EFFECTS OF COGNITIVE STRATEGY INSTRUCTION ON THE
MATHEMATICAL PROBLEM SOLVING OF
MIDDLE SCHOOL STUDENTS WITH
LEARNING DISABILITIES

Dissertation

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By

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* * * * *

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ABSTRACT

Recent pedagogical standards voiced by the National Council of Teachers of Mathematics (2000) have increasingly stressed the development of students' strategic knowledge of mathematics. Such knowledge extends students' procedural and declarative understanding of mathematics and stretches their cognitive application of mathematics to greater conceptual and applied heights. The value of learning and applying mathematics strategically is often not appreciated by students who are learning disabled. Representing mathematical problems, deriving goals for solving such problems, choosing among appropriate strategies for problem-solving, and engaging in self-monitoring processes are of greater challenge to students with learning disabilities. Further, students with learning disabilities often experience lower self-efficacy toward mathematics and attribute their mathematical learning outcomes to forces external to their effort and behavior.

This study assessed the efficacy of an intervention targeting the mathematical word problem-solving of middle school students with learning disabilities that is inclusive of cognitive, metacognitive, and motivational components. The intervention was based on the Mathematical Problem-Solving Model of Montague (1995, 1997, 2000) and taught
students a strategy based on principles of self-regulated learning. Additionally, the intervention encouraged students to attribute their learning outcomes to the application of the strategy and sought to foster the development of self-efficacy through the promotion of the strategy’s knowledge, use, and control.

Results of the present investigation are meaningful in terms of each of the four research objectives. First, strategy instruction was efficacious in improving the mathematical word problem solving of students with learning disabilities. Second, important and significant gains were evidenced in the LD students’ knowledge, use, and control of math word problem-solving strategies, such that their awareness of these domains approximated that of average-achieving students. Third, prior research was replicated and validated through the use of a comparison group. Fourth, although significant changes were not realized across the two motivational constructs - self-efficacy and attribution of effort - important insights were gained.
Dedicated to my father
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CHAPTER 1

INTRODUCTION

Since the National Council of Teachers of Mathematics (NCTM) adopted its new standards in 1989, a dramatic change has emerged in mathematics curricula. The direction of instruction, based on the new standards, is moving toward solving problems that are applicable to everyday life. Rather than focusing solely on rote memorization of declarative facts and procedural processes, the NCTM Standards (1989, 2000) advocate development of conceptual and strategic knowledge couched in a sociocultural framework.

While instruction of general mathematics is trending towards the development of problem solving proficiency, little is known about how students with learning disabilities perform in classrooms in which NCTM standards guide instruction (Fleischner & Manheimer, 1997). Recent literature on teaching students with learning disabilities to solve word problems rests largely on cognitive procedures (Fleischner, Nuzum, & Marzola, 1987; Hutchinson, 1993; Maccini & Hughes, 2000; Montague, 1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986; Naglieri & Gottling, 1997). Such research has shown promising trends towards improving the performance of higher-order
word problem-solving ability in middle- and high-school students and is increasingly addressing issues of transfer, or generalizability, to complex, ambiguous mathematical problems. Successful completion of non-routine problems has not, until recently (Maccini & Hughes, 2000), been persuasively evidenced in such research, particularly longitudinally.

Targeting the transfer problem inherent of many cognitive strategy interventions has been the cause of increasingly integrative, comprehensive strategy intervention models (e.g., Butler, 1998; Borkowski, 1992; Harris & Graham, 1994). These models conceptualize problem solving as a complex interplay between cognitive, metacognitive, and affective components. The models promote student’s development of self-regulation, metacognition, and strategic — or domain-specific — approaches to tasks (Butler, 1998). The application of these models to meaningful learning outcomes for students has been positive, though, understandably, not comprehensive across academic domains or grade levels.

Providing students with learning disabilities the skills needed to transfer strategic problem-solving skills across contexts is particularly relevant for the domain of mathematics. Secondary schools are increasingly requiring the successful completion of higher-order classes such as algebra and pre-algebra for all students. Such courses have traditionally been considered gatekeepers to further educational and occupational opportunities (Maccini & Hughes, 2000). Ironically, as expectations for student s
mathematical attainment increase with age, so too does the gap widen between students with learning disabilities and average-achieving students on measures of achievement.

Middle school, a time of emergence for higher-order cognitive processes, appears to be a critical time in the formation of mathematical self-efficacy and motivational constructs related to mathematical learning outcomes. A recent and particularly relevant finding by Pajares and Graham (1999) determined that students’ mathematics self-efficacy beliefs were solely predictive of mathematics performance at both the beginning and end of the first year of middle school. Additionally, by the end of this first year of middle school, students reported mathematics as less valuable and described decreased levels of effort and persistence in mathematics. This decline in motivational variables corresponded with declines in achievement indexes.

Pajares and Graham (1999) included average-achieving and gifted students in their study. What can be inferred about the self-efficacy and corresponding mathematical achievement of students with learning disabilities? An implication from Pajares and Graham’s findings is that the middle school period may open an invaluable window of opportunity to change motivational attitudes in the form of self-efficacy, attribution, and interest toward mathematics. Demonstrating improved achievement through strategic intervention which incorporates cognitive, metacognitive, and affective/self-regulatory components may have recursive effects on motivation, and in turn, improve achievement.
As such, this study seeks to work with middle-school students, specifically replicating and extending the findings of a major researcher of mathematical word problem-solving, Marjorie Montague. This study extends Montague’s research by building in a measure of self-efficacy to determine if participation in a strategic intervention affects this critical motivational construct. Further, this study incorporates direct attributional retraining — teaching students to ascribe their use of the strategy to their learning outcomes, rather than to uncontrollable forces such as ability or luck. The addition of attributional retraining holds implications for the maintenance of the strategy across time by explicitly teaching students that their use of the strategy is fundamental to optimal performance.

Montague’s research spans descriptive (Montague, 1991; Montague, 1995; Montague & Applegate, 1993; Montague & Applegate, 1997; Montague & Applegate, 2000) and empirical (Montague, 1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986) studies. Her empirical research on mathematical problem solving, even early in its development (Montague, 1992), has been described by Borkowski (1992) as being akin to a model of self-regulation in its classification of cognitive and metacognitive strategies (and) in the language used — strategy use, its regulation, and inspiration (p. 256).

Montague’s model of mathematical problem solving is derived from research in general problem solving, mathematical problem solving, metacognition, and affective
variables associated with problem-solving performance (Montague & Applegate, 1993). The cognitive strategies and processes (i.e., specific problem-solving strategies) are read, paraphrase, visualize, hypothesize, estimate, compute, and check. The metacognitive strategies and processes that develop awareness and regulation of the cognitive strategies include self-instruction, self-questioning, and self-monitoring. The instructional application of the model has four components: (a) assessing student performance and identifying students for whom the instructional program is appropriate; (b) explicitly instructing students in the acquisition and application of strategies for mathematical problem solving; (c) process modeling; and, (d) evaluating student outcomes, with an emphasis on strategy maintenance and generalization (Montague, 2000).

1.1 Problem Statement

Despite a call for intervention research targeting higher-level mathematical concepts (Maccini & Hughes, 1997), the number of studies investigating the efficacy of cognitive interventions has not kept pace with behavioral interventions in recent years. As such, additional research investigating the domain of strategic learning within mathematical learning is particularly time-relevant. The following research is further seen as pertinent in that additional evidence of parsimonious instruction enhancing learning-disabled students’ strategic learning in mathematical problem solving is relevant for educational professionals such as school psychologists and teachers. This research
further serves to add to existing research connecting when, where, and for whom particular skills and strategies result in improved learning for students with learning disabilities. Finally, this research strives to facilitate students’ learning and development in mathematical problem solving considering their motivations, including their interests and self-perceptions.

1.2 Purpose of the Study

The purpose of this study is to investigate the effects of cognitive strategy instruction on the mathematical problem-solving performance of middle school students with learning disabilities. The study assesses how participation in cognitive strategy instruction facilitates the students’ knowledge, use, and control of mathematical problem solving strategies. Finally, the study investigates how participation in strategy instruction affects two measures of motivation: students’ ratings of self-efficacy in solving mathematical word problems and students’ attribution of their effort in directing learning outcomes.
1.3 Objectives of the Study

The research objectives of this study are:

1. To assess the efficacy of Montague's cognitive/ metacognitive instructional strategy on the mathematical word problem-solving performance of learning disabled middle-school students.

2. To determine if engagement in such instruction results in qualitative changes in learning disabled students' knowledge, use, and control of mathematical problem solving strategies.

3. To replicate prior research on the impact of cognitive and metacognitive strategy instruction on mathematical word problem-solving ability through comparison of learning disabled students with a control group of average-achieving students.

4. To extend previous research by assessing change in two affective components of learning related to motivation, self-efficacy and attribution of effort, as a result of participation in an intervention enhancing strategic learning in mathematics that incorporates explicit attributional retraining.

1.4 Basic Assumptions

This study will be limited to middle school students within a suburban central Ohio school district from February 2002 to June 2002. It will include the implementation of an intervention at one identified middle school from the months of
March 2002 to May 2002. The intervention will include data from students whose parents granted permission for their participation in the study.

1.5 Limitations of the Study

Since participation in the strategy instruction treatment group and the comparison group is completely voluntary and is restricted to only those students who received parental consent for their participation in the study, one of the limitations of the study is that the results can only be generalized to the students who participated in the study. An additional limitation of the study results from the small number of subjects in the comparison and treatment groups. Finally, the study is limited by the inclusion of only students who attended one of four middle schools in a primarily middle to upper socioeconomic status school district.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The importance of mathematics is of increasing value in today's society. In fact, to be non-mathematical today is like being illiterate in the recent past: you become a passive spectator rather than an active participant in the modern world (Kirby & Williams, 1991, p. 107). Not only has mathematical proficiency grown in its importance but its very conception has dramatically changed from that of a generation ago. This change has been influenced by cognitive psychology and is reflected in policy-level events such as the National Council of Teachers of Mathematics (NCTM) Standards (NCTM, 1989, 2000), Goals 2000, and the New Standards (1995) project. The direction these standards have set for mathematics instruction has steered sharply away from traditional and conventional mathematics curriculum based on rote acquisition of declarative and procedural knowledge. The NCTM Standards advocate development of conceptual and strategic knowledge within the context of real-world mathematical experiences. Further, mathematics learning is now viewed from a sociocultural perspective in which the context
for learning and the relationship between social interactions and cognitive development are considered important factors (Montague, 1995).

As academic curricula adapt to meet the needs of a changing society in which technological competency, collaboration, communication, self-assessment, and problem solving are fundamental skills, so must instruction change (Goldman, Hasselbring et al., 1997; Montague, Warger, & Morgan, 2000). Mathematics instruction, under the new standards, uses problem solving proficiency as the centerpiece for mathematics learning. In so doing, instruction guides students to actively participate in their learning environments and make constructive, productive contributions to what they learn. Effective teaching includes understanding and facilitating the acquisition and application of knowledge as children solve mathematical problems (Montague, 1995, p. 39).

Despite recent shifts in theoretical paradigms and instructional methods in the mathematical domain, students with learning disabilities continue to be at risk for failure in mathematics (Montague, 1997). Such students experience a variety of problems related to learning and applying mathematics. For mathematical knowledge to be useful, students with learning disabilities must comprehend how procedures can function as tools for solving relevant problems. Yet, instruction for students with learning disabilities too often persists to focus on routines and memory instead of meanings and processes, or the relation between what is learned and the real world (Cawley & Parmar, 1992).
Successful learners are self-regulating (Butler, 1998). Self-regulated learners are characterized by their view of academic learning as something they do for themselves rather than as something that is done for them (Zimmerman, 1998). In academic contexts, self-regulation is a style of engaging with tasks in which students exercise a suite of powerful skills: setting goals for upgrading knowledge; deliberating about strategies to select those that balance progress toward goals against unwanted costs; and, as steps are taken and the task evolves, monitoring the accumulating effects of their engagement (Butler & Winne, 1995, p. 245).

A vital component to learning especially salient for children with learning disabilities is the engagement in self-regulatory practices. Research suggests that the strategic approaches to tasks by students with learning disabilities is often either deficient or actively inefficient (Butler, 1998, p. 161). As such, instruction that promotes a more strategic approach to academic tasks is particularly important and profitable for students with learning disabilities.

The purpose of this review is to explore the construct of self-regulation and its association with cognition, metacognition, and motivation from the perspective of mathematical problem solving. Self-efficacy and attribution - two measures of self-perception that are interwoven with learning and motivation - will be outlined. How all of these constructs interrelate will be discussed in reference to the instructional implications for teaching students with learning disabilities. Finally, empirical research will be
investigated regarding the impact of interventions that support the development of self-regulatory practices specific to mathematical problem-solving for students with learning disabilities.

2.2 Mathematical Problem-Solving, Self-Regulated Learning, and Cognitive, Metacognitive, and Motivational Constructs

Successful problem solving is dependent upon the interaction and influence of three components — cognition, metacognition, and affect. It is presumed that metacognition is central to problem solving because it manages and coordinates the other components (Mayer, 1998, p. 51). The pedagogical implications of these components are that they all deserve instructional attention. That is, as cognitive strategies are taught for effective problem solving, metacognitive information and motivational beliefs are imparted (Pressley, Woloshyn et al., 1995). The theoretical foundation underlying each component of successful problem solving is complex and warrants elucidation in the context of mathematical problem solving.

Cognitive science distinguishes among three basic types of mathematical knowledge: declarative, procedural, and conceptual. Each type of knowledge is critical to developing mathematical literacy. Declarative knowledge is best represented as mathematics facts (Goldman, Hasselbring et al., 1997). Easily drawing upon facts, and the relationships among them, is essential for the development of procedural knowledge.
Procedural knowledge can be defined as the rules, algorithms, or procedures used to solve mathematical tasks (Anderson, 1987). It is best represented as step-by-step instructions, executed in a linear sequence, in how to complete mathematical tasks (Goldman, Hasselbring et al., 1997). In using procedural knowledge, declarative knowledge is called upon to complete the procedures. The third type of knowledge, conceptual knowledge, determines understanding. It can be described as a connected web of information in which the linking relationships are as important as the pieces of discrete information that are linked (Goldman et al., p. 200). In fact, when previously unconnected information is understood as related, a significant cognitive reorganization takes place (Bruner, 1960).

Young children begin to acquire mathematical concepts at an early age through observation and interaction with their environment that allows for the construction of relationships between pieces of information (Baroody, 1987; Baroody & Ginsburg, 1986). As their conceptual understanding develops, declarative or factual knowledge is gained through informal and formal learning experiences. Children apply this declarative knowledge as they learn algorithms and become proficient in computational procedures.

With further immersion in experiential learning, real-world problem solving, and formal school learning, children draw on their cumulative conceptual, declarative, and procedural knowledge bases to construct a fourth type of knowledge, strategic knowledge (Montague, 1997). Strategic knowledge is that which allows a learner to describe or apply problem-solving strategies that are either specific to the task, or more general in
nature. The development of interactive relationships among declarative, procedural, and conceptual knowledge is essential for accessing and using strategic knowledge. If students do not understand why they are using specific procedures in particular contexts, then there can be little possibility of transfer to other contexts (Goldman et al., 1997, p. 201).

Strategic problem solvers must orchestrate and control the cognitive components of problem-solving tasks. Metaskills, or metacognitive knowledge, involve the knowledge of when to use, how to coordinate, and how to monitor various skills in problem solving (Mayer, 1998). Metacognitive proficiency allows learners to adapt to varying task demands and contexts. Further, it enhances the selection and use of techniques and strategies for successful task completion. Metacognitive information about strategies also plays a critical role in the generalization and maintenance of strategies (Pressley, Woloshyn et al., 1995).

Metacognitive theory is argued to be the overarching construct within which two major components, self-regulation and motivational beliefs, reside (Borkowski, Estrada, Milstead, & Hale, 1989). Self-regulation refers to the systematic direction of one’s thoughts, feelings, and actions toward the attainment of designated goals, whereas motivation involves processes that generate and maintain goal-directed actions (Schunk, 2000). Self-regulation and motivational beliefs interplay with metacognition in several important ways. Students are more likely to be motivated to engage in self-regulated
learning if they are aware that their choice to use certain strategic procedures improves performance (Pressley & Woloshyn, 1995). As a result of improved performance, feelings of self-efficacy, or personal beliefs about one's capabilities, emerge. Additionally, students need to be aware that their competent functioning is often due to using appropriate strategies, and in so realizing, attribute successful academic achievements to their effortful contributions and not to external forces.

2.2.1 Self-Regulation.

Self-regulatory skills form the foundation for adaptive, planful learning and problem solving (Borkowski, 1992). Such skills are not innate, fixed abilities, nor are they academic skills. Rather, self-regulated learning is the self-directive process through which learners transform their mental abilities into academic skills (Zimmerman, 1998, p. 2). Schunk (2000) describes that a critical element in self-regulation, and one that makes it distinctive, is that learners have some choice available (p. 356).

Self-regulated learning is a cyclical activity that occurs in three phases: forethought, performance or volitional control, and self-reflection (Zimmerman, 1998). Initially, in the forethought phase, self-regulation assists in the analysis of tasks for the purpose of outlining task-specific goals and selecting a problem-solving approach. The second self-regulatory phase, performance or volitional control, involves the learner in processes that affect concentration and the management of learning efforts. Self-regulated learners use motivation and volition-control strategies to keep themselves on task in the
face of task difficulty. Finally, as the course of learning progresses to the self-reflection
phase, self-regulation serves to monitor learning and adjust problem-solving strategies.
Self-reflection in turn influences forethought of subsequent learning efforts. As such, self-
regulated learners engage in a recursive cycle of cognitive activities (Butler, 1998).

Self-regulation posits that learners are dynamic contributors to their learning -
actively constructing knowledge versus passively receiving it (Schunk, 2000). The use of
strategies is an essential part of self-regulated learning in that strategy use allows learners
stronger control over information processing. Learning strategies are plans geared toward
academic task performance and include activities such as rehearsing new material to be
learned, selecting and organizing information, and relating new material to information
already learned. Schunk (p. 401) cautions that merely knowing how to use strategies does
not ensure that students will use them autonomously; learners need to be taught to use
learning strategies. As such, a vital aspect of strategy instruction is linking improved
performance with the use of the particular learning strategy. In so doing, key
motivational forces are tapped and the cycle of self-regulation is continued.

2.2.2 Motivational Constructs.

Models of strategic learning emphasize the management of cognitive, motivational,
and volitional processes during learning (Butler & Winne, 1995). Borkowski (1992)
succinctly described these mechanisms of motivational beliefs and self-regulation: Every
important cognitive act has motivational consequences, and furthermore, these
consequences potentiate future self-regulatory actions (p. 253). Motivation as a component of problem solving has been investigated from the vantagepoint of three interrelated approaches: interest theory, self-efficacy theory, and attributional theory.

According to Dewey (1913), interest theory posits that problem-solving skills need to be taught in context, consistent with the new standards set forth for modern mathematical instruction (NCTM, 1989, 2000). If the material holds interest for students, interest theory predicts that students will subsequently think harder and process the material more deeply. Even in the absence of tangible rewards, learners who have an intrinsic interest in a task will persevere in their learning efforts (Zimmerman, 1998). Research supports interest theory’s assumptions. Schiefele, Krapp, and Winteler (1992, p. 184) reviewed 121 studies and found a consistent correlation of approximately $r=0.30$ between interest and achievement. Anand and Ross (1987) found that elementary school children who learned how to solve mathematics problems using personalized examples that contained information about the individual student’s friends, interests, and hobbies performed better on solving transfer problems than students who learned from non-personalized examples. What remains to be determined from interest research is the specific mechanism by which interest affects learning (Mayer, 1998).

According to social cognitive theory (Bandura, 1986), self-efficacy refers to personal judgments about one’s own aptitude to learn or perform at a designated level on a particular task. The construct asserts that a person’s judgments of his or her own
capabilities to organize and execute courses of action influence task performance (Schunk, 2000). Self-efficacy is related to motivation in that motivation is augmented when students perceive they are making progress in learning (Schunk, 1991). Perceived self-efficacy influences the level of goal challenge people set for themselves, the amount of effort they mobilize, and their persistence in the face of difficulties (Zimmerman, Bandura, & Martinez-Pons, 1992, p. 664). The theory predicts that when students judge themselves as capable, they will work harder on a learning task. Schunk (1991) found a positive correlation between self-efficacy and persistence on exercise problems during arithmetic learning.

Self-efficacy theory predicts that students with high self-efficacy experience greater comprehension of material than those with low self-efficacy. Schunk and Hanson (1985) discovered that students' ratings of problem difficulty before learning were related to performance measures after instruction on solving arithmetic problems. In other words, students who anticipated that they would have less difficulty in learning to solve the problems tended to learn more than students who expected to have difficulty. Additionally, self-efficacy theory posits that students who improve their self-efficacy will improve their success in learning to solve problems. Pajares and Miller (1994) found that self-efficacy for mathematics problem-solving ability was more predictive of problem solving performance in post-secondary students than mathematical self-concept, perceived usefulness of mathematics, prior experience with mathematics, or gender.
Zimmerman, Bandura & Martinez-Pons (1992) further found self-efficacy to be more predictive of mathematics performance in ninth and tenth grade students than self-efficacy for self-regulatory practices. In a recent study, Pajares and Graham (1999) found that mathematics self-efficacy was the only motivation variable to predict mathematics performance for average-achieving and gifted middle school students. Across ability levels, students whose self-efficacy is higher are more accurate in their mathematics computation and show greater persistence on difficult items than do students whose self-efficacy is low (p. 125).

As learners engage in self-reflection, the third phase of self-regulation, they often attribute meaning to their outcomes, such as whether their performance was due to some intrinsic ability or to applied effort (Zimmerman, 1998). Self-regulated learners tend to attribute failures to correctable causes and attribute successes to personal competence (p. 5). Attributional theory holds that the kind of causes a student ascribes for successes and failures is related to academic performance (Weiner, 1986). In particular, students who attribute academic outcomes to effort are more likely to persist on academic tasks than students who attribute such outcomes to ability. Kurtz and Borkowski (1984) found that students who attributed success to their application of effort were more strategic following strategy training than students who attributed success to factors outside of their control, such as luck or ability. Further, strategic attributions aid in
clarifying the source of learning errors and adapting one's performance (Zimmerman & Martinez-Pons, 1992).

Cognitive, metacognitive and motivational factors are interdependent and often bidirectional in their influence on and relationship with problem-solving. How has research on these constructs lent insight into the strategic processes of students with learning disabilities? What are the implications of such research for the instruction of students with learning disabilities? Specifically, how has strategy instruction in the domain of mathematical word problem-solving influenced both the motivational tendencies and the learning outcomes of students with learning disabilities?

2.3 Cognitive, Metacognitive, and Motivational Processes of Students with Learning Disabilities: Implications for Mathematical Learning Outcomes

The development of strategic knowledge occurs naturally for most children with exposure to problem solving (Siegler, 1989). The opportunity to solve problems allows for the construction and generalization of increasingly effective and efficient problem-solving strategies. Some children, however, manifest disabilities that interfere with the acquisition of knowledge, thus affecting school performance and general problem-solving abilities (Montague, 1997, p. 166). Increasingly, research suggests that the mathematical difficulties faced by students with learning disabilities does not point to developmental differences, but rather to learning discrepancies or developmental delays...
(Cawley, 1984; Goldman, Hasselbring et al., 1997). The type of procedural errors made by these students is often akin to those made by younger, regular education students who have not yet developed higher level, abstract mathematical understanding. For example, students with learning disabilities often face difficulties predicting operations for solving problems, selecting appropriate algorithms to solve multi-step problems, and correctly completing problems after deciding how to solve them (Scheid, 1993).

Descriptive research investigating the strategic behavior of students with disabilities (Montague & Applegate, 1993, 2000; Montague, Bos, & Doucette, 1991; Swanson, 1990; Wong, Wong, & Blenkinsop, 1989) has shown that while the quantity of their strategies often does not differ from average-achieving students, significant qualitative differences exist. Students with learning disabilities may not apply their knowledge within the same context or in response to the same cues as normally developing problem solvers (Torgesen, 1982). In the domain of mathematical problem solving, students with learning disabilities reported greater reliance on solution strategies to solve mathematical word problems, such as rereading problems and shifting computations (Montague & Applegate, 1993; Montague, Bos, & Doucette, 1991). Further, the students showed difficulty in transforming linguistic and numerical information in word problems into appropriate mathematical equations and operations. In fact, the most salient difference between average-problem solvers and those with learning disabilities was associated with problem representation.
Students with learning disabilities additionally exhibit significant metacognitive differences in their approach to mathematical problem-solving when compared to average-achieving students (Borkowski et al., 1989; Fleishner & Garnett, 1987; Lucangeli, Coi, and Bosco, 1997; Montague & Applegate, 1993). Fleishner and Garnett found that students with learning disabilities who, despite demonstrated possession of the requisite skills for successful mathematical word problem solving, did not apply the appropriate strategies. Montague and Applegate found that, during a think-aloud task, gifted students verbalized more metacognitive strategies as the mathematical problems became increasingly more challenging. Conversely, students with learning disabilities verbalized fewer strategies on the more difficult problems, suggesting cognitive overload and a shutdown of their ability to verbalize processes and strategies (p. 165). Lucangeli, Coi, and Bosco found that Italian fifth grade students described as poor mathematical problem solvers (who equated to American definitions of students with learning disabilities) had overall lower metacognitive awareness than good mathematical problem solvers. Specifically, the poor mathematical problem solvers exhibited less conscious control over problem-solving processes. Further, the poor problem-solving students believed that the size of the numbers in a given problem was an indicator of the problem’s difficulty. The students’ expectations, in turn, influenced their problem-solving performance; the poor problem-solvers made more computational and procedural errors.
Lucangeli et al. (1997) later findings illustrate that academic performance is not solely dependent on cognitive factors, such as the ability to represent problems, or metacognitive factors, such as the ability to select an appropriate problem-solving strategy. Rather, academic performance is a complex derivative of cognitive, metacognitive, and noncognitive factors, such as self-perceptions of ability or academic competence. Measures of self-perception include perceived self-efficacy and attribution. Attributional theory holds that the kind of causes a student ascribes for successes and failures is related to academic performance (Weiner, 1986). Self-efficacy refers to personal beliefs about one's own aptitude to learn or perform at a designated level on a particular task (Bandura, 1986). The construct is based on the premise that a person's judgments of his or her own capabilities to organize and execute courses of action influence task performance (Schunk, 2000).

How students with learning disabilities approach a task and the amount of effort they put forth may be directly influenced by their self-perceptions (Montague, 2000). Learning-impaired children often develop motivational and personal problems as a consequence of their learning difficulties, including low self-esteem, inaccurate perception of their talents, and a tendency to attribute failure to diminished ability (Borkowski, Weyhing, and Carr, 1988, p. 46). Even if tasks are within the capability of students with learning disabilities, they may perceive them as being too difficult, lack the confidence to attempt them, and attribute their failure to deficient ability. As such, students with
learning disabilities tend to persist less in the face of challenge than average-achieving students. Persistence, which is highly correlated with problem-solving success, may be a contributor to poor problem solving for these students (Montague, p. 216).

Self-efficacy for cognitive competence is affected by students' actual performance outcomes on academic tasks and by environmental factors, such as students' social environment, teacher expectations, and task engagement variables (Schunk, 1989). Compared with their nondisabled peers, students with learning disabilities often hold a lower sense of their cognitive competence based on continued failures at achievement tasks (Schunk, 1985). Instruction that provides strategy instruction and metacognitive information has been effective in increasing students' self-efficacy by improving success at achievement tasks (Graham & Harris, 1989). Based on these findings, this study will investigate how participation in an instructional strategy intervention influences students' ratings of self-efficacy and how these ratings are associated with measures of performance after instruction.

A related goal of this study will be to determine if strategy training that includes direct attribution training influences students with learning disabilities' perception of control of their learning outcomes on mathematical problem solving. Through an intervention study with learning disabled upper elementary students, Borkowski, Weyhing, and Carr (1988) constructed an instructional program encouraging students to attribute failure to lack of effort rather than lack of ability. The program consisted of
instruction in how to summarize paragraphs and attribution training which stressed the importance of effort and the use of the strategy. Students who received both strategy and attributional training performed significantly better on answering transfer questions than students who received only strategy training.

2.4 Instructional Implications for Developing Mathematical Strategic Knowledge in Students with Learning Disabilities

Does merely increasing the amount of textbook or real-world practice in solving mathematical problems result in the development of strategic knowledge for students with learning disabilities? Mayer (1998) explains that although a focus on teaching basic skills may seem to be the most straightforward way to improve problem solving performance, the results of research clearly demonstrate that knowledge of basic skills is not enough (p. 51). Intervention research (Hutchinson; 1993; Montague & Applegate, 1993; Swanson, 1990) has decisively demonstrated that when children with learning disabilities are taught explicitly how and when to apply problem-solving strategies, in the context of guided learning, their mathematical problem solving significantly improves. Simply increasing the amount of exposure to mathematical problems for the purpose of practicing skills will not qualitatively change the nature of student’s problem-solving (Montague & Applegate). Such drill and practice may improve computational skills, but at the same time, deny students (with learning disabilities) opportunities to develop
problem-solving skills and engage in higher level mathematical thinking (Montague & Applegate, p. 193). Additionally, effective strategy instruction is inherently constructivist. That is, the teacher explanations and modeling are the start of a process by which students construct understanding of the strategy by using it (Pressley & Woloshyn, 1995, p. 8).

The significance of metacognition to mathematical problem solving is well acknowledged in the literature. Metacognitive deficits appear to adversely affect the development and use of effective strategies for representing problems and executing solutions; as a result, these deficits impede progress in academic tasks requiring considerable strategic activity, such as mathematical problem solving (Montague, 1997, p. 165). Intervention research focusing specifically on strategy instruction in this domain assists students who have a repertoire of problem-solving strategies but use them inefficiently or ineffectively. Metacognitive strategies (e.g., self-instruction, self-monitoring, self-evaluation) support the activation, selection, and monitoring of strategy use (Graham & Harris, 1994). While the content and duration of strategy instruction varies, several instructional principles are inclusive of such interventions: cognitive modeling, verbal rehearsal, guided practice, corrective and positive feedback, and mastery learning. Teacher’s understanding of these processes, that is, his or her implicit working model of children’s learning and problem solving, is essential for sustained, innovative, strategy-oriented instruction (Borkowski, 1992, p. 253).
2.5 Efficacy of Strategy Instruction for Students with Learning Disabilities Targeting Mathematical Word Problems

With the above instructional implications in mind for teaching students with learning disabilities mathematical concepts, what are the actual learning outcomes for students who receive such intervention? Are these outcomes maintained and transferred across domain-specific or domain-general content areas? Longitudinally, do students with learning disabilities show evidence of self-regulating their strategic approach to mathematical word problems?

Hutchinson (1993) demonstrated that the application of strategy instruction significantly improved the algebra word problem-solving performance of adolescents with learning disabilities (grade range = 8 to 10) and resulted in improvement on measures of thinking related to problem solving. Utilizing a combined repeated measures single-subject baseline design and a two-group pre- and posttest design, Hutchinson (1993) gained insight into both individual performance of the instructed students and the performance of a matched comparison group. The students who received strategy instruction met individually with Hutchinson on alternating days for four months. During the intervention sessions, students were instructed via scripted lessons that spanned an orientation phase and scripts specific to representation and solution for three problems.
types (relational problems, proportion problems, and two-variable, two-equation problems).

The treatment (application of strategy instruction) followed a set of general procedures. Initially, students were reminded of the day's purpose and discussed a graph charting their progress. Students were then given a task sheet of five problems and a prompt card for self-questioning. Examples of self-questions for representing algebra word problems included, Have I read and understood each sentence? Have I written down my representation on the worksheet? (Hutchinson, 1993, p. 39). Self-questions for solving problems consisted of prompts such as, Have I written an equation? Have I written out the steps of my solution on the worksheet? (p. 39). The students were asked to read (or say from memory) the self-questions and then read the first problem silently. The instructor provided a model of the strategy by thinking aloud for the first and second problems. As the student thought aloud on the third and fourth problem, the instructor gave the student prompts, encouragement, and corrective feedback. The student then completed the fifth problem independently and at its completion, the instructor gave the student corrective feedback. The student was then given an assessment task sheet to complete individually. Finally, the student charted their task performance on a graph and discussed with the instructor the concepts to be practiced in the next session. Sessions continued in a similar teach/test cycle until students reached the criterion of four out of five problems correct on three consecutive assessments. Over
the course of treatment, the external supports (e.g., prompts, encouragement, feedback) were gradually diminished.

Hutchinson’s (1993) single-subject, multiple baseline design illustrated that six of the twelve students obtained criterion on all three problem types (relational problems, proportion problems, and two-variable, two-equation problems). Four of the students reached criterion on two problem types, while the remaining two students reached criterion on only the first problem type. Two types of transfer problems were assessed — near (only surface structure altered) and far (mathematical structure altered). The students obtained criterion on the majority of the near-transfer questions. For far-transfer problems, the proportion of students reaching criterion was lower. In approximately two-thirds of the cases, criterion was attained. Maintenance data was collected six weeks after cessation of the treatment. Criterion performance was maintained by 10 of the 12 students on the relational and proportion problems. Five of the six students who reached criterion for the two-variable, two-equation problems maintained critical performance.

Pretest-posttest comparisons were made between the instructed students and a group of matched comparison students on scores of algebra performance and for three measures of thinking related to problem solving. The latter measures included a metacognitive interview, think-aloud protocols, and a classification task. All of the measures were statistically significantly higher for the students in the treatment group.
Hutchinson's (1993) findings demonstrated not only a gain in the immediate algebra word problem-solving performance of secondary students with learning disabilities who participated in a strategic intervention, but illustrated the adoption of strategic processes by these students that have applicability to both near- and far-transfer problems. Additionally, the treatment students improved in their awareness of strategies to use in domain-general situations, such as encountering an unknown word in reading problems (assessed in the Metacognitive Interview). The efficacy of the treatment intervention was given unique perspective by Hutchinson's (1993) use of a comparison group.

Utilization of a control group is not common practice across intervention studies. In a more recent investigation of a cognitive strategy designed to improve the algebra word problem solving performance of learning disabled students, Maccini and Hughes (2000) relied solely on a multiple-baseline, across subjects design. The effects of the treatment (problem-solving strategy) were visually analyzed for the six subjects (modal grade = 9). Maccini and Hughes measured the percentage correct on problem representation, percentage correct on problem solution and answer, and the percentage of strategy use. To assess problem representation and solution, instructional probes and generalization measures were scored using a holistic scoring guide to standardize procedures and involved points for correct or partially correct responses.
The treatment consisted of graduated instructional phases: concrete, semiconcrete, and abstract (C-S-A). The students applied a cognitive strategy known as STAR during these instructional phases with the aid of a worksheet containing the STAR strategy steps and substeps. The steps involved searching the problem, translating the words into an equation in picture form, answering the problem, and reviewing the solution. The substeps were metacognitive prompts to attend to self-processes, such as Ask yourself questions: What facts do I know? What do I need to find? (Maccini & Hughes, 2000, 12). In the first phase of concrete applications, students were taught to represent mathematical problems via manipulatives. The use of a workmat with positive and negative areas assisted the students in their representation of problems. Students progressed to subsequent phases of semiconcrete and abstract only after achieving 80% mastery on two consecutive probes in the preceding application phase. In the semiconcrete phase, students represented problems using drawings. The final phase of abstract application entailed the students representing problems using numerical symbols. Each lesson, regardless of instructional phase, was scripted to include six elements: (a) provide an advance organizer (e.g., identify the new skill or concept and provide a rationale for learning it), (b) describe and model (e.g., the think-aloud process was modeled by the researchers and then adopted by the students), (c) conduct guided practice, (d) conduct independent practice, (e) give posttest, and (f) provide feedback (p. 13). The researcher further provided up to five problems with guided practice and
presented five problems for students to solve independently. Far- and near- transfer
generalization tests similar in structure to those used by Hutchinson (1993) were
administered after students attained criterion performance on two consecutive probes at
the abstract level. A measure of maintenance was given up to 10 weeks following the
intervention. Maccini and Hughes additionally incorporated a social validation scale into
their dependent measures to assess students’ perceptions of the treatment, specifically its
effectiveness, its influence on their problem-solving skills and understanding of what it
means to solve word problems.

The multiple baseline across subjects results were displayed graphically and then
judged relative to stability of baseline conditions, changes in instructional variables
between conditions, and changes in mean performance between conditions. The
percentage of growth of students’ accuracy on problem representation increased
dramatically, spanning an initial range of approximately 10% - 33% correct prior to
treatment, to 93% - 97% correct during and after treatment (all mathematically functions,
including addition, subtraction, multiplication, and division were presented and analyzed
separately). Students maintained high mean percentage accuracy scores during
semiconcrete and abstract instruction (range = 90% - 100%). The percentage of growth of
students’ accuracy on problem solution improved from an initial range of approximately
40% - 60% to 91% - 98% before and after/ during treatment. As with problem
representation, students maintained high mean percentage accuracy scores during
semiconcrete and abstract instruction (90% - 100%). Students scored higher on the near-transfer generalization tasks than on the far-generalization tasks; findings similar to those of Hutchinson (1993). On a measure of maintenance, students mean percentage correct was 75% for problem representation and 91% for problem solution.

Maccini and Hughes (2000) results corresponded with those of Hutchinson's (1993) in finding a gain in the immediate algebra word problem-solving performance of secondary students with learning disabilities who participated in a strategic intervention. Maccini and Hughes research offers unique perspective into the specific tasks of problem representation and problem solution. It is noteworthy that the greatest improvement was made across problem representation accuracy. This finding corroborates prior research documenting that many students with learning disabilities exhibit problems with phases of the problem representation process (Montague, Bos, & Doucette, 1991). The significant gains witnessed in problem representation indicate that concentration on this skill may be particularly efficacious for students with learning disabilities in their efforts to solve word problems. Maccini and Hughes further illustrated that the adoption of strategic processes by these students has applicability to both near- and far-transfer problems and to maintenance problems. Finally, while the feedback given on the social validation measure generally was positive, it is difficult to report or evaluate this measure as Maccini and Hughes did not administer the measure in a
pretest/posttest format. Further, the results were reported informally, using descriptive statistics.

Across a series of three intervention studies, Montague (1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986) demonstrated the effectiveness of cognitive strategy instruction for improving the mathematical problem-solving performance of middle school students with learning disabilities. The goal of instruction was to teach students a comprehensive cognitive and metacognitive strategy to assist them in solving one-, two-, and three-step mathematical word problems. In two of the studies (Montague; Montague & Bos), the students were instructed individually; in the third study (Montague, Applegate, & Marquard) they were instructed in small groups of 8 to 12 students.

The mathematical problem-solving model Montague (1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986) adopted for application in the instructional strategy is based on research in general problem-solving, mathematical problem solving, metacognition, and affective variables associated with problem-solving performance, as discussed earlier in this review. The cognitive strategies (i.e., specific problem-solving strategies) and processes of Montague’s mathematical problem solving model are read, paraphrase, visualize, hypothesize, estimate, compute, and check. The metacognitive strategies and processes that develop awareness and regulation of the cognitive strategies include self-instruction, self-questioning, and self-monitoring. These processes and
strategies are admitted by Montague (1997) to need further validation, but are derived from prior qualitative research (Montague & Applegate, 1993) on the strategies of effective problem solvers.

The instructional application of the model has four components: (a) assessing student performance and identifying students for whom the instructional program is appropriate; (b) explicitly instructing students in the acquisition and application of strategies for mathematical problem solving; (c) process modeling; and, (d) evaluating student outcomes, with an emphasis on strategy maintenance and generalization (Montague, 2000).

Assessing student ability and performance is critical to application of Montague’s (1993) cognitive strategy instruction to ensure that students have certain fundamental skills in reading and mathematical computation that are necessary for meaningful involvement in the program. Further, students’ unique strengths and weaknesses with respect to strategic activity must be determined in order for appropriate goals to be developed in the context of the instructional program, tailored to meet the individual needs of students. Assessment at regular intervals during intervention treatment is also critical for ascertaining students’ progress as they acquire and apply strategies. The transfer and maintenance of strategies is measured via longer-range assessment.

To assess fundamental skills in reading and mathematical computation, Montague (1997) recommends standardized measures, such as the Woodcock-Johnson
Psychoeducational Battery (Woodcock & Johnson, 1989). Montague’s cognitive strategy instruction program presumes a reading grade equivalent of approximately 3.5; demonstrated skill in adding, subtracting, multiplying, and dividing whole numbers and decimals; and developmental readiness in metacognition and reasoning (1997, p. 168). Additionally, the program is recommended for students 12 years of age or older. Younger students, whose cognitive development has not yet reached the Piagetian formal operational stage of development, do not evidence gain from this integrative and comprehensive cognitive strategy instruction (Montague & Applegate, 1992).

To assess students’ attitudes towards mathematics and their knowledge, use, and control of strategies, the Mathematical Problem Solving Assessment-Short Form (MPSA-SF; Montague, 1996) is recommended. The MPSA-SF includes three word problems, five Likert-type items, and 35 open-ended questions. The MPSA-SF yields individual cognitive profiles that provide information about students’ perceptions of their mathematical ability, their attitude toward mathematics and mathematical problem-solving, and their knowledge, use, and control of problem-solving strategies.

Finally, a criterion-referenced test of approximately 10 one-, two-, and three-step mathematical word problems is recommended to be administered to determine baseline performance. Progress checks are conducted regularly throughout the instructional program to keep a continuous record of student performance. The end goal of the strategy is the students’ achievement of at least 7 out of 10 problems correct on four

The explicit instruction component of Montague's (1993) mathematical problem-solving strategy entails highly structured, organized lessons, appropriate cues and prompts, guided and distributed practice, immediate and corrective feedback on learner performance, and mastery. Montague (2000) explains that the core instructional procedures associated with explicit instruction promote active learning, acquisition of mathematical problem-solving processes and strategies within a reasonable time period, and effective and efficient application when solving mathematical problems (p. 112). To ensure that all of the instructional procedures are incorporated, each lesson is scripted. The lesson follows a general format in which the student is engaged in the learning process through an initial introduction to the importance of mathematical problem solving and discussion of their individual performance on a pretest, and performance goals that they set for themselves. After instruction on the processes and strategies, students are engaged in practice sessions. As students are guided through the scripted lessons and applications, they are given practice in verbalizing the cognitive processes and self-regulation strategies they use.

The process modeling component of Montague's (1993) strategy instruction involves thinking aloud while demonstrating a cognitive activity. The thinking aloud
strategy emphasizes learning by imitation and provides students with the opportunity to witness how to solve mathematical problems. The teacher models correct and incorrect strategies to allow students to observe both appropriate, successful application of the processes/strategies and identification and correction of errors in the problem-solving process. In so doing, process modeling fosters self-monitoring skills. Likewise, the self-regulation strategies inherent in the metacognitive activities and processes (i.e., self-instruction, self-questioning) facilitate thinking aloud. Through this engagement, it is foreseen that as students become more proficient problem solvers, they will progress to using covert self-regulation strategies when they solve problems independently (Montague, 2000, p. 113).

The fourth instructional application of Montague’s (1993) model is performance feedback. Specific feedback is advocated to explicate exactly which behaviors and responses are being reinforced. In so doing, students will understand those activities which can be repeated. Reinforcement demonstrates to students that they are successful and can become better problem solvers. Honest reflections of students’ responses will result in learners who are more independent, less fearful of being wrong, and more confident in themselves. The ultimate goal, Montague (2000, p. 113) describes, is to have students recognize that they have done well and praise themselves for doing well.

Attribution theory asserts that the kind of causes a student ascribes to their learning outcomes is related to academic performance. In other words, students self-
reflection on their performance, such as whether their performance was due to some intrinsic ability or to applied effort, subsequently gives meaning to their outcomes and impacts their later performance on a designated task (Zimmerman, 1998). Montague's aforementioned ultimate goal (2000, p. 113) adheres closely to the tenets of attribution theory. However, Montague's cognitive strategy instruction does not explicitly teach students to attribute their learning outcomes to their efforts. As such, an important extension of this research is to build into the process modeling and performance feedback components of Montague's strategy instruction the attributional retraining modeled by Borkowski, Weyhing, and Carr (1988). As such, students will be engaged in short lectures and dialogues about the role of effort in deploying strategies. Students will observe the instructor modeling self-talk that ascribes the instructor's successful completion of mathematical word problems to the use of the strategy. Unsuccessful performance, in turn, will be attributed to the failure to use the strategy. Measurement of attributional beliefs will take place in pre- and posttesting.

The groundwork for Montague's strategy instruction resulted from several descriptive studies designed to ascertain the characteristics of problem solvers (Montague, 1991; Montague, 1995; Montague & Applegate, 1993; Montague & Applegate, 1997; Montague & Applegate, 2000). These studies suggested that the most salient characteristic of students with learning disabilities was their inability to represent
problems. Problem representation processes include paraphrasing, visualizing, and hypothesizing or making a plan to solve a problem (Montague, 2000, p. 113).

The program's instructional format developed from the two initial intervention studies (Montague, 1992; Montague & Bos, 1986) using multiple-baseline designs. The problem-solving instructional strategy described and researched in Montague, Applegate and Marquard (1993) is predicated on findings from a mixed-effect, pretest/posttest quasi-experimental design. In this study, 72 students with learning disabilities in grades seven and eight were instructed in small groups of 8 to 12 students across three instructional periods, each ranging from five to seven days. The results of participating in the strategy instruction were significant. Not only did the students with learning disabilities significantly improve their mathematical word problem-solving but they approximated the performance of the comparison group, average-achieving students, who were not given instruction. Across three maintenance measures (3 weeks, 8 weeks, and 15 weeks after cessation of treatment), students displayed variable performance. All of the treatment students demonstrated continued improvement at the first maintenance assessment. However, the mean scores declined significantly at the second maintenance test, indicating that the students did not internalize the strategy and attribute its use to successful performance. In Montague's study, a brief, one-period booster (re-training) session was given prior to the third maintenance assessment. On this final measure, students' performance again increased to the criterion level. Montague et al. (1993) note
that this trend indicates that students will not maintain use of the strategy indefinitely without some form of a refresher program. Yet, it is the goal of strategy instruction that the strategy will become inherent to the cognitive response pattern of the students. The attributional retraining component of the present study explicitly teaches students that their effort in utilizing the strategy contributes to their learning outcomes. By inherently ascribing their own application of effort, that is, their self-directed behavior, the probability that students will make the strategy a part of their permanent repertoire is directly supported.

Montague's strategy instruction has recently been adapted for use in inclusive, general education classrooms under the name Solve It! (2000, p. 110). Field-based investigation on its efficacy in such a setting is forthcoming. One of the goals of the present research is to replicate the findings of prior research to add to the validity of Montague's strategic instruction program.

The basis of the present research is predicated on the expectation that an instructional intervention will provide the strategic knowledge students with learning disabilities need to function successfully in regular mathematics classes. It is believed that the findings from Montague et al. (1993) can be replicated, with differing procedures, incorporating a control group of average-achieving students. Extension of Montague's research to include explicit attributional retraining is seen as particularly important to promoting long-term use of the strategy. Additionally, direct assessment of self-efficacy
will lend support to existing research (Graham & Harris, 1988) that participation in strategy instruction with cognitive and metacognitive components improves feelings of self-efficacy. Further, use of the MPSA-SF will be incorporated for the first time as a pretest/posttest measure assessing change in students' affective responses based on participation in the intervention.
CHAPTER 3

PROCEDURE

3.1 Participants

Eighteen students between the ages of eleven and thirteen years of age were selected to participate in the study. Students attended a suburban middle school (grades 6, 7, 8) in central Ohio, and represented two levels of mathematics problem-solving ability, learning disabled and average-achieving. Students who were learning disabled and receiving resource room support in mathematics were selected to participate in the treatment condition. District eligibility criteria for learning disabled included evidence of (a) a disorder in one or more of the basic psychological processes including visual, auditory, or language processes; (b) academic achievement significantly below the student's level of intellectual functioning; (c) learning problems that are not due primarily to other disabling conditions; and (d) the ineffectiveness of general educational alternatives in meeting the student's educational needs. Eligible participants also had targeted mathematics skills on their Individualized Education Program and were receiving mathematics instruction in the learning disabilities resource room program.
A comparison group of students who were average achieving were selected from regular mathematics classes. These students did not have a history of having received remedial reading or mathematics instruction.

A cover letter, or letter of introduction (Appendix A), and consent for participation in social and behavioral research form (Appendix B) were distributed to all of the sixth-, seventh- and eighth-grade students who were learning disabled, received resource room support in mathematics, and had mathematics IEP goals. The cover letter explained the overall purpose of the study and the parameters of each child's involvement. The consent for participation in social and behavioral research form outlined that participation in the study was fully voluntary and could be ceased without repercussion at any time. Additionally, both the cover letter and the consent form stated that randomly selected sessions of the intervention would be audiotaped to ensure the integrity of the study.

Cover letters (Appendix C) and consent for participation in social and behavioral research forms (Appendix D), modified to include description of the control group involvement, were distributed to average-achieving students enrolled in sixth-, seventh-, and eighth-grade regular education mathematics classes (N=229). The cover letter explained the overall purpose of the study and the parameters of each child's involvement. The consent for participation in social and behavioral research form
outlined that participation in the study was fully voluntary and could be ceased without repercussion at any time.

Upon return of the endorsed consent letters (Appendices B and D), the academic records of both the treatment group students and average-achieving students were screened to gather student characteristic data. These data included age, estimated cognitive ability and estimated achievement in the area of reading comprehension. For the learning disabled students, the measure of cognitive ability was determined from the Wechsler Intelligence Scale for Children, Third Edition (WISC-III; Wechsler, 1991). All Full Scale IQ scores had been administered by school psychologists for all of the subjects within the past three years. IQ testing of the average achieving students was not allowed by the school system, therefore, the Otis-Lennon School Ability Test, Sixth Edition (OLSAT; Otis & Lennon, 1989), which yields a Total Student Ability Index (TSAI), was utilized as the measure of cognitive ability. The WISC-III Full Scale IQ and Otis-Lennon TSAI have a correlation of .73 (Wechsler, 1991, p. 203). Based upon the variability of achievement measures in the special education assessment of the learning disabled students, the Metropolitan Achievement Test, Seventh Edition (MAT), Reading Comprehension was utilized as an estimate of reading comprehension skills for both the treatment and comparison groups. The MAT is administered by the school district to all students every other school year.
Selection of the comparison group participants was based on the number of treatment group students, to ensure equal sample sizes. The selected comparison students were matched with the treatment group participants based on grade, the measure of general cognitive ability, and level of reading achievement.

Separate univariate analyses of variance (ANOVAs) were performed to determine whether significant differences existed between the two groups on important subject variables. Such measures ensured that outcome measures were due to the effects of the independent variable and not generated from significant differences in scores of ability and reading achievement. Results were presented in the form of group descriptive data and not as individual demographic information, so as to protect participant confidentiality.

3.2 Materials

3.2.1 Dependent Measures.

The first pre- and posttest dependent measure was a measure of self-efficacy that prompted the students to rate their perceived confidence in solving each of six word problems, as listed in Appendix E. The efficacy scale ranged from 10 to 100 in 10-unit intervals; the higher the scale value, the higher the perceived self-efficacy. As detailed by Bandura and Schunk (1981) and Graham and Harris (1989), verbal descriptors occurred at the following points: 10 (not sure), 40 (maybe), 70 (pretty sure), 100 (real sure). Practice with the self-efficacy assessment procedure was provided by having subjects judge their
capability to jump progressively longer distances, from a few inches to several yards (Graham & Harris, 1989). Following this introduction, subjects read each of the 6 word problems. After reading each problem, the students rated how sure they were of their ability to solve the problem. Subjects were asked to be honest and to mark privately the appropriate number on the scale for each item. The summed magnitude scores divided by the total number of questions provided the measure of strength of self-efficacy (Graham & Harris, 1989). The self-efficacy scores served as the first of two multiple measures of the time-series design.

The second pretest and posttest measure was actual performance on tests of six mathematics word problems. From a pool of one-, two-, and three-step mathematical word problems provided by Marjorie Montague (M. Montague, personal correspondence, February 24, 2002), six tests of six problems each were constructed. Problems were selected randomly from different pools of one-, two-, and three-step problems so that each test contained 2 one-step, 2 two-step, and 2 three-step problems requiring the four basic operations and using whole numbers or decimals. Items were not returned to the item pool. Appendix F lists an example of one of the six-word problem tests. The word problems were spaced so that only two problems appeared on one sheet of paper to allow adequate room for problem solving. The tests served as the second of two multiple measures of the time-series design. Three tests were administered pre-treatment and three tests administered post-treatment. The order of administration of the
six tests was random. The problems in each of the six tests were those from which the students rated their perceived self-efficacy (the first dependent measure).

The third pretest and posttest measure was a measure of students’ attributions toward problem solving, as depicted in Appendix G. Administration of the attributions test dependent measure took place one time pre- and post-treatment and occurred after the second administration of the multiple measures. Students, after rating how sure they were that they could solve each of the mathematics word problems and then actually solving each of the problems, estimated how well they believed they did on solving the problems. After rating their performance, students were asked to indicate the potential cause of their performance by marking an X on a barometer that represented the level of importance (e.g., none, a little, some, or a lot) the student attributed to each potential cause (e.g., effort, ability, task difficulty, luck). For example, How much was your performance due to effort (record a rating), ability (record a rating), task difficulty (record a rating), and luck (record a rating)? The rating scale was based on the antecedent attributions test developed by Borkowski, Weyhing, and Carr (1988). As outlined in Borkowski et al., the scores that attributed success or failure to uncontrollable causes (ability, task difficulty, and luck) were summed, then divided by 3, and subtracted from the total effort score to yield an effort attribution score. Changes in effort attribution score as a result of participation in the intervention treatment group were analyzed via ANOVA.
Measures of perceived mathematics ability, attitude toward mathematics and mathematics problem solving, and knowledge, use, and control of mathematical problem-solving strategies was assessed through administration of the Mathematical Problem Solving Assessment-Short Form (MPSA-SF; Montague, 1996), as listed in Appendix H. This instrument is an abridged version of a cognitive-metacognitive interview for mathematical problem solving (Montague & Applegate, 1991; Montague & Bos, 1990, Montague, Bos, & Doucette, 1991). The MPSA-SF consisted of 5 Likert-type and 35 open-ended questions. Administration of the MPSA-SF dependent measure was conducted one time pre- and post-treatment and occurred after the second administration of the multiple measures. It was administered in an interview format and in individual sessions with each participant.

Scoring of the MPSA-SF was conducted according to the developed guidelines outlined in Montague (1996) and utilized in Montague, Bos, and Doucette (1991). Appendix I lists the MPSA-SF scoring and interpretation directions. As outlined in Appendix I, the items probing perception of math performance, attitude toward math and attitude toward mathematical problem-solving were converted to a 5-point scale, (Very Poor, Poor, Average, Good, Very Good). The 35-open ended questions probing the students’ knowledge of mathematical problem-solving strategies and knowledge, use, and control of specific problem-solving strategies were operationalized according to a 3-point scale to reflect little, some, or much awareness or ability. For example, as noted by
Montague et al. (1991, p. 147), little knowledge indicated minimal awareness of strategies that are necessary for effective and efficient mathematical problem solving, as well as little awareness of one’s own problem-solving strategies or those used by others. Little use was operationalized as minimal ability to select and apply problem-solving strategies when solving mathematical problems. Little control indicated minimal ability to evaluate and modify strategy selection and application when necessary in order to reach a solution to a mathematical problem in the most expedient manner.

The students’ scores were then transferred to the MPSA-SF profile form (Appendix J). Analyses were conducted via ANOVA to assess change in the students’ answers across pre- and post-test measures.

3.2.2 Intervention Materials.

Materials included scripted lessons, as depicted in Appendix L, and a wall chart listing the cognitive processes and metacognitive strategies for mathematical problem solving, as depicted in Appendix K. A dry erase board was utilized to model the mathematical problem-solving process. Individual folders for each treatment participant held strategy study cards illustrating the cognitive and metacognitive strategies and individual graphs for recording practice problem scores. Approximately 70 practice problems were randomly selected from the word problem pool provided by Marjorie Montague (M. Montague, personal correspondence, February 24, 2002) to utilize for the lessons. Signs displaying positive attributional captions (e.g., I tried hard, used the
strategy, and did well) also comprised the needed materials. An audiotape recorder was utilized to record randomly selected intervention sessions.

3.3 Design and Analyses

A quasi-experimental, control group time-series design was used to provide comparisons between the treatment group of students with learning disabilities and a group of average-achieving students that did not receive strategy instruction. Through multiple measures of the pre- and posttest dependent measures to both groups, the treatment effect was in a sense twice demonstrated, once against the control and once against the pre-treatment values in its own series (Campbell & Stanley, 1963, p. 55). The control of factors threatening internal validity, particularly the non-random selection and assignment of treatment participants, was accommodated by virtue of the multiple data measures prior and subsequent to treatment. The non-equivalent control and treatment groups were matched on measures of ability and reading achievement, hence also controlling for selection of these groups from distinct populations.

The students participating in the treatment and comparison groups were assigned to three groups, respectively, consisting of approximately three students per group, for a total of six groups (three comparison groups, three treatment groups). Group assignment was determined by the scheduling of the participating students study center periods. One-way analyses of variance (ANOVAs) were performed to determine if the groups
were equivalent on achievement and ability measures. Assuming no significant differences existed between the treatment and comparison groups, the data were collapsed across the groups to form one overall treatment group and one overall comparison group.

Pretest measures were gathered from the treatment and comparison groups prior to the implementation of the strategy instruction intervention. Posttest measures were collected from both groups following completion of the strategy instruction. Three multiple measures were gathered pre- and post-treatment on two of the dependent measures, self-efficacy score and items correct on six-item mathematics word problem assessments. The remaining two dependent measures, the Mathematical Problem Solving Assessment, Short Form (MPSA-SF) and the questions of attributional perception, were administered only once pre- and post-treatment.

Analyses of the self-efficacy score and of the items correct on the six-item word problem tests were conducted linearly. Campbell and Stanley (1963, p. 43) explained that, statistical tests would probably involve, in all but the most extended time series, linear fits to the data, both for convenience and because more exact fitting would exhaust the degrees of freedom, leaving no opportunity to test the hypothesis of change. It was assumed that a treatment effect would result in changes in both intercept and slope on the two multiple dependent measures of self-efficacy score and word problem performance. In addition, exploratory analyses were conducted using ANOVA on these measures. The
MPSA-SF and attributional perceptions of problem performance, administered once at pre- and posttest measures, were assessed using ANOVA.

3.4 Procedure

Students in the treatment group received strategy training in mathematics problem solving by the investigator in a private classroom setting. Both instruction and testing occurred during the students' regularly scheduled 30-minute study center periods. Treatment participants met with the investigator in three groups consisting of 3 sixth grade, 2 seventh grade, and 4 eighth grade students. Group assignment was determined by the scheduling of participating students' study centers, which was dictated according to grade. Each group met two times per week.

Students in the comparison group met with the investigator in a private classroom setting. Assessment of the pre- and posttest dependent measures occurred during the students' regularly scheduled 30-minute study center periods in a small group format. Group assignment was determined by the scheduling of the comparison group students' study centers, which was dictated according to grade.

3.4.1 Pretesting

Pretesting of the multiple measures - the self-efficacy score and the tests of mathematical word problems - took place across three separate sessions for both the treatment and the comparison groups. The groups met with the investigator during their
regularly scheduled study center periods. They were first given the instructions orally — allowing for group discussion and clarification. Worksheets with the self-efficacy measure were then distributed and the students worked independently to complete their ratings. The students then completed the second dependent measure — actual performance on a test of six word problems.

After the second administration of the self-efficacy and mathematical word problem tests, the students were administered the attributions test and the MPSA-SF. Administration of the attributions test immediately followed administration of the test of mathematical word problems. Administration of the MPSA-SF was conducted individually; average administration time was 25 minutes.

3.4.2 Treatment.

Treatment in each session followed a set of general procedures. The treatment groups met with the investigator for 30-minute lessons twice a week for approximately two months, or 16 lessons. Treatment consisted of a) strategy acquisition training (lessons one through nine), b) strategy application practice (lessons ten through sixteen), and c) attributional retraining (comprehensive). The first four sessions of strategy acquisition training followed the script outlined by Montague, Applegate, and Marquard (1993) and Montague et al. (2000). The remainder of the strategy acquisition training and application practice sessions followed the guidelines provided by Montague et al., (1993, 2000). The attributional retraining formed an integral part of both the strategy acquisition
training and application practice lessons and was implemented as described in Borkowski, Weyhing, and Carr (1988). Appendix L outlines both the scripted lessons (one through four) and the guidelines for the subsequent training and practice sessions.

Lessons One and Two began with an overview of the strategy instruction. First, the investigator guided a discussion among students about mathematical problem solving in general and why it is important to be a good problem solver. Then, the strategy was described for the students (see Appendix K) — outlining the cognitive processes. The processes were presented on a wall chart. Students practiced verbalizing the processes and strategies by reading through the charts individually and as a group using choral reading techniques. The investigator demonstrated how to use the comprehensive strategy to solve typical mathematical word problems. The investigator introduced positive self-attribution, I need to try and use the strategy, while actually using the strategy to successfully perform the problem. Students had an opportunity to ask questions. Lesson One also involved the distribution of the students' folders, containing study cards listing the strategy and processes. The folders were kept by the examiner and distributed at the start of each subsequent lesson.

During Lessons Three and Four, volunteer students were tested for mastery of the seven cognitive processes. The investigator provided corrective and positive feedback and checked off the names of the students who were able to recite from memory the names and descriptions of the processes. Then the entire group practiced recitation of the
processes. Individual students again took turns reciting the processes from memory. Students were cued using the acronym RPV-HECC and the chart posted on the wall. Following group and individual recitation of the processes, the investigator reviewed the metacognitive strategies with the students and led the group as they recited the processes and strategies using the wall chart.

The investigator modeled problem solving for the students using process modeling. During this modeling, the investigator made a series of intentional errors. After each error, an attributional dialogue, designed to strengthen antecedent attributions about effort, was introduced (Borkowski, Weyhing, and Carr, 1988). First, the investigator noted the error and engaged the students in a discussion about the reasons for failure on school tasks. The importance of not attributing failure to uncontrollable factors (e.g., problem difficulty, ability, luck) was stressed. Second, the investigator combined the positive self-attribution, I need to try and use the strategy, with actual use of the strategy in revision of the previously failed item. When finished, the investigator checked herself, correctly recalled the failed item and reiterated the necessity of strategy use for good recall. In the third step, the relation between effortful strategy use and successful performance was discussed. A stuffed animal held a sign displaying the caption, I tried hard, used the strategy, and did well (Borkowski et al., p. 49). The caption was used to prompt discussion to highlight the role of effort in implementing a strategy. During subsequent practice problems, the investigator called on students to verbalize the
processes and strategies as the investigator worked through the problem. At the conclusion of Lessons Three and Four, students practiced reciting the processes and strategies.

At the start of Lessons Five through Nine, students were tested for mastery of the processes. Again, the investigator marked the names of students who had reached 100% criterion for recitation of the processes. The investigator led the group as they recited all the processes and the SAY, ASK, CHECK strategies. The students then solved a practice problem individually. They were instructed to think aloud and verbalize the processes and strategies as they solved the problem. The teacher or a student then modeled the correct solution. Students and the investigator assisted the problem solver who was modeling the correct solution by verbalizing the processes and strategies as he/she worked through the problem. Students were then paired for solving problems. Partners took turns telling one another what to do.

Errors made during the individual and small-group problem-solving exercises were used to highlight how the student’s activity was a controllable factor in performance outcomes. The student or student-dyad was asked to explain the reasons for each error. During each step, self-attributions about uncontrollable outcomes were rephrased with positive belief statements about the role of effort.

The criterion for moving beyond the format of Lessons Five through Nine included the following: (a) all students in the group met the mastery criterion of 100%
accuracy for recitation of the cognitive processes from memory; (b) students understood and were able to use the SAY, ASK, CHECK strategies; and (c) students worked through the example math problems utilizing the strategy and processes.

Lessons Ten through Sixteen involved strategy acquisition practice. The students were informed at the beginning of each lesson that the effortful use of the strategy would help them to understand and successfully solve the word problems. They were then checked for mastery of the strategy. The investigator led a short review on the importance of attributing success and failure to controllable factors and its importance to success on academic tasks. The students were then given their first set of three practice problems to solve individually. Students were cued to use the strategy, consult the wall chart or their booklets, and think aloud. Following completion of each problem, either the investigator or a student modeled the correct solution. During these modeling examples, any errors unintentionally made by the student, or intentionally made by the investigator, were used to highlight the role of a controllable force, that is, not using the mathematical word problem solving strategy. The importance of not attributing failure to uncontrollable factors was again stressed. The investigator combined the positive self-attribution, I need to try and use the strategy, with the actual use of the strategy, to guide the student toward successfully performance on the previously failed item, modeling the strategy aloud. The formula, strategy use equals success, was emphasized (Borkowski, Weyhing, and Carr, 1988, p. 49).
Lessons Ten through Sixteen involved the students monitoring their behavior by plotting their progress on individual performance graphs, held in their folders. The motto, strategy use equals success, was printed at the top of the graphs. Discussion of errors was conducted in the context of controllable factors, whereas successful performance outcomes were attributed to strategy use.

3.4.3 Posttesting.

Posttesting of the multiple measures - the self-efficacy score and the tests of mathematical word problems - took place across three separate sessions for both the treatment and the comparison groups. The administration procedures were identical to those followed for pretesting. The groups met with the investigator during their regularly scheduled study center periods. They were first given the instructions orally — allowing for group discussion and clarification. Worksheets with the self-efficacy measure were then distributed and the students worked independently to complete their ratings. The students then completed the second dependent measure — actual performance on a test of six word problems.

After the second administration of the self-efficacy and mathematical word problem tests, the students were administered the attributions test and the MPSA-SF. Administration of the attributions test immediately followed administration of the test of mathematical word problems. Administration of the MPSA-SF was conducted individually; average administration time was 25 minutes.
CHAPTER 4

RESULTS

4.1 Participant Characteristics

The consent letter (Appendix B) was returned by 100% of the potential treatment group participants, that is, all of the sixth-, seventh- and eighth-grade students who were identified as learning disabled, received resource room support in mathematics, and had mathematics IEP goals. Initially, 11 treatment group participants were slated to begin the study; however, two of the students moved out of the district prior to the implementation of the treatment. The control group participants were selected from a pool of 27 possible students who returned the endorsed consent letter (Appendix D). The return rate for the comparison group students was 12%. The comparison group total (n=9) was determined upon matching the treatment group participants based on grade, measure of general cognitive ability, and level of reading achievement. Student variable data, including grade; age; gender; and ability and achievement test scores are provided in Table 4.1.
Table 4.1: Student Variable Data

The mean scores and standard deviations of the cognitive ability and reading comprehension scores were analyzed via ANOVA to determine if significant between-group differences existed on the ability and reading achievement measures. Nonsignificant differences were evident between the treatment and control groups.
according to cognitive ability, $F(1, 16) = 2.42, p = .14$. However, significant differences between the two groups were apparent on the measure of reading comprehension, $F(1, 16) = 18.36, p < .01$. Group assignment was determined by the scheduling of the participating students’ study center periods, dictated by grade. Therefore, the respective treatment and control sixth graders each comprised one group (n=3), the seventh graders another (n=2), and the eighth graders the last (n=4).

According to ability, both the treatment and control groups may be considered equivalent. However, significant differences were evident between the treatment and control groups on the measure of reading comprehension. Thus, the difference in reading comprehension scores may be considered a possible confounding variable influencing three of the dependent variables (self-efficacy, math word problem solving, effort attribution), as reading comprehension was involved in these measures.

4.1.1 Treatment Session Attendance

Treatment attendance at the 16 strategy training and practice sessions averaged 82%. Average attendance rates, by treatment group, were sixth grade, 83%, seventh grade, 75%, and eighth grade, 84%. Attendance was typically not the result of absenteeism, but was due to the students’ teachers needing their presence in a required activity held during the designated study center.

4.2 Data Analysis
Pretest measures were gathered from the treatment and comparison groups prior to the implementation of the strategy instruction intervention and posttest measures were collected from both groups following completion of the strategy instruction. Three multiple measures were gathered pre- and post-treatment on two of the dependent measures, self-efficacy scores and items correct on six-item mathematics word problem assessments. The remaining two dependent measures, the MPSA-SF and the questions of attributional perception, were administered only once pre- and post-treatment.

Analyses of the self-efficacy score and of the items correct on the six-item word problem tests were conducted linearly. That is, the multiple data points collected at the three pre- and three post-testing measures for the dependent measures, self-efficacy and items correct on the six-item word problem tests, were plotted and described according to best-fit, or trendlines. It was assumed that a treatment effect would result in positive changes in both intercept and slope on the two multiple dependent measures of self-efficacy score and word problem performance. Comparison of the trendlines, intercept and slope were conducted descriptively. In addition, exploratory analyses were conducted using ANOVA on these measures. The MPSA-SF and attributional perceptions of problem performance, administered once at pre- and posttest measures, were assessed using ANOVA.
4.2.1 Self-Efficacy Ratings

Analyses of the participants' ratings on the self-efficacy measure were conducted linearly. Multiple data points collected at the three pre- and three post-testing measures of self-efficacy were plotted and described according to best-fit lines, or trendlines. It was assumed that a treatment effect would result in positive changes in both the intercept and slope of self-efficacy score for the LD students. Comparison of the trendlines, intercept and slope were conducted descriptively. In addition, exploratory analyses were conducted using ANOVA on these measures. Figure 4.1 presents the data according to the multiple time series design, where \(X\) represents the strategy intervention in mathematical word problem-solving experienced by the treatment group participants.
Figure 4.1: Linear Analysis of Self-Efficacy Ratings Across Measures

$y = 3.055x + 66.42$

$y = -2.035x + 83.28$

$y = -1.475x + 93.55$

$y = 3.97x + 73.94$

Trendlines
Pre Control:
\[ y = 3.97x + 73.94 \]

Post Control:
\[ y = -1.475x + 93.55 \]

Pre Treatment:
\[ y = 3.055x + 66.42 \]

Post Treatment:
\[ y = -2.035x + 83.28 \]
Table 4.2 provides the means and standard deviations of the treatment and control participants’ pre- and post-test ratings on the measure of self-efficacy (Appendix E).

Linear fits to the data resulted in trend lines, outlined in Table 4.3. It was assumed that a treatment effect would result in positive changes in both intercept and slope.

<table>
<thead>
<tr>
<th></th>
<th>Treatment n=9</th>
<th>Control n=9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Pretest 1</td>
<td>67.96</td>
<td>16.35</td>
</tr>
<tr>
<td>Pretest 2</td>
<td>75.56</td>
<td>17.66</td>
</tr>
<tr>
<td>Pretest 3</td>
<td>74.07</td>
<td>14.98</td>
</tr>
<tr>
<td></td>
<td>77.96</td>
<td>13.66</td>
</tr>
<tr>
<td>Posttest 2</td>
<td>67.47</td>
<td>17.11</td>
</tr>
<tr>
<td>Posttest 3</td>
<td>73.89</td>
<td>18.28</td>
</tr>
</tbody>
</table>

Table 4.2: Pre- and Post-Test Means and Standard Deviations of Self-Efficacy Ratings
Table 4.3: Linear Trendlines of Self-Efficacy Ratings

<table>
<thead>
<tr>
<th></th>
<th>Treatment ( n=9 )</th>
<th>Control ( n=9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest Trendline</td>
<td>( y = 3.06x + 66.42 )</td>
<td>( y = 3.97x + 73.93 )</td>
</tr>
<tr>
<td>Posttest Trendline</td>
<td>( y = -2.04x + 83.28 )</td>
<td>( y = -1.48x + 93.55 )</td>
</tr>
</tbody>
</table>

Visual analysis of Figure 4.1 indicates a decline in slope from pretest to posttest for both the treatment and control groups. It is apparent that consistent and meaningful change in self-efficacy ratings did not occur as a result of participation in treatment. However, across both the treatment and comparison groups, students rated their certainty about solving the six mathematics word problems favorably. The overall mean rating of the treatment group prior to the intervention was 72.53, indicating an approximate pretty sure score of self-efficacy. After treatment, the average rating across the three measures improved minimally for the treatment group, \( M = 73.11 \). The pretest comparison group rated themselves more favorably, \( M = 81.88 \), across the three multiple ratings. At posttesting, the comparison group’s self-efficacy ratings increased marginally overall, \( M = 86.17 \). In an exploratory analysis of the self-efficacy means represented in Table 4.2, it was confirmed via ANOVA that the change across the multiple measures
from pre- to post-testing for the treatment group was not significant, $F(1, 4) = 0.02, p = .89$. Likewise, the control group change across the three multiple pre- and post-test measures was nonsignificant, $F(1, 4) = 1.11, p = .35$. Although the three multiple pre-test measures for the treatment group did not differ significantly from those of the comparison group, $F(1, 4) = 4.15, p = .11$, the multiple post-measures were significantly different between the two groups, $F(1, 4) = 16.59, p = .02$. These results indicate that, across time, the comparison group’s ratings of self-efficacy in solving mathematical word problems increased to be significantly higher than those of the treatment group’s.

4.2.2 Word Problem-Solving Performance

Analyses of the participants’ performance on the six-item tests of math word problems were conducted linearly. Multiple data points collected at the three pre- and three post-testing measures of word problem-solving performance were plotted and described according to best-fit lines, or trendlines. It was assumed that a treatment effect would result in positive changes in both the intercept and slope for the LD students. Comparison of the trendlines, intercept and slope were conducted descriptively. In addition, exploratory analyses were conducted using ANOVA on these measures. Figure 4.2 presents the data according to the multiple time series design, where $X$ represents the strategy intervention in mathematical word problem-solving experienced by the treatment group participants.
Figure 4.2: Linear Analysis of Math Word Problem Performance Across Measure

Trendlines
Pre Control:
y = 0.67x + 1.59
Post Control:
y = -0.055x + 3.68
Post Treatment:
y = 0.17x + 1.78
Pre Treatment:
y = 0.055x + 1.67
Table 4.4 provides the means and standard deviations of the treatment and control participant's pre- and post-test performance on the six-item word problem tests (Appendix F). Linear fits to the data resulted in trend lines, outlined in Table 4.5. It was assumed that a treatment effect would result in changes in both intercept and slope on the multiple dependent measure of word problem performance.

<table>
<thead>
<tr>
<th></th>
<th>Treatment n=9</th>
<th>Control n=9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$SD$</td>
</tr>
<tr>
<td>Pretest 1</td>
<td>1.67</td>
<td>1.50</td>
</tr>
<tr>
<td>Pretest 2</td>
<td>1.89</td>
<td>1.54</td>
</tr>
<tr>
<td>Pretest 3</td>
<td>1.78</td>
<td>1.92</td>
</tr>
<tr>
<td>Posttest 1</td>
<td>2.33</td>
<td>1.22</td>
</tr>
<tr>
<td>Posttest 2</td>
<td>2.89</td>
<td>1.69</td>
</tr>
<tr>
<td>Posttest 3</td>
<td>2.67</td>
<td>1.12</td>
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</table>

Table 4.4: Pre- and Post-Test Means and Standard Deviations of Word Problem Performance
Table 4.5: Linear Trendlines of Word Problem Performance

<table>
<thead>
<tr>
<th></th>
<th>Treatment n=9</th>
<th>Control n=9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest Trendline</td>
<td>$y = 0.06x + 1.67$</td>
<td>$y = 0.67x + 1.59$</td>
</tr>
<tr>
<td>Posttest Trendline</td>
<td>$y = 0.17x + 1.78$</td>
<td>$y = -0.06x + 3.68$</td>
</tr>
</tbody>
</table>

Visual analysis of Figure 4.2, the linear trendlines illustrating word problem-solving performance, indicates an increase in slope (0.06 to 0.17) and intercept (1.68 to 1.78) from pretest measures to posttest measures for the treatment group. Thus, after receiving strategy instruction in math word problem-solving (X), improvement resulted in the LD student's math word problem-solving performance. Conversely, the average-achieving students appeared to be marking gradual improvement at pre-testing in their problem solving performance (slope = 0.67) across multiple measures, yet their performance at post-testing indicated more consistent performance (slope = -.06). Thus, comparison of the post measure trendlines indicates that the treatment students demonstrated gradual improvement in their word problem-solving performance. Yet, it appeared that the average-achieving students continued to out-perform the LD students in their word problem-solving performance.
An exploratory analysis compared the performance of the LD students across pretest measures with their performance across posttest measures via ANOVA. Results evidenced a significant difference from pretest performance to posttest performance, $F(1, 4) = 23.64, p < .01$, indicating improvement in the LD student’s word problem solving performance after implementation of the intervention. Similar analysis of pretest versus posttest performance conducted with the average-achieving control group students yielded nonsignificant results, $F(1, 4) = 1.45, p = .29$. Comparison of the pretest performance of the LD students and the average achieving students indicated, as expected, significantly different scores on the word problem-solving measure, $F(1, 4) = 8.27, p < .05$. Comparison of the posttest performance of the LD versus the average achieving students continued to yield significantly different results, $F(1, 4) = 21.45, p < .01$. Thus, while it appears that significant gains were made by the LD students in their word problem solving performance after receiving strategy instruction, their performance did not improve to that of their average achieving counterparts.

4.2.3 Attribution Ratings

The third pretest and posttest measure was a measure of students’ attributions toward problem solving, as depicted in Appendix G. Administration of the attributions test dependent measure took place one time pre- and post-treatment and occurred after the second administration of the multiple measures. Students were asked to indicate the potential cause of their performance by marking an X on a barometer that represented
the level of importance (e.g., none, a little, some, or a lot) the student attributed to each potential cause (e.g., effort, ability, problem difficulty, luck). The ratings were converted to a 4-point scale. The ratings representing uncontrollable causes (ability, task difficulty, and luck) were summed, then divided by 3, and subtracted from the total effort score to yield an effort attribution score. The maximum effort attribution score was 18. Changes in effort attribution score as a result of participation in the intervention treatment group were analyzed via ANOVA. The greater the effort attribution score, the more the students perceived their performance outcomes were due to forces within their control (i.e., effort), versus uncontrollable forces (i.e., ability, problem difficulty, luck). Table 4.6 provides the means and standard deviations of the effort attribution score across pre- and post-test measures for the treatment and control groups.

<table>
<thead>
<tr>
<th></th>
<th>Treatment n=9</th>
<th>Control n=9</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Pretest</td>
<td>2.85</td>
<td>4.19</td>
</tr>
<tr>
<td>Posttest</td>
<td>1.59</td>
<td>3.86</td>
</tr>
</tbody>
</table>

Table 4.6: Pre- and Post-Test Means and Standard Deviations of Effort Attribution Score
As evidenced in Table 4.6, change in effort attribution scores across pre- to post-testing for the treatment group was not significant, $F(1, 16) = 0.44, p = .51$. Likewise, the control group did not evidence significant change in effort attribution score across the pre- and post-test measures, $F(1, 16) = 0.08, p = .78$. Interestingly, the groups did not differ from one another significantly on the effort attribution score at pre-testing, $F(1, 16) = 0.62, p = .44$, or post-testing, $F(1, 16) = 1.24, p = .28$. Significant variability was evident across the groups, marking concern about the validity of the univariate analysis. Visual analysis of the mean effort attribution scores provided in Table 4.6 indicates that, while both the treatment and control groups' effort attribution scores declined from pre- to post-testing, this decline was more substantial for the treatment group.

4.2.4 Mathematical Problem Solving Assessment — Short Form (MPSA-SF)

Measures of perceived mathematics ability, attitude toward mathematics and mathematics problem solving, and knowledge, use, and control of mathematical problem-solving strategies was assessed through administration of the Mathematical Problem Solving Assessment-Short Form (MPSA-SF; Montague, 1996), as listed in Appendix H. The MPSA-SF consisted of 5 Likert-type and 35 open-ended questions. Administration of the MPSA-SF dependent measure was conducted one time pre- and post-treatment and occurred after the second administration of the multiple measures. It was administered in an interview format, in individual sessions with each participant.
Scoring of the MPSA-SF was conducted according to the developed guidelines outlined in Montague (1996). Appendix I lists the MPSA-SF scoring and interpretation directions. As outlined in Appendix I, the items probing perception of math performance, attitude toward math and attitude toward mathematical problem-solving were converted to a 5-point scale, (Very Poor, Poor, Average, Good, Very Good). The 35-open ended questions probing the students' knowledge, use, and control of mathematical problem-solving strategies were analyzed and ascribed a rating of Little, Some, or Much. The students' scores were then transferred to the MPSA-SF profile form (Appendix J). The ratings (Little, Some, or Much) of the students' knowledge, use, and control of mathematical problem-solving strategies were converted to a 3-point scale to allow for quantitative analysis of change in the students' answers across pre- and post-test measures. Analyses were conducted via ANOVA. Table 4.7 provides the means and standard deviations of the eight categories probed by the MPSA-SF.

The treatment students experienced no significant change in their perceptions of their math performance, $F(1, 16) = 2.04, p = .17$, nor in their attitude toward math, in general, $F(1, 16) = 0, p = 1$. However, significant improvement in their attitude toward mathematics word problem-solving did occur following treatment, $F(1, 16) = 6.4, p = .03$. Conversely, no significant change occurred in the ratings of the comparison students' perceptions of their math performance, $F(1, 16) = 1, p = .33$, attitude toward math, in
<table>
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<th>Treatment n=9</th>
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<th>Control n=9</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td></td>
<td><strong>M</strong></td>
<td><strong>SD</strong></td>
<td><strong>M</strong></td>
</tr>
<tr>
<td>Perception of Math Performance&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td>Pretest</td>
<td>3.33</td>
<td>1.00</td>
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<td></td>
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<td>Posttest</td>
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<td>1.01</td>
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<tr>
<td></td>
<td></td>
<td>Posttest</td>
<td>3.44</td>
<td>1.33</td>
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<tr>
<td>Attitude Toward Mathematics Problem-Solving&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>Pretest</td>
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<td>1.00</td>
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<td></td>
<td></td>
<td>Posttest</td>
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<td>Pretest</td>
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<td>Posttest</td>
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<td>Pretest</td>
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<td></td>
<td>Posttest</td>
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<td>0.71</td>
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<tr>
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<td></td>
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<td>Posttest</td>
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<td>Use&lt;sup&gt;b&lt;/sup&gt;</td>
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<td>Control&lt;sup&gt;b&lt;/sup&gt;</td>
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<td>Pretest</td>
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<td>0.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Posttest</td>
<td>1.90</td>
<td>0.26</td>
</tr>
</tbody>
</table>

<sup>a</sup> 5 points possible  \<sup>b</sup> 3 points possible

Table 4.7: Means and Standard Deviations of MPSA-SF Ratings
general, $F(1, 16) = 1.47, p = .24$, or attitude toward mathematics word problem-solving, $F(1, 16) = 0.31, p = .58$. Were there significant differences between the LD and average-achieving students to begin with on these measures? Analyses via ANOVA indicated that no significant differences existed between the LD and average-achieving groups prior to treatment in their perception of their math performance, $F(1, 16) = 0, p = 1$, attitude toward math, in general, $F(1, 16) = 0.30, p = .59$, or attitude toward math word problem-solving, $F(1, 16) = 3.6, p = .08$. Likewise, after treatment, no significant differences were evident on the measure of math perception, $F(1, 16) = 0.40, p = .53$, or either measure of attitude toward math, $F(1, 16) = 0.31, p = .59$, or attitude toward math word problem-solving, $F(1, 16) = 0.96, p = .34$.

The MPSA-SF (Appendix H) contains two sets of questions probing student’s overall knowledge of math word problem-solving strategies. The first set (KMPS1) appears at the beginning of the MPSA-SF open-ended questions and includes probes such as, Tell me what you remember being taught about how to solve math word problems, and, What do you do to solve math word problems like the examples I showed you? The second set (KMPS2) is the final question on the MPSA-SF and states, Now that you have thought about what you do when you solve math word problems, tell me about the problem solving strategies you use when you solve math word problems. The treatment group demonstrated deeper knowledge of mathematical word problem solving, as described by the KMPS1 and KMPS2 measures, following intervention.
The treatment group’s post-test responses to the KMPS1 included richer descriptions of their knowledge of problem solving strategies. For example, an eighth grade LD student initially described the following strategies she used in math word problem-solving on the pre-test KMPS1 probes as, I read the problem, get the numbers, bring them down and try to solve it. This description was classified as little according to the guidelines outlined by Montague (1996) and described in Appendix I. After receiving strategy instruction, the same student commented, I read it (the problem) and picture it in my head, underline the important words, draw or picture it in my head (again), and solve it. The numerous strategies the student commented on earned a descriptor of some for the posttest KMPS1. A sixth grade LD student initially shared a limited awareness of math word problem-solving strategies, I read the problem, look at the numbers, and see what makes sense. After receiving strategy instruction, he shared, I read the story and think of a strategy. I write out all the ways I could do it (the problem) and see which is right. I draw pictures and think of the different ways I could do it in my head. I find out the way that’s right and do it. His answers evidenced use of metacognitive strategies, as well as visualizing and planning strategies. As represented by these two examples, the treatment group evidenced a significant improvement on the KMPS1 probe of math word problem-solving strategies, $F(1, 16) = 20, p < .01$.

Similar evidence of improvement in the overall knowledge of math problem solving emerged on the second probe of strategy knowledge (KMPS2) for the treatment group,
A sixth grade LD student responded to the final probe with, I picture the answer in my mind or correct it. While this answer touched on some aspects of good math word problem solving strategies, it earned a classification of little knowledge demonstrated overall. After receiving strategy instruction, the same student earned a much rating as he listed all of the Mathematical Problem Solving strategies explicitly taught in the treatment, Read, paraphrase, visualize, hypothesize, estimate, compute, and check. To be expected, 8 out of the 9 (89%) treatment students responded with a listing of the strategies taught during the strategy instruction intervention. Additionally, four of the students (44%) specifically referenced the metacognitive processes (say, ask, check) in their responses to this final probe.

Unlike the treatment group students, the average-achieving students did not evidence significant change across either the first probe (KMPS1) of overall knowledge of problem solving strategies, $F(1, 16) = 0.31, p = .58$, or the second probe (KMPS2), $F(1, 16) = 0.13, p = .71$. For the KMPS1 probes, the mean overall rating for the comparison group at pretesting ($M = 1.89, SD = 0.60$) and posttesting ($M = 2.00, SD = 0$) spoke to solid, though not comprehensive, knowledge of math word problem solving strategies. For example, a sixth grade average-achieving student described adequate knowledge of math word problem-solving strategies. At pre-testing, given the KMPS1 probes, she commented, I read it (the problem), and figure out what it says to do. Then I find the keywords. This is my main strategy. I do my best. I used to underline the keywords,
but now I always write down everything I can. At post-testing, the same student remarked somewhat more succinctly, commenting, I look at the problem, underline the most important part, and solve it step-by-step.

The KMPS2 probe proved somewhat more difficult for the average-achieving students; they consistently earned a descriptive rating between little and some, with the pretesting KMPS2 mean falling at 1.56 ($SD = 0.53$) and the posttesting KMPS2 mean emerging slightly weaker, at 1.44 ($SD = 0.73$). For example, an eighth grade average-achieving student reflected on her math word problem-solving strategies at pretesting, I take it one by one. One step at a time, one problem at a time. At post-testing, she commented, I don't really have a strategy. I just attack and do it! A seventh grade average-achieving student remarked at pretesting, I use the strategy that the teacher writes on the board, or I try all the strategies out and figure out which one I like best. Again, the student referenced the classroom in his post-testing response to the KMPS2 probe, I usually use what I learned in class and hope that it is the kind of problem that we've done in class. Importantly, the average-achieving students did not have to reference the Mathematical Problem-Solving Strategy to earn a much rating on the KMPS2 probe. An eighth grade comparison student yielded strong knowledge of math word problem-solving strategies in her post-testing response to the KMPS2 probe, I read the problem and look at it closely. Then I make a picture in my mind. I solve it,
then check it. Her response indicated awareness of reading the problem for understanding, problem representation through visualization, and the use of checking.

Unexpectedly, the LD and average-achieving students did not differ significantly in their responses to either the KMPS1 probes, $F(1, 16) = 2.78, p = .11$, or the KMPS2 probe, $F(1, 16) = 0.84, p = .37$, at pre-testing, suggesting that the students generally conveyed similar problem-solving strategies in their responses. However, at post-testing, the LD students evidenced significantly more knowledge of word problem-solving strategies than their average-achieving counterparts on both the KMPS1, $F(1, 16) = 10, p < .01$, and the KMPS2, $F(1, 16) = 13.08, p < .01$.

The MPSA-SF questions assessed the students' knowledge, use and control of the different components of math word problem-solving strategies: reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking. The LD students demonstrated considerable growth in their range of understanding math word problem solving strategies (e.g., What is estimation? ), their implementation of such strategies (e.g., How do you estimate, imagine, or predict the answer before you complete the operations for a math word problem? ), and their regulation of the strategies (e.g., How do you compare your answer with your estimate? ). This growth was evidenced qualitatively and quantitatively. Change across pre- and post-testing measures emerged as significant in the areas of knowledge of strategies, $F(1, 16) = 5.06, p = .04$, use of strategies, $F(1, 16) = 9.64, p < .01$, and control of strategies, $F(1, 16) = 18.95, p < .01$. In
particularly, the LD students evidenced significantly increased use of self-regulatory practices (i.e., control) across the seven problem-solving strategies. For example, prior to the intervention, an eighth grade LD student stated that he did not ask himself any questions during or after reading word math problems. At post-testing, given the same probes, (e.g., What questions do you ask yourself while you are reading math word problems? What questions do you ask yourself when you finish reading math word problems?), the student stated that he asks himself, What is the question saying? Did I double-check it to make sure I understood? His response reflected good use of self-regulatory practices critical to self-assessing comprehension of math word problems.

When asked about his knowledge, use and control of visualization — a critical component to math word problem representation — another eighth grade LD student stated at pretesting that he did not know what he did to visualize a problem, but that he occasionally did visualize (depends on the problem). At post-testing, this student described his use of visualization, (e.g., Do you ever make a drawing of the problem or see a picture of the problem in your mind?) by stating, Yes, usually I just draw a picture. I do this for most problems — make a picture that goes along with the problem. This student’s knowledge of visualization was then probed by the question, What do you do to make a picture in your mind? The student stated, I draw a picture and try to picture myself there. Finally, when asked about his control of visualization, (e.g., How
do your pictures help you solve math word problems? he stated, They help me visualize a problem so I can see a problem. It makes it easier to solve.

The average-achieving students did not evidence similar growth in their knowledge, use, and control of math word problem-solving strategies. In fact, little change was evident from pretesting to posttesting in the domains of strategy knowledge, $F(1, 16) = 0.64, p = .44$, strategy use, $F(1, 16) = 0.28, p = .60$, and strategy control, $F(1, 16) = 0.06, p = .81$. However, the average-achieving students maintained solid performance in these domains across measures. For example, when queried about her knowledge of hypothesizing (e.g., How do you make a plan to solve a math word problem? ), one sixth grade control-group student stated at pretesting, I think what the problem says, what I need to do, and what the steps are. At posttesting, given the same question, the student stated, I read the problem, think what it's asking me, and look for the key words. Her response spoke to a strong general framework for planning her activities in solving math word problems and included mention of metacognition.

Importantly, the LD and average-achieving students differed significantly at pretesting in all three domains — knowledge, use, and control — of word problem-solving strategies. The average-achieving students outlined significantly more awareness of strategy knowledge, $F(1, 16) = 6.52, p = .02$, strategy use, $F(1, 16) = 24.12, p < .01$, and strategy control, $F(1, 16) = 34.01, p < .01$, than the LD students. However, at posttesting, the two groups approximated one another in their explication of the critical
components of word problem-solving strategies, across strategy knowledge, \( F(1, 16) = 0.91, p = .35 \), strategy use, \( F(1, 16) = 0.53, p = .48 \), and strategy control, \( F(1, 16) = 3.70, p < .07 \).

In sum, after participation in treatment, important and significant gains were evidenced in the LD students’ knowledge, use, and control of math word problem-solving strategies, such that their awareness of these domains approximated that of average-achieving students. Unlike the average-achieving control group, the treatment group experienced significant growth in their attitude toward solving math word problems and in their comprehensive knowledge of overall math word problem solving strategies. The relevance of these significant gains may perhaps best be reflected in the LD student’s actual improvement in math word problem solving, as indicated in Figure 2. The meaning of these findings, including the nonsignificant self-efficacy and attributions scores, will be outlined in Chapter 5.
CHAPTER 5

DISCUSSION

The purpose of this study was to investigate the effects of cognitive strategy instruction on the mathematical problem-solving performance of middle school students with learning disabilities. The study further assessed how participation in cognitive strategy instruction facilitated the students' knowledge, use, and control of mathematical problem solving strategies. Finally, the study investigated how participation in strategy instruction affected two measures of motivation: students' ratings of self-efficacy in solving mathematical word problems and students' attribution of their effort in directing learning outcomes.

The research objectives of this study were:

1. To assess the efficacy of Montague's cognitive/metacognitive instructional strategy on the mathematical word problem-solving performance of learning disabled middle-school students.
2. To determine if engagement in such instruction resulted in qualitative changes in learning disabled students' knowledge, use, and control of mathematical problem solving strategies.

3. To replicate prior research on the impact of cognitive and metacognitive strategy instruction on the mathematical word problem-solving ability through comparison of learning disabled students with a control group of average-achieving students.

4. To extend previous research by assessing change in two affective components of learning related to motivation, self-efficacy and attribution of effort, as a result of participation in an intervention enhancing strategic learning in mathematics that incorporates explicit attributional retraining.

5.1 Summary of Findings

Results of the present investigation are meaningful in terms of each of the four research objectives. First, strategy instruction was efficacious in improving the mathematical word problem solving of students with learning disabilities. Second, important and significant gains were evidenced in the LD students' knowledge, use, and control of math word problem-solving strategies, such that their awareness of these domains approximated that of average-achieving students. Third, prior research was replicated and validated through the use of a comparison group. Fourth, although
significant changes were not realized across the two motivational constructs - self-efficacy and attribution of effort - important insights were gained.

5.1.1 Math Word Problem-Solving

The effectiveness of participating in cognitive strategy instruction was demonstrated through significant improvement in the mathematical word problem solving performance of middle school students with learning disabilities. That is, significant growth resulted on posttest measures of math word problem-solving following treatment. This growth in performance did not approximate that of average-achieving students as the comparison students continued to outperform the students with LD on the posttest measures. However, analysis of the trendline summarizing the posttest performance of the treatment group suggested that the LD student’s performance was trending toward approximation of that of the average-achieving students.

These findings replicated those of Montague, Applegate, & Marquard (1993). While a significant improvement was demonstrated in the mathematical word problem solving of students with learning disabilities, a comparison group of average-achieving students outperformed the LD students on both the pretest and posttest measures (Montague et al.). It was only through an exploratory analysis that a between-group difference was not observed. That is, when the posttest performance of the average achieving students was compared with the performance of the LD students on the first of three maintenance measures (given only to the LD students), no significant differences
were observed. These findings suggest that had the present study administered a maintenance measure approximately one month after the last post-test measure, performance of the treatment and control groups may have been approximate. As previously noted, analysis of the trendline summarizing the posttest performance of the treatment group suggested that the LD student’s performance was trending toward approximation of that of the average-achieving students

5.1.2 Knowledge of Strategies and Attitudinal Attributes

A significant extension of the present research was the inclusion of the MPSA-SF as a pretest and posttest measure. This instrument assessed change in students perceived mathematics ability, attitude toward mathematics and mathematics problem solving, and knowledge, use, and control of mathematical problem-solving strategies. Results indicated that, unlike the average-achieving control group, the treatment group of LD students experienced significant growth in their attitude toward solving math word problems and in their comprehensive knowledge of overall math word problem solving strategies as a result of participation in cognitive strategy instruction. Additionally, after participation in treatment, important and significant gains were evidenced in the LD students’ knowledge, use, and control of math word problem-solving strategies, such that their awareness of these domains approximated that of average-achieving students.

Interestingly, as found in previous research (Montague, Bos, & Doucette, 1991; Montague & Applegate, 1993), the students with LD compared favorably with average-
achieving peers on general knowledge of mathematical problem-solving strategies at the pretest measure (e.g., KMPS1, KMPS2). This finding, of course, seems inconsistent; students with learning disabilities generally appear to be lacking in strategy knowledge compared with their more proficient peers. However, as indicated in Montague et al. (1991), it seems that although learning disabled students appear to possess a certain degree of strategy knowledge, it may be incomplete, insufficient, or inappropriately applied when solving mathematical word problems (p. 150). Based upon their equivalent general knowledge of math word problem-solving strategies at pretesting, it is perhaps not surprising to find that, at posttesting, the LD students exceeded their average-achieving peers on this domain. Importantly, however, this finding lends significant support to the efficacy of the cognitive strategy instruction in bolstering the general strategy knowledge of LD students. The question of application of this knowledge may best be reflected in the improved performance of the LD students on the word problem-solving measures.

Previous research (Montague, Bos, & Doucette, 1991; Montague & Applegate, 1993) conclusively indicates that the most salient differences between students with learning disabilities and their average achieving peers is in the knowledge, use, and control of problem representation strategies. Problem representation strategies are those needed to process linguistic and numerical information, form internal representations in memory, comprehend and integrate problem information, and develop solution plans.
(Silver, 1987) (Montague & Applegate, 1993, p. 176). As expected, in the present research the LD students differed significantly from the average-achieving students in their knowledge, use, and control of math word problem-solving strategies at pretest measures — indicating a deficit in problem representation and problem solution strategies. However, after receiving cognitive strategy instruction, the LD students approximated the average-achieving students in these domains. Again, the significance of these findings is relevant in consideration of the LD student's actual application of their knowledge, use and control of problem representation and solution strategies. With improvement gained on the posttest measures of word problem-solving, a correlation in the acquisition and application of strategy knowledge, use and control appears evident.

In comparison studies investigating the mathematical problem-solving characteristics of middle school students, Montague and Applegate (1993) and Montague, Bos and Doucette (1991) found that, like their average-achieving and gifted counterparts, students with learning disabilities generally displayed a positive attitude toward mathematics. However, students with learning disabilities differed significantly from average-achieving and gifted students in their perception of their own math performance — an affective variable believed to particularly influence student's confidence in solving problems.

In line with previous studies (Montage & Applegate, 1993; Montague, Bos & Doucette, 1991), the present research did not find significant differences between the LD
and average achieving students attitude toward mathematics or math word problem-solving. However, contrary to previous findings, no differences were evident between LD and average achieving students perception of their own math performance. Using the MPSA-SF at pre- and post-testing allowed for observation of change after participation in strategy instruction. Little change was evident in the students perception of their own math performance or in their attitude toward math in general. However, posttest measures indicated significant change in the LD students attitude toward mathematics problem solving. While this measure of attitude toward mathematical problem-solving continued to remain equivalent to that of the average-achieving students, the positive trend toward a more favorable attitude reflects well on the students perceptions of their experience participating in the cognitive strategy instruction and in their outlook on math word problem solving.

5.1.3. Self-Efficacy

Self-efficacy, or personal beliefs about one’s capabilities to learn or perform behaviors at designated levels, is thought to be a strong predictor of subsequent performance through its influence on persistence, expenditure of effort and choice of activities (Bandura, 1986; Graham & Harris, 1989; Pajares & Graham, 1999; Schunk, 2003). Teaching that provides strategy instruction and metacognitive information has been effective in increasing student’s self-efficacy by improving success at achievement tasks (Graham & Harris, 1989). The present research sought to replicate the findings of
Graham and Harris in measuring the effects of strategy training on students' sense of self-efficacy.

In contrast to previous research (Graham & Harris, 1989), the present study did not evidence change in students' self-efficacy as a result of participation in cognitive strategy instruction. Despite significant change in their achievement on measures of math word problem solving, the students' self-ratings of efficacy were stable. This stability was also evident for the comparison group of average-achieving students. Interestingly, self-efficacy ratings for the LD students approximated those of the average-achieving students at pretesting, indicating a relatively high level of self-efficacy among LD students. A similar observation was noted by Graham and Harris (1989); the ability to assess one's own capabilities, and particularly the ability to know that one has a problem, is an important metacognitive skill (Brown et al., 1981; Harris, Graham, & Freeman, 1988) (p. 360). Graham and Harris postulate that such overestimation may be due to LD students' misperception of task demands, faulty self-knowledge, comprehension deficiencies, or use of a self-protective coping strategy. Thus, while change in ratings of self-efficacy did not occur as a result of participation in strategy instruction, further research on the elevated expectancies of problem learners appears warranted.

5.1.4 Attribution of Effort

The attributional retraining component of the present study sought to explicitly teach students that their effort in utilizing the strategy contributed to their learning
outcomes. This retraining component was based on the work of Borkowski, Weyhing, and Carr (1988). Through an intervention study with learning disabled upper elementary students, Borkowski et al. constructed an instructional program encouraging students to attribute failure to lack of effort rather than lack of ability. The program consisted of instruction in how to summarize paragraphs and attribution training that stressed the importance of effort and the use of the strategy. Students who received both strategy and attributional training performed significantly better on answering transfer questions than students who received strategy training alone. However, little change was evidenced in the students antece dent attributional beliefs.

Interestingly, the present study found similar results to Borkowski, Weyhing and Carr (1988). Students with learning disabilities significantly improved their math word problem-solving performance, yet little change occurred in their effort attribution scores at post-testing. Borkowski et al. postulated that, because students with learning disabilities possess complex educational histories fraught with failures, it is not surprising that brief, program-specific retraining did not produce change. Simply training LD children to attribute success and failure to effort will not adequately address their dysfunctional attributional belief patterns (Borkowski et al., p. 52). Rather, more intensified, prolonged training may be required for meaningful transformation to occur in LD students attributional belief systems. Understanding the mediating effects of attributional retraining on students mathematical word problem-solving performance
gives direction for future research, as a treatment condition did not exist without attribution retraining in the present study.

5.2 Limitations

5.2.1 Impact of Sample Size

A significant limitation of the present study is the small sample size. Although effects of the independent variable were found across several dependent measures, the inclusion of a comparison group of learning disabled students who did not receive strategy instruction would have allowed for greater demonstration of treatment effect. Further differentiation across target treatment groups could have allowed for parallel differentiation across comparison average-achieving groups, such as the inclusion of average-achieving students in strategy instruction. As previously noted, an interesting variation of the present study, assuming larger sample size, would be strategy instruction without specific attributional retraining.

5.2.2 Impact of Non-Random Sampling and Assignment

Students selected for participation in the treatment group were not randomly selected from the population of students with learning disabilities in mathematics, nor were they randomly assigned to the treatment group, thus limiting the generalizability of the present results to only those students who participated in the study. Further, the results must be tempered by the inclusion of comparison students who were non-
randomly selected and matched to the treatment group participants. Although the population of average-achieving sixth, seventh and eighth graders enrolled in regular education mathematics classes were sent information and consent letters, only those who returned the consent letters were considered for inclusion in the study. It may be that parents of the average-achieving students returned the endorsed consent forms in the hope that their child would receive some type of tangential benefit in mathematics instruction.

5.2.3 Impact of Non-Equivalent Groups

Although the treatment and control groups were equivalent according to ability, significant differences appeared between the groups on a measure of reading comprehension achievement. In similar studies (Montague, 1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986), the treatment and control groups did not evidence such discrepancies. Thus, the impact of the treatment group’s lower reading comprehension skills may have mitigated the effects of the independent variable.

5.2.4 Impact of SES and Ethnicity

An additional limiting factor in the present research is the inclusion of only those students who attended one of four middle schools in a primarily middle to upper socioeconomic status school district whose student population is predominantly Caucasian (94%). Thus, the impact of treatment on students from diverse ethnic and socioeconomic backgrounds is unknown. Significantly more diverse student populations
were studied in Montague’s research (1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986).

5.2.5 Impact of Scheduling

A limiting factor to the implementation of the strategy instruction revolved around the use of students’ study centers to implement the intervention. Unlike typical 50-minute class instructional periods, study centers are limited to a 30-minute block of time. The time constraint primarily limited the amount of practice word problems the treatment students were able to complete in the later sessions of the strategy intervention. Woodward and Montague (2002) noted that, too often in special education, students are shortchanged in mathematics instruction because they have so many other pressing needs, including, but not limited to, academics (p. 91). Although the intervention did not replace mathematics instruction, the study center time was frequently utilized by teachers to catch up the treatment students in other academic domains.

5.2.6 Impact of Attendance

As noted, the 82% average attendance rate for participation in the strategy instruction is an unknown limiting factor. With greater consistency of implementation across subjects, the effects of treatment may have been changed.

5.2.7 Impact of Study Time Frame

A significant limitation of the present study is the measurement of the dependent variables only at pre- and post-testing indices. Little opportunity exists for decisive
statement of the generalizability and maintenance of the strategy instruction's effectiveness.

5.3 Implications

5.3.1 Relevance for Instruction of Students with Learning Disabilities

While instruction of general mathematics is trending toward the development of problem solving proficiency, little is known about how students with learning disabilities perform in classrooms in which the NCTM Standards (National Council of Teachers of Mathematics, 1989, 2000) guide instruction (Fleischner & Manheimer, 1997). Further, despite recent shifts in theoretical paradigms and instructional methods in the mathematical domain, students with learning disabilities continue to be at risk for failure in mathematics (Montague, 1997). For mathematical knowledge to be useful, students with learning disabilities must comprehend how procedures can function as tools for solving relevant problems. Yet, instruction for students with learning disabilities too often persists to focus on routines and memory instead of meanings and processes, or the relation between what is learned and the real world (Cawley & Parmar, 1992).

The present research served to specifically address those metacognitive activities essential to successful problem solving and proved effective in changing the mathematical problem solving of students with learning disabilities. Students with learning disabilities enhanced their problem-solving performance, yet perhaps more importantly, they gained
knowledge in critical metacognitive problem-solving activities. As Montague and Applegate (1993) outlined, Without explicit instruction in cognitive and metacognitive strategies necessary for solving mathematical word problems, it is doubtful that students with learning disabilities will learn to apply acquired mathematical skills and knowledge (p. 193-194). This application of acquired skills and knowledge was reflected in the LD students' responses to inquiries about their knowledge, use and control of critical problem-solving strategies. Meaningful growth occurred across these domains — evidencing significant promise for parsimonious instructional practices in mathematical problem-solving.

5.3.2 Relevance for Inclusive Classrooms

The overarching tenet of special education law (IDEA, 2000), least restrictive environment, is increasingly being practiced through the greater inclusion of students with disabilities in regular education classrooms. The success of inclusion perilously rests on the successful differentiation of instruction for all learners. New research (Mevarech, 1999) adds to the efficacy of embedding metacognitive training in cooperative learning settings, where learners span a diverse range of ability yet work together to solve problems and complete tasks. Mevarech found that middle school students in heterogeneous classrooms performed most successfully when metacognitive training was combined with cooperative learning. Cooperative learning alone was not sufficient for enhancing mathematical achievement. Rather, structuring group interactions around
metacognitive processes fostered greater mathematical reasoning for students, regardless of ability level. These findings, in combination with those of the present study, show significant promise for application to the inclusive middle school classroom and clearly point toward an area of future research.

5.3.3 Addition to Existing Research

Despite a call for intervention research targeting higher-level mathematical concepts (Maccini & Hughes, 1997), the number of studies investigating the efficacy of cognitive interventions has not kept pace with behavioral interventions in recent years. Recent meta-analyses of intervention research illustrated the paucity of research in mathematics for students with learning disabilities (Swanson, Hoskyn, & Lee, 1999). The present research supplements recent literature on teaching students with learning disabilities how to solve word problems (Hutchinson, 1993; Maccini & Hughes, 2000; Naglieri & Gottling, 1997) through the replication and extension of metacognitive strategy training research (Montague, 1995, 1997; Montague & Bos, 1986). With the successful replication of Montague’s seminal work in mathematics word problem-solving, promising trends towards improving the performance of higher-order word problem-solving ability in middle-school students with learning disabilities are validated.

Further, the present research adds depth to existing research (Montague, Applegate, & Marquard, 1993) through the use of a metacognitive assessment instrument at pre- and post-testing. With this exploratory step, new insights were gained into the
cognitive, metacognitive, and affective changes evidenced as a result of participation in strategy instruction.

5.4 Future Research Directions

Woodward and Montague (2002) declared, Although some special education researchers have investigated interventions consistent with math reform, others continue to focus on traditional topics in mathematics. What is missing is a synthesis of these approaches around topics that are commonly taught to students with learning disabilities (p. 95). Clearly, exciting new paths have been forged as a result of the present research. Application of metacognitive instruction to the classroom, and the resulting impact on learning, appears to be one of the most significant and pressing areas of future research. Understanding how metacognitive strategy instruction works in a cooperative learning environment offers the opportunity to marry social-constructivist theories (Vygotsky, 1978; Slavin, 1996) with that of mathematical problem-solving strategy instruction (Montague, 1995, 1997).

Further future research opportunities lay in understanding the transaction between cognitive, metacognitive, and affective components of problem solving. The present research did not serve to replicate the significant findings of Graham and Harris (1989) regarding the impact of strategy instruction on students' ratings of self-efficacy. Additional research replicating these findings is thus relevant. As Graham and Harris
noted, the inflated self-efficacy ratings of LD students may be due to their misperception of task demands, faulty self-knowledge, comprehension deficiencies, or use of a self-protective coping strategy. Future research relevant to understanding these possible causalities will in turn enhance our understanding of the complex processes by which effective learning takes place.

How students with learning disabilities approach a task and the amount of effort they put forth may be directly influenced by their self-perceptions (Montague, 2000). The present study found that, as self-efficacy ratings did not fluctuate, so too did perception of math performance (as measured on the MPSA-SF) remain stable. How these two constructs — perception of overall math performance and self-efficacy — relate and interface with math word problem-solving is basis for further correlational research.

Both strategy and error analysis of the LD students actual word problem performance, both before and after treatment, offers a potential wealth of information related to their application of strategies. For example, while the measure of accuracy alone speaks to the use of more effective problem-solving strategies, analysis of problem-representation strategies — such as underlining and visualizing (i.e., drawing) — are typically tangible signs of effective strategy use as students problem-solve. The operationalization of such tangible signs could offer the opportunity for correlation with students reported use of such strategies during the metacognitive interview.
Finally, future research on the maintenance and generalizability of cognitive strategy instruction in problem solving remains a venue for significant growth in the field of mathematics. Research indicates that metacognitive information about strategies plays a particularly critical role in the generalization and maintenance of strategies for LD students (Pressley & Woloshyn, 1995), yet conclusive longitudinal studies remain scarce in the literature.

5.5 Conclusions

The basis of the preceding research was predicated on the expectation that an instructional intervention would provide the strategic knowledge students with learning disabilities require to function successfully in regular mathematics classes. This expectation was met - students with learning disabilities gained and approximated average-achieving students in their knowledge, use, and control of effective problem-solving strategies. Further, their actual math word problem-solving performance increased significantly following strategy instruction. The implications of these findings lend strong evidence to the efficacy of strategy instruction in the inclusive classroom. Future research directions point to the relevance of incorporating cooperative learning into the differentiated mathematics classroom to facilitate problem-solving for middle school students across ability levels.
BIBLIOGRAPHY


Dear Parent:

I am pleased to have the opportunity to introduce myself. I am the intern school psychologist for Dublin City Schools for the 2001-02 school year. School psychologists are licensed professionals who work closely with teachers, parents, and students. We strive to make learning in the school environment a successful and meaningful experience for all children.

I am currently finishing my doctoral coursework in school psychology from The Ohio State University. As part of this process, I am conducting an intervention study. The study is a research-based intervention proven very effective with middle-school students who have learning disabilities in mathematics. The purpose of my study is to use this intervention to teach students with learning disabilities a strategy to solve mathematical word problems. Dublin City Schools has given me permission to conduct the study. Additionally, the Human Subjects Institutional Review Board has reviewed procedures regarding the protection of the rights and welfare of the human subjects involved in this research.

I ask for your consent to allow your child to be considered for participation in this valuable opportunity. Should you indicate interest for your child to participate, you will be contacted and asked for your involvement in scheduling a time for your child to work with me in a small group of students twice a week during their regularly scheduled study center or extension periods. Your child will not miss any of his/her classes or activities. Further, should your child participate in the study, full confidentiality will be guaranteed. The information on your child’s progress will be solely for the purpose of this study and will not be a part of your child’s educational records. However, you will be provided with a summary of your child’s progress at the conclusion of the study. Additionally, randomly selected intervention sessions in which your child participates will be audiotaped for the sole purpose of ensuring integrity of the intervention. These audiotapes will not be used as part of the data in the study and will not be reported in
any document. They will stored at my home and disposed of by me via shredding upon conclusion of my dissertation’s defense.

I look forward to hearing from you and appreciate your consideration of this opportunity for your child to be considered for additional support in the area of mathematics. If you are interested in learning more about this opportunity before signing the attached consent form, please do not hesitate to contact me, Teresa McCarthy, or Dr. Antoinette Miranda of The Ohio State University, with any questions. Our contact information is listed below. Thank you.

Sincerely,

Gretchen Daniel, M.A. Teresa McCarthy, M.A.
Intern School Psychologist School Psychologist, Davis Middle School
760-4623 718-8675
daniel_gretchen@msmail.dublin.k12.oh.us mccarthy_teresa@msmail.dublin.k12.oh.us

Antoinette Miranda, Ph.D.
Associate Professor, The Ohio State University
292-5909
miranda.2@osu.edu
CONSENT FOR PARTICIPATION IN SOCIAL AND BEHAVIORAL RESEARCH

Protocol Title: Effects of Cognitive Strategy Instruction on the Mathematical Problem Solving of Middle School Students with Learning Disabilities

Protocol Number: 01B0234

Principal Investigator: Dr. Antoinette Miranda

I consent to my child’s participation in research being conducted by Dr. Antoinette Miranda of The Ohio State University and her assistant.

The investigators have explained the purpose of the study, the procedures that will be followed, and the amount of time it will take. I understand the possible benefits, if any, of my child’s participation.

I know that my child can choose not to participate without penalty to him/her. If I agree to my child’s participation, my child can withdraw from the study at any time, and there will be no penalty.

I consent to the use of the following information from my child’s academic records:

- Age
- Student Ability Index
- Achievement Scores in the areas of mathematics and reading comprehension

I consent to allow randomly selected intervention sessions in which my child is a participant to be audiotaped for the purpose of ensuring the integrity of the study.
I have had a chance to ask questions and to obtain answers to my questions. I can contact the investigators at 760-4623 or 292-5909. If I have questions about my child’s rights as a research participant, I can call the Office of Research Risks Protection at (614) 688-4792.

I have read this form or I have had it read to me. I sign it freely and voluntarily. I understand that a copy will be given to me.

**Print the name of the participant: ________________________________**

Date:_______________  Signed: __________________________

*(Person authorized to consent for participant)*

Signed: __________________________

*(Principal Investigator or his/her authorized representative)*

HS-027 (Rev. 04/01)  *(To be used only in connection with social and behavioral research.)*
Dear Parent:

I am pleased to have the opportunity to introduce myself. I am the intern school psychologist for Dublin City Schools for the 2001-02 school year. School psychologists are licensed professionals who work closely with teachers, parents, and students. We strive to make learning in the school environment a successful and meaningful experience for all children.

I am currently finishing my doctoral coursework in school psychology from The Ohio State University. As part of this process, I am conducting an intervention study. The study is a research-based intervention proven very effective with middle-school students who have learning disabilities in mathematics. The purpose of my study is to use this intervention to teach students with learning disabilities a strategy to solve mathematical word problems. Dublin City Schools has given me permission to conduct the study. Additionally, the Human Subjects Institutional Review Board has reviewed procedures regarding the protection of the rights and welfare of the human subjects involved in this research.

In order to demonstrate progress by the students who receive the intervention, an important part of my study is the involvement of students to serve as a comparison group. Students who are part of the comparison group will not take part in the intervention. They will only be administered short measures regarding mathematical word problem solving during a regularly scheduled study center or extension time. Your child will not miss any of his/her classes or activities.

I ask for your consent to allow your child to be considered for selection as a control group participant in my study. By signing this form, your child will not automatically be a control group participant. You will be notified if your child is one of the selected participants. Should your child be selected, you will be contacted and given additional information. Further, if your child is selected to participate, full confidentiality will be
guaranteed. The information on your child’s performance will be solely for the purpose of this study and will not be a part of your child’s educational records.

I look forward to hearing from you and appreciate your consideration of this opportunity to help educators learn how best to teach students who struggle in the area of mathematics. If you are interested in learning more about this opportunity before signing the attached consent form, please do not hesitate to contact me, Teresa McCarthy, or Dr. Antoinette Miranda of The Ohio State University, with any questions. Our contact information is listed below. Thank you.

Sincerely,

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Antoinette Miranda, Ph.D.
Associate Professor, The Ohio State University
292-5909
miranda.2@osu.edu
APPENDIX D

CONSENT FORM: CONTROL GROUP

Short form HS-027

CONSENT FOR PARTICIPATION IN SOCIAL AND BEHAVIORAL RESEARCH

Protocol Title: Effects of Cognitive Strategy Instruction on the Mathematical Problem Solving of Middle School Students with Learning Disabilities

Protocol Number: 01B0234

Principal Investigator: Dr. Antoinette Miranda

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- Age
- Student Ability Index
- Achievement Scores in the areas of mathematics and reading comprehension

I have had a chance to ask questions and to obtain answers to my questions. I can contact the investigators at 760-4623 or 292-5909. If I have questions about my child’s rights as a research participant, I can call the Office of Research Risks Protection at (614) 688-4792.
I have read this form or I have had it read to me. I sign it freely and voluntarily. I understand that a copy will be given to me.

**Print the name of the participant:** ______________________________________

Date:______________  Signed: ____________________________________________

(Person authorized to consent for participant)

Signed: __________________________

(Principal Investigator or his/ her authorized representative)

HS-027 (Rev. 04/01)  (To be used only in connection with social and behavioral research.)
APPENDIX E

SELF-EFFICACY ASSESSMENT

Directions: Read the problem. Think about solving the problem. How sure are you that you can solve the problem? Circle the number that best describes how sure you are that you can solve the problem.

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<tr>
<th>Not Sure</th>
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1. Janice has a paper route. She delivers papers to 88 customers every day. Janice must also collect money from her customers each week. On Saturday, she collected money from 43 customers. How many customers must Janice collect money from on Sunday?

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<th>Not Sure</th>
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2. Four friends have decided they want to go to the movies on Saturday. Tickets are $2.75 for students. Altogether they have $8.40. How much more do they need?

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<th>Not Sure</th>
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</table>
3. Chain link fence sells for $1.23 a foot. How much will Farmer Jones have to spend for chain in order to enclose a 70 foot by 30 foot patch of ground, leaving a 4 foot entrance in the middle of each of the 30 foot sides?

<table>
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4. Bill and Shirley need to arrange the chairs for a play that the class is having. They took 252 chairs from the storeroom. Their teacher told them to make rows of 12 chairs each. How many rows will they have?

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<th>Not Sure</th>
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5. A group bought 52 tickets. Each ticket was $26 less than the $280 regular price ticket. How much did the group spend for the tickets?

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6. A store sells shirts for $13.50 each. On Saturday, it sold 93 shirts. This was 26 more than it had sold on Friday. How much did the store charge for all the shirts sold on both days?

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</table>
1. A jet flies 1,485 miles in 3 hours. What is the average distance each hour?

2. Jill saves $4 each week from her allowance. She wants to buy a school jacket that costs $24.59. She has been saving for 4 weeks. How much more does Jill need to save to buy the jacket?

3. Sam and Arturo have joined a Jog-a-thon to raise money for the public library. For every mile Sam jogs, his sponsors will pay $3.00. For every mile Arturo jogs, he will raise $2.75. Sam jogged 5 miles, while Arturo jogged 8 miles. Together how much money did they raise?

4. Radio Shack bought 48 radios at $59 each. The store then sold them at $79 each. How much profit did the store make?

5. An airline agent checked a bag that weighed 35 pounds, another that weighed 4 pounds less than the first, and a third that weighed 13 pounds less than the second. How many pounds were checked?

6. Lisa drives 400 miles in 8 hours. How many miles does she drive in one hour?
APPENDIX G

MEASURE OF ATTRIBUTIONS

Janice has a paper route. She delivers papers to 88 customers every day. Janice must also collect money from her customers each week. On Saturday, she collected money from 43 customers. How many customers must Janice collect money from on Sunday?

How sure are you that you solved this problem correctly? Circle your answer.

Not at All Sure  A Little Sure  Mostly Sure  Very Sure

Mark an x in one area for each of the four questions.

How much was your performance on this problem due to:

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</tr>
</tbody>
</table>

A group bought 52 tickets. Each ticket was $26 less than the $280 regular price ticket. How much did the group spend for the tickets?

How sure are you that you solved this problem correctly? Circle your answer.

Not at All Sure  A Little Sure  Mostly Sure  Very Sure

Mark an x in one area for each of the four questions.  
How much was your performance on this problem due to:

<table>
<thead>
<tr>
<th>Effort?</th>
<th>None</th>
<th>A Little</th>
<th>Some</th>
<th>A Lot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability?</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Problem Difficulty?</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Luck?</td>
<td></td>
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</tbody>
</table>

A store sells shirts for $13.50 each. On Saturday, it sold 93 shirts. This was 26 more than it had sold on Friday. How much did the store charge for all the shirts sold on both days?

How sure are you that you solved this problem correctly? Circle your answer.

Not at All Sure  A Little Sure  Mostly Sure  Very Sure

Mark an x in one area for each of the four questions.  
How much was your performance on this problem due to:

<table>
<thead>
<tr>
<th>Effort?</th>
<th>None</th>
<th>A Little</th>
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</tr>
<tr>
<td>Luck?</td>
<td></td>
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</table>
APPENDIX H

MATHEMATICAL PROBLEM SOLVING

ASSESSMENT-SHORT FORM (MPSA-SF)

PART A
Examiner: Here are three examples of math word problems. (Show problems 2, 3, 4). I will read them to you. You do not need to solve them. (Read the problems). Now I would like you to answer the following questions. I will write your answers.

<table>
<thead>
<tr>
<th></th>
<th>Very Poor</th>
<th>Poor</th>
<th>Average</th>
<th>Good</th>
<th>Very Good</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Describe your math skills</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2. Describe your math grades</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3. Describe how well you solve math word problems</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4. Do you like math?</td>
<td>Not at all</td>
<td>1/4 of the time</td>
<td>1/2 of the time</td>
<td>3/4 of the time</td>
<td>Always</td>
</tr>
<tr>
<td>5. Why or why not?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Do you like to solve math word problems?</td>
<td>Not at all</td>
<td>1/4 of the time</td>
<td>1/2 of the time</td>
<td>3/4 of the time</td>
<td>Always</td>
</tr>
<tr>
<td>7. Why or why not?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. How were you taught to solve math word problems?

9. What do you do to solve math word problems like the examples I showed you?

8. A strategy is a general plan or a specific activity people use to solve problems. Tell me about any strategies you use to solve math problems.

PART B
Examiner: Now I would like you to look at how you solved these problems. (Show the student the six word problems they completed at prior administration).

9. How do you read math word problems?

10. How many times do you read math word problems?
11. As you read, how do you help yourself understand the problem?

12. If you do not understand something about the problem, what do you do?

13. What questions do you ask yourself while you are reading math word problems?

14. What questions do you ask yourself when you finish reading math word problems?

15. What else do you do when you read math story problems?

16. How do you help yourself remember what the problem says?

17. Do you put what you read into your own words?

18. How do you do this? Now I would like you to put problem 3 into your own words.

19. When you put the problem into your own words, how do you know what you said is correct?

20. Do you ever make a drawing of a problem or see a picture of the problem in your own mind? (Have student clarify — See probes).

   Probes: What kind of picture? How often do you use drawings or pictures? When do you make drawings of problems? Under what conditions do you make drawings or see pictures in your mind? Which problems? (Have the student draw a picture of one of the problems).

21. What do you do to make a picture in your mind?

22. How do your pictures help you solve math word problems?

23. What else do you do when you visualize?

24. How do you make a plan to solve a math word problem?

25. How do you use your plan to help you solve math word problems?

26. How do you know which operations to use (such as adding, subtracting, multiplying, or dividing)?

27. How do you decide how many steps are needed to solve a math word problem?

28. What is estimation?

29. Estimation is making a prediction about the answer using the information in the problem. How does estimation help in solving math word problems?

30. How do you estimate, imagine, or predict the answer before you complete the operations for a math word problem?

31. How do you compare your estimate with your answer?

32. What goes on in your head as you compute?

33. What do you do when you compute answers to word problems?

34. How do you know your computation is correct?

35. What is checking?
36. How do you check that you have correctly completed a math word problem?

37. How do you check math word problems?

Examiner: Now I want you to (read a story, continue your class work, etc.) and in about 5 minutes I will ask you one more question.

38. Now that you have thought about what you do when you solve math word problems, tell me about the problem solving strategies you use when you solve math word problems.

OBSERVATIONS: Record any pertinent observations.
APPENDIX I

DIRECTIONS FOR ADMINISTRATION, SCORING AND INTERPRETATION OF THE MATHEMATICAL PROBLEM SOLVING ASSESSMENT (MPSA-SF)

Directions for Scoring and Interpreting the MPSA-SF:

Items 1,2,3  Perception of Math Performance (Average score and circle descriptor)
PMP:  VP  P  A  G  VG

Item 4  Attitude Towards Math (Score, circle descriptor, and write reason)
AM:  N  1/4  1/2  3/4  A

Item 6  Attitude Toward MPS (Score, circle descriptor, and write reason)
AMPS:  N  1/4  1/2  3/4  A

Items 8,9,10  Knowledge of MPS (Note strategies)
1.
2.
3.
4.
5.
(Score as little, some, or much knowledge)
KMPS1:  Little  Some  Much
Items 11,13, 17  **Knowledge of Reading**  
(Nota techniques, activities)  
1.  
2.  
3.  
4.  
5.  
(Score as little, some, or much knowledge)  
KR:  Little Some Much

Items 12,14  **Use of Reading**  
(Summarize responses in writing here.)  

(Score as little, some, or much use)  
UR:  Little Some Much

Items 15, 16  **Control of Reading**  
(List questions asked and note While reading or After reading)  
1.  W  A  
2.  W  A  
3.  W  A  
4.  W  A  
5.  W  A  
(Score as little, some, or much control)  
CR:  Little Some Much

Item 18  **Knowledge of Paraphrasing**  
(Write techniques, activities)  
1.  
2.  
3.  
4.  
(Score as little, some, or much knowledge)  
KP:  Little Some Much

Items 19,20  **Use of Paraphrasing**  
(Note quality of student s paraphrase)  

(Score as little, some, or much use)
Item 21  **Control of Paraphrasing**
(Summarize response)

(Score as little, some, or much control)

CP:  Little Some Much

Items 23, 25  **Knowledge of Visualization**
(Write activities)
1. 
2. 
3. 
(Score as little, some, or much knowledge)

KV:  Little Some Much

Item 22  **Use of Visualization**
(Summarize information given)

(Score as little, some, or much use)

UV:  Little Some Much

Item 24  **Control of Visualization**
(Write responses)
1. 
2. 
3. 
(Score as little, some, or much control)

CV:  Little Some Much

Item 26  **Knowledge of Hypothesizing**
(Write techniques, activities)
1. 
2. 
3. 
4. 
(Score as little, some, or much knowledge)

KH:  Little Some Much
Item 27  **Use of Hypothesizing**
(Write activities)
1. 
2. 
3. 
4. 
(Score as little, some, or much use)
UH:  Little  Some  Much

Items 28,29  **Control of Hypothesizing**
(Summarize responses)

(Score as little, some, or much control)
CH:  Little  Some  Much

Items 30,31  **Knowledge of Estimation**
(Write student's definition and understanding of usefulness of strategy)

(Score as little, some, or much knowledge)
KE:  Little  Some  Much

Item 32  **Use of Estimation**
(Summarize response)

(Score as little, some, or much use)
UE:  Little  Some  Much

Item 33:  **Control of Estimation**
(Write response)

(Score as little, some, or much control)
CE:  Little  Some  Much
Item 34: **Knowledge of Computation**  
(Summarize response)  
(Score as little, some, or much knowledge)  
KC: Little Some Much

Item 35: **Use of Computation**  
(Write activities)  
1.  
2.  
3.  
(Score as little, some, or much control)  
UC: Little Some Much

Item 36: **Control of Computation**  
(Write response)  
(Score as little, some, or much control)  
CC: Little Some Much

Item 37: **Knowledge of Checking**  
(Write definition)  
(Score as little, some, or much knowledge)  
KCH: Little Some Much

Item 39: **Use of Checking**  
(Write activities; look for consideration of both computation and process checks.)  
1.  
2.  
3.  
4.  
(Score as little, some, or much use)  
UCH: Little Some Much
Item 38  **Control of Checking**
(Write activities; again look for evidence of computation and process checks.)
1.
2.
3.
4.
(Score as little, some, much control)
CCH: Little Some Much

Item 40  **Knowledge of MPS**
(Write Strategies)
1.
2.
3.
4.
5.
6.
(Score as little, some, or much knowledge)
KMPS: Little Some Much

DIRECTIONS FOR INTERPRETATION:

**WORD PROBLEMS:** On the MPSA-SF Profile Form, write the strengths and weaknesses observed for the student’s performance on each of the three problems.

**INTERVIEW:** Transfer the scores for each item on the interview to the lower portion of the profile form. Connect points on the profile. Briefly summarize the student’s interview profile by writing a synopsis of his/her strengths and weaknesses.
# APPENDIX J

## MATHEMATICAL PROBLEM SOLVING ASSESSMENT — SHORT FORM (MPSA-SF)

### STUDENT PROFILE

| Student: _________________________   Date:________  Grade:______   Age: ______      Placement: _____    Gender: _____ |
| Problem 1:      Correct      Incorrect Problem 2:      Correct      Incorrect Problem 3:      Correct      Incorrect |
| Strengths:_______________________                                 |
| Weaknesses:_______________________                                 |
| Strengths: ________________________                                 |
| Weaknesses:_______________________                                 |
| Strengths:________________________                                 |
| Weaknesses:_______________________                                 |

| Very Good  -- | Good  -- | Average  -- | Poor  -- | Very Poor  -- |
| PMP ATTM ATTMPS |

**Interview Summary:**

**Strengths:**

**Weaknesses:**

**Recommendations:**

**Keys:**

- **PMP**: Perception of Math Performance
- **ATTM**: Attitude Toward Math
- **ATTMPS**: Attitude Toward Mathematical Problem Solving
- **KMPS**: Knowledge of Mathematical Problem Solving
- **K**: Knowledge of Strategies
- **U**: Use of Strategies
- **C**: Control of Strategies
APPENDIX K

MONTAGUE S COGNITIVE-METACOGNITIVE STRATEGY

FOR MATHEMATICAL PROBLEM SOLVING

READ (for understanding)
Say: Read the problem. If I don't understand, read it again.
Ask: Have I read and understood the problem?
Check: For understanding as I solve the problem.

PARAPHRASE (your own words)
Say: Underline the important information. Put the problem in my own words.
Ask: Have I underlined the important information? What is the question? What am I looking for?
Check: That the information goes with the question.

VISUALIZE (a picture of a diagram)
Say: Make a drawing or a diagram.
Ask: Does the picture fit the problem?
Check: The picture against the problem information.

HYPOTHESIZE (a plan to solve the problem)
Say: Decide how many steps and operations are needed. Write the operation symbols (+ - x /)
Ask: If I do____, what will I get? If I ____ then what do I need to do next? How many steps are needed?
Check: That the plan makes sense.

ESTIMATE (predict the answer)
Say: Round the numbers, do the problem in my head, and write the estimate.
Ask: Did I round up and down? Did I write the estimate?
Check: That I used the important information.

COMPUTE (do the arithmetic)
Say: Do the operations in the right order.
Ask: How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?
Check: That all the operations were done in the right order.

CHECK (make sure everything is right)
Say: Check the computation.
Ask: Have I checked every step? Have I checked the computation? Is my answer right?
Check: That everything is right. If not, go back. Then ask for help if I need it.
APPENDIX L

SCRIPTED LESSONS

The following scripted lessons were adapted from Montague's copyrighted Solve It! program, by Exceptional Innovations, Inc. Not for distribution.

LESSONS 1 THROUGH 4: COGNITIVE-METACOGNITIVE INSTRUCTION — STRATEGY ACQUISITION TRAINING

LESSON 1
Teacher: I am going to teach you to use a strategy for solving math word problems. Many of you told me that you are not very good at solving word problems and that you did not like to solve them. You also told me that you think it is important to be a good math problem solver and that you would like to improve your problem solving skills. Why do you want to improve your math problem solving?

Student: better grades, important skill, etc.

Teacher: You are right. (Good grades, managing money, you need it on the job, etc.) are good reasons for becoming a good math problem solver. You all have some good math skills. I will teach you how to use those math skills when you solve math word problems.

All right. Let's begin.

People who are good math problem solvers do several things in their head when they solve problems. They use several processes. Raise your hand if you know what a process is. (Call on students.) A process is a think skill. What is a process?

(DI TECHNIQUE) (All students respond) model (then ask same question and call on students individually to respond)
Good problem solvers tell us they use the following seven processes when they solve math word problems. I have these processes in a folder for you and also on a big chart to use in class while you are learning the strategy.

SHOW PROCESS CHART

(Show chart with only names of processes to students. Point to each process and read, explain, and question.)
First, good problem solvers read the problems for understanding.

Why do you read math word problems? (DI) model I read for understanding. (DI)

Then good problem solvers paraphrase the problem into your own words and remember the information.

What does paraphrase mean? (DI)

The third process is visualizing. When people visualize word problems, they use objects to show the problem, or they draw a picture or a diagram of the problem on paper, or they make a picture in their head.

How do people visualize? (DI)

Next good problem solvers hypothesize. Raise your hand if you know what hypothesize means. (Call on students)

Hypothesize means to set up a plan to solve the problem.

What does hypothesize mean? (DI)

Then people estimate the answer. Raise your hand if you know what estimation is. (Call on students)

Estimation means making a good prediction or having a good idea about what the answer might be using the information in the problem. Raise your hand if you know what a prediction is. (Call on students) People estimate or predict answers before they do the arithmetic. After they do the arithmetic and get the actual answer to the word problem, they compare their actual answer with the estimated answer. This helps them decide if the answer they got is right or if it is too big or too small.

What is estimating? (DI)
So, after good problem solvers estimate their answers, they do the arithmetic. We call this computing.

What is computing? (DI)

Finally, good problem solvers check to make sure that they have done everything right. That is, they check that they have used the right operations, completed all the necessary steps, and that their arithmetic is correct. People sometimes use addition to check subtraction problems and use multiplication to check division problems.

Why do you check math word problems (DI)

REVIEW PROCESS CHART

All right, here are the seven processes and the explanations for each one. (Review the chart with the processes.)

SAY ASK CHECK STRATEGIES

People who are good math problem solvers also do several things in their head when they solve problems. First, they SAY different things to tell themselves what to do. Second, they ASK themselves questions. Third, they CHECK to see that they have done what they needed to do to solve the math problems. I have put each SAY, ASK, CHECK activity with the right process on these charts.

SHOW FOLDERS

I have these activities written in your folders for you to study. I also wrote the activities on this big chart for you while you are learning the activities to solve word problems. We will call these processes and the activities that go with them strategies. Now I am going to read the entire strategy through once. Then we will read it as a group. Then I will call on each one of you to read the strategy.

(Show booklets and charts to students. Point to each activity and verbalization as you read and explain it.)

All right, now I would like you to read through the charts. I will help you with words if you need help. (Group reading — twice)
Now I would like you to read the process and the words say, ask, check, and I will read the activities. (Group)

Now I will read the process and the words say, ask, check, and you will read the activities. (Group)

Now I want you to read everything. (individual — one time each)

LESSON 2

Check strategy mastery.

You are beginning to learn the strategy. This strategy will help you improve your problem solving. It is important that you continue to improve and do even better. It is also important that you continue to use the strategy every time you need to solve problems in school and outside of school. When might you need to solve a math problem outside of school? (elicit responses — store, measuring, etc.)

Now I want you to watch me solve a problem using the entire strategy. Instructor verbalizes through-out, I need to try and use the strategy, while solving the problem.

PROBLEM

Jose and Nancy are selling greeting cards to raise money for the school camping trip. Together they sold cards totaling $88.50. Nancy sold $67.00 worth of cards. How much money did Jose make selling cards?

Use the following routine. During modeling, use the expressions now I will say to myself Use this routine when first modeling the strategy. Later, students will verbalize the words SAY etc. or the teacher will verbalize for the students. The goal is to eventually fade these cue words and have students automatically self-instruct, question, and check. Self-cueing will consist of remembering the processes, although students will appear to fade on certain processes as well.

READ: the problem for understanding. Now I will say to myself — Read the problem. (read the problem) If I don t understand it, read it again. (read the problem again) Now I will ask myself — Have I read and understood the problem? (yes) Now I will check myself by — checking for understanding as I solve the problem.
PARAPHRASE: Now I will say to myself — Put the problem into my own words. (Two kids sold cards for $88.50. One sold cards for $67.00. How much did the other kid make?) Underline the important information. (underline together $88.50 and Nancy sold $67.00 worth) Now I will ask myself — Have I underlined the important information? (yes) What is the question? (How much money did Jose make selling cards?) What am I looking for? (the amount of money that Jose made) Now I will check myself by — Checking that the information goes with the question. (I have the total amount and the amount that Nancy sold. I need to find the amount that Jose sold.)

VISUALIZE: Now I will say to myself — Make a drawing or a diagram.

Now I will ask myself — Does the picture fit the problem? (yes, I .) Now I will check myself by — Checking the picture against the problem information.

HYPOTHESIZE: Set up a plan to solve the problem. Now I will say to myself — Decide how many steps and operations are needed. (one step, subtraction) Write the operation symbols (write - ) Now I will ask myself — If I (subtract the amount that Nancy sold from the total amount sold), I will get (the amount that Jose sold.) Now I will ask myself — How many steps are needed? (one) Now I will check by myself by — Checking that the plan makes sense. If not, ask for help.

ESTIMATE: Predict the answer. Now I will say to myself — Round the numbers, do the problem in my head, and write the estimate. (Round up, 70, round up, 90 — 70 = 20.) Now I will ask myself — Did I round up and down? (rounded up only) Did I write the estimate? (yes) Now I will check myself by — Checking that I used the important information.

COMPUTE: Do the arithmetic. Now I will say to myself — Do the operations in the right order. (do the arithmetic) Now I will ask myself — How does my answer compare with my estimate? (very close) Does my answer make sense? (yes) Are the decimals or money signs in the right order? (check, yes) Now I will check myself by — Checking that all the operations were done in the right order.

CHECK: Make sure everything is right. Now I will say to myself — Check the computation. (check it) Now I will ask myself — Have I checked every step? (yes) Have I used the right numbers? Have I checked the computation? (yes) Is my answer right (yes)
Now I will check myself by — Checking that everything is right. If not, I will go back. Then I will ask for help if I need it.

Questions? Reinforce.

Now we will practice the strategy. I want you to memorize the seven processes for next lesson just as they appear on this card. Now tell me the strategy. (group and individual rehearsal until class ends)

LESSON THREE

Check strategy mastery

Model two-step problem.

PROBLEM

One Monday Dad bought 14 gallons of paint for the house. On Tuesday he bought 12 more gallons. After painting the house, returned 3 gallons to the store. How much paint was used?

READ: the problem for understanding. Now I will say to myself — Read the problem. (read the problem) If I don’t understand it, read it again. (read the problem again) Now I will ask myself — Have I read and understood the problem? (no, I will read it one more time.) Now I will check myself by — checking for understanding as I solve the problem.

PARAPHRASE: Now I will say to myself — Put the problem into my own words. ( Dad bought 14 gallons and 12 gallons of paint and then took back 3 gallons. How much paint did he use?) Underline the important information. (underline bought 14 gallons and bought 12 more, and returned 3 ) Now I will ask myself — Have I underlined the important information? (yes) What is the question? (How much paint was used?) What am I looking for? (the number of gallons of paint used) Now I will check myself by — Checking that the information goes with the question. (I have the number of gallons he bought and the number he returned. I need to find the total number of gallons used.)

VISUALIZE: Now I will say to myself — Make a drawing or a diagram.

Now I will ask myself — Does the picture fit the problem? (yes, I .) Now I will check myself by — Checking the picture against the problem information.
HYPOTHESIZE: Set up a plan to solve the problem. Now I will say to myself — Decide how many steps and operations are needed. (two steps, add and subtract) Write the operation symbols (write + and -) (two steps)
Now I will ask myself — If I (add the two amounts he bought and then subtract the gallons returned). I will get the (number of gallons used). Then what do I do next? (nothing, I am finished) Now I will ask myself — How many steps are needed? (two) Now I will check myself by — Checking that the plan makes sense. If not, ask for help.

ESTIMATE: Predict the answer. Now I will say to myself — Round the numbers, do the problem in my head, and write the estimate. (Round up, 15, down, 10 = 25 — 5 = 20 about 20 gallons used.) Now I will ask myself — Did I round up and down? (yes) Did I write the estimate? (yes) Now I will check myself by — Checking that I used the important information.

COMPUTE: Do the arithmetic. Now I will say to myself — Do the operations in the right order. (do the arithmetic) Now I will ask myself — How does my answer compare with my estimate? (very close) Does my answer make sense? (yes) Are the decimals or money signs in the right order? (none needed) Now I will check myself by — Checking that all the operations were done in the right order.

CHECK: Make sure everything is right. Now I will say to myself — Check the computation. (check it) Now I will ask myself — Have I checked every step? (yes) Have I used the right numbers? Have I checked the computation? (yes) Is my answer right? (yes)
Now I will check myself by — Checking that everything is right. If not, I will go back. Then I will ask for help if I need it.

SOLVE TOGETHER PRACTICE PROBLEM NUMBER 1

Now we will use the strategy and solve this problem together. I will help you if you need help. We will do it out loud. I will call on different people for each strategy.

(Practice problem #1). Group activity using direct instruction.

Now we will practice the strategy. I want you to memorize the seven processes for tomorrow just as they appear on this card. Now tell me the strategy. (group and individual rehearsal until class ends)
LESSON 4

Check strategy mastery.

Model problem.

In the Orange Bowl parade there were 50 marching bands. 28 bands had 660 members; and the remainder of the bands had 45 members each. How many band members were in all the marching bands?

READ: the problem for understanding. Now I will say to myself — Read the problem. (read the problem) If I don’t understand it, read it again. (read the problem again) Now I will ask myself — Have I read and understood the problem? (no, I will read it one more time.) Now I will check myself by — checking for understanding as I solve the problem.

PARAPHRASE: Now I will say to myself — Put the problem into my own words. (50 band altogether. 28 band had 660 members altogether. The rest had 45 members each. How many kids altogether?) Underline the important information. (underline 50 marching band, 28 band had 660, remainder of the bands had 45 members) Now I will ask myself — Have I underlined the important information? (yes) What is the question? (How many band members were in all the marching bands?) What am I looking for? (the total number of people in all the bands) Now I will check myself by — Checking that the information goes with the question. (28 with 660, the rest with 45 each — 50 bands in all, yes.)

VISUALIZE: Now I will say to myself — Make a drawing or a diagram.

Now I will ask myself — Does the picture fit the problem? (yes, I .) Now I will check myself by — Checking the picture against the problem information.

HYPOTHESIZE: Set up a plan to solve the problem. Now I will say to myself — Decide how many steps and operations are needed. (first subtract 28 from 50, multiply the answer by 45, add that to 660 .subtract, multiply, add) Write the operation symbols (write -, x, +) (three steps)

THREE STEPS -, x, +

Now I will ask myself — If I (subtract 28 bands from 50, I will get the remainder of bands, then what do I do next? Then I multiply that answer by 45 to get the number of people in the remainder of bands, than what do I do next? Then I ad that number to 660 and get the total number of band members, then what do I do next? nothing, I am finished)
Now I will ask myself — How many steps are needed? (three) Now I will check myself by — Checking that the plan makes sense. If not, ask for help.

ESTIMATE: Predict the answer. Now I will say to myself — Round the numbers, do the problem in my head, and write the estimate. (Round down 25 from 50 = 25 times 40 do the arithmetic) 1000 add 660 = about 1660 members
Now I will ask myself — Did I round up and down? (yes) Did I write the estimate? (yes) Now I will check myself by — Checking that I used the important information.

COMPUTE: Do the arithmetic. Now I will say to myself — Do the operations in the right order. (do the arithmetic) Now I will ask myself — How does my answer compare with my estimate? (very close)
Does my answer make sense? (yes) Are the decimals or money signs in the right order? (none needed, just a comma to separate the thousands from the hundreds)
Now I will check myself by — Checking that all the operations were done in the right order.

CHECK: Make sure everything is right. Now I will say to myself — Check the computation. (check it) Now I will ask myself — Have I checked every step? (yes) Have I used the right numbers? Have I checked the computation? (yes) Is my answer right (yes)
Now I will check myself by — Checking that everything is right. If not, I will go back. Then I will ask for help if I need it.

PRACTICE PROBLEM 2

Now I will tell you what to do as you solve this problem. Have students solve problem (practice problem #2) as a group at their seats. The teacher will say the process and the words SAY, ASK, CHECK aloud as they solve it.

PRACTICE PROBLEM 3

Now you tell me what to do. You will say the process and the words SAY, ASK, CHECK as I solve this problem (Practice problem #3). Have students take turns. Wait for me to say and do what I do.

Cover chart. Check verbalization of strategy to criterion. Check mastery chart.
LESSONS 5 THROUGH 9: STRATEGY ACQUISITION TRAINING, continued

Check strategy recitation mastery. Investigator marks the names of students who have reached 100% criterion for recitation of the processes

Group rehearsal. Individual rehearsal.

Students solve practice problem 1 individually. Instructor encourages them to reference the wall chart or their booklets and to think aloud during solving of a word problem

Instructor or a student models correct solution, using think aloud. Students and/or instructor assist the problem solver by verbalizing the processes and strategies as he/she works through the problem.

Student are paired for solving practice problems 2 and (if time) 3. Students take turns telling partner what to do.

Designated student dyad models the correct solution at the board. Students and/or instructor assist the problem solvers by verbalizing the processes and strategies as they work through the problem.

Check mastery for students who did not reach at beginning of class.

LESSONS 10 THROUGH 16: STRATEGY ACQUISITION PRACTICE

Instructor: Let s keep in mind our motto, strategy use = success. Using the strategy will help us to understand and successfully solve the word problems.

Mastery Check

Instructor provides short review on the importance of attributing success and failure to controllable factors and its importance to success on academic tasks:

Hedwig (stuffed animal) is here to remind us of the importance of using the strategy and trying hard in solving problems. Our success on problems is due to our own effort. If we make mistakes, are they beyond our control? No, failures are not b/c of the difficulty of
the word problem or to bad luck. Failures and successes are based on things within our control — like using the strategy and trying hard.

Students individually solve 3 word problems. Students are cued to use the strategy, consult the wall chart or their booklets, and think aloud.

Following completion of each problem, either the investigator or a student models the correct solution. During these modeling examples, any errors unintentionally made by the student, or intentionally made by the investigator, are used to highlight the role of a controllable force, that is, not using the mathematical word problem solving strategy. The importance of not attributing failure to uncontrollable factors is again stressed. The investigator combines the positive self-attribution, I need to try and use the strategy, with the actual use of the strategy, to guide the student toward successfully performance on the previously failed item, modeling the strategy aloud.

Students monitoring their behavior by plotting their progress on individual performance graphs, held in their folders

Check strategy mastery.