Design of Radiofrequency Coils for Magnetic Resonance Imaging Applications: A Computational Electromagnetic Approach

DISSERTATION

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By

Tamer S. Ibrahim, B.S.E.E., M.S.E.E

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Dissertation Committee:

Robert Lee, Advisor
Pierre-Marie L. Robitaille
Brian B. Baertlein

Approved by

Advisor
Department of Electrical Engineering
ABSTRACT

The advancement of MRI as a radiological instrument has been associated with a constant drive towards higher magnetic field strengths resulting in higher operational frequencies. More powerful magnets bring the promise of enhanced signal to noise ratio, exquisite resolution, and reduced scan times. At the same time however, MRI at higher frequencies adds significant engineering complexities to the MRI experiment, most notably in designing safe, versatile, and high-performance radio frequency (RF) coils.

In this work, computational and theoretical electromagnetic analysis of several RF coils used in MRI are presented at Larmor frequencies that range between 64 and 470 MHz representing clinical imaging at 1.5:11 Tesla. The electromagnetic interactions with phantoms and anatomically detailed head models, including a developed high-resolution human head mesh, are studied at different field strengths. The computational tool of choice here was the finite difference time domain (FDTD) method. Combined with measurements using an 8 Tesla MRI system, currently the most powerful clinical magnet in the world and a 1.5 Tesla system, the FDTD method is utilized to study, analyze, and eventually design RF coils. Innovative Engineering approaches using phased array techniques are presented to improve the performance of RF head
coils in terms of transverse magnetic field uniformity and reduction of specific absorption rate for operation at 4.7 and 8 Tesla. Novel analytical derivations are presented to explain the source of the MR signal. The combination of the analytical derivations, FDTD modeling, experiments, and infrared imaging gives a new prospective onto the electromagnetics associated with low and high field clinical imaging.
For my Mother, my Father, Nevine, Daniel, and the Truth
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VITA

November 24, 1972 .................. Born - Alexandria, Egypt

June 1995 - July 1996 .................. Student Research Assistant,
The ElectroScience Laboratory,
The Ohio State University.

March 1996 - January 2000 .............. Student Teaching Assistant,
The Ohio State University.

June 1996 .......................... B.S. Electrical Engineering,
The Ohio State University

August 1996 - January 2000 .............. Graduate Research Associate,
The ElectroScience Laboratory,
The Ohio State University

December 1998 ...................... M.S. Electrical and Computer Engineering, The Ohio State University

January 2000 - Present .................. Research Associate Engineer,
Department of Radiology The Ohio State University

PUBLICATIONS

REFEREED JOURNAL ARTICLES


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6.1 \[
\frac{\sin(\gamma |B_1^+|T)}{1-\cos(\gamma |B_1^+|T)}
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CHAPTER 1

INTRODUCTION

Magnetic resonance imaging (MRI) is a powerful medical diagnostic technique, which is based on the nuclear magnetic resonance (NMR) phenomenon. First discovered by Bloch [1], NMR is associated with the magnetic properties of specified nuclei [2]. In the medical diagnostic field, NMR is the absorption of radiofrequency (RF) energy by the nuclei of the human body’s atoms, which are in the presence of an external static magnetic field. The advantages of MRI among other medical diagnostic techniques are its non-invasive nature, versatility, excellent tissue contrast and sensitivity to flow and diffusion. As a result of its unprecedented power, MRI scanners now permeate the nation hospitals. Indeed, it can be argued that MRI is the premier diagnostic imaging technique, not only for its past achievements, but also for its unparalleled research potential.

The radio frequency (RF) coil is an essential element in MRI systems. It is a device which is used to transmit and receive the electromagnetic fields to nuclei such as $^1H$, $^{19}F$, $^{23}Na$, $^7Li$, $^{13}C$, $^{15}N$, and $^{31}P$ at the nuclei specific Larmor frequency which is governed by the relation:

$$f_0 = \gamma |B_0|$$  \hspace{1cm} (1.1)
where $\gamma \ (Hz/Tesla)$ is the gyromagnetic constant, $B_0 \ (Tesla)$ the static magnetic field intensity of the magnet known as the “external magnetic field”, and $f_0$ is the Larmor frequency. As such, every specific nuclei has a unique gyro magnetic constant and therefore a specific Larmor frequency for a specific $B_0$ field value. For instance, $\gamma$ for $^1H$ is equal to $(2\pi) \times 42.6 \times 10^6 \ (Hz/T)$. 

In this dissertation, a complete electromagnetic computational analysis of several RF coils used in magnetic resonance imaging is presented at Larmor frequencies that range between 64 and 500 MHz. The finite-difference time-domain (FDTD) method [3] combined with measurements using MRI clinical systems with $B_0$ fields equal to 8 Tesla, currently the most powerful clinical magnet in the world [4, 5], and 1.5 Tesla, are utilized to study, analyze, and eventually design RF head-, and extremity-coils.

1.1 Motivation

Ever since MRI was introduced as a medical diagnostic tool [6, 7], there has been a constant drive towards more powerful magnets and higher operational frequencies. Typically, magnets used in clinical MRI for whole body applications have field strengths which vary from 0.5 to 3.0 Tesla such as the 1.5 Tesla and 0.7 Tesla systems at The Ohio State University (OSU) Hospital. In 1998, An 8 Tesla (ultra high field) MRI system has been built at the same hospital [8]. There are many advantages to operating at higher frequencies (340 MHz in the 8 Tesla case) including the potential of exquisite resolution, reduced scan time, and increases in signal to noise ratio [9], chemical shift dispersion [10], susceptibility [11, 12], and related BOLD contrast [13]. With the most powerful clinical whole-body MRI system in the world,
it will be possible to obtain images with much greater details. This would help many diagnostic applications including early detection of small tumors.

At the same time, however, MRI at higher frequencies add significant technical complexities to the NMR experiment. In addition to obvious increases in the stored energy of the magnet [5], difficulties in fabricating suitable RF coils also mount [14, 15, 16]. Fortunately, technical difficulties can eventually be overcome in that they are dependent primarily on experimental techniques and innovative engineering approaches. More daunting, however, are fundamental physical limitations to imaging at ultra high field. These include potentially significant RF penetration problems [17, 18, 19], excessive RF power requirements [17, 20, 21, 22] and the presence of dielectric resonances [17, 18, 23, 24, 25, 26] or dielectric focusing effects [27].

The standard 1.5 Tesla clinical MRI uses RF coils such as the birdcage resonator [28], which is usually driven in quadrature (2 port) for head and whole body imaging. The resonance frequency for the 1.5 Tesla system is approximately 64 MHz for proton ($^1H$) imaging. The goal of the RF coil design at this frequency is to produce a uniform circularly polarized component of the transverse magnetic ($B_{1^+}$) field and low values of the specific absorption rate (SAR) in the imaged subject. These qualities are attainable at 64 MHz. The operating frequency of the 8 Tesla magnet rises approximately to 340 MHz for $^1H$ imaging. This in turn will affect the distribution of the $B_{1^+}$ field and the values of the SAR in the human body. In order to retain the desirable properties of a uniform magnetic field and a low SAR, redesigning the RF coils is essential. A computational tool based on electromagnetics can be very effective in conducting feasibility studies and in designing, and evaluating the performance of RF coils for use at 8 Tesla (340 MHz).
1.2 Overview of the Dissertation

This dissertation is organized as follows. In Chapter 2, a physical description of nuclear magnetic resonance (NMR) and its use in medical imaging is given. A review of MRI RF coils is also provided in Chapter 2. Chapter 3 gives a historical background of the use of electromagnetic numerical methods in medical applications in general and in MRI in particular. An extensive survey of the work done in this area is provided. In addition, a brief mathematical description of the FDTD [3] method is presented. Equations for the perfectly matched layer (PML), an absorbing boundary condition [29], are driven using the coordinate stretching approach [30].

Chapter 4 describes the implementation of the FDTD method for modeling the birdcage [28] and the TEM [15] head coils, and single strut extremity coil [31]. In addition, a description of a newly developed 18-tissue anatomically detailed human head model [32, 33, 34] is presented. To validate the numerical model, the FDTD results are compared to actual MRI measurements of phantoms at 1.5 and 8 Tesla. Detailed theoretical, experimental, and numerical analyses describing the operation of the birdcage and the TEM head resonators, and the single strut extremity coil are also provided. Chapter 4 also includes numerical and experimental comparisons of several head coils.

Chapter 5 provides an SAR analysis at 340 MHz and studies the dependence of RF power requirements on the frequency of operation (Larmor frequency) between 3 and 11.5 Tesla using head and extremity coils. The definition of a 90° flip angle is discussed from an electromagnetic perspective. The effect of the electromagnetic interactions between the RF excitation source(s) and the tissue on power requirements for the MR experiment is analysed numerically and with infrared imaging. Chapter
6 provides a thorough analysis of dielectric resonances [17, 18, 23, 24, 25, 26] at an 8 Tesla field strength. A novel approach based on the principle of reciprocity for driving the NMR signal is presented with experimental and numerical validations. Novel phased array techniques are proposed to improve the homogeneity of the $B_1^+$ field and to reduce the peak SAR values [35, 34] for operation at 4.7 and 8 Tesla. Conclusions and proposed future work appear in Chapter 7.
CHAPTER 2

MAGNETIC RESONANCE IMAGING: THE RADIOFREQUENCY PROSPECTIVE

The dual phenomena of electric polarization is magnetic polarization or what is more known as magnetization. Electric polarization exists in materials which exhibit a permittivity greater than that of free space ($8.854 \times 10^{-12} F/m$) when the material is subjected to an external electric field. By definition, these types of materials are called dielectrics. Magnetic materials (magnetics) exhibit magnetization when subjected to an external magnetic field. The explanation of the behavior of these types of materials under an external magnetic field is obtained from quantum theory. However, a classical yet accurate representation of the behavior of these materials could also be obtained using atomic models. Because magnetic resonance imaging (MRI) is associated with nuclear magnetic resonance (NMR) phenomena, the analysis presented is performed on protons (spins). A similar analysis could be obtained using electrons. There is an imaging modality known as electron precession resonance (EPR) which is based on electron magnetic resonance. It is noted however that the term “magnetic polarization” is usually associated with electron effects.
2.1 Atomic Models

The following analysis is considered to be classical, it can however be accurately applied to describe the operation of MRI. Consider a positive charge orbiting in its path, a circle with a small radius for simplicity. It is assumed that the positive charge does not possess spin character. An equivalent circuit of the positive orbiting charge is a small current loop where, by definition, the direction of the current is the same as the rotating charge (Figure 2.1). Using duality, the electromagnetic fields produced by a small current loop are equal to that of a thin linear magnetic dipole. Note that this representation is only valid outside the loop area. As such, the positive orbiting charge could be represented by a magnetic dipole which is perpendicular to the area of the small loop. The magnetic dipole consists of a pair of magnetic charges with equal magnitude and opposite signs. The magnetic moment associated with such dipole or current loop is [36]

\[
\vec{m} = |I|d\vec{s} = |q|d\vec{\eta}
\]  

(2.1)

where \(|I|\) is the current magnitude, \(|q|\) is the magnitude of any of the magnetic charges, \(ds\) is the area of the loop, and \(d\) is the distance between the magnetic charges of the dipole. With an applied static magnetic field \((B_0)\), the net torque on a current distribution (Figure 2.2) present under a constant (at least over the diameter of the current loop) \(B_0\) is given by

\[
d\vec{N} = \vec{R} \times d\vec{F}
\]  

(2.2)

where \(\vec{R}\) is a position vector; its origin could be anywhere. \(d\vec{F}\) is the force on each of the segments:

\[
d\vec{F} = |I|d\vec{l} \times \vec{B}_0
\]  

(2.3)
Figure 2.1: A positive charge rotating in a loop and its representation as an electric current loop.

The total force given by the line integral over $d\ell$ is equal to zero. The torque on $d\ell$ is given by

$$d\vec{\tau} = |I|d\ell (\vec{B}_0 \cdot \vec{R}) - |I|\vec{B}_0 (d\ell \cdot \vec{R})$$  \hspace{1cm} (2.4)

From Figure 2.2, $d\ell \cdot \vec{R} = 0$, therefore the torque on $d\ell$ is given by

$$d\vec{\tau} = |I|d\ell (|B_0||\vec{R}|\cos(g))$$ \hspace{1cm} (2.5)

By integrating the previous expression, the equation relating torque exerted on a dipole with a specified dipole moment is presented by

$$\vec{N} = \frac{\vec{n} \times \vec{B}_0}{(|\vec{n} \times \vec{B}_0|)} |I| |B_0| |\vec{R}|^2 \sin(w)$$ \hspace{1cm} (2.6)

$$\vec{N} = \vec{m} \times \vec{B}_0$$ \hspace{1cm} (2.7)
where $\vec{m}$ is the magnetic dipole moment.

It is clear that if the plane, in which the current loop exists, is perpendicular to the applied magnetic field, the total exerted torque is zero. Therefore, the exerted torque will tend to rotate the current loop such that its plane is perpendicular to the applied magnetic ($\vec{B}_0$) field. In other words, the magnetic dipole moment is rotated such that it is parallel to the $\vec{B}_0$ field.

Assume that the positive charge in question is a proton (spin). Experimentally, it was determined that the magnetic dipole moment is related to the angular momentum through the relation

$$\vec{m} = \gamma \vec{J}. \quad (2.8)$$

$\gamma$ is the gyromagnetic constant given by

$$\gamma = \frac{q}{2mm} \quad (2.9)$$

where $mm$ is the proton mass and $q$ is its charge. The angular momentum of a proton is given from quantum theory

$$J = \frac{h}{2\pi} \quad (2.10)$$

where $h$ is Planck’s constant. Our interest is to come with a differential equation where a solution of the magnetic moment could be obtained. To relate the angular momentum to the torque, let us assume that the mass $mm$ is moving with a velocity equals to $\vec{v}(t)$ and is positioned at $\vec{R}(t)$, the angular momentum relative to the origin (the starting point of $\vec{R}(t)$) is given by

$$\vec{J}(t) = \vec{R}(t) \times \vec{p}(t) \quad (2.11)$$

where $\vec{p}(t)$ is the moment given by

$$\vec{p}(t) = mm\vec{v}(t). \quad (2.12)$$
Figure 2.2: A current loop subjected to a static external magnetic \((B_0)\) field.
From Newton’s law

\[ \vec{F} = \frac{d\vec{p}}{dt}. \]  

(2.13)

By differentiating Eq. (2.11) with respect to time and substituting from Eqs. (2.13 and 2.2) into Eq. (2.11), the following is obtained

\[ \frac{d\vec{J}}{dt} = \vec{N} = \vec{m} \times \vec{B}_0. \]  

(2.14)

From Eqs. (2.8 and 2.14). The fundamental equation of motion is obtained

\[ \frac{d\vec{m}}{dt} = \gamma\vec{m} \times \vec{B}_0. \]  

(2.15)

To solve the fundamental equation of motion for a proton, consider a spinning magnetic moment \( \vec{m} \), i.e. a proton, subjected to a magnetic field \( \vec{B}_0 \) (Figure 2.3). Because the proton is a spin, a precession motion is introduced. For MRI to be feasible, a magnetic moment due to a proton has to exist; therefore, the total angular momentum must not equal to zero (Eq. (2.8)). Thus, even-even nucleus such as \(^{16}\text{O}\) can not be imaged [37]. Other nucleus including odd-odd and odd-even could be imaged. An important note is that only unpaired protons (protons have to be in the outer shell) can induce a magnetic moment.

From Eq. (2.15), the change in magnetic moment \( d\vec{m} \) in a time \( dt \) is given by

\[ d\vec{m} = d\tau\gamma\vec{m} \times \vec{B}_0. \]  

(2.16)

From Figure 2.3 and Eq. (2.16), a clockwise precession of the \( d\vec{m} \) around \( \vec{B}_0 \) is observed. As seen from Figure 2.3,

\[ |d\vec{m}| = |\vec{m}|sin(\theta)|d\phi|. \]  

(2.17)

Therefore, Eqs. (2.16 and 2.17) lead to

\[ |d\phi| = d\tau\gamma|B_0|. \]  

(2.18)
Figure 2.3: Precession of a proton around the $B_0$ field.
The Larmor precession equation can be easily obtained

\[ \left| \frac{d\phi}{dt} \right| = \omega_0 = \gamma |B_0|. \]  

(2.19)

The angular velocity vector by definition is counterclockwise (right-handed). For this particular situation, the magnetic moment is rotating clockwise. Therefore, the angular velocity vector is given by:

\[ \vec{\omega} = -\omega_0 \vec{z}. \]  

(2.20)

Images obtained using MRI are macroscopic, therefore the equation of motion will be considered in its macroscopic level. Thus, the summation of the magnetic moments over a small volume (in which the \( B_0 \) field is constant) is considered. Magnetic polarization or simply magnetization (not to be confused with magnetization due to electrons in their orbits) is given by

\[ \vec{M} = \sum \vec{m}_i. \]  

(2.21)

The equation of motion on a macroscopic level is defined as

\[ \frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B}_0. \]  

(2.22)

### 2.2 Relaxation Effects

In the previous analysis of the equation of motion, the interactions between the protons and their surrounding are ignored. Consider the interaction energy between a magnetic dipole and the applied magnetic field (Figure 2.3):

\[ w = -|\vec{m}| |\vec{B}_0| \cos(\theta) \]  

(2.23)
or for a magnetization

\[ W = -|\vec{M}| |\vec{B}_0| \cos(\theta) \]  
\[ W = -\vec{M}.\vec{B}_0. \]  

(2.24)  

(2.25)  

To reach minimum energy state \(-|\vec{m}| |\vec{B}_0|\), the magnetic dipole will align itself with the applied static field. This could also be seen mathematically from Eqs. (2.24, and 2.7), noting that

\[ -\frac{dW}{d\theta} = N. \]  

(2.26)  

At minimum energy state \(\frac{dW}{d\theta} = 0\), therefore \(N = 0\), which only occurs when \(\theta = 0\) (Eq. (2.7)). From quantum mechanics, the aforementioned process is performed through the thermal exchange of energy \((W)\) from the protons to the lattice. \(W\), is \(<<\) significantly less than the thermal energy, which is on the order of \(KT\) where \(K\) is the Boltzmann’s constant and \(T\) is the temperature in Kelvin. Therefore, the difference between the protons that align themselves parallel and the protons that align themselves anti-parallel is very small. This difference, which is proportional to the applied field strength, is the source of the MRI signal. The equilibrium magnetization is obtained using Curie’s laws [38]

\[ M_0 = 0.25\alpha_0 \gamma^2 \left(\frac{\hbar}{2\pi}\right)^2 \frac{B_0}{KT} \]  

(2.27)  

where \(\alpha_0\) is the density of spins available for excitation per unit volume.
To consider the relaxation effects, the equation of motion (2.22) is scalarized

\[
\frac{d\mathbf{M}_z}{dt} = 0 \quad (2.28)
\]

\[
\frac{d\mathbf{M}_{\text{transverse}}}{dt} = \gamma \mathbf{M}_{\text{transverse}} \times \mathbf{B}_0 \quad (2.29)
\]

\[
\frac{d\mathbf{M}_{\text{transverse}}}{dt} = \gamma |\mathbf{B}_0| \mathbf{M}_{\text{transverse}} \times \left| \frac{\mathbf{B}_0}{B_0} \right| \quad (2.30)
\]

\[
\frac{dM_x}{dt} = \omega_0 M_y \quad (2.31)
\]

\[
\frac{dM_y}{dt} = -\omega_0 M_x \quad (2.32)
\]

where

\[
\mathbf{M}_{\text{transverse}} = M_x \mathbf{x} + M_y \mathbf{y}. \quad (2.33)
\]

From the previous argument, the rate of change of the \(M_z\) (\(\frac{dM_z}{dt}\)) is no longer equal to zero as deduced from the Eq. (2.28); it is proportional to \(M_0 - M_z\), therefore

\[
\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1} \quad (2.34)
\]

where \(T_1\) is the spin-lattice relaxation time.

Unlike the spin-lattice interaction, the spin-spin interaction affects the magnetization in the transverse plane. The spin-spin interaction is attributed to the facts that a proton is not only subjected to the \(B_0\) field but also to the fields of neighboring protons. Because of the inhomogeneity of the local fields, different precessional frequencies are introduced and the magnetization tends to decrease due to this phenomena (dephasing). Therefore, Eqs. (2.31 and 2.32) are altered to

\[
\frac{dM_x}{dt} = \omega_0 M_y - \frac{M_x}{T_2} \quad (2.35)
\]

\[
\frac{dM_y}{dt} = -\omega_0 M_x - \frac{M_y}{T_2} \quad (2.36)
\]
where $T_2$ is the spin-spin relaxation time. $T_2$ is usually significantly less than $T_1$, therefore the magnetization phases out in the transverse plane at a much faster rate when compared to that in the longitudinal plane.

Eqs. (2.35, 2.36, 2.34) are called the Bloch equations [1]. The solution to Eq. (2.34) is obtained through the integration of the magnetization in time:

$$M_z(t) = M_z(t_0)e^{-t/t_0}/T_1 + M_0(1 - e^{-(t-t_0)/T_1}).$$ \hspace{1cm} (2.37)

It is clear that the steady state solution $M_z(\text{inf}) = M_0$.

If Eqs. (2.35, and 2.36) are all differentiated with respect of time while substituting $M_x = Ae^{-t/T_2}$ and $M_y = Be^{-t/T_2}$, scalar wave equations are obtained. The solution to which is given by

$$M_x(t) = e^{-(t-t_0)/T_2}(M_x(t_0) \cos(\omega_0(t-t_0)) + M_y(t_0) \sin(\omega_0(t-t_0)))$$ \hspace{1cm} (2.38)

$$M_y(t) = e^{-(t-t_0)/T_2}(M_y(t_0) \cos(\omega_0(t-t_0)) - M_x(t_0) \sin(\omega_0(t-t_0))).$$ \hspace{1cm} (2.39)

The steady state solution for the transverse magnetization = zero ($M_x(\text{inf}) = 0$ and $M_y(\text{inf}) = 0$).

### 2.3 The Rotating Frame vs. the Laboratory Frame

So far, it is established that the magnetization is precessing around the $\vec{B}_0$ field with precessional frequency equals to $\gamma B_0$, therefore, it is appropriate to introduce a rotating frame (Figure 2.4) as opposed to the standard frame of reference, called the laboratory frame. The definition of the coordinates of the rotating frame are given as $X', Y'$, and $Z'$ [39]. Note that the frame is rotating clockwise. A unit vector in any of the three directions of these coordinates is rotating around the vector $|\vec{G}|$ with an angular velocity equals to $|\vec{G}|$. If we examine Figure 2.2, it is observed that the rotating
frame concepts could be appropriate to our case where the magnetization is precessing around $B_0$ field in the same sense as in Figure 2.4 with an angular velocity equals to $\omega_0$. Substituting from the Larmor frequency relation (Eq. (2.19)) in Eq. (2.22):

$$\gamma|\vec{B}_0| = \omega_0 = |\vec{G}|$$

(2.40)

$$\vec{G} = -\omega_0 \frac{\vec{B}_0}{|\vec{B}_0|}$$

(2.41)

$$\frac{d\vec{M}}{dt} = \vec{G} \times \vec{M} + \frac{M_0 - M_T}{T_1} Z' - \left( \frac{\dot{M}_x}{T_2} + \dot{M}_y \right).$$

(2.42)

Note that Eq. (2.42) is still in the laboratory frame. Therefore, in the $X',Y'$, and $Z'$ coordinates, the equation of motion (with no relaxation effects) is defined as

$$\frac{d\vec{X}'}{dt} = \vec{G} \times \vec{X}'$$

(2.43)

$$\frac{d\vec{Y}'}{dt} = \vec{G} \times \vec{Y}'$$

(2.44)

$$\frac{d\vec{Z}'}{dt} = \vec{G} \times \vec{Z}'$$

(2.45)

Assume $\vec{A}$ is rotating around $\vec{G}$ with an angular velocity that is not equal to $|\vec{G}|$. The definition of $\vec{A}$ is given by:

$$\vec{A}(t) = A_x(t)\vec{x} + A_y(t)\vec{y} + A_z(t)\vec{z}$$

(2.46)

or in the rotating as

$$\vec{A}(t) = A_{x'}(t)\vec{X}'(t) + A_{y'}(t)\vec{Y}'(t) + A_{z'}(t)\vec{Z}'(t).$$

(2.47)

The derivative of $\vec{A}(t)$ is given by

$$\frac{d\vec{A}(t)}{dt} = \frac{dA_{x'}(t)}{dt}\vec{X}'(t) + \frac{dA_{y'}(t)}{dt}\vec{Y}'(t) + \frac{dA_{z'}(t)}{dt}\vec{Z}'(t) +$$

$$A_{x'}(t)\frac{d\vec{X}'}{dt} + A_{y'}(t)\frac{d\vec{Y}'}{dt} + A_{z'}(t)\frac{d\vec{Z}'}{dt}.$$
Figure 2.4: Rotating frame of reference. $X'$, $Y'$, and $Z'$ are rotating around $\vec{G}$ with an angular velocity equals to $|\vec{G}|$. 
From this equation and Eqs. (2.43,2.44,2.45)

\[
\frac{d\vec{A}(t)}{dt} = \frac{dA_X(t)}{dt}\vec{X}(t) + \frac{dA_Y(t)}{dt}\vec{Y}(t) + \frac{dA_Z(t)}{dt}\vec{Z}(t) + G \times \vec{A}(t) \tag{2.49}
\]

\[
\frac{d\vec{A}(t)}{dt} = \frac{d\vec{A}(t)}{dt} + G \times \vec{A}. \tag{2.50}
\]

Therefore the magnetization in the laboratory frame is related to the rotating frame by:

\[
\frac{d\vec{M}(t)}{dt} = \frac{d\vec{M}'(t)}{dt} + G \times \vec{M} \tag{2.51}
\]

\[
\gamma\vec{M} \times \vec{B}_0 = \frac{d\vec{M}'(t)}{dt} + G \times \vec{M} - \left(\frac{M_0 - M_0'}{T_1}\right)\vec{Z}' + \left(\frac{M_X'}{T_2}\vec{X}' + \frac{M_Y'}{T_2}\vec{Y}'\right) \tag{2.52}
\]

\[
\frac{d\vec{M}'(t)}{dt} = \gamma\vec{M} \times (\vec{B}_0 + \frac{\vec{G}}{\gamma}) + \left(\frac{M_0 - M_0'}{T_1}\right)\vec{Z}' - \left(\frac{M_X'}{T_2}\vec{X}' + \frac{M_Y'}{T_2}\vec{Y}'\right). \tag{2.53}
\]

### 2.4 RF Excitation

Using an RF coil, if we apply a magnetic ($B_1$) field in the transverse plane with an oscillating frequency equals to the Larmor frequency, the magnetization vector starts to precess around a vector that sums $B_0$ field and the $B_1$ field (in the laboratory frame). In the rotating frame, where the coordinates are precessing with the same angular velocity as the $B_1$ field frequency and the Larmor frequency, after applying the RF pulse the magnetization vector starts to precess around the $B_1$ field and eventually is rotated by 90° to the transverse plane after time = $t_{90}$. This process is known as “magnetic resonance”. The rotation of $\vec{M}$ from $Z'$ to $Y'$ when applying the RF field in $X'$ is shown in Figure 2.5.

As such, the RF coils are responsible for delivering the RF power to and receiving it from the object to be imaged; therefore there are 2 types of RF coils: transmitter and receiver. The transmitter coil creates the magnetic field that excites the nuclei of
the tissue. It is essential to have a uniform field over the region of interest to provide a spatially uniform excitation. If the coil does not provide a spatially uniform excitation (inhomogeneous $B_1$ field), some of the nuclei are either not excited or excited with different flip angles leading to poor image contrast and S/N. In Figure 2.6, dark spots (poor S/N) in an MRI image are present due to the inhomogeneity of the $B_1$ field.

The main function of the receiver coil is to detect the NMR signal and then induce a corresponding voltage. Another required characteristic in the RF coil is to have a large filling factor (volume of sample per volume of the coil). By having the size of the body part to be imaged and the size of the coil comparable, a good SNR is

![Diagram](image)

(a) $\vec{M}$ in the same direction as $\vec{B}_0$
(b) RF field in $X'$
(c) Rotation of $\vec{M}$ to $Y'$

**Figure 2.5:** Excitation of the magnetization with an RF field: the introduction of magnetic resonance.
achieved because the system is less sensitive to the external thermal noise and the receiver coil is close to the region of interest.

As the nuclei relax back to equilibrium, they emit RF energy, that has the same frequency as the applied RF. This signal is often referred to as free induction decay (FID) or the NMR signal. With different tissues, the nuclei relax at different rates leading to a difference in the NMR signal and consequently different tissue contrast in the image. Using magnetic gradients, the spatial location is determined and the measured signal is transformed to an image via signal processing tools.

Novel detailed analysis that describes excitation and reception using the principle of reciprocity will be presented in Chapter 6.

2.5 Free Induction Decay

The most basic MRI experiment involves the detection of a global signal from the entire object to be imaged. This is usually done without a slice selection. An RF pulse is applied to rotate the magnetization to the transverse plane. The sum of the RF field excited by the magnetization induces an \( emf \) on the receiver coil(s). This experiment, called free induction decay (FID), is usually utilized for several purposes including checking the tune of the RF coil, optimization of the system response, and determining the RF amplitude and duration in order to obtain a 90° flip. A new approach that deduces the formulation of the received signal strength is provided Chapter 6.

2.6 Spin Echo

It was shown earlier that the spin-spin interaction known as \( T_2 \) effects alters the magnetization in the transverse plane. \( T_2 \) is an inherent property of the sample and it
Figure 2.6: The inhomogeneity of the $B_1$ field in an MRI image. Image was provided by Dr. Chakeres.
can not be avoided. As a matter of fact, the weighing of the images could be dependent on this property. Inhomogeneities in the $B_0$ field could also lead to dephasing in the transverse plane. As such, a new parameter is $T^I_2$, which is presented to account for this inhomogeneity. $T^I_2$ could be much smaller than $T_2$ resulting in a significant drop in the signal to noise. The modified spin-spin interaction parameter is given by

$$\frac{1}{T^*_2} = \frac{1}{T_2} + \frac{1}{T^I_2}$$

(2.54)

where $T^*_2$ is a relaxation parameter that accounts for the total dephasing in the transverse plane.

The signal loss due to $T^I_2$ is recoverable by what is so called “spin echo” technique. A diagram of spin echo acquisition is shown in Figure 2.7. A $\pi/2$ pulse is applied in the $X'$ direction, causing the magnetization to precess around the $X'$ direction such that it finally rotates to the $Y'$ direction at time $= t90$. At $t = t90^+$, the magnetization starts to dephase in the transverse plane due to $T^*_2$ effects. At $t = t90 + \tau$, a $\pi$ pulse is applied in the $Y'$ direction, causing the magnetization to dephase in the opposite direction. At $t = t90 + \tau + t180 + \tau = T_E$ (echo time), the available magnetization becomes in the $Y'$ direction again and an echo is obtained. Note that the magnetization at echo time is given by

$$\bar{M}_{\text{transverse}}(T_E) = \bar{Y}'M_0e^{-(T_E)/T_2}.$$ 

(2.55)

In this equation, $T_2$ is used instead of $T^*_2$ because $T^I_2$ effects are fully recovered. It is noted however that the $T_2$ is irrecoverable. If the previous sequence is repeated with 2 different echo times, values for $T_2$ could be readily obtained [37].

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Figure 2.7: Spin echo Imaging technique. A $\pi/2$ pulse is applied in the $X'$ direction, causing the magnetization to rotate to the $Y'$ direction at time $t=90$. At time $t90^{+}$, the magnetization starts to dephase in the transverse plane. At time $t90^{+}+\tau$, a $\pi$ pulse is applied in the $Y'$ direction, causing the magnetization to dephase in the opposite direction. At time $t90^{+}+\tau+t180^{+}+\tau$, the available magnetization becomes in the $Y'$ direction again and an echo is obtained.
2.7 Inversion Recovery

The FID and spin-echo experiments deal mostly with the spin-spin relaxation effects (\(T_2^1, T_2^2\)). The inversion recovery method is usually used to determine the spin-lattice interactions (\(T_1\)). At time 0, a \(\pi\) pulse is applied in the \(X\) direction. At time \(t180\), the longitudinal magnetization aligns itself in the \(-z\) direction; the value of which is given by

\[
M_z(t180) = -M_0. \tag{2.56}
\]

At time \(t180 + tIR\), a \(\pi/2\) pulse is applied, the magnetization before and after the application of the \(\pi/2\) pulse is given by

\[
M_z(tIR + t180) = -M_0 e^{-(tIR)/T_1} + M_0 (1 - e^{-(tIR)/T_1}) \tag{2.57}
\]

\[
= M_0 (1 - 2e^{-(tIR)/T_1}) \tag{2.58}
\]

\[
M_{transverse}(t180 + tIR + t90 + t) = |M_0 (1 - 2e^{-(tIR+t90)/T_1})| e^{-(t)/T_2^*}. \tag{2.59}
\]

A null signal is received when \(1 = 2e^{-(tIR+t90)/T_1}\), or when \((tIR + t90) = T_1 \ln 2\). As such, it is possible to obtain a value for \(T_1\) using inversion recovery.

Of course all of the aforementioned methods assume a uniform excitation; i.e. the RF field is homogeneous across the sample. At low frequency, the RF field is fairly uniform across the sample. However, when imaging is performed at high fields such as that with the 8 Tesla system [8] at OSU, the fields become inhomogeneous rendering a major challenge in measuring \(T_1\) and \(T_2\) values [40, 41, 42]

2.8 Repetition of RF Pulses

Since signal to noise is a major issue in MRI, it may be important to repeat the experiment for number of times and then average the measurements for the final
results. For instance the RF pulses for a spin echo sequence could be repeated for several times. If the RF structure of a spin echo sequence is repeated every $T_R$ which is $>> T^*_2$, at $t = t90 + \tau$, the longitudinal magnetization is given by

$$
\tilde{M}_z(t90 + \tau) = \tilde{Z}'(M_z(t90)e^{-,(\tau)/T_1} + M_0(1 - e^{-(\tau)/T_1}))
$$

(2.60)

$$
\tilde{M}_z(t90 + \tau) = \tilde{Z}'(M_0(1 - e^{-(\tau)/T_1})).
$$

(2.61)

After the application of the first $\pi$ pulse:

$$
\tilde{M}_z(t90 + \tau + t180) = \tilde{Z}'(-M_0(1 - e^{-(\tau+t180)/T_1})).
$$

(2.62)

Therefore from Eq. (2.60), the longitudinal magnetization at $t = T_R$:

$$
\tilde{M}_z(T_R) = \tilde{Z}'(M_z(t90 + \tau + t180)e^{-(T_R-(t90+\tau+t180))/T_1} + M_0(1 - e^{-(T_R-(t90+\tau+t180))/T_1})))
$$

(2.63)

$$
\tilde{M}_z(T_R) = \tilde{Z}'M_0(1 - 2e^{-(T_R-(t90+\tau+t180))/T_1} + e^{-(T_R-t90)/T_1})).
$$

(2.64)

Since $T_R$ is $>> t90 + 2\tau + t180 = T_E$,

$$
\tilde{M}_z(T_R) \approx \tilde{Z}'M_0(1 - e^{-T_R/T_1}).
$$

(2.65)

After repetition $= nT_R$:

$$
\tilde{M}_z(nT_R) \approx \tilde{Z}'M_0(1 - e^{-T_R/T_1})^n
$$

(2.66)

while the transverse magnetization at echo time ($T_E$) is given by

$$
\tilde{M}_{\text{transverse}}(nT_R + T_E) \approx M_0(1 - e^{-T_R/T_1})^n e^{-T_E/T_2}.
$$

(2.67)

2.9 Types of Contrast in MRI imaging

2.9.1 Proton Weighted Imaging

As it was established earlier, the amplitude of the magnetization is proportional to the density of spins available for excitation per unit volume. Since different tissues
contain different densities of spins available for excitation, such densities could be used for tissue classification. These types of images are called proton weighted images. To avoid relaxation effects in these types of images, it is imperative that signal acquisition is performed at times \( \ll T_s^* \).

If a perfect \( \pi/2 \) pulse is applied across the sample (the flip angle = \( \pi/2 \) everywhere), the magnetization rotates to transverse plane with an initial value given by Eq. (2.27). The strength of the signal received (detailed derivations of the received signal is provided in Chapter 6), is given by

\[
S(t) = AA_0 \int B_{rec}(x,y,z,t)|M_{transverse}(x,y,z,t)|e^{j(\omega_0 t + \phi(x,y,z,t))} \, dv'
\]  

(2.68)

where \( B_{rec} \) is the receiver field (explained in the next section), \( AA \) is constant related to the electronics including gain, matching etc .., the angle \( \phi(x,y,z,t) \) is an accumulated phase given by

\[
\phi(x,y,z,t) = \int_0^t dt' \omega(x,y,z,t').
\]  

(2.69)

Now if the applied static field is constant, then \( \omega = \omega_0 \), but it is assumed that \( \omega \) is a function of \( x, y, z, \) and \( t \) because gradient fields will be applied later. The substitution of Eq. (2.27) into Eq. (2.68) results in

\[
S(t) = AA(0.25\gamma^2\left(\frac{\hbar}{2\pi}\right)^2\frac{B_0}{KT})\omega_0
\]

\[
\int B_{rec}(x,y,z,t) \alpha_0(x,y,z) e^{j(\omega_0 t + \phi(x,y,z,t))} \, dv'
\]  

(2.70)

\[
S(t) = \int \alpha(x,y,z) e^{j(\omega_0 t + \phi(x,y,z,t))} \, dv'
\]  

(2.71)

where \( \alpha(x,y,z) \) is a variable that is proportional to the \( \alpha_0(x,y,z) \) with the constant of proportionality includes temperature, frequency, electronics, and the receiver field.
The proportionality constant does not vary with space except for the receiver ($B_{rec}$) field and the transmitter ($B_1$) field which is already embedded in the magnetization. It will be demonstrated in Chapters 5 and 6 that the inhomogeneity of the $B_{rec}$ and $B_1$ fields can dramatically affect the quality of the image. This however will be ignored for now. Clearly, if the sampling time is $<< T_2^*$, the NMR signal given above is a function of the distribution of the spin density.

If multiple spin echo acquisitions are performed, Eqs. (2.67, 2.66) show that there are $T_1$ and $T_2$ dependencies in the magnetization. However if the repetition time ($T_R$) is $>> T_1$ and the echo time $T_E$ is $<< T_2$, the relaxation effects could be ignored. For the most general case however, the representation of the spin density is $\alpha(x, y, z, T_1, T_2)$ for a spin echo sequence.

After the completion of the RF pulse, a linear magnetic gradient is applied to the sample. The gradient field varies in space and its strength is $<<$ the strength of the $B_0$ field. We will assume 1D imaging. If a linear gradient field is applied in the $z$ direction, the total static field is given by

$$B_z(z, t) = B_0 + zG(t)$$

where

$$\phi(z, t) = -\int_0^t dt^\prime (\omega_0 + \omega_G(z, t^\prime)).$$

$$\omega_G(z, t) = \gamma zG(t).$$

This process is called frequency encoding: it relates the the position of the spins along some direction with their precessional frequency. Substituting Eq. (2.74) in
Eq. (2.71), the NMR signal is given by

\[
S(t) = \int \alpha(z)e^{j\Phi_G(z,t)} dz 
\]  \hspace{1cm} (2.75)

\[
S(k) = \int \alpha(z)e^{-j(2\pi k(t)z)} dz 
\]  \hspace{1cm} (2.76)

where

\[
k(t) = \frac{\gamma}{2\pi} \int_0^t dt' G(t'). 
\]  \hspace{1cm} (2.77)

\(k(t)\) is defined as the Fourier transform of the spin density of the sample, simply referred to as k-space. The inverse Fourier transform of the signal provides the spin density of the sample

\[
\alpha(z) = \int S(k)e^{j(2\pi k(t)z)} dk. 
\]  \hspace{1cm} (2.78)

The density can be reconstructed from the signal given that there are large set of \(k\) values (a good coverage over \(k\) space).

### 2.9.2 Gradient Echo Imaging

Consider a cylinder of an arbitrarily distribution of spins under static \(B_0\) field where the density only varies along the \(z\) direction (Figure 2.8a). A \(\pi/2\) pulse is applied along the \(X'\) direction such that a transverse magnetization is obtained along the \(Y'\) direction (Figure 2.8b). The FID signal represents the decaying signal in the transverse plane. At the end of the \(\pi/2\) pulse, a linear gradient field (negative) is applied in the \(z\) direction (Figure 2.8c). Since the spins are now subjected to inhomogeneous static (in addition to the inhomogeneity of the \(B_0\) field), the spins dephase in a much faster rate than that when the gradient field is not present.
Figure 2.8: Gradient echo in one dimension. The total magnetic field is plotted to the left of the images. In the top row, the total magnetic field equals to $B_0$. The total magnetic field in the middle row is adjusted by $-G_z$ and $G_z$. 
If the gradient field is applied between time \( t_{90} = t_1 \) and \( t_2 \), the phase associated with the gradient field at \( t_{2G} \) is given by

\[ \phi_G(z, t) = \gamma G z (t_{2G} - t_{90}). \]  

If another gradient (positive) is applied between times \( t_3 \) and \( t_4 \) where \( t_3 > t_2 \) (Figure 2.8d), the associated phase behavior is given by

\[ \phi_G(z, t) = \gamma G z (t_2 - t_{90}) - \gamma G z (t_3). \]  

Between \( t_2 \) and \( t_3 \), the phase does not change because the applied gradient field = 0. A gradient echo is constructed when \( \phi_G(z, t) = 0 \) or when \( t = t_{G_E} = t_3 + t_2 - t_{90} \). Similar to the spin echo time, the gradient echo time is defined as the time when the area under gradients in a time scale vanishes (Figure 2.8e). In general the time interval between \( t_4 \) and \( t_3 \) is chosen such as the echo time lies at the middle of the interval. The data are collected during the period associated with the second gradient. The signal during this period is given by

\[ S(t) = \int \alpha(z)e^{-j(\gamma G z (t - T_{G_E}))}dz \quad (2.81) \]

\[ S(k) = \int \alpha(z)e^{-j(2\pi k |t|)}dz \quad (2.82) \]

where \(-\frac{\gamma}{2\pi}(t_4 - t_3)/2 < k < -\frac{\gamma}{2\pi}(t_4 - t_3)/2\). As such it can be seen from the gradient echo experiment that the entire \( k \) space is filled. Between \( t_2 \) and \( t_{90} \), the \( k \) space is covered between 0 and \(-k_{max}\), and between \( t_4 \) and \( t_3 \), the \( k \) space is covered between \(-k_{max}\) and \( k_{max}\). The analysis above can be easily integrated into an arbitrarily direction instead of \( z \).
2.9.3 Phase Encoding and Two dimensional Imaging

2D imaging could be a thin slice within a 3D object or any thin 2D object. Consider one $\pi/2$ RF pulse with a sequence diagram shown in Figure 2.9. The 2D $k$ space ($k_x$ and $k_y$) is filled by acquiring 1D data ($k_x$) that is phase encoded by the gradient along $y$ ($G_y$). Note that the phase associated with the $y$ gradient is unchanged when the data sampling is along $k_x$ because $G_y = 0$ when $G_x \neq 0$ when $t \geq T G_E$. Note that along $x$, the read gradients ($G_x$) vary similar to that with the gradient echo sequence described in the previous section. The phase encoded gradient along $y$ is present only at the times where $G_x = 0$.

The signal measured for a general case where $G_x$ and $G_y$ can coexist is given by

$$S(k_x, k_y) = \int \alpha(x, y, z) e^{-j(2\pi k_x x + k_y y)} dx dy dz$$  \hspace{1cm} (2.83)

![Figure 2.9](image_url)

**Figure 2.9**: The sequence diagram for 2D imaging in the $x$ and $y$ directions. The read gradient is $G_x$ and the phase ending gradient is $G_y$. 
where

\[ k_x(t) = \frac{\gamma}{2\pi} \int_0^t dt' G_x(t') \] (2.84)

\[ k_y(t) = \frac{\gamma}{2\pi} \int_0^t dt' G_y(t') . \] (2.85)

### 2.9.4 \( T_1 \) Weighted Imaging

In proton density weighted imaging, \( T_E \) and \( T_R \) are chosen such that the effects of \( T_1 \) and \( T_2^* \) are neglected. Since \( T_1 \) values are different for normal soft tissue, \( T_1 \) weighted imaging can provide an excellent contrast for certain types of applications [37]. The effects of spin density however cannot be ignored for \( T_1 \) weighted imaging.

To eliminate the effects of \( T_2^* \), \( T_E \) is still chosen such that \( T_E \ll T_2(Tissue) \) where \( T_2(Tissue) \) is the \( T_2^* \) for each classified tissue. Consider two types of tissues given by A and B, the contrast equation for A and B is given by [37]

\[ C_{AB} = S_A(T_E) - S_B(T_E) \] (2.86)

where the \( S_A(T_E) \) and \( S_B(T_E) \) are the signals associated with the two specified tissues:

\[ S_A(T_E) = \alpha_{0A}(1 - e^{-(T_R)/T_{1A}}) \] (2.87)

\[ S_B(T_E) = \alpha_{0B}(1 - e^{-(T_R)/T_{1B}}) . \] (2.88)

The contrast equation, Eq. (2.86), follows from Eqs. (2.88,2.87):

\[ C_{AB} = (\alpha_{0A} - \alpha_{0B}) - (\alpha_{0A}(e^{-(T_R)/T_{1A}}) - \alpha_{0B}(e^{-(T_R)/T_{1B}})) . \] (2.89)

Therefore, a tissue with shorter \( T_1 \) provides higher signal than that with longer \( T_1 \). Our goal is to maximize the contrast between these two tissues by only altering \( T_R \).
To maximize $C_{AB}$:

$$\frac{dC_{AB}}{dt} = 0$$  \hspace{1cm} (2.90)

$$\frac{\alpha_{0A} (e^{-(T_R)/T_{1A}})}{T_{1A}} = \frac{\alpha_{0b} (e^{-(T_R)/T_{1B}})}{T_{1B}}.$$  \hspace{1cm} (2.91)

The optimum value of $T_R$ could be obtained from the previous equation. When there are more than 2 tissues, the choice of $T_R$ becomes much more difficult. From Eq. (2.91), a long $T_R$ (fully relaxed magnetization) would result in a spin density weighted image. When $B_1$ field extraction methods are utilized, the spins have to be fully relaxed such that a proton weighted image is obtained [43, 44]. When the sample used is homogeneous, the proton density factor is eliminated and the image becomes spatially dependent on the transmitter and the receiver fields.

**$T_2$ Weighted Imaging**

In most of the disease states, the $T_2$ values of such a pathology are higher than that with normal tissue. The relative difference in the $T_2$ values between disease and normal tissue is usually much higher than that with $T_1$ values. As such, $T_2$ weighted images are very useful for the diagnostic of disease. One can also have $T_2^*$ weighted images where there are local Inhomogeneities in the susceptibility values between tissues. This is different from the inhomogeneity associated with the $B_0$ field. In such a case however, the spins dephase very rapidly and the signal is lost faster. Therefore, $T_2^*$ weighted images are usually utilized for functional imaging applications.

To obtain a $T_2^*$ weighted image, the effects of $T_1$ must be avoided by choosing $T_R >> T_1(Tissue)$. The contrast for a gradient echo sequence is given by:

$$C_{AB} = \alpha_{0A} (e^{-(T_E)/T_{2A}}) - \alpha_{0B} (e^{-(T_E)/T_{2B}}).$$  \hspace{1cm} (2.92)
Similar to \( T_1 \) weighted images, the value of \( T_E \) could be optimized by differentiating Eq. (2.92) with respect to \( T_E \) and set the resulting equation to zero. When utilizing spin echo, \( T_2 \) is substituted for \( T_E^* \).

### 2.10 History of RF Coils in MRI

The first coil used in an NMR experiment [1, 2] was the multi-turn solenoid. As time progressed, the external magnetic field strength has increased leading to an increase in the required resonant frequency of operation. As a result, the coil’s stray capacitance and inductance rise and consequently the resonant frequency of the coil cannot surpass a certain limit given by

\[
f = \frac{1}{2\pi\sqrt{LC}}
\]

(2.93)

where \( f \) (Hz) is the resonant frequency; \( L \) (H) and \( C \) (F) are the inductance and the capacitance of the coil, respectively. While being limited by having an upper limit on the resonant frequency, the multi-turn solenoid has been used extensively for spectroscopy and imaging extremities such as knees and wrists. By distributing the capacitance instead of just one lumped one, Cook was able to resonate a coil at a higher frequency [45]. The disadvantage was introducing a different phase shift on each of the coil segments.

The first surface coil was introduced by Ackerman in 1980 [46]. The advantage of this coil is its ability to produce a strong and localized magnetic field that can provide a high SNR compared to other coils especially in the imaging of relatively small volumes (Figure 2.10). Several years later, multiple surface coils were introduced to image relatively large volumes such as a long section of the spine [47].
Although the geometry of saddle coils (Figure 2.11) has been investigated for other applications, it was first adapted by Ginsberg in 1979 to be utilized in NMR head coils [48]. The advantage of such a design is its ability to obtain a near ideal sinusoidal current distribution on the coil legs. However, low filling factor, low upper frequency limit, and low $B_1$ field homogeneity are the problems associated with this coil.

The next generation of the RF coils started with the slotted tube resonator introduced by Schneider in 1977 [49]. Using a quarter wave length tube with two slots and a surrounded shield, Schneider et al. were able to obtain a high sensitivity coil that can have multiple operating frequencies. By adding lumped capacitors to the two ends of the coil, Alderman and Grant significantly reduced the coil length to facilitate its use in spectroscopy [50].

The slotted resonator was also the main key of the next two head coils. The motorboard coil introduced by Willig et al. was invented by terminating the coil with a ground plane which leads to the reduction of the coil length by a half [51]. The second head coil was a hybrid design with slotted tube transmission line elements [23]. The

![Figure 2.10: A simple surface coil.](image-url)
advantage of this design is obtaining a homogeneous field, while a low Q and high electric field (high SAR) in the region of interest are the disadvantages of this coil.

The birdcage resonator (Figure 2.12) is considered to be one of the most famous RF coils used in MRI. It was first introduced by Hayes et al. at General Electric (GE) Corporate Research and Development Center in 1984 [28]. Nowadays, this coil is used in most of the MRI systems especially the GE systems. Compared to the saddle coil, slotted tube resonator, and Alderman-Grant coil, the birdcage coil provides a better SNR and a better overall $B_1$ field homogeneity.

In addition to the aforementioned coils, non-circular (elliptical) head resonators were also demonstrated by Leifer [52], and Bobroff and McCarthy [53]. After 1984, several designs similar to the birdcage coil were proposed. These include end-capped birdcage resonator [54] and the half-birdcage coil [55]. By using a physical ground plane, a higher filling factor was obtained with these coils [54, 55]. The next step in birdcage coil design was the double tuned [56] quadrature birdcage coil. These coils were designed such that imaging of two different nuclei with 2 different Larmor frequencies could be simultaneously obtained.

![Figure 2.11: Schematics of the saddle coil.](image-url)
Following the coils which were based on distributed, yet discrete lumped capacitive and inductive tuning elements, several coil designs based on distributed and non-discrete tuning elements were proposed. The transmission line resonator was introduced by Roschmann as an enhancement of the transmission line properties of conductors [14]. It is described as a thin coaxial cylinder where thin rods are surrounded by a continuous outer conductor and filled in between with a material that has a low dielectric constant. The rods slide closer or farther apart to change the impedance of the transmission line and hence, tune to the desired resonant frequency.
As such, the resonant frequency is dependent on the intrinsic impedance of the transmission line rather than discrete lumped elements (capacitors). With several of these resonators surrounding the object to be imaged and the appropriate feed, a magnetic field is generated around the outer conductor and the nuclei in the sample are excited.

Vaughn et al. introduced the TEM resonator as a resonant cavity with 16 of Roschmann’s resonators [15]. Figure 2.13 displays a 16-strut TEM resonator which is used for clinical imaging at 8 Tesla. The inner conductors are connected to the shield and one or two of the outer conductors is excited to obtain linear or quadrature drive, respectively. Compared to other coils, the TEM resonator is characterized by improved homogeneity, higher Q factor, multiple frequency of operations and not very significant droppage in SNR. Vaughn’s design was followed by the free element resonator [57]. Unlike the TEM resonator, there is no direct connections between the shield and the resonant elements and the non-driven elements are excited through inductive and capacitive coupling.

Dielectric resonators design was the next approach in improving the RF coils performance. These coils are based on hollow cylinders which are constructed of dielectric material that exhibits low conductivity. The intrinsic capacitive and inductive characters of the cylinder constitute the modes of the cavity. Another use of dielectric cylinders is surrounding the birdcage coil with a jacket of a high dielectric constant material to improve the homogeneity of the $B_1$ field [58]. This approach however leads to reduction in coil efficiency [58]. Finding a nuclei of interest with a suitable Larmor frequency and a dielectric material with low loss are the major hurdles for this approach. Wen et al. demonstrated the feasibility of the dielectric resonator approach with coils made of annular rings of deuterium oxide or water [59].
Figure 2.13: A 16-strut TEM resonator.
CHAPTER 3

THE FINITE DIFFERENCE TIME DOMAIN METHOD: A COMPUTATIONAL TOOL FOR THE ANALYSIS OF RF COILS

3.1 Background

When 1.5 Tesla MRI systems were first built, the design of the associated RF coils was not very well understood. For proton imaging, these coils were required to resonate at 64 MHz and to have a uniform circularly polarized component of the transverse magnetic ($B_1^+$) field distribution and low values of the specific absorption rate (SAR) in the human body. Also the quality factor of the coil was required to be relatively high. Several experimental and analytical tools have been used for analyzing the performance of RF coils. Analytically, circuit analysis concepts [60, 61] were applied to predict the resonance frequency of the coil and the Biot-Savart Law was utilized to determine the field distribution within the coil. These concepts invoke quasistatic field approximations [60]. Experimentally, infrared techniques [62, 63] have been explored as means in evaluating the performance of RF coils.

One of the greatest difficulties in designing RF coils is to determine how well the coil functions in the presence of the patient. The effect of the patient on the coil performance can be dramatic. At 64 MHz, the Q of the overall system is usually
dominated by the patient. Also the resonance frequency of the coil shifts in the presence of the patient. This shift may not be too significant at 64 MHz, and typically no re-tuning of the coil is required; however, from the experiments and simulations done in this work, it is observed that the sensitivity of the resonance frequency of the coil increases with frequency. Thus, re-tuning the coil to correct for the resonance frequency shift is always necessary. Another major difficulty is determining how the coil design affects the SAR within the patient.

At 1.5 Tesla, to determine the magnetic field distribution within the coil, the RF coil designers assume that the currents on the coil structure are uniform and then use the Biot-Savart Law to determine the magnetic field produced by these currents. The assumption of the uniform currents is realized from circuit models. The circuit model is a zero dimensional approximation for the three-dimensional electromagnetic resonance behavior in the coil. Such models can be very accurate for modeling relatively complex coil geometries; however, there are two major limitations. First, the circuit models approximation breaks down when the coil geometry is a significant fraction of the wavelength. This limitation is not critical at 64 MHz, since the wavelength is 4.7 m. Thus, even the diameter of the body coil (approximately 80 cm diameter) is less than 1/5 of a wavelength [60]. The second limitation is the difficulty of obtaining a circuit representation for all the coil geometries. In addition, circuit models breaks down when tissue is in a coil.

As the static magnetic ($B_0$) field of MRI scanners increases, the larger the ratio of the aligned nuclei in the human body becomes. Consequently, this leads to MRI images with much greater detail due to the higher SNR featured in these systems. However on the technical side, the Larmor frequency rises linearly with increasing $B_0$. 

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field. As such, the operational frequency for proton imaging rises to equal the Larmor frequency. As a result of the increase of the operational frequency, the applied RF field wavelength in tissue becomes smaller and the electrical dimensions of the RF coil and the electrically large anatomical structures of the head/body become comparable to the operating wavelength. Under these conditions the currents in the coil do not behave in a manner which circuit analysis can predict.

For instance at 8 Tesla, the operational frequency for proton imaging is 340 MHz. The wavelength inside the head (average dielectric constant of 64) is approximately 11 cm. Therefore, the distance between back and front of the head is about twice the operational wavelength. Given this fact and that the human head is asymmetrical and contains highly inhomogeneous lossy materials, strong electromagnetic interactions are expected between the RF coil, the excitation source(s) and the tissue. This can result in non-useful images, since the distribution of the $B_1^+$ field is now prone to be inhomogeneous in the human head. The interactions between the coil, excitation source(s) and the tissue can also lead to undesired local hot spots in certain organs. As a result, alternative approaches that consider full wave electromagnetic analysis with an accurate model of the tissue, the source, and the coil are necessary to address these problems and would be a great aid in redesigning RF coils.

### 3.2 Previous Work

There has been extensive use of electromagnetic simulation tools for medical applications. Full-wave methods are commonly used to model microwave hyperthermia systems for treating tumors [64, 65, 66]. Also, they have been used to predict SAR for humans in the presence of numerous electrical devices such as phones, power
lines, and antenna systems [67, 68]. Because of this, much of the electromagnetic properties of the human body have been carefully characterized for a large range of frequencies.

Until recently, full-wave numerical methods have seldom been used to model the fields in RF coils for MRI systems. There has not been much need for such an approach because most of the systems are at magnetic field strengths of 1.5 T and below. Thus, circuit approximations are appropriate for modeling RF coils. With the growth in the number of 4 Tesla [69, 70, 24, 18], 7 Tesla [71], and 8 Tesla [8] systems, electromagnetic modeling has become an essential tool for designing and evaluating RF coils in MRI. It is anticipated that the optimization of RF coils for high field systems will rely heavily upon numerical modeling [35, 34, 72, 73].

The finite difference time domain (FDTD) method has been used in calculating electromagnetic fields interaction between the human biological tissues and radiating objects such as surface MRI RF coils [74, 75]. Significant effort has been devoted to modeling the electrical characteristics of RF head coils. For instance, both the birdcage coil [28] and the TEM resonator [15] have been theoretically analyzed. The finite element method (FEM) was used to model the TEM resonator loaded with a phantom and human head model [15]. A 2-D FEM model has also been used to study the $B_1$ field and the SAR in a birdcage coil loaded with a human head model [76]. The FEM has also been utilized to calculate the SAR inside a human head model in a saddle shape MRI head coil [77]. Since the wavelength was electrically large compared to the largest dimension of the saddle coil at $64 \text{ MHz}$, Simunic et al assumed quasi-static conditions in calculating the fields outside the human head [77].
In 1996, Jin et al employed the conjugate gradient with fast Fourier transform method to evaluate the electromagnetic fields inside a human head model placed within the birdcage coil [25]. Recently, the finite difference time domain (FDTD) method was used to model the birdcage coil loaded with a human head model [78, 79].

The aforementioned studies shared the common assumption that the RF coil functions as an azimuthal transmission line at all the frequencies of interest [15, 76, 77, 25, 78, 79]. For instance, this was done by determining the current distribution in the coil without the head being present using the method of moments [79] or by replacing the lumped capacitors with voltage sources whose magnitudes varied sinusoidally [78]. As a result, the currents on the birdcage elements are required to have a sinusoidal distribution (the coil is solely operating in TEM mode of interest, the ideal mode of operation for the birdcage)

\[ I_i = I_{\text{max}} \cos \left( \frac{(i-1)\pi}{N} \right) \]

where \( I_i \) is the current in the \( i \)th element, \( I_{\text{max}} \) is the maximum current and \( N \) is the number of current elements.

Such assumptions however are not valid when the human head is positioned within the coil due to electromagnetic coupling between the coil and the head [80, 81, 82]. Although the resulting inaccuracies may not be too significant at 64 MHz, the field distribution calculations at higher frequencies are much more prone to be invalid. Even when the coil is empty, there are many cases where the ideal current distribution is not present [83, 82]. Chen stated that neglecting the effect of the head on the coil current distribution can be a major source of error and the most accurate simulation involves modeling the coil and the object to be imaged as a single system [79].
however was considered to be a difficult problem [79]. This approach, where the coil and the object to be imaged are treated as a single system, was considered in all the modeling work of this dissertation.

3.3 Motivation for Using the Finite Difference Time Domain

There are three major numerical methods used in electromagnetics: the finite element method (FEM), the FDTD method and the integral equation method (moments method (MM)). Although MM and FEM can be solved in the time domain, they are rarely used in this way; therefore, FEM and MM are usually associated with the frequency domain.

MM is different from FEM and FDTD in that MM can be formulated in terms of unknown surface currents on perfect conductors and unknown volume currents in materials, whereas the unknowns in FEM and FDTD are the fields values everywhere within the volume of interest. Because of this, MM has great advantages over FDTD and FEM when it is applied to geometries consisting of only perfect conductors, since the number of unknowns in MM is much less than for the other two methods.

For problems where large portions of the geometry are non-perfectly conducting, the number of unknowns for all three methods is comparable; however, the computation time is very different. Both MM and FEM require the solution of matrix equation. Since the number of unknowns to model the coil is very large, iterative methods offer the only viable way to solve the matrix equation. Assuming the number of unknowns in the problem is \( N \), the computation time is proportional to \( N^\Psi \), where \( \Psi \) is > 2 for MM and > 1.5 for FEM. It should be noted that when there are large permittivity and
conductivity contrasts in the geometry, which occurs whenever the human tissue is present, the values for $\Psi$ may be significantly larger than the nominal given values.

On the other hand, the FDTD method does not require a matrix solution, and its computation time is proportional to $N^4$. There is also a wide disparity in terms of memory requirements. The memory needed to solve an MM problem with 50,000 unknowns can be used to solve an FEM problem with 5,000,000 unknowns and an FDTD problem with 100,000,000 unknowns. The one disadvantage of FDTD relative to FEM is that it is less flexible for modeling arbitrary geometries, because FEM can be applied to unstructured grid. For the electrically large geometries that are encountered in high-field MRI, one can argue that it is better to use FDTD than FEM, because in many cases, the number of unknowns needed to solve the problem is relatively very large.

3.4 The Finite Difference Time Domain

The FDTD technique was introduced by Kane Yee in 1966 [3]. An algorithm, now referred to as the Yee algorithm, was developed to give a direct solution of Maxwell’s time-dependent curl equations [84]. Recently, this technique has become more popular among electromagnetic researchers to aid in the study of electromagnetic properties such as radiation, propagation, and the scattering of waves from either simple or complex structures. For instance, in the medical field, FDTD has been used in calculating electromagnetic fields interaction between the human biological tissues and radiating objects, including hand held communication devices [68, 67] and MRI RF coils [74, 75, 85, 86].
The FDTD method is essentially based on replacing the spatial and the time domain derivatives of Maxwell’s equations with finite difference approximations [87]. In other words, all the differential operators of the curl equations are replaced by second-order accurate central difference approximations [88].

3.5 Formulation

Maxwell’s equations [36] for isotropic, source free, and homogeneous media are given as:

\[
\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} - \sigma_m \vec{H} \quad (3.2)
\]

\[
\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \sigma_e \vec{E} \quad (3.3)
\]

\[
\nabla \cdot \vec{D} = \rho_e \quad (3.4)
\]

\[
\nabla \cdot \vec{B} = \rho_m \quad (3.5)
\]

given that the electric \( \vec{D} \) and magnetic \( \vec{B} \) flux densities are defined as

\[
\vec{B} = \mu \vec{H} \quad (3.6)
\]

\[
\vec{D} = \varepsilon \vec{E} \quad (3.7)
\]

where \( \vec{E} \) (V/m) and \( \vec{H} \) (A/m) are the electric and the magnetic fields, and \( \sigma_m \) (\( \Omega/m \)) and \( \sigma_e \) (\( S/m \)) are the electric and the magnetic conductivities, respectively. The dielectric parameters, permeability and permittivity are given by \( \mu \) (H/m) and \( \varepsilon \) (F/m), while \( \rho_e \) (C/m\(^3\)) and \( \rho_m \) (Wb/m\(^3\)) are the electric and magnetic charge densities, respectively. For three-dimensional structures, rewriting the above in the rectangular
coordinates will result in the following:

\[
\mu \frac{\partial H_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma_m H_x
\]  
(3.8)

\[
\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma_m H_y
\]  
(3.9)

\[
\mu \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma_m H_z
\]  
(3.10)

\[
\varepsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma_e E_x
\]  
(3.11)

\[
\varepsilon \frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma_e E_y
\]  
(3.12)

\[
\varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma_e E_z
\]  
(3.13)

where \( F_w \) is the \( F \) (\( H \) or \( E \)) field component in direction \( w \) (\( x, y \) or \( z \)). Following the Yee’s notation [3], a point in space is

\[
(i, j, k) = (i\Delta x, j\Delta y, k\Delta z)
\]  
(3.14)

and any function of space and time is defined as

\[
F^n(i, j, k) = F(i\Delta x, j\Delta y, k\Delta z, n\Delta t)
\]  
(3.15)

where \( \Delta x, \Delta y, \Delta z \) are the spatial steps in \( x, y, z \) directions, respectively, and \( \Delta t \) is the time step. The central difference approximation is obtained from Taylor’s theorem [89]

\[
\frac{\partial F(\tau)}{\partial \tau} = \frac{F(\tau + \Delta \tau/2) - F(\tau - \Delta \tau/2)}{\Delta \tau} + O((\Delta \tau)^2).
\]  
(3.16)

\( O((\Delta \tau)^2) \) is a function of \( (\Delta \tau)^2 \) and represents the error from the central difference approximations. To achieve second-order accuracy, we drop the error term and apply Eq. (3.16) on (3.15) to get

\[
\frac{\partial F^n(i, j, k)}{\partial x} \approx \frac{F^n(i + \frac{1}{2}, j, k) - F^n(i - \frac{1}{2}, j, k)}{\Delta x}
\]  
(3.17)
and

$$\frac{\partial F^n(i,j,k)}{\partial n} \approx \frac{F^{n+\frac{1}{2}}(i,j,k) - F^{n-\frac{1}{2}}(i,j,k)}{\Delta t}$$

(3.18)

for space and time derivatives, respectively. Substituting from Eqs. (3.17) and (3.18) into (3.8)-(3.13), the FDTD equations become:

$$E_{x}^{n+1}(i+\frac{1}{2},j,k) = \left( \frac{2\varepsilon - \sigma_e \Delta t}{2\varepsilon + \sigma_e \Delta t} \right) E_x^n(i+\frac{1}{2},j,k) + \left[ \frac{2\Delta t}{2\varepsilon + \sigma_e \Delta t} \right]$$

$$\left\{ \frac{1}{\Delta y} \left[ H_{z}^{n+\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) - H_{z}^{n+\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k) \right] - \frac{1}{\Delta z} \left[ H_{y}^{n+\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) - H_{y}^{n+\frac{1}{2}}(i+\frac{1}{2},j,k-\frac{1}{2}) \right] \right\}$$

(3.19)

$$E_{y}^{n+1}(i,j+\frac{1}{2},k) = \left( \frac{2\varepsilon - \sigma_e \Delta t}{2\varepsilon + \sigma_e \Delta t} \right) E_y^n(i,j+\frac{1}{2},k) + \left[ \frac{2\Delta t}{2\varepsilon + \sigma_e \Delta t} \right]$$

$$\left\{ \frac{1}{\Delta z} \left[ H_{x}^{n+\frac{1}{2}}(i,j+\frac{1}{2},k+\frac{1}{2}) - H_{x}^{n+\frac{1}{2}}(i,j,k+\frac{1}{2}) \right] - \frac{1}{\Delta x} \left[ H_{z}^{n+\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) - H_{z}^{n+\frac{1}{2}}(i-\frac{1}{2},j+\frac{1}{2},k) \right] \right\}$$

(3.20)

$$E_{z}^{n+1}(i,j,k+\frac{1}{2}) = \left( \frac{2\varepsilon - \sigma_e \Delta t}{2\varepsilon + \sigma_e \Delta t} \right) E_z^n(i,j,k+\frac{1}{2}) + \left[ \frac{2\Delta t}{2\varepsilon + \sigma_e \Delta t} \right]$$

$$\left\{ \frac{1}{\Delta x} \left[ H_{y}^{n+\frac{1}{2}}(i,j,k+\frac{1}{2}) - H_{y}^{n+\frac{1}{2}}(i,j,k+\frac{1}{2}) \right] - \frac{1}{\Delta y} \left[ H_{x}^{n+\frac{1}{2}}(i,j+\frac{1}{2},k) - H_{x}^{n+\frac{1}{2}}(i,j+\frac{1}{2},k) \right] \right\}$$

(3.21)

$$H_{x}^{n+\frac{1}{2}}(i,\frac{1}{2},j+\frac{1}{2}) = \left( \frac{2\mu - \sigma_m \Delta t}{2\mu + \sigma_m \Delta t} \right) H_{x}^{n-\frac{1}{2}}(i,\frac{1}{2},j+\frac{1}{2}) + \left[ \frac{2\Delta t}{2\mu + \sigma_m \Delta t} \right]$$

$$\left\{ \frac{1}{\Delta z} \left[ E_{z}^{n}(i,\frac{1}{2},j,k+1) - E_{z}^{n}(i,\frac{1}{2},j,k) \right] - \frac{1}{\Delta y} \left[ E_{y}^{n}(i,\frac{1}{2},j,k+1) - E_{y}^{n}(i,\frac{1}{2},j,k) \right] \right\}$$

(3.22)
Maxwell's time-dependent equations. For 3D problems the stability criterion is:

\[ H_n^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) = \left( \frac{2\mu - \sigma_m \Delta t}{2\mu + \sigma_m \Delta t} \right) H_n^{n-\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) + \left[ \frac{2\Delta t}{2\mu + \sigma_m \Delta t} \right] \]

\[ \left\{ \frac{1}{\Delta x} \left[ E_n^n(i + 1, j, k + \frac{1}{2}) - E_n^n(i, j, k + \frac{1}{2}) \right] \right\} - \frac{1}{\Delta z} \left[ E_n^n(i + \frac{1}{2}, j, k + 1) - E_n^n(i + \frac{1}{2}, j, k) \right] \]  

(3.23)

\[ H_n^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k) = \left( \frac{2\mu - \sigma_m \Delta t}{2\mu + \sigma_m \Delta t} \right) H_n^{n-\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k + \frac{1}{2}) + \left[ \frac{2\Delta t}{2\mu + \sigma_m \Delta t} \right] \]

\[ \left\{ \frac{1}{\Delta y} \left[ E_n^n(i + \frac{1}{2}, j + 1, k) - E_n^n(i + \frac{1}{2}, j, k) \right] \right\} - \frac{1}{\Delta x} \left[ E_n^n(i + 1, j + \frac{1}{2}, k) - E_n^n(i, j + \frac{1}{2}, k) \right] \]  

(3.24)

From Eqs. (3.19)-(3.24), it is apparent that the updated value of the field component is a function of its previous value (one time step before) and the previous values (half time step before) of the surrounding fields at half spatial steps away. For instance, \( E_n^{n+1}(i + \frac{1}{2}, j, k) \) depends on the previous value \( E_n^n(i + \frac{1}{2}, j, k) \) and the surrounding magnetic fields at a half time step before and half spatial steps away \( H_n^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k), H_n^{n+\frac{1}{2}}(i + \frac{1}{2}, j - \frac{1}{2}, k), H_n^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}), \) and \( H_n^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k - \frac{1}{2}) \).

The criteria for choosing time step \( \Delta t \) is based on Courant-Friedrichs (CFL) stability criterion [84], which is driven from time and space eigenvalue problems of Maxwell’s time-dependent equations. For 3D problems the stability criterion is:

\[ \Delta t v_{\text{max}} \leq \frac{1}{\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}} \]  

(3.25)

where \( v_{\text{max}} \) \((m/S)\) is the maximum velocity of the propagating waves.

\[ v_{\text{max}} = \frac{1}{\sqrt{\mu \varepsilon}} \]  

(3.26)

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For free space, the speed of light $= v_{max} = C = 3 \times 10^8 \ m/s$. In general, the choice of the spatial step is proportional to the minimum wavelength of the problem (maximum size of $1/10$ of a wavelength). This is only true if the wavelength dominates accuracy (the domain is a multiple of wavelength and each element in the domain can be represented by at least one spatial step). For a lossless case, if $\Delta x = \Delta y = \Delta z$, a good choice of the spatial step is:

$$\Delta x = \frac{\lambda_{min}}{20} = \frac{f}{20\sqrt{\mu_{max}\varepsilon_{max}}} \quad (3.27)$$
where \( \lambda_{\text{min}} \) (m) is the minimum wavelength and \( f \) (Hz) is the frequency of operation, while \( \mu_{\text{max}} \) and \( \varepsilon_{\text{max}} \) are the maximum relative permeability and permittivity, respectively.

The positioning of the field components \( (E_x, H_x, E_y, H_y, E_z, \text{and } H_z) \) is done by the Yee cell shown in (Figure 3.1). The Yee cell is a rectangular block where the electric and magnetic fields are staggered. The electric field values are sampled at the center of every edge on each block and the vector directions of the electric fields are the same as the directions of these edges. The magnetic field values are sampled at the centroid of each block face and the vector directions of the magnetic fields are perpendicular to each of these block faces. Note that the edges of the Yee cell coincide with the Cartesian coordinates.

3.6 The Outer Boundary Conditions

Special boundary conditions must be placed at the outer boundary of the computation domain to absorb the outward propagating energy. The development of appropriate outer boundary conditions is actually a major research topic within the electromagnetic modeling community. There are several techniques to account for the outer boundary conditions.

First is the brute force method where the computational domain was chosen to be large enough such that the reflections at the terminations planes do not reach the points of interest by the end of the simulations. The disadvantage of this method is that for electrically large problems, it is computationally expensive. Second is applying an absorbing boundary conditions (ABC) on the outer boundary. The application of this is more difficult than the first method. Instabilities and inaccuracy especially
when the waves are incident on the outer boundaries at oblique angles are the two dis-
advantages of this method. The third method is using a lossy material to simulate an
infinite region of free space. First introduced in 1983 by Holland and Williams [90],
this method of truncation has not been commonly used because of the reflections that
occur at the free space-material interface. Solutions such as the use of low loss mate-
rials were used to account for this problem; however, this lead to larger computational
domains because the lossy regions have to be significantly large.

In 1994, Berenger modified Maxwell’s equations to obtain material properties that
allows no reflections [29]. This material known as the perfectly matched layer (PML)
allows no reflections for a plane wave incident from free space at any angle given that
the free space-PML interface is infinite. At the same year, Chew and Weedon [30]
provided a different analysis of the PML based on “coordinate stretching”. Unlike, the
lossy material proposed by Holland [90], the PML is not required to be significantly
large. Because the PML is currently considered to be the best choice for the FDTD
method, it was selected to be used as the absorbing boundary conditions to account
for the RF radiation from the coil.

3.6.1 The PML Formulation: Coordinate Stretching Approach

For a general medium, the modified Maxwell’s equations in the frequency domain
with \(e^{j\omega t}\) dependence is given by [30]

\[
\nabla_e \times \vec{E} = -j \omega \mu \vec{H} \tag{3.28}
\]

\[
\nabla_h \times \vec{H} = j \omega \varepsilon \vec{E} \tag{3.29}
\]

\[
\nabla_h \cdot \varepsilon \vec{E} = \rho_e \tag{3.30}
\]

\[
\nabla_e \cdot \mu \vec{H} = \rho_m \tag{3.31}
\]

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where

\[
\begin{align*}
\nabla_e &= \hat{x} \frac{1}{e_x} \frac{\partial}{\partial x} + \hat{y} \frac{1}{e_y} \frac{\partial}{\partial y} + \hat{z} \frac{1}{e_z} \frac{\partial}{\partial z} \quad (3.32) \\
\nabla_h &= \hat{x} \frac{1}{h_x} \frac{\partial}{\partial x} + \hat{y} \frac{1}{h_y} \frac{\partial}{\partial y} + \hat{z} \frac{1}{h_z} \frac{\partial}{\partial z} \quad (3.33)
\end{align*}
\]

The coordinate-stretching variables \((e_x, e_y, e_z)\) and \((h_x, h_y, h_z)\) are used to stretch coordinates for \(\nabla_e\) and \(\nabla_h\), respectively [30]. A general plane-wave solution to Eqs. (3.28)-(3.31) is:

\[
\begin{align*}
\vec{E} &= \vec{E}_0 e^{i \vec{k} \cdot \vec{r}} \quad (3.34) \\
\vec{H} &= \vec{H}_0 e^{i \vec{k} \cdot \vec{r}} \quad (3.35)
\end{align*}
\]

where

\[
\begin{align*}
\vec{k} &= \hat{x} k_x + \hat{y} k_y + \hat{z} k_z \quad (3.36) \\
\vec{r} &= \hat{x} r_x + \hat{y} r_y + \hat{z} r_z \quad (3.37)
\end{align*}
\]

\(\vec{k}\) is the vector wave number and \((\vec{r})\) is the position vector. Substituting from Eqs. (3.34) and (3.35) into (3.28) and (3.29), we get

\[
\begin{align*}
\vec{k}_e \times \vec{E} &= \omega \mu \vec{H} \quad (3.38) \\
\vec{k}_h \times \vec{H} &= -\omega \epsilon \vec{E} \quad (3.39)
\end{align*}
\]

for

\[
\begin{align*}
\vec{k}_e &= \hat{x} \frac{k_x}{e_x} + \hat{y} \frac{k_y}{e_y} + \hat{z} \frac{k_z}{e_z} \quad (3.40) \\
\vec{k}_h &= \hat{x} \frac{k_x}{h_x} + \hat{y} \frac{k_y}{h_y} + \hat{z} \frac{k_z}{h_z} \quad (3.41)
\end{align*}
\]

From Eqs. (3.40) and (3.41), the following is obtained

\[
-\omega^2 \mu \epsilon = \vec{k}_e \times \vec{k}_h \times \vec{H} \quad (3.42)
\]

55
Using vector identity, for a homogeneous medium, the dispersion relation becomes

\[ \omega^2 \mu \varepsilon = \vec{k}_e \cdot \vec{k}_h \]  
\[ = k^2 \]  
\[ = \frac{k_x^2}{e_x h_x} + \frac{e_y h_y}{s_y^2} + \frac{e_z h_z}{s_z^2}. \]  

The above equation represents the general equation of an ellipsoid in 3D and has the solution:

\[ k_x = k \sqrt{e_x h_x \sin(\theta) \cos(\phi)} \]  
\[ k_y = k \sqrt{e_y h_y \sin(\theta) \sin(\phi)} \]  
\[ k_z = k \sqrt{e_z h_z \cos(\theta)}. \]  

For a \( TE_z \) polarized wave propagating from region 1 into region 2 where the normal between the two regions is in the \( \hat{z} \) direction. The fields values are

\[ \vec{E}_i = \vec{E}_0 e^{j \vec{k}_i \cdot \vec{r}} \]  
\[ \vec{E}_r = R^{TE} \vec{E}_0 e^{j \vec{k}_r \cdot \vec{r}} \]  
\[ \vec{E}_t = T^{TE} \vec{E}_0 e^{j \vec{k}_t \cdot \vec{r}} \]  

where \( E_i, E_r, \) and \( E_t \) are the incident, reflected and transmitted fields, while \( R^{TE} \) and \( T^{TE} \) are the reflection and the transmission coefficients, respectively. Now, the phase matching requires that \( \vec{k}_i = \vec{k}_r = \vec{k}_x \) and \( \vec{k}_y = \vec{k}_t = \vec{k}_y \) which will follow that \( E_{0i} = E_{0r} = E_{0t} \). The boundary conditions result in [30]:

\[ 1 + R^{TE} = T^{TE} \]
and

\[ \vec{k}_r \cdot \hat{z} = -\vec{k}_i \cdot \hat{z} \]  \hspace{1cm} (3.53)

\[ k_{iz}\mu_2 e_{2z}(1-R^{TE}) = T^{TE}k_{iz}\mu_1 e_{1z} \]  \hspace{1cm} (3.54)

Using Eqs. (3.52) and (3.54), the reflection and transmission coefficients are given by

\[ R^{TE} = \frac{k_{iz}\mu_2 e_{2z} - k_{iz}\mu_1 e_{1z}}{k_{iz}\mu_2 e_{2z} + k_{iz}\mu_1 e_{1z}} \]  \hspace{1cm} (3.55)

\[ T^{TE} = \frac{2k_{iz}\mu_2 e_{2z}}{k_{iz}\mu_2 e_{2z} + k_{iz}\mu_1 e_{1z}} \]  \hspace{1cm} (3.56)

for a TE case, and

\[ R^{TM} = \frac{k_{iz}\varepsilon_2 h_{2z} - k_{iz}\varepsilon_1 h_{1z}}{k_{iz}\varepsilon_2 h_{2z} + k_{iz}\varepsilon_1 h_{1z}} \]  \hspace{1cm} (3.57)

\[ T^{TM} = \frac{2k_{iz}\varepsilon_2 h_{2z}}{k_{iz}\varepsilon_2 h_{2z} + k_{iz}\varepsilon_1 h_{1z}} \]  \hspace{1cm} (3.58)

for a TM case.

Let us assume that region 1 is free space \((\mu_1 = \mu_0 \text{ and } \varepsilon_1 = \varepsilon_0)\). For a perfectly matched medium, we choose \(\mu_2 = \mu_1 = \mu_0, \varepsilon_2 = \varepsilon_1 = \varepsilon_0, e_x = h_x = s_y, \text{ and } e_y = h_y = s_y\). Using Eqs. (3.46) and (3.47) along with the phase matching conditions \((k_{1x} = k_{2x} \text{ and } k_{1y} = k_{2y})\), the following is obtained:

\[ s_{1x}\sin \theta_1 \cos \phi_1 = s_{2x}\sin \theta_2 \cos \phi_2 \]  \hspace{1cm} (3.59)

\[ s_{1x}\sin \theta_1 \sin \phi_1 = s_{2x}\sin \theta_2 \sin \phi_2. \]  \hspace{1cm} (3.60)

If \(s_{1x} = s_{2x} = s_x \text{ and } s_{1y} = s_{2y} = s_y\), then \(\theta_1 = \theta_2, \phi_1 = \phi_2\), and both \(R^{TE} = 0\), and \(R^{TM} = 0\) [30]. The previous result is for all angles of incidence and all frequencies.
By choosing \( s_y = s_y = s_{1z} = 1 \) and leaving \( s_{2z} \) to be a free variable, we obtain:

\[
\begin{align*}
    k_{1x} &= k_{2x} = k_0 \sin \theta \cos \phi \\
    k_{1y} &= k_{2y} = k_0 \sin \theta \sin \phi \\
    k_{1z} &= k_0 \cos \theta \\
    k_{2z} &= k_0 s_{2z} \cos \theta \\
    k_0 &= \omega \sqrt{\mu_0 \varepsilon_0}.
\end{align*}
\] (3.61-3.65)

By choosing \( s_{2z} = 1 - j \frac{\sigma_z}{\omega \varepsilon_0} \), the wave will attenuate in the \( \hat{z} \) direction.

The transformation into the time domain is straightforward. The stretch coordinates and the differential operators are defined as

\[
\begin{align*}
    s_x &= 1 - \frac{j \sigma_x}{\omega \varepsilon} \\
    s_y &= 1 - \frac{j \sigma_y}{\omega \varepsilon} \\
    s_z &= 1 - \frac{j \sigma_z}{\omega \varepsilon}.
\end{align*}
\] (3.66-3.68)

\[
\nabla_e = \nabla_e = \hat{x} \frac{1}{s_x} \frac{\partial}{\partial x} + \hat{y} \frac{1}{s_y} \frac{\partial}{\partial y} + \hat{z} \frac{1}{s_z} \frac{\partial}{\partial z}.
\] (3.69)

The modified Maxwell’s equations become:

\[
\begin{align*}
    - j \omega \mu \vec{H}_{sx} &= \frac{\partial}{s_x} \hat{x} \times \vec{E} \\
    - j \omega \mu \vec{H}_{sy} &= \frac{\partial}{s_y} \hat{y} \times \vec{E} \\
    - j \omega \mu \vec{H}_{sz} &= \frac{\partial}{s_z} \hat{z} \times \vec{E} \\
    j \omega \varepsilon \vec{E}_{sx} &= \frac{\partial}{s_x} \hat{x} \times \vec{H} \\
    j \omega \varepsilon \vec{E}_{sy} &= \frac{\partial}{s_y} \hat{y} \times \vec{H} \\
    j \omega \varepsilon \vec{E}_{sz} &= \frac{\partial}{s_z} \hat{z} \times \vec{H}
\end{align*}
\] (3.70-3.75)
for
\[ \vec{H} = \vec{H}_{sx} + \vec{H}_{sy} + \vec{H}_{sz} \]  \hspace{1cm} (3.76)
\[ \vec{E} = \vec{E}_{sx} + \vec{E}_{sy} + \vec{E}_{sz} \]  \hspace{1cm} (3.77)

Using Eqs. (3.66)-(3.69) and rewriting Eqs. (3.70)-(3.75), the modified time domain equations are defined as

\[ \mu \frac{\partial \vec{H}_{sx}}{\partial t} + \frac{\sigma_x \mu}{\varepsilon} \vec{H}_{sx} = - \frac{\partial}{\partial x} \hat{\mathbf{e}} \times \vec{E} \]  \hspace{1cm} (3.78)
\[ \mu \frac{\partial \vec{H}_{sy}}{\partial t} + \frac{\sigma_y \mu}{\varepsilon} \vec{H}_{sy} = - \frac{\partial}{\partial y} \hat{\mathbf{e}} \times \vec{E} \]  \hspace{1cm} (3.79)
\[ \mu \frac{\partial \vec{H}_{sz}}{\partial t} + \frac{\sigma_z \mu}{\varepsilon} \vec{H}_{sz} = - \frac{\partial}{\partial z} \hat{\mathbf{e}} \times \vec{E} \]  \hspace{1cm} (3.80)
\[ \varepsilon \frac{\partial \vec{E}_{sx}}{\partial t} + \sigma_x \vec{E}_{sx} = \frac{\partial}{\partial x} \hat{\mathbf{e}} \times \vec{H} \]  \hspace{1cm} (3.81)
\[ \varepsilon \frac{\partial \vec{E}_{sy}}{\partial t} + \sigma_y \vec{E}_{sy} = \frac{\partial}{\partial y} \hat{\mathbf{e}} \times \vec{H} \]  \hspace{1cm} (3.82)
\[ \varepsilon \frac{\partial \vec{E}_{sz}}{\partial t} + \sigma_z \vec{E}_{sz} = \frac{\partial}{\partial z} \hat{\mathbf{e}} \times \vec{H}. \]  \hspace{1cm} (3.83)

Replacing the spatial and the time domain derivatives of the above equations by the second-order accurate central difference approximations yields the 12 FDTD equations inside the PML:

\[ E_{sx(i+\frac{1}{2},j,k)}^{n+1} = \left( \frac{2\varepsilon - \sigma_y \Delta t}{2\varepsilon + \sigma_y \Delta t} \right) E_{sx(i+\frac{1}{2},j,k)}^n + \left[ \frac{2\Delta t}{2\varepsilon + \sigma_y \Delta t} \right] \]
\[ \frac{1}{\Delta y} \left\{ \left[ H_{sx(i+\frac{1}{2},j+\frac{1}{2},k)}^{n+\frac{1}{2}} - H_{sx(i+\frac{1}{2},j-\frac{1}{2},k)}^{n+\frac{1}{2}} \right] + \left[ H_{sx(i+\frac{1}{2},j,\frac{1}{2},k)}^{n+\frac{1}{2}} - H_{sx(i+\frac{1}{2},j,\frac{1}{2},k)}^{n+\frac{1}{2}} \right] \right\} \]  \hspace{1cm} (3.84)
\[ E_{x_{x_{c}}}^{n+1}(i + \frac{1}{2}, j, k) = \left( \frac{2\varepsilon - \sigma_{x_{c}} \Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right) E_{x_{x_{c}}}^{n}(i + \frac{1}{2}, j, k) - \left[ \frac{2\Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right] \]

\[ \frac{1}{\Delta z} \left\{ \left[ H_{x_{x_{c}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k) - H_{x_{x_{c}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k - \frac{1}{2}) \right] + \right. \]

\[ \left. \left[ H_{x_{x_{c}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) - H_{x_{x_{c}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k - \frac{1}{2}) \right] \right\} \] (3.85)

\[ E_{x_{y_{x}}}^{n+1}(i, j + \frac{1}{2}, k) = \left( \frac{2\varepsilon - \sigma_{x_{c}} \Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right) E_{x_{y_{x}}}^{n}(i, j + \frac{1}{2}, k) - \left[ \frac{2\Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right] \]

\[ \frac{1}{\Delta x} \left\{ \left[ H_{x_{x_{c}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k) - H_{x_{x_{c}}}^{n+\frac{1}{2}}(i - \frac{1}{2}, j + \frac{1}{2}, k) \right] + \right. \]

\[ \left. \left[ H_{x_{x_{c}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k) - H_{x_{x_{c}}}^{n+\frac{1}{2}}(i - \frac{1}{2}, j + \frac{1}{2}, k) \right] \right\} \] (3.86)

\[ E_{x_{y_{z}}}^{n+1}(i, j, k + \frac{1}{2}) = \left( \frac{2\varepsilon - \sigma_{x_{c}} \Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right) E_{x_{y_{z}}}^{n}(i, j, k + \frac{1}{2}) + \left[ \frac{2\Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right] \]

\[ \frac{1}{\Delta z} \left\{ \left[ H_{x_{x_{c}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) - H_{x_{x_{c}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k) \right] + \right. \]

\[ \left. \left[ H_{x_{x_{c}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) - H_{x_{x_{c}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k) \right] \right\} \] (3.87)

\[ E_{s_{x_{x}}}^{n+1}(i, j, k + \frac{1}{2}) = \left( \frac{2\varepsilon - \sigma_{x_{c}} \Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right) E_{s_{x_{x}}}^{n}(i, j, k + \frac{1}{2}) + \left[ \frac{2\Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right] \]

\[ \frac{1}{\Delta x} \left\{ \left[ H_{s_{y_{x}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) - H_{s_{y_{x}}}^{n+\frac{1}{2}}(i - \frac{1}{2}, j, k + \frac{1}{2}) \right] + \right. \]

\[ \left. \left[ H_{s_{y_{x}}}^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) - H_{s_{y_{x}}}^{n+\frac{1}{2}}(i - \frac{1}{2}, j, k + \frac{1}{2}) \right] \right\} \] (3.88)

\[ E_{s_{x_{y}}}^{n+1}(i, j, k + \frac{1}{2}) = \left( \frac{2\varepsilon - \sigma_{x_{c}} \Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right) E_{s_{x_{y}}}^{n}(i, j, k + \frac{1}{2}) - \left[ \frac{2\Delta t}{2\varepsilon + \sigma_{x_{c}} \Delta t} \right] \]

\[ \frac{1}{\Delta y} \left\{ \left[ H_{s_{x_{y}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) - H_{s_{x_{y}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k) \right] + \right. \]

\[ \left. \left[ H_{s_{x_{y}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) - H_{s_{x_{y}}}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k) \right] \right\} \] (3.89)
\[ H_{sx}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) = \left( \frac{2\epsilon - \sigma_{y_h}\Delta t}{2\epsilon + \sigma_{y_h}\Delta t} \right) \frac{H_{sx}^n(i, j + \frac{1}{2}, k + \frac{1}{2}) - \frac{2\epsilon\Delta t}{\mu(2\epsilon + \sigma_{y_h}\Delta t)}}{\Delta y} \left\{ \left[ E^n_{sz}(i, j + 1, k + \frac{1}{2}) - E^n_{sz}(i, j, k + \frac{1}{2}) \right] + \left[ E^n_{sy}(i, j + 1, k + \frac{1}{2}) - E^n_{sy}(i, j, k + \frac{1}{2}) \right] \right\} \] (3.90)

\[ H_{sz}^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) = \left( \frac{2\epsilon - \sigma_{z_h}\Delta t}{2\epsilon + \sigma_{z_h}\Delta t} \right) \frac{H_{sz}^n(i, j + \frac{1}{2}, k + \frac{1}{2}) + \frac{2\epsilon\Delta t}{\mu(2\epsilon + \sigma_{z_h}\Delta t)}}{\Delta z} \left\{ \left[ E^n_{sy}(i, j + \frac{1}{2}, k + 1) - E^n_{sy}(i, j + \frac{1}{2}, k) \right] + \left[ E^n_{sz}(i, j + \frac{1}{2}, k + 1) - E^n_{sz}(i, j, k + \frac{1}{2}) \right] \right\} \] (3.91)

\[ H_{sy}^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}) = \left( \frac{2\epsilon - \sigma_{y_h}\Delta t}{2\epsilon + \sigma_{y_h}\Delta t} \right) \frac{H_{sy}^n(i + \frac{1}{2}, j + \frac{1}{2}) + \frac{2\epsilon\Delta t}{\mu(2\epsilon + \sigma_{y_h}\Delta t)}}{\Delta y} \left\{ \left[ E^n_{sz}(i + 1, j + \frac{1}{2}, k + \frac{1}{2}) - E^n_{sz}(i, j, k + \frac{1}{2}) \right] + \left[ E^n_{sy}(i + 1, j + \frac{1}{2}, k + \frac{1}{2}) - E^n_{sy}(i, j, k + \frac{1}{2}) \right] \right\} \] (3.92)

\[ H_{sy}^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}) = \left( \frac{2\epsilon - \sigma_{y_h}\Delta t}{2\epsilon + \sigma_{y_h}\Delta t} \right) \frac{H_{sy}^n(i + \frac{1}{2}, j + \frac{1}{2}) - \frac{2\epsilon\Delta t}{\mu(2\epsilon + \sigma_{y_h}\Delta t)}}{\Delta z} \left\{ \left[ E^n_{sx}(i + \frac{1}{2}, j + 1) - E^n_{sx}(i + \frac{1}{2}, j) \right] + \left[ E^n_{sy}(i + \frac{1}{2}, j + 1) - E^n_{sy}(i + \frac{1}{2}, j) \right] \right\} \] (3.93)
\[ H_{szx}^{n+\frac{1}{2}}(i+\frac{1}{2}, j+\frac{1}{2}, k) = \left( \frac{2\varepsilon - \sigma_x \Delta t}{2\varepsilon + \sigma_x \Delta t} \right) H_{szx}^{n-\frac{1}{2}}(i+\frac{1}{2}, j+\frac{1}{2}, k) - \left[ \frac{2\varepsilon \Delta t}{\mu(2\varepsilon + \sigma_x \Delta t)} \right] \]

\[ \frac{1}{\Delta x} \left\{ \left[ E_{szx}^n(i+1, j+\frac{1}{2}, k) - E_{szx}^n(i, j+\frac{1}{2}, k) \right] + \left[ E_{szx}^n(i+1, j+\frac{1}{2}, k) - E_{szx}^n(i, j+\frac{1}{2}, k) \right] \right\} \] (3.94)

\[ H_{szy}^{n+\frac{1}{2}}(i+\frac{1}{2}, j+\frac{1}{2}, k) = \left( \frac{2\varepsilon - \sigma_y \Delta t}{2\varepsilon + \sigma_y \Delta t} \right) H_{szy}^{n-\frac{1}{2}}(i+\frac{1}{2}, j+\frac{1}{2}, k) + \left[ \frac{2\varepsilon \Delta t}{\mu(2\varepsilon + \sigma_y \Delta t)} \right] \]

\[ \frac{1}{\Delta y} \left\{ \left[ E_{szy}^n(i+\frac{1}{2}, j+1, k) - E_{szy}^n(i+\frac{1}{2}, j, k) \right] + \left[ E_{szy}^n(i+\frac{1}{2}, j+1, k) - E_{szy}^n(i+\frac{1}{2}, j, k) \right] \right\} \] (3.95)

where \( \sigma_{wj} \) is the \( f \) (magnetic or electric) conductivity in direction \( w \) (\( x \), \( y \) or \( z \)) and \( F_{sww1} \) is the \( w1 \) component of the \( F \) (\( H \) or \( E \)) field in direction \( w \) (\( x \), \( y \) or \( z \)).

One of the disadvantages of the PML introduced by Berenger and Chew is that it uses twice the number of unknowns as in regular FDTD method, requiring more RAM. In addition, the modification of Maxwell’s equations is outside the general framework of electromagnetics. In 1995, Sacks et al. proved that the PML properties could be obtained if the layer is assumed to be anisotropic (no modifications to Maxwell’s equations is needed) [91]. Unlike Berenger’s method, this Maxwellian approach requires the same number of unknowns as in regular FDTD method.

Chapter 4 of the dissertation describes how to set up an FDTD model for the MRI RF coils including the finite-difference grid of the coil structure and human head models.
Clinical MRI is now being performed over a tremendous range of magnetic field strengths, spanning from less than 0.2 Tesla up to 8 Tesla. Lower magnetic field strengths (< 0.5 Tesla) often offer the advantage of excellent patient access, technical simplicity and decreased financial burdens. Such systems are therefore vital to the dissemination of clinical MRI. Conversely, MRI systems with higher magnetic field strengths are associated with increased costs, inferior patient access and more significant technical and physical difficulties. Despite the challenges associated with higher magnetic fields however, these MRI systems remain in demand within academic medical centers as a result of the substantial enhancements in signal to noise which they provide. This advantage however can only be exploited in the presence of excellent RF coil performance.

At high magnetic fields (>3 Tesla), the performance of the RF coil becomes increasingly dependent on its interactions with the human head/body because the human head/body size becomes comparable to the wavelength.

Recognizing the increased role of tissue/coil interactions, experimental and numerical evaluations of RF coils can provide essential information for understanding
the behavior of these devices. In addition, such studies can help guide modifications in coil design required to retain reasonable homogeneity for the \( B_1 \) field distribution, and to minimize RF power deposition as monitored by the specific absorption rate (SAR).

In this chapter, theoretical, numerical, and experimental analyses of several MRI RF coils including the birdcage [28] and the TEM [15] head coils, and the single strut extremity coil [31] are presented. The coils are evaluated at frequencies that span 64 to 340 \( MHz \) representing proton imaging at 1.5 to 8 Tesla. Several samples were considered as loading to the RF coils. These include phantoms with different shapes, sizes, and contents and 2 anatomically detailed human head models.

4.1 The Anatomically Detailed Human Head Models

4.1.1 \((3\text{mm})^3, 6\text{-Tissue Model}\)

An anatomically detailed finite difference time domain (FDTD) mesh of a male human head and shoulders [92] was used in the simulations of the birdcage and TEM resonators. This data is obtained from MRI, CT and anatomical images [93]; hence, it is suitable for these types of simulations. The mesh consists of six tissue types: cartilage, muscle, eye, brain, dry skin, and skull bone. The electrical constitutive parameters of these tissue are dispersive. Thus, in any particular simulation, one must use the conductivity and the dielectric constant associated with the frequency of interest. This is done by tuning the coil to the frequency of interest and simultaneously using the constitutive parameters associated with this particular frequency. This particular head/shoulder mesh was used for birdcage and TEM resonators simulations spanning 64 \( MHz \) to 362.5 \( MHz \). To give a prospective on the dielectric properties
Table 4.1: Tissue properties of the human head/shoulder mesh.

<table>
<thead>
<tr>
<th>Tissue type</th>
<th>ρ (Kg/m³)</th>
<th>64 MHz</th>
<th>200 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartilage</td>
<td>1100</td>
<td>69.95</td>
<td>0.45</td>
</tr>
<tr>
<td>Muscle</td>
<td>1040</td>
<td>88.037</td>
<td>0.75594</td>
</tr>
<tr>
<td>Eye</td>
<td>1050</td>
<td>76.3</td>
<td>0.88</td>
</tr>
<tr>
<td>Brain</td>
<td>1030</td>
<td>105.82</td>
<td>0.59733</td>
</tr>
<tr>
<td>Dry skin</td>
<td>1100</td>
<td>75.31</td>
<td>0.43</td>
</tr>
<tr>
<td>Skull bone</td>
<td>1850</td>
<td>35.915</td>
<td>0.1274</td>
</tr>
</tbody>
</table>

of the 6 tissue that constitutes this model, the conductivity, dielectric constant, and density for these tissue types are given for two different frequencies (64 and 200 MHz) [94] in Table 4.1. The variables ρ, εᵣ = ε/ε₀, and σ are the density, relative permittivity, and conductivity, respectively. Figure 4.1 shows axial, sagittal, and coronal slices of the FDTD grid of the human head/shoulder model. At the interface between tissues, each cell can have up to 3 different conductivities (σₓ, σᵧ, σᶻ). This is critical for the SAR calculations due to the local hot spots that usually occur at these areas. However, for the dielectric constant an interpolation scheme is used to account for the discontinuity at these interfaces.
Figure 4.1: The FDTD grid of the (3mm)³, 6-tissue human head/shoulder model.
4.1.2 (2mm)$^3$, 18-Tissue Model

In addition to the Coarse (3mm)$^3$ 6-tissue model, a high resolution anatomically detailed human head mesh model was developed [32]. The mesh data was obtained from 0.5 mm $\times$ 0.5 mm $\times$ 2 mm 1.5 Tesla MR images. The model was constructed with the assistance of a physician who assigned tissue types in each image and encoded them on a digital image.

Several error-correction and validation procedures were performed. First, the digitally encoded tissue types were processed to remove voids in the data caused by human error in tissue delineation. Erroneous voids were distinguished from true voids (air spaces in the mouth and nasal passages) and were filled by assigning an adjacent tissue type. Automated image processing software was developed to accomplish this task. Next, differences from layer to layer (image to image) in the data set were reconciled by re-slicing the data along a different axis and re-examining the imagery to identify discontinuities in tissue boundaries. Some interpolation of the data was also required because of the difference in sample spacing within an image and between images. Finally, the image data were output as a single volumetric data set that specifies tissue types at each sample position. The tissue type information stored at each pixel is used with a look-up-table that provides dielectric constant and conductivity values for any frequency of interest.

To obtain an accurate detail of the internal human head structure, eighteen different tissue types, in addition to air, were identified in the images. By having 18 different tissues types and small pixel size, one can obtain very accurate results especially in modeling the internal electromagnetic fields within the biological tissues which greatly affect the SAR calculations and the $B_1$ field distribution in the human
Figure 4.2: 1.5 Tesla gradient echo images (left), encoded digital images with 18-tissue identified (middle), and the corresponding FDTD grids (right). The spatial resolution of the MR images was $0.5 \text{ mm} \times 0.5 \text{ mm} \times 2 \text{ mm}$ while the FDTD grid of the head model has a resolution of $(2\text{ mm})^3$. 
Figure 4.2 continued
Figure 4.2 continued
head. The identified tissue types are given as follow: blood, bone-cancellous, bone-cortical, cartilage, cerebellum, cornea, cerebro spinal fluid (CSF), dura, fat, gray-matter (GM), mucosa, muscle, nerve, skin, tongue, vitreous-humor, white-matter (WM), and mixed-GM-WM. Sample of the MRI images (4.2left), their encoded digital images (4.2middle), and the corresponding FDTD grids(4.2right) appear in Figure 4.2. This particular head model was used for simulations with the TEM resonator spanning 100 to 460 MHz. The density ($\rho$), dielectric constant ($\varepsilon_r = \varepsilon / \varepsilon_0$), and the conductivity ($\sigma$) for these tissue types are given at 340 MHz [94] in Table 4.2.

4.2 The Birdcage Coil

There are three typical birdcage coil configurations, low- (Figure 4.3a), high- (Figure 4.3b), and band-pass (Figure 4.3c) coils. The coils are composed of copper wires which join two copper circular rings. Depending on the configuration, the copper legs and/or circular rings at the top and the bottom of the coil contain gaps in which lumped capacitors have been inserted. An element is defined as a closed loop that includes four wire segments: two adjacent legs and two parallel segments from the top and bottom circular rings. By representing every wire segment as an inductor, Figure 4.4 shows that an equivalent circuit model can be obtained for each of these configurations. It is apparent that these circuit models represent low, high, and band pass filters, respectively.

The conductors and capacitors of the coils form a lumped element approximation of a transmission line around each end ring. The objective is to obtain a standing wave at the Larmor frequency. The resonant modes of the birdcage coil could be obtained using circuit analysis approximation at low frequencies [95]. For a high pass birdcage
<table>
<thead>
<tr>
<th>Tissue type</th>
<th>$\rho \ (Kg/m^3)$</th>
<th>$\varepsilon_r$</th>
<th>$\sigma (S/m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blood</td>
<td>1060.0</td>
<td>57.50</td>
<td>1.700</td>
</tr>
<tr>
<td>Bone-Cancellous</td>
<td>1850.0</td>
<td>21.84</td>
<td>0.209</td>
</tr>
<tr>
<td>Bone-Cortical</td>
<td>1850.0</td>
<td>13.91</td>
<td>0.100</td>
</tr>
<tr>
<td>Cartilage</td>
<td>1100.0</td>
<td>44.82</td>
<td>0.620</td>
</tr>
<tr>
<td>Cerebellum</td>
<td>1040.0</td>
<td>54.40</td>
<td>0.880</td>
</tr>
<tr>
<td>Cornea</td>
<td>1050.0</td>
<td>55.40</td>
<td>1.050</td>
</tr>
<tr>
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<td>1040.0</td>
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**Table 4.2**: Tissue properties of the human head mesh at 340 MHz.
Figure 4.3: Configurations of the birdcage coil.

Figure 4.4: Equivalent circuits of single elements in the birdcage coils.
coil (Figure 4.3b) with $N$ legs, consider the circuit model [95] of 3 segments adjacent to each other (Figure 4.5). If we assume that the index $n = n + N$ and $n$ varies form 0 to $N - 1$, the mesh equation for the segment that contain the current $I_n$ is given as [95]:

$$ (j\omega)^2 \sum_{m=0}^{N-1} (I_{n+m}M_m) + 2I_n\left(\frac{1}{C_1}\right) = 0 \quad (4.1) $$

where $\omega$ (rad/s) is frequency, $C_1$ (F) is the value of the lumped capacitor, $M_m$ (H) is the sum of mutual couplings between every pair of conductors that are parallel and are located in two meshes with $m$ separation. $M_m$ is defined as [95]:

$$ M_m = 2MLeg_m - MLeg_{m-1} - MLeg_{m+1} + 2(MRing_m - MRingOm) \quad (4.2) $$

where $MLeg_m$ is the mutual inductance between legs and $MLeg_0$ is equal to the self inductance of any leg. $MRing_sm$ and $MRingOm_m$ are the mutual inductance between the ring segments on the the same and opposite rings, respectively. The solution of the resonance frequencies is given from Eq. (4.1):

$$ \omega_r = \sqrt{\frac{2}{C_1\sum_{m=0}^{N-1}(M_m\exp\left((-j2\pi m r)/N\right))}} \quad r = 1, 2, \ldots \frac{N}{2} \quad (4.3) $$
In addition to the $\frac{N}{2} - 1$ modes described above there are two other modes [95] which are supported by the high pass design. These modes can not be obtained with the currents described in Figure 4.5. They are formed with equal currents flowing in the same or opposite direction in each of the two rings. The resonance frequency of one of these two modes is given by:

$$\omega_S = \sqrt{\frac{N}{C_1(L_{ring} + M_{ring})}}$$  \hspace{1cm} (4.4)

where $\omega_S$ is the resonance frequency of the mode with the same direction of currents on both of the coil rings, $L_{ring}$ is the self inductance of each ring, and $M_{ring}$ is the mutual inductance between the two rings. The other resonance frequency is given by:

$$\omega_O = \sqrt{\frac{N}{C_1(L_{ring} - M_{ring})}}$$  \hspace{1cm} (4.5)

where $\omega_O$ is the resonance frequency of the mode with opposite direction of the currents on both of the coil rings. Note that with a small mutual inductance between the coil rings, the frequency gap between $\omega_O$ and $\omega_S$ could be negligible. Unlike modes $\omega_r$ with $r = 1 : \frac{N}{2}$, modes $\omega_S$ and $\omega_O$ produce neither transverse magnetic field inside the the middle of the resonator nor currents on the coil legs.

MRI mode of interest is mode $\omega_1$ because at low frequency, the coil struts carry a sinusoidal current distribution

$$I_i = I_{max} \cos \left( \frac{(i - 1)\pi}{N} \right)$$  \hspace{1cm} (4.6)

where $I_i$ is the current in the $i$th element, $I_{max}$ is the maximum current and $N$ is the number of current elements. These currents are established by the standing wave (at the Larmor frequency) and create a linearly polarized transverse magnetic field within the coil. Note that in the previous analysis, the coil is assumed to function as an
azimuthal transmission line at all the frequencies of interest and the generated modes are purely TEM. In other words, the analysis given above are only valid when the coil is small compared to a wavelength. The circuit approximation begins to fail as the length of the coil approaches a quarter wavelength and/or when there is a significant interaction between the coil and the human head.

4.2.1 Excitations

For current MRI systems, the two main excitations which are used for the birdcage coils are linear and quadrature drives. For a high pass configuration, Figure 4.6 shows these excitations, respectively. The linear drive is a simple point source excitation. On the other hand the quadrature technique is much more involved than the linear. Basically, the input signal is split into two halves by using a hybrid. As in Figure 4.6, the first half is fed to the first excitation location (tangent to the $x$ axis), and the second half of the signal is adjusted by a $\pi/2$ phase shift and then is fed to another point which is one quarter on the circular ring away from the first feeding point (tangent to the $y$ axis).

For mode $M = 1$, it is clear that the quadrature excitation generates a circularly polarized field (only component of the RF field that excites the spins) which is more homogeneous than the linear case. This is due to fact that quadrature excitation produces currents with equal rather than sinusoidal varied magnitudes on the coil legs. In addition, the power required for a certain flip angle in a linear drive is twice as much as in the quadrature drive. Therefore, another advantage of the quadrature excitation is the enhancement of SNR by a factor of $\sqrt{2}$ because as the signal is increased by a factor of 2, the noise is only increased by a factor of $\sqrt{2}$. 
4.2.2 Numerical Model of the Birdcage Coil

Two FDTD models of two high pass birdcage coils were developed: one where the width of the coil conductors is infinitesimal and another where 0.3 in is the width of each conductor on the coil structure. In order to obtain accurate electromagnetic field calculations with the FDTD algorithm, an octagonal geometry was utilized in which lumped capacitive elements could be properly modeled. One of the models utilized four fold symmetry (Figure 4.7, infinitesimal conductors coil) while the other model used eight fold symmetry (Figure 4.8, 0.3 in conductors coil). The electrical performance of the coil was maintained since the modeled geometry did not deviate too far from the circular shape. For the four fold symmetry model, an FDTD grid was
generated both for the empty coil and for the case where the coil was loaded with a
cylindrical phantom (22 cm long with 19.64 cm diameter), an octagonal phantom (40
cm long with a 20.7 cm diameter), and the 6-tissue anatomically detailed human head
model. For the eight fold symmetry model, Brooks Air Force human head/shoulder
model (ftp://starview.brooks.af.mil/EMF/dosimetry_models) was used as the coil load.

A lumped element FDTD algorithm was used to model the tuning capacitors [96].
This algorithm requires that the capacitive lumped elements were positioned along the
Cartesian axes, namely x or y. Thus, in order to maintain symmetry, the orientation
of the capacitors along the four (eight) slanted edges of the rings may change. To get
quadrature excitation the same input was applied to the Ex and Ey components of the
electric field as shown in Figure 4.7 or in the back of the head or the front of the head
as shown in Figure 4.8. The only difference between the two ports is a 90 degrees
phase shift on the input excitation. The actual coil is 40 cm long and has a diameter
of 28 cm.

In order to minimize the errors cause by stair stepping, the Yee cells were chosen
to be small enough, 3 mm for the four fold symmetry coil and 2.509 mm for the eight
fold symmetry coil, to fully characterize the structure of the coil including the lumped
capacitors and the excitation source. The perfectly matched layer (PML) absorbing
boundary condition [29] was used to account for RF radiation from the coil. The
PML is composed of cells with a quadratic conductivity profile. 16 PML cells were
used and at least 10 cells separation between the PML surface and closest point on
the coil geometry. The best performance for the PML was achieved with a quadratic
Figure 4.7: Schematic representation of a high pass birdcage within the FDTD (3 mm) grid.
Figure 4.8: Schematic representation of a high pass birdcage within the FDTD (2.509 mm) grid.
conductivity profile with the maximum conductivity ($\sigma_{e\text{max}}$) equals to 1.06 $\text{S/m}$:

$$\sigma_h(i) = \sigma_{h\text{max}} \frac{(N_{PML} - i)}{N_{PML}}, \quad i = 0 : (N_{PML} - 1) \quad (4.7)$$

$$\sigma_e(i) = \sigma_{e\text{max}} \frac{(N_{PML} - (i + 0.5))}{N_{PML}} \quad (4.8)$$

where $N_{PML}$ is the number of the PML cells on one side of the domain. Having a different $\sigma_e$ and $\sigma_h$ at the same cell allows for more attenuation and less reflection as the waves propagate through the PML. A total of 4,000,000 cells in the case of the 3 $\text{mm}$ model or 25,000,000 cells in the case of the 2.509 $\text{mm}$ model were used in generating the complete grid. Note that the magnet shield was modeled in the 25,000,000 cells system. A differentiated Gaussian pulse was used to excite the coil in order to obtain the time domain solution.

FDTD calculated frequency spectrum results are shown in Figure 4.9 for 1.5 Tesla and 3 Tesla cases using the 2.509 $\text{mm}$ model. The capacitor values used along the end rings were the values of the actual capacitors used in the GE birdcage coil utilized for clinical operations (69 $\text{pF}$ for 1.5 Tesla and 15.5 $\text{pF}$ for 3 Tesla). Note that the resonance of mode 1 differs by less than 10% from what is actually obtained in the real coil. Also shown in Figure 4.9 is the shift in the resonance of mode 1 due to the presence of the human head/shoulder model. For the 1.5 Tesla case, the shift is not really noticeable, but for the 3 Tesla case, it is significant enough that it must be taken into account when designing the coil.

**Experimental Verification**

It is not possible to directly measure the magnetic field within the RF coil when the coil is loaded. However, its value can be indirectly determined by imaging a phantom inside the coil whose material properties are homogeneous. [97, 82] In this
Figure 4.9: FDTD calculated frequency spectrum of the birdcage coil loaded with the anatomically detailed human head/shoulder model at 1.5 (left) and 3 (right) Tesla. The capacitance values used in the FDTD model resemble the real values utilized in the MR system. With this complicated 3D model, mode (1) was obtained at the correct frequencies with less than 10% error. In the bottom two figures the solid line corresponds to the loaded coil while the dotted line corresponds to empty coil.
case [97, 82], the strength of the circularly polarized component of the transverse magnetic field at a specific location is a function of the ratio of two images obtained with a 2-fold change in nutation angles. Therefore for two gradient echo images with two flip angles \( \theta_1 \) and \( \theta_2 \), the ratio of the signals of the two images is given by [97, 82]:

\[
r = \frac{\sin(\theta_1)}{\sin(\theta_2)}.
\]

(4.9)

By choosing \( \theta_2 = 2\theta_1 \), the circularly polarized component of the transverse magnetic \( (B_1^+) \) field is given by [97, 82]:

\[
B_1^+ = \left( \frac{1}{\gamma t} \right) \cos^{-1} \left( \frac{1}{2r} \right)
\]

(4.10)

where \( \gamma \) (Hz/T) and \( t \) (S) are the gyromagnetic ratio and the pulse width, respectively.

The indirect method described above was used to test the simulation of the birdcage coil. A cylindrical phantom (length = 22.0 cm; diameter = 19.5 cm) was constructed to have the electrical properties of muscle at 64 MHz [98]. Using a 1.5 T system, two gradient echo images were obtained from this phantom with 60° and 120° flip angles. This is equivalent to applying four folds of the amount of power used for the 60° flip angle image in order to generate the 120° flip angle image. The two images were obtained while keeping all the other parameters unchanged: \( TE, TR, \) etc...

Using the same phantom size and electrical properties in the FDTD simulations, the numerical FDTD results were overlayed with the experimental MRI data. In general, the agreement between the FDTD solution and the MRI measurements was excellent everywhere in the phantom. Figures 4.10a and 4.10b show a comparison of the \( B_1^+ \) field as a function of \( y \) in the phantom. The results are displayed for slices...
Figure 4.10: Plot of the $B_1^+$ field as a function of: $y$ (a, b) located at 5 cm in the $+z$ (a) and 5.0 cm in the $-z$ (b) directions from the center of the coil for $x$ set to zero, and $x$ (c, d) located at 10 cm in the $+z$ (c) and 10 cm in the $-z$ (d) directions from the center of the coil for $y$ set to zero, The numerical FDTD solution (dashed curve) is compared to experimental MRI measurements (solid curve). The coil is operating at 64 MHz with quadrature excitation.
which are located at 5 cm in the +z (4.10a) and 5 cm in the -z (4.10b) directions from the center of the coil with x set to zero. In these figures, the dashed curve represents the numerical FDTD solution, while the solid curve corresponds to the MRI measurements. The sharp variations in signal intensity in the MRI curve was due to the presence of air bubbles in the phantom. These in turn generated inhomogeneities in the phantom that were unavoidable in its construction. Figures 4.10c and 4.10d provide a representation of the $B_1^+$ field as a function of $x$ in the top (10 cm in the +z direction from the center of the coil) section (4.10c) and bottom (10 cm in the -z direction from the center of the coil) section (4.10d) of the coil with y set to zero. From such comparisons, it is apparent that the agreement between the numerical FDTD solution and the experimental MRI measurements for this coil was excellent.

### 4.3 The Single Strut Extremity Coil

The use of a single TEM element has also been advanced [14, 31]. For instance, such an element could be used in a manner similar to a conventional surface coil. In this section the single TEM element (Figure 4.11) is analyzed experimentally and with FDTD method. It is demonstrated that the single TEM element, in conjunction with a RF shield, leads to an excellent extremity coil for high frequency MRI applications. Results are presented both in phantoms and on the human wrist.

The single strut coil could be viewed as a transmission line element where the inner strut is moved form the center of line towards its shield (Figure 4.12). The magnetic field produced inside a symmetric transmission line (Figure 4.12left) will vary as an function of $1/r$ where $r$ is the distance between the outer shield of the inner
The single strut coil will produce a magnetic field whose distribution is an approximate function of $1/r$ at least for the unloaded case.

An FDTD model of the coil was created (Figure 4.13). Using this model, the frequency dependence for this coil was calculated for an empty coil and a coil loaded with a cylindrical phantom filled with 0.125 $M$ saline. The results of these calculations are displayed in (Figure 4.13). Compared to the empty coil, it is apparent that the phantom load causes a resonance shift and a decrease in the quality factor. The variation of the fields inside an empty and loaded coils are shown in Figure 4.14. As expected, for the empty coil, the field variations approximate $1/r$, however deviations...
Figure 4.12: Transmission line model.

Figure 4.13: FDTD grid of the single strut coil loaded with a cylindrical phantom and frequency response of loaded and empty coils tuned to approx. 340 MHz. The solid line corresponds to the loaded coil while the dotted line corresponds to an empty coil.
Figure 4.14: Field variations inside an empty (left) and a loaded (right) single strut coils tuned to 340 MHz. The solid line corresponds to the solutions using the FDTD model while the dotted line corresponds to $1/r$ variation.

from $1/r$ are much more pronounced in the loaded case, especially at the location where the phantom is present.

The goal however is to create a TEM$_z$ field inside the coil. The phase distribution of the tangential magnetic flux density is shown in Figure 4.15(top row) for an empty coil (left) and a coil loaded with the saline phantom (right). It can be clearly seen that a TEM mode is excited in the empty coil while a hybrid mode is excited in the loaded coil. With a close examination however, the phase is constant in the direction that is parallel to the excited strut (up and down) for the loaded coil. This in return indicates homogeneous fields in the coronal slices. Indeed, these findings are verified from $B_{1z}$ field distribution shown in Figure 4.15(bottom row).

In Figure 4.16, an 8 Tesla high resolution wrist image [31] obtained using the single strut coil. This image reveals excellent signal to noise and excellent $B_{1z}$ homogeneity in the coronal slices.
Figure 4.15: Phase distribution of the tangential magnetic flux density (top row) and $B_{1}^{+}$ field distribution (bottom row) inside an empty single strut coil (left) and a coil loaded with a saline phantom (right).
Figure 4.16: 1k×1k image of a human wrist obtained using the single strut coil at 8 Tesla. Experimental image was provided by Dr. Robitaille.
4.4 The TEM Resonator

Experiences with ultra high field clinical imaging (340 MHz) have shown that distributed circuit RF coils such as the transverse electromagnetic (TEM) resonator [14, 15] have surpassed the performance of the conventional lumped circuit coils such as the birdcage resonator [28]. Beside the losses that the lumped elements, such as capacitors, possess at high frequencies, a major factor leading to the difference in performance between these coils is frequency tuning. At ultra high field, the human head causes a significant frequency shift when loaded in the coil. With different head sizes, flexible and robust techniques should be used to tune the RF head coils. Compared to the lumped circuit coils, these techniques are more realized when using distributed circuit coils.

In Figure 2.13, a photograph of the TEM resonator is displayed. This coil consists of 16 struts, which are contained in an open resonant cavity. Two circular rings are attached to the top and bottom of the open cavity. Each of the struts consists of coaxial line with a circular cross section. Teflon is used as a dielectric filler between the inner and outer rods of each strut. The coil is tuned by adjusting the gap between the two inner rods of each strut. In the experiment setting, this operation is done while the sample is loaded in the coil.

An analytical model based on multi-conductor transmission line theory [99] have shown that for an empty $N$ strut TEM coil, $N/2 + 1$ TEM modes exist. The second mode on the spectrum: mode 1 produces a linearly polarized field that can be utilized for imaging. The other modes produce nulls in the center of the coil rendering them ineffective for conventional imaging.
The TEM resonator and the object to be imaged (phantoms or human head models) were modeled as a single system with the FDTD method. The three-dimensional FDTD model of the TEM resonator consists of 8, 16, and 24 coaxial rods. Several resolutions were considered. However the highest resolution (2 mm) constituted a grid that is composed of approx. 8 million cells (193*193*193). A stair-step approximation was used to model the shield and the top and bottom rings of the coil. The coaxial rods were modeled in a similar manner while an FDTD algorithm was used to account for the curvatures of the rods to minimize the errors caused by stair-stepping. Figure 4.17 displays the FDTD grid of a 16-strut TEM resonator loaded with the 2 mm human head model. To account for the RF coil radiation from the top and bottom of the coil, the PML was used as an absorbing boundary condition.

The coil was numerically tuned by adjusting the gap between the TEM stubs until any mode of interest is resonant at the desired frequency of operation. This process was performed while the human head model was present within the coil. A dielectric constant of 2.2 (Teflon) was used for the filler between the inner and the outer rods. Figure 4.18 shows a comparison between the frequency response computed using the FDTD model and the response obtained using the analytical model [99] within an empty 8-strut coil. An excellent agreement can be observed in these results.

To experimentally verify the FDTD model of the TEM resonator, a 16-strut coil loaded with an 18.5 cm spherical phantom that has the same dielectric property as 0.5 mM Gd DTPA 0.125 M NaCl was modeled and numerically tuned by adjusting the gap between the TEM stubs until each of the modes of the TEM resonator is resonant at 340 MHz (8 Tesla). Experimental phantom gradient echo images were acquired for same sized coil and phantom. Figure 4.19 displays axial slices of low flip angle
Figure 4.17: Three-dimensional structure and axial, sagittal, and coronal slices of a 16-strut TEM resonator loaded with the 18-tissue anatomically detailed human head model in the FDTD grid. The cells represent FDTD Yee cell.
Figure 4.18: Spectral magnitude of magnetic fields present in an 8-element TEM resonator. Theoretical model is provided by Dr. Baertlein.

GE images (upper row) obtained at 8 Tesla and the corresponding FDTD images (lower row) calculated at 340 MHz of the spherical phantom. The coil is linearly excited where the excitation location is in the bottom left of each image. Images (0-5) correspond to modes (0-5) of the coil loaded with the specified Phantom.

Nine resonances were observed in the experiment and in the simulations. It should be noted that theoretically the aforementioned modes including normal mode of operation (1) are not transverse electromagnetic (TEM) due to two major factors. First, the coil is loaded with dielectric phantom. Second, the feed and termination loads introduce perturbations to the fields. These modes are hybrid modes. The results clearly demonstrate the accuracy of the full wave 3D model. It can also be seen that the location of the excitation source is critical on the field distribution and therefore the rigorous modeling of the source is essential.
Figure 4.19: Axial slices of low flip angle GE images (upper rows) obtained at 8 Tesla and FDTD calculated images (lower rows) at 340 MHz of an 18.5 cm sphere filled with 0.125 M NaCl. The sphere was loaded in a 16-strut head-sized TEM resonator in the experiment and in the FDTD model. Images 0-5 correspond to the first 6 modes (0-5) of the coil loaded with the specified phantom. Experimental images were obtained by Dr. Schmalbrock.
4.4.1 Analysis of $B_1$ Field Profiles and SAR Values for Multi-Strut TEM RF Coils in High Field MRI Applications

Three TEM resonators with 8, 16, and 24 coaxial lines or TEM elements were constructed. The surface of a cylindrical acrylic cavity (diameter = 34.6 cm; length = 21.2 cm) was lined with a thin copper foil. A pair of circular end plates (same diameter as the outside acrylic cavity) was made from acrylic sheets. A circular hole with a diameter of 23.8 cm was drilled in each of the end plates. The resulting end plates were lined with a copper foil. In one of the two end plates, two female BNC connectors were inserted for the excitation ports. Two additional holes were made to provide access for adjusting two single matching variable capacitors, one for each excitation port. The inner portion of each of the BNC connectors was mounted to a variable Teflon-based matching capacitor using a thin copper wire. The two circular plates were mounted on the two ends of the cavity to form the outside structure of the coil. A hollow acrylic cylinder was inserted in the coil as a separation between the sample and the TEM elements.

The TEM elements were constructed from hollow Teflon rods with an inner diameter of 0.635 cm and an outer diameter of 1.587 cm. The rods were machine cut to a final length of 21.2 cm and wrapped with a copper tape. Depending on the number of elements in the resonators, 8, 16, or 24 of the copper wrapped Teflon hollow rods were then inserted into the cavity. Each hollow rod was held in place with two copper rods. These copper rods were inserted from each end of the coil through the hollow Teflon rods. Figure 4.20 displays the constructed 8 (4.20a), 16 (4.20b), and 24-strut (4.20c) TEM resonators. The resonant frequency of the coil was determined using a Hewlett-Packard 4195A network analyzer. The length of the copper rods in each of
the TEM elements was adjusted such that the appropriate resonant condition (mode of interest) was achieved.

Images were acquired on an 8.0 Tesla, 80 cm superconducting magnet (Magnex Scientific, Abingdon, UK). The scanner was equipped with a BRUKER AVANCE console (Bruker, Billerica, MA, USA). All human studies were conducted under an investigational device exemption (IDE) granted by the Food and Drug Administration (FDA), which were also monitored by the IRB committee of The Ohio State University. Prior to image acquisitions, the subject was placed in a supine position on a movable cantilevered patient table. Each of the three constructed TEM resonators was used for imaging the same subject. The experimental settings were identical for all the three coils. The coil was positioned over the subject’s head such that the edge of the RF coil was aligned with the chin of the subject (full insertion). The RF coil was tuned to 340 MHz as monitored on each of two drive points (quadrature excitation). The subject was then advanced to the scan position while remaining on the table and without removal of the RF coil. Axial gradient echo images were acquired using

Figure 4.20: Photographs of three constructed TEM resonators with 8 (a), 16 (b), and 24 (c) struts.
the following parameters: \( TR = 300 \text{ msec}, TE = 5 \text{ msec}, FOV = 20 \times 20 \text{ cm}, \) matrix size = 256x256, number of slices = 16, slice thickness = 5 \text{ mm}, receiver bandwidth = 50 \text{ KHz}, excitation pulse = 2 \text{ msec} \) gaussian.

\( B_1^+ \) Field Comparison

The 90\(^\circ\) pulse power for the gradient recalled echo head images was determined by nulling the signal through the use of a 180\(^\circ\) pulse. Low flip angle (6\(^\circ\)) 8 Tesla axial images were obtained using 8, 16, and 24-strut TEM resonators (Figure 4.20). The coils were driven in quadrature with the two excitation ports, positioned roughly behind the ears. Simulations of the \( B_1^+ \) field in the human head were obtained using the FDTD model. To numerically tune the coils to 340 MHz, the 24-strut TEM resonator required gaps between the inner rods, which were 8 mm and 18 mm larger than those required for the 16-strut and 8-strut coils, respectively. Similar findings were also observed experimentally. Therefore, as the number of struts increases, the upper limit of the frequency of operation for the TEM resonator decreases.

The \( B_1^+ \) field distribution calculated using the FDTD model is shown in Figure 4.21a-c for 8 (4.21a), 16 (4.21b), and 24.21-strut (4.21c) TEM resonators at 340 MHz using back of the head quadrature excitation. The corresponding gradient recalled echo images are shown in Figures 4.21d-f. From these set of images, it is readily observed that an improvement in the field homogeneity is achieved when switching from 8 (Figures 4.21a,4.21d) to 16 (Figures 4.21b,4.21e) struts. This improvement, however, is hardly visible when switching from 16 to 24 strut coils (Figures 4.21c,4.21f). Stronger coupling between the TEM struts, and consequently a
better overall homogeneity, is observed as the number of coil elements (struts) increases. This is because the coaxial rods are closer to each other in the 16-strut and 24-strut coils. As a result, much of the energy is distributed among the coil elements rather than coupled from the excited element(s) directly to the tissue.

The results of the $B_1^+$ field homogeneity and the low flip angle gradient echo images reveal the potential of ultra high field for clinical imaging applications. While the images are not perfectly uniform, an acceptable homogeneity is observed especially in the 16-strut (Figures 4.21b and 4.21e) and 24-strut images (Figures 4.21c and 4.21f). It is observed that the intensity of the $B_1^+$ field fall from the portion of the images when moving toward the front of the head (Figures 4.21a-f). This is due the electromagnetic interactions between the excitation ports, which are positioned behind the head, and the tissue.

### 4.5 SAR Comparison

To study the effect of increasing/decreasing the number of struts on the power deposition in the head, SAR were calculated using the FDTD model. The SAR is given by

$$SAR = \frac{\sigma |E|^2}{2\rho}$$

(4.11)

where $\sigma$ (S/m) and $\rho$ (kg/m$^3$) are the conductivity and mass density of the tissue, respectively, and $|E|$ (V/m) is the magnitude of the electric field intensity in the tissue.

Figures 4.22a-c show axial slices of the SAR inside the head model loaded in 8 (4.22a), 16 (4.22b), and 24-strut (4.22c) TEM resonators driven in quadrature. Minimal difference in terms of SAR peak values is observed among these three coils. Upon closer examination, however, it is apparent that compared to the 16 or 24-strut
Figure 4.21: Axial slices of the $B_1^+$ field (a-c) calculated inside the anatomically detailed human head mesh using the FDTD model at 340 MHz and of gradient echo images (d-f) obtained at 8 Tesla. The results were obtained using 8-strut (a,d), 16-strut (b,e), and 24-strut (c,f) TEM resonators operating under 2-port back of the head quadrature excitation. The 6° MRI images were acquired using the following parameters: $TR = 300 \, msec$, $TE = 5 \, msec$, $FOV = 20 \times 20 \, cm$, matrix size = 256x256, number of slices = 16, slice thickness = 5 mm, receiver bandwidth = 50 KHz, excitation pulse = 2 msec gaussian. Experimental images were provided by Dr. Robitaille.
Figure 4.22: Axial slices of the SAR distribution inside the 18 tissue anatomically detailed human head model at 340 MHz. The results are obtained using the FDTD model and utilizing 8 (a), 16, and 24-strut (c) TEM resonator operating under back of the head 2-port quadrature excitation. The transmitted power to the coil is equal to 1 watt CW resembling a square pulse with 150 watt peak power, 2 ms width, and 300 ms time recovery (TR).

... coiled, the intensity of the local hot spot near the back of the head in each of the slices is higher in the 8-strut coil. Since the coil was excited behind the head, these results confirm the presence of stronger coupling between the struts as the number of coil elements increases. Unlike the 8-strut case, the 16 and 24-strut coil energy is distributed among the struts rather than coupled from the excited rod(s) directly to the tissue. Nonetheless, strong signs of head/strut coupling are revealed for the struts, which are excited, and/or closest to the front and back of the head versus those at the side of the head. As a result, it is clear that SAR values are highly affected by the local conditions in the coil (coil geometry and number of drive points) and the head/coil interaction. This can result in local hot spots that differ from one coil to another as demonstrated above.
4.6 The FDTD Code

The FDTD code starts by *dynamically allocating* the memory requirements for all the arrays including electric field vectors, magnetic field vectors, permittivity, conductivity, and capacitance. With different coil geometries, cell sizes, and coil loading (empty, phantoms, or human head models), *dynamic allocations* provide convenience and conservation of memory. The next step is assigning the electrical properties to the desired portions of the grid including the phantom or the biological tissues of the human head model.

The time loop then starts with no coil excitations, so the initial field values are set to be zero. The time is incremented by a time step. The excitation is turned on in the specified excitation location. The shape of the excitation is unimportant as long as its frequency spectrum contains the frequencies of interest.

The electric field values are then updated everywhere in the grid. If capacitors are used, at the capacitor locations, a lumped element FDTD algorithm is used to model the tuning capacitors. The electric field components which are tangent to a perfectly conducting surface (coil structure) are forced to zero. To avoid stair stepping errors, algorithms are used to create slanted perfect conductors. Using the calculated electric field values, the magnetic field values are then updated over the entire grid. This is the end of the time step. The time step procedure is repeated until the simulation has run a prescribed number of time steps. Because the updated field values are only functions of the previous field values, memory is conserved. At any cell, memory conservation is done by overwriting the updated field value into the same memory location which contains the previous value at the same cell.
In actuality, the goal is to obtain the field distribution within the coil at the resonance frequency where the lumped capacitors must be tuned or the gap between inner struts must be adjusted to set the resonance at the Larmor frequency. From the magnetic field distribution, one can extract the $B_1$ field distribution. From the electric field distribution, one can find the SAR as well as the total power absorbed by the phantom or the human head. Finding the field distribution is a two step process. In the first step, an initial guess is made for the capacitor values or for the gap between each of the two inner struts. An FFT is then applied to the FDTD solution at a few points within the grid to obtain the frequency response of the coil. The actual location of these points, where, the data is collected is not that important since the frequency response at any point within the coil should not differ significantly. The only difference should be due to variations in the field distribution of the coil. If the resonance frequency of the solution is not at the desired location, then, the capacitor values are changed, or the gap is adjusted and the FDTD program is rerun.

This step is repeated until the desired resonance frequency is obtained. In the second step, the FDTD solution is run with the correct capacitor values or the correct gap sizes, but instead of applying an FFT at a few points in the grid, a discrete Fourier transform is applied on-the-fly at all the points in the grid at the resonant frequency. Thus, the time data does not need to be stored, and at the end of the computer run, the field distribution is known at the resonant frequency.
5.1 Calculations of EM Interactions with Biological Tissue: Magnetic Resonance Imaging at Ultra High Field

Studies of microwave power absorption and heating [66, 64, 65] in human tissue have shown that local hot spots often develop in tissue. These hot spots commonly occur in areas close to the source and in regions where induced current flow is restricted by surrounding low conductivity objects, e.g., in the joints. In addition, there is a need to examine power deposition in certain small body features, including the lens of the eye, which is especially susceptible to thermal damage.

As magnetic resonance imaging (MRI) is now performed at fields in excess of 1.5 Tesla, violation of local specific absorption rate (SAR) FDA limits (8 watts/Kg in any gram over the head in a 5 minute exposure, taken from the web page of the FDA: www.fda.gov/cdrh/ode/magdev) can become a major concern. Nonetheless, accurate SAR prediction in these small regions requires a detailed model of the tissue. In addition, at high field strength, the RF coil performance, including efficiency, and the homogeneity of the transverse magnetic field ($B_1$) field, is dominated by the presence of the human body and its interactions with the RF coil.
In this section we will consider the \((2\text{mm})^3\) 18-tissue head model to provide SAR calculations inside the human head at ultra high field MRI (8 Tesla, 340 \(MH\)). The calculations were done in the TEM resonator with the FDTD method. The FDTD representation accurately models the coil structure including the coaxial rods, the shield, the top and bottom rings, and the excitation source(s). More importantly, the coil and the head are modeled together as a single system, which accounts for all the coupling effects between the TEM resonator and the human head. This includes the interactions between the coil, excitation source, and the head. It will be demonstrated that the number and the location(s) of the excitation port(s) significantly affect the SAR values at 340 \(MH\).

**Specific Absorption Rate Calculations**

An important aspect of RF coil design for magnetic resonance imaging applications is RF power absorption by the patient. Furthermore, the critical health concern is not RF power deposition but the rise in temperature that it produces. In practice, the most important concerns are the “hot spots” produced by small-scale tissue features.

Fields within the body are difficult to estimate accurately because of their dependence on the body’s complicated internal structure. Therefore, high resolution anatomically detailed human head/body models are essential to describe SAR in general, and local SAR maxima (hot spots) in particular for high frequency MRI applications.

From the last Chapter, the SAR is given by

\[
SAR = \frac{\sigma |E|^2}{2\rho}
\]  

(5.1)
where $\sigma$ (S/m) and $\rho$ (Kg/m$^3$) are the conductivity and the mass density of the tissue, respectively. $|E|$ (V/m) is the magnitude of the electric field in the tissue. By evaluating the SAR within specific tissues, one can obtain the power deposition for specific organs. Also, by summing the SAR from all tissues, an indication of the total power deposition can be obtained. The SAR in the above equation is normalized to the power transmitted from the coil.

The real input power of the coil is defined as [100]

$$P_{in} = P_{abs} + P_{rad}$$

$$= \frac{\sigma}{2} \iiint_V |\tilde{E}|^2 dV + \frac{1}{2} \iint_S (\tilde{E} \times \tilde{H}^*) \cdot dS$$

where $P_{abs}$ and $P_{rad}$ are the absorbed and radiated power, respectively while, $\iiint_V$ is the volume integral of the object to be imaged and $\iint_S$ is the integral of a closed surface that encloses the coil structure and the imaged object. The volume integration is done by numerically integrating $|\tilde{E}|^2$ over the human head model. The surface integration is done by choosing a surface that encloses the coil and the sample (a rectangular box which is 5 cells away from the PML surface) and then performing the numerical integration over that surface.

The following calculations are presented for 1 watt continuous input power. This is equivalent to a square pulse that has a 100-watt peak power, 5 ms width, and 500 ms time recovery (TR). Three excitations are considered: linear, 2-port quadrature, and 4-port quadrature. The 2-port excitation employs 0 and $\pi/2$ phase shifts on the drive points, while the 4-port excitation utilizes 0, $\pi/2$, $\pi$, and $3\pi/2$ phase shifts. The drive ports identification numbers are displayed in the Figure 5.1. X or Y linear excitation represents driving the coil in port 1 (X) or port 2 (Y), respectively. Back, side, or front
of the head 2-port quadrature excitation represents driving the coil in ports 1 and 4 (back), 1 and 2 (side), or 2 and 3 (front), respectively. 4-port quadrature excitation utilizes ports 1 to 4.

Figure 5.2 displays axial slices of the SAR distribution calculated inside the human head model using 16-strut TEM resonator tuned to 340 MHz. Note that each subfigure of Figure 5.2 has its own color scale with its unique minimum and maximum values. The grayscale however is still linear. Figures 5.2a and 5.2b correspond to X (a) and Y (b) linear excitation. Figures 5.2c, 5.2d, and 5.2e correspond to 2-port quadrature drives using back (c), front (d), and side (e) of the head excitation locations. Figure 5.2f displays the SAR distribution for 4-port quadrature excitation.

Figures 5.2a and 5.2b show that the SAR increases in the areas near the rods through which relatively high absolute values of currents are flowing. Figure 5.1 shows that X linear excitation utilizes rod 1 as a drive point. As such, the two rods, which are expected to carry the maximum amount of current, are 1 and 3, while rods 2 and 4 are expected to have minimal amount of currents flowing through them. Therefore, the areas with relatively high SAR values include the top right and bottom left of the axial slice shown in Figure 5.2a. In the case of Y linear excitation, the peak values of the SAR are largely concentrated in the bottom right and top left of the axial slice shown in Figure 5.2b. The absolute values of the currents on the rods between the struts that carry maximum currents and the struts that carry minimum currents progressively decrease. At low frequency, current values on the coil struts follow a sinusoidal distribution. This is not the case at 340 MHz.

The calculations also show that the SAR peak values do not significantly change from linear to 2-port quadrature excitation (Figure 5.2a-e). This confirms the finding
Figure 5.1: Axial and coronal slices of a 16-strut TEM resonator loaded with the 18-tissue anatomically detailed human head model in the FDTD grid. The cells represent FDTD Yee cells.
**Figure 5.2:** Axial slices of the SAR distribution inside the 18-tissue anatomically detailed human head model at 340 MHz. The results are obtained using the FDTD model and utilizing 16-strut TEM resonator operating under X (a) and Y (b) linear excitations, back (c), front (d), and side (e) of the head 2-port quadrature excitation, and 4-port (f) excitation. The description of the excitation locations is shown in Figure 5.1. The transmitted power to the coil is equal to 1 watt CW which is equivalent to a square pulse with a 100 watt peak power, 5 ms width, and 500 ms time recovery.
Figure 5.2 continued

(a)  
(b)  
(c)  
(d)  
(e)  
(f)
that at high frequency, the conventional 2-port quadrature excitation does not effectively produce circularly polarized fields [35]. Compared to linear excitation, the input power does not significantly decrease when using 2-port quadrature excitation. This is not the case at low frequency. For instance, with the commonly used 1.5 Tesla clinical magnets where the operating frequency for proton imaging is 64 MHz, images obtained using 2-port quadrature drive usually require half of the power used in linear excitation. At 8 Tesla (340 MHz), the interactions between the coil, excitation source and the head are significant. As such, linear excitation does not produce fields which are transverse electromagnetic or linearly polarized. This results in the ineffectiveness of the conventional 2-port quadrature excitation.

When using 4-port excitation (Figure 5.2f) however, the coil-tissue interactions are reduced which results in a noticeable decrease in the SAR peak values. Figure 5.2f shows that the SAR peak values have decreased by approx. 40% using the 4-port drive compared to the values obtained with 2-port quadrature excitations (Figures 5.2 c-e). The 4-port excitation distributes the energy among the struts more effectively than the conventional 2-port quadrature drive, which consequently leads to lower SAR peak values.

To study the electromagnetic interactions between the excitation source (s) and the human head, Figure 5.3 displays sagittal slices of the SAR values at 340 MHz. Similar to Figure 5.2, each sub figure of Figure 5.3 has its own color scale. The results are obtained using 16-strut TEM resonator operating under X (a) and Y (b) linear, back (c), front (d), and side (e) of the head 2-port quadrature, and 4-port (f) excitations. Due to the localization and the irregular shape of the nose, strong electromagnetic
interactions are expected between this structure and the coil in general or the excitation source(s) in particular. This is demonstrated in Figure 5.3b where the SAR peak values are observed in the nose. In this case (Y linear drive), the excitation source is close to the nose resulting in higher SAR peak values compared to the case where X linear drive is utilized (5.3a).

The 2-port quadrature excitations reveal similar results (Figures 5.3c-e). SAR peak values concentrated in the nose significantly increase from back (5.3c), 2 drive ports are furthest from the nose, to side (5.3e), 1 drive port is closest to the nose, to front (5.3d), 2 drive ports are closest to the nose, of the head quadrature excitation. The 4-port excitation (5.3f) displays similar results except the SAR peak values in the nose are less than those obtained with Y linear excitation (5.3b) or with front of the head (5.3d) quadrature excitation. As previously stated, 4-port drive effectively distributes the energy among the struts. As a result, lower SAR peak values are observed compared to that with linear or 2-port quadrature excitations.

Further demonstration of the effects of the excitation sources, tuning, and frequency of operation on SAR is considered [101, 102]. Figure 5.4 shows FDTD grids of axial and coronal slices of the 6-tissue model loaded in an 8-strut TEM resonator. SAR calculations were considered at 132 MHz (Figure 5.5a), 205 MHz (Figure 5.5b), 247 MHz (Figure 5.5c), 282 MHz (Figure 5.5d), and 362 MHz (Figure 5.5e). At 132 MHz, instead of numerically tuning the coil to mode 1, the coil was off-tuned (not tuned to a specific mode). The input power to the coil was normalized to 1 watt CW and the coil was driven linearly where the excitation location is positioned in front of the head (Figure 5.4). From 205 MHz to 362 MHz, the results show that there is a stronger coupling between the excitation source and the tissue as the frequency of
Figure 5.3: Sagittal slices of the SAR distribution inside the 18-tissue anatomically detailed human head model at 340 MHz. The results are obtained using the FDTD model and utilizing 16-strut TEM resonator operating under X (a) and Y (b) linear excitations, back (c), front (d), and side (e) of the head 2-port quadrature excitation, and 4-port (f) excitation. The description of the excitation locations is shown in Figure 5.1. The transmitted power to the coil is equal to 1 watt CW: a square pulse with a 100 Watt peak power, 5 ms width, and 500 ms time recovery.
Figure 5.3 continued
Figure 5.4: Axial and coronal slices of an 8-strut TEM resonator loaded with the 6-tissue anatomically detailed human head model in the FDTD grid. The cells represent FDTD Yee cells.
operation increases. This is clearly apparent in the increase of the local SAR values near the nose (the closest location to the excitation source). The SAR values have increased by 2.5 folds from 205 MHz (Figure 5.5b) to 362 MHz (Figure 5.5e).

When examining the 132 MHz case where the coil is off-tuned, the results show stronger coupling between the excitation source and the tissue. As it is shown, SAR peak values exceed 14 watts/kg, almost 3 folds higher than any other case shown in Figures 5.5b-e. These results demonstrate the significance of having an appropriate tune and of properly modeling the excitation source. When assuming uniform excitation [15, 76, 77, 25, 78, 79], the source-tissue coupling will not be observed.

Infrared imaging [63, 62] was utilized to study the interactions between the excitation sources and the loads. The technique involves placing into the RF coil a very thin film having a large, known surface resistivity and a low dielectric constant. Components of the field in the plane of the film produce ohmic currents, which deposit thermal energy in the film. For observation times that are short compared to the film’s thermal diffusion time, the local temperature rise will be proportional to the power in the local electric field. Thermal emissions from the surface of the film can be remotely sensed with an infrared camera and, with an appropriately calibrated camera, the film’s temperature distribution can be determined. Knowing the heat capacity, surface resistivity and emissivity of the film, one can determine from the measured temperature distribution the field magnitude in the plane of the film. A modest amount of image processing eliminates the effects of pre-existing background variations in temperature.

Example results are shown in Figure 5.6 for an empty (5.6 upper row) 16-strut TEM coil and a coil loaded with 18.5 cm 0.125 M NaCl sphere (5.6 bottom row). The
Figure 5.5: 3D surface plots of the SAR distribution inside the 6-tissue anatomically detailed human head model at 132 MHz (a), 205 MHz (b), 247 MHz (c), 282 MHz (d), and 362 MHz (e) The results are obtained using the FDTD model and utilizing 8-strut TEM resonator operating under linear excitation where the excitation location is positioned in front of the head (Figure 5.4). The transmitted power to the coil is equal to 1 watt CW. For the case at 132 MHz, the coil was intentionally off-tuned such that the coil is not numerically tuned to a specific mode.
Figure 5.5 continued

(a) 132 MHz

(b) 205 MHz

continue
Figure 5.5 continued

(c) 247 MHz

(d) 282 MHz

continue
Figure 5.5 continued

(e) 362 MHz
coil was driven in 2-port quadrature excitation. The outline of the coil is apparent in these images because (as a result of handling) the body of the coil is slightly warmer than the ambient air. Forming the difference image and reducing the noise through filtering, one obtains median filtered difference image (5.6 right). The measurement clearly shows the effect of RF excitation sources where the thermal heat distribution is much more uniform in the empty coil (Figure 5.6 upper row) compared to the loaded coil where the electromagnetic field deposition is mostly near the excitation sources (Figure 5.6 bottom row).

**Figure 5.6**: Infrared red imaging of a 16-strut TEM coil operating under 2-port quadrature excitation at 340 MHz. Upper row correspond to an empty coil while the bottom row corresponds to a coil loaded with an 18.5 cm sphere filled with 0.125 M NaCl.
5.2 Analysis of RF Power Requirements in MRI

In human imaging experiments, the dependence of RF power deposition on the frequency of operation (Larmor frequency) has been a topic of interest since MRI was introduced as a clinical imaging technique. Several electromagnetic approaches were utilized to solve this problem. Some of these approaches include quasi-static solutions [19] and full wave models [103, 104]. Now, as MRI human imaging is performed at very high field strengths [8, 105], currently as high as 8 Tesla and in the near future as high as 9.4 Tesla, the RF deposition-frequency dependence has drawn even more interest. In here, analysis of the relationship between the MRI frequency of operation and the RF power dissipated in the body is analyzed between 4 and 11.4 Tesla. This is accomplished through the use of the FDTD method applied to two anatomically detailed human head models [33, 92], and a cylindrical phantom placed within the RF coils.

For this study, three coils are considered, 8-strut, 16-strut TEM resonators [15, 43] designed for head and neck imaging, and a single-strut extremity coil [31]. The coil and the load were modeled together as a single system in all the cases considered above. As such, an accurate representation of the interactions between the coil and the biological tissue is obtained. The effects of linear and quadrature excitations on the power required in order to obtain a constant value of the circularly polarized component of the $B_1 \left( B_1^+ \right)$ field are studied at different frequencies.

Samples of the RF power-frequency dependence are presented. Let us consider the FDTD models of the TEM resonator and the single-strut coil. In the TEM resonator, two loads were considered: the 6-tissue, $3 \, mm \times 3 \, mm \times 3 \, mm$ and the 18-tissue $2 \, mm \times 2 \, mm \times 2 \, mm$ anatomically detailed human head models described in the previous
chapter. The size of the 6-tissue head model was relatively larger than the developed 18-tissue head model. Figure 5.7 shows the FDTD grids of the 6-tissue (5.7(top)) and the 18-tissue (5.7(bottom)) head models loaded inside TEM resonators. An 8-strut coil was used for the 6-tissue model while a 16-strut coil was used for the 18-tissue model. Calculations of the power required in order to obtain a constant value of the circularly polarized field (direction of propagation is along the coil axis) in the axial slices shown in Figures 5.7(top) and 5.7(bottom) were performed. For the single-strut coil described in the previous chapter, the power calculations were performed on a coil loaded with a cylindrical phantom filled with 0.125 $M\ NaCl$. Rather than a particular slice, the entire volume of the phantom loaded in the single-strut coil (Figure 5.8) was used for the power calculations.

For the single strut coil the power-frequency dependence was considered between 252 $MHz$ and 481 $MHz$. To show the variation limits of the circularly polarized field homogeneity, Figure 5.9 displays the FDTD solutions of the clock-wise (CW) (5.9a,5.9c) and counter clock-wise (CCW) (5.9b,5.9d) circularly polarized components of the $B_1$ field in a slice that is parallel to the coil strut. The results are presented in a coil computationally loaded with a 0.125 $M\ NaCl$ cylindrical phantom at 252 $MHz$ (5.9a,5.9b) and at 481 $MHz$ (11.4T) (5.9c,5.9d). The cylindrical phantom has a diameter and a length equal to 4.6 cm and 9.4 cm, respectively. Note that the coil strut was shifted 2 mm in Y direction to resemble some expected inaccuracy in the coil structure. The results show excellent homogeneity and almost no difference in the distribution of the circularly polarized field components at 252 $MHz$. Addressing this issue is given by a detailed analysis of MRI field polarization in the next chapter.
Figure 5.7: FDTD grids of the 6-tissue (top) and the 18-tissue (bottom), anatomically detailed human head models loaded inside an 8-strut (top) and a 16-strut (bottom) TEM resonators. The locations of the 4 slices of interest where the power calculations were performed are displayed on each figure.
Figure 5.8: Coronal slice of the FDTD grid of the single strut coil loaded with a 0.125 $M NaCl$ cylindrical phantom.
Figure 5.9: Sagittal slices of the FDTD calculated clock-wise (CW) (a,c) and counter clock-wise (CCW) (b,d) circularly polarized components of the $B_1$ field inside the single strut coil. The results are calculated in a coil loaded with a 0.125 M NaCl cylindrical phantom at 252 (a,b) and 481 (c,d) MHz.
Figure 5.10: The required power to obtain a constant value of the circularly polarized field (average value across the phantom volume) as a function of frequency using the FDTD method. The results are obtained in the single-strut coil. CW and CCW correspond to clock-wise and counter clock-wise fields.

Figure 5.10 displays the required power to obtain a constant value of the circularly polarized field (average value in the phantom volume) as a function of frequency. The power requirements are almost identical except near the high frequency values for both of the field components even with the slight asymmetry in the coil model. This is expected from the similarity of the field distribution plots of the CW and CCW components (Figure 5.9a-d).
To further analyse the previous result, it is important to consider the RF power analysis described earlier in MR literature [39]. The voltage induced in a cylindrical ring is given as

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  
\[ V = -j\omega \pi r^2 B. \]  

(5.4)  

(5.5)

In this equation, \( B \) is assumed to be the field that excites the spins. This can only be accomplished if the only magnetic field that exists in the coil is the \( B_1^+ \) field; a condition which is only valid when the coil is excited in quadrature at low frequency where the dimensions of coil and object to be imaged are small compared to operating wavelength. The conductance of the cylindrical ring is given as

\[ dG(r) = \frac{hdr\sigma}{2\pi r}. \]  

(5.6)

The total power into the coil is given by the sum of the dissipated power and the radiated power. If radiation is neglected under the conditions described above, the total power is equal to the dissipated power:

\[ dP(r) = \frac{V^2}{R} \]  
\[ = \frac{V^2}{d} G(r). \]  

(5.7)  

(5.8)

By integrating over the whole volume of the cylinder (radius = \( Rad \), length = \( l \)), the power dissipated is given by:

\[ P = \pi Rad^4 l \omega^2 \sigma B^2 \]  

(5.9)

Of course the above equation is only valid when the \( B \) field is homogeneous across the sample. However even with inhomogeneous field a similar expression could be
obtained. As such, it was established in the MR community that the power required for a certain flip angle varies with $\omega^2$ or in other words, it varies with the $B_0^2$. In addition to all the conditions described above for such conclusion to be deduced, it is also assumed that inhomogeneity and the strength of the $B$ field do not vary with frequency. For the single strut coil loaded with the electrically small-sized phantom, the above assumptions are valid at low frequency and starts to deviate as the frequency increases. This can be seen from the comparison of the power requirements and the $\omega^2$ curve shown in Figure 5.10.

Figure 5.11 shows axial slices of the $B_1^+$ field calculated inside a 16-strut TEM resonator loaded with the 18-tissue head model at 163 (5.11a), 183 (5.11b), 212(5.11c), 232 (5.11d), 258 (5.11e), 286 (5.11f), 315 (5.11g), 340 (5.11h), 368 (5.11i), 407 (5.11j), 431 (5.11k), and 458 (5.11i) MHz. The coil is operating under 2-port back of the head quadrature excitation. The slice considered for this study is ”slice 2” shown in Figure 5.7(bottom). Figure 5.12 displays the corresponding results for the 6-tissue model loaded in the 8-strut coil and operating under linear excitation. “Slice 3” shown in Figure 5.7(top) was considered for these calculations.

The $B_1^+$ field distribution results show that the homogeneity of the fields decreases as the operational frequency increases. As such, difficulties arise in predicting the power required for a constant $B_1^+$ field value. However, it was recently shown that the field inhomogeneity is surmountable and high quality images could be obtained at 8 Tesla [106]. As such, a reasonable assumption of the reference field could be the average value across the slice of interest. By applying such an assumption and using the FDTD method. Figure 5.13 displays the power required to obtain a constant
Figure 5.11: Axial slices of the FDTD calculated of the $B_1^+$ field inside a 16-strut TEM resonator operating under 2-port quadrature excitation. The excitation locations are positioned behind the head. The distribution is presented at slice #2 (Figure 5.7(bottom)). The results are calculated inside the 18-tissue anatomically detailed human head model at 163 (a), 183 (b), 212(c), 232 (d), 258 (e), 286 (f), 315 (g), 340 (h), 368 (i), 407 (j), 431 (k), and 458 (l) MHz.
Figure 5.11 continued
Figure 5.11 continued
Figure 5.11 continued

(i)  

(j)  

(k)  

(l)
Figure 5.12: Axial slices of the FDTD calculated $B^+_1$ field inside an 8-strut TEM resonator operating under linear excitation. The distribution is presented at slice #3 (Figure 5.7(top)). The results are calculated inside the 6-tissue anatomically detailed human head model at 205 (a), 254 (b), 282(c), and 362 (d) MHz.
Figure 5.12 continued
value of the $B_1^+$ field as a function of frequency for the 6-tissue (5.13a) and the 18-tissue (5.13b-d) models loaded in TEM resonators operating under linear excitation (5.13a-c), and quadrature excitation (5.13d). Figures 5.13b and 5.13c correspond to the power requirements using linear excitation with the correct sense (5.13b) and the opposite sense (5.13c) of the specified circular polarization. The results are presented for 4 axial slices spatially positioned in the brain region. The locations of these slices are shown in Figure 5.7.

At higher frequencies (> 200 MHz), with the size, asymmetry, and inhomogeneity of the head, the treatment of RF requirements presented earlier in MR literature is no longer valid. This is simply true since non of this analysis assumptions is valid in this frequency regime. Emphasizing this point, Figure 5.13 shows that the power required to obtain a certain circularly polarized component of $B_1$ field value increases with frequency plateauing at a certain value, and then it starts to drop as the frequency increases. This trend is apparent especially moving away from the sinus region towards the brain region. When considering linear excitation, Figure 5.13 shows that the peak power value has occurred in the 18-tissue head model at a frequency, which is approximately 120 MHz higher than that for the 6-tissue model. As previously mentioned, the 6-tissue model is larger, physically and consequently electrically, than the 18-tissue model. This is a major factor that can contribute to the differences between the results of the two head models. This result demonstrates that the power required for a certain flip angle depends on the coil load and on its position in the coil itself, and the type of excitation utilized. In addition, the electromagnetic interactions between the load and the coil, and between the load and the excitation source, also contribute to the power-frequency dependence.
Figure 5.13: The power required to obtain a constant value of the circularly polarized field (average value across the slice) as a function of frequency. The results are presented for the 6-tissue (a) and the 18-tissue (b,c,d) anatomically detailed head models loaded in a TEM resonator operating under linear excitation excitation (a,b,c), and quadrature excitation (d). Figures b and c correspond to the power requirement with the correct sense (b), and with the opposite sense (c) of circular polarization required for imaging.
Figure 5.13 continued

(a)

(b)

continue
Figure 5.13 continued

(c)

(d)
6.1 The RF Component Into MRI: A New Look at the NMR Signal

In Chapter 2, description of RF excitation in MRI was briefly stated. In this section, detailed analysis of excitation and reception are provided. A new and novel approach using the principle of reciprocity is introduced to describe the NMR signal strength.

Following the discussion in the section named “Relaxation Effects” from Chapter 2, it is noted that values for $T_1$ and $T_2$ are in the order of milliseconds for imaging at 1.5 Tesla. However, these values are not known at ultra high fields ($> 7$ Tesla). On the other hand, it is not expected for these values to be drastically different from those at lower fields. A fair assumption is the following: at any field strength, the magnetization vectors (represented by the sum of protons, magnetic dipoles, in the patient) are at steady state before applying an RF field. Therefore, at time $= 0$, where time $= 0$ is when the RF field is applied, the magnetization is given by

$$\vec{M} = M_0 \vec{z}. \quad (6.1)$$
6.1.1 RF Field Excitation

Assume a magnetic material with spin characteristics such as protons is subjected to an external magnetic field. The magnetic resonance phenomena is introduced when the magnetic material is applied to an alternating magnetic field which direction is not parallel to the external field. In order to create an image, the magnetization has to be flipped such that a component of which exists in the transverse plane. For this purpose an RF ($\vec{B}_1$) field is applied in the transverse plane.

From Eq. (2.53), the equation of motion will be modified such that $\vec{B}$ is substituted for the $\vec{B}_0$ field where

$$\vec{B} = \vec{B}_0 + \vec{B}_1. \quad (6.2)$$

Consider a rotating frame (Figure 2.4) where the coordinates are rotating clockwise around the $\vec{B}$ field with angular velocity equals to $2\pi*$ (the frequency of the applied RF field). For convenience, the transformation from/to rotating frame is given by

$$\dot{\vec{X}} = \vec{x}cos(\omega t) - \vec{y}sin(\omega t) \quad (6.3)$$

$$\dot{\vec{Y}} = \vec{x}sin(\omega t) + \vec{y}cos(\omega t) \quad (6.4)$$

$$\vec{x} = \vec{X}cos(\omega t) + \vec{Y}sin(\omega t) \quad (6.5)$$

$$\vec{y} = -\vec{X}sin(\omega t) + \vec{Y}cos(\omega t). \quad (6.6)$$

A circularly polarized $B_1$ field is given by

$$\vec{B}_1 = |B_1|(\vec{x}cos(\omega t) - \vec{y}sin(\omega t)) \quad (6.7)$$

where from looking into the paper, the field would be rotating clockwise. From an electromagnetic (EM) prospective of polarization definition, the wave associated with $\vec{B}_1$ must be traveling in the $-z$ direction. Note that polarization for EM fields is
applied with respect to the electric field intensity vector. In here, for convenience, the polarization sense will be applied with respect to the magnetic field density vector.

From Eq. (6.3),

$$\vec{B}_1 = |B_1|\vec{X}'.$$  \hfill (6.8)

Therefore, the amplitude of the $\vec{B}_1$ field in the laboratory frame is transferred completely to the rotating frame. Now let us consider a counterclockwise field given by

$$\vec{B}_1 = |B_1|(\bar{x}cos(\omega t) + \bar{y}sin(\omega t)).$$ \hfill (6.9)

The transformation in the rotating frame is given by

$$\vec{B}'_1 = |B_1|(\bar{X}'(cos(\omega t)cos(\omega t) - sin(\omega t)sin(\omega t)) +$$

$$\bar{Y}'(sin(\omega t)cos(\omega t) + cos(\omega t)sin(\omega t)))$$  \hfill (6.10)

$$\vec{B}'_1 = |B_1|(\bar{X}'cos(2\omega t) + \bar{Y}'sin(2\omega t)).$$ \hfill (6.11)

If RF period $T$ is $>>$ than $\frac{2\pi}{\omega}$, the average value of $\vec{B}_1$ over $T$ is given by

$$AVG(B_1) = \frac{1}{T}\int_{T} (\vec{B}_1 dt) = 0.$$ \hfill (6.13)

The previous analysis shows that the clockwise field is completely effective while the counterclockwise field is completely ineffective in the equation of motion in the rotating frame. It is noted that the rotation is defined when looking into the negative direction of the $\vec{B}_0$ field which is assumed to be the direction of propagation. Because a linearly polarized field can be decomposed into clockwise and counterclockwise circularly polarized fields, only 0.5 of a linearly polarized field strength is effective in the rotating frames [37].

In general, the electromagnetic field excited by an RF coil is elliptically polarized in the object to be imaged. Therefore, only the clockwise circularly polarized
component of this field, assuming that the direction of propagation is \(-z\), is effective in exciting the spins. The definition of the The \(\vec{B}_1^+\) field is extremely critical for understanding the RF excitation from an EM point view. For \(\vec{B}_0 = |B_0|\hat{z}\), the \(\vec{B}_1^+\) field is defined as, the circularly polarized component of the \(\vec{B}_1\) field in the clockwise direction if the direction of propagation is \(-z\) or the circularly polarized component of the \(\vec{B}_1\) field in the counterclockwise direction if the direction of propagation is \(+z\). If \(\vec{B}_0 = -|B_0|\hat{z}\), the sense of polarization would be reversed in the definition. In this analysis, it is assumed that the direction of \(\vec{B}_0\) field is always in the \(+z\) direction. The modification of the equation of motion with the substitution of Eq. (6.2) into Eq. (2.53) and considering the representation of \(\vec{B}_1^+\) as the only component of interest in the rotating frame:

\[
\frac{d\vec{M}'(t)}{dt} = \vec{M} \times (\vec{Z}'(\omega_0 - \omega) + \vec{Y}' \omega_1) + \left(\frac{M_0 - M_y'}{T_1}\right)\vec{Z}' - \left(\frac{M_x'}{T_2} \vec{X}' + \frac{M_y'}{T_2} \vec{Y}'\right) \tag{6.14}
\]

where \(\omega_1\) is the precessional frequency of the \(\vec{B}_1^+\) field \(= \gamma|\vec{B}_1^+|\). To eliminate confusion, \(\omega_0\) is the Larmor frequency and \(\omega\) is the RF frequency which is equal to the angular velocity of the rotating frame. If the relaxation effects are ignored, Eq. (6.14) reduces to

\[
\frac{d\vec{M}'(t)}{dt} = \vec{M} \times (\vec{Z}'(\omega_0 - \omega) + \vec{X}' \omega_1). \tag{6.15}
\]

If \(\omega_0 = \omega\), the magnetization \(\vec{M}\) precess around \(\vec{X}'\). From Eq. (6.1), \(\vec{M} = M_0 \vec{Z}' = M_0 \vec{Z}'\) before the RF field is applied. As such, Eq. (6.15) will cause all of \(\vec{M}\) to rotate to the \(Y'\) direction with the appropriate \(|B_1^+|\) strength. The requirement that \(\omega_0\) must equal \(\omega\) for maximum signal strength is demonstrated. If \(\omega_0 \neq \omega\), the magnetization will precess around \((\vec{Z}'(\omega_0 - \omega) + \vec{X}' \omega_1)\) instead of \((\vec{X}' \omega_1)\). Note for imaging at 1.5 Tesla and above, \(\omega_1\) is on an order of \(10^{-6} \times \omega_0\). Therefore a small difference between
\( \omega_0 \) and \( \omega \) results in an extreme degradation in signal to noise. It is also noted that it is impossible to obtain \( \omega_0 = \omega \) since the applied \( \tilde{B}_0 \) field contains some inhomogeneity and therefore \( \omega_0 \) is a function of space. In addition, the difference in susceptibility values (due to magnetic moments associated with the spinning electrons) between different tissue can lead to similar effects:

\[
B(x,y,z) = \mu_0(H_0 + MM(x,y,z))
\]

(6.16)

where

\[
B_0 = \mu_0H_0
\]

(6.17)

\[
MM(x,y,z) = \xi(x,y,z)H_0
\]

(6.18)

and \( \xi(x,y,z) \) is the susceptibility associated with a particular point in the subject.

The solution to Eq. (6.15) is given by

\[
M_X'(t) = M_X'(0)
\]

(6.19)

\[
M_Y'(t) = M_Y'(0)\cos(\omega_1t) + M_Z'(0)\sin(\omega_1t)
\]

(6.20)

\[
M_Z'(t) = -M_Y'(0)\sin(\omega_1t) + M_Z'(0)\cos(\omega_1t).
\]

(6.21)

Before the RF pulse is applied \( M_z'(0) = M_0 \) and \( M_X'(0) = M_Y'(0) = 0 \). Therefore,

\[
M_X'(t) = 0
\]

(6.22)

\[
M_Y'(t) = M_0\sin(\omega_1t)
\]

(6.23)

\[
M_Z'(t) = M_0\cos(\omega_1t).
\]

(6.24)
At $t = t_{90}$,

$$\omega_1 t = \gamma |B^\dagger_1| t_{90} = \pi/2$$ \hspace{1cm} (6.25)$$

$$M_{y'}(t_{90}) = M_0$$ \hspace{1cm} (6.26)$$

$$M_{z'}(t_{90}) = 0.$$ \hspace{1cm} (6.27)$$

After the RF is turned off ($t = toff$) and with the inclusion of the relaxation effects from Eq. (6.14), the magnetization at time $t$ is given by:

$$M_{y'}(t) = e^{-(t-toff)/T_2}(M_{y'}(toff))$$ \hspace{1cm} (6.28)$$

$$M_{z'}(t) = M_{z'}(toff)e^{-(t-toff)/T_1} + M_0(1 - e^{-(t-toff)/T_1}).$$ \hspace{1cm} (6.29)$$

**Excitation at High Field MRI**

In the previous analysis, the fields excited by the RF coil were considered to be circularly polarized in the correct sense, i.e. the $B^\dagger_1$ field that tips the spins. At high field MRI or at cases where the dimensions of the coil and/or the object to be imaged are significant fraction of the operating wavelength, the fields excited are neither circularly polarized nor TEM. As such hybrid modes rather than TEM modes exist in the coil and the $B_z$ component can no longer be ignored. In this analysis, the equation of motion will be solved without the assumptions described in the previous section.

Relaxation effects will not be included and as such it is assumed that the images are proton density weighted and excited by one pulse (gradient echo imaging). It is also assumed that the $B_0$ field is homogeneous, $\mu$ is constant across the object to be imaged and the Larmor frequency is matched to the RF frequency.
Let us assume that the $B_1$ field excited by the RF coil is given by:

$$\vec{B}_1 = \vec{x}B_{1x}\cos(\omega t + a1) + \vec{y}B_{1y}\cos(\omega t + a2) + \vec{z}B_{1z}\cos(\omega t + a3)$$  \hspace{1cm} (6.30)$$

where $B_{1x}$, $B_{1y}$, and $B_{1z}$ are the magnitude of the EM fields which are functions of space and not time; time harmonic electromagnetic fields are assumed. In the rotating frame where angular velocity $= \omega$, the corresponding magnetic field intensities are given by:

$$\vec{x}'B_{1x}' = B_{1x}\cos(\omega t + a1)\cos(\omega t) - B_{1y}\cos(\omega t + a2)\sin(\omega t)$$  \hspace{1cm} (6.31)$$

$$\vec{y}'B_{1y}' = B_{1x}\cos(\omega t + a1)\sin(\omega t) + B_{1y}\cos(\omega t + a2)\cos(\omega t)$$  \hspace{1cm} (6.32)$$

$$\vec{z}'B_{1z}' = B_{1z}\cos(\omega t + a3)$$  \hspace{1cm} (6.33)$$

$$\vec{B}_1' = \vec{x}'B_{1x}' + \vec{y}'B_{1y}' + \vec{z}'B_{1z}'$$  \hspace{1cm} (6.34)$$

The average value of $\vec{B}_1'$ over $T$ (duration of the pulse) is given by

$$AVG(\vec{B}_1') = \frac{1}{T}\int_{0}^{T} (\vec{B}_1' dt).$$  \hspace{1cm} (6.35)$$

For each of the coordinate components, the average values of the RF fields in the rotating frame are defined as:

$$AVG(B_{1x}') = \frac{1}{2}\frac{(-B_{1x}\sin(a1) - B_{1y}\cos(a2))(\sin(T\omega))}{T\omega}$$

$$+ \frac{1}{2}(B_{1x}\cos(a1) + B_{1y}\sin(a2))$$

$$+ \frac{1}{4}\frac{(B_{1x}\cos(a1) - B_{1y}\sin(a2))\sin(2T\omega)}{T\omega}$$  \hspace{1cm} (6.36)$$

$$AVG(B_{1y}') = \frac{1}{2}\frac{B_{1x}\cos(a1) - B_{1y}\sin(a2))(\sin(T\omega))}{T\omega}$$

$$+ \frac{1}{2}(-B_{1x}\sin(a1) + B_{1y}\cos(a2))$$

$$+ \frac{1}{4}\frac{B_{1x}\sin(a1) + B_{1y}\cos(a2))\sin(2T\omega)}{T\omega}$$  \hspace{1cm} (6.37)$$

$$AVG(B_{1z}') = \frac{1}{2}\frac{B_{1z}(\sin(T\omega)\cos(T\omega/2 + a3))}{T\omega}.$$

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Since $T$ is in the $ms$ range and $\omega$ is at least in the $10^8$ range, the average values are approximated as:

$$b_{1x} = AVG(B'_{1x}) \approx \frac{1}{2}(B_{1x}\cos(a1) + B_{1y}\sin(a2)) \quad (6.39)$$

$$b_{1y} = AVG(B'_{1y}) \approx \frac{1}{2}(-B_{1x}\sin(a1) + B_{1y}\cos(a2)) \quad (6.40)$$

$$b_{1z} = AVG(B'_{1z}) \approx \frac{B_{1z}(\sin(T\omega/2)\cos(T\omega/2 + a3))}{T\omega} \quad (6.41)$$

$$|b1| = \sqrt{b_{1x}^2 + b_{1y}^2 + b_{1z}^2} \quad (6.42)$$

Substituting from Eqs. (6.39, 6.40, 6.41) into Eq. (6.42), the magnitude of the total magnetic field is given by:

$$|b1| \approx \sqrt{\frac{B_{1x}^2 + B_{1y}^2 + 2B_{1x}B_{1y}\sin(a2 - a1)}{2}} \quad (6.43)$$

$$|b1| \approx |B_1^+|. \quad (6.44)$$

The equation of motion in the rotating frame with the inclusion of all the field components is defined as

$$\left( \frac{d\vec{M}(t)}{dt} \right)' = \vec{M} \times \left( \vec{Z}(\omega_0 - \omega + \gamma b_{1z}) + \vec{X}(\gamma b_{1x}) + \vec{Y}(\gamma b_{1y}) \right) \quad (6.45)$$

The scalarization follows:

$$\left( \frac{dM_x(t)}{dt} \right)' = \gamma (M'_x b_{1y} + M'_yb_{1z}) \quad (6.46)$$

$$\left( \frac{dM_y(t)}{dt} \right)' = \gamma (M'_y b_{1x} - M'_xb_{1z}) \quad (6.47)$$

$$\left( \frac{dM_z(t)}{dt} \right)' = \gamma (-M'_yb_{1x} + M'_xb_{1y}) \quad (6.48)$$

The solution of these equations is obtained using xmaple. After several simplifications, the magnetization in the rotating frame at time = $T$, the end of the RF pulse,
is given by:

\[ M'_x(T) \approx M_0 \frac{\left( b_{1z} b_{1x} (1 - \cos(\gamma |B_1^+| T)) - b_{1y} |B_1^+| \sin(\gamma |B_1^+| T) \right)}{|B_1^+|^2} \] (6.49)

\[ M'_y(T) \approx M_0 \frac{\left( b_{1z} b_{1y} (1 - \cos(\gamma |B_1^+| T)) + b_{1x} |B_1^+| \sin(\gamma |B_1^+| T) \right)}{|B_1^+|^2} \] (6.50)

\[ M'_z(T) \approx M_0 \frac{\left( b_{1z}^2 + b_{1x}^2 \cos(\gamma |B_1^+| T) + 2b_{1z}^2 \right)}{|B_1^+|^2} \] (6.51)

In the laboratory frame the magnetization is given as:

\[ M_x(T) \approx M'_x(T) \cos(\omega t) + M'_y(T) \sin(\omega t) \] (6.52)

\[ M_y(T) \approx -M'_x(T) \sin(\omega t) + M'_y(T) \cos(\omega t) \] (6.53)

\[ M_z(T) \approx M'_z(T). \] (6.54)

From Eqs. (6.39, 6.40, 6.41), \( b_{1z} \) is \( << b_{1x} \) or \( b_{1y} \) since \( T \omega \) is on the order of \( 10^5 \).

Figure 6.1 displays \( \frac{|\sin(\gamma |B_1^+| T)|}{(1 - \cos(\gamma |B_1^+| T))} \) as a function of \( \gamma |B_1^+| T \). Very interestingly, the range where \( b_{1z} \) has a significant effect on the magnetization is \( \chi \) where \( \chi \) is less than \( 0.04^\circ \) and \( 180^\circ \) is at the center of \( \chi \). As such the solution of the magnetization everywhere except in \( \chi \) reduces to

\[ M_x(T) \approx M_0 \left( -\frac{b_{1y} \sin(\gamma |B_1^+| T)}{|B_1^+|} \cos(\omega t) + \frac{b_{1x} \sin(\gamma |B_1^+| T)}{|B_1^+|} \sin(\omega t) \right) \] (6.55)

\[ M_y(T) \approx M_0 \left( \frac{b_{1y} \sin(\gamma |B_1^+| T)}{|B_1^+|} \sin(\omega t) + \frac{b_{1x} \sin(\gamma |B_1^+| T)}{|B_1^+|} \cos(\omega t) \right) \] (6.56)

\[ M_z(T) \approx M'_z(T) \] (6.57)
which simplifies to

\[ M_x(T) \approx M_0 \sin(\gamma |B_1^+| T) \left( \frac{B_{1x} \cos(\omega t + a_1) - B_{1y} \cos(\omega t + a_2)}{2|B_1^+|} \right) \]  

(6.58)

\[ M_y(T) \approx M_0 \sin(\gamma |B_1^+| T) \left( \frac{B_{1x} \cos(\omega t + a_1) + B_{1y} \cos(\omega t + a_2 - \frac{\pi}{2})}{2|B_1^+|} \right) \]  

(6.59)

\[ M_z(T) \approx M_z'(T). \]  

(6.60)

From this analysis, it can be shown that the magnetization is a time harmonic electromagnetic quantity. Now if the relaxation effects are included, the magnetization in transverse plane becomes a sinusoidal function of \( \omega \) multiplied by exponential function of \( \frac{1}{T_2} \), and the longitudinal component is an exponential function of \( \frac{1}{T_1} \) (see the previous section). In addition, \( \omega \) is at least \( 10^4 \) order of magnitudes larger than \( \frac{1}{T_2} \) or
It is known that magnetization is an electromagnetic current source where its derivative is equal to its strength, therefore the derivatives of $\frac{e_{1}}{T_{2}}$ and $e_{2}/T_{1}$ could be ignored when compared to the derivatives of $\sin(\omega_{0}t + A)$. And also the contribution to the MR signal from $M_{z}(T)$ is negligible when compared to that of $M_{x}(T)$ and $M_{y}(T)$ since $M_{z}(T)$ is not a function of $\sin(\omega_{0}t + A)$. As such, $M_{z}(T)$ will be ignored from this point on. In the frequency domain, the magnetization as a time harmonic electromagnetic quantity is given by:

$$MF_{x}(T) \approx M_{0}\sin(\gamma|B_{1}^{+}|T)\left(\frac{-jBF_{1x} - BF_{1y}}{2|B_{1}^{+}|}\right)$$  \hspace{1cm} (6.61)

$$MF_{y}(T) \approx M_{0}\sin(\gamma|B_{1}^{+}|T)\left(\frac{BF_{1x} - jBF_{1y}}{2|B_{1}^{+}|}\right)$$  \hspace{1cm} (6.62)

where

$$B_{1x}\cos(\omega_{0}t + a1) = \Re(BF_{1x}e^{j\omega t})$$  \hspace{1cm} (6.63)

$$B_{1y}\cos(\omega_{0}t + a2) = \Re(BF_{1y}e^{j\omega t}).$$  \hspace{1cm} (6.64)

Note the the applied RF field is given by Eq. (6.30) and in the frequency domain is given by:

$$\tilde{B}F_{1} = \tilde{x}BF_{1x} + \tilde{y}BF_{1y} + \tilde{z}BF_{1z}.$$  \hspace{1cm} (6.65)

The magnetization in Eqs. (6.61, 6.62) could be simplified to:

$$MF_{x} \approx -jM_{0}\sin(\gamma|B_{1}^{+}|T)\left(\frac{BF_{1}}{2|B_{1}^{+}|}\right)(\tilde{x} - \tilde{y}j)$$  \hspace{1cm} (6.66)

$$MF_{x} \approx -jM_{0}\sin(\gamma|B_{1}^{+}|T)\left(\frac{BF_{1}}{\sqrt{2}|B_{1}^{+}|}\right)(\tilde{B}_{1}^{+})^{*}$$  \hspace{1cm} (6.67)

$$MF_{x} \approx -\frac{jM_{0}\sin(\gamma|B_{1}^{+}|T)\exp(\angle B_{1}^{+})}{\sqrt{2}}$$  \hspace{1cm} (6.68)

$$MF_{y} \approx \frac{M_{0}\sin(\gamma|B_{1}^{+}|T)\exp(\angle B_{1}^{+})}{\sqrt{2}}$$  \hspace{1cm} (6.69)
By definition, the transverse magnetization is rotating clockwise (looking into the 
$-z$ direction) except with a $-\pi/2$ phase shift, i.e. the magnetization has the same 
sense of rotation as $\vec{B}_1^+$ field.

**Reciprocity During Transmission**

In order for the principle of reciprocity (to be discussed in the next section) to 
hold in a medium, its constitutive parameters must be reciprocal. For instance in the 
Cartesian coordinates, a reciprocal material is one in which

\begin{align}
\overline{\varepsilon} &= \overline{\varepsilon}^T \quad (6.71) \\
\overline{\mu} &= \overline{\mu}^T \quad (6.72) \\
\overline{\sigma} &= \overline{\sigma}^T \quad (6.73)
\end{align}

where $\overline{a}$ represents a matrix tensor quantity, and $^T$ is a transpose [107]. In the follow-
ing analysis of MR excitation, it is assumed that susceptibility effects are introduced 
due to nuclear magnetic resonance. In reality however the material tensors are af-
fected mostly by the electron magnetic resonance. The idea of this analysis however 
is to see the validity of relating the excitation of spins to the principle of reciprocity, 
electromagnetic theory.

To test the concept however, it will be assumed that imaging is performed at a low 
flip angle, therefore

\[ \sin(\gamma |B_1^+|T) \approx \gamma |B_1^+|T. \quad (6.74) \]

The magnetization is the frequency domain is given as

\[ MF_x \approx M_0 \gamma T \left( \frac{-jBF_{1x} - BF_{1y}}{2} \right) \quad (6.75) \]

\[ MF_y \approx M_0 \gamma T \left( \frac{BF_{1x} - jBF_{1y}}{2} \right) \quad (6.76) \]
The total magnetic flux density is given by

\[ \vec{B}F = \mu_0(\vec{H}F + \vec{M}F) \quad (6.77) \]

\[ \vec{B}F = \overline{\eta}.\vec{H}F. \quad (6.78) \]

The solution for the tensor \( \overline{\eta} \) follows

\[
\overline{\eta} = \begin{pmatrix}
1 - j \frac{\gamma \mu_0 M_0 T}{2} & - \frac{\gamma \mu_0 M_0 T}{2} & 0 \\
\frac{\gamma \mu_0 M_0 T}{2} & 1 - j \frac{\gamma \mu_0 M_0 T}{2} & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

(6.79)

It is apparent that the matrix is not symmetric. Therefore, the principle of reciprocity as it is defined in electromagnetics can not be applied to explain the excitation of the spins.

6.1.2 The Received Signal

Background

The issue of the received signal in MRI has been brought to attention lately by several groups around the world including Hershey Medical Center at the University of Pennsylvania [108], Center of Magnetic Resonance Research at the University of Minnesota [108]. In 2000, D. I. Hoult at National Research Council in Canada published an article describing the MRI signal strength using the principle of reciprocity [109]. Quite interestingly, Hoult does not prove analytically that reciprocity holds for an MRI experiment, however reciprocity was utilized based on a laboratory experiment that was conducted in his research lab [109].

Two surface coils were constructed from small current loops, where the magnetic flux density is assumed to be constant across its surface. One of the coils was
immersed in a flask filled with saline and the other was held in the air with some
distance away from the first coil. One of the coils was fed by a coaxial line and the
voltage induced on the other coil was measured using a network analyzer. The reverse
experiment was also conducted. The voltage induced in both cases was found to be
the same. Based on these results, Hoult asserted that reciprocity holds and it could be
applied to an MRI experiment [109].

Therefore, Hoult indicated that if a small loop is used as a transmitter coil, mag-
netic dipoles are created in the sample. The fields induced by these dipoles could
be approximated as fields produced by a small current loop where the magnetic flux
density is constant across its surface. From the experiment conducted earlier, Hoult
deduced that voltage induced in the pickup coil should be linearly scaled with that
induced in the current loop (proton magnetic dipole) when the pickup coil is used as
a transmitter carrying unity current. The scalar factor was simply magnitude of the
magnetization divided by the magnitude of current multiplied by area of the small
dipole. From these calculations, the strength of the received signal in the small loop
coil was found to be proportional to the rotating $B_1$ field produced by this coil where
the frame of rotation is opposite to that used for the forward problem (transmission).

In the above experiment, Hoult proves experimentally (not theoretically) that reci-
procity is holding for this particular problem (2 small loop coils used as transmitter
and receiver). In addition, from Hoult’s derivation to calculate the voltage using Farad-
day’s law, the assumption made is that the physical presence of the receiver coil does
not perturb the fields from the transmitter coil and that the magnetic flux density is
constant over the loop surface. In general in an MRI experiment especially at high
field, the receiver coil could be a volume coil with a complex structure, therefore
the magnetic flux density is not constant over the coil surface. Also unlike the small loop coil, the physical dimensions of the receiver coil are a significant fraction of the wavelength. As such, the physical structure of the receiver coil will perturb the electromagnetic fields induced by the magnetic dipoles. However, the assumption that magnetic moments (as a result of protons alignment) could be treated as small current loops is valid for this case.

In here, the theoretical derivation for the principle of reciprocity applied to an MRI experiment is presented. The physical structure of the receiver coil will be general. Experimental results and numerical simulations verifying the derivation will be provided.

**Principle of Reciprocity**

Consider Figure 6.2 where an infinite closed surface (Σ) contains material with 5 different materials given as \((\mu_0, \varepsilon_0, \sigma_0)\), \((\mu_1, \varepsilon_1, \sigma_1)\), \((\mu_2, \varepsilon_2, \sigma_2)\), \((\mu_3, \varepsilon_3, \sigma_3)\), and \((\mu_4, \varepsilon_4, \sigma_4)\). In addition, Σ also contains electric \((\tilde{J}_1, \text{and } \tilde{J}_2)\), and magnetic current \((\tilde{M}_1, \text{and } \tilde{M}_2)\) sources enclosed within the surfaces \((S_1, \text{and } S_2)\) [36]. Of course, magnetic current sources are fictitious quantities. It is assumed in here that the materials enclosed within Σ are inhomogeneous and dispersive; however, it must be linear, reciprocal, and the electromagnetic fields induced by the sources could be presented on a macroscopic level. Note that reciprocity is a method that is applied in the frequency domain. The fields radiated by sources \(\tilde{J}_1\), and \(\tilde{M}_1\), in the absence of \(\tilde{J}_2\), and \(\tilde{M}_2\) are given by Maxwell’s equations

\[
\nabla \times \tilde{E}_1 = -\tilde{M}_1 - j\omega \mu \tilde{H}_1 \quad (6.80)
\]

\[
\nabla \times \tilde{H}_1 = \tilde{J}_1 + j\omega \varepsilon \tilde{E}_1 + \sigma \tilde{E}_1 \quad (6.81)
\]
and the fields radiated by sources \( \mathbf{\tilde{J}}_2 \) and \( \mathbf{\tilde{M}}_2 \), in the absence of \( \mathbf{\tilde{J}}_1 \) and \( \mathbf{\tilde{M}}_1 \) are given by

\[
\nabla \times \mathbf{\tilde{E}}_2 = -\mathbf{\tilde{M}}_2 - j\omega \mu \mathbf{\tilde{H}}_2
\]

(6.82)

\[
\nabla \times \mathbf{\tilde{H}}_2 = \mathbf{\tilde{J}}_2 + j\omega \varepsilon \mathbf{\tilde{E}}_2 + \sigma \mathbf{\tilde{E}}_2.
\]

(6.83)

Note that the current sources specified are the only sources in the problem. If Eqs. (6.80) and (6.83) are dot multiplied by \( \mathbf{\tilde{H}}_2 \) and \( \mathbf{\tilde{E}}_1 \) respectively, and the resulting equations are subtracted, the following equation is obtained

\[
\mathbf{E}_1 \cdot \nabla \times \mathbf{\tilde{H}}_2 + \mathbf{\tilde{H}}_2 \cdot \nabla \times \mathbf{\tilde{E}}_1 = \mathbf{\tilde{E}}_1 \cdot \mathbf{\tilde{J}}_2 + \mathbf{\tilde{H}}_2 \cdot \mathbf{\tilde{M}}_1 + j\omega \varepsilon \mathbf{\tilde{E}}_1 \cdot \mathbf{\tilde{E}}_2 + j\omega \mu \mathbf{\tilde{H}}_2 \cdot \mathbf{\tilde{H}}_1. \quad (6.84)
\]

Using vector identities and applying the dot products on the other 2 equations, with mathematical simplifications, the reciprocity equation is obtained

\[
-\nabla \cdot (\mathbf{\tilde{E}}_1 \times \mathbf{\tilde{H}}_2 - \mathbf{\tilde{E}}_2 \times \mathbf{\tilde{H}}_1) = \mathbf{\tilde{E}}_1 \cdot \mathbf{\tilde{J}}_2 + \mathbf{\tilde{H}}_2 \cdot \mathbf{\tilde{M}}_1 - \mathbf{\tilde{E}}_2 \cdot \mathbf{\tilde{J}}_1 - \mathbf{\tilde{H}}_1 \cdot \mathbf{\tilde{M}}_2. \quad (6.85)
\]

By applying divergence theorem, the reciprocity theorem in integral form is obtained

\[
-\iiint_{\Sigma} (\mathbf{\tilde{E}}_1 \times \mathbf{\tilde{H}}_2 - \mathbf{\tilde{E}}_2 \times \mathbf{\tilde{H}}_1).\, ds' = \iiint_{V} (\mathbf{\tilde{E}}_1 \cdot \mathbf{\tilde{J}}_2 + \mathbf{\tilde{H}}_2 \cdot \mathbf{\tilde{M}}_1 - \mathbf{\tilde{E}}_2 \cdot \mathbf{\tilde{J}}_1 - \mathbf{\tilde{H}}_1 \cdot \mathbf{\tilde{M}}_2).\, dv'. \quad (6.86)
\]

If we assume that the surface \( \Sigma \) is at infinity, then the left hand side of the previous two equations is equal to 0 (Summerfield’s radiation condition). Therefore the equation of reciprocity reduces to [36]

\[
\iiint_{V} (\mathbf{\tilde{E}}_1 \cdot \mathbf{\tilde{J}}_2 - \mathbf{\tilde{H}}_1 \cdot \mathbf{\tilde{M}}_2).\, dv' = \iiint_{V} (\mathbf{\tilde{E}}_2 \cdot \mathbf{\tilde{J}}_1 - \mathbf{\tilde{H}}_2 \cdot \mathbf{\tilde{M}}_1).\, dv'. \quad (6.87)
\]
Figure 6.2: Generalized Reciprocity
Reciprocity in MRI Experiment

We have established so far that the signal that contributes to an MRI image is related to the induced magnetization, in other words, \( \vec{M}F_x \), and \( \vec{M}F_y \) provide the MRI signal. Note that the previous analysis has assumed that the fields could be represented on a macroscopic level (the classical representation of Maxwell equations is valid). Therefore at point \((x_2,y_2,z_2)\), the magnetization is represented by \( \vec{M}F_x(x_2,y_2,z_2), \vec{M}F_y(x_2,y_2,z_2), \) and \( \vec{M}F_z(x_2,y_2,z_2) \).

From Maxwell’s equations in the time domain [36], the derivative of the magnetic flux density \( \vec{B} \) with respect to time is equivalent to a magnetic current source, more precisely displacement magnetic current density. The magnetic flux density is given by

\[
\vec{B} = \vec{B}_1 + \mu_0 \dot{\vec{M}}
\]

(6.88)

Note that \( \vec{B}_1 \) field in here is the total field applied by the RF coil. Since the \( \vec{B}_1 \) field is only applied for a short period of time, one could consider \( \dot{\vec{M}} \) as the only source of RF signal. The resultant magnetic source will then excite electromagnetic fields. RF coil(s) is/are used to receive fields and induce a voltage (MRI signal). The magnetic current sources are given by

\[
MC_{2x} \approx \omega \frac{M_0}{\sqrt{2}} \sin(\gamma|B_1^+|T) \exp(\angle B_1^+) \]

(6.89)

\[
MC_{2y} \approx j\omega \frac{M_0}{\sqrt{2}} \sin(\gamma|B_1^+|T) \exp(\angle B_1^+) \]

(6.90)

\[
MC_{2x} = -jMC_{2y} = SIGNAL. \]

(6.91)

Note that the sense of rotation of this current is still equal to that of the \( \vec{B}_1^+ \) field. Again the \( \vec{B}_1^+ \) field is what originally excited this current.
In the MRI experiment, volume or surface coils are utilized for reception. The geometries of these coils such as volume RF head resonators could be of great complexity. Figure 6.3 displays the reciprocity problem for the MRI experiment, where a high pass birdcage resonator [28] is used as the receiver coil. The signal measured with the resonator induces voltage between the coil terminals. These terminals, enclosed in surface $Sexc$ (Figure 6.3) which is located in a transmission line used for excitation (if the coil was used for that purpose) or reception. In here, a general case is considered, a coil is used to excite the spins and a different coil is used for the reception of the NMR signal.

Let us shift to the frequency domain, the domain of reciprocity. At this point, is it possible to apply the reciprocity results from the previous section? For the forward problem if the birdcage coil (Figure 6.3) is used for transmission, the magnetic current sources (aligned protons) are not existent (Figure 6.3). Clearly, $\dot{M} = M_0 \tilde{z}$ before the RF coil is applied. A magnetic current source is equal to $\frac{d\tilde{M}}{dt} = 0$. Therefore, fields $\tilde{E}1$ and $\tilde{H}1$ are radiated by the birdcage coil in the absence of $MC2$. During reception however, the fields $\tilde{E}2$ and $\tilde{H}2$ are radiated by $MC2$ while the birdcage coil is present. Therefore the equations presented in the previous section can not be used. For this problem, let surface $Sv$ encapsulates volume $V$ which boundaries are defined by surfaces $\Sigma$, $Sexc$, and $Scoil$. Surface $Scoil$ tightly encapsulates the metal parts of the coil and the transmission line as shown in Figure 6.3.

The fields, radiated by the birdcage coil in volume $V$ ($\tilde{J}1$ is not enclosed in $V$), in the absence of $MC2$, are given by Maxwell’s equations

$$\nabla \times \tilde{E}1 = -j\omega \mu \tilde{H}1$$ (6.92)

$$\nabla \times \tilde{H}1 = \tilde{J}1 + j\omega \epsilon \tilde{E}1 + \sigma \tilde{E}1.$$ (6.93)
Figure 6.3: Reciprocity in MRI Experiment. A high pass birdcage coil is used as a receiver coil. The voltage induced by the magnetization is measured at $Sexc$. $Sexc$ is included in a transmission line that supports a TEM mode.
The fields, radiated by the source $\tilde{MC}2$ while the coil is present, are given by

\[
\nabla \times \vec{E}2 = -j\omega \mu \vec{H}2
\]

\[
\nabla \times \vec{H}2 = \tilde{MC}2 + j\omega \epsilon \vec{E}2 + \sigma \vec{E}2
\]

in volume $V$. From divergence theorem,

\[
\iiint_{V} \nabla \cdot (\vec{E}1 \times \vec{H}2 - \vec{E}2 \times \vec{H}1) = -\iiint_{Sv} (\vec{E}1 \times \vec{H}2 - \vec{E}2 \times \vec{H}1) \cdot \vec{n} ds
\]

(6.96)

where $Sv = Scoil + Sexc + \Sigma$. The integral vanishes at surface $\Sigma$ due to Summerfield’s radiation condition. If the coil is composed of perfect conductors, at surface $Scoil$,

\[
\vec{n} \times \vec{E}1 = 0
\]

\[
\vec{n} \times \vec{E}2 = 0.
\]

Utilizing Eq. (6.87), the reciprocity equation is obtained

\[
\int_{v} \tilde{MC}2.\vec{H}1 dv = -\iiint_{Sexc} (\vec{E}1 \times \vec{H}2 - \vec{E}2 \times \vec{H}1) \cdot \vec{n} ds
\]

(6.99)

Note that the fields at surface $Sexc$ are within the transmission line (middle of it). It is fair to assume a dominant TEM mode propagating within the line with dominant orthogonal vector mode functions $\vec{e}$ and $\vec{h}$ (unit vectors):

\[
\vec{e} \times \vec{h} = \vec{n}.
\]

(6.100)

With $\tilde{MC}2$ being infinitesimal and assuming $\iint_{Sexc} \vec{n} \cdot \vec{n} ds = 1$, the reciprocity equation is modified as follows

\[
\tilde{MC}2.\vec{H}1 = -\iiint_{Sexc} (VV_{tr} \vec{e} \times II_{rc} \vec{h} - VV_{rc} \vec{e} \times II_{tr} \vec{h}) \cdot \vec{n} ds
\]

(6.101)

\[
\tilde{MC}2.\vec{H}1 = VV_{tr} II_{rc} - VV_{rc} II_{tr}
\]

(6.102)
where $VV_{tr}$ and $II_{tr}$ are the voltage and current modal amplitudes in the transmission line when the coil is transmitting and $VV_{rc}$ and $II_{rc}$ are the same quantities when the coil is receiving. For the open circuit condition, $II_{rc} = 0$ [107]. The open circuit voltage is given by

$$VV_{rc} = VV_{oc} = -\frac{M\tilde{C}2.\tilde{H}1}{II_{tr}}. \quad (6.103)$$

$M\tilde{C}2$ is defined in Eqs. (6.89, and 6.90). It is noted again that $M\tilde{C}2$ would not exist if $\tilde{B}_1^+$ was not excited by a coil (any coil used is for excitation). The unique issue about antenna ($M\tilde{C}2$) is that it responds to clockwise fields if the associated direction of propagation is $-z$ direction and to counterclockwise fields if the associated direction of propagation is $+z$ (non-reciprocal antenna). As such, rotation sense of $M\tilde{C}2$ is always fixed. Eq. (6.103), is then given by

$$VV_{oc} \approx -\frac{\overline{xSIGNAL.\tilde{H}1 + j\overline{ySIGNAL.\tilde{H}1}}}{\sqrt{2}II_{tr}}. \quad (6.104)$$

$$VV_{oc} \approx -\frac{\overline{SIGNAL(\tilde{b}_1^+)} \cdot \tilde{H}1}{II_{tr}}. \quad (6.105)$$

$$VV_{oc} \approx -\frac{\overline{SIGNAL(\tilde{b}_1^-) \cdot \tilde{H}1}}{II_{tr}}. \quad (6.106)$$

$$VV_{oc} \approx -\frac{\overline{SIGNALH1_1^-}}{II_{tr}}. \quad (6.107)$$

The $B1_1^+$ field is a fictitious component of the $\tilde{B}1$ field, induced by $\tilde{J}1$. This component would excite the magnetization of interest if $\tilde{B}1$ field was used for excitation. The definition of $H1_1^-$ field is provided as the circularly polarized component of the $\tilde{H}1_1$ field in the clockwise direction if the direction of propagation is $z$ or the circularly polarized component of the $\tilde{H}1_1$ field in the counterclockwise direction if the direction of propagation is $-z$. In other words, the sense of rotation of $H1_1^-$ field is opposite to that associated with the $B1_1^+$. A note in here is that the current source $M\tilde{C}2$ could have
been excited by \( \tilde{J} \), any other coil, or any combination of coils. Therefore, regardless of the spatial configurations used in this problem, the received signal in MRI is a function of the magnetization and of a the \( H_{1}^{-} \) field described above.

Note that the previous analysis is expected from antenna theory. The voltage induced in antenna \( A \) by another \( B \) is related to a vector representing the polarization of the field that \( B \) transmits and the conjugate of a polarization vector that represents a field perfectly received by \( A \). Interestingly enough, in Hoult’s treatment of the problem, a negatively rotating frame field was introduced [109]

\[
\tilde{b}_{1}^{-} = (\bar{x}b_{1x} - j\bar{y}b_{1y})^* 
\]

(6.108)

where \( * \) denotes the complex conjugate. The receiving field from his calculations was found to be equal to \( (\tilde{b}_{1}^{-})^* \) [109] which in this case would be the circularly polarized field with the opposite sense of rotation when compared to the \( B_{1}^{+} \) field. This resultant field is what was introduced in here as the \( (H_{1}^{-}) \) field.

**Analysis of the Electromagnetic Polarization of Fields Induced by RF Coils at Ultra High Field MRI**

Experiences with clinical imaging at ultra high frequencies (>300 MHz) have shown the complexity of the electromagnetic fields induced within RF coils. Recognizing the increased role of tissue/coil interactions, it became clear that the use of numerical evaluations of RF coils provides essential information for understanding the behavior of these devices. As such, the FDTD method was utilized to model a single-strut extremity coil. The coil, excitation source, and the load were modeled together as a single unit. This is critical when analyzing the electromagnetic fields induced within the coil since the interactions between the excitation source and the
sample greatly influence the field polarization and distribution. Therefore, the inclusion of the proper model and location of the source is a must especially at high frequencies. A cylinder phantom with a diameter and length equal to 4.6 cm and 9.4 cm was considered. The phantom was filled with 0.125 M NaCl.

Figure 6.4 displays the 3D FDTD solutions of the magnetic field intensities inside a single-strut coil [31] operating under linear excitation. The results are presented for an empty coil (6.4a-d 1 and 3) and a coil computationally loaded with the 0.125 M NaCl cylindrical phantom (6.4a-d 2 and 4) at 254 MHz (6 Tesla) (6.4a-d 1 and 2) and 485 MHz (11.4 Tesla) (6.4a-d 3 and 4). Figures 6.4a and 6.4b represent the magnetic field intensity ($H$) in $x$ and $y$ directions respectively, while 6.4c and 6.4d correspond to the circularly polarized component of the $H_1$ field in clockwise and counterclockwise directions, respectively. The results show that the $H_y$ field clearly dominates the transverse magnetic field for the empty coil at both frequencies (6.4b 1 and 3) and for loaded coil at 254 MHz (6.4b 2). As such the induced fields in the coil are linearly polarized. This can be verified from the clockwise and counterclockwise fields where they are approximately identical for these three cases (6.4c and 6.4d 1-3). The clockwise and counter-clockwise components of a field $H$ are given as follows:

$$\vec{H} = H_y\vec{y} + H_x\vec{x}$$  \hspace{1cm} (6.109)

$$H_{cw} = \frac{(H_y + jH_x)}{\sqrt{2}}$$  \hspace{1cm} (6.110)

$$H_{ccw} = \frac{(H_y - jH_x)}{\sqrt{2}}$$  \hspace{1cm} (6.111)

Eqs. (6.111,6.110) demonstrate that with a negligible $H_x$ in the coil, the clockwise and the counterclockwise fields are equal in magnitude. Therefore, a linearly polarized
field can be decomposed into clockwise and counterclockwise fields:

\[ \vec{H} = H_y \hat{y} \]

\[ \vec{H} = \frac{(H_y + jH_x)}{2} + \frac{(H_y - jH_x)}{2}. \]

(6.112)  

This is clearly shown from the results at 254 MHz for the empty (6.4a-d 1) and loaded (6.4a-d 2) coils and at 485 MHz for the empty coil (6.4a-d 3).

When the dimensions of the coil and/or the load become a significant fraction of the operating wavelength, the electromagnetic interactions between the coil, excitation source, and the sample dominate the fields within the coil. Thus, linearly polarized fields are no longer feasible even though the coil is excited in the linear mode. This is demonstrated in Figure (6.4a-d 4) where the coil is operating at 485 MHz and loaded with the cylindrical phantom. \( H_x \) (6.4a 4) in this case is not negligible when compared to the \( H_y \) (6.4b 4). Thus from Eq. (6.111), the distribution of the resulting clockwise (6.4c 4) field is different from that of the counterclockwise (6.4d 4) field.

To verify the previous analysis an experiment was conducted using the 8 Tesla system at The Ohio State University. A 16-strut capped TEM resonator loaded with an 18.5 cm sphere filled with 0.5 mM Gd DTPA to shorten \( T_1 \) and 0.125 M NaCl for appropriate loading of the coil was used. Sagittal, axial, and coronal gradient echo images of the 18.5 cm sphere (\( T_1 = 370 \) ms) were acquired using linear excitation and with \( TR/TE = 2000/6.3 \) ms at 18 flip angles varying between nominal values of 6° and 190° (\( FOV = 20 \) cm, 256x1256 matrix, 10 slices of 5 mm thickness). A nominal 90° flip angle was defined for a 1 cm voxel near the isocenter of the phantom using STEAM voxel spectroscopy.
Figure 6.4: Calculated magnetic field intensities inside a single-strut TEM coil: empty (a-d 1 and 3) and loaded with the cylindrical phantom (a-d 2 and 4) at 254 MHz (6 Tesla) (a-d 1 and 2) and at 485 MHz (11.4 Tesla) (a-d 3 and 4). a and b represent the magnetic field intensity in x and y directions respectively while c and d correspond to the clockwise and counterclockwise fields, respectively.
Figure 6.4 continued

(a1) (b1) (c1) (d1)
Figure 6.4 continued
Figure 6.4 continued
Magnitude images for the 18 flip angles were subsequently fit pixel by pixel with a sine function using the *chi-square* minimization algorithm, *CURVEFIT*, present in *IDL* (Research Systems, Inc.). To fit the non-negative magnitude images with a sinusoidal function, a two-pass fit was employed in which a preliminary fit of the 5 smallest flip angles was used to determine the crossing point of the fitting function between positive and negative values. An absolute value function was not utilized because its derivative is not continuous over the range of nominal flip angles.

In terms of the FDTD model, the RF coil (TEM resonator) and the phantom were modeled as a single system with the FDTD method. This approach permits the electromagnetic interactions between excitation source and the sphere to be rigorously included. These source/sphere interactions are easily observed from the results presented in this study. The three-dimensional FDTD model of the TEM resonator consists of 16 coaxial rods. 4.79 \( \text{mm} \) was used for the spatial step. The coil was numerically tuned by adjusting the gap between the TEM stubs until any of the modes of the TEM resonator is resonant at the desired frequency of operation. The electromagnetic properties of the phantom were assigned according to the Debye theorem [110]. Figure 6.5 displays low flip angle images, measured transmit field, and receive field obtained at 8 Tesla (top row) and their corresponding simulated results obtained at 340 MHz using the FDTD model and Eq. (6.107) (bottom row) of mode 0. An excellent agreement is obtained between the experiment and the simulated results in terms of the image, the \( H_1^- \), and the \( B_1^+ \) fields. Note the difference between the distributions of \( H_1^- \), and the \( B_1^+ \) fields. Compared to the experiment, note that the physical location of the excitation source in the simulations differs by a small shift in azimuthal direction.
Since low flip angle axial images could be simply approximated by $H_1^- \times B_1^+$ (Eq. 6.107), it is expected that the axial images will be symmetric around the source location (Figure 6.5 left). However as the flip angle increases, the fields can no longer be approximated by $H_1^- \times B_1^+$ and the semi-mirror symmetry of the $H_1^-$ and $B_1^+$ fields will be lost due to the presence of the sine function. Consequently, the images will become asymmetric around the source location even for linear excitation. This can be verified from Figure 6.6 left where a high flip angle image is obtained numerically and with experiment. The experimental image was acquired at a $151^\circ$ while the calculated image is presented at $158^\circ$. This small discrepancy could be due to the imperfection of the experimental settings.

In terms of the coronal and sagittal slices, there is a very interesting point in this case. With the arrangement of the location of the excitation port, $45^\circ$ from the axis, the mathematics show that in the center of the coil, the $B_1^+$ field distribution for the coronal slice is equivalent to $H_1^-$ field distribution of the sagittal slice and vice versa is also correct. Therefore, both sagittal and coronal low flip angle images are expected to be identical. This can be observed from Figure 6.5, the small deviation in the experimental results is due to the fact that there is a small shift in source location. At high flip angles, these images will no longer have the same distribution due to the presence of the sine function as it can be verified in Figure 6.6.

When imaging the head at ultra high field ($< 300$ MHz), the distribution of the $|H_1^-|$ and the $|B_1^+|$ are expected to be different. From this analysis and the analysis presented earlier regarding the received signal in MRI, it is significant to optimize both the transmit and the receiver fields to achieve homogeneous and high signal to noise image. Therefore, it is imperative to design the transmitter and receiver coils
accordingly. It is noted however that the $B_{1}^{+}$ field is what excites the spins and thus it is what determines the power required for a certain flip angle.

Figure 6.7 displays axial, sagittal, and coronal slices of gradient echo cadaver images at 8 Tesla (lower row) and the corresponding simulated images (upper row) obtained using the FDTD model. The results were obtained using 16-strut TEM coils, which is 21.2 cm long. The head was positioned in the coil such that the edge of the coil aligns with edge of the chin. The coil was operating under 2-port back of the head quadrature excitation. The input power to the coil was adjusted such that a constant value for the digitizer filler was obtained in a particular axial slice through the anterior commissars. This was achieved while the following parameters remained constant at all the experiments: $TR = 600 \text{ msec}$, $TE = 9 \text{ msec}$, $FOV = 20x20 \text{ cm}$, matrix size = 256x256, number of slices = 20, slice thickness = 5 mm, receiver bandwidth = 50 KHz, and excitation pulse = 2 msec Gaussian. In addition to the phantom results, these results demonstrate the validity of the signal derivation and the effectiveness of the FDTD model in predicting the MR signal.

The Operation of Quadrature Excitation Using a Quad-Hybrid at Ultra High Field

Two-port quadrature excitation is the most common driving system utilized in MRI. It is almost used exclusively in all systems with field strength up to 3 Tesla for volume imaging. Two-port quadrature excitation is obtained through the use of a reciprocal 4-port quad-hybrid. The device operation is described in Figure 6.8. If A has an input $= 2X$, then the signal would split into $1X$ at C and $-jX$ at D, and B is isolated. At reception, the signal received at C is $G1$ and at D is $G2$ such that the total signal $= -jG1 + G2$, is available at port B while A is isolated.
Figure 6.5: Low flip angle images, measured transmit field, and receive field obtained at 8 Tesla (top row) and their corresponding simulated results obtained at 340 MHz using the FDTD model (bottom row) of mode 0. The coil used was TEM resonator. The high signal speckle in the experimental images is due to failure of the fitting algorithm in low SNR regions due to either low flip angle or low receive sensitivity. The experimental data were obtained by Dr. Schmalbrock.
Figure 6.5 continued

(a) Axial slices: Image (left), $B_1^+$ field (center), $H_{1-}$ field (right)

continue
Figure 6.5 continued

(b) Sagittal slices: Image (left) $B_1^+$ field (center), $H_1^-$ field (right)
Figure 6.5 continued

(c) Coronal slices: Image (left), $B^+_1$ field (center), $H^-_1$ field (right)
Figure 6.6: High flip angle images at 8 Tesla (top row) and their corresponding simulated results obtained at 340 MHz using the FDTD model (bottom row) of mode 0. The coil used was TEM resonator. The images were obtained by Dr. Schmalbrock.

If the quad-hybrid is assumed to excite a coil shown in Figure 6.9, the transverse electromagnetic fields induced inside the coil are given by

\[
\begin{align*}
\vec{B}_C &= \vec{x}B_{C_x} + \vec{y}B_{C_y} \\
\vec{B}_D &= \vec{x}B_{D_x} + \vec{y}B_{D_y} \\
\vec{B}_{tot} &= \vec{B}_C - j\vec{B}_D
\end{align*}
\]  

(6.114) 

(6.115) 

(6.116)
**Figure 6.7**: Axial, sagittal, and coronal slices displaying gradient echo images of a cadaver at 8 Tesla and the corresponding simulated images obtained using the FDTD model. The results were obtained using 16-strut TEM coil, which is 21.2 cm long. The head was positioned in the coil such that the edge of the coil aligns with the edge of the chin. The coils were operating under 2-port back of the head quadrature excitation. The images were obtained by Dr. Chakeres.
Figure 6.7 continued
Figure 6.7 continued
where $\vec{B}_C$ and $\vec{B}_D$ are the fields provided by the sources C and D (no phase addition), respectively. Let us assume that the $B_1^+$ field is in the direction provided by $\frac{(x-jy)}{\sqrt{2}}$.

The component of the $B_{tot}$ that would contribute to the excitation of magnetization is given by

$$B_1^+ = \frac{(x-jy)^*}{\sqrt{2}} . B_{tot} \quad (6.117)$$

$$B_1^+ = \frac{(BC_x - jBD_x + jBC_y + BD_y)}{\sqrt{2}} \quad (6.118)$$

$$B_1^+ = BC^+ - jBD^+ \quad (6.119)$$

where $BC^+$ and $BD^+$ are field components of $BC$ and $BD$ that would contribute to the excitation of magnetization.
If the operation is at low frequency, i.e., the electrical sizes of the coil and the object to be imaged are small compared to the wavelength, the sources D and C will produce linearly polarized fields and

\[ \mathbf{B}_1^+ = \mathbf{B}_{\text{tot}}. \] (6.120)

The voltage induced at C and D is proportional to the \( \mathbf{H}_1^- \) field produced by the C and D ports (if they were used as transmitters)

\[
\begin{align*}
HCr^- &= \frac{(\bar{x} + j\bar{y})^*}{\sqrt{2}} \cdot \mathbf{H}C \\
&= \frac{HC_x - jHC_y}{\sqrt{2}} \quad (6.121) \\
HDr^- &= \frac{(\bar{x} + j\bar{y})^*}{\sqrt{2}} \cdot \mathbf{H}D \\
&= \frac{HD_x - jHD_y}{\sqrt{2}} \quad (6.122)
\end{align*}
\]

The total voltage received at B is proportional to

\[
\begin{align*}
-jHCr^- + HDr^- &= -\frac{j(HC_x - jHC_y) + HD_x - jHD_y}{\sqrt{2}} \\
&= -\frac{jHC_x - HC_y + HD_x - jHD_y}{\sqrt{2}} \quad (6.125) \\
&= \frac{(\bar{x} + j\bar{y})^*}{\sqrt{2}} \cdot (-j\mathbf{H}C + \mathbf{H}D). \quad (6.126)
\end{align*}
\]

At low frequency, if the ports D and C are switched, not only the coil would be ineffective in exciting the spins, it would also be ineffective in receiving the fields. If different coils are used for transmission and reception, it is imperative to set the phase shifts appropriately. The transmitter coil phase shifts are set to excite the \( B_1^+ \) field and the receiver coil phase shifts are set as if the coil were to excite \( H_1^- \) field.
The previous analysis is demonstrated in Figure 6.10 where an axial slice of a cylinder filled with 0.125 M NaCl is imaged using a high pass birdcage coil at 1.5 Tesla (64 MHz). Figure 6.10a corresponds to quadrature excitation with appropriate phase shifts while 6.10b corresponds to switching the excitation ports. The signal of image 6.10a was 5291 and the noise was 79 (in air). The results when reversing the ports are quite different where the signal of image 6.10b was 407 and the noise was 75 (in air).

At high frequency, circularly polarized fields are not induced using quadrature excitation, i.e., linear excitation does not produce linearly polarized fields. As such, the fields induced in this case are elliptically polarized. If quadrature excitation is applied, the $B_1^+$ field is what excites the spins. The power associated with the rest of the field components ($B_1^-, B_{1z}$, and ..) are basically lost even though a large portion of it is deposited in the body. Unlike that at low frequency, if the two ports were reversed
Figure 6.10: Gradient echo images obtained using experiments conducted by transmitting from A and receiving from B (Figure 6.8) at 1.5 Tesla. Ports C and D were utilized in two different configurations: one where they were connected to the coil in a manner similar to that used at low frequency (6.10a) and the other where the locations of C and D were reversed (6.10b). The images were obtained by Dr. Abduljalil.
in terms of connection to the quad hybrid and quadrature excitation was reapplied, a
significant fraction of the spins would be excited due to the presence of a $B_1^+$ field.
The same concept also applies for the receiver field, if we were to receive using the
same port that was used for excitation (port A in Figure 6.9), the fictitious $H_1^-$ field is
also present.

The previous analysis was demonstrated using the 8 Tesla system and a 16-strut
TEM resonator loaded with a cylinder: 15.0 cm in diameter and 21.2 cm in length.
Four experiments were conducted. Consider the schematics shown in Figure 6.8 and
2-port excitation. Figure 6.11 displays gradient echo images obtained using experi-
ments conducted by transmitting from A and receiving from B. Ports C and D were
utilized in two different configurations: one where they were connected to the coil in
a manner similar to that used at low frequency (6.11a) and the other where the loca-
tions of C and D were reversed (6.11b). Figure 6.12 displays gradient echo images
obtained using experiments conducted by transmitting from A and receiving from A
and utilizing the same configurations of the previous two experiments.

The difference in the signal to noise between these configurations is not as dra-
matic as that at 1.5 Tesla. In addition, some bright spots on an image correspond to
dark spots on the other. At these locations, it is implied that $B_1^+$ field was recovered
using one excitation when the other excitation was ineffective (Figure 6.11), or the
$H_1^-$ field was recovered using one reception when the other reception was ineffective
(Figures 6.12). This is not the general case however.

The previous experiments and analysis open avenues for techniques that could be
utilized to obtain homogeneous images at ultra high field. Beside the phased array
techniques provided in the following sections [35, 34], the recovering of dark spots
Figure 6.11: Gradient echo images obtained using experiments conducted by transmitting from A and receiving from B (Figure 6.8) at 8 Tesla. Ports C and D were utilized in two different configurations: one where they were connected to the coil in a manner similar to that used at low frequency (6.11a) and the other where the locations of C and D were reversed (6.11b). The images were obtained by Dr. Abduljalil.
Figure 6.12: Gradient echo images obtained using experiments conducted by transmitting from A and receiving from A (Figure 6.8) at 8 Tesla. Ports C and D were utilized in two different configurations: one where they were connected to the coil in a manner similar to that used at low frequency (6.12a) and the other where the locations of C and D were reversed (6.12b). The images were obtained by Dr. Abduljalil.
in an image could utilized using some of the techniques presented above. If it is concluded that a signal loss in an area of ultra high field image is due to RF inhomogeneity, one has to determine whether this is due to the lack of spin excitation (inhomogeneous $B_i^+$ field effects), or due to the received signal ($H_i^-$ field effects). The decision could be based on numerical simulations or experiments such as these provided above. For instance, consider a case where A is used for transmission and reception and one where A is used for transmission and B is used for reception. This experiment could be done simultaneously using two T/R switches connected to A and B. The $B_i^+$ field is constant in both cases, but the $H_i^-$ field is not. In a specific configuration some of the signal suppressed due to low values of $H_i^-$ could be recovered using the other configuration.
6.2 Dielectric Resonances

With the advent of ultra high field magnetic resonance imaging [4], it became clear that imaging could be performed at these field strengths (≥ 7 Tesla) and that RF power [111, 112], RF penetration [113] and dielectric resonance [26] considerations would not be overwhelming in this frequency range (≥ 300 MHz) for head imaging. The ability to obtain MR images at ultra high field has now been confirmed using a 7 Tesla whole body system [105]. Nonetheless, while limitations in RF penetration appear to be non-existent for practical purposes [113], there remains considerable controversy surrounding RF power and the importance of dielectric resonance phenomena in ultra high field magnetic resonance imaging [114, 115, 116, 117].

When the first 4 Tesla high field systems were assembled by Phillips [18, 70], GE [118, 119], and Siemens [69, 120, 23], there was concern that dielectric resonances acted to distort image homogeneity at these field strengths. Indeed, dielectric resonances were largely held responsible as the source of inhomogeneity at 4 Tesla [18, 23, 24]. At the same time, difficulties in building suitable RF coils were recognized by these groups. Yet, the RF coils, the RF coil/sample interactions, and the manner in which the coils were driven, were not invoked as significant causes of inhomogeneity. With attention turned away from RF coils, the importance of so-called "dielectric resonance" phenomena in the human head gained increased prominence.

At the same time, it is well known that the solution of any electromagnetic boundary value problem can be expressed as a superposition of modes, where such modes include the effects of the body, the coil and the source [36]. The presence of "dielectric resonances" implies the excitation of a field distribution (possibly a single dominant mode) that has a high and insurmountable inhomogeneity.
In MRI, the presence of dielectric resonances implies a field distribution that is a function of the shape and dielectric properties of the body only. Dielectric resonances are independent of the RF coil and the location of the drive points. To be of importance in clinical MRI, they should be associated with significant fundamental inhomogeneities characterized by strong maxima or minima. Conversely, many inhomogeneities in MRI are not of fundamental nature. These include the nature of the RF coil, the exact driving configuration and the RF coil/sample interactions. Unlike dielectric resonances, the latter sources of inhomogeneity can be experimentally controlled.

In this section, the issue of dielectric resonances in magnetic resonance imaging is addressed using a combination of ultra high field images and FDTD \[3\] calculations. It is clear that with increasing operational frequency, the RF wavelength in tissue becomes smaller than the dimensions of large anatomical structures. Magnetic and electric fields also become more tightly coupled. These fields, given appropriate electrical properties (high permittivity and low conductivity), interact strongly with the global structure of the sample through wave propagation. This interaction, distinct from the spin-$B_1$ interaction, can give rise to dielectric resonances within the sample. The RF wavelength inside the human head at 8 Tesla is on the order of 12 cm, resulting in a half wavelength of only 6 cm. Such a short wavelength provides ample opportunity for generating local maxima and minima. Thus, prior to the acquisition of the first ultra high field images \[8\], it was thought that dielectric resonances might well produce highly inhomogeneous images at ultra high fields (with both high intensity and signal void regions), possibly rendering the resulting images unusable. However, now that successful human imaging has been performed at 8 Tesla \[8, 106\],
the relative importance of dielectric resonances in ultra high field magnetic resonance imaging has now been brought into question [117, 121, 122, 26]. Indeed, there is a need to differentiate between dielectric resonances and RF coil/sample interactions.

In here, the effects of dielectric resonances in ultra high field MRI are examined in three steps. First, the electromagnetic fields produced by a plane wave incident on spherical phantoms of pure water or 0.125 \( M \) NaCl are examined. Second, the electromagnetic fields produced by the TEM resonator [15, 16, 99, 14] are analyzed within the same spherical phantoms using the FDTD method. These results in turn, are compared to 8 Tesla images. Finally, the electromagnetic interactions between the TEM resonator and an anatomically detailed human head model are presented. The differences between dielectric resonances and inhomogeneity associated with the RF coil are also discussed.

6.2.1 Materials and Methods

Images were acquired on an 8.0 Tesla, 80 cm superconducting magnet (Magnex Scientific, Abingdon, UK). The scanner was equipped with a BRUKER AVANCE console (Bruker, Billerica, MA, USA). The system also includes an actively shielded asymmetric head gradient set [123]. The phantoms used in these studies were made of plastic spheres of 18.5 cm diameter filled either with distilled water or with 0.125 \( M \) NaCl. The phantoms were imaged using axial gradient echo (GRE) multi-slice acquisitions using the following parameters; \( TR = 1000 \text{ msec} \), \( TE = 8 \text{ msec} \), \( FOV = 20 \text{ cm} \), \( Matrix = 256x256 \), number of slices = 30, slice thickness = 5 mm, receiver bandwidth = 50 KHz, excitation pulse = 4 msec Gaussian.
All human studies were conducted under an investigational device exemption (IDE) granted by the Food and Drug Administration (FDA). Studies were also monitored by the IRB committee of The Ohio State University. Prior to image acquisition, the subject was asked to lie in a supine position on a movable cantilevered patient table. A 16-strut TEM resonator was then positioned over the subject’s head such that the face of the RF coil was aligned with the chin of the subject (full insertion). The RF coil was then tuned to 340 MHz as monitored on each of two drive points connected in quadrature. The patient was then advanced to the scan position while remaining on the table and without removal of the RF coil. The proton resonant position was then determined. After placing the transmitter on resonance, the 90° pulse power was determined by nulling the signal through the use of a 180° pulse. Human RARE images were then acquired using the following parameters: TR = 5,000 msec, TE = 18.6 msec, FOV = 20 cm, matrix = 512x512, number of slices = 12, echo train = 8, receiver bandwidth = 75 KHz, excitation pulse = 3 msec Gaussian. The power level for the exciting RF was well below SAR guidelines.

6.2.2 Results and Discussion

Many of the essential characteristics of dielectric resonances in MR systems can be illustrated using a spherical phantom in the presence of a circularly polarized plane wave. Over the working region of an empty TEM resonator operating in quadrature, the fields approximate a circularly polarized standing plane wave with underlying propagation in the axial direction [99].
The electromagnetic interactions between a homogeneous sphere and a linearly polarized plane wave can be described exactly [124, 121] if the permittivity and conductivity of the homogeneous sphere are known. Using the Debye model [110], the complex permittivity of spherical phantoms of pure water and 0.125 $M \text{NaCl}$ were calculated. At 340 MHz (8 Tesla), the relative complex permittivity ($\varepsilon_r$) is $80-i1.5$ for pure water and $78-i61$ for 0.125 $M \text{NaCl}$. The complex permittivity is given by:

$$
\varepsilon = \varepsilon_0 \left( \varepsilon' - i \frac{\sigma}{\omega \varepsilon_0} \right)
$$

(6.128)

where $\varepsilon_0$ is the free space permittivity ($8.854 \times 10^{-12} \ \text{F/m}$), $\varepsilon'$ is the dielectric constant, $\sigma$ ($\text{S/m}$) is the electric conductivity, and $\omega$ ($\text{rad}$) is the operational frequency. Using Eq. (6.128) and the values of the relative complex permittivity given above for the spherical phantoms, the dielectric constant and the conductivity for pure water and 0.125 $M \text{NaCl}$ are given by (80, 0.029 $\text{S/m}$) and (78, 1.154 $\text{S/m}$), respectively.

The analytical calculations for a plane wave excitation (the coil is not present) for an 18.5 cm spherical water phantom is displayed in Figure 6.13a where the simulated image intensity is presented. Note that when a plane wave excitation is utilized, it is impossible to account for the coupling with the RF source. In Figure 6.13b a low flip angle gradient echo measurement of distilled water (dielectric constant 80, conductivity 0.029 $\text{S/m}$) in an 18.5 cm spherical phantom is presented. The study was performed using a 16-strut TEM resonator with 2-port quadrature excitation. The distribution of the image intensity shows a clear dielectric resonance.

A general agreement between the two images (Figure 6.13a and Figure 6.13b) is apparent. Both images have a central bright region, but, there are notable differences in the regions far from the center of the sphere. A concentric null is apparent surrounding the central brightness in both images, and a faint outer ring is present in
both images. However, in Figure 6.13b, this faint outer ring is not perfectly uniform as in Figure 6.13a. This reflects the influence of the drive points in the MRI image. The general agreement between the two images (Figures 6.13a and 6.13b) is observed even though no RF coil was utilized in the analytical calculations, but detailed evaluation nonetheless reveals the slight influence of the RF coil/sample interaction.

The FDTD calculations of low flip angle images for the same spherical phantom are shown in Figure 6.13c. A 16-strut TEM resonator with the same geometry and dimensions as used in Figure 6.13b was modeled for these calculations. The simulations were done using 2-port quadrature excitation. The distribution in Figure 6.13c is near to that obtained in Figures 6.13a and 6.13b, but, the fields on the outer regions of the sphere more closely approximate the MRI image shown in Figure 6.13b reflecting the RF coil/sample interaction.

Based on these results, one can conclude that the electromagnetic fields are dominated by the sample (distilled water spherical phantom) and the presence of the coil has a minor effect on the image of the spherical phantom. As such, a dielectric resonance is clearly observed in this study.

In Figure 6.13e, an analytical result is presented for an 18.5 cm spherical 0.125 M\(NaCl\) phantom. In this analytical treatment (Figure 6.13e) the presence of a dielectric resonance is still evident, but it is reduced in intensity by more than an order of magnitude due to wave attenuation in the sample. This situation leads to somewhat more uniform images. Also, excellent circular symmetry still remains apparent in this result. In Figure 6.13f, a gradient echo image is presented for a 0.125 M\(NaCl\) (dielectric constant 78, conductivity 1.154 S/m) 18.5 cm spherical phantom loaded in a 16-strut TEM resonator with quadrature excitation at 8 Tesla. Note that the azimuthal
symmetry is not apparent in the experimental image. This is once again a result of RF coil/sample interactions. Note, in the 0.125 \text{M} \text{NaCl} case, that the experimental result does not agree with the analytical treatment. This is because the experimental result, unlike the analytical treatment, is also dominated by the RF coil/sample interaction and not solely by the presence of the dielectric resonance.

FDTD calculated low flip angle image in the 0.125 \text{M} \text{NaCl} spherical phantom is shown in Figure 6.13g. Note that in this figure, circular symmetry is distorted. The experimental finding (Figure 6.13f) is nearly identical to the FDTD result (Figure 6.13g). Note that the locations of the excitation sources in Figure 6.13f and Figure 6.13g differ by a small shift in azimuthal direction. The relative difference between these two images are likely due to experimental factors. This includes difficulty in tuning the loaded coil and the use of imperfect matching networks. In addition, it is also difficult to position the phantom exactly in the middle of the coil. It is also possible that a slight shift in the resonance frequency of mode 1 occurs when inserting the coil in the magnet. Conversely, the FDTD simulation involves perfectly symmetric sample/RF coil positioning and exact tuning situations, which could never be achieved experimentally. Nonetheless, it remains clear that the results in the 0.125 \text{M} \text{NaCl} phantom (Figures 6.13f and 6.13g) are dominated by the coupling between the (RF coil)/(excitation source) and the sample. Unlike the distilled water phantom, this coupling dictates the distribution of the excite and receive fields in the 0.125 \text{M} \text{NaCl} phantom much more than the presence of dielectric resonances.

To examine the effect of 4-port excitation on the homogeneity of image in general, and on the dielectric resonance phenomena in particular, the electromagnetic fields within the pure water and 0.125 \text{M} \text{NaCl} spherical phantoms were also calculated for
**Figure 6.13:** Computational (a,c,d,e,g,h) and experimental (b,f) studies of an 18.5 cm spherical phantom filled with either distilled water (a-d) or 0.125 M$\text{NaCl}$ (e-h) at 340 $MHz$. GRE images (b,f) were obtained at low flip angles using an 8 Tesla scanner. The spherical phantoms were loaded in a 16-strut TEM resonator, which was operating under quadrature excitation. Images a (pure water) and e (0.125M NaCl) correspond to the electromagnetic simulated intensity images using a plane wave excitation (the coil is not present) for a low flip-angle pulse exciting the 18.5 cm diameter spherical phantoms. Images c (pure water) and g (0.125 M$\text{NaCl}$) correspond to low flip angle simulated images obtained using the FDTD model for the same coil size, geometry, and excitation (2-port quadrature) as was used for the MRI images. Images d (pure water) and h (0.125 M$\text{NaCl}$) correspond to low flip angle simulated images obtained using the FDTD model for the same coil size and geometry and using 4-port quadrature excitation. The experimental images were provided by Dr. Robitaille, and the analytical calculations (e,f) were provided by Dr. Baertlein.
Figure 6.13 continued

(a)  

(b)  

(c)  

(d)  

continue
this case. The mathematical coil model used in these simulations was a 16-strut TEM resonator operating under 4-port quadrature excitation. The 4-port excitation was implemented in the FDTD model by applying a progressive $\pi/2$ phase shift on each drive point. Figures 6.13d and 6.13h display the FDTD calculated low flip angle image distribution inside pure water (6.13d) and inside 0.125 $M$ NaCl (6.13h) spherical phantoms using 4-port excitation.

As expected, the simulated image distribution in the pure water case (Figure 6.13d) is almost identical to the distribution obtained using 2-port quadrature excitation (Figure 6.13c). This is because the total transverse magnetic ($B_1$) field is dominated by the presence of dielectric resonances. Increasing the number of excitation ports would not significantly alter the $B_1$ field distribution. This is demonstrated in Figures 6.14a and 6.14b where the total transverse magnetic ($B_1$) field distribution is shown to be almost identical for a 16-strut TEM resonator using 2-port quadrature excitation (6.14a) and 4-port quadrature excitation (6.14b). In this case (pure water), the presence of the dielectric resonance is fundamental.

Unlike the case with the pure water phantom, the uniformity of the image in the 0.125 $M$ NaCl phantom significantly improves from 2-port quadrature excitation (Figure 6.13g) to 4-port quadrature excitation (Figure 6.13h). This confirms the findings that for a symmetric homogeneous load, the 4-port quadrature excitation reduces the load-coil interactions leading to more uniform circularly polarized components (transmit and receive) of $B_1$ field compared to a conventional 2-port quadrature excitation [35]. Figures 6.14c and 6.14d show that distribution of the total transverse magnetic ($B_1$) field has changed from 2-port quadrature excitation (6.14c) to 4-port quadrature excitation (6.14d). This result is different than what was obtained with the
pure water phantom (Figures 6.14a and 6.14b). Unlike the pure water case, increasing the number of excitation ports alters the $B_1$ field distribution, and the presence of a dielectric resonance is no longer the dominant factor that determines the $B_1$ field distribution.

Figure 6.15 gives axial slices of the $B_1$ field distribution inside an anatomically detailed human head model at 340 MHz. In Figure 6.15a, the $B_1$ field distribution is shown for a four point excitation without an RF coil. Figures 6.15b, 6.15c, and 6.15d correspond to the $B_1$ field distributions within the human head for excitation with a TEM resonator. In Figures 6.15b and 6.15c, 2-port quadrature (6.15b) and 4-port quadrature (6.15c) excitations with an 8 strut TEM resonator are utilized. Finally, in Figure 6.15d, the $B_1$ field profile is shown for a 24-strut TEM resonator using a 4 port quadrature excitation.

From the FDTD data, in which interactions between the source and the tissue are considered, it is apparent that the homogeneity of the fields improves significantly from no coil (Figure 6.15a), to 2-port quadrature 8-strut (Figure 6.15b), to 4-port quadrature 8-strut (Figure 6.15c) and finally to the 4-port quadrature 24-strut (Figure 6.15d) cases. Note that while the homogeneity in Figure 6.15d ranges from 0.6-1, there remains a significant improvement in homogeneity relative to the other cases. If dielectric resonances were a dominant factor, then whether the coil is present or not, or the changing geometry of the coil, or the way the excitation is performed would have little effect on the field distribution. But as demonstrated, the $B_1$ field distribution differed significantly in all four cases. This is computational evidence that dielectric resonances are not controlling the field distribution pattern in the human head at 340
Figure 6.14: Calculations of the $B_1$ field (linear gray scale) inside spherical phantom of pure water (a,b) and 0.125 $MNcCl$ (c,d) with diameter 18.5 cm using the FDTD model. The calculations were performed for spherical phantoms loaded in a 16-strut TEM resonator which was operating under 2-port (a,c) and 4-port (b,d) quadrature excitations.
Figure 6.15: Axial slices of the $B_1$ field distribution inside an anatomically detailed human head model. Results were obtained without an RF coil (a) and for 2-port quadrature excitation (b) and 4-port excitation (c) using an 8-strut TEM resonator and for 4-port quadrature excitation using a 24-strut TEM resonator (d) at 340 MHz.
$MHz$. Rather, it is the RF coil/head interactions, which play a dominant role in setting the homogeneity within the human head at 8 Tesla.

It is clear that gradient echo images are less sensitive to $B_1$ inhomogeneity than spin echo based images. As such, a series of human RARE images obtained with a 16-strut TEM resonator are displayed in Figure 6.16. Unlike previous 8 Tesla RARE images obtained with $120^\circ$ refocusing pulses [111], this series was acquired with $180^\circ$ pulses. Thus, they should be more sensitive to $B_1$ inhomogeneity than gradient echo images. While not perfectly uniform, these images nonetheless display acceptable homogeneity, with the RF field gradually falling in intensity from the lower left of each slice towards the front of the head. This inhomogeneity however is associated with the drive points of the RF coils as demonstrated in this work. It appears from such images, that image inhomogeneity is significantly better than would be predicted on the basis of dielectric resonance arguments only at 8 Tesla.
Figure 6.16: Series of 4 RARE images obtained at 8 Tesla using a 16-strut TEM resonator driven in quadrature. The images were provided by Dr. Robitaille.
6.3 Design of Birdcage RF Head Coils Using Multi-Port Excitations

At 1.5 Tesla, the birdcage resonator, with quadrature (2 port) excitation, is the preeminent RF coil design. At 64 MHz, circuit analysis and transmission line theory demonstrate that quadrature excitation generates a circularly polarized field within the birdcage coil. This has been confirmed within the MRI context. Thus, the birdcage resonator is known to generate a uniform $B_1^+$ field distribution in the human head while at the same time producing low SAR values. However, as the frequency increases, the homogeneity of the $B_1^+$ field deteriorates progressively. In order to recover the field homogeneity, the field distribution within the resonator must be modified. This can be accomplished by increasing the number of excitation ports feeding the coil. By properly exciting each port, the field distribution within the RF coil can be significantly altered in a manner similar to an antenna array concept. Nonetheless, the coil is operating in the near field rather than the far field. The use of arrays in the near field is not new. The array concept has been used for cancer treatment in microwave hyperthermia systems [66, 64]. In this case, the goal is to obtain a field distribution where the maximum occurs at the tumor and the minimum occurs at the vital organs which are susceptible to damage from heating. By contrast, in MRI a uniform $B_1^+$ field distribution is desired while the SAR is minimized.

In this section, the birdcage coil with four-port excitation is considered. Each port is driven with the same current amplitude. Two phase conditions are analyzed, the simple fixed phase and the variable phase (i.e. that which is optimized for a human head). Because of the complexity of this problem, such a coil design cannot
be achieved without the use of full-wave electromagnetic analysis approaches. Consequently, the FDTD method is utilized. The four-port driving system is presented along with results from the simulation for the one-port and two-port cases.

### 6.3.1 Performance of Four-Port Coil

The advantages of 4-port driving system with fixed phase condition have been presented by Bridges [125] at 85 MHz. The concept of a four-port drive is similar to the conventional quadrature excitation. The input signal is split into 4 quarters by using a hybrid. A π/2 phase shift is applied on each quarter of the signal as shown in Figure 6.17. The signal output of the hybrid is then fed to 4 points on the upper circular ring of the birdcage coil. Each of these drive points is located one quarter of the circular ring away from the neighboring drive point (Figure 6.17).

![Figure 6.17: Schematic representation of the four-port drive system for a high pass birdcage coil](image)

Figure 6.17: Schematic representation of the four-port drive system for a high pass birdcage coil
Results are shown for the coil loaded with a cylindrical phantom (22 cm long with 19.6 cm diameter), an octagonal phantom (40 cm long with a 20.7 cm diameter), and an anatomically detailed human head model. These two phantoms have the electrical properties of muscle tissue: dielectric constant of 84.7 and 56.6 and the conductivity is 0.76 S/m and 1.0 S/m at 64 MHz and 200 MHz, respectively [98].

In addition to a fixed drive relationship, the phased array concept can be utilized to excite the four-drive points on the birdcage coil with independent sources. In this case however, the experimentalist controls the amplitude and the phase of the RF excitation at each drive port. In this section, only the phase of the RF driving each element is adjusted to produce an optimum homogeneous circularly polarized $B_1$ field pattern. This process is done while both the RF coil and the human head are modeled as a single system. In essence, the coil with the load is tuned to the resonant frequency of operation, then the phased array concept is utilized. Using a numerical optimization process, the phase-optimized excitation is applied to the human head model within the coil. These results are then compared to the linear, conventional quadrature, and four-port excitations using the FDTD method.

### 6.3.2 The Muscle Phantoms

**Magnetic Field Homogeneity**

The homogeneity of the $B_1^+$ field is of central importance in the design of RF coils. If the RF coil does not provide a spatially uniform excitation (inhomogeneous $B_1^+$ field), some of the nuclei are either not excited, or are excited with different flip angles. This can lead to poor image contrast and SNR. In order to study magnetic field homogeneity in this work, linear, quadrature, and four-port drives are considered.
Figure 6.18: Frequency response of the birdcage coil for different loadings at 200 MHz for different coil loadings. Once the resonant frequency has been determined, the discrete Fourier transform is applied to the field solution within the coil at the resonant frequency. The birdcage coil considered for this study was structured of infinitesimal conductors. Numerically, it has been observed that 48.9 \( \text{pF} \) is used to obtain the mode of interest resonance frequency at 64 MHz for the 16 strut high pass birdcage coil. Figure 6.18 shows the capacitor values used to obtain the resonance frequency at around 200 MHz for different coil loadings. For linear excitation, the voltage source is applied at the Ex excitation location shown in Figure 4.7.
Figures 6.19a, 6.19c display the $B_1^+$ field inside the cylindrical phantom for axial slices at 64 $MHz$ for linear and quadrature excitations, respectively. Comparing the results with the empty coil (not shown), the $B_1^+$ field distribution remains homogeneous. For linear excitation (Figure 6.19a), the $B_1^+$ field is 60% homogeneous (the difference between the maximum and the minimum values of the $B_1^+$ field in this slice is 40% of the maximum value). A better overall homogeneity is obtained from linear to quadrature excitation (90%) in the axial slice (Figure 6.19c). Note that no improvement is observed in the sagittal and the coronal slices (not shown). Theoretically, this is valid because the quadrature drive provides a circularly polarized field in the transverse plane.

Figures 6.19b and 6.19d show the $B_1^+$ field at 200 $MHz$. Compared to the unloaded case at 200 $MHz$, the presence of the muscle phantom dramatically changes the field distribution ($< 19\%$ homogeneity). Unlike the 64 $MHz$ case, there is not much improvement in the homogeneity of the fields from linear to quadrature ($< 29\%$ homogeneity) drive. This is due to the fact that the ideal current distribution predicted from circuit analysis is not valid at these high frequencies. Hence, the conventional quadrature drive is not effective.

In addition, studies were performed for an octagonal phantom that fills the length and width of the RF coil. For this phantom, the $B_1^+$ field at 64 $MHz$ is shown in Figures 6.20a and 6.20c. The axial slices show that the $B_1^+$ field is about 39% homogeneous for the linear excitation. However, the homogeneity only improves to about 66% for the quadrature drive. The reduced homogeneity compared to the cylindrical phantom is due to several factors. First, the octagonal phantom is very large (in fact larger than any typical human head). As such, it is very close to the struts of the coil,
Figure 6.19: Series of axial slices displaying the calculated $B^*_r$ field of the high pass birdcage coil inside the cylindrical phantom at 64 $MHz$ and 200 $MHz$. The cylindrical phantom results correspond to linear excitation (a, b), quadrature excitation (c, d), and four-port excitation (e, f) at 64 $MHz$ and 200 $MHz$, respectively.
Figure 6.19 continued

(a)  

(b)  

(c)  

(d)  

continue
Figure 6.19 continued
and the strong coupling that results significantly disrupts the field distribution in the coil. Furthermore, unlike the cylindrical phantom, Figure 6.20c shows that quadrature excitation produces asymmetry in the $\phi$ direction because the phantom does not have circular symmetry. This in turn results in greater asymmetries in the induced currents. One can deduce from this discussion that at low frequency, within the birdcage coil, circularly polarized magnetic fields are only induced when the imaged object has a $\phi$ symmetry and is relatively small compared to the coil size.

Figures 6.20b and 6.20d display the $B_1^+$ field for the 16 strut birdcage coil loaded with the octagonal phantom at 200 MHz. Compared to the empty coil and the birdcage coil loaded with the cylindrical phantom, the $B_1^+$ field is very inhomogeneous ($< 16\%$ homogeneity in the axial slice). Similar to the cylindrical phantom, the quadrature drive produces a null in the field in the axial slice.

Figure 6.19f shows an axial slice of the $B_1^+$ field inside the cylindrical phantom for a coil driven in four ports at 200 MHz. One can observe that the homogeneity of the $B_1^+$ field has improved from 29% using the conventional quadrature excitation to 55% with a better overall homogeneity using the four-port drive. As expected, there is no improvement in the homogeneity of the $B_1^+$ field in the sagittal and coronal slices with the four-port drive over the conventional quadrature excitation. The four-port results for 64 MHz are displayed in Figure 6.19e for completeness.

The same axial slices are shown in Figures 6.20e and 6.20f for the octagonal phantom case at 64 and 200 MHz, respectively. At 64 MHz, the homogeneity of the $B_1^+$ field has improved from 66% using the conventional quadrature excitation to > 90% using the four-port drive. At 200 MHz, the improvement in the homogeneity is from 16% to 39%. The significant improvement in the $B_1^+$ field homogeneity with
Figure 6.20: Series of axial slices displaying the $B_1^+$ field of the high pass birdcage coil inside the octagonal phantom at 64 $MHz$ and 200 $MHz$. The cylindrical phantom results correspond to linear excitation (a, b), quadrature excitation (c, d), and four-port excitation (e, f) at 64 $MHz$ and 200 $MHz$, respectively.
Figure 6.20 continued

(a)  
(b)  

(c)  
(d)  

continue
Figure 6.20 continued
four-port drive is due to the fact that the currents on the coil legs are more uniform than the case where the coil is driven in quadrature. The four-port drive reduces the effects of the head-coil interactions leading to more uniform currents on the coil legs and consequently better $B_1^+$ field homogeneity.

Figures 6.21a and 6.21b show axial slices of the $B_1$ field contour plots at 64 MHz for a coil loaded with the octagonal phantom and driven using quadrature and four-port excitations, respectively. It is apparent that a four-port drive produces a more uniform circularly polarized $B_1$ field over the conventional quadrature excitation. Unlike the two-port quadrature drive, the $B_1$ field is symmetrical around an axis with $\phi$ of $45^\circ$ and $135^\circ$.

**Specific Absorption Rate**

Initial SAR calculations have been performed in cylindrical and octagonal muscle phantoms. This analysis is important since simple circular or octagonal structures, with homogeneous electrical properties, can provide insight on many issues including the interactions between the source and the imaged object. The latter in turn, can be used in the evaluation of the SAR inside the human head. The following calculations are presented for 1 watt CW absorption.

Figures 6.22a and 6.22b show axial slices of the SAR values inside the cylindrical phantom at 64 and 200 MHz for linear excitation and Figures 6.22c and 6.22d show the same results for quadrature excitation. For this particular axial slice with linear excitation, Figures 6.22a and 6.22b show that at 200 MHz (peak SAR<0.5 W/Kg), the SAR peak values have decreased by 25% compared with the peak values at 64 MHz (peak SAR<0.65 W/Kg). However for the quadrature excitation case, the situation
Figure 6.21: Series of axial slices for a high pass birdcage coil loaded with an octagonal phantom displaying the $B_1$ field contour plots at 64 MHz using quadrature (a) and four-port (b) excitations.
differs. The SAR peak values at 200 MHz (peak SAR < 0.58 W/Kg) have increased by 63% compared with that at 64 MHz (peak SAR < 0.37 W/Kg) as shown in Figures 6.22c and 6.22d, respectively.

The two-port quadrature excitation for the unloaded coil (or for the birdcage coil loaded with the cylindrical phantom) produces $B_1$ fields which are approximately circularly polarized at low frequency. For the cylindrical phantom, the simulations show that at 64 MHz the polarization of the tangential electric fields is almost circular when the excitation is done in quadrature. Because the SAR is an indication of the electric field for the homogeneous medium, Figure 6.22c shows that the SAR distribution is symmetric around the $\phi$ direction. Experiments have shown that quadrature excitation reduces the transmitting power and consequently the absorbed power by approximately a factor of 2 at 64 MHz (the ratio of the absorbed and radiated power is independent of the source at the same frequency). As the absorbed power (RF power deposition into the phantom) is an indication of the SAR, Figures 6.22a and 6.22c show that by applying quadrature excitation, the SAR values have decreased by approximately a factor of 2 compared to the case when the coil is linearly excited.

Because linear excitation does not give a sinusoidal current distribution on the coil elements at 200 MHz [83, 82], it is impossible to obtain circularly polarized fields using the conventional quadrature drive. Thus, the fields induced by each of the two sources, located at the Ex and Ey locations (Figure 4.7), can add up either constructively or destructively, depending on the geometrical shape and material properties of the head or phantom. As a result, it is expected that the SAR values will increase when switching from linear to the conventional quadrature excitation (unlike that at 64 MHz). This is demonstrated in Figures 6.22b and 6.22d.
Figures 6.22e and 6.22f show the SAR distribution for four-port excitation at 64 MHz and 200 MHz, respectively. Unlike the quadrature drive, and similar to the linear excitation case, the SAR peak values at 200 MHz are less than that at 64 MHz. Because the interaction between the cylindrical phantom and the coil is very minimal at 64 MHz, Figure 6.22e shows that when the excitation is done in four ports, the SAR peak values have furthered decreased compared to that with the conventional two-port quadrature drive (Figure 6.22c). At 200 MHz, the four-port drive reduces the effects of the head-coil interactions leading to more uniform currents on the coil legs. Therefore at 200 MHz, while the conventional quadrature excitation has lead to higher SAR peak values compared to that with linear excitation, driving the coil in four ports provides SAR distribution with peak values which are significantly less than those with linear or quadrature excitations.

6.3.3 The human Head Model

Magnetic Field Homogeneity

The magnetic field homogeneity is when imaging the human head can only be analyzed when the head and the RF coil are considered as a inseparable unit. In other words, they must be modeled together. Unfortunately, previous studies neglected this important fact [25, 78, 79]. It is clear that the presence of the head can significantly change the magnetic field homogeneity due to three major aspects. First, the RF wavelength in the head is approximately 8 times smaller than for the empty coil. Second, the head can induce additional non-uniform currents on the coil elements. Finally, eddy currents generated inside the head can produce additional magnetic fields within the head.
Figure 6.22: Series of axial slices displaying the SAR values for a high pass birdcage coil loaded with a cylindrical muscle phantom at 64 MHz and 200 MHz. The cylindrical phantom results correspond to linear excitation (a, b), quadrature excitation (c, d), and four-port excitation (e, f) at 64 MHz and 200 MHz, respectively. The color scale corresponds to SAR values in $W/Kg$ for a 1 watt CW absorption.
Figure 6.22 continued

(a) (b)

(c) (d)

continue
Figure 6.22 continued
A study of the $B_1^+$ field homogeneity was done on a coil loaded with a male human head and shoulders model which was purchased [92].

Figure 6.23a shows an axial slice of the $B_1^+$ field in the human head at 64 MHz for quadrature excitation. Within the brain region, where the axial slice is taken, the homogeneity of the $B_1^+$ field is around 90%. We observe a better overall homogeneity with quadrature drive compared to the linear drive.

Examples of non-uniform $B_1$ fields have been observed in experiments with 4 T magnets (170 MHz) [120, 24]. To examine the homogeneity at high frequency, the capacitor values are set to 3.4 pF so that the resonance frequency rises approximately to 200 MHz (Figure 6.18). An axial slice of the $B_1^+$ field is shown at 200 MHz in Figure 6.23b for quadrature excitation. The field homogeneity has dropped to 37%, making it much less homogeneous than the 64 MHz case.

To improve the homogeneity of the $B_1$ fields at 200 MHz, we applied the four-port drive (a fixed increment of $\pi/2$ phase shift on each of the four sources) and the phase-optimized four-port excitation. With the phase-optimized excitation, the coil loaded with the human head is first tuned. Then, a developed numerical optimization process that takes into account the dimensions, properties, spatial contents, and position of the head is applied. This process is based on the fact that with asymmetrical, inhomogeneous, and irregular shape loading (human head), 0, $\pi/2$, $\pi$, and $3\pi/2$ are not the necessary phase shifts needed on the excitation sources in order to obtain a homogeneous circularly polarized $B_1$ field. For instance, the phase shifts used to produce the phase-optimized four-port excitation results of this paper were 0, 0.714$\pi$, $\pi$, 1.52$\pi$. 

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Figure 6.23: Axial slices displaying the $B_1^+$ field inside the human head model. Figure a corresponds to the $B_1^+$ field at 64 MHz using quadrature excitation. Figures b, c, and d correspond to the $B_1^+$ field at 200 MHz for quadrature (b), four-port (c), and phase optimized four-port excitation (d), respectively.
Figures 6.23c and 6.23d show axial slices of the $B_1^+$ field distribution at 200 MHz using four-port drive (52% homogeneity) and the phase-optimized four-port excitation (63% homogeneity), respectively. It is apparent that the overall $B_1^+$ field homogeneity has improved from linear/quadrature excitation to the four-port drive.

**Specific Absorption Rate**

The following calculations are presented for a 1 watt CW absorption for an axial slice through the eyes and the nose inside the human head model. Figure 6.24a shows the SAR values in the aforementioned axial slice for quadrature excitation at 64 MHz. Compared to the linear excitation case, it is observed that SAR peak values have dropped by approximately 30% when the excitation is done in quadrature. Note that this value is less than that with the cylindrical phantom. This is because the human head model is heterogeneous and asymmetric; hence, the fields inside it are less circularly polarized than with the symmetric and homogeneous cylindrical phantom. Figure 6.24b shows the same slice at 200 MHz. It is apparent that the SAR distribution is different than that at 64 MHz. Note also that no reduction in the SAR peak values was observed for the human head from linear (results not shown) to quadrature excitation.

The SAR values at 200 MHz using the four-port quadrature drive (Figure 6.24c) and the phase-optimized four-port excitation (Figure 6.24d) must also be considered. Note that the optimized phased-array drive was used to obtain the most ideal $B_1$ field distribution, that is the most homogeneous circularly polarized component of the $B_1$ ($B_1^+$) field. At 200 MHz, Figure 6.24c demonstrates that the SAR peak values
have dropped by about 50% using a four-port drive when compared to the conventional quadrature excitation (Figure 6.24b). As previously demonstrated, the four-port quadrature drive reduces effects the head-coil interactions leading to more uniform currents on the coil legs. Figure 6.24d shows that the phase-optimized four-port excitation provides SAR peak values higher than those with the four-port quadrature drive. Note that the phase optimized method is used to obtain an optimum (homogeneous) $B_1^+$ field. As such with these phase shifts, the electric field components can add up constructively in some spots and consequently leads to higher peak SARs compared to that with the fixed phase four-port excitation case. However, from Figures 6.24b and 6.24d, and 6.24h, the SAR peak values using phase-optimized four-port excitation are still less than what was obtained with the linear or the conventional quadrature excitations at 200 MHz.
Figure 6.24: Axial slices displaying the specific absorption rate (SAR) inside the human head model. Figure a corresponds to the SAR at 64 MHz using quadrature excitation. Figures b, c, and d correspond to the SAR at 200 MHz for quadrature (b), four-port (c), and phase optimized four-port excitation (d), respectively. The color scale for the SAR values is in $W/Kg$ for a 1 watt CW absorption.
6.4 A Field Optimized TEM Resonator Using Phased Array Concepts

At ultra high field (≥ 7 Tesla), a major challenge is the design of RF coils that exhibit a good signal to noise ratio, $B_{1}^{+}$ field uniformity, and low SAR in the biological tissues. This is the case since the electrical dimensions of the human head and body are comparable to the operational wavelength. Given that and the facts that the human head is asymmetric, and contains highly inhomogeneous, lossy materials, strong electromagnetic interactions are expected between the RF coil, the excitation source(s) and the head. As such, nonuniform, asymmetric, and complex current distributions on the RF coil struts will be present which in turn can lead to insurmountable $B_{1}^{+}$ field homogeneity.

In order to recover the field homogeneity, the field distribution within the RF coil must be modified. Several approaches have been proposed to accomplish this task. For instance, Tincher et al. proposed a method to reduce body coil $B_{1}$ field inhomogeneity through three steps [126]. First, the inhomogeneity was modeled using polynomials and least squares approaches. Second, the modeled data was subtracted from the actual image. Finally, the compensated data was rescaled to reduce the $B_{1}$ filed inhomogeneity. This method assumes that the inhomogeneity of the RF coil is known from the Biot-Savart law for low frequency operations. The Biot-Savart law was also utilized to optimize the $B_{1}$ field by finding an optimal angular placement of the elements of a non-circular birdcage coil [127]. The Biot-Savart law can be very accurate for modeling relatively complex geometries at low frequency, but it is not valid when the coil geometry is a significant fraction of the wavelength. This is particularly true

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in the ultra high frequency (UHF) range where the electric and magnetic fields become highly coupled.

Simulated annealing has also been used to optimize 16-rung elliptical shielded and unshielded coils [128]. In 1997, Li et al. utilized Ohm’s law to obtain the optimum current distribution for an elliptical birdcage coil operating in the linear mode [129]. This technique, however, is only valid when the interactions between the coil and the sample is minimal.

The $B_1$ field can also be optimized by altering the manner in which the RF coil is driven. Indeed, modification of the excitation source(s) was found to be effective tool in improving $B_1$ field inhomogeneity, especially for high frequency applications. For instance, interrung feeding has been successfully used for an eight-capacitor high pass resonator [130]. A four-point excitation with progressive phase shifts of $\pi/2$ [125] has also been applied to a birdcage coil [28] at 85 MHz.

In this section, the FDTD method is applied to explore multiple drive concepts in the TEM resonator [15] for UHF MRI (340 MHz). Input excitations with both variable phase and magnitude are examined. This is accomplished for 2, 3, 4, 6, and 10 excitation ports in a 24-strut TEM resonator loaded with an 18-tissue anatomically detailed human head model. In addition, antenna array concepts are also applied to modify the phase and magnitude of all the 24 possible ports in the 24-strut TEM resonator. By properly exciting each drive port, significantly improved $B_1$ field homogeneity can be obtained.
6.4.1 The Variable Phase/Magnitude Excitation System

The use of complex excitation systems is not new in medical applications. For instance, array concepts have been utilized in microwave hyperthermia cancer treatments [66, 64, 65, 131]. By exciting sources with appropriate magnitudes and phases, a field distribution can be generated in which the maximum occurs at the tumor. This method is conceptually similar to an antenna array, with the exception that the near field is involved rather than the far field. This concept is also applicable in high frequency MRI. In this case, sources on a volume coil are adjusted in the near field in order to obtain a homogeneous $B_1^+$ field. In this section, three different feed strategies that involve the phase and the magnitude of the drive points are considered: (1) fixed phase and fixed (uniform) magnitude (FPFM), (2) variable phase and fixed (uniform) magnitude (VPFM), and (3) variable phase and variable magnitude (VPVM).

Consider Figure 6.25 and FPFM excitation. Conventional two-port quadrature excitation was utilized. This includes back of the head quadrature excitation (ports 4 and 22), side of the head quadrature excitation (ports 4 and 10), and front of the head quadrature excitation (ports 10 and 16). In these cases, the drive is split into two parts. A $\pi/2$ phase shift is applied to one part which is then fed to the first port while the other drive is fed directly to the other port without adding a phase shift. The three-port system uses ports 1, 17, and 9 or 13, 5, and 21 in Figure 6.25. The drive is split into three parts which comprise 0, $4\pi/3$, and $-4\pi/3$ phase shifts. These, in turn, are fed to the three drive points.

The four-port drive utilizes the 4 ports used for two-port quadrature excitation: 22, 16, 10, and 4. In this case, $\pi/4$, $3\pi/4$, $-3\pi/4$, and $-\pi/4$ phase shifts are applied on each segment of the incoming RF. These segments are then fed to ports 22, 16, 10,
Figure 6.25: An axial slice of the 24-strut TEM resonator loaded with the anatomically detailed human head model in the FDTD grid

and 4, respectively. The six-port system combines the 2 three-port drives described above. Phase shifts of $0, \pi/3, 2\pi/3, \pi, -2\pi/3,$ and $-\pi/3$ phase shifts are applied to ports 1, 21, 17, 13, 9, and 5 respectively. The ten-port drive combines the ports of the four-port and the six-port systems with the same phase shifts described for these configurations.
The VPFM and the VPVM driving schemes were used in the three-port, four-port, six-port, and ten-port systems using the drive points previously described. For these two drive systems, the phase or (phase and magnitude) of each excitation element was/were adjusted to produce an optimum uniform circularly polarized $B_1$ field.

In addition to the drive systems described above, a 24-port system was obtained by exciting all 24 struts of the resonator. FPFM excitation was used by applying progressive $\pi/12$ phase shifts on each port starting from port 24 (Figure 6.25) and proceeding in a counter-clock wise direction. VPFM and VPVM excitations were also examined.

The VPFM and the VPVM systems are developed as follows. The coil is first excited at a specified port. The excitation is performed with a specified gap size between the inner rods of each coaxial line, and a specified dielectric constant of the filler between the inner and outer rods. Conductivity and dielectric constant parameters, in our case the values at 340 MHz, are assigned to the human head tissues. This process (excitation) is accomplished while the TEM resonator and the human head are modeled as a single system. Time domain data is collected and a Fourier transformation is applied to find the resonance frequency of mode 1. If the resonance frequency of mode 1 lies at the frequency of interest (340 MHz), the code is re-executed and the frequency domain solution of the field is obtained at 340 MHz. If the previous condition is not satisfied, the entire process described above is performed with a different gap size between the inner rods of each coaxial line.

The previous process is repeated for all the excitation ports of interest (3, 4, 6, 10, or 24 ports). Super position concept of all the solutions obtained at all the excitation ports (with the desired phase or phase and magnitude) was then applied to compute
the total response (field distribution). Using a numerical optimization process, the phase or the phase and magnitude of the excitation ports was/were optimized such that an optimum $B_{1}^{+}$ field (lowest standard deviation of the field distribution) was obtained in a particular slice of interest in the head. This optimization process was computationally expensive due to the number of parameters which are varied to obtain an optimum $B_{1}^{+}$ field distribution. For instance, a VPVM 24-port system involves the variance of 47 parameters (23 phase shifts and 24 magnitudes). In addition to the large number of variable parameters, the optimization process is nonlinear, which significantly increases the computational resources required.

In order to ensure the validity of the proposed 24-port system, the resonance frequency of mode 1 must approximately lie at same location (340 MHz) regardless of which drive port is used for excitation. Note that the same material has to be used to fill all the coaxial lines, and the gap between the tuning rods within all the coaxial lines must be fixed for all (24) excitations. For the simulations shown here, these two conditions were satisfied. Frequency responses obtained using the FDTD model at one point inside the head loaded 24-strut TEM resonator are shown in Figure 6.26. The excitations were done using ports 1-13, and the frequency response of every excitation was collected at the specified point. It is clear that the resonance frequency of mode 1 lies at approximately 340 MHz regardless of which port is excited. The bandwidth is also similar in all the thirteen cases. Note that ports 14 to 24 are reciprocal to ports 12 to 2.
Figure 6.26: Frequency responses obtained using the FDTD model at one point inside the human head model loaded in a 24-strut TEM resonator. The excitations used ports 1-13 and the frequency response of every excitation was collected at the specified point. The resonance frequency of TEM mode 1 lies at approximately 340 MHz regardless of which port was excited. Note that Teflon was used to fill all the coaxial lines for all cases.

6.4.2 Distribution of the $B_1^+$ Field

Distributions of the $B_1^+$ field are displayed in Figure 6.27. Axial slices were obtained by using the FDTD model with 24-strut TEM resonator driven by a FPFM excitation at 340 MHz. Figures 6.27a, 6.27b, and 6.27c correspond to the $B_1^+$ field obtained with 2-port excitation at ports 4 and 22 (6.27a), 4 and 10 (6.27b), and 10 and 16 (6.27c). Figures 6.27d, and 6.27e correspond to the $B_1^+$ field with 3-port excitation and utilizing ports 1, 17, and 9 (6.27d), and 13, 5, and 21 (6.27e). Figures 6.27f, 6.27g, and 6.27h correspond to the results obtained with 4-port (6.27f), 6-port (6.27g), and
10-port (6.27h) excitations. The ports used for these excitations are described in the previous section. The standard deviation (SD) values of the $B_{1}^+$ field in these axial slices are shown in Table 6.1.

These results indicate that increasing the number of excitation ports improves the homogeneity of the $B_{1}^+$ field until a minimal SD value = 0.1306 (Table 6.1) was obtained using 6-port excitation. Increasing the number of ports to 10 and then to 24, did not produce further decreases in SD. In fact, the SD value increased to 0.1333 for 10-port excitation and then it slightly decreased to 0.1321 using 24-port excitation (Table 6.1). Therefore, a fixed-phase progression on the ports is not optimal for producing homogeneous circularly polarized fields. This is because the EM fields associated with the particular mode of operation are neither transverse electromagnetic (TEM) nor linearly polarized (using 1-port excitation). The previous two facts are true since (a) the coil dimensions constitute a large fraction of the operating wavelength and (b) the coil load is electrically large, inhomogeneous, lossy, asymmetric, irregular in shape. Such conditions make it impossible to generate a pure TEM mode or a pure linear polarization in a coil loaded with a human head operating at UHFMRI (> 7 Tesla).

Figure 6.28 displays the $B_{1}^+$ field distribution inside the human head model using the FDTD model. Results were calculated within a 24-strut TEM resonator using VPFM (6.28a-e) and VPVM (6.28f-j) excitations at 340 MHz. The results were obtained using 3-port (6.28a and 6.28f) and (6.28b and 6.28g), 4-port (6.28c and 6.28h), 6-port (6.28d and 6.28i), and 10-port (6.28e and 6.28j) systems. Ports 1, 17, and 9 were used in Figures 6.28a and 6.28f and ports 13, 5, and 21 were used in Figures 6.28b and 6.28g. The ports used for the 4-port, 6-port, and 10-port systems were
Figure 6.27: Axial slices of the $B_1^+$ field distribution inside the human head model at 340 MHz. Results were obtained with a 24-strut TEM resonator using fixed-phase and fixed-magnitude excitation. Figures a-c correspond to the $B_1^+$ field with 2-port excitation, utilizing ports 4 and 22 (a), 4 and 10 (b), and 10 and 16 (c). Figures d, and e correspond to the $B_1^+$ field with 3-port excitation and utilizing ports 1, 17, and 9 (d), and 13, 5, and 21 (e). Figures f-h correspond to the results obtained with 4-port (f), 6-port (g), and 10-port (h) excitations. The ports used were 4, 22, 16, and 10 for 4-port excitation and 1, 17, 9, 13, 5, and 21 for 6-port excitation; all of the previous ports were used for 10-port excitation. The ports identification numbers are given in Figure 6.25.
Figure 6.27 continued
Figure 6.27 continued
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Table 6.1: Calculated standard deviation values of the circularly polarized component of the $B_1$ ($B_1^+$) field in axial slices through the human head model. The calculations were done using the FDTD model where the mean value of the $B_1^+$ field in the slice is set to 1.
the same as those used in Figure 6.27. Table 6.1 displays the SD values of the $B_{1+}$ field using the aforementioned excitations.

Unlike the FPFM excitation, SD values for VPFM and VPVM monotonically decrease with increasing the number of excitation ports and by varying the phase or the phase and magnitude of each excitation port (Table 6.1). Table 6.1 shows that the SD value improved from 0.1361 using 3-port VPFM excitation to 0.0888 using 10-port VPVM excitation. These findings show that the EM interactions between the coil and the tissue in general and the interactions between the excitation source(s) and the tissue in particular dominate the $B_{1+}$ field distribution in the head.

Based on the previous results, it is clear that one can further improve the homogeneity of $B_{1+}$ field distribution by driving all the coaxial rods of the coil. Figure 6.29 shows the $B_{1+}$ field distribution inside the human head model using 24-port FPFM (6.29a), VPFM (6.29b), and VPVM (6.29c) excitations. Table 6.1 provides the SD values of the $B_{1+}$ field distribution for the 2-port and 24-port excitation systems. It is observed that the SD of the VPVM 24-port system is equal to 0.0685, almost a 2-fold improvement over the 24-port FPFM system and nearly 2.5-fold better than the two port system (Table 6.1). The VPVM solution is presented in Table 6.2.

The results in this section provide conclusive evidences that at 340 MHz, the $B_{1+}$ field distribution inside the human head is dominated by the interactions between the head, coil, and excitation source rather than the head by itself. As such, the limitations on achieving a homogeneous MRI image at ultra high field are due to sample-coil interactions and technological difficulties and not dominated by fundamental physical phenomenon such as “dielectric resonances”.

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Figure 6.28: Axial slices of the $B_1^+$ field distribution inside the human head model. Results were obtained with a 24-strut TEM resonator using variable-phase and fixed-magnitude (a-e) and variable phase-variable magnitude (f-j) excitations at 340 MHz. The figures correspond to 3-port (a and f) and (b and g), 4-port (c and h), 6-port (d and i), and 10-port (e and j) systems. Ports 1, 17, and 9 were used in Figures a and f and ports 13, 5, and 21 were used in Figures b and g. The ports used for the 4-port, 6-port, and 10-port systems are described in Figure 6.25.
Figure 6.28 continued

(a) 

(b) 

(c) 

(d) 

continue
Figure 6.28 continued

(e)  

(f)  

(g)  

(h)  

continue
Figure 6.28 continued
Figure 6.29: The calculated $B_1^+$ field distribution inside the human head model at 340 MHz. The human head model was loaded in a 24-strut TEM resonator excited at all the possible ports (24) (a-c). The results are presented for fixed-phase&fixed-magnitude (a), variable-phase&fixed-magnitude (b) and variable-phase&variable-magnitude (c) systems.
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<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>240.40</td>
<td>(0.058)</td>
</tr>
<tr>
<td></td>
<td>276.49</td>
<td>(0.024)</td>
</tr>
</tbody>
</table>

* The ports identification numbers are given in Figure 6.25.

**Table 6.2:** The relative phases and magnitudes of excitation ports 1-24 (Figure 6.25) required to obtain a standard deviation value equal to 0.0685 of the \(B_1^+\) field in an axial slice through the human head model at 340 MHz. These values correspond to the variable-phase variable-magnitude 24-port system used to obtain the results of Figure 6.29c.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Summary and Findings

In this dissertation, a complete electromagnetic computational analysis was presented for several RF coils used in magnetic resonance imaging at Larmor frequencies ranging between 64 and 500 MHz. The finite-difference time-domain (FDTD) method [3] combined with measurements using MRI clinical systems with $B_0$ fields equal to 8 Tesla, and 1.5 Tesla, were utilized to study, analyze, and eventually design RF coils. Novel analytical derivations were presented to explain the source of the MR signal. These derivations combined with FDTD modeling, experiments and infrared imaging provide significant insight into understanding of the electromagnetics associated with high field clinical imaging.

In Chapter 1 of this dissertation, an introduction was presented. The problems as well as the motivation for this work were stated. In Chapter 2, a brief physical description of nuclear magnetic resonance (NMR) and its use in medical imaging was given. A review of MRI RF coils was also provided. In Chapter 3, the FDTD method was derived from Maxwell’s time-dependent curl equations. The perfectly matched layer (PML) [29] was used as the absorbing boundary condition to account
for the RF radiation from the coil and its equations were derived using the coordinate stretching approach [30].

Chapter 4 described the implementation of the FDTD method for modeling the birdcage [28] and the TEM [15] head coils, and the single strut extremity coil [31]. This included the setup of the FDTD code. Chapter 4 also discussed the PML conductivity profiles, modeling the capacitors and slanted perfect electric conductors, obtaining the resonance frequency of a specific mode and the evaluation of the fields at this mode. Using transmission line and circuit theories, theoretical analyses of the birdcage coil, single strut coil and the TEM resonator were provided. In addition, a description of a newly developed 18-tissue anatomically detailed human head model [32, 33, 34] was presented. To validate the numerical model, the FDTD results were compared to actual MRI measurements of phantoms at 1.5 and 8 Tesla. Detailed theoretical, experimental, and numerical analysis describing the operation of the birdcage and the TEM head resonators, and the single strut extremity coil were also provided.

In Chapter 4, the performance of the 8, 16 and 24 strut TEM resonators were compared for the first time at 340 MHz using gradient recalled echo imaging of the human head in conjunction with FDTD analysis. The results confirmed the following: first, that increasing the number of TEM struts decreases the maximal achievable frequency while, at the same time, substantially increasing the experimental complexities associated with tuning and matching. Second, the circularly polarized component of the $B_1$ ($B_1^+$) field homogeneity in the head increases most when changing from 8 to 16 struts coils, with a lesser improvement between 16 and 24 strut coils. Third, a specific absorption rate (SAR) analysis in the human head revealed little difference between
these three coils, but stronger tissue/coil coupling was observed for the 8 strut case. As a result, it appears that the 16 strut coil provides the best compromise between construction complexity and tuning range.

Chapter 5 provided SAR analysis at 340 MHz. The coil was numerically tuned to 340 MHz, and SAR calculations were done inside human head models. The FDTD calculated SAR distribution in axial slices through the brain indicates that the peak SAR values are greatly reduced when switching from linear or 2-port quadrature drives to 4-port excitation. In addition, the FDTD simulations also show that the SAR peak values are affected by the excitation source(s) and their location(s). The numerical analysis was verified via infrared imaging techniques. Thus, for high frequency MRI applications, it is necessary to properly model the physical excitation source(s) of the coil rather than assume a current distribution on the coil struts in order to explain the electromagnetic phenomena.

Chapter 5 also studied the dependence of RF power requirements on the frequency of operation (Larmor frequency) between 3 and 11.5 Tesla using head and extremity coils. The effect of the electromagnetic interactions between the excitation source(s) and the tissue on power requirements was analysed. FDTD methods were used to obtain a general understanding of power requirements in human MRI. The results indicate clearly that the head/RF coil interaction is important in defining RF power requirements. For axial slices through the brain region, it was noted that the power required to obtain a specified circularly polarized field value increases with frequency, plateauing at a certain value, and then dropping as the frequency increases.

Chapter 6 introduced a novel approach based on the principle of reciprocity for driving the NMR signal. Experimental and numerical validations were presented.
The effectiveness of rigorous EM full wave modeling in describing RF coil operation at high field MRI was demonstrated. Chapter 6 also provides a thorough analysis of dielectric resonances [17, 18, 23, 24, 25, 26] at 8 Tesla. When true dielectric resonances are present, the modal field distributions are intrinsic properties of the object, unaffected by the excitation source. Thus, if dielectric resonances are present in UHFMRI, they must be unavoidable. Hence, a single homogeneous head image is sufficient to dispel their importance. At the same time, as the resonance frequency of operation rises, strong electromagnetic interactions are expected between the RF coil and the head. These interactions are dependent on several factors, such as the source location, the dielectric properties (conductivity and permittivity), the shape, and the position of the sample within the RF coil.

It is concluded that dielectric resonances are most strongly excited in objects comparable in size to the head when the conducting medium has a high dielectric constant and a low conductivity (see Figure 6.13a, 6.13b, 6.13c, and 6.13d). In addition, when the conductivity of the medium approaches levels found in tissue, there is strong attenuation of dielectric resonances. This fact leads to much more homogeneous images than are found in lossless materials. In MRI studies of the human head, the most important determinants of $B_1$ field homogeneity remain 1) the geometry of the RF coil and 2) the interaction of the RF coil (including the excitation source) and the sample (see Figures 6.13f, 6.13g, 6.13h, and 6.153a-d). For these reasons, in computational approaches it is imperative to model both the RF coil and the head as a single system. In addition, it is also imperative to model the excitation of the RF coil in a realistic manner rather than assuming an idealized current distribution [78].
The fact that $B_1$ homogeneity at 8 Tesla is dominated by RF coil/head interactions and not by dielectric resonances is an encouraging finding. The head simply has too high a conductivity, too little geometrical symmetry, and too irregular a structure to significantly support such resonances. This is fortunate, since had dielectric resonance fundamentally dominated this problem, UHFMRl may have been unfeasible.

In Chapter 6, the $B_1$ field homogeneity was examined for cylindrical and octagonal phantoms, in addition to the human head model inside a high pass birdcage coil. At 64 MHz, $B_1^+$ field was found to be homogeneous for a birdcage coil loaded with the aforementioned three objects. Compared to linear excitation, better overall homogeneity is obtained using quadrature excitation, except when the coil is loaded with the octagonal phantom. The octagonal phantom is relatively large, close to the coil elements, and does not have circular symmetry. This produces greater asymmetries in the induced currents. Therefore at 64 MHz, one can deduce that inside the birdcage coil, the circularly polarized magnetic fields are only induced when the imaged object has a $\phi$ symmetry and is relatively small compared to the coil.

At 200 MHz, the $B_1^+$ field is inhomogeneous for all three cases. Also, there is no improvement in the homogeneity of the fields from linear to quadrature excitation. A four-port excitation was developed to reduce the effects of the interactions between the coil and the object to be imaged and to consequently improve the $B_1^+$ field homogeneity. Compared to the conventional quadrature drive, a significant improvement in the $B_1^+$ field homogeneity was obtained using the four-port drive. For the human head, a significant improvement in the $B_1^+$ homogeneity was obtained using a phase-optimized four-port excitation.
SAR calculations were also presented at low and high frequencies for different excitations. At 64 MHz, peak SAR values have significantly decreased from linear to quadrature excitation and from quadrature to four-port quadrature excitation. At 200 MHz, compared to that with linear excitation, the SAR peak values have increased when the excitation is done in quadrature. Because linear excitation does not give a sinusoidal current distribution on the coil elements at 200 MHz, it is impossible to obtain circularly polarized fields using the conventional quadrature drive. At 200 MHz, unlike the two-port quadrature excitation, the four-port drive provides an SAR distribution with peak values that are significantly less than those obtained with linear or quadrature excitation.

In the last portion of Chapter 6, true phased array techniques were developed to improve the homogeneity of $B_{1}^{+}$ field at 8 Tesla. The $B_{1}$ field homogeneity was examined inside a 24-strut TEM resonator at 340 MHz using different excitations techniques namely, fixed-phase and fixed-magnitude (FVFM), variable-phase and fixed-magnitude (VPFM), and variable-phase and variable-magnitude (VPVM) systems. This was accomplished through the use of multi-port (2, 3, 4, 6, and 10 ports) excitation. It was observed that the resonance frequency and bandwidth of the mode of interest were consistent for each excitation port, which implies a resonance of the entire system as desired. In addition, a full multiple drive concept was utilized to drive all the possible (24) struts of the coil. The optimization criteria of the phases or the phases and magnitudes of the drive ports was the minimization of the standard deviation of the $B_{1}^{+}$ field.

In axial slices through the brain, a significant improvement in the $B_{1}^{+}$ field homogeneity was obtained by varying the phase (or the phase and magnitude) of each
excitation source beyond a linear phase progression. For instance, the standard de-
viation of the $B_1^+$ field decreased from 0.1617 using conventional back of the head
ear quadrature excitation to 0.0685 (2.5 fold improvement) using VPVM 24-port excita-
tion. Based on these results it is concluded that (1) highly homogeneous MRI images
at ultra high field are physically feasible, (2) electromagnetic fields within the head
are dominated by the interactions between the coil, excitation source(s) and the head.

7.2 Future Work

The work done in this dissertation provides a cornerstone for understanding the
electromagnetics associated with MRI at low and high fields. The results presented
can be of a tremendous aid in studies of $T_1$, $T_2$ relaxation measurements, suscepti-
bility effects, and biological hazards (RF power deposition), most especially at high
fields.

Numerically, developing highly efficient codes with accurate representation of
coil structures is still needed. This can be achieved through computational EM tech-
niques such as the finite elements method (FEM), implicit FDTD schemes, and hybrid
FDTD/FEM technique. Clearly, computational EM is the only viable tool in optimiz-
ing transmit and receive coils. Computational tools will play the dominant role in
designing the optimized head and body RF coils.

Designing RF coils a specific application at high fields is a clear avenue for suc-
cess. This includes abdominal imaging at 3 Tesla and all applications at fields higher
than 3 Tesla. In the past, coil designs were based on trial and error. A coil designer
would usually construct a coil and then test it for performance. Since the operational
frequencies of MR have increased, the coil dimensions have become electrically large
and the head/body possess the most influence on the coil performance. Therefore, such
approach is no longer viable and simulation techniques are the only realistic approach
for designing and testing these coils. The computational models developed in this
work could be utilized in the design of these high field coils. The coils would be very
much clinically driven because their designs are dependent on the specific clinical ap-
lication. The diagnosis of specific diseases such as multiple sclerosis, Alzheimer’s
(AD), and stroke at high fields would require specific coil designs. For instance in
AD, high fields bring the ability of visualizing microscopic plaques in the hypocam-
pus, a feature that is not feasible at 1.5 Tesla. The goal of designing such a coil would
be providing the highest signal to noise ratio in the particular area of interest, i.e. the
hypocampus.

The next step in terms of RF technology would be the implementation of the
multi-port VPVM system provided in this dissertation. This would require phased-
locked transmitter/receiver channels. These transceivers must be designed such that
the phase and the magnitude of each channel can be controlled independently of all
the other channels. Because of different head sizes, it is necessary to create a database
of the phases (VPFM) or the phases and magnitudes (FVFM) of the excitation ports
where the values of these parameters are computationally calculated for different head
models using the FDTD technique. These sets of phases and magnitudes could then be
applied to the multi-port system such that an optimal $B_1^+$ field distribution is obtained.
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