Exchange Rate Dynamics in a Continuous-Time Model of Uncovered Interest Parity with Central Bank Intervention

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

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ABSTRACT

Uncovered interest parity (UIP) is a simple model of international asset market equilibrium that continues to form a key building block in many open economy macro models. Econometric analyses, however, show that it is consistently rejected by the data. Regressions of the future home currency depreciation on the current interest differential yield a slope coefficient that is not only less than unity as predicted by UIP but is typically negative—a finding known as the forward premium anomaly.

Recent studies cast doubt on the reliability of the asymptotic inference of these regressions. They claim that autoregressive conditional heteroskedastic (ARCH) effects and nonnormality of the interest differential induce widely dispersed distribution for the OLS slope estimator in these regressions, but do not provide a theory for the basis of the nonstandard features. My thesis provides such a theory and considers whether the continued popularity of UIP is not misplaced by asking whether these and other features in the international financial market data can be consistent with UIP.

In continuous time, UIP is a stochastic differential equation. In the solution of this differential equation, the log of exchange rate is a nonlinear function of the exogenous interest differential. I consider the continuous-time model for the interest differential to take account of many nonstandard features in the data. To investigate the properties of the model and to examine its ability to explain the data I simulate
the model by setting the parameters of the continuous-time model to point estimates obtained by the method of simulated moments. I then discretize the observations to conform to the sampling intervals of the data and evaluate the ability of the model to match the moments of the exchange rate and interest differential data. A separate issue of interest is that the models provide a theory for the basis of ARCH effects in exchange rate returns.

Two alternative specifications of agent’s knowledge and beliefs of central bank intervention are considered in the simulation experiments. In the first specification, UIP holds continuously and market participants know and understand the intervention rule. In the second specification, intervention takes market participants by surprise and UIP is violated, but only at the instants of intervention. The model is capable of matching most of the moments of the data, such as the volatility and correlation function of exchange rate returns and the interest differentials. The implied distribution of the OLS slope estimator in regressions of the future depreciation on the current interest differential lies far away from the asymptotic distribution but this small sample distortion is not able to explain the forward premium anomaly. The model is able to explain the forward premium anomaly when interventions take the market by surprise.
To my parents
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CHAPTER 1

INTRODUCTION

Uncovered interest parity (UIP) is a simple model of international asset market equilibrium that continues to form a key building block in many open economy macro models. Econometric analyses, however, show that it is consistently rejected by the data. Regressions of the future depreciation on the current interest differential yield a slope coefficient that is not only less than unity but is typically negative – a finding known as the forward premium anomaly.

Recent studies suggest that autoregressive conditional heteroskedastic (ARCH) effects and nonnormality of the interest differential have led to sufficiently sizable small sample distortions of the distribution for the OLS slope estimator in these regressions to resolve the anomaly. The explanation for the anomaly is that it is a statistical artifact. UIP is actually true but the econometrics is flawed. Baillie and Bollerslev (2000) trace the problem to persistent conditional volatility in the interest differential whereas Schotman et al. (1997) point to the extreme-valued forecast errors that they call “big news.” Although these approaches are successful in explaining the forward premium anomaly as a statistical artifact of the data that induces fat-tailed small sample OLS distribution, they do not provide a theory for the basis of the nonstandard features. This thesis provides such a theory and considers whether the
continued popularity of UIP is not misplaced by asking whether these and other features in the international financial market data can be consistent with UIP.

In continuous-time, UIP is a stochastic differential equation and in its solution the log of exchange rate is a nonlinear function of the exogenous interest differential. The data, of course, are not available continuously so in order to assess the ability of the model to account for the empirical regularities of the international financial data, I need to discretize the continuous-time solution to conform to the sampling intervals of the data. The resulting aggregation between two points in time creates in the implied ex post discrete time deviations from UIP, a dependence on the lagged interest differential. This dependence forms the basis of the time-varying conditional variance which also provides a rationale for the “big news” representation of Schotman et al. (1997) in which the error in the regression of the exchange rate return on the forward premium contains both additive and multiplicative terms.

Although the model produces ARCH effects when UIP is constrained to hold continuously, including instants of central bank intervention, the statistical distortions that the model introduces are not sufficient to explain the forward premium anomaly. In this complete UIP world, market participants are fully rational and have full information about the central bank’s intervention rule. This may be a bit of a stretch in describing the market for the US dollar and other OECD country currencies over the post Bretton Woods period since the Fed does not announced exchange rate targets and intervention plans are ostensibly formulated in secret and conducted irregularly. To the extent that they exist, intervention bands are at best informal, and a reasonable alternative specification of the model allows for some degree of market participant ignorance of central bank intervention policy. In this alternative environment, UIP
holds at all times except during these brief episodes of foreign exchange intervention. The explanation of the forward premium anomaly that results is straightforward. Interventions create instantaneous but unexpected shifts in the stochastic process that governs the interest differential. The failure to correctly anticipate these sporadic shifts result in occasional UIP violations. OLS is adept at detecting these violations in samples formed by mixtures of observations drawn mostly from an urn where UIP holds and occasionally from the urn where it does not.

The remainder of the thesis is organized as follows. The next chapter presents literature survey on the forward premium anomaly. Chapter 3 presents the empirical regularities of exchange rates and interest differentials for which we seek to understand. They are i) weekly interest differentials and exchange rate returns possess ARCH effects but monthly returns do not, ii) both the level of the exchange rate and the interest differential are highly persistent, iii) the interest differential and the exchange rate exhibit discrete changes, iv) the exchange rate returns are fat-tailed, and v) the future exchange rate return is negatively correlated with the current interest differential.

In Chapter 4, I model the interest differential as a mean-reverting process. The mean-reverting model is less easily motivated on economic grounds, but it can be argued that it provides a reasonable statistical representation for the data since it can be made to be arbitrarily persistent while maintaining bounded variance. The mean-reverting model provides further insight into the idea of statistical artifact of small sample approach.

In Chapter 5, I build a simple continuous-time model of exchange market equilibrium based on UIP to understand both the forward premium anomaly and endogenous
generation of ARCH effects. To account for occasional central bank intervention in the foreign exchange market, the interest differential evolves according to a regulated Brownian motion process which is subject to Krugman (1992) styled marginal interventions.

In Chapter 6, I model the interest differential as a regulated jump-diffusion process to take account of some nonstandard features of the interest differential. The regulation of the process is again motivated by occasional foreign exchange intervention by central banks. The jump portion is also empirically motivated. Several studies report that the jump behavior of the exchange rate and the interest differential is important empirical feature of the data. Even though the regulated Brownian motion interest differential process shows potential, it is too “well behaved” in a sense that it did not produce sufficient nonnormality to explain the forward premium anomaly as an artifact of data.

In Chapter 7, I investigate and provide some evidence for the links between forward premium anomaly and regulations. Chapter 8 offers some concluding comments. Derivations of analytical results presented in the text are relegated to the Appendices.
CHAPTER 2

PREVIOUS STUDIES ON THE FORWARD PREMIUM ANOMALY

The financial market is efficient if no unexploited excess profit opportunities are available. What profit is excessive, however, depends on a model of exchange market equilibrium. The importance of foreign exchange market efficiency stems from various aspects and it generated a large volume of research. The Federal Reserve Bank of New York estimates that average daily volume of foreign exchange transaction was 405 billion dollars during April 1998 in the US alone. Assuming 260 business days in a year, this volume hikes up to 105.3 trillion dollars.\(^1\) Actual volume of foreign exchange transaction is much higher than this when we include other foreign exchange transactions in London, Tokyo, and Singapore market.

The foreign exchange market is a central component of the international asset market. Whether exchange rates are correctly priced or not is particularly important since the exchange rates simultaneously affect the prices of assets, factors, and commodities of both parties engaging in the international transaction. This efficiency issue becomes more important in relation to the monetary policy since whether the market is efficient or not generates huge difference in policy implications. Monetary

\(^1\)Mark (2001).
authority needs to explore the issue of the foreign exchange market efficiency to assess the performance of alternative financial market systems—fixed or floating exchange rate system. If the market is efficient then currency fluctuations reflect the long-run equilibrium exchange rate determination process under floating system. On the other hand, if the market is not efficient, foreign exchange speculation will make exchange market volatile and unstable. Firms and individual market participants evidently care about the forward and future exchange rate market efficiency when they make portfolio decision.

UIP is a simple model of international currency and money-market asset market equilibrium. If market participants are risk neutral and have rational expectations, then the future exchange rate movement is reflected in the interest differential. UIP implies that the current interest differential is an unbiased predictor of the future depreciation. This unbiasedness hypothesis is typically tested by regressions of the future home currency depreciation on the current interest differential,

\[ s_{t+1} - s_t = \alpha + \beta (i_t - i_t^*) + \epsilon_{t+1} \]  

(2.1)

where \( s_t \) is the log of spot exchange rate (dollar price of foreign currency), \( i_t \) is the one-period nominal interest rate on domestic currency denominated asset, and \( i_t^* \) is the nominal interest rate on the foreign currency denominated asset. The null hypothesis is that \( \alpha = 0 \) and \( \beta = 1 \). Under the null hypothesis interest differential is an unbiased predictor of the future depreciation. By covered interest parity we have, \( i_t - i_t^* = f_t - s_t \) where \( f_t \) is the log of one period forward exchange rate. Therefore, forward rate is an unbiased predictor of the future spot rate is equivalent to UIP. A very large volume of research tested the unbiasedness hypothesis and consistently rejected it. Regressions with actual data typically yield a negative slope coefficient,
not just less than one. This is the finding known as the forward premium anomaly, an empirical puzzle in international finance. Engel (1996), Froot and Thaler (1990), Hodrick (1987), and Lewis (1995) provide well documented surveys of research on the forward premium anomaly. Over 300 academic publications tells us that the importance of this market efficiency issue and how hard to generate full survey of existing studies. To limit the scope of this chapter, I will briefly discuss only three approaches here.

Different interpretations of the negative slope estimates provide several possible explanations. One line of research takes \( \beta < 1 \) as evidence of a time-varying risk premium in foreign exchange. The risk premium is a compensation that makes risk-averse market participant to hold risky asset—foreign exchange. With the existence of the risk premium, the forward rate is not an unbiased predictor of the future spot rate anymore. In his salient paper, Fama (1984) studies stochastic properties of the risk premium using the omitted variable bias problem. The risk premium is the expected excess nominal forward foreign exchange payoff

\[
p_t \equiv f_t - E_t[s_{t+1}],
\]

where \( E_t[s_{t+1}] = E[s_{t+1}|I_t] \) the expectation conditional on information set \( \{I_t\} \) available at time \( t \). Rearrange (2.2), subtract \( s_t \) from both sides and apply covered interest parity we have

\[
E_t(s_{t+1} - s_t) = r_t - p_t.
\]

Using risk premium, we can rewrite the forward regression as

\[
\Delta s_{t+1} = \beta r_t - p_t + u_{t+1}.
\]
Fama (1984) shows that the interest differential must be negatively correlated with risk premium $\text{Cov}(p_t, r_t)$ for negative $\beta$. This means when the dollar denominated assets interest rise, investment in dollar denominated assets become more risky. This also implies that the covariance of the expected depreciation and the risk premium is negative $\text{Cov}[p_t, E_t(\Delta s_{t+1})] < 0$. Fama (1984) also pointed out that the variance of the risk premium is greater than the variance of expected depreciation $\text{Var}(p_t) > \text{Var}[E_t(\Delta s_{t+1})]$. The ensuing challenge for the risk premium approach is then whether it can explain small changes in interest can generate huge changes in risk premium.

Researchers use Lucas model to understand pricing of the forward foreign exchange and deviations from UIP. Let $S_t$ be the nominal spot exchange rate expressed as dollar price of foreign currency and $F_t$ be the one period forward exchange rate then $F_t - S_{t+1}$ is the payoff from forward speculation. Let $P_t$ be th domestic price level and let $\beta$ be the subjective discount factor. Since speculation does not require investment at time $t$, Euler equation is given as

$$E_t \left[ \beta u'(C_{t+1}) \frac{F_t - S_{t+1}}{P_{t+1}} \right] = 0 \quad (2.5)$$

where $u'(C_{t+1})$ is the representative agent’s marginal utility evaluated at equilibrium consumption. Multiply $P_t/[u'(C_t)S_t]$ on both sides of (2.5) gives

$$E_t \left[ \left( \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \right) \left( \frac{F_t - S_{t+1}}{S_t} \right) \right] = 0. \quad (2.6)$$

Let

$$\mu^m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \quad (2.7)$$

which is the intertemporal marginal rate of substitution of money. In Lucas model the price of a one-period riskless domestic currency nominal bond is $(1 + i_t)^{-1} = \mu^m_{t+1}$. 

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Since \( F_t/S_t \) is known at time \( t \), it can be treated as a constant and use covariance decomposition we have

\[
E_t \left[ \frac{F_t - S_{t+1}}{S_t} \right] = (1 + i_t) \text{Cov}_t \left[ \mu_{t+1}, \frac{S_{t+1}}{S_t} \right]. \tag{2.8}
\]

This implies that \( \text{Cov} \left[ \mu_{t+1}, \frac{S_{t+1}}{S_t} \right] \) must be negative to relate risky asset to bigger risk premium. Market participants want to have big return on bad state, but for this the asset must be risky and to make agent hold risky asset it needs to pay risk premium.

After assuming constant relative risk aversion (CRRA) utility function, Mark (1985) estimates the Euler equation and tests the overidentifying restrictions using the generalized methods of moments (GMM) technique. He finds uncomfortably large coefficient of the relative risk aversion than generally considered value with large asymptotic standard error. However, the test of the overidentifying restrictions cannot be rejected even with the large (above 40) coefficient of risk aversion and subsequent research find similar result. Under CRRA utility and lognormality, test of the Euler equation using GMM technique takes form

\[
\ln \left( \frac{F_t - S_{t+1}}{S_t} \right) + \ln \left( \frac{P_t}{P_{t+1}} \right) = -\gamma \ln \left( \frac{C_t}{C_{t+1}} \right) + \ln w_{t+1} \tag{2.9}
\]

where \( w_{t+1} \) denotes Euler equation (2.6). Eq.(2.9) tells us that the large estimates of \( \gamma \) and its large standard error can be attributed to the high volatility of excess return with low volatility in consumption growth. This tells us that the difficulty of Lucas model does not stop where it requires large coefficient of relative risk aversion of market participants to generate sufficiently large and variable risk premium consistent with the volatility of exchange rate returns form actual data.

UIP involves market expectation of the participants about the future exchange rates. However, the test of UIP is typically conducted using realized exchange rate
returns. Instead of modeling the subjective expectations of the market participants as mathematical conditional expectations, one line of research uses survey data on what market participants actually think. Froot and Frankel (1989) study the forward premium anomaly using survey data of exchange rate forecasts by professional foreign exchange market participants. To capture the properties of the survey data, they run two regressions

\[
\Delta s^e_{t+1} - \Delta s_{t+1} = \alpha_1 + \beta_1 (f_t - s_t) + \epsilon_{1t+1}, \\
\Delta s^e_{t+1} = \alpha_2 + \beta_2 (f_t - s_t) + \epsilon_{2t+1}
\]

(2.10) (2.11)

where \(\Delta s^e_{t+1}\) is the median of the survey forecast of the log spot exchange rate \(s_{t+1}\) reported at date \(t\). If the market participants have rational expectations, the survey forecast error realized at time \(t+1\) will be uncorrelated with any information available at time \(t\), and the slope coefficient \(\beta_1\) in eq.(2.10) will be zero.

However, the survey forecast error regressions generally yield estimates of \(\beta_1\) that are significantly different from zero. Nonzero \(\beta_1\) provides evidence against rationality of the market participants. This \(\beta_1 > 1\) implies that survey respondents attach too much weight on the forward rate when predicting future spot rate. On the other hand, analyses of survey data find that the estimates of \(\beta_2\) are generally insignificantly different from one. This suggests that survey respondents do not believe that there is a risk premium in the forward exchange rate and they use the forward rate as a predictor of the future spot rate. Analyses of survey data from professional foreign exchange market participants find that the survey forecast error is systematic and this may be the reason for the empirical violation of UIP. However, some economists are skeptical about the accuracy of the survey data and the robustness of the results from the survey data. Their point is that survey respondents have no incentives to
reveal their predictions about the future spot rate and we should study what market participants do, not what they say.

Another line of approach involves quasi-rationality of the market participants. This noise trader approach by Mark and Wu (1998), and Jeanne and Rose (2002) start form the idea that some market participants are irrational in the sense that they believe that the value of an asset depends on extraneous information in addition to the fundamentals. Mark and Wu (1998) build a model in which a mixture of rational and irrational market participants generate exchange rate dynamics that are consistent with the findings from survey data. The model generates deviations of the exchange rate from its fundamental values. Their noise-trader model is a two-country constant-population partial equilibrium overlapping-generations model. Using this model, they study the determination of the foreign exchange rate in an environment where market participants have heterogenous beliefs that create the basis for trading volume and include systemic movements in the deviation from UIP.

Noise traders in their model are motivated by Black’s (1986) suggestion that the environment of the real world is so complex that noise traders are unable to distinguish between pseudo-signals and news. These irrational traders believe that the pseudo-signals contain information about economic fundamentals. They have distorted beliefs regarding prospective investment returns by waves of excessive optimism and pessimism. Due to the existence of distorted beliefs of market participants, the model generates equilibrium exchange rates that exhibit transitory deviations from their fundamental values. However, this charming approach has not gained widespread acceptance because it is impossible to distinguish what fraction of the market participants are irrational from data.
Recently, researchers raised questions about the robustness of the econometric evidence against UIP. This line of approach argues that UIP is true and the forward premium anomaly is a statistical artifact of data. Violation of regularity conditions and slow convergence of the slope estimator from the regressions of the future depreciation on the current interest differential induces widely dispersed slope estimator distribution and statistical inference drawn from the asymptotic distribution unreliable. Since this thesis is closely related with this approach, it is worth to look at it more closely.

Schotman et al. (1997) point to the extreme-valued forecast errors that they call “big news.” Schotman et al. (1997) provide a model of conditional heteroskedasticity that is consistent with the conventional efficient market hypothesis. Their specification of “big news” model is

$$s_{t+1} - s_t = r_t(1 + \epsilon_{t+1}) + v_{t+1}$$

$$= r_t + r_t\epsilon_{t+1} + v_{t+1} \quad (2.12)$$

where $r_t = i_t - i^*_t$ is the home-foreign interest differential, $v_{t+1} \overset{iid}{\sim} (0, \sigma_v^2)$, and $\epsilon_{t+1} \overset{iid}{\sim} (0, \sigma^2_{\epsilon})$. In (2.12), the additive error term $v_{t+1}$ is the small news, and the multiplicative error term $r_t\epsilon_{t+1}$ is the big news which is the omitted variable in the forward regression and induces heteroskedasticity in the errors. The model implies that even with the big news term the unconditional slope coefficient $\beta$ is still one. Their argument can be easily shown by rewrite (2.12),

$$s_{t+1} - s_t = r_t + \epsilon_{t+1}r_t + v_{t+1}. \quad (2.13)$$

The variance of the composite error term $w_{t+1} = r_t\epsilon_{t+1} + v_{t+1}$ is,

$$E_t(w_{t+1}^2) = r_t^2\sigma^2_{\epsilon} + \sigma^2_{v}. \quad (2.14)$$
With the empirical investigation of (2.14), they argue that the conditional heteroskedasticity and fat-tailed interest differential induce large small sample distortions of the distribution for the OLS slope estimator from its asymptotic distribution and any inference drawn from it unreliable. Table 2.1 presents 95 percentile range of OLS slope coefficient distributions from big news model. The 95 percentile range for OLS slope coefficient of \((-3.546, 5.503)\) with median value of 0.949.

Baillie and Bollerslev (2000) trace the problem to persistent conditional volatility in the interest differential. Assume

\[
E_t \left[ \frac{F_t - S_{t+1}}{P_{t+1}} \right] = 0
\]

(2.15)

where \(P_t\) is the date \(t\) domestic price. By Taylor’s expansion to second order terms we get,

\[
E_t(s_{t+1}) - f_t = \frac{1}{2} \text{Var}_t(s_{t+1}) + \text{Cov}_t(s_{t+1}, p_{t+1}).
\]

(2.16)

Their point is that even with rational expectations and risk neutrality, eq.(2.16) contains the two conditional second moment terms and they are attributable to the empirical violation of UIP. They build a daily model to explain the forward premium anomaly. Assuming fractionally integrated GARCH (FIGARCH) process for the volatility of the transitory part of the spot rate, and use this model for data generating process they claim to explain the forward premium anomaly. One simulation experiment gives them 90 percentile range for \(\hat{\beta}\) of \((-5.14, 10.9)\).

This statistical artifact of small sample approach is successful in providing empirical explain for the forward premium anomaly, however, it has pitfalls too. The models used in this approach are \textit{ad hoc} in a sense that they are not from economic theory. Schotman \textit{et al.} (1997)’s big new representation works well but it is just a
statistical representation to capture the nonstandard features of data. Baillie and Bollerslev (2000) need FIGARCH process for volatility. A stable GARCH(1,1) does not replicate the long memory feature in the forward premium, which is crucial in explaining the forward premium anomaly. However, economic justification for using FIGARCH is weak.

The plan of the thesis is first to further investigate along the ideas of the statistical artifact of small sample approach—UIP is actually true and the forward premium anomaly is a statistical artifact. The purpose here is to ask whether we can model an exchange rate model based on UIP that provides economic theory for the nonstandard features of international financial data. The answer that is obtained is no. Although the model produces ARCH effects when UIP is constrained to hold continuously, including instants of central bank intervention, the distributional distortion from the model is not sufficiently large to resolve the forward premium anomaly.

These results led me to explore a related but alternative avenue where I put my attention to the effects of the central bank intervention on the financial market. My first model is built on the assumption that all the market participants know and understand central bank intervention rule and UIP holds all the time even at the instant of intervention. This perfect UIP assumption may, however, be too strong. Since interventions in the exchange markets take place irregularly and informally, market participants cannot have full information about the intervention. I relaxed the assumption of perfect UIP and build a model in which the intervention always takes market by surprise and UIP is violated only at those instants of intervention. This central bank intervention approach provides a resolution to the forward premium anomaly.
Notes: Big news model is simulated following Schotman et al. (1997)’s specification

\[ s_{t+1} - s_t = r_t (1 + e_{t+1}) + v_{t+1}. \]

Sampling distributions of RHS variables are

\[ v \sim cN_1, \quad r \sim \frac{aM_t}{\sqrt{(N_2^2 + N_3^2 + N_4^2)/3}}, \quad M_t = \rho M_{t-1} + N_5, \quad \epsilon \sim bN_6/M_t \]

where \( N_i, i = 1, \ldots, 6 \) and \( M \) are independent standard normal distributed random variables with scaling constants \((a, b, c, \rho) = (1/1500, 10, 1/10, 0.7)\).

Table 2.1: OLS Slope Distributions of Big News Model (Percentiles of Monte Carlo Distributions from 5000 Replications of 1200 Weekly Observations)
CHAPTER 3

EMPIRICAL REGULARITIES OF EXCHANGE RATES
AND INTEREST DIFFERENTIALS

Let $s_t$ be the log dollar price of the foreign currency and $r_t$ be the corresponding 1-period “US-foreign” Eurocurrency rate differential. The data set consist of weekly and monthly observations of the spot exchange rates and the Eurocurrency rates for the US, Germany and the UK. Observations from 1/02/76 through 12/27/85 are Friday closings reported in the Harris Bank Weekly Review and observations from 1/03/86 through 12/25/98 are Friday quotations from Datastream. I end the sample one year before Germany irrevocably fixed the deutschmark to the euro. The weekly observations are plotted in Figures 3.1 and 3.2. Both figures provides visual evidence of jump behavior in the exchange rates and interest differentials as the number of

---

2Interest rates are stated in percent per annum. The log exchange rates are multiplied by 5200 to conform this normalization. I take the US as the home country throughout the paper.


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weekly changes in interest differential that exceed one percentage point are 87 and 166 times for US-German and US-UK interest differentials over the sample periods.

Tables 3.1 – 3.2 present some empirical regularities of the exchange rates and interest rate differentials upon which the thesis focuses. Table 3.1 shows the Lagrange-multiplier (LM) test for first-order ARCH, the skewness, and the excess kurtosis in the error from the regression of the future exchange rate return on the current interest differential,

\[ s_{t+1} - s_t = \alpha + \beta r_t + u_{t+1}. \]  

From the LM statistics, it is clear that weekly errors exhibit ARCH effects. The skewness coefficient offers little evidence that the error distribution is asymmetric while the excess kurtosis coefficient indicates that the error distribution is fat-tailed relative to the normal distribution.\(^4\) Figures 3.3 and 3.4 show visual evidence of volatility clustering in the US-German error terms and squared errors.

Variance ratio statistics for the exchange rate return at horizons 2, 12, and 24 provide an alternative characterization of the autocorrelation function and measure the persistence in the observations. All the variance ratio statistics lie near or above 1 and this indicates the presence of a high degree of persistence in the exchange rate.\(^5\) Exchange rate returns also show much higher volatility than the interest differential as the sample volatility of exchange rate returns \(\sigma_{\Delta s}\) are approximately 25 times that of

\(^4\)LM is \(T \cdot R^2\) from a regression of the squared residual regressed on its own lag and is distributed as \(\chi^2(1)\) under the hypothesis of no conditional heteroskedasticity. See p.664 of Hamilton (1994). Let \(\mu_j\) be the \(j\)–th central moment. Then the skewness coefficient \(\mu_3/\sigma^3\), is zero if the distribution is symmetric. The coefficient of excess kurtosis \((\mu_4/\sigma^4) - 3\), is zero if the distribution is Gaussian. If the underlying distribution is fat-tailed (thin-tailed) relative to the Gaussian distribution, this quantity will be positive (negative).

\(^5\)VR(\(k\)) is the variance of the \(k\)-period change in the log exchange rate relative to \(k\) times the variance of the one-period change–at horizons \(k = 2, 12, \text{and} 24\).
the interest differential $\sigma_r$. Exchange rate returns also exhibit trivial short-run serial
dependence as indicated by the very small first-order autocorrelation coefficients $\rho_{\Delta s}$.

The interest differential, on the other hand, exhibits a high degree of persistence
with high values of the autocorrelation coefficients $(\rho_r(k), k = 1, 12, 24).$ The LM
statistics for the interest differential suggest existence of strong ARCH effects in
weekly observations. The skewness coefficient provides small evidence of asymmetry
in distribution and the excess kurtosis coefficient indicates fat-tailed distribution.

The last two lines of Table 3.1 report the negative and asymptotically significant
non-unit estimates for the slope coefficient in regressions of eq.(3.1). These estimates
demonstrate that the forward premium anomaly is present in the sample (UIP predicts
that $\alpha = 0$ and $\beta = 1$). The asymptotic t-ratio uses heteroskedasticity consistent
standard errors to test the hypothesis that the slope coefficient is 1.

Table 3.2 compares the strength of the ARCH effects present in weekly and in
monthly observations. It is clear that the LM statistics for the first order ARCH
decline dramatically for the exchange rate excess return as the sampling horizon
expands from weekly to monthly. For a closer look at the conditional volatility in the
data, I fit the GARCH(1,1) model

$$
E_t u_{t+1}^2 = h_t = \omega + \delta u_{t-1}^2 + \gamma h_{t-1},
$$

(3.2)

where $u_{t+1} = \Delta s_{t+1} - \alpha - \beta r_t$ for weekly and monthly observations. Table 3.2 shows
declining strength of the ARCH effects in exchange rate return associated with the

---

6Whether the interest differential is I(1) or I(0) has been heavily tested by testing whether the spot
and forward exchange rates are cointegrated. Evans and Lewis (1995) cannot reject that the interest
differential is I(1) whereas Baillie and Bollerslev (1989), Choudhry (1999), Corbae et al. (1992), Hai
and Bollerslev (1994) conclude that the interest differential has long-memory but is mean reverting
with a fractional difference parameter between 1/2 and 1.
lengthening of the sampling horizon. A reduction in magnitude and significance of the estimated GARCH parameters can be seen as sampling period stretches from weekly to monthly.

To summarize, the thesis will focus on understanding following features of the data: i) weekly interest differentials and exchange rate returns possess ARCH effects but monthly returns do not, ii) both the level of the exchange rate and the interest differential are highly persistent, iii) the interest differential and the exchange rate exhibit discrete changes, iv) the exchange rate returns are fat-tailed, and v) the future exchange rate return is negatively correlated with the current interest differential.
<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM: $\chi^2(1)$</td>
<td>281.362</td>
<td>187.619</td>
</tr>
<tr>
<td>skewness</td>
<td>0.097</td>
<td>-0.298</td>
</tr>
<tr>
<td>excess kurtosis</td>
<td>1.523</td>
<td>3.997</td>
</tr>
<tr>
<td>VR(2)</td>
<td>1.001</td>
<td>1.009</td>
</tr>
<tr>
<td>VR(12)</td>
<td>1.113</td>
<td>1.174</td>
</tr>
<tr>
<td>VR(24)</td>
<td>1.202</td>
<td>1.205</td>
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<tr>
<td>$\sigma_{\Delta s}$</td>
<td>79.062</td>
<td>77.759</td>
</tr>
<tr>
<td>$\rho_{\Delta s}(1)$</td>
<td>0.001</td>
<td>0.009</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>3.229</td>
<td>3.122</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.985</td>
<td>0.962</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.847</td>
<td>0.717</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.789</td>
<td>0.513</td>
</tr>
<tr>
<td>LM($r$): $\chi^2(1)$</td>
<td>199.605</td>
<td>58.867</td>
</tr>
<tr>
<td>skewness($r$)</td>
<td>-0.576</td>
<td>-0.107</td>
</tr>
<tr>
<td>excess kurtosis($r$)</td>
<td>0.696</td>
<td>1.364</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.693</td>
<td>-1.506</td>
</tr>
<tr>
<td>t-ratio</td>
<td>-2.024</td>
<td>-3.227</td>
</tr>
</tbody>
</table>

Notes: Log exchange rates multiplied by 5200. Interest differential in percent per year.

Table 3.1: Features of Weekly Data
<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly</td>
<td>281.362</td>
<td>187.619</td>
</tr>
<tr>
<td>Monthly</td>
<td>86.580</td>
<td>64.767</td>
</tr>
</tbody>
</table>

**GARCH(1,1):** \( h_{t+1} = \omega + \delta u_t^2 + \gamma h_t \)

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly</td>
<td>0.080</td>
<td>0.083</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(5.673)</td>
<td>(15.989)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.914</td>
<td>0.898</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(26.035)</td>
<td>(23.262)</td>
</tr>
<tr>
<td>Monthly</td>
<td>0.050</td>
<td>0.029</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(2.508)</td>
<td>(2.750)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.916</td>
<td>0.934</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(4.586)</td>
<td>(3.448)</td>
</tr>
</tbody>
</table>

Table 3.2: ARCH Effects in the Exchange Return and Sampling Horizon
Figure 3.1: Weekly Exchange Rates

Figure 3.2: Weekly Interest Differentials
Figure 3.3: US-Germany Regression Error

Figure 3.4: US-Germany Regression Error Squared
CHAPTER 4

UIP IN CONTINUOUS-TIME: MEAN REVERTING INTEREST DIFFERENTIAL

Let \( r(t) \) be the instantaneous yield differential between domestic and foreign-currency denominated debt instruments with identical default risk. Then UIP in continuous time is

\[
E_t[ds(t)] = r(t) dt,
\]

where \( E_t[\cdot] \) is the expectation conditional on information available at instant \( t \) and \( ds(t) \) is the forward differential of \( s(t) \). Continuous-time UIP is a stochastic differential equation in \( s(t) \) and solution for eq.(4.1) requires knowledge on the underline process for the interest differential. This chapter explores exchange rate dynamics with mean-reverting interest differential model. The mean-reverting model of interest differential is less easily motivated on economic grounds, but it can provide a reasonable statistical representation for the data. Since the mean-reverting model provides desirable statistical properties of the interest differential, it can provide further insight into the idea of statistical artifact of small sample approach.
4.1 A Mean-Reverting Model of the Interest Differential

Let the interest differential to follow the mean-reverting process

\[ dr(t) = -\gamma r(t)dt + \sigma_r dz(t), \]  \hspace{1cm} (4.2)

where \( 1 > \gamma > 0 \) and \( dz(t) \) is a standard Wiener process. This model for the interest differential can be built up from the individual interest rates if each in turn follows mean-reverting processes

\[
\begin{align*}
di(t) &= \gamma_1(\mu_1 - i(t))dt + \sigma_1 dz_1(t) \\
di^*(t) &= \gamma_2(\mu_2 - i^*(t))dt + \sigma_2 dz_2(t),
\end{align*}
\]

with \( \gamma_1 = \gamma_2, \mu_1 = \mu_2, \) and \( \sigma_1 = \sigma_2. \) The symmetry is imposed to lessen the algebraic demands. Compared to the regulated process, the mean-reverting process for the interest differential is less easily motivated on economic grounds, but it can be argued that it provides a reasonable statistical representation for the data since it can be made to be arbitrarily persistent while maintaining bounded variance.

The Euler approximation for the continuous-time mean-reverting process with standard normal increments (4.2) is

\[
r_j = (1 - \gamma_r \delta_N)r_{j-1} + \sigma_r \epsilon_j \sqrt{\delta_N} \]  \hspace{1cm} (4.3)

where \( \epsilon_j \overset{iid}{\sim} N(0,1) \), for \( j = 1, \ldots, NT \), \( \sigma_r \) is the weekly standard deviation of the instantaneous yield differential \( r(t) \), and \( \gamma \) is mean-reversion parameter also known as speed of adjustment parameter. From (4.3) it can be easily seen that mean-reverting process is a continuous-time version of a first order autoregressive process.
The parameters $\sigma_r$ and $\gamma$ of the interest differential processes are estimated employing the simulated method of moments (SMM). Table 4.1 reports estimates obtained under two alternative sets of moment conditions. The first set consists of the 3 moments $E(\Delta r_t, \Delta r_t^2, r_t r_{t-1})$, whereas the second set consists of the 5 moments $E(\Delta r_t, \Delta r_t^2, r_t r_{t-1}, \Delta r_t^3, \Delta r_t^4)$. With p-value for the test of the over identifying restrictions of 0.934 for Germany, the mean-reverting interest differential process does a good job in matching the set of 3 moments. Since there is no truncation of observations compared to regulated process, the mean-reversion model for the interest differential is expected to perform well in matching the higher moments—skewness and the excess kurtosis. The model, however, does not perform well in matching higher-order moments with the p-value for the test of over identifying restrictions of 0.002 for the 5 moment estimation.

To elaborate tail property of the interest differential, I consider an alternative model that the increment follows student-$t$ distribution instead of standard normal distribution. This can be done easily by assuming that $dz(t)$ follows a student-$t$ distribution with $n$ degree of freedom. Schotman et al. (1997) propose that the interest differential is drawn from a fat-tailed distribution along with their “big news” representation for a story of poor small sample properties of OLS slope distribution. In a similar fashion, Baillie and Bollerslev (2000) demonstrate that the empirical distribution of the OLS estimate from the regressions of the exchange rate return on the interest differential is widely dispersed when the conditional volatility in the interest differential follows their calibrated fractionally integrated generalized ARCH process. The mean-reverting model for the interest differential provide further insight into the idea of statistical artifact of small sample approach.
The Euler approximation for the continuous-time mean-reverting process with student-$t(n)$ increments can be done by assuming $\epsilon_j$ in eq.(4.3) follows student-$t(n)$ distribution, $\epsilon_j \sim (0, \frac{\sigma^2}{\chi^2 (n-2)})$. Employing the same sets of moment conditions used before, the estimation results are reported in Table 4.2. Again the process is seen to do a good job in matching 3 moment conditions with p-values for the test of the over identifying restrictions of 0.937 for Germany when the increment follows student-$t(9)$ distribution. However, even with the non-normal increments, mean-reverting model for the interest differential does not perform well in matching higher-order moments. When matching the set of 5 moments, the model shows marginal improvement with p-value for the test of over identifying restrictions of 0.003.

Distributional properties of the estimated mean-reverting processes with different increments are shown in Tables 4.3 – 4.4. Both models perform well in matching volatility, however, the estimated processes appear to understate the persistence of the interest differential. This is perhaps due to the downward bias in estimation of the autocorrelation coefficient. Figures 4.1 – 4.2 show realizations of the mean-reverting process using the 3-moment estimates for Germany with observations sampled at weekly intervals.

### 4.2 Exchange Rate Solution

When the interest differential follows mean-reverting process, the exchange rate solution cannot be expressed in terms of elementary functions. We can, however, write the solution in integral form.
Proposition 4.1  If the interest differential follows mean-reverting process with standard normal increments, a family of the exchange rate solutions is

\[ s(t) = G[r(t)] = -\frac{r(t)}{\gamma} + Cr(t) \int_0^1 \exp \left( \frac{\gamma r(t)^2}{\sigma_r^2} u^2 \right) du, \quad (4.4) \]

where \( C \) is unknown constant.

Graphs of (4.4) for alternative values of \( C \) are displayed in Figure 4.3. The nonlinear relation between the exchange rate and the interest differential is symmetric and the volatility of the exchange rate exhibits sharp increase as absolute value of the interest differential increase. The curvature of the graph, however, varies according to the value of \( C \).

Also the exchange rate solution under mean-reverting interest differential with student-t increments takes similar form to the solution with standard normal increments.

Proposition 4.2  If the interest differential follows mean-reverting process with student-t increments, a family of the exchange rate solutions is

\[ s(t) = G[r(t)] = -\frac{r(t)}{\gamma} + Cr(t) \int_0^1 \exp \left( \frac{\gamma r(t)^2}{\delta \sigma_r^2} u^2 \right) du, \quad (4.5) \]

where \( \delta = \left( \frac{\nu}{\nu - 2} \right) \), and \( C \) is unknown constant.

Graphs of (4.5) for alternative values of \( C \) are displayed in Figure 4.4. Again, the nonlinear relation between the exchange rate and the interest differential is qualitatively the same as before and the curvature of the graph varies according to the value of \( C \).
Applying Ito’s lemma to eq.(4.4), we can obtain the instantaneous change in the log exchange rate as,

\[ ds(t) = r(t)dt - \left[ \frac{1}{\gamma} - C \exp \left( \frac{\gamma r^2(t)}{\sigma_r^2} \right) \right] \sigma_r dz(t). \]  

(4.6)

The implied error term displays nonlinear dependence on \( r(t) \), which will potentially exhibits volatility clustering, and leptokurtosis. It is difficult to discretize the process for the exchange rate analytically and I will rely on simulation methods for that purpose. I approximate the continuous-time process (4.4) and sample the implied exchange rate and interest rate differential at weekly intervals. Here again, \( r(t) \) is the instantaneous rate of return but scaled such that \( \sigma_r \) is the weekly volatility. The 1-week yield differential implied by the expectations theory of the term structure is

\[ R(0, 1) = -\frac{r(0)}{\gamma} (e^{-\gamma} - 1). \]  

(4.7)

### 4.3 Properties of the Simulated Observations

When the interest differential follows mean-reverting process, UIP by itself does not impose sufficient structure to allow determination of \( C \). To conduct simulation experiments, I estimate \( C \) by simulated method of moments (SMM) taking the parameters of the interest rate process fixed at their estimated values. I obtain two estimates of \( C \) using alternative moment conditions. I employ a set of three moments, \((E s_t^2, E \Delta s_t^2, E \Delta s_t r_{t-1})'\) and a set of five moments \((E s_t^2, E \Delta s_t^2, E \Delta s_t r_{t-1}, E \Delta s_t^3, E \Delta s_t^4)'\).

The estimation results are shown in Tables 4.5 – 4.6.

In the 5-moment estimation of constant with standard normal increments, the over identifying restrictions are rejected at tiny significance levels. This model evidently does a poor job of matching the high-order moments of the exchange rate. This
is also true even for the mean-reverting model with student-\(t\) increments that is expected to perform well in matching higher moments. I suspect that this poor performance of mean-reverting model is related to underestimation of persistence in interest differential.

The results of the Monte Carlo simulations for the model calibrated to US-German data estimates are reported in Tables 4.7 – 4.8. Both mean-reverting models also generates ARCH effects in the \textit{ex post} deviation from UIP as seen by the distribution of large LM statistics and persistent exchange rate dynamics as seen from the large median values of the variance ratio statistic. While the volatility in exchange rate returns relative to the volatility of the interest differential is quite high, it falls short of the volatility found in the data and only count for half of the actual volatility.

This result underscores the idea that the ARCH effects are induced by the continuous time market dynamics. The true regression error distribution appears symmetric for all models. The median excess kurtosis coefficients are close to zero, except for the 3-moment standard normal increments model, but distributions are heavily skewed to the right. It is not uncommon to obtain realizations that exhibit extreme values. Finally, in the mean-reverting model, since UIP holds at all instants, does not provide a resolution for the forward premium anomaly. The 95 percentile range of the OLS slope coefficient distribution does not contain zero. In fact, the distribution is skewed to the right.
<table>
<thead>
<tr>
<th>Moments</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_r$</td>
<td>0.562</td>
<td>0.368</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(8.411)</td>
<td>(15.086)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.016</td>
<td>0.008</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(3.804)</td>
<td>(5.636)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>0.007</td>
<td>14.624</td>
</tr>
<tr>
<td>p-value</td>
<td>0.934</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Notes: 3 moments used in estimation are $\Delta r(t), \Delta r^2(t), r(t) r(t - 1)$. 5 moments used in estimation, add $\Delta r^3(t), \Delta r^4(t)$.

Table 4.1: SMM Estimates of Interest Rate Process: Mean-Reversion with Standard Normal Increments
<table>
<thead>
<tr>
<th>Moments</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of Freedom</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.496</td>
<td>0.265</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(8.409)</td>
<td>(14.897)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.016</td>
<td>0.008</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(3.769)</td>
<td>(5.348)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>0.006</td>
<td>14.026</td>
</tr>
<tr>
<td>p-value</td>
<td>0.937</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Notes: 3 moments used in estimation are $\Delta r(t), \Delta r^2(t), r(t)r(t-1)$. 5 moments used in estimation, add $\Delta r^3(t), \Delta r^4(t)$. DF denotes degree of freedom in student-t distribution.

Table 4.2: SMM Estimates of Interest Rate Process: Mean-Reversion with Student-t Increments
Table 4.3: Properties of Interest Rate Processes: Mean-Reversion with Standard Normal Increments

<table>
<thead>
<tr>
<th>Moments</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_r$</td>
<td>0.562</td>
<td>0.368</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.016</td>
<td>0.008</td>
</tr>
<tr>
<td>$\sqrt{\text{Var}(r)}$</td>
<td>3.195</td>
<td>2.956</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.985</td>
<td>0.992</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.829</td>
<td>0.912</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.692</td>
<td>0.821</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>---------</td>
<td>----------</td>
</tr>
<tr>
<td>Moments</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>Degree of Freedom</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.496</td>
<td>0.265</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.016</td>
<td>0.008</td>
</tr>
<tr>
<td>$\sqrt{\text{Var}(r)}$</td>
<td>3.178</td>
<td>3.183</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.984</td>
<td>0.996</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.828</td>
<td>0.940</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.691</td>
<td>0.898</td>
</tr>
</tbody>
</table>

Table 4.4: Properties of Interest Rate Processes: Mean-Reversion with Student-t Increments
<table>
<thead>
<tr>
<th>Moments</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>-8.240</td>
<td>-2.222</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(-11.678)</td>
<td>(-1.145)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>24.368</td>
<td>87.889</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 4.5: SMM Estimation of Undetermined Constants in Exchange Rate Solution: Mean-Reverting Interest Differential with Standard Normal Increments
<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moments</td>
<td>3</td>
</tr>
<tr>
<td>Degree of Freedom</td>
<td>9</td>
</tr>
<tr>
<td>$C$</td>
<td>-2.888</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(-12.592)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>24.684</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 4.6: SMM Estimation of Undetermined Constants in Exchange Rate Solution: Mean-Reverting Interest Differential with Student-$t$ Increments
<table>
<thead>
<tr>
<th></th>
<th>3 Moments</th>
<th>5 Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5%</td>
<td>50%</td>
</tr>
<tr>
<td>LM: $\chi^2(1)$</td>
<td>39.071</td>
<td>306.028</td>
</tr>
<tr>
<td>skewness</td>
<td>-3.704</td>
<td>-0.002</td>
</tr>
<tr>
<td>excess kurtosis</td>
<td>-0.062</td>
<td>2.661</td>
</tr>
<tr>
<td>VR(2)</td>
<td>0.749</td>
<td>0.980</td>
</tr>
<tr>
<td>VR(12)</td>
<td>0.379</td>
<td>0.825</td>
</tr>
<tr>
<td>VR(24)</td>
<td>0.257</td>
<td>0.707</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}$</td>
<td>41.662</td>
<td>48.170</td>
</tr>
<tr>
<td>$\rho_{\Delta s}(1)$</td>
<td>-0.251</td>
<td>-0.020</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>2.173</td>
<td>2.946</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.966</td>
<td>0.982</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.648</td>
<td>0.801</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.387</td>
<td>0.636</td>
</tr>
<tr>
<td>LM($r$): $\chi^2(1)$</td>
<td>363.302$^{b/}$</td>
<td>402.032</td>
</tr>
<tr>
<td>skewness($r$)</td>
<td>-0.685</td>
<td>0.000</td>
</tr>
<tr>
<td>excess kurtosis($r$)</td>
<td>-0.922</td>
<td>-0.315</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.984$^{b/}$</td>
<td>1.559</td>
</tr>
<tr>
<td>Asy-t</td>
<td>-0.019</td>
<td>0.681</td>
</tr>
</tbody>
</table>

Note: $a/$—less than estimate from data. $b/$—greater than estimate from data.

Table 4.7: Properties of Exchange Rate Processes When Interest Differential Follows Mean-Reverting with Standard Normal Increments Calibrated to German 3-moment SMM Estimates (Percentiles of Monte Carlo Distributions from 5000 Replications of 1200 Weekly Observations)
<table>
<thead>
<tr>
<th></th>
<th>3 Moments</th>
<th>5 Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5%</td>
<td>50%</td>
</tr>
<tr>
<td><strong>LM: χ²(1)</strong></td>
<td>73.168</td>
<td>367.473</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.703</td>
<td>0.002</td>
</tr>
<tr>
<td>Excess kurtosis</td>
<td>-0.165</td>
<td>0.317</td>
</tr>
<tr>
<td>VR(2)</td>
<td>0.816</td>
<td>0.985</td>
</tr>
<tr>
<td>VR(12)</td>
<td>0.476</td>
<td>0.870</td>
</tr>
<tr>
<td>VR(24)</td>
<td>0.351</td>
<td>0.763</td>
</tr>
<tr>
<td>σ₅₈</td>
<td>36.855</td>
<td>39.455</td>
</tr>
<tr>
<td>ρ₅₈(1)</td>
<td>-0.182</td>
<td>-0.014</td>
</tr>
<tr>
<td>σₚ</td>
<td>2.142</td>
<td>2.912</td>
</tr>
<tr>
<td>ρₚ(1)</td>
<td>0.966</td>
<td>0.981</td>
</tr>
<tr>
<td>ρₚ(12)</td>
<td>0.644</td>
<td>0.797</td>
</tr>
<tr>
<td>ρₚ(24)</td>
<td>0.376</td>
<td>0.631</td>
</tr>
<tr>
<td><strong>LM(r): χ²(1)</strong></td>
<td>363.256</td>
<td>400.006</td>
</tr>
<tr>
<td>Skewness(r)</td>
<td>-0.667</td>
<td>0.005</td>
</tr>
<tr>
<td>Excess kurtosis(r)</td>
<td>-0.923</td>
<td>-0.331</td>
</tr>
<tr>
<td>β</td>
<td>0.784s</td>
<td>1.317</td>
</tr>
<tr>
<td>Asy-t</td>
<td>-0.472</td>
<td>0.603</td>
</tr>
</tbody>
</table>

Note: a/—less than estimate from data. b/—greater than estimate from data.

Table 4.8: Properties of Exchange Rate Processes When Interest Differential Follows Mean-Reverting with Student-𝑡 Increments Calibrated to German 3-moment SMM Estimates (Percentiles of Monte Carlo Distributions from 5000 Replications of 1200 Weekly Observations)
Figure 4.1: A Realization of the Mean-Reverting Process with Standard Normal Increments: Observations Sampled Weekly with $\sigma_r = 0.562$, $\gamma = 0.016$, $\delta = 1/84$.

Figure 4.2: A Realization of the Mean-Reverting Process with Student-$t(9)$ Increments: Observations Sampled Weekly with $\sigma_r = 0.496$, $\gamma = 0.016$, $\delta = 1/84$. 
Figure 4.3: Nonlinear Relation between Log Exchange Rate and Interest Differential: Plots of Equation (4.4) with Alternative Values of $C$.

Figure 4.4: Nonlinear Relation between Log Exchange Rate and Interest Differential: Plots of Equation (4.5) with Alternative Values of $C$. 

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Let \( r(t) \) be the instantaneous yield differential between domestic and foreign-currency denominated debt instruments with identical default risk. Then UIP in continuous time is

\[
E_t[ds(t)] = r(t)dt, \tag{5.1}
\]

where \( E_t[\cdot] \) is the expectation conditional on information available at instant \( t \) and \( ds(t) \) is the forward differential of \( s(t) \). The solution for eq.(5.1) requires knowledge on the underline process for the interest differential. For this purpose, I seek a model for the interest differential that exhibits a high degree of persistence with bounded variance. The regulated Brownian motion is one such model.

### 5.1 A Regulated Brownian Motion Model of the Interest Differential

Let \( \underline{r} < \bar{r} \) be fixed constants and suppose that the interest differential is constrained to lie within the bands \([\underline{r}, \bar{r}]\). When \( r(t) \) lies strictly within the bands, it
evolves according to the Brownian motion,

\[ dr(t) = \sigma_r dz(t), \quad (5.2) \]

where \( dz(t) \) is a standard Wiener process and \( \sigma_r \) is the weekly volatility in \( dr(t) \). To simplify the exposition, I assume that the reflecting barriers are symmetric (\( \underline{r} = -\bar{r} \)).

The process describes Krugman (1991) style infinitesimal central bank interventions that occur when \( r(t) = \bar{r} \) or \( r(t) = -\bar{r} \) to prevent \( r(t) \) from exiting the bands. In other times, when \( r(t) \) is interior to the bands, the authorities focus on domestic objectives and the interest differential evolves randomly, being subject to many different sources of shocks.

The Krugman-style regulated Brownian motion model assumes that the exchange rate bands (or equivalently the interest differential bands) are time invariant constants. While this cannot serve as a completely realistic representation of US exchange rate policy, the idea bears more than a shred of empirical plausibility. Although exchange rate bands for the US dollar during the post Bretton Woods era have never been formally established, both coordinated as well as uncoordinated foreign exchange interventions are frequently engineered by the major central banks, especially during

\[ \text{Under band symmetry, the unconditional mean of } r(t) \text{ is 0. The appendix shows how band symmetry can be relaxed. Recent research has exploited similar nonlinear models to study exchange rates [Michael, Nobay and Peel (1997), Kilian and Taylor (2001)]. Since interest differentials and exchange rates are functionally related, it is natural to also consider nonlinear adjustment in the interest differential. I note also that (5.2) is consistent with individual interest rate dynamics that evolve according to } di(t) = \sigma_1 dz_1(t) \text{ when } i \in [\bar{i}, \tilde{i}] \text{ and } di^*(t) = \sigma_2 dz_2(t) \text{ when } i^* \in [\bar{i}^*, \tilde{i}^*], \text{ where } dz_1(t) = \rho dz_1(t) + \sqrt{1 - \rho^2} dw(t) \text{ and } dw(t) \text{ and } dz_1(t) \text{ are independent standard Wiener processes. Then we have } dr(t) = d_i(t) - d_i^*(t) = \sigma_r dz(t) \text{ where } \sigma_r = \sqrt{(\sigma_1 - \rho \sigma_2)^2 + \sigma_2^2 (1 - \rho^2)}, \text{ and } dz(t) \text{ is a standard Wiener process. If we set } i = \bar{i} = \tilde{i} = 0, \text{ then we have } \bar{r} = \bar{i} \text{ and } r = -\bar{r}. \text{ In any finite sample, however, we may not have very many realizations of the event } \{i_t = \bar{i} \cap i_t^* = \bar{i}^*\} \text{ or of the event } \{i_t = \bar{i} \cap i_t^* = \tilde{i}^*\} \text{ so the standard error on the estimate of } \bar{r} \text{ is likely to be quite large.} \]
times of unusual dollar strength or weakness. The widespread practice of intervention at least suggests the existence of a set of informal bands.\textsuperscript{8}

I estimate the parameters \((\bar{r}, \sigma_r)\) of the interest rate processes using the simulated method of moments (SMM).\textsuperscript{9} I begin by dividing each of the \(T\) weekly observations into \(N\) subintervals, of length \(\delta_N = 1/N \approx dt\), and use Euler’s method to approximate the continuous-time model

\[
r_j = r_{j-1} + \sigma_r \epsilon_j \sqrt{\delta_N},
\]

where \(j = 1, \ldots, NT\), \(\epsilon_j \overset{iid}{\sim} N(0, 1)\), and \(\sigma_r\) is the weekly standard deviation of the instantaneous rate of return differential \(r(t)\). The parameters \(\bar{r}\) and \(\sigma_r\) are chosen such that the implied moments generated from simulations of (5.3) match a specified set of sample moments of the data.

Table 5.1 reports estimates obtained under two alternative sets of moment conditions. The first set consists of the 3 moments \(E(\Delta r_t, \Delta r_t^2, r_t r_{t-1})\), whereas the second set consists of the 5 moments \(E(\Delta r_t, \Delta r_t^2, r_t r_{t-1}, \Delta r_t^3, \Delta r_t^4)\). The regulated Brownian motion is seen to do a good job in matching the set of 3 moments with p-values for the test of the over identifying restrictions of 0.930 for Germany and 0.877 for the U.K. The model fares slightly less well in matching the higher-order moments. The p-values for the test of over identifying restrictions in the 5 moment estimation are 0.251 for the U.K. and 0.037 for Germany. Table 5.2 shows some of the properties implied by the estimated regulated Brownian motion models. As can be seen, the

---

\textsuperscript{8}See Baillie and Osterberg (2000) for a narrative of the Fed and the Bundesbank intervention history over the 80s and 90s. The interventions continue where on 9/22/2000, the European Central Bank (ECB), the Federal Reserve, the Bank of Japan (BOJ), the Bank of Canada, and the Bank of England engaged in a coordinated intervention to support the euro, the ECB engaged in subsequent purchases of euros on 6/30/1998, on 11/03/2000 the BOJ intervened to support the yen whereas on 4/03/2000, it intervened to support the dollar.

\textsuperscript{9}Lee and Ingram (1991), and Duffie and Singleton (1993). See Appendix for more description.
estimated models provide a reasonable account of the persistence and the volatility of the interest differential data. Figure 5.1 shows a realization of the process using the 3-moment estimates for Germany with observations sampled at weekly intervals.

5.2 Exchange Rate Solution

Proposition 5.1 If the interest differential evolves following regulated Brownian motion and lies within the bands, a family of solutions to eq.(5.1) is

\[ s(t) = A + Br(t) + \frac{r^3(t)}{3\sigma_r^2}, \]  

where \( A \) and \( B \) are constant coefficients to be determined by auxiliary conditions.

Proofs of propositions are shown in appendix. \( A \) will depend on initial conditions and on currency units, and the exchange rate will be decreasing in the interest differential if \( B \) is sufficiently negative. Figure 5.2 shows solutions for alternative values of \( B \). The nonlinearity in the solution—the manner in which the exchange rate function bends as the absolute magnitude of the interest differential increases—is qualitatively similar to the Krugman (1991) S-shape relationship between the exchange rate and the ‘fundamentals.’

Proposition 5.2 If the regulation bands are known to market participants and interventions are completely credible, then maintenance of UIP at the instant of the intervention gives

\[ B = -\left(\frac{\bar{r}}{\sigma_r^2}\right). \]  

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Substituting eq.(5.5) into eq.(5.4) gives

\[ s(t) = A - \left( \frac{\bar{r}}{\sigma_r} \right)^2 r(t) + \frac{r^3(t)}{3\sigma_r^2}. \] (5.6)

With a negative coefficient on \( r(t) \), the model predicts that high relative US interest rates are associated with dollar strength.\(^{10}\)

To see how UIP is maintained at times of interventions I apply Ito’s lemma to take the total differential of (5.4) to get the instantaneous change in the log exchange rate

\[ ds(t) = r(t)dt + \left( B + \frac{r(t)^2}{\sigma_r^2} \right) \sigma_r dz(t). \] (5.7)

When the interest differential lies strictly within the bands, \( dz(t) \sim N(0, \sqrt{dt}) \). Thus taking expectations on both sides of (5.7), it can be seen that \( E_t(ds(t)) = r(t) \) regardless of the value of \( B \). Conditional on the interest differential touching the upper band, the distribution of \( dz(t) \) becomes right truncated at zero with \( E(dz(t) | r(t) = \bar{r}) \approx -0.80 \) but because \( B = -\bar{r}^2/\sigma_r^2 \), the second term vanishes at this point which preserves UIP conditional on \( r(t) = \bar{r} \).

The restriction that UIP holds at all times including intervention episodes may, however, be unrealistically restrictive. Interventions in the dollar-deutschemark or

---

\(^{10}\)Altering the beliefs of market participants about the intervention rule results in modifications of the solution. For example, suppose participants believe that the authorities will intervene by setting the interest differential to 0 when one of the bands is hit, as in Flood and Garber (1991). Then maintaining UIP during instants of intervention gives \( B = -\frac{\bar{r}^2}{3\sigma_r^2} \), where the coefficient on \( r \) scaled down by 1/3. If the intervention rule lacks full credibility in the sense of Bertola-Caballero (1992), the coefficient is scaled down even further. In this setup, I begin with an initial band \([-\bar{r}, \bar{r}]\) of size \( b = 2\bar{r} \). Suppose that when the upper band \( \bar{r} \) is touched, there is a probability \( p \) that the authorities will realign instead of defending the initial band. \( 1 - p \) is the probability that they defend the initial band. If realignment occurs, the authorities establish a new band where the old upper band \( \bar{r} \) is now the lower band and the new upper band is \( \bar{r} + b \) and they place the interest differential in the middle of the new band. If defense takes place, the authorities place the interest differential back at the midpoint of the band as in the Flood-Garber intervention. In this environment, maintenance of UIP during instants of defense or realignment gives \( B = \left[ \frac{(8p-1)b^2}{3\sigma_r^2} \right] \).
dollar-pound markets take place irregularly and informally. They do not appear to conform to hard and fast rules or if so, the rules are quite complex. In the ensuing analysis, I will relax the assumption that market participants perfectly understand and completely believe the intervention policy. I do this by allowing values of $B \neq -\bar{r}^2/\sigma_r^2$. Conditional on being at the upper band, market participants believe ex ante that UIP will hold whereas the truth is

$$E[ds(t)|r(t) = \bar{r}] = \bar{r} dt - \left( B + \frac{\bar{r}^2}{\sigma_r^2} \right) \sigma_r(0.8).$$

(5.8)

However, only central bankers know the truth and the illegality of trading on such inside information prevents them from exploiting the potentially huge profit opportunities that they create.

In this alternative environment, I restrict market participants from learning the central bank rules so that interventions always take participants by surprise. An alternative strategy for incorporating this idea would be to build a model of nonsystematic interventions that are sufficiently irregular that agents maintain diffuse priors over the interventions.\textsuperscript{11}

### 5.3 Big News in Discretized Observations

Schotman \textit{et al.} (1997) argue that the forward premium anomaly is a statistical artifact. They propose and empirically motivate a parametric representation of UIP that stresses the role of heavy-tailed exchange rate returns in creating pitfalls for

\textsuperscript{11}Dominguez (2001) provides a narrative account of the Fed intervention policy and evidence on market discovery of intervention episodes. See also Klein and Lewis (1993) who present a model in which market participants update their prior probabilities about the interventions as Bayesian and learn about the bands over time. An analysis of learning is beyond the scope of this paper.
regression tests of UIP,
\[ s_{t+1} - s_t = r_t(1 + \epsilon_{t+1}) + v_{t+1}, \]
where \( \epsilon_{t+1} \) and \( v_{t+1} \) are conditionally zero-mean innovations. UIP can be seen to hold under this representation by taking expectations conditional on time-\( t \) information. The multiplicative error \( r_t\epsilon_{t+1} \) is called “big news” whereas the additive error \( v_{t+1} \) is regular news. OLS is consistent in this setting but Schotman et al. (1997) find that the big news contributes to poor small sample properties and note that a slow rate of convergence to the asymptotic distribution may cause inference based on the limit distribution to be undependable.\(^{12}\)

Notice also that the big news is conditionally heteroskedastic. A theory for big news is given by continuous-time UIP model. Integrating (5.7) gives the implied discrete-time depreciation
\[ s(1) - s(0) = r(0) + \sigma_r \left[ \int_0^1 z(t) dt - z(0) \right] + \frac{1}{\sigma_r} \int_0^1 r^2(t) dz(t) + B\sigma_r \int_0^1 dz(t). \quad (5.9) \]
The terms labeled (a), (b), and (c) are separate components of the true regression error. Further decomposition of the discrete-time change gives
\[ s(1) - s(0) = r(0)[1 + \epsilon(1)] + v(1), \quad (5.10) \]
where
\[ \epsilon(1) = \frac{1}{\sigma_r} \left[ r(0) - 2\sigma_r z(0) \right] \int_0^1 dz(t) + \frac{1}{2} \int_0^1 z(t) dz(t), \quad (5.11) \]
\(^{12}\) Big news is only one ingredient in Schotman et al. (1997)’s story of poor small sample properties of OLS. They also assume that the interest differential is drawn from a leptokurtic distribution. In work along similar lines, Baillie and Bollerslev (2000) demonstrate that the 95 percentile range of the OLS empirical distribution from regressing the exchange rate return on the interest differential is (-5.14,10.9) when the conditional volatility in the interest differential follows their calibrated fractionally integrated generalized ARCH process.
\[
v(1) = \sigma_r z^2(0) \int_0^1 dz(t) + \sigma_r \int_0^1 z(t) dt - 2\sigma_r z(0) \int_0^1 z(t) dz(t)
\]

\[
+ \sigma_r \int_0^1 z^2(t) dz(t) - \sigma_r z(0) + B\sigma_r \int_0^1 dz(t).
\]

(5.12)

The distribution of the big news is leptokurtic and conditionally heteroskedastic due to the dependence of \( E_0[r(0)^2 \epsilon(1)^2] \) on \( r(0) \). The conditional heteroskedasticity arises endogenously since the underlying process for \( r(t) \) is homoskedastic.\(^\text{13}\)

Several of the error components in eqs. (5.11) and (5.12) have more familiar representations. The term labeled (i) is \( \int_0^1 dz(t) \sim N(0,1) \), the term labeled (ii) is \( \int_0^1 z(t) dz(t) \sim \chi^2(1) - 1 \), which has a skewed distribution, and the term labeled (iii) is \( \int_0^1 z(t) dt \sim N \left( 0, \frac{1}{\tau} \right) \). The term labeled (iv) is \( \int_0^1 z^2(t) dz(t) \) which is nonstandard. I investigate its properties by simulation and find it to be zero-meaned with a symmetrically leptokurtic distribution with a coefficient of excess kurtosis equal to 86.56.

When we work with discrete-time sampled data, we are interested in regressing the weekly depreciation on the 1-week interest differential, \( R(0,1) \), and not on the instantaneous return differential \( r(0) \). Appealing to the expectations hypothesis of the term structure of interest rates, \( R(0,1) = E_t(\int_0^1 r(u) du) = r(0) \int_0^1 du = r(0) \), the discretized representation (5.9) corresponds to the regression run on the data even though \( r(0) \) is the instantaneous yield.

\(^\text{13}\)In their simulations, Schotman et al. (1997) assumed that innovations to the interest differential are drawn from a fat-tailed distribution.
5.4 Properties of the Simulated Observations

Qualitatively, the model predicts both ARCH effects and possibly a forward premium bias, if not an anomaly. To make a quantitative assessment of model’s capacity to explain these phenomenon I conduct a series of Monte Carlo simulations with parameter values set equal to the SMM estimates from US-German data (The results for the model calibrated to US-UK data estimates are qualitatively similar and are not reported to save space). Each experiment begins with a realization of the Euler-approximate continuous-time exchange rate and interest differential where each weekly time interval is divided into 84 subintervals. The initial value of the interest differential is drawn from the uniform distribution with support $[-\bar{r}, \bar{r}]$. Next, I draw 1200 observations at weekly intervals, which conforms to the number of data points in the sample, and use them to calculate the statistics used to characterize the data in Tables 3.1 – 3.2.

Two alternative specifications of agent’s knowledge and beliefs of central bank intervention are considered for the simulation experiments. For both specifications, I set $(\sigma_r, \bar{r}) = (0.576, 5.632)$ which are the 3-moment SMM estimates. In the first specification, which I call perfect UIP, market participants know and understand the central bank’s intervention rule. In this world, UIP holds continuously even at the instants of intervention as in eq.(6.6) and $B = -(\bar{r}/\sigma_r)^2$. This specification that UIP holds at all times including instances of intervention may, however, be too strong and unrealistic. I consider alternative specification to capture the facts that i) interventions in the exchange markets take place irregularly and informally, ii) regulation bands are not directly observable to market participants, and iii) spreading insider information about the regulation is illegal. In the second specification, which I call
imperfect UIP, intervention always takes market by surprise and UIP is violated only at those instants when intervention takes place.

For the imperfect UIP specification, I directly estimate $B$ from the data. Table 5.4 shows the estimation results using eq.(5.4) minimize the quadratic distance between the set of 3 moments conditions $(E \Delta s_t, E \Delta s_t^2, E \Delta s_t r_{t-1})'$ from the simulated observations and the data. As eq.(6.6) predicts, the negative value of the point estimate offers evidence favorable to UIP. In the imperfect UIP model, UIP holds most of the time but occasional violation of UIP occurs at a time of intervention. To capture occasional deviations from UIP, I set $B = 102.042$ which is the 3 moments condition SMM estimate.

Table 5.1 reports the median and 95 percentile range of the statistics from Table 3.1. Both models are seen to produce observations that exhibit strong ARCH effects in ex post deviations from UIP. The median values of the LM statistics are large, the deviations from UIP are symmetric, and they exhibit excess kurtosis. The models also produce persistent dynamics in the log exchange rate. The median volatility of the exchange rate return from the perfect UIP model is roughly 20 times larger than the volatility of the interest differential, but falls short of the volatility in the data. The volatility generated by the imperfect UIP model is better able to match the volatility in the data.

In the perfect UIP model, the potential role for big news to account for the forward premium is limited to its ability to generate sizable departures of the small sample distribution of the OLS estimator from its limit distribution. The distributional distortion from the big news is not sufficiently large to resolve the forward premium anomaly. The slope coefficient is median-biased, but the bias is upwards and in the
wrong direction to explain the forward premium anomaly. More importantly, the 95 percentile range of the OLS distribution does not include 0.

The simulated observations from the imperfect UIP model does generate the forward premium anomaly. The median of the OLS distribution with this model serving as the data generating mechanism lies below the point estimate from the data. With some experimentation, a value of $B$ can be found which allows an exact match. I note that the forward premium anomaly is heightened (indeed only appears) during these instances of intervention.\(^\text{14}\) The econometric implications are clear. If the sample contains observations drawn from dates in which central banks are managing foreign exchange interventions, the truncation of the error distribution creates omitted variables bias.

Table 5.5 reports results from a closer examination of the implied ARCH effects. The table shows the median and the 95 percentile range of the LM statistics and GARCH(1,1) estimates built from simulated observations sampled at weekly and at monthly intervals. As in the data, the simulated observations from the perfect UIP model exhibit strong ARCH effects at the weekly horizon. Here, the estimated value from the data $\hat{\delta} = 0.080$ lies below the median value of $\delta = 0.156$ but lies within the 95 percentile range. The implied ARCH effects also weaken under time addition as the median value of $\delta$ drops to 0.030 when the observations are sampled monthly.

The ARCH effects generated by the imperfect UIP model are less pronounced but more precise in a statistical sense. The 95 percentile range for $\delta$ is tighter and the median value 0.019 lies slightly closer to the point estimate from the data than was

\(^{14}\)The model’s prediction that deviations from UIP are heightened during intervention episodes is consistent with evidence in Baillie and Osterberg (2000) that central bank foreign exchange interventions have a significant effect on the deviation from UIP.
the case in the perfect UIP model. The implied ARCH effects in monthly sampled observations are again much less pronounced than those present in weekly sampled observations.

How frequently do violations occur in producing this result? Using the estimated model, we calculated the unconditional probability of touching either of the bands to be 0.081. Over the course of a sample of 23 years, this amounts to interventions in approximately 98 out of the total 1200 weekly observations. This is quite similar to the intervention record would suggest. Dominguez (2001) reports the record of Fed interventions from 1987 to 1995 from which we infer that the frequency of Fed interventions over that time period in either of two foreign exchange markets—dollar-yen or the dollar-deutschemark—was 0.084.
<table>
<thead>
<tr>
<th>Moments</th>
<th>Germany 3</th>
<th>Germany 5</th>
<th>UK 3</th>
<th>UK 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_r$</td>
<td>0.576</td>
<td>0.366</td>
<td>0.971</td>
<td>0.803</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(8.274)</td>
<td>(11.664)</td>
<td>(8.334)</td>
<td>(12.568)</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>5.632</td>
<td>5.115</td>
<td>5.268</td>
<td>5.288</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(8.761)</td>
<td>(8.369)</td>
<td>(9.233)</td>
<td>(11.712)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>0.008</td>
<td>8.474</td>
<td>0.024</td>
<td>4.095</td>
</tr>
<tr>
<td>p-value</td>
<td>0.930</td>
<td>0.037</td>
<td>0.877</td>
<td>0.251</td>
</tr>
</tbody>
</table>

Notes: 3 moments used in estimation are $\Delta r(t), \Delta r^2(t), r(t)r(t-1)$. 5 moments used in estimation, add $\Delta r^3(t), \Delta r^4(t)$.

Table 5.1: SMM Estimates of Interest Rate Process: Regulated Brownian Motion
<table>
<thead>
<tr>
<th>Moments</th>
<th>Germany</th>
<th>UK</th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_r$</td>
<td>0.576</td>
<td>0.366</td>
<td>0.971</td>
<td>0.803</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>5.632</td>
<td>5.115</td>
<td>5.268</td>
<td>5.288</td>
</tr>
<tr>
<td>$\sqrt{\text{Var}(r)}$</td>
<td>3.300</td>
<td>2.980</td>
<td>3.102</td>
<td>3.139</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.985</td>
<td>0.992</td>
<td>0.950</td>
<td>0.967</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.852</td>
<td>0.922</td>
<td>0.595</td>
<td>0.718</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.729</td>
<td>0.848</td>
<td>0.355</td>
<td>0.522</td>
</tr>
</tbody>
</table>

Table 5.2: Properties of Interest Rate Processes: Regulated Brownian Motion
Table 5.3: SMM Estimation of Undetermined Constants in Exchange Rate Solution: Regulated Brownian Motion

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th></th>
<th></th>
<th>UK</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J-statistic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>|     | 102.042 | 97.545 | 67.235 | 67.351 |
| J-statistic | 43.089 | 61.230 | 33.650 | 43.995 |
| p-value     | 0.000   | 0.000   | 0.000   | 0.000   |</p>
<table>
<thead>
<tr>
<th></th>
<th>Perfect UIP</th>
<th>Imperfect UIP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5% 50% 97.5%</td>
<td>2.5% 50% 97.5%</td>
</tr>
<tr>
<td>LM: $\chi^2(1)$</td>
<td>242.735 299.270 352.049</td>
<td>321.607(^a) 366.263 405.472</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.227 -0.002 0.230</td>
<td>-0.160 0.002 0.164</td>
</tr>
<tr>
<td>excess kurtosis</td>
<td>0.526 1.211 2.313</td>
<td>-0.047 0.276 0.717(^a)</td>
</tr>
<tr>
<td>VR(2)</td>
<td>0.927 0.992 1.060</td>
<td>0.895 0.956 1.017</td>
</tr>
<tr>
<td>VR(12)</td>
<td>0.712 0.920 1.174</td>
<td>0.602 0.767 0.957(^a)</td>
</tr>
<tr>
<td>VR(24)</td>
<td>0.586 0.846 1.179</td>
<td>0.471 0.663 0.898(^a)</td>
</tr>
<tr>
<td>$\sigma_{\Delta_s}$</td>
<td>34.503 39.912 44.673(^a)</td>
<td>69.858 74.912 79.798</td>
</tr>
<tr>
<td>$\rho_{\Delta_s}(1)$</td>
<td>-0.072 -0.007 0.061</td>
<td>-0.105 -0.043 0.018</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>2.270 3.088 3.628</td>
<td>2.270 3.088 3.628</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.969 0.983 0.989</td>
<td>0.969 0.983 0.989</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.682 0.828 0.896</td>
<td>0.682 0.828 0.896</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.447 0.688 0.819</td>
<td>0.447 0.688 0.819</td>
</tr>
<tr>
<td>LM($r$): $\chi^2(1)$</td>
<td>355.621(^b) 394.076 430.039</td>
<td>355.621(^b) 394.076 430.039</td>
</tr>
<tr>
<td>skewness($r$)</td>
<td>-0.877 0.008 0.881</td>
<td>-0.877 0.008 0.881</td>
</tr>
<tr>
<td>excess kurtosis($r$)</td>
<td>-1.428 -0.966 0.461(^a)</td>
<td>-1.428 -0.966 0.461(^a)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.670(^b) 1.105 2.104</td>
<td>-4.020 -2.156 -1.565(^a)</td>
</tr>
<tr>
<td>Asy-t</td>
<td>-1.732 0.399 2.511</td>
<td>-5.238 -4.156 -3.793(^a)</td>
</tr>
</tbody>
</table>

Note: \(^a\)–less than estimate from data. \(^b\)–greater than estimate from data.

Table 5.4: Properties of Exchange Rate Processes When Interest Differential Follows Regulated Brownian Motion Calibrated to German 3-moment SMM Estimates (Percentiles of Monte Carlo Distributions from 5000 Replications of 1200 Weekly Observations)
<table>
<thead>
<tr>
<th></th>
<th>Perfect UIP</th>
<th></th>
<th>Imperfect UIP</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Weekly</td>
<td>Monthly</td>
<td>Weekly</td>
<td>Monthly</td>
</tr>
<tr>
<td>LM: ( \chi^2(1) )</td>
<td>299.270</td>
<td>76.181</td>
<td>366.263</td>
<td>98.017</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GARCH(1,1): ( h_{t+1} = \omega + \delta u_t^2 + \gamma h_t )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.156</td>
<td>0.030</td>
<td>0.019</td>
<td>0.005</td>
</tr>
<tr>
<td>(5%:95%)</td>
<td>(0.074:0.288)</td>
<td>(0.0001:0.351)</td>
<td>(0.008:0.034)</td>
<td>(0.0003:0.027)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.834</td>
<td>0.947</td>
<td>0.977</td>
<td>0.994</td>
</tr>
<tr>
<td>(5%:95%)</td>
<td>(0.724:0.910)</td>
<td>(0.572:0.998)</td>
<td>(0.942:0.991)</td>
<td>(0.966:0.999)</td>
</tr>
</tbody>
</table>

Table 5.5: ARCH Effects and Return Horizon: Median Values and 90 Percentile Range
Figure 5.1: A Realization of the Regulated Brownian Motion: Observations Sampled Weekly with $\sigma_r = 0.576$, $\bar{r} = 5.632$, $\delta = 1/84$.

Figure 5.2: Nonlinear Relation between Log Exchange Rate and Interest Differential: Plots of Equation (5.4) with Alternative Values of $B$.  

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CHAPTER 6

UIP IN CONTINUOUS-TIME: REGULATED JUMP-DIFFUSION INTEREST DIFFERENTIAL

Let $r(t)$ be the instantaneous yield differential between domestic and foreign-currency denominated debt instruments with identical default risk. Then UIP in continuous time is

$$\mathbb{E}_t[ds(t)] = r(t)dt,$$

(6.1)

where $\mathbb{E}_t[\cdot]$ is the expectation conditional on information available at instant $t$ and $ds(t)$ is the forward differential of $s(t)$. In this chapter, I consider regulated jump-diffusion model for the interest differential to solve the stochastic differential equation given as eq.(6.1).

6.1 A Regulated Jump-Diffusion Model of the Interest Differential

The regulated Brownian motion interest differential in Chapter 5 shows potential in matching moments of the data. But it is too “well behaved” in a sense that it generates insufficient nonnormality to fully resolve the forward premium anomaly as an artifact of data. Here, I model the interest differential as a regulated jump-diffusion process to capture more of the empirical features of the data.

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Changes in the interest differential can be modeled with two components of the jump diffusion suggested by Merton (1976). The first component captures ‘normal’ variations in the process that is represented by fixed volatility. The second component captures ‘abnormal’ shocks that cause discrete jumps in the process due to abrupt changes in market conditions such as a shift in monetary policy. Again the regulation of the interest differential process is motivated by occasional foreign exchange intervention by central banks. The jump portion is also empirically motivated. Several studies report that the jump behavior of the exchange rate and the interest differential is important empirical feature of the data. Jorion (1988) finds that exchange rates exhibit systematic discontinuities and they are more significant in the exchange market than in the stock market. Chan, Karolyi, Longstaff and Sanders (1992) find that interest rates show level dependent volatility which is not captured by commonly used simple diffusion processes. Das (1998) argues that the jump effects are more prevalent in regulated intervention environments such as the interest rate and foreign exchange markets, and jumps can explain large portion of the interest rate variation.

The regulated jump-diffusion process captures both normal and abnormal variations by incorporating Brownian motion with Poisson process \( dq(t) \)

\[
dq(t) = \begin{cases} 
0 & \text{with probability } 1 - 2\lambda dt \\
1 & \text{with probability } \lambda dt \\
-1 & \text{with probability } \lambda dt
\end{cases}
\]  

(6.2)

where \( \lambda \) is the intensity parameter, which denotes the expected number of positive or negative jumps per unit time.

Let \( \underline{r} < \bar{r} \) be fixed constants and suppose that the monetary authorities target to maintain \( r(t) \) within the bands \([\underline{r}, \bar{r}]\). When \( r(t) \) lies inside the bands, it evolves
according to the regulated jump-diffusion,

\[ dr(t) = \sigma_r dz(t) + kdq(t) \]  

(6.3)

where \( dz(t) \) is a standard Wiener process, \( \sigma_r \) is the instantaneous volatility in \( dr(t) \), 
\( k \) is the jump size, and \( dq(t) \) is a Poisson process with intensity \( \lambda \). Also \( dz(t) \) and 
\( dq(t) \) are assumed to be independent. To simplify the exposition, I assume that the 
reflecting barriers are symmetric, \( \bar{r} = -\bar{r} \), the intensity parameter \( \lambda \) and jump size 
\( k \) are constants, and arrival rate for positive and negative jumps are the same.

In this chapter, I consider discrete intervention of monetary authorities rather than 
Krugman (1991) style infinitesimal intervention. The authorities engage in discrete 
intervention when \( r(t) \) exits the regulation bands. The intervention in the paper 
allows temporary exit of \( r(t) \) from the regulation bands. With the existence of jumps 
in \( r(t) \), the authorities cannot strictly maintain \( r(t) \) inside the bands all the time 
but allow temporary exit of \( r(t) \) at the instance of hitting the bands as a result of a 
jump. The monetary authorities engage in intervention and set \( r(t) \) to either upper 
or lower regulation bands when \( r(t) \) jumps and exits from the regulation bands. If the 
authorities observe that the current interest differential is equal or greater (smaller) 
than the upper (lower) regulation bands, they engage in intervention and set the 
interest differential equal to the upper (lower) bands.\(^{15}\) In other times, when \( r(t) \) is 
interior to the bands, the authorities focus on domestic objectives and the interest 
differential evolves randomly, being subject to different sources of shocks.

\(^{15}\)When current interest differential \( r(t) \) exits from the regulation bands, \( r(t) \geq \bar{r} \) or \( r(t) \leq -\bar{r} \), the 
intervention occurs by setting \( r(t + dt) = \bar{r} \) when \( r(t) \geq \bar{r} \) and \( r(t + dt) = -\bar{r} \) when \( r(t) \leq -\bar{r} \). The 
intervention rule in the paper coincides with the intervention mechanism of the European Monetary 
System (EMS). The EMS mechanism forces any two participating countries to maintain bilateral 
exchange rates within \( \pm 2.25 \) to \( \pm 15 \) percent about a central parity. If the exchange rate reaches or 
exits from either bands, central banks of both countries intervene to return the exchange rate back 
to the bands.
We need to consider two types of bands for intervention—regulation and trigger bands. The first bands are ‘regulation bands’ \([-\bar{r}, \bar{r}]\) where the authorities want to maintain the interest differential as described above. The idea is if \(r(t) > \bar{r} - k\), \(i.e.,\) \(r(t)\) is within \(k\) of \(\bar{r}\), a jump could place it outside of \(\bar{r}\) which will trigger an intervention. The second bands are ‘trigger bands’ where the monetary authorities conduct interventions when the interest differential lies in the bands with an occurrence of jump. They are \([\bar{r} - k, \bar{r} + k]\) for upper and \([-\bar{r} - k, -\bar{r} + k]\) for lower trigger bands.

Figure 6.1 illustrates intervention mechanism and three possible movements of \(r(t)\) associated with jumps at the upper bands when \(\bar{r} - k < r(t) < \bar{r}\).\(^{16}\) The first possibility is an arrival of positive jump. In this case, \(r(t)\) jumps outside the regulation bands at the instant of positive jump and the monetary authorities intervene immediately to set \(r(t + dt) = \bar{r}\). The second possibility is no occurrence of jump. In this case, \(r(t)\) evolves following regulated Brownian motion and the central bank engage in infinitesimal intervention to prevent \(r(t)\) from exiting the regulation bands. The last possibility is an occurrence of negative jump in \(r(t)\). In this case, however, even with the negative jump the interest differential remains inside the regulation bands and no intervention occurs.

The parameters \(\sigma_r, \bar{r}, k, \text{and } \lambda\) of the interest differential processes are estimated using the simulated method of moments (SMM). I begin by dividing each of the \(T\) weekly observations into \(N\) subintervals, of length \(\delta_N = 1/N \simeq dt\), and use Euler’s method to approximate the continuous-time model

\[
    r_j = r_{j-1} + \sigma_r \epsilon_j \sqrt{\delta_N} + kq_j, \tag{6.4}
\]

\(^{16}\)Due to symmetry of the regulation bands, it is sufficient to examine behavior of the interest differential at one of the bands.
where \( j = 1, \ldots, NT \), \( \epsilon_j \overset{iid}{\sim} N(0, 1) \), \( k \) is the jump size, \( q_j \) is Poisson event with intensity \( \lambda \), and \( \sigma_r \) is the weekly standard deviation of the instantaneous yield differential \( r(t) \). The parameters \( \sigma_r, \bar{r}, k, \lambda \) are chosen to minimize the quadratic distance between the implied moments generated from simulations of (6.4) and the sample moments of the data.

Table 6.1 reports estimates using 5 moments condition \( (E\Delta r_t, E\Delta r_t^2, Er_tr_{t-1}, E\Delta r_t^3, E\Delta r_t^4)' \). The estimates of Poisson arrival rate \( \hat{\lambda} \) from data are 0.0050 for Germany and 0.0035 for the UK and imply that mean frequency of 3.6 and 2.5 jumps per year for the German and the UK interest differentials respectively. The UK interest differential, however, has much higher jump size than German interest differential that the estimates of jump size \( \hat{k} \) are 1.748 percent for Germany and 4.684 percent for the UK. This also conforms with Figure 3.2 that the UK interest differential has bigger jumps than German interest differential before 90s. In summary, with p-values for the test of the over identifying restrictions of 0.396 for Germany and 0.990 for the UK, the regulated jump-diffusion does a good job in matching the set of 5 moments.

Table 6.2 shows some of the properties implied by the estimated regulated jump-diffusion models. The implied volatility of the interest differential is 3.200 for Germany and 3.140 for the UK while the volatility from the data is 3.229 and 3.122 for Germany and the UK respectively. As can be seen, the estimated models provide a good account for the persistence and the volatility of the interest differential data. Figure 6.2 shows a realization of the regulated jump-diffusion interest differential using the 5 moment estimates for Germany with observations sampled at weekly intervals.
6.2 Exchange Rate Solution

Proposition 6.1 If the interest differential follows regulated jump-diffusion, a family of solutions to eq.(6.1) is

$$s(t) = A + B[r(t - dt)]r(t) + \frac{r^3(t)}{3(\sigma_r^2 + 2\lambda k^2)},$$

(6.5)

where $A$ is constant and $B[r(t - dt)]$ is a function of $r(t - dt)$ to be determined by auxiliary conditions.

The constant $A$ will depend on initial conditions and on currency units, and the exchange rate will be decreasing in the interest differential if $B[r(t - dt)]$ is sufficiently negative.

Solving the differential equation driven by the continuous-time UIP and regulated jump-diffusion interest differential is made complicated by expected change in the exchange rate due to jumps in the interest differential. Existing studies impose auxiliary assumptions to maneuver jumps. Impossibility of perfect hedging is one downside of the jump-diffusion model. Black-Scholes type risk-free hedging is not possible with jump-diffusion, one needs to assume jumps should not be priced or take care of market price of risk to solve option pricing formula. Ball and Roma (1993) impose specific restrictions on the model parameters for the jump size to derive tractable representation for the transition densities for the exchange rates. Dumas et al. (1995) employ jump process to model regulation bands realignment mechanism. They impose homogenous exchange rate function assumption to derive exchange rate solution. With homogenous property, the various log exchange rate bands of fixed width are located along a 45 degree diagonal. However, imposing auxiliary assumptions tend to lose in a loss of tractability of the solution or encounter empirical difficulties as a result of
auxiliary assumptions. Instead of imposing such restrictions, I employ a second order Taylor expansion to maneuver the expected change in the exchange rate and provides a simple and tractable approximate solution technique.

**Proposition 6.2** Let \( g[r(t - dt)] \) be the distance between the interest differential at instant \((t - dt)\) and the regulation bands, \( g[r(t - dt)] = r(t - dt) - \bar{r} \) and \( 0 \leq g[r(t - dt)] \leq k \). If the regulation bands are known to market participants and intervention policy is completely credible, then maintenance of UIP at the instant of the intervention implies

\[
B[r(t - dt)] = -\left[ \frac{3\bar{r}^2 + 3\bar{r}g[r(t - dt)] + g^2[r(t - dt)]}{3(\sigma_r^2 + 2\lambda k^2)} \right].
\]

As we can see from eq.(6.6), the coefficient \( B \) is not uniquely determined but is a function of interest differential \( r(t - dt) \) and \( B[\cdot] \) is increasing in \( r(t - dt) \). Substituting \( B[r(t - dt)] \) into eq.(6.5) gives

\[
s(t) = A - \left[ \frac{3\bar{r}^2 + 3\bar{r}g[r(t - dt)] + g^2[r(t - dt)]}{3(\sigma_r^2 + 2\lambda k^2)} \right] r(t) + \frac{r^3(t)}{3(\sigma_r^2 + 2\lambda k^2)}. \tag{6.7}
\]

Figure 6.3 shows various exchange rate solutions for alternative values of \( \lambda \). Non-linearity in the solution exhibits an ‘inverted S-shape’ between interest differential (fundamental) and the exchange rate. The inverted S-shape implies that high US interest rates are associated with dollar strength and the exchange rate becomes more volatile when market participants expect intervention to occur. The intervention in this situation is a volatility ‘shaker’ not a ‘stabilizer’ even though the authorities are regulating exchange rates (equivalently fundamentals) not exiting from the bands.
6.3 Big News in Discretized Observations

The exchange rate solution under regulated jump-diffusion is similar to the solution with regulated Brownian motion interest differential. Both nonlinear exchange rate solutions are cubic function of the interest differential but coefficients on \( r(t)^3 \) is scaled down with the jump parameters \( \lambda \) and \( k \) under the regulated jump-diffusion. In Chapter 5, I show that discretization of the continuous-time exchange rate solution to conform with sampling intervals of the data induces a time aggregation error. In a similar fashion, the solution under regulated jump-diffusion induces aggregation error that exhibits ARCH effects and has features corresponds to the “big news” representation of Schotman et al. (1997).

To see how UIP is maintained at the instants of interventions, I apply Ito’s lemma to (6.5) to get instantaneous change in the log exchange rate

\[
 ds(t) = \left( \frac{\sigma_r^2}{\sigma_r^2 + 2\lambda k^2} \right) r(t) dt + \left( B[r(t - dt)] + \frac{r^2(t)}{3(\sigma_r^2 + 2\lambda k^2)} \right) \sigma_r dz(t) + \phi(\lambda, k, r, t),
\]

where the last term \( \phi(\lambda, k, r, t) \) denotes change in the exchange rate due to discrete jump in the interest differential. When the interest differential lies strictly within the bands, \( dz(t) \sim N(0, dt) \), it can be seen that

\[
 \mathbb{E}_t[ds(t)] = \left( \frac{\sigma_r^2}{\sigma_r^2 + 2\lambda k^2} \right) r(t) dt + \lambda \left( \frac{2k^2}{\sigma_r^2 + 2\lambda k^2} \right) r(t) dt = r(t) dt
\]

regardless of the value of \( B[r(t - dt)] \). Eq.(6.8) also shows a correlation between \( r(t) \) and \( \phi(\lambda, k, r, t) \).

Let \( \left( \frac{1}{\sigma_r^2 + 2\lambda k^2} \right) = \alpha \). Integration of (6.8) gives the implied discrete-time depreciation

\[
 s(1) - s(0) = \int_0^1 ds(t)
\]

66
Further decomposition of the discrete-time change gives

\begin{align*}
\text{Further decomposition of the discrete-time change gives}

s(1) - s(0) &= r(0) \\
&\quad + \alpha \sigma_r^2 \left( \sigma_r \left[ \int_0^1 z(t)dt - z(0) \right] + k \left[ \int_0^1 q(t)dt - q(0) \right] \right) \\
&\quad + \alpha \sigma_r \int_0^1 r^2(t)dz(t) + B[r(t - dt)]\sigma_r \int_0^1 dz(t) \\
&\quad + 2\alpha \lambda k^2 \left( \sigma_r \left[ \int_0^1 z(t)dt - z(0) \right] + k \left[ \int_0^1 q(t)dt - q(0) \right] \right) \\
&\quad + \eta(\lambda, k, r, t), \tag{6.11}
\end{align*}

where the terms \((a', b', c', d')\) are separate components of the regression error derived from \((a, b, c, d)\) in (6.10) respectively and \(\eta(\lambda, k, r, t)\) represents time aggregation error. After decomposition it can be seen that \(r(0)\) and error terms are correlated and generating conditional heteroskedasticity. However, further decomposition of the error term structure is technically demanding. I now turn to the simulation methods to explore the error dynamics.

### 6.4 Properties of the Simulated Observations

The primary concern of the study is to understand the basis of nonnormality in the interest differential and its relationship with the forward premium anomaly. To
explore the explanatory power of the model, I conduct Monte Carlo simulation experiments with parameter values set equal to the SMM estimates with US-German data. (The results for the model calibrated to US-UK data estimates are qualitatively similar). Each experiment begins with a realization of the Euler-approximate continuous-time exchange rate and interest differential where each weekly time interval is divided into 14 subintervals.\(^\text{17}\) The initial value of the interest differential is drawn from the uniform distribution with support \([-\bar{r}, \bar{r}]\). Next, I draw 1200 observations at weekly intervals, which conforms to the number of data points in the sample, and use them to calculate the statistics used to characterize the data in Tables 3.1 – 3.2.

Two alternative specifications of agent’s knowledge and beliefs of central bank intervention are considered for the simulation experiments. In the first specification of perfect UIP, market participants know and understand the central bank’s intervention rule. In this world, UIP holds continuously even at the instants of intervention as in eq.(6.6). In the second specification of imperfect UIP, intervention always takes market by surprise and UIP is violated only at those instants when intervention takes place.

For the imperfect UIP specification, I directly estimate \(B[r(t - dt)]\) from the data. Table 6.3 shows the estimation results using eq.(6.5) minimize the quadratic distance between the set of 5 moments conditions \((E\Delta s_t, E\Delta s_t^2, E\Delta s_t r_t, E\Delta s_t^3, E\Delta s_t^4)'\) from the simulated observations and the data. As eq.(6.6) predicts, the negative value of the point estimate offers evidence favorable to UIP. In the imperfect UIP model, UIP

\(^{17}\)I chose 14 subintervals because it is used to estimate parameters in interest differential model. Since regulated jump-diffusion model parameters are sensitive to the subinterval, particularly \(\lambda\), subinterval in simulation experiments should be equal to the subinterval in parameter estimation.
holds most of the time but occasional violation of UIP occurs at a time of intervention. To capture occasional deviations from UIP, I set \( B[r(t - dt)] = -120.993 \) which is the 5 moments condition SMM estimate. For both specifications, I set \( (\sigma_r, \bar{r}, k, \lambda) = (0.235, 5.228, 1.748, 0.005) \) which are the 5-moment condition SMM estimates.

Table 6.4 reports the median and 95 percentile range of the statistics presented in Table 3.1. Both perfect and imperfect UIP models are able to produce observations that exhibit ARCH effects in ex post deviations from UIP. The median values of the LM statistics show that the ARCH effects are more profound in the perfect UIP model than in the imperfect model. The deviations from UIP are symmetric, and they exhibit large excess kurtosis. The median coefficients of excess kurtosis from perfect and imperfect model are about 6 times larger than that of the data and indicate that the error distributions are heavily fat-tailed relative to the normal distribution. These fat-tails in the distribution conform with Schotman et al. (1997)’s “big news” representation of the extreme valued forecast error that induces ex post deviation from UIP.

The models also produce persistent dynamics in the log of exchange rate. The imperfect UIP model, however, generates smaller exchange rate return volatility than the perfect UIP model does. While the volatility of the exchange rate return from the perfect UIP model is roughly 45 times larger than that of the interest differential, the exchange rate return volatility from the imperfect model is only 14 times larger than the interest differential volatility and falls short of the actual volatility from the data. This higher volatility in the perfect UIP model is perhaps due to the inverted S-shaped exchange rate solution as the exchange rate shows high volatility when it approaches to the regulation bands.
The perfect UIP model does a good job in accounting volatility of the data, but does not produce sufficient small sample distortion of the OLS slope distribution to explain the forward premium anomaly. The slope coefficient from the perfect UIP model is median-biased and the bias is in the wrong direction to explain the forward premium anomaly. Not only the 95 percentile range of the OLS distribution does not include 1, but also the 95 percentile range of the slope coefficient from the perfect UIP model is heavily skewed to the right with median value of 4.103.\(^\text{18}\)

What is the cause of the upward bias? The higher upward bias implies that there should be a positive correlation between current interest differential and the forecast error in eq.(3.1). This positive correlation can be seen with expected movements of the exchange rate associated with jumps around the regulation bands. Figure 6.4 illustrates expected movements of the exchange rates associated with jumps in the interest differential when \(r(t) > \bar{r} - k > 0\), \(i.e., \ r(t)\) is within \(k\) of \(\bar{r}\). With positive jump, \(r(t)\) will exit from the upper bands which leads appreciation of domestic currency and negative forecast error. With positive jump, however, the monetary authority will intervene immediately and set \(r(t+dt) = \bar{r}\). In this case, the movement of the interest differential is upward-truncated and the magnitude of appreciation is also truncated. When negative jump occurs, the interest differential will fall and leads depreciation of the home currency which induces huge positive forecast error. However, negative jump will not trigger intervention and leads huge depreciation of the home currency. Although the probability of positive and negative jumps are the same, the effects of the negative jump is much bigger than the effect of the positive

\(^{18}\text{Monte Carlo experiments with 5,000, 100,000 and 1,000,000 replications produce similar results which serve evidence against the small sample bias.}\)
jump and the expected movement of $r(t)$ is downward. The changes in $r(t)$ are not equally transmitted to $s(t + dt)$ because i) upward (downward) jumps are truncated by intervention when the interest differential is close to upper (lower) bands, and ii) adjustment of the exchange rate according to the jumps in the interest differential is nonlinear and asymmetric as illustrated in Figure 6.4.

Using German data parameter estimates, Table 6.5 shows the correlation between $r(0)$ and the regression error components $(a')$, $(b')$, $(c')$, and $(d')$ and implied slope coefficient for the perfect UIP model. It is clear from table 6.5 that the major cause of the bias is discretization error $\eta(\lambda, k, r, t)$. The slope coefficients in projection of $(a', b', c', d')$ onto $r(0)$ are (-0.0041, -1.7392, 2.2341, -0.0023) and their contribution to the bias are small in aggregate while the remaining huge bias goes to the time aggregation error $\eta(\lambda, k, r, t)$.

In contrast to the perfect UIP model, the simulated observations from the imperfect UIP model does generate forward premium anomaly. Under imperfect UIP model, the forward premium anomaly is only heightened at the points of interventions.\(^{19}\) If the sample contains observations drawn from the dates in which central banks are conducting interventions, the truncation of the error distribution creates omitted variable bias. Without full information about the central bank intervention, market participants believe truncated portion of the distribution is still feasible even though it is not when central bank intervene in the market. The median of the OLS distribution from the imperfect model lies above the point estimate from the data. Although the 95 percentile range of the distribution lies above the point estimates

\(^{19}\)The model’s prediction that deviations from UIP are heightened during intervention episodes is consistent with evidence in Baillie and Osterberg (2000) that central bank foreign exchange interventions have a significant effect on the deviation from UIP.
from the data, it includes 0 and a proper value of $B[r(t - dt)]$ for exact fit can be found with some experiments.

Table 6.6 provides a closer look at the ARCH effects in the simulated observations. It reports the median and 95 percentile range of the LM statistics, and GARCH(1,1) estimates built from simulated interest differentials and exchange rates sampled at weekly and at monthly intervals. Table 6.6 shows weakening of ARCH effects in the exchange returns as the sampling horizon increases in terms of LM statistics. The simulated exchange returns exhibit ARCH effects at the weekly horizon but much weaker effects than in the data. Table 6.6 also shows parameter estimates for the GARCH(1,1) model. The median value of the GARCH parameters from the simulated observations are much smaller than that from the data as $\hat{\delta} = 0.0001$ for perfect UIP and $\hat{\delta} = 0.003$ for imperfect UIP compared to $\hat{\delta} = 0.080$ from the data. Furthermore, 95 percentile range of $\delta$ for both models fall short of the estimated value from the data. Unlike the interest differential, exchange rate returns show serial correlation between squared residuals due to the correlation between current interest differential and regression error as showed in (6.11).

To address the frequency of the intervention and violations to generate the result, I calculated the unconditional probability of touching either of the regulation bands to be 0.244. Over the 23 years of sample periods, this amounts to interventions in approximately 292 out of the total 1200 weekly observations which implies violation occurs every 4 weeks. This frequency is close to the intervention record would suggest. Dominguez (2001) reports the record of Fed interventions from 1977 to 1998 from which we infer that the frequency of Fed interventions over that time period in dollar-deutschemark was 0.188.
<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.0050</td>
<td>0.0035</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.235</td>
<td>0.481</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(8.188)</td>
<td>(4.925)</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>5.228</td>
<td>4.748</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(8.806)</td>
<td>(9.430)</td>
</tr>
<tr>
<td>$k$</td>
<td>1.748</td>
<td>4.684</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>(10.921)</td>
<td>(6.345)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>1.851</td>
<td>0.028</td>
</tr>
<tr>
<td>p-value</td>
<td>0.396</td>
<td>0.990</td>
</tr>
</tbody>
</table>

Note: 5 moments used in estimation are $\Delta r(t)^2$, $\Delta r^2(t)$, $r(t)r(t-1)$, $\Delta r^3(t)$, and $\Delta r^4(t)$.

Table 6.1: SMM Estimates of Interest Differential Process: Regulated Jump-Diffusion
<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.0050</td>
<td>0.0035</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.235</td>
<td>0.481</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>5.228</td>
<td>4.748</td>
</tr>
<tr>
<td>$k$</td>
<td>1.748</td>
<td>4.684</td>
</tr>
<tr>
<td>$\sqrt{\text{Var}(r)}$</td>
<td>3.200</td>
<td>3.140</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.985</td>
<td>0.954</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.872</td>
<td>0.596</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.773</td>
<td>0.350</td>
</tr>
</tbody>
</table>

Table 6.2: Properties of Interest Differential Processes: Regulated Jump-Diffusion
Table 6.3: SMM Estimation of Undetermined Constants in Exchange Rate Solution: Regulated Jump-Diffusion

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>-120.993</td>
<td>50.265</td>
</tr>
<tr>
<td>(t-ratio)</td>
<td>-3.692</td>
<td>10.891</td>
</tr>
<tr>
<td>J-statistic</td>
<td>222.547</td>
<td>104.697</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: 5 moments used in estimation are $s(t)^2$, $\Delta s^2(t)$, $\Delta s(t)r(t - 1)$, $\Delta s^3(t)$, and $\Delta s^4(t)$.
<table>
<thead>
<tr>
<th></th>
<th>Perfect UIP</th>
<th>Imperfect UIP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5%</td>
<td>50%</td>
</tr>
<tr>
<td>LM: $\chi^2(1)$</td>
<td>77.110</td>
<td>100.157</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.633</td>
<td>0.001</td>
</tr>
<tr>
<td>excess kurtosis</td>
<td>7.199$^b$/</td>
<td>9.152</td>
</tr>
<tr>
<td>VR(2)</td>
<td>0.925</td>
<td>0.982</td>
</tr>
<tr>
<td>VR(12)</td>
<td>0.679</td>
<td>0.846</td>
</tr>
<tr>
<td>VR(24)</td>
<td>0.543</td>
<td>0.753</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}$</td>
<td>141.581$^b$/</td>
<td>156.838</td>
</tr>
<tr>
<td>$\rho_{\Delta s}(1)$</td>
<td>-0.074</td>
<td>-0.017</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>2.104</td>
<td>2.978</td>
</tr>
<tr>
<td>$\rho_r(1)$</td>
<td>0.973</td>
<td>0.987</td>
</tr>
<tr>
<td>$\rho_r(12)$</td>
<td>0.711</td>
<td>0.854</td>
</tr>
<tr>
<td>$\rho_r(24)$</td>
<td>0.495</td>
<td>0.729</td>
</tr>
<tr>
<td>LM(r): $\chi^2(1)$</td>
<td>70.927</td>
<td>97.764</td>
</tr>
<tr>
<td>skewness(r)</td>
<td>-0.959</td>
<td>-0.007</td>
</tr>
<tr>
<td>excess kurtosis(r)</td>
<td>-1.467</td>
<td>-0.955</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.520$^b$/</td>
<td>4.103</td>
</tr>
<tr>
<td>Asy-t</td>
<td>1.197</td>
<td>1.981</td>
</tr>
</tbody>
</table>

Note: $^a$/less than estimate from data. $^b$/greater than estimate from data.

Table 6.4: Properties of Exchange Rate Processes When Interest Differential Follows Regulated Jump-Diffusion Calibrated to German SMM Estimates (Percentiles of Monte Carlo Distributions from 5000 Replications of 1200 Weekly Observations)
\[ \alpha \sigma_r^2 \left( \sigma_r \left[ \int_0^1 z(t) \, dt - z(0) \right] + k \left[ \int_0^1 q(t) \, dt - q(0) \right] \right) \]

\[ \alpha \sigma_r \int_0^1 r^2(t) \, dz(t) \]

\[ B \sigma_r \int_0^1 dz(t) \]

\[ 2\alpha \lambda k^2 \left( \sigma_r \left[ \int_0^1 z(t) \, dt - z(0) \right] + k \left[ \int_0^1 q(t) \, dt - q(0) \right] \right) \]

Bias from \( \eta(\lambda, k, \dot{r}, t) \)

Slope 3.3317

Table 6.5: Bias Components: Slope Coefficients in projection onto \( r(0) \)
<table>
<thead>
<tr>
<th></th>
<th>Perfect UIP</th>
<th>Imperfect UIP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Weekly</td>
<td>Monthly</td>
</tr>
<tr>
<td>LM: $\chi^2(1)$</td>
<td>100.157</td>
<td>62.907</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td></td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td>$\omega$ (95%)</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\omega$ (99%)</td>
<td>0.9999</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

Table 6.6: ARCH Effects in the Exchange Return and Sampling Horizon: Median Values and 95 Percentile Range
Figure 6.1: Intervention Mechanism with a Regulated Jump-Diffusion Interest Differential

Figure 6.2: A Realization of the Weekly Sampled Regulated Jump-Diffusion Process with $\lambda = 0.005$, $\sigma_r = 0.235$, $\bar{r} = 5.228$, $k = 1.748$, and $\delta = 1/14$. 
Figure 6.3: Nonlinear Relation between Log Exchange Rate and Interest Differential: Plots of Equation (6.7) with Alternative Values of $\lambda$.

Figure 6.4: Expected Movement of the Exchange Rate with Jumps in the Interest Differential.

(1) = arrival of negative jump
(2) = arrival of positive jump
CHAPTER 7

CENTRAL BANK INTERVENTION AND THE FORWARD PREMIUM ANOMALY

The plan of the thesis is first to further investigate along the ideas of the statistical artifact of small sample approach—UIP is actually true and the forward premium anomaly is a statistical artifact. I first build a model on the assumption that all the market participants know and understand central bank intervention rule and UIP holds all the time even at the instant of intervention. The purpose is to ask whether we can model an exchange rate model based on UIP that provides economic theory for the nonstandard features of international financial data. Although the model produces ARCH effects when UIP is constrained to hold continuously, including instants of central bank intervention, the distributional distortion from the model is not sufficiently large to resolve the forward premium anomaly.

These results led me to explore a related but alternative avenue where I put my attention to the effects of the central bank intervention on the financial market. To capture the nature of central bank’s foreign exchange intervention, I relaxed the assumption of perfect UIP and build a imperfect UIP model in which the intervention always takes market by surprise and UIP is violated only at those instants of intervention. This central bank intervention approach provides a resolution to the forward
premium anomaly. In this chapter, I investigate and provide evidence that the central bank intervention is in the center of the forward premium anomaly.

The simulation experiments with the imperfect UIP models find that if the observations are drawn from the instance of the central bank intervention, then these observations induce truncated error distribution that causes omitted variable bias. This omitted variable bias induced by the central bank intervention does generate huge deviations of OLS estimator distribution from its asymptotic distribution that is the main source of the forward premium anomaly. This omitted variable bias occurs due to the nature of the central bank’s foreign exchange intervention. Interventions in the exchange markets take place irregularly and informally during the post Bretton Woods era, and the regulation bands are not directly observable to market participants if they exist. Also spreading insider information about the regulation is illegal, market participants cannot have full information about central bank intervention.\textsuperscript{20} When monetary authority engages in market intervention, it will generate information asymmetry and consequently market participants are making inaccurate expectation about future. Due to this limited information about the intervention, market participants believe truncated portion of the distribution is still feasible even when it is not.

Whether the central bank intervention induces the forward premium anomaly can be tested in several ways. Table 7.1 presents probit regression results of the Fed and

\textsuperscript{20}This information asymmetry between monetary authority and market participants is frequently posited but rarely tested phenomenon. Recently, Romer and Romer (2000) tested existence of asymmetric information between the Fed and the market participants and find that the Fed has considerably more information about inflation than market participants. They also find that shift in Fed’s monetary-policy action provides signals to the Fed’s information and market participants are modifying their forecasts of future interest rates according to the signals.
Bundesbank intervention index on the interest differentials.\textsuperscript{21} The estimates represent that whether high home-foreign interest differential induces central bank intervention or not. The estimation results in Table 7.1 tell us that high home-foreign interest differential is associated with more chance of central bank intervention. The coefficient from the regression of the intervention index on level of interest differential is 0.220 and on the absolute interest differential is 0.208.\textsuperscript{22}

Figure 7.1 shows the number of interventions by the Fed and/or by the Bundesbank for the period from 1977 to 1998. It can be clearly seen that the number of interventions are greatly differ before and after 1980s. Intervention by the central banks was in its peak late 1970s then reduced when dollar became strong. From the start the Reagan administration adopt a policy of “being neglect” toward the foreign exchange market and do not intervene exchange market unless extreme circumstances. However, the strong dollar became a severe problem that cannot be ignored by 1985. This strong dollar results in the G-5 Plaza announcement in September 1985. Economic officials of the five countries—the US, UK, France, Germany, and Japan—met at New York’s Plaza hotel and announced that they will jointly intervene in the foreign exchange market to depreciate dollar. The dollar depreciated immediately responding this Plaza announcement and continued to fall through 1987.

In March 1979, the European Monetary System (EMS) start to operate and member countries mutually peg their bilateral exchange rates. Under EMS, participating

\textsuperscript{21}The Fed and the Bundesbank intervention index take value ‘1’ when either bank intervenes and ‘0’ when there is no intervention.

\textsuperscript{22}We cannot simply interprete the estimated coefficients as probability of intervention with higher level (or absolute value) of interest differential. However, at least we can interprete these positive coefficients as an indication that central bank intervention is more likely to occur with higher interest differential.
countries are enforced to maintain their exchange rates within specified fluctuation margins around central parity. An important characteristic of the EMS is that it involves more frequent and direct market intervention by the central banks to maintain the exchange rates within the bands. Many member countries are fixing their currencies to deutschmark to gain credibility by adopting the German Bundesbank’s monetary policy. This result in deutschmark to take a place of central currency of EMS and the Bundesbank frequently intervenes in the market to maintain credibility deutschmark. However, series policy conflicts between Germany and other countries led the EMS to adopt very wide currency bands (±15 percent) in August 1993 and this bands were maintained until the introduction of the euro in 1999.

Table 7.2 presents the subsample analysis of the forward regression before and after 1987. The first column of the Table 7.2 reports the negative and asymptotically significant non-unit slope coefficient estimate for the forward regression. This estimate of -0.657 demonstrates the presence of the forward premium anomaly in the 1977:1 to 1998:12 sample period. For 1977:1-1987:12 period, the slope estimate is -4.315 and statistically significantly differ from unity while the slope estimate for the 1988:1-1998:12 period is -0.458 and not significantly different from one. As can be seen from Figure 7.1 that both the Fed and the Bundesbank frequently engaged in intervention during the 1997:1-1987:12 period than the 1988:1-1998:12 period. This subsample analysis provides fragmentary evidence that the forward premium anomaly is more pronounced during the frequent central bank intervention period.

To see more closely whether central bank intervention heighten the forward premium anomaly, I separate German data into two groups—normal and abnormal period—according to the central bank intervention record and regress exchange rate
returns on the interest differentials.

\[ s_{t+1,n} - s_{t,n} = \alpha_n + \beta_n r_{t,n} + u_{t+1,n}, \quad (7.1) \]
\[ s_{t+1,a} - s_{t,a} = \alpha_a + \beta_a r_{t,a} + u_{t+1,a}. \quad (7.2) \]

Normal observations are drawn from a period of no intervention while abnormal observations are drawn from a period where intervention occur within window.\(^{23}\)

Regression results for the sample period of 1977:1-1998:12 are reported in Table 7.3. Scatter plots of German exchange rate returns on interest differentials are shown in Figures 7.2 – 7.3. One surprising result from the table is that slope coefficients are differ whether they are from the normal or abnormal period. The estimated slope are all positive in normal period. Slope estimates for 4-, 6-, 8-, and 10-days windows are 0.540, 1.206, 1.238, 1.196. Particularly, 10 days window slope estimate is close to one and asymptotically not different from unity that supports UIP. Compared to the estimates from normal period, slope estimates from abnormal periods are all negative and exhibit the forward premium anomaly. This also can be seen from the scatter plots. The scatter plot for the abnormal period exhibits negative regression line while the same plot for the normal period shows positive slope for the regression line.

I investigate whether the estimated model is robust across sub-period or not. Table 7.4 shows subsample analysis results for the normal and abnormal period. For the 1977:1-1987:12 period, slope coefficient estimates are negative for both normal and abnormal period. The slope coefficients for the normal period, however, are asymptotically insignificant for all windows while estimates for the abnormal period

\(^{23}\)To separate normal and abnormal periods, I run 4-, 6-, 8-, and 10-days window. 10-days window means one week lead and one week lag. Since the exchange rate and interest differential data are weekly sample while intervention indices are daily observations, 5 working days of lead results in a weekly lead and 5 working days of lag is equal to a weekly lag.
are all negative and significant. During the 1977:1-1987:12 period, central banks were actively engaged in the foreign exchange market and results in heightening of the forward premium anomaly under the period of central bank intervention. This heightening of the forward premium also can be seen in the less active central bank intervention period. For the 1988:1-1998:12 period, the slope coefficient estimates for the normal period are all positive and asymptotically not different from one while estimates for the abnormal periods are all negative. This heightening of the forward premium anomaly during the instances of intervention is consistent with the implications of my models in the previous chapters. Also this heightening of the forward premium anomaly during intervention episodes is consistent with evidence in Baillie and Osterberg (2000) that central bank foreign exchange interventions have a significant effect on the deviation from UIP.
Table 7.1: Binary Probit Regression of Intervention Index on Interest Differential

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>z-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.505</td>
<td>-8.269</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.220</td>
<td>12.115</td>
</tr>
<tr>
<td>constant</td>
<td>-0.727</td>
<td>-9.057</td>
</tr>
<tr>
<td>$</td>
<td>r_t</td>
<td>$</td>
</tr>
</tbody>
</table>
\[ s_{t+1} - s_t = \alpha + \beta r_t + u_{t+1} \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>2.814</td>
<td>20.272</td>
<td>-0.654</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-0.657</td>
<td>-4.315</td>
<td>-0.458</td>
</tr>
</tbody>
</table>

Note: Asymptotic t-ratios in parentheses are calculated using Newey-West (1987) standard errors to test the hypothesis that \( \alpha = 0 \) and \( \beta = 1 \).

Table 7.2: Regression of Exchange Rate Returns on the Interest Differential
<table>
<thead>
<tr>
<th>Window</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Period (77:1-98:12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>4.909</td>
<td>6.890</td>
<td>5.987</td>
<td>4.984</td>
</tr>
<tr>
<td>(1.451)</td>
<td>(1.982)</td>
<td>(1.691)</td>
<td>(1.354)</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.540</td>
<td>1.206</td>
<td>1.238</td>
<td>1.196</td>
</tr>
<tr>
<td>(-0.395)</td>
<td>(0.168)</td>
<td>(0.189)</td>
<td>(0.147)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Window</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abnormal Period (77:1-98:12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.772</td>
<td>-0.267</td>
<td>2.030</td>
<td>3.511</td>
</tr>
<tr>
<td>(0.313)</td>
<td>(-0.050)</td>
<td>(0.385)</td>
<td>(0.749)</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.274</td>
<td>-1.110</td>
<td>-1.440</td>
<td>-1.589</td>
</tr>
<tr>
<td>(-1.568)</td>
<td>(-1.512)</td>
<td>(-1.793)</td>
<td>(-2.105)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Windows are for equal daily leads and lags. 4 window means 2 leads and 2 lags. Asymptotic t-ratios in parentheses are calculated using Newey-West (1987) standard errors to test the hypothesis that $\alpha = 0$ and $\beta = 1$.

Table 7.3: Regression of Exchange Rate Returns on the Interest Differential: Normal vs. Abnormal Periods
<table>
<thead>
<tr>
<th>Window</th>
<th>Normal Period</th>
<th>Abnormal Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(1.610)</td>
<td>(1.822)</td>
</tr>
<tr>
<td></td>
<td>(-0.689)</td>
<td>(-0.937)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Window</th>
<th>Normal Period</th>
<th>Abnormal Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>α</td>
<td>2.950</td>
<td>1.813</td>
</tr>
<tr>
<td></td>
<td>(0.862)</td>
<td>(0.523)</td>
</tr>
<tr>
<td>β</td>
<td>0.129</td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td>(-0.648)</td>
<td>(-0.624)</td>
</tr>
</tbody>
</table>

Note: Asymptotic t-ratios in parentheses are calculated using Newey-West (1987) standard errors to test the hypothesis that $\alpha = 0$ and $\beta = 1$.

Table 7.4: Subsample Regressions of Exchange Rate Returns on the Interest Differential: Normal vs. Abnormal Periods
Figure 7.1: Number of Interventions by the Fed and the Bundesbank
Figure 7.2: Scatter Plot of Exchange Rate Returns on the Interest Differentials: Germany
Figure 7.3: Scatter Plot of Exchange Rate Returns on the Interest Differentials: Normal vs. Abnormal Periods
CHAPTER 8

SUMMARY AND CONCLUSIONS

Recent studies on the forward premium anomaly cast doubt on the reliability of the asymptotic inference from the regressions of the future depreciation on the current interest differential. They claim that ARCH effects and nonnormality of the interest differential induce widely dispersed distribution for the OLS slope estimator in these regressions and make asymptotic inference unreliable. However, they do not provide a theory for the basis of the nonstandard features of the interest differential. This thesis provides such a theory and considers whether the continued popularity of UIP is not misplaced by asking whether these and other features in the international financial market data can be consistent with UIP.

I presented a linear continuous-time model for the exchange rate whose solution is a nonlinear function of the interest differential. This nonlinear relation between the exchange rate and the interest differential forms the underlying basis of ARCH effects. This endogenous generation of ARCH effects in exchange rate returns is a condition called big news. I conduct simulation experiments to evaluate the ability of the model to match the moment of the exchange rate and interest differential data. For the simulation experiments, I consider two alternative specifications of agent’s knowledge and beliefs of central bank intervention. In the first specification, UIP
holds continuously and market participants know and understand the intervention rule. In the second specification, intervention takes market participants by surprise and UIP is violated, but only at the instants of intervention.

When UIP holds continuously, the econometric implications of big news by itself falls short of providing an account for the forward premium anomaly. The implied distribution of the OLS slope estimator in regressions of the future depreciation on the current interest differential lies far away from the asymptotic distribution but this small sample distortion does not explain the forward premium anomaly. On the other hand, even though UIP holds for the great majority of the time, inclusion of a few observations in which UIP is violated sufficiently corrupts the data set in a way that shows up as the forward premium anomaly. The evanescent violations of UIP occur as a result of the unfamiliarity of market participants with the intervention policy.

The simple linear model in this thesis does a good job in explaining several, if not all, empirical features of the exchange rate and interest differential. However, I am fully aware of that the model is not a perfect description of the foreign exchange market. Rather it should be viewed as a parable for a real world where the monetary authorities engage in informal and occasional intervention and market participants are attempt to learn about the underlying rules.
APPENDIX A

SIMULATED METHOD OF MOMENTS ESTIMATION OF THE INTEREST DIFFERENTIAL PROCESS

Let the simulated observations be denoted with a ‘tilde.’ For the discretized regulated Brownian motion, I divided each of the $T = 1200$ weekly time periods into $N = 14$ subintervals.* Experimentation using $N = 7$ and $N = 21$ subintervals produced little differences in the results. Setting $\delta_N = (1/N) \approx dt$, I simulate sequences of (5.3) by

$$\tilde{r}_j = \tilde{r}_{j-1} + \sigma_r \epsilon_j \sqrt{\delta_N} \tag{A.1}$$

where $\epsilon_j \overset{iid}{\sim} N(0, 1)$ and

$$\tilde{r}_j = \begin{cases} r_j & \text{if } \tilde{r}_{j-1} < r_j \\ \tilde{r}_j & \text{if } r_j \leq \tilde{r}_j \leq \bar{r} \\ \bar{r} & \text{if } \tilde{r}_{j-1} > \bar{r} \end{cases}$$

for $j = 1, \ldots, NMT$. The observations were then re-sampled at weekly intervals for a sequence of $MT$ weekly observations and I set $M = 30$.

SMM estimation of this model proceeds as follows. Let $\hat{\beta}$ be the vector of parameters to be estimated, $r^\prime = (r_1, r_2, \ldots, r_T)$ denote the collection of the actual time-series observations, and $\{\bar{r}_i(\hat{\beta})\}_{i=1}^M$ be the computer simulated time-series of

---

*SMM estimation of other interest differential processes are similar.
length $M$ which I generate according to (A.1). $\vec{r}(\beta) = (\vec{r}_1(\beta), \vec{r}_2(\beta), \ldots, \vec{r}_M(\beta))$ denotes the collection of these $M$ observations. To estimate $\sigma_r$ and $\vec{r}$ by matching $E(\Delta r_t), E(\Delta r_t)^2, E(r_t r_{t-1}), E(\Delta r_t)^3,$ and $E(\Delta r_t)^4,$ I let the vector function of the data from which to simulate the moments be $h(r_t) = (r_t, r_t^2, r_t r_{t-1}, r_t^3, r_t^4)'$ and the vector of sample moments be $H_T(r) = \frac{1}{T} \sum_{t=1}^{T} h(r_t)$. The corresponding vector of simulated moments is $H_M(\vec{r}(\beta)) = \frac{1}{M} \sum_{i=1}^{M} h(\vec{r}_i(\beta)),$ where the length of the simulated series is $M$. Now let $u_t = h(r_t) - H_T(r)$ be the deviation of $h$ from its mean, $\hat{\Omega}_0 = \frac{1}{T} \sum_{t=1}^{T} u_t u_t'$ be the sample short-run variance of $u_t,$ and $\hat{\Omega}_j = \frac{1}{T} \sum_{t=1}^{T} u_t u_{t-j}'$ be the sample cross-covariance matrix of $u_t,$ $\hat{W}_T = \hat{\Omega}_0 + \frac{1}{T} \sum_{j=1}^{m} (1 - \frac{j}{T})(\hat{\Omega}_j + \hat{\Omega}_j')$ is the Newey and West (1987) estimate of the long-run covariance matrix of $u_t.$

If we let $g_{T,M}(\beta) = H_T(r) - H_M(\vec{r}(\beta))$ be the deviation of the sample moments from the simulated moments, then the SMM estimator, $\hat{\beta}_S,$ is that value of $\beta$ that minimizes the quadratic distance between the simulated moments and the sample moments

$$g_{T,M}(\beta)' W_{T,M}^{-1} g_{T,M}(\beta),$$  \hfill (A.2)

where $W_{T,M} = \left[ (1 + \frac{T}{M}) W_T \right]$ and is asymptotically normally distributed with

$$\sqrt{T}(\hat{\beta}_S - \beta) \overset{D}{\to} N(0, V_S),$$

as $T$ and $M \to \infty$ where $V_S = \left[ B' \left[ (1 + \frac{T}{M}) W_T \right] B \right]^{-1}$ and $B = \frac{\partial g_{T,M}(\beta)}{\partial \beta}.$

I estimated $\sigma_r$ and $\vec{r}$ by minimizing (A.2) with respect to $\sigma_r$ and $\vec{r}.$
APPENDIX B

EXCHANGE RATE SOLUTION WHEN INTEREST DIFFERENTIAL FOLLOWS MEAN-REVERTING PROCESS WITH STANDARD NORMAL INCREMENTS

Proof of Proposition 4.1. I guess that the solution takes the form,

\[ s(t) = G[r(t)], \quad (B.1) \]

where \( G(\cdot) \) is a time-invariant continuous and twice differentiable function of \( r \). Using Ito’s lemma to take the total differential of (B.1) gives

\[ ds(t) = G'[r(t)]dr(t) + \frac{1}{2} G''[r(t)][dr(t)]^2 \quad (B.2) \]

where \( G' = dG(r)/dr \) and \( G'' = d^2G(r)/dr^2 \).

If the interest differential follows mean-reverting process with standard normal increments, then \( dr(t) = -\gamma r(t)dt + \sigma_r dz(t) \) and \( [dr(t)]^2 = \sigma_r^2 dt \). Substitute these expressions into (B.2), we get

\[ ds(t) = G'[r(t)](-\gamma r(t)dt + \sigma_r dz(t)) + \frac{\sigma_r^2}{2} G''[r(t)]dt \quad (B.3) \]

Now take expectations of both sides of (B.3) conditional on information known at instant \( t \),

\[ \mathbb{E}_t[ds(t)] = \frac{\sigma_r^2}{2} G''[r(t)]dt = r(t)dt \quad (B.4) \]
where the second equality is obtained by UIP. Now we seek to solve the differential equation,

$$\frac{\sigma^2_r}{2} G''[r(t)] - \gamma r G'[r(t)] = r(t).$$  \hspace{1cm} (B.5)

Let $y(r) = G'[r(t)]$ and write (B.5) as the first-order differential equation

$$\frac{\sigma^2_r}{2} y'(r) - \gamma r y(r) = r.$$ \hspace{1cm} (B.6)

The solution to (B.6) is

$$G'[r(t)] = y(r) = -\frac{1}{\gamma} + C \exp\left( \frac{\gamma r^2}{\sigma^2_r} \right),$$ \hspace{1cm} (B.7)

where $C$ is an arbitrary constant. Note that $G'(r)$ is a real-valued function. Now integrating (B.7) gives

$$G[r(t)] = \int y(r) dr = -\frac{r}{\gamma} - C \frac{i \sqrt{\pi} \sigma_r}{2 \sqrt{\gamma}} \left( \frac{2}{\sqrt{\pi}} \int_0^{i \gamma r} \exp \left( -u^2 \right) du \right).$$ \hspace{1cm} (B.8)

where $i = \sqrt{-1}$. The integral $\frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) du$ is known as the error function and is typically denoted by erf($x$). In this case, $x$ is a complex number. To avoid taking explicit path integrals, we use the fact that

$$\text{erf}(x) = \frac{2x}{\sqrt{\pi}} \int_0^1 \exp \left( -x^2 u^2 \right) du.$$ 

Then (B.8) can be written as

$$G[r(t)] = -\frac{r}{\gamma} + C r \int_0^1 \exp \left( \frac{\gamma r^2}{\sigma^2_r} u^2 \right) du.$$ \hspace{1cm} (B.9)

Next, we verify that (B.9) solves (B.6). Differentiating (B.7) gives,

$$G''[r(t)] = \frac{2C \gamma r}{\sigma^2_r} \exp \left( \frac{\gamma r^2}{\sigma^2_r} \right).$$ \hspace{1cm} (B.10)
That (B.9) is indeed the solution can be seen by substituting (B.10) and (B.7) into (B.6). ||
APPENDIX C

EXCHANGE RATE SOLUTION WHEN INTEREST DIFFERENTIAL FOLLOWS MEAN-REVERTING PROCESS WITH STUDENT-\( T(N) \) INCREMENTS

Proof of Proposition 4.2. Let \( G[ \cdot ] \) be the guess solution. By Ito’s lemma, we have

\[
ds(t) = G'[r(t)]dr(t) + \frac{1}{2}G''[r(t)][dr(t)]^2
\]

(C.1)

where \( G' = dG(r)/dr \) and \( G'' = d^2G(r)/dr^2 \).

If the interest differential follows mean-reverting process with student-\( t(n) \) increments, then \( dr(t) = -\gamma r(t)dt + \sigma_r dz(t) \) and \([dr(t)]^2 = \delta \sigma_r^2 dt\). \( dz(t) \) follows student-\( t(n) \) distribution, \( dz(t) \sim (0, \frac{n}{n-2} dt) \) with \( n \) degree of freedom and \( \delta = \left( \frac{n}{n-2} \right) \). Substitute these expressions into (C.1), we get

\[
ds(t) = G'[r(t)](-\gamma r(t)dt + \sigma_r dz(t)) + \frac{\delta \sigma_r^2}{2}G''[r(t)]dt
\]

(C.2)

Now take expectations of both sides of (C.2) conditional on information known at instant \( t \),

\[
E_r[ds(t)] = \frac{\delta \sigma_r^2}{2}G''[r(t)]dt = r(t)dt
\]

(C.3)

where the second equality is obtained by UIP. Now we seek to solve the differential equation,

\[
\frac{\delta \sigma_r^2}{2}G''[r(t)] - \gamma rG'[r(t)] = r(t) .
\]

(C.4)
Let $y(r) = G'[r(t)]$ and write (C.4) as the first-order differential equation

$$\frac{\delta \sigma_r^2}{2} y'(r) - \gamma y(r) = r.$$  \hfill (C.5)

The solution to (C.5) is

$$G'[r(t)] = y(r) = -\frac{1}{\gamma} + C \exp \left( \frac{\gamma r^2}{\delta \sigma_r^2} \right),$$  \hfill (C.6)

where $C$ is an arbitrary constant. Note that $G'(r)$ is a real-valued function. Now integrating (C.6) gives

$$G[r(t)] = \int y(r)dr = -\frac{r}{\gamma} + C \frac{\sqrt{\pi} \sigma_r}{2\sqrt{-\gamma/\delta}} \left( \frac{2}{\sqrt{\pi}} \int_0^{r/\sigma_r} \exp \left( -u^2 \right) du \right).$$  \hfill (C.7)

The integral $\frac{2}{\sqrt{\pi}} \int_0^r \exp(-u^2)du$ is known as the error function and is typically denoted by erf($x$). In our case, $x$ is a complex number. To avoid taking explicit path integrals, we use the fact that

$$\text{erf}(x) = \frac{2x}{\sqrt{\pi}} \int_0^1 \exp \left( -x^2 u^2 \right) du.$$

Then (C.7) can be written as

$$G[r(t)] = -\frac{r}{\gamma} + C \gamma \sigma_r \int_0^1 \exp \left( \frac{\gamma r^2}{\delta \sigma_r^2} u^2 \right) du.$$  \hfill (C.8)

Next, we verify that (C.8) solves (C.5). Differentiating (C.6) gives,

$$G''[r(t)] = \frac{2C \gamma r}{\delta \sigma_r^2} \exp \left( \frac{\gamma r^2}{\delta \sigma_r^2} \right).$$  \hfill (C.9)

That (C.8) is indeed the solution can be seen by substituting (C.9) and (C.6) into (C.5).
APPENDIX D

EXCHANGE RATE SOLUTION WHEN INTEREST DIFFERENTIAL FOLLOWS REGULATED BROWNIAN MOTION

I first derive the solution in the text following Krugman’s (1992) use of the method of undetermined coefficients. I guess that the solution takes the form,

\[ s(t) = G[r(t)], \quad (D.1) \]

where \( G(\cdot) \) is a time-invariant continuous and twice differentiable function of \( r \). Using Ito’s lemma to take the total differential of (D.1) gives

\[ ds(t) = G'[r(t)]dr(t) + \frac{1}{2}G''[r(t)][dr(t)]^2 \quad (D.2) \]

where \( G' = dG(r)/dr \) and \( G'' = d^2G(r)/dr^2 \).

**Proof of Proposition 5.1.** If the interest differential evolves according to (5.2), then \( dr(t) = \sigma_r dz(t) \) and \( dr(t)^2 = \sigma_r^2 dt \). Upon substitution into (D.2), we get

\[ ds(t) = G'[r(t)]\sigma_r dz(t) + \frac{\sigma_r^2}{2}G''[r(t)]dt \quad (D.3) \]

Now take expectations of both sides of (D.3) conditional on information known at instant \( t \),

\[ E_t[ds(t)] = \frac{\sigma_r^2}{2}G''[r(t)]dt = r(t)dt \quad (D.4) \]
where the second equality is obtained by UIP. Now I seek to solve the differential equation,

\[ \frac{\sigma_r^2}{2} G''[r(t)] = r(t). \]  \hspace{1cm} (D.5)

Let the solution to the homogeneous part of (D.5) be \( G_h \). This solution must satisfy \( G_h'' = 0 \) and is satisfied by setting \( G_h = A + Br \). Next, I guess that the solution to the nonhomogeneous part be \( G_n = kr^3 \). Then \( G_n' = 3kr^2, G_n'' = 6kr \). Upon substitution into (D.5), I obtain \( k = 1/3\sigma_r^2 \). The general solution is therefore \( s(t) = G_h + G_n = A + Br + r^3/(3\sigma_r^2) \), which is eq.(5.4).

**Proof of Proposition 5.2.** Here, I exploit knowledge of behavior at the bands to determine \( B \). Due to the symmetric nature of the bands, I need only examine behavior at one of the bands. Suppose that \( r(t) \) attains the upper band \( \bar{r} \). At that instant, \( G''[\bar{r}] = 0 = B + \bar{r}^2/\sigma_r^2 \) and solving yields \( B = -(\bar{r}^2/\sigma_r^2) \).

**Derivation of \( B \) for Flood-Garber Interventions in Footnote 9.** As in the derivation of eq.(5.6), the solution to the nonhomogenous part of the differential equation is given by \( G_n = r^3/(3\sigma_r^2) \). A general guess solution can be written explicitly in terms of the bands as,

\[ G(r|\bar{r}) = A + \frac{r^3}{3\sigma_r^2} + B(r - \bar{r}) + C(r + \bar{r}). \]  \hspace{1cm} (D.6)

Now suppose the upper band is hit at the instant \( t_0, r(t_0) = \bar{r} \), it follows that

\[ s(t_0) = A + \frac{\bar{r}^3}{3\sigma_r^2} + 2C\bar{r}. \]  \hspace{1cm} (D.7)
At the next instant, the interest differential is set to 0. Since these actions are known with certainty,

\[ s(t_0 + dt) = E_s(t_0 + dt) = G(0|\bar{r}) = A + (C - B)\bar{r}. \]  

(D.8)

Ruling out arbitrage profits requires that \( s(t_0 + dt) = s(t_0) \). Thus equating (D.7) and (D.8) gives \( (B + C) = -\bar{r}^2/(3\sigma_r^2) \). Due to the symmetry of the bands, we have \( B = C = -\bar{r}^2/(6\sigma_r^2) \). Substituting back into (D.6) gives,

\[ G(r|\bar{r}) = A - \frac{\bar{r}^2}{3\sigma_r^2}r + \frac{r^3}{3\sigma_r^2}. \]  

(D.9)

Derivation of \( B \) for Bertola-Caballero Interventions in Footnote 9. Again, begin by writing the guess solution explicitly in terms of the bands,

\[ G(r|\bar{r}) = A + \frac{r^3(t)}{3\sigma_r^2} + B(r - \bar{r}) + C(r - \bar{r}) \]  

(D.10)

where \( \bar{r} = -\bar{r} \), and where I have already made use of the solution to the nonhomogeneous part of the differential equation. Under symmetric intervention points, we know that \( B = C \). Let the bandwidth be \( b = \bar{r} - \bar{r} = 2\bar{r} \). It follows that,

\[ B(r - \bar{r}) + C(r - \bar{r}) = B(r - \bar{r} + r - \bar{r}) = B[2(r - \bar{r}) + b] \]

which I can use to rewrite (D.10) as,

\[ G(r|\bar{r}, \bar{r}) = A + \frac{r^3(t)}{3\sigma_r^2} + 2B\bar{r}. \]  

(D.11)

Now suppose that the upper band \( \bar{r} \) is attained at instant \( t_0 \), then

\[ s(t_0) = G(\bar{r}|\bar{r}, \bar{r}) = A + \frac{\bar{r}^3}{3\sigma_r^2} + Bb. \]

(D.12)
At the next instant, the authorities revalue with probability $p$ to $G(\bar{r}+(b/2)|\bar{r}, \bar{r}+b) = A + (\bar{r} + (b/2))^3/(3\sigma_r^2)$ or defend with probability $1-p$ by setting the exchange rate to $G(\bar{r}-(b/2)|\bar{r}, \bar{r}) = A + (\bar{r} - (b/2))^3/(3\sigma_r^2)$. That is,

$$s(t_0 + dt) = \begin{cases} A + (\bar{r} + (b/2))^3/(3\sigma_r^2) & \text{with probability } p \\ A + (\bar{r} - (b/2))^3/(3\sigma_r^2) & \text{with probability } (1-p). \end{cases} \quad (D.13)$$

To rule out expected arbitrage profits, we require $s(t_0) = \mathbb{E}_t [s(t_0 + dt)]$ from which it follows that,

$$A + \frac{\bar{r}^3}{3\sigma_r^2} + Bb = A + p\frac{(\bar{r} + (b/2))^3}{3\sigma_r^2} + (1-p)\frac{(\bar{r} - (b/2))^3}{3\sigma_r^2}. \quad (D.14)$$

Solving (D.14) for $B$ gives,

$$B = \frac{p(\bar{r} + (b/2))^3 + (1-p)(\bar{r} - (b/2))^3 - \bar{r}^3}{3b\sigma_r^2} = \frac{(8p-1)\bar{r}^2}{6\sigma_r^2} \quad (D.15)$$

where the second equality follows from the symmetry conditions.

**Exchange Rate Solution with Asymmetric Bands.** The symmetric band assumption is not key and can be relaxed. Here, I derive the exchange rate solution when $\underline{r} = -\alpha \bar{r}$. As above, the solution to the nonhomogeneous part of the differential equation is $G_n = r^3/(3\sigma_r^2)$. I write the general guess solution explicitly in terms as

$$s(t) = G(r|\underline{r}, \bar{r}) = A + \frac{r^3(t)}{3\sigma_r^2} + B[\bar{r} + r(t)/\alpha] + C[\underline{r} + \alpha r(t)]. \quad (D.16)$$

At $r(t) = \bar{r}$,

$$G(\bar{r}|\underline{r}, \bar{r}) = A + \frac{\bar{r}^3(t)}{3\sigma_r^2} + B[\bar{r} + \bar{r}/\alpha], \quad (D.17)$$

and

$$G'(\bar{r}|\underline{r}, \bar{r}) = 0 = B \left[\frac{1 + \alpha}{\alpha}\right] + \frac{\bar{r}^2}{\sigma_r^2}, \quad (D.18)$$
which gives

$$B = - \left[ \frac{\alpha}{1 + \alpha} \right] \frac{\bar{r}^2}{\sigma_r^2}. \quad \text{(D.19)}$$

Similarly, at $r(t) = \bar{r}$,

$$G(r|\bar{r}, \bar{r}) = A + \frac{r^3}{3\sigma_r^2} + C[r + \alpha r], \quad \text{(D.20)}$$

and,

$$G'(r|\bar{r}, \bar{r}) = 0 = C(1 + \alpha) + \frac{r^2}{\sigma_r^2} \quad \text{(D.21)}$$

which gives

$$C = - \left[ \frac{1}{1 + \alpha} \right] \frac{\bar{r}^2}{\sigma_r^2} = - \left[ \frac{\alpha^2}{1 + \alpha} \right] \frac{\bar{r}^2}{\sigma_r^2} = -\alpha B. \quad \text{(D.22)}$$

\[ \|

\text{\textit{Derivation of Eq.}(5.9).} \text{ Let’s begin with (5.7) which, for convenience, I reproduce here as}

$$ds(t) = r(t)dt + \left( B + \frac{r(t)^2}{\sigma_r^2} \right) \sigma_r dz(t).$$

Integration gives,

$$s(1) - s(0) = \int_0^1 ds(t) = \int_0^1 r(t)dt + \frac{1}{\sigma_r} \int_0^1 r^2(t)dz(t) + B\sigma_r \int_0^1 dz(t). \quad \text{(D.23)}$$

Since

$$r(t) - r(0) = \int_0^t dr(u) = \sigma_r \int_0^t dz(u) = \sigma_r [z(t) - z(0)], \quad \text{(D.24)}$$

It follows that

$$(a) = \int_0^1 r(t)dt = r(0) + \sigma_r \int_0^1 z(t)dt - \sigma_r z(0).$$
Next, squaring the interest differential using (D.24) gives

\[ r^2(t) = r^2(0) + \sigma_r^2[z^2(t) + z^2(0) - 2z(0)z(t)] + 2r(0)\sigma_r[z(t) - z(0)] \]  
(D.25)

Integrating (D.25) with respect to \( dz(t) \) gives,

\[
(b) = \frac{1}{\sigma_r} \int_0^1 r^2(t) dz(t) = \frac{1}{\sigma_r} \{ r^2(0) \int_0^1 dz(t) + \sigma_r^2 \int_0^1 z^2(t) dz(t) \\
+ \sigma_r^2 z^2(0) \int_0^1 dz(t) - 2\sigma_r^2 z(0) \int_0^1 z(t) dz(t) \\
+ 2r(0)\sigma_r \int_0^1 z(t) dz(t) - 2r(0)z(0)\sigma_r \int_0^1 dz(t) \}
= \frac{1}{\sigma_r} \left[ r(0) - \sigma_r z(0) \right]^2 \int_0^1 dz(t) + 2[r(0) - \sigma_r z(0)] \int_0^1 z(t) dz(t) \\
+ \sigma_r \int_0^1 z^2(t) dz(t).
\]

Now for part (c), I simply note that \( B\sigma_r \int_0^1 dz(t) = B\sigma_r[z(1) - z(0)] \). Substitute these expressions back into (D.23) to get

\[
s(1) - s(0) = r(0) + \sigma_r \int_0^1 z(t) dt - \sigma_r z(0) + \frac{1}{\sigma_r} \left[ r(0) - \sigma_r z(0) \right]^2 \int_0^1 dz(t) \\
+ 2[r(0) - \sigma_r z(0)] \int_0^1 z(t) dz(t) + \sigma_r \int_0^1 z^2(t) dz(t) + B\sigma_r \int_0^1 dz(t)
\equiv r(0) + \eta(1).
\]

Decomposing \( \eta(1) \) into terms that depend on \( r(0) \) and those that do not gives

\[ \eta(1) = r(0)\epsilon(1) + \nu(1) \]  
where \( \epsilon(1) \) is given by (5.11) and \( \nu(1) \) is given by (5.12).
APPENDIX E

EXCHANGE RATE SOLUTION WHEN INTEREST DIFFERENTIAL FOLLOWS REGULATED JUMP-DIFFUSION

Proof of Proposition 6.1. I guess that the solution takes the form,

$$s(t) = G[r(t)],$$  \hspace{1cm} (E.1)

where $G[\cdot]$ is a time-invariant continuous and twice differentiable function of $r$. Assume that the interest differential follows regulated jump-diffusion

$$dr(t) = \sigma_r dz(t) + k dq(t),$$  \hspace{1cm} (E.2)

where $\sigma_r$ is the instantaneous volatility of the interest differential conditional on no arrival of discrete jump, $dz(t)$ is a standard Wiener process, $k$ is jump-size, and $dq(t)$ is a Poisson process with intensity $\lambda$ for negative and positive jumps.

The upward change in function value of $G[r(t)]$ conditional on the arrival of the positive jump is $(G[r(t) + k] - G[r(t)])$ and the downward change in function value of $G[r(t)]$ conditional on the arrival of the negative jump is $(G[r(t) - k] - G[r(t)])$. From this we have expected change in $G[r(t)]$ as

$$\lambda dt E(G[r(t) + k] - G[r(t)]) + \lambda dt E(G[r(t) - k] - G[r(t)]) + (1 - 2\lambda dt)(0)$$

$$= \lambda E(G[r(t) + k] - G[r(t)])dt + \lambda E(G[r(t) - k] - G[r(t)])dt.$$  \hspace{1cm} (E.3)
Applying Ito’s lemma to (E.1) gives

\[ ds(t) = G'[r(t)]dr(t) + \frac{1}{2}G''[r(t)][dr(t)]^2 + (G[r(t) + k] - G[r(t)])dq(t) \]

\[ + (G[r(t) + k] - G[r(t)])dq(t) \]  

(E.4)

where \( G' = dG(r)/dr \) and \( G'' = d^2G(r)/dr^2 \).\(^1\)

If the interest differential evolves according to (E.2), we get instantaneous change in \( s(t) \) from (E.4) as

\[ ds(t) = G'[r(t)]\sigma_r dz(t) + \frac{\sigma_r^2}{2}G''[r(t)]dt + (G[r(t) + k] - G[r(t)]) dq(t) \]

\[ + (G[r(t) - k] - G[r(t)]) dq(t). \]

(E.5)

Since the size of jump and the probability of positive and negative jumps are the same, applying second order Taylor approximation to the jump part of (E.5) gives

\[ ds(t) = G'[r(t)]dr(t) + \frac{1}{2}G''[r(t)][dr(t)]^2 \]

\[ + \left( G'[r(t)]k + \frac{1}{2}G''[r(t)]k^2 \right) dq(t) \]

\[ + \left( G'[r(t)](-k) + \frac{1}{2}G''[r(t)](-k)^2 \right) dq(t). \]  

(E.6)

Take expectations of both sides of (E.6) conditional on information known at instant \( t \) gives

\[ E_t[ds(t)] = \frac{\sigma_r^2}{2}G''[r(t)]dt + \lambda k^2 G''[r(t)]dt = r(t)dt, \]  

(E.7)

where the second equality is obtained by UIP.

Now we seek to solve the differential equation,

\[ \left( \frac{\sigma_r^2 + 2\lambda k^2}{2} \right) G''[r(t)] = r(t). \]  

(E.8)

\(^1\)Kushner (1976) and Shimko (1992).
Let the solution to the homogeneous part of (E.8) be $G_h$. This solution must satisfy $G_h'' = 0$ and is satisfied by setting $G_h = A + Br$. Next, we guess that the solution to the non homogeneous part be $G_n = mr^3$. Then $G_n' = 3mr^2, G_n'' = 6mr$. Upon substitution into (E.8), we obtain $m = 1/3(\sigma_r^2 + 2\lambda k^2)$. The general solution is therefore

$$s(t) = G_h + G_n$$

$$= A + B[r(t - dt)]r(t) + \frac{r^3(t)}{3(\sigma_r^2 + 2\lambda k^2)}. \quad (E.9)$$

Next, we verify that (E.9) solves (E.8). Differentiating (E.9) gives

$$G''[r(t)] = \frac{2r(t)}{\sigma_r^2 + 2\lambda k^2}. \quad (E.10)$$

That (E.9) is indeed the solution can be seen by substituting (E.10) into (E.8). ||

*Proof of Proposition 6.2.* Here, determination of $B$ requires knowledge of exchange rate behavior at the bands. Due to the symmetric nature of the regulation bands, we need only examine behavior at one of the bands.

Let $g[r(t- dt)]$ be the distance between the interest differential at instant $(t - dt)$ and the regulation bands, $g[r(t- dt)] = r(t - dt) - \bar{r}$ and $0 \leq g[r(t- dt)] \leq k$. Now suppose the upper band is hit at the instant $t_0$, it follows that

$$s(t_0) = A + B(\bar{r} + g[r(t_0 - dt)]) + \frac{(\bar{r} + g[r(t_0 - dt)])^3}{3(\sigma_r^2 + 2\lambda k^2)}. \quad (E.11)$$

At the next instant, the interest differential is set to $\bar{r}$. Since these actions are known with certainty, we have,

$$s(t_0 + dt) = E_t s(t_0 + dt) = G[\bar{r}] = A + B\bar{r} + \frac{\bar{r}^3}{3(\sigma_r^2 + 2\lambda k^2)}. \quad (E.12)$$
Ruling out arbitrage profits requires that \( s(t_0 + dt) = s(t_0) \). Thus equating (E.11) and (E.12) gives

\[
B[r(t_0 - dt)] = -\left[ \frac{3\dot{r}^2 + 3\dot{r}g[r(t_0 - dt)] + g[r(t_0 - dt)]^2}{3(\sigma_r^2 + 2\lambda k^2)} \right].
\] (E.13)

Derivation of Eq.(6.11). Let’s begin with (6.8) which, I reproduce here as

\[
ds(t) = \left( \frac{\sigma_r^2}{\sigma_r^2 + 2\lambda k^2} \right) r(t)dt + \left( B[r(t - dt)] + \frac{\dot{r}^2(t)}{3(\sigma_r^2 + 2\lambda k^2)} \right) \sigma_r dz(t) + \phi(\lambda, k, r, t).
\] (E.14)

The jump part of the instantaneous change can be approximated as

\[
\phi(\lambda, k, r, t) \approx \lambda \left[ k^2 \left( \frac{d^2 s(r)}{dr^2} \right) \right] dt = \lambda \left[ \frac{2k^2}{\sigma_r^2 + 2\lambda k^2} r(t) \right] dt.
\] (E.15)

Thus

\[
ds(t) = \left( \frac{\sigma_r^2}{\sigma_r^2 + 2\lambda k^2} \right) r(t)dt + \left( B[r(t - dt)] + \frac{\dot{r}^2(t)}{3(\sigma_r^2 + 2\lambda k^2)} \right) \sigma_r dz(t)
+ \lambda \left[ \frac{2k^2}{\sigma_r^2 + 2\lambda k^2} r(t) \right] dt.
\] (E.16)

Let \( \left( \frac{1}{\sigma_r^2 + 2\lambda k^2} \right) = \alpha \), and integrate (E.16),

\[
s(1) - s(0) = \int_0^1 ds(t)
= \alpha \sigma_r^2 \int_0^1 r(t)dt + \alpha \sigma_r \int_0^1 \dot{r}^2(t)dz(t)
+ B[r(t - dt)] \sigma_r \int_0^1 dz(t) + 2\alpha \lambda k^2 \int_0^1 r(t)dt.
\] (E.17)
Since

\[ r(t) - r(0) = \int_0^t dr(u) = \sigma_r \int_0^t dz(u) + k \int_0^t dq(u) \]

\[ = \sigma_r [z(t) - z(0)] + k[q(t) - q(0)], \tag{E.18} \]

it follows that

\[ (a) = \alpha \sigma_r^2 \int_0^1 r(t) dt \]

\[ = \alpha \sigma_r^2 r(0) + \alpha \sigma_r^2 \left[ \int_0^1 z(t) dt - z(0) \right] + \alpha \sigma_r^2 k \left[ \int_0^1 q(t) dt - q(0) \right]. \]

Also part (d) gives

\[ (d) = 2\alpha \lambda k^2 \int_0^1 r(t) dt \]

\[ = 2\alpha \lambda k^2 r(0) + 2\alpha \lambda k^2 \sigma_r \left[ \int_0^1 z(t) dt - z(0) \right] + 2\alpha \lambda k^3 \left[ \int_0^1 q(t) dt - q(0) \right]. \]

Substitute these expressions back into (E.17) to get

\[ s(1) - s(0) = r(0) \]

\[ + \alpha \sigma_r^2 \left( \sigma_r \left[ \int_0^1 z(t) dt - z(0) \right] + k \left[ \int_0^1 q(t) dt - q(0) \right] \right) \tag{a'} \]

\[ + \alpha \sigma_r \int_0^1 r^2(t) dz(t) + B[r(t - dt)] \sigma_r \int_0^1 dz(t) \tag{b'} \]

\[ + 2\alpha \lambda k^2 \left( \sigma_r \left[ \int_0^1 z(t) dt - z(0) \right] + k \left[ \int_0^1 q(t) dt - q(0) \right] \right) \tag{c'} \]

\[ + \eta(\lambda, k, r, t). \tag{d'} \]

where the terms \((a', b', c', d')\) are separate components of the regression error derived from \((a, b, c, d)\) in (E.17) respectively and \(\eta(\lambda, k, r, t)\) represents error components from jump part. ||

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