Critical Vertical Deflection of Buried HDPE Pipes

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ABSTRACT

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Profile-wall high-density polyethylene (HDPE) pipes are used at increasing rates to convey surface drainage under roadways. This is because these products are cost effective, easy to handle and resistant to environmental elements. The HDPE pipes are derived from a thermoplastic material and much more flexible compared to the conventional pipes made of concrete and metals. Because of this inherent flexibility and other concerns that exist with thermoplastic materials, many highway agencies in the U.S. specify a limit on the HDPE pipe’s vertical deflection. For example, the Ohio Department of Transportation (ODOT) currently imposes a threshold vertical deflection of 7.5% on all HDPE drainage pipes installed in the ground. Many other state DOTs have similar vertical deflection limits for HDPE pipes. The origin of this vertical deflection limit is not well understood in the engineering community.

This study aims at locating the foundation of the threshold vertical deflection enforced by the highway agencies. In the study, different analytical methods and computer simulation models are applied to establish general guidelines concerning the critical vertical deflection for the profile-wall HDPE pipes. The analytical methods employed embrace theories of curved beams, elasticity, and viscoelasticity. In the finite element-based computer simulations, several different diameter sizes of the HDPE pipes are installed in varied installation modes and backfill materials to reflect the AASHTO Construction Specifications. The main outcome of the study makes a clear and reasonable statement about the critical vertical deflection for the commonly used thermoplastic drainage pipes.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>List of Tables</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>List of Figures</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>Chapter 1 : Introduction</td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>1.1</td>
<td>Background</td>
<td>19</td>
</tr>
<tr>
<td>1.2</td>
<td>Objective and Tasks</td>
<td>20</td>
</tr>
<tr>
<td>1.3</td>
<td>Outlines</td>
<td>21</td>
</tr>
<tr>
<td>Chapter 2 : Literature Review</td>
<td></td>
<td>23</td>
</tr>
<tr>
<td>2.1</td>
<td>Origin of Critical Vertical Deflection</td>
<td>23</td>
</tr>
<tr>
<td>2.2</td>
<td>Previous Field Tests</td>
<td>24</td>
</tr>
<tr>
<td>2.3</td>
<td>AASHTO Construction Specifications</td>
<td>36</td>
</tr>
<tr>
<td>2.4</td>
<td>Origin of Strain Limit</td>
<td>37</td>
</tr>
<tr>
<td>Chapter 3 : Methodology</td>
<td></td>
<td>39</td>
</tr>
<tr>
<td>3.1</td>
<td>Pipe Specifications</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>AASHTO Calculations</td>
<td>42</td>
</tr>
<tr>
<td>3.2.1</td>
<td>General Description</td>
<td>42</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Parameters in AASHTO Calculations</td>
<td>43</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Procedure of AASHTO Calculations</td>
<td>46</td>
</tr>
<tr>
<td>3.3</td>
<td>Theoretical Method</td>
<td>54</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Spangler Method</td>
<td>55</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Masada Method</td>
<td>57</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Burns and Richard Method</td>
<td>58</td>
</tr>
<tr>
<td>3.3.4</td>
<td>Hoeg Method</td>
<td>62</td>
</tr>
<tr>
<td>3.4</td>
<td>CANDE Simulations</td>
<td>63</td>
</tr>
<tr>
<td>3.4.1</td>
<td>General Descriptions</td>
<td>63</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Definitions in CANDE Simulations</td>
<td>64</td>
</tr>
</tbody>
</table>
Chapter 4: Results of AASHTO and Other Engineering Methods

4.1 AASHTO Calculation Results
4.1.1 Performances of Pipes under Dead Load Only
4.1.2 Performances of Pipes under Dead and Live Loads
4.1.3 Evaluation of AASHTO Calculation Results
4.1.4 Critical Parameters Determined by AASHTO Calculations

4.2 Theoretical Method Results
4.2.1 Performances of Pipes During Short-Term Service Condition
4.2.2 Performances of Pipes During Long-Term Service Condition
4.2.3 Evaluation of Theoretical Method Results
4.2.4 Critical Parameters Determined by Theoretical Methods

4.3 CANDE Simulation Results
4.3.1 Performances of Pipes Installed in the Embankment Mode
4.3.2 Performances of Pipes Installed in the Trench Mode
4.3.3 Evaluation of CANDE Simulation Results
4.3.4 The Critical Coefficients Determined by the CANDE Simulations

Chapter 5: Viscoelastic Analysis

5.1 Background
5.2 Methodology
5.2.1 The Chua and Lytton Method
5.2.2 Abaqus Simulations
5.2.3 Values of Viscoelastic Coefficients

5.3 Results of Viscoelastic Analysis
5.4 Evaluation of Viscoelastic Analysis Results
5.5 The Critical Coefficients of the Viscoelastic Analysis

Chapter 6: Summary and Conclusion

6.1 Summary
6.2 Conclusions
6.2.1 Task 1: Literature Review
6.2.2 Task 2: Mathematically Calculation
6.2.3 Task 3: Computer Simulations ................................................................. 167
6.2.4 Objective 1: Origin of the 7.5% Critical Vertical Deflection ................ 168
6.2.5 Objective 2: The Certain Critical Vertical Deflection of the HDPE Pipes ... 168
6.2.6 Objective 3: The Critical Vertical Deflection at the End of Construction .... 169

References ........................................................................................................... 170

Appendix A: The Plots of Results from Theoretical Methods ....................... 175
Appendix B: The Plots of Results from CANDE Simulations ........................... 203
LIST OF TABLES

Table 2.1: Conditions of HDPE Pipes in the U.S. ................................................................. 25
Table 2.2: Performance of HDPE Pipe Installed in Pennsylvania................................. 29
Table 2.3: Vertical Deflection of HDPE Pipe Installed in Pennsylvania .................. 29
Table 2.4: Diameters and Backfill Conditions of HDPE Pipes in Deep Burial Project... 30
Table 2.5: Vertical Deflections (%) of HDPE Pipes at Different Times ....................... 31
Table 2.6: Compressive Strains (%) of HDPE Pipes at Different Times ....................... 31
Table 2.7: Vertical Deflection Performance of Noble County Pipe Over Time............ 33
Table 3.1: Basic Dimensions of HDPE Pipes Analyzed ................................................... 40
Table 3.2: Detailed Dimensions of HDPE Pipe Analyzed ............................................. 41
Table 3.3: The Determination of Multiple Presence Factors Values............................. 47
Table 3.4: Shape Factor (Df) Based on Pipe Stiffness, Backfill, and Compaction Level. 53
Table 3.5: The Soil Types Correspondence between AASHTO and ASTM Classification ......................................................................................................................... 67
Table 4.1: The Comparisons of the Vertical Deflections from the Field Test and the AASHTO Calculations ........................................................................................................... 87
Table 4.2: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 1 ksi........................................ 88
Table 4.3: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 3 ksi............................................ 89
Table 4.4: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 5 ksi............................................ 89
Table 4.5: The Threshold Vertical Deflection and Threshold Cover Thickness of Each Diameter Pipe Based on Different Backfill Constrained Modulus.............................. 90
Table 4.6: The Comparison of the Vertical Deflection from the Field Test, The Masada Method and The No Slippage Case of the Hoeg Method ........................................... 109
Table 4.7: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 1 ksi............................................. 110
Table 4.8: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 3 ksi................................................................. 110
Table 4.9: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 5 ksi................................................................. 110
Table 4.10: The Vertical Deflection Observed from The Field Test and Estimated from The CANDE Simulations........................................................................................................ 125
Table 4.11: The Critical Vertical Deflection Reported from The Field Test and Estimated from The CANDE Simulations........................................................................................................ 126
Table 4.12: The Critical Cover Thickness Reported from The Field Test and Estimated from The CANDE Simulations ........................................................................................................ 126
Table 4.13: The Critical Vertical Deflection and Critical Cover Thickness for 12-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE).......................... 127
Table 4.14: The Critical Vertical Deflection and Critical Cover Thickness for 24-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE).......................... 128
Table 4.15: The Critical Vertical Deflection and Critical Cover Thickness for 30-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)...................... 128
Table 4.16: The Critical Vertical Deflection and Critical Cover Thickness for 36-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)...................... 129
Table 4.17: The Critical Vertical Deflection and Critical Cover Thickness for 42-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)...................... 129
Table 4.18: The Critical Vertical Deflection and Critical Cover Thickness for 48-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)...................... 130
Table 4.19: The Critical Vertical Deflection and Critical Cover Thickness for 60-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)...................... 130
Table 4.20: The Critical Vertical Deflection and Critical Cover Thickness for 12-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)................................. 131
Table 4.21: The Critical Vertical Deflection and Critical Cover Thickness for 24-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)................................. 131
Table 4.22: The Critical Vertical Deflection and Critical Cover Thickness for 30-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)................................. 132
Table 4.23: The Critical Vertical Deflection and Critical Cover Thickness for 36-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE).......................... 132
Table 4.24: The Critical Vertical Deflection and Critical Cover Thickness for 42-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE).......................... 133
Table 4.25: The Critical Vertical Deflection and Critical Cover Thickness for 48-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE).......................... 133
Table 4.26: The Critical Vertical Deflection and Critical Cover Thickness for 60-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE).......................... 134
Table 4.27: The Threshold Vertical Deflection of Each Diameter Pipe Based on Different Backfill Constrained Modulus ................................................................. 134
Table 4.28: The Threshold Cover Thickness of Each Diameter Pipe Based on Different Backfill Constrained Modulus ................................................................. 135
Table 5.1: Values of gi and ti in Abaqus Simulations .................................................. 147
Table 5.2: Comparisons of Vertical Deflections Determined by Viscoelastic Analysis and Field Test (5th Year after Pipe Installation) .......................................................... 159
Table 5.3: Maximum Cover Thickness and Threshold Vertical Deflections of HDPE Pipes Buried in 1-ksi Backfill (Viscoelastic Methods) .................................................. 159
Table 5.4: Maximum Cover Thickness and Threshold Vertical Deflections of HDPE Pipes Buried in 3-ksi Backfill (Viscoelastic Methods) .................................................. 160
Table 5.5: Maximum Cover Thickness and Threshold Vertical Deflections of HDPE Pipes Buried in 5-ksi Backfill (Viscoelastic Methods) .................................................. 160
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Vertical Deflection Performance of Test Pipe No. 8</td>
<td>32</td>
</tr>
<tr>
<td>3.1</td>
<td>Scanned Profile Wall Sections of HDPE Pipes</td>
<td>41</td>
</tr>
<tr>
<td>3.2</td>
<td>Typical and Idealized Shape of Pipe Profile Wall</td>
<td>42</td>
</tr>
<tr>
<td>3.3</td>
<td>Illustration of Embankment Mode (Left) and Trench Mode (Right)</td>
<td>64</td>
</tr>
<tr>
<td>3.4</td>
<td>Sketch of Half-Meshed Model</td>
<td>66</td>
</tr>
<tr>
<td>4.1</td>
<td>Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>70</td>
</tr>
<tr>
<td>4.2</td>
<td>Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>71</td>
</tr>
<tr>
<td>4.3</td>
<td>Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>71</td>
</tr>
<tr>
<td>4.4</td>
<td>Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>72</td>
</tr>
<tr>
<td>4.5</td>
<td>Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>72</td>
</tr>
<tr>
<td>4.6</td>
<td>Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>73</td>
</tr>
<tr>
<td>4.7</td>
<td>Short-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>74</td>
</tr>
<tr>
<td>4.8</td>
<td>Short-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>74</td>
</tr>
<tr>
<td>4.9</td>
<td>Short-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>75</td>
</tr>
<tr>
<td>4.10</td>
<td>Long-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>76</td>
</tr>
<tr>
<td>4.11</td>
<td>Long-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only</td>
<td>76</td>
</tr>
</tbody>
</table>
Figure 4.12: Long-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only............ 77
Figure 4.13: Long-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only77
Figure 4.14: Long-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only78
Figure 4.15: Long-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only78
Figure 4.16: Long-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only........................ 79
Figure 4.17: Long-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only........................ 79
Figure 4.18: Long-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only..................... 80
Figure 4.19: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft................................................................. 81
Figure 4.20: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft................................................................. 81
Figure 4.21: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft................................................................. 82
Figure 4.22: Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft................................................................. 82
Figure 4.23: Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft................................................................. 83
Figure 4.24: Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft. ................................................................. 83

Figure 4.25: Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft. .... 84

Figure 4.26: Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft. .... 84

Figure 4.27: Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft. .... 85

Figure 4.28: Short-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi .......... 92

Figure 4.29: Short-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi .......... 93

Figure 4.30: Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi ................................................................................................................. 93

Figure 4.31: Short-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi ................................................................................................................. 94

Figure 4.32: Short-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi .......... 94

Figure 4.33: Short-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi .......... 95

Figure 4.34: Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi ................................................................................................................. 95

Figure 4.35: Short-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi ................................................................................................................. 96
Figure 4.36: Short-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi .......... 96
Figure 4.37: Short-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi .......... 97
Figure 4.38: Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi ........................................................................................................................................ 97
Figure 4.39: Short-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi ........................................................................................................................................ 98
Figure 4.40: Long-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi ........ 100
Figure 4.41: Long-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi ........ 101
Figure 4.42: Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi ........................................................................................................................................ 101
Figure 4.43: Long-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi ........................................................................................................................................ 102
Figure 4.44: Long-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi ........ 102
Figure 4.45: Long-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi ........ 103
Figure 4.46: Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi ........................................................................................................................................ 103
Figure 4.47: Long-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi ........................................................................................................................................ 104
Figure 4.48: Long-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi ........ 104
Figure 4.49: Long-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi ........ 105
Figure 4.50: Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi .......................................................................................................................................................... 105
Figure 4.51: Long-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi .......................................................................................................................................................... 106
Figure 4.52: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode .................................................................................................................................................................................. 113
Figure 4.53: Short-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode .................................................................................................................................................................................. 114
Figure 4.54: Short-Term Buckling Ratios vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode .................................................................................................................................................................................. 115
Figure 4.55: Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode .................................................................................................................................................................................. 116
Figure 4.56: Long-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode .................................................................................................................................................................................. 116
Figure 4.57: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode .................................................................................................................................................................................. 117
Figure 4.58: Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode................................................................. 118
Figure 4.59: Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode................................................................. 118
Figure 4.60: Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode.................................................................................. 119
Figure 4.61: Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode.................................................................................. 119
Figure 4.62: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode .................................................................................. 120
Figure 4.63: Short-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode .................................................................................. 121
Figure 4.64: Short-Term Buckling Ratios vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode .................................................................................. 121
Figure 4.65: Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode .................................................................................. 122
Figure 4.66: Long-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode .................................................................................. 122
Figure 4.67: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode.................................................................................. 123
Figure 4.68: Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode........................................................................................................... 123
Figure 4.69: Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode........................................................................................................... 124
Figure 4.70: Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode........................................................................................................... 124
Figure 5.1: Maxwell and Kelvin Models .................................................................................. 138
Figure 5.2: Generalized Maxwell Model and Generalized Kelvin Model................................. 139
Figure 5.3: The Mesh of Each Instance .................................................................................... 144
Figure 5.4: The Sketch of Model Mesh .................................................................................... 145
Figure 5.5: Vertical Deflection of 30-in Diameter HDPE Pipes over 5-year Period beyond End of Construction (Constrain Modulus of Backfill 5 ksi) ......................................................... 148
Figure 5.6: Vertical Deflection of 30-in Diameter HDPE Pipes over 1-year Period beyond End of Construction (Constrain Modulus of Backfill 5 ksi) ......................................................... 148
Figure 5.7: Combined Compressive Strain in 30-in Diameter HDPE Pipes over 5-year Period beyond End of Construction (Constrained Modulus of Backfill 5 ksi) ...................... 149
Figure 5.8: Combined Compressive Strain of 30-in Diameter HDPE Pipes over 1-year Period beyond End of Construction (Constrained Modulus of Backfill 5 ksi) ...................... 149
Figure 5.9: Vertical Deflection vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi) ................................................................. 150
Figure 5.10: Vertical Deflection vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi) ................................................................. 151
Figure 5.11: Vertical Deflection vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 5 ksi) ................................................................. 151
Figure 5.12: Vertical Deflection vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi) ................................................................. 152
Figure 5.13: Vertical Deflection vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi) ........................................ 152
Figure 5.14: Vertical Deflection vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 5 ksi) ........................................ 153
Figure 5.15: Combined Compressive Strain vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi) .......................................... 155
Figure 5.16: Combined Compressive Strain vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi) .......................................... 155
Figure 5.17: Combined Compressive Strain vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 5 ksi) .......................................... 156
Figure 5.18: Combined Compressive Strain vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi) .......................................... 156
Figure 5.19: Combined Compressive Strain vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi) .......................................... 157
Figure 5.20: Combined Compressive Strain vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 5 ksi) .......................................... 157
CHAPTER 1 : INTRODUCTION

1.1 Background

Pipelines constitute an important part of infrastructure in the U.S. and other countries to transport drainage water as well as liquid and gas products. Most pipelines are buried underground, so it is critical to evaluate the structural conditions of the pipelines during and after construction. The pipelines can be broadly classified into flexible and rigid pipes. The flexible pipes are typically thin-wall metal or thermoplastic pipes, which show no structural distress when their vertical deflections are greater than 2%. On the other hand, the rigid pipes mainly consist of concrete and thick-wall metal pipes, which can sustain their original shapes with only small vertical deflections during their working life.

This dissertation is concerned with a class of thermoplastic pipes - high density Polyethylene (HDPE) pipe. It is a pipe product made from petroleum thermoplastic resin. HDPE has high strength-to-density ratio and is resistant to many chemicals and abrasive actions of drainage water. HDPE pipes are a member of the flexible pipe family, whose deflections may cause significant soil-pipe interaction, meaning that these pipes are flexible enough to transmit part of the external load to the soil envelope around it and the foundation soil underneath. HDPE pipes assume various shapes in the wall section, including straight wall, corrugated wall, and profile wall. For the straight wall, the interior and exterior sides of the pipe wall are both smooth. On the other hand, when the pipe wall is wavy throughout, it is said to be “corrugated.” Lastly, the profile wall is a combination of the straight and corrugated walls, with the interior side being smooth for the benefit of flow convenience, and the exterior side being wavy to provide extra stiffness. The installation modes of buried pipes include embankment installation and trench installation. When the pipe is placed on the natural ground and then enveloped with soil, it is installed in the embankment installation mode. Conversely, for the trench installation mode, the pipe is placed in a narrow trench excavated into the natural ground and then covered by a backfill material.

To assure the pipe’s initial structural integrity, most states permit a vertical deflection of 7.5% or smaller for thermoplastic pipes, although the origin and reasoning
underlying this threshold deflection value is unknown. There are many factors that can influence the vertical deflection limit, such as pipe diameter, geometric design of the pipe wall section, material properties of the resin used in the pipe, material properties of the backfill soil, pipe installation mode, and external loads. In general, the failure criteria for thermoplastic pipes are expressed in terms of the stress or strain level in the pipe material. Therefore, it is essential to find the origin of this threshold vertical deflection, and also to explicate the relationship between the vertical deflection and stress/strain magnitude in the pipe wall. The potential critical regions in the pipe wall are the crown, springline, and invert. Based on the rule of thumb, the highest stress and strain levels often appear at the pipe’s springline region.

1.2 Objective and Tasks

The primary objectives of this dissertation are to – 1) identify the possible origin of the traditional 7.5% vertical deflection limit, 2) establish the threshold vertical deflection values for HDPE pipes installed in a variety of installation conditions that conform to the AASHTO Construction Specifications, and 3) explore the critical vertical deflection upon completion of installation to keep the HDPE pipe performing well during its 50-year service period.

The four main tasks listed below are the detailed steps to accomplish the aforementioned objectives.

Task 1: Find the reasonable origin of the 7.5% vertical deflection limit through an extensive literature review, and also identify field studies related to the vertical deflections and wall stress/strain levels involving the profile-wall HDPE pipe installations. The field studies should include, but are not limited to, shallow and deep pipe burial studies by Ohio University, and studies of thermoplastic pipe installations carried out in Pennsylvania and Utah.

Task 2: Following AASHTO Construction Specifications, determine the vertical deflections and stress/strain levels in the profile-wall HDPE pipes utilizing the AASHTO LRFD design equations and the theoretical methods established by Spangler (1941), Masada (2000), and Burns and Richard (1964). Compare the results from these different
calculation methods to the performance limits listed in AASHTO to establish the theoretical critical vertical deflection of the HDPE pipe.

Since HDPE is a viscoelastic material, the stress/strain in the wall section and the vertical deflection of the buried HDPE pipe tend to change as a function of time. Through viscoelastic solution provided by Chua and Lytton (1989), identify the critical vertical deflections of the profile-wall HDPE pipes upon completion of installation that will ensure the pipe’s long-term (50-year) structural health.

Task 3: Study structural performance of the profile-wall HDPE pipes through available finite-element modeling programs. The computer simulations will examine the effects of various parameters, such as the pipe diameters, the pipe material’s elastic modulus, the backfill material type, the soil cover thickness, live loads, and the pipe installation mode. Evaluate the model results against the field data gathered during Task 1 (literature review) as an examination process.

Task 4: Combine the results from Tasks 1-3 and develop tables and charts for the profile-wall HDPE pipes. The tables and figures will illustrate the correlations among the pipe diameter, the pipe installation mode, the soil fill height, the pipe vertical deflection, wall stress/strain level, and loading condition.

1.3   **Outlines**

Chapter 1 provides the general information on the buried pipelines and the overview of this dissertation. The literature review that includes previous field tests, laboratory experiments and computer simulations on the profile-wall HDPE pipes is summarized in Chapter 2. Chapter 3 revolves around methodologies used to determine the structural performances of HDPE pipes under many different service conditions, and the methods applied in this study consists of elastic theory calculations and computer simulations. The outcomes from the theoretical calculations and computer simulations based on elastic theory are shown in Chapter 4. The content in Chapter 5 focuses on the analysis of theoretical calculations and computer simulations based on viscoelastic analysis. The results from various methods are compared to each other and calibrated to the data collected from the literature review if the installation conditions are similar. Chapter 6
provides an amalgam of the results from the proceeding chapters in a synthesized manner before arriving at several key conclusions of this dissertation.
CHAPTER 2 : LITERATURE REVIEW

2.1 Origin of Critical Vertical Deflection

An extensive literature review identified no experimental studies that tell us that the 7.5% vertical deflection is directly related to the performance limit of thermoplastic pipe. And, published field test results hardly discuss what damage could occur to the thermoplastic pipe at this level of vertical deflection. Despite all this, many specifications issued by engineering organizations and committees cite similar basis for the 7.5% vertical deflection limit. North American Pipe Corporation recommends that the maximum vertical deflection should be 7.5% within 30 days of installation. The basis for this is that it provides a safety factor of 4 before any reverse curvature develops. In Corrugated Polyethylene Pipe Design Manual & Installation Guide (2000), it is stated that 7.5% vertical deflection is a conservative value which is approximately a third of the vertical deflection that would induce development of reverse curvature. Janson and Molin (1981) proposed ideas about the vertical deflection limit in their book. According to them, the vertical deflection limit of thermoplastic pipes in the short term should be 5% with an acceptance of localized deflection reaching 7.5%. They also stated that diametric strains (or vertical deflections) considering creep should be less than 15% during the long term service condition. Petroff (1984) recognized that 20% vertical deflection was acceptable in certain circumstances, but he also advocated for 7.5% vertical deflection limit to assure overall stability of the pipe. Schluter and Shade (1999) suggested that a 7.5% vertical deflection limit can provide a generous safety margin against pipe collapse or curvature reversal. However, they also observed that the reverse curvature could be induced when diametric strains reach 22%, which is less than the 30% vertical deflection as commonly believed.

Since it was proposed that the metal pipe will experience structural failure when the vertical deflection reached 20%, NCHRP Report 429 (Hsuan & McGrath, 1999) described that the vertical deflection limit for metal pipe was 7.5% by considering an appropriate safety factor of slightly above 2.5. This philosophy could be suitable for thermoplastic pipe as well.
The hydraulic performance at pipe's joint section could be another potential basis for the 7.5% vertical deflection limit. According to a study in the AASHTO National Testing of Polyethylene Pipe (NTPEP) program, the liquid inside the pipe tends to leak from the joint section of most corrugated HDPE pipes when the vertical deflections approach 30%. So, the 7.5% limit on the vertical deflection can offer a safety factor of 4 to protect the pipes from water leakage at the joint sections.

2.2 Previous Field Tests

Hsuan and McGrath (1999) inspected many buried thermoplastic pipes in the U.S. and recorded their working conditions and structural conditions. For the purposes of this dissertation, only corrugated and profile wall HDPE pipes are selected from their report and listed in Table 2.1.
Table 2.1: Conditions of HDPE Pipes in the U.S.

<table>
<thead>
<tr>
<th>Install Year</th>
<th>Pipe Wall Type &amp; Dia.</th>
<th>Backfill Type &amp; Thickness</th>
<th>Pavement Type &amp; Thickness</th>
<th>Daily Traffic</th>
<th>Ver. Def.</th>
<th>Pipe Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>Cor., 24 in</td>
<td>gravel bank run ODOT 304, 1 ft</td>
<td>Asphalt, 15 in</td>
<td>light</td>
<td>14.6%</td>
<td>circumferential and longitudinal crack due to vehicle impact and mower damage at pipe ends</td>
</tr>
<tr>
<td>1982</td>
<td>Cor., 15 in</td>
<td>Gravel, 4.5 ~ 6 ft</td>
<td>Asphalt, 15 in</td>
<td>light</td>
<td>10%</td>
<td>Performing well with no signs of cracking or buckling</td>
</tr>
<tr>
<td>1982</td>
<td>Cor., 15 in</td>
<td>gravel and sand ODOT 304, 1 ft</td>
<td>Asphalt, 15 in</td>
<td>light</td>
<td>6.7%</td>
<td>circumferential crack at crest of corrugation</td>
</tr>
<tr>
<td>1983</td>
<td>Cor., 15 in</td>
<td>Mixed, 1 ft</td>
<td>asphalt</td>
<td>light</td>
<td>13.3%</td>
<td>Performing well with no signs of cracking or buckling</td>
</tr>
<tr>
<td>1983</td>
<td>Cor., 15 in</td>
<td>ash and rock, 3.5 ft</td>
<td>Asphalt, 15 in</td>
<td>light</td>
<td>13.3%</td>
<td>longitudinal crack at pipe inlet</td>
</tr>
<tr>
<td>1983</td>
<td>Cor., 15 in</td>
<td>gravel ODOT 8, 2 ~ 4 ft</td>
<td>Asphalt, 15 in</td>
<td>light</td>
<td>16.7%</td>
<td>circumferential crack at valley of corrugation</td>
</tr>
<tr>
<td>1983</td>
<td>Cor., 24 in</td>
<td>ODOT 304, 2 ft</td>
<td>asphalt</td>
<td>medium</td>
<td>max 8.3%</td>
<td>Performing well with no signs of cracking or buckling</td>
</tr>
<tr>
<td>1984</td>
<td>Cor., 12 in</td>
<td>ODOT 411 limestone, 5 ft</td>
<td>asphalt</td>
<td>light</td>
<td>8.3%</td>
<td>Performing well with no signs of cracking or buckling</td>
</tr>
<tr>
<td>&lt;1985</td>
<td>Cor., 18 in</td>
<td>granular &amp; native soil, 4 ft</td>
<td>asphalt</td>
<td>light</td>
<td>8.3%</td>
<td>circumferential and longitudinal cracks appear brittle</td>
</tr>
<tr>
<td>&lt;1985</td>
<td>Cor., 15 in</td>
<td>granular and native soil, 3 ft</td>
<td>asphalt</td>
<td>light</td>
<td>13.3%</td>
<td>circumferential crack appears slip at outlet</td>
</tr>
<tr>
<td>1985</td>
<td>Cor., 15 in</td>
<td>gravel and sand ODOT 304, 2 ~ 4 ft</td>
<td>Asphalt, 15 in</td>
<td>light</td>
<td>13.3%</td>
<td>circumferential and longitudinal crack near outlet</td>
</tr>
</tbody>
</table>
Table 2.1 (Cont’d)

<table>
<thead>
<tr>
<th>Install Year</th>
<th>Pipe Wall Type &amp; Dia.</th>
<th>Backfill Type &amp; Thickness</th>
<th>Pavement Type &amp; Thickness</th>
<th>Daily Traffic</th>
<th>Ver. Def.</th>
<th>Pipe Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>Profile, 18 in</td>
<td>Silt, 5 ft</td>
<td>asphalt &amp; unpaved</td>
<td>none</td>
<td>45% ~ 55%</td>
<td>many vertical and horizontal buckling along pipes</td>
</tr>
<tr>
<td>1991-1992</td>
<td>Pro., 36 in</td>
<td>ASTM D2487 #S3 stone, 2 ft</td>
<td>asphalt &amp; unpaved, 3 in</td>
<td>light</td>
<td>16.7%</td>
<td>12-o'clock buckling due to construction equipment damage; joint separated at bands</td>
</tr>
<tr>
<td>1993</td>
<td>Pro., 30 in</td>
<td>sand clay mixed, 1 ft</td>
<td>unpaved</td>
<td>heavy</td>
<td>0.7%</td>
<td>small separations at some joints</td>
</tr>
<tr>
<td>1994</td>
<td>Pro., 30 in</td>
<td>Mixed, 3 ~ 5 ft</td>
<td>Asphalt, 3 in</td>
<td>medium</td>
<td>3.3%</td>
<td>circumferential crack splits near flow line random throughout the pipe</td>
</tr>
<tr>
<td>1994</td>
<td>Pro., 42 in</td>
<td>Unknown, 3 ft</td>
<td>Asphalt, 6 in</td>
<td>medium</td>
<td>Ave. 9.5%</td>
<td>circumferential crack growth from 1 to 4.5 in in one year</td>
</tr>
<tr>
<td>1995</td>
<td>Pro., 30 in</td>
<td>Sand, 8 ft</td>
<td>unpaved</td>
<td>none</td>
<td>10%</td>
<td>circumferential crack; deflection at joints</td>
</tr>
<tr>
<td>1995</td>
<td>Pro., 24 in</td>
<td>Mixed, 1 ft</td>
<td>yard area</td>
<td>light</td>
<td>50%</td>
<td>12-in circumferential crack</td>
</tr>
</tbody>
</table>

[Note] "Cor." = corrugated wall HDPE pipe; "Pro." = profile-wall HDPE pipe; "Ver. Def." = vertical deflection of the pipe; and "Dia." = diameter of pipe.
The vertical deflections of the HDPE pipes installed in the U.S. are ranging from less than 1% to 55%. Although the information in the above table is not comprehensive, circumferential and longitudinal cracks typically appear in the pipes when their vertical deflections are greater than 7.5%. And, there are some pipes that are performing well even when their vertical deflections are above 7.5%. Without real-time monitoring, it is difficult to determine the exact point of the vertical deflection when the HDPE pipes start to develop circumferential and longitudinal cracks.

To examine the process of HDPE pipes' failure, Moser (2000) conducted three tests on profile-wall HDPE pipes. In each test, a 42-inch diameter HDPE pipe was installed in the embankment mode using a silty-sand (SM) backfill material. The preparations of all three tests were almost identical, with the only difference in the compaction level achieved on the backfill. No live load was applied in these tests, but the dead load was simulated by hydraulic cylinders located over the buried pipe specimen. The hydraulic pressure was increased slowly in a stepwise fashion during each test and was converted to equivalent soil cover thicknesses above the pipe. Moser (2000) concluded that the performance limit of the HDPE pipe should be determined by the cracking and hinging instead of dimpling.

In Test 1, the compaction level on the backfill was 75%. When the simulated dead load increased to a 40-ft cover thickness, the vertical deflection was 10% and a dimpling pattern started appearing at the 3 and 9 o'clock positions inside the pipe. The distance between adjacent dimples was almost identical to the corrugation rib spacing. When the simulated cover thickness was 55 ft, the vertical deflection increased to 14.5% and the hinging was noted at the springline of the pipe. Then, the simulated cover thickness increased to 60 ft, and vertical deflection reached 16.5%, at which point hinging was apparent and became more pronounced at the springline. The test ended at a 69-ft simulated cover thickness, and the ultimate vertical deflection was 20%.

In Test 2, the compaction level on the backfill rose to 85%. When the simulated cover thickness was 70 ft and 8.7% vertical deflection was observed, small dimples began to form near the 3 and 9 o'clock positions of the pipe. When the dead load was at an 82-ft simulated cover thickness, the vertical deflection reached 11% and waffling pattern formed on the pipe wall. The test ended at a 110-ft simulated cover thickness when the
vertical deflection was 16.5% and hinging appeared at the 4 and 9 o'clock positions of the pipe.

In Test 3, the compaction level on the backfill went to 95%. When the simulated cover thickness was 110 ft and the vertical deflection was only 3.5%, a dimpling pattern began to form at the springline of the pipe and spread toward the top of the pipe with the growth of the dead load. This pattern was like a wavy checkerboard in appearance, which might suggest the beginning of structural instability. When the simulated cover thickness was 140 ft, the vertical deflection increased to 5.2%, and dimples became pronounced but did not significantly affect the structural performance of the pipe. First signs of wall crushing were noted at the 10 o'clock position when the simulated cover thickness rose to 170 ft, and the vertical deflection was 6.7%. At a 225-ft simulated cover thickness, the vertical deflection was 9.4% and waffling and wall crushing became more pronounced. Test 3 ended at a cover thickness of 225 ft. These test results indicate that the critical vertical deflection decreases with the rise in the compaction level achieved on the backfill material.

Appointed by Pennsylvania Department of Transportation (Penn DOT), Hashash and Selig (1989) conducted a field test on a 24-inch diameter corrugated HDPE pipe. They built a 5-ft deep and 6-ft wide trench to install the pipe. Penn DOT 2A crushed rock material was selected as the backfill, and the backfill was compacted to 100% of the Standard Proctor Dry (SPD) unit weight. The ultimate cover thickness above the pipe was built up to 100 ft. During the soil embankment construction, sensors monitored the vertical deflections of the pipe and strains in the pipe wall. The collected data are listed in Table 2.2. When the soil cover thickness reached 100 ft, the maximum vertical deflection was 7.2% and the average vertical deflection was around 4% along the pipe length. No local distress and structural damage were noticed.
Table 2.2: Performance of HDPE Pipe Installed in Pennsylvania

<table>
<thead>
<tr>
<th>Height of Fill (ft)</th>
<th>Avg. Strain at Crest (%)</th>
<th>Avg. Strain at Centroid (%)</th>
<th>Avg. Circumf. Strain (%)</th>
<th>Avg. Ver. Def. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.000</td>
<td>0.000</td>
<td>0.21</td>
<td>0.00</td>
</tr>
<tr>
<td>15.0</td>
<td>0.228</td>
<td>0.115</td>
<td>0.32</td>
<td>N/A</td>
</tr>
<tr>
<td>17.0</td>
<td>0.287</td>
<td>0.118</td>
<td>0.32</td>
<td>0.67</td>
</tr>
<tr>
<td>20.0</td>
<td>0.153</td>
<td>0.020</td>
<td>0.32</td>
<td>N/A</td>
</tr>
<tr>
<td>36.0</td>
<td>0.218</td>
<td>0.097</td>
<td>0.38</td>
<td>0.97</td>
</tr>
<tr>
<td>55.0</td>
<td>0.271</td>
<td>0.264</td>
<td>0.47</td>
<td>1.58</td>
</tr>
<tr>
<td>63.0</td>
<td>0.442</td>
<td>0.292</td>
<td>1.06</td>
<td>2.27</td>
</tr>
<tr>
<td>76.0</td>
<td>0.775</td>
<td>0.718</td>
<td>1.15</td>
<td>3.22</td>
</tr>
<tr>
<td>95.0</td>
<td>1.455</td>
<td>1.149</td>
<td>1.26</td>
<td>4.02</td>
</tr>
<tr>
<td>99.0</td>
<td>1.662</td>
<td>1.333</td>
<td>1.30</td>
<td>4.04</td>
</tr>
</tbody>
</table>


Fifteen years after the initial installation of the HDPE pipe in Pennsylvania, Goddard (2002) resurveyed this pipe. He recorded the changes in the pipe’s vertical deflection with time, as shown in Table 2.3. Based on his observations, no signs of local distress and structural damage had been found inside the pipeline, other than cracking localized to the pipe's end, under the circumstance of high soil fill.

Table 2.3: Vertical Deflection of HDPE Pipe Installed in Pennsylvania

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Ver. Def. (%)</th>
<th>Time (days)</th>
<th>Ver. Def. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0.00</td>
<td>194</td>
<td>3.97 (end of construction)</td>
</tr>
<tr>
<td>55</td>
<td>0.67</td>
<td>391</td>
<td>3.89</td>
</tr>
<tr>
<td>68</td>
<td>0.97</td>
<td>721</td>
<td>4.55</td>
</tr>
<tr>
<td>98</td>
<td>1.58</td>
<td>1071</td>
<td>4.47</td>
</tr>
<tr>
<td>109</td>
<td>2.27</td>
<td>1437</td>
<td>4.43</td>
</tr>
<tr>
<td>138</td>
<td>3.22</td>
<td>5423</td>
<td>4.80</td>
</tr>
</tbody>
</table>

[Note] "Ver. Def." = vertical deflection of the pipe.

During the 20th service year of the HDPE pipe in Pennsylvania, Sargand, Masada, and Keatley (2007) of Ohio University examined the conditions of this pipe again. Some local cracks were found at joints located under the cover thickness of 70 ft or more. The pipe sections under less than 70 ft of cover thickness were functioning well without any signs of structural distress. Several positions of pipe wall were drilled to study the pipe
responses, and no creep and growth of slow crack occurred around each drilled location. Referring back to Goddard's observations, the authors reported that the horizontal deflection merely changed 0.3% over the past 17 years and the vertical deflection almost remained constant over the past 18 years. The ratio of the vertical deflection to the horizontal deflection increased steadily to the maximum value of 7.3 within the first 2 years after installation and then reduced to 4 at least in the last 5 years prior to this 20th year visit. Furthermore, circumferential shortening in the pipe wall increased by less than 0.2% over the last 18 years. The whole process of pipe deformations was completed during the initial 1000 days after installation.

To investigate the thermoplastic pipe's responses during and after installation, Sargand led a research team at Ohio University to conduct a comprehensive field test project for ODOT and FHWA in 1999. The project was titled "Field Verification of Structural Performance of Thermoplastic Pipe under Deep Backfill Conditions." The buried pipes in this project were under surveillance during construction and up to two years after construction. Among the thermoplastic pipes in this project, Test Pipe Nos. 7 through 9 and 13 through 15 were HDPE pipes from ADS N-12 series. The pipe diameters and relative backfill compaction levels are listed in Table 2.4. The vertical deflections of these pipes were small and stabilized within 3 to 6 months after the installation, and no structural distress exhibited at the beginning of the project.

<table>
<thead>
<tr>
<th>Pipe No.</th>
<th>Pipe Dia. (in)</th>
<th>Backfill Type</th>
<th>Comp. Lv.</th>
<th>Final Height of Fill (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>30</td>
<td>Sand</td>
<td>96%</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>Sand</td>
<td>96%</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>Crushed Limestone</td>
<td>86%</td>
<td>20</td>
</tr>
<tr>
<td>13</td>
<td>42</td>
<td>Sand</td>
<td>90%</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>42</td>
<td>Sand</td>
<td>96%</td>
<td>40</td>
</tr>
<tr>
<td>15</td>
<td>42</td>
<td>Crushed Limestone</td>
<td>90%</td>
<td>20</td>
</tr>
</tbody>
</table>

[Note] "Comp. Lv." = compaction level achieved on the backfill.

Five years after installation, Sargand and Masada (2007) continued monitoring the thermoplastic pipes' performances in their deep burial project. The subsequent vertical
deflections of the HDPE pipes are recorded in Table 2.5 and Figure 2.1. For the pipes installed in sand, the vertical deflection increased by 21% and the horizontal deflection increased by 2% over the 5-year period. For the pipes installed in crushed limestone, the vertical deflection increased by 25% and the horizontal deflection increased by 18% over the 5-year period. Most strain gages were dead soon after the construction of the pipe installations, so only a few strain readings are listed in Table 2.6.

Table 2.5: Vertical Deflections (%) of HDPE Pipes at Different Times

<table>
<thead>
<tr>
<th>Pipe No.</th>
<th>End of Constr.</th>
<th>1 Yr After Constr.</th>
<th>2 Yrs After Constr.</th>
<th>5 Yrs After Constr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.78</td>
<td>0.88</td>
<td>0.90</td>
<td>1.13</td>
</tr>
<tr>
<td>8</td>
<td>2.53</td>
<td>3.06</td>
<td>3.16</td>
<td>3.24</td>
</tr>
<tr>
<td>9</td>
<td>2.10</td>
<td>2.44</td>
<td>2.55</td>
<td>2.46</td>
</tr>
<tr>
<td>13</td>
<td>1.36</td>
<td>2.70</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>14</td>
<td>2.19</td>
<td>2.61</td>
<td>2.33</td>
<td>2.28</td>
</tr>
<tr>
<td>15</td>
<td>1.06</td>
<td>1.85</td>
<td>1.95</td>
<td>1.83</td>
</tr>
</tbody>
</table>

[Note] "Yr After Constr." = years after construction.

Table 2.6: Compressive Strains (%) of HDPE Pipes at Different Times

<table>
<thead>
<tr>
<th>Pipe No.</th>
<th>End of Constr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.06</td>
</tr>
<tr>
<td>8</td>
<td>0.40</td>
</tr>
<tr>
<td>9</td>
<td>N/A</td>
</tr>
<tr>
<td>13</td>
<td>0.25</td>
</tr>
<tr>
<td>14</td>
<td>0.35</td>
</tr>
<tr>
<td>15</td>
<td>N/A</td>
</tr>
</tbody>
</table>

A 24-inch diameter corrugated HDPE pipe was installed under SR 145 in Noble County in 1981, and this pipe was believed to be the oldest plastic pipe under roadways in Ohio. Masada and Sargand (2013) investigated this culvert which served for years under 1 to 1.3 ft of soil cover and low pH (4-5) drainage flow conditions. This pipe was still functioning well and exhibited no signs of structural distress in 2004. Table 2.6 summarizes the vertical deflection measurements taken inside this drainage structure at three times in its history. Only at the joint sections, the vertical deflection was greater than 7.5%. Several samples were collected from the pipe and tested in the laboratory to evaluate the density and tensile strength of the pipe material, and the test results indicated that no significant aging existed in pipe material even for more than 22 years.
Table 2.7: Vertical Deflection Performance of Noble County Pipe Over Time

<table>
<thead>
<tr>
<th>Location Along Pipeline</th>
<th>1982</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outlet End</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Middle of Section 1</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>End of Section 1</td>
<td>5.2</td>
<td>5.2</td>
</tr>
<tr>
<td>Joint</td>
<td>7.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Middle of Section 2</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Inlet End</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Average</td>
<td>2.3</td>
<td>2.3</td>
</tr>
</tbody>
</table>

[Note] "Ver. Def." = vertical deflection of the pipe.

To explore the allowable cover thickness under various pipe properties and backfill conditions, Katona (1988) simulated structural performances of buried HDPE pipes using the computer software CANDE. Since the software merely outputs the values of various pipe performance parameters, Katona had to set up several criteria to pinpoint whether the pipe was failing or not. The limit of thrust stress in the pipe wall was determined to be 1500 psi, and the limit of flexural strain was set to 5%, and the limit of vertical deflection was locked at 7.5%. The buckling soil pressure was chosen as half of the critical soil pressure estimated by the Chelapati-Allgood buckling theory. There were a total of 108 cases simulated by CANDE. According to the results, the pipe failure was controlled by the wall thrust stress limit in most cases. Only four simulations suggested that the vertical deflection limit caused pipe failure prior to any other limits did. In these four cases, the diameters of corrugated HDPE pipes were 4, 6, and 8 inches, and the pipes were buried in a fair quality backfill in short term service condition. Compared to the critical cover thickness calculated by the thrust stress limit, the pipes can afford on the average 25% more cover thickness if it were determined by the vertical deflection limit. The pipes can bear on the average 30% more cover thickness if it were controlled by the flexural strain limit. And, the pipes can hold on the average 55% more cover thickness if it were specified by the buckling limit alone.

Moore and Hu (1994) simulated the viscoelastic mechanical responses of a deeply buried HDPE pipe through 2-D finite element analysis. The name of the FE computer code was not mentioned in the report. In their simulation, a 24-in diameter corrugated HDPE
The pipe was buried under 60 feet of soil fill. The backfill was composed of a well-graded granular material either at 85% or at 95% relative compaction, and the in-situ soil surrounding the backfill was a low compressibility clay with 90% relative compaction. The viscoelastic behavior of the pipe was defined by the small deflection rheology, which was created by Moore in 1994. This rheological method was established through back-calculating the stiffness of the springs and the viscosities of the dashpots in a multi-Kelvin model, and this back-calculation was accomplished by curve fitting exercise based on a stress relaxation test performed on a HDPE pipe with small deflection. In this report, the rheological method was only validated by the results from several parallel plate tests. Thus, the FE simulation of the buried pipe's performances was merely the exploration of the pipe's long-term vertical deflection and its stress relaxation in 50 years after the end of the construction. The backfill was assumed to be an isotropic elastic material, but the in-situ clay was considered either as elastic or as viscoelastic material. The relaxation modulus of the viscoelastic clay was determined by a power-law time function. The authors also assumed two different construction procedures. One procedure was that the soil fill increased steadily to 60 feet over a 30-day period, and the other procedure had the soil fill being placed in only one day. According to the simulation results, the vertical deflections increased from 0.15% to 0.25% over 50 years, regardless of the backfill types and construction procedures. The stresses at the springline of the pipe were much higher than those at the crown of the pipe. The time spent on placing the soil fill can only affect the initial stresses in the pipe wall, but it had no effect on the tendencies of the viscoelastic performances of the pipe in the long term services condition. Furthermore, the viscoelastic behavior of the clay nearly had no influences on the pipe's viscoelastic vertical deflections and stresses.

Elzink and Molin (1992) surveyed buried plastic pipes in Europe, and they discovered that ultimate vertical deflections generally matured within two years after installation, and the final vertical deflection was about 1.5 times the initial vertical deflection, and the peak vertical deflection values were of a magnitude 1.5 to 2 times the average vertical deflection values. Hence, they recommended, in Scandinavia, that the
long-term maximum vertical deflection must remain less than 15%, and peak short-term vertical deflection shall not exceed 8% for PE pipes.

Hurd (1985) examined the field performances of corrugated PE pipes in Ohio. He found flattening appearing at vertical deflections exceeding approximately 15% and buckling formed at vertical deflections exceeding 25%. When the vertical deflection was smaller than 10%, the pipe had no apparent flattening, buckling and dent, and vertical deflection became steady after 2 to 4 years from the installation. Furthermore, the vertical deflections at the pipe's joints were slightly larger than the vertical deflections throughout the rest of the culvert, and shallow cover and highway loadings seemed to have no significant association with the structural performance and vertical deflection of the pipe.

Spangler (1941) derived a set of equations calculating the vertical deflections of the buried pipe compressed by the distributed vertical load at its top and bottom, and he also derived the solutions for the thrust force and bending moment in the pipe wall. However, Spangler assumed that the horizontal deflection and vertical deflection are identical to each other. Masada (2000) noticed that the vertical deflection is always higher than the horizontal deflection when pipes are under external compression load, so he created the relationship between the horizontal and vertical deflection of the buried pipe at the basis of the Spangler method. The equations of the thrust stress and bending moment from the Masada method are the same to those from the Spangler method, and these equations are applied to examine the limit of vertical deflection of the buried HDPE pipes.

Burns and Richard (1964) analyzed the distribution of stress in an elastic cylindrical shell covered by an elastic soil medium which is evenly compressed at the top and side boundaries. This comprehensive solution provides the vertical displacement, thrust stress and bending moment in the pipe wall. Moreover, there are two pipe-to-soil interface cases representing the friction conditions between the shell wall and the media, and the cases are no slippage and full slippage. Hoeg (1968) conducted another elastic analysis on the buried pipes, and the equations determining the performances of the pipes were similar to the equations from the Burns and Richard method, although some parameters and equations are different occasionally.
Chua and Lytton (1989) improved the elastic solutions from the Hoeg method by considering the viscoelastic characteristic of thermoplastic material. The solutions to the vertical deflection, the thrust stress and the moment from the Chua and Lytton method are completely homogenous to the equations from the Hoeg method. The viscoelastic solutions of the buried pipe were obtained by adding time factors into the parameters which are compressibility ratio and flexibility ratio.

2.3 AASHTO Construction Specifications

The construction specifications for thermoplastic pipes are detailed in AASHTO LRFD Bridge Construction Specifications, 3rd edition, 2010. The type, compacted density, and strength properties of the backfill soil should be according to AASHTO M-145. For thermoplastic pipes, the bedding and backfill materials should be built using A-1, A-2-4, A-2-5, or A-3 soil types. For any backfill material, a maximum of 50% of particle sizes may pass the No. 100 sieve and a maximum of 20% may pass the No. 200 sieve. The fine sands and silty and clayey materials are not recommended to be used as a backfill soil, because they are difficult to work with, are sensitive to moisture content changes, and do not provide support comparable to coarser or more broadly graded materials at the same percentage of maximum density. In some special cases, these materials could be used if moisture content and compaction procedures can be controlled.

For the trench installation mode, the minimum trench width should be the outside diameter of the pipe plus 16 inches or the outside diameter of the pipe multiplied by 1.5 plus 12 inches, whichever comes greater.

If the pipe is buried under unpaved areas, the minimum cover thickness over the top of the pipe should be the pipe’s inside diameter divided by 8 or 12 inches, whichever comes greater. If the pipe is installed under paved roads, the minimum cover thickness should be a half of the pipe’s inside diameter or 24 inches, whichever comes greater. The minimum cover thickness is measured from the top of the pipe to the top of the rigid pavement or the bottom of the flexible pavement.
2.4 Origin of Strain Limit

In NCHRP Report 438, McGrath and Sagan (2000) concluded that the relationship between the stress and the strain shows significant nonlinearity when the compressive strain in the polythene is greater than 4.1%. Zhang and Moore (1997) also reported the same observation about this behavior through the uniaxial compression tests on the cylindrical HDPE specimens. The uniaxial compression tests were terminated when the compressive strains of the specimens reached 15%, and no failures or stress declines were noted during the testing process. Besides, no failure was reported even when the strain in the specimen reached 21% in the creep test. Plastic Pipe Institute (2015) indicated that the primary consideration for the HDPE pipe is the stability of the thin element rather than the tear or cracking of the corrugated wall section when the HDPE material is in compression. In NCHRP Report 631, several stub compression tests were conducted on the specimens cut from profile wall HDPE pipes. According to the stress-strain curves obtained from these tests, the maximum stress appeared when the strains in the specimens varied from 4% to 6%, and most specimens buckled when the strains in the specimens raised to 7% or even to 10%. It can be extrapolated that the ultimate compressive strain of the HDPE material is able to achieve a higher level if the structural instability (buckling) can be refrained. Therefore, McGrath and Sagan set 4.1% as the thrust strain limit for the polythene material. Distinct from the loading of uniaxial compression and shape of HDPE specimen in the experiment, the buried HDPE pipe bears the combined compressive strain (sum of the strain due to the thrust and the strain due to the flexure) as a whole structure. Besides, the profile of pipe wall is meticulously designed as a trapezoid-like shape. Hence, the explanations in AASHTO Specifications said that a higher strain limit of combined compressive strain is allowed because the stress at the center of web element (sloping side) is low when the element is under flexure, and this stress condition can probably reduce the buckling and strengthen the stability at the crest and valley elements. NCHRP Report 631 also expressed that combined compressive strain limit is permitted to be 1.5 times higher than the thrust strain limit due to the stability of profile-wall pipe in the parallel plate test.
Under the tensile circumstance, HDPE shows significant nonlinear behavior when the tensile strain exceeds 4% to 6% in the material. Peggs et al. (2005) noticed that the ultimate tensile strain of the HDPE material can typically reach 30%. In order to protect the pipe from cracking in the long-term service condition, Janson (1985) recommended that the tensile strain limit should be 5%, which considered a certain factor of safety.
CHAPTER 3 : METHODOLOGY

This chapter describes the processes that were taken to determine the structural performances of buried profile-wall HDPE pipes using some notable elastic methods. The pipe performances include the vertical deflection, combined compressive strains in the pipe wall, and buckling ratios. In the AASHTO LRFD design specifications, the strain in the pipe wall is considered a main parameter controlling the pipe's failure, thus the pipe wall stresses derived from the elastic methods are transformed to pipe wall strains. To compare the results from all the elastic methods effectively, several assumptions were made concerning the material properties and installation conditions involved in the comprehensive analysis. These assumptions are as follows:

a. In light of Table 12.12.3.3-1 (AASHTO LRFD Bridge Design Specifications, 2013), the elastic modulus of HDPE material is set at 110 ksi and 22 ksi for the short-term and long-term service conditions, respectively.

b. In the AASHTO LRFD and other theoretical calculations, the constrained modulus of the pipe backfill material is assumed to take a value of 1 ksi, 3 ksi or 5 ksi, depending on the material type, the degree of compaction achieved, and the level of confining that exists. In the finite element computer simulations, the software (CANDE 2003) has predetermined the properties of many backfill types. Moreover, the overfill material placed above the pipe is treated as a homogeneous material in all the analysis cases. The unit weight of this overfill is 130 pcf, and the Poisson's ratio is set at 0.3 for both the backfill and overfill.

c. The short-term service condition refers to a relatively short time span past the completion of the installation of the pipe, and the long-term service condition implies up to 50 years after the end of the installation of the pipe.

d. Based on the AASHTO LRFD specifications, for the HDPE material the combined compressive strain limit is 6.15%, the tensile strain limit is 5.0%, and the buckling ratio limit is 1.0. If any pipe's performance surpasses its relative limit, the pipe is treated as a failed structure during the service period.

e. In this study, the shortening vertical deflections, compressive strains, and buckling ratios are all expressed in positive values.
f. The specifications of the profile-wall HDPE pipes analyzed are provided in Section 3.1.

### 3.1 Pipe Specifications

In this dissertation, the pipes adopted in the analysis are profile-wall HDPE pipes manufactured by Advanced Drainage Systems (ADS). The diameters of the pipe are 12 in, 24 in, 30 in, 36 in, 42 in, 48 in and 60 in. The basic geometric dimensions of each diameter pipe are extracted from ADS Product Note 3.107 (2005) and listed in Table 3.1. To procure more detailed dimensional features of the pipe's wall profile, a thin section was cut from the pipe's wall section along the pipe's longitudinal axis and sent to ODOT material laboratory for 3-D laser scan. Figure 3.1 shows the scanned images of all the pipe diameter sizes. The irregular shape of the pipe’s profile wall is transformed to the idealized trapezoid shape (see Figure 3.2) as per the AASHTO LRFD Specifications. Table 3.2 lists the detailed dimensions taken from the scanned images.

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>ID (in)</th>
<th>OD (in)</th>
<th>A (in²/in)</th>
<th>I (in⁴/in)</th>
<th>c (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>12.15</td>
<td>14.45</td>
<td>0.217</td>
<td>0.035</td>
<td>0.43</td>
</tr>
<tr>
<td>24</td>
<td>24.08</td>
<td>27.80</td>
<td>0.324</td>
<td>0.137</td>
<td>0.74</td>
</tr>
<tr>
<td>30</td>
<td>30.00</td>
<td>35.10</td>
<td>0.378</td>
<td>0.277</td>
<td>0.86</td>
</tr>
<tr>
<td>36</td>
<td>36.00</td>
<td>41.70</td>
<td>0.401</td>
<td>0.400</td>
<td>1.00</td>
</tr>
<tr>
<td>42</td>
<td>41.40</td>
<td>47.70</td>
<td>0.458</td>
<td>0.572</td>
<td>1.21</td>
</tr>
<tr>
<td>48</td>
<td>47.60</td>
<td>53.60</td>
<td>0.495</td>
<td>0.570</td>
<td>1.17</td>
</tr>
<tr>
<td>60</td>
<td>59.50</td>
<td>66.30</td>
<td>0.578</td>
<td>0.860</td>
<td>1.32</td>
</tr>
</tbody>
</table>

[Note] Dia = nominal diameter; ID = inside diameter; OD = outside diameter; A = wall area per unit length; I = wall moment of inertia per unit length; and c = distance from inside edge of pipe to the neutral axis.
Table 3.2: Detailed Dimensions of HDPE Pipe Analyzed

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>w-crest (in)</th>
<th>w-liner (in)</th>
<th>w-valley (in)</th>
<th>w-web (in)</th>
<th>t-crest (in)</th>
<th>t-liner (in)</th>
<th>t-valley (in)</th>
<th>t-web (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.787</td>
<td>1.181</td>
<td>0.591</td>
<td>0.984</td>
<td>0.079</td>
<td>0.039</td>
<td>0.098</td>
<td>0.098</td>
</tr>
<tr>
<td>24</td>
<td>1.378</td>
<td>2.165</td>
<td>0.787</td>
<td>1.575</td>
<td>0.098</td>
<td>0.079</td>
<td>0.118</td>
<td>0.118</td>
</tr>
<tr>
<td>30</td>
<td>1.575</td>
<td>2.559</td>
<td>1.181</td>
<td>1.969</td>
<td>0.118</td>
<td>0.079</td>
<td>0.197</td>
<td>0.177</td>
</tr>
<tr>
<td>36</td>
<td>2.165</td>
<td>2.953</td>
<td>0.984</td>
<td>1.772</td>
<td>0.197</td>
<td>0.157</td>
<td>0.236</td>
<td>0.197</td>
</tr>
<tr>
<td>42</td>
<td>2.362</td>
<td>3.543</td>
<td>0.984</td>
<td>2.362</td>
<td>0.157</td>
<td>0.118</td>
<td>0.197</td>
<td>0.197</td>
</tr>
<tr>
<td>48</td>
<td>2.362</td>
<td>3.543</td>
<td>1.181</td>
<td>2.362</td>
<td>0.197</td>
<td>0.118</td>
<td>0.236</td>
<td>0.217</td>
</tr>
<tr>
<td>60</td>
<td>2.756</td>
<td>4.134</td>
<td>1.378</td>
<td>2.756</td>
<td>0.236</td>
<td>0.157</td>
<td>0.256</td>
<td>0.256</td>
</tr>
</tbody>
</table>

[Note] “w” = width of each segment in the profile wall; and “t” = thickness of each segment in the profile wall.

Figure 3.1: Scanned Profile Wall Sections of HDPE Pipes
3.2 AASHTO Calculations

3.2.1 General Description

The version of the AASHTO LRFD Bridge Design Specifications selected in this study is the 2013 Interim Revisions to Six Edition 2012. Section 12 (Buried Structures & Tunnel Liners) of the AASHTO specifications provides performance limits as well as equations for determining the performances of thermoplastic pipes. The specifications regarding the magnitudes of external loads on the buried pipe and factors of safety for the load combinations are listed in Section 3 (Loads and Load Factors).

In the AASHTO LRFD calculations, the dead load is only generated by the overfill placed above the top of the pipe. The groundwater table is assumed to be located far below the bottom of the pipe. The AASHTO specifications address the live load applied by an H-20 truck. The rear axle of the truck is heavier than the front axle, and the distance between the two wheels at the rear axle and the distance between the front and rear axles are both much wider than the outside diameter of the 60-in diameter HDPE pipe. So, the worst loading condition can occur when the rear wheel on either side of the rear axle is positioned directly above the crown of the pipe. The weight exerted by one rear-wheel of the H-20 truck is 16000 lbs. The installation of the pipes doesn't allow the wheel to touch the pipes directly, as a certain height of soil cover is placed on top of the pipes. To study the relationship between the pipe performance and the soil cover thickness on top of the pipe, several cover thickness levels are examined in this dissertation: 1 ft, 2.5 ft, 5.0 ft, 7.5
The live load is only considered in the short-term service condition because it is impractical to park a truck above a pipe for 50 years.

3.2.2 Parameters in AASHTO Calculations

The following lists the resistance factors addressed in the AASHTO LRFD Specifications:

\( \phi_s = \) resistance factor for soil stiffness. According to Table 12.5.5-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \phi_s \) is 0.90.

\( \phi_f = \) resistance factor for flexure. According to Table 12.5.5-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \phi_f \) is 1.00.

\( \phi_T = \) resistance factor for thrust effects. According to Table 12.5.5-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \phi_T \) is 1.00.

\( \phi_{bck} = \) resistance factor for global buckling. According to Table 12.5.5-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \phi_{bck} \) is 0.70.

\( \eta_{EV} = \) load modifier applied to vertical earth loads on culverts. Assume the ductility and redundancy of pipe design is conventional, and the pipe is under a typical building. Then, the value of \( \eta_{EV} \) is 1.0 according to Article 1.3 in AASHTO LRFD Bridge Design Specifications (2013).

\( \gamma_{EV} = \) load factor for vertical pressure from dead load of earth fill. According to Table 3.4.1-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \gamma_{EV} \) is 1.00 under Service I.

\( K_{YE} = \) installation factor. Typical value is 1.5 to provide a traditional factor of safety. If the value is lower than 1.5, additional monitoring of the installation during construction and provisions for such monitoring shall be provided. The factor is ignored in the calculation, so this value is set as 1.0.

\( \gamma_{WA} = \) load factor for hydrostatic pressure. According to Table 3.4.1-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \gamma_{WA} \) is 1.00 under Service I.

\( \eta_{LL} = \) load modifier applied to live loads on culverts. Assume the ductility and redundancy of pipe design is conventional, and the pipe is under typical building. Then,
the value of $\eta_{LL}$ is 1.0 according to Article 1.3 in AASHTO LRFD Bridge Design Specifications (2013).

$\gamma_{LL}$ = load factor for live load. According to Table 3.4.1-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of $\gamma_{LL}$ is 1.00 under Service I.

$K_B$ = bedding coefficient, which varies from 0.083 for full support to 0.110 for no support (a point load reaction) at the invert. Haunching is always specified to provide good support. Typical value is 0.10 to account for inconsistent haunch support. In the calculation, typical value suggested by AASHTO is adopted.

$D_L$ = deflection lag factor. Recommended value is from 1.0 to 6.0, and the highest values are for installations with quality backfill and low initial deflections and do not generally control designs. Typical value is 1.5, which provides some allowances for an increase in deflection over time for installations with initial deflection levels of several percent. In the calculation, the effect of deflection lag is ignored, so the value of $D_L$ is assumed to be 1.0.

$LLDF$ = factor for distribution of live load through earth fills. Under the fill depth of 2.0 ft or greater, wheel loads may be considered to be uniformly distributed over a rectangular area with sides equal to the dimension of the tire contact area, and increased by either 1.15 times the depth of the fill in select granular backfill, or the depth of the fill in all other cases. In the calculation, the load distribution is assumed to be uniform over a rectangular area, so the value of $LLDF$ is 1.0.

$K_1$ = coefficient to consider for live load location (in). For live load at the crown of the pipe, the value is 0; for live load at the springline, the value is $0.5D_0$. In the calculation, the wheel load is located above the pipe crown, so the value of $K_1$ is 0.

$K_2$ = coefficient to account for variations of thrust around the circumference. For thrust at the springline, the value is 1.0; for thrust at the crown, the value is 0.6. In the calculation, the wheel load is located above the pipe crown, so the value of $K_1$ is 0.6.

$k$ = plate buckling coefficient. For supported elements, the value is 4; for unsupported elements, such as free standing ribs, the value is 0.43. In the calculation, the value of $k$ is 4 since the elements in profile-wall HDPE pipe are supported.
\[ E_p = \text{short- or long-term modulus of pipe material (ksi)}. \] According to Table 12.12.3.3-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( E_p \) is 110 ksi for initial condition (short-term) and 22 ksi for 50-year condition (long-term).

\[ L_0 = \text{length of live load contact area parallel to pipe diameter}. \] According to Article 3.6.1.2.5 in AASHTO LRFD Bridge Design Specifications (2013), the tire contact area of a wheel consisting of one or two tires shall be assumed to be a single rectangle, whose width is 20.0 in. and whose length is 10.0 in. So, the value of \( L_0 \) is 10 in.

\[ W_0 = \text{width of live load ground surface contact area parallel to the flow direction in pipe}. \] According to Article 3.6.1.2.5 in AASHTO LRFD Bridge Design Specifications (2013), the tire contact area of a wheel consisting of one or two tires shall be assumed to be a single rectangle, whose width is 20.0 in. and whose length is 10.0 in. So, the value of \( W_0 \) is 20 in.

\[ P_L = \text{service live load on culvert (psi)}. \] The value of \( P_L \) is set to be 0 at conditions only considering dead load.

\[ P = \text{design wheel load (lbs)}. \] According to Article 3.6.1.2.2 in AASHTO LRFD Bridge Design Specifications (2013), the heaviest axle load of design truck is 32 kips, so the value of \( P \) is set as 16 kips for one-wheel load.

\[ \omega = \text{spacing of corrugation (in)}. \] According to Figure 12.12.3.10.1b-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \omega \) is the period length of the pipe’s profile wall design.

\[ \varepsilon_{yt} = \text{service long-term tension strain limit of pipe wall material}. \] According to Table 12.12.3.3-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \varepsilon_{yt} \) is 5.0%.

\[ \varepsilon_{yc} = \text{service compression strain limit of pipe wall material}. \] According to Table 12.12.3.3-1 in AASHTO LRFD Bridge Design Specifications (2013), the value of \( \varepsilon_{yc} \) is 4.1%.

\[ P_w = \text{hydrostatic water pressure (psi)}. \] The value of \( P_w \) is set to be 0 under all conditions.
3.2.3 Procedure of AASHTO Calculations

According to the equations in the AASHTO specifications, the calculations for the HDPE pipe's performances are divided into ten steps. Step 1 is to calculate the effective area of pipe wall. Steps from 2 to 5 use equations to compute factors and serviced load. Step 6 is to determine the vertical deflection of the pipe. Step 7 through 10 calculate strains in the pipe wall.

Step 1: Profile wall pipe is idealized as straight elements. Each element shall be assigned a width (based on the clear distance between the adjoining elements) and a thickness (based on the thickness at the center of the element). The effective area is determined as:

\[ A_{\text{eff}} = A_g - \frac{\sum (w - b_e)t}{\omega} \]  \[1\]

in which:

\[ b_e = \rho w \]  \[2\]

\[ \rho = \frac{1}{\lambda} \left( 1 - \frac{0.22}{\lambda} \right) \]  \[3\]

\[ \lambda = \frac{w}{t} \cdot \frac{\varepsilon_{yc}}{\sqrt{k}} \geq 0.673 \]  \[4\]

where \( A_{\text{eff}} \) = effective area of pipe wall per unit length of pipe (in²/in)
\( b_e \) = element effective width (in)
\( \rho \) = effective width factor
\( \lambda \) = slenderness factor
\( \omega \) = spacing of corrugation (in)
\( \varepsilon_{yc} \) = factored compressive strain limit
\( A_g \) = gross area of pipe wall
\( t \) = thickness of element (in)
\( w \) = total clear width of element between supporting elements (in)
\( k \) = plate buckling coefficient; \( k = 4 \) for supported elements, \( k = 0.43 \) for unsupported elements, such as free standing ribs

\[ A_{\text{eff}} = \frac{P_{st}K_t}{F_u} \leq A_g \]  \[5\]
where \( P_{st} \) = stub compression capacity from AASHTO T 341 (kip/in)
\( K_1 \) = time factor
\( F_0 \) = material yield strength for design load duration (ksi)

Step 2: The live load shall be determined as a pressure applied to the pipe crown. The live load magnitude shall be based on the design vehicular live load, modifiers for multiple presence/overload, dynamic load allowance, and distribution through cover soils as:

\[
P_L = \frac{P \cdot (1 + IM / 100) \cdot m}{[L_0 + (12 \cdot H + K_1) \cdot LLDF] \cdot [W_0 + (12 \cdot H + K_1) \cdot LLDF]} \tag{6}
\]

\[
IM = 33 \cdot (1 - 0.125 \cdot H) \geq 0\% \tag{7}
\]

where \( P_L \) = service live load on culvert (psi)
\( P \) = design wheel load (lbs)
\( IM \) = dynamic load allowance
\( m \) = multiple presence factor (the values are shown in Table 3.3)
\( L_0 \) = length of live load contact area parallel to pipe diameter
\( H \) = depth of ill over top of pipe (ft)
\( K_1 \) = coefficient to consider for design live load location (in), \( K_1 = 0 \) for live load at the crown of the pipe, \( K_1 = 0.5D_0 \) for live load at the springline.
\( LLDF \) = factor for distribution of live load through earth fills
\( W_0 \) = width of live load ground surface contact area parallel to flow in pipe

<table>
<thead>
<tr>
<th>Number of Loaded Lanes</th>
<th>Multiple Presence Factors, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
</tr>
<tr>
<td>&gt;3</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Step 3: Hydrostatic pressure:

\[ P_w = \frac{\gamma_w K_{wa} H_w}{144} \]  \[\text{[8]}\]

where \( P_w \) = hydrostatic water pressure (psi)
\( \gamma_w \) = unit weight of water (lb/ft\(^3\))
\( K_{wa} \) = factor for uncertainty in level of groundwater table
\( H_w \) = depth of water table above springline of pipe (ft)

Step 4: The soil-prism load shall be calculated as a pressure representing the weight of soil above the pipe springline.

Condition 1: Water table is above the top of the pipe and at or above the ground surface.

\[ P_{sp} = \frac{(H + 0.11 \cdot \frac{D_o}{12}) \gamma_b}{144} \]  \[\text{[9]}\]

Condition 2: Water table is above the top of the pipe and below the ground surface.

\[ P_{sp} = \frac{1}{144} \left[ \left( H_w - \frac{D_o}{24} + 0.11 \cdot \frac{D_o}{12} \right) \gamma_b + \left( H - H_w + \frac{D_o}{24} \right) \gamma_s \right] \]  \[\text{[10]}\]

Condition 3: Water table is below the top of the pipe.

\[ P_{sp} = \frac{(H + 0.11 \cdot \frac{D_o}{12}) \gamma_s}{144} \]  \[\text{[11]}\]

where \( P_{sp} \) = soil prism pressure evaluated at pipe springline (psi)
\( H \) = depth of fill over top of pipe (ft)
\( D_o \) = outside diameter of pipe (in)
\( \gamma_b \) = unit weight of buoyant soil (lb/ft\(^3\))
\( H_w \) = depth of water table above springline of pipe (ft)
\( \gamma_s \) = wet unit weight of soil (lb/ft\(^3\))

Step 5: Factored and service loads

\[ P_u = \eta_{EV}(\gamma_{EV}K_EK_2P_{sp} \cdot VAF + \gamma_{WA}P_w) + \eta_{LL}Y_{LL}P_LC_L \]  \[\text{[12]}\]

\[ P_s = K_2P_{sp} \cdot VAF + P_LC_L + P_w \]  \[\text{[13]}\]
in which:

\[ VAF = 0.76 - 0.71 \cdot \frac{S_H - 1.17}{S_H + 2.92} \]  \[ [14] \]

\[ S_H = \frac{\phi_s M_s R}{E_p A_g} \]  \[ [15] \]

\[ C_L = \frac{L_w}{D_o} \leq 1.0 \]  \[ [16] \]

\[ L_w = W_0 + 12 \cdot H \cdot LLDF \]  \[ [17] \]

where \( P_u = \text{factored load (psi)} \)

\( \eta_{EV} = \text{load modifier applied to vertical earth loads on culverts} \)

\( \gamma_{EV} = \text{load factor for vertical pressure from dead load of earth fill} \)

\( K_{rE} = \text{installation factor. Typical value is } 1.0 \text{ to provide a traditional factor of safety} \)

\( K_2 = \text{coefficient to account for variations of thrust around the circumference. } K_2 = 1.0 \text{ for thrust at the springline, } K_2 = 0.6 \text{ for thrust at the crown} \)

\( P_{sp} = \text{soil prism pressure evaluated at pipe springline (psi)} \)

\( VAF = \text{vertical arching factor} \)

\( \gamma_{WA} = \text{load factor for hydrostatic pressure} \)

\( P_w = \text{hydrostatic water pressure (psi)} \)

\( \eta_{LL} = \text{load modifier applied to live loads on culverts} \)

\( \gamma_{LL} = \text{load factor for live load} \)

\( P_L = \text{live load pressure with dynamic load allowance (psi)} \)

\( C_L = \text{live load distribution coefficient} \)

\( S_H = \text{hoop stiffness factor} \)

\( \phi_s = \text{resistance factor for soil stiffness, } "\phi_s = 0.9 \text{ (mentioned in old version)}" \)

\( M_s = \text{secant constrained soil modulus (ksi)} \)

\( R = \text{radius from center of pipe to centroid of pipe profile (in)} \)

\( E_p = \text{short- or long-term modulus of pipe material (ksi)} \)

\( A_g = \text{gross area of pipe wall per unit length of pipe (in}^2/\text{in}) \)

\( L_w = \text{live load distribution width in the circumferential direction at the elevation of the crown (in)} \)
\[ \Delta_t = \frac{K_B(D_L P_{sp} + C_L P_L)D_o}{1000(E_p I_p / R^3 + 0.061 M_s)} + \varepsilon_{sc} D \]  

in which:

\[ \varepsilon_{sc} = \frac{T_s}{1000 A_{eff} E_p} \]  

\[ T_s = 0.5 P_s D_o \]

where  \( K_B \) = bedding coefficient, which varies from 0.083 for full support to 0.110 for line support at the invert. Haunching is always specified to provide good support. Typical value is 0.10 to account for inconsistent haunch support.

\( D_L \) = deflection lag factor. Recommended value is from 1.0 to 6.0, and the highest value is for installations with quality backfill and low initial deflections and do not generally control designs. Typical value is 1.5, which provides some allowances for an increase in deflection over time for installations with initial deflection levels of several percent.

\( P_{sp} \) = soil prism pressure evaluated at pipe springline (psi)

\( C_L \) = live load distribution coefficient

\( P_L \) = design live load pressure including vehicle load, dynamic load allowance and multiple presence effect (psi)

\( D_o \) = outside diameter of pipe (in)

\( E_p \) = short- or long-term modulus of pipe material (ksi)

\( I_p \) = moment of inertia of pipe profile per unit length of pipe (in^4/in)

\( R \) = radius from center of pipe to centroid of pipe profile (in)

\( M_s \) = secant constrained soil modulus (ksi)
\( \varepsilon_{sc} \) = service compression strain due to thrust, and taken as positive for compression

\( D \) = diameter to centroid of pipe profile (in)

\( T_s \) = service thrust per unit length (lb/in)

\( A_{eff} \) = effective area of pipe wall per unit length of pipe (in\(^2\)/in)

\( P_s \) = design service load (psi)

**Step 7: Strain due to thrust**

\[
\varepsilon_{uc} = \frac{T_u}{1000 A_{eff} E_p} \tag{21}
\]

\[
\varepsilon_{sc} = \frac{T_s}{1000 A_{eff} E_p} \tag{22}
\]

in which:

\( T_u = 0.5 P_u D_o \) \hspace{1cm} \tag{23}

\( T_s = 0.5 P_s D_o \) \hspace{1cm} \tag{24}

where \( \varepsilon_{uc} \) = factored compressive strain due to thrust

\( T_u \) = factored thrust per unit length (lb/in)

\( A_{eff} \) = effective area of pipe wall per unit length of pipe (in\(^2\)/in)

\( E_p \) = short-term modulus for short-term loading or long-term modulus of pipe material for long-term loading (ksi)

\( \varepsilon_{sc} \) = service compressive strain due to thrust

\( T_s \) = service thrust per unit length (lb/in)

\( \phi_T \) = resistance factor for thrust effects

\( \varepsilon_{yc} \) = factored compressive strain limit

\( P_0 \) = factored load

\( D_o \) = outside diameter of pipe (in)

\( P_s \) = service load

**Step 8: Strain due to flexure**

\[
\varepsilon_f = \gamma_{Ev} D_f \frac{c \cdot \Delta f}{RD} \tag{25}
\]
\[ \Delta f = \Delta A - \varepsilon_{sc} D \]  

where \( \varepsilon_f \) = factored strain due to flexure  
\( \varepsilon_{uc} \) = factored compressive strain due to thrust  
\( \phi_f \) = resistance factor for flexure  
\( \varepsilon_{yt} \) = service long-term tension strain limit of pipe wall material  
\( \phi_T \) = resistance factor for thrust effects  
\( \varepsilon_{yc} \) = service compression strain limit of pipe wall material  
\( \gamma_{EV} \) = load factor for vertical pressure from dead load of earth fill  
\( D_r \) = shape factor (the values are shown in Table 3.4). The shape factor for corrugated PE pipe can be reduced by 1.0 to account for the effect of the low hoop stiffness ratio  
\( c = \) the larger of the distance from neutral axis of profile to the extreme innermost or outermost fiber (in)  
\( R = \) radius from center of pipe to centroid of pipe profile (in)  
\( \Delta f = \) reduction of vertical diameter due to flexure (in)  
\( D = \) diameter to centroid of pipe profile (in)  
\( \Delta A = \) total allowable deflection of pipe, reduction of vertical diameter (in)  
\( \varepsilon_{sc} = \) service compression strain due to thrust
Table 3.4: Shape Factor ($D_f$) Based on Pipe Stiffness, Backfill, and Compaction Level

<table>
<thead>
<tr>
<th>Pipe Stiffness (ksi)</th>
<th>Pipe Zone Embedment Material and Compaction Level</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dumped to Slight</td>
<td>Moderate to High</td>
</tr>
<tr>
<td>0.009</td>
<td>5.5</td>
<td>7.0</td>
</tr>
<tr>
<td>0.018</td>
<td>4.5</td>
<td>5.5</td>
</tr>
<tr>
<td>0.036</td>
<td>3.8</td>
<td>4.5</td>
</tr>
<tr>
<td>0.072</td>
<td>3.3</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Note: 1. "Gravel" contains GW, GP, GW-GC, GW-GM, GP-GC and GP-GM per ASTM D2487 (includes crushed rock)
2. "Sand" contains SW, SP, SM, SC, GM, and GC or mixtures per ASTM D2487
3. "Dumped to Slight" means <85% of maximum dry density per AASHTO T 99, <40% relative density (ASTM D4253 and D4254)
4. "Moderate to High" means ≥85% of maximum dry density per AASHTO T 99, ≥40% relative density (ASTM D4253 and D4254)
5. Pipe stiffness (ksi) is determined as

$$\frac{F}{\Delta y} = \frac{EI}{0.149R^3}$$ [27]

6. Since secant constrained soil modulus are all assumed higher than 1 ksi, the compaction level of sand backfill should be higher than 85%, which means the compaction level of sand should be moderate to high. Through regression method, the function that reveals the relationship between pipe stiffness and sand backfill compacted in moderate to high level is:

$$D_f = 2.1842 \cdot PS^{-0.274}$$ [28]

Step 9: Strain limit
The combined strain at the extreme fiber where flexure causes tension shall satisfy:

$$\epsilon_f - \epsilon_{uc} < \phi_f \epsilon_{yt}$$ [29]

The combined strain at the extreme fiber where flexure causes compression shall satisfy:

$$\epsilon_f + \epsilon_{uc} < 1.5\phi_t \epsilon_{yc}$$ [30]

Step 10: Buckling strain limit

$$\epsilon_{uc} \leq \phi_{bck} \epsilon_{bck}$$ [31]

in which:
ε_{bck} = \frac{1.2C_n(E_pI_p)^{\frac{1}{3}}}{A_{eff}E_p} \left[ \frac{\phi_sM_s(1 - 2\nu)}{(1 - \nu)^2} \right]^{\frac{2}{3}} R_h \quad \text{[32]}

R_h = \frac{11.4}{11 + \frac{D}{12H}} \quad \text{[33]}

The theoretical method assumes that the top of the pipe is under uniformly distributed compression without any concentrated load, these methods can only address pipes under dead load.

3.3 Theoretical Method

By applying the theories of elasticity and curbed beam theories, some researchers established a set of equations to estimate the performances of buried pipes. These theoretical methods are fulfilled with reasonable hypotheses and complicated functions, and their applicability and practicability have been proven by their inventors. Therefore, these methods are adopted in this dissertation to explore the critical vertical deflection for HDPE pipes. The following are general descriptions of the various theoretical methods.

a. Because every theoretical method assumes that the top of the pipe is under uniformly distributed compression without any concentrated load, these methods can only address pipes under dead load.
b. Based on the nature of these theoretical methods, the pipe wall stress at the springline is always larger than the stress at the crown, when the pipe bears the dead load. Thus, the original equations in each theoretical method are ultimately developed to the functions revealing the relationship between the vertical deflection of the pipe and the stress at the springline of the pipe. The stresses are all converted to strains in the sections presenting the results.

c. None of the methods considered the problem of buckling conditions in the pipe, so the pipe's failure depends only on the magnitude of the combined compressive strains in the pipe wall.

d. In the original equations of these theoretical methods, the same symbol was not always used to represent the same mechanical properties. To avoid unnecessary confusions, some parameters in the original equations are relabeled with new symbols in this dissertation.

### 3.3.1 Spangler Method

Spangler (1941) established the original Iowa formula through elastic ring theory and empirical assumptions that he called the "fill-load hypotheses". The hypothesis included that:

1. The compressive vertical load acting on top of the pipe is uniformly distributed across the width of the pipe;

2. The vertical reaction force at the bottom of the pipe is uniformly distributed across the width of the pipe's bedding; and

3. Against each side of the pipe, the passive soil pressure is horizontally spread in the shape of a parabola over a 100-degree arc surrounding the springline of the pipe, with the maximum unit pressure \(h\) given by:

\[
h = \frac{e \cdot \Delta x}{2}
\]  

[34]

where  
\(e\) = modulus of passive soil resistance;

\(\Delta x\) = horizontal deflection of the pipe.

And, his resulting horizontal deflection \(\Delta x\) of the pipe or the "Iowa" formula is:
$$\Delta x = \frac{PD_t R^3 K}{EI + 0.061e R^4} = \frac{PD_t R^3 K}{EI + 0.061E'R^3} = \frac{P}{Q}$$

[35]

where \( P \) = vertical load on pipe;
\( e \) = modulus of passive soil resistance;
\( E \) = elastic modulus of HDPE material;
\( E' \) = modulus of soil reaction (= \( e \cdot r \));
\( D_t \) = Time lag factor, and the default value is 1.0;
\( Q \) = A symbol is created to simplify the equation;
\( K \) = bedding constant, and its equation is:

$$K = 0.5 \sin \alpha - 0.082 \sin^2 \alpha + 0.08 \frac{\alpha}{\sin \alpha} - 0.16(\pi - \alpha) \sin \alpha$$

$$- 0.04 \frac{\sin 2\alpha}{\sin \alpha} + 0.318 \cos \alpha - 0.208$$

[36]

Because the modulus of soil reaction (\( E' \)) cannot be easily measured in the laboratory, engineers try to replace \( E' \) by another parameter, which can be more readily measured in the lab. Fortunately, the constrained modulus (\( M_s \)) is a suitable alternative, and \( M_s \) is approximately equal to \( E' \) according to the research of Chambers et al. (1980) and Hartley and Duncan (1987). By changing the function with a parameter \( Q \), the relationship between the horizontal deflection and external vertical load can be rewritten as:

$$P = Q \cdot \Delta x$$

[37]

The vertex of the bedding angle (\( \theta \)) locates at the center of the pipe, and one-half of bedding angle is symbolized as \( \alpha \). Assume bedding angle \( \theta \) is 90° (= \( \pi/2 \)), so \( \alpha \) is 45° (= \( \pi/4 \)). The moment at the invert of the pipe (\( M_c \)) is:

$$M_c = -0.049qR^2 - 0.166hR^2 + q'R^2 \cdot [0.106 \sin^3 \alpha + 0.08\alpha - 0.04 \sin 2\alpha$$

$$- 0.159(\pi - \alpha) \sin^2 \alpha + 0.318(1 + \cos \alpha) \sin \alpha] = 0.157PR - 0.083M_s R \Delta x$$

[38]

where \( q \) = external vertical pressure distributed on the top of pipe (= \( P/(2R) \));
\( q' \) = pipe's bottom distributed reaction due to external vertical pressure (= \( P/(2R \cdot \sin \alpha) \)).
And, the thrust force at the invert of the pipe \((T_c)\) is:

\[
T_c = -0.106q'\sin^3 \alpha + 0.511hR + 0.106qR = 0.027p + 0.256M_s\Delta x \tag{39}
\]

So, by setting \(\phi = 90^\circ (= \pi/2)\), and the moment at the springline of the pipe \((M)\) is:

\[
M = M_c + T_cR(1 - \cos \phi) - q'R^2(\sin \phi - 0.5 \sin \alpha) \sin \alpha - hR^2(0.147 - 0.51 \cos \phi + 0.5 \cos^2 \phi - 0.143 \cos^4 \phi) - 0.5qR^2(1 - \sin \phi)^2 \tag{40}
\]

\[
= (0.099M_s - 0.139Q)R\Delta x
\]

The relationship between stress at the springline of the pipe and the horizontal deflection of the pipe is:

\[
\sigma = \frac{P}{2A} \pm \frac{Mc}{I} = \left[ \frac{Q}{2A} \pm \frac{(0.099M_s - 0.139Q)Rc}{I} \right] \Delta x \tag{41}
\]

Spangler only solved the horizontal deflection of the pipe, and he assumed that the deformation of the pipe is a perfect ellipse, which means that the vertical deflection and horizontal deflection of the pipe are the same in magnitude (opposite in directions or signs). Hence, the above equation can also represent the correlation between the stress at the springline of the pipe and the vertical deflection by replacing \(\Delta x\) with \(\Delta y\).

### 3.3.2 Masada Method

Some field tests and laboratory experiments revealed that the vertical deflection is always larger than the horizontal deflection, especially when the pipes are buried under moderate to deep soil cover. Following the theory and assumptions of Spangler (1941), Masada (2000) derived the vertical deflection as follows, based on the approach and assumptions suggested by Spangler (1941).

\[
\Delta y = \frac{R^2}{EI} \int_0^\pi M \sin \phi \, d\phi \tag{42}
\]

The resulting vertical deflection formula generated by Masada (2000) is:

\[
\Delta y = \frac{0.119hR^4 + JPR^3}{EI} \tag{43}
\]

The above can be modified to:

\[
\Delta y = \frac{1}{EI} \left( 0.119 \cdot \frac{e \cdot \Delta x}{2} \cdot R^4 + J \cdot Q \cdot \Delta x \cdot R^3 \right) = \frac{0.060M_sR^3 + JQ^3}{EI} \cdot \Delta x \tag{44}
\]
where constant J is:

\[ J = -0.434 + 0.25\alpha + 0.318\cos\alpha + \frac{0.08\alpha}{\sin\alpha} + \frac{0.167(\cos\alpha - 1)}{\sin\alpha} + \frac{0.04 + 0.125\sin\alpha}{\sin\alpha} \]

\[ -\frac{\sin 2\alpha}{\sin\alpha}(-0.25 + 0.159\alpha + 0.333\cos\alpha)\sin\alpha \]

So,

\[ \Delta x = \left(\frac{EI}{0.060M_sR^3 + JQR^3}\right) \Delta y = \left[\frac{EI}{(0.060M_s + JQ)R^3}\right] \Delta y \] \hspace{1cm} [45]

The relationship between the wall stress and vertical deflection is:

\[ \sigma = \left[\frac{QI}{2\pi A} \pm (0.099M_s - 0.139Q)Rc\right] \left[\frac{E}{(0.060M_s + JQ)R^3}\right] \Delta y \] \hspace{1cm} [46]

In the Masada method, the equations determining moment, thrust stress in the pipe wall remain identical to those given by Spangler (1941).

3.3.3 Burns and Richard Method

In contrast to the Iowa formula that involved some empirical elements, Burns and Richard (1964) studied the problem of buried pipes from a purely theoretical standpoint. They assumed that the pipe is an elastic circular shell embedded inside a uniform elastic medium. The boundary of the media is located sufficiently far away from the shell, so this analysis is also suitable for deeply buried pipes. The analysis of the pipe and backfill are based on the extensional shell theory and Michell's formulation of Airy's stress function, respectively. This method provides equations computing the stresses and displacements inside the shell, inside the media, and even at the interface between the shell and media. The compressive stress is evenly distributed over the top and bottom boundaries of the medium, and the lateral stress is located symmetrically on both sides of the medium. Because the horizontal diameter of the pipe expands due to the external vertical compression load, the pipe is also compressed by the lateral stress generated by the backfill. The ratio of the lateral stress to the vertical compression stress, symbolized as K, is dictated by the property of the soil medium as:

\[ k = \frac{\nu}{1 - \nu} \] \hspace{1cm} [48]
where \( \nu = \) Poisson's ratio of soil.

In order to express the elastic field solutions conveniently, the authors created several parameters. There are two constants \((B, D)\) related to the lateral stress ratio:

\[
B = \frac{1 + k}{2} = \frac{1}{2(1 - \nu)} \tag{49}
\]
\[
D = \frac{1 - k}{2} = \frac{1 - 2\nu}{2(1 - \nu)} \tag{50}
\]

And two non-dimensional parameters were devised to indicate the interactions between the shell and the medium. These parameters are extensional flexibility ratio \((U)\) and bending flexibility ratio \((V)\), given by:

\[
U = \frac{2BM_sR}{EA} = \frac{M_sR(1 + k)}{EA} \tag{51}
\]
\[
V = \frac{2D M_s R^3}{6EI} = \frac{M_s R^3 (1 - k)}{6EI} \tag{52}
\]

Two conditions were considered at the interface between the pipe and soil medium. One condition is the no slippage case, which means the pipe and the soil backfill are fully bounded to each other. Therefore, the radial and tangential displacements of the shell and the medium are continuous at the interface in this no slippage case. The other condition is the full slippage case, which indicates the backfill can slip along the outside edge of the pipe but cannot separate from the pipe wall. In the full slippage case, the tangential displacement of the shell is inconsistent with the tangential movement of the media, although the nominal displacements of the shell and the media are identical.

In the no slippage case, three non-dimensional constants \((a_0, a_1, b_1)\) are applied in the stress determination.

\[
a_0 = \frac{U - 1}{U + \frac{B}{D}} \tag{53}
\]
\[
a_1 = \frac{VD(1 - U) - UD}{V(1 + B) + UD \left( V + \frac{1}{B} \right) + 2(1 + D)} \tag{54}
\]
\[ b_1 = \frac{V(B + UD) - 2B}{V(1 + B) + UD\left(\frac{V}{B} + \frac{1}{B}\right) + 2(1 + D)} \]  \[55\]

The radial displacement of the crown and the invert of the pipe are equal to each other in magnitude but reverse to each other in direction, so the vertical deflection of the pipe is two times radial displacement at top of the pipe \((\theta = 90^\circ)\). The function of the vertical deflection \((\Delta y)\) is:

\[ \Delta y = \frac{PR}{M_s} [U(1 - a_0) + V(1 - a_1 - 2b_1)] \]  \[56\]

So,

\[ PR = \frac{M_s \Delta y}{U(1 - a_0) + V(1 - a_1 - 2b_1)} \]  \[57\]

The thrust force \((N)\) and the moment \((M)\) at the springline of the pipe \((\theta = 0^\circ)\) are given by:

\[ N = PR[B(1 - a_0) + D(1 + a_1)] \]  \[58\]

\[ M = \frac{PR^2 D}{6} \left[ \frac{U}{V}(1 - a_0) + 3(1 - a_1 - 2b_1) \right] \]  \[59\]

Therefore, the stress at the springline of the pipe is:

\[ \sigma = \frac{N}{A} \pm \frac{Mc}{I} \]

\[ = PR \left\{ \frac{B(1 - a_0) + D(1 + a_1)}{A} \pm \frac{RDc}{6I} \left[ \frac{U}{V}(1 - a_0) + 3(1 - a_1 - 2b_1) \right] \right\} \]  \[60\]

So,

\[ PR = \frac{\sigma}{B(1 - a_0) + D(1 + a_1)} \pm \frac{RDc}{6I} \left[ \frac{U}{V}(1 - a_0) + 3(1 - a_1 - 2b_1) \right] \]  \[61\]

Then, the relationship between the stress at the springline of the pipe and the vertical deflection of the pipe is:

\[ \sigma = \frac{B(1 - a_0) + D(1 + a_1)}{A} \pm \frac{RDc}{6I} \left[ \frac{U}{V}(1 - a_0) + 3(1 - a_1 - 2b_1) \right] M_s \Delta y \]  \[62\]
In the full slippage case, there are other two non-dimensional constants \((a_2, b_2)\) and an existing constant \((a_0)\).

\[
a_0 = \frac{U - 1}{U + B/D} \quad [63]
\]

\[
a_2 = \frac{2V - 1 + \frac{1}{B}}{2V - 1 + \frac{3}{B}} \quad [64]
\]

\[
b_2 = \frac{2V - 1}{2V - 1 + \frac{3}{B}} \quad [65]
\]

The vertical deflection \((\Delta y)\) of the pipe is:

\[
\Delta y = \frac{PR}{M_s}\left[U(1 - a_0) + \frac{2}{3}V(1 + 3a_2 - 4b_2)\right] \quad [66]
\]

The thrust force \((N)\), the moment \((M)\) and the stress at the springline of the pipe are:

\[
N = PR\left[B(1 - a_0) + \frac{D}{3}(1 + 3a_2 - 4b_2)\right] \quad [67]
\]

\[
M = \frac{PR^2D}{6}\left[\frac{U}{V}(1 - a_0) + 2(1 + 3a_2 - 4b_2)\right] \quad [68]
\]

\[
\sigma = PR\left\{\frac{B(1 - a_0) + \frac{D}{3}(1 + 3a_2 - 4b_2)}{A}\right\} + \frac{RDe}{6I}\left[\frac{U}{V}(1 - a_0) + 2(1 + 3a_2 - 4b_2)\right] \quad [69]
\]

Hence, the relationship between the stress at the springline of the pipe and the vertical deflection of the pipe is:

\[
\sigma = M_s\Delta y\left\{\frac{B(1 - a_0) + \frac{D}{3}(1 + 3a_2 - 4b_2)}{A}\right\} + \frac{RDe}{6I}\left[\frac{U}{V}(1 - a_0) + 2(1 + 3a_2 - 4b_2)\right] \quad [70]
\]

\[
\left[U(1 - a_0) + \frac{2}{3}V(1 + 3a_2 - 4b_2)\right] / \left[U(1 - a_0) + \frac{2}{3}V(1 + 3a_2 - 4b_2)\right]
\]
3.3.4 Hoeg Method

Hoeg (1968) independently made the same assumptions about the pipe and backfill as those made by Burns & Richard (1964) and carried out a similar elastic analysis of the buried pipe problems. The cross section of the pipe adopted in the analysis of this dissertation is a profile wall instead of a smooth wall, and the thickness of the pipe wall should be transformed to other parameters. If the magnitude is the only criterion used to select the alternative parameter, the thickness \( t \) is equal to the area \( A \) per inch in the longitudinal direction, and the cube of the thickness is equal to one-twelfth of the moment of inertia \( I \) per inch in the longitudinal direction. The Poisson's ratio of the HDPE material is smaller than 0.5, so the value of \( (1 - \nu_c^2) \) is approximated to 1. Another inconvenient aspect of the Hoeg's work is that only the performance of the backfill and the vertical deflection of the pipe are provided, while the equations of the pipe's mechanical performance are absent. Masada (1996) studied the Hoeg's paper and derived the equations for the thrust force and moment of the pipe under the no slippage case. This was because he considered this interface condition to be closer to the real situation.

Compressibility ratio \( C \) and flexibility ratio \( F \) are applied to express the stiffness ratios, and they are:

\[
C = \frac{M_sR(1 - \nu_c^2)}{Et(1 - \nu)} = \frac{M_sR}{EA(1 - \nu)} \quad [71]
\]

\[
F = \frac{2M_sR^3(1 - 2\nu)(1 - \nu_c^2)}{Et^3(1 - \nu)} = \frac{M_sR^3(1 - 2\nu)}{6EI(1 - \nu)} \quad [72]
\]

For the no slippage case, the three non-dimensional constants are:

\[
a_1 = \frac{(1 - 2\nu)(C - 1)}{C(1 - 2\nu) + 1} \quad [73]
\]

\[
a_2 = \frac{F(1 - 2\nu)(1 - C) - 0.5C(1 - 2\nu)^2 + 2}{F[(3 - 2\nu) + (1 - 2\nu)C] + C(2.5 - 8\nu + 6\nu^2) + 6 - 8\nu} \quad [74]
\]

\[
a_3 = \frac{F[1 + (1 - 2\nu)C] - 0.5C(1 - 2\nu) - 2}{F[(3 - 2\nu) + (1 - 2\nu)C] + C(2.5 - 8\nu + 6\nu^2) + 6 - 8\nu} \quad [75]
\]
The vertical deflection of the pipe \((\theta = 90^\circ)\) is:

\[
\Delta y = \frac{PR}{M_s} \left\{ (1 + k)(1 - v) \left( 1 + \frac{a_1}{1 - 2v} \right) + (1 - k) \left( \frac{1 - v}{1 - 2v} \right)[1 + a_2 + 4a_3(1 - v)] \right\} 
\]

[76]

And, the thrust force \((N)\), the moment \((M)\) and the stress \((\sigma)\) at the springline of the pipe \((\theta = 0^\circ)\) are:

\[
N = \frac{PR}{2C} \left\{ (1 + k) \left( 1 + \frac{a_1}{1 - 2v} \right) + \left( \frac{1 - k}{1 - 2v} \right) (1 - 3a_2 - 4a_3v) \right\} 
\]

[77]

\[
M = \frac{PR^2}{12F} \left\{ (1 + k)(1 - 2v) \left( 1 + \frac{a_1}{1 - 2v} \right) + 3(1 - k)[1 + a_2 + 4a_3(1 - v)] \right\} 
\]

[78]

\[
\sigma = \frac{N}{A} \pm \frac{Mc}{I} = PR \left\{ \frac{1}{2CA} \left[ (1 + k) \left( 1 + \frac{a_1}{1 - 2v} \right) + \left( \frac{1 - k}{1 - 2v} \right) (1 - 3a_2 - 4a_3v) \right] \right. \\
\pm \left. \frac{Rc}{12FI} \left( (1 + k)(1 - 2v) \left( 1 + \frac{a_1}{1 - 2v} \right) + 3(1 - k)[1 + a_2 + 4a_3(1 - v)] \right) \right\} 
\]

[79]

The relationship between the stress at the springline of the pipe and the vertical deflection of the pipe is:

\[
\sigma = M_s \Delta y \left\{ \frac{1}{2CA} \left[ (1 + k) \left( 1 + \frac{a_1}{1 - 2v} \right) + \left( \frac{1 - k}{1 - 2v} \right) (1 - 3a_2 - 4a_3v) \right] \right. \\
\pm \left. \frac{Rc}{12FI} \left( (1 + k)(1 - 2v) \left( 1 + \frac{a_1}{1 - 2v} \right) + 3(1 - k)[1 + a_2 + 4a_3(1 - v)] \right) \right\} 
\]

[80]

3.4 CANDE Simulations
3.4.1 General Descriptions

The theoretical methods described above may not be appropriate for analyzing the performances of underground pipes in a variety of installation and loading conditions, because the theoretical solutions were limited to homogeneous soil installations and dead soil loading. Also, the theoretical equations took only smooth-walled pipes when in reality most HDPE pipes are produced with the profile-wall design. While field tests
being time-consuming and expensive, a need existed for realistic and flexible computer simulation models. With steady improvements in the finite element method (FEM) and computer technology, engineers soon found that the FEM can be applicable to many research fields. Under the National Cooperative Highway Research Project (NCHRP) 15-28, a group of research engineers developed computer software exclusively dedicated for buried pipeline design and analysis.

The name of this software is "Culvert ANalysis and DEsign (CANDE)". This is a two-dimensional computer code based on the plain strain theory. The code can simulate construction sequences by building up a finite mesh step by step. Various pipe shapes (circular, ellipse, arc, etc.) and materials (thermoplastic, steel, concrete, etc.) can be handled by CANDE, and many soil types and their relative mechanical properties are stored in CANDE's database. Dead load and live load can be optionally added into the simulations. This powerful software was initially published in 1976 and was continually revised over the years. The version utilized in this dissertation is CANDE-2013, which can be downloaded from the official website of the Transportation Research Board (TRB).

3.4.2 Definitions in CANDE Simulations

Because CANDE can simulate different pipe installation modes, two installation modes are addressed in the dissertation - embankment mode and trench mode. Sketches of these two modes are provided in Figure 3.2. In the embankment mode, the pipe is placed on the in-situ soil and covered by the backfill. In the trench mode, the pipe is placed into a trench excavated into the in-situ soil, and the pipe is backfilled within the confinement of the trench.

Figure 3.3: Illustration of Embankment Mode (Left) and Trench Mode (Right)
According to the AASHTO Specifications, the bedding thickness under the pipe should be at least 6 inches, and the minimum width of the trench should be the pipe's outside diameter plus 16 inches or 1.5 times the outside diameter of the pipe plus 12 inches, whichever is greater. In reality, the trench width should be equal to the pipe's outside diameter plus a minimum of 4 ft. This is because the space between the pipe's exterior surface and the adjacent trench inside wall should be sufficient to permit a standard soil compaction machine whose width normally ranges from 1 ft to 1.5 ft. After giving some thoughts, in this dissertation the trench width is specified to be 4 ft wider than the outside diameter of the pipe, and this width is much greater than the minimum width required by the AASHTO specifications. The top of the trench is arranged 1 ft higher than the crown of the pipe.

CANDE provides the users with various levels of analysis tools. Level 1 offers the users to tap into the full-field elastic solutions of Burns and Richard (1964). Level 2 provides an FEM solution based on automated mesh generations for pipe and soil elements. Level 3 gives the user the most flexible FEM environment, where a complete freedom exists for constructing the FEM model mesh, material/loading selections, and arrangement of the construction sequences. In this dissertation, the Level 3 FEM was utilized to meet the assumptions listed at the beginning of this chapter. At this level, each discretized mesh is established by manually inputting the geometric coordinates of every node, node connectivity information to form elements, material properties of each element, and the boundary and loading condition statements. Because the pipe's installation and loading conditions are symmetric about the pipe's vertical centerline, the mesh needs to show only the right half of the buried pipe and soil. The pipe consists of ten beam-column elements connected together, and each soil element is represented by a four-node quadrilateral elements. To diminish the effect of boundary conditions on the pipe performance, the right edge and bottom edge of the soil are positioned three pipe diameters away from the pipe. The left and right boundaries of the half mesh can only move vertically, and the bottom edge is entirely fixed. A sketch of the FEM half-mesh model is illustrated in Figure 3.3.
The HDPE pipes simulated in CANDE contain all the diameter sizes listed in Table 3.1. The in-situ soil type is characterized as isotropic sand (constrained modulus 5 ksi), and the overfill is defined as isotropic silty-clay soil (unit weight 130 pcf). The backfill types include CA-90, CA-95, SM-85, SM-90, SW-85, SW-90 and SW-95. The letter symbols "CA", "SM" and "SW" represent coarse aggregate, silty sand, and well-graded (clean) sand, respectively. The number attached to each soil type here refers to the relative compaction achieved on the soil. Because CA-85 and SM-95 are absent from CANDE's material library, these two soil types cannot be selected. Based on AASHTO M145 and ASTM 2487, the correspondences between the AASHTO soil type and the ASTM soil type are listed in Table 3.5. The CA and SM soil types are applicable to the Duncan formulation, and the SW soil type is suitable for the Duncan/Selig formulation. In the Duncan formulation, the tangent elastic modulus of the soil is defined by the Duncan
hyperbolic model, and the tangent bulk modulus is set by Duncan's power law. The assumption of the tangent elastic modulus is included in the Duncan hyperbolic model as well in the Duncan/Selig formulation, but the tangent bulk modulus is simulated by Selig's hyperbolic law.

<table>
<thead>
<tr>
<th>AASHTO Soil Type</th>
<th>ASTM Soil Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>CA (coarse aggregate)</td>
</tr>
<tr>
<td>A-2</td>
<td>SM (silty sand)</td>
</tr>
<tr>
<td>A-3</td>
<td>SW (well-graded sand)</td>
</tr>
</tbody>
</table>

Interface elements are established to simulate frictional slippage interaction between the pipe wall and the backfill. The locations of interface elements are aligned along the borderline enclosed by the outside edge of the beam element (pipe wall) and the inside edge of the quadrilateral elements (backfill), and the pipe elements and backfill elements are connected with each other through interfacial elements. Because of the influence of gravity, the soil below the pipe's springline is unlikely to slide along the pipe wall. Thus, the interface elements exist only along the upper half of the pipe's outline. The friction coefficient of the interface element is set at 0.5, which means that the magnitude of the friction stress in the interface element is half of the normal stress transferred from the backfill.

In most pipe installation cases, the longitudinal axis of the pipe is perpendicular to the traffic directions. Even when the traffic moves along the longitudinal direction of the buried pipe, the space between the two rear wheels of an H-20 truck is 6 ft, which is greater than the outside diameter of the 60-in diameter HDPE pipe addressed in this dissertation. Therefore, the worst load condition that any buried pipe endures is thought to be the case when one rear wheel of the H-20 truck is placed on the top of the pipe's vertical centerline. As mentioned in the AASHTO specification, the load carried by one of the rear wheel is 16 kip, and it is uniformly distributed over a 10-in (length) x 20-in (width) rectangular contact area. Along the pipe's longitudinal direction, the live load pressure is 1.6 kip/in, which is uniformly spread across the 20-in width of the wheel. In a half mesh, an 800
lb/in live load pressure is distributed over a 10-in width expanding from the crown of the pipe. For the live load application cases, the soil cover thickness was varied first from 1 ft to 2.5 ft and then from 2.5 ft to 15 ft with an increment of 2.5 ft.
CHAPTER 4 : RESULTS OF AASHTO AND OTHER ENGINEERING METHODS

This chapter provides results obtained from all the methods outlined in Chapter 3. Before getting into the results, a few notes are needed concerning some performance trends observed and the way analytical results are presented. Compression failure occurs when the combined compressive strain in the pipe wall surpasses the thrust & bending moment-combined compressive strain limit (6.15%), and tension failure occurs when the combined tensile strain is higher than the tensile strain limit (5%). Based on the performances of pipes when all the methods are applied, the tensile strain in the pipe wall is always below 5% when the combined compressive strain reaches the 6.15% limit. Therefore, the tensile strain results are omitted in the illustrations of and discussion on the results because the combined compressive strains in the pipe wall are the primary parameters governing pipe failure. The buckling strain limit (AASHTO) and buckling stress limit (CANDE) vary with the pipe geometries and with the installation conditions of the pipes. Thus, it is convenient and straightforward to estimate the pipe wall buckling status using the buckling ratio, which is the maximum stress or strain values resulting from each method divided by the buckling stress or strain limits. The pipe will buckle if the buckling ratio is greater than 1.0. Wall buckling can sometimes occur before the appearance of compression failure in the short-term service condition. However, during the long-term service condition, the buckling ratio is found to be too small when compression failure is experienced. Therefore, the buckling ratios under the long-term service condition tend to be omitted for the pipes analyzed in this dissertation. A massive amount of data resulted from the analyses. Only a handful of them are included in this chapter, with the rest displayed in the appendixes.

4.1 AASHTO Calculation Results

All results derived from the AASHTO calculations are presented in this subsection, and the AASHTO calculation results are examined in light of the field tests listed in Chapter 2. In the legend used in the plots presenting the performances of HDPE pipes under dead and live loads, the letter "D" after each pipe's diameter indicates that only the dead soil load
is applied to each pipe, and the notation "D&L" indicates that each pipe is subjected to both
dead and live loads. The critical vertical deflection and cover thickness are determined
by the compression wall failure or buckling, whichever comes first. When the pipe bears
dead load only, the combined compressive strain tends to be higher at the springline than
at the crown. Thus, the combined compressive strains mentioned in Section 4.1.1 are the
strains at the pipe springlines. In contrast, the combined compressive strain tends to be
larger at the crown than at the springlines when live loads are applied to the pipes under a
shallow soil cover. This phenomenon is fully explained in Section 4.1.2.

### 4.1.1 Performances of Pipes under Dead Load Only

Figures 4.1 to 4.3 show the correlations between the vertical deflections and soil
cover thicknesses for all diameter sizes of the pipes installed with a backfill having a
constrained modulus of 1 ksi, 3 ksi, or 5 ksi. In these plots, vertical deflections are seen
increasing linearly with the soil cover thickness. The 12-inch diameter pipe behaves
stiffer than the other diameter-size pipes, regardless of the constrained modulus value
assigned to the backfill material and the amount of the soil cover placed over the pipe.

![Figure 4.1: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only](image)

- 12-in
- 24-in
- 30-in
- 36-in
- 42-in
- 48-in
- 60-in
Figures 4.2 to 4.6 show the correlations between the combined compressive strain and vertical deflection for each diameter pipe subjected to the soil dead loading. The plots are prepared for the three different constrained modulus values (1 ksi, 3 ksi, 5 ksi) specified for the backfill. The soil cover thickness is increased steadily until the combined compressive strain in the pipe wall reaches 6.15%. Therefore, the maximum vertical deflections illustrated in these figures reflect the critical vertical deflections of the pipes. According to the AASHTO calculations, the relationship between the combined compressive strain and the vertical deflection is linear. In addition, the critical vertical
deflections decrease with increasing constrained modulus of the backfill. Regardless of the value of the constrained modulus assigned to the backfill, a 7.5% vertical deflection appears to always protect these pipes from suffering from compression failure. Furthermore, the 12-inch diameter pipe seems to perform better than the other diameter pipes. At the same vertical deflection, the combined compressive strains in the 12-inch diameter pipe's wall section are slightly lower than those registered in the walls of the other diameter pipes, independent of the value given to the constrained modulus of the backfill.

![Figure 4.4](image4.4.png)

**Figure 4.4:** Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only

![Figure 4.5](image4.5.png)

**Figure 4.5:** Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only
The next group of graphical plots, Figures 4.7 to 4.9, present the relationships between the vertical deflection and the wall buckling ratio for different diameter HDPE pipes. The increment changes in the buckling ratio cease when the cover thickness become maximized in Figures 4.1 to 4.3. In Figure 4.7, the bucking ratio become nearly 1.0 when the pipes are subject to critical cover thicknesses controlled by the compression failure, indicating that compression failure and buckling may develop along the same vertical deflection when the constrained modulus of the backfill is 1 ksi. Hence, the critical vertical deflection of each diameter pipe determined by the buckling ratio is very close to the critical vertical deflection determined based on the combined compressive strain. Compared to the strain values for equivalent vertical deflection, the buckling ratio of each diameter pipe is substantially less than 1.0 if the backfill constrained modulus is 3 ksi or 5 ksi. Therefore, a pipe buried in a high-constrained-modulus backfill will not buckle even when compression failure has occurred in the pipe wall.
Figure 4.7: Short-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only

Figure 4.8: Short-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only
The figures mentioned so far present the short-term pipe performance under dead loading only. Figures 4.10 through 4.15 provide the pipe performances in the long-term service condition due to soil loading. Again, the soil cover thickness is increased incrementally until the compressive failure emerges. Regardless of the backfill material's constrained modulus, the governing failure mode of all diameter pipes in the long-term service condition is compression failure. According to Figures 4.13 to 4.15, the critical vertical deflection decreases as the backfill becomes stiffer. Furthermore, the critical vertical deflections of all diameter pipes (except for the 12-inch diameter pipe) are less than 7.5% when the constrained modulus of the backfill is equal to 1 ksi. In addition, when the pipes are installed in 3-ksi and 5-ksi backfill materials, the critical vertical deflections are approximately 6%. The critical cover thickness increases and the lower vertical deflections of the pipes decrease with increasing constrained modulus of the backfill. However, the influence of the intensive compaction of the backfill is not significant in the long-term service condition. Moreover, based on the observations of the pipes' performances during the short-term and long-term service conditions, the curves of each pipe tend to come together when the constrained modulus of the backfill increases. This tendency demonstrates that the larger pipes perform similarly to each other due to the larger confining pressure provided by the backfill with higher constrained modulus.
Figure 4.10: Long-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 1 ksi Constrained Modulus Soil, Dead Load Only

Figure 4.11: Long-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 3 ksi Constrained Modulus Soil, Dead Load Only
Figure 4.12: Long-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only

Figure 4.13: Long-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only
Figure 4.14: Long-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only

Figure 4.15: Long-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Dead Load Only
Figure 4.16: Long-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Dead Load Only

Figure 4.17: Long-Term Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Dead Load Only
The buckling ratios for the pipes under the long-term service condition are listed in Figures 4.16, 4.17, and 4.18. Even when subjected to the highest cover thicknesses determined by the combined compressive strains, the buckling ratios remain below 0.5 if the constrained modulus of the backfill is only 1 ksi. For the pipes surrounded by stiffer backfills, the buckling ratio becomes less noteworthy.

4.1.2 Performances of Pipes under Dead and Live Loads

Figures 4.19 through 4.27 present the performances of the pipes under dead and live loads. To demonstrate the impact of the live loads on the pipes' deformations and mechanical reactions, the performances of the pipes subjected to the dead loads only are also included in these plots. If the soil cover thickness is no more than 5 ft, the outcomes of the AASHTO calculations indicate that the combined compressive strain and buckling ratio at the springline of the pipes are less than the combined compressive strain and buckling ratio at the crown of the pipes. However, the combined compressive strain at the springline becomes primary when the cover thickness becomes greater than 5 ft. Because the largest mechanical reactions are more critical for the pipe structures, the combined compressive strains shown in the plots reflect the strains at the crowns of the pipes if they are greater than the strains at the springlines of the pipes. The buckling ratios in the figures also follow this rule.
Figure 4.19: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft

Figure 4.20: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft
Figure 4.21: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft

Figure 4.22: Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft
Figure 4.23: Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft

Figure 4.24: Short-Term Combined Compressive Strains vs. Vertical Deflections for Various Diameter HDPE Pipes under 5-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft
Figure 4.25: Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 1-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft

Figure 4.26: Buckling Ratios vs. Vertical Deflections for Various Diameter HDPE Pipes under 3-ksi Constrained Modulus Soil, Cover Thicknesses Ranges from 1 ft to 15 ft
In terms of the mechanical reactions of pipes buried in different-modulus backfill material, none of the pipes experiences overstraining or buckling failure even when the cover thickness is as little as 1 ft. When the pipes are buried in 1-ksi backfill and covered by 1 ft of overfill, the vertical deflections of 12-inch, 24-inch, and 30-inch diameter pipes are greater than 7.5%. However, the combined compressive strains and the buckling ratios for these pipes remain in the safe zone. Although the vertical deflections of the pipes never reach 7.5% in other cases, the combined compressive strains and buckling ratios in each case can be estimated by multiplying the factors, which equal 7.5% divided by the vertical deflection in this case. Then, the factored combined compressive strains and buckling ratios are all less than their corresponding limits. Therefore, a 7.5% vertical deflection represents a threshold that ensures that the pipes operate safely even when the pipes are subjected to dead and live loads. The influence of the live load on the pipe performances is substantial when the cover thickness is only 1 ft. Because the load tends to spread out during its transmission to the deeper soil layer, the proportion of live load sustained by the buried pipe will diminish with the increase of the soil cover thickness. The performances of the pipes under the two different (dead, live) loading conditions become more similar when the cover thicknesses increase. The diversity of performances of the pipes under two different load conditions is quickly lost when the cover thickness
increases to 5 ft and more, and the impact of the live loads on the pipe performance absolutely disappears from a cover thickness of 15 ft.

According to Figures 4.22 to 4.24, the combined compressive strain for each diameter of pipe decreases significantly with increasing constrained modulus of the backfill. The buckling ratio in Figures 4.25 to 4.27 indicates that a stiffer backfill can effectively prevent the pipe from buckling. The performances of the pipes significantly decrease when the constrained modulus of the backfill increases from 1 ksi to 3 ksi. However, the improvements in the pipe performances are not equally remarkable, when the constrained modulus of the backfill is increased from 3 ksi to 5 ksi.

4.1.3 Evaluation of AASHTO Calculation Results

The evaluation of AASHTO calculations can only be made using data acquired from actual field experiments. Numerous field tests are documented in Chapter 2, with some being readymade for the AASHTO calculations. Thus, the test data from these field tests are compared here to the results from the AASHTO calculations.

Sargand and Masada (2006) conducted a deep burial project using several HDPE pipes, and the performances of these pipes were recorded. The pipes utilized in the field test were ADS N-12 HDPE pipes, and the backfill materials were a crushed limestone at 90% and 96% compactions and a masonry sand at 86% and 90% compactions. The constrained modulus of the limestone at 90% and 96% compaction levels were in the range of 4 to 5 ksi. The constrained modulus of the sand was slightly higher than 1 ksi at 86% compaction and 3 ksi at 90% compaction level. The vertical deflections of the HDPE pipes measured in the field are compared to the vertical deflections calculated from the AASHTO calculations (see Figures 4.1 to 4.3) in Table 4.1 below. The table also compares the field versus the calculated in terms of the combined compressive strains experienced by the pipe wall.

With backfills having similar constrained modulus values, the vertical deflections from the AASHTO calculations are approximately the same as those in the field test data. Furthermore, the combined compressive strains (see Figures 4.4 to 4.6) and buckling ratios (see Figures 4.7 to 4.9) calculated using the AASHTO equations are all less than their
individual performance limits. No overstraining or buckling occurred in the field test, so the performances of the pipes based on these calculation results are in agreement with the observations at the deep burial project.

Table 4.1: The Comparisons of the Vertical Deflections from the Field Test and the AASHTO Calculations

<table>
<thead>
<tr>
<th>Pipe Dia. (in)</th>
<th>CT (ft)</th>
<th>Field Test Data</th>
<th>AASHTO Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Backfill Type</td>
<td>VD (%)</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
<td>Cl-86</td>
<td>2.10</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
<td>Cl-96</td>
<td>2.53</td>
</tr>
<tr>
<td>42</td>
<td>20</td>
<td>Sn-90</td>
<td>1.36</td>
</tr>
<tr>
<td>42</td>
<td>40</td>
<td>Sn-96</td>
<td>2.19</td>
</tr>
</tbody>
</table>

[Note] "Dia" = Diameter; "CT" = Cover Thickness; "VD" = Vertical Deflection; "CCS" = Combined Compressive Strains; "CMB" = Constrained Modulus of Backfill; "Cl" = Crushed limestone; "Sn" = Sand.

The comparisons of the combined compressive strains measured in the field tests and computed using the AASHTO calculations are listed in the above table, and the calculated strains are substantially higher than the observed strains. This may be due to the fact that the functions or factors used to estimate combined compressive strains are overly conservative in the AASHTO specifications.

Sargand and Masada (2013) investigated a 24-inch diameter corrugated HDPE pipe located in Nobel County, Ohio. This pipe had served State Route 145 since 1981, and it was installed in a backfill whose constrained modulus is 1 ksi. Many vehicles passed through this state route every day, and the cover thickness above the pipe was only 1 to 1.3 ft. The measured vertical deflection of the pipe was 8.3% in 2004, and no signs of structural distress were found. The AASHTO calculations estimate the vertical deflection of the pipe under dead and live loads to be 8.2% (see Figure 4.19), using the constrained modulus of 1 ksi and the cover thickness of only 1 ft. Figures 4.22 and 4.25 show that the maximum combined compressive strain and buckling ratio at the pipe's crown are 3.5% and 0.6, respectively. These calculated performance parameters are in the safe range, and this conforms the observations made in the field.
4.1.4 Critical Parameters Determined by AASHTO Calculations

Because no pipe failed under either the dead or live loads, the critical vertical deflection and critical cover thickness are only determined by the performances of pipes under dead load only. In Subsection 4.1.1, wall buckling never occurs when the pipes are subjected to pressure under the highest cover thicknesses, which are restricted by the limit of the combined compressive strain. Moreover, the vertical deflections of the pipes increase linearly with rising thicknesses of the soil cover. Hence, the critical cover thicknesses are the maximum values of the abscissa displayed in Figures 4.1 to 4.6. The relationships between the buckling ratios and vertical deflections are linear, and the critical vertical deflections and cover thicknesses can also be estimated via an interpolation method. The resulting critical vertical deflections and critical cover thickness values are displayed in Tables 4.2 to 4.4. In the tables, the abbreviation "VD" denotes the critical vertical deflections, and the abbreviation "CT" denotes the critical cover thicknesses.

Table 4.2: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 1 ksi

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>12</td>
<td>15.0</td>
<td>90</td>
</tr>
<tr>
<td>24</td>
<td>13.2</td>
<td>72</td>
</tr>
<tr>
<td>30</td>
<td>13.7</td>
<td>74</td>
</tr>
<tr>
<td>36</td>
<td>12.8</td>
<td>67</td>
</tr>
<tr>
<td>42</td>
<td>12.3</td>
<td>64</td>
</tr>
<tr>
<td>48</td>
<td>12.6</td>
<td>66</td>
</tr>
<tr>
<td>60</td>
<td>12.4</td>
<td>64</td>
</tr>
</tbody>
</table>
Table 4.3: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 3 ksi

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td>Dia. (in)</td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>12</td>
<td>10.9</td>
<td>149</td>
</tr>
<tr>
<td>24</td>
<td>9.4</td>
<td>120</td>
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<td>30</td>
<td>9.7</td>
<td>121</td>
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<td>9.1</td>
<td>109</td>
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<tr>
<td>42</td>
<td>8.8</td>
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<tr>
<td>48</td>
<td>8.8</td>
<td>108</td>
</tr>
<tr>
<td>60</td>
<td>8.7</td>
<td>104</td>
</tr>
</tbody>
</table>

Table 4.4: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 5 ksi

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td>Dia. (in)</td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>12</td>
<td>9.3</td>
<td>187</td>
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<tr>
<td>24</td>
<td>8.2</td>
<td>151</td>
</tr>
<tr>
<td>30</td>
<td>8.3</td>
<td>152</td>
</tr>
<tr>
<td>36</td>
<td>7.9</td>
<td>139</td>
</tr>
<tr>
<td>42</td>
<td>7.7</td>
<td>134</td>
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<tr>
<td>48</td>
<td>7.7</td>
<td>138</td>
</tr>
<tr>
<td>60</td>
<td>7.6</td>
<td>133</td>
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</tbody>
</table>

The critical vertical deflection and critical cover thickness generally decrease with increasing pipe diameter. During both short-term and long-term service conditions, the critical values dictated by the buckling ratio limit are higher than those dictated by the combined compressive limit. The smaller of the critical vertical deflections based on the two failure criteria will be treated as the true critical vertical deflection for each diameter pipe and for each constrained modulus assigned to the backfill type. The same procedure will be applied to identify the critical cover thickness for each diameter of pipe and for each constrained modulus of backfill. The threshold vertical deflections and cover
thicknesses for all pipes are listed in Table 4.5. According to this table, on the average a 7.5% vertical deflection can ensure that the pipes do not experience compression or buckling under the short-term service condition. Under the long-term service condition, a value of 6% can be treated as suitable for the critical vertical deflection. For all different diameter pipes surrounded by each of the three-moduli backfill type, the critical cover thicknesses in the long-term service condition are approximately half of those determined in the short-term service condition. In addition, an increase in the constrained modulus of the backfill leads to a decreased critical vertical deflection and an increased critical cover thickness both in the short-term and long-term service conditions.

Table 4.5: The Threshold Vertical Deflection and Threshold Cover Thickness of Each Diameter Pipe Based on Different Backfill Constrained Modulus

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Critical Vertical Deflection</th>
<th>Critical Cover Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ms = 1 ksi</td>
<td>Ms = 3 ksi</td>
</tr>
<tr>
<td>ST</td>
<td>LT</td>
<td>ST</td>
</tr>
<tr>
<td>12</td>
<td>15.0</td>
<td>8.2</td>
</tr>
<tr>
<td>24</td>
<td>13.2</td>
<td>7.3</td>
</tr>
<tr>
<td>30</td>
<td>13.7</td>
<td>7.5</td>
</tr>
<tr>
<td>36</td>
<td>12.8</td>
<td>7.1</td>
</tr>
<tr>
<td>42</td>
<td>12.3</td>
<td>6.9</td>
</tr>
<tr>
<td>48</td>
<td>12.6</td>
<td>7.0</td>
</tr>
<tr>
<td>60</td>
<td>12.4</td>
<td>6.9</td>
</tr>
<tr>
<td>Ave.</td>
<td>13.1</td>
<td>7.3</td>
</tr>
</tbody>
</table>

[Note] "Ms" = constrained modulus of backfill; "ST" = short-term service condition; "LT" = long-term service condition.

4.2 Theoretical Method Results

The structural responses of the buried pipes are addressed mostly in terms of stresses (instead of strains) in the pipe wall by the theoretical methods. So, the stresses resulting from these analytical methods need to be transformed to equivalent strains before applying the combined compressive strain limit specified by AASHTO. Moreover, none of the theoretical methods includes equations that calculate a safety factor for wall buckling failure mode. Therefore, only combined compressive strains and vertical deflections are presented in Appendix I for the theoretical methods. To make comparisons, the
performances determined by the theoretical methods and by the AASHTO methods are gathered together for each diameter size of the HDPE pipes. The critical vertical deflections and critical cover thicknesses resulting from the AASHTO calculations show that these values are both dominated by the combined compressive strains in the short-term and long-term service conditions. Hence, the combined compressive strains determined by the theoretical methods are sufficient and reliable for estimating the values of the pipes' critical parameters. The loading condition applied in the theoretical methods consist of uniform pressures distributed along the boundaries of the backfill domain. The live load has the effect similar to the concentrated load, so only dead load cases are considered for evaluating the theoretical methods. The AASHTO calculation results consistently show that the strains at the crowns of the pipes are smaller than the strains at the pipes' springlines when the pipes are subjected to the dead load. Therefore, the combined compressive strains at the springline are used as one of the parameters to assess the theoretical methods.

In Appendix I, the pipe performances predicted by each of the theoretical methods are denoted by the author's last name in the legends. "B&R NS" indicates no-slippage (or full-bond) solutions addressed in the Burns and Richard method. "B&R FS" indicates that the free-slip interface solutions contained in the Burns and Richard method. Similarly, "Hoeg NS" indicates the no-slippage part of the Hoeg method. The equations revealing the relationships between the vertical deflections and combined compressive strains (stresses) are identical between the Spangler and Masada methods, so the curves of the combined compressive strains vs. vertical deflections determined by the Spangler and Masada methods are entirely overlapped. However, Spangler assumed that the value of the vertical deflection equals the negative value of the horizontal deflection, and Masada found that the magnitude of the vertical deflection is higher than that of the horizontal deflection. Thus, the vertical deflection in the Masada method is greater than the vertical deflection obtained using the Spangler method when the cover thicknesses are consistent. If no slippage conditions prevail along the pipe-backfill interface, the Burns & Richard method and the Hoeg method generate nearly equal vertical deflection and combined compressive strain for any diameter pipe serving under a certain cover thickness in the short-term and long-term conditions, although these two methods are somewhat different
from each other in terms of their mathematical expressions. Therefore, only the Burns and Richard method (B&R method) is discussed in the following sections. For the convenience of reading, some typical results in Appendix I are shown in this subsection.

4.2.1 Performances of Pipes During Short-Term Service Condition

The short-term vertical deflections and combined compressive strains of the pipes buried in the 1-ksi constrained modulus backfill are shown in Figures I.1 through I.14 (e.g. Figure 4.28 through 4.31). The plots in Figures I.15 through I.28 (e.g. Figure 4.32 through 4.35) summarize the short-term pipe performances with the pipe structures embedded in the 3-ksi backfill soil. And, the short-term performance results with the 5-ksi constrained modulus are presented in Figures I.29 through I.42 (e.g. Figure 4.36 through 4.39). In these plots, the curves showing the combined compressive strains vs. vertical deflections end when the combined compressive strain extends to 6.15%. In the same manner, the vertical deflection vs. cover thickness curves are terminated by using the combined compressive strain limit.

![Figure 4.28: Short-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi](image-url)
Figure 4.29: Short-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi

Figure 4.30: Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi
Figure 4.31: Short-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi

Figure 4.32: Short-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi
Figure 4.33: Short-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi

Figure 4.34: Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi
Figure 4.35: Short-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi

Figure 4.36: Short-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi
Figure 4.37: Short-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi

Figure 4.38: Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi
When the pipes are buried in the 1-ksi backfill and deform to the same level of vertical deflection, the combined compressive strains derived from the full slippage case of the Burns and Richard method are always higher than the strains resulting from the Spangler and Masada methods. And, at the same vertical deflection value and using a constrained modulus of 3 ksi, the combined compressive strains based on the Spangler and Masada methods are larger than those given by the free-slip case of the Burns and Richard method (except for the 12-in diameter pipe) and smaller than those resulting from the no-slip case of the Burns and Richard method and the no-slip case of the Hoeg method. At the same vertical deflection level under 5-ksi for backfill, the combined compressive strains from most of the theoretical methods are approximately equal, except that the curves illustrating the Burns and Richard's full slippage case are lower than the other curves. Whenever a compression failure occurs, the vertical deflections forecasted by all the theoretical methods reach or exceed 7.5% independent of the backfill constrained modulus.

Regardless of the backfill material's constrained modulus, the curves of the vertical deflections determined by the Burns and Richard method's full slippage case almost overlap the lines of the vertical deflections deduced by the Burns and Richard method's no slippage case. This phenomenon demonstrates that the vertical deflection is minimally influenced by the interface friction condition between the pipe wall and backfill. If the
pipes are buried in the 1-ksi backfill and covered by a certain height of overfill, the vertical deflections calculated by the Burns and Richard method are the smallest. However, the vertical deflections derived from the Spangler method are consistently the lowest when the constrained modulus of backfill is equal to 3 ksi and 5 ksi. When installed in a 1-ksi backfill, the vertical deflections obtained from the Masada method are substantially larger than the vertical deflections computed by the free-slip case of the Burns and Richard method. When the constrained modulus of the backfill increases from 3 ksi to 5 ksi, the vertical deflection curves obtained from the Masada method and the Burns and Richard method remain adjacent to each other. However, regarding the 48-inch and 60-inch diameter pipes embedded in the 3-ksi or 5-ksi backfill, the vertical deflection curves are notably steeper by the Masada method than by the free-slip case of the Burns and Richard method. These indicate that the empirical coefficients contained in the Masada method may not be reasonable for large-diameter pipes. Compared to the critical values generated by the Burns and Richard methods, the critical vertical deflections and cover thicknesses computed by the Masada method are tremendous when the constrained modulus of the backfill is only 1 ksi. With the backfill having a 3-ksi constrained modulus, the critical vertical deflections obtained from the Burns and Richard method remain lower than the critical values determined by the Masada method, but the maximum cover thicknesses from the Burns and Richard method (except for the 12-in diameter pipe) surpass the maximum values determined by the Masada method. When the backfill constrained modulus is 5 ksi, the critical cover thicknesses from the Burns and Richard method are higher than the critical cover thicknesses from the Masada method, and the critical vertical deflections from the Burns and Richard method remain at the same or even slightly higher levels as the critical vertical deflections from the Masada method. Obviously, the largest cover thicknesses are obtained when using the Spangler method regardless of the backfill constrained modulus.

4.2.2 Performances of Pipes During Long-Term Service Condition

The long-term performances of the pipes buried in the 1-ksi backfill are presented in Figures I.43 through I.56 (e.g. Figure 4.40 through 4.43). The long-term performances
of the pipes installed in the 3-ksi backfill are illustrated in Figures I.57 through I.70 (e.g. Figure 4.34 through 4.37). For the pipes installed in the 5-ksi constrained modulus backfill, their long-term performances are presented in Figures I.71 through I.84 (e.g. Figure 4.38 through 4.51). In the figures showing the correlations between the combined compressive strains and vertical deflections, the combined compressive strains set the critical vertical deflections. In addition, the allowable maximum cover thicknesses are limited by the critical vertical deflections in the plots.

![Figure 4.40: Long-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi](image-url)
Figure 4.41: Long-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi

Figure 4.42: Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi
Figure 4.43: Long-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi

Figure 4.44: Long-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi
Figure 4.45: Long-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi

Figure 4.46: Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi
Figure 4.47: Long-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 3 ksi

Figure 4.48: Long-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi
Figure 4.49: Long-Term Vertical Deflections vs. Cover Thicknesses for 60-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi

Figure 4.50: Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 5 ksi
The curves in the short-term service condition are separated apart, but the curves of the combined compressive strains and vertical deflections developed by all the theoretical methods are substantially closer to each other in the long-term service condition, except for the curves determined by the free slip case of the Burns and Richard method. At any selected vertical deflection level, the combined compressive strains obtained from the Masada method are distinctly higher than the strains from the no-slip case of the Burns and Richard method for the small to moderate diameter (12 to 42 inches) pipes installed in the 3-ksi and 5-ksi constrained modulus backfills. With respect to the 48-inch diameter pipe, the correlations between the combined compressive strains and vertical deflections obtained by the Masada method almost agree with the correlations set up by the Burns and Richard method's no slippage case. For the 60-inch diameter pipe, the combined compressive strain values estimated by the Masada method are slightly lower than those resulting from the Burns and Richard method's no slippage case. The slopes of the curves correlating the combined compressive strains to the vertical deflections are the mildest for the Burns and Richard method's full slippage case.

Under equal cover thickness, the long-term vertical deflections calculated by the no slip case of the Burns and Richard method are the highest in most conditions, except for the 48-in and 60-in diameter pipes surrounded by the 3-ksi or 5-ksi backfill material. For
these cases, the vertical deflections originating from the Masada method are higher than those obtained from the Burns and Richard method. Under the long-term service condition, the vertical deflections resulting from the Spangler method are the smallest except for the 12-in diameter pipe buried in 1-ksi backfill. The largest critical vertical deflections are always created by the Burns and Richard method's full slippage case. With regards to the pipes with diameter sizes from 12 to 48 inches, the second largest critical vertical deflections are developed by the Burns and Richard method's no slippage case. However, the second highest critical vertical deflections for 60-in diameter pipes are obtained by the Masada method and Spangler method. The Burns and Richard method (no slippage and full slippage cases) ensures that the critical vertical deflections of all diameter pipes are greater than 6% for all three backfill types. Regarding the Masada method and Spangler method, the critical vertical deflections are greater than 6% when the pipes are installed in the 1-ksi backfill, but this critical value decreases to a range between 4% and 5% when the constrained modulus of the backfill increases to 3 ksi or 5 ksi, except that the critical vertical deflections of the 48-in and 60-in diameter pipes are around 6%. The critical cover thicknesses determined by the no slip case of the Burns and Richard method are always the highest for the small to moderate diameter (12 to 42-inch) pipes. With regard to the large diameter (48-in and 60-in) pipes, the critical cover thicknesses from the Spangler method are always the largest. As long as the pipes are enveloped in 1-ksi backfill material, the critical cover thicknesses obtained by the Masada method are always higher than the critical cover thicknesses obtained by the Burns and Richard method's no slippage case. When the pipes are embedded in a 3-ksi or 5-ksi backfill soil, the critical cover thicknesses obtained by the Masada method become smaller than the critical cover thicknesses obtained by the Burns and Richard method's no slippage case.

4.2.3 Evaluation of Theoretical Method Results

Because the AASHTO LRFD calculation method has been proven reasonable based on some field test results in the previous section, the performances (especially the vertical deflections) of the pipes derived from the AASHTO calculations may be used as standards to calibrate the data generated by the theoretical methods. According to the figures
presenting the combined compressive strains vs. vertical deflection relationships in the short-term and long-term service conditions, the curves resulting from the Burns and Richard elastic method's no-slippage interface case are closest to the curves created by the AASHTO method. However, in the short-term service condition the vertical deflections computed by the Burns and Richard method are notably smaller than those calculated by the AASHTO method when the cover thicknesses above the pipes are matching. During the long-term service condition, the differences between the vertical deflections obtained by the Burns and Richard method and through the AASHTO calculations become less notable. The mechanical responses calculated by the Burns and Richard method's full slippage case deviate further away from the strain values estimated by the AASHTO calculations.

Under the short-term service condition, the vertical deflections predicted by the Masada method are similar to the vertical deflections estimated by the AASHTO method for any given cover thickness. However, the long-term vertical deflections calculated by the Masada method are not close to the long-term vertical deflection determined by the AASHTO methods. Moreover, in the short term service condition, the relationships between the combined compressive strains and vertical deflections under the Masada method are acceptable for the pipes wrapped by backfill with 3 ksi or 5 ksi constrained modulus. When pipes are buried in backfill with 1 ksi constrained modulus, the differences in the short-term combined compressive strains generated by the Masada method and the AASHTO calculations are remarkable. Because the vertical deflections calculated by the Spangler method are quite different from the vertical deflections computed through the AASHTO calculations, the pipe performances determined by the Spangler method are not sufficiently precise.

To sum up, the best theoretical methods for forecasting the pipe performances are the Masada method and the Burns and Richard method's no slippage case, whose outcomes are identical to those derived from the Hoeg method's slippage case. To verify the accuracy of the Masada method and the Hoeg method's no slippage case, the experimental data observed in a deep burial project conducted by Sargand and Masada (2006) are compared. The calculated results and detected values are listed in Table 4.6.
Table 4.6: The Comparison of the Vertical Deflection from the Field Test, The Masada Method and The No Slippage Case of the Hoeg Method

<table>
<thead>
<tr>
<th>Pipe Dia. (in)</th>
<th>CT (ft)</th>
<th>Field Test Data</th>
<th>Masada Method</th>
<th>Hoeg Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Backfill Type</td>
<td>VD (%)</td>
<td>CCS (%)</td>
</tr>
<tr>
<td>30</td>
<td>20</td>
<td>Cl-86</td>
<td>2.07</td>
<td>N/A</td>
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<td>40</td>
<td>Cl-96</td>
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<td>0.40</td>
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<td>Sn-96</td>
<td>2.36</td>
<td>0.35</td>
</tr>
<tr>
<td>42</td>
<td>20</td>
<td>Cl-90</td>
<td>1.32</td>
<td>N/A</td>
</tr>
</tbody>
</table>

[Note] "Dia" = Diameter; "CT" = Cover Thickness; "VD" = Vertical Deflection; "CCS" = Combined Compressive Strains; "CMB" = Constrained Modulus of Backfill; "Cl" = Crushed limestone; "Sn" = Sand.

Considering the comparisons in Table 4.6, the vertical deflections calculated by the Masada method appear to be more similar to the field test data. Because the material properties of the HDPE pipes in the theoretical calculations are assumed according to the AASHTO Specifications, the combined compressive strains determined by the Masada method and the no-slippage case of the Hoeg Method are both much higher than the data collected from the field tests. However, the combined compressive strains from these two theoretical methods are both in the safe zone, and no buried pipes were found structural distresses during the tests. Compared to the combined compressive strains derived from the Masada method, the combined compressive strains computed by the Hoeg method are more similar to the strains detected in the field test.

4.2.4 Critical Parameters Determined by Theoretical Methods

The effectiveness and reliability of the Masada method and Hoeg method (Burns and Richard method) have been examined in Section 4.2.3. The critical vertical deflections and critical cover thicknesses are decided in light of the results obtained from these two methods. In addition, only the combined compressive strains and vertical deflections are examined in the theoretical methods, so the values of the critical parameters are only dominated by the compression failure. The critical values are listed in Table 4.7 through 4.9. In these tables, "VD" denotes the critical vertical deflections, and "CT" denotes the critical cover thicknesses.
Table 4.7: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 1 ksi

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Masada Method</th>
<th>Hoeg Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%) CT (ft)</td>
<td>VD (%) CT (ft)</td>
</tr>
<tr>
<td>12</td>
<td>20.8 175</td>
<td>9.2 78</td>
</tr>
<tr>
<td>24</td>
<td>26.7 187</td>
<td>8.1 57</td>
</tr>
<tr>
<td>30</td>
<td>24.7 174</td>
<td>7.9 56</td>
</tr>
<tr>
<td>36</td>
<td>25.3 170</td>
<td>7.5 51</td>
</tr>
<tr>
<td>42</td>
<td>26.4 175</td>
<td>7.5 49</td>
</tr>
<tr>
<td>48</td>
<td>24.2 142</td>
<td>8.3 49</td>
</tr>
<tr>
<td>60</td>
<td>22.3 121</td>
<td>8.9 48</td>
</tr>
</tbody>
</table>

Table 4.8: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 3 ksi

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Masada Method</th>
<th>Hoeg Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%) CT (ft)</td>
<td>VD (%) CT (ft)</td>
</tr>
<tr>
<td>12</td>
<td>14.6 274</td>
<td>4.8 90</td>
</tr>
<tr>
<td>24</td>
<td>11.3 170</td>
<td>5.3 79</td>
</tr>
<tr>
<td>30</td>
<td>11.0 168</td>
<td>5.1 77</td>
</tr>
<tr>
<td>36</td>
<td>10.2 145</td>
<td>5.1 73</td>
</tr>
<tr>
<td>42</td>
<td>10.0 139</td>
<td>5.1 71</td>
</tr>
<tr>
<td>48</td>
<td>10.5 120</td>
<td>6.3 72</td>
</tr>
<tr>
<td>60</td>
<td>10.8 108</td>
<td>7.1 71</td>
</tr>
</tbody>
</table>

Table 4.9: The Critical Vertical Deflection and Critical Cover Thickness of Each Diameter Pipe Based on Various Failure Mode, Ms = 5 ksi

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Masada Method</th>
<th>Hoeg Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%) CT (ft)</td>
<td>VD (%) CT (ft)</td>
</tr>
<tr>
<td>12</td>
<td>9.2 246</td>
<td>4.0 107</td>
</tr>
<tr>
<td>24</td>
<td>8.1 164</td>
<td>4.7 95</td>
</tr>
<tr>
<td>30</td>
<td>7.9 162</td>
<td>4.5 94</td>
</tr>
<tr>
<td>36</td>
<td>7.5 142</td>
<td>4.6 87</td>
</tr>
<tr>
<td>42</td>
<td>7.4 136</td>
<td>4.7 85</td>
</tr>
<tr>
<td>48</td>
<td>8.3 120</td>
<td>5.9 85</td>
</tr>
<tr>
<td>60</td>
<td>8.9 108</td>
<td>6.7 82</td>
</tr>
</tbody>
</table>
Based on the pipe performances derived from the Masada method, the short-term critical vertical deflections for the pipes buried in 1-ksi backfill appear to be extremely high, and the critical vertical deflections of pipes enveloped in 3-ksi or 5-ksi backfill are overly conservative in the long-term service condition. Whereas, the short-term critical cover thicknesses of the pipes show insignificant growths with stiffer backfill materials.

In the Hoeg method, the short-term critical vertical deflections are all above 7.5%, and the long-term critical vertical deflections are higher than 6%. This statement concurs in the comment issued in light of critical parameters in the AASHTO calculations. Furthermore, the critical vertical deflections decrease and the critical cover thicknesses increase with increasing backfill constrained modulus. Therefore, the best methods among all the theoretical methods for revealing the pipe performances are the Hoeg method's no slippage case and the Burns and Richard method's no slippage case.

4.3 CANDE Simulation Results

A total of seven backfill types, which are introduced in Chapter 3, are adopted in the CANDE simulations. The performances of all diameter pipes under various backfill types are illustrated in the 112 figures listed in Appendix II, and a few typical results are presented in this subsection. Because the 24-in diameter size HDPE pipe performs worst in the CANDE simulations, the typical results include the mechanical responses of the 24-in diameter pipe, as well as the plots showing the failures of the pipes. The pipe performances computed by CANDE software include the vertical deflections, thrust stresses, and combined compressive strains. The thrust stresses estimated by CANDE always remain large, which can induce extraordinarily conservative values of the critical vertical deflections and critical cover thicknesses. This special character of CANDE has been revealed by Katona, and his discussion is referred to in Chapter 2. Therefore, the thrust stresses are eliminated from the pipe performances in this dissertation. In addition, the critical vertical deflections and critical cover thicknesses are only controlled by the buckling ratios and combined compressive strains, whichever comes first. In the CANDE simulation, the buckling in the pipe wall is related to the thrust stress. Thus, the buckling ratios may be more crucial than the combined compressive strains. Similar to the results
in AASHTO calculations, the buckling ratios in the long-term service condition are too small to be considered. Therefore, the long-term buckling ratios are omitted from the results of the CANDE simulations. In the analysis considering dead and live loads, the highest combined compressive strains appear at the crowns of the pipes when the cover thicknesses are small. If the backfill is CA-90 or SW-85, the locations that exhibit the worst mechanical reactions are the crowns of the pipes when the overfill depths are no more than 10 ft. When CA-95 and SW-90 are adopted as backfill, the springlines of pipes experiences the greatest strains and buckling ratios when the cover thicknesses are deeper than 7.5 ft. With regard to other backfill types, the crowns of the pipes are primary concern if the overfill is no deeper than 5 ft.

4.3.1 Performances of Pipes Installed in the Embankment Mode

According to Figures II.1 through II.7 (e.g. Figure 4.52), the vertical deflections of the pipes increase linearly with increasing cover thickness when the pipes are buried in coarse aggregate (CA) or silty sands (SM). However, the relationship between the vertical deflection and cover thickness tends to be hyperbolic when the backfills around the pipes are coarse sand (SW). These different curves result from the fact that the material properties of the CA and SM backfill types are defined by the Duncan formulation, and the material properties of the SW backfill type are simulated by the Duncan/Selig formulation. In the Duncan formulation, the bulk modulus of the soil follows a power law, but a hyperbolic law is applied to the soil bulk modulus in the Duncan/Selig formulation. For pipes installed in the SW backfill, the growth rate of the vertical deflection gradually decreases with increasing cover thickness. Especially for the SW-95 backfill, such backfill can help pipes bear extremely high soil cover thickness while producing small deformations.
The combined compressive strains are displayed in Figures II.8 through II.14 (e.g. Figure 4.53). Equivalently, linear correlations exist between the strains and the vertical deflections when the pipes are buried in the CA and SW backfill. If the pipes are covered by the SW backfill, the strains will increase with the power function with increasing vertical deflection. Inferring from the rule whereby a backfill with a higher elastic modulus causes pipes to fail at lower vertical deflection, the SM-90 and SW-95 backfill types present the highest elastic moduli, the SM-85 and SW-90 backfill types present moderate elastic moduli, and the elastic moduli of the SW-85, CA-90 and CA-95 are relatively low. To prevent the occurrence of compression failure, a 7.5% vertical deflection limit is sufficient to protect pipes under most backfill types except for SW-95, because the constrained modulus of the SW-95 appears to be higher than 5 ksi in the CANDE material library.
Figure 4.53: Short-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Based on Figures II.15 through II.21 (e.g. Figure 4.54), the buckling ratio vs. vertical deflection curves are linear for the pipes in the backfill characterized by the Duncan formulation, but the relationships for the pipes surrounded by the SW backfill are irregular in these figures. This unreasonable demonstration implies that the Duncan/Selig formulation cannot properly estimate the buckling ratio. Therefore, the critical vertical deflection and critical cover thickness governed by the buckling ratio are only referential numbers instead of considerable values. When pipes are installed under the SW-85 or SW-90 backfill, buckling never occurs for any diameter pipe before the vertical deflection reaches 7.5%. Two CA backfill types can ensure that the critical vertical deflections of the small-diameter pipes (12 to 42-inch) remain greater than 7.5%. However, the critical vertical deflections of 48-in and 60-in diameter pipes are in the range of 6% to 7%. If the pipes are concealed by the SM-85, SM-90, or SW-95 backfill, the critical cover thicknesses of all diameter pipes are approximately 6%.
Under the long-term service condition, the universal patterns and tendency of the pipe performances are fairly consistent with the mechanical reactions of pipes serving in the short-term service condition. Compared with the maximum allowable cover thicknesses in the short-term service condition, the peak cover thickness of each diameter pipe and backfill type illustrated in Figures II.22 through II.28 (e.g. Figure 4.55) decreases by at least two thirds. In terms of the observations from Figures II.29 through II.35 (e.g. Figure 4.56), the combined compressive strains in the pipe wall tend to linearly increase with increasing vertical deflection. Moreover, the elastic modulus of the SW-85 backfill drops to a minimum in the long-term service condition; the CA-90, CA-95 and SW-90 backfill types exhibit moderate elastic moduli; and the elastic moduli of the SM-85, SM-90 and SW-95 remain at the high level. The critical vertical deflections under compressive failure vary between 6% and 8%.
The relationships between the vertical deflections and the cover thicknesses for the pipes under dead and live loads are presented in Figures II.36 to II.42 (e.g. Figure 4.57), the combined compressive strains under dead and live loads are shown in Figures II.43 to II.49 (e.g. Figure 4.58), and the plots from Figures II.50 to II.56 (e.g. Figure 4.59) illustrate the buckling ratios under dead and live load conditions. Under the dead and live loads, the 24-inch diameter pipe (see Figure 4.58) suffers a compression failure at its crown when
the backfill is CA-90 or SM-85 type and when the cover thickness is only 1 ft. Furthermore, when the cover thickness is 1 ft, the 12-inch diameter pipe (see Figure 4.59) buckles at the crown when it is buried in the CA-90 backfill and almost buckles when installed in the CA-95 or SW-85 backfill. Under a 1-ft cover thickness, buckling appears in the 24-inch diameter pipe (see Figure 4.60) installed in most of the backfill types except for SM-90, SW-90 and SW-95. Although the 48-inch diameter pipe (see Figure 4.61) under a 1-ft soil cover buckles when it is covered by the CA-90 backfill, no compression or buckling failure is discovered when the cover thickness is 2.5 ft or more, regardless of the backfill type. According to Figures II.51 through II.65, the performances of the 12-inch diameter pipe under both dead load only and dead and live loads may become consistent at a cover thickness of 25 ft, and all other diameter pipes seem to perform uniformly under two different load conditions when the cover thickness increases to 20 ft.

![Figure 4.57: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode](image-url)
Figure 4.58: Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Figure 4.59: Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
4.3.2 Performances of Pipes Installed in the Trench Mode

For the pipes under the dead soil load only, the short-term and long-term structural performances are shown in Figures II.57 through II.77 (e.g. Figure 4.62 through 4.64) and Figures II.78 through II.91 (e.g. Figure 4.65 through 4.66), respectively. Theoretically, the stiffer soil in the trench wall can help the pipe share the external load, but the results given by CANDE suggest that the assistance provided by the trench is not remarkable.
Covered by the same backfill types and cover thickness, the performances of each diameter pipe in the trench mode and in the embankment mode are roughly identical. Compared to the critical vertical deflections found in the embankment mode, the trench mode provides a slight increase in the critical vertical deflection values, except in the cases where the pipes are installed in the SW-90 or SW-95 backfill. Because the constrained modulus of the trench wall is assumed to be a constant value (5 ksi), even though the constrained moduli of the SW backfills increase with rising soil cover thickness, the ultimate constrained modulus of the SW-90 or SW-95 type surpasses 5 ksi when the pipes are subject to high overfill depth. This is why the embankment mode is stiffer than the trench mode when the SW-90 and SW-95 backfill types are selected.

Figure 4.62: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Figure 4.63: Short-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Figure 4.64: Short-Term Buckling Ratios vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Figure 4.65: Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Figure 4.66: Long-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

The performances of the pipes buried in the trench mode are shown in plots in Figures II.92 through II.112 (e.g. Figure 4.67 through 4.70) under dead and live loads. The combined compressive strain at the crown of the 24-inch diameter pipe (see Figure 4.68) continues to be higher than the combined compressive strain limit when this pipe is embedded in the CA-90 backfill. The buckling ratio of the 12-inch diameter pipe (see Figure 4.69) surpasses 1.0 when the pipe is covered by a 1-ft thick CA-90 backfill, and the
24-inch diameter pipe (see Figure 4.70) buckles under a 1-ft cover thickness when the backfill type is CA-90, CA-95 or SM-85. Based on these results, the trench mode is found to optimize the pipe performances compared to the embankment mode, but the benefits of the trench mode are not remarkable.

![Graph 1](Image)

**Figure 4.67: Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode**

![Graph 2](Image)

**Figure 4.68: Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under Various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode**
4.3.3 Evaluation of CANDE Simulation Results

Hashash and Selig (1989) conducted a deep burial field study on a 24-inch diameter corrugated HDPE pipe. The pipe was installed in a 5-ft-deep, 6-ft-wide trench, and the backfill was well-graded crushed limestone compacted to 100% of the standard Proctor maximum dry unit weight. Then, an embankment soil filled up to 99 ft was placed over the pipe. In the CANDE simulations, CA is appropriate for the backfill type utilized in
this study. However, CANDE's material library only offers 90% and 95% compaction levels for the CA material. Hence, the performances of the 24-inch pipe embedded in the CA-100 backfill are extrapolated linearly, based on the mechanical reactions of the same pipe buried in the CA-90 and CA-95 backfill soils. The estimated vertical deflection of the 24-inch diameter pipe under maximum cover thickness is listed in the following table.

Table 4.10: The Vertical Deflection Observed from The Field Test and Estimated from The CANDE Simulations

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Field Test</th>
<th>CANDE Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Backfill</td>
<td>Cover Thickness (ft)</td>
</tr>
<tr>
<td>24</td>
<td>99</td>
<td>100% compacted crushed limestone</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The vertical deflection values estimated in the CANDE simulations (see Figure 4.62) are higher than the vertical deflections measured in the field test. The discrepancy between them may be explained by the fact that the elastic modulus of the CA backfill in CANDE is lower than the elastic modulus of the actual crushed limestone material at the same compaction level. According to the discussions made in the previous two sections, the elastic modulus of the CA backfill appears to remain in the lower range. In the CANDE simulations, the combined compressive strain (see Figure 4.63) and buckling ratio (see Figure 4.64) are below their limits when the backfill is the CA-95 type. Because the strains and buckling ratio decrease with increasing compaction level, the 24-inch diameter pipe does not experience any failure under a 99-ft cover thickness, and the actual pipe in the field test exhibited no signs of failure.

Moser (2000) loaded a 42-inch diameter profile-wall HDPE pipe to failure and reported the maximum vertical deflection and cover thickness when the failure during his tests. The critical vertical deflection and critical cover thickness from the field test and CANDE simulations are displayed in the following two tables.
Table 4.11: The Critical Vertical Deflection Reported from The Field Test and Estimated from The CANDE Simulations

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Backfill</th>
<th>Critical Vertical Deflection (%)</th>
<th>Dia. (in)</th>
<th>Backfill</th>
<th>Critical Vertical Deflection (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>SM-85</td>
<td>16.5</td>
<td>42</td>
<td>SM-85</td>
<td>6.1 (buckling) 8.7 (compression failure)</td>
</tr>
<tr>
<td>42</td>
<td>SM-95</td>
<td>6.7</td>
<td>42</td>
<td>SM-90</td>
<td>5.4 (buckling) 7.6 (compression failure)</td>
</tr>
</tbody>
</table>

The estimated critical vertical deflections of the 42-inch diameter pipe buried in the SM-95 backfill are 4.7% for buckling and 6.5% for compression failure. In the field test, dimpling appeared at a vertical deflection of 3.5%, which is slightly lower than the critical vertical deflection controlled by buckling. However, wall crushing occurred at a vertical deflection of 6.7%, which is very close to the critical vertical deflection based on the compression strain limit. Unfortunately, the critical vertical deflection of the 42-inch pipe installed in the SM-85 backfill is approximate one third of the vertical deflection found in the field load tests. Therefore, it appears that the critical vertical deflection given by CANDE tend to be overly conservative.

Table 4.12: The Critical Cover Thickness Reported from The Field Test and Estimated from The CANDE Simulations

<table>
<thead>
<tr>
<th>Dia. (in)</th>
<th>Backfill</th>
<th>Critical Cover Thickness (ft)</th>
<th>Dia. (in)</th>
<th>Backfill</th>
<th>Critical Cover Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>SM-85</td>
<td>110</td>
<td>42</td>
<td>SM-85</td>
<td>105 (buckling) 147 (compression failure)</td>
</tr>
<tr>
<td>42</td>
<td>SM-95</td>
<td>170</td>
<td>42</td>
<td>SM-90</td>
<td>113 (buckling) 159 (compression failure)</td>
</tr>
</tbody>
</table>

The critical cover thickness for a 42-in diameter pipe installed in the SM-95 backfill is forecasted to be 121 ft based on the buckling limit and 171 ft based on the compression failure. Concerning the SM-85 backfill, the critical cover thickness determined by buckling is acceptable. However, for the cases in which the SM-95 backfill soil is specified, the critical cover thickness can be reliably considered based on the compression strain limit.
4.3.4 The Critical Coefficients Determined by the CANDE Simulations

The critical vertical deflection and critical cover thickness are computed according to the pipe performances under the dead load only. Although nonlinearity is the primary characteristics of the performances of the pipes installed in the backfill material whose stress-strain behavior is simulated by the Duncan/Selig formulation, the critical coefficients still can be determined by the interpolation method because the increment of the cover thickness is admissible. The values of the critical coefficients for each diameter pipe buried in disparate installation modes are listed from Tables 4.13 through Table 4.26. The abbreviation of the critical vertical deflection is "VD", and the critical cover thickness is abbreviated as "CT".

**Table 4.13: The Critical Vertical Deflection and Critical Cover Thickness for 12-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)**

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)  CT (ft)</td>
<td>VD (%)  CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.2   179</td>
<td>8.2   58</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.5   200</td>
<td>7.7   73</td>
</tr>
<tr>
<td>SM-85</td>
<td>9.6    250</td>
<td>6.7   98</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.9    283</td>
<td>6.4   125</td>
</tr>
<tr>
<td>SW-85</td>
<td>8.9    365</td>
<td>8.9   95</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.3    398</td>
<td>7.4   128</td>
</tr>
<tr>
<td>SW-95</td>
<td>6.3    634</td>
<td>6.0   267</td>
</tr>
</tbody>
</table>
Table 4.14: The Critical Vertical Deflection and Critical Cover Thickness for 24-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.2</td>
<td>127</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.1</td>
<td>145</td>
</tr>
<tr>
<td>SM-85</td>
<td>8.8</td>
<td>179</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.0</td>
<td>211</td>
</tr>
<tr>
<td>SW-85</td>
<td>9.7</td>
<td>270</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.6</td>
<td>294</td>
</tr>
<tr>
<td>SW-95</td>
<td>6.6</td>
<td>510</td>
</tr>
</tbody>
</table>

Table 4.15: The Critical Vertical Deflection and Critical Cover Thickness for 30-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>13.1</td>
<td>130</td>
</tr>
<tr>
<td>CA-95</td>
<td>12.0</td>
<td>156</td>
</tr>
<tr>
<td>SM-85</td>
<td>8.5</td>
<td>153</td>
</tr>
<tr>
<td>SM-90</td>
<td>7.5</td>
<td>181</td>
</tr>
<tr>
<td>SW-85</td>
<td>9.6</td>
<td>215</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.7</td>
<td>252</td>
</tr>
<tr>
<td>SW-95</td>
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<td>427</td>
</tr>
</tbody>
</table>
Table 4.16: The Critical Vertical Deflection and Critical Cover Thickness for 36-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%) CT (ft)</td>
<td>VD (%) CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.8 138</td>
<td>8.3 47</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.7 158</td>
<td>7.7 60</td>
</tr>
<tr>
<td>SM-85</td>
<td>9.2 196</td>
<td>6.5 80</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.3 231</td>
<td>6.2 106</td>
</tr>
<tr>
<td>SW-85</td>
<td>9.9 276</td>
<td>9.2 58</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.6 320</td>
<td>7.6 100</td>
</tr>
<tr>
<td>SW-95</td>
<td>7.0 553</td>
<td>6.6 245</td>
</tr>
</tbody>
</table>

Table 4.17: The Critical Vertical Deflection and Critical Cover Thickness for 42-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%) CT (ft)</td>
<td>VD (%) CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.3 113</td>
<td>7.1 37</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.1 132</td>
<td>6.7 47</td>
</tr>
<tr>
<td>SM-85</td>
<td>7.9 132</td>
<td>5.7 54</td>
</tr>
<tr>
<td>SM-90</td>
<td>7.2 157</td>
<td>5.3 65</td>
</tr>
<tr>
<td>SW-85</td>
<td>10.0 179</td>
<td>8.7 47</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.8 209</td>
<td>6.9 67</td>
</tr>
<tr>
<td>SW-95</td>
<td>7.3 385</td>
<td>6.6 142</td>
</tr>
</tbody>
</table>
Table 4.18: The Critical Vertical Deflection and Critical Cover Thickness for 48-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
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<td>124</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.7</td>
<td>143</td>
</tr>
<tr>
<td>SM-85</td>
<td>8.8</td>
<td>173</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.0</td>
<td>208</td>
</tr>
<tr>
<td>SW-85</td>
<td>10.4</td>
<td>241</td>
</tr>
<tr>
<td>SW-90</td>
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<td>282</td>
</tr>
<tr>
<td>SW-95</td>
<td>7.1</td>
<td>486</td>
</tr>
</tbody>
</table>

Table 4.19: The Critical Vertical Deflection and Critical Cover Thickness for 60-in Diameter Pipe Installed in Various Backfill, Embankment Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
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<td>125</td>
</tr>
<tr>
<td>CA-95</td>
<td>12.0</td>
<td>143</td>
</tr>
<tr>
<td>SM-85</td>
<td>8.9</td>
<td>172</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.0</td>
<td>205</td>
</tr>
<tr>
<td>SW-85</td>
<td>10.0</td>
<td>225</td>
</tr>
<tr>
<td>SW-90</td>
<td>9.1</td>
<td>281</td>
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<tr>
<td>SW-95</td>
<td>7.5</td>
<td>481</td>
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</table>
Table 4.20: The Critical Vertical Deflection and Critical Cover Thickness for 12-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)

<table>
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<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT(ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.6</td>
<td>183</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.7</td>
<td>206</td>
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<tr>
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<td>9.8</td>
<td>255</td>
</tr>
<tr>
<td>SM-90</td>
<td>9.0</td>
<td>285</td>
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<tr>
<td>SW-85</td>
<td>9.2</td>
<td>298</td>
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<tr>
<td>SW-90</td>
<td>8.6</td>
<td>357</td>
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<tr>
<td>SW-95</td>
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<td>494</td>
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</table>

Table 4.21: The Critical Vertical Deflection and Critical Cover Thickness for 24-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT(ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.8</td>
<td>151</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.4</td>
<td>167</td>
</tr>
<tr>
<td>SM-85</td>
<td>9.2</td>
<td>191</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.2</td>
<td>215</td>
</tr>
<tr>
<td>SW-85</td>
<td>9.9</td>
<td>226</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.7</td>
<td>265</td>
</tr>
<tr>
<td>SW-95</td>
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<td>337</td>
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</table>
Table 4.22: The Critical Vertical Deflection and Critical Cover Thickness for 30-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)

<table>
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<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
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<td>185</td>
</tr>
<tr>
<td>CA-95</td>
<td>12.0</td>
<td>195</td>
</tr>
<tr>
<td>SM-85</td>
<td>9.2</td>
<td>167</td>
</tr>
<tr>
<td>SM-90</td>
<td>7.9</td>
<td>182</td>
</tr>
<tr>
<td>SW-85</td>
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<td>230</td>
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<td>261</td>
</tr>
<tr>
<td>SW-95</td>
<td>7.3</td>
<td>385</td>
</tr>
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</table>

Table 4.23: The Critical Vertical Deflection and Critical Cover Thickness for 36-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>13.3</td>
<td>175</td>
</tr>
<tr>
<td>CA-95</td>
<td>12.1</td>
<td>195</td>
</tr>
<tr>
<td>SM-85</td>
<td>9.7</td>
<td>213</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.6</td>
<td>235</td>
</tr>
<tr>
<td>SW-85</td>
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<td>283</td>
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<tr>
<td>SW-95</td>
<td>7.0</td>
<td>347</td>
</tr>
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</table>
Table 4.24: The Critical Vertical Deflection and Critical Cover Thickness for 42-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%) CT (ft)</td>
<td>VD (%) CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.6 269</td>
<td>7.8 62</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.2 176</td>
<td>7.1 66</td>
</tr>
<tr>
<td>SM-85</td>
<td>8.7 147</td>
<td>6.0 59</td>
</tr>
<tr>
<td>SM-90</td>
<td>7.6 159</td>
<td>5.5 65</td>
</tr>
<tr>
<td>SW-85</td>
<td>9.7 200</td>
<td>8.0 69</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.4 220</td>
<td>7.0 88</td>
</tr>
<tr>
<td>SW-95</td>
<td>7.4 325</td>
<td>6.5 144</td>
</tr>
</tbody>
</table>

Table 4.25: The Critical Vertical Deflection and Critical Cover Thickness for 48-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%) CT (ft)</td>
<td>VD (%) CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.3 152</td>
<td>8.1 55</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.1 170</td>
<td>7.5 69</td>
</tr>
<tr>
<td>SM-85</td>
<td>9.3 192</td>
<td>6.5 77</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.3 214</td>
<td>6.1 97</td>
</tr>
<tr>
<td>SW-85</td>
<td>10.3 227</td>
<td>8.8 62</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.8 253</td>
<td>7.4 106</td>
</tr>
<tr>
<td>SW-95</td>
<td>7.2 310</td>
<td>6.5 167</td>
</tr>
</tbody>
</table>
Table 4.26: The Critical Vertical Deflection and Critical Cover Thickness for 60-in Diameter Pipe Installed in Various Backfill, Trench Mode (CANDE)

<table>
<thead>
<tr>
<th>Backfill</th>
<th>Combined Compressive Strain Limit (6.15%)</th>
<th>Buckling Ratio Limit (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td></td>
<td>VD (%)</td>
<td>CT (ft)</td>
</tr>
<tr>
<td>CA-90</td>
<td>12.3</td>
<td>154</td>
</tr>
<tr>
<td>CA-95</td>
<td>11.2</td>
<td>173</td>
</tr>
<tr>
<td>SM-85</td>
<td>9.5</td>
<td>195</td>
</tr>
<tr>
<td>SM-90</td>
<td>8.5</td>
<td>214</td>
</tr>
<tr>
<td>SW-85</td>
<td>10.3</td>
<td>230</td>
</tr>
<tr>
<td>SW-90</td>
<td>9.0</td>
<td>245</td>
</tr>
<tr>
<td>SW-95</td>
<td>7.4</td>
<td>309</td>
</tr>
</tbody>
</table>

Summarizing the data presented in the above tables, the minimum vertical deflection (and cover thickness) of each diameter pipe buried in each backfill type is treated as the threshold vertical deflection (and cover thickness), regardless of the installation mode. The values of the threshold coefficients are listed in Tables 4.27 and 4.28.

Table 4.27: The Threshold Vertical Deflection of Each Diameter Pipe Based on Different Backfill Constrained Modulus

<table>
<thead>
<tr>
<th>Backfill</th>
<th>12-in</th>
<th>24-in</th>
<th>36-in</th>
<th>48-in</th>
<th>60-in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ST</td>
<td>LT</td>
<td>ST</td>
<td>LT</td>
<td>ST</td>
</tr>
<tr>
<td>CA-90</td>
<td>7.8</td>
<td>8.2</td>
<td>7.4</td>
<td>7.9</td>
<td>7.9</td>
</tr>
<tr>
<td>CA-95</td>
<td>7.7</td>
<td>7.7</td>
<td>7.2</td>
<td>7.4</td>
<td>7.4</td>
</tr>
<tr>
<td>SM-85</td>
<td>6.5</td>
<td>6.7</td>
<td>6.0</td>
<td>6.3</td>
<td>6.1</td>
</tr>
<tr>
<td>SM-90</td>
<td>7.2</td>
<td>6.4</td>
<td>6.6</td>
<td>6.0</td>
<td>6.5</td>
</tr>
<tr>
<td>SW-85</td>
<td>8.4</td>
<td>8.9</td>
<td>9.5</td>
<td>9.1</td>
<td>9.5</td>
</tr>
<tr>
<td>SW-90</td>
<td>8.2</td>
<td>7.4</td>
<td>8.6</td>
<td>7.4</td>
<td>8.3</td>
</tr>
<tr>
<td>SW-95</td>
<td>5.6</td>
<td>6.0</td>
<td>6.0</td>
<td>6.2</td>
<td>6.1</td>
</tr>
<tr>
<td>Ave.</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.2</td>
<td>7.4</td>
</tr>
</tbody>
</table>

[Note] "ST" = short-term service condition; "LT" = long-term service condition.
Table 4.28: The Threshold Cover Thickness of Each Diameter Pipe Based on Different Backfill Constrained Modulus

<table>
<thead>
<tr>
<th>Backfill</th>
<th>12-in</th>
<th>24-in</th>
<th>36-in</th>
<th>48-in</th>
<th>60-in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ST</td>
<td>ST</td>
<td>ST</td>
<td>ST</td>
<td>ST</td>
</tr>
<tr>
<td>CA-90</td>
<td>110</td>
<td>58</td>
<td>75</td>
<td>44</td>
<td>80</td>
</tr>
<tr>
<td>CA-95</td>
<td>130</td>
<td>73</td>
<td>90</td>
<td>56</td>
<td>96</td>
</tr>
<tr>
<td>SM-85</td>
<td>170</td>
<td>98</td>
<td>124</td>
<td>74</td>
<td>132</td>
</tr>
<tr>
<td>SM-90</td>
<td>230</td>
<td>125</td>
<td>175</td>
<td>100</td>
<td>184</td>
</tr>
<tr>
<td>SW-85</td>
<td>298</td>
<td>91</td>
<td>226</td>
<td>59</td>
<td>226</td>
</tr>
<tr>
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<td>333</td>
<td>126</td>
<td>265</td>
<td>92</td>
<td>264</td>
</tr>
<tr>
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<td>433</td>
<td>234</td>
<td>290</td>
<td>186</td>
<td>293</td>
</tr>
<tr>
<td>Ave.</td>
<td>243</td>
<td>115</td>
<td>178</td>
<td>87</td>
<td>182</td>
</tr>
</tbody>
</table>

[Note] "ST" = short-term service condition; "LT" = long-term service condition.

In the AASHTO calculations, both the short-term and long-term critical vertical deflections are determined by the combined compressive strains. Thus, the long-term critical vertical deflections are less than the short-term values if the comparisons are based on the AASHTO results. In the CANDE simulations, the short-term critical vertical deflections are controlled by the buckling ratios in most cases, and the long-term critical vertical deflections are determined by the combined compressive strains. The buckling ratio in the CANDE simulations is based on the stress, which tends to be over-estimated by the software. That is why the critical vertical deflections are smaller in the short-term service condition than in the long-term service condition in the CANDE simulations.
CHAPTER 5: VISCOELASTIC ANALYSIS

Traditionally, the long-term performances of buried pipeline structures such as HDPE pipes are estimated by either applying an empirical factor to the short-term pipe performances or by reducing the elastic modulus of the pipe material. By this intuitive approach, the long-term pipe performance can be revealed only after a specific amount of time (ex. 50 years). The assessment of the pipe performance is often made only sporadically during the pipe's continued service life. With this argument, it is desirable to add the time factor as one of the parameters in analyzing the pipe performance. Then, the pipe's mechanical responses can be estimated continuously along its service life. None of the analytical methods utilized up to this point considered the time as a fundamental parameter. In this chapter, the HDPE pipe performance is analyzed through the viscoelastic method, and the critical vertical deflections and critical soil cover thicknesses for the pipes are both determined based on the combined compressive strains given by viscoelastic method. One of the key goals of this advanced analysis is to identify for each diameter size of the HDPE pipes the maximum vertical deflection at the end of construction, which will assure no failure in the long-term service life.

5.1 Background

The solid materials can be divided into two types, based on whether their elastic modulus are affected by time. One type belongs to elastic materials, such include steel, concrete, and intact rock. The other type forms a group of viscoelastic materials, which encompass thermoplastics, wood, and asphalt. For an elastic material, the value of its elastic modulus stays constant over time if the temperature is fixed and the external load is no near the ultimate load level. Thus, the elastic theory is sufficient to determine the stress/strain distributions in the elastic material. In contrast, the elastic modulus of a viscoelastic material may continue to decline as the time passes. So, the stresses or the strains in the material are not constant with time even when the external load stays steady. If the deformation of the body is fixed through time, the stresses in the viscoelastic material tend to gradually dissipate. This phenomenon is called “stress relaxation”. Oppositely,
the strains in the viscoelastic material will increase continuously (at ever decreasing rates) when a constant load is imposed. This is the circumstance of the “strain creep”.

In the mechanical analysis, there are two basic models which can be used to simulate the stress/strain behaviors of the viscoelasticity. One is the Maxwell model, and the other is the Kelvin model. The Maxwell model consists of a linear spring and a linear dashpot connected in series. When the Maxwell model is placed under a constant strain loading, the stress stored in the spring gradually dissipate due to the movement of the dashpot. Thus, the Maxwell model is suitable for the simulation of the stress relaxation. However, the strain in the Maxwell model increases linearly if the model is placed under a constant stress loading. At the instant when the constant-stress loading is removed, the strain in the model instantaneously drops to the initial level and then remains unchanged. This unrealistic behavior is due to the strain stored in the spring suddenly being released and the dashpot becoming static. Therefore, the Maxwell model cannot realistically simulate the behavior of the strain creep. The Kelvin model has a linear a spring and a linear dashpot connected in parallel. When the Kelvin model is placed under a constant stress, the stretch of the spring is held back by the dashpot, but the stress transfers from the dashpot to the spring gently with the increase of the strain in the whole system. That is why the creep strain in the Kelvin model shows up as a nonlinear curve instead of a linear line. After removing the constant stress, the strain of the Kelvin model slowly decreases to the initial level because the dashpot postpones the recovery of the spring. The stress relaxation will not happen if a constant deformation loading is applied to the Kelvin model, because all the stress is carried by the spring and the dashpot has no movement at all time. Therefore, the Kelvin model is only adept in forecasting the strain-creep behavior. The mathematical functions of the Maxwell model and the Kelvin model are listed below, and the sketches of these two models are displayed in Figure 5.1.

\[
\frac{d\varepsilon(t)}{dt} = \frac{d\varepsilon_s(t)}{dt} + \frac{d\varepsilon_d(t)}{dt} = \frac{\sigma_d(t)}{\eta} + \frac{1}{E} \frac{d\sigma_s(t)}{dt} \quad \text{(Maxwell Model)} \quad [81]
\]

\[
\sigma(t) = E\varepsilon_s(t) + \eta \frac{d\varepsilon_d(t)}{dt} \quad \text{(Kelvin Model)} \quad [82]
\]
where $\sigma(t)$, $\varepsilon(t)$ = the total stress and total strain of the model, respectively; $\sigma_s(t)$, $\varepsilon_s(t)$ = the stress and strain of the spring, respectively; $\sigma_d(t)$, $\varepsilon_d(t)$ = the stress and strain of the dashpot, respectively; $E$ = the elastic modulus of the spring; and $\eta$ = the viscosity of the dashpot.

Figure 5.1: Maxwell and Kelvin Models

The Maxwell model and Kelvin model merely represent the beginning stage of the viscoelasticity. They are not each powerful enough to simulate the mechanical behavior of the real viscoelastic material. The lengths of the molecular segments composing the viscoelastic material (e.g. HDPE) are diverse, so the mechanical characteristics of the viscoelastic material cannot be defined by only one set of viscoelastic coefficients. To present the various viscoelastic properties in one model, the best way is to construct a model by combining several Maxwell models or Kelvin models, and making the viscoelastic material property of each sub-model unique. There are two combined models (see Figure 5.2), with one being the generalized Maxwell model and the other being the generalized Kelvin model. The generalized Maxwell model basically consists of a series of Maxwell model connected in parallel, and the generalized Kelvin model are built by several Kelvin models attached in series. To simulate the instantaneous reactions that the HDPE material exhibit, an isolated spring is added outside the combination of the sub-models in these generalized models. Similar to the advantages of the basic models, the simulation of the stress relaxation relies on the generalized Maxwell model, and the behavior of the strain creep is determined by the generalized Kelvin model.
The best mathematical description of the generalized Maxwell model is the Prony series. The Prony method was developed by Gaspard Riche de Prony in 1795 and was not practicable until the invention of the computer. The general equation of the Prony Series is:

\[ G(T) = G_0 + \sum_{i=1}^{n} G_i e^{-T/t_i} \]  

where \( G(T) \) = shear modulus at specific time; \( G_0 \) = shear modulus of the independent spring; \( G_i \) = the shear modulus of each spring connected to the dashpot in series; \( t_i \) = the relaxation time of each combination of the spring and dashpot in series; and \( T \) = time.

Unfortunately, the closed form of the generalized Kelvin model in terms of the Prony series is still not solved. Therefore, the simulation of the viscoelastic behavior in the current computer software (e.g. Abaqus) is calculated on the basis of the generalized Maxwell model in terms of the Prony series. Although the generalized Kelvin model is suitable for the strain creep simulation and the generalized Maxwell model is adept to simulate the stress relaxation, the latter one can be deemed an approximate solution to the strain creep in the computer simulations.

5.2 Methodology

The lab or field experiments on the viscoelastic behavior of the HDPE material are very rare because such experiments are extremely time consuming. Moore and Hu (1994) proved the validity of the small deflection rheology in the simulation of the stress relaxation
based on the parallel plate test, let alone this method depends on the generalized Kelvin Model, so this method and the values of its viscoelastic parameters are not suitable for the Chua and Lytton method and Abaqus simulation in this study. To the author's knowledge, the only field data that looked into the viscoelastic nature of the buried HDPE pipes was collected by Sargand and Masada (2007) in their deep burial project. In this project, the Ohio university research group kept recording structural responses of thermoplastic pipes for five years after the end of embankment installations. In their long-term pipe performance data, only the vertical deflections were reliable because most strain gages became dead during the first year. The vertical deflections of the pipes increased for a certain period of time and then remained constant. This trend was also backed up by the fact that the strain gage readings taken on the pipe walls also increased a little during the first year. Because the soil dead loads above the pipes were constant beyond the end of the installation, the viscoelastic performances of these buried thermoplastic pipes can be treated as strain creep. In the previous elastic analysis, the pipe failure is forecasted by the strain limit. So, it is reliable to determine the pipe failure and relative values of the threshold vertical deflections by using the strain creep simulation instead of the stress relaxation. Therefore, the simulations of the HDPE pipes' viscoelastic behaviors focus only on the strain creep in this study. Moreover, the viscoelastic performances of the buried pipe are primary objectives in this study. Here, the backfill around each pipe is assumed to be an elastic solid. In the viscoelastic analysis, the pipes' geometric specifications and the material properties of the backfill are homogeneous to those in the elastic analysis. Because the viscoelastic behaviors belong to the long-term pipe performances, only soil dead load is considered in the Abaqus simulations. Lastly, the installation mode of the pipe is assumed to be the embankment mode.

5.2.1 The Chua and Lytton Method

Chua and Lytton (1989) imported the relationship between the time-dependent stress and relaxation modulus (Christensen 1971) into the elastic solutions established by Hoeg for buried pipe problems. By using the Laplace transformation technique, Chua and Lytton altered the viscoelastic governing equations to the equivalent elastic problems.
After solving the elastic problems, the exact solutions to the viscoelastic governing equations was achieved by inverting the Laplace transform back to the elastic solutions. According to the viscoelastic solutions provided by Chua and Lytton, only the compressibility and flexibility factors in the Hoeg solutions need to be changed, and all the other coefficients and equations remain the same. The viscoelastic forms of the compressibility (C) and flexibility (F) factors are:

\[
C = \frac{M_s R}{EA(1 - \nu)} \cdot \frac{\Gamma(1 - m)}{\Gamma(1 - m_c)} \cdot \left(\frac{1}{2T}\right)^{m-m_c} \tag{84}
\]

\[
F = \frac{M_s R^3 (1 - 2\nu)}{6EI(1 - \nu)} \cdot \frac{\Gamma(1 - m)}{\Gamma(1 - m_c)} \cdot \left(\frac{1}{2T}\right)^{m-m_c} \tag{85}
\]

where \( m \) = the exponent of the soil relaxation modulus; \( m_c \) = the exponent of the HDPE relaxation modulus; and \( T = \) time.

And, the function of the relaxation modulus \( E(t) \) is given by:

\[
E(t) = E_0 T^{-m} \tag{86}
\]

where \( E_0 = \) the initial elastic modulus of the material; \( t = \) time; and \( m = \) the exponent.

Since the soil around the pipe is assumed to be elastic, the value of \( m \) is set at 0. The determination of the value of \( m_c \) is described in subsection 5.2.3. The symbol \( \Gamma(n) \) represents the gamma function, which is defined by:

\[
\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx \tag{87}
\]

In the elastic analysis, the strain can be determined by the reciprocal of the elastic modulus. However, in the viscoelastic analysis, the creep strain is calculated by the creep compliance instead of relaxation modulus. Although the relationship between the creep compliance and the relaxation modulus is reciprocal in general, the exact relationship has to be determined based on the relaxation modulus function. For the case where the relaxation modulus \( E(t) \) is expressed by the power law equation, the relative creep compliance \( D(t) \) was found by Pipkin (1972) and referred by Chua and Lytton. The relationship between the creep compliance and relaxation modulus is given by:
\[ D(t) = \frac{\sin(m_c \pi)}{m_c \pi E(t)} \]  

[88]

5.2.2 Abaqus Simulations

Because the computer algorism applicable for the viscoelasticity is not yet well developed, only a few software packages available can simulate the viscoelastic performances of the buried thermoplastic pipes. Abaqus is selected as the tool to carry out the viscoelastic analysis. Abaqus is a powerful 3-D FEM computer code, and users can build and combine complicated shapes of the models and set various material properties. The version of Abaqus utilized in this study is 6.13. For each diameter HDPE pipe, the longitudinal depth of the pipe model is as long as four (4) corrugation pitches of the pipe-wall profile. The modeling in Abaqus can be divided into the following steps:

1. build parts of the pipe and soil;
2. define the material properties of the pipe and soil;
3. assemble all parts together and construct the model;
4. mesh each part independently;
5. apply the load and set up the boundary conditions on the model.

In the first step, the parts of the pipe and soil are built independently in different coordinate systems, and the dimensions of the parts are measured in inches. The complex shape of the pipe's profile-wall is transformed into a series of trapezoid (following the procedure outlined in the AASHTO LRFD specifications). The coordinates of each point consisting of one pipe wall profile are determined according to the detailed geometric information listed in Table 3.2, and the remaining three pitches of the pipe wall profile can be built by repeating the initial section three time. Then, the profile is revolved 360° degrees around the central longitudinal axis to create a 3-D section of the pipe. The part of the soil surrounding the pipe is divided into the inside part and outside part. The interior of the inside soil part precisely matches the exterior geometries of the pipe wall, and exterior of the inside soil part forms the surface of a cylinder which is 6 inches away the pipe wall. The outside soil part is a cuboid with a cylinder hollow located at the center,
the outside boundary of the outside soil part is 3 times pipe's diameter away the pipe wall. The longitudinal lengths of the inside soil part and outside soil part match that of the pipe.

The second step is to define the material properties for the soil and HDPE. The unit weight of soil should be expressed in pounds per cubic inch (pci), since the dimensions of the parts are measured in inches. The elastic modulus of the soil is set 1 ksi, 3 ksi or 5 ksi, and the Poisson's ratio of the soil is set to 0.3. The instantaneous elastic modulus of the HDPE material is entered 110,000 psi. The viscoelastic properties of the HDPE material are specified according to details given in subsection 5.2.3. When all the material properties are set up, two homogeneous solid sections are created, with one aligned with HDPE and another aligned with the soil material, respectively. Then, these parts are each assigned to the HDPE and soil sections, respectively.

To assemble all the parts together, instances are created for each independent part, so that the parts will be automatically moved into the same coordinate system. Initially, the instances may not be located where they should be. If that is the case, they need be moved and rotated to the correct locations. The connection type and relative contacted surfaces should be set up by the user. Otherwise, no connection can exist between the instances even if their boundaries are overlapped. Based on the results examined for the elastic analysis, the full-bond (no-slippage) results are close to the field pipe performance data. Therefore, the connection type between the pipe and soil instances is set as "tie" (fully bounded). Because the deformation of the pipe is induced by the soil pressures, the soil surface is set as master surface and the pipe surface as slave surface. The contact condition is mathematically controlled by the discretization method in Abaqus, and the option of this method is obviously "surface to surface".

When the model is constructed fully, the instances in the model are ready to be meshed independently. The pipe and soil instances are both meshed with the 20-node quadratic elements with reduced integration. This element type is appropriate for the calculations of strains in the deformable body. Because too many elements will consume much time in the calculation and only the pipe performances are primary, it is efficient to mesh the pipe instance with small-sized elements, and the soil instance can be meshed coarsely. The accuracy of the FEM analysis is satisfying when the total element number
of the pipe instance is around 5,000 for each diameter size of the pipe. The precision of the FEM analysis may deteriorate if the element sizes of the adjacent instances are excessively different, so the soil instances is divided into inside and outside soil instances. The element size for the inside soil instance is set twice as big as that of the pipe instance, and the element size of the outside soil instance is three times larger than that of the inside soil instance. The sketch of the mesh of each instance is shown in Figure 5.3, and the general view of the assembled model mesh is illustrated in Figure 5.4.

Figure 5.3: The Mesh of Each Instance
The extra soil dead load is applied by the pressure placed on top of the soil instance, whose upper thickness is sufficient to eliminate the boundary effect on the pipe instance. The bottom of the soil instance is totally fixed, and the left and right sides of the soil instance can only move vertically and longitudinally. The longitudinal length of the pipe and soil instances are not long enough, and the limitation of the longitudinal movement will cause unrealistic compression to the pipe instance, so the longitudinal deformation of the pipe and soil instances should be released. Because the pipe and soil instances deform vertically, horizontally and longitudinally, the front and back sides of the soil instance have no movement limitations.

5.2.3 Values of Viscoelastic Coefficients

In the Chua and Lytton method, the viscoelastic coefficient is only the exponent of the HDPE relaxation modulus ($m_c$). Based on the research work of Bilgin et al. (2007), the value of $m_c$ ranges from 0.086 to 0.098 when the temperature varies from -6.7 to 49 degrees Celsius. In the Abaqus simulation, the viscoelastic behavior of the HDPE
material is related to the coefficients $g_i$ and $t_i$. The coefficient $g_i$ is the ratio of each spring's shear modulus to the sum of all springs' shear moduli. In this study, the sum of all springs' shear moduli is determined by the instantaneous (short-term) elastic modulus of the HDPE pipe, which is 110 ksi listed in the AASSHTO LRFD specification. The shear modulus of the independent spring represents the ultimate shear modulus remained in the generalized Maxwell model. The ultimate shear modulus can be calculated by the long-term elastic modulus of the HDPE pipe, which is 22 ksi listed in the AASSHTO LRFD specification. The user need to input the shear-modulus ratio of each spring connected with dashpot in series, then Abaqus will automatically compute the shear-modulus ratio of the independent spring. Since the ultimate shear modulus is 20% of the instantaneous shear modulus, the sum of $g_i$ coefficient input in the Abaqus should be 0.8.

According to the material properties provided by the database in MatWeb and Springer Handbook of Condensed Matter and Materials Data (2005), the viscosity of the HDPE material is measured when it is melted into a viscous liquid. These experimental results are definitely different from the viscosity of the HDPE material in solid phase. Because the relaxation time ($t_i$) is determined by the viscosity, it is unreliable to simulate the viscoelastic performances of the buried pipes without reliable viscoelastic coefficients. Fortunately, the deep burial project conducted by Sargand and Masada (2007) can provided some data about the viscoelastic performances of the buried pipes. These data served as the guidance during the determination of the viscoelastic coefficient. Because most of the strain gages were dead soon after the installations of the pipes, only vertical deflections can be selected as the viscoelastic performance indicator for the buried pipes. Only several pipes tested in the deep burial project were HDPE pipes manufactured by ADS, and some of ADS pipes' vertical deflections fluctuated during the 5-year observations. In the end, only the vertical deflection of the No. 8 pipe is evaluated to be good enough for determining the values of the viscoelastic coefficients. Then, the pipe diameter, the backfill material properties and soil cover thickness in the Chua and Lytton method and Abaqus simulations are set identical to the installation conditions of the No.8 pipe in the deep burial project, which is referred to in Table 2.4.
In the deep burial project, the environmental temperature of the tests was about 0 degree Celsius. Based on the values provided by Bilgin et al. (2007), the value of $m_c$ is determined to be 0.087 by the interpolation method. However, the viscoelastic vertical deflection derived from the Chua and Lytton method will not match the realistic observations until the Poisson's ratio of the soil is changed to 0.35. The reliability of this value cannot be proved because the Poisson's ratio of the backfill is absent in the project report. In the Abaqus simulation, the Poisson's ratio of the backfill is set as 0.3 in order to remain consistent with the elastic analysis. The values of the viscoelastic properties in the small deflection rheology were found through the curve fitting exercise, so this method is applied in the determinations of the values of the $g_i$ and $t_i$ coefficients in the Abaqus simulation. The final values of $g_i$ and $t_i$ are listed in Table 5.1. The vertical deflections calculated by the Chua and Lytton method and by Abaqus are plotted against the vertical deflections of the No. 8 pipe (observed in the deep burial project) for a comparison in Figure 5.5 to 5.8.

<table>
<thead>
<tr>
<th>No.</th>
<th>$g_i$</th>
<th>$t_i$ (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.21</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0.18</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>1000</td>
</tr>
<tr>
<td>4</td>
<td>0.12</td>
<td>10000</td>
</tr>
<tr>
<td>5</td>
<td>0.09</td>
<td>100000</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>1000000</td>
</tr>
</tbody>
</table>
Figure 5.5: Vertical Deflection of 30-in Diameter HDPE Pipes over 5-year Period beyond End of Construction (Constrain Modulus of Backfill 5 ksi)

Figure 5.6: Vertical Deflection of 30-in Diameter HDPE Pipes over 1-year Period beyond End of Construction (Constrain Modulus of Backfill 5 ksi)
Figure 5.7: Combined Compressive Strain in 30-in Diameter HDPE Pipes over 5-year Period beyond End of Construction (Constrained Modulus of Backfill 5 ksi)

Figure 5.8: Combined Compressive Strain of 30-in Diameter HDPE Pipes over 1-year Period beyond End of Construction (Constrained Modulus of Backfill 5 ksi)

The vertical deflections derived from the Chua and Lytton method and the Abaqus simulation match very closely that of No. 8 pipe in the deep burial project during the first year of the post-construction period (see Figure 5.6), and these two simulated curves slightly diverge with the field test data from the start of the third year after the end of the construction (see Figure 5.5). Based on the curves in Figure 5.7 and 5.8, the combined compressive strains resulting from the Chua and Lytton method are always 0.2% higher than those derived from the Abaqus simulation over a 5-year period beyond the end of the construction.
5.3 Results of Viscoelastic Analysis

Consistent with the elastic analysis already performed, the long-term service period for the HDPE pipe in the viscoelastic analysis is also specified to be 50 years after the end of the pipe installation. The soil dead load on top of the pipe ensures that the maximum combined compressive strain in the pipe wall reaches 6.15% at the end of the pipe service period. The results of the viscoelastic analyses are shown in this subsection. For all diameter sizes of the HDPE pipes, their viscoelastic vertical deflections are shown in Figures 5.9 through 5.14, and the viscoelastic combined compressive strains are displayed in Figures 5.15 through 5.20. The results present not only the viscoelastic performances of the HDPE pipe during the 50-year service period, but also the viscoelastic performances in the first 5 years after the end of the pipe installation. This is because the increases of the vertical deflections and combined compressive strains are significant in this period.

In the legends used in these plots, "Chua" refers to the results calculated by the Chua and Lytton method, and "Abaqus" refers to the results determined by Abaqus.

![Graph](image)

**Figure 5.9: Vertical Deflection vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi)**
Figure 5.10: Vertical Deflection vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi)

Figure 5.11: Vertical Deflection vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 5 ksi)
Figure 5.12: Vertical Deflection vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi)

Figure 5.13: Vertical Deflection vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi)
According to the above figures, the diameter size of the pipe has little effect on the pipes' viscoelastic performances, but the influences of the backfill's constrained modulus on the pipe's mechanical responses are significant. If the constrained modulus of the backfill is 1 ksi, the viscoelastic vertical deflections of various diameter pipes in Figure 5.9 bundle with each other. For each diameter pipe, despite the initial vertical deflections simulated by Abaqus are about 0.7% higher than those calculated by the Chua and Lytton method, the viscoelastic vertical deflections determined by the two analysis methods become almost homogeneous beyond one month after the end of the construction. When the constrained modulus of the backfill is 3 ksi, the viscoelastic vertical deflections given by the Chua and Lytton method are slightly lower than those computed by Abaqus. But, the difference between the vertical deflections derived from the two analysis methods decreases from 0.7% to 0.3% when the time elapses from the end of the construction to the extent of service life (see Figure 5.10). Besides, a tiny gap appears between two groups of the curves created by the two analysis methods. When the constrained modulus of the backfill is increased to 5 ksi, the tendencies of the viscoelastic vertical deflections
generated by the two analysis methods are distinctly divided into two groups (see Figure 5.11). Furthermore, starting from the third year following the end of the construction, the viscoelastic vertical deflections from the Abaqus simulation are continuously 0.5% greater than those from the Chua and Lytton method. Thus, the rise of the backfill's constrained modulus will marginally enlarge the difference between the viscoelastic vertical deflections given by the theoretical method and computer simulation, but it can significantly decrease the divergence of the viscoelastic vertical deflections due to the diversity of the pipe diameters. One year after the pipes' installations, the viscoelastic vertical deflections of the pipes buried in a 1-ksi or 5-ksi constrained modulus backfill are approximately 83% or 94% of the peak values, respectively (see Figure 5.12 and 5.14). After the one-year mark, the viscoelastic vertical deflections keep growing to the maximum at the ends of the pipes' service lives. During this period, the increase rate of the viscoelastic vertical deflections is smaller than 0.01% per day, regardless of the pipe diameter size, backfill constrained modulus and analysis methods. Generally, the stiffer backfill can diminish the magnitude of the total deformation creeped in the pipe's service life, and it can also reduce the viscoelastic vertical deflection grew beyond one year after the end of the construction.
Figure 5.15: Combined Compressive Strain vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi)

Figure 5.16: Combined Compressive Strain vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi)
Figure 5.17: Combined Compressive Strain vs. Time for HDPE Pipes over 50 Years beyond End of Construction (Constrained Modulus of Backfill 5 ksi)

Figure 5.18: Combined Compressive Strain vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 1 ksi)
Figure 5.19: Combined Compressive Strain vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 3 ksi)

Figure 5.20: Combined Compressive Strain vs. Time for HDPE Pipes over 5 Years beyond End of Construction (Constrained Modulus of Backfill 5 ksi)
The viscoelastic combined compressive strains affected by the backfills with various constrained moduli are shown in Figures 5.15 through 5.17, and the curves in each figure overlap each other, regardless of the pipe diameters and the analysis methods specified. Similar to the impact of the backfill's constrained modulus to the viscoelastic vertical deflection, the stiffer backfill can restrain the total augmentation of the viscoelastic combined compressive strains in a 50-year service life of the HDPE pipe. In Figures 5.18 through 5.20, the increase rates of the combined compressive strains calculated by the two analysis methods decline to 0.01% per day at one year and a half after the pipe installations, regardless of the backfill's constrained modulus. At this time point, the viscoelastic combined compressive strain has respectively achieved 82%, 86% or 90% of the strain limit when the backfill constrained modulus is 1 ksi, 3 ksi or 5 ksi. The higher the backfill constrained modulus is, the less the viscoelastic combined compressive strains will develop beyond 1.5 years after the end of the construction.

5.4 Evaluation of Viscoelastic Analysis Results

The only available field test on the viscoelastic performances of the buried HDPE pipes came from the deep burial project conducted by Sargand and Masada (2007). Because it is a major challenge to provide reliable strain readings in the field, the vertical deflection measured in the field test served as the basis to verify the results determined by the viscoelastic methods. The calculated viscoelastic vertical deflections and observed field test data are listed in Table 5.2. Based on the comparisons of the vertical deflections of the buried pipes at the 5th year after the installation, the results given by the Abaqus simulations are more similar to the field test data, and the viscoelastic vertical deflections calculated by the Chua and Lytton method are not significantly different from the those measured in the deep burial project.
Table 5.2: Comparisons of Vertical Deflections Determined by Viscoelastic Analysis and Field Test (5th Year after Pipe Installation)

<table>
<thead>
<tr>
<th>Deep Burial Project</th>
<th>Abaqus</th>
<th>Chua</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe Dia.</td>
<td>Backfill Type</td>
<td>Ms (ksi)</td>
</tr>
<tr>
<td>30</td>
<td>Sn-96</td>
<td>5</td>
</tr>
<tr>
<td>42</td>
<td>Sn-96</td>
<td>5</td>
</tr>
<tr>
<td>42</td>
<td>Cl-90</td>
<td>3</td>
</tr>
</tbody>
</table>

[Note] "Ms" = the constrained modulus of the backfill; "CT" = soil cover thickness above the pipe; "VD" = vertical deflection at the fifth year after the construction of the pipes.

5.5 The Critical Coefficients of the Viscoelastic Analysis

In the viscoelastic analysis, the soil dead load above the pipe is increased until a compression failure happen in the pipe wall at the end of the pipe service period. Therefore, the soil cover thicknesses in the Chua and Lytton method are the critical cover thicknesses, and the pressure above the model in Abaqus can be transformed to the critical cover thicknesses by dividing the soil pressure by the unit weight of the overfill (130 pcf). It is not practicable to keep inspecting the vertical deflection of the pipe during its 50-year service period. So, ODOT uses the vertical deflection measured one month after the end of the pipe installation as the critical vertical deflection. The maximum cover thicknesses and critical vertical deflections are listed in Tables 5.3 through 5.5.

Table 5.3: Maximum Cover Thickness and Threshold Vertical Deflections of HDPE Pipes Buried in 1-ksi Backfill (Viscoelastic Methods)

<table>
<thead>
<tr>
<th>Pipe Dia. (in)</th>
<th>Chua and Lytton Method</th>
<th>Abaqus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max CT (ft)</td>
<td>VD at EoC (%)</td>
</tr>
<tr>
<td>12</td>
<td>49</td>
<td>4.8</td>
</tr>
<tr>
<td>24</td>
<td>43</td>
<td>4.9</td>
</tr>
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<td>30</td>
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<td>4.9</td>
</tr>
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</tr>
<tr>
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<td>39</td>
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</tr>
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<td>48</td>
<td>38</td>
<td>4.9</td>
</tr>
<tr>
<td>60</td>
<td>37</td>
<td>4.9</td>
</tr>
</tbody>
</table>

[Note]: CT = soil cover thickness; VD = vertical deflection; EoC = end of construction.
Table 5.4: Maximum Cover Thickness and Threshold Vertical Deflections of HDPE Pipes Buried in 3-ksi Backfill (Viscoelastic Methods)

<table>
<thead>
<tr>
<th>Pipe Dia. (in)</th>
<th>The Chua and Lytton Method</th>
<th>Abaqus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max CT (ft)</td>
<td>VD at EoC (%)</td>
</tr>
<tr>
<td>12</td>
<td>84</td>
<td>4.1</td>
</tr>
<tr>
<td>24</td>
<td>77</td>
<td>4.2</td>
</tr>
<tr>
<td>30</td>
<td>76</td>
<td>4.2</td>
</tr>
<tr>
<td>36</td>
<td>73</td>
<td>4.2</td>
</tr>
<tr>
<td>42</td>
<td>73</td>
<td>4.2</td>
</tr>
<tr>
<td>48</td>
<td>73</td>
<td>4.3</td>
</tr>
<tr>
<td>60</td>
<td>72</td>
<td>4.3</td>
</tr>
</tbody>
</table>

[Note]: CT = soil cover thickness; VD = vertical deflection; EoC = end of construction.

Table 5.5: Maximum Cover Thickness and Threshold Vertical Deflections of HDPE Pipes Buried in 5-ksi Backfill (Viscoelastic Methods)

<table>
<thead>
<tr>
<th>Pipe Dia. (in)</th>
<th>The Chua and Lytton Method</th>
<th>Abaqus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max CT (ft)</td>
<td>VD at EoC (%)</td>
</tr>
<tr>
<td>12</td>
<td>117</td>
<td>4.0</td>
</tr>
<tr>
<td>24</td>
<td>110</td>
<td>4.2</td>
</tr>
<tr>
<td>30</td>
<td>109</td>
<td>4.2</td>
</tr>
<tr>
<td>36</td>
<td>107</td>
<td>4.2</td>
</tr>
<tr>
<td>42</td>
<td>106</td>
<td>4.2</td>
</tr>
<tr>
<td>48</td>
<td>106</td>
<td>4.3</td>
</tr>
<tr>
<td>60</td>
<td>106</td>
<td>4.4</td>
</tr>
</tbody>
</table>

[Note]: CT = soil cover thickness; VD = vertical deflection; EoC = end of construction.

Based on the above tables, the increase in the backfill's constrained modulus can enlarge the maximum allowable soil cover thickness over the pipe. This observation is consistent with the elastic analysis. In the elastic analysis, the critical vertical deflection of the pipe decreased with the rise of the backfill material's constrained modulus. However, this phenomenon is not notable in the viscoelastic analysis, especially for the critical vertical deflections of the pipes buried in the 3-ksi and 5-ksi backfill soils. Because the vertical deflections need to settle some after the end of the construction, it may
be more suitable to adopt the vertical deflection taken one month after the end of the pipe installation as the critical vertical deflection. According to Tables 5.3 through 5.5, this critical vertical deflection may need to be set at 5% to protect the pipes from experiencing structure distress during their whole service life.
CHAPTER 6 : SUMMARY AND CONCLUSION

6.1 Summary

Due to the advantages of low-cost and durability, the thermoplastic pipes are widely used as the drainage conduits under roadways. High-density polyethylene (HDPE) is a common material used for manufacturing the thermoplastic pipes, so this study focuses on the HDPE profile-wall pipes. Because a great number of HDPE pipes are installed underground and their service lives could possibly span over several decades, it is not practically feasible to inspect every thermoplastic pipe during its continuous service period. The mechanical responses of the thermoplastic pipes are known to be related to the pipes' vertical deflections, so the pipe's structural failure can be prevented by limiting the vertical deflection. Many states DOTs specify that the vertical deflection of the HDPE pipe is to remain less than a threshold value (ex. 7.5%). No document published in the past clearly explains how this limit came to exist. In addition, no comprehensive studies were conducted to determine the critical vertical deflections for various diameter sizes of profile-wall HDPE pipe. Therefore, the objectives of this study are to: 1) find out why the conventional vertical deflection limit of the HDPE pipe is 7.5%; 2) determine the critical vertical deflection for various diameter sizes HDPE pipes buried in different backfill types and pipe installation modes that comply with the AASHTO Construction Specifications; and 3) discover the critical vertical deflection at the end of pipe installation to ensure the HDPE pipe operates normally in its 50-year service period.

The above objectives can be achieved effectively by carrying out the following four tasks, which are detailed below.

Task 1: Review literature to figure out the actual pipe performances in the field tests and identify reasonable origin of the vertical deflection limit. The literature review can examine technical papers published in ASCE and TRR journals, the standard pipe installation specifications listed in the AASHTO Construction Specifications, field test reports on shallow and deep pipe burial studies conducted by Ohio University, and other field test studies on the thermoplastic pipes done in Pennsylvania and Utah.

Task 2: Mathematically calculate the structural performances of the HDPE profile-wall pipes installed in different backfill materials and installation conditions that follow
the AASHTO Construction Specifications. The pipe performances here should include the vertical deflection, strains in the pipe wall, and the buckling ratio for the pipe wall. The elastic methods utilized in the analysis contain the AASHTO LRFD design equations and the other methods established by Spangler (1941), Masada (2000), Burns and Richard (1964), and Hoeg (1968). Because the stress/strain responses of the HDPE material could change with time, the viscoelastic analysis developed by Chua and Lytton (1989) as well as that based on the generalized viscoelastic model are also adopted in this study.

Task 3: Analyze mechanical responses of the profile-wall HDPE pipes through finite element method – based computer simulations. Simulations based on the elastic theory are conducted by CANDE, and viscoelastic simulations are handled by Abaqus.

Task 4: Summarize the HDPE pipe performances resulting from the elastic and viscoelastic methods. Examine these results in light of the field test data collected under Task 1. Then, establish charts and tables for the purpose of discovering the correlations among the pipe diameter, the pipe performances, the soil cover thickness, the backfill constrained modulus, the pipe installation condition, and the loading condition. The critical vertical deflections of the HDPE pipes are determined by comparing the results from the various calculation methods and computer simulations to the performance limits listed in the AASHTO LRFD specifications.

6.2 Conclusions

Based on the multifaceted analysis in the previous chapters, the conclusions for each task and objective are listed below in an organized approach. Specific conclusions derived for each task are outlined first, then more general conclusions are summarized at the end.

6.2.1 Task 1: Literature Review

- According to the inspections conducted by Hsuan and McGrath (1999), the circumferential and longitudinal cracks usually develop in the HDPE pipe wall when the vertical deflections were greater than 7.5%.
Moser (2000) conducted several full-scale tests on the profile-wall HDPE pipes, which were buried in the embankment mode with different soil compaction levels. The pipes failed when their vertical deflections reached 14.5%, 11%, and 6.7% for the backfills with 75%, 85%, and 95% compaction levels, respectively.

Appointed by Penn DOT, Hasash and Selig (1989) placed 100-ft-thick soil fill over a 24-inch diameter corrugated HDPE pipe. The maximum vertical deflection was 7.2%, and the average vertical deflection was around 4% along the pipeline length. In the 15th and 20th year after the end of the pipe installation, Goddard (2002) and Sargand et al. (2009) resurveyed this pipe and found the increase of the pipe's vertical deflection was not significant and no local distress and structural damage were noticed.

Several profile-wall HDPE pipes were investigated for 5 years in the deep burial project conducted by Sargand and Masada (2007). The soil cover thickness over these HDPE pipes was either 20 or 40 feet. Vertical deflections of these pipes never surpassed 4%. No structural distress/failure was observed in any HDPE pipes.

Masada and Sargand (2013) examined a corrugated HPDE pipe which has served Stated Route 145 (Noble County, Ohio) since 1981, the pipe was placed under a 1 to 1.3 ft of soil cover and low pH (4-5) drainage flow. The maximum vertical deflection of the pipe was 8.3%, but no signs of structural distress were found.

To explore the maximum allowable cover thickness for the HDPE pipes buried in various backfill conditions, Katona (1988) simulated the pipe performances by the computer software CANDE. He found that the pipe failure was typically controlled by the wall thrust stress limit in most cases.

Elzink and Molin (1992) surveyed many underground thermoplastic pipes installed in Europe. They recommended, in Scandinavia, that the long-term vertical deflection must remain less than 15%, and the peak short-term vertical deflection shall not exceed 8% for PE pipes.
- Hurd (1985) examined the field performances of many PE pipes in Ohio, and he concluded that the flattening appeared at the vertical deflections exceeding approximately 15% and buckling formed at vertical deflections exceeding 25%.
- In summary, the literature review identified many field studies in which profile-wall HDPE pipes continue to function well structurally as long as their vertical deflections remained less than 7.5%.

### 6.2.2 Task 2: Mathematically Calculation

AASHTO calculations:

- According to the pipe performances revealed by the AASHTO and theoretical methods, the compression failure always occurs in the pipe wall before the tension failure or buckling emerges.
- If the pipe is imposed by soil fill (dead) load only, the combined compressive strain and buckling ratio are always the largest at the springline of the pipe.
- In the short term service condition, the buckling ratio for the pipe wall is close to its limit when the compression failure happens, especially when the pipes are buried in the backfill with low constrained modulus. Compared to the occurrence of the compression failure, the buckling ratio is not notable anymore in the long term service condition.
- Regardless of the backfill material’s constrained modulus, a 7.5% vertical deflection can always protect the HDPE pipes from experiencing compression failure in the short term service condition, but the critical vertical deflection decreases to 6% in the long term service condition.
- A stiffer backfill can raise the critical cover thickness but reduce the critical vertical deflection of the HDPE pipe.
- When the pipe is loaded by live loads (from H-20 truck) through a soil cover of 5 ft or less, the combined compressive strain and buckling ratio at the crown of the pipes are the most significant. Even if the soil cover thickness is only 1 ft, no pipe experiences overstraining or buckling failure, despite the fact that the vertical
deflections of 12-inch, 24-inch, and 30-inch diameter pipes are greater than 7.5%. When the cover thickness is 15 ft or more, the impact of the live loads on the pipe disappears.

• Based on the comparison between the results from the AASHTO calculations and the data gathered in the field test, the AASHTO method is proved to be reasonable. However, the AASHTO calculations can address the structural performances of the pipes installed only in the embankment mode.

The theoretical methods:

• No significant differences exist between the vertical deflections determined by the full slippage case and no slippage case of the Burns and Richard method. Therefore, the pipe-backfill interface friction condition has little effect on the vertical deflection of the pipes.

• Based on the examination process applied to each theoretical method, the Masada method is suitable for forecasting the vertical deflection of the buried pipe, and the Burns and Richard method (the Hoeg method) is adept to estimate the combined compressive strain in the pipe wall.

• With the rise of the backfill constrained modulus, the growths of the critical cover thicknesses of the pipes determined by the Masada method are not remarkable. In the Burns and Richard method (the Hoeg method), the critical vertical deflections decline and the critical cover thicknesses grow with increasing backfill constrained modulus. Thus, in order to reveal the pipe performances, the no slippage case of the Burns and Richard method (the Hoeg method) are the preferred methods among all the theoretical methods.

• In the no slippage case of the Hoeg method, the short-term critical vertical deflections are 7.5%, and the long-term critical vertical deflections are approximately 6%. This statement agrees with the conclusion reached earlier by the AASHTO calculations.
6.2.3 Task 3: Computer Simulations

CANDE simulations:

- When the HDPE pipes are buried in the CA (coarse aggregate) or SM (silty sand) backfill material, the relationship between the vertical deflection and soil cover thickness is linear. Also, the combined compressive strain and buckling ratio both increase linearly with the rise of the vertical deflection. However, if the pipes are embedded in the SW (well-graded sand) backfill, these relationships mentioned above become somewhat nonlinear.

- Under the live loading condition, the highest combined compressive strains appear at the crown of the pipe if the cover thickness is 10 ft or less. None of the pipes shows compression failure or buckling problem when the soil cover thickness is 2.5 ft or more. However, if the soil cover thickness is only 1 ft, the 24-inch diameter pipe suffers a compression failure when the backfill is CA-90 or SM-85 type, and the 12-inch and 48-inch diameter pipe buckles when it is buried in the CA-90 backfill. The influence of the live load on the pipe performances thoroughly disappeared under a 25 ft of soil fill for the 12-in diameter pipe and under a 20 ft soil fill for other diameter pipes.

- Compared to the pipe buried in the embankment mode, the trench mode is somewhat helpful for reducing the load applied to the pipe and thus improving the pipe performances.

- Compared to the pipe performances measured in the field tests, the vertical deflections given by CANDE are sometimes reasonable. However, CANDE has a tendency to over-predict the thrust stress/strain and buckling ratio in the pipe wall. Because the buckling ratio in the CANDE simulations is determined based on the stresses, the short-term critical vertical deflection controlled by the buckling ratio tend to be overly conservative. If the combined compression strain limit is selected as the parameter for determining the short-term critical vertical deflection, a 7.5% vertical deflection limit is sufficient to protect pipes from compression failure. The buckling ratio in the long term service condition is too small to be
considered, and the long-term critical vertical deflections under compressive failure vary between 6% and 8%.

6.2.4 **Objective 1: Origin of the 7.5% Critical Vertical Deflection**

- Some technical reports and papers stated that the reverse curvature may happen in the thermoplastic pipe when the vertical deflection of the pipe is in the range of 22% to 30%. Therefore, these articles believed that the 7.5% vertical deflection limit can offer a factor of safety of 3 or 4 against reverse curvature and assure the overall stability of the pipe as well.

- The metal pipe will suffer from the structural failure when its vertical deflection approaches 20%, so the vertical deflection limit for the metal pipe needs to be at 7.5% by considering a safety factor close to 2.5. Some engineers believe this vertical deflection limit is also suitable for the thermoplastic pipe.

- According to the National Testing of PE Pipe (NTPEP) program, the flow inside the pipe is likely to leak at the joint sections of most corrugated HDPE pipes when the vertical deflections reach 30%. Thus, the vertical deflection limit of 7.5% can provide a safety factor of 4 to prevent the pipes from developing joint water leakage.

6.2.5 **Objective 2: The Certain Critical Vertical Deflection of the HDPE Pipes**

- In the AASHTO calculations, a 7.5% vertical deflection limit can protect the HDPE pipes from experiencing compression or buckling failures in the short term service, and a 6% vertical deflection limit can ensure the HDPE pipes will function well without any structural failure problems in the long term service.

- According to the pipe performances determined by the no-slip case of the Hoeg method, the critical vertical deflection is 7.5% for the short-term service condition, and is 6% for the long-term service condition.

- In the CANDE simulations, the short-term critical vertical deflection is sometimes controlled by the buckling criterion instead of the compression strains, making the critical vertical deflection vary from 6% to 8%. However, CANDE is known to have a tendency to overestimate the buckling ratios. Thus, the critical vertical
deflection in the short term service condition can be specified to be 7.5%. The buckling ratio is not notable in the long term service condition, and the long-term critical vertical deflection given by CANDE is 6%.

6.2.6 Objective 3: The Critical Vertical Deflection at the End of Construction

- According to HDPE pipes' viscoelastic performances given by the Chua and Lytton method and Abaqus simulations, the increase of the backfill's constrained modulus has a significant effect on the reduction in the development of the viscoelastic vertical deflection and viscoelastic combined compressive strains.

- The results given by the Abaqus simulations are more similar to the field test data, so Abaqus may be more suitable for forecasting the viscoelastic performances of the buried HDPE pipes. The Chua and Lytton method is also acceptable, because the difference between the viscoelastic vertical deflections calculated by the Chua and Lytton method and those recorded in the deep burial project are not significant.

- The critical vertical deflections determined by the Chua and Lytton method and by Abaqus are almost identical. ODOT defines that the critical vertical deflection to be the value of the vertical deflection measured one month after the end of the pipe installation. The critical vertical deflection may need to be set at 5% to protect the pipes from experiencing structure distress during their whole service life.
REFERENCES


APPENDIX A: THE PLOTS OF RESULTS FROM THEORETICAL METHODS

Short-Term Vertical Deflections vs. Cover Thicknesses for 12-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi

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**Graph 1:**
- Spangler
- Masada
- B&R NS
- B&R FS
- Hoeg NS
- AASHTO

Long-Term Vertical Deflections vs. Cover Thicknesses for 24-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi

**Graph 2:**
- Spangler
- Masada
- B&R NS
- B&R FS
- Hoeg NS
- AASHTO

Long-Term Vertical Deflections vs. Cover Thicknesses for 30-in Diameter HDPE Pipe by Various Theoretical Methods, Constrained Soil Modulus is 1 ksi

**Graph 3:**
- Spangler
- Masada
- B&R NS
- B&R FS
- Hoeg NS
- AASHTO
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APPENDIX B: THE PLOTS OF RESULTS FROM CANDE SIMULATIONS

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

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Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Short-Term Combined Compressive Strains vs. Vertical Deflections for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Short-Term Combined Compressive Strains vs. Vertical Deflections for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

![Graph showing combined compressive strain vs. vertical deflection for 48-in Diameter HDPE Pipes](image)

Short-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

![Graph showing combined compressive strain vs. vertical deflection for 60-in Diameter HDPE Pipes](image)

Short-Term Buckling Ratios vs. Vertical Deflections for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

![Graph showing buckling ratio vs. vertical deflection for 12-in Diameter HDPE Pipes](image)
Short-Term Buckling Ratios vs. Vertical Deflections for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Short-Term Buckling Ratios vs. Vertical Deflections for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Long-Term Combined Compressive Strains vs. Vertical Deflections for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode
Long-Term Combined Compressive Strains vs. Vertical Deflections for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Embankment Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Embankment Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Short-Term Combined Compressive Strains vs. Vertical Deflections for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Short-Term Combined Compressive Strains vs. Vertical Deflections for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Short-Term Buckling Ratios vs. Vertical Deflections for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Short-Term Buckling Ratios vs. Vertical Deflections for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Buckling Ratios vs. Vertical Deflections for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Long-Term Vertical Deflections vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Long-Term Combined Compressive Strains vs. Vertical Deflections for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode
Long-Term Combined Compressive Strains vs. Vertical Deflections for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Long-Term Combined Compressive Strains vs. Vertical Deflections for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead Load Only, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipelines under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipelines under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipelines under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode
Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Vertical Deflections vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode
Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode
Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode
Short-Term Combined Compressive Strains vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 12-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 24-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode
Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 30-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 36-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 42-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode
Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 48-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode

Short-Term Buckling Ratios vs. Soil Cover Thicknesses for 60-in Diameter HDPE Pipes under various Backfill Types and Compaction Levels, Dead and Live Load, Trench Mode