Automatic Prevention and Recovery of Aircraft Loss-of-Control
by a Hybrid Control Approach

A dissertation presented to
the faculty of
the Russ College of Engineering and Technology of Ohio University

In partial fulfillment
of the requirements for the degree
Doctor of Philosophy

Yue Zhao
April 2016
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This dissertation titled

Automatic Prevention and Recovery of Aircraft Loss-of-Control

by a Hybrid Control Approach

by

YUE ZHAO

has been approved for

the School of Electrical Engineering and Computer Science

and the Russ College of Engineering and Technology by

J Jim Zhu

Professor of Electrical Engineering and Computer Science

Dennis Irwin

Dean, Russ College of Engineering and Technology
ABSTRACT

ZHAO, YUE, Ph.D., April 2016, Electrical Engineering

Automatic Prevention and Recovery of Aircraft Loss-of-Control by a Hybrid Control Approach

Director of Dissertation: J. Jim Zhu

In this dissertation, an integrated automatic flight controller for fixed-wing aircraft Loss-of-Control (LOC) Prevention and Recovery (iLOCPR) is designed. The iLOCPR system comprises: (i) a baseline flight controller for six degrees-of-freedom (6DOF) trajectory tracking for nominal flight designed by trajectory linearization, (ii) a bandwidth adaption augmentation to the baseline controller for LOC prevention using the time-varying PD-eigenvalues to trade tracking performance for increased stability margin and robustness in the presence of LOC-prone flight conditions, (iii) a controller reconfiguration for LOC arrest by switching from the trajectory tracking task to the aerodynamic angle tracking in order to recover and maintain healthy flight conditions at the cost of temporarily abandoning the mission trajectory, (iv) a guidance trajectory designer for mission restoration after the successful arrest of a LOC upset, and (v) a supervisory discrete-event-driven Automatic Flight Management System (AFMS) to autonomously coordinate the control modes (i) - (iv). Theoretical analysis and simulation results are shown for the effectiveness of the proposed methods.
DEDICATION

To my grandparents,

Shushen Li, Yiwei Jin,

and my parents

Hong Jin, Yanyong Zhao
ACKNOWLEDGMENTS

Having this experience of studying at Ohio University is truly an amazing adventure in my life. It has been five years since the first day I walked into my lab.

I would like to express my deep gratitude to Dr. Jim Zhu for his endless patience, valuable guidance and great help for my research. It was him who introduced me to the field of flight control and let me know my potential capability. Wherever I will work in the future, I will always thank for the knowledge and skills that I learned from him. I would like to express my great appreciation to Dr. Douglas Lawrence for his constructive suggestions for my research. I gained a lot from talking to him every time. His is always my high standard of doing research. I am also indebted to my doctoral advisory committee members: Dr. Frank Van Graas, Dr. Robert Williams, Dr. Aili Guo and Dr. Sergiu Aizicovici for their reviewing my dissertation and providing helpful suggestions.

I wish to thank my parents for their confidence in me over these five years.

I wish to thank my boyfriend Pin Zhang for his patience and assistance.

I am very grateful to Ohio University.

I am very grateful to the manager John A. Tantzen in Seagate Technology for his patience during the last year of the completion of my degree.
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NOMENCLAURE

\[ \mathbf{P} = [x_e \ y_e \ z_e]^T \] = vehicle inertial position (m)
\[ \mathbf{V} = [u \ v \ w]^T \] = body frame vehicle velocity (m/s)
\[ \mathbf{A}_b = [a_u \ a_v \ a_w]^T \] = body frame acceleration (m/s^2)
\[ \mathbf{V}_a = [u_a \ v_a \ w_a]^T \] = body frame wind velocity (m/s)
\[ \mathbf{V}_i = [u_i \ v_i \ w_i]^T \] = body frame relative wind velocity (m/s)
\[ \mathbf{\Gamma} = [\phi \ \theta \ \psi]^T \] = Euler angles: roll, pitch, and yaw (rad)
\[ \mathbf{\Omega} = [p \ q \ r]^T \] = angular rates in the body frame (rad/s)
\[ \mathbf{\Theta} = [a_p \ a_q \ a_r]^T \] = angular acceleration in the body frame (rad/s^2)
\[ \mathbf{\Delta} = [\delta_a \ \delta_e \ \delta_r]^T \] = aileron, elevator and rudder deflection (rad)
\[ \mathbf{F} = [F_x \ F_y \ F_z]^T \] = total force on aircraft in the body frame (N)
\[ \mathbf{F}_a = [D \ Y \ L]^T \] = aerodynamic force: drag, side force, and lift in the wind frame (N)
\[ \mathbf{T}_m = [L_m \ M_m \ N_m]^T \] = total torque on aircraft in the body frame (Nm)
\[ \mathbf{T} = [T_x \ T_y \ T_z]^T \] = body frame thrust (N)
\[ \mathbf{\Lambda} = [\alpha \ \beta \ \mu]^T \] = aerodynamic angles: angle-of-attack, sideslip and bank angle (rad)
\[ \mathbf{R} = [R_N \ R_E \ R_D]^T \] = range vector (m)
\[ \mathbf{LOS} = [l_1 \ l_2 \ l_3]^T \] = Line-of-Sight vector
\[ h = -z_e \] = altitude (m)
\[ Q \] = dynamic pressure (N/m^2)
\[ \bar{\rho} \] = local atmosphere density (kg/m^3)
\[ \mathbf{V}_i \] = magnitude of vehicle velocity (m/s)
\[ \gamma, \chi \] = flight path angle, heading angle (rad)
\[ \delta \] = proportional thrust control effector
\[ T, \ T_{\text{max}} \] = thrust magnitude, maximum thrust magnitude (N)
\[ V_{t,\text{max}} \] = maximum allowable vehicle velocity (m/s)
\[ W \] = vehicle weight under conventional gravitational acceleration (N)
\[ m \] = vehicle mass (kg)
\[ g \] = gravitational acceleration (m/s^2)
\[ S \] = wing reference area (m^2)
\[ b \] = wing span (m)

\[ \bar{\sigma} \] = wing mean aerodynamic chord (m)

\[ n \] = load factor

\[ \rho \] = azimuth angle (rad)

\[ \sigma \] = elevation angle (rad)

\[ \bar{\varphi} \] = range vector magnitude

\[ \text{flag} \] = supervisory control flag

\[ K_p, K_i, K_D \] = proportional, integral and differential gain

\[ K_v, K_r \] = velocity and range regulation gains

\[ \alpha_{ij1} \] = \( \omega_n^2(t) \), where \( \omega_n(t) \) is the time-varying natural frequency of desired dynamics for \( i^{th} \) loop, \( i = 1, 2, 3, 4 \), \( j^{th} \) channel, \( i = 1, 2, 3 \), \( (1 = \text{roll channel}, 2 = \text{pitch channel}, 3 = \text{yaw channel}) \)

\[ \alpha_{ij2} \] = \( 2\zeta\omega_n(t) \), where \( \zeta \) is the constant damping ratio of desired dynamics for \( i^{th} \) loop, \( j^{th} \) channel

\[ k_a \] = bandwidth adaptation gain

\[ P(s) \] = plant transfer function

\[ L(s) \] = loop gain

\[ \phi(s) \] = phase response

\[ \omega_b \] = actuator system bandwidth (rad/s)

\[ \omega_{cg} \] = gain cross-over frequency of loop gain (rad/s)

\[ \varepsilon \] = singular perturbation parameter

\[ \omega_n \] = natural frequency (rad/s)

\[ C_{D_b}, C_{D_n}, C_{D\beta} \] = drag force coefficients

\[ C_{Y\beta}, C_{Yr} \] = side force coefficients

\[ C_{\lambda o}, C_{\lambda u}, C_{\lambda se} \] = lift force coefficients

\[ C_{l\beta}, C_{l\alpha}, C_{l\alpha r} \] = roll moment coefficients

\[ C_{m\alpha}, C_{m\mu}, C_{m\mu se} \] = pitch moment coefficients

\[ C_{n\beta}, C_{n\alpha}, C_{n\alpha r} \] = yaw moment coefficients

\[ I_{xx}, I_{yy}, I_{zz} \] = moment of inertia about the body frame axis (kg m^2)

\[ I_{yz}, I_{zx}, I_{xy} \] = products of inertia (kg m^2)

\[ I_{pq}, I_{pr}, I_{pq}, I_{qr}, I_{pr}, I_{qr} \] = inertia coefficients

\[ I_{rr}, g^p_r, g^q_r, g^q_r, g^r_r \] = moments of inertia
\( O_1, O_2, O_3 = \) operation box subset
\( P_1, P_2, P_3 = \) protection box subset
\( S_1, S_2, S_3 = \) safety box subset

**Subscript**
- \( \text{tgt} = \) target
- \( \text{nom} = \) nominal
- \( \text{ctrl} = \) feedback
- \( \text{com} = \) command
- \( \text{sen} = \) sensed
- \( \text{err} = \) state tracking error
- \( \text{arst} = \) arrest command
- \( \text{rstr} = \) restoration command
- \( \text{gui} = \) guidance command

**Shorthand**
- \( S_\theta, C_\theta, T_\theta = \) shorthand notation for \( \sin \theta, \cos \theta, \) and \( \tan \theta \)

**Acronyms**
- \( \text{LOC} = \) Loss-of-Control
- \( \text{iLOCPR} = \) integrated Loss-of-Control Prevention and Recovery
- \( \text{MNL} = \) Multiple-Time-Scale Nested-Loop
- \( \text{PID} = \) Proportional, Integral and Derivative
- \( \text{PI} = \) Proportional and Integral
- \( \text{FAA} = \) Federal Aviation Administration
- \( \text{NASA} = \) The National Aeronautics and Space Administration
- \( \text{EOM} = \) Equation-of-Motion
- \( \text{DOF} = \) Degree-of-Freedom
- \( \text{TLC} = \) Trajectory Linearization Control
- \( \text{PD} = \) Parallel Differential
- \( \text{LTI} = \) Linear-Time-Invariant
- \( \text{LTV} = \) Linear-Time-Varying
- \( \text{BTT} = \) Bank-to-Turn
- \( \text{A/D, D/A} = \) Analog-to-digital; Digital-to-analog
- \( \text{PM} = \) Phase Margin
- \( \text{AFMS} = \) Automatic Flight Management System
- \( \text{GON} = \) Guidance Outer-loop Nominal
GOF = Guidance Outer-loop Feedback
GIN = Guidance Inner-loop Nominal
GIF = Guidance Inner-loop Feedback
GAN = Guidance Control Allocation Nominal
GAC = Guidance Control Allocation Command
EON = Euler Attitude Outer-loop Nominal
EOF = Euler Attitude Outer-loop Feedback
EIN = Euler Attitude Inner-loop Nominal
EIF = Euler Attitude Inner-loop Feedback
EA = Euler Attitude Control Allocation
AON = Aerodynamic Attitude outer-loop Nominal
AOF = Aerodynamic Attitude Outer-loop Feedback
AIN = Aerodynamic Attitude Inner-loop Nominal
AIF = Aerodynamic Attitude Inner-loop Feedback
AA = Aerodynamic Attitude Allocation
LOS = Line-of-Sight
TWA = Tolerable Wind Amplitude
TWB = Time-varying bandwidth
NED = North-East-Down
CG = Center-of-Gravity
CM = Center-of-Mass
CHAPTER 1 INTRODUCTION

In this Introduction, a background about aircraft control and a significant issue for aerospace safety - Loss-of-Control (LOC) are presented based on surveys of the current literature in Section 1.1. An overview of previous work on LOC study and intervention study is made in Section 1.2. The problem of developing an automatic LOC prevention and recovery control system is formulated in Section 1.3. Objectives and approaches for this dissertation are proposed in Section 1.4. The main results of this research are previewed in Section 1.5. Section 1.6 provides the organization of this dissertation.

1.1. Background

There are three types of forces acting on an airplane simultaneously: the aerodynamic force, the gravitational force, and the propulsion. The aerodynamic force has three components: lift force, drag force and side force. These forces are generated by the relative motion of the airfoil with respect to the atmosphere as shown in Figure 1.1.

![Diagram of aircraft forces](image)

Figure 1.1 The aircraft forces in straight, level, un-accelerated flight.
Since dependencies of aerodynamic forces are usually very complex, approximations are made to determine the aerodynamic force by aerodynamic angles including angle-of-attack, side-slip angle, and bank angle as shown in Figure 1.2. Angle-of-attack $\alpha$ can be defined as the angle between the fuselage chord line and the relative wind. In normal operations, the lift is directly proportional to angle-of-attack. However, angle-of-attack cannot exceed an upper bound which is called the critical angle-of-attack, since beyond it flow separation induced stall may occur causing sudden reduction of the lift. In the situation of the stall, the airplane will lose altitude and the ability to control its attitude. The side-slip angle $\beta$ is the angle between the oncoming airflow and the direction towards which the aircraft is pointing. Similar to the angle-of-attack, side-slip angle determines the magnitude of side force. The Large side-slip angle may cause severe vibration of propellers or surge in turbo-engines, which may lead to LOC or cause damage to the vehicle. Sideslapping is undesirable since a lateral acceleration directed toward the airplane center makes passengers uncomfortable. Bank angle $\mu$ is the angle of the vehicle longitudinal axis inclines with respect to its velocity vector. It is preferred maneuver in carrying out a turn, known as a Bank-to-Turn (BTT). If the vehicle is overbanked, it may lose altitude and goes into spiral dive due to the inadequate lift. Therefore, the pilot or autopilot would lose the ability to fix the aircraft with the aerodynamic angles exceeding the safety boundaries and cause the vehicle to go into LOC. Here LOC means that the pilot will not be able to operate the plane, even though the control system of the aircraft may still be physically intact and functional.
Figure 1.2 The aircraft aerodynamic angles: $\alpha$ is the angle-of-attack; $\beta$ is the side-slip angle; $\mu$ is the bank angle.

Based on the Airplane Upset Recovery Training Aid [1] provided by the Federal Aviation Administration (FAA), “an airplane in flight unintentionally exceeding the parameters normally experienced in operations or training” is called airplane upset. LOC is described as motions that: 1) outside the normal operating flight envelopes; 2) not predictably controlled by pilot inputs; 3) high angular rates and displacements [2]. Statistics of airplane accidents show that aircraft LOC were associated with aircraft component failures (including jammed control surfaces, loss of engines, icing contamination), weather conditions (including turbulence, wind shear, mountain waves), and inappropriate crew control. The analysis also shows that the LOC accidents usually involve more than one contributing factors and consequently drive the aircraft into an inadvertent event with abnormal aircraft attitude, angular rate, acceleration, airspeed or flight trajectory.
According to the statistical report of commercial jet accidents occurring from 2003 through 2012 created by the Boeing Company, LOC-in-flight was the No. 1 cause in terms of both the numbers of accidents and the numbers of fatalities [3]. As indicated in Figure 1.3, there were 18 accidents and 1,648 deaths caused by LOC during that period. Moreover, not only for commercial transport airplanes, LOC is listed to be the No.1 cause of upset events also for general aviation [4].

Figure 1.3 The Boeing worldwide commercial jet fleet, 2003-2012 [3].
Due to the evidence above, the Commercial Aviation Safety Team (CAST) has spearheaded the effort to define the causes of LOC and to develop interventions to prevent these accidents. Even though the automatic flight control system (autopilot) is usually equipped in modern aircraft operation systems to reduce the pilot’s workload, the FAA upset recovery rules still require that the pilot needs to take control of the aircraft when LOC occurs. Several pilot training programs provide simulators to educate pilots so that they will have adequate skills to prevent and recover from upsets. Since pilot recovery depends on the pilot response time and may involve inappropriate operations due to human errors, it gives us motivations to develop effective interventions by designing an automatic system that has the capabilities of correctly and promptly responding to unanticipated events and recover from adverse LOC situations.

Accordingly, instead of pilot-in-the-loop, a technological solution to overcome LOC needs to be developed with automatic flight LOC prevention and recovery control.
systems, in which effective control laws can recover the airplane from LOC situations and regain control of the aircraft, and eventually restore normal flight.

1.2. Literature Review

This section provides an overview of the research being conducted for improving aircraft safety to reduce LOC accidents by automatic flight control system design and development across a broad spectrum.

1.2.1 Aircraft Modeling

Investigations of LOC required the dynamic airplane models that were outside of the normal flight envelope, therefore, a 5.5% scaled commercial transport model called Generic Transport Model (GTM) was built by NASA Langley Research Center for simulation and analysis in post stall and spin behaviors [5]. LOC scenarios could be simulated based on the GTM model built in Matlab/Simulink test environment.

Aerodynamic and flight dynamic characteristics during LOC have been investigated based on physical airplane models in [6]. Instead of using data-driven methods, the physically-based modeling approach derived flight dynamic characteristics in upset conditions based on nonlinear lifting line/strip theory analysis of the aerodynamic loads. The nonlinear strip theory was to define the distributed aerodynamic forces on discrete control surfaces such as a wing, horizontal and vertical tails. This method provided the ability to capture critical aerodynamics and resulting phenomena in classical upset conditions.

An aerodynamic model of a generic large transport airplane called SUPRA (Simulation of Upset Recovery in Aviation) was developed by the European Union
Research Project [7]. Prediction of post-stall aerodynamics was based on wind tunnel data and validated by pilot data for upset recovery training. Multiple stall conditions proved that such techniques were able to improve the fidelity and was useful for the ground–based simulations. Motion cue estimates for SUPRA stall scenarios contributed to better simulation performance for airplane recovery training [8].

1.2.2 Analysis Methods

LOC contributing factors were addressed by the review of 126 LOC accidents in [9]. Using scatterplots, the statistical analysis identified the worst case factor combinations and generated 52 LOC sequences based on data from the accidents. Another generalized LOC sequence consisting of vehicle impairment, inappropriate crew response, and vehicle upset was presented [10]. The author claimed that developing onboard integrated control systems that provide avoidance, detection, mitigation and recovery capabilities was an effective way to break a wide variety of LOC sequences [11].

Quantitative envelopes were created on the analytical data. In 2000, the Boeing Company and the NASA Langley Research Center developed a set of metrics to define LOC called Quantitative Loss-of-Control (QLC), comprising five envelopes: 1. The adverse aerodynamics envelop; 2. The unusual attitude envelope; 3. The structural integrity envelope; 4. The dynamic pitch control envelope; 5. The dynamic roll control envelope [2]. These criteria constituted a reliable quantitative indication of LOC such that any maneuver exceeded three or more QLC envelopes can be classified as LOC. A data-based predictive control theory was used to derive the minimum control input boundaries that would avoid LOC events in order to provide information to the pilot or control system [12].
A Safely Recoverable Flight Envelope (SRFE) was proposed in [13] by evaluating the time responses, stability and a limited set of flight upsets analysis. Flight states corresponding to a variety of control strategies were derived through the calculation of SRFE to determine the strengths and limitations of multivariable control strategies.

LOC nonlinearities were associated with two primary sources: the intrinsic nonlinear dynamics of the airplane and the state/input control constraints outside of the healthy flight envelopes. Nonlinearities affect the ability to control the aircraft and may contribute to LOC [14]. Considering the changes in the characteristics of the aircraft due to parameter variations, the bifurcation theory was applied to identify some fundamental airplane upset dynamic behaviors [15]. It indicated that natural rate damping would make the aircraft naturally stable and converge to steady state for small angle-of-attack and elevator deflections [16]. Furthermore, reducing the throttle to a low setting could avoid gravity effect when the aircraft is descending [17].

Based on the bifurcation analysis above, a multi-mode upset recovery flight control system was shown in [18]. The flight control recovery system was divided into several modes as follows: aerodynamic envelope, angular rate, attitude envelope, and speed. The natural rate damping and PID control laws were used to design the controllers in each mode.

1.2.3 Control Theory

Due to the limitations of actuators installed in the aircraft control system, “Anti-windup” techniques were applied to augment a manual flight control system [19]. The effect of actuator magnitude and rate saturation for the linearized aircraft model was
eliminated through a dynamic compensation scheme combined with dynamic modifications of the pilot command. The resulting anti-windup controller allowed a fast response to the pilot commands by employing the maximal effort of the constrained actuators. Therefore, the control system was able to guarantee the stability of aircraft for aggressive pilot commands and effectively overcome the actuator saturations induced LOC.

Robust control combined with intelligent learning techniques were used to supervise LOC recovery [20]. The Reinforcement Learning (RL) modules were applied to simulation data in order to discover the best recovery strategy under different optimization objectives. The autonomous recovery system was divided into two components: the angular rate arrest system and the unusual attitude recovery system. The actuators were directly controlled by the RL module in the angular rate arrest subsystem; feedback information was used to determine the best action in the unusual attitude subsystem.

Moments produced by gas turbine engines can be used for aircraft control, especially for the situations when an aircraft was facing jammed actuators, surface damage or in-flight icing. An integrated flight propulsion control design for LOC prevention was developed using engines to substitute the control surface and make an urgent attitude change in LOC prevention [21]. Such control law automatically made use of airplane engines as additional flight effectors in the event of LOC, therefore, avoiding possible crew faults.

Aircraft LOC was addressed in [22] for different failure modes that were associated with various categories of uncertain failures. A nonlinear disturbance rejection controller
was designed for fault tolerance based on impaired vehicle model. A constant disturbance characterizing the severity of the failure could be eliminated at jammed actuators.

A hybrid fault tolerant system with complete loss of actuator operations was addressed in [23]. Optimal supervisor strategy allowed the system to switch between different modes including system failures, maneuverability near stall and environmental-induced LOC. An extended aircraft model with nonlinear terms was used to design the nonlinear stabilizing regulator [24]. The system can be seen as having multi-regimes of operation with reconfigurable controller modes with fast responses. The recovery controller construction was solved by the Hamilton-Jacobi-Bellman partial differential equations. The constrained recovery controller design was based on the linear quadratic regulator (LQR) [25]. With the bounds of states and control inputs cascaded to the regulator design, the smooth recovery maneuverability was improved in the simulation results. A high order sliding mode observer was applied in the previous LOC recovery controller to realize decision making for different control modes [26].

1.2.4 Pilot Inspired Control

The Pilot Guide to Airplane Upset Recovery specifies the LOC as: the pitch attitude greater than 25 degrees, nose up; pitch attitude greater than 10 degrees, nose down; bank angle greater than 45 degrees; inappropriate airspeed. Therefore, the effective LOC recovery by pilot should include [1]: decrease the angle of attack by pushing the nose down so the aircraft can regain lift; smoothly increase power which increases the speed slowly while maintaining a full coordination of the controls; minimize altitude loss and an ideal and effective recovery procedure must keep altitude loss to within at most 30m (100feet).
Therefore, the pilot training strategies can be used to guide the design and validate automatic LOC control system.

“Pitch protection” logic in the pitch control system was used to facilitate stall recovery [27] because a “healthy” angle-of-attack will keep adequate lift to balance the gravity and possible disturbances. It is indicated that pitch down to a lower angle-of-attack could result in a quick recovery from stall upsets with minimal roll off and loss of altitude. In [28], an important solution to support LOC prevention and recovery was addressed under pilots’ operations. Sequential steps for recovery training from a stall were listed, which were: autopilot and auto-throttle, pitch, pitch trim, roll, thrust, speed brakes/spoiler, return to the desired flight path.

1.3. Problem Statement

Based on the survey of the research on LOC and the various control techniques used in automated LOC recovery, “LOC is a complex event, usually resulting from multiple causals and contributing factors that can occur individually or (more often) in combination. There is, therefore, no single intervention strategy that can be readily identified to prevent LOC accidents” [11]. The chain of a serious of complex events that cause LOC, and proper interventions can be subdivided into nominal, prevention, LOC arrest, and mission restoration as shown in Figure 1.5. Prevention is defined to be the control strategies and maneuvers to sustain normal flight under unexpected adverse events such as environmental hazards, system failures, vehicle damage or pilot errors. In the event that prevention fails and the aircraft still slips into upset situations, then LOC arrest becomes necessary, which is to activate control strategies to rescue the aircraft from a LOC event. A stabilized flight
path recovery should be initiated as soon as LOC arrest maneuvers are complete, which is the restoration. LOC arrest and restoration will eventually achieve full flight mission recovery.

![Figure 1.5 Automatic LOC prevention and recovery problem.](image)

According to the literature survey given in 1.2, it appears that the evaluation of an onboard automated LOC prevention and recovery system should be based on the criteria given in Table 1, which will be used to guide and substantiate the accomplishment of this research.

Table 1.1 LOC prevention and recovery criteria.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Effectiveness</th>
<th>Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prevention</td>
<td>Sustain long prevention time</td>
<td>Low-stress factors</td>
</tr>
<tr>
<td></td>
<td>Accommodate large disturbances</td>
<td>Low loading factors</td>
</tr>
<tr>
<td></td>
<td>Small tracking errors</td>
<td>Large stability margins</td>
</tr>
<tr>
<td></td>
<td>Small attitude deviations</td>
<td></td>
</tr>
<tr>
<td>Arrest</td>
<td>Small altitude loss</td>
<td>Low-stress factors</td>
</tr>
<tr>
<td></td>
<td>Small range loss</td>
<td>Low loading factors</td>
</tr>
<tr>
<td></td>
<td>Small course deviation</td>
<td>Large stability margins</td>
</tr>
<tr>
<td></td>
<td>Sustain short arrest time</td>
<td></td>
</tr>
<tr>
<td>Restoration</td>
<td>Optimal restoration time</td>
<td>Large stability margins</td>
</tr>
<tr>
<td></td>
<td>Optimal returning path</td>
<td></td>
</tr>
</tbody>
</table>
Even though the LOC problem can be formulated from different perspectives by the existing methods, the effectiveness of the methods and the performance of LOC prevention and recovery are still far from satisfactory. Small wind disturbance accommodation has been considered in the aircraft control system design [29, 30], but advanced aircraft control method should be applied for control system design to improve the robustness so that the aircraft can survive severe weather conditions. Given different LOC events (for example, jammed actuators or stuck aircraft surface), the existing LOC prevention methods focus on the computations of the safe set, especially in control constraints design of the flight control systems. However, it is difficult to anticipate all different adverse operating scenarios in the automatic control systems since the LOC occurs due to a combination of various factors, and detecting an impending LOC mode in real-time during an upset can be very challenging. In [18], up to 700 feet altitude loss occurred by linearized natural damping recovery for the aerodynamic envelopes. Such LOC recovery is more likely to experience a crash considering the terrain, especially during approach and landing. Such performance would not be considered an effective LOC recovery by the Pilot Guide to Airplane Upset Recovery [1], which requires that the altitude loss must be within 30m (100 feet).

Various control methods have been used striving to provide passengers with a smooth ride and high degree of safety. Order-reduction, Decoupling, Linearization and Frozen time techniques are commonly used to develop the control system for LOC prevention and recovery. However, flight control systems are designed typically for
nominal aircraft Equation-of-Motions (EOMs), which are invalid under upset conditions. In addition, such methods provide limited capabilities because the effectiveness or even stability of such designs rely heavily on the simplification assumptions pertaining to the specific LOC mode and aircraft models. In addition, perturbations caused by order-reduction (singular perturbation), decoupling (regular perturbation), linearization (vanishing regular perturbation), frozen time (non-vanishing regular perturbation) have typically been neglected during design relying on the stability margins to accommodate. In that case, the adequate stability margin and load/stress factors have not been given adequate attention in the controller design due to changes in the aircraft dynamics, possible excitation of un-modeled parasitic dynamics and parameter change by the upset. Furthermore, pilot LOC training requires the disengagement of autopilot when aircraft experiences upset, which clearly indicates the deficiencies and immaturity of the current automatic flight control systems in LOC recovery.

The analysis given above shows that significant improvements to the current LOC prevention and recovery performance by the existing techniques could be achieved with more advanced control techniques. A more intelligent and reliable automatic LOC prevention and recovery system needs to be developed to supplement the pilot operation or even substitute for the pilot to control the aircraft in an upset condition.

1.4. Objective and Approach

In order to address the problems identified above, in this dissertation we will tackle the root causes thereof, i.e. (i) exceedance of healthy aerodynamic envelope of the aircraft during LOC, and (ii) the design tradeoff between the tracking performance and stability
margins of the automatic flight controller, using a systematic approach to design an integrated LOC Prevention and Recovery (iLOCPR) flight control system. The proposed iLOCPR control system comprises a Nominal Mode, a Prevention Mode, a LOC Arrest Mode and a Mission Restoration Mode to prevent the airplane from entering upset; arrest LOC should protect the fail and finally restore normal flight mission post recovery. A supervisory discrete-event-driven Automatic Flight Management System (AFMS) is designed to autonomously coordinate the control modes. To this end, the following research objectives are identified.

1.4.1 Objective 1. LOC Investigation Platform Implementation

For this objective, a six Degree-of-Freedom (DOF), fixed-wing aircraft, trajectory tracking control system needs to be implemented in Matlab/Simulink for LOC investigation, test and simulation. With the consideration of the vehicle performance under upset conditions, a relatively high fidelity LOC investigation platform will be built with necessary flight control models such as aircraft dynamics, wind effects, stall characteristics, control effector saturations, aircraft upset simulation capabilities, etc.

1.4.2 Objective 2. Nominal Control Mode Algorithm Design

For this objective, the nominal mode is designed by the previous designed Trajectory Linearization Control (TLC) 6DOF flight trajectory tracking controller designed by Trajectory Linearization Control (TLC) [31]. Such baseline controller should be improved by a bank-to-turn guidance control law redesign in order to overcome the undesired sideslip force. Also, the wind effect on navigation needs to be considered for the wind-induced LOC study. Such redesigned controller can serve as the nominal controller
mode tuned by the desired tracking performance with reasonable robustness in the multi-modal controller for LOC prevention and recovery. Also, the performance of the nominal controller under various wind conditions can be treated as a baseline performance in order to evaluate the LOC prevention under the same wind conditions.

1.4.3 Objective 3. Multiple-Time-Scale Nested Loop System

For this objective, because the 6DOF aircraft Equation-of-Motion (EOM)s naturally exhibit modal time-scale separations, where the rotational dynamics is sufficiently faster than the translational ones, the previous baseline controller is designed based on the singular perturbation theory, by which the faster dynamics can be treated as a singular perturbation to the slower dynamics when designing the Multiple-Time-Scale Nested Loop (MNL) system. In order to guarantee the system stability, the inner-loop response is simply set to be four times faster than the outer-loop one according to the “rule-of-thumb” in industry applications. However, the wind effect increases the singular perturbation. Since the singular perturbation parameter $\varepsilon$ is by definition a measure of the time-scale separation, the results in the previous singular perturbation margin research enabled us to relate this objective - the time-scale separation problem with the phase margin for LTI systems, and with the SPM in general, whereby providing some quantitative guidelines in the design and synthesis of controllers for MNL systems. Thus, it is of great practical interest to establish a quantitative guideline for determining the modal time-scale separation in MNL controller design. Moreover, the dependency of the closed-loop robustness on the singular perturbation parameter $\varepsilon$ in the loop is important in MNL controllers tuning process in order to retain the system robustness. It is also critically
important to reveal how a change in the time-scale separation of an inner-loop affects the behavior of the outer-loops, as such changes can be effected by operating conditions while the system is in operation. And we will use this principle to guide our bandwidth adaptation design.

1.4.4 Objective 4. LOC Prevention Mode Algorithm Design

For this objective, an automatic aircraft loss-of-control prevention system by bandwidth adaptation is presented. The adaptation law employs the time-varying parallel differential eigenvalues for a real-time tradeoff between tracking performance and severe wind tolerance capability, which is defined as the maximum tolerable (locally uniform irrotational) wind amplitude (TWA) that the vehicle can sustain without experiencing LOC. It can be implemented as an augmentation to a six DOF trajectory tracking controller designed with constant gains by the singular perturbation (time-scale separation) principle in a MNL architecture. Theoretical analysis is presented to justify the design rationale. It relates the ratio of time-scale separation between the inner and outer loop (singular perturbation) to the phase margin of the outer-loop (perturbed) system, which is important in its own right. Simulation studies on tailwind, headwind, crosswind, downdraft and updraft will be presented to demonstrate the effectiveness of the proposed LOC prevention strategy.

1.4.5 Objective 5. LOC Arrest Mode Algorithm Design

For this objective, a reconfigurable flight control system for the autonomous arrest of aircraft loss-of-control (LOC) is presented. By employing a previously baseline 6 degree-of-freedom (DOF) trajectory tracking flight control system design for the nominal
mode, this objective focuses on the LOC arrest mode design, which switches the control from the flight trajectory tracking (nominal mode) to aerodynamic attitude tracking (LOC arrest mode) in order to maintain airborne and regain control authority of the aircraft at the cost of temporarily abandoning the mission trajectory. To successfully realize the proposed aerodynamic attitude tracking control design, Trajectory Linearization Control (TLC) can be employed for fixed-wing aircraft, which does not appear to have been attempted before. Such design is similar to 3 DOF Reusable Launch Vehicle (RLV) entry flight control system for tracking challenging aerodynamic attitude maneuvers during the entry operations. A supervisory control logic also needs to be designed according to the current LOC situations in order to automatically configure the controller modes. Simulation results will demonstrate the proposed LOC arrest controller’s capability to recover the normal flight, and its performance in terms of arrest time, altitude departure and aerodynamic loading.

1.4.6 Objective 6. Mission Restoration Mode Algorithm Design

For this objective, a mission restoration control system in an integrated automatic flight controller for aircraft Loss-of-Control Prevention and Recovery is presented, in which the two-phase Loss-of-Control recovery includes a LOC arrest mode and a mission restoration mode. For a trajectory tracking mission, a complete mission restoration mode is achieved by i) a close-in sub-mode of guiding the vehicle to catch up with the target position, and ii) a home-in sub-mode of restoring the mission. A pure pursuit guidance approach is applied in the proposed close-in sub-mode by aligning the velocity vector to the Line-of-Sight vector using proportional-integral-derivative linear regulation. The
home-in sub-mode is designed by bandwidth adaptation to gradually regain the tracking performance, and eventually, restore the mission. Simulation results demonstrate the effectiveness and the performance of the proposed mission restoration design.

A mission restoration controller mode is mainly to guide the vehicle back to the mission trajectory after the LOC arrest so that the tracking error is reduced to within the boundaries that the nominal trajectory tracking system is capable of accommodating. Since the current position on the mission trajectory can be viewed as a virtual target moving along a predefined trajectory, the mission restoration design can be formulated as a guidance problem of planning a restoration trajectory to chase and capture (rendezvous with) a moving target. In this case, the line-of-sight (LOS) pure-pursuit missile guidance approach can be borrowed for the guidance trajectory design [32]. Pure pursuit guidance (PPG) is to steer and maintain the vehicle velocity vector align with the LOS (range vector). Since the PPG is a closed-loop guidance, a closed-loop control law needs to be designed to stabilize the error between the vehicle velocity vector and the LOS. This method effectively turns the aforementioned large range error problem in the Cartesian frame into the regulation of small angular errors in the polar frame, and a simple maximum speed close-in control. Simulation results will show the effectiveness of this objective.

1.4.7 Objective 7. Automatic Flight Management System Algorithm Design

As mentioned above, the iLOCPR system comprises a set of operating modes including nominal mode, prevention mode, LOC arrest mode and restoration mode. As Objective 7, the algorithm for a top level supervisory flight management system will be designed which is capable of decision making and coordination of the different control
modes according to the situations of the aircraft. This objective entails developing a set of logical rules for switching the multi-mode dynamical controllers and to develop the initialization algorithms for the dynamical controllers to ensure smooth transient responses at the switching time.

1.5. Preview of the Main Results

For each specific objectives proposed in 1.4, the technical approaches and the main results are previewed below.

1.5.1 LOC Investigation Platform Implementation

In order to test the proposed LOC intervention strategies, a 6 Degree-of-Freedom (DOF) fixed-wing flight control platform is implemented in Matlab/Simulink based on the previous controller design. The wind effects on navigation and aerodynamics, stall characteristics and control effectors saturation are performed to ready the platform for the wind-induced LOC study.

1.5.2 Nominal Control Mode Algorithm Design

The nominal control mode algorithm in the iLOCPR system is designed using the MNL control architecture with time-scale separation by the trajectory linearization control (TLC) method. A baseline TLC controller for 6DOF fixed-wing flight trajectory tracking inherited from previous work [31] is redesigned by an improved bank-to-turn guidance control. Wind effect on navigation and aerodynamics are considered in the nominal mode design. Nominal controller is then tested by simulation to determine its maximum wind tolerance capability under a present condition, and the results are used as the baseline for assessing the effectiveness of the proposed iLOCPR system.
1.5.3 Multiple-Time-Scale Nested Loop System

The first new result presented in this chapter reveals the relationship of the time-scale separation with the phase margin deterioration of the singularly perturbed LTI system, which provides a quantitative guideline for selecting the time-scale separation between the fast and slow modes. The second result reveals that the dependence of the bandwidth of a singularly perturbed system on the singular perturbation need not be monotonic, which dictates that the bandwidth adaptation should be applied to all control loops simultaneously to ensure consistent transient response and robustness. The third result is an algorithm for approximating a high-order or unspecified (actuator) transfer function by a second-order one based on phase frequency response to facilitate singular perturbation analysis in practice.

The new results presented in this chapter not only provide practicing control engineers a quantitative guideline in selecting the time-scale separation in design and synthesis based on the dominant dynamics principle but also reveal the relationship of the time-scale separation with the PM deterioration in the presence of singular perturbation, which is of both practical and theoretical interest. In addition, the relationship between the closed-loop robustness (in terms of phase margin) and the time-scale separation obtained in this objective not only has practical significance in the design and synthesis of MNL controllers, more importantly, it can be used as the MNL system tuning criterion such as frequency response based adaptation control design.
1.5.4 LOC Prevention Control Mode Algorithm Design

An automatic aircraft loss-of-control prevention system by bandwidth adaptation is presented in this chapter. The adaptation law employs the time-varying parallel differential eigenvalues for a tradeoff between tracking performance and severe wind tolerance capability in real-time. It is implemented as an augmentation to a six DOF trajectory tracking controller designed with constant gains by the singular perturbation (time-scale separation) principle in a MNL architecture. Simulation studies on tailwind, headwind, crosswind, downdraft, and updraft are performed to demonstrate the effectiveness of the proposed LOC prevention strategy in inertial trajectory tracking, which is very challenging as the wind not only exerts aerodynamic forces and moments on the vehicle but also changes the inertial velocity and position of the vehicle. The baseline controller and the proposed LOC prevention by bandwidth adaptation are both tested for their wind tolerance capability. Simulation results show 160%–560% improvements in the TWA in all scenarios but the headwind, where the maximum headwind tolerance of the baseline controller is already at 78 m/sec, which is exceptionally high. These results demonstrate the effectiveness of the proposed adaptive control scheme. The time-varying Parallel Differential (PD)-eigenvalue principle is applied for adaptation law design in order to increase the wind tolerance capability.

Also, the bandwidth adaptation scheme using time-varying PD-eigenvalues has been applied to the challenging 3DOF attitude control of reusable launch vehicles in ascent and reentry flight to cope with large flight envelopes, engine, and control effector failure scenarios, accommodating the large model uncertainties in transonic flight, and severe
dispersion tests. It has also been proposed as a direct fault tolerant control technique, as well as a technique for coping with flexible structure modes. In this chapter, it is applied to the 6DOF fixed-wing aircraft for preventing LOC by postponing the onset of LOC, which is not a trivial extension of the previous works. Moreover, while the stability analysis of singularly perturbed system has been studied extensively, including multi-timescale singularly perturbed systems existing results are mostly for a single loop, and are qualitative in nature. It appears that (i) the proposed MNL singular perturbation problem formulation, which effectively reduces the multi-time-scale problem to the two-time-scale problem; and (ii) the quantitative result on the relationship between the time-scale ratio of the inner loop to the outer loop and the phase margin of the outer loop; are both original.

It is noted that the bandwidth adaptation scheme proposed herein can be further augmented the present design to cope with other adverse flight conditions, ranging from flight regimes with large aerodynamic uncertainties to expanding fault tolerance capability. It is also applicable as an augmentation to other types of baseline controller designed using the MNL architecture.

1.5.5 LOC Arrest Control Mode Algorithm Design

The LOC arrest control algorithm in the iLOCPR system is designed by aerodynamic attitude tracking using the LTC method in order to regain and maintain the healthy aerodynamic envelope, thereby regaining airborne and control authority of the aircraft for a full recovery from the LOOC. The mission trajectory tracking task under the nominal control mode is temporarily abandoned in favor of survival of the upset. The
supervisory AFMS on the top level will switch the control from the flight trajectory tracking (nominal mode) to aerodynamic attitude tracking (LOC arrest mode).

In order to demonstrate the capability of the proposed LOC arrest controller to recover the upset, simulation case studies on a fixed-wing aircraft model with post-stall aerodynamic characteristics are presented for two LOC scenarios: (i) a longitudinal stall upset and (ii) a lateral-directional spin. For each of the scenarios, three simulation tests are presented. In the first simulation, the nominal mode as a baseline controller is tested under a LOC event, which causes the instability of the system. Under the same flight situation, the second simulation shows that the proposed arrest controller is capable of recovering an imminent spin by immediately switching to the arrest control configuration. This operation serves as the intended normal LOC recovery control strategy, such that the arrest mode is automatically configured when the onset of LOC is detected. The 34m (100-foot) altitude loss specification can be met in this operation. The third simulation shows that the proposed LOC arrest controller can even rescue a severe LOC ensuing from a few seconds elapsed time after the onset of LOC when switch to the arrest mode controller. This delayed response not only serves to demonstrate the effectiveness of the proposed LOC arrest controller but also suggests that it can be used as a pilot assistant for manned flights. The performance in terms of the LOC recovery criterion such as arrest time, altitude departure and aerodynamic loading during the entire LOC arrest process is analyzed for all the simulations. The simulation results show that the proposed design serves as an effective way to protect aircraft against flight mishaps by correctly and promptly responding to LOC.
The proposed LOC arrest controller can be used either as a component in an autonomous flight control system or as a standalone emergency controller for piloted aircraft, which can be actuated by pressing a “panic button” or automatically overriding pilot control when an upset has exceeded a preset time limit.

1.5.6 Mission Restoration Control Mode Algorithm Design

The mission restoration mode algorithm in the iLOCPR system is designed by i) a close-in operation to guide the vehicle to close-in onto the target by pure pursuit guidance (PPG) strategy, and ii) a home in the operation of restoring the mission by reduced bandwidth adaptation to tradeoff tracking performance for increased robustness. Simulation results demonstrate that the proposed design method is capable of directing the vehicle back to the mission trajectory in the presence of a relatively large position tracking error, and restoring the original trajectory tracking mission with desired tracking precision.

1.5.7 Automatic Flight Management System Algorithm Design

The supervisory discrete-event-driven AFMS is designed to autonomously coordinate the control modes including the nominal mode, LOC prevention mode, LOC arrest mode and the mission restoration mode for the iLOCPR. The AFMS is designed as a Moore finite-state machine with the switching logic defined based on thresholding the continuous time dynamical state variables. Initialization of all 60 controller states at the switching times are also designed. Matlab/Stateflow is used to implement the AFMS. A comprehensive simulation entailing the nominal flight, LOC prevention, LOC arrest and mission restoration is presented to demonstrate the effectiveness of the proposed iLOCPR system.
1.6. Organization of Dissertation

This dissertation is organized as the follows. Following this introduction in Chapter 1, technical background about flight control fundamentals, flight control by TLC from the previous work and the theoretical basis are summarized in Chapter 2. In Chapter 3, the nominal control mode for the trajectory tracking system is established by improving the bank-to-turn control of the precious baseline controller design. Wind effect and stall characteristics are included in the LOC investigation platform. An simulation test study of various wind conditions under the nominal control mode is conducted to establish the baseline performance, which will be used to assess the effectiveness at the proposed LOC prevention and recovery control techniques. In Chapter 4, MNL system design principle is analyzed as a theoretical investigation for the LOC prevention design. Chapter 5 contains the LOC prevention control mode design using bandwidth adaptation, which is designed in order to improve the wind tolerance capability comparing to the nominal mode. Chapter 6 illustrates the LOC arrest mode design by protecting the aerodynamic attitude to save the vehicle from upset. Mission restoration mode is designed in Chapter 7 for restoring the original trajectory tracking mission. In Chapter 8, the iLOCPR is presented as a supervisory discrete-event-driven Automatic Flight Management System (AFMS), which is designed to autonomously coordinate the all the designed control modes. Chapter 9 concludes the dissertation with a summary of the main contributions of this dissertation. Future work is also proposed for the further research in this field in this chapter.
CHAPTER 2 TECHNICAL BACKGROUND

This chapter provides the technical background related to this dissertation. In Section 2.1, the flight control fundamentals are given including coordinate system, aircraft Equation-of-Motion, and the approximated aerodynamic forces and moments. Previous work on flight control by Trajectory Linearization Control (TLC) serving as the fundamental knowledge for the development of this dissertation is summarized in Section 2.2. Singular perturbation theory is introduced in Section 2.3 as the basic theoretical fundamentals of this research.

2.1. Fight Control

“The study of aircraft boils down to the determination of the position and velocity of an aircraft at some arbitrary time” [33]. Appropriate coordinate systems need to be defined in order to describe the flight states including the vehicle position, the vehicle velocity, and the forces and moments in the flight control research.

2.1.1 Coordinate System

In this document, we follow the conventional aircraft coordinate frames which are all right-handed and orthogonal. Figure 2.1 describes the four aircraft coordinate frames including the earth frame which is treated as the inertial frame $F_E$, the body frame $F_B$, the wind frame $F_W$, and the local atmospheric-fixed reference frame $F_A$. It is noticed that the atmospheric motion can be described by the frame $F_A$. Also, the center-of-gravity (CG) and center-of-mass (CM) are assumed to be collocated, which means that all the forces are applied at CG.
a. The earth frame, $F_E$.

The flat earth is assumed and $F_E$ is treated as the inertial frame. This coordinate system has its origin fixed to the defined home location with $x_E$ pointing to the North, $y_E$ pointing to East, and $z_E$ pointing down. The inertial frame is referred as the NED frame in the aerospace field.

b. The body frame, $F_B$.

The origin of the body frame locates at the center-of-gravity of the aircraft; $x_B$ points out the nose of the airframe, $y_B$ points out the right wing and $z_B$ points out of the belly.

c. The wind-axis system, $F_W$.

This is a body-carried frame in which $x_W$ axis is in the direction of airspeed vector; $z_W$ lies in the plane of symmetry with $y_W$ completing the right-handed orthogonal frame.
d. The local atmospheric-fixed reference frame, \( F_A \).

This coordinate system is describing the motion of the atmosphere with translational and rotational properties. This coordinate system has its origin fixed in the air mass surrounding the aircraft and aligns with the Earth Frame when there is no rotational movement.

2.1.2 Aircraft Equation-of-Motion

In order to execute the fly mission, aircraft must behave in the way according to the flight control design objectives. The control design problem needs to be formulated, designed and developed based on the aircraft dynamics, which is usually represented by the Equation-of-Motion (EOM).

Aircraft rigid-body dynamics models in terms of body components are given as follows [33], where the 12 ordinary differential equations represent the EOMs of the aircraft vehicle for the inertial position, velocity in the body frame, Euler attitude relative to the inertial frame, and the angular velocity in the body frame.
• Translational Kinematics:

\[
\begin{bmatrix}
\dot{x}_E \\
\dot{y}_E \\
\dot{z}_E
\end{bmatrix} = T_{E\rightarrow B} \begin{bmatrix} u \\ v \\ w \end{bmatrix}
\]

\[
\dot{P} = B_1(\Gamma)V
\]  

(2.1)

• Translational Dynamics:

\[
\begin{bmatrix} 
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}
\]

\[
\dot{V} = B_2(\Omega)V + \frac{1}{m} F
\]  

(2.2)

• Rotational Kinematics (Body Components):

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
1 & S_\phi T_\theta & C_\phi T_\theta \\
0 & C_\phi & -S_\phi \\
0 & S_\phi & C_\phi / C_\theta
\end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}
\]

\[
\dot{\Gamma} = B_3(\Gamma)\Omega
\]  

(2.3)

• Rotational Dynamics:

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
g^p_l & 0 & g^p_n \\
0 & g^q_m & 0 \\
g^r_l & 0 & g^r_n
\end{bmatrix} \begin{bmatrix} L_m \\ M_m \\ N_m \end{bmatrix} + \begin{bmatrix} I_{pq}^{p}pq + I_{pr}^{p}qr \\ I_{pq}^{q}q^2 + I_{qr}^{q}pr + I_{pr}^{q}qr \\ I_{pq}^{r}r^2 + I_{qr}^{r}qr + I_{pr}^{r}pr
\end{bmatrix}
\]

\[
\Omega = f_i(\Omega) + B_4T_m
\]  

(2.4)

where the inertial coefficients in the body frame are defined by the mass moments of inertia of the airframe in Appendix A.

Other than the rotational kinematics in the body frame in Eq. (2.3), the rigid-body rotational kinematics model in the wind frame is given as follows as
Rotational Kinematics (wind components):

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{\beta} \\
\mu
\end{bmatrix} = 
\begin{bmatrix}
-T_\beta C_\alpha & 1 & -T_\beta S_\alpha \\
S_\alpha & 0 & -C_\alpha \\
C_\alpha/C_\beta & 0 & S_\alpha/C_\beta
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} + 
\begin{bmatrix}
\Sigma_1 \\
\Sigma_2 \\
\Sigma_3
\end{bmatrix}
\]

where

\[
\Sigma_1 = -\frac{1}{mV_s C_\beta}(L + T S_\alpha) + \frac{g C_\gamma C_\mu}{V_t C_\beta}
\]

\[
\Sigma_2 = \frac{1}{mV_s}(D S_\beta + Y C_\beta - T C_\alpha S_\beta) + \frac{g C_\gamma S_\mu}{V_t}
\]

\[
\Sigma_3 = \frac{1}{mV_s}[D T_\gamma S_\beta C_\mu + Y T_\gamma C_\beta C_\mu + L(T_\beta + T_\gamma S_\mu)] - \frac{g C_\gamma C_\mu T_\beta}{V_t}
\]

where the aerodynamic attitude vector \( \mathbf{\Lambda} = [\alpha \beta \mu]^T \) denotes the body attitude relative to the velocity vector in the wind frame.

2.1.3 Aerodynamic Force and Moment

The aerodynamic forces components can be linearly approximated using nondimensional aerodynamic coefficients obtained by wind-tunnel tests. The calculated aerodynamic forces and moments are defined in terms of dynamic pressure \( \bar{Q} \), reference area \( S \), wing mean chord \( \bar{c} \), wingspan \( b \) and the dimensionless aerodynamic coefficients as below [34]:

\[\text{[Equation]}\]
\[ D = QS \left( C_D + C_{Dv} \alpha + C_{Dp} \beta \right) \]
\[ Y = QS \left( C_{yf} \beta + C_{y\delta_r} \delta_r \right) \]
\[ L = QS \left( C_{Lc} + C_{L\alpha} \alpha + C_{L\delta} \delta \right) \]
\[ L_m = QSb \left( C_{L_{m\alpha}} + C_{L_{m\delta}} \alpha + C_{L_{m\delta_r}} \delta_r \right) \]
\[ M_m = QSc \left( C_{m_{\alpha}} + C_{m_{\delta}} \alpha + C_{m_{\delta_r}} \delta_r \right) \]
\[ N_m = QSc \left( C_{n_{\beta}} + C_{n_{\delta}} \delta_a + C_{n_{\delta_r}} \delta_r \right) \]  

(2.6)

where \( D, Y, L \) are the three components of the aerodynamic force and \( L_m, N_m, M_m \) are the three components of the aerodynamic moment; \( C_D, C_Y, C_L, C_t, C_m, C_n \) primarily are functions of the aircraft system parameters and flight states, such as Mach number, aerodynamic angles, etc. The constant coefficients from wind-tunnel tests are referred to as static coefficients, which specifies the nominal flight conditions. On the other complicated flight conditions such as aircraft upset, the dynamic coefficients are more accurate to describe the force and moment generation under off-nominal test conditions. Look-up tables, polynomial fitting, and curve fitting are the typical techniques for establishing the dynamic coefficients models.

2.2. Flight Control by Trajectory Linearization Control

2.2.1 Trajectory Linearization Control

Among a broad range of advanced control method, the Trajectory Linearization Control (TLC) method can be viewed as an ideal gain-scheduling controller designed at every point on the trajectory, which provides good tracking performance with robust stability. TLC has been used for multiple applications for example: missile interception guidance control design [35, 36], tri-rotor helicopter trajectory tracking [37, 38], quadrotor
helicopter [39], fuzzy system based robust TLC approach and neural network based aerospace vehicle control [40] and fixed-wing flight control system [31] on multiple flight platforms such as NASA X-33 Launch Vehicle, Quanser UFO, Cessna 182 and OU UFO as shown in Figure 2.2.

Figure 2.2 TLC application in flight control: X-33, Quanser UFO, fixed-wing Cessna 182, OU UFO.

The typical TLC system design comprises two subsystems: an open-loop nominal controller by pseudo-inversion of the nonlinear plant; and a closed-loop stabilizing
controller of the linearized tracking error dynamics along the nominal trajectory as shown in Figure 2.3.

![Figure 2.3 Typical 2DOF TLC structure.](image)

Consider the nonlinear dynamic system as

\[
\begin{align*}
\xi(t) &= f(\xi(t), \mu(t)) \\
\eta(t) &= h(\xi(t), \mu(t))
\end{align*}
\]  

(2.7)

where \(\xi(t) \in \mathbb{R}^n\), \(\mu(t) \in \mathbb{R}^l\), and \(\eta(t) \in \mathbb{R}^m\) are the state, input, and output, respectively, and the nonlinear function \(f, h\) are bounded and locally Lipschitz. Let the nominal state, nominal input and nominal output to be defined as \(\overline{\xi}(t), \overline{\mu}(t)\) and \(\overline{\eta}(t)\), the nominal control satisfies the following equation

\[
\begin{align*}
\dot{\xi}(t) &= f(\overline{\xi}(t), \overline{\mu}(t)) \\
\overline{\eta}(t) &= h(\overline{\xi}(t), \overline{\mu}(t))
\end{align*}
\]  

(2.8)
By defining the state, input and output tracking error as 
\[ \xi(t) = \xi(t) - \xi(t), \]
\[ \mu(t) = \mu(t) - \mu(t) \]
and \[ \eta(t) = \eta(t) - \eta(t) \]
respectively, the nonlinear tracking error dynamics are written as
\[
\begin{align*}
\dot{\xi}(t) &= f(\xi(t) + \xi(t), \mu(t) + \mu(t), \theta(t)) - f(\bar{\xi}(t), \bar{\mu}(t), \theta(t)) \\
&= F(\xi(t), \mu(t), \theta(t), \bar{\xi}(t), \bar{\mu}(t)) \\
\dot{\eta}(t) &= h(\bar{\xi}(t) + \xi(t), \bar{\mu}(t) + \mu(t)) - h(\bar{\xi}(t), \bar{\mu}(t)) \\
&= H(\xi(t), \mu(t), \bar{\xi}(t), \bar{\mu}(t))
\end{align*}
\]
(2.9)

If: i) the nonlinear function \( f \) and \( h \) are both continuously differentiable with respect to \( \xi \) and \( \mu \); ii) the tracking error are assumed to be small, the nonlinear time-varying tracking error dynamics in Eq. (2.9) can be linearized along the nominal trajectory as
\[
\begin{align*}
\dot{x}(t) &= A(t)x(t) + B(t)u(t) \\
y(t) &= C(t)x(t) + D(t)u(t)
\end{align*}
\]
(2.10)
in which
\[
A(t) = \left. \frac{\partial f}{\partial \xi} \right|_{\xi(t), \mu(t)} \quad B(t) = \left. \frac{\partial f}{\partial \mu} \right|_{\xi(t), \mu(t)}
\]
\[
C(t) = \left. \frac{\partial h}{\partial \xi} \right|_{\xi(t), \mu(t)} \quad D(t) = \left. \frac{\partial h}{\partial \mu} \right|_{\xi(t), \mu(t)}
\]

where \( x(t) = [x_1(t) \cdots x_n(t)]^T \) is the tracking error state variable vector;
\( A(\xi, \mu) \in \mathbb{R}^{nxn} \), \( B(\xi, \mu) \in \mathbb{R}^{nxm} \), \( C(\xi, \mu) \in \mathbb{R}^{1xn} \), \( D(\xi, \mu) \in \mathbb{R}^{1xm} \) are uniformly bounded and Lipschitz in the operation region \( \Omega \), the nonlinear dynamics is exponential stable if and only if the linearized error dynamics is exponentially stable.
Based on Eq. (2.10), the tracking error stabilizer can be synthesized by Parallel Deferential (PD)-eigenvalues in order to guarantee the exponential stability [41]. As an illustration, the basic procedure of obtaining the TLC gain matrices for a 2nd-order system \((n = 2)\) is shown below.

### 2.2.1.1 Coordinate Transformation

In order to synthesize the control law, Eq. (2.10) is first to be transformed to the controller canonical form (CCF) by a Silverman-Wolovich transformation as \(x(t) = L(t)z(t)\). The details about \(L(t)\) can be found in the references [42, 43]. The linear-time-varying (LTV) tracking error dynamics is given as

\[
\dot{z}(t) = A_{CCF}(t)z(t) + B_{CCF}(t)u(t) \\
y(t) = C_{CCF}(t)z(t) + D_{CCF}(t)u(t)
\]

(2.11)

with

\[
A_{CCF}(t) = L^{-1}(t)[A(t)L(t) - \dot{L}(t)], \quad B_{CCF}(t) = L^{-1}(t)B(t) \\
C_{CCF}(t) = C(t)L(t), \quad D_{CCF}(t) = D(t)
\]

(2.12)

For a second-order system, Eq. (2.11) is written as below

\[
\dot{z}(t) = A_{CCF}(t)z(t) + B_{CCF}(t)u(t) \\
= \begin{bmatrix} 0 & 1 \\ -a_1(t) & -a_2(t) \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)
\]

(2.13)

where \(u(t)\) is scalar.

### 2.2.1.2 Parallel Differential (PD)-eigenvalue Synthesis

The single-input-single-output second-order differential equation representing the desired closed-loop tracking error dynamics as
where $e(t)$ is scalar; and the associated vector LTV system in companion canonical form is shown as

$$
\dot{z}(t) = A_{cl}(t)z(t),
$$

$$
z(t_0) = z_0, \quad t \geq t_0
$$

where $z(t) = [z_1(t) \ z_2(t)]^T = [e(t) \ \dot{e}(t)]^T$ and $A_{cl}(t)$ is continuous and bounded.

If the desired time-varying poles for the closed-loop are given as

$$
\rho_{1,2}(t) = \left(-\zeta_n \pm \sqrt{1 - \zeta_n^2}\right)\omega_n(t)
$$

where $\omega_n(t)$ is the time-varying natural frequency and $\zeta_n$ is the damping ratio, which is scalar; and the PD-spectral assignment are designed as

$$
\alpha_1(t) = \omega_n^2(t); \quad \alpha_2(t) = 2\zeta_n \omega_n(t) - \frac{\dot{\omega}_n(t)}{\omega_n(t)}
$$

by satisfying the associated PD-Characteristic Equation as

$$
\dot{\rho} + \rho^2 + \alpha_2(t)\rho + \alpha_1(t) = 0
$$

where the desired PD-eigenvalues are $\rho(t) = [\rho_1(t) \ \rho_2(t)]^T$.

### 2.2.1.3 Feedback Control Law

The closed-loop error dynamics can be obtained as

$$
\dot{z}(t) = \begin{bmatrix} 0 & 1 \\ -a_1(t) & -a_2(t) \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1(t) & k_2(t) \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & 1 \\ k_1(t) - a_1(t) & k_2(t) - a_2(t) \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}
$$
by the feedback control law

\[ u(t) = K(t)z(t) \]  

(2.20)

where

\[ k_1(t) = a_1(t) - \alpha_1(t), \quad k_2(t) = a_2(t) - \alpha_2(t) \]  

(2.21)

The control law is obtained as

\[ u(t) = K(t)z(t) = [k_1(t) \quad k_2(t)] \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = K(t)L^{-1}(t)x(t) \]  

(2.22)

The stabilization of the above feedback control law can be guaranteed according to the following theory [41].

2.2.2 6DOF Baseline Controller Design

To facilitate the exposition of the new results, the baseline control system designed in [31] will be summarized in this section. Taking advantage of the natural time-scale separation in the 6DOF rigid-body dynamics as shown in Eq. (2.1) - Eq.(2.4), the baseline controller for trajectory tracking is designed as a four-loop TLC structure, where each loop comprises an open-loop nominal controller designed by dynamic pseudo-inversion and a closed-loop proportional-integral (PI) feedback tracking error regulator designed by linearizing the nonlinear, time-varying tracking error dynamics along the nominal trajectory. Exponential stability of the tracking error dynamics is guaranteed by assigning the closed-loop (constant) eigenvalues to the left-half open complex plane by a time-varying coordinate transformation to the controller canonical form. Figure 2.4 shows the controller design diagram.
2.2.2.1 Guidance Outer Loop - Loop 1

In order to track command trajectory, the nominal velocity is calculated by inverting the Translational Kinematics model in Eq. (2.1) to obtain \( \mathbf{V}_{\text{nom}} = B_1^{-1}(\mathbf{\Gamma}_{\text{nom}})\dot{\mathbf{P}}_{\text{nom}} \).

The feedback control is implemented by design the \( \mathbf{V}_{\text{ctrl}} \) along the nominal trajectory to stabilize the linearized closed-loop error dynamics \( \dot{\mathbf{P}}_{\text{err}} = B_1(\mathbf{\Gamma}_{\text{nom}})\mathbf{V}_{\text{ctrl}} \), where \( \mathbf{P}_{\text{err}} = \mathbf{P}_{\text{sen}} - \mathbf{P}_{\text{com}} \). A Proportional-Integral (PI) control law

\[
\mathbf{V}_{\text{ctrl}} = -K_p \mathbf{P}_{\text{err}} - K_i \int_0^t \mathbf{P}_{\text{err}}(\tau) \, d\tau
\]  

(2.23)

is synthesized using constant PD-eigenvalue by the following procedure. Consider the second-order linear time varying (LTV) differential equation representing the desired closed-loop tracking error dynamics of the \( i^{th} \) loop, \( j^{th} \) channel \( (i = 1,2,3,4, \ j = 1,2,3) \) given by

\[
\ddot{e}_g(t) + \alpha_{y2}(t) \dot{e}_g(t) + \alpha_{y1}(t) e_g(t) = 0
\]  

(2.24)
The PD-eigenvalues are chosen by specifying the desired closed-loop dynamics as

$$\rho_{ij,1,2}(t) = -\zeta_j \omega_{n,ij}(t) \pm j \sqrt{1 - \zeta_j^2} \omega_{n,ij}(t)$$  \hspace{1cm} (2.25)

And the corresponding coefficients are synthesized by

$$\alpha_{ij1}(t) = \omega_{n,ij}^2(t), \quad \alpha_{ij2}(t) = 2\zeta_j \omega_{n,ij}(t) - \frac{\dot{\omega}_{n,ij}(t)}{\omega_{n,ij}(t)}$$  \hspace{1cm} (2.26)

where the term $\dot{\omega}_{n,ij}(t) / \omega_{n,ij}(t) = 0$ by choosing constant, $\omega_{n,ij}(t) = \omega_{n,ij}$. The ordinary synthesis formula for second-order Linear-Time-Invariant (LTI) systems is obtained.

Gain matrices $K_{p1}, K_{i1}$ together with the subsequent TLC gain matrices $K_{p1}, K_{i1}, i = 2, 3, 4$ can be found in Appendix B. And the $V_{com} = V_{nom} + V_{ctrl}$ will be the command for the Loop 2.

### 2.2.2.2. Guidance Inner Loop - Loop2

With the same TLC structure, the control law of Guidance Inner Loop is established on Translational Dynamics model in Eq. (2.2). The nominal force vector is calculated by the inversion as $F_{nom} = m \left[ V_{nom} - B_2 \left( \Omega_{nom} \right) V_{nom} \right]$, and the feedback control law by PD-eigenvalue theory is designed as

$$F_{ctrl} = -K_{p2} V_{err} - K_{i2} \int_{\tau_0}^{\tau} V_{err}(\tau) d\tau$$  \hspace{1cm} (2.27)

by linearizing the error dynamics as $\dot{V}_{err} = B_2 \left( \Omega_{nom,0} \right) V_{err} + F_{ctrl} / m$, where $V_{err} = V_{sen} - V_{com}$. 
2.2.2.3 Guidance Nominal/Command Allocation

The TLC guidance control yields a nominal force $F_{\text{nom}}$ in the open-loop path, and a closed-loop feedback control can be calculated by adding the closed-loop control force $F_{\text{ctrl}}$ as $F_{\text{com}} = F_{\text{nom}} + F_{\text{ctrl}}$. Due to the approximated aerodynamic forces given in Eq.(2.6), the total forces for both $F_{\text{nom}}$ and $F_{\text{com}}$ can be allocated to gravitational components, thrust and aerodynamic forces within the body frame as

$$F = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -QSC_{X_u} & -QSC_{X_\beta} & T_{\text{max}} \\ 0 & QSC_{Y_\beta} & 0 \\ -QSC_{Z_u} & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \delta_r \end{bmatrix} + \begin{bmatrix} -QSC_{X_\delta} - mg S_{\theta_{\text{nom},0}} \\ QSC_{Z_\beta} \delta_r + mg S_{\alpha_{\text{nom},0}} C_{\theta_{\text{nom},0}} \\ -QSC_{Z_\delta} - QSC_{Z_\alpha} \delta_\tau + mg C_{\alpha_{\text{nom},0}} C_{\theta_{\text{nom},0}} \end{bmatrix}$$

(2.28)

where the $\delta_{r,\text{com}}$ is one of the control inputs injected into the vehicle dynamics. Then the navigation for Euler angles command will be obtained by applying virtual control as

$$\alpha = \frac{F_z - mg C_{\delta} C_{\gamma} - QSC_{Z_u}}{QSC_{Z_u}}$$

$$\beta = 0$$

$$\delta_\tau = \frac{F_x + mg S_{\delta} - QSC(C_{X_\delta} + C_{X_\alpha} \alpha + C_{X_\beta} \beta)}{T_{\text{max}}}$$

(2.29)

using the following relations as
\[
\begin{bmatrix}
\phi \\
\theta \\
\psi
\end{bmatrix}
= \tan^{-1}\left(\frac{Y_\beta \left(a - b^2\right) + b \tan \alpha \sqrt{c \left(1 - b^2\right) + \gamma^2 S_\beta^2}}{a^2 - b^2 \left(1 + c \tan^2 \alpha\right)}\right)
\]

where the rate-of-climb and bank-to-turn (BTT) constraints are satisfied, and the parameters are given as

\[
a = 1 - Y_{com} \tan \alpha \sin \beta, \quad b = \frac{\sin \gamma}{\cos \beta}, \quad c = 1 + \gamma^2 \cos^2 \beta, \quad Y = \frac{\dot{V}_r}{g}
\]

\[
e = \cos \alpha \cos \beta, \quad f = \sin \phi \sin \beta + \cos \phi \sin \alpha \cos \beta
\]

\[
\mu = \tan^{-1}\left(\frac{uv \sin \theta + \left(u^2 + w^2\right) \sin \phi \cos \theta - vw \cos \phi \cos \theta}{V_{r, com} \left(w \sin \theta + u \cos \phi \cos \theta\right)}\right)
\]

Notice that the nominal \( \Gamma_{nom} \) and command \( \Gamma_{com} \) in this allocation are obtained by same relations given in Eq. (2.30) using the corresponding states.

### 2.2.2.4 Attitude (Euler) Outer Loop - Loop3

The nominal attitude control is achieved by inverting the Rotational Kinematics model in Eq. (2.3) and calculating the nominal angular velocity vector as 

\[
\Omega_{nom} = B_3^{-1} \left(\Gamma_{nom}\right) \dot{\Gamma}_{nom}
\]

Then, by defining the Euler angle tracking error \( \Gamma_{err} = \Gamma_{sen} - \Gamma_{com} \)

, the linearized tracking error dynamics is given as \( \dot{\Gamma}_{err} = B_3 \left(\Gamma_{nom}\right) \Omega_{ctrl} \) and the PI control law for this loop is written as
\[ \Omega_{\text{ctrl}} = -K_p \Gamma_{\text{err}} - K_I \int_0^t \Gamma_{\text{err}}(\tau) d\tau \] (2.31)

Similarly, \( \Gamma_{\text{com}} = \Gamma_{\text{nom}} + \Gamma_{\text{ctrl}} \) will be the command for the connected Loop 4.

### 2.2.2.5 Attitude (Euler) Inner Loop - Loop 4

The nominal moment vector is calculated by inverting Eq. (2.4) to obtain

\[ T_{m,\text{nom}} = B_4^{-1} \left[ \hat{\Omega}_{\text{nom}} - f_4 \left( \Omega_{\text{nom}} \right) \right], \]

and the PI control law for this loop is

\[ T_{m,\text{ctrl}} = -K_{p4} \Omega_{\text{err}} - K_{I4} \int_0^t \Omega_{\text{err}}(\sigma) d\sigma \] (2.32)

with tracking error \( \Omega_{\text{err}} = \Omega_{\text{sen}} - \Omega_{\text{com}} \).

\[ T_{m,\text{com}} = T_{m,\text{nom}} + T_{m,\text{ctrl}} \] (2.33)

### 2.2.2.6 Attitude (Euler) Allocation

By inverting the aerodynamic moment model given in Eq. (2.6), the moment equations are written in terms of the Jacobian, so that

\[
\begin{bmatrix}
L_m - QSbC_{1e} \beta_m \\
M_m - QS\bar{c}(C_{m_0} + C_{m_1} \alpha) \\
N_m - QSbC_{n_0} \beta
\end{bmatrix}
= 
\begin{bmatrix}
QSB_{C_{1e}} & 0 & QSbC_{1e} \\
0 & QS\bar{c}C_{m_0} & 0 \\
QSB_{C_{n_0}} & 0 & QSbC_{n_0}
\end{bmatrix}
\begin{bmatrix}
\delta_{a,\text{com}} \\
\delta_{e,\text{com}} \\
\delta_{r,\text{com}}
\end{bmatrix}
\]

\[ T_{\text{allo}} = J \Delta_{\text{com}} \] (2.34)

The inversion of Eq. (2.34) gives the control surface deflections command as

\[
\Delta_{\text{com}} = \begin{bmatrix}
\delta_{a,\text{com}} \\
\delta_{e,\text{com}} \\
\delta_{r,\text{com}}
\end{bmatrix} = \begin{bmatrix}
QSB_{C_{1e}} & 0 & QSbC_{1e} \\
0 & QS\bar{c}C_{m_0} & 0 \\
QSB_{C_{n_0}} & 0 & QSbC_{n_0}
\end{bmatrix}^{-1} \begin{bmatrix}
L_{m,\text{com}} - QSbC_{1e} \beta_{\text{com}} \\
M_{m,\text{com}} - QS\bar{c}(C_{m_0} + C_{m_1} \alpha_{\text{com}}) \\
N_{m,\text{com}} - QSbC_{n_0} \beta_{\text{com}}
\end{bmatrix}
\]

\[ \Delta_{\text{com}} = J^{-1} T_{\text{allo}} \] (2.35)
This section describes an autonomous, 6DOF, trajectory-tracking flight control design for a fixed-wing aircraft flight dynamics model. The details about the system design and implementation have been published in [31], which is serving as the baseline for the exposition of the new results. Six-DOF trajectory tracking results have been presented for a climbing, 360°, bank-to-turn maneuver showing the effectiveness of the TLC design in order to track a challenging mission trajectory as shown in Figure 2.5.

Figure 2.5 Helix climbing trajectory tracking performance.

2.3. Singular Perturbation Theory

Many dynamical systems naturally exhibit modal time-scale separations, where some dynamical modes have much faster transient than others, such as the rotational vs.
translational dynamics for 6-DOF rigid-body motions, actuator dynamics vs. plant dynamics, and many chemical production processes [44-48]. The slower dynamics are considered as the dominant modes (poles for LTI systems) as the faster modes will diminish quickly, leaving the motion to behave as if it was governed by the slow dynamics. Such time-scale separations have been utilized: (i) in model order reduction for effective controller design [49-52]; (ii) in synthesizing higher-order closed-loop dynamics by the dominant mode principle using a lower-order prototype dynamics, such as the all-pole second-order canonical model whose damping ratio and natural frequency have a unique correspondence with the overshoot and settling time in the unit step response [53-55]; (iii) in specifying actuator dynamics and synthesizing state observer dynamics in order to retain the direct state feedback performance [56-58].

Motivated by the modal time-scale separation control systems, the singular perturbation theory for ordinary differential equations has been employed to validate the dominant mode design principle. The standard singular perturbation model for two-time-scale nonlinear systems is given by [59]

$$\begin{align*}
\dot{x} &= f(x, z) \\
\varepsilon \dot{z} &= g(x, z)
\end{align*}$$

(2.36)

where state vectors $x \in R^n, z \in R^m$ represent the slow and fast dynamics, respectively; $\varepsilon$ is a small positive scalar representing the time-scale separation, which is called singular perturbation parameter. The key theoretical result about the stability of singularly perturbed system Eq. (2.36) is the celebrated Tikhonov Theorem, which has rigorously validated the widespread use of the dominant dynamics design and synthesis practice. However, that
validation is qualitative in nature, which does not provide a satisfactory answer to the problems above.

Consider the singular perturbation model Eq. (2.36), in particular, when \( \varepsilon = 0 \), it reduces to the nominal (slow) dynamics

\[
\dot{x} = f(x, \bar{z})
\]  

(2.37)

where \( \bar{z} = h(\bar{x}) \) is an isolated root of

\[
g(x, \bar{z}) = 0
\]

(2.38)

which vanishes at \( x = 0 \) and is bounded by a monotonically increasing function. Such a root is assumed to exist and typically does in practical control systems. When the full state \([x \ z]^T\) is perturbed from the nominal state \([\bar{x} \ \bar{z}]^T\), define the so-called boundary-layer variable

\[
\tilde{z} = z - \bar{z}
\]

(2.39)

Then the standard singular perturbation model can be rewritten in terms of the nominal motion Eq. (2.37) and the boundary-layer dynamics as

\[
\dot{x} = f(\bar{x}, \tilde{z} + h(\bar{x}))
\]  

(2.40)

\[
\varepsilon \dot{\tilde{z}} = g(\bar{x}, \tilde{z} + h(\bar{x})) - \varepsilon \frac{\partial h}{\partial \bar{x}} f(\bar{x}, \tilde{z} + h(\bar{x}))
\]

(2.41)

It is noted that the coordinate translation Eq. (2.39) preserves the stability of the two models, i.e. \([x \ z]^T = 0\) and \([\bar{x} \ \bar{z}]^T = 0\) have the same (exponential) stability. On the fast time-scale \( \tau = t / \varepsilon \), the nominal motion \([\bar{x}(\tau) \ h(\bar{x}(\tau))]^T\) can be treated as constant, so that the boundary-layer dynamics Eq. (2.41) can be further simplified as
\[ \frac{d\tilde{z}}{d\tau} = g(\bar{x}, \tilde{z} + h(\bar{x})) \]

(2.42)

With the above formulation, the key results on singularly perturbed systems provided by the celebrated Tikhonov Theorems can be succinctly summarized as:

Under appropriate conditions, there exists an \( \varepsilon^* \in (0,1) \) such that \( \forall \varepsilon \in (0, \varepsilon^*) \)

(i) The origin of the singularly perturbed system Eq. (2.36), is exponentially stable if both the origin of the nominal system Eq. (2.37) and that of the (simplified) boundary-layer dynamics Eq. (2.42) are exponentially stable, and

(ii) If the perturbed system Eq. (2.36) is exponentially stable, then the error between the perturbed state \( [x \quad z]^T \) and the nominal state \( [\bar{x} \quad \bar{z}]^T \) is of \( O(\varepsilon) \).

When specialized to LTI systems, the perturbed system Eq. (2.36), the nominal system Eq. (2.37) and the (simplified) boundary-layer dynamics Eq. (2.42) become, respectively

\[
\begin{align*}
\dot{x}(t) &= A_1 x(t) + A_{12} z(t) \\
\varepsilon \dot{z}(t) &= A_{21} x(t) + A_{22} z(t) \\
\hat{x}(t) &= \left[ A_{11} - A_{12} A_{22}^{-1} A_{21} \right] \bar{x}(t) \\
\frac{d\tilde{z}(t)}{d\tau} &= A_{22} \tilde{z}(t)
\end{align*}
\]

(2.43) \quad (2.44) \quad (2.45)

Recently the notion of Singular Perturbation Margin (SPM) has been introduced as a stability metric for nonlinear systems to gauge how much singular perturbation a nonlinear system can tolerate before losing exponential stability, which is defined as the supremum of the \( \varepsilon^* \) in Tikhonov Theorem [60]. In particular, when applied to LTI
systems, a bijective relationship between the SPM and the classical phase margin (PM) has been established [38, 61].
CHAPTER 3 NOMINAL MODE DESIGN

We choose the baseline controller design in Section 2.2.2 as our nominal controller. Three major redesigns are presented to that baseline control to ready it for the nominal controller and the wind-induced LOC prevention study: (i) an improved bank-to-turn guidance control allocation in Section 3.1, (ii) adding wind effects on navigation and aerodynamics in Section 3.2, and (iii) adding post-stall aerodynamic coefficients in Section 3.3. In Section 3.4, a LOC investigation platform is implemented in Matlab/Simulink. We will also present the simulation results on the maximum wind tolerance capability of the nominal mode controller to establish a baseline performance in Section 3.5 for a demonstration of the effectiveness of our proposed bandwidth adaptation law in extending the capability. Section 3.6 concludes the results of this chapter. This work is published in [62].

3.1. Improved Bank-to-Turn in Guidance Control

In the original design of the baseline controller, the Bank-to-Turn (BTT) guidance was not properly designed for the tracking error feedback control of the side force control allocation, since the side force was allocated into commanded sideslip angle $\beta_{\text{com}}$ so that the undesirable sideslip occurs. To minimize the undesired sideslip, a roll maneuver command defined by the *auxiliary roll* $\phi_{\text{aux}}$ is introduced in this BTT improvement design. By rolling the aircraft, lateral component $Y_{\text{com}}$ of the commanded lift $L_{\text{com}}$ is used to provided the desired side force, thereby eliminating the undesired sideslip by

$$Y_{\text{com}} = L_{\text{com}} \sin \phi_{\text{aux}}$$  \hspace{1cm} (3.1)
where we have assumed the body-frame roll angle equals to the aerodynamic bank angle \( \mu \) under small angle-of-attack \( \alpha \) and sideslip angle \( \beta \). Thus, the auxiliary roll can be obtained as

\[
\phi_{\text{aux}} = \arcsin \left( \frac{Y_{\text{com}}}{L_{\text{com}}} \right)
\]

(3.2)

Therefore, the commanded roll angle is to the following equation as

\[
\phi'_{\text{com}} = \phi_{\text{com}} + \phi_{\text{aux}}
\]

(3.3)

where \( \phi_{\text{com}} \) is obtained from Eq. (2.30).

3.2. Wind Effect

3.2.1 Wind Effect on Navigation

Since the weather factor plays a significant role in aircraft motion and aerodynamics, the wind effect needs to be considered. In order to describe the atmosphere properties and the airflow (wind) movement relative to the Inertial frame, the local atmospheric-fixed reference frame representing the air-mass surrounding the aircraft is added as a body-carried frame with its origin fixed at the center-of-gravity of the aircraft.

Considering the three-dimensional translational wind velocity \( \mathbf{V}_a = [u_a, v_a, w_a]^T \) in the body frame, the vehicle velocity \( \mathbf{V} = [u, v, w]^T \) and the relative wind velocity \( \mathbf{V}_r = [u_r, v_r, w_r]^T \) expressed in the body frame as

\[
\mathbf{V}_r = \mathbf{V} - \mathbf{V}_a = \begin{bmatrix} u_r \\ v_r \\ w_r \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} - \begin{bmatrix} u_a \\ v_a \\ w_a \end{bmatrix}
\]

(3.4)
This wind vectors relationship is called the wind triangle [63] as depicted in Figure 3.1 below.

Figure 3.1 Velocity vectors in the body frame.

Figure 3.2 The wind triangle: (a) horizontal and (b) vertical views.
In addition to the heading angle $\chi$ and flight path angle $\gamma$ describing the direction of the flight in the Inertial frame, the horizontal and vertical wind triangle components are defined as the crab angle $\chi_c = \chi - \psi$ and the air-mass-referenced flight-path angle $\gamma_a = \theta - \alpha$, respectively. These two wind-related angles are defined to navigate the vehicle in the Inertial frame for trajectory tracking in the presence of wind. In this case, the commanded Euler attitude described in Eq.(2.30) is redesigned as follows when considering the wind triangle

$$\Gamma_{\text{com}} = \begin{bmatrix} \phi_{\text{com}} \\ \gamma_a + \alpha \\ \chi - \chi_c \end{bmatrix}$$

(3.5)

where the wind triangle can be approximately calculated by [63]

$$\gamma_a = \gamma + \arcsin\left( \frac{1}{V_t} \begin{bmatrix} w_n \\ w_e \\ w_d \end{bmatrix}^T \begin{bmatrix} C_x S_\gamma \\ S_x C_\gamma \\ C_\gamma \end{bmatrix} \right)$$

$$\chi_c = \arcsin\left( \frac{1}{V_t C_{\gamma a}} \begin{bmatrix} w_n \\ w_e \end{bmatrix}^T \begin{bmatrix} -S_x \\ C_x \end{bmatrix} \right)$$

(3.6)

Note that the inertial wind velocity vector $V_a = [u_a \ v_a \ w_a]^T$ is estimated by subtracting the sensed relative wind velocity $V_{\text{sen}}$ from the sensed vehicle velocity $V_{\text{sen}}$. It is emphasized that these changes only apply to the feedback error stabilizers, whereas the open-loop nominal controller is unaffected as the nominal wind velocity is assumed to be zero.

3.2.2 Wind Effect on Aerodynamics

As a system parameter, the dynamic pressure depends on the local density of the atmosphere $\bar{\rho}$ and the airspeed $V_t$ as $Q = \frac{1}{2} \bar{\rho} V_t^2$, which is implemented according to the
relative wind velocity obtained by Eq. (3.4). Also, the aerodynamic angles are determined by the relative wind velocity as:

$$
\alpha = \arctan \left( \frac{w_i}{u_i} \right), \quad \beta = \arcsin \left( \frac{v_i}{V_i} \right), \quad \mu = \arctan \left( \frac{u_i v_i S_\theta + (u_i^2 + w_i^2) S_\theta C_\theta - u_i w_i C_\phi C_\theta}{V_i (w_i S_\theta + u_i C_\phi C_\theta)} \right)
$$

(3.7)

3.3. Stall Characteristics

When the aerodynamic forces are calculated in both the controller and the vehicle model, the nonlinear post-stall aerodynamic characteristics are required for aerodynamic force generation due to wind condition. Figure 3.3 provides the aerodynamic lift and drag coefficients in full flight envelope for typical fixed-wing aircraft according to the wind-tunnel test results [64].

![Figure 3.3 Lift and drag aerodynamic coefficients.](image)
3.4. LOC Investigation Platform Implementation

The aircraft LOC study platform is implemented in Matlab/Simulink as given in Figure 3.4 according to the Cessna 182 airplane flight data [65]. The command trajectory is provided by the Trajectory Planner block; the Controller block is implemented with the control design elaborated above; the Cessna 182 block simulates the 6DOF aircraft motions; The Wind block is used to emulate the presence of wind; Navigation Sensors block provides the onboard sensed data, which is a pass-through in the current study; Animation block provides a visualized 3D view of the 6DOF vehicle display. For simplicity, a proportional thrust law is assumed as $T = \delta_r T_{\text{max}}$, where the thrust coefficient is limited in the range of $\delta_r \in [0,1]$. The deflections of control effector are limited in the range of $\Delta = [\delta_a, \delta_e, \delta_r] \in [-20^\circ, 20^\circ]$. The above throttle and effectors saturations are applied in the simulation of dynamic aircraft model.

Figure 3.4 Matlab/Simulink platform for wind induced LOC.
Assuming the applied forces are acting at the center-of-gravity of the vehicle, 6DOF Vehicle Dynamics is implemented by a 6DOF (Euler Angles) Equations of Motion block in Simulink/Aerospace Blockset [66]. Such block is built according to the 6DOF rigid-body vehicle dynamics given in Eq. (2.1) - Eq. (2.4). Figure 3.5 shows the block inputs and outputs as

![Figure 3.5 6DOF Equation-of-Motion, Euler angles.](image)

in which 1) three-dimensional forces in body frame given in Eq. (3.8) and 2) three-dimensional moments in the body frame given in Eq. (3.9) are applied as inputs to obtain the outputs as 1) velocity in the Earth frame, position in the Earth frame, 2) Euler attitude (roll, pitch, yaw) in radians, 3) coordinate transformation from flat Earth axes to body frame, 4) velocity in the body frame, 5) angular rates in the body frame, 6) angular acceleration in the body frame, and 7) vehicle accelerations in the body frame.

\[
\mathbf{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} F_{g,x} \\ F_{g,y} \\ F_{g,z} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \left(-mg \sin \theta + T_{\text{max}} \delta - QSC_{x_0} - QSC_{\alpha - QSC_{\alpha \beta}} \right) \\ \left(mg \sin \theta \cos \phi \cos \delta + QSC_{\alpha} \beta + QSC_{\gamma} \delta \right) \\ \left(mg \sin \theta \cos \phi \sin \delta - QSC_{z_0} - QSC_{\alpha} \alpha - QSC_{\beta} \delta \right) \end{bmatrix}
\]  

(3.8)
Where the total applied force is the sum of the gravitational forces in the body frame, the thrust in body frame (zero thrusts in $y_b$, $z_b$ axis) and the aerodynamic force in body frame.

The total applied moment can be generated by the effectors deflection and the aerodynamic angles as according to Eq. (2.6)

$$\mathbf{T} = \begin{bmatrix} I_m \\ M_m \\ N_m \end{bmatrix} \begin{bmatrix} T_{allo, L} + QSB C \beta_{\sim m} \\ T_{allo, M} + QSC \left( C_{m_\alpha} + C_{m_\beta} \alpha_{\sim m} \right) \\ T_{allo, N} + QSB C_{n_\beta} \beta_{\sim n} \end{bmatrix}$$

(3.9)

where

$$\mathbf{T}_{allo} = \begin{bmatrix} T_{allo, L} \\ T_{allo, M} \\ T_{allo, N} \end{bmatrix} = \begin{bmatrix} QSB C_{i_\alpha} & 0 & QSB C_{i_\beta} \\ 0 & QSC C_{m_\beta} & 0 \\ QSB C_{n_\alpha} & 0 & QSB C_{n_\beta} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix}$$

3.5. Simulation Test Study

In order to exemplify the wind effects on the nominal controller performance, a wind gust is specified by the amplitude, wind speed increase time and wind start time [67]. Due to the transient and short-duration characteristics of the wind gust, our test wind gust profile is specified as:

a) the wind gust start time is $t = 20$ s;

b) the wind speed reaches its maximum amplitude at $t = 22.4$ s;

c) the total test time is 200 seconds.

The wind gust profile with the $u_a$ component is shown in Figure 3.6 as
The *Discrete Wind Gust Model* block in Simulink/Aerospace Blockset is applied for the implementation of a standard “1-cosine” shape of the wind gust as shown in Figure 3.7 [66].

By applying the proposed wind gust model, LOC has been induced under multiple wind conditions, including tailwind, headwind, crosswind, downdraft and updraft. The capability of wind tolerance capability for the nominal mode controller can be measured by the maximum Tolerable Wind Amplitude (TWA) at the onset of LOC. The simulated scenario is considered as the aircraft tracks a straight and level trajectory with constant
50 m/s inertial speed to the North from the initial position $\mathbf{P} = [0 \quad 0 \quad -1500]^T$ with well-trimmed initial states given in Appendix D.

3.5.1 **Tailwind**

The tailwind simulation test results are shown in Figure 3.8. With a 6 m/s tailwind occurrence at $t=20s$, the relative wind velocity component $u_r$ decreases to 44 m/s, so that the dynamic pressure is decreased, which causes the decrease of drag and lift. The controller effect pulls up the vehicle to overcome the loss of lift by increasing angle-of-attack. Then the rotation of body axis caused $w$ to rise since the nominal controller does not respond to the wind, $w$ tracking error increases, which calls for decreasing $\alpha$. However, the faster response of $\theta$ loop than $w$ loop caused oscillation of the control variables in this channel. Eventually, LOC occurs.
Figure 3.8 Baseline controller: LOC of 6m/s tailwind.
3.5.2 Headwind

For headwind LOC occurs at 76m/s as shown in Figure 3.9. The increasing $u_t$ causes a higher $Q$, so that drag and lift are increased. In response, the controller reduces the pitch angle and angle-of-attack to overcome the extra lift. In order to keep proper position tracking performance in a strong headwind, the throttle is quickly pushed to saturation. At around $t=27s$, integration windup causes LOC. The simulation results show that the baseline control will not enter LOC until 76m/s maximum amplitude of headwind (severe wind) since main consequences caused by headwind is slowing the vehicle motion rather than bring out the instability to the system.
Figure 3.9 Baseline controller: LOC of 76m/s headwind.
3.5.3 Crosswind

A crosswind from the port side is tested up to 8m/s when LOC occurs in Figure 3.10, which induced by a negative $v_t$ component, as well as a side force and sideslip angle. In order to compensate for non-zero sideslip angle, the controller banks to generate a counteractive lateral force. Then negative aileron and rudder command are saturated to cause LOC.

(a)  
\begin{tikzpicture}
\begin{axis}[
    xlabel={time (s)},
    ylabel={$v_t$ (m/s)},
    title={Airspeed-$v_t$ component},
    grid=major,
    xmin=0, xmax=25,
    ymin=-25, ymax=15,
    xtick={0,5,10,15,20,25},
    ytick={-25,-20,-15,-10,-5,0,5,10,15},
    legend entries={sen},
    legend style={at={(0.5,0.15)},anchor=north},
]
\addplot[blue,solid,mark=none,\label{fig:a}]{15};
\end{axis}
\end{tikzpicture}

(b)  
\begin{tikzpicture}
\begin{axis}[
    xlabel={time (s)},
    ylabel={$\beta$ (°)},
    title={Sideslip Angle},
    grid=major,
    xmin=0, xmax=25,
    ymin=-25, ymax=20,
    xtick={0,5,10,15,20,25},
    ytick={-25,-20,-15,-10,-5,0,5,10,15,20},
    legend entries={sen},
    legend style={at={(0.5,0.15)},anchor=north},
]
\addplot[blue,solid,mark=none,\label{fig:b}]{20};
\end{axis}
\end{tikzpicture}
Figure 3.10 Baseline controller: LOC of 8m/s crosswind.
3.5.4 Downdraft

In Figure 3.11, 4m/s downdraft causes LOC, where \( w_t \) increases at \( t=20 \)s so that angle-of-attack decreases. The controller then pitches up the vehicle, causing an abrupt jump of angle-of-attack, the vehicle climbs for a short time. In order to keep the level position, drag increases due to high angle-of-attack. After \( t=70 \)s, the \( x \) tracking error increases monotonically due to throttle saturation, which means the vehicle can not catch up the commanded trajectory. Therefore, the primary cause for continuous downdraft is throttle saturation.
Figure 3.11 Baseline controller: LOC of 4m/s downdraft.
3.5.5 Updraft

A 3m/s updraft brings about LOC shortly after $t=20s$ as shown in Figure 3.12. By neglecting the vehicle motion with the wind, the lift is increased by increasing the angle-of-attack. In response, the controller tries to maintain the level tracking by a nose-down pitch to decrease the angle-of-attack and then vehicle descends. Continuous updraft causes elevator and throttle to be saturated, which induces LOC.
Figure 3.12 Baseline controller: LOC of 3m/s updraft.

Table 3.1 summarizes the vehicle performances and controller responses under the specified wind conditions, along with the corresponding LOC cause.
Table 3.1 LOC analysis in various wind condition.

<table>
<thead>
<tr>
<th>TWA</th>
<th>Vehicle Performance</th>
<th>Control Effect</th>
<th>LOC Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tailwind 5m/s</td>
<td>( u_t \downarrow \Rightarrow \bar{Q} \downarrow \Rightarrow D, L \downarrow )</td>
<td>( \alpha \uparrow \Rightarrow w \uparrow \Rightarrow w_{err} \downarrow \Rightarrow \alpha \downarrow )</td>
<td>The faster response of ( \theta/\alpha ) loop than ( w ) loop causes oscillation of the control variables in the ( \alpha ) channel.</td>
</tr>
<tr>
<td>Headwind 75m/s</td>
<td>( u_t \uparrow \Rightarrow \bar{Q} \uparrow \Rightarrow D, L \uparrow )</td>
<td>( \alpha \downarrow \Rightarrow \delta_r \uparrow )</td>
<td>The throttle is quickly pushed to saturation in such a strong headwind. Integrator windup causes LOC.</td>
</tr>
<tr>
<td>Crosswind 7m/s</td>
<td>( v_t \uparrow \Rightarrow \beta, Y \uparrow )</td>
<td>( Y \downarrow )</td>
<td>The controller generates a counteractive lateral force, and then the aileron and rudder saturations cause LOC.</td>
</tr>
<tr>
<td>Downdraft 3m/s</td>
<td>( w_t \uparrow \Rightarrow \alpha \downarrow )</td>
<td>( \theta \uparrow \Rightarrow h \uparrow \Rightarrow D \uparrow \Rightarrow x_{err} \uparrow )</td>
<td>The ( x ) tracking error increases monotonically due to throttle saturation. Integrator windup causes LOC.</td>
</tr>
<tr>
<td>Updraft 2m/s</td>
<td>( \alpha \uparrow \Rightarrow L \uparrow )</td>
<td>( \theta \downarrow \Rightarrow \alpha \downarrow \Rightarrow h \downarrow )</td>
<td>Continuous updraft causes elevator and throttle to be saturated, which induces LOC.</td>
</tr>
</tbody>
</table>

3.6. Conclusion

The nominal controller is designed by three major redesigns to the baseline controller in the previous work rehashed in Section 2.2.2. The nominal controller mode in the iLOCPR is implemented in the Matlab/Simulink based LOC investigation platform. The simulation results on the wind tolerance capability, which is defined as the minimum LOC wind amplitude are obtained of the nominal mode controller to establish a baseline performance for the LOC prevention performance under the same wind conditions.
CHAPTER 4 QUANTITATIVE GUIDELINE FOR MULTIPLE-TIME-SCALE NESTED LOOP SYSTEM

By employing the Multiple-Time-Scale Nested-Loop (MNL) control system architecture, the nominal controller is designed by the time-scale separation principle; theoretical analysis is presented in this chapter to facilitate the synthesis, tuning and bandwidth adaptation using the singular perturbation theory, which is also the design rationale for the LOC prevention mode in Chapter 5. It relates the ratio of time-scale separation (singular perturbation) to the phase margin of the outer-loop (perturbed) system, which is important in its own right.

The chapter is organized as follows. After this introduction in Section 4.1, the MNL system is modeled in Section 4.2. In the MNL control system description is given based on singular perturbation model. The quantitative time-scale separation determination guideline is given in Section 4.3. Section 4.4 illustrated the guideline of bandwidth adaptation. Section 4.5 provides a high-order approximation approach in order to applicable for the proposed quantitative guideline. Section 4.6 concludes all the results in this chapter. This work is accepted by the conference as [68].

4.1. Introduction of Multiple-Time-Scale Nested-Loop System

Due to the fact that many dynamical systems are designed using modal time-scale separations, where some dynamical modes have much faster transient than others, there is one problem that has long been waiting to be settled, i.e. how much the fast modes have to be separated from the slow ones for the latter to be considered dominant. Some textbooks teach that the fast poles should be 3~5 times further to the left from the slower ones while
others say 8~10 times. Intuitively, if the separation is too small, the dynamical behavior predicted by the dominant modes would be inaccurate. On the other hand, large separation would significantly increase the implementation cost, as faster actuators, high-bandwidth analog devices, and high-speed processors and A/D, D/A converters must be used. This problem is, even more, acute in MNL systems that are designed based on modal time-scale separation. For instance, in our four-loop flight controller, the inner-most rotational dynamics loop would be 10,000 faster than the outer-most translational kinematics loop if a 10-time separation design was employed. Moreover, the resulting high-gain controller would be very sensitive to fast parasitic dynamics that are typically neglected in controller design, such as flex-modes, flutter modes, etc. Thus, it is of great practical interest to establish a quantitative guideline for determining, designing and tuning the modal time-scale separation in MNL controller design.

4.2. Multiple-Time-Scale Nested-Loop System Modeling

Figure 4.1 depicts a general architecture of MNL control system, where controller $C_i(s)$ and plant $P_i(s)$ in each loop are grouped together as the \textit{nominal loop gain} $L_i(s) = C_i(s)P_i(s)$ due to their slow dynamics; while the \textit{actuator} $A_i(s)$ is of different time-scale for its fast dynamics. The actuator dynamics within the $(i-1)^{th}$ loop are assumed exponentially stable and sufficiently faster by design than the desired nominal closed-loop dynamics of the $i^{th}$ loop without the actuator, which is also assumed exponentially stable, and. Therefore, they serve as the \textit{actuator} $A_i(s)$ for the $i^{th}$ loop.
Because of the same configuration in each control loop, without loss of generality, the first (innermost) singularly-perturbed module, which is composed of the actuator $A_j(j\omega)$ and loop gain $L_j(j\omega)$ will be studied to determine the quantitative time-scale relation between the actuator and nominal loop gain. The actuator dynamics are modeled as the singular perturbations (neglected fast dynamics) to the nominal loop gain (slow dynamics) according to Tikhonov’s Theorem. Such a system with zero-input $r_j = 0$ can be written into the standard singular perturbation model for two-time-scale linear system given by

$$\dot{x}(t) = A_1 x(t) + A_{22} z(t)$$
$$\epsilon \dot{z}(t) = A_{21} x(t) + A_{22} z(t)$$

(4.1)

where the state vectors $x \in \mathbb{R}^n, z \in \mathbb{R}^m$ represent the perturbed slow loop gain and fast actuator states, respectively, with $n$ being the order of the plant plus that of the controller, and $m$ is the order of the actuator. When $\epsilon = 0$, Eq. (4.1) is reduced to the closed-loop nominal (slow) system as

$$\dot{x}(t) = [A_{11} - A_{12} A_{22}^{-1} A_{21}] x(t)$$

(4.2)
and the fast mode is represented by the boundary layer equation

$$ \dot{z}(t) = -A_{22}z(t) $$

(4.3)

To facilitate the analysis, the singular perturbation parameter $\varepsilon$ which may be defined in one of the three possible and equivalent ways as

**Definition 1:** Following [61, 69], the singular perturbation parameter is defined by

(a) $\varepsilon = \frac{\omega_{cg,nom}}{\omega_{B_1}}$; (b) $\varepsilon = \frac{\omega_{B_2,nom}}{\omega_{B_1}}$; (c) $\varepsilon = \frac{\omega_{B_2,nom}}{\omega_{n_1}}$

(4.4)

where $\omega_{cg,nom}$ is the gain-crossover frequency of the nominal loop gain $L_{1,nom}(j\omega)$; $\omega_{B_1,nom}$ and $\omega_{B_2}$ are the bandwidth frequencies of the actuator $A_1(j\omega)$ and the nominal closed-loop frequency response $A_{2,nom}(j\omega) = L_{1,nom}(j\omega)/(1 + L_{1,nom}(j\omega))$, respectively; $\omega_{n_1}$ and $\omega_{n_2,nom}$ are the natural frequencies of the actuator $A_1(j\omega)$ and $A_{2,nom}(j\omega)$, respectively.

4.3. Quantitative Design Guideline

Our first result relates the time-scale separation (singular perturbation) parameter $\varepsilon$ defined by Eq. (4.4)-(a) with the phase margin ($PM$) deterioration, which is a direct consequence of Theorem 1 in [69].

**Theorem 1:** Consider the singularly-perturbed module in which the nominal loop gain is $L_{1,nom}(j\omega)$, and the actuator transfer function is $A_1(j\omega)$ with bandwidth $\omega_{B_1}$. Let $\varepsilon$ be defined by Definition 1(a), and assume that $A_1(j\omega)$ has monotonic phase lag $\phi_\omega(\omega)$ and unity DC gain. Then the following relation between $\Delta PM$ and $\varepsilon$ holds

$$ \Delta PM = PM_{nom} - PM_{pert} = \left| \phi_\omega(\varepsilon \omega_{B_1}) \right| + O(\varepsilon^2) $$

(4.5)
where $PM_{\text{nom}}$ and $PM_{\text{pert}}$ are the phase margin of the nominal $L_{1,\text{nom}}(j\omega)$ and perturbed loop gain $L_{1,\text{pert}}(j\omega) = L_{1,\text{nom}}(j\omega)A_1(j\omega)$, respectively.

In order to prove Theorem 1, we need the following Lemma 1 [69]:

**Lemma 1:** Consider the singularly perturbed LTI system, where $x(t) \in \mathbb{R}^n$ and $z(t) \in \mathbb{R}^m$; $A_{11}, A_{12}, A_{21}, A_{22}$ are constant matrices and $A_{22}$ is Hurwitz; $\varepsilon > 0$ is defined in Definition 1. If the fast system is a single-input-single-output (SISO) monotonic lag system with unity DC gain, then the following relation between $PM_{\text{pt}}$ and $PM_{\text{nom}}$ is satisfied

$$PM_{\text{pt}} = PM_{\text{nom}} + \phi_{\text{fast}}(\omega_{cg}, \varepsilon) + O(\varepsilon^2)$$

(4.6)

**Proof of Theorem 1:** In the singularly-perturbed module, the actuator system corresponds to the fast system in Lemma 1. By Lemma 1, we have

$$\Delta PM = PM_{\text{nom}} - PM_{\text{pt}} = -\phi_A(\omega_{cg}) + O(\varepsilon^2).$$

Since the actuator is assumed to be a monotonic phase lag system with unity DC gain, which means that $\phi_A(\omega_{cg}) < 0$, therefore,

$$\Delta PM = \left|\phi_A(\omega_{cg})\right| + O(\varepsilon^2).$$

By Definition 1(a), $\omega_{cg} = \varepsilon\omega_{hi}$. Thus,

$$\Delta PM = \left|\phi_A(\varepsilon\omega_{hi})\right| + O(\varepsilon^2).$$

Theorem 1 shows that the phase margin deterioration is approximately equal to the absolute value of the actuator phase frequency response at the frequency of $\varepsilon\omega_{hi} = \omega_{cg}$. Then $\varepsilon$ can be related to the decrease in phase margin. It is noted that the phase margin is roughly proportional to the damping factor $\zeta$ in under-damped responses, which provides a quantitative design guideline for each loop of MNL system.
Example 1: Suppose that the frequency-domain design specification of singularly-perturbed module is given as $PM_{\text{pert}} = 40^\circ$, and the actuator transfer function is given by

$$A(j\omega) = \frac{1}{(j\omega)^2 + 0.4242 j\omega + 1}$$

(4.7)

whose bandwidth frequency is $\omega_b = 1.5035 \text{ rad/s}$. The relation between $A(j\omega)$ and $\epsilon$ is shown in Figure 4.2 for $\epsilon$ from 0 to 1. If the nominal loop has been given with a phase margin $PM_{\text{nom}} = 60^\circ$, then the allowable phase margin deterioration by introducing the actuator in the loop is $\Delta PM \approx |\phi_A(\omega_{cg})| = 20^\circ$. Therefore, by Theorem 1, we can find $\epsilon = 0.39$ as shown from the data point to the left in Figure 4.2, which yields $\omega_{cg} = \epsilon \omega_b \leq 0.5864 \text{ rad/s}$, with which the nominal system should be designed. On the other hand, if the time-scale separation $1/\epsilon = 2$ is chosen first as shown from the data point to the right in Figure 4.2, such that the fast mode is placed twice the distance to the left of the dominant mode. Then by Theorem 1, $\Delta PM$ is found to be $36.25^\circ$ from Figure 4.2. Thus, either $PM_{\text{nom}} = PM_{\text{pert}} + \Delta PM = 76.25^\circ$ should be designed for or deterioration in $PM$ by $36.25^\circ$ should be expected when the actuator is reintroduced back into the nominal loop.
4.4. Bandwidth Adaptation Guideline

Since the closed-loop bandwidth of an inner loop can be used to define the singular perturbation parameter for the outer nested loop in an MNL control system, the dependence of the closed-loop bandwidth $\omega_{B_{i}, \text{pert}}$ of $A_2(j\omega, \varepsilon)$ (the actuator for its outer nominal loop gain) on the singular perturbation parameter $\varepsilon = \omega_{B_{i}} / \omega_{B_{i}, \text{nom}}$ in the inner loop needs to be investigated. The perturbed closed-loop transfer function is given by

$$A_{2, \text{pert}}(j\omega, \varepsilon) = \frac{1}{1 + L_{1, \text{pert}}(j\omega, \varepsilon)} \quad (4.8)$$

Then, the bandwidth of the perturbed system $\omega_{B_{i}, \text{pert}}$ can be found by the following equation

$$\left| A_{2, \text{pert}}(j\omega, \varepsilon) \right|^2 \bigg|_{\omega = \omega_{B_{i}, \text{pert}}} = \frac{1}{2} \quad (4.9)$$
Define \( f(\omega_{B_2,\text{pert}}, \epsilon) \) by

\[
f(\omega_{B_2,\text{pert}}, \epsilon) = \left| A_{n,\text{pert}}(j\omega, \epsilon) \right|^2 \bigg|_{\omega=\omega_{B_2,\text{pert}}} - \frac{1}{2} = 0 \tag{4.10}
\]

Since

a. \( \omega_{B_2,\text{pert}} \) is a function of the time-scale separation \( \epsilon \);

b. \( f(\omega_{B_2,\text{pert}}, \epsilon) \) is dependent on both \( \omega_{B_2,\text{pert}} \) and \( \epsilon \),

we take the derivative on both sides of Eq. (4.10) to obtain

\[
\frac{\partial f}{\partial \epsilon} d\epsilon + \frac{\partial f}{\partial \omega_{B_2,\text{pert}}} d\omega_{B_2,\text{pert}} = 0 \tag{4.11}
\]

which yields

\[
\frac{d\omega_{B_2,\text{pert}}}{d\epsilon} = -\frac{\frac{\partial f}{\partial \epsilon}}{\frac{\partial f}{\partial \omega_{B_2,\text{pert}}}} \tag{4.12}
\]

which can be used to determine if \( \omega_{B_2,\text{pert}}(\epsilon) \) is a monotonic function of \( \epsilon \) by its sign definiteness. Since Eq. (4.5) is, in general, a high order rational function, which eludes general analysis, Matlab symbolic toolbox is used for the existence of a real root of

\[
d\omega_{B_2,\text{pert}} / d\epsilon = 0 \quad \text{for} \quad \epsilon \in (0,1].
\]

Consider a second-order actuator transfer function and a second-order nominal loop gain.

\[
A_1(j\omega, \epsilon) = \frac{1}{\epsilon^2 s^2 + 2\epsilon\zeta s + 1}, \quad L_{1,\text{nom}}(j\omega, \epsilon) = \frac{1}{s^2 + 2\zeta s} \tag{4.13}
\]
which with normalized $\omega_{B_2,\text{men}} = 1$, $\omega_{n,1} = 1/\varepsilon$, and $\zeta_{A_1} = \zeta_{L_1} = \zeta$.

MATLAB symbolic analysis shows that $\omega_{B_2,\text{pert}}(\varepsilon)$ has a peak value for $\varepsilon \in (0,1]$, which can be seen from Figure 4.3. This discovery contradicts previous assumptions that an increase in the inner loop singular perturbation would monotonically reduce the bandwidth of the (perturbed) outer loop bandwidth, which is important to know in tuning an MNL controller gain, and in designing the bandwidth adaptation controller.

![Figure 4.3 Bandwidth frequency of singularly-perturbed module.](image)

4.4.1 Remarks 1

In previously published work on bandwidth adaptation [35, 36, 70], it had been assumed that an increase in the singular perturbation $\varepsilon$ will reduce the perturbed closed-
loop bandwidth. The above results show that this is not the case. The reason for this phenomenon can be explained by Theorem 1 because an increase of $\varepsilon$ reduces the phase margin of the perturbed system. However, it is well known in the classical linear system theory that the phase margin and the damping ratio $\zeta$ are positively related so that an increase in $\varepsilon$ will reduce the $\zeta$ for the perturbed system. It is also well-known in the classical linear system theory that the bandwidth of a 2nd-order system can be increased either by increasing the natural frequency $\omega_n$ or by reducing the damping factor $\zeta$. The former will increase the time response speed without affecting the quality of the transient such as the overshoot and ringing whereas the latter will speed up the rise-time but slow down the settling time while causing increased overshoot and ringing. It is noted that excessive overshoot may cause integrator windup in a practical nonlinear system, which may lead to a loss of stability. Therefore, we will design our bandwidth adaptation law by varying the natural frequencies of all loops simultaneously to maintain a constant phase margin during the adaptation, thereby maintaining the robustness and transient quality.

4.4.2 Remarks 2

It is also noted that the bandwidth adaptation by varying $\omega_n$ is a one-parameter gain adaptation. Another one-parameter adaptation scheme is to vary the loop gain [71]. From the classical root locus analysis, it is seen that increasing $\omega_n$ (gain) moves the closed-loop poles radially towards the left while maintaining the damping ratio $\zeta$, leading to decreased response time while maintaining consistent transient quality. Whereas increasing the loop gain pushes the closed-loop poles to the right while reducing the damping ratio $\zeta$, leading to decreased rise time but increased settling time and overshoot.
4.4.3 Remarks 3

We will implement the bandwidth adaptation using the time-varying PD-eigenvalues rather than the frozen-time eigenvalues evaluated from the frozen-time characteristic equation \( \det[\lambda(t)I - A(t)] = 0 \). For second-order oscillatory modes, these two types of eigenvalues defer by a term \( \dot{\omega}_n(t) / \omega_n(t) \). This term can be significant when \( \omega_n(t) \) is small or \( \dot{\omega}_n(t) \) is large, as reported in [72].

4.5. High-order Approximation

For each nominal loop in MNL, the actuator represents all the inner-loop dynamics, which can be complex and difficult to model. In addition, practical systems or devices may only provide some of the frequency response specifications, which are usually bandwidth \( \omega_b \) and resonant peak \( M_r \) of the actuator. These two system parameters can be used to predict the time-domain transient of the actuator system in terms of the response speed and the relative stability, which correspond to those for a 2nd-order system that we have full knowledge about its phase. Since the phase information of the actuator system should be known in order to evaluate the singular perturbation by Theorem 1, the actuator phase approximation can be formulated as an order-reduction problem as follows: within the frequency range \((0, \omega_b)\), given a high-order transfer function \( G(s) \) with its bandwidth \( \omega_b \) and resonant peak \( M_r \), find a 2nd-order transfer function \( G_2(s) \) whose phase can be used to approximate that of the high-order transfer function.

**Theorem 2**: Given a stable monotonic phase-lag mth-order transfer function \( G(s) \) with \( m > 2 \), whose bandwidth is \( \omega_b \) and resonant peak \( M_r \geq 1 \), the 2nd-order transfer function
\[
G_z(j\omega) = \frac{\omega_{n, G_z}^2}{(j\omega)^2 + 2\zeta_{G_z}\omega_{n, G_z}j\omega + \omega_{n, G_z}^2}
\] (4.14)

In which
\[
\zeta_{G_z} = \sqrt{\frac{M_r - \sqrt{M_r^2 - 1}}{2M_r}}, \omega_{n, G_z} = \frac{\omega_B}{\mu}, \mu = \sqrt{1 - 2\zeta_{G_z}^2 + \sqrt{2 - 4\zeta_{G_z}^4 + 4\zeta_{G_z}^4}}
\] (4.15)

satisfies
\[
|\phi_G(j\omega) - \phi_{\zeta_{G_z}}(j\omega)| = O(\omega), \quad \forall \omega \in (0, \omega_B)
\] (4.16)

Proof of Theorem 2 is given as follows.

**Proof of Theorem 2:** Without loss of generality, we assume \(\omega_B = 1\) by normalization. For all \(\omega \in (0,1)\), the normalized frequency response \(G(j\omega)\) can be written as
\[
G(j\omega) = \frac{\prod_{k=1}^{M_1} (j\omega + z_k) \prod_{i=1}^{M_2} \left((j\omega)^2 + 2\zeta_{j\omega_N}j\omega + \omega_N^2\right)}{\prod_{i=1}^{N_1} (j\omega + p_i) \prod_{j=1}^{N_2} \left((j\omega)^2 + 2\zeta_{j\omega_N}j\omega + \omega_N^2\right)}
\] (4.17)

whose normalized phase frequency response is given by
\[
\phi_G(\omega) = \sum_{k=1}^{M_1} \tan^{-1}\left(\frac{\omega}{z_k}\right) + \sum_{i=1}^{M_2} \tan^{-1}\left(\frac{2\zeta_{j\omega_N}\omega}{\omega_N^2 - \omega^2}\right) - \sum_{i=1}^{N_1} \tan^{-1}\left(\frac{\omega}{p_i}\right) - \sum_{j=1}^{N_2} \tan^{-1}\left(\frac{2\zeta_{j\omega_N}\omega}{\omega_N^2 - \omega^2}\right)
\] (4.18)

where \(m = N_1 + N_2\). The calculation of the first-order Taylor expansion of Eq. (4.18) about \(\omega = 0\) is given by:
\[
\phi_G(j\omega) = C \omega + O(\omega^2)
\] (4.19)

where
\[
C = \sum_{k=1}^{M_1} \left(\frac{1}{z_k}\right) + \sum_{i=1}^{M_2} \left(\frac{2\zeta_{j\omega_N}}{\omega_N^2}\right) - \sum_{i=1}^{N_1} \left(\frac{1}{p_i}\right) - \sum_{j=1}^{N_2} \left(\frac{2\zeta_{j\omega_N}}{\omega_N^2}\right)
\] (4.20)
Consider the 2nd-order transfer function given in Eq. (4.14), in which $\zeta_{G_2}$ and $\omega_{n,G_2}$ are chosen by Eq. (4.15) with $\omega_b = 1$. According to the linear system theory [73], it can be shown that $M_{r_2} = M_r$, $\omega_{b_2} = \omega_b$. Moreover,

$$
\left| \phi_{G_2}(j\omega) - \phi_{G_2}(j\omega) \right| = \left( C - \frac{2\zeta_{G_2} \omega / \mu}{\omega^2 - 1 / \mu^2} \right) \omega + O(\omega^2) = O(\omega) \quad (4.21)
$$

For the specialized non-resonant case when $M_r = 1$, the 2nd-order phase frequency response is given by

$$
\phi_{G_2}(j\omega) = \arctan \left( \frac{\sqrt{2} \omega}{\omega^2 - 1} \right) \quad (4.22)
$$

It can be verified that $M_{r_2} = M_r = 1$, $\omega_{b_2} = \omega_b = 1$. Furthermore,

$$
\left| \phi_{G_2}(j\omega) - \phi_{G_2}(j\omega) \right| = \left( C - \frac{\sqrt{2}}{\omega^2 - 1} \right) \omega + O(\omega^2) = O(\omega) \quad (4.23)
$$

Therefore, by denormalization, Eq. (4.16) is verified.

Example 2 is given here to show the performance of the order reduction technique regarding phase approximation given in Theorem 2. Example 3 will illustrate the MNL design when the actuator transfer function is unknown.

**Example 2:** Given two $3^{rd}$-order transfer functions as follows

\[
G_{h_1}(j\omega) = \frac{1}{((j\omega)^2 + 1.414 j\omega + 1)((j\omega)/5 + 1)}
\]

\[
G_{h_2}(j\omega) = \frac{1}{((j\omega)^2 + 0.707 j\omega + 1)((j\omega)/5 + 1)}
\]
with $M_{r1} = 1$, $\omega_{b_{1}} = 0.9799 \text{rad} / s$ and $M_{r2} = 1.4871$, $\omega_{b_{2}} = 1.3926 \text{rad} / s$, respectively.

By Theorem 3, the proposed method provides a good approximation of the phase frequency response of a 2nd-order systems with the same bandwidth and resonant peak, and the transfer functions are calculated to be

$$G_{i1}(j\omega) = \frac{0.9602}{(j\omega)^2 + 1.386 j\omega + 0.9602}$$
$$G_{i2}(j\omega) = \frac{0.9773}{(j\omega)^2 + 0.7127 j\omega + 0.9773}$$

respectively. The phase responses of the two systems are shown in Figure 4.4, in which it can be seen that the approximation errors are increasing with frequency $\omega$.

Example 3: Theorem 2 can be applied in singularly-perturbed module design by actuator phase approximation. Consider the actuator of $G_{h1}(s)$ in Figure 4.4(a), by Theorem 2 the
2nd-order approximation can be found in Eq. (4.14). Given the frequency design specification $PM_{\text{pert}} = 40^\circ$, if we choose $PM_{\text{nom}} = 90^\circ$, then the allowable phase margin deduction $\Delta PM = 50^\circ$. By Theorem 1, we can find $\omega_{cg} = 0.576 rad/s$ from the estimated phase curve in Figure 4.4 at the point of $\left| \phi_{c/g}(\omega_{cg}) \right| \approx \Delta PM = 50^\circ$. Therefore, $\varepsilon \approx \omega_{b_1} / \omega_{cg} = 0.6$ is the minimum time-scale separation.

4.6. Conclusion

The time-scale separation of Multiple-Time-Scale Nested Loop (MNL) control system is investigated by singular perturbation method. Our first main result relates the time-scale separation (singular perturbation) parameter $\varepsilon$ with the phase margin (PM) deterioration when the actuator dynamics are introduced back into the nominal design. The first new result presented in this chapter reveals the relationship of the time-scale separation with the PM deterioration in the presence of singular perturbation, which provides a quantitative guideline for selecting the time-scale separation between the fast and slow modes. The second result reveals that the dependence of the bandwidth of a singularly perturbed system on the singular perturbation need not be monotonic, which dictates that the bandwidth adaptation should be applied to all control loops simultaneously to ensure consistent transient response and robustness. The third result is an algorithm for approximating a high-order or unspecified (actuator) transfer function by a second-order one based on phase frequency response to facilitate singular perturbation analysis in practice.
The new results presented in M system do not only provide practicing control engineers a quantitative guideline for selecting the time-scale separation in design and synthesis based on the dominant dynamics principle, but also reveal the relationship of the time-scale separation with the PM deterioration in the presence of singular perturbation, which is of both practical and theoretical interest. In addition, the relationship between the closed-loop robustness (in terms of phase margin) and the time-scale separation obtained in this paper not only has practical significance in the design and synthesis of MNL controllers, more importantly it can be used as the MNL system tuning criterion such as frequency response based adaptation control design [74-76].
CHAPTER 5 LOC PREVENTION MODE DESIGN

In this chapter, we propose an automatic LOC prevention system by bandwidth adaptation using time-varying Parallel Differential (PD)-eigenvalues for a real-time tradeoff between tracking performance and severe wind tolerance capability, which is defined as the maximum tolerable wind amplitude (TWA) that the vehicle can operate in without encountering LOC. The time-varying PD-eigenvalue principle is applied for controller synthesis and adaptation law design for improving the wind tolerance capability. The adaptation law is an augmentation to the redesigned baseline (nominal mode) controller designed (in Chapter 3) with constant gains by the singular perturbation (time-scale separation) principle in a Multiple-Time-Scale Nested-Loop (MNL) architecture. Theoretical analysis presented in Chapter 4 is employed to establish the relationship between the time-scale separation with the phase margin of the singularly perturbed system, which not only provides a general guideline for designing and tuning MNL systems but also justifies the rationale for the bandwidth adaptation based LOC prevention design. The analysis also reveals that the dependence of the closed-loop bandwidth on the singular perturbation is not monotonic, which dictates that the bandwidth adaptation should be simultaneously applied to all loops in an MNL control system. This work is published in [62].

The wind-induced LOC is introduced in Section 5.1. In Section 5.2, the baseline controller is augmented with the bandwidth adaptation law using time-varying Parallel Differential (PD)-eigenvalues for a real-time tradeoff between the tracking performance and maximum wind tolerance. The effectiveness of the bandwidth adaptation for extending
the wind tolerance capabilities is demonstrated by simulation results in Section 5.3. Section 5.4 concludes this chapter with a summary of the main results and contributions.

5.1. Introduction of Wind LOC Prevention

Aircraft LOC is not a common occurrence, a predominant number of aviation accidents are related to severe wind variability (or wind shear), such as wake turbulence, thunderstorm, microburst and mountain wave, etc [77]. In those cases, the wind shear significantly affects the flight performances as well as the unpredicted aircraft motions and inadvertent pilot operations. Therefore, we can say that wind is the leading cause of aircraft LOC induced by the environment. “It should be possible to design better guidance and control aids to improve a pilot’s ability to avoid an accident in the event of an inadvertent wind-shear encounter,” which is stated by the Low-Altitude Wind Shear and Its Hazard to Aviation Book [78].

The wind shear is recognized as a sudden change in airflow speed or direction over a short distance with the duration typically for several minutes. The significant microburst involving horizontal and vertical wind shear, whose speed can be treated as a weak wind condition (below 10m/s) to hazard situations (above 25m/s) [79]. Such variations of airflow cause severe and unexpected changes in aircraft airspeed and aerodynamics, which may cause the autopilot or pilot to lose the ability to operate the vehicle, even though the control system of the aircraft may still be physically intact and functional. In order to conduct the LOC interventions, prevention is the first course of action for wind induced LOC. LOC prevention is defined as control strategies and maneuvers that sustain normal flight under
unexpected adverse events such as environmental hazards, system failures, vehicle damage or pilot errors.

It is noted that the existing TLC flight control systems provide in 2.2.2 assumed that there is no relative atmosphere motion. However, it is necessary to consider the wind effect for LOC prevention and recovery since the wind will not only change the inertial velocity of the aircraft, whereby producing large tracking errors that may cause control saturation leading to LOC; but also amend the dynamic pressure and aerodynamic coefficients, whereby producing instantaneous forces and causing sudden stall leading to LOC. In the former case, the wind existence can be treated as disturbances acting on the aircraft dynamics, while in the latter case, the wind can be modeled as a perturbation to the nominal system (without the wind). Since the wind only affects system parameters without increasing the order of the overall system. Such perturbation can be classified as regular. On the other hand, for flight control systems designed by the multi-loop time-separation (singular perturbation) method, the inner loop dynamics are treated as a singular perturbation (neglected fast dynamics) to the outer loop (slow dynamics) nominal design according to Tikhonov’s Theorem[60]. The standard singular perturbation model for two-time-scale nonlinear systems is given by

\[
\dot{x} = f(x,z)
\]

\[
\epsilon \dot{z} = g(x,z)
\]  \( (5.1) \)

where the state vectors \( x \in \mathbb{R}^n, z \in \mathbb{R}^m \) represent the slow and fast dynamics, respectively; \( \epsilon \) is a small positive scalar called singular perturbation parameter representing the time-scale separation. Moreover, parameter variations that reduced the effective gain or bandwidth as well as inner loop control saturation due to wind induced large tracking errors
can be viewed as an increase in the *singular perturbation*. When the increase of singular perturbation exceeds the singular perturbation margin, as in recently introduced stability metric that is essentially equivalent to the classical phase margin but applicable to nonlinear and time-varying systems\(^\text{17}\), the aircraft will lose stability and enter LOC.

The design of adaptive automatic flight control system gives us motivations to guarantee or extend the stability margin in the presence of increased singular perturbation. An effective LOC prevention automatic flight control system should provide robustness with respect to wind induced perturbation and adapt the control configuration in response to the adverse environment factors and correspondingly adjust control priority. The linearized, close-loop PD-eigenvalues formed by time-varying PD-spectrum synthesis allows real-time adjustment of system bandwidth, which has been employed in time-varying bandwidth filter (TVB) design for time-varying control requirements \cite{41, 80, 81} . In addition, PD-eigenvalue based TVB has been used in direct fault tolerant control to cope with increased singular perturbation induced by control effector failures \cite{82}.

5.2. Wind Induced LOC Prevention by Bandwidth Adaptation

As we can see from Table 3.1, the nominal controller is capable of accommodating moderate wind conditions to maintain the tracking task. Based on the singular perturbation theory, severe wind can be managed at the cost of reduced tracking performance by reducing the bandwidth (integral gain) of the closed-loop tracking error dynamics. Recall the 12 second-order linear time-varying closed-loop tracking error dynamics synthesized using the constant damping time-varying PD-eigenvalues in Eq. (2.26) are
where the index $i$ is the loop number counting from outer loop to inner loop and $j$ is the channel number; $\omega_{n,ij}(t)$ are time-varying natural frequencies and $\zeta_{ij}$ are the constant damping ratios of the desired closed-loop dynamics for each state variable $e_i(t)$ in Eq. (2.24). However, constant natural frequencies $\omega_{n,ij} = \omega_{n,ij,\text{nom}}$ were used for the nominal controller designed in Chapter 3 [31]. Therefore, Eq. (5.2) is reduced to the familiar Linear-Time-Invariant (LTI) synthesis formula, and the closed-loop dynamics were LTI in nature.

In this section, we will use time-varying natural frequency $\omega_{n,ij}(t)$, which is proportional and approximately equal to the closed-loop (instantaneous) bandwidth, for a real-time trade-off between tracking performance and robustness to prevent LOC. In this case, a based adaptive control is proposed in Figure 5.1, in which the aircraft model is subject to wind disturbance and perturbation; and the control system is augmented with a time-varying bandwidth adaptive controller. Both the baseline controller and the adaptive controller rely on the feedback from the on-board navigation system.
5.2.1 The General Bandwidth Adaptation Law

As discussed in the Introduction, for a multi-loop flight controller, wind effects that reduce the effective bandwidth of an inner-loop such as integrator windup can be treated as an increase in singular perturbation to its outer loops. Therefore, by the singular perturbation (time-scale separation) principle, the bandwidths of the outer loops should be reduced accordingly to preserve stability at the cost of reduced tracking performance. This bandwidth adaptation scheme can be readily realized with the time-varying bandwidth $\omega_{n,ij}(t)$ in Eq. (6.2). Based on the analysis in Chapter 4, a single adaptation gain $k_a(t)$ will be used for all bandwidths

$$\omega_{n,ij}(t) = k_a(t)\omega_{n,\text{nom},ij}$$

where the constant bandwidths $\omega_{n,\text{nom},ij}$ are those of the nominal controller as given in Table 5.1, which will be called nominal bandwidths. The $\omega_{n,\text{nom},ij}$ are first synthesized and tuned for the desired tracking performance with reasonable robustness in no wind condition.
Based on the singular perturbation principle, Table 5.2 gives the expected behaviors of tracking performance and wind tolerance under bandwidth adaptation, where TWA stands for Tolerable Wind Amplitude at the onset of that keeps from LOC and is determined by the same test wind gust profile shown in Table 5.2; \( \|x_{\text{error}}\|_1 \) is the cumulative state tracking error measured in \( L_1 \)-norm; \( k_{a,\text{min}} \) and \( k_{a,\text{max}} \) are the extreme values of \( k_a(t) \) beyond which stability will be lost due to an exceedance of stability margins.

### Table 5.1 Nominal TLC controller parameters.

<table>
<thead>
<tr>
<th></th>
<th>Loop1</th>
<th></th>
<th>Loop 2</th>
<th></th>
<th>Loop 3</th>
<th></th>
<th>Loop 4</th>
</tr>
</thead>
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<td>( x )</td>
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<td>( y )</td>
<td>0.2</td>
<td>( u )</td>
<td>0.8</td>
<td>( v )</td>
<td>0.8</td>
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<td>6</td>
<td>( q )</td>
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<td>( \phi )</td>
<td>4.4</td>
<td>( \theta )</td>
<td>4.4</td>
</tr>
<tr>
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<td>19.2</td>
<td>( r )</td>
<td>19.2</td>
<td>( r )</td>
<td>19.2</td>
<td>( r )</td>
<td>19.2</td>
</tr>
</tbody>
</table>

### Table 5.2 Bandwidth adaptation, expected TWA and tracking performance.

<table>
<thead>
<tr>
<th>Adaptation Constant</th>
<th>Bandwidth</th>
<th>TWA</th>
<th>( |x_{\text{error}}|_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{a,\text{min}} )</td>
<td>( \omega_{n,\text{min}} )</td>
<td>( V_{a,\text{max}} )</td>
<td>( |x_{\text{error}}|_{1,\text{max}} )</td>
</tr>
<tr>
<td>( k_{a,\text{nom}} )</td>
<td>( \omega_{n,\text{nom}} )</td>
<td>( V_{a,\text{nom}} )</td>
<td>( |x_{\text{error}}|_{1,\text{nom}} )</td>
</tr>
<tr>
<td>( k_{a,\text{max}} )</td>
<td>( \omega_{n,\text{max}} )</td>
<td>( V_{a,\text{min}} )</td>
<td>( |x_{\text{error}}|_{1,\text{min}} )</td>
</tr>
</tbody>
</table>

According to Eq. (6.3)

\[
\frac{\dot{\omega}_{n,ij}(t)}{\omega_{n,ij}(t)} = \frac{\omega_{n,\text{nom}}}{\omega_{n,\text{nom}} k_a(t)} = \frac{\dot{k}_a(t)}{k_a(t)}
\]

(5.3)

Therefore, the PD-spectral synthesis formula in Eq. (5.2) can be rewritten as
The bandwidth adaptation gain $k_a(t)$ and $\dot{k}_a(t)$ can be implemented using a first-order pseudo-differentiator as shown in Figure 5.2,

\[
\begin{align*}
\alpha_{y1}(t) & = \alpha_{y1,\text{nom}} k_a^2(t), \\
\alpha_{y2}(t) & = \alpha_{y2,\text{nom}} k_a(t) = \frac{\dot{k}_a(t)}{k_a(t)}
\end{align*}
\] (5.4)

Figure 5.2 Adaptive PI gain structure.

where $\omega_{LP}$ is a design parameter that determines the time constant of the $k_a(t)$ in response to a step command $k_{a,\text{desire}}(t)$. The time-varying coefficients $\alpha_{y1}(t)$ and $\alpha_{y2}(t)$ are then programmed into the baseline controller to replace the corresponding constant coefficients in the constant PI gain matrices $K_p$, $K_I$ as given in Appendix B.

5.2.2 Wind Adaptation Law Design

As shown in Table 5.3, with necessary modifications as described above, the extreme bandwidths $\omega_{n,\text{min}}$ and $\omega_{n,\text{max}}$, TWA along with the corresponding tracking error metric are determined under each wind case through the simulation test. Note that in Table 5.3, the tracking error metric $\|x_{\text{error}}\|$ does go up as $k_a$ decrease and go down as $k_a$ increases, in support of the rationale for the bandwidth adaptation. Notice that the
adaptation gain is reduced to 0.01 when downdraft is accommodated, which means the tracking is practically shut down temporarily to prevent the vehicle from wind-induced LOC.

Table 5.3 Bandwidth adaptation, TWA and tracking error under wind conditions.

<table>
<thead>
<tr>
<th>Wind Condition</th>
<th>$\alpha_{a,\text{min}}$</th>
<th>TWA</th>
<th>$|x_{\text{error}}|$</th>
<th>$\alpha_{a,\text{nom}}$</th>
<th>TWA</th>
<th>$|x_{\text{error}}|$</th>
<th>$\alpha_{a,\text{max}}$</th>
<th>TWA</th>
<th>$|x_{\text{error}}|$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tailwind</td>
<td>0.1</td>
<td>19</td>
<td>1.7e05</td>
<td>1</td>
<td>5</td>
<td>6.3e04</td>
<td>1.2</td>
<td>1</td>
<td>2.5e04</td>
</tr>
<tr>
<td>Headwind</td>
<td>0.1</td>
<td>80</td>
<td>434.4</td>
<td>1</td>
<td>75</td>
<td>1.2</td>
<td>1.2</td>
<td>15</td>
<td>0.3</td>
</tr>
<tr>
<td>Crosswind</td>
<td>0.1</td>
<td>25</td>
<td>6.1e04</td>
<td>1</td>
<td>7</td>
<td>3.2e04</td>
<td>1</td>
<td>4</td>
<td>3.2e03</td>
</tr>
<tr>
<td>Downdraft</td>
<td>0.01</td>
<td>25</td>
<td>9.9e04</td>
<td>1</td>
<td>3</td>
<td>2.9</td>
<td>1.2</td>
<td>2</td>
<td>0.9</td>
</tr>
<tr>
<td>Updraft</td>
<td>0.3</td>
<td>6</td>
<td>1.4e04</td>
<td>1</td>
<td>1</td>
<td>11.9</td>
<td>1.2</td>
<td>1</td>
<td>1.4</td>
</tr>
</tbody>
</table>

The adaptation laws are designed based on $V_{a,\text{sen}} = V_{\text{sen}} - V_{r,\text{sen}}$, where the onboard ground speed sensor and airspeed sensor will be required to estimate the wind velocity. Then the time-varying bandwidth adaptation laws can be described on each wind case as in Table 5.4:
### Table 5.4 Wind adaptation law.

<table>
<thead>
<tr>
<th>Wind Condition</th>
<th>Adaptation Gain</th>
</tr>
</thead>
</table>
| **Tailwind**   | \( k_a(u_a,0,0) = \begin{cases} 
-0.06u_a + 1 & 0 \leq u_a \leq 15 \\
0.1 & u_a > 15 \end{cases} \) |
| **Headwind**   | \( k_a(u_a,0,0) = \begin{cases} 
1 & -75 \leq u_a \leq 0 \\
0.1 & u_a < -75 \end{cases} \) |
| **Crosswind**  | \( k_a(0,v_a,0) = \begin{cases} 
-0.036v_a + 1 & 0 \leq |v_a| \leq 25 \\
0.1 & |v_a| > 25 \end{cases} \) |
| **Downdraft**  | \( k_a(0,0,w_a) = \begin{cases} 
-0.33w_a + 1 & 0 < w_a \leq 3 \\
0.01 & w_a > 3 \end{cases} \) |
| **Updraft**    | \( k_a(0,0,w_a) = \begin{cases} 
0.3 & w_a < -1 \\
0.7w_a + 1 & -1 \leq w_a < 0 \end{cases} \) |

For wind velocity vector with three non-zero components acting in the body frame, the bandwidth adaptation law is designed by

\[
\omega_n(t) = k_a(t)\omega_{n,nom}, \text{ where } k_a(t) = \min(k_a(u_a,0,0),k_a(0,v_a,0),k_a(0,0,w_a)) \quad (5.5)
\]

where the \( k_a(t) \) is inherited from the adaptation law design for individual wind case in Table 5.4. Such adaptation can ensure the minimum bandwidth requirement for longitudinal, vertical and lateral wind combination. Figure 5.3 shows the adaptation gain \( k_a \) for the three-dimensional wind speed when \( w_a = 0 \text{ m/s} \):
5.3. Simulation Results

5.3.1 One Direction Wind

The same test scenarios as the ones tested on the baseline controller shown in Section 3.5 are performed to demonstrate the effectiveness of the adaptation augmentation to prevent LOC in severer wind conditions. And the simulations results are plotted for the same flight states plus the bandwidth adaptation gain and the 3D trajectory.

In Figure 5.4, instead of LOC occurring at 6m/s tailwind for baseline controller, the augmented system can maintain stability with 13m/s tailwind by adapting $k_a$ to 0.22. In
addition, the tracking error is still in a limited range after 200s wind perturbation. Figure 5.5 shows that the adaptive controller can overcome 80m/s headwind by reducing $k_a$ to 0.1 when the headwind occurs, and its angle-of-attack is still in the normal flight envelope. The crosswind case is shown in Figure 5.6 where the sideslip angle is prevented from exceeding the safe region typically ($\pm 10^\circ$) with the crosswind increasing to 19m/s by adapting $k_a$ to be approximately 0.36. For downdraft case in Figure 5.7, the tracking control system is practically shut down by adapting $k_a$ to 0.01, while the vehicle can overcome as large as 20m/s downdraft albeit with an altitude loss. The updraft in Figure 5.8 also causes the vehicle to climb and descend; however, healthy angle-of-attack is kept after 200-second simulation.

(a)  

(b)
Figure 5.4  Adaptive control augmentation: TWA-13m/s tailwind.
Figure 5.5 Adaptive control augmentation: TWA-80m/s headwind.
Figure 5.6 Adaptive control augmentation: TWA-19m/s crosswind.
Figure 5.7 Adaptive control augmentation: TWA-20m/s downdraft.
Figure 5.8 Adaptive control augmentation: TWA-6m/s updraft.
Table 5.5 lists the wind tolerance capabilities of both the baseline controller and the adaptation augmentation in terms of TWA. The wind tolerance envelope of the aircraft has been extended by 160%-560% for all wind types except the headwind, for which the baseline controller could accommodate up to 75m/s already as shown in Table 5.5, which demonstrates the effectiveness of the proposed adaptive control scheme.

Table 5.5 Bandwidth adaptation results.

<table>
<thead>
<tr>
<th>Wind</th>
<th>Baseline TWA (m/s)</th>
<th>Adaptation TWA(m/s)</th>
<th>Percent Increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tailwind</td>
<td>5</td>
<td>13</td>
<td>160%</td>
</tr>
<tr>
<td>Headwind</td>
<td>75</td>
<td>80</td>
<td>7%</td>
</tr>
<tr>
<td>Crosswind</td>
<td>7</td>
<td>19</td>
<td>170%</td>
</tr>
<tr>
<td>Downdraft</td>
<td>3</td>
<td>20</td>
<td>560%</td>
</tr>
<tr>
<td>Updraft</td>
<td>1</td>
<td>6</td>
<td>500%</td>
</tr>
</tbody>
</table>

5.3.2 3D Wind

A 3D wind vector with three components as \( \mathbf{V}_{a,c} = [w_a \ w_e \ w_d]^T = [5 \ 2 \ -5]^T \) is tested. By the adaptation law given in Eq. (5.5). By adapting the \( k_a \) to the minimum value of the three pure wind law given in Table 5.4 to 0.1, the following simulation results shows that the vehicle deviates the command track with stability preserved.
Figure 5.9 Adaptive control augmentation: 3D wind.
5.4. Conclusion

A Loss-of-control (LOC) prevention controller is introduced as a part of the proposed integrated automatic flight controller for fixed-wing aircraft Loss-Of-Control Prevention and Recovery (iLOCPR). Such novel wind-induced LOC prevention controller is presented using bandwidth adaptation in the presence of winds in order to make a tradeoff between the tracking performance and the system robustness. The bandwidth adaptation law is designed to prevent LOC under various wind conditions by the time-varying PD-eigenvalue augmented in the baseline controller in order to increase the wind tolerance capability. By the proposed wind bandwidth adaptation design, the wind tolerance envelope of the aircraft has been extended by 160%-560% for all wind types except the headwind, for which the baseline controller could accommodate up to 75m/s already in Table 5.5, which demonstrates the effectiveness of the proposed adaptive control scheme. It is noted that the bandwidth adaptation can be applied not only to wind-induced LOC as demonstrated in this chapter but also to other baseline controller designed by MNL or other fault conditions to expand the fault tolerance capability.
CHAPTER 6 LOC ARREST MODE DESIGN

In this chapter, we present a reconfigurable flight control system for the LOC arrest controller, which is the first phase of the proposed two-phase LOC recovery intervention. By employing a previously published 6 DOF trajectory tracking flight control system design for the nominal mode, this chapter focuses on the LOC arrest mode design, which switches the control from the flight trajectory tracking (nominal mode) to aerodynamic attitude tracking (LOC arrest mode) in order to maintain airborne and regain control authority of the aircraft at the cost of temporarily abandoning the mission trajectory. To successfully realize the proposed aerodynamic attitude tracking control design, Trajectory Linearization Control (TLC) can be employed for fixed-wing aircraft, which does not appear to have been attempted before. Such design is similar to 3 DOF Reusable Launch Vehicle (RLV) entry flight control system for tracking challenging aerodynamic attitude maneuvers during the entry operations [36]. By introducing the mode transition condition, supervisory control design, and the automatic controller configuration (the details will be illustrated in Chapter 8), this chapter is focusing on the arrest controller design. Simulation results demonstrate the proposed LOC arrest controller’s capability to recover the normal flight, and its performance in terms of arrest time, altitude departure and aerodynamic loading.

Following the introduction 6.1, the proposed switching controller scheme is shown for multiple control configurations under a LOC supervisory controller in Section 6.2. Section 6.3 briefly introduces the supervisory control for switching the controller configuration use. The Section 6.4 presents the nominal controller configuration under the
supervisory control when LOC arrest mode is engaged. Section 6.5 describes the LOC arrest mode design based on 3DOF TLC for aerodynamic attitude tracking control. Simulation results of the proposed switching-mode control are shown in Section 6.6. Section 6.7 concludes this chapter with a summary of the main results and contributions. This work is accepted by the conference as [83].

6.1. Introduction of LOC Recovery

A variety of control strategies has been developed for LOC recovery. Fault-tolerant control techniques have been applied for malfunction by reconfiguring allocation of the available control effector [84]. In the consideration of vehicle physical limits, advanced safety envelopes such as flight envelope, service envelope or performance envelope were calculated to steer the vehicle states to the desired equilibrium condition from outside of the safety envelope [85]. Adaptive control has been applied to accommodate the nonlinear uncertainties and disturbances in the presence of LOC [86, 87]. Propulsion control for generating redundant force and moment have been applied to gain maneuverability especially when the control of surface functionality is lost [88, 89].

Figure 6.1 provides a depiction of LOC accident data which is related to angle-of-attack and sideslip angle, in which the red trace illustrates the abnormal aerodynamic angles during LOC accidents are usually outside of the normal flight envelope. Also, the following off-nominal conditions declare the aircraft LOC occurrence as [5]

- Pitch attitude greater than 25 degrees, nose up;
- Pitch attitude greater than 10 degrees, nose down;
- Bank angle greater than 45 degrees.
Within the above parameter, but flying at airspeeds inappropriate for the conditions.

Figure 6.1 Aerodynamic flight envelopes for current transport simulations [5].

Based on the study of the LOC data given above, our LOC arrest philosophy is to protect healthy aerodynamic attitude to regain and maintain airborne and control authority of the aircraft at the onset of a LOC upset, thereby stopping and recovering from a LOC should it inadvertently happen.

6.2. Multi-Modal Controller - Arrest

This section briefly introduces the framework of an integrated automatic flight controller for fixed-wing aircraft LOC Prevention and Recovery (iLOCPR). The integrated iLOCPR control system is characterized by a set of operating modes corresponding to the flight states: nominal mode for trajectory tracking mission (Chapter 3), LOC prevention mode (Chapter 5), LOC arrest mode (this Chapter) and mission restoration mode (to appear...
in Chapter 7), coordinated by a supervisory controller (details to appear in Section Chapter 8). It is noted that the contributions and merits of the present chapter can be evaluated independently of the other control modes.

The multi-modal controller configuration is shown in Figure 6.2. On the basis of the nominal mode is designed for tracking the nominal mission trajectory $P_{tgt}$ in the Earth frame which is assumed inertial. The prevention mode is designed in Chapter 5. In this chapter, a LOC arrest mode will be designed in order to switch from the mission trajectory tracking task to LOC arrest aerodynamic attitude trajectory tracking task in order to recover and maintain healthy flight condition at the cost of temporarily abandoning the mission trajectory. Table 6.1 shows the controller configuration including command trajectory, feedback variables, and controller outputs in order to reconfigure the controller structure according to the switching flag for each mode.

Figure 6.2 Multi-modal control system configuration.
Table 6.1 Multi-modal Controller configuration: nominal/prevention and arrest.

<table>
<thead>
<tr>
<th>Mode</th>
<th>flag</th>
<th>Trajectory</th>
<th>Controller</th>
<th>Feedback</th>
<th>Controller Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>0</td>
<td>$P_{\text{tgt}}$</td>
<td>6DOF Nominal</td>
<td>$P_{\text{set}^<em>}, V_{\text{set}^</em>}, \Gamma_{\text{set}^<em>}, \Omega_{\text{set}^</em>}$</td>
<td>$\Lambda_{\text{com}^*}, \delta_{r,\text{nom}}$</td>
</tr>
<tr>
<td>Prevention</td>
<td>1</td>
<td>$P_{\text{tgt}}$</td>
<td>6DOF Nominal</td>
<td>$P_{\text{set}^<em>}, V_{\text{set}^</em>}, \Gamma_{\text{set}^<em>}, \Omega_{\text{set}^</em>}$</td>
<td>$\Lambda_{\text{com}^*}, \delta_{r,\text{nom}}$</td>
</tr>
<tr>
<td>Arrest</td>
<td>2</td>
<td>$\Lambda_{\text{arst}}$</td>
<td>3DOF Arrest</td>
<td>$\Lambda_{\text{set}^<em>}, \Omega_{\text{set}^</em>}$</td>
<td>$\Lambda_{\text{arst}^*}, \delta_{r,\text{arst}}$</td>
</tr>
</tbody>
</table>

6.3. Supervisory Control

Designed on the top level, the supervisory controller has the switching logic variable $\text{flag}$ as its output, whose value is set according to the real-time flight states. Since upset is the direct consequence of aerodynamic attitude exceedance of the normal flight envelope, the aerodynamic attitude is essential as a LOC indicator. In addition, angular rates also constitute a LOC indicator, as they must be kept within a certain range to prevent abnormal airflow over the wings and control surfaces from inducing unbalanced stall. Another indicator is the airspeed, as stall will occur under the critical airspeed. Therefore, the vehicle safety can be described by three sets of thresholds, which consist of the extremities of aerodynamic attitude, angular rates and airspeed. An operation box is defined as the thresholds relate to nominal flight condition as

$$O_1 = \left\{ \Lambda: \alpha \in [-2^\circ, 5^\circ], \beta \in [-5^\circ, 5^\circ], \mu = [-20^\circ, 20^\circ] \right\}$$
$$O_2 = \left\{ \Omega: p, q, r \in [-20\text{deg/sec}, 20\text{deg/sec}] \right\}$$
$$O_3 = \left\{ V_r \in [1.5V_{r,\text{stall}}, V_{r,\text{max}}] \right\}$$

(6.1)

where $V_{r,\text{stall}}$ is the lowest speed below which, the vehicle is stall; and $V_{r,\text{max}}$ is the maximum allowable airspeed that the vehicle can achieve according to the throttle condition. A safety box can be specified according to the full flight envelope [5] as
And the supervisory control flag is determined by

\[
\text{flag} = \begin{cases} 
0, & \Lambda \in O_1 \text{ AND } \Omega \in O_2 \text{ AND } V_t \in O_3 \\
2, & \Lambda \not\in O_1 \text{ OR } \Omega \not\in O_2 \text{ OR } V_t \not\in O_3
\end{cases}
\]  

(6.3)

where \( \text{flag}(0) \) indicates the flag value at the last sampling time. The navigation sensors are capable of detecting the LOC condition and raise the flag. The details about the supervisory control for multi-mode design in terms of transition conditions and hysteresis will appear in Chapter 8. Thus, the controller configuration can be set by the value of the flag as shown in Table 6.1.

6.4. Nominal Controller Engagement

The nominal controller and LOC arrest controller in the switching mode control system configuration diagram Figure 6.2 are expanded in Figure 6.3, in which the upper path describes the configuration of the nominal mode controller. Note that the control objectives, controller configuration, and controller outputs are all supervised by the variable flag, which is determined by the flight condition given in Eq. (6.3). By default, the flight states are assumed to be contained in the operation box prescribed by Eq. (6.1). Thus, \( \text{flag} = 0 \) and the nominal controller is engaged. Mission trajectory is selected to be the control objective under nominal mode by applying \( \text{com tgt} = \text{P}_{\text{opt}} \). And the corresponding command trajectories in the upper path components are configured as

\[
\text{V}_{\text{com}} = \text{V}_{\text{nom}} + \text{V}_{\text{ctrl}}, \quad \Gamma_{\text{com}} = \Gamma_{\text{nom}} + \Gamma_{\text{ctrl}}, \quad \Omega_{\text{com}} = \Omega_{\text{nom}} + \Omega_{\text{ctrl}}, \quad T_{m,\text{com}} = T_{m,\text{nom}} + T_{m,\text{ctrl}}
\]  

(6.4)
where the command variables are obtained by adding the closed-loop PI feedback tracking error “ctrl” to the “nom” from dynamic pseudo-inversion [31]. In this case, the controller outputs $\left[ \Lambda_{\text{com}}, \delta_{r,\text{com}} \right]$ are selected.

Meanwhile, the LOC arrest mode is disarmed by setting the aerodynamic attitude tracking command and the ensuing body rate command equal to the sensed flight states as

$$\Lambda_{\text{com}} = \Lambda_{\text{sen}}, \quad \Omega_{\text{com}2} = \Omega_{\text{sen}}$$  \hspace{1cm} (6.5)

so that the integrators in the arrest mode controller are put on hold by setting zero tracking errors as $\Lambda_{\text{err}} = \Lambda_{\text{sen}} - \Lambda_{\text{com}} = 0$, $\Omega_{\text{err}2} = \Omega_{\text{sen}} - \Omega_{\text{com}2} = 0$. In this case, the arrest controller in Figure 6.2 is configured to track the sensed states of $\Lambda_{\text{sen}}$ and $\Omega_{\text{sen}}$ so that once the arrest mode is enabled, the integrators are properly initialized, thereby preventing switching transient or instability caused by excessive initial tracking error.
Figure 6.3 Nominal/Prevention and arrest controller configuration.

| GON  | = guidance outer-loop nominal           |
| GOF  | = guidance outer-loop feedback          |
| GIN  | = guidance inner-loop nominal           |
| GIF  | = guidance inner-loop feedback          |
| GAN  | = guidance allocation nominal           |
| GAC  | = guidance allocation command           |
| EON  | = Euler attitude outer-loop nominal     |
| EOF  | = Euler attitude outer-loop feedback    |
| EIN  | = Euler attitude inner-loop nominal     |
| EOF  | = Euler attitude inner-loop feedback    |
| LA   | = Euler attitude allocation             |
| AON  | = aerodynamic attitude outer-loop nominal |
| AOF  | = aerodynamic attitude outer-loop feedback  |
| AIN  | = aerodynamic attitude inner-loop nominal |
| AIF  | = aerodynamic attitude inner-loop feedback  |
| AA   | = aerodynamic attitude allocation       |
6.5. LOC Arrest Mode Design

The trajectory tracking mission for nominal mode controller has to be abandoned when LOC occurs. The LOC arrest philosophy is to protect healthy aerodynamic attitude in order to regain and maintain airborne and control authority of the aircraft at the onset of a LOC upset, thereby stopping and recovering from a LOC should it inadvertently happen.

In this section, LOC arrest mode controller is designed using the TLC technique for aerodynamic attitude tracking as shown in the lower path in Figure 6.3.

6.5.1 LOC Arrest Mode Controller Configuration

Once LOC is detected, the supervisory control is set as the following value according to Table 6.1

\[ \text{flag} = 2 \]  \hspace{1cm} (6.6)

Meanwhile, the LOC arrest mode is engaged to take control of the aircraft, and the control output are \([\Lambda_{\text{arst}}, \delta_{r,\text{arst}}]\).

6.5.2 LOC Arrest Command

The aerodynamic attitude command \(\Lambda_{\text{arst}} = [\alpha_{\text{arst}} \beta_{\text{arst}} \mu_{\text{arst}}]^T\) should be determined first for the LOC arrest mode controller design. By approximately balancing the weight of the aircraft and the Lift force as \(W = L = QS\left(C_{L_0} + C_{L_\alpha}\alpha\right)\), where the dynamic pressure \(Q = 0.5\rho v_{r,\text{arst}}^2\) is obtained using the airspeed \(v_{r,\text{arst}}\) at the time of switching, the arrest angle-of-attack command \(\alpha_{\text{arst}}\) can be obtained by solving the above equation. Since the desired attitude at the end of arrest is symmetric and level flight, both \(\beta_{\text{arst}}\) and \(\mu_{\text{arst}}\) are set to zero. Therefore, the arrest command can be determined as
In order to help restore the healthy aerodynamic angles, the maximum allowable throttle \( \delta_{r, \text{arst}} = 1 \) is applied to increase the airplane’s airspeed under LOC. A level flight path angle is commanded for minimum altitude loss as \( \gamma_{\text{arst}} = 0^\circ \).

### 6.5.3 LOC Arrest Controller Design

As shown in Figure 6.2, the control objective of the arrest mode controller is to track the aerodynamic attitude given as the arrest trajectory outlined in Eq. (6.7). The controller consists of aerodynamic attitude outer-loop, aerodynamic attitude inner-loop, and aerodynamic attitude control allocation. Each loop is designed according to the typical TLC design, which comprises an open-loop nominal control and a feedback tracking error regulator.

Since both of the nominal mode controller and the two-loop arrest mode controller are designed using TLC approach, the tracking error dynamics in each closed-loop are specified as \( \dot{x}_{ij} + \alpha_{y_{1j}}(t)\dot{x}_{ij} + \alpha_{y_{1j}}(t)x_{ij} = 0 \), where the coefficients \( \alpha_{y_{1j}}(t) \) and \( \alpha_{y_{2j}}(t) \) are synthesized using the constant damping time-varying \( v \)-eigenvalues by PD-eigenvalue as

\[
\alpha_{y_{1j}}(t) = \omega_{n_{ij}}^2(t), \quad \alpha_{y_{2j}}(t) = 2\zeta_{y_j}\omega_{n_{ij}}(t)\frac{\dot{\omega}_{n_{ij}}(t)}{\omega_{n_{ij}}(t)}, \quad i = 5, 6, \quad j = 1, 2, 3 \tag{6.8}
\]

where the index \( i = 5 \) is the loop number for the aerodynamic attitude outer-loop and \( i = 6 \) is the loop number for the aerodynamic attitude inner-loop, and \( j \) is the channel number; time-varying natural frequencies \( \omega_{n_{ij}}(t) \) are designed to be constant in this section, so are
\( \alpha_{q_1}(t) \) and \( \alpha_{q_2}(t) \); and \( \zeta_{ij} \) are the constant damping ratios of the desired closed-loop dynamics for each state variable \( e_{ij} \).

### 6.5.3.1 Attitude (Aerodynamic) Outer Loop - Loop 5

In AON (aerodynamic attitude outer-loop nominal) as shown in Figure 6.3, the nominal angular velocity is calculated by inverting the rotational kinematics (wind components) model given in Eq. (2.5) to calculate the body rate command for the corresponding inner loop. For command aerodynamic attitude tracking, the nominal body rate is obtained as

\[
\Omega_{\text{nom}2} = B_5^{-1}(\Lambda_{\text{nom}})(\dot{\Lambda}_{\text{nom}} - f_3(\Lambda_{\text{nom}}))
\]  

(6.9)

where the \( \dot{\Lambda}_{\text{nom}} \) is achieved by the pseudo-differentiator [90]. The feedback control is designed to stabilize the linearized closed-loop error dynamics \( \Lambda_{\text{err}} = \Lambda_{\text{sen}} - \Lambda_{\text{arst}} \) along the nominal trajectory given in Eq. (6.9). Then the PI feedback control law is obtained from the desired PD-eigenvalue based on the TLC design as shown below

\[
\Omega_{\text{ctrl}2} = -K_{\text{p5}} \Lambda_{\text{err}} - K_{15} \int \Lambda_{\text{err}}
\]

(6.10)

\[
K_{\text{p5}} = \begin{bmatrix}
k_{p5,11} & k_{p5,12} & k_{p5,13} \\
k_{p5,21} & k_{p5,22} & k_{p5,23} \\
k_{p5,31} & k_{p5,32} & k_{p5,33} 
\end{bmatrix},
K_{15} = \begin{bmatrix}
0 & \alpha_{321} S_\alpha & \alpha_{331} C_\alpha C_\beta \\
\alpha_{311} & 0 & \alpha_{331} S_\beta \\
0 & -\alpha_{312} C_\alpha & \alpha_{331} S_\alpha C_\beta
\end{bmatrix}
\]

where the expressions for the elements \( K_{\text{p5}}, K_{15} \) are given in the Appendix C. The output of the attitude outer-loop controller is then obtained as

\[
\Omega_{\text{com}2} = \Omega_{\text{nom}2} + \Omega_{\text{ctrl}2}
\]

(6.11)
6.5.3.2 Attitude (Aerodynamic) Outer Loop - Loop 6

The aerodynamic attitude inner loop control design is the same to the one of Euler attitude. The nominal moment vector is calculated by inverting Eq. (2.4) in order to obtain
\[ T_{m,\text{nom}} = B_6^{-1} \left[ \Omega_{\text{nom}} - f_6(\Omega_{\text{nom}}) \right], \]
and the PI control law for this loop is
\[ T_{m,\text{ctrl}} = -K_p \Omega_{\text{err}} - K_i \int_{t_0}^{t} \Omega_{\text{err}}(\sigma) d\sigma \]  \hspace{1cm} (6.12)
with tracking error \( \Omega_{\text{err}} = \Omega_{\text{sen}} - \Omega_{\text{com}} \).

\[ T_{m,\text{com}} = T_{m,\text{nom}} + T_{m,\text{ctrl}} \]  \hspace{1cm} (6.13)

The gain matrices are synthesized as in [31], which can be found in Appendix B, in which \( K_{p6} = K_{p4}, K_{i6} = K_{i4} \).

6.5.3.3 Attitude (Aerodynamic) Allocation

The aerodynamic attitude inner loop control allocation is achieved by inverting the approximated aerodynamic moment model as the same as the one in the nominal controller design. The moment equations are written in terms of the Jacobian so that Eq. (2.34) is obtained and inversion of Eq. (2.35) gives the control surface deflections command as:
\[ \Delta = \begin{bmatrix} \delta_{\alpha,\text{arst}} \\ \delta_{\epsilon,\text{arst}} \\ \delta_{\gamma,\text{arst}} \end{bmatrix} = \begin{bmatrix} Q\Sigma b C_{l_s} & 0 & Q\Sigma b C_{l_s} \\ 0 & Q\Sigma \Sigma C_{m_s} & 0 \\ Q\Sigma b C_{n_s} & 0 & Q\Sigma b C_{n_s} \end{bmatrix}^{-1} \begin{bmatrix} L_{m,\text{com}} - Q\Sigma b C_{l_s} \beta_{\text{arst}} \\ M_{m,\text{com}} - Q\Sigma \Sigma(C_{m_s} + C_{m_s} \alpha_{\text{arst}}) \\ N_{m,\text{com}} - Q\Sigma b C_{n_s} \beta_{\text{arst}} \end{bmatrix} \]  \hspace{1cm} (6.14)

6.6. Simulation Results

The LOC investigation platform in this chapter was slightly modified from the one in Figure 3.4 as given in Figure 6.4 below. The relatively high fidelity simulation platform
is built with a multi-mode *Controller* is configured by supervisory control, which is implemented using MATLAB/Stateflow; *Disruption* block is used to emulate the effector-induced upset condition.

![Diagram](image)

Figure 6.4 The Matlab/Simulink platform for effector disruption caused LOC.

The nominal controller parameters are set the same values of the ones in Table 5.1. The arrest controller parameters for synthesizing the gain matrices are given in Table 6.2 as

<table>
<thead>
<tr>
<th>Arrest Mode</th>
<th>$[\alpha \beta \mu]$</th>
<th>$[p \quad q \quad r]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{n,ij}$</td>
<td>[1.25 1.25 1.25]</td>
<td>[2.5 2.5 2.5]</td>
</tr>
<tr>
<td>$\zeta_{ij}$</td>
<td>[0.7 1.4 1.4]</td>
<td>[1.4 2.8 1.4]</td>
</tr>
<tr>
<td>$\omega_{n,ij,diff}$</td>
<td>[10 10 10]</td>
<td>[20 20 20]</td>
</tr>
<tr>
<td>$\zeta_{ij,diff}$</td>
<td>[1.4 1.4 1.4]</td>
<td>[1.4 1.4 1.4]</td>
</tr>
</tbody>
</table>
And the effector dynamics are implemented as a low-pass filter whose transfer function is

\[ H(s) = \frac{30}{s + 30} \]  

in which the bandwidth of the actuator is 30 rad/s, which indicates a much faster response than the innermost control loop.

Two simulation scenarios are performed in order to demonstrate the effectiveness of the LOC arrest controller including

(i) a longitudinal stall upset;

(ii) a lateral-directional spin upset.

Each scenario is tested by the following procedure:

- Nominal mode controller without arrest: The controller is in the nominal mode without reconfiguring to the arrest mode throughout the upset;

- Imminent LOC arrest: The arrest control mode is engaged at the onset of an upset, which is the intended use of the controller for autonomous upset recovery. Such operation may be used in conjunction with an upset prevention controller, such as the one reported in Chapter 5, for autonomous LOC prevention and recovery;

- Post LOC arrest: A 4sec time delay is introduced before turning on the arrest mode controller to allow the upset to fully develop, which demonstrates the controller’s capability to arrest a severely upset. This mode could be used as a standalone emergency controller for piloted aircraft, which can be actuated by pressing a “panic button” or automatically overriding pilot control when the
upset mode has exceeded a preset time limit (assuming the pilot is incapable of handling the upset or has been incapacitated).

6.6.1 Longitudinal Stall Upset

The aircraft is trimmed to a straight and level flight condition starting at the initial position of \( P_0 = [0 \ 0 \ -1500]^T \) flying to the North, and the vehicle’s airspeed is 50m/s with a trimmed angle-of-attack \( \alpha_{\text{trim}} = 1^\circ \). A pitch disruption induced by an excessive elevator deflection \( \Delta \delta_e = -30^\circ \) at \( t = 5 \text{sec} \) as shown in Figure 6.5.

![Figure 6.5 Effector disruption - elevator.](image)

6.6.1.1 Nominal Mode

As shown in Figure 6.6(a), in response to the \(-30^\circ\) elevator disruption at \( t = 5\text{sec} \), the controller immediately generates a \(+20^\circ\) elevator deflection, which is the maximum deflection allowed by the nominal controller to cancel the disruption. The remaining \(-5^\circ\) elevator deflection is enough to cause the pitch rate \( q \) and angle-of-attack \( \alpha \) to exceed the safety box (red dotted line) soon after as shown in Figure 6.6(b) and Figure 6.6(c),
respectively, which causes a stall, followed by steep bank and sideslip angles. All the control effectors go into saturation. The airspeed quickly drops to below the stall speed (in our case $V_{\text{stall}} = 28 \text{ m/s}$) at $t = 7 \text{ sec}$, with oscillating load factor, as shown in Figure 6.6(d).
6.6.1.2 Imminent LOC Arrest

As shown in Figure 6.7(a), after the negative elevator disruption is conducted at $t = 5\text{sec}$, the exceedance of pitch rate $q$ from the safety box shown in Figure 6.7(d) triggered the arrest controller at $t = 5.25\text{sec}$ by setting $\text{flag} = 1$ shown in Figure 6.7(b). The arrest controller is taking control of the system from this point. Figure 6.7(c) shows that after less than 1s, the arrest controller reduces the angle-of-attack to the arrest command $\alpha_{\text{arst}} = 1^\circ$. System response are returned to the operation box after $t = 7.5\text{sec}$, so that arrest is finished. The altitude change (gain) during the arrest is 23m as shown in Figure 6.7(g) while the maximum load factor is less than 4g, which is inherited from the onset of
the upset, as shown in Figure 6.7(f). It is noted that at the moment when $flag = 3$, a mission restoration controller can be actuated to steer that airplane back to the mission trajectory. But here the LOC arrest controller is simply left to run at maximum throttle so that the trajectory after the end of arrest should be ignored.
Figure 6.7 Longitudinal – imminent LOC system response.

6.6.1.3 Post-LOC Arrest

In Figure 6.8(a), LOC occurs due to the elevator disruption at $t = 5.24\sec$, since the pitch rate exceeds the safety box. Unlike the imminent LOC case studied above, the arrest mode is not armed until $4\sec$ time delay is elapsed, during which time the aircraft is controlled by the nominal controller. All the states responses are reacting as upset even with the full throttle effect under the nominal controller. The arrest mode is engaged at $t = 9.24\sec$ in Figure 6.8(b), excessive aerodynamic angles and angular velocities converge to the operation box at $t = 14.02\sec$ in Figure 6.8(c-d). The total arrest time after severe longitudinal upset is about $4\sec$. During the arrest period, airspeed is recovered to...
above the critical speed in Figure 6.8(f) and the maximum magnitude of the load factor $n$ is less than $4g$, again inherited from the upset and the altitude loss is 118m with a slightly lateral movement which is less than 2m, and a downrange loss of about 400m in Figure 6.8(g), which are to be recovered by a mission restoration controller.
(b) Switching flag

(c) Aerodynamic Attitude

- $\alpha$ (°)
- $\beta$ (°)
- $\mu$ (°)
6.6.2 Spin

The aircraft is under the same flight condition as in the above scenario initially. In order to emulate the spin upset, the following roll and yaw disruptions are considered as shown in Figure 6.9, where an excessive rudder deflection $\Delta \delta_r = -25^\circ$ for 1sec is conducted at $t = 5$sec with a moderate aileron deflection $\Delta \delta_a = 3^\circ$ of the same duration. Since the maximum effector deflections of the nominal controller are limited to $\pm 20^\circ$, such a rudder disruption is dominant in the total effector deflection.
Figure 6.9 Spin-induced by excessive rudder deflection and moderate aileron.

6.6.2.1 Nominal Mode

LOC occurs under the nominal mode when the effectors are disrupted after $t = 5\text{ sec}$ as shown in Figure 6.10. The sideslip angle $\beta$ goes beyond the safety box first as seen in Figure 6.10(c) and soon all angular rates and aerodynamic angles exceed the safety box as seen in Figure 6.10(b) and Figure 6.10(c) respectively. Under the nominal controller, control effectors are saturated in Figure 6.10(a). The load factor $n$ exceeds $4g$ at $t = 8\text{ sec}$ and then starts to oscillate; while theairspeed $V_i$ is reduced from 50m/s to below the critical stallairspeed 28m/s rapidly, as shown in Figure 6.10(d). The stability and control authority of the vehicle are lost.
(a) Actuator Deflection

(b) Angular Velocity
Figure 6.10 Spin – nominal system response.
6.6.2.2 Imminent LOC Arrest

Upon recognizing an upset caused by the excessive sideslipping, which exceeds its safety threshold at 10° as shown in Figure 6.11(c), the switching flag jumps to “2” while the controller is switched to the arrest mode to retain the control of the vehicle immediately as shown in Figure 6.11(b).

After $t = 6.32 \text{sec}$, the vehicle is under the control of the arrest mode controller that the nominal mode command whose subscript is “com1” are abandoned. Instead, the arrest command whose subscript is “arst” is picked up. Accordingly, the corresponding angular rates indicated by $\Omega_{\text{com2}}$ serving the aerodynamic attitude inner-loop command in Figure 6.11(d). In order to restore the healthy aerodynamic attitude, the tracking errors of aerodynamic attitude are decreasing by the control effect including the corresponding effectors and full throttle application as shown in Figure 6.11(e). There are no actuator saturations observed during the arrest process in Figure 6.11(a), and it only takes less than 2 seconds for the controller to recover the vehicle states to the operation box. At the end of arrest at $t = 8.28 \text{sec}$, the system stability and the control authority are regained. The maximum load factor during the arrest is $1.626 g$, and the airspeed is kept for only moderate loss in Figure 6.11(g-h). The altitude change from the nominal flight is only 6m when the LOC arrest mode regains the healthy and stable aerodynamic angles.
Figure 6.11 Spin – imminent LOC system response.
6.6.2.3 Post-LOC Arrest

Instead of initiating the LOC arrest immediately after the LOC occurrence is detected, a 4 sec delay is allowed before the switching flag changes. Under the nominal controller, a severe spin condition occurs since the aircraft rotates towards the direction of the rudder, the vehicle slips into a complex stall and spin situation, with saturated control effectors and a full throttle applied as shown in Figure 6.12(e). The spin is beyond the capability of the nominal controller to recover.

The arrest mode controller engages at $t = 10.32 \text{ sec}$ as shown in Figure 6.12(b). It is noticed that the control effectors saturate momentarily during the arrest process while the aerodynamic attitude errors for tracking the arrest command are decreasing. The angular rates converge to the operational flight condition with moderate rate values. The throttle serves as an additional control effector to recover the nominal aerodynamic attitude in Figure 6.12(e).

A $1.7g$ maximum value of load factor is experienced during the arrest, and the airspeed is recovered to be above the critical airspeed after $t = 14 \text{ sec}$. An attitude loss of 85m occurs during the arrest with a lateral deviation of 105m. A mission restoration controller will be taking control from this point to re-guide the aircraft back to the mission trajectory.
(a)

(b)
Figure 6.12 Spin – post LOC system response.
6.7. Conclusion

A Loss-of-control (LOC) arrest controller is presented as a part of the proposed integrated automatic flight controller for fixed-wing aircraft Loss-Of-Control Prevention and Recovery (iLOCPR). The multi-modal controller is configured by a supervisory control on the top level in order to switch the control from the flight trajectory tracking (nominal mode) to aerodynamic attitude tracking (LOC arrest mode). Two simulation scenarios are performed in order to demonstrate the effectiveness of the LOC arrest controller including (i) a longitudinal stall upset and (ii) a lateral-directional spin upset. When engaged at the onset of the upset, the arrest controller takes less than 2sec to recover the healthy aerodynamic attitude with the altitude loss/gain of 23m for scenario (i) and 6m for scenario (ii), respectively; such altitude departures satisfy the requirement of less than 34m (100-foot) bet forth by the FAA. Even with a severe stall (the LOC arrest is armed after 4sec time delay), about 100m altitude departure and lateral deviations occurs during less than 5sec arrest time both of the scenarios (i) and (ii), which meet the position tracking error allowance for the mission restoration mode using some general guidance designs.
CHAPTER 7 MISSION RESTORATION MODE DESIGN

In this chapter, a mission restoration control system in an integrated automatic flight controller for aircraft Loss-of-Control Prevention and Recovery is presented, in which the two-phase Loss-of-Control recovery includes a LOC arrest mode and a mission restoration mode. For a trajectory tracking mission, a complete mission restoration mode is achieved by i) a close-in sub-mode of guiding the vehicle to catch up with the target position, and ii) a home-in sub-mode of restoring the mission. A pure pursuit guidance approach is applied in the proposed close-in sub-mode by aligning the velocity vector to the Line-of-Sight vector using proportional-integral-derivative linear regulation. The home-in sub-mode is designed by bandwidth adaptation to gradually regain the tracking performance, and eventually, restore the mission. Simulation results demonstrate the effectiveness and the performance of the proposed mission restoration design.

Following an introduction in Section 7.1, Section 7.2 presents the multi-modal flight controller configuration, in which the configuration of the LOC mission restoration mode and the supervisory control for the mission restoration mode are illustrated. By applying the nominal controller designed in Chapter 3, Section 7.3 describes the complete mission restoration controller design. Simulation results of the proposed restoration strategy are shown in Section 7.4. Section 7.5 summarizes the main results and the contribution of this work.

7.1. Introduction of Mission Restoration

As shown in Figure 1.5, an automatic LOC recovery can be achieved by i) a LOC arrest controller for regaining healthy flight states at the cost of temporarily abandoning
the mission trajectory, and ii) a complete mission restoration guidance trajectory designer for mission recovery after the successful arrest of an upset. This chapter focuses on the mission restoration mode design within the framework of the iLOCPR.

Unlike the path following without the time constraints, trajectory tracking is the most challenging flight mission, which requires the vehicle to be at a given reference point at a prescribed time. However, the nominal trajectory tracking system is only capable of tolerating small tracking errors, which is subjected to the limitations of the local domain-of-attraction imposed by the nonlinear dynamics, such as control effector saturations [83]. In addition, the nominal trajectory tracking system is tuned for desired tracking performance, which is a trade-off between the system robustness so that a further limited tracking error tolerance capability is imposed on the nominal trajectory tracking mode. Therefore, the nominal trajectory tracking controller is incapable of regaining the trajectory tracking mission right after the LOC arrest, which usually experiences large position tracking errors such as flight course deviations and altitude loss.

A mission restoration controller mode is needed to guide the vehicle back to the mission trajectory after the LOC arrest so that the tracking error is reduced to within the boundaries that the nominal trajectory tracking system is capable of accommodating. Since the current position on the mission trajectory can be viewed as a virtual target moving along a predefined trajectory, the mission restoration design can be formulated as a guidance problem of planning a restoration trajectory to chase and capture (rendezvous with) a moving target. In this case, the line-of-sight (LOS) pure-pursuit missile guidance approach can be borrowed for the guidance trajectory design [32]. Pure pursuit guidance (PPG) is to
steer and maintain the vehicle velocity vector align with the LOS (range vector). Since the PPG is a closed-loop guidance, a closed-loop control law needs to be designed to stabilize the error between the vehicle velocity vector and the LOS. This method effectively turns the aforementioned large range error problem in the Cartesian frame into the regulation of small angular errors in the polar frame, and a simple maximum speed close-in control.

While the LOS based PPG is effective for close-in onto the target from afar, it is not practical for close-range home-in and rendezvous because small horizontal and vertical errors can cause large and rapid LOS angle changes at near range. This is why in missile guidance PPG is typically used only for mid-course navigation [32]. For applications where rendezvous rather than impact with the target is required, a home-in guidance law is required for close-range navigation, which is typically designed in the Cartesian frame.

Three major differences in the PPG for the aircraft mission restoration and the missile target tracking are noted. First, the missile guidance design typically generates the acceleration command, which is in turn allocated to the attitude controller (autopilot) and the throttle (if applicable), whereas here we use PPG to design a feasible position trajectory to take the advantage of the availability of a 6DOF baseline trajectory tracking flight controller. Second, a missile employing the PPG is typically equipped with a seeker to determine the LOS in real time, whereas in our case the virtual target trajectory is known, and the vehicle is assumed to have onboard inertial position sensors, such as the GPS or other ground-based navigation aids. The last difference is that typical missile PPG employs a proportional feedback law for stabilization of the LOS tracking error, whereas a proportional-integral-derivative (PID) control law is proposed. In addition, the home-in
sub-mode controller employs a bandwidth adaptation using parallel differential (PD) eigenvalues [41] (as the bandwidth adaptation strategy applied in Chapter 5) to facilitate a smooth transition into the nominal tracking mode. 6DOF simulation results for a Cessna 182 airplane model are presented to demonstrate the effectiveness of the proposed mission restoration mode controller.

7.2. Multi-Modal Controller - Restoration

As explained in the introduction, the position tracking error will increase to beyond the capability of the nominal controller after the LOC arrest. A restoration controller is needed in order to reduce the position tracking error to eventually restoring the mission. The restoration mode configuration in the multi-modal control system iLOCPR is shown in Figure 7.1, in which the mission trajectory is replaced by a restoration trajectory to direct the vehicle back to the mission trajectory after a successful arrest of upset.

In the mission restoration mode, a guidance trajectory $P_{str}$ is designed to direct the vehicle back to the mission trajectory after the successful arrest of a LOC upset. For each particular mode, Figure 7.1 shows the controller configuration including the commanded trajectory, active controller, and controller outputs including the actuator deflection $\Delta = [\delta_d \delta_e \delta_r]^T$ and the thrust coefficient $\delta_r$ to reconfigure the controller structure according to the switching flag.
It is noted that the multi-modal flight control scheme proposed herein can be applied to any six DOF nominal tracking controller.

7.3. Mission Restoration Mode Design

As explained in the Introduction, the position tracking errors may increase to beyond the capability of the nominal controller after the LOC arrest. A mission restoration mode is needed in order to reduce the position tracking error to a level that can be accommodated by the nominal controller to eventually restore the mission.
7.3.1 Mission Restoration Controller Configuration

The restoration mode configuration in the multi-modal control system iLOCPR is shown in Figure 7.1, in which the mission trajectory is replaced by a restoration trajectory as the command trajectory in order to direct the vehicle back to the mission trajectory after a successful arrest of upset.

Assuming a virtual aircraft that is moving along the mission trajectory at a prescribed time. Such virtual target can be referred as a target point, whose position is predefined in the inertial frame. At the end of LOC arrest, the flight envelope determining healthy flight states is recovered as

\[
\mathbf{\Lambda} \in O_1 \text{ AND } \mathbf{\Omega} \in O_2 \text{ AND } V_i \in O_3
\]

where

\[
O_1 = \{ \mathbf{\Lambda} : \alpha \in [-2^\circ, 5^\circ], \beta \in [-5^\circ, 5^\circ], \mu = [-20^\circ, 20^\circ] \} \\
O_2 = \{ \mathbf{\Omega} : p, q, r \in [-20^\circ / \text{sec}, 20^\circ / \text{sec}] \} \\
O_3 = \{ V_i \in [1.5 V_{\text{stall}}, V_{\text{max}}] \}
\]

where \( \mathbf{\Lambda} = [\alpha \ \beta \ \mu]^T \) are the aerodynamic angles; \( \mathbf{\Omega} = [p \ q \ r]^T \) are the angular rates; \( V_{\text{stall}} \) and \( V_{\text{max}} \) are the vehicle stall speed and the maximum achievable speed, respectively. The flight envelope \( O_1, O_2, O_3 \) indicates the success of the LOC arrest. At this moment, the restoration mode is engaged by setting the supervisory control \( \text{flag} = 3 \) as shown in Table 7.1. Under this controller mode configuration, the command trajectory is switched to \( \mathbf{P}_{\text{com}} = \mathbf{P}_{\text{rstr}} \); the feedback variables are the same as the ones designed for the
nominal mode controller as $\mathbf{P}_{\text{sen}}$, $\mathbf{V}_{\text{sen}}$, $\Gamma_{\text{sen}}$, $\Omega_{\text{sen}}$; and the controller outputs are the same to the nominal mode as $\Delta_{\text{con}}$, $\delta_{r,\text{con}}$.

By sensing the current position $\mathbf{P}_{\text{sen}}$ and reading the target point $\mathbf{P}_{\text{tgt}}$ at each sampling time, the range vector, which represents the position tracking error, can be obtained in real-time as

$$\mathbf{R} = \begin{bmatrix} R_N & R_E & R_D \end{bmatrix}^T = \mathbf{P}_{\text{tgt}} - \mathbf{P}_{\text{sen}} = \begin{bmatrix} x_{\text{tgt}} & y_{\text{tgt}} & z_{\text{tgt}} \end{bmatrix}^T - \begin{bmatrix} x_{\text{sen}} & y_{\text{sen}} & z_{\text{sen}} \end{bmatrix}^T$$  \hspace{1cm} (6.3)

where the range vector $\mathbf{R}$ is described in NED Cartesian coordinate. However, the position tracking error is normally very large, since the altitude loss and course deviations usually occur during the LOC arrest. As mentioned in the introduction, the nominal trajectory tracking controller designed and tuned for desired tracking performance cannot accommodate the large position tracking error. Therefore, the nominal controller cannot be engaged directly at the end of the arrest.

In order to restore the original mission, two sequential sub-modes are designed for the mission restoration mode. The first sub-mode is called close-in, in which a restoration trajectory $\mathbf{P}_{\text{rstr}}$ is planned to reduce the range given in Eq. (6.3) to a moderate level, which will be defined as a capture box. Once the range is in the capture box, a sub-mode called home-in is designed to gradually restore the original mission. The design of these two sub-modes will be described as follows.
7.3.2 Close-in Control

As mentioned in the Introduction, the PPG strategy is employed to cope with the large range vector Eq. (6.3) in the Cartesian coordinate by transforming it to the spherical coordinate as follows

\[
\rho = \arctan \left( \frac{R_E}{R_N} \right) \\
\sigma = -\arcsin \left( \frac{R_D}{r} \right) \\
r = \sqrt{R_N^2 + R_E^2 + R_D^2}
\]

(6.4)

where \( r \) is the range vector magnitude; \( \rho \) is the azimuth angle and \( \sigma \) is the elevation angle, respectively. Because of the limited capability of the design, the two angles are both assumed to be sufficiently small, which is satisfied with the vehicle behind the target with a large range and relatively small altitude and horizontal course deviation. Therefore, the range vector \( \mathbf{R} = [R_N, R_E, R_D]^T \) can be determined by the azimuth angle and the elevation angle as follows

\[
R_N = r \cos \rho \cos \sigma \\
R_E = r \sin \rho \cos \sigma \\
R_D = -r \sin \sigma
\]

(6.5)

As shown in Eq. (6.5), the components \( \rho \) and \( \sigma \) indicate the orientation in the inertial frame. Such orientation is called the LOS vector, which is defined as

\[
\text{LOS} = [l_1, l_2, l_3]^T
\]

whose three components are shown as below

\[
l_1 = \cos \rho \cos \sigma \\
l_2 = \sin \rho \cos \sigma \\
l_3 = -\sin \sigma
\]

(6.6)
where \( l_1 \), \( l_2 \) and \( l_3 \) are the three-dimensional normalized coordinates in the NED Cartesian frame. As shown in, the LOS vector is determined by the azimuth angle and the elevation angle of the range vector described in Eq. (6.5), but with the normalized magnitude.

![Figure 7.2 Line-of-sight vector and velocity vector.](image)

On the other hand, the aircraft inertial velocity \( \mathbf{V} = [\dot{x}, \dot{y}, \dot{z}]^T \) vector is represented using the flight course components as

\[
\begin{align*}
\dot{x} &= V_r \cos \chi \cos \gamma \\
\dot{y} &= V_r \sin \chi \cos \gamma \\
\dot{z} &= -V_r \sin \gamma
\end{align*}
\]

(6.7)
where the $\chi$ is the heading angle and $\gamma$ is the flight-path angle, respectively; $V$ is the inertial vehicle velocity magnitude. Figure 7.2 also shows the vehicle vector and its flight-path components in the inertial frame.

The PPG trajectory design is to align the vehicle velocity vector with the LOS by steering and accelerating the vehicle. The Restoration Trajectory block in Figure 7.1 can be expanded as shown in Figure 7.3, in which the first two channels are used for orientation regulation, and the third channel is dedicated to velocity control according to the distance and the target velocity, which are described below.

Figure 7.3 Block diagram of the close-in design.

### 7.3.2.1 Orientation Control

As shown in Figure 7.2, vehicle’s velocity vector is regulated to point the target direction indicated by LOS in Eq. (6.6). Therefore, the azimuth and elevation angles $\rho$ and $\sigma$ that determine the LOS are employed as the guidance commands for the close-in orientation design

$$\chi_{\text{gui}} = \rho, \quad \gamma_{\text{gui}} = \sigma$$

(6.8)

and the following PID control laws are designed as:
where $\chi_{err} = \chi_{g} - \chi_{s}$ and $\gamma_{err} = \gamma_{g} - \gamma_{s}$ are the feedback tracking errors for the heading angle channel and the flight-path angle channel, respectively; $\chi_{s}$ and $\gamma_{s}$ are the sensed vehicle heading angle and flight-path angle, respectively; and the $\chi_{rstr}$ and $\gamma_{rstr}$ are the feedback control variables for the close-in trajectory generation.

### 7.3.2.2 Velocity Control

The velocity command comprises two sources: the distance $r$ and the inertial speed of the virtual target $V_{tgt}$ [84]. At the end of the restoration, the vehicle velocity should be the same as the target speed $V_{tgt}$. However, if the distance $r$ is relatively large, the guidance law should provide a maximum allowable speed for the guidance trajectory to rapidly converge to the target. Additional proportional acceleration is provided at the range of $r > 0$. The velocity control is specified as

$$\dot{V}_{rstr} = K_V \left( V_{tgt} - V_{rstr} \right) + K_r r$$

(6.10)

where the inertial speed tracking error is obtained as $V_{tgt} - V_{rstr}$; $K_V$ and $K_r$ determine the relative distance feedback gain and the velocity feedback gain that contribute to the $V_{rstr}$, respectively. It is noted that the velocity speed should be bounded by the maximum achievable speed $V_{max}$, which represents the physical capability of the vehicle to chase the target, therefore,
Then the Velocity Control block in Figure 7.3 can be expanded as follows for the principle illustration and implementation as in shown in Figure 7.4.

\[
V = \begin{cases} 
V_{\text{rstr}}, & V_{\text{rstr}} < V_{\text{max}} \\
V_{\text{max}}, & V_{\text{rstr}} \geq V_{\text{max}} 
\end{cases}
\] (6.11)

7.3.2.3 Restoration Trajectory Generation

By the orientation \( \chi_{\text{rstr}} \) and \( \gamma_{\text{rstr}} \), and the velocity control \( V_{\text{rstr}} \) designs above, the guidance trajectory for mission restoration can be obtained by integrating the following inertial velocity in Cartesian coordinate as

\[
\begin{align*}
\dot{x}_{\text{rstr}} &= V_{t,\text{rstr}} \cos \chi_{\text{rstr}} \cos \gamma_{\text{rstr}} \\
\dot{y}_{\text{rstr}} &= V_{t,\text{rstr}} \sin \chi_{\text{rstr}} \cos \gamma_{\text{rstr}} \\
\dot{z}_{\text{rstr}} &= V_{t,\text{rstr}} \sin \gamma_{\text{rstr}}
\end{align*}
\] (6.12)

The restoration trajectory \( P_{\text{rstr}} = [x_{\text{rstr}} \quad y_{\text{rstr}} \quad z_{\text{rstr}}]^T \) in Eq. (6.12) is then planned for the nominal controller to track. When the position tracking error between the vehicle and the target converges to within the boundaries indicated by the capture range box defined in Eq. (6.13), the close-in control is finished.
\[
N = \{ R : R_N \in [-1,1], R_E \in [-1,1], R_D = [-1,1] \} 
\]

(6.13)

7.3.3 Home-in Control

After the capture box above is satisfied, the multi-modal controller is configured by switching the flag from value 2 to 0 as shown in Table 7.1, such configuration indicates the mission trajectory \( P_{tgt} \) is restored. This section presents the second phase of mission restoration called home-in, which is designed for restoring the original tracking precision.

For the nominal controller design in Chapter 3, the TLC 6DOF flight controller comprises four loops, and each loop has three channels, corresponding to the 6DOF in four loops. Recall the 12 second-order linear time-varying closed-loop tracking error dynamics for the nominal controller in Chapter 3 as

\[
\ddot{x}_{ij} + \alpha_{ij2}(t)\dot{x}_{ij} + \alpha_{ij1}(t)x_{ij} = 0, \quad i = 1, 2, 3, 4, \quad j = 1, 2, 3 \tag{6.14}
\]

are synthesized using the constant damping time-varying PD-eigenvalues as

\[
\rho_{ij}(t) = \left(-\zeta_{ij} \pm j\sqrt{1 - \zeta_{ij}^2}\right)\omega_{n,ij}(t) \\
i = 1, 2, 3, 4, \quad j = 1, 2, 3 \tag{6.15}
\]

By the PD-spectral synthesis formula

\[
\alpha_{ij1}(t) = \omega_{n,ij}^2(t), \quad \alpha_{ij2}(t) = 2\zeta_{ij}\omega_{n,ij}(t) - \frac{\dot{\omega}_{n,ij}(t)}{\omega_{n,ij}(t)} \\
i = 1, 2, 3, 4, \quad j = 1, 2, 3 \tag{6.16}
\]

where the index \( i \) is the loop number counting from outer loop to inner loop and \( j \) is the channel number; \( \omega_{n,ij}(t) \) are the time-varying natural frequencies and \( \zeta_{ij} \) are the constant...
damping ratios of the desired closed-loop dynamics for each state variable $x_{ij}$. Here we define $\omega_{n,ij}(t) = \omega_{n,\text{nom},ij}$ to be the nominal controller synthesis parameter for desired tracking performance and reasonable robustness.

However, the system tracking performance needs a transition to be restored since there is still an amount of tracking error which is beyond the capability of the nominal tracking precision indicated by $\omega_{n,\text{nom},ij}$, which is provided by desired tracking performance and reasonable system robustness. In this case, the robustness of the system needs to be increased to guarantee the system stability in order to accommodate the existing tracking error, then gradually restore the desired tracking performance. In order to trade off between the tracking performance and the system robustness, a single-parameter adaptation law is designed as

$$\omega_{n,ij}(t) = k_a(t)\omega_{n,\text{nom},ij}, \quad i = 1,2,3,4, \quad j = 1,2,3$$  \hspace{1cm} (6.17)

where $k_a(t)$ is the time-varying gain to adapt the nominal bandwidth. Table 7.2 shows the behaviors of the tracking performance and the robustness, which are controlled by the bandwidth adaptation law in Eq. (6.17). For instance, a minimum adaptation gain will induce a minimum bandwidth according to Table 7.2; however, a maximum system robustness is obtained, and a maximum tracking performance in terms of the infinity norm of the tracking error $\|x_{err}\|_{\text{inf}}$ can be coped with.
Table 7.2 Bandwidth adaptation law.

<table>
<thead>
<tr>
<th>Adaptation Gain</th>
<th>Bandwidth</th>
<th>Robustness</th>
<th>Tracking Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \min k_a(t) )</td>
<td>( \omega_{n,j,\min} )</td>
<td>( \max )</td>
<td>( \max | x_{err} |_{\infty} )</td>
</tr>
<tr>
<td>( \text{nom} k_a(t) )</td>
<td>( \omega_{n,j,\text{nom}} )</td>
<td>( \text{nom} )</td>
<td>( \text{nom} | x_{err} |_{\infty} )</td>
</tr>
<tr>
<td>( \max k_a(t) )</td>
<td>( \omega_{n,j,\max} )</td>
<td>( \min )</td>
<td>( \min | x_{err} |_{\infty} )</td>
</tr>
</tbody>
</table>

Based on the bandwidth adaptation philosophy shown in Figure 7.5, a bandwidth adaptation law is augmented to the nominal controller according to Eq. (6.17) for a smooth transition to the nominal configuration. As shown in Figure 7.5, the system bandwidth of the 6DOF nominal controller is adapted by \( k_a,\text{rstr} \), which is a moderate value in \((0,1)\) to ensure an adequate robustness and tracking performance for the close-in sub-mode. Such bandwidth adaptation gain is also the initial value for time-varying home-in gain \( k_{a,\text{homein}}(t_0) = k_a,\text{rstr} \), where \( t_0 \) indicate the moment that the system configuration switches to the home-in sub-mode. During the home-in sub-mode, the adaptation gain increases gradually in order to increase the tracking performance. Once \( k_{a,\text{homein}}(t) \) is increased to the value 1 at \( t_1 \), the nominal mission is restored completely. The system under the application of its tuned-up bandwidth for the trajectory tracking mission.
Figure 7.5  Bandwidth adaptation law for close-in, home-in and nominal.

7.4. Simulation Results

The LOC investigation platform implemented in Section 3.4 is revised for the proposed mission restoration application. As shown in Figure 7.6, a Restoration Trajectory block is used for the designed restoration trajectory output. And the 6DOF nominal controller designed in Chapter 3 is employed.

Figure 7.6 The Matlab/Simulink platform for mission restoration.
The PID gains of the close-in orientation guidance as given in Table 7.3 are tuned for the presented performance. The derivative control for both of the $\chi_{rstr}$ and $\gamma_{rstr}$ are implemented using a second-order pseudo-differentiator represented by the following transfer function

$$D(s) = \frac{\omega_n^2 s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(6.18)

where the parameter are chosen as $\omega_n = 0.2$ rad, $\zeta = 2$. The velocity control gain is also listed in Table 7.3.

<table>
<thead>
<tr>
<th>$K_{p_{\chi}}$</th>
<th>$K_{i_{\chi}}$</th>
<th>$K_{d_{\chi}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>0.011</td>
<td>0.3</td>
</tr>
<tr>
<td>$K_{p_{\gamma}}$</td>
<td>$K_{i_{\gamma}}$</td>
<td>$K_{d_{\gamma}}$</td>
</tr>
<tr>
<td>0.001</td>
<td>0.0001</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_v$</td>
<td>$K_r$</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

The mission trajectory is generated with an initial position

$P_{tgt0} = [1000 \quad 10 \quad -1520]^T$ with $V_{tgt} = 50$ m/s flying straight and level to the North. The initial vehicle position is $P_0 = [0 \quad 0 \quad -1500]^T$.

The simulation results are shown in Figure 7.7- Figure 7.15. Starting with a significant for all three directions in NED as $R = [100 \quad 10 \quad -20]^T$, the mission restoration mode is engaged according to the supervisory control value $flag = 3$ (in Figure 7.8). It is noticed that the bandwidth adaptation gain is $k_{a,rstr} = 0.6$, which provides a more robust system for the close-in control. As shown in Figure 7.9, at the beginning of restoration, the
azimuth angle $\rho$ and elevation angle $\sigma$ are slowly changing due to the relatively large distance between the vehicle and the target. After the finish of the closed-in, azimuth angle $\rho$ and elevation angle $\sigma$ are simply set to zero for the home-in sub-mode. As the vehicle is approaching the target, the heading angle and flight-path angle are regulated by the close-in control design to track the azimuth and elevation angle of the LOS, respectively as shown in Figure 7.11. The heading and flight-path angle errors are converging to very small value. The velocity increases rapidly to 55 m/s and then goes back to the target speed (50 m/s). In Figure 7.10, the tracking error is reducing by the restoration trajectory close-in control.

Once the position tracking error between the vehicle and the target (range vector) is reduced to within the boundaries indicated by Eq. (6.13), the switching flag is set to be 0 at $t = 68.79$ s (Figure 7.8), at which time, the home-in control starts with an initial bandwidth adaptation gain to be $k_a = 0.6$ in Figure 7.14. The controller is reconfigured accordingly, which indicates the mission trajectory is tracked instead of the restoration one. A short-term effectors saturations occur at the time of the reconfiguration at $t = 68.79$ s for thrust, aileron, elevator and rudder (Figure 7.13). However, the stability is maintained by the sufficient robustness provided by the bandwidth adaptation law. The bandwidth adaptation gain is ramping up till $k_a = 1$ is achieved at $t = 105$ s in Figure 7.14. Thereafter, the mission trajectory is completely restored with desired tracking performance. Figure 7.15 shows the 3-dimentional inertial position tracking performance, in which the “restoration track” is generated by the close-in guidance up to $t = 69.1$ s, and thereafter it is abandoned when the home in guidance takes over. The mission trajectory is restored eventually.
Figure 7.7 Range vector components.

Figure 7.8 Switching flag value.
Figure 7.9 Azimuth and elevation angles.

Figure 7.10 Inertial position tracking performance.
Figure 7.11 Heading angle, flight-path angle and vehicle speed.

Figure 7.12 Thrust coefficients.
Figure 7.13 Aileron, elevator, and rudder deflection, respectively.

Figure 7.14 Bandwidth adaptation gain.
If we further zoom in Figure 7.13 at the moment of switching (flag from 2 to 0) as shown in Figure 7.16, there exist fast transients for the actuator deflections after the moment of switching from the close-up mode to the home-in sub-mode.
As it is shown in Figure 7.16, at the moment \( t = 68.79 \text{ s} \) that the tracking trajectory is switched from the \( \mathbf{P}_{\text{str}} \) to \( \mathbf{P}_{\text{tgt}} \), since the capture box defined in Eq. (6.13) allows a maximum 1m position tracking error in the all three directions, such tracking error induced a transients for the aileron, elevator and rudder. Such transient can be further reduced by adding a trajectory smooth filter.

7.5. Conclusion

A complete mission restoration mode is achieved by designing i) a close-in sub-mode controller for guiding the vehicle to close-in onto the target by pure pursuit guidance, and ii) a home in the sub-mode controller for restoring the mission by bandwidth adaptation.
to minimize the transient at the guidance trajectory switching. By proper design of the
restoration mode, simulation results demonstrate that the proposed design method is
capable of directing the vehicle back to the mission trajectory in the presence of a relatively
large position tracking error; and restoring the original trajectory tracking mission with
desired tracking precision.

The pure pursuit restoration guidance is a closed-loop guidance, and currently the
PID controllers for the guidance errors are tuned heuristically. It is of theoretical as well as
practical importance to develop a proof of stability for our particular design. In particular,
the current guidance law is only applicable for small misalignment angles between the LOS
and the vehicle velocity vector, which means that at the end of the upset arrest, the vehicle
must be behind the target position with substantial distance, and the vertical and horizontal
deviation from the mission trajectory must be relatively small. An effective nonlinear
instead of the linear PID, guidance error regulator could significantly expand the
applicability of the current restoration controller.
CHAPTER 8 A HYBRID AUTOMATIC FLIGHT MANAGEMENT SYSTEM FOR LOC PREVENTION AND RECOVERY

In this chapter, the nominal mode controller, LOC prevention mode controller, LOC arrest mode controller and mission restoration mode controller developed in Chapters 2-7 are integrated within the iLOCPR architecture under a supervisory Discrete-Event-Driven (DED) Automatic Flight Management System (AFMS). In order to avoid confusions between a discrete-event state and a dynamic state, herein we will use the term mode for the former. The details of the AFMS design and the software implementation are presented. Simulation results demonstrate the effectiveness and performance of the proposed design.

This chapter is organized as follows. The architecture of AFMS, as the supervisory control of the proposed iLOCPR is described in Section 8.1. The details of the proposed iLOCPR finite-state machine design are elaborated in Section 8.2, including threshold quantization of the dynamic states as the input to the state machine, definition of the discrete modes, mode transition logic, and the output of the state machine, which equals to the current mode in this design. The proposed AFMS is implemented by Matlab/Stateflow. In Section 0, the integration of the finite-state machine with the multi-modal time-driven dynamical controller configuration is presented, including the switching of the dynamical controllers by the finite-state machine output, and the initialization of dynamic states of the dynamic controllers at the switching time in order to minimize the disruptive transient behaviors. Section 8.4 summarizes the state equations that constitute the overall multi-modal iLOCPR control algorithm, and the initialization algorithm of the controller.
dynamics at each discrete mode switching time. In Section 8.5, a longitudinal stall LOC scenario is tested by simulation to demonstrate the effectiveness and performance of the integrated iLOCPR system for LOC prevention, recovery and mission restoration. Section 8.6 concludes this chapter with a summary of the results and contributions, as well as suggestions for further research and development.

8.1. Automatic Flight Management System Architecture

The proposed integrated LOC prevention and recovery automatic control system scheme is shown in Figure 8.1. The system is characterized by a set of operating modes including the nominal mode, the prevention mode, the LOC arrest mode and the restoration mode. The functions of these modes are: (i) a baseline flight controller for 6DOF trajectory tracking as the nominal mode designed by TLC, (ii) a bandwidth adaption augmentation to the baseline controller for LOC prevention mode using the time-varying PD-eigenvalues to trade tracking performance for increased stability margin and robustness in the presence of LOC-prone flight conditions, (iii) a controller reconfiguration for LOC arrest mode by switching from the trajectory tracking task to the aerodynamic angle tracking in order to recover and maintain healthy flight conditions at the cost of temporarily abandoning the mission trajectory, (iv) a guidance trajectory designer for mission restoration mode after the successful arrest of a LOC upset.

Designed on the top level, the supervisory controller Automatic Flight Management System (AFMS) has the switching logic variable flag as one of its output, whose value is set according to the real-time flight states. Each of the control modes corresponds to one flag state in the hybrid system finite state machine and can be transferred automatically
from one to another under specific conditions. Another output of the AFMS is the bandwidth adaptation gain $k_a$, which is designed for the bandwidth augmentation under the specific control objectives for each mode. The bandwidth adaptation block in Figure 8.1 is designed based on the PD-eigenvalue theory, where the bandwidth of the MNL system can be adapted for the tradeoff between the tracking performance and the system robustness as illustrated in Chapter 5.

![Diagram](image)

**Figure 8.1** The integrated automatic flight controller for fixed-wing aircraft Loss-of-Control Prevention and Recovery (iLOCPR) system diagram.

The AFMS is acting as a supervisory control which is capable of decision making and coordination of the different control modes according to the situations of the aircraft. Such supervisory control design can be characterized as a Discrete-Event-Driven (DED) hybrid system in which the time-driven control modes are coordinated by a finite-state machine in the AFMS based on discrete events and a set of prescribed logical predicates.
When the flight is in the normal flight condition, the *nominal mode* is engaged by the supervisory control. When if there is an impending upset, *LOC prevention mode* is triggered by the satisfaction of the transition condition from the nominal mode to the LOC prevention mode. When if an upset occurs after the exceedance of the LOC prevention capability, *LOC arrest mode* is engaged. The *mission restoration mode* is triggered after a normal flight condition has recovered after a LOC. The AFMS must not only choose the appropriate control modes based on the LOC events, but also properly initialize the time-driven dynamic so as to minimize the dynamic transient.

8.2. Discrete-Event-Driven Finite-State Machine

As we can see from Figure 8.1, the AFMS is essentially a finite-state machine driven by discrete events. The design of the finite-state machine entails specifying on the threshold boxes for the flight mode determination and the state transition predicate. The finite-state machine for the AFMS design is implemented in the Matlab/Stateflow.

8.2.1 Quantification of the Discrete Flight Modes

Since an upset is the direct consequence of aerodynamic attitude exceedance of the normal flight envelope, the aerodynamic attitude is essential as a LOC indicator. In addition, angular rates also constitute a LOC indicator, as they must be kept within a certain range to prevent the wings and control surfaces from inducing unbalanced stall induced by abnormal airflow. Another indicator is the airspeed, as stall will occur below the critical airspeed. Therefore, the vehicle safety can be described by three sets of thresholds, which consist of the extremities of aerodynamic attitude, angular rates, and airspeed as shown in Figure 8.2.
In Figure 8.2, the operation box is defined by the thresholds as

\[ O_1 = \{ \mathbf{\Lambda} : \alpha \in [-2^\circ, 5^\circ], \ \beta \in [-5^\circ, 5^\circ], \ \mu = [-20^\circ, 20^\circ] \} \]
\[ O_2 = \{ \mathbf{\Omega} : p, q, r \in [-20 \text{deg/sec}, 20 \text{deg/sec}] \} \]
\[ O_3 = \{ V_t \in [1.5V_{\text{stall}}, V_{t_{\text{max}}} \} \] \hspace{1cm} (7.1)

which bound the healthy flight state variables under the nominal mode. The protection box defines a buffer (hysteresis) zone between the nominal and LOC arrest mode up to the maximum allowable flight conditions as shown in Eq. (7.2) as

\[ P_1 = \{ \mathbf{\Lambda} : \alpha \in 0.6 \times [-5^\circ, 15^\circ], \ \beta \in 0.6 \times [-10^\circ, 10^\circ], \ \mu = 0.6 \times [-45^\circ, 45^\circ] \} \]
\[ P_2 = \{ \mathbf{\Omega} : p, q, r \in 0.6 \times [-60 \text{deg/sec}, 60 \text{deg/sec}] \} \]
\[ P_3 = \{ V_t \in 0.6 \times [V_{\text{stall}}, V_{t_{\text{max}}} \} \] \hspace{1cm} (7.2)

The safety box is defined by the extremities of aerodynamic angles, angular rates and airspeed given in Eq. (7.3) such that LOC will be declared when any one of them is exceeded.

\[ S_1 = \{ \mathbf{\Lambda} : \alpha \in [-5^\circ, 15^\circ], \ \beta \in [-10^\circ, 10^\circ], \ \mu = [-45^\circ, 45^\circ] \} \]
\[ S_2 = \{ \mathbf{\Omega} : p, q, r \in [-60 \text{deg/sec}, 60 \text{deg/sec}] \} \]
\[ S_3 = \{ V_t \in [V_{\text{stall}}, V_{t_{\text{max}}} \} \] \hspace{1cm} (7.3)

It is noted that the specific values of aerodynamic angles, angular rates, and airspeed used in Eq. (7.1)-Eq. (7.3) to define the flight conditions are exemplify based on typical the wind-tunnel flight data and the LOC study [5]. Also, a small range (position error between the target and the vehicle) box for the home-in sub-mode and a large range box for the restoration mode are defined respectively.
In practice, these threshold boxes should be defined on a case-by-case basis for the specific aircraft model.

\[
\begin{align*}
R_{\text{small}} &= \{ \mathbf{R} : \sqrt{R_N^2 + R_E^2 + R_D^2} \leq 1 \} \\
R_{\text{med}} &= \{ \mathbf{R} : 1 < \sqrt{R_N^2 + R_E^2 + R_D^2} \leq 50 \} \\
R_{\text{large}} &= \{ \mathbf{R} : \sqrt{R_N^2 + R_E^2 + R_D^2} > 50 \}
\end{align*}
\] (7.4)

Figure 8.2 Vehicle health condition threshold boxes.

where \( x_i, i = 1, 2, 3 \) represent the flight states vector \( \Omega \), vector \( \Lambda \), scalar \( V_i \) and \( \text{range} \).

The supervisory control variable \( \text{flag} \) then is determined by

\[
\text{flag} = \begin{cases} 
0, & \Lambda \in O_1 \ \text{AND} \ \Omega \in O_2 \ \text{AND} \ V_i \in O_3 \ \text{AND} \ \text{range} \in R_{\text{small}} \\
1, & \Lambda \in P_1 \ \setminus O_1 \ \text{AND} \ \Omega \in P_2 \ \setminus O_1 \ \text{AND} \ V_i \in P_3 \ \setminus O_3 \\
2, & \Lambda \notin S_1 \ \text{OR} \ \Omega \notin S_2 \ \text{OR} \ V_i \notin S_3 \\
3, & \text{AND} \ \Lambda \in O_1 \ \text{AND} \ \Omega \in O_2 \ \text{AND} \ V_i \in O_3 \ \text{AND} \ \text{range} \in R_{\text{large}} 
\end{cases}
\] (7.5)
in which the $\text{flag} = 0$ value is set as long as the protection box defined in Eq. (7.1) is satisfied. The corresponding state for $\text{flag} = 0$ is the nominal mode. If any flight condition exceeds the operation box, while all of the indicating flight conditions still stay in the protection box defined in Eq. (7.2), $\text{flag} = 1$, and the LOC prevention mode is engaged. If any of the flight conditions exceeds the safety box defined in Eq. (7.3), $\text{flag} = 2$, which indicates the engagement of the LOC arrest mode. When all the flight conditions return to inside of the operation box in Eq. (7.1) and large position error $R_{\text{large}}$ occurs by Eq. (7.4), $\text{flag} = 3$ is set for the engagement of the mission restoration mode. When the $R_{\text{small}}$ is recovered and all the flight states are within the operation box, the nominal mode is reset with $\text{flag} = 0$.

8.2.2 Discrete Mode Transition Conditions

The AFMS is designed as a Moore finite-state machine. The interrelationships among each discrete mode are captured in the state transition diagram in Figure 8.3.
Figure 8.3 Finite-state machine design diagram.

Table 8.1 The transition and condition for the finite-state machine.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Exceed the protection box.</td>
</tr>
<tr>
<td>$b$</td>
<td>Inside the operation box; small range error.</td>
</tr>
<tr>
<td>$c$</td>
<td>Inside the safety box; large range error.</td>
</tr>
<tr>
<td>$d$</td>
<td>Exceed the safety box.</td>
</tr>
<tr>
<td>$e$</td>
<td>Inside the operation box.</td>
</tr>
<tr>
<td>$f$</td>
<td>Exceed the safety box.</td>
</tr>
<tr>
<td>$g$</td>
<td>Inside the operation Box; small range error.</td>
</tr>
</tbody>
</table>

Transition $a$: Nominal → Prevention

The *nominal* mode ($flag = 0$) is the default mode when the system is initialized. The LOC *prevention* mode ($flag = 1$) will not be turned on until the protection box is exceeded, in which case, the transition $a$ is made and the LOC prevention mode controller is enabled.
Transition $b$: Prevention $\rightarrow$ Nominal

By applying the LOC prevention controller, if the operation box is recovered and the range loss (position tracking error) is small, the transition $b$ is triggered to enable the nominal mode controller.

Transition $c$: Prevention $\rightarrow$ Restoration

If the prevention process causes a large range loss, the transition $c$ is triggered to enable the restoration mode controller.

Transition $d$: Prevention $\rightarrow$ Arrest

LOC is declared when the safety box is exceeded. And at that time, the transition $d$ is triggered to engage the arrest mode controller ($flag = 2$).

Transition $e$: Arrest $\rightarrow$ Restoration:

On the other hand, when the operation box is successfully recovered by the arrest controller, the transition $e$ is made to trigger the restoration mode ($flag = 3$). A large range error occurs at this time point.

Transition $f$: Restoration $\rightarrow$ Arrest

During the restoration mode, if the safety box is exceeded again, the arrest mode controller is triggered by the transition $f$.

Transition $g$: Restoration $\rightarrow$ Nominal:

After recovering the operation box, and range loss (position tracking error) is reduced by the restoration mode controller to within the $R_{small}$ set given in Eq. (7.4), the transition $g$ is made. The nominal mode is restored.
8.2.3 Matlab/Stateflow Implementation

The Matlab/Stateflow blockset provides the capability of implementing a finite state machine within a Simulink model, which includes Simulink blocks, Toolbox blocks and the Stateflow blocks. One of the Stateflow blocks that functions as a finite-state machine is the Stateflow chart, which contains the sequential decision logic based on the state machine. For the finite-state machine designed as in Figure 8.3 and Table 8.1, the Matlab/Simulink StateFlow chart is employed for the implementation as shown in Figure 8.4.

Figure 8.4 Matlab/Stateflow implementation.

Figure 8.4 shows the designed Stateflow chart including the input and the output ports that are used to design the supervisory controller within a simulation model. The
output of the AFMS finite-state-machine are flag and \(k_x\), which are utilized to configure the multi-modal controller. Table 8.2 shows the finite state machine transition conditions.

### Table 8.2 Finite-state machine transition condition implementation.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>(\alpha &gt; 0.6 \times 15^\circ \</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>(</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>(\alpha &lt; 15^\circ \ &amp;&amp;</td>
</tr>
<tr>
<td><strong>d</strong></td>
<td>(\alpha &gt; 15^\circ \</td>
</tr>
<tr>
<td><strong>e</strong></td>
<td>(</td>
</tr>
<tr>
<td><strong>f</strong></td>
<td>(\alpha &gt; 15^\circ \</td>
</tr>
<tr>
<td><strong>g</strong></td>
<td>(</td>
</tr>
</tbody>
</table>

### 8.3. Multi-Modal Controller Integration

The multi-modal controller design diagram is shown in Figure 8.5. The nominal mode is designed in Chapter 3, which is to execute the mission trajectory \(P_{tgt}\) tracking task.
The prevention mode is designed by a bandwidth adaptation augmentation to the nominal mode to trade off tracking performance with increased stability margin and robustness in the presence of LOC-prone flight conditions in Chapter 5. In Chapter 6, the LOC arrest mode is designed to switch from the mission trajectory tracking task to aerodynamic attitude trajectory tracking task for LOC arrest in order to recover and maintain healthy flight condition at the cost of temporarily abandoning the mission trajectory. In the restoration mode design in Chapter 7, a guidance trajectory $P_{\text{rstr}}$ is generated to direct the vehicle back to the mission trajectory after the successful arrest of a LOC upset and then restore the tracking performance.

Figure 8.5 Multi-modal controller diagram in iLOCPR.

Figure 8.5 shows the multi-modal controller configuration including the supervisory control flag value for configuring each mode, the bandwidth adaptation gain $k_a$ designed for each mode, the command trajectory, controller, feedback variables, and controller outputs. The system configuration can also be described as shown in Table 8.3.
Table 8.3 Multi-modal controller configuration.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Nominal</th>
<th>Prevention</th>
<th>Arrest</th>
<th>Restoration</th>
</tr>
</thead>
<tbody>
<tr>
<td>flag</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$k_a$</td>
<td>1</td>
<td>$k_a,\text{prev}$</td>
<td>$k_a,\text{arst}$</td>
<td>$k_a,\text{rstr}$</td>
</tr>
<tr>
<td>Trajectory</td>
<td>$P_{tgt}$</td>
<td>$P_{tgt}$</td>
<td>$\Lambda_{\text{arst}}$</td>
<td>$P_{\text{rstr}}$</td>
</tr>
<tr>
<td>Controller</td>
<td>Baseline</td>
<td>Baseline</td>
<td>Arrest</td>
<td>Baseline</td>
</tr>
<tr>
<td>Feedback</td>
<td>$P_{\text{sen}}, V_{\text{sen}}, \Gamma_{\text{sen}}, \Omega_{\text{sen}}$</td>
<td>$P_{\text{sen}}, V_{\text{sen}}, \Gamma_{\text{sen}}, \Omega_{\text{sen}}$</td>
<td>$\Delta_{\text{arst}}, \delta_{r,\text{arst}}$</td>
<td>$P_{\text{sen}}, V_{\text{sen}}, \Gamma_{\text{sen}}, \Omega_{\text{sen}}$</td>
</tr>
<tr>
<td>Controller Output</td>
<td>$\Delta_{\text{com}}, \delta_{r,\text{com}}$</td>
<td>$\Delta_{\text{com}}, \delta_{r,\text{com}}$</td>
<td>$\Delta_{\text{com}}, \delta_{r,\text{com}}$</td>
<td>$\Delta_{\text{com}}, \delta_{r,\text{com}}$</td>
</tr>
</tbody>
</table>

The multi-modal controller in Figure 8.5 is expanded in Figure 8.6 to show the details how each mode is configured in accordance with the value of the supervisory switching signal $\text{flag}$.
Figure 8.6 Multi-modal controller configuration.
8.3.1 Nominal Mode Configuration

As shown in Table 8.3, for nominal mode design, the baseline controller designed in Chapter 3 is engaged to achieve the mission trajectory tracking goal.

8.3.1.1 Supervisory control:

The nominal mode corresponds to the supervisory control \( \text{flag} = 0 \) and \( k_a = 1 \). The nominal controller aims to achieve the desired tracking performance with the tuned nominal bandwidth parameters shown in Table 5.1 without the bandwidth adaptation.

8.3.1.2 Baseline Controller Engagement:

When the baseline controller is engaged, the mission trajectory is selected to be the control objective under nominal mode by applying \( \mathbf{P}_{\text{com}} = \mathbf{P}_{\text{tgt}} \). The feedback states \( \mathbf{P}_{\text{sen}}, \mathbf{V}_{\text{sen}}, \Gamma_{\text{sen}}, \Omega_{\text{sen}} \) are utilized to calculate the feedback stabilizing TLC gain matrices. And the corresponding command trajectories for the baseline controller are configured as

\[
\begin{align*}
V_{\text{com}} &= V_{\text{nom}} + V_{\text{ctrl}}, \\
\Omega_{\text{com1}} &= \Omega_{\text{nom1}} + \Omega_{\text{ctrl1}}, \\
T_{m,\text{com1}} &= T_{m,\text{nom1}} + T_{m,\text{ctrl1}}
\end{align*}
\]  

(7.6)

where the command variables are obtained by adding the closed-loop PI feedback tracking error control with subscript “ctrl” to the nominal control with subscript “nom” from dynamic pseudo-inversion as shown in Figure 8.6. In this case, the controller outputs \( [\Delta_{\text{com}}, \delta_{\tau,\text{com}}] \) are selected.

8.3.1.3 Arrest Controller Disengagement:

Meanwhile, the arrest controller is on standby by setting the aerodynamic attitude tracking command and the ensuing body rate command equal to the sensed flight states as
so that the integrators in the arrest mode controller are put on hold by setting zero tracking errors as \( \Lambda_{\text{err}} = \Lambda_{\text{sen}} - \Lambda_{\text{com}} = 0 \), \( \Omega_{\text{err},2} = \Omega_{\text{sen}} - \Omega_{\text{com},2} = 0 \).

8.3.1.4 The initial value setting for the integrators in the controller.

The default controller mode is the nominal mode. The initial values of the integrators of the baseline controller \( \mathbf{P}_{\text{ini}}, \mathbf{V}_{\text{ini}}, \Gamma_{\text{ini}}, \Omega_{\text{ini}} \) and the arrest controller \( \Lambda_{\text{ini}}, \Omega_{\text{ini},2} \) are obtained by a trim flight condition.

8.3.2 LOC Prevention Mode Configuration

As shown in Table 8.3, the baseline controller is also employed for LOC prevention mode controller.

8.3.2.1 Supervisory control:

The prevention mode is in accordance with the supervisory control law as the flag = 1. The LOC prevention adaptation law \( k(t)_{a,\text{prev}} \) developed in Section 5.2 applied to the baseline controller in order to obtain a tradeoff between the tracking performance and the system robustness.

8.3.2.2 Baseline Controller Engagement:

The baseline controller is engaged. The mission trajectory, feedback states, command variables for each loop in Eq. (7.6), controller outputs are set to be the same as the ones for the nominal mode design as shown in 8.3.1.

8.3.2.3 Arrest Controller Disengagement:

The arrest controller remains on standby by Eq. (7.7).
8.3.2.4 The initial value setting for the integrators in the controller.

In the prevention mode design, all the state values for the integrator are inherited from the nominal mode operation. Integrators of the arrest controller are storing the feedback states from the sensing system in preparation for a LOC.

8.3.3 LOC Arrest Mode Configuration

As shown in Table 8.3, the baseline controller is disengaged for LOC arrest mode, while the arrest controller is armed.

8.3.3.1 Supervisory control:

Once LOC is detected, the supervisory control is set as \( \text{flag} = 2 \). The bandwidth adaptation \( k(t)_{\text{a,arst}} \) is applied for different LOC conditions.

8.3.3.2 Baseline Controller Disengagement:

The nominal mode controller is disarmed by the following setting

\[
P_{\text{com}} = P_{\text{sen}}, \quad V_{\text{com}} = V_{\text{sen}}, \quad \Gamma_{\text{com}} = \Gamma_{\text{sen}}, \quad \Omega_{\text{com1}} = \Omega_{\text{sen}}
\]

which implies that all the integrators in the nominal control feedback loop are frozen by taking in zero tracking errors, since

\[
P_{\text{err}} = P_{\text{sen}} - P_{\text{com}} = 0, \quad V_{\text{err}} = V_{\text{sen}} - V_{\text{com}} = 0, \\
\Gamma_{\text{err}} = \Gamma_{\text{sen}} - \Gamma_{\text{com}} = 0, \quad \Omega_{\text{err1}} = \Omega_{\text{sen}} - \Omega_{\text{com1}} = 0
\]

8.3.3.3 Arrest Controller Engagement:

The arrest controller is taking control of the system, in which \( \Lambda_{\text{arst}} \) is the command for arrest the LOC.

\[
\Lambda_{\text{com}} = \Lambda_{\text{arst}}, \quad \Omega_{\text{com2}} = \Omega_{\text{nom2}} + \Omega_{\text{ctrl2}}
\]
The control outputs $[\Delta_{\text{rst}}, \delta_{\text{rst}}]$ are selected.

8.3.3.4 The initial value setting for the integrators in the controller.

The arrest controller is properly initialized by the sensed states of $\Lambda_{\text{sen}}$ and $\Omega_{\text{sen}}$ at the switching time moment so that switching transient or instability caused by excessive inertial tracking error is avoided.

8.3.4 Mission Restoration Mode Configuration

As shown in Table 8.3, the baseline controller is engaged for LOC prevention mode.

8.3.4.1 Supervisory control:

The restoration mode corresponds to the supervisory control $\text{flag} = 3$. The bandwidth adaptation gain $k_{a, \text{rstr}}(t)$ is applied to increase the tracking error tolerance capability.

8.3.4.2 Baseline Controller Disengagement:

The baseline controller is engaged. The mission trajectory is set to $P_{\text{com}} = P_{\text{rstr}}$. Otherwise, all feedback states, command variables for each loop in Eq. (7.6), controller outputs are set to be the same as the ones for the nominal mode design as shown in Section 8.3.1.

8.3.4.3 Arrest Controller Disengagement:

The arrest controller is disarmed by satisfying the Eq. (7.7).

8.3.4.4 The initial value setting for the integrators in the controller.

At the restoration mode triggering moment when $t = t_r$, which is also the end of the arrest even though the position command is set to $P_{\text{com}} = P_{\text{rstr}}$, the internal states of the
controller are nor consistent with those induced by $P_{rst}$. Therefore, the velocity vector of the restoration guidance trajectory $P_{rst}$ is initialized at $V_{sen}(t_r)$ to guide the velocity for 5s, as shown in Eq.(7.11). This phase is called *coast*, which effectively avoids the throttle saturation induced by excessive controller transient.

\[
V_{\text{ref}} = \begin{cases} 
V_{\text{sen}}, & t_{rst} \leq t \leq t_{rst} + 5 \\
V_{\text{tgt}}, & t > t_{rst}
\end{cases}
\]  
\hspace{1cm} (7.11)

After the controller is initialized, vector $V_{\text{tgt}}$ is employed to guide the vehicle by the pure pursuit law given in Eq. (6.9), which allows the LOS tracking to converge first while maintaining a constant range.

Once the LOS errors are sufficiently reduced, the guidance velocity $V_{\text{ref}}$ is increased for the vehicle to close-in onto the target based on the range by

\[
V_r = \begin{cases} 
sat(r-1)k_r, & |v_e| < \delta & \& |z_e| < \delta \\
0, & \text{Otherwise}
\end{cases}
\]  
\hspace{1cm} (7.12)

where $\delta$ is assumed to be significant small and the saturation function on the range is set to of $2 \text{ m/s}$, which indicates the maximum increase in velocity from the in accordance with the range is limited; also, the range induced acceleration is applied only when the range is larger than 1m, therefore, the guidance velocity with decrease to $V_{\text{ref}}=V_{\text{tgt}}$ in preparation for home-in. Therefore, the guidance velocity control considering the mode switching mechanism is modified from the design of Chapter 7 as follows

\[
\dot{V}_{rst} = K_r \left( V_{\text{ref}} + V_r - V_{rst} \right)
\]  
\hspace{1cm} (7.13)
8.4. State Equations and Controller States Summary

The state equations for the baseline controller, arrest controller, and the restoration guidance are summarized in Table 8.4, respectively.

Table 8.4 Table of baseline controller state equations.

<table>
<thead>
<tr>
<th>Baseline Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{com}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$P_{nom}, \dot{P}_{nom}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>GON</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$P_{err}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>GOF</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$V_{com}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$V_{nom}, \dot{V}_{nom}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>GIN</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$V_{err}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>GIF</td>
</tr>
</tbody>
</table>
Table 8.4: continued.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GAN</td>
<td>$\Gamma_{\text{nom}} = f_2(F_{\text{nom}})$</td>
</tr>
<tr>
<td>GAC</td>
<td>$\Gamma_{\text{com}} = f_2(F_{\text{com}})$</td>
</tr>
<tr>
<td>$\Gamma_{\text{nom}}, \dot{\Gamma}_{\text{nom}}$</td>
<td>$\begin{bmatrix} \dot{\Gamma}<em>{\text{nom}} \ \dot{\Gamma}</em>{\text{nom}} \end{bmatrix} = A_{\text{diff,3}} \begin{bmatrix} \Gamma_{\text{nom}} \ \dot{\Gamma}<em>{\text{nom}} \end{bmatrix} + B</em>{\text{diff,3}} \Gamma_{\text{nom}}$</td>
</tr>
<tr>
<td>EON</td>
<td>$\Omega_{\text{nom}} = B_{3}^{-1}(\Gamma_{\text{nom}}) \dot{\Gamma}_{\text{nom}}$</td>
</tr>
<tr>
<td>$\Gamma_{\text{err}}$</td>
<td>$\Gamma_{\text{err}} = \Gamma_{\text{sen}} - \Gamma_{\text{com}}$</td>
</tr>
<tr>
<td>EOF</td>
<td>$\Omega_{\text{ctrl}} = -K_{p3} \Gamma_{\text{err}} - K_{I3} \int_{0}^{t} \Gamma_{\text{err}}(\tau) d\tau$</td>
</tr>
<tr>
<td>$\Omega_{\text{com1}}$</td>
<td>$\Omega_{\text{com1}} = \Omega_{\text{nom1}} + \Omega_{\text{ctrl1}}$</td>
</tr>
<tr>
<td>$\Omega_{\text{nom1}}, \dot{\Omega}_{\text{nom1}}$</td>
<td>$\begin{bmatrix} \dot{\Omega}<em>{\text{nom1}} \ \dot{\Omega}</em>{\text{nom1}} \end{bmatrix} = A_{\text{diff,4}} \begin{bmatrix} \Omega_{\text{nom1}} \ \dot{\Omega}<em>{\text{nom1}} \end{bmatrix} + B</em>{\text{diff,4}} \Omega_{\text{nom1}}$</td>
</tr>
<tr>
<td>EIN</td>
<td>$\begin{bmatrix} \dot{\Omega}<em>{\text{nom1}} - f_4(\Omega</em>{\text{nom1}}) \end{bmatrix}$</td>
</tr>
<tr>
<td>$\Omega_{\text{err1}}$</td>
<td>$\Omega_{\text{err1}} = \Omega_{\text{sen}} - \Omega_{\text{com1}}$</td>
</tr>
<tr>
<td>EIF</td>
<td>$T_{m,\text{ctrl1}} = -K_{p4} \Omega_{\text{err1}} - K_{I4} \int_{f_h}^{t} \Omega_{\text{err1}}(\sigma) d\sigma$</td>
</tr>
<tr>
<td>$T_{m,\text{com1}}$</td>
<td>$T_{m,\text{com1}} = T_{m,\text{nom1}} + T_{m,\text{ctrl1}}$</td>
</tr>
<tr>
<td>EA</td>
<td>$\Delta_{\text{com}} = J^{-1}T_{\text{all1}}$</td>
</tr>
</tbody>
</table>
Table 8.5 Table of arrest controller state equations.

<table>
<thead>
<tr>
<th>Arrest Controller</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_{rstr} )</td>
<td>( \Lambda_{nom} = \Lambda_{rstr} )</td>
</tr>
<tr>
<td>( \Lambda_{nom} ), ( \dot{\Lambda}_{nom} )</td>
<td>( \begin{bmatrix} \dot{\Lambda}<em>{nom} \ \dot{\Lambda}</em>{nom} \end{bmatrix} = A_{diff,5} \begin{bmatrix} \Lambda_{nom} \ \Lambda_{nom} \end{bmatrix} + B_{diff,5} \Lambda_{rstr} )</td>
</tr>
<tr>
<td>( \Lambda_{err} )</td>
<td>( \Lambda_{err} = \Lambda_{sen} - \Lambda_{rstr} )</td>
</tr>
<tr>
<td>AOF</td>
<td>( \Omega_{ctrl2} = -K_{p_d} \Lambda_{err} - K_{i_d} \int_{t_0}^{t} \Lambda_{err} (\tau) , d\tau )</td>
</tr>
<tr>
<td>( \Omega_{com2} )</td>
<td>( \Omega_{com2} = \Omega_{nom2} + \Omega_{ctrl2} )</td>
</tr>
<tr>
<td>( \Omega_{nom2} ), ( \dot{\Omega}_{nom2} )</td>
<td>( \begin{bmatrix} \dot{\Omega}<em>{nom2} \ \dot{\Omega}</em>{nom2} \end{bmatrix} = A_{diff,6} \begin{bmatrix} \Omega_{nom2} \ \Omega_{nom2} \end{bmatrix} + B_{diff,6} \Omega_{nom2} )</td>
</tr>
<tr>
<td>AIN</td>
<td>( T_{m,nom2} = B_{4}^{-1} \left[ \Omega_{nom2} - f_4 (\Omega_{nom2}) \right] )</td>
</tr>
<tr>
<td>( \Omega_{err2} )</td>
<td>( \Omega_{err2} = \Omega_{sen} - \Omega_{com2} )</td>
</tr>
<tr>
<td>AIF</td>
<td>( T_{m,ctrl2} = -K_{p_d} \Omega_{err2} - K_{i_d} \int_{t_0}^{t} \Omega_{err2} (\sigma) , d\sigma )</td>
</tr>
<tr>
<td>( T_{m,com2} )</td>
<td>( T_{m,com2} = T_{m,nom2} + T_{m,ctrl2} )</td>
</tr>
<tr>
<td>AA</td>
<td>( \Delta_{aux} = J^{-1} T_{allo2} )</td>
</tr>
</tbody>
</table>
Table 8.6 Table of restoration guidance state equations.

<table>
<thead>
<tr>
<th><strong>Restoration Trajectory Planner</strong></th>
</tr>
</thead>
</table>
| \( P_{err} \) Spherical coordinate | \[ \begin{align*} 
\rho &= \arctan \left( \frac{R_E}{R_N} \right) \\
\sigma &= -\arcsin \left( \frac{R_H}{r} \right) \\
r &= \sqrt{R_N^2 + R_E^2 + R_D^2} 
\end{align*} \] |
| \( \gamma_{err}, \dot{\gamma}_{err} \) | \[ \begin{bmatrix} \dot{\gamma}_{err} \\ \ddot{\gamma}_{err} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_{n,y}^2 & -2\zeta_j \end{bmatrix} \begin{bmatrix} \gamma_{err} \\ \dot{\gamma}_{err} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{n,y}^2 \end{bmatrix} \gamma_{err} \] |
| \( \chi_{err}, \dot{\chi}_{err} \) | \[ \begin{bmatrix} \dot{\chi}_{err} \\ \ddot{\chi}_{err} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_{n,x}^2 & -2\zeta_j \end{bmatrix} \begin{bmatrix} \chi_{err} \\ \dot{\chi}_{err} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{n,x}^2 \end{bmatrix} \chi_{err} \] |
| **Close-in** | \[ \begin{align*} 
\chi_{rstr} &= K_p \chi_{err} + K_I \int_0^t \chi_{err}(\tau)d\tau + K_D \dot{\chi}_{err} \\
\gamma_{rstr} &= K_p \gamma_{err} + K_I \int_0^t \gamma_{err}(\tau)d\tau + K_D \dot{\gamma}_{err} \\
\dot{V}_{rstr} &= K_V \left( V_{ref} + V_r - V_{rstr} \right) + K_r 
\end{align*} \] |
| \( P_{rstr} \) | \[ \begin{align*} 
\dot{x}_{rstr} &= V_{r,str} \cos \chi_{rstr} \cos \gamma_{rstr} \\
\dot{y}_{rstr} &= V_{r,str} \sin \chi_{rstr} \cos \gamma_{rstr} \\
\dot{z}_{rstr} &= -V_{r,str} \sin \gamma_{rstr} 
\end{align*} \] |

In which the coefficient matrices of the pseudo-differentiator is given as

\[
A_{diff,i} = \begin{bmatrix} O_3 & I_3 \\ A_{21,i} & A_{22,i} \end{bmatrix}, \quad B_{diff,i} = \begin{bmatrix} O_3 \\ -A_{21,i} \end{bmatrix}, \quad i = 1, 2, 3, 4, 5, 6
\]

\[
A_{21,i} = \text{diag} \begin{bmatrix} -\omega_{n,1}^2 & -\omega_{n,2}^2 & -\omega_{n,3}^2 \end{bmatrix}, \quad A_{22,i} = \text{diag} \begin{bmatrix} -2\zeta_j \omega_{n,1} & -2\zeta_j \omega_{n,2} & -2\zeta_j \omega_{n,3} \end{bmatrix}
\]  \hspace{1cm} (7.14)

where \( i \) represents the loop number.
The initial values of all the dynamic states (integrators) used in the controller design are summarized in Table 8.7.

Table 8.7 Integrator initial value setting.

<table>
<thead>
<tr>
<th>Integrator</th>
<th>Nominal</th>
<th>Prevention</th>
<th>Arrest</th>
<th>Restoration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t = 0$</td>
<td>$t = t_p$</td>
<td>$t = t_a$</td>
<td>$t = t_r$</td>
</tr>
<tr>
<td>$\dot{p}_{\text{nom}}$</td>
<td>$V_{\text{trim}}$</td>
<td>$\dot{p}_{\text{nom}}(t_p)$</td>
<td>$V_{\text{sen}}(t_a)$</td>
<td>$V_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$p_{\text{nom}}$</td>
<td>$p_{\text{trim}}$</td>
<td>$p_{\text{nom}}(t_p)$</td>
<td>$p_{\text{sen}}(t_a)$</td>
<td>$p_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$\int p_{\text{err}}d\tau$</td>
<td>0</td>
<td>$\int_0^{t_p} p_{\text{err}}(\tau)d\tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\dot{V}_{\text{nom}}$</td>
<td>$A_{b,\text{trim}}$</td>
<td>$\dot{V}_{\text{nom}}(t_p)$</td>
<td>$A_{b,\text{sen}}(t_a)$</td>
<td>$A_{b,\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$V_{\text{nom}}$</td>
<td>$V_{\text{trim}}$</td>
<td>$V_{\text{nom}}(t_p)$</td>
<td>$V_{\text{sen}}(t_a)$</td>
<td>$V_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$\int V_{\text{err}}d\tau$</td>
<td>0</td>
<td>$\int_0^{t_p} V_{\text{err}}(\tau)d\tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\dot{\Gamma}_{\text{nom}}$</td>
<td>$\Omega_{\text{trim}}$</td>
<td>$\dot{\Gamma}_{\text{nom}}(t_p)$</td>
<td>$\Omega_{\text{sen}}(t_a)$</td>
<td>$\Omega_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$\Gamma_{\text{nom}}$</td>
<td>$\Gamma_{\text{trim}}$</td>
<td>$\Gamma_{\text{nom}}(t_p)$</td>
<td>$\Gamma_{\text{sen}}(t_a)$</td>
<td>$\Gamma_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$\int \Gamma_{\text{err}}d\tau$</td>
<td>0</td>
<td>$\int_0^{t_p} \Gamma_{\text{err}}(\tau)d\tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\dot{\Omega}_{\text{nom1}}$</td>
<td>$\Theta_{\text{trim}}$</td>
<td>$\dot{\Omega}_{\text{nom1}}(t_p)$</td>
<td>$\Theta_{\text{sen}}(t_a)$</td>
<td>$\Theta_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$\Omega_{\text{nom1}}$</td>
<td>$\Omega_{\text{trim}}$</td>
<td>$\Omega_{\text{nom1}}(t_p)$</td>
<td>$\Omega_{\text{sen}}(t_a)$</td>
<td>$\Omega_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$\int \Omega_{\text{err1}}d\tau$</td>
<td>0</td>
<td>$\int_0^{t_p} \Omega_{\text{err1}}(\tau)d\tau$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 8.7: continued.

<table>
<thead>
<tr>
<th>$\dot{\Lambda}_{\text{nom}}$</th>
<th>0</th>
<th>$\dot{\Lambda}_{\text{nom}}(t_p)$</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_{\text{nom}}$</td>
<td>$\Lambda_{\text{sen}}(0)$</td>
<td>$\Lambda_{\text{nom}}(t_p)$</td>
<td>$\Lambda_{\text{arst}}(t_a)$</td>
<td>$\Lambda_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$\int \Lambda_{\text{err}} d\tau$</td>
<td>0</td>
<td>$\int_0^{t_p} \Lambda_{\text{err}}(\tau) d\tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\dot{\Omega}_{\text{nom}}$</td>
<td>0</td>
<td>$\dot{\Omega}_{\text{nom}2}(t_p)$</td>
<td>$\Theta_{\text{arst}}(t_a)$</td>
<td>$\Theta_{\text{es,sen}}(t_r)$</td>
</tr>
<tr>
<td>$\int \Omega_{\text{err}2} d\tau$</td>
<td>0</td>
<td>$\int_0^{t_p} \Omega_{\text{err}2}(\tau) d\tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\int \chi_{\text{err}} d\tau$</td>
<td>0</td>
<td>0</td>
<td>$\int_0^{t_p} \chi_{\text{err}}(\tau) d\tau$</td>
<td>$\int_{t_a}^{t_p} \chi_{\text{err}}(\tau) d\tau$</td>
</tr>
<tr>
<td>$\int \gamma_{\text{err}} d\tau$</td>
<td>0</td>
<td>0</td>
<td>$\int_0^{t_p} \gamma_{\text{err}}(\tau) d\tau$</td>
<td>$\int_{t_a}^{t_p} \gamma_{\text{err}}(\tau) d\tau$</td>
</tr>
<tr>
<td>$V_{\text{rstr}}$</td>
<td>$V_{\text{trim}}$</td>
<td>$V_{\text{sen}}(t_p)$</td>
<td>$V_{\text{sen}}(t_a)$</td>
<td>$V_{\text{sen}}(t_r)$</td>
</tr>
<tr>
<td>$P_{\text{rstr}}$</td>
<td>$P_{\text{trim}}$</td>
<td>$P_{\text{sen}}(t_p)$</td>
<td>$P_{\text{sen}}(t_a)$</td>
<td>$P_{\text{sen}}(t_r)$</td>
</tr>
</tbody>
</table>

8.5. Simulation Results

A simulation scenario is tested by considering the vehicle initially trimmed at a level, and straight flight condition at the initial position of $P_0 = [0 \ 0 \ -1500]^T$, and the flight states are provided in Table D.0.1. The mission trajectory is defined as a level and straight flight course to the North with the ground speed of 50.29 m/s (165 ft/s). An excessive elevator deflection $\Delta \delta_e = 30^\circ$ is introduced at $t = 10s$ as shown in Figure 8.7 in order to induce a LOC.
8.5.1 Nominal Mode

As shown in the Figure 8.8-Figure 8.22, the iLOCPR system is engaged by the default nominal mode at $t = 0$s. In this mode, the flag value is 0 in Figure 8.8, and the system bandwidth is $k_{u,\text{nom}}=1$ in Figure 8.9, and the vehicle is tracking the mission trajectory $P_{tgt}$ as shown in Figure 8.10. All the flight states are continuously changing from the trim condition.

8.5.2 Prevention Mode

At $t = 11.9$s as shown in Figure 8.8, the LOC prevention controller is engaged by flag = 1 so that the system bandwidth is immediately reduced to $k_{u,\text{prev}} = 0.7$ (Figure 8.9) in response to the disruption. The elevator deflection $\delta_e$ is being compensated to about $-5^\circ$. However, the remaining elevator deflection causes the pitch rate $q$ (Figure 8.17) and angle-of-attack $\alpha$ (Figure 8.18) to exceed the safety box, which is indicated by the red dotted line.
in the corresponding figures. In this case, a longitudinal stall occurs as the $V_t$ below the stall speed. In the period from 11.9s to 12.88s, the load factor $n$ goes to an extremely high value of 3.6 (Figure 8.13) because the increased drag (Figure 8.16) induced by the excessive $\alpha$. And both of the dynamic pressure $Q_{\text{sen}}$ in Figure 8.21 and the vehicle speed $V_{\text{sen}}$ in Figure 8.13 decrease during this period.

8.5.3 Arrest Mode

The arrest mode is engaged by $flag = 2$ (Figure 8.8) at $t=12.88s$. And the bandwidth is adapted by $k_{\alpha_{\text{arst}}} = 0.7$ (Figure 8.9). During the arrest, the controller reduces the angle-of-attack $\alpha$ to the arrest command $\alpha_{\text{arst}} = 1^\circ$ as shown in Figure 8.18. System responses are returned to the operation box after $t=26.18s$, which indicates the end of the arrest (Figure 8.8). The vehicle speed $V_{\text{sen}}$ in Figure 8.13 is lost at the beginning of the arrest and is recovered to exceed 50m/s at the end of the arrest. The dynamic pressure $Q_{\text{sen}}$ drops down to the lowest level of 1100 $N/m^2$ (Figure 8.21) according to the change of the vehicle speed $V_{\text{sen}}$. The thrust coefficient $\delta_r = 1$ is set in Figure 8.20, which indicates a full throttle engagement according to the arrest design algorithm. Euler attitude tracking (Figure 8.15) from the nominal controller is given up during the arrest. Instead, the aerodynamic attitude (Figure 8.18) is being tracked by the arrest controller. The range loss after the arrest for the restoration to catch up is $R = P_{\text{tgt}} - P_{\text{sen}} = [123.3 \ 0 \ -22.44]^T m$ as shown in Figure 8.11.

8.5.4 Mission Restoration Mode

The mission restoration mode is engaged according to the supervisory control value $flag = 3$ as shown in Figure 8.8. It is noticed that the bandwidth adaptation gain is
\(k_{a, \text{str}} = 0.7\) (Figure 8.9), which provides a robust system for the closed-in sub-mode. In Figure 8.12, as the vehicle is approaching with a decrease range, the heading angle \(\chi\) and the flight-path angle \(\gamma\) is converging to the azimuth angle \(\rho\) and the elevation angle \(\sigma\), respectively. As shown in Figure 8.20, the throttle is shut down for 5s and turned on with full throttle for 5s; then the throttle is set according to the restoration trajectory command.

Once the position tracking error between the vehicle and the target is reduced to within the capture box as shown in Figure 8.10, the switching flag = 0 is set at \(t = 78.7s\) (Figure 8.8), at which time, the home-in control starts with an initial bandwidth adaptation gain 0.6 (Figure 8.9). The mission trajectory is switched back for tracking. Then the bandwidth adaptation gain ramps up to \(k_{a, \text{nom}} = 1\) at \(t = 93s\) (Figure 8.9). Thereafter, the mission trajectory is completely restored with the desired tracking performance as shown in the 3D inertial position tracking performance in Figure 8.22.

![Supervisory Mode-Switching Control](image)

Figure 8.8 Supervisory control output-flag value.
Figure 8.9 System bandwidth adaptation gain.

Figure 8.10 Initial position tracking performance.
Figure 8.11 Catching position after arrest.

Figure 8.12 Guidance command for restoration mode.
Figure 8.13 Vehicle speed and load factor.

Figure 8.14 Body velocity tracking performance.
Figure 8.15 Euler attitude tracking performance.

Figure 8.16 Aerodynamic forces.
Figure 8.17 Angular velocity tracking performance.

Figure 8.18 Aerodynamic attitude tracking performance.
Figure 8.19 Effector deflection tracking performance.

Figure 8.20 Thrust coefficient tracking performance.
8.6. Conclusion

A supervisory discrete-event-driven, Automatic Flight Management System (AFMS) of the proposed integrated automatic flight controller for fixed-wing aircraft Loss-Of-Control Prevention and Recovery (iLOCPR) is presented. The multi-modal controller
is configured by the coordination of the supervisory control. The Matlab/Stateflow is employed to implement the AFMS. Simulation results of a longitudinal stall scenario demonstrate the effectiveness and performance of the proposed iLOCPR system, where the stall is arrested within 26.18s, and the desired tracking performance is recovered within 93s after the stall is arrested.

Future work will focus on verifying all possible event sequences and more complicated LOC scenarios. Hardware implementation and demonstration should also be performed.
CHAPTER 9 SUMMARY AND FUTURE WORK

9.1. Summary

In this dissertation, an integrated Loss-Of-Control (LOC) Prevention and Recovery (iLOCPR) automatic flight controller for fixed-wing aircraft is designed. The multi-modal controller in iLOCPR comprises the nominal mode, the LOC prevention mode, the LOC arrest mode and the mission recovery mode.

The accomplishment of this dissertation includes:

(i) A relatively high fidelity fixed-wing aircraft LOC investigation platform is implemented in Matlab/Simulink, in which the stall characteristics, effector saturation, and environment models are implemented for the research.

(ii) A nominal mode controller for six degrees-of-freedom (6DOF) trajectory tracking is established based on the previous TLC design by improving the bank-to-turn guidance law. Wind effect and stall characteristics are taken into account for LOC study.

(iii) The Multiple-Time-Scale Nested Loop (MNL) analysis results establish some quantitative guidelines for designing, synthesis and tuning of MNL control systems. Singular perturbation theory is employed to model the MNL system. Investigation of the phase margin reduction reveals a quantitative determination of the time-scale separation, which can be applied in MNL design and synthesis. Furthermore, an order reduction technique is developed in order to estimate the actuator phase frequency response so that the obtained quantitative relation is applicable even without the complete knowledge of the actuator transfer function.
(iv) A quantitative determination of the time-scale separation relates the ratio of time-scale separation (singular perturbation) to the phase margin of the outer-loop (perturbed) system, which is important in its own right. Theoretical analysis is presented to justify the LOC prevention design rationale.

(v) An automatic aircraft loss-of-control prevention system by bandwidth adaptation is presented. The adaptation law employs the time-varying parallel differential eigenvalues for a tradeoff between tracking performance and severe wind tolerance capability in real-time. It is implemented as an augmentation to the baseline controller designed with constant gains by the singular perturbation (time-scale separation) principle in an MNL architecture. Simulation studies on tailwind, headwind, crosswind, downdraft and updraft are presented to demonstrate the effectiveness of the proposed LOC prevention strategy.

(vi) A LOC arrest controller is designed by switching from the flight trajectory tracking (nominal mode) to aerodynamic attitude tracking (LOC arrest mode). Two simulation scenarios show the effectiveness of LOC arrest for both the cases at the onset of the upset and a severe stall. The proposed LOC arrest controller can be used either as a component of an autonomous flight control system or as a standalone emergency controller for piloted aircraft, which can be actuated by pressing a “panic button” or automatically overriding pilot control when an upset has exceeded a preset time limit.

(vii) A complete mission restoration mode is achieved by a close-up operation of guiding the vehicle close in onto the target by pure pursuit guidance, and a home in operation for restoring the mission by reduced bandwidth adaptation in gaining robustness from tracking performance. By proper design of the restoration mode, simulation results
demonstrate that the proposed design method is capable of directing the vehicle back to the mission trajectory in the presence of a relatively large position tracking error; and restoring the original trajectory tracking mission with desired tracking precision.

vii) A supervisory discrete-event-driven, Automatic Flight Management System (AFMS) is designed to autonomously coordinate the designed control modes. The AFMS is designed as a Moore finite-state machine and implemented in Matlab/Stateflow. The results of comprehensive LOC prevention and recovery simulation is presented to demonstrate the operation and the effectiveness of the iLOCPR system.

9.2. Future Work

The LOC investigation platform built by Matlab/Simulink can be further developed. Note that the stall characteristics of the wings are lumped together as implemented in the simulation platform. In order to simulate unbalanced wing stall and tail stalls, the stall characteristics must be implemented for each individual wing and the aerodynamic control surfaces. Moreover, the current aircraft model does not have the aerodynamic damping coefficients (stability coefficients), which cause the spin angular velocities to be unreasonably high and challenging to arrest. Such damping terms should be added in the future, and better LOC arrest results are to be expected. Future works include further testing the controller on a high fidelity simulation platform with stall characteristics on each wing and individual control surfaces, as well as with sensor delay and noises, such as the NASA AirSTAR-GTM. The controller can also be equipped with a more sensible throttle command during the arrest, and with bandwidth adaptation using PD-eigenvalues to further shorten the arrest time and altitude departure.
As shown in Figure 4.3, the current results for the nonmonotonic dependence of the closed-loop bandwidth of the perturbed system on the bandwidth variation of the actuator is only established numerically on the normalized 2nd-order system. A systematic proof is desired for more general or complex systems to obtain the bandwidth adaptation guidelines.

Future analysis of the trade-off between the tracking performance and robustness is needed for the MNL system and the bandwidth adaptation control. The current design also needs to be further refined and tested for complex and realistic wind conditions with sensor noise and errors and to minimize stress and load factors.

Future work could investigate the adaptation law for complex wind such as rotational wind, typical wind turbulence, microburst, etc. Other adverse flight conditions such as flight in transonic regime can be simulated on an appropriate aircraft model to test the effectiveness of the bandwidth adaptation principle in LOC prevention. On the other hand, other baseline controller (nominal controller) designed by different control methods can be augmented with bandwidth adaptation law for testing the effectiveness of the bandwidth adaptation principle in LOC prevention.

The current LOC arrest control algorithm can be further enhanced with a closed-loop aerodynamic angles and throttle command to future improve the arrest performance and to ready the aircraft for the mission restoration.

The pure pursuit restoration guidance is a closed-loop guidance, and currently the PID controllers for the guidance errors are tuned heuristically. It is of theoretical as well as practical importance to develop a proof of stability for our particular design. In particular, the current guidance law is only applicable for small misalignment angles between the LOS
and the vehicle velocity vector, which means that at the end of the upset arrest, the vehicle must be behind the target position with substantial distance, and the vertical and horizontal deviation from the mission trajectory must be relatively small. An effective nonlinear instead of the linear PID, guidance error regulator could significantly expand the applicability of the current restoration controller.

Future work for the integration of the iLOCPR will focus on verifying all possible event sequences and more complicated LOC scenarios. Hardware implementation and demonstration should also be performed.
REFERENCES


APPENDIX A: DEFINITION OF THE INERTIAL CONSTANTS

Based on the vehicle moment of inertia about the body frame axes, the inertial constants are defined as

\[
I_c = \frac{1}{(I_{xx}^2 - I_{xx}I_{zz})}
\]

\[
I_{pq} = I_{xx}(I_{yy} - I_{zz} - I_{xx})I_c
\]

\[
I_{qr} = -I_{pq}
\]

\[
I_{qr} = \frac{(I_{zz}^2 - I_{yy}I_{zz} - I_{xx}^2)}{I_c}
\]

\[
I_{pp} = -I_{xz}/I_{yy}
\]

\[
I_{rr} = -I_{pp}
\]

\[
I_{pp} = \frac{(I_{zz} - I_{xx})}{I_{yy}}
\]

\[
I_{pq} = (I_{xx}I_{yy} - I_{zz}^2 - I_{xx}^2)I_c
\]

\[
g^p = -I_{zz}I_c
\]

\[
g^n = -I_{xx}I_c
\]

\[
g^r = g^n
\]

\[
g^q = 1/I_{yy}
\]

\[
g^n = -I_{xx}I_c
\]
The TLC gain matrices for the 6DOF nominal controller design using PD-eigenvalue are synthesized by the symbolic calculation. The obtained gain matrices are attached as follows:

\[
\begin{align*}
K_{11} &= \begin{bmatrix} 
\alpha_{111}C_\varphi C_\varphi & \alpha_{121}C_\varphi S_\varphi & -\alpha_{131}S_\varphi \\
\alpha_{111} \left(S_\varphi S_\varphi C_\varphi - C_\varphi S_\varphi \right) & \alpha_{121} \left(S_\varphi S_\varphi S_\varphi + C_\varphi C_\varphi \right) & \alpha_{131} S_\varphi C_\varphi \\
\alpha_{111} \left(C_\varphi S_\varphi C_\varphi + S_\varphi S_\varphi \right) & \alpha_{121} \left(C_\varphi S_\varphi S_\varphi - S_\varphi C_\varphi \right) & \alpha_{131} C_\varphi C_\varphi 
\end{bmatrix} \\
K_{p1} &= \begin{bmatrix} 
\alpha_{112}C_\varphi C_\varphi & \alpha_{122}C_\varphi S_\varphi & -\alpha_{132}S_\varphi \\
\alpha_{112} \left(S_\varphi S_\varphi C_\varphi - C_\varphi S_\varphi \right) & \alpha_{122} \left(S_\varphi S_\varphi S_\varphi + C_\varphi C_\varphi \right) & \alpha_{132} S_\varphi C_\varphi \\
\alpha_{112} \left(C_\varphi S_\varphi C_\varphi + S_\varphi S_\varphi \right) & \alpha_{122} \left(C_\varphi S_\varphi S_\varphi - S_\varphi C_\varphi \right) & \alpha_{132} C_\varphi C_\varphi 
\end{bmatrix} \\
K_{p2} &= m \begin{bmatrix} \alpha_{212} & \bar{r} & -\bar{q} \\
-\bar{r} & \alpha_{222} & \bar{p} \\
-\bar{q} & -\bar{p} & \alpha_{232} \end{bmatrix}, \quad K_{12} = m \begin{bmatrix} \alpha_{211} & 0 & 0 \\
0 & \alpha_{221} & 0 \\
0 & 0 & \alpha_{231} \end{bmatrix} \\
K_{p3} &= \begin{bmatrix} 
\alpha_{312} & \bar{q} S_\varphi + \bar{p} C_\varphi & -\alpha_{332} S_\varphi \\
-\bar{r} & \alpha_{322} C_\varphi + \left(S_\varphi \left[\bar{q} S_\varphi + \bar{p} C_\varphi \right] S_\varphi \right) & \alpha_{332} S_\varphi C_\varphi \\
-\bar{q} & -\alpha_{322} S_\varphi + \left(S_\varphi \left[\bar{q} S_\varphi + \bar{p} C_\varphi \right] C_\varphi \right) & \alpha_{332} C_\varphi C_\varphi 
\end{bmatrix} \\
K_{13} &= \begin{bmatrix} 
\alpha_{311} & 0 & -\alpha_{331} S_\varphi \\
0 & -\alpha_{321} C_\varphi & \alpha_{331} S_\varphi C_\varphi \\
0 & 0 & -\alpha_{321} S_\varphi C_\varphi 
\end{bmatrix} \\
K_{p4} &= \begin{bmatrix} 
I_{xx} \left( \alpha_{412} + I_{pq} \bar{q} \right) - I_{pq} \bar{x} \bar{q} & I_{xx} \left( I_{pq} \bar{p} + I_{pr} \bar{r} \right) - I_{xx} \left( I_{pq} \bar{p} + I_{pr} \bar{r} \right) & I_{xx} I_{pq} \bar{q} - I_{xx} \left( I_{pq} \bar{q} + \alpha_{432} \right) \\
I_{yy} \left( 2 I_{pq} \bar{p} + I_{pr} \bar{r} \right) & \alpha_{422} I_{yy} & I_{yy} \left( 2 I_{pq} \bar{p} + I_{pr} \bar{r} \right) \\
\right. \\
\left. \right. \\
I_{xx} I_{pq} \bar{q} + I_{pq} \bar{q} \right) + I_{xx} I_{pq} \bar{q} + I_{pq} \bar{q} \right) & I_{xx} I_{pq} \bar{q} + I_{pq} \bar{q} \right) & I_{xx} I_{pq} \bar{q} + I_{pq} \bar{q} \right) & I_{xx} I_{pq} \bar{q} + I_{pq} \bar{q} \right) + I_{pq} \bar{q} \right) \end{bmatrix} \\
K_{i4} &= \begin{bmatrix} 
I_{xx} \alpha_{411} & 0 & -I_{xx} \alpha_{431} \\
0 & I_{yy} \alpha_{421} & 0 \\
-I_{xx} \alpha_{411} & 0 & I_{xx} \alpha_{431} 
\end{bmatrix} \\
(7.2) \\
(7.3) \\
(7.4) \\
(7.5)
where the state variables with an overline denotes the nominal state variable.
APPENDIX C: LOC ARREST CONTROLLER TLC GAIN MATRICES:

The 3DOF LOC arrest controller is designed by TLC, in which the outer-loop of the controller is synthesized based on the rotational dynamics model in the wind frame.

The gain matrix components of $K_{p5}$ in Eq. (6.10) can be found as follows

\[
k_{p5,11} = \frac{T}{mV_i} (C_\alpha C_\beta S_\mu T_\gamma + S_\alpha C_\alpha S_\beta C_\mu T_\gamma + S_\beta) + \bar{\rho}
\]

\[
k_{p5,12} = S_\alpha \left[ \alpha_{322} + \frac{1}{mV_i} (D_{nom} C_\beta - Y_{nom} S_\beta - TC_\alpha C_\beta) \right] + \bar{\rho} C_\alpha^2 C_\beta
\]

\[
+ \bar{\rho} S_\alpha C_\alpha T_\beta \frac{1}{mV_i C_\beta} \left[ C_\alpha (L + TS_\alpha - mg C_\mu C_\gamma + DC_\beta C_\mu T_\gamma - TC_\alpha C_\beta) - TS_\alpha S_\beta S_\mu T_\gamma \right]
\]

\[
k_{p5,13} = \frac{1}{mV_i} \left[ C_\alpha C_\beta (\alpha_{332} mV_i + LC_\mu T_\gamma + TS_\alpha C_\mu T_\gamma - DS_\beta S_\mu T_\gamma + Y C_\beta S_\mu T_\gamma + mg T_\mu S_\mu C_\gamma + TC_\alpha S_\beta S_\mu T_\gamma) - mg S_\alpha S_\mu C_\gamma \right] + \alpha_{312}
\]

\[
k_{p5,21} = \frac{T}{mV_i} \left[ -TS_\alpha C_\beta C_\mu T_\gamma - TC_\alpha C_\beta + TS_\alpha C_\mu T_\gamma + TC_\alpha S_\beta S_\mu T_\gamma \right] + \alpha_{312}
\]

\[
k_{p5,22} = \frac{T}{mV_i} \left[ -Y C_\mu + Y C_\beta C_\mu + DS_\alpha C_\beta C_\mu - TC_\alpha S_\beta C_\mu C_\mu \right] - \bar{\rho} C_\alpha - \bar{\rho} S_\alpha
\]

\[
k_{p5,23} = \frac{1}{mV_i} \left[ S_\beta (LC_\mu T_\gamma + TS_\alpha C_\mu T_\gamma - DS_\beta S_\mu T_\gamma - Y C_\beta S_\mu T_\gamma + mg T_\beta S_\mu C_\gamma + T_{nom} C_\alpha S_\beta S_\mu T_\gamma) - mg S_\mu C_\gamma / C_\beta \right] + \alpha_{332} S_\beta
\]

\[
k_{p5,31} = \frac{T T_\gamma}{mV_i} \left[ S_\alpha C_\alpha C_\beta S_\mu - S_\alpha S_\beta C_\beta C_\mu \right] - \bar{\rho}
\]

\[
k_{p5,32} = \frac{1}{mV_i} \left[ S_\alpha (L + TS_\beta - mg C_\mu C_\gamma + DC_\beta C_\mu T_\gamma - TC_\alpha C_\beta C_\beta T_\gamma - Y S_\beta C_\beta C_\mu T_\gamma) \right] + \bar{\rho} S_\alpha C_\alpha T_\beta + \bar{\rho} S_\alpha^2 T_\beta - C_\alpha \left[ \alpha_{322} = \frac{1}{mV_i} (YS_\beta - DC_\beta + TC_\alpha C_\beta) \right]
\]

\[
k_{p5,33} = \frac{1}{mV_i} \left[ S_\alpha C_\beta (LC_\mu T_\gamma + TS_\alpha C_\mu T_\gamma - DS_\beta S_\mu T_\gamma - Y C_\beta S_\mu T_\gamma + mg T_\beta S_\mu C_\gamma + TC_\alpha S_\beta S_\mu T_\gamma) + mg S_\alpha S_\mu C_\gamma \right] + \alpha_{332} S_\alpha C_\beta
\]

(7.6)
APPENDIX D: THE TRIM CONDITION

The trim conditions for level and straight flight simulation in this dissertation are given in the following table.

Table D.0.1 Level and straight trim conditions.

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