GIS Uses for Modeling Subsurface Conditions in Ohio Coal Mines

A thesis presented to
the faculty of
the Russ College of Engineering and Technology of Ohio University

In partial fulfillment
of the requirements for the degree
Master of Science

Kurt W. Kleski
December 2017

© 2017 Kurt W. Kleski. All Rights Reserved.
This thesis titled
GIS Uses for Modeling Subsurface Conditions in Ohio Coal Mines

by
KURT W. KLESKI

has been approved for
the Department of Civil Engineering
and the Russ College of Engineering and Technology by

R. Guy Riefler
Professor of Civil Engineering

Dennis Irwin
Dean, Russ College of Engineering and Technology
ABSTRACT

KLESKI, KURT W., M.S., December 2017, Civil Engineering

GIS Uses for Modeling Subsurface Conditions in Ohio Coal Mines

Director of Thesis: R. Guy Riefler

Commercial GIS can be used to model the coal seam using spatial interpolation of test hole data. Multilinear least squares regression (MSLR) is the most practical method of spatial interpolation, given the inaccuracies that can occur with test hole data. The coal industry has used the three-point method for years to establish the strike and dip of the coal seam. The 3-point method uses only 3 points of data rather than the entire test hole data set to establish the strike and dip of the coal bedding plane. A strike-dip is a description for geologic strata that implies that the lithologic unit lies on a planar surface. MSLR uses all the available data from test holes to spatially interpolate the elevation of the coal seam. The MSLR will be rationalized through comparison with typical mine design methods and requirements of Ohio law. The MSLR elevation model of the coal seam will then be used in a variety of analysis to show the practicality of modeling mines in the GIS environment. Models will be created in Python and GIS for simplicity to the end user.
DEDICATION

To Donald J. Trump for trying to make America great again!
ACKNOWLEDGMENTS

Thanks to Dr. Ben Stuart and Jennifer C. Chapman for giving me the encouragement to get started in the Civil Engineering Program 4 short years ago. Thanks to Deb McAvoy for providing the opportunity to teach and develop the Civil Engineering AutoCAD class and continue in the Civil Engineers Master’s Program at Ohio University. Thanks to B&N Coal, Inc. and Rosebud Mining, Co. for providing mine data to show the uses of the GIS models. Thanks to R. Guy Riefler for proof reading and helping with the editing of this thesis. Another thanks to Jennifer C. Chapman for the knowledge, support, and flexibility of work hours so that I may complete a Masters in Civil Engineering.
**TABLE OF CONTENTS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>3</td>
</tr>
<tr>
<td>Dedication</td>
<td>4</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>5</td>
</tr>
<tr>
<td>List of Tables</td>
<td>8</td>
</tr>
<tr>
<td>List of Figures</td>
<td>9</td>
</tr>
<tr>
<td>Introduction</td>
<td>11</td>
</tr>
<tr>
<td>Rationale for Research</td>
<td>11</td>
</tr>
<tr>
<td>Mining and the Environment</td>
<td>12</td>
</tr>
<tr>
<td>Climate</td>
<td>12</td>
</tr>
<tr>
<td>Surface Runoff</td>
<td>13</td>
</tr>
<tr>
<td>Groundwater</td>
<td>15</td>
</tr>
<tr>
<td>Ohio’s Geologic Description</td>
<td>17</td>
</tr>
<tr>
<td>Location</td>
<td>17</td>
</tr>
<tr>
<td>Pre-Cambrian and Cambrian (4500-500 MYA)</td>
<td>19</td>
</tr>
<tr>
<td>Ordovician, Silurian, and Devonian (500-350 MYA)</td>
<td>19</td>
</tr>
<tr>
<td>Carboniferous Periods (350-300 MYA)</td>
<td>21</td>
</tr>
<tr>
<td>Permian (300-250 MYA)</td>
<td>21</td>
</tr>
<tr>
<td>The Mesozoic Era (250-50 MYA)</td>
<td>22</td>
</tr>
<tr>
<td>Quaternary Period (2-0 MYA)</td>
<td>22</td>
</tr>
<tr>
<td>Summary of Ohio Geology and Coal Seams</td>
<td>23</td>
</tr>
<tr>
<td>History of Coal Mining in Ohio</td>
<td>25</td>
</tr>
<tr>
<td>Coal Mining Methods</td>
<td>28</td>
</tr>
<tr>
<td>Coal Mining Law</td>
<td>33</td>
</tr>
<tr>
<td>OAC 1501:13-4-04 (C)(2)</td>
<td>35</td>
</tr>
<tr>
<td>OAC 1501:13-4-04 (C)(3) Test Borings or Core Samples</td>
<td>36</td>
</tr>
<tr>
<td>OAC 1501:13-4-08 (A)</td>
<td>37</td>
</tr>
<tr>
<td>Subsurface Exploration</td>
<td>37</td>
</tr>
<tr>
<td>Strike and Dip</td>
<td>39</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>List of test holes from the D-2401 permit.</td>
<td>43</td>
</tr>
<tr>
<td>Table 2</td>
<td>Strike and dip for different combinations of 3 test holes on the D-2401 permit.</td>
<td>45</td>
</tr>
<tr>
<td>Table 3</td>
<td>Test hole data placed in an *xls spreadsheet for import into GIS.</td>
<td>60</td>
</tr>
<tr>
<td>Table 4</td>
<td>Inputs for the Existing Surface DEM model.</td>
<td>68</td>
</tr>
<tr>
<td>Table 5</td>
<td>Outputs for the Existing Surface DEM model.</td>
<td>69</td>
</tr>
<tr>
<td>Table 6</td>
<td>The inputs for the coal analysis model.</td>
<td>75</td>
</tr>
<tr>
<td>Table 7</td>
<td>The outputs for the coal analysis model.</td>
<td>76</td>
</tr>
<tr>
<td>Table 8</td>
<td>The inputs for the multilinear regression model.</td>
<td>85</td>
</tr>
<tr>
<td>Table 9</td>
<td>The outputs for the multilinear regression model.</td>
<td>86</td>
</tr>
<tr>
<td>Table 10</td>
<td>West Fork 3 model residuals</td>
<td>86</td>
</tr>
<tr>
<td>Table 11</td>
<td>Calculation results for the goodness of fit test.</td>
<td>87</td>
</tr>
<tr>
<td>Table 12</td>
<td>The results of the statistical analysis and t-test.</td>
<td>89</td>
</tr>
<tr>
<td>Table 13</td>
<td>Highwall optimization model inputs.</td>
<td>91</td>
</tr>
<tr>
<td>Table 14</td>
<td>Highwall optimization model outputs.</td>
<td>92</td>
</tr>
<tr>
<td>Table 15</td>
<td>Acid base accounting model’s inputs.</td>
<td>98</td>
</tr>
<tr>
<td>Table 16</td>
<td>Acid base accounting model’s outputs.</td>
<td>99</td>
</tr>
<tr>
<td>Table 17</td>
<td>Characteristic lithologic unit weights by material (ODNR,2009).</td>
<td>101</td>
</tr>
<tr>
<td>Table 18</td>
<td>Acid base accounting results for West Fork 3 permit.</td>
<td>101</td>
</tr>
<tr>
<td>Table 19</td>
<td>Description of inputs needed for the subsidence model.</td>
<td>105</td>
</tr>
<tr>
<td>Table 20</td>
<td>Description of outputs given by the subsidence model.</td>
<td>106</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Water budget for Ohio (ODNR-DSWR, 2011b).</td>
</tr>
<tr>
<td>2</td>
<td>The abandoned surface coal mines of Ohio (ODNR, 2013).</td>
</tr>
<tr>
<td>3</td>
<td>Underground mine extents in the Allegany region of Ohio (ODNR, 2013).</td>
</tr>
<tr>
<td>4</td>
<td>Physiographic regions of Ohio (ODNR, 1998).</td>
</tr>
<tr>
<td>5</td>
<td>Graph of eia data for the past 10 years shows a drastic decline in coal fired electrical generation.</td>
</tr>
<tr>
<td>6</td>
<td>Mine entry types (ODNR, 2014).</td>
</tr>
<tr>
<td>7</td>
<td>Room and pillar mining (Energy and Minerals Field Institute, 2012).</td>
</tr>
<tr>
<td>8</td>
<td>Longwall mining with room and pillar ingress and egress (Energy and Minerals Field Institute, 2012).</td>
</tr>
<tr>
<td>9</td>
<td>A typical cross section of subsidence (ODNR, 2010).</td>
</tr>
<tr>
<td>10</td>
<td>Typical modern rotary drilling rig.</td>
</tr>
<tr>
<td>11</td>
<td>A section of core sample, TY06-035, pulled from the ODNR permit D-2401.</td>
</tr>
<tr>
<td>12</td>
<td>A diagram of the three-point method.</td>
</tr>
<tr>
<td>13</td>
<td>Permit area and spatial relation of test holes for D-2401.</td>
</tr>
<tr>
<td>14</td>
<td>Cartesian to bearing diagram.</td>
</tr>
<tr>
<td>15</td>
<td>Driller’s report for TH-01 on the West Fork 3 permit.</td>
</tr>
<tr>
<td>16</td>
<td>Add x-y data tool in ArcGIS.</td>
</tr>
<tr>
<td>17</td>
<td>Test hole locations and test hole attribute table.</td>
</tr>
<tr>
<td>18</td>
<td>Model parameter input for the Existing Surface DEM model.</td>
</tr>
<tr>
<td>19</td>
<td>West Fork 3 digital elevation model.</td>
</tr>
<tr>
<td>20</td>
<td>Isopach raster for the West Fork 3 permit color gradation is from high (white) to low (cyan).</td>
</tr>
<tr>
<td>21</td>
<td>Aerial drone photography, contours (solid brown), coal outcrop (black dot), and permit (black dash) show the accuracy of the coal analysis model.</td>
</tr>
<tr>
<td>22</td>
<td>A quantile plot to test the normality of the residuals for the West Fork 3 test holes.</td>
</tr>
<tr>
<td>23</td>
<td>Final highwall analysis performed 3 separate ways.</td>
</tr>
<tr>
<td>24</td>
<td>ODNR acid base accounting spreadsheet that is filled out as part of the permitting process.</td>
</tr>
<tr>
<td>25</td>
<td>Acid base accounting area polygons placed at the middle elevation for the lithologic unit.</td>
</tr>
<tr>
<td>26</td>
<td>Frequency analysis of subsidence to overburden thickness (R.E. Gray, 1977).</td>
</tr>
<tr>
<td>27</td>
<td>Frequency distribution of the Box-Cox transform for overburden thickness.</td>
</tr>
<tr>
<td>28</td>
<td>Subsidence z-score raster with subsidence probability contours.</td>
</tr>
<tr>
<td>29</td>
<td>A simplified relationship between z-score and probability for use in a GIS model.</td>
</tr>
</tbody>
</table>
Figure 30: Subsidence probability raster, gradation form high (red) to low (blue) ... 111
INTRODUCTION

Rationale for Research

I have worked in the Ohio coal industry since 2011, and have authored and co-authored a variety of permits with the purpose of coal extraction. I started with co-authoring Clean Water Act (CWA) 401, 402, and 404 permits and later began to author sections of the Surface Mining Control and Reclamation Act (SMCRA) permits. One of my favorite sections was the geology section of the SMCRA application. The geology section of the SMCRA permit required a variety of generalized information based on a minimum lawful requirement of data. SMCRA and Ohio law set the minimum requirements for geologic data. The goal for coal companies is to only meet the minimum lawful requirement, since additional data equals additional money and time. The analyst is then forced to use sparsely sampled test hole data to perform a geologic analysis of the permit area. The data will be used to establish the pit floor, mine drainage, sediment pond locations, coal seam thickness, coal tonnage, and the list goes on. The reason why I enjoy this section of the SMCRA is due to the freedom of analysis. It is also challenging to try to provide detailed information with a minimum of information that can at times be speculative. Over the years, I have tried different approaches to refine the analysis and make the most of the data provided. For this thesis, I will use Geographic Information Systems (GIS) and Python to construct a series of tools and models that allows for a simpler, more time efficient, and more robust analysis of a geologic subsurface report and thus a better mine design.
Mining and the Environment

Climate

Ohio’s geographic location is between the 38 and 42nd parallel, in the mid-west of the United States, and south of Lake Erie. This geologic positioning gives Ohio its unique climate of large seasonal temperature fluctuations and frequent precipitation. The climate varies from north to south and is separated by 2 distinct climate descriptions. Ohio climate is described as hot-summer humid continental in the northwest and humid subtropical in the southeast. Ohio’s 50-year average annual rainfall is estimated as 37.5 inches by the Ohio Department of Natural Resources – Division of Surface Water (ODNR-DSWR, 2011a). Rainfall varies widely across the state with typically higher precipitation seen in the southeast and the drier part of the state in the northwest. The wettest months in Ohio are typically June and July and the driest months are October and February. A water budget is a tool used to estimate a region’s distribution of water resources based on the unique hydrologic conditions in the area. Ohio’s water budget estimates that 26 inches of precipitation infiltrates to the unsaturated zone, 6 inches infiltrates to the saturated zone, 2 inches is retained at the surface or evaporated, and 10 inches is runoff (ODNR-DSWR, 2011b). Figure 1 shows the water budget for the state of Ohio.
Ohio’s climate lends to the difficulty of mining coal with minimal impacts on the environment. One of the primary environmental focuses in surface mine design is the placement of sediment control structures. Sediment control structures control the surface runoff of precipitation across the disturbed ground within the mine complex. Sediment control structures consist of silt fence, rock or grass lined channels, and sediment ponds. Each structure serves the purpose of keeping the disturbed earth within the mine site. In other words, no precipitation runoff leaves the mine site without treatment. These structures are set at an elevation that is just below the coal outcrop to utilize gravity drainage. Keeping the sediment control structures as close to the coal outcrop as possible minimizes ground impacts, which keep reclamation costs low and increase profit.
margins. ODNR requires bond on a per acre of affectment basis. A company would only want to show the area to be impacted and minimize the permit area, so that less bond is required for the permitting of coal mine.

Disturbed soils that are not vegetated can easily be eroded and add to a receiving streams sediment load if not properly treated. Furthermore, exposed mine spoils contain coal fines and shales which can contain oxidized pollutants that lend to acid mine drainage (AMD). (ODNR, 2014) “Nonreclaimed spoil piles often contain pyrite-bearing rock. Pyrite is an iron disulfide (FeS₂) mineral. When it is exposed to oxygen and water, especially in the presence of Bacillus ferroxidans, a pyrite oxidizing bacterium, it produces sulfuric acid (H₂SO₄). Streams carrying acid mine drainage commonly have a reddish or yellowish sediment coating their bottoms. This coating, called yellow boy, is an iron sulfate precipitate (Fe₂SO₄). The acid mine drainage added to a stream severely affects aquatic wildlife and may adversely affect the quality of local groundwater aquifers.” Innumerable AMD research has been conducted by regulatory, academia, and industry. Nearly half a million acres of unreclaimed abandoned mine lands in Ohio contribute to the AMD problem (ODNR, 2017). Figure 2 shows the abandoned surface coal mines currently mapped in the ODNR database.
Figure 2: The abandoned surface coal mines of Ohio (ODNR, 2013).

*Groundwater*

One of the main hydrologic concerns in underground mining is infiltration rates of ground water from the overlying water-bearing strata. In down dip mining, it is important to ensure that the water entering does not exceed the rate at which the water can be economically pumped from the mine. The Meigs 31 mine could not be completed and was shut down in 1989 due to water infiltration exceeding the ability to pump water.
from the mine (20 F.3d 1418, 1994). In up dip mining, gravity drainage is used to allow water to simply flow out of the mine. The water can then be diverted to treatment ponds for sedimentation and chemical treatment before discharge to receiving streams. A good mine design takes the dip of the coal seam and the predicted water infiltration rates into account for placement of the mine entrance and modes of operation.

Water will eventually infiltrate into a mine whether during mining or post mining. It is assumed that water leaving an abandoned underground mine does not meet effluent standards. A typical post mining mitigative technique is to develop a mine pool by sealing the mine. Sealing the mine entrance keeps the mine pool from discharging to the environment (DOI-OSM, 2009). The sealed water in an underground mine is anoxic and does not allow the oxidized pollutants known as AMD to form. A mine pool is inevitable in below drainage mines and the quality of the water in the mine pool is assumed to be non-compliant to NPDES standards. The focus, when it comes to water, is containment and proper mine design from a during and post mining perspective. Mine pools have historically shown that containment is not indefinite and therefore will lead to AMD at some point. 600,000 acres of underground mines contribute to water pollution in Ohio (ODNR, 2017). Figure 3 show the Alleghany region of Ohio with known underground mines.
Ohio’s Geologic Description

Location

The study region is in the State of Ohio in the region known as the Appalachian plateau. The Glaciated Allegheny and Allegheny are the two major physiographic regions of the Appalachian plateau. Most coal mines in Ohio occur in the unglaciated Allegheny Plateau. The Appalachian Plateau lies west of the Appalachian Highlands, and
North west of the Appalachian Mountains. The Appalachian Plateau and Highlands nearly run the entire eastern seaboard of the United States. Figure 4 shows the major physiographic regions of Ohio and the portions of the state that are considered glaciated and unglaciated.

![Figure 4: Physiographic regions of Ohio (ODNR, 1998).](Image)

The Ohio Coal fields are estimated to be approximately 12,600 square miles. Most of coal fields in Ohio are in the unglaciated Allegheny Plateau. There are 11 economically viable coal fields that are currently economically minable; Sharon (No. 1), Quakertown (No. 2), Upper Mercer (No. 3a), Clarion (No. 4a), Lower Kittanning (No. 5), Middle Kittanning (No. 6), Upper Freeport (No. 7), Mahoning (No. 7a), Pittsburgh (No. 8), Pomeroy (No. Sa), and Meigs Creek (No. 9). Two other beds, the Brookville (No. 4) and Lower Freeport (No. 6a) (J.A. Bownocker, 1917). The minable coal fields of Ohio were formed during the Carboniferous Period or more specifically the Pennsylvanian era.
It was hundreds of millions of years of geologic evolution that created the terrain and subsurface conditions of Ohio.

*Pre-Cambrian and Cambrian (4500-500 MYA)*

The oldest and deepest known formations in Ohio consist of rock from the Precambrian period. Pre-Cambrian aged rock is igneous and metamorphic and is known for the first signs of life on Earth (Ohio Division of Geologic Survey, 1996). It was during the Pre-Cambrian period that the Grenville Mountains began to form. The Greenville Mountains were at the approximate location of the Appalachian Mountains. Mountains of metamorphic rock in eastern Ohio were upheaved in massive tectonic shifts in a process known as orogeny (Ohio Division of Geologic Survey, 1996). Two continents collided along the eastern portion of Ohio. At the end of the Pre-Cambrian Period the sea rose and covered much of Ohio in a shallow sea. The Cambrian Period covered the metamorphic rock of the Pre-Cambrian period in layers of sediment. Over the course of time and sea pressures these layers of sediment were compressed and hardened into sedimentary rock. The Cambrian period is historically known for the explosion of marine life that occurred during this time. This explosion of marine life during the Cambrian period buried under layers of sedimentary rock formed the earliest pockets of oil and gas (Ohio Division of Geologic Survey, 1996).

*Ordovician, Silurian, and Devonian (500-350 MYA)*

During the Ordovician, Silurian, and Devonian periods, Ohio was rising from the depth of the Ocean. These periods are segmented based on the fluctuating sea levels and the deposition of sedimentary layers. It is presumed that the oceans were rising and
falling due to the same reasons we see today, the melting of ice. The seas of these periods were rich in marine life (Ohio Division of Geologic Survey, 1996). The sea would recede leaving large isolated estuaries. Receding tidal waters also left behind blankets of sediment as tidal action eroded new shore lines. Eventually the sea would rise again and create pressure over the layers of sediment and biological material, turning them into rock. Ohio owes its large salt deposits to this period, since large pockets of sea water would be left behind to evaporate. As sea levels receded across Ohio, the sedimentary limestone deposits gave way to layers of carboniferous shale. Carboniferous shale forms from the compression of deposited mud and clay, and owes its black coloring due to organic inclusion. Carboniferous shale is indicative of an anoxic environment. If the sea water was well oxygenated, then the deposited biological material would decompose and not become part of the shale. Shales formed in this manner (i.e. under anoxic conditions) typically contain high concentrations of pyrite (fool’s gold). Pyrite, iron pyrite, or iron sulfide (fool’s gold) is typically associated with shales and coal in Ohio. The last of the Devonian period was marked by the formation of sandstone, which is a strong indicator that the sea had all but recessed from the Ohio landscape. Devonian Sandstone or better known as Berea Sandstone is well known for its oil and gas deposits. Berea Sandstone is coarse grained with large voids between particles of mica and sand, which makes it an excellent reservoir for oil and gas (Ohio Division of Geologic Survey, 1996). The Berea sandstone outcrops in the North along the boundary between the unglaciated and glaciated Appalachian Plateau.
**Carboniferous Periods (350-300 MYA)**

The economically minable coal layers of Ohio were formed during the Carboniferous Period. The Carboniferous Period gets its name from the abundance of marine and tertiary plant and animal life developed during this time. The Carboniferous period can be split into shorter time periods known as the Mississippian and the Pennsylvanian. The Pennsylvanian is the younger of the two sub-periods and is the rock where most of Ohio’s economically minable coal is extracted. The Pennsylvanian period provided a unique system, which allowed for the formation of Ohio’s minable coal seams (J.A. Bownocker, 1917). Over a period of approximately 18 million years, the ocean rose and fell across the Appalachian plateau. With each recess of the ocean it left a vast swamp of marine life. Peat formed in the vast swamps of dying vegetation. Rivers flowed down off the Appalachian Mountains into the swamps and deposit layers of sediment, which formed the sandstone, mudstone, clay, and limestone partings between coal layers. The sedimentary layers of sandstone, mudstone, clay, and limestone would compress the biologic material into coal. Over the course of 18 million years this process was repeated more than a dozen times to form the coal seams of the Appalachian Plateau. There are over 100 individually recognized bedding planes within the Pennsylvanian system (Ohio History Central, 2017).

**Permian (300-250 MYA)**

The end of the Permian period was marked by a major uplift, which moved Ohio’s bedrock upward. The Cincinnati arch and the Findlay Arch were two positive geologic structures formed (J.A. Bownocker, 1917). The uplift occurred most noticeably
in the Western part of the state. The uneven uplift pushed the bedrock off horizontal, creating the gradual south-easterly dip seen in many of Ohio’s coal seams. A typical dip seen in Ohio’s coal seams is 0.3% or about 15 feet per mile (J.A. Bownocker, 1917).

The Mesozoic Era (250-50 MYA)

This Era included one of the most notable and recognized periods of geologic time, the Jurassic Period. This period is known for the dinosaur, but little evidence of their existence has been left in Ohio. In fact, almost no geologic records exist in Ohio for this era (Ohio Division of Geologic Survey, 1996). No sedimentation occurred during the Mesozoic Era as in the periods that preceded this era. At the end of the Mesozoic Era there was large scale weathering and erosion that removed any deposition that occurred during the Mesozoic Era. There was downcutting through the Pennsylvanian Age stratigraphic layers as well. The massive erosion of the Mesozoic Era coupled with the bedrock uplift of the Permian Period created the north to south bands of geologic outcrops seen on Geologic Survey Maps of Ohio. The tertiary period followed the Mesozoic Era. “The bedrock in Ohio was complexly dissected over several million years by a pre-glacial drainage system known as the Teays River system.” (ODNR, 2001). The Teays River flowed South to North through the middle of Ohio at the approximate location of the current day Scioto River.

Quaternary Period (2-0 MYA)

The Quaternary Period is also known as the Pleistocene Epoch or more well known as the ice age. Massive glaciers, some as thick as 1 mile, moved down from Canada into the Northern parts of Ohio. The land beneath these massive glaciers would
compress. 10,000 years later, the rebound of the land from the compression of the glacial
activity is still ongoing (ODNR, 1995). Ice would build up in Canada and push south
carrying erratics and scarring the land below. Erratics are igneous and metamorphic
rocks carried by glacial activity. The ice would melt leaving the erratics behind. Till
filled the rivers and valleys of Ohio, leaving a much shallower relief in north and
northwest Ohio. The receding ice would leave behind nutrients and minerals that would
support an explosion of plant life. During the third progression of ice into Ohio, the
Teays River was destroyed. A chunk of ice dammed the Teays River and created Lake
Tight. Lake Tight rose to an elevation of 900 ft. before breaking down drainage divides
and creating what is now the modern-day Ohio River Drainage system (ODNR, 1995).
The ice fluctuated its reign across the state more than a dozen times during the
Quaternary Period. This period Shaped the surface of modern day Ohio, including the
Great Lakes. Glacial activity coupled with erosion removed the sedimentary layers of the
Carboniferous period in the North Western 2/3 of the state. The Appalachian Plateau is
the only region in Ohio that still bears coal from the Carboniferous Period. The glacial
erosion of Ohio during this period was severe enough to expose the top layers of rock
from the Ordovician Period in areas such as Cincinnati (J.A. Bownocker, 1917). That is
nearly 400 million years of geologic record removed.

Summary of Ohio Geology and Coal Seams

The economically recoverable coal seams in Ohio were formed during the
Pennsylvanian period. There are 11 economically mined coal seams in Ohio. They were
formed over the course of millions of years of tectonic activity and fluctuating ocean
levels. During this period, the seas rose and fell leaving behind vast beds of anoxic organic material that were trapped under sediment. Tectonic uplift in western Ohio, present day Cincinnati area, has caused the bedding planes of Ohio’s geography to dip to the southeast at about 15 feet per mile. Glacial melting and massive erosion during the Quaternary and Mesozoic Periods removed substantial portions of the geologic record in Ohio. The Appalachian plateau was not impacted by glacial activity, which has left the geologic layers from the Pennsylvanian period generally undisturbed. Incised valleys and steep drainage is indicative of the Appalachian plateau especially below the Allegheny escarpment. The escarpment is marked by the outcropping of the Berea Sandstone. Above the escarpment, the Pleistocene epoch had greater impact and is more prone to broader valleys and hilltops that have less topographic relief.

Ohio’s geology can be very irregular due to mountain building activity, continental collisions, plate tectonics, and massive erosion events in Ohio’s geologic history. The impacts of these geologic events had a significant impact on the coal bearing region of Ohio. The irregularities can take the form of unconformities, folds, and faults. Unconformities are periods of massive erosion or no deposition in which there is a missing geologic period is missing within the geologic record. Unconformities describe the appearance of unparallel bedding planes, interruption in the bedding plane, and missing bedding planes in the geologic record. Ohio’s unconformities are typically widespread over large areas. Highly localized unconformities also exist, but are usually limited in scale and can be easily viewed in a highwall cut face. Folds and faults are formed through stresses at the surface of the earth. Folding takes the form of anticlines
and synclines on the surface. The folds create a “wave” in the bedding plane. Faults are the displacement of the bedding plane along a near vertical shear plane. The displacement can occur in the horizontal or vertical direction. Ohio’s major faults are mapped by the USGS (USGS, 2017). The Cambridge anticline, the dome in Washington county, and the vertical bedding planes in Mineral City are a few of the most notable localized geologic irregularities in Ohio. Faults and folds vary widely in scale. Geologic irregularities are not easily recognized through exploratory drilling and go unnoticed until mining encounters them.

History of Coal Mining in Ohio

The Appalachian Valley was created over the course of 3.5 billion years of geologic evolution. The characteristics that are seen in Ohio’s coal can be traced back as far as the Pre-Cambrian Period. A series of biologic and geologic events created the rich abundance of resources seen in Ohio today. The most notable of these resources include sandstone, limestone, coal, oil, and gas. Ohio’s first pioneers settled in the regions of resource outcroppings, so that they could have easy access to the resources required for pioneering. Outcroppings in the Appalachian plateau are due to massive erosion events, glacial activity, and geologic activity. Deep incised valleys with high relief, allow for a wide range of exposed outcroppings in Appalachian Ohio.

Coal has played an influential role in the first colonization of Ohio and continues to play an influential role in Ohio as an economic resource. Coal mining in Ohio has been occurring for more than 200 years. The earliest mapping of coal outcrops was a map of the Middle British Colonies of America near the Hawking River published in
1755 to aid in the colonization of the Americas (ODNR, 2005). In the early 1800s, Ohio coal mining was limited to small personal adit mines that took advantage of the Appalachian plateau’s incised valleys. The incised valleys were created by hydrologic downcutting that exposed resource outcrops. The small adit mines were used to extract coal for heating homes during Ohio’s winters. By 2012, 61 mines produced 26.3 million tons of coal from 13 counties employing nearly 6,200 people (ODNR, 2005).

The industrialization of Ohio coal mining occurred during the mid to late 1800s. Coal was recognized as a cheap and readily accessible means of steam generation. Steam generation was the primary means for which to generate mechanical energy during this time. By the late 1800s coal mining began to use mechanized equipment in the recovery of coal to meet the demands of the nation. 80% of the demand for coal was used for steam power to generate electricity. Today, over 90% of the coal mined is burned to generate steam for the electrical power industry (ODNR, 2005). 4 billion Megawatts of electrical power was generated in America in 2016, and coal generated 1.24 billion MW of that power (EIA, 2017). Coal fired electricity is on a downward trend. Figure 5 shows a graph of power production from coal from the energy information administration (EIA) for the last 10 years. Extrapolation of the linear regression predicts the discontinued use of coal fired power plants in 2032. However unlikely it is for coal power to be eliminated in 15 years, coal could still be mined as an export commodity.
Ohio has been one of the leading producers of coal for the nation for the past 200 years. It is estimated that over 3.9 billion tons of coal have been mined in Ohio. The peak of the coal industry was in the 1970s when coal production reached a record 55 million tons produced (ODNR, 2005). This was due to the advent of more efficient means of mining. Larger earth moving equipment such as the Big Muskie Bucket were at the front of this drastic change in production. The number 8, 9, and 10 coal seams became more accessible and cheaper to extract due to the advent of larger equipment. It became cheaper to mine near surface coal as compared to deeper coal seams. This prompted a change for coal companies to extract coal using surface mining methods. By 1948, surface coal extraction became the economically preferred choice for coal extraction. Surface mining remained the dominant source of coal until 1995, when
underground mining produced more coal. The swing back to underground mining was due in part to the increasingly stricter regulations for reclamation placed on mining through the Surface Mine Coal and Reclamation Act (SMCRA).

Before the advent of coal mining regulation, there was no incentive for reclamation. This has left 450,000 acres of unreclaimed surface mines and 600,000 acres of underground mines that do not meet Ohio’s reclamation laws passed in 1972 (ODNR, 2017). Regulation and the coal industry have undergone an evolution over the past 200 years as technologies have improved and our understanding of how anthropologic disturbance can have a profound impact on the environment. Analysis of past failures lead to better methods and technologies for the future. Academia, industry, and regulatory should be using similar techniques to analyze the failures of the past and to permit mines of the future. It is imperative that coal extraction is environmentally safe, since coal mining will be a viable source of energy and an economic commodity for Ohio far into the future.

Coal Mining Methods

Surface mining and underground mining are the two methods available for the extraction of coal. Surface mining is the removal of all the layers of rock and soil (overburden) down to the coal seam, so that the coal can then be extracted. Surface mining becomes uneconomical when the overburden to coal thickness is greater than 20 to 1 (ODNR, 2014). When overburden to coal thickness is greater than 20 to 1, underground mining becomes the only viable alternative for coal recovery. For
underground mining to be technically feasible the coal seam thickness must be greater than 4 feet (ODNR, 2014).

Three types of underground mining techniques are used in the removal of coal reserves. The names of the methods are given based on the type of mine entry. Drift mines are horizontally cut into the side of a hill at the elevation of the coal seam. Shaft mines use a vertical entry point that descends to the elevation of the coal seam. Slope mines use an inclined entry that slopes down to the elevation of the coal seam. Figure 6 shows the three underground mining techniques and various surface mining techniques.

Room and pillar and longwall mining are two methods of underground coal extraction. Room and pillar mining is by far the most used underground mining method in Ohio. Room and pillar mining uses a series of intermittent coal support columns to support the overlying strata (See Figure 6 & 7). The coal pillars are typically 20 to 30
feet wide and up to 90 feet long. The areas of coal extraction are 20 to 30 feet wide and up to 400 feet long. Wood ties, roof bolts, and I-beams can be used to support the roof of the mine in addition to the coal pillars. The spacing of rooms and pillar is dependent on the quality of overburden above the coal seam. For example, if the overlying strata is known to be friable, then there would be less distance between columns, and the columns would be larger. This method only allows for approximately 60 to 70 percent of coal removal depending on column spacing. Retreat mining also known as full extraction mining uses room and pillar methods but removes the pillars as a last step. This last step causes an induced subsidence of the mine.

Figure 7: Room and pillar mining (Energy and Minerals Field Institute, 2012).
Long wall mining uses a piece of equipment known as a long wall miner that makes a single pass that can be up to 800 feet wide and up to 14,000 feet long. The longwall miner uses a system of supports to support the mine roof immediately behind the longwall miner and subsidence occurs where the extraction is complete. The longwall miner works between two room and pillar mines that allow ingress and egress from the working face. This method can extract up to 80% of the coal reserves. Long wall mining is not commonly used in the Appalachian region of the United States.

Longwall mining accounts for little more than 15% of deep mined coal in the United States (Younger, Banwart, & Hedin, 2002). Longwall mining has a significantly greater capital cost than room and pillar. Longwall mining is more popular in Europe than the United States, because coal mining is a state industry in Europe and therefore can afford the higher capital costs of longwall equipment (Younger et al., 2002). Figure 8 shows a conceptual view of the two coal remover techniques. A conveyor belt hauls coal away from the longwall miner through the room and pillar section.
Subsidence is the lowering in elevation of the surface of the earth. When talking about mining, it is due to the movement of roof rock into the mine cavity. Subsidence could be a product of all or some of the following factors; roof rock strength, underlying and overlaying material, water movement, mining methods, room and pillar dimensions, and geologic factors like joints and veins (ODNR, 2010). This complex problem is difficult to generalize between different mines or across the complex geology of Appalachia Ohio. Overburden thickness is the single most relatable factor. (Gray, 1977) The greater the vertical distance from the mine to the surface, the less effect mine subsidence will have at the surface. This is due to the 10 % bulk volume expansion of material as the compressed layers fall into the cavity and become disordered (Gray, R.E., Bruhn, R.W., & Turka, R.J., 1977). Figure 9 shows a typical cross section of subsidence.
Coal Mining Law

The Surface Mining Control and Reclamation Act (SMCRA) is defined as “An Act to provide for the cooperation between the Secretary of the Interior and the States with respect to the regulation of surface coal mining operations, and the acquisition and reclamation of abandoned mines, and for other purposes." (US Government, 1977) SMCRA is the federal law which governs the permitting, extraction, and reclamation of coal mines in the United States. The law was enacted by the US Congress August 3, 1977. The SMCRA law has undergone several revisions since 1977. The most current SMCRA revision was enacted July 6, 2012.

Four types of coal permits have been issued in Ohio since 1966. Each permit letter designation marks an evolution in the laws that regulate the coal mining industry. The A law permits were issued between 1966 through 1973. The B law permits were issued between 1973 to 1976. The C law permits were issued between 1976 to 1981.
The D law permits were issued between 1981 to present. In the context of earlier given definitions, A and B will be pre-law mining and C and D will be post law mining. Pre-law mining is the subject of much research due to the environmentally destructive nature of unreclaimed coal mines. The most effective mitigation for abandoned mine lands is remining (ODNR, 2017). Surface coal mines mined before 1976 did not have the advantage of large mining equipment. This means the economic recovery of coal had a lower overburden to coal thickness ratio (i.e. <20:1). Mines prior to 1976 can be remined for a profit and the land subsequently reclaimed.

“The supreme, absolute, and uncontrollable power by which an independent state is governed and from which all specific political powers are derived; the intentional independence of a state, combined with the right and power of regulating its internal affairs without foreign interference” (John Bouvier, 1856). In the context of coal mining, this allows states to create laws and regulate the coal mining industry within their state. The individual states are in the best position to be familiar with the unique conditions that are relevant to the mining of coal within their borders. However, Article VI, Section 2, of the U.S. Constitution is known as the Supremacy Clause, because it provides that the "Constitution, and the Laws of the United States … shall be the supreme Law of the Land." (John Bouvier, 1856) Regarding coal mining, the states must meet or exceed the baseline requirements stated in the SMCRA laws, subject to approval of the Department of the Interior. The Ohio Administrative code 1501:13 and Ohio Revised Code 1513 are the Ohio statutes that govern coal mining within the state of Ohio. For this paper, the focus will be on Ohio law and more specifically OAC 1501:13-04-04. OAC 1501:13-04-
04 is a list of requirements and methods for obtaining subsurface information for the area to be mined.

_OAC 1501:13-4-04 (C)(2)_

(a) The description shall include a general statement of the geology within the proposed permit area and adjacent areas down to and including the deeper of either the first stratum below the lowest coal seam to be mined or any aquifer below the lowest coal seam to be mined which may be adversely affected by mining. It shall also include the areal and structural geology of the permit and adjacent areas, and the other parameters which influence the required reclamation, and shall show how the areal and structural geology may affect the occurrence, availability, movement, quantity, and quality of potentially affected surface and ground waters. It shall be based on:

(i) The cross sections, maps and plans required by paragraph (B) of rule 1501:13-4-08 of the Administrative Code;

(b) Each application for a permit shall contain the results of tests conducted on the area of land to be mined. Unless the chief first approves a fewer number of test holes, such tests shall consist of test holes made by the boring or drilling method and be conducted at the rate of one test hole for each twenty-five acres of land or fraction thereof, which is underlain by coal on the area of land to be mined. At least one test hole shall be located on the highest elevation in the area of land to be mined. Holes shall be located as far apart as the size and shape of the area of land to be mined will allow. Such holes shall be drilled to the
bottom of the material underlying the lowest coal seam to be mined and shall be staked or otherwise marked at the time of filing the application for a permit to be clearly visible at the approximate location, and shall be numbered. Such stakes or other markers shall be maintained until the permit to conduct a coal mining operation is granted or denied. (Ohio Administrative Code, 2010)

OAC 1501:13-4-04 (C)(3) Test Borings or Core Samples

(a) Test borings or core samples from the proposed permit area shall be collected and analyzed down to and including the stratum immediately below the lowest coal seam to be mined. Individual drilling reports shall be furnished for each test boring or core sampling and shall contain the following information on forms prescribed by the chief:

(ii) Lithologic characteristics including physical properties and thickness of each stratum and each coal seam;

(iii) Chemical analyses to include pH, neutralization potential, potential acidity, total or pyritic sulfur, and calcium carbonate deficiency of each stratum;

(iv) Analyses of the coal seam for acid-forming or toxic-forming materials, including, but not limited to, an analysis of the total sulfur and the sulfur present in pyrite and marcasite;

(v) Identification of the test hole by the number assigned in paragraph (C)(2)(b) of this rule; and

(vi) Identification of all coal seams by name and number. (Ohio Administrative Code, 2010)
OAC 1501:13-4-08 (A)

(20) All coal crop lines and the strike and dip of the coal to be mined in the proposed permit area;… (Ohio Administrative Code, 2010)

Subsurface Exploration

Exploratory test holes to determine the subsurface condition of the proposed mine site are a requirement of OAC 1501:13-04-04(C)(2&3). The spatial distribution and number of test holes for each mine is stated under 1501:13-04-04(2)(b), “…one test hole for each twenty-five acres of land or fraction thereof…” (Ohio Legislature, 2010). The information required for this study is defined under subsections 1501:13-04-04: (3)(a)(ii) Lithology and thickness, (3)(a)(vi) identification of coal seams.

As previously stated the drilling reports will provide the lithologic description, the elevation of that lithologic unit, and neutralization potential of the strata. The test holes contained in the permit are the most localized data available to ascertain the subsurface conditions of the coal seam. Therefore, it should be the most accurate data available to represent the subsurface condition. This can be evidenced through Waldo Tobler’s (1970) first law of geography, “places that are closer together will be more alike than those that are further apart “(R. Webster & M.A. Oliver, 2007). It is therefore necessary to investigate thoroughly the error that can be associated with the exploratory drilling reports and establish methods to mitigate and control that error.

Auger drilling and core drilling are both typically used in exploratory test hole drilling. Auger drilling is the most commonly used, because it is the quickest and the cheapest method. Drilled particulates of the lithologic unit are forced up from the drill
head along the shaft to the surface. The material is then identified and logged by the field technician as it collects around the drill shaft at the surface. As the lithology changes the field technician notes the change and the thickness of the lithologic unit. The depth of the lithologic unit would be the total length of drill shafts minus the length of drill shaft above the surface. Figure 10 shows a small mobile drill rig for purchase at http://www.deeprock.com/frmcommercialrigs.aspx.

Figure 10: Typical modern rotary drilling rig.

The next method is the coring method. It is less common due to the greater amount of time and money required. A solid core of the earth is extracted and packaged in wooden boxes. Each length of core pipe is afforded its own box, where the lithologic
units are identified and measured. Figure 11 shows a core sampling of 1 length of core pipe (~10’) from KLM#7 (SMCRA Permit ID D-2401) and Test Hole TY06-035. Regardless of method, a “driller’s depth” is recorded by the field technician using the lengths of pipe logged in the pipe inventory and a hand-held measuring tool (e.g. yard stick) to measure the length of the pipe above ground.

Figure 11: A section of core sample, TY06-035, pulled from the ODNR permit D-2401.

Strike and Dip

OAC 1501:13-4-08(A)(20) states that “All coal crop lines and the strike and dip of the coal to be mined in the proposed permit area…”. The strike and dip can be thought of as a way of describing a bedding plane. The Editors of Encyclopedia Britannica (2017b) defines the strike as “the direction of the line formed by the intersection of a fault, bed, or other planar feature and a horizontal plane.” The strike line is considered an isoline at which the elevation of the bedding plane does not change. The Editors of Encyclopedia Britannica (2017a) defines the dip as “the angle at which a planar feature is inclined to the horizontal plane; it is measured in a vertical plane perpendicular to the
strike of the feature.” The dip is a measure of the steepness of the bedding plane in the
direction of the steepest slope.

OAC 1501:13-4-13(C)(2)(e)(i) says “At a minimum of three points, not in a
straight line, spaced to indicate the strike and dip of the coal seam.” To determine the
strike and dip, test holes must be drilled to determine the subsurface description. The
data from the test holes are then used to determine the strike and dip of the coal seam
using the 3-point method. The test holes are drilled at the points L, H, and I. Figure 12
shows the geometric analysis for the 3-point method. The length of LD can be found
using Equation 1. Equation 1 can be rearranged to show that the slope of line LH times
the change in elevation between LI is the distance of line LD. D marks the elevation
along line LH that has the same elevation as I. A line drawn from D to I is the strike line,
a line of equal elevation. The strike direction is the bearing angle that the strike line
makes with the North-South Cardinal Line. The dip is always perpendicular to the strike
line. The point E is the point on the strike line that allows a perpendicular line to be
drawn to L. The angle of the dip is found by using Equation 2. This is the angle the
bedding plane makes with the horizontal. Figure 12 shows the 3-point method of finding
the strike and dip. H, I, and L represent the High, Intermediate and Low elevations of the
bedding plane that is being characterized by strike-dip.
Figure 12: A diagram of the three-point method.

Equations provided by the WVDEP (2017)

\[ \frac{LD}{LH} = \frac{(Z_L - Z_I)}{(Z_L - Z_H)} \]  

(1)

\[ Dip = \tan^{-1}\left(\frac{Z_L - Z_I}{LE}\right) \]  

(2)

Where:

\( LD \) = The length of line from \( L \) to \( D \) (ft or m)

\( LH \) = The length of line from \( L \) to \( H \) (ft or m)

\( LE \) = The length of line from \( L \) to (ft or m)
\[ Z_L = \text{The elevation of the Low Elevation Test Hole (ft or m)} \]

\[ Z_I = \text{The elevation of the Intermediate Elevation Test Hole (ft or m)} \]

\[ Z_H = \text{The elevation of the High Elevation Test Hole (ft or m)} \]

Table 1 shows test hole locations and elevations underground mine permit, D-2401. The ID is the unique identifier assigned to each test hole, the X and Y are the spatial coordinates in NAD83 Ohio State Plane South feet, and Z is the elevation of the bottom of the coal seam in feet msl. There are 11 sites on the D-2401 permit, which can produce 165 separate combination of 3 test holes. There is no well documented method in law or ODNR policy directives that explain what to do for finding strike and dip when more than 3 test holes are available.
Table 1: List of test holes from the D-2401 permit.

<table>
<thead>
<tr>
<th>ID</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>TY_06_025</td>
<td>2314006.0</td>
<td>246666.0</td>
<td>891.5</td>
</tr>
<tr>
<td>TY_06_024</td>
<td>2314115.0</td>
<td>247710.0</td>
<td>890.8</td>
</tr>
<tr>
<td>TY_06_014</td>
<td>2314603.0</td>
<td>250369.0</td>
<td>888.5</td>
</tr>
<tr>
<td>TY_06_038</td>
<td>2314518.0</td>
<td>249325.0</td>
<td>884.1</td>
</tr>
<tr>
<td>TY_06_028</td>
<td>2316316.0</td>
<td>249230.0</td>
<td>883.6</td>
</tr>
<tr>
<td>TY_06_037</td>
<td>2317346.0</td>
<td>249224.0</td>
<td>883.1</td>
</tr>
<tr>
<td>TY_06_036</td>
<td>2318254.0</td>
<td>246373.0</td>
<td>877.5</td>
</tr>
<tr>
<td>TY_06_029</td>
<td>2317044.0</td>
<td>249179.0</td>
<td>877.3</td>
</tr>
<tr>
<td>TY_06_034</td>
<td>2320161.0</td>
<td>247321.0</td>
<td>871.2</td>
</tr>
<tr>
<td>TY_06_022</td>
<td>2319314.0</td>
<td>249131.0</td>
<td>870.8</td>
</tr>
<tr>
<td>TY_06_035</td>
<td>2317165.0</td>
<td>244525.0</td>
<td>866.0</td>
</tr>
</tbody>
</table>

Ohio law states that a test hole must be drilled for every 25 acres of minable coal within a permit. Most permit applications have more than 75 acres of coal, so more than 3 test holes of data are available for analysis. Figure 13 is a map of the D-2401 permit outline and the spatial orientation of the test holes within the permit. 3 test holes were chosen at random to calculate the strike and dip.
The strike is N 67 E with a Dip S 23 W at 0.216 degrees off horizontal. This is a strike and dip that is consistent with the typical geology of Ohio’s coal seams. Ohio coal typically drops 15’ per mile, 0.3% slope (Bownocker, 1917), or in other words a dip of 0.172 degrees. Choosing different combination of 3 test holes yields different results. Table 2 shows a few randomly selected test hole combinations with their respective strikes and dips.
Table 2: Strike and dip for different combinations of 3 test holes on the D-2401 permit.

<table>
<thead>
<tr>
<th>Test Hole Combination</th>
<th>Strike Direction</th>
<th>Dip Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>034-022-035</td>
<td>N17.0W</td>
<td>0.08</td>
</tr>
<tr>
<td>029-022-035</td>
<td>N40.0E</td>
<td>0.21</td>
</tr>
<tr>
<td>029-034-035</td>
<td>N78.1E</td>
<td>0.14</td>
</tr>
<tr>
<td>037-022-035</td>
<td>N32.6E</td>
<td>0.41</td>
</tr>
</tbody>
</table>

It can be seen in the table that depending on the test hole combination chosen, a radically different strike and dip can be obtained. Several methods were initially investigated to find a solution to permits with more than 3 test holes. The first thought was to average every combination of strike and dip to obtain the strike and dip for the area. This method assumes that the strike and dip for a localized portion of a coal seam is uniform and that the variation in strike and dip is due to error in the drill log. Averaging the strike and dip across the possible combinations of three test holes at a time should give an approximation of the localized strike-dip for the population. This would require 165 separate calculations for strike and dip, 11 samples chosen 3 at a time $11 \choose 3 = \frac{n!}{k!(n-k)!}$. A Python script was written to avoid doing 165 separate calculations individually. This code was incredibly inefficient due to the substantial number of “if statements” to account for every spatial orientation between the three test holes.

The strike and dip is a method used by geologists to describe a plane. Another method would be to use the general form of the plane equation. The general plane equation is given as Equation 3. The coefficients $a$, $b$, and $c$ represent the normal vector.
of the plane. \( X_0, Y_0, \) and \( Z_0 \) are the mean values for the data set. \( X, Y, \) and \( Z \) represent a point on the plane. The normal vector \( <a,b,c> \) can be found by taking the cross product of vectors \( HL \) and \( IL \). Vectors \( HL \) and \( IL \) can be found by taking the differences between the High-Low and Intermediate Low coordinate points respectively. Equation 4 shows the vector \( HL \). The cross product can then be written as Equation 5.

\[
a(X - X_0) + b(Y - Y_0) + c(Z - Z_0) = 0
\]

\[\bar{HL} = ((X_H - X_L), (Y_H - Y_L), (Z_H - Z_L)) = (HL_x, HL_y, HL_z)\]

\[
\langle a, b, c \rangle = \bar{HL} \times \bar{IL} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ HL_x & HL_y & HL_z \\ IL_x & IL_y & IL_z \end{vmatrix}
\]

The normal vector would then be calculated for every combination of 3 test holes. To average the normal vector from each test hole combination, the normal vector must be normalized. To normalize each normal vector, the normal vector must be divided by the magnitude as shown in Equation 6.

\[
\langle a, b, c \rangle = \frac{<a, b, c>}{\sqrt{a^2 + b^2 + c^2}}
\]

As before, 165 separate calculations for the normal vector must be performed to average the results together. Python was used to perform the calculations. To obtain consistency within the results an if statement was written to ensure that the triangles formed by the points were not too narrow. A narrow triangle (i.e. points that are nearly
collinear) require greater accuracy from the test hole data than is typical for geologic subsurface test hole data. Collinear points would cause erroneous results for that normal vector, which would then cause bias in the average. The if statement in the Python code complies with the statement in OAC 1501:13-4-13(C)(2)(e)(i) that the points should not be in a straight line and should have adequate spatial arrangement. 10 degrees between legs was arbitrarily chosen to ensure narrow triangles were not included in the statistical average of the final strike and dip. The strike angle of a Cartesian plane can be found from the average \(<a,b,c>\), by Equation 7.

\[ \theta = \tan^{-1}\left[ \frac{b}{a} \right] \]  

(7)

The direction of the strike is traditionally given in bearings. From Equation 7 a series of if statements can be used to place the strike in terms of bearings. The X and Y in Figure 14 represent the cardinal directions of East and North respectively. Theta is the Cartesian angle as measured traditionally counter-clockwise from the positive x-axis. The bearing is measured from the y-axis (i.e. North or South) in the direction of East and West. Each quad, listed in numerical order, represents the direction NE, NW, SW, and SE. This allows the stated angle to always be less than 90 degrees. Figure 14 shows the measurement of theta off the x-axis, and the bearing angle measured off the North axis. The sign of a and b in the normal vector decide which quadrant the strike is in.
Figure 14: Cartesian to bearing diagram.

The code was greatly simplified from the previous averaging of the strike and dip, but still lacks simplicity. The iterative process is time intensive, and the additional code required to exclude certain combinations is undesirable. The least squares regression method will use all the test holes to minimize the sum squared error between the test hole points and the regression plane.

Spatial Interpolation Techniques

Often scientists and engineers encounter spatially variable data sparsely taken at discrete spatially distributed points that do not align with a location of interest (Wang, Akeju, & Zhao, 2017). As in the case of the test holes with this study, the sampling points are spatially distributed over a large area with sparse test hole samples at 1 hole every 25 acres. To interpolate or extrapolate missing gaps of information, spatial interpolation can be performed using a variety of mathematical models. This allows for a
more complete understanding how the measured variable is distributed across the whole area. Several spatial interpolation techniques are used today, each with their own strengths and uses.

The two most commonly compared methods of spatial interpolation for environmental phenomena are inverse distance weighted and kriging (Li & Heap, 2014). Kriging is considered a geostatistical spatial interpolation method, while inverse distance weighted is not. Li & Heap (2014) review many of the variants for inverse distance weighted and kriging plus some of the less applied spatial interpolation methods in environmental studies.

Inverse distance weighted is one of the most used interpolation methods in GIS (Lu & Wong, 2008). This is probably due to the availability in almost all GIS platforms and the simplicity of the math used in the model. Inverse weighted methods used for interpolation of coal thickness and yield in surface mining are well documented in textbooks such as in (Bruce A. Kennedy, 1990). The Bureau of Land Management recognizes that inverse distance weighted is a viable method for the spatial interpolation and extrapolation of coal thickness and coal density (Richfield Field Office, 2008). One study used inverse distance weighted to spatially interpolate data for rock mass ratio, ground water level, and permeability from bore holes to correlate the data to subsidence events in South Korea (Kim, Lee, Oh, Choi, & Won, 2006).

Kriging was first developed by Daniel G. Krige for the spatial interpolation of gold deposits in the Witwatersrand in South Africa (1951). The accepted method of determining gold deposits in at the time was to use known values of gold deposits and
average them together across a mine area. The average height of the gold deposits multiplied by the area of the mine would be the volume of minable gold within the mine. This was a necessary step for the mine engineer, since the minable commodity, gold in Krige’s case, was the economic return on the expense of mining. Krige found the method of averaging to be lacking in both spatial detail and volume accuracy. Krige employed a spatial statistical approach to more accurately determine gold volumes in a mine. Since Krige wrote his paper, kriging has become the de facto standard for geostatistical interpolation. It has gained widespread popularity in environmental sciences, spawning a host of kriging variants to fit a variety of spatial interpolation needs.

Kriging is a statistical distance weighted interpolator. The method uses a distance autocorrelation, which takes into account the geostatistics mantra; things closer together are more alike than those that are far apart. Kriging has become the cornerstone of geostatistics and there is no shortage of documented kriging uses in the coal mining industry. Kriging was promoted for use in the coal industry in South Africa to determine deposit continuity (Jeffrey, 2015). Kriging analysis was shown to be a useful method of displaying the spatial variability of coal thickness and coal quality (Srivastava, 2013). Kriging was used in a case study on a mine in Indonesia to determine the spatial distribution of coal quality based on the heating value and sulfur content of coal by (Heriawan & Koike, 2008). Hohn & McDowell (2001) advocated kriging for severance taxes on coal and do away with the weight tickets.

Each spatial interpolation method has its own strengths and weaknesses. Choosing a spatial interpolation model often involves performing several different
methods and comparing the results. Li and Heap (2011) performed a comparative study of the various methods of spatial interpolation and of the post processing analysis of the models to determine the adequacy of the model to the data. The problem with the use of inverse distance weighted and kriging for use in this study was that they both honor the test hole data. As discussed in the test hole analysis section of this paper, it is believed that the test holes should not be taken as accurate. Inverse distance weighted or kriging should not be used unless knowledge of the methods and accuracies for the test holes are understood completely. Only with a complete understanding of the test hole data can a spatial interpolation method be chosen that best suits the data.

The market is slim when it comes to specialty software that performs subsurface modeling. AutoCAD and Carlson are the leading software companies that have specialized geotechnical and mining packages. AutoCAD and Carlson are considered CAD programming packages. CAD software is specifically designed with engineering and design in mind. Within the mining industry, Carlson and AutoCAD would be the software packages used for subsurface modeling, if subsurface modeling was performed.

AutoCAD has specialized CAD packages such as AutoCAD Civil to facilitate specific needs within engineering. Within the AutoCAD Civil software suite is the geotechnical module. The geotechnical module allows for the creation of sub-surfaces from test hole data. AutoCAD “surfaces” are three dimensional models generated from point data in AutoCAD Civil. AutoCAD does not have the ability to spatially interpolate grid based rasters like GIS software. Surfaces are created by creating a triangular plane between three points. (Louisa Holland, 2014) The AutoCAD geotechnical model creates
strata “surfaces” by joining identical lithologic units in each boring record with straight line segments (Gary Morin, 2014). This can be viewed as synonymous with the 3-point method for strike and dip. The difference being that the triangles are formed using the Delaunay method. The Delaunay method of triangulation ensures that no points are found within the area formed by the triangulation. The triangles are formed with the closest test holes. The area within the triangles would then have localized strike and dip. The strike and dip (slope) would change immediately between adjacent triangles and depending on the test hole data the slope could change drastically and suddenly.

Carlson software company specializes in CAD modules and offers one of the premier software suites for mining. The modules are built on top of a CAD programs like AutoCAD. The geology module is one of the modules in the Carlson mining suite and is promoted as a stratigraphic and modeling software package. The geology module in Carlson can perform grid based spatial interpolation using kriging, inverse distance weighted (inverse distance weighted), or least squares. Of course, Carlson has the same surface triangulation ability as AutoCAD (Carlson Software Company, 2015). Carlson (2015) recommends using triangulation for base elevations of bore hole data. Straight line slopes connect lithologic units, creating a triangulated irregular network between test holes. Strata thickness is more localized and is best modeled using inverse distance weighted or linear least square (Carlson Software Company, 2015).

The use of triangulation has many advantages that make the use of triangulation the preferred method among these software companies. The major advantage to triangulation is the simplicity of the model. The test hole data is honored as 100%
accurate. The test holes are connected via straight lines with constant slopes. This allows any desired point within the interpolated surface to be found with infinite resolution. Triangulation is advantageous to CAD software designers, since CAD’s advantage is that it is a vector based infinite resolution drawing platform. This advantage becomes obvious when compared with grid based interpolation methods. Each cell in grid interpolation has a single value. This means that no matter what point is defined within the cell the value will be the same. Triangulation would give different answers for each new location.

Triangulation also has many disadvantages. Triangulation honors the test hole data as 100% accurate. This was also listed in the advantages, but depending on the confidence the analyst has in the test hole data, this could be an advantage or disadvantage. If there is doubt in the spatial accuracy, depth measurement, or elevation of the test hole, the triangulated surface would be as inaccurate as the test hole data. Another disadvantage to triangulation is that mathematical operations cannot be performed on the triangulated surface. Grid based rasters can perform calculations and complex analysis. Triangulation has no means of extrapolation beyond the perimeter formed by the lines connecting the test holes. Test holes are drilled inside the permit boundary, but an analysis of the entire permit is needed. Kriging, inverse distance weighted, or linear least square would be used to extrapolate from the triangulation surface.
OBJECTIVES

Python and ArcGIS tools will be created for coal mine permitting and post mining research. ArcGIS (Esri Inc., 2015) is a commercial package that has three price points, depending on the included tools. Basic, Standard, and Advanced (formerly known as ArcView, Arc Editor, and Arc Info) are the labels that ArcGIS applies to the different tool packages. This analysis will be limited to the Basic version of tools and options. This benefits the users who does not have the resources for standard or advanced ArcGIS. The tools in the basic ArcGIS package are just that basic, and will come in most open source GIS platforms. Python is a free programming language that is used in some of the more popular commercial (ArcGIS) and open source (QGIS) GIS platforms. The tools will be developed for the end user, so that a minimum of GIS or Python knowledge will be required. Tools perform analysis automatically with a minimum of user inputs, which saves time and money while obtaining the maximum amount of information from the data in a reproducible and unbiased fashion.

Two permits were used to show the outputs in the results sections of the models. The first permit is West Fork 3, provided by B&N Coal, Inc. West Fork 3 is a 243-acre surface coal mine in Noble county, Ohio. The purpose of the mine is the extraction of the #9 coal seam (Meigs Creek Coal Seam). The second permit is KLM#7 provided by Rosebud Mining, Co. KLM# 7 has the SMCRA ID D-2401. KLM#7 is a 1,054-acre underground coal mine in Harrison County, Ohio. The intent of the KLM#7 mine is the extraction of the #7 (Upper Freeport Coal Seam).
Existing Surface Digital Elevation Model (DEM)

The existing surface DEM is a raster of the existing Earth’s surface. This model will serve as a base for the other models to be built from. The existing surface DEM of a permit will be used to assign surface elevations to the test hole data based on spatial location. The existing surface DEM will be used to perform calculations in other models. Because of the dependencies the other models have on the outcome of this model, the existing surface DEM model requires consistent and reliable elevation and position accuracy.

Coal Analysis Model using Multilinear Least Squared Regression

The outcome of this model will produce a DEM that approximates the middle of the coal seam using multilinear least squared regression (MLSR) on the elevation of the coal seam. The elevation of the coal seam was calculated using the depth measured for the test hole and the surface elevation at the test hole location. MLSR is believed to be the best interpolation method in the case where test hole data may be speculative. The error in the test hole data has unmanageable error mostly due to the location accuracies in the driller’s report. MLSR can be thought of as the average of the test hole sampling data. Just like in averaging the analyst could predict the result of the next sample value by the average or expectation. MLSR provides that same statistical analysis by predicting or interpolating the result of another test hole drilled in the area. If the analyzer has confidence in the test hole data, a more robust analysis using kriging should be used (Jeffrey, 2015). A less detailed and less sophisticated interpolation surface can be achieved using inverse distance weighted (Kennedy, 1990).
The coal thickness raster will be produced by using the coal thickness values in the test hole data. The coal thickness raster can provide the amount of coal in the permit using the thickness of coal in the raster and the area of the raster. The coal thickness raster will be created using an inverse distance weighted interpolation method (Kennedy, 1990; Richfield Field Office, 2008; Carlson Software Company, 2015). A DEM will also be created for the top and bottom of the coal seam using raster calculator and the coal thickness raster and the MLSR DEM. The top of the coal seam also indicated where the overburden begins. The bottom of the coal indicates the elevation of the pit floor upon completion of mining. An overburden thickness raster will give the height of material above the coal seam. The coal outcrop will be placed at the location of 0 overburden or the top of the coal.

Multi Linear Least Squared Regression Strike and Dip

ArcGIS does not have a tool that provides the strike and dip of a plane. This tool will be written in Python to fill fields in ArcGIS attribute tables. The output of this tool will provide the strike and dip as attributes in the coal outcrop attribute table. At this point residuals between the MLSR and the measured values taken at the test holes will be calculated and placed in the test hole attribute table.

Highwall Analysis and Optimization

The highwall analysis and optimization model will create highwall options for the mine engineer based on optimized coal recovery, average coal thickness, and highest test hole elevation. The ODNR estimated economic return for surface coal mining is 20 to 1, overburden to coal thickness ratio. The optimized recovery will use the overburden and
coal thickness rasters to place the final highwall. The average coal recovery will give a final highwall at 20 times the average coal thickness for the permit. The greatest cover will place a final highwall at the surface elevation of the highest test hole.

**Acid Base Accounting**

The Acid Base accounting model will provide a summation of the potential acidity of mine spoil for surface mines. The model will iterate through the lithologic units in the test hole report, providing the area of influence at each iteration. The area of influence will be derived from the middle elevation of the lithologic unit and the Existing Surface DEM. The model will then use density assumptions and the lab analysis to sum the potential acidity for the entire permit.

**Subsidence**

The subsidence model will show the probability of subsidence occurring in a permit based on the overburden thickness. The probability will be based on the frequency of subsidence reports at an overburden thickness. Probability calculations are complex and are usually performed through tables. GIS does not allow complex calculations such as integration or error function, so a unique solution is required to map probabilities. A z-score raster can be used as a preliminary step to bridge the gap between overburden and probability. To go from a z-score to probability an nth degree polynomial can be used as an approximation. This analysis uses only the frequency of reported subsidence events and does not try to over extend relationships with other measurable parameters as in (Kim, Lee, Oh, Choi, & Won, 2006).
MODELS

Before the models were ran, the project geodatabase was created, the “Project” polygon feature class was created, and the test hole point feature class was created. The models are designed for a minimum of user inputs. All data is stored in a geodatabase in ArcCatalog. ArcCatalog is the data management solution for ArcGIS. The geodatabase was used to store all required inputs and outputs for the following models. Inputs and outputs are not moved, renamed, or deleted between running different models. Some models depend on the outputs of other models. The same database was used for all models. The GDB parameter was used to reference the inputs and outputs to minimize user input. User created features and fields are named precisely, as they are given in this paper. This method benefits the user from having to input certain parameters and makes the entire process more efficient. This places rigidity in the naming of tables, headers, and features, but frees the end user of having to specify several parameters to use a model. Inputs and output names are provided in tables for easy reference in each of the model’s sections. The user can edit and move the files using ArcCatalog once analysis is complete. The “Permit” polygon feature class is a user created feature class that defines an area of interest. The “Permit” feature must be stored in the project database.

The test hole information is used in all the models except for the Existing Surface DEM. Test hole information is provided as a driller’s report. An example of a page from a driller’s report for the West Fork 3 site is provided in Figure 15. There are typically several pages for each test hole in a drillers report depending on the depth to the coal seam. Each test hole report is given a state plane coordinate in X and Y, a surface
elevation, test hole ID, drill depth, and thickness of lithologic unit. The location and surface elevation of the test hole are placed on these forms using a variety of methods as discussed in the previous section.

Figure 15: Driller’s report for TH-01 on the West Fork 3 permit.

Table 3 was created using the test hole ID, the state plane X and Y, thickness of the coal, thickness of the binder, and the depth down to the bottom of the coal seam. The binder is a thin shale parting of the coal seam that is not considered part of the sellable reserves. Notice that the elevation was not recorded. The file must be saved to the *.xls
file type if using Excel. The headers of the Spreadsheet are referenced by name in the models and must be identical to those in Table 3.

Table 3: Test hole data placed in an *.xls spreadsheet for import into GIS.

<table>
<thead>
<tr>
<th>TH_ID</th>
<th>X</th>
<th>Y</th>
<th>Thickness</th>
<th>Binder</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH-01</td>
<td>2268463.3</td>
<td>599791.971</td>
<td>7.75</td>
<td>1.25</td>
<td>113.75</td>
</tr>
<tr>
<td>TH-02</td>
<td>2267691.02</td>
<td>601400.835</td>
<td>9.5</td>
<td>2</td>
<td>123.5</td>
</tr>
<tr>
<td>TH-03</td>
<td>2268469.263</td>
<td>601872.425</td>
<td>12</td>
<td>2</td>
<td>126</td>
</tr>
<tr>
<td>TH-04</td>
<td>2269100.521</td>
<td>602798.651</td>
<td>9.5</td>
<td>2</td>
<td>138</td>
</tr>
<tr>
<td>TH-05</td>
<td>2268714.672</td>
<td>604937.215</td>
<td>9</td>
<td>1.5</td>
<td>68</td>
</tr>
</tbody>
</table>

The “add X-Y data” tool was used to import the test hole data. NAD 83 Ohio State Plane South (3402) are provided as the projected coordinate system. The test hole is created in ArcMap and was exported to project database as “TestHole”. All data from the spreadsheet is imported into the attribute table of the TestHole feature class and placed in a geographically known location defined by projected coordinate system 3402. Figure 16 shows the inputs required by the “add X-Y data” tool.
Figure 16: Add x-y data tool in ArcGIS.

Figure 17 shows the test hole locations per the spatial location provided by the driller’s report. The attribute table featured in the upper left of Figure 17 shows that the data were imported correctly. The 10 foot contours and contour annotations were provided by the Existing Surface DEM model.
Figure 17: Test hole locations and test hole attribute table.

Models

1. Existing Surface Digital Elevation Model (DEM)
2. Coal Analysis
The existing Earth’s surface of a permit is usually shown as contours. Contours can come from a variety of sources. The most common source is the United States Geological Survey (USGS). Topographic mapping used to be a very expensive and a long drawn out process. Coal companies could not afford the time or equipment to perform their own topographic surveys for a single permit. The USGS could use stereoscopy, a father to modern day photogrammetry, and later RADAR, a father to modern day LiDAR, to construct contours. The contours could then be ordered from the USGS. USGS maps are typically seen as 7.5-minute and 15-minute quadrangle maps. Quadrangle maps use 20’ contour intervals.

Before the introduction of GPS to the commercial market, test holes would be spatially located on a USGS map by a field technician. This could be done, with improved results, using laminar sheets of contours over an aerial photograph. The elevation of the test holes would then be interpolated from the contours. The elevation of the lithologic unit would then be estimated based on the difference between surface elevation and driller’s depth. The field technician could reasonably interpolate the surface elevation within 5’ from a 20’ interval contour map, assuming the location of the
test hole was accurate. Contour elevation interpolation is prone to random and systematic errors between permits. The random error is due to the interpolation between contours and the technician making the measurements. Systematic errors can be contributed to mapping errors (e.g. improper location of plotted contour lines or improper location of test hole). Test hole locations that have been improperly placed on the map could lead to errors of more than 5’ in the surface elevation.

In recent history, GPS has become more commonplace due to improved accuracy and affordability. Horizontal positional accuracy can range based on equipment used. In 2017, a wide variety of GPS devices are on the market. The software typically gives stated accuracies with each GPS point, but a GPS device manual can be used to provide estimated errors. A good representation in 2017 of the 2 opposite ends of accuracy are the I-Phone and Trimble R10. The horizontal error can range from 5 m (16’) for GPS enabled I-Phone 6 to 10 mm (0.03’) for a Trimble R10 survey grade GNSS receiver. Vertical error is typically double the horizontal error. For GPS enabled I-Phones the vertical error can be as high as 15m (50’), and a Trimble R10 survey grade GPS has a vertical error as low as 20mm (0.07’). The accuracy of the device used depends on the price that the user is willing to spend. Prices can range from $500 for a GPS enabled I-Phone to $30,000 for the Trimble R10 Survey kit. In the coal mining industry, survey grade GPS is not commonplace due to the high capital cost. Regulatory demands coupled with stiff market competition keep profit margins low for coal mines. Increasing positional accuracy of test holes is simply not a budget priority. It can be assumed for
most modern permits that low end GPS or USGS mapping is utilized for elevation and location.

Discussing potential error in location and surface elevation across many permits and permittees over decades of time can be difficult. It’s not the same equipment used, it’s not the same field technician, and the methods can vary. It would be a daunting task to control the error on an individual permit basis, let alone for all coal permits. This paper only discussed a portion of the possible spatial errors that exist in exploratory drilling reports. The purpose of pointing out these errors was to show the reader the necessity of developing a consistent, reliable, and recordable method that can be used across all coal permits.

Today LiDAR data is provided by the state of Ohio for free. LiDAR data can be processed by ArcGIS into any number of formats including contours and rasters. Ohio Statewide Imagery program (OSIP) provides topography and imagery for the entire state of Ohio. The topography comes in many forms for download, but LiDAR was used in this case. OSIP LiDAR data meets ASPRS 2014 Positional Accuracy Standards. The vertical accuracy of OSIP Lidar data has a root mean squared error of 0.5’ and a horizontal error of 2.5’ (OSIP, 2006). LiDAR is consistent across the entire state of Ohio and therefore creates a common thread across Ohio coal permits. Data is downloaded by tile or county. The map viewer and downloader can be found at [http://gis5.oit.ohio.gov/geodatadownload](http://gis5.oit.ohio.gov/geodatadownload). OSIP 3 is in production for 2017, so new and improved LiDAR data could be available soon.
Results

The Existing Surface DEM model takes the LiDAR data downloaded from OSIP and processes it into useful analytical and visual mapping elements. Figure 18 shows the input screen for the Existing Surface DEM model. The surface mine, West Fork 3 was used as an example to display the results of the Existing Surface Elevation model.

![Image of model parameter input](Image)

Figure 18: Model parameter input for the Existing Surface DEM model.

The input files are raw LiDAR data downloaded from OSIP. Choose the folder and name for the LAS dataset. The LAS dataset is a file format in ArcGIS for the storage and manipulation of LiDAR data. Choose the geodatabase that was created for the
Choose the coordinate system that was used for the permit feature. The permit feature is a feature class that provides the boundary of the permit (i.e. an area of interest). The Existing Surface DEM model takes the LiDAR files downloaded from the OSIP website and converts them into useful rasters and mapping features. Raster format is one of the most versatile and analytical tools in ArcGIS. Table 4 provides a synopsis of inputs and outputs for the model. The Elevation output will be the key raster from this model used in the other models.
Table 4: Inputs for the Existing Surface DEM model.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIDAR data</td>
<td>The LIDAR data was downloaded from the Ohio Statewide Imagery Program Website, <a href="http://gis5.oit.ohio.gov/geodatadownload">http://gis5.oit.ohio.gov/geodatadownload</a>. This is currently the best place to get free topographic LiDAR data for use in a GIS program.</td>
</tr>
<tr>
<td>LAS Dataset</td>
<td>The file location and name of the LAS dataset</td>
</tr>
<tr>
<td>GDB</td>
<td>The geodatabase to store files created by the tool or retrieve the files necessary to run the tool.</td>
</tr>
<tr>
<td>Permit</td>
<td>The permit feature must be in the GDB file location for this model to work. The Permit feature class is a boundary of the permit or an area of interest.</td>
</tr>
<tr>
<td>CS</td>
<td>The coordinate system to be used for the project. NAD 83 Ohio State Plane system in feet is suggested, 3402 if in the south and 3401 if in the North.</td>
</tr>
</tbody>
</table>
Table 5: Outputs for the Existing Surface DEM model.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAS file dataset</td>
<td>This is how ArcGIS stores LIDAR data. LIDAR data can be viewed in a variety of ways using the LAS Dataset toolbar in ArcMap.</td>
</tr>
<tr>
<td>Elevation</td>
<td>The Raster Digital Elevation Model with 2.5' square cells. This Raster is the base for all other topographic analysis.</td>
</tr>
<tr>
<td>Curvature</td>
<td>The second derivative of elevation. Useful for a variety of analysis such as soil erosion. 3 types of curvature are created; Curv, Profile Curv, Plan Curv</td>
</tr>
<tr>
<td>Slope</td>
<td>The first derivative of elevation. Slope measurements across the permit area are required on the application map.</td>
</tr>
<tr>
<td>HillShade</td>
<td>A purely cosmetic layer useful for accentuating large topographic relief.</td>
</tr>
<tr>
<td>Hydro</td>
<td>A 1000-foot buffer of the permit boundary used to limit the size of some of the created files.</td>
</tr>
<tr>
<td>Contour</td>
<td>10' contours with indexed 50' contours. Focal Statistics was used to smooth the contours slightly to eliminate LIDAR noise.</td>
</tr>
<tr>
<td>MaxElev</td>
<td>A points shapefile used to mark points of high elevation.</td>
</tr>
</tbody>
</table>

The DEM Existing tool creates a variety of outputs necessary for permit mapping and analysis. The Elevation raster is the conversion of LiDAR data to raster. The
Elevation raster was then used to obtain a variety of other mapping features used for analysis and mapping requirements. For a complete understanding of the tools, uses, and reasons, please see Appendix 2 for the model diagram (2.1), Python code (2.2), and model outputs (2.3). Figure 19 shows the Existing Surface DEM from high elevation (red) to low elevation (green), the permit as a black dashed line, and the hillshade provides a feeling of depth to the DEM.
Coal Analysis Model Using MLSR

Methods

A mobile drilling rig is used to perform the exploratory test holes. The rig requires relatively level ground to ensure the drill shaft is in the true vertical position when drilling begins. In the rolling, undeveloped hills of Appalachia, it is often
necessary for the drill rig to be accompanied by a bulldozer to clear and level the setup area. The vertical position of the drilling operation is positioned along the gravity vector, which is achieved using a bubble level. If the rig is not in the true vertical or the shaft begins to drift while drilling, this can lead to systematic error in driller’s depth. A 100-ft. drill hole would measure 0.38 ft. longer than the actual value, if it was 5 degrees off true vertical.

The precision to which test hole depth measurements are made vary across projects. Investigation of drilling reports shows that precision can range from 0.5 ft to 0.1 ft. However, due to field conditions the elevation of the lithologic unit or total depth from the surface cannot be given to have the same error. For auger drilling, adjacent lithologic units at the drill head blend together, and the material must be pushed to the surface. This mixing and delay can make it difficult for the field technician to identify the instant that the lithology changes. Literature on the error due to the time of operation required to move cut rock particles from the cutting head to the surface could not be found. Logical assessment of the problem can be easily used to determine that the error will increase linearly with depth. That is, the deeper the hole, the longer it will take for the drilled lithologic unit to rise to the surface and the more time available for the lithologic units to mix. 0.01 ft per vertical drilled foot is a conservative estimate of error. Using this assumption, a driller would be required to drill to a depth of 101 ft. to read the lithologic cuttings at 100 ft.

The Coal Analysis model will use the West Fork 3 test hole data that were imported from the drillers report. The surface elevations of the test hole data were filled
using the “Add Surface Information” tool. The original surface elevations were
discarded to use the OSIP data with known error of ± 0.5 ft. This will ensure that the
elevation for the test holes are reliable and the potential error is consistent between test
holes. Now the only sources of systematic error that can be questioned is the spatial
location of the test hole and the drill depth. The position of the test hole is used to
populate the surface elevation. The surface elevation is used to calculate the elevation of
the coal seam by subtracting the depth measurement. The elevation of the coal seam at
each test hole is used for raster interpolation. Since the error can be substantial and
unknown for each test hole, a statistical approach like multilinear least squared regression
(MLSR) is the most practical of the interpolation methods for establishing the coal plane.
If the test hole data is to be honored, use kriging to perform the spatial interpolation as in
(Lesley Jeffrey, 2015). Kriging provides for a robust statistical analysis of the test hole
data, where trend, anisotropy, and cross validation can be explored thoroughly.

MLSR is a tool that exists in ArcGIS known as trend. The trend tool can have a
polynomial order of 1 through 12 and perform linear or logistic regression. The
assumption for the coal bedding plane is that it has a trend that is linear or a polynomial
order of 1.

The thickness error of the lithologic unit is random measurement error since the
measurements are made relative to successive lithologic units. It is assumed the error in
thickness can be attributed to the precision to which the measurements were made. Most
drill reports show measurements between 0.1 and 0.25 feet. It is also assumed that the
coal seam thickness is the most accurate of all the measurements in the drill report, since
in the case of coal mine test holes, the coal is the reason for the drilling. For this reason, the test hole data for the thickness of coal seam will be honored. Spatial interpolation of a coal seam with inverse distance weighted is described in (Richfield Field Office, 2008).

The isopach and binder raster used inverse distance weighted interpolation. Isopach raster includes the bottom coal seam, binder, and rider coal seams (i.e. the entire stratigraphic thickness of the coal seam). In some coal seams, there is a thin layer of shale, known as binder, separates the rider coal and bottom coal. The coal thickness raster is the isopach raster minus the binder raster. The idea of the coal thickness raster is to be able to provide the information necessary to determine economic feasibility. In the IDW tool, I used a power of 2.5. A power of 2.5 was based on the desired effect of smooth transitions between test holes. Lower values produce bullseyes around the test holes (i.e. little hills or sinks with the test hole at the center). Higher powers produce sharp slopes between test holes. The top and bottom of the coal seam is found at each test hole by adding or subtracting half the thickness to the middle of the coal seam. The Raster Calculator tool is used to create the rasters.

Results

Before running the Coal Analysis model, the outputs from the Existing Surface DEM model must be in the project geodatabase. The only input required by the user is the Geodatabase location, GDB. Table 5 shows the inputs and outputs of the Coal Analysis Model. For a complete understanding of the tools, uses, and reasons, please see Addendum 3 for the model diagram (3.1), Python code (3.2), and model outputs (3.3).
Table 6: The inputs for the coal analysis model.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDB</td>
<td>The project geodatabase. This must be the same database that was used to collect the outputs from the Existing DEM model.</td>
</tr>
<tr>
<td>TestHole</td>
<td>A point Shapefile contained in the GDB geodatabase. The feature class must have the Thickness, Binder, and Depth field. The binder field can be specified as all zero if no binder exists.</td>
</tr>
<tr>
<td>Elevation</td>
<td>A raster digital elevation model contained in the GDB geodatabase created by the Existing_DEM model.</td>
</tr>
</tbody>
</table>
Table 7: The outputs for the coal analysis model.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TestHole</td>
<td>Adds the Z field and ElevMiddleCoal field to the existing TestHole feature. Z is populated using the add surface data tool and ElevMiddleCoal is calculated using ElevMiddleCoal+thickness/2.</td>
</tr>
<tr>
<td>CoalMid</td>
<td>A raster created using multilinear regression of the ElevMiddleCoal Field in the TestHole feature.</td>
</tr>
<tr>
<td>Binder</td>
<td>A raster that describes the thickness of a parting in the coal seam.</td>
</tr>
<tr>
<td>Isopach</td>
<td>A raster that describes the thickness from the top to the bottom of the coal seam.</td>
</tr>
<tr>
<td>CoalThickness</td>
<td>A raster that describes the thickness of the coal.</td>
</tr>
<tr>
<td>CoalBot</td>
<td>A raster that describes the elevation of the bottom of the coal</td>
</tr>
<tr>
<td>CoalTop</td>
<td>A raster that describes the elevation of the top of the coal</td>
</tr>
<tr>
<td>CoalOutcrop</td>
<td>A feature isoline that defines 0 overburden</td>
</tr>
<tr>
<td>Overburden</td>
<td>A raster that describes the depth from the surface to the coal seam.</td>
</tr>
</tbody>
</table>

Figure 20 shows the isopach raster created by the Coal Analysis model. Notice the center of the permit shown in white around TH-03. This indicates that the coal unit is
extremely thick in this area of the permit. The lower part of the permit shows thinner coal in a light cyan color around TH-01. This indicates that the coal unit is thinner in this region. A review of the test hole data shows that TH-01 had a thickness of 7.75 ft. and TH-03 had a thickness of 12 feet. Using this information, it is easy to see that the area between TH-02 and TH-04 will yield the most coal.
Figure 20: Isopach raster for the West Fork 3 permit color gradation is from high (white) to low (cyan).

The West Fork 3 permit was mined before SMCRA was enacted. The area was mined and left unreclaimed. Using today's large earth moving equipment, one or two more cuts can raise the highwall to the maximum economical extent. The unreclaimed
highwalls and large spoil piles have the benefit of showing the accuracy of the Coal Analysis Model. Figure 21 is a closeup of an aerial flown with a Phantom 3 Professional drone owned by Kleski Environmental Services, LLC. The coal seam created by the Coal Analysis model is shown as black dots. The permit line is shown as black dashes. The coal seam is placed near the base of the old highwall, exactly where you would expect. Small ringlets of coal outcrop (black dots) on the old pit floor that seem to be errors in the model. In fact, large spoil piles that are marked due to their elevation above the coal raster. Prelaw mining made the cut into the highwall and simply dumped the spoil off the edge of the pit floor. An analyst may use these data to get area and volume of the spoil piles, which would be important in any reclamation efforts.
Figure 21: Aerial drone photography, contours (solid brown), coal outcrop (black dot), and permit (black dash) show the accuracy of the coal analysis model.

MLSR Strike and Dip Model

Methods

MLSR is a superior method over the 3-point method for strike and dip, because MSLR uses all the test hole data to obtain the strike and dip. MSLR has not been used in the past due to the complexity of the calculations and the lack of documentation for its use. The Python model performs the calculations for the user, so that the user does not have to know differential calculus to achieve a strike and dip. There is no native way in ArcGIS to obtain the strike and dip of a coal seam.
Linear least squared regression was first introduced by Adrien-Marie Legendre, a French Mathematician. Least squared regression lines are a widespread practice in all the sciences. Least square regression minimizes the sum squared distance between the formulated line and the data points. The distance from the line to the data point is measured parallel to the y-axis. The minimization of sum squared error, S, (Equation 8 & 9) is achieved by setting the partial derivation of S with respect to a and b to zero (Equation 10 & 11). The linear least squared regression proof can be found in many calculus books (e.g. Larson and Edwards, (2010)). A full proof of the least squared regression line was conducted in the math software Maple (Maplesoft, 2015) and is attached as Appendix 1.1.

\[ S = \sum_{i=1}^{n} [f(x_i) - y_i]^2 \]  

(8)

\[ f(x) = a * x + b \]  

(9)

\[ a = \frac{n * \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} y_i * \sum_{i=1}^{n} x_i}{n * \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \]  

(10)

\[ b = \frac{1}{n} * \left( \sum_{i=1}^{n} y_i - a * \sum_{i=1}^{n} x_i \right) \]  

(11)

The method for least squared regression of a plane is not as well documented as the least squared regression of a line, and that is why it was necessary to step through the
well documented linear case. However, it is easily derived using similar methods. A proof was conducted in the math software Maple and is attached as Addendum 1.2. The general form of the equation of a plane is \( a \times x + b \times y + c \times z + d = 0 \). After some algebraic manipulation, the equation of the plane can be shown in the “slope intercept” form \( Z = q + m \times X + p \times Y \). This shows that \( Z \) is the dependent variable and \( X \) and \( Y \) are the independent variables. The distance (D) between a point and the plane would then be \( D = Zi - Z \), where \( Zi \) is the \( Z \) coordinate of any point \((Xi, Yi)\). Using the slope intercept equation of the plane, the partial derivative of \( D \) with respect to \( m \), \( p \), and \( q \) are set to 0 (Equations 12-14).

\[
m \times \sum_{i=1}^{n} [x_i] + p \times \sum_{i=1}^{n} [y_i] - \sum_{i=1}^{n} [z_i] + n \times q = 0
\]

(12)

\[
m \times \sum_{i=1}^{n}[x_i^2] + p \times \sum_{i=1}^{n}[x_i \times y_i] - \sum_{i=1}^{n}[x_i z_i] + q \times \sum_{i=1}^{n}[x_i] = 0
\]

(13)

\[
m \times \sum_{i=1}^{n}[x_i \times y_i] + p \times \sum_{i=1}^{n}[y_i^2] - \sum_{i=1}^{n}[y_i \times z_i] + q \times \sum_{i=1}^{n}[y_i] = 0
\]

(14)

Three equations with three unknowns are given in a linear system of equations. The summations can be interpreted as constants. The variables \( m \), \( p \), and \( q \) can be solved explicitly, but to make these equations more digestible to Python, matrix algebra was used. Manipulating the equations slightly, reveals the matrix format (Equation 15).
Equation 16 shows the sum of a data series divided by the number of data points in the series is the definition of the mean. Equation 17 shows that dividing the matrix equation by n further simplifies the equation to mean values, where n represents the number of values in the data set.

\[
\frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}
\]

(16)

\[
\begin{bmatrix}
\bar{x} & \bar{y} & 1 \\
\bar{x}^2 & \bar{xy} & \bar{x} \\
\bar{xy} & \bar{y}^2 & \bar{y}
\end{bmatrix}
\begin{bmatrix}
m \\
p \\
q
\end{bmatrix}
= \begin{bmatrix}
\bar{z} \\
\bar{xz} \\
\bar{yz}
\end{bmatrix}
\]

(17)

Solving the matrix equation will yield the coefficients of the slope intercept equation for a plane. The m is the slope in the X direction, the p is the slope in the Y direction, and the q is the Z intercept. Equations 18-20 are provided to convert from the slope intercept of the plane equation to the general form of the plane equation.

\[
a = -m \cdot c = -m \cdot 1
\]

(18)

\[
b = -p \cdot c = -p \cdot 1
\]

(19)
\[ c = c = 1 \] (20)

Treating Equations 18-20 as a system of linear equations, one of the solutions is \( c = 1 \). The choice of which solution is truly arbitrary, so any value of \( c \) can be used. \( c = 1 \) is chosen for the simplest solution. A proof that \( c = 1 \) yields a least squares regression plane is provided in Appendix 1.2 lines 21 through 30. The general form of the plane equation, \( a \times x + b \times y + c \times z + d = 0 \), was used to find the normal vector \(<a, b, c>\) to the plane. The normal vector is used to find the strike line and dip angle for the plane. The normal vector projected onto the X-Y plane (i.e. \(<a, b>\)) points in the direction of the dip. The strike line is considered 90 degrees from the direction of the dip. The dip angle is the angle between the z-axis and the normal vector, \(<a, b, c>\) and is provided as Equation 21.

\[
\cos^{-1} \left[ \frac{<a, b, c> \cdot <0, 0, 1>}{\sqrt{a^2 + b^2 + c^2}} \right]
\]

(21)

The MSLR strike and dip model used Python 2.7.8 to calculate the MLSR plane. The code can be added as a script file to ArcMap or executed in IDLE outside of the ArcGIS environment. Even though it is meant to be executed in Python 2, the code was written in Python 3 syntax for forward compatibility with ArcGIS Pro. This model uses the arcpy, numpy, csv, and math Python modules to perform a multilinear least squared regression to calculate the strike and dip of the coal plane and residuals for the test holes. A CSV file must be created containing the X, Y, and ElevMiddleCoal. This can be easily
created by copying the attribute table to excel, deleting all but the required fields, and saving as a *.csv. The Python script requires the file location of the TestHole.csv and the geodatabase (GDB). Table 6 shows the inputs and outputs of the Multilinear Least Squares Regression model. For a complete understanding of the tools, uses, and reasons, please see attachment Coal Analysis for the Python code, model diagram, and model outputs.

Results

Table 8: The inputs for the multilinear regression model.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TestHole.csv</td>
<td>A comma separated file used to pipe data into Python. The data is stored in the code as a matrix for analysis</td>
</tr>
<tr>
<td>GDB</td>
<td>The geodatabase that has the TestHole feature and CoalOutcrop feature.</td>
</tr>
</tbody>
</table>
Table 9: The outputs for the multilinear regression model.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CoalOutcrop</td>
<td>Adds the StrikeDirection, DipDirection, Strike, and Dip fields to the CoalOutcrop feature class and calculates their values.</td>
</tr>
<tr>
<td>TestHole</td>
<td>Adds the Residual field to the TestHole feature and calculates the difference between the multilinear regression plane and the measured elevation values (ElevMiddleCoal).</td>
</tr>
</tbody>
</table>

The Multilinear regression model calculated the strike to be N 22° 14' 27" E and have a dip of 0° 25' 39". This is characteristic of Ohio Coal. The residuals between the model and measured values are also calculated. Table 7 shows the residuals calculated by the model for the West Fork 3 permit. There are 2 indicators in Table 7 that the MLSR plane achieved the desired result. Notice that the residuals are relatively small and there are positive and negative numbers.

Table 10: West Fork 3 model residuals

<table>
<thead>
<tr>
<th>TH_ID</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH-01</td>
<td>-0.8118</td>
</tr>
<tr>
<td>TH-02</td>
<td>0.5932</td>
</tr>
<tr>
<td>TH-03</td>
<td>0.4814</td>
</tr>
<tr>
<td>TH-04</td>
<td>1.16</td>
</tr>
<tr>
<td>TH-05</td>
<td>-0.7811</td>
</tr>
</tbody>
</table>
The multilinear regression model must be analyzed to ensure that the model is appropriate for use with the data (Li & Heap, 2014). To determine how well the plane models the test hole data, the R squared goodness of fit was calculated. Adjusted R squared is unnecessary since it is a first order polynomial. The sum squared residual (SSr) is calculated using each test hole. The sum squared difference (SSd) between the average test hole and model was calculated for each test hole. The R squared is then one minus the ratio of SSr to SSd. Table 8 shows the results of the calculations for goodness of fit. Li and Heap (2014) discuss the primary way to discuss the adequacy of a multilinear regression model is through the goodness of fit method.

Table 11: Calculation results for the goodness of fit test.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SSr</td>
<td>3.198369</td>
</tr>
<tr>
<td>SSd</td>
<td>99.70888</td>
</tr>
<tr>
<td>R^2</td>
<td>0.967923</td>
</tr>
</tbody>
</table>

The t-test is another way to determine if the model adequately fits the test hole data. The null hypothesis is that the mean of the residual should be zero. T-test data should be normally distributed. Most analysis for normality does not make sense for a small dataset. The most practical tool is to use a quantile plot for normality even though the interpretation can be considered subjective. Figure 22 shows a residual quantile plot of the residuals. For most datasets, the residuals can be assumed to be normal since the intention of MLSR is to create a plane which minimizes sum squared error. Datasets
with an extreme outlier can cause non-normal distributions in the residual, but those would stand out in Table 7. Extreme outliers would have large residuals when compared to the other residual values and would be the only positive or negative number in the list of residuals. Li and Heap (2014) state that linear regression models must have normally distributed variance.

The test hole residual data appears normally distributed. The residuals seem a little sporadic off the line, but again such a small dataset cannot be overly scrutinized. The line comes sufficiently close to zero, which is desired when discussing the normal distribution of a residual dataset. Table 9 shows the results of the t-test. The Critical T value determined from Excel function, =T.INV.2T(0.05,5). The t-value was determined from $t = \frac{\bar{x} - \mu}{\text{std}/\sqrt{n}}$. Notice that the Critical t-value is larger than the t-value, which
means that the null hypothesis that the mean of the residuals is zero is confirmed. This means that the test hole measurement error is random. T-test should not fail when a plane is created using MLSR, unless an extreme outlier is present in the dataset.

Table 12: The results of the statistical analysis and t-test.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.12834</td>
</tr>
<tr>
<td><strong>Std. Dev</strong></td>
<td>0.88261159</td>
</tr>
<tr>
<td><strong>T-value</strong></td>
<td>0.325145248</td>
</tr>
<tr>
<td><strong>alpha</strong></td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Critical T</strong></td>
<td>2.570581836</td>
</tr>
</tbody>
</table>

Highwall Analysis and Optimization Model

*Methods*

The Highwall Analysis model uses three methods of establishing the area of minable coal. The first method is defined by law. OAC 1501:13-4-04 (C)(2)(b) “…At least one test hole shall be located on the highest elevation in the area of land to be mined….” This ensures that all stratigraphic contributors to the overburden are accounted for. This requirement of law can be waived by the chief through a test hole variance. The next 2 highwalls are placed for maximum economic recovery. Surface mining becomes uneconomical when the overburden to coal thickness is greater than 20 to 1 (ODNR, 2014). A more traditional method of placing the final highwall would be to take the average coal thickness, times by 20 to get a depth of overburden and then add to the elevation of the top of the coal to place the final highwall. A more optimized method uses GIS to get the true maximum economic recovery area. The optimized method uses
the 20 to 1 overburden to coal ratio, but does this using the overburden and coal thickness
rasters on a cell by cell basis. The con tool can make a Boolean decision of whether the
raster cell contains economically minable coal. This allows the mine to take full
advantage of where the coal seam is thicker. For a complete understanding of the tools,
uses, and reasons, please see Addendum 5 for the model diagram (5.1), Python code
(5.2), and model outputs (5.3).

Results

Table 10 shows the inputs and outputs of the highwall optimization model. Depth
of Cover is defined as a user defined input. For West Fork 3 a standard height of 100 feet
was chosen. This could have been any height or the recommended height of the highest
elevation on a test hole.
Table 13: Highwall optimization model inputs.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth Cover</td>
<td>A user defined numerical input of any value. Depth of cover is defined as the test hole with the maximum depth to the coal seam.</td>
</tr>
<tr>
<td>GDB</td>
<td>The workspace geodatabase for the project. This should contain the features and rasters obtained from the previous analysis.</td>
</tr>
<tr>
<td>Elevation</td>
<td>The raster digital elevation model of the surface topography created from OSIP LIDAR data</td>
</tr>
<tr>
<td>CoalTop</td>
<td>The raster digital elevation model of the top of coal</td>
</tr>
<tr>
<td>CoalThickness</td>
<td>The raster model of the coal thickness</td>
</tr>
<tr>
<td>Permit</td>
<td>A vector polygon feature that defines the area of interest for the surface coal permit.</td>
</tr>
</tbody>
</table>
Table 14: Highwall optimization model outputs.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden</td>
<td>A raster that has extents at the elevation above the coal seam. Each cell is given a value of depth from the surface down to the top of the coal.</td>
</tr>
<tr>
<td>Max Cover</td>
<td>A vector line feature that will be placed at the depth of overburden defined by the user in the Depth Cover Input.</td>
</tr>
<tr>
<td>MaxEconomicRecovery</td>
<td>A vector line feature that defines the maximum economic recovery of coal based on a 20 to 1 ratio of overburden to coal. The average coal seam thickness is calculated by taking the mean of the coal thickness raster.</td>
</tr>
<tr>
<td>CoalMined</td>
<td>A raster that defines the area suggested for mining based on the maximum economic recovery of coal of 20 to 1 overburden to coal seam thickness. Area suggested for mining is defined as 1 and not to be mined is 0.</td>
</tr>
<tr>
<td>CoalArea</td>
<td>A vector polygon feature that defines the area within the permit to be mined. The downslope should define the Coal Outcrop and the Upslope should define the final highwall.</td>
</tr>
</tbody>
</table>
Figure 23 shows the West Fork 3 permit with the 3 highwall options. The green and purple lines are constant in the overburden thickness. The contour tool can be used to simply place a line at a certain depth of overburden. The pink line however is done per raster cell by looking at the coal thickness in a cell multiplying the value by 20. Boolean logic is used against the overburden raster to determine if the coal should be mined at that location. All three final highwall lines are important to the mine engineer. This allows the engineer to make decisions in permitting and whether the additional coal is worth the additional paperwork of a test hole variance. In this case, it may be beneficial to mine the center peninsula between TH-02 and TH-04, since the coal is very thick through this area of the permit.
Acid Base Accounting Model

Methods

The Acid Base Accounting Model uses the outputs from the Surface elevation model calculate the potential acidity of the overburden. This model also helps to fulfill requirements of law. OAC 1501:13-4-04 (C)(3)(iv) “Analyses of the coal seam for acid-forming or toxic-forming materials...”. This analysis is important to determine if the extraction of coal poses unnecessary risk to the environment. Highly acidic overburden
is an indicator that the mine may generate AMD. ODNR provides an excel spreadsheet that is to be filled out to determine the Acid to Base ratio for the overburden of the Surface Coal mine. Figure 24 shows an example of the ODNR Acid Base accounting spreadsheet. The green columns are the data provided by the lab and drillers report. The yellow columns are calculation fields except for area. The main purpose of this model is to find the area of the lithologic unit to perform the acid base accounting calculations. The area is typically found by digitally hand drawing out the lithologic unit areas based on contour elevations. The process is very time consuming and the area boundaries for the lithologic unit is approximate.
Figure 24: ODNR acid base accounting spreadsheet that is filled out as part of the permitting process.
Results

The Acid Base Accounting model was created to fill out the ODNR excel spreadsheet. All the data required for the analysis and calculations is provided in the drillers report except for the area of the lithologic unit to be disturbed. This would be a very long drawn out tedious process if done by hand. This is where the model comes in handy. The ObjectID, TH_ID, Lab_ID, Lithology, AvgElev, PA, and NP were copied into a separate single spreadsheet containing all test holes. The file was named “lithology” and “Sheet1” was left as the tab name. A polygon feature class was also created, which defines the area of influence for each of the test holes. Voronoi polygons were used, since it defines the polygons through equal distances to the boundary between adjacent test holes. Table 11 shows the inputs and outputs of the Coal Analysis Model. For a complete understanding of the tools, uses, and reasons, please see Addendum 6 for the model diagram (6.1), Python code (6.2), model outputs (6.3), and acid base accounting spreadsheet (6.4). Addendum 6.4 has the required user inputs that lithology sheet 1 has before the model is ran and the red is the model outputs.
Table 15: Acid base accounting model’s inputs.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDB</td>
<td>The workspace geodatabase for the project. This should contain the features and rasters obtained from the previous analysis.</td>
</tr>
<tr>
<td>StrataData</td>
<td>A user defined spreadsheet in &quot;*.xls&quot; format. The spreadsheet headers should have ObjectID, TH_ID, Lab_ID, Lithology, AvgElev, PA, NP. The data needed for this should be copied from the Acid Base Accounting Spreadsheet after it is filled out using the Drill Report.</td>
</tr>
<tr>
<td>Elevation</td>
<td>The raster digital elevation model of the surface topography created from OSIP LIDAR data.</td>
</tr>
<tr>
<td>TH_Voronoi</td>
<td>A vector polygon feature that is required as input by the user. It gives the area of influence for each test hole within the boundary of the CoalArea Polygon. Using a Voronoi polygon analysis the areas will be based on halving the distance to the next adjacent test hole. The area can be user defined as well.</td>
</tr>
</tbody>
</table>
Table 16: Acid base accounting model’s outputs.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABA</td>
<td>A vector polygon feature that gives the area of each defined strata. The area is defined by using the mid elevation from the StratData to find the outcrop for that strata within the test hole area of influence defined by the TH_Voronoi. The Area is what is needed to complete the ODNR excel spreadsheet.</td>
</tr>
</tbody>
</table>

The model works by using Boolean logic to determine if the middle elevation of the lithologic unit is less than the Elevation raster. A polygon is created for each lithologic unit which creates an effect that looks like contours. Polygon feature classes have an area field that can be used to calculate the neutralization potential of the permit. One thing to notice about this method is that each area of the permit described by the test hole only goes to the elevation of the test hole. In other words, there is no accounting for the lithologic units that lay above the test hole elevation. This is a limitation of the methods using the ODNR spreadsheet. Figure 25 shows the areas for each lithologic unit (red) in the representative areas for each test hole (orange). Notice that the areas are not determined for elevations above the elevation of the test hole.
The acid base accounting used the labs values for neutralization potential and the potential acidity. Neutralization and potential acidity are measured in tons CaCO$_3$ per 1000 tons of material. The unit weight of the material is given a characteristic unit weight according to the drillers description. Table 12 shows the typical unit weights in tons per acre-foot for various Ohio lithologic units as given by ODNR PD (2009).
Table 17: Characteristic lithologic unit weights by material (ODNR, 2009).

<table>
<thead>
<tr>
<th>Material</th>
<th>tons/acre-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>3000</td>
</tr>
<tr>
<td>Acid Shale</td>
<td>3670</td>
</tr>
<tr>
<td>Alkaline Shale</td>
<td>3670</td>
</tr>
<tr>
<td>Siltstone</td>
<td>3500</td>
</tr>
<tr>
<td>Sandstone</td>
<td>3070</td>
</tr>
<tr>
<td>Limestone</td>
<td>3600</td>
</tr>
<tr>
<td>Coal</td>
<td>1800</td>
</tr>
<tr>
<td>Sandy Shale</td>
<td>3370</td>
</tr>
<tr>
<td>Spoil</td>
<td>3370</td>
</tr>
</tbody>
</table>

The volume is calculated as area in acres times the thickness in feet. The spoiled tonnage is a product of the unit weight and the volume of material. The potential acidity and neutralization potential are then multiplied to get the neutralization potential and potential acidity of the lithologic unit. The CaCO₃ deficiency is the difference between potential acidity and neutralization potential. Often the neutralization potential of a permit is expressed as a ratio. This is the ratio of the sum of neutralization potential over the sum of potential acidity for the permit. Table 13 shows the results of the acid base accounting for the West Fork 3 permit.

Table 18: Acid base accounting results for West Fork 3 permit.

<table>
<thead>
<tr>
<th></th>
<th>1000Tons NP</th>
<th>1000TonsPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Totals</td>
<td>5806094519</td>
<td>732352318.8</td>
</tr>
<tr>
<td>Deficiency</td>
<td>-5073742200</td>
<td></td>
</tr>
<tr>
<td>Ratio NP/PA</td>
<td>7.928007286</td>
<td></td>
</tr>
</tbody>
</table>
Mine subsidence is defined as the change in surface elevation due to overlying rock collapsing into a mine void. After reading a variety of papers about subsidence, it was determined that subsidence can happen for any number of reasons. The actual cause of subsidence is usually determined on a case by case basis. There is little evidence gained through the preliminary investigation of a mine that shows if and where subsidence will occur in a permit. Subsidence can be induced during mining as in the case with retreat mining (pillar robbing) and longwall mining. Room and pillar mining where the pillars are left intact will still undergo subsidence given sufficient time. If collapse of the mine cavity is inevitable given sufficient time regardless of actual cause, then the attention must be placed on keeping subsidence form translating to the surface. When overburden collapses into a mine it expands by 10% as the rock becomes disjointed. This expansion allows the cavity to be filled with unconsolidated material. Eventually, the unconsolidated material expands and provides sufficient support of the overlying strata. This indicates that subsidence is a product of the overburden thickness. In the R.E. Gray (1977) study, 354 cases of subsidence where examined for overburden, property damage, and area of surface impact. In his study, he performed a frequency of occurrence to overburden thickness, but never performed any real statistical analysis. Figure 26 shows Gray’s original frequency analysis of overburden thickness.
A statistical analysis was performed on the data provided. The data is obviously positively skewed from a normal distribution. Log and inverse were insufficient to normalize the data. A box-cox transform was used to transform the data (Equation 21). The Excel function “goal seek” was used to optimize lambda to a normal function. The lambda from the R.E. Gray (1977) dataset was determined to be -0.0129185 with correlation R value of 0.994936. Figure 27 shows the frequency distribution using the Box Cox transform of the overburden thickness. Addendum 7 provides a full detailed workup of the statistical analysis performed in Maple (2015).
Figure 27: Frequency distribution of the Box-Cox transform for overburden thickness.

Results

The outputs of the Existing Surface DEM and Coal Analysis models were used in the Subsidence model. Table 14 shows the Inputs required and the outputs for the subsidence model.
Table 19: Description of inputs needed for the subsidence model.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDB</td>
<td>The workspace geodatabase for the project. This should contain the features and rasters obtained from the previous analysis.</td>
</tr>
<tr>
<td>Elevation</td>
<td>The raster digital elevation model of the surface topography created from OSIP LIDAR data. The elevation raster must be in the GDB for the model to work</td>
</tr>
<tr>
<td>CoalTop</td>
<td>A raster that describes the elevation of the top of the coal. The elevation raster must be in the GDB for the model to work</td>
</tr>
<tr>
<td>Permit</td>
<td>The permit feature must be in the GDB file location for this model to work.</td>
</tr>
</tbody>
</table>
Table 20: Description of outputs given by the subsidence model.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overburden</td>
<td>A raster that has extents at the elevation above the coal seam. Each cell is given a value of depth from the surface down to the top of the coal.</td>
</tr>
<tr>
<td>BoxCox</td>
<td>A raster that is calculated using the Box Cox transform. The lambda was optimized by using excel goal seek and data from a 1977 study performed by R.E. Gray.</td>
</tr>
<tr>
<td>ZScore</td>
<td>A raster calculated by using the mean and standard deviation of the BoxCox transform and the BoxCox Raster cell values.</td>
</tr>
<tr>
<td>SubProb</td>
<td>A raster that shows the probability of subsidence based on a 3rd order polynomial approximation of the relationship between z-score and probability.</td>
</tr>
<tr>
<td>ProbContours</td>
<td>A vector shapefile that provides probability contours for the likelihood of subsidence.</td>
</tr>
</tbody>
</table>

Now that the dataset has been normalized the z-score and thus the probability can be determined for a provided overburden thickness. To put this into GIS, the raster calculator and the overburden raster was used to get the BoxCox Transform and then the z-score raster. Equations 21 and 22 show the equations used in the model.

\[
\text{BoxCox Transform} = \text{Overburden Thickness}^{\lambda} - \frac{1}{\lambda}
\]

(21)
\[
Z - \text{Score} = \frac{BCT_i - \mu_{BCT}}{\sigma_{BCT}}
\]

(22)

Where:

- \( BCT_i \) is the individual record for the Box Cox transform of overburden thickness
- \( \mu_{BCT} \) is the mean of the Box Cox transform for overburden thickness
- \( \sigma_{BCT} \) is the standard deviation of the Box Cox transform for overburden thickness

Mine subsidence is a unique problem to underground mining. For the subsidence model, the D-2401 underground mine permit was used in the example. Before running any of the models in this paper, the geodatabase, Test Hole point feature, and Permit polygon feature must be created. The Existing Surface elevation model and coal analysis outputs are required for the subsidence probability analysis. Figure 28 shows the z-score raster.
The subsidence model has the lambda, mean, and standard deviation as variables in the model. This allows the values to be changed out for a more localized dataset. The Grey (1977) dataset has subsidence records that are taken from northern West Virginia and southern Pennsylvania. If subsidence records are available for the Ohio county that is under analysis, then that dataset should be used and the values replaced. This follows the
geoscience axiom of things that are closer together are more alike than things that are farther apart.

ArcMap does not have the capability to perform complex mathematical operation such as integration. The Taylor series expansion assumption is that any complex equation can be sufficiently estimated using an nth degree polynomial. A list of was constructed relating probability to z-score. The relationship between z-score and probability can be found in a z-score table in most statistics textbooks. To get a simplified mathematical approximation for a relationship between the z-score and probability, a 3rd order polynomial regression was performed. Figure 29 shows the 3rd order polynomial regression performed in excel. The correlation coefficient is 0.99986, which shows a sufficient approximation of a binomial distribution with a 3rd order polynomial.
Figure 29: A simplified relationship between z-score and probability for use in a GIS model.

The 3rd order polynomial was used to show the probability of subsidence occurrence as a raster. Figure 30 shows the probability of subsidence occurring based on the Grey (1977) overburden dataset.
Figure 30: Subsidence probability raster, gradation form high (red) to low (blue).

The gradation from red to blue represent probability ranges from 67% to 5%. A mine engineer can look at the probability raster or probability contours to determine the risk versus reward of mining an area. If a residential home is built in an area that has 50% probability of subsidence then a mine engineer may decide it is not worth the risk to retrieve the coal under that property. Another likely scenario is the subsidence could cause interception of a stream. Analysis of the fluvial geomorphology for the permit would allow one to conclude that the high probability areas have a significant drainage
area. If subsidence occurs that intercepts a stream the mine could become flooded. The subsidence model allows the mine engineer to make informed decisions and weigh risk versus greater coal recovery.
CONCLUSION

It was shown that the coal seam can be established in ArcGIS using a multilinear regression approach. It is a native tool in ArcGIS called trend. This method proves to be a useful analysis method when the test hole data that is used has significant error. MLSR is neither better nor worse than IDW and kriging, but is a valuable interpolation method that is often overlooked as a valid option. As in all analytical methods, spatial interpolation requires the analyst to be knowledgeable of both the strength in the methods and short comings of the data.

Python can be used to apply useful information to attribute tables, such as the strike-dip of the coal seam and residuals. Statistical analysis of the model can be performed using the calculated residuals to affirm the adequacy of the regression model. Once the coal seam elevation model (MLSR) is established a variety of useful models can be constructed in ArcGIS and Python. This paper showed 3 tools that can be used in conjunction with the coal and DEM rasters. GIS tools allow for a consistent and repeatable process that can help in post mining analysis and future permitting work. The tools can be used to comply with Ohio law, while providing a more robust analysis of the permit area. In this paper, a few models showed the functionality of ArcGIS and Python and how they could be applied to coal mining in Ohio. These are a few of many possible applications that could be developed.
FUTURE WORK

The model known as Acid Base Accounting showed that the analysis is incomplete for elevations higher than the elevation of the test hole. The ODNR spreadsheet takes a test hole by test hole approach when performing the acid base accounting. This is probably sufficient given a reasonable margin of safety. Most strata that lays above the test holes are in the well weathered elevations of the overburden. Well weathered means that the strata are most likely very close to neutral. However, the Acid Base Accounting model could extrapolate a raster for the permit area at 5’ intervals where no data is available. A midpoint can be taken at each 5’ interval within the range of the drill data. A point would then be placed with a linear interpolated value at the test hole representing the neutralization potential at that test hole. A temporary raster could then be interpolated at each 5’ elevation interval. This would allow the entire permit to be analyzed up to the elevation of the highest cover. If only one test hole has data at that elevation the raster would be a constant value. All the rasters could then be summed together iteratively to get the total neutralization potential for the permit.
REFERENCES


Kennedy, Bruce A. (1990). Surface Mining (Second). SME.


APPENDIX: PROOFS, MAPS, RESULTS AND PYTHON CODE

1. Least Squared Regression Proof
   1.1. Linear
   1.2. Multilinear
2. OSIP Digital Elevation Model
   2.1. GIS Model Work Flow
   2.2. Python Script
   2.3. Map
3. Coal Analysis Model
   3.1. GIS Model Work Flow
   3.2. Python Script
   3.3. Map 1
   3.4. Map 2
   3.5. Attribute Table
4. Least Squared Regression Model
   4.1. Python Script
   4.2. Map
5. Highwall Analysis Model
   5.1. GIS Model Work Flow
   5.2. Python Script
   5.3. Map
6. Acid Base Accounting Model
   6.1. GIS Model Work Flow
   6.2. Python Script
   6.3. Map
   6.4. Attribute Table
7. Subsidence Probability Model
   7.1. Maple Statistics Grey (1977) dataset
   7.2. GIS Model Work Flow
   7.3. Python Script
   7.4. Map
APPENDIX 1

Least Square Regression Proof

1. Formulation of Least Squared Regression of Line proof used to determine LSR plane
2. Least Squared Regression of Plane to be used in analysis of coal seam
APPENDIX 1.1

\[ l := a \cdot x[i] + b \]

\[ S := \sum_{i=1}^{n} (l - y[i])^2 \]

\[ n \cdot b^2 + \sum_{i=1}^{n} \left( a^2 \cdot x_i^2 + 2 \cdot a \cdot b \cdot x_i - 2 \cdot a \cdot x_i \cdot y_i - 2 \cdot b \cdot y_i + y_i^2 \right) \]

(2)

\[ n \cdot b^2 + \sum_{i=1}^{n} \left( a^2 \cdot x_i^2 + 2 \cdot a \cdot b \cdot x_i \cdot \bar{x} - 2 \cdot a \cdot \bar{x} \cdot \bar{y} - 2 \cdot b \cdot \bar{y} + \bar{y}^2 \right) \]

(3)

\[ \text{differentiate w.r.t. } b \rightarrow \]

\[ 2 \cdot n \cdot b + n \cdot (2 \cdot a \cdot \bar{x} - 2 \cdot \bar{y}) \]

(4)

\[ \text{differentiate w.r.t. } a \rightarrow \]

\[ n \cdot (2 \cdot a \cdot \bar{x} + 2 \cdot b \cdot \bar{X} - 2 \cdot \bar{y}) \]

(5)

\[ 2 \cdot n \cdot b + n \cdot (2 \cdot a \cdot \bar{x} - 2 \cdot \bar{y}) = 0 \]

(6)

\[ \text{solve for } a \rightarrow \]

\[ \left[ a = \frac{-\bar{y} + b}{x} \right] \]

(7)

\[ n \cdot (2 \cdot a \cdot \bar{x} + 2 \cdot b \cdot \bar{X} - 2 \cdot \bar{y}) = 0 \]

(8)

\[ \text{solve for } b \rightarrow \]

\[ \left[ b = -a \frac{\bar{x} - \bar{y}}{x} \right] \]

(9)

\[ b := -a \frac{\bar{x} - \bar{y}}{x} \]

(10)

\[ a = \frac{-\bar{y} + b}{x} \]

(11)

\[ a = \frac{\bar{y} + a \cdot \bar{x} - \bar{y}}{x} \]

(12)
**APPENDIX 1.2**

```plaintext
restart
path := FileTools[AbsolutePath]("C:/Users/kurt/Desktop/StrikeDip/D2401.csv");
Mat := ImportMatrix(path):
with(Statistics):
infolevel[Statistics] := 1:
with(ArrayTools):
with(LinearAlgebra):

The ordinary least squares model derived from the hyperplane equation.
The sum of the error between the plane and measured values is minimized.

\[
Eq := \sum_{i=1}^{n} (q + m \cdot x[i] + p \cdot y[i] - z[i])^2
\]

\[n \cdot q^2 + \sum_{i=1}^{n} \left( m^2 \cdot x_i^2 + 2 \cdot m \cdot p \cdot x_i \cdot y_i + p^2 \cdot y_i^2 + 2 \cdot m \cdot q \cdot x_i + 2 \cdot m \cdot p \cdot y_i - 2 \cdot p \cdot q \cdot y_i - 2 \cdot q \cdot z_i + z_i^2 \right) \] (1)

Differentiating with respect to the coefficients \((a, b, c)\)
and setting equal to zero finds a local minimum for the OLE equation.

\[Eq1 := \frac{\partial}{\partial q} \; Eq \]
\[2 \cdot n \cdot q + \sum_{i=1}^{n} (2 \cdot m \cdot x_i + 2 \cdot p \cdot y_i - 2 \cdot z_i) \] (2)

\[Eq1 = 0 \]
\[2 \cdot n \cdot q + \sum_{i=1}^{n} (2 \cdot m \cdot x_i + 2 \cdot p \cdot y_i - 2 \cdot z_i) = 0 \] (3)

\[Eq2 := \frac{\partial}{\partial m} \; Eq \]
\[\sum_{i=1}^{n} (2 \cdot m \cdot x_i^2 + 2 \cdot m \cdot q \cdot x_i + 2 \cdot m \cdot q \cdot x_i - 2 \cdot x_i \cdot z_i) \] (4)

\[Eq2 = 0 \]
\[\sum_{i=1}^{n} (2 \cdot m \cdot x_i^2 + 2 \cdot p \cdot x_i \cdot y_i + 2 \cdot q \cdot x_i + 2 \cdot x_i \cdot z_i) = 0 \] (5)

\[Eq3 := \frac{\partial}{\partial p} \; Eq \]
\[\sum_{i=1}^{n} (2 \cdot m \cdot x_i \cdot y_i + 2 \cdot p \cdot y_i^2 + 2 \cdot q \cdot y_i - 2 \cdot y_i \cdot z_i) \]

\[Eq3 = 0 \]
\[\sum_{i=1}^{n} (2 \cdot m \cdot x_i \cdot y_i + 2 \cdot p \cdot y_i^2 + 2 \cdot q \cdot y_i - 2 \cdot y_i \cdot z_i) = 0 \] (7)
```
APPENDIX 1.2

The columns from the csv file are defined as single column vectors

\[ X := \text{Mat} \{1 \ldots 1\} \]
\[ Y := \text{Mat} \{1 \ldots 2\} \]
\[ Z := \text{Mat} \{1 \ldots 3\} \]
\[ n := \text{Count}(X) \]

11

The Summation of the ith term is equal to the Mean \( \hat{n} \)

\[ A := 2 \cdot n \cdot q + 2 \cdot m \cdot \text{Mean}(X) + 2 \cdot p \cdot \text{Mean}(Y) - 2 \cdot n \cdot \text{Mean}(Z) = 0 \]

\[ 22 \cdot q + 5.0965684 \cdot 10^7 \cdot m + 5.458106 \cdot 10^6 \cdot p - 19419.401832000 = 0 \]  
(8)

\[ B := 2 \cdot n \cdot q \cdot \text{Mean}(Y) + 2 \cdot m \cdot n \cdot \text{Mean}(X \sim Y) + 2 \cdot p \cdot n \cdot \text{Mean}(Y \sim Y) - 2 \cdot n \cdot \text{Mean}(Y \sim Z) = 0 \]

\[ 5.458106 \cdot 10^6 \cdot q + 1.26443559434900 \cdot 10^7 \cdot m + 1.354192767030 \cdot 10^{12} \cdot p - 4.81798990917851 \cdot 10^9 = 0 \]  
(9)

\[ C := 2 \cdot n \cdot q \cdot \text{Mean}(X) + 2 \cdot m \cdot n \cdot \text{Mean}(X \sim X) + 2 \cdot p \cdot n \cdot \text{Mean}(X \sim Y) - 2 \cdot n \cdot \text{Mean}(X \sim Z) = 0 \]

\[ 5.0965684 \cdot 10^7 \cdot q + 1.18068315113920 \cdot 10^{14} \cdot m + 1.26443559434900 \cdot 10^{13} \cdot p - 4.49871147693838 \cdot 10^{10} = 0 \]  
(10)

Solving \( a, b, c \) for the linear set of equations \( A, B, C \) defined in equation (9) - (11)

\[ \text{Eq4} := \text{solve} \{A, B, C, \{q, m, p\}\} \]
\[ \{m = -0.003122672723, p = 0.001338109332, q = 7784.773205\} \]

assign(Eq4)

An equivalent process to the one described above in Matrix format is \( \hat{A} \cdot A \cdot B = \hat{A} \cdot Z \)

\[ m_1 := \text{Matrix}\{1 \ldots n, 1, 1\} \]
\[ A := \{\{m_1\}\}\{X\}\{Y\} \]

11 x 3 Matrix
Data Type: anything
Storage: rectangular
Order: Fortran_order

\[ B := \text{MatrixInverse}(\text{Transpose}(A) \cdot A) \cdot \text{Transpose}(A) \cdot Z \]

\[ \begin{bmatrix} 7784.77320399033 \\ -0.0031226727290703 \\ 0.00133810933214731 \end{bmatrix} \]

\[ Z_1 := A \cdot B \]
\[ r := Z_1 \sim Z \]

1 .. 11 Vector_column
Data Type: float
Storage: rectangular
Order: Fortran_order

show all
APPENDIX 1.2

\[
\begin{array}{c}
1.23997391193916 \\
2.82644036112572 \\
-3.00606951346253 \\
-4.76706432592717 \\
-0.326287991716868 \\
1.2845826906705 \\
-2.48665032636393 \\
7.77289245878217 \\
-4.70437970965583 \\
-3.16071355230360 \\
5.32727495062375 \\
\end{array}
\]

Calculations (17) through (30) prove the relationship between coefficients a,b,c in Equation (20) and coefficients m,p,q used in the least squares Equation (1)

\[
a \cdot (x - xo) + b \cdot (y - yo) + c \cdot (z - zo) = 0
\]

\[a \ (x - xo) + b \ (y - yo) + c \ (z - zo) = 0 \tag{17}\]

manipulate equation

\[a \ (x - xo) + b \ (y - yo) + c \ (z - zo) = 0 \]
\[a \ x - a \ xo + b \ y - b \ yo + c \ z - c \ zo = 0 \]
\[a \ x + b \ y + c \ z = a \ xo + b \ yo + c \ zo \]
\[\frac{a \ x + b \ y + c \ z}{c} = \frac{a \ xo + b \ yo + c \ zo}{c} \tag{18}\]

manipulate equation

\[z = \frac{a \ xo + b \ yo + c \ zo}{c} - \frac{a \ x}{c} - \frac{b \ y}{c} \tag{19}\]

A value of unity was chosen since the choice for the value "c" is arbitrary

\[c := 1 \tag{20}\]

\[m = -\frac{a}{c} \tag{21}\]
\[a := -m \cdot c \tag{22}\]
\[p = -\frac{b}{c} \tag{23}\]
\[b := -p \cdot c \tag{24}\]
APPENDIX 1.2

-0.001338109332 \tag{25}

xo, yo, and zo represent any point in the plane. The mean values for each coordinate were used, since the mean values can be shown to exist on a linear regression plane.

\[ xo := \text{Mean}(X) \]
\[ yo := \text{Mean}(Y) \]
\[ zo := \text{Mean}(Z) \]

\[ q = \frac{a \cdot xo + b \cdot yo + c \cdot zo}{c} \]
\[ 7784.773205 = 7784.77320428147 \tag{29} \]

\[ c := \frac{(a \cdot xo + b \cdot yo + c \cdot zo)}{q} \]
\[ 0.999999999907701 \tag{30} \]

Substituting back into the original equation, an elevation value Z can be found on the regression plane at any coordinate (X,Y).

\[ z = q + m \cdot x + p \cdot y \]
\[ z = -0.003122672723 \times x + 0.001338109332 \times y + 7784.773205 \tag{31} \]
APPENDIX 2

OSIP Digital Elevation Model

1. GIS Model Work Flow
2. Python Script
3. Map
APPENDIX 2.1

DEM_Existing
APPENDIX 2.2

Existing_DEM

# -*- coding: utf-8 -*-
# Existing_DEM.py
# Created on: 2017-10-08 16:18:04.00000
# (generated by ArcGIS/ModelBuilder)
# Usage: Existing_DEM <Input_Files> <LAS_Dataset> <GDB> <CS>
# Description:
#
# Import arcpy module
import arcpy

# Script arguments
Input_Files = arcpy.GetParameterAsText(0)
if Input_Files == '#' or not Input_Files:
    Input_Files = r"D:\ArcGIS\LLCWaterline\LiDAR\s2075355.las"  # provide a default value if unspecified

LAS_Dataset = arcpy.GetParameterAsText(1)
if LAS_Dataset == '#' or not LAS_Dataset:
    LAS_Dataset = r"D:\ArcGIS\LLCWaterline\LiDAR\LLCWaterline.lasd"  # provide a default value if unspecified

GDB = arcpy.GetParameterAsText(2)
if GDB == '#' or not GDB:
    GDB = r"D:\ArcGIS\LLCWaterline\LLCWaterline.gdb"  # provide a default value if unspecified

CS = arcpy.GetParameterAsText(3)
if CS == '#' or not CS:
    CS = "PROJCS["NAD_1983_StatePlane_Ohio_South_FIPS_3402_Feet",GEOGCS["GCS_North_American_1983",DATUM["D_North_American_1983",SPHEROID["GRS_1980",6378137.0,298.25722101]],PRIMEM["Greenwich",0.0],UNIT["Degree",0.0174532925199433]],PROJECTION["Lambert_Conformal_Conic"],PARAMETER["False_Easting",1968500.0],PARAMETER["False_Northing",0.0],PARAMETER["Central_Meridian",-82.5],PARAMETER["Standard_Parallel_1",38.73333333333333],PARAMETER["Standard_Parallel_2",40.03333333333333],PARAMETER["Latitude_Of_Origin",38.0],UNIT["Foot_US",0.3048006096012192]]"  # provide a default value if unspecified

# Local variables:
LasDatasetLayer = "LasDatasetLayer"
Elevation = r"D:\ArcGIS\LLCWaterline\LLCWaterline.gdb\Elevation"
Curv = "%GDB%\Curv"
ProfileCurv = "%GDB%\ProfileCurv"
PlanCurv = "%GDB%\PlanCurv"
HillShade_2 = "%GDB%\HillShade"
SlopePct = "%GDB%\SlopePct"
APPENDIX 2.2

Existing_DEM

Aspect_2_ = "%GDB\Aspect"
Permit = "%GDB\Permit"
Hydro = "%GDB\Hydro"
Elevation_Clip = "C:\\Users\\kurt\\Documents\\ArcGIS\\Default.gdb\\Elevation_Clip"
ElevSmooth = "%GDB\ElevSmooth"
Contours = "%GDB\Contours"

Default_gdb = GDB
Contour_Labels = "ContourAnno1"
MaxElevations = "%GDB\MaxElevations"
rastercalc = "D:\\ArcGIS\\LLCWaterline\\LLCWaterline.gdb\\rastercalc"
MaxElev = "%GDB\MaxElev"
SlopeDeg = "%GDB\SlopeDeg"

# Set Geoprocessing environments
arcpy.env.outputCoordinateSystem = ""
arcpy.env.geographicTransformations = ""

# Process: Create LAS Dataset
tempEnvironment0 = arcpy.env.outputCoordinateSystem
arcpy.env.outputCoordinateSystem = CS
arcpy.CreateLasDataset_management(Input_Files, LAS_Dataset, "NO_RECURSION", "", CS, "COMPUTE_STATS", "RELATIVE_PATHS")
arcpy.env.outputCoordinateSystem = tempEnvironment0

# Process: Make LAS Dataset Layer
arcpy.MakeLasDatasetLayer_management(LAS_Dataset, LasDatasetLayer, "2", "", "true", "true", "false", "")

# Process: LAS Dataset to Raster
arcpy.LasDatasetToRaster_conversion(LasDatasetLayer, Elevation, "ELEVATION", "", "FLOAT", "CELLSIZE", "2.5", "1")

# Process: Curvature
arcpy.gp.Curvature_sa(Elevation, Curv, "1", ProfileCurv, PlanCurv)

# Process: Hillshade
arcpy.gp.HillShade_sa(Elevation, HillShade_2_, "315", "45", "NO_SHADOWS", "1")

# Process: Slope
arcpy.gp.Slope_sa(Elevation, SlopePct, "PERCENT_RISE", "1")

# Process: Aspect
arcpy.gp.Aspect_sa(Elevation, Aspect_2_)

# Process: Buffer
arcpy.Buffer_analysis(Permit, Hydro, "1000 Feet", "FULL", "ROUND", "NONE", "", "PLANAR")
APPENDIX 2.2

Existing_DEM

# Process: Clip
arcpy.Clip_management(Elevation, "2038449.06098793 341447.75607422 2082198.15848678 386477.155377924", Elevation_Clip, Hydro, "-3.402823e+038", "ClippingGeometry", "MAINTAIN_EXTENT")

# Process: Focal Statistics (2)
arcpy.gp.FocalStatistics_sa(Elevation_Clip, ElevSmooth, "Rectangle 5 5 CELL", "MEAN", "DATA")

# Process: Contour with Barriers
arcpy.gp.ContourWithBarriers_sa(ElevSmooth, Contours, "", "POLYLINES", "", "NO_EXPCLICT_VALUES_ONLY", "0", "10", "50", "", "1")

# Process: Contour Annotation
arcpy.ContourAnnotation_cartography(Contours, GDB, "Contour", "4000", Contour_Labels, "BLACK", "Type", "UPHILL", "ENABLE_LADDERING")

# Process: Focal Statistics
arcpy.gp.FocalStatistics_sa(Elevation_Clip, MaxElevations, "Rectangle 1000 1000 MAP", "MAXIMUM", "DATA")

# Process: Raster Calculator
arcpy.gp.RasterCalculator_sa("Con("%Elevation_Clip%" == "%MaxElevations%","%Elevation_Clip%")", rastercalc)

# Process: Raster to Point
arcpy.RasterToPoint_conversion(rastercalc, MaxElev, "Value")

# Process: Slope (2)
arcpy.gp.Slope_sa(Elevation, SlopeDeg, "DEGREE", "1")
APPENDIX 3

Coal Analysis Model

1. GIS Model Work Flow
2. Python Script
3. Map 1
4. Map 2
5. Attribute Table
APPENDIX 3.2

CoalAnalysis.py

Created on: 2017-09-18 19:48:31.00000
Usage: CoalAnalysis <GDB>
Description:

# Import arcpy module
import arcpy

# Script arguments
GDB = arcpy.GetParameterAsText(0)
if GDB == '#' or not GDB:
    GDB = "D:\ArcGIS\Thesis\D2401N\D2401_Rast.gdb"  # provide a default value if unspecified

# Local variables:
TestHole = "%GDB%\TestHole"
TestHole__2_ = TestHole
TestHole__5_ = TestHole__2_
Elevation = "%GDB%\Elevation"
TestHole__4_ = TestHole__5_
RMS_TXT = ""
CoalMid = "%GDB%\CoalMid"
Isopach = "%GDB%\Isopach"
CoalBot = "%GDB%\CoalBot"
Binder = "%GDB%\Binder"
CoalThickness = "%GDB%\CoalThickness"
CoalTop = "%GDB%\CoalTop"
Overburden = "%GDB%\Overburden"
CoalOutcrop = "%GDB%\CoalOutcrop"

# Process: Add Field
arcpy.AddField_management(TestHole, "ElevMiddleCoal", "FLOAT", "", "", "", "", "NON_NULLABLE", "NON_REQUIRED", "")

# Process: Add Surface Information
arcpy.AddSurfaceInformation_3d(TestHole__2_, Elevation, "Z", "BILINEAR", "", "1", "0", "NO_FILTER")

# Process: Calculate Field
arcpy.CalculateField_management(TestHole__5_, "ElevMiddleCoal", "[Z] - [Depth]+[Thickness]/2", "VB", "")

# Process: Trend
tempEnvironment0 = arcpy.env.extent
APPENDIX 3.2
CoalAnalysis

arcpy.env.extent = Elevation
tempEnvironment1 = arcpy.env.cellSize
arcpy.env.cellSize = Elevation
arcpy.gp.Trend_sa(TestHole__4_, "ElevMiddleCoal", CoalMid, "2.5", "1", "LINEAR", RMS_TXT)
arcpy.env.extent = tempEnvironment0
arcpy.env.cellSize = tempEnvironment1

# Process: IDW (2)
tempEnvironment0 = arcpy.env.extent
arcpy.env.extent = Elevation
arcpy.gp.Idw_sa(TestHole__4_, "Thickness", Isopach, "2.5", "3.5", "VARIABLE 12", ")
arcpy.env.extent = tempEnvironment0

# Process: Raster Calculator (2)
arcpy.gp.RasterCalculator_sa("\"%CoalMid\%-\"%Isopach\%/2", CoalBot)

# Process: IDW
tempEnvironment0 = arcpy.env.extent
arcpy.env.extent = Elevation
arcpy.gp.Idw_sa(TestHole__4_, "Binder", Binder, "2.5", "3.5", "VARIABLE 12", ")
arcpy.env.extent = tempEnvironment0

# Process: Raster Calculator (3)
arcpy.gp.RasterCalculator_sa("\"%Isopach\%-\"%Binder\", CoalThickness)

# Process: Raster Calculator
arcpy.gp.RasterCalculator_sa("\"%CoalMid\%+\"%Isopach\%/2", CoalTop)

# Process: Raster Calculator (4)
arcpy.gp.RasterCalculator_sa("\"%Elevation\%-\"%CoalTop\", Overburden)

# Process: Contour List
arcpy.ContourList_3d(Overburden, CoalOutcrop, "0")
## APPENDIX 3.5

<table>
<thead>
<tr>
<th>TH ID</th>
<th>Thickness</th>
<th>Binder</th>
<th>Depth</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Model/MidElev</th>
<th>Elev/MiddleCoal</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH-01</td>
<td>7.75</td>
<td>1.25</td>
<td>133.75</td>
<td>226863.3</td>
<td>599791.971</td>
<td>1025.007406</td>
<td>915.9442</td>
<td>915.1324</td>
<td>0.8118</td>
</tr>
<tr>
<td>TH-02</td>
<td>9.5</td>
<td>2</td>
<td>123.5</td>
<td>2267691.02</td>
<td>601400.835</td>
<td>1045.169507</td>
<td>925.8263</td>
<td>926.4195</td>
<td>0.5932</td>
</tr>
<tr>
<td>TH-03</td>
<td>12</td>
<td>2</td>
<td>126</td>
<td>2268469.263</td>
<td>601872.425</td>
<td>1042.262756</td>
<td>921.7814</td>
<td>922.2628</td>
<td>0.4814</td>
</tr>
<tr>
<td>TH-04</td>
<td>9.5</td>
<td>2</td>
<td>138</td>
<td>2269100.521</td>
<td>602798.651</td>
<td>1054.446722</td>
<td>920.0367</td>
<td>921.1967</td>
<td>1.16</td>
</tr>
<tr>
<td>TH-05</td>
<td>9</td>
<td>1.5</td>
<td>68</td>
<td>2268714.672</td>
<td>604937.215</td>
<td>991.464278</td>
<td>928.7454</td>
<td>927.9643</td>
<td>-0.7811</td>
</tr>
</tbody>
</table>
Least Squared Regression Model

1. Python Script
2. Map 1
# Define lists and dictionaries
X=[]
Y=[]
Z=[]
M=[]

# Import modules
import numpy as np
import numpy.linalg as la
import csv
import math

# Define the file location
CSV = input("Test Hole file path '*.csv' [X][Y][ElevMiddleCoal]")
GDB = input("geodatabase '*.gdb' ")
with open (CSV) as ifile:
    reader = csv.DictReader(ifile)
    for row in reader:
        X.append(float(row['X']))
        Y.append(float(row['Y']))
        Z.append(float(row['ElevMiddleCoal']))
        M.append(float(1.0))
Xm=np.mean(X)
Ym=np.mean(Y)
Zm=np.mean(Z)
A=np.matrix([[M,X,Y]])
B=np.matrix.transpose(A)*A
Z=np.matrix.transpose(np.matrix(Z))
B=B.getI()*np.matrix.transpose(A)*Z
q=B.item(0)
m=B.item(1)
p=B.item(2)
c=1
a=-m*c
b=-p*c
norm=np.array([a,b,c])
atan=math.fabs(math.atan(norm[1]/norm[0]))
if atan > math.pi/2:
    atan=math.pi
Dip=(math.acos(norm[2]/la.norm(norm)))*180/math.pi
    if norm[0]>0 and norm[1]>0:  # NE
        Azdip=(math.pi/2-atan)
        DD= ("North East\"")
        AzStrike=(Azdip+math.pi/2)*180/math.pi
        SD=("South East\"")
eelif norm[0]>0 and norm[1]<0:  # SE

APPENDIX 4.1
LSRegression
APPENDIX 4.1
LSRegression

Azdip=(math.pi/2+atan)
DD= ("South East")
AzStrike=(Azdip=math.pi/2)*180/math.pi
SD= ("North East")
eelif norm[0]<0 and norm[1]>0:  #SW
  Azdip=(3*math.pi/2-atan)
  DD= ("South West")
  AzStrike=(Azdip=math.pi/2)*180/math.pi
  SD=("South East")
eelse:  #NW
  Azdip=(3*math.pi/2+atan)
  DD= ("North West")
  AzStrike=(atan)*180/math.pi
  SD= ("North East")

# Import arcpy module
import arcpy

# Script arguments
CoalOutcrop = GDB + "\CoalOutcrop"
TestHole = GDB + "\TestHole"

# Process: Add Field
arcpy.AddField_management(CoalOutcrop, "StrikeDirection", "TEXT", "", "10", "NULLABLE", "NON_REQUIRED", "")

# Process: Add Field (2)
arcpy.AddField_management(CoalOutcrop, "DipDirection", "TEXT", "", "10", "NULLABLE", "NON_REQUIRED", "")

# Process: Add Field (3)
arcpy.AddField_management(CoalOutcrop, "Strike", "Float", "", "", "", "NULLABLE", "NON_REQUIRED", "")

# Process: Add Field (4)
arcpy.AddField_management(CoalOutcrop, "Dip", "Float", "", "", "", "NULLABLE", "NON_REQUIRED", "")

# Process: Add Field (5)
arcpy.AddField_management(TestHole, "Residual", "Float", "", "", "", "NULLABLE", "NON_REQUIRED", "")

# Process: Add Field (6)
arcpy.AddField_management(TestHole, "ModelMidElev", "Float", "", "", "", "NULLABLE", "NON_REQUIRED", "")

# Process: Calculate Field
APPENDIX 4.1
LSRegression

arcpy.CalculateField_management(CoalOutcrop, "StrikeDirection", SD, "PYTHON", "")
# Process: Calculate Field (2)
arcpy.CalculateField_management(CoalOutcrop, "DipDirection", DD, "PYTHON", "")
# Process: Calculate Field (3)
arcpy.CalculateField_management(CoalOutcrop, "Strike", AzStrike, "PYTHON", "")
# Process: Calculate Field (4)
arcpy.CalculateField_management(CoalOutcrop, "Dip", Dip, "PYTHON", "")
# Process: Calculate Field (5)
arcpy.CalculateField_management(TestHole, "ModelMidElev", '((-a*(!X!-Xm)-b*(!Y!-Ym))/c+Zm)', "PYTHON", "")
# Process: Calculate Field (6)
arcpy.CalculateField_management(TestHole, "Residual", '(!ElevMiddleCoal!-!ModelMidElev!)', "PYTHON", ",")
West Fork 3
Least Squared Regression
Highwall Analysis Model

1. GIS Model Work Flow
2. Python Script
3. Map
APPENDIX 5.1

Diagram showing a workflow for land use analysis with nodes such as Raster Calculator, DepthCover, Overburden, and Contour List.
APPENDIX 5.2

HighwallAnalysis

# -*- coding: utf-8 -*-
# HighwallAnalysis.py
# Created on: 2017-10-08 16:26:18.00000
# (generated by ArcGIS/ModelBuilder)
# Usage: HighwallAnalysis <GDB> <DepthCover>
# Description:
#--------------------------------------------------------
#
# # Import arcpy module
import arcpy

# Script arguments
GDB = arcpy.GetParameterAsText(0)
if GDB == '#' or not GDB:
    GDB = "D:\ArcGIS\WestFork3\WF3_Raster.gdb" # provide a default value if unspecified

DepthCover = arcpy.GetParameterAsText(1)
if DepthCover == '#' or not DepthCover:
    DepthCover = "100" # provide a default value if unspecified

# Local variables:
Elevation = "%GDB%\Elevation"
CoalTop = "%GDB%\CoalTop"
Overburden = "%GDB%\Overburden"
CoalThickness = "%GDB%\CoalThickness"
Permit = "%GDB%\Permit"
CoalAreaRas = "%GDB%\CoalAreaRas"
CoalArea = "%GDB%\CoalArea"
Property = "7.74234796076451"
output_value = "154.84695921529"
MaxEconomicRecovery = "%GDB%\MaxEconomicRecovery"
MaxCover = "%GDB%\MaxCover"
CoalBot = "%GDB%\CoalBot"
RasterCalc4 = "C:\Users\kurt\Documents\ArcGIS\Default.gdb\rastercalc4"
PitfloorRas = "%GDB%\PitfloorRas"
Pitfloor = "%GDB%\Pitfloor"

# Set Geoprocessing environments
arcpy.env.outputCoordinateSystem = ""
arcpy.env.snapRaster = Elevation
arcpy.env.extent = Elevation
arcpy.env.cellSize = Elevation
arcpy.env.geographicTransformations = ""
arcpy.env.mask = Permit

# Process: Raster Calculator (4)
APPENDIX 5.2

HighwallAnalysis
arcpy.gp.RasterCalculator_sa("Con("%Elevation%" > 
"%CoalTop%", "%Elevation%" - "%CoalTop%"), Overburden)

# Process: Raster Calculator (5)
tempEnvironment0 = arcpy.env.extent
arcpy.env.extent = Permit
tempEnvironment1 = arcpy.env.mask
arcpy.env.mask = Permit
arcpy.gp.RasterCalculator_sa("Con("%Overburden%"<"%CoalThickness%"*20,1)
CoalAreaRas)
arcpy.env.extent = tempEnvironment0
arcpy.env.mask = tempEnvironment1

# Process: Raster to Polygon
arcpy.RasterToPolygon_conversion(CoalAreaRas, CoalArea, "SIMPLIFY", "Value")

# Process: Get Raster Properties
arcpy.GetRasterProperties_management(CoalThickness, "MEAN", "")

# Process: Calculate Value
arcpy.CalculateValue_management("%Property%*20", "", "Double")

# Process: Contour List (2)
arcpy.ContourList_3d(Overburden, MaxEconomicRecovery, "154.84695921529")

# Process: Contour List (3)
arcpy.ContourList_3d(Overburden, MaxCover, "100")

# Process: Raster Calculator
arcpy.gp.RasterCalculator_sa("Con("%Elevation%" > 
"%CoalBot%", "%Elevation%" - "%CoalBot%"), rastercalc4)

# Process: Raster Calculator (2)
arcpy.gp.RasterCalculator_sa("Con("%rastercalc4%">0,1)", PitfloorRas)

# Process: Raster to Polygon (2)
arcpy.RasterToPolygon_conversion(PitfloorRas, Pitfloor, "SIMPLIFY", "Value")
Acid Base Accounting Model

1. GIS Model Work Flow
2. Python Script
3. Map
4. Attribute Table
APPENDIX 6.2

AcidBaseAccounting

# -*- coding: utf-8 -*-
#--------------------------------------------------------
# AcidBaseAccounting.py
# Created on: 2017-10-08 14:46:01.00000
# (generated by ArcGIS/ModelBuilder)
# Usage: AcidBaseAccounting <Elevation> <GDB> <TH_Voronoi> <Strata_Data>
# Description:
#--------------------------------------------------------

# Import arcpy module
import arcpy

# Load required toolboxes
arcpy.ImportToolbox("Model Functions")

# Script arguments
Elevation = arcpy.GetParameterAsText(0)
if Elevation == '#' or not Elevation:
    Elevation = "D:\ArcGIS\Thesis\D2218-2.gdb\Elevation" # provide a default value if unspecified

GDB = arcpy.GetParameterAsText(1)
if GDB == '#' or not GDB:
    GDB = "D:\ArcGIS\Thesis\D2218-2.gdb" # provide a default value if unspecified

TH_Voronoi = arcpy.GetParameterAsText(2)
if TH_Voronoi == '#' or not TH_Voronoi:
    TH_Voronoi = "D:\ArcGIS\Thesis\D2218-2.gdb\TH_Voronoi" # provide a default value if unspecified

Strata_Data = arcpy.GetParameterAsText(3)
if Strata_Data == '#' or not Strata_Data:
    Strata_Data = "D:\ArcGIS\Thesis\WF3_DBF\Lithology.xlsx" # provide a default value if unspecified

# Local variables:
StrataTable = "%GDB%\test_ExcelToTable1"
SingleRow = StrataTable
ABA_2 = StrataTable
RowNumber = "2"
TH = SingleRow
Elev = SingleRow
THVoronoi_Layer = "TH_Voronoi_Layer"
InfluenceAreaTH = "in_memory\Elevation_Clip"
RasterElev = "in_memory\rastercalc1"
PolygonArea = "in_memory\RasterT_Majorit1"
PolygonCombined = "in_memory\Polygon%RowNumber%"
APPENDIX 6.2

AcidBaseAccounting

Output_Values = PolygonCombined
ABA = "%GDB%\ABA"

# Process: Excel To Table
arcpy.ExcelToTable_conversion(Strata_Data, StrataTable, "Sheet1")

# Process: Iterate Row Selection
arcpy.IterateRowSelection_mb(StrataTable, "ObjectID ", "false")

# Process: Get Field Value (2)
arcpy.GetFieldValue_mb(SingleRow, "TH_ID", "String", "")

# Process: Make Feature Layer
arcpy.MakeFeatureLayer_management(TH_Voronoi, THVoronoi_Layer, ""TH_ID"" = "%TH%", ",", "OBJECTID OBJECTID VISIBLE NONE;Shape Shape VISIBLE NONE;TH_ID TH_ID VISIBLE NONE;X X VISIBLE NONE;Y Y VISIBLE NONE;Thickness Thickness VISIBLE NONE;Binder Binder VISIBLE NONE;Depth Depth VISIBLE NONE;ElevMiddle ElevMiddle VISIBLE NONE;Residual Residual VISIBLE NONE;SurfaceEle SurfaceEle VISIBLE NONE;Z Z VISIBLE NONE;Shape_Length Shape_Length VISIBLE NONE;Shape_Area Shape_Area VISIBLE NONE")

# Process: Clip
arcpy.Clip_management(Elevation, "2267354.14768214 598906.504807889 2270092.40992212 605444.275524978", InfluenceAreaTH, THVoronoi_Layer, "0", "ClippingGeometry", "MAINTAIN_EXTENT")

# Process: Get Field Value (3)
arcpy.GetFieldValue_mb(SingleRow, "AvgElev", "Double", "0")

# Process: Raster Calculator
arcpy.gp.RasterCalculator_sa("Con("%InfluenceAreaTH%" > %Elev%,1)", RasterElev)

# Process: Raster to Polygon
arcpy.RasterToPolygon_conversion(RasterElev, PolygonArea, "SIMPLIFY", "Value")

# Process: Dissolve
arcpy.Dissolve_management(PolygonArea, PolygonCombined, ",", ",", "MULTI_PART", "DISSOLVE_LINES")

# Process: Collect Values
arcpy.CollectValues_mb("in_memory\Polygon%RowNumber%")

# Process: Merge
arcpy.Merge_management(Output_Values, ABA, "]")

# Process: Join Field
arcpy.JoinField_management(ABA, "OBJECTID", StrataTable, "OBJECTID", ""OBJECTID_1;TH_ID;Lab_ID;Lithology;AvgElev")
West Fork 3
Acid Base Accounting

Project ID: D-2218

West Fork 3
Acid Base Accounting

Project ID: D-2218
Subsidence Probability Analysis Model

2. GIS Model Work Flow
3. Python Script
4. Map
APPENDIX 7.1

restart

path := FileTools[AbsolutePath]("C:/Users/kurt/Desktop/Thesis/12-Subsidence/Overburden.csv");
OB := ImportMatrix(path):
infolevel[Statistics] := 1:
with(ArrayTools):
OB1 := OB[1 .. 1, 1]:
Qm := Mean(OB1) = 69.4868421052632
σ := StandardDeviation(OB1) = 79.3168230353151
s := Skewness(OB1) = 2.40537679680105
Histogram(OB1)

ChiSquareSuitableModelTest(OB1, Normal(Qm, σ), bins = 10)

Chi-Square Test for Suitable Probability Model
---------------------------------------------
Null Hypothesis:
Sample was drawn from specified probability distribution
APPENDIX 7.1

Alt. Hypothesis:
Sample was not drawn from specified probability distribution
Bins: 10
Degrees of freedom: 9
Distribution: ChiSquare(9)
Computed statistic: 245.421
Computed pvalue: 0
Critical value: 16.918974487099
Result: [Rejected]
This statistical test provides evidence that the null hypothesis is false

\[ \text{hypothesis} = \text{false, critical value} = 16.9189774487099, \text{distribution} = \text{ChiSquare(9), pvalue} = 0, \text{statistic} = 245.4210526 \] (1)

\[ \text{OB2 := OB[1...2];} \]
\[ \text{Qm2 := Mean(OB2) = 78.3609531578070} \]
\[ \text{σ2 := StandardDeviation(OB2) = 0.0118630862656719} \]
\[ \text{s2 := Skewness(OB2) = -0.166373584971004} \]

Histogram(OB2)
Histogram Type: default
Data Range: 78.33290128 .. 78.3942651
Bin Width: .00204546066666751
Number of Bins: 30
Frequency Scale: relative
ChiSquareSuitableModelTest(\( OB2, Normal(Qm2, \alpha^2) \), bins = 10)

Chi-Square Test for Suitable Probability Model

Null Hypothesis:
Sample was drawn from specified probability distribution

Alt. Hypothesis:
Sample was not drawn from specified probability distribution

Bins: 10
Degrees of freedom: 9
Distribution: ChiSquare(9)
Computed statistic: 12.2632
Computed pvalue: 0.198877
Critical value: 16.9189774487099
Result: [Accepted]

This statistical test does not provide enough evidence to conclude that the null hypothesis is false

\[ hypothesis = \text{true}, \text{criticalvalue} = 16.9189774487099, \text{distribution} = \text{ChiSquare}(9), \text{pvalue} = 0.198877008807322, \text{statistic} = 12.26315790 \]
APPENDIX 7.3

Subsidence.py

Created on: 2017-10-08 16:46:24.00000
(generated by ArcGIS/ModelBuilder)

Usage: Subsidence <GDB>

Description:

Import arcpy module
import arcpy

# Script arguments
GDB = arcpy.GetParameterAsText(0)
if GDB == '#' or not GDB:
    GDB = "D:\ArcGIS\Thesis\D2401N\D2401_Rast.gdb" # provide a default value if unspecified

# Local variables:
Elevation = "%GDB%\Elevation"
CoalTop = "%GDB%\CoalTop"
Permit = "%GDB%\Permit"
Overburden = "%GDB%\Overburden"
Lambda = "-0.012918501762736"
BoxCox = "%GDB%\BoxCox"
Mean = "78.3609531580408"
Std = "0.011863086181836"
ZScore = "%GDB%\ZScore"
ProbContours = "%GDB%\ProbContours"
ProbContours__3_ = ProbContours
ProbContours__4_ = ProbContours__3_
SubProb = "%GDB%\SubProb"

# Set Geoprocessing environments
arcpy.env.extent = Permit
arcpy.env.snapRaster = Permit

# Process: Raster Calculator (4)
tempEnvironment0 = arcpy.env.snapRaster
tempEnvironment1 = arcpy.env.extent
arcpy.env.snapRaster = ""
tempEnvironment2 = arcpy.env.cellSize
tempEnvironment3 = arcpy.env.snapRaster
arcpy.env.cellSize = "MAXOF"
tempEnvironment3 = arcpy.env.mask
arcpy.env.snapRaster = Permit
arcpy.gp.RasterCalculator_sa("Con("%Elevation%" > \"%CoalTop\",\"%Elevation\"-\"%CoalTop\")", Overburden)
APPENDIX 7.3

Subsidance

```python
arcpy.env.snapRaster = tempEnvironment0
arcpy.env.extent = tempEnvironment1
arcpy.env.cellSize = tempEnvironment2
arcpy.env.mask = tempEnvironment3

# Process: Raster Calculator (2)
arcpy.gp.RasterCalculator_sa("Power("%Overburden%",float(%Lambda%)-1/float(%Lambda%))", BoxCox)

# Process: Raster Calculator
arcpy.gp.RasterCalculator_sa("("%BoxCox%"-float(%Mean%))/float(%Std%)", ZScore)

# Process: Contour List
arcpy.ContourList_3d(ZScore, ProbContours, "-1.645;-0.8416;-0.5244;-0.2533;0;0.2533;0.5244;0.8416;1.282;1.645;-1.2816")

# Process: Add Field
arcpy.AddField_management(ProbContours, "Probability", "FLOAT", "", "", "NULLABLE", "NON_REQUIRED", """)

# Process: Calculate Field
arcpy.CalculateField_management(ProbContours__3_, "Probability", "val\nif \n[Contour] <1.646 AND [Contour]>1.644 then\n  val = .04250 * "%ZScore%" ** 3 + 0.38655 * "%ZScore%" + 0.50000) *100", SubProb)
```
APPENDIX 7.4

Z-Score

-2.25803

0.446389

Probability

67%

5%

Rosebud Mine
Subsidence

Test Hole

Probability Contour

Permit

Project ID: D-2401