Analytical and Experimental Comparison of a Positive Displacement Water Pump Using an Infinitely Variable Transmission

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of the requirements for the degree
Master of Science

John A. Mullen
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This thesis titled

Analytical and Experimental Comparison of a Positive Displacement Water Pump Using

an Infinitely Variable Transmission

by

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has been approved for

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ABSTRACT

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Analytical and Experimental Comparison of a Positive Displacement Water Pump Using an Infinitely Variable Transmission

Director of Thesis: Timothy J. Cyders

Moving water quickly and efficiently has always been crucial to human development, especially in agriculture. However, this challenge still often goes unmet, especially in the developing world where access to infrastructure and mechanized pumping equipment is limited. In such locations human power is the most readily available source of power. The Beale Continuously Variable Transmission (CVT), modeled and experimentally examined by Cyders (2012), has potential to address this challenge. This thesis will adapt the work done by Cyders to a slider-crank simulation paired with a positive displacement pump. Predictions of system flowrate response at varying combinations of pressure and input shaft speed will be made using this model, and will then be compared against experimental results taken from a physical prototype of the system. Comparisons between these data sets will form the final result of this project, and will inform recommendations for any future work.
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1. INTRODUCTION

Moving water efficiently and effectively has always been a critical agricultural requirement. At the most rudimentary level, farmers carried water to their crops by bucket. Since these simple beginnings, irrigation technologies have improved and increased in both capacity and complexity. Today, while only about 20% of farmland in developing countries is irrigated, this irrigated land currently produces roughly 40% of all crops and nearly 60% of cereal crops in these nations (Bruinsma and Food and Agriculture Organization of the United Nations 2003). Irrigation development holds great potential to increase earning potential and close crop yield gaps in millions of additional smallholder farms across the developing world.

Irrigation is not equally utilized in all regions. South and East Asia contain 50% of the world’s irrigated land, while only about 3 to 4% of land in Sub-Saharan Africa (SSA) is irrigated (Earthscan 2011). In developing regions worldwide, and especially in SSA, a lack of infrastructure makes the implementation and maintenance of irrigation systems difficult. This difficulty perpetuates the cycle of food insecurity and ultimately regional hunger. While exact numbers are unobtainable, estimates place the total number of people affected by hunger worldwide at about one billion (GAFSP 2014).

In response to this reality, many governments and a variety of non-governmental organizations have become increasingly motivated in recent decades to pursue long-term solutions. Water source development in particular has received increasing attention (Food and Agriculture Organization of the United Nations 1993). Despite these efforts, however, lack of reliable water sources is still one of the most critical issues in many
developing countries. There is clearly a need for robust and affordable methods of water retrieval. This need is especially acute in SSA, as noted above. In this region, most farms are operated by a family or at most a village, typically cultivating only up to several acres at once and often less (“The State of Food and Agriculture 2014 In Brief (SOFA in Brief)” 2015). A comparison of average farm sizes worldwide is given in Figure 1:

**Average farm size (hectares) by world region** below.

![Figure 1: Average farm size (hectares) by world region (GAFSP 2014)](image)

Despite small cultivated plot sizes, lifting or pumping water is a significant portion of daily farm work where irrigation is practiced. Water delivery methods range in complexity, with hand-watering of individual plants not uncommon. The majority of all farm work in SSA is done without the aid of mechanized equipment (Food and Agriculture Organization of the United Nations 2013; Brian G Sims and Josef Kienzle
When pumps are used for lifting water from wells, those pumps are typically human powered and designed for relatively shallow use. To be successfully integrated into this environment, any new pump design must fulfill these criteria, as well as low cost, low level of complexity, robustness, and relatively simple maintenance requirements.

Pumps can be categorized as either positive displacement (PD) or dynamic. PD pumps operate by decreasing the size of a control volume filled with static fluid, thus applying pressure to the fluid and forcing an output flow. They can operate efficiently at any flowrate and pressure up to a maximum pressure dictated by seal quality (Elie Tawil 1993). In contrast, dynamic pumps move fluid by imparting a high velocity to the input flow, then increasing flow pressure prior to the outlet by slowing flow velocity. The most common type of dynamic pump is the centrifugal pump, which uses an impeller to impart fluid velocity and a volute (or set of volutes) to slow velocity and thus raise outlet pressure. Centrifugal pumps require a high, fairly constant impeller speed (typically thousands of rpm), and require more nearly constant operational parameters for efficient use. Additionally, they are generally inefficient for flowrates lower than 50 gallons per minute (gpm) (Herbert H. Tackett, Jr., James A. Cripe, and Gary Dyson 2008). Because they more closely match the pump implementation criteria listed above, PD pumps are used far more commonly in SSA than are centrifugal pumps.

There are two main categories of PD pumps: rotary and reciprocating. Rotary PD pumps use a constantly-rotating rotor to progressively create discrete pumping chambers (Parker 1994); rotary pumps may be either single- or multiple-rotor (Kreith and Goswami
Specific types include “vane, piston, flexible member, gear circumferential piston, or screw pumps” (Parker 1994). Some examples are shown in Figure 2 and Figure 3. Due to their complexity, progressive cavity PD pumps are not commonly used in developing regions.

Figure 2: Rotary vane pump (Elie Tawil 1993)

Figure 3: Rotary screw pump (Elie Tawil 1993)
Reciprocating PD pumps may be piston/plunger pumps (whether direct action or power pumps), or diaphragm pumps (Kreith and Goswami 2004). Direct action pumps harness linearly reciprocating power sources, while power pumps rely on rotational power converted into reciprocating motion via a slider-crank or camshaft (Herbert H. Tackett, Jr., James A. Cripe, and Gary Dyson 2008). Examples of direct action pumps are shown in Figure 4. These and similar designs are already commonly used in developing regions.

![Figure 4: Direct action pump examples ("Department of Public Health Engineering (DPHE)" 2015)](image)

One key difference between reciprocating and rotary PD pumps is that because reciprocating pumps rely on the expansion and contraction of a control volume to move discrete amounts of fluid, they have sinusoidal torque requirements over a complete pump cycle (one expansion and one contraction). This torque fluctuation can be smoothed by the inclusion of additional pumping chambers, but will never be totally constant. Figure 5 demonstrates this cycle. Rotary PD pumps, by contrast, have a constant input shaft torque requirement.
Fraenkel (1986) writes that the most common type of PD pump used in developing countries is the piston pump, in which a piston is reciprocated within a casing that rests inside the well. An alternative to the piston pump is the diaphragm pump, which operates by alternately compressing and expanding a diaphragm to force fluid motion. An advantage of this design is that the pumping mechanism can be kept totally separate from the pumped fluid, and can thus be located at the top of the borehole rather than submerged in the well.

While human-powered reciprocating PD pumps have been used successfully in developing countries, including SSA, they share a common limitation. All of the pressure developed in these pumps is directly transmitted to the user with no force reduction. A
design that allowed for passive selection of output flowrate based on back pressure in the system would have a significant advantage over current manual pumps.

Regardless of pump type, effective transmission of power is paramount. This challenge is clearly not new. Rotational transmission of power has been most prevalent for much of human history, but since the development of complex mechanical systems, by no means universal. A serious complaint against traditional rotational power transmission systems is that they are incapable of producing “any desired speed or torque level” (Cyders 2012). Conventional transmissions can provide a discrete set of ratios between input and output, gearing the output up or down as the speed and torque demands of the load are changed. But this solution only offers a finite set of ratios, and cannot perfectly match a continuously fluctuating demand.

The modern development of Continuously Variable Transmissions (CVTs) is one response to the issues faced by discrete ratio transmissions. This class of transmissions can provide a continuous range of gearing ratios, thus allowing a power source to operate at its peak efficiency (characterized by a specific shaft speed) over a wide range of torque or speed demands on the output. To date, most work on CVT systems has been done by automobile manufacturers, in an attempt to exploit an “optimized combination of performance and fuel consumption” (Kluger and Fussner 1997). Continued research in automobile CVTs gives hope for increased torque transmittance capabilities and decreased production costs; however, these designs are still limited by their absolute physical size. Thus, although they can transmit power at theoretically any ratio within a
given band, that band of ratios is still finite, constrained by the size of the housing and drivetrain elements.

The development of a new CVT free from these problems is already underway. Some companies and researchers are currently pursuing a new type of transmission, known as an Infinitely Variable Transmission (IVT). There are different approaches to creating transmissions with infinitely varying behavior, but they are consistent in that they allow for the continuous selection of gear ratios up to 1:0, or a stationary output shaft connected to a rotating input shaft. The torque multiplication advantage of IVTs as the gear ratio approaches this extremity is significant, provided that system efficiency is retained at that point. One system that shows promise is the CVT/IVT originally invented by William Beale (Beale 2006), shown in Figure 6.

![Figure 6: Beale CVT/IVT (Beale 2006)](image)

This design uses a rotating input link (88), connected via a two-force member (92) to a nonlinear spring (94), to incrementally rotate an output shaft via a one-way clutch or ratchet. The mechanism can be thought of as a four-bar mechanism in which one of the
connecting links is a sprung element. One advantage of this design is that the system can be expanded to contain multiple input links and ratchets, all connected to the same input shaft. As the number of ratchets on the output shaft is increased, the smoothness of the output shaft’s rotation is increased. Furthermore, the input shaft could be easily connected to a set of pedals, allowing the direct use of human power in a low-technology environment.

Previous research based on Beale’s design (Cyders 2012) included analytical modeling and physical testing of a crank-rocker adaptation of the original patented design. This work detailed the dynamic behavior of the system and the torque multiplication effect at the mechanism singularities (toggle or inversion points), concluding that the design was diversely applicable. One potential application proposed was a variable speed positive displacement pump, “using a slider-crank mechanism with a flexible member, a piston and a one-way valve” (Cyders 2012), as illustrated in Figure 7: Adaptation of transmission design to direct hydraulic pump (Cyders 2012). This proposed system was not modeled in that work, however. Such a system could provide the advantages of a positive displacement pump combined with passive flowrate selection across different system pressures. This capability would allow for high pressure pumping without the need for a variable speed power source. Such a quality would make the system well-suited to meet the agricultural water needs previously discussed.
This project had two main objectives. First, the modeling work done by Cyders (2012) was adapted to a slider-crank mechanism. This model was paired with a simulated PD hydraulic pump and used to make predictions of the ideal behavior of the combined system. Second, a physical prototype of the system was constructed, for the purpose of experimentation. This prototype included three critical subassemblies: the slider-crank IVT, the resistive load (pump), and the powered input. The experimental behavior of this prototype was compared to the theoretical predictions of the ideal model, and conclusions and future recommendations were drawn from this comparison.
2. LITERATURE REVIEW

Several fields of research were relevant when framing the scope and purpose of this thesis. First, the state of small-scale agriculture in SSA, and human-powered water retrieval methods in particular, were examined. Second, a discussion of human power was used to determine the ranges of input torque and input shaft speed over which to test the system. Third, previous modeling work conducted on reciprocating PD pumps, especially those driven by slider-crank or similar mechanisms, was studied. Fourth, research pertaining to CVT’s, and IVT’s in particular, was examined.

2.1 State of Agriculture

Water sources across SSA vary, but the most common are rivers or streams, or else a family or communal well. Wells are often hand-dug, although over the past several decades international non-profits such as the Practica Foundation have manually drilled thousands of small-bored wells (van der Wal 2008). While such projects have seen some success, the overarching goal of increasing irrigated land in SSA is recurrently hindered by lack of infrastructure and unavailability of repair parts. Installed hardware often fails prematurely for these reasons. Furthermore, despite some efforts to increase access to mechanized equipment, human power is likely to remain the predominant source of agricultural power over the next several decades (Brian G Sims and Josef Kienzle 2006).

A variety of human-powered pumps are currently being used in SSA; indeed, some thirteen different hand pump designs alone (Rural Water Supply Network 2015) were in use as of 2014. These pumps can be conveniently discussed in terms of maximum usable depth, as well as whether or not they are easily maintainable at the local
community level (Village Level of Maintenance, or VLOM). Figure 8 below introduces many different pump models currently in use across the developing world.

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Depth (meters) and corresponding Capacity (l/min)</th>
<th>Corrosion Resistant</th>
<th>Village Level Operation and Maintenance</th>
<th>Location of Origin or Successful Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rower pump</td>
<td>Suction-row</td>
<td>6m=80 l/min</td>
<td>Yes</td>
<td>Nigeria</td>
<td></td>
</tr>
<tr>
<td>New No. 6</td>
<td>Suction</td>
<td>7m=36</td>
<td>Yes</td>
<td>Bangladesh</td>
<td></td>
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<tr>
<td>Swiss Pedal</td>
<td>Suction-treadle</td>
<td>7m=60</td>
<td>Yes</td>
<td>Cambodia</td>
<td></td>
</tr>
<tr>
<td>NZ/Cansee</td>
<td>Direct/Plunger</td>
<td>12m=20</td>
<td>Yes</td>
<td>Ghana</td>
<td></td>
</tr>
<tr>
<td>Bucket Pump</td>
<td>Bucket pump</td>
<td>15m=10</td>
<td>Yes</td>
<td>Zimbabwe</td>
<td></td>
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<tr>
<td>Tara</td>
<td>Direct action</td>
<td>7m=24, 15m=23</td>
<td>Yes</td>
<td>Bangladesh</td>
<td></td>
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<tr>
<td>INTI</td>
<td>Direct action</td>
<td>15m=15</td>
<td>Yes</td>
<td>Bolivia</td>
<td></td>
</tr>
<tr>
<td>Afridev</td>
<td>Direct action</td>
<td>7m=26, 15m=22</td>
<td>Yes</td>
<td>Kenya</td>
<td></td>
</tr>
<tr>
<td>India Mark II</td>
<td>Piston</td>
<td>7m=12, 45m=12</td>
<td>No</td>
<td>India</td>
<td></td>
</tr>
<tr>
<td>India Mark III</td>
<td>Piston</td>
<td>45m=14, 45m=14</td>
<td>Yes</td>
<td>India</td>
<td></td>
</tr>
<tr>
<td>Consallen</td>
<td>Piston</td>
<td>7m=14, 45m=14</td>
<td>Yes</td>
<td>Liberia</td>
<td></td>
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<tr>
<td>U3</td>
<td>Piston</td>
<td>45m=10</td>
<td>Yes</td>
<td>Nigeria</td>
<td></td>
</tr>
<tr>
<td>Afridev</td>
<td>Piston</td>
<td>24m=22, 45m=15</td>
<td>Yes</td>
<td>Zimbabwe</td>
<td></td>
</tr>
<tr>
<td>Vergnet</td>
<td>Diaphragm</td>
<td>7m=24, 45m=15</td>
<td>No</td>
<td>France</td>
<td></td>
</tr>
<tr>
<td>Abi-ASM</td>
<td>Diaphragm</td>
<td>45m=15</td>
<td>No</td>
<td>Ivory Coast</td>
<td></td>
</tr>
<tr>
<td>Monolift</td>
<td>Progressive Cavity</td>
<td>25m=16, 60m=9</td>
<td>No</td>
<td>So. Africa</td>
<td></td>
</tr>
<tr>
<td>Rope pump</td>
<td>Rope &amp; disk</td>
<td>10m=40, 60m=8</td>
<td>Yes</td>
<td>Nicaragua</td>
<td></td>
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<tr>
<td>Bush pump-B</td>
<td>Piston</td>
<td>10m=30, 60m=9</td>
<td>Yes</td>
<td>Zimbabwe</td>
<td></td>
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<tr>
<td>India Mark IV</td>
<td>Piston</td>
<td>90m=14</td>
<td>Yes</td>
<td>India</td>
<td></td>
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<tr>
<td>Volanta</td>
<td>Piston</td>
<td>80m=4, 110m=4</td>
<td>Yes</td>
<td>Niger</td>
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</tbody>
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**Figure 8:** Categorization and comparison of human-powered pump models (Stewart 2003)

Shallow pumps such as the MoneyMaker Max (“KickStart :: Products :: MoneyMaker Max” 2015) and the Tany (Rural Water Supply Network 2015b), intermediate pumps such as the Afridev (Rural Water Supply Network 2015a), and deep well pumps such as the India Mark II (Rural Water Supply Network 2015c) have achieved widespread adoption because of their harnessing of human power, affordability, and robustness. Despite the success of these pumps, however, they all have the undesirable feature of transmitting all developed pressure in the system directly to the
user. This fact ultimately limits maximum pumping depth. The functionality of these pumps would be directly improved by the torque multiplication and passive flowrate selection that this project has the potential to offer.

2.2 Human Power

There are diverse methods of transferring power to a mechanical system. In light of the agricultural considerations discussed above, it was desired that the transmission method used in this project be compatible with the use of human power, regardless of whether human power was actually harnessed to complete the experimental portion of this project. The finished mechanism should maximize pumping flowrate at any given system back pressure, within the power generation capability of a typical adult human. In the modern world, the most recognizable and available method of transmitting human-generated power is the upright bicycle (Britannica 2017). As applied to this project, a set of pedals could be used to transmit power to the input link of the mechanism. Cycling science has also benefitted from thorough research over the past several decades; therefore, this project discussed human power limitations in the context of cycling power.

Reasonable power output expectations can be set with the aid of the human power generation curves given in Figure 9 below (Abbott and Wilson 1995). According to curve “C”, a conservative estimate of human power capacity for time durations greater than one hour is approximately 75 Watts (W).
Using this value of 75 W, and estimating low-speed torque capacity as equal to the person’s bodyweight multiplied by the bicycle crank arm length (about 130 N·m for a 170 lb. rider, using 170 mm crank arms), Cyders (2008) developed the following set of torque-speed curves, shown in Figure 10. This figure shows the low-speed limitations of torque production, due to the maximal muscular force that humans can develop.
Note the two vertical lines in this figure, bounding the pedaling rate between 60 rpm and 90 rpm. This envelope is based on Abbott and Wilson’s estimates of the most comfortable pedaling speed (Abbott and Wilson 1995), as well as work done by Kohler and Boutellier (2005). The latter work reexamined the Hill model of the force-velocity relationship in contracting muscles and applied it to find the ideal pedaling rate of a performance cyclist. Results of this analysis showed that the most efficient and most powerful pedaling speeds were not identical, as seen in Figure 11 below. While there are some differences in boundary values reported (50 rpm and 95 rpm vs. 60 rpm and 90 rpm), the general speed envelope concept remains valid as a constraint. One relevant exception is the chosen cadence of professional cyclists, which can often reach 120 rpm (Kohler and Boutellier 2005).
While cost and infrastructure constraints in SSA may well limit selection of power input to an upright bicycle, this project was not bound by such constraints. The goal was simply to test the performance of the system within the power limits of a human user, and thus an electric motor with equivalent characteristics was an option if advantageous.

2.3 Pump Modeling

Previous studies on reciprocating PD pumps driven by slider-crank mechanisms have exclusively examined models with rigid linkages. The use of a sprung link to enable passive selection of speed ratio and torque multiplication has not been considered. However, this prior work can still provide a useful starting point for this project. For example, Wilhelm and Van de Ven (2011; 2013) designed, modeled, and prototyped a six-bar crank-rocker-slider based PD pump with a constant top dead center (TDC) position. This system included an adjustable linkage that allowed for changing stroke.
length of the PD pump, producing a range of flowrates from zero to a known maximum. A concept of their prototype is shown in Figure 12.

**Figure 12:** Six-bar variable displacement linkage pump prototype (Wilhelm and Van de Ven 2013)

The modeling of the six-bar mechanism showed that transmission angles for the output slider (piston) could be kept above 60 degrees. Dynamic modeling of the designed system began with using planar vector loops to define the position equations of the mechanism, and then utilized an inverse dynamics approach to define the forces and torques in the system members over a complete revolution of the input link. The authors also examined pumping work in the system; work into the system was computed in the domain of torque and angular velocity, while work output from the system was calculated in the domain of pressure and flowrate. Energy losses considered in the system included “viscous friction between the piston and cylinder, coulomb friction of the linkage pins and crosshead bearing, and leakage across the piston cylinder gap” (S. Wilhelm and Van de Ven 2013).

The hydraulic system prototyped for the experimental portion of this study is shown in Figure 13; the experimental procedures included “a series of 6
experiments…conducted with the pressure varying between 1.2 MPa and 3.45 MPa at input speeds between 3 Hz and 9 Hz” (S. Wilhelm and Van de Ven 2013). These pressures and velocities are higher than those expected during experimental testing for this project, but are within the same order of magnitude.

![Figure 13: Hydraulic circuit diagram of experimental setup (S. Wilhelm and Van de Ven 2013)](image)

A comparison chart showing the relative significances of the different energy losses, based on experimental data, is given in Figure 14 (system pressure held constant at 2.4 MPa and input link rotating at 3 Hz for this result); it can be seen that coulomb friction is the largest energy loss observed.
Figure 14: Components of energy loss comparison in variable displacement pump system (S. Wilhelm and Van de Ven 2013)

Overall, the work and energy results showed good correlation between the analytical modeling techniques used and the experimental data from the prototype. The results of work input and output comparisons, showing modeling predictions and experimental results, are shown in Figure 15 and Figure 16 below. The discrepancies observed at low percentages of maximum displacement were attributed to losses in the check valves, and potentially linkage deflections in the mechanism. The authors also stated that high-friction plain bearings in the pin joints contributed to low-displacement efficiency losses; with roller bearings input into the analytical model, the frictional losses predicted were reduced to give the model efficiency shown in Figure 17. Finally, the authors recommended that active valves be implemented into future systems reduce slippage losses; for their work, they used passive check style check valves.
Figure 15: Work input comparison of model and experimental data (S. Wilhelm and Van de Ven 2013)

Figure 16: Work output comparison of model and experimental data (S. Wilhelm and Van de Ven 2013)
The recommendation for actively controlled check valves is found in other papers as well; Johnston (1991) states that for crankshaft (input link) rotational velocities of greater than approximately 500 rpm or about 8 Hz, the use of active valves becomes increasingly necessary. Another paper focuses on moderating the intensity of the pressure pulsations encountered in PD pump outputs, by replacing the typical constant speed drive with a drive capable of dynamically varying speed and torque (Josifovic, Corney, and Davies 2015). This was accomplished by using a torque converter as opposed to a conventional gearbox; this torque converter provided a hydrodynamic (and elastic) coupling between the engine and the piston. Such a coupling does not allow for automatic selection of the most efficient shaft speed, but it does allow for a variable speed selection; for low engine speeds, “torque multiplication will be small but as the Engine speed builds, torque multiplication will gradually increase up to its efficiency peak” (Josifovic, Corney, and Davies 2015). Note that a hydrodynamic torque converter still has a peak
efficiency linked to a certain speed; it is thus similar to, but distinct from, the research proposed in this project.

2.4 CVTs and IVTs

Finally, the state of CVT and IVT research was considered. This included more information on the range of CVT designs previously and currently explored, as well as a presentation of the work done in the field of IVTs by prominent researchers. Broadly speaking, a CVT is any transmission that allows for continuous selection of input to output speed ratios, whether for speed increase or torque multiplication. To date, the majority of applied CVT work has been undertaken in order to increase the efficiency of automobiles. A CVT allows an automobile engine to operate in its window of most efficient power generation across all speed and torque demands at the wheels, enabling “an optimized combination of performance and fuel consumption to be realized” (Kluger and Fussner 1997).

Designs implemented (or at least considered) by car manufacturing companies include three basic types: belt-driven, traction-based, and variable geometry designs (Kluger and Fussner 1997). Table 1 compares the average efficiencies of different types of transmission designs.
As shown in Table 1: **Mechanical efficiencies of various types of mechanical transmissions (Kluger and Fussner 1997)**, automobile CVT advantages are offset by their poor mechanical efficiency when compared to manual, and sometimes even automatic, discrete gear ratio transmissions. In addition, these designs often suffer from increased weight and greater production cost. Some of the types of CVTs introduced above are shown below.

<table>
<thead>
<tr>
<th>Transmission Type</th>
<th>Mechanism Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manual Transmission</td>
<td>97%</td>
</tr>
<tr>
<td>Automatic Transmission</td>
<td>80-86%</td>
</tr>
<tr>
<td>Belt CVT (steel)</td>
<td>90-95%</td>
</tr>
<tr>
<td>Belt CVT (rubber)</td>
<td>90-97%</td>
</tr>
<tr>
<td>Traction CVT (toroidal)</td>
<td>70-94%</td>
</tr>
<tr>
<td>Traction CVT (nutating)</td>
<td>75-96%</td>
</tr>
<tr>
<td>Variable Geometry CVT</td>
<td>85-93%</td>
</tr>
</tbody>
</table>
Figure 18: Steel push belt CVT (Kluger and Fussner 1997)

Figure 19: Variable diameter sheave traction drive CVT (Kluger and Fussner 1997)

Figure 20: Half toroidal traction drive CVT (Kluger and Fussner 1997)
Although CVTs are beginning to be implemented in automobiles as design efficiencies improve, widespread fuel efficiency and performance gains have yet to be realized. Another area of automotive transmission research being pursued is epicyclic CVTs. Like the Beale CVT, some of these designs are true IVTs, offering up to a 1:0 input to output speed ratio. Two key researchers in this field are Mangialardi and Mantriota; their work encompasses both type I and type II power flows, and both parallel and series style IVTs, shown below.

**Figure 21:** Nutating traction drive CVT (Kluger and Fussner 1997)

**Figure 22:** Series (left) and parallel (right) IVT setups (Mangialardi and Mantriota 1999)
Mantriota (2002) reported that series and parallel IVT setups are better adapted for different power flows: specifically, “series-IVT is more convenient to upgrade efficiency with type I power flow while a parallel-IVT ensures a better efficiency with the type II power flow”. Furthermore, the Type II parallel-IVT setup proved to have greater efficiency than the Type I series-IVT setup at low transmission ratio (ratio of input to output shaft speeds) values, while the inverse was true at high transmission ratios. The efficiencies of both a series IVT setup (paired with a Type I power flow) and a parallel IVT setup (paired with a Type II power flow) are presented in Figure 24 below.
As noted in the introduction, the mechanism design for this project was a slider-crank adaptation of the Beale CVT studied by Cyders (2012). Although the main purpose of this project was to build a simulation and compare it against a physical prototype, the finished design’s efficiency can be usefully compared against the previously developed CVTs and IVTs presented above. A power transmission efficiency equivalent to or greater than those of other CVTs and IVTs would be a promising indicator of design superiority.

Furthermore, the methods pursued by Cyders (2012) in developing static and dynamic analytical models of the Beale CVT informed the approach of the simulation and modeling done during this project. Cyders began with a kinematic model of the crank-rocker mechanism serving as the basis of the Beale CVT, and applied a pseudo-static analysis to this model. He also used the software program Modelica to build a component-based model of the Beale CVT; this simulation was used to dynamically
model the system in ways that the pseudo-static model could not. These modeling efforts were compared against a Finite Element Analysis (FEA) conducted on the system components. Each of these three approaches contributed unique value to the project. Finally, the predictions made from these models were compared against actual results from an equivalent physical prototype of the Beale CVT. Some of these techniques, especially the pseudo-static model, were used to good effect during the simulation stage of this project.
3. OBJECTIVES

The first objective of the proposed research included the complete kinematic and kinetic modeling of a power-sharing CVT pump with constant power input. This objective required the simulation of the full system, building from a basic rigid-link inverse dynamic model and incorporating more complex behavior of the sprung link during each rotation of the input link. The simulation was used to build a complete set of pressure-flowrate predictions that could be compared against the results of the second objective.

The second objective of the project was a comparison between simulation predictions made during the completion of the first objective and experimental data taken from a physical prototype of the system. This objective required the design, fabrication, and instrumentation of a physical manifestation of the slider-crank pumping system, followed by collection and analysis of data obtained from this prototype.

During the completion of both project objectives, a number of experimental variables were examined. For the predictive model, this examination was accomplished through defining input variables and solving for outputs. During the experimental comparison phase, variables were monitored through instrumentation and data signal recording. The variables were as summarized below, with the first three defined as system inputs and the remainder as system outputs:

- System back pressure (psi)
- Input speed (rpm)
- Sprung link stiffness (lb/in)
- Input torque (in-lbs)
- Spring deflection and piston displacement (in)
- Inter-linkage forces (lb, predictive model only)
- Pump flowrate (in$^3$/rev)
- Mechanism efficiency (%)
4. SIMULATION AND MODELING

The modeling process began with the development of an inverse dynamics model of a four-bar slider-crank mechanism using planar vector loops. This initial work was then completed by incorporating a sprung link of known stiffness as the connecting linkage between the crank and the piston. For this project, only linear force-displacement curves were considered for spring behavior, as this simplified both the analysis of the system and prototype design and construction.

The behavior of the system as the input link revolved was divided into four distinct regions via the use of piece-wise kinematic assumptions: (1) a compression phase during which the sprung link compresses while the piston remains stationary, until the sprung link force exceeds the external force imposed by the system back pressure; (2) a discharge phase in which the piston moves from its initial location through bottom dead center (BDC), where it comes once again to a stop; (3) a decompression phase during which the spring decompresses to its original (full) length, while the piston does not move; and (4) a recharge phase in which the piston moves back to its initial position at top dead center (TDC), priming the pump for the next cycle. These phases are shown graphically in Figure 25 below. Figure 26 shows an example of piston position, velocity, and acceleration plots for a system back pressure of 40 psi and an input link rotational velocity of 0.1 rpm.
Figure 25: Four phases of complete piston cycle
Figure 26: Piston position, velocity, and acceleration over a complete cycle

The periods of zero piston displacement, velocity, and acceleration above correspond to the compression and decompression phases; the other phases of each graph correspond to discharge and recharge, showing the sinusoidal nature of piston deflection, velocity, and acceleration when the system behaves as a rigid-linkage mechanism. Note the discontinuities in the velocity and acceleration plots. Theoretically, a velocity discontinuity such as was predicted here would create an infinite acceleration rather than a finite acceleration discontinuity.
The failure of the simulation to account for this phenomenon, and the velocity discontinuity itself, are results of the piece-wise, inverse dynamics approach used. They represent the results of transitioning instantly between two kinematic models: an infinitely stiff sprung link model and a finite stiffness model. These abrupt transitions were expected to be smoothed in the recorded experimental data. Also note that the inverse dynamics approach assumed constant piston engagement and disengagement angles at all speeds. A forward dynamics modeling approach, which would be necessary to analytically validate these assumptions, was beyond the scope of this work.

Once the simulation was complete, differences were examined between dynamic, pseudo-static, and static cases for the input link. First, the input link speed was set to zero as the input link was incrementally advanced around one complete revolution. Torques predicted during this initial run were compared against pseudo-static results generated from the model using an input link speed of 0.1 rpm. Finally, truly dynamic predictions were generated using an input link speed of 240 rpm, which was the maximum experimental speed of the prototype (initially intended to be 120 rpm, as discussed in a later chapter). Convergence to within less than 1% maximum divergence was observed between static and pseudo-static single-piston torque predictions, and convergence to within 3.1% maximum divergence was observed between static and dynamic torque predictions. All comparisons were made using an external pressure setting of 40 psi, an approximate pressure midpoint. The results of the static vs. dynamic torque predictions can be seen in Figure 27 below.
This comparison demonstrated that external forces predominated over inertial effects within the speed range intended for prototype testing, justifying the use of the simulation in a pseudo-static setting to generate a single predictive pressure-flowrate curve to eventually compare against experimental results. The use of a single pressure-flowrate simulation curve, rather than a separate simulation curve for each speed tested, significantly simplified the experimental data comparison.

Figure 27 was constructed using a single simulated piston. Completing the simulation required the addition of a second piston, which was accomplished by phase shifting the single-piston torque curve by 180 degrees and adding it to the original curve. The results of this combination are shown in Figure 28 below, using a back pressure of 40 psi and a rotational velocity of 0.1 rpm.
Once the model was capable of producing torque input requirement curves and had been verified for pseudo-static usage, the validity of the simulation was checked via an energy comparison, represented by Equation 1 and Equation 2 below. The single piston torque values were multiplied by the instantaneous change in input link angular position to find instantaneous mechanical energy input. Fluid energy imparted (boundary work done) during one rotation was calculated as the multiple of the instantaneous system pressure and the change in volume effected during the discharge phase. Sprung link potential energy was calculated at the point of maximum spring compression. The sum of boundary work and spring energy comprised the energy output during one rotation, since all sprung link energy would be released back into the system during the decompression phase.
A theoretical efficiency of approximately 100% was expected from the simulation results, as the simulation did not account for either significant friction losses or viscous fluid drag. It was found that input and output energy summations consistently converged to within 1% or better, thus validating the simulation. Once the model was designed and validated, it was then used to generate a pressure-flowrate predictive curve to compare against the experimental results from the prototype. Linkage lengths, sprung link stiffness, input link angular velocity (constrained at 0.1 rpm), and back pressure were set as independent variables during this generation, while resulting flowrate was the dependent output variable. Since a linear spring was used, this resulted in a linear pressure-flowrate curve. This pressure-flowrate curve and the energy comparison calculations comprised the completion of the first thesis objective, and were compared against the experimental results obtained.
5. EXPERIMENTAL SETUP

The completion of the second objective of this project began with the design and fabrication of the physical prototype. Fabrication of the final prototype design was completed utilizing the Ohio University engineering lab facilities. When designing the prototype, two design criteria were emphasized. The first objective was to design for the potential application of a similar device as a human-powered water pump. Using a maximum approximate well depth of 70 m, this produced a hydrostatic pressure of about 100 psi. To reflect the boundaries of human-selected pedaling cadences without truncating input speed range, motor speed was intended to be tested between 60-120 rpm. CVT mechanism link lengths and sprung link force-displacement behavior were designed around these constraints, using the simulated system model to generate maximum inter-link forces that the system would need to withstand at maximum speed.

The second objective considered during prototype design was maximizing simplicity and designing for ease of manufacturability, while ensuring that all necessary test variables could be measured at the requisite precision. Therefore, the prototype detailed below was designed to be the simplest feasible device that could successfully withstand the predicted forces in moderate-cycle fatigue loading conditions.

The core components of the prototype were the powered input, the sprung link, and the resistive load. The sprung link, shown as a cut-away CAD assembly in Figure 29 below, was designed as a shock absorber with two revolute joints (1 and 2 in the figure), containing a helical spring (3) constrained about a rod (4) that was allowed to slide within a bushing (5). This sliding joint was rotationally constrained by means of driving a pin
(not shown) through the outer casing (6) and bushing, into a slot (not shown) running along the length of the rod. The full extended length of this sprung link was set with a cap (7) bolted onto the end of the rod, which rested against the internal face of the bushing during full extension.

Figure 29: Schematic of sprung link (section view)

The other two crucial system components, the powered input and the resistive load, are both visible in the CAD assembly of the overall prototype shown in Figure 30 below. The resistive load was a positive displacement pump, which consisted of a reciprocating piston running inside a cylinder block, connected to a variable back pressure valve at the pump outlet. The powered input was connected to the sprung link and piston via two phase-offset input links connected to the same shaft. Not shown below are the fluid reservoir and the hoses connecting it to the pump inlet and outlet valves. A list of the numbered items in Figure 30 is given below the figure.
The key components called out in Figure 30 are as follows:

1. **Power supply**: Briggs & Stratton Etek 6 HP (continuous) brushed DC electric motor.
2. **Motor encoder**: a US Digital HD25 optical encoder, model HD25-200-F-S-L-NE-D. This encoder had 200 cycles per revolution (CPR), and was run in quadrature mode for a total of 800 pulses per revolution.
3. **LVDT (Linear Variable Differential Transformer)**: a Series 240 DC-DC Trans-Tek, model 0244-0000. This device was used to record the deflection of the sprung link during experimentation. Only one LVDT was used, as it was assumed that the two pistons would exhibit identical behavior during testing.
4. **Sprung link**: detailed previously.
5. **Piston**: the sliding surface of each piston was made of solid Teflon (not shown). Initial design of the piston called for brass turned on a lathe and fitted with a single
rubber buna-n o-ring; however, this design was modified after it was observed that static breakout friction was excessively high within the piston casing.

(6) Back Pressure Valve (BPV): two Griffco M-Series Back Pressure Valves, model BPM050P, with a stated back pressure range of 10-250 psi adjusted via a set screw.

(7) Outlet pressure transducer: two SSI Technologies 0-100 psig transducers, model P51-100-G-B-I36-5V-R, were used to measure instantaneous pump outlet pressure for both pistons. Negative pressures were not considered at either the inlet or the outlet because positive pressures were expected to predominate.

(8) Inlet valve: two backflow-prevention valves, McMaster Carr model 7768K22. Based on maximum motor speed, it was expected that passive check valves would be sufficiently quick-closing.

The Briggs & Stratton motor (48VDC max) was used instead of human power to ensure maximum repeatability of shaft speed and torque during trial runs. This particular motor was used because of its ready availability. Although it was designed for a much higher speed range than intended during this project, an approximation of available mechanical power at low speeds indicated that it would be sufficient. In practice, it was observed that the minimum sustainable motor speed during experimentation was approximately 80 rpm, and that finely controlling motor speed was difficult even above that lower bound. Due to these constraints, the motor speed during testing was adjusted from the initial plan of 60-120 rpm to a larger range of 80-240 rpm.

The power supply selected to power the motor was a Mean Well RSP-1500-48 1500 W single output power supply. The power supply and the motor were used in
tandem with a motor controller, to enable variation of motor supply voltage and therefore motor speed. The chosen controller was the Kelly KDZ 48200 24-48 V, 200 Amp Brushed Motor Controller.

Table 2 below shows each variable measured during testing, the sensor used for that measurement, the associated measurement uncertainty, and the approximate threshold of uncertainty acceptability for each variable being measured. In the case of the motor encoder, uncertainty depended on data sampling rate. The DAQ used in this experiment was the MCC USB-1608G DAQ, which had a sampling rate capability of 500 kHz. Seven differential-ended analog channels total were used during data collection, allowing for an individual channel sampling rate of over 71 kHz. The software used limited the maximum sampling rate to 16666 Hz when data was collected continuously for one minute. Recording 800 encoder steps per revolution at the maximum experimental motor speed of 240 rpm required a sampling rate greater than 3200 Hz, well under the achievable threshold. Thus, it was known with certainty that no encoder pulses were skipped during testing. As shown in Equation 3, the uncertainty of the encoder was calculated at the maximum motor speed used and the maximum data sampling rate available. In the equation, $\omega$ is the input link angular velocity in rad/sec, and $f$ is the data collection rate in Hz.

\[
\text{Angular Uncertainty, } \mu = \pm \frac{1}{2} \left( \omega_{\text{input}} \frac{360 \text{ deg}}{2\pi \text{ rad}} \right) * \frac{1}{f}
\]
The actual uncertainties of the LVDT and the pressure transducers were found from static calibration curves. Since changes in pressure and spring displacement have linear effects on pressure-flowrate simulation outputs, it was adequate to constrain the allowable uncertainty of these variables to within a couple percentage points. Because error in input link angle both directly affects calculated flowrate and also reduces the ability of the experimental data to describe system behavior near the toggle points, the uncertainty threshold for input link angle was lower than for either pressure or spring displacement.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measurement</th>
<th>Output Uncertainty (max. magnitude)</th>
<th>Uncertainty Acceptability Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input link angle [deg]</td>
<td>US Digital HD25-200-F-S-L-NE-D</td>
<td>±0.043 deg at 240 rpm motor speed</td>
<td>±1 deg (0.28% FSR*)</td>
</tr>
<tr>
<td>Sprung link compression [in]</td>
<td>DC-DC Trans-Tek LVDT, model 244-0000</td>
<td>0.012 in (0.6%)</td>
<td>±0.04 in (2% expected FSR)</td>
</tr>
<tr>
<td>System back pressure [psi]</td>
<td>2x SSI Tech. P51-100-G-B-I36-5V-R</td>
<td>0.65 psi (0.6%) and 1.2 psi (1.5%)</td>
<td>±2 psi (2% of expected FSR)</td>
</tr>
</tbody>
</table>

*Full Scale Range

5.1 Design of Experiments

During experimental prototype testing, pressure and shaft rotational velocity were independent variables and flowrate was the dependent variable. In order to collect sufficient data to fully describe the pressure-flowrate behavior of the system at each
setting, a full factorial breakdown table for was completed, shown in Table 3 below. The simplicity of the simulation pressure-flowrate curve allowed the number of testing points to be kept fairly low. As detailed above, the initial intention to take data at 60-120 rpm was infeasible due to the motor and controller configuration. An alternative range of 80-240 rpm was used instead, in intervals of about 20 rpm when possible.

<table>
<thead>
<tr>
<th>Back Pressure [psi]</th>
<th>30</th>
<th>60</th>
<th>70</th>
<th>85</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Shaft Speed [rpm]</td>
<td>80-110</td>
<td>130</td>
<td>150-160</td>
<td>170</td>
<td>190</td>
</tr>
</tbody>
</table>

Experimental testing began with several preliminary data runs to ensure correct sensor functionality. After these checks, actual tests were conducted at each combination of mean motor speed and back pressure prescribed in the table above. The order of mean shaft speed selection was semi-random, to minimize potential sources of cumulative user error. During each test, the pressures in the two back pressure valves were matched as closely as possible, by visually synchronizing the two maximum displacements of the piston. This method assumed equal spring constants and equal inlet valve behavior between the two pistons.
5.2 Data Analysis

During conditioning of the collected data, it was found that the encoder channels had a sufficiently great signal to noise ratio that no pre-analysis filtering or smoothing was required. The LVDT data signal and the two pressure transducer signals, however, required filtering. A low-pass IIR was used for each sensor, with a cutoff frequency of 1% of the maximum frequency observed. Filtering did not significantly distort any of the signals, as can be seen in Figure 31. In these plots, PT1 refers to the piston that was also equipped with the LVDT; PT2 refers to the opposite piston. Also, the solid black lines represent the unfiltered data, while the blue and green lines represent the filtered data. For this data run, the motor shaft speed was set as close to 170 rpm as possible, and the back pressure valves were nearly fully closed.

![Figure 31: PT1 and PT2 unfiltered and filtered comparison plots](image)
After the filtering was completed, the encoder channels data were used to create an array of input link angular positions. The data from each sensor was then divided into intervals corresponding to 360 degrees. These intervals were averaged together to produce aggregate data curves, which were then used to draw conclusions about the pressure and flowrate behavior of the system for that test. An example of these aggregate curves overlaid against individual interval data is given in Figure 32 below. Note that the curves are plotted over input link angle rather than time. Uncertainty for each point in the aggregate curves was calculated to the 95% CI using Equation 4, shown below (σ is standard deviation of each set of points taken from the individual intervals, and n is the number of points in each set). The maximum uncertainties and average uncertainties of each sensor for this example are shown in Table 4 below.

Figure 32: Aggregate cycle of PT1 and PT2 data (dashed), overlaid on individual cycles (solid)
\[ \text{Uncertainty, } \mu = \pm 1.96 \times \frac{\sigma}{\sqrt{n}} \]

<table>
<thead>
<tr>
<th>Aggregated Sensor</th>
<th>Maximum Uncertainty [±]</th>
<th>Average Uncertainty [±]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encoder</td>
<td>0.42 deg</td>
<td>0.00070 deg</td>
</tr>
<tr>
<td>LVDT</td>
<td>0.0016 in</td>
<td>0.19 in</td>
</tr>
<tr>
<td>PT1</td>
<td>0.57 psi</td>
<td>0.070 psi</td>
</tr>
<tr>
<td>PT2</td>
<td>0.62 psi</td>
<td>0.18 psi</td>
</tr>
</tbody>
</table>

Once the aggregate curves were found for each test, a single plot containing an overlay of LVDT and pressure transducer data was used to graphically check the assumption of consistent behavior between pistons. Results for the test referenced above are shown in Figure 33 below. In addition to the aggregate curve for the PT2 signal during this test, a curve of PT2 data shifted 180 degrees was also plotted in the same color (blue dashed curve). The PT1 maximum is circled in red (the vertical broken red line also corresponds to this location), as is the PT2 maximum. The location of the maximum LVDT displacement is marked by a broken vertical blue line.
Figure 33: Sample aggregate data curves comparison plot
6. RESULTS AND DISCUSSION

Some limitations of the experimental prototype setup were noticed during the post-testing data analysis. It was observed that a full range of pressure data from the PT2 transducer would not be available for a significant number of tests due to high-end saturation. This was the case for all closed-valve tests (maximum pressure setting), as well as for the first one or two tests below closed-valve position, at all motor speeds. Some truncation due to signal saturation was also an issue for the PT1 transducer, but this phenomenon was limited to the closed-valve tests as the PT1 transducer had a higher saturation point. The reason for these discrepancies from nominal manufacturer specifications was unknown. The data from the PT2 signal was still useful, however, primarily as a means of verifying the overall phenomena reported by the PT1 transducer during testing.

The uncertainties of the aggregate curve points for each of the measured variables (input link angle, pressure, and flowrate) as calculated via Equation 4 were considered when ascertaining the validity of the pressure-flowrate results from each test. Analysis revealed that the pressure transducers (especially PT2) typically displayed the largest aggregate point uncertainty during each test; this phenomenon is visible in Table 4 in the previous chapter. However, no sensor produced an uncertainty great enough to obscure the final results, as will be graphically demonstrated below.

Before pressure-flowrate graphs could be compiled for each motor speed setting, the aggregated sets of sensor data needed to be phase-shifted to match the phase of the analytical predictions. This step was necessary because of the lack of a position-indexed
encoder during testing. Simulation curves for LVDT compression, system pressure, and piston displacement were generated using the maximum experimental pressure from the corresponding test, and the decompression region of the predictions was matched with the corresponding experimental region. Since the LVDT data signal had the greatest signal to noise ratio and the least uncertainty of the sensors, it was used for this process. The results of phase shifting for the test used as an example above (170 rpm, 72 psi at engagement) are shown in Figure 34 below.

![Figure 34: Phase-shifted experimental LVDT data overlaid on predictive curve](image)

Note the initial discrepancy between the simulation and experimental LVDT curves in Figure 34. Initially, the cause for this discrepancy was unknown. High-speed video capture of the running prototype revealed that immediately past TDC of the input link, before the sprung link began to compress, the piston slipped forward by a consistent amount for that test. This phenomenon was deduced to be due to failure of the inlet valve...
to close immediately after recharge, as slippage occurred even when all air was removed from the inlet and outlet lines. Thus, rather than use the entire simulation LVDT curve as a reference during the phase shift, only the decompression phase was used as shown.

Once the aggregate data sets were phase-shifted, the position of the piston at every point around the cycle was calculated using the input link angular position and compression of the sprung link at each point. Figure 35 below shows the comparison of the calculated experimental piston displacement against the simulation predictions, again at 170 rpm and 72 psi. Note the rapid initial piston slippage in the experimental curve, showing the effects of the inlet valve closure delay. The unsteady region which follows (about 50-125 degrees) was deduced to show the piston slowly settling after initial slippage, prior to the opening of the outlet valve. Since there was no direct measurement of piston displacement during testing, this supposition could not be quantitatively verified; however, it is consistent with the results of the high-speed video capture taken.
Figure 35 was used to determine the experimental engagement and disengagement input link angles, and thus calculate the fluid flowrate during that cycle. The point whose coordinates are called out corresponds to the beginning of the transition from piston settling to displacement generally consistent with the predictive curve. This location was taken to be the point at which the outlet valve actually opened. Thus, judging from this plot, while the piston moved a total of nearly half an inch during the cycle, only about 0.1 in of that displacement occurred while the outlet valve was open.

Finally, the uncertainty of each calculated pressure and flowrate value was determined. The pressure uncertainty was already available, taken conservatively as the maximum value calculated over the aggregate cycle. The flowrate uncertainty (95% CI) was found using the LVDT uncertainty at the piston engagement and disengagement angles. For a conservative estimate, the maximum LVDT uncertainty over the aggregate cycle was used in both instances. For the test above, flowrate uncertainty was found to be
±0.0057 in³/rev. Note that this result assumes the correct selection of the piston engagement point using the piston displacement plot. The inability to directly measure piston position caused the greatest, and least quantifiable, source of uncertainty for the experimental results.

The first comparison made between simulation predictions and experimental results was made for the lowest sustainable motor speed, which ranged from about 80 to about 110 rpm depending on back pressure. Using the nominal sprung link stiffness of 46.1 lbs/in, as provided by the spring manufacturer, the results shown in Figure 36 below were obtained. The uncertainties associated with each experimental point are plotted; where difficult to discern, this is due to the small uncertainty of the point. Comparing these uncertainties against the sensor nonlinearities reported in Table 2, it can be seen that neither are significant enough to significantly obfuscate the pattern of the experimental data.
Figure 36: Low speed plot of volumetric flowrate vs. engagement pressure (original spring constant)

The x-axis of the graph above is the pressure at the engagement angle of the input link, which was always slightly less than the maximum pressure observed for each test. The smallest differences between the model predictions and the experimental data were found for pressure settings in the middle of the range. For most of the pressures tested in this speed range experimental flowrate corresponded with the analytical predictions, except for an offset difference of about 0.4 in³/rev between the experimental regression line and the predictive curve. The exception to this agreement is the lowest pressure data point.
Note that the application of a best-fit line to the experimental data in the figure was done to make comparisons against simulation predictions more convenient. More data points would need to be generated to increase the trustworthiness of the regression line, especially for making any extrapolations. Also note that the lowest pressure point was excluded from the regression line, as it simply would have decreased the quality of the fit with the other data points.

The lowest pressure data point in Figure 36 corresponded to the greatest amount of piston slippage, as demonstrated by Figure 37 and Figure 38 below. For this lowest pressure, the majority of the piston motion took place prior to outlet valve opening.

**Figure 37:** Low-speed LVDT results vs. simulation predictions (lowest pressure)
Figure 38: Low-speed piston displacement calculations against simulation predictions (lowest pressure)

Figure 39 below shows maximum sprung link compression plotted against the maximum pressure for all low-speed tests. This plot was used to confirm the underlying similarity of the simulation and the prototype that was obscured by check valve closure delay. The only significant difference between the simulation and experimental lines in Figure 39 is that the slope of the simulation regression line is slightly greater, indicating that the stiffness of the actual sprung link was greater than the nominal value of 46.1 lbs/in. Linear interpolation determined that the spring constant was closer to 108% of the nominal value, or 49.8 lbs/in. Using this value produced a closer match between predictions and experimental results, and was a reasonable adjustment as the spring manufacturer gives nominal stiffness with an uncertainty of ±10%.
Using the lowest-speed data, but with the sprung link stiffness used as 108% of its nominal specification, the pressure-flowrate and compression-pressure plots above were redrawn. It was found that changing the spring stiffness in the model changed both the magnitude and the position of the decompression portion of the LVDT simulation curve. Figure 40 below shows a comparison of simulation LVDT curves calculated using the original and the adjusted spring constants, for the same test. The experimental LVDT curve shown was phase shifted using the original spring constant simulation curve. The simulation curve produced using the increased spring constant has a decompression period offset from the original simulation curve by about 10 degrees, which would similarly change the experimental phase shift if it were used as the reference.
As discussed previously, piston displacement was calculated using both LVDT and encoder data, using Equation 5 below. Shifting the LVDT data affects the phase of the second term in this equation but not that of the first, thus causing a change in the shape of the piston displacement curve rather than a simple phase shift. This change in turn affects the selection of the piston engagement and disengagement angles in the piston displacement plot, and therefore the calculated flowrate. This effect can be seen in Figure 41, which was generated using the same test data.

\[
\text{Displacement, } x = l_2 \cos(\theta_2) + l_3 \cos(\theta_3)
\]
Figure 41: Comparison of piston displacement calculated using original spring constant and increased spring constant (low speed, 85 psi)

The results of reanalyzing the low speed data using an increased sprung link constant are given in Figure 42 and Figure 43 below. The experimental results for maximum LVDT compression vs. maximum pressure were improved to almost an exact match with the simulation predictions. Also note in Figure 42 that the differences between the original and the recalculated pressure-flowrate curves are minor, and that increasing the sprung link constant actually slightly improved the correlation of the simulation and experimental data.
Figure 42: Low speed plot of volumetric flowrate vs. engagement pressure (increased spring constant)
After the spring constant was changed from 46.1 lbs/in to 49.8 lbs/in, all other test data sets were analyzed and used to create pressure-flowrate plots for each motor speed. It was observed that as motor speed increased, experimental divergence from the simulation increased. This can be clearly seen in Figure 44 below, which presents pressure-flowrate data for all points tested. Uncertainty bars were not applied to this summary graph, but are available in the complete set of pressure-flowrate graphs in Appendix A: Complete Graphical Test Results.
The low speed experimental data regression line in Figure 44 shows that for the lowest speed range tested, the simulation model was an accurate predictor of experimental prototype behavior, for the majority of pressure settings. The data sets corresponding to 130 rpm and 150 rpm reveal steadily increasing divergence from the simulation results, showing a direct correlation between motor speed and piston slippage. The second experimental regression line in Figure 44 corresponds to the 170 rpm data set, and shows much greater divergence from simulation predictions than the low speed regression line. At speeds greater than 170 rpm, the experimental data was largely

\[ \text{Flow} = -0.0158 \times \text{Press} + 1.8602 \quad R^2 = 0.9262 \]

\[ \text{Flow} = -0.0006 \times \text{Press} + 0.149 \quad R^2 = 0.1494 \]
consistent between sets, suggesting that the piston slippage had reached a maximum
effect. Regression lines for 130 rpm and 150 rpm, as well as for 190 rpm and 200-240
rpm, were not given to avoid visual cluttering.

Observing the complete set of pressure-flowrate data for all motor speeds, it is
reasonable to predict that decreasing motor speed below the slowest range tested would
continue to improve the convergence of the simulation and experimental results. This
convergence would be the result of decreasing piston slippage rather than a fundamental
change in the behavior of the system.

After the pressure-flowrate data sets were completed for all motor speeds, an
efficiency study was performed to compare against the perfect efficiency predicted by the
simulation. This process began by finding the total boundary work (fluid energy
imparted) from the point of piston engagement through maximum piston displacement,
by summing the multiples of instantaneous pressure with instantaneous change in
displaced volume. Note that although the simulation model predicted that the maximum
piston displacement point would occur at BDC, this was not the case for experimental
results. Once boundary work was found, the potential energy in the spring was found at
the point of maximum piston displacement and added to the boundary work value, to find
the work output from the system over that cycle. This summation is shown in Equation 6
below; note that it is almost identical to Equation 2 above.

\[ W_{\text{exp}} = \frac{1}{2} k x_x^2 + \int_{v_1}^{v_2} P \, dV \]
Mechanical energy was then calculated for the cycle through maximum piston displacement. Torque was found as the cross product of input link position and sprung link force (calculated using LVDT compression data and spring constant). Each instantaneous torque value was then multiplied with the corresponding instantaneous change in angular position of the input link. For the example test used above (170 rpm, 72 psi at engagement), the torque curves and energy comparison plots are shown in Figure 45 and Figure 46. The energy comparison plot also shows simulation predictions for total energy required (original and normalized). Note that the equation used to find experimental work input is the same as that used for the simulation energy comparison, Equation 1 above.

Figure 45: 170 rpm (72 psi) experimental single-piston and double-piston torque curves overlaid on respective analytical predictive curves
The torque curve for the single-piston case in Figure 45 matches well with the predictions, except during the beginning of the cycle; this period corresponds to the LVDT compression delay induced by piston slippage. The decrease in required torque significantly affected the mechanical power magnitudes as compared against simulation predictions, as can be seen in Figure 46. The normalized simulation energy curve in this figure shows a slight shape difference between simulation and experimental energy curves, an effect of the divergence in torque curves as well as of the distortion caused by normalization. A comparison of input energy and output energy for the experimental results shows a relatively small difference (8.4%) at the point of maximum piston displacement.
The differences between simulation and experimental energy curves in Figure 46 were caused by the initial divergence in experimental torque from simulation torque. In addition, some of the discrepancy between the two experimental energy plots originated early in the cycle, when the inlet valve failed to close. This initial torque divergence, or torque lag, was more pronounced at lower pressures for any given speed. For example, Figure 47 below shows the torque curves for test data taken at 170 rpm and the lowest possible pressure. For all torque graphs, the double-piston experimental curves were estimated by phase-shifting the single-piston torque results by 180 degrees and adding them to the original torque curves.

**Figure 47:** 170 rpm single-piston and double-piston experimental vs. analytical torque curves (minimum achievable test pressure)
A summary of all energy calculations for the low-speed test data is presented below. Relatively little power loss was recorded for each test, with the exception of the open-valve test which had an efficiency drop of 50%. Complete efficiency results are available in Appendix B: Complete Experimental Efficiency Results.

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When examining the pressure-flowrate curves and the table above, uncertainty sources include lack of experimental spring constant knowledge, potential user error in selecting the angle of piston engagement, and of course the obscuration resulting from the undesirable behavior of the inlet check valves. However, it is clear that as back pressure decreased, efficiency losses increased. Table 5 shows that the efficiency of the prototype was typically 85% or greater, except for the low end of the pressure range. These results support the claim that a fundamental similarity between the simulation and the actual prototype behavior was obscured by check valve error. The amount of fluid successfully
pumped per piston revolution was less than predicted for all values, but the efficiency at which that fluid was pumped was typically very high.
7. CONCLUSIONS

This project successfully completed both objectives set. A prediction of pressure-flowrate behavior for the combined PD pump and CVT system was made using pseudo-static approximations in an inverse dynamics model. These predictions were then compared against experimental data taken from a complete system prototype that was run across a wide range of speeds and back pressures. This experimental data was collected using sensors that had sufficiently low uncertainties that confidence could be placed in the results, within the limitations denoted in the previous chapter.

7.1 Project Results

Despite pressure-flowrate divergence of the experimental results from the simulation predictions at higher speeds and lower pressures caused by check valve lag, it was confirmed that for lower speeds (80-110 rpm) the simulated model was an accurate predictor of prototype behavior. This conclusion is supported both by the good agreement of the compression-pressure graphs discussed in the previous chapter, as well as the high efficiencies shown by the prototype for all but the lowest-pressure test settings. The fit of the experimental data to the simulation predictions decreased in quality as motor speed was increased, reaching a maximum divergence at about 170 rpm. The divergence seen was deduced to be an effect of the passive inlet valves used for this project, and not a fundamental limitation of the simulation. As a result, it is expected that differences between the simulation and the prototype result would be reduced by implementing active check valves instead of the current passive ones.
In comparing the results of this project with the work done by Cyders (2012), two major parallels were the good pseudo-static simulation agreement with low-speed experimental results, and the ability of each design to idle by supplying power to the input while decoupling the output, without a loss in efficiency.

7.2 Future Recommendations

The analysis of the experimental data was affected by lack of direct measurement of multiple variables, most notably the sprung link stiffness and the piston displacement throughout the cycle. These limitations represent the greatest limitations of the results of this work. Thus, the first recommendation for future work is the implementation of active check valves, or faster-responding passive valves such as reed valves. Reed valves could even provide higher-cycle lifespans as compared to the current ball valve design. In addition, recommendations include directly measuring piston position and experimentally testing spring stiffness. An independent verification of average flowrate could also be established by using two reservoirs, an input and an output, and measuring the rate of output reservoir filling. This method would not necessarily be useful in validating instantaneous flowrate, however. In addition, in order to extend the range of usable tests, higher-saturation point pressure transducers are recommended for future work.

Furthermore, while the similar pressure data of the two pistons during testing validated the assumption of identical piston behavior, this assumption could be finally confirmed or rejected by implementing an LVDT on the other sprung link during testing. Future work could also easily incorporate different spring constants, including non-linear constants, in the sprung link. Simulation predictions indicate that this would change the
shape of the pressure-flowrate curve. Finally, future testing could examine the number of pistons required to approach a steady condition for torque requirement and output flow. Using two pistons in this project provided some diminution of the sinusoidal nature of input torque and a piece-wise output flow, but there were still significant fluctuations in both over a complete cycle.

Potential applications of the system tested in this project are diverse. The application discussed in previous chapters was as a pumping device for water retrieval from shallow wells in SSA. A larger-scale device could also be used to pump at a greater flowrate and from deeper wells. Since the system was intended to couple the benefits of PD pumps and those of centrifugal pumps, other potential applications include a myriad of industrial uses in the developed world. Examples include moving industrial waste water, providing a precisely controlled amount of coolant fluid in a thermally sensitive manufacturing environment, providing high-pressure flows of oil or gas, or evacuating a sealed chamber to form a vacuum. Any such application would benefit from the addition of more pistons to smooth the torque requirement curve and output flow cycle. In the broadest scale, this device is relevant in any application requiring precise control of flowrate across a spectrum of back pressures.
REFERENCES


Figure 48: Low speed plot of volumetric flowrate vs. engagement pressure (original spring constant)
Figure 49: Low speed plot of LVDT compression vs. pressure (original spring constant)
Figure 50: Low speed plot of volumetric flowrate vs. engagement pressure (increased spring constant)
**Figure 51:** Low speed plot of LVDT compression vs. pressure (increased spring constant)
Figure 52: 130 rpm plot of volumetric flowrate vs. engagement pressure (increased spring constant)
Figure 53: 130 rpm plot of LVDT compression vs. pressure (increased spring constant)
Figure 54: 150-160 rpm plot of volumetric flowrate vs. engagement pressure (increased spring constant)
Figure 55: 150-160 rpm plot of LVDT compression vs. pressure (increased spring constant)
**Figure 56:** 170 rpm plot of volumetric flowrate vs. engagement pressure (increased spring constant)
Figure 57: 170 rpm plot of LVDT compression vs. pressure (increased spring constant)
Figure 58: 190 plot of volumetric flowrate vs. engagement pressure (increased spring constant)
Figure 59: 190 rpm plot of LVDT compression vs. pressure (increased spring constant)
Figure 60: 200-240 plot of volumetric flowrate vs. engagement pressure (increased spring constant)
Figure 61: 200-240 plot of LVDT compression vs. pressure (increased spring constant)
### APPENDIX B: COMPLETE EXPERIMENTAL EFFICIENCY RESULTS

**Table 6:** Low speed (original spring constant) efficiency and energy loss data (single-piston)

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**Table 7:** Low speed (increased spring constant) efficiency and energy loss data (single-piston)

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