MODELING AND SENSORLESS CONTROL
OF
SOLENOIDAL ACTUATORS

DISSERTATION

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the Degree of Doctor of Philosophy in the Graduate
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Electromagnetic actuators (EMA), which incorporate solenoids, are increasingly becoming the actuator of choice in industry lately, due to their ruggedness, low cost, and relative ease of control. Latest applications of solenoid based EMA’s include Electromagnetic Valve Actuation (EMV) systems. This application presents challenges that require the improvement of the dynamic characteristics of the EMA. Some of these problems include, but are not limited to, quiet operation, reduced bounce, less energy consumption, trajectory shaping with a minimum number of measurements, and high actuation speeds. These demands, coupled with the nonlinear dynamics of the EMA, make the use of classical control strategies a less attractive option. A possible attempt to arrive at intermediate solutions to these problems should include some amount of model based robust control strategy. This includes the development of an accurate but simple control based model and a robust digital control strategy.

In this study a basic nonlinear model for a solenoidal EMA will be developed, and validated, which will include bounce, leakage inductance and temperature effects. The model is formulated for the linear legion (region before saturation) of the actuator dynamics, but validation will include operation in the saturation region as well. This
effectively means that a nonlinear model will be developed that is simple but accurate enough for control, neglecting hysteresis and magnetic saturation.

Next, an EMV will be designed and built. A nonlinear model for the EMV will be developed and validated. This model will include secondary nonlinearities like saturation, hysteresis, mutual inductance and bounce. In this study a variable that is easier and cheaper to measure, current, will be measured and the information of the position and velocity variables will be estimated from this measurement. The position estimate will be used for control. This is called *Sensorless Control*. The control objective is to reduce impact noise and seating velocity. The *sliding mode* methodology will be used here since it is nonlinear, robust to uncertainties, and easier to design and implement. The estimation and control algorithms will be validated in simulation and experimentally for the EMA and EMV, respectively.
Dedicated to:

“Pater noster, qui est in caeli”
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Electromagnetic actuators (EMA), which incorporate solenoids, ball screw actuators, and linear motors, are increasingly becoming the actuator of choice in industry lately, due to their ruggedness, low cost, reduced complexity, relatively high force density (force per unit mass) and relative ease of control. Solenoids have a relatively large force to weight ratio and high speed characteristics thus they have been useful in fuel injection actuation, Exhaust Gas Recirculation (EGR) systems, food dispensers, refrigerators, washing machines, etc.

Latest applications of solenoid based EMA’s include Electromagnetic Valve Actuation (EMV) systems. This application presents challenges that require the improvement of the dynamic characteristics of the EMA. Some of these problems include, but are not limited to, quiet operation, reduced bounce, less energy consumption, trajectory shaping with a minimum number of measurements, and high actuation speeds. The EMV is supposed to facilitate the concept of Variable Valve Actuation, which is intended to replace the conventional camshaft-driven valve train. The use of the EMV, coupled with electronic control would be capable of individual valve timing and individual valve lift, providing
an added degree of freedom and control over the entire breathing dynamics. The use of the EMV in production engines makes possible an additional optimization of cold start, warm-up and transient operation [54]. Also, coupled with a torque based control strategy, a wide range of advanced combustion techniques could be implemented, including throttleless operation, cylinder de-activation, and internal EGR. Such advanced methodologies are supposed to improve on engine efficiency, fuel consumption and tail pipe emissions.

It is therefore necessary to have a better control of the motion and force of this latest generation EMA, unlike had been the case in the past. Additionally it is imperative that the overall system be cost effective. This means that the system may need to have an optimal performance with fewer available sensors. These demands, coupled with the nonlinear dynamics of the EMA, make the use of classical control strategies a less attractive option.

A possible attempt to arrive at intermediate solutions to these problems should include some amount of model based robust control strategy. This includes the development of an accurate but simple control based model and a robust digital control strategy.

In this study a basic nonlinear model for a solenoidal EMA will be developed, and validated, which will include bounce, leakage inductance and temperature effects. The model is formulated for the linear legion (region before saturation) of the actuator dynamics, but validation will include operation in the saturation region as well. This effectively means that a nonlinear model will be developed that is simple but accurate enough for control, neglecting hysteresis and magnetic saturation.
Next, an EVA will be designed and built. This consists of two opposing electromagnets with a spring on the opposite sides, acting in parallel and a plunger which is attracted to the face of the electromagnet when energized. The device will be designed to produce linear motion when it is energized. The EMA is one the promising solutions to the challenge of reduction in fuel consumption and emissions. The idea of camless engines with “individual cylinder control” has reinforced interest in the concept of variable valve timing (VVT), or fully flexible valve actuation systems (FFVA). This effectively means controlling both individual valve timing and individual valve lift, providing an added degree of freedom and control to the intake and exhaust dynamics, hence engine torque. Different valve control strategies have been developed, including, but not limited to:

- Electro-magnetic actuation (EVA)
- Electro-hydraulic actuation (EHA)
- Electro-mechanical actuation (EVA)

This study is limited to the electromagnetic valve actuation system. The EMV is supposed to facilitate the concept of Variable Valve Actuation, which is intended to replace the conventional camshaft-driven valve train. The use of the EMV, coupled with electronic control would be capable of individual valve timing and individual valve lift, providing an added degree of freedom and control over the entire breathing dynamics. The use of the EMV in production engines makes possible an additional optimization of cold start, warm-up and transient operation [54]. Also, coupled with a torque based control strategy, a wide range of advanced combustion techniques could be implemented, including throttless operation, cylinder de-activation, and internal EGR. Such advanced
methodologies are supposed to improve on engine efficiency, fuel economy and tail pipe emissions.

Several issues have to be addressed with the design of the EMA, since its performance is greatly influenced by these factors. At the design stage, material properties, cost and performance of different metals have been investigated, so that a material with the best combination of these factors can be selected. Next, there are operational issues of the EMA that were taken into account, such as, noise, energy consumption, reliability, stability and fixed lift. The noise is caused by high velocity impact of the plunger against the face of the electromagnets. This is also responsible for structural failure of certain components. The noise can be reduced by slowing down the plunger just before it impacts each of the electromagnet. This problem can be reduced partly by mechanical design and entirely by electronic control. A nonlinear EMV model will be developed and validated. Since the forces and current consumption for the EMV are greater than that of the EMA, it means that nonlinearities and uncertainties will be more pronounced, hence it will be necessary to include all possible secondary nonlinearities to the model of the EMV. The secondary nonlinearities that will be included in this model of the EMV are saturation, hysteresis, mutual inductance and bounce. These nonlinearities are important in modeling the electromagnetic force to a reasonable degree of accuracy since the force exhibits these characteristics.

Usually, for feedback control of the EMA and EMV, the position of the plunger is measured and used for control. Measuring position requires a “noise free” and high resolution sensor, which are often expensive and will often malfunction in the presence of vibration. In this study, the position variable will not be measured, but will be estimated.
Another variable that is easier and cheaper to measure, current, will be measured and the information of the position and velocity variables will be estimated. The position estimate will be used as feedback signal for closed loop control. The *Sliding Mode* methodology will be used to design the state estimators and closed loop controller. The sliding mode methodology presents certain attractive characteristics like, robustness to uncertainties, ease of design and implementation. The dynamic models developed will be validated experimentally. Next, the estimation and control algorithms will be validated in simulation and experimentally for the EMA and EMV, respectively.
CHAPTER 2

LITERATURE REVIEW

Several models have been proposed to capture the dynamic characteristics of a solenoid. Odendaal et al. [26] derived a high frequency thermal model for transformers, based on the temperature measurements of a reference thermal configuration, from which the temperature rise of any other core is extrapolated. This method is relevant in optimizing transformer design for ferrite cores, but is not control oriented. Domanski [23] formulated a control-oriented thermal model by approximating the temperature rise of coil to the heat rise within an oven. The accuracy of the analysis is based on understanding the nature of the electrical power circuit and knowing the current profile, hence on line computation is not possible.

The Lequesne [14] model proposed the use of the Finite Element Method (FEM) to predict inductance, force levels and the effects of eddy currents, to arrive at a mathematical model that is truly representative of all physical dynamic characteristics of the solenoid. This approach was relevant as an analysis tool for coil design.
Vaughan [15] proposed a nonlinear magnetic model solenoid proportional valves, which involves identifying suitable coefficients experimentally to obtain a transient magnetic characteristic that bests represents saturation and magnetic hysteresis, which is useful for design and prototype manufacture, but thermal effects where neglected. His model assumed that the inductor current is made up of the restoring and the dissipation current. The restoring current is a function of flux and air-gap, and dissipation function represents the losses responsible for hysteresis. The dissipation function is a first order filter with one time constant. The parameters of the nonlinear function representing force and current were identified by least square regression, assuming force and flux have a square relationship. The solenoid was rated at 10V, 2.5Amps.

Masoud [22] proposed a model for solenoid controlled servo-valves based on the model in [15]. Validation was done by using different step voltages at different positions. A curve fitting technique was used on the steady state data of force/flux versus position for several armature positions. This method was used in identifying the force and restoring current polynomials. The dissipation current was modeled as a sigmoidal function and its parameters were adjusted to fit the s-shaped experimental curve of dissipation current against flux. The model took into account hysteresis, and saturation, but thermal effects and leakage inductance was not modeled.

Cheung [16] proposed an elegant method of arriving at a model that approximates the linear and saturation regions of a solenoid by a first order and a second order function, respectively. This involves identifying the constants in the empirical equation for reluctance when the current and flux linkage have a linear relationship. This is done using simple relationships derived from fundamental linear magnetic principles. A saturation
boundary is defined, above which the flux current relationship is nonlinear. This region is approximated as a parabola and parameters of the parabola are identified using curve fitting techniques. The model includes identifying leakage inductance as well. Experimental data was generated by feeding a sinusoidal voltage into the coil. The model does not include hysteresis. Saturation was not well modeled and the validation of the force was very sensitive to parameter variation. Thermal effects were neglected. His method was efficient in the design of proportional solenoid valves.

Lequesne [20] presented a different configuration of the EMV, where the magnetic force is produced by two permanent magnets and a pair of coils. Motion is produced by the oscillation of two springs supporting a steel plunger and the plunger is latched at the end of its travel by the permanent magnet, with no current flowing. The plunger is unlatched by energizing the coil with current so as to produce a magnetic field that opposes that produced by the permanent magnet. The expression for the magnetic force is represented as a surface integral of the magnetic field strength calculated by finite element method. Modeling of eddy currents was taken into account since this represented core losses. Their effect was modeled by finite element. The finite element model was validated with experimental data, but the match was not close.

Wang and Stefanopoulou [22] arrived at a model that takes into account the linear and saturation dynamics of the coil. The linear part of the magnetic characteristics was modeled using linear magnetic theory, and the parameters of system were identified using a curve fit method. The saturation region was modeled as a polynomial function dependent on current and position. The equation for magnetic force was derived from this polynomial and parameters were identified by linear regression. The inductance and rate
of change of flux were lumped as a nonlinear function dependent on position and current and this was identified by dynamic data and numerical calculation. Bounce, mutual inductance and thermal effects where not taken into consideration. The empirical model was validated with experimental data obtained from an engine equipped with an EMV. Numerical approximation of the velocity was not accurate enough due to sensor resolution. Also, the magnetic force was not validated.

Hoffman and Stefanopulou [6] used the validated model in [22] to design a control that will ensure accurate valve closing and opening with a small contact velocity. A tracking controller for valve position was designed and it was composed of a feedforward controller and feedback controller. The feedforward controller calculates the desired trajectory. The feedforward controller is updated after each cycle using the tracking error information using an iterative learning control methodology. The feedback controller was designed using state feedback method and the gains were obtained from the linear quadratic regulator. The plant was linearized and linear observer was designed to estimate position. The position controller was validated in simulation only and achieved good tracking but the control voltage peaked at 200V.

In 2002, Wang [3] attempted to solve the soft seating problem of an EMV actuator with a nonlinear controller and observer of unknown structure. Plunger position was measured and used for feedback and the velocity was estimated. The control algorithm was implemented experimentally and it was based on a linear gain scheduling controller that scheduled feedback gains based on the penalty set on the tracking error. Control voltages were as high as 100 V.
Chun Tai [2] proposed a model that was based on the linear magnetic model. The model was linearized and the parameters of the system transfer function were identified under closed loop control, since the open loop plant was unstable. The model included a valve lash that ensured complete closing even in the presence of thermal expansion of valve body. The position was measured and used for feedback and a tracking controller was designed. The controller consists of a feedforward controller that calculated the trajectory on-line, a notch filter to cancel the unstable modes of the plant, and an $H_{\infty}$ robust controller to stabilize the feedback system and achieve soft seating based on position feedback. Seating velocity was reduced to an acceptable value.

Butzman et al [4] attempted a non-observer based sensorless control algorithm based on current measurement. The model composed of a compact description of the mechanical and electrical dynamics in the current state equation. The chosen control method did not rely on the correct description of the magnetic model for small air-gaps, so magnetic saturation was not taken into account. Leakage flux was estimated as the difference between the core flux and the air-gap flux. A sensorless control strategy based on observing the ratio of the current through the coil and its derivative. This ratio is equivalent to that of velocity and air-gap. The quotient is compared to a predetermined value and the error determines the control voltage to be selected. A positive error will call for a decrease in coil current by applying a negative supply voltage, and vice versa. This has the effect of slowing down the coil. This method was sensitive to disturbances and it was improved by and adaptive algorithm where the electrical energy of the next cycle is determined by the landing performance of the previous cycle. The algorithm was tested on a table-top experiment and seating velocity was adequately reduced.
Giglio et. al. [7] presented some problems on the design of the EMV which are relevant to its functionality. The desired plunger rise time was met by selecting a spring with an appropriate spring constant. The magnetic force generated was designed to be large enough to pull the plunger up the first time, while minimizing voltage and energy consumption. The hold current was increased with increase in rise time. The number of turns of wire and the wire diameter were selected after a long period of trials and were judged to be the most useful selection process. All calculations to model the forces, eddy currents, and electrical forces were done numerically by a finite element solver called Flux 2D. This takes into account the material properties and then optimizes the equation parameters. The model presented utilized look up tables for force and inductance, derived by numerical steady state experiments at different currents and air-gap, carried out with a software called magnetic CAD. A control strategy based on current feedback was utilized to reduce seating velocity. An open loop controller was designed to apply voltages to the coil terminals based on a position sensor information. As the plunger crosses the equilibrium position, feedback control is switched on to reduce coil current at seating. Seating velocity was significantly reduced in simulation.

The control method that will be used in this study is Sliding Mode Control. This method of control originated when there was an interest in investigating systems with control as discontinuous functions of state, particularly, “on-off” relay control used for feedback systems. Flugge-Lotz [32] and Tyskin [34] used relay systems for a variety of designs. The control action switches at high frequency and has been proven to be efficient in controlling high-order nonlinear dynamic plants operating under uncertainty. Earlier applications of sliding mode could date back as far as the 1930’s.
Kulebakin V. [35] applied sliding mode control in the context of voltage control of a DC generator of an aircraft. The winding voltage was require to track a desired voltage by applying relay control as a function of voltage, and forcing the excitation windings to switch at high frequencies, minimizing tracking error. Also, in the 1930’s, Nikolski [33] used relay control to steer a ship. A. A. Andronov [12] indicated the possibility of non-ideal sliding mode in the presence of small time constants, time delays and hysteresis. The regularization method by Andre and Seibert [12] was used to prove that motion equations with one sliding surface will have the same motion equations for both hysteresis and time delay.

Filippov [30] developed a method which gives the solution of motion at the discontinuity surface as being complemented by a minimal convex set. This result matched that obtained by the regularization method.

Design of sliding mode for systems with vector control inputs and multidimensional sliding mode was proposed by Utkin [12], and the mathematical technique to describe this motion was developed. The boundary layer approach was applied to sliding modes at the intersection of several surfaces. This means that trajectories are not confined to the intersection of the surfaces, but to a neighborhood of the surfaces a proposed by Utkin [12]. Later, the discontinuous control was found to cause oscillation of state vector, called chattering. This control could be replaced by a piecewise smooth continuous approximation, called equivalent control, as proposed by Utkin in 1992 [12].

The problem of chattering was also solved by replacing the discontinuous sign function with a continuous approximation, the saturation function as proposed by Slotine and
Sastry [11]. This approximates the sign function in a boundary layer of the sliding surface.

Gutman and Leitmann [36] proposed other designs of the discontinuous function based on the Lyapunov function selected for the nominal system. This is the Unit Control. This was designed by selecting the control such that the Lyapunov function is decreasing along trajectories of the system.

Utkin and Drakunov [12] solved the problem of implementing sliding mode in discrete time, since the switching frequency of the discontinuous control is limited by the sampling rate making the system behave like an open loop system, hence, causing a phenomenon known as discretization chatter.
CHAPTER 3

THEORY OF SLIDING MODE

3.1 INTRODUCTION

An attractive method in designing the model based position estimator is by the Sliding Mode (SM) estimation technique, due to its robustness to uncertainties, speed of convergence and nonlinear structure. This involves obtaining a model of the system with its available measurements. The estimator is a copy of the system model, with its input being the system input and the mismatch between the measured output and the estimated output, namely the error. The mismatch is passed through a discontinuous function of the error and then multiplied by a feedback gain. This is then added to the observer equations, the intension being that the estimation error of all the unmeasured states could be reduced to some neighborhood of zero in finite time. Hence, the unmeasured states could be reconstructed from the available measurements and the system input.
3.2 SLIDING MODE ESTIMATION THEORY

The sliding mode theory is based on *Variable Structure System (VSS)* theory. In VSS theory, the feedback control gains could take several values depending on system state dynamics. These alternate state dynamics consist of different structures, which with a switching control term, could be forced to inherit certain desired characteristics that may not have been present in each of the individual structures. Usually, the state space is separated into individual planes, each with possibly unstable equilibrium points. With the addition of a discontinuous feedback control term, the system trajectories in each structure, is forced to be oriented towards an imaginary asymptotically stable manifold or *switching manifold*. When the state trajectories reach this manifold, they are constrained to remain on this manifold for all time, possessing the desired properties of the switching manifold. This is called *Sliding Mode*. Sliding modes may appear in a dynamic system governed by ordinary differential equations with discontinuous right hand sides. The control, designed as a function of system state switches at high (theoretically infinite) frequency.

In the case of the EMV and EMA, the sliding or *equilibrium manifold* is designed such that when the system error dynamics are confined to this manifold, the estimation and tracking errors would decay to some neighborhood of zero in finite time.
3.3 SLIDING MODE OBSERVERS

Consider a system of the form

\[ \dot{x}_1 = \alpha x_2 \]  
\[ \dot{x}_2 = B(t)u + d(t) \] \hspace{1cm} (3.0)

Where, \( \alpha \) is a parametric uncertainty and \( d(t) \) is a nonlinear disturbance that is not exactly known but both are positive and bounded by their upper bounds \( \alpha^+, d^+ \), respectively. \( B(t) \) is a continuous function of time, with a known sign and is also positive and bounded by an upper bound. Assume, that \( x_1 \) is measured and the value of \( x_2 \) is to be estimated from \( x_1 \). Let the system in (3.0) now be written as:

\[ \dot{x}_1 = \alpha^+ x_1 \]
\[ \dot{x}_2 = B^+ u + d^+ \] \hspace{1cm} (3.1)

An observer structure for the system in (3.1) could be designed as [10]:

\[ \dot{\hat{x}}_1 = \alpha^+ \hat{x}_2 - L_1 s - M_1 \text{sign}(s) \]
\[ \dot{\hat{x}}_2 = B^+ u - L_2 s - M_2 \text{sign}(s) + d^+ \] \hspace{1cm} (3.2)

\[ s = \bar{x}_1 = \hat{x}_1 - x_1 \]

Where, \( \bar{x}_1, \bar{x}_2 \) are the estimation errors of \( x_1 \) and \( x_2 \), respectively. \( L_1, L_2, M_1, \) and \( M_2 \) are positive constants to be chosen and \( \text{sign} \) is the signum function, defined as:

\[ \text{sign}(s) = \begin{cases} +1 & s > 0 \\ -1 & s < 0 \end{cases} \]
The error in $x_1$ is defined as $s$, the equilibrium manifold. The error dynamics is obtained by subtracting (3.1) from (3.2) to obtain (3.3).

$$\dot{x}_1 = \alpha^+ \tilde{x}_2 - L_1 s - M_1 \text{sign}(s)$$

$$\dot{x}_2 = -L_2 s - M_2 \text{sign}(s)$$

(3.3)

For sliding mode to occur, the reaching condition and the existence condition have to be satisfied. The reaching condition guarantees that the trajectories will reach the equilibrium manifold from any initial condition in finite time. The existence condition guarantees that the trajectories are oriented towards the discontinuity surface and will be confined to this surface for all time. To prove this, it is necessary to study the stability of motion on the manifold.

Sliding Mode exists on the equilibrium manifold, $s = 0$, if $s$ and its time derivative are of opposite signs in the vicinity of the discontinuity surface (Necessity) [12].

The motion projection on the space $s$ is governed by

$$\dot{s} = \alpha^+ \tilde{x}_2 - L_1 s - M_1 \text{sign}(s)$$

(3.4)

with $M_1 \geq |\alpha^+ \tilde{x}_2|$ and $M_1 + M_1^T > 0$

$$s\dot{s} < 0$$

(3.5)

Hence, the origin $s = 0$ is an asymptotically stable equilibrium manifold with finite convergence time.

The equilibrium manifold will be reached in finite time if the time derivative of a positive definite Lyapunov function is negative definite (sufficiency).

Let a positive definite Lyapunov function candidate be:

$$V = \frac{1}{2} s^T s$$

(3.6)
\[
\dot{V} = \frac{1}{2} \frac{\partial V}{\partial \dot{s}} \dot{s} = s(\alpha \dot{x}_2 - L_1 s - M_1 \text{sign}(s))
\]

\[
\dot{V} \leq \left| (\alpha \dot{x}_2) \right| s - L_1 s^T s - M_1 |s|
\]

If \( M_1 \geq |\alpha \dot{x}_2 | \) then \( \dot{V} < 0 \)

Hence, the derivative of \( V \) is negative definite, the Lyapunov function decays at a finite rate and vanishes, implying the error is continually decreasing and approaches zero. This ensures the equilibrium manifold is reached and sliding mode is enforced in the system. This implies asymptotic stability of the origin in the subspace, \( s \). The speed of convergence could be increased by increasing \( M_1 \).

The dynamics on the equilibrium manifold can be derived using Filippov’s method [8], which states that the end of all state velocity vectors in the vicinity of a point on the discontinuity surface should be complemented by a minimal convex set and the state velocity vector of sliding motion should belong to this set. Applying this to (3.3) and without loss of generality, assume that ideal sliding mode takes place and \( s = 0 \), hence \( L_1 s \) and \( L_2 s \) are zero.

\[
\dot{x} = \mu f^+ + (1 - \mu) f^-
\]

\[
\dot{x}_1 = \mu (\alpha \dot{x}_2 - M_1) + (1 - \mu)(\alpha \dot{x}_2 + M_1)
\]

\[
\dot{x}_2 = \mu (-M_2) + (1 - \mu)(M_2)
\]

(3.7)

On the equilibrium manifold

\[
\dot{s} = \dot{x}_1 = 0
\]
\[ \mu = \frac{(\alpha^+ \tilde{x}_2 + M_1)}{2M_1} \]

\[ \ddot{x}_2 = -\frac{\alpha^+ M_2 \tilde{x}_2}{M_1} \tag{3.8} \]

Hence, the estimation error in \(x_2\) will decay exponentially with a time constant of \(\frac{M_1}{\alpha^+ M_2}\) when it reaches the manifold. Thus, \(M_1\) is chosen to be the desired estimation error of \(x_2\) in sliding mode, and \(M_2\) is chosen such that the ratio \(M_2 / M_1\) is the desired eigenvalue in sliding mode [7], so far as \(M_2\) is selected such that:

\[ M_2 \geq d^+ \tag{3.9} \]

The gains \(L_2\) and \(L_1\) are chosen as in the design of the Luenberger observer of the linearized system, (with \(M_2 = M_1 = 0\)) so as to place the poles arbitrarily [7]. Their effect on the motion equation in sliding mode can be examined by applying (3.3) on the phase plane of the system error, as shown in Figure (3.1). For motion in the vicinity of \(s\), the domain of sliding mode is defined as:

\[ \tilde{x}_2 \leq \frac{1}{\alpha^+}(L_1 + M_1) \quad s > 0 \tag{3.10} \]

\[ \tilde{x}_2 \geq \frac{1}{\alpha^+}(-L_1 - M_1) \quad s < 0 \tag{3.11} \]

Thus \(L_1\) increases the domain of sliding mode, but \(L_2\) only affects the reaching time or speed of convergence, but the dynamics on the equilibrium manifold remains unchanged.

\[ \ddot{x}_2 = -\frac{\alpha^+ M_2 \tilde{x}_2}{M_1} \tag{3.12} \]
3.4 SYSTEM OBSERVABILITY

Convergence of sliding observer is dependent on system observability. A system is observable if its initial state, $x(t_0)$, can uniquely be determined from knowledge of its input $v(t)$, and measurements $y(t)$, regardless of the initial time $t_0$ and initial state $x(t_0)$. In this case, it is intended to reconstruct the final states from a knowledge of only the output, given an arbitrary initial condition. This is Reconstructibility.

A system is reconstructible if and only if it is observable [17]. Hence, system observability implies reconstructibility, but not the reverse. Henceforth, analysis of system observability will be emphasized instead.

The system in (3.0) is observable if the observability grammian is nonsingular.

This is given without proof. The observability grammian, $G_o$, is defined as:
\[
G_\phi(t_o, t_1) = \int_{t_o}^{t_1} \Phi^T(\tau, t_o)C^T(\tau)C(\tau)\Phi(\tau, t_o)d\tau
\] (3.13)

and
\[
\Phi(t_o, t_1) = e^{\mathcal{A}(t_1-t_o)}
\]

Where, \( t_1 \) is some final time, \( C \) is the output matrix and \( \Phi \) is the state transition matrix.

Hence, \( G_\phi \) should be of full rank, i.e. the number of nonzero singular values should be equal to the dimension of the states (invertibility of \( G_\phi \)). The singular values of \( G_\phi \) are the positive square roots of the eigenvalues of \( G_\phi^T G_\phi \).

In sensorless applications, the rank condition may be satisfied, but it may be important to know how strongly observable the system is. A knowledge of the condition number may be necessary to infer how close to singularity the observability grammian is. The condition number is obtained from the Singular Value Decomposition (SVD) of \( G_\phi \). It is defined as the ratio of the largest singular value, \( \lambda_1 \), to the smallest, \( \lambda_n \).

\[
cond(G_\phi) = \frac{\lambda_1}{\lambda_n}
\] (3.14)

A small condition number is desirable, since a condition number close to infinity means the system matrix is close to singularity.

### 3.5 THEORY OF SLIDING MODE CONTROL

Consider a second order system of the form
\[
\ddot{z} = F(z) + B(z)u + h(x, t)
\] (3.15)

\[
|F(z)| \leq f \quad \text{and} \quad |B(z)| \leq b \quad |h(x, t)| \leq h_o
\]
$F(z)$ is a possibly nonlinear system whose parameters are not exactly known, and it is estimated as $F_1(z)$ but its upper bound is known and is positive. The control is $u$, $h(x,t)$ is an unknown disturbance with an upper bound $h_o$, and $B$ is the control vector, which is not known, but its sign is known and it is bounded. The output $z$ is continuously differentiable and is required to track a time vary reference trajectory, $z_d(t)$, whose higher derivatives exist.

Let the tracking error be defined as:

$$\tilde{z} = z - z_d$$  \hspace{1cm} (3.16)\

And the initial condition of $z_d$ be such that:

$$z_d(0) = z(0)$$  \hspace{1cm} (3.17)\

It is important that (3.17) holds as well as its higher derivatives, so that tracking could be achieved using a finite control. In order to achieve good tracking such that $z(t) \equiv z_d(t)$, an equilibrium manifold $\sigma = 0$, is defined in $\mathbb{R}^n$ according to Slotine [11] as:

$$\sigma(\tilde{z},t) = \left( \frac{d}{dt} + \beta \right)^{n-1} \tilde{z} \text{ and } \beta > 0$$  \hspace{1cm} (3.18)\

Applying (3.18) to the system in (3.15) the equilibrium manifold is defined as:

$$\sigma = \dot{\tilde{z}} + \beta \tilde{z}$$  \hspace{1cm} (3.19)\

In sliding mode, the motion equation coincides with that of the manifold, hence,

$$\sigma(\tilde{z},t) = 0$$  \hspace{1cm} (3.20)\Rightarrow \dot{\tilde{z}} = -\beta \tilde{z}$$
The motion equation (3.20) is of reduced order (first order differential equation), with a solution as stated in (3.21).

\[ \ddot{z}(t) = \ddot{z}(t_1)e^{-\beta(t-t_1)} \]  

(3.21)

The motion in sliding mode is asymptotically stable, as shown in equation (3.21) and the error state trajectories will reach the equilibrium manifold at time \( t_1 \), and will be confined to this manifold for \( t > t_1 \). This motion, with trajectories of the error dynamics confined to the equilibrium manifold is called sliding mode.

Hence, the positive scalar, \( \beta \) is chosen such that desired eigen values in sliding mode could be assigned arbitrarily. The eigen values are chosen with negative real part so that the error dynamics are critically damped. It is chosen large enough to determine the desired rate of convergence of trajectories on to the manifold. It determines the reaching time. The motion in sliding mode is independent of the disturbance \( h(x,t) \) if the matching conditions are met. This means that \( h(x,t) \) should belong in the range space of \( B(x,t) \).

Sliding mode is invariant with respect to the vector \( h(x,t) \) if

\[ h(x,t) \in \text{range}(B(x,t)) \]  

(3.22)

hence, there exists a \( \rho(x,t) \) such that:

\[ h(x,t) = B(x,t)\rho(x,t) \]  

(3.23)

In tracking, it is desired to bring the tracking error and all its higher derivatives to zero, which is equivalent to having the error dynamics be confined to \( \sigma \) for all time. Taking the derivative of (3.19) the dynamics of the surface is defined as:

\[ \dot{\sigma} = F_i(z) + \beta \dot{e} + bu - \beta \dot{e}_d - \ddot{z}_d + h(x,t) \]  

(3.24)

23
The best continuous control that will maintain the system motion on the surface, despite imperfections, can be approximated by obtaining a solution for $u$ in (3.24) as:

$$\dot{\sigma} = F_i(z) + \beta \dot{z} + bu - \beta \dot{\sigma}_d - \ddot{z}_d + b \rho = 0 \quad (3.25)$$

$$u_{eq} = -\frac{1}{b} (F_i(z) + b \rho + \beta \dot{z} - \beta \dot{\sigma}_d - \ddot{z}_d) \quad (3.26)$$

If control, $u_d$ is selected as a discontinuous control term, the system structure could be varied across the equilibrium manifold $\sigma = 0$, and the error trajectories will be oriented towards this line, and sliding mode will be enforced in $\sigma = 0$ [12]. Hence the existence condition ($\sigma \dot{\sigma} < 0$) is satisfied (necessity). This can be shown to be

The case if the control be selected as:

$$u_d = -K \text{sign}(\sigma) \quad K > 0 \quad (3.27)$$

Where

$$\text{sign}(\sigma) = \begin{cases} +1 & \sigma > 0 \\ -1 & \sigma < 0 \end{cases}$$

Existence of sliding mode means, $\sigma$ and its time derivative should have opposite signs in the vicinity of the discontinuous surface, $\sigma = 0$ [12].

Or

$$\sigma \dot{\sigma} < 0 \quad (3.28)$$

Applying (3.24) - (3.28) to (3.19)

$$\sigma \dot{\sigma} = f \sigma - bK|\sigma| \quad (3.29)$$

Selecting $K > f + h_o$ guarantees

$$\sigma \dot{\sigma} < 0$$
Hence, the matching condition is satisfied and sliding mode is enforced, making $\sigma$ an \textit{invariant set}, despite parametric uncertainties. Thus, robustness is guaranteed on the equilibrium manifold.

Thus, control can be made up of two terms, as shown is (3.30):

$$u = u_{eq} + u_d$$

(3.30)
CHAPTER 4

A THERMAL MODEL OF THE EMA

Some applications of the EMA, like the EMV and fuel injection systems require the device to operate in environments with elevated temperatures ([1], [2], [5], [7], [20]). Actuation of the device requires activation of the coil with a current pulse. Some of the current is always lost in power dissipation as heat, due to the resistance of the coil. Excessive internal temperatures may cause the coil insulation to break down and hence lead to a catastrophic failure of the device. Also, the resistance of the coil has a positive temperature coefficient, which means it increases with temperature, before a steady state value is reached. This increase could be as much as 30% of the initial value. This effect increases model uncertainty and may cause unreliable operation, since current demand will have to increase. In the case of electronic control, a change in resistance increases model uncertainty and the controller performance may depreciate. Also, it is desirable to understand the thermal needs of the system in designing a reliable cooling system. A fundamental understanding of the heat transfer process from a thermodynamic point of view is necessary to adequately compensate for the primary and secondary thermal effects.
The approach used in this derivation is the *lumped capacitance* (uniform temperature distribution across the coil surface at any time during a transient and temperature varies only with time) method applied to the *control volume* [25]. A control volume is a thermodynamic idealization that identifies a region in space around the coil, whose boundaries are the *control surface*, through which there is an exchange of energy and matter into and out of the coil. This method of lumped capacitance can be used in this system because the *Biot Number* (*Bi*) is much less than 1.

\[
Bi = \frac{R_{\text{internal}}}{R_{\text{external}}} = \frac{h_C L_x}{k} < 0.1
\]  

(4.0)

Where, \(L_x(L_x = r_o/2)\) is the characteristic length of the coil, \(r_o\) is the external coil radius, \(h_C\) is the heat transfer coefficient due to convection, and \(k\) is the thermal conductivity of the coil.

This simply means the coil insulation thermal resistance (\(R_{\text{internal}}\)) is less than that due to convection and radiation (\(R_{\text{external}}\)) across the fluid boundary layer [25]. Virtually, all the temperature difference is between the coil and fluid and the coil temperature remains nearly uniform.

The following assumptions were made in this study.

*Assumptions:*

- The coil is an isothermal surface. This means the temperature within the body of the copper coil is spatially uniform at any instant during the transient process (no temperature gradients) since every turn has the same amount of heat generation effect.
- Temperature is only a function of time.
• Thermal conductivity within the copper wire is infinite, since the resistance to heat conduction within the solid coil is very small, compared to the resistance to heat transfer between the solid and its surroundings (Fourier’s Law of Heat Conduction).
• Constant properties \((\alpha, \rho, \varepsilon, c, h_R, h_C)\) [25].
• One dimensional steady-state heat conduction.
• The entire volume of the coil is contained within the control volume.
• The coil is thin such that the temperature on the surface of the coil in uniform and is equal to ambient temperature.
• Neglect electromagnetic, potential and kinetic energy affects.

To formulate an overall energy balance for this system, the Law of Conservation of energy is applied to the control volume. Heat transfer within the control volume is by conduction, convection and radiation. Energy is generated within the control volume at a uniform rate by the copper conductors, due to Ohmic losses. The rate of energy stored is defined as:

\[
\dot{E}_m = \dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} (J/s) \text{ or } (\text{Watts}) \tag{4.1}
\]
$E_{st}$ is the energy stored in the system that raises the temperature of the coil, $E_{gen}$ is the energy generated by the coil, and $E_{out}$ is the energy flow out of the system into the atmosphere by convection and radiation, and $E_{in}$ is the energy flow into the system (4.0). Since there is no heat input into the system, $E_{in}$ is zero.

\[ \dot{E}_{out} = \dot{E}_{gen} - \dot{E}_{st} \]  

(4.2)
4.1 HEAT GENERATION ($E_{\text{gen}}$)

$$\dot{E}_{\text{gen}} = i^2 R_u L_c = i^2 R(T_s)$$  \hspace{1cm} (4.3)

Where,

$$R(T_s) = R_o (1 + \alpha \theta)$$

$R_u$ is the resistance per unit length of control volume, and $L_c$ is the length of the copper coil, $R_o$ is the nominal coil resistance, $R(T_s)$ is the total coil resistance at some temperature, $T_s$. Since the heat flow is uniform within the control volume, the heat generation rate of the entire control volume is

$$\dot{E}_{\text{gen}} = \dot{Q} V_c \hspace{1cm} (W)$$  \hspace{1cm} (4.4)

$$\dot{Q} = \frac{i^2 R_u L_c}{\frac{\pi D_c^2 L_{coil}}{4}} \hspace{1cm} \left(\frac{W}{m^3}\right)$$  \hspace{1cm} (4.5)

Where, $Q$ is the heat generation per unit volume of coil.

4.2 ENERGY LOSS ($E_{\text{out}}$)

$$\dot{E}_{\text{out}} = \dot{E}_{\text{COND}} + \dot{E}_{\text{CONV}} + \dot{E}_{\text{RAD}}$$  \hspace{1cm} (4.6)

- **Convection:** Heat is lost by free convection due to the free motion of the bulk of air surrounding the coil insulation (Figure 4.1), caused by the temperature difference between the coil surface and the atmosphere.

  From Newton’s Law of cooling,

  $$\dot{E}_{\text{CONV}} = A_s \dot{Q}_{\text{CONV}} \hspace{1cm} (W)$$  \hspace{1cm} (4.7)
Where,

\[ \dot{Q}_{\text{CONV}} = h_c (T_s - T_{\text{amb}}) \left[W/m^2\right] \quad \text{and} \quad A_s = \pi D_c L_c \quad (m^2) \quad (4.8) \]

\( \dot{Q}_{\text{CONV}} \) is the rate of heat transfer, or convective heat flux which is expressed as Watts per unit surface area (W/m\(^2\)) and \( h_c \) is the convective heat transfer coefficient (W/m\(^2\).K). This constant depends on the conditions of the boundary layer, like surface geometry, fluid velocity and physical properties of fluid. \( \dot{Q}_{\text{CONV}} \) is positive since heat is transferred from the coil to the surroundings.

- **Radiation:** Since the coil surface temperature is finite, there is a net heat loss by radiation from the walls of the insulation. The net radiation heat exchange is expressed as

\[ \dot{E}_{\text{RAD}} = A_s \dot{Q}_{\text{RAD}} \quad (W) \quad (4.9) \]

Where,

\[ \dot{Q}_{\text{RAD}} = h_R (T_s - T_{\text{amb}}) \quad (W/m^2) \quad \text{and} \quad A_s = \pi D_c L_c \quad (m^2) \quad (4.10) \]

\[ h_R = \varepsilon \sigma (T_s + T_{\text{amb}})(T_s^2 + T_{\text{amb}})^2 \quad (4.11) \]

\( \dot{Q}_{\text{RAD}} \) is the radiative heat flux and it is expressed as Watts per unit surface area. It is convenient to express the radiative heat flux as being proportional to the difference in temperature between the two boundaries, rather than temperature to the forth power [25]. \( h_R \) is the radiative heat transfer coefficient (W/m\(^2\).K) which is strongly dependent on temperature.
• *Conduction*: There is constant rate of heat being conducted from the copper coils through the entire insulation thickness due to the existence of a temperature gradient. The rate of heat transfer by conduction can be formulated from Fourier’s Law [25]:

\[
\dot{E}_{\text{COND}} = A_s \dot{Q}_{\text{COND}} \quad (W)
\]  

(4.12)

Where,

\[
\dot{Q}_{\text{COND}} = -k \frac{(T_{\text{amb}} - T_s)}{L_c} \quad (W/m^2) \quad \text{and} \quad A_s = \pi D_c L_c \quad (m^2)
\]  

(4.13)

\(\dot{Q}_{\text{COND}}\) is the conductive heat flux which is expressed as Watts per unit surface area perpendicular to the direction of heat transfer. \(k\) is the thermal conductivity (W/m.K). It depends on the wall material characteristics. The negative sign is indicative of the transfer of heat from an area of high temperature to an area of low temperature \((T_s > T_{\text{amb}})\).

• *Contact Resistance* \((R_{\text{TOT}})\): Due to the fact the materials involved offer some resistance to heat flow and the copper conductor surfaces are physically touching the insulation, there is an effective contact resistance. Convection and radiation heat loss occur in parallel, while conduction occurs in series to both. Hence,

\[
\dot{E}_{\text{out}} = \frac{(T_s - T_{\text{amb}})}{R_{\text{TOT}}} = \frac{\Delta T}{R_{\text{TOT}}} = \frac{\theta}{R_{\text{TOT}}}
\]  

(4.14)

\[
R_{\text{TOT}} = R_{\text{COND}} + \frac{1}{R_{\text{CONV}}} + \frac{1}{R_{\text{RAD}}} \quad (m^2.K/W)
\]  

(4.15)

Where,


\[ R_{\text{COND}} = \frac{\ln \left( \frac{r_2}{r_1} \right)}{2 \pi L_c k} \]  
(4.16)

\[ R_{\text{CONV}} = \frac{1}{h_c A_S} \]  
(4.17)

\[ R_{\text{RAD}} = \frac{1}{h_r A_S} \]  
(4.18)

### 4.3 ENERGY STORED (Eₘ)

The rate of change of energy stored within the control volume is due to changes in internal thermal energy, since kinetic and potential energy effects are negligible.

\[ \dot{E}_{\text{ST}} = \frac{dU}{dt} = \dot{Q}_{\text{ST}} - W \]  
(4.19)

But there is no work done, so \( W = 0 \) (neglecting electromagnetic work which aligns the magnetic dipoles in the direction of the magnetic field),

\[ \frac{dU}{dt} = \frac{d}{dt}(\rho V c T) = \rho V_c \frac{dT}{dt} \]  
(4.20)

Where,

\( \dot{Q}_{\text{ST}} \) is the rate of energy stored per unit volume, \( c \) is the specific heat capacity of the insulation at constant pressure, \( \rho \) is the mass density and \( V \) is the volume of the coil \( (\pi D^2/4)L_c) \).

Substituting (4.3), (4.14) and (4.20) into (4.2), we have,

\[ \frac{\theta}{R_{\text{TOT}}} = i^2 R_o (1 + \alpha \theta) - \rho V_c \frac{d\theta}{dt} \]  
(4.21)

Where
\[ \frac{dT}{dt} = \frac{d\theta}{dt} = \frac{d}{dt}(T_s - T_{amb}) \]

The left hand side of (4.21) is a contribution from the insulation alone, while the right hand side is a contribution from the copper alone.

Rearranging (4.21)

\[ \frac{d\theta}{dt} + \left( \frac{1}{R_{TOT}} - \frac{i^2 R_o \alpha}{\rho V c} \right) \theta = \frac{i^2 R_o}{\rho V c} \tag{4.22} \]

or

\[ \tau \frac{d\theta}{dt} + \theta = \frac{i^2 R_o}{f} \tag{4.23} \]

Where,

\[ \tau = \left( \frac{\rho V c}{\frac{1}{R_{TOT}} - i^2 R_o \alpha} \right) \]

Equation (4.23) is a first order differential equation, with time constant \( \tau \), a forcing function \( i^2 \) and a steady state gain, \( R_o / mc \) (\( m = \rho V \)). This means that the material properties of the conductor insulation as well as the wire resistance (wire gauge or diameter) are instrumental in the thermal dynamics of the coil. An insulating material with a large thermal resistance (or small coefficient of thermal conductivity) will have a larger time constant, hence it will heat the coil faster, while a wire with a small diameter will tend to have a larger resistance (\( R = \rho c d l / A \)) hence higher losses, consume more energy and will run hotter. Hence, better energy consumption and low operating temperatures mean larger diameter wires, the use of insulators with higher thermal conductivity and a thin
layer of insulation. Insulations with a thick layer are only necessary in applications where the external temperatures are higher than the coil temperature. There is a tradeoff, since larger diameter wires have a higher inductance, hence higher electrical time constant \( \tau_{\text{elec}} = L/R \). To effectively develop and understanding of the model in (4.23), it is important to derive a method of computing the power losses which make up the right hand side of equation (4.23).

4.4 COMPUTATION OF POWER LOSSES \((i^2R_a)\)

This represents the total power loss \((P_{IR})\) in the EMA and the method of computation is significant since it is very sensitive to the current dynamics and the resistance effects. The losses are mainly due to three effects:

- Losses due to DC resistance.
- Losses due to skin effect
- Losses due to proximity effects.

\[
P_{IR} = P_{DC} + P_{SKIN} + P_{PROXIMITY}
\]

(a) DC resistance: The DC resistance is the low frequency resistance of the windings. The power dissipated per unit of winding volume due to its DC resistance \((P_{DC})\) is referred to as the Ohmic losses.

\[
P_{DC} = k_c a \rho_{cu} \left( J_{rms} \right)^2 \left( \frac{W}{m^3} \right)
\]
Where

\[ J_{rms} = \frac{i_{rms}}{A_{cu}} \quad V_{cu} = k_{cu} V_w \quad \text{and} \quad k_{cu} = \frac{NA_{cu}}{A_w} \]

\( J_{rms} \) and \( V_w \) (\( V_w = V \)) are the root mean square (rms) current density in the conductor and winding volume, respectively, while \( i_{rms} \) is the rms current in the winding. \( A_{cu} \) is the cross-sectional area of the bare copper wire, which if multiplied by the number of turns, \( N \), will give the total area in the winding window occupied by just copper. This does not include the effective area occupied by the copper insulation and the air spaces between the copper turns. However, \( A_w \) is the total winding area that includes the bare copper wire, the insulation on the conductors, and the air spaces. The ratio of the total copper area to the winding area, is the copper fill factor, \( K_{cu} \). \( \rho_{cu} \) is the resistivity of copper at some temperature.

(b) Skin Effect: These are secondary effects that contribute to losses due to the fact that the conductor is carrying a time varying current, \( i(t) \) as shown in Figure 4.2. The current generates eddy currents that flow inside the wire, in opposite direction to the energizing current. This prevents the applied current from flowing in the interior of the wire, so the magnetic field generated around the conductor does not penetrate into the wire. As a result, the current density is greatest at the surface of the wire, and decays as the center of the wire is approached. Hence, most of the current in the conductor is restricted to flow on a thin layer just close to the surface, known as the skin depth, \( \delta = \sqrt{2(2\pi\mu, k)^{-1}} \). A small skin depth means the effective cross-sectional area of current flow is smaller than the dimension of the wire, hence the effective resistance is larger and this results in more losses. The losses due to skin effect increase, as wire diameter becomes greater than the
skin depth and as the current switching frequency increases. This is because higher frequencies result in smaller skin depths. The increased value of the resistance is known as the AC resistance, $R_{AC}$. This effectively means that the power lost is increased by a factor equivalent to the ratio of the AC to the DC resistance. Thus, (4.26) is modified to give (4.27), the power loss due to the coil DC resistance and AC resistance due to the skin effect.

$$P_{DC} + P_{SKIN} = K_{cu} \frac{R_{AC}}{R_{DC}} \rho_{cu} (J_{rms})^2 \left( \frac{W}{m^2} \right)$$

(4.27)

Figure 4.2: Longitudinal sectional view of conductor showing Eddy current generation (above) and plot of current density (below)
(c) **Proximity Effect:** This is due to the fact that the current carrying conductors in the coil each has a magnetic field that induces local eddy currents in the conductors in its proximity. These secondary eddy currents will increase the coil resistance by an amount equal to the effective eddy current resistance, $R_{EC}$, and additional power will be dissipated. The power dissipated increases with the number of winding layers. The power dissipated by the proximity effect, $P_{PROXIMITY}$, is:

$$P_{PROXIMITY} = i_{rms}^2 R_{EC}$$  \hspace{1cm} (4.28)

The effective coil resistance is modified, by taking into account the eddy current effects by a factor called the **resistance factor**, $F_r$ [29]. This factor increases with frequency.

$$R_{AC} = F_r R_{DC} = R_{DC} + R_{EC}$$  \hspace{1cm} (4.29)

It is therefore necessary in high frequency applications, to find the optimum conductor diameter and number of layers that would minimize the losses due to eddy currents and DC resistance. This is obtained from the set of curves in Figure 4.3, which is a plot of the normalized power dissipated against $\Phi$. The number layers of coil, $m$ is a parameter. The normalized power dissipated is defined as [29]:

$$\frac{P_T}{(i_{rms})^2 R_{DC, h=\delta}} = \frac{R_{AC}}{R_{DC, h=\delta}} = \frac{F_r R_{DC}}{R_{DC, h=\delta}}$$  \hspace{1cm} (4.30)

Where, $R_{DC, h=\delta}$ is the winding DC resistance when the wire diameter is equal to the skin depth and $\Phi$ is given as:

$$\Phi = \frac{\sqrt{F_r}}{\delta} h_c, \quad \delta = \sqrt{\frac{2}{2\pi \mu_k}}$$  \hspace{1cm} and  \hspace{1cm} $h_c = \left( \frac{\pi}{\sqrt{4}} \right) d$  \hspace{1cm} (4.31)
Where, $h_e$ is the effective conductor height for round conductors [29], $f$ is the frequency of current in the conductor measured in Hz, $d$ is the wire diameter without insulation, and $F_i$ is the copper layer factor. This represents the fraction of the bobbin height that is occupied by copper alone (the rest of which would be occupied by the lumped insulation), and it is given by:

$$F_i = \frac{d_T}{d}$$ \hspace{1cm} (4.32)

Where, $d_T$ is the total conductor diameter (copper plus insulation). A value for $\Phi$ can be obtained and applied together with the value for the number of layers directly to Figure 4.3. It is desirable to obtain the maximum wire diameter that would result in the optimum resistance factor, $F_r$, having a value of 1.5 [29].

Figure 4.3: Curves of Normalized Power Dissipated
By applying (4.30) to Figure 4.3, the value of \( R_{AC} \) can be obtained. The total power loss can now be computed as:

\[
P_{ir} = i_{rms}^2 R_o = i_{rms}^2 R_{DC} + i_{rms}^2 R_{AC} \left( \frac{W}{m^2} \right)\tag{4.33}
\]

The first term on the right hand side of (4.33) is the power loss due to the coil DC resistance, and the second term is the power loss due to eddy current effects caused by skin and proximity effects.

(d) **RMS value of current:** Usually, in some EMA applications, like in the EMV, it is necessary to cycle the current at high frequencies so that the plunger is forced to follow a desired trajectory. The shape of the current profile becomes nonsinusoidal, but periodic and repeats in steady state with a period \( T \) and a frequency, \( f \). In Figure 4.4, the current is compared to an ideal sinusoid and it is approximated as a half sinusoid. This enables the rms value to be calculated using Fourier analysis [24]. Using waveform symmetry, the coefficients could be approximated by assuming odd half periodic waves.

\[
i(t) = i_o + \sum_{n=1}^{\infty} i_n(t) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left\{ a_n \cos(n \omega t) + b_n \sin(n \omega t) \right\} \tag{4.34}
\]

Where,

\[
i_o = \frac{1}{2} a_0 \tag{4.35}
\]

The rms value of current can be expressed as

\[
i(t)_{rms} = \left( i_o^2 + \sum_{n=1}^{\infty} L_n(t)^2 \right)^{\frac{1}{2}} \tag{4.36}
\]
Figure 4.4: Comparison Of Current And An Ideal Sinusoidal Wave

Where,

\[ i_n = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}} \]  

(4.37)

\( i_0 \) is the DC or average value of the current. The constants could be approximated, by assuming only the fundamental exists \( (n = 1) \) and that the current profile is equivalent to an odd and half-wave function. The constants of the time varying current can be expressed as [29]:

\[ a_n = 0 \quad \text{for all } n \]
\[ b_n = \frac{4}{\pi} \int_0^\frac{\pi}{2} i(t) \sin(n\omega t) d(\omega t) \text{ for odd } n \quad (4.38) \]

\[ b_n = 0 \quad \text{for even } n \]

From (4.23), (4.33) and (4.36), the differential equation for the thermal dynamics is:

\[ \tau \frac{d\theta}{dt} + \theta = \frac{i(t)_{\text{rms}}^2 (R_{DC} + R_{AC})}{f} \quad (4.39) \]

Where,

\[ i(t)_{\text{rms}} = \left( i_0^2 + \sum_{n=1}^{\infty} i_n(t)^2 \right)^{\frac{1}{2}} \quad (4.40) \]

\[ \tau = \left( \frac{\rho V c}{1 - i(t)_{\text{rms}}^2 \left( R_{DC} + R_{AC} \right) \alpha} \right) \quad (4.41) \]

\[ R(T_c) = (R_{DC} + R_{AC})(1 + \alpha \theta) \]

With the computation of current and the effective coil resistance, the surface temperature of the coil could be simulated using the coil thermal model in equation (4.39)-(4.41).

### 4.5 THERMAL MODEL VALIDATION

The experimental setup for the model validation consisted of the solenoid, a Hall effect current sensor, a thermocouple and a Dspace data acquisition system.
Using the formulation in (4.36) the thermal model in (4.22) was simulated in Simulink® and the results were compared to that acquired experimentally by a real time data acquisition system. Validation was done for coil temperature. The thermal data matches the experimental data well in the steady state (Figure 4.5). The discrepancy between the experimental and the simulated results during temperature transients was due to the assumptions made to minimize the complexity of the problem. Also certain important coil data used in the computation of the coil time constant had to be estimated, since it could not be obtained from the coil manufacturer.
CHAPTER 5

DYNAMIC MODEL OF THE EMA

The EMA works under the basic principle of electromagnetic induction. A current carrying conductor in a stationary coil produces a magnetic flux $\phi$, that links the coil turns, air gap, ferromagnetic core, and the plunger, with field of intensity $H$ (Amperes/meter). A force is generated which causes the plunger to move in a linear fashion, with a trajectory very much determined by the current dynamics.

Figure 5.1: Schematic of Solenoid Actuator
(a) Dynamic Equations of The Mechanical Subsystem

The schematic of a single actuator system is shown in Figure 5.1. For a single actuator, it is assumed that the spring is linear and damping is provided by mostly the spring and air. The spring is preloaded to set the maximum plunger displacement and to store enough potential energy to return the plunger to its equilibrium position when the coil is not energized. Applying a change of variable to take into account initial conditions, the new position variable, $z$ is:

$$z = x - x_0, \quad \dot{z} = \dot{x}, \quad \ddot{z} = \ddot{x} \quad \quad (5.1)$$

Hence mechanical dynamic equation of motion is:

$$\ddot{z} = \frac{1}{M} (-F_{sp} - B\dot{z} - F_n + F_{pl} + W + F_{mag}) \quad \quad (5.2)$$

Where,

$$F_{sp} = K_{sp}z + W + F_{pl} \quad , \quad B = B_{sp} + B_{air}$$

The variable $W$ represents the weight of the mass on the spring at equilibrium. The origin of the $z$ coordinate is chosen to coincide with the location of the mass at rest, with only the preload and the spring force acting on it. The spring is preloaded by a force $F_{pl}$, and $F_n$ is the force due to the hard nonlinearity, which is active when the plunger strikes the hard stops. For ease of implementation, it can be approximated as a piecewise linear function.

$$F_n(z, \dot{z}) = \begin{cases} 
0 & 0 < z < z_{\text{max}} \\
K_{st}z + B_{st}\dot{z} & z \leq 0, \quad z \geq z_{\text{max}} 
\end{cases} \quad \quad (5.3)$$

Where

$$K_{st} > 10^3 \quad \text{and} \quad B_{st} > 0$$
The degree of bounce is adequately modeled by iteratively varying the values of $K_{st}$ and $B_{st}$ to match that of the surface in question.

(b)  **Dynamic Equations of Magnetic Subsystem**

The magnetizing force produced in the plunger of the EMA is a direct consequence of the flux produced by the current carrying conductor. The flux couples the electrical and mechanical subsystems through the air gap and thus transfers energy between the two subsystems. The flux produced by the windings is made up of the magnetizing component and the leakage component. The magnetizing flux is common to the plunger and coil core. It links the turns of the windings and the plunger, while the leakage flux links the windings and the air gap.

From Ampere’s law, the line integral of the field intensity or field strength, $H$, about a closed path is equal to the net current enclosed within this closed path of integration. The closed path is made up of the ferromagnetic material and the air-gap. Field strength is mathematically defined as:

$$\oint H \cdot dL = i_n$$  \hspace{1cm} (5.4)

Where, $i_n$ is the net current enclosed. The net current is enclosed N-times, where $N$ is the number of turns. If the integration is carried out about the mean length of magnetic material, an expression for the ampere turns is obtained as in (5.5).

$$Ni = HL_i + HL_g$$  \hspace{1cm} (5.5)

Where $l_i$ is the mean length across the magnetic material and $l_g$ is the length across the air gap. The term, $Ni$ is usually referred to as Ampere turns or magnetomotive force (mmf)
and has the units of Amperes. It is analogous to electromotive force \((emf)\) in electrical circuits. Assuming an isotropic linear magnetic material and uniform distribution of flux over the cross-sectional area of the air gap and magnetic material, the surface integral of the flux density is the flux, \(\phi\).

\[
\phi = \int_B dA \quad \text{or} \quad \phi = BA \quad \text{and} \quad B = \mu H
\]

The flux linkage, \(\lambda\), is represented as:

\[
\lambda = N\phi \quad \text{and} \quad \phi = \phi_m + \phi_l
\]

Where, \(\phi_m\) is the magnetizing flux and \(\phi_l\) is the leakage flux. Neglecting magnetic saturation, the flux could be represented in terms of the reluctance, \(\mathcal{R}\). The reluctance is analogous to resistance and it tends to prevent the uniform distribution of flux over the cross-sectional area, i.e. it reduces the flux density. Hence, the air-gap reluctance is greater than the magnetizing reluctance, making the leakage flux to be representative of magnetic losses.

\[
\phi_m = \frac{Ni}{\mathcal{R}_m} \quad \text{and} \quad \phi_l = \frac{Ni}{\mathcal{R}_l}
\]

Where

\[
\mathcal{R} = \frac{l}{\mu A}
\]

and \(\mu\) is the relative permeability.

So

\[
\mathcal{R}_{tot} = \mathcal{R}_m + \mathcal{R}_l \quad \text{(5.9)}
\]

\[
\mathcal{R}_m = \mathcal{R}_{core} + \mathcal{R}_{plunger} + \mathcal{R}_{gap} \quad \text{(5.10)}
\]
\[ \Re_{\text{tot}} = \frac{1}{\mu_0} \left( \frac{l_c}{\mu_c A_c} + \frac{l_{pl}}{\mu_{pl} A_{pl}} + \frac{l_{gap}}{\mu_{gap} A_{gap}} \right) \]  

(5.11)

and

\[ \mu_c = \mu_0 \mu_{rc}, \quad \mu_{pl} = \mu_0 \mu_{rpl}, \quad \mu_{gap} = \mu_0 \mu_{rgap} \]  

(5.12)

\[ \frac{l_{gap}}{A_{gap}} = \frac{g}{A_{pl}} + \frac{g_a}{A_{ga}} = \frac{1}{A_{pl}} \left( \frac{g_a A_{pl}}{A_{ga}} + g \right) \]  

(5.13)

Where

\[ g = g_{\text{max}} - z \quad A_{gap} = A_g + A_{ga} \quad \text{and} \quad A_{g_a} = 2\pi s_a l_c \]

Generally, \( \Re_l \) and \( \Re_m \) are the reluctances of the leakage and magnetizing paths, respectively, and \( \mu_0 \) is the permeability of free space (\( 4\pi \times 10^{-7} \text{ Wb/A.m} \) or \( 4\pi \times 10^{-7} \text{ H/m} \)). Also \( \mu_{rc}, \mu_{rpl} \) and \( \mu_{rgap} (\mu_{rgap} = 1) \) are the relative permeabilities of core, plunger and air gap (g), respectively (Figure A.1).

The expression for mmf is:

\[ Ni = \phi (\Re_{\text{core}} + \Re_{\text{plunger}} + \Re_{\text{gap}}) \]  

(5.14)

The coil inductance is a function of plunger displacement, and is made up of the magnetizing inductance \( L_m \) and the leakage inductance, \( L_l \). The leakage inductance will be approximated from the magnetic force (Appendix, equation A4). Its actual value can only be obtained experimentally. The total inductance, is made up of the coil inductance and the inductance of the external circuit.

\[ L_{\text{tot}} = L_m(z) + L_{\text{leakage}} + L_{\text{circuit}} \]  

(5.15)

\[ L_m(z) = \frac{N^2}{\Re_m} \quad \text{and} \quad L_{\text{leakage}} = \frac{4\mu_0 N^2 w_b^2}{3h_b} \]
Where, $l_w (2\pi r_x)$ is the winding length at a distance $x$, $b_b$ is the bobbin width in the winding window and $h_b$ is the height of the bobbin in the winding window as shown in Figure (A.2). The magnetizing inductance is a function of plunger displacement and is represented as:

$$L_m(z) = \frac{k_3}{k - z} \quad \text{and} \quad z = z(t) \quad (5.16)$$

Where

$$k_3 = N^2 \mu_o A_g \quad k = A_g \left[ \frac{1}{\mu_{pl}} \left( \frac{l_c}{A_c} + \frac{l_{pl}}{A_{pl}} \right) + \frac{g_u}{A_{ga}} \right] + g_{max} \quad (5.17)$$

and

$$L(z) = L_m(z) + L_l \quad (5.18)$$

$A_g$ is the cross-sectional area of the air-gap and it is computed to approximate the effect of fringing or bulging that occurs at the sides.

$$A_g = \pi \frac{1}{4} (g + d_{pl})^2 + \pi \frac{1}{4} (d_{pl})^2 \quad (5.19)$$

Where $d_{pl}$, $d_{sh}$ are plunger and plunger shaft diameters, respectively. The flux linkage, $\lambda$ is a function of both current and displacement.

$$\lambda(i, z(t)) = L(z)i(t) \quad (5.20)$$

A derivation of the magnetic force necessitates an analysis of the energy balance of the system. The EMA consists of two interacting systems, the electrical and the mechanical subsystems and energy is transferred from one system to the other as result of the coupling provided by the magnetic field. Following the law of conservation of energy:

$$E_E = E_{EL} + E_{ES} + E_{ET}$$

$$E_M = E_{ML} + E_{MS} + E_{MT}$$
Where, $E_E$ is the total energy supplied by the current, $E_{EL}$ is the energy loss as heat due to coil resistance and hysteresis, $E_{ET}$ is the energy transferred to the coupling system and $E_{ES}$ is the energy stored in the electric field as e.m.f. Similarly, $E_M$ is the total mechanical energy supplied by the motion of the plunger across the magnetic field, and $E_{ML}$ are losses due to heat and $E_{MT}$ is the energy transferred to the coupling field.

Hence, following the law of conservation of energy (Figure 5.2):

$$E_{GS} + E_{GL} = (E_E - E_{EL} - E_{ES}) + (E_M - E_{ML} - E_{MS})$$

$$E_{GS} + E_{GL} = E_{ET} + E_{MT}$$

Neglecting magnetic losses,

$$E_{GS} = E_{ET} + E_{MT} = \int (V_{mag} \, i) \, dt - \int (F_{mag}) \, dz$$

(5.21)

Where, $V_{mag}$ is the voltage drop due to the coupling field and $F_{mag}$ is the electromagnetic force applied to the plunger, hence it is negative, according to the convention (energy supplied to the system is negative). Initially, when the electrical circuit is energized by a voltage, this results in a rate of change of flux linking the windings. $E_{MT}$ is zero, since there is no motion of the plunger, initially.

Hence, $dz = 0$

$$E_{GS} = \int i \, d\lambda \quad and \quad V_{mag} = \frac{d\lambda}{dt}$$

(5.22)
Figure 5.2: Schematic of Energy Block Diagram

Figure 5.3: Schematic of Stored Energy and Coenergy
For the linear region in Figure 5.3

\[ \lambda i = E_{gs} + E_c \]  \hspace{1cm} (5.23)

The electromagnetic force can be expressed as coenergy, \( E_c \), as shown in Figure 5.3, and for the linear magnetic region,

\[ E_{gs} = E_c = \frac{1}{2} \lambda i \]  \hspace{1cm} (5.24)

Thus, the displacement defines the influence of the mechanical subsystem on the coupling magnetic field. With \( z \) and \( i \) being independent variables. Taking the partial derivative of (5.20) results in (5.25):

\[ \delta \lambda = \frac{\delta \lambda(i,z)}{\delta i} di + \frac{\delta \lambda(i,z)}{\delta z} dz \]  \hspace{1cm} (5.25)

Substituting (5.20) into (5.24) and taking its partial derivative, yields the magnetic force, that represented as:

\[ F_{mag} = \frac{\delta E_c}{\delta z} = \frac{1}{2} \frac{\lambda^2}{L(z)^2} \frac{dL(z)}{dz} \]  \hspace{1cm} or \hspace{1cm} \[ F_{mag} = \frac{1}{2} \frac{dL(z)}{dz} i^2 \]  \hspace{1cm} (5.26)

Where

\[ L(z) = L_m(z) + L_{leakage} \]

(c) Dynamic Equations of Electrical subsystem

The electrical system inputs energy into the system by applying a voltage across the coil terminals, resulting in a current flow. The voltage equation may be written as

\[ V_s = R(T_s) i + L_c \frac{di}{dt} + V_{mag}(z,i) \]  \hspace{1cm} (5.27)
\[ V_s = R(T_s)i + \dot{\lambda}(z,i) \]

Where,
\[ R(T_s) = (R_{DC} + R_{AC})(1 + \alpha \theta) \text{ and } L_c \approx 0 \]

The term \( V_s \) is the supply voltage, while the first term on the right hand side of (5.27) is the voltage drop across the resistance, the second term is the voltage drop across the external circuit, and the last term is the voltage drop across the magnetic field. The coil resistance is function of the surface temperature, \( T_s \) and it is important in the modeling process since an increase in resistance due to temperature effects can be as much as 30%. The voltage drop across the external circuit could be neglected, since \( L_c \approx 0 \).

(d) **System State Dynamic Equations**

The dynamic model governing the operation of the EMA, with measurement, \( y \) is

\[ \dot{z}_1 = z_2 \]

\[ \dot{z}_2 = \frac{1}{M} \left( -K_q z_1 - B z_2 - F_n + \frac{\lambda^2}{2} \frac{dL_m(z)}{dz} \right) \]

\[ \dot{\lambda} = -R(T_s) \frac{\lambda}{L_{tot}} + V_s \]

\[ y = i = \frac{\lambda(i,z)}{L(z)} \]

The state dynamic equations are coupled, and strongly nonlinear.

The states are \( z_1, z_2, \) and \( \lambda \), plunger position, velocity and flux linkage, respectively, while the measurement is \( i \), current, and the input is \( V_s \), the supply voltage.
5.1 MODEL VALIDATION

The experimental setup for the model validation consisted of the solenoid, a position sensor, a Hall effect current sensor and a Dspace data acquisition system. The model was simulated in Simulink® and the results were compared to that acquired experimentally by a real time data acquisition system. Validation was done for the position, velocity and current. There is a close match between the simulated and experimental data.

![Figure 5.4: Validation of Electro-Mechanical Model](image)

The discrepancy between the experimental and the simulated results for velocity was due to the fact that the model neglected saturation (Figure 5.4) and hysteresis effects, hence the model uncertainty increases as the air-gap is reduced and impact is approached. Also
it was difficult to model the impact dynamics exactly, and the method of computing the velocity from the position measurements introduces an amount of lag in the velocity data.

5.2 SLIDING MODE OBSERVER FOR EMA

The estimate of flux linkage, can directly be obtained from the integration of the third equation of (5.28), assuming zero initial conditions. Hence, the formulation enables a reduced order observer to be designed on the nonlinear system, to estimate position, $z_1$ and velocity, $z_2$ from current measurement alone. The method of observing flux by integration will accumulate error depending on the solver and step size, as well as possible integrator windup. But it simplifies the observer design and reduces computational burden. An observer structure for this system similar to (3.2) is:

$$\dot{\hat{z}}_1 = \dot{\hat{z}}_2 - L_1 s - M_1 \text{sign}(s)$$

$$\dot{\hat{z}}_2 = \frac{1}{M} (-K_{s} \hat{x}_1 - B \hat{z}_2 - F_n + \frac{1}{2} \frac{dL_m(\hat{z})}{dz} - L_2 s - M_2 \text{sign}(s))$$  \hspace{0.5cm} (5.29)

Where

$$\dot{\hat{x}}(i, \hat{x}) = \dot{\hat{x}} = \int_0^t \left( -R(T) \frac{\hat{\lambda}}{L_{tot}} + V_s \right) dz \hat{\lambda} + \dot{\hat{\lambda}}(i_0, \hat{x}_0)$$  \hspace{0.5cm} (5.30)

The function for the equilibrium manifold should compare a variable dependent on the estimated position and a measurable variable. Let the equilibrium manifold be chosen as:

$$s = \frac{dL(\hat{z})}{dz} \left[ \hat{\lambda} - i \right] = \frac{dL(\hat{z})}{dz}\left[ \frac{\hat{\lambda}}{L(\hat{z})} - i \right]$$  \hspace{0.5cm} (5.31)
The error dynamics is:

\[ \ddot{z}_1 = \bar{z}_2 - L_1 s - M_1 \text{sign}(s) \]

\[ \ddot{z}_2 = \frac{1}{M} \left[ -K_S \bar{z}_1 - B \bar{z}_2 - F_n(\bar{z}) + \left( F_m(\hat{z}_1, \hat{\lambda}) - F_m(z_1, \hat{\lambda}) \right) \right] - L_2 s - M_2 \text{sign}(s) \quad (5.32) \]

Where

\[ \bar{z} = \bar{z} - z \quad \text{and} \quad F_m = \frac{1}{2} \frac{\lambda}{L(z)} \frac{dL_m(z)}{dz} \]

Assuming \( M_1 \) and \( M_2 \) are zero, and \( p_1 \) and \( p_2 \) are desired eigenvalues to be assigned, \( L_1 \) and \( L_2 \) were designed to place the system poles \( d_1 \) and \( d_2 \), at the desired locations.

The desired eigenvalues are designed such that the system is critically damped.

\[ p_1 \geq 10d_1 \quad \text{and} \quad p_2 \geq 10d_2 \quad (5.33) \]

Designing a positive definite Lyapunov function:

\[ V(\bar{z}_1, \bar{z}_2, \bar{\lambda}) = \frac{1}{2} \bar{z}_1^T \bar{z}_1 + \frac{1}{2} \bar{z}_2^T \bar{z}_2 + \frac{1}{2} \bar{\lambda}^T \bar{\lambda} \quad (5.34) \]

Taking the derivative of \( V \) along trajectories of the system:

\[ \dot{V} = \bar{z}_1^T (\bar{z}_2 - L_1 s - M_1 \text{sign}(s)) + \bar{z}_2^T (G - L_2 s - M_2 \text{sign}(s)) + \bar{\lambda}^T \bar{R} \quad (5.35) \]

If \( M_1 \) and \( M_2 \) are selected such that:

\[ M_1 \geq |\bar{z}_2|, \quad M_2 \geq G, \quad \text{then} \quad \dot{V} < 0 \quad (5.36) \]

\[ G = \frac{1}{M} \left( -K_S \bar{z}_1 - B \bar{z}_2 - F_n(\bar{z}) + \frac{\hat{\lambda}^2}{2} \frac{dL_m(\bar{z})}{dz} - \frac{\lambda}{2} \frac{dL_m(z)}{dz} \right) \quad (5.37) \]

The error dynamics become asymptotically stable:

\[ \ddot{z}_1 = \bar{z}_2 - L_1 s - M_1 \text{sign}(s) \]

\[ \ddot{z}_2 = -L_2 s - M_2 \text{sign}(s) \quad (5.38) \]
\[
\frac{dL(\dot{z})}{d\dot{z}} = \frac{k_i}{(k + g_{\text{max}} - z)^2} > 0
\]

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<tr>
<th>( \dot{z} &lt; z )</th>
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Table 5.1: Sliding Mode Existence condition
After reaching $s = 0$, $\ddot{z}_2$ decays exponentially to 0, with a time constant $\frac{M_1}{M_2}$. The error space in (5.38) will be asymptotically stable and will have dynamics similar to the observer in (3.2) if (5.36) holds such that $s$ and $\dot{s}$ are of opposite signs (existence condition). This condition is equivalent to equation (3.5). This simply means that the state trajectory describing deviation from the surface is oriented towards the discontinuity surface. As shown in Table 5.1, the existence condition is satisfied for all regions of operation. With (3.6) satisfied also, the error dynamics will decay at a desired rate and the manifold, $s$ will be attractive.

Applying SVD to the linearized system, the observability grammian is of full rank $(Rank = 3)$, meaning the system position and velocity can be reconstructed from current measurements alone. The condition number of the EMV system was large ($\text{cond.} = 1.12e6$), meaning the system has very little observable energy. This implies that for certain operating points, the observers will diverge to infinity, since the grammian is close to singularity. For this model, the observer will perform poorly as the air-gap is reduced and the observer may diverge just before impact. This is because at impact, the dynamics of the hard surface becomes part of the system and the large values of stiffness will increase the model uncertainty, hence $\lambda_1$ increases disproportionately to $\lambda_n$, resulting in an explosion of the condition number. This can be avoided by forcing the plunger to asymptotically approach the core or by ignoring the dynamics that occurs when the plunger strikes the core.
5.3 OBSERVER BASED SLIDING MODE CONTROLLER

In most applications of the EMA, the plunger position is required to follow a desired trajectory up to a certain lift and the return stroke is governed mostly by the stored spring energy. When the plunger impacts any of the hard stops, there is bounce and noise associated with the impact. These effects are undesirable and may result in a shorter fatigue life, wear and possibly failure. The control objective is to ensure that the plunger tracks the desired trajectory as closely as possible, and to eliminate bounce and impact noise.

In this study, a sensorless control strategy is proposed, based on an observer based sliding mode controller, using the estimated position for feedback, which is obtained from the current measurement only. The control problem is reduced to sensorless state estimation with exact output regulation.

(a) Design of Desired Trajectory: It is desired that the position estimate tracks a desired trajectory $z_d$ that approximates a cam profile for an intake / exhaust valve of an internal combustion engine. The function for the desired trajectory is continuously differentiable and it is a nonlinear function of $h$, where $h$ represents a vector of real discrete integers, $a$, and $z_{d \text{ max}}$ is the maximum plunger lift desired.

$$z_d(h) = z_{d \text{ max}} \exp\left(-\frac{h^2}{2}\right) \quad \text{where} \quad -a \leq h \leq +a \quad (5.39)$$
The desired trajectory is shown to have (Figure 5.5) no bounce and very low seating velocity and acceleration.

The separation principle will be applied here, where the observer and controller are designed separately.

Let the tracking error be defined as:

\[ e_1 = e(t) = \hat{z} - z_d \]

\[ e_2 = \dot{e}_1 = \hat{z}_2 - \dot{z}_d \]

\[ \dot{e}_2 = \ddot{e}_1 = \hat{z}_2 - \ddot{z}_d \]

\[ \ddot{e}_3 = \dddot{e}_2 = \hat{z}_2 - \dddot{z}_d \]  \hspace{1cm} (5.40)
The error space of the system becomes:

\[
\dot{e}_1 = e_2
\]

\[
\dot{e}_2 = \frac{1}{M} \left[ -K_y e_1 - B e_2 - F_n + \frac{1}{2} \left( \frac{\lambda}{L(\tilde{z})} \right)^2 \frac{dL_m(\tilde{z})}{dz} \right] + \frac{K_y}{M} \dot{z}_d + \frac{B}{M} \ddot{z}_d + \dddot{z}_d
\]

\[
\dot{e}_3 = \frac{1}{M} \left[ -K_y e_2 - B d_1 + d_2 + d_3 V_s^2 \right] + \frac{K_y}{M} \dot{z}_d + \frac{B}{M} \ddot{z}_d + \dddot{z}_d
\]

The upper bounds of unknown parameters are defined as \(d_1, d_2, d_3\).

\[
|\dot{e}_2| \leq d_1 \quad \text{and} \quad \left| \frac{1}{2} \left( \frac{1}{L(\tilde{z})} \frac{d^2L_m(\tilde{z})}{dz^2} \right) \right|^2 \leq d_3
\]

\[
\left| F_{pr}(\tilde{z}) - F_n(\tilde{z}_1, \tilde{z}_2) + \frac{1}{2} \left( \frac{1}{L(\tilde{z})} \frac{d^2L_m(\tilde{z})}{dz^2} \right)^2 \right| \leq d_2
\]

Let the equilibrium manifold be design as:

\[
\sigma(e_1, t) = \left( \frac{d}{dt} + \beta \right)^2 e_1 = \dot{e}_2 + 2\beta e_2 + \beta^2 e_1
\]

The control is designed as a discontinuous control voltage such that:

\[
V_s = -K\text{sign}(\sigma)
\]

Taking the product of (5.43a) and its derivative:

\[
\sigma \dot{\sigma} = \sigma \left( \frac{1}{M} \left[ -K_y e_2 - B d_1 + d_2 + K_y \dot{z}_d + B \ddot{z}_d \right] + \dddot{z}_d + 2\beta e_3 + \beta^2 e_2 + \frac{d_3}{M} V_s^2 \right)
\]

The ideal sliding Mode existence condition is satisfied if \(\frac{M}{d_3}\) is nonsingular and \(K\) is chosen such that:

\[
K \geq \tilde{F}
\]
Where \( F \geq \frac{M}{d_3} \left| \frac{1}{M} \left[-K_{sp}e_2 - Bd_1 + d_2 + K_{sp}z_d + B\gamma_z \right] + \dot{z}_d \right| \)

The error trajectories will coincide with the equilibrium manifold and sliding mode will be enforced in \( \sigma(e,t) = 0 \), such that the motion equation is of reduced order.

\[
\ddot{e} + 2\beta \dot{e} + \beta^2 e = 0
\]  \hspace{1cm} (5.44)

The motion equation is a second order differential equation with repeated roots. The solution of (5.44) is:

\[
e_1(t) = e_1(t_i)te^{-\beta(t-t_i)} + e_2(t_i)e^{-\beta(t-t_i)}
\]  \hspace{1cm} (5.45)

Where, \( t_i \) is the reaching time and \( e_1(t_i) \) and \( e_2(t_i) \) are the initial conditions.

The error dynamics is asymptotically stable, and \( \beta \) is chosen to give the desired rate of convergence.

![Figure 5.6: Boundary Layer Chattering (Left) and Saturation Function (right)](image-url)

Figure 5.6: Boundary Layer Chattering (Left) and Saturation Function (right)
Since saturation and hysteresis effects of the EMA are not modeled, these and other imperfections will prevent ideal sliding mode from occurring, and may lead to high frequency oscillations, since error trajectories may be confined to some vicinity of the equilibrium manifold. This might cause a finite frequency oscillation of the state vector in sliding mode. This phenomenon is called *chattering* (Figure 5.6). To avoid chattering, the control is implemented as:

\[ V_s = u_{eq} - K_{sat}(\frac{\sigma}{\phi}) \]  

(5.46)

The saturation function (*sat*) replaces the sign function and approximates the sign term in a boundary layer of the manifold. Where, \( \phi \) called the *boundary layer* and it is defined as (Figure 5.6):

\[ \|\sigma(e,t)\| \leq \phi \quad \|\sigma(e,t)\|=\left(\sigma^T \sigma\right)^{\frac{1}{2}} \]  

(5.47)

The term \( u_{eq} \) is the *equivalent control term*. It is a continuous control that replaces the fast switching within the boundary layer and forces the error trajectory to lie within a tangential manifold of the boundary layer, hence coinciding with the minimal convex set since the EMA is an affine system. Taking the derivative of (5.43) and solving for \( u \), the equivalent control term for this system is:

\[ u_{eq} = d^{-1}_3 \left[ -\beta^2 e_1 - \frac{1}{M} \left[ -K_{sp} e_2 + (2\beta - B)d_1 + d_2 \right] - \frac{K_{sp}}{M} \dot{z}_d - \frac{B}{M} \ddot{z}_d - \ddot{z}_d \right] \]  

(5.48)

From (5.48), the equivalent control is seen to be linear feedback control term that is dependent on system parameters. If \( K \) is selected greater than \( \tilde{F} \) (or large enough) as in (5.43c), all state trajectories should be oriented towards \( \sigma \), and sliding mode will be enforced.
\[ V_s(\text{max}) \geq K \geq \tilde{F} + \nu \quad \nu > 0 \quad (4.25) \]

\( V_s(\text{max}) \) is the rated coil voltage and \( \tilde{F} \) is and upper bound on all system parameter uncertainties. The saturation function (Figure 5.6) is defined as:

\[
\text{sat} \left( \frac{\sigma}{\phi} \right) = \begin{cases} 
+1 & \frac{\sigma}{\phi} \geq +1 \\
\frac{\sigma}{\phi} & 0 < \frac{\sigma}{\phi} < +1 \\
0 & \frac{\sigma}{\phi} \leq 0 
\end{cases}
\]

For real time control applications of the EMA, the minimum output from the saturation function was zero since a negative voltage into the coil would only de-flux the coil, and is meaningless in terms of desired actuator operation. Hence, control with the boundary layer is a linear feedback control that will maintain the error trajectories close to \( \sigma \).

### 5.4 SIMULATION RESULTS

The sliding estimator was simulated in open loop (Figure 5.7). Open loop control was necessary to verify if the estimates will converge to their actual values fast enough. Convergence of observer is asymptotic for full and variable lift. The estimation errors converged to the equilibrium manifold when sliding mode was enforced and were confined to this manifold for all time (Figure 5.8). The bounce of the system model is well captured by the observer (Figure 5.7). In closed loop control, the position signal and its estimate tracked the reference trajectory closely (Figure 5.9).
Figure 5.7: Simulation of Sliding Mode Estimator

Figure 5.8: Simulation of Estimation Error and Equilibrium Manifold
Comparing the controlled and the uncontrolled case (Figure 5.10), seating velocity is reduced from 0.5m/s to 0.04m/s and bounce is completely eliminated.
5.5 EXPERIMENTAL SETUP

The experimental setup consists of a single spring solenoid actuator, which is actuated by a linear amplifier unit (Figure 5.11). The linear amplifier receives control signals from the sliding mode controller embedded in the Digital Signal Processor (DSP) of the Dspace 1103 board. The coil current was measured using a current sensor, and this was the only signal used for control. The position signal measured by the laser sensor was for validating the performance of the position estimator. The only measurement into the observer was the coil current.
5.6 EXPERIMENTAL RESULTS

In the open loop, the position and velocity estimates converged to their actual values (Figure 5.12), except during impact, where the bounce of the observer had to be reduced to avoid divergence of observer. The match between the power supply output voltage and the open loop voltage commanded was close. The mismatch between current and its estimate is due to the fact that the current estimate was derived from a ratio of flux estimate and inductance estimate (Figure 5.13). There is an accumulation of error at each time step, due to numerical integration and the error in the position estimate from which the estimate of inductance is derived.

Figure 5.12: Validation of Sensorless Observer: Position and Velocity.
In the closed loop, the observer convergence is asymptotic and tracking of the reference signal was good (Figures 5.14). The sliding mode observer was stable and converged very fast. The estimates of position, velocity and current matched their actual values with very little error (Figure 5.14-5.15). The slight offset is due to a time delay that is evident even in the simulation results. This was due to magnetic time delay, actuator electrical time constant, mechanical inertial effects and the turnaround time (computational delay) of the Dspace 1103 processor.

Bounce was completely eliminated by feedback control as compared to the uncontrolled case, (Figure 5.16) and seating velocity is reduced from 0.3 m/s to about .04 m/s. The control was bounded and limited to operational range of the coil.

![Validation of Sensorless Observer: Current and Voltage](image)

Figure 5.13: Validation of Sensorless Observer: Current and Voltage
Figure 5.14: Tracking and Estimation of Position in Closed Loop

Figure 5.15: Current Estimate and Control Voltage in Closed Loop (Experimental)
Figure 5.16: Comparison of Bounce and Velocity for Open and Closed Loop Control (Experimental)
There is a need for a more accurate model for a class of EMA’s called the EMV. These devices may consume as much as 50 Amps of current and may be cycled at about 200 Hz. This means high speeds, high power consumption, and a much stronger flux field. A small uncertainty in modeling the flux field will lead to a pronounced error in estimating the magnetic force, which is crucial in estimating the actual position and velocity of the plunger as time evolves. Hence, it is important to model secondary nonlinearities like saturation, hysteresis, bounce and mutual inductance.

The existence of saturation and hysteresis is evident in the magnetization curve of magnetic materials. The B-H (magnetization) curve, is a plot of the flux density, B (Tesla [T]) versus the magnetic field strength, H (Ampere/meter [A/m]). The equations for B and H are:

\[
B = \frac{1}{NA} \lambda = C_1 \lambda \quad \quad H = \frac{N}{l} i = C_2 i
\]  

(6.1)

Where, \(N\) is the number of turns, \(A\) is the coil cross-sectional area, \(l\) is the mean length of magnetic path around the coil, \(i\) is the current, and \(C_1\) and \(C_2\) are constants. Since it is
difficult to obtain measurements of B and H, a scalable version of one of the family of magnetization curves could be obtained by plotting the flux linkage against the current ($\lambda$ versus $i$) which are easier to measure (Figure 6.2).

![Figure 6.1 Schematic (left) and Picture (right) of the EMV](image)

From Figure 6.2 and 6.3 it is evident that saturation and hysteresis are characteristics of this electromagnet in question and must be modeled.
Figure 6.2: Magnetization curve: 35.5V (left), Different Voltages same Air-gap (right)

Figure 6.3: Hysteresis: Force Diagram
6.1 MECHANICAL MODEL

The moving mass is composed of the plunger, shaft, spring and spring retainer (Figure 6.1), which are lumped since they constitute a rigid body (Figure 6.4). $F_{mu}$ and $F_{md}$ are the forces due to the upper and lower electromagnet respectively, $F_f$ is the force due to dry friction and $x$ is the air-gap. $F_{su}$ and $F_{sd}$ are the forces due to the upper and lower springs, respectively.

![Free-Body Diagram and Motion Space](image)

Figure 6.4: Free-Body Diagram (right) and Motion Space (left)
The springs are of the same stiffness, hence the forces are considered equal. The spring force is made up of the force due to motion of the plunger and the pre-compression force. Both springs are pre-compressed by an equal amount. It is necessary to have each spring in compression during the entire motion of the plunger so as to give it the ability to convert its stored potential energy into motion of the plunger.

Hence, the spring force is given as (Figure 6.4):

\[ F_{su} = F_{motion} + F_{precomp.} = K_s z - K_s x_s \]  \hspace{1cm} (6.2)
\[ F_{sd} = F_{motion} + F_{precomp.} = K_s z + K_s x_s \]  \hspace{1cm} (6.3)

Where, \( K_s \) is the spring constant, \( w \) is the width of the plunger and \( x_s \) is the length of spring pre-compression. It should be noted that the upper spring is pre-compressed in a direction opposite to that of motion and it is extended during motion upwards, while the bottom spring is compressed. Both springs will always oppose motion since there is a tendency to come back to their equilibrium positions.

Hence, the total spring force, \( F_{springs} \) is:

\[ F_{springs} = F_{su} + F_{sd} = K_s [(y - X) - x_s] + K_s [(y - X) + x_s] \]  \hspace{1cm} (6.4)
\[ F_{springs} = 2K_s z \]  \hspace{1cm} (6.5)

Where,

\[ z = y - X \text{ and } \frac{h - w}{2} = \text{const.} \]  \hspace{1cm} (6.6)
\[ 0 \leq z \leq z_{\text{max}} \text{ and } -X_{\text{max}} \leq X \leq X_{\text{max}} \]

Also, the total viscous damping force, \( F_B \) due to the springs is:
\[ F_B = F_{bu} + F_{bd} = B_u \dot{z} + B_d \ddot{z} = B\ddot{z} \]  \hspace{1cm} (6.7)

Where, \( F_{bu} \) and \( F_{bd} \) are the viscous damping forces due to the upper and lower springs respectively, \( B_u \) and \( B_d \) are the damping coefficient of the upper and lower springs respectively, and \( B \) is the lumped damping coefficient of both springs.

The plunger impacts the electromagnets at each end of travel. \( F_{bu} \) and \( F_{fd} \) are the forces due to the hard nonlinearities of the upper and lower electromagnet respectively, and are always opposing motion. They are equal since the stiffness and damping of both faces of the electromagnets are the same.

\[
F_h = \begin{cases} 
  K_u (\max(z,z_{\text{max}})) + B_{hu} \dot{z} & z \geq z_{\text{max}} \\
  K_d (\min(z,z_{\text{min}})) + B_{hd} \dot{z} & z \leq z_{\text{min}} 
\end{cases} \hspace{1cm} (6.8)
\]

Where, \( F_h \) is the force due to the hard nonlinearity, \( K \) is the stiffness of the electromagnet core face, \( B_h \) is its damping coefficient. The subscripts \( u \) and \( d \) represent the upper and lower electromagnet, respectively. These subscript notation for the upper and lower electromagnet will hold throughout this manuscript.

The differential equation describing the motion of the plunger is:

\[
M_i \ddot{z} = -2K_s z - B\ddot{z} - C \sgn(\dot{z}) - F_h - F_{md} + F_{mu} \hspace{1cm} (6.9)
\]

where, \( M_i \) is total moving lumped mass, \( C \) is the frictional force and \( \text{sign} \) is the signum function.
6.2 ELECTRICAL MODEL

The simplest form of an electromagnet is a resistor in series with an inductor. The simple relationship can be represented as:

\[ V_s = V_R + V_L = iR + \frac{d\lambda}{dt} \]  

(6.10)

Figure 6.5: Magnetization Curve Showing Relationship of current functions.
Since,
\[ V_L = L \frac{di}{dt} = N \frac{d\Phi}{dt} \quad \text{and} \quad N \frac{d\Phi}{dt} = \frac{d\lambda}{dt} \quad (6.11) \]

Where, \( V_S \) is the supply voltage, \( V_R \) and \( V_L \) are the resistive and inductive components of the supply voltage, respectively, \( \Phi \) is the flux, and \( R \) is the coil resistance. The current \( i \) is a nonlinear function of flux linkage, air-gap and the voltage across the inductor [15].

The current relationship can be described as (Figure 6.5):
\[ i = i_c + i_d = f(x, \lambda) + f_1(V_L, x) \quad (6.12) \]

Where, the air-gap, \( x \), is given as:
\[ x = y - X \quad (6.13) \]

The nonlinear functions \( f \) and \( f_1 \) are energy restoring and energy dissipation terms respectively. The component, \( i_r \), represents the fraction of current stored as coenergy in the coupling field that generates the magnetic force, and \( i_d \) is the fraction of current that is lost to hysteresis and is dissipated as heat. From Figure 6.5, the values of \( i \) and \( i_r \) differ only during transients, suggesting that hysteresis is a transient characteristics. A modified version of the function used in [15] will be used here. The dissipation current can be represented as:
\[ i_d = f_1(V_L, x) - \tau_j \frac{di_d}{dt} \quad j = 1,2,3 \quad (6.1) \]

Where,
\[ f_1(V_L, x) = \begin{cases} 
    d_1 |V_L|^{p_{12}(x)} \text{sgn}(V_L) & V_L \geq 0 \\
    d_2 |V_L|^{p_{12}(x)} \text{sgn}(V_L) & V_L < 0 
\end{cases} \]
\[ \tau_j = \begin{cases} 
    \tau_1 & V_L \geq 0 \\
    \tau_2 & V_L < 0, \quad x \leq x_v \\
    \tau_3 & V_L < 0, \quad x > x_v 
\end{cases} \quad (6.15) \]
The parameters \( d_{11}, d_{12}, d_{21}, d_{22}, \tau_j \) and \( x_v \) are to be determined experimentally.

Hence, the differential equation describing the electrical subsystem is:

\[
\frac{d\lambda}{dt} = -f_1(\lambda, x, V_L)R + V_s
\] (6.16)

### 6.3 MAGNETIC MODEL

The magnetic force produced by an electromagnet is proportional to the flux linkage and air-gap. Due to the fact that the magnetization curve of this electromagnet exhibits saturation and hysteresis (Figure 6.2), it was necessary to include these nonlinear effects in the model of the magnetic force. Also, due to the close proximity of the electromagnets (about 8mm apart), there is mutually induced flux on both electromagnets. These mutually induced flux generate opposing forces on each electromagnet.

The force of the upper electromagnet is modeled as:

\[
F_{ma} = h(\lambda_{u}, x_u) - h(\lambda_{d}, x_u)
\] (6.17)

Also, the force of the lower electromagnet is modeled as

\[
F_{md} = h(\lambda_{d}, x_d) - h(\lambda_{u}, x_d)
\] (6.18)

Where \( h(.,.) \) is some nonlinear function of several arguments. The second term on the right hand side of (6.17) and (6.18) is the force due to mutual induction. This term accounts for some of the losses that do not contribute in any useful mechanical motion of the plunger.

Thus the state dynamic equations of the EMA are:
The two electromagnets are similar, i.e. they are of the same size and rating, hence their resistances were considered to be the same. A simplified block diagram of the system dynamics for a single coil is shown in Figure 6.6.

Figure 6.6: A Simplified Block Diagram of The Nonlinear Electromagnet Model
6.4 EXPERIMENTAL CALIBRATION OF ELECTROMAGNET

The electromagnet was calibrated on the Material Testing System (MTS) machine. The MTS machine is a versatile dynamic workstation that could be configured for data acquisition and to perform various types of static and dynamic force testing. This machine was configured for calibrating the electromagnets by designing and manufacturing fixtures that enabled the electromagnets and plunger to be attached securely on the machine.

The experimental goal was obtain a nonlinear map of the coil flux field and magnetic force. This will be done by applying a known step voltage across the coil terminals and measuring the current flow and force generated by the electromagnet at different air-gaps. The data will be used in identifying parameters associated with nonlinear functions of force and current.

6.5 EXPERIMENTAL EQUIPMENT

The test platform was the MTS machine and Testar IIIs data acquisition system. The magnetic force generated by the electromagnet was measured by and MTS 5KN load cell and the data was sampled and acquired by the Testar system at 6KHz. The data was being instantaneously transferred in real time to the Dspace 1103 DSP rapid prototyping system. The Dspace board was also used to generate command voltages into a servo-amplifier. The position of the plunger was measured by a high resolution potentiometer.
integrated within the MTS machine. The current was measured by a Hall Effect current sensor and the voltage by a sensor integrated in the servo-amplifier.

6.6 EXPERIMENTAL PROCEDURE

The equipment and electromagnet were connected as shown in Figure 6.7.

1. The steel target or plunger is placed at a certain distance or air-gap from the face of the electromagnet.

2. A step voltage is applied to the coil by generating a command in Dspace.

3. The voltage, current and force are measured and the data acquired.

4. Steps 2-4 are repeated for different air-gaps.

Figure 6.7 Experimental Calibration Setup
6.7 DYNAMIC MODEL VALIDATION

6.71 Magnetic Subsystem

(a) Flux Linkage: The flux linkage is calculated using equations (6.10) and (6.11) from the experimental data. The magnetization curve for the electromagnet was plotted for different air-gaps and different voltages. The effect of increase in air-gap is to rotate the magnetization curve to the right, which means a reduction in flux linkage and force produced (Figure 6.2). The area under the curve also reduces with increase in air-gap. The area within the magnetization curve represents the energy lost in realigning the magnetic domains and it is called hysteresis loss. The effect of an increase in voltage is to lengthen the magnetization curve, thus increasing hysteresis loss (Figure 6.9).

Figure 6.8: Magnetization Curve for Different Air-gaps at 35.5 V
The flux linkage obtained from the empirical model is then plotted along with that obtained from the experimental data. This is to see how close the magnetic model matches the actual data. The experimental and simulated plots show a close match at different voltages and air-gaps, showing that saturation and hysteresis has been adequately modeled for all operating points of the electromagnet (Figures 6.8 and 6.9).

(b) **Force**: The magnetic force is produced as a result of the change in flux linkage in the magnetic field. A Least square curve fitting technique was performed on the force/flux experimental data and a good fit was obtained using a polynomial of the form:
The regression coefficients $m_2$ and $m_1$ are dependent on air-gap, so they were plotted against air-gap and the best fit was obtained using a polynomial of the form:

$$m_1 = p_{i8}x^8 + p_{i7}x^7 + p_{i6}x^6 + p_{i5}x^5 + p_{i4}x^4 + p_{i3}x^3 + p_{i2}x^2 + p_{i1}x + p_{i0}$$  

(6.21)

The regression coefficients were then used in a simulation. The data from the simulation was plotted with the experimental force data to validate the empirical model, as shown in Figure 6.10.

![Figure 6.10: Magnetic Force at Different Voltages, same Air-gap.](image)
6.72 Electrical Subsystem

(a) Current: The restoring current for each air-gap was obtained by fitting a mean magnetization curve to the already generated hysteresis curve, in the least square sense. A good fit was obtained using a polynomial of the form:

\[ i_r = g_4 \lambda^4 + g_3 \lambda^3 + g_2 \lambda^2 + g_1 \lambda + g_0 \]  

(6.22)

where, \( g_4, g_3, g_2 \) and \( g_1 \) are regression coefficients dependent on the air-gap. The effect of the coefficients of (6.22) can be represented by plotting them against the air-gap and fitting a curve to them in the least square sense. The best fit was then used in generating equation (6.23).

\[ g_i = n_{i6} \lambda^6 + n_{i5} \lambda^5 + n_{i4} \lambda^4 + n_{i3} \lambda^3 + n_{i2} \lambda^2 + n_{i1} \lambda + n_{i0} \]  

(6.23)

Combining equations (6.22) and (6.23), the restoring function could be obtained.

The dissipation current for each air-gap was estimated by first estimating the dissipation function \( f_1(V_L, x) \), and the time constant \( \tau_j \). Recall that the dissipation current is:

\[ i_d = f_1(V_L, x) - \tau_j \frac{di_L}{dt} \]  

(6.24)

Where,

\[ f_1(V_L, x) = \begin{cases} 
  d_{11}[V_L]^{d_{12}(x)} \text{sgn}(V_L) & V_L \geq 0 \\
  d_{21}[V_L]^{d_{22}(x)} \text{sgn}(V_L) & V_L < 0 
\end{cases} \]

\[ \tau_j = \begin{cases} 
  \tau_1 & V_L \geq 0 \\
  \tau_2 & V_L < 0 \quad x \leq x_v \\
  \tau_3 & V_L < 0 \quad x > x_v 
\end{cases} \]  

(6.25)

As shown in Figure (6.3), the effect of changing the air-gap is to move the magnetization curve to the right and to reduce the width of the hysteresis loop. This is evidence that displacement has an effect on the dissipation function. Using the restoring function already identified, \( i_d \) was calculated from equation (6.12) using experimental data.
Experimental data of $V_L$ was acquired and simulated in Simulink. The discontinuous function of (6.25) was used to simulate $V_L$. Arbitrary values of the parameters $d_{11}$, $d_{12}$, $d_{21}$, $d_{22}$, and $\tau_j$ were fitted to equation (6.24) and the simulation results were compared to the experimental results of $i_d$ for different air-gaps. This was done iteratively till a good fit was obtained. Hence, the parameters of equation (6.24) and (6.25) were then identified.

![Figure 6.11: Model Validation: Voltage (left), Current (right).](image-url)
Figure 6.12: Validation for Different Voltages at 0.3mm Air-gap: Voltage (left), Current (right).

Figure 6.13: Restoring and Dissipation Currents (left), Inductor Voltage (right).
These parameter values were then used in a simulation to generate $i_r$, $i_d$, $V_L$, and $i$. These were plotted along with the experimental data (Figures 6.11-6.13), and the match was very close.

6.73 Mechanical Subsystem:

From equation (6.9) the parameters to be identified from the mechanical dynamics include $M_t$, $B$, and $C$. The spring constant $K_{sp}$ was measured by placing the spring on the MTS machine and applying loads from zero to some maximum value. The force and displacement was recorded and plotted. The slope gave the value of the spring constant. An approximate value of the total moving mass, $M_t$ was calculated using the formula:

$$M_t = M_{plunger} + M_{shaft} + M_{retainers} + 2\left(\frac{2}{3}M_{spring}\right)$$

(6.26)

It was assumed that only two thirds of the mass of each spring effectively moves.

The damped period $T_d$ and the damped natural frequency $\omega_d$ of oscillation are:

$$T_d = t_3 - t_2 \quad \text{and} \quad \omega_d = \frac{2\pi}{T_d}$$

(6.27)

From [19] the logarithmic decrement (Figure 6.14), $\sigma_d$ is:

$$\sigma_d = \ln\left(\frac{x_1}{x_2}\right)$$

(6.28)

The damping ratio, $\xi$ is:

$$\xi = \frac{\sigma_d}{2\pi}$$

(6.29)

If the damping ratio is linear, it should be the same for any two successive peaks and the decay envelope is exponential. This is not the case for this system hence, an average
value was computed from the result got from all the peaks. The damping constant, B is
then estimated as:

\[ B = 2M_i \xi \omega_n = \frac{2M_i \xi \omega_d}{\sqrt{1 - \xi^2}} \] (6.30)

Figure 6.14: Experimental Data Showing Free motion (left) and Spring Calibration (right)

The oscillations stop in finite time and the response stops at a nonzero value, with an
offset, \( d \) from zero. This is because the oscillations become so small that the spring force
cannot overcome dry friction. Further, the decay envelope is linear and this indicates the
presence of dry friction. Hence, the friction force is equivalent to the spring force at a
displacement \( d \). The force due to friction, C is given as:
\[ C = K_s d \]  \hspace{3cm} (6.31)

The coefficient of friction, \( \mu \) can be further deduced from the slope, \( S \) of the decay envelope [18]. This is given as:

\[ \mu = \frac{\pi K_s}{2M g \omega_n} \]  \hspace{3cm} (6.32)

Where, \( g \) is the acceleration due to gravity.

The parameters obtained from equations (6.30) and (6.31) were used in a Simulink model of the mechanical subsystem and simulated from an initial condition. The results were plotted along with the experimental data of the vibration of the free system (Figure 6.15) and the match was close.

![Figure 6.15: Experimental Validation of Free Motion.](image)

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6.74 **EMV system:**

After validating each subsystem, the entire device was assembled and both coils were energized with an open loop voltage. The data was collected and plotted against that obtained from a Simulink model of the entire system as shown in Figure 6.16 and 6.17. There is a close match between the experimental and simulation data for current. There is an offset which increase as the plunger approaches the point of impact. The offset in the current validation is greatest at impact when the velocity is zero. This means the rate of change of current is zero as well. There is a dip in the current signal at this point (Figure 6.16). This dynamics could not be matched since it is hard to model the impact dynamics exactly.

There is a close match between the experimental and simulated data for position and velocity. The slight offset between the velocity model and the experimental data is at the point of maximum velocity and impact velocity. This was due to the fact that it is difficult to model the magnetic force and impact dynamics exactly, since the magnetic force model determines the plunger velocity. Also, the accuracy of the entire EMV model is dependent on how accurate the air-gap could be estimated, within a resolution of 0.01mm. This accuracy was difficult to obtain practically due to imperfections of the hydraulic system of the calibration machine, like backlash, hysteresis and temperature effects of the working hydraulic fluid. These affected the precision of the in built position sensor of the MTS machine.
Figure 6.16: Experimental Validation of Voltage and Current: Experimental (—), Simulation(----).

Figure 6.17: Model Validation of Position and Velocity: Experimental (—), Simulation(----).
6.8 SENSORLESS OBSERVER DESIGN FOR EMV.

6.81 Nonlinear observability theory

For linear systems, observability implies reconstructing the final states from information of the initial conditions and control. This is guaranteed if the Kalman rank condition is satisfied. In the case of nonlinear systems, observability is not enough to design an observer, because this property depends on the input of the system. For nonlinear systems, there are certain inputs for which two distinct initial conditions cannot be distinguished by using the knowledge of the available measurements. This conceptually means that the type of input selected may affect the observer gain, hence the properties of the observation error dynamics. Observability will be defined using the concept of indistinguishability [28].

Consider a nonlinear system defined as:

\[ z = f(z, u), \quad z \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \]  \hspace{1cm} (6.33)

\[ y = h(z), \quad y \in \mathbb{R}^p \]

Let \( Z_u(t, z_0) \) denote its solution at time \( t \), with initial condition \( z_0 \), at time \( t=0 \) and control \( u(t) \). Admissible inputs \( u(.) \) as assumed to be taken in some set \( U \) of measurable and bounded functions

**Definition 1.1: Indistinguishability**

A pair of initial conditions \((z_0, z'_0)\) are said to be indistinguishable by \( u \) if \( \forall t \geq 0 \), the corresponding outputs, \( h(Z_u(t, z_0)) \) and \( h(Z_u(t, z'_0)) \) are equal.

**Definition 1.2: Observability**
The nonlinear system in (6.33) is said to be observable if it does not have any indistinguishable pair of states [28].

This definition implies that a system can be observable, if there are no inputs which will render some states indistinguishable. This is because if any states are indistinguishable, they will be linearly dependent and the observability subspace will be rank deficient. Hence, it is desired to use inputs that cannot render any state indistinguishable for all combinations of pairs of initial states. These are known are universal inputs. These inputs will constrain the trajectories to remain in a given neighborhood in the observable subspace, hence, this yields a notion of local uniform observability, and is equivalent to an observability rank condition as follows [29]:

$$\text{rank} \left[ \frac{\partial (y, \dot{y}, \ldots, y^{(n-1)})}{\partial z} \quad \frac{\partial (u, \dot{u}, \ldots, u^{(n-2)})}{\partial z} \right] = n$$  \hspace{1cm} (6.34)

**Definition 1.3: Detectability**

The nonlinear system in (6.33) is said to be detectable if for every pair of initial conditions \((z_0, z_0')\) in \((\mathbb{R}^n \times \mathbb{R}^n) \times U\), there exists a \(t_0\) and \(u(.)\), for which \(\forall t \geq 0\), and the corresponding outputs, \(h(Z(t, z_0))\) and \(h(Z(t, z_0'))\) are equal, then

$$\left\| h(Z(t, z_0)) - h(Z(t, z_0')) \right\| \to 0 \quad t \to \infty$$  \hspace{1cm} (6.35)

If this is the case, the states which are indistinguishable will be unobservable, but stable, for those initial conditions and control for which they are indistinguishable, hence the system is detectable.
It can be recalled that the state dynamic equation of the EMV system is as follows:

\[ \dot{z}_1 = z_2 \]

\[ \dot{z}_2 = \frac{1}{M_t} \left( -2K_x z_1 - B z_2 - C \text{sgn}(z_2) - F_h - (h(\lambda_d, x_d) - h(\lambda_d, x_u)) + h(\lambda_u, x_u) - h(\lambda_d, x_u) \right) \]

\[ \dot{\lambda}_u = -f_{3u}(\lambda_u, x_u, V_{Lu}) R + V_{Su} \]

\[ \dot{\lambda}_d = -f_{3d}(\lambda_d, x_d, V_{Ld}) R + V_{Sd} \]

\[ y = \begin{bmatrix} y_u \\ y_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_u \\ i_d \end{bmatrix} \]  

(6.36)

Taking partial derivatives of the output of each electromagnet:

\[ \psi(z_1, V_{Su}) = \text{col} \left( y_u, \frac{\partial}{\partial z_1} y_u, \frac{\partial}{\partial z_1} y_u + \frac{\partial}{\partial V_{Su}} \left( \frac{\partial}{\partial z_1} y_u \right) \right) \]

\[ Q = \text{rank} \left( \begin{bmatrix} \frac{\partial}{\partial z_1} \psi(z_1, V_{Su}) \\ 0 \\ 0 \\ \frac{\partial}{\partial z_1} \psi(z_1, V_{Sd}) \end{bmatrix} \right) \left( z_1^0, y_u^0, i_u^0, y_d^0 \right) = 4 \]  

(6.37)

For the system to be locally observable, (6.37) should be of full rank [27] when evaluated at some point \( z_1^0, V_{Su}^0, V_{Sd}^0 \) at time \( t^0 \). Assuming (6.37) holds, \( Q = 4 \), hence, it is of full rank (the dimension of the state space is 4). The rank condition is satisfied and the EMA dynamics is \textit{locally uniformly observable} [37] for all values of time, \( t \), in a neighborhood of \( t = t^0 \).

It is assumed that only the current measurement is available and the position and velocity signals will be estimated from this measurement. An observer will be designed that will drive the estimation error of the position and the velocity to some neighborhood of zero in finite time, at which point the state estimates will approach their actual values.
6.82  Sensorless Sliding Mode Observer:

The sliding mode methodology will be used here, due to its robustness to uncertainty and ease of implementation. The observer structure for the system is (6.36) is designed as follows:

\[
\dot{z}_1 = \dot{z}_2 - M_1 \text{sign}(s_1) - M_2 \text{sign}(s_2) \\
\dot{z}_2 = \frac{1}{M_t}( -2K_s \dot{z}_1 - B\dot{z}_2 - C \text{sgn}(\dot{z}_2) - \hat{F}_h - \hat{F}_m - M_3 \text{sign}(s_1) - M_4 \text{sign}(s_2)) \\
\hat{\lambda}_u = -f_{3u}(\hat{\lambda}_u, \hat{x}_u, V_{Lu})R + V_{Su} - M_5 \text{sign}(s_1) \\
\hat{\lambda}_d = -f_{3d}(\hat{\lambda}_d, \hat{x}_d, V_{Ld})R + V_{Sd} - M_6 \text{sign}(s_2) \\
\]

(6.38)

Where the sliding manifolds are defined as:

\[
s_1 = \hat{i}_u - i_u \quad \text{and} \quad s_2 = \hat{i}_d - i_d \\
\]

(6.39)

Subtracting (6.36) from (6.38) yields the error dynamics as:

\[
\dot{\hat{z}}_1 = \hat{z}_2 - M_1 \text{sign}(s_1) - M_2 \text{sign}(s_2) \\
\dot{\hat{z}}_2 = -D + D_1 - M_3 \text{sign}(s_1) - M_4 \text{sign}(s_2) \\
\hat{\lambda}_u = -G_1 R - M_5 \text{sign}(s_1) \\
\hat{\lambda}_d = -G_2 R - M_6 \text{sign}(s_2) \\
\]

(6.40)

Where, the estimation errors are defined as:

\[
\tilde{z} = \hat{z} - z \quad \text{and} \quad \tilde{\lambda} = \hat{\lambda} - \lambda \\
\]

(6.41)

The constants, \(D, D_1, G_1, \text{ and } G_2\) are upper bounds on the nonlinear functions, defined as:
For sufficiently large gains, $M_i$ ($i=1,...,6$), sliding mode is enforced on the sliding surfaces, $s_1$ and $s_2$, convergence of the observer is attained, and the error dynamics become asymptotically stable. To study the stability of the error dynamics, a positive definite Lyapunov function is defined as:

$$V(\tilde{z}_1, \tilde{z}_2, \tilde{\lambda}_u, \tilde{\lambda}_d) = V_1(\tilde{z}_1, \tilde{z}_2) + V_2(\tilde{\lambda}_u, \tilde{\lambda}_d)$$

(6.46)

$$V_1(\tilde{z}_1, \tilde{z}_2) + V_2(\tilde{\lambda}_u, \tilde{\lambda}_d) = \frac{1}{2} \tilde{z}_1^T \tilde{z}_1 + \frac{1}{2} \tilde{z}_2^T \tilde{z}_2 + \frac{1}{2} \tilde{\lambda}_u^T \tilde{\lambda}_u + \frac{1}{2} \tilde{\lambda}_d^T \tilde{\lambda}_d$$

(6.47)

Taking the derivative of this function along solutions of the system:

$$\dot{V}_1(\tilde{z}_1, \tilde{z}_2) + \dot{V}_2(\tilde{\lambda}_u, \tilde{\lambda}_d) = \tilde{z}_1^T \dot{\tilde{z}}_1 + \tilde{z}_2^T \dot{\tilde{z}}_2 + \tilde{\lambda}_u^T \dot{\tilde{\lambda}}_u + \tilde{\lambda}_d^T \dot{\tilde{\lambda}}_d$$

(6.48)

$$\dot{V}_1(\tilde{z}_1, \tilde{z}_2) = \tilde{z}_1^T [\tilde{z}_2 - M_1 \text{sign}(s_1) - M_2 \text{sign}(s_2)] + \tilde{z}_2^T [-D + D_1 - M_3 \text{sign}(s_1) - M_4 \text{sign}(s_2)]$$

If $$M_1 > \max[\tilde{z}_2], \ M_2 > 0, \ M_3 > D, \ \text{and} \ M_4 > D_1,$$ then

$$\dot{V}_1 < 0$$

(6.49)

After some finite time, $\tilde{z}_1 \to 0$ and $\tilde{z}_2 \to 0$.

Similarly,
\[
\dot{V}_2(\tilde{\lambda}_u, \tilde{\lambda}_d) = \tilde{\lambda}_u \left[ -G_1 R - M_5 \text{sign}(s_1) \right] + \tilde{\lambda}_d \left[ -G_2 R - M_6 \text{sign}(s_2) \right] \tag{6.50}
\]

If \( M_5 \) and \( M_6 \) are selected such that:

\[
M_5 > G_1 |R|, \quad M_6 > G_2 |R|,
\]

then

\[
\dot{V}_2 < 0 \tag{6.51}
\]

Hence, if conditions (6.49) and (6.51) hold, then

\[
\dot{V}(\tilde{z}_1, \tilde{z}_2, \tilde{\lambda}_u, \tilde{\lambda}_d) < 0 \tag{6.52}
\]

The estimation errors will approach zero, and the surfaces, \( s_1=0 \) and \( s_2=0 \), become attractive and sliding mode is enforced.

### 6.9 SENSORLESS CONTROLLER DESIGN

An observer-based nonlinear controller will be designed for the nonlinear system and the position estimate will be used for feedback. The control objective is to control the impact noise and seating velocity. Noise control means reducing the impact velocity of the plunger before it impacts the upper and lower electromagnet. Seating velocity control simply means the plunger velocity is reduced to a desired value just before it contacts the face of the upper electromagnet’s core. This is also necessary for noise reduction, bounce and fatigue. The concept of controllability of the nonlinear system will be defined here.

**Controllability:** The system in (6.33) is said to be controllable from some initial state, \( z_0 \), if there exists a neighborhood, \( \Sigma \) of \( z_0 \), such that any state \( z_1 \) in \( \Sigma \) is reachable from \( z_0 \). This implies that there is a compact set of initial conditions that should be contained in the domain of attraction.
Since feedback control is not necessary during an entire cycle, except just before seating, the control strategy will be based on switching from open loop control to closed loop control just before the plunger impacts the electromagnet. This means that each actuator will have a controller assigned to it. Initially, an open loop voltage will drive the plunger from the start of travel to a predetermined point just before seating. At this instant, the feedback controller is switched on by and external command to force the estimate of the position signal to track a desired reference trajectory with a seating velocity lower than the desired seating velocity. In a real engine setting, this external command will be generated by the powertrain control unit, based on the crank angle information. The control problem is then reduced to an asymptotic regulation of the tracking error dynamics in finite time, with a bounded control signal.

6.91 Sliding Mode Controller: The dynamics to be controlled is:

\[
\dot{z}_1 = \dot{z}_2
\]

\[
\dot{z}_2 = \frac{1}{M} \left( -2K_s \dot{z}_1 - B \dot{z}_2 - C \text{sgn}(\dot{z}_2) - \dot{\hat{F}}_h - \dot{\hat{F}}_{md} + \dot{\hat{F}}_{mu} \right)
\]

\[
\dot{\lambda}_u = -f_{3u}(\lambda_u, \dot{x}_u, V_{Lu}) R + V_{Su}
\]

\[
\dot{\lambda}_d = -f_{3d}(\lambda_d, \dot{x}_d, V_{Ld}) R + V_{Sd}
\]

The tracking error for the upper electromagnet is defined as:

\[
e_1 = r_u - \dot{z}_1, \quad \dot{e}_1 = e_2, \quad \dot{\dot{e}}_1 = \ddot{e}_2 \quad (6.54)
\]

And for the lower electromagnet as:

\[
\bar{e}_1 = -(r_d - \dot{z}_1), \quad \dot{\bar{e}}_1 = \bar{e}_2, \quad \dot{\bar{e}}_3 = \ddot{\bar{e}}_2 \quad (6.55)
\]
Without loss of generality, only the tracking error dynamics of the upper electromagnet will be analyzed due to the symmetrical nature of the EMV.

The reference trajectory was chosen as to have a low seating velocity. The function used in designing the reference trajectory for the upper coil is \( r_u \):

\[
r_u(t) = -x_{\text{max}} \exp\left(-\frac{\rho_e t^2}{2}\right) + x_{\text{max}} + r_u(0)
\]  
(6.56)

The function for the reference trajectory of the lower coil, \( r_d \) was chosen as:

\[
r_d(t) = x_{\text{max}} \exp\left(-\frac{\rho_e t^2}{2}\right) - x_{\text{max}} - r_d(0)
\]  
(6.57)

The parameter \( r(0) \) is the initial condition of the reference trajectory and the constant, \( \rho_e \) is chosen to give the desired seating velocity. Large values of \( \rho_e \) increase the rate of convergence of the trajectory, hence the seating velocity.

The tracking error dynamics is defined as:

\[
\dot{e}_1 = e_2 \\
\dot{e}_2 = \frac{1}{M_t}\left[-2K_se_1 - Be_2 - C \, \text{sgn}(r_u - e_2) - F_{ms} + F_{mc} + 2K_s r_u + B_i u_u\right] + \ddot{r}_u
\]  
(6.58)

Where,

\[
\frac{d}{dt} \left(C \, \text{sgn}(\dot{r}_u - e_2)\right) \approx \frac{d}{dt} (C) = 0
\]  
(6.59)

\[
\Gamma = \frac{1}{M_t}\left[-2K_se_2 - Be_3 + 2K_s \ddot{r}_u + B_i u_u\right] + \ddot{r}_u
\]  
(6.60)

\[
B_u(\hat{z}_1) = 2m_2 \hat{\lambda}_u + m_1 \quad \text{and} \quad B_d(\hat{z}_1) = 2m_2 \hat{\lambda}_d + m_1
\]  
(6.61)
The parameters \( m_1 \) and \( m_2 \) are polynomial coefficients from (6.20). The control vectors \( B_u(\hat{z}_i) \) and \( B_d(\hat{z}_i) \) are positive and nonsingular \((m_i>0 \ \lambda>0)\) and are dependent on the air-gap estimate, hence on the position estimate. The equilibrium manifolds for the upper and lower electromagnets, respectively are defined as:

\[
\sigma_i(e_i) = c_i e_1 + c_2 e_2 + c_3 e_3
\]

(6.62)

\[
\sigma_2(\bar{e}_i) = c_4 \bar{e}_1 + c_5 \bar{e}_2 + c_6 \bar{e}_3
\]

(6.63)

The parameters, \( c_1-c_6 \) are designed to determine the desired rate of convergence of state trajectories on to the sliding surfaces. The objective is to design a control that would force the error trajectories to be oriented towards the equilibrium manifold. The conditions for the trajectories to converge on the manifolds, \( \sigma_1=0 \) and \( \sigma_2=0 \) and for sliding mode to exist in these manifolds may be derived based a Lyapunov function candidate. Defining the Lyapunov function for the upper coil controller as:

\[
V_u(\sigma_1) = \frac{1}{2} \sigma^T \sigma_1
\]

(6.64)

And for the lower coil controller as:

\[
V_d(\sigma_2) = \frac{1}{2} \sigma^T \sigma_2
\]

(6.65)

Without loss of generality, analysis will be done only for the upper coil, due to the symmetry of the EMV. When the upper coil controller is turned on \( V_{sd}=0 \), and vice versa, but \( f_{3d} \) may not be zero due to the large electrical time constant.

Differentiating (6.64) along trajectories of the system:

\[
\dot{V}_u(\sigma_1) = \sigma^T \dot{\sigma}_1 = \sigma^T (c_i \dot{e}_1 + c_2 \dot{e}_2 + c_3 \dot{e}_3)
\]

(6.66)
Since feedback control is switched on when \( x \neq 0 \) and \( f_{3d} \neq 0 \), it is assumed that \( B_d(\hat{z}_1) \) and \( B_u(\hat{z}_1) \) are nonsingular, hence control can be chosen as a discontinuous function of the error dynamics. Hence, the control is designed as:

\[
V_{s_u} = -U_u \text{sign}(\sigma_1)
\]  
(6.67)

where, \( U_u \) is a scalar gain. The derivative of the Lyapunov function becomes:

\[
\dot{V}_u(\sigma_1) = \sigma_1^T \left[ c_1 \dot{e}_1 + c_2 \dot{e}_2 + \Gamma - B_d(\hat{z}_1)Rf_{3d} - B_u(\hat{z}_1)Rf_{3u} - B_u(\hat{z}_1)U_u \text{sign}(\sigma_1) \right] \\
\leq \|\sigma_1\| \left\| c_1 \dot{e}_1 + c_2 \dot{e}_2 + \Gamma - B_d(\hat{z}_1)Rf_{3d} - B_u(\hat{z}_1)Rf_{3u} \right\| - B_u(\hat{z}_1)U_u \text{sign}(\sigma_1) 
\]  
(6.68)

If \( U_u \) is selected such that:

\[
U_u > B_u(\hat{z}_1)^{-1} \left\| c_1 \dot{e}_1 + c_2 \dot{e}_2 + \Gamma - B_d(\hat{z}_1)Rf_{3d} - B_u(\hat{z}_1)Rf_{3u} \right\| 
\]  
(6.69)

Then

\[
\sigma_1 \dot{\sigma}_1 < 0 \quad \text{and} \quad \dot{V}_u(\sigma_1) < 0
\]  
(6.70a)

Similarly, if the control of the lower coil is chosen as:

\[
V_{s_d} = -U_d \text{sign}(\sigma_2)
\]  
(6.70b)

And (6.65) is differentiated along trajectories of the system, then \( U_d \) can be selected as:

\[
U_d > B_u(\hat{z}_1)^{-1} \left\| c_1 \dot{e}_1 + c_2 \dot{e}_2 + \Gamma - B_d(\hat{z}_1)Rf_{3d} - B_u(\hat{z}_1)Rf_{3u} \right\| 
\]  
(6.71)

then

\[
\sigma_2 \dot{\sigma}_2 < 0 \quad \text{and} \quad \dot{V}_d(\sigma_2) < 0
\]  
(6.72)

If (6.69)-(6.72) hold, the trajectories will reach the surfaces in finite time (reaching condition) and sliding mode will be enforced in the system (existence condition). Hence, motion on the surfaces, \( \sigma_1 = 0 \) and \( \sigma_2 = 0 \) will be asymptotically stable. For practical stabilization of the error dynamics, the \text{sign} \ function, was replaced by \text{sat}, the saturation
function (6.73). This is to avoid the undesired finite frequency oscillations of the state vector caused by system imperfections. This is known as *chattering*. The saturation function is defined as:

\[
\text{sat}\left(\frac{\sigma}{\phi}\right) = \begin{cases} 
+1 & \frac{\sigma}{\phi} \geq +1 \\
\frac{\sigma}{\phi} & 0 < \frac{\sigma}{\phi} < +1 \\
0 & \frac{\sigma}{\phi} \leq 0 
\end{cases}
\]

(6.73)

Where, $\phi$ is a boundary layer around the manifold. System trajectories are confined to a $\phi$-vicinity of the sliding manifold $\|\sigma_1\| \leq \phi$ and $\|\sigma_2\| \leq \phi$ instead of exactly on $\sigma_1 = 0$ and $\sigma_2 = 0$ respectively, as suggested in ideal sliding mode. Hence, the trajectories in the vicinity of $\sigma_1$ and $\sigma_2$ will be uniformly ultimately bounded [36].

### 6.10 SIMULATION RESULTS

The observer in (6.38) was implemented in simulation with the validated model of the EMA, using numerical integration in Simulink. The observers converged asymptotically and the dynamics on the sliding surfaces were asymptotically stable (Figures 6.18 and Figure 6.19).
Figure 6.18: Open Loop Observer Simulation: Position and Velocity

Figure 6.19: Open Loop Observer Simulation: Current and Sliding Surfaces
The block diagram of the closed loop Simulink model is shown in Figure 6.20. The control designed in (6.67) and (6.70b) was implemented in a simulation of the validated model of the EMV. The control strategy was implemented as a combination of a closed loop and open loop control. The controller is embedded within a triggered subsystem. The open loop control accelerates the plunger inertia and the closed loop control is switched on just before the plunger contacts the electromagnets, and the plunger is held by the electromagnet.

Figure 6.20: Closed Loop Block Diagram Model of EMV
When closed loop control is switched off the plunger is released and it accelerates towards the opposing electromagnet under open loop control and the cycle is repeated. Figure 6.21 shows that the position estimate converged to its actual value and the desired trajectory was closely tracked by the position signal and its estimate. Comparing the controlled and the uncontrolled case, the seating velocity was reduced from about 0.5 m/s in the uncontrolled case, to about 0.065 m/s in the controlled case (Figure 6.22).
Less current was consumed under feedback control compared to open loop control (Figure 6.23). The average amount of current used for seating is about 11.0 Amps. Feedback was switched on at 0.09 seconds and at 0.15 seconds (Figure 6.24) for the upper and lower electromagnets, respectively. After these periods, dynamics on the surfaces converged to the origin and sliding mode was enforced on both surfaces.
Figure 6.23: Current and its Estimate in Closed Loop (Left). Current: Closed Loop (---) vs. Open Loop (----).

Figure 6.24: Closed Loop Control (Left). Sliding Surface (right)
6.11 EXPERIMENTAL RESULTS

6.11.1 *Equipment:*

The open loop and closed loop control strategy was implemented experimentally in the Dspace 1103 Digital Signal Processor (DSP) rapid prototyping system. The Simulink control block diagram was converted to C code by the Real Time Workshop code builder and downloaded on the Dspace DSP processor. The code in then internally optimized and runs on the processor at a sampling rate of 150 micro seconds. The Dspace board then generated command voltages into a servo-amplifier. The position of the plunger was measured by a high resolution laser sensor just to validate the accuracy of the position observer. The current was measured by a Hall Effect current sensor. The experimental setup is shown in Figure 6.25.

![Figure 6.25: Experimental Setup](image)
6.11.2 Open Loop Control (OL)

Figure 6.26: Open Loop Observer Validation: Position and Velocity.

To validate the observer experimentally, open loop control was implemented with only the coil currents measured. Sliding mode was enforced on both surfaces (Figure 6.27) and the current, position and velocity estimates converged to their actual values (Figures 6.26 and 6.27). The input voltage was 15V and 20 V, for the upper and lower electromagnet, respectively.
6.11.3 Closed Loop Control (CL)

(a) Noise and Seating Velocity Control: The metal-to-metal contact initiated when each electromagnet catches the plunger, causes a loud “banging” noise. Reducing the noise means slowing down the plunger before it contacts each electromagnet, i.e. at opening and at closing (seating). This was achieved by switching on each controller just before contact is made between the plunger and the face of each electromagnet. The controller forces the estimate of the position signal to track a desired trajectory with low seating velocity (Figure 6.28). Tracking was achieved but tracking becomes increasingly difficult with reduction in air-gap. Bounce is eliminated when contact is made with the lower and upper electromagnet and the seating velocity is reduced from 0.8 m/s to 0.3 m/s, as seen in the comparison of the controlled and uncontrolled cases (Figure 6.29).
Figure 6.28: Tracking Control (left) and Sliding Surfaces (right).

Figure 6.29: Open Loop (OL) vs. Closed Loop Control (CL): Lower Coil (left), Upper Coil (right).
Figure 6.30: Current: Open Loop (OL) vs. Closed Loop Control (CL) [left]. Control Voltage: Closed Loop [right].

Slightly less current was used in closed loop control than in open loop control (Figure 6.30 [left]), realizing some energy savings. Repeatability tests were carried out to see how uniform the controller and observer performance is for different runs (Figure 6.31). This was implemented by controlling both electromagnets. The seating velocity is reduced in all cases and the values range from 0.3m/s-0.05m/s. When the upper controller is switched on just before seating, the estimation error increases slightly due to a local perturbation of the observer. This is due to the discontinuity in control input to the observer when switching from open loop to closed loop control. This affects the estimated force and the position estimate, but the actual position is not affected and a low seating velocity is still obtained. Despite the excursion of the observer, good tracking is achieved and seating velocity is reduced to 0.05 m/s (Figure 6.32).
Figure 6.31: Repeatability Test: Position in Closed Loop

Figure 6.32: Repeatability Test: Desired Trajectory and Position [Left], Position [Right Top], Seating Velocity [Right bottom].
Having a low seating velocity does not depend on the asymptotic tracking of the desired trajectory or for the estimation error to be in some neighborhood of zero. The desired trajectory is designed to have a seating velocity lower than the desired seating velocity so it is a path designed for the position estimate to follow as closely as possible.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1  ELECTROMAGNETIC ACTUATOR

A control-based model of the EMA was presented and validated. An observer based sliding mode controller was designed and validated for this model. The sensorless estimation algorithm converged in closed loop and open loop control applications. Though hysteresis and saturation where not modeled, the observer based sensorless control strategy achieved almost exact regulation of the output, despite parametric uncertainties, hence it is robust.

Seating velocity was reduced to 0.04m/s and bounce was eliminated. A first order thermal model was presented based on basic thermodynamic, electrical and magnetic principles. This model was validated in real time. The coil temperature was estimated accurate enough during the transient and steady state regions of operation.

For future work, the flux linkage would be estimated by an observer instead of an integrator. It was implemented in this fashion because the estimate of position and velocity was of interest, and not flux linkage. Most importantly, it was desired to have a
7.2 ELECTROMAGNETIC VALVE ACTUATOR

A nonlinear model for the EMV was presented and validated experimentally. Transient characteristics of the EMV were taken into account in the model, including saturation, hysteresis, bounce and mutual inductance. Plunger position and velocity were estimated from current measurements and the estimates converged to their actual values asymptotically. A sliding mode based sensorless control strategy was implemented in simulation and validated experimentally. The control objective to reduce impact noise and seating velocity was obtained, with a peak and average control voltage of 30 V and 25 V, respectively. The minimum seating velocity was 0.05 m/s for an event time of 22 milliseconds (ms). The valve event time is much longer than the desired 10 ms because reduced order observer structure, to reduce the computational load on the processor. The turn around time (variable tracing time plus model execution time) in Dspace increases by about 3\( \mu s \) per additional variable. This adds up as delay in the control loop and increases the lag on the data. Hence, this puts a limit on how fast the processor can sample. In this application, it is necessary to sample at least 100 times faster than the current loop. This takes into account sampling at the Nyquist frequency (at least twice faster than the current loop) and the fact that the observer dynamics should be faster than the system dynamics (at least 6 times faster than the current loop). Due to the size of the model running on the processor, the processor could not sample faster than .0001 secs. The computational time would be reduced by coding the entire model as a C-based S-function. This of course, will increase development time.
the delay of the power amplifier running in voltage mode was 10ms. Also, a stiffer spring could have been used but this would have compromised the actuator design. This would have been time consuming since it would have meant editing the core drawings and remanufacturing the electromagnet cores.

Repeatability was acceptable and the actual position did not follow the desired trajectory exactly in every case due to noise. Also, this was because the accuracy of the model was dependent on the right estimation of the air-gap at the equilibrium position of the plunger, with none of the electromagnets energized. Hence this directly affected the accuracy of the estimated force and the state estimates. The equilibrium position is never at zero, due to the presence of residual flux on the electromagnets, though the model assumes that the plunger equilibrium position is zero. Also, the spring pre-compression displacement changes after cycling them at high frequencies. This is due to the fact that the spring rate is not constant, but a complex nonlinear function of several variables. The equilibrium position was observed to be anywhere from -0.3mm to +0.3mm. This value is crucial since an error of 0.1 mm might cause the estimated force to differ by as much as 200 N at small air-gaps. Such an offset has a considerable influence on the dynamics of the position observer just before seating. Alternatively, control could be designed based on using the current measurement for feedback control. The position estimate or measurement could be used only as a condition to switch from open loop control to closed loop control. This would reduce the model dependence on the air-gap. Also, the servo-amplifiers should be set to operate in current mode, instead of in voltage mode. This is because the amplifier dynamics in current mode is much faster and this will reduce the total delay, hence reducing the plunger rise time. However, it was much easier
to design and implement the current control strategy with the servo-amplifiers running in voltage mode. The degree of model fidelity is determined by how accurate the magnetic force and inductance or flux could be modeled. These could best be implemented as a look up table, with very fine grid, instead of a polynomial.

Also, control inaccuracies just before seating was also caused by a high rate of change of current and the control discontinuity introduced into the observer when switching from open loop to closed loop control. Implementing an observer based sensorless control means the entire model of the system is running on the processor. This adds a significant computational load on the processor and sampling rate is very much limited. It wasn’t possible to sample below 150 micro seconds.

The problem of modeling the mutual inductance could be solved by designing fixtures that could allow both electromagnets to be assembled in the configuration that they would be in service, before calibrating either of them. This eliminates mutual inductance since it will be implicitly taken into account in the experimental data. The feedback loop of the force transducer needs to be well-tuned to measure the right force profile. It is important to use a load cell that is non magnetic or shielded from the magnetic field of the electromagnets.

The hysteresis model presented in this study doesn’t account for minor hysteresis loops. The term “minor loop” is used to mean any closed B-H curve other than the fully saturated one, whether symmetrical or asymmetrical [55]. The minor loops represent losses that are due to harmonic effects of the energizing current, causing a flux reversal. Their shape is also significantly determined by non-local memories. This effect is difficult to model using analytical methods. Other theories explain the hysteresis effect as
being caused by the fact that the effective number of dipoles aligned in the direction of the applied field changes over time as domains switch under the action of an external electric field. This domain switching does not occur instantaneously, and it is this delay in response that gives rise to the hysteresis loop [56]. According to Mayergoyz [57], all hysteresis nonlinearities can be classified into two categories:

- **Hysteresis nonlinearities with local memories:** The future output path (flux linkage) depends uniquely upon the future input (electrical current) for any given output. This effect was modeled in this study.

- **Hysteresis nonlinearities with nonlocal memories:** The future output path depends not only upon the current output, and future input, but also on the past history of the input extremum values. For solenoidal actuators, this is responsible for residual flux effects. This effect was not taken into account by the hysteresis modeled presented in this study.

Because of the hysteresis effect, the response of solenoidal actuators to an applied input voltage becomes unpredictable. The hysteresis acts as an unmodeled phase lag [56] whose presence will cause instability in closed loop control if sufficient phase margin is not provided. This would degrade the tracking performance of most controllers in closed loop. The effect of hysteresis on the actuator response can be significantly reduced by closed loop control. But this necessitates modeling the hysteresis effect and incorporating the model in the controller design. In future work, it will be necessary to improve on the hysteresis model so that the size and shape of the minor loops can be predicted. This would be modeled using a numerical approach that can accumulate the past input
extremum values. Such an improvement on the model accuracy will add to the overall accuracy of the dynamic model and hence reduce model uncertainties.

The EMV referred to in this study is the first working prototype that was designed and built in-house and its performance could be enhanced if certain aspects of its design are improved. Primarily, the material of which the springs are made of should have stable dynamic characteristics. This means high tensile stress, very little plastic deformation and good thermal stability. Also, the pre-compressed springs should be in a rigid housing and any threads cut on the plunger shaft should be precision threads. Design precision and optimization using any desirable software should be performed for flux linkage, coil dimensions, wire size, plunger/shaft weight and size. This will give the best coil design that would optimize performance and energy consumption. Other magnetic materials with higher permeability and flux density should be considered since this will improve the energy density of the coil, though the cost will increase.

The major contributions of this work could be stated as follows.

- Design of an energy efficient EMV that consumes a peak power of 820W (30V, 27A)
- Modeling of the hysteresis effect to account for the effects of change in air-gap on the shape and width of the major hysteresis loop.
- Modeling of the magnetic force of solenoidal actuators to account for the effects of mutual inductance.
- Development of a thermodynamics based thermal model to predict temperature rise and resistance change in solenoidal actuators.
• Design and implementation of a nonlinear sensorless observer on the nonlinear dynamics of solenoidal actuators, using the sliding mode methodology. It should be noted that there is no unique way of designing observers for nonlinear systems.

• Design and implementation of a nonlinear sensorless controller on the nonlinear dynamics of solenoidal actuators, using the sliding mode methodology. This includes the Single Input, Single Output (SISO) and the Multi Input, Multi Output (MIMO) case. It should be noted that there is no unique way of designing controllers for nonlinear systems.
REFERENCES


APPENDIX

Figure A.1: Actuator Schematic, Showing Air-gaps, g and ga

A.1 DERIVATION OF LEAKAGE INDUCTANCE

From Figure A.2, the magnetic energy increases with $x$, the distance from the rib of the bobbin to the outermost coil, so does the leakage inductance $L_l$. The length of a turn of coil at a distance $x$ is $l_w$, with radius $r_x$. From (5.25), coenergy is given as:

$$E_c = \frac{1}{2} L_l i^2 = \frac{1}{2} \int \mu_0 H^2 dV$$  \hspace{1cm} (A1)
where \( 0 \leq x \leq b_b \)

\[
H = \frac{Ni}{l} \quad \text{for} \quad x = b_b
\]

\[
dV = l_w h_b dx \quad \text{for} \quad x = 2\pi \rho_x
\]

\[
H = \frac{Ni x}{l} \quad \text{for} \quad x \leq b_b
\]

For both halves of the bobbin
\[
\frac{1}{2} L_i i^2 = \frac{1}{2} \mu_0 \int_0^b \left( \frac{2Ni}{h_b l_w} \right)^2 dV = \frac{1}{2} \mu_0 \int_0^b \left( \frac{2Ni}{h_b l_w} \right)^2 h_b l_w x^2 dx
\]  (A2)

\[
\frac{1}{2} L_i i^2 = \frac{4}{6} \mu_0 \frac{N^2 i^2}{h_b} l_w b^2
\]  (A3)

Hence,

\[
L_i = \frac{4}{3} \mu_0 \frac{N^2}{h_b} l_w b^2
\]  (A4)

A.2 SIMULATION RESULTS

Figure A.3: Current Estimate, Sliding Surface and Control Voltage
A.3  DESIGN OF ELECTROMAGNETIC VALVE ACTUATORS

The main aim of using the EMA over the conventional camshaft is the comparative advantages that it presents in terms of reducing pumping losses due to throttling, by effectively adding an extra degree of freedom to the control of the breathing processes of the engine.

The EMA comprises two opposing electromagnets, with two pre-compressed springs, one at either end of the electromagnets, working in parallel. The moving part of the electromagnet, the plunger, is attached to the springs and valve body. When either electromagnet is not energized, the plunger is held in its equilibrium position, midway between the opposing faces of the electromagnet, with the valve half open. Energizing the upper electromagnet will attract the moving part upwards, against the spring force till it contacts the upper electromagnet. When deactivated, the electromagnet releases the plunger and the spring energy accelerates the masses downwards. The same happens if the lower electromagnet is energized instead. Hence, the electromagnets provide a linear displacement by generating a magnetic force over an air-gap. The force increases with current or flux, number of turns of coil and a reduction in air-gap, up to a certain point where it saturates. Proper design of the mechanical, electrical and magnetic system is necessary to reduce rise time, $t_r$, and energy consumption.

A.3.1  Mechanical Design: During motion, one of the springs is compressed while the other is extended. The rise time depends on the total moving mass $M_t$ and the spring stiffness, $K_s$. 

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In an ideal resonant system, \( t_r \) is half the period of oscillation, assuming there is no damping. The desired period is:

\[
T = 2\pi \left( \frac{M_t}{\sqrt{2K_s}} \right) = 2t_r
\]

(A5)

It is desired for the plunger to have a minimum period of 5ms. Hence a high spring stiffness and small mass is desired. The plunger shaft was made of aluminum and its diameter was made as small as is possible to avoid buckling. Springs of stiffness between 20-90N/mm were considered. High Spring Stiffness means faster rise time, increased moving mass. Higher magnetic forces are needed to overcome the spring force, which translates to higher voltages, bigger power supplies and an increase in cost and energy consumption. The plunger was made of mild steel, since this was the material available at that time. A thin plunger will increase \( t_r \), but this will compromise the structural integrity of the entire system since buckling may occur on impact with the electromagnet. A plunger was chosen that was thick enough to resist bending on impact, but with a mass small enough to maintain an acceptable value for \( t_r \). With the moving mass designed a spring of appropriate damping was selected to give an approximate desired rise time.

A.3.2 Electrical Design: The design of the coil, which comprises the inductor, is relevant in determining important coil characteristics, like rating, natural frequency, rise time of plunger, electrical time constant, heat dissipation and energy consumption. This reflects the total cost, performance quality and the relative ease of controlling the motion to the resonant system.
Primarily, the wire diameter or gauge is chosen. This choice is influenced by the skin depth, since a wire with a diameter less than the skin depth is desirable.

\[ d_w \leq 2\delta \] \hspace{1cm} (A6)

Where, \( d_w \) is the diameter of a round conductor and \( \delta \) is the skin depth.

This effectively minimizes eddy current loss that contributes to core temperature rise, hence increase in resistance. It was intended to limit the maximum temperature rise to 100-150\(^\circ\)C since most insulation will perform better below this range. Stranded wire or Litz wire was chosen to reduce the effects of eddy currents. This is because the wires comprising the conductor are twisted together and the direction of the eddy currents is opposite to that of the wire in proximity to it. Hence, the eddy currents cancel out. The final wire diameter was chosen to minimize the resistance and hence the current consumed, using the relationship:

\[ V_s = IR + L \frac{dI}{dt} \] \hspace{1cm} (A7)

Where \( V_s \) is the supply voltage, \( I \) is the current, \( R \) is the resistance and \( L \) is the inductance. A 12-gauge copper wire, round conductor was chosen together with a 12-gauge stranded conductor, made up 100 strands of 36-gauge wire each. The 12-gauge round conductor had less resistance per foot of wire than the Litz wire, was cheaper and more rugged. The total coil resistance cannot be too small since it will increase the electrical time constant \((L/R)\) of the coil. However, large electrical time constant increases the time delay of the system in response to an input. The increased delay diminishes the relative ease of using electronic control to influence the motion of the
resulting EMV system. Thus, the choice of the size and type of wire should be influenced by energy consumption, cost of wire, electrical time constant and ruggedness. The type of wire insulation and its thickness was chosen so as to determine the desired thermal rating of the coil. The coil insulation should be able to dissipate most of the heat that the coil will generate in a worst-case scenario without breaking down. The insulation chosen was a Class “C” polyurethane Nylon with a heavy build, rated for 200°C. The heavy build improves the scratch resistance, but will also increase the thermal resistance of the coil and prevent heat from being freely dissipated from the conductor. An extra layer of nylon surf was placed round the Litz conductor for extra ruggedness. A double or triple layer is necessary to avoid physical damage and shorting of the adjacent turns.

A.3.3 Magnetic Design: The most important components are the plunger material, core shape, core size, core material and number of conductor turns. The conductors were wound on a circular glass filled nylon core, rated at 120°C. The glass reinforcements increase the mechanical stiffness and prevent moisture absorption. The coil was then encapsulated with a high temperature electrical tape. It is important to wound the conductors very tightly to have a complete fill since air spaces will increase the leakage inductance and hence core losses. The number of turns, $N$ was approximated by the formula for the magnetic force, $F_m$ as:

$$ F_m = \frac{1}{2} N^2 \frac{d}{dx} \left( \frac{1}{\mathcal{R}} \right) I^2 $$  \hspace{1cm} (A8)

Where, $x$ is the air-gap and $\mathcal{R}$ is the total reluctance of the magnetizing path. Increasing the force means increasing the number of turns, increasing the current flow and
minimizing the total coil reluctance. The maximum current was determined by the rating of the power supply available. The total reluctance was reduced by using a magnetic material with a high permeability and relatively smaller hysteresis. The number of turns was limited by the size of the bobbin and the desired core dimensions. The maximum air-gap was fixed by the desired valve lift.

Selection of the magnetic alloy to be used for core material is influenced by several factors that affect the relative ease of using feedback control on the EMV. The factors that were taken into account are:

1. **High Saturation Induction or Flux Density**: This allows the development of strong magnetic fields for high force applications using little energy to generate a large flux density. A material with a high flux density translates to a decrease in weight and size of components.

2. **High Permeability**: This induces a high magnetic flux density per unit cross-sectional area and allows for the design of smaller and more efficient components. High permeability is ideal for low power consumption.

3. **Low Coercive Field Strength**: This permits rapid magnetization and demagnetization allowing for a high actuator bandwidth.

4. **Freedom From Magnetic Aging**: This enables the electromagnet to retain its magnetic properties for a long time.

5. **Electrical Resistivity**: A high resistivity increases coil DC resistance, increasing power loss due to heat generation.

6. **Corrosion Resistance**: Corrosion resistance is a must. Chrome core alloys are more corrosion resistant
7  *Machinability*: should be easy to machine.

8  *Cost*: It is desired to keep the cost low, without sacrificing performance.

The most important characteristics used in selecting a core material for this study were high flux density (strength), high permeability (sensitivity) and low cost. The material considered had to possess the best combination of these properties. The material selected was Carpenter Silicon Core Iron “B-FM” (Free Machinability). These are silicon alloys that offer a combination of high flux density (17500 Gaussses) and a superior magnetic permeability (5000). It costs $175 for a 4 inch diameter by 5 inch length. Usually, a material with an increased percentage of nickel, say 80% will have the highest permeability (Carpenter HyMu ‘80” Aloy:375000), while a high vanadium content, say in Hiperco 50 will increase flux density (24000 Gausses).

A coil was made of 12-gauge solid conductor copper wire, and another of Litz wire. They were each tested in cores made of mild steel and Carpenter iron respectively. This was necessary to determine the combination that is most efficient. This was done by putting each coil in turn in a mild steel core and increasing the current from zero to 10 Amps at a constant air-gap of 0.1mm. The same procedure was repeated for a constant air-gap of 2.0mm. The force was recorded at each instant and plotted against current as shown in Figure A.4.
Figure A.4: Magnetic Force of Litz wire vs. Solid Conductor (left). Magnetic Force of Iron vs. Mild Steel Core (right).

As shown, the force diagram for the coil made of a solid conductor saturated faster than that made of Litz wire. This could only mean that one coil exhibits more losses in its magnetic field with an increase in current. It was concluded that the eddy current losses are higher for the coil made of a solid conductor. The force produced when the same coil was put in a core made of silicon iron was larger, compared to that produced when it was in a core made of mild steel. The increase in force is significant for large currents and for small air-gaps (Figure A.4). Thus, material permeability is relevant for energy consumption. An EMV was assembled with mild steel core and another with an iron core.
and each was energized. The actuator with a mild steel core needed 3200W (40V and 80 Amps) for a lift of 10 mm and the force was not enough to catch the plunger. The actuator with a silicon iron core used 500W (20 V and 25A) for an 8mm lift and it had enough force to catch the plunger (Figure A.5). This demonstrates the relative importance of high core permeability and flux density as necessary characteristics for a large magnetic force output.

Figure A.5: Performance: Mild Steel Core (Left), Iron Core (right)
A.4 GLOSSARY OF SYMBOLS

\( z_1 \): position [m]

\( B_{qp} \): spring damping coefficient

\( x_s \): pre-tension distance [m]

\( z_{max} \): maximum valve lift = 6 mm

\( M \): sum total of moving mass [Kg]

\( d \): thickness of air film

\( A_a \): area of air film

\( \mu \): viscosity of air

\( F_{gas} \): intake/exhaust manifold force [N]

\( L_m \): magnetic inductance [Henries]

\( l_w \): mean turn length of conductor [m]

\( h_w \): height of wound copper conductors [m]

\( b_w \): width of wound copper conductors [m]

\( L_{tot} \): total inductance [Henries]

\( R_{th} \): total thermal resistance \([m^2.k/W]\)

\( L_{leakage} \): leakage inductance [Henries]

\( L_{circuit} \): external circuit inductance [Henries]

\( R_0 \): resistance at 25\(^\circ\)C [ohms]

\( R_{ac} \): A.C. resistance of coil [ohms]

\( R_{dc} \): D.C resistance of coil [ohms]
$k$: thermal conductivity \[\text{W/m.K}\]

$V$: coil volume \[\text{m}^3\]

$T_s$: copper conductor surface temperature \[\text{K}\]

$P_{tot}$: total heat energy released \[\text{W}\]

$g_{\text{min}}$: minimum air gap \[\text{m}\]

$K_{sp}$: stiffness of spring \[\text{N/m}\]

$K_{st}$: stiffness of hard nonlinearity or stop \[\text{N/m}\]

$R_{\text{cond}}$: thermal resistance due to conduction \[\text{m}^2\text{N/m}\]

$h_{\text{conv}}$: convective heat transfer coefficient \[\text{W/m.K}\]

$K_{\text{cu}}$: copper fill factor

$c_p$: specific heat capacity of cu at constant pressure \[386 \text{J/KgK}\]

$r_1, r_2$: inner, outer radius of effective insulation thickness \[\text{m}\]

$\rho$: density \[\text{Kg/m}^2\]

$\alpha$: resistivity temperature coefficient of copper \[0.00368 \text{Ohm/Ohm/}^{0}\text{C}\]

$\lambda$: flux linkage

$F_{\text{alin}}$: force due to hard nonlinearity \[\text{N}\]

$F_{\text{pl}}$: preload force \[\text{N}\]

$R$: resistance \[\text{ohms}\]

$\sigma$: Boltzmann constant \[5.67\times10^{-8} \text{W/m}^2\text{K}^4\]

$\varepsilon$: emissivity

$w_c$: core width

$l_c$: core face length

$\mu_c$: magnetic permeability of copper