Observational Signatures of the Macroscopic Formation of Strange Matter during Core Collapse Supernovae

DISSERTATION

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By

Juergen J. Zach, M.A.

* * * * *

The Ohio State University

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Dissertation Committee:

Richard N. Boyd, Adviser
Richard Furnstahl
Terrence Walker
Douglass Schumacher

Approved by

________________________
Adviser
Department of Physics
ABSTRACT

The consequences of a first order QCD phase transition in the protoneutronstar remnant of a core collapse supernova are presented with a special focus on the effects on neutrino transport. A secondary focus is the detection of these neutrinos in terrestrial detectors.

Hybrid stars are constructed such that a coexistence region of QCD-confined and deconfined phases forms in the protoneutronstar interior with possibly a pure deconfined phase in the center. The resulting Coulomb lattice (1D,2D and 3D) in the coexistence region is shown to crystallize for temperatures relevant in supernova cores seconds after bounce. Droplet deformation modes freeze out in the same range. For the outermost $\sim 1$ km of the coexistence region, the stability of the 3D lattice to shear stresses falls below the critical range of mechanical energy densities provided by hydrodynamical flow. This can lead to a non-spherical relief structure which, together with the enhanced neutrino opacity of the coexistence lattice, can result in anisotropic neutrino transport and therefore neutron star kicks. A computer model for neutrino diffusion coupled with quasistatic evolution of a solid lattice phase and hydrodynamical treatment of the confined matter envelope was developed to address the kick model and other problems. The state of newly formed hybrid stars is determined using a self-consistent approach of integrating the stellar structure equations with the constraint of heat flow equilibrium, resulting in relatively cool energy spheres.
(\(T \sim 1\text{ MeV}\)) compared to \(T \sim 10\text{ MeV}\) in the interior. Typical cooling timescales of hybrid stars are then \(\tau \sim 100\text{ sec}\). This is shown to result in a statistically significant signal in a Pb-neutron spallation detector. In exploratory calculations, observed kick speeds were reproduced and the presence of a sustainable convective flow pattern to maintain a crater in the coexistence region was verified.

The Pb and Fe components of a proposed neutron spallation neutrino detector concept were optimized with respect to cost-efficiency. DAMOCLES, a transport code for neutrons, capture \(\gamma\) rays and scintillation photons was developed for that purpose. The detection efficiency for liberated neutrons for the optimum configurations in both detectors is 38\%. The available sensitivity to sparse neutrino signals is \(\sim 1/(\text{sec} \times \text{kT})\) for expected radioactive background rates.
This work is dedicated to my parents Hans and Inge. And to my grandmothers Rosa (who sparked my interest in science by giving me the book “Cosmos” by Carl Sagan when I was seven years old) and Maria who both are able to read this from a better place now.
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VITA

March 16, 1974 ....................... Born - Kötzting, Bavaria, Germany

August 1996 .......................... Vordiplom Physik, Universität
                                      Würzburg, Bavaria, Germany

August 1997 .......................... M.A. in Physics, State University of
                                      New York at Buffalo, NY, USA

October 1997 - December 1999 ......... Graduate Teaching Associate, Depart-
                                      ment of Physics, The Ohio State Uni-
                                      versity, Columbus, OH, USA

January 2000 - September 2002 ........ Graduate Research Associate in the
                                      group of Professor R.N. Boyd, Depart-
                                      ment of Physics, The Ohio State Uni-
                                      versity, Columbus, OH, USA

October 2002 - present ................ Graduate Teaching Associate, Depart-
                                      ment of Physics, The Ohio State Uni-
                                      versity, Columbus, OH, USA

PUBLICATIONS

Research Publications

J.J. Zach, “Stability of the lattice formed in first-order phase transitions to matter

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Studies in:

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Prof. R.N. Boyd, advisor
Prof. G.M. Fuller, UCSD
Prof. R.N. Boyd and Prof. G.M. Fuller
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CHAPTER 1

INTRODUCTION

1.1 Astrophysical Site

Stars with masses greater than $\sim 8 - 10 \times M_\odot$ ($M_\odot = 1.989 \times 10^{30}$ kg being the mass of our sun) are believed to terminate their consecutive nuclear burning stages once an Fe core exceeding the Chandrasekhar limit $M_{ch} = 5.76 \times Y_e^2 M_\odot$ ($Y_e = n_e/(n_p + n_n)$ is the electron-to-baryon fraction) forms. At that point, (Fe, e$^-$)-capture and the disintegration of Fe by thermal $\gamma$ rays ensues, to be followed by supersonic gravitational collapse of the inner core [191]. The collapse, caused by the vanishing Fermi degeneracy pressure of captured electrons, occurs on free-fall timescales $t \sim 100$ msec and is only halted when the density exceeds the saturation density of nuclear matter, thus essentially by the degeneracy pressure furnished by nuclei [141]. The initial radius of the core is $\sim 100$ km, compared to a radius of the Chandrasekhar core of $\sim 3000$ km. Similar to a compressed bouncing-ball, the inner core rebounds from maximum central densities of up to a few times the density of nuclear matter, sending a shock wave through the still accreting outer core and into the surrounding matter [213]. Contrary to considerable initial optimism for having discovered a viable supernova explosion mechanism, it has been found later that for any realistic nuclear
equation of state (EOS), the binding energy of nuclei dissociated on the way (~8 MeV per nucleon) causes the shock to stall before it can cause the visible supernova explosion with a total mechanical energy of $E_{\text{exp}} \sim 10^{51}$ ergs [31]. A solution to this has been proposed in the form of neutrinos depositing energy into the region behind the stalled shock, thus reviving it [24]. The resulting high entropy bubble is also the most likely candidate for the r-process, thought to produce many of the isotopes beyond the mass of $^{56}$Fe [214]. Despite numerous computational studies being conducted over the past three decades [22, 213], the post-bounce evolution of the core is still one of the most intensely contested astrophysical sites, with the mechanism of core collapse, or type II, supernovae still not finally resolved. Cores beyond a certain mass limit, compressed into a Schwarzschild radius, are believed to proceed collapse to a black hole singularity [117, 191], the treatment of which is, however, not within the scope of the present work.

At least energetically, the supernova explosion is a side-show at best, as about 99% of the gravitational binding energy of the core $E_{\text{bd}} \approx 3 \times 10^{53}$ ergs is ultimately released in the form of neutrinos of all flavors ($\nu_e, \nu_\mu, \nu_\tau$) over a timescale of seconds [213]. The collapse starts to trap neutrinos at densities of $\sim 0.1 \times \rho_0$ ($\rho_0$ being the nuclear saturation density), freezing the total lepton number concentration at $Y_L = Y_e + Y_\nu \approx 0.4$ [93]. The core after bounce consists of protons, neutrons, electrons and neutrinos and is referred to as the protoneutron star (PNS). Its total mass depends on the mass of the Chandrasekhar core and thus on its chemical composition through $Y_e$, as well as on the amount of possible matter fallback from the supernova explosion [213]. However, it is believed to be in the range $1.2 - 2.0 \times M_\odot$. In the standard model of core collapse supernova theory [36, 170], the PNS deleptonizes on neutrino diffusion
timescales $\tau \sim 3$ sec and cools from an initial central temperature of a few $\sim 10$ MeV on slightly longer timescales [172], eventually resulting in a cold, deleptonized neutron star with a radius of $\approx 10$ km. This basic picture was confirmed by the observation of over a dozen neutrinos from supernova 1987-A in the Large Magellanic Cloud over $\sim 10$ sec[78, 76].

If the number density of nuclei in the interior of the PNS becomes comparable to one per the volume taken up by one nucleon, deconfinement of the constituent quarks has been predicted [173, 195]. If the associated phase transition is of first order, an extended confined/deconfined matter coexistence region will result, as both charge and baryon number can be exchanged between the phases. This will lead to the formation of a lattice of droplets/rods/slabs (for 3D/2D/1D geometry) of the respective minority phase [96]. For high enough central densities, the existence of a pure deconfined phase in the center is also possible. The term hybrid star has established itself over the past few years for this type of structure [60]. The study of such gravity-bound hybrid stars with a confined matter envelope is the subject of the present thesis. Deconfined strange quark matter is not assumed to be the true ground state of matter here. It would, however, result in self-bound (i.e., with no minimum mass) strange stars with, at most, only a thin normal matter envelope [4].

1.2 Organization of this Thesis

Chapter 2 covers the relevant equations of state for the confined (sec. 2.1) and deconfined EOS’s (sec. 2.2). For the former, the EOS’s of Lattimer/Swesty [143] and Shen et al. [192] were used, whereas for the latter, the standard [85] and the effective mass bag models [190, 188, 189] were incorporated. The Gibbs conditions for phase
equilibrium are invoked to describe the coexistence phase in sec. 2.2.4 in addition to
the pure phases (sec. 2.2.3). In sec. 2.3, the resulting EOS’s are integrated with the
stellar structure equations (sec. 2.3.1) for several EOS’s.

The possible consequences of the formation of an extended coexistence region
with free inter-phase exchange of both baryons and charge [96, 98] is discussed in
chapter 3. A structured lattice might form for a range of microphysical parameters
[47] (see sec. 3.1). The lattice might crystallize during the neutrino cooling phase
[216], which is discussed for the 3D case in sec. 3.2 and for the 1D and 2D cases in
sec. 3.4. Possible droplet deformation modes for the 3D case are discussed in sec. 3.3.

The resulting melting curves for the extended coexistence regions found in the hybrid
stars discussed in sec. 2.3 are summarized in sec. 3.5. Section 3.7 points out that the
critical shear stress for large parts of the solid lattice make it susceptible to fracture by
hydrodynamic flows discussed in sec. 3.6. Section 3.8 concludes the chapter, stressing
that the hydrodynamic paradigm of PNS evolution breaks down if the interior of a
hybrid star forms a solid lattice in an extended phase transition coexistence region.

Chapter 4 contains the relevant neutrino opacities, starting with a discussion
of the energy sphere region at densities $\sim 0.1\rho_0$ in sec. 4.1, where nucleon-nucleon
bremsstrahlung has recently been shown to play an important role [108, 177], ren-
dering the energy sphere into a fairly well-defined surface. The importance of this
for the neutrino emission spectra is pointed out in sec. 4.2. The transport opacities
in the deeper regions, however, are dominated by neutral current (NC) scattering
for $\nu_{\mu,\tau}$ neutrinos, whereas for $\nu_{e,\tilde{e}}$, charged-current (CC) neutrino capture reactions
by hadrons also play an important role [30, 183, 195], see sec. 4.3 for the confined
and sec. 4.4 for the deconfined matter phase. As it was first recognized by Reddy et
al. [181], the coexistence phase lattice provides permanent, temperature-independent scattering centers for neutrinos, see sec. 4.5, which turn out to dominate the opacity in that region, as is shown in sec. 4.8. Sections 4.6 and 4.7, respectively, summarize neutrino cooling and the diffusion approximation for neutrinos.

Chapter 5 treats the temperature and lepton number profiles after the formation of the hybrid star coexistence region. As a full hydrodynamical simulation up to the formation of the coexistence phase is not available at this point, a self-consistent approach is chosen based on chemical equilibration arguments during the phase transition and neutrino opacities in different hybrid star regions (sec. 5.1.1), as well as deleptonization timescales (sec. 5.1.2). Section 5.2 describes the self-consistent equilibrium flow procedure, together with the resulting temperature and opacity profiles.

Possible windows on the formation of a hybrid star are discussed in chapter 6. Section 6.1 starts with a summary of the numerical model used for neutrino transport in hybrid stars, together with the mechanical particularities of a normal, confined matter envelope which lends itself to a hydrodynamical description. A phase transition lattice (liquid or solid) in the coexistence phase can, once formed, extend the neutrino flux up to timescales significantly larger than those estimated for normal PNS's (see sec. 6.2). If the lattice is solid, it is able to support non-spherically symmetric structure and, given its strongly enhanced neutrino opacity, could lead to anisotropic neutrino transport (see sec. 6.3). An exploratory simulation with a 1 km deep crater in the coexistence phase is conducted in section 6.3.2, leading to neutron star kicks with magnitudes in the observed range (see sec. 6.3.1). Section 6.3.3 shows that hydrodynamical flow patterns could furnish a strong enough ordered kinetic energy density
in the crater to sustain it. The neutron star kick model presented is particularly attractive, as it does not require the extremely high magnetic fields necessary in other neutrino-driven models (sec. 6.3.4). Important information on the kick mechanism can also be gained from the relative direction of the neutron star angular momentum and the kick and the gravitational wave signal associated with an anisotropic matter and temperature distribution (sec. 6.3.5).

Chapter 7 (see also [217]) elucidates issues relevant for the terrestrial detection of supernova neutrinos using the example of a neutron spallation detector with lead or iron as targets. Section 7.1 defines the place of the OMNIS concept, which plans to make use of different spallation cross sections in different materials, within the framework of the worldwide supernova neutrino detection community. A description of the OMNIS features are given in section 7.2, followed by a detailed description of the Monte Carlo transport code DAMOCLES developed to simulate the detectors in section 7.3. The detector configurations are optimized with respect to maximum cost-efficiency and suppression of background radiation in section 7.4, whereas section 7.5 analyzes the ability of the OMNIS lead modules to detect the late-time low-temperature neutrino spectra relevant for fully formed hybrid stars. Lastly, chapter 8 summarizes the conclusions to be drawn from the present thesis and gives an outlook on future research directions to be pursued.

1.3 Progress of Physics by this Thesis

This thesis contains the first study of the mechanical properties of the lattice resulting during a first order phase transition to deconfined matter in PNS's, part of which was published in Phys. Rev. D [216] (see chapter 3). These, together with
the results for the enhanced neutrino opacity of the mixed phase lattice obtained by Reddy et al. [181] (chapter 4), are used to construct the first fully consistent computer model to simulate the evolution of a hybrid star. Its initial state upon the complete formation of the phase transition lattice is inferred (chapter 5). Possible observational signatures (chapter 6) are covered and include the first calculation of the cooling timescale of hybrid stars and a new model for neutron star kicks based on a crystallized mixed phase possibly supporting an anisotropic relief structure. The thesis concludes with the optimized design of a neutron spallation neutrino detector using a Monte Carlo particle transport code, which was recently published [217].
CHAPTER 2

EQUATION OF STATE AND STRUCTURE OF HYBRID STARS

2.1 Review: Confined Matter Equations of State

The nuclear equation of state (EOS) at densities beyond those found in nuclei has been the subject of intense experimental and theoretical investigation over the past decades. Its knowledge is important for nuclear and neutron star structure and high-density phenomena such as quark deconfinement in neutron stars and high energy ion collisions. For the transition to the deconfined quark phase and anisotropies in the large-scale energy transport in hybrid stars considered in the present study, only the bulk nuclear phase is of interest, and the transition from individual nuclei to bulk quark matter in the region below nuclear saturation density [143] need not be treated in detail. Several theoretical models were suggested and constrained by the known limits of nuclear matter properties, a detailed review of which and their interplay with neutron star structure can be found in [2, 189, 114, 113, 141, 142, 58, 106]. Due to the great variety of available models and parameter sets, no comprehensive review is attempted, rather combinations of parameters are chosen such that a mixed confined/deconfined and/or a pure deconfined phase exists in the center of the star.
Two models were chosen, both of which are well established in numerical astrophysics and easy to build into any code as subroutines. One EOS relies on a momentum-dependent potential model (Lattimer/Swesty [143]), whereas the other model (Shen et al. [192]) was constructed using the relativistic mean field (RMF) theory. Also, the present focus is on a transition to deconfined quark matter with macroscopic strange quarks. Any other forms of strange matter which might soften the equation of state prior to deconfinement and possibly compete with the formation of strange quark matter are neglected. Examples would be the extension of the Lattimer/Swesty EOS by strange hyperons [12] or the formation of a macroscopic Kaon condensate [169]. The pressure as a function of density in the EOS’s used (for a temperature of $T = 10\text{ MeV}$) is plotted in figure 2.1. An important parameter for all nuclear equations of state is the compressibility modulus at nuclear saturation density $\rho_0$, which has been predicted to vary in the range $K \sim 220 - 380\text{ MeV}$ [55]. The Shen et al. EOS is based on a value of $K = 281\text{ MeV}$, whereas $K$ is adjustable as a free parameter in the Lattimer/Swesty EOS. In order to represent the stiffer end of the possible range, a value of $K = 375\text{ MeV}$ is adopted for the latter EOS in the present study.

2.2 Review: Deconfined Quark Matter Equations of State

2.2.1 Conditions for the Formation of Quark Matter

For sufficiently high temperatures, or densities leading to high enough Fermi degeneracy pressures, asymptotic freedom will lead to quark deconfinement, the dissolution of nucleons into their constituent quarks [107, 132]. If the resulting chemical potentials of the up and down (u,d) quarks exceed the effective mass of the strange
Figure 2.1: Bulk nuclear equations of state for $Y_e = 0.05$ and $Y_e = 0.2$ after Lat-timer/Swesty (compressibility modules $K = 220$ MeV and $K = 375$ MeV) and Shen et al ($K = 281$ MeV).

(s) quark, the leptonic

$$d/s + e^+ \leftrightarrow u + \nu_e$$ \hspace{1cm} (2.1)

$$u + e^- \leftrightarrow d/s + \nu_e,$$ \hspace{1cm} (2.2)

and the nonleptonic

$$d + u \leftrightarrow s + u$$ \hspace{1cm} (2.3)

weak reactions will proceed in equilibrium, leading to quark matter with macroscopic strangeness content. It has even been conjectured [212] that this might be the absolute ground state of matter, turning any dense enough distribution of matter into strange matter. This hypothesis will not be adopted in the present work. The nature of the associated matter phase transition and the transition temperature at low densities is one of the main objectives of lattice QCD. To what extent lattice QCD
results for the low-density, high-temperature scenarios which are the subject of most theoretical investigations at this point can be applied to high-density and, relatively, low-temperature neutron star matter is not clear. There are, however, strong indications for a first order deconfinement phase transition for densities and temperatures found in PNS’s [107, 132]. The approach chosen in this work is to use a phenomenological deconfined EOS and assume the phase transition to be first order, which naturally leads to the existence of an extended confined/deconfined matter coexistence region.

2.2.2 Examples for Deconfined Equations of State

The most widely used models for deconfinement phase transitions in neutron stars are based on the MIT bag model, which describes quark confinement in nuclei with a negative pressure contribution in the deconfined EOS relative to the confined phase, the phenomenological bag constant $B_{MIT}$. Most models for deconfined quark matter are based on that basic idea, and two of them were adopted here. Both a description with non-density dependent bag constant [85] and an extended model with a density dependent bag constant, the effective mass bag model [190, 188, 189], were used, both including the one-gluon exchange term linear in the QCD fine structure constant $\alpha_{QCD}$.

The thermodynamic potentials in the former model are, as functions of the quark chemical potentials $\mu_{q=u,d,s}$:

$$\Omega_{u/d} = -\frac{\mu_{u/d}^4}{4\pi^2} \left( 1 - \frac{2\alpha_{QCD}}{\pi} \right)$$

$$\Omega_s = -\frac{1}{4\pi^2} \left[ \mu_s \sqrt{\mu_s^2 - m_s^2} \left( \mu_s^2 - \frac{5}{2} m_s^2 \right) + \frac{3}{2} m_s^4 \ln \left( \frac{\mu_s + \sqrt{\mu_s^2 - m_s^2}}{m_s} \right) \right]$$

$$-\frac{2\alpha_{QCD}}{\pi} \left[ 3 \left( \mu_s \sqrt{\mu_s^2 - m_s^2} - m_s^2 \ln \left( \frac{\mu_s + \sqrt{\mu_s^2 - m_s^2}}{\mu_s} \right) \right)^2 \right]$$

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\[-2(\mu_s^2 - m_s^2)^2 + 3m_s^4 \ln^2 \left( \frac{m_s}{\mu_s} \right) + 6 \ln \left( \frac{\mu_{\text{renorm}}}{\mu_s} \right) \left( \mu_s m_s^2 \sqrt{\mu_s^2 - m_s^2} - m_s^4 \ln \left( \frac{\mu_s + \sqrt{\mu_s^2 - m_s^2}}{m_s} \right) \right) \right] \tag{2.5}\]

where \(\mu_{\text{renorm}} = 313 \text{ MeV}\) is the renormalization scale \([85]\). For the light quarks, \(m_{u,d} \approx 0\) is assumed, whereas \(m_s \approx 100 - 250 \text{ MeV} \) \([189]\) is treated as a free parameter. From these, the equation of state is determined as

\[
\begin{align*}
n_q & = -\frac{d\Omega_q}{d\mu_q} \quad (2.6) \\
\rho & = \Sigma_q (\Omega_q + \mu_q n_q) + B_{MIT} \quad (2.7) \\
P & = \Sigma_q (n_q \frac{\delta \rho}{\delta n_q}) - \rho, \quad (2.8)
\end{align*}
\]

where \(q = (u, d, s)\), \(n_q\) is the quark number density, \(\rho\) the energy density, \(P\) the pressure and \(B_{MIT}\) the bag constant which is treated as a free parameter allowed to vary in the range \(\approx 60 \text{ MeV}/\text{fm}^3 < B_{MIT} \ll 200 \text{ MeV}/\text{fm}^3\) \([4, 189]\].

In the effective mass bag model, the quark effective mass is \([190]\]

\[
m_q^* = \frac{m_q}{2} + \sqrt{\frac{m_q^2}{4} + \frac{g^2 \mu_q^2}{6\pi^2}}, \tag{2.9}
\]

where \(\alpha_{QCD} = g^2/(4\pi)\) defines the strong coupling parameter which is used as a free parameter, but assumed to be \(g \sim 0.1\), consistent with the perturbative treatment of the effective mass. With the Fermi momentum,

\[
k_{q,F} = \sqrt{\mu_q^2 - m_q^2}, \tag{2.10}
\]

and the dispersion relation,

\[
\omega_q^*(k) = \sqrt{k^2 + m_q^*}, \quad (2.11)
\]

the equation of state is given as \([190]\]

\[
n_q = \frac{1}{\pi^2} k_{q,F}^3 \tag{2.12}
\]
\[
\epsilon(\mu_q) = \frac{3}{\pi^2} \int_{k=0}^{k_{q,F}} dk \left( k^2 (\omega_q^*(k)) \right) + B^*(\mu_q) + B_{MIT} \tag{2.13}
\]

\[
= \frac{3}{8\pi^2} \left[ \mu_q k_{q,F} (2\mu_q^2 - m_q^2) - m_q^4 \ln \frac{k_{q,F} + \mu_q}{m_q^*} \right] + B^*(\mu_q) + B_{MIT}
\]

\[
P(\mu_q) = \frac{3}{\pi^2} \int_{k=0}^{k_{q,F}} dk \left( k^2 (\mu_q - \omega_q^*(k)) \right) - B^*(\mu_q) - B_{MIT} \tag{2.14}
\]

\[
= \frac{1}{6\pi^2} \left[ \mu_q k_{q,F} (\mu_q^2 - \frac{5}{2} m_q^*) + \frac{3}{2} m_q^* \ln \frac{k_{q,F} + \mu_q}{m_q^*} \right] - B^*(\mu_q) - B_{MIT}.
\]

\(B^*(\mu_q)\) is a momentum-dependent bag term necessary to satisfy the thermodynamical requirement that \((\delta P_{\mu_q}/\delta m^*)_{\mu_q} = 0\) [190].

For finite temperatures, which in the physical environment under study are limited to \(T \sim 50\text{ MeV}\), a first-order thermal contribution with a heat capacity \(C \propto T\) was used (see sec. 4.6). Since the quark chemical potentials are \(\mu_q > M_N/3 \sim 1/3\text{ GeV}\), higher-order effects from a finite temperature on the quark chemical potentials were neglected. Electrons and \(\nu_e\)-neutrinos cannot be necessarily considered as degenerate for all temperatures. However, for a largely deleptonized neutron star with trapped lepton number concentrations at or below \(Y_L = (n_e + n_{\nu_e})/n_B \sim 0.1\) (\(n_B\) being the baryon number concentration), electron and electron neutrino degenerate pressures yield a small contribution to the total pressure and energy density in the EOS. On the other hand, as \(k_{e/\nu_e} \sim n_e^{1/3}\), the chemical potentials of electrons and electron neutrinos are still larger than thermal energies, so that the expressions for relativistic degenerate Fermi gases were used for the electron and electron neutrino sections of the EOS:

\[
\epsilon_e = \frac{\mu_e^4}{12\pi^2} \tag{2.15}
\]

\[
\mu_e = (3\pi^2 n_e)^{1/3}
\]

\[
\epsilon_{\nu_e} = \frac{\mu_{\nu_e}^4}{12\pi^2} \tag{2.16}
\]

\[
\mu_{\nu_e} = (6\pi^2 n_{\nu_e})^{1/3},
\]

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Figure 2.2: Pressure as a function of energy density for the constant bag constant model [85].

in addition to thermal contributions for degenerate matter from sec. 4.6. The equations of state for matter in \( \beta \)-equilibrium (reactions 2.1, 2.2) are plotted in figures 2.2 and 2.3 for different combinations of the bag constant \( B_{MIT} \) and the strange quark mass \( M_S \).

2.2.3 Pure Confined and Deconfined Matter Phases in Hybrid Stars

In order to integrate the stellar structure, the equation of state \( P = P(\rho) \) (\( \rho \) being the total mass energy) has to be known for all occurring densities. Both the confined and deconfined phases will be subject to \( \beta \) equilibrium

\[
\begin{align*}
\mu_n + \mu_e &= \mu_p + \mu_{\nu_e} \quad \text{(confined)} \\
\mu_d + \mu_e &= \mu_u + \mu_{\nu_e} \quad \text{(deconfined)}
\end{align*}
\]
Figure 2.3: Pressure as a function of energy density for the effective mass bag model [190].

\[ \mu_s = \mu_d \text{ (deconfined).} \]  

(2.18)

Local charge neutrality in the pure phases leads to

\[ n_p = n_e \text{ (confined)} \]  

(2.19)

\[ \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s = n_e \text{ (deconfined).} \]  

(2.20)

Core collapse initially traps a lepton number per baryon number \( Y_L \approx 0.4 \), which, over typical neutrino diffusion timescales of a few seconds, decreases below \( Y_L \approx 0.1 \). This yields the additional constraint that

\[ n_e + n_{\nu_e} = Y_L(n_n + n_p) \text{ (confined)} \]  

(2.21)

\[ n_e + n_{\nu_e} = Y_L(n_u + n_d + n_s)/3 \text{ (deconfined.)} \]  

(2.22)

Strictly, \( n_{\nu_e} \) should be written as \( n_{\nu_e} - n_{\nu_x} \), however, as the electron neutrino concentration is the same order of magnitude as the electron concentration, which results
in the electron neutrino chemical potential to be larger than typical temperatures \( \mu_{\nu_e} > T \), the electron neutrinos can be approximated as a degenerate Fermi gas and \( n_{\sigma} \approx 0 \) (exact for \( T = 0 \)).

2.2.4 Coexistence Phase in a First Order Phase Transition

The quark deconfinement transition is assumed to be first order and therefore can be described using the Gibbs equilibrium conditions for two conserved charges (representing baryon number and electric charge, as was first correctly pointed out by Glendenning [96]):

\[
\begin{align*}
\mu_n &= \mu_u + 2\mu_d \\
\mu_p &= 2\mu_u + \mu_d.
\end{align*}
\]

(2.23) (2.24)

Within the quark phase, the weak reactions 2.1 through 2.3 occur on timescales fast compared to PNS evolution times during the deleptonization or neutrino cooling phases (\( \tau \sim \) seconds), hence

\[
\begin{align*}
\mu_d &= \mu_s \\
\mu_u + \mu_e &= \mu_d + \mu_{\nu_e}.
\end{align*}
\]

(2.25) (2.26) (2.27)

The latter, together with \( \beta \)-equilibrium in the confined phase

\[
\mu_p + \mu_e = \mu_n + \mu_{\nu_e}
\]

(2.28)

and a constant \( \mu_{\nu_e} \) over length scales many orders of magnitude larger than the phase transition lattice constant, enforces a constant electron number concentration \( n_e \) in both phases.
Pressure and temperature equilibrium between the strange and non-strange phases will apply:

\[ P_S = P_{NS} \]
\[ T_S = T_{NS}, \]

as well as global charge conservation

\[ (1 - \chi) n_p - n_e + \chi \left( \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s \right) = 0, \]

and a trapped lepton number relative to the baryon number:

\[ n_e + n_{\nu_e} = Y_{ir} \left[ (1 - \chi)(n_n + n_p) + \chi/3(n_u + n_d + n_s) \right], \]

where \( \chi \) is the volume fraction of the deconfined phase.

One of the most vibrant areas of research in QCD is the possibility of the formation of a color-superconducting state [5], associated with a BCS-style [13] energy gap \( \Delta_0 \). For high enough gap energies \( \Delta_0 \) and low enough strange quark masses, this has been predicted to result in a CFL (color-flavor locked) phase, where all quark number concentrations \( n_q \) and chemical potentials \( \mu = \mu_q \) are identical [178]. Its effect on the pressure is suppressed by a factor of \( (\Delta_0/\mu)^2 \), given the approximate EOS [8]

\[ P \approx \frac{3}{4\pi^2} \mu^4 + \frac{3}{4\pi^2} \Delta^2 \mu^2 - B_{MIT}. \]

However, it has been shown to lead to a deconfinement phase transition at lower densities than without the CFL phase [8]. It might also inhibit the existence of a mixed confined/deconfined phase, as the free energy of a mixed CFL and confined phase could exceed the free energy for completely separated phases for \( \Delta_0 > m_s^2/(4\mu) \sim 30 \text{ MeV} \).
Energy gaps were predicted in the interesting range of $\Delta_0 \sim 10 - 100$ MeV [7]. For finite temperatures, BCS theory predicts an energy gap [40]

$$\Delta(T) = \Delta_0 \sqrt{1 - \left(\frac{T}{T_C}\right)^2}, \quad (2.34)$$

with a transition temperature of $T_C \approx 0.57 \times \Delta_0$, which indicates that a cooling PNS might cross the temperature boundary into the CFL locked phase, promising complex new phenomena. Since the theoretical understanding of color superconductivity is still rapidly evolving, it is not treated in the present thesis.

### 2.3 Hybrid Star Structure

#### 2.3.1 Equations of Stellar Structure

The structure of a spherically symmetric stellar object consisting of a structureless fluid or gas is uniquely determined by the central density $\rho_C$, the equation of state $P = P(\rho)$ and the relativistic hydrostatic structure equations according to Tolman, Oppenheimer and Volkoff (TOV) [191]:

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{\rho m}{r^2} \left(1 + \frac{P}{\rho}\right) \left(1 + \frac{4\pi P r^3}{m}\right) / \left(1 - \frac{2m}{r}\right), \quad (2.35)$$

in geometrized ($G_{grav} = c = 1$) units.

#### 2.3.2 Integration of the Stellar Structure Equations

The foci of the present study are anisotropies in large-scale energy transport in neutron/hybrid stars rather than neutrino spectra formation or surface structure. Since the density profile of a neutron star is particularly steep near its surface, the hybrid star radius is defined as the distance from the center to a density cutoff point
Figure 2.4: Schematic drawing of the structure of a hybrid star with confined matter envelope, confined/deconfined matter coexistence region and (possibly) a pure deconfined matter core.

where the mass energy density is equal to the saturation density of symmetric nuclear matter, $\rho_0 = 0.155 \text{ fm}^{-3}$. The resulting hybrid star radii of $R_s \sim 9 - 12 \text{ km}$ and masses $M \sim 1.3 - 1.8 \times M_\odot$ are well within observational limits [203, 202, 141, 142] and could, even if accurately measured, not be used in themselves to distinguish the hybrid star from a neutron star without deconfined quark matter. The solution of the equations of state described in sec. 2.2.3 and 2.2.4 and the subsequent integration is performed numerically for different equations of state and central densities, such that a hybrid star with a mixed confined/deconfined region and, possibly, a pure quark core results, as schematically shown in figure 2.4.

All EOS’s used in this thesis are summarized in table 2.1. The resulting density and pressure profiles for cold ($T = 1 \text{ MeV}$) and deleptonized hybrid stars are shown
in figures 2.5 and 2.6, respectively. The small discontinuities in some density profiles are due to an unknown numerical error in the EOS of Lattimer and Swesty. The figures include the profiles for EOS 1-5 for $\rho_C = 1000 \text{ MeV/fm}^3$ and, for comparison, the profiles for EOS 1 and 4 for $\rho_C = 700 \text{ MeV/fm}^3$. The lower enclosed masses for the lower central density leads to a slightly larger radius, as is also observed in normal neutron stars for most nuclear EOS’s. Increasing either the bag constant or the strange quark mass slightly above the values used in EOS’s 1-4 ($B_{MIT}, M_s = (130 \text{ MeV/fm}^3, 150 \text{ MeV})$) will prevent the formation of a pure deconfined phase at the center for $\rho_C = 1000 \text{ MeV/fm}^3$, as can be seen from the lack of an inflection point in the modified density profiles for EOS 1 in figure 2.7, as opposed to the sudden change in the slope in the original curve at a radius of $\sim 1.5 \text{ km}$. Figures 2.8-2.11 show the chemical composition of hybrid stars with $\rho_C = 1000 \text{ MeV/fm}^3$ for cold ($T = 1 \text{ MeV}$) EOS’s 1-4. The formation of an extended coexistence phase is fairly independent upon the exact EOS used, as long as appropriate values are chosen for ($B_{MIT}, M_s$). For probable central temperatures after hybrid star formation of

<table>
<thead>
<tr>
<th>EOS</th>
<th>confined EOS</th>
<th>deconfined EOS</th>
<th>$B_{MIT} (\text{MeV/fm}^3)$</th>
<th>$M_s (\text{MeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOS 1</td>
<td>Lattimer/Swesty</td>
<td>standard bag model</td>
<td>130</td>
<td>150</td>
</tr>
<tr>
<td>EOS 2</td>
<td>Shen et al.</td>
<td>standard bag model</td>
<td>130</td>
<td>150</td>
</tr>
<tr>
<td>EOS 3</td>
<td>Lattimer/Swesty</td>
<td>eff. mass bag model</td>
<td>130</td>
<td>150</td>
</tr>
<tr>
<td>EOS 4</td>
<td>Shen et al.</td>
<td>eff. mass bag model</td>
<td>130</td>
<td>150</td>
</tr>
<tr>
<td>EOS 5</td>
<td>Lattimer/Swesty</td>
<td>standard bag model</td>
<td>100</td>
<td>170</td>
</tr>
<tr>
<td>EOS 6</td>
<td>Lattimer/Swesty</td>
<td>eff. mass bag model</td>
<td>100</td>
<td>170</td>
</tr>
</tbody>
</table>
Figure 2.5: Mass density in MeV/fm$^3$ versus radius in m for $\rho_C = 1000$ MeV/fm$^3$ with the EOS’s 1-5 and $\rho_C = 700$ MeV/fm$^3$ with the EOS’s 1 and 4 (see table 2.1).

At $T \sim 50$ MeV, the thermal energy contribution will push the pure deconfined phase beyond the maximum central density assumed here (see chapter 5).
Figure 2.6: Pressure in MeV/fm³ versus radius in m for $\rho_C = 1000$ MeV/fm³ with the EOS's 1-5 and $\rho_C = 700$ MeV/fm³ with the EOS's 1 and 4 (see table 2.1).

Figure 2.7: Mass density in MeV/fm³ versus radius in m for $\rho_C = 1000$ MeV/fm³ with EOS 1 and $(B_{MIT}, M_s) = (130$ MeV/fm³, 150 MeV), (150 MeV/fm³, 150 MeV) and (130 MeV/fm³, 170 MeV).
Figure 2.8: Number densities of constituents in fm$^{-3}$ and strange volume fraction $\chi$ versus radius for $\rho_C = 1000$ MeV/fm$^3$. EOS: standard bag model for deconfined, Lattimer/Swesty EOS for confined phase (EOS 1 in table 2.1).

Figure 2.9: Number densities of constituents in fm$^{-3}$ and strange volume fraction $\chi$ versus radius for $\rho_C = 1000$ MeV/fm$^3$. EOS: standard bag model for deconfined, Shen et al. EOS for confined phase (EOS 2 in table 2.1).
Figure 2.10: Number densities of constituents in fm$^{-3}$ and strange volume fraction $\chi$ versus radius for $\rho_C = 1000$ MeV/fm$^3$. EOS: effective mass bag model for deconfined phase, Lattimer/Swesty EOS for confined phase (EOS 3 in table 2.1).

Figure 2.11: Number densities of constituents in fm$^{-3}$ and strange volume fraction $\chi$ versus radius for $\rho_C = 1000$ MeV/fm$^3$. EOS: effective mass bag model for deconfined phase, Shen et al. EOS for confined phase (EOS 4 in table 2.1).
CHAPTER 3

MECHANICAL PROPERTIES OF HYBRID STAR MATTER

3.1 Formation of a Coulomb Lattice in the Coexistence Phase

As I assume the quark deconfinement phase transition to be first order, the exchange of both charge and baryon number is allowed between the confined and deconfined phases in regions where they coexist. The global rather than phase-localized nature of charge conservation was only realized as late as 1992 [96], whereas up to then, both phases had been assumed to be charge-neutral (see, e.g., [16, 23]). This - essentially artificial - constraint predicts, using the MIT bag model for the deconfined phase, densities for the phase transition higher than even the most optimistic estimates for the center of a neutron star. Due to the hydrostatic stability condition for stellar structure $\delta P/\delta R < 0$ [191], a first-order phase transition would then lead to the spatial separation of both phases, with the higher-$\rho$ deconfined phase descending to smaller radii $R$ and a density discontinuity at some radius. However, charge exchange will result in a lattice with a geometric structure which is determined by the strange volume fraction $\chi$, as the minimum of the sum of Coulomb and surface
energy of a unit cell [97]:

\[
\frac{E}{V} = \frac{E_C}{V} + \frac{E_S}{V} = C(\chi) r^2 + \frac{S(\chi)}{r},
\]  

(3.1)

leading to the condition \(E_S = 2E_C\). The bulk approximation used here is only strictly valid for length scales equal to or smaller than the screening lengths of the hadrons exchanged between the phases. If the deconfined phase is the minority phase, its absolute charge density will be higher because the majority phase can significantly lower its isospin by pushing negative charge into the minority phase [47], more than compensating for the opposing effect of a higher repulsive Coulomb energy within the latter. The reverse is true for the deepest layers of the mixed phase, where the normal PNS matter is the minority phase and a high positive charge density resides in that phase, whereas the Coulomb interaction is compensated by the condensation energy of hadrons. The minimum energy geometry for the minority phase changes from droplets to rods to platelets as the minority phase volume fraction is varied from zero to 1/2. The resulting lattice constant is for a given lattice dimensionality \(d\) [98, 180]:

\[
a = \left( \frac{\sigma \varepsilon_0}{(\Delta \rho_C)^2 f_d(\chi)} \right)^{1/3},
\]  

(3.2)

where \(\Delta \rho_C\) is the net charge density of the minority phase and

\[
f_{1,3}(\chi) = \frac{1}{d+2} \left( \frac{1}{d-2} (2 - d \chi^{1-2/d} + \chi) \right)
\]

\[
f_2(\chi) = \frac{1}{4} (\chi - 1 - \ln \chi).
\]  

(3.3)

The geometric dimension \(d\) the lattice will assume is determined by the minimum of the lattice energy density

\[
\left( \frac{E}{V} \right)_d = 6\pi \chi \left[ \sigma^2 d^2 (\Delta \rho_C)^2 f_d(\chi)/(16\pi^2) \right]^{1/3}.
\]  

(3.4)
Although the exact boundary values between different lattice dimensions depend on the EOS parameters, 3D geometry is given for a minority volume fraction of $\chi < 1/5$ ($\chi > 4/5$), 2D geometry between that and $\chi = 1/3$ ($\chi = 2/3$), and 1D geometry around $\chi = 1/2$. The microphysics to accurately describe the interface between the two phases is not well known at this point. An important quantity to determine the lattice geometry is the surface tension $\sigma$ between confined and deconfined quark matter. It can be estimated to lie in the range $10 \text{ MeV/fm}^2 < \sigma < 100 \text{ MeV/fm}^2$ [85, 21, 115]. A more accurate prediction of $\sigma$, by nature a phenomenological quantity, would require a deeper knowledge of the strong interaction on all energy scales. In the present study, $\sigma = 50 \text{ MeV/fm}^2$ is used unless specified otherwise. Screening effects, which become relevant for droplet radii larger than the quark screening length ($\lambda_q \sim 5 \text{ fm}$, see [115, 112]), have so far not been fully accounted for in a self-consistent manner. As was pointed out by [210] (see also [157], which treats a first order Kaon condensate; however, the discussion of screening effects qualitatively applies to deconfined quark matter as well), screening effects lead to opposite charges accumulating at the phase boundary, raising the effective surface tension and thus favoring larger droplets. Further, higher order energy terms beyond the surface tension are not known to any accuracy, but would be desirable in the future for their possible effects on the conditions for the phase transition and the resulting lattice structure [47]. Hence, any charge densities given are to be understood as effective values, already taking into account screening effects.
3.2 Melting Behavior of the 3D - Phase Transition Lattice [216]

As a 3D lattice only results for $\chi < 1/5$, due to global charge conservation the minority phase at zero temperature will be a solid Coulomb lattice of minority (negative deconfined or positive confined) charges immersed in an oppositely charged background with relatively small charge density. Assuming rigid, structureless minority phase droplets, the lattice can then be regarded as a one-component plasma (OCP). Its treatment in the harmonic approximation is well-established [147, 3, 45, 44], and the thermal deviation of the droplets from their equilibrium lattice sites can be calculated analytically. The equation of motion of the $\alpha$-component of the displacement $u$ on a lattice site $l$ for a Bravais lattice (defined as having one particle per unit cell) can be written as

$$M \ddot{u}_\alpha(l) = -\frac{\delta \Phi}{\delta u_\alpha(l)} = -\sum_{\beta} \frac{\delta^2 \Phi}{\delta u_\alpha(l) \delta u_\beta(l')} u_\beta(l'),$$  \hspace{1cm} (3.5)

where $\Phi$ is the electrostatic potential and the displacement amplitude $u_\alpha$ is

$$u_\alpha = \sqrt{\frac{\hbar}{2NM}} \sum_{\tilde{k}, j} \frac{e_\alpha(\tilde{k}_j)}{\omega(\tilde{k})} e^{i\tilde{k}_j(l)} A_{\tilde{k}_j},$$  \hspace{1cm} (3.6)

with $A_{\tilde{k}_j} = a_{-\tilde{k}_j}^\dagger + a_{\tilde{k}_j}$, where $a_{-\tilde{k}_j}^\dagger$ and $a_{\tilde{k}_j}$ are the phonon creation- and anti-phonon annihilation operators, respectively. The three characteristic polarization modes, two transverse and one longitudinal, of the Bravais lattice are denoted by the index $j$, completely defining the unit vector $e_\alpha(\tilde{k}_j)$ for a given wave vector $\tilde{k}$. $N$ is the droplet number, $M$ the droplet mass and $\omega_j(\tilde{k})$ the dispersion relation.
In this formalism, the mean square displacement relative to the distance between nearest neighbors \( d = (3\pi^2)^{1/6}a \) for a BCC (body-centered cubic) lattice is

\[
\frac{\langle u^2 \rangle}{d^2} = \frac{\hbar}{d^2} \frac{2M}{\sum_{k_j} \coth(\beta \hbar \omega_j(k))}{\omega_j(k)}
\]

\[
= \frac{3\hbar}{d^2} \frac{2M}{\alpha \omega_p} \left( 1 + \frac{4}{\alpha \eta} D_1(\alpha \eta) \right),
\]

(3.7)

where \( \eta = \hbar \omega_p/k_B T \) is the degeneracy parameter and the Debye integral is defined as

\[
D_n(x) = \frac{n}{x^n} \int_0^x dt \left( \frac{t^n}{e^t - 1} \right).
\]

(3.8)

The dispersion relation for the transverse modes has the acoustic Debye form [45, 44]

\[
\omega_T(k) = \alpha \omega_p \frac{k}{k_D}
\]

(3.9)

with the Debye wavenumber \( k_D = (6\pi^2 N/V)^{1/3} \), the plasma frequency \( \omega_p = (Ze/\epsilon_0 \times N/MV)^{1/2} \) and a constant \( \alpha = 0.393 \), which was obtained from the Monte Carlo calculations of the excitation spectrum in the classical limit [45, 44]. For the longitudinal branch, the Einstein model has been suggested with a constant frequency of \( \omega_L \propto \omega_p \) [45]. However, for a cubic lattice in the harmonic approximation, the symmetry condition \( \langle u^2 \rangle = 3 \times \langle u_T^2 \rangle = 3 \times \langle u_T^2 \rangle \) [147] makes only one branch necessary to calculate the average square displacement amplitude. For typical lattice constants of \( a \approx 10 \text{ fm} \), we obtain \( k_D \approx 0.4 \text{ fm}^{-1} \) and \( \hbar \omega_p \approx 5.8 \text{ MeV} \). Typical values for the strange (minority) phase were used for the latter, a charge density of \( \rho_C = 0.4 \text{ fm}^{-3} \), a mass density of \( \rho_M = 0.4 \text{ fm}^{-3} \) and a droplet radius of \( R = 3.0 \text{ fm} \). The plasma frequency is an important quantity characterizing the lattice, because \( \eta \) determines the role quantum effects play and, ultimately, the freeze-out of the OCP. In the present case, for “typical” protoneutronstar evolution temperatures, \( T \sim 10 \text{ MeV} \), we get for
the degeneracy parameter $\eta = (\hbar Z e)/(\sqrt{M\epsilon_0 k_B T a^3/2}) \sim 0.5$. The problem at hand can, a priori, therefore be treated neither in the zero temperature- (quantum-) nor in the classical limit.

The Lindemmann parameter $\gamma^2$ is defined as the value of the quantity $\langle u^2 \rangle / d^2$ at the solid-liquid transition. For an OCP, it has been determined using Monte Carlo simulations in both the classical (high temperature) limit $[167, 159, 197]$ and in the quantum case (zero temperature) $[43]$, for both fermions and bosons. The droplets are macroscopic systems in that they contain at least dozens of charges, and individual intra-droplet spin interactions are much stronger than the interaction between, say, two proton spins in different minority droplets. Energetically, spin pairing effects on the droplets will therefore drive their total spin to zero, which motivated the present treatment as bosons. However, when the Lindemmann criterion for fermions was used, the results only differed significantly for coexistence curves with very low transition temperatures below 1 MeV. For the intermediate degeneracies $\eta \sim 1$ given here, the interpolation formula for the Lindemmann parameter of a bosonic OCP by Chabrier $[44]$ is used:

$$\gamma(\eta) = \gamma_0 - \frac{0.096 + 4.31 \times 10^{-3} \eta^2}{1 + 0.05 \eta^2 + 2.092 \times 10^{-4} \eta^4},$$

(3.10)

with $\gamma_0 = 0.249$ being the quantum limit.

The melting curve for a surface tension of $\sigma = 50$ MeV/fm$^2$ and typical relative charge densities $\Delta \rho_C$ between 0.1 fm$^{-3}$ and 0.6 fm$^{-3}$ is shown in figure 3.1. Remarkable are the decreasing melting temperatures with increasing charge densities, which, however, are also observed for the 1D - and 2D - geometries. This can be understood such that in plasma theory, melting is usually observed to occur around a critical
Figure 3.1: Melting curve for the 3D case for different droplet charge densities: $\rho_C = 0.1 \text{ fm}^{-3}$ (top curve) to $\rho_C = 0.6 \text{ fm}^{-3}$ (bottom curve) with $\rho = 0.5 \text{ fm}^{-3}$. The 3D lattice is only the minimum energy configuration for $\chi < 1/5$ (or $\chi > 4/5$).

The coupling constant

$$\Gamma_c = \frac{q^2}{4\pi\epsilon_0 a} / kT_c = \text{const.},$$

(3.11)

where the droplet charges are

$$q = \frac{4}{3} \pi r_d^3 \times \Delta \rho_C.$$  

(3.12)

For a given $\chi$, the droplet radius $r_d \propto a$ and, using eq. 3.2, $r_d \propto (\Delta \rho_C)^{-2/3}$, so that $q \propto 1/\rho_C$ and hence, from eq. 3.11, the melting temperature $T_c \propto (\Delta \rho_C)^{-4/3}$. It can be seen that an initially liquid phase transition lattice crystallizes for PNS temperatures of $T \sim 10 \text{ MeV}$, which lies well within the temperature range through which PNS's cool via neutrino emission in the standard PNS paradigm.

### 3.3 Droplet Deformation Modes

The OCP assumes inherently rigid point charges in a level sea of constant charge background. However, since the surface tension is only $\sigma \sim 10 - 100 \text{ MeV/fm}^2$, small
compared to strong interaction energy scales $\epsilon_{\text{strong}} \approx 10^9 \text{MeV/fm}^3$ over strong interaction length scales of $\sim \text{fm}$, it is clear that the minority phase droplets cannot necessarily be considered as rigid. It is therefore important to know whether deformation modes have to be taken into consideration in the treatment of lattice vibrations.

Consider a droplet of strange matter which is slightly elongated along the x-direction to $R + dR$, yielding an ellipsoid:

$$(a, b, b) \sim \left( R + dR, R/\sqrt{1 + \frac{dR}{R}}, R/\sqrt{1 + \frac{dR}{R}} \right).$$

(3.13)

Its surface area is

$$S = 2\pi \left( b^2 + \frac{a^2b}{\sqrt{a^2 - b^2}} \arcsin\left(\frac{a^2 - b^2}{a^2}\right) \right),$$

(3.14)

which, when expanded to second order in $dR$, yields

$$S \simeq 4\pi R^2 + 2\pi \frac{9}{8} dR^2 = S_0 + \Delta S,$$

(3.15)

from which the elastic constant $k_S$ for the deformation energy can be deduced:

$$\Delta E_S = \sigma \Delta S = 2\pi \frac{9}{8} \sigma dR^2 = \frac{1}{2} k_S dR^2.$$

(3.16)

The inertial term $m_S$ can be found via the kinetic energy

$$\int_{-R}^{R} dx \int_{0}^{\sqrt{R^2 - x^2}} d\rho (2\pi \rho \left| \frac{1}{2} \rho M x^2 \left( \frac{\omega_S}{2\pi} \right)^2 \right|)$$

$$= \frac{1}{30\pi} \rho M \omega_S^2 R^5 = \frac{1}{2} \left( \frac{4\pi \rho M R^3}{15} \right) \dot{R}^2 = \frac{1}{2} m_S \dot{R}^2.$$  

(3.17)

The characteristic vibration energy can therefore be estimated as

$$\omega_S = \sqrt{\frac{k_S}{m_S}} = \sqrt{\frac{9\pi\sigma/2}{M/5}}.$$  

(3.18)

A typical value, for $\rho_M = 0.4 \text{fm}^{-3}$, $R = 3 \text{fm}$ and $\sigma = 50 \text{MeVfm}^{-2}$, is $\hbar \omega_S \sim 20 \text{MeV}$, which is comparable to the plasma frequency. Hence, deformation modes
freeze out at about the same temperature as lattice vibrations. The OCP can therefore be considered as a valid description of the melting curve of the phase transition lattice, since no other degrees of freedom are relevant once it becomes a crystal. At temperatures above the transition between a liquid and a solid droplet phase, the lattice- and deformation modes will be in thermal equilibrium.

3.4 Melting Behavior of the 1D and 2D - Phase Transition Lattices

Liquid-solid phase transitions in 1D and 2D are fundamentally different from the 3D case. True long-range order is limited to three dimensions, as the equivalent for the lattice sum in eq. 3.5 would be divergent for the lower-D cases. However, field theoretical studies [207] and earlier studies of the 2D OCP [186, 187] predict the existence of a crystalline phase at low temperatures. For two dimensions, the Kosterlitz-Thouless-Halperin-Nelson-Young (KTHNY) theory (see, e.g., [196]) predicts a phase transition in two stages. Cooling a disordered 2D lattice down, quasi-orientational order obtains at a temperature $T = T_{c2}$, in the intermediate “hexatic” phase, described by algebraic decay of orientational order. After further cooling, the solid phase with quasi-translational ordering and long-range orientational ordering is reached at $T = T_{c1}$, where the final recombination of dislocation pairs occurs. The latter temperature is given by [155]:

$$kT_{c1} = \frac{a^2 \mu_L (\mu_L + \lambda_L)}{4\pi (2\mu_L + \lambda_L)},$$

(3.19)

where $\mu_L$ and $\lambda_L$ are the Lamé elastic constants for a given charge density. Monte Carlo studies predict a critical coupling constant for melting in the range $\Gamma_c = (q^2/4\pi\varepsilon_0 l)/kT_{c1} \approx 130 - 200$ [86, 145], where $q$ is the charge on the rods and $l$
Figure 3.2: Melting curve for the 2D case for different droplet charge densities: $\rho_C = 0.1 \text{fm}^{-3}$ (top curve) to $\rho_C = 0.6 \text{fm}^{-3}$ (bottom curve) with $\rho = 0.5 \text{fm}^{-3}$. The 2D lattice is only the minimum energy configuration for $1/5 < \chi < 1/3$ (or $4/5 > \chi > 2/3$).

their length. In the present study, I will adopt a critical value of $\Gamma_c = 150$. Since $q \sim l$, $T_{c1} \sim l^{-1}$, which will prevent crystallization of long rods and therefore lead to intra-rod collisions, the rod length will be limited to a value commensurate with a solid crystal. In the present, an upper limit of $l = 10 \times r_d$ is assumed. The resulting melting curves for a 2D phase transition lattice are plotted in figure 3.2.

For the 1D case, the restrictions on long-range order are even more severe and no solid phase with quasi-translational ordering exists [127]. It has recently been suggested that 1D-melting might be possible if it occurs along the slip line of a 2D crystal [127], resulting in effectively 2D melting with a critical temperature determined by equation 3.19. This slip line might be associated with grain boundaries between solid 1D zones in the appropriate region of the hybrid star. Since no further analytical or numerical analyses of such melting behavior has been conducted to date, I will assume the same critical coupling constant as in the 2D-case, hence
Figure 3.3: Melting curve for the 1D case for different droplet charge densities: $\rho_C = 0.1 \text{ fm}^{-3}$ (top curve) to $\rho_C = 0.6 \text{ fm}^{-3}$ (bottom curve) with $\rho = 0.5 \text{ fm}^{-3}$. The 1D lattice is only the minimum energy configuration for $\chi > 1/3$ (or $\chi < 2/3$).

$\Gamma_c = q^2 a / (2 A \epsilon_0 k T_c) \approx 150$ (where $A$ is the area of the platelets $A = \pi l^2$ and $l = 10 r_d$ is assumed, based on the same argument as in the 2D-case). Examples of resulting melting curves are shown in figure 3.3.

3.5 Results: Melting Curves for the Hybrid Star Coexistence Phase

The resulting melting temperature, together with the temperature profile as a function of radius is shown in figures 3.4 and 3.5 (based on hybrid stars with $\rho_C = 1000 \text{ MeV/fm}^3$ for the EOS’s 1 and 4 discussed in sec. 2 and $\sigma = 20/50/100 \text{ MeV/fm}^2$). The temperature profile is obtained with $T = 2 \text{ MeV}$ as neutron star surface temperature and the condition of heat flow equilibrium, as outlined in 5.2. The vertical solid lines mark the boundaries between the different lattice geometries (1D,2D,3D) for the case $\sigma = 50 \text{ MeV/fm}^2$. Different surface tensions lead to different lattice constants which modify the elastic scattering of neutrinos off the droplets (see sec. 4.5)
Figure 3.4: Melting temperature of the phase transition lattice for different lattice dimensions (1D, 2D, 3D) following sec. 3.4, 3.2 and temperature profile for EOS 1 (sec. 2). $\sigma = 20/50/100$ MeV/fm$^2$ (from below).

and hence stellar structure under the condition of equilibrium heat flow in the star. Under the conditions shown, the coexistence phase will be a solid lattice for surface tensions above $\sigma \sim 50$ MeV/fm$^2$ (with the exception of the inner regions for the case of EOS 4).

3.6 Sources of Mechanical Stress in Hybrid Stars

Besides thermodynamic criteria for the existence of a crystalline mixed phase, it is important to know how the lattice will behave under typical shear stresses present. The most important sources of shear stresses are convection and differential rotation.

A negative gradient in the lepton concentration has been shown to lead to convection during the Kelvin-Helmholtz cooling phase of a PNS [62]. More recently,
Figure 3.5: Melting temperature of the phase transition lattice for different lattice dimensions (1D,2D,3D) following sec. 3.4, 3.2 and temperature profile for EOS 4 (sec. 2). $\sigma = 20/50/100 \text{ MeV/fm}^2$ (from below).

Hydrodynamics simulations including convection indicate that the Ledoux criterion for convective instability

$$C_L \equiv \left( \frac{\delta \rho}{\delta S} \right)_{p,\nu_l} \frac{dS}{dr} + \left( \frac{\delta \rho}{\delta Y_l} \right)_{p,\nu_l} \frac{dY_l}{dr} > 0$$

(3.20)

is true in most of the PNS for times of more than $\sim 1$ s after bounce [124]. In a possible pure deconfined phase in the center of a hybrid star, any strong negative lepton or entropy gradients will be stratified. This is due to the thick (several km), relatively neutrino-opaque mixed phase (see chapter 4) enclosing it and the fact that the transport of both heat and lepton number in the interior is dominated by neutrinos. If the coexistence phase is a solid lattice, hydrodynamic flow will be inhibited there, as long as it is not fractured by shear stresses. Hence, the most violent convection will take place in the matter exterior to the coexistence phase. Although various authors
disagree on the extent and strength of convection in PNS’s, convective velocities of $v_c \sim 10^6 \text{ms}^{-1}$ are reported in many studies [124, 32, 116]. Given similar temperature and lepton number gradients in hybrid stars compared to “normal” PNS’s, large scale convective flows in the same velocity range can be expected, which is equivalent to an ordered kinetic energy density of $E_{\text{conv}}/V \sim 10^{-3}\text{MeV/fm}^3$.

The discovery of a number of millisecond pulsars in recent years [52] indicates that some of the angular momentum residing in the core collapse supernova might remain in the PNS. The resulting rotation is likely to be differential, at least initially, and has been studied by Goussard et al.[101, 102] who solved the relativistic stellar structure equations for rotating PNS’s and a simplified equation of state ($p \propto \rho^\Gamma$) for the different epochs in the PNS evolution. The rotation period $\Omega$ as a function of radius $r$ assumed in that study is (in the Newtonian limit) [102]

$$\Omega = \frac{R_0^2 \Omega_C}{R_0^2 + r^2 \sin^2(\theta)},$$

(3.21)

$R_0 \sim 1\text{ km}$ being the characteristic scale of variation of $\Omega$, $r \sin(\theta)$ the distance from the rotation axis and $\Omega_C$ the central rotation period. The rotation period a PNS can acquire without additional accretion has been shown to be limited by the increase of the minimum neutron star mass for times up to $\sim 100\text{ms}$ post-bounce and by the mass shedding limit beyond that, resulting in $P_{\text{min}} \approx 1.7\text{ms}$ [102]. This corresponds to velocities of $v \sim (r/1\text{km})(\Omega/\Omega_C)10^6\text{ms}^{-1}$, which could also lead to kinetic energy densities comparable to convective flows.

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3.7 Mechanical Stability of the Solid Phase Transition Lattice [216]

The shear constant of a cubic Coulomb lattice is [92]

\[ c_{44} = \frac{d^2 W_l}{d\gamma_{xy}^2}, \]  

(3.22)

where \( W_l \) is the lattice energy and \( \gamma_{xy} \) the angle of distortion. The Coulomb lattice energy for 3D lattices can be calculated using Ewald’s method [84, 59]:

\[ W_l = \frac{1}{2} \sum \frac{e^2}{4\pi \epsilon_0 r(l)} = \frac{1}{2} \sum \frac{e^2}{4\pi \epsilon_0} \left( \sum \frac{\text{erfc}(gr(l))}{r(l)} + \sum \frac{4\pi \exp(-G_l^2/4g^2)}{G_l^2} \right), \]  

(3.23)

where the complementary error function \( \text{erfc}(x) \equiv \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-y^2)dy \), \( \Omega \) is a unit cell volume and \( g \) is a parameter to be adjusted for fast numerical convergence of both real (vectors \( \vec{r}(l) \)) and inverse (vectors \( \vec{G}(l) \)) lattice sums. The result for a bcc-lattice is [92]

\[ c_{44} = 0.7423 \times \frac{(1/3) \pi R^3 \rho c e^2)^2}{2 \times 4\pi \epsilon_0 a} \]

\[ = 2.985 \text{ MeV} \times \left( \frac{R^3 \rho_c}{1.0 \text{ fm}} \right)^2, \]  

(3.24)

\( a \) being the lattice constant. The critical shear stress is the force per unit area necessary to maintain two planes of the crystal distorted against each other by an angle corresponding to a displacement of \( a/4 \) perpendicular to a lattice plane [11] (see illustration figure 3.6), which is in the linear approximation

\[ \sigma_{\text{crit}} = \frac{1}{NA} \frac{dU}{dx} \approx \frac{1}{A} \frac{d}{dx} \left( 2c_{44} \left( \frac{x}{a} \right)^2 \right) \]

\[ = \frac{c_{44}}{a^3} = 2.985 \text{ MeV} \times \left( \frac{r_d \Delta \rho_c}{1.0 \text{ fm}} \right)^4, \]  

(3.25)
Figure 3.6: Schematic drawing of lattice shear force in the harmonic approximation.

which, for a set of typical values, a lattice constant $a = 15\text{fm}$, droplet radius $r_d = 3.0\text{fm}$ and charge density $\Delta \rho_C = 0.4\text{fm}^{-3}$, gives $\sigma_{\text{crit}} = 6.9 \times 10^{-3}\text{MeVfm}^{-3}$. The resulting critical shear stresses as a function of radius for the outer 3D lattice regions of hybrid stars based on EOS’s 1 and 4 from chapter 2, $\sigma = 20/50/100\text{MeV/fm}^2$ and $\rho_C = 1000\text{MeV/fm}^3$ are shown in figures 3.7 and 3.8.

Convection or differential rotation can cause stresses in the range $> 10^{-3}\text{MeV/fm}^3$ (see 3.6). Hence, a region of $\sim 1 - 2\text{ km}$ wide from the confined matter envelope into the coexistence region consisting of a solid phase transition lattice is susceptible to breakup caused by hydrodynamic stresses, in the sense that its formation could be prevented in the first place or that they break up an already formed solid lattice. A strong enough convective cell forming outside the coexistence phase layer after its crystallization during a transient quiet period might generate a crater in the solid lattice and mix matter from the envelope of pure confined matter into that crater below the convective cell. For the duration of the convective flow, this would result in a localized solid angle region with a substantially lowered neutrino opacity (see chapter 4) compared to the still intact solid lattice in all other directions, hence in
Figure 3.7: Critical Shear Stress of the 3D-phase transition lattice on the outer surface of the confined/deconfined matter coexistence region for EOS 1 (sec. 2) and different QCD surface tensions in MeV/fm$^2$.

Figure 3.8: Critical Shear Stress of the 3D-phase transition lattice on the outer surface of the confined/deconfined matter coexistence region for EOS 4 (sec. 2) and different QCD surface tensions in MeV/fm$^2$.
anisotropic neutrino transport through the mixed strange/non-strange layer. This will be explored in more detail in chapter 6.

In a situation when differential rotation in a region of a fast rotating PNS or hybrid star is strong enough to prevent the crystallization of a lattice, it is not likely that this region goes through a transient phase with low rotation period during which the phase transition lattice could crystallize, possibly to be broken up at later times by a larger gradient in the rotation period. Rather, for PNS’s with rotation periods below ~ 100 ms, the timescale for the stratification of differential rotation might be of significant importance for the melting curve of the phase transition lattice, possibly comparable to the cooling timescale. However, unless the transport of angular momentum in core collapse supernovae and in particular within the PNS matter is finally resolved, the mechanical properties presented here only apply to stars (or regions therein) exhibiting low differential rotation.

3.8 Resulting Constraints on the Hydrodynamics of Hybrid Stars

A solid phase transition lattice forces a modification of the traditional hydrodynamical paradigm of PNS evolution. Temperatures are on the order of $T \sim 10$ MeV, small enough to ensure a crystalline lattice inhibiting hydrodynamic flow in the co-existence region. The hydrodynamics of the pure deconfined quark phase is of no interest for energy transport, since it will be surrounded by a thick, crystalline shell which is more than an order of magnitude more opaque to neutrinos than either pure confined or deconfined matter (chapter 4), making its temperature almost constant. Only the outer layers consisting of “normal” neutron matter can and should then be treated using a full hydrodynamical formalism. The Eulerian equations of
hydrodynamical flow are [49]

\[
\begin{align*}
\delta_t \rho + \nabla (\rho \vec{u}) &= 0 \\
\delta_t (\rho \vec{u}) + \nabla (\rho \vec{u} \cdot \vec{u}) + \nabla p &= \rho g \\
\delta_t (\rho E) + \nabla (\rho E \vec{u}) + \nabla (p \vec{u}) &= \rho u g, 
\end{align*}
\] (3.26)

where \( \vec{u} \) and \( p \) are the three-velocities and momenta of the fluid in a Eulerian grid element and \( g \) is the gravitational acceleration. The hydrodynamics code used in this study is VH-1 [111] which is built around a PPM solver [49] in Lagrangian coordinates which are remapped onto the Eulerian grid after each step. VH-1 assumes a generic ideal gas EOS

\[
\epsilon = \rho u^2 + \frac{1}{\gamma - 1} p, \tag{3.27}
\]

for which a wrapper routine was written to interface the code with the adopted nuclear equation of state using an effective adiabatic coefficient \( \gamma \). Radiation transport has to be treated separately from VH-1 using operator splitting.
CHAPTER 4

NEUTRINO TRANSPORT IN HYBRID STARS

4.1 Neutrino Spectra Formation

The neutrino energy spectrum is determined by the last few creation/annihilation and thermalizing scatter events in the outer PNS regions. Deeper regions merely provide the necessary neutrino flux, their history and structure does not affect the neutrino emission spectrum. For electron and electron antineutrinos, the dominant processes to determine both flux and energy in the region of the neutrino sphere are the charged current (CC) absorptions $\nu_e + p \leftrightarrow n + e^+$ and $\nu_e + n \leftrightarrow p + e^-$ [206]. At energies up to $\sim 10$ MeV, the $\mu$- and $\tau$-neutrinos only interact via neutral-current (NC) processes. Important number-changing NC channels are nucleon bremsstrahlung $N + N \leftrightarrow N + N + \nu + \bar{\nu}$ and pair annihilation $e^- + e^+ \leftrightarrow \nu + \bar{\nu}$ [108, 200]. Inelastic scattering off electrons $e + \nu \rightarrow \nu + e$ and nucleons $N + N + \nu \rightarrow \nu + N + N$ (the latter being the crossed version of nucleon bremsstrahlung) as well as nucleon recoil in $N + \nu \rightarrow \nu + N$ are important as thermalizing channels [108, 177].

The neutrino energy sphere is defined as the radius $R_\nu$ where neutrino radiation of a given flavor decouples from matter, which is defined through the optical depth $\tau$
via the condition \[191\]

\[
\tau(R_e) = \int_{R_e}^{\infty} (dr / \lambda_{\text{eff}}) = 2/3, \\
\lambda_{\text{eff}} = \sqrt{\lambda_{\text{total}} \lambda_{\text{energy}}} \tag{4.1}
\]

\(\lambda_{\text{energy}}\) is the mean free path for energy-changing reactions, whereas \(\lambda_{\text{total}}\) is the resulting mean free path. For \(\nu_{\mu/\tau}\) neutrinos, elastic neutrino scattering dominates the total opacity, so that \(\lambda_{\text{total}} \approx \lambda_{\text{scatter}}\). This leads to interesting consequences for the location of the energy sphere, together with the peculiar neutrino energy dependence of nucleon-nucleon bremsstrahlung. The latter has recently been predicted to define the energy sphere for \(\mu\) and \(\tau\) neutrinos for certain evolution phases in PNS's \[108, 200\].

The relative importance of bremsstrahlung and inelastic scattering versus recoil effects depends on the density, as \(\lambda_{\text{brems}}^{-1} \sim \rho^2\). No energy sphere exists for recoil effects, as they become irrelevant only for the free-streaming region. For evolved neutron star surfaces with their sharply decreasing surface density profile (\(\rho \sim r^{-10}\)), recoil effects can then be neglected. Indeed, their relative impact on the neutrino spectrum has been shown to be in the \(\sim 10\%\) range \[177\]. The same can be said about inelastic electron scattering due to the deleptonized state of the surface, leading to the conclusion that \(\lambda_{\text{energy}} \approx \lambda_{\text{brems}}\). Its corresponding mean free path goes linear with the neutrino energy \(\epsilon\), \(\lambda_{\text{brems}} \sim \epsilon\), whereas \(\lambda_{\text{scatter}} \sim \epsilon^2\), leading to \(\lambda_{\text{eff}} \sim 1/\sqrt{\epsilon}\). This results in a much more narrow range of energy spheres for different neutrino energies than previously assumed (if the effective mean free path would behave as \(\lambda_{\text{eff}} \sim \epsilon^2\).

In fact, a single blackbody surface at the energy sphere, located at densities about one order of magnitude below nuclear saturation density, has been shown to correctly describe the freezeout from number-changing reactions \[177\].
In this spirit, the present study focuses on neutrino fluxes in the PNS/hybrid star interior, whereas the emitted spectrum is considered to be determined by the matter temperature at the energy sphere, which is assumed to be at a well-defined radius for any given flavor. For high enough temperatures ($T \sim$ MeV) and deep enough regions ($\rho > \rho_0 = 0.155 \text{ u/fm}^3$) for neutrinos to be in thermal equilibrium with matter, no specific absorption or emission processes (which would have a complex dependence on temperature and nucleon interaction effects, see e.g. [183]) have to be treated in detail, since the spectrum will always be thermal. The neutrino (and, hence, energy) transport opacity in these density and temperature regimes is dominated by scattering off baryons (N,q). In addition, temperatures in the inner regions are small compared to typical nucleon and quark chemical potentials, $E_\nu << \mu_{q,N}$, so that opacities there can be mostly taken in the limit of degenerate matter.

## 4.2 Description of Neutrino Emission Spectra

Generally, neutrino spectra obtained with various simulation methods are fit to a modified black-body (Fermi-Dirac) spectrum [122]

$$\frac{dN_\nu}{d\epsilon} = \frac{1}{2\pi^2(c h)^3} \frac{\epsilon^2}{1 + \exp\left(\frac{E_\nu}{T_{\nu}} - \eta\right)},$$

(4.2)

where the effective degeneracy $\eta$ describes qualitatively deviating high and low energy behavior. The deeper in the star a given neutrino flavor decouples, the smaller the neutrino sphere radius, the higher the average neutrino energy, which leads to the expected hierarchy $T_{\nu_{\mu,\tau}} > T_{\nu_\tau} > T_{\nu_e}$.

Detailed spectra were obtained in previous studies for times $t_{pb} < 1 \text{s}$, using Boltzmann transport [149] and Multi Group Flux Limited Diffusion [154, 39] as well as Monte Carlo transport [122] simulations. None of the earlier studies, however, took
into account nucleon bremsstrahlung. In the deleptonized outer core of a partially evolved PNS at intermediate times $t_{ph} \sim 1$ sec, resulting spectral temperatures are $T_{\nu_e} \approx 4.2$ MeV versus $T_{\nu_\mu} \approx 3.2$ MeV with a much harder spectrum for the other flavors, $T_{\nu_{\mu,\tau}} \approx 8$ MeV [154]. The latter result was obtained neglecting nucleon-nucleon bremsstrahlung, though, rendering it an upper limit to the true value. The resulting effective degeneracies were $\eta_{\nu_e} \approx 1.2$, $\eta_{\nu_\mu} \approx 2.5$ and $\eta_{\nu_\tau} \approx 3.5$. In a more recent study including nucleon bremsstrahlung and recoil effects [39], a considerably lower value for the $\mu$ and $\tau$ neutrinos of $T_{\nu_{\mu,\tau}} \approx 6 - 7$ MeV and a negative effective degeneracy $\eta_{\mu/\tau}$ was reported, as well as lower temperatures for $\nu_{e/\mu}$ -neutrinos ($(T_{\nu_e}, \eta_{\nu_e}) = (3.2$ MeV, 3.48) and $(T_{\nu_\mu}, \eta_{\nu_\mu}) \approx (2.2$ MeV, 3.16)), although for earlier times post-bounce. To this date, no groups have attempted to predict the neutrino spectra for times $t_{ph} \sim 3 - 10$ sec likely to be relevant for the formation and evolution of a hybrid star. However, the largely neutronized outer regions and the mentioned effects of bremsstrahlung will drive $T_{\nu_{\mu,\tau}} \approx T_{\nu_e}$. Due to the largely neutronized star surface, only $T_{\nu_e}$ might deviate from the former, although the steep density gradient of the settled surface will make this effect less pronounced than at the earlier times studied in past numerical simulations.

4.3 Mean Free Path in the Pure Confined Phase

4.3.1 Neutral Current Scattering

Interaction cross sections of neutrinos in asymmetric nuclear matter, in particular the influence of medium effects, are the subject of intense current research. Many-body correlations mediated by the strong force have been shown to possibly increase the mean free path by a factor of $\sim 2$ in the mean-field approximation [182, 183].

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Correlation functions including the strong interaction and Fermi statistics obtained through the random phase approximation (RPA) [37, 38] even predict an increase by a factor of up to $\sim 5$. Nucleon-spin fluctuations have, so far only for densities well below $\rho_0$, also been shown to possibly influence the neutrino-nuclear matter interaction in the range of several $\%$ [176].

Clearly, more work is still needed to fully resolve the effects of correlations and the correct treatment of these effects is computationally hard to tract in a macroscopic transport code. Hence, the effect of correlations is neglected and the approximation of non-interacting nucleons is used [206, 30, 184].

$$\lambda_{N,nc}^{-1} = C_N \sigma_0 n Y_{NN} \left( \frac{T}{m_e c^2} \right)^2 \frac{\mathcal{F}_5(\eta_\nu)}{\mathcal{F}_4(\eta_\nu)}, \quad (4.3)$$

where $n = n_p + n_n$ is the nucleon number density, $\sigma_0 = 1.76 \times 10^{44}$ cm$^2$, the neutrino degeneracy $\eta_\nu = \mu_\nu / T$, $C_{(N=N)} = (1 + 5\alpha^2)/24$ and $C_{(N=p)} = (4(C_V - 1)^2 + 5\alpha^2)/24$ with $C_V = 1/2 + 2\sin^2(\theta_W)$, $\alpha \approx 1.25$ and $\sin^2(\theta_W) \approx 0.23$ for the Weinberg angle $\theta_W$. The Fermi integrals $\mathcal{F}_j(\eta)$ are given by

$$\mathcal{F}_j(\eta) = \int_0^\infty dx \frac{x^j}{1 + \exp(x - \eta)}, \quad (4.4)$$

and $Y_{NN}$ represents an interpolation between degenerate and non-degenerate nucleons suggested in [30]:

$$Y_{NN} = \frac{Y_N}{1 + \frac{2}{3} \max(\eta_N, 0)}. \quad (4.5)$$

The nucleon degeneracy parameter $\eta_N = (\mu_N - m_N)/T$ and $Y_N = n_N/(n_n + n_p)$. The cross section for scattering off electrons has been shown to be small compared to the nucleon contributions for densities $\rho > \rho_0$ [183] and will therefore be neglected, especially as the net lepton number concentration in the mostly deleptonized stars considered here is below $Y_L = Y_e + Y_\nu_e \sim 0.1$. 

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4.3.2 Charged Current Absorption

Strong interaction mediated correlations are also neglected in the expressions for the CC neutrino capture cross sections for \((p, \nu_e)\) and \((n, \nu_e)\). The importance of CC absorption as an opacity source depends on the neutrino degeneracy \([183]\). For non-degenerate neutrinos \(\mu_{\nu_e} \approx 0\), absorption is kinematically suppressed compared to NC scattering by the difference between neutron and proton chemical potentials for temperatures below \(T \sim 15\,\text{MeV}\). As the expected temperatures in the confined matter regions are \(T < 10\,\text{MeV}\), absorption as an opacity source is neglected for non-degenerate neutrinos, resulting in an opacity inside the neutrino energy sphere independent upon neutrino flavor.

For neutrinos with finite degeneracy, the usual expression using final-state Fermi blocking of the outgoing electrons/positrons is used \([206, 30, 184]\):

\[
\lambda^{-1}_{\nu_e} = \frac{1 + 3\alpha^2}{4} \sigma_{0n} Y_{np} \left( \frac{T}{m_e c^2} \right)^2 \frac{\mathcal{F}_5(\eta_{\nu_e})}{\mathcal{F}_4(\eta_{\nu_e})} (1 - f_{e^{-}}(\epsilon))
\]

(4.6)

\[
\lambda^{-1}_{\bar{\nu}_e} = \frac{1 + 3\alpha^2}{4} \sigma_{0n} Y_{pn} \left( \frac{T}{m_e c^2} \right)^2 \frac{\mathcal{F}_5(\eta_{\bar{\nu}_e})}{\mathcal{F}_4(\eta_{\bar{\nu}_e})} (1 - f_{e^{+}}(\epsilon)),
\]

(4.7)

where the blocking of the nucleon phase space is accounted for by

\[
Y_{np} = \frac{2Y_e - 1}{\exp(\eta_p - \eta_n) - 1}
\]

\[
Y_{pn} = \exp(\eta_p - \eta_n) Y_{np},
\]

(4.8)

and for the fermion blocking factors, the expressions \([184]\)

\[
\langle 1 - f_{e^{-}}(\epsilon) \rangle = \left( 1 + \exp \left( - \left( \frac{T}{m_e c^2} \right)^2 \frac{\mathcal{F}_5(\eta_{\nu_e})}{\mathcal{F}_4(\eta_{\nu_e})} - \eta_e \right) \right)^{-1}
\]

(4.9)

\[
\langle 1 - f_{e^{+}}(\epsilon) \rangle = \left( 1 + \exp \left( - \left( \frac{T}{m_e c^2} \right)^2 \frac{\mathcal{F}_5(\eta_{\bar{\nu}_e})}{\mathcal{F}_4(\eta_{\bar{\nu}_e})} + \eta_e \right) \right)^{-1}
\]

(4.10)

were used.
4.4 Mean Free Path in the Pure Deconfined Phase

4.4.1 Neutral Current Scattering

As in the confined phase, scattering off electrons is neglected, since \( n_e < n_q \). For neutrino interactions with quarks, the cross sections obtained in [195] were largely adopted. Typical neutrino energies \( E_\nu \sim \pi T \) are small compared to quark chemical potentials, given densities beyond those for which free quarks and nuclei are in thermal equilibrium (\( \mu_q > M_N/3 \)). In that limit, the inverse mean free path for scattering of non-degenerate neutrinos off the quark species \( q \) is

\[
\lambda^{-1}_\nu = \frac{G_F^2 \mu_q^2}{5\pi^3 E_\nu^3}. \tag{4.11}
\]

The net electron neutrino concentration in the central regions is, given a trapped lepton number of \( Y_L \sim 0.1 \), still \( n_{\nu_e} \sim 0.1 \text{ fm}^{-3} \), which leads to a Fermi energy of \( E_{F\nu_e} = 197.57 \times (6\pi n_{\nu_e})^{1/3} \text{ MeV} \sim 150 \text{ MeV} \). Hence, \( \nu_e/\nu_\ell \) - neutrinos cannot necessarily be considered as non-degenerate, even towards the end of the deleptonization phase. As an accurate treatment using the concepts from [195] would be numerically prohibitive, their result for degenerate neutrinos is adopted for finite neutrino degeneracies:

\[
\lambda^{-1}_{\nu_\ell/\nu_e} = \frac{G_F^2 \mu_q^3}{5\pi^3} [(E_\nu - \mu_\nu)^2 + \pi^2 T^2] \sqrt{\frac{x E_\nu}{\mu_q} [\nu_q^2 + A_q^2]} \{10 + x^2 + 5(2\nu_q A_q)x\}, \tag{4.12}
\]

where \( x = \min(E_\nu, \mu_q)/\max(E_\nu, \mu_q) \) and

\[
\begin{align*}
\nu_u &= \frac{1}{2} - \frac{4}{3} \sin^2(\theta_W) \\
\nu_d &= \frac{1}{2} + \frac{2}{3} \sin^2(\theta_W) \\
\nu_s &= \frac{1}{2} + \frac{2}{3} \sin^2(\theta_W) \\
A_u &= -A_d = -A_s = \frac{1}{2}.
\end{align*}
\]
4.4.2 Charged Current Absorption

The possible CC reactions in the deconfined phase are

\[ \nu_e + d \rightarrow e^- + u \]
\[ \nu_e + s \rightarrow e^- + u \]
\[ \nu_x + u \rightarrow e^+ + d/s. \]  \hspace{1cm} (4.13)

Analogous to CC processes in confined matter, CC absorption is kinematically suppressed compared to NC scattering \[195\], so that these processes are neglected for non-degenerate neutrinos. Neutrinos with finite degeneracies are again treated in the degenerate limit \[195\]:

\[ \lambda^{-1}_{\nu_e} = \frac{2G_F^2 \mu_e}{5\pi^3 \mu_{\nu_e}} (10\mu_u^2 + 5\mu_u\mu_e + \mu_e^2)[(E_{\nu_e} - \mu_{\nu_e})^2 + \pi^2 T^2] \]
\[ \lambda^{-1}_{\nu_x} = \frac{2G_F^2 \mu_x}{5\pi^3 \mu_{\nu_x}} (10\mu_d/s + 5\mu_d/s\mu_x + \mu_x^2)[(E_{\nu_x} - \mu_{\nu_x})^2 + \pi^2 T^2]. \]  \hspace{1cm} (4.14)

4.5 The mixed strange/non-strange phase

The standard contribution from neutral current scattering off (individual) baryons in the mixed strange/non-strange phase is

\[ \sigma_{nc} = \sigma_h + \sigma_q. \]  \hspace{1cm} (4.15)

Elastic scattering off the net weak charge on the minority phase droplets has been predicted to result in a major contribution to the total cross section \[181\]:

\[ \frac{1}{V} \frac{d\sigma_{sd}}{d(\cos \theta)} = n_D \frac{E_F^2}{16\pi} G_F^2 N_W^2 (1 + \cos \theta) F^2(q), \]  \hspace{1cm} (4.16)

where \( n_D \) is the droplet density, \( N_W \) the net weak charge on the droplet \( N_W = 4/3\pi r_D^3 \rho_W \) with given weak charge density

\[ \rho_W = [(n_u - (n_d + n_s)) - (n_n - n_p)] - 4/3 [n_u - (n_d + n_s)/2 + 3n_p] \sin^2(\theta_W). \]  \hspace{1cm} (4.17)
The quark number densities in the deconfined phase are \( n_{u,d,s} \) and the nucleon densities in the hadronic phase \( n_{n,p} \), respectively. \( \tilde{F}(q) = F(q) \times S(q) \) is the form factor as a function of momentum transfer \( q = \sqrt{2}E_\nu(1 - \cos \theta) \), consisting of the term for individual droplets \( F(q) \) and the lattice form factor \( S(q) \). The former is, for a homogenous charge distribution and neglecting any screening effects,

\[
F(q) = \frac{1}{N_w} \int_0^{r_p} d^3x \rho_W(x) \frac{\sin qx}{qx}.
\] (4.18)

The lattice form factor is defined as

\[
S(q) = 1 + N_D \int_0^\infty d^3x (g(r) - 1) \exp iq\vec{x},
\] (4.19)

where for the distribution function of the other droplets, a random charge distribution (liquid lattice) was assumed:

\[
g(r) = \Theta(r - R_W).
\] (4.20)

The resulting lattice form factor is

\[
S(q) = 1 - \frac{3}{F(q)} \sin qR_W - \frac{(qR_W) \cos qR_W}{(qR_W)^3}.
\] (4.21)

The resulting transport mean free path is obtained by integrating over \( \cos \theta \) [181]

\[
\sigma_{el} = \int_{-1}^{1} d\cos \theta (1 - \cos \theta) \frac{d\sigma_{el}}{d(\cos \theta)}.
\] (4.22)

In principle, as typical neutrino wavelengths are on the order of lattice constants \( a \sim 10 \text{fm} \), Bragg scattering would have to be taken into account. This, however, would be numerically intractable since it would involve 3D transport with a large number of angular bins. For 1D and 2D geometries, the differential cross section \( d\sigma_{el}/d\cos \theta \) will depend on the orientation of the rods or platelets relative to the
neutrino flight path, with the component perpendicular to the lattice orientation being larger. Due to the mean free path (at least 10\(^{-3}\) m) being many orders of magnitude larger than the lattice constants, even in directions parallel to the 1D or 2D structures, the orientation of rods or platelets relative to the neutrino flight paths can be considered as random, so that an effective “droplet” radius for the scattering off the weak droplets \( r_d = (4/(3\chi))^{1/3} \times a \) is assumed for the 1D and 2D geometries. Structures with longitudinal dimensions comparable to neutrino mean free paths can only be expected for temperatures small compared to lattice melting temperatures, \( T < 1 \text{ MeV} \), at which point the neutrino cooling phase will be completed. Bragg scattering is neglected for the same reason, since lattice defects, whether thermally or structurally induced, can be expected to cause the lattice orientation to be random on length scales comparable to typical neutrino mean free paths.

### 4.6 Neutrino Cooling

Neutrino transport and cooling were done in separate, independent steps for each time interval, a procedure which is called operator splitting [26]. For the heat capacity \( C \), the expression from [138] for a degenerate Fermi gas was adopted,

\[
\frac{C}{V} = \frac{\mu k_F T}{3},
\]

(4.23)

where \( \mu \) is the chemical potential and \( k_F \) the Fermi momentum. For the constituent species, this results in (see also [215])

\[
\frac{C_{n,p}}{V} = \frac{\mu_{(n,p)} k_{F,(n,p)} T}{3},
\]

(4.24)

\[
\frac{C_{u,d,s,e,\nu_{\tau}}}{V} \approx \frac{\mu_{u,d,s,e,\nu_{\tau}}^2 T}{3}.
\]

(4.25)
4.7 Neutrino Transport in the Diffusion Limit

The neutrino energy flux $\vec{F}$ can be written as [168]

$$\vec{F}(\vec{r}, t) = \int_{0}^{\infty} d\epsilon \int_{4\pi} d\Omega \vec{I}(\vec{r}, \epsilon, t),$$  \hspace{1cm} (4.26)

where $I = I(\vec{r}, \epsilon, t)$ is the distribution function of the neutrino radiation, which is, in the Eddington approximation [168, 151] and assuming thermal equilibrium with matter,

$$I(\vec{r}, \epsilon, t) = B_F(\epsilon, \mu, T) - \frac{V}{\sigma_{\nu}(\vec{r}, \epsilon, t)} \nabla B_F(\epsilon, \mu, T)$$  \hspace{1cm} (4.27)

where $B_F(\epsilon, \mu, T)$ is the Fermi distribution function. This approximation relies on the relevant length scales over which the system varies to be large compared to the mean free path. The density scale height is several km above $\rho = \rho_0$ (see fig. 2.5), large compared to the longest mean free paths encountered ($\lambda_{\text{R, max}} \sim 10^2$ m). The use of a flux limiter [26] to prevent apparent neutrino diffusion velocities larger than $c$ was considered but turned out to be unnecessary.

The spectra for the degenerate $(\mu, \tau)$ neutrino species are completely determined by the temperature profile, whereas the transport of $\nu_e/\nu_{\bar{e}}$ neutrinos also depends on the trapped lepton number through the neutrino chemical potential $\mu$. The resulting flux is

$$\vec{F}(\vec{r}, t) = -\frac{c}{3} \left( \frac{V}{\sigma_{RT}(\vec{r}, t)} \frac{\delta \rho_{\nu}(\mu, T)}{\delta T} \nabla T(\vec{r}, t) + \frac{V}{\sigma_{R,\mu}(\vec{r}, t)} \frac{\delta \rho_{\nu}(\mu, T)}{\delta \mu} \nabla \mu(\vec{r}, t) \right),$$  \hspace{1cm} (4.28)

where $\sigma_{RT/\mu}$ is the Rosseland mean opacity [151]

$$\frac{V}{\sigma_{(RT/\mu)}} = \int_{0}^{\infty} d\epsilon \left( \lambda(\epsilon, T, \mu) \frac{\delta B_F(\epsilon, \mu, T)}{\delta T(\mu)} \right) / \int_{0}^{\infty} d\epsilon \left( \frac{\delta B_F(\epsilon, \mu, T)}{\delta T(\mu)} \right),$$  \hspace{1cm} (4.29)
$\lambda(\epsilon, T, \mu)$ is the neutrino mean free path and the neutrino energy densities are defined through the Fermi integrals (eq. 4.4)

$$\rho_\nu = \frac{T^4}{2\pi^2 (\hbar c)^3 F_3(\mu/T)}. \quad (4.30)$$

It has been shown previously [172] that at early times, when significant lepton number is trapped in the PNS, heat transport will be dominated by the lepton number gradient (second term in equation 4.28) and therefore by $\nu_e$ neutrinos. This deleptonization phase, accompanied by significant contraction of the PNS, is then followed by the neutrino cooling phase, when the temperature gradient term in eq. 4.28 will take over.

### 4.8 Comparison of Neutrino Opacity Sources in the Hybrid Star Interiors

In order to illustrate and compare the main opacity sources in the hybrid star interior, three representative radii were chosen in the hybrid star following from EOS 5 (table 2.1) for which the Rosseland mean free paths are plotted versus temperature. The plots are for a constant trapped lepton number concentration $Y_L = 0.1$ in the regions carrying macroscopic strangeness and for a linearly decreasing value through the confined matter envelope to a value of $Y_L = 0.05$ at the hybrid star surface.

Point 1 is in the deconfined matter core at $r = 1.5$ km at a density of $\rho = 960$ MeV/fm$^3$, and $\lambda_R$ for the different neutrino flavors is shown in figure 4.1. The neutrino Fermi energy is about $k_{F, \nu_e} \approx 230$ MeV, rendering the $\nu_e, \bar{\nu}_e$ degenerate for temperatures up to $\sim 50$ MeV. The Rosseland opacity for $\nu_{\mu/\tau}$ neutrinos is more than two orders of magnitude lower than for $\nu_e$ neutrinos, showing that the CC channels dominate for degenerate neutrinos. The high opacity for $\nu_{\bar{e}}$ neutrinos is a
consequence of low lying states forbidden by degeneracy, whereas energies beyond the Fermi energy are strongly thermally suppressed. Point 2 is in the coexistence region at a radius of $r = 6.4\text{ km}$ with a density of $\rho = 432\text{ MeV/fm}^3$ and a strange volume fraction of $\chi = 0.3$. The $\lambda_R$-plots are shown in figure 4.2 for scattering off individual baryons and in figure 4.3 for scattering off the minority phase lattice. Scattering of $\nu_{\mu/\tau}$ neutrinos off lattice sites is $1 - 3$ orders of magnitude larger than regular NC scattering off baryons, making it the dominant opacity source for $\mu$ and $\tau$ flavors. As that opacity is in the general range of the standard $\nu_{e,\bar{e}}$ neutrino opacity for the degenerate case, degenerate electron neutrinos are not significantly affected by the presence of a phase transition lattice. The Rosseland mean free paths for $\nu_{e,\bar{e}}$ are very close for the cases of thermally and degeneracy induced diffusion, in particular for the pure confined phase envelope, see figure 4.4 for point 3 at $r = 10.3\text{ km}$ ($\rho = 176\text{ MeV/fm}^3$) in the confined matter envelope.
Figure 4.1: Rosseland mean free paths (bulk baryon matter) for different neutrino flavors as a function of temperature at a radius $r = 1.5 \text{ km}$ ($\rho = 960 \text{ MeV/fm}^3$) in the deconfined matter core for EOS 5 (see table 2.1) with $\rho_C = 1000 \text{ MeV/fm}^3$. Also included are the mean free paths for neutrino degeneracy gradients (see sec. 4.7).

Figure 4.2: Rosseland mean free paths (bulk baryon matter) for different neutrino flavors as a function of temperature at a radius $r = 6.4 \text{ km}$ ($\chi = 0.3, \rho = 432 \text{ MeV/fm}^3$) in the deconfined/confined matter coexistence region for EOS 5 (see table 2.1) with $\rho_C = 1000 \text{ MeV/fm}^3$. Also included are the mean free paths for neutrino degeneracy gradients (see sec. 4.7).
Figure 4.3: Rosseland mean free paths for elastic scattering off the minority phase lattice with different neutrino flavors as a function of temperature at a radius $r = 6.4\text{km}$ ($\chi = 0.3, \rho = 432\text{MeV/fm}^3$) in the deconfined/confined matter coexistence region for EOS 5 (see table 2.1) with $\rho_C = 1000\text{MeV/fm}^3$. Also included are the mean free paths for neutrino degeneracy gradients (see sec. 4.7).

Figure 4.4: Rosseland mean free paths (bulk baryon matter) for different neutrino flavors as a function of temperature at a radius $r = 10.3\text{km}$ ($\rho = 176\text{MeV/fm}^3$) in the confined matter envelope for EOS 5 (see table 2.1) with $\rho_C = 1000\text{MeV/fm}^3$. The mean free paths for neutrino degeneracy gradients (see sec. 4.7) are negligibly different from the thermal ones in this case.
CHAPTER 5

HYBRID STAR EVOLUTION

5.1 Formation History of the Hybrid Star

5.1.1 Limits on Formation Dynamical Timescales

To date, the full formation of a hybrid star in core collapse supernovae has not been described yet in a full hydrodynamical simulation including all feedback effects. The phase transition has a tendency to shut itself off both through heating and lepton number transport, possibly resulting in a “burning front” for the deconfinement transition, expanding out from the center of the PNS. The study by Gentile et al.[95] focused on the possibility that the formation of a deconfined QCD phase might aid the supernova explosion by enhancing the shock energy from core bounce. However, its deconfined EOS does not take into account the formation of an equilibrium coexistence phase by falsely enforcing phase-localized charge neutrality, rendering the formation of a lattice impossible. Further, it only extends until a few msec after bounce. Clearly, a full hydrodynamical treatment starting from a progenitor until several seconds post-bounce is presently not feasible, especially as the explosion mechanism has not been fully resolved yet [22, 213]. The present study is therefore

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limited to a time of a few seconds after core bounce, when the lepton number concentration will be significantly below the initial $Y_L \sim 0.4$, for which the appearance of strange quarks is pushed beyond reasonable densities through a significant stiffening of the EOS [173, 105].

The dynamics of the phase transition is determined by two effects, equilibration of the weak reactions in equations 2.1-2.3 and the transport of the resulting heat and lepton number. The former has been shown to proceed on timescales typically expected for weak interactions. Using the standard V-A theory of weak interactions, one obtains for the formation of three-flavor out of two-flavor quark matter a timescale of $\tau \sim 10^{-7} - 10^{-8}$ s [54, 165]. The deconfinement of nucleons into two-flavor quark matter [50] which precedes the weak equilibration is driven by the strong force and is therefore fast compared with weak timescales.

However, a significant temperature increase of up to $\Delta T \sim 50$ MeV [165] goes along with the weak equilibration due to the liberation of kinetic energy in the conversion of $d$ to $s$ quarks, the latter having an initial chemical potential close to their current mass $m_s \sim 150$ MeV, whereas $\mu_{d,\text{initial}} > M_N/3 \sim 300$ MeV. This energy heats and expands neighboring hadronic regions, inhibiting further growth of the deconfined strange phase. At the beginning of the phase transition, in a given localized region of the PNS, all reactions including the semileptonic reactions 2.1,2.2 and

$$d/s \rightarrow u + e^- + \nu_e$$

$$u \rightarrow d/s + e^+ + \nu_e$$

will run out of equilibrium, as the neutrinos are only coupled to matter over regions on the order of the mean free path, which is, for the conditions present, $\lambda_\nu \sim 10^{-3} - 1$ m and therefore large compared to the strong interaction length scales over which quarks
interact. Due to the (pre-deconfinement) numerical superiority of down quarks over up quarks in the neutronized matter, the initial conversion of \( d \) into \( u \) quarks will result in net electron antineutrino emission, raising the total lepton number in the region where the phase transition occurs, again locally inhibiting it. The equilibrating reactions involving strange quarks are both Cabibbo and phase space - suppressed and therefore slower than the \( d \rightarrow u \) conversions.

Hence, the weak interaction timescales obtained in previous works have to be considered as a lower limit to the formation timescale of the deconfined and mixed phases, since both heat and lepton number are transported on neutrino diffusion timescales of \( < 10^2 \) seconds, which are then an upper limit for the complete formation of the strange regions. This range is consistent with earlier estimates on the conversion speed of cold neutron star matter to strange matter [4, 160]. Further studies are necessary to determine the exact timescale for the formation of macroscopic strangeness in PNS’s.

### 5.1.2 Initial Deleptonization State

As was pointed out in [172], the post-bounce evolution of a standard model PNS can be roughly divided into a deleptonization and a neutrino cooling phase. Heat transport is clearly lepton driven for high lepton number concentrations \( Y_L \sim 0.4 \) found immediately after bounce. A deconfinement phase transition can be safely excluded for the associated low matter densities and high lepton concentrations at these early times. However, hybrid stars with a fully deconfined core can be found for some equations of state for \( Y_L < 0.15 \). The degree of deleptonization and the timescale for
further deleptonization from $Y_L \sim 0.1$ is relevant for the determination of the subsequent neutrino cooling phase, since it was pointed out in chapter 4 that electrons and electron neutrinos cannot be considered as non-degenerate yet for $n_{e\nu_e} \sim 0.01 \text{fm}^{-3}$. Assuming a constant $Y_L = 0.1$ for the deconfined and mixed phases and a linearly decreasing distribution from the mixed-confined interface to $Y_L = 0.05$ at the cutoff density (star surface), the ratio of degeneracy gradient driven heat transport to thermal emission from the surface was plotted versus radius in figure 5.1. Lepton number driven heat transport dominates for a temperature profile obtained using the condition of equilibrium thermal heat flow (using a surface temperature of $T = 2 \text{MeV}$ as boundary condition) as well as for a constant temperature profile through the entire hybrid star (using $T = 10 \text{MeV}$). The smaller ratio for the constant temperature profile was due to its higher surface temperature, as the thermal emission from the surface goes as $\sim T^4$. The vertical lines show the boundaries between the confined, coexistence and deconfined matter regions which depend on the temperature profile through the total energy density in the TOV equations (sec. 2.3.1). It is then clear that even for relatively low $Y_L$, the hybrid star will further deleptonize on timescales fast compared to further cooling timescales. Hence, $\mu_{\nu_e} \approx 0$ will be assumed by the time the phase transition lattice has completely formed. As the assumed, rather simplistic radial lepton number profile can, in lack of a complete evolution model, only considered to be qualitatively true, no further conclusions about the late phases of deleptonization are possible from figure 5.1.
Figure 5.1: Ratio of lepton driven heat flux to thermal radiation off the surface for a trapped $Y_L = 0.1$ in a hybrid star with $\rho_C = 1000 \text{ MeV} / \text{fm}^3$ and based on EOS 5 (see table 2.1) for both a constant temperature profile and heat flow equilibrium (sec. 5.2).

5.2 Initial State of the Hybrid Star for the Neutrino Cooling Phase

The strange and mixed phases are assumed to be completely formed with the temperature consistent with a solid phase transition lattice. The temperature profile depends on the dynamics of the phase transition. However, its general shape follows from the short neutrino mean free path for the mixed versus the pure phases and the release of heat during the conversion of $d$ into $s$ quarks. Quark deconfinement occurs starting at small radii, and the transition heats up regions both inside and outside the conversion front. Elastic scattering off the weak charges on the lattice sites dominates the neutrino opacity in the mixed phase, as is shown in figures 4.2 and 4.3. The mixed phase lattice will therefore tend to retain the heat generated and will exhibit the largest temperature gradient. The resulting temperature gradient in the confined matter envelope will be much smaller due to its higher heat conductivity and will be
completely determined by the interface to the mixed phase and black body neutrino emission from the neutrino energy sphere. The pure deconfined matter phase at the center, if the density profile allows for its existence, is expected to show an essentially constant temperature set by the surrounding opaque mixed region. The temperature distribution within the mixed phase is more dependent on the formation dynamics of the deconfined and mixed phases. However, it should be well reproduced by assuming initial equilibrium flow everywhere. The dominance of the neutrino opacity by permanent scattering centers in the mixed phase also renders any opacity-decreasing effects ineffective. Examples of the latter are correlation effects and the possible formation of a color-superconducting phase, which has been shown to decrease the opacity for NC scattering by up to an order of magnitude for energy gaps of $\Delta_0 \sim 10 - 100$ MeV ([40], see also the remarks in sec. 2.2.4).

The initial temperature profile for the spherically symmetric hybrid star is then obtained self-consistently using the following procedure:

- 1 Choose an energy sphere temperature $T_{ES}$ and the associated blackbody radiation as boundary condition. The saturation density for symmetric nuclear matter was used as radiation boundary surface. The actual energy sphere has previously been found to be at densities about an order of magnitude below that (see sec. 4.1), however, the temperature profile in the confined matter envelope falls off very slowly compared to the density profile of a neutron star surface at late times.

- 2 Choose an initial guess for the temperature profile of the hybrid star.
• 3 Integrate the TOV equations (sec. 2.3.1) with the appropriate equation of state (sec. 2) and the current temperature profile.

• 4 Update the neutrino opacities (sec. 4) for the current temperature profile.

• 5 Update the temperature profile interior to the energy sphere for a vanishing net energy flux into any volume: \( \mathcal{F}(r^*, t) = 0 \).

• 6 Repeat 3 through 5 until convergence of the temperature profile.

The resulting equilibrium flow profile corresponds to a point in time when the phase transition has just finished heating the star’s interior, and the hybrid star resumes cooling via neutrino emission. Figure 5.2 shows the resulting temperature profile for a hybrid star with \( \rho_C = 1000 \text{ MeV/fm}^3 \) and EOS 4 (see table 2.1). The knee between 2—1 km below the density cutoff \( \rho = \rho_0 \) (marked by the termination of the curve) is a consequence of the interface between the coexistence and confined matter regions, as the lattice contribution in the phase transition region leads to an increase in the total opacity by at least one order of magnitude throughout the entire coexistence region, see figures 5.3, 5.4. The short vertical lines in figure 5.2 label the beginning of the pure deconfined matter region in the hybrid star core, which can only only exist up to a limiting temperature (here corresponding to a surface temperature of 2.0 MeV).
Figure 5.2: Temperature profile for a hybrid star ($\rho_C = 1000\ MeV/fm^3$) based on EOS 4 (see table 2.1) for surface temperatures from $T = 1.25\ MeV$ (bottom) to $T = 2.5\ MeV$ (top) in steps of 0.25 MeV.

Figure 5.3: Rosseland mean free path for degenerate neutrinos for baryon and lattice contributions in a hybrid star ($\rho_C = 1000\ MeV/fm^3$) based on EOS 4 (see table 2.1) with heat flow equilibrium and a surface temperatures of 1.5 MeV.
Figure 5.4: Rosseland mean free path for degenerate neutrinos for baryon and lattice contributions in a hybrid star ($\rho_C = 1000\text{ MeV/fm}^3$) based on EOS 4 (see table 2.1) with heat flow equilibrium and a surface temperatures of 2.25 MeV.
CHAPTER 6

NEUTRINO EMISSION FROM HYBRID STARS

6.1 Numerical Methods for Hybrid Star Evolution

In order to obtain a platform for the study of hybrid star structure and transport phenomena, a code package was developed containing the following components:

1. Arbitrary combinations of equations of state for different phases of matter.

2. Equation solver for chemical equilibrium conditions in the confined/deconfined coexistence phase, see sec. 2.2.4 (D=7 with trapped electron neutrinos, D=6 for \( \mu_{\nu_e} = 0 \))

3. Integrator for stellar structure equations (sec. 2.3.1) for a given equation of state.

4. Finite difference neutrino diffusion code in two dimensions.

5. Capability to track three types of neutrinos, \( \nu_{\mu/\tau} \) and their antiparticles with zero chemical potential and degenerate \( \nu_e/\nu_\bar{e} \), with arbitrary cross sections.

6. Interface to the VH-1 hydrodynamics code in two dimensions (sec. 3.8) for sections of the hybrid star.
7. Run-time graphical output of physical quantities using the Open-GL library.

Despite of its larger overhead for numerical calculations compared to other programming languages, the code was mainly written in C++ because the modularity its object-oriented style encourages makes upgrades and debugging easy. Where necessary, such as with the VH1 hydrocode or the Lattimer/Swesty EOS, FORTRAN components were built in as subroutines.

For most special functions, such as Fermi or Debye integral, the Gnu Science Library (GSL) was used. As most neutrino cross sections have to be numerically integrated over and need to be updated every few timesteps, their computation can take up the bulk of the processing time. For that reason, 19-point Milne integration was used to integrate the Rosseland mean free paths, an approach which was determined to be accurate to within $\sim 2\%$.

Simulations were run via operator splitting [26], where the effect of different physical processes (such as heat transport, hydrodynamical flow) on quantities such as temperature and density was computed separately and summed up. Simulation time was evolved in explicit timesteps. Feedback effects in implicit methods, which would allow longer timesteps, would have to be addressed semi-analytically before numerical integration. The explicit method was therefore chosen because of its transparency and flexibility in its ability to include new effects [26]. Heat diffusion can be evolved using either linear timesteps or second or fourth order Runge-Kutta steps, where the timesteps ($\sim 10^{-4} - 10^{-3}$ s) are determined by decreasing them until convergence of an extended run ($\sim$ seconds). If the hydrodynamics of the envelope was included, timesteps were set by the Courant limiter of VH-1 (that range being $\sim 10^{-7}$ s). Material and neutrino transport properties were updated as appropriate (usually every
\( \sim 10 \) timesteps), the frequency was again determined by convergence of longer runs. Since the star radiates away energy in the form of neutrinos, it will contract over time, which necessitates a reintegration of the TOV equations at regular intervals. This was done every \( \sim 500 \) timesteps, which turned out to be more frequent than necessary.

### 6.2 Neutrino emission timescales of Hybrid Stars

Without the formation of a permanent lattice of scattering centers in the coexistence phase, traditional estimates of the neutrino cooling timescale range up to \( \tau \equiv \frac{d \log(L_\nu)}{dt} \sim 10 \text{ sec} \) [36, 170] (where \( L_\nu \) is the neutrino luminosity). This would in itself have to be considered as an upper estimate due to the opacity-decreasing effects of correlations (see sec. 4.3). With a neutrino mean free path of \( \lambda_\nu \sim 1 \text{ cm} \) in the presence of a phase transition lattice, however, a simple estimate of the neutrino diffusion timescale yields \( \tau \approx (R/\lambda_\nu)^2 \times \lambda_\nu/c = 33 \text{ sec} \). Results from a series of diffusion calculations with a hybrid star based on EOS 6 (table 2.1) for different temperature profiles (surface temperatures \( T = 1.25 - 2.25 \text{ MeV} \) in steps of 0.25 MeV, see sec. 5.2) are shown in figure 6.1. The cooling timescale \( \tau \) is shown in figure 6.2, with obtained results of \( \tau = 1 - 2 \text{ min} \). The “glitches” in the curves are numerical relics caused by the periodic reintegration of the hybrid star structure, as outlined in sec. 6.1, which can add or cut off the respective outermost mass zone. In general, the luminosity fairly well reproduces an exponential decay with a timescale only weakly decreasing with increasing temperature, which is not a priori clear considering the fairly complex interdependence of neutrino cooling and opacity.
Figure 6.1: Luminosity of a hybrid star based on EOS 6 (table 2.1) versus time of completion of the coexistence phase for different initial surface temperatures (see sec. 5.2).

Figure 6.2: Exponential decay timescale for the luminosity of a hybrid star based on EOS 6 (table 2.1) versus time of completion of the coexistence phase for different initial surface temperatures (see sec. 5.2).
6.3 Neutrino Transport for Anisotropic Coexistence Phases in Hybrid Stars

6.3.1 Neutron Star Kicks - Astronomical Observations

There is an increasing body of observational data pointing towards neutron stars obtaining natal kicks with $v_K$ up to $\sim 1000 \text{ km/sec}$, well above normally observed average stellar velocities of $v \sim 30 \text{ km/sec}$. Major discrepancies among different inferred kick velocity distributions are whether the distribution is bimodal and the fraction of neutron stars in the low- and high-velocity tails. Among the objects of observation are isolated pulsars, surveys of which have been interpreted with roughly Maxwellian- shaped distributions [146, 109, 25, 171] around $v_K \sim 200 - 300 \text{ km/sec}$ as well as double peak distributions $(90 - 175 \text{ km/sec}, 500 - 700 \text{ km/sec})$ [53, 10] with a considerable high-velocity tail $> 1000 \text{ km/sec}$ in the latter case. Explanations based on kicks imparted during binary breakup following the supernova were dismissed [57, 209]. Data from neutron star binaries exist both on the orbital parameters of individual binaries and in population surveys of classes of binary stars. The former revealed orbital parameters which appeared to be consistent with neutron star kicks of several $100 \text{ km/sec}$. The latter include surveys of low mass X-ray binaries (LMXB) [123], high mass X-ray binaries (HMXB) [164] and double neutron star systems (DNS) [89] and a comprehensive Monte Carlo population survey including all classes of neutron star binary systems [88]. The results point to a bimodal distribution at $(0 \text{ km/sec}, 600 \text{ km/sec})$. A rather severe limit, $< 1 \%$, on the low energy tail of the distribution is set by the number of observed isolated neutron stars in the accreter phase [171]. This might conflict with the pulsar retention in globular clusters, unless neutron stars born in certain binaries (e.g. HMXB, see [163, 164]) receive a lower kick

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velocity. Despite some uncertainties in the detailed distribution, it can be considered as well-established at this point that core collapse supernovae can be asymmetric, leading to neutron star kicks which can exceed several 100 km/sec.

6.3.2 Energy Transport through Craters in the Coexistence Phase - A Model for Neutron Star Kicks

Since about 99% of the neutron star binding energy in a core collapse supernova is transported away by neutrinos, a small asymmetry of \( \alpha \approx 1\% \) is sufficient to account for kicks of several 100 km/sec \( (\alpha = \frac{Mv_K c}{E_{\text{total}}}) \) [133]. The possibility of a hybrid star with a crystalline confined/deconfined matter coexistence region as outlined in chapter 3 that has a strongly enhanced neutrino opacity (sec. 4.8) suggests a neutron star kick scenario via a non-spherically symmetric surface “geo”-graphy in the coexistence region. In order to assess the degree of asphericity, a series of exploratory calculations was conducted. A rotationally symmetric crater, extending over 60° in a 2D-projection of the hybrid star and 1 km deep, was cut into the coexistence region, see the illustration figure 6.3. The depth is motivated by the range of the lattice susceptible to fracture by shear stresses typically associated with hydrodynamical flows (see sec. 3.7), whereas deeper layers should remain stable. The lateral size corresponds to the size of convective cells typically observed in hydrodynamical studies of PNS convection [124, 116]. The void is filled with material from the deep sections of the confined matter envelope. Since the timesteps to numerically track hydrodynamical flow for densities \( \rho > \rho_0 \) need to be below \( \sim 10^{-7} \) sec., and the relevant cooling timescales are \( \sim 100 \) sec, a full hydrodynamical simulation is prohibitive for reasons of both numerical accuracy and complexity. Energy transport with heat diffusion alone can be accurately done with timesteps of \( \sim 10^{-4} \) sec and is justified
Figure 6.3: Schematic drawing of a “crater” in the solid coexistence phase lattice of a hybrid star.

by any hydrodynamical flow patterns, such as convective cells, being in steady state compared to cooling timescales, as $\tau_{\text{hydro}} \sim 10^{-3}$ sec.

The resulting net acceleration versus time after the crater formation for a hybrid star based on EOS 6 (table 2.1) with different surface temperatures is shown in figure 6.4. The secondary peaks in some plots after several seconds are caused by the periodic update of the stellar structure using the TOV equations (sec. 2.3.1), as the hybrid star cools. The crater is always kept at a depth of 1 km, and can therefore descend to a lower radius. It then encompasses a high-temperature region with previously high neutrino opacity, liberating a considerable amount of thermal energy which then escapes, artificially boosting the hybrid star. These secondary acceleration peaks contribute up to 20% to the terminal kick speed, which is then assumed to be the numerical uncertainty in the present calculations. The terminal kick speed
Figure 6.4: Acceleration of a hybrid star based on EOS 6 (table 2.1) for a crater (depth: 1 km, lateral extent: 60°) in the solid coexistence phase for different initial surface temperatures.

versus surface temperature is plotted in figure 6.5. The results are consistent with observed neutron star kick speeds. There is a natural upper limit to the kick speeds in the framework of the present model. An equilibrium heat flow configuration (see sec. 5.2) with some limiting surface temperature > 2.25 MeV will eventually correspond to temperatures in the interior which are beyond the melting temperature of the underlying phase transition lattice (see sec. 3.5), in which case no crater can be supported any more. This limits neutron star kicks to the range below ~ 1000 m/s for the specific crater depth chosen.

The kick timescale is ~ 10 sec., which is small compared to the cooling timescale of the hybrid stars determined in sec. 6.2. The reason for this becomes obvious when comparing the evolution of the temperature profiles in opposite radial directions, one
Figure 6.5: Terminal kick speed for hybrid star versus initial surface temperature (see fig. 6.4).

of which points through the middle of the crater, see figure 6.6 showing superimposed snapshots of the profiles $\Delta t = 6$ sec apart for an initial surface temperature of $T = 2$ MeV. The kick timescale reflects the obtaining of a new equilibrium temperature profile after the cooling of the regions of solid mixed phase below the bottom of the crater.

6.3.3 Hydrodynamical Flow Patterns in Coexistence Phase Craters

Following the stability analysis in section 3.7, a shear stress of $\sim 10^{-3}$ MeVfm$^{-3}$, corresponding to hydrodynamical flow with velocities of $\sim 10^6$ m/s is necessary to prevent the recrystallization of a fractured lattice. With deleptonization timescales fast compared to neutrino cooling and kick timescales, as pointed out in section 5.1.2,
any sustained convection thereby has to be temperature gradient driven (see the Ledoux criterion in sec. 3.6). The presence of strong enough convective flows has been verified in a crater 1 km deep for up to $\sim 100$ msec. As turnover timescales are $\sim 1 - 10$ msec, it can be concluded that the convective flows can exist for times comparable to the kick timescales obtained in the previous section. Figure 6.7 shows a snapshot of the lateral convection velocities within the crater 10 ms after its formation.

### 6.3.4 Other Suggested Models for Neutron Star Kicks

Models for intrinsic neutron star kicks developed so far can be classified into three categories: hydrodynamical, neutrino/magnetic field driven and the “EM rocket effect” (see the review in [133]). A consistent hydrodynamically driven neutron star
Figure 6.7: Snapshot of lateral (in $\theta$-direction) flow velocities at $t = 10$ ms within the crater discussed in sec. 6.3.2 (surface temperature of $T = 2$ MeV).
kick model is still lacking to date. In full-scale hydrodynamical core collapse supernovae, the treatment of possible kicks has only been in the framework of statistical fluctuations in the distribution of convective cells. Net kick speeds were all found to be well below \( v_K = 200 \text{ km/sec} \) \[119, 124, 35, 120\]. Any hydrodynamical explanation for neutron star kicks must therefore rely on presupernova perturbations which grow as \( \Delta \rho \sim r^{-1/2} \) and \( \Delta v \sim r^{-1} \) as the star collapses \[135\]. Kick velocities of \( v_K \sim 500 \text{ km/sec} \) have been shown to result if the density in a wedge of the outer Chandrasekhar core of the supernova progenitor is artificially reduced by 15\% \[34\]. Suggested sources of such perturbations include convective burning in the oxygen shell \[17\] and overstable g-mode oscillations in the progenitor core \[99\], none of which were verified to produce anisotropies sufficient for the observed kicks, though. The statistical nature of hydrodynamical perturbations would lead to a Maxwell-type distribution of kick speeds (centered at \( \bar{v}_K = 0 \text{ km/sec} \) in 3D). Any bimodal distribution could not be explained by the standard core collapse model. More comprehensive surveys of the kick speed distribution in the future will therefore help constrain hydrodynamical models. Extensions of the classical core collapse supernova picture include bipolar jets produced either by a magnetorotational mechanism \[125\] or as byproducts of rotational collapse \[90\]. It is however, not clear how the necessary asymmetry in the polar jets of \( \sim 2 \) could be produced. Another class of models involves the fragmentation of a massive collapsing core which might impart a significant kick by subsequent fragment mergers \[56\] or the explosion of one of the fragments \[51\].

All the neutrino-driven models suggested so far require the presence of a strong magnetic field. Anisotropies in the cross section will lead to anisotropic momentum
transport around the neutrino energy sphere [131]. One possibility is the parity violating nature of the weak interaction [137], which has been shown to possibly lead to kick speeds in the observed range for magnetic fields of \( B \sim 10^{15-16} \text{ G} \) [9]. Fields of more than an order of magnitude above that were reported to be required to polarize nucleons and electrons sufficiently to distort final state Landau levels in neutrino scattering sufficiently [136, 46]. Strong magnetic fields also distort the MSW resonance spheres for possible neutrino oscillations, which might lead to anisotropic momentum transport if they are located between the energy spheres of the neutrino flavors involved. Kick speeds of several 100 km/sec were predicted for oscillations between \( \nu_x \) and \( \nu_e \) neutrinos for fields between \( 10^{14} \text{ G} \) and \( \sim 10^{16} \text{ G} \) [129, 130, 175, 14]. However, this would require \( m_{\nu_x} \sim 100 \text{ eV} \), which would overclose the universe. Oscillations involving a postulated sterile neutrino were suggested [103], predicting required fields of \( \sim 10^{15-16} \text{ G} \). More exotic models include a matter-enhanced transition between left- and right-handed neutrinos of different flavors by spin-flavor precession for neutrino transition magnetic moments \( \mu_{\nu} > 10^{-15} \mu_B \) [1] and an extension of the Standard Model by two new singlet leptons for each generation of leptons [104], both of which would require magnetic fields beyond \( \sim 10^{15} \text{ G} \). The argument has been made that all studies concerning the production of neutron star kicks via a distortion of resonance spheres were inherently flawed, because they failed to treat the temperature gradient in the resonance regions in a self-consistent manner with the changing neutrino flux in case of a shifted resonance sphere. It has been suggested that no net emission anisotropy would be generated to leading order, while the required magnetic fields for higher order effects to be important were \( B \sim 10^{17} \text{ G} \) [121].
Recent observations point towards the existence of magnetic fields of $B \sim 10^{14-15}$ G in some neutron stars (magnetars) [61, 128]). These might be due to magnetorotational effects subsequent to core collapse, in which case magnetized regions the size of convective cells $\sim 1$ km with $B \sim 10^{14-15}$ G were predicted [199]. Whereas this might be just sufficient to cause some anisotropic neutrino emission (although the models clearly need quantitative refinement), the nature of the random formation of convective cells fails to reproduce a bimodal kick velocity distribution. Further, observations point towards most neutron stars being born with kick speeds of at least several 100 km/s, which would mean that most neutron stars would have to be born as magnetars, for which there is no evidence at this point.

The “EM rocket effect” [110, 134] describes a kick imparted to a spinning neutron star with off-centered magnetic dipole moment via electromagnetic radiation. Its ability to explain neutron star kicks depends on the competition of gravitational and electromagnetic wave emission for angular momentum loss over timescales of $> 100$ sec, which is not well known to date.

### 6.3.5 Other Observational Signatures of Anisotropic Neutrino Transport

**Spin-Kick Alignment and the Kick Timescale**

A conclusive survey of the degree of alignment between the kick direction and the spin axis would greatly constrain the various kick models. Linear and spin angular momentum are aligned if either the kick is imparted along the spin axis or the kick timescale is long compared to the rotation period, so that spin averaging can occur [194, 134]. For rotation periods $\ll 1$ sec, the latter is always true for the EM rocket effect and for neutrino-driven kick models on account of deleptonization and neutrino
cooling timescales of at least a few seconds. However, it might be difficult to satisfy for hydrodynamically driven models which are likely to operate on timescales of the shock travel time after core bounce, $\tau_{\text{shock}} \sim 100\ \text{msec}$. With typical radii for the formation of convective patterns during collapse $\sim 100\ \text{km}$ and a final neutron star radius $\sim 10\ \text{km}$, a spin period of the formed compact object of $P_{NS} < 1\ \text{msec}$ would have to result, which very likely can be ruled out for stability reasons [101, 102]. Recent indications for spin-kick alignment of some pulsars (e.g. Vela and Crab, see [134]) therefore appear to support neutrino-driven collapse models. The kick model suggested in sec. 6.3.2 would likely exhibit a tendency to have its spin and rotation axes aligned for fast rotating PNS’s, even without spin averaging, as it was shown [90] that the polar regions are strongly preferred sites for the formation of convective cells.

**Gravitational Waves from a Rotating Anisotropic Hybrid Star**

On the grounds of angular momentum conservation alone, most neutron stars are expected to be rotating upon formation. Several ms -pulsars have been observed [52] which, based on estimates of the dissipation of angular momentum during rotating collapse, represent the fastest rotating pulsars [101, 102]. For any neutrino driven kick with all but the longest $\tau_{\text{ref}}$, rotational averaging will align the rotation with the kick axis over several times the rotation period [194] and thus provide another window on core collapse supernovae via gravitational waves, in addition to the ones discussed in [208, 91]. The signature of an anisotropic “neutrino jet” which is initially unaligned with the poles would be similar to that of a decaying bar mode.
The gravitational wave amplitude in the TT ("transverse-traceless") projection of the quadrupole radiation field is [201, 218]

\[ h^{TT} = \frac{1}{8} \sqrt{\frac{15}{\pi}} \sin^2(\theta) \frac{A_{20}}{R}, \]  

(6.1)

where \( A_{20} \) is the only non-vanishing quadrupole term, \( R \) is the distance to the observer, and \( \theta \) is the orientation of the rotation plane (I assume \( \sin \theta = 1 \)). The quadrupole moment is, assuming axisymmetry [91]

\[ A_{20} = \frac{G_{\text{grav}}}{c^4} \frac{16 \pi^{3/2}}{\sqrt{15}} \int_{-1}^{1} \int_{0}^{\infty} d r d z r^2 \rho \]
\[ \times [v_r^2(3z^2 - 1) + v_\theta^2(2 - 3z^2) - v_\phi^2] \]
\[ -6v_r v_\theta z \sqrt{1 - z^2} - r \frac{\delta \Phi}{\delta r}(3z^2 - 1) \]
\[ +3 \frac{\delta \Phi}{\delta \theta} z \sqrt{1 - z^2}], \]  

(6.2)

where \((v_r, v_\theta, v_\phi)\) is the velocity vector in polar coordinates. For rotation periods fast compared to the neutrino diffusion timescale, the motion of mass-energy will be dominated by \( v_r \), and I will neglect all terms containing \( v_\theta \) and \( v_\phi \). Further, anisotropies in the gravitational force can be neglected, hence all terms with partial derivatives of the Newtonian gravitational potential \( \Phi \) will be dropped. The result is the trivial integral

\[ A_{20} = -\frac{G_{\text{grav}}}{c^4} \frac{16 \pi^{3/2}}{\sqrt{15}} \int_{-1}^{1} \int_{0}^{\infty} d r d (\cos \theta') (r^2 \rho \sin^2(\theta') r^2 \omega_{\text{rot}}^2), \]  

(6.3)

\( \omega_{\text{rot}} \) being the rotation period. If indeed the model proposed in sec. 6.3.2 causes the observed neutron star kicks, the quadrupole moment will be dominated by the low mass density in the crater. Due to both the anisotropic thermal energy distribution itself and a larger radius for hotter hybrid stars (given the same central density), the gravitational wave signal will, however, be modified by the temperature. Figure 6.8
shows $h^{TT}$ versus the surface temperature using the same model as in 6.3.2 (EOS 6 and surface temperatures from 1.25 MeV to 2.25 MeV). The distance and rotation period assumed were 1 kpc and 10 msec, respectively. The gravitational wave strain is only weakly dependent on the temperature, and the dependence is dominated by the hybrid star radius increasing from 10.3 km to 10.9 km as the surface temperature increases from 1.25 MeV to 2.25 MeV. The thermal contribution to the $h^{TT}$ alone should be of similar magnitude than in other neutrino-driven kick models; it is, however, an order of magnitude smaller than the total strain in the present model. Figure 6.9 shows the thermal wave strain versus time for different surface temperatures. Its increase in magnitude versus time reflects the lower opacity of the material in the crater. Unless the crater is located exactly perpendicular to the rotation axis, the signal would decay over several rotation periods (i.e., within $\sim 1$ sec). This is not shown in figure 6.9, as it depends on the initial orientation of the hybrid star rotation axis.

Gravitational wave astronomy has attracted considerable attention in recent years, and hopefully, several of the running or planned detectors [144] will be online during the next core collapse supernova. The detection limits in the $\tau \sim 1 - 100$ ms regime are, with future ground-based observatories, $h < 10^{-22}$ (enhanced LIGO, see [91]), sufficient to detect the signal in fig. 6.8. For a useful measurement of the thermal contribution only (fig. 6.9), the supernova would have to be significantly closer ($\sim 100$ pc).
Figure 6.8: Gravitational wave strain $h^{TT}$ for a hybrid star based on EOS 6 (table 2.1) with crater (depth: 1 km, lateral extent: 60°) in the solid coexistence phase for different initial surface temperatures.

Figure 6.9: Thermal gravitational wave strain $h^{TT}_{therm}$ versus time for a hybrid star based on EOS 6 (table 2.1) with crater (depth: 1 km, lateral extent: 60°) in the solid coexistence phase for different initial surface temperatures (neutrino jet assumed perpendicular to rotation axis, i.e. not considering spin averaging).
CHAPTER 7

DETECTION OF SUPERNOVA NEUTRINOS

7.1 General Overview over Supernova Detector Designs

7.1.1 Existing and Planned Supernova Neutrino Detectors

The only direct observational confirmation of the general core collapse supernova model of trapped neutrinos leaving the PNS on timescales much longer than weak interaction timescales are the neutrinos detected during supernova 1987-A. However, both the Kamiokande [78] and the IMB [76] observatories were only sensitive to electron-antineutrinos via their capture on protons in water:

\[ p + \bar{\nu}_e \rightarrow n + e^+ , \]  

and the subsequent detection of Cerenkov radiation from \( e^+ \). Experiments utilizing this water-Cerenkov principle account for the bulk of the currently available data on the disappearance of atmospheric \( \mu \)-neutrinos [87] (Super-Kamiokande [70], IMB [71] and Kamiokande [77]) and oscillations of solar electron neutrinos (Homestake [63], Super-Kamiokande [83], SNO [81, 82] and Kamiokande [68]). The neutrino detectors LVD [69, 66] and MACRO [80, 79] at Gran Sasso rely on the capture of electron antineutrinos by free protons in liquid scintillator and the neutrino telescope AMANDA [65, 75] uses protons in Antarctic ice for the same purpose.
Spallation detectors are based on reactions such as the neutral-current neutron spallation
\[ \frac{A}{Z} X + \nu_x \rightarrow \frac{A-1}{Z} X + ^1_0 n + \nu_x, \]  
(7.2)
or the charged-current reaction
\[ \frac{A}{Z} X + \nu_e \rightarrow \frac{A}{Z+1} X' + e^- \rightarrow \frac{A-1}{Z+1} X'' + e^- + ^1_0 n. \]  
(7.3)
Examples for spallation detectors are the gallium solar neutrino experiments GALLEX [72] and SAGE [67] and the iron calorimeter Soudan2, which was used for atmospheric neutrinos [74]. The high energy physics community also uses spallation detectors for its short- and long-baseline neutrino oscillation experiments, e.g. MINOS [73] and LSND [64].

An observation of the fluxes of different neutrino flavors is vital for the understanding of core-collapse supernovae. In particular, for effects related to the occurrence of strangeness, a detection rate of \( \sim 1 \text{ sec}^{-1} \) for each neutrino flavor at late times, \( t_{pb} \sim 10 \text{ sec} \), is desirable. Further, the neutrino luminosities and distances of Galactic core collapse supernovae provide a unique opportunity for extremely long baseline measurements for the study of new neutrino physics beyond the standard model. In this context, the need for a large neutral-current neutrino detector has become increasingly obvious in recent years, where a spallation detector concept was suggested based on the use of different nuclei [48] to extract information on the neutrino energy spectra.

### 7.1.2 OMNIS and other Neutral Current Neutrino Detectors

The Observatory for Multiflavor NeutrInos from Supernovae (OMNIS) [48, 27] is planned to utilize neutron spallation thresholds in various nuclei, such as \(^{208}\text{Pb}, ^{56}\text{Fe}\,
or $^{37}$Cl. OMNIS is being designed to provide a large sample of $\mu$- and $\tau$-neutrinos and their anti-neutrinos as well as electron antineutrinos with a time resolution which is only limited by the event rate (and, hence, the distance to the supernova). Recently confirmed oscillations between $\mu$ and $e$ - neutrinos [82] in PNS atmospheres will result in a high energy electron neutrino flux comparable to the $\mu/\tau$ - neutrino fluxes. With the planned quantities of material and the detection strategy outlined below, the lead modules of OMNIS alone will be able to detect several thousand $\mu$- and $\tau$-neutrino events from a typical supernova at the center of the Galaxy [28, 217].

There are other major experiments which are expected to detect significant numbers of neutrinos through similar spallation reactions. The Gran Sasso experiments LVD and MACRO with carbon as a target will yield a detectable signal in their scintillator. Super-Kamiokande is expected to see some $e$-neutrinos and some $\mu$- and $\tau$-neutrinos through the detection of $\gamma$-rays produced in the neutral-current channels $^{16}$O$(\nu, \nu'p/n\gamma)$ [139]. Their total number for a core-collapse supernova at a distance of 8 kpc has been estimated to between 300 and 560, depending upon the neutrino spectrum. SNO would be expected to produce around 750 break-up events of deuterium following neutrino neutral-current scattering for a supernova at the same distance [19, 20]. A significant number of charged-current capture events of high-energy electron neutrinos ($\nu_\mu \leftrightarrow \nu_e$, due to the more energetic $\mu$- neutrino spectrum) can also be expected. The data from these detectors represent different thresholds and will therefore complement the energy resolution and counting statistics of OMNIS.

These detected neutrinos will provide a window for looking well beyond the peak of the supernova neutrino burst to study the evolution at times up to $\sim 1$ minute after the start of the pulse, an important diagnostic of the cooling mechanism and
the possible macroscopic appearance of strange matter. A few thousand events are also adequate for accurate time-of-flight measurements of the mass difference of different neutrino flavors with a resolution well below the $\Delta m^2_{\nu} \approx 50/30$ eV for Super-Kamiokande and SNO, respectively (based on the counting statistics arguments presented in [19, 20]).

A possible scenario is a core-collapse supernova proceeding to a black hole, which should produce an abrupt termination of neutrino flux as well as several remarkable measurements. The timing and shape of the signal preceding the cut-off can help shine light on the protoneutron star evolution and the high-density equation of state [15]. As the black hole expands outward from the center of the star it will envelope successive neutrinospheres [15]. One might therefore expect the $\nu_{\mu/\tau}$-flux to be terminated earlier than the $\bar{\nu}_e$-flux which would in turn occur before the termination of the $\nu_e$-flux. The difference in time between these terminations has been estimated to be of order 1 ms [15]. If the supernova is close enough, OMNIS would be able to observe these differences, and thus could infer the structure of the neutrinospheres. Lastly, the termination of the luminosity provides a well defined feature that would provide an opportunity for time-of-flight mass measurements of the neutrino mass with unparalleled accuracy, down to limits of 2 eV for $\bar{\nu}_e$ in Super-Kamiokande and as low as 4 eV for $\mu$- and $\tau$-neutrinos from OMNIS [18].

### 7.1.3 Neutrino Cross Sections Relevant to OMNIS

Part of OMNIS is planned to consist of several $\frac{1}{2}$ kT modules with lead and possibly also modules with iron as target materials. Some of the incoming neutrinos from a supernova will undergo charged-current or neutral-current interactions with
the composite nuclei that result in neutron emission. The cross sections for such reactions depend on the neutrino energy, the Q-value for the reaction and the availability of suitable energy levels in the target nucleus. There are no available experimental data for the cross sections governing these reactions, with the exception of that for $^{56}\text{Fe}(\nu_e, e^-)^{56}\text{Co}$ measured by the KARMEN collaboration [148], but theoretical studies have been conducted by two groups [94, 126]. Despite qualitative agreement between the two groups, some uncertainty remains, due to the largely unknown location of the Gamow-Teller resonance relative to the neutron separation energies. These separation energies are 10.67 MeV for $^{56}$Fe and 7.19 MeV for $^{208}$Pb for single-neutron emission and 19.78 MeV and 13.31 MeV, respectively, for double-neutron emission. The single-neutron channels in lead and in iron and the two-neutron channel in lead constitute three observables closely related to the neutral-current thresholds that can be used to deduce the relevant contributions of the various neutrino flavors via the different energy dependencies of their cross sections.

7.1.4 Expected Number of Events in OMNIS

The results presented in chapter 7 were completed and published before the most recent oscillation results from SNO showing oscillations between $\mu$ or $\tau$ and $e$ neutrinos [82]. In core collapse supernovae, the harder spectrum of the latter results in a strongly enhanced charged current event rate in the case of oscillations of that type. The efficiency of matter-enhanced oscillations depends on the structure and composition of the outer layers of a PNS. Different groups have just recently begun to incorporate the SNO results, including possible feedback effects on the structure due to oscillations [162, 198]. A full treatment including the fluxes for the different flavors is not yet
Table 7.1: Comparison for single- and double-neutron events liberated from iron and lead for the different reaction channels (per kT)

<table>
<thead>
<tr>
<th>Material, Event Type</th>
<th>CC-(\nu_e)</th>
<th>CC-(\bar{\nu}_e)</th>
<th>NC-(\nu_e)</th>
<th>NC-(\bar{\nu}_e)</th>
<th>NC-(\nu_x)</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb, single-n, no osc.</td>
<td>59</td>
<td>0</td>
<td>8</td>
<td>37</td>
<td>677</td>
<td>781</td>
</tr>
<tr>
<td>Pb, double-n, no osc.</td>
<td>26</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>20</td>
<td>47</td>
</tr>
<tr>
<td>Fe, single-n, no osc.</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>6</td>
<td>146</td>
<td>163</td>
</tr>
<tr>
<td>Pb, single-n, full osc. (\nu_\mu \leftrightarrow \nu_e)</td>
<td>826</td>
<td>0</td>
<td>184</td>
<td>35</td>
<td>516</td>
<td>1563</td>
</tr>
<tr>
<td>Pb, double-n, full osc. (\nu_\mu \leftrightarrow \nu_e)</td>
<td>1852</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>15</td>
<td>1874</td>
</tr>
<tr>
<td>Fe, single-n, full osc. (\nu_\mu \leftrightarrow \nu_e)</td>
<td>57</td>
<td>5</td>
<td>37</td>
<td>6</td>
<td>112</td>
<td>217</td>
</tr>
</tbody>
</table>

Available at present. Accordingly, all expected rates of other detectors quoted in sec. 7.1.2 might have to be modified in future estimates. As limiting cases, the total yields of neutrons liberated in the Fe and Pb modules are calculated both for full oscillations \(\nu_e \leftrightarrow \nu_\mu\) and their complete absence. However, none of the results other than the total yields presented in this thesis depend on the specific neutrino oscillation scenario. The yields are based on a “standard” supernova in the center of the Galaxy (distance = 8 kpc, \(E_{tot} = 3 \times 10^{53}\) ergs). The number of neutrons liberated per kT in OMNIS modules of lead or iron was calculated by Alexander Murphy [152] using luminosities provided by the group of Anthony Mezzacappa [150] and is summarized in table 7.1. The cross sections for lead were taken from Fuller et al. [94] and for iron from Kolbe/Langanke [126]. The strong enhancement of the overall yields, in particular for double neutron events, is evident.
7.2 Features of the OMNIS detector

7.2.1 Gd loaded Liquid Scintillator for Neutron Detection

Once neutrons are emitted, they interact with the neutron source material (Fe or Pb) and the neutron detector, assumed in the present study to be organic liquid scintillator, which serves three functions: neutron moderator, sink and detector. The use of Gd in liquid organic scintillators is well established [193], it has the highest known neutron capture cross section [179] and its solubility is high enough to make it the preferred neutron sink at this time. The possible alternatives Li and B have not been found to be soluble in organic liquids in sufficiently high concentrations and there are no known organic molecules containing Li or B that have a light yield similar to Gd loaded scintillator.

7.2.2 Particle Propagation and Event Identification

The neutrons produced by the neutrino interactions are expected to have initial energies of $\sim 1$MeV [126], hence moderation will primarily occur through elastic scattering off the protons in the scintillator. Our Monte Carlo transport calculations [217] show that the neutrons lose nearly all of their kinetic energy within $\propto 100$ns (see fig. 7.1) after emission, resulting in the production of a significant number of scintillation photons. With a 0.1% loading of Gd in the scintillator, capture occurs after $\approx 30$ $\mu$s, see also figure 7.2, predominantly on $^{155/157}$Gd. Figures 7.1 and 7.2 were generated with the optimized design for the lead modules presented below, but these time-scales are not very dependent on the detector material and geometric configuration used. Further, the capture time scales are consistent with a recent study [28, 153] performed using a prototype scintillator vessel and with similar Monte Carlo
Figure 7.1: Neutron Moderation Time-scale

studies conducted in the past [204, 205]. To enable discrimination against cosmic rays and $\gamma$-rays produced by the decay of radioactive impurities in the lead or iron or other detector components, a double-pulse technique with a time window between prompt and capture signals of $\approx 50$ $\mu$s will be used. The characteristic second pulse, caused by the emission of $\gamma$-rays following capture on Gd or H in the scintillator vessels, is used to identify the neutron pulse. The time of neutron liberation is then known to within a few 100 ns. Although pulse shape discrimination might be considered as a means to identify the neutron induced events, this would be impractical for detectors as large as those required for a facility of the size of OMNIS.

### 7.2.3 Design Constraints

Generally, the more fine-grained neutron sources and detection elements are, the higher will the neutron detection efficiency be, since neutron energy losses and captures before they reach the detector will be minimized [193]. Structural stability,
though, dictates lead walls with thicknesses of at least $\sim 10$ cm, interlaced by layers of scintillator with a thickness determined by the neutron attenuation length in organic liquids, which is several cm. Rectangular acrylic tubes are assumed as scintillator containment vessels, which guide the scintillation light via total internal reflection off the acrylic/air interface. Their length will be limited by the attenuation length for photons in the scintillator, which is about 450 cm [166]. The rectangular shape maximally fills out the scintillator walls, and can be manufactured for a considerably lower price than comparable units with different geometries and materials. For the regions on the tank ends not occupied by the photomultiplier tubes, a highly reflective coating of aluminum or $\text{Ti}_2\text{O}$ is planned. A similar design was used for the Palo Verde [166] reactor experiment, which utilized acrylic vessels with $0.1\%$ Gd loaded pseudocumene (PC) as a liquid scintillator in a solution with 60% mineral oil. The
present analysis has assumed a similar material which has a high proton concentration \((\frac{\beta}{\alpha} \approx 1.64)\), making it a good moderator. However, to increase the range over which the thickness of the detector walls can vary, stacking of the scintillator vessels in single-, double- or triple-width columns has been considered. To provide shielding from external radiation sources and reflect escaping neutrons back into the detector, each module is encased in an external lead or iron hull. For the lead, it is important to be free of antimony, which acts as a significant neutron sink in concentrations above \(\sim 1\%). Figures 7.3 and 7.4 show a schematic design for a \(\frac{1}{2}\)kT lead module.

7.3 DAMOCLES - Model for OMNIS

7.3.1 Basics of Monte Carlo Code Development

The Monte Carlo code for OMNIS was designed to provide:

1. Modularity: The detector consists of three principal regions, the scintillator vessels, the lead walls and the outer hull which have to be simulated in a large number of different configurations, necessitating easy control of the detector configuration. In particular, the code handles the operation of a large number of identical cells.

2. Parameter Control: The development of OMNIS requires the control of all parameters regarding geometry, data acquisition and the materials used, including the composition of the scintillator and possible sources of radioactive background in all detector components. That includes the cost of a given detector configuration, which is necessary to optimize cost efficiency.
Figure 7.3: Front View of an OMNIS Lead Module
Figure 7.4: Top View of Part of an OMNIS Lead Module
3. Data Flow Control: In order to design the best trigger mechanism, extraction of any kind of information about particle positions and energy deposited in the detector was accommodated.

4. Portability: Since the simulations were performed on both single- and parallel-processor architectures, the code had to be portable among different platforms.

7.3.2 Chronology of Events in the Monte Carlo Simulation

The treatment of events within the Monte Carlo code follows their chronology. Neutrons are first generated with appropriate energies from the spallation following neutrino interactions. Each neutron is then tracked through the detector, taking into account scattering reactions and eventual capture in one of the components or its escape from the detector. In the former case, capture $\gamma$-rays are traced through the detector until their eventual capture or escape. Neutrons leaving the detector are not tracked any further. The effects of including an additional external reflecting hull were tested beyond that required for shielding out external background radiation from the surrounding rock, but the resulting changes were negligible. Energy deposited into the scintillator via neutron or $\gamma$-interactions is converted into a number of scintillation photons, each of which is then traced until its absorption in the liquid, failure to reflect from a surface, or transmission to a photomultiplier tube.

7.3.3 Energy Spectrum of Double-Neutron Events

In the case of a double-neutron emission event, the available energy is distributed to both neutrons [126], and phase space arguments tend to equalize the energy of both neutrons. A Gaussian energy distribution was therefore assumed with the energy for each neutron limited to the total available energy, centered at and with a width of half
the total energy. The influence of the width of the Gaussian on the detection efficiency for double-neutron events in lead was investigated. The narrower the Gaussian, the higher the efficiency, since the probability for one neutron having very low energy is then small. However, the double-neutron efficiency was found not to depend very sensitively on the width; the efficiency changed by only 1%, if the width is changed to either 10% or 200% of the original value.

7.3.4 Neutron Interactions

Neutron Scattering

The total elastic scattering and absorption cross sections have been approximated using polynomials which were fitted to data from the ENDF/B file [41]. Resonances were averaged over and not treated in detail, since they are limited to narrow energy ranges. At the neutron energies with which OMNIS will be dealing, \( \sim 1 \text{ MeV} \), inelastic scattering can be neglected. The expansion of the elastic differential scattering cross section in Legendre polynomials can be written as

\[
f(\cos(\theta),E) = \Sigma_\ell \left( \frac{2\ell + 1}{2} a_\ell(E) P_\ell(\cos(\theta)) \right),
\]

where \( a_\ell \) are the (energy dependent) Legendre coefficients. Step functions were used based on the known Legendre coefficients as functions of the energy, and the series was terminated at \( \ell = 5 \) for Fe and \( \ell = 10 \) for Pb, beyond which contributions are negligible [41]. Both C and H (scintillator) exhibit an s-wave distribution in the angular dependence of elastic scattering in the center of mass system for the energy ranges considered [41], thus terminating the series in eq. (7.4) after \( \ell = 0 \). Since its concentration is only 0.1%, and its scattering cross section is comparable to the host material, we neglected scattering from Gd.
**Neutron Capture**

Neutron capture on Gd is well described by four photons with a total energy of 7.937 MeV (\(^{157}\)Gd) or 8.536 MeV (\(^{155}\)Gd) [204] with individual energies of between 50 keV and 3.5 MeV [42]. Capture on a proton leads to the emission of a 2.2 MeV photon. The effects of capture on carbon were negligible, as was confirmed by tests. A random \(\gamma\)-ray of up to 4 MeV was assumed to be produced by neutron capture on iron or lead. However, our results did not change at all if this process was left out completely, due to the high absorption cross section in Fe or Pb for \(\gamma\)-rays. For the same reason, any secondary \(\gamma\)-rays produced in inelastic scattering off Pb or Fe were found to be negligible.

**7.3.5 \(\gamma\)-ray Interactions**

The main energy loss channel for \(\gamma\)-rays in the energy range of interest here, up to a few MeV, in both the scintillator and the lead or iron, is Compton scattering, although all possible effects were included. The cross sections for the photoelectric effect, pair production, coherent scattering and Compton scattering were obtained from the XCOM database [156] and fitted for energy bins of 10 keV (1 keV for the first 100 keV in the scintillator). For the angular dependence in the differential cross section for Compton scattering the Klein-Nishina equation for photons scattering off electrons was used [118]. Immediate photoelectric absorption was assumed for \(\gamma\)-rays below an energy of 5 keV in Fe and Pb and 0.5 keV in the scintillator. Below these respective energies, all interaction channels except photoelectric absorption are negligible, and the attenuation length is \(< 0.6\) mm in the scintillator, \(< 0.12\) mm in lead and \(< 0.9\) mm in iron [156].

100
7.3.6 Scintillation Photons

The liquid scintillator used in the Palo Verde experiment had a light yield of 56% of that of anthracene [166], the latter being 16 photons per 1 keV [185]. The neutron energy conversion efficiency due to the higher charge density caused by recoil protons is about 40% of that of the γ-ray conversion efficiency [174]. The phototubes were assumed to detect a given photon with an efficiency of 20%. Since the tracking of the scintillation photons constitutes the bulk of computing time, only 20% of the actual number of photons were simulated, but a detection efficiency of 100% was adopted in the phototube. Scintillation photons are created with random polarization along either of two possible axes. The absorption cross section in the fluid is determined by an exponential law with an attenuation length of 450 cm. If the incident angle on a surface is larger than the limit for total reflection, the photon is reflected with 100% efficiency. The validity of this assumption was confirmed by introducing an additional finite absorption probability when a scintillation photon traverses the acryllic wall. Even for a transmission of only 95% for every reflection, the efficiency decreases by only 1%. For all other incident angles, every encounter with the walls was tested for possible transmission, with the reflection probability being a function of polarization and incident angle [118]. If the photon hits one of the scintillator vessel ends, it is either transmitted through the photomultiplier glass face with 98% efficiency or reflected off the highly reflective rim surrounding the phototube with an efficiency of 88% for aluminum. The latter value was confirmed by measurements conducted by Dr. Alex Murphy [152].
7.3.7 Event Identification

In order to discriminate against noise in the photomultiplier tubes and the electronics, a threshold was set requiring that one vessel has to register at least five photo-electrons on each end within a timebin. The length of a timebin was 100 ns, in accordance with the neutron moderation timescale and the expected resolution of the data acquisition system. The threshold of five photoelectrons was chosen because it is the lowest pulse that can reliably be detected in a photomultiplier tube, but the effect of changing that value was studied as well. If two such pulses occur in the same vessel within the time-to-amplitude converter time window, i.e. if both ends of one vessel therefore fire twice each, an event is registered in that vessel. Double-neutron events are identified based on two such events registering in separate vessels with the first pulse in both events happening within 100 ns.

7.3.8 Intrinsic Background Radiation

Assuming that only virgin lead will be used in OMNIS, all major internal background sources in the lead modules will be elements in the natural decay series or their daughters. In lead from the DOE RUN company, a survey of samples exhibited background ranging from $< 0.001$ to $\sim 10 \frac{dpm}{kg}$ [161], with most samples towards the lower end of the range. The dominant background source is $^{210}\text{Pb} \left( T_\frac{1}{2} = 22.2 \text{ yr} \right)$ [29]. One of its daughters, the short-lived $^{210}\text{Bi}$, emits 1.17 MeV $\gamma$-rays [140]. The absence of significant contributions from other isotopes, in particular the $^{226}\text{Th}$ series, was recently confirmed by CEMRC in Carlsbad [211]. Bremsstrahlung from the various beta-decays can be neglected, because its intensity peaks at energies below $\sim 0.25$ MeV, where lead absorbs the photons efficiently.
The total radioactivity in iron ranges between 0.00 and 0.17 decays per second per kg above 1 MeV [100]. Most of the background in iron is due to $^{60}$Co (1.17 MeV and 1.33 MeV, respectively; 60% of the background) and $^{40}$K (1.46 MeV; 30% of the background). An admixture of 10% $^{226}$Th (2.6 MeV $\gamma$-ray from $^{208}$Tl) was assumed for higher energy $\gamma$-ray emissions.

Another background source is the decay of $^{40}$K in the glass faces of the photomultiplier tubes. The available purity ranges from 1 to 100 decays per second per kg per face [158].

The detected frequency of false events $f_F$ should scale with the background rate $b$ as

$$f_F = k_1 b f_R + k_2 b^2,$$

(7.5)

where $k_1$ and $k_2$ are constants depending on the trigger time window and $f_R$ is the frequency of real neutron events. The linear term in $b$ enhances the detection efficiency by providing a missing second pulse to go with either the prompt (neutron-) or delayed ($\gamma$-) pulse. The quadratic term in $b$ describes coincident double-pulse triggers from the background, in which both pulses within the trigger window are caused by background radiation. It is important for the latter term to be small at all times, even when there is no supernova, in order to avoid false alarms. During the actual supernova pulse, the background contribution is then dominated by the linear term.

The number of background $\gamma$-rays occurring in a time bin, as implemented in the code, is Poisson distributed:

$$p_k = \frac{\lambda^k}{k!} e^{-\lambda},$$

(7.6)

where $k$ denotes the resulting number of background events and $\lambda$ is the number of events per time bin.
7.4 Efficiency Optimization

7.4.1 Optimization of Detector Dimensions

The basic philosophy adopted was to optimize the cost efficiency of the OMNIS modules rather than the absolute efficiency. Cost efficiency here is defined as efficiency divided by cost times US$ $10^7$, which ranges between 0 and 2. The dependence of the detection efficiency and cost efficiency on the parameters scintillator vessel dimensions, lead/iron wall thickness, hull thickness, photon threshold and neutron energy was investigated.

The Lead Modules

The most cost efficient way to detect single-neutron events was found to be a design with single scintillator columns with photomultiplier tubes (PMTs) of 10 in (25.4 cm) diameter. Double and triple columns of photomultiplier tubes with smaller PMTs were also considered. However, scintillator vessels with a diameter that is considerably larger than the attenuation length of both neutrons and $\gamma$-rays dramatically increase the detection efficiency and decrease the number of scintillator vessels needed for a given mass of lead. This leads to a 50% better cost efficiency for single columns with 25.4 cm-PMTs compared to double columns of 12.7 cm-PMTs. For double-neutron events, the greater stopping power for larger vessels also leads to a higher cost efficiency, more than compensating for the detrimental effect of a less fine-grained detector on the detection of two neutrons in two separate detectors.

The cost efficiency for both single- and double-neutron event detection rises as the lead wall thickness increases, then flattens out for a thickness of 50 cm (fig. 7.5
and 7.6), although from a pure efficiency-point of view, somewhat thinner lead walls would be preferable.

As expected from the bulk photon attenuation length in the scintillator, a vessel length of 300 cm maximizes both efficiency and cost efficiency for single- and especially double-neutron event detection (fig. 7.7, 7.8).

The cost efficiency for single-neutron events might be optimized for vessels with widths slightly larger than required by the size of the PMTs, the fewer number of PMTs necessary might more than compensate for the lower efficiency for each single scintillator vessel. There is a slight decrease in the cost efficiency with increasing vessel width (by 5 – 10%) and a slight increase (by ~ 3%) with vessel height, when increased from 30 cm to 45 cm. However, the cost efficiency for two-neutron events requires that the vessel dimensions should only exceed the PMT dimensions by a few
Figure 7.6: Efficiency and cost efficiency vs. lead wall thickness for double-neutron events.

Figure 7.7: Efficiency and cost efficiency vs. scintillator vessel length in the OMNIS lead module for single-neutron events.
Figure 7.8: Efficiency and cost efficiency vs. scintillator vessel length in the OMNIS lead module for double-neutron events.

cm at most, since a decrease of the cost efficiency for two-neutron events by $\sim 20\%$ is observed when both vessel width and height are increased from 30 cm to 45 cm.

The calculation does not consider the cost for the electronics, although it will be small compared to the cost for the scintillator vessels and the PMTs. However, it increases with the number of scintillator vessels. We have considered using four PMTs with diameters of 12.7 cm instead of one 25.4 cm-PMT per scintillator vessel. This would provide the same fractional coverage of the vessel face area, but on top of the approximately four-fold cost for the electronics, one larger PMT is less than four times as expensive as four smaller PMTs.

A thicker lead hull slightly decreases both the detection efficiency and the cost efficiency of the detector, since more of the neutrons that are produced are lost. We therefore plan to make the hull only as thick as necessary to attenuate the external background sources.
Figure 7.9: Efficiency vs. neutron energy in the OMNIS lead module for single-neutron events.

The detection efficiency for single-neutron events is constant for neutron energies beyond a few hundred keV (fig. 7.9). For double-neutron events, the available energy gets distributed to two neutrons. Hence, the detection efficiency for those events starts showing an asymptotic behavior only for energies above $\sim 1$ MeV (fig. 7.10).

The detection efficiency is still satisfactory, if the pulse threshold has to be set higher than the five photoelectrons assumed (fig. 7.11). However, as expected, the double-neutron efficiency decreases more rapidly for higher thresholds (fig. 7.12).

**The Iron Modules**

The iron modules have to be optimized only with respect to the cost efficiency for single-neutron events. The optimum dimensions of the scintillator containment vessels and the wall thicknesses are similar to those for the lead modules. As in the case of lead, an iron detector with fewer and larger scintillator vessels is both more
Figure 7.10: Efficiency vs. neutron energy in the OMNIS lead module for double-neutron events.

Figure 7.11: Efficiency vs. photoelectron threshold in the OMNIS lead module for single-neutron events.
efficient and cost efficient. The dependence of the cost efficiency on the vessel width and height suggests more elongated vessels along the vertical direction than in the case of lead. The detection efficiency becomes flat for higher neutron energies than in the case of lead (fig. 7.13), which is due to the fact that more energy is lost in an average elastic collision with an iron nucleus.

**Summary for Optimized Dimensions**

Table 7.2 summarizes the optimized dimensions for $\frac{1}{2}$kT lead and iron modules, stacked in single scintillator vessel columns (with photomultiplier tubes with diameters of 25.4 cm on each end).
Figure 7.13: Efficiency vs. neutron energy in the OMNIS iron module.

Table 7.2: Summary of optimized detector dimensions for the OMNIS lead and iron modules.

<table>
<thead>
<tr>
<th>Material</th>
<th>wall (cm)</th>
<th>hull (cm)</th>
<th>vessel (L x W x H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb</td>
<td>50</td>
<td>15</td>
<td>300 x 30 x 30</td>
</tr>
<tr>
<td>Fe</td>
<td>50</td>
<td>25</td>
<td>300 x 30 x 45</td>
</tr>
</tbody>
</table>
7.4.2 Influence of the Background

The Lead Modules

The detected false event rate in OMNIS in the absence of emitted neutrons is shown in fig. 7.14. Radiological analyses (see sec. 7.3.8 and ref. [29]) show that the bulk impurity rate in lead can be maintained below 0.1 Bq/kg, hence the false event rate in OMNIS will be less than \( \sim 1 \) per second. The ability to detect neutrino events at late times after the core bounce is determined by the influence of the radioactive background on a sparse neutron signal, with a typical time interval between two neutrons being long compared to the dead time of the detector, taken to be 80 \( \mu \)s. The apparent efficiency of a sparse neutron signal relative to the efficiency without background is shown in fig. 7.15, which shows that a tolerance of 0.1 Bq/kg is also acceptable in this respect.

Figures 7.16 and 7.17 show the false event rate due to \(^{40}\text{K}\) decays in the photomultiplier glass faces without neutrons and the relative apparent efficiency in the presence of a sparse neutron signal. A tolerance limit of 100 Bq has been adopted for each PMT.

The Iron Modules

Due to the higher energy of the background \( \gamma \)-rays and their longer attenuation length in iron, the requirements for its purity have to be more stringent than for lead. The resulting tolerance limit for the bulk background in the iron used is \( \sim 0.05 \) Bq/kg (see figures 7.18 and 7.19). This also appears to be consistent with radiological analyses [100] (see also sec. 7.3.8). The allowed background rate in the photomultiplier glass faces is, however, the same as in lead, 100 Bq per PMT (see figures 7.20 and 7.21).
Figure 7.14: False Event Rate per kT due to Bulk Background in OMNIS Pb Modules.

Figure 7.15: Apparent Enhancement of Sparse Neutron Signal due to Bulk Background in OMNIS Pb Modules. The two horizontal lines indicate a ±5% change relative to zero background.
Figure 7.16: False Event Rate per kT due to $^{40}$K decays in PMT faces in OMNIS Pb Modules.

Figure 7.17: Apparent Enhancement of Sparse Neutron Signal due to $^{40}$K decays in PMT in OMNIS Pb Modules. The two horizontal lines indicate a ±5% change relative to zero background.
Figure 7.18: False Event Rate per kT due to Bulk Background in OMNIS Fe Modules.

Figure 7.19: Apparent Enhancement of Sparse Neutron Signal due to Bulk Background in OMNIS Fe Modules. The two horizontal lines indicate a ±5% change relative to zero background.
Figure 7.20: False Event Rate per kT due to $^{40}$K decays in PMT faces in OMNIS Fe Modules.

Figure 7.21: Apparent Enhancement of Sparse Neutron Signal due to $^{40}$K decays in PMT faces in OMNIS Fe Modules. The two horizontal lines indicate a ±5% change relative to zero background.
Table 7.3: Number of detected neutron events versus supernova distance for 16 \( \frac{1}{2} \)kT Pb modules (assuming no neutrino oscillations).

<table>
<thead>
<tr>
<th>Dead Time (( \mu )s)</th>
<th>0.20 kpc</th>
<th>0.50 kpc</th>
<th>1.0 kpc</th>
<th>2.0 kpc</th>
<th>4.0 kpc</th>
<th>8.0 kpc</th>
<th>16 kpc</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.888 \times 10^6</td>
<td>0.293 \times 10^6</td>
<td>95400</td>
<td>26200</td>
<td>6740</td>
<td>1730</td>
<td>440</td>
</tr>
<tr>
<td>0</td>
<td>2.86 \times 10^6</td>
<td>0.458 \times 10^6</td>
<td>112000</td>
<td>27500</td>
<td>6860</td>
<td>1740</td>
<td>440</td>
</tr>
</tbody>
</table>

7.4.3 Dependence on Supernova Distance

If one event strobe is used for an entire detector module, there is a minimum distance of the supernova for which its dead time becomes critical for reliable event detection. Assuming one detected neutron event to trigger a strobe with 80 \( \mu \)s for a lead module of \( \frac{1}{2} \)kT (with 60 scintillator vessels), dead time losses become significant for supernovae closer than 2 kpc, which represents only a few percent of the Galaxy. Table 7.3, generated with a simulated supernova neutrino burst [33], shows the number of detected events versus supernova distance. The case without dead time is shown for reference. We intend to utilize a sufficiently sophisticated data acquisition system such that it can handle the higher neutron liberation rates associated with close supernovae in at least one module, thus precluding the excessive loss of high resolution data obtainable from a close supernova.
7.5 Late Time Measurement of a Core Collapse Supernova Neutrino Burst

7.5.1 Observational Signatures of a Confined/Deconfined Co-existence Phase

The most unambiguous signature of the formation of a first order phase transition lattice would be a significant increase in the neutrino cooling timescale at late times. Since the presented neutrino opacities for the pure phases have to be considered as upper limits due to the opacity lowering effects of many-body correlations (see sec. 4.3), neutrino mean free paths more than one order of magnitude shorter than expected would be a “smoking gun” for permanent scattering centers as opposed to random, thermal fluctuations. The challenging aspect of detecting neutrinos at the relevant late times is their relatively soft energy emission spectrum compared to energies of $\bar{E} > 10$ MeV, for which most supernova neutrino detectors are designed. However, the high energy tails might still lead to a statistically significant signal in a large enough detector, especially as the analysis can be done off-line after the detector has already been triggered by the more energetic early burst. Neutrino spectra after $t_{pb} \sim 5 - 10$ s will tend to converge for different neutrino flavors (sec. 4.2), hence neutrino oscillations among active flavors do not have to be taken into account to first order. According to the results from chapter 5.2, surface temperatures will be in the range $T_{surf} \sim 1 - 3$ MeV for Fermi spectra with zero effective degeneracy, and emission times up to $\sim 100$ s.

7.5.2 Interaction of Low Energy Neutrinos with Lead

For typical supernove neutrino energies, the cross section for their interaction with lead has been shown to be dominated by the allowed and first-forbidden transitions
for both the relevant neutral current interaction of $\nu_{\mu/\tau}$ neutrinos with $T \sim 8$ MeV and charged current interactions of $\nu_e$ neutrinos with $T \sim 3.5$ MeV relevant for times less than $t_{pb} \sim 3\,\text{s}$ [94, 126]. For softer neutrino spectra, the allowed transitions start to dominate. For that reason and because there is still disagreement within an order of magnitude about the relevant first-forbidden cross sections, only the allowed transitions are considered here. The treatment closely follows [94].

The charged current cross section consists of the transition to the isobaric analog state carried by the Fermi (F) strength

$$M_F^2 = \frac{1}{2J_i + 1} \langle J_f \| \Sigma_{i=1}^A \tau_+(i) \| J_i \rangle^2, \quad (7.7)$$

and the transition to the Gamow-Teller states carried by the GT strength

$$M_{GT}^2 = \frac{1}{2J_i + 1} \langle J_f \| \Sigma_{i=1}^A \sigma(i) \tau_+(i) \| J_i \rangle^2. \quad (7.8)$$

Hence,

$$\sigma(E_{\nu_e}) = \frac{G_F \cos^2(\theta_C)}{\pi} k_e E_e F(Z + 1, E_e) \times [M_F^2 + (g_A^{eff})^2 M_{GT}^2], \quad (7.9)$$

where $g_A^{eff} \sim 1$ is the empirical effective axial-vector coupling constant, $\theta_C$ the Cabibbo angle, $E_e$ and $k_e$ are the energy and three-momentum of the outgoing electron, and $F(Z + 1, E_e)$ accounts for the Coulomb distortion of the outgoing electron wave function by the daughter nucleus with $Z + 1$ (in this case $^{208}\text{Bi}$). With the energy transfer to the daughter nucleus $E = E_{\nu_e} - E_e$, the matrix elements are

$$M_F(E)^2 = \delta_{EE_{IAS}}, \quad (7.10)$$

$$(g_A^{eff})^2 M_{GT}(E)^2 = \frac{96.2}{\Delta_{CC} \sqrt{\pi}} \exp \left(-\frac{(E - E_{CC,GT})^2}{\Delta_{CC}^2} \right), \quad (7.11)$$

where the IAS is (relative to the parent ground state) at $E_{IAS} = 17.53\,\text{MeV}$ and the GT resonance is at $E_{CC,GT} = 17.9\,\text{MeV}$ with a width of $\Delta_{CC} = 2.4\,\text{MeV}$. As
the inverse direction \((\nu_e, e^+)\) is largely blocked in a neutron-rich nucleus like lead, especially for the low energies considered here, the full Ikeda sum rule \(3(N - Z)\) for the total GT strength was applied for the \((\nu_e, e^-)\) reaction and corrected by an empirical factor of 0.46.

The neutral current cross section is completely governed by the Gamow-Teller Matrix elements

\[
\sigma(E_{\nu_i}) = \frac{G_F^2}{\pi} E_{\nu_i}^2 (g_A^{\text{eff}})^2 (M_{GT}^{NC})^2
\]

\[
(M_{GT}^{NC})^2 = \frac{1}{2J_i + 1} (J_f \parallel \sum_i \sigma(i) \frac{\tau_3(i)}{2} \parallel J_i)^2.
\]

A fit to the associated isovector M1 response yields a centroid at \(E_{NC,GT} = 7.32\) MeV with a width of \(\Delta_{NC} = 0.6\) MeV:

\[
(g_A^{\text{eff}})^2 M_{GT}^{NC}(E)^2 = \frac{6.1}{\Delta_{NC} \sqrt{\pi}} \exp\left(-(E - E_{NC,GT})^2/\Delta_{NC}^2\right),
\]

where \(E = E_{\nu_i} - E_{\nu_f}\) is the inelastic energy transfer.

In heavy nuclei like lead, neutron emission dominates over all other emission channels even slightly above the emission threshold. The transition between one and two neutron emission has been experimentally verified and reported to occur relatively sharply at 2.2 MeV above the double neutron emission threshold [94], which will be adopted here.

### 7.5.3 Expected Signal in the OMNIS Lead Modules

For neutrino spectrum temperatures \(T < 3\) MeV, the signal from the Fe modules will be negligible compared to Pb, especially as the effect of flavor oscillations are less pronounced at late times due to the expected convergence of different flavor spectra. Hence, only Pb modules are considered in the following. The reaction rate in a
detector with $N_{ Pb}$ lead atoms is

$$R_{1n,2n} = F_\nu N_{ Pb} \sigma_{1n,2n},$$

(7.15)

where $F_\nu$ is the neutrino number flux

$$F_\nu = \frac{\dot{E}}{4\pi d^2 \bar{E}_\nu (T)},$$

(7.16)

$\dot{E}$ is the rate at which the star radiates energy, $\bar{E}_\nu (T)$ is the average energy for a non-degenerate Fermi spectrum, and the thermal average of the cross section is

$$\sigma_{1n,2n} = \frac{\int dE_\nu (E_\nu^2 \times \sigma_{1n,2n}(E_\nu/(1 + \exp(E_\nu/T))))}{\int dE_\nu (E_\nu^2 \times 1/(1 + \exp(E_\nu/T)))},$$

(7.17)

with the thresholds for one and two neutron emission $E_{1n,NC} = 7.19$ MeV, $E_{2n,NC} = 15.5$ MeV, $E_{1n,CC} = 6.89$ MeV and $E_{2n,CC} = 17.2$ MeV (see sec. 7.5.2). The rate at which neutrons are liberated in Pb per 1 kT is

$$R = 1.516 \times 10^3 \text{s}^{-1} \frac{\dot{E}}{10^{51} \text{ergs/s}} \frac{M_{ Pb}}{10^{-44} \text{m}^2} \left( \frac{1 \text{kpc}}{d} \right)^2 \frac{1 \text{MeV}}{\bar{E}_\nu (T)}. \quad (7.18)$$

For the total neutrino luminosity $\dot{E}$, the temperature dependent value found in sec. 6.2 was used. The expected rate for one and two neutron events for CC reactions and for one neutron events for NC events are shown in figure 7.22. The two neutron - NC rate was negligibly small for the allowed transitions and therefore not plotted. The expected number of neutrinos detected versus the hybrid star surface temperature is shown in table 7.4. Mainly due to the process’ low detection efficiency, the detected number of double neutron events will be only statistically significant for surface temperatures above $\sim 2.5$ MeV. The single neutron events, however, will not only produce an unambiguous signal, but also provide a more effective thermometer for the neutrino spectral temperature than for times sooner after the bounce, when
Table 7.4: Number of neutrinos liberated $R_{lib}$ and detected $R_{det}$ for late time neutrino cooling phase of a hybrid star (per kT lead; assuming detection efficiencies of $\sim 38\%$ for single- and $\sim 13\%$ for double-neutrons).

<table>
<thead>
<tr>
<th>Temperature (MeV)</th>
<th>$R_{lib}^{1n}$ (1/min)</th>
<th>$R_{det}^{1n}$ (1/min)</th>
<th>$R_{lib}^{2n}$ (1/min)</th>
<th>$R_{det}^{2n}$ (1/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>5.58</td>
<td>2.12</td>
<td>0.039</td>
<td>0.005</td>
</tr>
<tr>
<td>1.75</td>
<td>25.0</td>
<td>9.5</td>
<td>0.44</td>
<td>0.06</td>
</tr>
<tr>
<td>2.0</td>
<td>86.9</td>
<td>33.0</td>
<td>2.9</td>
<td>0.4</td>
</tr>
<tr>
<td>2.25</td>
<td>253.8</td>
<td>96.5</td>
<td>13.8</td>
<td>1.8</td>
</tr>
<tr>
<td>2.5</td>
<td>650.2</td>
<td>247.1</td>
<td>50.0</td>
<td>6.5</td>
</tr>
</tbody>
</table>

the neutrino spectra are more flavor-sensitive. From that, the cooling evolution of the hybrid star can be fairly accurately tracked.
Figure 7.22: Rate of neutrons liberated by soft, late-time spectra with zero effective degeneracy per kT of lead, assuming six identical spectra.
CHAPTER 8

CONCLUSION

Historically, supernovae were always among the most spectacular light-shows in the heavens. Despite some of their remnants being among the most-studied astronomical objects, their explosion mechanism is still not fully resolved. This is in part caused by the complexity and the elusive nature of the protoneutron star (PNS), the compact object thought to form in the center of a core collapse supernova. The neutrinos which cool the core and transport away almost all of its gravitational binding energy are also thought to be instrumental in the actual explosion mechanism. Their interaction cross section with matter is the lowest among all components of the standard model, making them the only window into the center of a core collapse supernova, but also turning their detection in terrestrial detectors into a formidable technological challenge which it has only become possible to meet over the past two decades. PNS’s have interior densities that might go well beyond nuclear densities, an area of physics otherwise only explored in recent heavy-ion collision experiments. It also involves number densities for nuclei which might be high enough for their constituent quarks to loose their confinement, complementary to the low-density, high-temperature QCD phase transition which is the subject of lattice-QCD. Temperatures relevant for PNS evolution are small compared to Fermi chemical potentials
present, the associated QCD phase transition is hence likely to be of first order. The resulting spatial separation of confined and deconfined phases, together with free mutual exchange of baryon number and electrical charge, leads to the arrangement of the respective minority phase into a charged lattice. The droplet sizes are determined by the strong interaction scale and therefore range up to a few fm, with lattice constants $a \sim 10 - 30$ fm. A structured lattice forces us to apply the concepts of condensed matter physics with its possible consequences on mechanical properties and radiation transport to the interior of - now - hybrid stars, which is what most of the present thesis is focusing on. As until now, the interior of PNS’s was treated as a homogeneous fluid and studied with the methods of numerical hydrodynamics, this might well force the nuclear astrophysics community to modify the accepted paradigm of PNS evolution, at least for evolution stages late enough for the QCD phase transition to occur.

Using the TOV equations, hybrid stars were integrated using a QCD-deconfined phase beyond a certain density (chapter 2). EOS parameters in the deconfined equations of state were, within the known constraints, adjusted so that an extended deconfined/confined coexistence phase region and possibly a pure deconfined core, results. The EOS’s of Lattimer/Swesty or Shen et al. for the confined matter phase were combined with the standard or effective mass bag model for the deconfined phase, using the range $B_{MIT} = 100 - 130 \text{ MeV/fm}^3$ for the bag constant and $M_S = 150 - 170 \text{ MeV/fm}^3$ for the strange quark mass.

In chapter 3, the properties of the structured lattice resulting in the coexistence phase are discussed. For the 3D case, the vibrations of a crystalline lattice can be calculated analytically using OCP theory, resulting in melting temperatures between
10 – 100 MeV (decreasing with increasing lattice charge densities) for an effective surface tension between the confined and deconfined phases of $\sigma = 50 \text{MeV/fm}^2$. Deformation modes for a 3D lattice are shown to freeze out in the same temperature range. Due to divergent lattice sums in 1D and 2D lattices, crystallization in lower dimensional lattices is limited to the obtaining of orientational order, as described by the KTHNY theory, with similar resulting effective melting temperatures. For typical temperature profiles in the hybrid star, the phase transition lattice is predicted to crystallize for most of the constrained range for the surface tension. The critical shear stress for the (3D) lattice is a sensitive function of the lattice constant, resulting in the outer $\sim 1 - 2 \text{km}$ of a crystallized lattice being susceptible to fracture due to shear stresses typically obtained for hydrodynamical flow.

As it was pointed out first by Reddy et al., a phase transition lattice forming in the coexistence region would provide temperature-independent targets for elastic neutrino scattering, resulting in a strongly enhanced (by 1-2 orders of magnitude for $T \sim 10 \text{MeV}$) neutrino opacity for neutral current scattering compared to the pure phases (chapter 4).

The state of the hybrid star towards the end of the formation of the coexistence phase is determined using self-consistent equilibrium heat flow in chapter 5. The coexistence phase will retain much of the heat produced in the phase transition, resulting in a high-temperature deconfined and mixed region (a few 10 MeV) being sustained against a relatively cool confined surface (1–3 MeV). As long as a significant net electron neutrino concentration is trapped, heat transport is caused by the lepton number gradient, leading to deleptonization fast compared with temperature gradient induced heat diffusion, even for relatively low trapped lepton numbers $Y_L \sim 0.1$. 

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Hence, the electron neutrino chemical potential will be close to zero by the time the hybrid star is completely formed.

Chapter 6 presents exploratory transport simulations using a newly developed 2D diffusion code, coupled with the hydrodynamical treatment of the confined envelope and quasistatic evolution of the stellar structure of a solid coexistence phase. Due to the enhanced opacity of the coexistence phase, the neutrino cooling timescale is shown to lie in the range of \( \sim 100 \text{ sec} \) as opposed to \( \tau \sim 3 \text{ sec} \) in the standard Kelvin-Helmholtz cooling model. The relative mechanical fragility of the outer \( \sim \text{ km} \) of the phase transition lattice suggests the possibility that a non-spherical relief structure might be formed by hydrodynamical flow, yielding a possible model for neutron star kicks. A crater with angular extent of \( 60^\circ \) and a depth of 1 km is shown to reproduce observed kick speeds of up to \( \sim 1000 \text{ km/sec} \) over a kick timescale of \( \sim 10 \text{ sec} \). Hydrodynamical flow patterns in the crater are observed in a separate exploratory calculation, showing persisting convective flows with kinetic energy densities sufficient to keep the crater open. In the case of a rotating hybrid star, the crater would result in a gravitational wave strain corresponding to a decaying bar mode, as spin and kick axes align over several spin periods. The resulting wave strain \( h^{TT} \sim 10^{-22} \) for a rotation period of 10 msec and a distance of \( \sim 1 \text{ kpc} \) could be detected by present and future ground-based gravity wave observatories. The asymmetric heat flow alone would result in a signal about an order of magnitude lower. The most unambiguous observational confirmation of hybrid star observation would, however, be a slowly decaying neutrino cooling curve at late times.
Using the example of a planned neutron spallation detector (OMNIS), it is shown in chapter 7 that a large enough detector (a few $10^3$ T of lead) can result in a statistically significant signal rate for energy sphere temperatures above $T \sim 1.75$ MeV. Chapter 7 represents another focus of the present thesis, which is the detection side of supernova neutrino physics. The OMNIS concept involves the use of different neutral and charged current cross sections and neutron separation thresholds to gain information on the supernova neutrino emission spectra, where iron and lead are used as examples. A particle transport code was developed to track neutrons liberated from lead, the $\gamma$ -rays they produce when capturing on the neutron sink used (in this case $^{155,157}$Gd), as well as scintillation photons produced in the scintillator when a neutron or $\gamma$ -ray deposits energy into the scintillator modules used to extract the signal. The detector design was optimized with respect to cost-efficiency, resulting in an efficiency of 38% for single neutron events in lead and iron and 13% for double neutron events in lead. The false event rate caused by radioactive impurities is thereby limited to less than 1/sec for random signals and 5% of an observed supernova signal for impurities below $\sim 0.1$ Bq/kg with commercially available materials.

A new venue has been opened up for nuclear astrophysics, and its relevant new basic properties and observational signatures in hybrid stars have been worked out in the present thesis. Further, the groundwork has been laid for future investigations involving more large-scale numerical calculations. The code package developed is sufficiently versatile to accommodate extensions such as detailed neutrino transport and hence an accurate determination of the neutrino spectrum for densities below $\rho = 0.1\rho_0$, where the neutrino spectrum obtains its final shape. The basic ingredients for a full dynamical simulation of the formation of a deconfined phase are also present in the
form of the coupling of hydrodynamical and crystallized layers. Another interesting aspect would be the timescale over which a fractured lattice can regrow, possibly in the presence of hydrodynamical flows (below the critical shear stress). Before that can be accomplished, however, more work is clearly needed on the microphysics of the coexistence phase, in particular the dynamical expansion of a conversion front and the mechanical properties (viscosity) of a liquid coexistence phase. The full effects of screening, higher order curvature terms and zero point energy effects in the phase transition lattice have not been accounted for in a comprehensive treatment of all second-order effects. Other aspects of the microphysics which would also be of great interest to many other areas of physics are a numerically economical treatment of transport with Bragg scattering in a crystal, which does not exist yet to my knowledge, and a more predictive theory of 1D/2D melting, which is, however, under intense investigation by many groups at this time. Neutron stars are, indeed, among the most complex objects in the universe - especially if they do not actually turn out to be “neutron” stars.
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