CONTROL AND OBSERVATION OF ELECTRIC MACHINES
BY SLIDING MODES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

Zhang Yan, B.S.E.E., M.S.E.E.

* * * * *

The Ohio State University

2002

Dissertation Committee:

Professor Vadim I. Utkin, Co-Adviser
Professor Giorgio Rizzoni, Co-Adviser
Professor Longya Xu

Approved by

Co-Adviser
Co-Adviser
Department of Electrical Engineering
© Copyright by
Zhang Yan
2002
ABSTRACT

The objective of this dissertation is to develop control and estimation methods for electric machines based on sliding mode control theory. Major attention is paid to two types of AC machines, i.e. the induction machine (IM) and the synchronous machine, including the permanent magnet synchronous machine (PMSM). This choice may be explained by the fact that AC drives are gradually superseding DC ones for many dynamic plants in modern industrial applications. The method proposed in this dissertation for both control and observation is the so-called sliding mode approach chosen because of its robustness and ability to reduce the order of the motion models. A further advantage is that the average values of discontinuous inputs (i.e. the so-called equivalent control) in sliding modes are algebraic functions of unknown state components and parameters. These equivalent control values can be easily obtained by using low pass filters and they are useful in calculation and estimation.

As real-time computation costs continually decline, both mechanical robustness and economic considerations increasingly stimulate the replacement of mechanical sensors by software-based observation methods. These so-called sensorless systems are free of maintenance and exhibit high reliability and low cost. Elimination of encoders or resolvers on induction machine drives is a prime example.

Due to the above reasons, many sensorless control schemes have been developed and described in literature. High order models of AC machines, nonlinearities in
motion equations, uncertainties in model parameters and disturbances are the main obstacles hindering the development and rigorous mathematical analysis of such systems. However, their efficiency has been demonstrated by experiments and real applications. In contrast to conventional approaches, where control and observation are handled independently, the core idea of the approach proposed in this dissertation implies that they are treated as one interconnected system. This approach facilitates control system analysis and design since the speed is not an arbitrary time function any more but the solution to the known differential equations. As a result, the new structure of the observer is offered and the convergence of the observation is proven.

There is one very important issue in the framework of the studies: varying of the model parameters in a wide range, in particular the rotor resistance, which may be within 30 – 40% because of heating. New approach is developed to identify speed, flux and rotor resistance simultaneously under the common assumption that the electromagnetic processes are faster than the mechanical ones.

The developed control and estimation algorithms are tested experimentally for different types of induction machines. The sensorless systems demonstrate high accuracy of tracking reference inputs for speed and torque.
To my wife and our families ...
ACKNOWLEDGMENTS

First of all, I am indebted to my advisors, Prof. Vadim Utkin and Prof. Giorgio Rizzoni, for their advice, guidance, encouragement, and support throughout my Doctoral program of study. It has always been educational, stimulating, and fun. I would also like to thank Prof. Longya Xu for being on both my candidacy exam and dissertation exam committees. Thanks also go to Prof. Hooshang Hemami and Prof. Donald Kasten for serving on my candidacy exam committee.

I would like to thank the many professors and teachers that I have had during all my years of education. It would be impossible to list them all here but their help are deeply appreciated.

I would like to thank the sponsor of my Ph.D. research work, Ford Scientific Research Laboratory for the financial support it provided.

I also wish to thank all the fellow students in our Controls Group at OSU for the friendship we shared and the assistance they gave me in many ways.

Finally, I am grateful to my wife Jia and my parents for their endless support and understanding. To them I owe everything I attained and will accomplish.
VITA

October, 1973 .......................... Born - Nanjing, P.R.China

1995 ................................. B.S. Electrical Engineering,
                        Southeast University
                        Nanjing, P.R.China

1997-1998 ............................. University Fellowship,
                        The Ohio State University

1999 ................................. M.S. Electrical Engineering,
                        The Ohio State University

2000-present ........................... Ford Fellowship

FIELDS OF STUDY

Major Field: Electrical Engineering

Studies in:
    Control Systems
    Electric Power Engineering
    Computer and Information Science
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Vita</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xi</td>
</tr>
<tr>
<td>Chapters:</td>
<td></td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Literature Review</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1 Control and Observation of Induction Machines</td>
<td>1</td>
</tr>
<tr>
<td>1.1.2 Control and Observation of Synchronous Machines</td>
<td>12</td>
</tr>
<tr>
<td>1.2 Objectives of This Dissertation</td>
<td>14</td>
</tr>
<tr>
<td>1.3 Dissertation Organization</td>
<td>16</td>
</tr>
<tr>
<td>2. Dynamic Modelling of Electric Machines</td>
<td>18</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>18</td>
</tr>
<tr>
<td>2.2 DC Machine Modelling</td>
<td>19</td>
</tr>
<tr>
<td>2.3 Induction Machine Modelling</td>
<td>21</td>
</tr>
<tr>
<td>2.4 Synchronous Machine Modelling</td>
<td>24</td>
</tr>
<tr>
<td>2.5 Permanent Magnet Synchronous Machine (PMSM) Modelling</td>
<td>30</td>
</tr>
<tr>
<td>2.6 Modelling of a Synchronous Generator Connected to an Infinite Bus</td>
<td>33</td>
</tr>
<tr>
<td>2.7 Summary</td>
<td>37</td>
</tr>
</tbody>
</table>
3. DC Machine Sliding Mode Control and Observation .......................... 38
  3.1 Introduction .................................................. 38
  3.2 Sliding Mode Controller ....................................... 39
  3.3 Sliding Mode Observer ......................................... 41
  3.4 Control of DC Machines with Observers ......................... 45
  3.5 Summary ........................................................ 46

4. Induction Machine Sliding Mode Control and Observation .............. 47
  4.1 Current Regulation by Sliding Mode Controller ................... 47
    4.1.1 Control System Analysis and Simulation .................. 48
    4.1.2 Experimental Results .................................... 54
    4.1.3 Chattering Problem ...................................... 62
    4.1.4 Experiments with Observer ................................ 66
  4.2 Sensorless Sliding Mode Control ................................ 72
    4.2.1 Control Objective ....................................... 72
    4.2.2 Sliding Mode Flux/Speed Observer ......................... 72
    4.2.3 Analysis of Current Tracking ............................ 73
    4.2.4 Sliding Mode Torque Regulation .......................... 76
    4.2.5 Composite Observer-Controller Analysis .................. 79
    4.2.6 Simulation Results ...................................... 82
    4.2.7 Experiment Setup and Results ............................ 90
    4.2.8 Sliding Mode Controller in Low Speed Range .............. 94
  4.3 Sliding Mode Speed and Rotor Time Constant Observer ............. 97
    4.3.1 Observer Design and Analysis ............................ 97
    4.3.2 Simulation and Experiment Results ....................... 104
  4.4 Implementation Issues ......................................... 113
    4.4.1 Sliding Mode Control Scheme in terms of Phase Voltages
          $u_i (i = a, b, c)$ ........................................ 113
    4.4.2 Sliding Mode Control Scheme in terms of $g_i$ with $i = a, b, c$ 115
  4.5 Summary ........................................................ 116

5. Synchronous Machine Sliding Mode Control and Observation .......... 118
  5.1 Current Control of Synchronous Machines ........................ 118
    5.1.1 Synchronous Machine Field Oriented Control .............. 118
    5.1.2 Sliding Mode PWM ........................................ 121
    5.1.3 Direct Sliding Mode Current Control ...................... 125
  5.2 Torque/Speed Control of Synchronous Motors ...................... 126
    5.2.1 Sliding Mode Torque Control ............................. 126
5.2.2 Sliding Mode Speed Control .......................... 128
5.3 Control of a Synchronous Generator in Power Systems ........ 128
  5.3.1 Power Angle Control of Synchronous Generators .......... 128
  5.3.2 Terminal Voltage Control of Synchronous Generators ...... 129
5.4 Sliding Mode Observers for PMSM ............................. 130
  5.4.1 Sliding Mode Position Observer of PMSM ................. 130
  5.4.2 Sliding Mode Speed Observer for Non-Saliency PMSM ....... 131
  5.4.3 Simulation Results .................................... 134
5.5 Summary .................................................. 135

6. Conclusions and Future Research Directions ......................... 149
  6.1 Contributions ............................................ 149
  6.2 Future Research Directions ................................ 151

Bibliography .................................................. 153
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Nomenclature in induction machine FOC</td>
<td>2</td>
</tr>
<tr>
<td>2.1</td>
<td>Nomenclature in DC machine</td>
<td>20</td>
</tr>
<tr>
<td>2.2</td>
<td>Nomenclature in induction machine</td>
<td>22</td>
</tr>
<tr>
<td>2.3</td>
<td>Nomenclature in synchronous machine</td>
<td>26</td>
</tr>
<tr>
<td>4.1</td>
<td>Inductive load data</td>
<td>52</td>
</tr>
<tr>
<td>4.2</td>
<td>Induction machine test data</td>
<td>52</td>
</tr>
<tr>
<td>4.3</td>
<td>Experiment data of inductive load #2 on 60 Hz bench</td>
<td>67</td>
</tr>
<tr>
<td>4.4</td>
<td>Experiment data of inductive load #2 on inverter bench</td>
<td>68</td>
</tr>
<tr>
<td>4.5</td>
<td>Parameters used in the simulation</td>
<td>83</td>
</tr>
<tr>
<td>4.6</td>
<td>Real induction motor data</td>
<td>91</td>
</tr>
<tr>
<td>4.7</td>
<td>Base system data</td>
<td>92</td>
</tr>
<tr>
<td>4.8</td>
<td>Real induction motor data</td>
<td>104</td>
</tr>
<tr>
<td>5.1</td>
<td>Permanent magnet synchronous machine parameters</td>
<td>135</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Field Oriented Control system block diagram</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>Direct FOC method 1</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Direct FOC method 2</td>
<td>4</td>
</tr>
<tr>
<td>1.4</td>
<td>Indirect FOC</td>
<td>4</td>
</tr>
<tr>
<td>1.5</td>
<td>DTC system block diagram</td>
<td>8</td>
</tr>
<tr>
<td>2.1</td>
<td>Two-winding model of dc machine</td>
<td>19</td>
</tr>
<tr>
<td>2.2</td>
<td>Equivalent circuits of dc machine</td>
<td>20</td>
</tr>
<tr>
<td>2.3</td>
<td>Two-pole, 3-phase, Y-connected induction machine</td>
<td>23</td>
</tr>
<tr>
<td>2.4</td>
<td>Magnetic axes of a three phase induction machine</td>
<td>23</td>
</tr>
<tr>
<td>2.5</td>
<td>Magnetic axes and winding arrangement in synchronous machine</td>
<td>25</td>
</tr>
<tr>
<td>2.6</td>
<td>Permanent magnet synchronous machine</td>
<td>31</td>
</tr>
<tr>
<td>2.7</td>
<td>Synchronous generator loaded by an infinite bus</td>
<td>33</td>
</tr>
<tr>
<td>4.1</td>
<td>System block diagram for current control of inductive load</td>
<td>50</td>
</tr>
<tr>
<td>4.2</td>
<td>Simulink block diagram for current control of inductive load</td>
<td>51</td>
</tr>
<tr>
<td>4.3</td>
<td>Simulation results of current tracking on inductive load #1</td>
<td>51</td>
</tr>
</tbody>
</table>
4.24 Torque tracking ............................................ 84
4.25 Magnitude of flux tracking ............................. 84
4.26 Flux convergence ............................................. 85
4.27 Speed estimation ............................................ 86
4.28 Current convergence ....................................... 86
4.29 Torque tracking ............................................ 87
4.30 Magnitude of flux tracking ............................. 87
4.31 Flux convergence ............................................. 88
4.32 Speed estimation ............................................ 89
4.33 Current convergence ....................................... 89
4.34 System block diagram .................................... 90
4.35 Torque control with speed sensor ...................... 93
4.36 Flux estimation ............................................. 94
4.37 Sensorless torque control ............................... 95
4.38 Sensorless speed control ............................... 96
4.39 Sensorless speed control ............................... 97
4.40 Low speed control ......................................... 98
4.41 System block diagram .................................... 105
4.42 Actual and estimated speed ............................ 105
4.43 Actual and estimated currents ......................... 106
4.44 Actual and estimated $\left(L, \hat{L} \right)$ .................. 106

xiii
4.45 $\eta$ ................................................................. 107
4.46 Measured and estimated speed ........................................ 108
4.47 Measured and observed currents (between 7.5-9.5 sec.) ........ 108
4.48 Calculated and observed states $L$ and $\dot{L}$ ....................... 109
4.49 Enlarge of Figure 4.48 in between (6-8 sec.) ....................... 109
4.50 Measured and estimated speed ........................................ 110
4.51 Calculated and observed states $L$ and $\dot{L}$ ....................... 110
4.52 Enlarge of Figure 4.51 in between (1.2-2.4 sec.) ................... 111
4.53 Measured and observed currents ..................................... 111
4.54 Enlarge of Figure 4.53 in between (7.5-9.5 sec.) ................... 112
4.55 Estimated $\dot{\eta}$ .................................................. 112
4.56 PWM implementation ................................................. 113
5.1 System block diagram .................................................. 136
5.2 Field oriented speed control .......................................... 137
5.3 Speed tracking error ................................................... 138
5.4 Sliding mode speed observation ...................................... 139
5.5 Sliding mode current observation $i_\alpha$ ............................ 140
5.6 Sliding mode current observation $i_\alpha$ (enlarged) ................... 141
5.7 Sliding mode current observation $i_\beta$ ............................ 142
5.8 Sliding mode current observation $i_\beta$ (enlarged) ................... 143

xiv
5.9 Back EMF observation $e_\alpha$ ................................. 144
5.10 Back EMF observation $e_\alpha$ (enlarged) ...................... 145
5.11 Back EMF observation $e_\beta$ ................................. 146
5.12 Back EMF observation $e_\beta$ (enlarged) ...................... 147
CHAPTER 1

INTRODUCTION

This chapter presents a review of the literature in the areas of control and observation of two major kinds of AC machines, i.e. induction machines and synchronous machines. We are especially concerned with the following subjects: Field Oriented Control (FOC), Direct Torque Control (DTC), adaptive speed observer, observer-based flux estimation, sliding mode flux and speed observer and feedback linearization control of synchronous machines. The objectives and contributions of the dissertation are also discussed in the last section of this chapter.

1.1 Literature Review

1.1.1 Control and Observation of Induction Machines

Field Oriented Control
Field Oriented Control (FOC) [1] has made possible the application of induction motors for high performance applications, where only DC motors had previously been used. By FOC, the induction motor can be controlled in the same manner as are separately-excited DC motors. The system block diagram of FOC is shown in Figure 1.1. The nomenclature is listed in Table 1.1. The basic idea is that torque control is decoupled into two tasks: the control of the torque current component, \( i_{q*} \), and of the
Figure 1.1: Field Oriented Control system block diagram

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{qs}, i_{ds}$</td>
<td>two phase stator currents in $(d, q)$ rotational frame</td>
</tr>
<tr>
<td>$i^<em>_{qs}, i^</em>_{ds}$</td>
<td>two phase stator reference currents in $(d, q)$ rotational frame</td>
</tr>
<tr>
<td>$u_{qs}, u_{ds}$</td>
<td>two phase stator voltage command in $(d, q)$ rotational frame</td>
</tr>
<tr>
<td>$u_{as}, u_{bs}, u_{cs}$</td>
<td>stator voltage command in $(a, b, c)$ three phase</td>
</tr>
<tr>
<td>$T(\theta)$</td>
<td>the reference frame transformation from stationary to rotational</td>
</tr>
<tr>
<td>$T^{-1}(\theta)$</td>
<td>the reference frame transformation from rotational to stationary</td>
</tr>
<tr>
<td>$\theta_e$</td>
<td>angle of the rotor flux</td>
</tr>
</tbody>
</table>

Table 1.1: Nomenclature in induction machine FOC
flux current component, $i_{ds}$, independent of one another. Decoupling is achieved in the synchronous rotating frame when the rotor flux vector is aligned with the d-axis of the rotating frame, i.e., the so-called field orientation. This decoupling control between the flux and torque makes quick dynamic response possible. According to the approach used to achieve field orientation, there are two classes in FOC: Direct FOC (DFOC) and Indirect FOC (IFOC).

- In Direct FOC (DFOC), there are two ways to obtain the rotor flux position. One is to install a flux sensor to measure the air gap flux, then algebraically compute the rotor flux. The block diagram of this method is shown in Figure 1.2. The disadvantage of this approach is that special flux sensors are necessary; this installation is not possible in commercial off-the-shelf motors. As shown in Figure 1.3, the second method is to first integrate the sensed stator voltage and current to calculate the stator flux, and then compute the rotor flux from the stator flux. The drawback of this method is its parameter sensitivity (especially with respect to the stator resistance) and integration problems at a low rotor speed due to the stator $IR$ voltage drop.
• In Indirect FOC (IFOC), the rotor flux position is obtained by adding the slip frequency position calculated from the torque and flux commands to the rotor position. The calculation of the rotor flux angle $\theta_e$ in IFOC is shown in Figure 1.4. Thus a rotor position sensor is needed in Indirect FOC, and the slip frequency calculation is directly related to the rotor time constant, $L_r/R_r$, which is changing with temperature and with varying flux level.

In addition to the limitations mentioned above, it should also be observed that it is customary to employ PI controllers in conjunction with FOC methods. Such controllers require careful tuning for good control performance.

**Sliding Mode Control**

Reference [2] reports the first application of sliding mode control (SMC) method to
the induction motor for control of torque, speed and position. The paper's main contribution is that it formulates the switchover functions in general form according to different control objectives and proposes the generation of phase voltages to satisfy the sliding mode condition. In [3], a unified approach for control of AC machines by sliding mode is presented. Three types of AC machines are under consideration, i.e. induction, synchronous and PM synchronous machines. Unlike in [2], the state variables for the machine model are selected to be the currents and fluxes, so that all the sliding surfaces can be selected in terms of current to eliminate the requirement of flux measurement or observation. Another basic difference between [2] and [3] is that in [2] the sliding mode is enforced by the control input in the form of the three phase input voltages $u_a$, $u_b$ and $u_c$, so they can be implemented directly by the inverter; in [3], the control to enforce the sliding mode consists of the two voltages $u_d$ and $u_q$ in the synchronous rotating frame so a mapping rule is necessary to find $u_a$, $u_b$ and $u_c$ for every pair of $u_d$ and $u_q$. In terms of control complexity, there is a tradeoff between the effort to calculate $u_a$, $u_b$ and $u_c$ in [2] and that to be spent on the mapping process in [3].

Another sliding mode control method is reported for induction motor position control in [4]. However, what is really under consideration in this paper is the simplest second order speed and position system with torque as the control input. Sliding mode control is only used to produce the torque command, while the inner loop is just the traditional indirect field oriented control.

Some general guidelines to design sliding mode controllers for electric drives are given in [5], including DC motors, induction motors and synchronous motors. The chattering problem which is often met in practice is discussed, and possible solutions
are given. For the induction motor, the control methodology is formulated in the model based on the stationary frame. This makes it necessary to use a machine-parameter-dependent transformation to obtain the control input, i.e. the three phase terminal voltages.

A novel sliding mode speed and rotor flux control strategy for induction motor is proposed in [6]. The model used is similar to that in [2]. The control inputs are formulated in the two voltage components $u_\alpha$, $u_\beta$ in the stationary frame. They are specially designed so that there is a direct relation between $u_\alpha$, $u_\beta$ and $u_a$, $u_b$, $u_c$, i.e. no mapping is needed. But the disadvantage is that only four switching states in the inverter are used among all eight possible states. To compute the rotor flux, the traditional integration method is used, so that problems inherent to numerical integration are unavoidable.

A nonlinear sliding surface is selected in [7] to reduce the complexity in designing sliding mode controllers. This approach has the feature that the surface becomes linear when rotor flux reaches the steady state. The controller design is based on the model in the synchronous rotating frame with rotor flux, speed and their derivatives as state variables. The use of the synchronous rotating frame has the advantage that it avoids the nonsingular transformation in the diagonalization method [5], which is machine-parameter-dependent. On the other hand, because of the use of synchronous rotating frame, a mapping table is needed for the inverter voltage control. The flux information is obtained in the same way as [6].

Another sliding mode controller using nonlinear sliding mode surfaces is presented in [8][9]. [9] compares sliding mode control performances with FOC and Input-Output Linearization Control by experiments. The results show SMC gives better results with
respect to low speed and rotor resistance variations. In these two papers, the boundary layer method is used for chattering reduction. The boundary-layer approach avoids generating sliding mode by replacing the discontinuous switching actions with a continuous saturation function. Thus the voltage inputs from the proposed sliding mode controller cannot drive the inverter directly, instead they have to be implemented by PWM algorithm. Also note that in boundary-layer approach no real sliding mode takes place and the system trajectories are only confined to a vicinity of the sliding manifold $s = 0$ instead of exactly to $s = 0$ as in ideal sliding mode.

**Direct Torque Control**

Direct Torque Control (DTC) of induction motors was developed more than ten years ago [10][11]. The system block diagram of DTC is shown in Figure 1.5. Unlike FOC, DTC does not tend to reproduce the electromechanical behavior of a DC motor, but is aimed at a complete exploitation of the flux and torque producing capabilities of an induction motor fed by a PWM inverter. The principle of DTC is based on hysteresis control using an optimal PWM output. The control system is designed based on the machine model in terms of stator current and flux in a stationary reference frame. The instantaneous torque and flux values are calculated from the stator voltage and current. They are controlled directly and independently by selecting optimal inverter switching modes. In DTC, no speed information is needed and it can be regarded as one kind of sensorless control. In spite of its quick dynamic response and implementation simplicity, there are two main restrictions. One is how to select the best inverter switching pattern; sometimes a compromise has to be made since there is no scheme that is optimal for all motor operating conditions. The second drawback is how to obtain the stator flux. The traditional way is to integrate over stator voltage
and current. Obviously, this approach has the usual problems inherent to integration and is heavily influenced by the variability in stator resistance. Further, how to determine an optimal switching scheme is still an open research topic. **Adaptive Speed Observer**

A speed identifier by the approach of Model Reference Adaptive Systems (MRAS) is proposed in [12]. However, the identifier design is based on a linearized induction motor model, so the time constant in the identifier needs to be varied online to reduce the influence of different machine operating points. Reference [13] also employs MRAS on the rotor flux model in stationary frame for speed estimation in Indirect FOC, which uses the errors between the rotor flux from the reference and adjustable model to drive the speed estimation. Unfortunately, the reference model has a pure
integrator, which causes problems with initial conditions and drift. Besides, although
the rotor time constant has little effect on the speed estimation, the stator resis-
tance does have some influence. To avoid the need for pure integration, [14] presents
a MRAS scheme in terms of the auxiliary state variables (the counterelectromotive
force), which is completely independent of stator resistance variance. The estimated
speed is used for both field orientation and speed feedback in the Indirect Speed FOC.
It is shown that the rotor resistance variance does not destroy the field orientation,
it only produces an error in the speed feedback.

Reference [15] uses the same MRAS as in [13] for Direct Speed FOC. Since flux
estimation is necessary in Direct FOC, a Gopinath flux observer [16] is constructed,
which in turn requires speed information from the MRAS. Complete insensitivity to
rotor resistance is obtained in this Direct FOC using the estimated speed. Another
adaptive sensorless scheme is proposed for Direct FOC in [17]. Instead of MRAS,
the adaption law is designed based on a Lyapunov approach. The rotor flux and
speed are estimated at the same time. However, both stator and rotor resistance
affect the estimation. A compensation is given, but this result is not accompanied by
theoretical analysis.

Observer-Based Flux Estimation

Rotor flux observers can provide an attractive means for achieving Direct FOC of
induction machines. Traditionally, flux observers are constructed in the stationary
reference frame so that rotor position or speed information is not necessary.

Reference [18] studies the flux observers assuming stator voltage, current and rotor
speed known. By studying different observers based on rotor, stator circuits, it is
shown the feedback of a corrective prediction error term can speed up the estimation
convergence and reduce the sensitivity to parameter variations compared with the pure real-time simulation of the dynamic models.

A physical approach for rotor flux observer design is proposed in [19]. In this paper, real-time model simulation is denoted as open-loop observers while those with feedback correction term are called closed-loop observers. By the proposed use of frequency response functions (FRF), the traditional current model and voltage model open-loop observers are analyzed, where their sensitivity to machine parameters and limitation of use are shown. A superior closed-loop observer combining the current and voltage model with speed invariant dynamics is proposed. The price for getting better performance is the need for rotor position which is not necessary in traditionally used voltage model open-loop observers. Following a similar approach, a closed-loop stator flux observer is presented for the stator-flux-regulated stator-flux-oriented DFO system in [20]. Different performances are shown in this observer compared with that in [19] when the machine is in different operating conditions.

Reference [21] presents a combination of the closed-loop flux observer in [19] with MRAS speed estimator [13][15]. In this sensorless scheme, the flux estimates are used by MRAS for speed estimation. However, the mechanical system model has to be included for the rotor position information which is needed in the flux observer [19]. The difficulty to acquire an accurate mechanical model is a major disadvantage of this scheme.

**Sliding Mode Flux and Speed Observer**

A sliding mode rotor flux observer is proposed in [8][9]. The error between the measured currents and estimated currents is used to construct sliding mode surfaces so that after sliding mode happens, the estimated flux values are driven to converge to
real ones exponentially. Proof of the observer stability is given and robustness against modeling uncertainties and measurement errors are shown in [8]. Current sensors and a position encoder for speed information are needed.

In [22], a sensorless control scheme is presented, in which the rotor flux and speed are observed by an adaptive sliding mode observer. After sliding mode is enforced in the switching surface, i.e. current estimation error, the flux and speed estimation convergence is shown by analysis of Lyapunov function. However, the authors are not able to prove the stability of the global control system. Further, the theoretical analysis is based on the assumption that the machine parameters are exactly known and the speed variation is slow, although the experimental results claim to show the robustness with respect to the machine parameter variation.

**Other Approaches**

There exist other technologies for speed- or position-sensorless control of induction machines. One is utilization of the anisotropic properties of the induction machine such as magnetic saliency for rotor position information [23]. Unfortunately this approach requires a custom-designed rotor construction, which limits its practical application. Another algorithm [24] is based on harmonic signal injection to the machine. By measuring the difference of the terminal impedance between the flux axis and quadrature axis at the injected high frequency, FOC can be achieved.

Since our research focuses on control and observation of induction machines from a control engineer’s point of view, these methods are not analyzed in detail here.
1.1.2 Control and Observation of Synchronous Machines

Large synchronous machines are beginning to see utilization with variable frequency power supplies from static inverters. These drives are used in high speed, high power applications, for which DC drives cannot be built [25][26]. For DC drives, the limitations generally relate to the delivery of large amounts of electrical energy to the rotor through the commutator. On the other hand, since a synchronous motor does not have to be magnetised through the inverter, as is the case with induction motors, it has a larger airgap. This is advantageous and desirable for mechanical reasons. This mechanical robustness makes synchronous motors well suited for high-power and high-speed drives [27]. Compared with a given induction machine, it is pointed out in [26] that a synchronous motor requires less volume, has lower weight and inertia, and the lowest losses, hence minimizing the need for mechanical auxiliaries.

One type of adjustable synchronous motor drive employs machines of conventional design having one excitation winding and two damper windings. Their typical applications are in those high power drives at low speed such as rolling mills [25]. Moreover, all synchronous generators follow this kind of structure. Generally speaking, control of permanent magnet synchronous machine (PMSM) is simpler than that of the synchronous machine with damper and excitation windings because of its much simpler models [25]. Our research on synchronous machine control concentrates on the machine with two damper windings and one excitation winding. Unfortunately, few references are found in this area. In [25], a simplified model was taken for synchronous machines excitation and damper windings. In [1], the complete 5th order model was given and the field oriented control principles were formulated.
The control of synchronous generators is one of the most important and widely studied problems in the area of power system control, among which are power angle and terminal voltage control. These two directly influence power system stability. Power angle stability is the power system’s ability to preserve its synchronism after large disturbances. As to voltage stability, the generator terminal voltage should be maintained a constant under normal operating conditions and regulated to the prefault steady value quickly and effectively after a fault occurs.

Traditionally, linear controllers such as PID controllers, are used in PSS (Power System Stabilizers) and AVR (Automatic Voltage Regulators) for power system excitation control [28]. These linear controllers are designed based on models linearized around the system’s operating point and work effectively under small disturbances. However, in the case of large events which cause the system states to leave the normal operating point, linearized models are no longer valid, and therefore linear controllers do not provide good control results. Therefore, there arises the necessity to develop nonlinear controllers for power system which are based on nonlinear system models.

Reference [29] is the first paper reporting feedback linearization techniques for power system control. A reduced 5th order generator model is used and the turbine model is not taken into account. Mechanical power is directly used as one of the control inputs, but it is not an available input in practice. A so-called Direct Feedback Linearization approach was proposed in [30] for power angle stability and post-fault voltage regulation, but this method is still based on a simplified generator model. In [30], it was assumed that the mechanical power inputs to the generator is a constant. At the beginning, the controller regulated the power angle and then switched to the control of generator terminal voltage. But it was not clear how to determine the
switching time, which was crucial for this method. Reference [31] designed a variable structure controller for the generator. The model used was a 4th order linearized model and linear control was used for the main voltage regulator. Sliding mode control was only used for the damping of the auxiliary signals. Nonlinear feedback linearization was also utilized in [32] for an excitation controller design in a single machine infinite-bus power system (SMIB). Here, an even more simplified 3th order model was used. An observer was necessary because feedback linearization required knowledge of all the state variables. The control objective was to only control power angle by the excitation voltage \( u_{fd} \). In [33], exact stochastic feedback linearization was used for SMIB to deal with disturbances of a random nature. Again the control was based on a much simplified 3th order system model. In [34], the authors developed an exact feedback linearization controller based on the most complete 9th order nonlinear modeling of the power system (2th order turbine model plus the 7th order generator model) to control the power angle and terminal voltage at the same time. The control inputs were the excitation voltage \( u_{fd} \) and the turbine valve input. However, this approach assumed that all the states needed for the nonlinear controller were available, which is not realistic. Besides, the feedback linearization scheme was very computationally intensive and the control law was a function of all plant parameters, hence the control performance was sensitive to the plant parameters variations.

1.2 Objectives of This Dissertation

The objective of this dissertation is to develop control and observation methods for electric machines based on sliding mode control theory, with emphasis on induction and synchronous machines.
For the induction machines, more specific objectives are to

- decouple control design based on sliding mode current controller when the currents are handled as intermediate controls for the output variables such as torque, speed and position;

- develop a sensorless control scheme by integrating controller and observer design to guarantee convergence of the estimates to the real states;

- propose a sliding mode observer for estimating the machine speed and rotor time constant simultaneously;

- apply sliding mode control to implement power converter control by pulse width modulation.

For the synchronous machine, the research objectives are to

- design current control systems following two current control schemes using sliding mode, i.e.: sliding mode PWM, and sliding mode direct current control;

- develop methods for designing torque and speed controllers;

- design power angle control and terminal voltage control for synchronous generators used in electric power system;

- develop position and speed observers for permanent magnet synchronous machines.

Finally, one last objective of this dissertation is also to experimentally validate the developed control and estimation algorithms.
1.3 Dissertation Organization

Within this dissertation, the sliding mode control approach is explored for the control and observation of electric machines, with particular interests in the sensorless control of induction machine (IM) and synchronous machine. Sliding mode control is one approach to nonlinear control that has the advantages of low sensitivity to disturbances and plant parameter variations. Also, expressing a system in sliding mode form leads to reduced order and simplified formulations. Control of electric machines is a challenging problem for control engineers due to the internally nonlinear and high-order nature of these plants. The application of sliding mode algorithms in electric machine control can, in principle, offer a very versatile and efficient solution.

Chapter 1 is the introduction section, where the state-of-the-art literature review and dissertation organization are given.

In Chapter 2, the modeling of the electric machine within our research scope is discussed, including the DC machine, the induction machine, and the synchronous machine. Modeling of the synchronous generator in power systems is also considered. It should be emphasized that it is the dynamic modelling that is studied, rather than modelling in steady state.

Chapter 3 serves as an introductory part for the sliding mode methodology. The objective of this chapter is to demonstrate the sliding mode approach for both control and observation.

Chapter 4 is the main part of this dissertation and constitutes the major contribution. Induction machine sliding mode control and observation are investigated in
detail. Four topics are presented: sliding mode current control, sliding mode sensorless control, speed and rotor time constant observer, and sliding mode control in PWM implementations.

Chapter 5 is devoted to the study of the synchronous machine. First, current control by two different sliding mode approaches is discussed. Then torque/speed sliding mode control is formulated. For the synchronous machine in generation mode, its operation in power system is considered, and power angle control and terminal voltage control are presented. Moreover, an analysis of observers for permanent magnet synchronous machine is also provided. This chapter does not include experimental results.

Finally, Chapter 6 gives the conclusions and future research directions.
CHAPTER 2

DYNAMIC MODELLING OF ELECTRIC MACHINES

2.1 Introduction

The aim of this chapter is to present dynamic models of various families of electric machines, including DC machines, induction machines and synchronous machines, before we proceed to design the control and observation algorithms for them. All the control and observation algorithms in the later chapters are based on these machine models.

First, the modelling of a separately excited DC machine is provided. Then the models are simplified to the case of constant excitation DC machine. Next, a detailed model of the induction machine is given. The resulting differential equations will be useful later for various observer and controller design. Note that all of the modelling is done in the stationary ($\alpha, \beta$) reference frame instead of the rotating ($d, q$) reference frame. After this, the modelling of synchronous machine in rotating ($d, q$) frame is given, which is followed by the modelling of permanent magnet synchronous machines (PMSM). In the end, a particular case of synchronous generator is considered, where the modelling of the synchronous generator connected to an infinite bus is discussed.
2.2 DC Machine Modelling

In this chapter, separately excited DC machines as shown in Figure 2.1 are under consideration. The corresponding machine equivalent circuits are shown in Figure 2.2. All the parameter notations are listed in Table 2.1. The differential equations describing this kind of DC machine are[35][25][36]:

\[ u_a = R_a i_a + L_a \frac{di_a}{dt} + c\lambda_f \omega \]  
(2.1)

\[ u_f = R_f i_f + L_f \frac{di_f}{dt} \]  
(2.2)

\[ J \frac{d\omega}{dt} = T - T_i \]  
(2.3)

where the electromagnetic torque

\[ T = c\lambda_f i_a \]  
(2.4)
Figure 2.2: Equivalent circuits of dc machine

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>electrical rotor speed</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>exciting flux linkage</td>
</tr>
<tr>
<td>$i_a, i_f$</td>
<td>armature and exciting currents</td>
</tr>
<tr>
<td>$u_a, u_f$</td>
<td>armature and exciting voltages</td>
</tr>
<tr>
<td>$R_a, R_f$</td>
<td>armature and exciting circuit resistances</td>
</tr>
<tr>
<td>$L_a, L_f$</td>
<td>armature and exciting circuit inductances</td>
</tr>
<tr>
<td>$T, T_l$</td>
<td>machine and load torque</td>
</tr>
<tr>
<td>$J$</td>
<td>inertia of the rotor</td>
</tr>
<tr>
<td>$E$</td>
<td>back-EMF induced in armature circuit</td>
</tr>
</tbody>
</table>

Table 2.1: Nomenclature in DC machine
and \( c\lambda_f \omega \) is usually denoted as the so-called back-EMF \( E \), i.e.

\[
E = c\lambda_f \omega
\]  

(2.5)

Also note the excitation flux \( \lambda_f = L_f i_f \), where \( L_f \) is the self-inductance of the excitation winding.

For a DC motor with constant excitation (\( \lambda_f = \text{const.} \)), (2.2) can be disregarded, so (2.1)(2.3) are sufficient to describe the system behavior. After defining \( c\lambda_f \) as a new constant \( k \), the model equations for constant excitation DC motor is

\[
L_a \frac{d i_a}{dt} = -R_a i_a - k \omega + u_a
\]  

(2.6)

\[
J \frac{d \omega}{dt} = T - T_i = k i_a - T_i
\]  

(2.7)

All the following design and analysis are based on the model (2.6)(2.7).

### 2.3 Induction Machine Modelling

The winding arrangement of a 2-pole, 3-phase, wye-connected, symmetrical induction machine is shown in Figure 2.3 and Figure 2.4. Under the commonly used assumptions, equations for an induction machine in the orthogonal stator frame ((\( \alpha, \beta \)) coordinate) can be expressed as

**Mechanical equations**

\[
\frac{d \omega}{dt} = \frac{P}{J}(T - T_i)
\]  

(2.8)

\[
T = \frac{3P}{2} \frac{L_m}{L_r} (i_{\beta r} \lambda_{ar} - i_{\alpha r} \lambda_{\beta r})
\]

**Rotor flux equations**

\[
\frac{d \lambda_{ar}}{dt} = -\eta \lambda_{ar} - \omega \lambda_{\beta r} + \eta L_m i_{\alpha s}
\]  

(2.9)
| \( \omega \) | electrical rotor speed |
| \( \lambda_{ar}, \lambda_{br} \) | \( \alpha, \beta \) axis fluxes |
| \( i_{as}, i_{bs} \) | \( \alpha, \beta \) axis stator currents |
| \( u_{as}, u_{bs} \) | \( \alpha, \beta \) axis stator voltages |
| \( i_{as}, u_{bs}, i_{cs} \) | \( a, b, c \) three phase stator currents |
| \( u_{as}, u_{bs}, u_{cs} \) | \( a, b, c \) three phase stator voltages |
| \( T, T_l \) | machine and load torque |
| \( R_r, R_s \) | rotor and stator resistance |
| \( L_r, L_s, L_m \) | rotor, stator and mutual inductance |
| \( J \) | inertia of the rotor |
| \( P \) | number of pole pairs |

Table 2.2: Nomenclature in induction machine

\[
\frac{d\lambda_{br}}{dt} = -\eta \lambda_{br} + \omega \lambda_{ar} + \eta L_m i_{bs}
\]  

(2.10)

Stator current equations

\[
\frac{di_{as}}{dt} = \beta \eta \lambda_{ar} + \beta \omega \lambda_{br} - \gamma i_{as} + \frac{1}{\sigma L_s} u_{as}
\]  

(2.11)

\[
\frac{di_{bs}}{dt} = \beta \eta \lambda_{br} - \beta \omega \lambda_{ar} - \gamma i_{bs} + \frac{1}{\sigma L_s} u_{bs}
\]  

(2.12)

with stator voltage and current vector defined as

\[
\begin{bmatrix}
  u_{as} \\
  u_{bs}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
  e_{aa} & e_{ba} & e_{ca} \\
  e_{a\beta} & e_{b\beta} & e_{c\beta}
\end{bmatrix} \begin{bmatrix}
  u_{as} \\
  u_{bs} \\
  u_{cs}
\end{bmatrix}
\]  

(2.13)

\[
\begin{bmatrix}
  i_{as} \\
  i_{bs} \\
  i_{cs}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
  e_{aa} & e_{ba} & e_{ca} \\
  e_{a\beta} & e_{b\beta} & e_{c\beta}
\end{bmatrix} \begin{bmatrix}
  i_{as} \\
  i_{bs} \\
  i_{cs}
\end{bmatrix}
\]  

(2.14)

where

\[
\begin{bmatrix}
  e_{aa} & e_{ba} & e_{ca} \\
  e_{a\beta} & e_{b\beta} & e_{c\beta}
\end{bmatrix} = \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix}
\]  

(2.15)

\( \omega \) is the electrical rotor angle velocity; the two-dimensional vectors \( \lambda^T = (\lambda_{ar}, \lambda_{br}) \), \( i^T = (i_{as}, i_{bs}) \) and \( u^T = (u_{as}, u_{bs}) \) represent rotor fluxes, stator currents and voltages
Figure 2.3: Two-pole, 3-phase, Y-connected induction machine

Figure 2.4: Magnetic axes of a three phase induction machine
in the \((\alpha, \beta)\) coordinate system respectively; \(T\) and \(T_l\) are the torques developed by the motor and the load respectively; \(J\) is inertia of the rotor; \(P\) is the number of pole pairs; \((u_{as}, u_{bs}, u_{cs})^T\) and \((i_{as}, i_{bs}, i_{cs})^T\) are three phase voltage and current respectively.

\[
\eta, \beta, \sigma \text{ and } \gamma \text{ are positive constants defined as } \eta = \frac{R_r}{L_r}, \sigma = 1 - \frac{L_s^2}{L_s L_r}, \beta = \frac{L_m}{\sigma L_s L_r}, \\
\gamma = \frac{1}{\sigma L_s} (R_s + \frac{L_s^2}{L_r} R_r), \text{ where } R_r \text{ and } R_s \text{ are rotor and stator resistances, } L_r \text{ and } L_s \text{ are rotor and stator inductances, } L_m \text{ is the mutual inductance.}
\]

All the above parameter and machine state nomenclature is listed in Table 2.2.

2.4 Synchronous Machine Modelling

The analysis and design of a control system for an electric drive calls for a dynamic model of the machine. With a synchronous machine, this may be of considerable complexity. As shown in Figure 2.5, the synchronous machine under consideration here is assumed to have the most general form, i.e. three stator windings, one field winding (excitation winding) and two damper windings. It has non-uniform air gap and rotor windings are non-symmetric. The machine parameter notation is listed in Table 2.3.

The voltages across the three stator phases of the synchronous machine can be written

\[
\begin{bmatrix}
u_{as} \\
u_{bs} \\
u_{cs}
\end{bmatrix} = R_s
\begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix} + \frac{d}{dt}
\begin{bmatrix}
\lambda_{as} \\
\lambda_{bs} \\
\lambda_{cs}
\end{bmatrix},
\]

(2.16)

where \(u_{as}, u_{bs}\) and \(u_{cs}\) are the three phase stator voltages, \(i_{as}, i_{bs}\) and \(i_{cs}\) are the three phase stator currents, \(\lambda_{as}, \lambda_{bs}\) and \(\lambda_{cs}\) are the flux linkages in the three stator windings. \(R_s\) is the stator resistance.
Figure 2.5: Magnetic axes and winding arrangement in synchronous machine
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>electrical rotor speed</td>
</tr>
<tr>
<td>( \lambda_{a_s}, \lambda_{b_s}, \lambda_{c_s} )</td>
<td>three phase stator flux linkage</td>
</tr>
<tr>
<td>( i_{a_s}, i_{b_s}, i_{c_s} )</td>
<td>three phase stator currents</td>
</tr>
<tr>
<td>( u_{a_s}, u_{b_s}, u_{c_s} )</td>
<td>three phase stator voltages</td>
</tr>
<tr>
<td>( u_{fd}, u_{kd}, u_{kq} )</td>
<td>voltages on field, ( d )-axis damper and ( q )-axis damper windings</td>
</tr>
<tr>
<td>( i_{fd}, i_{kd}, i_{kq} )</td>
<td>currents in field, ( d )-axis damper and ( q )-axis damper windings</td>
</tr>
<tr>
<td>( \lambda_{fd}, \lambda_{kd}, \lambda_{kq} )</td>
<td>flux linkages in field, ( d )-axis and ( q )-axis damper windings</td>
</tr>
<tr>
<td>( u_{ds}, u_{qs} )</td>
<td>two-phase stator voltages in (( d, q )) rotationaly frame</td>
</tr>
<tr>
<td>( i_{ds}, i_{qs} )</td>
<td>two-phase stator currents in (( d, q )) rotationaly frame</td>
</tr>
<tr>
<td>( \lambda_{ds}, \lambda_{qs} )</td>
<td>two-phase stator flux linkages in (( d, q )) rotationaly frame</td>
</tr>
<tr>
<td>( T )</td>
<td>machine torque</td>
</tr>
<tr>
<td>( R_s, R_{fd} )</td>
<td>resistances in stator, field windings</td>
</tr>
<tr>
<td>( R_{kd}, R_{kq} )</td>
<td>resistances in ( d )-axis and ( q )-axis damper windings</td>
</tr>
<tr>
<td>( L_{ds}, L_{qs}, L_f )</td>
<td>inductances in ( d, q ) stator axis, field windings</td>
</tr>
<tr>
<td>( L_{kd}, L_{kq} )</td>
<td>inductances in ( d )-axis and ( q )-axis damper windings</td>
</tr>
<tr>
<td>( L_{ds}, L_{kd} )</td>
<td>leakage inductances in stator, field windings</td>
</tr>
<tr>
<td>( L_{kq}, L_{kq} )</td>
<td>leakage inductances in ( d )-axis and ( q )-axis damper windings</td>
</tr>
<tr>
<td>( L_{rd}, L_{mq} )</td>
<td>inductances in ( d )-axis and ( q )-axis</td>
</tr>
<tr>
<td>( \theta_r )</td>
<td>angle of rotor position</td>
</tr>
<tr>
<td>( P )</td>
<td>number of pole pairs</td>
</tr>
</tbody>
</table>

Table 2.3: Nomenclature in synchronous machine
For the field, d-axis damper and q-axis damper windings, the voltage equations can be modeled as

\[
\begin{align*}
    u_{fd} &= R_{fd}i_{fd} + \frac{d}{dt}\lambda_{fd} \\
    u_{kd} &= R_{kd}i_{kd} + \frac{d}{dt}\lambda_{kd} = 0 \\
    u_{kq} &= R_{kq}i_{kq} + \frac{d}{dt}\lambda_{kq} = 0,
\end{align*}
\]  

(2.17) (2.18) (2.19)

where \( u_{fd}, u_{kd} \) and \( u_{kq} \) are the voltages on the field winding, d-axis damper winding and q-axis damper winding respectively, \( i_{fd}, i_{kd} \) and \( i_{kq} \) are the currents in the three windings, \( \lambda_{fd}, \lambda_{kd} \) and \( \lambda_{kq} \) are their corresponding flux linkages. \( R_{fd}, R_{kd} \) and \( R_{kq} \) are the windings’ resistances, which are of different values.

Note that the zero sequence voltage is zero in a balanced system. So \( u_{as}, u_{bs} \) and \( u_{cs} \) can be regarded as either phase voltages or terminal voltages. Also note \( u_{kd} \) and \( u_{kq} \) equal zero because these damper windings are short circuited. Because most of the inductances are functions of the rotor position, the flux linking each winding heavily depends on the rotor position, which makes the expressions for the winding voltages quite complicated.

As in the modeling of induction machines, the synchronous machine can be modeled in the orthogonal stationary reference frame \((\alpha,\beta)\) after the following transformation

\[
\begin{bmatrix}
    f_{\alpha s} \\
    f_{\beta s}
\end{bmatrix} = \Gamma_{\alpha\beta}^{abc} \begin{bmatrix}
    f_{as} \\
    f_{bs} \\
    f_{cs}
\end{bmatrix}
\]  

(2.20)

where

\[
\Gamma_{\alpha\beta}^{abc} = \frac{2}{3} \begin{bmatrix}
    1 & -1/2 & -1/2 \\
    0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix}
\]  

(2.21)

and \( f \) can be voltage, current or flux in the stator winding. For synchronous machines, this transformation does not exist in the rotor winding because the rotor does not
have the necessary symmetry. Furthermore, the so-called Park’s Transformation [1] is performed on the stator variable to get the following Park’s Equations

\[
\begin{align*}
    u_{ds} &= R_s i_{ds} + \frac{d}{dt} \lambda_{ds} - \omega \lambda_{qs} \\
    u_{qs} &= R_s i_{qs} + \frac{d}{dt} \lambda_{qs} + \omega \lambda_{ds}
\end{align*}
\]  

(2.22)

(2.23)

The Park’s Transformation is defined as

\[
\begin{bmatrix}
    f_{qs} \\
    f_{ds}
\end{bmatrix} = \Gamma_{qd} \begin{bmatrix}
    f_{\alpha s} \\
    f_{\beta s}
\end{bmatrix}
\]

(2.24)

where

\[
\Gamma_{qd} = \begin{bmatrix}
    \cos \theta_r & \sin \theta_r \\
    \sin \theta_r & -\cos \theta_r
\end{bmatrix}
\]

(2.25)

where \( \theta_r \) denotes the angle of rotor position. From the following, we can find that a great simplification in the mathematical description of the synchronous machine is obtained after the Park’s Transformation. The basic reason is that the Park’s Transformation makes the inductance matrix of the flux linking become a matrix of constants, not dependent on the rotor position any more.

After Park’s Transformation, the synchronous machine is modeled in the rotating reference frame \((d,q)\) by

\[
\begin{align*}
    u_{ds} &= R_s i_{ds} + \frac{d}{dt} \lambda_{ds} - \omega \lambda_{qs} \\
    u_{qs} &= R_s i_{qs} + \frac{d}{dt} \lambda_{qs} + \omega \lambda_{ds} \\
    u_{jd} &= R_d i_{jd} + \frac{d}{dt} \lambda_{jd} \\
    u_{kd} &= R_d i_{kd} + \frac{d}{dt} \lambda_{kd} = 0 \\
    u_{kq} &= R_d i_{kq} + \frac{d}{dt} \lambda_{kq} = 0
\end{align*}
\]  

(2.26)

(2.27)

(2.28)

(2.29)

(2.30)
where

\[
\lambda_{ds} = L_{ds}i_{ds} + L_{rd}(i_{fd} + i_{kd}) \quad (2.31)
\]

\[
\lambda_{qs} = L_{qs}i_{qs} + L_{rq}i_{kd} \quad (2.32)
\]

\[
\lambda_{fd} = L_{fd}i_{fd} + L_{rd}(i_{ds} + i_{kd}) \quad (2.33)
\]

\[
\lambda_{kd} = L_{kd}i_{kd} + L_{rd}(i_{ds} + i_{fd}) \quad (2.34)
\]

\[
\lambda_{kq} = L_{kq}i_{kd} + L_{rq}i_{qs} \quad (2.35)
\]

Note that \(L_{rd}\) and \(L_{rq}\) are the \(d\)-axis and \(q\)-axis inductances, \(L_{ds} = L_{rd} + L_{ds}\), \(L_{qs} = L_{rq} + L_{qs}\), \(L_f = L_{rd} + L_{fd}\), \(L_kd = L_{rd} + L_{kd}\), \(L_{kq} = L_{rq} + L_{kd}\) in which \(L_{ds}\), \(L_{qs}\), \(L_{kd}\), \(L_{kq}\) are the leakage inductances of the stator, the field, \(d\)-axis damper and \(q\)-axis damper windings respectively.

The torque output for the synchronous machine can be shown to be

\[
T = \frac{3}{2}P(\lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}) = \frac{3}{2}P[(L_{ds} - L_{qs})i_{qs}i_{ds} + L_{rd}i_{qs}i_{fd} + L_{rd}i_{qs}i_{kd} - L_{rq}i_{ds}i_{kq}] \quad (2.36)
\]

Again \(P\) is the number of pole pairs. From physical analysis, this produced torque is composed of three components:

1) reluctance torque = \(\frac{3}{2}P(L_{ds} - L_{qs})i_{qs}i_{ds}\). This part exists due to the protruding (salient) rotor poles.

2) excitation torque = \(\frac{3}{2}P(L_{rd}i_{qs}i_{fd}\). This part is produced because of the excitation of the field winding with dc current.

3) damping torque = \(\frac{3}{2}P(L_{rd}i_{qs}i_{kd} - L_{rd}i_{ds}i_{kq}\). This torque appears because of the damper windings.
2.5 Permanent Magnet Synchronous Machine (PMSM) Modelling

The permanent magnet synchronous machine (PMSM) is shown in Figure 2.6. It can be modelled in \((\alpha, \beta)\) stationary reference frame [1] and [36] as

\[
\frac{d\lambda_{\alpha_s}}{dt} = -R_s i_\alpha + u_\alpha \quad (2.37)
\]

\[
\frac{d\lambda_{\beta_s}}{dt} = -R_s i_\beta + u_\beta \quad (2.38)
\]

where

\[
\lambda_{\alpha_s} = L_0 i_\alpha - L_1 (i_\alpha \cos 2\theta_r + i_\beta \sin 2\theta_r) + \lambda_m \sin \theta_r \quad (2.39)
\]

\[
\lambda_{\beta_s} = L_0 i_\beta - L_1 (i_\alpha \sin 2\theta_r - i_\beta \cos 2\theta_r) - \lambda_m \cos \theta_r \quad (2.40)
\]

where \(\theta_r\) is the electrical angle of the rotor with respect to the stator position, \(\lambda_m\) is the amplitude of the flux linkage established by the permanent magnet, \(R_s\) is the stator resistance, \(L_{ds}\) and \(L_{qs}\) are \(d, q\) axis inductances. And we define \(L_0 = \frac{1}{2}(L_{ds} + L_{qs})\) and \(L_1 = \frac{1}{2}(L_{ds} - L_{qs})\) with \(L_1 < 0\). Note we are using the stator flux here instead of the rotor flux.

In order to get the PMSM model with the stator currents as state variables, let us differentiate (2.39) and (2.40):

\[
\begin{bmatrix}
\dot{\lambda}_{\alpha_s} \\
\dot{\lambda}_{\beta_s}
\end{bmatrix}
= \begin{bmatrix}
L_0 - L_1 \cos 2\theta_r & -L_1 \sin 2\theta_r \\
-L_1 \sin 2\theta_r & L_0 + L_1 \cos 2\theta_r
\end{bmatrix}
\begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix}
+ \begin{bmatrix}
2L_1 \omega_r \sin 2\theta_r & -2L_1 \omega_r \cos 2\theta_r \\
-2L_1 \omega_r \cos 2\theta_r & -2L_1 \omega_r \sin 2\theta_r
\end{bmatrix}
\begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix}
+ \lambda_m
\begin{bmatrix}
\omega_r \cos \theta_r \\
\omega_r \sin \theta_r
\end{bmatrix}
\]
Figure 2.6: Permanent magnet synchronous machine
Substitution of the above into (2.37)(2.38) produces

\[
\begin{bmatrix}
\dot{i}_\alpha \\
\dot{i}_\beta \\
\end{bmatrix} = \begin{bmatrix}
L_0 - L_1 \cos 2\theta_r & -L_1 \sin 2\theta_r \\
-L_1 \sin 2\theta_r & L_0 + L_1 \cos 2\theta_r \\
\end{bmatrix}^{-1} \begin{bmatrix}
u_\alpha \\
\nu_\beta \\
\end{bmatrix} - R_s \begin{bmatrix}
\dot{i}_\alpha \\
\dot{i}_\beta \\
\end{bmatrix} - \lambda_m \begin{bmatrix}
\omega_r \cos \theta_r \\
\omega_r \sin \theta_r \\
\end{bmatrix} \\
\begin{bmatrix}
2L_1\omega_r \sin 2\theta_r & -2L_1\omega_r \cos 2\theta_r \\
-2L_1\omega_r \cos 2\theta_r & -2L_1\omega_r \sin 2\theta_r \\
\end{bmatrix} \begin{bmatrix}
\dot{i}_\alpha \\
\dot{i}_\beta \\
\end{bmatrix}
\]}

Note that

\[
\begin{bmatrix}
L_0 - L_1 \cos 2\theta_r & -L_1 \sin 2\theta_r \\
-L_1 \sin 2\theta_r & L_0 + L_1 \cos 2\theta_r \\
\end{bmatrix}^{-1} = \frac{1}{L_0 - L_1} \begin{bmatrix}
L_0 + L_1 \cos 2\theta_r & L_1 \sin 2\theta_r \\
L_1 \sin 2\theta_r & L_0 - L_1 \cos 2\theta_r \\
\end{bmatrix}
\]

Then we have the PMSM modelling in terms of stator current

\[
\begin{bmatrix}
\dot{i}_\alpha \\
\dot{i}_\beta \\
\end{bmatrix} = \begin{bmatrix}
f_1 & f_2 \\
f_5 & f_6 \\
\end{bmatrix} \begin{bmatrix}
\dot{i}_\alpha \\
\dot{i}_\beta \\
\end{bmatrix} + \begin{bmatrix}
f_3 & f_4 \\
f_7 & f_8 \\
\end{bmatrix} \begin{bmatrix}
\nu_\alpha \\
\nu_\beta \\
\end{bmatrix} - \begin{bmatrix}
g_1 \\
g_2 \\
\end{bmatrix} \lambda_m
\]

(2.41)

where \(d = L_0^2 - L_1^2 \neq 0\) and

\[
\begin{align*}
f_1 &= \frac{1}{d}[(L_0 + L_1 \cos 2\theta_r)(-2L_1\omega_r \sin 2\theta_r - R_s) + L_1 \sin 2\theta_r(2L_1\omega_r \cos 2\theta_r)] \\
f_2 &= \frac{1}{d}[(L_0 + L_1 \cos 2\theta_r)(2L_1\omega_r \cos 2\theta_r) + L_1 \sin 2\theta_r(2L_1\omega_r \sin 2\theta_r - R_s)] \\
f_3 &= \frac{1}{d}[(L_0 - L_1 \cos 2\theta_r)(2L_1\omega_r \sin 2\theta_r) + L_1 \sin 2\theta_r(-2L_1\omega_r \sin 2\theta_r - R_s)] \\
f_4 &= \frac{1}{d}[(L_0 - L_1 \cos 2\theta_r)(2L_1\omega_r \sin 2\theta_r - R_s) + L_1 \sin 2\theta_r(2L_1\omega_r \cos 2\theta_r)] \\
f_5 &= \frac{1}{d}[(L_0 + L_1 \cos 2\theta_r)\omega_r \cos \theta_r + L_1\omega_r \sin 2\theta_r \sin \theta_r] \\
f_6 &= \frac{1}{d}[(L_0 - L_1 \cos 2\theta_r)\omega_r \cos \theta_r + (L_0 - L_1 \cos 2\theta_r)\omega_r \sin \theta_r] \\
g_1 &= \frac{1}{d}[(L_0 + L_1 \cos \theta_r)\omega_r \cos \theta_r + L_1\omega_r \sin 2\theta_r \sin \theta_r] \\
g_2 &= \frac{1}{d}[(L_0 - L_1 \cos \theta_r)\omega_r \cos \theta_r + (L_0 - L_1 \cos 2\theta_r)\omega_r \sin \theta_r]
\end{align*}
\]

As can be seen, \(f_1, f_2, f_3, f_4, f_5, f_6, f_7, g_1\) and \(g_2\) are functions of \(\sin 2\theta_r, \cos 2\theta_r, \omega_r \sin 2\theta_r, \omega_r \cos 2\theta_r, \sin \theta_r\) and \(\cos \theta_r\).
2.6 Modelling of a Synchronous Generator Connected to an Infinite Bus

The system of a synchronous generator with connection to an infinite bus is shown in Figure 2.7. In generator modeling, the positive direction of stator currents are defined as the direction flowing out of the generator terminals. So we will change the signs of $i_{ds}$ and $i_{qs}$ in (2.26)-(2.35) for the generator modeling, i.e. replace $i_{ds}$ and $i_{qs}$ with $-i_{ds}$ and $-i_{qs}$. Another change we make in the generator modeling is that the dynamic model is represented in terms of $i_{ds}$, $i_{qs}$, $\lambda_{fd}$, $\lambda_{kd}$ and $\lambda_{kq}$ instead of $i_{ds}$, $i_{qs}$, $i_{fd}$, $i_{kd}$ and $i_{kq}$. Although the currents are a good choice of variables in many types of analysis, it has been determined that use of flux linkages results in a more compact, stable solution of machine transients since the flux linkages can not change instantaneously and they generally vary much slower than the currents. Furthermore, the sensitivity of the fluxes with respect to the parameter variations is lower than that of the currents.

After substituting (2.31)-(2.35) into (2.26)-(2.30), we have

$$u = Gi + L \frac{d}{dt}i$$  \hspace{1cm} (2.42)
where \( \mathbf{u} = [u_{ds} \ u_{qs} \ u_{fd} \ 0 \ 0]^T \), \( i = [i_{ds} \ i_{qs} \ i_{fd} \ i_{kd} \ i_{kq}]^T \) and

\[
G = \begin{bmatrix}
R_s & -\omega L_{qs} & 0 & 0 & -\omega L_{mq} \\
-\omega L_{ds} & R_s + \omega L_{rd} & \omega L_{rd} & 0 & 0 \\
0 & 0 & R_{fd} & 0 & 0 \\
0 & 0 & 0 & R_{kd} & 0 \\
0 & 0 & 0 & 0 & R_{kq}
\end{bmatrix}
\] (2.43)

\[
L = \begin{bmatrix}
L_{ds} & 0 & 0 & 0 & 0 \\
0 & L_{qs} & 0 & 0 & 0 \\
0 & 0 & L_{rd} & 0 & 0 \\
0 & 0 & 0 & L_{kd} & 0 \\
0 & 0 & 0 & 0 & L_{kq}
\end{bmatrix}
\] (2.44)

To make the variable transformation, define \( \mathbf{x} = [i_{ds} \ i_{qs} \ \lambda_{fd} \ \lambda_{kd} \ \lambda_{kq}]^T \)

\[
\mathbf{x} = T \mathbf{i} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
L_{rd} & 0 & L_f & L_{rd} & 0 \\
L_{rd} & 0 & L_{rd} & L_{kd} & 0 \\
0 & L_{mq} & 0 & 0 & L_{kq}
\end{bmatrix} \mathbf{i}
\] (2.45)

From (2.42) and (2.45), the model equations in terms of \( \mathbf{x} \) are obtained

\[
\frac{d}{dt} \mathbf{x} = -T L^{-1} G T^{-1} \mathbf{x} + T L^{-1} \mathbf{u}
\]

After calculation, the inverse of matrix \( T \)

\[
T^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
-\frac{L_{rd} L_{rd} L_{fd}}{k_1} & 0 & \frac{L_{kd}}{k_1} & \frac{L_{rd}}{k_1} & 0 \\
-\frac{L_{rd} L_{rd} L_{fd}}{k_1} & 0 & \frac{L_{kd}}{k_1} & \frac{L_{rd}}{k_1} & 0 \\
0 & -\frac{L_{mq}}{L_{eq}} & 0 & 0 & 1
\end{bmatrix} \Delta = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
-b_1 & 0 & b_3 & -b_4 & 0 \\
-b_2 & 0 & -b_3 & b_5 & 0 \\
0 & -b_6 & 0 & 0 & b_7
\end{bmatrix}
\]

where \( k_1 = L_{kd} L_f - L_{rd}^2 \). Note \( b_1, b_2, b_3, b_4, b_5, b_6 \) and \( b_7 \) are all positive constants.

Finally, the model equations are represented as

\[
\frac{d}{dt} \begin{bmatrix}
i_{ds} \\
i_{qs} \\
\lambda_{fd} \\
\lambda_{kd} \\
\lambda_{kq}
\end{bmatrix} = \begin{bmatrix}
c_1 & \omega c_2 & c_3 & c_4 & \omega c_5 \\
\omega c_6 & c_7 & \omega c_8 & c_9 & c_{10} \\
c_{11} & 0 & c_{12} & c_{13} & 0 \\
c_{14} & 0 & c_{15} & c_{16} & 0 \\
0 & c_{17} & 0 & 0 & c_{18}
\end{bmatrix} \begin{bmatrix}
i_{ds} \\
i_{qs} \\
\lambda_{fd} \\
\lambda_{kd} \\
\lambda_{kq}
\end{bmatrix} + \begin{bmatrix}
-a_1 u_{ds} - a_2 u_{fd} \\
a_4 u_{qs} \\
0 \\
0
\end{bmatrix}
\] (2.46)
Note $a_1$, $a_2$ and $a_4$ in the control inputs change to opposite signs because of the opposite current directions in the generator. It will be shown later that the model equations in terms of $i_{ds}$, $i_{qs}$, $\lambda_{fd}$, $\lambda_{kd}$ and $\lambda_{kq}$ are really simplified significantly compared with the model equations in terms of $i_{ds}$, $i_{qs}$, $i_{fd}$, $i_{kd}$ and $i_{kq}$.

After modelling the synchronous generator, it is necessary to model the relationship between the synchronous generator and the power system. Consider the system of Figure 2.7 where a synchronous generator is connected to an infinite bus through a transmission line having resistance $R_e$ and inductance $L_e$. By inspection, we can write

\[
\begin{bmatrix}
  u_{as} \\
  u_{bs} \\
  u_{es}
\end{bmatrix} = L_e \frac{d}{dt} \begin{bmatrix}
  i_{as} \\
  i_{bs} \\
  i_{es}
\end{bmatrix} + R_e \begin{bmatrix}
  i_{as} \\
  i_{bs} \\
  i_{es}
\end{bmatrix} + \begin{bmatrix}
  u_{a\infty} \\
  u_{b\infty} \\
  u_{e\infty}
\end{bmatrix}
\]  

(2.47)

Let us define $\delta$ to be the power angle of the generator. Physically, $\delta$ denotes the angular position of the generator rotor in electrical radians with respect to the synchronous rotating reference from the power system (i.e. the infinite bus). So

\[
\delta = \omega t - \omega_s t
\]

(2.48)

where $\omega_s$ is the synchronous angular frequency of the infinite bus in $rad/s$. After differentiation, the so-called swing equation is obtained from (2.48), which describes the dynamics of the power angle $\delta$

\[
\frac{d\delta}{dt} = \omega - \omega_s
\]

(2.49)

Applying the Park’s transformation on (2.47), we have

\[
\begin{bmatrix}
  u_{qs} \\
  u_{ds}
\end{bmatrix} = L_e \Gamma_{qd} \frac{d}{dt} \begin{bmatrix}
  i_{\alpha s} \\
  i_{\beta s}
\end{bmatrix} + R_e \begin{bmatrix}
  i_{qs} \\
  i_{ds}
\end{bmatrix} + \begin{bmatrix}
  u_{q\infty} \\
  u_{d\infty}
\end{bmatrix}
\]  

(2.50)

Differentiation of $i_{qds} = \Gamma_{qd} i_{\alpha s}$ produces

\[
\frac{d}{dt} \begin{bmatrix}
  i_{qs} \\
  i_{ds}
\end{bmatrix} = \Gamma_{qd} \frac{d}{dt} \begin{bmatrix}
  i_{\alpha s} \\
  i_{\beta s}
\end{bmatrix} + \omega \begin{bmatrix}
  -i_{ds} \\
  i_{qs}
\end{bmatrix}
\]  

(2.51)
For the infinite bus, assuming it is a balanced three phase system, i.e.

\[
\begin{bmatrix}
u_{a\infty} \\
u_{b\infty} \\
u_{c\infty}
\end{bmatrix}
= V_{\infty}
\begin{bmatrix}
\cos \omega_s t \\
\cos(\omega_s t - \frac{2\pi}{3}) \\
\cos(\omega_s t + \frac{2\pi}{3})
\end{bmatrix}
\]

After Park’s Transformation, it becomes

\[
\begin{bmatrix}
u_{q\infty} \\
u_{d\infty}
\end{bmatrix}
= V_{\infty}
\begin{bmatrix}
\cos \delta \\
\sin \delta
\end{bmatrix}
\]

(2.52)

Combining (2.50)(2.51)(2.52), we have the following equation, which gives the constraint between the generator terminal voltage \(u_{q\infty}\) and the generator current \(i_{q\infty}\) for a given power angle \(\delta\):

\[
\begin{bmatrix}
u_{qs} \\
u_{ds}
\end{bmatrix}
= L_c \frac{d}{dt} \begin{bmatrix} 
i_{qs} \\
i_{ds}
\end{bmatrix}
+ L_c \omega \begin{bmatrix} 
i_{ds} \\
-\ni_{qs}
\end{bmatrix}
+ \begin{bmatrix}
R_c 
i_{qs} \\
R_c 
i_{ds}
\end{bmatrix}
+ V_{\infty}
\begin{bmatrix}
\cos \delta \\
\sin \delta
\end{bmatrix}
\]

(2.53)

After substituting (2.53) into (2.46) and combining with (2.48) and the rotor speed dynamics, we have a complete 7-th order model of synchronous generator connected to an infinite bus

\[
\begin{bmatrix}
\dot{\delta} \\
\dot{\omega} \\
\dot{i_{ds}} \\
\dot{i_{qs}} \\
\dot{\lambda_{fd}} \\
\dot{\lambda_{kd}} \\
\dot{\lambda_{kq}}
\end{bmatrix}
= \begin{bmatrix}
c_1 & \omega c_2 & c_3 & \omega c_4 & c_5 & \omega c_6 & c_7 & c_8 & \omega c_9 & c_{10}
\end{bmatrix}
\begin{bmatrix}
i_{ds} \\
i_{qs} \\
\lambda_{fd} \\
\lambda_{kd} \\
\lambda_{kq}
\end{bmatrix}
+ \begin{bmatrix}
-a_1' V_{\infty} \sin \delta - a_2' u_{fd} \\
-a_3' V_{\infty} \cos \delta \\
0 \\
0
\end{bmatrix}
\]

(2.54)

where

\[
c_1 = \frac{c_1 - R_c a_1}{1 + a_1 L_c}, \quad c_2 = \frac{c_2 + L_c a_1}{1 + a_1 L_c}, \quad c_3 = \frac{c_3}{1 + a_1 L_c}, \quad c_4 = \frac{c_4}{1 + a_1 L_c}, \quad c_5 = \frac{c_5}{1 + a_1 L_c}, \quad c_6 = \frac{c_6 - L_c a_4}{1 + a_4 L_c},
\]

\[
c_7 = \frac{c_7 - R_c a_4}{1 + a_4 L_c}, \quad c_8 = \frac{c_8}{1 + a_4 L_c}, \quad c_9 = \frac{c_9}{1 + a_4 L_c}, \quad c_{10} = \frac{c_{10}}{1 + a_4 L_c}.
\]

Note (2.54) is in a nonlinear state space form, where \(V_{\infty}\) is considered a constant and \(u_{fd}\) is the only control input for the system. \(T_m\) denotes the mechanical driving
torque and is regarded as a constant. The sign convention of the torque in the rotor speed dynamics is different from that in motor equations. Here a positive $T_m$ accelerates the generator shaft, whereas a positive $T$ is a decelerating torque.

Finally, we need to rewrite the torque equation (2.36) in terms of $i_{ds}$, $i_{qs}$, $\lambda_{fd}$, $\lambda_{kd}$ and $\lambda_{kq}$. Combining (2.33)(2.34)(2.35) and (2.36), we have

$$T = \frac{3P}{2} \left( d_1 i_{ds} i_{qs} + d_2 i_{ds} \lambda_{kq} + d_3 i_{qs} \lambda_{fd} + d_4 i_{qs} \lambda_{kd} \right) \tag{2.55}$$

where $d_1 = L_{ds} - L_{qs} + \frac{L_m^2}{L_{kq}} - \frac{L_{md}}{L_{f}L_{kd} - L_{md}} (L_{kd}L_{md} + L_{fd}L_{md} - 2L_{md}^2)$, $d_2 = \frac{-L_{md}}{L_{kq}}$, $d_3 = \frac{-L_{md}L_{kd} - L_{md}^2}{L_{f}L_{kd} - L_{md}}$, $d_4 = \frac{-L_{md}L_{fd} - L_{md}^2}{L_{f}L_{kd} - L_{md}}$.

### 2.7 Summary

In this chapter, we analyzed the models of the electric machines, including DC machine, induction machine, synchronous machine, permanent magnet synchronous machine and synchronous generator. All the work in the later chapters is based on the models provided here. From the DC machine modelling, sliding mode approach for controller and observer design will be shown as a demonstration. Then in the chapter on induction machine, both the sensorless control and parameter/state estimation are designed in the framework of the induction machine modelling in stationary ($\alpha$, $\beta$) reference frame. In the synchronous machine area, the control and observation issues for both motor and generator will be studied, which employs the modelling provided here.
CHAPTER 3

DC MACHINE SLIDING MODE CONTROL AND OBSERVATION

3.1 Introduction

Direct Current (DC) motors have been dominating the field of adjustable speed drives for a long time. Even today, they are still very popular because of the excellent operational properties and control characteristics, although they have the disadvantage that the mechanical commutator restricts the power and speed of the motor, increases the inertia and requires periodic maintenance. With the development of AC adjustable speed drives, the commutators can be eliminated, but it is at the cost of the increased complexity. This is part of the reasons why the fast advancing AC drives cannot supplant DC motors completely till now.

DC motor control by sliding mode was investigated in [37][38]. This chapter serves as an introductory part for the control and observation by sliding mode approach. Different sliding mode control strategies are formulated for different control objectives, e.g. torque control, speed control and position control. The ideas of sliding mode observers are also clarified with several different observers designed for the above different control requirements.
3.2 Sliding Mode Controller

Here, three sliding mode controllers (torque, speed, position) are designed for DC motors. In this section, all the state variables are assumed available. Later, we will extend to the situation with observers.

**Torque Control**

Let us consider the torque control problem by defining switching function

$$ s = T^* - T $$

(3.1)

as the error between the reference torque $T^*$ and the real torque $T$ developed by the motor. Our objective here is to design the discontinuous control

$$ u_a = U_0 sgn(s) $$

(3.2)

where $U_0$ is high enough to enforce the sliding mode in $s = 0$, which implies that the real torque $T$ tracks the reference torque $T^*$. To satisfy the existence condition for sliding mode $s \dot{s} < 0[39]$, we follow the analysis

$$ \dot{s} = \dot{T}^* - k_i \dot{a} \\
= \dot{T}^* + \frac{k_R}{L_a} i_a + \frac{k_i^2}{L_a} \omega - \frac{k}{L_a} u_a \\
= f(t) - \frac{k}{L_a} U_0 sgn(s) $$

where $f(t) = \dot{T}^* + \frac{k_R}{L_a} i_a + \frac{k_i^2}{L_a} \omega$ depending on the reference signal and system states but not on the control input. For $U_0 > \frac{k}{k} |f(t)|$,

$$ s \dot{s} = s f(t) - \frac{k}{L_a} U_0 |s| < 0 $$

(3.3)

so sliding mode can be enforced in $s = 0$. 

39
Speed Control

In speed control, the sliding surface and discontinuous control are designed as

\[
s(t) = c(\omega^* - \omega) + \frac{d}{dt}(\omega^* - \omega) \tag{3.4}
\]

\[
u_a(t) = U_0 \text{sgn}(s) \tag{3.5}
\]

This design makes the speed tracking error \(\omega^* - \omega\) converge to zero exponentially after sliding mode occurs in \(s = 0\), where the positive constant \(c\) decides the converging rate.

Following a similar analysis, combining \((2.6)(2.7)(3.4)\) produces

\[
\dot{s} = c\dot{\omega}^* + \ddot{\omega}^* - \frac{c}{J}(k_i - T_i) + \frac{1}{J}\dot{T}_i + \frac{k}{JL_a}(R_i + k\omega) - \frac{k}{JL_a}u_a
\]

\[
= g(t) - \frac{k}{JL_a}u_a
\]

where \(g(t) = c\dot{\omega}^* + \ddot{\omega}^* - \frac{c}{J}(k_i - T_i) + \frac{1}{J}\dot{T}_i + \frac{k}{JL_a}(R_i + k\omega)\). If \(U_0 > \frac{JL_a}{k}|g(t)|\), \(s\dot{s} < 0\), then sliding mode will happen.

Position Control

To consider the position control issue, it is necessary to augment the motor equations \((2.6)(2.7)\) with

\[
\frac{d\theta}{dt} = \omega \tag{3.6}
\]

where \(\theta\) denotes the rotor position.

Similarly, the switching function \(s\) for the position control is selected as

\[
s(t) = (\ddot{\theta}^* - \ddot{\theta}) + c_1(\dot{\theta}^* - \dot{\theta}) + c_2(\theta^* - \theta) \tag{3.7}
\]

and the discontinuous control is

\[
u_a(t) = U_0 \text{sgn}(s) \tag{3.8}
\]
Combining (2.6)(2.7)(3.6)(3.7), we have

\[
\dot{s} = h(t) - \frac{k}{JL_a} \dot{u}_a
\]

(3.9)

where \( h(t) = \dot{\omega}^* + c_1 \dot{\omega}^* + c_2 \omega^* - \frac{c_T}{J} (k i_a - T_i) - c_2 \omega + \frac{k}{JL_a} (R_a i_a + k \omega) \).

Choice of \( U_0 \) as \( U_0 > \frac{J L_a}{k} |h(t)| \) makes \( ss < 0 \), which means that sliding mode can happen in \( s = 0 \). In sliding mode \( s = 0 \), and with properly chosen \( c_1 \) and \( c_2 \), we can make the velocity tracking error \( \omega^* - \omega \) converge to zero. The converging transient behavior is influenced by \( c_1 \) and \( c_2 \).

**3.3 Sliding Mode Observer**

As already mentioned, in the above sliding mode controller design all the state variables needed in the control implementation are assumed to be available. Obviously, this is not practical or desirable in a real control systems. In this section, we use the sliding mode observers to estimate unavailable system states. Of course, other kinds of observers may also fulfill the task. Here, we are only to demonstrate the basic ideas of the sliding mode observers.

**Observer I: Speed Observer based on Current Measurement**

Design the current observer as

\[
L_a \frac{d\hat{i}_a}{dt} = -R_a \dot{i}_a - u_a - l_1 \text{sgn} (\bar{i}_a)
\]

(3.10)

where \( \hat{i}_a \) is the estimated current, \( \bar{i}_a \) denotes the estimation error, i.e. \( \bar{i}_a = \hat{i}_a - i_a \) and \( l_1 \) is the positive observer gain to be chosen. From (2.6)(3.10), the error dynamics is obtained

\[
L_a \frac{d\bar{i}_a}{dt} = -R_a \bar{i}_a - k \omega - l_1 \text{sgn} (\bar{i}_a)
\]

(3.11)
To enforce the sliding mode in $\tilde{\tau}_a = 0$, let us check the existence condition $\tilde{\tau}_a \dot{\tilde{\tau}}_a < 0$,

$$\tilde{\tau}_a \dot{\tilde{\tau}}_a = \frac{1}{L_a}(-R_a \dot{\tilde{\tau}}_a^2 + k\omega \tilde{\tau}_a - l_1 \tilde{\tau}_a)$$

If $l_1 > |k\omega - R_a \tilde{\tau}_a|$, then $\tilde{\tau}_a \dot{\tilde{\tau}}_a < 0$ so that $\tilde{\tau}_a$ will decay to zero in sliding mode.

Now let us investigate the system motion in sliding mode by looking at the error dynamics (3.11). Using the equivalent control concept [38], after setting $\tilde{\tau}_a = 0$ and $\dot{\tilde{\tau}}_a = 0$, we have $k\omega - (l_1 \text{sgn}(\tilde{\tau}_a))_{eq} = 0$, from which we obtain the speed information

$$\omega = \frac{(l_1 \text{sgn}(\tilde{\tau}_a))_{eq}}{k}$$

(3.12)

As in [38], the equivalent control $(l_1 \text{sgn}(\tilde{\tau}_a))_{eq}$ can be obtained by passing the discontinuous value $l_1 \text{sgn}(\tilde{\tau}_a)$ through a low pass filter.

**Observer II: Current Observer based on Speed Measurement**

Assuming the speed is measured, the current observer is designed as

$$L_a \frac{d \tilde{i}_a}{dt} = -R_a \dot{\tilde{\tau}}_a - k\omega + u_a$$

(3.13)

From (2.6), we get the error dynamics

$$L_a \frac{d \tilde{\tau}_a}{dt} = -R_a \tilde{\tau}_a.$$ 

(3.14)

So the estimation error $\tilde{\tau}_a$ will converge to zero exponentially and the converging rate is decided by the system parameter $\frac{R_a}{L_a}$.

To increase the converging rate of the above observer, let us define another variable $\dot{i}_a = i_a + l\omega$, where $l$ is the parameter to be chosen. After combining (2.6)(2.7), we can get the differential equation with respect to the new variable $\dot{i}_a$,

$$L_a \frac{d \dot{i}_a}{dt} = -(R_a - \frac{lk}{J})\dot{i}_a - (k + \frac{l^2k}{J} - R_a l)\omega + u_a - \frac{IT_i}{J}$$

(3.15)
• For the simplicity, we assume the load torque $T_l$ is available. Then the observer for $\dot{i}_a'$ is designed as

$$L_a \frac{d\dot{i}_a'}{dt} = -(R_a - \frac{lk}{J})\dot{i}_a' - (k + \frac{\dot{\omega}k}{J} - R_a\dot{\omega}) + u_a - \frac{T_l}{J}. \quad (3.16)$$

And the corresponding observation error dynamics is

$$L_a \frac{d\tilde{i}_a}{dt} = -(R_a - \frac{lk}{J})\tilde{i}_a \quad (3.17)$$

Since $l$ is a parameter to be chosen, then compared with (3.14), we can see that as long as $l < 0$, the observer (3.16) is able to converge at a faster rate than (3.13). After $\dot{i}_a'$ is observed, $i_a$ can be obtained by $i_a = \dot{i}_a' - l\dot{\omega}$.

• In practical situation, the load torque $T_l$ is unknown, then the observer (3.16) is not implementable any more. However, it makes sense to assume that $T_l$ is varying slowly, i.e. $\dot{T}_l = 0$. So the system equations can be obtained with $\dot{i}_a'$, $\omega$, $T_l$ as state variables and $\omega$ as the output.

$$\begin{bmatrix} \dot{i}_a' \\ \dot{\omega} \\ \dot{T}_l \end{bmatrix} = \begin{bmatrix} -R_a + \frac{kl}{J} & R_a l - \frac{l^2 k}{J} - k & -\frac{l}{J} \\ \frac{k}{J} & -\frac{l^2 k}{J} - \frac{k}{J} & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{i}_a' \\ \omega \\ T_l \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_a$$

$$y = \omega$$

It is easy to show that the above system is observable and the parameter $l$ can be chosen to speed up the observer converging rate.

**Observer III: Load Torque Observer based on Current and/or Speed Measurement**

• Assuming both the speed and current information are available,
- A load torque observer can be designed as

\[ J \frac{d\hat{\omega}}{dt} = ki_a - l_2sgn(\omega) \]  

(3.18)

From (2.7), the error dynamics for \( \bar{\omega} \) is

\[ J \frac{d\bar{\omega}}{dt} = -l_2sgn(\omega) + T_i \]  

(3.19)

From the sliding mode existence condition \( \bar{\omega}\hat{\omega} < 0 \), we conclude that if \( l_2 > |T_i| \), the sliding mode occurs in \( \bar{\omega} = 0 \). Once again, employing the equivalent control concept, we obtain the load torque

\[ T_i = (l_2 sgn(\omega))_{eq} \]  

(3.20)

- Assuming \( T_i \) is varying slowly, i.e. \( \dot{T}_i = 0 \), then the dynamic system can be represented in standard state space equations with \( x = [\omega \ T_i] \), \( u = i_a \), \( y = \omega \).

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= cx
\end{align*}
\]

where

\[
A = \begin{bmatrix}
0 & -1/J \\
0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
k \\
0
\end{bmatrix}, \quad C = [1 \ 0]
\]

Since \( \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1/J \end{bmatrix} \) is nonsingular, an observer can be designed for the above system to obtain \( T_i \).

- Assuming only speed \( \omega \) is available and \( \dot{T}_i = 0 \), the state space equations can be obtained with respect to \( x = [i_a \ \omega \ T_i] \), \( u = u_a \), \( y = \omega \) in which

\[
A = \begin{bmatrix}
-k_2/a & -k_1/\tau & 0 \\
-k_2/\tau & 0 & -1/\tau \\
0 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
1/L_a \\
0 \\
0
\end{bmatrix}, \quad C = [0 \ 1 \ 0]
\]
It is easy to check that the observability matrix
\[
\begin{bmatrix}
C \\
CA \\
CA^2
\end{bmatrix}
\]
is nonsingular, so an observer for \( T_i \) can be designed.

- Assuming only current \( i_a \) is available and \( \dot{T}_i = 0 \), the state space equations can be obtained with respect to \( x = [i_a \quad \omega \quad T_i] \), \( u = u_a, y = i_a \) in which \( C = [1 \quad 0 \quad 0] \).

This system is also observable by checking the observability matrix.

### 3.4 Control of DC Machines with Observers

Unlike Section 3.2, we do not assume that all the state information needed in the controller is available. Then the observers are necessary to be used with the controllers together.

**Torque Control**

Since \( T = ki_a \), the only information needed in torque control is the armature current \( i_a \). There are two situations:

- If the current sensor is available, then no observers are needed.
- If only the speed sensor is available, then Observer II is used.

**Speed Control**

For speed control, both the speed \( \omega \) and speed derivative \( \dot{\omega} \) are required. From (2.7), \( \dot{\omega} \) can be calculated by \( \dot{\omega} = \frac{1}{J}ki_a - \dot{T}_i \), which makes the load torque \( T_i \) necessary to observer. Three different situations are as follows:

- If the current sensor is available, first use Observer I for speed, then use Observer III for load torque so that \( \dot{\omega} \) can be calculated.
• If the speed sensor is available, first use Observer II for current, then use Observer III for load torque.

• If both the speed and current sensors are available, use Observer III directly for load torque observation, then calculate \( \dot{\omega} \).

Position Control

We assume the position sensor is available always. So in position control, the situation is exactly the same as speed control.

3.5 Summary

The objective of this chapter is to demonstrate sliding mode approach for both control and observation. We take the simplest DC motor control and observation issue just as an example although the problem here has been simplified. First sliding mode controllers are constructed for different control objectives such as torque, speed, position control. Then the observers by sliding mode approach are proposed, after which the combination of the controllers and observers are given for the practical control systems.
CHAPTER 4

INDUCTION MACHINE SLIDING MODE CONTROL AND OBSERVATION

4.1 Current Regulation by Sliding Mode Controller

The reason why we pay much attention to current tracking is because the basic block of any high performance drive system is the current regulator. In our opinion, any system should have good current tracking regardless what control methodology is applied. Current regulators for ac drives are more complex than dc drives because ac current regulators must control both the amplitude and phase of the stator current. In addition, the steady state currents are ac, not dc, currents so the straightforward application of conventional PI controllers as applied in dc drives can not be expected to provide performance comparable with that of PI current regulators in dc drives. So we started our work from the current regulator by sliding mode approach.
4.1.1 Control System Analysis and Simulation

Sliding mode current regulator for the inductive load

The three phase inductive load can be modeled as follows:

\[ u_a = Ri_a + L \frac{di_a}{dt} \]  \hspace{1cm} (4.1)

\[ u_b = Ri_b + L \frac{di_b}{dt} \]  \hspace{1cm} (4.2)

\[ u_c = Ri_c + L \frac{di_c}{dt} \]  \hspace{1cm} (4.3)

In the orthogonal stationary reference frame ((\(\alpha, \beta\)) coordinate), the voltage and current vectors are defined as:

\[
\begin{bmatrix}
    u_{\alpha s} \\
    u_{\beta s}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
    1 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
    0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
    u_a \\
    u_b \\
    u_c
\end{bmatrix}
\]  \hspace{1cm} (4.4)

\[
\begin{bmatrix}
    i_{\alpha s} \\
    i_{\beta s}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
    1 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
    0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix}
\]  \hspace{1cm} (4.5)

So the three phase inductive load in the stationary reference frame can be modeled in state space as:

\[ \dot{i}_\alpha = -\frac{R}{L}i_\alpha + \frac{1}{L}u_a \]  \hspace{1cm} (4.6)

\[ \dot{i}_\beta = -\frac{R}{L}i_\beta + \frac{1}{L}u_\beta \]  \hspace{1cm} (4.7)

Let us consider the current control problem by defining switching functions as the error between the real current \(i\) and the reference current \(i^*\), i.e.

\[ s_\alpha = i^*_\alpha - i_\alpha \]  \hspace{1cm} (4.8)

\[ s_\beta = i^*_\beta - i_\beta \]  \hspace{1cm} (4.9)

Design the discontinuous control as

\[ u_\alpha = U_0 \text{sign}(s_\alpha) \]  \hspace{1cm} (4.10)

\[ u_\beta = U_0 \text{sign}(s_\beta) \]  \hspace{1cm} (4.11)
In practical implementation, the discontinuous function $\text{sign}(.)$ is replaced by continuous approximation and $U_0$ should be selected such that DC bus voltage is used to full extent. To enforce the sliding mode, the DC bus voltage should be high enough such that the conditions for sliding mode to exist $s_\alpha s_\alpha^* < 0$ and $s_\beta s_\beta^* < 0$ hold. Since

$$s_\alpha s_\alpha^* = s_\alpha(i_\alpha^* - \frac{R}{L} i_\alpha) - \frac{U_0}{L}\left|s_\alpha\right|$$

$$s_\beta s_\beta^* = s_\beta(i_\beta^* - \frac{R}{L} i_\beta) - \frac{U_0}{L}\left|s_\beta\right|,$$

(4.12) (4.13)

if $U_0$ satisfies

$$U_0 > \max\{L_\alpha^2 i_\alpha^* + R_\alpha i_\alpha, L_\beta^2 i_\beta^* + R_\beta i_\beta\}$$

(4.14)

then $s_\alpha s_\alpha^* < 0$ and $s_\beta s_\beta^* < 0$, i.e. sliding mode can be enforced, which makes real current $i_\alpha, i_\beta$ track $i_\alpha^*, i_\beta^*$.

### Sliding mode current regulator for induction machines

The same approach can be applied to induction machines. For the induction machines, the stator current equations are:

$$\frac{di_{\alpha s}}{dt} = \beta \eta \lambda_{\alpha r} + \beta \omega \lambda_{\beta r} - \gamma i_{\alpha s} + \frac{1}{\sigma L_s} u_{\alpha s}$$

(4.15)

$$\frac{di_{\beta s}}{dt} = \beta \eta \lambda_{\beta r} - \beta \omega \lambda_{\alpha r} - \gamma i_{\beta s} + \frac{1}{\sigma L_s} u_{\beta s}$$

(4.16)

Design the discontinuous control similar to (4.10)(4.11), then

$$s_\alpha s_\alpha^* = s_\alpha(i_\alpha^* - \beta \eta \lambda_{\alpha r} - \beta \omega \lambda_{\beta r} + \gamma i_{\alpha s}) - \frac{U_0}{\sigma L_s}\left|s_\alpha\right|$$

(4.17)

$$s_\beta s_\beta^* = s_\beta(i_\beta^* - \beta \eta \lambda_{\beta r} - \beta \omega \lambda_{\alpha r} + \gamma i_{\beta s}) - \frac{U_0}{\sigma L_s}\left|s_\beta\right|$$

(4.18)

If $U_0$ satisfies

$$U_0 > \max\{\sigma L_s(i_\alpha^* - \beta \eta \lambda_{\alpha r} - \beta \omega \lambda_{\beta r} + \gamma i_{\alpha s}), \sigma L_s(i_\beta^* - \beta \eta \lambda_{\beta r} - \beta \omega \lambda_{\alpha r} + \gamma i_{\beta s})\}$$

(4.19)

then current tracking in induction machines is achieved.
Simulation of current regulator for inductive load

The system block diagram is shown in Figure 4.1 with the Simulink block diagram displayed in Figure 4.2. The inductive load #1 parameters are taken as $R = 0.234\Omega$, $L = 150\mu H$ as shown in Table 4.1. In order to simulate the real experiment situation, we set the simulation with fixed step $100\mu s$.

The simulation results in Figure 4.3 show that the current regulator works well.

Simulation of current regulator for induction machines

The same simulation is conducted for the induction machine. The machine parameters obtained by testing are shown in Table 4.2.

The system block diagram and simulink block diagram for current control of induction machine are shown in Figures 4.4 and 4.5. The simulation results are shown in Figure 4.6.
Figure 4.2: Simulink block diagram for current control of inductive load

Figure 4.3: Simulation results of current tracking on inductive load #1
### Table 4.1: Inductive load data

<table>
<thead>
<tr>
<th>Inductive load #1</th>
<th>$R = 0.234 \Omega$</th>
<th>$L = 150 \mu H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductive load #2</td>
<td>$R = 0.0008 \Omega$</td>
<td>$L = 400 \mu H$</td>
</tr>
</tbody>
</table>

### Table 4.2: Induction machine test data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \text{ KW}$</td>
<td>$R_s = 0.00587 \Omega$</td>
</tr>
<tr>
<td>$16 \text{ poles}$</td>
<td>$R_r = 0.00695 \Omega$</td>
</tr>
<tr>
<td>$J = 2.6068 N.m.s^2$</td>
<td>$L_s = 0.19776 mH$</td>
</tr>
<tr>
<td>$\text{Max speed } 6000 rpm$</td>
<td>$L_r = 0.19522 mH$</td>
</tr>
<tr>
<td>$\text{DC bus voltage } 48v$</td>
<td>$L_m = 0.18510 mH$</td>
</tr>
</tbody>
</table>

### Figure 4.4: System block diagram for current control of induction machine
Figure 4.5: Simulink block diagram for current control of induction machine

Figure 4.6: Simulation results of current tracking on induction machine
4.1.2 Experimental Results

We tried the proposed current regulator with both dSPACE DS1102 and DS1103 controllers. Since DS1103 has a much more powerful processing capability, the sampling time can be reduced substantially using DS1103. However, because of the limitation of PWM inverter switching frequency, the highest sampling time for the system is chosen as 50μs. No significant difference is observed in the experimental results between sampling times of 50μs and 100μs.

The chattering phenomenon, shown in Figures 4.7 and 4.8 happens in both the inductive load and induction machine. In Figure 4.7, channels 3 and 4 show the measured current signals $i_α$ and $i_β$, respectively. In Figure 4.8, channels 2 and 3 show the reference and real currents, respectively. It is important to point out that this chattering is not caused by the load or machine parameter inaccuracy because the parameters are not needed in the control system at all. It follows from the existence
Figure 4.8: Chattering phenomenon in current control of induction machine

conditions for sliding mode (inequalities (4.14) and (4.19)), that the parameter inaccuracy does not play any role in this chattering issue if we know their ranges. Even in the sensorless torque control we propose in the later sections, inaccurate parameters will only produce inaccuracy in the estimation. They will not destroy the tracking property.

The only possible reason for the chattering are the unmodeled dynamics in the system, e.g. the lag or transport delay in the inverter or the sensors. For verification of this hypothesis, a simulation was run with a delay included. Simulation of the system with delay (shown in Figures 4.9 and 4.10) confirmed our assumption. The only difference from Figure 4.5 is that a transport delay block is added after the controller block to simulate the delay in the real system. As shown in Figure 4.11, chattering does happen in the simulation.
Frame Transformation \( \Gamma_{abc} \)

\[
\begin{align*}
    &i_\alpha^* \\
    &i_\beta^* \\
    &\rightarrow \quad u_\alpha^* \\
    &\rightarrow \quad u_\beta^* \\
    &\rightarrow \quad u_\gamma^*
\end{align*}
\]

\[
\begin{align*}
    &i_\alpha \\
    &i_\beta \\
    &\rightarrow \quad u_\alpha \\
    &\rightarrow \quad u_\beta \\
    &\rightarrow \quad u_\gamma
\end{align*}
\]

\[
\begin{align*}
    &\Gamma_{\alpha\beta} \\
    &\Gamma_{abc}
\end{align*}
\]

**Figure 4.9:** System block diagram for delayed induction machine system

**Figure 4.10:** Simulink block diagram for delayed induction machine system
Figure 4.11: Chattering in induction machine current tracking simulation
We may observe that the results above have high level of chattering and low accuracy. Our explanation is that it may be caused by imperfection of the inverter. Usually, the inverter resistance and dynamics could be disregarded. Unfortunately, this is not the case here because the special test motor has very low inductance.

To further study the chattering problem, we worked on another inductive load (Inductive load #2) with higher inductance. The parameters are listed in Table 4.1. We found that on inductive load #2, the current tracking results in Figure 4.12 are much better compared with Figure 4.7, i.e. the chattering is much less. The reason is because these two loads have different $R$ and $L$ parameters. Inductive load #1 has $R = 0.234 \Omega, L = 150 \mu H$, while inductive load #2 has $R = 0.0008 \Omega, L = 400 \mu H$. It is obvious that the time constant of inductive load #2 $\tau_2 = 0.5$ is much larger than that of inductive load #1 $\tau_1 = 6.4 \times 10^{-4}$. Actually this is the basic reason for the chattering to happen in our system. If the system time constant is so small that the delay in the system can not be neglected, the chattering will probably happen.

As before, we also try the simulation to verify our points. The simulation system is as shown in Figure 4.13. The above two sets of parameters ($R = 0.234 \Omega, L = 150 \mu H$ vs. $R = 0.0008 \Omega, L = 400 \mu H$) are tested respectively, whose results are shown in Figures 4.14 and 4.15. As we can see, the different parameters really affect the chattering.

This testing confirms our assumption regarding unmodeled dynamics/delay of the inverter. Since the inductance here is much higher than that in the first experiment, the unmodeled dynamics of the inverter may be disregarded. Indeed, in the second load, when trying the current controller directly without the observer, we can increase
Figure 4.12: Current tracking experiment on inductive load #2

Figure 4.13: Simulink diagram for delayed inductive load system
Figure 4.14: Current tracking on inductive load #1
Figure 4.15: Current tracking on inductive load #2
Figure 4.16: Control loop with auxiliary observer loop

considerably the gain in the continuous approximation of the \( \text{sign}(.) \) function. It does not result in chattering, which happens for the first load with low inductance.

4.1.3 Chattering Problem

We propose an asymptotic current observer in the control loop to eliminate chattering. In addition to chattering suppression, from practical point of view, it is desirable to design a control system with no current transducers because the transducers always introduce noise into the system.

For chattering prevention by observers, the key idea is to generate ideal sliding mode in an auxiliary observer loop rather than in the main control loop. Ideal sliding mode can be enforced in the observer loop since it is entirely implemented in the control software and thus does not contain any unmodeled dynamics. The main loop follows the observer loop according to the observer dynamics. The basic idea of chattering suppression using state observers is demonstrated in Figure 4.16. As it is shown in the block diagram, an asymptotic observer serves as bypass for high frequency component of the control.
Figure 4.17: Simulation of the delayed system with observer

The simulation system for the current regulator with an asymptotic observer is shown in Figure 4.17. The principal difference between the results of Figure 4.10 and of Figure 4.17 lies in that we use the current output from the observer for the feedback instead of the real current from the machine. Of course, the transport delay block still exists. The simulation results are shown in Figures 4.18 and 4.19, which comply with our theoretical analysis. The conclusion is that as long as the observer can work properly and sliding mode exists in the observer loop, we can make the real and reference currents close to each other. The main objective of this simulation is to demonstrate that state observers can be used to suppress chattering (compare Figure 4.18 and Figure 4.11).
Figure 4.18: Simulation results of the delayed system with observer
Figure 4.19: Simulation results of the delayed system with observer
4.1.4 Experiments with Observer

We conducted the experiments for the induction machine with the current regulator closed-loop with the observed currents. As shown in Figure 4.20, the results are not good enough. Channel 1 is the reference current $i_a^r$. Channel 2 is the observed current $i_a$. Channel 3 is the real current $i_a$. And channel 4 is the real current $i_b$.

One possible reason is the machine parameter inaccuracy. The other reason is unmodeled dynamics of the inverter. As we know, the voltage signal used by the observer is taken from the controller in the software directly, i.e. free of the inverter nonideal factors such as IGBT voltage drop. Actually the real voltage outputs are different from the command voltage signal because of the inverter.
<table>
<thead>
<tr>
<th>voltage</th>
<th>current</th>
<th>calculated load inductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.46V</td>
<td>20A</td>
<td>0.4594mH</td>
</tr>
<tr>
<td>6.10V</td>
<td>40A</td>
<td>0.4042mH</td>
</tr>
<tr>
<td>8.59V</td>
<td>60A</td>
<td>0.3795mH</td>
</tr>
<tr>
<td>11.03V</td>
<td>80A</td>
<td>0.3658mH</td>
</tr>
<tr>
<td>13.44V</td>
<td>100A</td>
<td>0.3565mH</td>
</tr>
</tbody>
</table>

Table 4.3: Experiment data of inductive load #2 on 60Hz bench

At first, we assumed the discrepancy between the observed current and the real current is mainly caused by the parameter inaccuracy of the induction machine. So we tried to tune the parameters in the observer. Unfortunately, this effort did not help much. Under this circumstance, we realized that the imperfection of the inverter modeling plays the main role. To get better observer performance, it is necessary to take the inverter dynamics/parameters into account.

After the above experiment, we came to the conclusion that the main reason of lower observer accuracy is the neglected inverter dynamics and parameters. By this reasoning, we improved our modeling by taking into account the resistance of the inverter, which resulted in much improvement of observer accuracy.

Since the IGBT voltage drop is considered as the main nonideal factor in the inverter for the observer, it is reasonable to bring an equivalent resistor into the inverter model. It is important that the resistance should be varying with the current.

At first, we tested the inductive load #2 on 60Hz bench to evaluate its resistance and inductance. In addition to $R = 0.0008\Omega$, the experiment data is collected in Table 4.3, from which we can calculate the load inductance.
<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Calculated Inverter Resistance ($R_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.37</td>
<td>20</td>
<td>0.2046 Ω</td>
</tr>
<tr>
<td>8.13</td>
<td>40</td>
<td>0.1341 Ω</td>
</tr>
<tr>
<td>10.11</td>
<td>60</td>
<td>0.08816 Ω</td>
</tr>
<tr>
<td>13.51</td>
<td>80</td>
<td>0.07854 Ω</td>
</tr>
<tr>
<td>16.46</td>
<td>100</td>
<td>0.09082 Ω</td>
</tr>
</tbody>
</table>

Table 4.4: Experiment data of inductive load #2 on inverter bench

Then we switched to the inverter experiment bench. By taking the same experiment, we get the corresponding voltage and current data, from which we can calculate the equivalent inverter resistance $R_i$ in Table 4.4.

After the inverter equivalent resistance $R_i$ is obtained, it is included into the machine stator resistor such that the modified stator resistance is taken as $R'_s = R_s + R_i$ and used in the observer.

Figure 4.21 shows the experiment results for the inductive load #1 with and without the inverter resistance. In both figures, channel 3 is the real phase current, channel 1 is the observed phase current and channel 2 is the phase voltage. Here we are just checking how the observer works with the inverter resistance so the observer works out of the controller loop independently. As we can see, with the inverter resistance, the observer works much better, i.e. the observed current is much closer to the real current.

Then we closed the controller loop for the inductive load #1 with feedback signals from the observer output. Channel 1, 2, 3 are the observed, reference, real currents respectively. From Figure 4.22, it is shown that on one hand, chattering is suppressed effectively since the sliding mode is happening in the observer loop, on the other
Figure 4.21: Current observation without and with inverter resistor (on inductive load #1)

Channel 1: observed phase current
Channel 2: phase voltage
Channel 3: real phase current
Figure 4.22: Controller/observer closed-loop experiment (on inductive load #1)
Channel 1: observed current
Channel 2: reference current
Channel 3: real current

hand, with the inverter resistance, the observer works well enough (compared with Figure 4.21). So the sliding mode current regulator works much better after using the observer and including the inverter equivalent resistance. We also tested the system in different frequency and amplitude for the reference current. All the experimental results are acceptable.

Finally, we tested the control system on the induction machine instead of the inductive load. One of the results is demonstrated in Figure 4.23. As we can see, the results are better than those without observers (Figure 4.20). By the combination of controller and observer, the chattering problem is solved. The set of experiments have confirmed that the main reason for the chattering and low accuracy is the imperfection
Figure 4.23: Observer results for the induction machine
Channel 1: observed current
Channel 2: reference current
Channel 3: real current
Channel 4: phase voltage

of the inverter, which can not be disregarded for low inductance machines. The experiments also let us develop the method to solve these problems.
4.2 Sensorless Sliding Mode Control

4.2.1 Control Objective

It is known that the rotor flux is needed for the implementation of torque or speed control. Unfortunately, the rotor flux can not be measured directly. If the angular speed is available, the flux can be estimated with a second order observer and its convergence is guaranteed for any speed [5]. But if no information about the mechanical variables is acquired, the design of the observer is no more a trivial problem. Our objective is to design a torque or speed tracking controller for the induction motor given by (2.8)-(2.12) without measurement of mechanical variables. So we have the following problem statement: Design a flux/speed observer to estimate the flux and speed simultaneously based on the measurement of the stator currents and voltages and then design a corresponding controller to guarantee that the real torque or speed tracks the desired torque or speed.

4.2.2 Sliding Mode Flux/Speed Observer

Applying the same structure of (2.9)(2.10)(2.11)(2.12), the sliding mode rotor flux, stator current and speed observer is proposed as

\[
\begin{align*}
\frac{d\hat{\lambda}_r}{dt} &= -\eta \hat{\lambda}_r - \omega \hat{\lambda}_{\beta r} + \eta L_m i_{\alpha s} + C \hat{\lambda}_{\beta r} \mu \\
\frac{d\hat{i}_{\alpha s}}{dt} &= \beta \eta \hat{\lambda}_r + \beta \omega \hat{\lambda}_{\beta r} - \gamma i_{\alpha s} + \frac{1}{\sigma L_s} u_{\alpha s} - \beta \hat{\lambda}_{\alpha r} \mu \\
\frac{d\hat{i}_{\beta s}}{dt} &= \beta \eta \hat{\lambda}_{\beta r} - \beta \omega \hat{\lambda}_r - \gamma i_{\beta s} + \frac{1}{\sigma L_s} u_{\beta s} - \beta \hat{\lambda}_{\beta r} \mu
\end{align*}
\]

where continuous time function $\dot{\lambda}_r, \dot{\lambda}_{\beta r}$ represent the estimated rotor flux and $\dot{i}_{\alpha s}, \dot{i}_{\beta s}$ represent the estimated stator currents. $C$ is a parameter to be selected. The estimate
of the angle speed $\dot{\omega}$ and the auxiliary variable $\mu$ are discontinuous parameters given by

$$\dot{\omega} = \omega_0 \text{sign} s_n, \quad \mu = \mu_0 \text{sign} s_\mu, \quad \omega_0, \quad \mu_0 \text{ are constants}$$

where $s_n$ and $s_\mu$ are nonlinear functions of the stator current errors and estimated rotor flux $s_n = (i_{\beta s} - i_{\beta s}) \hat{\lambda}_{ar} - (i_{\alpha s} - i_{\alpha s}) \hat{\lambda}_{\beta r}$, $s_\mu = (i_{\alpha s} - i_{\alpha s}) \hat{\lambda}_{ar} + (i_{\beta s} - i_{\beta s}) \hat{\lambda}_{\beta r}$. The method of selecting $s_n$ and $s_\mu$ is nothing but the conventional approach to control design based on equation in the frame rotating with rotor flux [1]. It enables one to decouple the design problem of making the estimates $\hat{i}_{\alpha s}$ and $\hat{i}_{\beta s}$ track $i_{\alpha s}$ and $i_{\beta s}$ into two independent ones.

First, it will be shown that there exist constant values $\omega_0$ and $\mu_0$ such that sliding mode occurs in the surfaces of $s_n = 0$ and $s_\mu = 0$ and as a result, the estimation errors

$$\tilde{\lambda}_{\alpha s} = \hat{i}_{\alpha s} - i_{\alpha s}, \quad \tilde{\lambda}_{\beta s} = \hat{i}_{\beta s} - i_{\beta s}$$

are equal to zero (see Section 4.2.3). Then a sliding mode controller will be designed (Section 4.2.4) and it will be shown that the flux estimation errors

$$\tilde{\lambda}_{ar} = \hat{\lambda}_{ar} - \lambda_{ar}, \quad \tilde{\lambda}_{\beta r} = \hat{\lambda}_{\beta r} - \lambda_{\beta r}$$

will tend to zero and the average value of the discontinuous function $\dot{\omega}$ tends to the real speed $\omega$.

4.2.3 Analysis of Current Tracking

To analyze convergence of the estimates to the real values for the proposed observer structure, we first need to analyze the stator current tracking property. As it follows from the equations (2.9) (2.10) (2.11) (2.12) and (4.20) (4.21) (4.22) and (4.23), the
sliding mode observer equations with respect to the errors $\tilde{\lambda}_{\alpha s}, \tilde{\lambda}_{\beta s}, \tilde{\lambda}_{\alpha r}$ and $\tilde{\lambda}_{\beta r}$ can be written as

$$\frac{d\tilde{\lambda}_{\alpha r}}{dt} = -\eta \tilde{\lambda}_{\alpha r} - \omega \tilde{\lambda}_{\beta r} - \omega \tilde{\lambda}_{\alpha r} + C \tilde{\lambda}_{\beta r} \mu$$ (4.26)$$

$$\frac{d\tilde{\lambda}_{\beta r}}{dt} = -\eta \tilde{\lambda}_{\beta r} + \omega \tilde{\lambda}_{\alpha r} + \omega \tilde{\lambda}_{\alpha r} - C \tilde{\lambda}_{\alpha r} \mu$$ (4.27)$$

$$\frac{d\tilde{\alpha}_s}{dt} = \beta\eta \tilde{\lambda}_{\alpha r} + \beta \omega \tilde{\lambda}_{\beta r} + \beta \omega \tilde{\lambda}_{\alpha r} - \beta \tilde{\alpha}_r \mu$$ (4.28)$$

$$\frac{d\tilde{\beta}_s}{dt} = \beta\eta \tilde{\lambda}_{\beta r} - \beta \omega \tilde{\lambda}_{\alpha r} - \beta \omega \tilde{\lambda}_{\alpha r} - \beta \tilde{\beta}_r \mu$$ (4.29)

Combining (4.20) (4.21) (4.28) and (4.29) yields

$$\dot{s}_n = \dot{\tilde{\lambda}}_{\beta s} \tilde{\lambda}_{\alpha r} - \dot{\tilde{\lambda}}_{\alpha s} \tilde{\lambda}_{\beta r} + \dot{\tilde{\lambda}}_{\alpha s} \tilde{\lambda}_{\alpha r} - \tilde{\alpha}_s \tilde{\lambda}_{\beta r}$$ (4.30)$$

$$\dot{\tilde{\lambda}}_{\beta s} \tilde{\lambda}_{\alpha r} - \dot{\tilde{\alpha}}_s \tilde{\lambda}_{\beta r} = -\beta \omega \| \tilde{\lambda} \|^2 + \beta \omega \| \tilde{\lambda} \|^2 + \beta \eta e_2 - \beta \omega e_1$$ (4.31)$$

$$\dot{\tilde{\alpha}}_s \tilde{\lambda}_{\alpha r} - \tilde{\alpha}_s \dot{\tilde{\lambda}}_{\beta r} = -\omega s_\mu + \eta L_m(\tilde{\beta}_s \tilde{\alpha}_s - \tilde{\alpha}_s \tilde{\beta}_s) + C s_\mu \mu$$ (4.32)

Then

$$\dot{s}_n = -(\beta \| \tilde{\lambda} \|^2 + s_\mu) \omega_0 \text{sign}s_n + f(\omega, \tilde{\alpha}_s, \tilde{\beta}_s, e_1, e_2)$$ (4.33)

where $\| \tilde{\lambda} \| = \sqrt{\tilde{\lambda}_{\alpha r}^2 + \tilde{\lambda}_{\beta r}^2}$, $f(\omega, \tilde{\alpha}_s, \tilde{\beta}_s, e_1, e_2) = \beta \omega \| \tilde{\lambda} \|^2 + \eta L_m(\tilde{\beta}_s \tilde{\alpha}_s - \tilde{\alpha}_s \tilde{\beta}_s) + \beta \eta e_2 - \beta \omega e_1 + C s_\mu \mu$ and

$$e_1 = \tilde{\alpha}_s \dot{\tilde{\alpha}}_r - \dot{\tilde{\alpha}}_s \tilde{\alpha}_r$$ (4.34)$$

$$e_2 = \tilde{\beta}_s \dot{\tilde{\beta}}_r - \dot{\tilde{\beta}}_s \tilde{\beta}_r$$ (4.35)

It follows from (4.33) that if the condition

$$\beta \| \tilde{\lambda} \|^2 + s_\mu > 0 ,$$ (4.36)

holds, then for high enough $\omega_0$, $s_n \dot{s}_n < 0$, i.e., sliding mode will occur on surface $s_n = 0$. 74
Similarly, for $s_\mu$, we have

$$
\dot{s}_\mu = \dot{i}_{\alpha_s} \lambda_{\alpha r} + \dot{i}_{\beta s} \lambda_{\beta r} + \alpha_{\alpha_s} \dot{\lambda}_{\alpha r} + \alpha_{\beta_s} \dot{\lambda}_{\beta r} - \beta \eta e_1 + \beta \omega e_2 + \eta L_m (i_{\alpha_s} \tilde{i}_{\alpha r} + i_{\beta s} \tilde{i}_{\beta r}) - \beta \mu_0 \| \lambda_r \|^2 \text{sign } s_\mu \quad (4.37)
$$

If $\mu_0$ is high enough, $s_\mu \dot{s}_\mu < 0$, and sliding mode will occur on the surface $s_\mu = 0$.

After sliding mode arises on the intersection of both surfaces $s_n = \tilde{i}_{\beta s} \lambda_{\alpha r} - \tilde{i}_{\alpha_s} \lambda_{\beta r} = 0$ and $s_\mu = \tilde{i}_{\alpha s} \lambda_{\alpha r} + \tilde{i}_{\beta s} \lambda_{\beta r} = 0$, then $\dot{i}_{\alpha s} = 0$ and $\dot{i}_{\beta s} = 0$ under the assumption $\| \lambda_r \|^2 \neq 0$, which means that the estimated currents $\dot{i}_{\alpha s}, \dot{i}_{\beta s}$ converge to the real currents $i_{\alpha s}, i_{\beta s}$.

The sliding mode equations on $s_n = 0$, $s_\mu = 0$ can be derived by replacing the discontinuous functions $\omega_0 \text{sign } s_n$ and $\mu_0 \text{sign } s_\mu$ by the equivalent values $\omega_{eq}$ and $\mu_{eq}$, which are the solutions of the algebraic equations $s_n^* = 0$ and $s_\mu^* = 0$ [5]. For our case,

$$
\omega_{eq} = \omega - \frac{\omega}{\| \lambda_r \|^2} e_1 + \frac{\eta}{\| \lambda_r \|^2} e_2 \quad (4.38)
$$

As seen from (4.38), if the estimated rotor flux converges to the real flux also, the equivalent rotor speed will tend to the real speed. However, $\omega_{eq}$ can not be evaluated by equation (4.38), since it contains unknown real rotor flux in the errors $e_1$ and $e_2$ (see equation (4.34) (4.35)). $\dot{\omega}$ has slow and high frequency components, of which the slow component is equal to $\omega_{eq}$. So $\omega_{eq}$ may be obtained through a low-pass filter [39] with discontinuous value $\dot{\omega}$ as the input, i.e.,

$$
\tau \ddot{z} + z = \omega, \quad z \approx \omega_{eq} \quad (4.39)
$$

where $\tau$ is the time constant of the low pass filter. $\tau$ should be chosen small enough as compared with the slow component of the real control $\dot{\omega}$ but large enough to filter out the high rate component [39]. The output $z$ of this low pass filter is taken as
\( \omega_{eq} \). Since \( z \) can be measured directly, in the following analysis we assume that \( \omega_{eq} \) is available.

**Remark 1:** The condition \( \beta\|\hat{\lambda}_r\|^2 + s_\mu > 0 \) for sliding mode to occur on the surface \( s_n = 0 \) is not very restrictive. The reason is that the stator currents \( i_{\alpha s} \) and \( i_{\beta s} \) are measurable. We can always choose the initial conditions \( \hat{i}_{\alpha s}(0) \) and \( \hat{i}_{\beta s}(0) \) close enough to the true stator currents \( i_{\alpha s}(0) \) and \( i_{\beta s}(0) \) such that the initial errors \( \tilde{\alpha}(0) \) and \( \tilde{\beta}(0) \) are small enough to guarantee that the condition (4.36) holds.

**Remark 2:** Although the structure of the observer is selected in the framework of [40], modifications are made to guarantee the convergence of the observer. We will show in section 4.2.5 that under certain conditions, the asymptotic stability of the sliding mode observer can be guaranteed with the flux errors \( \tilde{\alpha} \) and \( \tilde{\beta} \) converging to zero and \( z \) tending to the real speed \( \omega \).

**Remark 3:** Although the flux/speed observer is of the fourth order, after sliding mode arises on the surfaces \( s_n = 0 \) and \( s_\mu = 0 \), the error equations of the sliding mode observer is actually of the second order. This order reduction property of the sliding mode is very helpful for the asymptotic stability analysis of the nonlinear time-varying error system.

### 4.2.4 Sliding Mode Torque Regulation

The control objective in this section is to design a torque tracking controller for the electromechanical system given by (2.8)-(2.12). Specifically, based on the rotor flux and speed estimation strategy described in the previous section, we design a corresponding sliding mode torque controller to guarantee the asymptotic stability of the sliding mode observer and the torque tracking controller. The additional goal of
control dictated by technological requirements is to make the flux track the reference flux input. In this paper, the designs of the observer and the controller are integrated rather than be performed separately.

From the above discussion, three sliding surfaces are designed as:

\[
\begin{align*}
    s_1 &= T_0 - \dot{T} \\
    s_2 &= c_2(\lambda_0 - \|\dot{\lambda}_r\|) + \frac{d}{dt}(\lambda_0 - \|\dot{\lambda}_r\|) \\
    s_3 &= \int_0^t (u_{\alpha_s} + u_{b_s} + u_{c_s})dt
\end{align*}
\]

where \( \dot{T} = \frac{3P}{2L_r}(i_{\beta r}\dot{\lambda}_{ar} - i_{\alpha r}\dot{\lambda}_{br}) \) is the estimated torque, \( T_0 \) and \( \lambda_0 \) are the reference torque and the reference magnitude of flux, \( c_2 \) is a positive constant parameter which determines the converge speed of the error \( (\lambda_0 - \|\dot{\lambda}_r\|) \) in the sliding mode.

In this controller scheme, should sliding mode occur on manifolds \( s_1 = 0 \) and \( s_2 = 0 \), the estimated torque and the magnitude of the rotor flux converge to the reference values. Indeed, \( s_1 = 0 \) means that \( T_0 = \dot{T} \) and \( s_2 = 0 \) means \( \lambda_0 - \|\dot{\lambda}_r\| \) tends to zero exponentially. In the next section we will also show that if the estimated flux and torque converge to the reference values, the estimated values will also converge to the real values. Thus, the real flux and torque are forced to the desired value. Manifold \( s_3 \) is used just to constitute a three-phase balanced system if all three phase voltage may be selected arbitrarily. Note that this requirement may be redundant.

The design task is reduced to enforcing sliding mode in the manifolds \( s = 0 \), \( s^T = (s_1, s_2, s_3) \) with control \( u^T = (u_{\alpha_s}, u_{b_s}, u_{c_s}) \). Equations of the observer/controller
motion projection on the subspace \( s = (s_1, s_2, s_3)^T \) can be written as

\[
\dot{s}_1 = f_1 + a_1 (u_{\alpha s} \hat{\lambda}_{\beta r} - u_{\beta s} \hat{\lambda}_{\alpha r}) \\
\dot{s}_2 = f_2 + a_2 (u_{\alpha s} \hat{\lambda}_{\alpha r} + u_{\beta s} \hat{\lambda}_{\beta r}) \\
\dot{s}_3 = u_{\alpha s} + u_{\beta s} + u_{c s}
\] (4.43) (4.44) (4.45)

where \( f_1 \) and \( f_2 \) are continuous state functions,

\[
a_1 = \frac{3P}{2} \frac{L_m}{L_s L_r - L_r^2}, \quad a_2 = -\frac{1}{\|\lambda_r\|} \frac{R_r L_m}{L_s L_r - L_r^2}
\]

Rewriting (4.43) (4.44) (4.45) in matrix form and taking into account the transformation (2.15) yield

\[
\dot{s} = F + Du
\] (4.46)

where \( F^T = (f_1, f_2, 0) \), \( u^T = (u_{\alpha s}, u_{\beta s}, u_{c s}) \),

\[
D = \begin{bmatrix}
\frac{2}{3} a_1 (e_a \times \hat{\lambda}) & \frac{2}{3} a_1 (e_b \times \hat{\lambda}) & \frac{2}{3} a_1 (e_c \times \hat{\lambda}) \\
\frac{2}{3} a_2 (e_a \cdot \hat{\lambda}) & \frac{2}{3} a_2 (e_b \cdot \hat{\lambda}) & \frac{2}{3} a_2 (e_c \cdot \hat{\lambda}) \\
1 & 1 & 1
\end{bmatrix}
\]

which is nonsingular and \( e_i \times \hat{\lambda} = e_{ia} \hat{\lambda}_{\beta r} - e_{ib} \hat{\lambda}_{\alpha r}, e_i \cdot \hat{\lambda} = e_{ia} \hat{\lambda}_{\alpha r} + e_{ib} \hat{\lambda}_{\beta r}, \ i = a, b, c. \)

To find the discontinuous controls such that sliding mode is enforced in the manifold \( s = 0 \), select Lyapunov candidate function \( v = \frac{1}{2} s^T s \geq 0 \). Its time derivative on the state trajectories of system (4.46):

\[
\dot{v} = s^T (F + Du)
\] (4.47)

Following the design methodology introduced in [5], select the discontinuous control

\[
u = -U_0 \text{signs}^s
\]

where \( s^T = (s_1^s, s_2^s, s_3^s), (\text{signs}^s)^T = (\text{signs}_1^s, \text{signs}_2^s, \text{signs}_3^s), s^s = D^T s \) and \( U_0 \) is a positive constant. Then (4.47) can be rewritten as

\[
\dot{v} = (s_1^s f_1^s - U_0 |s_1^s|) + (s_2^s f_2^s - U_0 |s_2^s|) + (s_3^s f_3^s - U_0 |s_3^s|)
\] (4.48)
where \((f_1^*, f_2^*, f_3^*) = (D^{-1}F)^T\). Note that \(\text{det}(D) \neq 0\).

From (4.48), it is obvious that if the DC-link voltage \(U_0\) satisfies

\[
U_0 > \max_{i=1,2,3} |f_i^*| \tag{4.49}
\]

then the time derivative of Lyapunov function \(dv/dt\) is negative definite and hence the origin in the space \(s^*\) is asymptotically stable. Note that since matrix \(D\) is nonsingular, sliding mode also arises in the manifold \(s = 0\), which enables one to steer the estimated variables to the reference values.

4.2.5 Composite Observer-Controller Analysis

As mentioned before, although the sliding mode observer is of the fourth order, due to the order-reduction of sliding mode equation, the error system of the observer is of the second order. To facilitate the analysis of the composite observer-controller tracking error system, we choose the errors \(e_1\) and \(e_2\) defined in (4.34)(4.35) as the state variables of the error system.

Calculate the time derivative of the transformed flux errors \(e_1\) and \(e_2\) to build the error system of the rotor flux estimation:

\[
\dot{e}_1 = \dot{\lambda}_{ar} \lambda_{ar} + \dot{\lambda}_{br} \lambda_{br} + \lambda_{ar} \dot{\lambda}_{ar} + \lambda_{br} \dot{\lambda}_{br} \tag{4.50}
\]
\[
\dot{e}_2 = \dot{\lambda}_{br} \lambda_{ar} - \dot{\lambda}_{ar} \lambda_{br} + \lambda_{br} \dot{\lambda}_{ar} - \lambda_{ar} \dot{\lambda}_{br} \tag{4.51}
\]

and find \(\bar{\lambda}_{ar}\) and \(\bar{\lambda}_{br}\) from (4.34) (4.35),

\[
\bar{\lambda}_{ar} = \frac{\lambda_{ar}}{\|\lambda_r\|^2} e_1 - \frac{\lambda_{br}}{\|\lambda_r\|^2} e_2, \quad \bar{\lambda}_{br} = \frac{\lambda_{br}}{\|\lambda_r\|^2} e_1 + \frac{\lambda_{ar}}{\|\lambda_r\|^2} e_2 \tag{4.52}
\]

79
Substituting the right hand sides of equations (4.20)(4.21)(4.26)(4.27) and the above two equations into (4.50) and (4.51) yields

\[ \dot{e}_1 = (-2\eta + \eta \frac{L_m i_{ds}}{\|\lambda_r\|}) e_1 + \eta \frac{L_m i_{qs}}{\|\lambda_r\|} e_2 + \bar{\omega} e_2 - C\mu e_2 \]  
\[ \dot{e}_2 = (-\eta + \eta \frac{L_m i_{ds}}{\|\lambda_r\|}) e_2 - (\omega + \eta \frac{L_m i_{qs}}{\|\lambda_r\|}) e_1 + \bar{\omega} e_1 - C\|\lambda_r\|^2 \mu + C\mu e_1 \]  

(4.53)

(4.54)

where

\[ i_{ds} = \frac{\lambda_{\alpha r}}{\|\lambda_r\|} i_{\alpha s} + \frac{\lambda_{\beta r}}{\|\lambda_r\|} i_{\beta s}, \quad i_{qs} = \frac{\lambda_{\alpha r}}{\|\lambda_r\|} i_{\beta s} - \frac{\lambda_{\beta r}}{\|\lambda_r\|} i_{\alpha s} \]  

(4.55)

\( i_{ds} \) and \( i_{qs} \) are the flux and torque component of the stator currents projected on the \((d, q)\) coordinate frame which is aligned with the estimates of the rotating field flux, \( \bar{\omega} = \dot{\omega} - \omega \). After sliding mode occurs, \( \dot{\omega} \) should be replaced by \( \omega_{eq} \) in the motion equations. So the speed deviation \( \bar{\omega} \) can be calculated from (4.38),

\[ \bar{\omega} = -\frac{\omega}{\|\lambda_r\|^2} e_1 + \frac{\eta}{\|\lambda_r\|^2} e_2 \]  

(4.56)

Substitution of (4.56) into (4.53) and (4.54) results in the error dynamics in matrix form

\[ \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -2\eta + \eta \frac{L_m i_{ds}}{\|\lambda_r\|} & \eta \frac{L_m i_{qs}}{\|\lambda_r\|} \\ -\eta + \eta \frac{L_m i_{ds}}{\|\lambda_r\|} & -\eta + \eta \frac{L_m i_{qs}}{\|\lambda_r\|} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} -C\mu e_2 \\ -C\|\lambda_r\|^2 \mu + C\mu e_1 \end{bmatrix} + \begin{bmatrix} \frac{\omega}{\|\lambda_r\|^2} e_1 e_2 + \frac{\eta}{\|\lambda_r\|^2} e_2 \\ \frac{\omega}{\|\lambda_r\|^2} e_1 e_2 - \frac{\eta}{\|\lambda_r\|^2} e_2 \end{bmatrix} \]  

(4.57)

Assuming that sliding mode also arises on manifold \( s_{\mu} = 0 \), and solving the algebraic equation \( s_{\mu} = 0 \) (4.37) with respect to \( \mu \),

\[ \mu_{eq} = \frac{\eta}{\|\lambda_r\|^2} e_1 + \frac{\omega}{\|\lambda_r\|^2} e_2 \]  

(4.58)

we can eliminate the discontinuous parameter \( \mu \) from the error system.
Since the estimates \( \|\dot{\lambda}_r\| \) and \( \dot{T} \) track the reference inputs,

\[
\lambda_0 = \|\dot{\lambda}_r\| \quad (4.59)
\]

\[
\frac{d}{dt} \frac{\|\dot{\lambda}_r\|}{\lambda_0} = \frac{d\lambda_0}{dt} \quad (4.60)
\]

\[
T_0 = \frac{3P L_m}{2} (i_{\beta s}\dot{\lambda}_{\alpha r} - i_{\alpha s}\dot{\lambda}_{\beta r}) \quad (4.61)
\]

To express (4.20) (4.60) (4.61) in the \((d, q)\) frame, find \(i_{\alpha s}, i_{\beta s}\) from (4.55),

\[
i_{\alpha s} = \frac{\dot{\lambda}_{\alpha r}}{\|\lambda_r\|} i_{ds} - \frac{\dot{\lambda}_{\beta r}}{\|\lambda_r\|} i_{qs}, \quad i_{\beta s} = \frac{\dot{\lambda}_{\alpha r}}{\|\lambda_r\|} i_{qs} + \frac{\dot{\lambda}_{\beta r}}{\|\lambda_r\|} i_{ds} \quad (4.62)
\]

For constant \(\lambda_0\), after substituting the above two equations and equations (4.20) (4.21) (4.59) into (4.60) and (4.61), we have

\[
i_{qs} = \frac{2L_r T_0}{3P L_m \lambda_0} \quad (4.63)
\]

\[-\eta + \eta L_m \frac{i_{ds}}{\|\lambda_r\|} = 0 \quad (4.64)
\]

Finally, substituting equations (4.58) (4.59) (4.63) and \(i_d\) from (4.64) into the error dynamic system (4.57), and performing linearization, we get

\[
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix} =
\begin{bmatrix}
-\eta \\
-(\omega_{eq} + a) - C\eta & a \\
C\eta & -C\omega_{eq}
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix} \quad (4.65)
\]

where \(a = \frac{2P R_b}{3L_m \lambda_0}\).

Note that the linearized error system does not depend on the real speed, real rotor flux and real stator current. It depends only on the reference torque, the magnitude of the reference flux, the equivalent angle speed and the adjustable parameter \(C\). As follows from the equation (4.65), for large \(C\), the motion of the error dynamic system can be decomposed into the slow and fast motion [41]. The fast component \(\eta e_1 + \omega_{eq} e_2\) decays rapidly with

\[
\lim_{t \to \infty} e_2 = -\frac{\eta}{\omega_{eq}} e_1 \quad (4.66)
\]
The slow motion is governed by

\[ \dot{e}_1 = -\eta(1 + \frac{a}{\omega_{eq}})e_1 \]  \hspace{1cm} (4.67)

One of the sufficient conditions for the asymptotic stability of the slow motion for any time-varying speed is \( 1 + \frac{a}{\omega_{eq}} > 0 \). It is clear that

\[ \frac{a}{\omega_{eq}} > 0 \]  \hspace{1cm} (4.68)

is a sufficient condition as well. It means that the solution to (4.67) is stable and \( \lim_{t \to \infty} e_1 = 0 \) if the reference torque and the equivalent speed have the same sign. We have described qualitatively the design procedure based on motion separation approach. The details may be found in [41]. According to (4.66), \( \lim_{t \to \infty} e_2 = 0 \).

Remark 4: According to [41], the asymptotic stability of the fast motion of the error dynamic (4.65) requires that the parameter \( C \) and \( \omega_{eq} \) have the same sign. Since the equivalent speed can be calculated through equation (4.39) on-line, the sign of the parameter \( C \) can be adapted to the sign of the equivalent speed. At the instants when \( \omega_{eq} = 0 \), according to (4.68), the sign of parameter \( C \) can be adapted to the sign of the reference torque, and then the stability of the system can also be guaranteed.

Remark 5: The proposed torque regulation is insensitive to the variation of the load torque. Even the sign of the load torque will not affect the stability of the observer-controller.

### 4.2.6 Simulation Results

The proposed control scheme was first simulated using MATLAB. In the simulation, PWM technique is not used and the dynamics in the voltage source inverter is also ignored.
\[
L_s = 590 \times 10^{-6} \, H \\
L_r = 590 \times 10^{-6} \, H \\
L_m = 555 \times 10^{-6} \, H \\
R_s = 0.0106 \, \Omega \\
R_r = 0.0118 \, \Omega \\
U_0 = 12.0 \, V \\
T_0 = 1.0 \, N \cdot m \\
J = 4.33 \times 10^{-4} \, N \cdot m \cdot s^2 \\
c_2 = 400
\]

Table 4.5: Parameters used in the simulation

One point that should be noted is the calculation of the derivative of the rotor flux amplitude \( d\|\lambda_r\| / dt \). To avoid too much noise, it is not desirable to differentiate directly to obtain the time derivative of \( \|\lambda_r\| \). And even though \( d\lambda_{\alpha r} / dt \) and \( d\lambda_{\beta r} / dt \) are discontinuous functions, the derivative \( d\|\lambda_r\| / dt \) should be a continuous function of time \( t \), otherwise the sliding surface \( s_2 \) would be a discontinuous function and sliding mode can not be enforced in such a manifold. Note that

\[
\frac{d\|\lambda_r\|}{dt} = \frac{\lambda_{\alpha r} d\lambda_{\alpha r}}{\|\lambda_r\|} + \frac{\lambda_{\beta r} d\lambda_{\beta r}}{\|\lambda_r\|}.
\]

It can be seen that the discontinuity introduced by the discontinuous parameter \( \mu \) is cancelled. We can eliminate another discontinuous parameter \( \dot{\omega} \) by replacing \( \dot{\omega} \) by \( \omega_{eq} \) which is continuous. This will not destroy the sliding mode because sliding mode is enforced through the stator current.

Parameters used in the torque controller, in the flux/speed observer and of the induction motor model for the simulation are listed in Table 4.5. Figure 4.24-4.33 show the simulation results. Figure 4.24-4.28 show the piecewise continuous situation while Figure 4.29-4.33 show the sinusoidal situation for the torque tracking. From these simulation results, we can see that the proposed sliding mode speed/flux observer
Figure 4.24: Torque tracking

Figure 4.25: Magnitude of flux tracking
Figure 4.26: Flux convergence
Figure 4.27: Speed estimation

Figure 4.28: Current convergence
Figure 4.29: Torque tracking

Figure 4.30: Magnitude of flux tracking
Figure 4.31: Flux convergence
Figure 4.32: Speed estimation

Figure 4.33: Current convergence
exhibits high accuracy. Note that convergence of estimations to real values of flux and speed takes place if torque estimate tracks the reference input. This condition cannot be fulfilled for piecewise continuous input and discontinuities result in converging transient processes. This phenomenon may be observed in the simulation.

4.2.7 Experiment Setup and Results

The proposed control scheme was also implemented in the laboratory. The real-time control and estimation program was written in C language. The motor is a Westinghouse 5-hp, 220 V, Y connected four pole induction machine with the parameters are listed in Table 4.8. The block diagram of the torque control system is
shown in Figure 4.34. The part within the dashed line is implemented by a DSP system. Instead of the fixed point DSP system, we used a PC plug-in DS1102 dSPACE system, which includes TMS320C31 32 bit floating point Digital Signal Processor as the main processing unit and a set of on-board peripherals frequently used in digital control systems.

Main components of the experiment environment include: a DSP system; an induction motor and associated voltage source inverters; an optical encoder attached to the motor shaft for speed estimation verification and comparison; cables for connecting the whole analog/digital signals; ac current sensors.

The torque control is executed every 100 μs. Since the system was tested with no load, we had to apply a sign-varying torque reference input.

To verify and compare the estimated flux of our sliding mode observer, a reduced-ordered observer is designed with the state vector \((\hat{\lambda}_a, \hat{\lambda}_\beta)\) as the estimate of rotor flux components. In this observer, the required information is stator current \(i_a, i_\beta\) and the rotor speed \(\omega\), which are obtained from the sensor and encoder, respectively.
\[
\begin{align*}
V_b &= 500V \\
I_b &= 50A \\
R_b &= 10\Omega \\
f_b &= 100Hz \\
\omega_b &= 2\pi f_b \\
\lambda_b &= \frac{V_b}{\omega_b} \\
\bar{M}_b &= 1.5P_b\lambda_b
\end{align*}
\]

Table 4.7: Base system data

The observer model is as follows:

\[
\begin{align*}
\frac{d\hat{\lambda}_{ar}}{dt} &= -\eta \dot{\lambda}_{ar} - \omega \dot{\lambda}_{\beta s} + \eta L_m \dot{i}_s \\
\frac{d\hat{\lambda}_{\beta r}}{dt} &= -\eta \dot{\lambda}_{\beta r} + \omega \dot{\lambda}_{\alpha s} + \eta L_m \dot{i}_s
\end{align*}
\]

It is easily proven that this observer guarantees the exponential convergence of the estimated stator flux to the real flux [5].

All the experiments are conducted in the so-called per-unit system, whose base parameters are listed as in Table 4.7. All the experiment results obtained are expressed in the per-unit system.

Figure 4.35 shows the result of the experiment in which only sliding mode controller is implemented while the speed is from the optical sensor. \( \tilde{T}_{real} \) is defined as follows: \( \tilde{T}_{real} = \frac{3P}{2L_r}(i_{\beta r}, \dot{\lambda}_{ar} - i_{\alpha s}, \dot{\lambda}_{\beta s}) \). The flux is from the above reduced-order observer. Figure 4.36 shows the flux estimations of our fourth-order sliding mode observer. We also compare them with the flux estimation from the reduced-order observer. Figure 4.37 shows the results for the system with both the sliding mode controller and sliding mode observer working. In this case, we do not need to use the optical sensor to obtain the speed, i.e. sensorless control. All the variables needed
for torque or speed control, such as flux components, their derivatives and speed, are obtained from the sliding mode observer.

Speed control can also be achieved through torque control. In our experiments, with zero load, $\frac{d\omega}{dt} = \frac{k_f}{P} T$. As shown above the estimate $\omega_{eq}$ tends to $\omega$, therefore if the torque is set as $k(\omega_{eq} - \omega_0)$, then $\frac{d\omega}{dt} = \frac{k_f}{P} (\omega_{eq} - \omega_0)$ and the motor speed converges to $\omega_0$ exponentially with the rate decided by the parameter $\frac{k_f}{P}$. Figure 4.38-4.39 show the results for speed control. Figure 4.38 shows the case in which the speed is constant, while Figure 4.39 shows the case with time-varying speed reference input.
4.2.8 Sliding Mode Controller in Low Speed Range

Currently low speed control is still a challenging issue for induction motors. The potential of sliding mode control and estimation methodology was tested experimentally for low speed case with the speed measurement. We still use the speed control method discussed above but the speed information is obtained from the sensor. The results are shown in Figure 4.40. We can see even for very low speed 1.5rpm, the controller demonstrates good performance.
Case 1: period time is 4 seconds

Case 2: period time is 2 seconds

Curve 1: \( n_{\text{real}} \) from sensor
Curve 2: \( n_{\text{eq}} \) from observer
Curve 3: \( M_{\text{ref}} \times 10 \)
Curve 4: \( M_{\text{real}} \times 10 \)

Figure 4.37: Sensorless torque control
Case 1: $n_{ref} = 0.2$ \hspace{1em} (600rpm)

Case 2: $n_{ref} = 0.1$ \hspace{1em} (300rpm)

Curve 1: $n_{real}$
Curve 2: $n_{eq}$

Figure 4.38: Sensorless speed control
4.3 Sliding Mode Speed and Rotor Time Constant Observer

4.3.1 Observer Design and Analysis

After rewriting the machine parameter $\gamma$, the induction machine model equation (2.11)(2.12) can be rewritten as

$$\begin{align*}
\frac{di_{\alpha s}}{dt} &= \beta \eta \lambda_{\alpha r} + \beta \omega \lambda_{\beta r} - \left( \frac{R_s}{\sigma L_s} + \beta L_m \eta \right) i_{\alpha s} + \frac{1}{\sigma L_s} u_{\alpha s} \quad (4.69) \\
\frac{di_{\beta s}}{dt} &= \beta \eta \lambda_{\beta r} - \beta \omega \lambda_{\alpha r} - \left( \frac{R_s}{\sigma L_s} + \beta L_m \eta \right) i_{\beta s} + \frac{1}{\sigma L_s} u_{\beta s} \quad (4.70)
\end{align*}$$

We propose a nonlinear robust observer with discontinuous parameters and stator currents and voltages as its input, so that the motor speed and rotor time constant can be estimated simultaneously without the measurement of mechanical variables. This observer is designed based on the assumption that the stator resistance $R_s$ and
Case 1: $n_{ref} = 0.00125$  \((3.75 \text{rpm})\)

Case 2: $n_{ref} = 0.0005$  \((1.5 \text{rpm})\)

Curve 2: $20 \times n_{real}$
Curve 3: $20 \times n_{eq}$

Figure 4.40: Low speed control
motor speed $\omega$ are constants since they are varying much slower than other electrical variables. This assumption makes the observer design much simplified.

The sliding mode observer is designed as follows:

\[
\frac{d\hat{i}_{\alpha}}{dt} = -\frac{R_s}{\sigma L_s} i_{\alpha} + \frac{1}{\sigma L_s} u_{\alpha} + V_{\alpha} \tag{4.71}
\]

\[
\frac{d\hat{i}_{\beta}}{dt} = -\frac{R_s}{\sigma L_s} i_{\beta} + \frac{1}{\sigma L_s} u_{\beta} + V_{\beta} \tag{4.72}
\]

where $\hat{i}_{\alpha}$, $\hat{i}_{\beta}$, are estimates of the stator current components. $V_{\alpha}$ and $V_{\beta}$ are discontinuous functions of the current errors

\[
V_{\alpha} = -V_0 sign(s_\alpha) = -V_0 sign(\hat{i}_{\alpha} - i_{\alpha}) \tag{4.73}
\]

\[
V_{\beta} = -V_0 sign(s_\beta) = -V_0 sign(\hat{i}_{\beta} - i_{\beta}) \tag{4.74}
\]

It will be shown that there exists constant values $V_0$ such that sliding mode arises in the surfaces of $s_\alpha = 0$ and $s_\beta = 0$ and as a result, the estimation errors

\[
\tilde{i}_{\alpha} = \hat{i}_{\alpha} - i_{\alpha}, \quad \tilde{i}_{\beta} = \hat{i}_{\beta} - i_{\beta} \tag{4.75}
\]

tend to zero.

Comparing model and observer equations, the sliding mode observer equations with respect to the current estimation errors can be written as

\[
\frac{d\tilde{i}_{\alpha}}{dt} = V_{\alpha} - \beta \eta \lambda_{\alpha x} - \beta \omega \lambda_{\beta x} - \frac{R_s}{\sigma L_s} \tilde{i}_{\alpha} + \beta L_m \eta \hat{i}_{\alpha} \tag{4.76}
\]

\[
\frac{d\tilde{i}_{\beta}}{dt} = V_{\beta} - \beta \eta \lambda_{\beta x} + \beta \omega \lambda_{\alpha x} - \frac{R_s}{\sigma L_s} \tilde{i}_{\beta} + \beta L_m \eta \hat{i}_{\beta} \tag{4.77}
\]

To find the discontinuous controls such that sliding mode is enforced in the manifold $s_\alpha = 0$ and $s_\beta = 0$, let us select Lyapunov candidate function $V = \frac{1}{2}(s_{\alpha}^2 + s_{\beta}^2)$. 

99
Its time derivative on the state trajectories of system \((4.76)(4.77)\) can be written as

\[
\dot{V} = s_\alpha \dot{s}_\alpha + s_\beta \dot{s}_\beta \\
= s_\alpha (-V_0 \text{sign}(s_\alpha) - \beta \eta_\lambda_\alpha \omega_\lambda_\beta \beta \omega_\lambda_\beta - \frac{R_s}{\sigma L_s} i_{\alpha s} + \beta L_m \eta i_{\alpha s}) \\
+ s_\beta (-V_0 \text{sign}(s_\beta) - \beta \eta_\lambda_\beta \omega_\lambda_\alpha + \beta \omega_\lambda_\alpha \beta \omega_\lambda_\beta - \frac{R_s}{\sigma L_s} i_{\beta s} + \beta L_m \eta i_{\beta s}) \\
= -V_0 (|s_\alpha| + |s_\beta|) - \frac{R_s}{\sigma L_s} (s_\alpha^2 + s_\beta^2) + s_\alpha f_\alpha + s_\beta f_\beta,
\]

where \(f_\alpha\) and \(f_\beta\) are continuous functions of the motor states \(i_{\alpha s}, i_{\beta s}, \lambda_\alpha, \lambda_\beta\) and \(\omega\) but they do not depend on the control signal \(V_\alpha\) and \(V_\beta\).

It is obvious that if \(V_0\) is large enough, then \(\dot{V} < 0\), i.e. sliding mode will occur in the intersection of the surfaces \(s_\alpha = 0\) and \(s_\beta = 0\). So the estimated currents \(\dot{i}_{\alpha s}\) and \(\dot{i}_{\beta s}\) will converge to the real ones when sliding modes happen.

The sliding mode equations on \(s_\alpha = 0\) and \(s_\beta = 0\) can be derived by replacing the discontinuous functions \(V_\alpha\) and \(V_\beta\) by their equivalent control component \(V_{\alpha eq}\) and \(V_{\beta eq}\) obtained by setting \(\dot{s}_\alpha = 0, s_\alpha = 0\) and \(\dot{s}_\beta = 0, s_\beta = 0\) [39].

\[
V_{\alpha eq} = \beta \eta_\lambda_\alpha + \beta \omega_\lambda_\beta \beta \omega_\lambda_\beta - \beta L_m \eta i_{\alpha s} \\
V_{\beta eq} = \beta \eta_\lambda_\beta - \beta \omega_\lambda_\beta \beta \omega_\lambda_\alpha - \beta L_m \eta i_{\beta s}
\]

(4.78)  \hspace{1cm} (4.79)

Note that \(V_{\alpha eq}\) and \(V_{\beta eq}\) can not be calculated by the above two equations, since they contain unknown real rotor flux \(\lambda_\alpha\) and \(\lambda_\beta\). In fact, the two discontinuous functions \(V_\alpha\) and \(V_\beta\) have slow and high frequency components, of which the slow components are equal to \(V_{\alpha eq}\) and \(V_{\beta eq}\) respectively. So \(V_{\alpha eq}\) and \(V_{\beta eq}\) may be obtained through a low-pass filter [5] with discontinuous value \(V_\alpha\) and \(V_\beta\) as the inputs, i.e.,

\[
\tau \dot{z}_\alpha + z_\alpha = V_\alpha, \quad z_\alpha \approx V_{\alpha eq} \\
\tau \dot{z}_\beta + z_\beta = V_\beta, \quad z_\beta \approx V_{\beta eq}
\]

(4.80)  \hspace{1cm} (4.81)
where $\tau$ is a small time constant of the low pass filter. $\tau$ should be chosen small enough as compared with the slow component of the real value $V_\alpha$ and $V_\beta$ but large enough to filter out the high rate component [5]. The output $z_\alpha$ and $z_\beta$ of the low pass filters are taken as $V_{\alpha eq}$ and $V_{\beta eq}$. Since $z_\alpha$ and $z_\beta$ can be obtained directly, from now on we assume $V_{\alpha eq}$ and $V_{\beta eq}$ are available.

For the notation convenience, let us define $L_\alpha = V_{\alpha eq}$ and $L_\beta = V_{\beta eq}$. It is reasonable to assume that $\dot{\omega} = 0$ and $\dot{\hat{\eta}} = 0$ if their variations are very slow compared with the electrical variables such as stator currents and rotor flux. Then we have

$$\dot{L}_\alpha = \beta \eta \dot{\lambda}_r - \beta \omega \dot{\lambda}_r - \beta L_m \dot{\eta}_\alpha \tag{4.82}$$

$$\dot{L}_\beta = \beta \eta \dot{\lambda}_r - \beta \omega \dot{\lambda}_r - \beta L_m \dot{\eta}_\beta \tag{4.83}$$

From (2.9)(2.10)(4.78)(4.79), it is obvious that

$$\dot{\lambda}_r = -L_\alpha / \beta, \quad \dot{\lambda}_r = -L_\beta / \beta \tag{4.84}$$

Then combining these two with (4.82) and (4.83), we have the dynamics of the equivalent control

$$\begin{bmatrix} \dot{L}_\alpha \\ \dot{L}_\beta \end{bmatrix} = -\begin{bmatrix} \eta & \omega \\ -\omega & \eta \end{bmatrix} \begin{bmatrix} L_\alpha \\ L_\beta \end{bmatrix} - \beta L_m \eta \begin{bmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{bmatrix} \tag{4.85}$$

The observers for $L_\alpha$ and $L_\beta$ are designed as

$$\begin{bmatrix} \dot{\hat{L}}_\alpha \\ \dot{\hat{L}}_\beta \end{bmatrix} = -\begin{bmatrix} \hat{\eta} & \hat{\omega} \\ -\hat{\omega} & \hat{\eta} \end{bmatrix} \begin{bmatrix} L_\alpha \\ L_\beta \end{bmatrix} - \beta L_m \hat{\eta} \begin{bmatrix} \dot{\hat{i}}_\alpha \\ \dot{\hat{i}}_\beta \end{bmatrix} - K \begin{bmatrix} \hat{L}_\alpha \\ \hat{L}_\beta \end{bmatrix} \tag{4.86}$$

where $\hat{\eta}$, $\hat{\omega}$ are the estimates of $\eta$, $\omega$ and $K$ is a positive constant to be chosen. $\hat{L}_\alpha = \hat{L}_\alpha - L_\alpha$ and $\hat{L}_\beta = \hat{L}_\beta - L_\beta$ denote the errors. Note that in the real implementation $\dot{\hat{i}}_\alpha$ and $\dot{\hat{i}}_\beta$ in the observer can be obtained from (4.71)(4.72) since the sliding mode current observer works. Then the error dynamics equations are:

$$\begin{bmatrix} \dot{\hat{L}}_\alpha \\ \dot{\hat{L}}_\beta \end{bmatrix} = -\begin{bmatrix} \hat{\eta} & \hat{\omega} \\ -\hat{\omega} & \hat{\eta} \end{bmatrix} \begin{bmatrix} L_\alpha \\ L_\beta \end{bmatrix} - \beta L_m \hat{\eta} \begin{bmatrix} \dot{\hat{i}}_\alpha \\ \dot{\hat{i}}_\beta \end{bmatrix} - K \begin{bmatrix} \hat{L}_\alpha \\ \hat{L}_\beta \end{bmatrix} \tag{4.87}$$

101
Let us select the Lyapunov candidate function
\[
V = \frac{1}{2} \dot{L}_\alpha^2 + \frac{1}{2} \dot{L}_\beta^2 + \frac{1}{2} \dot{\omega}^2 + \frac{1}{2} \dot{\eta}^2 \geq 0 \quad (4.88)
\]

Then combining with (4.87), the derivative of \( V \) is obtained
\[
\dot{V} = L_\alpha \dot{L}_\alpha + L_\beta \dot{L}_\beta + \dot{\omega} \dot{\omega} + \dot{\eta} \dot{\eta}
\]
\[
= \dot{\omega} \dot{\omega} + \dot{\eta} \dot{\eta} - K \dot{L}_\alpha^2 - K \dot{L}_\beta^2
\]
\[
+ \dot{L}_\alpha ( -\eta L_\alpha - \dot{\omega} L_\beta - \beta L_m \dot{\eta}_{\alpha_s} ) + \dot{L}_\beta ( \dot{\omega} L_\alpha - \eta L_\beta - \beta L_m \dot{\eta}_{\beta_s} ).
\]

Choose the adaptive law as
\[
\begin{bmatrix}
\dot{\eta} \\
\dot{\omega}
\end{bmatrix} =
\begin{bmatrix}
L_\alpha + \beta L_m \dot{\eta}_{\alpha_s} & L_\beta + \beta L_m \dot{\eta}_{\beta_s} \\
-\dot{L}_\alpha & \dot{L}_\beta
\end{bmatrix}
\begin{bmatrix}
L_\alpha \\
L_\beta
\end{bmatrix}
\quad (4.89)
\]

then
\[
\dot{V} = -K(\dot{L}_\alpha^2 + \dot{L}_\beta^2) \leq 0,
\]

which means that under the adaptive law (4.89), the Lyapunov function \( V \) is delaying until
\[
L_\alpha = 0, \quad L_\beta = 0. \quad (4.90)
\]

From (4.89)(4.90), we know \( \dot{\omega} = 0 \) and \( \dot{\eta} = 0 \), which means \( \omega \) and \( \eta \) are constant values. On the other hand, substituting (4.90) into (4.87), we get
\[
\begin{bmatrix}
\dot{\eta} & \dot{\omega} \\
-\dot{\omega} & \dot{\eta}
\end{bmatrix}
\begin{bmatrix}
L_\alpha \\
L_\beta
\end{bmatrix} + \beta L_m \dot{\eta} \begin{bmatrix}
\dot{\eta}_{\alpha_s} \\
\dot{\eta}_{\beta_s}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\quad (4.91)
\]

i.e.,
\[
\begin{bmatrix}
L_\alpha + \beta L_m \dot{\eta}_{\alpha_s} & L_\beta + \beta L_m \dot{\eta}_{\beta_s} \\
L_\beta + \beta L_m \dot{\eta}_{\beta_s} & -\dot{L}_\alpha
\end{bmatrix}
\begin{bmatrix}
\dot{\eta} \\
\dot{\omega}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\quad (4.92)
\]

Since \( L_\alpha + \beta L_m \dot{\eta}_{\alpha_s}, L_\beta, L_\beta + \beta L_m \dot{\eta}_{\beta_s} \), and \(- \dot{L}_\alpha \) are all functions with respect to time and not proportional to each other (to be analyzed below), they are linearly independent.
time functions. Moreover \( \bar{\omega} \) and \( \bar{\eta} \) are constants, so it is concluded that \( \bar{\eta} = 0 \) and 
\( \bar{\omega} = 0 \), which means that \( \dot{\bar{\eta}} = \eta \) and \( \dot{\bar{\omega}} = \omega \).

The system is summarized as: For the observer (4.86), under the adaptive law (4.89), the estimated rotor speed \( \hat{\omega} \) and rotor time constant reciprocal \( \hat{\eta} \) will converge to their real values when the observed equivalent control components \( \hat{L}_\alpha \) and \( \hat{L}_\beta \) converge to \( L_\alpha \) and \( L_\beta \).

- To show \( L_\alpha + \beta L_m \dot{i}_\alpha \) and \( L_\beta, L_\beta + \beta L_m \dot{i}_\beta \) and \(-L_\alpha \) are not proportional to each other, let us consider in this way. Suppose they are in a proportion, i.e.

\[
L_\alpha + \beta L_m \dot{i}_\alpha = k_1 L_\beta \\
L_\beta + \beta L_m \dot{i}_\beta = k_2 L_\alpha
\]

Combining with (4.84), we have

\[
-\dot{\lambda}_{ar} + L_m \dot{i}_\alpha = -k_1 \dot{\lambda}_{br} \\
-\dot{\lambda}_{br} + L_m \dot{i}_\beta = -k_2 \dot{\lambda}_{ar}
\]

After substitution of the model equations (2.9)(2.10)(2.11)(2.12), the following algebraic equations are obtained

\[
L_m(\beta \eta \lambda_{ar} + \beta \omega \lambda_{br} - \gamma i_\alpha + \frac{1}{\sigma L_s} u_\alpha) \\
= -\eta \lambda_{ar} - \omega \lambda_{br} + \eta L_m \dot{i}_\alpha - k_1(-\eta \lambda_{br} + \omega \lambda_{ar} + \eta L_m \dot{i}_\beta) \\
L_m(\beta \eta \lambda_{br} - \beta \omega \lambda_{ar} - \gamma \dot{i}_\beta + \frac{1}{\sigma L_s} u_\beta) \\
= -\eta \lambda_{br} + \omega \lambda_{ar} + \eta L_m \dot{i}_\beta - k_2(-\eta \lambda_{ar} - \omega \lambda_{br} + \eta L_m \dot{i}_\alpha)
\]

As we can see, these two are algebraic equations. However, according to the model equations, the relations between current, flux and voltage should be
differential instead of algebraic. Moreover, the voltage is obtained from the controllers as the control inputs, so it does not make sense that they are able to satisfy the above equations. All these analysis show that $L_\alpha + \beta L_m i_{\alpha s}$ and $L_\beta$, $L_\beta + \beta L_m i_{\beta s}$ and $-L_\alpha$ are not proportional to each other.

### 4.3.2 Simulation and Experiment Results

In this section the performance of the proposed observer structure is presented via simulation and experimental results. The block diagram of the indirect field oriented induction machine drive system with observer structure is given in Figure 4.41. The machine parameters are listed in Table 4.8. The performance of the observer is analyzed in the simulation and implementation by operating the observer without using it in the closed loop, i.e., in the feedback the actual speed from encoder is used and observer structure works parallel to the overall system without affecting the closed loop system at all. The closed loop system follows different trajectory, and in parallel with closed loop system, the observer operates and estimates the speed of the machine. In the speed regulation loop, the PI controller is used.

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>$R_s$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.6Ω</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>$L_m$ (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>0.41Ω</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$L_m$ (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.8 A</td>
<td>0.043H</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>$L_m$ (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.043H</td>
</tr>
</tbody>
</table>

Table 4.8: Real induction motor data
Figure 4.41: System block diagram

Figure 4.42: Actual and estimated speed
Figure 4.43: Actual and estimated currents

Figure 4.44: Actual and estimated \((L, \hat{L})\)
Simulation Results

The validity of the observer structure is verified by the simulation with the results given in Figure 4.42-4.45. First step for the speed estimation is the current observation. The estimated and actual speeds, d-axes currents, states and rotor time constant are given in the figures. It is obvious from these results that current, speed and rotor time constant convergence is satisfied.

Experiment Results

The proposed control scheme was implemented in the laboratory. The experimental results for different speed trajectories are given in Figure 4.46-4.55.
Figure 4.46: Measured and estimated speed

Figure 4.47: Measured and observed currents (between 7.5-9.5 sec.)
Figure 4.48: Calculated and observed states $L$ and $\dot{L}$

Figure 4.49: Enlarge of Figure 4.48 in between (6-8 sec.)
Figure 4.50: Measured and estimated speed

Figure 4.51: Calculated and observed states $L$ and $\hat{L}$
Figure 4.52: Enlarge of Figure 4.51 in between (1.2-2.4 sec.)

Figure 4.53: Measured and observed currents
Figure 4.54: Enlarge of Figure 4.53 in between (7.5-9.5 sec.)

Figure 4.55: Estimated $\hat{\eta}$
4.4 Implementation Issues

4.4.1 Sliding Mode Control Scheme in terms of Phase Voltages $u_i$ ($i = a, b, c$)

In this section, it will be shown that we can use sliding mode method for direct control of a converter.

Figure 4.56 shows the system configuration for the PWM implementation. There are three control signals $g_a$, $g_b$, $g_c$, which control the on-off switches in the three phases. We only assign $+1$ or $-1$ to these three signals, which corresponds to voltage value $\frac{u_{dc}}{2}$ or $-\frac{u_{dc}}{2}$ (with respect to ground level). From the circuit analysis, the following circuit equations for the three phases are obtained:

\[
\begin{align*}
    i_a \cdot Z &= u_a = g_a \cdot \frac{u_{dc}}{2} - u_0 \\
    i_b \cdot Z &= u_b = g_b \cdot \frac{u_{dc}}{2} - u_0 \\
    i_c \cdot Z &= u_c = g_c \cdot \frac{u_{dc}}{2} - u_0
\end{align*}
\]
From the physical configuration of the circuit, we know the sum of the left sides is equal to zero, which gives us: \( 0 = (g_a + g_b + g_c) \frac{u_{dc}}{2} - 3u_0 \). Then the voltage of the central point is obtained \( u_0 = \frac{1}{3} \sum_{i=a,b,c} g_i \frac{u_{dc}}{2} \), from which the phase voltage is obtained:

\[
\begin{align*}
\begin{bmatrix}
  u_a \\
  u_b \\
  u_c \\
\end{bmatrix} = \frac{1}{3} \Gamma \begin{bmatrix}
  g_a \\
  g_b \\
  g_c \\
\end{bmatrix} \frac{u_{dc}}{2} \quad \text{with} \quad \Gamma = \begin{bmatrix}
  2 & -1 & -1 \\
  -1 & 2 & -1 \\
  -1 & -1 & 2 \\
\end{bmatrix}
\end{align*}
\] (4.93)

Note that for any control commands \( u_i \), the balance condition \( \sum_{i=a,b,c} u_i = 0 \) holds and we may implement three phase voltages \( f_a(t), f_b(t), f_c(t) \) only if they satisfy \( \sum_{i=a,b,c} f_i = 0 \). PWM implementation algorithm is designed as follows:

\[
z_i = f_i - U_0 \text{sign}(z_i), \quad i = a, b, c
\]

It is clear that if scalar \( U_0 \) is large enough, sliding mode occurs on the surfaces \( z_i = 0 \). Using equivalent control concepts, we know that \( [U_0 \text{sign}(z_i)]_{eq} = f_i \).

After assigning \( g_i = \text{sign}(z_i) \) and substituting them into voltage equation, we get:

\[
\begin{align*}
\begin{bmatrix}
  u_a \\
  u_b \\
  u_c \\
\end{bmatrix} = \frac{u_{dc}}{6} \Gamma \begin{bmatrix}
  \text{sign} z_a \\
  \text{sign} z_b \\
  \text{sign} z_c \\
\end{bmatrix}
\end{align*}
\]

In sliding mode, the motion equations become:

\[
\begin{align*}
\begin{bmatrix}
  u_a \\
  u_b \\
  u_c \\
\end{bmatrix} \big|_{eq} &= \frac{u_{dc}}{6} \Gamma \begin{bmatrix}
  \text{sign} z_a \\
  \text{sign} z_b \\
  \text{sign} z_c \\
\end{bmatrix} = \frac{u_{dc}}{6U_0} \Gamma \begin{bmatrix}
  f_a \\
  f_b \\
  f_c \\
\end{bmatrix} = \frac{u_{dc}}{2U_0} \begin{bmatrix}
  2f_a - f_b - f_c \\
  2f_b - f_a - f_c \\
  2f_c - f_a - f_b \\
\end{bmatrix}
\end{align*}
\]

If \( u_{dc} = 2U_0 \), then \( (u_i)_{eq} = f_i \), which shows that this algorithm provides a feasible scheme for PWM implementation.

For the voltage equation (4.93), we have the following observations:

1. There are altogether eight situations for the on-off states of the three switches \( g_a, \)
$g_b, g_c$. For the situations in which all $g_i = 1$ or $g_i = -1$, each phase voltage $u_i = 0$.

2. For the other six situations, the magnitude of one phase voltage is $\frac{4}{3}\left|\frac{u_{dc}}{2}\right|$ while the magnitude of the other two is $\frac{2}{3}\left|\frac{u_{dc}}{2}\right|$.

Obviously, all eight combinations satisfy the balance condition although it never holds for $g_i$.

### 4.4.2 Sliding Mode Control Scheme in terms of $g_i$ with $i = a, b, c$

In this part, it is shown that sliding mode control can be implemented in terms of $g_i$ directly instead of $u_i$. We design the sliding surfaces as:

$$
\begin{align*}
    s_1 &= T_0 - T \\
    s_2 &= c_2(\lambda_0 - \|\lambda_r\|) + \frac{d}{dt}(\lambda_0 - \|\lambda_r\|) \\
    s_3 &= \int_0^t (g_a + g_b + g_c)dt
\end{align*}
$$

After sliding mode occurs, $(g_a + g_b + g_c)_{eq} = 0$, then:

$$
\begin{bmatrix}
    u_a \\
    u_b \\
    u_c \\
\end{bmatrix}_{eq} = \frac{u_{dc}}{6} \Gamma \begin{bmatrix}
    g_a \\
    g_b \\
    g_c \\
\end{bmatrix}_{eq} = \frac{u_{dc}}{2} \begin{bmatrix}
    g_a \\
    g_b \\
    g_c \\
\end{bmatrix}_{eq}
$$

This means $(u_i)_{eq} = k(g_i)_{eq}$. So the voltage balance condition can be satisfied automatically, i.e. $(u_a + u_b + u_c)_{eq} = 0$. From here, it is concluded that if one of the sliding surfaces is designed as the balance condition for $g_i$, we can design the control in terms of $g_i$. In other words, $g_i$ can be handled as phase voltages.

We can also design two dimensional sliding mode in terms of three switches command $g_i$. In this case, there are only two sliding mode surfaces:

$$
\begin{align*}
    s_1 &= T_0 - T \\
    s_2 &= c_2(\lambda_0 - \|\lambda_r\|) + \frac{d}{dt}(\lambda_0 - \|\lambda_r\|)
\end{align*}
$$

115
\[
\begin{bmatrix}
\dot{s}_1 \\
\dot{s}_2
\end{bmatrix} = \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix} + D \begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} = F + \frac{u_d}{6} D \Gamma \begin{bmatrix}
g_a \\
g_b \\
g_c
\end{bmatrix} = F + D \Gamma G
\]

where \( D = \begin{bmatrix}
\frac{2}{3} a_1(e_a \times \lambda) & \frac{2}{3} a_1(e_b \times \lambda) & \frac{2}{3} a_1(e_c \times \lambda) \\
\frac{2}{3} a_2(e_a \cdot \lambda) & \frac{2}{3} a_2(e_b \cdot \lambda) & \frac{2}{3} a_2(e_c \cdot \lambda)
\end{bmatrix} \) and \( G = \frac{u_d}{6} \begin{bmatrix}
g_a \\
g_b \\
g_c
\end{bmatrix} \).

Choose the Lyapunov function as \( V = \frac{1}{2} s^T s \), then \( \dot{V} = s^T \dot{s} = s^T F + s^T D \Gamma G \).

Let \((s^*)^T = s^T(D \Gamma)\), then \((s^*)^T(D \Gamma)^{-1} = s^T\). Note that \((DT)^{-1}\) denotes the pseudo inverse of \((DT)\), which is a \(3 \times 2\) matrix. Since \(\dot{V} = s^T F + (s^*)^T G\), implement the three PWM switches by the command \( g_i = -sign s_i \), i.e. \( G = -U_0 sign s^* \).

Then \(\dot{V} = (s^*)^T (D \Gamma)^{-1} F - U_0 (s^*)^T sign s^* \leq \| (s^*)^T \| \| (D \Gamma)^{-1} \| \| F \| - U_0 \| s^* \| \). If \( U_0 \geq \| (s^*)^T \| \| F \|, \dot{V} \leq 0 \) so that sliding mode can occur.

### 4.5 Summary

In this chapter, the control and observation issues for induction machine are widely explored.

First, current regulation of an induction machine with very low inductance is dealt with by sliding mode controller. Some experiment results with both inductive load and the induction machine are also included. The way to suppress the chattering is the utilization of current observers. The experiments with the inductive load and induction machine confirm that this way is promising.

Secondly, a nonlinear sliding mode observer/controller is proposed for the induction machine sensorless control. The stability analysis of the observer/controller shows that the real flux converges to the estimated flux and the average value of the discontinuous parameter of the observer is equal to the real speed. The simulation
and experiment work are also conducted to verify the proposed sensorless control scheme.

Besides, a novel sliding mode observer for induction machine is developed by combining the variable structure systems and Lyapunov designs. A Lyapunov function is chosen to determine the speed and rotor resistance of an induction motor simultaneously based on the assumption that the speed is an unknown constant parameter. This method uses measurement of the stator currents and stator voltages to estimate speed and rotor resistance of the motor and does not need differentiation of measured states to get additional information. The rotor time constant estimation is useful to overcome the rotor resistance variation, especially in indirect field orientation control. The performance of the proposed method is investigated and verified experimentally.

Moreover, more work is completed concerning use of sliding mode methodology to control the inverter directly and design the sliding mode control in terms of inverter switching commands instead of voltage command.

For the torque control in our sensorless control, the three phase voltage balance condition is taken as one sliding surface to be satisfied. In practice, the balance requirement is often met automatically by the inverter itself. Then we have one degree of freedom for the voltage control signals. How to formulate this condition and deal with that one degree of freedom is also discussed.
CHAPTER 5

SYNCHRONOUS MACHINE SLIDING MODE CONTROL
AND OBSERVATION

5.1 Current Control of Synchronous Machines

In this section, we propose a PWM technique using sliding mode principle for the current regulated pulse width modulated (CRPWM) inverter used in synchronous machine field oriented control.

5.1.1 Synchronous Machine Field Oriented Control

Steady state analysis of field orientation

In steady state, all $d$, $q$ variables are constant DC components and the damper winding currents $i_{kd}$ are $i_{kq}$ are zero. So the model equations under steady state become

$$\dot{V}_{ds} = R_s I_{ds} - \omega L_{qs} I_{qs}$$  \hspace{1cm} (5.1)

$$\dot{V}_{qs} = R_s I_{qs} + \omega (L_{ds} I_{ds} + L_{rd} I_{fd})$$ \hspace{1cm} (5.2)

$$V_{fd} = R_f d I_{fd}$$ \hspace{1cm} (5.3)
and

\[
\begin{align*}
\lambda_{ds} &= L_{ds}I_{ds} + L_{md}I_{fd} \\
\lambda_{qs} &= L_{qs}I_{qs} \\
\lambda_{fd} &= L_{f}I_{fd} + L_{md}I_{ds} \\
\lambda_{kd} &= L_{md}(I_{ds} + I_{fd}) \\
\lambda_{kq} &= I_{mq}I_{qs}
\end{align*}
\]

(5.4) (5.5) (5.6) (5.7) (5.8)

The steady state torque becomes

\[
T = \frac{3}{2} P\left[(I_{ds} - I_{qs})I_{qs}I_{ds} + L_{md}I_{qs}I_{fd}\right]
\]

(5.9)

It only includes the reluctance torque and excitation torque.

In field orientation, the stator current is all in the q axis, i.e. \(i_{ds} = 0\), so there is no reluctance torque any more. The produced torque becomes

\[
T = \frac{3}{2} PL_{md}I_{qs}I_{fd}
\]

(5.10)

Since \(I_{fd}\) is fixed by field winding voltage, i.e. \(I_{fd} = V_{fd}/R_{fd}\), the steady state torque under field orientation is decided by \(I_{qs}\) only. From here, we can see for synchronous machine field orientation control, the key part is to design a current regulator for \(I_{ds}, I_{qs}\). And the reference currents are usually chosen as

\[
\begin{align*}
I_{ds}^* &= 0 \\
I_{qs}^* &= \frac{T^*}{\frac{3}{2} PL_{md}I_{fd}}
\end{align*}
\]

(5.11) (5.12)

Next, we will design a PWM algorithm using sliding mode control for the current regulator. Before going to the sliding mode PWM, it is necessary to investigate the dynamics of synchronous machines under field orientation.

119
Dynamics of synchronous machine field orientation

Here we assume that the orientation of the stator \(d, q\) currents is maintained during the whole transient period, i.e. \(i_{ds} = 0\). After substituting \(i_{ds} = 0\) into (2.28)-(2.30) and (2.33)-(2.35), the dynamics equations under field orientation are obtained

\[
\begin{align*}
    u_{fd} &= R_{fd}i_{fd} + \frac{d}{dt}\lambda_{fd} = R_{fd}i_{fd} + \frac{d}{dt}(L_{fd}i_{fd} + L_{md}\dot{i}_{kd}) \quad (5.13) \\
    0 &= R_{kd}\dot{i}_{kd} + \frac{d}{dt}\lambda_{kd} = R_{kd}\dot{i}_{kd} + \frac{d}{dt}(L_{kd}\dot{i}_{kd} + L_{md}\dot{i}_{fd}) \quad (5.14) \\
    0 &= R_{kq}\dot{i}_{kq} + \frac{d}{dt}\lambda_{kq} = R_{kq}\dot{i}_{kq} + \frac{d}{dt}(L_{kq}\dot{i}_{kq} + L_{mq}\dot{i}_{qs}) \quad (5.15)
\end{align*}
\]

and the produced torque becomes

\[
T = \frac{3}{2} PL_{md}\dot{i}_{qs}(i_{fd} + i_{kd}) \quad (5.16)
\]

With a constant excitation voltage, i.e. \(u_{fd} = V_{fd}\), we can get the solutions to (5.13)(5.14) under the initial conditions \(i_{fdo} = V_{fd}/R_{fd}\) and \(i_{kdo} = 0\)

\[
\begin{align*}
    i_{kd}(t) &= 0 \quad (5.17) \\
    i_{fd}(t) &= \frac{V_{fd}}{R_{fd}} \quad (5.18)
\end{align*}
\]

and the torque

\[
T = \frac{3}{2} P\frac{V_{fd}}{R_{fd}} L_{md}\dot{i}_{qs} \quad (5.19)
\]

So it can be concluded that for constant excitation field, the dynamics of the field oriented synchronous machine is the same as the steady state situation. From (5.19), it can be seen that the torque response under field orientation is instantaneous and follows \(i_{qs}\) exactly. This also shows that the performance of current regulator is very important for the field oriented control.
5.1.2 Sliding Mode PWM

In this section, we propose one PWM algorithm using sliding mode for the current regulator (CRPWM) in the synchronous machine field oriented control.

We have the reference currents \( i_{dq}^* \) and \( i_{d}^* \) as

\[
\begin{align*}
    i_{dq}^* &= 0 \\
    i_{q}^* &= \frac{T^*}{\frac{3}{2} P L_{md} I_{fd}} = \frac{T^* R_{fd}}{\frac{3}{2} P L_{md} V_{fd}}
\end{align*}
\]

From (2.42), we have

\[
\frac{di}{dt} = -L^{-1} Gi + L^{-1} u
\]

The inverse of the matrix \( L \) is calculated out as

\[
L^{-1} = \begin{bmatrix}
    a_1 & 0 & a_2 & a_3 & 0 \\
    0 & a_4 & 0 & 0 & a_5 \\
    a_2 & 0 & a_6 & a_7 & 0 \\
    a_3 & 0 & a_7 & a_8 & 0 \\
    0 & a_5 & 0 & 0 & a_9
\end{bmatrix}
\]

where

\[
\begin{align*}
    a_1 &= \frac{-L_{kd} L_f + L_{md}^2}{K_2}, \\
    a_2 &= \frac{L_{md}(L_{kd} - L_{md})}{K_2} \\
    a_3 &= \frac{L_{md}(L_f - L_{md})}{K_2} \\
    a_4 &= \frac{L_{kq}}{K_3}, \\
    a_5 &= \frac{L_{mq}}{K_3} \\
    a_6 &= \frac{-L_{kd} L_d + L_{md}^2}{K_2} \\
    a_7 &= \frac{L_{md}(L_d - L_{md})}{K_2} \\
    a_8 &= \frac{-L_d L_f + L_{md}^2}{K_2} \\
    a_9 &= \frac{L_d}{K_3}
\end{align*}
\]
Finally, the machine model (5.22) can be represented as

\[
\frac{d}{dt} \begin{bmatrix}
  i_{ds} \\
  i_{qs} \\
  i_{fd} \\
  i_{kd}
\end{bmatrix} =
\begin{bmatrix}
  a_1 R_s & -\omega L_{qs} a_1 & a_2 R_{fd} & a_3 R_{kd} & -\omega L_{mq} a_1 \\
  \omega L_{ds} a_4 & R_s a_4 & \omega L_{md} a_4 & \omega L_{md} a_4 & R_{kd} a_5 \\
  a_2 R_s & -\omega L_{qs} a_2 & a_6 R_{fd} & a_7 R_{kd} & -\omega L_{mq} a_2 \\
  \omega L_{ds} a_5 & a_5 R_5 & a_7 R_{fd} & a_8 R_{kd} & -\omega L_{mq} a_3
\end{bmatrix}
\begin{bmatrix}
  i_{ds} \\
  i_{qs} \\
  i_{fd} \\
  i_{kd}
\end{bmatrix}
+ \begin{bmatrix}
  a_1 u_{ds} + a_2 u_{fd} \\
  a_4 u_{qs} \\
  a_2 u_{ds} + a_6 u_{fd} \\
  a_3 u_{ds} + a_7 u_{fd} \\
  a_5 u_{qs}
\end{bmatrix}
\]  

(5.24)

Design the sliding surfaces as

\[
s_q = i_{qs}^* - i_{qs} \\
s_d = i_{ds}^* - i_{ds} = -i_{ds}
\]

(5.25)(5.26)

The projections of the system motion on the subspace \(s_q, s_d\) are

\[
\dot{s}_q = \dot{i}_{qs}^* - \dot{i}_{qs} = i_{qs}^* - f_1(i, \omega) - a_4 u_{qs}
\]

(5.27)

\[
\dot{s}_d = -\dot{i}_{ds} = -f_2(i, \omega, u_{fd}) - a_1 u_{ds}
\]

(5.28)

where

\[
f_1(i, \omega) = \omega L_{ds} a_4 \dot{i}_{ds} + R_s a_4 \dot{i}_{qs} + \omega L_{md} a_4 \dot{i}_{fd} + \omega L_{md} a_4 \dot{i}_{kd} + R_{kd} a_5 \dot{i}_{kd}
\]

\[
f_2(i, \omega, u_{fd}) = a_1 R_s \dot{i}_{ds} - \omega L_{qs} a_1 i_{qi} + a_2 R_{fd} \dot{i}_{fd} + a_3 R_{fd} \dot{i}_{kd} - \omega L_{mq} a_1 \dot{i}_{kd} + a_2 u_{fd}
\]

Then

\[
s_q \dot{s}_q = s_q [\dot{i}_{qs}^* - f_1(i, \omega)] - a_4 s_q u_{qs}
\]

(5.29)

\[
s_d \dot{s}_d = -s_d f_2(i, \omega, u_{fd}) - a_1 s_d u_{ds}
\]

(5.30)
According to the sliding mode existence conditions for discontinuous systems, the control strategy is proposed as

\[ u_{qs} = U_{q0} sgn(a_4 s_q) \]  \hspace{1cm} (5.31) \\
\[ u_{ds} = U_{d0} sgn(a_1 s_d) \]  \hspace{1cm} (5.32)

where \( U_{q0} > |f_1 - i_{qs}^*|/|a_4| \) and \( U_{d0} > |f_2|/|a_1| \). Then

\[ s_q \dot{s}_q = s_q [i_{qs}^* - f_1(i, \omega)] - U_{q0} a_4 s_q < 0 \]  \hspace{1cm} (5.33) \\
\[ s_d \dot{s}_d = -s_d f_2(i, \omega, u_{fd}) - U_{d0} a_1 s_d < 0 \]  \hspace{1cm} (5.34)

so the values of \( s \) and \( \dot{s} \) have opposite signs and the state reaches the sliding surface \( s = 0 \) after a finite time interval.

As defined in (2.20)(2.24),

\[ u_{qds} = \Gamma_{qdl}^{abc} u_{abc} \]  \hspace{1cm} (5.35)

where \( u_{qds} = [u_{qs} \ u_{ds}]^T \), \( u_{abc} = [u_{a_1} \ u_{b_1} \ u_{c_1}]^T \) and \( \Gamma_{qdl}^{abc} = \Gamma_{qdl}^{\alpha \beta} \Gamma_{\alpha \beta}^{abc} \). From (2.21)(2.25), \( \Gamma_{qdl}^{abc} \) can be obtained

\[ \Gamma_{qdl}^{abc} = \Gamma_{qdl}^{\alpha \beta} \Gamma_{\alpha \beta}^{abc} = \frac{2}{3} \begin{bmatrix} \cos \theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin \theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \]  \hspace{1cm} (5.36)

Since \( \Gamma_{qdl}^{abc} \) is a \( 2 \times 3 \) matrix, the inverse transformation from \( (q, d) \) to \( (a, b, c) \) can not be decided uniquely. We have the pseudoinverse \( \Gamma_{qdl}^{abc^+} \) as

\[ \Gamma_{qdl}^{abc^+} = (\Gamma_{qdl}^{abc})^T (\Gamma_{qdl}^{abc}) (\Gamma_{qdl}^{abc})^T \]  \hspace{1cm} (5.37)

so

\[ u_{abc} = \Gamma_{qdl}^{abc^+} u_{qds} \]  \hspace{1cm} (5.38)

Combining (5.31)(5.32) with (5.38), the desired terminal voltages are obtained

\[
\begin{bmatrix}
  v_{a_1} \\
  v_{b_1} \\
  v_{c_1}
\end{bmatrix}
= \Gamma_{qdl}^{abc^+} \begin{bmatrix}
  U_{q0} sgn(a_4 s_q) \\
  U_{d0} sgn(a_1 s_d)
\end{bmatrix}
\]  \hspace{1cm} (5.39)
Unfortunately, because of the inverter, the real terminal voltages $u_{as}$, $u_{bs}$, $u_{cs}$ can only take two possible values: $U_0$ or $-U_0$. Hence it is not possible to implement (5.39) directly in the inverter. Here we propose a PWM algorithm using sliding mode method.

Define the sliding mode surfaces

$$s_a = \int_0^t (u_{as}^* - u_{as})dt$$

(5.40)  

$$s_b = \int_0^t (u_{bs}^* - u_{bs})dt$$

(5.41)  

$$s_c = \int_0^t (u_{cs}^* - u_{cs})dt$$

(5.42)

and the control strategy for the terminal voltages

$$u_{as} = U_0 sgn(s_a), \quad u_{bs} = U_0 sgn(s_b), \quad u_{cs} = U_0 sgn(s_c)$$

(5.43)

where $U_0$ is DC bus voltage in the inverter.

Then we have

$$s_a \dot{s}_a = u_{as}^* s_a - U_0 |s_a|$$

(5.44)  

$$s_b \dot{s}_b = u_{bs}^* s_b - U_0 |s_b|$$

(5.45)  

$$s_c \dot{s}_c = u_{cs}^* s_c - U_0 |s_c|$$

(5.46)

If $U_0 > \max(|u_{as}^*|, |u_{bs}^*|, |u_{cs}^*|)$, sliding modes can be enforced in $s_a = 0$, $s_b = 0$ and $s_c = 0$. The equivalent control values for $u_{as}$, $u_{bs}$, $u_{cs}$ can be obtained by setting $\dot{s}_a = 0$, $\dot{s}_b = 0$, $\dot{s}_c = 0$

$$u_{as}^{eq} = u_{as}^*, \quad u_{bs}^{eq} = u_{bs}^*, \quad u_{cs}^{eq} = u_{cs}^*$$

(5.47)

This means the desired terminal voltages in (5.39) are implemented successfully using the proposed sliding mode PWM (5.43).
5.1.3 Direct Sliding Mode Current Control

In this section, another sliding mode control approach is proposed for the terminal voltages \( u_a, u_b, u_c \). The main difference from the above approach is that the terminal voltages can be directly implemented by the inverter. No additional PWM is needed.

Design the sliding mode surfaces

\[
\begin{align*}
    s_q &= \frac{1}{a_4} (i_{qs}^* - i_{qs}) \\
    s_d &= \frac{1}{a_1} (i_{ds}^* - i_{ds}) = \frac{1}{a_1} i_{ds}
\end{align*}
\]

Similarly, after differentiation, we have

\[
\dot{s}_{qd} = F_{qd} - u_{qds} = F_{qd} - \Gamma_{qd}^{abc} u_{abc}
\]

where \( s_{qd} = [s_q \ s_d]^T \) and \( F_{qd} = \begin{bmatrix} -1/\alpha_4 (i_{qs}^* - f_1(i, \omega)) \\ -1/\alpha_1 f_2(i, \omega, u_{fd}) \end{bmatrix} \)

Design the Lyapunov function candidate \( V = \frac{1}{2} s_{qd}^T s_{qd} \), the derivative is

\[
\dot{V} = s_{qd}^T \dot{s}_{qd} = s_{qd}^T (F_{qd} - \Gamma_{qd}^{abc} u_{abc})
\]

The control signal i.e. three terminal voltages is designed as

\[
u_{abc} = U_0 \text{sgn}(s_{abc})
\]

in which we define

\[
\begin{align*}
    s_{abc} &= [s_a \ s_b \ s_c]^T = \Gamma_{qd}^{abc} s_{qd} \\
    \text{sgn}(s_{abc}) &= [\text{sgn}(s_a) \ \text{sgn}(s_b) \ \text{sgn}(s_c)]^T.
\end{align*}
\]

So (5.51) is rewritten

\[
\dot{V} = s_{qd}^T (F_{qd} - \Gamma_{qd}^{abc} u_{abc}) = (\Gamma_{qd}^{abc} s_{abc})^T (F_{qd} - \Gamma_{qd}^{abc} u_{abc})
\]

\[
= s_{abc}^T \Gamma_{qd}^{abc} F_{qd} - s_{abc}^T \Gamma_{qd}^{abc} U_0 \text{sgn}(s_{abc})
\]
After defining $\Gamma_{qd}^{acT} F_{qd} = [F_a \ F_b \ F_c]^T$, 
\[
\dot{V} = (s_a F_a + s_b F_b + s_c F_c) - \frac{4U_0}{9} [s_a \ s_b \ s_c] \left[ \begin{array}{ccc} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{array} \right] [\text{sgn}(s_a) \ \text{sgn}(s_b) \ \text{sgn}(s_c)]
\]

From (5.53), it can be found that $s_a$, $s_b$ and $s_c$ cannot be of the same signs simultaneously. Without loss of generality, suppose $\text{sgn}(s_a) \neq \text{sgn}(s_b) = \text{sgn}(s_c)$, we have 
\[
\dot{V} = (s_a F_a + s_b F_b + s_c F_c) - \frac{4U_0}{9} (2|s_a| + |s_b| + |s_c|)
\]

If DC bus voltage satisfies $U_0 > \frac{2}{9} \max(|F_a|, |F_b|, |F_c|)$, then $\dot{V} < 0$, which means that sliding mode happens after finite time in $s_{qd} = 0$.

5.2 Torque/Speed Control of Synchronous Motors

In the previous section, the control system is of cascade structure actually, i.e. it has several control loops. In the usual field orientation control, the outer loop is torque or speed control loop using PID controller, which produces the reference current $i_{ds}^*, i_{qs}^*$. Then in the inner current control loop, different techniques can be applied for PWM inverter, including our sliding mode PWM method.

In this section, we propose a different control methodology: to apply sliding mode control directly to the machine model for torque or speed control. In this methodology, we do not have the current control block any more. In fact the current control is implemented in the sliding mode torque/speed control implicitly.

5.2.1 Sliding Mode Torque Control

(5.3)(5.9) show that in steady state, for a constant excitation machine ($V_{jd} = constant$), the produced torque is decided by two stator current components $I_{ds}$, $I_{qs}$.
This motivates us to design the sliding mode surfaces as

\[ s_1 = \frac{4}{3P}(T^s - T) \]
\[ s_2 = i_{ds}^s - i_{ds} \]

Projections of the system motions on the subspace \((s_1, s_2)\) give

\[ \dot{s}_1 = \frac{4}{3P}\dot{T}^s - [(L_{ds} - L_{qs})(i_{qs}i_{ds} + i_{qs}i_{ds}) + L_{rd}(i_{qs}i_{fd} + i_{qs}i_{fd})] + L_{rd}(i_{qs}i_{kd} + i_{qs}i_{kd}) - L_{mq}(i_{ds}i_{kq} + i_{ds}i_{kq}) \]
\[ \dot{s}_2 = i_{ds}^s - i_{ds} \]

Substituting machine model (5.24), then

\[ \dot{s}_{12} = F - W_{qds} \quad (5.55) \]

where \(s_{12} = [s_1 \ s_2]^T\), \(F = [f_1 \ f_2]^T\) and

\[ W = \begin{bmatrix} a_4(L_{ds} - L_{qs}) + 2a_4L_{rd} - a_5L_{mq} & a_1(L_{ds} - L_{qs}) + (a_2 + a_3)L_{rd} - a_1L_{mq} \\ 0 & a_1 \end{bmatrix} \]

only depends on the machine parameters and is nonsingular. \(f_1\) and \(f_2\) are functions of \(T^s, \omega, i_{ds}^s, i_{ds}, i_{qs}, i_{fd}, i_{kd}, i_{kq}\) and \(u_{fd}\).

Define the new sliding mode surfaces \(s_{12} = [s_1^s \ s_2^s]^T\) after the transformation 

\[ s_{12}^s = W^T s_{12}, F^s = [f_1^s \ f_2^s]^T = W^{-1}F, \text{ then} \]

\[ s_{12}^T \dot{s}_{12}^s = s_{12}^T W^{-1}(F - W_{qds}) = s_{12}^T F^s - s_{12}^T W_{qds} \]

Design control strategy

\[ u_{qs} = U_{q0} \text{sgn}(s_1^s) \]
\[ u_{ds} = U_{d0} \text{sgn}(s_2^s) \quad (5.56) \]

where \(U_{q0} > |f_1^s|\) and \(U_{d0} > |f_2^s|\). So

\[ s_{12}^T \dot{s}_{12}^s = s_1^s f_1^s + s_2^s f_2^s - U_{q0}|s_1^s| - U_{d0}|s_2^s| < 0 \]

127
i.e. sliding modes will happen in \( s_{12} = 0 \). Then the terminal voltages \( u_{qs}, u_{bs}, u_{cs} \) are implemented from \( u_{qs}, u_{ds} \) using the approach in Section 5.1.2.

### 5.2.2 Sliding Mode Speed Control

For speed control, the sliding mode surfaces are proposed as

\[
\begin{align*}
    s_1 &= c(\omega^* - \omega) + \frac{d}{dt}(\omega^* - \omega) \\
    s_2 &= i_s^* - i_{ds}
\end{align*}
\]

So

\[
\begin{align*}
    \dot{s}_1 &= \frac{4J}{3F_2}[c(\omega^* - \omega) + \dot{\omega}^* + \frac{P}{J}\dot{T}_1] - [(L_{ds} - L_{qs})(\dot{i}_{qs}i_{ds} + i_{qs}\dot{i}_{ds}) + L_{md}(\dot{i}_{qs}i_{fd} + i_{qs}\dot{i}_{fd}) \\
    &+ L_{md}(\dot{i}_{ds}i_{kd} + i_{ds}\dot{i}_{kd}) - L_{md}(\dot{i}_{ds}i_{kq} + i_{ds}\dot{i}_{kq})] \\
    \dot{s}_2 &= \dot{i}_{ds} - \dot{i}_{ds}
\end{align*}
\]

then following the same approach proposed in sliding mode torque control, it is easy to prove sliding mode will happen in \( s_{12} = 0 \).

### 5.3 Control of a Synchronous Generator in Power Systems

#### 5.3.1 Power Angle Control of Synchronous Generators

Here, we propose a sliding mode control scheme to control the power angle \( \delta \) by the excitation voltage \( u_{fd} \).

Based on the model (2.54), the sliding mode surface is designed as

\[
    s = \frac{d^2}{dt^2}(\delta^* - \delta) + k_1 \frac{d}{dt}(\delta^* - \delta) + k_2(\delta^* - \delta) \tag{5.57}
\]

where \( \delta^* \) is the reference power angle. \( k_1 \) and \( k_2 \) are chosen for the convergence behavior of \( \delta \) after sliding mode happens in \( s = 0 \).
Projection of the system motion (2.54) on the subspace $s$ is

$$\dot{s} = k_1(\ddot{\delta} - \dot{\delta}^* + k_2(\dot{\delta} - \ddot{\delta}) + \dot{\delta}^* - \underbrace{\omega_i}_+ + \underbrace{\frac{P_1}{f}} + \underbrace{\frac{P_2}{f}}$$

$$= f + \frac{3}{2} P_2 f (d_3 - d_1 a'_2 - d_2 a'_2) u_{fd}.$$

where $f$ is a function related to all system states. Design the control input

$$u_{fd} = -U_0 sgn[(d_3 - d_1 a'_2 - d_2 a'_2)s]$$

If $U_0 > \frac{22J}{3P_2} \left( \frac{1}{|d_3 - d_1 a'_2 - d_2 a'_2|} \right)$,

$$s\dot{s} = sf - \frac{3}{2} P_2 U_0 (d_3 - d_1 a'_2 - d_2 a'_2)s < 0$$

so sliding mode will be enforced by $u_{fd}$ in $s = 0$, i.e. $(\ddot{\delta} - \dot{\delta}) + k_1(\dot{\delta} - \ddot{\delta}) + k_2(\dot{\delta}^* - \delta) = 0$.

With properly chosen $k_1$ and $k_2$, the power angle $\delta$ will converge to the reference value $\delta^*$.

### 5.3.2 Terminal Voltage Control of Synchronous Generators

Consider Equation (2.53) for the generator terminal voltage. In steady state, $i_{ds} = 0$ and $i_{qs} = 0$, then the sum of the square of (2.53) produces

$$u^2_g = u^2_{ds} + u^2_{qs} = (L^2 \omega^2 + R^2_e)(i^2_{ds} + i^2_{qs}) + V^2$$

Using this relationship between terminal voltages and currents, we can transform the terminal voltage control issue into current control. Suppose we have $V^*_g$ as the reference terminal voltage, from (5.59) the corresponding reference current can be obtained

$$i^*_s = \sqrt{(i^2_{ds} + i^2_{qs})} = \sqrt{\frac{V^2 - V^2}{L^2 \omega^2 + R^2_e}}$$

Let us design the sliding mode surface

$$s = i^*_s - \sqrt{i^2_{ds} + i^2_{qs}}$$

129
Differentiate (5.61) and combine with (2.54), we have

\[ \dot{s} = i_s^* - \frac{1}{\sqrt{i_d^2 + i_q^2}} (i_d \dot{i}_{d_s} + i_q \dot{i}_{q_s}) \]

\[ = f + \frac{a_d^2 i_{d_s} + a_q^2 i_{q_s}}{\sqrt{i_d^2 + i_q^2}} u_{fd} \]

Under the control strategy \( u_{fd} = -U_0 \text{sgn}(a_d^2 i_{d_s}s) \) and \( U_0 > \frac{f\sqrt{i_d^2 + i_q^2}}{|a_d^2 i_{d_s}|} \),

\[ s \dot{s} = sf - \frac{U_0}{\sqrt{i_d^2 + i_q^2}} |a_d^2 i_{d_s}s| < 0 \]

This means the current control to (5.60) can be achieved using sliding mode control, which brings us the terminal voltage control objective finally.

### 5.4 Sliding Mode Observers for PMSM

#### 5.4.1 Sliding Mode Position Observer of PMSM

Based on the model equation (2.41), the sliding mode current observer is designed

\[ \dot{i}_\alpha = -h \text{sign}(\dot{i}_\alpha - i_\alpha) \]  \hspace{1cm} (5.62)

\[ \dot{i}_\beta = -h \text{sign}(\dot{i}_\beta - i_\beta) \]  \hspace{1cm} (5.63)

Combination with (2.41) produces

\[ \dot{i}_\alpha = -h \text{sign}(\dot{\vec{i}}_\alpha - f_1 i_\alpha - f_2 i_\beta - f_3 u_\alpha - f_4 u_\beta + g_1 \lambda_m) \]

\[ \dot{i}_\beta = -h \text{sign}(\dot{\vec{i}}_\beta - f_3 i_\alpha - f_4 i_\beta - f_5 u_\alpha - f_6 u_\beta + g_2 \lambda_m) \]

For the Lyapunov function \( V = \frac{1}{2}(\vec{i}_\alpha^2 + \vec{i}_\beta^2) \), we have

\[ \dot{V} = -h(|\vec{i}_\alpha| + |\vec{i}_\beta|) \]

\[ + \vec{i}_\alpha(g_1 \lambda_m - f_1 i_\alpha - f_2 i_\beta - f_3 u_\alpha - f_4 u_\beta) \]

\[ + \vec{i}_\beta(g_2 \lambda_m - f_3 i_\alpha - f_4 i_\beta - f_5 u_\alpha - f_6 u_\beta) \]
If \( h \) is chosen to satisfy \( h > \max \{ |g_1 \lambda_m - f_1 i_\alpha - f_2 i_\beta - f_3 u_\alpha - f_4 u_\beta|, |g_2 \lambda_m - f_5 i_\alpha - f_6 i_\beta - f_7 u_\alpha - f_8 u_\beta| \} \), it can be concluded that sliding mode can be enforced in \( \tilde{i}_\alpha = 0 \) and \( \tilde{i}_\beta = 0 \), i.e. current estimations converge to the real values.

After sliding mode happens, we can obtain those two equivalent control

\[
\begin{bmatrix}
-h \text{sign}(\tilde{i}_\alpha) \\
-h \text{sign}(\tilde{i}_\beta)
\end{bmatrix}_{eq} =
\begin{bmatrix}
 f_1 & f_2 \\
 f_5 & f_6 \\
 f_3 & f_4 \\
 f_4 & f_7
\end{bmatrix}
\begin{bmatrix}
 i_\alpha \\
 i_\beta
\end{bmatrix}
+ \begin{bmatrix}
 f_3 & f_4 \\
 f_4 & f_7
\end{bmatrix}
\begin{bmatrix}
 u_\alpha \\
 u_\beta
\end{bmatrix}
- \begin{bmatrix}
 g_1 \\
 g_2
\end{bmatrix} \lambda_m
\]  

\hspace{1cm} (5.64)

It is assumed here that \((-h \text{sign}(\tilde{i}_\alpha))_{eq}, (-h \text{sign}(\tilde{i}_\beta))_{eq}, i_\alpha, i_\beta, u_\alpha, u_\beta \) and \( \lambda_m \) are known. The rotor position \( \theta_r \) and rotor speed \( \omega_r \) are to be found from the above two equations.

Let us define \( x_1 = \sin \theta_r, x_2 = \omega_r \cos \theta_r \), then we have

\[
\begin{align*}
\sin \theta_r &= x_1 \\
\cos \theta_r &= \sqrt{1 - x_1^2} \\
\omega_r \cos \theta_r &= x_2 \\
\omega_r &= \frac{x_2}{\sqrt{1 - x_1^2}} \\
\sin 2\theta_r &= 2x_1 \sqrt{1 - x_1^2} \\
\cos 2\theta_r &= 1 - 2x_1^2.
\end{align*}
\]

So all of the functions above \( f_1, f_2, f_3, f_4, f_5, f_6, f_7, g_1 \) and \( g_2 \) can be represented in terms of \( x_1 \) and \( x_2 \). From the two algebraic equations (5.64), we are able to calculate \( x_1 \) and \( x_2 \), from which the rotor position and speed information \( \theta_r \) and \( \omega_r \) can be obtained.

### 5.4.2 Sliding Mode Speed Observer for Non-Saliency PMSM

In this section, the PMSM with non-saliency is under consideration. In this case, \( L_1 = 0 \). Then the model equation will be much simpler compared with that with
saliency effect. The model becomes

\[
\dot{i}_\alpha = -\frac{R_s}{L_0} i_\alpha + \frac{1}{L_0} u_\alpha - \frac{\lambda_m}{L_0} \omega_r \cos \theta_r \tag{5.65}
\]

\[
\dot{i}_\beta = -\frac{R_s}{L_0} i_\beta + \frac{1}{L_0} u_\beta - \frac{\lambda_m}{L_0} \omega_r \sin \theta_r \tag{5.66}
\]

A speed observer by sliding mode approach is proposed for this kind of PMSM. Let us define the back-EMF as

\[
e_\alpha = \lambda_m \omega_r \cos \theta_r \tag{5.67}
\]

\[
e_\beta = \lambda_m \omega_r \sin \theta_r \tag{5.68}
\]

Then we have the dynamics of the back-EMF based on the assumption that the speed is varying slowly in contrast with those electrical variables, i.e. \(\dot{\omega}_r = 0\),

\[
\dot{e}_\alpha = -\omega_r e_\beta \tag{5.69}
\]

\[
\dot{e}_\beta = \omega_r e_\alpha. \tag{5.70}
\]

As before, the sliding mode observer is designed for the stator current as

\[
\dot{i}_\alpha = -\frac{R_s}{L_0} \tilde{i}_\alpha + \frac{1}{L_0} u_\alpha - \frac{l}{L_0} \text{sign}(\tilde{i}_\alpha - i_\alpha) \tag{5.71}
\]

\[
\dot{i}_\beta = -\frac{R_s}{L_0} \tilde{i}_\beta + \frac{1}{L_0} u_\beta - \frac{l}{L_0} \text{sign}(\tilde{i}_\beta - i_\beta), \tag{5.72}
\]

then the error dynamics for the current estimation is obtained

\[
\dot{\tilde{i}}_\alpha = -\frac{R_s}{L_0} \tilde{i}_\alpha + \frac{e_\alpha}{L_0} - \frac{l}{L_0} \text{sign}(\tilde{i}_\alpha - i_\alpha) \tag{5.73}
\]

\[
\dot{\tilde{i}}_\beta = -\frac{R_s}{L_0} \tilde{i}_\beta + \frac{e_\beta}{L_0} - \frac{l}{L_0} \text{sign}(\tilde{i}_\beta - i_\beta) \tag{5.74}
\]

To show the convergence of the above observer, the Lyapunov candidate function is chosen \(V = \frac{1}{2}(\tilde{i}_\alpha^2 + \tilde{i}_\beta^2)\), then

\[
\dot{V} = -\frac{R_s}{L_0} (\tilde{i}_\alpha^2 + \tilde{i}_\beta^2) + \frac{1}{L_0} (e_\alpha \tilde{i}_\alpha + e_\beta \tilde{i}_\beta - l|\tilde{i}_\alpha| - l|\tilde{i}_\beta|)
\]

132
If \( l \) is large enough, \( V \) is decaying to zero until \( \tilde{i}_\alpha \) and \( \tilde{i}_\alpha \) are equal to zero, which means that the estimated currents will converge to their true values.

After sliding mode occurs, the equivalent control information of two discontinuous control components can be obtained by using the low pass filter, i.e.

\[
(l\text{sign}(\tilde{i}_\alpha))_{eq} = e_\alpha
\]

\[
(l\text{sign}(\tilde{i}_\beta))_{eq} = e_\beta
\]

From the back-EMF dynamics (5.69) (5.70), the corresponding observers are designed as

\[
\dot{e}_\alpha = -e_\beta \omega_r - l_2(\dot{e}_\alpha - e_\alpha)
\]

\[
\dot{e}_\beta = e_\alpha \omega_r - l_2(\dot{e}_\beta - e_\beta)
\]

Then for the error \( e_\alpha, \bar{e}_\beta \), we have

\[
\dot{\bar{e}}_\alpha = -e_\beta \bar{\omega}_r - l_2 \bar{e}_\alpha \tag{5.75}
\]

\[
\dot{\bar{e}}_\beta = e_\alpha \bar{\omega}_r - l_2 \bar{e}_\beta \tag{5.76}
\]

To guarantee the stability of the above error system, the Lyapunov design approach is employed here. Let us choose the Lyapunov candidate function \( V = \frac{1}{2}(e_\alpha^2 + e_\beta^2 + \bar{\omega}_r^2) \), then

\[
\dot{V} = -l_2(e_\alpha^2 + e_\beta^2) + \bar{\omega}_r \dot{\bar{\omega}}_r + \bar{\omega}_r (-e_\alpha e_\beta + \bar{e}_\beta e_\alpha).
\]

To cancel the last two terms, the adaptive law is chosen as

\[
\dot{\bar{\omega}}_r = -(-e_\alpha e_\beta + \bar{e}_\beta e_\alpha),
\]

so \( \bar{e}_\alpha \to 0 \) and \( \bar{e}_\beta \to 0 \) with the Lyapunov function decaying, which makes \( \bar{\omega}_r \) a constant value. By taking (5.75)(5.76) into consideration, we can conclude that the estimation error \( \bar{\omega}_r \) will converge to the constant value zero.
5.4.3 Simulation Results

In this section, the simulation is conducted on a permanent magnet synchronous machine to validate the observer algorithm proposed in Section 5.4.2. The PMSM under consideration is assumed to be non-salient, i.e. \( L_{ds} = L_{qs} \triangleq L_s \), so \( L_1 = 0 \) and \( L_0 = L_s \).

The Field Oriented Control (FOC) is used to control the machine speed while our sliding mode speed observer is applied for speed estimation. As we know, the sliding mode speed estimator is constructed in \((\alpha, \beta)\) stationary reference frame shown in Section 5.4.2. Compared with the observer, the FOC scheme is under synchronous rotating \((d, q)\) reference frame. To clarify FOC, the permanent magnet synchronous machine modeling in \((d, q)\) frame is given:

\[
\begin{align*}
  u_{ds} &= R_s i_{ds} + \hat{\lambda}_{ds} - \omega_r \lambda_{qs} \\
  u_{qs} &= R_s i_{qs} + \hat{\lambda}_{qs} + \omega_r \lambda_{ds}
\end{align*}
\]  

(5.77)  

(5.78)

where

\[
\begin{align*}
  \lambda_{ds} &= L_s i_{ds} + \lambda_m \\
  \lambda_{qs} &= L_s i_{qs}
\end{align*}
\]  

(5.79)  

(5.80)

After substitution, we have

\[
\begin{align*}
  u_{ds} &= R_s i_{ds} + L_s \dot{i}_{ds} - \omega_r L_s i_{qs} \\
  u_{qs} &= R_s i_{qs} + L_s \dot{i}_{qs} + \omega_r L_s i_{ds} + \omega_r \lambda_m
\end{align*}
\]  

(5.81)  

(5.82)

and

\[
T = \frac{3}{2} P (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) = \frac{3}{2} P \lambda_m i_{qs}
\]  

(5.83)
<table>
<thead>
<tr>
<th>1.1 kW</th>
<th>220 V</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000 rpm</td>
<td>P = 4</td>
</tr>
<tr>
<td>$\lambda_m = 0.175\ Wb$</td>
<td>$J = 0.0008\ Kg.m^2$</td>
</tr>
<tr>
<td>$R_s = 2.875\ \Omega$</td>
<td>$L_s = 8.5\ mH$</td>
</tr>
</tbody>
</table>

Table 5.1: Permanent magnet synchronous machine parameters

In field orientation, $i_{ds} = 0$ and the torque $T$ will be directly controlled by $i_{qs}$.

The machine parameters are listed in Table 5.1. And the system block diagram is shown in Figure 5.1. All the simulations results are shown in Figures 5.2-5.12. Figure 5.2 demonstrates the speed control achieved by field oriented control method with Figure 5.3 showing the speed tracking error. As can be seen, the speed control exhibits high accuracy. Figure 5.4 shows the speed estimation done by our proposed sliding mode speed observer, which works quite well. Figures 5.5 and 5.6 are the results of sliding mode current observation of $i_\alpha$ while Figures 5.7 and 5.8 are the results of sliding mode current observation of $i_\beta$. Figures 5.9-5.12 demonstrate the back EMF $e_\alpha$ and $e_\beta$ obtained from the equivalent control components of $lsign(\bar{i}_\alpha)$ and $lsign(\bar{i}_\beta)$ by using low pass filters. For comparison, we are showing $e_\alpha$ and $e_\beta$ obtained from the low pass filters with those calculated from (5.67)-(5.68).

5.5 Summary

In this Chapter, control of synchronous machines with damper and excitation windings by sliding mode approach is proposed. Both the motor and the generator are considered, including the single generator infinite bus system.

The following work is done:
Figure 5.1: System block diagram
Figure 5.2: Field oriented speed control
Figure 5.3: Speed tracking error
Figure 5.4: Sliding mode speed observation
Figure 5.5: Sliding mode current observation $i_\alpha$
Figure 5.6: Sliding mode current observation $i_\alpha$ (enlarged)
Figure 5.7: Sliding mode current observation $i_\beta$
Figure 5.8: Sliding mode current observation $i_\beta$ (enlarged)
Figure 5.9: Back EMF observation $e_\alpha$
Figure 5.10: Back EMF observation $e_\alpha$ (enlarged)
Figure 5.11: Back EMF observation $e_\beta$
Figure 5.12: Back EMF observation $e_{\beta}$ (enlarged)
• Synchronous Motor Control:

  – a sliding mode PWM algorithm is proposed for the current regulator in synchronous motor field orientation control.

  – a direct current control strategy by sliding modes is designed.

  – a direct torque control strategy by sliding modes is designed.

  – a direct speed control strategy by sliding modes is designed.

• Synchronous Generator Control:

  – a sliding mode controller for power angle is proposed.

  – a sliding mode controller for terminal voltage is proposed.

Permanent magnet synchronous machines (PMSM) observation issues are also studied. The sliding mode position and speed observers are proposed for the PMSM and non-saliency PMSM respectively.
CHAPTER 6

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

This chapter will summarize the contributions of the dissertation and provide suggestions for future research directions.

6.1 Contributions

This dissertation primarily discusses the development of control and observation strategies for AC electric machines, especially induction machines and synchronous machines. Specifically,

- Induction machine control and observation
  
  - A sliding mode current controller is proposed for induction machine. Both theoretical and implementation analysis are conducted, including the simulation and real experiments. Besides, the chattering problems, encountered often in sliding mode control area, are dealt with by using an asymptotic observer.
  
  - A complete sensorless control system by using sliding modes is developed. Sliding mode control is used for both the control and observation sides
of the system. The experiments show the validity and efficiency of the system. Moreover, experimental results of the system working at very low speeds prove its high performance.

- A speed and rotor time constant observer is designed to improve the performance of the Indirect Field Oriented Control system, which is widely used in industry for induction machines. The system is experimentally demonstrated for its validity.

- The sliding mode methods used in PWM implementations are also discussed. Here they are used to control the power inverter directly, i.e. inverter switching commands instead of voltage commands. It should be noted that these methodologies are not limited to induction machine control only and can be expanded to be used for synchronous machines.

- Synchronous machine control and observation

  - Synchronous machine current control is discussed. Both sliding mode PWM and direct current control schemes are proposed.

  - The sliding mode torque/speed controllers are designed.

  - The synchronous generator working in the power systems is considered. And the sliding mode controllers are designed for power angle and terminal voltage respectively.

  - The observation problems for permanent magnet synchronous machines are dealt with using sliding mode control methods. Both the speed and position observers are designed.
6.2 Future Research Directions

The results of this dissertation open some interesting and challenging problems of great importance. In what follows, we point out some of the possible future research directions:

- Hybrid Electric Vehicle (HEV) is a promising direction for the automotive industry and research in this area is held at almost all the world famous companies. The core problem for HEV is design of an efficient electric drive based on AC electric machine. Usually an induction machine or permanent magnet synchronous machine is used in HEV. This dissertation opens one possible research direction for the electric drive system in HEV development: to investigate and develop a robust sliding mode drive system for HEV. Currently there are some projects like ISA (Integrated Starter Alternator) project going on in Center for Automotive Research at The Ohio State University.

- This dissertation proposes a sliding mode observer of speed and rotor time constant for induction machine control to improve the performance of an Indirect Field Oriented Control system. Open-loop experiments have been conducted showing the validity of the control methods. Experiments with closed-loop systems are expected to implement in the future. Also the system robustness to the so-called detuning effects (e.g. when the machine parameters deviate from their nominal values) is an interesting topic to be researched.

- The observation methodology is only outlined for the position observer of permanent magnet synchronous machine. However, the problem of solving those
algebraic equations to get the position information in an efficient way is still worth researching.
BIBLIOGRAPHY


