to affect the marginal product of capital to affect growth.

Another class of models that are popular are those that assume a cash-in-advance (CIA) constraint, building off the work of Clower (1967). Proponents of CIA models often argue that they provide better microfoundations than money in the utility (MIU) function models, although this is not as obvious as they would suggest. By assuming the constraint, the demand for money may be as ad-hoc as assuming MIU. In either case, they have been widely used and provide useful results. In practice, though, the results are similar to money in the utility function: if CIA is on consumption goods
only than it does not affect output, whereas if investment goods are subject to CIA then the steady-state level of output is lower (Stockman 1981).

In practice, CIA models often distinguish between cash goods and credit goods where credit goods are anything that is not subject to the CIA constraint. This is difficult to interpret in practice because there is no explicit credit in the model so it is not clear what this means. In fact, because leisure is not subject to the CIA constraint it is often referred to as a credit good. In any case, these models allow an escape from the criticism that early CIA models produces a consumption velocity of one. A popular recent approach is to assume some fraction of consumption or investment is subject to the CIA constraint and thus velocity will be different from one and comparative statics can be analyzed regarding changes in the fraction of goods subject to the constraint.

Because of the influence of endogenous growth models, many recent papers ask how inflation may affect the long run growth rate. The above models all deal with the level of output and not growth affects. The present paper also relies on level effects that will only deliver growth affects as the economy makes the transition to it’s long run equilibrium. Since there is much debate empirically whether endogenous growth models perform better or not this isn’t too much of a shortcoming. It does however provide a challenge to see if the model can explain the relevant magnitudes of inflation’s affect on growth via the transitional dynamics.

2.2 Costly Intermediation and Cash-in-Advance

In this section I lay out a model that is a modification of the one developed by Ireland (1994). Before getting to the formal model, let me state the nature of the
problem and stress where I take a different approach than Ireland. The model is driven by the possibility of multiple means of payments\textsuperscript{2}. Households face a cash-in-advance constraint on consumption goods, but can instead, if they choose, use costly financial intermediation. Markets are segmented spatially and the cost of intermediation is proportional to consumption and increasing in the distance from the home market. On the other hand, using cash is costly but is proportional to the nominal rate of interest which is identical in all markets. As will be seen, there exists a cutoff at some location such that credit will be used in that location and all of those closer to the home market, and cash will be used in all markets further away. This cutoff will depend on the nominal interest rate and, through the Fisher equation, on inflation. Because using credit is costly, there will be an explicit financial services sector, whose size also will depend on the nominal interest rate and inflation.

Ireland’s model is driven by two assumptions, that the cost of intermediation is a fixed amount of output and that production is of the AK type. The fixed cost assumption implies that the cost of credit is becoming a smaller fraction of the growing amount of output. This allows credit to be used in more markets as the economy grows. As less cash is used velocity is increasing which fits recent data. Although more credit is being used, because the cost is a decreasing fraction of output the financial services sector as a share of output is asymptotes toward zero. The AK assumption implies that the marginal product of capital (MPK), and therefore the real interest rate, is invariant to the level of capital. Investigating higher inflation the main conclusions are: (1) velocity is higher for higher rates of inflation, (2) the financial services sector’s share of output is increasing in inflation, and (3) an increase

\textsuperscript{2}Multiple means of payments models are developed by Prescott (1987) and Schreft (1992) but they do not consider questions of growth.
inflation reduces the amount of consumption, increases the accumulation of capital and thus temporarily the growth rate, which asymptotically approaches it’s long run level.

In contrast, because I allow the cost of intermediation to be a proportional cost and the production function to be neoclassical I will get typical steady-state results. Although in this section there will be no growth effect, I will get implications that the steady-state fraction of output going to financial services does not asymptote toward zero and velocity does not approach infinity without continual financial innovations.

2.2.1 The Model

The model consists of an infinitely lived representative household, where time is discrete. Markets are indexed by $i \in [0, 1]$. This index is interpreted as distance from market zero, where all households reside. Goods can be purchased with either cash or credit. There is a cash-in-advance constraint which is assumed to be always binding so that holding money incurs an opportunity cost equal to the nominal rate of interest. On the other hand, credit can be used to buy consumption goods but there is a proportional cost of intermediation. This cost is different in each market $i$ and assumed to be increasing in $i$. In the spatial interpretation this means that intermediation is essentially verifying someone’s identity, which becomes more expensive in markets that are farther away. The specific assumptions, which are detailed below, produce the result that credit will be used in markets closer than some threshold location and cash used in markets further away than the threshold
Setting up the model formally, households choose their lifetime sequence of consumption to maximize utility

\[ U(c_t) = \sum_{t=1}^{\infty} \beta^t \left\{ \int_0^1 u[c_t(i)] \, di \right\} \]  

(2.1)

where the instantaneous utility function \( u[c_t(i)] \) is strictly concave and \( \beta \in (0, 1) \) is the discount factor. The production technology is neoclassical

\[ y_t = f(k_t) \]  

(2.2)

where \( y \) and \( k \) are output and capital, measured per efficiency unit per capita. It is assumed that technology is growing exogenously at the gross rate \( \gamma \), that is \( A_{t+1} = \gamma A_t \) where \( \gamma \) is one plus the rate of productivity growth. In what follows money and bonds will also be denoted in per efficiency unit, per capita terms, that is \( M \) is the money supply per capita divided by the productivity parameter \( A \) and similarly for bonds. The household begins in period 0 with \( H_0 \) of money from the government and can carry into period 1 either money \( M_1 \) or bonds \( B_1/R_0 \). At time 1 and beyond the household has five sources of funds. A lump sum transfer of money from the government \( H_t \), income \( y_t = f(k_t) \), where \( k_t \) is given from the prior period, and the amount of money, bonds and undepreciated capital carried over from the last period, \( M_t, B_t \) and \( (1 - \delta)k_t \) respectively. The household can use these funds to purchase consumption goods (using either money or credit), or carry into the next period money, capital and/or bonds. If the household chooses to use credit in some market \( i \) then it incurs a proportional cost of intermediation equal to \( \phi(i) \). Formally the budget constraint faced by the household at each period in time \( t \geq 1 \) is

\[ \frac{B_t + M_t + H_t}{p_t} + f(k_t) + (1 - \delta)k_t \geq \gamma k_{t+1} + \int_0^1 c_t(i) di \]

\[ \quad + \int_0^1 \xi_t(i) \phi(i) c_t(i) di + \frac{\gamma M_{t+1}}{p_t} + \frac{\gamma B_{t+1}}{R_t p_t}. \]  

(2.3)
Because the decision to use credit is endogenous, $\xi_t(i)$ is an indicator variable taking the value 1 if the good in market $i$, at time $t$, is purchased on credit and 0 if by cash. The amount $\phi(i)c_t(i)$ is the total cost for intermediation for purchasing amount $c_t(i)$ in market $i$ and the integral over 0 to 1 represents the total credit costs to the household. The proportional cost $\phi(i)$ is assumed to be strictly increasing in $i$ with $\phi(0) = 0$ and $\phi(1) = \infty$. This, along with the assumption that the gross nominal interest rate is greater than 1 at all times, guarantees that money and credit will both be used in equilibrium. The cash in advance constraint is

$$\frac{M_t}{p_t} \geq \int_0^1 [1 - \xi_t(i)] c_t(i) di. \quad (2.4)$$

This says that total consumption of goods at time $t$ using cash ($\xi_t(i) = 0$) must be less than or equal to real cash balances held at the beginning of the period, where $p_t$ is the money price of output. Money balances and capital must be non-negative and the appropriate no-ponzi condition on bond holdings must be satisfied. The money supply is simply the sum of the money transfers from time zero to the present.

The competitive equilibrium of this economy is as follows. The household takes the money supply process, prices, and interest rates as given for all periods. It also takes the initial capital stock as given. It solves the time zero problem by choosing the allocation of money and bonds to carry forward into period one. Taking these and the initial capital stock, the households maximizes it’s utility subject to the budget constraint and CIA constraint in each period $t = 1, 2, 3...$ The equilibrium is a sequence of the households choice variables that solves it’s maximization problem.
such that markets clear. The market clearing conditions are

\[ f(k_t) = \gamma k_{t+1} - (1 - \delta)k_t + \int_0^1 c_t(i) \, di + \int_0^1 \xi_t(i) \phi(i) \, c_t(i) \, di \]  \hspace{1cm} (2.5) \\
M_{t+1} = M_{t+1}^s \]  \hspace{1cm} (2.6) \\
B_{t+1} = 0. \]  \hspace{1cm} (2.7) \\

The first is goods market clearing, the second is money demand equal to money supply. The last, clearing of the bond market, follows from the representative agent assumption: Because all agents are identical, the interest rate must be such that bonds are in zero net supply. The maximization problem is solved by differentiating the following lagrangian

\[ L = \lambda_0 \left[ H_0 - \frac{M_1}{p_t} - \frac{B_1}{R_1 p_t} \right] + \sum_{t=1}^{\infty} \beta^t \left\{ \int_0^1 u[c_t(i)] \, di + \lambda_t \left[ \frac{B_t + M_t + H_t}{p_t} + f(k_t) - \gamma k_{t+1} \right.ight. \\
+ (1 - \delta)k_t - \int_0^1 c_t(i) \, di - \int_0^1 \xi_t(i) \phi(i) \, c_t(i) \, di - \frac{\gamma M_{t+1}}{p_t} - \frac{\gamma B_{t+1}}{R_t p_t} \\
\left. \left. + \mu_t \left[ \frac{M_t}{p_t} - \int_0^1 [1 - \xi_t(i)] \, c_t(i) \, di \right] \right\} \right. \]

where \( \lambda_t \) and \( \mu_t \) are lagrangian multipliers. The lagrangian is written in terms of the indicator function \( \xi \) because whether credit is used or not is a remaining choice variable. From the households perspective the consumption good in some market \( i \) is the same, irrelevant of how it is purchased. However, the quantities demanded will differ and the location will matter for financial services expenditures, so we introduce the notation \( c_t^0(i) \) and \( c_t^1(i) \) to distinguish consumption purchased with cash and consumption purchased using credit respectively. Solving the maximization problem yields the first order conditions with respect to consumption, capital, money and
bonds

\[ u' [c^0_t (i)] = \lambda_t + \mu_t \quad \text{for all } i \in [0, 1] \text{ and } t \geq 1 \tag{2.8} \]

\[ u' [c^1_t (i)] = \lambda_t (1 + \phi (i)) \quad \text{for all } i \in [0, 1] \text{ and } t \geq 1 \tag{2.9} \]

\[ \gamma \lambda_t = \beta \lambda_{t+1} (1 + f'(k_{t+1}) - \delta) \quad t \geq 1 \tag{2.10} \]

\[ \lambda_0 = \beta (\lambda_1 + \mu_1) / p_1 \quad t = 0 \tag{2.11} \]

\[ \gamma \lambda_t / p_t = \beta (\lambda_{t+1} + \mu_{t+1}) / p_{t+1} \quad t \geq 1 \tag{2.12} \]

\[ \lambda_0 / R_0 = \beta \lambda_1 / p_1 \quad t = 0 \tag{2.13} \]

\[ \gamma \lambda_t / R_t p_t = \beta \lambda_{t+1} / p_{t+1} \quad t \geq 1. \tag{2.14} \]

Equations (2.8) and (2.9) are true for all \( i \in [0, 1] \) taking the mode of payment as given, i.e. if consumption is purchased with cash then (2.8) will hold. The solution to which consumption goods are actually purchased with cash is spelled out below. Equations (2.11) and (2.13) pertain to the time zero choice and are included for completeness, but will play little role in the rest of the analysis. Equations (2.12) and (2.14) can be combined to show that \( \lambda_t + \mu_t = R_{t-1} \lambda_t \). This shows, with concavity of the utility function, that the quantity of consumption goods purchased using cash (2.8) is decreasing in the nominal interest rate. It is also immediate from (2.9) that the amount of consumption when purchased using intermediation is not affected by the nominal interest rate. These equations determine the dynamics of the main variables of the model.\(^3\)

Because (2.8) does not depend on \( i \) the same amount of \( c^0_t (i) \) is purchased for all \( i \) where cash is used. However the amount of \( c^1_t (i) \) is decreasing in \( i \), because the

\(^3\)There are also 3 transversality conditions that will be satisfied by the optimal program, pertaining to the capital stock, money holdings and bond holdings as follows: \( \lim_{T \to \infty} \beta^T \lambda_T k_{T+1} = 0, \lim_{T \to \infty} \beta^T \lambda_T M_{T+1} = 0 \) and \( \lim_{T \to \infty} \beta^T \lambda_T B_{T+1} = 0. \)
cost is increasing due to $\phi(i)$ increasing in $i$. The key remaining result is for the household to choose the threshold value of $i$, determining when to use cash or credit. Again we use the result $\lambda_t + \mu_t = R_{t-1}\lambda_t$. Look at (2.8) and (2.9) and consider the household’s choice before the method of payment is determined. The household has the choice $u'[c_t(i)] = \lambda_t R_{t-1}$ or $u'[c_t(i)] = \lambda_t(1 + \phi(i))$. Using the concavity of utility the household maximizes utility by choosing the minimum of $\lambda_t R_{t-1}$ and $\lambda_t(1 + \phi(i))$ for each market $i$. By the assumptions that the interest rate is strictly greater than one and that $\phi(i)$ goes from zero to infinity over the interval $[0, 1]$ there will be some market $i$ such that $1 + \phi(i)$ exactly equals $R_{t-1}$. Thus the cutoff market, at time $t$, $\bar{\tau}_t$, solves the equation

$$\phi(\bar{\tau}_t) = R_{t-1} - 1. \quad (2.15)$$

Credit will be used in all markets $i$ less than or equal to $\bar{\tau}_t$ and cash will be used in all markets $i$ greater than $\bar{\tau}_t$. It is also true that all purchases using credit will be larger than those using cash as in Prescott (1987). Thus the last choice variable is solved by

$$\xi_t(i) = \begin{cases} 
1 & \text{if } 1 + \phi(i) \leq R_{t-1} \\
0 & \text{otherwise.} 
\end{cases} \quad (2.16)$$

Since we have assumed that the cash-in-advance constraint is binding in all periods, the demand for real money balances will be equal to the amount of consumption purchased with cash. Using the results above that the quantity of consumption purchased with cash is independent of the market $i$ and that cash will be used in all markets $i > \bar{\tau}_t$ we have the equilibrium solution for real money balances at each period $t$

$$\frac{M_t}{p_t} = \int_{\bar{\tau}_t}^1 c_t^0(i) \, di = [1 - \bar{\tau}_t] c_t^0 \quad (2.17)$$
where we have simplified notation by denoting \( \bar{i}_t = \bar{i}(R_{t-1}) \) and \( c_t^0 = c^0(\lambda_t, R_{t-1}) \).

Because the demand for money equals supply in equilibrium we can use \( M_t^s \) in (2.17).

Let the exogenous growth of the nominal money supply be \( g_t \) and recall that \( M_t \) is per efficiency unit. Using (2.17) at period \( t \) and \( t + 1 \) and solving for the ratio of prices we get

\[
\frac{p_{t+1}}{p_t} = \frac{g_t [1 - \bar{i}_t] c_t^0}{\gamma [1 - \bar{i}_{t+1}] c_{t+1}^0},
\]

(2.18)

which describes the evolution of the price level over time. Finally the nominal interest rate can be solved from (2.14), (2.10) and (2.18) as

\[
R_t = (1 + f'(k_{t+1}) - \delta) \frac{p_{t+1}}{p_t} = \frac{(1 + f'(k_{t+1}) - \delta) g_t [1 - \bar{i}_t] c_t^0}{\gamma [1 - \bar{i}_{t+1}] c_{t+1}^0}.
\]

(2.19)

This is the Fisher equation which states that the nominal interest rate equals the real interest rate plus the rate of inflation. Rearranging the market clearing equation, the law of motion for the capital stock can be seen to be

\[
\gamma k_{t+1} = f(k_t) + (1 - \delta) k_t - [1 - \bar{i}_t] c_t^0 - \int_0^{\bar{i}_t} c_t^1(i) \phi(i) \, di - \int_0^{\bar{i}_t} c_t^1(\bar{i}) \phi(\bar{i}) \, d\bar{i}.
\]

(2.20)

As described, the economy fits the mold of optimal growth models studied in any graduate macro textbook. The dynamics are governed by the equilibrium paths of consumption and capital, (2.8), (2.9) and (2.20). Thus taking the money supply process as given, the household chooses an initial consumption level such that the capital stock follows the law of motion and the transversality condition is satisfied. Since there is no uncertainty we can take the money supply process as given, or we can allow the monetary authority to peg the nominal interest rate or a specific inflation rate. We will follow Ireland and assume that the monetary authority chooses a path of money \( g_t \) such that the rate of inflation in (2.18) is constant.
At this point the above model is very near that of Ireland’s with two significant changes. The cost of intermediation is modeled as a proportional cost rather than a fixed cost, and the production function is neoclassical rather than $AK$. The gain is that the model now fits squarely into the optimal growth framework which is the workhorse of modern macroeconomic theory. As this framework has a well defined steady-state solution, we now turn to that characterization.

2.2.2 Stationary Equilibrium

In the previous section the dynamics of the economy are determined by the first order conditions with respect to consumption and the law of motion of capital. The solution is for the representative household to choose the initial value of consumption taking the initial capital stock as given, such that the transversality conditions are satisfied. The dynamics are not that different from the typical optimal growth problem, in which the convergence properties are now pretty standard. As is typically the case we want to simplify our task by looking at the steady-state of the economy and consider the comparative statics with respect to several parameters.

To begin we observe that in a steady-state $\lambda_t = \lambda_{t+1} = \lambda$. Letting steady-state values be denoted simply without time subscripts things fall into place quite quickly. It is easy to see from (2.10) that the steady-state capital stock satisfies

$$f'(k) = \frac{\gamma - \beta (1 - \delta)}{\beta}. \quad (2.21)$$

Using this value of $k$ we get steady-state output $y = f(k)$. If we assume the money growth rate is a constant, $g_t = g$, then the growth of prices (and therefore the nominal rate of interest) will be constant if the cutoff $\bar{r}_t$ is a constant. But $\bar{r}_t$ is only a function of the nominal interest rate, so it will be constant as long as the nominal interest

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rate is constant. Thus we can see that the steady-state rate of inflation and nominal interest rate are the constants

\[
\frac{p_{t+1}}{p_t} = \frac{g}{\gamma} \tag{2.22}
\]

\[
R = (1 + f'(k) - \delta) \frac{p_{t+1}}{p_t} = \frac{\gamma}{\beta} \frac{p_{t+1}}{p_t} = \frac{g}{\beta}. \tag{2.23}
\]

Thus the steady-state has the usual results that the rate of inflation is approximately the rate of money growth minus the growth rate of productivity and that the nominal interest rate is the real interest rate plus the inflation rate. The real interest rate is the inverse of the discount factor modified to allow for positive economic growth. I write the nominal interest rate in (2.23) as both a function of the inflation rate and the money growth rate to be precise in allowing for the monetary authority to choose either instrument. In particular, later when considering a change in the growth rate of an economy I assume that the inflation rate is being held constant and thus the \(\frac{\gamma p_{t+1}}{\beta p_t}\) term is used rather than \(\frac{g}{\beta}\). In either case, the constant nominal interest rate implies that the cutpoint \(\bar{r}_t\) is also a constant, the implicit solution to

\[
\phi(\bar{r}) = R - 1. \tag{2.24}
\]

Consumption of cash goods is a function of \(\lambda\) and \(R\) while consumption of credit goods is a function of \(\lambda\) and \(i\), thus the steady-state levels of consumption can be found by solving the \(\lambda\) that solves the market clearing equation for goods

\[
f(k) = \gamma k - (1 - \delta)k + [1 - \bar{r}] c^0 + \int_{0}^{\bar{r}} c^1(i)di + \int_{0}^{\bar{r}} c^1(i)\phi(i)di. \tag{2.25}
\]

And finally equilibrium money demand is

\[
\frac{M}{p} = \int_{\bar{r}}^{1} c^0 di = [1 - \bar{r}] c^0 \tag{2.26}
\]
which will be used to calculate velocity. In essence we have a typical CIA model where some fraction \( \bar{r} \) of consumption goods are credit goods and some fraction \( 1 - \bar{r} \) are cash goods. The difference is that \( \bar{r} \) is an endogenous variable that will depend on the nominal interest rate and via the Fisher equation on the rate of inflation. Also of interest is that the credit goods are associated with a cost and thus there is a well defined credit sector, which fits nicely with papers emphasizing finance and growth.

2.2.3 Parametric Example

We turn now to simulating a specific example and seeing how the model performs. In particular I try to look at issues regarding the cost of credit and share of output going to financial services that cannot be addressed in a typical CIA model where \( \bar{r} \) is exogenous. I take the parametric example

\[
\begin{align*}
    u [c_t(\bar{r})] &= \ln c_t(\bar{r}) \\
    y_t &= k_t^\alpha \\
    \phi (i) &= \frac{i}{\theta (1 - i)}.
\end{align*}
\]

Allowing utility to take the logarithmic form simplifies the analysis and is a common assumption in simulations. The cost of intermediation differs from Ireland’s formulation in that it is a proportional cost and includes the additional parameter \( \theta \). This is used for two purposes. One, it is a fit parameter that allows me to adjust velocity and the size of the financial services sector to be reasonable, i.e. so that the financial sector is not 60% of output. Second, I can proxy an improvement in the financial services sector as an increase in \( \theta \), which would simply scale down the proportional cost of intermediation in all markets \( i \) by the same amount. Ireland doesn’t need
this device because the fixed cost assumption allows the cost of intermediation to be falling continuously toward zero as a share of output.

The steady-state capital stock is determined from (2.21) to be

\[ k = \left[ \frac{\alpha \beta}{\gamma - \beta (1 - \delta)} \right] \frac{1}{1 - \alpha}. \] (2.27)

From this, the steady-state output and nominal interest rate are given by

\[ y = k^\alpha \quad \text{and} \quad R = (1 + \alpha k^{\alpha-1} - \delta) \frac{p_{t+1}}{p_t}. \]

I assume the monetary authority is setting \( \frac{p_{t+1}}{p_t} \) at some constant rate rather than setting \( g \). The nominal interest rate is constant and thus the cut-off level of \( \bar{i} \) is constant and equal to \( \frac{\delta (R-1)}{1 + \delta (R-1)} \). Assuming that the nominal interest rate is greater than 1, this value is strictly between zero and one for all \( R < \infty \). The market clearing equation (rearranged so that all the \( k \) terms are on the left) becomes

\[ k^\alpha + (1 - \delta - \gamma)k = [1 - \bar{i}] c^0 + \int_0^{\bar{i}} c^1(i) di + \int_0^{\bar{i}} c^1(i) \phi (i) di \] (2.28)

where \( c^0 = (R\lambda)^{-1} \) and \( c^1 (i) = (\lambda (1 + \phi (i)))^{-1} \). The left hand side is output minus investment and the right hand side is consumption and the cost of credit used for consumption purchases requiring intermediation. I wrote the market clearing equation in this manner to emphasize that the capital stock and other parameters on the LHS are known and fixed and that it only remains to determine the \( \lambda \) that sets the RHS to the same level. As can be seen from the left hand side, the money growth rate (or inflation rate) does not affect the steady-state capital stock or output. It does however affect the cut-off level \( \bar{i} \), the quantities of all consumption goods purchased with cash, \( c^0 \), and spending on credit services. Thus the composition of consumption and financial services on the RHS is affected.

Using the above parametric formulation we can see how an increase in the inflation rate affects the share of output to the financial services sector and velocity as in figure
Figure 2.1: Velocity and Share of Financial Services of GDP at Different Levels of Inflation. Constraint on Consumption Goods.

1. These are both issues that Ireland addresses and let me stress that where he looked at the time series property, figure 1 is plotting steady-state values against alternative inflation rates. The specific parameter values are fixed at common levels, $\beta = .985$, $\delta = .05$ and $\gamma = 1.025$ which imply that the discount factor is 0.985, i.e. the real risk free rate is 1.5%, the average life of capital is 20 years and the long run growth rate is 2.5%, which is not too far from US experience. The credit parameter is fixed at $\theta = 2$, simply to make the quantities of velocity and financial services reasonable. The production parameter $\alpha$ is set equal to 0.6. This is based on the conclusion of Mankiw, Romer and Weil (1992) that for the neoclassical growth model to match observed rates of convergence the elasticity on capital needs to be close to 0.6. They interpret this as including a broad measure of capital, including human capital.
The intuition behind the results of figure 1 are pretty straight forward. A higher rate of inflation leads to a higher nominal interest rate through the Fisher effect, which has two effects. It has the usual affect of decreasing the amount of consumption of cash goods, $c^0$, but also leads to a higher cutpoint $\bar{r}$. Thus consumption goods in more markets are bought with credit and fewer with money which strengthens the usual result of generating higher velocity of M1. On the other hand, M2 consists of money plus credit. The total amount of consumption, using both cash and credit, plus the cost of financial services does not change, as this is the entire RHS of (2.28). Therefore the velocity of M2 does not change. All consumption and financial services are paid for with cash or credit and these together comprise approximately 50% of gross output, leading to a M2 velocity of about 2\(^4\). As already stressed, the LHS of (2.28) is unaffected, what is happening is that consumption is being offset one-for-one with financial services costs. If you were to plot aggregate output, consumption and financial services, output is a horizontal line, unaffected by inflation while financial services is increasing and exactly offsetting the decreasing consumption.

Two things worth looking at are the effects of different productivity growth rates and also improvements in the intermediation technology on velocity and financial services share. Note first from (2.27) that the steady-state capital stock in efficiency units is decreasing in the growth rate of output. This represents that more investment is needed to keep the capital stock growing at the constant rate $\gamma$ and is intuitively similar to an increase in the depreciation rate $\delta$. An increase in the exogenous growth rate $\gamma$ will increase the real rate of interest $\gamma/\beta$ and therefore the nominal rate $\frac{\gamma}{\beta} \frac{\pi \nu}{\nu}$.

\(^4\)While Ireland is particularly interested in the relationship between M1 and M2, all the action is in M1. In his model M2 doesn’t move much and M1 is driving the results. Here M2 literally doesn’t move at all, thus we only look at M1 in what follows.
Figure 2.2: Velocity and Financial Services Share of GDP at Different Growth Rates. Constraint on Consumption Goods.

via the Fisher equation, assuming the monetary authority keeps inflation constant. Since we are concerned with the effects of inflation we will assume that the monetary authority is holding the inflation rate constant. The increase in the nominal interest rate leads to a higher cutpoint $\bar{i}$, and thus higher velocity and more resources to financial services as shown in Figure 2. Figure 2 shows the steady-state velocity for M1 and the share of financial services in GDP for growth rates of 1%, 2.5% and 5%, which cover the long run averages of most countries.

At least according to the parametric example here, the growth rate seems to have a significant affect on the share of financial services while velocity is hardly changed. Less consumption is purchased using cash and more using credit which would tend to increase velocity. However this change is small, because the increase in the nominal rate due to an increase in $\gamma$ from 1 to 2.5% is the same as an increase in inflation
Higher theta is a lower cost of credit.

Figure 2.3: Velocity and Financial Services Share of GDP at Different Levels of Financial Innovation. Constraint on Consumption Goods.

of 1.5%. On the other hand, the share of GDP spent on financial services nearly doubles even at low levels of inflation when productivity growth is 2.5% rather than 1%. This is particularly interesting considering the enormous interest (both recent and old) in finance and growth. It would appear that higher financial services are associated with higher growth, even though it is not finance causing growth. This is also interesting because this is just financial services as a means of payment whereas the usual emphasis is on better intermediation of investment.

Related to the idea that finance does promote growth, we can ask what happens if there is financial innovation. Of course in this setup we already know that it doesn’t affect output or growth because it only affects consumption, but we can still look at how a change in the cost of intermediation affects velocity and financial services
share of output. Increasing the parameter $\theta$ reduces the cost of intermediation in all markets $i \in [0, 1]$. Figure 3 shows M1 velocity and finance’s share, now holding the growth rate constant at 2.5% and allowing $\theta$ to take the values 2, 3, and 4. In contrast to the growth effects, the change in credit technology has a significant affect on velocity and the opposite affect on finance’s share, i.e. financial services share goes down despite more markets using credit. Credit is used in more markets and cash in fewer and since steady-state output is unchanged velocity increases significantly. Even though more markets are using credit, the total cost of credit has decreased and thus the share of GDP going to financial services has actually decreased. Since I am not opening up the black box of $\phi(i)$, I don’t attempt to analyze the realism of the magnitudes of this example. For example it is not clear what an increase in $\theta$ from 2 to 3 really signifies. It is clear that this is a 50% decrease in the cost of credit in all markets and this is large.

2.3 Cash in Advance Requirement on Capital Goods

The above model is close in spirit to Ireland’s (1994) and the results are not too surprising. Ireland predicts that inflation does not affect growth much but that economic growth will affect the transactions in the economy. The results with respect to the use of money, credit and velocity in the above model are similar. There is no effect of inflation on growth, yet productivity growth affects the means of payment and the development of financial technology has significant effects on velocity and financial services. As already mentioned, I believe the drawback of the above model and Ireland’s is that they do not reflect the negative affect on growth often found in empirical studies.
In this section I extend the model of section 2 by allowing capital goods also to be subject to a cash-in-advance constraint, as in Stockman (1981). Stockman’s results were to show that cash-in-advance on investment goods would decrease the steady-state capital stock, overturning the superneutrality results when CIA is only applicable to consumption goods. Thus I can obtain the negative implication of inflation on growth and keep the proceeding relationships with respect to velocity and financial services. It is harder to justify the assumption of CIA on investment goods than on consumption goods, but I don’t think this should be an obstacle for proceeding with the analysis. There are many reasons that firms hold money, and some such as having cash on hand to buy inputs, and the difficulties of many firms to obtain trade credit seem relevant.

2.3.1 The Model

The interpretation here is that output is produced with capital which is differentiated across \( j \) markets, \( j \in [0, 1] \), as represented by the production function 

\[
y_t = \int_0^1 f(k_t(j)) dj.
\]

The function \( f(k_t(j)) \) is a concave function in effective capital\(^5\). The interpretation is that there are differentiated inputs across markets and that they are imperfect substitutes in production\(^6\). Also, for notational purposes I index markets \( j \in [0, 1] \) when discussing capital goods and continue to use \( i \) for consumption goods. In the spatial setup this is irrelevant, but for other interpretations it may very well be that the attributes increasing the cost of credit for households is very

\(^{5}\)For example, the aggregate production function below is \( Y = (AL)^{1-\alpha} \int K(j)^{\alpha} dj \) which becomes \( y = \int k(j)^{\alpha} dj \).

\(^{6}\)From the households perspective, output is a homogenous good that can be used as consumption or investment. The story could be that each market performs some costless transformation on the output differentiating it from other capital.
different from those affecting firms. The model proceeds in much the same fashion as in section 2.2, so some details will be omitted.

As in the consumption case, households can choose to use cash or financial intermediation to purchase capital goods and again the intermediation costs are increasing across \( j \). Although in this model households own both the consumption goods and capital, it is reasonable that in a model with firms the cost of intermediation for capital goods is different than that for consumption. For example, firms may have sizable cash management departments\(^7\) and are also required to report relevant information to various agencies\(^8\). Thus we shall distinguish between the cost of credit for consumption purchases \( \phi (i) \) and for investment purchases \( \psi (j) \).

The household’s problem is still to choose sequences of consumption, capital, money and bonds, to maximize utility

\[
U (c_t) = \sum_{t=1}^{\infty} \beta^t \left\{ \int_0^1 u [c_t(i)] \, dt \right\}
\]

subject to the budget constraint and cash in advance constraint. Here the budget constraint is almost identical to that of section II

\[
\frac{B_t + M_t + H_t}{p_t} + \int_0^1 f (k_t(j)) \, dj + \int_0^1 (1 - \delta) \, k_t(j) \, dj \geq \frac{\gamma M_{t+1}}{p_{t+1}} + \frac{\gamma B_{t+1}}{R_{t+1}} + \int_0^1 \gamma k_{t+1}(j) \, dj + \int_0^1 c_t(i) \, di + \int_0^1 \xi_t(i) \phi (i) \, c_t(i) \, di + \int_0^1 \zeta_t(j) \psi (j) [\gamma k_{t+1}(j) - (1 - \delta) \, k_t(j)] \, dj.
\]

The left hand side is the sources of funds. The uses of funds are as follows: Money and bonds to carry forward; additional capital for each type of capital good, \( j \in [0, 1] \); consumption of output \( c_t(i) \) for each type of consumption good \( i \in [0, 1] \); and the

\(^7\)Marquis and Witte (1989) investigate optimal cash management innovations by firms and the affect on money demand and its interest rate sensitivity.

\(^8\)Prescott (1987) uses the analogy that a fixed cost of intermediation for bank drafts is a record-keeping cost.
payment of intermediation services on both consumption purchases \( \phi (i) c_t(i) \) and investment purchases \( \psi (j) \left[ \gamma k_{t+1}(j) + (1 - \delta) k_t(j) \right] \) if the household chooses to use credit rather than cash. Thus the cash-in-advance constraint is

\[
\frac{M_t}{pt} \geq \int_0^1 [1 - \xi_t(i)] c_t(i) di + \int_0^1 \left[ 1 - \zeta_t(j) \right] \left[ \gamma k_{t+1}(j) - (1 - \delta) k_t(j) \right] di. \tag{2.29}
\]

Again there is the relevant time zero problem, the non-negativity requirements on money and capital, the no-ponzi condition, and the initial capital stock of each type \( j \) is given. Here \( \zeta_t(j) \) is another indicator variable taking the value 1 if the capital good is purchased on credit and 0 if by cash and \( \psi (j) \) is the proportional cost of intermediation in capital goods. As before it is assumed that \( \psi (j) \) is increasing in \( j \) with \( \psi (0) = 0 \) and \( \psi (1) = \infty \). The first order conditions with respect to consumption are unchanged

\[
u' \left[ c^0_t(i) \right] = \lambda_t + \mu_t, \quad i \in [0, 1] \text{ and } t \geq 1
\]

\[
u' \left[ c^1_t(i) \right] = \lambda_t(1 + \phi(i)), \quad i \in [0, 1] \text{ and } t \geq 1.
\]

As with consumption we will distinguish capital purchased with cash as \( k^0_{t+1}(j) \) and capital purchased with credit as \( k^1_{t+1}(j) \). The other first order conditions, with respect to capital, money and bonds are

\[
\gamma \left( \lambda_t + \mu_t \right) = f'(k^0_{t+1}(j)) \beta \lambda_{t+1} + \beta \left( \mu_{t+1} + \lambda_{t+1} \right) (1 - \delta) \tag{2.30}
\]

\[
\gamma (1 + \psi(j)) \lambda_t = f'(k^1_{t+1}(j)) \beta \lambda_{t+1} + \beta (1 + \psi (j)) (1 - \delta) \lambda_{t+1} \tag{2.31}
\]

\[
\gamma \lambda_t / p_t = \beta \left( \lambda_{t+1} + \mu_{t+1} \right) / p_{t+1} t \geq 1 \tag{2.32}
\]

\[
\gamma \lambda_t / R_t p_t = \beta \lambda_{t+1} / p_{t+1} t \geq 1. \tag{2.33}
\]

The time zero conditions are left out for simplicity, so the above hold for all periods \( t \geq 1 \) and (2.30) and (2.31) hold for all \( j \in [0, 1] \). Note that the amount of consumption
using cash will be the same for all $i$ and similarly the amount of capital will be the same in all markets $j$ if cash is used. But the amount purchased using credit will be decreasing in $i$ and $j$ respectively. Thus we can determine the cut-off market, $\bar{i}$, for consumption goods in the same way as in section 2 and we can determine the cut-off \( \bar{j} \) for capital goods. There is nothing changed from the households problem regarding consumption purchases so the value of $\bar{i}$ is the same as in section 2, i.e. it is the value that solves $\phi (\bar{i}) = R_{t-1} - 1$ and $\xi_{t}(i) = 1$ for all $i < \bar{i}_t$ and zero otherwise.

In a similar manner we need to determine the threshold value of $j$ determining where cash and credit are used for purchasing capital goods. It is still the case that (2.32) and (2.33) imply that $R_{t-1} \lambda_t = \lambda_t + \mu_t$. Substituting this into (2.30) and (2.31) yields

\[
\begin{align*}
 f'(k^0_{t+1}(j)) & = \frac{\gamma R_{t-1} \lambda_t}{\beta \lambda_{t+1}} - R_t (1 - \delta) \quad (2.34) \\
 f'(k^1_{t+1}(j)) & = \frac{\gamma (1 + \psi (j)) \lambda_t}{\beta \lambda_{t+1}} - (1 + \psi (j)) (1 - \delta) \quad (2.35)
\end{align*}
\]

Thus the household will set the MPK equal to the cost of capital. If using cash to purchase capital this cost depends on the nominal rate of interest no matter what market $j$ the capital is purchased from, thus (2.34) is independent of $j$. On the other hand, if capital is purchased using credit, the cost will depend on the market $j$ and thus the equilibrium quantity will also depend on $j$. As with the consumption case, the household will want to choose the least expensive method of making its purchases and will choose to use credit when it is cheaper than cash. Because of the assumptions on $\psi (j)$ it will be the case that credit will be cheaper than cash is some strictly positive measure of markets, and there will exist some $\bar{j}_t$ such that they are exactly equal. Setting the above two equations equal to each other gives the critical
value $\bar{j}_t$, which is the implicit solution to

$$
\psi(\bar{j}_t) = \frac{\gamma R_{t-1} \lambda_t - \beta \lambda_{t+1} R_t (1 - \delta)}{\gamma \lambda_t - \beta \lambda_{t+1} (1 - \delta)} - 1. \tag{2.36}
$$

The cost of credit is increasing in $j$ so capital will be purchased using cash for all $j > \bar{j}_t$. As can be seen, as long as the gross nominal interest rate is strictly greater than 1, the cutpoint will be strictly between 0 and 1. Thus the solution to the households choice problem is

$$
\zeta_t(j) = \begin{cases} 
1 & \text{if } 1 + \psi(j) \leq \frac{\gamma R_{t-1} \lambda_t - \beta \lambda_{t+1} R_t (1 - \delta)}{\gamma \lambda_t - \beta \lambda_{t+1} (1 - \delta)} \\
0 & \text{otherwise.} \end{cases} \tag{2.37}
$$

Note that the cost of capital is increasing in $j$ and thus it is clear that optimal quantity of $k_{t+1}^1(j)$ will be decreasing in $j$ up to the point $\bar{j}_t$. Larger investment projects will be financed with credit. After this point it is cheaper to use cash and the cost is the same form all $j > \bar{j}_t$ which is just a function of the nominal interest rate. From (2.35) it can be seen that the marginal product of capital relative to the cost of credit

$$
\frac{f'(k_{t+1}^1(j))}{1 + \psi(j)} = \frac{\gamma \lambda_t}{\beta \lambda_{t+1}} - (1 - \delta) \tag{2.38}
$$
is the same for all $j$. In particular at $j = 0$ this implies $f'(k_{t+1}^1(0)) + (1 - \delta) = \frac{\gamma \lambda_t}{\beta \lambda_{t+1}}$ which is the solution to the problem with no credit cost and no cash-in-advance requirement. This is just the real interest rate and noting the above condition (2.38), the real interest rate is equal across all sectors that use credit and is a function of the MPK, the rate of depreciation and the cost of credit. For all other $j > 0$ the quantity of capital purchased is less than this, and after $\bar{j}$, the quantity is $f'(k_{t+1}^0) = \frac{\gamma R_{t-1} \lambda_t}{\beta \lambda_{t+1}} - R_t (1 - \delta)$. When the gross nominal interest rate is greater than one and therefore the cash-in-advance constraint is binding at every time period, the demand
for real balances is

\[ \frac{M_t}{p_t} = \int_{\tilde{\iota}_t}^1 c_t^0 di + \int_{\tilde{\iota}_t}^1 k_t^0 di = [1 - \tilde{\iota}_t] c_t^0 + [1 - \tilde{\iota}_t] k_t^0 \]

where the arguments \( \lambda_t, R_t \) and \( R_{t-1} \) are suppressed for ease of exposition. Money demand is equal to the purchases of consumption and capital goods using cash. Taking the same approach as in section 2, the dynamics of prices become

\[ \frac{p_{t+1}}{p_t} = \frac{g_t \left( [1 - \tilde{\iota}_t] c_t^0 + [1 - \tilde{j}_t] k_t^0 \right)}{\gamma \left( [1 - \tilde{\iota}_{t+1}] c_{t+1}^0 + [1 - \tilde{j}_{t+1}] k_{t+1}^0 \right)} \]

where \( g_t \) is again the exogenous growth of the money supply. The nominal interest rate is solved using equations (2.31) and (2.33) and depends on the term \( f' (k^1_{t+1}(j)) \) \( (1 + \psi(j)) \) + 1 + \( \delta \). But as mentioned above, this term is the same for all \( j \in [0, \tilde{j}] \). Thus we can write the nominal interest rate as

\[ R_t = (1 + f' (k^1_{t+1}(0)) - \delta) \frac{p_{t+1}}{p_t} = \frac{[1 + f' (k^1_{t+1}(0)) - \delta] g_t ([1 - \tilde{\iota}_t] c_t^0 + [1 - \tilde{j}_t] k_t^0)}{\gamma ([1 - \tilde{\iota}_{t+1}] c_{t+1}^0 + [1 - \tilde{j}_{t+1}] k_{t+1}^0)}. \]

In this case the market clearing for final output becomes

\[ \int_0^1 f(k_t(j)) dj = \int_0^{\tilde{j}_t} [\gamma k^1_{t+1}(j) - (1 - \delta) k_t(j)] dj + (1 - \tilde{j}_t) [\gamma k^0_{t+1} - (1 - \delta) k^0_t] \\
+ \int_0^{\tilde{\iota}_t} c_t^0(i) di + (1 - \tilde{\iota}_t) c_t^0 + \int_0^{\tilde{i}_t} \phi(i) c^1_t(i) di \\
+ \int_0^{\tilde{j}_t} \psi(j) [\gamma k^1_{t+1}(j) - (1 - \delta) k_t(j)] dj \]

which says that output is equal to investment plus consumption plus the cost of financial services used on consumption and investment purchases.

### 2.3.2 Stationary Equilibrium

Again the stationary equilibrium can be solved and comparative statics exercises performed. As the above equations make obvious, adding the cash-in-advance constraint to investment complicates the dynamic solution to the problem, but it is still
the case that the convergence properties of the traditional optimal growth framework hold. In principle it is still the task of choosing a $\lambda_0$ such that taking the law of motion of the capital stock, with initial capital given, satisfies the transversality equations. But now the law of motion of the capital stock depends on the rate of money growth and the dynamics will be affected by the inflation rate$^9$.

It is however still fairly easy to solve the stationary equilibrium. Using the fact that in a steady-state $\lambda$ is a constant (2.31) can be used to show that

$$f'(k^1(j)) = (1 + \psi(j)) \left( \frac{\gamma - \beta(1 - \delta)}{\beta} \right)$$

for all $j \leq \tilde{j}$. This includes $j = 0$ and thus it will be convenient to work with

$$1 + f'(k^1(0)) - \delta = \frac{\gamma}{\beta}$$

which is the familiar result for the real interest rate discussed in section 2. This means that the real interest rate is the same as in the model without the costs of intermediation, the difference is made up by decreasing the amount of capital in each market $j$ to account for the credit costs.

So for capital goods purchased with credit, the steady-state quantities of capital in each market $j$ are determined by parameters of the production technology $f(\cdot)$, the credit technology $\psi(\cdot)$, the growth rate of productivity, $\gamma$, and the discount factor $\beta$. From this it is easy to see the equilibrium nominal interest rate is

$$R = \frac{\gamma}{\beta} \frac{p_{t+1}}{p_t} = \frac{g}{\beta}$$

$^9$For analyses of the transition paths in CIA models applying to consumption and investment one should see Fischer (1979) and Abel (1985) respectively.
which is also unchanged. On the other hand the quantity of capital purchased using cash will depend on the rate of inflation
\[
f'(k^0) = R\left(\frac{\gamma - \beta (1 - \delta)}{\beta}\right) = \frac{\gamma \rho t_{t+1}}{\beta} \left(\frac{\gamma - \beta (1 - \delta)}{\beta}\right) = \frac{g}{\beta} \left(\frac{\gamma - \beta (1 - \delta)}{\beta}\right).
\]

Thus we have that for all markets \(j > \tilde{j}\) the amount of capital purchased depends on the money growth rate, and the last term is identical to Stockman (1981) except we allow for exogenous growth \(\gamma\).

Thus we can denote the average capital stock as
\[
k = \int_0^{\tilde{j}} k^1(j) dj + (1 - \tilde{j}) k^0.
\]

Here the average capital stock is decreasing in inflation. An increase in inflation will increase the cutpoint \(\tilde{j}\) and thus the measure of markets which are purchased with credit. In these markets the quantity purchased is unaffected by inflation although decreasing in \(j\). The increase in \(\tilde{j}\) decreases the number of markets where capital is purchased using cash and the quantity in each of those markets decreases.

Finally we have our market clearing equation given by
\[
\int_0^1 f(k(j)) dj = \int_0^{\tilde{j}} \gamma k^1(j) dj + (1 - \tilde{j}) k^0 - \int_0^{\tilde{j}} (1 - \delta) k^1(j) dj - (1 - \tilde{j}) (1 - \delta) k^0 + \int_0^i c^1(i) di + (1 - \tilde{\gamma}) c^0 + \int_0^i \phi(i) c^1(i) di + \int_0^{\tilde{j}} \psi(j) [\gamma k(j - (1 - \delta) k(j)] dj
\]

where investment is \((\gamma + \delta - 1) k\) and \(\gamma\) is one plus the rate of growth of productivity.

And lastly, money demand is given by
\[
\frac{M}{p} = [1 - \tilde{i}] c^0 + [1 - \tilde{j}] k^0
\]

which will determine the velocity of money.
2.3.3 Parametric Example

To compare the results to the example in section 2, I consider a similar parametric example

\[ u[c_i(i)] = \ln c_i(i) \]
\[ y_t = \int_0^1 f(k_t(j))dj = \int_0^1 k_t(j)^\alpha dj \]
\[ \phi(i) = \frac{i}{\theta(1-i)} \]
\[ \psi(j) = \frac{j}{\eta(1-j)}. \]

For this Cobb-Douglas production function we have from (2.40) and (2.39) the steady-state values of capital per-capita, per-efficiency-unit are

\[ k^1(j) = \left[ \frac{\alpha\beta}{(1 + \psi(j))(\gamma - \beta)(1 - \delta)} \right]^{\frac{1}{1-\alpha}} \] (2.41)
\[ k^0 = \left[ \frac{\alpha\beta}{(\frac{\gamma}{\beta}\frac{p_t+1}{p_t})(\gamma - \beta)(1 - \delta)} \right]^{\frac{1}{1-\alpha}} \] (2.42)

which are the same as usual except they are decreasing in the cost of credit \( \psi(j) \) and the nominal interest rate \( \frac{\gamma}{\beta}\frac{p_t+1}{p_t} \) respectively. We already know that the real interest rate is \( \frac{\gamma}{\beta} \) and the nominal interest rate is \( R = \frac{\gamma}{\beta}\frac{p_t+1}{p_t} \), from the previous section. The cutpoints are \( \bar{i} = \frac{\theta(R-1)}{1+\theta(R-1)} \) and \( \bar{j} = \frac{\eta(R-1)}{1+\eta(R-1)} \). The remaining variable is the \( \lambda \) such that the equilibrium quantities of consumption clear the goods market.

It is now the case that higher rates of inflation affect the steady-state level of output and investment as can be seen in figure 4. We did not consider a similar figure in section 2, but mentioned the lines for output and investment would simply be horizontal and all the action would take place between consumption and credit services. Here, on the other hand, output and investment are decreasing in the rate
Figure 2.4: Aggregate Output, Consumption, Investment, and Financial Services at Different Levels of Inflation.

of inflation. Interestingly, the inflation rate seems to have much lower effects after rates of around 40 to 50%. The reason the inflation effect on capital decreases is because the cutpoint $\bar{j}$ is also increasing with the inflation rate. That is, while $k^0$ is decreasing, it is also being purchased in fewer markets, $1 - \bar{j}$, and the total fraction of capital purchased with cash $(1 - \bar{j})k^0$ is decreasing faster than if the fraction of investment purchased with cash were fixed exogenously. That inflation has less of an effect at higher rates agrees with Fischer (1993) but contradicts other findings that inflation is detrimental to growth only at higher levels.

The fact that aggregate output is decreasing in inflation has some interesting implications for the results presented in section 2. An increase in the growth rate has very similar effects as the case when the CIA constraint only applied to consumption
Figure 2.5: Velocity and Share of Financial Services of GDP at Different Levels of Inflation and Different Growth Rates. Constraint on Capital Goods.

goods. M1 velocity is affected slightly and the share of financial services is affected quite a bit. It is interesting however that for a given growth rate, i.e. for a particular curve in figure 5, say when growth is 2.5%, the mechanism affecting the share to financial services from higher inflation rate is quite different from the previous section. The reason financial services share increased in inflation before was because there was a one-for-one increase in financial services for the decrease in consumption. Thus more credit being used to purchase the same amount of output. In this case the aggregate level of financial services is hardly changed but the steady-state level of output is decreasing in the inflation rate. Thus rather than having a higher amount of finance out of a fixed amount of output, here we have roughly the same amount of finance out of a lower amount of output.
Higher $\eta$ is lower cost of credit.

Figure 2.6: Velocity and Financial Services Share of GDP at Different Levels of Financial Innovation. Constraint on Capital Goods.

We can consider an exogenous decrease in the cost of credit on consumption goods, $\theta$, as in the previous section. The results are qualitatively the same as before. Less money will be used and more credit, leading to higher velocity but a smaller total share of finance in the economy. The main difference is that the absolute levels change less because a large fraction of the money supply and financial services are being used for investment goods, which are unaffected by $\theta$.

On the other hand, a change in the cost of credit for investment goods, $\eta$, not only affects velocity and financial services but also output. Figure 6 shows the effects of a decrease in the cost of intermediation. Here the differences are smaller than the case of consumption goods. With a decrease in the cost of credit, the cutoff $\bar{J}$ is higher and thus cash is used for investment in fewer markets, which would tend to increase
velocity. In each market \([0, j]\), where credit is used, the amount of investment is strictly higher now that the cost of capital has decreased in each location and leads to the increase in average capital and output. This increase in output also leads to an increase in consumption, yet the threshold \(i\) has not changed. Thus cash is used in the same measure of consumption markets as before and they use more of it, thus counteracting some of the increase in velocity.

![Graph of Output and Consumption](image)

Figure 2.7: Output and Consumption at Different Levels of Financial Innovation. Constraint on Capital Goods.

### 2.4 Critique of Cross-country Evidence

With the formal models set out above and the examples simulated to give reasonable results, we turn in the next two sections to discuss the implications for existing studies. This section addresses the question of inflation and growth in empirical studies, such as those surveyed in the introduction. The next section briefly discusses
welfare issues which have taken up more of the research agenda. In this section I focus on the model with CIA on investment since it provides growth results.

None of the empirical tests mentioned in the introduction, nor any others that I know of, take credit into account as an alternative means of exchange. Yet the above analysis indicates that the elasticity of output with respect to inflation will depend on the level of development of credit markets to provide intermediation. There are two key issues I would like to bring up. First, it is without a doubt that a broad sample of countries will have a diverse range of intermediation technologies. Not only due to technological reasons, but institutional and demographic. Consider a large sample of countries $l$ with the technology parameters $\theta_l$ and $\eta_l$. These are not only going to affect their steady-states, but also their rates of convergence. Yet most empirical studies assume that the slope coefficients are the same across countries and time. In a recent study by Grier and Grier (2003) looking at the US and Mexico they reject the null hypothesis that inflation has the same affects across countries. Thus it should not be surprising that many tests of inflation are non-robust.

Also the non-linear affect of inflation on growth will likely hamper empirical tests. This is magnified by the prospect that credit market development and a government’s propensity to have excessive inflation are probably negatively correlated. More developed countries are likely to have lower inflation but also lower credit costs which suggests their output is less elastic with respect to inflation. Countries with higher inflation, although having higher credit costs, are likely to be in the relatively flat portion of the output inflation curve in figure 7.

It is also important to consider changes in credit in addition to simple differences across countries. While we looked above only at steady-state paths with exogenously
different levels of credit technology, it’s not hard to imagine transitional dynamics with some adjustment costs on capital. Cross-country studies that do not account for differences in credit obviously don’t account for changes in credit. Thus it is possible that one country has much higher inflation than another but has deregulated its credit industry and is growing faster than normal to its now higher steady-state path.

These last two points emphasize that omitting the interaction of finance and inflation may seriously hamper empirical tests. On the other hand the results above do not help much in providing a solution. As shown above, there is a key difference in whether an innovation affects the household or business sector. For example, the rise of ATM machines would affect velocity and the share of financial services in output as well as total consumption, but it would not affect output or growth. On the other hand, the widespread use of electronic transfers for sweeping business accounts may have a sizeable affect on output. Yet how do you take these things into account in an aggregate cross-country data set? The above figures suggest you cannot use finances share of output as a proxy because it could be high or low for many reasons, with opposite implications for growth. Thus an appropriate measure of financial efficiency is left unanswered.

One question still unanswered is whether the model can account quantitatively for the inflation-growth effects that are often found in the empirical literature. Although the increase in the financial sector from higher inflation will decrease measured productivity somewhat, it is the case that decreased investment will be the driving force behind the results. Empirically inflation seems to be working mostly through productivity rather than investment, so the model may need to be modified to match these features. However, the empirical results on inflation and growth are quite mixed with
many results not necessarily robust, so it is not easy to try to calibrate a model to match the data, since there is little agreement on what the data in fact say.

2.5 Welfare Costs of Inflation

It seems that the majority of the literature utilizing CIA models and differentiating between cash goods and credit goods are concerned with welfare costs of inflation rather than growth effects. It is common to assume some exogenous fraction of consumption is credit goods and the rest cash goods. Then the model is simulated and welfare affects estimated (Cooley and Hansen (1991)). The above analysis has two implications for these models. First, by assuming the fraction of cash goods is fixed, the results are overstating the effects of inflation on consumption, because they ignore the opportunity to switch to credit. Whether this difference is large or not is not clear, but should be easy to check. Second, what happens to the resource cost of using credit is important. In section 2 it is assumed that intermediation uses up real resources. In that section the welfare affect would be the decrease in consumption which was reflected one-for-one with the increase in financial services. These effects seem quite large, as can be seen in figure 1, where the financial services sector is 1.7% of GDP at an inflation rate of 50%. It may be reasonable to assume instead that the resources used in intermediation are refunded back to households in a lump sum manner.

Of course adding CIA to capital increases the cost of inflation due to of the decrease in output, whereas most research on welfare effects ignore this possibility.
2.6 Conclusion

At the least, the above model shows that endogenizing the choice of credit and cash goods will have some, though possibly small, effect on the conclusions of papers that try to measure welfare or growth effects of inflation by taking the fraction of cash goods subject to CIA as fixed. More optimistically, it suggests that empirical tests trying to find the long-run relationship between inflation and growth need to be much more precise. The relationship also seems to be significantly non-linear. It suggests that the level of development of the finance sector may have significant effects, and in fact these will be difficult to proxy, as finance share of output may not be appropriate. Clearly the non-robustness of inflation in growth equations could be affected by the omission of country relevant measures of financial development.

Whether the model can quantitatively match those found in the empirical literature is yet to be done. The non-linear results seem promising on the one hand, but whether the affects of inflation are decreasing at higher rates is unclear. It is also unclear whether countries with more developed financial markets are more or less affected by inflation, whereas the model suggests they would be affected less.
CHAPTER 3

RE-EXAMINING THE ROBUSTNESS OF INFLATION AND GROWTH

3.1 Introduction

This paper takes a new look at the inflation-growth literature from the perspective of model uncertainty. In particular, the method of Bayesian Model Averaging is used to examine both the robustness of inflation’s effects on long run economic growth as well as the size of the effect in growth regressions. Bayesian Model averaging has been used in recent attempts to find robust determinants of growth by Fernandez, Ley and Steel (2001) and Doppelhofer, Miller and Sala-i-Martin (2001), however, inflation has not been considered. Unlike these previous studies, this paper applies these methods to panel data and finds evidence that panel data provides valuable information to researchers and policy-makers that is not available in pure cross-sectional data. The possibility of parameter heterogeneity is addressed, as emphasized in a recent paper by Brock and Durlauf (2000). This is the problem that the coefficients on a particular variable are not the same for different observations. Whether the results from a general set of countries can be used to make inferences to specific countries is critical for policy-makers. As far as the effects of inflation, while it does not seem significant in the cross-sectional data, it is one of the most robust variables when panel data are
considered. Similar results hold when we allow for non-linear effects from moderate to high inflation. Considering parameter heterogeneity, it appears that the effect of inflation, and nearly all the other factors studied, are sensitive to the selection of countries included. Specifically, the coefficients obtained for OECD versus non-OECD countries are quite different. This suggests that although policymakers should take the inflation-growth results seriously, they should consider there own particular situation. Before going into detail of the methods and results, the previous literature is quickly reviewed to emphasize some of the points that are made later.

Over the last 20 years, many economists have found an association between inflation and slow growth, for instance Kormendi and McGuire (1985), Grier and Tullock (1989), Fischer (1993), and Barro (1996,1997). The typical approach in these papers has been to run linear regressions with the growth rate of per capita GDP as the dependent variable, and numerous possible factors including inflation as the independent variables. There have been extensions to these results indicating the effect on growth rates is non-linear in inflation. These papers tend to support that inflation is bad for growth, at least beyond a moderate level, such as 10%. Sarel (1996) tests for a structural break and finds that inflation is negatively related to growth after 8%. The point estimate for inflation at rates less than 8% is positive but statistically insignificant. Fischer (1993) uses a spline regression and finds a negative relationship at all levels of inflation. Barro (1996) found inflation to be harmful to growth, but showed the results were driven by the observations where inflation exceeds 20%. For inflation below 20% the point estimate would be negative, but statistically insignificant. Khan and Senhadji (2000) use recently developed methods on determining threshold effects

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and find a threshold at 11%. Looking separately at OECD and non-OECD countries they find the threshold to be 1% and 11% respectively.

Many of the early papers are based on cross-sectional data covering a large number of countries and looked at averages over long periods of time, say 35 years. Recent work has also taken into consideration the time dimension of inflation and growth and utilized panel data. To avoid the influence of business cycles the usual approach is to take 5 or 10 year averages. The results are usually similar, see Fischer (1993) and Barro (1996) for example. Rather than using linear regressions over long period averages, Bruno and Easterly (1998) take a time series approach. They find that inflation ‘crises’, episodes of over 40% inflation, have a negative effect on output but that economies are able to rebound rather quickly, suggesting that the inflation-growth relationship “is only present with high frequency data and with extreme inflation observation.”

Most of the results cited above find a negative relationship between inflation and long run growth rates. However there are a few instances where a positive relationship is found and others that find the relationship to be statistically insignificant. Despite these alternative findings, what really makes the empirical findings difficult to accept is the fragility of the inflation-growth relationship. The seminal work of Levine and Renelt (1992) finds that only initial GDP and investment are “robust” using Leamer’s extreme bounds analysis. In fact inflation is notoriously one of the least robust variables. The extreme bounds method runs many possible combinations of variables and if the variable becomes statistically insignificant even once, the variable is labeled “fragile”. More recently, Sala-i-Martin (1997), Doppelhofer, Miller and Sala-i-Martin (2000) and Fernandez, Ley and Steel (2001) examined the robustness
of growth variables using alternative methods and find several variables are likely to be important in growth regressions. Sala-i-Martin (1997) runs a large number of regressions and tries to measure what percentage of the distribution lies to the relevant side of zero. Doppelhofer, Miller and Sala-i-Martin (2000) and Fernandez, Ley and Steel (2001) use methods similar to Sala-i-Martin (1997) but rest on the theoretical results of Bayesian Model Averaging. Bayesian Model Averaging also utilizes a large number of possible regressions, but the models are weighted by a Bayesian posterior probability. Since their results will be cited often, these two sources will simply be denoted as DMS and FLS through the remainder of the text. In contrast to Levine and Renelt’s (1992) findings, several variables are found to be significant, but inflation is not one of the forty-one variables they consider most likely to affect growth.

The paper proceeds as follows: Section 2 provides an overview of the Bayesian Model Averaging approach and section 3 proceeds to the application on inflation and growth. This section begins by applying Bayesian model averaging to growth regressions using cross-sectional data, similar to DMS and FLS, but includes inflation as an explanatory variable and examines its robustness. Then the method is extended to panel data, which provides quite different results regarding the robustness of many variables, including inflation. As a further extension, we allow for non-linear effects of inflation in both the cross-section and panel. Section 4 examines how much parameter heterogeneity may affect the results. Using a similar procedure, a large number of regressions are run allowing for different variations in the sample of countries. Arbitrarily using OECD and non-OECD as a guideline to differentiate countries, the coefficients on the regressors are quite different. Section 5 discusses the usefulness of the results to policymakers and Section 6 concludes.
3.2 Accounting for Model Uncertainty

Model uncertainty addresses the question of what variables to include in a regression. Usually one would rely on past research and theory as a guide. The usual approach has been to run a reasonable regression and then check for robustness by adding and omitting a few variables on the right hand side. If the coefficients of interest do not change much the results are labeled robust. However, the difficulty for growth economists is that past research has included an enormous number of possible variables and theory does not offer as much guidance as it might in more traditional macroeconomic applications. For example, Brock and Durlauf (2000) note that there have been more proposed variables than there are country observations, and a theory can be developed that can support nearly all of them. Bayesian Model Averaging provides a formal way to test the importance of variables under model uncertainty. It allows the right hand side variables to vary over all of the possible combinations and then looks at the probability that these individual models are appropriate.

This approach has been used to study the possible determinants of economic growth by DMS, FLS and Brock and Durlauf and each of these papers devotes a few pages to explain the methodology. There are also good references to the procedure in general terms, rather than specifically economic growth, that are available, such as Raftery (1995) and Hoeting, Madigan, Raftery, and Volinsky (1999). Therefore, the next subsection only sets out the basics of the approach and directs the reader to other sources for more technical details.
3.2.1 Bayesian Model Averaging in Theory

Bayesian Model Averaging (BMA) is a method to account for model uncertainty, having been utilized in the statistics literature for some time but only recently emerging in economics. Economists most often run a regression on what they propose to be the proper, or best, model and then report these results. Often robustness is considered by running a few reasonable alternatives and verifying the original results, but the fact that the model may not be adequate is hardly addressed. It is true that a high measure of goodness of fit, such as adjusted $R^2$, may suggest that the model is indeed appropriate, but the coefficients and standard errors do not reflect the added uncertainty with regards to the unknown model. BMA accounts for model uncertainty by considering all possible models and using the posterior probability that each of the models is the true model, conditional on the data.

This section borrows heavily from Raftery (1995) to lay out the essential ideas. To be precise, let $\mathcal{M} = \{M_1, M_2, \ldots, M_K\}$ represent the set of all $K$ possible models, and $\beta_o$ represent a coefficient of interest. For example, below the interest will focus on the effect of inflation on growth, so $\beta_o$ will be the coefficient on inflation in a linear growth regression. A model $M_i$, $i = 1 \ldots K$, is simply a combination of variables. The Bayesian approach is concerned with the probability of the coefficient conditional on the data, $p(\beta_o|D)$. From probability theory the posterior distribution of $\beta_o$ is

$$p(\beta_o|D) = \sum_{i=1}^{K} p(\beta_o|D, M_i) p(M_i|D)$$

(3.1)

where $p(\beta_o|D)$ is the posterior distribution of $\beta_o$, $p(\beta_o|D, M_i)$ is the probability distribution of observing $\beta_o$ conditional on the data $D$ and the model $M_i$, and $p(M_i|D)$ is the posterior probability that the model is $M_i$ conditional on the data. The formula
shows that the posterior distribution of $\beta_o$, conditional on the data, depends on all $K$ possible models. Note that, if the variable whose coefficient is $\beta_o$ is not included in the model $M_i$, then $p(\beta_o = 0|D, M_i) = 1^{10}$. There are difficulties in calculating this expression. The posterior probabilities of the models are

$$p(M_i|D) = \frac{p(D|M_i) p(M_i)}{\sum_{j=1}^{K} p(D|M_j) p(M_j)} \quad (3.2)$$

The problems arise because of the need for prior information. The posterior, $p(M_i|D)$, depends on the likelihood of the data $p(D|M_i)$, as well as the prior probability assigned to the model $p(M_i)$. Of course these problems can be a virtue, since most Bayesian’s would want to take advantage of prior information. But the question, and point of skepticism for non-Bayesians, is where do the priors on $\beta_o$ and $M_i$ come from? We adopt the simplifying assumption, as in Brock and Durlauf (2000), that all models are equally likely a priori, and therefore each model has a prior probability of inclusion of $1/K$. If there are $q$ variables of interest, allowing for all possible combinations of these variables leads to $K = 2^q$ possible models.

There are three features of the distribution $p(\beta_o|D)$ that are the main points of interest: the expected value of $\beta_o$, the standard deviation of $\beta_o$, and the probability that $\beta_o$ is not equal to zero$^{11}$. The main quantities needed to derive these values are the posterior probabilities of each model $i$. Here $p(M_i|D)$ is approximated using the Bayesian Information Criterion (BIC). The BIC is an approximation for $p(M_i|D)$ derived by a Taylor Expansion of an integrated likelihood function (see Raftery (1995))

$^{10}$For example, if the model $M_n$ includes only capital growth and initial GDP, and the coefficient on inflation is $\beta_o$, then the conditional distribution $p(\beta_o|D, M_n)$ has mass one at the point zero.

$^{11}$These calculations are done for each parameter $\beta$, where the description in the text is simplified to the scalar $\beta_o$. 

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for the details). For linear models, with normal errors, it is given by

\[ BIC_i = -\frac{1}{2} \left( n \log(1 - R_i^2) + k_i \log n \right) \]  

(3.3)

where \( n \) is the number of observations, \( k_i \) is the number of variables in model \( i \), not including the intercept, and \( R_i^2 \) is the measure of \( R^2 \) for model \( i \). Once this approximation is in hand it is easy to calculate the quantities of interest. The probability that the coefficient \( \beta_o \) is not equal to zero, i.e. \( \Pr(\beta_o \neq 0 | D) \), corresponds to the measure of robustness reported in FLS. This is the sum of all the model probabilities \( p(M_i | D) \), for models that include \( \beta_o \). Thus if we denote the set of \( K/2 \) models that contain \( \beta_o \) as \( A_o = \{ M_i : i = 1, ..., K; \beta_o \in M_i \} \) we can calculate

\[ \Pr(\beta_o \neq 0 | D) = \sum_{A_o} p(M_i | D). \]  

(3.4)

This is the quantity that FLS look at in attempting to determine the importance of many variables on growth. Whereas they are trying to address the robustness of variables, it is also important to consider the economic significance of the variables and look at the magnitude and sign of the coefficients. The BMA approach is also useful in this regard, in that it provides an estimate of the coefficient that takes into account the additional uncertainty due to the many possible models. Raftery (1995) discusses the weakness of t-statistics and p-values when the econometrician has not controlled for the additional uncertainty of not knowing the true model. Heeding those arguments, we estimate the expected value and standard deviation of \( \beta_o \), conditional on it being included in the model. Note that the summations below are over the set \( A \) described above. Under some regularity conditions, such as normal errors and non-stochastic regressors, these statistics can be calculated as (see Leamer (1978),
Ratery (1995))

\[ E(\beta_o|D, \beta_o \neq 0) = \sum_{A_0} E(\beta_o|D, M_i) p(M_i|D) [\Pr(\beta_o \neq 0|D)]^{-1} \quad (3.5) \]

\[ \text{var}(\beta_o|D, \beta_o \neq 0) = \sum_{A_0} \text{var}(\beta_o|D, M_i) p(M_i|D) [\Pr(\beta_o \neq 0|D)]^{-1} + \sum_{A_0} E(\beta_o|D, M_i)^2 p(M_i|D) [\Pr(\beta_o \neq 0|D)]^{-1} - E(\beta_o|D, \beta_o \neq 0)^2 \quad (3.6) \]

where \( E(\beta_o|D, M_i) \) and \( \text{var}(\beta_o|D, M_i) \) are the OLS estimates for \( \beta_o \) and \( \text{var}(\beta_o) \) in model \( i \). As Brock and Durlauf (2000) point out, the variance of \( \beta_o \) is the sum of the variance within models and the variance across models.

### 3.2.2 Bayesian Model Averaging in Practice

The difficulty in implementing the BMA approach in practice occurs when there are a large number of possible variables. The number of possible models is \( 2^K \), where \( K \) is the number of possible variables, which becomes computationally infeasible when \( K \) is large. Many of the statistics papers cited above deal with these problems, as does FLS. With 41 possible growth regressors FLS calculate that it would take 115 years to run all possible models on a modern computer. Ratery (1995) presents an example of economic factors influencing crime where there are 32,678 possible models arguing that “direct evaluation of the sums over all models is not feasible.” These problems are addressed elsewhere by trying to limit the number of models considered, first by sampling the model space and second by omitting models that perform much worse (by some pre-set measure) than others.

However, computing power has increased so as to make a difficult problem 5 years ago trivial now. This paper chooses to deal with a number of possible variables
that are manageable by utilizing prior results. In particular, the results of FLS and DMS are used to eliminate variables that they find highly unlikely to affect growth. Thus the regressions for all possible models are run and the results from each are recorded. Then the model posterior probabilities are calculated using the BIC approximation. Using the stored regression results and the BIC approximation we calculate the expected coefficients and t-statistics, taking model uncertainty into account, and also calculate the probability that each coefficient is not equal to zero, as derived in the formulas of the previous subsection.

Further, since the coefficients from all the regressions are stored, they can be used to plot a distribution of the coefficients. This provides a visual demonstration of the expected magnitude of the coefficient as well as the dispersion across models. If the distribution is quite precise than the usual method of running a few reasonable regressions may be adequate. However, a high variance distribution suggests that a wide range of estimates of the coefficient are likely in to be found in practice.

\section*{3.2.3 Growth and Model Uncertainty}

Fernandez, Ley and Steel (2001) and Doppelhofer, Miller and Sala-i-Martin (2000) use Bayesian Model Averaging to calculate the posterior probability that certain variables are related to economic growth. Rather than trying to find the “true” coefficients they are concerned with robustness, although DMS do provide the expected value of the coefficients. Basically both are trying to address what they see as shortcomings in the results of Levine and Renelt (1992). The extreme-bounds test labels a variable as fragile if it becomes insignificant even once in thirty-two thousand regressions and does not put any weight on how well a specific regression performs relative to others.
<table>
<thead>
<tr>
<th></th>
<th>BMA post prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GDP level 1960</td>
</tr>
<tr>
<td>2</td>
<td>Fraction Confucian</td>
</tr>
<tr>
<td>3</td>
<td>Life Expectancy</td>
</tr>
<tr>
<td>4</td>
<td>Equipment Investment</td>
</tr>
<tr>
<td>5</td>
<td>Sub-Saharan dummy</td>
</tr>
<tr>
<td>6</td>
<td>Fraction Muslim</td>
</tr>
<tr>
<td>7</td>
<td>Rule of Law</td>
</tr>
<tr>
<td>8</td>
<td>Number of Years Open Economy</td>
</tr>
<tr>
<td>9</td>
<td>Degree of Capitalism</td>
</tr>
<tr>
<td>10</td>
<td>Fraction Protestant</td>
</tr>
<tr>
<td>11</td>
<td>Fraction GDP in Mining</td>
</tr>
<tr>
<td>12</td>
<td>Non-Equipment Investment</td>
</tr>
<tr>
<td>13</td>
<td>Latin American dummy</td>
</tr>
<tr>
<td>14</td>
<td>Primary School Enrollment 1960</td>
</tr>
<tr>
<td>15</td>
<td>Fraction Buddhist</td>
</tr>
<tr>
<td>16</td>
<td>Black Market Premium</td>
</tr>
<tr>
<td>17</td>
<td>Fraction Catholic</td>
</tr>
<tr>
<td>18</td>
<td>Civil Liberties</td>
</tr>
<tr>
<td>19</td>
<td>Fraction Hindu</td>
</tr>
<tr>
<td>20</td>
<td>Primary Exports 1970</td>
</tr>
<tr>
<td>21</td>
<td>Political Rights</td>
</tr>
<tr>
<td>22</td>
<td>Exchange Rate Distortions</td>
</tr>
<tr>
<td>23</td>
<td>Age</td>
</tr>
<tr>
<td>24</td>
<td>War Dummy</td>
</tr>
</tbody>
</table>

Other variables included (in decreasing order of probability) size of labor force, fraction speaking a foreign language, fraction speaking English, ethno-linguistic fractionalization, Spanish colony dummy, standard deviation of black market premium, French colony dummy, absolute latitude, ratio of workers to population, higher education enrollment, population growth, British colony dummy, outward orientation, fraction Jewish, revolutions and coups, public education share, and Size area.

Table 3.1: Bayesian Average Probabilities from Fernandez, Ley and Steel (2001).
Considering 41 variables (from Sala-i-Martin (1997)) FLS find 24 variables that have more than 5% posterior probability of not being zero. These results are shown in Table 1, where the remaining 17 variables are listed in the footnote for completeness. The data are separated the by two lines to emphasize those that have a posterior probability greater than ninety percent and those less than ten percent. Initial GDP and Equipment Investment are highly likely to affect growth, as found in Levine and Renelt (1992) and other studies. The number of years open to trade does better than most, again consistent with previous research that suggest trade is an important variable in growth regressions. Primary Schooling is at 18.4% whereas Secondary School is below 5% (see the footnote to the Table). A peculiar result is that the fraction of the population that is Confucian is the second most significant variable, although DMS point out that this is essentially a dummy for Hong Kong and some East Asian Tigers. Other variables that seem to do moderately well are dummy variables for Sub-Saharan Africa and Latin America as well as other religious variables. The conclusion of these authors is that the approach finds several variables that are important for explaining long run growth, and is thus more “optimistic” than the conclusion of Levine and Renelt (1992).

3.3 Inflation and Growth

This section investigates the inflation-growth relationship using the methods developed in the preceding section. Because of the need for panel data, the data set used is not the same as FLS and DMS, but includes variables similar to those in Table 1 which, according to those papers, seem to be significantly related to economic growth. The analysis proceeds with three approaches in trying to determine if an
inflation-growth relationship exists, and if so to what extent. First inflation is added to a cross-sectional data set similar to FLS and DMS. Both the sign of the coefficient on inflation as well as the likelihood that inflation is relevant are emphasized. Second the same analysis is performed using a panel data set, taking into account the importance of fixed country effects. Possible advantages that panel data provide, both for the empirical results and policy concerns, are discussed. Third a threshold effect, which has found support in previous research, is examined.

3.3.1 Cross-Section Results

This section considers inflation and growth in a cross-sectional data set. The data set is different from DMS and FLS so some attention is paid to the significant similarities and differences between their results and those below. The question of interest in this paper is whether inflation has a robust effect on long run growth, and if so in what direction. Thus these other variables are only control variables in regards to this primary concern. As long as we have not omitted a variable that is correlated with inflation, the results should be valid. This is not the case for the papers by DMS and FLS who were asking the much broader question of what, if any, are the important determinants of growth. In that case the selection of possible variables is much more important. The data set is from Levine et. al. (2000) (available at the World Bank web site)\textsuperscript{12} and provides both cross-sectional and panel data for similar variables. The aim is to use as many as possible of the 18 variables that FLS found to have posterior probabilities greater than 10%. The 16 variables used for the cross-section regressions are: Inflation, Capital Growth, Initial GDP, Black Market Premium, Average Years Schooling, Average Years Secondary Schooling, Fraction of

\textsuperscript{12}Except Years Open to Trade which is from Sachs and Warner (1995).
the Population Catholic, Muslim and Protestant, Index of Lack of Civil Liberties, Number of Years Open to the World, Government Share of GDP, Private Credit, Population Growth and dummy variables for Latin America and Africa. The time period is 1960 through 1995. This leads to 65,536 possible combinations of variables, and all of these regressions are run and a constant term is included in every regression.

To begin, some differences between these variables and FLS/DMS should be noted. Government Share of GDP is included as it would tend to be of interest to the sort of policymakers who are concerned with inflation. It also may be one cause of higher inflation, and may bias the effects of inflation if it is omitted. Private credit is included because it has been emphasized in recent literature on finance and growth and also may be related to inflation’s impact. We include Population Growth, which was not important according to FLS’s results, because it is relevant in many theoretical models of growth. We should note that whereas we have Capital Growth, FLS/DMS have separate variables for Share of GDP for Equipment and Non-Equipment Investment. And finally their religious fractions are an average for the entire period whereas the ones in this data set are for 1980.

The first question addressed is whether inflation is a robust determinant of growth? The last column of Table 2 presents the BMA posterior probabilities of all the variables included. The Table is sorted in decreasing order of BMA probability and the constant term is not shown. Inflation, is the least likely variable to have an important contribution to growth according to Table 2. Thus there is no evidence that inflation is strongly relevant to long run economic growth using the cross-sectional data. However, it is nearly as likely as some variables that economists might a priori expect to be significant, such as population growth and secondary schooling. The
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
<th>Post. Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Growth</td>
<td>0.3526</td>
<td>3.6390</td>
<td>0.0006</td>
<td>98.5</td>
</tr>
<tr>
<td>Years Open</td>
<td>0.0142</td>
<td>2.2680</td>
<td>0.0269</td>
<td>67.0</td>
</tr>
<tr>
<td>Lack of Civil Liberties</td>
<td>-0.0026</td>
<td>-1.9555</td>
<td>0.0552</td>
<td>55.2</td>
</tr>
<tr>
<td>Fraction Catholic</td>
<td>-0.0083</td>
<td>-1.6076</td>
<td>0.1132</td>
<td>38.7</td>
</tr>
<tr>
<td>Africa Dummy</td>
<td>-0.0113</td>
<td>-1.5541</td>
<td>0.1254</td>
<td>37.4</td>
</tr>
<tr>
<td>Fraction Protestant</td>
<td>-0.0122</td>
<td>-1.5614</td>
<td>0.1237</td>
<td>35.7</td>
</tr>
<tr>
<td>Latin America Dummy</td>
<td>-0.0077</td>
<td>-1.4784</td>
<td>0.1445</td>
<td>35.4</td>
</tr>
<tr>
<td>Government Share GDP</td>
<td>-0.0009</td>
<td>-0.8498</td>
<td>0.3988</td>
<td>18.8</td>
</tr>
<tr>
<td>Initial GDP*</td>
<td>-0.0026</td>
<td>-0.6770</td>
<td>0.5010</td>
<td>18.2</td>
</tr>
<tr>
<td>Avg. Years Secondary School</td>
<td>0.0014</td>
<td>0.6320</td>
<td>0.5298</td>
<td>15.4</td>
</tr>
<tr>
<td>Fraction Muslim</td>
<td>0.0003</td>
<td>0.0363</td>
<td>0.9712</td>
<td>15.2</td>
</tr>
<tr>
<td>Black Market Premium**</td>
<td>-0.0055</td>
<td>-0.6578</td>
<td>0.5132</td>
<td>15.0</td>
</tr>
<tr>
<td>Average Years School</td>
<td>0.0002</td>
<td>0.1520</td>
<td>0.8797</td>
<td>13.2</td>
</tr>
<tr>
<td>Private Credit</td>
<td>0.0008</td>
<td>0.0970</td>
<td>0.9231</td>
<td>13.1</td>
</tr>
<tr>
<td>Population Growth</td>
<td>-0.0403</td>
<td>-0.1567</td>
<td>0.8760</td>
<td>12.9</td>
</tr>
<tr>
<td>Inflation**</td>
<td><strong>-0.0022</strong></td>
<td><strong>-0.1530</strong></td>
<td><strong>0.8789</strong></td>
<td><strong>12.7</strong></td>
</tr>
</tbody>
</table>

Bayesian Model Averaging using cross-sectional data. Variables in decreasing order of BMA Probability. * are measured in logs, ** measured as log (1 + variable).

Table 3.2: Cross Sectional Results.

The first three columns present the estimated coefficients and t-statistics, conditional on the variable being included in the model, accounting for model uncertainty. Thus for inflation, these values are for the regressions that only made up 12.7% of the posterior probability of models. The point estimate for inflation is small and negative, -0.0022, and statistically insignificant from zero.

Although the emphasis is on inflation, it is worth looking at some of the other results. The variable most likely to affect growth is capital growth, at 98.5% which is slightly higher than Equipment Investment Share of GDP in FLS. The next variable is Years Open to Trade at 67%. This is slightly higher than the 50.2% in FLS. Note
that there are several variables higher than this in FLS, but most of them are not in our data set, e.g. fraction Confucian and Life expectancy The dummy variable for Sub-Saharan Africa in Table 1 does better than the variable simply for Africa in Table 2. The next variable is the Lack of Civil Liberties Index, which is much higher than in FLS but we do not have Rule of Law which does well in Table 1 and are probably highly correlated. Initial GDP is only at 18.2%, in the middle of the table, whereas it is the most relevant variable in FLS, at 100% due to rounding. This variable has received much attention in the empirical literature as a negative coefficient indicates convergence. One possible reason for the lower importance here is that the growth series is different from FLS, as well as the sample of countries is different. It is the case that initial GDP will be highly significant in the panel data and there is evidence in the section on parameter heterogeneity that the effects are driven by the OECD countries, which is commonly found. The remaining variables are plausible and in line with FLS, e.g. religious composition and years schooling followed by black market premium and population growth. In Table 2 these range between 12% and 40% whereas theirs lie closer to 9-50% in Table 1.

The BMA posterior probability, which is the focus of FLS and DMS, does not indicate anything concerning the sign of the effect. The expected values of the coefficients, taking model uncertainty into account, make up the first column of table 2. Looking at the coefficient values, most results seem reasonable. Capital Growth is by far the most important and the signs all seem to go in the direction expected.

Perhaps as interesting to researchers is the distribution of the coefficients, conditional on their inclusion. Although these do not indicate the probability that the variable is significant, they do show what estimates might be achieved in various
Figure 3.1: Cross Sectional Distributions. Distributions of coefficients, conditional on variable included in model.
Figure 3.2: Cross Sectional Distributions (continued).
regressions. Figures 1 and 2 show the distributions of the estimated coefficients, conditional on the variable being included in the model, i.e. the distribution for the 32,768 models that includes the variable for that sub-figure. Inflation is plotted in the upper left of figure 1. It is nearly symmetric and centered close to zero. Thus, as Levine and Renelt (1992) found, there are many combinations of variables that would cause the coefficient on inflation to not only become insignificant, but the point estimate would switch signs. Looking at this plot, it is somewhat surprising that few studies have found a positive affect of inflation, although it may be that these positive point estimates are from models that are far from what empirical researchers would use as their baseline model.

Considering some of the other variables, every estimated coefficient on capital growth and years open is positive, and those on capital growth are quite large as might be expected. Whereas capital growth has few estimates even close to zero, years open has some point estimates close to zero which would surely be within two standard deviations of zero, and therefore be labeled fragile under the extreme bounds criteria. Many other variables have a lot of mass on both sides of zero and thus researchers are likely to find either positive or negative relationships, depending on the choice of control variables. Note also that black market premium has most of the distribution to the left of zero, but has a low BMA probability. This highlights the shortcoming of these illustrations mentioned above; although most of the regressions that included black market premium produced negative point estimates, the probability of these models (and their R²) of these regressions were significantly lower than other regressions without black market premium. For example, in this case the
32,768 models including black market premium make up 15% of the mass of model probability, whereas the models excluding black market premium make up 85%.

3.3.2 Panel Results

This section carries out the same exercise as the previous section but using panel data. One advantage of using the panel is the increased number of observations. In this particular case the panel data has 428 observations compared to only 60 in the cross-section. Another advantage, especially in the case of inflation is that there is increased variation by using the panel, which should allow the coefficients to be estimated more precisely. For example, very few countries have experienced high inflation for the entire 35 year period, whereas many countries have experienced high inflation over one of the five year periods. Of the papers cited earlier which investigate inflation and growth, Fischer (1993) uses both cross-sectional data and a panel with annual observations, whereas Barro (1996) considers a panel using decade averages. Both of these papers use pooled panel data. There are arguments, however, that there are country specific factors that cannot be ignored, which suggests the use of fixed effects methods (Evans (1998) and Knight, et. al. (1993)). These arguments are persuasive, and we proceed by allowing for fixed country effects in the remainder of this section. There are some problems that arise in the fixed effects framework. It is known that if there is a lagged dependent variable on the right hand side, such as initial GDP in this case, the coefficient will be biased. This has led to the use of dynamic panel data methods, for example Levine, et. al. (2000). However, it is not clear that the small sample properties of these estimators are an improvement when the time dimension is small, so we proceed with the more traditional fixed effects
method. Another problem that is relevant to inflation is the obvious endogeneity problem. Several steps are taken to try to alleviate this criticism. First, we use data for 5 year averages which hopefully removes most of the cyclical relationship. Further we try to use additional control variables such as time dummies and terms of trade to account for aggregate supply shocks. The endogeneity problem is addressed further at the end of this section.

In the transition from the cross-sectional data to the panel there are some changes in our available data. This is because some variables are not available for the panel and some variables are not relevant once fixed effects are allowed for, because they do not vary over the time periods. For example, in the panel regressions we no longer include any religious composition variables. This is due to both reasons: we do not have them for the panel, but also because they probably have little variation over time and thus would show up in the country specific terms. The regional dummy variables for Latin America and Africa are also dropped. The Lack of Civil Liberties is not included because we did not have it for the panel. As mentioned above, a variable for growth in terms of trade was added to try to account for any supply shocks that would be causing growth to decrease and inflation to increase. This gives us 12 variables and thus 4,096 regressions. The panel data is for 5 year non-overlapping periods from 1960 to 1995 and we include a time dummy for each period, where the time dummies are always included.

The results are in Table 3 and are quite surprising in two regards. First, inflation is now highly likely to affect growth (99.9% probability), to the same extent as capital growth and initial GDP. Second, the expected value of the coefficient, taking into account model uncertainty but conditional on inflation being included, is much larger
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
<th>Post. Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial GDP*</td>
<td>-0.0503</td>
<td>-9.4968</td>
<td>0.0000</td>
<td>100.0</td>
</tr>
<tr>
<td>Capital Growth</td>
<td>0.3166</td>
<td>7.8845</td>
<td>0.0000</td>
<td>100.0</td>
</tr>
<tr>
<td>Inflation**</td>
<td>-0.0334</td>
<td>-4.4264</td>
<td>0.0000</td>
<td>99.9</td>
</tr>
<tr>
<td>Government Share GDP</td>
<td>-0.1057</td>
<td>-2.7098</td>
<td>0.0070</td>
<td>67.1</td>
</tr>
<tr>
<td>Avg. Years Secondary School</td>
<td>0.0048</td>
<td>1.5324</td>
<td>0.1262</td>
<td>14.2</td>
</tr>
<tr>
<td>Average Years School</td>
<td>0.0025</td>
<td>1.1417</td>
<td>0.2542</td>
<td>9.5</td>
</tr>
<tr>
<td>Growth Terms of Trade</td>
<td>0.0219</td>
<td>0.9978</td>
<td>0.3189</td>
<td>7.5</td>
</tr>
<tr>
<td>Private Credit</td>
<td>0.0066</td>
<td>0.8898</td>
<td>0.3741</td>
<td>6.8</td>
</tr>
<tr>
<td>Years Open</td>
<td>-0.0004</td>
<td>-0.8621</td>
<td>0.3891</td>
<td>6.6</td>
</tr>
<tr>
<td>Population Growth</td>
<td>0.0288</td>
<td>0.1353</td>
<td>0.8924</td>
<td>4.7</td>
</tr>
<tr>
<td>Black Market Premium**</td>
<td>-0.0003</td>
<td>-0.0727</td>
<td>0.9421</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Bayesian Model Averaging using panel data and allowing for fixed country effects. Variables are in decreasing order of BMA Probability. * are measured in logs, ** measured as log (1 + variable).

Table 3.3: Panel Results.

(in absolute value) than before, -0.0334 compared to -0.0022. It is also significantly different from zero, according to the t-statistics which include the additional variation from model uncertainty.

Some of the other variables change in significant ways. Initial GDP and government share of GDP have much higher BMA probabilities whereas years open to trade is much lower. Also the signs of the expected value of the coefficient change for years open to trade and for population growth. This is less surprising for population growth which is near the bottom in both the cross-sectional and panel results. Both schooling variables remain with positive signs but also do poorly under both methods.

The distribution of the coefficients are plotted in Figure 3. Note that in the subplot for inflation, upper left corner, the entire distribution is less than zero with
Figure 3.3: Fixed Effects Distributions. Distributions of coefficients, conditional on variable included in model.
few point estimates even close. Thus it seems that it would be very unlikely to find a positive coefficient on inflation, regardless of the model chosen. It may also be that inflation would even be deemed robust under an extreme bounds analysis. Note also that this distribution is fairly narrow, suggesting that there should not be a wide range of estimates in papers using panel data, even if they differ in various ways.

It is also the case that initial GDP, capital growth, government share of GDP and years open have distributions that lie entirely to one side of zero. The distributions for Initial GDP and capital growth have relatively little variance. Also even though years open shows nearly all of the point estimates are negative, the BMA probability was low.

To emphasize the results for inflation Figure 4 shows the distributions of the coefficients for both data sets. To summarize the above results, under cross-section regressions the average value of \( \beta \) is very close to zero and the distribution is fairly symmetric around zero. Using panel data we get a negative mean coefficient (-0.0334), and in fact all the point estimates are negative and the variance has decreased.

### 3.3.3 Threshold Effects

This subsection allows for possible non-linearities, which have been found in several studies on the inflation-growth relationship. Sarel (1996) allows for a structural break and determines that inflation below 8% does not affect growth, but higher inflation does. Fischer (1993) uses a spline regression and estimates coefficients for inflation from 0-10\%, 10-40\% and 40+\% and finds a negative but diminishing effect on growth. Barro (1996,1997) also finds a possible threshold at 20\%. However Barro
Figure 3.4: Distribution of Inflation Coefficients.

simply looks at the results for the observations with low and high inflation separately, excluding the other observations. The spline method utilizes all of the data.

Another advantage of the spline method is that if forces the relationship to be continuous. It is possible to run regressions with two separate variables, one for low inflation and one for high. This will obviously fit the data better, because it is unconstrained, but lacks economic reasoning. The spline approach allows the relationship to be non-linear but requires the effect to be continuous. Fischer (1993) chooses the possible kink points arbitrarily, while Sarel (1996) tests for a possible structural break. This paper uses the results of a recent paper by Khan and Senhadji (2000), which estimates the most likely threshold level in a spline regression of inflation and growth. They use non-standard tests to determine the most likely threshold level, utilizing recent econometric developments by Hansen (1999,2000). They find and find a point estimate for the threshold of 11% for their sample of all countries which has a 95%
confidence interval of 1 to 20%. The approach below is to simply adopt a threshold of 10% because it is close to their point estimate and agrees with prior studies, and the concern here is not with the most likely threshold value per se, but rather the effects of low and high inflation. The nature of the threshold regression is different from some of the earlier empirical papers on non-linear effects. Unlike Barro’s method of allowing separate coefficients on high and low observations, the threshold method relies on a spline regression. The important distinction is that the effect is allowed to change but is forced to be continuous. Thus the regression includes two inflation variables: Inflation at all levels and an additional effect for inflation above 10%. This second variable is constructed as the level of inflation, less 10%, times a dummy variable equal to 1 if inflation is greater than 10% and zero otherwise. Thus the regression amounts to

\[ \gamma = \alpha + \beta_1 \pi + \beta_2 [\pi - \pi^*] \delta + \Gamma X + \varepsilon \]

where \( \gamma \) is the per-capita growth rate, \( \pi \) is the rate of inflation, \( \pi^* \) is the threshold, \( \delta \) is equal to 1 when inflation is greater than the threshold, and \( X \) are the control variables. The coefficient \( \beta_2 \) is the additional impact that inflation has on growth when it is greater than the threshold level. That is, the effect of inflation on growth, when inflation is greater than 10% is \( \beta_1 + \beta_2 \).

Table 4 presents the results using the spline regression for cross country data and panel data with fixed effects. The control variables are not reported because their point estimates and BMA probabilities were hardly changed. Here all possible variable combinations are run but include inflation and high inflation together whenever inflation is included. Thus inflation is considered as one variable and the spline is an approximation to the non-linear relationship on growth. The results are similar to
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
<th>BMA Post. Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>cross-section</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation*</td>
<td>0.1091</td>
<td>0.9861</td>
<td>0.3280</td>
<td>3.5</td>
</tr>
<tr>
<td>High Inflation*</td>
<td>-0.1152</td>
<td>-1.0125</td>
<td>0.3154</td>
<td>3.5</td>
</tr>
<tr>
<td><strong>Panel Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation*</td>
<td>-0.0209</td>
<td>-0.4309</td>
<td>0.6667</td>
<td>97.7</td>
</tr>
<tr>
<td>High Inflation*</td>
<td>-0.0128</td>
<td>-0.2600</td>
<td>0.7950</td>
<td>97.7</td>
</tr>
</tbody>
</table>

Bayesian Model Averaging using cross-sectional and panel data with fixed-effects. Threshold level is 10 percent. * measured as log (1 + inflation).

Table 3.4: Threshold Results.

the previous sections. In the cross-section, inflation is not important at all according to the BMA posterior probability. The posterior probability is 3.5% which is far less than the next variable, population growth with 12.9%. The expected value of the coefficient on inflation is positive, 0.1091 but imprecisely estimated. The expected value of the coefficient for inflation over 10% is negative, -0.1152, and also imprecisely estimated. Recall that the total effect of inflation over 10% is the sum of the two coefficients which is negative, but small, at -0.0061. Note that the individual coefficients are very large compared to the estimates from the results without allowing for the threshold.

For the panel data, inflation still does well with a BMA probability 97.7%. Here both coefficients, inflation and the additional effect for inflation over 10%, are negative. Thus inflation is harmful to growth and the effect is greater when inflation is over 10%. The total effect for inflation over 10% is -0.0337 which is very close to the estimates using panel data and not allowing for the threshold, -0.0334. Figure 5 which
shows the distribution of the coefficients for inflation and the additional effect of high inflation for both methods. As can be seen, the distribution is more precise for the fixed-effects versus the cross-section estimates. Looking at the cross-section results, the distribution is centered further away from zero, but there is also a significant mass to the opposite side of zero. Specifically the mean for low inflation is clearly positive and the additional effect for higher inflation is clearly negative. Again the cross-sectional data does not indicate any strong effect of inflation on growth. Not only is there high variance in these point estimates, but inflation does not help specific models perform well, as indicated by the low BMA probability. Looking at the panel data, both distributions have more probability on negative estimates. For the coefficient on low inflation the distribution is still concentrated around zero, but the high inflation coefficient distribution is almost entirely below zero. Thus the panel data supports a negative relationship between inflation and growth at least at levels of inflation above 10%.

3.3.4 Interpreting the Evidence

Now that the empirical results have been presented, what do they mean? One of the purposes of using panel data is to increase the number of observations and thus get more precise estimates of the parameters of interest. In the sections above there are 60 observations using cross-sectional data and 428 using the panel data thus it is not surprising that the panel data produces more precise estimates. However the change in the results with regard to inflation are so great that it is hardly likely to be due simply to the increase in sample size. Another argument made above is that there is more variation in inflation and growth rates in the panel data, and this
Figure 3.5: Distribution of Inflation Coefficients with Threshold. Coefficient on Inflation represent marginal effect on inflation up to 10 percent. Coefficient on High Inflation represents additional effect for inflation over 10%.
would improve the estimates, but it is hard to imagine this is the cause of the large differences. Trying to find the true cause, if possible, is left for future research but there is the obvious candidate of reverse causation. If aggregate supply shocks are driving inflation and output in opposite directions then the direction of causation is reversed and the regressions above simply are detecting supply shocks. Karras (1993) demonstrates that this can explain some of the inflation-growth results in the literature. One way to try to distinguish this is through instrumental variables estimation. However the instruments that have most often used seem flawed. In particular, lagged inflation has been used in panel data, but this is probably not a good instrument (Kocherlakota (1996)). In unreported results, lagged inflation was used as an instrument and the results were not changed significantly\textsuperscript{13}. Rather the approach taken above was to try to include variables that would proxy for aggregate supply shocks, specifically the terms of trade variable and the time dummies. It is also the case that using 5 year averages should lessen this argument.

Another common belief is that inflationary problems are simply a proxy for other institutional problems. However this possibility is also partially addressed. First, some control variables such as civil liberties and black market premium are included. More importantly, allowing for fixed effects eliminates any unobservable factors that are invariant over time. Thus if the institutional problems are not changing over time, they are captured in the fixed country effect. Of course there can be more complicated relationships. In the study of openness and growth by Sachs and Warner (1995) note that many regime changes towards openness follow an inflationary period.

\textsuperscript{13}Andres and Hernando (1997) use instrumental variables in a study of OECD countries and find the causation runs from inflation to growth.
3.4 Parameter Heterogeneity

The previous section determined how robust the inflation-growth relationship is when using cross-section and panel data, as well as allowing for thresholds. But this ignores the possibility that countries may have significantly different responses to inflation. Brock and Durlauf (2000) point out that if country observations are draws from different distributions, typical statistical inference will be incorrect. The practical importance of parameter heterogeneity in growth has been demonstrated in the literature on convergence clubs and testing exogenous versus endogenous growth theories (Evans (1998)). Sala-i-Martin (2001) argues that it is known how to take into consideration such heterogeneity, for example by interacting a dummy variable with the relevant slope variable and testing whether the difference is significant. However, the problem may be more difficult. It may not be just a dummy variable for Latin America added to the slope coefficient on inflation, but rather the slope coefficient may be different for all countries and indeed all variables.

The idea of heterogeneity and inflation was considered by Khan and Senhadji (2000). After applying their method for threshold econometrics to their full data set, they split the data into developed and developing countries. Again, testing for a threshold effect, they find that inflation decreases growth for developed economies at all levels of inflation, but for developing economies only at a higher rate. But the question remains, what is it about developed and undeveloped that makes the inflation effect different?

This section is unable to answer what the underlying cause is between these distinct effects, however the choice of OECD and non-OECD provides a division of developed and less-developed that provides a reasonable first step. The approach is
similar to the mechanics of BMA: numerous regressions are run with different samples of the data and plots of the distributions of the coefficients are presented. These plots show clearly how different the coefficients may be and demonstrates that this is the case for most of the variables considered. However, this approach does not rely on the theoretical foundations of BMA, so more precise statistics are not presented. For example, the mean and standard deviations of the coefficients for the two subsamples could be calculated, but it is not clear that this would offer more than the plots deliver.

Unlike the case of model uncertainty, there is no obvious choice of possible regressions to consider. It is not possible to run all possible combinations of countries because a minimum sample size is needed to get reasonable estimates. The approach taken is to run samples of the full data set such that a fixed number of observations are available. It is infeasible to run all possible combinations of observations for most sample sizes of interest. For example, running all the possible combinations of 68 countries from the 73 total countries in the panel data would require over 15 million regressions. Instead, a large number of samples of fixed size are drawn randomly from the full data set and regressions run for each draw. The fixed size of the sample is arbitrary, but hopefully is not crucial to the results.

For the cross-section data there are 63 countries of which 41 are non-OECD and 22 are OECD members. Because there are 17 variables to estimate it is not possible to use all of them if we are going to use the OECD data. Thus only 7 variables are used, inflation, initial GDP, capital growth, years open, government share of GDP, lack of civil liberties and average years schooling. Obviously the estimates will be imprecise because of the small sample-size but this will be not be such a problem
when the panel data is used. The procedure is to take 200,000 random draws of the data and run regressions for each draw for the full data set, and then repeat this for the OECD and non-OECD subsets. The arbitrary sample sizes are 58 for the full data set, 37 for the non-OECD countries and 17 for the OECD subset. Note that there are less than 200,000 combinations for the OECD and non-OECD countries so most of the possibilities have been estimated. For the whole sample there are over 5 million combinations so only a small number of the possibilities are estimated in that case.

The results are presented in Figure 6. There are sub-plots for each variable in the regression, and each sub-plot has three distributions. The distributions are for the point estimates for the coefficient on the variable corresponding to the sub-plot. The three distributions are for the full data set, the OECD countries and the non-OECD countries as indicated. The impact of parameter heterogeneity is emphasized by how the distributions for the OECD and non-OECD differ, especially how far the means of the two distributions are apart. It is also the case that the distribution of one subset may be mostly around zero while for the other subset it is mostly positive or negative, suggesting that any results different from zero are being driven by one subset of the data. The distribution of the coefficients from the full data set do not indicate anything with respect to parameter heterogeneity, but rather demonstrate what coefficients are likely if the subsamples are pooled together. This is the usual case in empirical work although it may not be the intention of the researcher. Since there are more non-OECD countries they dominate the results for the full sample, and this would be intensified as larger data sets are used. Thus it is not clear what growth regressions on a large sample of countries implies for the developed countries.
Looking at the details of Figure 6, inflation is in the upper left sub-plot. While inflation does not seem to be significantly different from zero for the full sample, most of the distribution for the OECD countries is negative. This is surprising if one expects that the inflation-growth results are being driven by less-developed countries. However, that inflation is negatively related to growth in the OECD is supported by Kahn and Senhadji (2000) and Andres and Hernando (1997). Note also that the distribution for the OECD has a large variance, which could be due to both the small sample size and the relative lack of variation in inflation and growth rates among the OECD countries.

The distribution of the coefficients for the OECD countries lies closer to zero for years open, civil liberties, average years schooling and capital growth. In fact, there is no evidence that years open, or average years schooling contribute significantly to growth in the OECD countries by looking at these figures. Government share of GDP is very imprecisely estimated for the OECD countries, the distribution barely visible, although many of the point estimates are positive. The coefficient on initial GDP is more negative for the OECD countries supporting evidence that convergence is often found for OECD countries but not larger data sets.

The procedure above is repeated for the panel data set. Since there are now more observations, the regressions are run with all the variables that were included in section 3. For the panel data there are 73 countries, 52 non-OECD, and 21 OECD. Again regressions are run on each of 200,000 draws from the such that there are 68 observations from the full sample, 48 that are non-OECD and 18 OECD members. The results are presented in Figure 7. The negative effect of inflation is still coming strongly from the OECD countries, and again the distribution for the OECD countries
Figure 3.6: Parameter Heterogeneity Using Cross Sectional Data. Plots are distributions of coefficients over random samples of the cross-sectional data for the full sample, OECD and non-OECD countries.
Figure 3.7: Parameter Heterogeneity with Panel Data and Fixed Effects. Plots are distributions of coefficients over random samples of the panel-data for the full sample, OECD and non-OECD countries.
is much less precise than the non-OECD. Also, the mean of the distribution for the OECD countries indicates a very large negative affect of inflation.

Looking at the other variables provides some interesting findings. Government share of GDP, growth in terms of trade and average years schooling all indicate coefficients different from zero for the OECD sample but less support when using the non-OECD countries. Also private credit demonstrates the extreme result where the coefficients seem to be positive for the non-OECD countries and negative for the OECD, and when the countries are pooled the coefficient appears to be close to zero.

The main point of this section is that the distribution of some coefficients is quite different for the country sub-samples, supporting the idea that parameter heterogeneity is an important factor. It is the case that the distribution for the OECD and non-OECD are quite far apart, and thus it is clear that policymakers should be weary of drawing inference from results that include a pooled sample of diverse countries. Of course the division into OECD and non-OECD is arbitrary, and research along these lines should focus on the underlying causes driving the parameter heterogeneity. In the case of inflation, it may be financial markets, and this division may proxy well. However, for other issues such as the effects of trade and growth, other divisions may be more suitable. It is also the case that these results are less rigorous than the model averaging methods, suggesting further research into the question of parameter heterogeneity.

3.5 Policy Implications

This section briefly addresses Brock and Durlauf’s (2000) call for policy relevant growth empirics. There are two important points to be stressed. One is that for
growth empirics to be policy relevant it is necessary to investigate variables that are policy relevant. Thus to look at such variables as inflation, government share of GDP, years schooling, etc. These variables present endogeneity problems that may present difficulties to the researcher, but still reveal important information, that may not be available by focusing on strictly exogenous factors. The fact that fraction Confucian is exogenous, does not make it interesting in a policy setting. It also seems that for policy decisions, panel data provides useful information. Looking simply at 35-year time spans rules out many policy relevant variables. It is not surprising that high inflation matters little for relatively long periods. High inflation does not usually persist for 35 years, but this does not lessen the fact that it may be detrimental to growth for 5 or 10 years. It is also the case that economies may adjust to permanent inflation and thus not suffer any permanent growth effects, but again policymakers are probably concerned with the adjustment period.

The second idea from Brock and Durlauf’s paper is to focus on decision theoretic econometrics. How is the policymaker supposed to use empirical results, such as the ones presented in this paper? The answer depends on the loss function, and it may be the case that t-statistics are irrelevant and the point estimate of a coefficient drives policy\textsuperscript{14}. In this situation, accounting for model uncertainty is all the more important. The above analysis has shown that it is likely that inflation is bad for growth, at least over 5 year horizons. However, the skeptic can still point out that for the cross-sectional results or simply looking at t-statistics one cannot reject a zero coefficient. Even though the entire distribution of estimates lies below zero, it is still

\textsuperscript{14}Brock and Durlauf (2000) show that under risk neutrality, i.e. a linear and increasing utility function, the policymaker cares only about the mean of the coefficient. Under a specific mean/variance utility function the policymaker acts only if the t-statistic is greater than 2.
true that a 95% confidence interval might include zero much, or all, of the time. For academics interested in model testing, it is true that the above results may not reject a null hypothesis that inflation does not affect growth. But for policymakers, the evidence may seem convincing of a negative correlation. Whether this is from reverse causation remains to be seen, but a zero or positive relationship seems unlikely.

Further, the evidence on parameter heterogeneity is relevant for policymakers. What is driving the differences in the estimates for OECD and non-OECD countries? For aid organizations, such as the IMF and Worldbank, should there be a one size fits all approach? From the panel data results, figure 7, it seems that inflation is negatively related to growth for both sets of countries. But how much of a problem is parameter heterogeneity within the non-OECD countries? As they have found out with the Asian Financial crises in the late 1990’s, applying general austerity measures fitted to one set of countries may not have the intended effects. This just emphasize the need to study these problems in more detail.

For national economies the problem may be worse. Even in the OECD, what is driving this large negative coefficient on inflation? Does it apply to the US? National policymakers surely have a difficult time drawing inferences from cross country results such as these and rely heavily on prior beliefs. This suggests the need to study precisely what channels inflation is affecting growth, and thus individual countries need to study their own inflationary experiences.

3.6 Conclusion

This paper set out with the goal of testing the robustness of inflation and growth using recent techniques of Bayesian Model Averaging. In particular to extend these
methods to panel data, which might improve the results on inflation. The principle finding is that there is scarce evidence that inflation affects growth over long horizons such as found in 35-year cross-sectional data, even if we allow for non-linearities. On the other hand the evidence indicates that inflation is harmful to growth looking at 5-year periods of panel data allowing for fixed country effects. It also seems that it is inflation over 10% that is driving the negative relationship. The regressions allow for capital growth, private credit, terms of trade shocks and some institutional factors, all of which may be thought to be related to the effects of inflation and growth. That the effects are still so strong suggests that the transmission mechanism of inflation to growth is not well understood. This corroborates previous results that finds inflation affects productivity rather than investment.

Looking at possible heterogeneity problems it is demonstrated that most of the variables in the growth regressions can deliver a wide array of estimates simply based on the sample of countries considered. If researchers are worried about the possibility of data mining with respect to variable choice, the problem is as real in the choice of country observations. The fact that the distributions of the coefficients are quite different for OECD and non-OECD countries demonstrates that parameter heterogeneity could be a significant problem. The methods used are not as rigorous as the Bayesian Model Averaging used in section 3, thus further work needs to be done in accounting for these differences.

The results presented throughout should be useful to policymakers, although they do not suggest that policymakers should infer that inflation is necessarily harmful to their specific country or area of interest. They do suggest that inflation is a robust and important variable in determining long run growth on average for a large data
set. And we can conclude that there is no evidence of a positive relationship between inflation and growth. The serious problem of causality is not completely addressed, but the use of relevant control variables and prior studies suggests this is not driving the results.

It seems that contrary to the claim that inflation is not a robust determinant of growth, there is ample future research to be done. The direction of causality is an obvious start, but there is much more. Looking more rigorously at parameter heterogeneity and the causes of such differences remains particularly interesting. Further extensions of times series investigations into inflation and growth may provide answers to some outstanding questions.
CHAPTER 4

LONG-RUN IMPACTS OF INFLATION ACROSS SECTORS IN A SMALL CROSS SECTION OF COUNTRIES

4.1 Introduction

Much work has been done recently to address the question of whether inflation affects economic growth. Two distinct approaches have been taken in the empirical research. Many articles have been motivated by the work of Barro (1996,1997), which renewed interest in the problem and popularized growth regressions, although some earlier work had addressed the problem in the same manner, such as Kormendi and Meguire (1985) and Fischer (1993). Additional work along these lines attempted to address specific issues such as whether the relationship is non-linear, see Sarel (1996) and Khan and Senhadji (2000, or whether the results are robust, as in Levine and Renelt (1992). The general result from the growth regression approach is that inflation is harmful to long-run growth, with this conclusion often amended with the disclaimer that the results may not be robust or may depend mainly on high inflation countries. There is little evidence of a positive effect using growth regressions. Although these growth regressions can be criticized for their lack of formal structure, they at least suggest that the correlation is negative while leaving questions of causation unsettled.
The other approach is to use time series data and vector autoregressions (VAR), often with some structural identification. These methods utilize annual or quarterly data and may contain information that is omitted from the growth regressions, which usually use averages over long periods of time such as 10 to 30 years. They are also able to address causation by a variety of identifying restrictions, rather than trying to find appropriate instruments, which is difficult in practice. Interestingly, some recent papers using time series find a positive relationship between inflation and long-run output levels. Bullard and Keating (1995) use bivariate VARs including inflation and GDP and long-run restrictions proposed by Blanchard and Quah (1989) on 58 countries and find a positive long-run effect of inflation for several countries, with the effect larger for low-inflation countries such as Germany. They also find that there is no long-run effect in a large number of countries. They do not find a statistically significant negative effect for any country. Rapach (2003) adds the nominal interest rate and estimates a trivariate VAR with the same identifying restrictions. For the 14 countries he investigates, he finds inflation has a positive and significant effect on output for two countries, a positive but statistically insignificant effect for ten countries and a negative but statistically insignificant effect for two countries. King and Watson (1997) consider alternative identifying restrictions and find a positive relationship is plausible with similar long-run restrictions.

This paper uses a similar time series approach to investigate whether inflation affects output levels in separate sectors of the economy in significantly different ways across a small sample of countries. If this is indeed the case, it could suggest why these overall effects of inflation and GDP differ across countries. Other related papers try to better understand the transmission of monetary shocks (changes in the level of money
rather than the growth rate of money) by looking at different industries, Dedola and Lippi (2000), and different states in the US, Carlino and DeFina (1998). Looking at different sectors and seeing if they are affected differently may also determine what types of models are needed to study the inflation output relationship. There are numerous models that predict that certain sectors may have important implications with regard to output growth for the entire economy. The endogenous growth models of the last decade emphasize the importance of sectors that are relatively intensive in research and development. If industries that perform R&D need to hold more cash than other industries, for whatever reason, then they may be more affected by an increase in inflation. This would be important to policy makers, yet would not be discernible by only looking at aggregate GDP. Both old and recent work on financial development and growth suggests that the financial sector is especially important to the long-run growth of countries, and this sector may be particularly affected by the rate of inflation. It is also possible that as in a simple two-sector neoclassical model of growth with money the capital intensity of the consumption good is different than for the investment good and inflation may be good or bad for growth, see Huo (1997). Thus how inflation affects these different sectors matters. These arguments are meant to be suggestive that investigating how inflation affects the sectors of the economy may have important implications. Unfortunately, because of data limitations, none of these models can be tested in this paper. Although sector level output data is available, there is no additional information on the capital intensities of these sectors or their reliance on R&D, etc. On the other hand, if inflation shocks do impact sectors differently and in interesting ways then this suggests
that one sector growth models probably will not produce representative and reliable results, and will legitimize further research using disaggregated data.

The paper proceeds as follows. The next section describes the methodology for investigating whether inflation has a long-run effect on output using vector autoregressions. This involves the role of unit roots in the data and also the identification of structural shocks that can be given the interpretation of exogenous monetary shocks. The third section describes the results, beginning with a description of the data, then proceeding to the details of the unit root tests and presentation of impulse response functions and long-run derivatives. The conclusion attempts to put the results in perspective with previous findings and the possibilities of further research concentrating on sectoral differences.

4.2 Methodology

There are two methodological issues in this paper. The first is the framework for evaluating whether inflation has an effect on the long-run growth of an economy. This follows Fisher and Seater (1993) and is concerned with the order of integration of the data. The second is the identifying restrictions in the VAR, which follows Blanchard and Quah (1989) and imposes long-run restrictions rather than the contemporaneous restrictions used in much of the research on monetary policy. This section lays out the essentials needed for the subsequent analysis, whereas the reader interested in further details is referred to the original sources.

4.2.1 Unit Roots

Fisher and Seater (1993) lay out a framework for studying questions of whether changes in the level of money have long-run effects on economic variables of interest
such as GDP and interest rates (long-run neutrality (LRN)), and similarly whether
changes in the growth rate of money have long-run effects on similar variables (long-
run superneutrality (LRSN)). This section will only discuss the implications for LRSN
and real output variables, whereas Fisher and Seater (1993) show the implications for
any real or nominal variable in general. Additionally, although LRSN is concerned
with the rate of growth of the money supply this paper will consider inflation with the
assumption that in the long-run inflation is driven by money growth. The key insight
is that since the question of LRSN is concerned with whether a permanent shock to
inflation has a permanent effect on output, a permanent shock must be present in
both series in order to study the long-run effect. A permanent shock to inflation and
none to output indicates a zero effect. The presence of a permanent shock implies the
existence of an unit root. This can be seen by considering a univariate AR(1) process

\[ x_t = \rho x_{t-1} + \varepsilon_t \]  

(4.1)

where the constant is omitted without loss of generality. Substituting in the similar
equation for \( x_{t-1} \) as a function of \( \varepsilon_{t-1} \) and continuing indefinitely we arrive at the
moving average representation

\[ x_t = \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i}. \]  

(4.2)

If the parameter \( \rho \) is less than 1 then the shocks die out over time and any shock
will have only a temporary effect. In this case \( x_t \) is stationary, and after any shock
the variable will tend to revert to its mean. On the other hand, if \( \rho \) is exactly one
then shocks do not die out and any shock \( \varepsilon_t \) will permanently affect the level of \( x_{t+j} \),
\( j \geq 0 \). In this case \( x_t \) has a unit root and is integrated of order 1. This means that
while \( x_t \) is non-stationary and the first difference, \( \Delta x_t \), is stationary. Thus the critical
question is whether there are permanent shocks present in the inflation and output series. If there are permanent shocks in the inflation series, then it is informative to ask whether these have a long-run effect on output. If the output series does not have any permanent component, then the answer is immediate that there is no long-run effect. If output does have a permanent component, then further analysis needs to be done to examine whether the permanent shock to inflation is related to the permanent shock to output. It is also important to note that if inflation does not have any permanent shocks, then the data is uninformative on the question. It may be that at a permanent inflation shock would permanently effect output, but none have occurred.

The first step, then, is to test all the series for unit roots. If the country’s inflation has no permanent shocks then we cannot investigate the effects for this country any further. If the country’s inflation has a permanent shock, then we can examine how this affects output in the various sectors of the economy. The next step is to look at whether each individual output series (measured in logs) has a permanent shock. If the sector does not have a unit root then we conclude that inflation does not have any permanent effect on this sector. Lastly, if the sector does have a unit root then proceed to examine how the permanent inflation shock affects output. This is performed by analyzing the impulse response function and long-run derivative from a VAR. The long-run derivative of output with respect to inflation is the effect a shock to inflation has on output at time approaches infinity, divided by the effect the inflationary shock has on the level of inflation itself as time goes to infinity. This is
stated as
\[ LRD_{y,\pi} \equiv \lim_{k \to \infty} \frac{\partial \pi_{t+k}}{\partial \pi_{t}} \frac{\partial y_{t+k}}{\partial \pi_{t+k}} \]  
where \( \pi_{t} \) is an inflationary shock at time \( t \). Note that other LRD’s can be calculated, such as \( LRD_{\pi,y} \), but here we are only interested in the long-run effect of inflation on output.

### 4.2.2 Structural Identification

Although the Fisher and Seater (1993) framework considers a general ARIMA model, recent work attempts to make identifying restrictions to pin down structural monetary shocks. In general to identify monetary shocks there are various restrictions that can be made. This paper only considers annual data and is concerned with long-run effects of inflation, thus the long-run restrictions proposed by Blanchard and Quah (1989) are particularly well suited.\(^{15}\) Consider a bivariate VAR that contains inflation and output at the sector level, and thus two unique shocks. The long-run restriction is that shocks to the output series do not have a long-run effect on the inflation rate, although they may have a permanent effect on the output series itself. The shocks to inflation are not restricted and can have temporary or permanent effects on inflation and the output series. This is consistent with generally held beliefs that monetary shocks can have permanent effects on inflation whereas output shocks, such as oil or technology shocks, only have temporary effects on inflation. Once these structural inflation shocks are identified the structural model is estimated and items of interest such as the impulse response function and/or long-run multipliers are calculated.

\(^{15}\) Other restrictions often concern contemporaneous restrictions, such as one variable only responding to the other with a lag. This could be the case if policy makers only have knowledge of output growth with a lag, or output only responds to policy after a lag. These restrictions make sense when using monthly data or maybe even quarterly data, but for annual data they are implausible.
To see this consider the bivariate structural model, with the constant omitted,

$$A \Delta x_t = B(L) \Delta x_{t-1} + \varepsilon_t$$  \hspace{1cm} (4.4)

where $\Delta x_t$ is the first difference of the variables $x_t$, which have a unit root, and $B(L)$ is a polynomial in the lag operator. Specifically, throughout the remainder of the paper $x_t = (\pi_t, y_t)^T$ where $\pi_t$ is the inflation rate for a country and $y_t$ is the natural log of output for a specific sector. The structural shocks $\varepsilon_t = (\varepsilon_t^\pi, \varepsilon_t^y)^T$ are assumed to be uncorrelated and normalized to have variance 1, making the variance-covariance matrix the identity matrix, $I$. This structural model is solved for its reduced form as

$$\Delta x_t = C(L) \Delta x_{t-1} + e_t$$  \hspace{1cm} (4.5)

where $C(L) = A^{-1} B(L)$ and the reduced form shock is $e_t = A^{-1} \varepsilon_t$, with variance covariance matrix

$$\Sigma_e = A^{-T} \Sigma_\varepsilon A^{-1}. \hspace{1cm} (4.6)$$

The reduced form model provides observable estimates of $C(L)$, $e_t$ and $\Sigma_e$. However, the parameters of interest are those from the structural model and are unobserved. Note, from the linear relationships, that if $A$ can be identified then the structural parameters and structural shocks, $\varepsilon_t$, can be identified.

The problem of identification stems from the fact that the reduced form regression gives us an estimate of $\Sigma_e$ which is symmetric and provides us three unique elements, whereas the transformation $A^{-T} \Sigma_\varepsilon A^{-1}$ has four unknown elements. As mentioned above, one way to identify $A$ is to make an assumption regarding the contemporaneous effect of one of the variables on the other, which would impose a zero restriction on one of the off-diagonal elements of $A$. Then one could compute the Cholesky
decomposition of $A^{-T} \Sigma \varepsilon A^{-1}$. An alternative is to impose a long-run restriction. Consider the moving average representation of (4.5)

$$\Delta x_t = [I - C(L)I]^{-1} \varepsilon_t = [I - C(L)I]^{-1} A^{-1} \varepsilon_t. \quad (4.7)$$

If there is a shock to the system at time $t$, then the long-run effect of the shock is

$$\lim_{k \to \infty} x_{t+k} = [I - C(1)]^{-1} A^{-1} \varepsilon_t = D \varepsilon_t, \quad (4.8)$$

where $D = [I - C(1)]^{-1} A^{-1}$ is a matrix containing the elements representing the long-run multipliers of the structural shocks on the respective variables\(^{16}\). The structural matrix $A$ can be solved as a function of $C(1)$ and $D$, and after substituting into (4.6) and rearranging produces

$$[I - C(1)]^{-1} \Sigma \varepsilon [I - C(1)]^{-T} = D \Sigma \varepsilon D^T = DD^T. \quad (4.9)$$

This equation also relates the reduced form model to the structural parameters, where the structural parameters are in terms of $D$ rather than $A$. The left-hand-side (LHS) is symmetric and again provides three unique elements from the estimated VAR. The right-hand-side (RHS) contains four unknowns and some restriction needs to be imposed to identify the structural model. In this case, a zero restriction on one of the off diagonal elements of $D$ implies that a particular shock does not have a long-run effect on the other variable. The long-run effects of the shocks can be seen as

$$\lim_{k \to \infty} \begin{pmatrix} \pi_{t+k} \\ y_{t+k} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \begin{pmatrix} \varepsilon^\pi_t \\ \varepsilon^y_t \end{pmatrix} \quad (4.10)$$

where, for instance, $d_{11}$ measures the long-run effect of an inflation shock, $\varepsilon^\pi_t$, on inflation and $d_{12}$ measures the long-run effect of an output shock, $\varepsilon^y_t$, on inflation.

\(^{16}\)If the lag polynomial is $C(L) = C_1 L + C_2 L^2 + C_3 L^3$, then the long-run effect depends on $C(1) = C_1 + C_2 + C_3$. For a slightly different, element by element, derivation of the Blanchard and Quah (1989) restrictions see Enders (1995).
The identifying restriction $d_{12} = 0$ implies that output shocks have no long-run effect on the level of inflation. With this zero restriction there are now only three unknowns on the RHS of (4.9) and since $D$ is lower diagonal it can be found by the Cholesky Decomposition of the LHS, which is estimated from the reduced form VAR. With this estimate of $D$ and the estimates of $C(1)$, the matrix $A$ can be calculated and the structural disturbances $\varepsilon_t$ derived from the reduced form errors $\epsilon_t$. The long-run derivative of primary interest in this case is $LRD_{y,\pi} = \frac{d\varphi}{d_{11}}$.

The reduced from errors $\epsilon_t$ are not economically meaningful, although the reduced from VAR may be good at forecasting. The structural errors can be given economic interpretations, which are meaningful only as far as the identifying restrictions are appropriate. The restriction imposed here is that inflation shocks have permanent effects on the level of inflation and may or may not have permanent effects on levels of output. This suggests the “monetarist” interpretation that these are monetary shocks, in the sense of Friedman’s dictum that “inflation is anywhere and everywhere a monetary phenomenon”. The output shocks only have a temporary, or short-run, effect on the level of inflation, although they may permanently affect the price level. These output shocks may have a permanent or temporary effect on the level of output. Thus these shocks can be given the economic interpretation of aggregate supply or preference shocks.

4.3 Results

This section of the paper contains all the results of the analysis, using the methods discussed in the previous section. The first subsection describes the data. The second subsection explains the details of the unit root tests performed and the results of these
tests on the inflation and output series. Once the order of integration is determined, those series that are informative toward the question of LRSN are investigated using the structural VAR. The third subsection estimates the impulse response functions of a permanent shock to inflation on the level of output over a 10-year horizon. Because the impulse response function is necessarily limited to a finite horizon the long-run derivative is calculated in the fourth subsection.

4.3.1 Data

The question of this paper is how permanent inflation shocks permanently affect levels of output in different sectors of the economy, if such effects are present. As mentioned in the introduction, there are several theoretical models that suggest that these differences may be important. However, the data available are not detailed enough to test these specific theories, and the hope is that the results indicate whether these types of differences are important empirically. If so, then the question of what theories are consistent with the data can be explored further in future research.

The output data comes from the Groningen Growth & Development Centre’s web site, which has gathered the data from various issues of the OECD national accounts. The series are Agriculture, Mining, Manufacturing, Construction, Public Utilities, Retail and Wholesale Trade, Transport and Communication, Finance and Business Services, Community and Public Services, and Government Services. Total GDP is also included so that the results across sectors can be contrasted to what the result would be if we only looked at aggregate output, as in earlier papers. This paper uses data for seven countries that have reliable data and are among of the larger industrialized countries: the United States, Japan, France, Germany, Italy,
Spain and the United Kingdom. The inflation series comes from the International Monetary Funds International Financial Statistics database. The years covered are 1950-1996 for most countries with the following exceptions: Japan begins in 1953, Italy begins in 1951 while Spain ends in 1994.

4.3.2 Unit Root Analysis

As discussed, the presence or absence of a unit root is critical in assessing long-run superneutrality. This paper follows a relatively simple approach to investigating the existence of a unit root. All tests are standard Dickey-Fuller (DF) or Augmented Dickey-Fuller (ADF) tests, which take the following form:

\[ \Delta x_t = \mu + \rho x_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta x_{t-i} + \beta t + \varepsilon_t \quad (4.11) \]

where the series \( x_t \) is univariate, e.g. it is either the inflation rate or one of the output series. The null hypothesis is a unit root in which case the parameter \( \rho \) is zero. The test statistic, \( \tau \), is the \( t \)-value on \( \hat{\rho} \), but this statistic is not distributed student’s-\( t \). The distribution was computed by Dickey and Fuller (1979). The test is “augmented” if there are additional lagged values of \( \Delta x_t \) included in the regression. This will be the case if there is additional autocorrelation, which if not accounted for would lead to the residuals exhibiting serial correlation. Serial correlation in the residuals would invalidate the \( \tau \)-statistics. Further, the whether the parameters \( \alpha \) and \( \beta \) are included or not affect the distribution of \( \tau \), unlike in the case of stationary variables. The constant term \( \mu \) represents drift if the process is a unit root, whereas it is a deterministic constant if the process is stationary. If the series is known to be trended then the appropriate question is whether the process is a unit root with drift or a stationary process with deterministic time trend, thus the trend term, \( \beta \cdot t \), needs
to be included to nest the alternative hypothesis in the test. The test statistic when the linear time trend is included is denoted \( \tau_\beta \), and when only a constant is included it is \( \tau_\mu \).

The first step is to determine if additional lags are necessary in order for the residuals to be free of serial correlation. Four additional lags of \( \Delta x_t \) are included in the ADF regression and serial correlation in the residuals is tested for at lags one through four using Box-Ljung Q-statistics at the 5% significance level. If there is no evidence of serial correlation with 4 lags, the ADF regression is run with only 3 lags and serial correlation is tested for in the same manner. If there is evidence of serial correlation, the process ends and 4 lags are included, which did not exhibit serial correlation. If there is no serial correlation at 3 lags, proceed with only 2 lags and keep 3 lags if there is now serial correlation, and so on. The number of lags needed in each ADF regression to remove serial correlation are included in table 1.

Once the lag length is determined, the appropriate DF or ADF test is performed and the estimated test statistics compared to the critical values at the 5% significance level. For the output series the null hypothesis is that output follows a random walk with drift. The appropriate alternative hypothesis is that there is a deterministic trend and therefore a linear time trend is included. If the series fails to reject the null then that variable is considered to have a unit root and will be included further in the analysis of inflation and growth. If the null is rejected, I look at the coefficient on the time trend. If the deterministic time trend is significant then the alternative hypothesis that the variable is stationary is supported, whereas if it is not statistically significant this suggests that the alternative hypothesis is incorrect and the test is inappropriate. If the trend is not significant, I proceed to test the null of a unit
root without drift against the alternative of a stationary process with a deterministic mean. If the unit root without drift is not rejected, the series is included in the VAR analysis.\textsuperscript{17} If the series is found to be stationary then there are no permanent shocks and it cannot be the case that permanent inflation shocks, if they exist, permanently effect output.

For the output series there are three series where a unit root with drift is rejected: Agriculture in Japan and Agriculture and Finance in Germany. These are indicated by the asterisk next to the estimates of the $\tau_\beta$ statistic. For the Agriculture series in Germany, the coefficient on the time trend is statistically significant and thus the alternative hypothesis that the data generating process is stationary with a deterministic trend is preferred. The coefficient on the time trend is not significant for the Agriculture series in Japan and the Finance sector in Germany, thus this does not support the alternative hypothesis of a deterministic trend. The null of a unit root without drift is tested and also rejected in favor of a stationary process with deterministic mean, indicated by the asterisks on the $t_\mu$ statistic. It can be concluded without further analysis that permanent inflation shocks in Japan and Germany do not permanently affect the level of output in these three output series.

For the inflation series there are no obvious trends so the test is the null of a unit root without drift against the alternative of a stationary process with deterministic mean. The statistics from the ADF test reject the null of a unit root only for France’s

\textsuperscript{17} The appendix addresses the issue of testing the output series for a unit root without drift, even if the null of a unit root with drift is not rejected. Also the results if a 10\% significance level are used rather than 5\% is discussed in the appendix.
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Results for Inflation

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* indicates rejection at 5% confidence level, critical values are -3.5449 for $\tau_9$ and -2.9493 for $\tau_5$.

Table 4.1: Dickey-Fuller Tests for Null of a Unit Root in Inflation and Output Series.
inflation series. The sections that follow proceed to see whether a permanent inflation shock affects output in those sectors that do exhibit a unit root in countries other than France. France is uninformative to this question since there seems to have been no permanent shocks to inflation, so no further analysis is performed using the French data.

4.3.3 Impulse Response Functions

According to the unit root tests, the six countries other than France have incurred permanent inflation shocks and only three sectors seem not to have incurred permanent shocks, Agriculture in Japan and Agriculture and Finance in Germany. Using the Blanchard and Quah (1989) long-run restriction, the structural VAR is estimated and the impulse response functions (IRFs) are computed. Figures 1 through 6 show the IRFs for these 6 countries, where each subplot represents an individual sector and the last subplot shows total GDP. Only the effect of inflation shocks on the sector outputs are shown, and 90% confidence bands are computed using the bootstrap with 1000 repetitions, see Runkle (1987). The missing plots for Japan and Germany represent the output series that rejected the unit root and the impulse response function does not need to be estimated to conclude inflation does not effect these sectors.

As in Bullard and Keating (1995) and Rapach (2003) almost all of the point estimates for the impulse response functions indicate a positive effect of inflation on output, although some of these are statistically insignificant according to the confidence bands. This contrasts sharply with the results from most growth regressions, which tend to find a negative relationship and seldom find a positive relationship.

I also check for a unit root with drift against a deterministic time trend just in case, and this also does not reject the null of a unit root except for France.
Evidence that inflation has a permanent negative effect is weak using this structural identification. Those series that suggest a negative relationship, i.e. those where the IRF is below zero for a significant portion of the 10 year horizon\textsuperscript{19}, are: Agriculture, Construction and Wholesale/Retail trade in the US, Mining in Japan, Mining and Community/Personal Services in Italy, and Agriculture, Mining and Construction in the United Kingdom. Also the IRF for the total GDP series for the United Kingdom lies below zero, but only slightly. Most of these have wide confidence bands and thus do not reject a zero long-run effect, the exceptions being Agriculture in the UK, where the IRF is significantly different from zero, and Mining in the UK, where the IRF is “nearly” significant. Note also that Agriculture in Japan and Germany were two of the three series that rejected a unit root and therefore were not affected by permanent inflation shocks. Thus it does seem possible that in many countries Agriculture and possibly Mining are either hurt by inflation, or at least may be affected differently than other sectors.

Some previous work that has focused on the level of money and output has found that the results are much more precise when using Industrial Production data rather than GDP. Assuming that the Manufacturing data is similar, it is interesting to note that the IRF for the manufacturing sector is positive in all countries and either significant or “nearly” significant in all countries except the United Kingdom. Thus, it does seem that the effects of inflation on manufacturing output do provide more

\textsuperscript{19}Here the intuition is whether output is still affected by inflation at the 10 year horizon, and also if the IRF lies sometimes above and sometimes below what is the overall effect at this horizon. The next subsection that calculates the long-run derivatives will be able to precisely show whether the long-run effects are positive, negative and statistically significant or not.
Figure 4.1: Impulse Response Functions - Effect of Inflationary Shock on Output Series - United States.
Figure 4.2: Impulse Response Functions - Effect of Inflationary Shock on Output Series - Japan.
Figure 4.3: Impulse Response Functions - Effect of Inflationary Shock on Output Series - Germany.
Figure 4.4: Impulse Response Functions - Effect of Inflationary Shock on Output Series - Italy.
Figure 4.5: Impulse Response Functions - Effect of Inflationary Shock on Output Series - Spain.
Figure 4.6: Impulse Response Functions - Effect of Inflationary Shock on Output Series - United Kingdom.
precise estimates, suggesting future work investigating the details of the inflation-output relationship may be better off focusing on manufacturing where the results are statistically significant.

Another sector that may be particularly affected by inflation is Finance. Inflation is a tax on holding money, and thus will affect the allocation of assets between money and other financial instruments. The Mundell-Tobin effect emphasizes the shift from money to more productive assets. Other theories, such as cash-in-advance models, suggest a positive relationship between money holdings and investment; thus higher inflation leads to a higher cost of investment goods. It may also be the case that higher inflation leads to more frequent transactions; thus whether higher inflation leads to higher or lower output is uncertain. These models focus on the end result of inflation on output, but do not model the finance sector itself, so it is not clear whether inflation is increasing or decreasing the size of the financial sector. We have already determined that Finance is not permanently affected by inflation in Germany, but what of the remaining countries? The IRF is positive and significant in the US, Japan, and Italy, while it is positive but insignificant in Spain and the United Kingdom. This suggests that the effect of inflation on the financial sector is not an homogenous result across countries, and pooling countries together to investigate such effects may be misleading. To emphasize this, not that while Italy has only four of ten sectors that exhibit a statistically significant effect from inflation, one of them is finance, whereas Spain has seven sectors that have statistically significant IRFs, two sectors that are near significant at the 90% level, and only finance is clearly not significantly different from zero.
Considering countries as a whole it is interesting that Bullard and Keating (1995) emphasized the large and significant positive effect of inflation on German GDP. Every sector of the German economy, except the two eliminated because they did not contain a unit root, have positive and significant effects from long-run inflationary shocks. On the other hand, the United Kingdom has a negative point estimate for GDP, although clearly not statistically significant, and has very few sectors with statistically significant IRF. Only Agriculture, which is negative, and Government Services, which is positive, are significant.

4.3.4 Long-Run Derivatives

The long-run derivatives are computed from the structural VAR estimates and 90% confidence bounds are calculated using the bootstrap method with 1000 repetitions. Note these are the long-run changes in output due to an inflation shock divided by the long-run change in inflation due to an inflation shock. The long-run effect on output is suggested by many of the impulse response functions that seem to asymptote to a level different than zero within the 10 year horizon considered. The denominator is the long-run effect of an inflationary shock on inflation. Although these other IRF’s are not shown, the long-run effect is close to one in most cases. The long-run derivatives formalize our intuitive arguments of what the IRFs appeared to do as time extended past the 10 year horizon plotted.

The LRD on GDP is positive and significant in every country except the United Kingdom. Also, it is clear that all the sectors in Germany that are affected by inflation have a positive and significant relationship, whereas all the sectors in the UK are either not significantly affected or the affect is numerically small.
Figure 4.7: Long-Run Derivatives - Unites States, Japan and Germany.
Figure 4.8: Long-Run Derivatives - Italy, Spain and United Kingdom.
4.4 Conclusion

This paper investigates whether different sectors of the economy are affected in significantly different ways by permanent shocks to inflation, and, if so, are there any regularities that reveal interesting implications for theoretical models of inflation and growth. Also, do the results across sectors support the findings using GDP or are there important distributional effects that would matter for policy makers. To answer the first question, different sectors are clearly affected to a different degree by inflation with no particular pattern across countries. For example, in the US the Transportation and Communication sector has the largest point estimate and is statistically significantly affected by permanent inflation, while in Germany it also is positive and significant but has the smallest point estimate of the sectors in the German economy. More interestingly while the majority of sectors are positively affected by inflation, there are some sectors in the same economies that may be hurt by inflation, such as Agriculture and Mining although the statistical significance is weak. This supports arguments that in practice there are often winners and losers from higher inflation and often the conflict is between manufacturing and agricultural interests.

There are few regularities across countries, although the consistency of inflation and Manufacturing output is interesting. Another item of interest, is the actual lack of regularity of inflation and the Finance sector. The relationship is permanent and significant for the US and Japan, insignificant for Italy, Spain, and the UK and there is no permanent effect on Germany’s Finance sector.

It would clearly be interesting to test some specific theoretical implications, but the data limits the complexity of the questions that can be dealt with. It would
be interesting to extend the analysis beyond the bivariate regressions, but with only annual data the additional parameters exhaust the degrees of freedom rapidly. It would also be interesting to look at more details of the sectors such as the level of finance, the average size of firms, etc. but the aggregate level of the sectors prevents looking at more intricate details of the sectors.

The results do, however, confirm previous studies finding a positive relationship when similar long-run restrictions are assumed. They also suggest that there are interesting differences across sectors that would be useful to policymakers with regard to who is most hurt or helped by inflation.
CHAPTER 5

CONCLUSION

Does inflation affect long-run economic growth? Tests using growth regressions suggest that inflation is indeed harmful to long-run growth, although this is robust only in the case of panel data and not cross-sectional data. On the other hand, the relationship seems to be reversed, not only for aggregate GDP, but for most of the individual sectors of the economy when using structural vector autoregressions where long-run restrictions are imposed. Thus the answer to whether inflation matters seems to be yes, however the direction of the effect and the channels through which inflation works is uncertain. This is important since there is a large literature that tries to dismiss such monetary causes and focus only on real factors. While the results don’t offer a clear direction and causal mechanism to be useful for policy-makers, it is clear that future research on inflation and growth is important to better understanding long-run growth.

Accepting the evidence that inflation has significant effects on output, the previous essays offer some interesting results and suggest several directions for future research. Inflation is found to have a robust negative relationship using panel data in growth regressions. This can be extended to trying to determine the channels through which inflation is operating. It is often the case that inflation is found to be detrimental
to productivity rather than investment, even though investment is the channel most often analyzed in theoretical models. Are the effects of inflation on investment and productivity robust? Is has also recently been emphasized that inflation may work through effects on the financial sector. Are these results robust?

The time series evidence does not indicate a significant negative effect from inflation to growth, in fact the bulk of the findings are in favor of a positive effect. These results are dependent on the identification scheme, and even though it has been argued that the long-run restrictions used seem appropriate, investigating alternative identification methods is one of the more pressing needs for future research. Further research can also investigate the specific channels that inflation may be working through. Rapach (2003) analyzes the failure of the Fisher equation to hold and the subsequent real interest rate effect. This can be applied to the sectoral data, since it is probably the case that some sectors are more interest sensitive than others. Also trying to analyze the finance sector in more detail would help test some of the most recent theories of inflation and financial sector activity.

It is also clear that there are large differences in the effects that inflation has across countries. This opens the door for research into the best way to handle these differences, such as simply allowing for differences in the slope coefficients as suggested by Sala-i-Martin (2001) or some other way. In fact, maybe a re-emphasis on country case studies is needed. From a policy standpoint, a good analysis of the country or region is question is probably worth more than many studies using growth regressions or time series studies pooling several diverse countries together.
In any case, there is plenty of research to be done and it seems unlikely that inflation has only negligible effects on output. It also seems unlikely that policy-makers understand the effects of inflation on output well enough to pursue policies that are optimal.
APPENDIX A

ALTERNATIVE UNIT ROOT RESULTS

The results in chapter 4 rely crucially on the results of the unit root tests. There are two important departures with respect to the unit root tests in that essay and those in other papers, such as Bullard and Keating (1995) and Rapach (2003). The first is the use of several alternative unit root tests and rejecting the null if the series fails any of these tests, and the second is the appropriate significance level to reject the null hypothesis. The first difference makes quite a difference to the conclusions of whether inflation is affecting output, whereas the second has only minor implications.

The approach taken in this essay is that the appropriate test for the output series is the null of a unit root with drift versus the alternative of a stationary process with a deterministic time trend. If the unit root is not rejected, then the analysis proceeds to the VAR results with the differenced data. In Bullard and Keating (1995) and Rapach (2003) they perform multiple unit root tests, i.e. with a drift term and without, and if the unit root is rejected in any of these tests they eliminate it from further analysis. The problem is that the test that most often rejects the null is the unit root without a time trend included, whereas the unit root with drift and the time trend included rejects much less frequently. This goes back to the appropriate null hypothesis and whether it is nested in the test procedure. If I follow the same procedure then I also
find that many series which do not reject the null of a unit root with a time trend included do reject the unit root when only a constant term is included. There are 30 output series that would then reject the unit root and the immediate conclusion is that permanent inflation shocks do not affect the long-run level of these output series, see Table 2. The method in this essay allows more series be included in the VAR and then investigates whether inflation is really having long-run effects. Observing the impulse response function and/or long-run multiplier demonstrates if that is indeed case. If there is not an effect, then had that series been thrown out to begin with, little would have been missed. On the other hand, if output is significantly affected, then this information would have been missed had that series been eliminated because it was incorrectly concluded there was no permanent component. Much of this hinges on unit root tests that are plagued with problems, so I choose to err on the side of including more series.

Obviously, the level of confidence placed in the conclusions will be less if the results are conflicting and the alternative hypothesis that output series are not trending upwards is reasonable. For example, looking at the LRDs for Italy, the evidence that inflation affects growth is not strong. The test statistics in Table 1, where the time trend was included, failed to reject the null for any of the sector outputs. On the other hand, in Table 2, where the trend is not included, the unit root is rejected for six sectors and the immediate conclusion is that inflation has no long-run effect in these series. This doesn’t contradict the weak results from the LRDs. Considering Germany the case is quite different. Only two series rejected a unit root in Table 1, whereas six sectors reject the unit root in Table 2, as well as total GDP. According to the LRDs there are statistically significant effects from inflation to output in these
series and GDP, whereas according to Table 2 the analysis would have stopped and concluded that there was no long-run effect.

Regarding the second difference, the unit root tests in this paper were evaluated at the 95% confidence level. In some of the previous papers the tests were done at the 90% level, most likely because of the low power of unit root tests. Again, the bias was to keep as many series as possible in the VAR analysis rather than jumping to the implication that there is no long-run effect. If instead a significance level of 90% were used to reject the null of a unit root, using the $\tau_\beta$ statistic, 7 additional output series would reject a unit root: Wholesale/Retail Trade and Transportation/Communication in the US, total GDP in Germany, Construction in Italy, Government Services in Spain, and Wholesale/Retail Trade as well as total GDP in the UK. Although the individual series do not alter the results much, the fact that total GDP in Germany and the UK reject a unit root is important. In the case of the UK, few sectors had a significant effect nor did GDP so this does not alter the conclusion. On the other hand, all individual sectors that had a unit root in Germany (and still do at a 90% level) had a significant and positive effects as did total GDP. But if GDP does not have a unit root then we immediately conclude that permanent inflation does not have permanent effects on GDP. Thus the conclusion is completely different if we fail to reject the null at the 95% level or if we reject at the 90% level. The true effect is obviously not known, but if German GDP was eliminated immediately then this contradicting result would never have been observed.
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Table A.1: Dickey-Fuller tests for output series without deterministic time trend.
BIBLIOGRAPHY


