Spectral Variability Analysis of BL-Lacertae

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This thesis titled
Spectral Variability Analysis of BL-Lacertae

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Abstract

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Spectral Variability Analysis of BL Lacertae

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BL-Lacertae is the prototype of BL Lac objects. It has been observed during the months of May, October and December 2011 with the aim of studying the intranight color variations on short timescales using the method of Color-Magnitude Diagram analysis and to look for the time-lag between the variations at different optical wavelengths using the method of Discrete Correlation Function analysis. Quasi-simultaneous measurements in the UBVRI bands have been performed using the 1.3 m optical telescope at MDM observatory at Kitt-Peak, AZ. A flare is observed during May 2011 in optical data that is also seen in the same time period in the gamma ray data, which is taken directly from the Fermi LAT website. BL Lacertae showed the trend Redder when Brighter and the transition to Bluer when brighter in the higher flux states, i.e. when the magnitude of R band is less than 13.5. A time-lag between the variations at optical wavelength band B and R is found to be $(0.01 \pm 0.0056)$ day (less than $2\sigma$). Using the value of time-lag, the lower limit on the magnetic field in the jet is found to be $B \geq 1.24 \delta_{1}^{-1/3}$ G.
This thesis is dedicated

To Isaac, Rashmi and my Papa

Family....

Where life begins and

Love never ends.
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Chapter 1: Introduction

AGN

Active Galactic Nuclei (AGNs) are among the most powerful and spectacular objects in the universe. These are the compact sources of energy at the center of the galaxy, assumed to be super massive black holes and whose luminosity outshines the stellar luminosity of the host galaxy. In some AGNs, the luminosity is $10^4$ times the luminosity of the host galaxy within the volume of smaller than 1 pc$^3$. These compact objects produce the radiation over the whole electromagnetic spectrum, from radio to gamma rays. The radiation coming from the AGNs is normally non-thermal radiation, as it is stronger than the thermo-nuclear reactions, which powers the stars. The most interesting aspect of AGNs is that when they were seen with an optical telescope, they appear as point like objects. The most powerful AGNs are known as Quasars or Quasi-Stellar Objects. Not all the AGNs are observed to have similar properties; in fact only 10% of AGNs are observed to have jets. AGNs are also variable and weakly polarized but very few of them are strongly variable and polarized.

**Basic components**

The basic components of AGN are (Ghisellini, 2012):

- A super massive body at the Center of the Galaxy named Black Hole.
- An accretion disk, which is the major source of radiation in the UV/X-ray regime. Matter with even a small amount of angular momentum, attracted by the black hole's gravity, spirals in and forms a disk.
- It is believed that the accretion disk is covered by the corona containing thermal material. The light coming from the accretion disk, when scattered by the thermal material present in the corona, produces the radiation, which is observed as UV/X-rays.

- There is also a dust-torus located at several parsecs from the Black Hole. The radiation coming from the accretion disk is absorbed by the particles present in the torus and re-emitted as infrared light.

- There are also small clouds present at less than one parsec from the black hole. The ionizing radiation coming from the accretion disk is absorbed and re-emitted by the particles present in these clouds in the form of lines. Because the Doppler shift broadens these lines, they constitute a part of the spectrum known as the Broad Line Region (BLR).

- At some larger distance, there are less dense and slow clouds present. Again, the light is absorbed by the particles present in these less dense clouds and re-emitted as narrow lines, so this region is known as the Narrow Line Region (NLR).

- The Black Hole's gravity attracts material from the accretion disk, after which this material is ejected violently and relativistically in directions perpendicular to the disk. These streams of luminous relativistic material are known as jets.

**Jets**

Almost 10% of all the AGNs known till now are discovered with jets. Jets are the collimated outflows from the super massive black holes. It is believed that jets
are composed of relativistic electrons and/or positrons, protons, photons from the synchrotron radiation, BLR or accretion disk with the magnetic field of at least $\geq 1\mu G$ (Boettcher, Harris and Krawczynski, 2012). Synchrotron radiation is the convincing proof for having magnetic fields in the jets. Jets can travel long distances up to 1 Mpc. As all the particles are moving with relativistic speed, there will be modification in energy and direction of the propagation of photons as a result. The observed energy of the photon ($E^{rec}$) i.e. the energy in the receiver frame is related to photon energy ($E^{em}$) i.e. in the emitter frame is:

$$E^{rec} = \frac{E^{em}}{\Gamma(1 - \beta \cos \theta^{rec})}$$

Here $\frac{1}{\Gamma}$ and $\frac{1}{(1 - \beta \cos \theta)}$ are the terms from the special relativity effect. The combined term $\frac{1}{\Gamma(1 - \beta \cos \theta)}$ determines the boost of photon energy and is known as Doppler factor ($\delta$).

Also, $\beta = \frac{v}{c} = Normalized \ speed$

$\Gamma = Bulk \ Lorentz \ factor \ of \ the \ emission \ region,$
Fig 1. Geometry of Emission region and the observer’s frame (Boettcher, Harris and Krawczynski, 2012).

$\theta^{em/rec}$ is the angle with respect to the jet axis (z-axis) as shown above in the Fig 1.

If the photons are received in the backward direction ($\cos \theta^{rec} = -1$) then photons will be de-boosted i.e. red-shifted by a factor $\frac{1}{\Gamma}$. In the forward direction the photons will be boosted i.e. blue-shifted by a factor $2\Gamma$. The photons are emitting at right angle to the jet axis would receive at an angle $\theta = \frac{1}{\Gamma}$ in the receiver frame.

By using the four vector scalar product, the expression for the photon energy $E^{rec} = \frac{E^{em}}{\Gamma(1-\beta \cos \theta^{rec})}$ can be written as $E^{rec} = E^{em} \Gamma (1 + \beta \cos \theta^{rec})$. As the two expressions for the photons energy are equivalent, after combining these two, a relation between the two angles of propagation with respect to jet axis can be found to be $\mu' = \frac{\mu - \beta}{1 - \mu \beta}$; $\mu = \cos (\theta)$, where $\mu'$ is the emitter frame and $\mu$ is the receiver frame.

If we plug in the values for the angles of the photons emitted in the backward or
forward direction ($\mu' = \pm 1$), we see that it will receive in the same direction ($\mu = \pm 1$). In addition to that, the photons emitting at right angle to the jet axis ($\mu' = 0$) will be received in the receiver frame at an angle $\mu = \beta$. As discussed earlier also, assuming that the source is emitting photons isotropically, the previous results imply that half of all the photons will be beamed into a narrow cone of opening angle $\theta^{rec} = \frac{1}{r}$.

This distortion of the direction of propagation of photons is given by $\mu' = \frac{\mu - \beta}{1 - \mu \beta}$ is known as the Relativistic Aberration. Using the same photon energy transformation, transformation between the time intervals can also be found in the emitter and receiver frames. As the energy is proportional to frequency and frequency is inversely proportional to time, as a result, the time interval in the emission frame and received frame is given by: $\frac{\Delta t^{rec}}{\Delta t^{em}} = \frac{E^{em}}{E^{rec}} = \frac{1}{\delta}$. Hence, according to this equation, the time interval in the emission frame will be shortened by one factor of $\delta$ when observed in the receiver frame known as Time Dilation (Boettcher, Harris and Krawczynski, 2012).

**Classification of AGN**

Active Galactic Nuclei are among the most energetic sources of radiation in the universe. These are very luminous and variable objects, on scales from minutes to years. The classification of AGNs is based on main properties like optical spectroscopy (the presence or absence of broad or narrow emission lines),
variability, and radio properties. Based on these properties, they are grouped into categories like Radio-loud/Radio Quiet, Seyfert galaxies and Quasars.

**Seyfert galaxy**

Seyfert galaxies were the earliest class of AGN to be identified. They are characterized by a very bright nucleus at the center of the galaxy, whose luminosity is more than the luminosity of the host galaxy. At first, Seyfert galaxies were subclassified as Type 1 and Type 2. Type 1 Seyfert galaxies include both broad and narrow emission lines, indicating the presence of both high and low velocity sources, whereas Type 2 shows only narrow lines. Now, it is believed that Seyfert galaxies are the same in essence and only differ because of the angle at which they have been observed as can be seen in Fig 1. The nucleus of the seyfert galaxy is surrounded by a dusty torus shaped region and the observed properties are determined only by the angle between the torus and the line of sight of an observer to the seyfert nucleus. In type 1 nuclei, the axis of the torus is close to the line of sight and the observer can observe the AGN directly with its connected broad line region in full view. In type 2 nuclei, the orientation of the dusty torus is such that it shields the seyfert nucleus from view and only the more extended narrow line clouds are observed (S.J.Curran, 2000).

**Radio Galaxies**

These are also one kind of active galaxies, which are very luminous at radio wavelengths due to the synchrotron process. The main characteristic of radio galaxies is the presence of symmetrical jets and radio lobes on either side of the
active nuclei. These lobes are often very symmetrical. Fanaroff and Riley divided these radio sources into two categories, FRI (compact source of luminosity) and FRII (extended source of luminosity).

**Quasars**

This is the most energetic and luminous subclass of AGN. In 1963, the radio source 3C 273 was discovered with broad emission lines, appearing as a star with a jet pointing away from the star. Astronomers called it a quasi-stellar object, or quasar. Most quasars have similar properties as Type 1 Seyfert galaxies; few of them are radio sources. Depending on the radio emission luminosity, quasars can be subcategorized into Radio Quiet and Radio Loud. As the name suggests, Radio loud has more dominance of radio emission in the spectrum than in Radio Quiet. When the observer’s line of sight coincides with the jet, the object is known as a Blazar, a class that also includes BL Lac objects as can be seen in Figure 2.
Fig 2. Unified model of Active galactic Nuclei. Credits: C.M. Urry and P. Padovani,
(http://www.cv.nrao.edu/course/astr534/ExtraGalactic.html)
Blazars

Blazars (blazing quasi-stellar object) are a subclass of AGN. These are very compact objects with a super massive black hole at the center of the host galaxy. The main characteristic of the blazar is that the direction of the jet is coincident with the line of sight of the observer at earth. The beaming effect makes the jet brighter, so it is not possible to see the host galaxy if the angle of the jet is coinciding with the line of sight of the observer.

- The blazar family comprised BL Lac objects and Optically Violent Variable quasars (OVV quasars), which are highly variable objects with one difference; that is BL Lac objects do not have broad emission lines in the observed spectrum while OVV objects generally have strong broad emission lines. Astronomer Edward Spiegal first suggests the name “Blazars” in 1978. Due to the expansion of the universe, Blazars are known for their high red shift on the basis of observed emission lines which means blazars are generally very distant quasars. These are also known for high polarization and variability.

- The emission from radio to UV/X rays is due to the synchrotron process, and the gamma ray emission is due to Compton scattering or leptonic/hadronic processes, which will be discussed later in the Chapter 2. The evidence for the synchrotron origin comes from the measurement of a very high polarization degree. As such, synchrotron emission can explain a higher polarization degree than any other emission process. Blazars are variable at all frequencies but they are most highly variable at high energies. The most significant feature of Blazars
is their unique spectral energy distribution curve (SED) in flux density. Spectral energy distribution means flux density or brightness as a function of frequency. The spectral energy distribution curve of Blazars shows two broad peaks, one is between Infra Red and soft X-rays and another one is at high energies like MeV-GeV i.e. gamma ray band. Depending on the location of the synchrotron peak in the SED, one can define Red Blazars and Blue Blazars. If the first peak is observed between Infrared to Ultra Violet band in the SED, it comes under the category called “Red Blazars” (which includes Low-energy peaked BL Lacs (LBLs) and Flat Spectrum Radio Quasars (FSRQs)). If the first peak is observed in the Ultra Violet or soft X-ray band, these are so-called Blue Blazars (includes High-energy peak BL Lacs (HBLs) as can be seen in Figure 3. The spectral energy distribution curve of blazars is interpreted as the first bump, due to the synchrotron emission. The inverse Compton scattering between the low energy photons and the same electrons, which produced the synchrotron emission, produces the second bump as can be seen in Figure 3.
Fig3. Spectral Energy distribution curve for blazars. FSRQ and LBLs have synchrotron peak between radio and optical and the Compton peak is at GeV energies. HBLs have synchrotron peak between UV and soft X-rays and the Compton peak at higher energies. ([http://ned.ipac.caltech.edu/level5/Urry/Urry5.html](http://ned.ipac.caltech.edu/level5/Urry/Urry5.html))

**Blazar Models**

As pointed out before, the process of synchrotron emission explains the first hump in the SED, but little general agreement exists on the origin of the high-energy
hump. Also, the rapid variability and the apparent super-luminal motions observed in blazars suggest that there is non-thermal continuum emission produced in approximately less than a light day sized emission region, which is propagating along the jet directed at smaller angle with respect to the line of sight to the observer at earth (Boettcher (2010). In order to explain the origin of high energies and the rapid variability, theorists suggest two models: The Leptonic Model and The Hadronic Model. An overview is provided here but a detailed discussion can be found in Chapter 2.

**Leptonic Model**

In the leptonic model, theorists give an explanation of the high-energy hump present in the SED of Blazars. According to this model, the hump in the SED is due to the process of Compton Scattering. In this scattering the electrons participating are the same electrons that are responsible for the synchrotron emission. The target photons in this scattering are the soft photons that can potentially come either from synchrotron emission in the jets, the emission from the central accretion disk, Broad Line Region or the dust torus. The Synchrotron-Self-Compton process is the process if the target photons are from the synchrotron emission.

**Hadronic Model**

The hadronic model deals with the presence of hadrons in the jets. A significant amount of energy from the jet is used to accelerate the protons present in the jets in a strong magnetic field environment. Much more energy is required to accelerate protons than electrons, to allow them to participate in any emission
process and reach the threshold energy to undergo pγ pion production. After gaining energy, these relativistic protons may interact with the photons present either in the jet or photons coming from the accretion disk or BLR to produce secondary particles and develop a cascade. Due to this pγ interaction, a cascade will be produced and the radiation generated is in the gamma ray band, which will be discussed in detail in Chapter 2.

**BL-Lacertae**

BL-Lacertae is the prototype of the “BL-Lac Objects” blazar class. As the name suggests, BL-Lacertae is located in the Lacertae constellation. BL-Lacs are characterized by the properties of high variability and high degree of linear polarization that are similar to FSRQs. Because of high variability of BL-Lac objects, they were believed to be a variable star in the beginning. The emission is due to the relativistic jet seen at an angle coinciding with the line of sight. The main difference between the two sub-classes of Blazars is the presence or absence of broad emission lines. Enlightenment on the emission lines is given by the concept of Equivalent Width (EW), which is defined as:

$$\text{EW} = \int \frac{(F_\lambda - F_0)}{F_0} d\lambda$$

Where $F_\lambda$ is the total flux and $F_0$ is the flux of the continuum.

The EW is positive for absorption lines and negative for emission lines. BL-Lacertae is observed to have weak or absent emission lines. The EW for BL-Lac objects in general is $< 5 \text{ A}^\circ$ (Ghisellini, 2012).
Classification of BL-Lac Objects

Low Frequency Peaked BL Lac (LBL)

The Synchrotron peak observed in the case of LBLs is in the infrared regime at $v_s \leq 10^{14}$ Hz (M. Boettcher, 2010). OJ 287 and BL-Lacertae are the examples of LBLs.

Intermediate BL-Lac (IBL)

The synchrotron peak observed in this is between optical and UV frequencies at $10^{14}$Hz < $v_s$ ≤ $10^{15}$Hz. One example of IBL is 3C66A (M. Boettcher, 2010).

High Frequency Peaked BL Lac (HBLs)

In the case of HBLs, the synchrotron peak is observed in X-ray regime in the SED with $v_s$ > $10^{15}$ Hz (M. Boettcher, 2010). For HBLs, The X-ray flux is observed to be greater than the Radio flux. In general, LBLs are more luminous than HBLs. RGB J0710+591 is an example of HBL.
Chapter 2: Radiation Processes

Light (electromagnetic radiation) emitted by astrophysical objects like stars is thermal radiation, but the radiation coming from AGNs, when analyzed is mostly non-thermal. Non-thermal radiation includes synchrotron radiation or radiation produced by Compton scattering. Also in the non-thermal radiation, the emitting particles do not have the Maxwell-Boltzmann distribution. It is now well known that most of the astrophysical objects are magnetized and have relativistic leptons. In this chapter, I am going to discuss both processes that are responsible for non-thermal emission.

Synchrotron radiation

Lorentz force is the kind of force that is responsible for the particle to gyrate around the magnetic field lines and follow a helical path. According to classical electrodynamics, charged particles accelerated by the magnetic field will radiate. For the non-relativistic particles, the radiation emitted is known as cyclotron radiation. On the other hand, if the particles are relativistic, that is moving with a speed near the speed of light, the radiation emitted is called as synchrotron radiation. The frequency of cyclotron radiation is the same as the gyration frequency, whereas the synchrotron frequency spectrum is more complex and can extend to many times the gyration frequency. The relationship between the synchrotron ($\nu_s$) and cyclotron frequency ($\nu_c$) in C.G.S units is (Rybicki & Lightman, 1979):
Where $\gamma$ is the Lorentz Factor: 
\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

$\alpha$ = angle which a velocity vector makes with the magnetic field line and known as Pitch angle.

The gyration frequency is: 
\[ \omega = \frac{qB}{mc\gamma} \]

and total emitted radiation Power (per electron) is given by:
\[ P = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_B = -\frac{dE}{dt} \] (Energy Loss Rate)

$\sigma_T = \frac{8\pi r^2}{3}$; Thomson cross section

\[ r = r_c = \frac{e^2}{mc^2} \]

$U_B = \frac{B^2}{8\pi}$; Magnetic energy density

After considering the emission due to a single particle, power law distribution can be used to consider the emission from many relativistic electrons. Generally a power law can approximate the particle spectrum over a limited range of frequency. Power law distribution is:
\[ n(\gamma) = n_0 \gamma^{-p} \quad \gamma_1 \leq \gamma \leq \gamma_2 \]

In the above equation, $n(\gamma)$ is the number of particles per unit volume in the Lorentz factor interval ($\gamma$, $\gamma + d\gamma$) and the quantity $n_0$ can vary with the pitch angle and ‘p’ is known as power-law index which is the negative slope on plot of log ($n(\gamma)$) and log ($\gamma$). The relation between synchrotron radiation and particle spectral indices
is given by $\alpha = \frac{p-1}{2}$. The emissivity, which is $j_\nu \propto \nu^{-\alpha}$ is defined as the power per unit solid angle per unit frequency produced within 1 cm$^3$.

**Synchrotron Self-Absorption**

All emission processes have their absorption counterpart. So, Synchrotron emission is also accompanied by absorption (synchrotron self-absorption (SSA)). In this process, a photon from synchrotron emission gets absorbed by the relativistic electron and gives up all its energy to the electron. The absorption coefficient is given by:

$$\alpha(\nu) = -\frac{1}{8\pi m c^2} \int_{\nu_1}^{\nu_2} d\nu P_\nu (\gamma)\gamma^2 \frac{d}{d\nu} \left(\frac{n(\gamma)}{\gamma^2}\right),$$

(Rybicki & Lightman 1979). In the above equation, $P_\nu (\gamma)$ is the synchrotron power per electron per unit frequency. With the delta approximation, the absorption coefficient reduces to $\alpha(\nu) \propto \nu^{-(p+4)/2}$. Thus, below a critical break frequency called synchrotron self- absorption frequency $\nu_{SSA}$ where $\tau_{\nu_{SSA}} = 1$, the nonthermal synchrotron sources tend to be optically thick. (Boettcher, Harris, Krawczynski, 2012). Here $\tau_{SSA}$ (optical depth) is defined as $\tau_{SSA} = \int_0^l \alpha(s)ds$, where $\alpha$ is the absorption coefficient (Boettcher, Harris, Krawczynski, 2012). This means that for the non-thermal electron distribution, the opacity to SSA increases with decreasing frequency. The peak spectral output occurs around the critical break frequency where $\tau_{SSA} = 1$. 
Compton Scattering

The interaction between the photons and free electrons is scattering. In the jets, we are dealing with relativistic electrons, which were responsible for synchrotron radiation. The photons can be the same photons, which were emitted during the synchrotron process, and also there would be more photons coming from accretion disk, BLR or in the jets itself. Taking every case into consideration, the energy of the photons can be less than the energy of the electrons or vice-versa. In the electron rest frame, if the energy of the photon is less than the rest mass energy of the electron, the scattering is known as Thomson scattering. On the other hand, if the energy of the incoming photon is greater or comparable to the rest mass energy of the electron, this regime is known as Klein-Nishina regime.

Consider $\varepsilon_1$ is the energy of a scattered photon and $\varepsilon$ is the energy of the incoming photon before scattering, thus, the expression for the scattered photon energy after Compton Scattering is:

$$\varepsilon_1 = \frac{\varepsilon}{1 + \varepsilon(1 - \cos\theta)}$$

Here, $\theta$ is the scattering angle.

Now $\langle \cos\theta \rangle \sim 0$,

For $\varepsilon \gg 1 \rightarrow \varepsilon_1 \sim 1 \rightarrow Klein - Nishina \ region$

For $\varepsilon \ll 1 \rightarrow \varepsilon_1 \sim \varepsilon \rightarrow Thomson \ Regime$

In the Thomson scattering, the energy of the incident and scattered photon is found to be approximately same and hence the scattering in the rest frame of an electron is elastic, because the recoil of an electron is negligible. On the other hand,
in the Klein-Nishina regime, where the transfer of energy from the photon to the electron becomes substantial, the scattering is inelastic (Rybicki & Lightman 1979).

In the Laboratory Frame (Boettcher, Harris and Krawczynski, 2012)

\[ \epsilon_1 \equiv \gamma^2 \epsilon \rightarrow \text{in the Thomson Regime} \]

The Compton Energy Loss rate in the Thomson limit is (Boettcher, Harris and Krawczynski, 2012):

\[ -\frac{d\gamma}{dt} \approx \frac{4}{3} c \sigma_T \frac{u_{ph}}{m_e c^2} \gamma^2 \]

which is the same as the synchrotron energy loss rate, if \( U_B \) is replaced by \( u_{ph} \). Thus, the luminosity in synchrotron and Compton radiation is related in the Thomson limit:

\[ \frac{L_{\text{Thomson}}}{L_{\text{Syn}}} = \frac{\dot{\gamma}_{\text{Thomson}}}{\dot{\gamma}_{\text{Syn}}} = \frac{u_{ph}}{U_B} \]

The Compton energy loss rate is suppressed with respect to the Thomson Limit for higher energy photons and electrons. Thus, for high energy electrons if their Compton cooling occurs in the Klein-Nishina regime, \( |\dot{\gamma}| \propto \gamma^a \) with \( a < 2 \) which leads to flatter energy dependence (Boettcher, Harris and Krawczynski, 2012).

**Leptonic Model**

The leptonic model gives one of the possible explanations behind the high-energy hump in the spectral energy distribution of blazars. This model suggested the high-energy emission is due to the Compton scattering. The electrons participating in scattering are the same relativistic electrons, which were responsible for the synchrotron emission. There are several radiation fields in a typical blazar environment, which can serve as a source of photons for the
scattering. The possibilities are: these photons can be the same which were emitted during the synchrotron emission in the same jet, they can come from the accretion disk emission or line emission from the broad line or narrow line region can be the another possibility. In addition to this scattered radiation, $\gamma\gamma$ absorption can also be responsible for the high energy hump in the SED.

Relativistic electrons in the magnetic field are responsible for the synchrotron process and produce low energy photons. The photons produced through this emission would have the probability to interact with the same relativistic electrons. As the energy of photons is less than the high energy electrons, in this interaction the energy is transferred from the high-energy electron to the photon. This process is known as Inverse Compton scattering. As the same relativistic electrons of synchrotron emission are the participants in this interaction, this process is known as Synchrotron self-Compton emission.

Electrons lose energy while moving along the jets through radiation processes. The steepening in the energy spectrum shows the energy loss through radiation. The radiative energy loss rate per electron is given by:

$$\frac{dy}{dt} = -\frac{4}{3} \sigma_T c \gamma^2 \frac{u}{m_e c^2}$$

Here, ‘$u$’ is the sum of energy densities of magnetic field and all radiation fields (Boettcher, Harris and Krawczynski, 2012). The time taken by an electron to lose a substantial amount of energy is known as Cooling Time ($t_c$) and is given by:

$$t_c = \frac{\gamma}{|\dot{\gamma}|}$$
After Substituting $\dot{\gamma}$ in the above cooling time expression, it is inferred that $t_c \propto \gamma^{-1}$. Thus, the highest energy electrons will cool off most quickly.

The two peaks in the SED, one because of Synchrotron and another one because of the Compton Effect are comparable, thus, considering the cooling time for both the effects.

The observed cooling time is given by: 

$$t_c^{obs} = t_c \left( \frac{1+z}{\delta} \right)$$

Where $\delta = \frac{1}{\Gamma(1-\beta \mu)}$ is the Doppler factor and $z$ is the red-shift. Plug in the values

$$u = \frac{B^2}{6\pi} (1 + k), \text{where } k = \frac{u_{ph}}{u_B} \approx 1, \text{ in } \gamma \text{ and also } \dot{\gamma} \text{ in the expression of cooling time}$$

$t_c$. As a result, the observed cooling time becomes: 

$$t_c^{obs} = \frac{6\pi m_e c^2}{c \sigma_T B^2 \gamma} \left( \frac{1}{\delta} \right) \left( \frac{1+z}{1+k} \right) \text{ s}$$

Also, $v^{obs} = v_0 B \gamma^2 \frac{\delta}{(1+z)} \text{ Hz}$, where, $v_0 = \frac{q}{2\pi m_e c}$ and $\gamma = \left( \frac{v^{obs}(1+z)}{v_0 B} \right)^{1/2}$; thus the observed cooling time is now looks like 

$$t_c^{obs} = \frac{6\pi m_e c^2}{c \sigma_T B^{3/2}} \left( \frac{v_0}{v^{obs}} \right)^{1/2} \left( \frac{1+z}{\delta} \right)^{1/2} \left( \frac{1}{1+k} \right) \text{ s}.$$ 

As BL-Lac objects are known for their high variability, it is possible to find the time lag between variations at various wavelengths. The time lag between two emissions can be interpreted as the difference in cooling time of electrons emitting at those frequencies. It is possible to estimate the magnetic field if an estimate of the Doppler factor ($\delta$) is available.

The observed cooling time difference is now:

$$\Delta t_c^{obs} = t_c^{obs} - t_c^{obs} = \frac{6\pi m_e c^2 v_0}{c \sigma_T B^{3/2}} \left( \frac{1+z}{\delta} \right)^{1/2} \left( v_1^{-1/2} - v_2^{-1/2} \right) \text{ s}.$$ 

(Takahashi et al., 1996).
After plugging in the respective frequency values for the respective wavelength bands, one can find the value of B (magnetic field) in terms of the Doppler factor. For example, the frequency of Red and Blue is $\nu_R = 4.68 \times 10^{14} Hz$, and $\nu_B = 6.849 \times 10^{14} Hz$ respectively and $\nu_0 = 6.849 \times 10^6 Hz$ and $z = 0.069$ for BL Lacertae. After plugging all the values, the Magnetic Field is given by:

$$B = 241 (\Delta t_c^{obs})^{-2/3} \delta_1^{-1/3} (1 + k)^{-2/3} \text{ G.}$$

Here, $\delta_1 = \frac{\delta}{10}$

**Hadronic Model**

The Hadronic Model gives another theoretical explanation of the high-energy hump present in the SED of blazars. There is a general agreement that the primary relativistic electrons produce the first low energy hump via synchrotron radiation. In the jets, protons are also present, but much more energy is needed to accelerate the protons. If a significant amount of jet energy is converted to accelerate the protons, then these relativistic protons can interact with the photons present in the jet and produce secondary particles. But a very high magnetic field (several tens of Gauss) is required to accelerate the protons to the relativistic energies and reach the threshold for $p\gamma$ pion production.

In the presence of sufficiently high magnetic field, relativistic protons can also emit radiation through the synchrotron process. Electromagnetic cascades can be initiated by the photons from the $\pi^0$-decay (Boettcher, Harris, Krawczynski, 2012). There are two major processes resulting in gamma rays.
Photo-Pion Production

If relativistic proton is interacting with the photon, it will create an electron-positron pair: \[ p + \gamma \rightarrow e^+ + e^- + p'. \] And then electrons and positrons again interact with soft photons to produce gamma rays.

In the proton gamma interaction:

\[ p + \gamma \rightarrow \pi^0 + p \rightarrow p + 2\gamma \] and then \( \gamma \) rays will produce electron positron pairs by \( \gamma - \gamma \) absorption.

\[ p + \gamma \rightarrow \pi^+ + n \] and then

\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \]
\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ n \rightarrow p + e^- + \bar{\nu}_e \]
\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \]
\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]

And thus, again electrons and positrons interacting with the soft photons will produce gamma rays.

\( \gamma - \gamma \) absorption

Photons with high energy can interact with each other and also with the lower energy photons called soft photons to produce electron-positron pairs and the process is called \( \gamma - \gamma \) absorption. If the two photons have enough energy known as threshold energy to interact, it will start a multiplicative cascade of secondary particles; the process is known as pair-production. Once this pair-production
shower starts, it will expand with multiple productions and also the magnetic field
deflection as can be seen in Figure 4.

Fig4. Pair-Cascade
Chapter 3: Observations and Data Analysis

Observations

Optical Astronomy incorporates a variety of observations through telescopes that are sensitive in the range of visible light. The 1.3 m McGraw-Hill telescope of the MDM observatory at Kitt-Peak, Arizona is also an optical telescope. This telescope is principally a reflecting telescope with a concave mirror for gathering and focusing light from the astronomical bodies. In order to bring the image to a convenient point, a hyperboloid convex mirror is used as a secondary mirror, where the Charged Coupled Device (CCD) camera can record the image.

BL-Lacertae was observed in a multiwave-length operation at ultraviolet (U), blue (B), visual (V), red (R), infra-red (I) bands during the 3 observing runs, in May 2011, October 2011 and December 2011. The CCD camera recorded the data.

![Reflecting Telescope](http://www.universetoday.com/45905/cassegrain/)

Fig5. A Reflecting Telescope with primary concave mirror and secondary convex mirror. ([http://www.universetoday.com/45905/cassegrain/](http://www.universetoday.com/45905/cassegrain/))
The main purpose of these observing runs was to collect the optical data for BL-Lacertae to look for variability and also time lags between the variations at different wavelengths. To characterize the intranight variability, the Color-Intensity diagram has been studied, which is an indicator of how the brightness and spectral hardness of BL-Lac are related during a certain time-interval.

The observations of BL-Lac were carried out at MDM observatory at Arizona using the 1.3 m reflecting telescope, which consists of UBVRI filters. As the name suggests, every filter has its own wavelength band. So, these filters transmit an object’s light only within a certain wavelength band. The filtering of light helps us measure the brightness of an object in a particular bandpass that falls within the wavelength range of the filter being used. The measurement of brightness helps to look for any variability that might be taking place inside the object at that time. As filters have their own defined wavelength band, likewise they also have their own defined energy ranges. In the optical bands used here, U band filters the highest energy optical photons through it whereas I filter passes the lowest energy optical photons to pass through it. The exposure times were between 40 seconds in R band to 120 seconds in U band. Observing other blazars like OJ287, 1ES0414+009, S50716+714, 1ES1959+650, 1ES0502+67.5, 1ES0229+200 were also part of the observing runs. As the main project was BL-Lac, it was observed frequently. Each observation run lasts for about a week with ~12 hours of nighttime observing depending on the weather conditions.
In addition to Optical data the Fermi (γ-ray) data has been taken directly from the website [http://fermi.gsfc.nasa.gov/ssc/data/access/](http://fermi.gsfc.nasa.gov/ssc/data/access/). The Fermi Gamma Ray Space Telescope studies the outer space in the energy range 30 MeV to 300 GeV. The Fermi Large Area Telescope (LAT) consists of a $4 \times 4$ array of identical towers, each comprises tracker, calorimeter and data acquisition module. In the Fermi LAT, the incident radiation is first passing through the anticoincidence shield, which is sensitive to charged particles. The primary interaction of γ-ray photons with matter is pair-conversion, resulting in producing electron positron pairs. After the conversion, the trajectories of electrons and positrons are measured by the tracking detector and their energies are measured by the calorimeter ([http://fermi.gsfc.nasa.gov/science](http://fermi.gsfc.nasa.gov/science)).

**Data Reduction**

The raw data obtained from the 3 observing runs, which were held in May 2011, October 2011 and December 2011, needs to be reduced in order to interpret. Using the software Image Reduction and Analysis Facility (IRAF), it has been carried out. The recipe of data reduction includes removing the electronic noise on the pixels of the CCD (charged coupled device) by dividing the flat and subtracting the bias fields. Bias level subtraction is the first step in the data reduction using the procedure of averaging the sample of bias frames which has taken with the zero exposure time pixel by pixel in order to get the mean bias frame. Then subtracting these frames from image and with `ccdproc` package in the IRAF, finally bias level can be subtracted. Secondly, pointing the telescope to the homogeneously
illuminated area inside the closed dome and begin to take the exposures called Flat exposures. It is necessary to take the flat exposure on each filter before taking the actual exposure. After the bias subtraction, dividing the Flat exposures from the object’s image will result in correction of pixel to pixel variation of the CCD. After all the reduction, the reduced image of the object is comparatively clearer.

Once the basic reduction of the photographic plate of the CCD camera is done, the photometry can be carried out. Photometry gives us the measurement of the relative brightness of the object with respect to well-calibrated stars present in the field, known as comparison stars. The measured brightness is quantified by the magnitude of an object and this magnitude is related to the flux according to the formula:

\[ m_1 - m_2 = -2.5 \log_{10} \left( \frac{F_1}{F_2} \right). \]

Here, \( m_1 \) is the magnitude of the object, \( m_2 \) is the magnitude of the comparison star and \( F_1 \) and \( F_2 \) are the corresponding fluxes. By the definition of magnitude, smaller the magnitude means brighter-appearing the star. Using the DOAPHOT package of IRAF, the instrumental magnitude of an object can be obtained in a particular filter. The physical magnitude of an object can then be obtained from the instrumental magnitude by calibrating the instrumental magnitude of an object with the magnitudes of comparison stars present in the field of view using the equation:

\[ m_{\text{obj}} - m_{\text{comp.star}} = m_{\text{inst.obj}} - m_{\text{inst.comp.star}} \]
Here, $m_{obj}$ and $m_{comp.star}$ are the apparent magnitude of the object and the comparison star and $m_{Inst.obj}$ and $m_{Inst.comp.star}$ are the instrumental magnitudes of the object and comparison star in one of the UBVRI filters used in the telescope.

Using the physical magnitudes for each particular band over the entire observation time, lightcurves can be plotted of an object. These plots between the physical magnitude of an object and the observing time are used to study the variability of an object. As discussed above, the formula between the magnitudes and the corresponding fluxes can then be used to calculate the flux in each band. To study how the object's brightness would vary through each wavelength band, the Spectral Energy Distribution (SED) graph is very helpful. SED is a log-log plot of flux density as a function of frequency. Spectral Energy Distribution curves for each run can be seen in fig 6.
For the Fermi Data, the fits files are available online, which are downloadable. For the reduction of data, Fermi Software has to be downloaded, which is also available on the FERMI LAT website (http://fermi.gsfc.nasa.gov/). In order to get the LAT (large area telescope) light curves, there are two basic ways, Aperture Photometry and Likelihood Analysis. Likelihood analysis has the potential for greater sensitivity and flux measurement with all the background subtraction. In the procedure of likelihood analysis, there are series of commands which needs to be followed and I am going to discuss those commands in brief here.
gtselect - filters the data based on time, position and energy

gtmktime - create good time intervals

gtlcube, gtexpmap – will calculate the exposure

After going through all the commands, a python script which is also available on the Fermi LAT website is used to create the model files. These model files are created by using isotropic and diffuse models in order to subtract the photon noise coming from interstellar medium. Finally, the command gtlike will give the light curve of a source with all the background subtracted.
Chapter 4: Correlation Studies

Light Curves

The Light curve is a plot of intensity of light coming from any astrophysical object in a particular frequency or band as a function of time. Fig. 7, 8 show the optical light curves of BL-Lacertae in U, B, V, R, I bands for the two observing runs in May and October 2011.

![BLLac May](image)

Fig 7. Optical Light curve of the May Observing Run
Small variations can be seen in the October observing run, but the averaged brightness of BL-Lacertae was quite stable in this period. In the May 2011 run, variations on the nights JD-2455000-707, 710 and 711 are more significant. To find if there is any statistical relationship between the data of different wavelength bands, correlation studies have been done, which will be discussed in the next section. Also, the Optical data has been compared with the Fermi data, which can be seen in fig 9. Likewise, the Fermi light curves for May and October 2011 have been
compared with an Optical light curves as can be seen in fig 10 and 11. By looking at fig. 10, a flare is visible on MJD 55706.8 in the Fermi as well as in the optical light curve. Similarly, by looking at fig. 11, a flare on MJD 55848.4 is visible in both the light curves.
Fig 9. The middle plot is Fermi data of Year 2011 with binning of 2 days. The upper plot is Optical data of May 2011 and the Lower plot is the Optical data of month October 2011.
Fig 10. The above plot is Optical light curve of May 2011 and lower one is Fermi LAT light curve of May 2011 with the binning of 1 day.
Fig 11. The above plot is Optical light curve of October 2011 and lower one is Fermi LAT light curve of October 2011 with the binning of 1 day.
**Color-Magnitude Correlation**

The Color-Magnitude diagrams are used to interpret the color behaviors of an object. The difference in the brightness between two wavelengths is referred to as color.

The color-index is the difference between magnitudes at different wavelengths such as (B – R), (B – V). If the difference is less in B – R, the object would be brighter in the Blue wavelength band compared to Red wavelength band, as a result the spectrum would be harder or “bluer’. In the case of BL-Lacertae, the color-index (B-R) and (B-V) was calculated and plotted against R and B respectively as can be seen in fig 12, 13, 14, 15. The plot between B-V and B shows no apparent correlation overall, but the plot between B-R and R for the October and May run indicates the color behavior of BL- Lacertae. After careful analysis of the color-magnitude plots (B-R vs R) of May and October data runs, by looking at the data of nights 1, 4, 6 and 7 in the October run, it indicates that BL Lacertae is Redder when brighter. Because as the color B-R is increasing, the magnitude R is decreasing, resulting in the negative slope, indicating the object is redder when brighter (RWB). In a similar way, when BL-Lacertae is even brighter, the slope containing the data of nights 2, 3, and 5 in the October run and the data of nights 1 and 3 in the May run is positive, indicating the object is Bluer when Brighter (BWB) as can be seen in fig 13 and 14.
Fig 12. A plot between color B-V versus B-Band magnitude. Here, the data of all the observing runs are included.
Fig 13. Plot between color B-R and R -Band magnitude. This is the data of the May run only.
Fig 14. A plot between color $B$-$R$ versus $R$-Band magnitude. It is the data of the October run only. As can be seen from the plot, the data of day 1, 4 and 7 is showing that BL-lacertae is Redder when Brighter.
Fig 15. This is a plot between color B-R and the R-Band magnitude. This data is of December.

In the color-magnitude plot, a variation is seen between Day 1, 4, 6, and 7 in the fig 7. In order to see, how strongly the variations of different nights are correlated, the Pearson Coefficient ‘r’ has been calculated. It is defined as:

\[
r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}
\]
Here x and y are two variables. The Pearson Coefficient is ranging from -1 to +1; +1 signifies a perfect positive relationship between two variables and -1 signifies a perfect negative relationship between two variables and Zero means No relationship.

The Pearson correlation coefficient (r) has been calculated for the three runs of every wavelength band UBVRI. In addition to that, the correlation coefficient is calculated for the color-magnitude diagram B-R vs R and B-V vs B.

In the Color-Magnitude Diagram B-R versus R for the October Run, there are main two groups: Days 1467 and Days 235. The Pearson Correlation coefficient has been calculated for both the groups, the values are as follows:

- B-R versus R (days 1467): r = -0.70 → weak negative correlation
- B-R versus R (days 235): r = 0.20 → weak positive correlation
- B-R versus R October run (total): r = 0.427 → medium positive correlation
- B-V versus V October run (total): r= No Association.

The values for the May run:

- B-R versus R May run (total): r = 0.57 → medium negative correlation
- B-R versus R May (days 13): r = 0.25 → weak positive correlation
- B-R versus R May (days 24): r = -0.21 → weak negative correlation
- B-R versus R December run (total): r = -0.582 → medium negative correlation
Consequently, the slopes are negative for nights 1, 4, 6 and 7 in the October run and on night 1 in the December run, indicating the color of BL Lacertae is Redder when Brighter, i.e. when the magnitude of R band $\geq 13.5$. Likewise, the slopes are positive for nights 1 and 3 in May run and nights 2, 3 and 5 in October run, as the correlation coefficient is positive. This positive slope is indicating the color of BL Lacertae is Bluer when even Brighter, i.e. when the magnitude of R band $\leq 13.5$ as discussed in section 4.2 under color-magnitude correlation.

**Cross-Correlation Analysis and Time Lags**

BL-Lacertae is known for its variability in time. As we have seen from the spectral energy distributions and color-magnitude diagrams, the signal is varying in time. The main goal is now to determine if there is any time lag between the variations at different optical wavelength bands. In order to find the time lag, generally the discrete correlation function is calculated, which I am going to discuss here.

The Cross Correlation function is generally calculated for two input functions which vary in time. Considering two continuous functions, one “$a(t)$” at time ‘$t$’ and another one “$b(t+\tau)$”, where $\tau$ is the time lag applied to the second function. Thus, according to definition of the cross correlation function:

$$CCF(\tau) = \int \frac{(a(t) - \bar{a})(b(t + \tau) - \bar{b})}{\sigma_a \sigma_b} dt$$

Here, $a$ and $b$ the two continuous data trains, $\bar{a}$ and $\bar{b}$ are the two corresponding averages and $\sigma_a$ and $\sigma_b$ are the corresponding standard deviations.
All cross-correlation methods work well for evenly spaced data. Our data is not evenly spaced. For the unevenly or coarsely spaced data, the Discrete Correlation Function (DCF) was introduced and is given by:

$$DCF_{i,j} = \frac{(a_i - \bar{a})(b_j - \bar{b})}{\sqrt{(\sigma_a^2 - e_a^2)(\sigma_b^2 - e_b^2)}}$$

Here, $a_i$ and $b_j$ the two discrete data trains, $\bar{a}$ and $\bar{b}$ are the two corresponding averages. In order to preserve the proper normalization, $\sigma_a$ and $\sigma_b$ are replaced in the CCF with $\sqrt{(\sigma_a^2 - e_a^2)(\sigma_b^2 - e_b^2)}$ in DCF (Edelson and Krolik, 1988).

Binning the result in time allows the directly useful function $DCF(\tau)$ to be measured and averaging over the N pairs for which $\tau - \Delta \tau/2 \leq \Delta t < \tau + \Delta \tau/2$, DCF is now given by (Edelson and Krolik, 1988):

$$DCF(\tau) = \frac{1}{M} \sum DCF_{i,j}$$

The inter-band correlation analysis has been performed and thereafter by fitting the Gaussian curve on the DCF function, a search has been done for the possible inter-band time delay. A positive lag means that the former variation leads the latter one. In order to look for a time lag between two variations at different wavelength bands, it is useful to find the DCF between the wavelength bands whose wavelength value is not very near to each other, because of larger the wavelength separation, the more easily the lag will be detected. Thus, B-R or B-I should be performed rather than B-V or V-R.

The discrete correlation function has been calculated between the different wavelength bands. The discrete Correlation function analysis has been performed.
on the wavelength bands B and R with binning of 1.5 hour as can be seen in fig 16. A time lag of $0.01 \pm 0.0056$ day ($< 2\sigma$) is found after fitting Gaussian to the DCF plot. It is a positive lag, thus variation in wavelength band B is leading the R band by $(0.01 \pm 0.0056 )$ days, but this lag is not statistically significant.

Fig 16. Discrete correlation function between B band and R band for BL Lacertae binned every 1.5 hours. The green line is the symmetric Gaussian curve is drawn on the DCF points in the plot and giving the time lag of $0.01 \pm 0.0056$ day.
Chapter 5: Interpretation

Discussion of Color Behavior

BL Lacertae showed Redder when Brighter (RWB) chromatism during the October run on nights 1, 4, 6, 7 and on December night. By looking at the fig. 14, as the value for color-index is increasing, the value for the R band magnitude is decreasing for the data of nights 1, 4, 6 and 7 which is showing the color Redder when the BL Lacertae is brighter. Also, the correlation coefficient on these nights is weakly negative, i.e. -0.587; which means the slope is negative on the plot. On the other hand, BL Lacertae is showing the Blue chromatism when even brighter, i.e. on nights 2, 3 and 5 in October and 1and 3 in May, as the value for color index is decreasing with the magnitude of R band is increasing as discussed in Chapter 4, Section 4.2.

In the past, the redder when brighter trend has been observed in Flat Spectrum Radio Quasar 3C454.3 and the gamma ray quasar PKS 0528+134. One possible explanation for this color change could be UV excess from the accretion disk due to the thermal emission. The bump in the SED after turnover to high frequency is called Big Blue Bump (BBB). As can be seen in the fig 17, the power law becomes harder and harder as going towards the lower frequencies. On the other side if moving towards the higher frequencies, at the point when the accretion disk comes into play, the power law becomes negligible. Accretion disk luminosity begins to dominate the flux in the SED at larger frequencies and this explains why the color changes to blue when the accretion disk luminosity dominates.
Fig17. SED showing the domination of accretion disk and the power law slope is becoming harder as going towards the synchrotron peak or lower frequencies.

**Discussion of Time Lag**

The variation in time is found by observations at multi wavelength. After calculating the discrete correlation function between variations at wavelength band B and R, a lag of $(0.01\pm0.0056)$ day has been found. This observation can be used to discuss the physical processes in relativistic jets, like acceleration of electrons and the strength of the magnetic field.
This rapid variability of emission is implying that observed flux is Doppler-boosted as a result of relativistic motion of the radiating material. The coincidence of the optical flare and the Fermi flare at the same time in May suggests that both have arisen from the same population of electrons. The flare in optical and at Fermi energy level is comparable, assuming the magnetic field and the seed photons are relatively steady and the variability is due to the variation in the highest energy end of a single relativistic particle population. As discussed before, the loss in energy is directly proportional to $\gamma^2$; for larger $\gamma$, the amplitude of the light curve declines. The observation of time lag between the B band and R band is not significant. So, this will allow estimating a limit of the magnetic field.

According to the equation discussed in Chapter 2 under section 2.3 Leptonic model, the expression for magnetic field is:

$$B = 241 (\Delta t_{\text{obs}}^{\text{opt}})^{-2/3} \delta_1^{-1/3} (1 + k)^{-2/3} \text{ G.}$$

Here $\Delta t_{\text{obs}}^{\text{opt}}$ is $(0.01 \pm 0.0056) \text{ day or } (864 \pm 483.8) \text{ seconds}$ and considering $k = 1$, the lower limit on the magnetic field becomes:

$$B \geq 1.24 \delta_1^{-1/3} \text{ G.}$$

Once the Doppler factor is known, the magnetic field can be calculated.

The magnetic field that usually considers to model blazar SEDs is $\sim 0.1 - 1 \text{ G}$ for BL Lac objects and $\sim 1 - a \text{ few G}$ for Flat Spectrum Quasars. The $\delta$ is considered to be typically between 10 - 20, thus, the lower limit on the magnetic field can be estimated.
Chapter 6: Summary

The purpose of this research was to discuss the spectral variability of BL Lacertae and also look for the time-lag between variations at different optical wavelengths. The optical observations were taken at the MDM observatory, Kitt-Peak, AZ. In addition to the optical data, the Fermi data was taken from the Fermi LAT website. After all the data reduction procedure for both Optical and Fermi data, a flare during the month of May was seen in optical as well as gamma-ray data, which indicates that the electron population that is responsible for the synchrotron emission is same that will initiate the Compton effect.

When the reduced data was analyzed, the color behavior of BL Lacertae is found to be Redder when the magnitude of wavelength band R > 13.5 and Bluer when R< 13.5. Also, using the Discrete Correlation Function analysis, a time-lag between the variations at wavelength band B and R was found to be (0.01 ± 0.0056 ) day, which is not statistically significant, i.e. < 2σ. However, using the value of time-lag, the lower limit on the magnetic field in the jet was calculated to be $B \geq 1.24 \delta_1^{-1/3} \text{ G}$. This limit on the magnetic field is consistent with the limit of magnetic field that consider for modeling BL Lacertae Spectral Energy Distribution
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