How Mathematical Disposition and Intellectual Development Influence Teacher Candidates’ Mathematical Knowledge for Teaching in a Mathematics Course for Elementary School Teachers

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The dissertation titled
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Abstract
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How Mathematical Disposition and Intellectual Development Influence Teacher Candidates’ Mathematical Knowledge for Teaching in a Mathematics Course for Elementary School Teachers

Director of Dissertation: Gregory D. Foley

Several prominent educators cite developing mathematical proficiency as the primary goal of mathematics education at all levels, and the RAND Mathematics Study Panel included developing teachers’ mathematical competence as a key to improving mathematics education. To accomplish these goals, many colleges and universities require elementary school teacher candidates to complete one or more courses in the foundations of elementary school mathematics. Although there have been several studies on the mathematical development of prospective teachers in school settings, there have been far fewer studies focusing on teacher candidates’ mathematical development in mathematics for elementary school teachers (MEST) courses.

This dissertation was a case study of how teacher candidates learn mathematics while enrolled in such a course. Specifically, the investigation focused on the mathematical experiences of five teacher candidates enrolled in a first quarter MEST course in number and operation concepts at a large Midwestern university. This research examined how their participation in this course influenced the teacher candidates’ mathematical knowledge for teaching within the context of (a) their mathematical
disposition and (b) their intellectual development. This study used Ball’s framework mathematical knowledge for teaching, Perry’s theory of intellectual development, and an adaptation to the National Research Council’s definition of productive disposition toward mathematics.

By the end of MEST 1, each participant increased their understanding of the algorithms they used to solve mathematical tasks, improved their reasoning about the mathematical principles they used when applying algorithms, and became more flexible in their problem-solving strategies. Each participant began to view mathematics as a logical and understandable discipline. This realization improved the mathematical dispositions of participants with relatively unproductive initial dispositions as they came to see mathematics as something they could understand, and lowered the mathematical dispositions of participants with relatively productive initial dispositions as they came to see mathematics as more complicated than they previously thought. With regard to mathematical intellectual development, each participant began the course thinking of mathematics as a purely dualistic subject, and four participants ended the course viewing mathematics from a multiplistic position.

Approved: ______________________________________________________________

Gregory D. Foley
Robert L. Morton Professor of Mathematics Education
To my wife Lisa:

for believing in me even when I did not believe in myself.

And to my daughter Alexandra:

for teaching me all that I know that I could not learn from a book.
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Chapter 1: Introduction

Many mathematics education researchers have stated that elementary school teachers in the United States are not adequately prepared to increase their students’ mathematics achievement (Association of Mathematics Teacher Educators, 2009; National Center for Research on Teacher Education [NCRTE], 1991; National Council of Teachers of Mathematics [NCTM], 2000; National Mathematics Advisory Panel, 2008). Most notably, in the U.S. many practicing teachers at this level lack the depth of mathematical content knowledge or the mathematical self-confidence to successfully teach children mathematics (Wu, 2009). Ma (1999) notes this shortage of depth in mathematical understanding in her study of elementary mathematics teachers in the United States. Further, Ma states that this deficiency in teachers negatively impacts their students learning mathematics beyond memorizing standard algorithms. The RAND Mathematics Study Panel, in light of these findings, places developing teachers’ mathematical knowledge “in ways directly useful for the teaching of mathematics” as a key area for improving K–12 mathematics (2003, p. 14). The National Council for Accreditation for Teacher Education (NCATE, 2009) also recognizes this problem and requires teacher preparation institutions to include in their elementary school teacher education programs a mathematics component designed to educate prospective teachers in the mathematical knowledge necessary to teach elementary school mathematics. Although institutions of teacher education the United States meet this requirement using a variety of approaches, one commonly used method is to require prospective elementary school teachers to complete a mathematics course or sequence of mathematics courses.
specifically designed to address mathematical content knowledge. As Zazkis (2011) points out

A Mathematics course for prospective elementary school teachers is the main avenue in which they experience mathematics. Most teacher education programs require such a course, or sequence of courses, either for certification or as a prerequisite for admission in [the] teacher education program. (p. viii)

Targeting teacher education programs as a means to reform K–12 schools is a common method and can be a powerful tool to effect changes in teacher candidates’ mathematical knowledge, disposition toward mathematics, and eventually their beliefs about teaching mathematics (NCRTE, 1991). However, successfully changing prospective teachers’ mathematical knowledge and beliefs about mathematics requires a substantial and systematic effort:

Teacher education must take mathematics and mathematics pedagogy as an explicit focus in order to make a difference for students. Only in programs which focused directly on mathematics did we see changes in teachers’ understanding of mathematics, their notion of mathematical pedagogy, or in their dispositions related to the teaching and learning of mathematics… Our data show that simply requiring prospective teachers to [take mathematics courses] at the university is unlikely to make a significant difference. Yet, the usual course for teachers or math methods courses seems inadequate as an intervention. (NCRTE, 1991, p. 44)
To alter teacher candidates’ mathematical knowledge or beliefs about mathematics, teacher candidates should be taught in ways designed to alter their beliefs and knowledge bases using current best practices which are informed by thoughtful research on mathematics education (RAND Mathematics Study Panel, 2005).

College mathematics courses provide a fertile ground to increase prospective teachers mathematical knowledge and positively change their beliefs about mathematics—essential components to successfully teaching children mathematics (Ball, Thames, & Phelps, 2008; Delaney et al., 2008; Stylianides & Ball, 2008). This dissertation will focus on the learning experiences of prospective teachers enrolled in mathematics courses designed to strengthen their mathematical knowledge for teaching. Further, this study investigates prospective teachers’ experiences taking these courses with respect to the impact of the courses on the teacher candidates’ mathematical knowledge for teaching, their mathematical disposition, and their intellectual development.

Although there is a wide variety of research on the mathematical development of prospective teachers while in teaching internships or in mathematics teaching methods courses, there is little research dedicated to the teaching of mathematics to and learning of mathematics by prospective elementary school teachers within mathematics courses. This dissertation serves as a contribution to this developing knowledge base.

**Statement of the Problem**

Research has shown a link between the mathematical knowledge of elementary school mathematics teachers and their classroom practices (Ma, 1999; National Research
Council, 2001a; NCTM, 2007) as well as a positive correlation between (a) teachers’ beliefs and emotions regarding mathematics and (b) their students’ beliefs and emotions regarding mathematics (Beilock, 2011; Beilock et al., 2010; National Mathematics Advisory Panel, 2008; Oldfather, 1991). Therefore, it is important to foster in prospective elementary school teachers a strong foundation of mathematical content knowledge as well as positive beliefs and emotional responses to mathematics—so that they can take that knowledge and those beliefs into their mathematics classroom once they become practicing teachers.

This dissertation examines five students’ experiences in a mathematics course designed for prospective elementary school teachers, as well as how their intellectual development and mathematical disposition affect their learning of mathematics in the course. Specifically, this study examines students taking a course focusing on number and operation concepts by examining what the teacher candidates’ experiences in the course are, how those experiences influence their overall learning in the course, and potentially if and how they can translate that learning into mathematical knowledge for teaching.

**Definition of Terms**

First, for the purposes of this study, the terms preservice teacher, prospective teacher, candidate, and teacher candidate are used interchangeably. By those I mean any individual either enrolled in a teacher education program or taking classes in preparation to apply to such a program. Further, this dissertation refers to all courses geared to teach the foundations of elementary school mathematics to teacher candidates as Mathematics
for Elementary School Teachers (MEST). The first course in the sequence deals with numbers and operations, and this course will be called MEST 1 and will be the focus of the study. When referring to the entire series of courses, this study will say the MEST series or MEST courses.

**Mathematical Knowledge for Teaching (MKT).** Building from Shulman’s work on pedagogical content knowledge (1986, 1987), Ball et al. (2008) define *mathematical knowledge for teaching* as, “the mathematical knowledge needed to carry out the work of teaching mathematics… [and] by *teaching* we mean everything a teacher must do to support the learning of their students” (p. 395). Further, Ball, Thames, and Phelps (2008) assert that the demands of MKT are greater than the mathematics demanded of an average non-teaching adult because teachers not only have to know mathematical concepts necessary for teaching, but also how concepts interrelate, how to best present those concepts to students, and the nature of student errors and how to provide feedback that corrects those errors.

Ball, Thames, and Phelps (2008) understand mathematics knowledge for teaching (MKT) as six related domains partitioning the whole of mathematical content knowledge. Four of these domains are *common content knowledge*, *specialized content knowledge*, *knowledge of content and students*, and *knowledge of content and teaching*. Ball et al. (2008) also list *horizon content knowledge* and *knowledge of content and curriculum* as factors in MKT, but see them as less significant than the previous four.

Common content knowledge (CCK) is mathematical content knowledge as it would be used in a field other than teaching. This is the knowledge that anyone
approaching a mathematical task would use to complete that task. This is not to assert that the knowledge is common, rather that it is commonly used by anyone successfully completing a mathematical task. It is important for teachers to possess this type of knowledge, for example, to know whether a student responded correctly to a question, to know if textbook is factually and conceptually correct in its mathematical statements, or to use correct notation when teaching. In short, a teacher must be able to successfully perform the mathematical tasks that they are assigning to their students.

Ball et al. (2008) define specialized content knowledge (SCK) as content knowledge exclusive to the profession of teaching and not typically needed for non-teaching purposes. This type of knowledge includes noticing patterns in student errors or creating tasks or scenarios to best illustrate mathematical concepts. It is important to realize that this is still mathematical knowledge, distinct from pedagogical implementations. Ball and her associates see this as subservient to horizon content knowledge, which they define as “mathematical knowledge [that is] not commonly used to solve mathematics problems” (Ball, Thames, & Phelps, 2008, p. 401).

The next domain is knowledge of content and students (KCS). Ball, Thames and Phelps (2008) see KCS as a combination of knowledge of mathematics and knowledge of students. A teacher must know what concepts the students find simple or confusing or how best to address mathematical concepts taking into account the developmental needs of their students. When assigning a mathematical task, teachers must anticipate how their students might approach it, how to address difficulties students may have with the task, and how to best enrich the task for their students. Teachers also must identify students’
emerging or incomplete understanding of a concept as the student expresses the concept in his or her own terms. Most importantly, teachers must possess an understanding of students’ conceptions and misconceptions of a mathematical idea and how to promote accurate mathematical ideas and redress misconceptions. Note that this knowledge is distinct from SCK because KCS combines mathematical knowledge and knowledge relating to student learning and SCK is focused on how mathematical is used in the context of teaching. In other words, SCK is still mathematical knowledge while KCS is pedagogical knowledge.

The final domain of MKT discussed here is knowledge of content and teaching (KCT). KCT combines mathematical knowledge with pedagogical knowledge (Hill et al., 2008). “Much of the process of teaching mathematics requires knowledge about how to design instruction…Teachers’ must sequence content for instruction, choose examples to best fit their students’ knowledge of the concept, evaluate instructional advantages and disadvantages of representations used” (Ball, Thames, & Phelps, 2008, p. 402), and identify what different teaching methods best advance their students’ understanding of the topic. These teaching tasks require combining (a) mathematical understanding with (b) teachers’ knowledge of pedagogical issues and student learning. Within this, Ball et al. (2008) place knowledge of content and curriculum, which they see as the knowledge of where a mathematical concept fits into the larger subject of mathematics—what mathematical skills are needed to understand a concept, how that concept relates to other mathematical concepts being taught, and how to present future concepts best by understanding what concepts with which the students already are familiar.
Mathematical Disposition (MD). The National Research Council (NRC, 2001a) states that for students to have a “productive disposition” toward mathematics means that the student possesses a “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 116). Because a teacher candidate’s disposition toward mathematics may not be productive, for the purpose of this dissertation, let a candidate’s mathematical disposition be the candidates beliefs and attitudes about mathematics, the candidates mathematical self-efficacy, and the role the candidate believes diligence plays in solving mathematical tasks. Further, for the purposes of this study, a productive mathematical disposition is a productive disposition toward mathematics as defined by NRC (2001a), and a productive mathematical disposition element (or more simply a productive element) is a factor that supports a prospective teacher’s productive mathematical disposition. For example, we would say that a candidate’s belief in their ability to do mathematical tasks is a productive element of their mathematical disposition. On the other hand, an unproductive mathematical disposition is the habitual inclination to see mathematics as futile, impractical, useless, coupled with a lack of belief in diligence and a low mathematical self-efficacy, and an unproductive mathematical disposition element (or unproductive element) is a factor that influences an unproductive mathematical disposition in a candidate. For example, we would say that a candidate’s belief that only certain people are capable of learning mathematics is an unproductive element in their mathematical disposition.
It is important to note that mathematical disposition is not a single sliding scale, but rather a multi-dimensional manifold of overlapping intellectual and emotional factors related to completing tasks that require mathematical thinking or processes. Thus, an individual may have a productive mathematical disposition in one area and an unproductive one in another. For example, a candidate may be reluctant to attempt a task that he or she sees as mathematical in nature (an unproductive response) but upon reflection see their mathematical skills as being above average. Alternatively, a prospective teacher may possess an enthusiasm toward doing mathematics and confidence in his or her ability to do mathematics, but believes that mathematics is a skill only a few people possess. Neither of the above dispositions are completely productive or unproductive, but both contain an element that is productive and an element that is unproductive.

**Research Questions**

This dissertation examines how prospective teachers learn mathematics in the context of college mathematics courses. Using the method of research questions recommended by Marshall and Rossman (2006) this study lists a primary research question separated into several smaller sub-questions. The primary research question is as follows:

*How do prospective elementary school teachers experience college mathematics courses designed to teach them the foundational concepts of elementary school mathematics?*

In order to problematize the research question to a form more suitable to this process, the primary research question will be divided into the following three sub-questions:
In what ways do college mathematics courses designed to teach the foundations of elementary school mathematics help develop mathematical knowledge for teaching (MKT) in prospective teachers?

In what ways do prospective elementary school teacher’s mathematical disposition influence their learning while taking college mathematics courses designed to teach the foundations of elementary school mathematics and how do such courses influence their mathematical disposition?

In what ways do prospective elementary school teacher’s intellectual development affect their learning in college mathematics courses designed to teach the foundations of elementary school mathematics and how does their participation in those courses affect their intellectual development?

Although there have been several studies on the mathematical development of prospective teachers in school settings (e.g. Steffe, 1991; Lin & Gorrell, 1997; Ball, 1993; Mewborn, 1999; Foss & Kleinsasser, 1996), this study will focus on the mathematical development of prospective teachers in the university mathematics classroom. Thus, this dissertation is different from the previous studies as it focuses on prospective teachers’ learning in a different learning environment and examines prospective teachers’ mathematical learning at a much earlier point in their academic careers. The prospective teachers enrolled in this course are typically first-year college students, and much different developmentally from prospective teachers in classroom internships at the end of their degree programs (Evans et al., 2010). And although some
prospective teachers may chose to put this course off until later in their academic career, the course is designed to be taken in the prospective teachers’ first year of college.

**Theoretical Framework**

The primary objective of MEST 1 is to develop the prospective teachers’ mathematical knowledge for teaching (MKT), specifically in terms of common content knowledge (CCK) and specialized content knowledge (SPK) as it relates to number and operation. Students enter the course with an existing history with mathematics, a previously developed mathematical disposition, and developing adult intelligences. In order to acquire this knowledge, students are presented with a wide variety of mathematical experiences in class, which include (but are certainly not limited to) lectures, computer-based lab activities, activities with manipulatives, and a variety of informal and formal assessments.

In MEST courses, prospective elementary school teachers are presented with their first experiences with the mathematical knowledge that will be required of them to be effective mathematics teachers. Before taking these courses, the prospective teachers most likely only have their beliefs about what knowledge will be required of them to teach mathematics in elementary schools. These beliefs are influenced by a number of factors, for example: their mathematical knowledge, their memory of their mathematical experiences in elementary school, and conversations they may have had with practicing teachers. Although MEST courses have several other goals, one of the course’s main focuses is taking prospective mathematical beliefs and forming them into a foundation on which they can build their mathematical knowledge for teaching. See Figure 1.
Figure 1. Interactions of intellectual development, mathematical disposition and mathematical knowledge for teaching in a MEST course.

Although MEST courses primarily deal with MKT, they also address the prospective teacher’s overall intellectual development—as the course strives to convince students that mathematics is a logical system rather than a list of rules handed down by authority figures—and their mathematical disposition. Potentially MEST 1 will be less effective influencing these two latter areas. Both intellectual development and mathematical disposition are long developing positions that are not easily permanently influenced by one college course. So although the course may have some interaction with these two structures, due to its brevity MEST 1 does not affect great change on those positions.
It is important to consider the impact of MEST courses on the prospective teachers’ intellectual development, mathematical knowledge for teaching, and their mathematical disposition. At many colleges and universities, prospective elementary school teachers are only required to take one elementary mathematics methods class, with the assumption that the prospective elementary school teachers made the necessary gains in CCK and SCK when taking MEST courses. Potentially, a prospective elementary school teacher could enter their teaching internship with only these courses to influence their mathematical knowledge and their mathematical disposition. Further, because the emphasis of the mathematics methods course is teaching mathematics to children and not mathematical knowledge, MEST courses are the only courses that directly influence the candidates’ mathematical knowledge for teaching in terms of CCK and especially SCK.

**Delimitations and Limitations**

The primary delimitation in this study is limiting the participant pool to one university. First, the university where this study was conducted divides its academic year into 10-week quarters instead of the much more common 15-week semesters. Future research should be done to examine and analyze the differences between how a quarter–system and a semester–system address these needs, and if there are any differences what effect they may have. Further, only recruiting participants from one university limits exposure to the broad diversity of how other colleges and universities handle meeting the pedagogical content knowledge needs of prospective teachers (NCRTE, 1991). Although NCATE (2009) is quite clear on the need for prospective teacher education programs to address pedagogical content knowledge, it gives the individual institutions of teacher
education a great deal of latitude on how to address this need. However, in spite of that broad spectrum, MEST 1 uses standard best practices in the course: as with courses at other teacher education institutions, MEST 1 uses the tenants of current research in cognitively-guided, student-centered instruction (Carpenter, Fennema, & Franke, 1999; O’Connell 2005; Van de Walle & Lovin, 2006; Van de Walle, 2007) as its basis to begin implicitly teaching prospective elementary teachers the power of such methods and how to integrate them into a mathematics classroom. However, the intent of this research is to examine closely the experiences of a select few participants and how they experience MEST 1. Therefore, it makes sense to limit the variability of the context of the course to better examine how the teacher candidates’ mathematical disposition and developmental position affects their experiences and their perception of those experiences.

A second important delimitation to the dissertation is the assumption that neither the participants’ mathematical disposition nor their intellectual development position will change significantly during the study. Although there is much support that, in most cases, a prospective teacher’s mathematical disposition will not change significantly in such courses (Wilson & Cooney, 2002; Phillipou & Christou, 2002; Clemens & Sarama, 2009), Libeskind (2011) argues that

If we model the kind of teaching we would like prospective teachers to engage in, we capture and spark their interest in learning mathematics and teaching mathematics in a way that does the same for their students. (p. 484)

And although Perry’s (1970; 1981) theory states that transitioning from one position to another is a slow process, Evans et al., (2010) acknowledges that college is a time that
many students can begin transitions from one position to another—both moving to higher
positions and regressing to lower ones—as many other elements in their lives are
changing.

The primary limitation of the study is the use of the mathematics disposition
survey to determine roughly the mathematical dispositions of the participants before the
study. Mathematical disposition is a broad, complex, and nuanced part of the human
psyche influenced by a student’s experiences with mathematics, the mathematical
disposition of his or her family and his or her previous mathematics teachers, and many
cultural influences, both implicit and explicit (Blanzieri et al., 2001; Brinkley, Morris, &
Simonson, 2007). Thus, a single numeric scale cannot capture the subtle nuance of
mathematical disposition and trying to distill such a complex psychological devise into a
numeric score is, at best, problematic. However, because the researcher attempted to
select participants with a broad spectrum of mathematical dispositions and is not as
concerned with their linear order except to select a participant with a productive
disposition, an unproductive disposition, and a middling disposition, these linear
gradations are not an important part of the study.

Other limitations to the study are the small number of participants studied. Every
prospective teacher has a vast array of unique mathematical experiences and diverse
opinions about mathematics, and although every attempt was made to select participants
who represented a broad range of dispositions, these cannot represent every possible
mathematical disposition that a prospective teacher might have. Also, all of the
participants recruited in the study identified themselves as Caucasian, limiting the
dissertations ability to see differences that may occur because of the participants’
ethnicity or culture.

Another possible limitation is if a participant chooses to seek assistance for the
course in some outside-of-class environment, such as seeking a tutor. Although it can be
argued that outside-of-class learning opportunities, if done well, enhance the classroom
experience, this dissertation is unable to differentiate changes to MKT that occur because
of classroom experiences or outside-of-class learning experiences. At best, the outside-
of-class learning will enhance the classroom experiences and positively affect the
prospective teacher’s MKT, and at worst, it will undermine the in-class experience or
reinforce negative stereotypes about mathematics. And although two of the five
participants shared that they did seek help from sources outside of class, this dissertation
cannot say for certain if the remaining three did not and, if they did, either actively or
passively, chose to not share that with the researcher.

**Educational Significance of the Study**

This dissertation contributes to several developing knowledge bases in education
research. As stated previously, there has been little research on the role mathematics
courses play in developing prospective elementary school teachers. In that, this study
contributes to an underdeveloped knowledge base in mathematics education research. In
addition, this study uses Perry’s Theory of Intellectual Development, a commonly used
theory in college student development, to examine classroom learning in a mathematics
research context, where the theory has been used much less. Although Perry (1970) has
been thoroughly researched and applied to the development of programs for college
students, there is less research on how these theories can benefit the university mathematics classroom—especially on how Perry can inform the teacher-education process. Also, this study develops the idea mathematical disposition as a construct of students’ beliefs, affective responses, and emotional reactions to mathematics and mathematics education.
Chapter 2: Review of Relevant Literature

One of the major advancements in mathematics education in K–12 schools is the incorporation of children’s development theory into curriculum, teaching practice, and application. The National Council for Accreditation of Teacher Education (NCATE, 2009), the National Council of Teachers of Mathematics (NCTM 2000, 2006, 2007), and the National Research Council (NRC, 2001a, 2009) strongly advocate knowledge of children’s development in hopes of preparing a developmentally appropriate curriculum for students. The Ohio Department of Education makes a stronger statement, listing as its first element of its first standard in its Standards for Professional Educators that “teachers display knowledge of how students learn and the developmental characteristics of age groups” (2005, p. 16). To understand the development of teacher candidates as students, this dissertation will use Ball’s theories of Mathematical Knowledge for teaching, the an adaptation of NRC’s (2001) definition of productive disposition toward mathematics, and Perry’s Theory of Intellectual Development, as well as current assessment practices used in mathematics education in 2012.

Mathematical Knowledge for Teaching

Paul Ernest (1977) argued that accurate mathematical knowledge is essential to a student’s (and thus a prospective teacher’s) personal appropriation of knowledge. However, because of individual sense-making in the ikonic phase (Bruner, 1972), a students’ appropriation of mathematical knowledge is unique to that individual.

Of course the acquisition of mathematical competence by individuals and its use in socially situated performances are irrevocably interwoven. For only through the individual performances are the individual constructuals or their consequences
made public and confronted with the alternate, extensions, corrections, or corroborations. Continual participation in dialogue… is necessary for the personal appropriation and internalization of mathematical knowledge, if it is to mesh with the utterances of others, and hence, with their knowledge. Ultimately, such interactions are what allows individual’s personal knowledge of mathematics to be regarded as an interiorization of collective knowledge. (Ernest, 1977, p. 221)

**Shulman’s Development of Pedagogical Content Knowledge.** Although many agree that a *mastery of content knowledge* is important to successful teaching, there existed no definitive statement of what *mastery of content knowledge* entailed. To help correct this, Shulman (1986) proposed that there exists an exclusive domain for teaching knowledge, which he termed *pedagogical content knowledge* (PCK). Shulman describes PCK as “a specialized knowledge distinctive to teaching… a kind of subject-matter-specific professional knowledge” (Ball, Thames, & Phelps, 2008, p. 389). PCK provides a link between “mastery of content knowledge and the practice of teaching” (Shulman, 1987, p. 5). Unfortunately, although this term is in wide use, as of this writing the concept of pedagogical content knowledge still remains underdeveloped (Ball et al., 2008).

Two central tenants of Shulman’s (1986, 1987) work were his redefining what it means to have mastery of a concept appropriate for teaching and his representation of content knowledge as a key component for teaching. He reframed the question of content knowledge in a way that focused on the context of teaching. This was radically and controversially different from the research of his time that saw content knowledge as
merely the context for education. Because of this de-emphasis in content knowledge, Shulman referred to the study of PCK as a “missing paradigm” (1986, p. 4) in educational research. Further, he called for the creation of a national board system “that would focus upon the teacher’s ability to reason about teaching and to teach specific topics, and to base his or her actions on a premise that can bear the scrutiny of the professional community” (Shulman, 1987, p. 20).

Specifically, “Shulman was concerned with the prevailing conceptions of teacher competencies, which focused on generic teaching behaviors” (Ball et al., 2008, p. 390). Shulman argued “the currently incomplete and trivial definitions of teaching held by the policy community comprises a far greater danger to good education than does a more serious attempt to formulate that knowledge base” (1987, p. 20). Shulman believed that quality teaching requires a refined, specialized knowledge base that transcends simple teaching procedures, so much so that “mere content knowledge is likely to be as useless pedagogically as content-free skill” (1986, p. 8).

In order to describe knowledge necessary for teaching, Shulman created seven key categories for PCK:

- General pedagogical knowledge, with special references to those broad principles and strategies of classroom management and organization that appear to transcend the subject matter,
- Knowledge of learners and their characteristics,
• Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, and the character of communities and cultures,

• Knowledge of educational ends, purposes, and values and their philosophical and historical grounds,

• Content knowledge,

• Curriculum knowledge, with particular grasp of the materials and programs that serve as ‘tools of the trade’ for teachers, and

• Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding. (Shulman, 1987, p. 8)

These categories “highlighted the important role of content knowledge and situate content-based knowledge” within the broader perspective of professional teacher development (Ball, Thames, & Phelps, 2008, p. 398). “The first four categories address general dimensions of teaching knowledge that served as the mainstay of teacher education at that time” (Ball, Hill, & Bass, 2005, p. 17). The last three categories define the knowledge of course content necessary for teaching.

The first category, content knowledge, refers simply to knowledge of the subject matter being taught as well as the concepts which are foundational to that subject matter (Grossman, Wilson, & Shulman, 1980; Wilson, Shulman, & Rikart, 1987). The second of these categories, curricular knowledge, is
Represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of a particular curriculum or program materials in particular circumstances. (Shulman, 1987, p. 10)

The final of these three content knowledge categories is pedagogical content knowledge (PCK).

The most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstrations—in a word, the most ways of representing and formulating the subject that makes [it] comprehensible to others…

Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (Shulman, 1987, p. 9-10)

PCK represents that given two accurate representations of an idea, one may be easier for a learner to understand and for a different learner the other may be easier—and that a teacher must not only understand both models, but also which model will be more appropriate for a given student. Further, representations of a concept should be formed from the teachers’ “content-specific knowledge of student conceptions” (Grossman, 1990, p. 172).
By emphasizing the conceptions and representations used in teaching, pedagogical content knowledge brought new light to how content knowledge may matter to teaching. Ball, Thames, and Phelps (2008) suggest “that it is not only knowledge of content, on the one hand, and knowledge of pedagogy on the other hand, but also a kind of amalgam of knowledge of content and pedagogy that is central to the knowledge needed for teaching” (p. 392). Further, “pedagogical content knowledge is the category most likely to distinguish the knowledge of the content specialist from the pedagogue” (Shulman, 1987, p. 8).

**Ball’s Mathematical Knowledge for Teaching (MKT).** Argument over Shulman’s broad definitions of *content knowledge, curricular knowledge,* and *pedagogical content knowledge* has led to debate about how to construct narrower and more specific definitions (Ball, 1999). In order to better focus these topics, the Conference Board of the Mathematical Sciences (CBMS, 2001) problematized the concept of the role of content knowledge in education by mathematicians and mathematics educators. Their goal was “to identify the core mathematical concepts and skills needed to teach mathematics effectively” (CBMS, 2001, p. 19). In concert with CBMS, Stylianidias and Ball (2004) approached the problem from the opposite direction: by analyzing research on students to discover which mathematical concepts that the students generally had difficulty learning. It quickly became evident that mathematics teachers needed to have mastery of the mathematical concepts they teach; Stylianidias and Ball (2004) focused on how teachers needed to understand the concepts rather than what concepts they needed to understand. From this research, a practiced-based model
for *mathematical knowledge for teaching* developed. Ball, Thames, and Phelps (2008) define *mathematical knowledge for teaching* (MKT) as, “the mathematical knowledge needed to carry out the work of teaching mathematics… [and] by *teaching* we mean everything a teacher must do to support the learning of their students” (p. 395). Further, Ball and her research teams assert that the demands of MKT are greater than the average adult, as teachers not only have to know the concept, but also

(a) how concepts interrelate,

(b) how to best present those concepts to students,

(c) the nature of student errors and

(d) how to use provide feedback that corrects those errors.

This is a difficult process, and many teachers and teacher candidates are reluctant to embrace these additional knowledge demands (RAND Mathematics Study Panel, 2005). Foss and Kleinsasser (1996) show that in spite of taking a course designed to model constructivist principles, many preservice elementary teachers see mathematics as “a fixed set of rules” (p. 437) and more damning that “the preservice teachers rely on the belief that children must possess innate knowledge or have a certain type of mind in order to understand mathematics, thus relinquishing their responsibility for applying methods used to teach and learn challenging mathematics” (Foss & Kleinsasser, 1996, p. 439).

**The Structure of Mathematical Knowledge for Teaching.** Ball et al. (2008) understand mathematics knowledge for teaching as six associated domains partitioning the whole of mathematical content knowledge. These domains are *common content knowledge* (CCK), horizon content knowledge, *specialized content knowledge* (SCK),
knowledge of content and students (KST), knowledge of content and teaching (KCT), and knowledge of content and curriculum. In her work, Ball focuses on CCK, SCK, KST, and KCT, and sees horizon content knowledge and knowledge of content and curriculum as interrelated to other domains. This study will follow her lead and focus on those four domains. Ball et al. (2008, p. 339) sees the theory overlapping with the work of Shulman as illustrated in Figure 2:

Common content knowledge (CCK) is simply mathematical content knowledge, as it would be used in a field other than teaching. This is the knowledge that anyone approaching a mathematical task would use to complete said task. This is not to assert that the knowledge itself is common, rather that it is commonly used by anyone attempting to complete a task. It is important for teachers to possess this knowledge to know if a student or a textbook has given a correct or incorrect answer, or to employ correct notation when teaching. In other words, a teacher must be capable of performing the mathematical tasks they assign to students.

The second domain is specialized content knowledge (SCK). Ball et al. (2008) define this as “content knowledge unique to the profession of teaching” (p. 393) and not typically needed for non-teaching purposes. This type of knowledge includes noticing patterns in student errors or creating tasks or scenarios to best illustrate mathematical concepts. For example, Ma (1999) presented the following task to 23 U.S. preservice and inservice teachers:

People seem to have different approaches to solving problems involving division with fractions. How do you solve a problem like this one?

\[ 1 \frac{3}{4} \div \frac{1}{2} = ? \]

Imagine that you are teaching division with fractions. To make this meaningful for kids, something that many teachers do is relate mathematics to other things. Sometimes they try to come up with real-world situations or story-problems to show the application of a particular piece of content. What would you say is a good story or model for \( 1 \frac{3}{4} \div \frac{1}{2} \)? (p. 55)
Sadly, a meager 10 of the U.S. teachers Ma surveyed were able to obtain the correct answer, with four participants “either unclear about the procedure or obviously unsure about what they were doing” (p. 56). Further, only one participant was able to come up with a correct setting for the problem, although 16 teachers and preservice teachers came up with representations with misconceptions, and six could not come up with a setting at all. This demonstrates a failing in the teachers’ and teacher candidates’ specialized content knowledge (SCK).

Ball et al. (2008) see horizon content knowledge as any mathematical knowledge known by an individual but, unlike CCK or SCK, is not needed in the individual’s everyday lives. Interestingly, Ball theorizes that horizon content knowledge can be used by mathematics teachers to find interesting and relevant mathematical tangents, but when the teachers do this, that knowledge moves from the realm of horizon content knowledge to SCK.

The third domain is knowledge of content and students (KCS). This domain combines knowledge of mathematics and knowledge of students. A teacher must know what concepts the students find simple or confusing, and how to best address mathematical concepts taking into account the developmental needs of their students. When teachers assign a mathematical task, they must anticipate (a) how students may approach it, (b) how to address difficulties that students may have with the task, and (c) how to best enrich the task for their students. Teachers must use their students’ language to identify the students’ emerging or incomplete understanding of a concept. Most importantly, teachers must possess an understanding of students’ conceptions and
misconceptions of a concept, and how to best address and teach those conceptions and re-teach those misconceptions.

Knowledge of content and teaching (KCT) combines mathematical knowledge with pedagogical knowledge. Many mathematical teaching tasks require mathematical knowledge related to the design of instruction, such as sequencing concepts, designing lessons and activities, choosing which of several models to present to demonstrate a mathematical concept, and evaluation on instruction. Each of these tasks interacts between the understanding of pedagogical concepts and the understanding of mathematical concepts. Knowledge of content and curriculum works with KCT as it allows teachers to best present concepts in an order that the students can understand, as well as emphasize mathematical concepts which will be important to the students in subsequent mathematics courses.

Implications of Mathematical Knowledge for Teaching to MEST Courses.
Because teacher candidates at this point in their education have not formally started studying student development or classroom management, the bulk of the content of MEST courses is geared to developing CCK and SCK. However, education is most meaningful when it is placed in a useful context (Bruner, 1971; Dewey 1916, 1939) and thus it is useful to show preservice teachers explicitly how they can potentially use the content knowledge and skills gained in their future classrooms. Further, these should be authentic applications of concepts as opposed to half-hearted, overly generic fictitious ones that the preservice teachers can easily see through (Dewey, 1938; NCTM, 2007; NRC, 2001a).
Mathematical Disposition

Newer theoretical perspectives of classrooms as distinct cultures, and of student and teacher behavior as practices situated within those cultures, have made students’ and teachers’ views on the nature and usefulness of mathematics a central object of study and a target for instructional innovation (Gainsberg, 2007). Mathematics education researchers (e.g., Boaler, 1997; Burton, 2004, Povey & Burton, 2004; Schoenfeld, 1992) recognize that the mathematics disposition of adults and children can be as influential on their mathematical learning and behavior as is their mastery of content. Further, new standards for teaching prospective mathematics teachers list dispositions as a key factor to successfully learning mathematics (NCATE, 1999; NCTM, 2007; NRC, 2001a).

There is no greater influence on the mathematical disposition of a student than the disposition of the classroom and the largest influence on the mathematical disposition of a classroom is its teacher (White, 2003). Therefore, when considering the mathematics education of prospective teachers, it is imperative to also discern what effect that education has on their mathematical disposition.

“Every individual has a unique set of values, experiences, and predetermined beliefs entering into the educational experience… And as educators we must consider these individual needs when we teach” (NRC, 2001a, p. 86). It is important to realize that every student who enters a mathematics classroom does so with a belief in the value of mathematics, perceptions of their own mathematical ability, and an emotional reaction to mathematics. NRC (2001a, 2009) first introduces the concept of mathematical disposition when they define a productive disposition towards mathematics as a “habitual
inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 116). The NRC (2001a) lists productive disposition along with strategic competence, adaptive reasoning, procedural fluency and conceptual understanding as the five interwoven strands of mathematical proficiency. However, a productive disposition occupies a different role from the other components. The four other components are tools that a student can use to solve a mathematical task. They are intellectual components that mathematics educators can and do assess in their students. Unlike the others, productive disposition is an affective component—rather than an intellectual means to approach or solve a mathematical task, it is the belief that such tasks can and should be solved, and the corresponding appropriate emotional responses that allow that belief.

With respect to the NRC (2001a) pictorial representation of mathematical proficiency as strands in an interwoven rope, this dissertation offers an alternative model. In this model, mathematical proficiency is separated into two distinct groups: intellectual elements—procedural fluency, adaptive reasoning, strategic competence, and conceptual understanding—and the affective element—productive mathematical disposition (see Figure 3). When a candidate attempts a mathematical task, he or she uses the four intellectual elements as tools in order to successfully complete that task. Mathematical disposition acts as field in which the other four intellectual elements can operate. When a candidate has a productive mathematical disposition, it allows them the ability to access the four intellectual elements freely, quickly, and frequently, as well as transition between those elements when appropriate. A productive mathematical disposition also
allows a candidate to not become frustrated to the point of failure, because part of having a productive mathematical disposition includes mathematical self-efficacy and a belief in the role of diligence in mathematics. However, when faced with a mathematical task a candidate with an unproductive mathematical disposition may experience fear or anxiety that may not allow them to access their intellectual tools. Further, a candidate with an unproductive mathematical disposition may stop attempting to solve a mathematical problem when they become frustrated; because the candidate may believe that they cannot do mathematics no matter how much effort they put in the problem. In short, mathematical disposition is a space that allows the intellectual elements to operate and to interact in mathematical problem solving. A candidate who possesses a productive mathematical disposition has the ability to use their procedural fluency, adaptive reasoning, strategic competence, and conceptual understanding when solving a mathematical task. However, a candidate with an unproductive mathematical disposition can experience problems accessing those elements, and thus difficulty solving mathematical problems.
Therefore, it is imperative to the development of a productive mathematical disposition to create an environment in which students are challenged to grow and given the support that they need to help them grow (Gardener, Upcraft, & Barefoot; 2005). Giving students both the challenge and support they need to grow is one of the key tenants of the field of college-student development. Students need a learning environment where they feel comfortable enough to experiment, fail, receive constructive feedback, and try again. According to Beyers, (2005),

A student’s beliefs and attitudes about mathematics support the inclination to see mathematics as sensible, useful, and worthwhile. Mathematics self-concept
constitutes the student’s belief in his own efficacy and influences his belief that diligence leads to successful learning. (p. 1)

The American Mathematical Association of Two-Year Colleges (AMATYC, 2006) focuses on limiting the amount of unproductive mathematical disposition present in the class. “Attitudes toward mathematics can create either a feeling of confidence or anxiety that may have a positive or negative effect on mathematical behavior” (p. 23).

Cooney and Hirsch (1990), NCTM (1989, 1991, 2000, 2006, 2007) and NRC (2001a, 2009) recognize that for success in mathematics, teachers must address both the cognitive and affective mathematical development of students. However, there is evidence to show that mathematics teachers may possess negative mathematical dispositions. Usiskin (2003) notes that many mathematics teachers in the high school setting do not feel confident in teaching mathematics at the highest levels taught in high school, with

• 23% reporting that they believe they are “not well qualified” to teach statistics,

• 39% reporting that they believe they are “not well qualified” to teach calculus, and

• 44% reporting that they believe they are “not well qualified” to teach discrete mathematics.

Lin and Gorrell (1997) report more severe disposition issues among elementary school and middle school teachers, with teachers at those levels not only reporting that they are uncomfortable performing and thinking mathematically, but also some
questioning the “value of advance[d] mathematics” for “the average person” (p. 12).

Further, Foss and Kleinsasser (1996) show that in spite of taking a course designed to model constructivist principles, many preservice elementary teachers see mathematics as “a fixed set of rules” (p. 437) and more damning.

The preservice teachers rely on the belief that children must possess innate knowledge or have a certain type of mind in order to understand mathematics, thus, relinquishing their responsibility for applying methods used to teach and learn challenging mathematics. (p. 439)

Hart and Walker (1993) break mathematical affect into four areas: confidence in learning mathematics, persistence in the mathematics classroom, perceived usefulness of mathematics, and student motivation to learn. In order to increase a student’s perceived usefulness of mathematics, a mathematics educator should present tasks that are meaningful and relevant to the student (Bruner, 1996). By using mathematics to solve problems that may arise in the real world rather than problems that exist in some artificially generated setting, students can see mathematics as a practical and useful endeavor rather than an artificially created mental exercise. Hart and Walker (1993) suggest that the reason students lack persistence in the mathematics classroom is that they are not faced with opportunities to struggle.

Continuing to struggle by themselves in the face of difficulty was rarely encouraged or observed…The high-achieving students had no opportunity to struggle with a mathematics problem because the work was easy for them, and the
low achieving students... had no opportunity to struggle because of the pace of instruction. (pp. 10–11)

It is therefore important to ask students to struggle with a mathematical problem that the student perceives as useful and to encourage and reward both the process of struggling with the problem as well as the outcome of that struggle. This is true of both mathematical problems and mathematical concepts and connections.

Also, it is imperative that mathematics educators and mathematics teacher educators to foster intrinsic motivation to learn mathematics rather than artificial extrinsic motivators and punishments. Malone and Leper cited four sources of intrinsic motivation in learning activities: (a) provide and appropriate level of challenge, (b) appeal to the learner’s sense of curiosity, (c) provide the learner with a sense of control, and (d) encouraging the learner to be involved with a world of fantasy (as cited in Hart & Walker, 1993). Oldfather (1991) developed the concept of the continuous impulse to learn. The source of motivation in this impulse is the pleasure of learning rather than seeking approval for learning. Oldfather sees the continuous impulse to learn as

Linked, above all else, to activities involving self-expression. The depth of thought and feeling and action that were part of their experience of self-expression brought the meaning of self-expression to a new level: a level that involved deep engagement, self-discovery, and empowerment. [The students] found their own voices... Not only did students gain their own voices, their voices were heard, taken seriously, acted upon, and honored. (pp. 226–227)
McLead and Orteiga (1993) recommend that it is better to confront these issues with students rather than ignore them.

By allowing students to air their grievances and give voice to their frustration, they realize that they are not alone… and that their views are valid. By confronting these negative stereotypes, [the mathematics teacher] moves from being a further source of frustration and begins to be a mentor and counselor. It is no longer math and [the teacher] working against the student, but the teacher and the students working together to learn mathematics. (p. 98)

There is some research that suggests that certain groups of students may be culturally biased into having a negative disposition toward mathematics, specifically women (Shulman, 1996; Stigler & Heibert, 1986; Bolar, 2008), minority groups (Gutstein, 2006; Applebaum, 1999; Bolar, 2008, Moses & Cobb, 2001), or English language learners (Applebaum, 1999).

Mathematics disposition is formed by four key interactions in the student’s mathematical life: experiences, their family’s mathematical disposition, their mathematics teachers’ mathematical disposition, and the implicit views of mathematics by the cultures with which the student (or preservice teacher) is a member. See Figure 4.
“At most colleges and universities in the United States, the mathematics requirements for students majoring in elementary education are minimal” (Malzahn, 2002, p. 19)—MEST courses are typically intended as first-year courses with no college mathematics prerequisites and, as previously mentioned, are potentially the only mathematics courses a prospective elementary teacher would take before beginning their teaching internship. “As a result, prospective teachers can successfully pursue a career as in elementary education if they have a propensity to avoid mathematics” (Beilock et al., 2010, p. 1850). Interestingly, elementary education majors “have the highest levels of math-anxiety of any college major” (Hembree, 1990, p. 35).

Not only do individuals with unproductive mathematical dispositions avoid mathematics, but

They also tend to perform more poorly on mathematics assessments than their mathematical ability would indicate they should… This is because [unproductive
mathematical dispositions are] not simply a proxy for poor math ability. Rather, the fears that people with [unproductive mathematical dispositions] experience when they are called on to do math—whether it is working through a problem at the chalkboard as an entire class looks on, taking a math test, or even calculating a restaurant bill—prevent them from using the math knowledge they possess to show what they know. When worries and self-doubt occur, thinking and reasoning can be compromised. (Beilock et al., 2010, p. 1850)

Unproductive mathematical dispositions have been recognized as an obstruction to mathematical achievement (National Mathematics Advisory Panel, 2008). However, fears and anxiety about math in early childhood teachers has particularly pervasive consequences than merely impacting the teacher’s personal mathematical achievement. If teachers who have an unproductive mathematical disposition are charged with teaching mathematics to early grade elementary students, the teachers’ anxieties about mathematics has the effect of transmitting their unproductive mathematical disposition to their students (Beilock et al., 2010).

**Implications of Mathematical Disposition to MEST Courses.** First, it is necessary to acknowledge that many elementary teachers do not have a productive disposition toward mathematics (Ma, 1999). Further, the MEST sequence may be the only mathematics courses prospective elementary teachers take at the college level.

Prospective teachers at the university where the research was conducted also take a mathematics methods course in the college of education, but that is focused on more knowledge of content and teaching (KCT) and knowledge of content and students (KCS)
and less on the more pure mathematical content courses (CCK and SCK). Thus, although there is a distinct partition of the sorts of knowledge taught in these two courses, a candidate's mathematical disposition could still be affected.

This is one of, if not the last opportunity for candidates' mathematical disposition to be influenced in a mathematics class; it is important that interventions are presented to prospective teachers to improve their mathematical disposition before entering the mathematics methods course, the teaching experiences related to that methods course, and their professional teaching internship.

**Perry’s Theory of Intellectual Development in Young Adults**

Perry’s scheme is comprised of nine distinct *positions*. The term position is used as opposed to stage because “stage refers to a relatively stable and enduring form, pattern, or structure of meaning making that pervades a person’s experience” (Perry, 1970, p. 21). Perry did not see the positions as stable, and did not make any assumptions to the length of time a person may spend in a position, thus not assuming there would be a stable or predictable pattern. Further,

Amid the variety and range of structures a particular student uses to make sense of the various aspects of the world at any particular point in time, position could express a central tendency in the person’s meaning making… Also, the term *position* implies the *place*, or vantage point, from which the student views the world. (Perry, 1970 p. 97-99)

Perry uses the term *Absolute* in conjunction with *Truth*—by which he means “unchanging, universal, timeless facts and knowledge” (Perry, 1970, p. 36). *Truth*
resides in the possession of the Authorities in what Perry terms the Absolute. Authorities are seen as having been granted their authority from myriad sources: i.e. knowledge, wisdom, influence, power or skill. It is important to note that Perry capitalizes terms like Truth, Absolute, and Authority to emphasize that the nature of these terms is derived from a power both external to and higher than the student.

**Definition of Terms.** Perry (1970) identified three alternatives to progression through the model: temporizing, retreat, and escape. Temporizing would occur when a student stalled in a position and intentionally delays forward progress. In this case, the student was usually aware of the requirements of the position ahead, but would not proceed. Retreat is “a backward movement to the relative safety and security of dualism—a world where right and wrong were clear and ambiguity did not exist” (Knefelkamp, 1999, p. 4). This occurred most often as a negative reaction to the complexities presented by multiplicity. Escape describes when students avoided moving past relativism to making commitments. “Students who took the escape route realized that it was easier to remain in a relativistic stance than to face the difficulty of making commitments and personal choices” (Knefelkamp, 1999, p. 4).

Knefelkamp (1999) found another alternative to progression: functional regression. Functional regression is the “process where students who were undertaking new learning in a new environment regressed to previous positions until they felt comfortable in the new environment” (p. 5). Interestingly, Knefelkamp saw functional regression as developmentally appropriate; since to progress to the next position, it was necessary for
that student to retreat to previous sense making patterns as to become comfortable with
the demands of the next position.

Different researchers have partitioned and organized Perry’s theory in myriad ways.
As indicated earlier in the chapter, Perry’s original schema contains nine separate
positions. However, many times these positions are grouped for easier use and
understanding. Kloss (1994) suggests

That the dominant model is four groups within the nine positions: dualism
(Positions 1 and 2), multiplicity (Positions 3 and 4a), relativism (Positions 4b, 5,
and 6), and commitment in relativism (Positions 7, 8, and 9). (p. 81)
Perry (1970) originally organized the positions into three groups: (a) Positions 1, 2, & 3
is the transition from dualistic thinking to the beginnings of relativistic thinking; (b)
Positions 4, 5, & 6 is the development of relativism; and (c) Positions 7, 8, & 9 as
developing commitments in relativism. However, Love and Gutherie (1999) organize
Perry’s Theory into two orientations: pre–Position 5 (dualism and multiplicity) and
see the most significant difference in the student’s sense making, as students transition
from knowledge and truth as being universal (as a dualistic or multiplistic thinker would)
to being relative. Due to the nature of the study and its participants, this dissertation
focuses on Position 2 through Position 5 and the transitions among them as “providing
information on the transitions between positions allows for clearer application of the
For added clarification, Knefelkamp’s (1999) labels will be used as her terms are more prevalently used in contemporary research (Love & Gutherie, 1999; Evans et al., 2011).

**Position 1: Basic Dualism.** For students that see their experiences from Basic Dualism, the world is separated into two conflicting absolutes—good versus evil, right versus wrong, etc. *Authorities* know everything and define the absolute and unquestionable *Truth*. *Authorities* possess the correct answers to every possible question and are responsible for identifying what is good and what is bad (Perry, 1970). The universe of the *Authorities* has no disagreement, and in order to know *Truth*, a student need only listen to the *Authorities*. Every problem is solvable, and the answer to any question or problem can be found by obeying and conforming to *Authorities*.

Transitioning to the next position is prompted when the student begins to recognize that some *Authorities* have differing opinions and do not always agree on what is right and wrong (Perry, 1981). Research has confirmed that it is rare for college-aged students to still think from this position (Perry, 1970; Knefelkamp, 1999; Love & Gutherie 1999; Evans et al., 2011) at the end of their first year. In fact, in Perry’s (1970) original study found that many students who were still in Basic Dualism when they started college began to transition to the next position when confronted by their peers, and that living in residence halls helped facilitate this process. However, the expectation of the existence of right-wrong answers persist as an integral assumption that underlye the first four positions of Perry’s scheme. “It is only in Position 5 that the discovery of a simple right
answer becomes recognized as the exception rather than as the rule” (Love & Gutherie 1999, p 9).

**Position 2: Strict Dualism.** This stage is characterized by students recognizing the existence of multiplicity but are oppose to it. *Multiplicity* is defines as an acknowledgement of the existence to multiple and conflicting ideas, opinions, answers, and points of view. “In this position the student remains loyal to *Authority*, still seeking truth from professionals,” such as professors, religious leaders, advisors, or from books written by *Authorities* and oppose abstractness, complexity, pluralism, diversity, and interpretation (Love & Gutherie, 1999, p. 9). Perry (1970) also noticed that students in Strict Dualism can “express fear, stress, and sadness when they realize that the way they have known (that is, the world as absolutely known and knowable) is at times no longer valid” (p. 86). Since multiplicity is not seen to be legitimate, students must try to reconcile its existence. For example, students may acknowledge that differences of opinions exist, but view those differences as temporary as the *truth* will be revealed later; or students may reason that questions which do not have definitive answers are artificially constructed by *Authorities* to make students think. In this position, students differentiate between *good Authorities*—whose *Truths* align with the world-views which they are familiar—and *bad Authorities*—who either support multiplicity or whose *Truths* differ from the students’ world-views.

Students may begin to transition when they see *good Authorities* admit “that they do not have all the answers” (Perry, 1981, p.125). This can lead the students to see multiplicity as a legitimate world-view, and transition to the next position. However,
students still struggling to transition could categorize subjects into *definite subjects* where there are right and wrong answers (i.e., sciences and mathematics) and *vague subjects* without right and wrong answers (i.e., humanities and social sciences) (Perry, 1981).

**Position 3: Early Multiplicity.** Students at this position accept there is uncertainty and pluralism; “however this uncertainty does not affect the nature of *Truth* itself” (Love & Gutherie, 1999, p. 11). In this position, students view uncertainty as temporary, and given enough time the one universal *Truth* will be discovered for all situations. Students can tolerate much higher levels of uncertainty than they could in Strict Dualism, but students are still uncomfortable with uncertainty. However, in cases when the universal *Truth* is not known, then there is also no wrong answer. Rather, when the *Truth* is uncertain, each person can have different, personal opinion. And because everyone has their own personal opinion, there is no one right answer; and thus rightness cannot be used as a means of evaluation. For example, a student whose worldview stems from this position may question grades or systems of grading, with the reasoning that if there are no universally right answers, then there is nothing left as a basis to judge schoolwork except style and good expression. In these cases, many students attempt to discern what the *Authorities* want and then try to conform their work to the perceived desires of *Authorities* (Perry, 1970).

Transitioning from this position begins as students realize that uncertainty is not isolated to certain questions and ideas, but is instead pervasive. Further, students begin to see that the chance of universally right answers being found for many questions is
minimal. With uncertainty being unavoidable, the students’ formerly pervasive mooring Authorities and Truth is undermined further (Perry, 1981).

**Position 4: Late Multiplicity.** In previous positions, students widely varied in how they experienced those positions and transitioned to new positions. “However, in Position 4 the differences were so dichotomized that Perry and his colleagues identified two different paths students took. Basically, students split into two groups in Position 4, only to be reunited in Position 5” (Love & Gutherie, 1999, p. 10). Perry (1981) found that on rare occasion a few students “actually proceed from Position 4a to Position 4b before moving on to Position 5” (p. 81). Further, he “found that the path students took seemed to be dictated by their relationship and identification with Authority—that is, the balance between a student’s tendency toward opposition on the one hand and adherence on the other” (p. 82).

**Position 4a: Oppositional Alternative.** Perry (1981) describes students’ world-view from this position as follows:

These students create the double dualism of a world in which the Authority’s right-wrong world is one element and personality diversity is the other. The students have thus succeeded in preserving a dualistic structure for their absolute freedom. (p. 84)

By *double-dualism* Perry means that a student divides all issues that they encounter into one of two categories. For many questions and issues, Authorities were still the sole proprietors of the Truth. In this position, students maintain the dualistic, right-wrong world-view. In the cases of questions and issues where there are no certain answers, then
the student believes everyone had a right to their opinion, and that no opinion is superior or inferior to another. Because of this, “in these issues no one has the right to call anyone wrong” (Love & Guthrie, 1999, p. 11). However, unlike in Position 3, in the Oppositional Alternative Position, students now claim ownership of multiplicity, with their opinion on unanswered topics “equal in legitimacy to the Authority’s” (Perry, 1970, p. 83). Perry (1981) reasoned

That multiplicity should not be dismissed as mere license to discount others who disagree with one’s opinions… Instead, the belief that all opinions have equal worth and validity expresses a respect for others through a respect for their views. (p. 85)

At times, when a student begins to confront Authorities (such as college professors) about the validity of their claims, they may become trapped by their own argumentativeness. “Unable to leave well enough alone, [students] demand that Authority justify itself by reasons and . . . by evidence in order to prove this or that opinion any more worthwhile than their own” (Perry, 1970, p. 99). Inadvertently then, the students begin to develop their relativistic sense making by their need to rationalize their opinions, world-views, and judgments

“The establishment of a domain separate and equal to that of Authority, in which the self takes a stand in chaos, will provide (once contextual thought is discovered to provide some order) a platform from which Authority may be viewed with entirely new eyes. . . . The bridge to the new world is the distinction between an opinion and a supported opinion.” (Perry, 1970, pp. 99–100)
Position 4b: Adherence Alternative. From this position, students can recognize and accept ambiguity, diversity, and differences in opinion. Although students could recognize these characteristics at earlier positions, in Adherence Alternative, “issues of context and rules of evidence are included in order to allow for analysis, comparison, and evaluation of opinions, points of view, and interpretations” (Perry, 1970, p. 103). For students reasoning from this position,

Knowledge is viewed as contingent and contextual; ideas are better or worse rather than right or wrong. Students who make sense of the world from position 4b recognize these aspects of relativism but still see relativism as subordinate to the overall multiplistic nature of the world. (Love & Guthrie, 1999, p. 11).

The major change in Adherence Alternative is from the knowledge of the Authority to the practices of the Authority—from “what they want” (Position 3) to “the way they want us to think” (Love & Guthrie, 1999, p. 11). For example, students begin to think of course instructors as not looking for a right or wrong answer, but rather believe those instructors seek to have students provide the rationale behind their thinking. This leads to the great paradox in this position: “that these students were trying to learn to think independently and critically out of a desire to conform to the expectations of Authorities” (Perry, 1970, 112). In this position ‘Reasoning’ provides the lever that will move knowledge from the dualistic realm to the qualitative… The requirement that an answer or opinion be reasonable raises the possibility that some questions may have [several] legitimate answers and thus that some answers will be more legitimate than others. (Perry, 1970, p. 102)
As students become cognoscente of the influences of context in all decision-making, they transition from “seeing relativistic thought as a special case to recognizing that relativistic thinking will be required more frequently and will work more frequently both in coursework and outside of academics” (Love & Guthrie, 1999, p. 11).

Perry (1970) and his research team found that the preponderance of college students transitioned through this position.

**Position 5: Contextual Relativism, Relational Knowing.** Perry (1970) noted, “Up to this point students have been able to assimilate [new ways of thinking] to the fundamentally dualistic structure with which they began” (p. 109). However, transitioning from either Position 4a or Position 4b to Position 5 “involves adopting a way of understanding, analyzing, and evaluating that requires a radical re-perception of all knowledge and values as contextual and relativistic” (Perry, 1970, p. 119). The thought processes necessary for Position 5 also promote metacognition—the ability for an individual to examine and reason about one’s own thinking and thought processes.

In the transition to Position 5, relativistic thinking is made routine. These thought processes first must be done consciously, but after practice and refinement they become automatic (Perry, 1970).

“Complexity is expected, and the simplicity of dualism is consigned to the subordinate status of a special case. The notion of Authority becomes authority, that is, authority loses its status as not being open to challenge. Instead, authority’s assertions are now open to analysis, evaluation, and the requirements of contextualized evidence. Students recognize the existence of multiple (and often
conflicting) authorities. Authorities are recognized as groping in a relativistic world along with the students, though they may be more advanced in their experience and in their expertise in groping.” (Perry, 1970, p. 89-90).

Perry (1970) found that this transition was at once one of the most difficult accommodations for students while also being the least eventful. Where students were aware of and remembered the transitional processes between and within the other positions, many times they were not able to recollect their transition to Position 5 sense making. The prominent characteristics of this position are as follows:

- A collapse of the student’s old structure of identity,
- A realization of growth and competence in a relativistic world;
- An altered relationship to authority as peers and equals;
- A new capability for detachment; and

Perry (1970, 1981) did not observe a student who had accepted relativism show any evidence of regression back to positions where absolutism was the dominant sense-making scheme.

**Positions 6–9: Commitment in Relativism.** Perry (1970) stated “in relativism one is threatened with unbearable disorientation and that students had three alternatives: to go limp, become an active opportunist, or transcend the disorientation through commitment” (p. 128). In Position 6 (commitment foreseen) students begin to see they must make commitments in order to establish themselves in a relativistic world. Here, many
students experience a need to begin making personal choices, as “to remain undefined or uncommitted would be irresponsible” (p. 133). However, students in this position cannot actually make decisions, establish commitments, or adequately narrow their possible commitments. Positions 7–9 (initial commitment, orientation in implications in commitment, and developing commitments) endeavor to diagram the diffusion of commitments during the person’s lifetime.

**Applications of Intellectual Development Theory to MEST Courses.**

Although it is difficult to speak about the whole of teacher candidate enrollment in MEST courses, many of these candidates are traditional college-age students, and thus according to Perry are most likely in Position 3 or Position 4 (a or b). Therefore it is important to organize the learning environment to take advantage of a transition from *Authoritative Truth* to multiplicity and relativism. This can be especially tricky in a subject as *Truth* oriented as mathematics is, at times, seen to be. One strategy to engage prospective teachers transitioning from 4a would be to ask candidates to analyze the strategies and algorithms they have been taught to use to solve problems. Another way to aid in this transition is to allow the prospective teachers to use multiple methods to solve the same exercise and find the same answer. Then encourage discussion to discern each method’s strengths, weaknesses, and how that one method is not necessarily superior to another. Finally, instructors should welcome questioning and present multiple (even disagreeing) opinions on mathematics and mathematics education to encourage classroom discussion.
Problem-Based Learning

Problem-based learning (PBL) is a “student-centered pedagogy in which students learn about a subject in the context of a complex, multifaceted, and realistic problems” (Hmelo & Evensen, 2000, p. 1). In this style of learning, students are taught by working in groups to solve problems that they may face outside of the classroom. “Working in groups, students identify what they already know, what they need to know, and how and where to access new information that may lead to resolution of the problem” (Abshire, 2010, p. 39). Originally designed for use in medical school classes (Barrows, 1996), PBL has expanded to be used in multiple different subjects in both the K-12 and college settings (Amador, Miles & Peters, 2006; Barrows, 1996; Duch, Gruh, & Allen, 2001; Gasser, 2011). Implementation of PBL—and other student-centered methods of teaching—have been motivated by a deepening understanding of how student’s learn (NRC, 2000).

Although PBL can take many different forms, Schmidt, Rotgans, and Yew (2011) see PBL as defined by six foundational characteristics.

- Learning is student-centered,
- Learning occurs in small groups,
- The teacher acts as a facilitator of learning
- Problems act as the organizational focus and primary stimulus for learning
- Solving the problems help students develop new problem-solving skills, and
- Learning is student-directed.
In PBL, the primary role of the instructor is to prepare and present thoughtful problems, provide scaffolding for the students, facilitate discussions around the problem and solution(s), and model the problem solving process for students. Also, the instructor “must build students confidence to take on the problem, encourage the student, while also stretching their understanding” (Wheatley, 2010, p. 4). Problems used in class ideally should be able to be solved in many different ways and may have more than one viable solution. “A good problem is authentic, meets students’ level of prior knowledge, engages students in discussion and is interesting” (Schmidt, Rotgans, & Yew, 2011, p. 799).

Sweller (2006) points out that while PBL is an effective strategy as learners become more competent, it may be a less effective strategy for training novices. Certainly active problem solving is useful as learners become more competent, and better able to deal with their working memory limitations. But early in the learning process, thus the rigors of active problem solving may become an issue for novices. Once learners gain expertise the scaffolding inherent in problem-based learning helps learners avoid these issues. (p. 166)

Sweller (1988) propose that for PBL to be effective for novices, it is necessary for the instructor in the beginning of the course to use other forms of instruction to model the tenants of PBL to gradually be replaced by problems that the students should solve on their own.
Implications for MEST courses. Although MEST instructors at various colleges and universities take differing approaches to teaching MEST students, the MEST 1 course being analyzed chooses to use this model in its classroom.

Assessment in Mathematics Education

For the proposes of this dissertation, we will define assessment in mathematics education as Niss, (2010) defines the term:

Assessment in mathematics education is taken to concern the judging of mathematical capability, performance, and achievement—all three notions taken in their broadest sense—of students whether as individuals or in groups, with the notion of students ranging from Kindergarten pupils to PhD students. Assessment thus addresses the outcomes of mathematics teaching at the student level. (p. 3)

While the nature of assessment in mathematics education is a broad and constantly evolving topic, the National Research Council does not see it as changing quickly enough to meet the needs of education in the 21st century.

The principles and practices of educational assessment have changed over the last century, but not sufficiently to keep pace with the substantial developments that have accrued in the understanding of learning and its measurements. (NRC, 2001b, p. 313)

Even though assessment is more complex than the following categories may suggest, for convenience assessment is sometimes described using the following distinctions.

Formative Assessments and Summative Assessments. Assessment is often divided into formative or summative as a means to understand the goals of the
Formative assessment is any assessment whose primary goal is informing the teacher or the student of the student’s learning of a concept or topic (Black & Wiliam, 2001; Black et al., 2003). Formative assessment is generally carried out during the course of instruction and is used to inform and enhance instruction.

An assessment activity can help learning if it provides information to be used as feedback by teachers, and by their students in assessing themselves and each other, to modify the teaching and learning activities in which they are engaged. (Black et al., 2003, p. 2)

Summative assessments “are tests given at the end of a lesson, semester, or school year to determine what has been learned” (Caffrey, 2009, p. 6). These assessments do not influence the teaching of that material to that group of students. Stake (as cited in Dirksen, 2011) describes a simple example to distinguish these two paradigms. “When the cook tastes the soup, that’s formative. When the guests taste the soup, that’s summative” (p. 26).

**Formal Assessments and Informal Assessments.** Formal assessments are assessments used primarily to assess student learning in an official manner (Caffrey, 2009). These typically include exams, quizzes, essays, homework, or any other assessment that produces documentation that the teachers may access. These assessments are usually used in summative assessments, for instance computing a student’s final grade for a course.

An informal assessment, on the other hand, is usually much more casual and does not necessarily produce a document. Informal assessments can include observations,
discussions, self-evaluations, peer-evaluations, or ratings scales. Many students with math anxiety may benefit from the more casual nature of informal assessments (Beilock, 2010).

**Norm-Referenced Assessments and Criterion-Referenced Assessments.** Assessments may be measured against the population of the students being assessed, or against a set of standard concepts that the student is expected to have mastered. In the former, norm referenced tests “produce raw scores that are transformed into standard scores using calculations involving the mean and standard deviation. The standard score is used to report how a student performed relative to peers” (Caffrey, 2011, p. 7). In the case of criterion-referenced assessments, the student’s knowledge is compared against set standards that all students are held. Whereas in norm-referenced assessment the student’s learning is compared to other student’s learning, in a criterion-referenced assessment, the student’s learning is compared to a predetermined criterion.

**Goals for Assessment.** Assessment of students’ mathematical learning is a multi-faceted concept with equally multi-faceted goals and objectives.

Current assessment modes and practices involve conflicting interests, divergent aims, and unintended or undesired side effects. In particular, it is difficult to devise assessment modes which at the same time: (a) allow us to assess, in a valid and reliable way, the knowledge, insights, abilities, and skills related to the understanding and mastering of mathematics in its essential components, (b) provide genuine assistance to the individual learner in monitoring and improving his or her acquisition of mathematical insight and power; (c) help the individual
teacher in monitoring and improving his or her teaching, guidance, supervision, and counseling [sic]; [and] (d) assist curriculum planners and authorities, textbook authors, and in-service teacher trainers in adequately shaping the framework of mathematics instruction. (Niss, 2010, p. 5)

Assessment is useful as it gives the student information about his or her learning—in relative or absolute terms, gives the teacher information about the learning of the individual student and of the class, and information to other interested parties (i.e. parents, school administrators, or other educational institutions). However, educational assessment is also used determine what Ginsberg, Jacobs, & Lopez (2010) refer to as a social reality. Assessments, and the grades which stem from them, create a stratified system that places a value on one student over another. This social reality can factor in determining which organizations the students can belong, in which future educational intuitions the student can attend, or which career opportunities will be available for the student.

In general, feedback given as rewards or grades enhances ego rather than ask involvement—that is, it leads students to compare themselves with others and focus on their image and status rather than think about the work itself and how they can improve it…Feedback which focuses on needs to be done can encourage all to believe that they can improve. Such feedback can enhance learning, both directly through the effort that can ensue and indirectly by supporting the motivation to invest such effort. (Black et al., 2003, p. 46)
Ultimately, these high stakes for assessment places additional pressure on the students to succeed, which may compromise learning objectives (Webb, 2010).

**Implications of Assessment in MEST 1.** MEST 1 is a course that seeks to examine mathematics much more deeply than other mathematics courses the prospective teachers may have taken. However, in doing so MEST 1 clashes with mathematics education standards, practices, and assessment paradigms to which the prospective teachers have become accustomed.

The ‘old’ way is working with textbooks, mainly solving textbook problems in order to obtain adequate skills with respect to examination. The ‘new’ way is what is recommended by the sort of teacher education which in various ways transcends the problem solving in textbooks. (Mellon-Olsen, 2010, p. 152)

In MEST courses, prospective teachers are asked to write explanations to accompany each of their solutions, which is the first time many of the prospective teachers have been asked to reason about their mathematical procedures and processes. These explanations are roughly 40% of the teacher candidates’ grade in the course, and because such a significant portion of their grade is crafting these explanations, there is a substantial amount of pressure for the prospective teachers to quickly adapt to this difficult new aspect of mathematics assessment, as the candidates’ final summative grades influence the prospective teachers’ lives beyond their mathematics learning in MEST 1. And this added pressure might negatively impact the prospective teachers learning in MEST 1.
Chapter 3: Research Design and Procedures

In order to reach a sufficient depth of understanding of the participants’ experiences related to the research questions, this study employs qualitative methodology. Denzin and Lincoln (2005) offer the following definition that recognizes the cross-disciplinary nature of qualitative research:

Qualitative research is multimethod in focus, involving an interpreting, naturalistic approach to its subject matter. This means the qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. Qualitative research involves the studied use and collection of a variety of empirical materials—case study, personal experience, introspective, life story, interview, observational, historical, interactional, and visual texts—that describe routine and problematic moments and meanings in individuals’ lives. (p. 2)

By journeying with the prospective teachers as they proceeded through these courses, this researcher is better able to understand how the course affects candidates’ Learning Mathematics, their intellectual development, and their mathematical disposition. By using qualitative methods to travel with the participants, the researcher gains an understanding of the effects of MEST 1 and experience of being in MEST 1 from the perspective of the prospective teacher. Further, this dissertation is a case study. “Case study research involves the study of an issue explored through one or more cases within a bounded system (Creswell, 2007, p. 57). Creswell (2007 goes on to describe case-study research as
A methodology, a type of design in qualitative research, or an object of study, as well as the product of the inquiry. Case study research is a qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth analyses involving multiple sources of information… and reports a case description and case-based themes (Creswell, 2007, p. 57)

The Case

This research project is set in a large Midwestern university. This university is a large PhD–granting institution with a strong tradition of academic and educational research. The university has an increased emphasis on recruiting top graduating high school students and improving their academic standing at all levels—undergraduate, graduate, and professional.

According to the undergraduate course catalogue, the university requires that all students who wish to enter the elementary school teacher education program take the three-course MEST sequence—MEST 1, MEST 2, and MEST 3—from the Mathematics Department prior to applying to the teacher education program. According to online course descriptions, MEST 1 focuses on number, numeration, and operation, MEST 2 focuses on geometry and spatial reasoning, and MEST 3 focuses primarily on algebraic reasoning. The MEST courses at this university are taught emphasizing new standards and current research on what consists of best practices in the elementary mathematic classroom.
The course coordinator of the MEST courses is a mathematics educator housed in the Mathematics Department. However, because of the great demand for these courses (especially MEST 1 and MEST 2), MEST courses are not just taught by mathematics educators. Occasionally, mathematics professors with a wide variety of specializations, full-time and part-time non-PhD mathematics instructors, and graduate teaching assistants (in both mathematics and mathematics education) working toward their PhD teach these courses. Although the university takes care to standardize instruction by standardizing syllabi, creating common exams, and offering meetings for those inexperienced with the MEST course, instructors enter the course with a broad spectrum of experience with mathematics education—ranging from instructors who are elementary mathematics education professors and graduate teaching assistants, instructors with experience teaching K–12 mathematics, and instructors with no formal education training and only experience teaching college mathematics.

The course is divided into lectures and recitation sessions. There were three sections offered the Fall quarter being studied. The lectures meet three times a week (Monday, Wednesday, and Friday) for about an hour and enrolled a median of 53 students at the beginning of the quarter the study was conducted. The lecture sections being investigated were taught by two mathematics educators (one of whom served as the coordinator for the course) who were had taught MEST 1 and other MEST courses multiple times over their academic careers.
One lecturer declined participation in the study, so that instructor, the teaching assistant for the course, and all students in the class were not included in the investigation.

Each lecture section was divided into two smaller recitations with a median student enrollment of 25.5 students the studied was conducted. Each lecturer was paired with a recitation leader, both of whom are graduate students at the university studying mathematics education.

The candidates typically take this course during their freshman year. However, it is not uncommon to find students taking these course later in their academic career because of not receiving a high enough grade for advancement in the sequence (or admission to the teacher education program) and are repeating the course or the student has avoided taking the course because of the candidate’s math-anxiety (Beilock, 2011). This study does not discriminate based on the point in the candidate’s academic career when they are taking MEST 1. Rather, this study chose to select participants to represent a cross section of MEST 1 sections taught during the Fall academic quarter, as well as participants with a variety of academic positions, mathematical dispositions, intellectual positions, ethnicities (see Freire, 1970/2006; Spring, 2007), and genders (see Damarin & Erchick, 2010; Glasser & Smith, 2008; World Health Organization, 2010).

In order to help students succeed in the course, the university offers tutoring specifically for MEST courses in a dedicated room staffed by instructors and teaching assistants currently teaching MEST courses, as well as students who have successfully passed (with a B or better) the MEST courses.
Research Questions

This dissertation examines how prospective teachers learn mathematics in the context of college mathematics courses. Using the method of research questions recommended by Marshall and Rossman (2006) this study lists a primary research question separated into several smaller sub-questions. The primary research question is as follows:

How do prospective elementary school teachers experience college mathematics courses designed to teach them the foundational concepts of elementary school mathematics?

In order to problematize the research question to a form more suitable to this process, the primary research question will be divided into the following three sub questions:

- In what ways do college mathematics courses designed to teach the foundations of elementary school mathematics help develop mathematical knowledge for teaching (MKT) in prospective teachers?
- In what ways do prospective elementary school teacher’s mathematical disposition influence their learning although taking college mathematics courses designed to teach the foundations of elementary school mathematics and how do such courses influence their mathematical disposition?
- In what ways do prospective elementary school teacher’s intellectual development affect their learning in college mathematics courses designed to teach the foundations of elementary school mathematics and how does their participation in those courses affect their intellectual development?
Subjectivity—Self as Researcher

Kilborn (2006) suggests that “it is appropriate to comment on one’s own biography as it relates to the study because this is too an issue of perspective—personal perspective” (p. 546). Rather than assume that I can remove my subjectivity from my study, Glesne (2006) suggests that qualitative researchers should be aware of their own subjectivity—from the “qualities that will enhance [my] research” to the “biases that could skew the interpretation of results” if I remained ignorant of them (p. 42). Indeed, each person has a different set of experiences and thus a different worldview from which to analyze the results. However, “this unique signature is not a liability but a way of providing individual insight to a situation.” (Kilborn, 2006, p. 547)

I have always seen myself on the precipice of being a mathematician and an educator. From my second semester as an undergraduate, I decided to peruse both a degree in education as well as the mathematics courses required for a bachelor’s degree in mathematics. At my school, this put me into a unique place among my peers: all the prospective teachers taking education classes with me saw me as more mathematician than teacher, and all the students taking upper-level mathematics courses with me saw me as more an educator than a mathematician. Although this duality set me apart in both of my peer groups, it also provided me unique opportunities that I would not have been able to access without that duel role. On one hand, I was able to do mathematical research as an undergraduate and present that research at small regional conferences. On the other hand, I was able to enter public school classrooms, teach as a teaching intern and student teacher, and consider mathematics education issues—both in terms of the individuals
learning of mathematics as well as issues involving educational politics and the practicalities needed to efficiently manage a mathematics class—that many of my peers in mathematics classes never confronted before being asked to teach a class as a teaching assistant.

I continued to bump into this dichotomy when I began my master’s degree and later in my doctoral career. My masters’ thesis was on the use of pseudoprime numbers in modern public key ciphers (Feldhaus, 2004), however since I had a background in education, I was tasked with teaching that school’s MEST 1 courses. After finishing my master’s degree, I taught at a public high school for a year. I had originally intended my break from higher education to be longer, but due to budget shortfalls my position was eliminated. This hastened my return to higher education, and renewed my original goal of finishing my PhD and teaching at a college. As part of my teaching assistantship, I was once again tasked to teach the department’s equivalent of the MEST courses, which I did happily. In both schools, I enjoyed the unique opportunities that MEST courses offered me as a teacher—chances to teach underlying mathematical principles over algorithms, to examine reasons behind mathematical concepts most of my students had taken for granted, and to explore mathematics that was wholly and completely relevant to the careers of those students in the class. At both schools, every student in those classes was a prospective teacher, and would eventually need to use the models, principles, and ideas discussed in MEST courses in their eventual classrooms.

However, as I said before, I live in a unique place as a mathematician and an educator. I found that at both schools there was a wide array of teaching styles, methods,
and techniques used by instructors in these courses. Some instructors focused on group-
work while others focused on lectures. Some instructors relied heavily on manipulatives,
while others simply drew pictorial representations of those manipulatives. And some
instructors were excited to teach MEST courses because of their unique nature while
others viewed the different focus of the courses as an annoyance. It seemed that in spite
of the departments’ attempts to standardize the curriculums and teaching practices in
MEST courses, each instructor brought a unique teaching style to their classroom, and
thus each MEST class was different.

This led to teacher candidates having different educational experiences in these
classes. Candidates, after completing MEST courses, had varying levels of success with
the mathematical concepts taught in the courses or applying those concepts in the
elementary school classroom. I am interested in understanding what teaching strategies
and techniques are effective in teaching foundational mathematics in MEST courses.
Although I believe that techniques that are closely aligned to those considered best
practice by current education research will be more effective than those that are not, I am
interested to see if different techniques may be more or less effective for a given
candidate based on their previous mathematical disposition or their intellectual
development. This interest eventually led me to alter my career path from being an
algebraist to return to educational issues and peruse this passion as my dissertation.

**Participant Selection**

The initial call for participants was comprehensive within the MEST 1 sections at
the participating university. The researcher had planned to select five to six participants
from a large number of volunteers based on their course enrollment, preexisting mathematical disposition, and other differentiating factors. However, only five eligible individuals volunteered, and thus each was selected as a participant. Happily, these participants did have a wide variety of mathematical dispositions and represented both sections of MEST 1 being studied (see Ch. 5 through Ch. 9).

The primary concern for this study is working with participants with a wide spectrum of mathematical dispositions and previous mathematics course experiences, to best see the totality of how the course is taught in the quarter. Before the start of the Fall academic quarter, the researcher e-mailed the MEST 1 instructors to build entrée and asked for permission to speak to the lecture sessions after class to recruit participants. The course coordinator offered to instead use the university’s Learning Enhancement System to send e-mails to all students enrolled in MEST 1 (see Appendix A), and the researcher accepted the suggestion. The researcher asked for volunteers who would not mind being interviewed several times during the quarter, and as reciprocity offered the participants $100 for their time. These funds came from the Graduate Dissertation Fund from Patton College of Education at Ohio University. The researcher intended to recruit no more than 40 volunteers and analyze their mathematics disposition as determined by a survey that the mathematics department gave all MEST 1 students. This survey was originally an instrument to be used exclusively with this study, but has been adopted by the mathematics department for wider use among its MEST classes potentially as part of a separate study (adapted from Ferdinand, 1999). From these volunteers, the researcher intended to select 8-10 participants as the prospective teachers
with the intent to gain a wide range of mathematics dispositions, backgrounds, ethnicities, genders, and course enrollments, with the ideal being to have a least one prospective teacher enrolled in each MEST 1 course offered, and to represent a broad spectrum of pre-existing mathematical dispositions. In the case that the person selected from this process declined participation in the study, the researcher would have use the same process to select the appropriate replacement. As it happened, only five eligible candidates responded to the initial call for participants. Without request from me, the MEST 1 instructors also mentioned the study in class and the course coordinator resent the original call for participants to MEST 1 students. However, no new students chose to participate.

After conducting the introductory interviews, the researcher intended to select 4-5 participants who offered diverse a) preexisting mathematical dispositions, b) developmental positions, and c) course enrollment, as well as potential for gathering useful data (Patton, 2002). Thankfully, as will be discussed in later chapters, the five participants did offer diverse mathematical dispositions, were divided between both sections where the lecturers volunteered to participate, and met the expected range of intellectual positions.

Gaining Entrée

Entrée is defined as “gaining entrance, acceptance, and trust with those with whom you will be researching” (Glesne, 2006, p. 43). It is essential to gain the trust of the participants: both the prospective teachers and college instructors, in order to gain
“honest participant feedback freed from the constraints of nervousness, uneasiness, or the inherent researcher-participant power imbalance” (Glesne, 2006, p. 44).

In order to gain entrée with the participants, before the initial interview the researcher had a brief phone conversation with each participant in order to introduce myself and answer any questions they had regarding the study. While the researcher was researching with the participants, the researcher gave them his personal contact information (e-mail and cell phone) and had several personal conversations that did not involve the research topic.

**Data Sources**

For the purposes of triangulating the data (Patton, 2002; Glesne, 2006; Creswell, 2007), as well as understanding the changes in the participants’ mathematical disposition, intellectual development, and knowledge for teaching, this study used multiple data sources (Rubin & Rubin, 2006). Of primary use are semi structured interviews with the participants. These interviews were used to gauge the prospective teachers’ changes in mathematical knowledge for teaching, as well as to discern their intellectual stage and gain supplemental data regarding their mathematical disposition. To create a holistic view of the university mathematics classroom in which the candidates were learning, the researcher also included in-class observations and semi structured interviews with the course instructors and the course coordinator (who is also a participating instructor) for MEST 1.

**Mathematical Disposition Survey.** The first data collected were the participants’ responses to the Mathematical Disposition Survey (see Appendix C). This
survey attempts to determine the prospective teachers’ mathematical disposition and opinions regarding teaching mathematics. This is an amalgam survey adapted from Fennema and Sherman (1976) and the Mathematics Views and Opinions survey used by the Better Math Through Literacy program at Ohio University. Better Math Through Literacy is a professional development institute for in-service early childhood teachers designed to increase students’ mathematical performance by incorporating children’s literature into mathematics lessons. The survey is given at the beginning and end of the institute and attempts to measure changes in the teachers’ beliefs about teaching mathematics.

The Mathematical Disposition Survey was administered to every student in MEST 1 as part of a separate study being conducted by the University. The researcher is listed on that study as a primary researcher and thus has access to those scores. For ease of analysis, the survey questions are coded into four subcategories:

- Preparedness to Teach Mathematics (1, 6, 12, 15, 17, 19, 20, 25),
- Mathematical Beliefs and Attitudes (2, 3, 10, 11, 13, 16, 21, 23),
- Mathematical Self-Efficacy (5, 7, 14, 24) and
- Diligence in Learning Mathematics (4, 8, 9, 18, 22).

Each question is scored 1–5, with 1 representing the response that would indicate the most negative mathematical disposition element and 5 representing the response that would indicate the most positive mathematical disposition element.
Because of the small number of participants, it is not feasible to employ high-level statistical analysis on the sample. None of the participants in the study responded “NA” to any questions.

**Semi Structured Interviews with Participants.** The researcher conducted two semi-structured interviews with the participants: one at the beginning of the course and one at the end. The first interview served three research functions: to gain a more thorough mathematical history from the participant (see Appendix D), to gauge the participants’ intellectual development on Perry’s scheme (see Appendix E), and to administer the first set of mathematical tasks (see Appendix G). The researcher used the survey adapted from Ferdinand (1999) to gain greater knowledge of the participants’ mathematical history, disposition, and attitudes. To determine the participant’s intellectual position, the researcher used the instrument adapted from Baxter Magolda (1987), which has been regularly used and adapted (Baxter Magolda, 1995; Baxter Magolda & Porterfield, 1987; Steinke & Fitch, 2007). The researcher also asked the participants to complete three mathematical tasks adapted from Ma (1999). Two versions of these tasks were created (labeled Task Set A and Task Set B), and one set was administered as a pretest, while the other was reserved for a posttest. Whether the individual participant received Task Set A or Task Set B as their pretest was determined by the “Coin Flipper” option on the random number generator from random.org, with heads representing Task Set A and tails representing Task Set B.

The second semi-structured interview was during the final week of the course, and explored the effects the course had on the participants, especially relating to any changes
in Mathematical Knowledge for Teaching. The interview followed two parts, an interview designed to gauge the participants overall impressions of the course (see Appendix F) and the set of mathematical tasks which the participant did not complete during the first interview (see Appendix G).

These interviews were recorded digitally using a .mp3 recorder and were transcribed by the researcher. Once the interviews were transcribed, the digital recordings were deleted. The researcher also kept field notes taken during the interview as a supplement to the transcript. These notes were hand written on a legal pad and were later typed and stored digitally. Once typed, the original paper documents were shredded.

After each interview, the researcher recorded an audio log to serve as a reflection on the interview immediately after its conclusion.

This data served to help answer each of the three research questions.

**Unstructured Interviews.** The researcher met with the participants five times during the course apart from the initial and final interviews to discuss their experiences in the course. These interviews were unstructured and focused primarily on the participants’ experiences in MEST 1 and how the participants interpreted and understood those experiences. As recommended by Kvale and Brinkman (2009) the researcher used the following questions as primers for those interviews:

- How is the course going for you?
- What have you learned so far in the course?
- What topics are you struggling with? Why?
- What topics are coming easy for you? Why?
• What aspects of the class do you think help you learn?
• What aspects of the class are not helpful to your learning?

These interviews were transcribed by the researcher, with field notes and audio logs kept for data analysis.

**Participant Work.** The researcher collected ungraded samples of participant work to better understand the assessments being used by the course instructors. This was done to see how the participants were using the concepts taught in their class, were assessed on those concepts, and how the participants performed on those assessments. The researcher made copies of all work that was analyzed and returned the originals to the participants, and used the work samples to gain understanding of the prospective teachers’ gains in mathematical knowledge for teaching and overall intellectual development. This work included course exams (both quizzes and midterms), homework, and in-class activities. The collection of this work directly related to the students’ developments in mathematical knowledge for teaching (MKT).

**Semi Structured Interviews with the Course Instructors.** The researcher conducted semi structured interviews with the course instructors (both lecturers and recitation leaders) for MEST I (see Appendices H). These are adapted from a course instructor interview created by Ferdinand (1999). The researcher also adapt these questions to reflect themes emerging from other data sources.

During the academic quarter, the researcher interviewed the instructor of each MEST 1 course in which the participants were enrolled. From these interviews, the study gained a broader perspective of what the classroom culture was for the participants, how
each instructor viewed their role in the class, and how these views affected the in-class experiences of the prospective teachers.

After the final interviews, the researcher conducted an unstructured interview with the two participating lecturers to discuss themes that arose during the course of the research.

The researcher, who also typed notes taken during the interview as a supplement to the transcript, transcribed these interviews. Once transcribed, the original documents (both digital recordings and interview notes) were destroyed in the same manner as similar documents.

Interviews with the instructors (both lecturers and teaching assistants) served to help frame the context with which the researcher analyzed the remaining data. This information also formed a picture of the classroom culture, a key component to understanding participants’ mathematical dispositions and any changes within those dispositions (Feldhaus, 2010).

**Classroom Observations.** The researcher conducted classroom observations of the MEST classes in which the participants were enrolled—both the lecture sections and the recitation sessions. The intent of the researcher was to act as more of an observer as not to interfere with the classroom culture or power dynamics which may be present in the classroom. The primary data collection device for this data source was a field journal, which recorded the events that transpired in the classroom. These notes were first written and then later typed for organization and ease-of-use purposes.
Field Notes. These were the notes that the researcher collected while conducting interviews and classroom observations, as well as notes made after each interview or observation. The former were handwritten notes made during those instances, and were typed for analysis. The latter were audio-logs, and were partially transcribed when shown relevant to answering the research questions. By their nature, the field notes serve to supplement and preserve the authenticity of the experience (Patton, 2002), and were used in answering each of the three research questions.

Data Collection and Management

The primary data source is the semi structured interviews with the participants. These interviews were recorded using a digital audio recorder, dated, and saved as an .mp3 file in a folder identified by, first a randomly generated number (from 1-5) assigned to each participant and later, their pseudonym. These interviews were transcribed and de-identified by the researcher, dated, and saved in the participant’s folder. The original audio recordings were then deleted to better protect the anonymity of the participants.

Semi structured interviews with the course instructors were digitally recorded, transcribed, dated, de-identified, and saved to folders dedicated to collecting data regarding the culture of the course. Similarly, the original audio recordings were then deleted to better protect the anonymity of the participants.

Any work collected from participants was scanned and turned into a .pdf and saved into the participant’s folder. The original documents were promptly returned to the participant.
During the entirety of the data collection processes, the researcher kept a field journal to aid in the analysis of the data (Glesne, 2005; Bloomberg & Volpe, 2008; Desimone, 2009). These field notes were typed, de-identified and stored electronically. Field notes that correspond to participant interviews were saved in the folder labeled for each participant. Field notes for interviews with instructors were saved in that folder along with the audio recording and transcription. Observation field notes were saved in a folder dedicated specifically to them labeled “Classroom Observations.” The original hard copies were shredded.

After preliminary analyses, the researcher saved copies of data which go to answering the research questions in folders labeled “MKT” for data related to the participants’ mathematical knowledge for teaching, “MD” for data related to the participants’ mathematical disposition, and “CSD” for data related to the participants’ college student development. When themes emerged, new folders were created and dedicated to that theme, with copies of any data relevant to that theme redundantly saved in the new corresponding folder.

**Data Analysis**

To analyze the data, the researcher used a simplified version of the Stevick-Colazzi-Keen method as discussed by Moustakas (1994) and later Creswell (2007):

- **Describe personal experiences.** First, the researcher described the personal experience of the participants within the case studied—in this case MEST 1. This served as an attempt to remove any own personal experiences and biases (which cannot be done completely) to focus more on the experiences of the participants.
• **List significant statements.** Next, the researcher collected significant statements—from interviews and other data sources—concerning how the participants experienced mathematics in MEST 1. This was first done within participants to create the narrative experiences, and later across participants. Next, the researcher *horizontalized* the data by “organizing the data into a list of non-repetitive, non-overlapping statements” (Creswell, 2007, p. 159).

• **Creating themes.** Next, the researcher organized the statements and grouped them into meaningful units, better known as themes (Saldaña, 2009). Again, this was first done within participants and then across participants.

• **Create a textual description.** Then, the researcher wrote a description of *what* the participants experienced within the class.

• **Creating a structural description.** After that, the researcher created a “description of *how* the experiences happened… [in which], the researcher reflects on the setting and the context in which the phenomenon was experienced” (Creswell, 2007, p. 160).

• **Create a composite description.** Finally, the researcher created a composite narrative description that incorporates both the textual and structural descriptions. This composite description captures the essential components of the experience and serves as the culmination of the study.

**Credibility and Trustworthiness**

Although it is imperative to remain open to the ideas and methodologies of other researchers, research must meet standards to be considered credible. "Traditions are
important, even when one is taking an open stance, because they provide a set of orientating assumptions” (Jacob, 1987, p. 40). To undertake research, the researcher must have specific, justifiable goals that are based on theoretical reasoning. If research is to be considered valuable to educators and researchers unfamiliar with qualitative approaches, it must offer some evidence that the traditional notions of validity and reliability are either applicable or have direct analogues within the qualitative mathematical education research paradigm (Eiserhart & Howe, 1992). Unfortunately, as of 2012, no neat guidelines for credibility in qualitative research exist. Indeed, by the nature and intent of the research, it is unlikely that these standards or tests will or can exist (Silver & Kilpatrick, 1994). At the most, only general guidelines for credibility can be applied across all research. However, it is appropriate and important to consider design-specific standards that relate judgments of quality to particular methodological approaches. Further, making these criteria explicit aids in differentiating the perspectives needed to judge individual research. A crucial element in this process that researchers must address is that of openness, both in carefully reporting the research design and in making explicit the subjective nature of the researcher’s role in collecting and analyzing qualitative data (Patton, 2002). Ultimately, researchers must be concerned not only with the credibility and trustworthiness of their work, researchers must also establish themselves as a credible and trustworthy source for the reader, who is the final judge of the research and the theories extrapolated from it.
The researcher used a variation of Creswell’s (2007) questions to self-check for credibility within a case study. To that end, I as the researcher, repeatedly asked the following questions of myself:

- Am I remaining true to the philosophical tenants of qualitative research upon which the study is based?
- Have I clearly stated the case that is being studying?
- Am I clearly and explicitly using the data collection and analysis methods to which the research has ascribed (namely Mousakas, 1994)?
- Am I clearly and explicitly conveying the experiences of the participants? Does this include the context in which the experience occurred?
- Am I being appropriately reflexive throughout the study?

**Instructor and Participant Checks.** To add trustworthiness to the presentation of the data, this study chose to ask participants and instructors similar questions, as well as creating new interview questions to test statements gathered in interviews for accuracy. By *cross-comparing* (Wolcott, 2009) these different perspectives on in-class experiences, the research gains a holistic view of the in-class experiences, classroom culture, and assessments, as well as triangulates collected data for accuracy in participant reporting.

**Research Timetable**

Because of the nature of the study, it took place over one 10-week academic quarter. In brackets, the researcher mentions what data were collected and to which research question(s) the data are related.
• Before start of Fall Quarter: Gained entrée with the mathematics education professionals who coordinated and facilitated the MEST courses. [Gaining entrée].

• Before start of Fall Quarter: Gained approval to conduct the study from all appropriate authorities. This included the dissertation committee, the Mathematics Department at the university where the research was conducted, the Institutional Review Board (IRB) at Ohio University, and the IRB at the university where the research was conducted.

• Before start of Fall Quarter: E-mailed instructors and asked for permission to introduce myself and the study to the class. [Gaining entrée].

• Beginning of Fall Quarter: E-mail students enrolled in MEST 1, and asked for volunteers for study. Graded MD surveys and selected five participants. [Gaining entrée, Mathematical Disposition Surveys (RQ 3)].

• Fall Quarter: Beginning of Study: Conducted introductory interviews with participants. [Semi structured interviews (RQ 1, 2, 3), Field notes (RQ 1, 2, 3)].

• Fall Quarter: Weeks 2-3: Conducted first unstructured interviews with participants. E-mailed instructors and asked permission for classroom observations in Week 4/5 and Week 8/9, begin looking for convenient time for course instructor interviews. [Unstructured Interviews (RQ 1, 2, 3), Instructor interviews (context), Observations (RQ 2, 3), Field notes, (RQ 1, 2, 3)].
- Fall Quarter: Week 4-5: Conducted first classroom observations. Began preliminary data analysis. Began searching for emergent themes. [Observations (RQ 2, 3) Field notes, (RQ 1, 2, 3)].
- Fall Quarter: Week 5-7: Conducted second unstructured interviews, completed instructor interviews. [Unstructured Interviews (RQ 1, 2, 3), Instructor interviews (context), Observations (RQ 2, 3), Field notes, (RQ 1, 2, 3)].
- Fall Quarter: Week 8-9: Conducted third unstructured interviews, Completed second course observation. [Unstructured Interviews (RQ 1, 2, 3), Observations (RQ 2, 3), Field notes (RQ 1, 2, 3)].
- Fall Quarter: Week 10-Finals Week: Conducted second semi structured interviews, began organizing data into themes. [Semi structured interviews (RQ 1, 2, 3), Field notes (RQ 1, 2, 3)].
- Winter Break: Organize data collected, organize data into themes.
- Winter Quarter: Weeks 1-2: Created textual descriptions.
- Winter Quarter: Weeks 2-3: Created composite descriptions.
- Winter Quarter: Weeks 4-7: Organized descriptions into “Data Analysis” chapters.
- Winter Quarter: Week 8: Summarized results into “Chapter 10: Results.”
- Spring Quarter: Finalize first draft of dissertation.
- End Spring Quarter: Prepared dissertation for distribution to the committee.
- Summer Quarter: Defended dissertation.
Research in Action

Writing qualitative research is “a method of inquiry, a way of finding out about yourself and your topic” (Richardson, 2000, p. 923) and a “personal tale of what went on in the backstage of doing research” (Ellis & Bochner, 2000, p. 741). Qualitative research makes no claims of objectivity as good qualitative researchers are themselves the primary instrument used for analysis (Glesne, 2006). As Watt (2006) points out,

A retrospective examination of my own research permitted me to make meaningful connections between theory and practice. This inquiry thus provoked a depth of learning which may not have been possible through any other methodological means. By reconsidering my pilot study in this way, I experienced the extent to which reflection is an essential mediator in the research process. Reflective writing allowed me to meaningfully construct my own sense of what it means to become a qualitative researcher. (p. 84)

Exploring the Power Dynamic in the Researcher/Participant Relationship.

One of the first things I noticed during the research was the constant interplay of the power dynamics. To the lecturer-participants, I was simply a graduate student looking for a venue to conduct research. To the recitation leaders, I was more a peer: a fellow graduate student who was slightly more advanced in his degree. And to the student participants and other students in the MEST 1 class, I was another mathematics expert like their instructors and recitation leaders. For the sake of the research, I knew that I would need to minimize those power imbalances to best work with each of the participants.
Long before I met student-participants, I worked with Dr. Jones and Dr. Smith (pseudonyms) to finalize some of the details of my study. They were instrumental in helping me understand the culture of the MEST 1 course as well as helping me navigate many of the logistic details (rooms for interview, parking, etc.) of my study. In looking back through my notes, I now realize that any concerns about the power dynamic between Dr. Jones and myself or Dr. Smith and myself were purely an invention of my own nervousness.

I found my interviews with Faith and Lara much more akin to the conversations I would have with my peer graduate students than formal interviews. This is perhaps not surprising, as a graduate student myself, I occupy a position much closer to theirs than I do with the lecturers or the student participants. In both cases, once they realized I was more concerned with learning their perspectives than judging them against the perspectives of their lecturer, they began to open up to me.

The undergraduate students, both the student-participants and the other students in the MEST 1 classes, initially saw me as an expert and, in some cases, as another teacher. Many times when I was trying to observe the students working in groups, a student who was not a participant in the study would ask me if I could help them with a problem. Frequently, both the lecturers and recitation leaders would comment that the class behaved differently when I observed them.

When I first began interviewing them, my participants all saw me as another authority figure. Someone who was not teaching MEST 1, but could easy be another lecturer or recitation leader. As I look back on my research notes, I see myself making a
constant effort to self-identify as a student—dressing more casually, talking about my recent experiences in college, etc. In the end though, I do not believe that had any great effect on my research: my participants began to know me as a person and put those biases aside, and I did not interact with the other students enough to change their perception of me.

**Researcher, Not Teacher.** Although there was been much written on the paradigm of teacher-researcher (see Kincheloe, 2003; Mills, 2006; Morse et al., 2002; Hubbard, Shagoury, & Power, 2003) that was not the intent of this study. I was neither the lecturer nor the recitation leader for any of my student-participants. However, I am a mathematics educator, and during the research process, both my student participants and other students in the class would ask me for help understanding a concept or completing a task. And this put me in an unenviable situation: I could either alienate myself by not answering the question (and violate most of my instincts as a teacher) or answer the question and risk coloring my results. In almost every situation I was presented with this choice, I chose the latter. I did not want to risk the relationships I was developing, especially with the student-participants, and I can reduce my influence on the research by careful reflection on the data and disclosing those concerns here.

Also, I had to put aside all my inclinations about how I would teach MEST 1. I have taught similar courses in the past, and constantly I would compare and contrast how I would approach teaching a concept to the approach being used in MEST 1. As a teacher, I enjoyed this comparison, and in this process I learned several new strategies for the next time I teach a course similar to MEST 1. However, as a researcher, I made a
conscious attempt to not let my own experiences as a teacher influence my research, and let the course stand on its own and my participants speak for themselves.

The next several chapters will report the results of that research. Chapter 4 will describe MEST 1 as it is viewed by the university, the course instructors, and teaching assistants, as well as the daily routine of the course. Chapters 5–9 will chronicle the experiences of the participants as they progressed through MEST 1. Finally, Chapter 10 will describe the emergent themes and results of the research and Chapter 11 will discuss the significance of the results.
Chapter 4: The Culture of MEST 1

MEST 1 is different from many other college mathematics courses, and many other mathematics courses that the preservice teachers have taken earlier in their education. First, MEST 1 emphasizes both learning mathematics and changing the students’ (prospective teachers’) negative attitudes and beliefs about mathematics. Both the course documentation and the instructors recognize that many students enrolled in MEST courses have beliefs and opinions about mathematics that harm their ability to learn and teach mathematics, and thus have made improving those attitudes and beliefs a goal for the course. Second, MEST 1 is designed as a problem-centered course. Although many mathematics courses teach theories and give examples and exercises to illustrate the theories, MEST 1 gives students problems and uses those problems to generate discussion and discover theoretical principles in the problems’ solution processes. Finally, MEST 1 stresses that prospective teachers be able to write and speak succinct and precise explanations of the mathematical principles and steps used to solve problems. The preservice teachers were required to explain each of their mathematical processes when solving a problem, and which underlying mathematical principle allowed them to do that process. Because of these differences, many students initially struggle in MEST 1.

Overview of MEST 1

As stated earlier, this research took place at a large Midwestern university and focused on two lecture sections of MEST 1 taught in the Fall Quarter, 2011. Each lecture section was led by an experienced mathematics educator, namely Dr. Smith and
Dr. Jones (pseudonyms). Although one lecturer is listed as the course coordinator for the MEST sequence, they shared most of those duties equally. Each lecture section was divided into two recitation sections, and each lecturer was paired with one teaching assistant who led the recitation sessions. Dr. Smith worked with Faith (a pseudonym) and Dr. Jones worked with Lara (a pseudonym). Faith was a graduate student seeking an MA in mathematics with an emphasis in mathematics education, and Lara was a part-time instructor with a masters’ degree in mathematics education. Both Faith and Lara have experience teaching high school mathematics, but neither had teaching experience at the elementary school level.

The university’s course catalogue describes MEST 1 as a course designed for the “Development of basic ideas of arithmetic as appropriate for early elementary school teachers” and more specifically to develop an appreciation of mathematics; as well as developing of basic competency in the use of analytical thought and developing a cohesive body of useful mathematical knowledge with regard to topics encountered in elementary and middle school mathematics programs. Specifically, MEST 1 address the following topics:

- Fractions
- Addition and subtraction
- Multiplication
- Multiplication of fractions, decimals, and negative numbers
- Division, and
• Combining multiplication and division: proportional reasoning (University Online Course Catalogue, 2011).

Key in the course catalogue description is the development of both students’ mathematical skills and their knowledge necessary for teaching (i.e. MKT) and the development of an appreciation for mathematics and mathematical ideas (i.e. MD).

**Development of MKT.** MEST 1 is the first in a sequence of three courses. These courses focus on the foundational mathematics taught in elementary schools as well as mathematical concepts, connections between those concepts, and how the algorithms used to solve problems are related to those concepts. Further, MEST courses develop prospective teachers’ ability to explain those ideas to novices. To emphasize this idea, Dr. Jones and Dr. Smith included the following as the purpose of the course in the common syllabus given to all MEST 1 students,

This is the first course in the three-course sequence…[MEST 1] focuses on concepts of number systems and operations. The goal of this course is to prepare you to become teachers of elementary and middle school students. Knowing the mathematics for yourself is not the same as knowing the math for teaching. To that end, we emphasize explanations of mathematical ideas. To make this point very clear: Full credit will NOT be given for correct mathematical answers without an explanation that is clear and complete… Explaining your thinking verbally in small and large groups will prepare you to explain mathematics to your students. It will also help you clarify your own ideas and/or questions. (Syllabus for MEST 1, Fall 2011)
The key difference between these courses and other lower-level undergraduate mathematics courses taught at the university is the MEST sequence’s emphasis on explanation. It is not enough for a student to have mastery of the mathematical algorithms; students must also be able to explain their work clearly and concisely and connect the mathematical ideas used in their work.

This course, [MEST 1] even though it is a lower-level course could be considered a graduate course for the level of thinking we ask of them, especially compared to what they’ve done before. Obviously, in terms of content it isn’t a graduate level course, but in terms of the students’ thinking and thought processes and the explanations that we require of our students, I would put it up there. (Interview with Dr. Jones)

And in this process of crafting explanations, students are required to learn mathematics in a way that may be much different than how they have previously learned the subject, both in elementary school and in college mathematics classes. In reflecting on the differences between her experiences as a recitation leader for MEST 1 and her other experiences as a recitation leader for other courses, Faith shared the following insight:

[Being a recitation leader for MEST 1 is] very different from my experiences teaching other courses like Calculus or [College Algebra] or anything really. It seems like [the course’s] goals are different in that students are being encouraged to learn how to think about math instead of just come up with answers… In teaching [other college mathematics courses], I felt like my job [as a recitation
leader] was to present solutions; but in [MEST 1], my job is to encourage students to come up with solutions, discuss why or why not those solutions may be correct, and compare different solutions and processes that the students used to get to their solutions. So I find myself in the front of the class asking a lot of “why” or “how” as opposed to just telling them. (Interview with Faith)

In the process of having prospective teachers craft explanations, the designers of the course endeavor to teach mathematics in a more complete and thorough way than most of the students had previously experienced in hopes that (a) the preservice teachers will have a deep and thorough understanding of the mathematics being taught, and (b) they will endeavor to teach mathematics in that same way to their students. Dr. Smith states the problem and solution:

What we do, and we don’t like to say this too loudly, is very much about remediation. We are trying to stop a cycle where [elementary school mathematics] teachers do not know the mathematics and perpetuate errors and misconceptions and rule-oriented beliefs about mathematics with their students. What they come in with is what they go out and teach, and they come in that way because that’s what they were taught. It’s just terribly circular, and we feel that if we can help these students really learn mathematics and help them learn how to learn mathematics… then they can go out there and [pause]. At least the rock bottom we can hope for is that they don’t stop teaching math because they hate it so much. And maybe they may actually teach math because they believe in it, or maybe they have their kids try to do a problem instead of another set of exercises,
or maybe they have their [students] do some mental arithmetic. And any of that may potentially help break the cycle… On my worst days I think it’s all about remediation and on my best days I know that we are visiting these topics much more deeply than most of these students have ever seen them. (Interview with Dr. Smith)

The idea that MEST 1 acts as a remedy for the prospective teachers’ insufficient mathematics education background repeated itself multiple times throughout the course of the interviews. Further, the instructors of the course see MEST courses and similar courses taught at other universities as a means to break the “vicious cycle” Dr. Smith describes. For example, Lara recounted the following tales,

I would get a lot of students typing two times zero into their calculator—that showed me that they didn’t have a fundamental understanding of the math. And students would admit as such—one student told me she never wanted to teach anything higher than kindergarten because she couldn’t handle the math.

(Interview with Lara)

She went on to theorize

What happens is you have a lot of elementary teachers who don’t like math, so they teach it using this very rote method. So the kids who want to be teachers are scared of the math, and they want to teach the math that they were taught—the easy way. What we want them to do is teach it the hard way—the way that requires the students to think, but to do that the teachers have to understand the
math—have to be comfortable with it. And that is our struggle—getting students to be good math teachers and to understand the math. (Interview with Lara)

Dr. Jones put it more directly,

When I first started teaching MEST 1, I noticed a lot of those people who were going to become teachers eventually didn’t know a whole lot about mathematics. Not only skill-wise, but they couldn’t think, they couldn’t understand the concepts behind it. And this is elementary school mathematics. I had college students who didn’t know how to do elementary school math! (Interview with Dr. Jones)

Dr. Jones sees the students’ lack of mathematical knowledge and the need for MEST courses as a comment on the state of elementary school education in the United States:

Students shouldn’t have to take this course. Students should have these ideas, the [NCTM] standards integrated into their education…In other nations, I say $\frac{2}{3}$ times $\frac{5}{7}$ then people try to draw a model, relate the operation to an idea. If I go to Americans and I ask this in class, I say ‘$\frac{2}{3}$ times $\frac{5}{7}$, what do you remember about this’? And I am lucky if they remember an algorithm for computing it. I have not had one student from the US who tried to relate that operation to a picture. We shouldn’t need these courses. We shouldn’t have to try to get people to change their views on mathematics this late in the game. But they’ve had this bad math taught to them for 12, 13 years in spite of the standards. Teachers teach the way they were taught, and then we get more teachers teaching bad math. It’s a vicious cycle. But the only way to break that cycle is through courses [like MEST 1]. But that may not be enough. You need an absolute cultural change.
But ultimately, I can only do so much. It’s up to these kids to make this course obsolete, and make me find something else to teach. (Interview with Dr. Jones)

**Mathematical Disposition.** Interestingly, in the online course catalogue, “develop[ing] an appreciation of …useful mathematical knowledge” is listed as the first goal in the course, before even developing mathematical competencies. Dr. Smith commented on the mathematical dispositions of the students enrolled in MEST 1. “In some ways, [MEST students] are the most motivated students I’ve ran across. But the problem is they don’t like the math, and that can really derail them” (Interview with Dr. Smith).

Both of the teaching assistants noticed negative mathematical dispositions in their recitation sessions:

I think about half or more of the students in my class just don’t like math, maybe somewhere between 50% to 80% of the class. I think the rest don’t actively hate it, but it isn’t their favorite subject either. It may not be something they disliked, but it may not have been something they had a specific interest in. (Interview with Faith)

And again Lara,

A lot of [my MEST 1] students waited to take the course. Part of it is a lot of students just put it off because they hate math—they’re afraid of it. I can’t say why so many juniors and seniors are in the course [even though MEST 1 is designed to be taken as a first-year course], but I think math phobia and math anxiety play a big role in it. Students have even told me that they waited till the
last possible minute to take the class because they didn’t want to do math and they wanted to not do math as long as they could. (Interview with Lara)

However, negative mathematics dispositions are reflected in more than just mathematics anxiety (which this dissertation will examine further in the later chapters). Faith shares her experiences about how some of the greater cultural concerns may affect the prospective teachers.

There’s a stigma against math—from a teacher who was just a bad math teacher, or maybe a teacher that used math as a punishment, or maybe just some really bad experiences with math, for some reason a lot of people just don’t like math. And it is something in the culture as well, people think math is hard and that only super-smart people are good at math. There is a wall [between people who like mathematics and people who don’t], and that’s one of the things that [MEST] wants to eliminate. (Interview with Faith)

The idea that well-taught MEST courses can positively influence a prospective teachers’ mathematical disposition is one that is shared by both lecturers in the course. Dr. Jones sees positively influencing the prospective teachers’ mathematical disposition as more important than covering content in the course.

We have a goal for these students… to change their disposition about mathematics. I don’t care if we don’t get through everything—I don’t care!

What I want to do is [to] change these students’ dispositions to mathematics and change the way these kids think about mathematics. And I want students in [MEST courses] to expect to have to think about math, to expect to reason it out.
To expect to ask why and question what we’re doing. I want them to learn how to do mathematics, maybe really for the first time, and I want them to enjoy doing it.

(Interview with Dr. Jones)

Dr. Smith described a broader picture—that positively influencing the prospective teachers’ mathematical disposition will eventually lead to their students having more positive mathematical dispositions.

I want to encourage my students to have a can-do attitude about math. I want them to think that they can do it, so they can pass that can-do attitude on to their [students]…. But sometimes the students come into [MEST 1] with such a helpless and defeated attitude—that there isn’t anything to understand, I just need to copy and memorize. They don’t think that they can understand it, and that if they could then they would be some kind of a math genius or math professor. We want to show them they can understand this math, and they can teach this math so that one day their students can understand [it]. (Interview with Dr. Smith)

Both Faith and Dr. Smith commented on their students seeing mathematics as being a walled garden which only “a math genius or math professor” (Interview with Dr. Smith) or “super-smart people” (Interview with Faith) can enter. This, along with the students negative emotional reactions to mathematics and mathematical tasks (as commented on by Dr. Smith, Dr. Jones, Faith, and Lara) are the two biggest negative elements in the prospective teachers’ mathematical disposition, and thus the two elements the course seeks to most improve.
However, in spite of their negative disposition to mathematics, the prospective teachers are interested not only in their success but also in the success of their peers.

The students in [MEST courses] I think are a little more personable. Maybe that’s not the right word, but because they have to work in groups and present at [the] board, they pretty early on made some friends and are working with each other outside of class. In some of the other classes I worked with, that doesn’t happen until at least the first exam, if ever for some students. They are also more outgoing and dedicated than some of the other students I’ve seen. These students want to succeed, but they want everyone to succeed. And sometimes, when a student doesn’t get something, if someone else in the group does, that student [who gets it] will work their butt off until everyone [in the group] gets it. They seem to be emotionally invested in how they’re doing in the course, and how their friends are doing in the course. (Interview with Faith)

**The Day-to-Day Routine of MEST 1**

As discussed earlier, MEST 1 has different objectives and goals from many other college mathematics classes, and the “problem centered approach [MEST 1] uses is different than many other college mathematics courses” (Interview with Dr. Smith). Thus many students need to be prepared for and adjust to the different nature of MEST courses.

It became well explicated among us that we have to say things to address the nature of these courses: I give a speech and [Dr. Jones] sends a letter. I tell [the students] that [MEST 1] is different; I tell them that they will be unhappy at
times; I tell them I want to hear about it. But I try to give them examples of why we would need to do this, and try to get them to realize that when we don’t quite understand something that it’s okay to not quite understand. And once we [Dr. Smith and the class] break that barrier, they are comfortable admitting that they don’t understand, and then we can work together to help them understand. And that draws them in to the course, and hopefully to mathematics. (Interview with Dr. Smith)

In order to help ease this transition, Dr. Jones sends the following e-mail (as addressed in the above quote) to the MEST 1 classes before the quarter begins:

Welcome to [MEST 1]!

In this course, you'll work on learning the "why's" behind the mathematics that you'll someday teach! We will do this through activities and discussions each day as well as your learning to write about mathematical concepts and skills on your homework and exams. The goal of this course (and sequence) is to make itself obsolete in that, through the experiences you provide them, your students will expect to and be able to conceptually reason about mathematics (and, thus, will not need to take such a course if they want to teach).

The course will hopefully encourage you to see mathematics as not an area full of just rules and skills to follow (as it is often and deceptively presented in American classrooms), but one that springs from reasoning that all students can understand. It is hoped that you'll see that you and your students can have ownership in mathematics for life instead of just temporarily borrowing it from a
teacher. For example, we should look at a sum of fractions as more than just something to "do." Rather, we should be concerned for what it means (both the numbers as well as the "plus" sign). We should be able to present "story problems" that are modeled by the sum. And, finally, we should, using knowledge of fractions and "+", come up with a method or methods for computing the sum as one number.

Each day in class, you will be doing activities... During the activity, you will participate in solving the problem(s) at hand through reading the problem, understanding what it is asking, attempting a solution, and then discussing your work with your group members. Following the group work, we usually will have a whole class discussion about the activity, sometimes led by the instructor and sometimes by you, the students. The discussion will hopefully bring out the important mathematical issues that the activity unearths and maybe carry us further to more ideas.

Be careful in that the instructor's role in this course is not to tell you the answers or solution process or even to verify that you are correct or incorrect. Rather, [the instructor] is there to clarify issues you have with the problem at hand as well as provide questions that might lead you to the solution or solution process.

Following (or, if you prefer, before) the activity, you should go home and read the corresponding section(s) of the textbook that relates to the activities of the day. This you might consider the "lecture" for that material. Then, you
should begin to attempt the homework problems. Always assume that you should
not only write what you did to solve the problem, but also explain why your
solution process makes sense. You might do this as if you're talking with a
student or with a peer. Your homework (and exams) will be graded according to
a 10-point scale (see the syllabus). Note that correct mathematics alone only
earns you 6 out of the 10 points. Your explanation of your reasoning is the most
important part and thus accounts for the rest of the grade. (E-mail correspondence
with Dr. Jones)

This e-mail not only explicates one of the primary course objectives, it sets many of the
day-to-day activities in which the students will take part, and outlines some of the
grading procedures used commonly throughout the MEST sequence.

Textbook. The MEST sequence uses Beckmann (2011a) as the primary text for
the class and supplements the text heavily with Beckmann (2011b). Beckmann believes
Teachers must know more than just how to carry out basic mathematical
procedures; they must be able to explain why mathematics works the way it
does...By learning to explain why mathematics works the way it does, teachers
will learn how to make sense of mathematics. I hope they will carry this ‘sense of
making sense’ into the classroom.” (2011a, p. xvi)

This aligns with several of the MKT development goals Dr. Smith and Dr. Jones used in
their class, especially the emphasis on using student-crafted explanations of mathematical
concepts to enhance, reinforce and assess learning.
Also of interest, the book organizes itself around operation instead of around number systems, which Beckmann (2011a) states is the more common approach to textbooks for MEST courses. She sees two advantages to her organizational strategy:

The first advantage is a more advanced, unified perspective, which emphasizes that a given operation (addition, subtraction, multiplication, or division) retains its meaning across all the different types of numbers. Prospective teachers who have already studied numbers and operations in the traditional way for years will find this method enables them to take a broader view and to consider a different perspective. A second advantage is that fractions, decimals, and percents—traditional weak spots—can be studied repeatedly throughout the course, rather than only at the end. The repeated coverage of fractions, decimals, and percents allows students to gradually become used to reasoning with these numbers, so they aren’t overwhelmed when they get to multiplication and division with fractions and decimals. (Beckmann, 2011a, p. xviii)

Both Dr. Smith and Dr. Jones agreed with Beckmann’s approach, and in the role of course coordinator, Dr. Smith mandates using the order of the book to structure the course calendar (so Chapter 1 is taught first, followed by Chapter 2, etc.). Dr. Jones is especially complimentary of how Beckmann treats the structure of her textbook:

One of the places I love the Beckmann [text] is how she deals with operations. The day after we learn about addition, we take it through all the numbers. We add fractions; we read story problems and ask if the problems are modeled by
addition. We learn what the operations mean, not just what they do to certain sets of numbers.

MEST 1 covered Chapters 1 through Chapter 8 in order, with only a few sections omitted for time. Over the span of the MEST sequence, the majority of the textbook is covered. Further, the book features problems that examine common misconceptions regarding mathematics, examines nonstandard calculation processes, and presents real-world scenarios that the mathematics can be used (Beckmann, 2011a). During observations, both lecturers used problems from the textbook to branch into think-pair-share class discussions. Also, homework problems came from the textbook.

Although the lecturers used exercises out of the main text, both the lectures and recitation leaders used the activity manual extensively in class. Beckmann sees the activity manual as an integral part of the classroom experience:

I wrote these activities because I wanted my students to be actively engaged in mathematics in class. Few students seem to get much out of long lectures, and every teacher of math knows math is not a spectator sport: To learn math, you have to do math, and you have to think deeply about math. (Beckmann, 2011b, p. xiv)

And indeed, every class that the researcher observed, the instructor (be they recitation leader or lecturer) used some activity to structure a discussion. These activities came from the activity manual, the main textbook, or were a supplemental activity created by one of the instructors of the course (See Appendix I for examples).
Although both lecturers are fond of many aspects of the book, Dr. Jones also noted some weaknesses to the text.

Don’t get me wrong, the book [Beckmann, 2011] does a lot of stuff really well. It has some great stuff on Cavalieri’s principle, and I love some of what she does with division and relating that to real-life principles, but I think her book has some serious flaws. We used to use a lot more manipulatives, but because of the new textbook we don’t use them as much... We have this whole treasure trove of them—dice, Cuisenaire rods, GeoBoards—all those things we used to use in this course and I loved them. The kids got to see the mathematics and feel the mathematics, like I hope their kids will. I try to at least bring them in a few days…but it isn’t the approach that [the textbook] uses.

Not that any book is perfect, and I think hers is really good, but it is deficient in some areas. But that’s why we have our own activities. In truth, I would like to move away from textbooks altogether and just create our own course. (Interview with Dr. Jones)

This non-reliance on manipulatives forced the students to rely on pictorial representations and written explanations: both in class and on assessments, which Lara also saw as a barrier to reaching some prospective teachers.

We need to reach these students in as many different ways as we can, both because they are different styles of learners and learn best differently, and because we want them to be able to use a bunch of different stuff in their classroom…In the class now, we draw a lot of pictures and write a lot of words, which is great if
you learn best by drawing pictures or writing. But I feel like we are losing the
students who would learn best by touching the math, by physically manipulating
things to demonstrate mathematics.

Lectures. The lecture sections for MEST 1 met three times per week (Monday,
Wednesday, and Friday) for slightly less than an hour per meeting. Each lecture has an
enrollment limit of 50 students, but slightly more than that are admitted to account for
student attrition. And although Dr. Smith admitted this isn’t an ideal situation for MEST
courses, this state is viewed as a significant improvement to the course from when Dr.
Smith started working with the MEST sequence.

I would much rather have smaller classes taught by one instructor [with a research
emphasis in mathematics education], but we are a lot better than where we were…

When we started teaching this course, there was one lecture with 200 people in it
with the 20 to 25 person recitations… Now we are down to 50 person lectures and
that is a lot better. You can have a discussion with 50 people. You can do
activities with 50 people. So it isn’t an ideal situation, but I think we are moving
in the right direction. (Interview with Dr. Smith).

And as Dr. Smith alluded, lecturing was rarely used as a teaching method. Both
lecturers preferred to introduce a problem to the class, have the students work in small
groups (which usually involved grouping students in near proximity) and then try to
discuss the activity as a class. At times both lecturers had groups of students come to the
board to present their work to the class.
We really want to start each class with a problem, before there is any instruction, before there is any formal lecture. We want our students to wrestle with problems, with mathematics problems, and to be able to solve those problems, and explain both what they did to solve the problems and why they did what they did to solve those problems. (Interview with Dr. Smith)

Dr. Smith further explicated the importance of selecting good problems to generate discussion, which both lecturers admit is difficult in a class of 50 students.

Discussion is difficult. You have to have problems that are good enough to generate discussion—you can’t have discussion with a bad problem. But the problem isn’t enough; you have to, as the instructor, ask good questions. You have to be able to surrender control of the class to the class and in some ways direct it, but in others let [the discussion] just go where it will. It takes a certain kind of instructor to say, ‘Oh, I don’t know, what do you think?’ And to be out of control of your class in that way… I work hard to create a collegial attitude in the class… I want my students to go up to the blackboard and make mistakes, and they won’t do that if they are afraid of harsh critique from me or their classmates. (Interview with Dr. Smith)

Dr. Jones summed the teaching style used in the lectures more succinctly:

We make up a lot of our own activities to go along with what is in the textbook… And a lot of what we try to do is give them the activities and say, ‘Here you go, get to it’ and leave them alone as much as I can. I sometimes am not as good at leaving them alone as I want, but that is my goal. (Interview with Dr. Jones)
Both Dr. Smith’s and Dr. Jones’s lectures were in large classrooms with tables facing forward and movable chairs, which allowed the students to easily transition from working in small groups to focusing on the front of the class.

**Recitation.** Recitation sessions met for slightly under an hour the two days of the week that there was not a MEST lecture (Tuesday and Thursday). Although the university studied used a lecture-recitation model for MEST 1, both lecturers used a teaching model in their lecture that was different than the traditional lecture. In many ways, the lectures tried to use the teaching methods that in the past were reserved for recitation sessions.

Recitation is a lot better than the large lectures—we try to do the same things but recitation is almost like heaven. [Having the smaller class] lets you go around to each group, to not only learn everyone’s name but their knacks. Kids can get to know each other and be comfortable talking to the class and in front of the class. In the large lectures, you can’t go around to each group. It’s hard to get kids to talk sometimes. It’s just harder. (Interview with Dr. Jones)

Although there was a difference in the size of the classroom and the instructor, the recitation sessions replicated the teaching style used in the lectures: a recitation leader would use problems (some new, some which were began in the previous lecture) to instigate class discussions and introduce new ideas and concepts. Faith described the differences between her recitation experiences in other courses and her experience as a recitation leader for MEST 1:
I think it is really different from my other recitation experiences, in that I am presenting new stuff. I sometimes lecture, and some of the activities I do present new, if not distinct, ideas to what was done in lecture. Also, I am not presenting solutions… In [my recitations for other mathematics courses] I helped students out by presenting solutions; here I am still helping them out, but I am not presenting solutions. I feel like my job here isn’t to answer the students’ questions, but to push and prod them until they come up with the solution on their own. (Interview with Faith)

Because of the additional responsibility of introducing new material, generating discussions, and acting as a second-instructor for MEST 1 as opposed to only a recitation leader, the Mathematics Department makes a special effort to select its best teaching assistants to be MEST 1 recitation leaders.

In choosing our TAs, the department has always tried to look for students getting degrees in mathematics education, or students with some significant math ed. background. Apart from those, we have always received very senior TAs—students who were finishing their PhDs [in mathematics] and were very successful teaching more standard [college mathematics] classes. (Interview with Dr. Smith)

And as mentioned earlier, both Lara and Faith have high school mathematics teaching experience. Lara has a master’s degree in Mathematics Education, while Faith is perusing a Masters of Mathematics with an emphasis in Education.
Even with this high-level of mathematics education experience, the lecturers were in regular contact with their recitation leaders: recitation leaders often sat in on lecture (throughout the course Faith sat in on ten lectures and Lara sat in eight lectures), there were weekly meetings between instructors and teaching assistants, and there were monthly meetings among all the recitation leaders and lecturers.

Because of this specialization and the support of the lecturers, at times the recitation leaders were allowed to have more freedom in how they chose to organize the recitation sessions. One example of this was the way Faith handled forming her small groups. Although Dr. Smith, Dr. Jones, and Lara chose to form groups out of convenience by grouping students sitting near one another, Faith has a much more elaborate system she used for her recitations.

I have a variety of methods for assigning groups, each with distinct advantages. In the first week and a half of classes, I allow students to pick their own groups; this allows me to see who the students already know or with whom they are already comfortable working. I make a note on the attendance sheet of the students in each group. After the first assignment is graded, I record homework grades in an excel spreadsheet which fills the cells in a color scheme based on score. I keep those (usually few) students with a perfect score and excellent communication skills to place into groups last. All other scores between 90% and 100% are evenly distributed between 6 groups. All scores between 80% and 89% are distributed next, followed by 70% to 79% and 60% to 69%. For students that scored below 60%, but turned in the assignment, they are paired with those
students with perfect scores and excellent communication skills. Then, these pairs of students are distributed between the groups. Finally, Students that did not complete the assignment are distributed into groups.

I will repeat this process after each midterm grade is recorded and sometimes when there is a dramatic change in the category in which students are placed. Clearly, many students fall into the same categories each time. So, when scores are mostly consistent, I take the set of students in a category and trade them between the groups.

Between the first designed set of groups and the second, I have them count off to create a more sporadic set of students. In the end, I would like every student to have worked in a group with every other student at least once. So I always keep track of the students in each designed group.

This system of designing groups helps to promote a variety of talents within a group. I developed it while instructing elementary and middle school students. They tend to have more group work in a classroom setting than my high school students. I find the system very effective in assisting students that are not doing well, while developing the talents and teaching skills of students that are doing well. I have found that students are often more receptive to correction from classmates than from teachers; I assume that this is perceived by students as collaboration instead of criticism. (E-mail correspondence with Faith)

The recitation sessions observed were of the same format as the lectures, but although the lectures seemed to focus on class discussion, the recitation sessions focused
more on work in the small group setting. And because these were problems designed to generate discussion around the foundations of arithmetic, some of the students initially resisted the activities.

[Some of the early activities] were upsetting for the students. They were like ‘This does not seem like math, why are we doing this!’ So it was upsetting in the sense that the students weren’t sure where the course was going and didn’t see some of it as math. Perhaps unnerving is a better word… In fact one student asked ‘Why are we doing all this the hard way?’ To which I responded something like ‘It is only hard because it isn’t the way we learned it, me included. But if we learn this now, then you can develop these skills in your students one day, and it won’t be hard for them.’ (Interview with Faith)

However, because of the time crunch, some of the activities had to be shortened or extended into later classes (usually the next class period, whether it be a lecture or recitation). “We have [less than an hour], and that’s not a lot of time to get into stuff… a student may have five minutes to make a profound discovery about mathematics. And that’s not a lot of time” (Interview with Lara).

**Assessment in MEST 1**

MEST 1 uses multiple forms of formal assessment: homework and quizzes for formative assessment and exams for summative. The homework is due weekly and quizzes happen during recitation periods the weeks when there is not an exam. During the quarter, there were two mid-term exams that took place during Week 5 and Week 9, and a cumulative final exam during the university’s designated final exam week. All the
exams took place during a common-hour setting with all MEST 1 sections taking the same exam at the same time. If a student had a conflict with an exam time (e.g. due to another class), then it was up to the student and their lecturer to come up with alternate arrangement to take the exam.

As stated earlier, each individual problem was graded both for creating a mathematically correct solution and for crafting a clear and concise explanation of their solution. For each graded problem, 60% of the credit was awarded for determining the correct answer and 40% of the credit was awarded for a clear explanation of the mathematical concepts used by the prospective teacher in crafting the solution. On homework and exams, problems were 10 points (6 for the mathematics and 4 for the explanation), and for quizzes a problem would be 15 points (9 for the mathematics and 6 for the explanation). As Faith put it,

I think MEST is distinct from other college math classes… [In other college math courses] I would like to think that there is a burden on understanding a concept, but it is much, much less than in [MEST 1]. The [homework] papers I’m grading are like paragraphs, and my job is to read through this long explanation of what they think [for example] multiplication of fractions means and to determine how accurately they understand and have explained the concept. (Interview with Faith)

Because of the focus on crafting mathematically correct explanations for their solutions, it can be a difficult transition for both the prospective teachers and the teaching assistants tasked to grade the work. To help students and teaching assistants adjust to
this, the first homework assignment was a “practice homework… that gives the students practice on how to craft explanations and TAs practice on how to grade them” (Interview with Dr. Smith). The first assignment was collected and assessed by the teaching assistants, but those grades were not recorded and would not be a factored into the student’s final grade. The lecturers helped their teaching assistants grade and comment on the assignments, and the comments were designed to help the student improve their written work and explanations for future assignments.

In spite of the assistance given by the practice homework, both students and TAs had a difficult time adjusting to what was required of them in the assessment process. This is because the emphasis on crafting complete explanations, correct and complete mathematics work looks different in MEST than in other mathematics courses.

I think a lot of students I have don’t like having to explain stuff or prove stuff. I think the [mathematics] curriculum up to this class really rewards students [for] coming up with the right answer, and [MEST 1] wants to go beyond that to why the answer is right. And I think a lot of students are resistant to that, especially in the beginning. I mean, if the mathematics is correct you will still get points, but to get all the points and to pass the class, you need to know the whys. (Interview with Faith)

And at times the recitation leaders (who are responsible for grading homework, quizzes, and in part the exams) had difficulty adjusting to the different standard of assessment used in MEST 1.
Some [first-time instructors and teaching assistants] don’t like the grading—they say it’s too much work. But when I talk to other TAs, they say that it isn’t more work than in other math classes, but it is very different work. There is a grading rubric, and some have said it’s too qualitative—it’s too loose. But there is a lot of reading, and a lot of grading essays. (Interview with Dr. Smith)

However, in spite of that difficulty, there is inherent value to assessing students understanding of mathematical concepts based on their explanations of those concepts (Black et al., 2003). Lara gave an example of how explanation can show mastery of a concept more than merely following a procedure:

If you don’t understand it, you can’t explain it. And it’s really hard to get students to [pause] We’re talking about why we need to get a common denominator [to add or subtract] fractions. Most students know they need to get a common denominator, and they know how to find one, but they don’t know why. It seems like focusing on the why’s and trying to focus on the understanding really trips them up. (Interview with Lara).

The one problem with high stakes assessment is the amount of pressure it places on the students. Grades in MEST 1 can affect the students’ ability to proceed in the MEST sequence, admittance to the Teacher Education program, and their admittance into graduate school. Because of this, there was a great deal of pressure on the students to receive a high grade, and that pressure, at times, can hinder learning. The pressure was also compounded by the different nature of MEST 1 from many other mathematics
courses and the difference in the expectations for work and assessment. Lara recounted a common experience that has happened during her recitation sections:

I don’t know how many times a student has asked me to just tell them EXACTLY what I want them to say to get an A. And what I keep telling them over and over is to pretend that I am a fifth–grader, pretend (Dr. Jones) is a third–grader. What would you say to them? And if you don’t get full credit, it isn’t because you used this word or didn’t use that word. It’s that your explanation wasn’t good enough—if I didn’t know what was going on and you gave me that explanation, I still wouldn’t know what’s going on. (Interview with Lara)

Dr. Jones lamented the unfortunate situation both instructors and students find themselves because of the reliance on grades for advancement beyond the course.

I really wish we didn’t have to give them grades. The students worry too much about them, and not enough about learning the math. And I hate it. Assessment should be about learning what the students have learned and trying to fix problems—either re-teaching the students or adjusting the course to better address the students’ needs. Instead, it becomes a value-judgment on these kids [prospective teachers], and they need a good grade. They need the grade to get into their program and they need the grade to get into grad school; the grade is important, but not for the reason it should be…I think if we didn’t have to assign grades to students, it would solve a lot of problems [students in MEST 1] have. It would undoubtedly create new problems, but it would solve many of the ones we have. (Interview with Dr. Jones)
Although this is not a problem specifically reserved for the MEST sequence, it is compounded by the unique expectations MEST 1 places on students, especially when compared to more traditional college mathematics courses.

**Long-Term Goals of the MEST Sequence**

All of the MEST 1 instructors view the MEST sequence as a way to improve mathematics education in elementary schools by improving future teachers’ knowledge (MKT) and opinions (MD) relating to mathematics. It is their implicit hope that these prospective teachers will take their improved mathematical dispositions and more complete understanding of the mathematics into their eventual classrooms, and that both of those pieces will improve the prospective teachers mathematics teaching.

We want our teachers to know mathematics, and we want them to know it in a more flexible way than many currently do. We want to open up the conversation about the mathematics the children are doing and we want to show [the prospective teachers] that the mathematics is quite complicated. We want our students to know that the math that [their future elementary school] students are doing and saying is related to the math that they are teaching, and how to bridge the gaps in understanding rather than force students to abandon their thinking and focus on the way the teacher thinks about it. So [MEST courses] try to build up their mathematical understanding, but we also have the very strong feeling that, in part by the way we work on the math understanding, that we want to work on their attitudes toward math. We want students to see math isn’t just rules, it’s not just follow the leader and salute, it’s about reasoning and thinking…It is our [the
instructors] hope that when they see this they will be more positively inclined toward math. It’s a course that leads to a career—the students see themselves on the path to teaching. The more the students see the connections to teaching, the more they stay with you. (Interview with Dr. Smith)

Dr. Jones gave a vision for MEST 1 as a chance to begin re-awakening the prospective teachers’ dormant curiosity about mathematics.

You reason at all levels of mathematics, you think at all levels of mathematics. There is a certain logic here…In this course we don’t have that with these students. It’s not their fault; they had it when they were born. But unfortunately, our K–12 system beat it out of them. So we are trying to bring it out again. It was there, we had them shelf it for 13 years, and now we are trying to bring it out again in the hopes that they won’t have their students put it away when they teach them math… These students aren’t taking this course for themselves; they are taking it for their [future] students. (Interview with Dr. Jones)

Lara took that sentiment a step further,

I want to talk about the mathematics, I want to talk about the algorithms and where they come from. Let’s talk about the ways that we can talk about them and explain them. And with that, play around with this thing and see what it tells you about the math. I want students to have some deep discovery and learn why mathematics makes sense and why it is fun. (Interview with Lara)
And Faith wanted the prospective teachers to examine how the teaching methods used in MEST 1 may be superior to the teaching methods perhaps used in the prospective teachers’ elementary school mathematics classrooms.

A student asked me why we aren’t just learning how to do this, and I asked them how that way of teaching worked for you? And if it worked for you, how did it work for everyone else. Right now, you are a student in a mathematics class at a prestigious university, but what about everyone else? Maybe what worked for you didn’t work for everyone else, and maybe that what you think worked for you didn’t work as well as you think. Maybe this is a better way [to teach and learn mathematics], and if it is, shouldn’t we use it? (Interview with Faith)

Although these are lofty goals, both lecturers acknowledged that this is a difficult process, both in terms of what the MEST 1 course can accomplish and the rigors that MEST 1 students must endure to break mathematics education patterns that many students were taught with for the majority of their education lives.

And a lot of times over the course of [the MEST Sequence] I see them change.

But a lot of times they resist it. They want to think mathematics is a list of rules, as a set of instructions. I have 10 weeks, maybe more if they keep going with me, to undo 13 plus years of bad mathematics. (Interview with Dr. Jones)

However, in spite of the difficulty, Dr. Smith observed that the process works for many students.

I know the press is out on [MEST 1] and I know what the students say. And they say that it is really, really hard. But those same students who say MEST 1 is so
hard also say that they learned a lot, that I never learned as much in a math class as I did in that class. (Interview with Dr. Smith)

Summary

MEST 1 is a different mathematics course than most mathematics courses that the prospective teachers have taken, either in elementary school or at the college level. MEST 1 has different goals, methods, and assessment strategies than traditional mathematics courses, and in many ways it is these differences that cause students to struggle. However, these differences are important, as they begin to address the chasm between best practices (as of 2012) for teaching elementary school mathematics and the teacher candidates’ expectations for a mathematics classroom. The instructors of the course realize how different MEST 1 is from the teacher candidates’ previous experiences taking mathematics courses, and tried to ease that transition period. However, each instructor was adamant that in spite of how difficult this transition is for many prospective teachers, ultimately the transition is necessary to facilitate the teacher candidates’ mathematical learning at the foundational level that is necessary to teach mathematics in elementary schools.
Chapter 5: Amber

Amber [a pseudonym] began MEST 1 with an unproductive mathematical disposition influenced primarily by (a) her own lack of faith in her ability to do mathematics, (b) her views of mathematics as a rule-based system that did not connect meaningfully to logic or reason, (c) and her fears regarding fractional operations. And even though she did not make the gains in MKT that some of the other participants did, Amber showed the biggest positive change in her MD by the end of the course. She began to see mathematics as connected concepts from which algorithms were formed rather than algorithms without meaning. Because Amber began to see from where the algorithms were derived, she became more confident in using them and reconstructing the algorithms if she could not remember how to use them. This led to her being more confident in her own mathematical ability and increased her opinion about the usefulness of mathematics and perhaps her movement from mathematics being an autocratic discipline to a multiplistic one.

Mathematical Disposition and Mathematical Experiences Before MEST 1

Amber began MEST 1 with an extremely unproductive mathematical disposition, the lowest starting MD of the participants. She scored a 42 (out of a possible 125) on her introductory mathematics disposition survey (see Table 1), with a mean response of 1.68 to positively phrased questions about her mathematical disposition (after recoding). These responses included giving the most negative possible response to “8. I generally do worse in mathematics courses than other courses,” “16. I have trouble understanding ideas based on mathematics,” and “4. I am not the type of person who is good at
mathematics.” In fact, when asked what she thought when she heard the word *mathematics*, she responded

[Deep groan… then laugh] I guess that pretty much explains it: uugh! I switched majors a few times, and I wanted to make my math as easy as possible. When I took [the university’s mathematics placement exam], I didn’t take it seriously at all. Most of the time, if I didn’t know something I just guessed ‘c,’ and I did that a lot. So, no surprise, I got placed in [developmental mathematics]. And I did ok in that, but when I switched majors, I found out I had to take [the MEST sequence], I just went ‘uugh.’ Math is just something I don’t enjoy doing because I have never been very good at it. And while I have improved, I’ve never been as good at math as the people around me. So it is frustrating. (Introductory Interview)

Amber felt that her negative mathematics disposition began in elementary school when she started learning about fractions.

Amber: I remember liking math at first, but when we started doing fractions (in elementary school), that was when I stopped liking math. So I guess I haven’t liked math since about third grade.

Specifically, she recalled liking mathematics until she began working with fractions.

Researcher: What was it you liked about math before you started into fractions?

Amber: I don’t know. It was something new and something fresh. It was always fun for me to learn new concepts and take something you didn’t know before and be like ‘Look, oh yeah I did it!’
Researcher: And what was it about fractions that made you stop liking math?
Amber: It was probably the fact that you weren’t dealing with whole numbers; and that they are these complex ideas that don’t really make sense. Like two different fractions can represent the same thing, or the weird rules to add and subtract them. bring in models or manipulatives to help you understand fractions? 
Amber: Not really. I mean if there was ever an example the teacher would draw a pie on the board sometimes. (Introductory Interview)
Amber also pointed to the nature of some of her early mathematics instruction as being a contributing factor to her negative views of mathematics.

I feel like in early math I was just kinda told ‘It is what it is’ and no one ever really would tell me why. I was never really given a background…And that continued really throughout high school. And maybe that’s why I don’t like math. And it’s unfortunate, because I like reading and looking for whys in books or stories. But math was never like that. It was always just follow the leader. (Introductory Interview)
Table 1

*Amber’s Pre-Mathematical Disposition Survey*

<table>
<thead>
<tr>
<th>Category</th>
<th>Prescore</th>
<th>Mean Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preparedness to Teach Mathematics (of 40)</td>
<td>17</td>
<td>2.13</td>
</tr>
<tr>
<td>Mathematical Beliefs and Attitudes (of 40)</td>
<td>10</td>
<td>1.25</td>
</tr>
<tr>
<td>Mathematical Self-Efficacy (of 20)</td>
<td>7</td>
<td>1.75</td>
</tr>
<tr>
<td>Diligence in Learning Mathematics (of 25)</td>
<td>8</td>
<td>1.60</td>
</tr>
<tr>
<td>Total (of 125)</td>
<td>42</td>
<td>1.68</td>
</tr>
</tbody>
</table>

*Note.* Mean response determined after recoding to reflect the participants’ response to a positively stated question.

In spite of her negative views on mathematics, Amber spoke highly of her previous mathematics teachers.

Amber: I’ve never had a horrible teacher, and all my math teachers were good. It was just a matter of whether or not they could explain the concept enough to me for it to get through. And some teachers could reach me in seconds and others it took half a year.

Researcher: What do you mean by ‘explain the concept enough’?

Amber: Like showing me the process of how it works. A lot of the times I would be able to do the problem and get to the answer, but I had no idea what I was actually doing or why I was doing it. (Introductory Interview)
Specifically, she pointed out a strong positive experience she had with her high school geometry teacher.

I had this really good math teacher in high school, [name omitted], he was really about making math fun. But he wasn’t afraid to push you and your limits. He would explain things really well, was very patient, and made math seem not so bad… We would come into the class and he would have all these problems written on these two big chalkboards. And we would go down the board one-at-a-time and do these problems. And though it was a really fast pace, I feel like it taught you to not be afraid to ask questions, or else you would get left behind.

(Introductory Interview)

One of the projects from that class made a particularly resounding impact on her.

I remember doing this picture collage [in high school geometry]. We had this list of items: shapes, areas, figures, and other stuff. I really enjoyed it—it was nice to see how the stuff we were learning in class was actually out there, in the real world. (Introductory Interview)

In spite of this positive experience, Amber responded to the statement “21. I see mathematics as practical and useful” mostly false, which reinforces that although this was a positive experience and a positive element toward her MD, it did not successfully alter her previously held beliefs formed from her negative experiences earlier in her education.

For example, a key experience during middle school reinforced several of Amber’s beliefs about mathematics, namely her belief that mathematics is a rigid set of algorithms which are not connected to underlying principles.
I had this other math teacher [in middle school] who would explain things one way: what he called the right way. Well, we had this student teacher for a little while, and he would try to explain stuff different ways. He had to be careful or [the supervising teacher] would get pissed at him for teaching it to us wrong, but he tried. And after he left, we would ask [the supervising teacher] to explain some stuff how [the student teacher] did it, and he got really pissed off. He said that ‘While [the student teacher’s] way was ok, there is one way to do math in my class: MY WAY’! (Introductory Interview)

Amber also has a lack of confidence in her own mathematical ability that she primarily attributes to negatively comparing her mathematical ability to that of her friends.

All of my friends are engineering students, so it’s hard for me to say how good I am [in mathematics]. I’m not horrible [at mathematics], but I am definitely not the best. Math has never really been easy for me. (Introductory Interview)

She said further

Amber: I was always in the ‘dumb’ math course: while my friends were in Algebra II, I was in Algebra I. When my friends were in AP Calculus, I was struggling to get through Algebra II. And it’s not like they looked down on me or anything, but I always felt like the dumb girl. We were in all the same [college-prep] courses apart from math, and I always did as well as my friends. But I was always behind everyone else in my math.

Researcher: Did that bother you?
Amber: Yeah. Like I said, in the classes we were in I did as well as my friends, but because I wasn’t even close to them in math, I always felt like I was the stupid one. (Introductory Interview)

In fact, these experiences have contributed to Amber having an extreme lack of confidence in her own mathematics and the belief that there are people who are inherently poor mathematics students.

Researcher: Do you think there are people who just will never be good at math?
Amber: Yeah, absolutely. Not everyone can be good at everything, and some people are just wired in a way that means that they aren’t going to be good at math.

Researcher: Do you think you are one of those people?
Amber: Yeah, I think maybe I am. (Introductory Interview)

Views on MEST 1 entering the course. Amber initially had a naïve view of the demands of an elementary school mathematics classroom. In her MD Survey, she responded “Mostly True” to the first statement—that “1. Generally, [she] feels secure about the prospect of teaching children mathematics.” This is in spite of her previous statements regarding her lack of confidence in her own mathematical ability. At the beginning of the course, Amber believed that the mathematical demands for teaching young children would not be too strenuous.

Researcher: How do you feel about the prospect that someday you will teach children math?
Amber: It’s [long pause] interesting. I want to teach elementary school, so I think that as long as I learn the basic stuff I am probably ok. (Introductory Interview)

In spite of her answer to the questions posed by the researcher and on the MD survey, Amber was beginning to lose confidence in her belief that teaching elementary school mathematics would be straightforward. In fact, the presence of a mathematics content course and her early experiences in MEST 1 began to demonstrate to her the depth and complexity that exist in elementary school mathematics.

[MEST] really excites me, because I want to put the reasons stuff works with the math I have learned. And it’s also scary, because math is hard for me, and now I am going to have to relearn all of it, but now with the reasons stuff works. I barely passed just trying to get by with how math worked; I don’t know if I can understand why math works. (Course Interview 1)

These experiences were both enlightening and frightening for Amber. On one hand, Amber was beginning to see mathematics as something more than a list of rules and algorithms; and that there may be a way to learn mathematics that asks for deeper understanding than how she remembers being taught. However, Amber was intimidated by this revelation, because she saw her previous struggles in mathematics classes as potentially foreshadowing her struggles in MEST 1, a class that endeavors to look more deeply at mathematics content than any of her previous courses. In spite of this, she was excited to learn mathematics at that deeper level, and how to integrate those deeper mathematical concepts into her eventual classroom teaching.
Amber: It’s neat to know that there are all these reasons why this stuff works. I always learned math was like ‘it is what it is,’ but now [in MEST 1] I get to find out why it is what it is, and that’s exciting. But it also is completely mind-boggling. It’s really neat being in [MEST 1], but it is really confusing. Math was always just ‘do what I do,’ so I never thought about it this way, and I feel like I need to relearn everything. *I wish someone would have taught math to me like this when I was little.*

Researcher: When you are a teacher, do you think you would want to teach math like that, with the reasons and such, to your students?

Amber: Yeah, I hope I can. (Introductory Interview)

**Position on Perry’s Scheme**

Amber saw the world from the perspective of Position 5: Contextual Relativism, Relational Knowing. She has accepted the pluralistic nature of the world, and views knowledge and truth as contextual.

A lot of people see knowledge as what you take in, but it is also your ability to re-apply it and see past what you know to expand your horizons. You want to gain as much as possible, and we all have different bases of knowledge, and the more of those bases you can tie yourself into the better a person you will be.

(Introductory Interview)

Amber has demonstrated she was comfortable with viewing the world as multiplicitic.
Researcher: How would you relate your views of knowledge to truth?

Amber: That’s a good question! [Laugh, then long pause] I guess truth is open to interpretation. There are some things that people think differently are true, so maybe knowledge is your own version of what is true.

Researcher: What do you mean by that?

Amber: Well, your knowledge comes from other people and what their world-views are, so maybe your truth comes from the culmination of all the world views of the people you encounter and listen to. I guess truth is open to interpretation, I mean there are things people think is true and someone else might not think it is true.

Researcher: So do you think that there are universal truths that apply to everyone or do you think that everyone has their own truths?

Amber: I think probably everyone has their own versions of truth, I mean some may be similar to other peoples, but ultimately everyone has their own unique truths. (Introductory Interview)

Later, Amber showed evidence that viewed the world relativisticly and was beginning to seek context to determine rightness or wrongness of decisions. As evidence, she gave an example of a complicated decision that many at an earlier position would view as simply a right-wrong choice.

Researcher: Do you think there are decisions that are definitely right and wrong?

Amber: No, well sometimes. I mean there are places where you can have a definite consequence for a decision. For instance, you could sleep in and skip
your morning class. But you do that and you lose points. Or you could also get up, get a cup of coffee and make it to class.

Researcher: So in that, do you always think it is the better decision to go to class than to not go to class?

Amber: If you have a good excuse, a death in the family or you’re sick, then of course you should miss class. That’s why there are excused and unexcused absences. But if you are sleeping in because you were out til 3am partying, then no.

Researcher: So do you think that there are legitimate reasons to not attend class that would not be considered university excused absences?

Amber: I think there can be. I mean, there is so much that can happen in college that could be a good reason to miss class that no one would ever think of until it happens. I have a friend who was going through some [omit expletive] and I was really the only person she had. Well, a few mornings we stayed up til 6 and I had class at 7, so there was no way I could make it. I emailed my prof and let him know, and thankfully he understood and excused me. But a lot of students don’t have that good relationship. (Introductory Interview)

Also note that she acknowledged having a good relationship with her professor, which indicates that she saw herself as existing at the same level as authority (with a lower-case a).
Amber viewed herself as a member of a relativistic world, rather than a dualistic or multiplistic one. She saw her college experience as a means to improve her sense-making in a relativistic world to aid her transition to Position 5.

I feel like I am learning how to think about ideas and about life itself—a lot of different classes make me think about ideas that I wouldn’t have thought of on my own. And I feel like it is preparing me to help expose me to a lot of different things that I will see in real life outside of college. (Introductory Interview)

Amber was also beginning to reject her previous sense-making patterns, both in terms of distancing herself from her past sense-making and rejecting the sense-making of those who are at lower positions of development. First, Amber recounted a significant difference in her “maturity” between her present self and her past self.

When I compare myself now to when I was in high school: I have really matured. I originally thought that college would be high school with more partying, but it really is more than that. There are so many new ideas and perspectives that I just never thought existed. College should change you, and if it doesn’t you missed something. You are here for at least four years and you go through so many different experiences. I know I have seen so many different things that I never knew since I got here, and while I have had some bad stuff happen, I feel like I have grown-up a lot since coming to [college]. (Introductory Interview)

And Amber rejected the sense-making of those at lower positions.

I have always been an open-minded person, and I have tried to be open to new ideas and opinions. I am from a small town and there are some people who aren’t
accepting of new things, and even here there are those people. They tend to wall themselves off and only do things that they are comfortable with. And it is uncomfortable at times. Especially in [class discussions] there are these people who have to defend their position no matter what it is when no one else cares what they think or what they have to say. I always try to look at someone’s perspective from what they have been through and who they are. It makes no sense to judge someone if you don’t know who they are. (Introductory Interview)

Amber first describes people she knows from her hometown who she characterized as viewing the world in a dualistic manner, and then describes peers who are, at highest, in Position 4a: Late Multiplicity, Oppositional Alternative. In both cases, Amber was extremely frustrated with the people she observes as being at a lower intellectual position than her.

**MKT Before MEST 1**

By the random selection process described in Chapter 3, Amber was given Exercise Set B as her pre-test and later Exercise Set A as her post-test.

**Exercise 1.** When given Task 1, she began by approaching each subtraction problem using the traditional algorithm for subtraction. And when asked about her reasoning, Amber was not able to give a mathematically sound justification for her processes, nor name what she was doing.

Researcher: The thing you just did, crossing out the 2, do you have a name for it?

Amber: No, I am sure there is one, but I don’t know it.

Researcher: And why did you do that?
Amber: Well, if we try to take the two from the seven we will get a negative number, and we can’t do that in this problem.

Researcher: And why is that?

Amber: Because you can’t have them with these [problems]. (Introductory Interview)

When asked how she would approach teaching this concept (subtraction with regrouping) to children, Amber gave the following description:

Amber: If I was teaching this, I would say that the top number can’t be smaller than the bottom number because they [the students] don’t know negative numbers yet. So I would focus on the 2 and you don’t have 7 to take away from the 2, so you need to make the 2 a bigger number. So you put a one in front of the two and then subtract one from the number right before it [in the ten’s place].

Researcher: Why do you do that subtraction?

Amber: Because you’re taking the one from the six and putting it over here [in the one’s place].

Researcher: Then why can you subtract 1 from here [the ten’s place] and just stick the one in front of the 2 [in the ones place]?

Amber: Because you are borrowing the 1 from the 6 to the 2. (Introductory Interview)

Notice that now that Amber was focused on teaching the concept to others rather than simply performing the task, she used the term *borrowing*. When she was concentrating on performing the task, she could not discern any term for what she was doing, she was
merely performing the activity which she was taught. But when Amber focused on
teaching others how to perform that activity, she re-discovered a term which was
probably used to teach her the algorithm.

   Researcher: You mentioned borrowing and used a borrowing technique to solve
these problems. Do you think it is possible to solve these problems without
borrowing?

Amber: Ummmm… I don’t know.

Researcher: Do you think you could solve this problem without borrowing?

Amber: No, I think even if I weren’t doing the borrowing on paper I would still be
doing that in my head. (Introductory Interview)

Also of note, the above quote showed Ambers’ lack of adaptive reasoning with these
types of problems, admitting that any procedure she could use to solve the problems
would eventually devolve to *borrowing*.

**Exercise 2.** Amber immediately found the student error on the multiplication
problem and correctly linked the student error with the most probable conceptual error
made by the *student* who did the work.

   Amber: This is a place value thing. So you start out over here with the ones, but
once you get through with that then you have to move the number over here to the
tens spot. I always remember putting a 0 in here [the one’s digit in the second
line of multiplication] as a placeholder.

Researcher: And why would you put a 0 there?
Amber: Just to act as a placeholder and keep everything in the right spot for when you add the lines up. (Introductory Interview)

Although Amber found the error and correctly surmises the underlying cause for the error, she was unable to discern why she would place zeros to represent place value. Amber knew that the issue with the work is the student’s misconception of place value, knew that using a zero as a place-holder will fix the algorithmic issues, but failed to relate the two concepts.

Researcher: You mentioned ‘place value’ previously, what do you mean by that?
Amber: Just where the number lives in terms of how big it is. Sort of the scale of the number.
Researcher: What do you mean by that?
Amber: Well, it really is just how big the number is—if I have 0 in the tens place and 5 in the ones place I have five, but if I switch them I have fifty.
Researcher: So what does this mean for the multiplication?
Amber: That you need to put the zero here or you will add stuff wrong.

(Introductory Interview)

Further, when asked how she would teach this concept, she still sees the zeros as related to place-value, but can still not connect them appropriately.

Researcher: So if you found one of your students making this mistake during multiplication, how would you address it?
Amber: I would just emphasize the place value—this is your one’s place, this is your ten’s place, this is your hundred’s place. So you need to account for that when you’re doing the multiplication, and put your 0s in to hold the place.

In spite of some of her struggles with adaptive reasoning on the other exercises, Amber was able to use estimation as a way to verify the incorrectness of the first answer.

Amber: I mean, just looking at the two answers, you can tell which is right and which is wrong.

Researcher: How can you tell that?

Amber: Well, you’re multiplying two three-digit numbers, and the answer for one of them is only a four-digit number. It isn’t big enough to be the correct answer. I mean, if you take 456 and add it to itself 123 times, you are going to get a big number. (Introductory Interview)

**Exercise 3.** Amber was not able to successfully complete the division problem. She began immediately by converting both fractions to decimals, but was unable to recall the algorithm for dividing decimals. When asked why she took that approach, she responded “I hate fractions, so I would rather work with the decimals” (Introductory Interview). After trying for several minutes to recall the algorithm for dividing decimals, she reverted to working with the fractions. However, instead of using the traditional algorithm, she tried to reason about what it means to divide. Nevertheless, she was once again unsuccessful, and instead of dividing by 1/2, Amber divided by 2.

When asked to create a story problem which modeled the division, she once again confused dividing by 1/2 and dividing by 2.
Researcher: How would you create a real life situation or a story problem for that exercise?

Amber: I guess a cooking scenario. Maybe a recipe calls for 2 1/4 cups of sugar and you need to divide that between two batters?

Researcher: So what would your problem be?

Amber: Say you have sugar for a cookie recipe. You have 2 1/4 cups of sugar that is required for a batter, but you want to make two batters. So how much sugar are you going to need to make two batters?

Researcher: How does that model the division problem?

Amber: Well, you have the 2 1/4 cups of sugar and you want to divide that into the two batters for the recipes. (Introductory Interview)

**Discussion.** While performing these Exercises, Amber consistently showed a low level of conceptual understanding and adaptive reasoning. In all the activities, Amber could only see (at most) one way to correctly complete the exercise, and was often unable to link the procedures she uses to solve the problems with the mathematical principles central to those procedures. Also, it is interesting to note that although Amber is at Position 5 for her general sense-making, she exhibits a much lower level of sense-making for her mathematical reasoning. Although she was comfortable with relativism in her general knowledge and acceptance of truth, she is much more dualistic in her mathematics reasoning.
Experiences Within MEST 1

Amber initially had conflicting emotions regarding the nature of MEST 1 and the nature of several of the activities in MEST 1.

MEST 1 is weird. It’s weird relearning stuff you already know, but I feel like I understand this stuff a lot better than when I was in elementary school. I feel like now we are examining the problem more than the process. And yeah, we are learning about fractions, and doing story problems with sheep, but we are really learning about a bunch of new concepts and ideas. There are a lot of why questions. (Course Interview 1)

Amber immediately grasped the mathematical intent of MEST 1: to learn the foundational elements of elementary school mathematics at a deeper level than most elementary school classes reach.

Classroom Activities. Amber saw many of the activities done in MEST 1 as different when compared to some of the teaching techniques used in her previous mathematics classes. However, she really enjoys the overall collaborative and collegial nature of the class.

Amber: It’s really neat to hear other people’s explanations about things. It’s not like other college classes [Dr. Smith] actually has us participate and come to the board and give out our own explanations. And even if we are wrong, [Dr. Smith] has us go through our thinking and tries to work with us on our thinking and explanations. I really feel involved in the class.

Researcher: And what about the recitations?
Amber: Recitation is the same. We haven’t done as much talking as a class, but we have done a lot of group work and working with each other. (Course Interview 1)

Although Amber found value in the way the class operates, some activities she found esoteric and does not immediately see the direct application to her future teaching career.

Amber: We did this thing with, like, super-bundles, and bundles, and did this weird thing were we like made numbers without using numbers; instead we used letters and stuff. We had, like, little sticks to represent the numbers. It was basically trying to get rid of that numerical system that we are used to and do something else. So we had to figure out what place value meant and other stuff. It was really confusing… When we were doing the bundles and super-bundles, it kinda threw me for a loop because they weren’t numbers. It was hard to think of a number as an object and set up a specific order to group them in, and name them without calling them the numbers I know.

Researcher: Why do you think you did that activity in class?

Amber: I don’t know, maybe because we are used to numbers, but kids aren’t. So maybe doing this helps us realize what it’s like for kids to not know numbers.

(Course Interview 2)

However, as evidenced above, even in the activities that Amber failed to see the immediate connection to elementary school mathematics, Amber tried to apply each lesson directly to a skill she would need to develop to become a better teacher. MEST 1
refocused those efforts by asking prospective teachers questions that directly relate to their future careers as mathematics teachers.

Researcher: Do you have a lot of questions where you are asked to look at what a student did and analyze whether what they did was right or wrong?

Amber: Yeah, I think we get asked stuff like that a lot. Like, a student thought of the work this way, or a student got this answer to the problem, is he right or wrong? If he is right, what did he do, and if he is wrong, what could you do to point out his errors. And in [lecture] even if we do have it perfect, [Dr. Smith] will say ‘Okay, but I’ve seen some students do this, so what do you think that student is doing’? And we will have to think, ok, is that student right or wrong, and why are they right or wrong. (Course Interview 2)

Of all the activities, Amber usually focused on the ones that relate to fractions and operations with fractions, the mathematical concept she views as her weakest area going into MEST 1. She described how MEST 1 approaches fractions, and how that is more appealing for her than the way fractions were taught to her in her past.

Amber: We are learning about fractions and how to model fractions differently.

Researcher: How are you enjoying that? I know you have said fractions can be a sticky spot for you sometimes.

Amber: The class itself and dealing with the fractions is really neat and I am kinda grasping the new concepts. I mean, fractions are this way, but kids don’t understand that, so we need to come up with different ways to explain the problems.
Researcher: How do you mean?

Amber: There is how they [fractions] work, and we know how to work them because we are in college, but a kid seeing them for the first time would have an easier time looking at fractions with pictures or circles or other manipulatives. We want to explain the concepts, but use these math toys to help reach the students to connect math with something tangible and real.

Researcher: How does that compare to the way you were taught fractions before?

Amber: It really doesn’t. I remember just being taught rules that looked the same and never made sense. Now, I feel like we are learning how fractions work, and why those rules [fraction operations] give you the right answers. (Course Interview 1)

One of the strengths of organizing MEST 1 by discussing each group of numbers and then taking each group through the operations is that instead of teaching fractions in a large block, fractions are spread throughout the entire term. Even though Amber had yet to master fraction concepts when they were first introduced, she had opportunities to review and relearn many of the same concepts when the course re-encountered fractions when learning number operations. So later in the quarter, Amber had more experiences and confidence with her fraction skills.

We then moved [to] fractions, and we did this activity with pies. So we had one pie, but then we made a second pie, so now the whole was two pies instead of just one. So we were really trying to figure out what the whole of each problem was. (Course Interview 2)
This led to what is perhaps Amber’s biggest mathematics disposition improvement in the course.

Amber: I really like what we’ve been doing lately. Really, we’ve been talking a lot about multiplying and dividing fractions, and I used to not like them. Now, I think I have a better handle with what’s going on.

Researcher: So you mentioned liking fractions, and also understanding fractions.

Amber: I think I like them better because I understand them. Don’t get me wrong, I don’t like them, but I definitely don’t hate them as much as I used to.

Researcher: How do you think your understanding of them has changed?

Amber: I think I know what’s going on better. Before, [fractions] were just a bunch of weird rules. Now, like, I see how they work, and why some of the rules work. (Course Interview 5)

Not only did Amber use the term “like” to describe an activity with fractions, she also felt much more confident in her ability to do operations with fractions. Not surprisingly, Amber linked her improved disposition toward fractions and her improved understanding of fractions.

**Explanations and Assessment.** Another element of the course which had an immediate first impression on Amber is the emphasis on explanations in MEST 1 as both a teaching tool and means for assessment. Particularly, Amber viewed the explanations as a means to evaluate her own learning in the course.

We are supposed to explain everything, why things are the way they are. But when you are coming up with these explanations, you begin to see problems with
the way you’re thinking about stuff. Maybe it’s that you know it, but not in a way that you could explain what you’re doing to someone else, and for me, I have had several times that I tried to write what was going in and I realized I had no idea what I was doing. (Course Interview 1)

In fact, Amber saw the explanation required in MEST 1 as something different than anything that had been asked of her in any other mathematics course.

Researcher: How are the explanations in [MEST 1] different from some of the other math classes you’ve had?

Amber: It’s really, really different. I mean, most math teachers want you to show your work and that’s it. There usually wasn’t an explanation, and if there was it would usually be a sentence or two. But in [MEST 1] you have to show your work, and then explain everything you did in as much detail as you can. So instead of a sentence or two, it could be a paragraph or two.

Researcher: So instead of ‘show your work,’ you have to ‘explain your work’?

Amber: Yeah, but it’s more than that too. It’s more like evaluate what you’re actually doing… I really enjoy thinking about this, and I find that I am doing better in [MEST 1] than I do other [mathematics] courses, and I am definitely enjoying [MEST 1] more than other math courses. (Course Interview 2)

However, even though Amber believed the explanation portion of MEST 1 would help her understand the material at a higher level, she struggled to create explanations.
Researcher: How are you doing with the explanation portion of the assignments?
Amber: I’m doing better. I did my practice homework and my first quiz and I did horribly on those, but I’ve been getting better. I am getting used to the requirements, and I have been trying to practice explaining things a lot in class. And I did a lot better on my second quiz.

Researcher: What do you think Dr. Smith and Faith are looking for in your explanations?
Amber: Mostly that you really know what you’re doing, that you know what the processes are called, and that you aren’t just doing stuff without knowing what you are doing. [Dr. Smith] says that when we are writing explanations, we should treat it like we were writing them for a student.

Researcher: What do you think [Dr. Smith] means by that?
Amber: Just to write clearly and to use the right words for things. You [the prospective teachers] should be able to tell a student learning this stuff what you [the prospective teacher] are doing in the class so that they [the students] can learn from what you’re [the prospective teacher] doing and do it themselves [the students]. (Course Interview 2)

In particular, Amber viewed herself as trying to make the explanations more difficult than they needed to be and becoming confused by her own thought processes.

Sometimes I get confused and make things more difficult than they need to be.

But I feel like I am starting to grasp the concepts. In the beginning, I was trying to but things weren’t really clicking. (Course Interview 2)
It remains unclear whether Amber is struggling because she is over complicating her thought process due to math anxiety, or because there is something fundamentally flawed in her understanding of the concepts.

After performing poorly on her first few quizzes, homework assignments, and first midterm, Amber decided to speak to Dr. Smith to determine whether there was anything she could do to improve.

I think I did most of the math correct, but I guess my explanations weren’t good enough. I didn’t think there was a lot of feedback on the exam, so that’s why I want to talk to [Dr. Smith] to see what exactly I did wrong. I think I got bits and pieces of each problem, but I think I had trouble putting the bigger picture together from the pieces. Maybe I am missing pieces, or maybe I just am not putting them together, I don’t know. (Course Interview 3)

Amber later elaborated on what she took from this meeting and if she agrees with Dr. Smith’s assessment of her problems.

Researcher: Do you think you are having trouble getting the ‘big picture’ ideas that are being talked about in [MEST 1]?

Amber: Yeah, I feel like I am getting the details, but the things I have issues with is the explanation. I know how to do most of the math and I can explain the steps by themselves, but I think I am having trouble seeing the broader picture and explaining that.

Researcher: What exactly do you mean by ‘the bigger picture’?
Amber: A lot of problems will say to explain why [the concept works] after the problem, and that’s where I think I end up getting really confused. Like, for example, in fractions: If we are talking about fractions. I know what the parts are, I know what the whole is, but I have trouble understanding what a fraction is.

(Course Interview 3)

However, when asked further, Amber admitted that she was still unsure what she was missing from her explanations.

It’s really frustrating—I want to do well in this class and I am really motivated to do well in the class. But I am missing something, and I really don’t know what I am missing. (Course Interview 3)

In reflecting on her issues creating mathematically correct explanations, Amber believed that her previous mathematics courses did not prepare her for the expectation standards held by the MEST 1 instructors.

You’re taught how to do math by being told how it’s done, and no one ever asked me why this stuff worked. And now, it seems like all [Dr. Smith] and [Faith] want to know is why stuff works, and I really feel like I wasn’t prepared for this… There were never any reasons to math [concepts], and like I said before, I think if I knew the why’s I would be better in this class. (Course Interview 4)

This statement reflected many of the concerns with mathematics education in the United States brought forth by Dr. Smith and Dr. Jones in the previous chapter. This increase in expectation, although necessary to improve elementary school mathematics, is not easy
for the prospective teachers. Amber seemed to be especially negatively affected by this sudden increase in expectations.

To try to gain a better understanding of the expectations instructors have of her graded work, Amber met separately with both Dr. Smith and Faith before taking the second midterm.

I was talking to [Dr. Smith], and she went over the [first] midterm with me, and went over how she and the other graders grade the midterms. So now I think I will have a better idea of what they’re looking for. And that is an issue for me in a lot of classes, I won’t do well on the first midterm, but once I know the expectations of the instructors I do a lot better on the next midterm. And on the first midterm, I am usually really nervous and I do a lot of silly mistakes and I am a bit frazzled. But come the second midterm, I am more confident and I don’t make as many mistakes.

Researcher: Is this something that happens in all your classes, or just math classes?

Amber: Mostly math. I mean, it happens some in other classes, but much more I feel like in my math classes. (Course Interview 4)

After meeting with Faith, Amber believed that not only does she have a better understanding of what was required of her, but also she knew how to craft better mathematical explanations.

I met with [Faith] yesterday and I feel a lot better [about my explanations]. We went through my [first] midterm and some of my quizzes and homeworks, and we
figured out that I am getting the parts correct, but I am struggling to show that I understand how the parts relate—I am having trouble with the whole picture. And I am getting it in parts, but I sometimes mix things up in my explanations and I sometimes I don’t explain things as well as she thinks I know them… One of the big things [Faith] pointed out was that she thinks what I have are good explanations, but sometimes I don’t put in everything I need to. (Course Interview 4)

When asked directly about her explanation skills in the final interview, Amber admitted that only toward the end of MEST 1 did she began to fully grasp what was expected of her.

Amber: Well, I think I am finally starting to get the explanation thing. It isn’t that I don’t understand the [mathematical] concepts, it’s that I am missing pieces: words, phrases, stuff like that. I think once I get those words down, I will do a lot better on [the final exam].

Researcher: So what pieces do you feel you are missing?

Amber: Just the right words. Like, sometimes I will know the steps, but not know how to put them together, and sometimes the words help. (Course Interview 5)

Amber believed that the explanation portion of the assessments was a major impediment to her success in the course—that her poor performance on assessments was not due to a lack of mathematical understanding, but rather reflected her inability to craft well-written explanations of the mathematics. “I think I know a lot more about the stuff
we’re doing [than my grade would indicate] and I think I have had real problems putting everything I know together” (Course Interview 4).

**MD After MEST 1**

Amber increased her overall score from 42 to 60 (out of 125) with her mean response increasing from 1.68 to 2.40 when responding to positively phrased questions about her mathematical disposition (after recoding). This increase was reflected in minor changes to her responses to several items, but of interest is Amber’s mostly positive response to the statements “10. I enjoy learning about mathematics” and “16. I have trouble understanding ideas that are based on mathematics.” In her previous survey, Amber gave the most negative responses, so these changes signal a large impact on her overall mathematical disposition.
Table 2

*Amber’s Post-Mathematical Disposition Survey*

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<td>+18</td>
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<td>+0.72</td>
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</tbody>
</table>

*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

Amber described how MEST 1 impacted her beliefs about mathematics and strengthened her confidence in her own mathematical ability.

I would say this course has improved my overall attitude to math. I was really scared and nervous. But now that I am in the class and we are looking at why math works, it is really interesting. I feel like I am more comfortable with math and I believe in myself and my math skills a bit more. I feel like I can look at a problem and start asking myself a bunch of good questions about the problem and
try to analyze what the problem may be asking, whereas before I would just see a problem and if I didn’t know how to do it I would just freeze… I think if I was taught [mathematics previously using the techniques used in MEST 1] I think I would be a lot better at mathematics. (Final Interview)

Interestingly, Amber theorized that if mathematics had been taught to her using the principles of MEST 1—emphasis on mathematical principles instead of algorithms, problem-centered mathematics, etc.—she would have a higher mathematical disposition.

**MKT After MEST 1**

Amber showed significant improvement on each of the three exercises. Also, she chose different approaches to solve each problem than the one she used during the initial interview, and further each of the new approaches models an idea or technique discussed in MEST 1.

**Exercise 1.** Unlike in the first interview where Amber tried to use the traditional algorithm to solve these problems, this time Amber chose to draw base-10 blocks to model solving the problems. When asked why she chose this method, she acknowledged having a preference for this algorithm to solve the problems because she feels it better connects to the mathematical concept of subtraction.

Researcher: You know that’s not how you solved these problems before.

Amber: Yeah, but I like this way better. It lets me see what’s going on.

Researcher: What do you like better about it?

Amber: It just makes more sense. I mean in the standard algorithm you are really just moving stuff from one place to another. Here, you can actually take
something away from something else, you can actually do subtraction. (Final Interview)

Amber has also refined the concept of borrowing to the concept of regrouping, which she once again believed connected more to the mathematical concepts underpinning the operation than her previous conception of borrowing.

Researcher: And what are you doing here?
Amber: I am breaking up one of the tens so that I have enough ‘1s’ to take five away from it.

Researcher: What would you call that?
Amber: Regrouping. Some people call it borrowing, but we talked about how regrouping is the better word. You see, you aren’t just borrowing a ‘1’ to put somewhere else; it’s more like regrouping bundles.

Researcher: And how do you see borrowing and regrouping as different?
Amber: Like I said, borrowing is just moving a ‘1’ around, it really doesn’t make sense why you subtract ‘1’ from one number [digit] and then add 10 to another [digit].

Researcher: And you think regrouping makes more sense?
Amber: Yeah. When you are regrouping, you are dealing with real stuff. I can physically see what is going on with that ‘1’. I am breaking a ten [block] up into ten individual pieces so I have enough to take away. (Final Interview)
Further, rather than replacing the standard algorithm, Amber viewed manipulatives as a means to teach the standard algorithm in a manner that connected the mathematical concepts to the algorithm.

Researcher: So if you were teaching this to a class, how would you teach it?

Amber: Ideally, I would have some blocks or Popsicle sticks or something we can bundle. But if we didn’t have that, I would start by having them draw the picture. I think the kids need to see what is going on.

Researcher: What about the traditional algorithm, the way you did it when we first met. Do you think that it still has a role being taught?

Amber: Yeah, I think it does. I mean, we should give kids as many ways to do stuff as we can. But I would start with these models. I think this shows what is going on more [than the traditional algorithm]. And I really want kids to see what is going on. (Final Interview)

**Exercise 2.** Much like the previous exercise, Amber rejected the standard algorithm she first used to solve the problem, and instead used the *partial product method*, which was one of several algorithms taught to the prospective teachers to multiply integers with two digits or more. And once again, Amber sees her new algorithm as a means to connect the mathematical concepts being used in the problem to the standard algorithm.

Researcher: You learned a bunch of different ways to multiply these two numbers. Which did you prefer?
Amber: I think I like partial products best. I think that it better shows students exactly what they are doing when they are multiplying the numbers. I mean, it’s like the way I learned as a kid, but longer.

Researcher: So would you have your students multiply large numbers using partial products?

Amber: Yeah, but not exclusively. I think that the students would need to know the real way to multiply the numbers for when they get to the next grade. Even if I do this, there is no guarantee the [student’s] next teacher will. (Final Interview)

Amber was also able to use partial products to refine her thinking about place-value and the use of zero “as a placeholder.”

Researcher: When you did this problem before, you said the zero was ‘Just a placeholder.’ Do you still feel that way?

Amber: No, it is really important; the zeroes represent the place value. So the student didn’t understand that instead of multiplying by five [in the tens place] he was multiplying by fifty. And that shows in the answer. One is way too low, and the other seems reasonable. (Final Interview)

As before, Amber chose to use estimation to help explain to a student why the answer given by the incorrect algorithm was incorrect. However, this time Amber used alternative strategies to find an estimate closer to the true answer.

I think one way to get students to realize their mistake here is to get them to do some estimation before they start doing the problem. I mean, I looked at the problem and I knew that the number should be over 10,000, because both
numbers were larger than 100. So if $100 \times 100 = 10,000$, there is no way my answer should be less than that. And really, you could do $100 \times 600 = 60,000$ and get a better estimate. (Final Interview)

**Exercise 3.** Just as in the first interview, Amber struggled with the fraction concepts presented in this problem. However, instead of incorrectly using standard algorithms, Amber tries to reason with the fractions and operations using models drawn on the page.

Amber: What does it mean to divide? Well, it could mean how many pieces of 1/2 could we make from 1 3/4 or it could mean how many groups of 1/2 could we make from 1 3/4?

Researcher: So which one makes more sense here?

Amber: Well, if 1 3/4 is the whole amount, I should figure out how many groups I can make with each having 1/2 things in them. (Final Interview)

Unlike her first encounter with this problem, Amber correctly solved the division problem.

However, in spite of her success understanding the problem, Amber still struggles to create a story-problem that models division with fractions.

Researcher: How would you create a real life situation or a story problem for that exercise?

Amber: I would make a problem using recipes. So say a recipe needs 1 3/4 cups of flower and you only want to make half of the recipe, how much do you need?
Researcher: You mentioned estimation earlier, does that story problem fit the answer you computed earlier.

Amber: [Pauses]. No, I don’t guess it does, because if you only needed to make half the recipe, you wouldn’t need as much flower, but the answer I found was 3 1/2.

Researcher: So where do you think the error is, is it in the model you drew or in your story problem?

Amber: I donno. (Final Interview)

Even though she did not successfully create a story-problem, Amber did show improvement. Unlike before, Amber is now able to reason about the mathematics behind the problem and see the error in her model.

Discussion. Amber improved her performance on each of the three exercises from before the course, showing an increased understanding of the algorithms she used to solve problems and the mathematical principles on which the algorithms are constructed. Interestingly, she chose different approaches to solve many of the problem than the ones used during the initial interview, and further each of the new approaches models an idea or technique discussed in MEST 1.

Reflections on MEST 1

On reflecting about her time in MEST 1, Amber expressed both excitement for the progress she has made toward understanding mathematics and remorse that she was not more prepared for the course. First, she recounted how her mathematical disposition and mathematical understanding had developed.
You know, I like math a lot more now. Before, I never thought to question the reasons we did what we did [in mathematics]. Which is weird, now that I think about it… I think I know more about math now. Maybe I am not a better math student, but I feel like I know more about it, even than some of my engineer[ing student] friends. I mean, I couldn’t do the stuff that they’re doing, but I bet that they couldn’t tell me what it means to divide fractions, or why a negative [number] times a negative [number] is [a] positive [number]. (Final Interview)

As much as Amber saw her mathematical ability and her mathematical disposition had improved as a result of MEST 1, she regretted the amount of time it took for her to fully grasp the explanation requirement for MEST 1. Ultimately, Amber felt her previous mathematics courses did not adequately prepare her for the level of thought required to craft mathematically sound explanations of the concepts discussed in MEST 1.

Amber: I really feel like I wasn’t prepared for [MEST 1]. I would love to go back, with what I know now, and re-do a lot of my early homework and midterms. I feel like I have gotten so much better with explanations, and with understanding what’s going on.

Researcher: So if you could go back [to the start of the quarter], and give yourself one piece of advice to succeed in [MEST 1], what would it be?

Amber: To make sure you understand everything. Every word, every idea, and every problem. I feel like in the beginning, I was getting some of the stuff, but I was leaving it at that. I wish I had really focused on understanding everything, and how things fit together.
Researcher: Do you think that may have been why you were having trouble with your early explanations?

Amber: Yeah, I wasn’t understanding the words and the concepts, and I don’t think I was using them correctly. That probably was in part why I was getting docked in my explanations. I could explain the steps I was doing to solve the problems, but not necessarily why I was doing them.

Researcher: Did the practice homework help at all?

Amber: Yeah, it did, but not enough. It was really a few weeks before I understood exactly what everyone wanted in their explanations, and by then I had done some real homework and quizzes and was about to take a midterm. I donno, I never felt like I didn’t know what was going on, I just felt like I couldn’t figure out what I needed to do in the explanations. (Final Interview)

In spite of the warnings presented to her by Dr. Smith on the first day of class and the presence of the practice homework assignment, Amber believed she was not prepared for the level of rigor that MEST 1 required.

However, Amber still viewed MEST 1 as a positive experience that positively influenced how she thought about mathematics and the prospect of teaching mathematics.

I really liked the class. I feel like we got to see math in a way that I hadn’t before. I really wish someone would’ve taught me [mathematics] like this when I was in [elementary] school, maybe I wouldn’t have so many problems now. At times, I feel like I have to re-learn everything I thought I knew about math, and that is really frustrating. But I feel like now I am getting to see the inner-workings of
math. It’s like watching a movie and then seeing all the how-it’s-made special features. I finally feel like I am getting the behind-the-scenes view of math.

(Final Interview)

Amber realized that MEST 1 should not have been the first time she learned about mathematics by examining its foundational principles. She saw this revelation as something that would influence her future mathematics teaching.

I want to be able to give my students as much background as I can on how to do math. I don’t want to be one of those teachers who just tell students ‘It is the way it is.’ I don’t think that works for students, and it didn’t work for me… Kids want to know the whys and hows, they want to ask questions. And I want to be able to give them the answers, because I know that there are answers that I can give them. (Final Interview)

**Summary**

Amber showed considerable productive changes in her views of mathematics, her belief in her own mathematical ability, and her mathematical sense-making. Amber went beyond seeing mathematics as a collection of nonsensical algorithms to see mathematics as connected concepts from which algorithms were formed. Because Amber began to see from where the algorithms were derived, she became more confident in her mathematical ability and raised her opinion about the usefulness of mathematics and helped foster her movement from mathematics being an autocratic discipline to a multiplistic one.
Amber’s Themes

- Amber’s mathematical knowledge for teaching increased as she was better able to explain the mathematics underlying the algorithms she used to solve problems. Similar themes emerged with Brandy, Carl, Dawn, and Ed.

- Amber’s mathematical disposition improved as she began to see mathematics as a subject that she could understand. Similar themes emerged with Brandy.

- Although Amber could accept relativism in her general sense-making, she began MEST 1 thinking of mathematics as a dualistic discipline. By the end of the course, she could accept multiplicity in her mathematical thinking. Similar themes emerged with Carl, Dawn, and Ed.

- Amber initially struggled to explain her mathematics and was often confused as to why her explanations were not sufficient. This led to her struggling in the course. Similar themes emerged with Brandy, Carl, Dawn, and Ed.

- Amber repeatedly expressed her intent to use teaching techniques from MEST 1 in her eventual elementary school mathematic classroom. Similar themes emerged with Dawn and Ed.
Chapter 6: Brandy

Brandy [a pseudonym] struggled to let go of her belief that learning mathematics was simply memorizing algorithms, and had difficulty accepting that mathematics could be about concepts and the foundations from which algorithms are built. And in many ways, this struggle epitomizes her experiences in MEST 1: Brandy had trouble moving past her desire to simply do more exercises to learn and reason about the mathematics she was doing. Even toward the end of the course, Brandy still focused on the algorithms she knew—and trusted—to the exclusion of her own thought process, which she did not trust at the end of MEST 1.

Mathematical Disposition and Mathematical Experiences Before MEST 1

Brandy sought both certification as an elementary school teacher and middle school certification in English and Science. In fact, she regarded the elementary school certification as her fall-back option—preferring to find a job at a middle school teaching one or both of those other subjects.

I donno, when I was looking at [the course requirements to seek elementary school certification], it seemed that there was a ton of overlap in the classes that I was already taking for my middle school certification. So I figured why not. I mean, worst case, I can’t find a job in a middle school, maybe I can take a job at an elementary school for a year or two and get to know the schools [in the area].

(Introductory Interview)

Because of this, Brandy was the only participant who when taking MEST 1 did not necessarily see herself teaching mathematics at any level. “I donno, there is a ton of math
in science, but if I never have to teach math, I’m okay with that. I kinda don’t want to” (Introductory Interview).

Brandy scored 60 (out of 125) on the mathematics disposition survey with a mean response of 2.40 to positively phrased questions about her mathematical disposition (after recoding). Unlike Amber, Brandy chose many options that would indicate a slightly negative mathematical disposition, but only gave the most negative response to “13. At school, my friends would always come to me for help in mathematics,” “22. I tend to not ask questions in math classes because I am afraid I will look dumb,” and “23. I get frustrated when I do mathematics” (see Table 3). However, some of these responses were at odds with what she reports in her introductory interview. When asked if she thought she was good at math, Brandy responded

I think I am a really dedicated math student. [For example], Senior year [of high school] I was still paying attention, doing homework, taking notes, et cetera, when all of my friends had already checked out for the year. (Introductory Interview)
Table 3

*Brandy’s Pre-Mathematical Disposition Survey*

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</tr>
<tr>
<td>Total (of 125)</td>
<td>60</td>
<td>2.40</td>
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*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

In fact, Brandy reported having success in her previous mathematics classes both in terms of her grades and her learning in the course.

Researcher: Do you usually do well in math classes?

Brandy: Yeah, I will usually get an A or B.

Researcher: So do you think you’re good at math?

Brandy: Yeah. Sometimes it takes me a little bit [to learn math] if it is hard, but I am pretty good. I really like doing homework. (Introductory Interview)

And her learning,

I have always tried to understand everything I learn [in mathematics]. I mean, I didn’t get very far in math courses [in high school], and I’m not going to take crazy math courses here [in college], but I always wanted to understand
everything. If I was in Geometry, I wanted to know everything we talked about. If I was in Pre-Calculus, I wanted to know how to do everything we did in class.

(Introductory Interview)

Brandy attributed much of her success in mathematics to her diligence in completing assignments, even going so far as to say that she enjoyed mathematics homework. However, Brandy later revealed that it is not the process of doing mathematics homework she enjoys, but rather that she sees it as directly linked to her success in mathematics.

Researcher: What do you like about homework?

Brandy: I like the practice. I was in [a College Algebra course] and the homework was never graded, and I never did it. We would just have tests, and I, of course, did really bad on them.

Researcher: You said you enjoyed homework, though?

Brandy: I do, yeah.

Researcher: So why didn’t you do the homework for the class?

Brandy: I donno. I mean if it counted for something I would have done it, but it didn’t. And there was just so much stuff that quarter that I had to do [for other classes] and I felt that I could put off doing the homework till the test, but when it got around to test time I just didn’t do it. (Introductory Interview)

Brandy clearly was not doing mathematics homework because she enjoyed it, but rather because she saw it as related to her success; and when she believed she could be successful not doing homework, Brandy did not do it. However, Brandy believed that
her failure to do homework leads to her not being as successful in her mathematics courses.

Researcher: What does someone do to make themselves better at mathematics?
Brandy: Practice. Do extra problems, do more work, practice. I mean you could go to tutoring or talk to your teacher, but really, the best way to get better at something is to practice. (Introductory Interview)

In fact, she took the sentiment a step further to say that a person’s enjoyment of mathematics is interrelated to their mathematical ability.

Researcher: Why do you think so many people say they don’t like math?
Brandy: Because they aren’t good at it. I mean sometimes numbers and steps confuse people, but ultimately, I think that people who don’t like math don’t like it because they are bad at it.

Researcher: Do you think that there would be a way to teach math to those people who are confused by the steps and numbers so that they would be better at it?
Brandy: No. I think that most people should just work harder. I think if most people would put more work into it [their mathematics courses], then they would learn it [mathematics] and wouldn’t mind it so much. I mean maybe there are some people who won’t be good at math no matter how much work they put in, but most people just need to work harder. (Introductory Interview)

Brandy theorized that it is the individual’s responsibility to learn mathematics regardless of how it is being taught, and that if a person is good at mathematics that person will not dislike—which she saw as different from liking—mathematics. Thus, she believed that if
someone does not like mathematics, it is because they did not work hard enough to learn it.

In spite of this belief in the role of diligence in learning mathematics, Brandy viewed mathematics as two disparate disciplines: hard math and easy math.

Researcher: Do you think there are people who, no matter what, will never be good at mathematics?

Brandy: Yeah, I guess. I mean some people are just not capable of thinking like that. Wait, do you mean hard math or easy math?

Researcher: Mathematics in general.

Brandy: I donno, maybe if someone would learn the math differently then maybe they would be better at it. (Introductory Interview)

When asked to elaborate on these differences, Brandy shared that she believed that her hard math and easy math concepts could just as easily be described as practical math and academic math.

Researcher: Let’s go back for a second to something you said. When I asked you if there are people who no matter what are bad at math, you asked me about hard math versus easy math.

Brandy: Yeah.

Researcher: What did you mean by hard math and easy math?

Brandy: Like easy math would be the stuff we are learning now to teach elementary school kids, and hard math would be like Pre-Calculus and Calculus and stuff.
Researcher: So fundamentally, what is the difference between the two?

Brandy: What do you mean?

Researcher: If I were to give you a concept or a topic, how would you know if it was a hard math topic or an easy math topic?

Brandy: By how much it relates to real life, I guess. I mean, [MEST 1] deals with easy math: math that everyone should know and needs to know to live. But something like Calculus, only a few people can actually use that in their lives, and I bet most of that stuff doesn’t even relate to real life. Hard math deals with ideas, and easy math deals with stuff, I guess. (Introductory Interview)

Brandy viewed easy math as something that is used in the individual’s everyday life and hard math as mathematics that is divorced from most practical applications that a non-specialist would need.

Brandy also has a gender-diffused view of the role of women in mathematics, one which precludes her from reaching the highest echelons of the subject.

Researcher: What do you think of when you hear the word mathematics?

Brandy: I think of a bunch of numbers and letters and variables. I think of scientists at a college working on a big blackboard on a problem that takes up the whole board.

Researcher: What do those scientists look like?

Brandy: I donno, a bunch of old guys with glasses and messed-up hair and lab-coats.

Researcher: You said guys; do you think there are women in that picture as well?
Brandy: I think there can be, but that’s not what I would think of. (Introductory Interview)

Further,

Researcher: What does someone who is good at mathematics do when they are trying to solve a problem?

Brandy: First, he would understand the problem, because if you don’t understand what the problem is then you can’t possibly understand how to solve it. Then he would try to come up with a strategy to see if it works. If it works, great, if not, he would just keep trying.

Researcher: I notice you keep saying he when you talk about people who are good at mathematics.

Brandy: Yeah, I probably shouldn’t do that, because girls can be good at math too. It’s just that most people who are really good at math are guys.

Researcher: Why do you think that is?

Brandy: I think it is easier for a guy to be good at math or science. I mean that’s what we are taught, guys do math and science, girls do humanities.

Researcher: What if you had a student who was a girl and was really good at math and science?

Brandy: I would try to encourage her, but I would want her to realize that it would be hard for her to be in certain fields. For instance, she would have an easier time being a nurse than a doctor.

Researcher: Why do you feel that is?
Brandy: That’s just the role we are supposed to play. (Introductory Interview)

Brandy described the world of mathematics and sciences as a male-dominated field, and one that is hard for women to penetrate. Although this dissertation will yield to others to comment more thoroughly on Brandy’s gender-related beliefs, it is important to note them here to show context for statements she will make later in the study.

**Views on MEST 1 entering the course.** Brandy viewed getting a good grade in the course as her primary objective, with any sort of learning goals as secondary. However, in spite of her desire to not teach mathematics, Brandy still believed that the emphasis on crafting thoughtful explanations will benefit her teaching other subjects.

Researcher: What do you want to get out of [MEST 1]?

Brandy: I just want to pass it. I mean I am going to try to learn the math, because you have to learn it to pass it, but other than that I don’t really care.

Researcher: Do you think you need to take [MEST 1]?

Brandy: Yeah, I guess.

Researcher: Why?

Brandy: I think I need to learn how to explain things better. I mean the math is going to be easy, but this course says it’s going to be about explaining things, and I think I could use as much practice with that as I can get. (Introductory Interview)

**Position on Perry’s Scheme**

Brandy was at Position 4a: Oppositional Alternative. Brandy accepted a multiplistic world, as evidenced in the following.
Researcher: How would you define knowledge?

Brandy: I think having knowledge is not just knowing book stuff, but also having common sense. [For example] One of my roommates is really book smart and gets good grades but she can’t talk to people, so she’s just dumb. Also, my boyfriend’s sister is into English [class] and takes AP English and wants to be a writer, and I am just like ‘You know that’s not a real job.’ She is just so dumb—well, not dumb, but dumb. So I think knowledge isn’t just knowing stuff, but knowing how to use stuff.

Researcher: How does college play into that?

Brandy: It should be about expanding what you know and developing your common sense. (Introductory Interview)

She went on to say

Researcher: So how are knowledge and truth related?

Brandy: I think knowledge is the stuff written in books, while truth is more like beliefs.

Researcher: Do you think knowledge and truth are the same for everyone?

Brandy: Knowledge is. I mean the facts are the facts, and unless someone is going out of their way to distort them, everyone has the same facts. Beliefs and wisdom though everyone has a different take on. I mean, with my experiences I can take those same facts and come up with a different belief than someone who doesn’t have my experiences. Or maybe if someone had a completely different set of experiences, they would have a different belief. (Introductory Interview)
Brandy also demanded that every belief that is presented to her be justified, especially by the Authority and those she sees as aligned with Authority—in the above example her boyfriend’s sister and her roommate. Further, when she saw a belief that she does not believe accurately reflects the state of the world, she was quick to challenge that belief. During the introductory interview, Brandy viewed the university structure, her professors, and those that align themselves with the first two as the Authority, specifically questioning the validity of education for education’s sake as opposed to education as career preparation.

The only thing that really bugs me is when people try to shove their opinions down my throat. I mean, I have my opinions and if you have yours, more power to you. But I hate it when people try to shove their opinions onto you and think that they are right just because that’s what they think. And you get that a lot here, people who think they’re right because they are Dr. So-and-So, or Prof. Big-Shot, PhD. But they aren’t any better than I am, they’re not any better than you [the researcher] are. But they think they are. I mean some will try to just tell you they are right, and some will try to woo you over to their side by pretending to be your friend, but they all are just shoveling crap hoping you’ll pick it up. (Introductory Interview)

For a more concrete example, Brandy described something that happened to her in her Freshman Spanish class.

I was in Spanish class today and we have to make this stupid family tree. I mean, come on. Worst part was that not everyone had their own stuff, we had to share
tape and scissors. We aren’t in fifth grade, we shouldn’t have to cut stuff out—and no one carries scissors. (Introductory Interview)

Brandy also directly commented on her views of college education and what the college experience should entail, again placing herself opposed to the perceived views of the Authority.

Researcher: What would your view of an ideal college education be?

Brandy: To get outa here in four years and not have to take all these dumb classes you shouldn’t need to take. I donno, I really question this college stuff in general. All these people I see who have college degrees didn’t learn anything that they use. I mean, a friend of mine got their degree in computer sciences, but all she does now is enter stuff in a spreadsheet. I mean, you [the researcher] work at a community college and that is valuable, because those kids will actually use what they learn in college in their real lives.

Researcher: So you think college should be about career prep?

Brandy: Yeah, even though—yeah.

Researcher: What were you going to say?

Brandy: Colleges always say that they want you to have this broad view of the world, but I took History of Rock and Roll and Sociology last quarter and I am not going to remember any of that. I will be lucky to remember the stuff I have to so that I can be a good teacher. I don’t need to know about the Beatles or about rural counties anymore than I already do. (Introductory Interview)
Again, Brandy is vocally opposed to what she sees as discrepancies between the views of authorities and her world view. However, it is important to note that she chose to sign up for those classes, and that other courses (which were perhaps more pertinent to her education) could have filled the same General Education requirements which History of Rock and Roll and Sociology did. Brandy was not so much lamenting Authority requiring her to take classes that do not directly relate to her career, but rather she was questioning *why* the Authority wanted her to take these classes. In this, Brandy was looking for answers, not permission to take or not take classes.

**MKT Before MEST 1**

By the random selection process described in Chapter 3, Brandy was given Exercise Set A as her pre-test and later Exercise Set B as her post-test.

**Exercise 1.** Brandy initially struggled to find any similarity between the exercises, and only after completing all three exercises and examining her processes did she find the commonality. Brandy used the traditional algorithm, and eventually referred to her procedure as *borrowing*.

Researcher: So is there anything that is the same in all these problems?
Brandy: Well, they are all two-digit subtraction problems, and they all *<pause>*
Wait, I had to borrow in each of them! Is that what you mean?
Researcher: Yeah, but what do you mean by *borrowing*?
Brandy: Well, in each case this number (pointing to the one’s digit of the larger number) is smaller than this number (pointing to the ones digit of the smaller
number). So I have to borrow one from over here (pointing to the tens digit of the larger number).  (Introductory Interview)

When she was questioned as to why borrowing worked, Brandy was not able to explain the concepts underlying the algorithm.

Researcher: So you subtracted one from here (pointing to the tens digit of the larger number) and then stuck the one in front of the ones digit.

Brandy: Right.

Researcher: Why does that work like that?

Brandy: [Pauses] What do you mean?

Researcher: Well, you are subtracting one from this number, why then do you not just add one over here. Why can you stick that one in front of that number?

Brandy: I donno. Because it works.

Researcher: How do you know it works?

Brandy: You know, I don’t know. (Introductory Interview)

Similar to Amber, Brandy did not know and was not able to invent an alternative algorithm which did not use borrowing.

**Exercise 2.** Brandy immediately noticed the error in the student work, saying that the student forgot to “zero-fill.”

Researcher: What do you mean by zero-fill?

Brandy: Well after you multiply the first line, you put a zero here (points to under the ones place) and then you put two zeros in here. It looks like this other person just left empty spaces, which I guess is ok too.
Researcher: Why do you put a zero here?

Brandy: I think I remember someone saying it was a place-holder.

Researcher: What do you think you are holding the place of?

Brandy: I don’t know. [Pauses] I have no idea. I don’t think they [previous mathematics teachers] ever said why. It was always just ‘Put a zero here.’

(Introductory Interview)

Brandy was unable to discern the mathematical truths underpinning the algorithms she employed, nor was she able to invent an alternative strategy using mathematical principles that she better understood.

**Exercise 3.** Brandy did not successfully solve the division problem. Unlike the previous examples, however, Brandy did not try to use the traditional algorithm for division with fractions, admitting later that she “didn’t remember it” (Introductory Interview). Without the traditional algorithm to use, Brandy attempted to reason through the division process. At first, Brandy tried to model division as inverse of multiplication \((a ÷ b = x \text{ then } a = x * b)\) but fails to solve for \(x\) correctly. Instead of dividing by 1/2, Brandy ultimately divided by two. Brandy realized her mistake, but was unable to find her error and eventually abandoned the strategy.

Next, Brandy tried to devise a strategy where she modeled division as

\[
\text{Whole ÷ # Groups = # Items per Group,}
\]

However, Brandy was unable to discern how to model 1/2 of a group. With scaffolding by the researcher, she was able to better understand the second model, but was still unable to calculate the correct value.
Brandy was also unable to create a story problem which models fractional division, instead creating one that models multiplication by 1/2.

Researcher: Can you think of a story problem for this exercise?

Brandy: Say you have 1 and 3/4 boxes of chocolates, but you only want half of them because you are on a diet. How many boxes will you have? (Introductory Interview)

**Discussion.** Brandy was tied to the standard algorithms for most exercises. In Exercise 1 and Exercise 2, Brandy applied the standard algorithm with little to no thought of the mathematical principles being used in the algorithm. And when asked to try to reason about them, she could not, and reverted to trying to remember what was taught previously rather than trying to find new and different understanding. However, in Example 3, Brandy did not remember the traditional algorithm for solving problems with division by fractions. Instead of trying to remember an algorithm, she approached the exercise as an unseen problem, and invented several creative (if unsuccessful) strategies to solve the problem. When the traditional algorithm was not available for her, Brandy tried to find the answer by reasoning about the concepts of division. Interestingly, the fact that she did not remember the algorithm led Brandy to think more deeply about the mathematics and to be more creative in her problem-solving strategies than when she did know the traditional algorithm.

**Experiences Within MEST 1**

Brandy quickly grasped the nature of the course as looking deeply at mathematical concepts taught in elementary school.
Researcher: Describe someone who is good at mathematics.

Brandy: Someone who is good at mathematics can take the concepts and explain them to someone who doesn’t know them, can teach them to people.

Researcher: But what about someone who is a professor? I mean I know I have met people who were brilliant at coming up with original research and solving really hard math problems, but weren’t as good at teaching math to students.

Brandy: I donno, I feel like that’s how a lot of people are. I mean in our class now that’s kinda what we’re doing. [Dr. Jones] will give us some really easy concept and ask us to explain it and everyone is like ‘I donno, because it is.’

(Introductory Interview)

This approach would be especially challenging for her, as Brandy relied heavily on using mathematical procedures to solve problems without giving much thought to the mathematics used to formulate those algorithms.

**Classroom Activities.** Brandy quickly had her eyes opened to the complexity of the mathematics that underlie even the simple concept of place value.

Brandy: Right now we are learning place values, and it’s like I am learning place values all over again. We are learning it like we would have to teach it, and there is just so much detail and so many things that I just never thought of that go into it.

Researcher: Can you tell me more about that?

Brandy: [Dr. Jones] said that we could only use the words zero-through-nine to describe 697. So this is seven sticks, nine bundles of sticks, and six super-
bundles. Then we had to use letters A-through-F. And no one in the class got it. It didn’t make sense, but we had to just slow down and take it a step at a time.

Researcher: Why do you think [Dr. Jones] did this in class?

Brandy: Because this is the kind of thing that kids will have to think of when they are first learning about place value. I mean, we didn’t do anything like that, we just learned ones, tens, hundreds, thousands, etc. I don’t remember doing anything like that when I was in school.

Researcher: Did anyone go into place value that much with you when you were in elementary school?

Brandy: I don’t think so, no.

Researcher: Do you think that doing stuff like this would’ve helped you?

Brandy: I donno, maybe. (Course Interview 1)

While looking at place value, Brandy was exposed briefly to base-10 blocks. However, due to limited supplies, most people in the class were not able to actually handle the manipulative.

Brandy: The other day [Dr. Jones] brought in some blocks. And [Dr. Jones] said that the little pieces represented one, and the sticks were 10, and then the flat pieces were 100, and then there were big squares that were 1,000. And [Dr. Jones] showed us how we can use those to help us with place value.

Researcher: Do you know what those were called?

Brandy: I think [Dr. Jones] called them base-10 blocks.

Researcher: Did you get to use them?
Brandy: No, there was only one set. But [Dr. Jones] showed us how to use them, and then we would just draw them on the paper. (Course Interview 2)

Later,

Researcher: So what sort of things were you doing with your base-10 block models?

Brandy: We were comparing numbers. Like, say we were trying to convince a kid that 3,092 was bigger than 985. Well, we would have three cubes, nine lines, and two dots for the first one, and the second one we would have 9 flats, eight lines, and five dots.

Researcher: How does that show that the first number is larger than the second?

Brandy: Well, the first number has three cubes, but the second number doesn’t even have one. (Course Interview 2)

Assuming Brandy’s memory is accurate, this was the first time she had been exposed to place-value as a complex mathematical idea, but she still struggled to relate this concept and the mathematical algorithms she is familiar with. Brandy was counting the cubes as a formula to find the correct answer, rather than making the conclusion that 1 cube is 1,000 dots. Although with this particular concept they are similar, the arguments she uses is much more algorithmic and does not necessarily express an understanding of the meaning of the numbers or the representations of the numbers.

Brandy’s struggle became clearer as MEST 1 transitioned from whole numbers and began discussing rational numbers.

Researcher: Can you tell me about what you’ve been doing with fractions?
Brandy: Well, all the problems we’ve been doing are like: Bob ordered 3/4 of a ton, but he only received 2/3 of his order, so how much stuff did he get?

Researcher: And how would you go about solving that problem?

Brandy: [Dr. Jones] tells us to draw 3/4 of a whole vertically and shade them going left to right; then divide that into thirds horizontally and shade two of them going right to left. So we want then to count the total number of pieces and the number of pieces shaded both ways. So we end up getting 6/12. I don’t know if that’s right.

Researcher: Does that relate to any other concepts you may know about.

Brandy: No, not that we’ve talked about. I’m sure it does, but I don’t see it.

(Course Interview 2)

Brandy still tried to find the algorithm or pattern that she could use to solve multiple identical problems.

Brandy: I struggled a little bit with [fraction concepts]. I donno, once I know what I’m doing then I do alright, but it takes me awhile to figure out what’s going on. It’s like every problem we do I have trouble starting, but once I get started I do fine. And most problems are the same, so once I figure one or two out, then I can get the rest… [Dr. Jones] really tries to narrow down what we are doing to one simple process. I mean all the problems are different, but they are similar to each other, and once you know how to do one, you pretty much know how to do them all.
Researcher: So is it that [Dr. Jones] does a few examples in class and then you have to go home and do more examples like you did in class for homework?

Brandy: Yeah, that’s exactly it. (Course Interview 2)

This became a struggle for Brandy, as the problems presented in class, on the homework, and on assessments were too dissimilar for her to be able to construct a procedure to generalize the solutions.

Brandy also struggled with the problem-focused nature of the individual assignments. Because she was accustomed to having algorithms to apply to exercises, Brandy had difficulty accepting the open nature of the class and trying to use mathematical concepts to reason about problems.

Brandy: Well, [Dr. Jones] will give us a worksheet and [Dr. Jones] will expect us to work in our groups, but won’t tell us what to do. But then after we look at it for a bit, we will get back together as a class and [Dr. Jones] will tell us how to do it, so we can finish it.

Researcher: So how often does class work like this?

Brandy: Most every day. I mean [Dr. Jones] will say we are working on percents or whatever, pass around an attendance sheet and a worksheet, and then we will be expected to figure it out. But with about 20 minutes left in the class we talk about the worksheet as a class and put some problems on the board, and from there we can figure out the rest. (Course Interview 2)

Even much later in the course, Brandy still tried to find algorithms to complete exercises, rather than reason mathematically about problems. As the example below
indicates, Brandy was successful in repeating algorithms, but has a much more difficult
time generalizing those algorithms or discussing the concepts underpinning her
algorithms.

Researcher: So you’ve been talking about multiplying integers?

Brandy: Yeah.

Researcher: So something that sometimes confuses people is why a negative
number times a negative number is a positive number. Could you explain that for
me?

Brandy: Yeah, but I would have to use a story problem, because that’s the way we
did it in class.

Researcher: That’s fine.

Brandy: So the way we did it was thinking of sending and receiving checks and
bills. So think of sending a letter as a negative and receiving a letter as a positive.
Then think of a check as a positive and a bill as a negative. And think of your
total bank account. So if you send out three bills for $15 each, you would get in
your bank account $45 when those people pay their bills. So \((-3)(-15) = +45\).

Researcher: So do you think you could take that model and put it in a different
story problem?

Brandy: No, that’s the way we did it in class. We also did something with green
chips and yellow chips, but I donno, it was really confusing the way we did that in
class, so I don’t think I could do it differently. (Course Interview 4)
Even at the end of the course, Brandy continued to cling to her belief that mathematics is merely a set of algorithms to solve problems that do not necessarily have their foundations in understandable mathematical principles.

Brandy: We talked about four or five different ways to multiply large [whole] numbers, but we didn’t really go into any reasons why they worked.

Researcher: So if you were going to show someone how to multiply large numbers, what algorithm do you think you would use?

Brandy: I would just teach them the one I learned in school [the traditional algorithm]…Because that’s how I was taught to do it. I mean all those other algorithms are confusing, and I think they [students] would just get confused.

(Course Interview 5)

**Explanations and Assessment.** Because Brandy tended to apply algorithms without examining the mathematical concepts that form them, Brandy had trouble adjusting to the explanation portion of assessments.

Researcher: What have you been struggling with [in the assessments]?

Brandy: I think part of it is bad luck. I mean they [the instructors] only grade a few problems out of the homework, and if they happen to pick the one you were confused on, then your grade suffers no matter how good you did on the rest.

Researcher: Did that happen to you?

Brandy: Yeah. I donno, I guess that I have had some problems with the whole ‘explanation’ part of the course. I think that once I get that down, then I should be fine.
Researcher: What do you mean?

Brandy: Well, they want us to write explanations for everything. And sometimes that can be a bit tricky. I am not sure exactly what they are looking for. (Course Interview 2)

It is important to note that she began by blaming Authority for her lack of success (Perry Position 4a), but later admitted that some of the struggle could be due to her own lack of understanding. This dichotomy between her need to question Authority and her need to think of mathematics as a set of repeatable algorithms is pervasive in any conversations regarding her assessments or the explanation portion of the course.

For example, while reflecting on one of her midterms, Brandy deflected some of her struggles onto the unseen Authority, the graders of the midterm.

Brandy: On several points on the test, it would say things like ‘Needed to say blah, blah, blah.’ Like, for one of them, [the grader] said I needed to say these two things. But I said them up here; I just didn’t use those words.

Researcher: So you think if you used those terms you would’ve got a better score.

Brandy: Yeah, which sucks. I mean, they harp on us about how each kid is going to think about this stuff differently, but when we don’t do these explanations exactly how they want, then we lose a bunch of points. (Course Interview 3)

It is largely because of her developmental position that she easily sees the inconsistency between the goals of the course and the way she perceives her midterm to have been assessed.
However, Brandy also struggled with some of the mathematical principles used in the course as well as the explanations. These struggles compounded and created an issue with her time management when Brandy took the first mid-term.

Brandy: I donno, it is really frustrating. I knew what I was doing on three of the problems. But two of them I got points taken off because of explanation stuff, one I made a little mistake on and got a lot taken off, and then I just didn’t have enough time. I missed [a lot of] points because the last problem I didn’t get to until there was about five minutes left in the midterm, and then there wasn’t enough time to draw a good picture or write an explanation.

Researcher: So time was really an issue for you?

Brandy: Yeah.

Researcher: How much more time do you think you would have needed?

Brandy: I donno. I just got kinda panicky at the end. I think if I knew I had as much time as I needed, I could have got it done [in the allotted time]. But because I knew there was a clock I started to get nervous and rushed. (Course Interview 3)

Even towards the end of the course, Brandy was searching for algorithmic solutions to the problems she was assigned.

Brandy: We have the lecture and the recitation and the homework. And I donno, it’s like no one really teaches us how to do the homework. Nobody ever just tells us how to do the homework, so I have to look it up in the book, how to do the
homework. It actually explains what the processes are of what we are doing in class.

Researcher: And you don’t feel like you are learning that from class?

Brandy: No, not at all. (Course Interview 5)

Not receiving the support she wanted from the class (i.e. the correct way to solve the problems), Brandy began to look toward the books examples to discern the patterns that the text uses to solve its example problems. And although it was encouraging that Brandy read the textbook for the course, Brandy’s need to do this further emphasizes her reliance on algorithms and her refusal to view mathematics as anything more than unconnected and unrelated algorithms.

**MD After MEST 1**

In spite of some of her struggles with the course material, Brandy increased her mathematics disposition score to 71 with a mean response of 2.71 (increased from 2.40) to positively phrased questions about her mathematical disposition (after recoding). In general, see saw herself as being more confident in her own mathematical ability, and slightly less confident in her ability to teach mathematics in elementary school. (See Table 4). Brandy also responded definitely true to the statements “2. I find many mathematical problems interesting” and to “9. I am confident in my ability to solve mathematical problems” as well as responding definitely false to the statement “4. I am not the type of person who is good at mathematics.”
Table 4

*Brandy’s Post-Mathematical Disposition Survey*

<table>
<thead>
<tr>
<th>Category</th>
<th>Prescore</th>
<th>Postscore</th>
<th>Change</th>
<th>Prescore Mean</th>
<th>Postscore Mean</th>
<th>Change in Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preparedness to Teach Mathematics (of 40)</td>
<td>21</td>
<td>25</td>
<td>+4</td>
<td>2.63</td>
<td>3.13</td>
<td>+0.50</td>
</tr>
<tr>
<td>Mathematical Beliefs and Attitudes (of 40)</td>
<td>15</td>
<td>21</td>
<td>+6</td>
<td>1.88</td>
<td>2.63</td>
<td>+0.75</td>
</tr>
<tr>
<td>Mathematical Self-Efficacy (of 20)</td>
<td>10</td>
<td>13</td>
<td>+3</td>
<td>2.50</td>
<td>3.25</td>
<td>+0.75</td>
</tr>
<tr>
<td>Diligence in Learning Mathematics (of 25)</td>
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<td>12</td>
<td>–2</td>
<td>2.80</td>
<td>2.40</td>
<td>–0.40</td>
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<tr>
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<td>71</td>
<td>+11</td>
<td>2.40</td>
<td>2.84</td>
<td>+0.44</td>
</tr>
</tbody>
</table>

*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

**MKT After MEST 1**

Brandy showed improvement when doing the exercises. Brandy still used algorithmic thinking to solve many of the problems, but she is now better able to state some of the mathematical reasons why her algorithms are effective.
Exercise 1. Brandy was immediately able to recognize that each subtraction problem models subtraction by regrouping. However, she still favored the term borrow to describe the process.

Researcher: What do these problems have in common?
Brandy: They are all subtraction with bor…err…regrouping.
Researcher: What were you going to say?
Brandy: Well, I still think of it as borrowing, but we are supposed to say regrouping now. It is a better term, I guess.
Researcher: Do you think there is a difference between borrowing and regrouping?
Brandy: No, they are just different words, but they mean the same thing. (Final Interview)

Brandy once again used the traditional algorithm to complete the exercises, but this time was better able to describe why her process of borrowing works.

Researcher: So you subtracted one from here (pointing to the tens digit of the larger number) and then stuck the one in front of the ones digit.
Brandy: Yeah.
Researcher: Why did you do that?
Brandy: Well, you can take one from here (the tens place) but you really aren’t taking one—you’re taking away ten. So when you add it to the digit in the ones place, you need to add ten. (Final Interview)
Exercise 2. Once again, Brandy used the same term that she used in the initial interview, zero-fill. But similarly to Exercise 1, she is better able to express the mathematics behind the term and the procedure.

Researcher: What do you mean when you say zero-fill?

Brandy: Well after you multiply the first line, you put a zero here (points to under the ones place) because you aren’t just multiplying 5, you are really multiplying 50. And then you put two zeros in here, because you are really multiplying by 600.

Researcher: When we first talked, you mentioned something about a place-holder. Do you remember that?

Brandy: I think I remember.

Researcher: Can you tell me what that means?

Brandy: I don’t know. I guess it means that you are holding the place for the digit. Like if five is in the tens place then it is 50. So I guess you are holding the place of the place-value. (Final Interview)

However, even though Brandy is better able to explain this algorithm for multiplying whole numbers, she still does not have enough confidence to use an alternative algorithm.

Researcher: You mentioned earlier that of all the methods for multiplying numbers, you preferred this one. Do you still?

Brandy: Yeah. I mean, it’s the one I used, it’s the one all my friends know. And even if I were to teach a different one [to an elementary school student], their
teacher next year would probably want them to use this one. I mean if I have to teach this one anyway, then I might as well just teach this one.

Researcher: If that weren’t the case, say you knew the student’s teacher next year used a different model, like lattice multiplication or partial-products, would you still teach this method.

Brandy: Yeah. I mean, maybe not their next year teacher, but someone is going to ask [the student] to know this algorithm. Someone has to teach it. (Final Interview)

Exercise 3. Brandy correctly used the flip-and-multiply method to solve the problem. However, when asked why her algorithm worked, she attempted to change algorithms to explain her process.

Researcher: I donno, did you ever think it was strange that the whole flip-and-multiply thing actually works?

Brandy: Yeah, I guess it is kinda weird.

Researcher: Do you know how it works?

Brandy: Hold on. (Final Interview)

Brandy then attempted to solve the problem using a pictorial representation of the fractions. She completed the answer, but got a different (and incorrect) answer using the model.

Researcher: So you have two answers there, which one do you think is right?

Brandy: I hope the second one, because that’s what I will need to do on the final.

Researcher: Can you convince me that one of them is right or not?
Brandy: Well, (checks work) I don’t think I made any mistakes when I did *flip-and-multiply*, so that’s probably right.

Researcher: Can you tell me what happened then here? (Points to pictorial representation)

Brandy: (Pause). I think this is multiplying by 1/2.

Researcher: That’s right, so how can you fix it?

Brandy: I don’t remember how to do that, I will have to go back and look it up.

(Final Interview)

Once again, Brandy relied on her algorithms almost to the exclusion of thinking about the problems and the mathematical principles. Ironically, this represented a significant step backward from her thinking when she first encountered the problem. When she did not know any algorithm to solve the problem, she was forced to try to reason about the mathematics and what the mathematical statement meant. But now that Brandy knew multiple algorithms, she relied on those to the exclusion of reasoning about the mathematics.

When asked for a story problem, Brandy gave a problem which modeled dividing by 2 instead of 1/2.

Say we [Brandy and the Researcher] have 2 1/4 feet of yarn and we need to split it into half so we each have an equal piece. How long a piece do each of us get?

(Final Interview)

**Discussion.** Brandy’s continued to rely on traditional algorithms to solve problems, though she understood the mathematical principles underlying those
algorithms later. And even though she was hostile to the new way to do mathematics taught in MEST 1, that is a necessary step to her intellectual development, as is discussed in Chapter 10.

**Reflections on MEST 1**

Brandy left MEST 1 with an improved disposition toward mathematics as well as a deeper understanding of the algorithms she uses to solve mathematical problems. In fact, Brandy viewed her reinforced knowledge of mathematics as one of the key elements she gained from MEST 1, along with an improved ability to craft explanations, which Brandy believed would benefit her whether or not she teaches mathematics.

Researcher: At the beginning of the quarter, I asked if you thought you needed to take [MEST 1], and you said ‘yes.’ Do you still feel that way?

Brandy: Yeah. I mean for starters there was a lot of stuff I either didn’t learn in elementary school or learned and forgot. So it was nice to get the refresher. Second, I think that taking [MEST 1] really got me to think about my explanations. How I phrase things, how exact and precise I need to be. And that will help me be a better teacher period, if I teach math or science or English or something else.

Researcher: So how would you say your explanation technique is different now than it was at the beginning [of MEST 1]?

Brandy: I try to be more careful about what words I use and to make sure I write and say exactly what I mean with my explanations. I think before I would just try to explain things the way I thought about them, but now I know that it is more of
a process—that you have to think and work to make a good explanation. (Final Interview)

Ironically, although Brandy viewed her improved explanation process as one of the key skills learned in MEST 1, she also saw that as something that would negatively affect her overall grade in the course. She regretted not being able to craft explanations using sound mathematical concepts sooner.

Researcher: If you can go back and change one thing about [MEST 1], what would it be?

Brandy: I think I would have taken that letter [Dr. Jones] sent at the beginning of the quarter more seriously.

Researcher: What do you mean?

Brandy: At the beginning of the quarter [Dr. Jones] sent us this letter, saying that [MEST 1] wasn’t like most other math classes and that we wouldn’t learn as much math but learn how to think and explain stuff. But I just kinda laughed it off. I mean, I figured it was just one of those things teachers say to scare kids.

Researcher: So what would you have done differently?

Brandy: I would have tried to take better notes and try to get concepts sooner. I feel like I am a lot better at explanations now, but when I started the class I missed a lot of points because I wasn’t as good at explanations as I thought I was. So I guess what I would really want is a way to get better at explaining stuff quicker, so I wouldn’t have lost those points. (Final Interview)
Brandy showed improvement in many of the explicitly stated objectives of the course—Brandy improved her (a) mathematical disposition, (b) she improved her mathematical thinking and increased her mathematical knowledge for teaching, (c) and she is better able to craft explanations that are both sound mathematically and readily understandable by a novice. However, Brandy did not improve in the one objective that, although implicit, is unanimously shared by all the instructors of MEST 1. Brandy does not see the techniques and learning strategies modeled in MEST 1 as something she would integrate into her future mathematics classroom.

Researcher: So overall, what do you think you will take away from [MEST 1]?
Brandy: To use in my class?
Researcher: If that’s how you want to think of it, yeah.
Brandy: I donno. I mean, I think that it is neat to see where some of the rules come from, but I don’t know if I will ever really do anything with them.
Researcher: Do you think it helps you to know that the math rules aren’t just rules? That they were formed from mathematical reasons and thinking?
Brandy: Yeah, it’s nice, but I don’t see how knowing that is ever going to affect the way I teach.
Researcher: I know you don’t want to teach math, but if you ever do, do you think you would use anything you may have picked up in [MEST 1]?
Brandy: Probably not. I mean, maybe some stuff. I think some of the thinking that goes into the algorithms may be good for the students to have. But as far as
the new algorithms, no. I mean, the students need to learn the right way to do these problems.

Researcher: What do you mean by the right way?

Brandy: Not that one way is better than another, but if I am teaching third–grade math, and I know the fourth–grade math teacher wants her students to solve problems one way, I should teach them that way. (Final Interview)

Summary

Throughout the entire course, Brandy remained tied to her algorithmic belief structure, so much so that it hindered her learning during the course. Interestingly, she actively rejected mathematics as a multiplistic discipline in spite of clear evidence to the contrary. This may be due to her developmental position, or may be due to the fact that she could not progress past her authoritarian views of mathematics. Brandy’s thought was from a perspective of Position 4a, and by being oppositional to those she presently views as authority figures (the instructors for MEST 1) she was clinging to the views of previous, now unseen authorities (her previous mathematics teachers).

Brandy’s Themes

- Brandy’s mathematical knowledge for teaching increased as she was better able to explain the mathematics underlying the algorithms she used to solve problems.
  Similar themes emerged with Amber, Carl, Dawn, and Ed.

- Brandy was reluctant to engage in activities that she did not see as directly relevant to her perceptions of the demands of teaching elementary school mathematics. Similar themes emerged with Amber.
• Brandy struggled to move from thinking of mathematics as an algorithm-based discipline to a concept-based discipline. Similar themes emerged with Carl.

• Brandy had trouble adapting to the explanation portion of assessments, and at times blamed authorities (graders) for her lack of success. Similar themes emerged with Amber, Carl, Dawn, and Ed.

• Brandy began the course in Perry’s 4a: Oppositional Alternative and thinking of mathematics dualistically. By the end of the course, Brandy’s MID moved to Position 2: Strict Dualism. This provided an interesting synergy where Brandy could at one time be opposed to her current authorities (MEST 1 instructors championing mathematics as a multiplistic discipline) and aligned to past authorities (old math teachers who taught mathematics as a dualistic subject). Similar themes emerged with Carl.
Chapter 7: Carl

Carl [a pseudonym] is a senior who, by his own admission, postponed taking MEST 1, almost jeopardizing his graduation. This is in spite of his previous success in mathematics courses, including traditional college mathematics courses. Even though Carl was successful in MEST 1 by the metric he values most (his grades), his interpretation of many of his experiences served to reinforce negative stereotypes about the nature of mathematics. Namely, Carl’s success on graded materials reinforced the notion that even complex mathematics problems have a sole right answer, that the answer is possessed by the Authority (in this case, his professor, Dr. Jones and his recitation leader, Lara) and that mathematics is about students finding the way the Authority wants them to solve the problem.

Mathematical Disposition and Mathematical Experiences Before MEST 1

Carl comes to MEST 1 with the notion that mathematical ability and mathematical tolerance are two interrelated concepts.

Researcher: I noticed several times that I asked you about ‘mathematical ability’ and your answer had something to do with how much someone liked mathematics. Do you see those ideas interrelated?

Carl: Yeah, I do. I mean, I don’t think that anyone really loves mathematics, but I think that there are people who are really good at it and don’t mind it as much. So I think if you are good at math, you will like it. And I think if you aren’t very good at math, then you don’t like it as much, and if you don’t like it, you won’t do it and you won’t get better at it.
Researcher: So you don’t think anyone really loves mathematics?

Carl: No… I mean maybe someone does, but I think most people just put up with it. (Introductory Interview)

Interestingly, Carl confuses his own views of mathematics with the views of everyone else: because Carl tolerates mathematics only up to the point he stops being good at it, so too must everyone else.

Carl was hesitant to take MEST 1, in part because of his negative attitudes about mathematics, and in part due to his trepidation to take a college mathematics course after not successfully completing calculus.

Yeah, I put off taking this course. I started out undecided, and I took [Pre-Calculus] my freshman year, and then the next quarter I tried to take [Calculus 1] but I dropped it. It was too much. I chose to go into early childhood last year, but I didn’t really want to take another math course. (Introductory Interview)

This negative experience in calculus soured his enjoyment of the course, but did not necessarily after his overall mathematical disposition.

Researcher: What are some of your feelings about mathematics?

Carl: It’s annoying. I mean, it takes more thought than a lot of the other subjects I have to do, so I guess that’s why I don’t like it as much. But if I am in the right mood I kinda enjoy doing it. I like solving the puzzle and finding the right answer. I donno, it is really about success for me: if I am doing well in the math I will enjoy it and if I am not then I think it’s terrible.

Researcher: What do you mean by ‘doing well’?
Carl: If I am understanding it and getting a good grade in it. Like I really hated Calculus because I was struggling with it and I saw no way that I would ever apply or use that stuff in my real life. But now [in MEST 1], we are learning about the kind of math that I am going to teach to kids, so that is important. Like basic subtraction, basic addition, stuff that if the kids don’t know this there is no way they can be successful in math later on. (Introductory Interview)

Before he took calculus in college, Carl described himself as being good at mathematics and enjoying the subject.

I always kinda enjoyed math, but that was because I was good at it. Like I always got As and Bs in math up until that Calculus class [in college]…My favorite subject was Spelling. I really enjoyed the straight memorization of it, and learning how words were spelled. I liked that there was a right and a wrong, and that it was clear whether you spelled a word right or not. (Introductory Interview)

Carl found that the skills he developed for spelling also applied to mathematics, and he could be successful in both subjects. Carl viewed both as topics that could be mastered through rote memorization, and both should be preformed quickly without the need for deep thought.

Carl: I remember we had these sheets of math problems, like 50 or so problems. And my teachers would just make [the class] do them as fast as possible. I eventually got really good at those—I could get them all done and correct. I liked
finding the patterns and memorizing the patterns: things like 4+7=11 or anything divided by 0 it always equals 1, or that anything times 1 is 1.

Researcher: Is it fair to say that that is the part of math you enjoy, memorizing and applying those rules?

Carl: Yeah, because I was good at it. I could see that I was doing it faster than everyone else and it made me feel good.

Researcher: Did you ever explore where those rules came from?

Carl: No, it was just memorizing the rules and do [sic] them as fast as possible.

Researcher: What if you didn’t remember a rule?

Carl: Then you got that problem wrong.

Researcher: But what if you may have known it in elementary school but had since forgotten a rule or maybe remember the rule wrong?

Carl: Then I guess you didn’t really learn it that well. (Introductory Interview)

It is important to realize that even though Carl places a great value on rules and computational algorithms, at times he does not correctly remember the algorithms (e.g. “Anything times 1 is 1”).

Throughout middle school and high school, Carl would continue to equate the speed which he could learn mathematics with his mathematics success. In that, he viewed his best mathematics teachers as the ones who could facilitate his fast learning of a topic, as opposed to those teachers who may have asked him to think deeply about mathematics.
Researcher: Can you tell me about some the math teachers you had in school that you really liked?

Carl: Well, there was [my 8th grade algebra teacher] and my [12th grade trigonometry teacher], they were really good.

Researcher: Why do you think those math teachers were really good?

Carl: I’m not sure. I just remember when they would explain stuff I would usually get it the first time, so that later when they were re-explaining stuff I didn’t have to pay attention because I already understood it. (Introductory Interview)

Although his experience in calculus did not improve his mathematical disposition, Carl still maintains a positive mathematical disposition (see Table 5). Carl scored 94 (out of 125) on the mathematical disposition survey, with a mean response of 3.76 to positively phrased questions about his mathematical disposition (after recoding). Namely, Carl believes in his own mathematical ability and his ability to teach elementary school mathematics, and Carl also correlates diligence to mathematical success. On the mathematical disposition survey, Carl responded definitely true to “5. I have always done well in mathematics classes” and to “24. Overall, I feel confident in my mathematical ability.” Carl also responded definitely true to “1. Generally, I feel secure about the idea of teaching young children mathematics” and “25. Overall, I feel confident in my ability to teach mathematics,” meaning that Carl believes that his mathematical ability will transition to him becoming a successful mathematics teacher.
Carl believes that diligence and practice make someone good at mathematics; however he, at times, conflates diligence with repetition.

Researcher: What does someone do to become really good at mathematics?

Carl: Drill. I mean do flashcards, do speed-sheets, and try to get as fast as you can with the stuff.

Researcher: And what if someone is really good at the speed tasks, what do you think is the next step for them?

Carl: Just make the speed-sheets harder, more advanced. I mean if you can just do things really fast that is always going to be good. (Introductory Interview)
Further, Carl views that the best way to approach solving a new problem is to model it after a previously solved problem, and the best way to prepare for a mathematics exam is a “practice test.”

Researcher: What do you think someone who is really good at math does when he or she comes across a math problem she doesn’t know how to solve?

Carl: Just try to relate it back to a problem that they have solved before, or seen solved before.

Researcher: And what does that person do when they are studying for a math test?

Carl: I donno, I have never really figured out the whole how-to-study-for-a-math-test thing. I mean if the teacher gave me a practice test I would do it, but that is the only time I ever really studied for a math test. (Introductory Interview)

He goes on to describe how important practice tests were for him in his previous mathematical success, and how the lack of practice test may be partially at fault for some of his lack of success in college mathematics courses.

One of the things I loved in high school and middle school was that my math teachers would usually give us practice tests before the real test. And then the real test was just the practice test with different numbers. But in college, my [pre-calculus] teacher would give a practice test, but the real test would have stuff that wasn’t on the practice test, and my [calculus] teacher wouldn’t give [practice tests] at all. So I really liked having good practice tests. (Introductory Interview)

Carl sees being an elementary school teacher as both a rewarding profession and a stable career.
Researcher: When did you decide you wanted to become an elementary school teacher?

Carl: Well, it was the end of my sophomore year, and the [the university] said I had to declare something. So I started looking around and I had taken some classes in human development and I thought they were interesting. So since I had to declare something, I figured this was as good a thing to declare as anything.

Researcher: So what made you decide to go into elementary education as opposed to something else in human development?

Carl: Well, I think that being an elementary school teacher would be a cool job. I mean, it’s a lot of work getting there, but I think being an elementary school teacher would be a secure job and would be fun.

Researcher: What makes you say that?

Carl: I donno, I guess just my vision of what it was like when I was in elementary school and the teachers that I had. I mean, a lot of those same teachers are teaching the same grade at the same school. (Introductory Interview)

Even though Carl sees career stability and the job being “fun” as important factors in his decision to become a teacher, Carl nonetheless sees teaching mathematics to elementary school students as an important task.

Carl: People have told me ‘Man, you’re just going into elementary school, you’re going to be a baby-sitter.’ But this is the most important time for these kids. If they don’t learn addition, then they aren’t going to get multiplication. If they don’t learn multiplication, how are they going to understand polynomials. So I
see the math that I am going to be teaching as very important to these kids and their futures. (Introductory Interview)

Researcher: So does that put a lot of pressure on you as their teacher?

Carl: No, because it shouldn’t be very hard. I mean, in the end it is just basic addition and subtraction. (Introductory Interview)

**Views on MEST 1 entering the course.** Carl sees himself as a strong candidate to teach elementary school mathematics. Carl’s early experiences with the course material have left him confused, as success in MEST 1 seems radically different than the way he would view success in many other math courses.

This new math that we’re doing [in MEST 1] is weird. I mean the math is really simple, but having to be able to explain it to a student is going to be the hard part. The math is easy—anyone can figure out these math problems. They are straightforward and common sense, but they aren’t common sense to a first-grader.

(Course Interview 1)

**Position on Perry’s Scheme**

During the quarter, Carl was transitioning to Position 3: Early Multiplicity. Carl can accept other people’s differing views, however he does not prioritize those views as a relativistic thinker might. For instance, when asked what he thought the ideal college education may be, Carl stated his belief is that the ideal education can only be determined by the individual.

[An ideal college education] is about making it whatever you want it to be. This is the time in your life where you will have the most free time and the least
responsibility, and this is the time to explore anything you would want to explore. You should find what you like, what you’re good at, and what you want to do for the rest of your life. (Introductory Interview)

Further, when asked about his decision-making processes, Carl reiterated that he would seek the perspectives from many different people with differing backgrounds.

I like to ask for advice from a lot of different people: my parents of course. But sometimes I like to ask stuff from my friends—I feel that they usually understand better what I am going through. But I usually will ask advice from a lot of different people and try to make a decision from that advice. I don’t think I take one person’s advice more than another, and sometimes what I do isn’t really anyone’s advice, but something that is an amalgam of the opinions, or something I just made up myself. (Introductory Interview)

Although in most of his answers Carl responded from Position 3, with some of his responses he still struggled with multiplicity. And although this could be evidence of a transition to one of the Position 4a Oppositional Alternative, it is more likely remnants of thinking from Position 2: Strict Dualism.

Carl: I think everyone is entitled to their opinion. I mean, I am not a religious person, I have never gone to church, I think most of that stuff is dated. I mean people used to believe in gods or whatever because they couldn’t explain stuff. But now we have science, we know why it rains and it has nothing to do with Thor or Poseidon or whatever.
Researcher: So how would you interact with someone you know who holds strong religious convictions?

Carl: Well I definitely wouldn’t bring up abortion or anything else that is super religious around them. I mean, everyone can believe what they want, but there are just so many little variations between so many beliefs. And sometimes I like to ask people what they believe so I can try to understand some of the variations and what it means to believe Catholic stuff or Christian stuff or Jewish stuff.

(Introductory Interview)

Also, when evaluating his own knowledge, Carl placed a priority on grades, which are highly dualistic—as they are handed down from authorities.

Carl: [Knowledge is] being able to recognize patterns. And it’s like building a foundation of stuff, ideas, so you can combine them into different ideas. Kinda like math, you learn about addition and then you learn about multiplication, and eventually you learn about variables and stuff…

Researcher: So how would you evaluate knowledge?

Carl: Grades. I know some people wouldn’t agree with that, but I think that is the only true objective measure of what you know.

Researcher: How do you think the people who don’t use grades would evaluate their knowledge?

Carl: How practical their knowledge is. I mean maybe someone didn’t get an A, but they got a C and they learned what they thought they needed, so for them their knowledge is sufficient. (Introductory Interview)
Again, here Carl is speaking from a position of absolutism, which may indicate that his thinking has not completely transitioned to Position 3. This belief could be a reflection of the reality of the importance of grades in Carl’s future—as grades will determine if he can successfully complete MEST 1 or what graduate school he will enter—but may also show him still transitioning to multiplistic thinking.

**MKT Before MEST 1**

By random selection, Carl received Exercise Set A for his Pre Test. Carl relied heavily on algorithms when looking at the exercises. When Carl was familiar with the algorithm, he was able to quickly and successfully complete the problem. However, in those cases, Carl relied heavily on the algorithm, and was not able to reason about the algorithm or use a second algorithm to check his answer. When Carl did not remember the algorithm, he was unable to reason about the problem at all, and refused to complete the tasks.

**Exercise 1.** Carl quickly recognized the similar nature of the subtraction problems and correctly described the algorithm to solve those problems.

Researcher: What do the three subtraction problems have in common?

Carl: You have to take one away from each problem and add it to the ones.

Researcher: What do you mean by that?

Carl: I think you call it ‘borrowing.’ You have to take one from the tens place and add it to the ones.

Researcher: So you add one to the two to get three?

Carl: No, you stick it in front of the two and get 12.
Researcher: Why can you do that?

Carl: You know, I am not real sure, I don’t know really.

Researcher: How would you teach those ideas to students?

Carl: I donno, that’s a real good question. (Introductory Interview)

However, Carl was not able to reason why the algorithm work, he did not connect the concept of place value to the algorithm, and he could not think of a strategy to teach the solution process to students.

Exercise 2. Carl could not remember the correct way to multiply two large whole numbers, and because of that did not reason about the students’ probable confusions.

Carl: I remember learning how to do this before in elementary school, but I have no idea how to do this now.

Researcher: Can you tell me anything about the process?

Carl: I remember it has something to do with putting 0s in to keep your place, but I really don’t remember how to do this. I remember place value and 0s, but I wouldn’t know how to do this.

Researcher: How would you solve the problem?

Carl: My calculator. I mean I always have one on me: either my graphing one or the one on my phone, but man, I completely forget how to multiply large numbers like this. I mean, who would do this in your head, everyone I know would pull a calculator out to do this kind of problem. (Introductory Interview)

Also of note, Carl could not recognize which answer was “probably correct” and which was not until he computed the answer with his calculator.
**Exercise 3.** Carl did not engage with the problem once again until encouraged to solve using a calculator. Even then, Carl opted to convert the fractions to their decimal equivalents to compute the answer more easily.

Carl: [Audible groan] Division with fractions.

Researcher: Well, first, let’s just focus on how you would solve this problem.

Carl: Well, I know there is a reciprocal in there somewhere, but I would use a calculator to do this.

Researcher: Can you do this on your calculator?

Carl: Sure. [Divides 1.75÷0.5 to find 3.5]

Researcher: Why did you change them to decimals?

Carl: It’s just easier to put them into the calculator.

Researcher: What if it were a repeating decimal, like 1/3 would give?

Carl: I would just round it. (Introductory Interview)

Also, Carl did not attempt to create a story problem to model division with fractions (decimals).

Researcher: So can you come up with a good story problem to model the problem?

Carl: That is a good question. [Pauses] Dividing by 1/2 is such a difficult thing to think about, because dividing you think the number should be smaller, but when you are dividing by a number less than 1 you get a bigger number. I remember learning that and it blowing my mind.

Researcher: Why do you think that is?
Carl: Well, you have to multiply by the reciprocal.

Researcher: I have always thought that was an odd rule, do you have any idea why you do that?

Carl: I donno, it’s a mystery. Like the meaning of life.

Researcher: Would you be comfortable teaching a rule like that?

Carl: Yeah, I mean it works, and it is an easy thing to do, but whoever came up with that is a genius. (Introductory Interview)

**Discussion.** Although it may be easy to see Carl’s lack of engagement in these activities as counter to his previous statements regarding the role of diligence in mathematical learning, it speaks more to how Carl viewed mathematical learning. In Carl’s view, mathematics was a subject where the right answers could be quickly computed, and the responsibility to teach those algorithms lay on the teacher. It was the student’s responsibility to memorize the taught algorithms and apply them when the situation called for the algorithms to be applied. So when Carl could not recall an algorithm to multiply large numbers, divide fractions, or create word-problems to model division, he abdicated responsibility for those solutions. Carl did not exert effort to solve these problems not because he did not value effort in learning mathematics, but rather because that was not how he believed effort needed to be exerted in mathematical learning.

**Experiences Within MEST 1**

Carl entered the course with many conflicting and contradictory views on the course. First, Carl acknowledged that he was good at mathematics and has had success in
mathematics courses apart from the one calculus class. However, that calculus class was such a bad experience that Carl hesitated to take any other mathematics courses.

I’ve heard this course is really hard, but I don’t see how. I mean, it is just elementary school math. But everyone I talk to says that this course, man it’s a lot harder than you think coming in. So, I donno. I mean, I don’t want to take this class, but I figure I need to. (Introductory Interview)

Interestingly, although Carl acknowledged that he has heard from other students that the course is difficult, he did not know why the course was difficult, as it focused on elementary school mathematics content.

**Classroom Activities.** Carl began the course struggling with the necessity of the explanations and the seeming triviality of the mathematics being explored. However, upon consideration, Carl began to realize that although he was familiar with the mathematical concepts being addressed, he did not understand how the concepts were related, or why the concepts were true.

Sometimes I think [Dr. Jones] is talking to us like we’re idiots, but then I realize that [Dr Jones] is talking to us like we are first graders. And then I realize that [Dr. Jones] is asking us questions that I think are obvious, but when I think about it, I don’t know the answer to this stuff. I never thought about it before—it just was because it was. (Course Interview 1)

Carl found that there were distinct differences between the way MEST 1 addressed mathematical concepts and how he understood those concepts entering the course. However, rather than see these differences as a deficiency in his mathematical
knowledge, Carl viewed these differences as largely pedagogical—that he was learning mathematics as it should be taught to elementary school students.

We actually started talking about [place value] in [MEST 1], but instead of using *ones* and *tens* we would use things like *cubes* and *longs*. I guess that is the best way for the kids to learn this stuff, to have physical objects that they can see and touch in the class. To show the kids what is really going on in real-world stuff that they can touch. (Course Interview 1)

Like many of the other participants, Carl struggled to accept the assignments that did not directly relate to how he remembered solving problems in elementary school. For example, Carl related a frustrating assignment from recitation.

Today [in recitation] I had to do a problem where I have 25 of something and I need to divide it into 10 equal parts. So I did 25 ÷ 10, but [Lara] said that my operation was too complex and the age group you are trying to teach wouldn’t understand that. But why are we doing a division problem for a group of students who don’t know division? (Course Interview 2)

Carl’s reaction to this activity stemmed from his algorithm-first approach to mathematics. Carl could not envision asking students to attempt a problem without first teaching them the algorithm to solve the problem.

Once the course moved to multiplication, Carl was better able to apply his algorithm-first approach to learning to the content being discussed.

We just started multiplying. And we talked about four different ways to model multiplication. Like 3×5 is three sets with five individuals per set. And that is
different than $5 \times 3$, which is five sets with three individuals per set. And we have
to be careful to use the word ‘per’ like we are doing ratios. (Course Interview 3)

Carl even began to use his algorithmic thinking to aid him in crafting models for
multiplication problems.

Carl: We also emphasized using pictures, and the different types of pictures we
can use for multiplication, like an array versus an area.

Researcher: When would you use one or the other?

Carl: Like, an array model is when you have individuals, but an area model is like
when you are measuring stuff. They are pretty similar, except one is like pieces
of candy and the other is like pouring chocolate.

Researcher: So which is which?

Carl: The array [model] would be [used for] the individual pieces of candy, and
the area [model] would be [used for] pouring chocolate. But there are similar
models, just one is Xs and the other uses boxes. (Course Interview 3)

Although Carl began to use the correct terminology in his work, he did not see
why Dr. Jones preferred one term over another.

Carl: I donno, whenever we are talking about addition, [Dr. Jones] wants us to use
the word ‘combine.’ I haven’t been doing so well with that.

Researcher: What word do you use?

Carl: Different stuff. Like here I use ‘count them,’ or sometimes I will use ‘add
them together’ or ‘add them up.’
Researcher: Do you think the language makes sense—why [Dr. Jones] prefers ‘combine’ to those other terms.

Carl: No, not really. I think it has something to do with physical objects and how that relates to using physical objects to solve problems. (Course Interview 4)

**Explanations and Assessment.** Carl initially struggled with the explanation portions of the assessments. Carl first noticed that his explanations may not have been as thorough as some of his peers when he recognized the lack of volume of the homework he turned in compared to the volume of the homework of his classmates.

I don’t think I did too hot on my explanations. I noticed that some of the other people were turning in their homework and they would have pages and pages. I mean, I think I explained stuff well enough for someone who knows math to get it, but I don’t know if I explained it well enough for someone who doesn’t know math to get it. And that’s what I have to remember: a first grader doesn’t know how to reduce fractions; a first grader doesn’t know how to do things like that. So I need to use pictures and start thinking about how I can explain something to someone who doesn’t already know stuff. (Course Interview 1)

Carl did not see the explanations as a tool to learn mathematics, but rather related the explanations to what he believed would be required of him as an elementary school mathematics teacher.

Carl continued to struggle with the explanation portion of assessments. And although he admitted that in the beginning of MEST 1 he did not put as much effort into
the explanations as he ought, Carl did not find the feedback he received on assignments as constructive as he felt he needed the feedback to be.

Carl: I probably haven’t taken the homework as seriously as I should, but the teachers are kinda ridiculous about what they want.

Researcher: How do you mean?

Carl: They don’t…like… I am getting points off on my homework and quizzes and the notes that they write aren’t clear. They don’t write stuff like this is what you did wrong and this is what you can do better. The notes they give are really vague. (Course Interview 2)

The impetus of Carl’s frustration was how he viewed mathematics and learning mathematics. Carl believed mathematics instruction should be teaching algorithms, and Learning Mathematics should be practicing the algorithm to the point of automation. In that, Carl was not getting what he expected from the course. The instruction was predicated around asking questions and exploring underlying mathematical concepts, and the homework often asked Carl to solve mathematics problems with which he was not familiar.

There is a general confusion in the class about what was being asked. I mean, if [Dr. Jones] would have given us a [quiz] question like we had done before, then I think everyone would have gotten it—because [Dr. Jones] would have shown us how to do it. But the quiz question was not like anything we had ever seen before. (Course Interview 2)
This difference, especially on graded homework and quizzes, frustrated Carl. Because he could not practice the activity, he viewed the quiz problem as unfair.

Researcher: Do you have that quiz question?
Carl: Yeah. [Retrieves quiz].

Researcher’s Note: The quiz question was ‘Jerry the Giraffe’s weight is 3/7 the weight of Egbert the Elephant’s weight. Write Egbert’s weight as a fraction of Jerry’s weight. Draw a picture, label the picture, and use the picture to explain your answer.’

Researcher: So what did you do to solve that?
Carl: [Audible sigh] Well when I first got it I was mad because we hadn’t done anything like this in class before. So I was pissed and confused. I started out by drawing this diagram with 10 pieces. I donno, I wrote that… I donno, it’s just such a confusing question to figure out what the whole is. I [omit expletive] it up.

Researcher: So how would you do it now?
Carl: [Dr. Jones] gave us this key, but it was really confusing, [Dr. Jones] drew this picture, but [Dr. Jones] has this page of explanation—it even goes on the back here.

Researcher: Do you think [Dr. Jones] was looking for that explanation from you or do you think he was being overly pedantic?
Carl: I donno, maybe. When I first got it I thought [Dr. Jones] was just over-explaining it, but maybe that’s what they expect from us. But I sit next to this girl, and her answer made less sense to me than my work, and she got a better grade
than I did. So I don’t know. I don’t know how they give partial credit on this stuff. (Course Interview 2)

Carl became frustrated that he could not discern the correct pattern to solve the problems presented on quizzes, exams, and homework. Carl eventually asked Lara if there was a pattern for the exercises that he was missing.

Carl: I wanted a rule that I could follow. Like, give me one rule that I can use to follow to get all the answers I need. And [Lara] couldn’t—she couldn’t think of one single rule to give me.

Researcher: What do you mean by a rule: like a grading rubric, or a place to start, or something else?

Carl: Like a rule that I can use for most problems that I know if I do this rule, then I am going to get the right answer. Like for most of these problems it isn’t that you got the right answer, but how you get to that answer…I’m frustrated because everything seems so subjective so far. You have to do everything exactly how they want it, and if you don’t you can’t get a good grade. They are telling us that there is no one right way, but from my grades and a lot of other peoples grades there seems to be plenty of wrong ways. (Course Interview 2)

When asked to reason what Dr. Jones and Lara were looking for in explanations, Carl gave the following description.

Researcher: What do you think [Dr. Jones and Lara] are looking for in your explanations?
Carl: The diagram is really important. And for these problems we have to identify what the whole is. So if we are talking about 3/5, then 3/5 of what? So I am starting to pick up on little things, but it is only Week 3 [of the quarter].
(Course Interview 2)

Before the midterm, however, Carl received exactly what he wanted from the mathematics course, or any mathematics course: a guide.

I did really well on the midterm...The day before [Dr. Jones] went over some questions that were exactly on the test, so then it was easy. I mean, there was a midterm study guide, but I didn’t do any of it, I just learned the problems we did in class...I am doing a lot better on the homework too—I am looking at the example problems [in the textbook] and modeling my explanations on what they do. (Course Interview 3)

Because he now had a model for explaining the problems, he was able to memorize the pattern for the explanations and do well on his midterm. However, doing this reduced the level of thought Carl put into any given problem from synthesis to application.

Although Carl was drastically improving his explanations and grades because of the newly discovered template, he has not noticed the same improvement in his peers.

Researcher: I know you are doing well in the class now, do you think most of your classmates are doing better too?

Carl: No—I mean I am sure some are, but most of the other students are still struggling.

Researcher: Why do you think that is?
Carl: I think that [Dr. Jones] is looking for very specific strategies and explanations when grading. I think that it is important to use the right words, to draw the right pictures, and to really give the graders the explanations that they would do themselves.

Researcher: So what if you gave a good explanation that wasn’t necessarily the one that they were looking for, but was still mathematically correct and simple?

Carl: I think that there were specific things that the graders were looking for, and if you didn’t have those things, you got points off. I mean, look at how much underlining there is on my test. These are just short words or phrases that the graders are looking for. And I was looking at some other [student’s exam] papers, and they did the exact same thing I did, but didn’t use the same words I did, and they got points off and I didn’t.

Researcher: Can you think of a specific example where that happened?

Carl: Well, there is this one problem: Explain why

\[
\frac{3}{4} = \frac{3 \times 4}{4 \times 4}
\]

So you draw a diagram and you split it into the three fourths, and then you do vertical lines. And then you split it not with just horizontal lines, but with \textit{horizontal dashed lines}. The dashed lines imply the process of doing it. So the shaded part in the picture doesn’t actually change. And you have to actually say in the explanation that the shaded part didn’t change. And when I said that it didn’t on my test, you see it’s like underlined three times.
Researcher: And you think that if you didn’t say that you would have had points removed?

Carl: I know I would have, because the girl I sit next to did exactly that. She had my picture, my explanation, but she left out those words, and she got like [three points less than me]. (Course Interview 4)

Carl eventually found what he wanted from MEST 1: a model from which to craft his explanations. And from this model, he was able to refine his process and create explanations that were suitable for those that were assessing his work. However, by finding this model and repeating it, Carl no longer had to think of the problems as unique and different, but rather as exercises that he knew the algorithm to solve.

Carl: I [did really well] on the second midterm. Now that I know what kinds of problems are going to be on the test, all I have to do is practice them. I make sure I use the right words, draw a good picture, and it’s really easy after that.

Researcher: What if you found a problem on an exam that you didn’t know how to solve?

Carl: I don’t know. It’s never really happened. I mean, especially on the midterms, everything that was on them was on that study guide, and if not [Dr. Jones] goes over it before the test. So all I had to do was do those problems that [Dr. Jones] did or were on the study guide and I was good. (Course Interview 5)
Carl showed a slight decrease in his overall mathematical disposition score (see Table 6), lowering his score from 94 to 90, with his mean response of to positively phrased questions about his mathematical disposition decreasing from 3.76 to 3.60 (after recoding). However, this minimal change disguises some significant differences. First, Carl showed a large decrease in his views regarding his preparedness to teach mathematics. When asked, Carl gave the following:

Carl: You know, coming into MEST 1 I thought teaching math to kids would be easy. I mean teach them to add, subtract, multiply, and divide, and that’s about it. But now I see that there is a whole mess of stuff that goes into teaching math, even to little kids. I mean you have to know which models to use, what the kids are going to get wrong, what it means if a kid makes this mistake, there’s just a lot to it.

Researcher: So before you thought you were all but ready to teach kids math, what about now?

Carl: I donno. I mean, I am still pretty good, but math is just a lot bigger than I thought before. (Final Interview)

It would seem that as Carl’s views on the nature of elementary school mathematics matured, he began to see teaching the subject as a more difficult task (see Table 6). Of note, Carl responded only mostly true to the statements, “1. Generally, I feel secure about the idea of teaching mathematics to young children,” “12. Teaching mathematics doesn’t
scare me in the least,” and “25. Overall, I feel confident in my ability to teach mathematics,” all to which he previously responded definitely true.

Table 6

*Carl’s Post-Mathematical Disposition Survey*

<table>
<thead>
<tr>
<th>Category</th>
<th>Prescore</th>
<th>Postscore</th>
<th>Change</th>
<th>Prescore Mean</th>
<th>Postscore Mean</th>
<th>Change in Mean</th>
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<tbody>
<tr>
<td>Preparedness to Teach Mathematics (of 40)</td>
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<td>3.38</td>
<td>−1.12</td>
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<td>Mathematical Beliefs and Attitudes (of 40)</td>
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<td>31</td>
<td>+6</td>
<td>3.13</td>
<td>3.88</td>
<td>+0.75</td>
</tr>
<tr>
<td>Mathematical Self-Efficacy (of 20)</td>
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<td>15</td>
<td>−2</td>
<td>4.25</td>
<td>3.75</td>
<td>−0.50</td>
</tr>
<tr>
<td>Diligence in Learning Mathematics (of 25)</td>
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<td>17</td>
<td>+1</td>
<td>3.20</td>
<td>3.40</td>
<td>+0.20</td>
</tr>
<tr>
<td>Total (of 125)</td>
<td>94</td>
<td>90</td>
<td>−4</td>
<td>3.76</td>
<td>3.60</td>
<td>−0.16</td>
</tr>
</tbody>
</table>

*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

**MKT After MEST 1**

Carl showed much greater flexibility with the algorithms and an increased understanding of the algorithms than he did in the introductory interview. Carl was able
to successfully complete each exercise without the aid of his calculator, and had ideas about how he would teach the topics to elementary school students.

**Exercise 1.** Just as before, Carl was immediately able to recognize the pattern in the exercises. However, unlike last time, Carl was able to explain the mathematics behind the algorithms.

Carl: These are all subtraction with regrouping.

Researcher: What do you mean by that?

Carl: That the number in the ones place in the bigger number is smaller than the number in the ones place for the smaller number. So you need to take a digit from the tens place, subtract one, and then add ten to the ones place.

Researcher: And why do that?

Carl: Because every number in the tens place represent a group of tens. So if we move it to the ones place, we aren’t just moving one, we are moving ten. (Final Interview)

Carl even had some well-formed ideas on how to teach the concept.

If I were teaching this to a class, I would probably start with some beads or buttons or some other counting tools, and then have the kids just physically do a few problems to show it can be done. Then I would use base-10 blocks to demonstrate what exactly happens when you do the regrouping. And finally, we would move to the algorithm. (Final Interview)
**Exercise 2.** Carl still struggled with the traditional algorithm for multiplication. However, instead of relying on a calculator, this time Carl used several alternative strategies to solve the problem.

Carl: Man, I still don’t like that way to [solve large number multiplication] problems.

Researcher: How would you do it?

Carl: I would still use a calculator, but I would probably teach it to kids using partial products or lattice multiplication.

Researcher: Can you do the problem using one of those two methods?

Carl: Yeah. [Solves the problem by using lattice multiplication].

Researcher: Why did you choose that method?

Carl: Well, I think lattices help preserve place value, and it helps kids keep stuff organized, because everything is in its own box. (Final Interview)

**Exercise 3.** Carl began the problem by talking through one interpretation of division in order to better understand what the problem was asking.

Carl: So this means we have two and a quarter items, and we want to group them into groups that are half an item big, right?

Researcher: Then what?

Carl: So we need to figure out how many groups we have. [Draws a model.] So here we have four groups of a half each, and then we have a quarter left over. But since the quarter is half of the half, we really have a half left.

Researcher: So what is your answer?
Carl: Four and a half. (Final Interview)

However, when asked to create a story problem, Carl instead created a story problem that modeled multiplication by 1/2.

Researcher: So what would be a good story problem to model this?

Carl: Say you have 2 1/4 pounds of sugar. [Pauses]. No say a recipe takes that much sugar. But you only want to make half of that recipe. So how much sugar do you need?

Researcher: Does your story problem model that operation?

Carl: Yeah.

Researcher: Why?

Carl: Because when we do division problems, [Dr. Jones] said to do recipe problems like that. (Final Interview)

Discussion. Carl improved in his ability to solve the problems presented, his understanding of how the algorithms he used operated, and his ideas of how he may eventually teach these concepts to students. However, especially in his last answer, it became clear that Carl was still heavily tied to processes and algorithms, and at times did not bother to analyze his answer to see if it made sense. In many cases, Carl still relied as much on the processes he was taught in class to get the correct answer without examining the underlying concepts that form those processes. In spite of the process he uses being more complex, and the fact that he demonstrated he had more knowledge of the foundations of his algorithms, Carl lacked the ability to question the algorithms he used,
instead choosing to rely on them without consideration of the appropriateness of the answers they gave.

**Reflections on MEST 1**

Carl viewed MEST 1 as a unique experience, especially when he compared MEST 1 to the other mathematics courses he had taken in college.

Researcher: Can you compare [MEST 1] to the other math courses you’ve taken in college?

Carl: It’s different—it’s very different.

Researcher: How do you see MEST 1 as being different?

Carl: Like in other math classes, the content was really hard, but what they asked you to do with the hard content was kinda easy. I mean, you do a problem like on the homework like what was done in recitation. But here, the math is easy math, it’s stuff you do in grade school. But you have to explain it, and to explain it you have to know it inside and out and get all the little picky details down. I don’t think I have learned as much math [in] as much detail in any math class I have ever before [MEST 1]. (Final Interview)

Key in this difference is the role explanations play in MEST 1, and how Carl adjusted to the cognitive demands that accompany explaining his mathematics in such great detail.

Researcher: If I gave you a magic button and you could change anything about your time in [MEST 1], what do you think you would change?

Carl: Can I save it for the final? [Laughs] I guess what I would do is look more carefully at all the explanations early on. I mean, once I figured out where to
look, I think I was better able to follow their examples and get better explanations and get better grades on that early stuff.

Researcher: Do you think you write better explanations now than before?

Carl: Yeah. I mean, I think I had all the stuff there before, I just didn’t word it right, or I would use the wrong word, or I wouldn’t label my picture well enough. I think I was missing a lot of little things, and I think now I don’t miss those points. (Final Interview)

Carl viewed his time in MEST 1 as a success because he felt that he learned a great deal about mathematics and how to best teach mathematics to elementary school students. However, when asked what he would take away from the course, Carl responded that the most important lesson he learned regarded the nature of mathematics.

Researcher: What do you think was the most important thing you took from being in [MEST 1]?

Carl: Really that there are reasons for everything. Like, math works, and it works because there is stuff there to make it work. I mean, I use to think really smart people came up with all this stuff, and I am sure that the people who came up with it were really smart, but I don’t think you have to be really smart to get where it comes from. (Final Interview)

Summary

By the end of the quarter, Carl showed marked improvement in his ability to correctly apply algorithms and his understanding of the foundational mathematics that underpinned those algorithms. However, Carl’s methods to success only served to
reinforce some of the negative stereotypes that he previously held. Carl’s success on graded materials reinforced the notion that even complex mathematics problems have a sole right answer, that the answer is possessed by the Authority (in this case, his professor, Dr. Jones and his recitation leader, Lara) and that mathematics is about students finding the way the Authority wants them to solve the problem. Carl began to improve his explanations and his performance in the course when he was able to discern what types of problems would be graded on homework, or given on a quiz or exam, and memorized what he believed those who graded his work would want to see in a correct answer. And in doing so, Carl reduced what were intended to be unique mathematical problems with multiple correct solution strategies to exercises with one solution strategy. And although it remains open to whether Carl’s interpretation reflected the situation accurately, Carl’s grades improved when he employed this strategy. Regardless of whether or not Carl accurately captured the reality of assessment in MEST 1, it cannot be argued that the manner in which Carl was successful in the course reinforced this negative stereotype, one that the course instructors specifically sought to eliminate in MEST 1.

**Carl’s Themes**

- Carl’s mathematical knowledge for teaching increased as he was better able to explain the mathematics underlying the algorithms he used to solve problems. Similar themes emerged with Amber, Brandy, Dawn, and Ed.

- Carl moved from thinking about mathematics as a dualistic discipline to a relativistic discipline. Similar themes emerged with Amber, Dawn, and Ed.
• Carl struggled to view mathematics as anything other than an algorithmic system, so much so that he would use study guides to “find the algorithm” to solving assessments. Similar themes emerged with Brandy.

• Carl saw differences between the way mathematics was approached in MEST 1 and the way mathematics was approached in his other mathematics courses were mostly pedagogical—that enabled him to not challenge his own notions about the nature of mathematics. Similar themes emerged with Brandy.

• Carl believed that the use of proper vocabulary in an assessment is as important as—if not more important than—the content of his mathematical explanations. Further, he believed that the graders were specifically looking for those terms during assessed problems, and this was especially true on large formal assessments (midterm exams and final exams). Similar themes emerged with Amber, Brandy, Dawn, and Ed.
Chapter 8: Dawn

Dawn [a pseudonym] came to MEST 1 with a strong mathematics background and experience working with an elementary school mathematics class, both of which helped ease her transition to the course. Dawn quickly acclimated to the problem-centered nature of the assignments and to thoroughly explaining her mathematics. In fact, although other participants initially had explanations that were lacking details, Dawn would consistently add superfluous or repeated details, making her early explanations significantly longer than her later ones. Dawn’s previous experiences as well as her quick adjustment to explaining her mathematics helped her succeed in MEST 1.

Mathematical Disposition and Mathematical Experiences Before MEST 1

Dawn entered the course having been a successful mathematics student throughout her school career. Dawn scored 96 (out of 125) on the mathematics disposition survey, with a mean response of 3.84 to positively phrased questions about her mathematical disposition (after recoding). However, in spite of this, Dawn did not necessarily see herself as being good at mathematics. Dawn constantly compared her mathematical ability negatively to her brother, and that comparison led her to devalue her own skill. On her mathematical disposition survey (see Table 7), Dawn responded definitely true to the statement “4. I am not the type of person who is good at mathematics” and definitely false to the statement “24. Overall, I feel confident in my mathematical ability.” However, she also responded definitely true to the statements “I have always done well in mathematics classes” and “14. At school, my friends would
always come to me for help in mathematics” and “5. I have always done well in mathematics classes” which indicates Dawn’s mathematical disposition is nuanced.

Table 7

*Dawn’s Pre-Mathematical Disposition Survey*

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<tr>
<th>Category</th>
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<th>Prescore Mean</th>
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<td>Mathematical Self-Efficacy (of 20)</td>
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<td>Total (of 125)</td>
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*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

Dawn experienced success in her previous mathematics classes, and by her own admission, is a good mathematics student.

Dawn: I’ve never struggled at math, but I don’t think it is one of my best subjects either. Like, I have always done well in math classes, but I have had to work harder in math classes than some of my other classes to get the same [good] grades.

Researcher: And what do you mean when you say you got good grades?
Dawn: Until high school I never got below an A in anything, any report card, any test, anything. That changed a bit when I got to high school and started taking harder math classes, then I get a B or two, and even some Cs in AP Calculus. But overall, I have always gotten As in my math classes. (Introductory Interview)

Dawn’s success in mathematics began when she was in elementary school and would continue throughout her college career.

Mathematics was never my favorite subject. I was always more of a reading–writing–art kind of person. But, I think my favorite memory of math was in third–grade. We would play Around-the-World, where the teacher would hold up a big flash-card and they would have the multiplication facts on it, and whatever student could answer first got to move up. And I was really good at it, and I am a super competitive person anyway, so it was good to win at something.

(Introductory Interview)

However, every time Dawn would mention her mathematical success, she also mentioned her perceived weakness regarding mathematics.

Researcher: If I were to give you a magic button and you could change anything about your mathematics education past, what would you change?

Dawn: Oh my gosh! [Pause] Starting my freshman year, I would have gotten credit for all my high school math and never had to take it. [Laughs]

Researcher: But you did good in your math classes.

Dawn: Yeah, but they were hard, and I definitely would have liked to not had to worry about them in high school.
Researcher: Alright, apart from that, what would you change about your math past?

Dawn: I would have liked to have done better on the AP [Calculus] exam. I mean [I did well] and tested out of the first two calculus courses here, but I would have liked to have done better. (Introductory Interview)

This is, in part, due to Dawn negatively comparing her skill to her brother’s, whom Dawn saw as being much better at mathematics than she is.

Researcher: How would you characterize your mathematical ability?

Dawn: I would say I am not above average in math. I took the GRE a few months ago and I did the best on math, really well on the math part actually. But that was what I studied the most for. I would just consider myself average in math.

Researcher: So can I clarify something? You say you are average in math, but you always got As and Bs in math classes, took the AP exam and tested out of two college calculus classes, and got a high score on the quantitative portion of the GRE?

Dawn: [Laughs] Yeah, but you have to understand where I am coming from. My older brother just finished his degree in mechanical engineering. He is super-smart. So whenever I think to compare my mathematical skill I compare myself to him, and compared to him I am pretty average.

Researcher: What about when you compare yourself to some of the other students in [MEST 1]?
Dawn: I donno, we’ve only met a few times, but I think it’s safe to say I am above average. (Introductory Interview)

When she is asked to compare herself to other students in her class (and not to her brother), Dawn shows much more confidence in her mathematical ability.

Dawn characterizes herself as someone who may not be as naturally gifted in mathematics as others, but as someone who uses diligence to make up for what she perceives as a lack of innate ability.

Researcher: What would your previous math teachers say are some of your strengths as a math student?

Dawn: I think my old math teachers would say that I was one of the hardest working students in the class. I would work and work and work until I got a concept or a problem right. I think they would also mention my leadership, because if I understood something and someone else didn’t, I would try to work with them and help them understand it.

Researcher: And what about your weaknesses?

Dawn: I am really narrow-minded. In that, I mean, I like to do things one way. So if, like in college, there is a lecture with an example problem then I am really good at following that example to do other problems. But if a problem has multiple parts where you could look at them multiple different ways, then I get confused sometimes. (Introductory Interview)
Dawn also expressed a fondness for mathematics teachers who could reach her in multiple different ways until she and the teacher found the way that she could best understand the mathematics concept.

Dawn: My favorite [mathematics] teacher was my 9th grade Geometry teacher and my AP Calculus teacher. In fact he was the reason I signed up for AP Calc[ulus], I originally didn’t want to take it.

Researcher: What do you think made him such a good teacher?

Dawn: Well, he was younger and he was able to relate better to the students than some of the older teachers… But he was also really able to explain stuff well.

Researcher: What do you mean by that?

Dawn: I consider myself a very visual learner, and he would always draw pictures, create diagrams, and try to create stuff to go along with the example. Plus, since he was so nice, I was never afraid to ask him questions when I didn’t understand something. He was always very welcoming to kids who didn’t understand something, and many times after class or after school I would go back to his room and he would have like three or four different ways to explain the same concept. And one of those ways usually clicked with me. (Introductory Interview)

One of the key experiences that led Dawn to want to peruse a career in elementary education was working with an elementary school classroom.

I’ve worked with some school kids during math, one of my mom’s friends is a third grade teacher. So I have helped her work with her kids in math for the past
three years for their state assessments. And I really love it when the kids get that a-ha moment and they finally understand something. (Introductory Interview)

Particularly, Dawn cites working with this classroom as one of the “keys to [her] wanting to be an elementary school teacher” (Introductory Interview).

**Views on MEST 1 entering the course.** Dawn began MEST 1 eager to learn mathematics in a similar style to what she saw while she was working in an elementary school classroom.

I am really excited to take [MEST 1]. I have been told it is completely re-learning math and how to teach it, so I am really excited to see how to better explain stuff to kids and how to really use all of those blocks and toys that I see in [her mother’s friend’s] classroom… I would really like to know more about Everyday Math… They explain stuff really differently, and I never learned it that way, so I struggled with it. I guess the thing I really want from [MEST 1] is to be able to know how to teach math in ways that I didn’t learn it. (Introductory Interview)

Dawn recognized that how she remembered being taught mathematics in elementary school did not necessarily mirror the way she would be asked to teach elementary school mathematics, and was eager to learn those differences.

**Position on Perry’s Scheme**

Dawn’s thinking was at Position 4b: Adherence Alternative. Dawn was beginning to think about decisions as being contextual as opposed to right or wrong, and was beginning to function with relativistic thought, even though she still at times prefers
a multiplistic world view. In fact, in many of her answers, Dawn struggled with this conflict. For example,

Dawn: I think the ideal education, college education, is about getting the knowledge you need for your career.

Researcher: Do you think there is room for gaining knowledge that isn’t related to a career goal?

Dawn: Yeah, definitely. I think college should be about your career first, but also if you are curious about something or want to learn something then you should have the freedom to pursue it. But I think that there should be a balance. Knowledge for fun and purely for education’s sake is good, but ultimately you can’t be in college forever. It is important then to have something that you can do once you graduate, so you can be productive. (Introductory Interview)

And

I donno, I am pretty conservative, but everyone has the right to their own opinion. I just wish some people would think about their opinions themselves more. I mean, a lot of people believe stuff because someone tells them to believe it, like a professor. And some people just believe the opposite of someone just to be different. I mean, no one is right or wrong 100% of the time, I just wish more people would think and reason for themselves rather than just accept or denounce something because someone said it is true. (Introductory Interview)

In the first quote, Dawn focused on how different people can view a college education, but also the intrinsic value of college to many people. In that, she reasoned that although
different people can view a college education differently, a student should ultimately be able to take something from college that they could apply to their post-college life. And in the second quote, Dawn attempted to understand how people can make decisions without reasoning. In both quotes, Dawn attempted to balance her views with the potential views of others. And although she was hesitant to place a preference on either, she recognizes that in both the relative truths and similarities.

Dawn also struggles with the transition to relativism in her overall views of knowledge and truth.

I would say knowledge and truth are highly contextual. I mean, what I know in one situation is informed by all my past experiences, but someone else in that same situation would do something different, because her knowledge is from all her past. But, I think truth is slightly more absolute. Where knowledge is a collection of facts and how we interpret those facts, which can be different for everyone, I think truth is the over-riding principles which govern our lives. But because my life is different from your life, I think our truths may be different as well. (Introductory Interview)

When asked about her decision-making, Dawn also shows signs of early relativism, as she looks for experts to help guide her opinions.

Unfortunately, Dawn linked people who are good at mathematics with people who were dualistic thinkers.

Dawn: I think that there are some people who are good at math, people who think more black and white, right or wrong. I mean, there are a lot of subjects that
aren’t that way, Art, Reading, Writing, which are more shades-of-grey kinda courses. And I think a lot more people think that way, that there isn’t just a right and a wrong, and I think that’s part of the reason more people aren’t good at math.

Researcher: Can you describe what you mean by a ‘black and white’ mindset?

Dawn: Someone who there is a right and a wrong. I don’t want to say narrow-minded, but someone who if you pose a question to them then there is there answer and everything else is wrong. (Introductory Interview)

It is typical for those at higher positions on Perry’s theory to be frustrated with those at a lower position; however it is unfortunate that Dawn saw mathematical success as tied to a lower position.

**MKT Before MEST 1**

Dawn performed well on the pre-test, due in part to her high mathematical aptitude and in part to her experience working with elementary school students. Dawn’s high level of mathematical success in other mathematics courses allowed her to reason about some of the activities rather than relying on memory, and Dawn’s experiences helping teach elementary school mathematics made her more prepared for these exercises than other participants, especially when asked questions that directly related to how she would teach content to elementary school students.

By random selection, Dawn received Task Set A as her pre-test and Task Set B as her post-test.

**Exercise 1.** Dawn quickly recognized the pattern in the subtraction problem, and explained the algorithm more completely than any other participant.
Researcher: Do you notice any similarities among these problems?

Dawn: They are all subtraction problems, they are all two-digit numbers, they all involve borrowing.

Researcher: How would you solve these problems? How about this one?

[Researcher points to 42 -27].

Dawn: So, since you can’t subtract two from seven, you cross out the four, so [the four] becomes a three and [the two] becomes a twelve… Then you have three minus two, which you can do.

Researcher: Why do you subtract one from the four and then stick it in front of the two?

Dawn: Because the four is really 40, so when you subtract one you are really subtracting 10. So you can add 10 to the 2 to get 12. But I always just thought of sticking it in front. (Introductory Interview)

Dawn also showed her experience working with elementary school students when crafting a teaching strategy that used manipulatives.

Researcher: So if you were teaching this to a second grade class, how would you go about that?

Dawn: Well, I would assume the kids know how to add double digit numbers and subtraction without borrowing, so then I would probably use base-10 blocks to show them how to break apart the numbers in the tens spot to reorganize them.

(Introductory Interview)
Exercise 2. Unlike other participants, in the multiplication problem, rather than point out the student’s algorithmic error, Dawn instead pointed to the student’s conceptual error.

Researcher: So this is a common mistake that many elementary school students make when multiplying large integers.

Dawn: They don’t preserve the place value.

Researcher: Right, so how would you convince the student who did this that they didn’t get the right answer?

Dawn: Well, I would ask them to estimate, but their estimate may be bad because it doesn’t look like their algorithm is right. So I may get out a calculator and show them the answer on that.

Dawn also chose to use technology to demonstrate the student’s error, reasoning that the student may also have conceptual errors in other algorithms.

When asked how she would re-teach this concept to the student, Dawn continued to focus on the student’s misunderstandings regarding the concept of place-value.

Researcher: How would you show the student how to do the problem differently?

Dawn: Well, they have the algorithm mostly right, but they need to put 0s in to represent that they are multiplying different place values.

Researcher: What do you mean?

Dawn: So, when you are multiplying first, you are multiplying in [the] ones spot. But the next line here you are multiplying from the tens spot, so you need to add a zero at the end to account for that place value. (Introductory Interview)
Exercise 3. Dawn initially struggled remembering some of the terminology and added an unnecessary step to her algorithm. However, in her verbal explanation, Dawn gave the most detail of all the participants while still using the traditional algorithm.

Dawn: Ok, first thing I would do is put the mixed fractions to…what is it called… an uneven fraction, no. Is it called improper fractions?
Researcher: Yeah, improper factions. Can you do that?
Dawn: Well, I don’t know if this is the best way to explain it, but this is how I visualize it in my head. What I would do is say you have 1 3/4 which is (4/4) plus (3/4) so you have 7/4. That is your improper fraction. So the next thing you have to do is find the greatest common denominator between the two denominators, which is 4. So I will change 1/2 to 2/4. So now we have 7/4 ÷ 2/4, and now what do I do. [Pause] Don’t you multiply by the reciprocal?
Researcher: Why don’t you try that?
Dawn: So you have 7/4 ×4/2. So that is 28/8.
Researcher: Can you do anything else to that?
Dawn: Well, we can reduce.
Researcher: Can you do that?
Dawn: Well, we can divide both [the numerator and denominator] by 2, so that gives us 14/4.
Researcher: Can we go further?
Dawn: I don’t think so, unless you want the answer in a mixed number?
Researcher: 14 and 4 are both what kind of numbers?
Dawn: Even.

Researcher: So if we have two even numbers…

Dawn: [Interrupts]. Oh, we can divide that by two, which means we could have divided first by four.

Researcher: Exactly. Now if we wanted to change that back to a mixed number, what would we do?

Dawn: A mixed fraction?

Researcher: Yeah.

Dawn: So we have 7/2, that is 3 and 1/2. (Introductory Interview)

However, when probed, Dawn admitted to not knowing how the flip-and-multiply algorithm works.

Researcher: Can we go back and talk about from this step to this step. [Points to where Dawn used the flip-and-multiply strategy].

Dawn: Multiplying by the reciprocal?

Researcher: Yeah, do you know why that works?

Dawn: I have no idea! [Laughs]. I just remember doing that when I was in elementary school. (Introductory Interview)

Also, when trying to come up with a story problem, Dawn incorrectly models division by 2. When asked to reason about her problem, she realizes that her model is incorrect, but cannot find a good solution to correct the error.
Researcher: So how would you make a story problem to model 1 3/4 ÷ 1/2?

Dawn: Hmmm. [Pause]. You could say that ‘Mom made two pies for me and my friend, but my brother ate some, so now there is only 1 3/4 pies left. If me and my friend wanted to split the pies up evenly, how much pie do we both get?

Researcher: Look at your answer from when you computed the problem.

Dawn: Ok.

Researcher: Does that answer make sense with the story problem you set up?

Dawn: [Pauses]. I donno. I mean it doesn’t because that means we each get three and a half pies, but I don’t know how to fix my problem so that it makes sense.

(Introductory Interview)

**Discussion.** Dawn showed a remarkable understanding of the concepts being taught, both in terms of common content knowledge (CCK) and specialized content knowledge (SCK). In many ways, Dawn’s mathematical prowess combined with her experience working with elementary school students made her uniquely prepared for these exercises. Dawn was frequently able to think mathematically about each problem presented as well as demonstrate a basic knowledge of how that problem could be addressed in an elementary school classroom.

**Experiences Within MEST 1**

Unlike other participants, Dawn typically focused more on the activities being taught in class and how she interpreted them, and not as much on the assessments and explanations used in the class. This may be because she more quickly adapted to the explanation portion of assessments than the other participants, and thus this was less of
an issue for her than for other students. In fact, when Dawn talks most about assessment was when she struggled with a midterm.

Dawn was also accepting of non-traditional activities, as she saw them directly applying to what she will teach in her eventual classroom. Dawn’s views of teaching mathematics to elementary school students were more influenced by her recent experience in an elementary school classroom rather than her memories of learning mathematics in elementary school.

**Classroom Activities.** Dawn was initially excited to begin the class, because she saw it as a means to help her improve her explanations of mathematical concepts to students—some concepts which she struggled to explain to students in the past.

We are doing a lot with fractions right now, which is good for me. I have had trouble explaining fractions to kids before, so I am excited to see what we are going to be doing with them. (Course Interview 1)

For example, Dawn recounted a particular type problem involving rational numbers that initially confused her.

There have been a lot of questions [involving rational numbers] where we change the whole, and sometimes that is confusing. Like we had this one question where we had to find 2/3 of 3/4 cups of water, and I mislabeled the whole on that one. There was this other question that was like ‘Greg’s height is 3/7 of Jerry’s, so what is Jerry’s height as a proportion to Greg’s’? And it was just like that the whole is changing, and that sort of thing. So when I am messing up my explanations, that’s the stuff I mess up. (Course Interview 2)
And later

Dawn: Right now we are doing more stuff with rational [numbers] and with ratios
and percents. Like today, we worked in our activity book and did this problem.
‘So say there are 60 blue marbles and that is 30 percent of the marbles in the bag,
so how many marbles are in total in the bag’?
Researcher: How did you solve it?
Dawn: Well, I drew a picture.
Researcher: What did it look like?
Dawn: Well, I started by saying that if we have 100 [percent] of the marbles, well
that is three groups of 30 percent and a group of 1/3 of 30 percent. So I put in my
three groups of 60, and then my last was a third of 60, so 20. (Course Interview
2)

However, for most of the class, Dawn expressed frustration over the course not moving
as quickly as she believed she could handle.

Researcher: How is [MEST 1] so far?
Dawn: Good, I like it a lot. It’s really different than the other math classes I’ve
had so far, but I expected that—I was told to expect that from some of my friends
[who had previously taken MEST 1]. I feel like we are moving kinda slow.
Researcher: What do you mean?
Dawn: Well yesterday we spent the entire class on one problem. I realize that
classes are only [less than an hour] long and that is going to happen from time-to-
time, but it was really frustrating for me.
Researcher: How so?

Dawn: Well, I understood [the concept] from the very beginning, and I know that some other students were still struggling to get it, it just got really boring for me. (Course Interview 1)

In fact, when asked what she felt was the most important thing the researcher should note, she said it was the slow nature of the course.

Dawn: I think we are going really, really slow. I mean, I am used to math classes going fast, but sometimes we will spend the entire [lecture] class on one problem and I will just be sitting there bored because I got it in like five minutes.

Researcher: Do you think other students feel the class is going too slow?

Dawn: Well, there is this girl who sits with me in lecture and recitation, and we both feel it is real slow. But there are some [students] who will constantly ask questions and will really hold the lecture back, and [Dr. Smith] tries to answer everyone’s questions and it just feels like we are going slow.

Researcher: Do you think there are more students in the class like you and your friend who are getting it and feel the class is moving too slow, or do you feel that more students are like that one girl who is asking questions and kinda struggling?

Dawn: I donno, I think there are a lot more people like her [the student who was struggling], but I just wish we could go faster for me. (Course Interview 1)

Dawn realized that she may be one of the students grasping concepts more quickly and that other students may need the additional practice; however she was still frustrated by what she perceived as the sluggish nature of MEST 1.
Dawn continued to be especially interested in problems that challenged the way she thought about mathematics. For example, because of her understanding of the commutative property, Dawn had equated $5+7$ and $7+5$. However, when creating models, she discovered that the two problems are different even though they give the same answer.

Dawn: One of the things we have been talking about is how even though two problems look similar and give you the same answer, they are actually two different things.

Researcher: What do you mean?

Dawn: Like $5+7$ and $7+5$. One is having a group of five and then adding seven to it, and the other is starting with a group of seven and adding five to it.

Researcher: Can you give me an example of that?

Dawn: Well, let’s say Cary has five apples and Suzie gives her seven more. How many apples does Cary have? So that is $5+7$, you have five and add seven to it. Now, let’s say Cary has seven apples and Suzie gives her five more. How many apples does Cary have? So that is $7+5$, you have seven and add five to it.

(Course Interview 3)

Dawn also appreciated the nature of the course, and what she thought was a similar manner that the class would approach each new concept.

Dawn: I really like the class, because even though we are learning different things, we are learning them the same way.

Researcher: How do you mean?
Dawn: So, even though we’re adding and subtracting fractions, we are learning it the same way we learned comparing fractions: by drawing pictures. And I think it makes it easier for a lot of people in the class to get the concept. It’s not a whole different thing you are learning when you go from chapter to chapter.

Researcher: For you too?

Dawn: Yeah, I guess so. (Course Interview 4)

Although Dawn appreciated the similar way her MEST teachers would approach a topic, she also enjoyed looking at a concept in a way she had not before. For example, Dawn was particularly interested in how the course used the distributive property to multiply mixed numbers without converting them to improper fractions.

Dawn: We did this interesting thing [when multiplying mixed numbers] where we draw the models and divided them up, and it looked like foiling. So then we actually had to foil some mixed [numbers].

Researcher: What do you mean by ‘foiling’?

Dawn: Like you would do when you [multiply] binomials. So Front, Outer, Inner, Last. And then you add the pieces together.

Researcher: Can you show me?

Dawn: Sure.
Researcher’s note: Dawn then created and completed the following exercise.

While doing the exercise, Dawn carefully explained where she was getting her numbers for FOIL.

\[
\left(1\frac{1}{2}\right) \times \left(2\frac{2}{3}\right)
\]

\[
= (1 \times 2) + \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{2}{3}\right)
\]

\[
= 2 + \frac{2}{3} + 1 + \frac{1}{3}
\]

\[
= 3 + \frac{3}{3} = 4
\]

Researcher: So how did you know to break up 1 1/2 and 2 2/3 like you did?

Dawn: Because that is just 2 + 2/3.

Researcher: And do you know what mathematical properties you used to solve that problem?

Dawn: I think it’s called the distributive. (Course Interview 5)

Explanations and Assessment. Like the other participants, Dawn initially struggled with the explanation portion of assessments. However, unlike the other participants, Dawn’s explanations were often too long and included superfluous, repeated, or meaningless details.

Researcher: How are you adjusting to having to explain everything you do in [MEST 1]?
Dawn: The homework takes me *forever*. Like, math homework used to never take me long, but now it takes me a long time, and it ends up being like eight or nine pages when I turn it in.

Researcher: Why do you think it takes you longer in this class than other math classes?

Dawn: It’s the explanations. I mean, they have to be long for me to say everything I think I need to say about a problem, but they take a while. (Course Interview 2)

Dawn admitted that the most difficult part of the homework was creating succinct explanations, and that much of her time is spent in crafting her explanations.

Researcher: What part of the homework takes you longer—do you think you are spending more time figuring out answers or writing explanations?

Dawn: I mean, every homework has maybe two or three questions that I have to think for awhile about, but it is definitely the explanations. (Course Interview 2)

These struggles transitioned to her first exam, where Dawn struggled to create her lengthy explanations in the time allotted for the exam.

Dawn: I was really surprised about how bad I did on my midterm, and I don’t know what happened. Like… and I got most of my points taken off because of my explanations, and I thought I explained that stuff exactly like we did them in class.

Researcher: So what are you going to do?
Dawn: Well, I turned my exam back into [Dr. Smith] and asked for further explanations and clarifications, and maybe I won’t get any points back, but at least she knows I had those concerns.  (Course Interview 3)

Although Dawn did receive more credit after reviewing the exam with Dr. Smith, she still had difficulty choosing what details were necessary for her explanations, and which were unnecessary.

As the course progressed, Dawn became better able to choose important details and construct her explanations in a more succinct manner.

Researcher: I know earlier said you were turning in very long explanations.

Dawn: Yeah.

Researcher: Is that still the case for you?

Dawn: Not as much.  I mean, in the beginning, my explanations were maybe three times as long as they needed to be, but [other students’] explanations were maybe half of what they needed.  So I think most of the class kind of met in the middle.  I think my explanations are a lot shorter, and everyone else’s [explanations] are a bit longer.  So my homework isn’t massively bigger than most everyone else’s, but I think my explanations are still slightly longer than they need to be.  (Course Interview 4)

These shorter explanations were much simpler, more direct, and easier to produce in a timed exam setting.  This led to Dawn’s grade increasing.
However, like other participants, Dawn pointed out what she perceived as a discord between the ideals being explicitly taught in lecture and recitation and those being implicitly reinforced during exams, quizzes, and graded homework.

Dawn: There was one day that [Dr. Smith] filled in for [Faith] during recitation… and I didn’t like [Dr. Smith] in that setting at all.

Researcher: Why do you say that?

Dawn: Well, I felt like [Dr. Smith] never gave me a right way to do the problem. Like, we worked a problem out of the activity book, and then [Dr. Smith] had us put each of our answers on the board. And in the end [Dr. Smith] just said that each of these is a right way to do this problem.

Researcher: That’s interesting, because a few minutes ago, we were talking and you said that you thought if we gave a group of students the same problem, each of them would come up with their own way to solve it.

Dawn: Yeah, but we have a midterm and a final coming up, and I need to know the way that the graders want me to do the problem so I get it right. (Course Interview 5)

MD After MEST 1

Dawn’s score on the mathematical disposition survey lowered from 96 to 91, with her mean response decreasing from 3.84 to 3.64 when asked positively phrased questions about her mathematical disposition (after recoding). Dawn’s biggest change in her mathematical disposition came in her Preparedness to Teach Mathematics score and her Mathematical Self-Efficacy (see Table 8). Dawn’s Preparedness to Teach Mathematics
score dropped, in part, because she changed her answers on “6. I am nervous about having to teach mathematics” and “15. I am confident in my ability to teach mathematics” from definitely true to neutral. However, Dawn also increased her mathematical self-efficacy. When asked about this, Dawn replied

Dawn: I donno. I think I understand math a lot better now, a lot more now. I feel like I know what’s going on, not just how to do something but how something works. I feel like I know more about math. But at the same time, I know there is a lot of stuff in math that I don’t know that I need to know. I mean, I feel like I learned a lot, and I am sure I will learn a lot more by the end of [the MEST sequence], but will I know everything I need to know to teach [mathematics]? I donno.

Researcher: What would you do if you were asked to teach something you weren’t taught in [MEST 1]?

Dawn: Well, I think I could find out how. I mean the book would have a way, and there are other teachers, and I think I could figure it out. I really worry less about that than what would happen if I thought I knew something [but did not know it in as much detail as the topics discussed in MEST 1]. Would I be a bad teacher with that [topic]? Would I just teach it how I know it? Do I know it as well as I think? I donno. (Final Interview)
Table 8

*Dawn’s Post-Mathematical Disposition Survey*

<table>
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<th>Postscore</th>
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<th>Postscore Mean</th>
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<td>+1</td>
<td>4.40</td>
<td>4.60</td>
<td>+0.20</td>
</tr>
<tr>
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<td>91</td>
<td>−5</td>
<td>3.84</td>
<td>3.64</td>
<td>−0.20</td>
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</tbody>
</table>

*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

**MKT After MEST 1**

Dawn originally gave many mathematically sound answers to the Ma (1999) exercises. However, after MEST 1, her answers were much better formed and showed a refined understanding of the mathematics content being asked in each question.
Exercise 1. Just as before, Dawn immediately recognized the pattern in the subtraction problems. However, unlike during the introductory interview, Dawn relied less on the standard algorithm to compute the answers.

Dawn: These are all subtraction problems with regrouping.

Researcher: What do you mean by that?

Dawn: Well, all of these problems the ones place in the larger number is smaller than the ones place in the smaller [number]. (Final Interview)

Like before, Dawn successfully used the standard algorithm to solve the problem. However, when asked how she would teach this concept to students, she relied less on content she expected students to know and more on manipulatives.

I would probably use base-10 blocks or have the [students] draw the base-10 blocks. That way I can show the students how to change a ten into ten ones and then combine them with the number in the ones place. Once they were comfortable with that, then I can show them the [standard algorithm]. (Final Interview)

Note that Dawn has slightly refined her language, using the terms *regrouping* as opposed to *borrowing* and *combining* as opposed to *add them*.

Exercise 2. Once again, Dawn pointed to what she perceived as the student’s conceptual error instead of the student’s algorithmic error. This time, Dawn postulated that the algorithm the student used to solve the multiplication problem as perhaps the source of the student’s confusion.
Dawn: I don’t think the student really understands multiplication if he thinks that the answer to [that question] is so small.

Researcher: How would you address that?

Dawn: Well, in [MEST 1] we talked about four or five different algorithms to do problems like this. Maybe the student needs to try to use a different algorithm.

Researcher: Do you have one you would show the student?

Dawn: Yeah. I would probably use a picture model, with areas. That way the student could see what was actually going on with the numbers and the place value. I may also want to use partial products. I think it allows students to see place value better [than the standard algorithm]. (Final Interview).

**Exercise 3.** Dawn approached the division with fraction problem with a bit more trepidation, and inquired as to whether she should solve the problem using MEST 1 techniques or how she would if she encountered it in another setting.

Dawn: So, I know how I would do this, and I know how we are doing this in [MEST 1]. Which way should I do it?

Researcher: Can you show me both [ways]?

Dawn: Okay.

Dawn correctly used the standard algorithm to solve the problem. Unlike last time, Dawn did not do the superfluous step of finding a common denominator.

Researcher: You mentioned you were doing this in class, but in a different way?

Dawn: Yeah.

Researcher: Can you show me?
Dawn: Well. [Pauses] Let me think. So if we think of division as how many groups. So we have $2 \frac{1}{4}$ and we want $\frac{1}{2}$ in each group. So then we have four $\frac{1}{2}$ s in the two, and we are left with $\frac{1}{4}$, which is half of $\frac{1}{2}$.

Researcher: So what is your answer?

Dawn: It’s $4 \frac{1}{2}$ again. (Final Interview)

When asked to create a story problem, Dawn consciously tried to create a problem where each piece could be a fraction, including the answer.

Dawn: Well let’s say you have two and a quarter bottles of [soda] pop. And we want to give half a bottle to each kid… wait.

Researcher: What is it?

Dawn: Well, if I do that, then we end up having a half of a kid, and that isn’t good. So I need to make sure everything is continuous.

Researcher: What do you mean by continuous?

Dawn: That the thing makes sense as a fraction. [Pauses] Well, we always do recipe examples in class, so let’s see. [Pauses] Say we have a recipe that requires half a cup of flower, and we have $2 \frac{1}{4}$ cups. How many times can we make the recipe?

Researcher: Is that a better [story] problem?

Dawn: Yeah, I think so. We can talk about each of those things as fractions, the recipe, the flower, and they all make sense. (Final Interview).

**Discussion.** Dawn was uniquely prepared for this course. Dawn comes in with a much stronger mathematics background than many other MEST students, having taken
Calculus and she also has experience helping teach mathematics to elementary school students. These benefits helped her perform well on the instruments before starting MEST 1, and she continued to refine those skills in MEST 1. Dawn demonstrated a thorough mathematical understanding of each of the concepts taught, and was able to use her experiences both in class and working with students to form her ideas regarding how to best teach the content to students.

**Reflections on MEST 1**

Dawn initially entered MEST 1 with the idea that she would be “re-learning” mathematics (Introductory Interview). When asked if she still felt that way, Dawn responded

Dawn: I think the main thing I learned in [MEST 1] was to question what I know and why I think I know it. I think the way I was taught math [in elementary school] really caused me to stop thinking about math. I didn’t ever need to ask why things worked, I just did them. And looking back, some of what we do in math is weird. I mean, look at fractions. Sometimes [when doing operations with fractions] we need a common denominator, sometimes we don’t. Sometimes we just add the numerators; sometimes we multiply both the top and bottom of the fractions. There are just a lot of rules, and until [MEST 1] I never thought they were anything more than that. I mean, I figured there were reasons, but I didn’t figure I could understand them. I just thought that it worked, so do it.

Researcher: And now?
Dawn: I want to know why something works, especially if it is some weird thing. And I want to show that to students, so maybe they think it’s weird and interesting. Or maybe they just won’t think it’s weird at all, maybe it will just make sense for them. (Final Interview).

Earlier in the course, Dawn made a similar comment.

Dawn: Its kinda like unlearning math. I mean, trying to teach someone the concepts of the difference between 3/4 and 2/3 and then how to explain it. It really is like I am unlearning the way I learned math and relearning it this new way. This way that I know I can explain it to someone.

Researcher: How do you feel about that?

Dawn: I donno, I mean I was taught math this old way and I thought I knew what was going on. And I guess I knew how to do math, but I didn’t know why math worked. I think now we are really getting into why math works.

Researcher: Why do you think you weren’t taught math this way?

Dawn: I think because it is hard. I mean, it is a lot easier to just memorize what to do and do it and not have to worry about why it works. But in that, I think this is a better way to learn math. I mean, I shouldn’t have to do something just because someone says it works, that person should be able to convince me that it works.

Researcher: So when you teach, do you think you will integrate some of the [MEST 1] stuff—go beyond how to do something but also teach why you do it this way?
Dawn: Yeah, absolutely. Math should make sense, and it does make sense. So why do we need to hide that from kids? (Course Interview 2)

In both cases, Dawn reiterated that her learning experiences in MEST 1 were different from her learning experiences in other mathematics courses.

Dawn also responded that she would like to allow students to learn mathematics using many of those innovative strategies, but she hesitated when she was asked if she planned on using them in her class.

Researcher: How comfortable would you be allowing your students to try some of these different strategies, or even just give the students a problem and see how they try to solve it?

Dawn: I think I would be [comfortable doing that], but I don’t know that I will. It just isn’t like the real world.

Researcher: How do you mean?

Dawn: I think everyone is going to expect the students to know how to solve the problems in a certain way, and if they don’t know that, school will be hard for them…

Researcher: Do you think that doing one method precludes doing the other?

Dawn: No, but there is only so much time, and if I teach kids what they don’t need to know, or even if I take a long time teaching them what they need to know, I won’t get through everything I need to cover. (Final Interview)
Summary

Dawn was uniquely experienced entering MEST 1, both in terms of her strong mathematics background and her experience working with elementary school students with mathematics. That, combined with Dawn’s heeding the advice of those informing her of the different nature of MEST 1, made her transition to the course relatively easy. Dawn quickly adapted to the different level of rigor in learning the course content and to the explanations required in assessments. In fact, although most participants initially struggled with not having enough detail to their explanations, Dawn’s mistakes were that she added too much detail or superfluous to her explanations. However, Dawn quickly learned to pare her explanations while the other participants had trouble learning to expand their explanations. This could be because Dawn had a richer initial understanding of the material than the other participants: Dawn’s explanations included irrelevant details that she knew, and it was simply a matter for her to learn which details were relevant to her process.

Dawn’s Themes

- Dawn’s mathematical knowledge for teaching increased as she was better able to explain the mathematics underlying the algorithms she used to solve problems. Similar themes emerged with Amber, Brandy, Carl, and Ed.
- Dawn believed that the course was not progressing quickly enough to keep her academically engaged. Although she appreciated that she grasped many of the concepts more quickly than her peers, she was still frustrated by the pace of the course.
• Dawn also rejected the problem-centered nature of MEST 1, saying that she would have preferred the lectures were more traditional lectures. Similar themes emerged with Brandy.

• Even though Dawn could accept relativism in her general sense-making, she began MEST 1 thinking of mathematics as a dualistic discipline. By the end of the course, she could accept multiplicity in her mathematical thinking. Similar themes emerged with Amber, Carl, and Ed.

• Dawn struggled with explanations as she initially included many repeated or superfluous details. By the end of MEST 1, Dawn was better able to pare her explanations to a minimum. Similar themes emerged with Amber, Brandy, Carl, and Ed.

• Dawn’s mathematical disposition decreased as she became less confident that she understood mathematics as completely as she previously thought. Similar themes emerged with Carl and Ed.

• Dawn expressed her intent to use teaching techniques from MEST 1 in her eventual elementary school mathematic classroom, but questioned if those techniques would be viable in the students future mathematics courses. Similar themes emerged with Brandy and Ed.
Chapter 9: Ed

Ed [a pseudonym] entered the course with a strong mathematical background, a productive mathematical disposition, and experience in the Mathematics for Middle School Teachers sequence. Ed would become proficient with his explanations and broaden his views on mathematics and teaching mathematics. However, as Ed began seeing that mathematics is a broader and more interconnected discipline than he previously believed, Ed’s mathematical disposition decreased, especially in his confidence in his ability to teach mathematics to elementary school students.

Mathematical Disposition and Mathematical Experiences Before MEST 1

Ed viewed himself as a good math student and by his own admission, had always been successful in mathematics classes.

I have always just had a brain for mathematics; I have always been good with numbers. I donno, I am just really good with that stuff. I always feel good when I am doing mathematics. It is relaxing and it gives me a lot of confidence. I mean, I think I have a long way to go before I will be a good math teacher, explaining stuff was never my strong suit, but I think I will be good in the class.

(Introductory Interview)

In elementary school, Ed said mathematics was one of his favorite classes, and that one of his favorite things about elementary school mathematics was the timed tests.

Researcher: What were some of your favorite classes in elementary school?

Ed: I really liked math and science.

Researcher: What did you enjoy about them?
Ed: I really liked the timed tests. Like, especially in math, but even in science, where we would just have a bunch of stuff to memorize, like times tables, and then we had to answer questions about the stuff we learned as quick as we could. I really liked those—I was good at them. (Introductory Interview)

Ed’s mathematical success continued throughout his school career, going into college.

Ed: I have always gotten pretty much straight As in my math classes.

Researcher: Even in the middle school math sequence?

Ed: Yeah, the only time I really got below an A was when I took [Calculus 1] at [a different university]. I donno… I really should have done better, but once I knew I was transferring here and wasn’t going to need Calculus, I kinda slacked off. I still got a [good grade], but I could have gotten an A. (Introductory Interview)

As mentioned in the previous quote, before declaring as an early childhood major, Ed considered a career as a middle school math and science teacher.

I was originally in the middle childhood program, so I have already taken the middle childhood mathematics sequence, but when I switched to early childhood, none of that stuff switched over. So now I have to go through and take all the [MEST] classes. (Introductory Interview)

When asked if he felt if that experience would help him, Ed gave the following reply:

I donno. I have heard that [MEST 1] is really all about explaining stuff, where the middle child math courses were really about content and not as much explaining.
In fact, that is one of the things I really want to learn in MEST 1, how to break-down math for kids. (Introductory Interview)

On the mathematical disposition survey, Ed answered almost every question as positively as possible, with the lone exception of answering neutral to “20. If I taught in a team or with a teaching partner, I would like to have another teacher teaching the mathematics” (see Table 9). This yielded Ed scoring 123 (out of 125) on the mathematics disposition survey with a mean response of 4.92 to positively phrased questions about her mathematical disposition (after recoding).

Table 9

*Ed’s Pre-Mathematical Disposition Survey*

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<thead>
<tr>
<th>Category</th>
<th>Prescore</th>
<th>Prescore Mean</th>
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<td>Mathematical Beliefs and Attitudes (of 40)</td>
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<td>Mathematical Self-Efficacy (of 20)</td>
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<tr>
<td>Diligence in Learning Mathematics (of 25)</td>
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</tr>
<tr>
<td>Total (of 125)</td>
<td>123</td>
<td>4.92</td>
</tr>
</tbody>
</table>

Note. Mean response determined after recoding to reflect the participants response to a positively stated question.

However, when probed, Ed would reveal that he believed mathematics was an innate talent and not so much a learned skill.
Researcher: Why do you think so many people dislike mathematics?
Ed: I donno, I think that there are some people whose brains are just wired to make them good at mathematics. Being able to process numbers and being able to understand numbers. I think if you are one of those people, then you like mathematics and if not, you don’t.
Researcher: Do you think that you have to be good at mathematics in order to like it?
Ed: Yeah, I mean if you are just beating your head against it, you aren’t going to like it. I think a lot of it is like, natural. You are either born with it or you aren’t.
(Introductory Interview)

And

Researcher: What do you think makes a person good at mathematics?
Ed: Well, I think that those people are really organized. Not that they aren’t creative, but someone who is good at math looks at things logically and in an organized, step-by-step fashion.
Researcher: Do you think that you can only be one or the other—creative or organized?
Ed: I think that most people are one or the other. I mean, I am sure there are super-creative people who are really organized too, but I think most people are largely one or the other. (Introductory Interview)

Later in the interview, Ed would slightly weaken his stance.
Researcher: Do you think being good at mathematics is something you are born with, or is it something you learn?

Ed: Kinda both. I mean, I think you are probably born with a certain amount of [mathematical] ability, but I think you can learn to think mathematically, especially if you are taught how to think logically early on.

Researcher: What about later on?

Ed: I mean, it’s kinda like learning a new language. Can you pick it up later in life, sure. But it is a lot easier if you learn it when you’re a little kid.

(Introductory Interview)

Even in this quote, Ed revealed that he believed math was less a skill that could be practiced and perfected, but more a talent that was either innate or developed “very early on.”

**Views on MEST 1 entering the course.** In spite of his Ed’s positive mathematical disposition and previous mathematical success, he is worried about whether or not he will succeed in the course.

Researcher: Do you think you need to take [MEST 1]?

Ed: Yeah. To be honest, I am kinda worried about [the course]. I mean, in [MEST 1] it is really less about how to do the math, but how to explain the math, and I am kinda weak in that area.

Researcher: Is that similar to what you saw in the middle school sequence?

Ed: No, that course was a lot more about doing content. (Introductory Interview)
When asked what he really wants to develop or learn in the course, Ed continued to focus on how he wanted to improve his mathematical explanations.

Researcher: What do you want to take-out of [MEST 1]?

Ed: I really want to get better at explaining stuff. I want to understand how to take a math concept and break it down to where a little kid can understand it.

(Introductory Interview)

Ed realized that it to succeed in MEST 1 he would have to develop his ability to simply explain mathematics to novices, and he worried that he was not prepared to craft those explanations.

**Position on Perry’s Scheme**

During the quarter, Ed was transitioning to Position 3: Early Multiplicity. Ed could accept other people’s differing views; however he did not prioritize those views as a relativistic thinker might. Ed saw this as a transition that happened directly as a result of his attending college.

Ed: I think that college has opened my mind to other people’s views. I mean, when I was in high school, I was pretty closed-minded and a lot of my friends were that way too. But now that I am here, almost every day I meet someone with really different views from me. So it has really helped open my mind to what is really out there. And I think that is important, because as a teacher, I am going to encounter students and parents who have some very different beliefs and lifestyles, and I don’t think the me from high school could have been open to that.

Researcher: And you think you’re more open to that now?
Ed: Yeah. Don’t get me wrong, I still have my beliefs and I don’t want to change those. But I don’t necessarily think my way is the only way anymore. I can accept another person without accepting their beliefs. (Introductory Interview)

Ed also believed that knowledge and truth are individualized.

Researcher: Do you think knowledge and truth are the same for everyone, or do you think those are different for everyone?

Ed: I think everyone has their own, definitely. I mean, there may be some similarities, but I think ultimately everyone is entitled to their own views on the world. Everyone has their own beliefs, and those inform what they consider knowledge and truth. (Introductory Interview)

However, Ed still showed some signs of dualistic thinking. For example, Ed had a narrow view of what the college experience should be, saying that he believed that college should prepare the individual for their career after college. However he is open to the idea that not all majors do or should have a direct career path.

Researcher: Describe for me the ideal college education.

Ed: I think college should ultimately prepare you for your career.

Researcher: What about a major that doesn’t necessarily have a direct career path, like maybe mathematics?

Ed: [Long pause] In that case, I think that college should prepare you for a wide variety of things you could do after you finish.

Researcher: So, would you say the primary goal of college is to prepare you for what you are going to do next?
Ed: Yeah, definitely. (Introductory Interview)

Ed also show dualistic thinking in his decision making process.

Researcher: If I asked you to make a decision but you didn’t know all the information, how would you go about making that decision?

Ed: I donno, I have a group of people whose opinions I really trust: friends, parents, [and] my pastor. So I would probably ask them and see what they say.

Researcher: Would you follow their advice?

Ed: Yeah, probably.

Researcher: And what if they disagreed?

Ed: I donno. I would probably ask some more people and try to figure out from the answers what I think is right. (Introductory Interview)

**MKT Before MEST 1**

Ed struggled to move beyond what he immediately knew. When Ed immediately knew an answer, he shared it completely and correctly. However, Ed struggled anytime was asked to think about a problem differently or more deeply than he had originally.

By random assignment, Ed was given Exercise Set B as his pre-test and Exercise Set A as his post-test.

**Exercise 1.** Unlike the other participants who solved the problem using the traditional algorithm, Ed interpreted each subtraction problem using a missing-addend model. Because of this, Ed did not notice the pattern of each subtraction problem using regrouping.
However, when he was asked if he would teach his way to students, he hesitated, saying that his way was less accepted and thus a less valid way to solve the problem.

Ed: I donno, I was taught using the other way, with regrouping and stuff. But my brain works with my way [the missing addend model], so I would use that.

Researcher: So what would you do if you were teaching this concept to students?

Ed: I would use the other way.

Researcher: But your way got you to the right answer?

Ed: Yeah, but after my class my students are going to have different teachers, and when that happens those teachers will probably expect students to do it the right way. And that’s not going to be the way I taught them, that’s going to be the other way.

Researcher: Why do you think that way is the right way, and you’re way isn’t?

Ed: Just that the other way is the way everyone else does it. (Introductory Interview)

Exercise 2. Even though Ed was not as familiar with the traditional algorithm for subtracting large integers, Ed was familiar with the procedures needed traditional algorithm for multiplying integers. However, Ed was not able to reason about the mathematics underlying that algorithm.

Researcher: Where did the student go wrong?

Ed: Well, he didn’t move his lines over when he multiplied here [the tens place] and here [the hundreds place]. I mean, I was always taught to put zeroes in there.

Researcher: Why put zeroes there?
Ed: Just as a place-holder, so you keep the lines straight, and it doesn’t matter adding zero.

Researcher: What do you mean by it ‘doesn’t matter adding zero’?

Ed: Well, anything plus zero is itself.

Researcher: Do you think the zeroes have any mathematical meaning beyond just holding the place?

Ed: No, probably not. (Introductory Interview)

Ed did not understand that the zeroes he inserted were associated with place-value, and instead thought of them simply as an organizational tool.

**Exercise 3.** Ed correctly solved the division problem using the *flip-and-multiply* method, even going so far as to convert the answer from an improper fraction to a mixed number. However, when asked to create a story problem, Ed was not able to make an attempt.

Researcher: Can you come up with a word problem that models $2 \frac{1}{4} ÷ \frac{1}{2}$?


Researcher: Take your time.

Ed: [Long pause]. I really don’t know. I mean, I can’t think of a situation where it makes more sense to divide by $1/2$ rather than multiply by $2$. (Introductory Interview)

Ultimately, Ed was not able to divorce the concept of division with the algorithm he used to solve the original problem.
**Discussion.** In all three problems, Ed quickly displayed everything he knew about each problem, but was unable to reason about any of the problems beyond what was immediately apparent to him. In Exercise 2 Ed correctly reasoned about how to use the traditional algorithm. In Exercise 3, Ed correctly used the traditional algorithm to solve the division with fraction problem. However, in each problem, when Ed was asked to reason about the mathematics and mathematical principles beyond what was immediately evident, Ed could not even begin to reason more deeply about the mathematics.

**Experiences Within MEST 1**

Ed entered MEST 1 with a stronger mathematical background (with the exception of perhaps Dawn) and a more positive mathematical disposition than the other participants. However, in spite of these advantages, Ed had a clear view regarding what exactly he wanted to focus on learning in MEST 1.

I really want to learn how to explain stuff better in [MEST 1]. Like, something that a lot of people have told me is that I am too smart to explain stuff to them. And I don’t know if that’s something that makes sense. I mean, if I were really smart, I should be able to explain the stuff I know to other people, right? And that is especially important if I am going to be a teacher. I mean, if I can’t explain [mathematics] to students, how can I be a good math teacher? (Introductory Interview)
Ed is a strong mathematics student who has excelled in his previous, algorithm-based mathematics courses, and now seeks to understand mathematics beyond those algorithms, and in such a way as he can explain mathematics to his future students.

**Classroom Activities.** One of Ed’s strengths in the course was how quickly he was able to integrate the course’s vocabulary into his mathematics and his explanations of mathematics. For example, early on in the course, Ed was able to construct a simple and accurate—if incomplete—definition of *the whole* as the concept related to comparing fractions.

Ed: We have been talking a lot about doing story problems with fractions. Like how to compare them, what the whole part represents in each, and how to determine which is bigger or smaller?

Researcher: What do you mean by *the whole part*?

Ed: Just if you had one whole of a thing, what would you have. Like you can’t subtract 2/3 of a gallon from 4/5 of a liter.

Researcher: And how have you been doing?

Ed: It’s kinda hard. I mean, it is just a lot of thinking. And it’s a lot of stuff I am not used to thinking about in math. (Course Interview 1)

Ed would apply this concept to problems he encountered later in the course.

We have been working a lot of word problems, like taking about five or six problems and deciding whether or not that could be solved by, [for example] 1/2 + 1/3, or something like that. Mostly it really involves looking at the whole parts
of each fraction and looking at whether or not the whole parts are the same or not.

(Course Interview 4)

Later, Ed would show a similar ability to craft a simple explanation of the distributive property and demonstrate how a student was using it.

Ed: So we did this problem where there was this student who did 123 – 58. And what she did was took 120–60, and then add 3 because she took 120 instead of 123 and then add 2 because she took 60 instead of 58. So we had to write up this explanation so that it would make sense to everyone in the class.

Researcher: What did you do?

Ed: Well, we pretty much wrote down the equations, and use the distributive property to explain what the students were doing.

Researcher: What do you mean by the distributive property?

Ed: Just that if you are adding two things and then multiplying something to them, it is the same as if you multiply that thing to each of them first and then add together the [results].

Researcher: How does that apply here?

Ed: Well, you have to think of subtraction as adding a negative, and then that negative as multiplying a positive number by –1. (Course Interview 2)

Even the times when Ed would forget the exact vocabulary, he was able to use the concepts behind the vocabulary to strengthen his explanations.
Ed: We’ve been doing multiplication word problems. Basically looking at different problems [that] can be solved with multiplication, but that they are all multiplication problems.

Researcher: What do you mean?

Ed: Like say you have five boxes of candy bars with seven bars in each box. So that is five times seven. But you could also be asking the area of a rectangle room that is five feet long and seven feet wide. Well that is also five times seven.

Researcher: What is the difference between those situations?

Ed: I forget the words, but it has to do with, in the first one you are looking at groups and how many things you have in total and in the second you are looking at things like areas. (Course Interview 3)

Ed’s adjustment to the mathematical vocabulary used in MEST 1 helped ease his transition to MEST 1 and the different expectations placed on him in MEST 1 as opposed to his other mathematics courses.

Ed has also tried to take note of the mathematical pedagogy MEST 1 instructors bring to the course. For example,

Ed: We have been talking a lot about writing story problems and [the instructors] really have tried to get those points home to us, because we will be writing a lot of story problems for our classes.

Researcher: Why do you say that?
Ed: Well, we will have a lot of resources and books and stuff, but not everything we want to do will be in a book, so we need to be able to come up with story problems on our own, that we can have students solve.

Researcher: Do you think you are going to do that when you are a teacher?
Ed: Yeah. I mean, I will use what I have, but it is always nice to be able to create math problems that are tailored to the [future] class, to my [future] students and to the content I am teaching the way I am teaching it. (Course Interview 3)

Ed also plans on using manipulatives in his future teaching. However, like modeled by the MEST 1 course, Ed believed that drawing pictures of the models might be an effective substitute to having actual physical manipulatives.

Ed: We have done a lot with positive and negative numbers. And every time we talk about those, we say we have black chips and red chips, and draw them. And we say that a black chip is +1 and a red chip is −1.

Researcher: So have you actually had black chips and red chips in class, or have you just drawn them?
Ed: We have just been drawing them.

Researcher: Why do you think that is?
Ed: I mean it’s a big class and we are all in college, so I think everyone can imagine that. But classes are only so long, and by the time we all would get out a bunch of red chips and black chips, it is just wasted time.

Researcher: So when you are teaching this to elementary school kids, would you want to have physical chips, or do you think that the kids should just draw them?
Ed: I think it depends on the class and the kids. I think for some [kids], it would definitely be better to have real chips, but for others, it may be better to just draw them. (Course Interview 5)

Explanations and Assessment. Like the other participants, Ed initially struggled with the explanations that accompanied his mathematics. For instance,

Ed: Right now we are learning how to write explanations—how to draw pictures and use words to explain math concepts. Like today, we were talking about how a student might think 6/7 and 7/8 are equal because they are both one away from the whole, so we had to explain why two fractions being one away from the whole didn’t make them equal.

Researcher: What did you do?

Ed: Well, I forgot exactly. But my group boiled it down to that the one piece left over in one was smaller than the one piece left over in the other. (Course Interview 1)

In this example, Ed could not re-invent his explanation, which demonstrates that he did not have a thorough understanding of either the problem or his method to solve it.

However, Ed quickly acclimated to the mathematical explanations he used to solve problems, and this view quickly improved his explanation skills.

Researcher: What do you think has improved in your explanations between when you started the course and now?

Ed: I think it is practice. I mean in the beginning I was having trouble coming up with the right words to explain stuff or coming up with a good picture. But since
I have been explaining every problem I come across, I have gotten a lot better at finding those right words and coming up with those necessary details. (Course Interview 2)

Key to Ed’s improvement is that he has practiced explaining mathematics, but also that he practiced using the correct vocabulary in his explanations.

One aspect of Ed’s explanations that was lacking was his integration of diagrams with his written work. Although Ed written mathematical prose is concise and well thought, he continued to struggle understanding and implementing pictorial diagrams to enhance his work.

[Dr. Smith] and [Faith] are really big into having pictures with the explanations. I think they make the explanations more clear, because then you can reference what you are explaining with the pictures. Like on my first quiz, I ended up misreading the question, but I ended up not doing too bad on it because I explained the problem I solved pretty well, it just wasn’t the problem that they [the instructors] were asking. (Course Interview 2)

And later,

Ed: I did pretty well on the midterm… I really did perfect except for one problem.

Researcher: Which one was it?

Ed: It was… using pictures to explain why \((3/5) = (3/4) / (5/4)\).

Researcher: And what do you think you did wrong on it?
Ed: I didn’t explain it in enough detail, which was odd because we did the exact same problem in our review and we didn’t go into that much detail, so I thought my explanation was fine. (Course Interview 3)

Even toward the end of the course, Ed would continue to struggle with the pictorial representations used in mathematical explanations.

Ed: My explanations are usually about two or three sentences per problem, maybe a paragraph tops. [Dr. Smith] and [Faith] have both said that there is no need to write pages and pages, and I don’t. I think a lot of students try to over-think some of these concepts and over-explain their thinking. I hear what some of the other groups are doing and I see what [some students are] turning in, like, pages and pages of homework, when I think that there is usually a simple way to do and explain these problems.

Researcher: And what about the pictures?

Ed: I donno, I guess I still need to do better with those. I think it is important that I be able to draw something for kids to see, I think I just need to pay more attention to them—be more mindful of them, you know. (Course Interview 5)

Even though Ed struggled with his diagrams, his prose explanations were succinct and used the correct vocabulary.

**MD After MEST 1**

Ed had the largest change in his mathematical disposition from the beginning of MEST 1 to the end (see Table 10). In the beginning of the course, Ed gave responses that almost unanimously pointed to a productive mathematical disposition. However, when
he was given the same instrument again, his responses were much more middling. Ed’s score lowered from 123 to 88 (out of 125), with his mean response to positively phrased questions lowering from 4.92 to 3.52.

Table 10

*Ed’s Post-Mathematical Disposition Survey*

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<thead>
<tr>
<th>Category</th>
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<th>Postscore</th>
<th>Change</th>
<th>Prescore Mean</th>
<th>Postscore Mean</th>
<th>Change in Mean</th>
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<td>−10</td>
<td>5.00</td>
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<tr>
<td>Mathematical Self-Efficacy (of 20)</td>
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<td>−7</td>
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<tr>
<td>Total (of 125)</td>
<td>123</td>
<td>88</td>
<td>−35</td>
<td>4.92</td>
<td>3.52</td>
<td>−1.40</td>
</tr>
</tbody>
</table>

*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

When asked about this, Ed gave a thoughtful response, reflecting both on his views on mathematics entering the course and his views during the final interview.
I donno, I think part of it was I didn’t take that first [survey] very seriously. I mean, it was almost a game: find and mark the right answers. And I guessed at the time, I believed that stuff too. I knew I was good at math, so I thought that would be all that [it] would take to make me a good math teacher. But now, I know math is a lot bigger, a lot more complicated, than I thought back then. I mean, I am still pretty good at math, at least what I know. But there is a lot [of mathematics] that I don’t know. And even if I know all of math, that doesn’t mean I will automatically be a good math teacher. It isn’t as simple as I thought it was. (Final Interview)

MEST 1 broadened Ed’s views of mathematics and what it meant to be a good mathematics teacher. And these new revelations made Ed more nervous about his own ability to do and teach mathematics, even to elementary school students.

MKT After MEST 1

Unlike in Ed’s pre-test, during the post-test Ed was more easily able to reason about these problems and do the operations in multiple different ways. Also, when asked to reason beyond what he immediately knew, Ed was able to think mathematically about the problem and not impotently surrender.

Exercise 1. Unlike the pre-test, when Ed used a missing addend model to solve the subtraction problem, Ed opted to use regrouping to solve the subtraction problems.

Ed: These are all subtraction by regrouping.

Researcher: And how would you solve them?
Ed: I would subtract 1 from the tens place of the larger number, turn it into ten singles, and regroup them with the ones digit.

Researcher: Can you show me?

Ed: Yeah. [Successfully solves the first problem]. (Final Interview)

Interestingly, when asked about how he solved the problem originally, Ed was originally unable to recall how he used to think about the algorithm.

Researcher: So this isn’t the way you solved these problems when we first met.

Ed: No, I don’t guess it was.

Researcher: Do you remember how you used to solve these?

Ed: [Pause] You know, not really. Wait. [Pause]. I think I would think about these as addition problems. I would be like ‘what plus this gives me that.’

Researcher: That’s right. So can you tell me which way you like better?

Ed: I think I like regrouping. I mean it sorta shows what’s going on behind the math more.

Researcher: What do you mean?

Ed: Well, it’s like my old way I was just adding up, I was just thinking about subtraction as not addition. But in regrouping, you are actually doing subtraction. I think I like thinking of subtraction like that. (Final Interview)

**Exercise 2.** When Ed approached the multiplication problem the second time, he questioned the student’s use of the standard algorithm.

Ed: Well, it looks like the student didn’t preserve his place value.

Researcher: What do you mean?
Ed: Well, here the student is really multiplying by 50, not 5, so that number should have a zero on the end, to keep the place value. To be honest, I don’t know why the student is using this algorithm.

Researcher: What do you mean?

Ed: Well, we learned about five or six algorithms to [multiply large integers] and I think they all are better than this.

Researcher: So if you were teaching the student to do this problem, what algorithm would you use?

Ed: I would probably use partial products, that way it reinforces the notion of place value. (Final Interview)

**Exercise 3.** When Ed attempted the division with fractions problem, he first asked how the researcher wanted him to approach the problem.

Ed: So, do you want me to do this like I would do it, or should I do it like we would in [MEST 1]?

Researcher: How would you do the problem?

Ed: I would multiply by the reciprocal.

Researcher: And how would you do it in [MEST 1]?

Ed: We would draw some pictures. To be honest, I am still trying to get that under my belt before the final.

Researcher: Can you solve it by multiplying by the reciprocal?

Ed: Sure. [Correctly solved problems using the *flip and multiply* algorithm].

Researcher: So now, can you do it like you would on the final?
Ed: I’ll try. [After a pause Ed begins to draw two rectangles, divide them into quarters, and then correctly divides the quarters into groups of one half].

Ed: Is that right?

Researcher: You tell me.

Ed: Well, I got the same thing [using \textit{flip and multiply}], so it must be right. (Final Interview)

When asked to construct a story problem to model the division with fractions, Ed still struggled, but was able to successfully create a model.

Ed: So we always use recipes for these kinds of problems. So, lets see. [Pause] Say a recipe calls for one and [pause]. No. Say the recipe calls for… I donno.

Researcher: So what are your division models?

Ed: So division is either how many groups or how many in a group. So I should use 1/2 to be the chips in the recipe?

Researcher: Why?

Ed: Because I am using the recipe as the amount in the group.

Researcher: So what is your problem?

Ed: We have a recipe that calls for half a cup of chocolate chips. We have 1 3/4 cups, so how many times can we make the recipe? (Final Interview)

\textbf{Discussion.} Ed showed much greater versatility in his mathematical thinking after MEST 1. Ed demonstrated in each problem that he was familiar with multiple ways to solve the given problem, and unlike during the pre-test, Ed understood the mathematical principles he was using to solve each problem.
Reflections on MEST 1

Ed entered MEST 1 with the desire to improve his mathematical explanations. In that, Ed viewed MEST 1 as a success.

Researcher: You came into [MEST 1] and you really wanted to improve your ability to explain mathematics. Do you think you did that this quarter?

Ed: Yeah, definitely. I think part of my problem before was I really didn’t know the math, I knew what I was doing but I didn’t know why I was doing it. So if I was trying to explain something to someone and they didn’t get what I was saying, I didn’t have anything to fall back on.

Researcher: And now?

Ed: I think I have a better grasp of how math works, and have many different ways to approach a problem. So even if a student doesn’t get the way I would do it at first, I can explain how to do the problem and why it works, not just try to yell the same stuff that the student didn’t get. (Final Interview)

However, when Ed was asked what he considered the most valuable thing he learned in MEST 1, he responded that MEST 1 broadened his views on what it meant to know and teach mathematics.

Researcher: What do you think are the most valuable things you learned in [MEST 1]?

Ed: That math is really, really big. And complex. But not in a bad way. I mean, there are rules and stuff, but all the rules come from something. And all the rules make sense. They do. I used to think math should be about memorizing rules.
`But if they are just rules, kids will forget them. But if the rules make sense, then it will be easier for the kids to remember them and use them. It will be easier for them to think about math mathematically. (Final Interview)

**Summary**

Ed entered MEST 1 with a strong mathematical background, having taken Calculus I and the Mathematics for Middle School Teachers sequence. These experiences helped Ed quickly adapt to the explanation portion of the course, especially correctly using the vocabulary to create prose explanations.

Of greater interest is perhaps Ed’s significant change in mathematical disposition between his pre- and post-MD surveys. This change is due in part to two major factors. First, Ed admitted to not taking the pre-MD survey seriously, and looking for right answers. Second, MEST 1 broadened Ed’s views regarding mathematics from a simple family of algorithms to a complex system of interrelated concepts and ideas. In spite of Ed understanding the mathematics more after the end of MEST 1 than before, he is more intimidated by the prospect of teaching mathematics to students.

**Ed’s Themes**

- Ed’s mathematical knowledge for teaching increased as he was better able to explain the mathematics underlying the algorithms he used to solve problems and use multiple strategies in solving problems. Similar themes emerged with Amber, Brandy, Carl, and Dawn.
• Ed initially struggled to explain his mathematics on assessments, but improved by the end of the course. Similar themes emerged with Amber, Brandy, Carl, and Dawn.

• Carl moved from thinking about mathematics as a dualistic discipline to a relativistic discipline. Similar themes emerged with Amber, Carl, and Dawn.

• Ed’s mathematical disposition decreased as he became less confident that he understood mathematics as completely as he previously thought. Similar themes emerged with Carl and Dawn.

• Ed expressed his intent to use teaching techniques from MEST 1 in his eventual elementary school mathematic classroom. Similar themes emerged with Brandy and Dawn.
Chapter 10: Themes

For each of the participants, MEST 1 posed a way of looking at mathematics that none of their previous mathematics courses explored. MEST 1 is a course that was designed to explore mathematics beyond the rote application of algorithms to exercises, and examine the mathematical underpinnings of those algorithms and the concepts being used to solve mathematical problems. Prospective teachers were asked to go from simply repeating algorithms on similar exercises to solve new problems using the ideas discussed in class and to be able to explain their solutions to novices in ways that were both concise and mathematically complete. MEST 1 attempted to change prospective teachers’ negative opinions about mathematics by showing them that mathematics is a discipline of interconnected ideas and logical principles and not just simply a list of nonsensical algorithms which mysteriously give correct answers when performed.

Because of the different nature of MEST 1 to any other mathematics course the participants had taken, they each had an adjustment period as they acclimated to the new demands of this mathematics course. These adjustments included (a) participants initially struggling to craft explanations, (b) struggling with the problem-centered nature of the course, and (c) questioning the validity of activities that the participants did not see as directly related to improving their ability to teach elementary school mathematics.

Mathematical Knowledge for Teaching in MEST 1

Each participant showed gains in their mathematical knowledge for teaching (MKT). After MEST 1, each participant was able to explain the concepts in Ma’s (1999) exercises beyond simply the process of performing the algorithm. However, when the
participants were presented a concept that still confused them, they tried to mimic vocabulary and processes that they recalled from class rather than apply new mathematical thinking to the question. When the participants learned a concept in MEST 1, they understood that concept in a deep and through way. However, when the participants were still struggling to understand a concept or were presented with a new idea, they tended to avoid original mathematical thinking and instead tried to create a mathematical statement that the participant viewed as similar to the ones that were used in MEST 1, but ultimately these explanations had little to no mathematical value.

**Exercise 1.** During the introductory phase, four of the five participants were able to find the pattern in the subtraction problems, and identified them as subtraction problems with *borrowing*. Only Ed, who instead of using the traditional algorithm used an invented strategy, did not see the pattern. However, of those four, only Dawn was able to correctly reason about the algorithm itself. Amber, Brandy, and Carl each were able to use the algorithm, but none could reason why they were subtracting one from the tens place and adding ten to the ones place.

After MEST 1, each participant was able to solve the problem. Four participants, Brandy, Carl, Dawn, and Ed, still used the standard algorithm to solve the problem. Of these, Brandy was the only one not comfortable using the term *regrouping*, and still preferred to use the term *borrow*. However, each of these four participants were able to mathematically explain the algorithm, and in particular how the regrouping worked.

Amber chose to model the problem using base-10 blocks to assist her thinking. Amber correctly explained the model she was using, and pointed out how splitting a long
into ten singles equates to subtracting one from the tens column and adding ten to the ones column. Each participant was aware of this model, and Amber and Dawn both stated that they planned to use this in their mathematics classroom when they become teachers.

**Exercise 2.** When first encountering the exercise, only Amber and Dawn correctly associated the student’s error with the concept of place-value. Both Amber and Dawn took the concept a step further, suggesting that the student should use estimation to see that their answer was not numerically close to the correct answer. Brandy and Ed both found the computational error in the student’s algorithm, but attributed that mistake to the student not zero-filling. However, neither Brandy nor Ed were able to associate the concept of zero-filling with place value. Carl was not able to reason about the algorithm, as he could not recall the correct algorithm to solve multiplication problems. He would later state that if presented this problem, he would simply use a calculator to solve the problem.

It should be noted that each of the four participants that were familiar with the standard algorithm noted that the student should have put zeroes in the answer, instead of leaving a space as Ma (1999) does in her exercises. In future studies, it may be necessary to revise Ma’s interpretation of the standard algorithm to include 0s instead of blank spaces.

After MEST 1, only Brandy still saw the student’s error as algorithmic and not conceptual. Amber, Carl, Dawn, and Ed each decided to reinforce the concept of place value with the student, and each believed that the student should be shown an alternative
algorithm to aid the student’s learning. Each of the four participants mentioned partial-products as an algorithm to help the student solve the problem and reinforce the concept of place-value. Dawn and Ed both noted that using an area model or lattice model may also help make the concept of place-value more concrete for the student.

**Exercise 3.** Unlike the previous exercises, each participant had a unique method for attempting this problem. Amber first converted the fractions to decimals, reasoning she would rather divide decimals than divide fractions. However, after several minutes, Amber gave up on her decimals and tried once again to reason about the fractions. She was unsuccessful in solving the problem using either strategy. Brandy, not remembering the traditional algorithm, attempted to rewrite the division problem as a multiplication problem with a missing factor. However, Brandy did not solve for the missing factor correctly. Carl converted the fractions to decimals and used a calculator to compute the answer. Both Dawn and Ed correctly solved the problem using the *flip-and-multiply* method, but neither had any knowledge of why that algorithm gave the correct answer, or even if the answer computed was a reasonable answer to the exercise. No participant was able to successfully create a story problem that modeled division by 1/2, instead creating models which were either division by 2 (Amber and Dawn), multiplication by 1/2 (Brandy), or simply giving up on the task (Carl and Ed).

After MEST 1, several of the participants still struggled with the concept of division with fractions. Even though Amber was not able to recall any algorithm to solve the problem, she did reason about the process of division, created a diagram, and was able to use the diagram to solve the problem. When asked to create a story problem,
Amber incorrectly used the *recipe model* and created a story problem that modeled multiplication by 1/2. Amber could not reason why her answer for the division problem did not make sense as the answer to the story problem.

Brandy correctly used *flip-and-multiply* to compute the answer to the division problem, but could not explain why the algorithm worked, nor could she invent an alternative algorithm. Brandy did not create a word problem using the recipe model, preferring to reason about lengths of yarn. However, she did not correctly model the problem, instead modeling division by 2.

Like Amber, Carl chose to create a model to solve the division with fractions problem. And again, like Amber, Carl incorrectly created a story problem that modeled multiplication by 1/2 instead of division by 1/2, and was not able to reason why the answer he computed for the division problem did not make sense for the multiplication problem.

Both Dawn and Ed correctly used *flip-and-multiply* as well as created a model to solve the problem. Interestingly, both of them asked whether they should solve the problem using the standard algorithm or how they believed they should do it in MEST 1. Both were also able to correctly create a story problem with division by 1/2, both using similar *recipe* problems.

**Discussion.** The participants showed a greater understanding of the algorithms that they used to solve problems, especially as those concepts relate to place value. In Example 1, each participant was able to identify the role place-value played in subtraction with regrouping (or borrowing), and in Example 2, each participant
understood that the student’s error was perhaps conceptual as well as algorithmic. In both of the first two examples, the participants understood both the mechanics of the algorithm and the mathematical concepts forming that algorithm more than they did previous to MEST 1.

However, in Example 3, the participants had a much wider range of success. Even at the end of the quarter, only two participants were able to both correctly solve the division with fractions problem and create a correct story problem to model the operation. However, four of the participants used a similar setting in their story problem (Amber, Carl, Dawn, and Ed) where they attempt to reason about a recipe. Namely, the model they either used or tried to use was

A person has $x$ quantity of an ingredient. A recipe requires $y$ quantity of that ingredient. How many times can that person make the recipe?

which is a how many groups model for division. However, the fact that four of the five participants chose to use this exact scenario indicates that there was a preference in MEST 1 to use recipe problems to model fractional division.

More concerning is that when asked if they could think of a model for division with fractions without using recipes, neither Dawn nor Ed could adjust their model to a new scenario. Essentially, the two participants who could successfully create a model for division could not generalize the recipe formula other scenarios which use how many groups model for division. Amber, Carl, Dawn and Ed were simply parroting a practiced exercise and could not think more generally about real-world applications for fractional division.
Although these changes are promising, it is important to realize that when the interviews were conducted, each of the participants had just completed a mid-term exam and were beginning to prepare for their cumulative final exam. In that, it is difficult to understand the level of permanence of these changes. It remains an open question to see if the changes in MKT that were observed at the end of MEST 1 will carry through the entire MEST sequence, and will benefit the prospective teachers in their future mathematics methods course, teaching internship, and their eventual classroom.

Mathematical Disposition in MEST 1

MEST 1 endeavors to alter prospective teachers’ negative views and perception regarding mathematics. The course does this with the hope that altering those views and perceptions will cause the prospective teachers to (a) become better mathematics teachers, and (b) provide a better mathematics education experience for the prospective teachers’ future students. This goal is stated in the university documentation for MEST 1, and was reiterated by both of the lecturers for the course as well as both of the recitation leaders. The key change that MEST 1 seeks to engender in prospective teachers is that they see mathematics as more than algorithms, but as a series of interconnected ideas. As Dr. Jones put it:

We have a goal for these students… to change their disposition about mathematics. I don’t care if we don’t get through everything—I don’t care. What I want to do is [to] change these students’ dispositions to mathematics and change the way these kids think about mathematics. And I want students in [MEST courses] to expect to have to think about math, to expect to reason it out. To
expect to ask why and question what we’re doing. I want them to learn how to do mathematics, maybe really for the first time, and I want them to enjoy doing it.

(Interview with Dr. Jones)

Dr. Smith gave a similar sentiment:

I want to encourage my students to have a can-do attitude about math. I want them to think that they can do it, so they can pass that can-do attitude on to their [students]…. But sometimes the students come into [MEST 1] with such a helpless and defeated attitude—that there isn’t anything to understand, I just need to copy and memorize. They don’t think that they can understand it, and that if they could then they would be some kind of a math genius or math professor. We want to show them they can understand this math, and they can teach this math so that one day their students can understand. (Interview with Dr. Smith)

Namely, the instructors and recitation leaders of MEST 1 desire to use the course to change students’ perception of the nature of mathematics—from a discipline of memorizing algorithms which may or may not make logical sense to the student, to a discipline of logic, interconnected concepts, and algorithms which are logically derived from those concepts.

Entering the course, each participant viewed mathematics as the previous—as a set of algorithms largely divorced from context. During the initial interview, each participant attempted to use or successfully used an algorithm that they did not understand, with Brandy and Carl showing little understanding of any of the
mathematical problems beyond the algorithm they were applying. Also, when asked to reason about those algorithms or to reason beyond them, each participant struggled.

As stated earlier, each participant grew in their understanding of mathematics as a logically connected set of concepts, and each was better able to approach and reason about the problems they were asked to solve. However, this change had differing effects on each participants’ mathematical dispositions (see Table 11).

Table 11

*Participants’ Change in Mathematical Disposition*

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<tr>
<th>Participant</th>
<th>Prescore</th>
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<th>Change</th>
<th>Prescore Mean</th>
<th>Postscore Mean</th>
<th>Change in Mean</th>
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<tr>
<td>Ed</td>
<td>123</td>
<td>88</td>
<td>–35</td>
<td>4.92</td>
<td>3.52</td>
<td>–1.40</td>
</tr>
</tbody>
</table>

*Note.* Mean response determined after recoding to reflect the participants response to a positively stated question.

From Table 11, we see that Amber and Brandy, the two participants with more unproductive initial mathematical dispositions, raised their mathematical disposition score after MEST 1, while Carl, Dawn, and Ed, the three participants with more productive mathematical disposition scores, scored lower on the mathematical disposition
survey after completing MEST 1. Each participant reacted differently to MEST 1’s assertion that mathematics was more than memorizing algorithms.

**Preparedness to Teach Mathematics.** Both Amber and Brandy believed they were more prepared to teach mathematics to elementary school students after taking MEST 1. This is in large part due to their new understanding of the mathematical algorithms being used in elementary school. Amber especially wants to take the views of mathematics she learned in MEST 1 and apply them to her mathematics teaching.

I want to be able to give my students as much background as I can on how to do math. I don’t want to be one of those teachers who just tell students ‘it is the way it is.’ I don’t think that works for students, and it didn’t work for me… Kids want to know the whys and hows; they want to ask questions. And I want to be able to give them the answers, because I know that there are answers that I can give them. (Amber: Final Interview)

Although Brandy stated she was less likely to use those same principles, she also believed that understanding them and being able to explain them would help her should she ever teach mathematics.

Researcher: So overall, what do you think you will take away from [MEST 1]?

Brandy: To use in my class?

Researcher: If that’s how you want to think of it, yeah.

Brandy: I donno. I mean, I think that it is neat to see where some of the rules come from, but I don’t know if I will ever really do anything with them.
Researcher: Do you think it helps you to know that the math rules aren’t just rules? That they were formed from mathematical reasons and thinking?

Brandy: Yeah, it’s nice, but I don’t see how knowing that is ever going to affect the way I teach.

Researcher: I know you don’t want to teach math, but if you ever do, do you think you would use anything you may have picked up in [MEST 1]?

Brandy: Probably not. I mean, maybe some stuff. I think some of the thinking that goes into the algorithms may be good for the students to have. But as far as the new algorithms, no. I mean, the students need to learn the right way to do these problems.

Researcher: What do you mean by the right way?

Brandy: Not that one way is better than another, but if I am teaching third-grade math, and I know the fourth-grade math teacher wants her students to solve problems one way, I should teach them that way. (Final Interview)

Carl, Dawn, and Ed each feel less prepared to teach mathematics after taking MEST 1. This is in large part due to the realization that mathematics is not simply memorizing algorithms. Each participant viewed mathematics as something larger than they did before they began MEST 1, and that lessened their confidence to teach the subject. Carl summarizes his views of teaching mathematics after having completed MEST 1.

Carl: You know, coming into MEST 1 I thought teaching math to kids would be easy. I mean teach them to add, subtract, multiply, and divide, and that’s about it.
But now I see that there is a whole mess of stuff that goes into teaching math, even to little kids. I mean you have to know which models to use, what the kids are going to get wrong, what it means if a kid makes this mistake, there’s just a lot to it.

Researcher: So before you thought you were all but ready to teach kids math, what about now?

Carl: I donno. I mean, I am still pretty good, but math is just a lot bigger than I thought before. (Final Interview)

Dawn put her beliefs more succinctly.

Researcher: So when you teach, do you think you will integrate some of the [MEST 1] stuff—go beyond how to do something but also teach why you do it this way?

Dawn: Yeah, absolutely. Math should make sense, and it does make sense. So why do we need to hide that from kids? (Course Interview 2)

However, Dawn also expressed concern over the possibility of not understanding a topic to the level she believed she would need to successfully teach it.

I donno. I think I understand math a lot better now, a lot more now. I feel like I know what’s going on, not just how to do something but how something works. I feel like I know more about math. But at the same time, I know there is a lot of stuff in math that I don’t know that I need to know. I mean, I feel like I learned a lot, and I am sure I will learn a lot more by the end of [the MEST sequence], but
Mathematical Beliefs and Attitudes. After MEST 1, each participant believed mathematics was a discipline with reasons that were accessible, even if some of the participants were still struggling to master those reasons. As Dr. Smith and Dr. Jones supposed, that belief—that mathematics was something that the prospective teachers can and should understand—played a key role in the disposition of each participant. As Dawn put it

Dawn: I think the main thing I learned in [MEST 1] was to question what I know and why I think I know it. I think the way I was taught math [in elementary school] really caused me to stop thinking about math. I didn’t ever need to ask why things worked, I just did them. And looking back, some of what we do in math is weird. I mean, look at fractions. Sometimes [when doing operations with fractions] we need a common denominator, sometimes we don’t. Sometimes we just add the numerators; sometimes we multiply both the top and bottom of the fractions. There are just a lot of rules, and until [MEST 1] I never thought they were anything more than that. I mean, I figured there were reasons, but I didn’t figure I could understand them. I just thought that it worked, so do it.

Researcher: And now?

Dawn: I want to know why something works, especially if it is some weird thing. And I want to show that to students, so maybe they think it’s weird and
interesting. Or maybe they just won’t think it’s weird at all, maybe it will just
make sense for them. (Final Interview)

**Mathematical Self-Efficacy.** Although only two of the five participants saw
themselves as better able to teach mathematics after MEST 1, four of the five participants
saw themselves as better able to do mathematics after the course. Amber, Brandy, Carl,
and Dawn each raised their mathematical self-efficacy, largely due to their new-found
understanding of the algorithms that they previously used to solve problems.

I would say this course has improved my overall attitude to math. I was really
scared and nervous. But now that I am in the class and we are looking at why
math works, it is really interesting. I feel like I am more comfortable with math
and I believe in myself and my math skills a bit more. I feel like I can look at a
problem and start asking myself a bunch of good questions about the problem and
try to analyze what the problem may be asking, whereas before I would just see a
problem and if I didn’t know how to do it I would just freeze… I think if I was
taught [using what we are learning in MEST 1] I think I would be a lot better at
mathematics. (Amber: Final Interview)

Each of the four participants expressed a similar idea—that they each had the ability to
approach mathematics problems that that were not familiar and use logic and reason to
understand and solve the problem.

**The Role of Diligence in Learning Mathematics.** In the introductory interview,
each participant characterized himself or herself as a diligent mathematics student, and
each attributed what success they had in mathematics to their diligence. However,
mathematical tasks (problems) in MEST 1 were different than the tasks (exercises) the participants were familiar with from other mathematics classes. In the past, the participants were asked to memorize mathematical rules, algorithms, and to be able to apply those rules to groups of similar exercises. In MEST 1, many times the participants were given new and unique mathematical problems, and had to reason about those problems using the concepts and principles discovered in previous problems.

This change had a two-fold effect on the participants. First, because the mathematics problems demanded greater cognitive skill than simple application, repeated memorization was less effective. Participants could no longer simply use brute force to memorize the algorithm and how to apply it. However, because of the higher cognitive nature of the tasks, participants were more likely to experience mastery of the content, and gain a more thorough understanding. Although this was a difficult transition for many of the participants, each believed that their hard work was being rewarded. As Amber pointed out

I really liked the class. I feel like we got to see math in a way that I hadn’t before. I really wish someone would’ve taught me [mathematics] like this when I was in [elementary] school, maybe I wouldn’t have so many problems now. At times, I feel like I have to re-learn everything I thought I knew about math, and that is really frustrating. But I feel like now I am getting to see the inner-workings of math. It’s like watching a movie and then seeing all the how-it’s-made special features. I finally feel like I am getting the behind-the-scenes view of math.

(Final Interview)
Summary. Dr. Smith and Dr. Jones endeavored to change the way prospective teachers taking MEST 1 thought about mathematics and their mathematical skills. In that, they were largely successful, each participant transitioned from seeing mathematics as a system of algorithms to a set of logically connected mathematical concepts. And this changed perspective tended to make more positive the mathematical dispositions of participants with more unproductive MDs as well as lower the mathematical dispositions of participants with more productive MDs. For the participants with unproductive MD scores, they were able to transition from a place where mathematics made little sense to one where mathematics was understandable and mathematical ideas should make sense. The transition removed one of their major impediments to mathematics learning: the overreliance on the application of algorithms that the participants did not understand. However, the participants with productive mathematical dispositions saw mathematics as a much broader topic than they previously believed.

Researcher: What do you think are the most valuable things you learned in [MEST 1]?

Ed: That math is really, really big. And complex. But not in a bad way. I mean, there are rules and stuff, but all the rules come from something. And all the rules make sense. They do. I used to think math should be about memorizing rules. But if they are just rules, kids will forget them. But if the rules make sense, then it will be easier for the kids to remember them and use them. It will be easier for them to think about math mathematically. (Final Interview)
This realization made those participants less sure of their mathematical skills, but at the same time broadened their perspective of what it may mean to know, learn, and teach mathematics.

As Feldhaus (2010) points out, mathematical disposition is formed from a lifetime of mathematical experiences and cultural influences. As such, it is difficult to determine what lasting effect on mathematical disposition MEST 1 and the MEST sequence will have. In the best possible scenario, students with unproductive MDs will continue to become more productive over the course of the sequence, and prospective teachers who experienced dips in their mathematical confidence will recover through the sequence as they become more comfortable with their new perspectives on mathematics. Further, in the ideal setting, these new productive dispositions will supplant any unproductive elements and benefit the prospective teachers in their eventual classroom. In the worse-case scenario, none of the changes in mathematical disposition will be permanent, and prospective teachers after completing the MEST sequence will revert back to their unproductive mathematical dispositions or naïve views of mathematics.

**Mathematical Intellectual Development in MEST 1**

One of the more surprising findings of the research was that the participants were not at the same intellectual position in their mathematical thinking as they were in their general sense-making. Even though no participant thought below Position 3: Early Multiplicity in their general thought processes, each showed signs of thinking at Position 1: Basic Dualism for their mathematical thought when they began MEST 1. This study proposes that Perry’s Theory of Intellectual Development must be adapted when
discussing a student’s mathematical thinking, which we will call Mathematical Intellectual Development (MID). And further, that although Perry’s theory will be used as a model to discuss this new theory, there is not a necessarily a direct correlation between a student’s position on Perry’s scale and their position on MID.

Table 12

*Participants’ MID Positions Before and After MEST 1.*

<table>
<thead>
<tr>
<th>Name</th>
<th>Perry Position</th>
<th>MID Position Before MEST 1</th>
<th>MID Position After MEST 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amber</td>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Brandy</td>
<td>4a</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Carl</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Dawn</td>
<td>4b</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Ed</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

**Position 1: Basic Dualism.** Students who think mathematically from this position hold a dualistic, right-wrong approach to mathematics. Mathematical truths are held by experts (teachers, scientists, etc.) and are largely not understandable by the average student, even the most gifted student. It is therefore the student’s job to simply memorize the algorithms and procedures that are being used by the teacher.

Mathematical knowledge is seen as the quick application of the algorithms, and someone thinking from this position sees the two most accurate measures of mathematical knowledge as the speed and accuracy that one can apply the algorithm. Further, it is not
necessary to know from where the algorithms are derived, as that knowledge is only accessible by the *Authorities* who invented the algorithm.

Each participant in the study began thinking mathematically from this position.

**Position 2: Strict Dualism.** As in Perry’s model, students from this position begin to see mathematics in a multiplistic way, but are hostile to that change. Similarly, the students tend to divide the world into two groups of *Authorities*, those they see as supporting dualism and those they see as opposed to it. Students often argue that the majority of mathematics is dualistic, and that they need to know how to do mathematics to please the dualistic *Authorities*. In terms of preparing prospective teachers, someone arguing from this position might say

Researcher: I know you don’t want to teach math, but if you ever do, do you think you would use anything you may have picked up in [MEST 1]?

Brandy: Probably not. I mean, maybe some stuff. I think some of the thinking that goes into the algorithms may be good for the students to have. But as far as the new algorithms, no. I mean, the students need to learn the right way to do these problems.

Researcher: What do you mean by the right way?

Brandy: Not that one way is better than another, but if I am teaching third–grade math, and I know the fourth–grade math teacher wants her students to solve problems one way, I should teach them that way. (Final Interview)

From the example above, we see that Brandy is beginning to be confronted with the idea of multiplicity in her mathematical thinking, but she rejected that form of thinking,
saying that it was something few teachers used. In that, she believed it was her responsibility to teach students in a dualistic way to prepare them for other teachers who will expect the students to think dualistically about mathematics.

At the end of MEST 1, Brandy was the only participant still at Position 2.

**Position 3: Early Multiplicity.** In this position, students begin to grasp the multiplicative nature of mathematics—that even though many mathematics problems have one correct answer, those problems have many mathematically correct processes to arrive at that solution. Students begin to apply their own thinking to solve mathematical problems and not simply rely on the rote application of memorized algorithms. In that, students are beginning to appreciate that they may be able to both grasp mathematical concepts and understand that other students’ have valuable insights to mathematical problems.

However, in this position, students still believe that there is a best-possible solution process, and **Authorities** possess that process. Often times, students struggling in this position will become frustrated with their lack of progress, and yearn for the simplicity of dualistic thinking. These students will often wish the **Authority** would tell them how to do the problem. This points out a key distinction between this position and Position 2: in Position 2, a student will not engage in an activity that they do not know the way the **Authority** would want them to do it, while in Position 3, the students will struggle with that activity, even if begrudgingly so.

Amber, Carl, and Ed each reached this position during MEST 1.
**Position 4: Late Multiplicity.** Students viewing mathematics from this position have fully embraced mathematics as a multiplistic discipline and are beginning to understand mathematics as a relativistic one. When in this position, the student views themselves much more confidently in both their ability to do mathematics and the algorithms that they construct to solve mathematical tasks. Because of this, they demand authority give equal respect to their methods as the authority does its own. It is in this phase that students will question most loudly authorities’ monopoly on correct solution processes. Also, the student will question what is seen as inconsistencies in Authorities treatment of mathematics.

Dawn: There was one day that [Dr. Smith] filled in for [Faith] during recitation… and I didn’t like [Dr. Smith] in that setting at all.

Researcher: Why do you say that?

Dawn: Well, I felt like [Dr. Smith] never gave me a right way to do the problem. Like, we worked a problem out of the activity book, and then [Dr. Smith] had us put each of our answers on the board. And in the end [Dr. Smith] just said that each of these is a right way to do this problem.

Researcher: That’s interesting, because a few minutes ago, we were talking and you said that you thought if we gave a group of students the same problem, each of them would come up with their own way to solve it.

Dawn: Yeah, but we have a midterm and a final coming up, and I need to know the way that the graders want me to do the problem so I get it right. (Course Interview 5)
In the above quote, Dawn shows elements of Perry’s 4a: Oppositional Alternative (when she questions Dr. Smith’s teaching strategy) and Perry’s 4b: Adherence Alternative (when she says that she just wants to know how the graders want her to solve the problem). It remains unclear if there are multiple paths through Position 4 as there are in Perry’s model.

Of the participants, only Dawn reached this position.

**Position 5: Contextual Relativism.** As no participant reached this position during the course of MEST 1, this remains a theoretical construct based on mirroring Perry’s theory. In this position, a student fully embraces the relativistic nature of mathematics. The student is able to approach a mathematical task in a number of different ways, can analyze the strengths and weakness of each approach, and proceed forward with the most appropriate strategy for the given problem and the context in which it was presented. Students see their mathematical reasoning as on par with authorities, and see themselves and others as having unique perspectives on how to approach mathematical tasks.

Again, no participant reached this position during the course of MEST 1

**Discussion.** Although the researcher has chosen to use Perry’s theory as a model for MID, some tenants of Perry’s theory may or may not apply to MID. For instance, it is unclear if this is a theory that can be generalized to broader populations than prospective elementary school teachers in the US. Each of these students received what was considered a traditional education in the US, which has been described as being “a mile wide and an inch deep” (Roth et al., 2006, p. 58) trying to teach too many different topics
without exploring any of the topics in depth (Schmidt et al. 1997; Schmidt et al. 2001).

Each participant characterized their previous mathematics classes as algorithm-centered, with little attention paid to the mathematical concepts being used in the algorithm. In that, it is possible that an elementary school student being taught in mathematics with problem-centered approaches may score beyond their Perry position in MID, but also beyond the position college students taught mathematics in the more traditional way. Although the researcher suspects that a similar pattern may be present for students in other mathematics courses which are problem-centered (i.e. a course taught using the Moore Method, or a course designed to introduce students to the concepts of proof and logic), it remains open to the effectiveness of MID in settings beyond working with prospective elementary school teachers in mathematics courses.

Further, it remains open to whether or not students from other countries with different standards for and cultures of mathematics education would experience similar developmental patterns to the US students who participated in this research.

It is also unclear as to whether or not students need to pass through the positions of MID in order is as necessary in Perry’s model. If we once again visit our theoretic elementary school student being taught mathematics in a problem-centered approach, it is unclear if he or she would ever enter any earlier positions, or if they would automatically view mathematics as a multiplistic discipline. Also, as it is likely that a student would be in Perry’s fist position, it remains open to ask how would the student’s MID position influence the student’s Perry position. It may also be useful to explore competing theories of student intellectual development to perhaps find a better match for MID.
Although Perry’s theory is still commonly used in education research (see Evans et al., 2010; Love & Gutherie, 1999), there are several competing theories that address concerns other researchers had with Perry’s work (see Baxter-Magolda, 1999; Knefelkamp, 1999). One of these theories may be better able to address or be more easily adapted to address the mathematical intellectual development of prospective mathematics teachers.

**Participants Becoming Acclimated to MEST 1**

Both the instructors and the participants saw MEST 1 as a different mathematics course. This is due to (a) the problem-based instruction used in MEST 1 and (b) the use of mathematical explanations as a means to assess learning. Because of these differences, each participant saw MEST as both a mathematics course and an education course, believing that the instructors were teaching mathematics the way the instructors believed mathematics should be taught in elementary schools.

Amber: It’s really neat to hear other people’s explanations about things. It’s not like other college classes [Dr. Smith] actually has us participate and come to the board and give out our own explanations. And even if we are wrong, [Dr. Smith] has us go through our thinking and tries to work with us on our thinking and explanations. I really feel involved in the class.

Researcher: And what about the recitations?

Amber: Recitation is the same. We haven’t done as much talking as a class, but we have done a lot of group work and working with each other. (Amber: Course Interview 1)
These differences were initially difficult for four of the participants to adjust to, with only Dawn reporting that she felt the course moved too slowly for her liking. More often than not, the participants were frustrated with the differences between their expectations of what a mathematics course should be and what MEST 1 actually was. The instructors of MEST 1 realized that this is a difficult transition for most students, and try to address these differences and ease the transition to the course.

It became well explicated among us that we have to say things to address the nature of these courses: I give a speech and [Dr. Jones] sends a letter. I tell [the students] that [MEST 1] is different; I tell them that they will be unhappy at times; I tell them I want to hear about it. But I try to give them examples of why we would need to do this, and try to get them to realize that when we don’t quite understand something that it’s okay to not quite understand. And once we [Dr. Smith and the class] break that barrier, they are comfortable admitting that they don’t understand, and then we can work together to help them understand. And that draws them in to the course, and hopefully to mathematics. (Interview with Dr. Smith)

Some of the devices that are built into the course to ease the transition are the practice homework, warnings, and the beginning of the course including more direct instruction than the rest of the course. However, this transition was still difficult for the participants.

Carl: Like in other math classes, the content was really hard, but what they asked you to do with the hard content was kinda easy. I mean, you do a problem like
on the homework like what was done in recitation. But here, the math is easy math, it’s stuff you do in grade school. But you have to explain it, and to explain it you have to know it inside and out and get all the little picky details down. I don’t think I have learned as much math [in] as much detail in any math class I have ever before [MEST 1]. (Carl: Final Interview)

Amber did not feel prepared for the level of understanding that would be asked of her in MEST 1.

You’re taught how to do math by being told how it’s done, and no one ever asked me why this stuff worked. And now, it seems like all [Dr. Smith] and [Faith] want to know is why stuff works, and I really feel like I wasn’t prepared for this… There were never any reasons to math [concepts], and like I said before, I think if I knew the why’s I would be better in this class. (Amber: Course Interview 4)

Brandy felt unprepared for the problem-centered nature of the course.

Brandy: We have the lecture and the recitation and the homework. And I donno, it’s like no one really teaches us how to do the homework. Nobody ever just tells us how to do the homework, so I have to look it up in the book, how to do the homework. It actually explains what the processes are of what we are doing in class.

Researcher: And you don’t feel like you are learning that from class?
Brandy: No, not at all. (Course Interview 5)
Each participant had a difficult transition to MEST 1, marked by struggling with in-class activities, lower grades on assessed activities than they expected, or both.

Each participant also viewed the activities being done in class as something that will benefit the participants future students as much as it benefits the college-students enrolled in MEST 1. Carl described his views on the course

This new math that we’re doing [in MEST 1] is weird. I mean the math is really simple, but having to be able to explain it to a student is going to be the hard part. The math is easy—anyone can figure out these math problems. They are straightforward and common sense, but they aren’t common sense to a first-grader.

(Carl: Course Interview 1)

Amber described an activity and its consequences using this,

Amber: We did this thing with, like, super-bundles, and bundles, and did this weird thing were we like made numbers without using numbers; instead we used letters and stuff. We had, like, little sticks to represent the numbers. It was basically trying to get rid of that numerical system that we are used to and do something else. So we had to figure out what place value meant and other stuff. It was really confusing… When we were doing the bundles and super-bundles, it kinda threw me for a loop because they weren’t numbers. It was hard to think of a number as an object and set up a specific order to group them in, and name them without calling them the numbers I know.

Researcher: Why do you think you did that activity in class?
Amber: I don’t know, maybe because we are used to numbers, but kids aren’t. So maybe doing this helps us realize what it’s like for kids to not know numbers.

(Amber: Course Interview 2)

By the end of the course, each participant had adjusted to the teaching style used in MEST 1, and Amber, Dawn, and Ed felt that they benefitted from both the structure and the different pedagogy of the course.

Dawn: I really like the class, because even though we are learning different things, we are learning them the same way.

Researcher: How do you mean?

Dawn: So, even though we’re adding and subtracting fractions, we are learning it the same way we learned comparing fractions: by drawing pictures. And I think it makes it easier for a lot of people in the class to get the concept. It’s not a whole different thing you are learning when you go from chapter to chapter.

Researcher: For you too?

Dawn: Yeah, I guess so. (Course Interview 4)

And Amber.

Sometimes I get confused and make things more difficult than they need to be.

But I feel like I am starting to grasp the concepts. In the beginning, I was trying to but things weren’t really clicking. (Amber: Course Interview 2)

Discussion. The transition from a traditional mathematics course to MEST 1—and the MEST sequence—was difficult for each participant. Only Dawn did not find the new activities difficult, and each participant initially struggled to perform as
well as they would like on assessed work. These results are consistent with Sweller (1988, 2006), as are some of the remedies used by the MEST 1 instructors. However, by the time each participant became acclimated to the course, each participant had a graded assessment that they felt did not represent their best work, which ultimately may undermine their ability to progress in the sequence.

**Interplay Between Explanations and Assessment in MEST 1**

The syllabus for MEST 1 clearly articulates that creating mathematical explanations will be an integral part of both learning and assessment in the course. This is the first course in the three-course sequence…[MEST 1] focuses on concepts of number systems and operations. The goal of this course is to prepare you to become teachers of elementary and middle school students. Knowing the mathematics for yourself is not the same as knowing the math for teaching. To that end, we emphasize explanations of mathematical ideas. To make this point very clear: Full credit will NOT be given for correct mathematical answers without an explanation that is clear and complete… Explaining your thinking verbally in small and large groups will prepare you to explain mathematics to your students. It will also help you clarify your own ideas and/or questions. (Syllabus for MEST 1, Fall 2011)

Each problem graded on any assessment (homework, quiz, midterm, or final) is graded with 60% of the allotted points for the problem being given for the student correctly solving the mathematic problem and 40% of the allotted points for the accompanying mathematical explanation.
The instructional teams see the explanation portion of each mathematical task as both a learning tool and a tool for the graders to better assess student learning. The mathematical explanations serve as a tool for the students to refine and master the underlying mathematical concepts being used to solve problems.

Dr. Smith views the explanations as an integral part of any mathematics problem attempted in MEST 1.

We really want to start each class with a problem, before there is any instruction, before there is any formal lecture. We want our students to wrestle with problems, with mathematics problems, and to be able to solve those problems, and explain both what they did to solve the problems and why they did what they did to solve those problems. (Interview with Dr. Smith)

Dr. Smith also describes precisely what is expected in student explanations, namely the students’ problem solving methods and why those methods are mathematically valid.

Faith described how the mathematical explanations required in MEST 1 force students to examine mathematical concepts in greater detail than they perhaps have done in previous mathematics classes.

I think MEST is distinct from other college math classes… [In other college math courses] I would like to think that there is a burden on understanding a concept, but it is much, much less than in [MEST 1]. The [homework] papers I’m grading are like paragraphs, and my job is to read through this long explanation of what they think [for example] multiplication of fractions means and to determine how
accurately they understand and have explained the concept. (Interview with
Faith)

Carl makes a similar comparison between MEST 1 and other mathematics courses he had
taken.

Researcher: Can you compare [MEST 1] to the other math courses you’ve taken in college?

Carl: It’s different—it’s very different.

Researcher: How do you see MEST 1 as being different?

Carl: Like in other math classes, the content was really hard, but what they asked you to do with the hard content was kinda easy. I mean, you do a problem like on the homework like what was done in recitation. But here, the math is easy math, it’s stuff you do in grade school. But you have to explain it, and to explain it you have to know it inside and out and get all the little picky details down. I don’t think I have learned as much math [in] as much detail in any math class I have ever before [MEST 1]. (Final Interview)

The explanations also serve as a document to record the student’s thinking about a
given task. In that, they serve as a primary tool to assess any problem that a student submits to be graded. However, because the students previous to MEST 1 have not been asked to think about mathematics deeply enough to craft mathematically sound explanations nor explain their work in any meaningful way in the past, crafting explanations can be difficult for many students in MEST 1.
Participants’ Struggle with Explanations. Each instructor attempts to deal with this transition. Both Dr. Smith and Dr. Jones attempt to warn students that explanations will be a significant portion of their grade on each assignment.

I think a lot of students I have don’t like having to explain stuff or prove stuff. I think the [mathematics] curriculum up to this class really rewards students coming up with the right answer, and [MEST] wants to go beyond that to why the answer is right. And I think a lot of students are resistant to that, especially in the beginning. I mean, if the mathematics is correct you will still get points, but to get all the points and to pass the class, you need to know the whys. (Interview with Faith)

The instructors built into the syllabus a “practice homework” assignment designed to allow students the opportunity to do an assignment in MEST 1 and practice explaining their mathematics with no negative repercussions to their final grade. The practice homework is submitted, graded as it would be normally, but students are given full credit upon completion. This also gives recitation leaders the opportunity to practice giving feedback to students’ explanations, something many recitation leaders new to MEST have not done in the past.

Unfortunately, even after the practice homework, many of the participants still struggled to craft simple, succinct mathematical explanations. At times, the participants’ struggles reflected an insufficient knowledge of the mathematics content.

We are supposed to explain everything, why things are the way they are. But when you are coming up with these explanations, you begin to see problems with
the way you’re thinking about stuff. Maybe it’s that you know it, but not in a way
that you could explain what you’re doing to someone else, and for me, I have had
several times that I tried to write what was going in and I realized I had no idea
what I was doing.  (Amber: Course Interview 1)

And in other cases, the participant was having difficulty crafting an orderly explanation
from the concepts.

Brandy: Yeah. I donno, I guess that I have had some problems with the whole
‘explanation’ part of the course. I think that once I get that down, then I should
be fine.

Researcher: What do you mean?

Brandy: Well, they want us to write explanations for everything. And sometimes
that can be a bit tricky. I am not sure exactly what they are looking for. (Course
Interview 2)

As one might expect, four of the five students initially crafted explanations that
did not sufficiently explain their solution process and the mathematical concepts being
used in that process. However, Dawn, when learning how to craft the mathematical
explanations initially wrote explanations that were too long.

Researcher: How are you adjusting to having to explain everything you do in
[MEST 1]?

Dawn: The homework takes me forever. Like, math homework used to never
take me long, but now it takes me a long time, and it ends up being like eight or
nine pages when I turn it in.
Researcher: Why do you think it takes you longer in this class than other math classes?

Dawn: It’s the explanations. I mean, they have to be long for me to say everything I think I need to say about a problem, but they take a while…

Researcher: What part of the homework takes you longer—do you think you are spending more time figuring out answers or writing explanations?

Dawn: I mean, every homework has maybe two or three questions that I have to think for awhile about, but it is definitely the explanations. (Course Interview 2)

Unfortunately, some participants were still struggling to explain their mathematics well into the quarter, and that struggle significantly impacted their performance in the course.

Amber: I really feel like I wasn’t prepared for [MEST 1]. I would love to go back, with what I know now, and re-do a lot of my early homework and midterms. I feel like I have gotten so much better with explanations, and with understanding what’s going on.

Researcher: So if you could go back [to the start of the quarter], and give yourself one piece of advice to succeed in [MEST 1], what would it be?

Amber: To make sure you understand everything. Every word, every idea, and every problem. I feel like in the beginning, I was getting some of the stuff, but I was leaving it at that. I wish I had really focused on understanding everything, and how things fit together.
Researcher: Do you think that may have been why you were having trouble with your early explanations?

Amber: Yeah, I wasn’t understanding the words and the concepts, and I don’t think I was using them correctly. That probably was in part why I was getting docked in my explanations. I could explain the steps I was doing to solve the problems, but not necessarily why I was doing them.

Researcher: Did the practice homework help that at all?

Amber: Yeah, it did, but not enough. It was really a few weeks before I understood exactly what everyone wanted in their explanations, and by then I had done some real homework and quizzes and was about to take a midterm. I donno, I never felt like I didn’t know what was going on, I just felt like I couldn’t figure out what I needed to do in the explanations. (Final Interview)

**Expectations for Explanations.** Each of the participants also believed that, at times, the graders were either unclear or contradictory. This could also have been due in part to the participant misunderstanding the feedback given on assignments. Lara describes the metric she used to determine if a student should receive full-credit on an explanation or not.

I don’t know how many times a student has asked me to just tell them EXACTLY what I want them to say to get an A. And what I keep telling them over and over is to pretend that I am a fifth–grader, pretend (Dr. Jones) is a third–grader. What would you say to them? And if you don’t get full credit, it isn’t because you used this word or didn’t use that word. It’s that your explanation wasn’t good
enough—if I didn’t know what was going on and you gave me that explanation, I still wouldn’t know what’s going on. (Interview with Lara)

Although none of the graders are looking for language developmentally appropriate for children, Lara painted a clear picture of the style of explanation. Explanations should be mathematically complete, appropriate for novices, and succinct.

When asked, each participant had a slightly different idea of what was ultimately expected of him or her. For instance, when pressed Amber, Brandy, Carl, and Ed each point to the use of specific vocabulary as a key component to a good explanation.

Researcher: What do you think Dr. Smith and Faith are looking for in your explanations?

Amber: Mostly that you really know what you’re doing, that you know what the processes are called, and that you aren’t just doing stuff without knowing what you are doing. [Dr. Smith] says that when we are writing explanations, we should treat it like we were writing them for a student.

Researcher: What do you think [Dr. Smith] means by that?

Amber: Just to write clearly and to use the right words for things. You [the prospective teachers] should be able to tell a student learning this stuff what you [the prospective teacher] are doing in the class so that they [the students] can learn from what you’re [the prospective teacher] doing and do it themselves [the students]. (Course Interview 2)
And Brandy,

Researcher: So how would you say your explanation technique is different now than it was at the beginning [of MEST 1]?

Brandy: I try to be more careful about what words I use and to make sure I write and say exactly what I mean with my explanations. I think before I would just try to explain things the way I thought about them, but now I know that it is more of a process—that you have to think and work to make a good explanation. (Final Interview)

Carl believes that the instructors are actually combing through explanations simply looking for key words and phrases.

Carl: I think that [Dr. Jones] is looking for very specific strategies and explanations when grading. I think that it is important to use the right words, to draw the right pictures, and to really give the graders the explanations that they would do themselves.

Researcher: So what if you gave a good explanation that wasn’t necessarily the one that they were looking for, but was still mathematically correct and simple? Carl: I think that there were specific things that the graders were looking for, and if you didn’t have those things, you got points off. I mean, look at how much underlining there is on my test. These are just short words or phrases that the graders are looking for. And I was looking at some other [student’s exam] papers, and they did the exact same thing I did, but didn’t use the same words I did, and they got points off and I didn’t. (Course Interview 4)
Ed sees both the correct vocabulary and a good diagram as keys to creating good mathematical explanations.

Ed: I think it is practice. I mean in the beginning I was having trouble coming up with the right words to explain stuff or coming up with a good picture. But since I have been explaining every problem I come across, I have gotten a lot better at finding those right words and coming up with those necessary details. (Course Interview 2)

Using correct vocabulary (and using the correct vocabulary correctly) is a key component to explaining mathematics. What is at issue is that the participants did not have a clear understanding of the expectations of the instructors when it came to their explanations. Many times, this just confused the participants, and other times, it frustrated them.

One of the things that both Brandy and Carl struggled with was the lack of a consistent pattern or algorithm that they could apply to multiple problems.

Brandy: We have the lecture and the recitation and the homework. And I donno, it’s like no one really teaches us how to do the homework. Nobody ever just tells us how to do the homework, so I have to look it up in the book, how to do the homework. It actually explains what the processes are of what we are doing in class.

Researcher: And you don’t feel like you are learning that from class?

Brandy: No, not at all. (Course Interview 5)
And Carl,

Carl: I wanted a rule that I could follow. Like, give me one rule that I can use to follow to get all the answers I need. And [Lara] couldn’t—she couldn’t think of one single rule to give me.

Researcher: What do you mean by a rule: like a grading rubric, or a place to start, or something else?

Carl: Like a rule that I can use for most problems that I know if I do this rule, then I am going to get the right answer. Like for most of these problems it isn’t that you got the right answer, but how you get to that answer…I’m frustrated because everything seems so subjective so far. You have to do everything exactly how they want it, and if you don’t you can’t get a good grade. They are telling us that there is no one right way, but from my grades and a lot of other peoples grades there seems to be plenty of wrong ways. (Course Interview 2)

Both Brandy and Carl wanted a simple formula or pattern that they could use to solve multiple problems, like they had in previous mathematics classes. Because MEST 1 sought to look at the subject differently and more deeply than those other courses, there were no simple algorithms or techniques to universally apply. It was this realization that vexed both Brandy and Carl for most of the quarter.

These misunderstandings also confused the participants struggling to incorporate multiplicity in their mathematical thinking. Dawn and Brandy both struggled to know when an explanation was acceptable but different, and when an explanation was simply incorrect.
Dawn: There was one day that [Dr. Smith] filled in for [Faith] during recitation… and I didn’t like [Dr. Smith] in that setting at all.

Researcher: Why do you say that?

Dawn: Well, I felt like [Dr. Smith] never gave me a right way to do the problem. Like, we worked a problem out of the activity book, and then [Dr. Smith] had us put each of our answers on the board. And in the end [Dr. Smith] just said that each of these is a right way to do this problem.

Researcher: That’s interesting, because a few minutes ago, we were talking and you said that you thought if we gave a group of students the same problem, each of them would come up with their own way to solve it.

Dawn: Yeah, but we have a midterm and a final coming up, and I need to know the way that the graders want me to do the problem so I get it right. (Course Interview 5)

And Brandy

Brandy: On several points on the test, it would say things like ‘Needed to say blah, blah, blah.’ Like, for one of them, he [the grader] said I needed to say these two things. But I said them up here; I just didn’t use those words.

Researcher: So you think if you used those terms you would’ve got a better score.

Brandy: Yeah, which sucks. I mean, they harp on us about how each kid is going to think about this stuff differently, but when we don’t do these explanations exactly how they want, then we lose a bunch of points. (Course Interview 3)
Formal Summative Assessment Undermining MEST 1 Goals. At times, the practical needs to assess students conflicted with the stated objectives of the course. For example, MEST 1 administered three summative exams to the students: two midterms and a final. Each exam covered many different concepts, and each exam was a significant percentage of the student’s final grade. In order to help prepare the prospective teachers for the exams, students received a study guide before each exam, and instructors reviewed several of those problems in the class before the exam. Carl describes this process

I did really well on the midterm...The day before [Dr. Jones] went over some questions that were exactly on the test, so then it was easy. I mean, there was a midterm study guide, but I didn’t do any of it, I just learned the problems we did in class…I am doing a lot better on the homework too—I am looking at the example problems [in the textbook] and modeling my explanations on what they do. (Carl: Course Interview 3)

And before the second midterm,

Carl: I [did really well] on the second midterm. Now that I know what kinds of problems are going to be on the test, all I have to do is practice them. I make sure I use the right words, draw a good picture, and it’s really easy after that.

Researcher: What if you found a problem on an exam that you didn’t know how to solve?

Carl: I don’t know. It’s never really happened. I mean, especially on the midterms, everything that was on them was on that study guide, and if not [Dr.
Jones] goes over it before the test. So all I had to do was do those problems that
[Dr. Jones] did or were on the study guide and I was good. (Course Interview 5)

Although the course goal was to present students with unique problems, it is also difficult
to expect students, many of whom have an aversion to mathematics, to understand a new
problem in a high-pressure situation, successfully solve it, and accurately explain their
reasoning. However, this does show prospective teachers that during high-stakes
assessments, the principles being taught by the course should be replaced by the
principles of more traditional mathematics courses.

**Discussion.** MEST 1 endeavors to explore mathematics content more deeply than
the prospective teachers have previously seen, in the hopes that when the MEST 1
students become elementary school teachers they might explore mathematics with their
students in a similar way. Unfortunately, this means MEST 1 needs to teach mathematics
differently in the space of one academic term than most of the students have been taught
mathematics in the course of their previous K–12 careers. Ideally, it would be better for
both the prospective teachers and the instructors to prepare students for the demands of
MEST 1 before the course started, however both Dr. Smith and Dr. Jones made efforts to
do just that, with warnings and the practice homework. In many ways, the difficulty the
prospective teachers experience in MEST 1 is completely natural and unavoidable—they
are being asked to look at mathematics more deeply than they have been asked to in the
past.

Ultimately, one of the unfortunate realities of college is that assessment in college
classrooms is used for more than just assessing student learning.
I really wish we didn’t have to give them grades. The students worry too much about them, and not enough about learning the math. And I hate it. Assessment should be about learning what the students have learned and trying to fix problems—either re-teaching the students or adjusting the course to better address the students’ needs. Instead, it becomes a value-judgment on these kids [prospective teachers], and they need a good grade. They need the grade to get into their program and they need the grade to get into grad school; the grade is important, but not for the reason it should be…I think if we didn’t have to assign grades to students, it would solve a lot of problems [students in MEST 1] have. It would undoubtedly create new problems, but it would solve many of the ones we have. (Interview with Dr. Jones)

Many of the problems with the explanations and how they are used in assessment are tied to this paradigm. In theory, if grades were not as important to the prospective teachers, they could concentrate on how to craft good explanations instead of worrying about immediately finding the right answer. The summative assessments could maintain the philosophy of the course without need to sacrifice ideals to help the students get a good grade. Undoubtedly, as Dr. Jones noted, unmooring assessment from grades would cause myriad new and unexpected problems, but it would solve many of the problems currently present with assessment in MEST 1.
Chapter 11: Analysis of Results

As stated previously, mathematics education in elementary schools in the US is a concern for many prominent educational researchers and research organizations, and a common means to address this concern is to require prospective elementary school teachers take a course or sequence of courses that teach the foundations of elementary school mathematics—Mathematics for Elementary School Teacher (MEST) courses. And while these courses are commonly taught in college mathematics departments, there has been little research on their effectiveness.

This study investigated prospective elementary school teachers’ experiences when taking a mathematics course designed to teach the foundational elements of arithmetic. Specifically, this study focused on the prospective teachers’ mathematical knowledge for teaching, mathematical disposition, and intellectual development. During the research, it was discovered that the manner the course used mathematical explanations in assessments played a pivotal role in understanding the experiences of the participants.

The prospective teachers each made substantial gains in mathematical knowledge for teaching. By the end of the course, each participant showed a greater understanding of the algorithms they used to solve mathematical tasks, were better able to reason about the mathematical principles they used when applying algorithms, and more flexibility in their problem-solving strategies.

At the end of the quarter, the mathematics disposition of the prospective teachers changed. Participants with more unproductive mathematical dispositions raised them at the end of the quarter. This was due, in part, to the prospective teachers increased
mathematical understanding at the end of the course. However, participants with more productive mathematical dispositions lowered them at the end of the course. This was due, in part, to the participants seeing mathematics as a much larger and more complex discipline than they had originally thought.

In terms of the participants’ mathematical intellectual development, each participant began the course thinking of mathematics as a purely dualistic subject. By the end of the course, four participants were able to view mathematics from a multiplistic position.

Many of the assessment practices in MEST 1 heavily influenced the participants’ mathematical experiences and understanding in the course, some of which acted against the stated goals of the course. Although MEST 1 was mostly problem-centered and focused on presenting students’ new problems to apply their thinking, two participants (Brandy and Carl) reported mastering topics by repeating processes over similar problems, turning the unique problems into repeatable exercises. This was especially true during the higher-stake summative assessments.

**Reflexivity**

Bloomberg and Volpe (2008) define reflexivity as “reflecting, or thinking critically, carefully, honestly and openly, about the research experience and process” (p. 37). To that end, this section will detail my critical, careful, honest, and open reflection regarding my research experience and process.

I first became interested in MEST courses after having taught them as a graduate student and a part-time instructor at various colleges and universities. In truth, until
teaching mathematics to prospective elementary school teachers, I would not have guessed I would be interested in the mathematical principles being discussed in the class, nor would I have assumed that I would become interested in the experiences prospective teachers have leaning—or relearning—elementary school mathematics. But I indeed became interested in those ideas, and because of that interest in the topic as a teacher, I will also consider the results from a teaching perspective. This section will describe how I understand my results, and how I will incorporate those results the next time I teach a MEST course.

Each participant made strides in their mathematical knowledge for teaching as they began to piece together the mathematical principles that formed the algorithms that they previously used. In fact, four of the participants become much more open to understanding mathematics, with only Brandy rejecting this newly introduced mathematical paradigm. I believe that elementary school students are naturally curious to why these algorithms work, and over time schools in the US stifle this curiosity by not exploring those reasons. The next time I teach a MEST course, I want to explore those questions more and try to have the participants’ remember a time that they were curious about those reasons to help re-ignite that curiosity.

Each participant altered their disposition to mathematics, and that had divergent effects on the participants’ attitudes toward mathematics. For prospective teachers who enter the course with more unproductive dispositions, I would want to continue to nourish their burgeoning belief that mathematics is both (a) understandable by the average person and (b) something that they themselves can understand. For those entering the course
with more productive dispositions, I will try to relate their previous success in mathematics to future mathematical success—that given time they will be as comfortable with this mathematics as they were in their previous courses. For both groups, I would give the students ample opportunities throughout the time of the course, to express their frustrations—to me as the teacher and to each other—to help the prospective teachers realize that they are not alone in struggling with mathematics and/or with the MEST class.

Finally, I would endeavor to create a plan to assess students that minimized the negative effects of the large formal assessments of the MEST 1 course studied. While I doubt those assessments can be removed entirely, nor can their negative effects be completely excised, I would endeavor to minimize them. I would also allow students the opportunity to resubmit homework assignments with the intent of having students think about their explanations and how they can improve their explanations, rather than trying to conform their work to some standard they believe the instructor holds.

**Opportunities for Further Research**

The nature of case-study research leads to both finding new research questions to ask and creating hypotheses to be tested by further research (Ferdinand, 1999). In that spirit, the results of this study suggest that the following topics be investigated.

- How typical was the role, structure, and pedagogy used in this MEST 1 course studied? If the course were proven to be atypical, what results would change by studying a different MEST 1 course?
• How would altering the assessment practices used in this MEST 1 class impact the prospective teachers’ mathematical knowledge for teaching, mathematical disposition, and mathematical intellectual development?

• How would the study have been altered if the researcher studied the participants through the entirety of the MEST sequence, and not just the first course?

• How specific are these results to prospective elementary school teachers? Would the results hold when studying prospective middle or high school mathematics teachers in the United States? Would they hold for studying any group of college students from the US enrolled in a mathematics course with similar educational goals (namely, to change students’ views on the nature of mathematics and improve students disposition to mathematics)?

• Would these results hold when studying prospective elementary school teachers who are not in the United States? And if not, what are the cultural factors that influence the results?

• What could be done to further aid prospective teachers’ transition to creating mathematically correct and concise explanations to problems?

• Can mathematical intellectual development be further refined as a theory to go beyond describing trends in prospective elementary school teachers in the United States to describe trends in general student thinking?

• Does the tendency for prospective teachers with more unproductive mathematical dispositions to increase after taking MEST 1 still hold when analyzed quantitatively? Does the tendency for prospective teachers with more productive
mathematical dispositions to decrease after taking MEST 1 still hold when analyzed quantitatively?

Conclusion

MEST 1 was a transformative experience for each of the participants. Each participant improved their proficiency for teaching mathematics, four participants moved from seeing mathematics as a dualistic, right-wrong, discipline to a multiplistic one, and each participant began to see mathematics as a discipline that was formed from interconnected concepts understandable by a layperson. This was a difficult transition, due in part to the different nature of the course—MEST 1 was a problem-based course that used explanations as a primary means to assess students. However, the different nature of MEST 1 that caused the participants to struggle with the course was the key to the participants’ changes in MKT, MD, and MID.
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Appendix A: Copy of Recruitment E-mail

Dear Student,

My name is Adam. I am a doctoral-level math education student at Ohio University looking for future elementary school teachers taking MEST 1 this quarter to participate in a research project to better understand how future elementary school teachers learn mathematics. If you would participate, I would need to meet with you about every other week (most of these can be done over the telephone), answer my questions honestly, and allow me to occasionally observe you in your MEST 1 course and in recitation (not more than 5 times in the quarter).

If selected, you will receive a $100 stipend* as a compensation for your time! If you are interested, please e-mail me (e-mail address) or call me at (phone number). Thank you for your time, and I look forward to working with you in the Fall.

Thank You,

C. Adam Feldhaus
Doctoral Candidate, Ohio University

*You will receive $100 as compensation for your time and effort participating in this study. If for any reason you choose not to complete the study, you will be compensated commensurate with the amount of time involved in the study.
Appendix B:  
Participant Research Consent Form (Student Participant)  

Title of Research: How Intellectual Development and Mathematical Disposition Influence Teacher Candidates’ Mathematical Knowledge for Teaching in a Mathematics Course for Elementary School Teachers: A Case Study.

Researcher: C. Adam Feldhaus  
You are being asked to participate in research. For you to be able to decide whether you want to participate in this project, you should understand what the project is about, as well as the possible risks and benefits in order to make an informed decision. This process is known as informed consent. This form describes the purpose, procedures, possible benefits, and risks. It also explains how your personal information will be used and protected. Once you have read this form and your questions about the study are answered, you will be asked to sign it. This will allow your participation in this study. You should receive a copy of this document to take with you.

Explanation of Study

This study is being done to better understand the unique learning needs of prospective elementary teachers taking mathematics courses. This study will analyze the intellectual development students go through in the course, the knowledge students gain from the course which will be useful to the participant in their role as a teaching intern/inservice teachers at the elementary school level, and how the course affects the participants disposition toward mathematics as a topic of study.

If you agree to participate, you will be asked to conduct at most bi-weekly interviews with the researcher in which you discuss what questions/difficulties you are having in the course. You also agree that the researcher has your permission to observe your class no more than five times per quarter.

Your participation in the study will last for the entirety of your time involved within this class.

Risks and Discomforts

No risks or discomforts are anticipated.

Benefits

This study is important to science/society because it will help researchers, course designers, and instructors how to best meet the needs of preservice elementary teachers in mathematics courses.

Confidentiality and Records

Your study information will be kept confidential by using pseudonyms and by keeping the master key inaccessible except for those on the research team. Additionally, while every effort will be made to keep your study-related information confidential, there may be circumstances where this information must be shared with:

* Federal agencies, for example the Office of Human Research Protections, whose responsibility is to protect human subjects in research;
* Representatives of Ohio University (OU), including the Institutional Review Board, a committee that oversees the research at OU;

**Compensation**

You will receive $100 for your participation in this study.

**Contact Information**

If you have any questions regarding this study, please contact C. Adam Feldhaus (researcher) at cafeldhaus@gmail.com (304-916-0416) or Gregory D. Foley (research advisor) at foleyg@ohio.edu (740-593-4430).

If you have any questions regarding your rights as a research participant, please contact Jo Ellen Sherow, Director of Research Compliance, Ohio University, (740)593-0664.

By signing below, you are agreeing that:

- you have read this consent form (or it has been read to you) and have been given the opportunity to ask questions and have them answered
- you have been informed of potential risks and they have been explained to your satisfaction.
- you understand Ohio University has no funds set aside for any injuries you might receive as a result of participating in this study
- you are 18 years of age or older
- your participation in this research is completely voluntary
- you may leave the study at any time. If you decide to stop participating in the study, there will be no penalty to you and you will not lose any benefits to which you are otherwise entitled.

Signature__________________________________________ Date____________

Printed Name________________________________________

Version Date: 01/01/2011
Participant Research Consent Form (Faculty Participant)

Title of Research: How Intellectual Development and Mathematical Disposition Influence Teacher Candidates’ Mathematical Knowledge for Teaching in a Mathematics Course for Elementary School Teachers: A Case Study.

Researcher: C. Adam Feldhaus

You are being asked to participate in research. For you to be able to decide whether you want to participate in this project, you should understand what the project is about, as well as the possible risks and benefits in order to make an informed decision. This process is known as informed consent. This form describes the purpose, procedures, possible benefits, and risks. It also explains how your personal information will be used and protected. Once you have read this form and your questions about the study are answered, you will be asked to sign it. This will allow your participation in this study. You should receive a copy of this document to take with you.

Explanation of Study

This study is being done to better understand the unique learning needs of prospective elementary teachers taking mathematics courses. This study will analyze the intellectual development students go through in the course, the knowledge students gain from the course which will be useful to the participant in their role as a teaching intern/inservice teachers at the elementary school level, and how the course affects the participants disposition toward mathematics as a topic of study.

If you agree to participate, you will be asked to conduct at most bi-weekly interviews with the researcher in which you discuss what questions/difficulties you are having in the course. You also agree that the researcher has your permission to observe your class no more than five times per quarter.

Your participation in the study will last for the entirety of your time involved within this class.

Risks and Discomforts

No risks or discomforts are anticipated.

Benefits

This study is important to science/society because it will help researchers, course designers, and instructors how to best meet the needs of preservice elementary teachers in mathematics courses.

Confidentiality and Records

Your study information will be kept confidential by using pseudonyms and by keeping the master key inaccessible except for those on the research team. Additionally, while every effort will be made to keep your study-related information confidential, there may be circumstances where this information must be shared with:

* Federal agencies, for example the Office of Human Research Protections, whose responsibility is to protect human subjects in research;
* Representatives of Ohio University (OU), including the Institutional Review Board, a committee that oversees the research at OU;

**Contact Information**
If you have any questions regarding this study, please contact **C. Adam Feldhaus (researcher) at cafeldhaus@gmail.com (304-916-0416) or Gregory D. Foley (research advisor) at foleyg@ohio.edu (740-593-4430).**

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- you have been informed of potential risks and they have been explained to your satisfaction.
- you understand Ohio University has no funds set aside for any injuries you might receive as a result of participating in this study
- you are 18 years of age or older
- your participation in this research is completely voluntary
- you may leave the study at any time. If you decide to stop participating in the study, there will be no penalty to you and you will not lose any benefits to which you are otherwise entitled.

Signature __________________________________________ Date __________
Printed Name ____________________________________________

Version Date: 01/01/2011
Appendix C: Mathematics Disposition Survey for Prospective EC Teachers

Name: ___________________________ e-mail: _________________________________

Campus address: ____________________ Campus Phone: _______________________

For each statement, chose one option that best describes your feelings toward that statement. Options range from Definitely False (DF) to Definitely True (DT).

<table>
<thead>
<tr>
<th>Attitude Statement</th>
<th>Not Applicable (NA)</th>
<th>Definitely False (DF)</th>
<th>Mostly False (MF)</th>
<th>Neutral (N)</th>
<th>Mostly True (MT)</th>
<th>Definitely True (DT)</th>
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<tbody>
<tr>
<td>1. Generally, I feel secure about the idea of teaching mathematics to young children.</td>
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<td>2. I find many mathematical problems interesting.</td>
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<td>3. Mathematics makes me feel inadequate.</td>
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<td>4. I am not the type of person who is good at mathematics.</td>
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<td>5. I have always done well in mathematics classes.</td>
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<td>6. I am nervous about having to teach mathematics.</td>
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<td>7. I think I am good at mathematics.</td>
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<td>8. I generally do worse in mathematics courses than in other courses.</td>
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<td>9. I am confident in my ability to solve difficult mathematics problems.</td>
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<td>10. I enjoy learning about mathematics.</td>
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<td>11. I have hesitated to take a course that is mathematics based.</td>
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<td>12. Teaching mathematics doesn’t scare me in the least.</td>
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<td>Attitude Statement</td>
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<td>Definitely False (DF)</td>
<td>Mostly False (MF)</td>
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<td>13. I would get a sinking feeling if I came across a hard problem while teaching</td>
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<td>mathematics.</td>
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<td>14. At school, my friends would always come to me for help in mathematics.</td>
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<td>15. I am confident in my ability to teach mathematics.</td>
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<td>16. I have trouble understanding ideas that are based on mathematics.</td>
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<td>17. It would not bother me to teach a lot of mathematics.</td>
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<td>18. I do not do well on tests that require mathematical reasoning.</td>
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<td>19. Of all the subjects, mathematics is the one I worry about teaching.</td>
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<td>20. If I taught in a team or with a teaching partner, I would like to have another</td>
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<td>teacher teaching the mathematics.</td>
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<td>21. I see mathematics as practical and useful.</td>
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<td>22. I tend to not ask questions in math classes because I am afraid I will look</td>
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<td>23. I get frustrated when I do mathematics.</td>
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<td>24. Overall, I feel confident in my mathematical ability.</td>
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<td>25. Overall, I feel confident in my ability to teach mathematics.</td>
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Appendix D: Introductory Interview Protocol (adapted from Ferdinand, 1999)

0. Introduction/Need for audio taping /confidentiality

1. What were some of your favorite subjects in elementary school? What do you remember about your elementary school experiences with mathematics? (Probe for teachers, memorable experiences, assignments, etc). Who was your best mathematics teacher? What attributes did he/she have that made them a good mathematics teacher? If I could give you a magic button to push that would change anything about your mathematics education experiences, how would you use it?

2. How would you characterize your mathematical ability? (Probe for grades, evidence of mathematical ability). What would your previous teachers describe as your mathematical strengths? Your weaknesses?

3. What do you think when you hear the word “mathematics?” What would you say your feelings are about mathematics? How do you feel about the prospect of teaching children mathematics? Why do you think so many people think mathematics is difficult?

4. How would you describe a person who is good at mathematics? What does a person do to make himself/herself good at mathematics? What does a person who is good at mathematics do when solving a math problem? Studying for an exam? What reasons would you give for a person being good at mathematics? Are those reasons the same for everyone or does it vary from person to person? Do you think there are people who, no matter what, will never be good at mathematics?

5. What are your expectations/goals going into MEST 1? What do you hope to do/learn in the course? Do you think you need to take it? Why or why not?
Appendix E: Perry’s Intellectual Position Interview Protocol

1. What is your view of an ideal college education? How, if at all, should a student change as a result of that educational experience?

2. Have you encountered any significant differences in beliefs and values in your peers in college or other people you've met in your experiences here? What is your reaction to this diversity; how do you account for these differences? How do you go about evaluating the conflicting views or beliefs you encounter? How, if at all, do you interact with people who have views different from your own?

   [NOTE: The focus here is on the process of evaluating and/or interacting, not on specific beliefs or reactions]

3. Facing an uncertain situation in which you don't have as much information as you'd like and/or the information is not clear-cut, how do you go about making a decision about what you believe? Is your decision in that situation the right decision? Why or why not? If so, how do you know?

   [NOTE: Try to get the student to describe the process of coming to a judgment in that kind of situation, which in many cases will involve generating a concrete example of some personal relevance but not too emotionally-charged--preferably an academic-related context, related if possible to their major field.]

4. How would you define "knowledge"? How is knowledge related to what we discussed earlier in terms of a college education? What is the relationship between knowledge and your idea of truth? What are the standards you use for evaluating the truth of your beliefs?
or values? Do your personal beliefs/values apply to other people--in other words, are you willing to apply your standards to their behavior? Why or why not?

**Possible follow-up probes in each area:**

- How have you arrived at this particular view of these issues?

- Can you remember a time when you didn't think this way and recall how your view changed over time?

- To what extent do you think the view you have expressed is a logical and coherent perspective you've defined for yourself?

- What, if any, alternative perspectives have you considered?

- How likely is it that your view will change in the future?

- If you think it's likely to change, what kind of experiences or situations might produce such change?
Appendix F: End of Course Interview Protocol (adapted from Ferdinand, 1999)

1. What is your overall feeling about MEST 1? What useful things did you take away from it? What do you think were some of the strengths of the course? What about some of the areas MEST 1 could improve?

2. If you could have a “magic button” to change anything about MEST 1, what would you change? (Probe: Concepts taught, individual assignments, grading, groups…) If you could use that magic button to go back and change something that YOU did in MEST 1, what would you change and why?

3. What skills did you develop while taking MEST 1? Do you feel these skills will be useful to you in your future as a teacher? What specific things have you learned in MEST 1 that you think will make you a better elementary mathematics teacher?

4. How do you think opinion about mathematics has changed as a result of what you learned in MEST 1? Now, how do you feel about the prospect of teaching children mathematics?

5. What mathematics (or math methods) course are you planning to take next? What are your expectations for that course?
Appendix G: Mathematical Tasks (Ma, 1999)

Set A (original)

Exercise 1: (p. 1)

Let’s spend some time thinking about one particular topic you may work with when you teach, subtraction with regrouping. Look at these questions:

\[
\begin{array}{ccc}
52 & 91 & 42 \\
-35 & -79 & -27 \\
\end{array}
\]

How would you approach these problems if you were teaching second grade? What would you say students would need to understand or be able to do before they could start learning subtraction with regrouping?
Exercise 2: (p. 28)

Some sixth-grade teachers noticed that several of their students were making the same mistake in multiplying large numbers. In trying to calculate

\[
\begin{array}{c}
123 \\
\times 645
\end{array}
\]

the students seemed to be forgetting to “move the numbers” (i.e., the partial products) over each line. They were doing this:

\[
\begin{array}{cccc}
1 & 2 & 3 \\
\times & 6 & 4 & 5 \\
6 & 1 & 5 \\
4 & 9 & 2 \\
7 & 3 & 8 \\
\hline
1, & 8 & 4 & 5
\end{array}
\]

instead of:

\[
\begin{array}{cccc}
1 & 2 & 3 \\
\times & 6 & 4 & 5 \\
6 & 1 & 5 \\
4 & 9 & 2 \\
7 & 3 & 8 \\
\hline
7 & 9, & 3 & 3 & 5
\end{array}
\]

While teachers agreed this was a problem, they did not agree on what to do about it.

What would you do if you were teaching sixth-grade and you noticed several of your students doing this?
Exercise 3: (p. 55)

People seem to have different approaches to solving problems involving division with fractions. How do you solve problems like this one?

\[ 1 \frac{3}{4} \div \frac{1}{2} = \]

Imagine you are teaching division with fractions. To make this meaningful for kids, something many teachers try to do is relate mathematics to other things. Sometimes they try to come up with real world situations or story-problems to show the application of a particular piece of content.

What do you say would be a good story or model for \( 1 \frac{3}{4} \div \frac{1}{2} \)?
Set B (adapted from Ma 1999)

Exercise 1:

Let’s spend some time thinking about one particular topic you may work with when you teach, subtraction with regrouping. Look at these questions:

\[
\begin{array}{ccc}
62 & 85 & 42 \\
-47 & -48 & -24 \\
\end{array}
\]

How would you approach these problems if you were teaching second grade? What would you say students would need to understand or be able to do before they could start learning subtraction with regrouping?
Exercise 2:

Some sixth-grade teachers noticed that several of their students were making the same mistake in multiplying large numbers. In trying to calculate

\[
\begin{array}{c}
123 \\
\times 456
\end{array}
\]

the students seemed to be forgetting to “move the numbers” (i.e., the partial products) over each line. They were doing this:

\[
\begin{array}{c}
1 \ 2 \ 3 \\
\times 4 \ 5 \ 6 \\
7 \ 3 \ 8 \\
6 \ 1 \ 5 \\
4 \ 9 \ 2 \\
1 \ 8 \ 4 \ 5
\end{array}
\]

instead of:

\[
\begin{array}{c}
1 \ 2 \ 3 \\
\times 4 \ 5 \ 6 \\
7 \ 3 \ 8 \\
6 \ 1 \ 5 \\
4 \ 9 \ 2 \\
5 \ 6, \ 0 \ 8 \ 8
\end{array}
\]

While teachers agreed this was a problem, they did not agree on what to do about it.

What would you do if you were teaching sixth-grade and you noticed several of your students doing this?
Exercise 3:

People seem to have different approaches to solving problems involving division with fractions. How do you solve problems like this one?

\[ 2 \frac{1}{4} \div \frac{1}{2} = \]

Imagine you are teaching division with fractions. To make this meaningful for kids, something many teachers try to do is relate mathematics to other things. Sometimes they try to come up with real world situations or story-problems to show the application of a particular piece of content.

What do you say would be a good story or model for \( 2 \frac{1}{4} \div \frac{1}{2} \)?
Appendix H: Interview Protocol for Course Instructors
(adapted from Ferdinand, 1999)

1. Can you tell me a little about your history with mathematics and mathematics education? What about your history teaching MEST courses?
2. Why do you feel it is important to include mathematics in elementary education? Describe an excellent elementary school mathematics teacher? What qualities make a good elementary school mathematics teacher? How does MEST 1 help impart those qualities to your students (teacher candidates)?
3. How do you define your role when teaching MEST 1? What do you feel are some of the unique challenges teaching MEST 1? How would you change the course to better meet the needs of your students? If you had complete control of the course, how would you design it?
4. How do you feel about the required text for the course? How much do you use it in your course? How do you use it? What do your students tell you about the text? What do you think are some of the strengths of the text? If you could change anything about the text, what would you change?
5. How do you feel about the integrated lab activities? What comments do students have about the activities? What do you think are some of the strengths of these activities? Weaknesses? How do (would) you change them to improve them for your class?
6. How do you feel about using hands-on manipulatives in class? How often do you say you use them in class? What have students said to you about using them in class?
7. Overall, what do you think are the strengths of MEST as a series of courses? What do you feel the series of courses could improve upon? If you were overhearing one of your students (prospective teachers) talking about your course after taking it, what would you hope that they would say?
8. It has been said that teaching teaches most the teacher. With that understanding, what lessons have you learned teaching MEST 1. What lessons would you hope a first-time MEST 1 instructor would learn from the teaching course?
Appendix I: Examples of Activities Used in MEST 1

Calculations with Percents

1. Suppose that at a certain location, the average daily rainfall in September is $\frac{5}{8}$ of an inch. Last year, the average daily rainfall at that location in September was only $\frac{3}{8}$ of an inch. What percent of the average September daily rainfall fell last year in September? First solve the problem by drawing a picture. Explain how your picture helps you solve the problem. Then solve the problem numerically.

2. Lenny has received 6 boxes of paper, which is 30% of the paper he ordered. How many boxes of paper did Lenny order? First solve the problem by drawing a picture. Explain how your picture helps you solve the problem. Then solve the problem numerically.

3. If 95% of 40,000 voters voted for a referendum, then how many people voted for the referendum? First solve the problem by drawing a picture. Explain how your picture helps you solve the problem. Then solve the problem numerically.

4. A company has bought 3.4 acres of land out of the 4 acres of land that it plans to buy. What percent of the land has the company already bought? First solve the problem by working with common fractions and by drawing a picture. Explain how your picture helps you solve the problem. Then solve the problem numerically.

5. The mayor says that $3.6$ million dollars have been spent and that this represents 75% of the money allocated for a project. What was the total amount of money that was allocated for the project? First solve the problem by drawing a picture. Explain how your picture helps you solve the problem. Then solve the problem numerically.

6. Sixty percent of a city’s population of 84,000 people live within 5 miles of the library. How many people live within 5 miles of the library? First solve the problem by drawing a picture. Explain how your picture helps you solve the problem. Then solve the problem numerically.
Multiplying Integers

1. “MailTime” Problems: Assume that we consider “plus” to be receiving something in the mail, “minus” to mean sending something in the mail, “positive” to mean a check, and “negative” to mean a bill.

   a. Write a story problem modeled by $3 \times 5$. Be careful to phrase your question so you don’t give away the sign of the answer!

   b. Write a story problem modeled by $3 \times (-5)$. How could you model this with chips?

   c. Explain why it would be difficult to write a “mailtime” (or chips) problem that is modeled by $(-3) \times 5$ or $(-3) \times (-5)$. Explain why it would be difficult to write any “repeated addition” problem for these two products.

2. In 1c, we were faced with the difficulty of finding contexts in which one or both of the factors in a product could plausibly be negative. Let’s see whether we can find one. Back in school, you probably learned that “distance = rate x time”. We’ll look at it as “distance = time x rate” in this exercise (so that the number of groups is the first factor). For example, if you traveled for 3 hours at a rate of 50 miles per hour, you would have traveled a distance of $3 \times 50 = 150$ miles (objects are miles and groups are hours). In other words, your position has shifted 150 miles from where you started.

   Let’s expand this idea and speak of time in terms of time in the past (“ago”: considered to be negative time) and time in the future (positive time). We can also speak of position in terms of west (negative) and east (positive). In this scheme, we can rewrite our “$3 \times 50$” problem in the following way: Let’s assume we are currently at home, which sits on a west-east highway. Let’s say we’ve been traveling eastward at 50 miles per hour for awhile and will continue to do so. Where will we be 3 hours from now (in the future)?

   In this scheme, what would the story problem (and answer) be for:

   $3 \times (-50)$?  $(-3) \times 50$?  $(-3) \times (-50)$?
The Shepherd’s Necklace

A small shepherd boy was responsible for taking to pasture each day 23 sheep. These sheep would graze freely all day, and in the evening the boy was to round them all up and drive them home. Unfortunately, the boy could not count, so he had difficulty each day deciding whether he had found all the sheep before returning home.

a) Can you devise a method (other than teaching the boy to count) that would make it possible for him to be sure he had them all? If so, describe it.

b) The sheep's owner came up with a simple scheme. It involved making, for each sheep, a string necklace that would fit around the sheep's neck. In the morning, the boy would take the necklaces and place them all around his own neck. What should he do with them in the evening?

c) The "necklace idea" spread throughout the region and soon all the young people tending sheep went to the meadows with string necklaces around their necks. One day, an argument arose between a shepherd and a shepherdess as to which was tending the largest number of sheep. Resolving the argument was made difficult since neither could count. Can you describe a method by which they could resolve their argument while leaving their sheep grazing undisturbed?

d) What is possible to determine about two sets if one doesn’t know how to count?

e) What does it mean to “know how to count”? How does it relate to the sheep and necklaces? How could we define the number “seven” if we knew no other quantities?

f) What is possible to determine about two sets if one does know how to count?