Math Teachers' Circles: The Effects of a Professional Development Community on Mathematics Teachers' Identities

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This dissertation titled
Math Teachers' Circles: The Effects of a Professional Development Community on Mathematics Teachers' Identities

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Abstract

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Math Teachers' Circles: The Effects of a Professional Development Community on
Mathematics Teachers' Identities

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Math Teachers’ Circles are content-focused professional development for K–12 mathematics teachers that engage teachers and mathematicians in intensive, collaborative problem solving. Typically, Math Teachers’ Circles begin with a weeklong summer immersion workshop. This study explored the effects of participation in such professional development on elementary and middle school teachers’ mathematical identities, their mathematics teaching identities, and the interaction of these identities.

This investigation used an explanatory multiple-case study methodology. Extreme cases were identified from first-time participants at three Math Teachers’ Circle sites across the United States. Shifts in these teachers’ identities were explored through open-ended interviews, pre- and post-workshop surveys, and written reflections.

Teachers’ identities were understood as the extent to which the teachers’ personal identities aligned with the normative identity of the Math Teachers’ Circle. The teachers’ mathematical identities evolved most significantly as a result of their participation. During the immersion workshop, the teachers found that perseverance and collaboration assisted in their success at solving challenging and open-ended mathematics problems, and their confidence and motivation increased over the week. As a result, teachers’ sense
of self, including mathematics self-concept and self-efficacy, became stronger, and their understanding of the nature of mathematics evolved to include patterns, connections, and open-ended problems.

The immersion workshop also changed teachers’ perceptions of effective mathematics pedagogy. The teachers in this study found that collaborating and struggling through nonroutine problems was useful to their understanding of the problems and of teaching and learning mathematics. The teachers intended to use similar problems and pedagogy in their classes. However, the teachers’ perceptions of their teaching abilities remained relatively stable after the immersion workshop.

This study found that teachers’ experiences doing mathematics are influential to their mathematics teaching identities. Thus, teachers’ active participation in the Math Teachers’ Circle immersion workshops, through which they became mathematics learners, was beneficial to their growth as teachers.

These findings suggest that the role of identity may be critical to understanding the ways in which teachers learn from professional development and enact new practices in the classroom. This study also suggests that teachers’ experiences engaging with content as learners have implications for their teaching.
Dedication

To my family,

For always believing in me.
Acknowledgments

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Chapter 1: Introduction

Education policy documents in the United States have increasingly urged for an emphasis on science, technology, engineering, and mathematics in schools. Skills in mathematics, problem solving, and critical thinking are widely agreed to be important for students entering college or careers (e.g., National Governors Association, Council of Chief State School Officers, & Achieve, 2009; Partnership for 21st Century Skills, 2008). The Common Core Standards for School Mathematics, recently adopted and implemented in a majority of states, emphasize student learning of mathematical practices, such as making sense of problems, abstract reasoning, communication, and mathematical modeling (Common Core State Standards Initiative [CCSSI], 2010).

However, students in the United States are not adept at solving complex problems or thinking conceptually (Stigler & Hiebert, 1999; Stigler & Hiebert, 2009). Many teachers teach in the ways that they were taught mathematics, requiring students to memorize facts and apply memorized procedures without understanding why those procedures work (Burns, 2004). Students do not often engage in thinking and reasoning, and when problem solving is included in the curriculum, it is often shallow and lacking in rich applications (Ben-Hur, 2006). This type of teaching is not conducive to developing problem-solving skills. In order for deep mathematical learning to occur, students need the opportunity to struggle and engage with mathematical concepts (Hiebert & Grouws, 2007).
Rationale

Teacher content knowledge is widely agreed to be one of the most important parts of teacher knowledge (Ball, 1999; Conference Board of the Mathematical Sciences [CBMS], 2012; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Shulman, 1986, Taton, 2015). Recent descriptions of the knowledge necessary to teach mathematics has centered on the intersection of content knowledge and pedagogical knowledge, known as “pedagogical content knowledge” (Shulman, 1986), or “mathematical knowledge for teaching” (Ball, 1990; Ball & Bass, 2000; Thompson & Thompson, 1996). Mathematical knowledge for teaching has been conceptualized as the mathematical understandings that teachers should have in order to understand student thinking and misconceptions and how best to explain concepts to students (Ball & Bass, 2000).

Because of the importance of mathematical knowledge for teaching, there is an abundance of literature on teaching methods that engage students in the exploration of problems and sharing of solutions (e.g., Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; McClaine, 2003; M. S. Smith & Stein, 2011). However, despite these efforts over the past few decades, the teaching of problem solving has yet to improve (Lesh & Zawojewski, 2007). One problem may be that many elementary and middle school teachers have weak mathematics backgrounds and lack experience with problem solving (Ball, 1999; C. T. Cross, Woods, & Schweingruber, 2009; National Research Council [NRC], 2001). As a result, there is general consensus that in order to improve teaching and learning of mathematics, teacher professional development should focus on content knowledge for teachers (Arends & Kilcher, 2010; Guskey, 2000; Sparks, 2000).
Teachers’ mathematics beliefs may also hinder their teaching of rich mathematics. Schoenfeld (1985) observed that college students occasionally failed to solve problems correctly despite possessing the necessary mathematical knowledge, heuristics, and metacognitive skills. To explain this disparity, Schoenfeld included belief systems, or mathematical worldview, in his framework on problem solving. A person’s mathematics belief system may inhibit their problem-solving ability if they hold the belief that certain mathematics knowledge will not be useful in a particular situation, or negative beliefs about problem solving, such as, “only geniuses are capable of discovering or creating mathematics,” or “mathematics problems are always solved in less than 10 minutes, if they are solved at all” (Schoenfeld, 1985, p. 43). According to Schoenfeld, mathematics belief systems and mathematics self-concept have tangible effects on a person’s ability to solve problems. Other researchers have also noted the influence of beliefs and environment on cognition and achievement (M. L. Anderson, 2003; J. S. Brown, Collins, & Duguid, 1989; Leron & Hazzan, 2006; Nunez, Edwards, & Matos, 1999; Schunk & Pajares, 2007).

**Math Teachers’ Circles**

One model for mathematics-focused professional development is the Math Teachers’ Circle (MTC) (White, Donaldson, Hodge, & Ruff, 2013). Begun in 2006, MTCs are communities of teachers who engage collaboratively in problem solving and mathematical explorations with the facilitation of mathematics professors (White, Donaldson, Hodge, et al., 2013). Typically, middle school teachers and mathematics professors meet for a weeklong summer immersion, followed by monthly sessions.
throughout the school year. MTCs are typically formed apart from a specific school or district and focus on community building across grade levels and schools.

While these are fairly new groups, MTCs come from a long tradition of Math Circles, which are groups focused on improving middle and high school students’ problem-solving skills. Math Circles for students originated in Bulgaria and Russia over 100 years ago and eventually migrated to the United States in the late 1990s (Shubin, 2006; Vandervelde, 2009). In promotional materials, Math Circles in the United States have been described as “a form of outreach that bring mathematicians into direct contact with pre-college students…to work on interesting problems or topics in mathematics” for the purpose of getting “the students excited about the mathematics they are learning; to give them a setting that encourages them to become passionate about mathematics” (Vandervelde, 2009, p. 3).

After attending student Math Circles, a teacher in California wanted to bring the excitement of problem solving to teachers. MTCs were officially created by the American Institute of Mathematics (AIM) in 2006 (Fernandes, Koehler, & Reiter, 2011; Math Teachers’ Circle [MTC] Network, 2015). MTCs typically consist of elementary and middle school teachers and mathematicians or university mathematics faculty, although they are also accessible to high school mathematics teachers, preservice teachers, and mathematics coaches or consultants. Small teams of teachers and mathematicians interested in starting MTCs in their communities contact the AIM to obtain training and resources. AIM also runs the MTC Network, which maintains a website with resources
for starting and sustaining MTCs, uses an e-mail list-serv to communicate with leaders of MTCs, and publishes a newsletter twice per year.

MTCs typically hold a weeklong summer immersion workshop, plus monthly follow-up sessions throughout the school year. Each session lasts about three hours and consists of one or two rich problems. Facilitators can be mathematicians, teachers, or mathematician and teacher pairs. Each session is attended by, on average, 15 to 20 participants. According to the MTC Network (2015), as of 2015, there were 71 MTCs in 36 states. New teams are encouraged to form each year. MTCs are meant to be long-term, sustained communities, holding an immersion workshop each summer and recruiting new participants to join each year. Past participants are also encouraged to continue attending each year, and teachers often participate in these groups for multiple years.

The goals of MTCs are to “encourage teachers as mathematicians, connect mathematics professors with K–12 education, and to build a K–20 community of mathematics professionals committed to fostering a love and understanding of mathematics among all students” (MTC Network, 2015, para. 1). In an ideal MTC session, teachers develop greater mathematical knowledge, improve problem-solving skills, work with rich mathematical tasks and persevere in solving problems, and increase their enjoyment of and appreciation for mathematics and mathematical problem solving. Key aspects of MTC sessions are the appropriate level of challenge for all participants (from nonspecialist elementary school teachers to mathematicians), a social learning and risk-taking atmosphere, the “exploratory component of the problems,” and facilitation of the problem-solving process (Fernandes et al., 2011, p. 112).
Typically, after a short introduction, a problem is posed, and the majority of the time is dedicated to individual exploration of the problem. The group discussion emphasizes solution paths and multiple strategies, rather than the correct answer. For example, one problem that has engaged teachers in many MTCs for a few hours is Operation Cookie Jar:

There are 15 cookie jars, numbered consecutively from 1 to 15. The number of cookies in each jar is equal to the number of the jar. A “move” consists of choosing one or more jars, then removing one or more cookies from the chosen jars—but the same number of cookies from each jar. What is the minimum number of moves that are necessary to empty all the jars? (Pinter, 2012).

This problem above is a model MTC problem because it allows for entry at a variety of mathematical levels. For example, participants may begin by acting it out and recording their moves. Participants may begin to look for patterns. The problem may seem simple, but deep mathematics is embedded within it. The two-dimensional analog of this activity models the algorithm for optimizing delivery of radiation therapy for cancer patients. It also allows for extensions, such as, what if you have 20 cookie jars, or $n$ cookie jars?

MTCs meet the criteria of several frameworks for effective professional development. The CBMS (2012) recommends the formation of MTCs as an avenue for teacher professional development and teacher–mathematician partnerships. The CBMS explain that teachers of mathematics “need experiences that renew and strengthen their interest in and love for mathematics” that include “being able to place themselves in the position of mathematics learners” (p. 68). Desimone (2009) lists following five criteria for effective professional development: content focus, active learning, coherence,
duration, and collective participation. White, Donaldson, Hodge, et al. (2013) argue that MTCs meet all five criteria due to the focus on rich mathematics problems, active participation by teachers, the direct connection to the mathematical habits of mind described in the Standards for Mathematical Practice (CCSSI, 2010), the multiyear nature of MTCs, and the building of community between mathematicians and teachers. J. S. Brown et al. (1989) suggest that other important aspects of teacher professional development are collaborative learning and social interaction, two characteristics of MTCs. Guskey (2000) argues that professional development must be intentional, ongoing, and systematic. MTCs meet all three criteria: they have clearly defined purposes and intent; they take place throughout the school year and over multiple years; and they are systematic because they are intended to create long-term change. Finally, MTCs share many characteristics with Professional Learning Communities, an extensively studied, effective, and high quality form of professional development (Arends & Kilcher, 2010).

To determine the effectiveness of teacher professional development, Guskey (2000) recommends focusing on changes in student learning and enjoyment of the subject, or changes in teacher knowledge and teacher practice (which affect student learning). But most evaluations of teacher professional development have been uninformative, typically listing what happened or surveying teacher levels of enjoyment. Other evaluations have examined the gains in teacher knowledge, but it is important to note that gains in knowledge do not transfer automatically into changes in the classroom. More accurate evaluations of the effectiveness of professional development include how the workshop affected the climate or procedures in the school district, whether the
participants used their new knowledge in the classroom, and the changes in students’ learning and positive feelings toward the subject (Guskey, 2000). Few studies, however, have focused on this transfer of knowledge into the classroom (Joyce & Showers, 2002).

Although Math Circles have existed for almost a century, MTCs have a relatively short history. Therefore, their effects on teacher content knowledge and classroom practice have not yet been extensively studied. White, Donaldson, Hodge, et al. (2013) examined the effect of MTCs on mathematical knowledge for teaching and found that, at multiple sites, teachers’ mathematical knowledge for teaching, particularly in number concepts and operations, increased significantly.

This study examines outcomes related to teacher identity development, which is potentially an important element of MTCs and which contributes to changes in teacher knowledge and classroom practice.

**Research Questions**

The purpose of this study is to explore the effects of participation in MTCs on mathematics teachers’ identities.

In particular, this study examines the following research questions:

RQ1. In what ways does participation in a Math Teachers’ Circle affect teachers’ mathematical identity?

RQ2. In what ways does participation in a Math Teachers’ Circle affect teachers' professional identity as a mathematics teacher?

RQ3. In what ways does participation in a Math Teachers’ Circle affect the interaction between teachers’ mathematical identities and their professional identities?
This study addresses changes in mathematics teachers’ identities as a result of participating in an MTC weeklong summer immersion workshop. A qualitative research design was used to gather data related to teachers’ identities as a doer of mathematics and as a teacher of mathematics prior to, during, and after participating in the MTC immersion.

**Significance of the Study**

The research literature on teacher professional development is lacking in analyses of teacher identities, transference into the classroom, and teacher mathematics belief systems. Teacher professional development is rarely evaluated in one of the high-quality ways recommended by Guskey (2000). This study advances the teacher professional development literature by providing an analysis of the effect on teacher identity.

Further, although MTCs satisfy the recommendations for high-quality teacher professional development from a variety of sources (e.g., Arends & Kilcher, 2010; J. S. Brown et al., 1989; Desimone, 2009; Guskey, 2000), they constitute a fairly new form of professional development. Recent research studies of MTCs examine teachers’ mathematical knowledge for teaching. The author found no studies on MTCs that have examined the changes in teachers’ identities, how teachers transfer their new knowledge into classroom practice, or how they share their new knowledge with colleagues. This study begins to fill this current gap in the literature.

This research study may be beneficial to groups that intend to start a new MTC, as well as leaders of existing MTCs. The results could be used to help justify grant funding for new or existing MTCs. Further, existing and future MTCs could use this research to
plan and provide experiences with the best possible outcomes for teachers and students. This study could be helpful to leaders of MTCs who are trying to recruit participants or sustain their circle.

The research may also have implications for school district administrations, university faculty, and others seeking to design professional development for in-service and preservice teachers. Teacher identity development is a key aspect of the study, so leaders who are planning professional development could see how teachers’ identities change or develop as they participate in professional development. MTCs may provide a model for a transformative professional development program for mathematics teachers. The effective practices of MTCs could also be included in teacher education programs for preservice teachers.

**Researcher Positionality**

In 2012, I picked up a brochure about the “How to Run a Math Teachers’ Circle” workshop sponsored by the American Institute of Mathematics. MTCs sounded like an enjoyable way to spend some time doing mathematics, so I spoke to some elementary school mathematics teachers and university mathematics professors and put together a team to attend the workshop and to start an MTC in our region. At the weeklong workshop, we experienced MTC sessions and planned the development of our circle back home.

The following summer, I helped organize and facilitate the summer immersion workshop of the Southeast Ohio Math Teachers’ Circle. The problems were perplexing, engaging, and reminded me of the fun that can be had while doing mathematics. I also
found that I had more energy and excitement about teaching problem solving. I felt more capable of letting my students struggle because I learned how to facilitate productive struggle. The support from the other teachers gave me encouragement to try new problems and teaching strategies with my students.

As I spoke to my mathematics teacher colleagues who also attended the MTC summer immersion, I realized that the workshop had profound effects on those who had a fear or dislike of mathematics. These teachers had shifted their ways of thinking about the nature of mathematics beyond calculations and procedures and were reconsidering their ideas of how to teach mathematics effectively. One teacher told me that she realized that mathematics problems did not have to be short; in fact, her class could spend an entire class period wrestling with one problem. She also realized that she could leave a problem unsolved at the end of the class period and go back to it the next day, or the next week if necessary. Other teachers I informally spoke with had started to think deeper about the various methods students could use to solve the same problem, and they tried to understand their students’ thinking in different ways.

Teachers were also undergoing shifts personally, regarding who they were in relation to their new understanding of mathematics. During a visual activity with beautiful patterns based on oddness and evenness of numbers in Pascal’s triangle, I heard comments like, “I never knew that math could look like this!” At other times, I saw previously timid MTC participants gain confidence when explaining their thinking. Through informal conversations, I learned that some teachers began to see themselves as doers of mathematics for the first time.
Conversations with teachers and the shifts I heard and saw throughout the next few years cemented my interest in studying the types of shifts that teachers experience as a result of their MTC participation.

**Definition of Terms**

For the purpose of this study, the following terms are operationally defined as follows:

*Belief:* A belief is “any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase ‘I believe that’ “ (Rokeach, 1968, p. 113). Beliefs about motivation, mathematics, and teaching and learning are relevant to this study.

*Identity:* Identity is “the way we define ourselves” (Sfard & Prusak, 2005, p. 15) and being recognized by others as “a certain ‘kind of person’ in a given context” (Gee, 2001, p. 99).

*Math Teachers’ Circle (MTC):* An MTC is a specific “form of professional development that emphasizes developing teachers’ understanding of and ability to engage in the practice of mathematics, particularly mathematical problem solving, in the context of significant mathematical content” (White, Donaldson, Hodge, et al., 2013). In this study, professional development is only considered an MTC if it is affiliated with the MTC Network.

*Mathematical identity:* Mathematical identity refers to how individuals see themselves as mathematically capable, or as successful in mathematics (Martin, 2000). Mathematical identities “include stories related to how one interacts with mathematics
both in and out of school,” including factors such as persistence, motivation, and interest in mathematics (McCulloch, Marshall, DeCuir-Gunby, & Caldwell, 2013, p. 380; see also Cobb, Gresalfi, & Hodge, 2009).

It is important to note that in the literature on identity, the terms mathematics identity and mathematical identity are used, seemingly interchangeably. The difference between these two terms is nuanced and worthy of further exploration. For the purposes of this study, I have chosen to use mathematical identity to refer to this construct; however, when the literature about mathematics identity is being discussed, that term will be used.

**Mathematics teaching identity:** Mathematics teaching identity consists of how teachers define themselves as mathematics teachers and how they want to be recognized by others (Luehmann, 2007).

**Problem solving:** Problem solving is the act of engaging in making sense of problems using mathematics. See the definition of “rich problems (or problems)” for the type of problems used in problem solving.

**Productive disposition:** Productive disposition refers to one of the five strands of mathematical proficiency described by the *Adding It Up* report from the NRC (2001). Productive disposition describes “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (p. 131). In this work, productive disposition is assumed to refer to productive disposition towards mathematics.
Professional identity: Teacher professional identity is “how teachers define themselves to themselves and to others” (Lasky, 2005, p. 1350) and involves “being recognized by self or others as a certain kind of teacher” (Luehmann, 2007, p. 827).

Rich problems (or problems): In this dissertation, the terms “problem” and “rich problem” are considered synonymous. A problem is a mathematical task in which “the problem solver does not have easy access to a procedure for solving the problem” (Schoenfeld, 1985, p. 11). Rich problems are generally agreed to be cognitively challenging and nonroutine: they allow for multiple entry points, have multiple solution paths, and encourage making sense of a situation mathematically.

Teacher: In this study, the word teacher refers to a classroom teacher of kindergarten through Grade 12 in the United States.
Chapter 2: Review of the Literature

The recent focus on science, technology, engineering, and mathematics (STEM) fields and 21st-century skills has led to an increased interest in mathematical problem solving and the skills students develop in mathematics (Lesh & Zawojewski, 2007). Many education groups also emphasize the importance of mathematical literacy for all students, not just those going into STEM-related careers (e.g., National Council of Teachers of Mathematics [NCTM], 2014). Skills such as making sense of situations using mathematics, communicating mathematically, and understanding a solution in terms of its context are important for all students, for both career preparation and for an informed citizenry with basic mathematics literacy (Lesh & Zawojewski).

If we are to allow all students to see mathematics as a discipline that values thinking deeply about problems, analyzing available information critically, and making sense of their results, then the teaching of mathematics needs to include opportunities for critical problem solving (Ben-Hur, 2006; Goldsmith & Schifter, 1997). In fact, an abundance of publications related to mathematics education recommend the activities and skills that are important in mathematics, such as reasoning, sense-making (Stein, Grover, & Henningsen, 1996), communication, and connections between mathematical ideas (Jackson et al., 2013).

However, many mathematics teachers, particularly those without strong mathematics backgrounds, place an inordinate emphasis on rote knowledge and procedures (Ball, 1999; C. T. Cross et al., 2009; NRC, 2001). As Ben-Hur (2006) put it, many classrooms “treat mathematical problem solving as an appendix to the curriculum
and fail to consider problem solving as a rich context for developing mathematical concepts” (p. vi). This is particularly evident in elementary school classrooms, in which teachers are often generalists and have never studied mathematics in depth. Some teachers also had negative experiences with mathematics themselves and bring their own fears or dislike of mathematics into the classroom (Ball, 1996; Schifter, 1996; Schoenfeld, 1992).

There is an opportunity for professional development to bridge this gap and have a profound influence on teachers and the way they think about and teach mathematics. MTCs help teachers develop mathematics knowledge and ways of teaching problem solving. At the same time, the activities in MTCs can be fun and engaging for teachers, increasing their positive feelings for the subject and changing their perceptions of the discipline.

Before diving into the literature related to MTCs and their outcomes, it is important to consider the foundations of the structure and ideas of MTCs. The next section explores the theories of learning and professional development that undergird MTC activities, and the subsequent sections provide the lenses through which teacher learning is viewed: theories of identity, mathematics and mathematical identity, professional identity, mathematics teaching identity, and the interaction of identities.

Theories of Learning

MTC activities often engage teachers in collaboration, active exploration, and making sense of problems. The use of such instructional practices is supported by constructivist theories of learning, including social constructivism, experiential learning,
the zone of proximal development, and situated learning. Constructivist theorists such as Bruner, Dewey, Flavel, Kolb, Piaget, and Vygotsky describe the ways in which learners construct understanding of concepts by building actively and collaboratively on prior understandings.

It is worth mentioning here that some of the literature related to theories of learning and motivation focuses primarily on students. This literature is reported as it was intended. However, the ways in which teachers experience mathematics learning are important as well. The literature on teacher learning in professional development settings is reported separately, but the general theories of learning and motivation have relevance for teachers who are in mathematics learning situations.

**Constructivism.** According to constructivism, humans cannot be given knowledge; they build it through experience, constructing schemas of information in their minds (Flavell, 1996). Two processes are used to internalize or construct knowledge: accommodation, or the changing of mental structures, and assimilation, or incorporating new knowledge into existing schemas (Flavell). According to Piaget, cognitive development is a “gradual, step-by-step process of structural acquisition and change, with each new mental structure growing out of its predecessor through the continuous operation of assimilation and accommodation” (Flavell, p. 200). For example, consider a child who is learning about two-dimensional shapes. If the child’s previous understanding of squares was that all figures with four sides are squares, and the child is confronted with trapezoids, then the child will have to adapt their mental schema to account for this new information. Accommodation would occur when the child changes their
understanding of four-sided figures to include both squares and trapezoids. Assimilation occurs when new information is simply added to existing schema; extending the previous example, assimilation may occur when the child learns about perimeter, if the concept of perimeter is added to the child’s existing schema and understanding of four-sided figures.

The theory of constructivism incorporates discovery or active learning as an act that takes place using assimilation and accommodation. Bruner (1961) explained, “Discovery…is in its essence a matter of rearranging or transforming evidence in such a way that one is enabled to go beyond the evidence so reassembled to additional new insights” (p. 22). That is, discovery learning takes places as the learner uses the processes of assimilation and accommodation to create new knowledge and understandings. According to Bruner (1961), learning occurs when one is able to organize information so as to “discover regularity and relatedness” (p. 26). Discovering such patterns allows the mental schema around an idea to change and adapt. Further, Bruner argued that the only method of improving inquiry-related skills and heuristics is engaging in discovery and inquiry. Continued experiences with discovery learning, then, lead to the development of skills and strategies for uncovering patterns and organizing information, which are useful in further discovery learning activities.

Social constructivism. Social constructivism is a theory of learning in which cultural and social contexts are important factors in the learning process (Vygotsky, Cole, John-Steiner, Scribner, & Souberman, 1980). It builds on constructivism, in that the individual constructs knowledge by incorporating new information into their existing schema. However, social constructivism extends the constructivist theory to include the
importance of cooperation with others for learning. The role of the teacher, then, is to create an environment that enables students to work together, sharing their knowledge and information with one another. Students thrive when they are in an environment where they can experience and interact with curriculum and each other.

**Experiential learning.** Kolb’s theory of experiential learning theory draws on the theories of John Dewey, Kurt Lewin, and Jean Piaget. Experiential learning theory examines “linkages among education, work, and personal development” (Kolb, 1983, p. 4). This theory takes into account the realities of life and the real-life contexts in which people learn. Kolb summarized this theory of learning in the following way: “Learning is the process whereby knowledge is created through the transformation of experience” (p. 38).

According to experiential learning theory, learning is a process rather than an outcome. Consider the example of the child learning about four-sided figures. Under the experiential learning view, “learning” is the process of the child accommodating and assimilating new information (the existence of trapezoids) into a preexisting schema, rather than the result of that process.

Another aspect of experiential learning is that “all learning is relearning” (Kolb, 1983, p. 28). In other words, we do not come to each learning experience as blank slates, so part of learning is to modify or change preexisting beliefs. Experiential learning, thus, results from the process of resolving conflicts between preexisting beliefs and new ideas.

**Zone of proximal development.** Vygotsky’s zone of proximal development describes how a teacher or more advanced peer can scaffold a task for someone until they
reach a greater level of knowledge and understanding (Bruner, 1984). It requires knowledge of the students’ current level of understanding as well as the next step in advancement of that students’ knowledge. Learners are challenged slightly above their current level of development, and they gain confidence and motivation when they successfully complete challenging tasks. Piaget incorporated a similar concept into his theories of development, conceptualizing children as “active, constructive thinkers who learn only what they are structurally ready to learn,” so the timing of new material is key (Flavell, 1996, p. 202). Learners need to have the mental schemas capable of accommodating and assimilating the new material or ideas; otherwise, learning will not occur.

**Situated learning.** The concept of situated learning, according to Lave and Wenger (1991), describes the process of learning as participating in “communities of practitioners” (p. 29). Situated learning involves social interaction with others in the community. Further, there is no defined “center” of activity, so all activity is varied and peripheral. As a learner moves “toward full participation in the sociocultural practices of a community,” the learner is able to master the knowledge and skills of the community (Lave & Wenger, p. 29). Engaging in legitimate peripheral participation, then, is one way that people can become members of a community and acquire knowledge from that community.

The theory of situated learning is built from the process of apprenticeship, wherein a person learns a trade working alongside an experienced mentor (Lave & Wenger, 1991). However, in learning and education, apprenticeship is meant
metaphorically rather than literally. In situated learning, the apprentice learns by being involved in the community and learning from everyone around, not solely or directly from the master. According to Lave and Wenger (1991), the learning curriculum for such a community “unfolds in opportunities for engagement in practice” rather than as “a set of dictates for proper practice” (p. 93). Participants learn the practices of the community as they engage with those practices. Mastery itself is improvised and “resides not in the master but in the organization of the community of practice of which the master is part” (p. 94).

But situated learning means more than simply learning by doing (Lave & Wenger, 1991). Learners gain greater knowledge as they increase their participation in the community of practice. Remaining on the periphery of the community allows learners extended periods of time to incorporate the “culture of practice” into their own practices (p. 95). As learners on the periphery gradually begin to understand the practices of the community, they learn how to become full practitioners in the community. However, the relations between learners and individual identity are constantly evolving as people transition from newcomers to old-timers.

**Summary of learning theories.** These theories of learning can be applied to the instructional structure of MTCs and the activities of teachers participating in MTCs. According to constructivist theories, learning takes place through experience and active engagement with concepts. Similarly, MTCs follow a “guide on the side” rather than a “sage on the stage” model, with individual or small-group work consuming the majority of the time in a session. Collaborative learning, a tenet of social constructivism, is also a
key feature of MTCs. In addition, MTCs encourage experiential learning by focusing on the process of solving a problem, sometimes emphasizing the process over the answer. The problems posed in an MTC session and the support provided by the facilitator attempt to support all participants’ learning of appropriately challenging problems through the zone of proximal development. Finally, MTCs attempt to develop a community of problem solvers with shared norms, thus echoing situated learning. These constructivist theories, taken together, inform the instructional tenets of MTCs.

**Teacher Learning**

Teachers learn throughout their careers in both informal and formal environments. Formal learning environments include professional development, which often seeks to increase teacher knowledge or change teacher practice. However, the outcomes of professional development vary, so it is important to consider elements of effective professional development and the process of teacher change.

**Teacher knowledge.** According to Shulman (1987), the knowledge base of teachers forms the foundation of the act of teaching. A significant amount of research in recent years has focused on teacher knowledge, what it includes, and how it develops (Ball & Bass, 2003; Hill et al., 2005; Lampert, 1990; Shulman, 1986; Shulman, 1987). It is generally agreed that teachers possess various types of knowledge, including *content knowledge* (i.e., the subject matter knowledge that might be obtained in college coursework) and *pedagogical knowledge*, which are teaching practices that can be applied to the teaching of various subjects. Other types of knowledge have been theorized and formulated, but one of the first and most significantly explored types of knowledge for
teaching is *pedagogical content knowledge* (Shulman, 1986), which is a “special kind of teacher knowledge that links content and pedagogy” (Ball, 1999, p. 26).

Shulman (1987) defined pedagogical content knowledge as “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (p. 8). Within the teaching of mathematics, pedagogical content knowledge includes knowing the most useful examples and representations for commonly taught mathematical content, understanding student preconceptions, and anticipating possible misconceptions (Shulman, 1987).

The term *mathematical knowledge for teaching* is sometimes used interchangeably with pedagogical content knowledge and is sometimes thought to be a subset of pedagogical content knowledge (Ball, 1999). According to Hill et al. (2005), mathematical knowledge for teaching is the “mathematical knowledge used to carry out the work of teaching mathematics” and includes knowing “how to use pictures or diagrams to represent mathematics concepts and procedures to students, provide students with explanations for common rules and mathematical procedures, and analyze students’ solutions and explanations” (pp. 372–373). Mathematical knowledge for teaching is the knowledge of mathematics “that teachers use in classrooms to produce instruction and student growth” and is specific to the teaching profession, that is, it is generally not knowledge held by mathematicians (Hill et al., 2008, p. 374).

Early research on teacher knowledge focused on subject-matter knowledge as measured by courses taken, degrees obtained, or mathematical knowledge as measured
by an instrument, but more recent research has begun to explore pedagogical content knowledge (Ball & Bass, 2003; Hill et al., 2005; Lampert, 1990; Shulman, 1986; Shulman, 1987). According to Hill et al., (2005), this group of scholars has “begun to conceptualize teachers’ knowledge for teaching differently, arguing that teacher effects on student achievement are driven by teachers’ ability to understand and use subject-matter knowledge to carry out the tasks of teaching” (p. 372). Shulman (1987) believed that pedagogical content knowledge had a greater effect on student outcomes than any other category of teacher knowledge. Ball and Bass (2003) agree that teaching students to reason mathematically requires specialized teaching practices and specific knowledge beyond “simply posing open-ended mathematical problems” or “merely asking students to explain their thinking” (p. 43). According to Ball and Bass (2003), teachers need explicit opportunities to learn how to support this type of student thinking.

**Teacher professional development.** It is necessary for teachers to keep their knowledge up to date, as knowledge about how students learn develops rapidly (Guskey, 2000). It is also useful when the adoption of new content standards or ways of teaching require changes in the ways that teachers teach (E. Green, 2014). Teacher professional development plays a critical role in both increasing teacher knowledge and affecting change in teacher practices (E. Green).

Professional development is the general term used for activities that seek to increase teachers’ knowledge and positively change their teaching practices. Effective professional development, then, results in increased teacher knowledge as well as changes in teacher beliefs and behaviors for improved support of student learning.
Effective professional development ultimately impacts student outcomes, such as achievement. The most immediate result of professional development, according to a meta-analysis of studies, is changes in teacher learning. A meta-analysis of professional development research found that it had a significant effect on teacher learning (Cohen’s $d = 0.90$) (Hattie, 2009). Professional development was found to have a smaller effect on teacher behavior in the classroom ($d = 0.60$) and even less of an effect on student learning ($d = 0.37$) (Hattie, 2009, p. 120).

Increasing teacher knowledge, then, may be one of the main goals of professional development. For mathematics teachers in particular, content knowledge and teaching strategies are of major concern (Ball, 1999, CBMS, 2012; Hill et al., 2005; Hill et al., 2008; Taton, 2015). However, teachers are unlikely to incorporate new teaching strategies into their classroom practice unless they have opportunities to experience those strategies and change their beliefs (e.g., Desimone, 2009; Wilkins, 2008). It is important to understand the process of teacher change and factors that are conducive to such change.

**Changing teacher beliefs and practices.** The purpose of professional development is to create changes in the instruction delivered by teachers. Thus, teachers should be able to transfer the understandings they gain in a professional development workshop to their classroom practices. However, according to McDonald (2012), until recently, little attention has been given to teacher learning from professional development. Much of the past professional development for teachers was transmission style with little active involvement from the teachers. While more recent professional
development has begun to involve active learning and a focus on beliefs, transfer of learning is still not adequately explored (McDonald, 2012).

Pajares (1992) asserts that the study of teachers’ beliefs is “critical to education” and can be “the single most important construct in educational research” (p. 327). In a review of the literature around beliefs, Pajares (1992) summarized several common assumptions that can be made about beliefs. Beliefs perpetuate themselves and are difficult to change. That is, people are more likely to remember information that fits with their current belief system and less likely to adapt to information that contradicts their belief system. This makes it difficult to change those core beliefs. Further, although “knowledge and beliefs are inextricably intertwined” (p. 325), beliefs have affective and evaluative components that filter and interpret new information. This screening process means that new events or phenomena are interpreted through the lens of the current belief system, thus redefining or distorting the processing of new information. Pajares (1992) asserts, “Beliefs play a critical role in defining behavior and organizing knowledge and information” (p. 325).

In order for teachers to change their practices, their beliefs must change first (Ball, 1996; Chapman, 1999; Desimone, 2009; Fullan, 1991; Goldsmith & Schifter, 1997; McDonald, 2012). Sowder (2007) reports that changing teacher beliefs is essential for teachers to experience growth in knowledge and practice. Desimone (2009) describes the outcome of professional development as a three-part process: increasing teacher knowledge and/or leading to changes in attitudes and beliefs; which lead to improved
classroom practice as teachers implement their new knowledge, skills, attitudes, or beliefs; which in turn leads to increased student learning.

Models of teacher change focus on the first step in Desimone’s (2009) process: increasing knowledge and changing beliefs. The process of increasing knowledge, changing beliefs, and changing actions is complex. Changing teacher beliefs requires “conditions in which the individual is faced with new information and experiences that come in conflict with established beliefs” (Philippou & Christou, 2002, p. 213). However, the process is more complex than “simply exposing mathematics teachers to alternative beliefs” and involves either deconstruction or construction (Chapman, 2002, p. 192). The first process, deconstruction, involves dismantling prior sets of beliefs that are incompatible with the new beliefs and replacing them with the new beliefs. The second process, construction, involved adding the new beliefs onto a compatible preexisting set of beliefs (Chapman, 2002). Similarly, Lappan et al. (1988) theorized that teachers underwent a cycle of unfreezing, changing, and refreezing of their beliefs. As teachers’ thoughts changed, their actions began to change (unfreezing), which further helped their beliefs to change (changing), which finally helped to change their established behaviors (refreezing).

Another model of teacher change, based on student teachers’ experiences during a methods course, involves a five-step process: “problematising current beliefs and practices,” “becoming aware of a new approach,” “exploring and testing alternative beliefs and practices,” “reflectively analysing benefits of the new approach,” and “changing one’s views of mathematics and one’s teaching practices” (Kaasila & Lauriala,
2010, p. 856). This model begins with teachers realizing the need for their current beliefs and knowledge to change and concludes with teachers analyzing the effects of their changed practices. Arends and Kilcher (2010) describe a similar model, in which learning new teaching skills involves “initial learning of the skills involved; experimenting with the new approach; extended practice; and opportunities for reflection” (p. 9).

Kaasila and Lauriala (2010) see the process of teachers’ mathematical learning and teacher change as both individual, via individual construction, and collective, via enculturation or socialization into a wider community’s mathematical practices. In the social and cultural context, teacher change occurs based “on mutual interaction and transitions from the group (community) level to the individual level” (p. 859–860). Thus, teacher change can follow from organizational change or changes to the social norms in the community. The authors theorize teacher change as a process of unfreezing at the group level then the individual level, moving or changing at the group level and then at the individual level, and then refreezing, again at the group level first and then at the individual level. During the study, peer mentors and novices created shared understandings, which enabled the novice to rethink their individual understanding. It was also important for emerging leaders amongst the peer group to internalize the changes.

Warford (2011) applies the concept of the zone of proximal development to teacher education to create a new concept, the zones of proximal teacher development. According to Warford, the first zone is that of self-assistance, or teacher reflection “on prior experiences and assumptions” (p. 253). The second is “expert other assistance” or
teacher assistance, during which the teacher experiences teaching in a classroom combined with research and theories on education (p. 254). The third stage is internalization (or automatization of new pedagogies), and the fourth is recursion (or de-automatization), the stage at which teachers confront the differences between their prior assumptions and current knowledge.

**Qualities of effective teacher professional development.** Professional development experiences vary widely. Many professional development experiences for teachers are top-down (Arends & Kilcher, 2010; Guskey, 2000), disconnected from the reality of the classroom (Guskey, 2000), and completed within a single session (Fullan, 1991). However, these types of professional development are typically not effective at increasing student learning (Fullan, 1991; Guskey, 2000).

Guskey (2000) defines high quality professional development as “a process that is (a) intentional, (b) ongoing, and (c) systematic” (p. 16). A review of recommendations for teaching professional development based on the research literature identifies several commonly cited elements of effective teacher professional development: focus on content knowledge, long-term sustainability, administrative support, collaborative learning, and active learning (e.g., Desimone, 2009; Doerr, Goldsmith, & Lewis, 2010; Hattie, 2009; Hill, Beisiegel, & Jacob, 2013; Joyce & Showers, 2002). Each of these elements is explored in more detail below.

**Content knowledge.** Most recommendations for mathematics teacher professional development agree that it should have a strong focus on content knowledge in order to improve the teaching and learning of mathematics (e.g., Arends & Kilcher, 2010; Ball,
Professional development that focuses on content knowledge has been shown to lead to increased teacher knowledge, changes in teaching practices, and increased student achievement (C. A. Brown & Borko, 1992; Desimone, 2009), more so than professional development that has a greater focus on pedagogy (Doerr et al., 2010). Further, increasing teachers’ content knowledge also “can help them develop the confidence to teach mathematical topics that they previously avoided” (Doerr et al., 2010, p. 2).

Increased teacher learning and skill is associated with opportunities to engage in inquiry and problem solving (Doerr et al., 2010). Similarly, Ball (1996) recommends that “teachers need opportunities to delve into some mathematics in depth, to engage in mathematical reasoning and problem solving, to compare and validate alternative solutions and methods, and…to experience a mathematical learning environment” (p. 39).

The type of content knowledge needed by teachers may also include a focus on pedagogical content knowledge and a deep understanding of concepts. For example, Doerr et al. (2010) recommends that content-focused professional development should help teachers develop their skills to “choose appropriate mathematical tasks, judge the advantages of particular representations of a mathematical concept, help students make connections among mathematical ideas, and grasp and respond to students’ mathematical arguments and solutions” (p. 1), which are all aspects of pedagogical content knowledge.
Often, content-focused professional development includes a focus on pedagogical content knowledge or mathematical knowledge for teaching.

**Long-term sustainability.** The most effective professional development experiences require sustained effort over a long period of time (C. A. Brown & Borko, 1992; Hattie, 2009). The more hours teachers spend in a given professional development setting, the greater the effect (Desimone, 2009; Doerr et al., 2010; Scher & O’Reilly, 2009). Many summaries of teacher professional development recommend that it is ongoing throughout the school year and includes frequent opportunities for teacher reflection (Desimone, 2009; Guskey, 2000; Lieberman & Miller, 2008b; Scher & O’Reilly, 2009). Specifically, Desimone (2009) reports that the greatest teacher outcomes occur as a result of professional development with 20 or more contact hours and either semester-long activities or intense summer experiences with follow-up throughout the year.

**Administrative support.** Several researchers report the importance of alignment with school district policies, which is sometimes termed coherence (Desimone, 2009; Hill et al., 2013). This includes support from school leadership (Hattie, 2009), such as embedding the learning within the school, connecting it to other professional development initiatives (Guskey, 2000), encouraging the teachers’ work, or providing collaboration time (Doerr et al., 2010). It should align with state standards and assessment (Doerr et al., 2010) and provide a clear vision (James & McCormick, 2009). It should also include collective participation by teachers from the same school or grade level (Desimone, 2009) to allow them to work together to change the school atmosphere.
and develop shared norms and practices (Lieberman & Miller, 2008b; McLaughlin & Talbert, 2006). It should build on staff expertise (James & McCormick, 2009).

**Collaborative learning.** Research has found that collaborative learning and social interaction are important aspects of successful professional development (Borko, 2004; Desimone, 2009; Doerr et al., 2010; Hattie, 2009; Hill et al., 2013; Lieberman & Miller, 2008b). Borko (2004) explains that teacher learning consists of enculturation, or learning the practices of a particular group, in addition to individual knowledge construction. Joyce and Showers (2002) recommend that teachers study together, apply what they learn, and share the results with one another. James and McCormick (2009) recommend that teachers have time to learn “from research and also working together to plan, try out and evaluate new ideas” (p. 982).

**Active learning.** Another element of effective professional development is active learning, meaning that the teachers are actively engaged in the workshop (Desimone, 2009; Doerr et al., 2010), such as examining and reflecting on their teaching practices (Arends & Kilcher, 2010). Teachers should be engaged in the learning process by extending their skills and challenging their ideas about teaching and learning (Hattie, 2009).

**Other elements of effective professional development.** Several other elements of effective professional development have been reported. According to Arends and Kilcher (2010), professional development communities tend to be more successful in the long term when they extend beyond a single school or district because they draw from a broader audience and periodically bring in fresh talent and ideas (also, Borko, 2004;
McLaughlin & Talbert, 2006). Professional development should be created with clear goals and outcomes in mind (Guskey, 2000) and incorporate teacher leadership (Arends & Kilcher, 2010). It should nurture productive habits of mind, beliefs, and dispositions, such as inquiry, curiosity, and self-monitoring, because “teachers’ beliefs about mathematics… influence what teachers learn from professional development” (Doerr et al., 2010, p. 2). Professional development should be practical (Hattie, 2009), relate to the classroom (Guskey, 2000), and focus on teacher actions such as noticing and responding to student thinking (Doerr et al., 2010). Larger effect sizes also came from programs that were developed outside of the school district and included both high school and elementary school teachers (Hattie, 2009).

**Professional learning communities.** One type of teacher professional development that includes many of the qualities of effective professional development is the professional learning community (PLC). PLCs are an extensively studied, effective, and high-quality form of professional development for K–12 educators (Arends & Kilcher, 2010; Beckmann, 2010). They also share many features with MTCs, and because they have been extensively studied, the research base may be applicable to MTCs.

PLCs are defined as “ongoing groups of teachers who meet regularly for the purpose of increasing their own learning and that of their students” (Lieberman & Miller, 2008b, p. 32). PLCs focus on critically examining teaching practices in an “ongoing, reflective, collaborative, inclusive, learning-centered, growth-promoting way” by relying on the shared expertise within a cohesive and supportive group (Stoll & Louis, 2007, p. 2). These communities provide a supportive environment for teachers to examine new
ideas, reflect, collaborate and learn from one another. PLCS can take place within a single school, a single district, or across schools and districts. They can focus on a single content area or grade range, or a variety.

Penlington (2008) describes two critical components for effective PLCS: teacher-teacher dialogue and building culture. Drawing on the philosophical theory of practical reason, Penlington (2008) argues that dialogue is effective for teacher change because “dialogues between teachers might have an effect on the practical reasoning within teachers” (p. 1314). The most effective teacher-teacher dialogue might occur when teachers feel comfortable questioning one another and when the dialogue evaluates teaching activities rather than simply recounting them. Because some theorize that effective professional development should focus on building culture, this culture should be developed in order to consider dissonant views and reflection of teaching practice.

PLCs have important implications for the classroom. Strong PLCS that led to increased student achievement tended to have the following characteristics: Teachers had shared norms about teaching, there was a focus on student learning, teachers engaged in reflective dialogue, teachers shared what happened in their classrooms, and teachers collaborated (Arends & Kilcher, 2010). According to Lieberman and Miller (2008b), PLCs have great potential to change teaching practices and improve student learning because they “challenge long-held ideas about how teachers learn; they enact professional development in nontraditional ways; and they position teachers as the leaders of their own professional growth and development” (p. 2). PLCS provide an opportunity for teachers to focus their development on increasing student learning.
Because PLCs focus on the development of relationships, they can also reduce feelings of isolation and promote the development of a teacher identity from “seeing themselves as ‘just a teacher’ to being part of a larger community” (Lieberman & Miller, 2008b, p. 2). Teachers learn from one another through conversations and forge new identities as members of the group (Lieberman & Miller, 2008a).

There are also several important implications of PLCs for mathematics instruction in particular. Borko (2004) suggests that teachers need to be part of their own nurturing community before they can create a similar community of learners in their classrooms. Teachers who engage in professional learning communities have also shown increased use of rich tasks and instructional strategies that support students’ deep learning (Borko, 2004).

PLCs may also provide the experience of being part of a mathematical community. According to Schoenfeld (1992), “‘having a mathematical point of view’ and ‘being a member of the mathematical community’ are central aspects of having mathematical knowledge” (p. 344) and developing a competent and successful mathematical identity. It is important for teachers to experience mathematics in the ways that mathematicians experience mathematics. Mathematicians often work collaboratively, and this type of collaborative work helps teachers to feel as though they have membership and belonging in a mathematical community.

**Mathematics teacher professional development.** Content-focused professional development for mathematics teachers at all levels is particularly important. According to the NRC (2001), students in the United States “tend to have a limited understanding of
basic mathematical concepts” and are “notably deficient in their ability to apply mathematical skills to solve even simple problems” (p. 4). Elementary school education is particularly important, as students often fail to develop mathematical proficiency in these early grades. In response to this growing concern about inadequate mathematical preparation of students, there has been a call for “early childhood educators to take up the challenge associated with mathematics achievement,” particularly with high-risk students (Benner & Hatch, 2009, p. 307).

However, an analysis of existing research and curricula for early childhood education found that “many early childhood settings do not provide adequate learning experiences in mathematics” (C. T. Cross et al., 2009, p. 2). Beyond early childhood, many K–8 teachers of mathematics are not certified in mathematics or have a “shaky grasp of mathematics” (NRC, 2001, p. 4). Research has found that some elementary school teachers have shallow mathematics content knowledge (Ball, 1999; NRC, 2001; Schifter, 1996). An analysis of research revealed that the preparation of preservice elementary school teachers was significantly lacking in mathematics content and pedagogy (C. T. Cross et al., 2009). Even experienced teachers with greater mathematical knowledge have difficulty with tasks that require strong pedagogical content knowledge, such as “translating their mathematical knowledge into tasks” for students to think and reason mathematically (Cooney, 1999, p. 166). Often, mathematics teachers use teaching methods that rely on the teacher as the provider of knowledge, but students develop deeper mathematical understanding when teachers are facilitators of learning instead (McLaughlin & Talbert, 2006).
It is important to note that these issues are systemic and are not the fault of the individual teachers. Elementary teachers’ lack of content knowledge and use of teaching strategies can be attributed to multiple factors, including their preparation programs, curricular materials, or being asked to teach outside of their primary certification area (NRC, 2001). Elementary teachers’ experiences with mathematics begin when they are students themselves and continue through their teacher preparation programs and professional development.

Researchers agree that teachers are heavily influenced by their experience as students, which often results in limited views about mathematics and teaching mathematics in ways that focus on computation and memorizing facts rather than as a subject for students to explore and debate (Ball, 1996; Hannula, 2002; Lutovac & Kaasila, 2011; Phillip, 2007; Schifter, 1996; Schoenfeld, 1992; Swars, 2005). Ball (1996) believes that changing teachers’ negative dispositions about mathematics requires “returning to mathematics themselves, to rebuild their sense of themselves as mathematical knowers and to make connections to mathematics as an arena of ideas and human activity” (p. 38). Similarly, Benner and Hatch (2009) recommend that teacher educators need to help preservice teachers “reconceptualize their own understanding of mathematics and what constitutes appropriate mathematical experiences for young children, while concurrently addressing their own uncertainties about basic mathematics content” (p. 308).

Professional development for mathematics teachers is another avenue for increasing the content knowledge and changing mathematics teaching strategies of
elementary school teachers. Taton (2015) argues that much like art teachers produce their own art, English teachers often read for pleasure, and music teachers play instruments on their own time, mathematics teachers should have experiences in doing mathematics themselves. According to Taton, “doing mathematics” is “like wrestling with a brain-teaser or creating a new dish. Real mathematics involves imagination, questioning, testing, and exploring” (p. 9).

Simply focusing professional development on content is not enough, however. Facilitators need to probe the teachers’ mathematical ideas, engage in deep discussion of the problems, and reflect on the process of solving problems (P. D. K. Cohen & Hill, 2001; Goldsmith & Schifter, 1997). Benner and Hatch (2009) recommend that professional development focus primarily on problem-solving strategies and how to help children build an understanding of mathematical concepts and engage in mathematical thinking. Successful experiences in doing mathematics experienced by elementary school teachers during professional development “fostered the confidence they needed to begin changing their approach to teaching mathematics” (Ball, 1996, p. 38). An immersive experience in mathematics problem solving and learning through inquiry can prepare teachers to incorporate problem solving in the classroom themselves (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2009), by providing a pedagogical model (Ball, 1996) or examples of teaching practices that can be used in the classroom (Loucks-Horsley et al., 2009).

In addition to increasing content knowledge and developing teaching practices, one of the goals of professional development is often for teachers to develop an
empowered, confident identity as a mathematics teacher (Sowder, 2007). Taton (2015) argues that mathematics teachers “require different PD opportunities than their peers” due to the “deeply-rooted negative perceptions of mathematics that reside in our culture” (p. 5). Teachers of mathematics may have mathematics anxiety from negative past experiences in mathematics. Mathematics anxiety in prospective elementary school teachers was mostly caused by their experiences in mathematics classes as elementary school students (Harper & Daane, 1998). Specifically, preservice teachers reported experiences that emphasized finding the right answers and using the right method, not making mistakes, not having enough time to work, and being afraid of word problems or problem solving (Harper & Daane, 1998). According to Sowder (2007), a confident mathematics teaching identity “is particularly difficult for elementary teachers to develop because of the mathematical anxiety many of them experience” (p. 168).

Identity development can be facilitated through professional development “that allows teachers to experience learning as an intellectual endeavor in which mathematics is explored together with others” (Sowder, 2007, p. 168); that is, positive experiences being successful in mathematics and working with others can lead to confidence in doing mathematics and, in turn, confidence as a teacher of mathematics. Sowder reports that empowered feelings of doing and teaching mathematics can lead teachers to more positive outcomes, such as working collaboratively and seeking to further their knowledge and seeking improvement in their teaching practice. Further, teachers who feel passionate about teaching mathematics often see a moral component in their teaching: They feel that promoting mathematical exploration with students will lead
toward a better society (Sowder, 2007). In other fields, content-focused professional development has been shown to change teachers’ beliefs. In one study, professional development focused on inquiry and science changed teachers’ beliefs about science and lead to more investigation-focused lessons (Luft, 2009). Thus, mathematics teacher professional development with the specific aim of developing mathematics teacher identity and mathematical identity are crucial.

**Identity**

The process of collaborative learning, according to situated learning theory, includes becoming increasingly more enculturated in the group. This process of change can have a profound effect on how a person thinks about her or himself. For example, a teacher engaged in an MTC might begin to think about her or himself as a problem solver. These experiences can begin to have an effect on a person’s identity and, because identities shape actions, can change teachers’ teaching practices.

Teacher change can be examined through the lens of identity. Ketelaar, Beijaard, Boshuizen, and Den Brok (2012) explore teacher change through three concepts closely related to teacher identity: ownership, sense-making, and agency. Ownership is “a facilitator of expressing who one is as a teacher and what one finds important” (p. 273). Sense-making “involves the interaction between one’s identity and the innovation, resulting in maintenance or alteration of one’s identity” (p. 273). And agency is “a vehicle to give direction to one’s career as a teacher and stay true to oneself” (p. 273). These three concepts relate closely to both teacher identity and changing teacher practice.
**Definitions of identity.** Identity is a complex concept that is difficult to define. It is generally agreed that identity is strongly related to interactions with other people, but it also has an internal component. It is “man-made and...constantly created and re-created in interactions between people” (Sfard & Prusak, 2005, p. 15). It is both “the way we define ourselves and how others define us” (R. Anderson, 2007, p. 8). It involves recognition by others as being “a certain ‘kind of person,’ in a given context” (Gee, 2001, p. 99). It includes our perceptions of past interactions as well as our aspirations of ourselves in the future and who we want to become (R. Anderson, 2007; Beijaard, Meijer, & Verloop, 2004). Identities are dynamic and malleable, changing as a result of experiences and interactions with other people (Beijaard et al., 2004; Wenger, 1998). Identity has been defined as “collections of stories about persons” that are repetitive and reifying, endorsed by the individual, and significant enough to affect someone else’s feelings about the individual (Sfard & Prusak, 2005, p. 16; see also Beijaard et al., 2004). These stories or narratives are told and retold until they become ubiquitous and self-evident to the individual, making them part of the person’s sense of identity. Identities may or may not be desired by the individual: On one hand, they may have been sought out, and on the other, they may have been imposed by others (Gee, 2001).

While identities are malleable, they are also self-reinforcing. Once an identity is developed, a person is able automatically and without reflection to endorse or reject statements that are or are not in agreement with his or her identity (Sfard & Prusak, 2005). Identities shape people’s actions (Sfard & Prusak, 2005). Further, identities are important to consider in relation to teaching, because there is a strong relationship
between teachers’ identities and their classroom practices (Gresalfi & Cobb, 2011). Changes in teacher identities are a necessary precursor to changing teaching practices (Gresalfi & Cobb, 2011).

**Theories of identity.** Gee (2001) differentiates among four types of identities: nature identity, discourse identity, institutional identity, and affinity identity. The nature identity relates to characteristics determined by nature, such as being a twin. This type of identity is the least relevant to this study. Discourse identity is an individual trait that is recognized via interactions or discourse with others, such as being recognized as a charismatic person. Institutional identity relates to one’s position as recognized by authorities within a given institution, such as being recognized by school officials as a teacher. This type of identity depends on recognition in a particular defined role, such as mathematics teacher, by a legitimized authority. The final type of identity is affinity identity. Affinity identity relates to experiences undertaken by an individual and how those practices are recognized and shared by members in affinity groups. This type of identity is relevant for people who participate in groups, such as professional teaching communities. In this case, a teacher might be recognized as a good teacher based on their classroom practices and the value that the members of the community place on those practices.

Cobb et al. (2009) developed a framework for identity development that consists of two interrelated identities: normative identity and personal identity. In this framework, normative identity is jointly developed by members of a group. It consists of a set of desirable practices for people in that group. Individuals develop personal identities in
relation to the normative identity as they participate with the group. In the context of mathematics classrooms, teachers and students construct the normative identity for doers of mathematics in that classroom, and individual students develop their personal identities as they participate in the class. Personal identity “concerns the extent to which individual students identify with, merely comply with, or resist their classroom obligations” (Cobb et al., p. 44). Normative and personal identity together describe the ways in which students “come to understand what it means to do mathematics as it is realized in their classroom and with whether and to what extent they come to identify with that activity” (Cobb et al., 2009, p. 42).

Another aspect of identity to consider is whether the identity is actual or desired. Sfard and Prusak (2005) differentiate identity narratives between actual identities, which consist of “stories about the actual state of affairs,” and designated identities, which consist of “narratives presenting a state of affairs which, for one reason or another, is expected to be the case, if not now then in the future” (p. 18). Learning is necessary for individuals to evolve from their actual identities to the designated identities that they hope to embody in the future (Sfard & Prusak, 2005).

**Aspects of identity.** The construct of identity is broad and includes a variety of factors. The following aspects of identity are constructs that are related to identity: beliefs, attitudes, motivation, self-concept, and self-efficacy. One’s identity as a doer of mathematics is likely to contain some of these beliefs and dispositions; however, it may not include all of them. For example, some peoples’ mathematical identities include self-efficacy beliefs, but others’ mathematical identities do not. The constructs considered
here are also interrelated and affect one another. For example, several of the constructs related to motivation also have a beliefs component. Although these constructs could be considered separately and in their own right, for the purpose of this study, I consider these constructs as they fall within the larger construct of identity. These descriptions focus on how these components are part of mathematical identity and mathematics teaching identity. The descriptions often include studies of students, as teacher mathematical identity and mathematics teaching identity have not been fully developed.

**Beliefs.** Beliefs, attitudes, intentions, and behavior together form a hierarchy, in which beliefs are the most deeply held and most difficult to change. Beliefs influence attitudes, which influence intentions, which finally influence behavior (Fishbein & Ajzen, 1975). Thus, while all four aspects of this hierarchy are important for changing behaviors, beliefs undergird the other three. The notion that people act in accordance with their beliefs is well accepted (e.g., Felbrich, Muller, & Blomeke, 2008; Leatham, 2006; Rokeach, 1968).

A *belief* is defined as “any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase ‘I believe that’ ” (Rokeach, 1968, p. 113). Beliefs are the information that a person has about an object, person, or issue; and they are based on experiences, feelings, and emotions (Petty & Cacioppo, 1996; Philippou & Christou, 2002). They may or may not be based on fact (Petty & Cacioppo, 1996; Philippou & Christou, 2002). Beliefs have “an orienting as well as explaining function,” which is to say that they describe an orientation toward a belief as good or bad, but they also provide a reason or explanation for that orientation (Felbrich
et al., 2008, p. 763). The content of a belief may evaluate an object as “true or false, correct or incorrect…good or bad…desirable or undesirable” (Rokeach, 1968, p. 113). Beliefs are difficult to change (Rokeach, 1968).

Beliefs “always occur in sets or groups,” or belief systems (T. F. Green, 1971, p. 41). Belief systems represent “the total universe of a person’s beliefs about the physical world, the social world, and the self” (Rokeach, 1968, p. 123). Belief systems are organized in a structure such that some beliefs are primary and others are derived from other beliefs (T. F. Green, 1971).

It is also possible to hold conflicting beliefs, which helps to explain why people sometimes appear to act contrary to their beliefs. Beliefs are clustered, with little interaction between clusters (T. F. Green, 1971). Different clusters of beliefs may be contradictory to one another. Although at times it may appear that a person acts counter to their beliefs, that person is simply acting in accordance with a different cluster of beliefs that took precedence at that moment (Leatham, 2006; Rokeach, 1968).

Further, not all beliefs hold equal importance to the individual (Rokeach, 1968). Beliefs may be central or peripheral (T. F. Green, 1971). Beliefs that are more central hold the “greatest psychological strength” (p. 46), are typically accepted without questioning, and are least likely to be debated and/or changed. More peripheral beliefs, then, are held with less strength and the individual is “more prepared to examine, discuss, and alter” these beliefs (T. F. Green, 1971, p. 46). Finally, beliefs may be held evidentially (with the support of evidence) or nonevidentially (T. F. Green, 1971). Beliefs that are held nonevidentially are much more difficult to change, because they are not
modified by the introduction of evidence or reasons to support or contradict them. Beliefs based on evidence are more easily modified in the presence of rational criticism.

Attitudes. While beliefs ultimately influence actions, attitude is more directly related to action. Attitude is defined as the favorable or unfavorable predisposition a person has toward an object, person, or issue. An attitude toward a particular object is based upon an organization of multiple beliefs around the object. In most instances, people have both positive and negative beliefs about objects, and attitude combines these belief orientations into a single effect (Petty & Cacioppo, 1996). Attitudes predispose a person “to respond in some preferential manner,” either positive or negative (Rokeach, 1968, p. 112). Attitude changes occur as a result of either a change in the content of a belief or a change in the belief structure or organization (Rokeach, 1968).

Motivation and persistence. Motivation can be a key factor in explaining why students engage in mathematics learning and why they are successful in mathematics. Middleton and Spanias (1999) define motivations as “reasons individuals have for behaving in a given manner in a given situation” (p. 66). In terms of learning, the experience of success in mathematics can have “a powerful influence on the motivation to achieve” (Middleton & Spanias, 1999, p. 68). Students who experience success are more likely to enjoy mathematics and will be more likely to expend effort into trying to succeed again.

A synthesis of the research in education found that much of the research on motivation reports on internal versus external beliefs (Hattie, 2009). Students with internal beliefs take personal responsibility for their learning and attribute success to their
ability, while students with external beliefs believe that their learning is beyond their control, and they attribute success (or failure) to luck or chance. Internal motivation tends to arise from experiences that provide feelings of competence, autonomy, and relatedness; while external motivation develops in environments that people perceive to be externally controlling (Beckmann, 2010). Mathematics learning environments can help develop internal motivation by encouraging autonomy, supporting sense-making, and providing challenging experiences at the appropriate level for learners to experience success.

Internal and external factors have been described in various theories of motivation. Concepts related to internal and external beliefs are intrinsic motivation, flow theory, attribution, goal theories, and implicit theories, each of which is described in greater detail below.

**Intrinsic motivation.** Intrinsic motivation “refers to doing something because it is inherently interesting or enjoyable” while extrinsic motivation “refers to doing something because it leads to a separable outcome” (Ryan & Deci, 2000, p. 55). In academic settings, students with intrinsic motivation engage in learning for learning’s sake; they enjoy learning and it relates to their self-image. Students with extrinsic motivation are motivated to learn in order to obtain rewards, such as good grades or approval from adults, or to avoid negative experiences such as bad grades or disapproval.

Students with intrinsic motivation achieve better outcomes than those with extrinsic motivation. For example, these students possess greater mathematics self-efficacy and are more likely to display persistence, creativity, risk taking, and other

It is important to remember that motivation is not dichotomous. People are simultaneously motivated by both intrinsic and extrinsic factors. According to Cognitive Evaluation Theory (Ryan & Deci, 2000), “feelings of competence during action,” even when the action is extrinsically motivated, “can enhance intrinsic motivation for that action” as long as learners have a sense of autonomy about the action (p. 58). Effective teaching strategies that draw from constructivist learning theory, then, can help students to develop intrinsic motivation regardless of their initial motivations.

Mathematics education researchers and theorists have explored intrinsic motivation specific to mathematics learning. Some recommend particular classroom experiences to assist students’ development of intrinsic motivation in mathematics. According to Middleton and Spanias (1999), “students must feel comfortable about mathematics, must be challenged to achieve, and must expect to succeed before the development of intrinsic motivation can begin” (p. 67). Further, students will be more likely to enthusiastically engage in an activity if they find it interesting, and they base their interest on the “stimulation (challenge, curiosity, fantasy) it provides and the
personal control (free choice, not too difficult) the activity affords” (Middleton & Spanias, 1999, p. 75). This effect may be explained by flow theory, a theory of motivation related to intrinsic motivation (Csikszentmihalyi, 2000).

**Flow theory.** According to flow theory, intrinsically motivated people find activities rewarding when several criteria are met: First, they have the skills necessary to accomplish a task, the task is at a high level of challenge, and they have positive feelings toward the task (Csikszentmihalyi, 2000). In general, the challenge of the task must be proportional to the skill level of the student (or within the zone of proximal development) in order to achieve flow. When achieving flow, students have an optimal, joyful experience in a learning situation. In this theory, flow increases motivation and engagement.

**Attribution theory.** Attribution theory is particularly important for mathematics educators, as Middleton and Spanias (1999) claim it is one of “the most widely held of the theoretical orientations” (p. 69) in mathematics education. Attribution theories relate to the factors to which students attribute their success or failure (Weiner, 1972). Students hold either dispositional or situational attribution. Students with dispositional attribution beliefs attribute outcomes to internal factors like their ability and effort. Students who hold situational attribution beliefs attribute outcomes to external factors which they cannot control, such as the difficulty of the task (Petty & Cacioppo, 1996).

Similarly, Dweck (2000) has described people’s responses to failure as either helpless or mastery oriented. The helpless reaction occurs when students believe that if they fail, it was out of their control and they cannot do anything to achieve success. On
the other hand, mastery-oriented reactions focus on “achieving mastery in spite of their present difficulties” (Dweck, 2000, p. 6). Learned helplessness is a consequence of attribution theory. When students believe that “success is out of their grasp and attribute failure to internal factors, learned helplessness often becomes perceived as a trait” that they cannot change (Middleton & Spanias, 1999, p. 71).

The judgments that students make about the causality of success or failure have an effect on their future academic effort and persistence. According to Weiner (1972), “causal attributions influence the likelihood of undertaking achievement activities, the intensity of work at these activities, and the degree of persistence in the face of failure” (p. 213). These behaviors will then influence learning and achievement (Weiner, 1972). In general, “when students attribute their successes to ability, they tend to succeed; when they attribute their failures to lack of ability, they tend to fail” (Middleton & Spanias, 1999, p. 70). Alarmingly, girls are more likely to attribute successes to luck and failures to lack of ability.

In mathematics classes, it is important that students experience success in problem solving. Problems should be structured so that students are challenged at an appropriate level and given opportunities to draw on their abilities and knowledge. This can help them to see that their success is not due to luck but to persistence and hard work. Further, problems at an appropriate level of complexity may provide opportunities for students to experience the cycle of failure and success that is typical of mathematical modeling. Experiencing setbacks can help students to see that failure can be a positive experience in learning and in problem solving. The implications of attribution theory are that students
would develop stronger dispositional attribution theories for their successes and, as a result, perform better in mathematics in the future.

*Goal theories.* According to goal theories, students may have either performance goals or learning goals. Achievement goals are the goals that students have that help to determine if their response to outcomes will be helpless or mastery oriented (Dweck, 2000). Performance goals are goals students have to achieve external rewards, such as positive judgments from peers and teachers about their performance. Learning goals relate to increasing skill and competence. While both types of goals are motivating and can lead to achievement, students who place more importance on performance goals are more likely to have helpless responses to failure, and they tend to not learn as much. However, students with learning goals tend to embrace challenges, have greater persistence, and have mastery-oriented responses. Mathematics learning environments, then, should promote learning goals rather than performance goals in order to improve student learning, persistence, and risk taking.

*Implicit theories.* The implicit theory of motivation describes the different belief systems that people have about intelligence and other personal characteristics (Davis, Burnette, Allison, & Stone, 2011). It is similar to attribution theory but relates specifically to character traits and abilities (rather than the attribution of success and failure). Dweck’s (2000) work on implicit theories describes two different sets of beliefs people may hold about intelligence: entity theories and incremental theories. First, people who believe that intelligence (and other characteristics) are fixed traits hold entity theories. They believe that intelligence is innate and cannot change. Students with entity
theories are concerned with appearing stupid, so they seek out easy tasks in order to succeed and appear to be smart. On the other hand, people who believe that intelligence (and other characteristics) can be developed through learning hold incremental theories. They believe that effort can help improve their intelligence. These people are more motivated to face challenges or problems that are unfamiliar and believe that persistence pays off. These beliefs about intelligence are closely tied to people’s self-concepts and self-efficacy beliefs. In particular, students who believe that mathematics ability can be achieved through learning (that is, incremental beliefs), tend to have greater self-efficacy and stronger identities as mathematics learners, and vice versa.

The implicit theories that are held by learners have an effect on self-efficacy and feelings of helplessness. Undergraduate students with incremental beliefs are more likely to have higher emotional self-efficacy (Tariq, Qualter, Roberts, Appleby, & Barnes, 2013). When positioned in an “underdog” status in a mathematics competition, college students with incremental theories of ability demonstrated greater mathematics self-efficacy, whereas students with entity theories of ability experienced greater feelings of helplessness and lower mathematics self-efficacy (Davis et al., 2011). Entity beliefs can lead students with low or negative mathematics self-concept to attribute their success in mathematics to luck and attribute their failure in mathematics to their lack of ability (Schoenfeld, 1989).

The type of implicit theories held by students ultimately affects their motivation and mathematics achievement. Students possessing incremental theories of intelligence at the start of middle school achieved increased grades in mathematics classes over two
years, while students with entity theories of intelligence maintained a flat trajectory of mathematics achievement (Blackwell, Trzesniewski, & Dweck, 2007). The teaching of implicit theories has been shown to have a positive effect, as well. Middle school students who were explicitly taught incremental theories of intelligence increased both motivation and mathematics achievement, while a control group experienced a decline in motivation and achievement (Blackwell et al., 2007).

Adolescents’ academic achievement can be predicted by their implicit theories and beliefs about intelligence (Romero, Master, Paunesku, Dweck, & Gross, 2014). Students with incremental beliefs of intelligence earn higher grades in middle school mathematics and eventually take more advanced mathematics courses. Further, students who possess entity theories of mathematics intelligence and view tests as measuring their ability are less likely to take risks by tackling challenging problems and more likely to be discouraged by failure (NRC, 2001). Because persistence and risk taking are important skills for mathematical problem solving, it is important to help students develop incremental theories. Some research has shown that students can be taught to change their beliefs from entity theories to incremental theories (Blackwell et al., 2007). Environments in which students see that their success comes from effort and hard work and that learning can help them to succeed may help students to develop incremental theories.

Summary of motivation and persistence. A learner’s motivations to engage in doing mathematics can be part of mathematical identity when it influences how they think about themselves when it comes to doing mathematics. They may have intrinsic motivation to persist in solving problems. They may make judgments about their
successes or failures being due to their ability or their hard work, which influence their perceptions of themselves as being capable mathematics learners. The types of goals that learners set for themselves can help them to embrace challenges. And, perhaps most importantly, whether a learner holds entity or incremental theories about mathematics intelligence or skill can have an effect on how they see themselves as mathematics doers.

**Self-concept.** Self-concept is defined as “one’s collective self-perceptions formed through experiences with and interpretations of the environment” and is heavily influenced by the perceptions of others (Schunk & Pajares, 2007, p. 88). These beliefs are oriented towards the past and based on past experience. Self-concept is considered to be more stable than self-efficacy because these beliefs are more general and much less task specific (Schunk & Pajares, 2007). Beliefs such as “I am very good at mathematics” or “I enjoy doing mathematics” relate to self-concept.

Aspects of students’ mathematical identities related to self-concept can affect their effort and motivation in mathematics, thereby reinforcing their identities and becoming self-fulfilling. For example, students’ decisions for whether or not to study or do their mathematics homework have been shown to be based on their mathematical identities as being either gifted at mathematics or bad at mathematics (Browne, 2009). Similarly, student behavior in group work can be based on self-concept and become self-fulfilling. In one study, a student who saw herself as smart in mathematics behaved with greater confidence in her abilities, drawing attention to her ideas while dismissing others’ ideas, while a student who saw herself as dumb in mathematics waited for others to make decisions and did not press her ideas on others (Bishop, 2012). Similarly, another study
found that mathematics self-concept is significantly correlated with motivation ($R^2 = .63$) for students in Kenyan secondary schools (Githua & Mwangi, 2003). In another study, students transitioning from secondary school to college saw college mathematics as a challenge they could overcome and an opportunity to grow and take personal responsibility for their mathematics learning (Hernandez-Martinez et al., 2011). These studies are both examples of the self-fulfilling aspect of mathematical identities and self-concept beliefs.

**Self-efficacy.** Self-efficacy, on the other hand, is defined as “one’s perceived capabilities to learn or perform behaviors at designated levels” (Schunk & Pajares, 2007, p. 85). Self-efficacy is very task dependent or context dependent and oriented toward the future, so these beliefs may not transfer to other situations at other points in time (Schunk & Pajares, 2007). These beliefs are also considered to be fairly malleable (Schunk & Pajares, 2007).

Mathematics self-efficacy is specific to future-oriented beliefs about one’s ability to solve mathematics problems. It is defined by Hackett and Betz (1989) as “a situational or problem-specific assessment of an individual’s confidence in her or his ability to successfully perform or accomplish a particular task or problem” (p. 262). Mathematics self-efficacy is thought to be very specific to the content and/or mathematics problem and does not provide a general level of confidence in one’s ability or understanding of a concept. The research literature strongly indicates that students’ mathematics self-efficacy is positively correlated with positive outcomes, including motivation, mathematics achievement, and problem-solving performance.
According to Bandura (1989), “people’s self-efficacy beliefs determine their level of motivation, as reflected in how much effort they will exert in an endeavor and how long they will persevere in the face of obstacles” (p. 1176). People with greater senses of self-efficacy tend to visualize success, which then “provide positive guides for performance” (Bandura, 1989, p. 1176). People with greater self-efficacy beliefs have greater resilience and perseverance, and hold the belief that they have control over events that happen to them. They are much more adept at overcoming failure.

In general, students with greater mathematics self-efficacy beliefs have higher mathematics achievement (Hackett & Betz, 1989; Pajares & Graham, 1999; Schoenfeld, 1989). Mathematics self-efficacy was shown to be a significant predictor of mathematics achievement for fifth grade students in Taiwan (Chang, 2012) and gifted middle school students in the United States (Pajares & Graham, 1999). Mathematics self-efficacy was positively correlated with academic performance for middle school (Cupani, de Minzi, Pérez, & Pautassi, 2010), high school (Schoenfeld, 1989) and college students (Hackett & Betz, 1989).

Similarly, mathematics self-efficacy has a positive effect on problem-solving performance for high school students (Pajares & Kranzler, 1995), gifted middle school students (Pajares, 1996), and undergraduate students (Pajares & Miller, 1995). Other research revealed that students with greater self-efficacy beliefs engage in problem-solving actions such as reflecting, generating hypotheses when experiencing difficulty, and generating questions and novel problems (Cifarelli & Goodson-Espy, 2007). Students with lower self-efficacy beliefs did not reflect on previous work, did not know what to do
when stuck, and were only able to replicate simple results (Cifarelli & Goodson-Espy, 2007).

Mathematics self-efficacy may play a more significant role in students’ mathematics achievement and motivation than other variables. For example, mathematics self-efficacy had a greater effect on mathematical literacy scores than interest in and enjoyment of mathematics, anxiety in mathematics, disciplinary climate for mathematics lessons, self-concept in mathematics, sense of belonging at school, instrumental motivation in mathematics (Güzel & Berberoğlu, 2010), race, gender, or homework-related variables (Kitsantas, Cheema, & Ware, 2011). Pajares and Miller (1994) found that “math self-efficacy was more predictive of problem-solving than was math self-concept, perceived usefulness of mathematics, prior experience with mathematics, or gender” (p. 193). Further, Wolters and Pintrich (1998) found that mathematics self-efficacy has a greater effect on grades than interest in mathematics or the use of cognitive and self-regulatory strategies.

In any discussion of self-efficacy, it should be noted that self-efficacy must be self-reported, and thus, it may not reflect actual mathematics ability. In fact, when students are assessed on mathematics self-efficacy for specific mathematics problems and then are asked to complete the mathematics problems, they tend to be overconfident about their abilities. Overconfidence is reported when a student marks high self-efficacy for a given problem and then answers the problem incorrectly. Similarly, underconfidence is reported when a student marks low self-efficacy for a problem and then answers the problem correctly (Pajares & Graham, 1999). High school students
(Pajares & Kranzler, 1995), middle school gifted students (Pajares & Graham, 1999), kindergarten students (Tirosh, Tsamir, Levenson, Tabach, & Barkai, 2013), and undergraduate students (Tariq et al., 2013; Young & Ley, 2002) all displayed overconfidence. However, while middle school students in general had overconfidence, girls typically displayed more underconfidence in their abilities (Pajares, 1996). Middle school girls’ self-efficacy have been found to be misaligned with their actual achievement in mathematics; in one study, girls tended to have higher grades in mathematics than boys, but their self-efficacy scores were generally lower (Wolters & Pintrich, 1998).

Overall, the research indicates that students with greater mathematical self-efficacy have greater mathematics achievement and perform better on problem-solving tasks. Mathematics self-efficacy was a stronger predictor than mathematics self-concept, interest in mathematics, or beliefs about mathematics, but these factors also have an effect on problem solving and achievement. Implicit theories also have important implications for mathematics learning. Students who possess entity theories within their personal or normative mathematical identity have worse outcomes than students who possess incremental theories. Those with incremental theories of mathematics ability have greater motivation, self-efficacy, and mathematics achievement, and lessened feelings of helplessness. More importantly, it is possible for teachers and classroom environments to promote the development of incremental theories.

**Summary of aspects of identity.** Each of the above aspects of identity—beliefs, attitudes, motivation and persistence, self-concept, and self-efficacy—contribute to the understanding of identity as a whole. Each of these elements is also part of developing a
strong identity. The next section explores these elements in the context of mathematical identity.

**Mathematical Identity**

Identity development can take place within learning situations. According to R. Anderson (2007), “learning mathematics involves the development of each student’s *identity* as a member of the mathematics classroom community” (p. 7). Teachers in MTCs are engaged in learning mathematics and thus may be developing their own identities as members of the MTC community. These identities may relate to doing mathematics, as the teachers engage in problem solving during the workshop, or to teaching mathematics, as the teachers reflect on their own current or future teaching practice. This section focuses primarily on mathematical identity, or the identities that teachers have in relation to themselves as doers of mathematics, and the next section explores mathematics *teaching* identity.

Definitions of mathematical identity draw from definitions and frameworks of identity in general, and they include emotions and beliefs related to doing mathematics. As discussed in the definition, some authors use the phrase *mathematics* identity rather than mathematical identity to describe a similar construct. Mathematics identities “include stories related to how one interacts with mathematics both in and out of school” (McCulloch et al., 2013, p. 380). Cobb et al. (2009) include a range of affective factors in their definition of mathematics identity, including “persistence and interest in mathematics and their motivation to learn mathematics” (p. 41). It includes the ways in which “students think about themselves in relation to mathematics and the extent to
which they have developed a commitment to, and have come to see value in, mathematics as it is realized in the classroom” (Cobb et al., 2009, 40–41). Another definition of mathematics identity incorporates beliefs about “(a) their ability to perform in mathematical contexts, (b) the instrumental importance of mathematical knowledge, (c) constraints and opportunities in mathematical contexts, and (d) the resulting motivations and strategies used to obtain mathematics knowledge” (Martin, 2000, p. 19). Many people may include feelings of mathematics anxiety, or “apprehension and anxiety toward mathematics” in their mathematics identity (Harper & Daane, 1998, p. 30). People with high mathematics anxiety may feel helpless, insecure, or fearful when asked to work on mathematics. Students develop their mathematics identities as they experience mathematics in school and as they think about the role of mathematics in their futures (R. Anderson, 2007).

In addition to the classroom environment, interactions with teachers and peers can contribute to the development of a strong identity as a mathematics learner for both students and teachers (R. Anderson, 2007; Lim, 2008; Martin, 2000). Students’ mathematics identities are partially formed “based on the perception of their peers viewing them as mathematically competent” (Keck-Staley, 2010, p. 27) and their beliefs about the way their peers treat them (Martin, 2000), as well as their teachers’ perceptions of their mathematical ability (Heyd-Metzuyanim, 2013). Teachers often ground their mathematics identities in their experiences as students. Teachers who wrote mathematics autobiographies during a professional development workshop focused on the people who influenced their mathematics identities (McCulloch et al., 2013). For the teachers in the
study, “relationships with self, family, peers, and most importantly, teachers, seemed to be the most influential aspects” on their mathematics identities (McCulloch et al., 2013, p. 388). Further, the amount of support provided by people in their lives was an influential factor on whether these teachers developed positive or negative attitudes toward mathematics and about themselves as mathematics learners, and those teachers whose poor self-images evolved into empowered mathematics identities all identified a specific supportive teacher. For female university students, relationships and interactions with tutors were particularly important to their confidence in mathematics (Solomon, Lawson, & Croft, 2011).

While the concept of mathematics or mathematical identity has not been fully explored for teachers, mathematics identity has been explored in greater deal for students. Strong versus weak mathematics identities have been connected to mathematics learning and motivation. A strong, empowered mathematics identity incorporates beliefs about the importance and application of mathematics and “personal identities and goals” (Martin, 2000, p. 119). Similarly, “the ways students see mathematics in relation to the broader context,” such as mathematics in everyday life, in future college education, or in future careers, “can contribute either positively or negatively to their identity as mathematics learners” (R. Anderson, 2007, p. 9). If students cannot make connections between school mathematics and their lives, then their identity does not include the need for mathematics learning.

**Mathematical identity formation.** Mathematical identity for students has been theorized in two different ways. Martin (2000) created a hierarchy for the influences on

Martin’s (2000) four-level framework for the mathematics identity formation among African Americans was based on interviews with high school students, parents, and teachers. The levels are sociohistorical, community, school, and intrapersonal. Although this framework is specific to African American students, it has potential for adaptation to other demographic groups. According to Martin (2000), students’ mathematics identities are partially based on their beliefs about their mathematics ability, motivation to learn, and whether or not they believe that mathematics ability is something you are born with and have no control over.

The first part of the framework, sociohistorical, “refers to the historically based discriminatory policies and practices that have prevented African-Americans from becoming equal participants in mathematics and other areas of society” (Martin, 2000, p. 29). It is a broad notion that influences students’ personal mathematics identity as they go through school. The next part of the framework, community, focuses on peoples’ “beliefs about mathematics abilities, their motivations for obtaining mathematical knowledge, their beliefs about the instrumental importance of mathematics, their relationships with school officials and teachers, and their socioeconomic goals and expectations for themselves and their children” (p. 31). This level of the framework focuses primarily on beliefs that are held by parents and community members, but they influence students’
mathematics identity formation. The third level, school, is primarily focused on “the negotiation of mathematical and social norms” that takes place in classrooms (Martin, 2000, p. 31). This level is influenced by the teachers’ beliefs about mathematics as well as relevant aspects of the student culture, including low achievement and motivation. The fourth level is intrapersonal or “agency and mathematics success among African-American students” (Martin, 2000, p. 31). This level focuses on the beliefs of successful African-American students that contributed to their success in mathematics in light of the negative forces of the other four levels. Themes in this level include “personal identities and goals,” “perceptions of school climate, peers, and teachers,” “beliefs about mathematics abilities and motivation to learn,” “beliefs about instrumental importance of mathematics knowledge,” and “beliefs about differential treatment from peers” (p. 119).

These four levels of the framework interact to influence students’ mathematics identity development. The framework posits that identity development depends upon a variety of factors that are internal and specific to the student, as well as in the larger historical and social contexts. Other research has also shown the influence of family and cultural backgrounds on the development of adolescent girls’ mathematics identity (Lim, 2008), as well as the influence of socioeconomic status and parenting style on fifth grade students’ mathematics identities (Chang, 2012), lending support to Martin’s framework and applications for diverse groups.

R. Anderson (2007) developed an alternative framework to describe the mathematics identity development of rural high school students and how they “come to know who they are relative to mathematics” (p. 7). R. Anderson (2007) draws upon
Wenger’s (1998) discussion of three modes of belonging—engagement, imagination, and alignment—and adds a fourth, nature. These four modes of belonging, or “faces” of mathematics identity, are ways in which students come to see themselves as capable or incapable mathematics learners.

The first face, engagement, is described as the view students have of themselves as someone who has or has not learned mathematics through their engagement with mathematics, teachers, and peers. If students are successful when they engage with mathematics, they begin to develop an identity as capable mathematics learners. On the other hand, if the students engage with mathematics and do not experience success, these students may identify as “marginally part of the mathematics learning community” (R. Anderson, 2007, p. 8). The style of classroom environment and type of learning that is prioritized in the class affect this view. For example, if the class encourages rote learning, students begin to see themselves as good at mathematics only if they are successful at memorizing pieces of information or calculating correct answers quickly. The students who cannot calculate as quickly see themselves as incapable of doing mathematics, even though they may be skilled at other aspects of mathematics.

The second face of mathematics identity is imagination. This face includes the ways that students see mathematics fitting in with their lives at the present and in the future. Some students, for example, believe that high school mathematics classes will be useful in further education or in future careers, so they choose to enroll in more mathematics classes. But other students do not see mathematics as important for their future careers: Either they do not see mathematics beyond simple arithmetic present in
careers, or they do not know people who formally studied advanced mathematics or appear to use it in their careers. Thus, the view that students have of mathematics in the broader context of the world influences their vision of themselves as mathematics learners.

The third face of mathematics identity is the alignment of students’ actions to their imagination face of identity. For example, students who see mathematics as necessary for college or careers may choose to align their actions with college admission requirements. These students see themselves as mathematics learners based upon those requirements. On the other hand, students who meet only the minimal mathematics requirements for graduation are less likely to see themselves as mathematics learners.

Nature is the fourth face of identity, and it concerns the beliefs that students have about their innate and unchangeable traits. This relates directly to entity theories and incremental theories (Dweck, 2000), in which students may believe that people who are good at mathematics are genetically gifted and that mathematics abilities are unchangeable. These beliefs directly contribute to the ways in which students see themselves as capable or incapable of learning mathematics. R. Anderson (2007) describes the nature face as the most “unsound and unfounded” (p. 11) views of identity as a mathematics learner.

The four faces defined by R. Anderson (2007) describe four separate ways to view oneself as a learner or doer of mathematics, but the four faces interrelate and affect one another. For example, a student’s imagination face can influence their alignment face if the student believes that mathematics is important for their future career and they
subsequently enroll in more mathematics classes. Similarly, a student’s nature face can influence their engagement face. If, for example, a student believes that their mathematics abilities are fixed, then they may choose not to explore a challenging mathematics problem. Together, a student’s views of each face combine to create the mathematics identity.

McGee (2015) describes high-achieving Black college students’ mathematical identities as “fragile” or “robust.” These identities were described as the result of racialized experiences. Three components of these identities are “(a) central motivations to succeed in mathematics, (b) the use of coping strategies in response to students’ racialized mathematical experiences, and (c) dispositions associated with one’s successful outcomes in mathematics” (p. 604). First, students with fragile mathematical identities defend themselves against negative stereotypes related to mathematics success by proving the stereotypes wrong and achieving success. On the other hand, students with robust mathematical identities define (rather than defend) themselves in ways that show an enjoyment of doing mathematics. Second, students with fragile mathematical identities use reactive, or “in the moment,” strategies to cope “with racially unsettling situations” (p. 605). Students with robust mathematical identities use “stable and clever coping strategies” instead—strategies that they developed and adapted over time, and that are “clever” and “more sophisticated retorts” (p. 604). Third, students with fragile mathematical identities describe their mathematics successes as bittersweet—typically because it is externally focused on proving their intellect or gaining achievement but not for enjoyment of mathematics. Those with robust mathematical identities associate their
mathematics success as internally focused and “based on reasons associated with one’s affinity with the discipline and one’s own self-satisfaction” (p. 606). According to McGee (2015), “a robust mathematical identity partially shields one from other people’s judgments, thereby allowing one to maintain positive attitudes and behaviors” (p. 606). The mathematical success of the student, then, is sustainable.

Other researchers agree that mathematical identities are not static. New identities can be constructed, but it may take time and significant effort. In one study of teenage boys who were not interested in academics, some of the boys constructed new identities based on new experiences with mathematics outside of the classroom, such as in the workplace, but other boys maintained a negative mathematical identity even after improving mathematics performance (Browne, 2009). Other students’ mathematics identities shifted in relation to the way that mathematics related to their future goals (Black et al., 2010). In the McCulloch et al. (2013) study, teachers who described changing their mathematics identity from negative to positive described “a particular teacher who made them feel comfortable, cared about, and believed in” (p. 388), which facilitated that change.

**Normative and personal mathematical identity.** Normative and personal identity construction are important for students and classrooms, as well as teachers and professional development settings. Although there is little research on teacher’s mathematical identity development, studies of students have explored student mathematical identity, and studies of teacher professional development have explored mathematics teacher identity development in a broader context. Thus, the research
described here about identity construction includes relevant studies of students in mathematics classrooms.

Boaler and Greeno (2000) explored the mathematical identity of high school calculus students \( (n = 48) \) and the ways in which they understood mathematics and their selves in relation to mathematics. Much of students’ mathematical identity related to the ways of knowing taught in their mathematics class; that is, whether mathematics was taught in a connected, separate, subjective, or received manner.

In classrooms that relied on received knowing, students saw their knowledge as dependent on the authority of the teacher or textbook and had little agency. Lessons emphasized procedures and memorization. The students played a passive role in class, believed that there was always only one right answer, and did not believe that mathematics required them to think. In order to be successful in these classes, students needed to develop mathematical identities that aligned with being a received knower. Students whose identities aligned with connected knowing and wanted to understand and think about mathematics chose not to continue their study of math. Their mathematical identities were incompatible with the pedagogical practices of the classes.

On the other hand, classrooms that relied on connected knowing were more discussion based. Students in these classes “identified more positively with mathematics and many of them did so because they were able to be thoughtful and to develop connected, relational understanding” (Boaler & Greeno, 2000, p. 188). Students saw themselves as active learners and engaged in deep thinking. In these classrooms, students’ mathematical identities as connected knowers aligned with the classroom
environment, and these students were more interested in continuing their study of mathematics.

There is also evidence to support the idea that a personal mathematical identity develops in relation to the normative identity of a particular environment. In one comparison of students enrolled in two pedagogically different mathematics classes (one based on lecture and practice, the other based on problem solving and student collaboration), the students co-constructed different normative identities for each class and described their identities for each class differently, based on the activities in which they engaged (Cobb et al., 2009).

These studies exemplify the interactions between personal identity and normative identity and describe the implications of the normative identity on both teaching and learning. Further, these examples show the ways in which personal identities evolve in relation to the normative identity of a given environment and the effects that they have on the individual’s future actions. These are both important considerations for MTCs, in which teachers’ personal identities may interact with normative identity of the group. This may be particularly apparent for teachers who are not accustomed to experiencing mathematics problem solving in the ways presented in MTCs.

**Productive disposition.** Productive disposition provides a lens through which mathematical identity can be viewed. Productive disposition is one of the five strands that the *Adding It Up* report recommended that students should develop to achieve mathematical proficiency. According to the NRC (2001), mathematical proficiency is the ability “to cope with the mathematical challenges of daily life” and enable students “to
continue their study of mathematics in high school and beyond” (p. 116). The five strands, then, are considered to be the necessary requirements for one to successfully learn mathematics. These five strands are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The last strand, productive disposition, is the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 116). All five of the strands are mutually dependent: They depend on one another, and they support one another. What this means is that in order for students to develop the other four strands, they must have developed their self-efficacy and beliefs about mathematics that enable them to be successful—and vice versa (productive disposition can assist the development of the other four strands). For example, as students learn more mathematics and develop strategic competence, mathematics begins to make more sense and they develop more positive beliefs about themselves as competent doers of mathematics. Similarly, as students gain more confidence in their abilities, they are confident in approaching challenging problems and develop their adaptive reasoning.

Productive disposition describes “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (NRC, 2001, p. 131). Further, people who are mathematically proficient “believe that mathematics should make sense, that they can figure it out, that they can solve mathematical problems by working hard on them, and that becoming mathematically proficient is worth the effort” (p. 133). To develop productive disposition, students need
“frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics” (NRC, 2001, p. 131).

It is also important for teachers to develop a productive disposition toward mathematics. Research has shown that teachers’ affect toward the content (including measures such as beliefs about mathematics, positive feelings toward mathematics, and self-efficacy) has an effect on students’ feelings toward the subject and on student learning outcomes (e.g., NRC, 2001). On another level, identity affects teachers’ classroom actions, and it makes sense that the ways in which they view themselves in relation to mathematics would influence the actions they take while teaching mathematics. The literature around productive disposition has typically focused on students as learners rather than teachers as learners, but the concept can be applied to anyone doing or experiencing mathematics.

As a descriptor of mathematical identity, productive disposition aligns with the views of Martin (2000) and R. Anderson (2007). Productive disposition towards mathematics incorporates beliefs about mathematics, motivation and persistence, attitudes toward mathematics, self-concept, and self-efficacy. These ideas were included in the intrapersonal and school levels of Martin’s framework and all four of the faces of identity in R. Anderson’s framework. Each of these ideas contributes to one’s mathematical identity, and each idea is further described in the following section.

**Beliefs about mathematics.** An important aspect of productive disposition, and one relevant to identity and teacher change, is beliefs about mathematics. Mathematics
beliefs are defined as the “implicitly or explicitly held subjective conceptions students hold to be true, that influence their mathematical learning and problem solving” (Op ’T Eynde, De Corte, & Verschaffel, 2002, p. 16). Numerous categories of beliefs have been shown to influence mathematics learning: beliefs about the subject of mathematics itself, beliefs about mathematics education, beliefs about the learning of mathematics, beliefs about problem solving, beliefs about the teaching of mathematics, self-efficacy beliefs, beliefs related to motivation (such as control, task value, and goal orientation), beliefs about mathematics within a social context, and beliefs related to the social norms in a mathematics classroom (Op ’T Eynde et al., 2002). Some beliefs are associated with greater mathematics achievement, and others are associated with lessened mathematics achievement. Examples of mathematics beliefs that may impede problem solving progress include: not viewing one’s formal mathematical knowledge as applicable to a problem at hand, believing that all mathematics problems can be solved in a matter of minutes, or believing that mathematics is a set of rules or procedures to follow (Schoenfeld, 1989). On the other hand, beliefs that are associated with successful problem solving include the belief that mathematics is useful in real-life situations, the belief that perseverance will pay off, and the belief that different concepts in mathematics are related. In this study, mathematics beliefs are defined solely as beliefs about the nature and subject of mathematics, and other categories of beliefs (such as motivation, self-concept, and self-efficacy) are defined as separate but related constructs.

For students, beliefs about mathematics matter for multiple reasons. One reason is that beliefs “play an important role in facilitating problem solving success” and the
methods and strategies students use in solving problems (Ernest, 2008, p. 11; see also Schoenfeld, 1989). Beliefs have an effect on students’ motivation and how they react to difficulties. Students’ decisions about which subjects to study or which career to choose can be influenced by their beliefs about mathematics (Philippou & Christou, 2002).

**Fallibilist and absolutist philosophies.** Ernest (2008) describes two distinct philosophies of mathematics: fallibilist and absolutist. A fallibilist philosophy of mathematics “emphasizes the practice of, and human side of mathematics, and characterizes mathematical knowledge as historical, changing and corrigible” (Ernest, 2008, p. 2). This philosophy is parallel with connected values of mathematics. Connected values are beliefs about mathematics that emphasize “relationships, connections, processes, empathy, caring, feelings and intuition, holism and human-centredness” and “foreground the role of human activity in mathematics” (p. 4). An absolutist philosophy views mathematics as an “objective, absolute, certain and incorrigible body of knowledge, which rests on the firm foundations of deductive logic” (Ernest, 2008, p. 2). This philosophy is commonly paralleled with separated values of mathematics. Separated values are the beliefs about mathematics that emphasize “rules, abstraction, objectification, impersonality, dispassionate reason, analysis, atomism and object-centredness” and view “mathematics as a product, a body of knowledge with the role of humans minimized or factored out” (p. 4).

While acknowledging the importance of the balance between absolutist and fallibilist philosophies, Ernest (2008) argues that many people have a fear or dislike of mathematics because of its absolutist qualities. For example, some people had negative
experiences with mathematics in school because they were expected to perform quick calculations and procedures without understanding what they were doing. These people might believe that mathematics is absolute, abstract, and difficult. But they did not have an opportunity to experience the fallibilist side of mathematics, which they may have enjoyed and understood.

According to Ernest (2008), mathematics educators generally agree that school mathematics should promote a connected view of mathematics in order to counter the negative image of mathematics that many people have. The fallibilist philosophy and associated connected beliefs result in positive views of mathematics as approachable, accessible, artistic, and beautiful. According to Cooney, Shealy and Arvold (1998), recently mathematics philosophers and the mathematics reform movement have begun to embrace the fallibilistic philosophy of mathematics and the notion that mathematics is “a subject that constructs meaning” (p. 309).

Felbrich et al. (2008) describe two similar types of beliefs about the nature of mathematics: static beliefs and dynamic beliefs, which align with absolutist and fallibilist philosophies, respectively. Static beliefs are grounded in formalism or scheme orientations, which view mathematics as “an exact science” or “a collection of terms, rules and formulae” (Felbrich et al., p. 674). Dynamic beliefs are grounded in process or application orientations of mathematics, which understand mathematics as problem solving and discovery, or “a science which is relevant for society and life” (Felbrich et al., 2008, p. 674). While most people favor one view or the other, Felbrich et al. caution
that mathematicians express beliefs about mathematics that reflect both views, and the two types of beliefs are not mutually exclusive.

*Productive disposition and beliefs.* Much of the research on mathematics beliefs and student outcomes focuses on two characterizations of beliefs. One is the set of beliefs that are positively correlated with positive learning outcomes, like higher academic achievement, deep understanding of certain mathematical concepts, effective studying strategies, and effective problem-solving practices (Muis, 2004). The other set of beliefs are those that have “no influence on learning outcomes or negatively influence learning outcomes” (Muis, 2004, p. 323). These two sets of beliefs can be viewed as the ends of the productive disposition continuum: Learners with beliefs that are correlated with positive learning outcomes have a highly developed productive disposition, and learners with beliefs that do not correlate with positive learning outcomes do not have a well-developed productive disposition.

These two sets of beliefs have been referred to as “positive” versus “negative” beliefs, “reform-oriented” versus “traditional” beliefs, “sophisticated” versus “naïve” beliefs, or “availing” versus “nonavailing” beliefs (e.g., Briley, Thompson, & Iran-Nejad, 2009; Goldsmith & Schifter, 1997; Hart, 2002; Muis, 2004). For the purpose of this study, productive disposition has been chosen to represent “positive,” “reform-oriented,” “sophisticated,” and “availing” beliefs.

*Summary of mathematics beliefs.* Mathematical identities include multiple facets. Productive disposition is the lens used here to describe a well-developed mathematical identity that will assist learners in developing mathematical proficiency. The following
key elements of mathematical identity, when taken together, can contribute to the development of productive disposition. Beliefs about mathematics that result in improved learning outcomes, such as believing that mathematics is sensible and useful, are part of productive disposition. Productive attitudes toward mathematics include the belief that doing mathematics is worthwhile and that motivation and persistence are key to believing that “steady effort in learning mathematics pays off” (NRC, 2001, p. 131). Other elements of productive disposition, self-concept and self-efficacy, were also described above as elements of identity.

**Changing teachers’ mathematical identities.** Mathematical identities are not static and can change over time. Although teachers’ mathematical identities are influenced by their experiences as students, professional development can also have an influential effect on teachers’ mathematical identities. Specific types of experiences have been shown to help teachers’ mathematical identities evolve toward greater productive disposition. These experiences are constructivist learning environments and increased mathematics knowledge.

**Constructivist learning environments.** Teachers (both preservice and in-service) who participate in constructivist or problem-solving focused mathematics courses or professional development develop greater mathematics self-efficacy, more positive attitudes toward mathematics, and greater productive disposition. As an example of a constructivist mathematics course typical of those described here, one course emphasized active learning and preservice teacher involvement in which groups of preservice teachers worked on problems of their choosing and presented the solutions to the class for
discussion (Alsup, 2004). A second example of a popular, research-based set of strategies for effective mathematics instruction is Cognitively Guided Instruction (CGI). CGI consists of “basing instruction on children’s intuitive understandings about math, helping students build on what they already know, and helping them learn from each other” (Bonner, 2006, p. 32). Practicing and preservice teachers who take constructivist or problem-solving focused mathematics courses or professional development experience decreased mathematics anxiety (Alsup, 2004; Harper & Daane, 1998; Sloan, 2010), increased positive attitudes toward mathematics (Bonner, 2006; Chapman, 1999; Gujarati, 2013; Lutovac & Kaasila, 2014; Zambo & Zambo, 2008), and increased mathematics self-efficacy (Chapman, 1999). These factors are important to the development of productive disposition and an empowered mathematical identity.

Correlations have been found between teachers with problem-solving orientations of teaching mathematics and holding dynamic or connected beliefs about mathematics. Beswick (2006) reported that teachers possessing a constructivist view of learning believed that mathematics was fun. Teachers who learned how to teach problem solving and understanding students thinking using CGI experienced greater enjoyment doing and teaching mathematics (Bonner, 2006) and changed their beliefs about mathematics instruction (Vacc & Bright, 1999). Conversely, teachers with more static or separated beliefs about mathematics (e.g., that mathematics is a set of operations, that there is one right answer to mathematics problems) enjoyed mathematics less and had less self-confidence in teaching mathematics (Stipek, Givvin, Salmon, & MacGyvers, 2001).
These orientations to mathematics appear to affect teachers’ enjoyment and beliefs related to mathematics.

Certain teaching and learning strategies in preservice courses resulted in decreased mathematics anxiety and changed beliefs. Mathematics anxiety in preservice elementary school teachers decreased after experience working in pairs or cooperative learning groups, using manipulatives, writing in journals about mathematics, and doing fieldwork (Harper & Daane, 1998). Similarly, preservice teachers’ mathematics anxiety significantly decreased after taking a methods course that was based on the principles of the NCTM standards, valuing conceptual understanding over memorizing procedures, and emphasizing manipulatives and small group work (Sloan, 2010). Preservice teachers enrolled in a mathematics methods course that promoted reform mathematics, such as the practices recommended by the NCTM standards, “changed their beliefs in a way that was more consistent with current mathematics education reform” (Wilkins & Brand, 2004, p. 231). The teachers focused on conceptual understanding; the use of manipulatives, investigations, and hands-on exploration; and the use of cooperative groups (Rethlefsen & Park, 2011; Wilkins & Brand, 2004). Another study reported that preservice elementary school teachers who took a college mathematics course that was constructivist in nature significantly increased autonomy when compared to a control group (enrolled in a traditional college mathematics course), although both groups experienced lessened mathematics anxiety at the end of the course (Alsup, 2004). The results of this study may have implications for increasing overall mathematics knowledge, which are discussed in a later section.
Problem solving focused in-service workshops have been shown to increase positive attitudes towards mathematics and empowered mathematical identity in teachers with weak mathematics backgrounds. In particular, one such workshop asked teachers to “examine personal meanings of themselves as mathematics problem solvers” and experience “self-reflection…as mathematics teachers” (Gujarati, 2013, p. 645). One problem-solving workshop for teachers focused on research-based mathematics instruction and teaching strategies, including problem-solving strategies, math journaling, and CGI (Bonner, 2006). At the end of the workshop, these teachers reported greater confidence and self-efficacy in teaching mathematics (Bonner, 2006). Similarly, teachers who participated in a two week summer workshop focused on improving teachers’ mathematics problem-solving skills and problem-solving instruction reported greater mathematics teaching self-confidence and self-efficacy (Zambo & Zambo, 2008).

Teachers in collaborative in-service programs experienced positive changes as they realized that they were not the only ones who had past negative mathematics experiences (Chapman, 1999; Lutovac & Kaasila, 2014). As teachers who believed that they were poor problem solvers saw that others also struggled with problem solving, they realized that their personal struggles were “not something unique to them as individuals” (Chapman, p. 132). After the program, the teachers “portrayed themselves more positively in relation to mathematical problem solving,” felt more confident in their problem-solving abilities, and were more confident in their problem-solving classroom teaching (Chapman, p. 132).
Teachers also reported increased mathematics self-efficacy after successful problem-solving experiences during the in-service program that focused on problem solving, personal meaning and experiences. Prior to the in-service, teachers believed “that they were deficient in the type of thinking one should have to be successful with mathematical problem solving” (Chapman, 1999, p. 130). However, as the teachers found that they could be successful by relying on their thinking, the teachers reported more confidence in attempting problems they may have otherwise avoided. At the same time, teachers began to understand that the process of problem solving is cyclical, involving both success and failure, and that frustration and barriers encountered in problem solving are “integral features of problem solving and not as situations they created because something was wrong with them” (Chapman, p. 132). Further, the teachers broadened their definitions of problems from “word problems that usually contained numbers” (p. 130) and they changed their thinking of teaching problem solving from “getting the students to identify the right strategy/algorithm and to execute it” (p. 131) to encouraging students to engage in the problem-solving cycle.

*Increased mathematics knowledge.* Many studies have connected gains in teacher mathematics knowledge to changing beliefs and instructional practices. After taking mathematics courses, preservice teachers improved their attitudes towards mathematics, particularly their satisfaction from doing mathematics and about the usefulness of mathematics (Philippou & Christou, 1998); increased their mathematics self-efficacy (Isiksal, 2005); increased their confidence in teaching mathematics (Holm & Kajander, 2012); and developed more reform-oriented beliefs about mathematics (Hart, 2002).
The results of the effects of general mathematics courses are mixed. Some studies report that students in constructivist-based courses focused on student thinking have more positive outcomes than students in a typically taught mathematics course. Others report no difference, implying that increased mathematics knowledge, no matter how it is received, results in more positive outcomes for in-service and preservice teachers. For example, preservice teachers who took courses that involved discussion of student thinking in mathematics reported greater mathematics teaching self-efficacy, greater mathematics knowledge, and greater productive disposition towards mathematics, in contrast to regular mathematics courses or additional fieldwork (Philipp et al., 2007; M. E. Smith, Swars, Smith, Hart, & Haardorfer, 2012). Another study found that from the beginning of their university program to the end of their program, mathematics preservice teachers in Germany developed more dynamic beliefs about mathematics and fewer static beliefs about mathematics (Felbrich et al., 2008). However, another study reported that mathematics teaching self-efficacy improved for preservice teachers who took an additional college mathematics course, regardless of whether the teaching style was traditional or constructivist (Alsup, 2004).

Elementary school teachers with greater mathematics knowledge also develop more cognitively oriented beliefs (M. E. Smith et al., 2012), beliefs in inquiry-based instruction (Wilkins, 2008), and lessoned approval of instructional methods that rely on rote memorization and practice (Clark et al., 2014). One teaching strategy that the literature shows supports students’ conceptual understanding is “the engagement of students in struggling or wrestling with important mathematical ideas” (Hiebert 

Grouws, 2007, p. 387). Note that this type of cognitive struggle is not frustrating or despairing but instead an expended effort to understand something that is within reach and yet “not immediately apparent” (Hiebert & Grouws, 2007). Elementary school teachers with greater mathematics knowledge tend to hold the belief that students should struggle with problems (Clark et al., 2014). However, there is also some data that this trend (increased mathematics knowledge is related to more productive beliefs for conceptual understanding) is reversed for middle school teachers. Middle school teachers with greater mathematics knowledge were less likely to believe that students should engage in struggle, and teachers who had completed more college mathematics courses were more likely to believe in rote memorization and practice (Clark et al., 2014).

Because the outcomes from regular mathematics content courses appear to be mixed, content-focused professional development should rely on other strategies to change beliefs, such as a constructivist orientation, emphasis on problem solving, and allowing the teachers opportunities to share their experiences and beliefs with other teachers.

Mathematics Teaching Identity

A second identity that is crucial to mathematics teacher development is the identity as a mathematics teacher. This identity relates specifically to the action of teaching mathematics rather than the action of doing mathematics. Although these identities may influence one another, they are distinct.

Mathematics teaching identity is described here as a part of a teachers’ professional identity. Because the construct of mathematics teacher identity is not well
developed in the literature, this section also draws from the research and theories of teacher professional identity in general and from the science education literature.

Teacher professional identity is “how teachers define themselves to themselves and to others” (Lasky, 2005, p. 901). Luehmann (2007) defines “‘teacher professional identity’ as being recognized by self or others as a certain kind of teacher” and that recognition (for science teachers) “occurs in the interpretations of common everyday experiences as a science teaching” (p. 827). Professional identity is important because it is “central to the beliefs, values, and practices that guide their [teachers’] engagement, commitment, and actions in and out of the classroom” (J. L. Cohen, 2010, p. 473). Teachers’ professional identities are “connected to their willingness to implement innovations in teaching and grow within a changing professional environment” (J. L. Cohen, 2010, p. 480). Teacher professional identity can be understood through “teachers’ representations of their experiences through talk” (J. L. Cohen, 2010, p. 473).

Past experiences are critical to understand in relation to professional identity development. For example, preservice teachers’ experiences in school, influential teachers, and prior formal and informal teaching experiences are important aspects of their identity development (Beijaard et al., 2004).

**Personal and normative identity.** In a review of the literature, Beijaard et al. (2004) summarize the various definitions of professional identity as “an ongoing process of integration of the ‘personal’ and the ‘professional’ sides of becoming and being a teacher” (p. 113). In fact, most descriptions of teacher professional identity include two dimensions: The first is often termed “personal,” and the second is referred to as
“normative” or “professional.” According to Beijaard et al. (2004), the personal aspect of identity involves agency and recognition of the self as a teacher. The professional side of identity consists of being seen as a teacher by others; in particular, the identity is given a social legitimacy within a particular context. Identity formation, as a whole, involves both dimensions of identity; it is “a process of practical knowledge-guiding characterized by an ongoing integration of what is individually and collectively seen as relevant to teaching” (p. 124).

Gresalfi and Cobb (2011) similarly see two sides of teachers’ professional identity. They view teachers’ identities as “the ways in which teachers conceive of and recount their work experiences” (p. 273). In this view, identities are influenced by the norms and values of specific environments and contexts in which teachers work. Gresalfi and Cobb (2011) distinguish between the normative identity for teaching and the personal identity for teaching, drawing from the normative and personal identities developed by Cobb et al. (2009), which was described in the earlier section on identity. Gresalfi and Cobb (2011) report on identity construction by nine middle school mathematics teachers as they participated in an ongoing professional development experience, and how they came to identify with the normative identity of the professional development community. The teachers’ personal identities for teaching mathematics evolved as they began to “identify with the vision of high-quality mathematics instruction” (p. 271) that was encompassed in the normative identity of the professional development. The teachers’ evolution involved reconciling the normative identity for teaching in the community with the conflicting normative identity in their schools. The teachers decided to change their
practice and formulated plans for changing the normative identity of the school district to align with their new personal identities for teaching.

The *normative identity for teaching* “comprises a set of obligations that a teacher would have to fulfill to be recognized as a competent mathematics teacher” in a particular setting (Gresalfi & Cobb, 2011, p. 274). These normative identities are collective notions particular to a setting, so the normative identity for teaching in a given school district may be different from the normative identity for teaching in a professional development community. The definitions of high quality teaching and instructional practices that reflect good teaching may be different in different contexts.

The *personal identity for teaching* is linked to the normative identity, because it develops as a reaction to the pre-established normative identity. It is developed by an individual teacher and “concerns the extent to which he or she identifies with others’ expectations for competent teaching in that context” (Gresalfi & Cobb, 2011, p. 275). The personal identity might develop in one of three ways: a teacher disagrees with the normative identity but nevertheless attempts to align with it (fulfilling “obligations-to-others”), a teacher agrees with the normative identity and attempts to align with its practices (fulfilling “obligations-to-oneself”), or a teacher disagrees with the normative identity and develops opposing practices (Gresalfi & Cobb, 2011, p. 275). This notion of development gives agency to the individual while also acknowledging the predetermined social context. Identity forms, then, based on the teacher accepting or opposing “the ways in which her instructional practices are recognized” (Gresalfi & Cobb, 2011, p. 274).
Similarly, Trent and Lim (2010) found that teachers positioned themselves in multiple environments: their schools, their professional development partnership, and the context of education in the country as a whole. That is, they related their experience to current education reforms and “new ways of teaching” (p. 1616). Trent and Lim found that the context of the partnership provided “a key institutional structure that shaped teacher identity by, for example, positioning Blaxland teachers as learners…and as the acquirers of new competencies” (p. 1616). Further, the teachers’ identities included aspects of imagination, or “seeing beyond the immediate context, making connections across space and time,” in which the teachers viewed their own growth alongside education reform in the country and in their schools (p. 1616). On the other hand, teachers whose power was marginalized within their school felt a lack of ownership over decisions, and so they did not buy into or participate fully in the partnership. The schools’ decision to participate in the partnership and the teachers’ identities in relation to their schools influenced their level of engagement with the partnership and ultimately influenced the amount of change that was able to take place as a result of their participation. Further, the teachers engaged in othering, creating distinctions between themselves and the school leadership, and between themselves and the partnership consultant.

Professional identity formation is thought to have a personal as well as a public component. Beijaard et al. (2004) envision the formation of identity on one continuum between the individual and the collective, and on a second continuum from public to private. Similarly, Lee and Luft (2008), in their work with science teachers, found that
teachers develop pedagogical content knowledge that is both individual and personalized and which changes and develops throughout their careers. They report on pedagogical content knowledge as a type of identity that varies from teacher to teacher and in different environments. While all of the teachers in their study held the same three core aspects of pedagogical content knowledge, the teachers emphasized different areas. J. L. Cohen (2010) found that “being a learner has been shown to be a key aspect of teacher professional identity” and in conversations with one another, teachers acted as learners “by linking their actions, values, and dispositions as teachers to the implicit identity claim of learner” (p. 479). In discussion, teachers re-affirmed one anothers’ identities as learners through reflective talk.

In order to experience growth in teaching practice and professional identity, student teachers need to experience “productive friction” (Ward, Nolen, & Horn, 2011). Productive friction takes place when teacher candidates experience conflict between two social worlds—in Ward et al.’s (2011) study, the “worlds” are their teacher preparation program and their experience in the classroom. Ward et al. argue that this friction can be productive or unproductive, depending on whether the teacher candidates have appropriate support.

**Four dimensions of professional identity.** Helms (1998) developed a four-dimensional framework for teacher identity based on interviews with secondary science teachers. Helms explored the identity development of science teachers based on the “ways in which science teachers obtain a sense of personal or professional identity from their subject matter” (p. 811). Helms argues that dimensions of teachers’ identities are
defined by their subject matter. According to Helms (1998), “the self comes not just from what a person does, or his or her affiliations, but also from what a person believes, what a person values, and what a person wants to become” (p. 813). Helms found that teachers’ sense of self or identity fell along four dimensions: “(a) actions; (b) institutional, cultural, and social expectations, or what people think others expect; (c) values and beliefs; and (d) where people see themselves going, or the kind of people they want to become” (p. 829).

Each dimension is affected by external factors and, according to Helms (1998), identity is “the experienced self in context” (p. 829). The four dimensions describe how the subject area “has played a role in their thinking about what kind of person they are and what kind of person they want to become” (Helms, 1998, p. 830). Further, these identities are in a constant state of shifting and flow as teachers work towards becoming the imagined future self.

The first facet, actions, relates to the deliberate actions that a person takes. According to Helms (1998), “actions may or may not reflect directly what is believed or valued, or even the kind of person one wants to become” (p 829). However, conscious and committed action has an effect on becoming a reflective person and developing a strong sense of self.

The second facet, others’ expectations, relates to the perceived expectations of others. “Others” in this case refers to “the institutions within which we work, professional communities, [and] the cultures and societies we live in” (Helms, 1998, p. 829). Note that this dimension relates to what a person thinks others expect, which may differ from their
actual expectations. In some cases, these perceptions result in action, such as when teachers feel “the need to act on policies that they did not agree with that their school or district imposed on them, or, in some cases, on perceived constraints, such as the need to cover the textbook” (p. 830).

The third facet, values and beliefs, relates to “their view of science and their beliefs about what is worth teaching” and “beliefs about what makes science special and what science has value” (Helms, 1998, p. 830). Values and beliefs mediate the interactions of actions and others’ expectations, and the effects of these two facets on the idea of the future self.

The fourth facet, the imagined future self, relates to “where people see themselves going, or the kind of people they want to become” (Helms, 1998, p. 829). Helms saw the three other facets as “working toward an imagined future self,” and so this facet is “interacting and responding to shifts in the other three dimensions” (p. 830). Teachers often express their feelings about this dimension “more in broad ideals than specific stories or concrete images” (p. 830). Hamman et al. (2010) calls this possible-selves theory. Possible-selves theory examines future-oriented thoughts of new teachers in relation to their expected and feared possible selves (Hamman et al., 2010).

Helms (1998) sees values and beliefs at the center of identity, with a strong link to the future self. Actions and others’ expectations, in addition to flowing through values and beliefs, also influence one another. According to Helms (1998), “deliberate actions and what is believed others…expect—flow through (or collide with) values and beliefs, contributing to (or obstructing) the strength and import of this link” (p. 829). Helms
argues that teachers feel an association with their subject area such that “the teachers in this study felt a sense of personal identification with science; that is, their sense of what makes science special is rooted in their own sense of themselves as science teachers and individuals in the world” (p. 812).

**Beliefs about teaching.** Beliefs about teaching are important. It is currently believed that beliefs influence actions and that the reverse is also true—that events and experiences can change beliefs (Richardson, 1996). From a constructivist perspective, preservice teachers have already formed strong opinions about teaching based on their experiences as students. These opinions and beliefs influence what preservice teachers learn from teacher education, and beliefs held by in-service teachers influence their approaches to professional development and their resulting changes. Many studies of teachers’ beliefs suggest that teachers’ beliefs about their subject matter and about education practice influence teachers’ actions in the classroom (Richardson, 1996; Roehrig & Luft, 2004; Simon & Tzur, 1999)

Although the focus of *Adding It Up* is students as learners of mathematics, the text includes a section about teaching proficiency. Teaching proficiency includes several components, one of which is “a productive disposition about one’s own knowledge, practice, and learning” (NRC, 2001, p. 384). Teachers “should think that mathematics, their understanding of children’s thinking, and their teaching practices fit together to make sense and that they are capable of learning about mathematics, student mathematical thinking, and their own practice themselves” (p. 384).
Teachers’ beliefs, according to a review by Richardson (1996), are based upon their personal experiences (as a more general idea of worldview and beliefs about the self), their experiences with schooling and instruction (as students), and their experiences with formal knowledge (both in and outside of school). Experiences growing up—former teachers, and family experiences in early childhood—had a strong influence on teacher role identity formation, whereas teacher education programs had little immediate influence (Richardson, 1996). However, a few studies suggested that lessons from teacher education may have an effect several years later, after a lag time.

Luehmann (2007) argues that “becoming a science teacher who values and engages in reform-based practices involves much more than acquiring a new set of knowledge and skills, and that this process could be better understood and supported once we think of it as developing a new professional identity as a ‘reform-minded science teacher’ ” (p. 823). In this case, reform-minded means aligned with the NRC’s vision for science, including grounding science teaching in learning sciences research. Luehmann (2007) sees the development of this professional identity as important for preparing teachers to implement reform-based practices; in other words, teachers need to develop a new professional identity related to these practices in order to implement them.

One aspect of teacher beliefs related to professional identity is mathematics teaching self-efficacy, a construct related to teachers’ confidence in teaching mathematical concepts. Mathematics teaching self-efficacy is typically made up of two dimensions: teaching efficacy, which “represents a teacher’s belief in his or her skills and abilities to be an effective teacher;” and teaching outcome expectancy, which is “a
teacher’s belief that effective teaching can bring about student learning regardless of external factors” (Swar, Hart, Smith, Smith, & Tolar, 2007, p. 326).

**Changing teacher beliefs.** According to Richardson (1996), in-service programs have been shown to be more successful at changing teacher beliefs than pre-service programs. In fact, Richardson (1996) wrote, “Recent studies of the effects of in-service programs on teachers’ changes in beliefs are quite encouraging” (p. 112). Several studies revealed that teachers changed their beliefs as a result of professional development. Others also found that such changes in teachers’ beliefs were reflected in the teachers’ practices. Richardson, however, “calls for research that examines both beliefs and actions” as well as research on constructivist-focused teacher education (p. 114).

Katz, McGinnis, Riedinger, Marbach-Ad, and Dai (2013) explored identity development for beginning elementary school science teachers. The teachers experienced an informal science education program in their teacher preparation program. Prior to the information science education internship, the teachers had a fear of teaching science, but at the time of the study, they were becoming more confident in their identities as science teachers. The researchers were interested in seeing how their identities persisted after they had been teaching for a few years. The experience was memorable and served “as a meaningful experience in their thinking about how they themselves could learn and teach science to their students” (p. 1373). The teachers continued to think about the experience and referenced it even after they began teaching. The teachers referred to their informal science education internships when they spoke about their identity development as science teachers, specifically, aspects of their identities that included affective benefits
such as persistence, excitement, resilience, and motivation. Similarly, Katz et al. (2011) found that science teacher candidates’ experiences in an informal science setting had potential to “complement and enhance” (p. 1174) their professional identity development. Their beliefs about the teaching and learning of science related to their “view of themselves as a future science teacher” (p. 1173). The preservice teachers’ drawings of themselves teaching science before and after their internships demonstrated shifts in their thinking about student-centered pedagogy and broadening their views of teaching and learning. They also gained enthusiasm for science.

Luehmann (2007) recommends that teacher preparation programs seeking to develop reform-minded teachers consider several factors related to identity: identities that beginning or preservice teachers currently hold are based in their experiences and beliefs, can be contradictory to reform-minded science learning, and these identities can be difficult to change. Luehmann recommends that “safe and supportive contexts” can help these teachers take the risks to “trying on a new identity” (p. 828). Although participation in a community of practice is important, Luehmann emphasizes the essence of identity is “the interpretation, narration, and thus recognition” (p. 828) of participation in the community of practice. Luehmann posits that out-of-school experiences teaching can “offer prospective teachers opportunities to experience meaningful success with reform-based practices” (p. 830). Such experiences can be low-risk as they are in a safe environment, allowing the teachers to try new methods with support from peers and the professor. Katz et al. (2011) posit that these informal experiences can enhance the teachers’ identity development gained from formal experiences.
Research on teachers’ beliefs has shifted from large-scale quantitative studies to qualitative studies and “the attempt to understand how teacher make sense of the classroom,” partly because multiple-choice surveys are constraining (Richardson, 1996, p. 107). Teachers’ beliefs about teaching and theories of teaching and learning do not necessarily map to established theories in the literature, and often teachers’ beliefs include disparate elements in the literature.

**Changing professional identity.** Professional identities can change over time. For example, new teachers’ professional identities shift during the first year of teaching, from being ready to begin teaching and focused on students, to becoming more focused on themselves and survival mode (Thomas & Beauchamp, 2011). Thomas and Beauchamp suggest that teachers have difficulty developing their professional identity during their first year of teaching, that developing a professional identity is a gradual and complex process, and that there is a connection “between a strong professional identity and a sense of efficacy in the classroom” (p. 767). Preservice teacher identity is theorized to change through the processes of identification and negotiation (Horn, Nolen, Ward, & Campbell, 2008). Preservice teacher identities shape the ways in which they engaged in their coursework, and their identities evolved as they gained experience in their coursework and in the field. Identification occurred when the pre-service teachers integrated styles of teaching from their coursework or fieldwork into their existing ideas of themselves as teachers. Negotiation occurred when they “modified their identities based on what they encountered” (Horn et al., 2008, p. 67).
Collopy (2003) found that teacher learning was interrelated with teacher identity. Identity was seen as “the constellation of a teacher’s beliefs and knowledge about subject matter, learners, pedagogy, and self as a teacher and learner” (p. 308). The ways in which teachers learned from professional development and changed their instructional practice was related to the combinations of their beliefs and the particular beliefs that were most integral to their identities. For example, teachers’ beliefs about learning and their own mathematics self-efficacy had an effect on their changing practice. Similarly, if a teacher’s most closely held or integral beliefs are compatible with the professional development, then the teacher was more likely to learn from the professional development. If they conflict, it may be more difficult for the teacher to change their beliefs to align with the professional development.

Future anticipated learning can be important when thinking about teacher change. It “provides the link between self and identity over time by including anticipatory, or future-oriented, goal-directed behavior” (Hamman et al., 2010, p. 1357). Possible-selves theory can provide information about the teachers’ current identities as well as motivate the teachers to pursue their goals related to future improvement (Hamman et al., 2010). These possible selves can be hoped for, or they can be feared. Thus, possible selves can reveal the intersection between emotion and identity, and they can provide “a way of conceptualizing the process of change” (p. 1358). Other theories suggest that teachers’ beliefs about science teaching can be changed by focusing “on new teachers’ images of their ‘ideal self as teacher’ ” (Katz et al., 2011, p. 1173). The image of their ideal self
“may be influenced by experiences of teaching and learning both in and out of school” (Katz et al., 2011, p. 1173).

**Summary of mathematics teaching identity.** Teacher professional identity as a mathematics teacher may be considered a subidentity of their overall professional identity, but it is distinct from their mathematical identity. Professional identity involves both an internal and external recognition. Similar to other constructs of identity, it relates to being recognized by others as a mathematics teacher, consists of stories one tells about oneself, and is related to the environment’s normative identity for teaching. The literature on science teacher identity provides a framework for mathematics teacher identity, which includes actions, others’ expectations, values and beliefs related to mathematics teaching, and the imagined future self.

**Interaction of Identities**

Although empirical work on identity often silos identities from different contexts, in reality, the identities that a person holds are rarely held in isolation and often interact. In particular, the close ties between the actions of doing mathematics and teaching mathematics lends facility to the interactions of these identities. The theories of Van Zoest and Bohl (2005) and Beijaard et al. (2004) attempt to describe and explain how multiple identities develop and interact in a teaching context.

According to Van Zoest and Bohl (2005), teachers often convey aspects of their identity and their learning differently in different settings; for example, as they move from learning in a college classroom to teaching in their school classroom. Van Zoest and Bohl built a framework in order to explain the relationship between learning that takes
place in one community and the actions that take place in another context. They draw
from Wenger’s ideas of learning as participation in a community of practice and consider
both individual cognition and community membership. The two components of this
framework, on opposite sides of a continuum, are “aspects of self-in-mind” and “aspects
of self-in-community” (p. 333). Along this continuum, from the direction of the mind to
the community, the components of identity are listed as knowledge, beliefs,
commitments, and intentions. Thus, Van Zoest and Bohl theorize that knowledge is more
tightly aligned with the self-in-mind and intentions are more tightly aligned with the self-
in-community.

Beijaard et al. (2004) also describe the multiple subidentities that people hold, and
they explain that in different situations, people take on different identities. The closer
these different identities are aligned, the more similar the actions will be in different
contexts. In their words, “the better the relationships between the different identities, the
better the chorus of voices sounds” (p. 113). Further, some identities are more closely
held, while others are peripheral, like beliefs. According to Beijaard et al. (2004),
preservice teachers often experience these conflicts between their subidentities as they
explore and develop their identity (or identities) as a teacher. On the other hand,
practicing teachers may also experience conflict when their teaching context or
environment changes. It seems to be important for these subidentities to harmonize, so
conflicting subidentities are grappled with until they are again cohesive.

The notion of multiple identities are seen in studies of teacher identity. J. L.
Cohen (2010) found that teacher identity recognition required “contextualising
professional identity in terms of related identities” (p. 480) including that as learner or that as a reader (when the teacher taught reading). Similarly, Ulvik, Smith, and Helleve (2009) found that novice secondary teachers have conflicting identities: they wanted to be recognized as fully qualified, but they also wanted to be recognized as new and inexperienced teachers. Skott (2001) found that for a novice mathematics teacher, the relationship between his beliefs and practices about the teaching and learning of mathematics “was very different in different situations” (p. 23). The difference typically depended on his priority in a given situation: when his priority was related to general education goals, his practices did not align with his beliefs, but when his priority was related to teaching mathematics, his practices did align with his beliefs.

Williams (2010) explored the experience of a career changer becoming a teacher and developing a new professional identity while student teaching. For this new teacher, developing a teacher identity meant reconciling her expertise in other situations with being a novice in teacher education. The identity reconciliation and formation process was assisted by social support from peers and recognition by others of the skills and expertise she had from her previous career.

The identities and beliefs that teachers have about mathematics interact with the professional identities that teachers have related to mathematics teaching. These interactions have influential effects on students. The research suggests that teachers’ mathematical identities influence student outcomes and teachers’ actions (an aspect of their mathematics teaching identities) influence student outcomes.
**Teacher mathematical identity and student outcomes.** Although there is a general lack of research on teacher mathematical identity and its influences in the classroom, there does exist research related to various aspects of identity, including beliefs about mathematics and attitudes toward mathematics.

It is widely agreed that the beliefs that teachers have about a subject, such as mathematics, affect how they teach it (Felbrich et al., 2008; Gresalfi & Cobb, 2011; Lappan & Theule-Lubienski, 1994; Leatham, 2006; Philippou & Christou, 2002; Singletary, 2015). Most of the research on teacher beliefs and actions support this conclusion, as teacher actions typically reflect teacher beliefs (D. I. Cross, 2009; Stipek et al., 2001; Thompson, 1984; Voss, Kleckmann, Kunter, & Hachfeld, 2013; Wilkins, 2008). T. F. Green (1971) notes, “The acquisition of beliefs or their modification is a major concern in the activity of teaching” (p. 42). Teachers’ beliefs about mathematics affect multiple aspects of teaching, particularly pedagogical beliefs and decisions, but also “the learning climate they will contribute to, and specifically their choices of teaching strategies and learning activities” (Philippou & Christou, 2002, p. 212).

According to Philipp (2007), “while students are learning mathematics, they are also learning lessons about what mathematics is, what value it has, how it is learned, who should learn it, and what engagement in mathematical reasoning entails” (p. 257). Teachers’ beliefs, enjoyment of, and knowledge of mathematics consciously and unconsciously influence how and what they teach.

Ernest (1989) argues that teachers’ beliefs about the nature of mathematics, mathematics teaching, and mathematics learning are as important as teachers’
Wilkins (2008) reported that teacher's mathematics beliefs had a greater influence on teaching practice than content knowledge. Teachers need to have productive dispositions toward mathematics and believe that mathematics is more than memorizing rules before they can “help students obtain more authentic and productive notions about mathematics” (Lappan & Theule-Lubienski, 1994, p. 254). Preservice and novice teachers’ experiences with learning mathematics in school strongly influenced the formation of their beliefs about the nature of mathematics and how mathematical learning occurs (S. Brown, Cooney, & Jones, 1990). According to Philipp (2007), “the existing research shows that the feelings teachers experienced as learners carry forward to their adult lives, and these feelings are important factors in the ways teachers interpret their mathematical worlds” (p. 258). Novice teachers whose experiences with school mathematics were based on traditional instruction “may be socialized to this manner of teaching” and it is important to “help them develop different conceptualizations of mathematics teaching” (C. A. Brown & Borko, 1992, p. 223).

This is important because generally, formal mathematics education is an influencing factor on students’ mathematics beliefs (Muis, 2004). The structure of the classroom and the type of engagement students have with mathematics can affect high school students’ mathematical identities: When success in a mathematics classroom depends on obtaining correct answers quickly, students who take longer to solve problems do not see themselves as capable mathematics learners (R. Anderson, 2007). On the other hand, these students do see themselves as mathematically competent in classrooms that value different strategies and making meaning of problems (R. Anderson,
For example, students tend to hold beliefs associated with negative learning outcomes in classrooms where students work individually, are expected to memorize formulas, the teacher is the authority figure and relays information to the students, work is dominated by teacher instruction and demonstration, and connections among mathematical concepts are not made (Muis, 2004). On the other hand, students tend to develop greater productive disposition in classrooms where students work together with their peers, are presented with novel problems to solve, are actively engaged in learning (rather than passively following or memorizing procedures), and use multiple resources beyond the textbook (Muis, 2004). By adopting particular classroom practices, teachers can influence the development of their students’ productive dispositions.

Teachers’ attitudes toward mathematics have been found to positively influence students’ attitudes toward mathematics. Several studies have found that as teachers change their attitudes and begin to develop more positive attitudes toward mathematics, their students also begin to have more positive attitudes towards mathematics. Similar results have been reported for teachers who develop greater confidence in teaching mathematics. Students begin to enjoy mathematics more, experience increased mathematics self-efficacy, and decrease their apprehension of mathematics (Bagaka’s, 2011; Bonner, 2006; Clark, Badertscher, & Napp, 2013; Stipek et al., 2001). Students are also negatively affected by negative teacher attitudes. Beilock, Gunderson, Ramirez, and Levine (2010) reported that, after spending one year in the classroom, first and second
grade girls’ mathematics achievement was affected by their female teachers’ mathematics anxiety.

**Teacher actions and student outcomes.** As we have seen, the classroom or learning environment can have an effect on the development of mathematical identity, leading to productive disposition or not.

Burns, Pierson, and Reddy (2014) found that as teachers in India implemented more learner-centered instruction (including project-based learning and open-ended questioning techniques) and collaborative learning, and used less lecture and focus on rote practice, students reported higher levels of engagement and increased confidence in mathematics. Students were more interested in class material and no longer had “the ‘fear’ associated with working alone—the fear of making a mistake; of being solely responsible for learning; of not having someone to help them if they didn’t understand” (Burns et al., 2014, p. 27). Students were more confident in asking the teacher questions and became more confident in explaining mathematics concepts to their peers. Their confidence in their ability to do the problems in class also increased. At the same time, the teachers’ beliefs about their students evolved. As the teachers implemented collaborative learning and other learner-centered strategies, the teachers developed more incremental theories about learning and increasingly viewed their students as capable, independent learners (Burns et al., 2014).

Students in constructivist classroom environments view themselves and others as successful in mathematics class, while students in lecture-based, procedurally-focused classes are not as likely to view themselves as successful (Clark et al., 2013). Students in
a constructivist classroom felt that they were successful because they were “able to make substantial contributions to classroom discussions” (Clark et al., 2013, p. 62). Those same students, in a second mathematics class that was traditionally taught and focused on procedures and calculations, viewed themselves as less successful, because success depended on their ability to “produce correct answers by enacting prescribed methods on written notations,” and the judgment of the teacher on the correctness of their answers (Clark et al., 2013, p. 61).

According to Boaler (1999), a school that used “project-based methods of teaching” (p. 260) (including open problems, little teacher exposition, and extensive group discussions) in mathematics classrooms “encouraged the students to develop mathematical beliefs and practices that were more consistent with the demands of both the classroom and the ‘real world’ ” (p. 260). The students “developed the idea that mathematics was a thinking, flexible subject” (p. 264). On the other hand, students in a school with traditional methods of teaching mathematics believed that “school mathematics was made up of numerous rules, formulas and equations that needed to be memorized” and that “their role in the mathematics classroom was to memorise the different rules they had been given and many believed that school mathematics was incompatible with thought” (Boaler, 1999, p. 263). These students generally did not believe that the mathematics they learned in school could be useful outside of school, and they did not believe that their knowledge of the real world was useful in the mathematics classroom.
The classroom environment and jointly developed normative identity for mathematics learning can affect various aspects of student identities and ultimately, student learning and achievement. It is critical that teachers have the skills to create classroom environments and facilitate mathematics activities to promote empowered mathematical identities.

**Interaction of mathematics teachers’ identities.** It is important that students develop productive disposition in their mathematical identities, but it is also important for teachers to have strong mathematical identities that include well-developed productive disposition. Teachers who have productive disposition are better able to create learning opportunities for students to develop it themselves.

It is important for teachers to develop beliefs about mathematics that will allow them to facilitate mathematical problem solving. Teachers’ beliefs about mathematics ultimately influence the ways in which they teach and the subconscious messages they pass along to their students (e.g., Ernest, 1989). Thus, teachers need to develop productive dispositions towards mathematics, which include availing beliefs about mathematics, positive mathematics self-concepts, and positive mathematics self-efficacy beliefs. Teachers’ beliefs and overall mathematical identities influence student beliefs and identities, which influence, among other outcomes, student achievement and problem-solving performance (e.g., Ernest, 1989; Lappan & Theule-Lubienski, 1994).

Teacher experiences are critical to the evolution of these beliefs (e.g., Gresalfi & Cobb, 2011). Leaders of teacher professional development, then, ought to be concerned with research-supported strategies for developing teacher beliefs and productive
disposition. It is worth noting that the literature on in-service teacher mathematics beliefs and productive disposition is sparse, so some of what is known comes from research on preservice teachers instead.

A significant amount of research has shown that preservice teachers’ beliefs about mathematics and mathematical identities are shaped by years in school as a student (Hannula, 2002; Lutovac & Kaasila, 2011; Mapolelo, 1998). In particular, teachers lacking productive disposition often had negative experiences with school mathematics. For example, teachers who score low on a scale of mathematics teaching self-efficacy report negative early experiences in mathematics (Swarz, 2005). Studies of mathematical identity development report that students frequently describe the effects of negative experiences in mathematics (R. Anderson, 2007; Lim, 2008).

However, as previously discussed, mathematical identities can continue to change and evolve as a result of further experiences with mathematics (Gresalfi & Cobb, 2011). Thus, content-focused professional development that gives teachers positive experiences with rich mathematics is crucial for developing teacher productive disposition and mathematical identities.

**MTCs: A Type of Professional Development**

MTCs are a specific type of content-focused professional development for teachers, sometimes referred to as the mathematician’s version of a book club. At an MTC, K–12 teachers and mathematicians work collaboratively on mathematical exploration and problem solving. According to White, Donaldson, Hodge, et al. (2013), “the core activity of MTCs is regular meetings focused on mathematical exploration” (p.
4) and the development of problem-solving skills. Sessions typically end with a debriefing about what the participants learned about problem solving and teaching problem solving. The hope of an MTC is that “over time, this excitement [of solving problems] will contribute to the belief that problem solving, and mathematics in general, can be a fun and worthwhile endeavor” (Fernandes et al., 2011, p. 112).

MTCs are meant to be long-term, sustained communities, so teachers often participate in these groups for multiple years. MTCs typically include elementary and middle school teachers working alongside mathematicians or mathematics professors. Some MTCs are also open to all K–12 teachers, pre-service teachers, and others in the mathematics education community. Each session is attended by, on average, 15 to 25 participants.

MTCs typically meet for three to five days in the summer for an immersion workshop, with meetings occurring about once per month throughout the following school year. The school year sessions typically last about two to three hours and include the exploration of one or two problems. Facilitators are frequently mathematicians or university mathematics faculty, although MTCs encourage teacher/mathematician pairs to present, and other groups have regular teacher facilitators.

Social learning and community building are important aspects of MTCs. Problems are typically explored in groups, and insights are frequently shared among groups. Teachers and mathematicians work side-by-side on problems, explaining their thinking to one another. The collegial atmosphere and emphasis on multiple strategies supports risk taking.
The sessions are typically facilitated with a “guide on the side” outlook rather than a direct instruction or “sage on the stage” focus, which allows for extensive individual and small-group exploration. Often, after the problem is posed, the facilitator circulates among small groups, providing support or guidance when needed, but rarely explaining the problem or telling a participant what to do next.

The problems posed are typically high-quality, non-routine, rich problems. Because participants have a range of mathematical backgrounds, the problems posed in a session have a low floor and high ceiling, providing an entry point for people with minimal mathematical backgrounds but also containing sophisticated mathematical extensions for experienced mathematicians. Thus, the sessions are engaging for both elementary school mathematics teachers and mathematics professors, and they have the potential to result in new mathematical knowledge and understanding for both groups.

MTCs tend to emphasize the “exploratory component of the problems” (Fernandes et al., 2011, p. 112). Although the same question is typically posed to the entire group, individual exploration is encouraged and participants often take the problem in different directions. During the debriefing session at the end, the focus is on multiple strategies and solution paths rather than a single answer.

MTCs have multiple goals, but there are two goals that relate to mathematics content: to develop teacher mathematical knowledge, and to improve teachers’ skills in teaching problem solving. The first goal, to develop teacher mathematical knowledge, intends to bolster the content knowledge of teachers, particularly elementary or middle school teachers who lack strong mathematics backgrounds. An MTC “emphasizes
developing teachers’ understanding of and ability to engage in the practice of mathematics, particularly mathematical problem solving, in the context of significant mathematical content” (White, Donaldson, Hodge, et al., 2013, p. 3-4). By participating in an MTC, teachers experience new mathematics and stretch their thinking. They learn new strategies for problem solving, discover new connections among mathematical topics, and expand their personal definitions of mathematics. The MTCs focus on developing middle school mathematics teachers’ content knowledge and allow teachers to experience doing mathematics the way that mathematicians do mathematics.

The second mathematics content goal, improving teachers’ skills in teaching problem solving, is accomplished by exposing teachers to rich mathematical tasks and providing them with the experience of persevering in problem solving and being successful. At MTCs, experienced facilitators guide teachers through explorations of interesting problems, providing a model for how to teach problem solving. Teachers discuss the strategies used by the facilitators to push their thinking forward, and they relate those strategies to methods that they can use in the classroom. Together, these experiences increase teachers’ comfort in bringing problem solving to their classes.

Others, however, see different purposes of MTCs. The CBMS (2012) described MTCs as “opportunities for teachers to develop their mathematical habits of mind while deepening their understanding of mathematical connections and their appreciation of mathematics as a creative, open subject” (p. 68). MTCs also provide opportunities for teachers to participate in professional learning communities and develop more positive attitudes toward mathematics. Shubin (2006) explained that one of the goals of her MTC
is to “free teachers from the fear of tackling difficult problems” (p. 65) by engaging them in problems that require a long period of time to solve, hoping that they learn through example that problem solving can provide a powerful intrinsic motivation for studying mathematics.

MTCs are a fairly new development, with the first one begun in 2006 and new groups forming every year. To understand how and why MTCs developed, it is important to understand the history of their precursor, Math Circles.

The development of Math Circles. Math Circles are groups for students, typically middle or high school students, to engage in collaborative problem solving with rich, interesting problems. Math Circles in the United States are a fairly modern endeavor based on a rich tradition in Hungary and the Soviet Union. Their development is intertwined with the development of Math Olympiad competitions.

Math Circles are widely thought to have been originated by “a loose tradition in the Soviet Union of informal, often covert, meetings of students with teachers” (Kaplan & Kaplan, 2008, p. 140). However, because Math Circles in the Soviet Union are intertwined with the Math Olympiad and “a spirit of competition,” their origin can be traced back to Hungary (Kaplan & Kaplan, 2008, p. 140). In 1894, mathematics competitions known as the Hungarian Nursery involved students sitting down to individually work through advanced problems (Kaplan & Kaplan, 2008). The two best student papers were selected and those students given awards. While the competition may have been a motivating factor, one Hungarian problem book suggested that students were more stimulated by the mathematical problem-solving process itself: “The students who
were so deeply involved in the long and hard work of solving the problems were probably spurred on not by the possibility of a medal” but instead by the powerful feeling of satisfaction of expending metal effort on working and solving problems (Kaplan & Kaplan, 2008, p. 140). This is the belief that fuels Math Circles in the United States today: Mathematical problem solving is an interesting and engaging endeavor that can result in deep satisfaction.

The style of mathematics competition in Hungary then migrated to the Soviet Union, becoming increasingly similar to current practices at U.S. Math Circles. The mathematician Andrea Nikolaevich Kolmogorov, in the early part of the twentieth century, posed problems in his lectures, providing the students “complete freedom to attack them as they saw fit” (Kaplan & Kaplan, 2008, p. 141). The tradition of competition in Hungary, combined with Kolmogorov’s method of instruction, led to the development of Math Circles and mathematics competitions in Russia in the 1930s. These different types of events often attracted the same organizers and the same student participants. The first Moscow Math Olympiad and the first Math Circle were organized at Moscow State University in 1935. The Math Olympiad winners were determined by the solutions that showed the most “mathematical originality, thoughtfulness, and rigor,” and not necessarily correctness (Kaplan & Kaplan, 2008, p. 142). The format of Math Circles varied from place to place, often taking the form of a lecture or reading biographies of mathematicians. However, Math Circles began incorporate student exploration and discussion under the leadership of David Shklyarskiy, a student at Moscow State University at the time. In 1938, Shklyarskiy’s students swept the
competition at the Math Olympiad, and his Math Circle format began to be adopted throughout the nation (Kaplan & Kaplan, 2008).

Math Circles in the United States are a fairly recent phenomena, with the first ones appearing in the late 1990’s (Shubin, 2006; Vandervelde, 2009). In fact, Math Circles began when “émigrés who had received their inspiration from math circles as teenagers” in the Soviet Union and were now working at universities in the United States “decided to initiate math circles within their communities to preserve the tradition which had been so pivotal in their own formation as mathematicians” (Vandervelde, 2009, p. 6).

One of the first Math Circles in the United States was the Berkeley Math Circle in 1998, begun by Zvezdelina Stankova, a former Bulgarian Mathematics Olympiad competitor (Shubin, 2006). The Berkeley Math Circle produced some of the members of the 2001 U.S. International Mathematical Olympiad team (Vandervelde, 2009). Also in 1998, the San Jose Math Circle for middle school students was established by Tatiana Shubin, another former participant in Math Circles in her youth (Shubin, 2006). In 1994, one of the most successful Math Circles, known as The Math Circle, began in Boston (Kaplan & Kaplan, 2008).

Math Circles in the United States are typically geared toward middle school students. Vandervelde (2009) argues that high school students who join Math Circles do so because they have already been successful at mathematics, while elementary and middle school students “are usually in the midst of exploring this question [‘am I good at math?’], and attendance at the local math circle can be part of the process of finding the answer” (Vandervelde, 2009, p. 10). Vandervelde explains that most Math Circles, even
those that intend to serve high school students, eventually find a natural focus at the middle school level. However, this varies from location to location, and The Math Circle in Boston hosts classes for children as young as four and even allows adults to enroll in classes along with the children.

Math Circles have a variety of formats and purposes. Some are informal collaborative explorations of games or hands-on activities, some provide students with problem sets to work independently, others take the form of a formally structured environment, and still others prepare students for Mathematics Olympiad competitions. Note that Math Olympiad competitions, like the first competitions in Moscow, typically place an emphasis on explanations rather than correctness (Vandervelde, 2009). The purposes of Math Circles vary, from helping to prepare students for competitions, to encouraging talented students in mathematics, to establishing and maintaining interest in mathematics for all students (Cavalieri & Hartenstine, 2005). At the University of Utah, one Math Circle exposes high school students to “interesting or more advanced mathematics that they would not normally see in high school” and provides these students with “the opportunity to explore these topics by working on problems with faculty and graduate students (and each other)” (Cavalieri & Hartenstine, 2005, p. 1). Kaplan (1995) maintains the success of Math Circles is because “the appetite for real math, done neither competitively nor scholastically but as the most exciting of the arts, is enormous” (p. 978).

Features of a Math Circle. What happens during a Math Circle? Cavalieri and Hartenstine (2005) explain, “The best Math Circles are those in which the students
experiment, discuss ideas, make conjectures, try to prove them and explain their discoveries and solutions of problems to their peers” (p. 3), and this is the process that is replicated in MTCs. Similarly, Kaplan (1995), cofounder of The Math Circle, explains that his approach “is to pose questions and let congenial conversation take over” (p. 976). According to Kaplan and Kaplan (2008), “the most fundamental message of The Math Circle is that you learn math by inventing or discovering it yourself” (p. 204). The goal of all Math Circles is to engross the students in problems that they can then tease apart, play around with, and reassemble collaboratively (Kaplan & Kaplan, 2008).

According to Kaplan and Kaplan (2008), the work that children do in Math Circles is intended to be similar to the work that mathematicians do, which is “using reason to understand the underlying structure of things” (p. 145). In order for this to occur, the topic for a Math Circle needs to be chosen carefully. The goal of a Math Circle is to “reward our students’ efforts by having this work move them steadily and significantly beyond where they found themselves in the mathematical landscape” (Kaplan & Kaplan, 2008, p. 160). Thus, the selection of a topic at the appropriate level and accessibility for students is critical (Kaplan & Kaplan, 2008).

In a Math Circle, “a roomful of egos becomes absorbed in It rather than I, and the surprising pleasures open up of forgetting yourself in the play” (Kaplan & Kaplan, 2008, p. 162). Students (and adults) at Boston’s The Math Circle are so engaged that they are not ashamed to admit when they are unable to follow along. They ask for someone to explain it to them and then once they are able to rejoin the conversation, they are able to explain it to others.
It should be noted that the format of a typical Math Circle in Russia is different from that in the United States. While the U.S. Math Circles typically involve the collaboration of the whole group sharing insights and working toward a common goal, the Russian circles typically consist of a problem set worked individually (Dorichenko, 2011). The instructor may begin a two-hour Math Circle with a 20 minute explanation to set up the theme of the problem set, but “the rest of the time is devoted to students solving the problems and discussing them individually with instructors” (Dorichenko, 2011, p. xv). As a result of observing and learning about the Russian Math Circles, the San Jose Math Circle began implementing Russian-style Math Circles twice per year, in which students are given a problem set a week ahead of time to work individually. They bring their work to the circle where they explain their process to a mathematician in a one-on-one setting (Shubin, 2011). The key to these sessions is that the students are the ones who are talking, not the mathematicians, and the mathematicians listen and pose questions to guide the students’ understanding.

**The development of Math Circles for teachers.** Proponents of Math Circles believe that the type of learning that takes place in the circles is the type of learning that should be happening in the everyday mathematics class. Math Circles have inspired “a successful model for math instruction in public school districts” when mathematics teachers “create similar environments in classrooms” (Sheperd & Sakashita, 2009, p. 13). Kaplan and Kaplan (2008) believe “the best way, certainly, would be for The Math Circle’s approach to replace that in regular school classes—and this will surely happen one day” (p. 224). But there is “the problem of developing teachers able to run such
courses” as elementary school teachers “would have to master a lot more math, not just a new methodology” (Kaplan & Kaplan, 2008, p. 224). Creating a Math Circle for teachers, or an MTC, is a proposed strategy to scale up the Math Circle model to more classrooms (Tanton, 2006).

The idea of a Math Circle for teachers was first discussed at the Mathematical Sciences Research Institute Conference on Mathematical Circles and Olympiads in 2004 (Shubin, 2006). In 2006, the American Institute of Mathematics (AIM) officially started the first MTCs in northern California (Fernandes et al., 2011). The AIM hosted a summer workshop for local mathematics teachers and mathematicians. The goal of the workshop was “to demonstrate that problem solving is an effective mechanism for learning, since we believe that it provides the strongest motivation for studying mathematics” (Shubin, 2006, p. 64–65).

Since that summer, AIM has continued to host workshops for teams interested in developing their own MTCs. There are currently 57 MTCs in 31 states, and new teams are encouraged attend the AIM training each year (MTC Network, 2014).

The Mathematical Education of Teachers II (CBMS, 2012), a report from mathematics and mathematics education experts, makes recommendations for K–12 teaching and professional development. The document specifically references MTCs as a type of professional development that should be more widespread, because they foster mathematics growth and opportunities for discussion about teaching.

Research on MTCs. The MTC model is still young and so very little research has been completed on these groups and the outcomes for teachers. However, two separate
groups of researchers have each published a series of studies related to MTC outcomes. Other publications have reported anecdotal or informal survey results that support the research in this area. Overall, the outcomes for teachers have been positive. White, Donaldson, Hodge, et al. (2013) reported that teachers who participated in a weeklong MTC workshop increased their Mathematical Knowledge for Teaching in the area of Number Concepts and Operations. Khaliqi, Marle, and Decker (2013) reported increases in several teacher beliefs, including preparedness to teach inquiry mathematics and personal mathematics teaching efficacy.

One set of research results has been reported by White, Donaldson, Conrey, Umland, and Nakayame (2013); White, Donaldson, Hodge, et al. (2013); White and Donaldson (2011); and White (2011). Results of these studies are described in the next section—this paragraph focuses on the methodology and context of the studies. White, Donaldson, Hodge, et al. (2013) reported the results of pre- and post-surveys from 50 teachers at three MTC summer immersion sites focused on middle-level (grades 5–9) mathematics. The authors administered the Learning Math for Teaching instrument, which measures teachers’ Mathematical Knowledge for Teaching in different content areas. Subsections from two content areas were given to the teachers: Middle School Number Concepts and Operations, and Middle School Geometry. White and Donaldson (2011) reported the preliminary results of teacher surveys administered to 169 teachers in the fall after participating in an MTC summer immersion from 13 different sites. This survey, modeled after the Student Assessment of Learning Gains survey, asked teachers to “rate their gains in: mathematical content knowledge, attitudes and dispositions toward
mathematics, classroom instructional practices, [and] professional activities,” and included both numerical ratings and open-ended items (White & Donaldson, 2011, slide 3). White, Donaldson, Conrey, et al. (2013) reported on data collected and analyzed from nine teachers from three MTC sites. The data included videotapes from three days of classroom instruction, semi-structured interviews after the videos, and lesson plans. While the videos did not provide enough information for generalization, they did allow for descriptive case studies of the teachers.

A second series of related studies reported the outcomes after one, two, and four years of MTC cohorts (Khaliqi et al., 2013; Marle, Decker, & Khaliqi, 2012; Szarka, Marle, Decker, Khaliqi, & Abrams, 2012). Each of these studies relied on the administration of a survey to teachers before and after yearlong MTC participation. The surveys included the Local Systemic Change through Teacher Enhancement-Math Questionnaire and the Mathematics Teaching Efficacy Beliefs Instrument, and assessed teachers’ personal mathematics teaching self-efficacy, teaching outcome expectancy, preparedness to provide guidance in mathematics, preparedness to teach inquiry mathematics, investigative classroom culture, classroom investigative practices, feelings of freedom from teaching to standards, and use of traditional teaching practices. Two of these studies also include pre and post observation and interview data (Khaliqi et al., 2013; Marle et al., 2012). Szarka et al. (2012) reported on survey data obtained from a group of primarily female teachers (N = 16) who participated in a yearlong MTC. Marle et al. (2012) reported on two years of data from 52 (primarily female) teacher participants, and Khaliqi et al. (2013) similarly reported data from four cohorts of
primarily female MTC participants ($N = 80$). These studies did not find significant results for teaching outcome expectancy, feelings of freedom from teaching to standards, and use of traditional teaching practices. The significant results from these studies are described below.

Due to the small amount of research on MTCs, anecdotal reports are also included here. Anecdotal evidence from Fernandes et al. (2011) was gathered via a questionnaire, focus group, and personal interactions with the MTC participants. Some of these teachers were new to the MTC while others had been participating for a while and had already modified their teaching based on the MTC sessions. Similarly, Shubin (2006) relates anecdotal evidence from surveys of MTC participants, but each of these participants were surveyed at the end of their first weeklong workshop.

**Increased mathematics knowledge.** Most studies of MTCs report an increase in teachers’ mathematics knowledge. Teachers’ average scores for Mathematical Knowledge for Teaching for Middle School Number Concepts and Operations, as measured by the Learning Math for Teaching instrument, significantly increased ($p < .001$) between pre- and post-tests (White, Donaldson, Hodge, et al., 2013). For the other section of the Learning Math for Teaching instrument, Middle School Geometry, the average scores increased slightly, but no single site nor the average of the sites showed a significant increase in scores. The authors concluded that while a significant portion of the workshop time was spend on Number Concepts and Operations (approximately 24% to 40%), the amount of time spent on these concepts was fairly minimal (6 to 9.5 hours) and most likely did not result in the acquisition of additional content knowledge. Instead,
the authors hypothesize that “the intensive time spent and hence experience gained at engaging in the practice of doing mathematics” helped the teachers to “reason more deeply about content knowledge that they already possessed” (p. 19), which resulted in the increased scores.

Similarly, White and Donaldson (2011) found that 60% of teachers reported “good” or “great” gains in topics related to their mathematical content knowledge, and over 75% of teachers reported moderate or greater gains. Teachers reported that they were able to see the logic, order, and the bigger picture of mathematics. A teacher in a case study also reported learning more mathematics from participation in the MTC (White, Donaldson, Conrey, et al., 2013). Other teacher survey results indicated that after participation in a yearlong MTC cohort, they had significantly ($p < .05$) increased preparedness to provide guidance for mathematical concepts (Szarka et al., 2012).

Observation and interview data in similar studies revealed that teachers’ “understandings of mathematical concepts” increased (Khaliqi et al., 2013, p. 3; see also Marle et al., 2012).

In the anecdotal reports, Fernandes et al. (2011) explain that the “biggest benefit reported has been to their mathematical learning” (p. 113). One teacher reported the benefit of feeling uncomfortable when challenged with high-level mathematics and explained that it helped her/him to understand where middle school mathematics eventually ends up. Similarly, teachers in Shubin’s (2006) report found it valuable to understand “the compatibility of ‘hard’ mathematics with the middle school curriculum”
(p. 66). All of the teachers responding to Shubin’s (2006) survey reported that they found the mathematics content valuable.

**Attitudes towards mathematics.** Teachers have also changed their attitudes towards mathematics or perceptions of themselves as mathematics learners as a result of MTC participation. In survey results reported by White and Donaldson (2011), over 80% of respondents reported moderate gains or greater in their positive attitudes about mathematics, including enthusiasm and interest in discussing mathematics with their colleagues and mathematicians, and over 33% reported great gains in this area. Half of the respondents reported that they were more likely to work on mathematics problems in their spare time. In the open-response section, one teacher mentioned an increase in confidence in working on nonroutine problems. Three other teacher cohorts significantly increased ($p < .05$) their reported feelings of preparedness to teach inquiry mathematics and personal mathematics teaching efficacy after participating in an MTC (Khaliqi et al., 2013; Marle et al., 2012; Szarka et al., 2012).

Teachers also began to see themselves as learners of mathematics. White and Donaldson (2011) reported that many of the teachers mentioned that they began to see themselves as mathematicians for the first time. In a case study, the teacher saw value in being a learner while also being a teacher (White, Donaldson, Conrey, et al., 2013).

Anecdotal evidence revealed that teachers found value in feeling uncomfortable and lost when the mathematics was over their heads or incomprehensible. Fernandes et al. (2011) reported that at least one teacher empathized with students who experience discomfort when faced with challenging mathematics. Shubin (2006) reported that some
teachers also explained that they felt overwhelmed or didn’t understand some of the MTC topics, but that the experience helped to put them in the position of their students who struggled.

**Changes in teaching practices.** Many teachers reported changes in their teaching practices. In survey results reported by White and Donaldson (2011), at least 65% of teachers reported increases in their time in class spent on problem solving, letting students work collaboratively on rich problems, exploring student questions, and helping students understand problem-solving strategies. Teachers also reported using more student-centered approaches in the classroom, providing more opportunities for students to problem solve, introducing more challenging problems, encouraging meaningful mathematical discourse in the classroom, and having increased expectations for all students (Morrison & Karakok, 2015; White & Donaldson, 2011). After participating in a yearlong MTC cohort, teachers survey results indicated ($p < .05$) increased classroom investigative or reform-based culture and practices, and observation and interview data revealed that teachers increased their use of reform-based methods (Khaliqi et al., 2013; Marle et al., 2012; Szarka et al., 2012). One teacher in a case study from White, Donaldson, Conrey, et al. (2013) reported that she learned instructional practices by seeing them modeled at the MTC sessions.

Anecdotal evidence supports the conclusion that teachers changed their teaching after MTC participation. Participants reported that they plan to have students think deeply about problems rather than merely performing calculations, focus on incorporating problem solving in the class, pose challenging questions to their students, and balance
their emphasis on the process and the answer (Shubin, 2006). Similarly, teachers in Fernandes et al.’s (2011) MTC report that they hope to find problems that engross students in fun, inspiring problems that enable them to “shut out the whole rest of the world” and “give kids a sense of mathematics as a living discipline that engages their minds no matter where they are or what level they are” (p. 114). These teachers want to encourage student excitement and engagement in mathematics, and felt more freedom to focus on students’ understanding. Participants in both groups explained that they learned to appreciate the value of letting students struggle with uncertainty in the problem-solving process (Fernandes et al., 2011; Shubin, 2006). One teacher reported giving students problems that she had yet answered so that she could explore them alongside her students (Fernandes et al., 2011). Some teachers reported that they were beginning to understand the connections among mathematical topics and they planned to incorporate these connections and multiple dimensions of mathematics concepts in their teaching (Shubin, 2006).

Many teachers mentioned incorporating more collaborative learning in their teaching (Shubin, 2006). Fernandes et al. (2011) describes one teacher whose positive experience working collaboratively helped him to realize that student collaboration could result in more actively engaged students who learn from one another.

Teachers also reported that the MTC sessions provided them with good problems that they could use or adapt for their classrooms (Fernandes et al., 2011; Morrison & Karakok, 2015; Shubin, 2006). Teachers used problems they experienced in the MTCs, and they also used the online MTC resources to find problems used in other MTCs. One
teacher explained that the problems made his students excited about mathematics and shared their excitement with their parents (Fernandes et al., 2011). This piqued the parents’ interest in reform-oriented curriculum and helped them understand the purpose of different curricula.

One common theme reported by White and Donaldson (2011) in the open-response section was that teachers believed that changes in their attitudes and understanding resulted in changes in their teaching. One teacher explained that s/he began to see mathematics as a creative endeavor and attempts to pass this belief onto her/his students.

**Community.** Teachers also frequently mentioned the value of becoming part of a professional community (White & Donaldson, 2011) and attribute their professional growth to the supportive community provided by MTCs (White, Donaldson, Conrey, et al., 2013). MTC participation has also been the catalyst for numerous leadership opportunities for at least one teacher (White, Donaldson, Conrey, et al., 2013).

Fernandes et al. (2011) reported that teachers who had been participating in the MTC for a while experienced benefits beyond changing classroom practice. These teachers found support in the MTC community that they did not receive in their schools. One teacher “felt validated” when she attended MTCs because they “gave her the belief structure and courage to carry on the problem-solving focus that she had in her classroom” (p. 114). Similar, teachers in Shubin’s MTC explained that they found valuable “the variety of educators present (middle school teachers, college professors, etc.), the opportunity to work with other teachers that allowed them to weigh and solidify
ideas, the time to explore and work on mathematics with a partner or small group, [and] the developing network of teachers with whom they are now able to share ideas” (Shubin, 2006, p. 66). For these teachers, working with a community of educators was a valuable aspect of the MTC experience.

**Reported challenges and future directions.** Shubin (2006) explains that the long-term impacts for teachers participating in MTCs are yet unknown. MTCs are difficult to study because of the variety of activities taking place at different sites. Although the MTC Network offers training to leaders of MTCs, the leaders may structure their MTC in a slightly different way, or the MTC at a particular site may evolve over time. Further, each MTC has a relatively small number of participants, each of whom is involved for a varying number of years: Some participants only attend the weeklong immersion, others attend the immersion and follow-up meetings throughout the year, and some repeat this cycle every year, becoming long-term participants. Thus, the long-term effects of MTCs may be difficult to determine for teachers who only attended for one year. It may also be difficult for a study to isolate the effects of the MTC if teachers are engaged in other professional development activities simultaneously.

Despite these challenges to research, MTCs are a promising site for teacher development. Shubin (2006) suggests that the positive outcomes for students participating in Math Circles may hint at positive outcomes for students enrolled in mathematics classes taught by MTC participants. Emerging research on MTCs has suggested that teachers begin to see themselves as mathematicians (White & Donaldson,
2011) and that teachers believe that the changes in their beliefs and attitudes towards mathematics resulted in shifts in their teaching (White & Donaldson, 2011).

Because MTCs are a fairly recent development, they remain largely unresearched. Future research can explore the effects on outcomes for teachers such as beliefs and teaching practices, and the effects on outcomes for students. Further, in-depth qualitative studies and/or case studies of sites or participants can provide a greater understanding of the effects reported from surveys and assessments.

Moving Forward

While a considerable amount of research exists in the affective domain of mathematics learning, there are still some significant gaps in the literature.

Much of the research on teachers’ mathematics self-efficacy was completed with preservice teachers. There is much less information about the self-efficacy beliefs of practicing mathematics teachers. Further, while there is a lot of research on mathematics teaching self-efficacy, or preservice teachers’ confidence about their abilities to teach mathematics, there is less research about their mathematics self-efficacy, or their confidence about their ability to solve mathematics problems. There was also very little research identified related to teachers’ mathematical identity as learners or doers of mathematics. But because teachers’ confidence about mathematics ability can affect their classroom teaching practices and their students’ mathematics beliefs, this is an area in which further research is needed. Further, mathematics content knowledge and mathematical knowledge for teaching are major areas of focus for mathematics teacher professional development, as it is generally agreed that content knowledge and
specialized content knowledge for teaching has an effect on teachers’ teaching practices (e.g., Ball, 1999; Shulman, 1986). Beliefs about mathematics are likely to influence teachers’ content knowledge, specialized content knowledge for teaching, and the way they apply this knowledge in classroom practice.

The literature on mathematics teacher beliefs and professional development reveals that the current understanding of mathematics teachers’ identities as doers of mathematics is very limited. The ways in which these identities and mathematics beliefs evolve as a result of content-focused and discovery-oriented professional development would make an important addition to this literature and would provide valuable information for leaders of professional development.

MTCs, as a promising new professional development model, are also in need of research. MTCs contain many of the qualities of high-quality professional development and show potential for positively influencing teachers’ mathematical knowledge for teaching and classroom practices. As constructivist-focused professional development that seeks to change teachers’ attitudes toward and perceptions of mathematics, MTCs may have an influence on teachers’ mathematics beliefs and identities. Future research is needed on MTCs to better understand the ways in which MTCs affect teachers and their students.

**Scholarly Commentary**

This section provides a synthesis of the literature review and how it relates to this study and instruments.
The basis of prior MTC research. The prior research on MTCs, while limited, suggests promising outcomes and areas for further research. MTCs are fairly new but are growing. New MTCs are begun each year, and the MTC Network has begun to establish regional centers to support MTC growth and collaboration across the country. MTCs have also been referenced by the CBMS as a high-quality form of PD, and there is interest in MTCs from state-level organizations, such as the Ohio Council of Teachers of Mathematics and the Ohio Department of Education. This type of support suggests that MTCs will continue to grow for a number of years.

Prior research on MTCs found that increased content knowledge is an outcome of teacher participation. Several studies have reported increases in teachers’ mathematics knowledge for teaching (e.g., White, Donaldson, Hodge, et al., 2013) and mathematics content knowledge (e.g., Khaliqi et al., 2013; Marle et al., 2012; White & Donaldson, 2011). Anecdotal evidence reveals another promising area of outcomes related to MTCs; that is, that changes in attitudes and understanding as a result of MTC participation changed the ways in which teachers taught (e.g., White & Donaldson, 2011). Other research revealed promising but limited results related to teachers changing their attitudes towards mathematics (Khaliqi et al., 2013; Marle et al., 2012; Szarka et al., 2012; White & Donaldson, 2011) and beginning to see themselves as learners of mathematics (White, Donaldson, Conrey, et al., 2013; White & Donaldson, 2011). These promising preliminary results suggest that future research related to attitudes towards mathematics and identity as a learner of mathematics may provide further insight regarding teacher changes.
The outcomes from prior research and informal surveys suggest that there is a need for in-depth case studies of teachers to determine what shifts take place and how they occur. Because of the established links between facets of identity and teacher practice (e.g., Collopy, 2003; Felbrich et al., 2008; Gresalfi & Cobb, 2011; Lappan & Theule-Lubienski, 1994; Leatham, 2006; Luft, 2009; Philippou & Christou, 2002), exploring teacher identity in relation to MTCs is a potentially important area of analysis.

**Teacher identity as a doer of mathematics.** The first research question relates to teacher identity as a doer of mathematics. In this case, identity is understood as a series of stories that a teacher tells about themselves, in relation to how others see them and how they see themselves (e.g., R. Anderson, 2007; Beijaard et al., 2004; Gee, 2001; Sfard & Prusak, 2005). The framework for understanding this type of identity draws primarily from the construct of productive disposition (NRC, 2001), but also considers established frameworks for student mathematics identity (R. Anderson, 2007; Martin, 2000), as well as a variety of affective factors that contribute to identity.

Productive disposition incorporates elements of beliefs, attitudes, motivation, self-concept, and self-efficacy. Two of these characteristics have been refined further within the domain of mathematics: beliefs and motivation. In particular, beliefs about mathematics are taken from the work of Schoenfeld (1985), fallibilist versus absolutist philosophies of mathematics (Ernest, 2008), and static versus dynamic views of mathematics (Felbrich et al., 2008). Research on these aspects of mathematics beliefs has related types of beliefs to positive or negative outcomes for students. Motivation, in this case, draws mainly from implicit theories of motivation; in particular, entity theories and
incremental theories (Dweck, 2000), and the beliefs that people have about intelligence and mathematics ability.

The research question concerns how teachers see themselves as doers of mathematics, what they think it means to “do mathematics” (including their beliefs about the nature of mathematics), how they enjoy (or not) mathematics and see it as useful (or not), and how they think others see them as capable or not capable of doing mathematics.

The instruments used for this research question are the pre- and post-immersion surveys, daily reflections, and interview questions. The sections of the survey titled “Learning Math,” “You and Learning Math,” and “Mathematics Autobiography” address these aspects of identity. In particular, the Mathematics Autobiography is specifically intended to elicit stories from teachers about their most impactful experiences with mathematics. Interview questions that address these aspects of identity ask the participant to reflect on their experience as a student, what parts of mathematics they enjoy, whether they consider themselves to be good at mathematics, and how their MTC experience aligns with their self-perceptions as a doer of mathematics.

**Professional identity as a mathematics teacher.** The second research question concerns aspects of teacher professional identity as it relates to teaching mathematics. The professional identity integrates the personal and professional sides of being a teacher (Beijaard et al., 2004); that is, viewing oneself as a mathematics teacher and being viewed by others as a mathematics teacher. This research question is further informed by the notion of identity as it relates to personal and normative identity development (Boaler & Greeno, 2000; Gresalfi & Cobb, 2011). The framework for this research question

The research question concerns the ways in which teachers see themselves as mathematics teachers (or as teachers of mathematics), their role as a mathematics teacher, how they think others see them as effective (or not) mathematics teachers, their roles in the MTC community and in their school, and their desired future teacher self.

The instruments used to explore this research question are the pre- and post-immersion surveys and the interviews. In particular, survey questions in the sections “Teaching Math” and “Mathematics Autobiography” address these aspects of identity. Relevant interview questions ask teachers how and why they became mathematics teachers, what they enjoy about teaching mathematics, what their successful mathematics lessons are like, how their school culture sees successful mathematics lessons, how their MTC experiences align with how they see themselves as a mathematics teacher, and how they plan to teach in the future.

Interaction of mathematical identity and mathematics teacher identity. The third research question explores the interaction of the teachers’ mathematical identity (as a doer of mathematics) and professional identity as a teacher of mathematics. Van Zoest and Bohl (2005) and Beijaard et al. (2004) describe how people hold multiple identities simultaneously. According to Beijaard et al. (2004), people hold multiple identities that are tied to different contexts. Van Zoest and Bohl (2005) describe these different identities as self-in-mind and self-in-community. These more closely that these identities align with one another, the more similar teachers’ actions will be in different contexts
(Beijaard et al., 2004). The literature on science teacher identity contributes ideas of personalized pedagogical content knowledge (Lee & Luft, 2008) and personal content knowledge (Helms, 1998). This research question explores the alignment and interactions of teachers’ different identities.

This research question explores the ways in which teachers’ past and current experiences with doing mathematics, including at the immersion, influence and are influenced by the ways that they see themselves as mathematics teachers.

The instruments that address this research question are the daily reflections, the pre- and post-immersion surveys, and the interviews. Specifically, the survey questions under the headings “You and Teaching Math” and “Mathematics Autobiography” address this question. In the interview, relevant questions include relating their experiences in the MTC to their descriptions of themselves as a teacher, to their perceptions of mathematics, and to their views of themselves as a mathematics teacher. Questions also ask the teachers to relate their experiences as a student to the ways that they currently teach, and their experiences as a doer of mathematics to their role as a mathematics teacher.
Chapter 3: Methodology

This chapter describes the methodological choices for this study. This qualitative study used explanatory multiple-case study methodology, and data collection included surveys, interviews, and teacher reflections. This chapter describes the research design, including data analysis and efforts to increase data reliability, validity, and transparency. Limitations of the study and ethical considerations are also explained.

Research Design

This study employed an explanatory multiple-case study methodology. A case study “involves the study of a case within a real-life, contemporary context or setting” (Creswell, 2012, p. 97) and explains how a phenomena or circumstance works (Yin, 2014). In this study, the cases are the teachers participating in the MTC, and the setting is the MTC. The choice of case study as a methodology draws upon Creswell (2012), who considers case study to be a methodology, rather than a research strategy or object of study. According to Creswell (2012):

Case study research is a qualitative approach in which the investigator explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audiovisual material, and comments and reports), and reports a case description and case themes. (p. 97)

The purpose of this study is to explain how teachers’ identities evolve during and after an MTC immersion workshop. An explanatory case study method is appropriate due to the focus on explanation (Creswell, 2012; Yin, 2014). Further, my study “account[s] for and describe[s] contextual conditions,” which is also appropriate for the purpose of a case study (Zeldin & Pajares, 2000, p. 221).
In particular, I chose to use a collective case study, as each case provides insight into the phenomenon. In collective case study, multiple cases are selected in order to illustrate a single issue, concern, or phenomenon (Creswell, 2012; Stake, 2003). Those cases may or may not share common characteristics, and “they are chosen because it is believed that understanding them will lead to better understanding, perhaps better theorizing, about a still larger collection of cases” (Stake, 2003, p. 138). I studied multiple cases and examined the similarities among the cases as well as the differences (that is, themes that are particular to some cases). A multiple-case design is considered more robust and the evidence “is often considered more compelling” than that from a single-case design (Yin, 2014, p. 57). However, it is important not to neglect the differences between the cases. I attempted to balance generalization and comparison with the specific features of individual cases (Stake, 2003) in order to maintain the depth of the individual cases (Creswell, 2012).

To gain a deep understanding of teachers’ beliefs resulting from participation in an MTC, I administered surveys with Likert and open-ended items, obtained documents related to the content of the MTC workshop, asked selected participants to complete a nightly reflection, and conducted semi-structured interviews with selected participants. Multiple types of evidence or data, such as interviews, reflections, and surveys, provide the information needed to deeply understand a case (Creswell, 2012; Denzin & Lincoln, 2003; Yin, 2014). Each type of evidence has strengths and weaknesses (Yin, 2014), but taken together, they can provide a more robust overall picture of each case.
A qualitative research design “describes a flexible set of guidelines” (Denzin & Lincoln, 2003, p. 36). That is, I determined in advance a detailed methodology; however, I remained flexible while implementing my design, adapting to unforeseen circumstances. In the interest of transparency, this chapter describes in detail both my intentions for the study as well as any deviations from my intended methodology (Yin, 2014).

Case Study

Setting. Participants for pre- and post-immersion surveys were recruited from a subset of the MTCs in the United States holding summer immersions in 2015. With the help of the MTC Network, I identified new MTCs holding immersions that lasted at least four days and that included large numbers of new participants. Three MTC sites met these criteria. I contacted the leaders of those MTCs, requesting that they contact their participants on my behalf and allow me to send my surveys out to their participants. I also asked the leaders to answer a demographic survey about the workshop and share their agenda and workshop materials with me.

MTC Site A. MTC Site A is located in the Midwestern region of the United States (according to the U.S. Census Bureau definition). The MTC took place in a meeting room on a university campus with a non-classroom feel and round tables. Teachers pay a $95 registration fee to attend and have the option of paying for graduate continuing education credit. About 70-80% of teachers paid for the credit. The MTC provided complimentary swag to participants, including a math game, paper, pencils, a university-branded cup, and other items. They provided a continental breakfast, lunch, afternoon snacks, coffee,
tea, and water. They also provided one evening dinner, which was attended by about two thirds of the participants. The days lasted from 8:30 am to 4 pm Monday through Thursday and 8:30 to noon on Friday. Teachers from several school districts attended, including from both public and private schools. About 20 teachers attended the MTC. The facilitators have led similar MTC workshops for the previous two summers. The MTC advertised that the teachers would “expand mathematical awareness and enrich problem-solving skills, create connections to the Common Core math standards, and establish a network of secondary math teachers.”

During the workshop, teachers spent the majority of time (about 70%) working in pairs or small groups, with the remaining time split between working alone and listening to facilitators. The sessions were fairly typical of an MTC. They used some of the standard activities, such as Liar’s Bingo and Exploding Dots. A nationally known and experienced MTC leader facilitated some of the sessions, including one session that lasted the entire day. Other sessions included mathematical games and spirographs; mathematics content like circuits, probability, Pythagorean triples, and algebra; and sessions on pedagogy like inquiry-based learning and planning for the next school year. The agenda included problem solving each day during breakfast. Informal networking was built into the schedule during two breaks, lunch, breakfast, and wrap up each day.

This MTC did not have as much debriefing as the typical MTC and as the others in the study—the facilitator said, “We forgot to debrief as much as we’d like to do.” They held two or three 10-minute debriefing sessions during the week, and a 30-minute session at the end of the week, during which they discussed “what they learned about problem
solving, the teaching of problem solving, and some of the connections to the Mathematical Practice Standards of the Common Core.” The majority of the facilitators were mathematics professors and mathematics education professors, although one session was led by engineers and another was team-led by a religion professor and a librarian.

**MTC Site B.** MTC Site B is located in the Southern region of the United States. The workshop took place on a college campus, Monday through Friday, from 8:30 am to 12:30 pm each day. Teachers received a $250 stipend for attending ($50 per day), parking passes, 20 professional development hours, daily breakfast and beverages, a SET game, and a t-shirt. Multiple breaks were built into the workshop days for networking, and breakfast was also used as an informal networking time. Participants came from various schools and districts in the region. Twenty teachers participated in the MTC. The facilitators estimated that about 80% of the workshop time was spent collaborating in a group or with a partner and about 20% was spent listening to a facilitator. Facilitators included mathematics professors, a statistics professor, middle school mathematics teachers, and a mathematician from industry. The two mathematics teachers who facilitated sessions were part of the leadership team. Each led one session, and they led the debriefing sessions together daily.

Sessions included some of the standard MTC activities, such as Liar’s Bingo, Exploding Dots, SET, and the Brownie Problem. Other problems were taken from the MTC website’s list of activities, including KenKen and Pick’s Theorem, and others included mathematical content like probability, functions, and polygons. Debriefing sessions were held after every session. During the debriefing sessions, the facilitators
“asked participants to tie each session to Common Core if possible. We asked for similarities and differences among the sessions. We discussed problem-solving strategies used to help in each session.”

**MTC Site C.** MTC Site C is located in the Midwestern region of the United States. It took place at a hotel: The hotel ballroom was used for the workshop and the teachers stayed in rooms at the hotel for the week. The workshop days lasted from 8:30 to 4:30 Monday through Friday and 7 to 9 pm Monday through Thursday (this was the lengthiest MTC of the three sites). Teachers received a stipend of $30 per day and $150 to spend on materials. The MTC also provided lunch, coffee, water, and snacks. The hotel provided breakfast and heavy appetizers for dinner. Participants came from about eight different school districts. Nineteen teachers participated in this MTC. Of the five facilitators, two were mathematics education professors and three were classroom teachers. The teachers lead some sessions and debriefed other sessions.

The agenda built networking time in during lunch and evening sessions. The MTC facilitator reported that there was also a scheduled hour for teachers who work in the same school district to “brainstorm next steps for including more problem solving and the mathematical practices in their classrooms.” They spent about equal time working alone, in pairs or small groups, and as a large group, and a little less time listening to facilitators. The MTC included standard MTC sessions such as the Brownie Problem, Cookie Jar, Introduction to Problem Solving, and the 1–100 Machine. Other problems included Math Bracelets, Hexominoes, Paper Bear’s Bakery, the Perfect Shuffle, and Math Walks. Some sessions were pedagogically focused, like “How to Make a Rich
Problem” or “Engage in SMP Practices.” During each session, participants worked on problems alone first, then shared with a partner or their table, and finally they looked at the problem as a large group, “sharing ideas for solving the problem and identifying the problem-solving strategies used to solve the problems.”

Each problem solving session was followed by a debriefing session: “One facilitator would lead the presentation of the problem and another facilitator would debrief by asking questions such as ‘what did you learn about problem solving?’ or ‘what did you learn about teaching problem solving?’”

Participants. The target population for this study was all K–8 mathematics teachers participating in an MTC Immersion Workshop for the first time during summer 2015. The participants in the reflections and interviews were elementary or middle school mathematics teachers who attended an MTC Immersion Workshop for the first time and elected to participate. Because MTCs are open to all teachers regardless of grade level, survey participants were elementary, middle, or high school mathematics teachers who were attending the MTC Immersion Workshop for the first time and elected to complete the survey.

Because teachers can attend MTCs every year, new participants are defined as those who had never attended a weeklong summer immersion, although they may have attended individual MTC sessions (for example, a kickoff event or a Saturday session). As a result, seven survey participants had attended a single MTC session in the past, two of whom became interview participants.
Once MTC immersion sites were obtained, I e-mailed all new participants of the MTCs with information about the study and a link to the informed consent and the pre-immersion survey. Those who agreed to the online informed consent form were provided access to the pre-immersion survey, and those who completed the pre-immersion survey were invited to complete the post-immersion survey. Overall, 33 teachers completed the pre-immersion survey and 26 completed the post-immersion survey. Table 1 provides more information about the number of participants from each site at each phase of data collection.

Table 1.

<table>
<thead>
<tr>
<th>Site</th>
<th>Pre-Immersion Survey</th>
<th>Post-Immersion Survey</th>
<th>Reflections</th>
<th>Interviews</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site A</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Site B</td>
<td>14</td>
<td>11</td>
<td>3 (4 invited)</td>
<td>2</td>
</tr>
<tr>
<td>Site C</td>
<td>12</td>
<td>9</td>
<td>3</td>
<td>2 (4 invited)</td>
</tr>
<tr>
<td>Total</td>
<td>33</td>
<td>26</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Case studies begin “with the identification of a specific case” or cases (Creswell, 2012, p. 98). Based on the pre-immersion survey, I used purposeful sampling to select ten teachers who reported beliefs associated with low or undeveloped productive disposition to record their impressions of each day of the immersion using a guided format. I anticipated that these teachers with the least productive disposition had the most potential for change, and I hoped to see the most extreme changes. The number of teachers selected is based on Guest, Bunce, and Johnson (2006), who found that six to twelve
cases were sufficient for saturation. One teacher did not respond to my request, so nine teachers completed reflections. After the post-immersion survey, I again purposefully selected teachers for interviews. These were teachers who demonstrated the greatest positive change in beliefs and identities, based on the pre- and post-immersion survey data. Six of the teachers who completed reflections were also selected for interviews. Eight teachers were selected and contacted for interviews, but two were unable to participate in interviews, so six teachers were interviewed (five of whom had completed reflections).

One goal of using purposeful sampling is to “select your sample to deliberately examine cases that are critical for the theories that you began the study with, or that you have subsequently developed” (Maxwell, 1996, p. 72). The teachers selected for interviews were those whose pre- and post-immersion survey responses indicated the most dramatic shift toward productive disposition; that is, from beliefs associated with negative outcomes (such as separated or static views of mathematics, nonavailing beliefs, and lower self-concept or self-efficacy) to beliefs associated with positive outcomes (such as connected or dynamic views of mathematics, availing beliefs, and greater self-concept or self-efficacy). Further, the selection of extreme cases “often provide[s] a crucial test of these theories and can illuminate what is going on in a way that representative cases cannot” (Maxwell, 1996, p. 72). I selected what I anticipated to be similar and extreme cases based on impact of the workshop rather than representative cases. Ideally, extreme cases contribute the most toward understanding the effect of MTCs on teacher identity and beliefs. These extreme changes help to conceptualize the
potential for change in mathematics teachers’ identities. I hoped that the changes in teacher identity or beliefs would be more apparent in these extreme cases, making these cases information rich.

The extreme cases for surveys and interviews were selected through a combination of several factors. First, negative Likert-scale survey items were reverse coded such that a higher number corresponded to greater productive disposition. I calculated the sum of the pre-survey items. This, along with the number of negative responses and analysis of short answers, was used to select participants to complete reflections. After collecting pre- and post-survey data, interview participants were selected based on short-answer responses, the mathematics autobiography, and changes from the pre- to post-survey. I calculated the sum of the post-survey items and the difference between the pre- and post-survey totals. I counted the number of items that changed from a negative to positive response (and vice versa) for each participant and the magnitude of the change. I also counted the number of individual items that increased from pre- to post-survey. From these data, I identified seven teachers who displayed the greatest changes in at least two of the criteria above.

Once the interview participants were selected, I reviewed the demographics of the interview participants compared to the larger group of teachers who responded to the survey. Demographics are summarized in Appendix A. If the demographics of one group was very different from the other, then I would be concerned that my survey questions were biased in some way, and that I was inadvertently selecting participants based on something other than changes in beliefs, attitudes, and other aspects of identity. After
reviewing the demographics of these teachers, I realized that I did not have any male teachers, so I looked back over the participants and identified a male who had fairly significant changes. I did not find any other substantial differences between the demographics of the interview participants and of the survey participants.

**Data Collection**

The data collection procedures and participants are summarized in Table 2.

<table>
<thead>
<tr>
<th>Table 2.</th>
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</thead>
<tbody>
<tr>
<td><strong>Summary of Data Collection Procedures</strong></td>
</tr>
<tr>
<td><strong>Population</strong></td>
</tr>
<tr>
<td>MTC leaders ((N = 3))</td>
</tr>
<tr>
<td>All participants of new MTCs ((N = 24))</td>
</tr>
<tr>
<td>Selected MTC participants ((N = 9))</td>
</tr>
<tr>
<td>Selected MTC participants ((N = 8))</td>
</tr>
</tbody>
</table>

**Teacher surveys and reflections.** Participating teachers completed up to three different written responses. First, all consenting participants completed a pre- and post-immersion survey. In addition to collecting demographic information at the end of the post-immersion survey (including gender, college degree, license, grade level taught,
subject(s) taught, years teaching experience, and whether the teacher had attended an MTC session or an MTC summer immersion workshop in the past), these surveys contained two types of questions. One type of items were Likert scale and drawn from existing instruments intended to measure beliefs about mathematics. The other type of questions were open-ended items that asked teachers to reflect on their beliefs about mathematics, their experiences with mathematics, and their mathematical identity. These questions occasionally asked the teachers to expand upon their responses to Likert-scale questions. The pre- and post-survey questions are contained in Appendix B.

Although a survey that measures the mathematical identity or mathematics teaching identity (as operationally defined in this study) for practicing teachers does not exist, the survey questions used in this study draw from instruments that measure similar constructs. Similar instruments include the Mathematics Belief Instrument (Hart, 2002), the beliefs survey (Schoenfeld, 1989), the Beliefs Questionnaire (Beswick, 2006), the Attitudes and Beliefs Regarding Mathematics and its Teaching (Beswick, 2006), the Beliefs About Mathematics and Teaching survey (Stipek et al., 2001), and the Conceptions of Mathematics Inventory-Revised (Briley et al., 2009).

Nine selected teachers also completed a written reflection each night of the immersion workshop. I asked those teachers to write one to two paragraphs responding to the prompt: “Describe how you felt doing the mathematics in the workshop today.” Each evening, the teachers e-mailed their reflections to me. I responded to the teachers individually after I received each reflection, to encourage their continued reflections and to guide their reflection if necessary. A few times, a teacher’s first reflection was very
general, and so I attempted to encourage a deeper reflection in my response. For example, one of my emails included, “It sounds like you had quite an interesting first day. For tomorrow’s reflection, I’d love to hear more about the specific situations where you felt a particular way.”

The process of data collection is displayed in Figure 1.

**Figure 1.** Process of data collection.

**MTC quality.** Because the activities of MTCs can vary, I wanted to verify that the MTCs attended by participants aligned with the goals and activities of MTCs as outlined in this study and engaged the participants in “doing” mathematics. I used the daily reflections from the participants to gauge the activities and the active involvement of the participants, and I also obtained information from the leaders of the MTC.

I asked the leaders of the MTCs to share with me their agendas, planned activities, and any handouts. I also sent a short demographic survey about the workshop (Appendix
C) to the leaders to obtain more contextual information about the workshop. These questions were drawn from the structured observation protocol developed by Brackett and Hurley (2004) to describe the activities of the MTC session and provide a snapshot of the session.

**Interview protocol.** I conducted one 45- to 60-minute interview with each of the teachers selected for the study. After a few rapport-establishing questions, Creswell (2012) recommends five to seven scripted open-ended questions. Specific questions that I asked are listed in Appendix D. The questions followed up on the pre- and post-immersion surveys, and drew from other interview protocols developed for tracking changes in preservice teachers’ knowledge, soliciting stories about people’s prior mathematics successes and failures, and understanding teachers’ views about mathematics (D. I. Cross, 2009; Kennedy, Ball, & McDiarmid, 1993; McCulloch et al., 2013).

The rich information and detailed stories, examples, and experiences gained from in-depth interviews were necessary for my analysis and understanding of changes in teacher identities. According to Yin (2014), “one of the most important sources of case study evidence is the interview” (p. 110). In-depth interviews “explore in detail the experiences, motives, and opinions of others and learn to see the world from perspectives other than their own” (Rubin & Rubin, 2011, p. 3). Such interviews can help me reconstruct situations and events that I have not experienced and can “create portraits of complicated processes” by combining interview data from multiple participants (Rubin & Rubin, 2011, p. 3).
The interviews in my study were semi-structured and responsive. In semi-structured interviews, “the researcher has a specific topic to learn about, prepares a limited number of questions in advance, and plans to ask follow-up questions” (Rubin & Rubin, 2011, p. 31). I used a list of interview questions to guide my line of inquiry, but I was also open to probing teachers’ responses and gaining more information about their stories. Open-ended interviews allow the respondents to answer in any way they choose and provided more flexibility for me to ask follow-up questions about new information or to skip questions that do not apply (Rubin & Rubin, 2011). This flexibility also allowed me the opportunity to ask my participants to elaborate on particular responses on their survey or in their reflections. Rubin and Rubin (2011) describe responsive interviewing as a particular style of interviewing in which the researcher can adjust the questions “in response to what he or she is learning” (p. 7). As a result, “responsive interviewing brings out new information often of startling candor, and often suggests unanticipated interpretations” (Rubin & Rubin, 2011, p. 7). This method is suitable for talking with participants about sensitive topics, such as beliefs or identity in relation to a feared subject. It requires the building of a relationship between interviewer and participant, mutual trust, a friendly tone, and flexible questions.

I audio recorded the interviews and had them transcribed using a professional transcription service. I then listened to each interview while reading the transcription to ensure that everything was properly recorded, including mathematical terms. I did this as soon as possible after the interview so that the interview was still fresh in my mind. I shared the transcriptions with the participants to verify their meaning and intent, and I
followed up with one participant via email with a clarification question. All of the teachers who responded to the transcription wrote that they had nothing to add and that their responses were accurately represented.

**Data Analysis**

In qualitative designs, the researcher is the instrument (Janesick, 2003). According to Denzin and Lincoln (2003), “qualitative interpretations are constructed” (p. 37). From the documents (interview transcripts, reflections, and survey responses), known as “field texts,” I created “notes and interpretations based on the field text,” or a “research text” (Denzin & Lincoln, 2003, p. 37). This research text is “re-created as a working interpretive document that contains the writer’s initial attempts to make sense of what he or she has learned” (p. 37). In the creation of this research text, I used three main types of data analysis strategies: memos, coding and other categorizing strategies, and contextualizing strategies (Maxwell, 1996).

**Memos.** According to Maxwell (1996), “the initial step in qualitative analysis is reading the interview transcripts, observational notes, or documents that are to be analyzed” (p. 78). While reading and listening to the audiotaped interviews, I began making memos in which I noted tentative ideas about relationships and patterns. As I continued with the process of coding and data analysis, I continued to use memoing as a strategy to keep track of ideas.

**Coding.** I coded interview transcripts, written survey responses, and reflections using a priori and emergent codes. The goal of coding, according to Maxwell (1996), is to decompose the data and rearrange it in order to create categories and compare data within
and across cases. I coded the interview transcriptions, teacher survey responses, and
teacher reflections by marking key phrases or passages with a key word that described the
passage. The coding system used themes from the relevant literature as a priori codes, but
I also sought out emergent codes and created new codes as they emerged from the data
(Yin, 2014). According to Maxwell (1996), “the key feature of most qualitative coding is
that it is grounded in the data” (p. 79) and so I used the participants’ words when naming
a code whenever possible. For example, initial codes included the words “struggling” and
“challenge,” because these were words commonly used by the participants.

The coding process was iterative. After an initial pass through the data and coding
line-by-line, I went back through the data and examined quotations at the sentence and
paragraph level. I attempted to keep codes “linked…to the data that gave rise to them, in
order not to lose the original context from which they developed” (Maxwell, 1996, p. 79).
I revised codes based on the larger context of the quotations. For example, the quotation
“the problem solving wasn’t over my head” was initially coded “confident doing
mathematics.” Closer examination of the paragraph, however, revealed that this statement
was given in the context of a tool (geoboards) that the teacher did not have prior
experience with and that the hints provided by the MTC leaders helped her to feel
successful. I also added more quotations to any codes that emerged during the process of
coding. For example, when a code emerged during the process of coding the third
interview, I read back through the first two interviews to see if I missed any instances
where that code was applicable. I added a few quotations in this process.
After two passes through all of the data, I sorted the codes with ATLAS.ti (2015), collapsing and organizing the codes. I flagged codes that had only one or two associated quotations, or that were unique to only one participant. This exercise helped me to note things that were unique about the cases. Some of the codes with one or two associated quotations were collapsed into larger code groups. I also flagged codes that had an extreme number of associated quotations and read through the quotations to find ways to split the code into multiple, more specific codes. For example, the code “perseverance” was split into two codes to reflect teaching students perseverance and experiencing perseverance as an MTC participant. As I read through each set of quotations, I sometimes found that a quotation did not fit with the others. I was able to refine the definitions of the codes and unlinked them from some quotations. Some quotations had three or more associated codes, so this process helped me to limit the codes to only the most salient ones for the greater context of the quotation.

I began to group the codes into themes, and then I read through the data once more, paying close attention to the major themes and confirming or disconfirming any emerging theories. The emerging themes appeared to highlight aspects of specific research questions, so I examined each code in relation to the theme and the related research question, and I tested each code. I explored whether it fit better with a different theme or a different research question. Reading the associated quotations helped to clarify each of the codes and themes. I wrote a summary of each code based on the associated quotations and reflected on whether the name of the code was accurate and descriptive. Similarly, I wrote a summary of each theme and checked that each code and
quotations accurately reflected the summary and that the summary accurately reflected the codes. A list of the codes and subcodes associated with each theme is in Appendix E.

Throughout this process, I read through the data multiple times.

**Contextualizing strategies.** The third type of data analysis strategies used were contextualizing strategies. These strategies attempt “to understand the data…in context, using various methods to identify the relationships among the different elements of the text” (Maxwell, 1996, p. 79). This took place simultaneously and iteratively alongside the process of coding. I attempted to make “connections between categories and themes” (p. 79) by drawing on and building upon the contextual ties in the data to describe a coherent whole. I used the constant-comparative method throughout data collection and analysis to find themes that recurred in different data sources and from different participants. According to Janesick (2003), this means to “continually reassess and refine concepts while conducting the fieldwork” and then during data analysis, to “develop working models or theories in action that explain the behavior under study” (p. 61). I identified relationships between themes and the significance of each theme, and I tested the working models with other data.

I first completed a within-case analysis, in which I analyzed each case individually and wrote a summary of each case (Creswell, 2012). Once I had this summary of each individual case, I began to compare the cases using a cross-case analysis. I examined how the themes in one case may or may not be present in another case. I tracked similarities and differences between the cases. I also noticed similarities as I constructed the single cases, so I used memoing to keep track of these ideas as I noticed
them, and I later referred back to these memos. Finally, I developed assertions about the case study in general (Creswell, 2012). I wrote a general description of the changes in identities that were supported by all the cases.

I also attempted to explore plausible rival explanations for the teachers’ changing beliefs and mathematical identities (Yin, 2014). I was careful to refrain from assuming that the changes were due to the MTC workshop, and if I was uncertain about influencing factors, I followed up with the interview participant. In order to be reasonably certain that changes were due to the MTC, I actively sought out influences on teacher beliefs and mathematical identity outside of the MTC.

**Transparency of Data Collection and Analysis**

According to Hiles and Čermák (2007), “the notion of transparency is the overarching concern in establishing the quality of qualitative research” (p. 2). Transparency relates to the clarity and accuracy of the data collection and analysis. In order to have transparency, I have described in detail each step of data collection, theory, and analysis (Moravcsik, 2014). While “there is no value-free or bias-free design,” it is important to minimize bias and be open and forthright about my interpretations and potential biases that may have influenced these interpretations (Janesick, 2003, p. 56). Therefore, I explained my positionality and potential biases in Chapter 1 and the framework I drew from in Chapter 2 in order for the reader to understand my orientation and interpretations.

This chapter (chapter 3) describes in detail each step and procedure of data collection and analysis, so that these can be replicated by others (Hiles & Čermák, 2007). I described my process for selecting cases in this chapter, and in Chapter 4, I justify my
selection of particular cases for examination (Moravcsik, 2014). My description of the
data analysis describes any factors that may have affected the survey responses,
reflections, or interview responses. Additionally, I described any variation in data
collection procedures that occurred, as data collection “in naturalistic settings requires
compromises and adjustments to procedures” (Hiles & Čermák, 2007, p. 3).

I also described each step of data analysis and the interpretive process that I used,
including direct quotes from the interview transcripts, teacher reflections, and survey
responses (Moravcsik, 2014). The direct quotes assist the reader in making their own
conclusions about the data and determining if their conclusions align with mine
(Moravcsik, 2014). However, it is important that the reader remember that according to
qualitative research, there is no “single interpretative truth” and different interpretations
can be constructed (Denzin & Lincoln, 2003, p. 37–38). I describe any contradictions that
I found in the data and how I incorporated those contradictions into my analysis and
conclusions.

I also tracked saturation of data. I noted that with each new case, fewer new codes
were added. If multiple new codes were added upon coding the final few cases, I would
have been concerned that the selected cases did not adequately represent the breadth of
possible experiences. The sixth complete case resulted in three new codes. The first
incomplete case added one additional code. The final incomplete case resulted in no new
codes. After coding all eight cases, I felt confident that the chosen cases reflected a
variety of extreme case experiences.
Data Reliability and Validity

Reliability. Reliability considerations include replicability of the data collection process and consistency of data analysis.

Replicability means that another researcher would obtain the same results if they followed my procedure with the same case (Yin, 2014). In order to increase replicability, my data collection procedures have been thoroughly described. I also attempted to increase reliability by including “low-inference descriptors” into the finished work (Silverman, 2004). Essentially, this means that actual quotes or parts of transcriptions are included in my summary. Longer excerpts of data are preferred, such as “the question preceding a respondent’s comments as well as the interviewer’s ‘continuers’ (e.g. ‘mm hmm’) which encourage a respondent to enlarge a comment” (Silverman, 2004, p. 221). I include these longer data excerpts as appropriate in Chapter 4.

I made every effort to analyze the data in a consistent manner. I did this by following the same set procedure for coding each document (Creswell, 2012).

However, Janesick (2003) argues that “qualitative researchers do not claim that there is only one way of interpreting an event” (p. 69) and so the description that I have formulated based on the data is based on my interpretation only. Others with different backgrounds and theoretical frameworks may form different interpretations.

Validity. Different types of validity that should be considered are internal validity, external validity, and construct validity. Several methods were used to increase validity of this study.
To address internal validity, I used pattern matching and explanation building, I am transparent about discrepancies in the data, and I use rich, thick descriptions. Pattern matching involves comparing the patterns found in a case study with predicted patterns made prior to data collection. I also used pattern matching to determine if a result was found over multiple cases, thus making the result more robust (Yin, 2014). Similarly, I used explanation building by attempting to build a general explanation that explains all of the cases. This is an iterative process by which I created an initial explanation, then would compare this explanation to one case and revised the explanation. I continued by comparing other details of the same case or different cases to the explanation and revising the explanation accordingly until the explanation fit the information from all the cases (Yin, 2014). It is important to be transparent about discrepancies in the data and cases that do not confirm the emergent themes. The constant-comparative method was used to determine if all cases share relevant themes, and deviant cases were identified and analyzed (Silverman, 2004). I identified and analyzed “discrepant data and negative cases” to determine how they fit into or modified my framework, and I have reported any negative findings in my analysis (Maxwell, 1996, p. 93). I have carefully considered whether any other factors may have led to the observed changes (Yin, 2014). A final way to address validity is to use rich, thick descriptions about the participants and settings (Creswell, 2012). I engaged in memoing or reflective journaling throughout the data analysis process, recording questions about my analysis, my thinking about the analysis, and reflexivity. I used this process to help organize my thoughts as I analyzed the data,
and I also went back through these journals and memos at several times throughout the process to ensure that I did not neglect any important emergent themes.

To address external validity, I used multiple cases. Further, the participants were selected from multiple sites (Yin, 2014). I also addressed external validity by relating the cases back to established theory (Yin, 2014). However, it is important to note that this study does not make any broad claims for generalizability; but rather, reports possible outcomes for teachers.

To address construct validity, I used data triangulation, conducted member checks, and established a chain of evidence. Data triangulation ensures that relevant themes are found in multiple data sources, such as interviews, surveys, and/or teacher reflections (Maxwell, 1996; Yin, 2014). For example, many of the stories that participants wrote about in their pre- and post-surveys were repeated in the interviews. This suggests that these stories are part of their identities, as they are told over and over again. I used member checks by e-mailing the interview participants transcripts of the interviews and my interpretations, and I requested their feedback (Janesick, 2003; Yin, 2014). Member checks helped to ensure that I was attempting to learn from the participants directly, rather than “pigeonholing their words and actions” into my framework (Maxwell, 1996, p. 90). This helps to ensure that the phenomenon occurred as I explained it and my description is not solely based on my impressions.

**Potential bias.** The researcher has been a leader of an MTC in the past, and the dissertation advisor has been active in MTCs and has a stake in the success of MTCs. To reduce potential bias in this study, we specifically excluded from consideration in the
sample MTCs with which the advisor had a direct relationship. However, the researcher and advisor are aware of potential bias towards MTCs that may exist if particular measures to reduce bias were not put into place. Therefore, measures such as triangulation and member checks were an essential part of the research design and methodology.

**Delimitations**

Interview participants were selected from multiple MTC sites. The sites have been established as typical MTC sites, but the results of data analysis may not encompass changes in teacher beliefs and identities that occur at other sites. Therefore, the results should not be considered to represent the experiences of all teachers participating in an MTC summer immersion. Also, because extreme and similar cases were selected for in-depth study, these cases may not show the range of experiences of MTC participants. The pre-immersion questions used to identify the extreme cases were chosen based on the relevant literature, but these criteria may have failed to identify some teachers who experienced great changes in their beliefs or identities.

The study is also restricted to K–8 teacher participants who have never before participated in an MTC summer immersion and does not explore the long-term outcomes of MTC participation. Therefore, these results are limited to the more immediate evolution of teacher identities and cannot make any assumptions about long-term changes.
Limitations

Due to the sensitive nature of beliefs, identity, and potential mathematics anxiety, the accuracy of the data is limited by the extent to which participants were forthright and open with me. Teachers may have been reluctant to share some of these feelings and beliefs with me. For example, they may have been concerned that if they shared their fears about teaching mathematics with me, then I would look badly upon them as teachers, or I would share their weaknesses with their supervisors. This research is dependent on participants sharing their beliefs and experiences openly and honestly. To address this concern, I attempted to establish rapport with the participants through my emails and my conversations with them prior to and during the interview. I assured them that everything they said in the interviews would be held in confidence. I also shared my experiences with them, including my own weaknesses and fears related to mathematics and my own experience as an MTC participant. I attempted to affirm their responses that they gave during the interview.

More interview participants were selected from MTC Site C than the other two sites. This may have been due to the much greater number of contact hours of this MTC as compared to the others (this MTC met for full days plus evenings), or it may have been due to chance.

There are a few limitations due to participant demographics. A few of the teachers who were selected for interviews or reflections were unable to participate. For example, I was unable to conduct any interviews with male MTC participants. I included the data from one male as an incomplete case, but the analysis may be limited in that the results
may only apply to female MTC participants. All of the participants were also White or Caucasian. Only one Black teacher completed the pre- and post-surveys, and this teacher was not selected to complete reflections or an interview. I did not collect participant demographic information from the MTC leaders, so I do not know if the pre- and post-survey participation was representative of the MTC participation.

Pre- and post-survey data revealed that several teachers chose to attend the MTC because they recently began teaching more mathematics classes, or they would be in the upcoming year. Their choice to attend the MTC based on these reasons may have made them more amenable to changing their identities, as their job roles were changing as well. Overall, teachers self-selected to attend the MTC, so they may have already been on the path to rethinking their mathematics teaching, or they may have been particularly interested in learning more mathematics.

**Ethical Considerations**

All participants were provided with informed consent forms that they read electronically prior to their participation in the study. The informed consent form explained the purpose and risks of the study and explained that participants could opt out of the survey or interview at any time they chose. Participants acknowledged that they consented to participate by typing their name.

Teachers were not pressured to participate, and their decisions to participate or not to participate had no effect on their involvement in the immersion. Personally identifiable information was not shared with anyone, including the teachers’ supervisors or the MTC leaders. Every effort was taken to be responsible to anonymity and
confidentiality for participants. All data that are reported have been depersonalized. All records and transcripts were stored under a pseudonym and I conducted the interviews in a private location where they could not be overheard.

Participants may have experienced benefits from reflecting on their experiences and thinking deeply about their beliefs and identities related to mathematics. Talking about their MTC experience may have enriched their experience and caused them to develop more productive beliefs and identities. These may be considered potential benefits to the participants of the study.

Subjects were selected for the study with disregard to race, gender, age, socioeconomic status, or any other factors other than their first-time participation in an MTC Immersion workshop, grade level taught, and their survey responses in relation to the constructs under study. Participant demographics were not asked until the final section of the post-immersion survey. Therefore, these demographics could not be considered in the selection of participants to complete reflections, and the teachers were also not inadvertently thinking about these identity markers during completion of the surveys.

Approval for the study was obtained from the Ohio University Institutional Review Board prior to participant recruitment and data collection.
Chapter 4: Individual Cases

Introduction

This chapter contains the results of data analysis of the eight participant cases. There were six complete cases and two incomplete cases. The six complete cases were constructed from survey data, interview data, and in all but one case, reflection data. The two incomplete cases included only the analysis of survey and reflection data, as these participants were unable to participate in an interview. I have chosen to include these data in my analysis because these participants had demographic characteristics that were not represented in the other cases (i.e., one teacher was male, and the other teacher was very new to teaching). Many of the teachers referenced specific MTC activities or problems. Appendix F contains links to the most common activities so the reader can gain some context for the problems.

Each case begins with a description of the participant demographics, followed by sections related to the participants’ mathematical identity, teaching identity, and interaction of identities. Although the MTC is the focus of the study, teachers’ identities are complex and interweave experiences from the past and present, and further incorporate elements related to their anticipations of the future. Therefore, the case descriptions include stories from the teachers’ pasts in addition to their perceptions of the MTC experience.

Case 1: Anne

Anne attended MTC Site A. She was a 51-year-old Caucasian female with a bachelor’s degree in elementary education and mathematics emphasis. Anne taught in an
urban, high-needs school district with a high population of English as Second Language (ESL) students. Anne originally did not intend to be a mathematics teacher. She wrote, “I was going into special ed, learned what an IEP was (in the 80’s) and ran to the next idea. That was, I was doing well in my math classes so I decided, well, how about a middle school emphasis.” She clarified that she enjoyed teaching multiple subject areas (including math), but that at the time of the MTC, her school district needed mathematics teachers. She taught sixth- through eighth-grade mathematics, although in the past she taught reading, science, and social studies. She taught math for 18 out of the 28 years she has been teaching. She did not know anything about the MTC when she signed up for it, and signed up with colleagues because the timing fit their schedules. As a result, she said that she was “happy that we happened to luck out and get into” the MTC for the summer and that she was very likely to attend another MTC workshop.

**Mathematical identity.** Anne considered herself to be average at doing mathematics because she could not get enthusiastic about mathematics without real world connections:

Researcher: Would you consider yourself to be good at math, or not good at math?
Anne: Average. I’m not extremely good at it, no.
Researcher: Why not?
Anne: Again, it’s more the…once it gets out of the realm of useful, I tend to lose enthusiasm for it. I get fractions. I get linear equations. I’m okay. I get the thought pattern behind it. I get why you kids would think it’s boring to get into, finding the derivative, you know? Why would I need that? I have a hard time exciting myself about it, let alone, the kids, other than, “Ooh, it’s neat. Okay now, move on.”

In the survey, Anne explained that she was anxious about higher level mathematics, particularly when she had not used it for a while or she needed to help
students extend their understanding: “Problems that I need to ‘stretch’ for. Having not taught above the eighth grade, some of the higher math has not been used for a VERY long time so I am always concerned that I can help students reach an understanding while learning it myself sometimes.” Her anxiety about mathematics was related to having to teach it to students. She said that other than teaching, she did not often do mathematics in daily life: “Mostly just teaching it. A little bit of financing, a little bit of…predicting, you know, ‘I need to do this for the kids. Each kid needs some of that and some of this, but how much do I need to make sure,’ so there’s some predicting, some ratio proportion kind of stuff, but not a lot.”

In the past, Anne did not enjoy doing mathematics, but she had a different experience in the MTC: “That’s the first time I’ve enjoyed doing math in a long time.” She also explained that although she would not be able to bring everything to her classroom, she said that it was “good because I didn’t want to do everything that was just on their level, you know, giving me a challenge was nice, too.” Her reasons for enjoying the MTC also related to the problem-solving approach and the group collaboration. Both helped her to feel successful. One of her favorite parts of the MTC was “the group dynamics.” She enjoyed one particular problem because “there was a lot of block moving, there was a lot of talking. I had an exceptionally good group that was really on it that day so everybody was tuned in.” She also felt like she could approach the problems from her skill set: “That was exciting, not to be the dumb kid in class, not to be the one that was going, ‘Um, wait a minute. I don’t get that. How’d that happen?’ Where the way circle was run, it was a lot more, ‘Here’s the idea. You come up with a way to solve it
using whatever math skills you have right at this moment.’ ” She was pleased that the problems were accessible to “us and the high school teachers; we’re all able to attack it at our own level” and that some problems didn’t rely “heavily on previous knowledge.”

Anne also enjoyed when she learned something new and interesting to her. She had some “a-ha” or “wow” moments during the MTC where she was amazed by the connections or the patterns in the mathematics problems. For example, she said, “It mostly just blew my mind that you could have an exponent, or a base that was a two-thirds or a three-halves, you know like, ‘Whoa! Nobody ever told me that.’ ” Another time, she had an epiphany with the help of another group:

Probability spoons was another time that I got that wow moment. The circle I was working in was doing such hard work and had hit a wall. I went to another circle and asked if they had found a third way to meet the parameters, they had an additional combination that worked. With that small hint, we were able to come back to the problem and work to find the next and next and then the pattern… wow!

Some of the mathematics was new to her, and some of the mathematics that was not new to her was “interesting” in the ways that it “connect[ed] to other things.” She explained, “some things [we]re new, some things just reinforced.”

However, Anne also had some moments of anxiety during the MTC. One instance that she described in detail was an activity in which the facilitator led the teachers through a problem using graphing calculators. In her post-survey, she wrote, “I am anxious about problems that need advanced technology that is used.” She explained in the interview, “I got so lost on the graphing calculator I never got anything else.” She similarly “checked out” during explanations when problems were extended past her understanding and “at the last extension he/she would sometimes leave me behind.”
typically added that even though she “did not follow all the way to the higher level math,” she was “tickled to have gotten as far as I had.” She added, “Some people like just the basics; some people like going a little deeper” and that “some of the insight was great.”

Anne often based her level of understanding on comparison to others in the MTC. For example, the following description of one problem focused on her perception of her abilities in relation to the other teachers:

I was excited I could keep up with most everybody. I can get lost too often. A few times I was able to move the thought along so that we weren’t stuck in one area. I figured out real quick I had remembered something from, who knows, high school, middle school, about how to find a number adding from one to a hundred and you don’t have to add one plus two plus three. You add the one plus the ninety nine, the two plus the, you know, so it was half of whatever the number is plus the number gives you…Anyways, I was able to put that into one of the equations to make the math that we were doing a lot easier. We were looking for variations of perfect whaters, who knows, but the fact that I was able to put a couple of things in that other people had not thought about made me feel like I was, ‘Okay, I’m not as iffy as I thought I was going to be.’ A lot of times I would be listening to somebody else and they’d come up with an idea and it was like, ‘Why didn’t I think of that? That’s a better way of doing it.’ I felt like I was right in there with the other teachers. I hadn’t stunted my growth in seventh grade.

In this instance, Anne was excited to be able to contribute to her group but also disappointed when she did not think of a strategy or idea on her own. Several of her other statements compared herself to other teachers in the way she was able to follow a discussion or in the ways they were able to approach problems at different levels. For example, she said about not being able to understand a problem, “It was kind of good because then I also watched everybody working, and some of the people got into it. The higher level teachers basically got into it more than we did…they were probably going, ‘Finally, well I can use another tool.’ ” She was very conscious of the high school
teachers being able to use different “tools” or strategies to solve problems. She was also conscious of her own understanding—or lack of understanding—and whether she was able to follow a discussion of a problem through the extensions.

**Mathematics teaching identity.** Although she did not initially choose to teach mathematics, when asked if she considered herself to be a mathematics teacher or a teacher who happened to be teaching mathematics right now, Anne said, “I can be considered a math teacher. I’ve been in math now for six years straight.” She considered her teaching abilities to be “in the top half of the population” but said, “I’m not the best. I do reach a lot of kids, but I could be better, and I’m trying to get better.” She said that there was a “disconnect between what I’d like to be able to give kids and do, and my skill level of being able to drop something on them like that and stand back while the chaos goes on might not be exactly the same.” She was excited by the possibilities of using problems from the MTC but said, “It’s a little on the scary side when you haven’t done it, but it looks exciting.”

She believed that she changed over her teaching career and that she would continue to change and evolve. She described herself as “always adapting and learning… have not taught math exactly the same in 28 years… new is good.” In particular, Anne did “a lot more [instructional] differentiation nowadays” and more “interactive hands-on stuff, the active engagement, that kind of idea, a lot more scaffolding because I am more aware of the differences between the kids than I was when I first started.” Another change in her teaching was small group work. She said, “30 years ago when I was ‘new’ to this, I was more into each kid needed to work by themselves at all times. Made my
teaching a lot harder and less enjoyable.” Her current teaching philosophy focused on collaboration: “If I could teach the rest of my career in a collaborative classroom where students could be supported would be my dream. At the moment I achieve it about 26% of the time.”

Some of the changes in her teaching were due to larger changes in the field of mathematics education and curricula. The curriculum included more advanced material than it used to, and so not only did she “expect more from [her] students,” but she said, “They can reach farther than I was taught they could.” She struggled with this particular change, though, and had difficulty balancing computation with more advanced problems. She said she “used to really push the basics,” and “nowadays, it’s ‘give them a calculator.’ I’m not a hundred percent against that. It’s just sometimes swinging a little too far to the right and it was a little too far to the left.” She had a “hard time letting go of the computation skills and just allowing calculators take over the algorithm parts. It does free the students to explore deeper and feel more successful. Still trying to find a balance here.” She seemed to be internally conflicted about how best to help students learn mathematics under these changes: “I wish some of the kids knew some of the basic facts like multiplication facts when they got to me, since I do seventh grade, but if they don’t, do I really have the time to go back and teach it? Is that what my main goal is?”

She also perceived differences between the values of her school district administration and her own beliefs about effective mathematics teaching (which tended to align with the MTC activities and pedagogy). She wanted to continue to use more instructional differentiation to “reach kids on whichever level they need to be reached,”
but she felt constrained by the school district’s “lockstep” requirements, in which the teachers had a “mandated need to ‘be here by this time and there by this time.’ ” Further, she did not want to teach students to be dependent on her: She wanted to teach them to be dependent on themselves. This meant that sometimes students struggled or were “mad at me because I’m not giving him an answer,” and she was not sure that the administration understood the pedagogical reasoning behind productive struggle. She said that the administration looked “for a number on a piece of paper some days” but that “there’s a lot of math going on that they don’t see. There’s a lot of learning situations that aren’t just paper/pencil. I’m looking for more things: Did they connect it to anything else? Can they tell me why it works and not just do it? I don’t think they look for that.” But she also said that her principal “really trusts me and doesn’t really question” her pedagogy, and that the district supported mathematics teachers by providing them with longer class periods and smaller class sizes. So whereas the administration might have held different perspectives on pedagogy, they supported the teacher.

The MTC “added to” or reinforced Anne’s ideas about teaching mathematics. She spoke about specific aspects of the pedagogy, and she compared it to her own teaching and the ways that she wanted to change her instruction to be more similar to that of the MTC. She said she:

Watched very closely how they [MTC facilitators] were able to…walk around and become part of the conversation without guiding it. I’m not sure I’d be that good at it. I’d tend to want to go in and say, “Have you thought about this?” and [the facilitator] was very good about letting the conversation continue down the wrong path for a little while before he would, you know, to see if it would come back around or make a very subtle suggestion which wasn’t, “Have you thought about this?” It was, “Hmm…so that’s 15? Interesting,” and then move on,
where…I was very impressed with that with him. He was able to do that and demonstrated that very well for me.

During the MTC, she actively thought about how to incorporate the activities in her classroom. This was “the fun part” of the MTC to her. She did not think of ways to incorporate all of the activities, but she said, “About a third of it I went, ‘Hey, I could use it in the classroom.’” Anne and her colleagues were able to “immediately implement” a few of the shorter activities “that aren’t so long.” She also planned to use the Exploding Dots activity in her classroom and explained both when it would fit into the curriculum and why she thought it would be useful for the students:

I like the idea of teaching base ten using that idea. I’m about ready to do long division. I have so many kids that think they can’t do long division. I was thinking about that would be a good way to show them what they’re doing with taking the blocks and moving them from box to box. Or just, “We have this one left over. He can’t stay here so we’re going to knock him to the one below, but remember, he breaks into ten.” I think it’d catch a couple of kids that don’t get why you’re doing the certain steps.

However, she was having trouble finding time to use some of the problems. She said, “The big picture problems we haven’t done as much of” and she was uncertain about how to incorporate “investigations that will be successful with the goals that are ‘pre directed’ by the district.” She was concerned that the administrators “outside of math might not see the connections. We’re dealing with principals and people that are not math teachers so trains [a problem from the MTC] to them might not equal teaching ratio proportions.” She was “trying to figure out a time that I’m not being observed/mandated beyond the ‘chapter one at this time’” and thought that she might fit it in during “a little wiggle room right at the end of the school year and also right before the breaks.” Another concern was the time that it would take to teach with multi-day problems:
Hoping to get a lot more use out of the open-ended group activity like the train problem that they gave the kids that took more than one day to solve, but yet, it’s not lockstep. It’s something that they have time to figure out and they have days to figure out and not just, “In 15 minutes this needs to be done.” Haven’t been able to figure that out yet, but it’s something I’m looking forward to trying.

Although she had the support of a peer to use the problem, she seemed hesitant to fully incorporate it into the curriculum. She said, “The time limits are a big one, trying to get time in to do things like that. I mean at the circle, adults some of that, it was taking a day to figure out some of the stuff we were doing. It was like, this should be at the kids’ level, but it’s going to take them longer to figure the math out than it did us.” She would be able to “cut down” some of the problems, though, and would not need to go as “deep” with her students as they did during the MTC.

She was “impressed with how hard [the participants worked] and the buy-in that the circles achieved,” in contrast to some of the “learned helplessness” and students giving up in her classroom. Anne believed that it was important for students to “figure out that they do like math, that they do like the challenge, that they do have the skills.” She was excited about “the possibilities of letting the kids have a little bit more of that leash.” She said the MTC “may have made me better at the guidance instead of trying to direct. Instead of trying to be the backseat driver I need to be the trunk driver, get back and let people try a little bit more before I… I mean, I’m pretty good about not jumping and saving somebody, but I even need to be better about it, let them struggle a little bit.”

The MTC also reinforced that “a lot of the things I do aren’t a waste of time” and gave her ideas for how to gain administration support. For example, she said, “I do a lot of gaming. I do a lot of teaching kids new games or teaching kids how to do a logic
something or other and it reminded me that’s important, too. Sometimes you don’t know how to get the support for that through administration, but there’s a lot of payoff to being able to logically think two steps ahead.” She explained that she liked the “explore part” of the MTC problems, and that she liked “giving the kids a chance to be frustrated, get themselves out of a situation where the teacher is not just saying, ‘Here, do this.’ It’s ‘How would you like to solve this?’ or ‘How do you think we’re going to go about solving this?’ ” She also wanted to incorporate more extensions for students who figure something out quickly to “encourage them to go in more depth.”

Although Anne spoke of misalignment between her ideas and the district administration, she perceived a closer alignment with her fellow teachers. In particular, some of her colleagues attended the MTC, and she talked about how they all planned to incorporate some of the MTC problems into their classes. When she talked about using specific problems from the MTC in her classes, she always referred to her colleagues: “The other teacher I teach with also went to the same workshop and is interested in trying it [the train problem].” When talking about some of the shorter activities, like the 24 game, she said, “We brought that back to the kids, and we've done the sub squares where they have seven different pieces and they have to put it together to make a square, cube, or other things can be made together. All the teachers that I went with have made sets of those for their classroom. Some of that kind of stuff was come back and immediately implemented.”

**Interaction of identities.** Anne’s identity as a doer of mathematics seemed to be very intertwined with her identity as a teacher of mathematics. When asked about doing
mathematics, she did not talk much about doing mathematics growing up or doing mathematics in her life; instead, she mainly spoke about teaching mathematics. She said that most of the time that she did mathematics in her life was when she was teaching, and she was anxious about doing mathematics problems when she felt unable to help her students understand. When asked about the instances she enjoyed doing mathematics, she wrote that she enjoyed doing “the ‘diagnostics’ of mathematics. Trying to analyze and find ways to reach students with misconceptions or ‘holes’ and being able to help them reach or sometimes even try…This is the part of math I enjoy. The actual math is a way for me to reach them.” All three influential moments in her mathematics autobiography took place during teaching: a student who made an advanced connection between mathematical topics, working with students when concepts “click,” and the frustration of working with students with “learned helplessness.” For Anne, the teaching of mathematics was the same thing as doing mathematics.

Further, Anne mentioned some experiences in learning mathematics that influenced her teaching. She was a visual learner, and so she used more visual examples with students than when she was in school. She tried to “make sure I’m doing it a lot of different ways” and “show it to them on pictures or I’d give an example of it instead of just, ‘learn it.’ ” She also made an effort not to judge students by their past, because of a teacher who gave her the benefit of the doubt when she was a student. She was failing mathematics when she moved to a new school, and her new teacher—who misread her previous F as a B—took the time to help her get caught up, saying, “Oh, you had a B at the other school, you must not have gotten to this point yet. Let me teach it to you.” Anne
said that experience “really impacts how I teach—that doesn’t mean that they’re never going to learn it.”

Anne also talked about experiences learning in the MTC that she related to teaching. When she was unable to solve a problem on her own but was able to follow someone else’s explanation, she said, “It was kind of good because then I also watched everybody working, and some of the people got into it.” She wrote that these experiences were useful because she “also got a glimpse at the kid who might not have come up with the idea but by seeing what others were thinking grew in their understanding. As adults we were more likely to say, ‘wait, where did you get that from?’ I think teaching kids to be comfortable doing this would be of great benefit.” She also felt empathic with students; she “felt that I was the student” when she was confused about one of the extensions.

**Case 2: Beverly**

Beverly attended MTC Site C. She was a 57-year-old Caucasian female with a bachelor’s degree in education and a license to teach all subjects in Grades 1 through 8. She taught fourth grade mathematics, religion, reading, and language arts in a suburban school district. Beverly taught third grade—all core academic subjects—for five years at the start of her career and then became a stay-at-home mother. When she began teaching again 16 years ago, her new school district had recently adopted a new mathematics curriculum. She attended two summer workshops in consecutive years related to the new mathematics curriculum and teaching mathematics using discovery methods and hands-on materials. She “really did not enjoy the math” when she taught third grade but became
“excited about the math” after attending those workshops. She found the MTC to be “very motivational” and she was very likely to attend another MTC meeting.

**Mathematical identity.** Beverly “hated math” as a student and “found it difficult and painful.” She was confused by the “new math” taught in her elementary school. Beverly also had difficulty with mathematics in middle school when she had a series of substitute teachers in mathematics class. About high school geometry, she said, “I could not understand those proofs,” and in Algebra II, she “sat there everyday with rocks in my stomach thinking, ‘don’t call on me, don’t call on me, don’t call on me.’ ” Because of these experiences in high school, she only took the minimum required mathematics courses in high school and college. She said, “I was so glad that back when I went to school, you only had to take three years of math in high school, so I never took it my senior year. Then when I was in college, not much was required of teachers either. I think we had one basic math class, which was very basic, and then math methods. I really didn’t have much beyond high school. Math was not a favorite subject of mine.”

However, despite her negative experiences in mathematics, she currently considered herself to be “fair” at doing math:

Researcher: Would you consider yourself to be good at doing math?
Beverly: I don’t know a lot of higher math. I mean, I never took calculus or anything like that. Even now I don’t know how much I could get through high school algebra. I’m not…I don’t do much higher math. But definitely I’m sure I could do fifth and sixth grade. It’s a little above what I teach.
Researcher: Yes.
Beverly: You would expect that. You asked if I was good at it? If I thought I was good at it?
Researcher: Yes.
Beverly: I’m fair. The higher levels I don’t know.
Beverly did mathematics “just for fun” sometimes. For example, after the MTC, she described a situation when she shared a puzzle from the MTC with her family: “I might share a problem. One of the problems that we did in the Math Teachers’ Circle, the other day, we visited our son in [city] and we were sitting at the restaurant and I said, ‘Oh, try this.’ We just had fun with it. I like to do Sudoku puzzles and things, or problem solving. I might do things for fun in math.” She also talked about some instances in which she used mathematics in real life: “I do things like balance the checkbook and that stuff.”

She learned a lot about herself, her mathematical abilities, and her mathematical preferences in the MTC. She wrote, “I think I am learning as much about myself this week as I am about math—that’s a lot.” She had some anxiety about doing mathematics, but post-MTC, she wrote, “I don’t think I feel anxious about math as I did when I was a student.” She also focused more on types of problems after the MTC. In the pre-survey, her comments about mathematics problems that she was excited about solving focused on her skill level: “I am NOT excited when it is so far over my head that I haven’t a clue how to begin” and “I am excited to solve a math problem when it is a little challenging, but I have the skills to figure it out.” However, after the MTC, her responses focused more on the types of problems she preferred, which tended to be “problems that have real world application, such as MEAs [Model-Eliciting Activities], as opposed to pure math problems, such as the 1–100 challenge.” She liked one problem because of the “human side” of the problem:

I really enjoyed the MEA problem about Papa Bear’s retirement. I think one of the reasons I liked it was because there were no right or wrong formulas, but it
brought out the human side of business. I loved the fact that Patrick “Papa” Bear cared enough about his employees to have set aside so much money to share with them at the time of his retirement.

She also liked problems that had patterns: “I feel excited about a math problem when it has a real world application or when I begin to see a pattern in the solution.” She realized that she enjoyed finding patterns in mathematics, and she was frustrated when she could not find a pattern. It seemed that finding patterns helped to reinforce that she was on the right track and provided motivation to persevere. She said:

I realized the other night at our Math Teachers’ Circle meeting, I like things where there’s a pattern and I can find a pattern. Because we were doing a problem and I could not find the pattern. The problem was, “If you had three containers and they each had a different capacity and you had a certain amount of water, how could you get one gallon or two gallons or three?” I didn’t know how to start it and even after I got started, I didn’t know if I was on the right track. I found that hard whereas when we did the problem of summing consecutive numbers, there I could see a pattern and after a while it got exciting. It’s like, I could go on and do this and go up to 100. When I’m not sure that I’m one the right track, it’s like, okay I don’t know, should I keep going, should I try a different way?

Beverly also learned new things about mathematics that were amazing to her. She said she learned that there were patterns everywhere, and she was amazed by the patterns she was able to find in various problems: “I learned that there’s a lot more patterns in the world than I ever imagined. Some of the problems, they just seemed so random to me until someone pointed, discovered a pattern—wow!”

She was frustrated by a few aspects of the MTC. One source of frustration was when she found the mathematics to be difficult. She described one mental math problem when she “had a difficult time keeping all the numbers in my head. I wanted to write it down on paper.” She also talked about wanting to give up when she struggled: “I guess wanting to give up. The pencil thing was a little frustrating…I guess when…if I worked
at it a while and I couldn’t see how to solve it, it was tempting to give up. But it doesn’t happen too often.” A second source of frustration for Beverly was making mathematical connections to problems or activities that did not include mathematics on the surface. She said:

There was one activity that I understand was a geometry thing. It was like…I really don’t know what I learned about math by doing this. I’ll show it to you…we did this pencil sculpture. It was the last day and I was having such a hard time with it. Finally our time was up. I thought, “I’m just taking these pencils home and I’ll give them to the kids.” Then [MTC leader] says, “I want you to bring that back the next time, finished.” Oh, shoot. I was just going to forget about it. I persevered. I said, “I am going to do this until I get it done.” I did not enjoy doing it. But when I showed it to my students, first of all they thought it was cool. I said, “This shows a lot of perseverance. This is what we have to do in math. We have to persevere. We have to stick to it.” I said, “Even though I didn’t enjoy doing it, I do like the finished product.” Then I told them, I said, “Now if you need to borrow a pencil, do you think I’m going to let you borrow from here?” They go, “No.” I said, “You’re right. I put a lot of work in here.” That was one thing I really questioned. What am I really learning from this other than persevering with something. It also…it was interesting. I was listening to NPR on the way driving somewhere a couple days later…after I did this and they were talking about this new discovery somebody made about a pentagon that could be made into a dodecahedron. I’m going, “Oh my gosh, that’s what I did.” I mean I don’t…I’m still not 100 percent certain why we did this.

Although she suggested that the activity related to perseverance and dodecahedrons, she was still uncertain about the greater mathematics connections. She later talked about a few other activities that “I never would have thought of that as math” because “it was more problem solving than math problem solving, some of it.” She thought to herself, “Okay, I understand this is solving a problem, but it’s not really a math problem…I was trying to solve a mystery, but I didn’t see it so much as why are we doing this at a math thing, even though it was fun.”
Perhaps as a reaction to these problems, she said that she did not learn new mathematics in the MTC: “As far as learning new math things, then no. Calculus or algebra, I don’t think I learned that.” On the other hand, she did learn “a lot” about mathematics and about problem-solving strategies.

Beverly said that the MTC “helped improve my image of myself as a person who can do math.” She said:

I know I will never be able to come up with all the formulas like the high school and college teachers were able to. But at the same time, I was able to solve some of the problems in my own way. We did some card…yeah I guess they’d be called card tricks. I was able to figure out patterns even though I couldn’t put a formula to it. Yes. I guess it helped me to see myself as a mathematician.

She felt satisfied and competent when she was able to solve problems, and the experience of success during problems, such as the card trick activity, gave her motivation to continue trying: “Being able to figure it out. I guess when I got partway through it and I was experiencing success even though I didn’t have all the answers to finish the card trick, I wanted to keep going. I didn’t want to give up because I had experienced a little bit of success. It gave me motivation to keep going. I can figure this out. I can do this.” A few other times, she mentioned feeling successful. For example, in her reflection on the first day, she wrote, “When we solved the circuit board problem I was proud of myself for being able to find an answer so quickly. I didn’t get it on the first try, but on the second.” She wrote that one of the things she learned from the MTC was “to persevere in problem solving. It is far easier to give up and say, ‘I can’t do this,’ but it is far more rewarding to stick with a problem until it is solved.”
When she struggled with problems, Beverly found that collaboration with her group was beneficial: “There was one I didn’t understand. It was near the beginning of the week when we did a problem…I can’t even remember what it was, but I could not see a pattern right away until somebody told me. After they told me the pattern, then I could figure it out.” In her reflections, she described how the collaborative process was particularly helpful for solving a problem:

We were trying to see how many arrangements of hexominoes there are. At first I was trying to draw them with paper and pencil, but then I realized that the other people in my group were able to find a lot more by working together with tiles. So I abandoned my paper and pencil tool to work with them. We were very successful working together, but I can understand why one or other of my students might find group work challenging. Even though it requires effort, working together is well worth the effort one puts into it. Together we accomplished more than we could have on our own.

She also reflected, “It was very exciting to be with people who are excited about doing math.”

Despite these positive experiences with collaboration, Beverly struggled to work with her group for much of the week. She reflected, “Group work is hard, especially for an introvert.” She wrote:

I found it difficult to connect with the other people at my table today. They are very nice people, but two of them are from the same school and often talked together about whatever. (Actually, if someone else had been here from my school I probably would have done the same.) The third person in the group liked to work independently and always had a thousand more examples on her paper than I did. (Not that I was trying to copy off of her; I just happened to notice.) I started thinking, “It would be fun to be part of group B,” or “Group C would be more interesting.” Then I thought of my classroom in which I assign groups. I told myself that I just needed to make sure that I was being the best group member that I could be.
Beverly also negatively compared herself to the others in the MTC. She felt slower than the others, and sometimes she commented that she was the only person to not solve a problem:

Sometimes I feel a little foolish because I take a lot longer to solve a problem than my peers do. When we began the day with a “number talk” (mental math problem), I knew how I would solve the problem, but I had a difficult time keeping all the numbers in my head. I wanted to write it down on paper. Still, I wasn’t discouraged because other students couldn’t see how “slow” I was. I enjoyed the discussion that followed because someone explained the way I was trying to solve the problem, and my partner chose to solve it another way that made sense to me. I’m hoping that if we do another mental math problem tomorrow I can come up with two ways to solve the problem in a reasonable amount of time.

She later said about a different problem, “I think I was the only one who could not figure out the pattern of the numbers. Still, I didn’t let that get me down.”

**Mathematics teaching identity.** Beverly was enthusiastic about teaching mathematics. She said, “It’s one of my favorite subjects to teach. I wish we had more time to do it. I only had 43 minutes. I wish I had an hour at least. But, yes, it’s definitely one of my favorites.” However, she “really did not enjoy the math” during her first five years of teaching, when she taught third grade. She said, “It was boring for me as well as for the students” because “it was all individual” and “it was more memorization.” She had shifted her way of teaching from memorization to understanding: “You have to memorize these basic multiplication and division facts, where now it’s more understanding and I emphasize a lot, ‘Go from what you know to help you figure out what you don’t know.’” She also explained that her methods of teaching had changed: “I used to teach by ‘show and tell.’ Now my teaching is much more about having the students discover and actually learn mathematical thinking.”
Her teaching changed when she returned to the classroom after spending seven years as a stay-at-home mom. Teaching mathematics was “not intentional,” but it was part of the open fourth grade position she applied for. She attended an in-service for the new curriculum, and she “really got interested in it” and was “convinced that this was an excellent way to teach math.” After teaching the new curriculum for a year, she said, “I was still overwhelmed with it and not sure I was doing a good job. The following year I enrolled in the Success program led by [facilitator]. That program gave me the confidence to teach math and to help students to get excited about learning math.” As a result of the program, she said, “I got excited about the math” and “the hands-on and the different way of learning.” Specifically, the ways that she taught at the time of the MTC included “quite a bit of talking in the classroom” by students, and students “figuring it out for themselves, discovery, investigating it.” Beverly described one lesson that she recently taught:

Now I try to get kids to discover things on their own rather than telling them. For example, yesterday we were making arrays. Then they put their posters up and tried to see why did some only have 2 factors, and others had an odd numbers of arrays, and others had an even number of arrays. Some arrays could be made into a square. That helped me introduce that vocabulary, but it came through their discovery rather than me just telling it to them and having them copy it down.

Beverly also believed that student feelings towards mathematics were important. She considered a lesson successful if “the kids are excited when they’ve discovered something, they want to come and show me what they did or go home and talk about it.” She believed in developing perseverance and “that they can learn even if it’s hard.” She told her students a story about a mathematician who would solve the same problem in a different way every day for a week, and she tried to encourage them to “think of different
ways we can approach a problem and share that with the rest of the class. I think it helps them to be flexible in their thinking, that there isn’t just one way of doing things.” She also told her students that “mathematicians make mistakes” and that “we actually learn more from our mistakes than if we had gotten it right the first time maybe by accident.” She believed all students could be good at mathematics and that “maybe those who aren’t better didn’t have the same opportunities that those who are did.” She also believed that “making math fun to learn makes it easier to learn” because “if math is enjoyable to the students they are more likely to stick with it.”

Beverly believed that she was still learning and growing as a teacher. She said, “I can always do better” and gave her teaching practices “maybe a B or a B-.” She seemed to see growth and improvement as ongoing. She wrote, “It seems like I am always rethinking and learning new ways of doing things. I will hopefully be learning until I retire.” She said, “There’s a lot I need to do to improve. Being more mindful of the mathematical practices. I mean they’re posted, but actually keeping them in mind. I wouldn’t say I’m a model teacher.” In addition to the mathematical practices, she wanted “to learn how to do differentiated instruction in the math classroom.” Being an effective teacher was important to her. Prior to the MTC, she wrote, “I want the trust that the parents of my students place in me to be well founded. Furthermore, I will be having a student teacher in my room this year, and I want to give her the best learning experience possible.”

Beverly had planned to integrate pedagogical elements of the MTC into her instruction. In her post-survey, she wrote that she planned “to discuss and post class
norms with the students at the very beginning of the year and to review them frequently.”

The interview took place very soon after the school year began, and she confirmed that her classes set up norms on the first day:

Beverly: I did that with my students but instead of calling them norms, I called them guidelines. I figured they would understand that term a little better. They came up with pretty much the same things that we adults had come up with.

Researcher: What kinds of things were those?
Beverly: That we need to respect one another and persevere, not to give up if you can’t think of the answer the first time. We’re all working at this together. If we’re talking in a group, listening to one another or waiting. If someone is thinking, say, “Are you ready to talk about it now?” We’re not interrupting their thoughts. A lot of being respectful of one another in the classroom and listening to one another.

She also said that the MTC “reminded me I should be doing this type of thing with my students” and reminded her what successful mathematics lessons could look like. The idea of using problem solving was not new, but it did serve as “a refresher” for her because “it’s easy to fall back into old ways.” She also planned to use specific MTC problems and activities in her classroom: the math walk, finding patterns in consecutive number sums, the fraction track game, and number talks. She said, “I also like the fact that many of the problems we have worked on this week can be adapted to different grade levels.”

However, she also talked about some challenges to using the MTC problems in her classroom:

For one thing, we had lots of time. When we did our Math Teachers’ Circle problems, we had two hours. And I have 43 minutes and usually the first three minutes is, ‘Okay copy your assignment and yes you may use the restroom and yes you may get a drink but hurry back.’ Then it’s just…it seems like so much takes away from the time I have. I feel like I want to stop the clock. Whereas Math Teachers’ Circle, we had the time to just work on the problem. That’s a challenge. I’ve been thinking ahead about this fraction game I want to do with the
kids when we get to fractions. Making sure that we do enough examples at the board first so they understand the rules of the game and then giving them time to actually do it and learn from it. It seems like so often I’m rushing. Okay, we’re not finished with the game, but our time is up. That’s a challenge. That’s the difference between Math Teachers’ Circle and sometimes in my classroom.

Beverly found time to be the greatest challenge in using MTC problems in her classroom.

**Interaction of identities.** Beverly’s mathematical identity and mathematics teaching identities were very tightly intertwined. She had negative mathematics experiences growing up and only began to enjoy teaching mathematics after taking a workshop related to teaching inquiry mathematics. Consequently, she began to enjoy doing mathematics as well. She wrote, “My best experience has been learning by doing. Having struggled through the first year of Investigations, then taking part in the Success project I learned to love math and the teaching of math.” She also did not seem to distinguish between doing mathematics and teaching mathematics. When asked questions about doing mathematics, her responses often described her teaching of mathematics. For Beverly, when she taught mathematics, she was also doing mathematics.

During the MTC, she had some experiences in which she felt like a student, and as a result, she was better able to relate to her students. She said that this was “a good learning experience for me too because we were in groups and it helped me to understand how some of my students may be feeling at times.” For example, when she found herself wishing she could switch to a different group, she reminded herself to focus on being a good group member herself. This is something that she “frequently ask[s] of my students, but never really had to do myself.” Similarly, she took what she learned from the
experience and drew upon that when she introduced her students to working in groups this school year:

I explained to my students at the beginning of the year, I was at this math teacher’s meeting this summer and we were in groups. I was there thinking, “Oh, that group looked like they’re be more fun. Those people in that group? I’d rather be over there.” But I had to learn how to get along with the people in my group. The first couple days were a little rough for me because two of them were from the same school and they really knew each other. Then the third person, she was a very nice person, but she just had a very different personality than I was used to. Finally, I just told myself, “You’re not going to change them, you’re not going to change groups, so you better change yourself and your attitude.” By the end of the week, I really…we were able to work together, but it took some adjustment on my part. I know when I assign groups here in the classroom, it helps me understand maybe how some of them are feeling at times.

During the MTC, she said that she was actively trying to make connections between the MTC and her classroom. She wrote, “During all of this I keep thinking how I can use this in my classroom. That is my main purpose for being here.” She used a variety of problem-solving strategies in the MTC, which helped her to realize that she needed to reinforce these skills with her students: “I guess maybe it reminded me of different things that I should be doing in math” like “working backwards or making an easier problem, even trial and errors, giving multiple examples.” After her experience with an engaging problem, she wrote, “During the discussion I suddenly realized how I cheat my students by teaching them how to solve certain problems instead of helping them to discover it for themselves. I made a note to change that in my lessons this year.” She was also considering starting an after-school Math Circle for the students: “It will be for any student; not just those who are at the top of the class or those who need remediation, but anyone who would like to have fun playing math games and solving fun problems.”
Case 3: Carol

Carol attended MTC Site B. She was a 35-year-old Caucasian female with a bachelor’s degree in music and a master’s degree in elementary education. She taught fifth grade in a rural, high-need, low-income, Title I designated school. She had a pre-K through five license, and she had taught kindergarten and Grades 3 and 4 in the past. She had taught for seven and a half years. Mathematics had always been one of several subjects that she teaches. However, she taught more mathematics over time, and the upcoming year would be her first as a dedicated mathematics teacher. The administration had made this assignment for her, but she clarified, “I don’t have any problem with it…I teach everything and I love teaching everything, but I know I’ve always enjoyed teaching math.” The MTC was not what she expected, but she enjoyed it and was likely to attend an MTC in the future.

Mathematical identity. Carol enjoyed mathematics in middle school but found it boring in elementary school, high school, and college. Her middle school mathematics teacher “really engaged me as a math learner” and “really got me to the point where I was exploring and experiencing math in a completely different way.” One example of this was when her teacher gave the class an “extra credit challenge” that she would work to solve for hours every night. She remembered “persevering through that and continuing to try and try and try until I, finally, I was the only person who got it and solved it.” But in elementary and high school, she said her mathematics classes were “very dry, boring, and ‘kill and drill.’ ” She described her classes as, “Here’s the text book, we’re going to start with chapter one, read it, do the work, I’ll check your work.” As a result, she “only took
the required math classes in college.” At the moment, she felt anxious about mathematics only “when there are specific things that just have to be memorized rather than it being something I can work through.”

However, she enjoyed doing mathematics, particularly “solving the problems and really finding different solutions to things. At the same time I like seeing the different paths I can take to get to the same answer.” She also enjoyed the “feeling of success and accomplishment” when she solved a problem correctly. She enjoyed logic puzzles and playing math games. She said, “I have a tendency to sit and play KenKen and different other math type games. I lose myself in those games.” Although she saw instances of doing mathematics in the real world, they were situations that she did not enjoy: “besides obviously my job, really all the standard things like taking care of my checking account and doing all of that maintenance type of stuff, which isn’t necessarily fun.”

She believed that she was “naturally good” at mathematics because she was “very methodical, very logical, and very organized,” which helped her when working through problems. She recently read an article about growth mindset, and she believed that having a growth mindset was important in mathematics. Her comments about success and failure in the MTC workshop reflected this belief: “Getting things wrong during the workshop allowed me to delve deeper into why and how.” Some of the mathematics problems in the MTC required knowledge of algebra or probability in which “the background knowledge I used to have has been mostly lost and it thus takes longer for me to get there simply because I’ve not used that skill in decades.” She found some of these problems challenging because although she was able to partially work through problems using
logic, she needed assistance from others to help her remember some of the formulas she
needed. As a result, she wanted “to go back to college level math courses and just take
them for fun to build that knowledge and just collect that knowledge back and
reinvigorate my brain that way.” She learned about the nature of mathematics in the
MTC, particularly the connections between mathematical topics. She “immediately saw
the binary connection” in the Exploding Dots problem, but “I’d never made that
connection with the base ten system, or the place value system that we have. It was really
great to see that expansion.” She assumed that one problem using geoboards would
involve geometry, but her group “quickly discovered we were using geometric shapes but
working more closely with algebraic formulas. We even discussed at our table that when
we learned these subjects in school they were completely ‘separate’ topics within
mathematics but once we were older and teaching it the ‘lightbulb’ flashed on with the
connection between the two seemingly separate units.” She also discovered connections
between “patterning, binary coding, and calculation, again connecting seemingly
‘separate’ units.”

She also learned about herself and her preferred learning styles in the MTC. For
example, she said, “This week it became clear to me how much of a difference it makes
what type of learner I am. I excelled with spatial and visual questions.” She realized that
when she was “learning or developing something new,” she needed complete silence and
time to “sit and think to myself before sharing out.” She realized that this was also the
way that she learned and processed when she was reading. “I clearly realize (only after
participating in this workshop) how I work the same way in mathematics.” She said that
the MTC “helped remind me of the things that I haven’t learned or I haven’t used in some time.” For example, near the end of the week, she realized that she was using strategies to solve problems that “I’d learned throughout the week.” She also realized that she “hate[d] being unsuccessful with something,” and that she would persevere “until I get it right.” She believed that the MTC helped her to build perseverance because “it allowed me the space and the time to really work with other people to grow my processes.” Collaboration was a key part of her experience successfully working through problems: “It was exciting to struggle with math and persevere through to success or even not to get there but then learn from others!”

Collaboration was “a huge important piece” of the MTC for Carol. When asked, “Why was that such an important part of what you were doing?” she replied, “I think because we learn from each other. And someone, the way their brain works and the way mine works, I might see something one way, but someone else might see something that helps me process in a different way. That collaboration really helps to develop your way of thinking and your processes.” She enjoyed problems when her group “bounced off of each other’s ideas very quickly.”

Mathematics teaching identity. Carol had always seen herself as a mathematics teacher. She said, “I’ve been a gen-ed teacher since the beginning, so math was always in there. It was always part of the curriculum that I had to teach.” The upcoming year would be her first year as a dedicated mathematics teacher, and although that decision was made by an administrator, she said, “I’ve always enjoyed teaching math.” This somewhat contradicted what she wrote in her survey, which was, “When I first went into teaching I
was scared to teach math. I quickly discovered that it’s an area that I excel in teaching.”
And later, when she moved up to teach fourth and fifth grade, she was “very nervous…because I didn’t want to teach them something that was wrong.” Her confidence improved as she became more familiar with the mathematics content at that level.

She believed that she had gotten better as a teacher over the years, but said, “It’s the natural progression of growth” to “get better at what you do and adjust what you do. You see things that don’t work and things that do and then you gravitate towards the things that do work.” She was now “able to read the students” and “make connections and explain things in many different ways for them, that they can latch onto whichever way makes the most sense for their own little brains.” She wanted to continue to improve, and was “open to learning and changing how I do things.” Her goal was “to really be the best teacher that I can for the kids that are in my room.” She mentioned taking more college mathematics courses, and although her main purpose “would be just for my own personal growth,” “at the same time the deeper knowledge I have of where they’re going to go in math in their life, the better I can incorporate the basic level skills into them now so that they have that real strong foundation.” These statements are further examples of the growth mindset that Carol valued.

Carol’s teaching style was fairly traditional and teacher-centered. The structure of her lessons typically followed “I do, we do, you do,” but she attempted to engage the students as much as possible with “hands-on and interactive work.” She incorporated “small group partners,” and “some kind of hook—a little video clip or a little interactive
activity, a game of some kind” into her “standard math lessons.” She believed that students gained a lot from sharing their work and their strategies with one another. She said it was “necessary” to “incorporate some of the drill and kill” because it helped students “get better with their fast facts and things like that.” But she limited fast fact practice to five minutes each class period and tried to keep it engaging to her students “so that they’re not having that ‘sit and read from a book.’ ” After a lesson that she recently taught, she said, “They left my class talking about rounding and how rounding is easy because of that. That to me is a successful lesson because we did the ‘I do, we do, you do’ and they understood it and they were engaged in it and really pushed themselves to master that concept. I think that’s key to an interactive, good class.” She believed that it was important for students to be “engaged in what they’re doing” in order “to retain the knowledge that they’re gathering.”

Carol believed that student attitudes towards mathematics and excitement in mathematics class were important. She also recently learned about growth mindset and did some activities with her students the prior year to work on developing a growth mindset. She described it as, “not thinking you’re good or bad or something, but that you can become good at anything as long as you keep trying and practicing with it.” She tried to exemplify “persevering through” to her students, by showing them how she thought through mistakes: “Okay, I messed up here and this is where I can go back and work on it until I do get it right.” She also wanted her students to see that she enjoyed playing math games and puzzles: “I tell my students, I may be a big nerd, but I’m okay with that. We joke about that in my classroom. It’s a good way for them to see that I enjoy doing what
we’re doing. I do those games and such outside of school.” Her favorite part of teaching was seeing students “get really excited” when “the lightbulbs start popping on” and they transitioned from confusion to understanding.

**Interaction of identities.** Two of the influential moments that Carol described in her mathematics autobiography took place during teaching: when she started having students share their work with one another and how she enjoyed “engaging students in different strategies to see the light bulbs turn on behind their eyes.” These are examples of the ways in which she saw her mathematics teaching as part of her mathematical identity. Further, she was most familiar with the mathematics that she taught, and explained that she did not remember some of the topics in the MTC from high school because she had never had to teach the topics. Similarly, when she moved up from the lower grades to teach fourth and fifth grade students, she said, “I had to go back and relearn some of those things.” Part of the reason she was interested in taking more mathematics classes was “the deeper knowledge I have of where they’re going to go in math in their life, the better I can incorporate the basic level skills into them now so that they have that real strong foundation.”

During the MTC, she felt like a student, and said, “I tried to take in what I try to tell my students. I took my own advice and just got started and tried something to begin with and then see if that sparks where to go from there. There were definitely times where we had some things, I’m not sure where to go with that, and it’s really important to jump in.” She told her students about this strategy in the past, but it was not something she had experienced for herself recently.
Carol also planned to take some of the things she learned in the MTC back to her classroom. She said, “I plan on having more collaborative group work where the students are guided to productively struggle and converse about the math they’re working on. That collaboration piece really makes a huge difference on feeling excited and successful with math in ways our students desperately need.” She became more conscious about different learning styles in the MTC, and wanted to keep in mind that “everyone learns in a different way” because “some are more visual, other people are more formula based.”

She also described how collaboration can take advantage of those differences:

It was fun and interesting to see where different people’s strengths are despite it being the same subject. In reflection I see how easily we can tailor math to utilize the different learning styles and really engage all learners. The word problems allowed us to solve things using our strengths and also build off of other people’s thoughts as well. Collaborative math activities really allow for everyone to shine. She said that the MTC “helped really drill home having that [different learning styles] in the classroom.” Because she realized that she sometimes needed a silent environment in order to think through problems, she said, “I have to be very conscious of the fact that I have to give the kids silent work time. In addition to their group work or partner work and all that, they really do need to have that silent time. That way all the different learning styles are accommodated.” She also realized that “talking, if it’s guided, it can really be a useful tool.”

She also wanted to try to make connections between mathematical topics more explicit to the students. She was surprised and amazed by some of the connections she made between different topics in mathematics, and said, “These connections really stuck me as something we need to be more transparent about with our students to really
encourage that knowledge bridge between math units.” She also felt more “open to where I’m okay with kids doing things in a way that I’d never thought of.”

However, it seemed that pedagogy was the only part of the MTC that she would be using in her classroom. She said, “There were a lot of things that I wouldn’t necessarily take the whole thing to my classroom, but I could definitely adapt it to fifth graders.” On the other hand, she did not intend to use any of the problems in her classes because “the fifth grade standards don’t go to the higher level that we’re working with.”

**Case 4: Deborah**

Deborah was a 40-year-old Caucasian female who attended MTC site C. Deborah had a bachelor’s degree in elementary education and early childhood education with a minor in psychology and a master’s degree in reading. She had taught for 16 years. She taught in a rural area with high rates of free and reduced lunch. For the upcoming school year, she would be a dedicated mathematics teacher for fifth and sixth grades for the first time, although she had taught mathematics her entire career. In the past, she taught Grades 1 and 2 (all subjects) and Grade 5 (mathematics and English/language arts). She said about the new position, “It was brought to me saying, ‘Would you be willing to?’ I was like, ‘Well, sure. I could use a change, and that would be good.’” She was very likely to attend a future MTC meeting.

**Mathematical identity.** Deborah struggled in mathematics growing up. She first fell behind when her family moved from Michigan to Indiana when she was in fourth grade. She was “lost from day one” and “it was downhill from that day forward.” She said that most teachers would “show it to you once and then expect you to do the rest of
the page on your own,” and never used manipulatives. She said, “With a struggling student like I was, that didn’t work.” She similarly remembered falling further behind in pre-algebra. She said, “I used to feel very anxious about math and solving problems. The way I was taught was full of ‘exercises,’ instead of actual problems.” However, her mother “was a great support and advocate” for her, finding tutors for her and reading her “textbooks into a tape recorder to help with my comprehension issues.”

Deborah considered herself to be “adequate” at mathematics, and only at the grade level that she taught. She said, “I would not consider myself at a collegiate level. I am a good problem solver but if you put a calculus problem in front of me, I would not have a clue. It would depend on the level of math. I feel like I’m a really good sixth grade math student. But when it goes beyond…up through algebra, we’re good. Beyond that, no.” Her perception of being good at mathematics seemed to rely on the amount of content she knew rather than her skill at solving problems. She said, “I never thought of myself as being good at math” and was “really nervous” about attending the MTC. But after attending, she said, “When going there and I was able to actually solve some of the problems and do different things, I was like, ‘Oh, hey. This isn’t so bad.’ It helped boost confidence in regards to the ability to be a problem solver.”

She enjoyed doing mathematics for fun and at home with her children. She said, “I’m constantly doing logic problems and most of the apps on my phone are math related.” At home, she used activities like cooking or building as mathematics learning opportunities for her children. Prior to the MTC, she usually did not get excited about
doing mathematics problems, but instead enjoyed seeing the creative ways that her students solved problems.

During the MTC, she experienced “excitement, frustration, and renewed interest.” She described several of the problems that she enjoyed working on. One problem in particular drew her in when she experienced early success: “I was excited because we first started with eight cards and then I got that one really quick. I was able to see the patterns and different things and then I was trying with bigger numbers. I even tried it with the whole deck. That was interesting and that held my attention. Time flew by as we were doing that problem.” Another problem that she enjoyed “lacked the frustration of some of the other problems,” and “dealing with the ‘caring family’ factor and being real world helped draw me in.” She enjoyed problems with multiple approaches: One problem she enjoyed had “many angles to look at it,” and she enjoyed her role during one activity in which she wrote a memo explaining her group’s process because, “I was able to think through the process we used and see other variations that could have been used.”

Deborah “learned so much” during the week. The mathematics was “completely different than the way I ever had it in school.” She had seen similar problems and experienced them “here and there” but the MTC mathematics was “more higher level and just a lot more in depth” than most mathematics she had experienced in the past. She mostly did computation in mathematics classes growing up, and the MTC helped her to realize that mathematics problems may not necessarily have a right or wrong answer. She said about one problem, “There is the strong possibility that four is the smallest number for the 15 cookie jars but I’m going to continue to work and see if I can find a way to do
it in three. Even though all of my conjectures and everything else goes to four.” During the MTC, the facilitators never provided the “answer” to the problems: “In the end, nobody would ‘steal anybody’s ice cream’ so they wouldn’t ever give us a total, what the actual answer should be. It was more, ‘these are the things we came up with.’ There’s a lot of people who came up with the same thing but does that mean that that’s the only answer? Does that mean that’s best option?” She said that although some problems may have “cut and dry answers,” she had shifted her thinking about problem-solving activities. “I used to think, ‘Well, okay. There’s one way to solve it,’ or you might have a couple different strategies but you’re going to come up with same answer. But even my students today showed me outside of the box creative thinking that I never thought of, but that they did in order to solve and figure out different ways to do the cookie jar problem.”

She was occasionally frustrated with problems but said that it usually “challenges me to do more.” For example, she “had no problem finding all of the available options” for one problem but felt challenged to continue thinking about the problem: “I keep feeling like there has to be some sort of pattern in regards to the number of edges showing with the number of tiles and total options of shapes.” Often, collaboration helped to ease the frustration of problems: “That one was a little frustrating but then when we were able to talk it through with our partner or then our team, we were able to bounce ideas off each other and collaborate and come up with more ideas.” Most of the MTC activities took the structure of individual think time, partner work, and finally whole group discussion. She said, “The collaboration was huge because the way I think of things might be totally different than my shoulder partner and our ideas together then
helped form even more conjectures and thoughts. Then, it was a manner of going back and as we’re solving, can we rule out any of our conjectures? Can we create any more?”

Deborah occasionally felt insecure during the MTC but ultimately, being “immersed in those problems” was exciting to her and “had a positive impact on my future mathematical progress.” She wrote:

Throughout the week I was often reminded of my struggles I had as a child. Every time a new problem was presented, my immediate reaction was an overwhelming feeling. Sure that I would be unable to successfully do the problem. Then during a self pep talk I reminded myself that I am an adult and a math teacher, and that I should be able to handle the problems that were being presented. As the week went on the pep talks were less frequent. By the end of the week my sense of math security had greatly improved.

Her description of what it meant to be good at mathematics included risk taking and critical thinking after the MTC. Prior to the MTC, she wrote, “To be good at math means that you are able to solve the numerical problems in the world around you.” And after the MTC, she wrote, “To be good at math one needs to be a good problem solver and a critical thinker. They must also be a risk taker, so that they can test out their ideas.”

After the MTC, Deborah developed more confidence and was more likely to attack challenging problems and persevere in solving them. She wrote, “Now I feel very excited when faced with problems, whether real life or not. I like the challenge. But, I will confess that there are times that I get frustrated and still get tempted to give up.” This confidence may have been a result of experiencing success, as she said, “Especially if I can actually get the problem solved, it’s a really good feeling. I mean it gives you a sense of accomplishment and helps you to feel a little stronger in your ability to be a problem
solver or whatever it is the actual problem asking you to do.” In the following exchange, she talked about her development of perseverance:

Deborah: Just the perseverance of it and even if you think of a problem as being too difficult at the beginning, if you break it apart and look at it in smaller chunks and look for patterns and sequences and different things, then it can make a difficult problem easier, where I’d be surprised the biggest thing that it came through for the week for me.

Researcher: Is that something you’ve thought about before the workshop?
Deborah: A little bit. Before the workshop, I’d see certain problems or certain things and I was like, “Yeah. I don’t think I’m going to try that.” Now, I’m trying everything that comes my way in regards to math.

Researcher: Like if you’d see a problem and you weren’t sure immediately what to do, then you would not do it?
Deborah: Yeah. I would skip it and go to different one, because I have a lot of like logic puzzle-type books and different things like that and apps. I would just skip it if it didn’t make sense right away instead of spending time and putting effort in.

Mathematics teaching identity. The upcoming year would be Deborah’s first year teaching mathematics all day. She said that in the past, “Reading was my choice because I have my reading endorsement and my master’s in reading, but then math was secondary at that point. Now, at this point, math is definitely my top subject.” She thought that the change happened because she “got more secure in my ability to teach it, especially coming up to fifth and sixth grade.” Deborah suspected that part of the reason she felt more confident teaching mathematics is that “when you’re in primary, you are self-contained” and “responsible for everything” but when she began teaching higher grade levels, she could “really focus on the math.” She said, “I think that plays a big part in it in regards to prep and being able to explore other avenues and different things but I think also doing the Math Teachers’ Circle has helped that.” Although she “wanted to do things” like problem solving, she “didn’t even know where to start to implement, especially when I was down in primary.”
She taught differently from how she was taught when she was a student. The curriculum “expects more from younger kids now” compared to when she went through school. And her own teaching changed over time: “[I] evolved in regards to how I teach math. When I started out, there was a lot of whole group work and right from the book. Now, we still have to use the book but there will be a lot more problem solving done independently, with partners and small groups.” She had manipulatives available for the students and tried to give “more in-depth, real-life, rich problems that they can really sink their teeth into,” while avoiding “just doing exercises through the workbooks” as much as possible. She said, “Every year I’m changing and growing” and “enjoy[ing] it more.” She wanted to “implement more of the problem-solving and hands-on” activities. When asked if she had always taught this way with problem solving or if it was something she started doing recently, she said, “Every year, I’m trying to do more and more,” and that “as I’ve done more, I saw the benefit of it.” She wished that there was more problem solving in the school’s required curricula.

Deborah also thought that it was important for students to be “excited about the new discovery of a type of problem or strategy.” She said, “I want to guide students through the world of math and problem solving and help them avoid the hazards of becoming frustrated and losing confidence.” She enjoyed seeing “the discovery” and “the lightbulbs going off” as students worked on problems and discovered patterns.

She planned to use a lot of the pedagogy from the MTC in her class. After the MTC, she reflected, “Sometimes, especially last year, I think I hand held a little too much. I need to step back a little more this year and let them be a little more independent
and just intervene with the students who need and enrich the students who already have the concept.” She planned to give the students “more think time” and to institute group norms at the beginning of the school year. As a result of the MTC, she had “reworked my first two weeks of school” and was “institutioning a lot of problem solving in the math principles before we even get out our textbooks.” She and the other mathematics teacher were going to use problem-solving activities and group work from the MTC. She planned to use the structure of group work used in the immersion: “Students would be immersed in problem solving as an individual, then with a partner and then with a team.” At the time of the interview, she said she was “loving” the way her team started the school year “without even touching the books.” She had already used a few problems from the MTC, used math talks, focused her students on perseverance, and used wait time by having students hold their thumbs at their chest when they have an answer (a pedagogical technique used in the MTCs).

Deborah talked about some of the difficulties with incorporating more problem solving in her classroom. She said, “It is expected that we follow the GO Math! curriculum because they put the money into the program.” She did not think the students need the workbooks, but “since they’ve bought the workbooks, we need to use them.” She was the mathematics team leader at her school and was working with the other teachers to “figure out a way to work around it.” She felt an obligation to use the workbooks to “appease the district and the families who paid for the workbooks,” and was trying to find a balance. She already had a plan for integrating math talks and the workbooks: “I think instead of having them do it on the workbook page the way they
have it set up, we’re going to have them cross that page out and we’re just going to do it as a math talk. So it’s more they’re doing it in their head and then being able to share out how they solved.”

**Interaction of identities.** Deborah’s feelings about teaching mathematics are related to her feelings about doing mathematics. She would feel “anxious anytime I have to teach a new strategy to my students.” And she most enjoyed mathematics problems “when my students can find multiple ways to solve a problem and share them with me. Sometimes, they find a way to solve it much better than I thought of solving it.” She also believed that her struggle with mathematics as a student “has led me to become a more confident and adept teacher of math” because she “understand[s] the frustration and exhaustion that math can cause.” Her experience needing (and not receiving) extra assistance from her teachers made her “strive to be the opposite of that” and intervene with students who need help.

She also related her experience in the MTC to her classroom. At the end of the week, she wrote that she was “excited to get my school year started. I have a renewed thrill for problem solving.” She also learned a lot about teaching mathematics by experiencing mathematics in a new way during the MTC:

My experience with the Math [Teachers’] Circle, all we were doing, we were engaged in all of that. You think that you have drill them with all the facts and drill them with all the computations and different things like that, but I learned so much more just from that week of problem solving and I think my students will do the same. It’s not all just computation and getting through the book. It’s a matter of covering your standards obviously but also getting all the problem solving in to help them to cover the standards and to do all that you can to immerse them in all the math principles and all kinds of problem-solving activities and things.
Deborah also had more ideas about how to adapt problems to her grade level after the MTC. She said during the MTC, she was thinking about “how I could use one [problem] in my fifth and sixth grade class and that same problem could be used with high school with just a little adaptation here and there.” In the past, she said she would think about problems: “This is a cool problem. I really like it but I would never use it.” But after her experience in the MTC solving problems and “talking it through,” she was able to see how she could use various problems in her class. She said it was “neat because there’s a lot of problems I’ve come across along the way” that she thought were interesting “but I never thought about bringing it down to where it would be appropriate.” She was already looking at problems and figuring out which standards they address so that she could work them into her curriculum. She said, “I think I’ll hopefully do a better job at that, too” as the year progressed.

**Case 5: Erica**

Erica attended MTC site B. She is a 50-year-old White female. She taught Grades 5–8 mathematics, religion, and computers in an urban school district. She had taught for 27 years and taught mathematics each of those years, although as time went on, the number of sections of mathematics gradually increased. At the time of the MTC, she taught four sections of mathematics. She had both bachelor’s and master’s degrees in elementary education and was certified to teach Grades 1 through 8. The concentration areas for her master’s degree were mathematics and science. She originally planned to teach high school mathematics but said, “Two semesters of calculus kind of changed my
mind, so then I was thinking more elementary.” She was likely to attend an MTC in the future.

**Mathematical identity.** Erica’s mathematics autobiography included both positive and negative experiences in mathematics. Overall, she “had always liked math when [she] was in school.” In second grade, she “struggled with understanding that \(2 + 2 = 4\) and that \(2 \times 2 = 4\). I can remember my dad helping me. It was one of the few times that happened because I always did homework on my own. I don’t think I really understood the connection between addition and multiplication at that time, but I’ll always remember that my dad tried to help me.” In her high school algebra class, she “felt clueless until about after Christmas break. It was like a lightbulb turned on and I was understanding. It was because of the language the teacher was using. Instead of \(x\) or \(y\), he would say oranges or apples. This helped me to see the connection between the English language and the language of math.” This experience was impactful to her understanding of mathematics and continued to influence the way that she thought about math: “I love how visual math is and how it is connected to our language. Math is just another way to write what we can say in English.”

Despite some of her negative experiences in school mathematics, she wrote, “I often feel excited about doing a math problem. I love it when I can point out that math is being used in the everyday world.” Her children joked about this: “Oh, we’re using math.” She believed, “The thinking skills you learn in math are used daily.” She was motivated to solve problems when “I know I have a lot of experience and background information about that kind of problem. In other words, I am more excited if I think I am
going to be successful.” She usually did not feel anxious about doing mathematics, unless “I have to do simple mental math in front of someone, fearing that I will look foolish if I make a basic fact error in front of them.” She elaborated, “It makes me feel not as smart and less confident because I am thinking that wasn’t a mistake a math teacher should make.”

The mathematics in the MTC was different from mathematics Erica had done in the past because it was “more problem solving than just doing straight algebra problems or solving equations.” She wrote several times that she felt simultaneously challenged and confident with the mathematics. She said, “Since I have a lot of experience with Sudoku, I really caught on to the KenKen game and felt very confident.” And about the geoboard activity, she said, “The problem solving wasn’t over my head with the hints our group was given.” She was most proud of solving the Missing Brownie problem: “I showed on the whiteboard a few examples of cuts that would work. The guest speaker asked me what they all had in common. I had no idea at that moment, but after going back to my seat and thinking about it more, I was able to figure out the answer. I felt very confident at that moment.” The MTC gave her “more confidence” because “we would spend like an hour, hour and a half working on a problem and we would always get to a solution, so that made me feel successful.”

She had trouble with some problems and “felt I was in a little over my head” or “overwhelmed with all the things to keep track of.” Although the Presidential Debate problem involved fractions (with which she felt confident), she said, “This was my first
experience with this type of problem,” and “I just didn’t feel like I understood how to solve it like I did the pizza problem.”

She learned some new mathematics during the MTC, such as “other bases besides base 10.” But she felt that connections between mathematical topics were the most powerful: “I was amazed to see a connection between geometry and algebra that I had not seen before,” and “There were just a lot of the connections that were branched from one area of math to another that was neat.”

Erica focused on the atmosphere of collaboration with other teachers. She was particularly energized by all of the “sharing amongst teachers” and she “will probably continue to have contact with those people.” During the times that she was overwhelmed by a problem, collaboration was helpful to her: “I felt validated by the people in my group and was glad to have their contributions, especially when I was stuck.” She said, “There were definitely times where I felt like I’m so glad I don’t have to attack this on my own, I can work with other people. I would be feeling overwhelmed and like how do I even start this but knowing that I was there at a table with other people and we could talk to each other and work together, it was fine.” When she “had a really hard time seeing the patterns,” hearing from others helped her to understand the problem: “I was able to understand other people’s explanations, but I never would have been able to see the connection to place value and base two on my own. Even though I don’t think I could have solved the problems on my own, I was happy that I understood as things were being explained.” At the same time, collaboration helped to validate her hard work: “The group work showed me also that I could explain to other people, too.”
Erica often compared her skills to the others in the group. She was pleased and “felt the most confident” on a day that she solved a problem first in her group. She realized that some problems were challenging to her when she “saw other people being able to start sooner.” She was sometimes concerned that “because I’m older…I’m not figuring these things out as quickly” as the others in her group, but that ultimately, “I was just very glad to have other people that could explain it to me.”

**Mathematics teaching identity.** Erica wanted to become a mathematics teacher due to the influence of her teachers (both good and bad) growing up. She decided to become a teacher in third grade when “I was afraid of my teacher and I decided nobody should have to be afraid to go to school.” A few of her mathematics teachers recommended that she become a mathematics teacher, too. She said, “I just got things better by how they taught me and I wanted to be able to do that.” Although she started her career as an elementary education generalist, over time, “I was in a position where I had two sections of math,” and eventually four sections of mathematics. She had “passion” to teach mathematics, it was “where [her] heart [wa]s,” and she was “happiest when I’m teaching those subjects [math and religion].”

Her instruction had changed from more teacher-centered to more student-centered. She said, “I think I have changed from a ‘let me tell you, let me show you,’ to ‘let the student have more input’ in the class.” She believed, “Effective math teaching shows that math is all around us,” and she enjoyed when students “see the connection between math and English.” However, her description of student activities in her class were fairly teacher-centered: “There are a variety of activities happening during the
lesson such as mental math, review of basic skills, explanation of concepts by teacher and students, and opportunity for individual practice.” She said that a lesson is successful when students “can work the same kind of problems.” She said, “I probably don’t focus as much on the problem solving, as teaching them the algorithms and this is how you do it and giving them practice with that. I think I’ve always felt that you have to have a good foundation in math before you move on to those hard problems.”

After the MTC, she realized that “persistence and perseverance are very important” to being good at mathematics. She still believed that it was important to emphasize the basics, but she “doesn’t feel as strongly” that way anymore. She did not plan to incorporate more problem solving but instead wanted to focus on changing some of the pedagogical strategies she used. She wanted to give her students “more opportunity to talk through problems and give them a chance to work together to problem solve.” She wanted to see “a lot of thought sharing and either confirmation of thoughts or sharing disagreement with support of either.” She intended “to talk less and let the students talk more, showing what they know and guide them to make connections. I need to be more of a coach.” She planned “to have groups of three or four, instead of two, for daily problem solving” to encourage more talking, and she planned to ask “each group to be able to share how they found the solution.” She said, “Now I can see some advantage” to students working in groups rather than pairs, and that she was now more able to “take whatever the concept is to a higher level in problem solving.”

She explained that she struggled with balancing basic facts and problem solving in her classes:
Figuring out how long things are going to take and just fitting it all in is still an issue and trying to have a balance between drilling so that they know their basic facts and then with having eighth grade, making sure that they’re going to be ready for high school math…Just trying to balance not just the facts of math but also pulling in the problem solving, it’s just a lot to fit in. I don’t know if I’ll ever have long enough classes to be able to do that.

**Interaction of identities.** Erica’s experiences teaching mathematics were impactful on her mathematical identity. In her mathematics autobiography, Erica described a moment that took place when a former student came back to visit her classroom. The student told the current students to listen to Erica’s advice about preparing for high school mathematics. Erica reflected, “it was very gratifying that she came back to visit and share that.” This moment, which is more closely related to teaching than to mathematics, was important to both of her identities.

Erica was only anxious about mathematics when she thought her errors might reflect poorly on her as a mathematics teacher. She said if she makes calculation errors, “I am thinking that wasn’t a mistake a math teacher should make.” Her perception of what a mathematics teacher is and does, and her desire to be seen as an effective mathematics teacher, contributed to her anxiety about doing mathematics calculations.

She also used teaching strategies of using language and manipulatives based on the strategies that worked for her as a student. During the MTC, she said, “It was good to be in the position of ‘student’ to remember what that feels like, especially when I was struggling.” Being in the role of student helped her to “better think about how I need to teach it. While I was doing the math, things that other people did that helped me, those are the kinds of things I want to do while I’m teaching it.” For example, she said:
There were definitely times there where I didn’t feel like I could do it on my own. It definitely helped being able to discuss it in groups and to get help from other people. In the past, when I had the kids working on problem solving, I would partner them up and there would usually just be two of them but now I can see some advantage to having times where it’s maybe double that, maybe a group of four.

**Case 6: Faye**

Faye attended MTC site A. She was a 54-year-old White female who taught for 26 years. She held a bachelor’s degree in music education with mathematics certification, a master’s degree in music education, and licenses for K–12 vocal and instrumental music and seventh to ninth grade mathematics. She originally taught music exclusively, then taught both mathematics and music, and then mathematics exclusively for 15 years.

At the time of the MTC, she taught eighth grade mathematics, including algebra and general mathematics. She taught in a school district that was “a combination of a rural community school with also suburban students from the nearby city.” She always enjoyed mathematics and was told by her high school mathematics teacher to consider becoming a mathematics teacher. She chose to pursue music in college but took enough mathematics classes that she was able to add a mathematics certificate. She said that it was somewhat likely that she would attend an MTC activity in the future.

**Mathematical identity.** Faye said, “Math had always come easy to me” in school. Her fifth grade mathematics teacher allowed her to work through both fifth and sixth grade mathematics books independently: “Her confidence in my abilities gave me a lot of self esteem, both in mathematics and in my personal life.” Faye said she had “always had an aptitude for math. It is something in which I have always excelled and enjoyed.” However, even though she was always good at mathematics, she “had never
been confident enough in my skills” to consider that she was “good enough to teach it.” She also said, “I don’t think I’m brilliant, but I think I’m definitely a higher math student. I wouldn’t consider myself a mathematician. There are definitely smarter people in math than me, but I think I’m a very good student. Sometimes I have to work at it.”

Faye was “always excited to do math problems.” She loved puzzles, and “even book problems… I love application problems and just trying to figure them out, and maybe something that I’m not familiar with the challenge of trying to figure out how to do it.” She enjoyed learning new things because of the “sense of accomplishment when you figure it out.” She said, “Doing math is many times like putting together puzzles. I like the challenge of calculating each piece and putting the pieces together to find a solution.” She enjoyed working “through a problem to see if I obtain a correct solution” because she “always enjoy[s] solving problems correctly.” During the MTC, she discovered that “even though I am a math teacher and am supposed to be a mathematical ‘expert’ to some extent…I still enjoy learning new concepts and that I am still intrigued and challenged by problems.”

She felt anxious about mathematics in “timed or in a competitive situation.” She liked winning but was “afraid that the time limitation will cause me to overlook steps or make mistakes.” After the MTC, she became more specific about types of problems that make her anxious: “I feel anxious about math problems when I am unclear as to what a problem is asking. I am also anxious when I do not know how to proceed in order to formulate a solution. I am sometimes anxious about math problems when they are new or unfamiliar to me or when I do not have enough prior knowledge to solve them.” When a
particular open-ended activity was frustrating to her, she realized that she found open-ended explorations to be challenging and difficult.

In the MTC, Faye was initially “nervous” after the MTC leaders “told us that the purpose of this week was to challenge us mathematically and that we would have moments where we would be lost and frustrated.” She felt lost at certain times, such as when “my group came to a point where we were out of strategies to solve the problem. We were ready to give up. It wasn’t until someone from another group suggested that the entire class combine our resources that we were able to collectively throw out ideas and work collaboratively to finally solve the activity.” She was also frustrated by a problem when she “couldn’t see the point of it. I didn’t know where we were going with it at all. I really didn’t understand truly what we were supposed to be doing and that was very frustrating to me.”

Throughout the week, Faye’s confidence in her skills increased and she was “proud of my accomplishments.” She was pleased when she was “making some connections that were taking others longer and I enjoyed helping my peers.” She felt “pride when I was able to come up with strategies and solutions that I could share with the group.” At the end of the second day, she “was mentally exhausted” but “felt accomplished that I stuck it out and finished the activity.” She also “learned that it is acceptable to not immediately know an answer and to have to immerse myself into a problem in order to solve it.”

She learned some new things about mathematics, like making “connections between the base ten and base three-halves systems” and discovering “shortcuts that
could be derived from the patterns in division and multiplication with the mod numbers.”
She was also excited to make connections between two of the problems, including one problem that was frustrating and challenging to her at the time. She said, “This showed me how sometimes we don’t always know the objective of a problem and that sometimes just the exploration of the problem in and along the way is the actual objective of the activity, not necessarily formulating a solution.” She realized, “A problem doesn’t necessarily need to be solved in a certain time frame. It can be extended over several sessions and several days.” She also realized that mathematics “doesn’t have to just be with pencil, paper, and calculating,” because none of the MTC mathematics was like this, and that “math can be hands on, doing, experiencing.”

Faye often compared herself to the other teachers, and she was concerned about their perceptions of her. Prior to the MTC, she wrote that she was nervous when “problem solving with and/or competition with peers” because “I am sometimes afraid that my strategies are not the correct ones.” It was important to her that others perceived her as competent. She was not “real self-confident about my skills” at the beginning of the week, and she “felt a little apprehensive at first being put in situations where I wasn’t comfortable and not knowing the answers to problems.” During independent work, she “found myself anxious at times as some of my tablemates were sometimes solving problems sooner than me,” and thinking, “I bet everyone else is smarter and getting this faster than me.” But she said, “As the week progressed, I realized that everyone felt that same way at one time or another. We all possess strengths in different areas and it was through collaboration and having an open mind that we are able to solve challenging
problems.” She also realized that collaboration could decrease her anxiety: “Later, as we played mathematical games, I once again felt apprehensive as I hadn’t played any of the games before and did not want to look like I didn’t know what I was doing. I found my peers to be helpful and supportive which decreased my anxiety and I was able to enjoy learning the new games.” For Faye, realizing that “the strategies that I had to offer were as valuable as anyone else’s” and making “great contributions to problem solving as much as anyone else” helped to increase her confidence in herself and realize, “I guess I am pretty smart.”

Collaboration with her peers was a significant part of the MTC experience for Faye. She “enjoy[ed] working in groups and learning from others” and wrote, “I could not have survived the week without the support and help from my peers.” She was amazed at how much they were able to figure out when they worked together, “going from having absolutely no clue how he [the facilitator] was doing this to figuring it out.” She said, “When we all got together as a class and worked on it, then it just started falling into place. I just found that amazing that we could even solve it. It happened but it took all of us together.” She explained, “The reason that we problem solve is because we all work together. None of us could problem solve on our own, but by working together we were really able to come up with good strategies and good ideas.” She realized that she “learned much more through collaborating with my peers.”

**Mathematics teaching identity.** Faye chose to teach mathematics because she enjoyed doing mathematics as a student. She first began thinking about teaching mathematics in high school, when her mathematics teacher “told me that he thought that I
should consider teaching mathematics, that I would be a good mathematics teacher.” She did not immediately pursue mathematics education, but later on in college, her “department dean there just in passing had suggested to me that I go ahead and get my certification in math since I had several hours anyway; I think I needed one more class to be certified.”

She enjoyed teaching mathematics and believed that she was good at “leading students to understanding why math works, rather than just teaching them mindless steps with no understanding.” She was good at “explain[ing] problems in many different ways” and enjoyed seeing how she “can help kids get it.” However, she “sometimes feel[s] anxious about teaching problems/concepts that I have either not taught before or problems/concepts that I have not been exposed to in many years.”

Faye had gained confidence in teaching mathematics over her career. Because she did not major in mathematics education, she felt like she was “entering it through the backdoor” and “didn’t feel as self-assured as I felt maybe some of peers” did. She wrote that she had “become more knowledgeable just in the pedagogy of teaching and the strategies that help kids learn math best” every year. She believed that “every year you grow as a teacher and if you’re not you shouldn’t be teaching. You learn more things to help kids better. You find better resources, try new strategies.” She was interested in learning more about “how to best utilize the new technology in my classroom to enhance and deepen student understanding.”

Faye had changed her teaching over the past 20 years in response to the emphasis of “having students understand why algorithms work and how rules in mathematics were
derived.” Prior to that, she said, “Teachers basically taught an algorithm, showed several examples to students and then had students practice.” She also “saw a shift from teacher-directed learning to a more discovery approach where students discover concepts through experimentation and investigation and the teacher becomes more of a facilitator. The teacher has to learn to ask the appropriate questions of students to guide student inquiry.”

More recently, with the Common Core Standards, she perceived “much more emphasis has been placed on writing about and speaking the language of mathematics.” In general, she believed, “The shift in my teaching has led to students having a deeper understanding of the mathematics” compared to the students she had in the past. However, she wanted to “continue to align curriculum with real-life applications so that students will understand the importance of mathematics in society.”

Her teaching was also heavily influenced by a Connected Mathematics curriculum workshop in which she learned about teaching the discovery-oriented curricula. She said, “That training, along with teaching that series for many years, really taught me how to teach kids the ‘why’ of math and how to facilitate learning and not just teach concepts.” She said that the MTC “definitely reinforces” the teaching of discovery and “bring[s] it to light again, the importance of it.” She believed that mathematics teachers should provide the direction for discovery, but that “there could be many directions to go also.”

Faye believed that it is important for students to “become more confident in their skills.” During a successful lesson, the students would be “engaged and really participating,” and they “talk with each other” or even “argue with each other about the math.” She said, “I like students to ask lots of questions and to challenge each other and
even to challenge me.” Her daily routine involved a bell ringer or problem of the day to review, grading the students’ assignment, some teacher-led instruction, and “some activity to get the kids engaged in practicing the skill, whether it would be a game or scavenger hunt or something where they can have some movement and some collaboration and work together.” Finally, she assigns “some independent practice for the next day.” When asked to compare the problems she uses in class, the structure of the activities, and the teaching pedagogy to that of the MTC, she said, “I think similarities are probably not that much, except on occasion.” She explained that she did not use “a lot of activity-based learning” due to “all of the standards that we’re required to teach” and a lack of time.

Because of her MTC experience, though, she wanted to make some changes in her classroom. She wanted to incorporate more collaboration and “give my students more time to collaborate to process information, talk about the math together.” She was interested in integrating “more challenging problems into my curriculum,” and wanted to “devote more time to reflecting at the conclusion of activities.” She said, “The inquiry-based learning—I was somewhat familiar with that, but I think I learned more about that on how to utilize that in my classroom and just different strategies I guess, more in terms of how to structure a challenging problem, how to set it up before teaching it.” She explained:

I could try some more challenging problems and realize that it doesn’t have to be something that has a definite start. It doesn’t have to encompass one class period or it doesn’t have to even encompass consecutive class periods. That it could be something that we work on a little bit each day for an extended period of time. I think I’m more open to trying something like that and definitely having kids collaborate more to solve problems and experience math.
But she explained, “The most difficult part of that is finding enough time to allow students to adequately explore and solve problems.” Mathematics teachers have “to teach so many indicators that it makes it difficult to explore any one topic as deep as we would like. Also, having to cover so much material makes it difficult to devote enough time to adequately explore and solve problems.” Because of the required standards, she said, “I’m having to teach so much new content all of the time” and she did not have the time to use “activity-based learning.” She used short activities when she could, but she had not used “these long challenging really being immersed in the problem” activities from the MTC.

**Interaction of identities.** Faye’s mathematics and mathematics teaching identities overlapped in various areas. Her enjoyment of doing mathematics was what led her to become a teacher. And her anxieties about teaching were related to her comfort with the mathematics: “I sometimes feel anxious about teaching problems/concepts that I have either not taught before or problems/concepts that I have not been exposed to in many years.”

Faye believed that the MTC “put me into the role of a mathematical student once again,” which she believed “will make me a more empathetic and understanding teacher.” She said, “I have a better understanding of a student’s perspective as it relates to learning new material” which will allow her “to become a better teacher.” She said she learned “how important collaboration is” and “the value of working with peers in solving challenging problems.” As a result, she said:
I plan to give my students more time to solve challenging problems and to work together collaboratively. I plan to give my students more time to reflect upon their learning and I want to incorporate more inquiry and open-ended questions in my teaching. I want to do this because I learned this week how much it expanded my own learning and confidence in my mathematical abilities.

She picked up on this pedagogy not through explicit instruction about the pedagogy, but “by experiencing it through the modeling.”

Because group processing and collaboration helped her so much in the MTC, she said:

I hope to give them maybe more challenging problems at a higher level than I maybe would have attempted previously and that I think they can do it with collaboration. I think that they would probably have the same experience I did, be apprehensive at first. If they stick with it and endure, and are successful, I think they can build a lot of confidence that way.

She also realized that “a problem doesn’t necessarily need to be solved in a certain time frame. It can be extended over several sessions and several days.”

**Case 7: Jeff (Incomplete Case)**

Jeff represented an incomplete case. He was unable to participate in an interview, so the analysis drew from reflections and pre- and post-survey data. Jeff was a 35-year-old White male who attended MTC site C. He had a bachelor’s degree in middle childhood education (mathematics and science), was licensed to teach fourth through ninth grades mathematics and science, and held a reading endorsement. He taught mathematics for nine years and taught in an urban/suburban district. He was teaching mathematics because it was the first open position in his school district. He said, “I love science too, but I’ve been so happy that I’m teaching math and that I’m part of the best department in our building.” He was likely to attend an MTC activity in the future.
Jeff enjoyed mathematics in high school and attributed this to the teachers who “made math interesting and fun.” As an adult, he enjoyed doing mathematics for fun: “I also like to ‘nerd out’ with friends and family and work math problems into casual conversations.” He also enjoyed doing puzzles like Sudoku and “really enjoyed” an MTC session that focused on “different kinds of puzzles.” He was eager to return to the website shared by the presenter to explore it further. On the other hand, Jeff expressed that he “would feel slightly anxious about extremely high level problems” like calculus, but that he generally enjoyed challenging problems and “would like to relearn how to solve” calculus problems. Jeff, like most of the other teachers in this study, used the word “challenge” with a positive connotation. This statement reflected his confidence in his capabilities to learn and do mathematics.

Jeff enjoyed the mathematics in the MTC. He stated a few times that he “was really looking forward to the problem” and wrote about one problem, “Working with such a cool, real-world example made it seem less daunting.” Jeff “felt comfortable” with the level of mathematics in the MTC and said, “It’s not as if I was struggling with advanced calculus concepts or anything, yet the problems themselves were still challenging.” This suggested that he was surprised to find the problems to be so challenging when the mathematics content itself was simple to him.

Several of Jeff’s reflections related to his desire, as a learner, to obtain the solution to problems. One evening, he wrote in his reflection that he was “still grinding” over a problem. He wrote, “I’m still REALLY wanting to do a proof that shows the solution was correct and will work every time. Hopefully I can just fall asleep…”
Another day, he reflected, “The math from our activities today, at least on the surface, seemed a bit more ‘easy’ than problems in past days,” and he attributed this perception to the fact that “I actually came up with ‘solutions’ to these problems.”

Collaboration was an important aspect of Jeff’s problem solving in the MTC. Jeff wrote that he enjoyed collaborative work: “I like to engage in problems where I feel like I can take the lead, at least to get the group going, and then step back into the group and work together.” Jeff described one instance in which his group members were all stuck after working individually, but after talking as a group, they were able to make progress. Jeff felt successful after solving the problem but was simultaneously frustrated with himself for not solving the problem earlier on his own because he believed he had the background necessary to solve it:

I tried lots of different ways to solve the problem, and came up with some new ways after talking with my group members. I started to get frustrated a bit because I felt like there was a solution that I hadn’t come up with and that it was probably something minor I had overlooked, and the rest of my group felt the same way. We had a whole group discussion and got some fresh perspectives that really helped to get us going again and we got very close to the main solution. And after one final take from a classmate I figured it out. I was both relieved and ticked off! I couldn’t believe that I had completely missed what was essentially right in front of me and that I had kind of been beating around the bush the whole time! I was glad to have figured things out but upset with myself that I didn’t see it sooner.

Jeff was often “motivated to find solutions” but reflected, “I had to remember though that the problem solving aspect and fostering discussion was really paramount compared to actually finding a solution.” He seemed to value the problem-solving process and discourse as pedagogical techniques for learning, but as a learner himself, he was driven to find a solution.
Mathematics teaching identity. Jeff enjoyed teaching mathematics. He wrote, “I love teaching math and, in one-on-one tutoring situations, I love working through the issues and seeing the growth.” He enjoyed doing mathematics, and he believed that his enthusiasm was important for students to see: “I think that the teacher has to show the most enthusiasm in the room. You always want the students to be enthusiastic too and show that, but if the kids can see and feel that love of math from you, they are going to be more involved and want to succeed more.” He was most confident in teaching or tutoring when he was familiar with the content; he wrote, “When helping someone, I like to feel as though I know what I’m doing.”

He had not changed his teaching “substantially” since he began, but he wrote that his views had changed a little “as part of the natural course of teaching and learning.” In fact, the “biggest change” he experienced had been that he gained “more confidence in the students’ creativity when it comes to problem solving. It happens often enough that the kids will come up with some strategy for solving a problem that I hadn’t considered, and I make sure to add it to my repertoire for the future.” Although he did not believe his teaching had evolved much since he started, he expressed his desire to learn and develop his teaching, partially in response to curricular changes at his school and partially in response to what he learned in the MTC. He originally chose to attend the MTC to help him prepare to teach the new curriculum:

My district is going to implement the EngageNY curriculum in the junior high this year and I think that the collaboration with others and the practice with the rich tasks and application questions will be extremely helpful to me in my efforts to effectively use the new curriculum. Even though I’m entering my 10th year in the district, the new curriculum and teaching styles are making me feel like a
rookie again and I’m hoping that the Immersion workshop will help assuage those feelings.

Jeff’s philosophy of teaching and learning was student centered and focused on discourse and mindset. He wrote, “My vision is that all students are engaged in the task and that there is discussion taking place correctly using mathematical vocabulary. Students are inquisitive and explore all possibilities for solving the problem.” And he wrote, “I think the biggest indicator [of a successful lesson] is the discussion that goes on. If students are discussing math and their strategies and using math vocabulary, I think that it shows a deeper understanding and growth as a young mathematician.” He also frequently wrote about student confidence and mindset. He believed that being good at mathematics “means to be comfortable with what you are doing. I tell my students that math is a ‘confidence game’ and that if they try hard and start to feel confident that they will be ‘good at math.’” He further explained that being good at mathematics “means to have enough confidence to try and fail repeatedly, while examining your procedures and results critically each time until finding the solution you desire.” He told his students, “Don’t be afraid to try and fail. You know more than you think and just attacking the problem is a step in the right direction. Being good at math means having confidence in your abilities and feeling comfortable enough to try and find solutions without hesitation.”

He wanted to move “away from just worksheets and drills and tried to move more towards problem solving and projects, but it is a hard transition to make. I know that it is definitely what is best for the students but getting them rewired to want to persevere with a problem and get excited about math rather than looking for an immediate solution is a
tough road.” After the MTC experience, he wrote that he had begun to “retool” his intervention class “to be more problem based and am trying to move away from only strictly reteaching what was done in the regular math class. I think it will be much more beneficial to incorporate the problem-solving skills and will make the students more successful in their regular class.” He also expressed that he was “less intimidated” by trying problem solving in his class now and felt “better equipped to handle the change in my class paradigm from ‘textbook’ to ‘problem solving’ and other critical thinking activities.”

Jeff’s interactions with others played a significant role in identity as a mathematics teacher. For example, his alignment with the other teachers in his school had influenced his enjoyment of teaching mathematics. He explained, “I love science too, but I’ve been so happy that I’m teaching math and that I’m part of the best department in our building. The department gets along really well and I think that makes a huge difference when it comes to doing our jobs in our classrooms.” He also wrote that peer observation was beneficial for his teaching because “I was able to see some new processes and procedures and adopt some new practices for my class.” Similarly, Jeff enjoyed being part of the MTC community and found working with teachers to be beneficial to his learning:

We were asked to think of questions that would help deepen our understanding and lead possible discussions. And for me, coming up with questions to discuss and having the discourse with other math teachers is probably the most beneficial thing for me. One of the biggest reasons I joined the group was that I like to do math problems and talk shop, so the fact that I have been able to do that is extremely enjoyable.
**Interaction of identities.** Jeff perceived that teaching mathematics and doing mathematics were related. For him, doing mathematics influenced his teaching of mathematics, and teaching mathematics had influenced his mathematical identity, to the extent that his mathematics autobiography included multiple mathematics teaching moments. For example, he wrote in his mathematics autobiography, “Getting the opportunity to teach math and ‘show off’ some of my skills on an everyday basis made me really happy and focused me to be the best I could be. Being able to share my knowledge and experiences with students and colleagues is one of my favorite things about the job.” Another impactful moment in his mathematics autobiography related to peer observation of another teacher’s classroom and learning new teaching strategies. Jeff also believed it is valuable for him to refresh his higher-mathematics skills when tutoring students in advanced mathematics courses: “I take what I already know and apply that as we ‘learn together.’ It’s a nice refresher!”

In his reflections, Jeff often wrote about how he would incorporate MTC problems in his classes. He explained:

I figured that diving into these problems and experiencing them firsthand would be a great way to help me decide how to incorporate things like this into my own classroom. Teaching math intervention, I’m always looking for new and different activities to engage in to help the students feel more comfortable in math while challenging them and making them think outside the box. These problems (most of them so far) are definitely ones that I could modify to fit in my seventh and eighth grade classes. I’m looking forward to trying some things out!

He explained that he was “constantly examining” each problem “for pieces that I could bring back to my junior high classroom,” “no matter how challenging the activity or the seemingly advanced nature of the problem at hand.” He wrote, “I was definitely
happy with what I discovered too! I was able to take something from every single problem we attacked and am looking forward to seeing them in action in my own classroom!” He enjoyed talking with the other teachers about different ways they could incorporate math walks into their classrooms (mathematics activities relevant to a geo-tagged location on Google Earth). He was excited to learn about new “invaluable” resources, such as a website with math puzzles: “I really would love to get into that site more and figure out how and when to use it in my class. There was a lot of geometry and spatial reasoning involved and the puzzles could be used for problem solving or geometry or group icebreakers…there are tons of possibilities! I’m very excited to delve more into that website!”

**Case 8: Kathy (Incomplete Case)**

Kathy represented an incomplete case. She was unable to participate in an interview, so only pre- and post-survey data were included in the analysis. Kathy attended MTC site C. She was a 28-year-old female who had taught mathematics for two years. She had a bachelor’s degree in elementary education and was certified to teach fourth and fifth grades. She taught in an urban school district. She was studying mechanical engineering when a conversation with her goddaughter led her to become a teacher. She wrote:

> My elementary-aged goddaughter needed help with math homework and she told me girls were supposed to be bad at math and that her teachers didn’t really like teaching math. And that was the spark, but long story short I realized that if we wanted to get young people excited about STEM careers it would take elementary teachers that loved mathematics, science, technology.

She was very likely to attend an MTC activity in the future.
**Mathematical identity.** Kathy did well in mathematics growing up. She remembered working on proportional reasoning problems in middle school with her father: “My dad was always busy but he loved those problems and he would always make time to sit down and work on them with me.”

However, when she got to college, “the math was terribly difficult for the first time in my life.” She felt “overwhelmed, and isolated,” but because of that experience, “I can empathize with people who claim to ‘hate math’ because I remember looking at a page of problems wide eyed and feeling hopelessly powerless like there was no way I’d ever figure it out. And I had no idea who to go to for help.” Despite this negative experience, she said, “In general I enjoy mathematics and I truly love teaching it to students.” She said, “I don’t very often feel anxious” about doing mathematics, although she felt disinterest in some problems. She enjoyed doing mathematics problems “that have a real world connection, or connection to my own life.”

Her description of what it meant to be good at mathematics was more descriptive after the MTC than it was prior to the MTC. She included collaborative actions after the MTC. Before participating in the MTC, she wrote, “Persevering in problem solving and logically working towards a solution.” Post-MTC, she wrote, “To persevere in problem solving. Communicate your ideas effectively. Could take the reasoning of others. And engage in all the standards of mathematical practice that allow for a student to be a mathematically proficient student.” Her answers on the post-MTC survey signaled that she valued communicating ideas effectively and understanding the reasoning of others,
and it also included language similar to the Standards for Mathematical Practice from the Common Core standards (CCSSI, 2010).

Mathematics teaching identity. Kathy enjoyed teaching mathematics and said, “I truly love teaching it to students.” She also recognized ways that her teaching had changed and the ways in which she wanted to continue to grow as a teacher. She was working at being more student centered in her instruction after the MTC: “I think I’m more open minded about students driving their own education, math talks, and not being the ‘driver’ all the time. I’m trying to become more student led. And I need help.” Although she wanted to make these changes, she recognized that she was not completely comfortable with it yet. Kathy also wrote, “I admittedly struggle with determining how to structure my classes/curriculum to create the best environment for student growth.” In particular, she intended to implement “increased focus on group norms,” math talks, and “more time spent problem solving” in her classes after the MTC.

She believed that student enjoyment and teacher enjoyment of mathematics were important. She became a teacher because she realized that students needed to have teachers who enjoyed STEM subjects themselves. She also believed that student-centered instruction was important. She described her successful lessons in the following way: “Students are challenged to find solutions to mathematical problems. Encouraged to engage in multiple solutions and strategies. Encouraged to work with peers, use manipulatives, and talk about and explore their own thinking and understanding.” She also said, “Students are engaged in rich problem-solving activities. And the teachers
facilitating the learning, restructuring questions and problems that address areas of student need.”

Kathy was eager to become part of the MTC community and work collaboratively with other teachers. Prior to the MTC, she wrote, “I’m excited for the opportunity to commiserate with likeminded mathematics teachers and discuss curriculum and what is best practice to encourage student learning.”

**Interaction of identities.** Kathy enjoyed doing mathematics and teaching mathematics, and her enjoyment of the two appeared to be related. When asked what she liked about doing mathematics, her answer was mostly about teaching math: “I love the kinds of problems that require students to persevere and problem solving. And their unique solutions get me excited about doing and teaching math.” It seemed that her love of mathematics led her to enjoy teaching mathematics, and teaching mathematics, in turn, invigorated her excitement for doing mathematics.

She also expressed that experiences as a learner were beneficial for her as a teacher. Her experience struggling in mathematics in college helped her to empathize with people who dislike mathematics or feel mathematics anxiety. She also said that during the MTC, the “rich problem-solving strategies that we investigated were helpful both as a teacher and student.” As a teacher, she appreciated learning more about the Standards for Mathematical Practice from the Common Core (CCSSI, 2010) and learning about pedagogy for teaching problem solving. But as a learner, she enjoyed “the variety of problems that we got to attempt.”
Summary of Individual Cases

Teachers’ mathematical identities were explored through the lens of productive disposition (NRC, 2001), including facets of identity such as beliefs about mathematics, attitudes toward mathematics, motivation and persistence, self-concept, and self-efficacy. Identity is considered to include both personal and normative components (Cobb et al., 2009). Teachers’ mathematics teacher identities were explored using Helms’s (1998) four-dimensional framework of identity: actions, expectations, values and beliefs, and the future. The interaction and alignment of teachers’ identities were explored through the lenses of the theories of Van Zoest and Bohl (2005) and Beijaard et al. (2004).

The MTC experience was powerful for these teachers in different ways. Each teacher’s mathematics and mathematics teaching identities were unique prior to the MTC, and they were uniquely influenced by their MTC participation. Through the experience of struggling with challenging mathematics and learning to rely on their peers to problem solve collaboratively, the teachers strengthened aspects of their mathematical identities and mathematics teaching identities. But the extent to which these identities shifted varied.

The takeaways that teachers got from MTCs differed. Some teachers were able to adapt problems to different grade levels, and others believed they could not. Some decided to use general strategies like group work. Others used specific problems from the MTC in their classrooms. Some decided to use specific pedagogical strategies used by the facilitators. Some made vague connections to their classrooms as they experienced the
problems, and others made specific plans for what they were going to use in their classrooms.

On the other hand, teachers’ perceptions about mathematics and about their own abilities to do mathematics problems changed more drastically. Teachers’ self-concepts and self-efficacies improved. They learned about the nature of mathematics and began to see characteristics of mathematics that they did not know about before. They saw themselves as mathematics doers and as mathematics learners.

It is incredibly difficult to disentangle teachers’ identities, particularly ones that are as closely related as mathematics and mathematics teaching identities. From the pre-immersion to post-immersion survey, teachers had very few changes to statements related to core identities, like “math is my favorite subject to teach,” but the most drastic changes were related to doing mathematics. Identity is relatively stable, and identity changes can take time. However, the teachers were already displaying evidence of change in markers of identity such as beliefs and perceptions of themselves.

Perhaps the teachers’ identities related to being mathematics teachers are more deeply held and difficult to change than their mathematical identities. It also may take time for teachers to incorporate these changes into their teaching. The process of changing identity and changing practices are cyclical, and changing one facilitates the changing of the other. Perhaps over time the teachers’ mathematics teaching identities will continue to shift.
The next chapter explores seven themes identified across the cases in relation to aspects of teachers’ mathematical identities, mathematics teaching identities, and the interaction of these identities.
Chapter 5: Cross-Case Comparison

Seven prevailing themes were identified across the cases. These themes are described in Table 3. While not every theme is exemplified in every case, these themes were found to be common among most of the cases. Each theme is described as a combination of the individual cases and includes salient quotations.

Table 3.

Themes and Associated Research Questions

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<td>Perceptions of mathematics</td>
<td>Relates to beliefs about and values of mathematics, including connected values of mathematics</td>
<td>RQ 1</td>
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<td>Perceived mathematics teaching abilities</td>
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<td>Enacting new teaching methods</td>
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<td>Sense of group belonging</td>
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<td>Conflation of doing and teaching mathematics</td>
<td>Relates to perception of “teaching mathematics” and “doing mathematics” as the same</td>
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<td>Relates to connections between mathematics experiences and classroom actions</td>
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Theme 1: Beliefs About the Self in Relation to Mathematics (RQ1)

Teachers’ beliefs about themselves in relation to mathematics were closely related to their past experiences doing mathematics. The teachers also reported experiences in the MTC that influenced their beliefs about themselves as doers of mathematics. These included experiences being successful in doing mathematics, realizing that others had similar anxieties about mathematics, and feeling increased confidence when confronted with mathematics problems. This theme is closely related to productive disposition (NRC, 2001), but contains elements of motivation (Czikszentmihalyi, 2000; Deci & Ryan, 2008; Dweck, 2000; Middleton & Spanias, 1999), persistence (Cobb et al., 2009), self-concept and self-efficacy (Schunk & Pajares, 2007), intrapersonal aspects of identity (Martin, 2000), and engagement as a doer of mathematics (R. Anderson, 2007). It is viewed through the lens of normative and personal identity (Cobb et al. 2009).

Pivotal moments of struggle and success in mathematics. The teachers’ mathematics autobiographies included pivotal moments of struggling in mathematics or being unsuccessful as well as stories of success in mathematics. In all of the surveys, including teachers who were not interviewed, about one third of the stories (24) described positive experiences in mathematics, slightly fewer were negative (18), and another third described experiences teaching mathematics. The remaining stories gave no positive or negative value judgment to the experiences they related. Six of the interviewed teachers relayed at least one moment related to being successful in mathematics: times when a light bulb seemed to switch on (Erica), or when they were recognized for their achievement or accomplishment. For Carol, it was when she was the only student to solve
an extra credit problem. Three teachers (Erica, Faye, and Kathy) connected their perceived mathematics abilities to their decisions to become teachers. These three teachers chose to teach mathematics because they were good at and enjoyed doing mathematics (which is explored in more detail in the theme “perceived mathematics teaching abilities”). On the other hand, Carol and Deborah each believed that struggling in mathematics led them to become better teachers of mathematics.

Four teachers (Beverly, Deborah, Erica, Kathy) mentioned a specific mathematics course or grade level where they remembered struggling with mathematics: Calculus, algebra, geometry, and middle school were common areas of difficulty. Two of these teachers—Deborah and Erica—attributed their struggles to specific teachers who taught in ways that did not work for them. Beverly related her struggles in geometry to specific content. Both Deborah and Erica relayed stories about falling behind their peers and needing extra help at home. Carol, Kathy, and Beverly disliked mathematics growing up and reported that mathematics class was boring (Carol, Kathy) or “difficult and painful” (Beverly). Erica enjoyed mathematics and did well in it until college. When she experienced difficulty in her college mathematics class, she decided to change her major. Similarly, Beverly and Carol chose not to continue their study in mathematics by only taking the minimum mathematics requirements in high school or college.

**Mathematics anxiety.** Seven of the eight teachers reported feeling what they described as mathematics anxiety. Anne and Beverly were anxious when the mathematics was too far over their heads. Carol and Faye felt anxious when problems required memorization or mental mathematics. Faye also felt anxious in competitive situations or
when she did not know how to begin to solve a problem. Deborah related her anxiety in mathematics to the ways that it was taught when she was a student—with no manipulatives and a focus on routine mathematics exercises. Jeff and Kathy felt anxious about calculus, and Anne felt anxious about mathematics above the grade level she taught. Three teachers felt anxious about doing mathematics in teaching situations (to be explored in the “perceived mathematics teaching abilities” theme), such as when they had to teach a mathematics concept that they did not feel confident about, or when they did not want to make a basic calculation error. However, both Beverly and Deborah mentioned that they no longer felt anxious about mathematics like they did in the past. And Erica was the only participant who reported not feeling anxious about mathematics.

Mathematics abilities. When speaking about their mathematics abilities in the interviews, the teachers generally reported that they were average, but their perceptions of their abilities varied from good to average or bad to average. Half of the teachers reported that they were average or adequate (Anne, Beverly, Deborah, and Faye), although Deborah also said she thought of herself as bad at mathematics, and Anne and Faye reported that they were good at doing mathematics when they were students. Both Beverly and Deborah explained that they were good at mathematics at the level that they taught, but not higher. Comments about their mathematics abilities revealed the ways in which these teachers saw the nature of mathematics, being a mathematician, and what it means to be good at mathematics. For example, Deborah noted, “I am a good problem solver but if you put a calculus problem in front of me, I would not have a clue.” To Deborah, being good at mathematics means being good at advanced mathematics, like
calculus, rather than problem solving. Faye’s description of herself also revealed her beliefs about the nature of mathematics and doing mathematics. She had to work hard to be successful in math: “I don’t think I’m brilliant, but I think I’m definitely a higher-math student. I wouldn’t consider myself a mathematician. There are definitely smarter people in math than me, but I think I’m a very good student. Sometimes I have to work at it.” To Faye, being good at mathematics means that you do not have to work at it. Because she worked hard to be successful in mathematics, she did not believe that she was exceptionally good at it. Carol and Jeff both felt that they had been successful in mathematics, and both talked about how being good at mathematics is “a confidence game” (Jeff) or related to growth mindset (Carol). Similarly, Erica and Kathy said that perseverance is an important part of being good at mathematics.

**Enjoyment of doing mathematics.** All of the teachers reported instances in which they enjoyed doing mathematics. Both Carol and Deborah enjoyed the challenge of mathematics problems, and they, along with Beverly and Faye, enjoyed the sense of accomplishment or reward when they were successful. Beverly and Erica enjoyed doing mathematics when it was at the right level for them, it was appropriately challenging, or they had adequate support to be successful. For example, Erica talked about a time in algebra class when it finally “clicked” for her because of the support and strategies provided by her teacher.

The teachers generally enjoyed the mathematics in the MTC—all eight teachers reported finding it fun or enjoyable. Anne explained that she had not enjoyed doing mathematics for a long time prior to the MTC. All eight teachers described specific
problems that they enjoyed, and some described general attributes of problems that they enjoyed, like finding patterns (Anne and Beverly). Beverly, Carol, and Deborah enjoyed feeling successful, and Faye particularly enjoyed making connections between topics. Carol, Deborah, and Jeff used the word “challenge” to describe what they enjoyed about the MTC. The word typically had a positive connotation—that the level of challenge was appropriate and exciting, and that collaboration helped them to work through challenging problems. Two of the teachers were motivated when they were drawn into a problem and experienced flow: Deborah mentioned time flying, and Jeff wrote about becoming so engrossed in a problem that he was having trouble putting the problem down that evening. Anne, Beverly, Deborah, and Erica described specific problems when they felt very engaged. For the most part, these problems were experienced without frustration, although Jeff wrote he was still “grinding” about a problem that had been frustrating to him that day.

**Moments of success and frustration in the MTC.** Teachers also had moments of feeling unsuccessful in mathematics during the MTC. Anne felt left behind during the explanation of problems, often during the extensions. Carol and Faye were frustrated when they were challenged to access their prior knowledge and had difficulty recalling the relevant information. Three of the teachers felt frustrated with the way that particular problems were presented in the MTC, such as not knowing the point of the exercise and being told to explore or look for patterns (Beverly, Carol, Faye). Beverly was frustrated by a problem that required the use of mental math. Beverly, Deborah, and Erica felt overwhelmed by problems when they did not know where to begin, when a solution path
was not immediately apparent, or when they had too many things to keep track of during a problem. Seven of the teachers (all except Kathy) talked about how they coped with this frustration or uncertainty, and each mentioned the collaboration with their group members as critical to easing this frustration for at least one problem (various aspects of this collaboration are explored in the theme “sense of group belonging”). Four teachers—Beverly, Deborah, Erica, and Faye—also talked about perseverance and pushing themselves to work hard when they were faced with a difficult or frustrating problem. Anne was the only teacher who felt anxious about using technology in an MTC session, and she coped by giving up on the technology and focusing on following along with the mathematics.

Most of the teachers (all except Kathy) talked about feeling successful in the MTC. These moments in which they felt or experienced success—like when they figured out a pattern, were able to contribute to the group, or solved a problem—helped them to feel accomplished, competent, and motivated. Five of the teachers (Beverly, Deborah, Erica, Faye, and Jeff) spoke about feeling confident in doing mathematics. Deborah and Faye both talked about their confidence increasing throughout the week: Early in the week, when presented with a problem, they immediately felt anxious or overwhelmed; but later on in the week, they felt more secure in their abilities to handle problems that came up. This increased confidence may have been buoyed by experiences of success and improved mathematics self-confidence. These two teachers, along with three others (Beverly, Carol, Deborah, Erica, and Faye), all said that persevering through problems that were challenging helped to improve their self-concept and self-efficacy. These
feelings not only related back to the sense of self (e.g., seeing oneself as good at doing mathematics or capable of doing math) but also to the future (e.g., feeling motivated when faced with a similarly challenging problem in the future).

**Seeing self as successful.** More significantly, three of the teachers began to see themselves as successful doers of mathematics for the first time. Solving problems helped them to feel greater self-concept and self-efficacy related to doing mathematics. For example, Beverly said, “I suppose it helped improve my image of myself as a person who can do math…I guess it helped me to see myself as a mathematician.” Similarly, Deborah, who reported thinking of herself as bad at doing mathematics, explained that her experience in the MTC helped to change this perception. Deborah said:

> I think when I originally went into the Math [Teachers’] Circle, I was really nervous because I’ve often thought of myself as bad at math or with me struggling in math, I thought it helped me become a better math teacher but I never thought of myself as being good at math. When going there and I was able to actually solve some of the problems and do different things, I was like, ‘Oh, hey. This isn’t so bad.’ It helped boost confidence in regards to the ability to be a problem solver.

Anne felt similarly; she reported, “That was exciting, was not to be the dumb kid in class, not to be the one that was going, ‘Um, wait a minute. I don’t get that. How’d that happen?’ Where the way the Circle was run, it was a lot more, ‘Here’s the idea. You come up with a way to solve it using whatever math skills you have right at this moment.’ ”

Four of the teachers’ feelings of success related to the others in the MTC. Anne and Erica felt positively about mathematics problems that they were unable to solve on their own when they were able to follow along with others’ explanations. Discourse and
collaboration in the MTC were important for these teachers to feel successful in mathematics (which is discussed in depth in the “sense of group belonging” theme). Faye and Beverly also reported feelings of pride when they were able to contribute to their group or help others. They felt proud of their accomplishments when they were able to solve a problem quickly or before the others in their group. These teachers’ accomplishments were felt as successes in relation to the rest of the group.

**Others felt similarly.** Another aspect of the group environment that helped to increase teachers’ confidence was the realization that others felt similarly. For example, Anne and Faye both gained confidence when they realized that they were not the only ones who saw themselves as bad at mathematics or inadequate when presented with a mathematics problem. The realization that others have these feelings too helped them to see that their feelings were normal, and that it did not mean that they would be unable to do the mathematics.

**Insights about self.** The MTC also gave teachers different types of insights about themselves as mathematics learners. Four teachers (Beverly, Carol, Deborah, and Faye) talked about their learning styles, realizing that they learn better visually, or that they needed a silent environment in order to process problems. Teachers also learned new problem-solving strategies. When asked what they learned, five of the teachers focused on problem-solving strategies rather than specific mathematics content (particularly Anne, Beverly, Erica, Faye, and Jeff). While they did learn some new mathematics content, and four teachers (Anne, Carol, Erica, and Faye) mentioned specific connections or problems, these were eclipsed by the focus on problem solving. Every teacher learned
something about solving problems. In fact, Beverly reported that she did not learn any new mathematics, but instead learned new strategies for approaching problems. In the post-survey, five teachers said that people who are good at mathematics are able to use various problem-solving strategies. Three teachers (Carol, Deborah, and Erica) also talked about the strategy of “jumping in” and getting started on problems by trying something. This was a problem-solving strategy, but it also related to teachers’ self-efficacy and confidence that they would be successful in solving a problem. Four teachers realized that perseverance is a problem solving strategy, and it helped them learn how to break an unapproachable problem into pieces (Beverly, Carol, Deborah, and Faye).

**Theme 2: Perceptions of Mathematics (RQ1)**

Teachers’ beliefs about mathematics included the connections that they saw between mathematical topics and between mathematics and other areas. The instances in which teachers enjoyed doing mathematics were often related to connections between mathematical topics, doing mathematics in real life, or mathematical puzzles. During the MTC, teachers encountered mathematics that they described in ways related to connected values of mathematics (Ernest, 2008). When describing mathematics in the MTC, the teachers tended to focus on problem solving and connections, and insights that they had, rather than on specific mathematical concepts that they learned. This theme relates closely to beliefs about mathematics (Schoenfeld, 1985), fallibilist and absolutist beliefs (Ernest, 2008), and static and dynamic beliefs (Felbrich et al., 2008). It also includes elements of mathematical identity such as imagination (R. Anderson, 2007) and community (Martin, 2000).
Using mathematics in daily life. When asked in the interviews, all six of the teachers gave examples of when they use mathematics in their daily lives. For example, teachers mentioned banking, checkbooks, and finances, cooking at home with their kids, or home improvement projects (Anne, Beverly, Carol, Deborah). Five of the teachers enjoyed playing puzzles or games like Sudoku, logic puzzles, or KenKen; or they enjoyed doing challenging mathematics puzzles that they could share with their families or friends (Beverly, Carol, Deborah, Faye, and Jeff). Jeff enjoyed “nerding out” with friends. Four teachers—Anne, Beverly, Erica, and Kathy—specifically mentioned that they enjoyed mathematics with connections to the real world. Four teachers enjoyed doing mathematics when they could find patterns (Anne, Beverly, Deborah, and Faye). However, five of the teachers said that the mathematics that they did in their daily lives was the mathematics that they did while teaching—an example of the closely held relationship between doing mathematics and teaching mathematics (which is described more fully in the theme “conflation of doing and teaching mathematics”).

Descriptions of MTC mathematics. Teachers’ descriptions of mathematics in the MTC related most closely to connected values of mathematics (Ernest, 2008). For example, three teachers made connections between mathematical topics for the first time or understood connections between multiple problems in the MTC (Anne, Carol, Erica). Three of the teachers were amazed by the connections that were revealed to them as a result of the MTC problems—they had very positive feelings about the math—it was “amazing” or “cool” (Anne, Beverly, Carol). Carol and Erica also made connections between the mathematics that they were doing and other content areas like English,
language, or generalized learning. Three teachers mentioned feeling great enjoyment from a problem that focused on “the human side of math,” which they said made the problem more interesting and “less daunting” (Beverly, Deborah, Jeff). Erica talked about how the mathematics was different from other mathematics that she had done in the past. Anne and Faye were amazed to see that mathematics could consist of problem solving and could be ambiguous, and that it did not solely consist of straightforward exercises. Three teachers realized that problems did not need to have a single solution, and that the instructor did not need to know the solution in advance (Carol, Deborah, Faye). For example, Carol said, “I enjoy solving the problems and really finding different solutions to things. At the same time I like seeing the different paths I can take to get to the same answer.” Three of the teachers also realized that mathematics problems could be approached via different solution paths (Beverly, Deborah, Kathy). These types of problems were enjoyable and motivating to Anne and Deborah, because they could use different skills to approach the problems in different ways and by using their strengths. When speaking about the MTC problems they enjoyed, four teachers frequently mentioned problem solving and finding patterns in mathematics or seeing them emerge (Anne, Beverly, Deborah, Faye). Beverly struggled to find the mathematics in a few of the problems. After the MTC, she was still trying to understand the mathematical connections and seemed to settle with the idea that these problems taught “soft skills” like perseverance or general problem solving. However, she was still unsure and was still working through her understanding of these connections.
Teachers talked about enacting these connected values in the classroom. As a result of the MTC, three of the teachers felt as though they were capable of presenting problems that did not have an “answer” or problems to which they (the teachers) did not know the answer (Anne, Beverly, and Faye). Faye explained that the exploration could be more important than finding the correct solution: “This showed me how sometimes we don’t always know the objective of a problem and that sometimes just the exploration of the problem in and along the way is the actual objective of the activity, not necessarily formulating a solution.”

**Theme 3: Perceived Mathematics Teaching Abilities (RQ2)**

Teachers’ mathematics teaching self-concept, self-efficacy, and enjoyment of teaching mathematics were salient aspects of their mathematics teaching identities. However, this theme appeared to be the least changed as a result of the MTC. In fact, from the pre-immersion survey to the post-survey, responses changed the least for questions about teaching mathematics, including “Math is my favorite subject to teach” and “I’m good at communicating math material to students” (average increases of 0.25 and 0.247, respectively, on a six-point scale). Thus, teachers’ beliefs about their abilities related primarily to past experiences and did not often draw from their MTC experience. Their decisions to teach mathematics often (in four cases) related to their mathematics self-concept and their enjoyment of doing mathematics growing up, or to increased enjoyment and self-efficacy as their teaching career evolved (for two teachers). Half of the teachers felt their confidence in teaching mathematics increase over their careers. All of the teachers enjoyed teaching mathematics at the moment, although some of the
teachers originally did not enjoy the subject earlier in their careers. All of the teachers saw growth and change as natural aspects of being a teacher, and five teachers spoke of a desire to improve their teaching abilities in order to be more effective teachers in the future (their self-efficacy). Although this theme was least connected to the MTC experience, it was an important aspect of teachers’ identities, and so it is included as a theme. This theme relates most closely to the future-oriented facet of professional identity described by Helms (1998) as “where people see themselves going, or the kind of people they want to become” (p. 829), desired identities (Sfard & Prusak, 2005), the ideal self (Katz et al., 2011), or possible-selves theory (Hamman et al., 2010). It also contains elements of self-concept and self-efficacy (Schunk & Pajares, 2007), personal and professional identity (Beijaard et al., 2004), and beliefs (Richardson, 1996). It is viewed from the lens of teacher change (Kaasila & Lauriala, 2010; Warford, 2011).

**Becoming a mathematics teacher.** Teachers’ decisions to become mathematics teachers are somewhat related to their mathematics self-concept and enjoyment of mathematics. Four teachers chose to become certified to teach mathematics because they enjoyed doing mathematics or were doing well in their mathematics classes at the time: Anne, Faye, Jeff, and Kathy. For example, Faye said, “I knew that I liked math and that I was pretty good at it, but I never thought I was good enough to teach it” until she was encouraged by a professor. And the teachers who did not enjoy or were not doing well in mathematics chose to become generalists or focus on another subject. For example, Erica originally wanted to teach mathematics when she entered college but changed her major after a challenging mathematics course. The other three teachers, Beverly, Carol, and
Deborah, did not intend to teach mathematics initially, other than what was part of being a generalist. It was only later on that they became interested in teaching mathematics.

Over time, some of the teachers’ roles changed, and they began teaching more mathematics classes. Four teachers—Anne, Carol, Deborah, and Erica—were teaching multiple mathematics courses at the time of the MTC because it was needed in their school. Erica made a conscious decision to focus on mathematics while earning her master’s degree.

**Enjoyment of teaching mathematics.** All of the teachers enjoyed teaching mathematics at the time of the MTC. Erica said, “I feel like I like the subjects that I teach, that’s where my heart is. I have passion to teach those subjects, moreso than other subjects and I’m probably happiest when I’m teaching those subjects.” Four teachers—Anne, Beverly, Carol, and Deborah—talked about enjoying the excitement students feel when they discover something new. Beverly and Deborah did not enjoy teaching mathematics in the past, and they were anxious or nervous about teaching it. However, their experiences as teachers changed their perceptions of teaching math: they did not enjoy teaching mathematics until something changed in their teaching, after which they began to enjoy it. For Deborah, the change was in moving from teaching the early elementary school grades to the later elementary school grades. Beverly was not interested in teaching mathematics until she took an inquiry-based mathematics teaching workshop: “That program gave me the confidence to teach math and to help students to get excited about learning math.”
**Confidence to teach mathematics.** Four teachers had gained confidence in teaching over the years. Beverly, Carol, Deborah, and Faye all reported feeling nervous or insecure about teaching mathematics, but as they gained more experience, attended mathematics pedagogy workshops, or changed grade levels, their mathematics teaching self-efficacy increased. Faye did not feel as competent as her peers when she began teaching mathematics. Three teachers still felt anxious about teaching mathematics in certain situations: Anne was anxious about teaching higher-level mathematics, Deborah was anxious about teaching new strategies, and Faye was anxious about teaching mathematics that she had not taught or used in a long time.

Teachers’ mathematics teaching self-concept or perceived mathematics teaching abilities seemed to be related to their perception of continuous improvement. Because the teachers saw growth and change as a natural part of teaching, they wanted to continue to improve every year. In general, they saw themselves as good but not exceptional teachers, and they wanted to get better (Anne, Beverly, Carol, and Faye). Anne, Carol, and Jeff spoke of a desire to improve in order to reach students better or help students learn how to make sense of mathematics. For example, Anne said, “I’d say I’m in the top half of the population. I’m not the best. I do reach a lot of kids, but I could be better, and I’m trying to get better.”

**Theme 4: Enacting New Teaching Methods (RQ2)**

The teachers talked about teaching methods that they believed to be effective for student learning, and their adoption of new teaching methods in the past and anticipated in the future. All of the teachers believed that increasing student self-efficacy and
perseverance are important. All of the teachers also talked about student-centered methods, although their interpretation of this phrase appeared to differ (for example, three of the teachers described teacher-centered methods). The teachers all described ways that they intended to change their teaching after the MTC, and all but one teacher wanted to incorporate teaching methods from the MTC (such as collaboration, math talks, and class norms). This theme aligns closely with the action facet of teacher identity (Helms, 1998) and theories of beliefs and action (Collopy, 2003; T. F. Green, 1971; Leatham, 2006; Rokeach, 1968). It incorporates the notion of integrating professional and personal identities (Beijaard et al., 2004), teacher change based on negotiation or identification (Horn et al., 2008), and productive friction (Ward et al., 2011). It also incorporates teacher beliefs and action (Richardson, 2006). It is viewed primarily through the lenses of normative and personal identities for teaching mathematics (Gresalfi & Cobb, 2011) and the process of teacher change (Kaasila & Lauriala, 2010; Warford, 2011).

**Encouraging student self-efficacy.** All of the teachers believed that building student self-efficacy and perseverance was an important aspect of teaching mathematics. Two teachers—Jeff and Faye—talked about building up students’ confidence. Faye thought that using some of the problems from the MTC might be challenging for her students, but that their success with the problems could increase their confidence. Several teachers also talked about encouraging student perseverance (Beverly, Carol, Erica, Jeff, and Kathy), such as by letting them know that we learn from our mistakes (Beverly). Carol said it was important to relay information on growth mindset to her students, to encourage the belief that they “can become good at anything as long as you keep trying
and practicing with it.” Anne, Beverly, and Carol enjoyed helping kids figure out that they enjoy doing mathematics and that they can persevere when it is hard. After the MTC, three teachers (Deborah, Erica, and Jeff) added perseverance to their previous definition of what it means to be good at mathematics. Carol, Jeff, and Kathy said that it was important that their students see their teachers doing mathematics for fun. Beverly liked making mathematics enjoyable and fun because her belief was that it helped students stick with it and learn it. Anne was the only teacher who talked about being frustrated by students’ “learned helplessness” and giving up.

**Student-centered methods.** All of the teachers believed that student-centered methods were effective. All of the teachers used phrases like student centered, inquiry, discovery, problem solving, engagement, collaboration, and discourse. For example, five teachers (Beverly, Carol, Erica, Jeff, and Kathy) talked about getting kids to discover things on their own and engage with one another. Erica talked about how her teaching had become more student centered, whereas Kathy still wanted to become more student centered. All of the teachers also mentioned engagement as an important aspect of successful mathematics teaching. Anne and Deborah talked about getting students to make connections between mathematical topics. Anne said she looked for students to make connections; Deborah described a successful day as one in which students incorporated ideas from prior lessons. On the other hand, teachers often meant different things by these terms (particularly “student centered,” “engagement,” and “discovery”), and three of the teachers described activities or beliefs that underlie teacher-centered strategies. Carol and Faye taught using mostly teacher-centered methods, like “I do, we
do, you do.” Erica similarly believed that a strong foundation of the basics is important for students to master before they could engage in problem solving. These teachers’ descriptions of their activities and beliefs contradict their stated beliefs about engaging students in problem solving and other student-centered methods. Although the teachers used phrases like “student engagement,” their descriptions of what that looked like varied.

**Changes in teaching pedagogy.** Every teacher talked about learning new ways to teach over their career—such as using group work, more hands-on activities and investigations, or more instructional differentiation. Anne and Faye had changed their teaching in response to outside pressures, like changing from emphasizing the “basics” to using calculators for basic skills. In contrast to other teachers’ changes, Deborah used group work less often than she did in the past. She cited increasing pressures related to standards and testing. Seven of the teachers talked about their intention to use specific pedagogical techniques from the MTC. These teachers talked about becoming more student centered, using collaboration, various groupings, math talks, and class norms (Anne, Beverly, Carol, Deborah, Erica, Faye, and Kathy). Discourse and collaboration were frequently mentioned. Anne, Beverly, and Erica talked about specific actions from the MTC leaders related to encouraging collaboration, such as facilitating student conversations. Five of the teachers talked about the use of discourse in the MTC and their intention to incorporate more discourse into their classes in the future (Beverly, Carol, Deborah, Erica, and Faye). These teachers realized the need to have students collaborate and work in groups. For example, Faye said:
I certainly hope to give my students more time to collaborate to process information, talk about the math together. I hope to give them maybe more challenging problems at a higher level than I maybe would have attempted previously and that I think they can do it with collaboration. I think that they would probably have the same experience I did, be apprehensive at first. If they stick with it and endure, and are successful, I think they can build a lot of confidence that way.

Three teachers—Beverly, Deborah, and Kathy—intended to introduce class norms to the students at the beginning of the school year. Beverly called these norms “what we expect from one another” and allowed the activity to be student-led, with the students coming up with the list of norms. This was an activity that the teachers completed at the beginning of the MTC, and Beverly’s students came up with “pretty much the same things that we adults had come up with.” Anne, Erica, and Faye talked about specific strategies used by the MTC leaders, like not giving too much assistance to students (Anne), giving a small hint at the right time (Anne and Erica), scaffolding (Faye), and using a game format (Anne). Carol wrote that the MTC helped her to realize how to address all learning styles.

All of the teachers talked about teaching with different types of problems or approaching mathematics content in different ways, such as problem solving and discovery. Some of the teachers talked about their evolution from how they used to teach to how they currently teach, and other teachers focused on their future ideas based on the MTC problems. Anne and Beverly both used to emphasize basics and memorization but had changed somewhat since they started (although Anne still struggled to balance some of the basics with more advanced understanding). Deborah, Faye, Jeff, and Kathy all talked about incorporating more problem solving into their classroom and discussed specific strategies such as allowing multiple days for a problem, backing off and
becoming more student-centered, and using instructional differentiation. Four of the teachers talked about the importance of letting students discover mathematical connections for themselves and understand why something works (Anne, Carol, Deborah, Faye). They related this to specific MTC problems that made explicit connections between mathematical topics or that led them to understand the underlying mathematics. Three teachers—Anne, Beverly, and Erica—explained that the MTC may not have changed their ideas about successful mathematics teaching, but that it instead supported or reinforced their ideas about teaching problem solving. Anne was impressed by the buy-in achieved by the facilitators, and by the strategies the facilitators used to engage everyone at different levels. Faye mentioned the difference between the facilitator’s skill and her own at facilitating problem solving. Beverly said during the MTC that she “suddenly realized how I cheat my students by teaching them how to solve certain problems instead of helping them to discover it for themselves.”

**Difficulty with teaching problem solving.** All of the teachers spoke about their struggles or difficulties implementing teaching strategies like problem solving. Their difficulties included having to teach required standards, not having enough time for longer problems, and feeling anxious about teaching problem solving. Most of the teachers—Anne, Beverly, Carol, Deborah, Erica, Kathy, and Jeff—were struggling with finding a balance between their beliefs about effective teaching and their district’s requirements, or between basic facts and problem solving. For example, Anne said, “it doesn’t seem to ever have a happy medium” between “push[ing] the basics” and focusing on problem solving, and that “there’s a lot of give and take and pull” between the two.
Other teachers, Deborah and Beverly, believed problem solving was effective but they struggled to balance it with requirements from their school district, such as required textbooks, curriculum, or pacing guides. Time was mentioned as a challenge by half of the teachers, particularly in comparison with the longer chunks of time in the MTC (Anne, Beverly, Erica, and Faye). These four teachers reported their main challenge with using problem solving or specific MTC problems as their limited class times. One solution undertaken by Anne and Deborah was to add problem solving into their classroom at the beginning of the year or in the time before breaks. They did not plan on incorporating problem solving or problems from the MTC into relevant units during the year, but rather teaching “problem solving” as a singular standalone unit. Anne was also unsure of effective pedagogy for implementing longer problems, but she was the only teacher to talk about this. In contrast, both Deborah and Jeff felt that the MTC experience helped them feel confident in teaching longer problems.

**Theme 5: Sense of Group Belonging (RQ1 and RQ2)**

Teachers spoke about their sense of group belonging, both in their school districts and in the MTC. In their districts, teachers tended to feel that their beliefs and teaching styles were similar to those of the other teachers but out of sync with the school district administrators. In the MTC, the teachers were generally excited to be part of the MTC and to collaborate with other educators. The focus on collaboration in the MTC played a significant role in the teachers’ mathematical progress and in their feeling of belonging to a supportive community. However, collaboration also played a role in teachers’ negative self-concepts, as four teachers negatively compared themselves and their mathematics
abilities to the other teachers in the MTC. This theme aligns with recognition by others (Gee, 2001) and the *expectations of others* facet of teacher professional identity described by Helms (1998). It relates to the development of normative identity for doing mathematics (Boaler & Greeno, 2000; Cobb et al., 2009; Gresalfi & Cobb, 2011) and the professional side of identity (Beijaard et al., 2004). It also incorporates elements of mathematical norms (Martin, 2000) and the process of teacher change (Kaasila & Lauriala, 2010; Warford, 2011).

**Being seen as a teacher.** Three of the teachers spoke of wanting to be seen as an effective mathematics teacher by others. Deborah wanted the district administration and families of her students to see her as effective. In the pre-survey, Beverly said that she chose to attend the MTC because she wanted the trust of the parents to be well founded. She was also hosting a student teacher the following year and wanted to give the student teacher a good experience. Faye explained that she was supposed to be a mathematical expert to others but was excited that she was still able to learn new mathematics in the MTC. Four teachers experienced mathematics anxiety related to others: their fear of making mistakes in front of others, in competitive situations, or when they had to process quickly or memorize something (Anne, Beverly, Erica, Faye). For Erica, this was related to being a mathematics teacher because she perceived that a mathematics teacher would not make a simple mistake.

**Alignment with colleagues.** For the most part, teachers perceived that their colleagues had similar views about effective mathematics teaching (Anne, Beverly, Deborah, Erica, and Jeff). A few teachers—Anne, Deborah, and Erica—said that their
colleagues attended the same professional development or use the same curriculum, and they worked together to plan effective lessons and make decisions. Jeff preferred teaching mathematics over every other subject, not because of the content, but because of the teacher team he worked with. Deborah said that when she taught primary grades, she did not feel like she had a community of likeminded mathematics teachers, but when teaching the later elementary school grades, she did. On the other hand, Anne and Deborah reported that the expectations of their administrations were different from the teachers’ ideas about effective mathematics teaching. Anne felt that her instruction was mandated by the curriculum and “lock-step” requirements, and she said that the district administration seemed to overvalue test scores. Beverly believed that her administration was beginning to understand their inquiry-based curriculum, because the administrators sat in on some professional development. However, both Anne and Beverly were concerned that the administrations would not see the mathematical applications in problem-solving activities. Anne and Deborah both said that one of their challenges is finding investigations or problems that met the district requirements and standards.

**Collaboration in the MTC.** Collaboration was an important aspect of teachers’ experiences in the MTC. Every teacher mentioned collaboration when they spoke about their positive experiences. They enjoyed collaborating with others, and they also related specific experiences or mathematics problems when collaboration helped them solve or understand the mathematics. Only one teacher, Beverly, reported that she had difficulty engaging in collaboration with her group. Although this was detrimental to her problem solving, it was beneficial in that it helped her to learn about herself and to feel empathetic
with her students when she assigns them to groups. Most of the teachers enjoyed the
problems in the MTC because of the collaboration. For example, Deborah said, “just
being able to bounce ideas off of each other and work as partners, as individuals, as teams
throughout the whole week” was powerful to her. Working in groups also helped the
teachers to feel recognition by others for their mathematics skills (Carol, Erica, Faye, and
Jeff). Six teachers learned from watching other teachers in the MTC: They learned about
both teaching and doing mathematics. Carol, Deborah, Erica, and Faye learned about how
others approached problems in different ways and how everyone had different strengths
(which some of these teachers connected to the classroom). Six of the teachers also
explained that by listening to others’ solutions, they were able to understand problems
they did not figure out on their own. Collaboration assisted in the growth of teachers’
mathematical understanding, pedagogical understanding, and mathematics self-concept.

However, as a result of working closely together, teachers also compared
themselves negatively to the others in the MTC. Some of this was elaborated in the theme
“beliefs about the self in relation to mathematics.” Four of the teachers (Anne, Beverly,
Erica, and Faye) compared their processing speed to the others in the MTC, which made
them feel anxious. Anne and Beverly specifically compared themselves to the high
school teachers in the MTC, saying that the high school teachers were able to engage
with problems in different ways, or that they were able to come up with formulas or
attack problems at a higher level. However, these feelings did not remain negative
through the week. Faye, although feeling insecure in comparison to her peers, said, “As
the week progressed, I realized that everyone felt that same way at one time or another.”
Beverly was worried that others could see how slowly she worked, and so she was relieved during a mental math exercise that others could not see how slow she was. Anne, Beverly, and Erica felt pleased when, although they did not come to a solution on their own, they were able to keep up with the group during the discussion.

**Theme 6: Conflation of Doing and Teaching Mathematics (RQ3)**

In general, for these teachers, doing mathematics and teaching mathematics were closely related. The teachers perceived “doing mathematics” and “teaching mathematics” to be very similar, if not the same. Often, when asked about situations in which they enjoyed doing mathematics or what types of mathematics problems they enjoyed doing, the teachers responded with answers related to teaching mathematics. Further, several teachers had impactful moments in their mathematics autobiographies that took place while they were teaching. And the teachers found it valuable to be in the position of learner during the MTC and took aspects of their experience as learners back to the classroom. Overall, teachers’ mathematical identities were linked to their teaching identities, and it was difficult to separate the two. This theme relates to the development of personalized pedagogical content knowledge (Lee & Luft, 2008), the interaction of identity and subject matter (Helms, 1998), and holding multiple subidentities (Beijaard et al., 2004).

For six teachers, their mathematics autobiography included moments of teaching mathematics. That is, events that took place while they were teaching had been impactful to their mathematical identity. For example, Anne told about a moment when she was amazed by a connection made by a student. Others talked about pivotal teaching
moments when they realized that collaborative or investigative learning was working for their students (Carol, Erica, and Jeff), or when they learned about a different way to teach (Beverly and Faye). For example, Beverly said, “My best experience has been learning by doing. Having struggled through the first year of Investigations, then taking part in the Success project I learned to love math and the teaching of math.”

During three teachers’ interviews (Anne, Beverly, and Carol), when asked when they did mathematics in their daily life, or when they enjoyed doing mathematics, the teachers responded by talking about the teaching of mathematics. Three of the teachers also wrote that they enjoyed teaching mathematics when they could work through it with students and see students make progress, or when they used inquiry in mathematics (Anne, Carol, Kathy). And these connections were not only made during positive experiences. When asked when they felt anxious about doing mathematics, Anne and Faye felt anxious when they had to teach concepts they had not taught before or seen in a while. Carol had to reteach herself mathematics prior to teaching it, and another teacher had a colleague help her understand it. In these cases, teachers’ mathematical identities (and confidence in doing mathematics) were based on their mathematics teaching identities (and their abilities to teach math).

A few of the teachers talked about being nervous about learning mathematics alongside the students. Carol was nervous when she started teaching mathematics at later grades for the first time. Anne and Carol were concerned about being able to help students make extensions when they were learning it themselves. Jeff sometimes felt this way when tutoring students in advanced classes, but he talked about using this as a
learning experience for the student to show that the teacher did not have all the answers. He saw this as a positive outcome. In these cases, the teachers’ mathematics teaching identity (and confidence in teaching mathematics) was partially based upon their mathematical identity (and confidence in understanding and doing the mathematics).

**Theme 7: Connections Between Mathematics Experiences and Classroom Actions (RQ3)**

This final theme was interwoven into many of the previous themes and codes. For these teachers, their experiences learning and doing mathematics had an influential effect on their mathematics teaching. The converse was also true: Teachers’ experiences teaching mathematics had an effect on their mathematical identities. Teachers connected their mathematics experiences in the MTC to their mathematics teaching and to their anticipated ways of teaching in the future. This theme relates to the idea of multiple subidentities (Beijaard et al., 2004) and to the relationship between identities in different settings (Van Zoest & Bohl, 2005). It also includes elements of feeling like a learner (J. L. Cohen, 2010; Collopy, 2003), personal content knowledge (Helms, 1998), and pedagogical content knowledge (Lee & Luft, 2008), as well as theories of changing identity (Gresalfi & Cobb, 2011) and beliefs (Ernest, 1989).

**Using aspects of mathematics experiences in the classroom.** Teachers made connections between their past experiences and their current teaching. Most of the teachers (all except Kathy) gave specific examples of how their mathematics autobiography and past experiences doing mathematics had influenced their current teaching. Five of the teachers (Carol, Deborah, Erica, Faye, Jeff) became teachers
partially due to the influences of teachers that they had growing up. Some teachers—Anne, Deborah, and Erica—mentioned that struggles or successes they had as students influenced some of their teaching actions. For example, Anne had a positive experience as a student, when she moved and her F was carried over as a B, so the new teacher gave her extra assistance. She learned from this not to judge students by past performance, and that everyone can do mathematics with time, patience, and support. Anne compared her teaching style (very visual) to the nonvisual teaching styles of her teachers growing up, and Deborah explained how she uses manipulatives (in contrast to her experience growing up). Erica uses visuals and language in mathematics based on her algebra teacher’s use of visuals and language. Deborah struggled with mathematics growing up and used the knowledge from that experience to help her with her interventions with struggling students. Deborah, Erica, and Kathy all believed their struggles with mathematics as students made them better mathematics teachers. They were more aware of how students might act when confused. Other teachers—Beverly and Faye—had influential experiences with mathematics in workshops that influenced the ways in which they taught. These experiences helped the teachers to finally enjoy both doing and teaching mathematics.

At some point in their reflections or the interview, three of the teachers described how an event in the MTC was similar to something that happens in the classroom. For example, Anne noted how some teachers were really engaged by the extensions and others were not, “just like with the kids,” or when going into depth on a problem took time away from the next problem, “which makes you think of yourself when you teach.”
Beverly intended to tell her students about perseverance by giving an example of her own perseverance with a problem when she wanted to give up. During the MTC, Erica realized that a successful mathematics lesson was one that could be extended. And four teachers talked about new strategies they planned to use, such as more collaborative work, more investigations, fun games, and encouraging students to make connections—all things that they experienced during the MTC and were looking forward to bringing into their classroom for the upcoming school year (Beverly, Carol, Deborah, and Faye).

Four of the teachers also gained more confidence in teaching mathematics after attending the MTC. More specifically, they gained confidence in teaching in particular ways or using specific strategies. Jeff was more confident to try problem solving in his classroom and was “less intimidated” by it. Two other teachers said that establishing norms would help them to feel more confident about group work and classroom management (Anne and Deborah). The modeling of the MTC facilitators and the strategies used in the MTC (like group norms) helped Faye to see how she could implement something similar.

Value of being a learner. The teachers also talked about various benefits of becoming learners themselves. Faye and Kathy believed that doing mathematics makes them a better teacher. Four teachers felt that it was beneficial that they experienced higher-level mathematics during the MTC. Anne enjoyed that some of the MTC mathematics was above the level that she teaches: She enjoyed the challenge. The MTC prompted Carol’s memory of higher level algebra she had not done in a long time, which was “really interesting” to her. She wanted to go on to take more advanced mathematics
classes because it could help her understand where her students were going in mathematics and how she could better prepare them. Two teachers found it valuable to be in the perspective of learner. For example, Erica said, “Getting to experience having to do the math, it puts you in the role of student so I’m able to better think about how I need to teach it. While I was doing the math, things that other people did that helped me, those are the kinds of things I want to do while I’m teaching it.” Other teachers—Carol and Faye—became aware of their learning styles or preferred ways to do mathematics. They connected this experience to the classroom and how they could structure their classrooms to accommodate different ways of learning. Five of the teachers (Beverly, Carol, Deborah, Erica, Faye) also made connections to how they could help students learn problem solving, including using various types of groupings (solo thinking time, pairs, and groups).

During the MTC, three teachers talked about feeling empathy with their students. Anne, Beverly, and Erica talked about how struggling to work in groups helped them to understand how their students feel working in groups. Anne and Faye also said they felt like students when they were able to follow along with others’ explanations and advanced their understanding by listening to others, or when they felt less confident and anxious about not getting the answer right away. One teacher also realized how certain behaviors of the teacher (like proximity) could make students more anxious, and Erica realized how manipulatives could help students feel more confident. Beverly talked specifically about how she could share her experience with the students. Two teachers mentioned that they
had to remind themselves of advice that they gave their students—like to jump in and try a strategy (Carol) or to be the best group member that she could be (Beverly).

Using MTC problems in class. Four of the teachers intended to use specific problems from the MTC in their classroom (Anne, Beverly, Deborah, Jeff). When they described the problems, they tended to talk about the level of inquiry or problem solving, rather than specific mathematics content. Deborah, Jeff, Erica, and Faye believed that the MTC pedagogy and problems could help students build confidence, get excited about mathematics, and have fun with mathematics, because that was their experience with those same problems. Three of the teachers talked about adapting the problems for the grade level they teach. Anne and Carol said that they would probably not use the exact MTC problems but that they could adapt them to their grade level. Deborah explained that not only could she adapt the MTC problems, but she had more confidence that she would be able to adapt other problems she encountered to her grade level and standards.

However, four teachers also mentioned challenges that prevent them from using MTC problems in the classroom. Several of these concerns were similar to the challenges the teachers encountered related to teaching problem solving in general. Time was most frequently mentioned, as teachers compared their short class time to the lengthier sessions in the MTC (Anne, Beverly, and Faye). Anne worried that her district administration might not see the value in teaching problem solving, and she was unsure how to fit the problems into the required curriculum. Anne was also concerned that she would not have the skill level to implement the problems in the ways in which she wanted.
Discussion of Findings

The seven themes described above were identified in relation to the changes that the teachers experienced and described. Most of the themes spoke directly to a single research question, with the exception of the theme “sense of group belonging,” which applied to the first two research questions. The following sections make connections between the research questions, the literature, and the themes.

Mathematical identity. The first research question was, “In what ways does participation in a Math Teachers’ Circle affect teachers’ mathematical identity?” The teachers’ mathematical identities evolved in three dimensions as their personal identities began to align with the normative identity of the MTC. These three dimensions are reflected in the themes: beliefs about the self in relation to mathematics, perceptions of mathematics, and sense of group belonging. Although the three themes were discussed separately, they interacted in ways that affected the development of the teachers’ mathematical identities. For example, teachers who began to change their perceptions of mathematics from static to fallibilist (Felbrich et al., 2008) experienced changes in their beliefs about their abilities to do mathematics and their anxiety in doing mathematics.

Beliefs about the self in relation to mathematics. Teachers’ mathematical identities were grounded in their past experiences doing mathematics. The MTC experience provided an opportunity for teachers to experience doing mathematics with a focus on problem solving and collaboration. How did this experience influence each teacher’s sense of self in relation to mathematics? Because a large part of identity includes the way that we define ourselves (R. Anderson, 2007), the teachers’ definitions
or perceptions of themselves were important. Teachers’ self-concepts were reflected in
the ways that they saw themselves (Martin, 2000). For example, Beverly began to see
herself as a mathematician for the first time, and Deborah talked about her increasing
confidence in herself as a problem solver. These outcomes are similar to the findings of
White and Donaldson (2011) of math teachers’ views of themselves after MTC
participation.

Many of the mathematics problems in the MTCs involved problem solving,
discovery, and application. These problems were consistent with fallibilist philosophies
of mathematics, or dynamic beliefs about mathematics, which focus on the process rather
than the solution (Felbrich et al., 2008). Some of the teachers were frustrated by MTC
problems in which the goal was unclear, or a solution path was not immediately apparent.
It is possible that the teachers’ static beliefs about mathematics, that mathematics is an
exact science and made up of terms to know and rules to follow (Felbrich et al., 2008),
contributed to their frustration. Jeff admitted that he realized that the process was the
main point of the activity but had difficulty letting go of his desire to find a single
solution.

Despite some frustrations, the teachers had mostly positive experiences with these
types of mathematics problems. Being able to approach problems from different
perspectives and different skillsets helped the teachers feel confident in their abilities.
Teachers learned a lot about problem-solving strategies during the MTC, and they
generally strengthened or developed their beliefs that people who are good at
mathematics are able to use various problem-solving strategies. Their experiences with
approachable or open-ended problems may have helped them to develop more dynamic beliefs about mathematics (Felbrich et al., 2008) and about the strategies that can be used to solve problems, similar to findings by White, Donaldson, Hodge, et al. (2013). The teachers began to enjoy the challenge of problems in which the solution path was not immediately known, an important aspect of flow and motivation (Czikszentmihalyi, 2000). They also gained self-efficacy beliefs and beliefs in the value of perseverance (Deci & Ryan, 2008; Middleton & Spanias, 1999). These beliefs are consistent with productive disposition (NRC, 2001). After the MTC, a few teachers no longer felt anxious about mathematics like they did as students. This is consistent with the preservice elementary school teachers in Sloan’s (2010) study, in which the preservice teachers’ mathematics anxiety decreased after they took a methods course with emphases on conceptual understanding, using manipulatives, and small group work.

The process of persevering through a challenging problem, and then experiencing success, was beneficial to teachers’ mathematics self-concept and self-efficacy and helped the teachers to develop growth mindsets about their own learning (Dweck, 2000). Developing perseverance is important for mathematical identity and productive disposition (Cobb et al., 2009). Increased self-efficacy, motivation, and perseverance are connected, with gains in any of these influencing the other two. Teachers in the MTC developed perseverance throughout the week, increasing their motivation and self-confidence, and feeling more capable of solving problems (R. Anderson, 2007). This is consistent with the literature that people with greater self-efficacy are better at overcoming failure (Bandura, 1989) that self-efficacy has been shown to play a role in
mathematics achievement (for example, Pajares & Miller, 1994), and that teachers in MTCs experience an increase in confidence (White & Donaldson, 2011). The successes that teachers experienced helped to keep them motivated to solve new problems throughout the week. Such feelings of competence can enhance intrinsic motivation (Ryan & Deci, 2000), and positive feelings of success in mathematics specifically can lead to confidence doing and teaching mathematics (Sowder, 2007).

**Perceptions of mathematics.** The teachers’ perceptions of mathematics, or beliefs about the nature of mathematics, played a role in their increasing confidence. Teachers’ self-concepts are closely related to their beliefs about what it means to be successful in mathematics (Stipek et al., 2001). If a teacher believes that solving problems quickly indicates that one is good at mathematics, then the teacher will feel good about working through a problem quickly. The teachers’ perceptions were related to their beliefs about mathematics, revealing that they valued (or saw that others valued) speed in doing mathematics.

Many of the teachers seemed to be developing connected values of mathematics (Ernest, 2008). Connected values of mathematics emphasize the relationships and connections in mathematics, and the humanistic, fallibilistic side of mathematics. For example, teachers talked about making or understanding connections between mathematical topics for the first time, mirroring the findings of White and Donaldson (2011). They were amazed or wowed by some of the mathematics in the MTC, seeing it as beautiful or fascinating. Some teachers made connections between mathematics and other content areas. And several enjoyed the MTC problems that emphasized the human
side of mathematics or real-world connections. These problems were motivating to the teachers. Similarly, Boaler and Greeno (2000) found that students felt more positively about mathematics in classrooms that emphasized connected knowing.

The teachers also conveyed more dynamic beliefs rather than static beliefs about mathematics after the MTC (Felbrich et al., 2008). The teachers said that the MTC mathematics was different from the mathematics that they had done in the past—it was not focused on exercises like they typically did in school when they were students. Instead, they realized that mathematics could be ambiguous, that it did not have to lead to a single solution, and that the instructor did not have to know the solution in advance. These realizations reflect dynamic beliefs about mathematics (Felbrich et al., 2008) and are consistent with the findings of Fernandes et al. (2011). The teachers enjoyed experiencing problems that could be solved using multiple solution paths or by finding patterns. They made connections between these new beliefs about the nature of mathematics and what they wanted to do in the classroom, like the teachers in Shubin’s (2006) study. Generally, teachers began to feel more confident presenting problems to students when they did not know the answer or when there was not a single answer.

**Sense of group belonging.** A large part of identity is recognition by others (Gee, 2001). Mathematical identities are partially based on the perception that their peers view them as competent (Keck-Staley, 2010) and on the mathematical and social norms of an environment (Martin, 2000). Teachers’ anxieties and feelings of success in the MTC often related to the teachers’ perceptions of what the others perceived. And a significant and influential part of teachers’ experiences in the MTC was the collaboration with their
peers. This is consistent with other research on MTCs that reported collaboration and community as important to teachers’ experiences (Shubin, 2006; White & Donaldson, 2011; White, Donaldson, Conrey, et al., 2013). Finding their place in the group (their strengths and weaknesses and the others’ strengths and weaknesses) was important to the teachers, and can be important for learning and enculturation (Borko, 2004). Teachers’ personal identities can be influenced by how these identities align with the normative identity in a given environment (Boaler & Greeno, 2000; Cobb et al., 2009).

Some of the teachers’ mathematics self-concepts were based on their comparison to their peers in the MTC (Schunk & Pajares, 2007). The teachers felt confident when they were able to contribute to their group and help others, like the learners in Clark et al. (2013). They also felt anxious that they would make a mistake in front of the others. At the beginning of the week, the teachers compared themselves negatively to the others. Some teachers felt like they were not as quick as the others or wondered if it was because they were older than the others. But as the week went on, they began to feel more confident. Like the teachers in Chapman’s (1999) study, they gained confidence in problem solving as they realized that others felt anxious or uncertain about mathematics. The teachers in Chapman’s study realized that the problem solving process included cycles of success and failure, which helped them to realize that feelings of frustration or failure were natural. In the MTC, the teachers got to know the other teachers and realized that they were not alone in their feelings of confusion or incompetence, which helped to decrease their anxiety and increase their confidence.
The teachers also felt confident and successful when they earned the recognition of their group or the MTC facilitator, or when they answered others’ questions. The teachers were pleased to keep up during group discussions even when they did not solve the problems themselves. By listening to others, they learned about teaching and how different people have different problem-solving strengths, and they also learned about the mathematics by understanding problems through a peer’s explanation. This is consistent with the theory of normative identity (Cobb et al., 2009) in the context of the MTC.

When the teachers were frustrated with problems or got stuck, they coped by collaborating with their group members. Many of the teachers were surprised to realize how much their groups could accomplish by working together. Positive interactions with peers can contribute to strong identity as a mathematics learner (R. Anderson, 2007; Lim, 2008; Martin, 2000). Similarly, experiences working in groups on mathematics problems can lead to decreased mathematics anxiety (Harper & Daane, 1998).

**Summary of teachers whose mathematical identity evolved.** Collaboration, beliefs about themselves, and beliefs about the nature of mathematics were all critical to the evolution of the teachers’ mathematical identities. They typically began the week feeling nervous and comparing themselves to the others in the MTC. The mathematics was new and different from mathematics that they had done in the past. By experiencing mathematics in this new way—approaching problems using their strengths, focusing on problem solving and the process rather than the solution, exploring multiple solution paths, and making connections between areas of math—and through the process of collaboration, the teachers began to experience success. This success fed their confidence
and helped them to develop perseverance and persistence with problem solving. At the same time, collaboration with their peers helped them to realize that their anxieties about problem solving were not unique to them.

For teachers whose mathematical identities did not evolve, their perceptions of themselves or of others’ views of them remained static. When talking about doing mathematics in the MTC, they talked about learning some things about problem solving, such as the importance of persistence or collaboration, but their beliefs about the nature of mathematics did not fundamentally change.

Mathematics teaching identity. The second research question was, “In what ways does participation in a Math Teachers’ Circle affect teachers’ professional identity as a mathematics teacher?” Mathematics teacher professional identity was observed in the ways teachers saw themselves as mathematics teachers, in their enactment of their beliefs related to mathematics teaching, and in their sense of normative identity or alignment of their beliefs with those of others. These facets of identity were explored through three themes: perceived mathematics teaching abilities, enacting new teaching methods, and sense of group belonging.

Perceived mathematics teaching abilities. This facet of mathematics teachers’ professional identity was least influenced by the MTC. Instead, teachers’ perceptions of themselves as mathematics teachers were based primarily on their past experiences. This is consistent with theories of teacher change, in which the process of change involves learning about new practices and implementing those practices (e.g., Kaasila & Lauriala, 2010; Warford, 2011). Because the teachers had not yet had opportunities to test out new
ideas related to teaching, their mathematics teaching identity had not fully integrated these new ideas. Teachers’ perceptions of themselves as teachers were partially based on their experiences as students, which is consistent with Beijaard et al. (2004) and Richardson (1996), in that experiences in school as students are important to teachers’ identity development. Their identity was also based somewhat on their experiences since they began teaching. Teachers’ mathematics teaching self-efficacy, an important aspect of teacher identity (Swaras et al., 2007), typically increased as they gained experience teaching, as their careers evolved, and as they learned new methods of mathematics teaching.

Teachers also tended to see growth and change as naturally occurring during the course of a teaching career, consistent with literature that reports shifts in teachers’ identities over time (Thomas & Beauchamp, 2011). The teachers all wanted to improve and become more effective teachers in the future, consistent with the theory of desired identities (Sfard & Prusak, 2005). The literature on teacher change supports the notion of possible-selves theory for teachers (Hamman et al., 2010) and focusing on the ideal self (Katz et al., 2011) or imagined future self (Helms, 1998) as ways for teachers to envision and enact growth and change.

**Enacting new teaching methods.** Richardson (1996) explains that the relationship between beliefs and action is bidirectional: Beliefs influence action, but events and experiences can also influence beliefs. Luehmann (2007) found that science teachers needed to develop a professional identity as a “reform-minded science teacher” (p. 823) in order to implement reform-oriented changes to their teaching. Similarly, the MTC
experience influenced the teachers’ beliefs about effective teaching methods, and these new beliefs had potential to influence the actions the teachers took in the classroom.

The teachers believed that encouraging student self-efficacy and perseverance were important to mathematics teaching. They also believed in the effectiveness of student-centered methods like discovery, engagement, collaboration, and discourse. However, the teachers’ definitions of these terms varied. Their identities depended on the extent of the alignment between the teachers’ use of these strategies in the classroom and their perception of the strategies used in the MTC. And when confronted with new styles of teaching, teachers may integrate these new ideas into their existing identity—a process of identification—or they may modify their identities based on their experiences—the process of negotiation (Horn et al., 2008).

Some of the teachers did not see many similarities between the MTC and their classrooms. Although they described their classroom as student centered or prioritized active engagement, some of the teachers did not use problem solving or the strategies used by the MTC facilitators. Because they did not use these strategies in the classroom, they closely watched the MTC leaders’ facilitation of discussions. This type of “productive friction”—or experiencing differences in two environments—can be productive for teacher change (Ward et al., 2011).

Not all of the teachers felt such a disconnect between their teaching styles and the MTC. Some of the teachers said that the emphasis on problem solving in the MTC reinforced or reminded them of how they should be teaching mathematics, like the teacher in Fernandes et al.’s study (2011). These teachers’ identities may already be fairly
closely aligned with the types of problems and types of pedagogical strategies used in the MTC, and they may have added these strategies to their teaching identities using identification (Horn et al., 2008). According to Collopy (2003), if a teacher’s closely held beliefs are compatible with the professional development, then the teacher is more likely to learn from the professional development. The teachers’ desires to use more problem solving in their classes, to use pedagogy from the MTC, or to incorporate more student discourse into their teaching align with the findings from other studies of MTCs (Fernandes et al., 2011; Morrison & Karakok, 2015; Shubin, 2006; White & Donaldson, 2011).

The teachers described some of the difficulties they faced when they implemented or planned to implement problem solving in their classrooms, like issues of time, working with district mandates, and balancing problem solving with computation basics. Skott (2001) described how teachers’ priorities in the classroom could influence which beliefs were reflected in their actions. And theories of beliefs support the idea that actions can be counter to beliefs (T. F. Green, 1971; Leatham, 2006; Rokeach, 1968). For example, although teachers may believe that effective mathematics teaching includes particular elements, they may not enact them in the classroom because their conflicting beliefs about classroom management are prioritized at that moment. Some of the teachers in this study explained that they were unsure how to use problem solving or inquiry in the classroom, or that they were not confident in their skills at facilitating group work on an open-ended activity. These teachers’ beliefs about the importance of inquiry and problem solving conflicted with their beliefs about their skills at facilitating such an activity while
maintaining effective classroom management. These teachers may also be at an earlier stage of the change process, in which they are figuring out how to incorporate new pedagogies and beliefs into the classroom (Kaasila & Lauriala, 2010; Warford, 2011).

**Sense of group belonging.** Identity involves recognition by others (Gee, 2001) and contains a professional component alongside the personal component (Beijaard et al., 2004). The professional side of identity (Beijaard et al., 2004), or the normative identity for teaching (Gresalfi & Cobb, 2011), includes being seen as a competent teacher by others. Some of the teachers in this study were concerned about being seen as an effective mathematics teacher.

Teacher change is both individual and collective (Kaasila & Lauriala, 2010). The collective part of change takes place through enculturation or socialization into a community of practice. For many of the teachers in this study, their colleagues had similar views about mathematics teaching. This was important for these teachers to feel part of their peer group, or what could be considered the normative identity for teaching (Cobb et al., 2009). The collective or normative identity for teaching was important for teachers when considering changing the ways that they taught mathematics. Some of the teachers’ colleagues attended the same MTC and planned to implement the same changes. This may have been an important feature of the MTC for these teachers, because in addition to considerations of personal and normative identity, Penlington (2008) explains that teacher-teacher dialogue is important for change.

Another context for normative identity for teachers is the context of their school district. Some of the teachers perceive that their values and beliefs differ from those of
the school administration, and what teachers think others expect is an important part of identity (Helms, 1998). And some teachers are concerned about how to incorporate problem-solving activities within the district requirements or approval. This aligns with Gresalfi and Cobb (2011), in which teachers who began to identify with the normative identity of the professional development group had to reconcile this identity with their district’s normative identity. The teachers’ takeaways from the MTC were influenced by the context of their school and their district: whether their colleagues also attended the MTC and they could co-plan activities, or whether their district supported problem solving and inquiry. This appeared to be easier for teachers whose colleagues attended the MTC.

The teachers in this study seemed to be most influenced by collaboration with their colleagues. This is in contrast to Trent and Lim (2010), who found that teachers’ identities in relation to their school—whether they feel like they have agency or ownership over decisions in the school—can impact how much they buy into or participate in professional development. In this study, it seems that support from the school district was less important than support from fellow teachers. It presented challenges, but the teachers were generally able to work around these issues easier than other issues.

**Summary of teachers whose mathematics teaching identity evolved.** Teachers who were able to incorporate elements of the MTC into their identities by the process of negotiation (Horn et al., 2008) are considered to have changed their mathematics teaching identities the most. The teachers who changed the most had not only changed their beliefs
about effective teaching, but also had descriptive plans for what they intended to change about their teaching. They were also able to think through solutions to roadblocks, like how to adapt new problems to their grade level, or how to align the problems with the school district’s mandated curriculum. Often they had the support of their colleagues, either because the colleagues attended the same MTC, or because they shared ideas and collaborated often. Their plans for their classrooms aligned more closely with the MTC than their own experiences in school. They planned to fully integrate aspects of the MTC problems and pedagogy into their daily teaching. Other teachers incorporated elements of the MTC into their identities via the process of identification, integrating new ideas into their existing identities (Horn et al., 2008).

On the other hand, teachers whose mathematics teaching identities evolved less were only beginning to think about the ideas from the MTC. Some thought about how the facilitators presented problems but did not yet make the connection to their classrooms. Others began to make those connections but foresaw insurmountable challenges with implementing these types of problems in their classes. Some of these teachers made plans to use some of the shorter MTC activities in their teaching, to try one or two of the pedagogical strategies, or to teach problem solving only at the beginning or end of the school year.

Interestingly, teachers who attended MTC Site C all had the clearest ideas of how they would incorporate aspects of the MTC into their teaching. This may have been due to unique features of MTC Site C: It was the longest of the three workshops (comprising five full days plus four evening sessions), and it included extensive time for teachers to
debrief and collaborate on teaching plans for the upcoming year. Some of the sessions, rather than focusing on doing mathematics, were focused on pedagogy.

In general, however, teachers’ mathematics teaching identities did not evolve as much as their mathematical identities. Kaasila and Lauriala’s (2010) five-step process of change may help to explain this discrepancy. The change process consists of problematizing current practice, learning about new approaches, testing out the new approaches, analyzing the benefits of the new approaches, and finally changing their view of their teaching practices. Considering this process of teacher change, the teachers in this study were at the first, second, or third stage. Some of the teachers had already tested out new practices and beliefs from the MTC, but others were still at the earlier stages of problematizing their current practices or learning about alternative practices. This type of change takes longer and involves cycles of new beliefs, incorporating the beliefs into action, and then solidifying these new beliefs based on those experiences and putting them into action.

Further, the teachers’ methods appeared to have begun to change, but their perceptions of themselves had not yet evolved. This may be explained by Warford’s (2011) zones of proximal teacher development, in which teachers first engage in reflection, then experience teaching with the new ideas, internalize the new pedagogies, and then finally analyze the difference between the new pedagogies and the teachers’ earlier practice. It is likely that the teachers were at the early part of this cycle, engaging in reflection, as they had not yet had time to try out the new ideas in the classroom. Their beliefs had changed, but their actions had not yet changed. So rather than changing their
perceptions of themselves as teachers, the teachers’ desired future identities shifted. Rather than seeing themselves as “a mathematics teacher who is skilled at using inquiry,” they instead saw themselves as “a somewhat skilled mathematics teacher who wants to improve in their use of inquiry.” Warford’s theory also helps to explain why some of these changes in beliefs take longer to manifest in teachers’ practice.

**Interaction of mathematics and mathematics teaching identities.** The third research question was, “In what ways does participation in a Math Teachers’ Circle affect the interaction between teachers’ mathematical identities and their professional identities?” The data suggested that teachers’ experiences doing mathematics influenced their mathematics teaching identities, and teachers’ experiences teaching mathematics influenced their mathematical identities. In fact, it was difficult to tease apart the two identities. Two themes emerged in the interaction of the two identities: teachers equated doing and teaching mathematics, and teachers made connections between their experience doing mathematics and their teaching actions.

**Conflation of doing and teaching mathematics.** Teachers’ mathematical identities were influenced by their experiences teaching. When asked about their mathematics autobiography, or instances that they enjoyed or were anxious doing mathematics, the teachers often included examples from their teaching. For these teachers, teaching mathematics included the act of doing mathematics, and their experiences teaching it impacted their perceptions of and attitudes toward mathematics itself. Beijaard et al. (2004) explain that when people’s different identities are closely aligned, their actions are similar in different contexts.
For some teachers, doing mathematics while teaching, and doing mathematics in the real world were the same thing: The instances that they described as doing mathematics in their lives were instances in which they were teaching. These teachers were doing mathematics on a daily basis, and although the mathematics was at an elementary or middle school level, the teachers had to engage with it deeply. They needed to understand student misconceptions and think about ways to present new topics to students so that students could make connections. This type of mathematical thinking can be very sophisticated, and it is typically what is included in mathematical knowledge for teaching or pedagogical content knowledge—the specific mathematics knowledge that is essential to and unique to the teaching profession (e.g., Ball & Bass, 2000). The way that the teachers talked about doing mathematics while teaching, however, provides a nuanced perspective of how teachers perceived and were influenced by these actions and this knowledge. This mathematics that they did while teaching—pedagogical content knowledge—both influenced how they felt about mathematics in general and shed new light on their experiences as students, similar to findings related to science teacher content knowledge and identity (Helms, 1998; Lee & Luft, 2008).

After the MTC, it seemed that teachers were somewhat more able to separate the actions of doing and teaching mathematics. For example, when asked about when they enjoy or are anxious doing mathematics, teachers gave specific examples of problems outside of the teaching context. Five of the teachers talked about the mathematics that they did while teaching in the pre-immersion survey, but afterwards, none of the eight mentioned teaching contexts.
**Connections between mathematics experiences and classroom actions.** This theme is the converse of the previous theme, in that teachers’ mathematics teaching identities were influenced by their experiences learning and doing mathematics. Being a learner and feeling like a learner are important parts of teacher professional identity (J. L. Cohen, 2010; Collopy, 2003) and were important outcomes of other MTC research (Fernandes et al., 2011; Shubin, 2006; White, Donaldson, Conrey, et al., 2013).

The teachers found it valuable to feel like a learner again, to experience higher-level mathematics, and to feel empathy with their students. Helms’s (1998) study of science teachers suggested that teachers feel a personal connection to their content knowledge, and that their understanding of and personal identification with this content influences their sense of self as teachers. Similarly, the pedagogical content knowledge developed by teachers is personalized and can change over time (Lee & Luft, 2008), and mathematical identities can change and evolve as a result of further experiences with mathematics (Gresalfi & Cobb, 2011). Theories of identity support the idea that experiences are critical to beliefs (Ernest, 1989; Gresalfi & Cobb, 2011). And other studies of MTCs found that teachers’ changes in attitudes changed their teaching (White & Donaldson, 2011).

The teachers in this study generally seemed to be able to describe aspects of what they learned and experienced that they intended to take back to their classrooms. The teachers felt that experiencing the problems and pedagogies helped them to become more confident in trying problem solving or group work in their classrooms. This is consistent with the findings of other research (Szarka et al., 2012; White, Donaldson, Conrey, et al.,
Some teachers described the specific problems from the MTC that they intended to use in their classes, similar to teachers in other studies of MTCs (Fernandes et al., 2011; Morrison & Karakok, 2015; Shubin, 2006). This may contradict Van Zoest and Bohl’s (2005) theories of identity that describe how teachers may learn in one setting, but their actions in another setting fail to reflect what they learned. According to Van Zoest and Bohl, teachers have difficulty applying the knowledge in a new setting. The teachers in this study, however, have not been observed or asked what they implemented; so perhaps their intentions did not align with their practice.

**Summary of interaction of identities.** Teachers whose mathematical and mathematics teaching identities were most closely intertwined should act fairly consistently in different settings. According to Beijaard et al. (2004), people take on different subidentities in different situations. When their identities are more closely aligned, their actions in different settings will be similar.

In some respects, the MTC helped the teachers to see the distinction between doing and teaching mathematics. It gave them experiences doing mathematics outside of the teaching context, which many teachers had not done for many years. But on the other hand, the MTC also helped the teachers make connections between their experiences doing and teaching mathematics. The teachers felt like their students and felt empathetic with their students.

The teachers’ identities also changed in tandem. Some teachers had positive experiences doing mathematics that changed their mathematics teaching identities. And some teachers had positive teaching experiences that helped to change their mathematical
identities. Teachers’ anxieties in one context influenced their anxieties in the other, and their positive feelings and perceptions in one similarly influenced the other.
Chapter 6: Discussion and Implications

This study explored elementary and middle school mathematics teachers’ identities as they participated in a mathematics-focused professional development experience. Because one of the goals of these professional development workshops is to “encourage teachers as mathematicians” (MTC Network, 2015, para. 1), it is important to understand the ways in which teachers’ mathematical identities develop. Because the goal of professional development more generally is to impact teachers’ practice, it is important to understand the impacts that MTCs have on teachers’ professional identities as mathematics teachers. And because identities do not exist in isolation (Beijaard et al., 2004), it is important to understand the ways in which teachers’ mathematics teaching identities and mathematical identities interact.

Implications for Practice

This work has important implications for providers of professional development, particularly those who lead MTCs or similar mathematics-focused workshops. With professional development a universally accepted part of the teaching profession, and with many states adopting professional learning standards for teachers, it is important to consider lasting and quality changes from professional development. This study suggests that teachers’ identities are valuable for understanding what teachers “get” out of professional development.

In particular, professional development providers could consider the following recommendations.
Incorporate pedagogical sessions as well as content-focused sessions. The teachers in this study learned from both types of sessions. The content-focused sessions were important for their mathematical identities, and the pedagogical sessions were important for their mathematics teaching identities. The teachers (Beverly, Deborah, and Jeff) who attended MTC Site C, which included specific sessions dedicated to pedagogy, were better able to describe the ways that they planned to use the MTC content and pedagogy in their classrooms. Both types of sessions are important for teachers to gain knowledge and change practice (Van Zoest & Bohl, 2005), as evidenced by Beverly’s, Deborah’s, and Jeff’s plans for changing their teaching practice.

Teacher learning influences the development of teacher identity (Collopy, 2003), so it is important that professional development leverage the learning that takes place. Content-focused professional development sessions that don’t include elements of mathematical content knowledge could include ten minutes of debriefing at the conclusion of content-focused activities, as is common in MTC sessions. This debriefing can focus on instructor moves, or strategies used by the instructors. Teachers could be asked to reflect on how they might use these moves or strategies in their own classes. And longer, multi-day workshops could spend a half-day on the teaching problem solving, like MTC Site C. The workshop facilitators could share strategies and techniques for engaging all learners in problem-solving activity. For example, the most commonly cited challenge of incorporating MTC problems into the classroom (by Anne, Beverly, Erica, and Faye) was the lack of time. Professional development providers could spend a session working with teachers on how to structure problem solving activities over
multiple days and class periods, or how to use these problems to supplement the regular curricula.

Most teacher professional development, however, is focused on pedagogy or on pedagogical content knowledge. This study and others (e.g., Hart, 2002; Holm & Kajander, 2012; Isiksal, 2005; Philippou & Christou, 1998) suggest that content-focused professional development can have a significant impact on teachers. This is consistent with recommendations for professional development from Arends and Kilcher (2010), Desimone (2009), Guskey (2000), and others. Teachers such as Beverly and Deborah began to see themselves in relation to mathematics in a new way—seeing themselves as competent, capable doers of mathematics for the first time. Professional development that focuses on pedagogy can incorporate some content into the sessions, particularly content that the teachers can engage with as mathematics learners. This experience is valuable for teachers to feel empathy with their students and to experience teaching strategies from the perspective of learner. It is also valuable for the development of mathematical identity (Browne, 2009; Gresalfi & Cobb, 2011), which can shift based on new experiences with mathematics, as evidenced by Beverly’s and Deborah’s experiences.

Engage the teachers as learners. The teachers found it valuable to experience perplexity and feelings of frustration and success. The mathematics content should be above the level that they teach so that the teachers can experience being mathematics learners again. Teachers found it valuable to feel like their students and experience the things they ask their students to do. This is influential for the teachers’ beliefs about themselves, beliefs toward mathematics, and building connections between the
professional development and their teaching. Being a learner is an important part of teacher professional identity (J. L. Cohen, 2010) and was part of Beverly’s, Faye’s, and Kathy’s professional identities.

Professional development that seeks to influence teachers’ teaching strategies should include content for the teachers to engage with as learners. This content should be novel to the teachers in order for them to experience nonroutine problems. This does not necessarily mean that the content is advanced, but instead, it could be deep. Engaging with elementary or middle school content at a deep level can be impactful for teachers, as they gain experience as learners in addition to deeper knowledge of the content they teach. Jeff realized this when he said the mathematics was challenging even though it was not covering advanced topics.

Professional development should also include processes that teachers can engage with as learners. If professional development aims to encourage teachers’ use of problem solving in their teaching, the teachers should have opportunities to engage in problem solving processes during the professional development. Doerr et al. (2010) found that engaging in problem solving during professional development is associated with increased teacher learning. One facet of mathematical identity, according to R. Anderson (2007), is engagement, or the ways in which the learner experiences mathematics. Erica described the experience of problem solving and struggling, and she said that it will help her to be a better teacher. Teachers who engage in rich problem solving tasks will begin to see themselves as good at mathematics in those situations (Muis, 2004) and will develop greater agency and self-confidence in mathematics, as did the students in Boaler
and Greeno’s (2000) study of classrooms emphasizing connected knowing, and as suggested by Deborah’s and Faye’s increased confidence throughout the week.

**Hold intensive immersion-style professional development.** Identity takes time to evolve. The weeklong MTCs immersed the teachers in problem solving, which was beneficial to their learning. It took time for teachers to begin to feel comfortable with problem solving and confident in their skills. Professional development that takes place over a single session or a single day does not allow enough time for teachers to fully engage in the activities and begin to internalize the teaching methods.

Professional development that takes place over the course of a full week (or longer) can be structured to help the teachers feel like part of a community and feel comfortable taking risks. Lengthy, in-depth programs also facilitate the development of normative identity with a group (Cobb et al., 2009). Repeated experiences with nonroutine problems and problem solving are valuable for teachers to begin to feel confident and successful, as experienced by Deborah and Faye. And the experience of success is important for developing intrinsic motivation and enjoyment of doing mathematics (Middleton & Spanias, 1999), which are important aspects of productive disposition (NRC, 2001).

**Hold follow-up sessions throughout the school year.** The teachers in this study had begun to make significant changes to their beliefs and identities. Holding regular MTC meetings would be beneficial for encouraging the teachers to continue their growth and would provide support for teachers with questions about implementing MTC problems or pedagogy into their classes. Many MTCs plan monthly or bi-monthly
problem solving sessions throughout the school year, but not all do. Some of the teachers in the study, like Carol, left the MTC without solid plans or ways to incorporate what they learned into the classroom, and follow-up would be useful to them. It is widely agreed that professional development needs to be ongoing to be most effective (e.g., Desimone, 2009; Guskey, 2000; Hattie, 2009).

Professional development that takes place over the summer should also plan follow-up sessions throughout the school year. These sessions would help to refresh the teachers’ understanding of teaching problem solving and would help to support the continued evolution of their beliefs and identities. These sessions could also include time for teachers to share the ways that they are incorporating new ideas into their classes and to share the challenges they have faced. This can be an opportunity for teachers to support and learn from one another, and it can help to encourage classroom implementation. Gresalfi and Cobb (2011) found that teachers’ personal identities evolved as they began to reconcile the normative identity of the professional development with the normative identity of their schools. Similarly, some teachers in the MTC, like Anne and Beverly, identified differences between the MTC and their classroom or school culture. These teachers may experience growth as they juxtapose the normative identities of the MTC and their schools, or experience productive friction between their identities (Ward et al, 2011). Faye may have been experiencing productive friction when she talked about the difference between her teaching and the facilitator’s. Repeated MTC sessions throughout the school year could facilitate that growth.
Build in support and instructional differentiation. The teachers in the study differed in terms of what problems and situations made them frustrated or anxious during the MTC. Some teachers, like Carol and Faye, were frustrated by open-ended problems or by mental math (like Beverly), whereas others enjoyed being challenged in a new way (Deborah and Jeff) or feeling successful in a familiar context (Erica). Professional development providers should consider these differences when selecting multiple types of problems and building in layers of scaffolding and support for the teachers. Adequate support is important for teachers to experience productive (rather than unproductive) friction between their identities in different settings (Ward et al., 2011). Thus, as teachers in an MTC experience mathematics in a new way, adequate support can help them to productively manage the conflict between the MTC context and their teaching context.

In addition to selecting a variety of problem types, professional development providers can sequence activities appropriately, so that teachers build their confidence over the week. Adequate support is important for building dispositional attribution, or attributing success to one’s effort or ability (Middleton & Spanias, 1999; Petty & Cacioppo, 1996; Weiner, 1972). Problems should allow for a variety of entry points and allow teachers to progress toward a solution on a variety of solution paths. Facilitators can provide adequate support by monitoring for teachers or groups who are stumped and providing hints or tips to get them moving again, as described by Anne. Facilitators can also normalize the feelings of confusion or feeling stumped. For example, they could explain that this is a normal part of the mathematical problem solving process, or their actions and responses to teachers who are stuck can celebrate the process of productive
struggle. The process of productive struggle, if ultimately successful, can help learners develop incremental theories about learning (Dweck, 2000). In this study, Anne and Faye felt more confident when they realized that others had similar feelings of inadequacy or struggle.

Collaboration with peers can also be useful for teachers to feel supported and successful. Having teachers work on problems in pairs or groups of three or more can provide teachers with support and fresh ideas as they work through the problems. Anne and Faye both described situations in which collaboration was essential to their problem solving.

**Engage teachers in collaborative groupings.** Collaborative groupings were beneficial to the teachers for numerous reasons. The teachers (particularly Deborah and Jeff) appreciated the time to talk to their peers about teaching, which is part of developing professional or normative identities for teaching (Beijaard et al., 2004). They learned about teaching, as they observed different learning styles and strengths that others brought to the problems. And they were able to reach success on challenging problems from using the skills of all of the group members. The teachers, particularly Faye and Beverly, also felt successful when they were able to help others and when others were able to help them extend their understanding. Being seen as a competent doer of mathematics by peers is an important part of mathematical identity (Keck-Staley, 2010; Martin, 2000), which was experienced by these teachers.

Professional development should engage teachers in collaborative talk, both about teaching strategies and during the act of problem solving. This is an important part of
Effective professional development (Borko, 2004; Desimone, 2009; Hattie, 2009) and was valuable to teachers like Erica. Teachers learn a lot about how to present and facilitate problems when they work collaboratively. Facilitators of professional development should also monitor the groups’ productive struggle to ensure that the experience of the group is not soured by one teacher’s negative experience with frustration.

**Consider identity as an important outcome.** Changes to beliefs and attitudes are important for teachers, but in order to change their actions, their identity needs to evolve to incorporate those new beliefs (Chapman, 2002; Desimone, 2009). Professional development should consider various factors of identity and multiple conflicting identities.

Normative identity can be facilitated by actively creating a community of practice (Cobb et al., 2009), such as a PLC (Arends & Kilcher, 2010). This helps teachers to see themselves as part of the group and to align (or see the differences between) their personal identity and the normative identity. Like the teachers in Gresalfi and Cobb (2011), Deborah and Faye shifted identities as they began to identify with the normative identity of the MTC.

The process of change takes time and several steps, including processes of learning about new teaching strategies, modifying their beliefs, trying out the new strategies, and cementing those beliefs (Kaasila & Lauriala, 2010). In order to be effective in the long term, professional development should consider incorporating elements that ultimately help teachers’ identities to evolve, including intensive
immersion-style professional development, follow-up sessions over the school year, and incorporating collaboration and support into the sessions. It is important for teachers to experience discord between their identities and new ideas in order to evolve (Ketelaar et al., 2012).

Providers should be sensitive to these potential effects of professional development. Because there is a close relationship between identity and teachers’ actions in the classroom (Gresalfi & Cobb, 2011), identity change can be a desired outcome of professional development. Understanding that good professional development can involve identity changes, providers should endeavor to create opportunities for teachers to experience identity growth.

**Recommendations for Further Research**

This study was an attempt to add to the scant literature related to teacher mathematics and mathematics teaching identities, and MTC professional development workshops. However, future research is needed in order to more fully understand both areas and to understand whether the MTC model is unique in its influence on teacher identity, or if aspects can be replicated in other professional development programs.

**Teacher mathematical identity.** The knowledge about teacher identity is still underdeveloped, particularly about teachers’ mathematical identities and mathematics teaching identities. Teachers hold multiple interacting identities, particularly teachers who are generalists and teach multiple content areas. These identities, and the interactions of them, are valuable areas of study. Further, identity takes time to develop and evolve. Longer studies are needed to understand the ways in which teachers’ identities develop
over time. More studies are needed in order to fully understand the identities held by teachers of mathematics at all levels.

For example, how do teachers’ mathematical identities evolve over their careers? How do high school teachers’ mathematical identities differ from elementary school teachers’ mathematical identities? And how do high school teachers’ mathematical identities interact with their mathematics teaching identities?

**Professional development and identity.** Professional development can have a profound impact on teacher identity. However, work thus far has not adequately considered the role of identity in teacher professional development. Future studies can explore teacher identity in relation to other types of professional development. What types of experiences and professional development are influential in changes to teachers’ mathematical identities? How do teachers’ mathematical identities influence their mathematics teaching identities, and vice versa?

MTCs are a unique form of professional development. For example, in all of the sites identified for this study, teachers opted to participate voluntarily. These teachers who self-selected to attend an MTC over the summer may be unique or already working on aspects of their identities. Often, teachers are mandated to attend professional development. Because the literature suggests that teachers’ experiences of mandated professional development are different based on the context of their school district (Trent & Lim, 2010), it is important to understand teachers’ identity development as a result of mandated professional development. What does identity development look like in mandated professional development?
MTCs and teacher outcomes. Future studies of MTCs could also focus on the environments that facilitate positive outcomes by teachers. This study only examined teacher identity, but future studies could explore other effects of this professional development, such as teacher actions in the classroom after MTC participation. Although self-reports are valuable for a study of identity, other methods are useful to understand actions. In what ways do teachers use what they gained from the MTC in their classroom? How do teachers’ classrooms change after their MTC participation? How does long-term MTC participation affect teachers’ identities, beliefs, and teaching actions?

This study focused on a particular population and extreme cases, but this only provides one level of the change that can occur during participation in an MTC. It is important for future research to explore what is typical or what is common by surveying larger quantities and a cross-section of MTC participants. For example, what are the effects of MTCs on high school teachers? What are the experiences of teachers who attend MTCs for multiple years?

MTCs also differ from site to site, so it would be interesting to compare teacher and facilitator outcomes from different sites. What MTC features are most impactful to teachers’ identities? What MTC features are most impactful to teachers’ practice? Is there a difference in teacher outcomes based on the length of the MTC or the number of follow-up sessions? Future studies could examine MTC sites that do not hold summer immersions and only meet during the school year, MTC sites that hold shorter summer immersions, or MTC sites that have been in existence for multiple years.
**MTCs and facilitator outcomes.** This study only explored outcomes for teacher participants of MTCs. However, this left out the facilitators: some of whom are teachers and some of whom are mathematics educators, university faculty, or mathematicians. What is the experience for teacher facilitators of MTCs? Does facilitating an MTC have an effect on their classroom teaching?

MTCs are meant to be a collaborative community of mathematics teachers and mathematics professors. Two of the three goals of MTCs involve mathematics professors: “connect mathematics professors with K–12 education” and “build a K–20 community of mathematics professionals.” In what ways do the identities of mathematics professors (and other facilitators) evolve after participating in an MTC? In some MTCs, mathematics professors serve as facilitators, and the impacts on the facilitators would be important to study. In other MTCs, mathematics professors attend MTC sessions on the same level as the teachers. In these cases, mathematics professors are full MTC participants. What is the experience of professors attending MTCs, and in what ways do their identities evolve as a result of participation? How does this compare to teacher participants’ changes in identity?

**Conclusion**

Teachers hold multiple identities, particularly regarding the content that they teach, and those identities are tightly entwined and difficult to separate. The experiences, beliefs, and expectations related to doing mathematics have an influence on teachers’ experiences, beliefs, and expectations related to teaching mathematics, and vice versa.
The process of development and evolution of one identity is influenced by other identities, sometimes simultaneously.

Ultimately, teachers’ identities influence their beliefs and actions. Their perceptions of mathematics and mathematics teaching are carried out in their teaching actions, and the priorities and beliefs that they portray to students. If we want students to engage in critical thinking, develop problem-solving skills, and develop productive disposition, then mathematics teachers must engage in these activities themselves.
References


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## Appendix A: Interview Participant Demographics

<table>
<thead>
<tr>
<th></th>
<th>Selected interview participants</th>
<th>All survey participants</th>
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<tbody>
<tr>
<td></td>
<td>((N = 8))</td>
<td>((N = 24))</td>
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<tr>
<td>Gender</td>
<td>7 female, 1 male</td>
<td>22 female, 2 male</td>
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<tr>
<td>Age</td>
<td>44 (mean)</td>
<td>42 (mean)</td>
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<td>Ethnicity</td>
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<td>1 Black, 23 White/Caucasian</td>
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<td></td>
<td>High: 0</td>
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<td>Years teaching math</td>
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<td>Mathematics education: 1</td>
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<td>Music education: 1</td>
<td>Middle childhood education: 7</td>
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<td>(\text{(includes 2 with concentrations in mathematics)})</td>
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</tr>
<tr>
<td>Past participation in mathematics PD</td>
<td>Yes: 7</td>
<td>Yes: 19</td>
</tr>
<tr>
<td></td>
<td>No: 1</td>
<td>No: 6</td>
</tr>
</tbody>
</table>
Appendix B: Pre- and Post-Immersion Surveys

1. Pre-immersion survey
This was administered electronically using Google Forms. In addition to collecting some demographic information, the survey contained both reflection questions and Likert-scale items. These questions and items were drawn from existing measures related to mathematical identity and beliefs. Some questions asked teachers to expand upon their responses to the Likert-scale items.

Directions:
After completing this survey and the post-immersion survey, you will be automatically entered into a raffle for one of two $25 Amazon gift cards.

- Name:
- Preferred e-mail address (to receive the follow-up survey):
- Name and/or location of the Math Teachers’ Circle you will be attending:
- Have you ever attended a Math Teachers’ Circle session?
  o Yes, attended a single session (e.g., Saturday morning event)
  o Yes, attended multiple sessions (e.g., one weeknight each month throughout the year)
  o Yes, attended a weeklong summer immersion
  o No
  o Unsure
- Why did you choose to sign up for the Math Teachers’ Circle? (multiple answer)
  o I was told by an administrator that I had to attend
  o I was asked to attend by an administrator but it was not required
  o My colleagues were attending so I signed up with them
  o I heard about Math Teachers’ Circles from someone else
  o I wanted to obtain professional development for my license or LPDC
  o It sounded like something I would find interesting
  o Other (please describe)
- What do you hope to get out of the Math Teachers’ Circle Summer Immersion?
  o In terms of your professional growth:
  o For your classroom:
  o For your school:
  o Other:

I am interested in your thoughts about the field of mathematics, teaching mathematics, and learning mathematics. Read each item carefully and select the response that best describes your beliefs about each item. There are no right or wrong answers.

Scale: 1=Strongly Disagree  4=Slightly Agree
2=Disagree       5=Agree
3=Slightly Disagree  6=Strongly Agree
Learning Math: RQ3
- Some people are good at mathematics and some aren’t.
- In mathematics something is either right or it's wrong.
- In mathematics, you can be creative and discover things on your own.
- Knowing why an answer is correct in mathematics is as important as getting a correct answer.

What does it mean to be “good at math”?

You and Learning Math: RQ1
- I don't enjoy doing math.
- When my answer to a math problem doesn't match someone else's, I usually assume that my answer is wrong.
- I’m pretty good at math and I enjoy the challenge of it.
- Mathematics makes me feel anxious and nervous.

Have you ever, or do you ever, feel anxious about math problems? What types of problems make you feel anxious? Think about math problems you encounter while teaching, as well as math problems you encounter in your life or your own education.

Do you ever feel excited to do a math problem? When, in what situations, or what types of problems?

Teaching Math: RQ2
- Good math teachers show you the exact way to answer the math questions you'll be tested on.
- It is important for students to master the basic computational skills before studying problem solving.
- An important aspect of mathematics teaching is engaging children in interesting mathematical investigations.

What does it look like when mathematical learning is taking place in a classroom?

You and Teaching Math: RQ3
- Math is my favorite subject to teach.
- I'm good at communicating math material to students.
- I feel confident that I understand the math material I teach.
- I am comfortable with students suggesting solutions to math problems that I had not thought of.

What led you to teach math?

Have your views about teaching mathematics changed substantially since you started teaching? Explain how your views have changed and what influenced the change.

Are you currently rethinking your ideas about teaching mathematics? Explain.

Is there anything else you want to say about effective mathematics teaching and/or learning? Feel free to mention any of the statements above or elaborate on your answers.
Mathematics Autobiography: RQ1, RQ2, RQ3

• Tell me about three pivotal or memorable moments in your “mathematics autobiography.” Please think about your past experiences in math that affected you the most, both positively and negatively, from the earliest you can remember, through school and non-school activities, in college, and since college. Think about the experiences that shaped the mathematics learner and mathematics teacher you became. Please use specific examples—such as people, places, events, or activities, and be honest about how you felt at critical moments. What was your mathematical journey like? Choose the three most pivotal or memorable moments to describe here.

2. Post-immersion survey

This was administered electronically using Google forms. This contained questions similar to the questions in the pre-workshop survey, including open-ended reflection items and Likert-scale items, but it also included demographic questions. Some questions directly recalled the workshop activities.

Directions:
After completing this survey, you will be automatically entered into a raffle for one of two $25 Amazon gift cards.

• Name:
• How likely are you to attend a Math Teachers’ Circle activity in the future?
  o Monthly meetings
    ▪ Very likely, somewhat likely, not very likely, not at all likely
  o The weeklong immersion next summer
    ▪ Very likely, somewhat likely, not very likely, not at all likely
• Think about the pivotal moments of your mathematics autobiography that you wrote about in the survey before the immersion week. (RQ1)
  o Write a new paragraph in your mathematics autobiography about the Math Teachers’ Circle immersion workshop. Use specific examples of events or activities and how you felt at those moments.
  o Do you think anything that you experienced during the immersion week explains something from your mathematics autobiography in a different light? If so, explain.
• If anything, what do you plan to change about your math instruction this year, and why? (RQ2)

I am interested in your thoughts about the field of mathematics, teaching mathematics, and learning mathematics. Read each item carefully and select the response that best describes your beliefs about each item. There are no right or wrong answers.
Scale: 1=Strongly Disagree   4=Slightly Agree
      2=Disagree            5=Agree
      3=Slightly Disagree   6=Strongly Agree

Learning Math: RQ3
• Some people are good at mathematics and some aren’t.
• In mathematics something is either right or it's wrong.
• In mathematics, you can be creative and discover things on your own.
• Knowing why an answer is correct in mathematics is as important as getting a correct answer.
➢ What does it mean to be “good at math”?

You and Learning Math: RQ1
• I don't enjoy doing math.
• When my answer to a math problem doesn't match someone else's, I usually assume that my answer is wrong.
• I’m pretty good at math and I enjoy the challenge of it.
• Mathematics makes me feel anxious and nervous.
➢ Have you ever, or do you ever, feel anxious about math problems? What types of problems make you feel anxious? Think about math problems you encounter while teaching, as well as math problems you encounter in your life or your own education.
➢ Do you ever feel excited to do a math problem? When, in what situations, or what types of problems?

Teaching Math: RQ2
• Good math teachers show you the exact way to answer the math questions you'll be tested on.
• It is important for students to master the basic computational skills before studying problem solving.
• An important aspect of mathematics teaching is engaging children in interesting mathematical investigations.
➢ What does it look like when mathematical learning is taking place in a classroom?

You and Teaching Math: RQ3
• Math is my favorite subject to teach.
• I'm good at communicating math material to students.
• I feel confident that I understand the math material I teach.
• I am comfortable with students suggesting solutions to math problems that I had not thought of.
➢ What led you to teach math?
➢ Have your views about teaching mathematics changed substantially since you started teaching? Explain how your views have changed and what influenced the change.
➢ Are you currently rethinking your ideas about teaching mathematics? Explain.
Is there anything else you want to say about effective mathematics teaching and/or learning? Feel free to mention any of the statements above or elaborate on your answers.

Demographics
1) Gender:
2) Age:
3) Ethnicity/Race:
4) Other markers of diversity that are important to you:
5) What was your undergraduate degree(s)? Major(s)? Minor(s)?
6) Do you have any graduate degree(s), such as a Master’s degree? If so, please list the degree(s) and field(s) of study.
7) What type of license(s), certification(s), and/or endorsement(s) do you currently possess? Include grade level and content area, if applicable.
8) What grade level(s) and subject(s) do you currently teach?
9) Are there other grade level(s) or subject(s) that you taught in the past but do not teach now?
10) How many years have you taught mathematics?
11) Describe the demographics of the school in which you teach. Would you describe it as urban, suburban, rural, high need, etc.? What is the name of the school where you teach?
12) Other than the Math Teachers’ Circle summer immersion, have you ever participated in other mathematics-focused professional development?
   a. Yes
   b. No
   If yes, what is the amount of time spent in the longest professional development you attended?
Appendix C: Workshop Demographics Survey

These questions were sent to the facilitators/leaders of the Math Teachers Circles under study.

1. What is the location of the immersion workshop (e.g., school classroom, hotel conference room, other community space)?
2. What are the benefits or compensation provided to participants (e.g., stipend, course credit, teaching kits, “swag”)?
3. Are meals or drinks (e.g., coffee, lunch) provided to participants?
4. Does the agenda build in time for networking or social time?
5. Are your participants all from the same school/district or different schools/districts?
6. Approximately what proportion of time did participants spend working alone, working in pairs or small groups, and listening to a facilitator?
7. Did you hold debriefing sessions after each content session? Briefly describe the structure of the debriefing sessions.
8. Who were the facilitators (e.g., math professors, math education professors, teachers)?
   a. If teachers facilitated content sessions or debriefing sessions, please explain how many sessions were facilitated, the type, and whether they were members of the leadership team or general participants.
9. Is there anything else unique or interesting about your workshop?
10. Are you using any of “standard” or “canon” MTC activities? Please list these activities.
11. Please provide an agenda for the workshop, and, if possible, any workshop materials or handouts used (or a link to a website where these will be posted)
Appendix D: Interview Protocol

Semi-structured, open-ended interviews. These questions followed up on the pre- and post-immersion surveys and loosely followed existing interview protocols for tracking changes in teacher knowledge, beliefs, and identity.

1. How long have you been teaching? Have you always taught math?

2. What made you decide to become a teacher, and a math teacher in particular? (RQ2)
   P: Did you decide to become a teacher first or a math teacher first?
   P: At what point did you start to see yourself as a math teacher?

2.A How have you grown as a teacher? (RQ2)

2.B How would you describe yourself as the teacher you are, and the teacher you want to be? (RQ2)
   P: What was it like for you to teach math for the first time? How is it going for you now?

2C. How do you think this description of yourself relates to your experiences in the Math Teachers’ Circle? (RQ3)

3. You teach [elementary/middle/high] school, is that right? When you think back to your own experience in elementary/middle/high school math class, what stands out to you? (RQ1)

3.A. Reflecting on what you just said, compare your experience to the way that you teach now. How it is similar or different? (RQ3)

4. Overall, would you say that you enjoy doing math? (as opposed to teaching it) (RQ1)
   P: In what settings or instances do you do math in your life? [What does it mean to “do math” to you?]
   P: Are there some parts of doing mathematics that you especially like or enjoy? Are there some parts of doing mathematics that you especially dislike? [What is it about … that you like or dislike?]

4A. Still thinking about yourself as a doer of math, do you consider yourself to be good at math or not good at math? Why? (RQ1)
   P: What does it look like to be “good at math”?

4B. Thinking about the days that you taught a really successful math lesson- what do those days look like? (RQ2)
5C. Thinking about the culture of your school and your administration, what do you think they see as qualities of an effective math lesson? Are they similar to your beliefs? (RQ2)

6. Tell me about your experience with the Math Teachers’ Circle workshop. What was it like for you? (RQ3)
   P: Did you learn anything new from the workshop?
   P: Would you say that your perceptions of math changed due to your MTC experience?

6A. Let’s talk explicitly about your experience in the MTC related to you as a doer of math. How did your experience align, or not, with your perceptions of yourself as a doer of mathematics? (RQ1)
   P: Give me an example of a problem that really engaged you. What did it look like when you felt like you were really actively involved in doing the math during the workshop? [What does it mean to you to “do math”?]?
   P: Was the math in the workshop different from the math that you’ve done before, in school or in your life? [In what ways did it differ? In what ways was it the same?]

6B. Now let’s do the same thing related to yourself as a math teacher. How did your experiences align, or not, to how you think of yourself as a math teacher? (RQ2)
   P: Was the math in the workshop different from the math that you teach? [In what ways did it differ? In what ways was it the same?]
   P: What about the structure of the activities or the pedagogy? If someone flew a drone into your classroom to take video of your class, and then flew a drone into the Math Teachers’ Circle to take video there, what differences would be seen?

6C. How do you think your experiences as a math doer and your role as a math teacher are aligned? Are there any disconnects? (RQ3)
   P: Reflecting on the Math Teachers’ Circle and thinking ahead to next school year, are there any new things you want to try in your classroom this year, or any aspects of the MTC that you see influencing your practice?
   P: Has the MTC changed your ideas about what a successful math lesson looks like? [What happened during the Math Teachers’ Circle that made you think that?]

7. I noticed that before the workshop you said/rated ….. but afterwards your response shifted to …… Can you tell me more about that? (RQ1, RQ2, RQ3)
   P: [Additional questions that follow up on interesting answers in the pre and post surveys. Focus on items in which the responses contradict one another, or change from pre to post. Ask for elaboration on open-ended items.]
### Appendix E: Themes, Codes, and Subcodes

#### Theme 1: Beliefs about the self in relation to mathematics

<table>
<thead>
<tr>
<th>Code</th>
<th>A Priori or Emergent</th>
<th>Subcode</th>
<th>Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Being successful in mathematics growing up</td>
<td>A priori</td>
<td>Disliked doing mathematics (a priori), stopped taking mathematics (a priori)</td>
<td>3</td>
</tr>
<tr>
<td>Struggling with mathematics growing up</td>
<td>Emergent</td>
<td>Good at doing mathematics (a priori), bad at doing mathematics (a priori), average at doing mathematics (emergent)</td>
<td>18</td>
</tr>
<tr>
<td>Anxious about doing math</td>
<td>A priori</td>
<td>Enjoyment of challenge (emergent)</td>
<td>11</td>
</tr>
<tr>
<td>Mathematics self-concept</td>
<td>A priori</td>
<td>Mathematics self-efficacy- a priori, being successful in MTC mathematics – a priori, confident doing mathematics- emergent, pleased to have understood the mathematics-emergent, proud- emergent, pleased to solve problem quickly- emergent, Seeing self as a doer of mathematics (a priori)</td>
<td>38</td>
</tr>
<tr>
<td>Mathematics is rewarding</td>
<td>Emergent</td>
<td>Enjoy challenge in MTC (emergent), drawn into the problem/flow (emergent), patterns in mathematics (emergent)</td>
<td>26</td>
</tr>
<tr>
<td>Enjoyment of MTC math</td>
<td>Emergent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Struggling with mathematics in MTC</td>
<td>Emergent</td>
<td>Felt overwhelmed (emergent), left behind (emergent), difficulty with problem solving (emergent), challenge (emergent)</td>
<td>33</td>
</tr>
<tr>
<td>Improved mathematics self-concept</td>
<td>A priori</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Learned about self</td>
<td>Emergent</td>
<td>Seeing how others feel anxious about mathematics (emergent), learned problem solving strategy (emergent), jumped in (emergent), perseverance (a priori)</td>
<td>16</td>
</tr>
</tbody>
</table>
**Theme 2: Perceptions of mathematics**

<table>
<thead>
<tr>
<th>Code</th>
<th>A Priori or Emergent</th>
<th>Subcode</th>
<th>Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enjoy doing math</td>
<td>A priori</td>
<td>Doing puzzles for fun (emergent), enthusiasm for mathematics with connections to the real world (emergent)</td>
<td>26</td>
</tr>
<tr>
<td>Connected values of math</td>
<td>A priori</td>
<td></td>
<td>34</td>
</tr>
<tr>
<td>Learned something new about math</td>
<td>Emergent</td>
<td>Connecting mathematics learning to generalized learning (emergent), didn’t learn new mathematics in MTC (emergent)</td>
<td>21</td>
</tr>
</tbody>
</table>

**Theme 3: Perceived mathematics teaching abilities**

<table>
<thead>
<tr>
<th>Code</th>
<th>A Priori or Emergent</th>
<th>Subcode</th>
<th>Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Becoming a mathematics teacher</td>
<td>Emergent</td>
<td>Decision to become a mathematics teacher (a priori), teaching more mathematics over time (emergent)</td>
<td>17</td>
</tr>
<tr>
<td>Confidence to teach math</td>
<td>A priori</td>
<td>Anxious about teaching mathematics (emergent), increased mathematics teaching self-efficacy (a priori)</td>
<td>10</td>
</tr>
<tr>
<td>Enjoyment of teaching math</td>
<td>A priori</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>Desire to improve</td>
<td>a priori</td>
<td>Mathematics teaching self-concept (a priori), MTC provides desire to improve (emergent)</td>
<td>19</td>
</tr>
</tbody>
</table>
### Theme 4: Enacting new teaching methods

<table>
<thead>
<tr>
<th>Code</th>
<th>A Priori or Emergent</th>
<th>Subcodes</th>
<th>Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encouraging student self-efficacy</td>
<td>Emergent</td>
<td>Perseverance (emergent)</td>
<td>21</td>
</tr>
<tr>
<td>Student-centered methods</td>
<td>A priori</td>
<td>Teacher-centered methods (a priori), student engagement (emergent)</td>
<td>67</td>
</tr>
<tr>
<td>Changes in teaching pedagogy</td>
<td>A priori</td>
<td>Discourse (emergent), collaboration (emergent), using teaching methods from MTC (a priori)</td>
<td>37</td>
</tr>
<tr>
<td>Incorporating more problem solving</td>
<td>Emergent</td>
<td>Discovery (emergent)</td>
<td>35</td>
</tr>
<tr>
<td>Difficulty with teaching problem solving</td>
<td>Emergent</td>
<td>Time (emergent), balancing aspects of curriculum (emergent)</td>
<td>29</td>
</tr>
</tbody>
</table>

### Theme 5: Sense of group belonging

<table>
<thead>
<tr>
<th>Code</th>
<th>A Priori or Emergent</th>
<th>Subcode</th>
<th>Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Being seen as an effective mathematics teacher</td>
<td>Emergent</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Alignment with colleagues</td>
<td>A priori</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Collaboration in MTC</td>
<td>A priori</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>Comparison to others in MTC</td>
<td>Emergent</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
### Theme 6: Conflation of doing and teaching mathematics

<table>
<thead>
<tr>
<th>Code</th>
<th>A Priori or Emergent</th>
<th>Subcode</th>
<th>Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impactful mathematics moment occurred while teaching</td>
<td>Emergent</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Past mathematics experiences have effect on current teaching</td>
<td>A priori</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Perceptions of doing and teaching mathematics intertwined</td>
<td>Emergent</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>Learning mathematics alongside the students</td>
<td>Emergent</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

### Theme 7: Connections between mathematics experiences and classroom actions

<table>
<thead>
<tr>
<th>Code</th>
<th>A Priori or Emergent</th>
<th>Code</th>
<th>Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connecting experience to the classroom</td>
<td>Emergent</td>
<td></td>
<td>23</td>
</tr>
<tr>
<td>Confidence to teach mathematics gained</td>
<td>A priori</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Value in being a student/experiencing higher level math</td>
<td>A priori</td>
<td>Empathy with students (a priori)</td>
<td>13</td>
</tr>
<tr>
<td>Using problems from MTC</td>
<td>A priori</td>
<td></td>
<td>29</td>
</tr>
</tbody>
</table>
Appendix F: Example MTC Activities

The following is a sample of MTC activities mentioned by the teachers and links to the activity. Many of the activities can be found linked at www.mathteacherscircle.org.

The Brownie Problem:
http://www.mathteacherscircle.org/assets/toolkits/beginning/mathematics/ProblemsforSessionDesign.pdf

Card tricks:

Exploding Dots (or Exploding Blocks):
http://www.mathteacherscircle.org/assets/session-materials/JTantonExplodingDots_EducatorsVersion.pdf

Hexominoes:
http://www.ms.uky.edu/~lee/mathcircle/combined.pdf

KenKen:
http://www.geometer.org/mathcircles/kenken.pdf

Liar’s Bingo: p. 18–19 here:
http://issuu.com/mathteacherscircle/docs/mtcircular_winter_spring_2015

Math talks: descriptions are many places, e.g.,
http://www.mathperspectives.com/pdf_docs/number_talks.pdf

Math walks:
https://sites.google.com/site/gearthwalks/home

Pencilcosa:
### Appendix G: Quotations for Each Theme

**Theme 1: Beliefs about the self in relation to mathematics**

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Supporting Quotation From the Interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anne</td>
<td>“That was exciting, was not to be the dumb kid in class, not to be the one that was going, ‘Um, wait a minute. I don’t get that. How’d that happen?’ Where the way circle was run, it was a lot more, ‘Here’s the idea. You come up with a way to solve it using whatever math skills you have right at this moment.’”</td>
</tr>
<tr>
<td>Beverly</td>
<td>“I suppose it helped improve my image of myself as a person who can do math. I know I will never be able to come up with all the formulas like the high school and college teachers were able to. But at the same time, I was able to solve some of the problems in my own way…I guess it helped me to see myself as a mathematician.”</td>
</tr>
<tr>
<td>Carol</td>
<td>“This week it became clear to me how much of a difference it makes what type of learner I am. I excelled with spatial &amp; visual questions…It was exciting to struggle with math and persevere through to success or even not to get there but then learn from others!”</td>
</tr>
<tr>
<td>Deborah</td>
<td>“I think when I originally went into the Math [Teachers’] Circle, I was really nervous because I’ve often thought of myself as bad at math or with me struggling in math. I thought it helped me become a better math teacher, but I never thought of myself as being good at math. When going there and I was able to actually solve some of the problems and do different things, I was like, ‘Oh, hey. This isn’t so bad.’ It helped boost confidence in regards to the ability to be a problem solver.”</td>
</tr>
<tr>
<td>Erica</td>
<td>“I had no idea at that moment, but after going back to my seat and thinking about it more, I was able to figure out the answer. I felt very confident at that moment.”</td>
</tr>
<tr>
<td>Faye</td>
<td>“I think I alluded to earlier I went in maybe not being real self-confident about my skills and thinking I bet everyone else is smarter and getting this faster than me, but then I realized that really that wasn’t the case and that the strategies that I had to offer were as valuable as anyone else’s. I made great contributions to problem solving as much as anyone else. I think it increased my confidence in myself and realizing that I guess I am pretty smart.”</td>
</tr>
<tr>
<td>Kathy</td>
<td>“I also remember being in college and having my first experience where the math was terribly difficult for the first time in my life. Feeling overwhelmed, and isolated. I can empathize with people who claim to ‘hate math’ because I remember looking at a page of problems wide eyed and feeling hopelessly powerless like there was no way I’d ever figure it out. And I had no idea who to go to for help.”</td>
</tr>
</tbody>
</table>
### Theme 2: Perceptions of mathematics

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Supporting Quotation From the Interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anne</td>
<td>“It mostly just blew my mind that you could have an exponent, or a base that was two thirds or three halves, you know like, ‘Whoa! Nobody ever told me that.’”</td>
</tr>
<tr>
<td>Beverly</td>
<td>“I learned that there’s a lot more patterns in the world than I ever imagined. Some of the problems, they just seemed so random to me until someone pointed…discovered a pattern- wow!”</td>
</tr>
<tr>
<td>Carol</td>
<td>“We did the Exploding Dots and I immediately saw the binary connection as did one other teacher who was also technological, but I’d never made that connection with the base ten system, or the place value system that we have. It was really great to see that expansion.”</td>
</tr>
<tr>
<td>Deborah</td>
<td>“Yes, there are problems that have cut and dry answers. If you’re doing straight computations, yes. But when there’s a problem-solving type activity, I used to think, ‘Well, okay. There’s one way to solve it,’ or you might have a couple different strategies but you’re going to come up with same answer.”</td>
</tr>
<tr>
<td>Erica</td>
<td>“The connection that we made, like with that SET game, the connection that we made to geometry with that, there were just a lot of the connections that were branched from one area of math to another. That was neat.”</td>
</tr>
<tr>
<td>Faye</td>
<td>“This showed me how sometimes we don’t always know the objective of a problem and that sometimes just the exploration of the problem in and along the way is the actual objective of the activity, not necessarily formulating a solution.”</td>
</tr>
<tr>
<td>Jeff</td>
<td>“The first problem, writing a proposal for dividing a retirement bonus, had us present our proposal to the boss and working with such a cool, real-world example made it seem less daunting.”</td>
</tr>
</tbody>
</table>
### Theme 3: Perceived mathematics teaching abilities

<table>
<thead>
<tr>
<th>Teacher</th>
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<tbody>
<tr>
<td>Anne</td>
<td>“I’d say I’m in the top half of the population. I’m not the best. I do reach a lot of kids, but I could be better, and I’m trying to get better.”</td>
</tr>
<tr>
<td>Beverly</td>
<td>“That program gave me the confidence to teach math and to help students to get excited about learning math.”</td>
</tr>
<tr>
<td>Carol</td>
<td>“I was very nervous when I first got back into the fourth and fifth grade group because I didn’t want to teach them something that was wrong. I was very, very nervous…I see where I’ve gone from being very nervous to actually being quite confident in the way I teach math.”</td>
</tr>
<tr>
<td>Deborah</td>
<td>“I guess I just got more secure in my ability to teach it especially coming up to fifth and sixth grade.”</td>
</tr>
<tr>
<td>Erica</td>
<td>“I feel like I like the subjects that I teach, that’s where my heart is. I have passion to teach those subjects, more so than other subjects and I’m probably happiest when I’m teaching those subjects.”</td>
</tr>
<tr>
<td>Faye</td>
<td>“I’ve definitely grown in my confidence for one thing. I think entering it through the backdoor maybe I didn’t feel as self-assured as I felt maybe some of my peers…I just think every year you grow as a teacher and if you’re not you shouldn’t be teaching. You learn more things to help kids better. You find better resources, try new strategies.”</td>
</tr>
<tr>
<td>Jeff</td>
<td>“I love teaching math and, in 1-on-1 tutoring situations, I love working through the issues and seeing the growth.”</td>
</tr>
<tr>
<td>Kathy</td>
<td>“I truly love teaching it to students.”</td>
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</tbody>
</table>
### Theme 4: Enacting new teaching methods

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Anne</td>
<td>“Just the presenting it and the possibilities behind it, possibilities of letting the kids have a little bit more of that leash, a little bit more of that, being able to direct it. Again, it’s a little on the scary side when you haven’t done it, but it looks exciting.”</td>
</tr>
<tr>
<td>Beverly</td>
<td>“I won’t say this is exactly new, but it reminded me I should be doing this type of thing with my students. It’s like, what are some problem-solving strategies that we use? Like working backwards or making an easier problem, even trial and errors, giving multiple examples. I guess maybe it reminded me of different things that I should be doing in math.”</td>
</tr>
<tr>
<td>Carol</td>
<td>“I think it helped me to see another way of incorporating that collaboration piece in the classroom. So often we’re telling the kids to stop talking. That talking, if it’s guided, it can really be a useful tool.”</td>
</tr>
<tr>
<td>Deborah</td>
<td>“Especially this year, with doing the Math [Teachers’] Circles and everything else, we’re instituting a lot of problem solving in the math principles before we even get out our textbooks.”</td>
</tr>
<tr>
<td>Erica</td>
<td>“I need to talk less and let the students talk more, showing what they know and guide them to make connections. I need to be more of a coach.”</td>
</tr>
<tr>
<td>Faye</td>
<td>“I certainly hope to give my students more time to collaborate to process information, talk about the math together. I hope to give them maybe more challenging problems at a higher level than I maybe would have attempted previously and that I think they can do it with collaboration.”</td>
</tr>
<tr>
<td>Jeff</td>
<td>“I am re-tooling my intervention class to be more problem based and am trying to move away from just strictly reteaching what was done in the regular math class. I think it will be much more beneficial to incorporate the problem solving skills and will make the students more successful in their regular class.”</td>
</tr>
<tr>
<td>Kathy</td>
<td>“Increased focus on group norms. Math talks fully implemented. More time spent problem solving.”</td>
</tr>
</tbody>
</table>
### Theme 5: Sense of group belonging

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Anne</td>
<td>“I felt like I was right in there with the other teachers. I hadn’t stunted my growth in seventh grade.”</td>
</tr>
<tr>
<td>Beverly</td>
<td>“I really appreciated the time we had today to get together with other teachers on our level and talk about how we can apply what we have learned this week. I am looking forward to getting together with everyone at the end of August.”</td>
</tr>
<tr>
<td>Carol</td>
<td>“I felt like, we were working in teams and I really felt strongly about the idea that some of us really jumped in and took control and bounced off of each other’s ideas very quickly. I saw that collaboration as being a huge important piece of what we’re doing, and that was a lot of fun to me.”</td>
</tr>
<tr>
<td>Deborah</td>
<td>“Being able to talk with like-minded people, and some that weren’t necessarily like-minded, but just being able to bounce ideas off of each other and work as partners, as individuals, as teams throughout the whole week, we were able to experience it all.”</td>
</tr>
<tr>
<td>Erica</td>
<td>“I felt validated by the people in my group and was glad to have their contributions, especially when I was stuck.”</td>
</tr>
<tr>
<td>Faye</td>
<td>“I felt a little apprehensive at first being put in situations where I wasn’t comfortable and not knowing the answers to problems, but then knowing that my peers all felt the same way I had comfort in that.”</td>
</tr>
<tr>
<td>Jeff</td>
<td>“We were asked to think of questions that would help deepen our understanding and lead possible discussions. And for me, coming up with questions to discuss and having the discourse with other math teachers is probably the most beneficial thing for me.”</td>
</tr>
<tr>
<td>Kathy</td>
<td>“I’m excited for the opportunity to commiserate with like-minded mathematics teachers and discuss curriculum and what is best practice to encourage student learning.”</td>
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</tbody>
</table>
**Theme 6: Conflation of doing and teaching mathematics**

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Anne</td>
<td><em>In response to a question about she enjoys about doing math:</em> “I like to do the ‘diagnostics’ of math. Trying to analyze and find ways to reach students with misconceptions or ‘holes’ and being able to help them reach or sometimes even try. I have a lot of learned helplessness and defeatist attitudes when students arrive in my class. This is the part of Math I enjoy. The actual math is a way for me to reach them.”</td>
</tr>
<tr>
<td>Beverly</td>
<td>“My best experience has been learning by doing. Having struggled through the first year of Investigations, then taking part in the Success project I learned to love math and the teaching of math.”</td>
</tr>
<tr>
<td>Carol</td>
<td>“I would say when I got to the fourth and fifth grade level and started teaching math to them it was, I had to go back and relearn some of those things cause you don’t use some of those skills. I was like, ‘oh crud, I don’t remember how this is done.’ I had to go back on my own time and reteach myself.”</td>
</tr>
<tr>
<td>Erica</td>
<td><em>In mathematics autobiography:</em> “I had a student come back and visit while I was teaching an eighth grade class. She told the students that they would need everything they were learning in math now for math classes in high school. She warned them to pay attention in class, do their work, ask questions, etc. all things I had tried to convey to them as an eighth grade teacher preparing them for high school. It was very gratifying that she came back to visit and share that.”</td>
</tr>
<tr>
<td>Faye</td>
<td><em>In response to a question about when she feels anxious doing math:</em> “I sometimes feel anxious about teaching problems/concepts that I have either not taught before or problems/concepts that I have not been exposed to in many years.”</td>
</tr>
<tr>
<td>Jeff</td>
<td><em>In mathematics autobiography:</em> “I had an experience a few years ago where I was able to go to a fellow teacher’s classroom and observe her teaching. That was a great deal for me in that I was able to see some new processes and procedures and adopt some new practices for my class. Peer observation is a fantastic tool.”</td>
</tr>
<tr>
<td>Kathy</td>
<td><em>When writing about enjoying doing math:</em> “I love the kinds of problems that require students to persevere and problem-solving. And their unique solutions get me excited about doing and teaching math.”</td>
</tr>
</tbody>
</table>
**Theme 7: Connections between mathematics experiences and classroom actions**

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<tr>
<td>Anne</td>
<td>“I also got a glimpse at the kid who might not have come up with the idea but by seeing what others were thinking grew in their understanding.”</td>
</tr>
<tr>
<td>Beverly</td>
<td>“During the discussion I suddenly realized how I cheat my students by teaching them how to solve certain problems instead of helping them to discover it for themselves. I made a note to change that in my lessons this year.”</td>
</tr>
<tr>
<td>Carol</td>
<td>“It’s really going to help me keep in mind the different ways of learning. I saw a lot of that when someone in our team in the math teachers’ circle, people who were better at different processes. Some are more visual, other people are more formula based. Really keeping in mind that everybody learns in a different way.”</td>
</tr>
<tr>
<td>Deborah</td>
<td>“My experience with the Math [Teachers’] Circle, all we were doing were engaged in all of that. You think that you have drill them with all the facts and drill them with all the computations and different things like that but I learned so much more just from that week of problem solving and I think my students will do the same.”</td>
</tr>
<tr>
<td>Erica</td>
<td>“Getting to experience having to do the math, it puts you in the role of student so I’m able to better think about how I need to teach it. While I was doing the math, things that other people did that helped me, those are the kinds of things I want to do while I’m teaching it.”</td>
</tr>
<tr>
<td>Faye</td>
<td>“I plan to give my students more time to solve challenging problems and to work together collaboratively. I plan to give my students more time to reflect upon their learning and I want to incorporate more inquiry and open-ended questions in my teaching. I want to do this because I learned this week how much it expanded my own learning and confidence in my mathematical abilities.”</td>
</tr>
<tr>
<td>Jeff</td>
<td>“I am definitely going to incorporate more problem solving activities in my class. I feel like I am better equipped to handle the change in my class paradigm from ‘textbook’ to ‘problem solving’ and other critical thinking activities.”</td>
</tr>
<tr>
<td>Kathy</td>
<td>“The rich problem-solving strategies that we investigated were helpful both as a teacher and student.”</td>
</tr>
</tbody>
</table>