Family Formation, Loading and Batch-Cyclic Flowshop Scheduling in Cellular Manufacturing Systems

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This thesis titled
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Abstract

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Family Formation, Loading and Batch-Cyclic Flowshop Scheduling in Cellular Manufacturing Systems

Director of Thesis: Gürsel A. Süer

In many cellular manufacturing systems, cells have flowshop scheduling characteristics. Most flowshop scheduling methods appeared in the literature assume that products are processed in batch or mixed mode. However, it has been observed that processing products in mixed mode (Batch-Cyclic Scheduling) improves the production rates as long as setup times are negligible. In this study, three phases are proposed to solve this problem. The first phase determines the product families based on processing similarity coefficients between products. Then in the second phase a new dissimilarity coefficient among products is used to divide the product family into subfamilies. Even though products in the same family are similar, we are attempting to create subfamilies where products in a subfamily have different bottleneck machines and wide difference between processing times. The production rate improvement observed when running products in batch-cyclic-mode was higher when bottleneck machines shifted and difference in processing times was higher. Finally, in the third phase, the products in each subfamily are scheduled in their corresponding cells using batch-cyclic and batch scheduling methods. However, the results show the efficiency of the batch-cyclic scheduling method to minimize the makespan, reduce the number of cells, and improve capacity parameter in the system.
Dedication

To my parents, who enriched my life with their LOVE.

To my advisor (Dr. Suer), who helped and encouraged me to be a better person.
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Chapter 1: Introduction

This chapter introduces the general definition of a Manufacturing System, Group Technology, Cellular Manufacturing System, and Loading and Scheduling of Cellular Manufacturing Systems briefly. Later, Minimal Part Set, and Mathematical Model, are also briefly described. Finally, the Objectives of this Thesis and Justification are given in this chapter.

1.1 Manufacturing System

Manufacturing system is the place where the raw materials/components are processed by using machines to produce the final products. Manufacturing system typically consists of three parts: inputs, manufacturing processes and outputs. Figure 1.1 shows these parts:

Based on production volume and product variety, manufacturing systems are divided into four types of layout (Product Layout, Process Layout, Cellular Layout, and Fixed Layout). Dissimilar machines are grouped together into production lines in the product layout, and each production line meets the high volume of demand. The flexibility of this layout is low since only one product can be processed in the system, and whole production line shuts down if any machine in the system breaks down. On the other hand, production line efficiency is high compared to other layouts because the volume of demand
is very high. In addition, the line is designed almost for the perfection of a single product, so modification in the line is very costly and difficult. Therefore, product layout is usually in the form of flowshop or transfer line. In the cellular layout, similar products are grouped together into a family based similarity coefficient. Each cell has dissimilar machines required to run the product family. The system flexibility and machine utilization are between medium and high, while the product variety and production volume are medium. In process layout, similar machines are grouped together into different departments. The products move from one department to others before completing the final product, so this type of layout is called departmental layout, functional layout or job shop configuration. While the fixed layout is different than the other three types; the product remains in a fixed position and machines and workers move around the product (Askin, &Standridge, 1993; Süer, Huang, & Maddisetty, 2010). Figure 1.2 explains types of manufacturing system layouts.

![Figure 1.2 Manufacturing System Layouts](image-url)
1.2 Group Technology

Group technology is a method used to group similar products together in order to make it easier to process them in the system based on the processing similarity. When the group technology is applied, products are assigned into families and their corresponding machines are collected in a cell.

Many researchers have studied group technology, and they explained the importance of grouping dissimilar machines together to process similar products. Group technology has many benefits: improve the machine utilization, reduce the total cost and increase the production rate (e.g., Abedinnia, Glock, & Brill, 2016; Alhourani, 2013; Aydilek, Aydilek, & Allahverdi, 2015). Assigning products into the family and their corresponding machines into a cell can be done based on different factors: processing similarity coefficient, processing time similarity coefficient, product similarity coefficient, and processing time dissimilarity coefficient. In this research, the processing similarity coefficient is utilized to formulate the product families, while the processing time dissimilarity coefficient divides the product family into subfamilies.

1.3 Cellular Manufacturing Systems

Cellular manufacturing system is one of the main applications of Group Technology (GT). It is a group of dissimilar machines that is brought together to produce similar products in a family. Each cell produces a family of products because the demand of these individual products is between low and medium. Grouping products into a cell reduces the total cost (inventory cost, material handling cost) and setup time of machines,
improves the cell utilization, decreases the intercellular and intra-cell worker movement, and increases the production rate (Alhourani, 2016).

Usually cellular manufacturing systems consist of different products required to be processed on several machines. According to the similarity between the products, some of the machines can be grouped together into the cells. The cells can be categorized into either independent cells or dependent cells. In the independent cells, each cell has its own machines because the machine sharing is not allowed. The product movements between the machines inside an independent cell are called intra-cellular movements. On the other hand, the machine sharing is allowed between dependent cells to improve and enhance the machine utilization. Machine utilization in the dependent cell is greater than the machine utilization in the independent cell due to machine sharing; the products move from one cell to others. Therefore, product movements from one cell to others are called inter-cell movements.

A cell in the cellular manufacturing systems can be classified as a dedicated cell, shared cell, or remainder cell. Dedicated cell (DC) has only one product family to process on its corresponding machines because of the demand value is high. Sometimes the demand of two product families is low, so they can be processed on one cell. This cell is called a shared cell (SC) which is a group of dissimilar machines required to meet demand of two relatively similar product families. Shared cell reduces the total cost of the system and improves machine utilization. Finally, three or more product families can be processed in a remainder cell (RC). The demand value of families in this cell is low, so it can be met in one cell. The machine utilization is high compared to the machine utilization in the
independent cell, due to machine sharing between the cells (Erenay, Suer, Huang, & Maddisetty, 2015; Süber, Huang, & Maddisetty, 2010).

Another classification of cells is based on the directional flow of processing, so it can be either flowshop cell or job shop cell. In a flowshop cell, there is only one direction of flow for products and all products in product families are processed in the same order. On the other hand, in a job shop cell there are at least two directions of flow (Greene & Sadowski, 1983). Fig 1.3 shows flowshop and job shop cells in the cellular manufacturing systems in relation to dedicated cell, shared cell and remainder cell.

![Figure 1.3 Control of Cellular Manufacturing System (Greene & Sadowski, 1983)](image)

**1.4 Loading and Scheduling in Cellular Manufacturing Systems**

Cell loading deals with the assignment of product families into cells (Süber, Ates, & Mese, 2014; Suer, Saiz, & Gonzalez, 1999). On the other hand, determining the sequence
of products on the machines, starting times and ending times of products in machines is called cell scheduling. The cell loading and cell scheduling are the main problems that face the cellular manufacturing system designers (Greene & Sadowski, 1983; Süer & Sáiz, 1993). The complexity of system depends on the number of machines and products, processing times, due dates, setup times and ready times of products in the system. Therefore, control of a manufacturing system is difficult because cell loading and cell scheduling have to be handled simultaneously (Saad, Baykasoglu, & Gindy, 2002).

Many heuristic methods were reported in the literature for cell loading and cell scheduling. Some heuristic methods aimed to improve the cell loading procedure, while others handled the scheduling issue. Most scheduling heuristic methods depend on batch scheduling to meet the demand. In this method, the entire batch of a product is completed, then the line switches to the next product. This method is widely adapted for scheduling the products in a cell when the setup times of machines are high. Cyclic method is another method that can be utilized to schedule the products in the system. This method is an appropriate method when the setup times of machines are negligible. In this method, the system covers the demand of all products almost simultaneously because it relies on mixed mode to cover the demand at the same time based on the minimal product set. In this research, the batch-cyclic method is used to schedule products in mixed mode based on the minimal product set concept, so the batch method and cyclic method are mixed together to schedule products in the system.
1.5 Minimal Part Set

The minimal part set (MPS) concept appeared in different situations in the literature. Creating the minimal part set can be done based on different factors: number of products (N), demand for product “i” (D_i) and the greatest common divisor of demand to all products (L) in the system. The minimal part set can be \{(D_1/L), (D_2/L), \ldots, (D_N/L)\} (Sawik, 2011; Soltani, & Karimi, 2015). Figure 1.4 explains the concept of batch method, cyclic method and minimal part set concept, when all products in the system have the same demand.

![Diagram of batch scheduling, cyclic scheduling, and minimal part set](image)

Figure 1.4 Batch Scheduling Method and Cyclic Scheduling Method

1.6 Mathematical Model

The mathematical model is used to find the optimal solution based on the objective function and different constraints. Sometimes, it targets to either minimize or maximize objective function. The constraints determine the relationship between the decision variables to achieve the main objective function. These constraints and objective function
can be linear or nonlinear depending on the relation between the decision variables. Therefore, different types of mathematical models are formulated to develop the relation between the decision variables: linear programming, nonlinear programming, and dynamic programming (Arora, Haleem & Singh, 2013).

In this thesis, three linear mathematical models are developed for family formation, loading and scheduling the products in the flowshop system: the first model determines the product families based on the processing similarity coefficient. The second capacitated P-median model divides the main product family into subfamilies based on the processing time dissimilarity coefficient. Finally, the third mathematical model schedules the products using batch-cyclic method in a subfamily. Typically, the ILOG/OPL software solves the linear programming model. It is utilized to solve the mathematical models in this research.

1.7 **Objectives of the Thesis**

The objectives of this thesis are listed below:

- Developing mathematical models to determine the product families based on processing similarity, and product subfamilies based on the processing time dissimilarity coefficient, and scheduling products in each cell based on the batch-cyclic method.

- Determining the main factors that affect the scheduling processes.

- Determining the sequence of products in the batch-cyclic scheduling.

- Comparing the results of the batch method and batch-cyclic method under the effect of the shifting bottleneck machines.
- Determining the effect of the total processing time per machine and per product in the production rate of the minimal product set.
- Determining the minimal product sets that might be used to meet the demand of the product.
- Minimizing the number of cells in the system
- Developing the rules to define the processing time dissimilarity coefficient between products in the system
- Increasing the production rate of the system and minimizing the makespan of products in the system.

1.8 Justification

Many methods in the literature have discussed the loading the flowshop in a cellular manufacturing system. Most of these methods relied on the similarity coefficient to create the product family. In this study, the processing similarity coefficient would be utilized to find the product family, while the processing time dissimilarity coefficient assigns the products in the product family into subfamilies. Also, the shifting bottleneck machine is taken into consideration. It means that each product has its own bottleneck machine. The main assumption in this research is that the setup time of machines is negligible, therefore the system switches processing from one product to others without any concern.

Other studies in the literature tried to schedule the products in a cell based on different rules such as the shortest processing time (SPT), longest processing time (LPT), and earliest due date (EDD). Also, several heuristic methods were applied to accomplish different objectives, such as minimizing the total tardiness, and number of tardy jobs. Some
of these methods schedule the products in batch mode, while others depend on the cyclic method. In this research, the batch-cyclic method would be applied to increase the production rate of the system, reduce the number of cells, and minimize the makespan of products in the system, and improve capacity parameter.

The objectives of this research are handled by developing three mathematical models. The first model is the P-median model to define the product family. The second product family is the capacitated P-median model to determine the subfamilies based on the processing times dissimilarity coefficient. Finally, the last mathematical model schedules the products in the subfamily based on the batch-cyclic scheduling method. However, the possibility of mixing products in the batch-cyclic model is high because the subfamily is created based on the processing time dissimilarity coefficient and the shifting of bottleneck machine concept. Also, the production rate of the system increases once the subfamilies are created based on maximizing the processing time dissimilarity coefficient.

1.9 Organization of the Thesis

This thesis is organized into five chapters as follows:

- Chapter 1 covers the general definition of manufacturing systems and its issues. It also clarifies objectives of the thesis and justification.
- Chapter 2 provides some of the previous work in loading and scheduling jobs in manufacturing systems, flow shop scheduling, and scheduling methods.
- Chapter 3 describes the problem statement in details and shows the main methods that will be used to solve the problem.
Chapter 4 explains three mathematical models that are used for loading and scheduling the products in flowshops in a cellular manufacturing system.

Chapter 5 illustrates and analyzes the results obtained in details.

Chapter 6 concludes the thesis and suggests future research to develop and enhance the proposed scheduling method.
Chapter 2 : Literature Review

Modeling and creating manufacturing systems are the main areas of study that have appeared in the recent studies. It is considered an active area of research because of the dynamic changes of demand and technology that are used to design systems.

2.1 Cellular Manufacturing Systems

A cellular manufacturing system typically is a combination of the process and the product systems. Based on that, many approaches can be implemented to design the system (e.g., Imran, Kang, Lee, Jahanzaib, & Aziz, 2017; Selim, Askin, & Vakharia 1998; Shiyas, and Madhusudanan, 2014; Sofianopoulou 1999).

Flexibility and importance of a cellular manufacturing system make it an active area of study to many researchers. They considered different objectives such as reducing the total cost of products, material handling costs, and increasing the production rate of the system (Alhourani & Seifoddini 2007; Ganesh & Srinivasan, 1994; Won, & Logendran, 2015). Bhandwale and Kesavadas studied the way to improve machine utilizations and reduce the total cost of products. In doing so, they created a new method to assign a new product to existing manufacturing cells. They concluded that it increases in machine utilization as well as a decrease in total cost of the system (Bhandwale, & Kesavadas, 2008). P-median approach, and genetic algorithm are the other methods that might be used to improve the system performance kusiak (1987).

The P-median approach is considered one of the main methods to design a cellular manufacturing system. Therefore, P-median aimed to implement different objectives such as assigning machines to cells and parts to families (e.g. Alhourani & Seifoddini 2007;
Other researchers aimed to minimize the distance between machines by using the P-median approach (Ganesh & Srinivasan 1994). Wang and Roze (1997), depended on the similarity coefficient between the machines to create cellular manufacturing systems. In their model, they added constraints to determine the number of machines and products in the cells. This significantly reduced the number of machines and resources needed. Finally, Ilić (2014) used the similarity coefficient between the parts and machines based on the machine-part relationship, demand values, and total processing time in the cell. Also, Ilić determined the number of machines in each cell precisely to meet the demand of product family in a cell and to reduce intercellular and intracellular movements of product families. However, this thesis develops two rules to define the processing times dissimilarity coefficients which are used to create the subfamilies. Also, the processing similarity coefficients are used to define the product families.

Genetic Algorithm is another method that is used to design the cellular manufacturing system. Some researchers depended on the combined local search heuristic and the genetic algorithm to design the cellular manufacturing systems. For example, Gonçalves and Resende relied on local search heuristic method and the genetic algorithm to determine the number of machines and product families in a cell (Gonçalves & Resende 2004). Arora, Haleem and Singh compared several methods to design the cellular manufacturing system such as genetic algorithm, P-median, neural network, mathematical programming, fuzzy clustering, and artificial intelligence. The results showed that the genetic algorithm is a sufficient method to design the cellular manufacturing system (Arora, Haleem, & Singh, 2013).
In addition, Shiyas and Madhusudanan Pillai (2014) suggested a new algorithm to assign the machines into cells and to improve the grouping efficacy. Grouping efficacy is used to determine the utilization of the diagonal block in the matrix relying on the total number of 1 or 0 inside and outside the diagonal box. The results show that using the genetic algorithm improved the results 33% compared to the other techniques.

However, researchers have not only studied the deterministic data, but have also examined stochastic data. In this case, the arrival and processing time of demand are stochastic data. Baykasoglu and Gorkemli explained in their research how the agent-based method is used to create product families in the systems and assigned them into the cell when the arrival times of products are dynamically changing with time (Baykasoglu & Gorkemli, 2015).

2.2 Flowshop Scheduling

The flowshop scheduling continues to be the main area of study to many researchers in order to execute objectives such as minimizing makespan, average flow time, maximum tardiness, number of tardy jobs and total tardiness (Laha & Sarin, 2009; Reeves 1995; Yu & Seif 2016). Many methods are suggested in recent studies in order to implement many objectives such as the batch scheduling method and the cyclic scheduling method. Sawik (2012) explained in his article three methods to schedule the product families in the hybrid flowshop with parallel machines to minimize the completion time. The first method is batch scheduling, which means processing each product separately than the other products, and then the system switches to process the other products. The second method, the cyclic scheduling method, covers the demand of all products simultaneously based on the minimal
part set. The minimal part set creates a proportion between products to provide demands at the same time. The third method, the batch-cyclic scheduling method, a combination of batch method and cyclic method are used to cover the demand of products. The CPLEX Software was used to solve the mathematical models. The results have shown that processing the products in the mixed mode improved the production rate of the system.

2.2.1 Batch Flowshop Scheduling

The batch flowshop scheduling method is an active method for scheduling processes in the cellular manufacturing system. It is based on covering the demand of a product separately than the other products. Some researchers have shown how GA could minimize the makespan of product families in several machines in a cell (e.g., Reeves 1995). Other researchers’ proposed alternative heuristic methods to minimize the total completion time of different jobs assigned to be processed in the flowshop (Laha, & Sarin, 2009). They have proposed a new method to minimize the total completion time in the system. They assumed that job with the highest processing time in the system has the highest priority. Once there is more than one job with the same priority, they are assigned to be processed in the system concurrently. This method was repeated to schedule all jobs. Then the makespan was calculated. However, this method is used to deal with different problems and the final results show that the efficiency of systems is higher when the number of jobs is small.

The batch scheduling method has also been used to minimize the total tardiness and earliness (i.e., Yu & Seif 2016). Researchers have demonstrated the importance of reducing tardiness and earliness to reduce the total cost of the system by using the GA heuristic
method. For example, İşler, Toklu, and Çeli (2012) clarified in their paper how they minimized the total tardiness and earliness for different jobs at two machines in the flowshop with a common due date. They used a Genetic Algorithm (GA), Tabu Search (TS), Random Search (RS) heuristic method, and mathematical model to minimize the total tardiness and earliness. Then they tested these methods for different problems with different numbers of jobs. The final results proved that the GA heuristic method and TS are the appropriate methods to minimize both tardiness and earliness.

2.2.2 Cyclic Flowshop Scheduling

In the cyclic scheduling, the system meets the demand of all product families simultaneously, so it is considered a useful method exclusively when the parts have different arrival times, same priorities, and zero setup time. The cyclic scheduling method also incorporates the concept of the minimal part set (MPS). The MPS is the smallest possible set of product in the same quantity as the overall production demand. For example, assume that demand of A and B is 20, and 10 units respectively. The unit production cyclic (AB) repeats ten times, then A repeats ten times to cover the whole demand (McCormick, & Rao, 1994; Sawik, 2011; Soltani, & Karimi, 2015). However, this thesis deals with batch-cyclic flowshop scheduling to minimize the makespan based on minimal product set instead of minimal part set.

Scheduling jobs in the flowshop using the cyclic scheduling method has many advantages such as minimizing the cycle time and improving machine utilization. Recent studies have shown how the cyclic scheduling improves the machine utilizations and minimizes the cycle time (Bożejko, Gniewkowski, Pempera, & Wodecki, 2014; Bożejko,
Uchroński, & Wodecki 2015). Researchers have also shown that the tabu search method minimized the cycle time. Bożejko, Gniewkowski, Pempera, and Wodecki, (2014) explained that using tabu search method improves the system efficiency.

Cyclic scheduling also minimizes the total production cost. Bahroun, Campagne, and Moalla (2006) discussed the minimization of the inventory holding cost by using cyclic scheduling under variable demand and cyclic deliveries to three machines with three products. They depended on four steps to create the final schedule. The first step verifies the carrying and set-up cost for each product in different orders with different deliveries. The second step decides the initial schedule based on the production frequencies and the kind of products. The third step combines two phases to determine the best schedule, so the best results depends on minimizing the inventory cost. Finally, after creating the schedule for the third machine they scheduled second machine and then first machine. The ending time to the product in the third machine equals the starting time to this part in the second machine, and the ending time to part in machine two equal to the starting time to this part in machine one. This scheduling process is a complex process because the scheduling process is different from one product to another. As a result, the local optimal solution is determined based on this method.

El-Najdawi (1997) proposed a new heuristic method for multi-cyclic scheduling to defind the near optimal solution to minimize the total cost and flow time for products in the flowshop system. The author developed three stages to create part families and schedule part families in the system. The first stage creates a sequence for part families based on minimizing the setup time. The second stage tries to finalize the scheduling of jobs
depending on the priority of the product instead of cost. Finally, in the third stage, the scheduling process is based on the ratio between cost and setup time. In other words, the product with the highest ratio has the highest priority to process in the system. Moreover, this algorithm is applied to minimize costs and the flow time of products in the system for several problems.

2.2.2.1 Cyclic Flowshop Scheduling with Blocking and Bottleneck Machine

The raw materials in flowshop system are processed on different machines before the final products are created. Sometimes the absence of a buffer and differences in the processing time of products at machines create the blocking machine concept. This concept was explained by Hall, and Sriskandarajah, (1996). They illustrated that a machine is called the blocking machine when it couldn’t process any a new item because the next machine is not available to process the new product for different reasons. For example, it still works on the other products. Also, they mentioned that there are different reasons for an absent buffer in the flowshop system. For example, parts have to save at certain environmental conditions (temperature and pressure) which like the machine condition, and the cost of the buffer is high. In addition, they demonstrated several applications to the blocking machine concept in different fields such as the flowshop, and job shop.

Soltani, and Karimi, (2015) have minimized the cyclic time for different products when they are processed in the hybrid flowshop with specific constraints, buffer size and machine eligibility. Also, the authors expounded the benefits of the buffer size in the scheduling, and how it made the scheduling problem more complex. For instance, the machine becomes blocked when the subsequent buffer is full. In this case, the machine
remains still idle until the buffer becomes unoccupied. In addition, they used two stages in heuristic method to schedule the jobs. The first stage creates the heuristic method by determining the initial sequence using different rules such as shortest processing time (SPT), Longest Processing time (LPT), NEH algorithm, Plamer, Genetic Algorithm (GA) Simulated annealing (SA), and Greedy algorithm. NEH algorithm suggested by Nawaz et al (1983) to schedule the products in the flowshop based on the two steps. The first step orders jobs based on the SPT, and then the second step determines the best sequences for the first and second jobs based on the algorithm, and then the previous two steps repeat to schedule the other jobs. Plamer algorithm depended on sorting jobs based on the slope which was defined based on the specific equation that are suggested by Palmer (1965). The second stage enhances the schedule to improve the results. They concluded that a new method helped them to make the problem more realistic and less complex.

In the flowshop system, several machines that are grouped together into a cell process several product families. The processing time of product families is different from one product family to others and from machines to others, so the production rate is different from one product family to others. In addition, the production rate of a family is based on the processing time of the machines which has the highest processing time in each product family when the batch scheduling method is utilized to schedule the products in the system. Typically, this machine is called the bottleneck machine and many recent studies have illustrated the concept of bottleneck machine in the flowshop (Raaymakers, & Hoogeveen, 2000; Paternina-Arboleda, Montoya-Torres, Acero-Dominguez & Herrera-Hernandez 2008).
Furthermore, Abadi, Hall, and Sriskandarajah (2000) suggested a new method to schedule product families in flowshop to minimize cycle time. Cycle time of parts in the system equals to the processing time of bottleneck machine. Also, Abadi et al. depended on a minimal part set to schedule parts in the system in the cyclic scheduling method. Finally, the method efficiency was improved when applied to solve problems with small number machine and jobs.

Researchers have studied the flowshop cyclic scheduling with setup time and limited buffer size to implement different objectives such as minimizing the total completion time and makespan. Zandieh and Rashidi (2009) proposed a new technique to schedule several jobs with different setup times and limited buffer sizes to minimize the total completion time and the makespan, which is called hybrid GA. It is a combination of the NP-hard (non-deterministic polynomial-time hard), and GA.

2.2.3 Bottleneck Concept in Flowshop

In flowshop systems, the product families have to be processed in the production line at several stages. Sometimes, each stage has one machine, the processing time of a part is different from one machine to others with different bottleneck machines. While, other flowshop systems have two or more machines in each stage. In this case, flowshop system is known as the hybrid flowshop so product can be processed in any machine in each stage. These machines can be unrelated parallel machines, uniform machines or identical parallel machines. In this case, the bottleneck stage concept is replaced with the bottleneck machine term. This replacement has developed different techniques in recent the studies to define
the bottleneck stage concept in the hybrid flowshop system (Lee, Kim, & Choi, 2004; Phadnis, Brevick, & Irani, 2003).

Paternina-Arboleda et al. (2008) proposed a heuristic method in their article to schedule different jobs to be processed in several stages in the flexible flowshop to minimize makespan depended on the bottleneck stage. Each stage has identical parallel machines. The workload, the ratio between the total processing times of parts divided by the number of machines, determines the bottleneck stage. Then, the stage which has the highest workload in the system is considered the bottleneck stage. Paternina-Arboleda et al. scheduled the jobs at the bottleneck stage, and subsequently scheduled jobs on other stages based on the decreasing the stage workload.

Moreover, Phadnis, Brevick, and Irani (2003) relied on the identical parallel machines in each stage at the hybrid flow shop to create a new method in the scheduling process. This method is a Progressive Bottleneck Improvement to schedule several jobs and minimize the makespan in a cell. They defined the bottleneck stage based on the lower bound of each machine. They clarified the lower bound for each stage. A stage which has high lower bound called bottleneck stage. Phadnis et al. have reported that calculated the lowest bound for each stage before scheduling any jobs because the bottleneck stage is different from one job to others. Therefore, using the Progressive Bottleneck Improvement helped them to avoid the bottleneck problem. This problem resulted from shifting the bottleneck stage so they solved this problem by determining the bottleneck stage before scheduling process.
Lee, Kim, and Choi (2004) suggested in their article a new algorithm to schedule several jobs in an identical parallel machine in a hybrid flowshop to minimize the total tardiness. They determined the bottleneck stage as the stage which has the highest workload in the system. They depended on scheduling the jobs in the bottleneck stage, then scheduling the job on another stage based on the scheduling on the bottleneck stage. Also, they tested their method by using different sources of data and results show the efficiency of this method to solve the problems.

On the other hand, Chen, and Chen (2008) proposed in their article a heuristic method to minimize the number of tardy jobs in the unrelated parallel machines in the flexible flowshop. They defined the bottleneck stage as the stage which has the highest workload in the production line. Several steps are used to schedule the products; identify the bottleneck stage, define the workload, schedule the jobs in the stage before the bottleneck stage, schedule jobs at the bottleneck stage based on certain rules, and then depending on the dispatching rules to schedule the jobs at the remaining stages.

However, the shifted bottleneck machine concept is used in this thesis which means that the bottleneck machine is different from one product family to others. Therefore, each product family has its own bottleneck machine. Also, the batch-cyclic scheduling method will be used to minimize the makespan and increase the production rate of the flowshop in the cellular manufacturing system, enhance and improve capacity parameter, and reduce the number of cells in the system.
2.3 Flowshop Scheduling by Different Methods

Different techniques are used to schedule jobs in the flowshop such as shortest processing time, longest processing time, and early due date. These methods provide best results when the data are deterministic data, but there are many restrictions that appear when the data are stochastic data. Many methods are suggested in the recent studies to implement these restrictions such as combined simulation and the heuristic method. Gourgand, Grangeon, and Norre (2003) have shown a new algorithm to schedule different jobs in the flowshop system with stochastic data and unlimited buffer size to minimize mean makespan. This algorithm depended on two strategies: Markov chain and discrete-event simulation. Also, they have applied discrete-event simulation to estimate the mean makespan because the stochastic data have numerous random variables. Markov chain calculates the mean makespan.

Additionally, Azadeh, Jeihoonian, Shoja, and Seyedmahmoudi (2012) proposed a new algorithm to schedule different jobs with stochastic data. Two-stage of assembly flowshop with setup time were used to minimize the makespan and mean completion time of jobs in the system. Visual SLAM simulation Software and Artificial Neural Network (ANN) are used to deal with this problem. Simulation software was used to deal with stochastic data to provide the results, so simulation software was used to determine the scheduling of jobs and the ANN was utilized to define the makespan of the system and the average completion time. Azadeh, Maleki-Shoja, Sheikhalishahi, Esmaili, Ziaeifar, and Moradi, (2015) used in the computer simulation software and Artificial Neural Network (ANN) to schedule jobs in the flowshop system with identical parallel machines to
minimize the total cost, tardiness, earliness, inventory, and backlogged cost. As a result, the integration between the computer simulation and artificial neural network was able to schedule the jobs in the stochastic flowshop system and implement the main goals of scheduling.

Moreover, researchers have solved the stochastic flowshop scheduling problems by mixing the simulation software with the metaheuristic (e.g., Lin & Chen, 2015). They have explained how computer simulation software and GA heuristic method are applied to minimize the flow time of parts in the stochastic hybrid flowshop system. Juan, Barrios, Vallada, Riera, and Jorba (2014) proposed a new algorithm to schedule different jobs in the stochastic flowshop replied on the Monto Carlo Simulation Software and the NEH heuristic method to minimize the mean makespan. Therefore, these studies illustrated the benefits of using the combination of computer simulation and metaheuristic to schedule the parts in the stochastic flowshop system.

On the other hand, other researchers have explained that it is possible to schedule the jobs in the stochastic flowshop system without using the computer simulation. Baker and Trietsch (2011) presented three heuristic methods to schedule the jobs at two machines in the flowshop system with random processing times to minimize the average makespan. It was based on three heuristic methods to implement their goals. The first heuristic method (Talwar’s rule) is sorting jobs by decreasing the differences in processing rates. The second heuristic method (Johnson’s Heuristic) is changing the stochastic values of processing time to mean values. Therefore, the stochastic data are replaced with a deterministic data. Finally, adjacent pairwise interchange (API) aims to find a stable schedule. The position
of the job is changed to create a stable job. Overall, Baker, and Trietsch, mentioned that these heuristic methods are effective to schedule the jobs in the stochastic flowshop systems without by using computer simulation.
Chapter 3 : Problem Statement

In this research, family formation, loading and scheduling flowshop systems in cellular manufacturing systems are discussed in detail. In a flowshop cell, products are processed in the same order on dissimilar machines. The products are assigned into product families and their corresponding machines are assigned into a cell based on the processing similarity coefficient. In order to identify the product families, subfamilies and schedule the products in each subfamily, the product-machine matrix, processing times and the volume of demand should be determined. This section discusses the above issues in detail, and the processing time dissimilarity coefficient and batch-cyclic scheduling method are described in depth.

3.1 Product – Machine Matrix

An example is introduced in this section to clarify three phases of work that are implemented to load and schedule several products in a flowshop cell. Table 3.1 explains the product-machine binary matrix of 12 products and 14 machines. Each cell in the table illustrates the possibility of processing a product in the machine. Binary number ‘1’ at cell “ij”, the “i” and “j” indices represent product and machine respectively, shows that product “i” will be processed on machine “j”, while, “0” at cell “ij” product “i” will not be processed on machine “j”. For example, P_{1} will be processed on M_{1}, M_{3}, M_{5}, M_{7}, M_{8}, M_{9}, M_{11}, and M_{13}. The products are not processed on all machines. Therefore, the machine skipping is one of the main assumptions in this research.
Table 3.1 Product-Machine Binary Matrix

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3.2 Processing Times and Demand Values

Typically, the processing times and the demand values can be either deterministic data or stochastic data. Table 3.2 shows the deterministic processing times of products, and each product is processed on several machines with different processing times. In addition, the production rate of cell depends on the processing time of the bottleneck machine when the batch method is utilized to schedule the products in the cell. The bottleneck machine is the machine which has the highest processing time in the system/cell. For example, M1 is the bottleneck machine for product two (P2), while M8 and M9 are two bottleneck machines for product one (P1) because they have the highest processing times in the system. Therefore, the production rate of cell equals (60 min/0.4 min = 150 parts/hour) when product one is processed in the cell, and (60 min/0.24 min = 250 parts/hour) when product two is manufactured in the cell. Therefore, the shifting bottleneck is another main assumption in the research.
Table 3.2 Unit Processing Times (Minutes)

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Table 3.3 Monthly Demand (Units)

<table>
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<th>Product</th>
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<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
<th>P11</th>
<th>P12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>4256</td>
<td>14854</td>
<td>11601</td>
<td>7343</td>
<td>12646</td>
<td>5527</td>
<td>13127</td>
<td>9930</td>
<td>7986</td>
<td>9933</td>
<td>4765</td>
<td>4903</td>
</tr>
</tbody>
</table>
The demand values for the products represent the amount of product that has to be produced in the cell during the time period. Table 3.3 shows the monthly demand values for products which are generated randomly in the interval of [4100, 15000] by using Microsoft Excel worksheet.

Setup time can be defined as the time required to change from processing one product to another. In this study, setup times of machines are assumed to be zero. The system is available 8 hours per day, one shift per day, five days per week, and four weeks per month, so the available system capacity is 160 hours per month.

3.3 **Product Family**

According to the literature, many methods can be used to identify the product families and product subfamilies. However, the processing similarity coefficient and the processing time dissimilarity coefficient are utilized in this study in order to perform two phases of family formulation and cell loading. The first phase depends on the processing similarity coefficient to create the product family. In the second phase, each product family is divided into subfamilies depending on the processing time dissimilarity coefficient for each product family by using the capacitated P-medial model.

3.3.1 **Product Family Formation**

Product family formation is the first phase in the cellular manufacturing system design. Many studies have discussed the product families’ formation. In this study, P-median model is used to assign products into product families based on the processing similarity coefficient. In this example, two product families are created. The first product family consists of P₂, P₃, P₅, P₇, P₈ and P₁₀, which are assigned to be processed in the first
cell (M_1, M_2, M_4, M_5, M_6, M_7, M_8, M_10, M_11, M_12, and M_14). The second cell processes P_1, P_4, P_6, P_9, P_{11}, and P_{12} at machines (M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8, M_9, M_{10}, M_{11}, M_{12}, M_{13}, and M_{14}). The machine duplication is allowed and machine shared is not acceptable so each cell has its own machines. Therefore (M_1, M_2, M_4, M_5, M_6, M_7, M_8, M_{10}, M_{11}, M_{12}, and M_{14}) are duplicated in two cells because they process the products in two cells. Figure 3.1 shows two product families and their corresponding machines.

Figure 3.1 Product Families Formations

3.3.2 Cell Loading and Identifying Product Subfamilies

Cell loading and identifying product subfamilies represent the second phase of cell loading in this research. The bottleneck machine shifts from one product to others. Similarly processing times vary from one machine to others, and from one product to others. The products in each product family are assigned into subfamilies based on the processing time dissimilarity coefficient. Considering this, two methods are developed to
define processing time dissimilarity coefficients. The first method determines the dissimilarity coefficients based on the average of the difference between the processing times between product pairs on all machines in the cell. Equation 3.1 shows the general formula for the first rule when it is utilized to find the processing time dissimilarity coefficient between product pairs \((i, j)\) in the \(M\) machines:

\[
\text{Processing Time Dissimilarity Coefficient} = \frac{\sum_{m=1}^{M} \text{Abs}(P_{im} - P_{jm})}{M} \quad (3.1)
\]

The following formula shows the processing time dissimilarity coefficient between the product pairs \((P_2 \text{ and } P_3)\) in the first product family when they are processed on 11 machines. The absolute value is taken into consideration per each machine to avoid a negative value in the results:

\[
= \text{Abs}(P_{2,m1} - P_{3,m1}) + \text{Abs}(P_{2,m2} - P_{3,m2}) + \text{Abs}(P_{2,m4} - P_{3,m4}) + \text{Abs}(P_{2,m5} - P_{3,m5}) + \text{Abs}(P_{2,m6} - P_{3,m6}) + \text{Abs}(P_{2,m7} - P_{3,m7}) + \text{Abs}(P_{2,m8} - P_{3,m8}) + \text{Abs}(P_{2,m10} - P_{3,m10}) + \text{Abs}(P_{2,m11} - P_{3,m11}) + \text{Abs}(P_{2,m12} - P_{3,m12}) + \text{Abs}(P_{2,m14} - P_{3,m14}) / 11
\]

\[
= (\text{Abs} (0.24-0.13) + \text{Abs} (0.1-0) + \text{Abs} (0-0.12) + \text{Abs} (0-0) + \text{Abs} (0.19-0.14) + \text{Abs} (0-0) + \text{Abs} (0.19-0.2) + \text{Abs} (0.08-0) + \text{Abs} (0.21-0.17) + \text{Abs} (0.06-0.09) + \text{Abs} (0.11-0.07))/11
\]

\[
= 0.0527
\]

In the second method, the processing time dissimilarity coefficient is calculated based on the average of the difference between the processing times of machines processing either product ‘i’ or ‘j’. The machines process either product ‘i’ or ‘j’ “mc” as given in the equation 3.2.

\[
\text{Processing Time Dissimilarity Coefficient} = \frac{\sum_{m=1}^{mc} \text{Abs}(P_{im} - P_{jm})}{mc} \quad (3.2)
\]
For example, the total number of machines in the first cell is 11 machines, while \( P_2 \) and \( P_3 \) are processed in 9 machines out of 11 machines. In this case, the second method is developed to define the processing time dissimilarity coefficient based on 9 machines. The following equation shows the result of the second method:

\[
\frac{((\text{Abs}(P_{2,m1} - P_{3,m1}) + \text{Abs}(P_{2,m2} - P_{3,m2}) + \text{Abs}(P_{2,m4} - P_{3,m4}) + \text{Abs}(P_{2,m6} - P_{3,m6}) + \text{Abs}(P_{2,m8} - P_{3,m8}) + \text{Abs}(P_{2,m10} - P_{3,m10}) + \text{Abs}(P_{2,m11} - P_{3,m11}) + \text{Abs}(P_{2,m12} - P_{3,m12}) + \text{Abs}(P_{2,m14} - P_{3,m14}))}{9}
\]

\[
= ((\text{Abs}(0.24 - 0.13) + \text{Abs}(0.1 - 0) + \text{Abs}(0 - 0.12) + \text{Abs}(0.19 - 0.14) + \text{Abs}(0.19 - 0.2) + \text{Abs}(0.08 - 0) + \text{Abs}(0.21 - 0.17) + \text{Abs}(0.06 - 0.09) + \text{Abs}(0.11 - 0.07))}{9}
\]

\[
= 0.0644.
\]

Capacitated P-median model is utilized to define the subfamilies. The capacity restriction is added to the model based on the hourly production rate of the bottleneck machine. Two subfamilies are formed per each product family, when the CPLEX software solves the mathematical model. Figure 3.2 and Figure 3.3 show a particular example of subfamily formation based on the processing time dissimilarity coefficient using the first rule of processing time dissimilarity coefficient. The production rate of the system improves once the product subfamilies are created based on maximizing the processing time dissimilarity coefficients between products. Using the processing time dissimilarity coefficient increases variation in the processing times between products in the subfamily. Therefore, the possibility of mixing the products is increased once the batch-cyclic scheduling method is utilized to schedule the products in the cell.
Having determined the product families and product subfamilies, the products are ready to schedule in machines. The scheduling process can be implemented by using different methods that were discussed in recent studies such as the batch method and the cyclic method. In this study, two methods are combined together to schedule the products in the cell. This method is called batch-cyclic method. The proposed scheduling method is utilized because it is easy to implement, and it minimizes the makespan of products in the system at the same time. In this method, three steps are used to schedule the products: the first step determines the minimal product sets or the cyclic set. The second step defines the
production rate for each minimal product set based on the manual calculation. Finally, the mathematical model is developed in the third phase to minimize the makespan of products in the system. However, in this research the minimal product set is used instead of minimal part set.

As explained before, the minimal part set is calculated depending on the number of products (N), demand of each product “i” (D_i) and the greatest common divisor of demand for all products (L) in the system. Minimal part set equals \{(D_1/L), (D_2/L)… (D_N/L)\}. For example, A, B, C, D products are required to be processed in the flowshop cell machines with different demands. The demand values are 200, 300, 400, and 550, respectively. In this case, the system produces four parts of product A, six parts of product B, eight parts of product C and, eleventh parts of product D. Then this set is repeated a 50 times to meet the demand of products almost simultaneously.

Several steps are followed to schedule the product by using batch-cyclic method. The first step determines the size of the minimal product set based on the number of products in the system. The second step covers the lowest demand of products. Once the lowest demand is covered a new minimal product set is generated depending on the new
number of products and demands. Then, these three steps are repeated over and over until the entire demand values have been met. For instance, $P_1$, $P_2$ and $P_3$ are assigned to be processed by flowshop cell machines with the demand values are 100, 200, and 350, respectively. The first minimal product set based on cover the demand for one part on each product ($P_1 P_2 P_3$). Then this set is repeated a hundred times to cover the whole demand of $P_1$ and 100 parts of $P_2$ and $P_3$, once the $P_1$ has the lowest demand in the system. After that, another minimal product set is created to meet the reset demands of $P_2$ and $P_3$, which is ($P_2P_3$). Now, the set would be also repeated a hundred times to meet a hundred parts of each product $P_2$ and $P_3$. In this case, the system covers the second lowest demand in the system. Finally, the system produces 150 parts of $P_3$ to cover the demand of $P_3$ as well. Figure 3.5 illustrates a minimal product set in the batch-cyclic scheduling method.

Figure 3.5 Minimal Product Set in the Batch-Cyclic Scheduling Method

Figure 3.6 shows a particular example of batch-cyclic scheduling of four products in the flowshop cell with different demand values. However, there are many benefits of using the batch-cyclic scheduling method to schedule several products in flowshop cell such as improving the cell efficiency, ease of implementation, reducing the inventory and holding cost, and increasing the production rate of the system.
This study aims to maximize the production rate of the system, reduce the number of cells, and improve the capacity parameter of system, therefore, three phases are developed to implement these targets. The first phase deals with family formation, the second phase aims to identify the subfamily and cell loading, while the third phase schedules the products in the cell based on the batch-cyclic scheduling method. Using the batch-cyclic scheduling method improves and enhances the machines’ utilization because it maximizes the production rate, reduces the number of cells, minimizes the makespan of products in a subfamily and improve capacity parameter.
Figure 3.6 Batch-Cyclic Flowshop Scheduling
Chapter 4  : Methodology

In this research, three phases are developed to load and schedule flowshop cells in cellular manufacturing systems. The first phase creates the product families based on the processing similarity coefficient. The second phase divides the product family into subfamilies depending on the processing time dissimilarity coefficients. In this case, the output rate of system improves, once the subfamilies are created based on maximizing the processing time dissimilarity coefficients. In phase three, the products are scheduled in subfamilies based on the batch-cyclic scheduling method. However, the product families are defined in phase one and subfamily formations and cell loading are explained in the second phase, while phase three illustrates the cell scheduling. Figure 4.1 represents the general description of the methodology utilized in this research. In this study, several assumptions are made, deterministic data, zero setup times, shifting bottleneck machines, machine skipping, unit transfer, and batch-cyclic scheduling mode.

4.1 General Overview of Three Phases

In this thesis, mathematical models are utilized to develop the three phases of family formation, cell loading and scheduling the products in the flowshop cell in a cellular manufacturing system. During the first phase, the mathematical model is implemented to maximize the processing similarity coefficient between products in product families. In the
second phase, the mathematical model is used to maximize the processing time dissimilarities coefficient between products in subfamilies. Since the maximizing dissimilarity coefficient between products is used to create the product subfamilies, the variations in the processing times between products in the subfamily are expected to be high. Then, the batch-cyclic scheduling method is utilized to schedule products depending on the minimal product sets/cyclic set by using the mathematical model. In addition, manual calculation is used to define the production rate of the system for each minimal product set.

4.2 Phase One: Product Family Determination

Literature discussed different algorithms to create the product families. In this study, the mathematical model is used to define the product families based on maximizing the processing similarity coefficient (similarity coefficient of the product pairs). The Jaccard’s similarity coefficient is employed to define the similarity coefficient matrix as suggested by McAuley (1972).

\[
S_{ij} = \frac{\text{Number of products processed on machines ‘i’ and ‘j’}}{\text{Number of products processed on either machine ‘i’ or ‘j’}}
\]

In this study, the similarity coefficient matrix is calculated based on the similarity of products instead similarity of machines. It is more important to form families first since the machines are inexpensive. Therefore, we use modified Jaccard’s similarity coefficient.

\[
MS_{ij} = \frac{\text{Number of machines processing products ‘i’ and ‘j’}}{\text{Number of machines processing either product ‘i’ or ‘j’}}
\]
The product-machine matrix shows the relation between the products and machines as given in Table 4.1. Each cell has a value of ‘1’ or ‘0’, where ‘1’ in cell ‘ij’ represents that product ‘i’ should be processed at machine ‘j’, while ‘0’ shows that the product ‘i’ isn’t processed by a machine ‘j’.

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</table>

The similarity matrix is created based on the processing similarity coefficient for product pairs. For example, the processing similarity coefficient between P_9 and P_1 is $MS_{91} = \frac{6}{9} = 0.667$.

In this phase, the mathematical model is built to deal with 12 products -14 machines in cellular manufacturing system. The objective of the mathematical model is to maximize the sum of processing similarity coefficient for products as given in equation (4.1), and the penalty is added if the cell (family) is opened. Equation (4.2) guarantees that a product is
assigned to one family, and the last constraint ensures that each product belongs to a family if it is formed as given in equation (4.3).

Objective Function:

$$\text{Maximize } Z = \sum_{i=1}^{n} \sum_{k=1}^{n} S_{ik} \times X_{ik} - \sum_{k=1}^{n} P \times X_{kk}$$  \hspace{1cm} (4.1)$$

Subject to:

$$\sum_{i=1}^{n} X_{ik} = 1 \hspace{1cm} i = 1, \ldots, n$$  \hspace{1cm} (4.2)$$

$$X_{ik} \leq X_{kk} \hspace{1cm} i = 1, \ldots, n \text{ and } k = 1, \ldots, n$$  \hspace{1cm} (4.3)$$

Indices

i  Product index
k  family (cell) index

Parameters

n  number of products
$S_{ik}$  Similarity coefficient between product $i$ and $k$

P  penalty for opening a new family (cell)

Decision variable

$X_{ik}$  1 if product $i$ belongs to family $k$, 0 otherwise

However, different product families can be obtained based on the penalty factor as the data given the Appendix C. The data show that the number of product families are decreased when the penalty factor increases. In this study, we assumed that the penalty factor equals “1”. The CPLEX software is used to solve the model and to determine the Product Families. Two product families are generated as a result of the mathematical model. First product family consists of $P_2$, $P_3$, $P_5$, $P_7$, $P_8$, and $P_{10}$, while $P_1$, $P_4$, $P_6$, $P_9$, $P_{11}$,
and \( P_{12} \) form the second product family. Table 4.2 shows the products and their corresponding machines.

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Corresponding Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>( P_2, P_3, P_5, P_7, P_8, P_{10} ) ( M_1, M_2, M_4, M_5, M_6, M_7, M_8, M_{10}, M_{11}, M_{12}, M_{14} )</td>
</tr>
<tr>
<td>PF2</td>
<td>( P_1, P_4, P_6, P_9, P_{11}, P_{12} ) ( M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8, M_9, M_{10}, M_{11}, M_{12}, M_{13}, M_{14} )</td>
</tr>
</tbody>
</table>

### 4.3 Phase Two: Subfamily Identifications and Cell Loading

In this section, the processing time dissimilarity coefficients, demand, capacity restriction, and the product subfamily mathematical model are discussed.

#### 4.3.1 Processing Time Dissimilarity Coefficient

Once the product families are created in the first phase, the subfamilies have to be determined in the second phase. The results of the first mathematical model show that two product families are generated in the flowshop system. Table 4.3 and Table 4.4 show processing times for the first product family and the second product family on their machines. In addition, the processing time dissimilarity coefficient is calculated to group the products in a product family into subfamilies.
### Table 4.3 Processing Times for the First Product Family (Minutes)

<table>
<thead>
<tr>
<th></th>
<th>M₁</th>
<th>M₂</th>
<th>M₄</th>
<th>M₅</th>
<th>M₆</th>
<th>M₇</th>
<th>M₈</th>
<th>M₁₀</th>
<th>M₁₁</th>
<th>M₁₂</th>
<th>M₁₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₂</td>
<td>0.24</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.19</td>
<td></td>
<td>0.08</td>
<td>0.21</td>
</tr>
<tr>
<td>P₃</td>
<td>0.13</td>
<td></td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2</td>
<td>0.17</td>
<td>0.09</td>
</tr>
<tr>
<td>P₅</td>
<td></td>
<td>0.27</td>
<td>0.14</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.15</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>P₇</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.22</td>
<td>0.24</td>
<td>0.19</td>
<td></td>
<td></td>
<td>0.29</td>
<td>0.07</td>
</tr>
<tr>
<td>P₈</td>
<td></td>
<td></td>
<td></td>
<td>0.23</td>
<td></td>
<td>0.1</td>
<td>0.14</td>
<td></td>
<td></td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td>P₁₀</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3</td>
<td>0.32</td>
<td>0.13</td>
<td>0.21</td>
</tr>
</tbody>
</table>

### Table 4.4 Processing Times for the Second Product Family (Minutes)

<table>
<thead>
<tr>
<th></th>
<th>M₁</th>
<th>M₂</th>
<th>M₃</th>
<th>M₄</th>
<th>M₅</th>
<th>M₆</th>
<th>M₇</th>
<th>M₈</th>
<th>M₉</th>
<th>M₁₀</th>
<th>M₁₁</th>
<th>M₁₂</th>
<th>M₁₃</th>
<th>M₁₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁</td>
<td>0.29</td>
<td>0.21</td>
<td></td>
<td>0.08</td>
<td></td>
<td>0.11</td>
<td>0.4</td>
<td>0.4</td>
<td></td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>P₄</td>
<td>0.29</td>
<td></td>
<td>0.12</td>
<td>0.06</td>
<td>0.09</td>
<td>0.1</td>
<td>0.22</td>
<td>0.31</td>
<td></td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P₆</td>
<td>0.29</td>
<td></td>
<td></td>
<td></td>
<td>0.16</td>
<td>0.12</td>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P₉</td>
<td>0.29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.42</td>
<td>0.4</td>
<td></td>
<td></td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>P₁₁</td>
<td></td>
<td>0.7</td>
<td>0.61</td>
<td>0.2</td>
<td>0.32</td>
<td>0.41</td>
<td></td>
<td></td>
<td>0.3</td>
<td>0.51</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P₁₂</td>
<td></td>
<td>0.6</td>
<td>0.23</td>
<td>0.15</td>
<td>0.42</td>
<td>0.42</td>
<td>1</td>
<td>0.6</td>
<td></td>
<td>0.21</td>
<td>0.33</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.5 Monthly Demand (Units)

<table>
<thead>
<tr>
<th></th>
<th>P₁</th>
<th>P₂</th>
<th>P₃</th>
<th>P₄</th>
<th>P₅</th>
<th>P₆</th>
<th>P₇</th>
<th>P₈</th>
<th>P₉</th>
<th>P₁₀</th>
<th>P₁₁</th>
<th>P₁₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>4256</td>
<td>14854</td>
<td>11601</td>
<td>7343</td>
<td>12646</td>
<td>5527</td>
<td>13127</td>
<td>9930</td>
<td>7986</td>
<td>9933</td>
<td>4765</td>
<td>4903</td>
</tr>
</tbody>
</table>
Tables 4.3 and 4.4 clarify that the products will be processed in machines within the same order. Also, they explain the machine skipping concept in the flowshop cell, which means the product isn’t processed in all machines in the cell. For instance, P_7 would be processed in 7 machines out of 14 machines. Based on that, two rules are used to define the processing time dissimilarity coefficient. The first rule determines that the processing time dissimilarity coefficient depends on the average of the processing time difference for product pairs (k & l) in all machines in a cell, so all machines are taken into consideration as explained in the equation 3.1. It is worth mentioning that some machines do not process product pairs because the processing time for the product pairs equals “0”.

The second rule calculates the processing time dissimilarity coefficients depending on the average of the difference between the processing times of machines which processing either product “i” or “j” as explained in equation 3.2. For example, P_4 and P_9 are processed on 8 machines out 14 machines (Total Number of Machines in the Cell). In this case, the dissimilarity coefficient is calculated based on the eight machines only.

4.3.2 Deterministic Demand and Capacity Restriction

Typically, in the industrial environment, the demand is unstable, and it varies from one season to others. In this study, the demand for the product is assumed constant. Using MS Excel random distribution function in the interval [4100, 15000] to generate the monthly demand, Table 4.5 explains the monthly demand for all products.

The transfer line or flowshop line is a group of dissimilar machines arranged in the series of line to process different products. The products should be processed in the machines before being ready in the final shape. The system builds each product separately
from the other products in the batch scheduling method. In this case, the production rate of the system depends on the processing time of the bottleneck machine when the bottleneck machine is a common machine for all products in the system. The bottleneck machine is the machine that has the highest processing time in the system.

Even though, the bottleneck machine is a shifting bottleneck machine, the production rate of products is determined based on the processing time of the bottleneck machine in the second phase. Shifting bottleneck machine means that each product has its own bottleneck machine. For instance, M_1 is the bottleneck machine for P_2, P_5, and P_8, M_5 is the bottleneck machine for P_{10}, M_8 is the bottleneck machine for P_3, and the M_{11} is the bottleneck machine for P_7. Therefore, the production rate is different from one product to others based on the different bottleneck machines and their processing times. For instance, the processing time of the bottleneck machine for P_1 is 0.4 min. Then, the hourly production rate of the system is (60 min/0.4 min = 150 units per hour). Accordingly, the system should be run (4256 units/ 150 units per hour = 28.4 hours) to cover the demand of P_1. Table 4.6 shows the production rate of the system for each product based on the processing time of the bottleneck machine, when the first technique of batch scheduling method is used to schedule the products in the system, and the capacity required to meet the demand of the all products.

However, two techniques of the batch scheduling method are utilized to schedule the product in the system. The first technique depends on the processing time of the bottleneck machine to define the production rate of the system. Also, the system processes the next product when the last item of the previous product is completed in the system. The
second technique depends on the Gantt chart to define the makespan of product in the system.

Table 4.6  Processing Time of Bottleneck Machine, Production Rate, and Capacity Required

<table>
<thead>
<tr>
<th></th>
<th>Processing Time of BM (min)</th>
<th>Bottleneck Machine</th>
<th>Production Rate (Units/Hour)</th>
<th>Capacity Required (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P&lt;sub&gt;1&lt;/sub&gt;</td>
<td>0.4</td>
<td>M&lt;sub&gt;8&lt;/sub&gt;, M&lt;sub&gt;9&lt;/sub&gt;</td>
<td>150</td>
<td>28.373</td>
</tr>
<tr>
<td>P&lt;sub&gt;2&lt;/sub&gt;</td>
<td>0.24</td>
<td>M&lt;sub&gt;1&lt;/sub&gt;</td>
<td>250</td>
<td>59.416</td>
</tr>
<tr>
<td>P&lt;sub&gt;3&lt;/sub&gt;</td>
<td>0.2</td>
<td>M&lt;sub&gt;8&lt;/sub&gt;</td>
<td>300</td>
<td>38.67</td>
</tr>
<tr>
<td>P&lt;sub&gt;4&lt;/sub&gt;</td>
<td>0.31</td>
<td>M&lt;sub&gt;9&lt;/sub&gt;</td>
<td>194</td>
<td>37.939</td>
</tr>
<tr>
<td>P&lt;sub&gt;5&lt;/sub&gt;</td>
<td>0.27</td>
<td>M&lt;sub&gt;1&lt;/sub&gt;</td>
<td>222</td>
<td>56.907</td>
</tr>
<tr>
<td>P&lt;sub&gt;6&lt;/sub&gt;</td>
<td>0.4</td>
<td>M&lt;sub&gt;8&lt;/sub&gt;</td>
<td>150</td>
<td>36.847</td>
</tr>
<tr>
<td>P&lt;sub&gt;7&lt;/sub&gt;</td>
<td>0.29</td>
<td>M&lt;sub&gt;11&lt;/sub&gt;</td>
<td>207</td>
<td>63.447</td>
</tr>
<tr>
<td>P&lt;sub&gt;8&lt;/sub&gt;</td>
<td>0.23</td>
<td>M&lt;sub&gt;1&lt;/sub&gt;</td>
<td>261</td>
<td>38.065</td>
</tr>
<tr>
<td>P&lt;sub&gt;9&lt;/sub&gt;</td>
<td>0.42</td>
<td>M&lt;sub&gt;8&lt;/sub&gt;</td>
<td>143</td>
<td>55.902</td>
</tr>
<tr>
<td>P&lt;sub&gt;10&lt;/sub&gt;</td>
<td>0.32</td>
<td>M&lt;sub&gt;5&lt;/sub&gt;</td>
<td>188</td>
<td>52.976</td>
</tr>
<tr>
<td>P&lt;sub&gt;11&lt;/sub&gt;</td>
<td>0.7</td>
<td>M&lt;sub&gt;3&lt;/sub&gt;</td>
<td>85.7</td>
<td>55.592</td>
</tr>
<tr>
<td>P&lt;sub&gt;12&lt;/sub&gt;</td>
<td>1</td>
<td>M&lt;sub&gt;9&lt;/sub&gt;</td>
<td>60</td>
<td>81.717</td>
</tr>
</tbody>
</table>

4.3.3 Mathematical Model

In the second phase, the mathematical model is utilized to determine the product subfamilies in each product family through maximizing the variation in processing times of products in a subfamily.

The objective function used in the mathematical model is maximizing the processing time dissimilarity coefficient between product pairs as shown in the equation (4.4). The penalty is added to the objective function for each additional subfamily open. The penalty factor is assumed “1”. Equation (4.5) is used as capacity restriction as given
in Table 4.11. Also, equation (4.6) ensures that each product belongs only to one subfamily. Finally, equation (4.7) guarantees that a product is assigned to subfamily if it is open.

Maximize \[ Z = \sum_{i=1}^{m} \sum_{k=1}^{m} D_{ik} \times X_{ik} - \sum_{k=1}^{m} P \times X_{kk} \] \tag{4.4}

Subject to:

\[ \sum_{i=1}^{m} u_{i} \times X_{ik} < = T_{i} \quad k = 1, \ldots, m \] \tag{4.5}

\[ \sum_{k=1}^{m} X_{ik} = 1 \quad i = 1, \ldots, m \] \tag{4.6}

\[ X_{ik} \leq X_{kk} \quad i = 1, \ldots, m \text{ and } k = 1, \ldots, m \] \tag{4.7}

**Indices**

i Product index

k Product index and family cell index

**Parameters**

m number of products in a subfamily

D_{ik} Dissimilarity coefficient between product i and k

P Penalty for opening a new cell

u_{i} Capacity requirement for Product i (hr)

T_{i} Overall available capacity in the system (hr)

**Decision variable**

X_{ik} 1 if product i belongs to family k, 0 otherwise.

The CPLEX software is used to solve the model. Two subfamilies are created per each product family as a result of the mathematical model.
4.4 Phase Three: Batch-Cyclic Scheduling Method

In this phase, the batch scheduling method, cyclic set/minimal product set, batch-cyclic scheduling method, manual calculation, and the mathematical model are explained.

4.4.1 Batch Scheduling Method

Cell scheduling usually comes after cell loading. Scheduling literature is very rich and active area. In the batch method, the system processes each product separately from the other products. However, in this study two techniques of the batch method are used to calculate the makespan of products in the subfamily when the batch scheduling method is utilized to schedule the products in the system. The first technique depends on the processing time of the bottleneck machine to define the production rate of the system. In this case, the production rate of the system is different from one product to others due to different bottleneck machine processing times. For example, the system must be processed $P_2$, $P_3$, and $P_{10}$ ($59.42 \text{ hrs} + 38.67 \text{ hrs} + 52.98 \text{ hrs} = 151.06 \text{ hrs}$) to satisfy the demand for all products as shown in the following Gantt chart. However, the makespan of the subfamily ($P_2 P_3 P_{10}$) can be 151.06 hours based on the first technique of the batch scheduling method.

<table>
<thead>
<tr>
<th></th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>59.2 hr</td>
<td>97.87 hr</td>
<td>151.06h</td>
</tr>
</tbody>
</table>

The second technique relies on the Gantt chart to define the makespan of products in the system. This technique is more accurate because it considers all machines in the systems. For example, the makespan for the subfamily ($P_2 P_3 P_{10}$) can be 139.475 hours.
Based on this result, the second technique of the batch scheduling method, provides lower makespan compared to the first technique of the batch scheduling method. This difference between two techniques depends on shifting bottleneck machine, which means each product has its own bottleneck machine. However, the results show that the production rate of system depends on the processing time of the bottleneck machine, when the bottleneck machine is a common bottleneck machine.

However, this study introduces a new method (batch-cyclic scheduling method) to schedule the products in the flowshop system instead of the batch scheduling method and cyclic scheduling method. This method depends on minimal product set or cyclic set to cover the demand for products. Also, the mathematical model is developed based on the production rate of the minimal product set, which is calculated by using the manual calculation, to minimize the makespan and maximize the production rate, reduce the number of cells and improve the capacity parameter of the system.

4.4.2 Cyclic Set or Minimal Product Set Formation

Cyclic set (CS) or the minimal product set (MPS) is the smallest possible set of products in the same quantity as the overall production demand, which is to be respectively processed in a cyclic mode. The number of products and the volume of demand determine the number of sets required to meet the volume of demand. In addition, each product has its own sets. For example, the minimal product sets that might be required to produce $P_1$ are different than the minimal product sets for other products in the subfamily. Therefore, “Hasse Diagram” is used to define the Cyclic set (CS)/ the minimal product set (MPS) for each product (Godin, Pichet, & Gecsei, 1989; Hollert, Heise, Pudenz,
Brüggemann, Ahlf, & Braunbeck, 2002), which might be used to meet the demand of product at subfamily. Figure 4.2 shows a particular example, when the “Hasse diagram” is utilized to define the cyclic sets (MPS) required to meet the demand of P₂ at subfamily (P₂ P₃ P₁₀). As shown in the Figure 4.2, four cyclic sets might be used to cover the demand of P₂ (P₂, P₂P₁₀, P₂P₃, and P₂P₃P₁₀) based on the demand values. On the other hand, the demand value and the production rate for each minimal product set determine minimal product sets needed to meet the demand of products.

On the other hand, the number of cyclic sets required to meet the demand for a product depends on the number of products. The number of cyclic sets increases when the number of products increases. Figure 4.3 shows the number of cyclic sets needed to cover the demand for P₁ when the number of products in the subfamily equals four products (P₁, P₂, P₃, and P₄). Eight cyclic sets are constructed to cover the demand of P₁ once the batch-cyclic method is utilized to schedule the products in the flowshop system.
Table 4.7 explains the cyclic sets required to meet the demand of $(P_1, P_2, P_3, \text{and} P_4)$ when the batch-cyclic scheduling method is applied.

<table>
<thead>
<tr>
<th>Product</th>
<th>Cyclic Sets or Minimal Product Sets</th>
<th>Number of Cyclic Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$P_1P_1, P_1P_2, P_1P_3, P_1P_4, P_1P_2P_3, P_1P_2P_4, P_1P_3P_4, P_1P_2P_3P_4$</td>
<td>8</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$P_2, P_1P_2, P_2P_3, P_2P_4, P_1P_2P_3, P_1P_2P_4, P_2P_3P_4, P_1P_2P_3P_4$</td>
<td>8</td>
</tr>
<tr>
<td>$P_3$</td>
<td>$P_3, P_1P_3, P_2P_3, P_2P_4, P_1P_2P_3, P_1P_2P_4, P_2P_3P_4, P_1P_2P_3P_4$</td>
<td>8</td>
</tr>
<tr>
<td>$P_4$</td>
<td>$P_4, P_1P_4, P_2P_4, P_2P_4, P_1P_2P_4, P_1P_2P_4, P_2P_3P_4, P_1P_2P_3P_4$</td>
<td>8</td>
</tr>
</tbody>
</table>

In scheduling process in this research a combination of two methods, cyclic method and batch method, is used to schedule the products. The cyclic method determines the cyclic set or the minimal product set of products while repeating this cyclic set (minimal product set) over and over to cover the demand of the product is called a batch method. The Figure 4.4 explains the batch-cyclic scheduling method for three products when they are assigned into the flowshop system.
In this study, the manual calculation determines the production rate for each minimal product set. Table 4.8 shows the processing times for two products in three machines at flowshop system. Figure 4.5 and Figure 4.6 show the hourly production rate of the system when the manual calculation is utilized to schedule the product A and product B in the system based on the batch scheduling method. Figure 4.7 represents the hourly production rate of the system when the manual calculation schedules the product A and product B in mixed mode at 3-Machine flowshop.

<table>
<thead>
<tr>
<th>Table 4.8 Three Machines Flowshop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Product A</td>
</tr>
<tr>
<td>Product B</td>
</tr>
</tbody>
</table>
Three Machines Flowshop Running Product A (A1, A2, A3, A4…)

<table>
<thead>
<tr>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>15</td>
<td>A2</td>
<td>25</td>
</tr>
</tbody>
</table>

Hourly Production Rate = (480 min per day/10 min) = 48 Units/Day

Figure 4.5 Three Machines Flowshop Batch Scheduling Method

Three Machines Flowshop Running Product B

<table>
<thead>
<tr>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>10</td>
<td>B2</td>
<td>16</td>
</tr>
</tbody>
</table>

Hourly Production Rate = (480 min per day/15 min per unit) = 32 Units/Day

Figure 4.6 Three Machines Flowshop Batch Scheduling Method
4.4.4 Mathematical Model

A mathematical model is developed based on the production rate of the minimal product sets which are defined by using the manual calculation as mentioned before. The objective function of the model is minimizing the makespan of product in the system as given in equation (4.8). Makespan should be greater than zero as shown in equation (4.9). Equation (4.10) makes sure that system meets the whole demand of each product also the capacity restriction is added to the system and cyclic sets used to cover the demand for each product.

Minimize \( Z = ms \) \hspace{1cm} (4.8)

Subject to:

\[ ms - \sum_{nc=1}^{N} X_{nc} \geq 0 \] \hspace{1cm} (4.9)

\[ \sum_{k=1}^{S_i} P_{ik} * X_{ik} \geq D_i \quad i = 1, \ldots, n \] \hspace{1cm} (4.10)
Indices

i        Product index, 1 …. n
nc      Combination index
k        Cyclic set index

Parameters

D_i     Demand of Product i
n       Number of products
P_{ik}  Hourly production rate of ik
N       Total number of combinations including all products
S_i     Total number of combinations including product ‘i’
{ik}    Set of the combinations including product i

Decision variable

X_{ik}  Processing time of cyclic set “ik” in the system

CPLEX software is used to solve the mathematical model. Table 4.9 shows the results of the mathematical model to schedule the products of the subfamily (P_2 P_3 P_{10}) in a subfamily. Final results illustrate that the system runs the cyclic set P_2 P_3, P_2 P_{10}, and P_2 P_3 P_{10}, respectively 31.986 hours, 22.229 hours, and 64.573 hours, respectively to meet the demand of the products in the system as given in Figure 4.16. Also, the code is found in Appendix B.

<table>
<thead>
<tr>
<th>MPS</th>
<th>P_2</th>
<th>P_3</th>
<th>P_{10}</th>
<th>P_2 P_3</th>
<th>P_2 P_{10}</th>
<th>P_3 P_{10}</th>
<th>P_2 P_3 P_{10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (Hour)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>31.986</td>
<td>22.229</td>
<td>0.00</td>
<td>64.573</td>
</tr>
</tbody>
</table>

Table 4.9 Processing Time for each Set in the System (Hours)
The makespan of the system when it produces three products in the batch-cyclic scheduling method equals 118.788 hours, while the makespan for $P_2$, $P_3$, and $P_{10}$ for the first technique and the second technique of the batch scheduling method, can be 151.06 hours and 139.475 hours respectively. As a result, the batch-cyclic scheduling method provides minimum makespan time to process different products in the system compared to two techniques of the batch method. In this case, different factors are taken into consideration to implement this method such as the shifting bottleneck machine, zero setup time, and increasing the gap between the processing times between products in the system.
Chapter 5  Results and Discussion

In this chapter, three phases are discussed in detail. Firstly, the product family formulation based on maximizing the processing similarity coefficient is explained. Secondly, the product families are divided into subfamilies, where the subfamilies are formed based on maximizing or minimizing the processing time dissimilarity coefficient. In order to define the dissimilarity coefficient, two rules are utilized. Thirdly, a new scheduling method, batch-cyclic scheduling method, is developed to schedule the products in a subfamily at the flowshop cell. Besides batch-cyclic scheduling, the two techniques of the batch scheduling method are also utilized to schedule the products in the subfamilies. Also, the effect of shifting bottleneck machine is studied. The effect of the variation on the processing time on the machines will be explained. Modifying the capacity parameter also is discussed to improve the overall capacity parameter in the system and to reduce the number of cells. Table 5.1 shows the Sections in this chapter.

<table>
<thead>
<tr>
<th>Sec</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Product Family Formulation</td>
</tr>
<tr>
<td>5.2</td>
<td>Product Subfamily Identification and Cell Loading</td>
</tr>
<tr>
<td>5.3</td>
<td>Batch-Cyclic Scheduling Method</td>
</tr>
<tr>
<td>5.4</td>
<td>Minimize Dissimilarity Coefficient vs. Maximize Dissimilarity Coefficient</td>
</tr>
<tr>
<td>5.5</td>
<td>Batch-Cyclic Scheduling Method vs. Batch Scheduling Method</td>
</tr>
<tr>
<td>5.6</td>
<td>Variations around Bottleneck Machine with Batch-Cyclic Scheduling Method</td>
</tr>
<tr>
<td>5.7</td>
<td>Variations in terms of Total Processing Time per Machine with Batch-Cyclic Scheduling Method</td>
</tr>
<tr>
<td>5.8</td>
<td>Experiment with Another Example</td>
</tr>
<tr>
<td>5.9</td>
<td>Modify Capacity Parameter</td>
</tr>
</tbody>
</table>
5.1 Phase One: Product Family Formulation

The product-machine matrix consisting of 12 products and 14 machines as the data given in the Table 3.2. Based on the machine skipping concept, the products aren’t processed by all machines in the flowshop cell. For example, P₁ must be processed in 8 machines (M₁, M₃, M₅, M₇, M₈, M₉, M₁₁, and M₁₃) out of 14 machines in the system. Each product has its own bottleneck machine based on the shifting bottleneck machine. Also, some products have one bottleneck machine, while others have two bottleneck machines. For example, P₃, P₆, and P₉ have the M₈ as a bottleneck machine, and P₁ has two bottleneck machines M₈ & M₉. Therefore, the system has 6 bottleneck machines and 12 products. The processing similarity coefficients between products are calculated as shown in Table 5.2 and later used to form product families.

<table>
<thead>
<tr>
<th></th>
<th>P₁</th>
<th>P₂</th>
<th>P₃</th>
<th>P₄</th>
<th>P₅</th>
<th>P₆</th>
<th>P₇</th>
<th>P₈</th>
<th>P₉</th>
<th>P₁₀</th>
<th>P₁₁</th>
<th>P₁₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₂</td>
<td>0.23</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₃</td>
<td>0.25</td>
<td>0.67</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₄</td>
<td>0.78</td>
<td>0.33</td>
<td>0.36</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₅</td>
<td>0.17</td>
<td>0.75</td>
<td>0.44</td>
<td>0.27</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₆</td>
<td>0.56</td>
<td>0.27</td>
<td>0.3</td>
<td>0.56</td>
<td>0.33</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₇</td>
<td>0.17</td>
<td>0.75</td>
<td>0.63</td>
<td>0.27</td>
<td>0.50</td>
<td>0.20</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₈</td>
<td>0.25</td>
<td>0.67</td>
<td>0.75</td>
<td>0.36</td>
<td>0.63</td>
<td>0.30</td>
<td>0.44</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₉</td>
<td>0.67</td>
<td>0.36</td>
<td>0.40</td>
<td>0.88</td>
<td>0.30</td>
<td>0.63</td>
<td>0.30</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₁₀</td>
<td>0.27</td>
<td>0.40</td>
<td>0.18</td>
<td>0.27</td>
<td>0.20</td>
<td>0.09</td>
<td>0.33</td>
<td>0.18</td>
<td>0.18</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P₁₁</td>
<td>0.33</td>
<td>0.23</td>
<td>0.36</td>
<td>0.33</td>
<td>0.17</td>
<td>0.27</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.17</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>P₁₂</td>
<td>0.42</td>
<td>0.21</td>
<td>0.14</td>
<td>0.31</td>
<td>0.25</td>
<td>0.25</td>
<td>0.15</td>
<td>0.23</td>
<td>0.23</td>
<td>0.36</td>
<td>0.55</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.2 Processing Similarity Coefficient Matrix
Once the similarity coefficient is calculated, the P-median model is used to determine product family to maximize the processing similarity coefficients between products. Several product families are obtained by using different penalty factors as a data shown in the Appendix C. The number of product families is decreased, when the penalty factor increases. In this study, the penalty factor assumes to be “1”. Two product families are created as the result of the model, when it is solved by using the CPLEX software. Table 5.3 shows the two product families and their corresponding machines.

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Products</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF₁</td>
<td>P₂, P₃, P₅, P₇, P₈, P₁₀</td>
<td>M₁, M₂, M₄, M₅, M₆, M₇, M₈, M₁₀, M₁₁, M₁₂, M₁₄</td>
</tr>
<tr>
<td>PF₂</td>
<td>P₁, P₄, P₆, P₉, P₁₁, P₁₂</td>
<td>M₁, M₂, M₃, M₄, M₅, M₆, M₇, M₈, M₉, M₁₀, M₁₁, M₁₂, M₁₃, M₁₄</td>
</tr>
</tbody>
</table>

5.2 Phase Two: Product Subfamily Identification and Cell Loading

Two topics will be explained in this phase: two rules to define the processing time dissimilarity coefficient, and the capacitated P-median model to maximize the processing time dissimilarity coefficient.

5.2.1 Two Rules to Define the Processing Time Dissimilarity Coefficient

In this phase, the subfamilies are created by dividing the product family into subfamilies with maximizing processing time dissimilarity approach. It is believed that increasing the dissimilarity coefficient between products in each subfamily affects the performance of system positively, where the batch-cyclic scheduling method is adopted to
schedule the products in the flowshop cell. Also, the subfamily formations based on the minimizing the processing time dissimilarity coefficient will be studied to show the affect of maximizing and minimizing in the processing time dissimilarity coefficient in the system performance. In this regard, two rules are utilized to define the processing time dissimilarity coefficient. The first rule relies on the average of a difference for the processing time for two products including all machines in the system based on the Equation 3.1 since the machine skipping is the one of the assumption in this study. Some machines are included in the calculation while the processing time for the product pairs in these machines equals zero. Table 5.4 and Table 5.5 show the processing time dissimilarity coefficients for the first and the second product family using the first rule.

Table 5.4 Processing Time Dissimilarity Coefficient for the First Product Family Using the First Rule

<table>
<thead>
<tr>
<th></th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_5$</th>
<th>$P_7$</th>
<th>$P_8$</th>
<th>$P_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_2$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.0527</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_5$</td>
<td>0.0455</td>
<td>0.0727</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_7$</td>
<td>0.0473</td>
<td>0.08</td>
<td>0.0764</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_8$</td>
<td>0.0427</td>
<td>0.0336</td>
<td>0.0445</td>
<td>0.0773</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$P_{10}$</td>
<td>0.1464</td>
<td>0.1682</td>
<td>0.1645</td>
<td>0.13</td>
<td>0.1655</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.5 Processing Time Dissimilarity Coefficient for the Second Product Family Using the First Rule

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_4$</th>
<th>$P_6$</th>
<th>$P_9$</th>
<th>$P_{11}$</th>
<th>$P_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.0414</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_6$</td>
<td>0.0586</td>
<td>0.0629</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_9$</td>
<td>0.0286</td>
<td>0.0343</td>
<td>0.0457</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P_{11}$</td>
<td>0.2286</td>
<td>0.2286</td>
<td>0.2557</td>
<td>0.2386</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$P_{12}$</td>
<td>0.275</td>
<td>0.2907</td>
<td>0.3164</td>
<td>0.3036</td>
<td>0.2721</td>
<td>0</td>
</tr>
</tbody>
</table>
In order to machine skipping, the processing times for some product pairs in some machines equal zero. In this case, the second rule is developed to avoid including idle machines in calculating the dissimilarity coefficient. In other words, this rule uses the average of the differences between the processing times of machines which processing either product “i” or “j”. Table 5.6 and Table 5.7 represent the processing time dissimilarity coefficient based on the second rule of the dissimilarity coefficient.

Table 5.6 Processing Time Dissimilarity Coefficients for the First Product Family Using the Second Rule

<table>
<thead>
<tr>
<th></th>
<th>P2</th>
<th>P3</th>
<th>P5</th>
<th>P7</th>
<th>P8</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P3</td>
<td>0.0644</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P5</td>
<td>0.0625</td>
<td>0.0889</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P7</td>
<td>0.065</td>
<td>0.11</td>
<td>0.105</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P8</td>
<td>0.0522</td>
<td>0.0463</td>
<td>0.0613</td>
<td>0.0944</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>P10</td>
<td>0.161</td>
<td>0.1682</td>
<td>0.181</td>
<td>0.1589</td>
<td>0.1655</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.7 Processing Time Dissimilarity Coefficients for the Second Product Family Using the Second Rule

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P4</th>
<th>P6</th>
<th>P9</th>
<th>P11</th>
<th>P12</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P4</td>
<td>0.0644</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P6</td>
<td>0.0911</td>
<td>0.0978</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P9</td>
<td>0.0444</td>
<td>0.06</td>
<td>0.08</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P11</td>
<td>0.2667</td>
<td>0.2667</td>
<td>0.3255</td>
<td>0.2783</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>P12</td>
<td>0.3208</td>
<td>0.3131</td>
<td>0.3692</td>
<td>0.3269</td>
<td>0.3464</td>
<td>0</td>
</tr>
</tbody>
</table>
5.2.2 Capacitated P-Median Model

Having determined the processing time dissimilarity coefficients, the capacitated P-median mathematical model is developed to divide the product family into subfamilies with maximizing the processing time dissimilarity coefficient. The gap between processing times for products in a subfamily is large since the subfamilies are created considering maximizing the processing time dissimilarity principle. Table 5.8 and Table 5.9 show the subfamilies and their corresponding machines, which are created by using capacitated P-median model when processing time dissimilarity coefficients are calculated using the first rule and the second rule of dissimilarity coefficient respectively. Each product family is divided into two subfamilies in two rules with three products per each subfamily. Also, each product has its own bottleneck machine. For example, the bottleneck machine for products in the subfamily (P₂ P₃ P₁₀) is M₁, M₈, and M₅, respectively. While the bottleneck machine is a common machine for the products in the subfamily (P₁ P₄ P₁₂), which is M₈.

Table 5.8 First Rule-Product Subfamilies and their Corresponding Machines

<table>
<thead>
<tr>
<th></th>
<th>Products</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF₁</td>
<td>SPF₁ P₂, P₃, and P₁₀</td>
<td>M₁, M₂, M₄, M₅, M₆, M₇, M₈, M₁₀, M₁₁, M₁₂, M₁₄</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₅, P₇, and P₈</td>
<td>M₁, M₂, M₄, M₆, M₈, M₁₀, M₁₁, M₁₂, M₁₄</td>
</tr>
<tr>
<td>PF₂</td>
<td>SPF₁ P₁, P₉, and P₁₁</td>
<td>M₁, M₃, M₄, M₅, M₆, M₇, M₈, M₉, M₁₁, M₁₂, M₁₃, M₁₄</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₄, P₆, and P₁₂</td>
<td>M₁, M₂, M₃, M₄, M₅, M₆, M₇, M₈, M₉, M₁₀, M₁₁, M₁₂, M₁₃</td>
</tr>
</tbody>
</table>
5.3 Phase Three: Batch-Cyclic Scheduling Method

In this phase, minimal product set, manual calculation and the mathematical model will be discussed in detail to schedule the products in the subfamilies based on the batch-cyclic scheduling method. The “Hasse diagram” is utilized to determine the minimal product sets. The manual calculation is used to define the production rate for each minimal product set. Finally, the mathematical model is adopted to obtain the makespan of products in the cell, and to define the processing time for each minimal product set in the system.

5.3.1 Minimal Product Set (MPS)/Cyclic Set (CS)

Based on demand values and the number of products in the system, several minimal product sets have to be used to meet the demand of products. In this regard, the “Hasse Diagram” is used to determine the minimal product sets which might be utilized to cover the demand value of a product in the flowshop cell. Figure 5.3, Figure 5.4 and Figure 5.5 illustrate the minimal product sets which might be used to cover the demand for P₂, P₃ and P₁₀ respectively in the subfamily (P₂ P₃ P₁₀).

<table>
<thead>
<tr>
<th></th>
<th>Products</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF₁</td>
<td>SPF₁ P₂, P₈, and P₁₀</td>
<td>M₁, M₂, M₄, M₅, M₆, M₇,M₈,M₁₀, M₁₁, M₁₂, M₁₄</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₃, P₅, and P₇</td>
<td>M₁, M₂, M₄, M₆, M₈, M₁₀, M₁₁, M₁₂, M₁₄</td>
</tr>
<tr>
<td>PF₂</td>
<td>SPF₁ P₁, P₄, and P₁₂</td>
<td>M₁, M₃, M₄, M₅, M₆, M₇, M₈, M₉, M₁₁, M₁₂, M₁₃, M₁₄</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₆, P₉, and P₁₁</td>
<td>M₁, M₃, M₄, M₅, M₆, M₇, M₈, M₉, M₁₁, M₁₂, M₁₃, M₁₄</td>
</tr>
</tbody>
</table>
Figure 5.1 Minimal Product Sets to Meet the Demand for $P_2$ at Subfamily $(P_2 \ P_3 \ P_{10})$

Figure 5.2 Minimal Product Sets to Meet the Demand for $P_3$ at Subfamily $(P_2 \ P_3 \ P_{10})$

Figure 5.3 Minimal Product Sets to Meet the Demand for $P_{10}$ at Subfamily $(P_2 \ P_3 \ P_{10})$
Four Minimal Product sets might be utilized to cover the demand for each product in the subfamily. Sometimes, four minimal product sets are used to cover the demand, while others three, two or one minimal product sets are appropriate to meet the demand. However, the minimal product set \((P_2P_3P_{10})\) is a common minimal product set for all products in the subfamily, because three products \((P_2, P_3 \text{ and } P_{10})\) are processed in this set. Seven minimal product sets \((P_2, P_3, P_{10}, P_2P_3, P_2P_{10}, P_3P_{10}, P_2P_3P_{10})\) are utilized to cover the demand values for three products in the subfamily \((P_2 \ P_3 \ P_{10})\).

### 5.3.2 Manual Calculation

In this section, the manual calculation is utilized to determine the production rate for each minimal product set, since the processing time and demand are deterministic data. Table 5.10 and Table 5.11 show the cycle time for the minimal product sets based on the manual calculation when the batch-cyclic method schedules the products in the flowshop cell. Cycle time represents the time required to produce a minimal product set in the system in mixed mode.
Table 5.10 Cycle Time for the First Rule-Minimal Product Sets with Maximizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₂, P₃, and P₁₀</td>
<td>P₅, P₇, and P₈</td>
</tr>
<tr>
<td>P₂</td>
<td>0.24</td>
</tr>
<tr>
<td>P₃</td>
<td>0.20</td>
</tr>
<tr>
<td>P₁₀</td>
<td>0.32</td>
</tr>
<tr>
<td>P₂P₃</td>
<td>0.39</td>
</tr>
<tr>
<td>P₂P₁₀</td>
<td>0.41</td>
</tr>
<tr>
<td>P₅P₁₀</td>
<td>0.37</td>
</tr>
<tr>
<td>P₂P₃P₁₀</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 5.11 Cycle Time for the Second Rule-Minimal Product Sets with Maximizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁, P₉, and P₁₁</td>
<td>P₄, P₆, and P₁₂</td>
</tr>
<tr>
<td>P₁</td>
<td>0.4</td>
</tr>
<tr>
<td>P₉</td>
<td>0.42</td>
</tr>
<tr>
<td>P₁₁</td>
<td>0.7</td>
</tr>
<tr>
<td>P₁P₉</td>
<td>0.82</td>
</tr>
<tr>
<td>P₁P₁₁</td>
<td>0.91</td>
</tr>
<tr>
<td>P₉P₁₁</td>
<td>0.87</td>
</tr>
<tr>
<td>P₁P₉P₁₁</td>
<td>1.21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₂, P₈, and P₁₀</td>
<td>P₃, P₅, and P₇</td>
</tr>
<tr>
<td>P₂</td>
<td>0.24</td>
</tr>
<tr>
<td>P₈</td>
<td>0.23</td>
</tr>
<tr>
<td>P₁₀</td>
<td>0.32</td>
</tr>
<tr>
<td>P₂P₈</td>
<td>0.47</td>
</tr>
<tr>
<td>P₂P₁₀</td>
<td>0.41</td>
</tr>
<tr>
<td>P₈P₁₀</td>
<td>0.37</td>
</tr>
<tr>
<td>P₂P₈P₁₀</td>
<td>0.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁, P₄, and P₁₂</td>
<td>P₆, P₉, and P₁₁</td>
</tr>
<tr>
<td>P₁</td>
<td>0.40</td>
</tr>
<tr>
<td>P₄</td>
<td>0.31</td>
</tr>
<tr>
<td>P₁₂</td>
<td>1.00</td>
</tr>
<tr>
<td>P₁P₄</td>
<td>0.71</td>
</tr>
<tr>
<td>P₁P₁₂</td>
<td>1.40</td>
</tr>
<tr>
<td>P₄P₁₂</td>
<td>1.31</td>
</tr>
<tr>
<td>P₁P₄P₁₂</td>
<td>1.71</td>
</tr>
</tbody>
</table>
5.3.3 Batch-Cyclic Scheduling- Mathematical Model

The mathematical model is utilized in this phase to schedule the minimal product sets in the system. As mentioned before, the objective function of the model is minimizing the makespan of products in the system. The proposed model is solved in OPL-CPLEX for all subfamilies.

The mathematical model is developed to find the makespan and to define the processing time for each minimal product set. When the program is run, the processing time for each minimal product set and the makespan of products in the system are obtained. The solution obtained is in terms of the “Real Numbers” and “Zero”, where real numbers represent the processing time for the minimal product sets and makespan for products in the system, and “Zero” represents that the processing time for the minimal product set in the system equals “Zero”. Table 5.12 shows the makespan for first rule-subfamilies. Table 5.13 illustrates the makespan for the second rule-subfamilies.

Makespan for the subfamily \((P_2 \ P_3 \ P_{10})\) equals 118.788 hours, where 64.573 is processing time for the minimal product set \(P_2P_3P_{10}\), 22.229 hours is processing time for \(P_2P_{10}\), and 31.986 hours is processing time for \(P_2P_3\). The system produces 4921 units of \(P_2 \ P_3\) when the system runs the minimal product set \((P_2P_3)\) for 31.986 hours. The system produces 6678 units of \(P_2 \ P_3 \ P_{10}\) when the minimal product set \((P_2P_3P_{10})\) is processed in the system for 64.573 hours, and the system produces 3254 units of \(P_2 \ P_{10}\), when the minimal product set \((P_2P_{10})\) is processed in the system for 22.229 hours. In this case, the system depends on the three minimal product sets \((P_2P_3, P_2P_{10}, \text{and } P_2P_3P_{10})\) out of seven minimal product sets to cover the demand for three products in the system. Therefore, the
system depends on the mixed mode to cover the whole demand values for three products, 14854 units for P_2, 11601 units for P_3 and 9933 units for P_{10} in the subfamily (P_2 P_3 P_{10}). Additionally, the makespan of the system equals 143.18 hours and 122.88 hours for the first rule- first product family and the second rule- second product family respectively. The makespan 130.734 hours and 148.03 hours respectively for the first and the second product family when the second rule of the dissimilarity coefficient is utilized to define the processing time dissimilarity coefficient.
Table 5.12 Makespan for the First Rule-Subfamilies with Maximizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Product Family</strong></td>
<td><strong>Second Product Family</strong></td>
</tr>
<tr>
<td>P2, P3, and P10</td>
<td>P5, P7, and P8</td>
</tr>
<tr>
<td>P2</td>
<td>0</td>
</tr>
<tr>
<td>P3</td>
<td>0</td>
</tr>
<tr>
<td>P10</td>
<td>0</td>
</tr>
<tr>
<td>P2 P3</td>
<td>31.986</td>
</tr>
<tr>
<td>P2 P10</td>
<td>22.229</td>
</tr>
<tr>
<td>P3 P10</td>
<td>0</td>
</tr>
<tr>
<td>P2 P3 P10</td>
<td>64.573</td>
</tr>
<tr>
<td>Makespan</td>
<td>118.788</td>
</tr>
</tbody>
</table>

Makespan 118.788

Table 5.13 Makespan for the Second Rule-Subfamilies with Maximizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Product Family</strong></td>
<td><strong>Second Product Family</strong></td>
</tr>
<tr>
<td>P2, P8, and P10</td>
<td>P3, P5, and P7</td>
</tr>
<tr>
<td>P2</td>
<td>19.684</td>
</tr>
<tr>
<td>P8</td>
<td>0</td>
</tr>
<tr>
<td>P10</td>
<td>0</td>
</tr>
<tr>
<td>P2 P8</td>
<td>0</td>
</tr>
<tr>
<td>P2 P10</td>
<td>0.0205</td>
</tr>
<tr>
<td>P8 P10</td>
<td>0</td>
</tr>
<tr>
<td>P2 P8 P10</td>
<td>95.99</td>
</tr>
<tr>
<td>Makespan</td>
<td>115.695</td>
</tr>
</tbody>
</table>

Makespan 148.03

Makespan 115.7561

Makespan 115.7561
5.4 Minimize Dissimilarity Coefficient vs. Maximize Dissimilarity Coefficient

In this section, the mathematical model developed earlier is used to divide a product family into subfamilies with a different objective. The objective in this section minimizes the processing time dissimilarity coefficient instead of maximizing the processing time dissimilarity coefficient. In this case, the differences between the processing times of products in each product family is expected to be low, once the subfamilies are created based on minimizing the dissimilarity coefficient.

Table 5.14 and Table 5.15 show the results of the mathematical model when the subfamilies are created based on minimizing the dissimilarity coefficient with their corresponding machines. The results show that each product family is divided into two subfamilies with different products per each subfamily. For example, \(P_1, P_4, P_6,\) and \(P_9\) in the first and second rule belong to first subfamily, and \(P_{11},\) and \(P_{12}\) represent the second subfamily in two rules of dissimilarity coefficient. Also, \(P_1,\) and \(P_4\) have \(M_9\) as a bottleneck machine, and \(P_1, P_6,\) and \(P_9\) have \(M_8\) as a common bottleneck machine.

<table>
<thead>
<tr>
<th>(PF_1)</th>
<th>Products</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF</td>
<td>(P_2, P_5,) and (P_8)</td>
<td>(M_1, M_2, M_4, M_6, M_8, M_{10}, M_{11}, M_{12}, M_{14})</td>
</tr>
<tr>
<td>SPF</td>
<td>(P_3, P_7,) and (P_{10})</td>
<td>(M_1, M_2, M_4, M_5, M_6, M_7, M_8, M_{10}, M_{11}, M_{12}, M_{14})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(PF_2)</th>
<th>Products</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF</td>
<td>(P_1, P_4, P_6,) and (P_9)</td>
<td>(M_1, M_3, M_5, M_6, M_7, M_8, M_9, M_{11}, M_{13})</td>
</tr>
<tr>
<td>SPF</td>
<td>(P_{11},) and (P_{12})</td>
<td>(M_2, M_3, M_4, M_5, M_6, M_7, M_9, M_{10}, M_{12}, M_{13}, M_{14})</td>
</tr>
</tbody>
</table>
Table 5.15 Second Rule-Subfamilies and their Corresponding Machines with Minimizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th></th>
<th>Products</th>
<th>Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF₁</td>
<td>SPF₁ P₂, P₃, and P₅</td>
<td>M₁, M₂, M₄, M₆, M₈, M₁₀, M₁₁, M₁₂, M₁₄</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₇, P₈, and P₁₀</td>
<td>M₁, M₂, M₄, M₅, M₆, M₇, M₈, M₁₀, M₁₁, M₁₂, M₁₄</td>
</tr>
<tr>
<td>PF₂</td>
<td>SPF₁ P₁, P₄, P₆ and P₉</td>
<td>M₁, M₃, M₅, M₆, M₇, M₈, M₉, M₁₁, M₁₃</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₁₁, and P₁₂</td>
<td>M₂, M₃, M₄, M₅, M₆, M₇, M₉, M₁₀, M₁₂, M₁₃, M₁₄</td>
</tr>
</tbody>
</table>

Additionally, several minimal product sets might be used to meet the demand of products. As mentioned before, the “Hassa Diagram” is utilized to define the minimal product sets/Cyclic sets to cover the demand of subfamily. Figure 5.4 represents the minimal product sets that might be used to cover the demand for P₁ in the subfamily (P₁ P₄ P₆ P₉).

![Figure 5.4 Minimal Product Sets to Meet the Demand for P₁ in Subfamily (P₁ P₄ P₆ P₉)
The manual calculation is utilized to define the production rate for the minimal product sets of a subfamily. Table 5.16 and Table 5.17 show the cycle time for the minimal product sets for four subfamilies in the system based on the first rule and the second rule of the processing time dissimilarity coefficient. The number of the minimal product sets required increases when the number of products in a subfamily increases. For example, 15 minimal product sets are generated to meet the demand values for products in the subfamily \((P_1, P_4, P_6, P_9)\), while 3 minimal product set are generated to meet the demand values for the subfamily \((P_{11}, P_{12})\).

<table>
<thead>
<tr>
<th>First Main Product Family</th>
<th>Second Main Product Family</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF1</td>
<td>SPF1</td>
</tr>
<tr>
<td>P_2, P_5, and P_8</td>
<td>P_3, P_7, and P_10</td>
</tr>
<tr>
<td>P_2</td>
<td>0.24</td>
</tr>
<tr>
<td>P_5</td>
<td>0.27</td>
</tr>
<tr>
<td>P_8</td>
<td>0.23</td>
</tr>
<tr>
<td>P_2P_5</td>
<td>0.51</td>
</tr>
<tr>
<td>P_2P_8</td>
<td>0.47</td>
</tr>
<tr>
<td>P_5P_8</td>
<td>0.50</td>
</tr>
<tr>
<td>P_2P_5P_8</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Table 5.17 Cycle Time for the Second Rule-Minimal Product Sets with Minimizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>First Main Product Family</th>
<th>Second Main Product Family</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPF1</td>
</tr>
<tr>
<td>P₂, P₃, and P₅</td>
<td>P₇, P₈, and P₁₀</td>
</tr>
<tr>
<td>P₂</td>
<td>0.24</td>
</tr>
<tr>
<td>P₃</td>
<td>0.20</td>
</tr>
<tr>
<td>P₅</td>
<td>0.27</td>
</tr>
<tr>
<td>P₂P₃</td>
<td>0.39</td>
</tr>
<tr>
<td>P₂P₅</td>
<td>0.51</td>
</tr>
<tr>
<td>P₃P₅</td>
<td>0.40</td>
</tr>
<tr>
<td>P₂P₃P₅</td>
<td>0.64</td>
</tr>
<tr>
<td>P₁, P₆, P₉, and P₁₁</td>
<td>P₁₁ and P₁₂</td>
</tr>
<tr>
<td>P₁</td>
<td>0.40</td>
</tr>
<tr>
<td>P₄</td>
<td>0.31</td>
</tr>
<tr>
<td>P₆</td>
<td>0.4</td>
</tr>
<tr>
<td>P₉</td>
<td>0.42</td>
</tr>
<tr>
<td>P₁P₄</td>
<td>0.71</td>
</tr>
<tr>
<td>P₁P₆</td>
<td>0.8</td>
</tr>
<tr>
<td>P₁P₉</td>
<td>0.82</td>
</tr>
<tr>
<td>P₄P₆</td>
<td>0.62</td>
</tr>
<tr>
<td>P₄P₉</td>
<td>0.71</td>
</tr>
<tr>
<td>P₆P₉</td>
<td>0.82</td>
</tr>
<tr>
<td>P₁P₄P₆</td>
<td>1.02</td>
</tr>
<tr>
<td>P₁P₄P₉</td>
<td>1.11</td>
</tr>
<tr>
<td>P₁P₆P₉</td>
<td>1.22</td>
</tr>
<tr>
<td>P₄P₆P₉</td>
<td>1.04</td>
</tr>
<tr>
<td>P₁P₄P₆P₉</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Once the minimal product sets are determined for each subfamily, the mathematical model is developed based on the manual calculation results. The results of the mathematical model determine the makespan of the subfamily and the processing time for minimal product sets as the data given in Table 5.18, Table 5.19 and Table 5.20. Table 5.18 shows that the makespan for the first rule- first product family equals 154.387 hours. Table 5.19 illustrates that the makespan for the second rule- first product family equals 141.4583 hours. Table 5.20 explains that the makespan for the second product family based on the first and second rule of processing time dissimilarity coefficient can be 150.16 hours. Table 5.21 shows the makespan for the products in the system based on the first and the second rule of the dissimilarity coefficient when the batch-cyclic scheduling method is utilized to
schedule the products in the subfamily. As a result, the creating the subfamilies based on maximizing the processing time dissimilarity coefficient provides less makespan in the two rules.

Table 5.18 Makespan for the First Rule-First Product Family with Minimizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>First Main Product Family</th>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2, P5, and P8</td>
<td>P3, P7, and P10</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>0</td>
<td>P3</td>
</tr>
<tr>
<td>P5</td>
<td>0</td>
<td>P7</td>
</tr>
<tr>
<td>P8</td>
<td>29.601</td>
<td>P10</td>
</tr>
<tr>
<td>P2P5</td>
<td>107.49</td>
<td>P3P7</td>
</tr>
<tr>
<td>P2P8</td>
<td>17.296</td>
<td>P3P10</td>
</tr>
<tr>
<td>P3P8</td>
<td>0</td>
<td>P7P10</td>
</tr>
<tr>
<td>P2P5P8</td>
<td>0</td>
<td>P3P7P10</td>
</tr>
<tr>
<td>Makespan</td>
<td>154.387</td>
<td>Makespan</td>
</tr>
</tbody>
</table>

Table 5.19 Makespan for the First Rule-Second Product Family with Minimizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>First Main Product Family</th>
<th>SPF1</th>
<th>SPF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2, P3, and P5</td>
<td>P7, P8, and P10</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>55.236</td>
<td>P7</td>
</tr>
<tr>
<td>P3</td>
<td>0</td>
<td>P8</td>
</tr>
<tr>
<td>P5</td>
<td>0</td>
<td>P10</td>
</tr>
<tr>
<td>P2P3</td>
<td>0</td>
<td>P7P8</td>
</tr>
<tr>
<td>P2P5</td>
<td>8.8823</td>
<td>P7P10</td>
</tr>
<tr>
<td>P3P5</td>
<td>77.34</td>
<td>P8P10</td>
</tr>
<tr>
<td>P2P3P5</td>
<td>0</td>
<td>P7P8P10</td>
</tr>
<tr>
<td>Makespan</td>
<td>141.4583</td>
<td>Makespan</td>
</tr>
</tbody>
</table>
Table 5.20 Makespan for the Second Product Family Using the First and the Second Rule with Minimizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>Second Main Product Family</th>
<th>SPF₁</th>
<th>SPF₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁, P₆, P₉, and P₁₁</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P₁</td>
<td>0</td>
<td>P₁₁</td>
</tr>
<tr>
<td>P₄</td>
<td>0</td>
<td>P₁₂</td>
</tr>
<tr>
<td>P₆</td>
<td>0</td>
<td>P₁₁P₁₂</td>
</tr>
<tr>
<td>P₁₀</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>P₉</td>
<td>13.398</td>
</tr>
<tr>
<td></td>
<td>P₁₀P₄</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₁₀P₆</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₁₀P₉</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₄P₆</td>
<td>13.134</td>
</tr>
<tr>
<td></td>
<td>P₄P₉</td>
<td>21.489</td>
</tr>
<tr>
<td></td>
<td>P₆P₉</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₁₀P₄P₆</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₁₀P₄P₉</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₁₀P₆P₉</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₄P₆P₉</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>P₁₀P₄P₆P₉</td>
<td>102.14</td>
</tr>
<tr>
<td>Makespan</td>
<td>150.16</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.21 Makespan for Products in the Flowshop Based on Batch-Cyclic Scheduling Method in the First and the Second Rule (Hours)

<table>
<thead>
<tr>
<th>First Rule of the Dissimilarity Coefficient</th>
<th>Second Rule of the Dissimilarity Coefficient</th>
<th>Makespan (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximize the Dissimilarity Coefficient</td>
<td>Minimize the Dissimilarity Coefficient</td>
<td>143.18</td>
</tr>
<tr>
<td>Minimize the Dissimilarity Coefficient</td>
<td></td>
<td>154.387</td>
</tr>
<tr>
<td>Maximize the Dissimilarity Coefficient</td>
<td>Minimize the Dissimilarity Coefficient</td>
<td>148.03</td>
</tr>
<tr>
<td>Minimize the Dissimilarity Coefficient</td>
<td></td>
<td>150.16</td>
</tr>
</tbody>
</table>
5.5 Batch-Cyclic Scheduling Method vs. Batch Scheduling Method

Generally, in the batch method the system depends on the covering the whole demand for each product separately than other products. In this regard, several assumptions effect of this mechanism such as, start time for products once the first product is processed in the system, common bottleneck machine and shifting bottleneck machine. However, in this section, two groups will be developed to test the system performance. The first group determines the makespan of products in the system when the bottleneck machine shifts, and the second group defines the makespan of products in the system when the bottleneck machine is a common machine.

5.5.1 Batch Scheduling Method with Shifting Bottleneck Machine

The bottleneck machine is the slowest machine in the system. The shifting bottleneck machine is the main assumption in this study. Shifting bottleneck machine means different products have different bottleneck machines. Table 5.22 explains the original data for Subfamily (P₂ P₃ P₁₀).

<table>
<thead>
<tr>
<th></th>
<th>M₁</th>
<th>M₂</th>
<th>M₄</th>
<th>M₅</th>
<th>M₆</th>
<th>M₇</th>
<th>M₈</th>
<th>M₁₀</th>
<th>M₁₁</th>
<th>M₁₂</th>
<th>M₁₄</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₂</td>
<td>0.24</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0.19</td>
<td>0</td>
<td>0.19</td>
<td>0.08</td>
<td>0.21</td>
<td>0.06</td>
<td>0.11</td>
<td>14854 Unit</td>
</tr>
<tr>
<td>P₃</td>
<td>0.13</td>
<td>0</td>
<td>0.12</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0.20</td>
<td>0</td>
<td>0.17</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>P₁₀</td>
<td>0</td>
<td>0.30</td>
<td>0</td>
<td>0.32</td>
<td>0</td>
<td>0.13</td>
<td>0</td>
<td>0.21</td>
<td>0.20</td>
<td>0</td>
<td>0.25</td>
<td>9930</td>
</tr>
</tbody>
</table>

Table 5.22 Processing Times and Demand for P₂, P₃, and P₁₀ with Shifting Bottleneck Machine

Two techniques of the batch scheduling method can be used to schedule the product in the system and to define the makespan of products in the system. The first technique
depends on the processing time of the bottleneck machine to schedule the products and to define the makespan of product in the system. Figure 5.5 shows the scheduling for the subfamily based on the first technique of the batch scheduling method. In this technique, the ending time of the last item on the product equals to the starting time for the first item in the next product. As a mentioned before, the makespan for the subfamily \((P_2 \ P_3 \ P_{10})\) equals to 151.063 hours, when the first technique of the batch scheduling method is utilized to schedule products in the system. The second technique relies on the Gantt chart to schedule the products and to define the makespan of products in the system. Figure 5.6 explains the Gantt chart for \(P_2\), \(P_3\), and \(P_{10}\) when they are processed in the system using the second technique of the batch scheduling method. The makespan for three products \((P_2, P_3, \text{ and } P_{10})\) is 139.475 hours. Therefore, the bottleneck machine does not determine the production rate of the flowshop system when the bottleneck machine shifts and each product has its own bottleneck machine.

However, the second technique of batch scheduling method obtains less makespan compared to the first technique when the bottleneck machine shifts form one product to others.
Figure 5.5 Gantt Chart for Scheduling for the P₂, P₃, and P₁₀ in the Flowshop Cell Using the First Technique of the Batch Scheduling Method with Shifting Bottleneck Machine (Hours)
Figure 5.6 Gantt Chart for Scheduling for the P₃, P₆, and P₁₀ in the Flowshop Cell Using the Second Technique of Batch Scheduling Method with Shifting Bottleneck Machine (Hours)
5.5.2 Batch Scheduling Method with Common Bottleneck Machine

The common bottleneck machine is the main assumption in this group. In this case, all products in the system have the same bottleneck machine with different processing times. Based on the data given in Table 5.22, three products must process in the M_{11} and M_{14} beside the other machines. In this case, the M_{11} and M_{14} are common machines between three products in the subfamily, therefore, they are chosen to be the bottleneck machines in the system. For example, the M_{11} can be the bottleneck machine for the subfamily (P_{2} P_{3} P_{10}) by swapping the content of the processing time for the bottleneck machine in the original problem with the M_{11} as data given in Table 5.23.

<table>
<thead>
<tr>
<th></th>
<th>M_1</th>
<th>M_2</th>
<th>M_4</th>
<th>M_5</th>
<th>M_6</th>
<th>M_7</th>
<th>M_8</th>
<th>M_{10}</th>
<th>M_{11}</th>
<th>M_{12}</th>
<th>M_{14}</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_2</td>
<td>0.21</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0.19</td>
<td>0</td>
<td>0.19</td>
<td>0.08</td>
<td><strong>0.24</strong></td>
<td>0.06</td>
<td>0.11</td>
<td>14854 Unit</td>
</tr>
<tr>
<td>P_3</td>
<td>0.13</td>
<td>0</td>
<td>0.12</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
<td><strong>0.20</strong></td>
<td>0.09</td>
<td>0.07</td>
<td>11601</td>
</tr>
<tr>
<td>P_{10}</td>
<td>0</td>
<td>0.30</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0.13</td>
<td>0</td>
<td>0.21</td>
<td><strong>0.32</strong></td>
<td>0</td>
<td>0.25</td>
<td>9930</td>
</tr>
</tbody>
</table>

The makespan of products in the system based on the first technique of batch scheduling method can be 151.063 hours. Also, the makespan of products in the system equals to 151.063 hours using the second technique of batch scheduling method as given in Figure 5.7. In this case, the processing time of the bottleneck machine, M_{11}, determines the production rate of the system. Therefore, the processing time of the bottleneck machine determines the makespan of the system when the bottleneck is a common machine between products in the system.
Figure 5.7 Gantt Chart to Schedule for the $P_2$, $P_3$, and $P_{10}$ Using the Second Technique of the Batch Scheduling Method with Common Bottleneck Machine
Table 5.24, Table 5.25, Table 5.26 and Table 5.27 show the Makespan for the first rule-subfamilies and second rule-subfamilies with maximizing and minimizing the dissimilarity coefficient using two techniques of the batch scheduling method. The results explain that the second technique of batch scheduling method obtains less makespan compared to the first technique of batch scheduling method, when the bottleneck machine shifts. Otherwise, the two techniques provide the same results.

Table 5.24 Makespan for the First Rule-Subfamilies by Using the Batch Scheduling Method with Maximizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>Products</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF1</td>
<td>P2, P3, and P10</td>
<td>151.062</td>
</tr>
<tr>
<td>SPF2</td>
<td>P5, P7, and P8</td>
<td>158.4192</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Products</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF1</td>
<td>P1, P9, and P11</td>
<td>150.457</td>
</tr>
<tr>
<td>SPF2</td>
<td>P6, P9, and P11</td>
<td>148.34</td>
</tr>
</tbody>
</table>

Table 5.25 Makespan for the Second Rule-Subfamilies by Using the Batch Scheduling Method with Maximizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>Products</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF1</td>
<td>P2, P8, and P10</td>
<td>150.457</td>
</tr>
<tr>
<td>SPF2</td>
<td>P3, P5, and P7</td>
<td>159.0242</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Products</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF1</td>
<td>P1, P4, and P12</td>
<td>148.0288</td>
</tr>
<tr>
<td>SPF2</td>
<td>P6, P9, and P11</td>
<td>148.34</td>
</tr>
</tbody>
</table>
Table 5.26 Makespan for the First Rule-Subfamilies by Using the Batch Scheduling Method with Minimizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th></th>
<th>Products</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PF₁</strong></td>
<td>PF₁ SPF₁</td>
<td>154.39</td>
<td>154.39</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₂, P₅, and P₈</td>
<td>155.0932</td>
<td>143.517</td>
</tr>
<tr>
<td><strong>PF₂</strong></td>
<td>SPF₁ P₁, P₄, P₆, and P₉</td>
<td>159.0608</td>
<td>148.83</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₁₁, and P₁₂</td>
<td>137.30833</td>
<td>137.354</td>
</tr>
</tbody>
</table>

Table 5.27 Makespan for the Second Rule-Subfamilies by Using the Batch Scheduling Method with Minimizing the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th></th>
<th>Products</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PF₁</strong></td>
<td>PF₁ SPF₁</td>
<td>154.993</td>
<td>141.47</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₂, P₃, and P₅</td>
<td>154.4882</td>
<td>139.192</td>
</tr>
<tr>
<td><strong>PF₂</strong></td>
<td>SPF₁ P₁, P₄, P₆, and P₉</td>
<td>159.0608</td>
<td>148.83</td>
</tr>
<tr>
<td></td>
<td>SPF₂ P₁₁, and P₁₂</td>
<td>137.30833</td>
<td>137.354</td>
</tr>
</tbody>
</table>

Table 5.28 shows the makespan for products with maximizing and minimizing the processing time dissimilarity coefficient based on the two techniques of batch for the first and the second rule of dissimilarity coefficient.
Table 5.28 Makespan for Products Based on the Two Techniques of the Batch Scheduling Method for the First and the Second Rule (Hours)

<table>
<thead>
<tr>
<th>Rules to Define the Dissimilarity Coefficient</th>
<th>Processing Time Dissimilarity Coefficient</th>
<th>Makespan Using the First Technique of Batch the Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Rule to Define the Dissimilarity Coefficient</td>
<td>Maximize the Dissimilarity Coefficient</td>
<td>158.4192</td>
<td>148.5</td>
</tr>
<tr>
<td></td>
<td>Minimize the Dissimilarity Coefficient</td>
<td>159.0608</td>
<td>154.39</td>
</tr>
<tr>
<td>Second Rule to Define the Dissimilarity Coefficient</td>
<td>Maximize the Dissimilarity Coefficient</td>
<td>159.0242</td>
<td>148.066</td>
</tr>
<tr>
<td></td>
<td>Minimize the Dissimilarity Coefficient</td>
<td>159.0608</td>
<td>148.83</td>
</tr>
</tbody>
</table>

As a result, the makespan of the second technique is lower than the makespan of the first technique of the batch scheduling method when the shifting bottleneck machine concept is applied. Also, creating the subfamilies based on the maximizing the dissimilarity coefficient between products provides less makespan compared to create the subfamilies depending on minimizing the dissimilarity coefficient.

5.6 Variations around Bottleneck Machine with Batch-Cyclic Scheduling Method

The results in section 5.5 explain the effect of the shifting bottleneck machine on the makespan of the system when the batch-cyclic scheduling method is utilized to schedule the product in the system. In this section, several groups will be discussed to explain the effect of the shifting bottleneck machine, common bottleneck machine, and machine skipping in the system performance when the batch-cyclic scheduling method is
utilized to schedule the products. The first group studies the influence of the common bottleneck machine with machine skipping on the makespan of products in the subfamily. The second group shows the effect of the shifting bottleneck machine with machine skipping on the makespan of products in the subfamily. The third group illustrates the system performance, when the bottleneck machine is a common machine and the machine skipping not allowed. Finally, the fourth group studies the system behavior with shifting bottleneck machine and machine skipping not allowed.

5.6.1 Common Bottleneck Machine and Machine Skipping

The common bottleneck machine and machine skipping are the main assumptions in this group. As the data given in Table 5.22 two machines (M_{11} & M_{14}) might be common bottleneck machines for the subfamily (P_2 P_3 P_{10}) because three products must be processed in these two machines beside other machines. Therefore, two cases will be developed to test the system performance in this group. These two cases are created by swapping the content of the bottleneck machines on the original problem with M_{11} and M_{14} respectively as the data given in the Appendix D. Table 5.29 explains the Cycle Time for the MPS/CS based on the manual calculation results. The results show that the cycle time for the minimal product sets depends on total processing times of the bottleneck machine in a minimal product set.
As the data given in Table 5.29, the cycle time for the minimal product set \((P_2 P_3 P_{10})\) equals to the total processing times of the bottleneck machine. The makespan of the system for two cases equals 151.063 hours when the batch-cyclic scheduling method is utilized to schedule the products in the system, and the makespan of system can be 151.063 hours when the batch scheduling method is applied to schedule the products in the subfamily. Therefore, the makespan for the two methods, batch scheduling method and batch-cyclic scheduling method, is the same when the bottleneck machine is a common machine in the system.

### 5.6.2 Shifting Bottleneck Machine and Machine Skipping

As discussed before, the shifting bottleneck machine affects on the production rate of the system. Therefore, 15 cases are developed to evaluate the system performance in this group of the subfamily \((P_2 P_3 P_{10})\) as the data given in the Appendix E. Table 5.30 shows the cycle time for the minimal product set for the subfamily \((P_2 P_3 P_{10})\) with different locations to the bottleneck machines. The results show the change makespan of products in the system when the position of the bottleneck machine is changed.

<table>
<thead>
<tr>
<th>Minimal Product Sets</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_2)</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>(P_3)</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>(P_{10})</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>(P_2 P_3)</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>(P_2 P_{10})</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>(P_3 P_{10})</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>(P_2 P_3 P_{10})</td>
<td>0.76</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Table 5.30 Cycle Time for the Minimal Product Set with Shifting Bottleneck Machine and Machine Skipping (Minutes)

<table>
<thead>
<tr>
<th>Case</th>
<th>P_2</th>
<th>P_3</th>
<th>P_{10}</th>
<th>P_{2P_3}</th>
<th>P_{2P_{10}}</th>
<th>P_{3P_{10}}</th>
<th>P_{2P_3P_{10}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
<td>0.41</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>2</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.42</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>3</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.42</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>4</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.53</td>
<td>0.49</td>
<td>0.70</td>
</tr>
<tr>
<td>5</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
<td>0.41</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.54</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>7</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
<td>0.41</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>8</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.41</td>
<td>0.4</td>
<td>0.61</td>
</tr>
<tr>
<td>9</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.53</td>
<td>0.49</td>
<td>0.70</td>
</tr>
<tr>
<td>10</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.41</td>
<td>0.45</td>
<td>0.58</td>
</tr>
<tr>
<td>11</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
<td>0.43</td>
<td>0.39</td>
<td>0.58</td>
</tr>
<tr>
<td>12</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
<td>0.41</td>
<td>0.37</td>
<td>0.58</td>
</tr>
<tr>
<td>13</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.53</td>
<td>0.49</td>
<td>0.70</td>
</tr>
<tr>
<td>14</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.41</td>
<td>0.45</td>
<td>0.58</td>
</tr>
<tr>
<td>15</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.43</td>
<td>0.39</td>
<td>0.58</td>
</tr>
</tbody>
</table>

However, the results show that the cycle time for each minimal product set equals to the total processing time of the machine which has the highest total processing times in the minimal product set.

Table 5.31 shows the mathematical results when the proposed model in the phase three is utilized to schedule the products in the system by using batch-cyclic scheduling method for 15 cases. Several minimal product sets are utilized to cover the demand of products with different processing times per each MPS.
Table 5.31 Mathematical Model Results for Subfamily (P₂P₃P₁₀) with Shifting Bottleneck Machine and Machine Skipping (Hours)

<table>
<thead>
<tr>
<th>Case</th>
<th>P₂</th>
<th>P₃</th>
<th>P₁₀</th>
<th>P₂P₃</th>
<th>P₂P₁₀</th>
<th>P₃P₁₀</th>
<th>P₂P₃P₁₀</th>
<th>Makespan</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<td>64.545</td>
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<td>27.264</td>
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<td>137.818</td>
</tr>
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<td>0</td>
<td>64.545</td>
<td>118.78</td>
</tr>
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<tr>
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<td>58.225</td>
<td>27.264</td>
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<td>137.818</td>
</tr>
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<td>64.545</td>
<td>117.959</td>
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<td>64.545</td>
<td>119.864</td>
</tr>
</tbody>
</table>

The results show that the makespan of products in the system is lower than the makespan when the bottleneck machine is common machine.

5.6.3 Common Bottleneck Machine and Machine Skipping Not Allowed

The machine skipping is not allowed in this section, the processing times for the idle machines are created by using the random number between (0.01 and the processing time of the bottleneck machine -0.10). For example, the processing time for P₂ at M₄, M₅, and M₇ equals “0”, so the random numbers between (0.01 and 0.23) are utilized to create
the processing times for $P_2$ at $M_4$, $M_5$, and $M_7$. Table 5.32 illustrates the processing time for the three products when the machine skipping is not allowed.

Table 5.32 Processing Time for the $P_2$, $P_3$, and $P_{10}$ when the Machine Skipping Not Allowed (Minutes)

<table>
<thead>
<tr>
<th></th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_4$</th>
<th>$M_5$</th>
<th>$M_6$</th>
<th>$M_7$</th>
<th>$M_8$</th>
<th>$M_{10}$</th>
<th>$M_{11}$</th>
<th>$M_{12}$</th>
<th>$M_{14}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_2$</td>
<td>0.24</td>
<td>0.10</td>
<td>0.05</td>
<td>0.22</td>
<td>0.19</td>
<td>0.22</td>
<td>0.19</td>
<td>0.08</td>
<td>0.21</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.13</td>
<td>0.06</td>
<td>0.12</td>
<td>0.07</td>
<td>0.14</td>
<td>0.13</td>
<td>0.20</td>
<td>0.04</td>
<td>0.17</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>$P_{10}$</td>
<td>0.07</td>
<td>0.30</td>
<td>0.12</td>
<td>0.32</td>
<td>0.10</td>
<td>0.13</td>
<td>0.30</td>
<td>0.21</td>
<td>0.20</td>
<td>0.28</td>
<td>0.25</td>
</tr>
</tbody>
</table>

In this section, three cases are developed based on the location of the bottleneck machine in the system. The first case determines the makespan of the system when $M_1$ is a common bottleneck machine. In this case, the content of the bottleneck machines and the $M_1$ in Table 5.32 is swapped. In the second case, $M_7$ is chosen to be a common bottleneck machine, and in the third case, $M_{14}$ is selected to be the common bottleneck machine as the data shown in Appendix F. Table 5.33 shows the cycle time for the three cases.

Table 5.33 Cycle Time for the System with Common Bottleneck Machine and Machine Skipping Not Allowed (Minutes)

<table>
<thead>
<tr>
<th></th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_{10}$</th>
<th>$P_2P_3$</th>
<th>$P_2P_{10}$</th>
<th>$P_3P_{10}$</th>
<th>$P_2P_3P_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.44</td>
<td>0.56</td>
<td>0.52</td>
<td>0.76</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.44</td>
<td>0.56</td>
<td>0.52</td>
<td>0.76</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.44</td>
<td>0.56</td>
<td>0.52</td>
<td>0.76</td>
</tr>
</tbody>
</table>

The results illustrate that the cycle time for each minimal product set depends on the total processing times of the bottleneck machine in the set once the bottleneck machine
is a common bottleneck machine. Therefore, the makespan for the system in this case is 151.063 hours when the batch-cyclic scheduling method and the batch scheduling method are utilized to schedule the products in the system. As a result, two methods, batch method and batch-cyclic method, provide the same results when the bottleneck machine is common machine in the system.

5.6.4  **Shifting Bottleneck Machine and Machine Skipping Not Allowed**

In this section, the shifting bottleneck machine is allowed, while the machine skipping is not allowed based on data that given in Table 5.32. Fifteen cases are created based on the different locations of the bottleneck machine in the system to test the effect of shifting bottleneck machine on the cycle time, production rate and the makespan of products in the system as the data given in the Appendix G. Each case is created separately from the others by swapping the processing times of the machines in the original problem. Table 5.34 illustrates the cycle time for the minimal product sets when they are processed in the system in mixed mode based on the batch-cyclic scheduling method. Shifting bottleneck machine affects clearly on the cycle time of products in the system. However, cycle time for each minimal product set in this group depends on the total processing times of the machine which has the highest total processing times in the set.

In addition, the sequence of product inside the minimal product set is not important, once the cycle time for the minimal product set depends on total processing time of the machine which has the highest total processing time in the set. For example, the production rate for the minimal product sets (P₂P₃ & P₃P₂) is the same as explained in the Table 5.34
with little difference in the completion time of products in the cell. This difference returns to the first and the last part in the sequence. For example,

\[ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \]

This sequence represents the sequence of the Minimal product set \((P_2P_3)\) in the system when it is used to run the \(P_2 \& P_3\) in the system in mixed mode. The cycle time of the system in this case can be 0.38 min. On the other hand, if the first and the last part are removed from the sequence, a new sequence is created. This sequence represents the sequencing of products in the system when the minimal product set \((P_3P_2)\) is utilized to schedule the products in the subfamily.

\[ P_2 / P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 \ P_3 \ P_2 / P_3 \]

Therefore, the sequence of products in the minimal product set doesn’t affect the in cycle time of products in the system as long as the setup time of the machine is “0”. Table 5.34 shows the cycle time for the several minimal product sets with different sequences. It is important to mention that the cycle time of products in the minimal product set relies on the total processing times of machine which has the highest total processing times in the set.
Table 5.34 Cycle Time for the Minimal Product Set with Shifting Bottleneck Machine and Machine Skipping Not Allowed (Minutes)

<table>
<thead>
<tr>
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<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
<th>Case 9</th>
<th>Case 10</th>
<th>Case 11</th>
<th>Case 12</th>
<th>Case 13</th>
<th>Case 14</th>
<th>Case 15</th>
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<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
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<td>0.20</td>
<td>0.20</td>
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<td>0.20</td>
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</tr>
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<td>0.32</td>
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<td>0.32</td>
<td>0.32</td>
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<td>0.54</td>
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<td>0.61</td>
<td>0.66</td>
<td>0.69</td>
<td>0.65</td>
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</table>
Table 5.35 illustrates the results of the mathematical model when it is solved by using the CPLEX software. The results show obvious the change in the Makespan of products for 15 cases based on the positions of the bottleneck machines. Also, the makespan for all cases is less than the makespan of the system when the bottleneck machine is a common machine.

<table>
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<th>P2P10</th>
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</tbody>
</table>
5.7 Variations in Term of Total Processing Time per Machine with Batch-Cyclic Scheduling Method

The total processing time per each machine is another factor that has influence on the production rate of system beside the shifting bottleneck machine. Two sets are performed in this section to explain the system performance when the batch-cyclic scheduling method is utilized to schedule the products in the system. The first set is based on the shifting bottleneck machine, different processing times per each machine, and the machine skipping not allowed. The second set depends on the shifting bottleneck machine, different processing times per each machine and machine skipping.

5.7.1 Shifting Bottleneck Machine and Machine Skipping Not Allowed

Changing and adjusting the total processing time per each machine are studied in this section to test the effect of the total processing time per each machine in the production rate for each minimal product set. In this regard, several cases are developed to test the changing and adjusting total processing time per each machine in the makespan subfamily in the system. Appendix H shows the processing time for all cases which are developed to test the system performance in this section. Table 5.36 illustrates the cycle time for the minimal product sets, when they are used to cover the demand for products in mixed mode based on the batch-cyclic scheduling method. The results clearly show the influence of the total processing time per each machine on the cyclic for each minimal product set in the system. Based on the results, the cycle time for each minimal product set depends on the total processing time of the machine which has the highest total processing time in the system.
Table 5.36 Cycle Time for the Minimal Product Set with Shifting Bottleneck Machine, Machine Skipping Not Allowed and Adjusting Processing Time per each Machine (Minutes)

<table>
<thead>
<tr>
<th></th>
<th>P_2</th>
<th>P_3</th>
<th>P_{10}</th>
<th>P_{2P_3}</th>
<th>P_{2P_{10}}</th>
<th>P_{3P_{10}}</th>
<th>P_{2P_3P_{10}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.49</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.49</td>
<td>0.43</td>
<td>0.61</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.49</td>
<td>0.44</td>
<td>0.58</td>
</tr>
<tr>
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<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.49</td>
<td>0.44</td>
<td>0.58</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.34</td>
<td>0.43</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>Case 6</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.34</td>
<td>0.47</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>Case 7</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.34</td>
<td>0.47</td>
<td>0.43</td>
<td>0.54</td>
</tr>
<tr>
<td>Case 8</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.34</td>
<td>0.52</td>
<td>0.42</td>
<td>0.61</td>
</tr>
<tr>
<td>Case 9</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.5</td>
<td>0.38</td>
<td>0.54</td>
<td>0.66</td>
</tr>
<tr>
<td>Case 10</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.34</td>
<td>0.44</td>
<td>0.43</td>
<td>0.53</td>
</tr>
<tr>
<td>Case 11</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.34</td>
<td>0.44</td>
<td>0.43</td>
<td>0.53</td>
</tr>
<tr>
<td>Case 12</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.54</td>
<td>0.47</td>
<td>0.71</td>
</tr>
<tr>
<td>Case 13</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.54</td>
<td>0.47</td>
<td>0.71</td>
</tr>
<tr>
<td>Case 14</td>
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<td>0.20</td>
<td>0.32</td>
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<td>0.44</td>
<td>0.43</td>
<td>0.53</td>
</tr>
<tr>
<td>Case 15</td>
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<td>0.32</td>
<td>0.34</td>
<td>0.44</td>
<td>0.43</td>
<td>0.53</td>
</tr>
</tbody>
</table>

As mentioned before, the CPLEX software is used to solve the proposed mathematical model in the phase three. Table 5.37 explains the processing time for each minimal product set and the makespan of the system when the batch-cyclic scheduling method is used to schedule the products in the system. In the first case, the processing time for the minimal product set $P_2$, $P_2P_3$, and $P_2P_3P_{10}$ can be 13.012 hours, 10.583 hours, and 99.3 hours, respectively, while the processing times for the other minimal product sets ($P_3$, $P_{10}$, $P_2P_{10}$, $P_3P_{10}$) equal “0”. Therefore, three minimal product sets out of seven minimal product sets are appropriate to cover the demand for the three products ($P_2P_3&P_{10}$) in the
subfamily. Also, the makespan for all cases is less than the makespan of the system when the bottleneck machine is a common machine in the system.

Table 5.37 Mathematical Model Results for the Minimal Product Set with Shifting Bottleneck Machine, Machine Skipping Not Allowed and Adjusting Processing Time per each Machine (Hours)

<table>
<thead>
<tr>
<th>Case</th>
<th>P_2</th>
<th>P_3</th>
<th>P_10</th>
<th>P_2P_3</th>
<th>P_2P_10</th>
<th>P_3P_10</th>
<th>P_2P_3P_10</th>
<th>Makespan</th>
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<td>11.419</td>
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<td>0</td>
<td>100.95</td>
<td>125.381</td>
</tr>
<tr>
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<td>0</td>
<td>10.583</td>
<td>0</td>
<td>0</td>
<td>95.99</td>
<td>119.585</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>10.583</td>
<td>0</td>
<td>0</td>
<td>95.99</td>
<td>119.585</td>
</tr>
<tr>
<td>5</td>
<td>13.012</td>
<td>0</td>
<td>0</td>
<td>9.469</td>
<td>0</td>
<td>0</td>
<td>87.715</td>
<td>110.196</td>
</tr>
<tr>
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<td>0</td>
<td>89.37</td>
<td>111.8515</td>
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<td>0</td>
<td>0</td>
<td>9.4695</td>
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<td>0</td>
<td>89.37</td>
<td>111.8515</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>11.419</td>
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<td>0</td>
<td>100.95</td>
<td>125.381</td>
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<td>9.469</td>
<td>0</td>
<td>0</td>
<td>87.715</td>
<td>110.196</td>
</tr>
<tr>
<td>11</td>
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</tr>
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<td>9.469</td>
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<td>0</td>
<td>87.715</td>
<td>110.196</td>
</tr>
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<td>0</td>
<td>0</td>
<td>9.469</td>
<td>0</td>
<td>0</td>
<td>87.715</td>
<td>110.196</td>
</tr>
</tbody>
</table>

5.7.2 Shifting Bottleneck Machine and Machine Skipping

The total processing time per each machine effects on the production rate of the system and the makespan of products in the subfamily as explained in the previous section.
Fifteen cases are tested in this group to illustrate the system performance with the shifting bottleneck machine, changing the total processing time per each machine, and machine skipping as the data shown in Appendix I. Table 5.38 illustrates the cycle time for each minimal product set.

<table>
<thead>
<tr>
<th></th>
<th>P_2</th>
<th>P_3</th>
<th>P_{10}</th>
<th>P_{2P_3}</th>
<th>P_{3P_10}</th>
<th>P_{2P_3P_{10}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.38</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>Case 4</td>
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<td>0.32</td>
<td>0.36</td>
<td>0.53</td>
<td>0.5</td>
</tr>
<tr>
<td>Case 5</td>
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<td>0.41</td>
<td>0.42</td>
<td>0.33</td>
</tr>
<tr>
<td>Case 6</td>
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<td>0.32</td>
<td>0.38</td>
<td>0.49</td>
<td>0.37</td>
</tr>
<tr>
<td>Case 7</td>
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<td>0.32</td>
<td>0.41</td>
<td>0.51</td>
<td>0.33</td>
</tr>
<tr>
<td>Case 8</td>
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<td>0.32</td>
<td>0.41</td>
<td>0.51</td>
<td>0.33</td>
</tr>
<tr>
<td>Case 9</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.51</td>
<td>0.33</td>
</tr>
<tr>
<td>Case 10</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.51</td>
<td>0.34</td>
</tr>
<tr>
<td>Case 11</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.51</td>
<td>0.38</td>
</tr>
<tr>
<td>Case 12</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.51</td>
<td>0.44</td>
</tr>
<tr>
<td>Case 13</td>
<td>0.24</td>
<td>0.20</td>
<td>0.32</td>
<td>0.41</td>
<td>0.51</td>
<td>0.44</td>
</tr>
<tr>
<td>Case 14</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.39</td>
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<td>0.33</td>
</tr>
<tr>
<td>Case 15</td>
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<td>0.32</td>
<td>0.37</td>
<td>0.51</td>
<td>0.39</td>
</tr>
</tbody>
</table>

In this group, the production rate for each minimal product set also depends on the total processing times of machine which has the highest total processing time in the
minimal product set. Table 5.39 explains the mathematical model results for each case. The
Makespan of products, and the processing time for each minimal product set in the system
are determined by using the CPLEX software.

Table 5.39 Mathematical Model Results for the Minimal Product Set with Shifting
Bottleneck Machine, Machine Skipping and Different Processing Times per each
Machine (Hours)

<table>
<thead>
<tr>
<th></th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_10</th>
<th>P_2P_3</th>
<th>P_2P_10</th>
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<th>P_2P_3P_10</th>
<th>Makespan</th>
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</thead>
<tbody>
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</tr>
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<td>0</td>
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<td>108.836</td>
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</tr>
<tr>
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<td>84.405</td>
<td>108.836</td>
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</tr>
<tr>
<td>Case 10</td>
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<td>84.405</td>
<td>108.836</td>
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</tr>
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<td>122.617</td>
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</tr>
</tbody>
</table>

It is worth mentioning that the production rate for all minimal product sets in two
groups in this section depends on the total processing time of the machine which has the
highest total processing time in the minimal product set. Also, adjusting the total processing
time per each machine clearly effects on the cycle time of the minimal product set.
5.8 Experiment with another Example

In this section, another example is developed to test the performance of the batch-cyclic scheduling method. The flowshop system in the cellular manufacturing system consists of 39 products with 18 machines as data given by (Erenay, Suer, Huang, & Maddisetty, 2015). The processing time represents that the time is taken by machine to process the product. Based on the shifting bottleneck machine, the processing time of the bottleneck machine in the original problem is swapped with the processing time of other machines randomly because the bottleneck machine is a common bottleneck machine in the original problem. Also, the machine skipping is applied in this section. In this regard, a product isn’t processed on all machines in the system, for example, $P_1$ is processed in 5 machines out of 18 machines. Table 5.40 provides the processing times for all machines.

Monthly demand for each product is assumed to follow the random number between 40 and 210 which are created by using Microsoft Excel sheet. Table 5.41 shows the monthly demand values for all products in the system.

It is worth mentioning that the setup time of machines is zero, and the time taken for a product to move from one machine to others is assumed to be zero. It is assumed that the products move in units between machines. It is considered that the cell is independent cell which means each cell has its own machines and sharing cell not allowed. Also, the cell is available 8 hours per day, five days per week, and four weeks per month. Therefore, the total number of business hours in a cell can be 160 per month.
Table 5.40 Unit Processing Times for the Second Example (Minutes)

<table>
<thead>
<tr>
<th></th>
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<th>M2</th>
<th>M3</th>
<th>M4</th>
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<th>M14</th>
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<th>M16</th>
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</table>
Table 5.40 Continued

|       | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 | M9 | M10 | M11 | M12 | M13 | M14 | M15 | M16 | M17 | M18 |
|-------|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|
| P_{27} | 18 | 20 | 14 |    | 17 |    |    |    |    | 13  | 13  | 13  | 13  | 13  | 13  | 13  | 13  | 13  |
| P_{28} |    | 19 | 17 |    |    | 13 |    |    |    |     |     |     |     |     | 12  |    |    |    |
| P_{29} | 19 | 16 | 20 |    |    |    |    |    |    | 10  | 10  |     | 19  |     |     |     |    |    |
| P_{30} |    | 17 |    |    |    | 16 |    |    |    |     |     | 19  |     | 16  |     |    |    |    |
| P_{31} | 16 | 18 | 10 |    |    |    |    |    |    | 16  | 12  | 16  | 16  | 16  | 16  | 16  | 16  | 16  |
| P_{32} |    | 16 | 17 | 16 |    | 16 | 16 | 18 |    | 16  | 16  | 16  | 16  | 16  | 16  | 16  | 16  | 16  |
| P_{33} | 15 | 17 | 18 |    |    | 15 | 15 | 15 | 15 | 10  | 17  | 17  | 17  | 17  | 17  | 17  | 17  | 17  |
| P_{34} | 19 | 19 | 17 | 17 |    | 18 | 18 | 18 | 18 | 18  | 18  | 18  | 18  | 18  | 18  | 18  | 18  | 18  |
| P_{35} | 19 | 19 | 15 | 15 |    | 18 | 18 | 18 | 18 | 18  | 18  | 18  | 18  | 18  | 18  | 18  | 18  | 18  |
| P_{36} | 17 | 19 | 18 | 16 | 18 | 18 | 18 | 18 | 18 | 18  | 18  | 18  | 18  | 18  | 18  | 18  | 18  | 18  |
| P_{37} | 17 | 17 | 18 | 13 |    | 17 | 17 | 17 | 17 | 17  | 17  | 17  | 17  | 17  | 17  | 17  | 17  | 17  |
| P_{38} | 13 | 17 | 18 | 19 |    | 19 | 19 | 19 | 19 | 19  | 19  | 19  | 19  | 19  | 19  | 19  | 19  | 19  |
| P_{39} | 18 | 10 | 16 |    |    |    |    |    |    | 19  | 17  | 17  | 17  | 17  | 17  | 17  | 17  | 17  |

Table 5.41 Monthly Demand for the Second Example (Units)

<table>
<thead>
<tr>
<th>Product</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
<th>P_5</th>
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<th>P_7</th>
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<th>P_9</th>
<th>P_{10}</th>
<th>P_{11}</th>
<th>P_{12}</th>
<th>P_{13}</th>
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<th>P_{16}</th>
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<th>P_{18}</th>
<th>P_{19}</th>
<th>P_{20}</th>
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</thead>
<tbody>
<tr>
<td>Demand</td>
<td>135</td>
<td>60</td>
<td>191</td>
<td>87</td>
<td>98</td>
<td>167</td>
<td>140</td>
<td>197</td>
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<table>
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<th>Product</th>
<th>P_{21}</th>
<th>P_{22}</th>
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<th>P_{24}</th>
<th>P_{25}</th>
<th>P_{26}</th>
<th>P_{27}</th>
<th>P_{28}</th>
<th>P_{29}</th>
<th>P_{30}</th>
<th>P_{31}</th>
<th>P_{32}</th>
<th>P_{33}</th>
<th>P_{34}</th>
<th>P_{35}</th>
<th>P_{36}</th>
<th>P_{37}</th>
<th>P_{38}</th>
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<td>Demand</td>
<td>81</td>
<td>120</td>
<td>64</td>
<td>191</td>
<td>97</td>
<td>76</td>
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5.8.1 Product Family Formation

As mentioned before, the P-median mathematical model is utilized to define the product family based on maximizing the processing similarity coefficients between products. The modified Jaccard’s similarity coefficient is applied to define the similarity coefficients. Once the processing similarity coefficient matrix is determined the mathematical model is developed to assign the products into product families based on maximizing the processing similarity coefficient. The data is entered to the CPLEX Software to solve the problem. Three product families are generated by the mathematical model when the penalty factor assumed to be “1” as the data given in the Table 5.42. Appendix C shows different product family formations based on the different penalty factor values.

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>P₃, P₇, P₀, P₁₁, P₁₅, P₂₀, P₂₁, P₂₄, P₂₇, P₂₉, P₃₁, P₃₉</td>
</tr>
<tr>
<td>PF₂</td>
<td>P₄, P₆, P₁₀, P₁₃, P₁₆, P₁₈, P₂₂, P₂₈, P₃₂, P₃₅</td>
</tr>
<tr>
<td>PF₃</td>
<td>P₁, P₂, P₅, P₈, P₁₂, P₁₄, P₁₇, P₁₉, P₂₃, P₂₅, P₂₆, P₃₀, P₃₃, P₃₄, P₃₆, P₃₇, P₃₈</td>
</tr>
</tbody>
</table>

5.8.2 Subfamily Formulation and Cell Loading

In this section, each product family is divided into subfamilies based on maximizing the processing time dissimilarity coefficients. As mentioned before, two rules are used to define the processing time dissimilarity coefficient, where the shifting bottleneck machine and machine skipping are the main assumptions in this research. Also, the capacitated P-
median mathematical model is developed based on the processing time dissimilarity coefficient to obtain the subfamilies per each product family. Table 5.43 and Table 5.44 illustrate the subfamilies for three product families based on the first and the second rule of the processing time dissimilarity coefficient respectively. The first product family and the second product family in two rules have the same subfamilies, while the third product family is divided into five subfamilies with different products per each subfamily in two rules.

Table 5.43 First Rule-Subfamily and their Products

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Subfamily</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF₁</td>
<td>SPF₁</td>
<td>P₃, P₉, P₁₅, P₂₀, and P₃⁹</td>
</tr>
<tr>
<td></td>
<td>SPF₂</td>
<td>P₇, P₂₄, P₂₇, and P₂₉</td>
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<tr>
<td></td>
<td>SPF₃</td>
<td>P₁₁, P₂₁, and P₃₁</td>
</tr>
<tr>
<td>PF₂</td>
<td>SPF₁</td>
<td>P₄, P₂₂, and P₃₂</td>
</tr>
<tr>
<td></td>
<td>SPF₂</td>
<td>P₆, P₁₃, and P₃₅</td>
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<td></td>
<td>SPF₃</td>
<td>P₁₀, P₁₆, P₁₈, and P₂₈</td>
</tr>
<tr>
<td>PF₃</td>
<td>SPF₁</td>
<td>P₂, P₁₇, P₃₀, P₃₄, and P₃₇</td>
</tr>
<tr>
<td></td>
<td>SPF₂</td>
<td>P₁₄, P₂₅, and P₃₃</td>
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<tr>
<td></td>
<td>SPF₃</td>
<td>P₁, and P₃₆</td>
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<td></td>
<td>SPF₄</td>
<td>P₁₂, P₁₉, and P₃₈</td>
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<tr>
<td></td>
<td>SPF₅</td>
<td>P₅, P₈, P₂₃, and P₂₆,</td>
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</table>
Table 5.44 Second Rule-Subfamily and their Products

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Subfamily</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>SPF1</td>
<td>P_3, P_9, P_{15}, P_{20}, and P_{39}</td>
</tr>
<tr>
<td></td>
<td>SPF2</td>
<td>P_7, P_{24}, P_{27}, and P_{29}</td>
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<td></td>
<td>SPF3</td>
<td>P_{11}, P_{21}, and P_{31}</td>
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<tr>
<td>PF2</td>
<td>SPF1</td>
<td>P_4, P_{22}, and P_{32}</td>
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<td>SPF2</td>
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<td>SPF3</td>
<td>P_{10}, P_{16}, P_{18}, and P_{28}</td>
</tr>
<tr>
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<td>SPF1</td>
<td>P_2, P_{17}, P_{30}, P_{34}, and P_{37}</td>
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<td></td>
<td>SPF2</td>
<td>P_1, P_{8}, P_{23}, and P_{26}</td>
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<td></td>
<td>SPF3</td>
<td>P_{25}, P_{33}, and P_{36}</td>
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<td>SPF4</td>
<td>P_{12}, P_{19}, and P_{38}</td>
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<td>SPF5</td>
<td>P_5, and P_{14}</td>
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</table>

5.8.3 Batch-Cyclic Scheduling Method

As mentioned before, three steps have to be followed to schedule the products in the flowshop system when the batch-cyclic scheduling method is utilized to schedule the products. Minimal product set/Cyclic set is determined by using “Hasse Diagram. The manual calculation is used to define the production rate for each minimal product set. Finally, the mathematical model is developed based on the results of the manual calculation with the objective of minimizing the makespan of products in the system.

5.8.3.1 Minimal Product Set/ Cyclic Set

Different minimal product sets are created per each subfamily to meet the demand values of products. In this case, “Hasse Diagram” is used to define the minimal product sets required to meet the demand values for each product. In the Figure 5.8, “Hasse Diagram” shows the minimal product sets required to meet the demand for the P_1 in the subfamily (P_1, P_8, P_{23}, P_{26}).
Figure 5.8 Minimal Product Sets to Meet the Demand Values for $P_1$ in the Subfamily $(P_1, P_8, P_{23}, P_{26})$
Table 5.45 Minimal Product Sets for the First Rule-Subfamilies

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Subfamily</th>
<th>Minimal Product Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF2 SPF1</td>
<td></td>
<td>P4, P22, P32, P4P22, P4P32, P22P32, and P4P22P32</td>
</tr>
<tr>
<td>PF3 SPF1</td>
<td></td>
<td>P2, P17, P30, P34, P37, P2P7, P2P30, P2P34, P2P37, P17P30, P17P34, P17P37, P30P34, P30P37, P34P37, P2P17P30, P2P17P34, P2P17P37, P2P30P34, P2P30P37, P2P34P37, P17P30P34, P17P30P37, P30P34P37, P2P17P30P34, P2P17P30P37, P2P30P34P37, P17P30P34P37, and P2P17P30P34P37</td>
</tr>
<tr>
<td>PF3 SPF3</td>
<td></td>
<td>P1, P36, and P1P36</td>
</tr>
<tr>
<td>PF3 SPF4</td>
<td></td>
<td>P12, P19, P38, P12P19, P12P38, P19P38, and P12P19P38</td>
</tr>
<tr>
<td>PF3 SPF5</td>
<td></td>
<td>P5, P8, P23, P26, P5P8, P5P23, P5P26, P8P23, P8P26, P23P26, P5P8P23, P5P8P26, P1P23P26, P8P23P26, and P5P8P23P26</td>
</tr>
<tr>
<td>Product Family</td>
<td>Subfamily</td>
<td>Minimal Product Sets</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------</td>
<td>----------------------</td>
</tr>
<tr>
<td></td>
<td>SPF₂</td>
<td>P₇, P₂₄, P₂₇, P₂₉, P₆P₂₄, P₇P₂₇, P₇P₂₉, P₂₄P₂₇, P₂₄P₂₉, P₂₇P₂₉, P₇P₂₄P₂₇, P₇P₂₄P₂₉, P₇P₂₇P₂₉, P₂₄P₂₇P₂₉, and P₇P₂₄P₂₇P₂₉</td>
</tr>
<tr>
<td><strong>PF₂</strong></td>
<td>SPF₁</td>
<td>P₄, P₂₂, P₃₂, P₄P₂₂, P₄P₃₂, P₂₂P₃₂, and P₄P₂₂P₃₂</td>
</tr>
<tr>
<td></td>
<td>SPF₂</td>
<td>P₆, P₁₃, P₃₅, P₆P₁₃, P₆P₃₅, P₁₃P₃₅, and P₆P₁₃P₃₅</td>
</tr>
<tr>
<td><strong>PF₃</strong></td>
<td>SPF₁</td>
<td>P₂, P₁₇, P₃₀, P₃₄, P₃₇, P₂P₇, P₂P₃₀, P₂P₃₄, P₂P₃₇, P₁₇P₃₀, P₁₇P₃₄, P₁₇P₃₇, P₁₇P₃₀P₃₄, P₁₇P₃₀P₃₇, P₁₇P₃₄P₃₇, P₂P₁₇P₃₀, P₂P₁₇P₃₄, P₂P₁₇P₃₇, P₂P₁₇P₃₀P₃₄, P₂P₁₇P₃₀P₃₇, P₂P₁₇P₃₄P₃₇, P₂P₁₇P₃₀P₃₄P₃₇</td>
</tr>
<tr>
<td></td>
<td>SPF₂</td>
<td>P₁, P₈, P₂₃, P₂₆, P₁P₈, P₁P₂₃, P₈P₂₆, P₁P₂₃P₂₆, P₁P₈P₂₆, P₁P₂₃P₂₆, P₈P₂₃P₂₆, and P₁P₈P₂₃P₂₆</td>
</tr>
<tr>
<td></td>
<td>SPF₃</td>
<td>P₂₅, P₃₃, P₃₆, P₂₅P₃₃, P₂₅P₃₆, P₃₃P₃₆, and P₂₅P₃₃P₃₆</td>
</tr>
<tr>
<td></td>
<td>SPF₅</td>
<td>P₅, P₁₄, and P₂P₁₄</td>
</tr>
</tbody>
</table>
Table 5.45 and Table 5.46 show the minimal product sets for subfamilies in the first and the second rule of the dissimilarity coefficient respectively.

### 5.8.3.2 Production Rate for Minimal Product Sets

Having determined the minimal product sets, the production rate is defined by using the manual calculation. Table 5.47 shows the processing times for the subfamily \((P_{12}, P_{19}, P_{38})\) in the flowshop system.

<table>
<thead>
<tr>
<th></th>
<th>(M_1)</th>
<th>(M_2)</th>
<th>(M_3)</th>
<th>(M_4)</th>
<th>(M_6)</th>
<th>(M_8)</th>
<th>(M_9)</th>
<th>(M_{13})</th>
<th>(M_{15})</th>
<th>(M_{16})</th>
<th>(M_{18})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{12})</td>
<td>16</td>
<td>18</td>
<td>17</td>
<td>11</td>
<td>0</td>
<td>19</td>
<td>0</td>
<td>18</td>
<td>11</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(P_{19})</td>
<td>10</td>
<td>19</td>
<td>15</td>
<td>18</td>
<td>20</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>0</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>(P_{38})</td>
<td>13</td>
<td>17</td>
<td>0</td>
<td>18</td>
<td>0</td>
<td>0</td>
<td>19</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>15</td>
</tr>
</tbody>
</table>

The manual calculation is utilized to define the cycle time for a minimal product set in the subfamily. Table 5.48 shows the manual calculation results for each minimal product set in the system. The results show that the cycle time for the minimal product set depends on the total processing time of the machine which has the highest total processing time in the minimal product set.
### Table 5.48 Cycle Time for the Minimal Product Sets in Subfamily (P₁₂ P₁₉ P₃₈)

<table>
<thead>
<tr>
<th>Minimal Product Set</th>
<th>Production Rate (Minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁₂</td>
<td>19</td>
</tr>
<tr>
<td>P₁₉</td>
<td>20</td>
</tr>
<tr>
<td>P₃₈</td>
<td>19</td>
</tr>
<tr>
<td>P₁₂P₁₉</td>
<td>37</td>
</tr>
<tr>
<td>P₁₂P₃₈</td>
<td>35</td>
</tr>
<tr>
<td>P₁₉P₃₈</td>
<td>36</td>
</tr>
<tr>
<td>P₁₂P₁₉P₃₈</td>
<td>54</td>
</tr>
</tbody>
</table>

#### 5.8.3.3 Mathematical Model

In the section, the mathematical model developed earlier in phase three is utilized to schedule the minimal product sets in the system based on the production rate of the minimal product sets and the demand values. The problem is solved based on full coverage demand. As explained previously, the model developed is solved using CPLEX for three product families. Table 5.49 and Table 5.50 explain the processing time for the minimal product sets and the makespan for the subfamilies in the system based on the first rule and the second rule of processing time dissimilarity coefficient. The makespan for the products in the first rule and the second rule of dissimilarity coefficient equals to 144.633 hours, and 144.633 hours, respectively based on batch-cyclic scheduling method.
Table 5.49 Processing Time for the Minimal Product Sets Required to Meet the Demand Values for Product Family and Makespan for the First Rule-Subfamilies Using the Batch-Cyclic Scheduling Method (Hours)

<table>
<thead>
<tr>
<th>PF1</th>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
<th>SPF4</th>
<th>SPF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>XP3</td>
<td>43.75</td>
<td>XP7</td>
<td>33.6</td>
<td>XP11</td>
<td>3.3333</td>
</tr>
<tr>
<td>XP3</td>
<td>25.799</td>
<td>XP7</td>
<td>21.6</td>
<td>XP11</td>
<td>68.4</td>
</tr>
<tr>
<td>XP3</td>
<td>9.35</td>
<td>XP7</td>
<td>50.0</td>
<td>XP11</td>
<td>72.9</td>
</tr>
<tr>
<td>XP3</td>
<td>26.134</td>
<td>XP7</td>
<td>31.45</td>
<td>XP11P</td>
<td>144.6333</td>
</tr>
<tr>
<td>XP3</td>
<td>32.999</td>
<td>Makespan</td>
<td>136.65</td>
<td>Makespan</td>
<td>144.6333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF1</td>
<td>SPF2</td>
<td>SPF3</td>
</tr>
<tr>
<td>SPF1</td>
<td>SPF2</td>
<td>SPF3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PF2</th>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>XP32</td>
<td>13.5</td>
<td>XP36</td>
<td>40.30</td>
</tr>
<tr>
<td>XP52P32</td>
<td>64.0</td>
<td>XP53</td>
<td>8.00</td>
</tr>
<tr>
<td>Makespan</td>
<td>77.5</td>
<td>XP5P35</td>
<td>50.567</td>
</tr>
<tr>
<td>Makespan</td>
<td>98.867</td>
<td>Makespan</td>
<td>122.084</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
<th>SPF4</th>
<th>SPF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF1</td>
<td>SPF2</td>
<td>SPF3</td>
<td>SPF4</td>
<td>SPF5</td>
</tr>
<tr>
<td>SPF1</td>
<td>SPF2</td>
<td>SPF3</td>
<td>SPF4</td>
<td>SPF5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PF3</th>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
<th>SPF4</th>
<th>SPF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>XP2P30P34P37</td>
<td>12.0</td>
<td>XP14</td>
<td>13.617</td>
<td>XP12P19</td>
<td>54.883</td>
</tr>
<tr>
<td>XP2P30P34P37</td>
<td>61.602</td>
<td>XP14P33</td>
<td>14.4</td>
<td>XP12P38</td>
<td>18.083</td>
</tr>
<tr>
<td>XP1P30P34P37</td>
<td>38.235</td>
<td>XP14P35P33</td>
<td>77.6</td>
<td>XP19P38</td>
<td>62.399</td>
</tr>
<tr>
<td>XP3P37</td>
<td>2.4</td>
<td>Makespan</td>
<td>105.617</td>
<td>Makespan</td>
<td>103.5</td>
</tr>
<tr>
<td>Makespan</td>
<td>114.237</td>
<td>Makespan</td>
<td>135.365</td>
<td>Makespan</td>
<td>120.216</td>
</tr>
</tbody>
</table>
## Table 5.50 Processing Time for the Minimal Product Sets Required to Meet the Demand Values for Product Family and Makespan for the Second Rule-Subfamilies Using the Batch-Cyclic Scheduling Method (Hours)

<table>
<thead>
<tr>
<th>PF</th>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>XP₃P₉</td>
<td>XP₉P₂₄</td>
<td>XP₁₁P₃₁</td>
</tr>
<tr>
<td>PF₁</td>
<td>43.75</td>
<td>33.6</td>
<td>3.3333</td>
</tr>
<tr>
<td></td>
<td>XP₃P₂₀</td>
<td>XP₂₄P₂₇</td>
<td>XP₁₁P₃₈</td>
</tr>
<tr>
<td></td>
<td>25.799</td>
<td>21.6</td>
<td>68.4</td>
</tr>
<tr>
<td></td>
<td>XP₃₄₃₁₅</td>
<td>XP₂₄P₂₉</td>
<td>XP₁₁P₃₈₃₁</td>
</tr>
<tr>
<td></td>
<td>9.35</td>
<td>50.0</td>
<td>72.9</td>
</tr>
<tr>
<td></td>
<td>XP₁₅₃₉</td>
<td>XP₂₄P₂₇</td>
<td>Makespan</td>
</tr>
<tr>
<td></td>
<td>26.134</td>
<td>31.45</td>
<td>144.6333</td>
</tr>
<tr>
<td></td>
<td>XP₉P₁₃₃₉</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>32.999</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makespan</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>138.032</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PF</th>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF₂</td>
<td>XP₃₂</td>
<td>XP₈P₃₃</td>
<td>XP₁₀P₃₀₈</td>
</tr>
<tr>
<td></td>
<td>13.5</td>
<td>40.3</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>XP₄₂₂₃₂</td>
<td>XP₁₃₃₅</td>
<td>XP₁₀P₂₈</td>
</tr>
<tr>
<td></td>
<td>64.0</td>
<td>8.0</td>
<td>48.417</td>
</tr>
<tr>
<td></td>
<td>Makespan</td>
<td>XP₆₁₃₃₅</td>
<td>XP₁₈₃₂₈</td>
</tr>
<tr>
<td></td>
<td>77.5</td>
<td>50.567</td>
<td>16.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Makespan</td>
<td>48.667</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Makespan</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>122.084</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PF</th>
<th>SPF1</th>
<th>SPF2</th>
<th>SPF3</th>
<th>SPF4</th>
<th>SPF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF₃</td>
<td>XP₂₃₇₃₀₃₇₃₇₃₇₉</td>
<td>XP₁₈₃₆</td>
<td>XP₁₃₃₆</td>
<td>XP₁₃₉₃₈</td>
<td>XP₁₄P₄₄</td>
</tr>
<tr>
<td></td>
<td>12.0</td>
<td>22.667</td>
<td>14.0</td>
<td>54.883</td>
<td>20.9</td>
</tr>
<tr>
<td></td>
<td>XP₂₃₇₃₀₃₄₃₇₃₇₉</td>
<td>XP₂₆₅₂₆₃₆</td>
<td>XP₁₃₃₈</td>
<td>XP₁₃₉₃₈₃₈</td>
<td>XP₃₄₅₈₈</td>
</tr>
<tr>
<td></td>
<td>61.602</td>
<td>40.95</td>
<td>18.083</td>
<td>62.399</td>
<td>58.8</td>
</tr>
<tr>
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<td>XP₁₅₃₇₃₄₃₇₃₇₉</td>
<td>XP₁₃₃₆</td>
<td>XP₂₅₃₃₃₆</td>
<td>Makespan</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38.235</td>
<td>13.333</td>
<td>82.45</td>
<td>135.365</td>
<td></td>
</tr>
<tr>
<td></td>
<td>XP₄₃₇₉</td>
<td>XP₂₆₅₂₆₃₆</td>
<td>Makespan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.4</td>
<td>19.733</td>
<td>Makespan</td>
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<td></td>
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<tr>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makespan</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>114.237</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.51 The Makespan for the Three Product Families in the System Based on the Batch-Cyclic Scheduling Method

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Makespan (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>144.6333</td>
</tr>
<tr>
<td>PF2</td>
<td>122.084</td>
</tr>
<tr>
<td>PF3</td>
<td>122.084</td>
</tr>
</tbody>
</table>

First Rule to Define the Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Makespan (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>144.6333</td>
</tr>
<tr>
<td>PF2</td>
<td>122.084</td>
</tr>
<tr>
<td>PF3</td>
<td>135.365</td>
</tr>
</tbody>
</table>

Second Rule to Define the Dissimilarity Coefficient

Table 5.51 shows the makespan for the three product families, when the batch-cyclic scheduling is utilized to schedule the subfamilies. However, the makespan of the product family is depended on the makespan of the subfamily which has the highest makespan in the product family.

5.8.4 Batch-Cyclic Scheduling Method vs. Batch Scheduling Method

As illustrated in section 5.5, two techniques of the batch scheduling method are utilized to schedule the products in the subfamilies. Also, the shifting bottleneck machine clearly effects on the makespan of products in the system. Table 5.52 and Table 5.53 explain the makespan for subfamilies in the first rule and the second rule of the dissimilarity coefficient respectively based on batch-cyclic scheduling method and two techniques of the batch scheduling method.
<table>
<thead>
<tr>
<th>Product Family</th>
<th>Sub-family</th>
<th>Products</th>
<th>Makespan Using the Batch-Cyclic Method (Hours)</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>SPF1</td>
<td>P₃ P₉ P₁₅ P₂₀ P₃₉</td>
<td>138.03</td>
<td>156.08</td>
<td>154.75</td>
</tr>
<tr>
<td></td>
<td>SPF2</td>
<td>P₇ P₂₄ P₂₇ P₂₉</td>
<td>136.65</td>
<td>155.33</td>
<td>141.55</td>
</tr>
<tr>
<td></td>
<td>SPF3</td>
<td>P₃₁ P₂₁ P₃₁</td>
<td>144.63</td>
<td>157.08</td>
<td>153.63</td>
</tr>
<tr>
<td>PF2</td>
<td>SPF1</td>
<td>P₄ P₂₂ P₃₂</td>
<td>77.5</td>
<td>112.5</td>
<td>82.32</td>
</tr>
<tr>
<td></td>
<td>SPF2</td>
<td>P₆ P₁₃ P₃₅</td>
<td>98.867</td>
<td>144.52</td>
<td>118.92</td>
</tr>
<tr>
<td></td>
<td>SPF3</td>
<td>P₁₀ P₁₆ P₁₈ P₂₈</td>
<td>122.08</td>
<td>155.53</td>
<td>128.75</td>
</tr>
<tr>
<td>PF3</td>
<td>SPF1</td>
<td>P₂ P₁₇ P₃₀ P₃₄ P₃₇</td>
<td>114.24</td>
<td>148.55</td>
<td>136.1</td>
</tr>
<tr>
<td></td>
<td>SPF2</td>
<td>P₁₄ P₂₅ P₃₃</td>
<td>105.62</td>
<td>122.98</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>SPF3</td>
<td>P₁ P₃₆</td>
<td>103.5</td>
<td>108.02</td>
<td>108.017</td>
</tr>
<tr>
<td></td>
<td>SPF4</td>
<td>P₁₂ P₁₉ P₃₈</td>
<td>135.37</td>
<td>145.08</td>
<td>145.03</td>
</tr>
<tr>
<td></td>
<td>SPF5</td>
<td>P₅ P₈ P₂₃ P₂₆</td>
<td>120.22</td>
<td>140.83</td>
<td>125.63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product Family</th>
<th>Sub-family</th>
<th>Products</th>
<th>Makespan Using the Batch-Cyclic Method (Hours)</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1</td>
<td>SPF1</td>
<td>P₃ P₉ P₁₅ P₂₀ P₃₉</td>
<td>138.03</td>
<td>156.08</td>
<td>154.75</td>
</tr>
<tr>
<td></td>
<td>SPF2</td>
<td>P₇ P₂₄ P₂₇ P₂₉</td>
<td>136.65</td>
<td>155.33</td>
<td>141.55</td>
</tr>
<tr>
<td></td>
<td>SPF3</td>
<td>P₁₃ P₂₁ P₃₁</td>
<td>144.63</td>
<td>157.08</td>
<td>153.63</td>
</tr>
<tr>
<td>PF2</td>
<td>SPF1</td>
<td>P₄ P₂₂ P₃₂</td>
<td>77.5</td>
<td>112.5</td>
<td>82.32</td>
</tr>
<tr>
<td></td>
<td>SPF2</td>
<td>P₆ P₁₃ P₃₅</td>
<td>98.87</td>
<td>144.517</td>
<td>118.92</td>
</tr>
<tr>
<td></td>
<td>SPF3</td>
<td>P₁₀ P₁₆ P₁₈ P₂₈</td>
<td>122.08</td>
<td>155.53</td>
<td>128.75</td>
</tr>
<tr>
<td>PF3</td>
<td>SPF1</td>
<td>P₂ P₁₇ P₃₀ P₃₄ P₃₇</td>
<td>114.24</td>
<td>148.55</td>
<td>136.1</td>
</tr>
<tr>
<td></td>
<td>SPF2</td>
<td>P₁ P₈ P₂₃ P₂₆</td>
<td>126.82</td>
<td>152.55</td>
<td>138.95</td>
</tr>
<tr>
<td></td>
<td>SPF3</td>
<td>P₂₅ P₃₃ P₃₆</td>
<td>119.12</td>
<td>134.05</td>
<td>130.63</td>
</tr>
<tr>
<td></td>
<td>SPF4</td>
<td>P₁₂ P₁₉ P₃₈</td>
<td>135.37</td>
<td>145.08</td>
<td>145.03</td>
</tr>
<tr>
<td></td>
<td>SPF5</td>
<td>P₅ P₁₄</td>
<td>79.699</td>
<td>83.83</td>
<td>83.83</td>
</tr>
</tbody>
</table>
The makespan of product family depends on the makespan of subfamily which has the highest makespan in the product family. Table 5.54 shows the makespan for the product families when the first and the second rule are utilized to define the processing time dissimilarity coefficient. The results show that the batch-cyclic scheduling method obtains the lowest makespan in the three product families compared to two techniques of the batch scheduling method.

Table 5.54 Makespan for Three Product Families Based on the Batch-Cyclic Scheduling Method and Two Techniques of the Batch Scheduling Method

<table>
<thead>
<tr>
<th>Processing Time Dissimilarity Coefficient</th>
<th>Product Family</th>
<th>Makespan Using the Batch-Cyclic method (Hours)</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Rule of Dissimilarity Coefficient</td>
<td>PF1</td>
<td>144.63</td>
<td>157.08</td>
<td>154.75</td>
</tr>
<tr>
<td></td>
<td>PF2</td>
<td>122.08</td>
<td>155.53</td>
<td>128.75</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
<td>135.37</td>
<td>148.55</td>
<td>145.03</td>
</tr>
<tr>
<td>Second Rule of Dissimilarity Coefficient</td>
<td>PF1</td>
<td>144.63</td>
<td>157.08</td>
<td>154.75</td>
</tr>
<tr>
<td></td>
<td>PF2</td>
<td>122.08</td>
<td>155.53</td>
<td>128.75</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
<td>135.37</td>
<td>152.55</td>
<td>145.03</td>
</tr>
</tbody>
</table>

Table 5.55 explains the makespan for the products when the batch-cyclic scheduling method, and two technique of batch scheduling method are utilized to schedule the products in the flowshop cell. The results show that the batch-cyclic scheduling method provides lowest makespan compared to the two techniques on the batch scheduling method.
Table 5.55 Makespan of Products Based on the Batch-Cyclic Scheduling Method and Two Techniques of the Batch Scheduling Method

<table>
<thead>
<tr>
<th>Processing Time Dissimilarity Coefficient</th>
<th>Makespan Using the Batch-Cyclic method (Hours)</th>
<th>Makespan Using the First Technique of the Batch Scheduling Method (Hours)</th>
<th>Makespan Using the Second Technique of the Batch Scheduling Method (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Rule of Dissimilarity Coefficient</td>
<td>144.63</td>
<td>157.08</td>
<td>154.75</td>
</tr>
<tr>
<td>Second Rule of Dissimilarity Coefficient</td>
<td>144.63</td>
<td>157.08</td>
<td>154.75</td>
</tr>
</tbody>
</table>

The results have shown that the batch-cyclic scheduling method provides less makespan in the system compared to the two techniques on the batch scheduling method when the bottleneck machine shifts. However, the batch-cyclic scheduling method is an appropriate method to apply when the shifting bottleneck machine is applied. Otherwise, batch-cyclic scheduling method and two techniques of the batch scheduling method will provide the same results. Even though, the dissimilarity coefficient between products is large. Also, the production rate for a minimal product set depends on total processing time of the machine which has the highest total processing times in the minimal product set based on the results of the manual calculation, therefore, the sequence of products in the minimal product set is not important. In this regard, this machine is called the bottleneck machine for the minimal product set.

5.9 Modify Capacity Parameter

As a mentioned before, the production rate of the flowshop system depends on the processing time of the bottleneck machine when the bottleneck machine is a common
machine for all products in the system. Otherwise, the second technique of the batch scheduling method, Gantt chart, has to use to define the makespan of products in the system. The results in the previous sections show that the makespan of system decreases when the shifting bottleneck machine concept is applied. Even though the bottleneck machine is not a common machine and each product has its bottleneck machine, the production rate of system in the second phase is calculated based on the first technique of batch scheduling method, which is based on the processing time of the bottleneck machine to define the production rate of the system. The first technique is used to define the production rate of system in the second phase because it obtains the exact production rate of system, while the second technique doesn’t obtain exact solution because it based on the processing time of bottleneck machine. The results in the previous sections show the importance of using the second technique of batch scheduling method, when the bottleneck machine is shifted because it obtains less makespan compared to the first technique of the batch scheduling method.

Using the first technique of batch schedule method to schedule the products with shifting bottleneck machine increases the idle times of machines. Also, using the batch-cyclic scheduling method to schedule products in the subfamily reduces the makespan of products in the system, which means increasing the idle time of machines in the system. However, in this section capacitated P-median model runs several times to form the subfamilies with modifying capacity parameter in phase two. Different subfamilies can be obtained by modifying capacity parameter in the system. The main objectives of this section are reducing the number of cells in the system and improving the capacity
parameter of the system. Table 5.56 and Table 5.57 show the 6 configurations are generated by modifying the capacity parameter of the system in the second phase starting with 160 hours in the first and the second rule respectively. Each configuration is created by modifying capacity parameter 5%.
<table>
<thead>
<tr>
<th>Capacity</th>
<th>160 Hours</th>
<th>176 Hours</th>
<th>192 Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product Family 1</strong></td>
<td>SPF</td>
<td>P₃ P₉P₁₅P₂₀ P₉ P₃₉</td>
<td>SPF</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₇ P₂₄P₂₇ P₂₉</td>
<td>SPF₂</td>
<td>P₉P₁₁P₃₁</td>
</tr>
<tr>
<td>SPF₃</td>
<td>P₁₁ P₂₁ P₃₁</td>
<td>SPF₃</td>
<td>P₇P₂₄P₂₉</td>
</tr>
<tr>
<td><strong>Product Family 2</strong></td>
<td>SPF</td>
<td>P₄ P₂₂ P₃₂</td>
<td>SPF</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₆ P₁₃ P₃₅</td>
<td>SPF₂</td>
<td>P₄P₆P₁₅P₃₅</td>
</tr>
<tr>
<td>SPF₃</td>
<td>P₁₀ P₁₆P₁₈ P₂₈</td>
<td>SPF₃</td>
<td>P₁₀P₁₆P₁₈P₂₈</td>
</tr>
<tr>
<td><strong>Product Family 3</strong></td>
<td>SPF</td>
<td>P₂P₁₇P₃₀P₃₄P₃₇</td>
<td>SPF</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₁₄ P₂₅ P₃₃</td>
<td>SPF₂</td>
<td>P₁₇P₃₀P₃₆P₃₇</td>
</tr>
<tr>
<td>SPF₃</td>
<td>P₁ P₃₆</td>
<td>SPF₃</td>
<td>P₁₈P₁₂P₂₆</td>
</tr>
<tr>
<td>SPF₄</td>
<td>P₁₂ P₁₉ P₃₈</td>
<td>SPF₄</td>
<td>P₂P₂₄P₃₄P₃₈</td>
</tr>
<tr>
<td>SPF₅</td>
<td>P₅ P₈ P₂₃ P₂₆</td>
<td>SPF₅</td>
<td>-</td>
</tr>
<tr>
<td><strong>Capacity</strong></td>
<td>168 Hours</td>
<td>184 Hours</td>
<td>208 Hours</td>
</tr>
<tr>
<td><strong>Product Family 1</strong></td>
<td>SPF</td>
<td>P₃P₁₅P₂₀P₂₁P₂₉ P₃₉</td>
<td>SPF</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₉P₁₁P₃₁</td>
<td>SPF₂</td>
<td>P₁₁P₃₁</td>
</tr>
<tr>
<td>SPF₃</td>
<td>P₂P₂₄P₂₇</td>
<td>SPF₃</td>
<td>P₂P₂₄P₂₉</td>
</tr>
<tr>
<td><strong>Product Family 2</strong></td>
<td>SPF</td>
<td>P₄P₂₂P₃₂</td>
<td>SPF</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₆P₁₅P₃₅</td>
<td>SPF₂</td>
<td>P₄P₆P₁₃P₃₅</td>
</tr>
<tr>
<td>SPF₃</td>
<td>P₁₀P₁₆P₁₈P₂₈</td>
<td>SPF₃</td>
<td>P₁₀P₁₆P₁₈P₂₈</td>
</tr>
<tr>
<td><strong>Product Family 3</strong></td>
<td>SPF</td>
<td>P₁₂P₃₀P₃₆P₃₇</td>
<td>SPF</td>
</tr>
<tr>
<td>SPF₂</td>
<td>P₁₈P₁₇P₂₆</td>
<td>SPF₂</td>
<td>P₃P₁₄P₂₅</td>
</tr>
<tr>
<td>SPF₃</td>
<td>P₂P₃P₁₄P₁₉</td>
<td>SPF₃</td>
<td>P₁₈P₁₂P₂₆</td>
</tr>
<tr>
<td>SPF₄</td>
<td>P₂₃P₂₅P₃₃P₃₄P₃₈</td>
<td>SPF₄</td>
<td>P₂P₁₇P₃₀P₃₆P₃₇</td>
</tr>
<tr>
<td>SPF₅</td>
<td>-</td>
<td>SPF₅</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 5.57 Product Family and their Products for each Configuration Based on the Second Rule of Dissimilarity Coefficient

<table>
<thead>
<tr>
<th>Capacity</th>
<th>Sub-family</th>
<th>Products</th>
<th>Sub-family</th>
<th>Products</th>
<th>Sub-family</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>160 Hours</td>
<td></td>
<td>176 Hours</td>
<td></td>
<td>192 Hours</td>
<td></td>
</tr>
<tr>
<td>Product</td>
<td>SPF1</td>
<td>P3 P9 P15 P20 P39</td>
<td>SPF1</td>
<td>P3 P9 P20 P21</td>
<td>SPF1</td>
<td>P3 P9 P15 P20 P21 P29</td>
</tr>
<tr>
<td>Family 1</td>
<td>SPF2</td>
<td>P7 P24 P27 P29</td>
<td>SPF2</td>
<td>P24 P27 P31 P39</td>
<td>SPF2</td>
<td>P7 P11</td>
</tr>
<tr>
<td>SPF3</td>
<td>P11 P21 P31</td>
<td>SPF3</td>
<td>P7 P11 P29</td>
<td>SPF3</td>
<td>P24 P27 P31 P39</td>
<td></td>
</tr>
<tr>
<td>SPF4</td>
<td>P12 P19 P38</td>
<td>SPF4</td>
<td>P2 P17 P26 P30 P34 P37</td>
<td>SPF4</td>
<td>P1 P14 P25 P33</td>
<td></td>
</tr>
<tr>
<td>SPF5</td>
<td>P5 P14</td>
<td>SPF5</td>
<td>-</td>
<td>SPF5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacity</td>
<td>168 Hours</td>
<td></td>
<td>184 Hours</td>
<td></td>
<td>208 Hours</td>
<td></td>
</tr>
<tr>
<td>Product</td>
<td>SPF1</td>
<td>P3 P9 P20 P21 P39</td>
<td>SPF1</td>
<td>P3 P9 P15 P20 P21 P29</td>
<td>SPF1</td>
<td>P3 P9 P15 P20 P21 P29</td>
</tr>
<tr>
<td>Family 1</td>
<td>SPF2</td>
<td>P15 P24 P29 P31</td>
<td>SPF2</td>
<td>P7 P11</td>
<td>SPF2</td>
<td>P7 P11</td>
</tr>
<tr>
<td>SPF4</td>
<td>P14 P25 P33</td>
<td>SPF4</td>
<td>P2 P17 P30 P34 P36</td>
<td>SPF4</td>
<td>P2 P17 P30 P34 P36</td>
<td></td>
</tr>
<tr>
<td>SPF5</td>
<td>P5 P18 P28</td>
<td>SPF5</td>
<td>-</td>
<td>SPF5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Product</td>
<td>SPF1</td>
<td>P2 P26 P30 P34 P36</td>
<td>SPF1</td>
<td>P1 P14 P25 P33</td>
<td>SPF1</td>
<td>P1 P5 P8 P23 P37</td>
</tr>
<tr>
<td>Family 2</td>
<td>SPF2</td>
<td>P14 P25 P33</td>
<td>SPF2</td>
<td>P2 P17 P30 P34 P36</td>
<td>SPF2</td>
<td>P12 P19 P38</td>
</tr>
<tr>
<td>SPF3</td>
<td>P17 P23 P38</td>
<td>SPF3</td>
<td>P5 P23 P26 P37</td>
<td>SPF3</td>
<td>P2 P17 P26 P30 P34 P36</td>
<td></td>
</tr>
<tr>
<td>SPF4</td>
<td>P5 P8 P12 P37</td>
<td>SPF4</td>
<td>P12 P19 P38</td>
<td>SPF4</td>
<td>P14 P25 P33</td>
<td></td>
</tr>
<tr>
<td>SPF5</td>
<td>-</td>
<td>SPF5</td>
<td>-</td>
<td>SPF5</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
In the second configuration, where the capacity parameter is 168 hours the subfamilies are reduced from five subfamilies to four subfamilies in the product family three as shown in the Table 5.56 and Table 5.57. Also, the subfamilies are reduced from three subfamilies to two subfamilies in the second product family when the capacity parameter is 208 hours. However, each subfamily has to schedule in the cell within 160 hours (monthly available capacity) in the phase three when the batch-cyclic scheduling method is utilized to schedule the products in the system. Otherwise, the system is considered violating the capacity parameter if the makespan of subfamily is greater than 160 hours.

5.9.1 Batch-Cyclic Scheduling Method

Using the batch-cyclic scheduling method to schedule the products in the system with shifting bottleneck machine decreases the makespan of products in the subfamily. In this section, the second technique of the batch scheduling method is used beside the batch-cyclic method to schedule the subfamilies. Table 5.58 and Table 5.59 show the makespan for subfamilies in all configurations when the batch-cyclic scheduling method and the second technique of the batch scheduling method are utilized to schedule the first rule-subfamilies and the second rule-subfamilies.

As the data given Table 5.58 and Table 5.59, the makespan for SPF₁ in the first product family in the configuration 4 (184 hours) is greater than 160 hours, when the batch-cyclic scheduling method is utilized to schedule the products in the system. In this configuration, the system violates the available capacity, which is monthly demand 160 hours. Also, the SPF₄ in the product family three violates the available capacity in the
configuration five (208 hours) when the batch-cyclic scheduling method is used to schedule the products, where the monthly available capacity is 160 hours.
<table>
<thead>
<tr>
<th>Capacity Parameter</th>
<th>Product Family</th>
<th>Sub-family</th>
<th>Products</th>
<th>MS Using Batch-Cyclic (Hours)</th>
<th>MS Using BM (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>160 (0%)</td>
<td>PF1</td>
<td>SPF1</td>
<td>P_1 P_3 P_5 P_9 P_10 P_12 P_14 P_16</td>
<td>138.03</td>
<td>156.08</td>
</tr>
<tr>
<td></td>
<td>PF2</td>
<td>SPF2</td>
<td>P_1 P_3 P_5 P_7 P_9</td>
<td>136.65</td>
<td>155.33</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
<td>SPF3</td>
<td>P_1 P_3 P_5</td>
<td>144.63</td>
<td>157.08</td>
</tr>
<tr>
<td>168 (5%)</td>
<td>PF1</td>
<td>SPF1</td>
<td>P_1 P_3 P_5 P_9 P_10 P_12 P_14 P_16</td>
<td>148.47</td>
<td>166.35</td>
</tr>
<tr>
<td></td>
<td>PF2</td>
<td>SPF2</td>
<td>P_1 P_3 P_5</td>
<td>155.13</td>
<td>166.82</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
<td>SPF3</td>
<td>P_1 P_3 P_5</td>
<td>122.65</td>
<td>135.53</td>
</tr>
<tr>
<td>176 (10%)</td>
<td>PF1</td>
<td>SPF1</td>
<td>P_1 P_3 P_5 P_9 P_10 P_12 P_14 P_16</td>
<td>153.6</td>
<td>171.35</td>
</tr>
<tr>
<td></td>
<td>PF2</td>
<td>SPF2</td>
<td>P_1 P_3 P_5</td>
<td>155.13</td>
<td>166.82</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
<td>SPF3</td>
<td>P_1 P_3 P_5</td>
<td>115</td>
<td>130.33</td>
</tr>
<tr>
<td>184 (15%)</td>
<td>PF1</td>
<td>SPF1</td>
<td>P_1 P_3 P_5 P_9 P_10 P_12 P_14 P_16</td>
<td>159.07</td>
<td>173.45</td>
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<tr>
<td></td>
<td>PF2</td>
<td>SPF2</td>
<td>P_1 P_3 P_5 P_9 P_10 P_12 P_14 P_16</td>
<td>163.33</td>
<td>183.08</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
<td>SPF3</td>
<td>P_1 P_3 P_5 P_9 P_10 P_12 P_14 P_16</td>
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<td>P_1 P_3 P_5 P_9 P_10 P_12 P_14 P_16</td>
<td>136.65</td>
<td>155.33</td>
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Table 5.58 Makespan for Subfamilies in All Configurations for the First Rule-Subfamilies
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<th>Capacity Parameter</th>
<th>Product Family</th>
<th>Sub-family</th>
<th>Products</th>
<th>MS Using Batch-Cyclic (Hours)</th>
<th>MS Using BM (Hours)</th>
<th>Capacity Parameter</th>
<th>Product Family</th>
<th>Sub-family</th>
<th>Products</th>
<th>MS Using Batch-Cyclic (Hours)</th>
<th>MS Using BM (Hours)</th>
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<tbody>
<tr>
<td>160 (0%)</td>
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<td>P1 P6 P15 P20 P29</td>
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<td>156.0833333</td>
<td>SPF1</td>
<td></td>
<td>SPF1</td>
<td>P1 P6 P15 P20 P29</td>
<td>162.83</td>
</tr>
<tr>
<td></td>
<td>PF2</td>
<td></td>
<td>SPF1</td>
<td>P4 P12 P17</td>
<td>77.5</td>
<td>112.5</td>
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<td></td>
<td>SPF2</td>
<td>P5 P11</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>SPF2</td>
<td>P1 P6 P15 P20 P29</td>
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<td>155.3333333</td>
<td>SPF2</td>
<td></td>
<td>SPF3</td>
<td>P2 P12 P15 P20 P29</td>
<td>151.05</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
<td></td>
<td>SPF3</td>
<td>P1 P21 and P31</td>
<td>144.63</td>
<td>157.0833333</td>
<td>SPF3</td>
<td></td>
<td>SPF3</td>
<td>P2 P12 P15 P20 P29</td>
<td>135.07</td>
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<td>SPF1</td>
<td>P2 P17 P20 P24 P27</td>
<td>97.233</td>
</tr>
<tr>
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<td>SPF2</td>
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<td>126.82</td>
<td>152.55</td>
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<td>SPF2</td>
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<td></td>
<td>SPF3</td>
<td>P2 P17 P20 P24 P27</td>
<td>119.12</td>
<td>134.05</td>
<td>SPF3</td>
<td></td>
<td>SPF3</td>
<td>P2 P17 P20 P24 P27</td>
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<td></td>
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<td>SPF5</td>
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<td>79.699</td>
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<td>SPF5</td>
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</tr>
<tr>
<td>168 (5%)</td>
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<td></td>
<td>SPF1</td>
<td>P3 P4 P6 P20 P29 P30</td>
<td>144.93</td>
<td>162.4</td>
<td>SPF1</td>
<td></td>
<td>SPF1</td>
<td>P3 P4 P6 P20 P29 P30</td>
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</tr>
<tr>
<td></td>
<td>PF2</td>
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<td>P3 P4 P6 P20 P29 P30</td>
<td>144.93</td>
<td>162.4</td>
<td>SPF2</td>
<td></td>
<td>SPF2</td>
<td>P3 P4 P6 P20 P29 P30</td>
<td>97.233</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
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<td>SPF3</td>
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<td>144.93</td>
<td>162.4</td>
<td>SPF3</td>
<td></td>
<td>SPF3</td>
<td>P3 P4 P6 P20 P29 P30</td>
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</tr>
<tr>
<td>176 (10%)</td>
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<td></td>
<td>SPF1</td>
<td>P3 P4 P6 P20 P29 P30</td>
<td>144.93</td>
<td>162.4</td>
<td>SPF1</td>
<td></td>
<td>SPF1</td>
<td>P3 P4 P6 P20 P29 P30</td>
<td>144.93</td>
</tr>
<tr>
<td></td>
<td>PF2</td>
<td></td>
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<td>P3 P4 P6 P20 P29 P30</td>
<td>151.05</td>
<td>171.0833333</td>
<td>SPF2</td>
<td></td>
<td>SPF2</td>
<td>P3 P4 P6 P20 P29 P30</td>
<td>150.07</td>
</tr>
<tr>
<td></td>
<td>PF3</td>
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<td>P3 P4 P6 P20 P29 P30</td>
<td>129.08</td>
<td>135.07</td>
<td>SPF3</td>
<td></td>
<td>SPF3</td>
<td>P3 P4 P6 P20 P29 P30</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.59 Makespan for Subfamilies in All Configurations for the Second Rule-Subfamilies
Table 5.6 illustrates the improvement capacity parameter in all configurations based on the first rule of dissimilarity coefficient. The results show that the improvement capacity parameter in the first product family is lower than 15% when the batch-cyclic scheduling method is utilized to schedule products in the cell. In the configuration 4, where capacity parameter is 184 hours, the first subfamily violates the available capacity, the makespan in this subfamily is 162.38 hours. In addition, the improvement capacity parameter in the second product family is greater than 30% and the number of cells is reduced from three cells to two cells. Also, the improvement capacity parameter in the third product family is less than 20%, and the number of cells in this case is also reduced from five cells to four cells. However, using the batch-cyclic scheduling method is improved capacity parameter in the system and reduced the number of cells in the system.

It is worth mentioning that the overall improvement capacity parameter in the system depends on the improvement capacity parameter in the first product family because it has the lowest improvement capacity parameter in the system. In this regard, several configurations are developed to determine the exact improvement capacity parameter in the first product family. The difference between the configurations is 0.6%, which is equivalent to 1 hour. The results illustrate that the improvement capacity parameter in the first product family is 14.375%, where the capacity parameter is 183 hours. Three subfamilies, P_3P_9P_{15}P_{20}P_{29}P_{39}, P_{21}P_{24}P_{31}, and P_{7}P_{11}P_{27}, are created, and the makespan for all subfamilies equals to 154.85 hours, 136.92 hours and 130.69 hours respectively. However, the overall improvement capacity parameter in the system when the batch-cyclic
scheduling method is utilized to schedule the products in the system is 14.375%. Also, the number of cells is decreased to 10 cells.
### Table 5.60 Modifying Capacity Parameter in the System for First Rule-Subfamilies

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<tbody>
<tr>
<td>5%</td>
<td>PF1</td>
<td>SPF1</td>
<td>P_3 P_5 P_{13} P_{20} P_{39}</td>
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<td>PF1</td>
<td>SPF1</td>
<td>P_3 P_{13} P_{20} P_{21} P_{27} P_{39}</td>
<td>10%</td>
<td>PF1</td>
<td>SPF1</td>
<td>Violate Capacity</td>
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<tr>
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<td>SPF2</td>
<td>P_{7} P_{24} P_{27} P_{29}</td>
<td>SPF3</td>
<td>P_{12} P_{21} P_{31}</td>
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<td>P_{7} P_{24} P_{29}</td>
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<td>SPF2</td>
<td>P_{6} P_{13} P_{35}</td>
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<td>P_{10} P_{16} P_{18} P_{28}</td>
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<td>P_{10} P_{16} P_{18} P_{28}</td>
<td>SPF3</td>
<td>P_{12} P_{19} P_{48}</td>
<td>SPF3</td>
<td>P_{2} P_{3} P_{4} P_{34} P_{38}</td>
<td>SPF3</td>
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<td>SPF6</td>
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<td></td>
</tr>
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<td>5%</td>
<td>PF1</td>
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<td>P_{3} P_{13} P_{20} P_{21} P_{39}</td>
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<td>PF2</td>
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<td>P_{2} P_{22} P_{32}</td>
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<td>PF1</td>
<td>SPF1</td>
<td>Violate Capacity</td>
</tr>
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<td>P_{12} P_{19} P_{48}</td>
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<td>P_{10} P_{16} P_{18} P_{28}</td>
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<td>SPF1</td>
<td>P_{3} P_{13} P_{20} P_{21} P_{39}</td>
<td>20%</td>
<td>PF2</td>
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<td>SPF2</td>
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<td>P_{4} P_{8} P_{13} P_{35}</td>
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Table 5.61 shows capacity parameter for subfamilies in all configurations when the second rule of dissimilarity coefficient is utilized to create the subfamilies. The results show that using the batch-cyclic scheduling method improves capacity parameter in the first product family to less than 15%. The improvement capacity parameter in the second product family can be more than 30%, and the number of cells will be reduced from three cells to two cells, while the improvement capacity parameter in the third product family is less than 20%, and the number of cells is reduced from five to four cells.

On the other hand, the first product family determines the overall improvement capacity parameter in the system, because it has the lowest improvement capacity parameter in the system. In this regard, several cases are created to determine the exact improvement capacity parameter in the system. The difference between the cases is 0.6%, which is equivalent to 1 hour. The results show that the exact improvement the capacity parameter in the system equals 13.75%. In this case, first product family is divided into three subfamilies (P_3 P_9 P_20 P_21, P_24 P_27 P_31 P_39, and P_7 P_11 P_29), with makespan 146.83 hours, 151.05 hours, and 129.08 hours respectively, when the batch-cyclic scheduling method is utilized to schedule the products in the system. Also, the number of cells is reduced from 11 cells to 10 cells.

However, the number of cells can be reduced to 9 cells in two rules of dissimilarity coefficient when the improvement capacity parameter for each product family is calculated separately from the other product families. For example, the improvement capacity parameter for the first, the second and the third product family is less than 15%, more than 30% and less than 20% respectively in two rules of dissimilarity coefficient.
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<td>SPF1</td>
<td>P3 P9 P15 P20 P29</td>
<td>10%</td>
<td>PF1</td>
<td>SPF1</td>
<td>P3 P9 P20 P21</td>
<td>20%</td>
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<td>SPF1</td>
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<td>P10 P16 P18 P28</td>
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<td>PF3</td>
<td>SPF1</td>
<td>Violate Capacity</td>
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Table 5.61 Modifying the Capacity Parameter in the System for Second Rule-Subfamilies
Using the batch-cyclic scheduling method to schedule the products with shifting bottleneck machine is an appropriate method when the bottleneck machine shifts and the gap between the processing times between products is large. It reduces the number of cells and the same time improvement capacity parameter in the cell. However, using the first rule of the processing time dissimilarity coefficient increases the overall improvement capacity parameter in the system to 14.375%, and the number of cells is reduced from 11 cells to 10 cells. The second rule of the processing time dissimilarity coefficient improves capacity parameter to 13.75%, and the number of cells is reduced from 11 to 10 cells. In addition, the system performance is increased when the gap between the processing times of products in the subfamilies is large. In this case, the number of cell decreases and the capacity parameter is improving.
Chapter 6  : Conclusions and Future Research

This thesis focuses on family formation, cell loading and batch-cyclic flowshop scheduling in the cellular manufacturing system with shifting bottleneck machine, machine skipping, zero setup time, and deterministic processing times and demand. The proposed methodology depends on three phases. Product families are created by using a similarity coefficient based on the mathematical model in the first phase. Each product family is divided into subfamilies based on maximizing dissimilarity coefficient in the second phase. In this regard, two rules are developed to define the dissimilarity coefficient. The first rule relies on the average of the difference between the processing times of the product pairs. The second rule adopts on the average of the difference between the processing times for the product pairs in the machines which processing either the first product or the second product in the product pairs. Finally, the third phase schedules the products in the subfamilies based on the batch-cyclic method. The mathematical model is developed to define the makespan of subfamily in the system, and to determine the processing time for the minimal product sets required to cover the demand values of products in the subfamily. Also, the two techniques of batch scheduling are utilized to determine the makespan of products in the system beside the batch-cyclic scheduling method.

However, the results show the importance of batch-cyclic scheduling method to maximize the production rate of the system, minimize the makespan of products in the system, reduce the number of cells and improve capacity parameter of the system with shifting bottleneck machine and zero setup time.
6.1 Conclusions

In this thesis, two rules were developed to define the dissimilarity coefficient between the products in the product family. These rules are applied in the P-median model, which is depended on maximizing the dissimilarity coefficient between the products in each subfamily. Different subfamilies were created per each product family and per each rule. In addition, using maximizing dissimilarity coefficient to divide the product family into subfamilies increase the gap between the processing times of products in each subfamily. Increasing the gap between the processing times in the subfamily reflects on the system performance positively. Therefore, the probability of mixing products in the system increases, when the batch-cyclic scheduling method is utilized to schedule the products in the system.

The scheduling process in this thesis depends on the mixed mode, batch-cyclic method, to schedule the products in the system. This method depends on the many steps. The first step defines the minimal products set in the system. In this regard, the “Hasse Diagram” is used to define the minimal product sets for each product, and subfamily. Since the minimal product sets are defined for each subfamily, the manual calculation is utilized to define the production rate of the system when the set is processed in the system. However, the results of the manual calculation show that the cycle time for each minimal product set depends on the total processing time of a machine which has the highest total processing time in the minimal product set. Based on that the sequence of products in each minimal product set is not important since the production rate of the system depends on the total processing time of a machine which has the highest processing time in the set.
Having determined the production rate for each minimal product set the data enter into the mathematical model and the CPLEX software is used to solve the mathematical model. The results explained the efficiency of the batch-cyclic scheduling method to schedule the products in the subfamilies with shifting bottleneck machines.

Beside the batch-cyclic scheduling method, the two techniques of the batch scheduling method are also utilized to schedule the products in the flowshop cells. The first technique relies on the processing time of the bottleneck machine to determine the makespan of products in the subfamily and define the production rate of system for each product. In this technique, the processing time for the first item in the product equals the completion time for the last item in the previous product. The second technique relies on the Gantt chart to define the makespan of products in the system. The results show the importance of using the second technique of the batch-cyclic method when the shifting bottleneck machine concept is applied and each product has its own bottleneck machine, while the two can be used to schedule the product in the system when the bottleneck machine is a common machine for all products in the system.

Minimizing the makespan in the system means more idle time of machines. In this case, it is recommended to develop more cases with modifying capacity parameter 5% per each case. Different subfamilies are created per each product family with different products. However, the results shown that the overall improvement capacity parameter in the system equals 14.375%, where the first rule of the dissimilarity coefficient is utilized to create the subfamilies. Also, the number of cells is reduced from 11 cells to 10 cells. On the other hand, the number of cells in the system can be reduced from 11 cells to 9 cells,
when the improvement capacity parameter in each product family is calculated separately than the other product families. In this regard, the improvement capacity parameter in the system also increases. For example, the improvement capacity parameter and the number of cells for the first product family, the second product family and the third product family with their cells equals 14.375% with three cells, more than 30% with two cells and less than 20% with four cells respectively. It is worth mentioning that overall modifying the capacity parameter in the system in this rule depends on the first product family because it has the lowest improvement capacity parameter in the system.

Moreover, the overall improvement capacity parameter in the system when the second rule of dissimilarity coefficient is utilized to form the subfamily is 13.75%, and the number cell also is reduced from 11 cells to 10 cells. The first product family also determines the overall improvement capacity parameter in the system in this rule. On the other hand, the improvement capacity parameter and the number of cells for the first product family, the second product family and the third product family equals 13.75% with three cells, greater than 30% with two cells, and less than 20% with four cells. Therefore, the improvement capacity parameter increases, and the number of cells decreases when the improvement capacity parameter for each product family is calculated separately from the other product families.

In short, using the batch-cyclic scheduling method with zero setup time, shifting bottleneck machine and skipping machines decreases the makespan of product of system, reduces the number of cells and improves the capacity parameter in the system. Also, the
efficiency of this method increases when the gap between the processing times in the system increases.

6.2 Future Research

This thesis can be extended by considering the following issues:

- Developing the Java code to define the minimal product sets and their production rate.
- Determining the factor to define the improvement capacity parameter of the capacitated P-median model in the second phase.
- Defining the sequence of the minimal product set in the system.
- Experimenting with stochastic processing time and demand values.
- Developing the simulation program when the data are stochastic data.
- Setup time is greater than zero when the system switch processing from one minimal product set to others.
- Setup time is greater than zero when the system switch processing from one product family to others.
- Comparison the batch-cyclic scheduling method results with the cyclic scheduling method results.
- Allowing the sharing machine between cells
- Calculating the machine efficiency
- Creating layered layout, dedicated cell, shared cells and remainder cells instead of classical layout.
Developing a new method to calculate the dissimilarity coefficient in the second phase and adding the weight to the processing time. In this case, the bottleneck machines have the higher weight.
References


Appendix A: OPL Code for Family Formulation and Subfamily Formation

Product Family Formulation:

Short Cut Form:

```opl
int n = ...;
int m = ...;

range parts = 1.. n;
range Cells = 1.. m;

float b[parts][Cells] = ...;

dvar int+ x[parts][Cells];

maximize sum(i in parts, j in Cells) b[i][j] * x[i][j] - sum(j in Cells) (p * x[j][j]);
subject to {
    forall (i in parts)
        sum(j in Cells) x[i][j] == 1;

    forall (i in parts)
        forall (j in Cells)
            x[i][j] <= x[j][j];
}
```
Open Form:

```plaintext
dvar int+ X11 in 0..1;
dvar int+ X21 in 0..1;
dvar int+ X31 in 0..1;
dvar int+ X41 in 0..1;
dvar int+ X51 in 0..1;
dvar int+ X61 in 0..1;
dvar int+ X71 in 0..1;
dvar int+ X81 in 0..1;
dvar int+ X91 in 0..1;
dvar int+ X101 in 0..1;
dvar int+ X111 in 0..1;
dvar int+ X121 in 0..1;

dvar int+ X12 in 0..1;
dvar int+ X22 in 0..1;
dvar int+ X32 in 0..1;
dvar int+ X42 in 0..1;
dvar int+ X52 in 0..1;
dvar int+ X62 in 0..1;
dvar int+ X72 in 0..1;
dvar int+ X82 in 0..1;
dvar int+ X92 in 0..1;
dvar int+ X102 in 0..1;
dvar int+ X112 in 0..1;
dvar int+ X122 in 0..1;

dvar int+ X13 in 0..1;
dvar int+ X23 in 0..1;
dvar int+ X33 in 0..1;
dvar int+ X43 in 0..1;
dvar int+ X53 in 0..1;
dvar int+ X63 in 0..1;
dvar int+ X73 in 0..1;
dvar int+ X83 in 0..1;
dvar int+ X93 in 0..1;
dvar int+ X103 in 0..1;
dvar int+ X113 in 0..1;
dvar int+ X123 in 0..1;

dvar int+ X14 in 0..1;
dvar int+ X24 in 0..1;
dvar int+ X34 in 0..1;
dvar int+ X44 in 0..1;
dvar int+ X54 in 0..1;
dvar int+ X64 in 0..1;
dvar int+ X74 in 0..1;
dvar int+ X84 in 0..1;
dvar int+ X94 in 0..1;
dvar int+ X104 in 0..1;
dvar int+ X114 in 0..1;
dvar int+ X124 in 0..1;
```

dvar int+ X15 in 0..1;
dvar int+ X25 in 0..1;
dvar int+ X35 in 0..1;
dvar int+ X45 in 0..1;
dvar int+ X55 in 0..1;
dvar int+ X65 in 0..1;
dvar int+ X75 in 0..1;
dvar int+ X85 in 0..1;
dvar int+ X95 in 0..1;
dvar int+ X105 in 0..1;
dvar int+ X115 in 0..1;
dvar int+ X125 in 0..1;

dvar int+ X16 in 0..1;
dvar int+ X26 in 0..1;
dvar int+ X36 in 0..1;
dvar int+ X46 in 0..1;
dvar int+ X56 in 0..1;
dvar int+ X66 in 0..1;
dvar int+ X76 in 0..1;
dvar int+ X86 in 0..1;
dvar int+ X96 in 0..1;
dvar int+ X106 in 0..1;
dvar int+ X116 in 0..1;
dvar int+ X126 in 0..1;

dvar int+ X17 in 0..1;
dvar int+ X27 in 0..1;
dvar int+ X37 in 0..1;
dvar int+ X47 in 0..1;
dvar int+ X57 in 0..1;
dvar int+ X67 in 0..1;
dvar int+ X77 in 0..1;
dvar int+ X87 in 0..1;
dvar int+ X97 in 0..1;
dvar int+ X107 in 0..1;
dvar int+ X117 in 0..1;
dvar int+ X127 in 0..1;

dvar int+ X18 in 0..1;
dvar int+ X28 in 0..1;
dvar int+ X38 in 0..1;
dvar int+ X48 in 0..1;
dvar int+ X58 in 0..1;
dvar int+ X68 in 0..1;
dvar int+ X78 in 0..1;
dvar int+ X88 in 0..1;
dvar int+ X98 in 0..1;
dvar int+ X108 in 0..1;
dvar int+ X118 in 0..1;
dvar int+ X128 in 0..1;

dvar int+ X19 in 0..1;
dvar int+ X29 in 0..1;
dvar int+ X39 in 0..1;
dvar int+ X49 in 0..1;
dvar int+ X59 in 0..1;
dvar int+ X69 in 0..1;
dvar int+ X79 in 0..1;
dvar int+ X89 in 0..1;
dvar int+ X99 in 0..1;
dvar int+ X109 in 0..1;
dvar int+ X119 in 0..1;
dvar int+ X129 in 0..1;
dvar int+ X110 in 0..1;
dvar int+ X210 in 0..1;
dvar int+ X310 in 0..1;
dvar int+ X410 in 0..1;
dvar int+ X510 in 0..1;
dvar int+ X610 in 0..1;
dvar int+ X710 in 0..1;
dvar int+ X810 in 0..1;
dvar int+ X910 in 0..1;
dvar int+ X1010 in 0..1;
dvar int+ X1110 in 0..1;
dvar int+ X1210 in 0..1;
dvar int+ Y111 in 0..1;
dvar int+ X211 in 0..1;
dvar int+ X311 in 0..1;
dvar int+ X411 in 0..1;
dvar int+ X511 in 0..1;
dvar int+ X611 in 0..1;
dvar int+ X711 in 0..1;
dvar int+ X811 in 0..1;
dvar int+ X911 in 0..1;
dvar int+ X1011 in 0..1;
dvar int+ X1111 in 0..1;
dvar int+ X1211 in 0..1;
dvar int+ Y112 in 0..1;
dvar int+ X212 in 0..1;
dvar int+ X312 in 0..1;
dvar int+ X412 in 0..1;
dvar int+ X512 in 0..1;
dvar int+ X612 in 0..1;
dvar int+ X712 in 0..1;
dvar int+ X812 in 0..1;
dvar int+ X912 in 0..1;
dvar int+ X1012 in 0..1;
dvar int+ X1112 in 0..1;
dvar int+ X1212 in 0..1;

maximize
\[
\begin{align*}
0.23 & \times X12 + 0.25 & \times X13 + 0.78 & \times X14 + 0.17 & \times X15 + 0.56 & \times X16 + \\
0.17 & \times X17 + 0.25 & \times X18 + 0.67 & \times X19 + 0.27 & \times X110 + 0.33 & \times Y111 + \\
0.42 & \times Y112 + \\
0.23 & \times X21 + 0.25 & \times X31 + 0.78 & \times X41 + 0.17 & \times X51 + 0.56 & \times X61 + \\
0.17 & \times X71 + 0.25 & \times X81 + 0.67 & \times X91 + 0.27 & \times X101 + 0.33 & \times X111 + \\
0.42 & \times X112 + \\
0.67 & \times X23 + 0.33 & \times X24 + 0.75 & \times X25 + 0.27 & \times X26 + 0.75 & \times X27 + 0.67 & \times X28 + 0.36 & \times X29 + 0.40 & \times X310 + 0.23 & \times X311 + 0.25 & \times X312 + \\
0.67 & \times X32 + 0.33 & \times X42 + 0.75 & \times X52 + 0.27 & \times X62 + 0.75 & \times X72 + 0.67 & \times X82 + 0.36 & \times X92 + 0.40 & \times X102 + 0.23 & \times X112 + 0.25 & \times X122 + \\
0.36 & \times X34 + 0.44 & \times X35 + 0.3 & \times X36 + 0.63 & \times X37 + 0.75 & \times X38 + 0.4 & \times X39 + 0.18 & \times X310 + 0.36 & \times X311 + 0.17 & \times X312 + \\
0.36 & \times X43 + 0.44 & \times X53 + 0.3 & \times X63 + 0.63 & \times X73 + 0.75 & \times X83 + 0.4 & \times X93 + 0.18 & \times X103 + 0.36 & \times X113 + 0.17 & \times X123 + \\
0.27 & \times X45 + 0.56 & \times X46 + 0.27 & \times X47 + 0.36 & \times X48 + 0.88 & \times X49 + 0.27 & \times X410 + 0.33 & \times X411 + 0.31 & \times X124 + \\
0.27 & \times X54 + 0.56 & \times X64 + 0.27 & \times X74 + 0.36 & \times X84 + 0.88 & \times X94 + 0.27 & \times X104 + 0.33 & \times X114 + 0.31 & \times X124 + \\
0.33 & \times X56 + 0.5 & \times X57 + 0.63 & \times X58 + 0.3 & \times X59 + 0.2 & \times X510 + 0.17 & \times X511 + 0.25 & \times X512 + \\
0.33 & \times X65 + 0.5 & \times X75 + 0.63 & \times X85 + 0.3 & \times X95 + 0.2 & \times X105 + 0.17 & \times X115 + 0.25 & \times X125 + \\
0.2 & \times X67 + 0.3 & \times X68 + 0.63 & \times X69 + 0.09 & \times X610 + 0.27 & \times X611 + 0.25 & \times X612 + \\
0.2 & \times X76 + 0.3 & \times X86 + 0.63 & \times X96 + 0.09 & \times X106 + 0.27 & \times X116 + 0.25 & \times X126 + \\
0.44 & \times X78 + 0.30 & \times X79 + 0.33 & \times X710 + 0.27 & \times X711 + 0.15 & \times X712 + \\
0.44 & \times X87 + 0.30 & \times X97 + 0.33 & \times X107 + 0.27 & \times X117 + 0.15 & \times X127 + \\
0.40 & \times X89 + 0.18 & \times X810 + 0.25 & \times X811 + 0.23 & \times X812 + \\
0.40 & \times X98 + 0.18 & \times X108 + 0.25 & \times X118 + 0.23 & \times X128 + \\
0.18 & \times X910 + 0.25 & \times X911 + 0.23 & \times X912 + \\
0.18 & \times X109 + 0.25 & \times X911 + 0.23 & \times X129 + \\
0.17 & \times X1011 + 0.36 & \times 1012+ \\
0.17 & \times X1110 + 0.36 & \times 1210+ \\
0.55 & \times X1112 + 0.55 & \times X1211 - 1*X11 - 1*X22 - 1*X33 - 1*X44 - 1*X55- \\
1*X66 - 1*X77 - 1*X88 - 1*X99 -1*X1010 - 1*X1111 - 1*X1212 \\
\end{align*}
\]

subject to

\[
\begin{align*}
X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X110 +Y111 + Y112 & = 1; \\
X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X210 +X211 + X212 & = 1; \\
X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X310 +X311 + X312 & = 1; \\
X41 + X42 + X43 + X44 + X45 + X46 + X47 + X48 + X49 + X410 +X411 + X412 & = 1; \\
X51 + X52 + X53 + X54 + X55 + X56 + X57 + X58 + X59 + X510 +X511 + X512 & = 1; \\
X61 + X62 + X63 + X64 + X65 + X66 + X67 + X68 + X69 + X610 +X611 + X612 & = 1;
\end{align*}
\]
X71 + X72 + X73 + X74 + X75 + X76 + X77 + X78 + X79 + X710 + X711 + X712 == 1 ;
X81 + X82 + X83 + X84 + X85 + X86 + X87 + X88 + X89 + X810 + X811 + X812 == 1 ;
X91 + X92 + X93 + X94 + X95 + X96 + X97 + X98 + X99 + X910 + X911 + X912 == 1 ;
X101 + X102 + X103 + X104 + X105 + X106 + X107 + X108 + X109 + X1010 + X1011 + X1012 == 1 ;
X111 + X112 + X113 + X114 + X115 + X116 + X117 + X118 + X119 + X1110 + X1111 + X1112 == 1 ;
X121 + X122 + X123 + X124 + X125 + X126 + X127 + X128 + X129 + X1210 + X1211 + X1212 == 1 ;

X11 <= X11 ;
X21 <= X11 ;
X31 <= X11 ;
X41 <= X11 ;
X51 <= X11 ;
X61 <= X11 ;
X71 <= X11 ;
X81 <= X11 ;
X91 <= X11 ;
X101 <= X11 ;
X111 <= X11 ;
X121 <= X11 ;

X12 <= X22 ;
X22 <= X22 ;
X32 <= X22 ;
X42 <= X22 ;
X52 <= X22 ;
X62 <= X22 ;
X72 <= X22 ;
X82 <= X22 ;
X92 <= X22 ;
X102 <= X22 ;
X112 <= X22 ;
X122 <= X22 ;

X13 <= X33 ;
X23 <= X33 ;
X33 <= X33 ;
X43 <= X33 ;
X53 <= X33 ;
X63 <= X33 ;
X73 <= X33 ;
X83 <= X33 ;
X93 <= X33 ;
X103 <= X33 ;
X113 <= X33 ;
X123 <= X33 ;
X14 <= X44 ;
X24 <= X44 ;
X34 <= X44 ;
X44 <= X44 ;
X54 <= X44 ;
X64 <= X44 ;
X74 <= X44 ;
X84 <= X44 ;
X94 <= X44 ;
X104 <= X44 ;
X114 <= X44 ;
X124 <= X44 ;
X15 <= X55 ;
X25 <= X55 ;
X35 <= X55 ;
X45 <= X55 ;
X55 <= X55 ;
X65 <= X55 ;
X75 <= X55 ;
X85 <= X55 ;
X95 <= X55 ;
X105 <= X55 ;
X115 <= X55 ;
X125 <= X55 ;
X16 <= X66 ;
X26 <= X66 ;
X36 <= X66 ;
X46 <= X66 ;
X56 <= X66 ;
X66 <= X66 ;
X76 <= X66 ;
X86 <= X66 ;
X96 <= X66 ;
X106 <= X66 ;
X116 <= X66 ;
X126 <= X66 ;
X17 <= X77 ;
X27 <= X77 ;
X37 <= X77 ;
X47 <= X77 ;
X57 <= X77 ;
X67 <= X77 ;
X77 <= X77 ;
X87 <= X77 ;
X97 <= X77 ;
X107 <= X77 ;
X117 <= X77 ;
X127 <= X77 ;
X18 <= X88 ;
X28 <= X88 ;
X38 <= X88 ;
X48 <= X88 ;
X58 <= X88 ;
X68 <= X88 ;
X78 <= X88 ;
X88 <= X88 ;
X98 <= X88 ;
X108 <= X88 ;
X118 <= X88 ;
X128 <= X88 ;
X19  <= X99 ;
X29  <= X99 ;
X39  <= X99 ;
X49  <= X99 ;
X59  <= X99 ;
X69  <= X99 ;
X79  <= X99 ;
X89  <= X99 ;
X99  <= X99 ;
X109 <= X99 ;
X119 <= X99 ;
X129 <= X99 ;
X110 <= X1010 ;
X210 <= X1010 ;
X310 <= X1010 ;
X410 <= X1010 ;
X510 <= X1010 ;
X610 <= X1010 ;
X710 <= X1010 ;
X810 <= X1010 ;
X910 <= X1010 ;
X1010 <= X1010 ;
X1110 <= X1010 ;
X1210 <= X1010 ;
Y111 <= X1111 ;
X211 <= X1111 ;
X311 <= X1111 ;
X411 <= X1111 ;
X511 <= X1111 ;
X611 <= X1111 ;
X711 <= X1111 ;
X811 <= X1111 ;
X911 <= X1111 ;
X1011 <= X1111 ;
X1111 <= X1111 ;
X1211 <= X1111 ;
Y112 <= X1212 ;
X212 <= X1212 ;
X312 <= X1212 ;
X412 <= X1212 ;
X512 <= X1212;
X612 <= X1212;
X712 <= X1212;
X812 <= X1212;
X912 <= X1212;
X1012 <= X1212;
X1112 <= X1212;
X1212 <= X1212;
X1212 <= X1212;
X1212 <= X1212;
}

Product Subfamily Identification and Cell Loading:

Short Cut Form:

```plaintext
int n = ...;
int m = ...;

range parts = 1.. n;
range Cells = 1.. m;

float b[parts][Cells] = ...;
float u[parts] = ...;

dvar int+ x[parts][Cells];

maximize sum(i in parts, j in Cells) b[i][j] * x[i][j] - sum(j in Cells) (p * x[j][j]);
subject to{
    forall (i in parts)
        sum(j in Cells) x[i][j] == 1;

    forall (j in Cells)
        sum(i in parts) u[i] * x[i][j] <= 160;

    forall (i in parts)
        forall (j in Cells)
            x[i][j] <= x[j][j];
}
```
Open Form:

dvar int+ X11 in 0..1;
dvar int+ X21 in 0..1;
dvar int+ X31 in 0..1;
dvar int+ X41 in 0..1;
dvar int+ X51 in 0..1;
dvar int+ X61 in 0..1;

dvar int+ X12 in 0..1;
dvar int+ X22 in 0..1;
dvar int+ X32 in 0..1;
dvar int+ X42 in 0..1;
dvar int+ X52 in 0..1;
dvar int+ X62 in 0..1;

dvar int+ X13 in 0..1;
dvar int+ X23 in 0..1;
dvar int+ X33 in 0..1;
dvar int+ X43 in 0..1;
dvar int+ X53 in 0..1;
dvar int+ X63 in 0..1;

dvar int+ X14 in 0..1;
dvar int+ X24 in 0..1;
dvar int+ X34 in 0..1;
dvar int+ X44 in 0..1;
dvar int+ X54 in 0..1;
dvar int+ X64 in 0..1;

dvar int+ X15 in 0..1;
dvar int+ X25 in 0..1;
dvar int+ X35 in 0..1;
dvar int+ X45 in 0..1;
dvar int+ X55 in 0..1;
dvar int+ X65 in 0..1;

dvar int+ X16 in 0..1;
dvar int+ X26 in 0..1;
dvar int+ X36 in 0..1;
dvar int+ X46 in 0..1;
dvar int+ X56 in 0..1;
dvar int+ X66 in 0..1;

maximize

//Z ==
0.053 * X12 + 0.045 * X13 + 0.047 * X14 + 0.043 * X15 + 0.146 * X16 +
0.053 * X21 + 0.045 * X31 + 0.047 * X41 + 0.043 * X51 + 0.146 * X61 +
0.073 * X23 + 0.080 * X24 + 0.034 * X25 + 0.168 * X26 +
0.073 * X32 + 0.080 * X42 + 0.034 * X52 + 0.168 * X62 +
0.076 * X34 + 0.045 * X35 + 0.165 * X36 +
0.076 * X43 + 0.045 * X53 + 0.165 * X63 +
0.077 * X45 + 0.130 * X46 +
0.077 * X54 + 0.130 * X64 +
0.165 * X56 +
0.165 * X65 - X11 - X22 - X33 - X44 - X55 - X66;

subject to
{
59.42 * X11 + 38.67 * X21 + 56.91 * X31 + 63.45 * X41 + 38.07 * X51 +
52.98 * X61  <= 160;
59.42 * X12 + 38.67 * X22 + 56.91 * X32 + 63.45 * X42 + 38.07 * X52 +
52.98 * X62  <= 160;
59.42 * X13 + 38.67 * X23 + 56.91 * X33 + 63.45 * X43 + 38.07 * X53 +
52.98 * X63  <= 160;
59.42 * X14 + 38.67 * X24 + 56.91 * X34 + 63.45 * X44 + 38.07 * X54 +
52.98 * X64  <= 160;
59.42 * X15 + 38.67 * X25 + 56.91 * X35 + 63.45 * X45 + 38.07 * X55 +
52.98 * X65  <= 160;
59.42 * X16 + 38.67 * X26 + 56.91 * X36 + 63.45 * X46 + 38.07 * X56 +
52.98 * X66  <= 160;

X11 + X12 + X13 + X14 + X15 + X16   == 1 ;
X21 + X22 + X23 + X24 + X25 + X26   == 1 ;
X31 + X32 + X33 + X34 + X35 + X36   == 1 ;
X41 + X42 + X43 + X44 + X45 + X46   == 1 ;
X51 + X52 + X53 + X54 + X55 + X56   == 1 ;
X61 + X62 + X63 + X64 + X65 + X66   == 1 ;

X11 <= X11 ;
X21 <= X11 ;
X31 <= X11 ;
X41 <= X11 ;
X51 <= X11 ;
X61 <= X11 ;

X12 <= X22 ;
X22 <= X22 ;
X32 <= X22 ;
X42 <= X22 ;
X52 <= X22 ;
X62 <= X22 ;

X13 <= X33 ;
X23 <= X33 ;
X33 <= X33 ;
X43 <= X33 ;
X53 <= X33 ;
X63 <= X33 ;
X14 <= X44;
X24 <= X44;
X34 <= X44;
X44 <= X44;
X54 <= X44;
X64 <= X44;
X15 <= X55;
X25 <= X55;
X35 <= X55;
X45 <= X55;
X55 <= X55;
X65 <= X55;
X16 <= X66;
X26 <= X66;
X36 <= X66;
X46 <= X66;
X56 <= X66;
X66 <= X66;
Appendix B: OPL Code for the Batch-Cyclic Flowshop Scheduling

Subfamily \((P_2 P_3 P_{10})\)

dvar float+ XP2 in 0.. maxint;
dvar float+ XP3 in 0.. maxint;
dvar float+ XP10 in 0.. maxint;
dvar float+ XP2P3 in 0.. maxint;
dvar float+ XP2P10 in 0.. maxint;
dvar float+ XP3P10 in 0.. maxint;
dvar float+ XP2P3P10 in 0.. maxint;

dvar float+ Z in 0.. maxint;
dvar float+ MS in 0.. maxint;

minimize

\[ Z; \]

subject to

\{ 
250\times XP2 + 153.846\times XP2P3 + 146.3415\times XP2P10 + 103.448\times XP2P3P10 \geq 14854; \\
300\times XP3 + 153.846\times XP2P3 + 162.162\times XP3P10 + 103.448\times XP2P3P10 \geq 11601; \\
187.5\times XP10 + 146.3415\times XP2P10 + 162.162\times XP3P10 + 103.448\times XP2P3P10 \geq 9933; \\
Z == MS; \\
MS - ( XP2 + XP3 + XP10 + XP2P3 + XP2P10 + XP3P10 + XP2P3P10 ) \geq 0; 
\}
Subfamily \((P_3P_9P_{15}P_{20}P_{29}P_{39})\)

dvar float+ XP3 in 0.. maxint;
dvar float+ XP9 in 0.. maxint;
dvar float+ XP15 in 0.. maxint;
dvar float+ XP20 in 0.. maxint;
dvar float+ XP29 in 0.. maxint;
dvar float+ XP39 in 0.. maxint;
dvar float+ XP3P9 in 0.. maxint;
dvar float+ XP3P15 in 0.. maxint;
dvar float+ XP3P20 in 0.. maxint;
dvar float+ XP3P29 in 0.. maxint;
dvar float+ XP3P39 in 0.. maxint;
dvar float+ XP3P9P15 in 0.. maxint;
dvar float+ XP3P9P20 in 0.. maxint;
dvar float+ XP3P9P29 in 0.. maxint;
dvar float+ XP3P9P39 in 0.. maxint;
dvar float+ XP3P15P20 in 0.. maxint;
dvar float+ XP3P15P29 in 0.. maxint;
dvar float+ XP3P15P39 in 0.. maxint;
dvar float+ XP3P20P29 in 0.. maxint;
dvar float+ XP3P20P39 in 0.. maxint;
dvar float+ XP9P15P20 in 0.. maxint;
dvar float+ XP9P15P29 in 0.. maxint;
dvar float+ XP9P15P39 in 0.. maxint;
dvar float+ XP9P20P29 in 0.. maxint;
dvar float+ XP9P20P39 in 0.. maxint;
dvar float+ XP9P29P39 in 0.. maxint;
dvar float+ XP9P15P20P29 in 0.. maxint;
dvar float+ XP9P15P20P39 in 0.. maxint;
dvar float+ XP9P15P29P39 in 0.. maxint;
dvar float+ XP9P20P29P39 in 0.. maxint;
dvar float+ XP3P9P15P20P29 in 0.. maxint;
dvar float+ XP3P9P15P20P39 in 0.. maxint;
dvar float+ XP3P9P15P29P39 in 0.. maxint;
dvar float+ XP3P15P20P29P39 in 0.. maxint;
dvar float+ XP9P15P20P29P39 in 0.. maxint;
dvar float+ XP3P15P20P29P39 in 0.. maxint;
dvar float+ XP3P9P15P20P29P39 in 0.. maxint;
dvar float+ XP3P9P15P20P29P39 in 0.. maxint;
dvar float+ MS in 0.. maxint;

minimize
Z;

subject to
{
3\* XP3
+ 1.714285714\* XP3P9
+ 1.764705882\* XP3P15
+ 1.666666667\* XP3P20
+ 1.714285714\* XP3P29
+ 1.666666667\* XP3P39
+ 1.176470588\* XP3P9P15
+ 1.111111111\* XP3P9P20
+ 1.153846154\* XP3P9P29
+ 1.111111111\* XP3P9P39
+ 1.153846154\* XP3P15P20
+ 1.2\* XP3P15P29
+ 1.224489796\* XP3P15P39
+ 1.090909091\* XP3P20P29
+ 1.090909091\* XP3P20P39
+ 1.32075472\* XP3P29P39
+ 0.869565217\* XP3P9P15P20
+ 0.895522388\* XP3P9P15P29
+ 0.909090909\* XP3P9P15P39
+ 0.845070423\* XP3P9P20P29
+ 0.821917808\* XP3P9P20P39
+ 0.869565217\* XP3P9P29P39
+ 0.857142857\* XP3P15P20P29
+ 0.869565217\* XP3P15P20P39
+ 0.882352941\* XP3P15P29P39
+ 0.821917808\* XP3P20P29P39
+ 0.697674419\* XP3P9P15P20P29
+ 0.705882353\* XP3P9P15P20P39
+ 0.714285714\* XP3P9P15P29P39
+ 0.674157303\* XP3P9P20P29P39
+ 0.681818182\* XP3P15P20P29P39
+ 0.576923077\* XP3P9P15P20P29P39 >= 191;
3.157894737* XP9
+ 1.714285714* XP3P9
+ 1.818181818* XP9P15
+ 1.621621622* XP9P20
+ 1.621621622* XP9P29
+ 1.621621622* XP9P39
+ 1.176470588* XP3P9P15
+ 1.111111111* XP3P9P20
+ 1.153846154* XP3P9P29
+ 1.111111111* XP3P9P39
+ 1.176470588* XP9P15P20
+ 1.2* XP9P15P29
+ 1.224489796* XP9P15P39
+ 1.090909091* XP9P20P29
+ 1.071428571* XP9P20P39
+ 1.132075472* XP9P29P39
+ 0.869565217* XP3P9P15P20
+ 0.895522388* XP3P9P15P29
+ 0.909090909* XP3P9P15P39
+ 0.845070423* XP3P9P20P29
+ 0.821917808* XP3P9P20P39
+ 0.869565217* XP3P9P29P39
+ 0.857142857* XP9P15P20P29
+ 0.869565217* XP9P15P20P39
+ 0.882352941* XP9P15P29P39
+ 0.821917808* XP9P20P29P39
+ 0.697674419* XP3P9P15P20P29
+ 0.705882353* XP3P9P15P20P39
+ 0.714285714* XP3P9P15P29P39
+ 0.674157303* XP3P9P20P29P39
+ 0.681818182* XP9P15P20P29P39
+ 0.576923077* XP3P9P15P20P29P39
>=116;

3.529411765* XP15
+ 1.764705882* XP3P15
+ 1.818181818* XP9P15
+ 1.714285714* XP15P20
+ 1.764705882* XP15P29
+ 1.818181818* XP15P39
+ 1.176470588* XP3P9P15
+ 1.153846154* XP3P15P20
+ 1.2* XP3P15P29
+ 1.224489796* XP3P15P39
+ 1.176470588* XP9P15P20
+ 1.2* XP9P15P29
+ 1.224489796* XP9P15P39
+ 1.111111111* XP15P20P29
+ 1.132075472* XP15P20P39
+ 1.153846154* XP15P29P39
+ 0.869565217* XP3P9P15P20
+ 0.895522388* XP3P9P15P29
+ 0.909090909* XP3P9P15P39

3.529411765* XP15
\begin{align*}
+ 0.857142857 \times & \text{XP3P15P20P29} \\
+ 0.869565217 \times & \text{XP3P15P20P39} \\
+ 0.882352941 \times & \text{XP3P15P29P39} \\
+ 0.857142857 \times & \text{XP9P15P20P29} \\
+ 0.869565217 \times & \text{XP9P15P20P39} \\
+ 0.882352941 \times & \text{XP9P15P29P39} \\
+ 0.697674419 \times & \text{XP3P9P15P20P29} \\
+ 0.705882353 \times & \text{XP3P9P15P20P39} \\
+ 0.714285714 \times & \text{XP3P9P15P29P39} \\
+ 0.681818182 \times & \text{XP3P15P20P29P39} \\
+ 0.681818182 \times & \text{XP9P15P20P29P39} \\
+ 0.576923077 \times & \text{XP3P9P15P20P29P39} \\
+ 0.833333333 \times & \text{XP15P20P29P39} \geq 73; \\
3 \times \text{XP20} \\
+ 1.666666667 \times & \text{XP3P20} \\
+ 1.621621622 \times & \text{XP9P20} \\
+ 1.714285714 \times & \text{XP15P20} \\
+ 1.538461538 \times & \text{XP20P29} \\
+ 1.578947368 \times & \text{XP20P39} \\
+ 1.111111111 \times & \text{XP3P9P20} \\
+ 1.153846154 \times & \text{XP3P15P20} \\
+ 1.090909091 \times & \text{XP3P20P29} \\
+ 1.090909091 \times & \text{XP3P20P39} \\
+ 1.176470588 \times & \text{XP9P15P20} \\
+ 1.090909091 \times & \text{XP9P20P29} \\
+ 1.071428571 \times & \text{XP9P20P39} \\
+ 1.111111111 \times & \text{XP15P20P29} \\
+ 1.132075472 \times & \text{XP15P20P39} \\
+ 1.052631579 \times & \text{XP20P29P39} \\
+ 0.869565217 \times & \text{XP3P9P15P20} \\
+ 0.845070423 \times & \text{XP3P9P20P29} \\
+ 0.821917808 \times & \text{XP3P9P20P39} \\
+ 0.857142857 \times & \text{XP3P15P20P29} \\
+ 0.869565217 \times & \text{XP3P15P20P39} \\
+ 0.821917808 \times & \text{XP3P20P29P39} \\
+ 0.857142857 \times & \text{XP9P15P20P29} \\
+ 0.869565217 \times & \text{XP9P15P20P39} \\
+ 0.821917808 \times & \text{XP9P20P29P39} \\
+ 0.697674419 \times & \text{XP3P9P15P20P29} \\
+ 0.705882353 \times & \text{XP3P9P15P20P39} \\
+ 0.674157303 \times & \text{XP3P9P20P29P39} \\
+ 0.681818182 \times & \text{XP3P15P20P29P39} \\
+ 0.681818182 \times & \text{XP9P15P20P29P39} \\
+ 0.576923077 \times & \text{XP3P9P15P20P29P39} \\
+ 0.833333333 \times & \text{XP15P20P29P39} \geq 43; \\
3 \times \text{XP29} \\
+ 1.714285714 \times & \text{XP3P29} \\
+ 1.621621622 \times & \text{XP9P29} \\
+ 1.764705882 \times & \text{XP15P29} \\
+ 1.538461538 \times & \text{XP20P29}
\end{align*}
\[ + 1.621621622 \times \text{XP29P39} + 1.153846154 \times \text{XP3P9P29} + 1.2 \times \text{XP3P15P29} + 1.090909091 \times \text{XP3P20P29} + 1.132075472 \times \text{XP3P29P39} + 1.2 \times \text{XP9P15P29} + 1.090909091 \times \text{XP9P20P29} + 1.132075472 \times \text{XP9P29P39} + 1.111111111 \times \text{XP15P20P29} + 1.153846154 \times \text{XP15P29P39} + 1.052631579 \times \text{XP20P29P39} + 0.895522388 \times \text{XP3P9P15P29} + 0.845070423 \times \text{XP3P9P20P29} + 0.869565217 \times \text{XP3P9P29P39} + 0.857142857 \times \text{XP3P15P20P29} + 0.882352941 \times \text{XP3P15P29P39} + 0.821917808 \times \text{XP3P20P29P39} + 0.857142857 \times \text{XP9P15P20P29} + 0.882352941 \times \text{XP9P15P29P39} + 0.821917808 \times \text{XP9P20P29P39} + 0.69764419 \times \text{XP9P15P20P29} + 0.714285714 \times \text{XP9P15P29P39} + 0.674157303 \times \text{XP9P20P29P39} + 0.681818182 \times \text{XP9P15P20P39} + 0.681818182 \times \text{XP9P15P29P39} + 0.576923077 \times \text{XP3P9P15P20P39} + 0.833333333 \times \text{XP15P20P29P39} \geq 60; \]

3 \times \text{XP39} + 1.666666667 \times \text{XP3P39} + 1.621621622 \times \text{XP9P39} + 1.818181818 \times \text{XP3P9P15P29} + 1.621621622 \times \text{XP29P39} + 1.111111111 \times \text{XP3P9P39} + 1.224489796 \times \text{XP3P15P39} + 1.090909091 \times \text{XP3P20P39} + 1.132075472 \times \text{XP3P29P39} + 1.224489796 \times \text{XP9P15P39} + 1.071428571 \times \text{XP9P20P39} + 1.132075472 \times \text{XP9P29P39} + 1.132075472 \times \text{XP15P20P39} + 1.153846154 \times \text{XP15P29P39} + 1.052631579 \times \text{XP20P29P39} + 0.900900909 \times \text{XP3P9P15P39} + 0.821917808 \times \text{XP3P9P20P39} + 0.869565217 \times \text{XP3P9P29P39} + 0.869565217 \times \text{XP3P15P20P39} + 0.882352941 \times \text{XP3P15P29P39} + 0.821917808 \times \text{XP3P20P29P39} + 0.869565217 \times \text{XP9P15P20P39} + 0.882352941 \times \text{XP9P15P29P39} + 0.821917808 \times \text{XP9P20P29P39} + 0.705882353 \times \text{XP3P9P15P20P39}
\[ + 0.714285714 \times XP3P9P15P29P39 \\
+ 0.674157303 \times XP3P9P20P29P39 \\
+ 0.681818182 \times XP3P15P20P29P39 \\
+ 0.681818182 \times XP9P15P20P29P39 \\
+ 0.576923077 \times XP3P9P15P20P29P39 \\
+ 0.833333333 \times XP15P20P29P39 \geq 62; \]

\[
Z == MS;
\[ + \text{XP}3\text{P}9\text{P}20\text{P}39 \\
+ \text{XP}3\text{P}9\text{P}29\text{P}39 \\
+ \text{XP}3\text{P}15\text{P}20\text{P}29 \\
+ \text{XP}3\text{P}15\text{P}20\text{P}39 \\
+ \text{XP}3\text{P}15\text{P}29\text{P}39 \\
+ \text{XP}3\text{P}20\text{P}29\text{P}39 \\
+ \text{XP}9\text{P}15\text{P}20\text{P}29 \\
+ \text{XP}9\text{P}15\text{P}20\text{P}39 \\
+ \text{XP}9\text{P}15\text{P}29\text{P}39 \\
+ \text{XP}9\text{P}20\text{P}29\text{P}39 \\
+ \text{XP}3\text{P}9\text{P}15\text{P}20\text{P}29 \\
+ \text{XP}3\text{P}9\text{P}15\text{P}20\text{P}39 \\
+ \text{XP}3\text{P}9\text{P}15\text{P}29\text{P}39 \\
+ \text{XP}3\text{P}9\text{P}20\text{P}29\text{P}39 \\
+ \text{XP}3\text{P}15\text{P}20\text{P}29\text{P}39 \\
+ \text{XP}3\text{P}15\text{P}20\text{P}29\text{P}39 \\
+ \text{XP}3\text{P}9\text{P}15\text{P}20\text{P}29\text{P}39 \\
+ \text{XP}15\text{P}20\text{P}29\text{P}39 \\
\} \geq 0; \\
\}
## Appendix C: Product Family Formation Based On the Penalty Factor

Product Family and their Corresponding Products Based on the Penalty Factor for the First Example

<table>
<thead>
<tr>
<th>Penalty Factor</th>
<th>Product Family</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1.20</td>
<td>PF₁</td>
<td>P₂, P₃, P₅, P₇, P₈, P₁₀</td>
</tr>
<tr>
<td></td>
<td>PF₂</td>
<td>P₁, P₄, P₆, P₉, P₁₁, P₁₂</td>
</tr>
<tr>
<td>Penalty Factor&gt;=1.21</td>
<td>PF₁</td>
<td>P₁, P₂, P₃, P₄, P₅, P₆, P₇, P₈, P₉, P₁₀, P₁₁, P₁₂</td>
</tr>
</tbody>
</table>
Product Family and their Corresponding Products Based on the Penalty Factor for the Second Example

<table>
<thead>
<tr>
<th>Penalty Factor</th>
<th>Product Family</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 0.12</td>
<td>PF₁</td>
<td>P₂, P₈, P₁₄, P₁₇, P₁₉, P₃₃, P₃₄, P₃₆, P₃₇, P₃₈</td>
</tr>
<tr>
<td></td>
<td>PF₂</td>
<td>P₁, P₃, P₅, P₁₂, P₂₃, P₂₅, P₂₆, P₃₀, P₃₅</td>
</tr>
<tr>
<td></td>
<td>PF₃</td>
<td>P₇, P₉, P₂₀, P₃₉</td>
</tr>
<tr>
<td></td>
<td>PF₄</td>
<td>P₄, P₆, P₁₀, P₁₃, P₁₆, P₁₈, P₂₂, P₂₈, P₃₂</td>
</tr>
<tr>
<td></td>
<td>PF₅</td>
<td>P₁₁, P₁₅, P₂₁, P₂₄, P₂₇, P₂₉, P₃₁</td>
</tr>
<tr>
<td>0.13 - 0.57</td>
<td>PF₁</td>
<td>P₂, P₈, P₁₄, P₁₇, P₁₉, P₃₃, P₃₄, P₃₆, P₃₇, P₃₈</td>
</tr>
<tr>
<td></td>
<td>PF₂</td>
<td>P₁, P₃, P₅, P₁₂, P₂₃, P₂₅, P₂₆, P₃₀, P₃₅</td>
</tr>
<tr>
<td></td>
<td>PF₃</td>
<td>P₄, P₆, P₁₀, P₁₃, P₁₆, P₁₈, P₂₂, P₂₈, P₃₂</td>
</tr>
<tr>
<td></td>
<td>PF₄</td>
<td>P₇, P₉, P₁₁, P₁₅, P₂₀, P₂₁, P₂₄, P₂₇, P₂₉, P₃₁, P₃₉</td>
</tr>
<tr>
<td>0.58 - 1.89</td>
<td>PF₁</td>
<td>P₃, P₇, P₉, P₁₁, P₁₅, P₂₀, P₂₁, P₂₄, P₂₇, P₂₉, P₃₁, P₃₉</td>
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<tr>
<td></td>
<td>PF₂</td>
<td>P₄, P₆, P₁₀, P₁₃, P₁₆, P₁₈, P₂₂, P₂₈, P₃₂, P₃₅</td>
</tr>
<tr>
<td></td>
<td>PF₃</td>
<td>P₁, P₂, P₅, P₈, P₁₂, P₁₄, P₁₇, P₁₉, P₂₃, P₂₅, P₂₆, P₃₀, P₃₃, P₃₄, P₃₆, P₃₇, P₃₈</td>
</tr>
<tr>
<td>1.90 – 2.17</td>
<td>PF₁</td>
<td>P₁, P₂, P₅, P₆, P₈, P₁₀, P₁₃, P₁₄, P₁₆, P₁₇, P₁₈, P₁₉, P₂₂, P₂₃, P₂₅, P₂₆, P₂₇, P₂₈, P₃₀, P₃₂, P₃₄, P₃₅, P₃₆, P₃₇, P₃₈</td>
</tr>
<tr>
<td></td>
<td>PF₂</td>
<td>P₃, P₄, P₇, P₉, P₁₁, P₁₂, P₁₅, P₂₀, P₂₁, P₂₄, P₂₉, P₃₁, P₃₃, P₃₉</td>
</tr>
</tbody>
</table>
Appendix D: Processing Times for the Subfamily \((P_2 \; P_3 \; P_{10})\) with Common Bottleneck Machine and Machine Skipping

Case 1:

<table>
<thead>
<tr>
<th></th>
<th>(M_1)</th>
<th>(M_2)</th>
<th>(M_4)</th>
<th>(M_5)</th>
<th>(M_6)</th>
<th>(M_7)</th>
<th>(M_8)</th>
<th>(M_{10})</th>
<th>(M_{11})</th>
<th>(M_{12})</th>
<th>(M_{14})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_2)</td>
<td>0.21</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0.19</td>
<td>0</td>
<td>0.19</td>
<td>0.08</td>
<td>0.24</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>(P_3)</td>
<td>0.13</td>
<td>0</td>
<td>0.12</td>
<td>0</td>
<td>0.14</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
<td>0.20</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>(P_{10})</td>
<td>0</td>
<td>0.30</td>
<td>0</td>
<td>0.20</td>
<td>0</td>
<td>0.13</td>
<td>0</td>
<td>0.21</td>
<td>0.32</td>
<td>0</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Case 2:

<table>
<thead>
<tr>
<th></th>
<th>(M_1)</th>
<th>(M_2)</th>
<th>(M_4)</th>
<th>(M_5)</th>
<th>(M_6)</th>
<th>(M_7)</th>
<th>(M_8)</th>
<th>(M_{10})</th>
<th>(M_{11})</th>
<th>(M_{12})</th>
<th>(M_{14})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_2)</td>
<td>0.11</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0.19</td>
<td>0</td>
<td>0.19</td>
<td>0.08</td>
<td>0.21</td>
<td>0.06</td>
<td>0.24</td>
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Appendix E: Processing Times for the Subfamily (P2 P3 P10) with Shifting

Bottleneck Machine and Machine Skipping

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## Appendix H: Processing Times for the Subfamily \((P_2\ P_3\ P_{10})\) with Shifting

Bottleneck Machine, Changing Time per Each Machine, And Machine Skipping Not Allowed

### Case 1:

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Appendix I: Processing Times for the Subfamily (P₂ P₃ P₁₀) with Shifting Bottleneck Machine, Changing Time per Each Machine, and Machine Skipping

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