Inventory Policy for a Hospital Supply Chain with Perishable Inventory

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Mandana Sakhaii
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This thesis titled

Inventory Policy for a Hospital Supply Chain with Perishable Inventory

by

MANDANA SAKHAI

has been approved for

the Department of Industrial and Systems Engineering

and the Russ College of Engineering and Technology by

Dale T. Masel

Associate Professor of Industrial and Systems Engineering

Dennis Irwin

Dean, Russ College of Engineering and Technology
Abstract

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In a hospital, there are many factors that must be considered when determining inventory levels and frequency of ordering. Many supplies in a hospital are perishable, so they must be disposed of if they are not used before their expiration date. In addition, multiple hospitals in an area are often owned by the same organization, so they could potentially benefit from joint purchasing in larger quantities and redistributing to the individual hospitals.

This inventory model determines the inventory level and reorder point for supplies to minimize total cost. The total cost includes holding cost, distribution cost, purchasing cost, stockout cost and discarding expired product cost. To reflect the fact that hospitals can jointly take advantage of bulk purchasing, a discount cost structure is utilized. To validate the model small-sized numerical examples are solved using the CPLEX solver within GAMS software. And for large-sized problems a Genetic Algorithm is presented. The model is also tested on a case based on data from the catheterization laboratory of a large hospital. Also, a sensitivity analysis of optimal solution is conducted and some important observations are drawn.
Dedication

This thesis work is dedicated to my mother and father who always offered love and support although I was too many miles away from them. I am very thankful for having them in my life. This work is also dedicated to my boyfriend who has been a constant source of support and encouragement during the challenges of graduate school and life.
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Chapter 1: Introduction

1.1. Background

Unlike process operations and management, only a small number of supply chain optimization studies are directly dedicated to the health care sector. The health care supply chains need optimization techniques to increase their efficiency and responsiveness [1, 2]. Therefore, designing more efficient supply chain methods can result in significant cost savings in this area that can provide people with access to more affordable health care.

Health care supply chain costs include the costs for the flow of goods from the point of origin to point of consumption, including purchasing, distribution, warehousing, inventory holding, staff and obsolescence cost. By scrutinizing each of these costs more closely, methods can be developed to reduce them and in result, reduce the whole supply chain cost. For example, for purchasing cost, hospitals can reap the benefits of high volume purchasing with volume-based cost discounts to reduce products’ prices. For transportation cost, one of the ways to reduce it is to consolidate the orders. Instead of sending products from each supplier to each hospital monthly, they can be consolidated and in result, the supplier shipment frequency decreases.

Unlike other industries, in the health care industry stock outs are more dangerous, since running out of certain products may threaten the patients’ health and cause serious problems for hospitals. Due to long lead times, the hospitals may stockpile products to avoid the service disruption. By this stockpiling, inventory holding cost increases and the hospitals would need to expand their facilities to keep these larger inventories.
Additionally, products’ deterioration is also an important consideration. Most of the pharmaceutical products are perishable, so holding too much inventory may result in an additional cost when passing the expiration dates which results in product loss. Based on a survey conducted in 2003, in the branded segment, the cost incurred due to expiration was over $500 million at supermarkets and drugstores in the U.S. \[3\].

1.2. Motivation

In 2015, health care spending in the USA increased by 5.8%, reaching $3.2 trillion, which accounts for nearly 20% of gross domestic product \[4\], so cost savings can be obtained through distribution and inventory management \[5\]. Based on a survey, one hospital can lower its costs by at least 2% by applying better product inventory management and distribution \[6\].

There exist studies \[7, 8\] focused on inventory management in health care, and each tries to model some aspects of decisions need to be made in this area to increase efficiency. Most of the studies done in inventory management area assumed that the products have an unlimited shelf life which is not applicable in the real world since in some cases using expired products may jeopardize patients’ health. Some of these works considered that product demand rate is constant over time at hospitals or only modeled single product. Furthermore, to model the deterioration cost, a few studies \[22\] and \[24\] assumed that a fixed percentage of the order quantity deteriorated each period. This cannot be a precise modeling of the product deterioration.

Therefore, to cover all vital above-mentioned decisions, an inventory model for multiple products while considering product perishability is developed.
1.3. Objectives

Within hospitals, fulfilling patients’ needs at the right time and right manner is a must. Hence, designing an effective inventory management system that can fulfill these needs is vital.

The objective of this thesis is to develop a two-echelon supply chain model to find the optimal replenishment interval for multiple items at hospitals, while minimizing overall system costs, including inventory holding cost, distribution cost, purchasing cost, stockout cost and obsolescence cost. Due to the perishability of items used in health care system, each item is considered to have a shelf life, after which it cannot be used.

Moreover, hospitals can take advantage of bulk purchasing based on the modified all-unit discount cost structure. To verify the behavior of the proposed model, numerical examples are solved using CPLEX solver and for larger problems a Genetic Algorithm is used to examine the model on a real case.
Chapter 2: Literature Review

The supply chain network in any industry encompasses all activities in fulfilling a customer request from product development to customer service, and management of each function is crucial for companies. Considerable numbers of studies have been conducted in supply chain management (SCM), trying to address key activities and concerns, though the topic is still evolving. More importantly, other industries such as health care systems are trying to apply supply chain models to strengthen and improve their efficiency. In this study, a two-echelon health care supply chain is considered — suppliers and retailers (hospitals) — here, hospitals deal with multiple perishable items.

The literature review is organized into three main parts. The first part is dedicated to studies which consider inventory policy for multi-item and multi-retailer systems. The second part is devoted to studies conducted in inventory management in health care system, and the third part is about studies that model perishable items.

2.1. Multi-item, multi-retailer models

Multi-item and multi-retailer models, try to coordinate the purchasing and shipping plan of multiple items in a multiple retailer inventory system. These types of models can be applied to health care sector since hospitals can be considered as retailers in which multiple items are used.

Muckstadt and Roundy [9] developed a mathematical model and algorithm to find the reorder interval for each item stored in one warehouse serving multiple retailers. The algorithm minimizes the sum of holding costs; fixed costs for ordering and shipping each item between the supplier and the warehouse; and a fixed joint item order cost at each
retailer. In this model, it is considered that the reorder intervals follow from nested and stationary policies. In the nested policy, the retailers receive an item whenever the warehouse receives that item from a supplier while in a stationary policy, the number of items sent and the time intervals from supplier to warehouse or warehouse to retailers are the same for each type of item. To formulate this model, a base planning period was considered and each time interval was considered to be a power of 2 times this base period: 

\[ T_n = M_n \cdot B \]  

where \( M_n = 2^j, j=0,1,2,... \) and where \( B \) is base time.

The model was developed based on the directed and noncyclic graph, where two node types were introduced: inventory node and joint-replenishment node. Inventory node represents receiving and storing each item at a specific location in which holding cost and fixed order cost are incurred. Joint-replenishment node represents receiving of items at a retailer in which the ordering cost at the retailer is incurred. Based on this structure, an algorithm was proposed to find the optimal solution for the model, which required solving the relaxed form of the original model from the power of 2 constraint (RP model) first. Then it was demonstrated how an optimal solution to RP can result in an optimal solution in the original model.

In another study, Cha et al. [10] formulated a more flexible joint replenishment and delivery scheduling model for a two-stage supply chain including one warehouse and multiple retailers to minimize the sum of the warehouse ordering cost; outbound transportation; warehouse inventory holding cost; and retailer inventory holding cost. In this model, the warehouse delivers items after being replenished jointly from suppliers. It
was assumed that each retailer only has demand for one type of item and they are restocked every integer multiple of the basic period at the warehouse.

In this study they introduced two efficient algorithms: joint replenishment and delivery – simple heuristic (JRD–SH) and joint replenishment and delivery – RAND algorithm (JRD–RAND) and also developed a hybrid genetic algorithm (GA) to optimize the model. Comparing these three algorithms, JRD–RAND outperformed the other algorithms, and the hybrid GA was very good with an average difference of 0.0268% from optimal. However, in this study, joint replenishment of multi-items was not considered and it was assumed that multiple hospitals are supplied by multiple independent suppliers.

Chan et al. [11] designed simple inventory policies in a class of finite time planning horizon. In this study, one warehouse serves as a cross-dock facility for multiple retailers. Transportation cost structures (incremental and all-unit discount cost functions) were used and by these cost structures, they took the advantage of quantity discounts, giving customers special pricing when they buy products in bulk.

The authors use a zero-inventory-ordering policy, in which replenishments were made only when inventory at the retailer reaches zero. They demonstrated that there is a zero-inventory-ordering policy whose total cost (inventory and transportation) is less than 1.33 times the optimal cost. To find the best zero-inventory-ordering policy, they formulated two algorithms. The first one was an exact algorithm with polynomial running time for a fixed number of retailers. Hence, by increasing the number of retailers, its computational complexity grows exponentially. To remedy this computational burden, the authors developed the second algorithm that was a heuristic based on linear programming
that runs in polynomial time. Then they applied the proposed algorithm to two single-
warehouse and multi-retailer problems with different cost structures, one with a concave
transportation cost function and the other with a modified all-unit discount cost function.
The Modified all-unit discount cost structure changes between increasing and flat sections,
while starting with a flat section denoting the minimum cost. It was observed that there is
an optimal zero-inventory-ordering policy for problem type one.

Yang et al. [12] extended the model of Chan et al. [11] to determine an optimal
restocking policy for shipping items from suppliers to a warehouse and from the warehouse
to a retailer, to minimize the inventory holding cost, backlogging cost and transportation
cost. The warehouse in this study is a cross dock facility and items are sent to retailers
without being stored in it, so the warehouse doesn’t incur any inventory holding cost.
Moreover, the backlogging cost is considered only for retailers. Also, the all-unit discount
cost function was used for transportation cost and no limitation was considered on the
quantity sent to retailers. Due to the model complexity (NP-hard), the authors applied GA
to solve the model. Steps used in their approach are as follows:

- Generation of random individuals
- Evaluation of fitness function
- Genetic operations

Genetic operations were performed through 3 steps: selection, crossover and mutation.

- Selection: The roulette wheel selection method in which individuals are
  assigned a probability of being selected based on their fitness.
• Crossover: Single-point crossover in which given two parents, a cut-point is generated and recombines the first part of the first parent with the second part of the second parent to create one offspring.

• Mutation: Swap mutation in which the mutation operator works by swapping two randomly selected genes.

The results showed that the GA is robust and effective in terms of computational time in generating replenishment policy.

Wang et al. [13] also developed a multiple objective joint replenishment and delivery decision of a single warehouse and multiple retailers to find an efficient restocking interval, travel routing and safety stock factor to minimize total cost. Three categories of cost were considered: Inventory holding cost associated with holding cycle stock and safety stock; distribution cost associated with visiting the suppliers and traveled distance; and replenishment cost (fixed and variable ordering cost).

Jointly replenishing the items can result in a decrease in the unit cost of each item at both replenishment and distribution levels. At the distribution level, the authors utilized a traveling salesman problem (TSP) to find the shortest path among suppliers that should be visited at a specific time. To deal with this multi-objective model, two approaches, linear programming (LP) and multi-objective evolution algorithm (MOEA) were designed using the hybrid differential evolution algorithm (HDEA). In LP, a multi-objective model was changed into a single objective model but in MOEA the model was solved directly. Moreover, HDEA was proposed based on combining differential evolution (DE) algorithm and GA. It was observed that HDEA outperformed DE and GA in regardless of whether
LP or MOEA was used. DE and HDEA are more appropriate than GA, and HDEA and GA are more acceptable than DE when LP and MOEA were adopted, respectively.

Schwarz [14] examined a one warehouse and multi-retailer system to obtain a system-optimal policy instead of locally-optimal policies and demonstrated that finding the optimal policy can be very sophisticated; thus to simplify the model he considered that the order quantity is not time variant and determined the optimal policy conditions. Additionally, the author provided a method to solve a single warehouse and multi-retailer problem and proposed heuristic solutions for the general problem. It was demonstrated that an optimal policy can be found in the set of “basic” policies; that is, any feasible policy in which items are delivered to the warehouse when the warehouse inventory and at least one retailer’s inventory is zero. Any given retailer received items only when that retailer’s inventory is zero. He also developed single-cycle and myopic policies. The single-cycle policy is stationary and nested while the myopic policy considers a single warehouse and $N$ retailers as $N$ single warehouse and one retailer problems.

Duan and Liao [15] developed centralized and decentralized models that considered a two-echelon supply chain. It was assumed that both warehouse and retailers adopt minimum/maximum inventory policy ($s, S$).

To evaluate the performance of these two models, both were simulated and optimal restocking policies were obtained by adopting a hybrid meta-heuristic algorithm that was comprised of a DE algorithm, a harmony search (HS) algorithm and one local search (LS). In this study, ten demand patterns under four capacity constraint levels and two control strategies were covered to examine their effect on unit inventory cost. The obtained
solution was close to an optimal result, with maximum optimality gap of 0.2%. Moreover, results indicated that unstable demand patterns (i.e. exponential and uniform patterns) and tighter capacity constraints result in a higher unit cost. Also, it was observed that a capacity constraint resulted in a change in ordering patterns for demand with high variations.

On the other hand, Abdul-Jalbar et al. [16] analyzed the centralization versus decentralization effect on the total cost and found the optimal reorder policy that resulted in minimum total cost, which was the sum of inventory holding cost and replenishment cost. In the centralized case, the warehouse and retailers are branches of one corporation and in the decentralized case, the warehouse and retailers are independent. Backlogging and lead time were not considered. A two-echelon supply chain including one warehouse and $N$ retailers was considered. To find ordering plans, an algorithm was adopted for the decentralized case and two procedures were developed for the centralized case (common replenishment time and different reorder times).

After performing different numerical tests consisting of 100 instances, it was observed that as the number of retailers increased, the decentralized case outperformed centralized case.

2.2. Inventory management in health care system

Lappierre and Ruiz [7] presented a mathematical model for the general supply problem in a health care organization. Their main goal was to organize the procurement and distribution processes considering hospital’s inventory capacities.

The model comprised two parts (Model I & II). The model I tried to minimize the total inventory cost with a restriction on manpower time and available storage capacity.
(weight and volume) and considering safety stock. While Model II also tried to balance the workload over the planning horizon. For Model II, two decision variables $G(t)$ – the total manpower time for period $t$ – and a coefficient $\alpha$ that allows the decision maker to adjust the importance of balanced workload, are defined. Model I was inventory-cost-oriented while Model II was both inventory-cost-oriented and service-oriented. Due to quadratic terms in the objective function of Model II, it was more difficult to solve than Model I.

To solve these models, a Tabu Search metaheuristic that searched four neighborhoods was presented. The first two neighborhoods focused on replenishments to care units while the two other neighborhoods worked around the schedule of the supplier. Comparing these two models, it was observed that for almost the same working time and inventory level, Model II yielded a more balanced schedule which is not more expensive than Model I.

In another study, Kelle et al. [17] formulated a mathematical model to apply inventory management practice at a hospital storage unit. Their objective was to find the optimal reorder point and order-up-to-level in such a way that the inventory holding cost, shortage cost and replenishment cost were minimized.

To avoid exceeding the available storage for inventories, a volume constraint was considered as well. However, due to the difficulty of finding shortage cost factor quantities, this cost was replaced with a service level constraint, in which the service level $\alpha$ is a probability that no shortfall happens. In spite of this replacement, the model was still complex. Therefore, to remedy this, they introduced several approximations and then solved the model iteratively. The reorder point and order-up-to-level that were obtained
provided a uniform service level, decreased emergency replenishments by allocating safety stock and decreased the daily replenishment by allocating space for cycle stock.

Nagurney et al. [18] studied the distribution of a blood bank supply chain network. In this network, nodes represented facilities (blood collection sites, blood centers, component labs, storage facilities, distribution centers and demand points) and arcs connecting these nodes denoted supply chain activities (blood collection, shipment of collected blood, testing and processing, storage, shipment and distribution). Their main goal was to find the optimal blood levels on each arc while minimizing operations, discarding outdated blood and blood shortage costs as well as risk related to activities.

To formulate the shortage section, a large penalty factor was considered, to satisfy essentialness of blood availability. Their formulation only considered a single period and the Euler method was applied to find the optimal solution.

Nicholson et al. [19] compared total inventory costs and service levels of two different inventory management scenarios in a health care system: “an in-house three-echelon network” (first scenario) and “an outsourced two-echelon network” (second scenario). In the first scenario, the health care system owned a warehouse, hospitals and departments, but in the second one third party logistics were used. For both scenarios, in the case of shortage, sending out products either from one department to another one or from one hospital to another one was not allowed. Also, it was assumed that only one non-critical product type was held in the network. To compare these two networks the authors applied the greedy algorithm and argued that inventory cost associated with a three-echelon
network is higher than a two-echelon network. Additionally, the service level for the second scenario was higher than the first scenario.

Hoyaya and Tsadikovich [20] presented a network flow model of an influenza vaccine supply chain. The supply chain studied consisted of a manufacturer and a health care organization. The health care organization was composed of a distribution center and clinics. The DCs were responsible for the cost of shipping vaccines from the manufacturer and inventory holding cost; clinics’ cost included shipping cost from DCs, services cost such as injection, inventory holding cost and shortage cost.

The objective of the study was to find the optimal inventory level at DCs and clinic, shipping vaccine quantity from the manufacturer to DCs and from DCs to clinics and shortage quantity in clinics by minimizing the vaccination program cost while considering public benefit standard. After applying the real life data provided by CLALIT Health Services – the largest of Israel's four state-mandated health service organizations – in the developed model, it was revealed that by using the suggested model, CLALIT Health Services can reduce the vaccination program cost by 12%.

Mustafa and Potter [21] investigated collaborative arrangements in the health care supply chain. The authors’ main focus was on inventory management and replenishment at a company in Malaysia that had two echelons in the supply chain: wholesaler, which includes an administration depot, a warehouse and clinics.

After collecting the required data through process mapping, interviews and data analysis, the obtained data were analyzed and it was observed that there are two issues in the company’s supply chain: urgent orders and stock availability at the wholesaler. To
solve these issues, a vendor-managed inventory (VMI) approach was adopted in which the wholesaler takes full responsibility for maintaining inventory at clinics. This study also discussed that VMI application resulted in higher customer service and fewer stock-outs.

2.3. Inventory management considering deterioration

Alizadeh et al. [22] developed a modified form of (S-1, S) inventory system for perishable items with the objective of profit maximization. In this study, continuous items were volatile liquids such as alcohol, hydrogen peroxide, and essence which were subject to vaporization and were kept in packages. To remedy the volatility problem, the warehouse ordered $\alpha$ more units than the estimated customers’ demand. The authors focused on five cost elements: holding cost, shortage cost, deterioration cost, penalty cost of delivering less than one unit of the item, and purchasing cost.

The assumption was made for the deterioration cost element was that a fixed fraction of the item disappears per time. In addition, it was considered that customers’ arrival rate and the elapsed time between a customer putting in an order and the warehouse fulfilling it follow Poisson and Erlang distributions, respectively, with non-negative lead time. Likewise, due to the complexity of the proposed model, an upper bound and algorithm were introduced and used to find the optimal solution. The obtained numerical results showed that the modified model outperformed the classic policy (s, S).

Agrawal et al. [23] investigated an inventory model for perishable items with ramp-type demand. This demand pattern applied to new items which have low demand at the beginning and increased demand when introduced.
The authors assumed that the model uses an owned warehouse with a specific capacity. If the capacity was not sufficient, another rented warehouse with infinite capacity can be used. Deterioration rates at both warehouses are constant. The main objective of this study was to find the optimal replenishment policy when minimizing the sum of holding cost, lost sale opportunity cost, shortage cost and deterioration cost. This enabled the decision maker, to choose whether to rent another warehouse or not. Obtained results showed that when the owned warehouse has a high deterioration rate, the total cost can be reduced by increasing inventory at rented warehouse. Also, they demonstrated that the manager should not adopt the second warehouse when the rented warehouse has a high deterioration rate.

Most importantly, Uthayakumar and Priyan [24] developed an inventory model for the single pharmaceutical company and single hospital with the aim to obtain a prescribed customer service level and minimum total inventory cost. In this study, six categories of cost were considered: holding, transportation, production, set up, expiration and opportunity interest losses.

To model the expiration cost, it was assumed that each product type has expiration probability of \( d_i \); as a result, the total amount of expired products is equal to \( d_i \) multiplied by order quantity. Moreover, it is considered that raw material needed for a production cycle in a pharmaceutical company is delivered in one shipment without considering any lead time between supplier and company.

The authors used a perpetual review inventory model in which the order is placed when the inventory level reaches a level of \( r \). In order to determine the optimal lead time
and batch size of the proposed model, an algorithm in reference to Lagrangian multiplier methodology was proposed.

Fries [25] and Nahmias [26] independently studied the multi-period perishable inventory system for a single product with zero lead time and identically distributed demand. Both of these authors had ordering, holding, disposal and shortage costs. Fries considered unfilled demands as lost sales and assumed that the disposal costs are charged at the period that the products expire while Nahmias considered that unfilled demands would be backlogged and the disposal costs are charged at the period that the product arrives. Their policies were difficult since considering perishability of products, complicated the base stock policy. They showed that by dynamic programming approach, that has variables with \( m-1 \) dimension (\( m = \) product’s lifespan) the base stock policy can be a good approximation of the optimal solution.
Chapter 3: Problem Description

In this study, a two-echelon supply chain with multiple hospitals as retailers and multiple suppliers is considered. Hospitals submit orders at the end of the allowed period and receive their products at the beginning of the following period. Products are assumed to have a known shelf-life after which they will be expired and discarded. The main concern of this study is to find the optimal time interval between orders. The mathematical model is developed based on following assumptions:

- If demand exceeds current inventory, excess demand is borrowed and filled through the nearest hospital.
- Hospital’s demand distribution for each product type is known.
- The shelf-life of each product type is known and measured from when the product arrives at the hospital.
- FIFO inventory management is used.

3.1 Mathematical description of the problem

3.1.1 Notations

3.1.1.1 Indices

\( P \)  Number of products
\( I \)  Number of hospitals
\( J \)  Number of suppliers
\( H \)  Number of periods
\( p \)  Index for products (\( p = 1,\ldots,P \))
\( i \)  Index for hospitals (\( i = 1,\ldots,I \))
3.1.1.2 Input parameters

- $j$ Index for suppliers ($j = 1, \ldots, J$)
- $h$ Index for periods ($h = 1, \ldots, H$)
- $T$ Base time
- $d_{p ij}$ Demand (usage rate) per period for product $p$ from supplier $j$ at hospital $i$
- $c_{p i}$ Inventory holding cost for keeping one unit of product $p$ at hospital $i$
- $S_{ij}$ Distance between hospital $i$ and supplier $j$
- $\delta$ Transportation cost per mile
- $\rho_p$ Backorder cost per unit of product $p$
- $\varrho_{p i}$ Disposal cost per unit of product $p$ at hospital $i$
- $L_p$ Lifespan for product $p$
- $\beta_p$ Quantity breakpoints for price discounts
- $\alpha_p$ Purchasing cost of one unit of product $p$
- $M_i$ Storage capacity of hospital $i$

3.1.1.3 Variables

- $Q_{p ij}^h$ Order quantity of product $p$ sent from supplier $j$ to hospital $i$ at the end of period $h$
\[ B_{p_{ij}}^h \] Quantity of product \( p \) at hospital \( i \) at the beginning of period \( h \) that was sent from supplier \( j \)

\[ E_{p_{ij}}^h \] Quantity of product \( p \) kept at hospital \( i \) at the end of period \( h \)

\[ X_{p_{ij}}^h \] Quantity of product \( p \) expired at hospital \( i \) at the end of period \( h \) that was sent from supplier \( j \)

\[ Pen_{p_{ij}}^h \] Quantity of product \( p \) back ordered at hospital \( i \) that was sent from supplier \( j \) at period \( h \)

\[ b_{p_{ij}}^h \] Auxiliary binary variable used for linearization

3.1.1.4 Decision variables

\[ K_{p_{ij}} \] Number of base time periods between orders of product \( p \) at hospital \( i \) that was sent from supplier \( j \)

3.1.2 Mathematical model

The objective function shown in equation (1) tries to minimize the sum of five cost elements including:

- Inventory holding cost (1a)
- Transportation cost (1b)
- Purchasing cost (1c)
- Backorder cost (1d)
- Discarding cost of expired products (1e)
Min $Z = \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{p=1}^{P} \frac{1}{2} c_{pi} d_{p} K_{pij} T_{ij}$ (1a)

$+ \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{h=1}^{H} 2 \cdot \delta_{ij} S_{ij} b_{h}^{pij}$ (1b)

$+ \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{h=1}^{H} F(Q_{pij}^{h})$ (1c)

where $F(Q_{pij}^{h}) = \begin{cases} 
\alpha_{p,1} Q & \text{if } 0 \leq Q < \beta_{p,1} \\
\alpha_{p,2} Q & \text{if } \beta_{p,1} \leq Q < \beta_{p,2} \\
\alpha_{p,3} Q & \text{if } \beta_{p,2} \leq Q < \beta_{p,3} \\
\alpha_{p,4} Q & \text{if } Q \geq \beta_{p,3}
\end{cases}$ (1d)

$+ \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{h=1}^{H} \rho_{p} \cdot \text{Pen}_{pij}^{h}$ (1e)

$+ \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{h=1}^{H} \varrho_{pij} X_{pij}^{h}$ (1f)

Subject to:

$Q_{pij}^{h} = 0 \quad d_{pij}^{h} = 0 \quad K_{pij} T_{ij} \quad \forall p, i, j$ (2)

$X_{pij}^{h} = \begin{cases} 
\sum_{h=0}^{h-L_{p}} Q_{pij}^{h} - \sum_{h=0}^{h} d_{pij}^{h} - \sum_{h=1}^{h-1} X_{pij}^{h} & \text{If } h \geq L_{p} \text{ and } h > 1 \\
0 & \text{If } h \geq L_{p} \text{ and } h = 1 \\
\sum_{h=0}^{h-L_{p}} Q_{pij}^{h} - \sum_{h=0}^{h} d_{pij}^{h} & \text{Otherwise}
\end{cases}$ (3)

$B_{pij}^{h} = Q_{pij}^{h-1} + E_{pij}^{h-1}$ (4)

$E_{pij}^{h} = B_{pij}^{h} - d_{pij}^{h} - X_{pij}^{h}$ (5)

$Q_{pij}^{h} = \left( Q_{pij}^{0} - E_{pij}^{h} \right) b_{h}^{pij}$ (6)
where \( b_{pij}^h \) =
\[
\begin{cases}
1 & \text{If } \text{MOD}(h, K_{pij}) = 0 \\
0 & \text{Otherwise}
\end{cases}
\quad \forall h, p, i, j
\]

\( \sum_{p=1}^{P} Q_{pij}^h \leq \text{Capacity}_{ij} \quad \forall h, i, j \) (7)

\( \text{Pen}_{pij}^h = \begin{cases}
-(B_{pij}^h - d_{pij}^h) & \text{If } (B_{pij}^h - d_{pij}^h) \leq 0 \\
0 & \text{Otherwise}
\end{cases} \quad \forall h, p, i, j \) (8)

\( K_{pij} \leq L_p \quad \forall p, i, j \) (9)

\( X_{pij}^h \geq 0, B_{pij}^h \geq 0, Q_{pij}^h \geq 0, K_{pij} > 0 \text{ and int. } \forall p, i, j, h \) (10)

\( b_{pij}^h \in (0,1) \quad \forall p, i, j, h \) (11)

Total inventory holding cost (1a) is associated with physically holding stock in hospitals’ storage area. The average inventory level is equal to \( \frac{1}{2} \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{p=1}^{P} d_{pij} \cdot K_{pij} \cdot T \) (halfway between empty and full).

Total transportation cost (1b) is involved in trucks traveling from suppliers to hospitals to deliver the order. In this element, transportation cost per mile includes fuel, labor, maintenance, repairs and other expenses incurred on the road.

Total purchasing cost (1c) is associated with purchasing products from suppliers, in which economies of scale are considered and modeled using a discount cost structure. It is considered that \( \alpha \) is the purchasing cost rate, and \( \alpha_1 > \alpha_2 > \ldots > 0 \), so the hospitals may pay
more for per unit of the product when the order quantity batch size ‘Q’ is $\beta_1 \leq Q < \beta_2$ than when $\beta_2 \leq Q < \beta_3$ or $\beta_3 \leq Q < \beta_4$.

Total backorder penalty cost (1d) is incurred when the suppliers are unable to fill a demand and must be satisfied from another facility and pay to expedite.

Total obsolescence cost (1e) is involved in discarding expired products. It includes labor and all the services needed for disposing.

First constraint (Eq. (2)) specifies that order quantity batch size at period $h = 0$ (or Par level) is the product of expected demand over the time interval between orders.

The second constraint (Eq. (3)) calculates the expired inventory level ($X$) at period ($h$) for each product ($p$) with life span of ($L_p$) in different scenarios. At period ($h$), if $h < L_p$, none of the products could have passed their corresponding shelf life. If $h \geq L_p$ and $h = 1$, the expired inventory level equals the sum of all order quantity batch sizes that have passed their shelf life minus the cumulative filled demand of hospitals and already expired inventories. In addition, for $h \geq L_p$ and $h = 1$, the expired inventory level is the same as when $h > 1$ except for no inventory is expired at period $h = 0$.

The third constraint (Eq. (4)) balances the beginning inventory with order quantity and end inventory of previous period. The fourth constraint (Eq. (5)) balances end inventory with beginning inventory, hospitals’ demand and expired inventory. The fifth constraint (Eq. (6)) forces the model to order based on time interval up to the Par level. The sixth constraint assures that the quantity sent from supplier $j$ to hospital $i$ does not exceed the pre-established capacity limits (Eq. (7)). The seventh constraint (Eq. (8)) calculates the amount
of shortage for each product. The eighth constraint (Eq. (9)) forces the time interval between orders for product \( p \) to be less than or equal to its lifespan to reduce products’ deterioration. Constraints nine and ten (Eq. (10) and Eq. (11)) define the decision variable types.

3.2 Linearization

The developed mathematical model is linearized for two reasons [27]. First, it is difficult to distinguish a local optimal solution from a global optimal solution in nonlinear models. Second optimal solutions are not restricted to extreme points in nonlinear problems and the optimal solution can be anywhere. However, in linear problems we only have to check corner or extreme points.

Here, Eq. (6) made the model nonlinear. To linearize this constraint, all possible combinations of \( K_{p_{ij}} \) (\( p \cdot i \cdot j \) combinations) were generated and given as input parameters to the model to calculate the related cost. Then amongst computed costs, the lowest one was chosen and thereby the best combination of \( K_{p_{ij}} \) (the optimal number of base time periods) was obtained.

Exact resolution methods provide optimal solutions for small-sized problems but their computational time is high when solving real-world problems. On the other hand, metaheuristics such as Genetic Algorithm are able to produce a close to an optimal solution in a reasonable time.

3.3 Genetic Algorithm methodology

Genetic algorithm (GA) is a search and optimization methodology that mimics the natural selection process and was invented by John Holland in the early 1970's [28]. GA
starts with generating a set of any possible solutions called the preliminary population. A preliminary population is a group of all individuals and a chromosome is a blueprint of an individual that contains information about the solution. Each chromosome has a fitness, which is an indicator of how well the chromosome solves the problem. In each generation of the GA, the fitness associated with each chromosome is evaluated and the fittest ones (parents) are chosen for reproduction. Neighborhood search operators such as crossover and mutation are applied to the best-fitted ones (parents) to give birth to new chromosomes (offspring). Then, the fitness function associated with offspring is evaluated and the least fit chromosomes are replaced with newly reproduced offspring within the population. Therefore, the population develops toward the best possible solution. This process is iterated until the stopping criteria are reached. Some of the commonly used stopping criteria in GA are as follows:

- Improvement Limit: A satisfactory improvement level is reached.
- Generations: A maximal number of iterations is reached.
- Time Limit: A maximal CPU time is reached.

3.3.1 Chromosome representation

Chromosome representation shows the structure of the solution in GA. As mentioned in section 3.1.2, the decision variables for this problem are the number of base time periods between orders of product \( p \) at hospital \( i \) that was sent from supplier \( j \) (\( K_{pij} \)). Here, a chromosome is represented by an array of integers in which genes are the decision variables. Therefore, the number of genes in each chromosome is equal to the number of products \( \times \) number of hospitals \( \times \) number of suppliers. As shown in Figure 1, for a problem
with 3 products, 2 hospitals and 2 suppliers, a chromosome has 12 genes \((3 \times 2 \times 2)\) in which the 2\(^{\text{nd}}\) gene \((p,i,j=1,1,2)\) represents the time interval of product 1 at hospital 1 that was sent from supplier 2.

<table>
<thead>
<tr>
<th>(p, i, j)</th>
<th>1,1,1</th>
<th>1,1,2</th>
<th>1,2,1</th>
<th>1,2,2</th>
<th>2,1,1</th>
<th>2,1,2</th>
<th>2,2,1</th>
<th>2,2,2</th>
<th>3,1,1</th>
<th>3,1,2</th>
<th>3,2,1</th>
<th>3,2,2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_{pij})</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 1. Sample chromosome

### 3.3.2 Population initialization

The preliminary population is randomly generated from a discrete uniform distribution with bounds of 1 and \(L_p\).

### 3.3.3 Selection strategy

The important intention of selection is to allow the fittest chromosomes to pass on their genes to the successive generation. Here, the roulette wheel selection method is used in which probabilities are assigned to each chromosome based on their fitness function value (cost) and chromosomes with the lowest cost have a higher chance to be selected as parents. The selection probability of chromosome \(m\) is obtained by the following formula, where \(N\) is the population size.

\[
Probability_m = 1 - \frac{\text{Cost(} chromosome_m \text{)}}{\sum_{n=1}^{N} \text{Cost(} chromosome_n \text{)}}
\]  

A pair of parents is selected based on the forenamed selection.
3.3.4 Genetic operators

Reproduction is performed using a combination of crossover and mutation (genetic operators) to obtain a new population. The crossover operator merges the genetic information of two parents to form offspring. In this thesis, single point crossover is used, where one crossover point is selected randomly between \([1, 2, \ldots, N_{\text{var}} - 1]\) and the variables before and after that point is exchanged between the two parents to produce two new children.

![Chromosome reproduction example](image)

The mutation operator induces diversity to the population and prevents premature convergence. The mutation rate is the probability that a gene in a chromosome might flip to another value. A high mutation rate may inhibit the population from converging to an optimal solution and very low rates may lead to premature convergence. Here, the uniform mutation is used in which a gene value might be altered with a random value between the gene’s upper and lower limits. Since the life span of the product should not be violated, an
upper limit equal to lifespan of each product is considered. Figure 3, shows an example of uniform mutation to generate an offspring

Before mutation

| 2 | 2 | 2 | 1 | 2 | 1 | 3 | 2 | 1 | 1 | 1 | 2 |

After mutation

| 2 | 2 | 2 | 1 | 2 | 3 | 3 | 2 | 1 | 1 | 1 | 2 |

Figure 3. Offspring mutation example

In Figure 3 the mutation operator changed the 6\textsuperscript{th} gene in offspring from 1 to 3.

3.3.5 Termination criteria

The process of developing toward a better solution is iterated until either a maximal number of iterations is reached or the improvement percentage between the last iteration and its previous 1000\textsuperscript{th} iteration is less than 2%.

Figure 4 depicts a flowchart with the steps of genetic algorithm runs.
Figure 4. The flowchart of Genetic Algorithm
This iterative process mentioned in Figure 4 starts from creating the initial populations and after each generation, the population evolves toward a better solution and it continues until one of the possible termination criteria is met.
Chapter 4: Results

This chapter describes a series of experiments to test and analyze the performance of the model developed in Chapter 3. There are four main sets of tests completed.

- The first set discusses the optimality of the obtained solution.
- The second set includes tests to investigate the impact of different model components on total cost.
- The third set tests the effectiveness of the Genetic Algorithm (GA) to obtain an optimal solution.
- The fourth set includes the discussion of solving proposed model with a real-world data obtained from a large hospital in Pennsylvania.

For the first three sets of tests, instances are created to consider a wide range of situations. Up to 26 different instances of tests which vary in terms of the number of hospitals, suppliers and products are generated. The testbed contains instances that are generated with the input parameters is shown in Table 1. Some of these input parameters such as inventory holding cost, backorder cost, disposal cost per unit of product are set based on the obtained information from the case study. Transportation cost per mile is set based on RTS Financial website [29].
Table 1. Computational study parameters

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Parameter Letters</th>
<th>Value (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hospitals</td>
<td>I</td>
<td>2, 4, 6</td>
</tr>
<tr>
<td>Number of products</td>
<td>P</td>
<td>3, 6, 9</td>
</tr>
<tr>
<td>Number of suppliers</td>
<td>J</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>Number of periods</td>
<td>H</td>
<td>12 months</td>
</tr>
<tr>
<td>Base Time</td>
<td>T</td>
<td>1 month</td>
</tr>
<tr>
<td>Demand</td>
<td>$d_{pij}$</td>
<td>randomly selected from the interval [0, 200]</td>
</tr>
<tr>
<td>Inventory holding cost</td>
<td>$c_{pi}$</td>
<td>7% of unit purchasing cost</td>
</tr>
<tr>
<td>Distance</td>
<td>$s_{ij}$</td>
<td>randomly selected from the interval [20, 300]</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>$\delta$</td>
<td>$2 per mile</td>
</tr>
<tr>
<td>Backorder cost</td>
<td>$\rho_p$</td>
<td>randomly selected from the interval [4, 7]</td>
</tr>
<tr>
<td>Disposal cost</td>
<td>$\sigma_p$</td>
<td>2% of unit purchasing cost</td>
</tr>
<tr>
<td>Lifespan</td>
<td>$L_p$</td>
<td>randomly selected from the set {2,3,4}</td>
</tr>
<tr>
<td>Quantity breakpoints</td>
<td>$\beta_{p,1}, \beta_{p,2}, \beta_{p,3}$</td>
<td>randomly selected integers from the intervals [25, 35], [45, 55] and [65, 75], respectively</td>
</tr>
<tr>
<td>Purchasing cost</td>
<td>$\alpha_p$</td>
<td>randomly selected from the interval [0.5, 50]</td>
</tr>
<tr>
<td>Storage capacity</td>
<td>$M_i$</td>
<td>randomly selected integers from the interval [400, 600]</td>
</tr>
</tbody>
</table>

In this study, random values are first generated based on Table 1 for the smallest instance with the size of 2-2-3 (Hospital-Supplier-Product). Then larger instances utilize
the values from the previous instance and values for the larger dimensions are generated. For example, to generate the instance with the size of 2-4-6, the instance with the size of 2-4-3 is used and for the remaining combinations, values are generated based on Table 1.

4.1 Optimal solution

In this section, the performance of the model in generating logical results is examined. To investigate this, the lower and upper limits of the time interval (K) that are equal to product lifespan 1 and (K=Max=LS), respectively, are entered into Objective Function (OF) presented in Chapter 3 and the OF elements are calculated. The OF elements for those upper and lower limits are then compared with the optimal solution for three different combinations of Hospital-Supplier-Product ($i \times j \times p$).

It is expected to incur higher inventory holding cost when orders are placed less frequently, since in that case a larger number of units are ordered each time which results in keeping more inventory in stock during the planning period. Moreover, ordering less frequently results in less shortage and more deterioration because products stay longer at the storage area before being used. Also, when the products are bought in larger quantities, the unit price for each product reduces, thus the purchasing cost decreases when orders are placed less frequently (larger amount). On the other hand, ordering less frequently implies fewer trips, so there is less transportation cost.

Table 2, Table 3, and Table 4 show the results of comparing 3 different scenarios (K=1, K=Max, K=Optimal) for 2-3-3, 2-3-6 and 2-3-9 (Hospital-Supplier-Product) combinations.
Table 2. Cost elements comparison for $2 - 3 - 3$ combination

<table>
<thead>
<tr>
<th>Cost Elements</th>
<th>K= 1</th>
<th>K= Optimal</th>
<th>K = Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory holding cost</td>
<td>$8,539</td>
<td>$12,737</td>
<td>$29,481</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>$68,759</td>
<td>$50,629</td>
<td>$49,350</td>
</tr>
<tr>
<td>Backorder cost</td>
<td>$428</td>
<td>$21</td>
<td>$0</td>
</tr>
<tr>
<td>Discarding cost</td>
<td>$0</td>
<td>$101</td>
<td>$938</td>
</tr>
<tr>
<td>Purchasing cost</td>
<td>$58,732</td>
<td>$45,325</td>
<td>$44,119</td>
</tr>
<tr>
<td><strong>Total Cost</strong></td>
<td>$136,458</td>
<td><strong>$108,813</strong></td>
<td>$123,888</td>
</tr>
</tbody>
</table>

Table 3. Cost elements comparison for $2 - 3 - 6$ combination

<table>
<thead>
<tr>
<th>Cost Elements</th>
<th>K= 1</th>
<th>K= Optimal</th>
<th>K = Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory holding cost</td>
<td>$16,147</td>
<td>$16,921</td>
<td>$40,924</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>$124,905</td>
<td>$95,732</td>
<td>$94,603</td>
</tr>
<tr>
<td>Backorder cost</td>
<td>$1030</td>
<td>$96</td>
<td>$0</td>
</tr>
<tr>
<td>Discarding cost</td>
<td>$0</td>
<td>$331</td>
<td>$1,910</td>
</tr>
<tr>
<td>Purchasing cost</td>
<td>$164,231</td>
<td>$121,060</td>
<td>$119,612</td>
</tr>
<tr>
<td><strong>Total Cost</strong></td>
<td>$306,313</td>
<td><strong>$234,140</strong></td>
<td>$257,049</td>
</tr>
</tbody>
</table>
Table 4. Cost elements comparison for 2 – 3 – 9 combination

<table>
<thead>
<tr>
<th>Cost Elements</th>
<th>K= 1</th>
<th>K= Optimal</th>
<th>K = Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory holding cost</td>
<td>$28,657</td>
<td>$30,985</td>
<td>$71,746</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>$187,905</td>
<td>$160,732</td>
<td>$159,603</td>
</tr>
<tr>
<td>Backorder cost</td>
<td>$1,790</td>
<td>$221</td>
<td>$0</td>
</tr>
<tr>
<td>Discarding cost</td>
<td>$0</td>
<td>$1,060</td>
<td>$3,634</td>
</tr>
<tr>
<td>Purchasing cost</td>
<td>$256,761</td>
<td>$224,872</td>
<td>$221,984</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$475,113</td>
<td>$417,870</td>
<td>$456,967</td>
</tr>
</tbody>
</table>

Table 2, Table 3, and Table 4 all show that the time intervals obtained from optimization approach return the lowest or optimal Total Cost. Therefore, considering K=1 or K=Max do not provide the lowest cost and the optimal time interval is somewhere between these two extreme values. Indeed, the optimal time interval is associated with the best tradeoff between different cost elements.

More importantly, all of the tables show that as the time interval increases from its minimum value to its maximum value:

- The inventory holding cost decreases
- Transportation cost increases
- Backorder cost decreases
- Discarding cost increases
- Purchasing cost decreases
Based on what was discussed earlier, these observed trends for cost elements are the same as what was expected.

4.2 Testing model components

This section is divided into three subsections; each investigates the impact of components that are incorporated in the model on total cost. The three components are as follows:

- Considering different time intervals for different products.
- Considering a discount structure.
- Considering hospitals’ storage capacity.

4.2.1 Considering different time interval for different products

As mentioned in the previous chapter, in this study each supplier ships the products based on the time interval of each product. Also, there is a fixed storage capacity at each hospital and this capacity constraint cannot be violated.

In this section, the scenario of delivering different products to hospitals using the same replenishment frequency is compared with the scenario that different products have different time intervals as in the developed model developed in Chapter 3. To perform this investigation, these two scenarios are compared through 9 different combinations of Hospital-Supplier-Product.

The results of this comparison are shown in Figure 5 in the form of percentage of decrease in cost. Each bar shows how much cost is decreased when different replenishment frequencies are allowed for different products than when the same replenishment frequency is considered.
As shown in Figure 5, the scenario of delivering different products to hospitals requiring the same replenishment frequency does not provide a better result in terms of cost. The decrease in total cost when using the optimal frequency for each product, for the 10 combinations ranges from 6% up to 15% which indicates the significant impact of this component.

4.2.2 Considering discount structure

As discussed in Chapter 3, one of the components of this study is the exploitation of economies of scale in the purchasing cost structure. This component has a major impact on the replenishment plan and total cost. Indeed, it is one of the model’s components that forces the model to order less frequently to take the advantage of savings in bulk purchases.

In order to examine the impact of this component, 27 different instances (combinations of Hospital-Supplier-Product) are compared when the discount structure is
incorporated into the model, versus when no discount structure is considered. The results of this comparison are shown in Figure 6 in the form of percentage of decrease in cost. Each bar shows how much cost is decreased regarding this comparison.

![Figure 6: Impact of discount structure on total cost](image)

As expected, incorporating a discount structure results in choosing better replenishment plans with lower cost. The decrease in cost ranges from 8% up to 18%, which indicates the significant impact of this component. This decrease is due to taking the advantage of paying a low unit price for products. Then ANOVA test was applied to see which factors (Hospital, Supplier, Product) have significant effect on this decrease. The P-value for Hospital, Supplier and Product are 0.94, 0.54 and 0.214 respectively, all of them above 0.05, so these factors didn't have a significant effect on this decrease.

Moreover, in the real world, many suppliers offer a discount for bulk purchases and considering this component makes the model more applicable.
4.2.3 Considering storage capacity at hospitals

As mentioned in Chapter 3, hospitals are often replenished by several suppliers. Practically, hospitals have a finite storage area that can potentially affect the replenishment plan and associated cost. In this section, the impact of adding the storage capacity for hospitals is investigated. To test this impact, 27 different combinations of Hospital-Supplier-Product are compared when the supply chain has capacity constraints at hospitals versus when hospitals have an infinite storage area. The results of this comparison are shown in Figure 7 in the form of percentage of increase in cost.

![Figure 7. Impact of capacity on total cost](image-url)
Based on the obtained results, including the capacity constraint leads to a cost increase for all of the combinations, starting from 3% up to 17%. Then just the same as previous component, ANOVA test was applied to see which factors (Hospital, Supplier, Product) have significant effect on this increase. The P-value for Hospital, Supplier and Product are 0.065, 0.856 and 0.073 respectively, all of them above 0.05, so these factors didn't have a significant effect on this increase.

To further investigate the reason for this increase, the cost element percentages related to this increase is examined and the results are shown in Figure 8.

Figure 8. Impact of capacity on cost elements

Figure 8 shows that adding the capacity constraint mainly has high impact on increasing transportation and backorder costs. Transportation cost increased due to an
increase in the number of trips done from suppliers to hospitals, which indicates that due to the limited storage area, the hospitals are replenished more frequently. On the other hand, shortages happen more because of finite inventory. Although the capacity constraint increases total cost, it cannot be ignored since hospitals do not have access to infinite warehouses.

4.3 Genetic Algorithm vs. Cplex (global optimum)

Exact solution methods provide optimal solutions for small-sized problems but their computational time is high when solving real-world problems. On the other hand, metaheuristics such as Genetic Algorithms are able to produce close to an optimal solution in a reasonable time. Therefore, metaheuristics are a worthwhile option to consider.

In order to verify the effectiveness of the applied Genetic Algorithm (GA), the total cost obtained from the GA is recorded and compared with Cplex (global optimum) for 12 small-sized instances (combinations of Hospital-Supplier-Product) in Figure 9 and Table 5.
Figure 9 and Table 5 show that the total costs obtained from Cplex and GA for 12 instances are almost the same. The differences between these two approaches over these
12 instances are less than maximum 0.01%. Therefore, the applied GA is reliable to be used in real-world problems instead of Cplex when the size of the problem becomes large.

After verifying the GA is obtaining a solution close to optimal, the proposed model is solved for different 24 combinations of Hospital-Supplier-Product, five times per mutation probability of: 0.1, 0.2, 0.3, 0.4 and 0.5. The objective is to find the best mutation probability range in terms of a number of generation needed to reach the optimal solution.

Figure 10 shows that with what mutation probability, the average number of generations needed to reach the optimal solution is minimum.

![Figure 10. Mutation probability vs. number of generations for optimal solution](image)

In Figure 10, the number of generations range indicates the smallest and largest number of generations to fulfill the stopping criteria among different 27 combinations of Hospital-Supplier-Product. On the other hand, the average indicates the number of generations where most of the combinations fulfill the stopping criteria. It can be seen that
when the mutation probabilities are equal to 0.2 and 0.1 the average number of generations needed to reach a satisfactory improvement percentage and also the ranges are smaller than other probabilities. Therefore, from now on either one of these two mutation probabilities are utilized.

4.4 Case study

4.4.1 Data description

In this section, the proposed model is applied on the cardiac catheterization laboratory (cath lab) of a large hospital in Pennsylvania, to find the best replenishment plan. The medical center supply system is relatively organized but it is felt that stronger inventory model is needed to improve their efficiency.

The data used represents the cath lab products’ monthly usage rate for years 2015 and 2016. Here, due to extensity of database, the products that have low demands are eliminated and the data for those products that have average monthly usage rate (demand) of 20 or higher are kept. After this cut-off, 144 products are remaining, which is large enough to test the proposed model. The average monthly usage rate for each product is calculated over years 2015 and 2016.

In this case study, a supplier distributes 144 types of products to the cardiac cath lab.

Some of the assumptions are as follows:

- In the case of stock-outs, the unsatisfied demand is borrowed and filled through the nearest hospital at an additional cost and they are returned as soon as a new order arrives.
• The shortage cost per product is 20% of its purchasing cost.
• The discarding cost per expired product is 2% of its average purchasing cost.
• The inventory holding cost for keeping one unit of product is 7% of its average purchasing cost.
• Due to lack of information on some of the model’s features such as available storage capacity at cardiac cath lab and complexity in their transportation system, those features are ignored and the model is updated.

4.4.2 Results and discussion

After updating the model for cath lab case study the adopted GA is implemented on the model with collected input parameters. This procedure is coded in Matlab with the mutation probability of 0.2.

Table 5 presents the obtained optimal total cost and its elements for ordering the 144 products for a planning horizon of one year. Also, the best replenishment schedule for cath lab is shown in Appendix A.

Table 6. Values of cost elements under optimal plan

<table>
<thead>
<tr>
<th>Cost Element</th>
<th>Value</th>
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<tbody>
<tr>
<td>Inventory Holding Cost</td>
<td>$61,810</td>
</tr>
<tr>
<td>Backorder Cost</td>
<td>$15,999</td>
</tr>
<tr>
<td>Discarding expired products Cost</td>
<td>$1,054</td>
</tr>
<tr>
<td>Purchasing Cost</td>
<td>$724,186</td>
</tr>
<tr>
<td>Total Cost</td>
<td><strong>$797,892</strong></td>
</tr>
</tbody>
</table>

Table 6 shows that the purchasing cost has the highest share among other cost elements in total cost.
To investigate the obtained replenishment plan of 144 products, the total number of the products are broken down and grouped by lifespan and optimal replenishment plan in Table 7.

Table 7. Number of the products breakdown by lifespan and optimal replenishment plan

<table>
<thead>
<tr>
<th>Optimal K</th>
<th>Lifespan</th>
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<tbody>
<tr>
<td></td>
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<td>3</td>
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<td>4</td>
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</table>

Table 7 shows that for all three lifespans, number of products increases for larger K. In other words, at each lifespan most of product types are ordered less frequently over planning horizon of one year. To further investigate this observation, the average usage rates for all three lifespans are calculated and shown in Table 8.

Table 8. Average usage rate breakdown over a year by lifespan and optimal replenishment plan

<table>
<thead>
<tr>
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<td>1</td>
<td>86.09</td>
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<td>2</td>
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<td>73.54</td>
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</table>
Table 8 shows that for all three life span, the average usage rate increases for larger K. Considering the high share of purchasing cost in total cost, this trend might be due to taking the advantage of bulk purchasing.

To further investigate the cath lab results, the obtained total cost with the GA is compared with the total cost of 2 different scenarios in Table 9.

- Scenario I: The cath lab puts in an order every month (K=1).
- Scenario II: The cath lab puts in an order based on the lifespan of products (K=Max=LS).

Table 9. Values of cost elements under optimal plan

<table>
<thead>
<tr>
<th></th>
<th>Scenario I</th>
<th>Obtained Result</th>
<th>Scenario II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inventory Holding Cost</strong></td>
<td>$29,104</td>
<td>$61,810</td>
<td>$87,652</td>
</tr>
<tr>
<td><strong>Purchasing Cost</strong></td>
<td>$796,688</td>
<td>$724,186</td>
<td>$720,245</td>
</tr>
<tr>
<td><strong>Shortage Cost</strong></td>
<td>$33,358</td>
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<td><strong>Discarding Cost</strong></td>
<td>$157</td>
<td>$1,054</td>
<td>$2,123</td>
</tr>
<tr>
<td><strong>Total Cost</strong></td>
<td>$859,308</td>
<td>$797,892</td>
<td>$820,659</td>
</tr>
</tbody>
</table>

Table 9 shows that the same trends are obtained for cost elements of larger problem as are observed for smaller problems in Table 2, Table 3, and Table 4.

4.4.2.1 Sensitivity analysis of usage rate

In this section, a sensitivity analysis is conducted to investigate the effect of changes in the usage rate on replenishment plan and cost. Sensitivity analysis is performed by changing the initial usage rate of each product by -50%, -25%, +25%, +50% and +100%. The results are displayed in Figure 11, and Figure 12.
Figure 11 shows that as the usage rate increases the number of trips done between the cath lab and supplier decreases. This means that, in higher usage rates, it is more efficient to order less frequently, since the model can take advantage of bulk purchasing discount.
From Figure 12 it is observed that as the usage rate increased the cost spent per unit of product decreased. This can be concluded that increase in usage rate results in spending less money for each product. Again this is due to taking advantages of bulk purchasing.
Chapter 5: Conclusion

5.1 Summary

Unlike process operations and management, only a few studies on supply chain optimization studies have been done in health care systems, so optimization techniques are needed to improve the efficiency of the health care supply chain [1, 2].

This thesis was successful in developing a mathematical model for a two-echelon supply chain to find the optimal replenishment interval of multiple perishable products at hospitals while minimizing overall system costs. Overall system costs included inventory holding cost, distribution cost, purchasing cost, backorder cost, and discarding cost. Due to the perishability of products used in a health care system, each product was considered to have a shelf life, after which it becomes spoiled. Also, in order to make the model more practical, it was considered that the hospitals could take advantage of bulk purchasing by paying less per unit of the product when they bought bigger quantities from suppliers.

After formulating the model mathematically, it was verified by programming it within GAMS software package and then solved for different small-sized combinations of Hospital-Supplier-Product using the CPLEX solver. The global optimum replenishment intervals for those instances were obtained by considering an optimality gap of 0.0001.

In order to analyze the performance of the model and its components, two sets of tests were completed. The First set compared the total cost and cost elements at the upper and lower limits of the time interval (K) with the optimal time interval for three different instances. It was observed that inventory holding cost and discarding cost were directly related to the time interval. And transportation cost, purchasing cost and backorder cost
were inversely related to the time interval. This meant that considering $K=1$ or $K=LS$ did not guarantee the lowest cost and the optimal time interval was somewhere between these two values.

The second set investigated the impact of three components that were incorporated into the model:

- Considering different replenishment interval for different products resulted in 6% to 14% decrease in cost.
- Considering discount structure resulted in choosing better replenishment plans with lower cost. The cost decrease ranged from 8% up to 18%.
- Considering hospitals' storage capacity led to cost increase from 3% up to 17%. Adding the capacity constraint had mainly high impact on increasing transportation and backorder cost.

To cope with the limitation of Cplex in solving the large-sized problems, a Genetic Algorithm (GA) was presented and then validated to produce a near-optimal solution through comparing with the Cplex global optimum in small-sized problems. The difference percentages between GA and Cplex were less than 0.01%. Therefore, the applied GA was reliable to be used in real-world problems.

To find the best mutation probability for GA, 24 combinations of Hospital-Supplier-Product, were tested 5 time with a mutation probability of 0.1, 0.2, 0.3, 0.4 and 0.5. The results showed that when the mutation probabilities are equal to 0.2 and 0.1 the average number of generations needed to reach the stopping criteria was minimal.
Finally, using GA, the practical case coming from a large hospital in Pennsylvania was analyzed and best optimal replenishment plan was obtained. It was observed that for all three lifespans, number of products and average usage rates increased for larger K. Considering the high share of purchasing cost in total cost, this trend might be due to taking advantage of bulk purchasing discount.

Then, the obtained result was compared versus when K=1 or K=LS and it was observed that the obtained replenishment plan is more efficient. Furthermore, a sensitivity analysis showed that replenishment plan and cost per unit of product are sensitive to usage rate.

It was observed that as the usage rate increased the number of trips done between cath lab and supplier and the cost spent per unit of product decreased. This trend also could be due to taking advantage of bulk purchasing discount.

5.2 Application

Although the current study was developed to model the healthcare supply chain, it can also be used for other industries in which their products lose their market value overtime. Examples of such products are dairy products, fresh produce, blood products and certain chemicals.

5.3 Future research and extensions

Based on the scope of this research, there are some extensions for attention in future research. One of the extensions would be developing a three-echelon supply chain in which there is a distribution center between suppliers and hospitals and then comparing this
system with the current model to find out whether it is reasonable to add a distribution center to the chain or not.

Another extension would be incorporating other concepts such as safety stock, lead time, supplier production capacity, transit capacity and truck-load or less-than-truckload shipments. These extensions make the model more applicable to be used in real world problems.

Lastly, it would be interesting to use other metaheuristics or develop a new hybrid metaheuristic and compare them with adopted GA results to find out whether they can outperform the GA or not.
References


## Appendix A: Optimal Replenishment Plan of Cath Lab

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</table>
Appendix B: GAMS Code

Sets
  p 'Number of Products' /p1*p3/
i 'Number of retailers' /i1*i2/
j 'Number of suppliers' /j1*j2/
h 'Number of periods' /h0*h12/
Count
  tt /t1*t12/
;

ALIAS   (h,hh);

scalars
  T 'Base Time' /1/
delta 'Transportation cost per mile'
Beta1 'Break Point 1'
Beta2 'Break Point 2'
Beta3 'Break Point 3'
Alpha1 'Purchasing Cost 1'
Alpha2 'Purchasing Cost 2'
Alpha3 'Purchasing Cost 3'
Alpha4 'Purchasing Cost 4'
M 'Big value'
C_Pen
;

Parameters
  d(h,p,i,j) 'Demand'
K(p,i,j) 'Time interval'
Obj(Count)
Rest(h,p,i,j)
b0(h,p,i,j)
    LS(p) 'Life span for product P'
;

Table  dis(i,j) 'Distance'
Table  c(p,i) 'Inventory holding cost'
Table noe(p,i) 'Discarding cost'
;
Variables
   OF          'Objective variable'
   EI(h,p,i,j)  'Number of expired products'
   Pen(h,p,i,j)
   BI(h,p,i,j)  'Number of expired products'

Integer variables
   Q(h,p,i,j)   'Order quantity (#Carton/month)'
   EX(h,p,i,j)  'Number of expired products'

Positive variable
   F(h,p,i,j)

Binary variable
   b1(h,p,i,j)
   b2(h,p,i,j)
   b3(h,p,i,j)
   b4(h,p,i,j)
   b5(h,p,i,j)
   b6(h,p,i,j)
   b7(h,p,i,j)
   b8(h,p,i,j)
   bp1(h,p,i,j)
   bp2(h,p,i,j)
   bp3(h,p,i,j)
   bp4(h,p,i,j)
   bp5(h,p,i,j)
   bp6(h,p,i,j)
   bp7(h,p,i,j)
   bp8(h,p,i,j)

BI.fx('h0',p,i,j)=0;
EI.fx('h0',p,i,j)=0;
EX.fx('h0',p,i,j)=0;
Equations

Objective Function

Aux_Cons2_2_1(h,p,i,j)
Aux_Cons2_2_2(h,p,i,j)
Aux_Cons2_2_3(h,p,i,j)

Aux_Cons2_3_1(h,p,i,j)
Aux_Cons2_3_2(h,p,i,j)
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Aux_Cons2_6_4(h,p,i,j)

Cons3_1(h,p,i,j)
Cons3_2(h,p,i,j)

Cons4(h,p,i,j)

Cons5(h,p,i,j)

Cons6(h,p,i,j)

Cons7_1(h,p,i,j)
Cons7_2(h,p,i,j)
Objective Function.. \[ \text{OF} = \sum(p,i,j) 0.5c(p,i)d(p,i,j)K(p,i,j)T \]
\[ + \sum(h,p,i,j)\text{ord}(h)>1 2\delta \text{dis}(i,j)b0(h,p,i,j) \]
\[ + \sum(h,p,i,j)\text{ord}(h)>1 F(h,p,i,j) \]
\[ - \sum(h,p,i,j)\text{ord}(h)>1 C_{Pen} Pen(h,p,i,j) \]
\[ + \sum(h,p,i,j)\text{ord}(h)>1 c(p,i)EX(h,p,i,j) \]

Aux_Cons2_2_1(h,p,i,j)\text{ord}(h)>1.. \[ (\text{Beta1} - Q(h,p,i,j)) = g = \]
\[ b1(h,p,i,j) - Mbp1(h,p,i,j) \]

Aux_Cons2_2_2(h,p,i,j)\text{ord}(h)>1.. \[ (\text{Beta1} - Q(h,p,i,j)) = l = \]
\[ Mb1(h,p,i,j) \]

Aux_Cons2_2_3(h,p,i,j)\text{ord}(h)>1.. \[ (b1(h,p,i,j) + bp1(h,p,i,j)) = e = 1 \]

Aux_Cons2_3_1(h,p,i,j)\text{ord}(h)>1.. \[ (Q(h,p,i,j) - \text{Beta1} + 1) = g = \]
\[ b2(h,p,i,j) - Mbp2(h,p,i,j) \]

Aux_Cons2_3_2(h,p,i,j)\text{ord}(h)>1.. \[ (Q(h,p,i,j) - \text{Beta1} + 1) = l = \]
\[ Mb2(h,p,i,j) \]

Aux_Cons2_3_3(h,p,i,j)\text{ord}(h)>1.. \[ (b2(h,p,i,j) + bp2(h,p,i,j)) = e = 1 \]

Aux_Cons2_3_4(h,p,i,j)\text{ord}(h)>1.. \[ (\text{Beta2} - Q(h,p,i,j)) = g = \]
\[ b3(h,p,i,j) - Mbp3(h,p,i,j) \]

Aux_Cons2_3_5(h,p,i,j)\text{ord}(h)>1.. \[ (\text{Beta2} - Q(h,p,i,j)) = l = \]
\[ Mb3(h,p,i,j) \]

Aux_Cons2_3_6(h,p,i,j)\text{ord}(h)>1.. \[ (b3(h,p,i,j) + bp3(h,p,i,j)) = e = 1 \]

Aux_Cons2_4_1(h,p,i,j)\text{ord}(h)>1.. \[ (Q(h,p,i,j) - \text{Beta2} + 1) = g = \]
\[ b4(h,p,i,j) - Mbp4(h,p,i,j) \]

Aux_Cons2_4_2(h,p,i,j)\text{ord}(h)>1.. \[ (Q(h,p,i,j) - \text{Beta2} + 1) = l = \]
\[ Mb4(h,p,i,j) \]

Aux_Cons2_4_3(h,p,i,j)\text{ord}(h)>1.. \[ (b4(h,p,i,j) + bp4(h,p,i,j)) = e = 1 \]

Aux_Cons2_4_4(h,p,i,j)\text{ord}(h)>1.. \[ (\text{Beta3} - Q(h,p,i,j)) = g = \]
\[ b5(h,p,i,j) - Mbp5(h,p,i,j) \]

Aux_Cons2_4_5(h,p,i,j)\text{ord}(h)>1.. \[ (\text{Beta3} - Q(h,p,i,j)) = l = \]
\[ Mb5(h,p,i,j) \]
Aux_Cons2_4_6(h,p,i,j)$\text{ord}(h) > 1$.. \((b5(h,p,i,j)+bp5(h,p,i,j)) = e = 1;\)

Aux_Cons2_5_1(h,p,i,j)$\text{ord}(h) > 1$.. \((Q(h,p,i,j)-Beta3+1) = g = \)

\(b6(h,p,i,j)-M*bp6(h,p,i,j)\);

Aux_Cons2_5_2(h,p,i,j)$\text{ord}(h) > 1$.. \((Q(h,p,i,j)-Beta3+1) = l = \)

\(M*b6(h,p,i,j)\);

Aux_Cons2_5_3(h,p,i,j)$\text{ord}(h) > 1$.. \((b6(h,p,i,j)+bp6(h,p,i,j)) = e = 1;\)

Aux_Cons2_6_1(h,p,i,j)$\text{ord}(h) > 1$.. \(F(h,p,i,j) = g = \)

\(Alpha1*Q(h,p,i,j)-M*(1-b1(h,p,i,j))\);

Aux_Cons2_6_2(h,p,i,j)$\text{ord}(h) > 1$.. \(F(h,p,i,j) = g = \)

\(Alpha2*Q(h,p,i,j)-M*(2-b2(h,p,i,j)-b3(h,p,i,j))\);

Aux_Cons2_6_3(h,p,i,j)$\text{ord}(h) > 1$.. \(F(h,p,i,j) = g = \)

\(Alpha3*Q(h,p,i,j)-M*(2-b4(h,p,i,j)-b5(h,p,i,j))\);

Aux_Cons2_6_4(h,p,i,j)$\text{ord}(h) > 1$.. \(F(h,p,i,j) = g = \)

\(Alpha4*Q(h,p,i,j)-M*(1-b6(h,p,i,j))\);

Cons3_1(h,p,i,j)$\text{ord}(h) > 2$ and $\text{ord}(h) > \text{LS}(p)-1$.. \(EX(h,p,i,j) = g = \)

\(\text{sum}(hh)$\text{ord}(hh) > 0$ and $\text{ord}(hh) < \text{ord}(h)-\text{LS}(p)+1$,$Q(hh,p,i,j)) - \text{sum}(hh)$\text{ord}(hh) > 0$ and $\text{ord}(hh) < \text{ord}(h)+1$,d(hh,p,i,j)) - \text{sum}(hh)$\text{ord}(hh) > 1$ and $\text{ord}(hh) < \text{ord}(h))$,EX(hh,p,i,j));

Cons3_2(h,p,i,j)$\text{ord}(h) = 2$ and $1 > \text{LS}(p)-1$.. \(EX(h,p,i,j) = g = \)

\(\text{sum}(hh)$\text{ord}(hh) > 0$ and $\text{ord}(hh) < 2-\text{LS}(p)+1$,$Q(hh,p,i,j)) - \text{sum}(hh)$\text{ord}(hh) > 0$ and $\text{ord}(hh) < 2+1$,d(hh,p,i,j));

Cons4(h,p,i,j)$\text{ord}(h) > 1$.. \(BI(h,p,i,j) = e = Q(h-

1,p,i,j)+EI(h-1,p,i,j);\)

Cons5(h,p,i,j)$\text{ord}(h) > 1$.. \(EI(h,p,i,j) = e = BI(h,p,i,j)-\)

d(h,p,i,j)-EX(h,p,i,j);

Cons6(h,p,i,j)$\text{ord}(h) > 1$.. \(Q(h,p,i,j) = e = (Q('h0',p,i,j)-\)

EI(h,p,i,j)*b0(h,p,i,j);

Cons7_1(h,p,i,j)$\text{ord}(h) > 1$.. \(Pen(h,p,i,j) = l = EI(h,p,i,j);\)

Cons7_2(h,p,i,j)$\text{ord}(h) > 1$.. \(Pen(h,p,i,j) = l = 0;\)
model Problem /All/;
Problem.OptCR = 1e-6;
Option MIP=Cplex;

loop(Count,
  K('p1','i1','j1')=Status(Count,'t1');
  K('p1','i1','j2')=Status(Count,'t2');
  K('p1','i2','j1')=Status(Count,'t3');
  K('p1','i2','j2')=Status(Count,'t4');
  K('p2','i1','j1')=Status(Count,'t5');
  K('p2','i1','j2')=Status(Count,'t6');
  K('p2','i2','j1')=Status(Count,'t7');
  K('p2','i2','j2')=Status(Count,'t8');
  K('p3','i1','j1')=Status(Count,'t9');
  K('p3','i1','j2')=Status(Count,'t10');
  K('p3','i2','j1')=Status(Count,'t11');
  K('p3','i2','j2')=Status(Count,'t12');

  Rest(h,p,i,j)=mod(ord(h)-1,K(p,i,j));

  b0(h,p,i,j)=1$(Rest(h,p,i,j)=0)+0$(Rest(h,p,i,j)>0);

  Q.fx('h0',p,i,j)=d('h0',p,i,j)*K(p,i,j)*T;

solve Problem minimizing OF using MIP;

);