Teacher Knowledge That Supports Student Processes in Learning Mathematics:

A Study at All-Female Middle Schools in Saudi Arabia

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This dissertation titled
Teacher Knowledge That Supports Student Processes in Learning Mathematics: A Study at All-Female Middle Schools in Saudi Arabia

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Abstract

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Teacher Knowledge That Supports Student Processes in Learning Mathematics: A Study at All-Female Middle Schools in Saudi Arabia (206 pp.)

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Teachers in Saudi Arabia are attempting to advance their teaching in mathematics to address specific reforms by the Ministry of Education. Saudi teachers must improve their students’ thinking through engagement in problem solving. This qualitative study investigated how teachers use knowledge of student mathematical learning and how they promote students’ experiences of various mathematical processes. The research investigated the process standards of the National Council of Teachers of Mathematics (2000) that teachers should encourage students to experience during instruction.

The study gathered data from 12 teachers at female-only middle schools in a city in eastern Saudi Arabia. The investigation used, in sequence, three methods: classroom observations, initial interviews, and scenario-based interviews. The observations used Instructional Quality Assessment rubrics (Boston & Wolf, 2006) and field notes. The initial interviews linked to the observations and asked teachers about their instructional philosophy and how they supported student learning processes. Delving deeper, the scenario-based interviews helped the researcher to understand how the teachers might respond to novel mathematics teaching situations.
Most of the findings related to the process of problem solving. The participants overtly guided students while solving problems. Teachers perceive that discovery problem solving can be achieved through gradual hints and reminders to the students. Saudi teachers appear to understand the power of various features of problem solving such as discovery, high-level tasks, and encouraging multiple solutions. Further, the study revealed that teachers encourage students’ verbal and written expression. The participants believed that their students can be taught about mathematical connections in a declarative way.

The observed and indicated problem-solving approaches lack many of the necessary features cited in the literature on promoting students’ mathematical processes and supporting students’ understanding of mathematics. The teachers’ declarative approaches for helping students form mathematical connections do not align with practices cited in research literature and reform documents, which recommend engaging students in mathematical activities that allow them to think of and experience mathematical connections.

An important interpretation of the findings is that teachers in Saudi Arabia tend to avoid students’ struggling while solving complex problems even if such struggle could deepen their mathematical thinking and learning.

Approved: _____________________________________________________________

Gregory D. Foley

Robert L. Morton Professor of Mathematics Education
To the ones who never gave up waiting for this day to happen

To my parents

Saad Alsaeed and Fuzia Alsaeed

To the one who sacrifices his comfortable life to make this work possible

To my beloved husband

Abdullah Alodail
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Chapter 1: Introduction

Overview

The government in Saudi Arabia has given education the highest priority for development. More than a quarter of the total economic budget has been designated for education and vocational training for the year of 2012. This priority given to education aligns with the Saudi Arabia’s long-term development plan that will lead to a knowledge-based economy by 2022 (Ministry of Education in Saudi Arabia, 2010).

The current development plans in Saudi Arabia to transform toward a knowledge-based society mainly will be achieved through advancing the regional education system. Although education in Saudi Arabia has been receiving more government support than any other sector, education continues to attract much criticism and calls for improvement. One of these criticisms is that Saudi education does not prepare citizens for job skills: “The country needs educated young Saudis with marketable skills and a capacity for innovation and entrepreneurship. That’s not generally what Saudi Arabia’s educational system delivers, steeped as it is in rote learning” (Lindsey, 2010, para. 2). Mathematics education is one of the areas that consistently has been attached with continued calls for improvement. Evidence has repeatedly indicated the unsatisfactory mathematical competency of Saudi students. These criticisms of mathematics education obligated the Ministry of Education to undertake immediate efforts to improve the standard mathematics curricula and school programs.

One call for developing mathematics education followed results of the Trends in International Mathematics and Science Study (TIMSS). Students in Saudi Arabia have
not performed competitively in mathematics compared to international standards. The 2007 TIMSS reported that Saudi students were at the bottom of the scoring list; their low mathematics performance placed them in an unsatisfactory position. The achievement average of Saudi Arabia is significantly less than the TIMSS scale average (Martin, Mullis, & Foy, 2008). This relatively low performance suggested the need for improved planning for teaching and learning mathematics in Saudi Arabia.

The recommendation resulting from the 2007 TIMSS report was that students in Saudi Arabia should develop the strategic competence and adaptive reasoning to allow them to solve nonroutine problems, such as the ones being assessed on the TIMSS tests. Another recommendation was that students should experience mathematics problem-solving skills that can be transferred to new situations and problems (Alnatheer, 2009). Traditionally, students in Saudi Arabia have received direct instruction from teachers that encourage them to mimic problem-solving procedures with limited concerns for their creative thinking and inventing of solutions. Alnatheer recommended that teachers should emphasize helping students to engage in a variety of mathematical processes. Students should be required to participate in problem solving, to integrate multiple mathematical representations, and to experience how mathematics concepts are connected with their real lives. Such mathematical activities would be a way to support students in developing a variety of problem-solving strategies that would help them to work through and adapt to new mathematical situations, such as those being assessed through international tests.

As a response to major educational criticism regarding the placement of students’ mathematical proficiencies, the Ministry of Education in Saudi Arabia has undertaken
major reform efforts to improve mathematics education and to integrate and address recent reform recommendations. The ministry is undertaking an overall advancement program to reform mathematics and science education. This program will target mathematics school curricula as well as the improvement of teaching (Ministry of Education in Saudi Arabia, 2010). Complete descriptions of these reform plans will be discussed in Chapter 3.

The objective of the current study was to understand the teachers’ knowledge of students’ mathematical learning and how teachers engage students in a variety of mathematical processes during instruction. It should be noted that the role of the teachers is critical in addressing reform requirements that are being undertaken by the Ministry of Education in Saudi Arabia. In addition, achieving the goal of giving Saudi students the opportunity to experience problem-solving strategies and other mathematical processes requires the teacher to orchestrate the classroom in a way that helps students to experience mathematical thinking and investigate problem solving (National Council of Teachers of Mathematics [NCTM], 2007). Teachers’ responsibilities cannot be overlooked in addressing and supporting the learning processes. This study is an initial step toward understanding the level of knowledge and skills teachers have in order to develop and engage the students in the mathematical processes.

The definition and role of mathematical processes in mathematics instruction and the framework used for teacher knowledge will be discussed in the following sections drawing on literature from the United States. From an international perspective the United States, one of the largest leading countries in technology and industry, has also
confronted serious issues with the current quality of mathematics education among their students (Zhao, 2009). In addition, the unsatisfactory mathematics results from international comparison tests, such as TIMSS and Program for International Student Assessment (PISA), indicated the limited competence of the students of the United States (Hiebert & Stigler, 2000; National Mathematics Advisory Panel, 2008). As such, the United States system of mathematics education is facing similar patterns as Saudi Arabia yet different challenges regarding improving students’ mathematical proficiency. The following section will discuss the role of the mathematical processes in the classroom using literature from the U.S.

**Learning Processes in Mathematics Education**

The focus on students’ learning processes in mathematics in the United States was one of the themes during the “New Math” movement that started in the mid-1950s (Fey & Graeber, 2003). From the mid-1950s to the publication of the NCTM’s document *An Agenda for Action* (1980), many political and national events caused a demanding movement to reform mathematics education in the United States. Initially, this movement was a reaction to the threats of Soviet technological advancement compared to that of the United States (Fey & Graeber, 2003).

Mathematics education reform recommendations during the New Math movement reflected new insights into mathematics pedagogy. At that time, innovative pedagogical ideas that arose were inspired by educational psychologists who advocated mathematics reform movements with variety of new contents. For example, the work of psychologists called for the constructivist approach, optimal use of students’ cognitive processes,
student engagement and learning discovery, which stemmed from psychological ideas such as those of Piaget and Bruner. There was great attention on the role of mathematical problem solving approach and its cognitive processes instead of focusing on a specific product of the problem (Bruner, 1960). The product of problem solving in this case refers to the emphasis on particular answers or on achieving specific goals, whereas the cognitive processes refer to thinking, reasoning and other the strategies the student employs in order to achieve these goals.

During the New Math era, innovative proposals for reforming mathematics education were guided by the “leading voice” of the famous psychologist Jerome Bruner (Fey & Graeber, 2003, p. 525). Bruner’s work (1960) turned educators’ attention to the learning processes students use in an instructional environment. The power of process education is that it promotes transfer of learning. Its essence is helping students adapt to new mathematical situations by acquiring problem-solving skills. Bruner indicated that engaging students in active learning processes and discovery should be a teacher’s educational objective, and this can be accomplished through active learning that promotes transfer of learning for new situations. Such active learning is key to “learning how to learn.” In transfer, current learning results in later performance being more active and more successful. Bruner advocated for education that values teaching with general ideas and conceptual structural understanding that are the essence for empowering transfer for later learning situations. Bruner (1960) explained the power of the relationship between the learning process and transfer:
It is interesting that around the turn of the last century the conception of the learning process as depicted by psychology gradually shifted away from an emphasis upon the production of general understanding to an emphasis on the acquisition of specific skills. The study of “transfer” provides the type case—the problem of the gain in mastery of other activities that one achieves from having mastered a particular learning task. (p. 5)

Bloom and Broder (1950) conducted a study that investigated whether instructional emphasis on the problem-solving process was a useful tool for transferring learning. The subjects of the study encountered different problem-solving strategies and processes such as thinking aloud and explaining their mathematical thought processes. One of the main goals of this classic study was to assess different strategies and habits of the problem-solving learning process; Bloom and Broder (1950) argue that “habits of problem solving, like other habits, could be altered by appropriate training and practice” (p. 67). The preliminary findings of this study indicated that students who engaged in instruction that stressed the problem-solving learning process outperformed the control group (who did not focus on problem-solving processes) in their ability to work through and achieve nonroutine mathematical tasks.

The NCTM’s publication of *An Agenda for Action: Recommendations for School Mathematics of the 1980s* (1980) was influenced by the work of psychologists and educators in the 1960s, especially with regard to the emphasis on the process of problem solving in mathematics learning. This agenda for action established reform recommendations for educators to enhance the learning of mathematics and assure
mathematical competence among students. Such position statements provided broad base for future reforms in mathematics education in the United States and beyond. This vision raised the goal of incorporating the learning process in all parts of mathematics instruction to a higher level. The agenda also established recommendations for enacting the learning process. It advocated for evaluating the learning process in instruction and considered the learning process a necessary aspect of curriculum development. The following statements from the NCTM (1980) demonstrate these recommendations:

The evaluation of the use of problem-solving processes must be given special attention by schools, teachers, researchers, test developers, and teachers educators. (p. 14)

And further,

The curriculum that stresses problem solving must pay special heed to the developmental sequence best suited to achieving process goals, not just content goals. (p. 21)

The mathematical educational reform era initiated by the publication of the NCTM’s Curriculum and Evaluation Standards (1989) and Principles and Standards for School Mathematics (2000) provides a definition for addressing the learning processes in mathematics instruction. According to the NCTM (2000), the process standards of communication, reasoning and proof, problem solving, representation, and connections are not meant to be isolated from the content standards; instead, they should be enacted through the content (p. 29). One of the main goals of the reform efforts of the NCTM publication is heavy emphasis on problem solving and the power of engaging students in
learning processes. Through the standards discussed in both publications, NCTM supports teacher acknowledgment of the processes that students utilize, from making errors or making connections, to explaining and justifying their answers, to solving the problem.

Current trends in mathematical education continue to confirm the benefits of problem-solving processes of learning over focusing on a specific product or solution. For example, the standards in the NCTM’s (2007) *Mathematics Teaching Today* still support the role of teachers in engaging students through the learning mathematical processes. Teachers should have adequate knowledge to assess students’ unusual solutions and to guide their mathematical thinking. The NCTM (2007) advocates for teaching methods that respect students’ thinking. As the following passage discusses, considering students’ learning processes requires teachers to be active listeners who allow for classroom discussion, a variety of unexpected solutions, and creativity:

Although students should engage in a variety of mathematical processes throughout school, those processes may look very different in early elementary grades than they do in high school. . . . Likewise, to effectively orchestrate a class discussion during which students share a variety of solution strategies, teachers must be aware of the common strategies used to solve problems as well as the distinctive elements and connections among them. (NCTM, 2007, p. 26)
Pedagogical Knowledge for Students and Mathematics and the Role of Teachers in Attending to the Processes of Learning

As mentioned earlier, to support students’ mathematical processes in the classroom, teachers need a special kind of knowledge that helps them engage their students in mathematical learning processes. The teachers’ pedagogical content knowledge, first proposed by Shulman (1986), could be the initial step in understanding and conceptualizing the domain of teachers’ knowledge in the areas that are directly related to the students’ learning processes. Such knowledge is meant to guide teachers in understanding students’ learning and thinking processes, which may include: errors, uncommon solutions, or verbal explanations.

Shulman’s (1986) discussion of pedagogical content knowledge addresses the importance of teaching with respect to students’ knowledge. Shulman emphasized that teachers should have pedagogical content knowledge in order to provide effective instruction. With respect to mathematics, such knowledge would go beyond what mathematics specialists know; mathematics teachers must also be able to recognize how they can support students’ conceptual understanding. Pedagogical content knowledge can support students in improving both their thinking and their lifelong learning. Shulman stated, “pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conception and the preconceptions that students of different ages and backgrounds bring with them to the learning of the most taught topics and lessons” (1986, p. 9). This comment suggests that it is not only
important for teachers to teach the content, but also to consider the life experiences students bring into the class with them when they plan and deliver their instruction.

Based on Shulman’s theory of pedagogical content knowledge, Deborah Ball and colleagues have conducted systematic research studies to explore *pedagogical mathematical knowledge* that can be used to teach mathematics. Building on Shulman’s work, their proposed framework includes knowledge of content and students as a part of the knowledge domain for pedagogical mathematical knowledge. According to Ball, Thames, and Phelps (2008), teaching knowledge includes two subdomains: pedagogical mathematical knowledge and mathematical knowledge. Pedagogical mathematical knowledge includes three categories: knowledge of content and students, knowledge of content and curriculum, and knowledge of content and teaching. Hill, Ball, and Shilling (2008) defined the teachers’ knowledge of content and students as “teachers’ knowledge of students’ mathematical thinking” (p. 373). Their work categorizes knowledge of content and students into four major groups:

- Knowledge about common errors that students can make in each mathematical concept.
- Understanding how students understand concepts such as realizing what part of a solution indicates students’ understanding.
- Knowledge about the students’ current cognitive development such as in each grade level what part of knowledge students can understand.
- Understanding of and familiarity with students’ usual solutions. (p. 381)
As a part of pedagogical mathematical knowledge, knowledge for content and students would be appropriate for measuring and evaluating teachers’ proficiency in attending to the students’ learning processes for mathematics when students are actively engaged in mathematical thinking.

In accordance to the NCTM’s 2007 teaching standards, “teachers’ knowledge of students' mathematical learning” is the appropriate set of standards that can capture teachers’ abilities to support and address students’ mathematical learning processes. Teachers’ knowledge of students' mathematical learning corresponds to knowledge for content and students as defined by Hill et al. (2008). Although NCTM uses different terminology, as shown through their standards, they conceptualize knowledge of student mathematical thinking similarly to Hill et al. definition of the knowledge of content and students. Further, the NCTM (2007)’s standards of teachers’ knowledge of student mathematical learning obligate teachers to be proficient in engaging students in mathematics processes. The NCTM (2007) standards for teachers’ knowledge of student mathematical learning are as follows:

Teachers of mathematics must know and recognize the importance of–

- what is known about the ways students learn mathematics;
- methods of supporting students as they struggle to make sense of mathematical concepts and procedures;
- ways to help students build on informal mathematical understandings;
- a variety of tools for use in mathematical investigation and the benefits and limitations of those tools; and
• ways to stimulate engagement and guide the exploration of the mathematical processes of problem solving, reasoning, and proof, communication, connections, and representations. (p. 25)

Carpenter, Fennema, Peterson, and colleagues conducted a series of studies that explored teachers’ knowledge of students’ thinking and problem-solving learning processes. In one of these studies, Carpenter, Fennema, Peterson, Loef, and Chaing (1989) examined how providing teachers with knowledge about differences among mathematical problems, students’ strategies of solving problems, and students’ thinking processes can influence instruction. Participants in the research who were part of the experimental group attended a professional development program called Cognitively Guided Instruction (CGI) that gave them access to research about students’ thinking and development. On the other hand, the control group did not attend any such workshop. The study showed that the program enhanced experimental group participants’ beliefs about teaching; specifically, that these teachers facilitated students’ involvement in the process of problem solving, promoted students’ ability to verbally express their learning processes, and encouraged them to use multiple ways to problem solve. Peterson, Fennema, Carpenter, and Loef (1989) found a significant correlation between teachers’ knowledge of students’ problem-solving processes and teachers’ improved attitudes about how students learn mathematics. Teachers of students who achieved well in addition and subtraction word problems tended to practice instruction that value students’ thinking and their preexisting knowledge. Another study further suggested that there is a significant correlation between teachers’ knowledge of students’ thinking and learning
processes, and the students’ achievement (Carpenter, Fennema, Peterson, & Carey, 1988).

**Statement of the Problem**

For students to achieve mathematical understanding, classroom instruction that provide students the opportunity to engage in various mathematical processes should be incorporated into lessons (Stein, Grover, & Henningsen, 1996). Although this is an area that has already generated much discussion, it is still addressed in the NCTM’s most recent teaching standards. Teachers need to have adequate knowledge to engage students in mathematical processes by properly assessing students’ errors and guiding their multiple solutions (Hill, Ball, & Shilling, 2008). Teachers need to have the knowledge for listening to students’ thought processes and their solutions. Most importantly, teachers require the necessary knowledge to engage students in the process of high cognitive thinking and problem solving (NCTM, 2007; Schoenfeld & Kilpatrick, 2008).

Knowledge of student mathematical learning, discussed by NCTM (2007), is a suitable framework for addressing and evaluating teachers’ proficiency that helps them engage students’ processes. This framework was implemented throughout this research to help define the knowledge domain “teachers’ knowledge of student mathematical learning”.

Research studies have shown that pedagogical mathematics knowledge plays an important role in predicting students’ achievement (Hill, Rowan, & Ball, 2005). In addition, many dimensions, previously discussed, of teachers’ pedagogical mathematical knowledge have been identified and explored. This study focused on a specific dimension of pedagogical mathematical knowledge that deals with teachers’ understanding of
students’ intuitive thinking and their engagement in mathematical processes. The NCTM (2007) framework indicates that for teachers to have knowledge about students’ mathematics learning they should be proficient about how “to stimulate engagement and guide the exploration of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representations” (p. 25).

**Research Questions**

The following research questions were used to examine Saudi teachers’ knowledge of supporting students’ engagement in mathematics processes. The first two questions addressed teachers’ knowledge and understanding about students’ mathematical learning; more specifically about to what extent the teachers encourage the students’ involvement in the processes of problem solving, reasoning and proof, representation, communication, and connections. Question 1 in particular aimed at understanding the Saudi teachers’ knowledge of student mathematical learning as they practice in the classroom. The last question was aimed at a general understanding of the teachers’ knowledge of student mathematical learning.

1. To what extent do Saudi middle school teachers exhibit their knowledge of student mathematical learning as they engage students in practicing the mathematical processes of communication, reasoning, and proof, problem solving, representation, and connections during instruction?

2. What is the Saudi middle school teachers’ knowledge of student mathematical learning regarding how to engage and guide students in the mathematical
processes of communication, reasoning and proof, problem solving, representation, and connections?

3. What is the Saudi middle school teachers’ knowledge of student mathematical learning?

**Significance of the Study**

Many research studies have been conducted to understand teachers’ pedagogical mathematical knowledge and to investigate their values on students’ achievements. An extensive body of literature discusses how to engage students in the mathematical learning process, including many studies by learning theorists and cognitive psychologists. However, a limited number of studies explore teachers’ knowledge of student mathematical learning and how they use such knowledge to support students’ engagement in mathematical processes such as problem solving, reasoning and representations. The teachers’ knowledge of students’ intuitive thinking can be a useful foundation for teachers to reform their philosophy by allowing instruction that gives students the opportunity to be active and creative in the classroom.

In Saudi Arabia, this study will be significant in contributing to the understanding of Saudi teachers’ knowledge of students’ learning. The research will explore how teachers in Saudi Arabia understand the processes of students’ cognitive thinking, along with their classroom discussion and reasoning about the mathematics they experience through instruction. In Saudi Arabia, the Ministry of Education is undertaking reform strategies to enhance mathematics education and help students meet international standards of mathematics. These efforts will be discussed in more length in Chapter
Three. Such reform in the mathematics education system in Saudi Arabia provokes high expectations of teachers to enhance the mathematical literacy among students and prepare them for international and national assessment. Teachers in Saudi Arabia need to help students develop problem solving abilities so they can cope with new mathematical situations and problems, such as those presented in the TIMSS. As such, one of the goals of this study will be to support these reform efforts by exploring the Saudi teachers’ knowledge of student mathematical learning.

**Middle Childhood Education System in Saudi Arabia**

This study spotlighted on middle school teachers’ knowledge of student mathematical learning. For the reader not familiar with the education system in Saudi Arabia, I provide a brief overview of the country’s middle childhood education system to clarify the sampling population of the study.

The education system in Saudi Arabia is centralized. The curriculum, syllabus, and education standards are unified across Saudi Arabia schools. The Ministry of Education is authorized to create and develop education resources in Saudi Arabia. Further, the schools system is under gender segregation. All the facilities, teachers and administrators in the female schools are women. The education system in Saudi Arabia offers 6 years of elementary education, 3 years of middle school education, and 3 years of secondary school education.

In Saudi school, middle childhood education officially crosses over 7, 8, and 9 grades as compared to the United States K–12 school system. Mathematics is a main subject in middle school where students study mathematics for 45 min, five times a week.
In addition, students in mathematics are required to attend summative assessment, which is a compulsory requirement in middle school education. Each year, especially in mathematics, students are required to pass two examinations in order to acquire the middle school certificate and to verify their eligibility to move to high school.

Teachers in Saudi Arabia are required to obtain a minimum four year bachelor’s degree to be eligible to teach in middle school. In addition, teachers in Saudi Arabia are required to obtain a minimum score in a competency test that is designed for teachers before they are accepted to teach in schools. The Ministry of Education legislations does not require teachers in Saudi Arabia to obtain or update a teaching certificate. The only requirement for teaching is a bachelor’s degree in mathematics education and acquiring a specific score in the competence test.

**Limitation and Delimitation**

The research study focuses on investigating teachers’ knowledge of student mathematical learning, specifically that which promotes students’ mathematics processes. This study is aimed at sampling from schools at Alahsa, a small city in Saudi Arabia that is located in the eastern part of the country. The study did not attempt to sample from other regions in Saudi Arabia. Also, the study did not attempt to study teachers other than middle school teachers. The sampling pool provides delimitations of the study.

Another delimitation that was considered in the study is related to the number of participants. The study conducted a qualitative approach to investigate the research questions. The qualitative approach requires in-depth investigation of each subject in the study. As such, the researcher planned to have a small number of participants.
An important limitation of the study is related to gender choice. Due to the Saudi legislation to have a gender-segregated school system, I was not able to include male teachers in my sample. Having mixed genders in the study could have added to the quality of its findings.

Research biases can be included as a source of limitations in the study. This is because the researcher has been exposed to mathematics education experiences that might be different than the study subjects. The researcher’s experiences can influence choices of what to consider as effective teaching aspects in mathematics compared to the ineffective ones. The discussion of the researcher biases will be presented in Chapter 3.

**Definition of Terms**

**Knowledge of student mathematical learning.** Throughout the study, this knowledge domain is defined according to the NCTM (2007) framework.

**Mathematics processes.** The term “processes of mathematics” implies various connotations that all aim to address students’ thinking and the stages of their meaningful work that can contribute to either complete or incomplete mathematical understanding (Stein, Grover, & Henningsen, 1996). It can show that students are practicing certain activities such as making conjectures, looking for patterns, connecting ideas, abstracting, inventing solutions, or explaining thinking (Schoenfeld, 1992). To limit these activities into certain specific and meaningful mathematical practices, this study defined mathematical processes as referred to by the NCTM’s process standards of problem solving, reasoning and proof, communication, connections, and representations.
Summary

This study explored middle school teaching in Saudi Arabia. The main goal of the study was to examine the Saudi teacher’s knowledge of student mathematical learning as defined by the NCTM (2007) teaching standards. This construct of knowledge was used because it captured how teachers have the proficiency to attend to students’ processes of thinking and learning during classroom instruction. The study mainly focused on the knowledge of students’ learning that supported teachers to address mathematics processes in instruction. The mathematics processes are defined as the processes of problem solving, reasoning and proof, communication, and connections (NCTM, 2000).
Chapter 2: Review of Literature

Overview

This chapter frames the proposed study within related literature. As discussed previously, the aim of this study was to learn about Saudi middle school teachers’ knowledge of student mathematical learning especially regarding how to support them and guide their engagement in the mathematical processes. Accordingly, to provide clear background on the area related to the study, three areas of research are extensively reviewed. The first section analyzes literature about the nature of classroom instruction and the characteristics that foster students’ mathematical learning. Through the second section of the chapter, discussion about the importance of mathematical processes is examined. The last section further explores research studies on the area of teachers’ knowledge of student mathematical learning.

It was discussed in Chapter 1 that the study investigated middle school teachers in Saudi Arabia. As such, the researcher attempted through the study’s review to provide literature that is applicable for middle school education. Most of the reviewed literature is grade level independent and discusses issues in mathematics education for the general school K–12 of mathematics. A portion of literature used investigates research aspects in middle school education. Further, minimum research was used that explores mathematics education issues that came from elementary and high school.

Characteristics of Mathematical Instructional Practices That Foster Learning

The following section addresses the nature of mathematics classroom instructional practices, and the imperative role of examining such teaching practices to
recognize the extent of students’ mathematical learning. Further, specific examination of
students’ mathematical thinking and understanding will be examined in a separate portion
of this section as one of the major reform requirements for effective classroom
instructional practices (Carpenter & Leher, 1999; Franke, Fennema, & Carpenter, 1997).
Another section will examine literature on the role of students’ struggle while learning
mathematics as a key characteristic of instruction that can foster learning.

The classroom learning environment is mainly discussed via research to highlight
instructional practices that facilitate students’ learning and foster their engagement in the
mathematical process. In evaluating everyday teaching practices, Franke, Fennema, and
Carpenter (1997) characterize three dimensions of the classroom instruction environment
that must be present to conclude that the instructional environment supports the students’
learning process. Classroom instruction must:

- Provide students opportunities to use multiple and nonalgorithmic methods to
  solve a problem
- Ensure ample understanding when listening to students’ reasoning during
  classroom discussion
- Use students’ thinking to shape classroom practice.

The first two dimensions in particular can be used when informing researchers about
students’ roles in the classroom and the extent of their involvement during the thinking
and learning process. The National Research Council (NRC) defines such “opportunities
to learn” as “circumstances that allow students to engage in and spend time on academic
tasks such as working on problems, exploring situations, and gathering data, listening to
explanations, reading text or conjecturing and justifying” (2001, pp. 333–334). According to Franke, Fennema, and Carpenter, providing students the opportunity to learn mathematics allows them to use their knowledge and efforts and thinking in order to listen to their peers’ ideas during problem solving, thus students become better able to engage in active learning and thinking. The following studies highlight the importance of students’ learning opportunities in mathematics instructional practices as discussed by Franke, Fennema, and Carpenter (1997).

Providing students opportunities to engage in meaningful thinking in the instruction is a key variable that was addressed by research in mathematics education. The nature of classroom instruction practiced by teachers significantly affected how students experience the learning of mathematics. Hiebert and Grouws (2007) extensive literature review establishes such hypotheses. There are many variables that can mediate such connections between classroom practice and students’ achievement, such as teaching beliefs, interests, culture, practices, as well as choice of and implementation of academic tasks. After recognizing a need for a more valuable factor, however, the authors cited “opportunities to learn” as better able to explain this connection between classroom and student achievement. This key factor describes the extent to which teachers are providing students the opportunity to engage in mathematical learning. As stated previously, the NRC indicated that an “opportunity to learn” is one of the best predictors of students’ learning (p. 333).

Hiebert and Grouws use such terminology to refer to classroom instruction that facilitates circumstances for students that value their thinking, abilities and previous
knowledge. This facilitating classroom environment relies on the appropriateness of the presentation of mathematical topics to particular groups of students with consideration to their previous knowledge and possibility of students’ active engagement. The “opportunity to learn” is not similar to the opportunity to expose students to knowledge of mathematics, but rather acknowledges such knowledge as useful, interesting and built upon students’ previously learned concepts to conclude that students have sufficient learning opportunities.

Researchers further established the connection between students’ achievement and the nature of classroom instruction that provides students with the opportunity to engage in thinking and problem solving with the ability to reason about their thinking. The results of the study conducted by the Center for Study of Mathematics Curriculum (CSMC) contribute to the understanding that classroom practice that values students’ learning process is linked with student achievement. The researchers investigated the impact of curriculum type (conventional vs. standards-based curriculum), fidelity of curriculum implementation, and the nature of learning environments. They also examined the achievement of 2,533 students from 10 middle schools. The study findings reveal that students’ achievement is not predicted solely by the type of curriculum used nor by the fidelity of the implementation. Nonetheless, the findings of the study suggest that learning environments that support classroom practice provide students with the power to pose questions, discuss their solutions, predict and engage in problem solving, all of which are factors that strongly contribute to students’ learning (Tarr et al., 2008). Thus, the reviewed study demonstrated that the classroom environment and instruction
that encouraged students to engage in the mathematical processes through “opportunities to learn” positively related to student achievement.

The research group of the National Mathematics Advisory Panel was given a task to review data about the mathematical instruction practice. The report of the panel was presented in the publication The Final Report of the National Mathematics Advisory Panel (2008) which classified two extreme instructional practices specific to student-centered and teacher-directed instruction. The panel emphasized the danger of putting forth such classifications that do not really exist without necessary high-quality research. The panel indicated that during the past 25 years of mathematics education research, effective instructional practice was more associated with cooperative learning to student-centered approaches that give students the opportunity to participate with peers in a complex learning environment. The panel stated:

Instructional practice should be informed by high-quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers. High-quality research does not support the contention that instruction should be either entirely “student-centered” or “teacher-directed.” Research indicates that some forms of particular instructional practices can have a positive impact under specified conditions. (2008, p. 11)

In summary, the evaluation of classroom norms and everyday teaching practices is important that to draw understanding and connection between teaching to learning of mathematics. Stigler and Hiebert (1999) have elaborated in their book The Teaching Gap, the critical effectiveness of teachers’ everyday practices in the classroom is defined and
shaped by the effectiveness of their daily teaching methods, thus making classroom norms an important variable to evaluate. Yackel and Cobb (1996) indicated that classroom environment that support students’ opportunity to learn by helping them to engage in inquiry form of instruction that value students’ thinking, students can develop since of positive beliefs and autonomy in learning mathematics.

**Current expectations about students’ learning, thinking and understanding.**

In Chapter 1, the fact that the NCTM’s *Standards* require expectations for educational programs to help students be accountable for their learning by involving them in the mathematical process was discussed. In addition, this accountability of students over their learning is presented through various research topics and trends. Students are required to be mathematically competent and are asked to experience different kinds of roles in the classroom, as well as to learn new types of mathematical knowledge (Ball, Lubienski, & Mewborn, 2002; NCTM, 2009). The new knowledge creates greater demand for students to explore complex thinking as they experience non-routine problems.

In addition, the presentation of a technology workforce that increases focus on international assessments among nations requires more advanced expectations of students in their classrooms. Both middle school students, like high school students, should experience the same higher demands of knowledge and skills to prepare both groups for future career challenges (ACT, 2008). The ACT research report *The Forgotten Middle Ensuring That All Students Are on Target for College and Career Readiness Before High School* (2008) provides evidence regarding the critical role of middle schooling and how eighth graders’ accomplishments can influence their readiness for college-level education
and their future career. The following quote stated by ACT (2008) serves to demonstrate why the higher demands and expectations must be required of middle school students:

The process of preparing students to make successful transitions from middle school to high school is just as important as the process of preparing them to make successful transitions from high school to postsecondary education. Obstacles to college and career readiness must be met head on. A challenge for educators is to integrate activities into the curriculum that promote behaviors that enhance college and career readiness, such as academic discipline. . . . The earlier that students develop these behaviors, the more likely that the behaviors will become habitual and the more likely that students will be ready for college and career by the end of high school. (p. 40)

It can thus be understood that mathematics instruction should reflect these demands among middle school students. Among the higher demands for students in the school of mathematics and to specify the promising characteristics of mathematics instructional practices that foster student’s learning, this section will address characteristics which specifically focus on students’ comprehensive abilities. There is no doubt that mathematics classroom norms should encourage students’ thinking and their conceptual understanding as two substantial mentalities that the research has extensively highlighted (Carpenter & Leher, 1999; Wood, 1999; Yackel & Cobb, 1996). From the cognitive psychological point of view, students’ thinking is an imperative process that demonstrates students’ mathematical growth and construction of knowledge, which should then lead to greater achievement of mathematical understanding (Rasmussen,
Zandieh, King, & Teppo, 2005; Carpenter & Leher, 1999). The following section analyzes literature on the area of students’ mathematical thinking and learning from both current and classic research to provide insight to student roles in mathematical classrooms.

In a recent article, Harel and Sowder (2005) cited the differences between two types of mathematical knowledge involving methods of mathematical thinking and methods of mathematical understanding or comprehension. The authors articulate that mathematical understanding can be asserted within three categories:

- The particular meaning/interpretation a person gives to a concept, relationship between concept, assertions, or problems.
- The particular solution a person provides to a problem
- The particular evidence a person offers to establish or refute a mathematical assertion (Harel & Sowder, 2005, p. 30).

These categories indicated that one’s mathematical understanding can convey their ways of solving or dealing with particular mathematical activities, such as students’ mathematical reasoning and interpretations. On the other hand, Harel and Sowder indicate that students’ thought process can reveal how they best understand. Such methods of thinking can be expressed through at least three nonexclusive categories:

- **Beliefs.** Students’ beliefs about mathematics learning can influence their ways of thinking which in turn impact their mathematical understanding. Phrases such as “formal mathematics have little or nothing to do with real thinking or problem solving,” “only geniuses are capable of discovering or creating mathematics”
(Schoenfeld, 1985, p. 43) or “understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures” (Ambrose et al., 2003, p. 33) are examples that indicate student’s mathematical beliefs that can impact their ways’ of thinking.

- **Problem-solving approaches.** The ways in which students solve particular problems demonstrate their approaches of thinking. For example, comparing the problem to a simpler problem, examining particular cases, or looking for key words in the problem statement are some problem-solving approaches that students tend to use to illustrate their ways of thinking about problems.

- **Proof schemas.** Harel and Sowder (2005) also indicated that the processes of proving, such as informal justification or formal reasoning, reveal specific thinking methods that students demonstrate in the instruction. Discussion about students’ proof schemas will be presented in a later section of this chapter.

Responses from major mathematics education reform demand that greater emphasis is placed on students’ thinking and understanding, which was well researched by education psychologists such as Richard Skemp. Skemp’s work (1987) contributed significantly to the longstanding debate presented in mathematics education literature that distinguishes two types of understanding: conceptual understanding and procedural understanding (Hiebert & Carpenter, 1992). The ideas of relational and instrumental comprehension were also central to one of Skemp’s most cited articles regarding the concept of understanding (Kieran, 1994). The instrumental understanding of mathematics involves learning that requires memorizing certain procedures to be recalled whenever
the learner needs to apply them in solving mathematical problems. Memorization of certain procedures or rules in mathematics falls into the category of instrumental understanding. The other method of learning is mathematical relational understanding, which requires grasping of the conceptual structure and creation of a connection between concepts to be able to produce plans that can be used to reach a certain goal. Though relational understanding can be difficult to achieve and will often take a long time to understand, when achieved it will facilitate solving any related problem. In contrast, instrumental understanding is usually easier for students to grasp, but such understanding would be inadequate when applied to further adaptation of such learning to solve new tasks.

The NRC provide a general position toward the role of conceptual understanding by incorporating Skemp’s ideas of relational and instrumental understanding with other strands to complete *Mathematical Proficiency* framework that set expectations for student learning. Mathematical understanding is considered as a necessary aspect to achieve successful learning of mathematics (NRC, 2001). These strands are as following:

- **Conceptual understanding**—comprehension of mathematical concepts, operations, and relations.
- **Procedural fluency**—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic competence**—ability to formulate, represent, and solve mathematical problems
• *Adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification

• *Productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy. (p. 5)

Achieving students’ understanding and thinking implies that the classroom environment is able to help students engage in the mathematical process when problem solving, reasoning, justifying or inventing their mathematical solutions (Schoenfeld, 1992). Students’ thinking and their perceived understanding plays an imperative role in mathematics instruction. Teaching practices in mathematics should be directed toward support of students’ understanding and learning. The previous section also illustrates that the extent of learning opportunities that teachers grant their students within the learning environment is an important factor that works to highlight the effectiveness of teaching–practices in term of fostering student learning (NRC, 2001).

**Current expectations about students’ struggle in instruction while learning important mathematics.** To achieve students’ thinking and conceptual understanding, instruction should require students to struggle with mathematics ideas and problems that are challenging for them (Hiebert & Grouws, 2007). Hiebert and Grouws review extensive literature in mathematics education that advocates for students’ efforts and struggle while learning important mathematics ideas. They suggest that struggling with mathematics means that students are engaged in activities that are within the reach but yet have not been demonstrated before by their teachers. These activities in the lesson are a
considered as a “natural part of doing mathematics” as practiced by mathematicians in the discipline (Hiebert & Grouws, 2007, p. 388). Classroom environments that encourage struggling and making effort while learning have been found to increase students’ understanding. From a psychological point of view, students must be taught to accept challenges and to make productive mistakes, which will help them stay motivated for a future life full of difficulties. Dweck (2006) discusses two kinds of students’ mindsets—a fixed mindset or a growth mindset—that provide implications of mathematics educators and beyond, based on this classification. Students with a fixed mindset tend to belief that intelligence and being smart are the main reasons for success. These students think that effort and mistakes that come with their learning are threatening their level of intelligence because smart people should be able to solve any problem without noticeable effort. Students with a growth mindset, in contrast, can see that struggle is a positive indicator for their learning. Effort is appreciated and necessary because they think that learning occurs by seeking challenges and struggling with difficult academic tasks and that these are ways for them to get smarter.

Dweck (2008b) found in a collection of studies that when the two groups of students with the two types of mindsets enter seventh grade with the same achievement, students with a fixed mindset tend to become less motivated in learning and have less achievement in mathematics than their peers with a growth mindset. This collection of studies, based on Dweck’s (2006) work of understanding students’ mindsets, offers many important applications to teachers and teacher educators in mathematics. Dweck (2008a) indicates that students should be taught through their classroom to value hard work and
challenging classroom material and that their mistakes are of value to their achievements. Educators and teachers should supplement students with teaching that is based in developing growth mindset by valuing students’ mistakes and errors not just basing their achievement on their inherited intelligence learning. Praising students for their level of intelligence might shut them off when they encounter difficult tasks, which they could see as beyond their abilities. In contrast, praising for their hard work and effort can help them to learn to appreciate any difficult tasks they encounter (Dweck, 2008b).

Collectively, Dweck’s work addresses mathematics instruction that values students’ error and struggle while engaged in tasks and how this impacts their motivation to learn mathematics. Providing students with challenging tasks that encourage cognitive effort is a way to increase their conceptual understanding (Hiebert & Grouws, 2007; Skemp, 1987; Stein, Grover, & Henningsen, 1996).

**Research Studies on the Importance of the Mathematical Processes**

This section will address the power of engaging students during the mathematical processes as necessary to learning expectations. The NCTM indicated in the documented *Standards* why the process of problem solving, reasoning and proof, communication, connections, representations should be part of all mathematical contents and grade levels. In addition, various research studies indicated that students demonstrated interrelatedness among the coherent nature between each of the aforementioned process standards. The mathematical process standards as described by NCTM (2000) should be considered as a coherent whole in regards to mathematical activity. For example, the activity of problem solving can generate other sets of mathematical processes such as communication,
reasoning and comprehension of mathematics (Harel & Sowder, 2005). In addition, reasoning and proof are examined in research as interrelated processes that should be coupled with the process of mathematical communication (Yackel & Cobb, 1996; Stylianides, 2009). This section will review literature that addresses the mathematical processes and its relation to student learning. The discussion will begin with a presentation of literature detailing the process standard of problem solving since problem solving in classroom tend to generate and interrelate with other processes. The discussion will then continue to elaborate on other process standards. The reader of the following discussion will notice that each topic will relate to various other topics and processes. However, each topic of discussion will be presented in a separate section to further the understanding of each component of the mathematical processes.

**Problem solving.** In Chapter 1, an analysis of how problem solving has become a major theme for the mathematics reform was presented. Hiebert et al. (1996) have incorporated the important role of mathematical problem solving when reforming mathematics instruction and curriculum when stating “reform in curriculum and instruction should be based on allowing students to problematize the subject rather than mastering skills and applying them, students should be engaged in resolving problems… that elicit their curiosities and sense-making skills” (p. 12).

Hiebert et al. (1996) elaborated on the role of teachers to present and implement classroom materiel that encourages students’ problem solving. Additionally, students’ engagement in problem solving depends on the nature of academic work, as well as the students’ level of involvement during the instruction (Doyle, 1983). Consequently,
because of the imperative role of classroom task in the development of students’ problem solving, discussion about the process of problem solving in following paragraphs will be drawn from literature regarding how classroom academic work influences students’ level of understanding through active engagement when completing mathematical tasks.

Mathematical tasks or instructional tasks are important constructs that tend to relate to the effectiveness of teaching to encourage greater learning (Doyle, 1983; Hiebert & Wearne, 1993; Stein et al., 1996). Academic tasks should provide students with opportunities to experience high level thinking through problem solving while also helping them improve their conceptual understanding. To engage students in thinking and reasoning, such opportunities are “often more complex and more time consuming than routine activities” (Doerr & English, 2006, p. 9). However, not all instructional tasks provide an equal amount of benefits to students nor do instructional tasks equally engage students in the mathematical process. Doyle (1988) proposed that tasks differ in their ability to stimulate student learning at the cognitive level and varying tasks tend to encourage different levels of learning. The richness and complexity of instructional tasks introduce different levels of students learning.

Researcher’s growing interest in the cognitive level of academic tasks is built upon Doyle’s (1988) work in which he advocates for the use of instructional tasks that arm students with a deeper understanding of mathematics and allow them to think and reason. The use of academic tasks receives much attention from the literature because such tasks are cited as providing students with greater opportunities to experience the mathematical process. There has been a growing body of literature that seeks to connect
the association between cognitive tasks implemented in the mathematics classroom and
the nature of students’ engagement during the learning process.

Other research efforts related to academic tasks in teaching mathematics came
from the work of Stein and her colleagues when completing the QUASAR project.
Central to their work regarding a conceptualization of the domain of cognitive level tasks
is how often students have the opportunity to engage in the learning process when
recognizing or searching for patterns, thinking, reasoning and using their own non-
algorithmic solutions. According to the Instructional Quality Assessment rubric for the
implementation of academic tasks (IQA), for a task to be scored as high-level cognitively
demanding, it must meet some of the following components:

- Solving mathematical problems through use of complex non-algorithmic thinking
- Applying procedures with connections
- Students may have solved a genuine, challenging problem for which students’
  reasoning is evident in their work in the task
- Students may have made conjectures and supported conclusions with
  mathematical evidence (Boston & Smith, 2009, p. 145).

Stein, Remillard, and Smith (2007) describe two patterns of learning processes
associated with tasks implemented during mathematics instruction. The first pattern is
aligned with NCTM (2000) process standards. This pattern emerged when teachers set up
high-level cognitively demanding tasks that include characteristics that allow students to
engage in the learning process. Teachers and students implemented these tasks in such a
way that maintains the high cognitive demand and engagement in the learning process.
Through use of this pattern, students experience meaningful thinking, are required to provide mathematical reasoning and can draw conceptual connections between mathematical topics. The second pattern of implementation occurs when teachers set up tasks that demand high-level cognitive thinking and active engagement in the learning process. Teachers and students implement these tasks in a way that leads to a decline in cognitive and process demand. For example, the decline of the cognitive level of task may accrue when solving the high level task becomes only a way of mimicking mathematics procedures. In this manner, students experience a different set of learning processes, such as recalling previously memorized facts and procedures.

The previous discussion on task implementation demonstrates how observation of mathematical tasks can provide greater understanding of the nature of the learning process that students have been involved with when working through mathematical tasks. Stein et al. (2007) indicated that during task implementation, the learning process might not have value in supporting students’ thinking and learning.

**Communication.** Mathematical communication is one of the major reform requirements that is known to foster students’ learning. Communication processes help students develop mathematically appropriated language through use of shared thinking, reasoning about mathematical situations, or gradual development of their formal mathematics vocabulary (NCTM, 2000). Classroom norms and teaching practices are also important aspects in developing and encouraging mathematical communication among students (Hiebert & Wearne, 1993; Wood, 1999).
Social learning theorists and cognitive psychologists have advocated for the accountabilities of the process of communication in their learning frameworks (Cobb, Wood, Yackel, & McNeal, 1992; Hiebert, 1992). That is to say that the reinforcement of communication in mathematical instructional involves cognitive aspects by supporting the development of students’ understanding and thinking because “the event that happen within the interaction serves to invoke one’s reflective consciousness” (Wood, 1999, p. 174). In addition, to cognitive aspect, mathematical communication during instruction helps students to adapt to social mathematical interaction by increasing their positive beliefs and autonomy when discussing mathematics topics (Yackel & Cobb, 1996).

Research study conducted by Hiebert and Wearne (1993) acknowledges links between the nature of classroom communication and the process of problem solving, as well as demonstrates how both processes influence the relation between mathematics teaching and learning. The research study highlights the connection between teaching and learning by examining different patterns of instructional practices and classroom discourse as related to students’ learning. The first pattern of instruction is similar to traditional teaching methods of mathematics that explore adding and subtracting multi-digit numbers. The second type of instruction allows students to construct their own procedures by helping them understand the idea of grouping by ten in order to improve their conceptual understanding. The students in the experimental group were encouraged to discuss their solutions using longer explanations in order to influence them to think more deeply about tasks instead of simply recalling information. The research results illustrated that instructional tasks and classroom discourse tend to impact the relationship
between teaching and learning of mathematics, while instructional tasks and classroom
discourse employ methods that involve students in the process of mathematics by
encouraging them to think, reason and make mathematical connection in order to effect
the nature of students’ learning.

Mathematical connections. The reform call of the school of mathematics has
advocated that students should be able to make connections between various
mathematical concepts or procedures, mathematics concepts and life situations, relate
mathematics to other subject matters, as well as associate various kinds of mathematical
representations.

This strand of mathematical processes interrelates with other processes and
connection between mathematical concepts. “Connecting mathematical ideas means
linking new ideas to related ones and solving challenging mathematical tasks by seeking
familiar concepts and procedures that may help in new situations”(Leikin & Levav-
Waynberg, 2007, p. 350). The concept of mathematical connections can be described
very well through use of Skemp’s idea of relational understanding in which the author
indicates such understanding does not depend on applying specific procedures, such as
instrumental, but rather implicates that students will understand why specify procedures
should be used and how such procedures are connected with mathematical concepts.

Hiebert and Carpenter’s (1992) framework of mathematical understanding has
also incorporated the role of mathematical connection. In their framework they proposed
two kinds of mathematical connections and both are sufficient processes to foster
students’ understanding: external and internal connections. External connection is
established when students are connecting two concepts within the same representation or connecting two distinct representations within the same concepts. An example of external connection is connecting between graphic and algebraic representation of the same function. Another example of the same representation is the connection between algebraic representations of the area and perimeter of quadrilaterals. External connection involves students’ abilities to mentally construct relatedness between mathematical concepts.

An imperative role of mathematical connection can be achieved through authentic mathematical modeling tasks (Zbiek & Conner, 2006). Engaging students with the modeling process occurs, for example, when providing students with a real-life problem that is translated into mathematical language that aids them in discovering a solution for the particular problem. Afterword, the solution is translated back to the real-world situation to testify the validity and applicability of the mathematical solution. The process is not linear and the math solution might be expanded or modified to reflect the real-world system (Schoenfeld, 1994). From an educational perspective, engaging students with modeling tasks that connect mathematics with their real-life experience is not only an imperative element to motive students to learn mathematics (Zbiek & Conner, 2006). It can also deepen students’ thinking and understanding of mathematical concepts (Zbiek & Conner, 2006).

**Reasoning and proof.** Reasoning and proof are additional aspects of the mathematics process that are highly interconnected with other mathematical processes. Reasoning can be used as a method of verification and justification in instruction and in
problem solving situations such as “analyzing problem situations … explaining strategies and checking the reasonableness of the results” (Kim & Kasmer, 2006, p. 90). Usually reasoning and proof take formal and informal dimensions. Informal dimensions are more often presented during the early years of schooling and employ verification whereas the process of formal reasoning is mostly introduced during the later years of schooling and is mostly based on rigorous mathematical explanation (Francisco & Maher, 2005). These processes of reasoning in mathematics instruction are used to enhance students’ understanding. Stylianides defines both the reasoning and proof processes as related to the formation of mathematical knowledge through “identifying patterns, making conjectures, providing non-proof arguments, and providing proofs” (Stylianides, 2009, p. 259).

In a longitudinal study, Francisco and Maher (2005) describe how to encourage students’ process of reasoning during problem solving situations. Results were drawn from data collected when analyzing a classroom-video episode of the students’ open-ended problem solving and exploratory investigation of mathematical topics. The study data, which are based on research at Rutgers University, reflect insights into how specific classroom environments and problem solving activities can promote students’ reasoning and proof. General themes of the study contributed to prompting reasoning, such as helping students build a “sense of ownership” over problem-solving activities, as well as reinforcing their mathematical understanding and reasoning skills. In addition, aspects of collaboration and level of complexity of the tasks was found to support students in sustaining the level of their engagement and reasoning process.
Another finding came from the Francisco and Maher (2005) study indicated that informal reasoning, such as “justification,” was found to be more successful than rigorous and formal mathematical proof to enhance the engagement of reasoning activities. Supporting elementary and middle school students to develop informal justification to solve problems can help them to enhance their proof making in later grade levels.

**Representations.** Pape and Tchoshanov (2001) defined representation in mathematics as terms of the internal mental process that involve abstraction of mathematical ideas by learners in relation to external manifestations of mathematical concepts such as graphs, tables, algebraic equation or charts. Helping students develop fluency and flexibility in translating among representations also allows them to improve their mathematical understanding. In addition, this tool by itself helps students better support their solutions, as well as improves student communication and justification skills that are necessary to problem solving.

Advocating for supporting students to develop a single kind of external representation, such as visual graph, does not necessary guarantee that students will develop mathematical understanding (Stylianou & Pitta-Pantazi, 2002). Students need to develop flexibility to use representations. Also, teachers’ abilities to sequence mathematical representations and combine issues play further roles in developing student understanding (Lesser & Tchoshanov, 2005; Pape & Tchoshanov, 2001). Classroom instruction should help students develop connections among external representations and their meaning to mathematical concepts, as well as allow multiple external
representations to occur in a single problem. The following statement explains the appropriate role of mathematical representation:

Representations must be thought of as tools for cognitive activity rather than products or the end result of a task. For example, the models (e.g., graphs or other pictorial representations) produced may be used to help students explain or justify an argument. (Pape & Tchoshanov, 2001, p. 124)

Many research studies have explored the significant role of mathematical representations in regards to students’ learning (Goldin & Shteingold, 2001; Lesser & Tchoshanov, 2005). In a recent study that investigated the function of encouraging students to use external multiple representation during problem solving, Akkus and Cakiroglu (2010) explored the power of multiple representation-based instruction. During the study four seventh-grade classes from two public schools were assigned to either traditional instruction or the experimental group both of which were taught using multiple representation-based instructions. Through use of this type of instruction and 21 lessons on algebra, students were required to construct different kinds of representations, such as tables and graphs, to work through algebraic situations. Using three types of assessment instruments, the researchers illustrated the significant impact of the treatment on the experimental group during employment of these instruments.

Research Studies on the Teachers’ Knowledge of Student Mathematical Learning

The first section in this chapter discusses the characteristics of instructional practices demonstrated by teachers that facilitate students’ opportunities to learn, their active engagement in thinking, and the classroom norms teachers establish to support
students’ reasoning, discussion and other mathematical processes. Through the second portion of the chapter, the process of mathematics that should be encouraged was presented. The following section examines literature that concentrates on the teachers’ knowledge of student mathematical learning in general, and in specific, that support students’ mathematical process. Teachers’ knowledge about how to facilitate classroom norms that support mathematical processes is not easy to achieve even if such teachers are knowledgeable of the benefits of using this knowledge in their own mathematical work. “Unless teachers have good understanding of proof, we cannot expect that they will be able to effectively promote proving among their students” (Stylianides & Ball, 2008, p. 309)

In Chapter 1, the definition of the knowledge of student mathematical learning was described using two frameworks of the NCTM (2007) and Hill et al. (2008). Because understanding teachers’ knowledge of student mathematical learning is the main aim of the presented study, this section will be devoted to approaching literatures that investigate such knowledge domains. Most of the presented literature uses various definitions and standards to discuss knowledge of student mathematical learning domain. Nevertheless, this literature commonly explains how teachers’ understanding of students’ mathematical thinking should be valued in instruction to improve the proficiency of teaching mathematics (Allen et al., n.d).

The aim of this study is to identify teachers’ knowledge of student mathematical learning in order to accomplish engaging and guiding students in the mathematical processes. In general many literatures have explored such knowledge by exceeding this
definition. While there are various definitions and frameworks that explain the construct of teachers’ knowledge of students’ mathematics learning, these definitions explain “[the] content knowledge intertwined with knowledge of how students think about, know, or learn this particular content” (Hill et al., 2008, p. 377). In addition, researchers use different terminology to explain the construct of teachers’ knowledge of understanding student’ thinking and learning, such as “knowledge about students’ mathematical thinking” (Carpenter, Fennema, Peterson, & Carey, 1988; Hill et al., 2008); knowledge about students’ mathematical conceptions and misconceptions” (Even & Tirosh, 1995; Hill et al., 2008); or knowledge about how to provide alternative strategies for teaching to guide students’ thinking (NCTM, 2007; Graeber, 1999). As such, the following paragraphs present literature that investigates the domain of teachers’ knowledge about students’ mathematical learning.

Teachers’ knowledge about their students’ thinking and learning affects the nature of the relationship between the teaching and learning and also directs the level of classroom instruction. Schoenfeld and Kilpatrick (2008) discuss the framework for proficiency in teaching mathematics that consists of the following dimensions: “knowing school mathematics in depth and in breadth, knowing students as thinkers, knowing students as learners, managing learning environments, crafting and managing learning environments, developing classroom norms …, building relationships that support learning, and reflecting on one’s practice” (p. 322). In this framework, the teachers’ knowledge about the students as thinkers and as learners serve as overlapping categories that together correspond to the NCTM Framework of “knowledge of students
Knowing students as thinkers means “being aware of [teachers’] theory of learning and how that plays out in terms of classroom activities and interactions with individuals” (p. 13). Their discussion indicates how much teachers’ perspective theory on students’ learning impacts and shapes their teaching, classroom norms and the nature of classroom discourse the teachers are generating in the classroom.

In an example given by the researchers to verify how the knowledge of students as thinkers shapes their instructional practice, a teacher asks a student to solve a problem on the board and the student provides the wrong answer. The teachers’ responses often vary from letting the student sit down and calling for another student, walking the student through the steps of the problem to help him or her to arrive at the solution, or using the student’s misunderstanding to ask the question to the whole class and refine the instruction based on their responses. The three perspectives indicate how greatly responses to students during the instruction vary in regards to teachers’ knowledge of students’ ‘mathematical learning.’ This variation among teachers’ knowledge condition their action in the classroom, as well as guide the direction of their instruction.

Sometimes teachers’ knowledge can negatively affect their knowledge of student mathematical learning toward how to engage students in the mathematical process of learning. As such, teachers’ knowledge tends to control how the teachers engage students in the learning process, which in turn contributes to their knowledge about students’ mathematics. For example, Nathan and Koedinger (2000) investigated the relationship between teachers’ mathematical expertise knowledge and their beliefs about students’ role in classrooms as reflected by a reform-based view of mathematics education. The
participants vary between elementary, middle and high school teachers. These participants were asked to rank-order six problems for beginning-level high school algebra students according to the problems’ levels of difficulty. The participant teachers also completed a beliefs survey that assessed teachers’ beliefs on: algebraic best procedures, the effect of student-invented solution, the symbolic precedence view, students’ reasoning in the classroom and the importance of correct answer versus answer process during an evaluation of students’ alternative solutions.

The findings of the previous study demonstrated that the more advanced mathematics knowledge teachers tend to largely disagree with the reform-based view of mathematics instruction. In contrast, the group with less advanced content knowledge (specifically middle and elementary school teachers) believed that instruction should build on students’ reasoning and help them invent their own procedures. The term “expert blind spot” which was presented in the article, exhibits how the advanced level of mathematical knowledge can influence the teachers’ pedagogical instructional approach. These findings could be interpreted as indication that teachers’ knowledge can result in the development of negative beliefs toward the role of students’ mathematical learning process in the classroom. These findings correspond to what Putnam and Leinhart (1986) explored when they determined that teachers’ expert knowledge could negatively impact their understanding of students’ difficulties and process of thinking in learning mathematics.

Cognitively Guided Instruction (CGI). The approach of the knowledge-based children’s mathematical thinking process through the CGI professional development was
mainly discussed in Chapter 1. Most of the CGI work is related to elementary school mathematics education. However, their collection of research studies has been widely cited beyond elementary mathematics education. The collection of research studies conducted by the researchers of CGI demonstrates how the knowledge of students’ mathematical thinking is a powerful avenue that affects their teaching and also guides teachers’ knowledge of curriculum, subject matter and pedagogy. The collection of the studies (Carpenter et al., 1989; Fennema et al., 1993; Fennema et al., 1996; Peterson et al., 1989; Steinberg et al., 2004) that was undertaken over the past decades by the CGI research group found that “learning to understand the development of children’s mathematical thinking could lead to fundamental changes in teachers’ beliefs and practices and that these changes were reflected in students’ learning” (Carpenter et al., 2000, p. 1). These research studies stress the importance of how this knowledge among teachers will help them understand their students’ thinking and be able to better guide conceptual understanding.

The work that was conducted by the CGI research group demonstrated how the program succeeded in improving teachers’ practice and beliefs as a result of enhancing the teachers’ understanding of students’ mathematical thinking. In the CGI professional development, teachers are encouraged to employ research-based knowledge about students’ mathematical thinking when planning their instruction. The CGI professional program gives the teachers the opportunity to learn about children’s difficulties regarding specific mathematics concepts, as well as to learn about students’ most common approaches to solutions. During the program, the teachers had the opportunity to examine
and classify their students’ solutions to learn about various reasoning and solution strategies that the students would use when working on particular tasks which would enhance their knowledge about students’ thinking and reasoning of specific tasks and concepts. The students’ work and various thinking processes became the groundwork for the teachers’ to examine their instructional practices and to look for the best ways to improve their practices to accommodate different levels of thinking and ability in their classrooms. The program helped the teachers to revise and improve their beliefs about how students learn mathematics which in turn would be reflected in their teaching practice (Steinberg, Empson, & Carpenter, 2004).

In a three year longitudinal study of 21 teachers, the CGI research group examined the levels of teachers’ improvement and changes in instructional practices and beliefs about how to best approach student learning. Fennema et al. (1996) illustrated that at the end of the program, 90% of the participants of the program were observed to provide classroom instruction that support students’ engagement in problem solving while attending to their thinking processes. Further, a follow up study was conducted to gather information about the sustainability of participants’ implementation of the principles of the program in their teaching (Franke et al., 2001). The study used 22 samples of teachers who all continued to engage in instructional growth as a result of implementing their knowledge of children’s thinking in their instruction. In the study, ten teachers were distinguished in their implementation by seeking support from their colleagues to sustain the improvement of their teaching. These teachers were not only trying to improve their teaching but also looked for support from their colleagues to help
them understand their current students’ thinking and to improve their instruction and understanding.

Results of the collection of the studies conducted by the CGI research group demonstrated how the improvement of teaching practices and beliefs greatly extended the impact to students’ achievement (Carpenter et al., 1989; Fennema et al., 1996). These studies indicated that when teachers have guidance about how to plan instruction around students’ current level of knowledge, knowledge of students’ difficulties in specific conceptual problem solving, and information about how to implement such knowledge to reform their instruction around students’ learning needs, this body of information helps the teachers to make instruction accessible for students and improve their achievement. In addition, improvement of teachers’ instructional practice has a great impact into students’ problem solving strategies and their thinking and discussion in the classroom. The findings of the CGI studies contribute to the current reform recommendation of pressing into students’ mathematical thinking and reasoning in the classroom. Fennema et al. (1996) stated that, “developing an understanding of children's mathematical thinking can be a productive basis for helping teachers to make the fundamental changes called for in current reform recommendations” (p. 403).

Empson and Junk (2004) explored the knowledge of children’s mathematics the teachers gained after the implementation of standards-based curriculum. Thirteen teachers were interviewed through pedagogical scenarios that involve non-standards algorithmic ideas the children would come up when investigating student-centered and non-standards tasks. These scenarios are designed to help teachers report on their
knowledge gained of students’ nonalgorithmic thinking and their beliefs on the implementation of standards-based instruction. The findings of the research indicated how the implementation of standard-based curriculum supported the teachers to learn to respect students’ mathematics’ capabilities and the disposition to use nonalgorithmic strategies when solving problems. This productive knowledge was found to be disconnected with teachers’ knowledge of mathematics. In the study, the teachers’ saw how their students experience a variety of problem that foster deep learning of the content. These teachers see the power of how their students experience a variety of nonalgorithmic problem solving as a motivational factor for them to respect their thinking and problem solving strategies.

**Emphasis on students’ conceptions and misconceptions.** Understanding students’ conception and misconception is an important aspect of the knowledge of student mathematical learning. As discussed in Chapter 1, Ball et al. (2008) included it as important categories for measuring teachers’ further knowledge of student mathematical learning and thinking. NCTM (2007)’s framework also incorporates that teachers should have the capacities to support students’ difficulties in learning. The presentation of the following two studies literature related to students’ conception and misconception. Studying students’ conception and misconception domain of teachers’ knowledge does not mean to be isolated part of the knowledge about student’ mathematical learning. As discussed, research tends to provide various definitions and frameworks for the knowledge domain that does not agree with each other but it explains the same construct. Nevertheless, one unique thing that was found from reviewing the scholarly work on the
area of students’ conceptions and misconceptions is that, these research studies tend to associate this topic with specific mathematics content areas such as difficulties about fractions, decimal points, and addition and subtraction.

Stacey, Helme, Steinle, Baturo, Irwin, and Bana (2001) assert that including teachers’ knowledge of students’ common difficulties and misconception should be significant focus in teacher education programs. In this research, the authors investigated teachers’ knowledge of students’ difficulties and misconception in decimal numeration. A number of 553 participated in this study by completing a Decimal Comparison Test (DCT) that was designed by the researchers. In addition to this test, the participants were asked to identify the difficulties and misconceptions that they believe their students would face in working through problems that involve decimal numeration. A high percent of the research participants have the awareness and the understanding that their students have the misconceptions when comparing two decimal numbers by thinking that longer decimals demonstrate that the quantity of the number to be larger. As opposed to that, a small number of the participant did not recognize the students’ difficulties.

Similar results came from the work of Even and Tirosh (1995). The research study illustrates that instructional approaches of teachers and their responses about students mistakes regarding the concept of “slope” is impacted largely by their mathematical knowledge of the learners and not much influenced by their mathematical knowledge. Participants of this research tended to respond to students mistakes correctly without thinking about why the students have these particular misconceptions or considering what kind of difficulty the students have. For example, the students’
misconception that “twice the angle means twice the slope” (p. 13) did not initiate classroom discussion to address the students’ conceptual misunderstanding. This indicates the inadequate knowledge of the participant teachers about how to address their students’ misconceptions.

**Knowledge About How to Engage Students in Mathematical Processes**

This last portion of the literature review discusses literature that specifically aim to address the issue of teachers’ knowledge about students’ learning about how to engage student in the mathematical processes. However, it should be noted one major goal of the previously reviewed collection of the research that was undertaken by various researchers, as well as CGI research group, demonstrated role of teachers’ knowledge of student mathematical learning in general stating that “learning to understand the development of children’s mathematical thinking could lead to fundamental changes in teachers’ beliefs and practices and that these changes were reflected in students’ learning” (Carpenter et al., 2000, p. 1). As such, the following research will describe knowledge about how to engage students’ in the mathematical processes. A minimum number of research studies were found to investigate the knowledge of teachers in terms of their abilities to engage students in the mathematical process. The following two studies were found to address this area.

Stylianides and Ball (2008) and Francisco and Maher (2011) both implemented frameworks that use knowledge about engaging students in the process of proving and reasoning. Francisco and Maher (2011) articulated that observing students’ processes of problem solving and habits of reasoning as well as their abilities to provide mathematical
justifications can support teachers in acquiring knowledge about their students in multiple ways. These include their understanding of the abilities of their students, their appreciation that students can develop solid mathematical justifications, their proficiency in giving students the opportunities to use informal justification and formal proof. Stylianides and Ball (2008) classify teachers’ knowledge of proving as “knowledge of different kinds of proving tasks and knowledge of the relationship between proving tasks and proving activity” (p. 307).

Summary

This chapter began by reviewing studies on the current mathematics education instructional characteristics as advocated by current reform requirements in the field. In addition, the review studies include research on how we can view students’ learning in mathematics.

The reviewed research studies demonstrated how knowledge about understanding students’ learning, thinking process, conception and misconception, as well as engaging students in the mathematical process, can be a powerful tool to support educational reform. Advocating for helping teachers to improve their knowledge about students’ learning can be a way to allow teachers to stay professional and current in their understanding with a variety of mathematical topics and concepts (Rine, 1998). Rine noted the powerful implications of professional projects that base their learning on the knowledge about students’ learning and understanding such as the CGI project. The imperative role of such professional programs is clear because, as long as the mathematical topics that are taught are changing and improving, teachers who have the
adequate knowledge and awareness about students’ thinking will be armed with professional tools to constantly update their knowledge about students and always fully examine their misconceptions on varieties of topics.

To improve teachers’ knowledge about students’ learning and thinking, it is important to understand the teacher’s current position on how students come to learn mathematics. The presented study aimed to achieve such an understanding of the teachers’ knowledge.
Chapter 3: Methodology

This chapter will discuss the qualitative research method used to address the three research questions presented in Chapter 1. The main goal of the study was to understand the Saudi teachers’ knowledge of student mathematical learning with regard to their approaches of how they support their students’ experiences of the mathematical processes. A description of the research setting, including a description of the current educational system, will be provided to facilitate a clear understanding of the presented research questions. Next, the rationale of the qualitative design and the choice of grounded theory will be discussed. Finally, the data collection procedures and the instruments that were employed will be explained.

The Setting

The research study was conducted in the Alahsa, Saudi Arabia. The city of Alahsa is considered as a traditional oasis that is located in the eastern part of Saudi Arabia. Alahsa has a total population of 1,300,300. The study was conducted to understand Saudi middle school teachers’ knowledge of student mathematical learning of how to engage students in the mathematical processes. There is a significant rationale as to why the research questions of the study were investigated in Saudi Arabia. The Ministry of Education in Saudi Arabia is currently undertaking reform efforts within mathematics and science education in order to align with the current mathematics education standards in more economically advanced countries, such as the United Kingdom and United States. The Ministry is committed to these efforts and aims to prepare citizens who possess life skills and knowledge in mathematics and science. The educational
advancement project is part of a larger initiative that will also include other neighboring countries, such as Kuwait and the Kingdom of Bahrain (Ministry of Education in Saudi Arabia, 2010). As part of the project, the Ministry and the other countries have signed a contract with a national publication company, Obeikan, that will supplement the Ministry with translated and adapted versions of international mathematics and science textbooks. Recently, Obeikan Company has signed for agreement with the company McGraw-Hill Education to translate and adapt international mathematics and science textbooks for the Ministry of Education. These textbooks are considered to address current mathematics standards for the 21st century and are aligned with NCTM standards. As part of the contract, the selected company will undertake systematic professional development programs for educators to support them in implementing these reform materials.

Participants and Sampling Procedure

Since 2008, when the Saudi educational advancement project was established, mathematics teachers in Saudi Arabia have been confronting challenges as they transform their own teaching and adjust to the new textbooks. Accordingly, Saudi middle school teachers are expected to gain further exposure to the reform-style practice. As a result of the advancement project, teachers are expected to be familiar with student-centered education and are in the position to implement these reform strategies. Studying participants in such an environment is and will continue to be crucial in helping educators understand this transformation. The influence of such expectations can be investigated by investing in studies that question teachers’ practices and how such practices are related to students’ active engagement in learning through instruction.
The purpose of sampling from the Saudi teachers’ population is to find “information-rich cases” (Patton, 2002, p. 230) who would be willing to share insight about their current teaching practices. The challenge of implementing new styles of teaching could reveal the targeted teachers’ increased willingness to share their stories and insight about their new experiences in the scope of this research study. This increased desire to share insight occurred because such drastic educational reforms often initiate discussion regarding teachers’ perceptions of student engagement and receptiveness. In the research that targeted Saudi middle school students, Razeq (2007) reviewed literature that demonstrated that students in Saudi Arabia were only exposed to direct traditional instruction during the time in which she conducted her research study. Recent research has continued to generate the same results. Al-Shahrani (2010) revealed that Saudi teachers are investing the majority of their mathematics teaching lessons in an authority-based teaching style that is based upon training students to memorize and implement procedures that are meaningless for them without instilling creativity in the students or promoting critical thinking. As such, the relatively newly reformed requirements that the participants are confronting encouraged them to share their thoughts and stories to enrich the study data.

The focus of the study was limited to middle school mathematics teachers, though all grade levels of mathematics in Saudi Arabia are under similar circumstances. The three targeted grade levels of Saudi middle schools (seventh, eighth, and ninth) have successfully implemented the initial phases of the advancement project. This means that
middle school teachers being sampled are all experiencing the same reformed teaching requirements.

I received an approval from the ministry’s branch in Alahsa to conduct my research study (see Appendix A for the Arabic and English translated version of the approval letter from the Ministry). I also received an IRB (Institutional Review Board) approval letter from the Office of Research Compliance at Ohio University to carry on my research study (see Appendix B). I contacted the Research and Project Administration department at the Ministry of Education in Saudi Arabia to help me recruit the 12 teachers who volunteered in sharing their teaching experiences. This initial number of participants was identified by the researcher and additional participants can be added if new information regarding the data is needed or if a category in one area does not reach the level of saturation.

Because I am not claiming to make a general statement about the teachers’ knowledge of student mathematical learning, the sample does not have to be representative to the general demographic characteristics of the Saudi teachers’ population or to meet a specific sample size. The research excluded male teachers from the sample because the qualitative design requires direct interviews with the participants and a direct examination of classroom instruction. Saudi Arabian schools are segregated by gender and researchers are prohibited to visit opposite gender schools. These conditions made it impossible for me, as a female researcher, to interact with male teachers.
Nonetheless, I obtained some demographic information of the teachers from the schools’ administrators to provide context for the study and to ensure having various characteristics of participants in the research sample. This information includes teachers’ years of experience and their current teaching academic ranking. Further, because identifying the academic yearly ranking is considered to be surreptitious information that participants might feel is offensive to reveal, the Research and Project Administrator at Alahsa agreed to provide me with two lists. One of the lists has the names of teachers who obtained a perfect (A) score at academic ranking in the year of collecting the data. The other list contains all other middle school teachers who had less than A in their academic ranking. The sample of the study was drawn equally from both lists. Additional information on the participants and the schools will be discussed in Chapter 4.

Such information helped me to identify various characteristics of teachers to include in the sample of the study. In addition, such information aimed to strengthen the depth and credibility of the study because such information helped me to explore new connections and patterns between teachers’ demographic information and their responses during the other data collection procedures. In addition, the results of the study showed to some cases that demographic information relates to teachers’ current view of how students are engaged in the processes of learning mathematics. In either case, the use of this information helped me to choose varied characteristics of the study’s participants and assisted me in discovering patterns and connections.
**Researcher Subjectivity**

As the primary instrument of the study, researchers come to any study with their cultural background, beliefs, biases, and expectations. In the modern qualitative ideology, this does not have to be considered as a negativity of the whole investigation (Strauss & Corbin, 1998). It is equally imperative for the researcher to recognize these biases and assumptions, which researchers from the same culture of the participants usually have. In this study, I had to show high sensitivity in questioning to the actions of the participants. This is because my culture is similar to the participants’ culture and many of their words and actions oftentimes are normal for me to hear and do not cause me to question them.

As a consequence of that, I am forwarding my personal beliefs, biases, and perspectives regarding how I view the Saudi female teachers’ knowledge of students’ learning. My view of this matter has been impacted by my personal academic journey in my homeland Saudi Arabia as well as through my educational journey in the United States.

My formal education in elementary, middle, high, and undergraduate school was achieved in my home town Alahsa in Saudi Arabia, which is in fact the setting of the presented study. I was fortunate to experience elementary and middle school mathematics teachers who were able to recognize my mathematics interests. I am confident to acknowledge that teachers in my past schooling were knowledgeable about me as a student and were able to guide me to experience extra mathematics homework and rich learning activities. I remembered the struggling time in my high school calculus when my teacher always challenged me with solving problems related to finding the volume
rotating a specific area. I had to come up with solutions to these challenging math problems and present them to my classmates.

However, the experience was not equal for other students who were left behind since they did not have the potential math interest. Thus, I do not hold the same position with how the same teachers practice their knowledge of other students’ mathematical learning. The typical teaching for them was centered on the teacher who supplements them with mathematical formulas and roles that might be meaningless for them. In terms of the current study’s focus, my contemporary belief about teachers in Saudi Arabia reveals that they represent limited knowledge about how to help students become active thinkers in the classroom in case the student her or himself does not have the potential mathematical interests.

I was fortunate to experience different mathematics teaching ideology from what I saw other students experience in my schooling in Saudi Arabia. My personal opinion has been shaped initially with my own learning experiences in schooling including my academic journey in master’s and PhD classes at Ohio University. Throughout my graduate school, which I attended in the U.S., I came to realize that teaching of mathematics needs special concerns and that it is not easy for teachers to support students in learning mathematics actively with engagement in problem solving. I am committed to the motto: “Interests can be created and stimulated.” This is true even if the students do not believe that they have mathematical interest or the positive energy toward learning mathematics. I have a personal belief that students’ knowledge and understanding cannot be achieved by the simple “telling” way of teaching. Learning should be formed through
the work and struggle that students have to engage through. The students should have learning environments that allow them to engage in mathematical activities and mathematical processes of thinking. I appreciate the ideological constructivism view and disclose mathematical processes of problem solving, reasoning and proof, communication, and connections. Students should have the opportunity to experience these processes in a way that helps to stimulate and discover their mathematical interest. Yet, because I was raised in a teacher-centered system, I still feel skeptical about how I will apply my current beliefs in my teaching of mathematics. In my future teaching experiences, I will have to find the balance between when to tell my students and when to encourage them to explore mathematics concepts.

Acceptance and high awareness of my existing biases and understanding about teaching in Saudi Arabia assured me that such beliefs and assumptions would not negatively impact my interpretation of the results. Acknowledging my existing knowledge about teachers in Saudi Arabia helped me to be open in evaluating their responses to my questions even if these responses are contradicted with what I previously know about these teachers and their way of teaching.

Rationale for a Qualitative Design

The purpose of the research was to begin to understand the current level of Saudi teachers’ knowledge of student mathematical learning with regard to how they guide students’ engagement in the mathematical processes. In order to understand the teachers’ knowledge of student mathematical learning, a qualitative design was conducted to provide a rigorous and in-depth analysis of the research situation. Qualitative research
methods are usually used when the purpose of the research study is to understand some social phenomena within its socio-cultural context from the perspectives of those who are involved in such social constraints (Glesne, 2006). The design facilitates studying phenomena in depth in order to inductively generate a theoretical perspective from the fieldwork.

Teachers’ knowledge of student mathematical learning is difficult to investigate with the traditional hypothesis-testing techniques. Understanding such knowledge must involve close examination of teacher-student interactions within the natural classroom setting. The use of qualitative research helped me to record and understand the teaching experience using varying methods and sources of data collection. Direct interviews consisting of open-ended questions and prompts encouraged the participants to share their stories and experiences, as well as helped me learn about the extent of their knowledge of student mathematical learning.

Without the consideration of the cultural context, the research would provide little information about teachers’ knowledge of how their students learn mathematics. This is because such knowledge is not solely influenced by teachers’ formal education, which can be measured through numerical procedures. Many factors impact students’ learning opportunities in the classroom. Both students’ and teachers’ culture, school policies, and educational national polices determine the extent to which students have control over their learning experience. Such surrounding criteria necessitate the research study to be investigated through the path of a qualitative design.
Grounded Theory

Grounded theory is a method that “consist[s] of a set of inductive strategies for analyzing data” (Charmaz, 2004, p. 497). The final goal for such a method is to develop a theoretical framework that fits the given research situation faithfully and that can be applied to explain various contexts related to the given phenomena.

As Glaser and Strauss (1967) have proposed, the uniqueness of grounded theory, as opposed to other qualitative descriptive methods, stemmed from the fact that it presents to the researcher a set of logical scientific procedures to follow in a way similar to quantitative research methods. The method requires simultaneous involvement in collecting, note taking, coding, selecting, and categorizing emergent themes. Using various methods of data collection, the researcher eventually moves to another research step once collecting data about a specific category and does not add anything new to that theme. This strategy is referred to as saturation.

The main purpose of grounded theory is to develop a theoretical perspective that can explain the research phenomena and enlighten the relation between variables when such variables or concepts and relations between them have not been yet identified by previous research. This requires the researcher to investigate the study phenomena inductively, allowing more freedom and flexibility to bring into discussion new variables and relations that would contribute to the understanding of the whole research situation. On the other hand, grounded theory research statements, as opposed to other qualitative research methods, are required to be narrow and more focused toward the research situation and oriented to study the participants’ process and action (Strauss & Corbin,
The data collection procedures in this study were implemented to achieve this orientation. I investigated teachers’ knowledge of student mathematical learning during classroom instruction to give the research the power to study teaching and learning process and action. I conducted face-to-face interviews to provide the depth required of the research and to verify the findings gathered from classroom observation. An extensive discussion of the data collection procedures is presented in the following section.

**Data Collection Procedures**

Interviews and classroom observations were conducted to learn about practicing middle school teachers in Saudi Arabia. In this section, the method of interviewing and the nature of observation protocol will be described. The data collection procedures will go through three successive stages:

I. Classroom Observation

II. Initial Interview

III. Scenario-Based Interview

These stages of data collection will be discussed in the following sections.

**Classroom observation.** In keeping with the first research question, the aim of this question is to learn about teachers’ knowledge of student mathematical learning with regard to how they engage students’ in the mathematical processes during instruction. Accordingly, one of the aims of the classroom observation is to answer the first research question. The classroom observation offered non self reported data that helped the
researcher to get deep insights into teachers’ knowledge of student mathematical learning by comparing the findings derived from other data sources.

In this study I performed one classroom observation with each participant. Classroom observation of middle school teachers functioned as the first stage of the data collection process. This stage occurred after meeting with the potential participants to explain my study to them and seeking their written and oral involvement approval. After they signed their consent forms, I arranged classroom visits to do my one-step-observation. The main purpose was to observe how teachers practice their knowledge of student mathematical learning as they deal with their students and stimulate their engagement in the mathematical learning processes. Such determination can be transformed through many venues and themes including, but not limited to:

- Examining the nature and quality of mathematics tasks the students are doing during the course of the instruction
- Examining how teachers approach and encourage nonalgorithmic solutions
- Investigating how they approach student misconceptions
- Observing the nature of students’ mathematics communication and informal reasoning with their peers and with the teachers, and the extent of its role in their learning
- Exploring the extent to which the instruction gives students the opportunity to experience how mathematics is connected to their real life and with other mathematics topics; and
Noting the extent that the learners are engaged into multiple mathematical representations (i.e., graphical and numerical), and their opportunities to experience the flexibility of dealing with multiple representations in a single mathematical task.

Recording detailed field notes helped me to gather information regarding teachers’ knowledge and teaching styles, which permitted me to explore the above aspects of teachers’ knowledge of student mathematical learning and the nature of mathematical processes that students are practicing during classroom instruction. Additionally, during the observation, the researcher used observation protocol to examine the teachers’ knowledge of student mathematical learning and how they support students to engage in mathematics processes. Discussion about the protocol is presented subsequently.

**The instructional quality assessment (IQA) observation protocol.** The classroom observation made use of the IQA observation protocol to provide understanding of the middle school Saudi teachers’ knowledge of student mathematical learning in terms of their knowledge about how to engage students in the learning processes (Adapted from Boston & Wolf, 2006). The protocol aimed to assess the possible students’ cognitive level demand of the mathematical tasks and activities that teachers presented during my observation visit. From the IQA observation rating package, I applied two rubric functions for scoring the potential of the mathematical task and the implementation of the mathematical task (see Rubric 1 and 2 in Appendix C). According to the IQA rubric for implementation of academic task for a task to be scored as high-level cognitively demanding, it must meet some of the following attributes
• Doing mathematics using complex non-algorithmic thinking;
• Applying procedures with connections;
• Students may have solved a genuine, challenging problem for which students’ reasoning is evident in their work in the task;
• Students may have made conjectures and supported conclusions with mathematical evidence (Boston & Smith, 2009, p. 145).

The purpose of implementing this protocol was not solely to assign numerical measures to the teachers’ quality of teaching. The purpose of using this protocol was also to understand the extent and level of teachers’ support of the student in learning mathematics and how they exhibit such knowledge of student mathematical learning. Further, the administration of the IQA facilitated the recognition of the broaden themes listed on the previous page that I observed during the course of the study.

As it was previously discussed, the theoretical purpose behind the implementation of the IQA rubric is to evaluate the cognitive level of academic tasks. Furthermore, the examination of cognitive demands of instructional tasks can give us similar insights into how the students’ learning processes has been engaged. The protocol connects the cognitive level of academic tasks that the teachers implement during their instruction and the extent to which students are engaged in the mathematical learning processes. This will also feed into the understanding of the level of teachers’ knowledge of student mathematical learning of the teachers.

**Interviewing.** The second stage of the data collection was meeting one-on-one with the participants for interviewing. The primary purpose of the interview is to uncover
teachers’ current knowledge and understanding of how students learn mathematics. I followed the NCTM (2007) definition for teachers’ knowledge of student mathematical learning as a framework that guided my investigation about teachers’ abilities of how to involve the students in the processes of learning mathematics. I chose this framework, among others, because it presents a practical tool that will inform me about teachers’ knowledge of student mathematical learning. The process of interviewing was done through two main forms: initial interviewing and scenario-based interviewing.

**Initial interview.** This part of the interview gave me initial access to the teachers’ current knowledge of student mathematical learning in terms of their capacity to allow students to practice mathematical processes during instruction. For this purpose, I developed an interview protocol that was administrated during the initial meeting with the participants (Appendix D). During this meeting with the participants, they had the opportunities to articulate their teaching practices. Meeting with them helped me understand their responses and their opinions about students’ roles in the classroom in terms of giving students the opportunity to experience the mathematical processes.

I designed the interview questions to address the research questions of the study. I included open-ended questions that give teachers the opportunities to discuss their current teaching practices and their understanding of students’ learning. Since the study implements the NCTM (2007) definition of teachers’ knowledge of student mathematical learning, I used it to design questions that incorporate this theoretical framework. Further, to understand the teachers’ view of student’ active involvement in the mathematical processes, I include in the protocol questions that explore teachers’ knowledge in how to
involve students in classroom mathematical activities such as: discussion, problem solving, reasoning, and mathematical connections.

The other purpose of this initial interview was to establish rapport between me and the participants. The interviewees and I were elected to introduce ourselves to each other which facilitated the administration of the follow up stages of the data collection.

*Scenario-based interview.* In a previous discussion, knowledge of student mathematical learning is defined as the “teachers’ knowledge of students’ mathematical thinking” (Hill, Ball, & Shilling, 2008, p. 373). Hill et al (2008) and NCTM (2007) previously discussed that classifications of such knowledge provoke the consideration of mathematical content when attempting to explore such part of teaching knowledge. Accordingly, implementing scenario-based interviews that are based on mathematical content gave the teachers the opportunity to talk about their knowledge of content as it interacts with both their understanding of the students’ role in the classroom and understanding of mathematical pedagogy.

Teachers were given teaching scenarios that consist of teaching situations related to students’ action in the classroom and their processes of learning mathematics (Appendix E). The participant teachers were asked to respond and comment about these scenarios. Such scenarios stimulated discussion of their teaching styles, their knowledge of students’ mathematical thinking and their perspectives of students’ roles in the classroom. I consider three criteria in developing or adapting these scenarios:

- The teaching scenarios discuss mathematical content that is widely known to be conceptually problematic for middle school teachers and students.
• Similar versions of the tasks that I choose are presented in the current textbooks the middle school Saudi teacher participants are using in their instruction.

• Also, I will use other teaching scenarios that involve mathematical topics that can be dealt with using cognitively demanding instruction or using procedural solutions.

These previous criteria helped me to stimulate various discussion topics among the participants related to their knowledge of student mathematical learning to involve students in the processes of learning mathematics.

**Data Analysis**

Grounded theory method was used to guide the analysis of the data during the data collection. To answer the research questions, interviewing and observation were implemented. Grounded theory provides a logical series of processes that, if implemented correctly, produce a theory that can fulfill the research phenomenon. This theory requires a simultaneous method of data analysis that can establish a rigorous understanding of the research question.

Analysis in grounded theory begins with open coding which refers to a “process of breaking down, examining, comparing, conceptualizing and categorizing data” (Strauss & Corbin, 1990, p. 61). Open coding begins as soon as the first interview is conducted, and is carried out by conceptualizing and labeling what is being observed during the data collection. Constant comparison is important in relating interview to interview which will facilitate labeling, the grouping of these labels, and allowing for the creation of categories. During the course of observing and interviewing the participants, I
coded the data gathered in order to identify similarities and differences between the open
codes of each interview and each observation. In addition to how open coding allows
patterns to emerge, this procedure helped me to alter or add more questions to ask the
participants in a follow up interview or observation.

Following the open coding, the categories that were identified during the stage
open coding, these categories were grouped and connections between them were
identified to reach axial coding (Patton, 2002). Strauss and Corbin (1998) defined axial
coding as, “the process of relating categories to their subcategories, termed ‘axial’
because coding occurs around the axis of a category, linking categories at the level of
properties and dimensions” (p. 123). Open coding and axial coding were developed side
by side until the information in the category reaches the level of saturation and no further
data collection is needed to explain this category. I used memoing during the course of
data collection to help me identify the relationship between the categories and
subcategories that were recognized and help to reach axial coding.

From these two processes of coding the researcher developed selective coding,
which refers to the process of selecting the core categories of the research that canexplain and answer the research question (Strauss & Corbin, 1990). I used selective codes
that are dominant across most of the participants to develop themes that can explain the
research questions.

The method of analyzing the data of the study described above was carried out for
several times in this study. The classroom observations and initial interviews were coded
and analyzed together side by side using the same analysis method. The transcript of each
teaching scenario was coded separately. Consequently, this coding process was conducted five times, once for the observation and initial interview and four times for each teaching scenario. The discussion of the rational of the specific coding procedures of each data source will be presented in Chapter 4.

Translation and Transcription

All the instruments that were administered during the course of data collection were translated in Arabic to be used for the participants. The researcher performed back-translation to all instruments. McKay et al. (1996) indicated that this translation method is the most effective practice for culturally sensitive instruments. The process of back-translation was divided into two parts. First, a native Arabic speaker doctoral student was asked to translate the instruments from the English version presented in the appendices to an Arabic version. The doctoral student is in fact a student at the Patton College of Education in Ohio University. Second, I applied my own translation to the instruments from the Arabic version, translated by the doctoral student, to a new English version. To reach the best consensus translated version of the instruments, I cross checked between the two English versions to see how they differed and I then edited my Arabic translation accordingly to better reach accuracy of content in the instrument. In addition, after reaching a final Arabic version of the instrument, I gave my instruments to the same doctoral student to read them and give feedback regarding language usage and meaning. I edited my last version to address the new feedback.

During the course of data collection, I collected my data and recorded my interviews in the Arabic language. Consequently, all of the data was transcribed in the
participants’ tongue language (Arabic). To facilitate the process of analyzing, the author had worked with the data in the Arabic language. However, for the next chapters, the quotes that were presented in this study were the only English-translated segments of the transcripts. To reach accuracy and credibility of the results, the researcher used back translation for all of the quotes that presented in Chapters 4 and 5.

**Credibility of the Qualitative Findings**

Credibility in qualitative inquiry deals with the level of truth established in the data collected in a study and how much the reader can trust the associated results. Brantlinger, Jimenez, Klingner, Pugach, and Richardson (2005) discuss methods that can be used by researchers to assure such credibility. During this research some of these methods were used to ensure the credibility of the study. These included careful record keeping, triangulation of data from multiple sources, and examining negative cases.

Throughout the data collection stages, the author of study kept careful records of the quotes presented in Chapters 4 and 5, and maintained audio and written record of the interviews to keep track of the actual data. The quotes presented in the study are dated according to when they took place. The author included detailed descriptive quotes and other evidence to support the findings of the study. Brantlinger et al. (2005) state that these methods of careful treatment of the quotes can be used by researchers to establish the credibility and quality of the study.

The researcher implemented “method triangulation” as a way to support the credibility of the study. In other words, the author verified “The consistency of findings generated by different data collection methods” (Patton, 1999, p. 1193). This
triangulation used classroom observations, initial interviews, and scenario-based interviews to support the findings presented in Chapter 5. The data generated from the three data sources were compared, contrasted, and discussed in Chapter 5. Performing triangulation has been advocated by many researchers to be used as a means of enhancing credibility (e.g., Brantlinger et al., Patton, 1999).

Both Brantlinger et al. (2005) and Patton (1999) advocate for the use negative cases to enhance credibility. When a portion of the data revealed specific patterns, the researcher looked for negative cases that did not fit the emerging patterns. For example, the findings related to how teachers support students’ mathematical connections presented in Chapter 5, some contradictions were shown between classroom observations and scenarios-based interviews.

In addition, Brantlinger et al. (2005) suggest that there are many other methods that can be implemented to ensure credibility and that researchers should use these methods with caution depending on whether they fit the purposes and methods of the investigation. For example, Brantlinger and colleagues suggest that member check can be used to establish credibility, and discuss incidences where member checks are not used because participants might not tolerate hearing about their actual biases and beliefs. Thus, member checking was not used in this study because the researcher thought that sharing with the participants the findings or part of them might offend the participants. Based on the prior knowledge of the researcher, teachers in Saudi Arabia are not accustomed with opening up and sharing information about their teaching. Volunteering for this study was a big step for them that was already beyond their comfort zone. As such, the researcher
did not want to push the participants harder by cross checking their beliefs that they had shared during interviews.

**Summary**

The purpose of conducting this study was to learn about teachers’ knowledge of student mathematical learning that will verify their position on how to support the students’ involvement in the mathematical processes. The study undertook three data collection procedures to answer the research questions. These procedures include: initial interview, scenario-based interview and observation. Grounded theory was implemented to guide the analysis of the data gathered from the three data collection methods.

Conducting three methods of data collection helped me to establish triangulation and helped to strength the depth of the research results. This research attempted to enhance its credibility by focusing on recording detailed field notes during the course of observation and interview. In addition, as required by grounded theory, data collection and coding were occurred in the same time to ensure the depth and the quality of the data. Such method ensured that all the necessary details are recorded and coded which would enhance the credibility of the qualitative analysis.
Chapter 4: Data Analysis and Findings

The research mainly investigated the Saudi teachers’ knowledge of student mathematical learning with regard to how they promote students experience of the mathematical processes. This chapter discusses the themes discovered during the analysis of the research data. The researcher conducted observations, initial interviews, and teaching scenario interviews with each of 12 middle school teachers. The findings derived from these data sources are presented.

This research was conducted in the city of Alahsa in Saudi Arabia. In keeping with the sampling procedure discussed in Chapter 3, the 12 participants in the study vary in terms of their yearly academic ranking, which is evaluated by the Ministry of Education in Saudi Arabia. As a result of this sampling procedure, the volunteer participants in this study came from a total number of seven middle schools. The schools also diverge in terms of their financial resource availability, as well as the financial resources and educational background of the families of the children attending the schools. This chapter will provide general context to the study by explaining the schools and demographic information of the studied subjects.

The study implemented grounded theory to guide the analysis of the data derived from the three sources. Grounded theory procedure requires comparison of each data source. Interviews and observation field notes were consistently compared and contrasted. This procedure was carried on to arrive at themes that can explain the Saudi teachers’ knowledge on how to promote students’ mathematical processes. As a result of the evaluation of the data, the researcher has decided to perform two stages of data
analysis. Through the first stage, the classroom observation field notes and the initial interviews and were investigated. Themes emerged from both data sources helped to generate a grounded theory that explain how teachers in Saudi Arabia are using their knowledge about students learning to promote students’ mathematics processes. The second stage of data analysis consisted of evaluating the scenario-based interview transcription, which helped me to continue to investigate the common themes derived from the first stage as well as new ideas that did not emerge from the observation and initial interview. The findings derived from each stage of data analysis will be presented as well as the coding procedures that were embedded in each stage of analysis.

**Description of Schools and Teachers**

The 12 teachers who participated as subjects of the study taught at seven schools in Alahsa, Saudi Arabia. This section provides background for the research in regards to the financial and educational resources of the seven schools, as well as the physical structure of the school buildings. Throughout this research, shortened symbols are used to refer to the schools and the teachers to protect their privacy and to assure that the identity of the teachers cannot be tracked in any way. The schools were designated by numbers ranging from 1 through 7. The teachers’ names were chosen by a combination of their first and last initials and the number of their school. For example, AL.3 is a teacher from school 3.

All of the seven schools are under the supervision of the Ministry of Education in Saudi Arabia. However, some of the schools were specially built by a private company
contracted by the government. These schools are distinct from the others in their level of resources and the building facilities.

School 1. This school is located in a relatively poor area as compared with the other schools in this research. The middle school is located on the third floor of a building that also includes an elementary school. There were 191 students at this school and 21 teachers. The teaching staff includes three mathematics teachers. The average number of students in each classroom is 28. The sampled teachers from this school are TH.1 and MO.1. As it is located on the third floor of the building, this is a relatively small school; thus, it has no library, or mathematics and computer labs. This small amount of space results in teachers complaining about the lack of resources within their school. This has an impact on their manner of teaching and limits the use of effective teaching strategies. TH.1 asserts this condition as follows

I love doing activities. The custom here [in this school] is to do PowerPoint presentations but I only like doing activities such as discovery activities. For example, if we had a problem we could normally go to the library and read about it…but we do not have a library. Also too many students means little interaction.

(TH.1, December 11, 2011)

In the following quote, MO.1 illustrates that the level of the students in this school is unsatisfactory and is preventing her from attaining her teaching goals and enjoying teaching. In her opinion, the parents and the elementary school are not doing their job in preparing children to enter middle school.
I used to love teaching but with these students it is very difficult because they hate this subject [mathematics], as do the parents because they do not understand it ...Students’ level of understanding becomes worse than before because when they were in elementary school their teachers did not give them accurate assessments and they gave them higher grades than they deserved. They went to middle school when they should not have been eligible. (MO.1, December 11, 2011)

**School 2.** This middle school is located in a new building that was recently constructed by the government. The average number of students in each classroom is 28. The school has a total of 196 students with 14 teachers, two of whom are mathematics teachers. Many known business investors of Alahsa and their families live in the region where the school is located. In addition, wealthy families are traditionally interested in building their houses in this region because it is a peaceful area. Teacher DA.2 teaches at this school. There are other three middle schools in the same community, one of which is School 3.

**School 3.** There are 376 students at this school, with an average of 28 students in each class. The school has 29 teachers, three of whom are mathematics teachers. As opposed to Schools 1 and 2, this school was built by Saudi Aramco¹, a company contracted by the Saudi government. AL.3 is one of the teachers in School 3. AL.3 discusses the pressure of having the mathematics resources needed but not being able to implement them in her teaching: “I wish that we had responsible proctors for the

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¹Aramco is the Saudi Arabia’s national oil company. The company has several investments, one of those investments is within the educational sector in Saudi Arabia. There are numerous schools in Alahsa that are built by this company and are distributed evenly throughout the city. Teachers and students are considered fortunate to be part of this community of schools because of its excellent conditions and extensive educational recourses.
mathematics resources center in the school, in the same way that the science lab does” (AL.3, December 13, 2011).

**School 4.** It is the biggest middle school in this research, both in terms of the size of the building and the number of students. It is located in a middle-class community. School 4 has a total number of 510 students with 35 teachers, four of whom teach mathematics. The typical classroom contains 32 students. Participants HE.4, NO.4, and MA.4 teach in this school. Although School 4 was also built by Saudi Aramco, teachers explained that there is a lack of mathematics resources and that the computer labs are often full. This prevents them from doing creative mathematics activities in their teaching, as shown in this quote:

> For example, there is the problem of lack of availability of educational resources. Posters with graphs are not available so I provide them for students by copying graphs on the copy machine and then enlarging them. Hands-on manipulatives are not available for each individual. (HE.4, November 30, 2011)

**School 5.** This school is located within a low- to middle-class socioeconomic community. As opposed to all other schools in the study, this school was in the transition of changing principals and during this research, there had not yet been a principle assigned. In terms of the school’s physical layout, it occupies the majority of a three-floor building. There are 349 students and an average of 32 students in each classroom. There are 27 teachers, 2 of whom teach mathematics.

In this school, mathematics teachers HI.4 and MA.4 participated in this study. Being that they were the only two mathematics teachers in the school, they had
overloaded schedules, which prevented them from trying different approaches to teaching mathematics. Due to their busy schedules, they were concerned that they would not have much to add to this research. The teachers did share their concerns about the socioeconomic level of the students and how that impacted their level of appreciation of the value of education and specifically the value of learning mathematics, as stated:

Students are not thinking and they do not try and they depend either on the teacher or on their group leader (during group activity). There is no solution for that...maybe the socioeconomic status of the area around us. Students only want to move to the next grade and do not want to think (MA.5, December 10, 2011).

**School 6.** This school was also built by Saudi Aramco and has the same excellent educational resources as Schools 3 and 4. The school has a total number of 376 students with an average of 32 students in each class. There are three mathematics teachers from a total of 27 teachers. Teachers in the school indicated that they are satisfied with the overall school condition and the availability of educational resources.

**School 7.** This school has a total of 398 students, with an average of 35 students in each classroom. There are four mathematics teachers in the school. Three of them were highly rated in terms of their yearly academic ranking. Participant AR.7 was evaluated relatively lower than her colleagues in the same school.

**The studied teachers.** As was indicated in Chapter 3, the teachers vary in terms of their yearly academic ranking, which is evaluated by the Ministry of Education in Alahsa. However, because of the confidentiality of such information, the only information that was able to be obtained was who received an A in the year of 2011 and
who did not. Furthermore, the teachers’ educational background is not included in the table since all of the interviewed teachers hold a bachelor’s degree in mathematics. Table 1 provides description of the demographic information of the participants.

Table 1
Demographic Information of the Participants

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Experience (years)</th>
<th>Academic Ranking</th>
<th>Grade Levels Taught</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH.1</td>
<td>Missing</td>
<td>Not A</td>
<td>MS only</td>
</tr>
<tr>
<td>MO.1</td>
<td>19</td>
<td>A</td>
<td>MS only</td>
</tr>
<tr>
<td>DA.2</td>
<td>6</td>
<td>A</td>
<td>ES, MS, and HS</td>
</tr>
<tr>
<td>AL.3</td>
<td>14</td>
<td>Not A</td>
<td>ES, MS, and HS</td>
</tr>
<tr>
<td>MA.4</td>
<td>9</td>
<td>Not A</td>
<td>ES, MS, and HS</td>
</tr>
<tr>
<td>HE.4</td>
<td>5</td>
<td>A</td>
<td>MS only</td>
</tr>
<tr>
<td>NO.4</td>
<td>4</td>
<td>Not A</td>
<td>ES, MS, and HS</td>
</tr>
<tr>
<td>MA.5</td>
<td>16</td>
<td>A</td>
<td>MS only</td>
</tr>
<tr>
<td>HI.5</td>
<td>10</td>
<td>A</td>
<td>ES, MS, and HS</td>
</tr>
<tr>
<td>RA.6</td>
<td>5</td>
<td>Not A</td>
<td>MS and HS</td>
</tr>
<tr>
<td>TA.6</td>
<td>16</td>
<td>A</td>
<td>MS and HS</td>
</tr>
<tr>
<td>AR.7</td>
<td>14</td>
<td>Not A</td>
<td>MS only</td>
</tr>
</tbody>
</table>

Note. ES = Elementary School, MS = Middle School, HS = High School

I. Findings from Observation and Initial interviews

**The coding procedure.** The data resulting from the initial interviews transcription and the observation field notes were analyzed using open coding. I started my open coding during the data collection process and continued it after completing the data collection process. Because grounded theory procedure involves constant comparison of data, the comparison of the two data sources resulted in finding common
open codes that dominate across the field notes and the interview transcription. This commonality results in my decision to continue with the open coding process for both data sources.

After reading each interview three times (one of which was during the transcription phase), I started by listing the overall codes that might have the potential for being included in the grounded theory. Initially I did line by line open coding. During this phase, many codes appeared consistently throughout the interview transcription and the field notes. The early coding stage unfolded categories such as *semi-discovery problem solving approach*, *declarative connection*, *limitation of resources*, *presentation technology*, *parental anxiety*, *problem solving approaches*, *knowledge in formal proof*, and *reasoning in terms of formula writing*.

After completing the early coding stage, I began working on axial coding. In this stage, axial coding was used to discover relationships between the open codes. Axial coding, as defined by Strauss and Corbin (1998), is “the process of relating categories to their subcategories, termed ‘axial’ because coding occurs around the axis of a category, linking categories at the level of properties and dimensions” (p. 123).

During the axial coding stage, I worked on connecting major open codes with each other to develop themes and their categories. In the early stages of open coding, the category *guided problem solving* dominated throughout the transcriptions and field notes. Many properties that were coded are centered on this category. These properties are *semi-discovery problem solving approach*, *limitation of problem solving multiple solutions*, and *students’ understanding level*.
Themes. The researcher performed multiple stages of axial coding. These stages helped me to link and group categories that produce similar ideas. The last stage of axial coding resulted in four main themes. These themes can provide understanding of the Saudi teachers’ knowledge of students’ learning and how they use this knowledge to promote students’ engagement in the mathematics processes, as developed from the observation and initial interviews.

Theme 1. guided problem solving. Three major categories are related to the first theme. The findings under this theme explain the Saudi teachers’ problem solving approach as they indicated through observing them and as they explained during the interview. This theme is composed of three categories. The overall nature of the categories is to give students guidance into solutions when engaged in problem solving.

Semi-Discovery Problem Solving Approach. One of the main categories that was frequently addressed throughout the data is the teachers’ overwhelming use and emphasis on what I called “semi-discovery problem solving approach.” In this regard, participants demonstrate the importance of engaging students in problem solving activities by giving them problems to work on and discovering their solutions on their own. However, observing this method first-hand, the data reveals that teachers are skillful in giving students a set of questions, hints and reminders to support students working their way through the given problem. In this way, students are able to deductively solve any problem because of their interaction with the teachers in answering simple questions that build upon one another to arrive at a more complex mathematical solution. The students’ direction of thinking is controlled by these simple statements that are discussed with
them; thus, the entire class is guided in the same direction toward the solution to the problem.

Through the observation, most of the participants appear to support this method for discovery problem solving. Through one of the observations, TA.6 was asking her students to work on a discovery problem that is presented in Figure 1. The following activity was given by TA.6, who is considered an excellent teacher in School 6. When I was arranging my class visit with this teacher, the school administrator helping with the arrangements commented that I would be lucky to observe such a class because of the creative way that TA.6 teaches. In my interview with her, she told me that she is interested in providing various discovery activities and believes that the goal of teaching mathematics is to help students discover solutions to real-life problems. She pointed out the advantage of developing mathematics teaching in Saudi Arabia: “Teaching mathematics has changed because now students should be working on activities in our lessons… They should be doing the activities by themselves, and discovering the solutions. Discovering is a very nice thing to do…” (TA.6, December 12, 2011).

I attended an eighth grade lesson that was about discovering the formula of the sum of an interior angle in a polygon with \( n \) sides. In the lesson, TA.6 asked students to open the book and work through the problem presented by Figure 1, which was to discover the patterns or the formula of the sum of the interior angles with any given polygon.
TA.6 presented the lesson and gave students five min to work on the problem individually. Afterward, the teacher gave the students a set of statements, definitions, and reminders on how to arrive at the final formula. These are some of her questions:

TA.6: How can we find the number of triangles in a given shape?

What is the name of the segment that we can use to divide the polygon and then arrive at the number of triangles?

With the teacher helping the students with questions, they were able to complete the table. Some of these questions were used to arrive at the formula.

TA.6: What is the relationship between the number of sides and number of triangles?

S [student]: I see that the number of sides is always less than the number of triangles by two.
TA.6: Given that, if you have a polygon with six sides, how many triangles can you find?

S: 6 – 2 = 4

Similar questions were asked until the whole class was able to deduce that the sum of the interior angles in a polygon with \( n \) sides is 180°(\( n – 2 \)).

The interview transcriptions disclosed similar points. Teachers in Saudi Arabia develop students’ problem solving abilities by supplying them with sequential sets of simple questions and supporting mathematical statements to arrive at a complex solution. NO.4 illustrates this problem solving approach as well. She argues that the more teachers ask questions, the more she is reminding them of past similar problems, and thus the more students will be able to solve the current problem. In her words:

The role of the student has changed from a recipient of information to participant. She becomes a helper in deducing the information and arriving at the final answer [of a mathematical problem]. The role of teachers is only to ask questions and the students’ is to answer them. (NO.4, November 28, 2011)

The following quotes are illustrated by participant teachers regarding their use of semi-discovery problem solving:

- AL.3: When they solve a problem ... I arrange the questions that will be asked to the students. These questions help students arrive at the conclusion and solution. (December 13, 2011)

- DA.2: Deductive method and self-learning helps middle school students a lot and enhances their confidence in themselves. (December 17, 2011)
• AR.7: Of course, I enjoy teaching, depending on the lesson – if the lesson contains many conclusions and deductions to make from students, I find the lesson to be fun. However, if the lesson is new to students, I do not like this lesson. (December 5, 2011)

RA.6: I encourage mathematical reasoning. If one student comes to the board, I critique what she is writing. I ask the students to solve the problem part by part and I discuss each part of the solution with them. (December 14, 2011)

• MA.5: Deductive method helps students and it is the most important method. Also, I encourage interactive learning. Often I use the deductive method because it helps students to gradually understand the lesson. (December 10, 2011)

Partial opportunities of alternative solutions. The transcription of the interviews discloses how teachers hold themselves accountable for giving their students the opportunity to choose their own ways of solving particular mathematics activities. Teachers in Saudi Arabia believe in the power of having the freedom to devise the students’ own way of thinking and problem solving. MA.5 demonstrated how she stands firm that her students should have this opportunity of thinking and solving problems in their own way:

I give them a chance, when a student asks a question or offers an idea. I give her chances to say what is on her mind. Many of the students devise their own ways for the solution, which happens a lot. I love the student who writes what she understands. Mathematics does not depend on one way, so, I give her a chance. (MA.5, December 10, 2011)
The data of the study reveals two properties of this category: specific students and specific methods. These two properties describe the category of *partial opportunities of alternative solutions*.

The property *specific students* expresses how teachers allow students to use multiple methods to solve problems but students also have the freedom to use the solution taught by their teacher. My observations showed that students are welcome at any time during the lesson to comment on their own way of solving a particular problem. However, because it is something totally dependent on the students’ initiative, teachers think that it is something extra that they can do in their free time. The following quote illustrates this point.

I told her [the student] that she is creative because she brought a new way to the solution. I gave the students with creative answers extra points and at the end of the month, I calculate how many points they received. The first who gathers 40 points received special recognition. (RA.6, December 12, 2011)

The property *specific methods* demonstrates how the students are provided with alternative solutions, but these multiple solutions are chosen by the teacher. In one of the interviews, AL.3 is a teacher with 14 years’ experience and, according to the school principal, always scores below her colleagues in terms of her yearly teaching rating. When I asked her about how she gives the students the ability to use their own way of thinking in order to develop their problem solving abilities, she agreed that she is always giving her students the power to improve their problem solving by choosing their own methods when working on a particular problem. She accomplishes that by demonstrating
to the students a set of algorithmic methods, from which they can choose one to work with. The following quote is from my AL.3 interview, where she discusses how her students prefer to devise their own ways of solving mathematics problems:

I give them more than one idea of the lesson. I want them to be encouraged and I say to the student that her mistake can be resolved so she can be encouraged and discuss it [the solution] with us. Sometimes I like to compromise with the student in order to complete her answer. Sometimes students choose the way it is easier for them. (AL.3, December 13, 2011)

In one observation, MO.1, an excellent highly experienced mathematics teacher, was giving a lesson on a proportional reasoning chapter for seventh grade students. The topic was about testing whether a statement is proportional or not. She started with a warm up that illustrated the definition of equivalent ratio. Further, MO.1 provided students with opportunities to choose between two solutions to verify whether two ratios are equivalent or not. Cross product and simplifying the ratio by multiplying or dividing were the two tests that were taught by the teacher in the lesson.

At a moment, students were able to use both ways to solve the same problem. The students in her class were interested in using both tests when working through classroom activities that involved the concept. After a while, many students had realized the difficulty of one way of testing proportion or the other. That was clear when some of the students sitting next to each other insisted on using one direction for them.

MO.1 gave her students the ability to use two ways to test equivalent ratios. She insisted on teaching them both ways and spent a substantial amount of time to work with
both ways. When she gave them some activities to work with, she was directing her students’ attention by saying “try both tests please, but at the end, I want you to be able to tell me why you chose one way or another. I also want you to justify your choice”

Limited implementation of cognitively demanding tasks. This third property describes Saudi teachers’ knowledge of how to support students’ problem solving. The data of the study has revealed certain limitations on how the teachers are considering implementing high level tasks.

The property selectivity indicates teachers’ knowledge in implementing cognitively demanding tasks as something to be available for students as an option and that is something only supplementary for their learning. Some of the teachers indicated that students should be accountable to look up some high level activities and work with them by themselves to improve their mathematical thinking while other teachers see it available in their textbooks for students to work with and then can show it to the teachers in their free time. The interview questions did not include direct questions about high level tasks but teachers elected to indicate it as something important but not essential for their students’ learning. The following quotes that came from various teachers regarding high-level tasks illustrate this point:

- RA.6: I sometimes give them [the students] problems that provide connection of mathematics with their real life. These are excellent elective questions and these are not obligatory. They come to me outside of the class period to discuss their answer with me and they will gain extra points and become distinguished students. (December 12, 2011)
• MA.5: In the book there are higher-order thinking problems. The students love them. International tests need training from the students and do not depend on the teachers’ efforts. (December 10, 2011)

• AL.3: The students now have several classroom activities they can work on…there are activities such as higher-order thinking problems and real-life application problems. Students interact with them. In the past we did not know these things. (December 13, 2011)

The property inconsistent implementation describes how some teachers in Saudi Arabia have the propensity of lowering the cognitive level when implementing high thinking tasks. The discussion of this element of the findings is derived mostly from the classroom observations, not from the interview. However, many places in the interview transcription show that teachers are leaning towards lowering the cognitive level of task that is presented in their curriculum, as stated:

When they solve a problem ... I arrange the questions that will be asked to the students. These questions help students arrive at the conclusion and solution.

(AL.3, December 13, 2011)

Through one of the observed lessons, AR.7 asked her students to work individually at the problem presented in Figure 2. AR.7 is a low rated academic teacher based on the evaluation of the Ministry of Education, yet talking to her through the interview revealed her high experience in teaching mathematics. AR.7 presented this problem to her students who were just finished reviewing tasks on similar triangles. The nature of the problem
was newly introduced to the students. The potential of this task is rated as level 4 based on the IQA observation protocol.

In the problem, the teacher directed her students to use the specific formula that they have studied before. AR.7 also directed her students to use a sketch that showed how the length of the tree, the length of the dog and the length of the shadows for both objects are related to each other. Consequently, the implementation of the problem was rated as level two because the students were applying a procedure that was developed previously by the teacher. Clearly, the cognitive level of the presented task declined during the implementation stage.

![Figure 2. High-level task presented by AR.7.](image)

On a sunny day, suppose that the length of a dog’s shadow was measured to be 0.3 m and the height of the dog was measured to be 0.45 m. At the same time, supposed that you have also measured the shadow of a tree standing next to the dog. You found that the shadow of the tree was 2.2 m. Using this information, how can you calculate the height of the tree?

Participant AR.7 is not alone in this situation; seven out of twelve observed tasks in the study were rated as Level 3 or 4 for potential. This shows that teachers have the knowledge to locate activities for students that are cognitively demanding for them. However, all of these tasks were rated in the implementation as level 1 or 2. The diagnosing reasons for the decline of the level of task vary between making the problem algorithmic in nature to giving very limited time to work with the tasks.
For many of the teachers, it takes time for the students to think about these problems and to solve them on their own. The teacher always made herself accessible to her students right when the students struggled with individual activities and needed the most support. In one classroom observation, students have asked RA.6 politely to stop giving them hints and let them work with their tasks individually because they could answer their tasks without any help.

**Theme 2. declarative connections.** The interview and the observation data reveal many venues about how the Saudi teachers exhibit their knowledge with regard to supporting their students understanding of mathematical connections. This theme shows how teachers in Saudi Arabia support students’ engagement in the process of mathematical connections using a declarative way of learning about mathematics connections. Two categories describing this theme are *declarative by the teacher* and *declarative by the textbook*.

During the interview, participants reported on their ability and appreciation to help students understand the connections between mathematics topics and how these topics have real-life applications. Students can understand mathematical connections through listening to the teacher talking about them briefly at any part of the lesson or by reading their textbooks.

*Declarative by the teacher* category refers to the Saudi teachers’ approach to supporting students’ connections where they view that their students would understand it solely by listening to their teachers reporting about it during the lesson. The interview transcription shows that seven out of twelve participants demonstrate the same concept.
In this regard, teachers are promoting mathematical Connections among various mathematical concepts as well as connecting concepts with life situations. Mostly, at the beginning of the lesson, they encourage their students to see how the studied concept is going to be relevant to them by direct oral reporting about how various mathematical situations are connected together. The observations indicate that when teachers are going in this direction, there is a minimum or absence of students’ interaction with the connected topic. Three teachers have brought up an example of the real-life application of calculating percentage during the sales seasons. Other teachers have mentioned to the students that specific topics will be connected with other topics they will study in future lessons. These examples can be interesting and relevant to students’ lives but the teachers are reporting these examples without requiring their students’ engagement.

The following is a selection of quotes that discuss how teachers encourage mathematics connections among students through oral reporting:

- **NO.4**: [I encourage] the development of scientific thinking for the student where she can apply mathematics in all life situations and development of problem solving. At the beginning of every lesson I tell them that they have previously taken a concept, and now we will investigate it in a broader way. For example, students who start middle school [are reminded] that they studied the lesson in elementary school. Now we will build upon that but in a broader sense in ninth grade. (November 28, 2011)

- **HE.4**: The link to real-life application is through telling them examples. I try that and the students give me examples from their own lives. At the beginning of each
lesson I recalled and linked information to previous lessons. (November 30, 2011).

Declarative by the textbook\(^2\), on the other hand, describes how some teachers in Saudi Arabia rely on their reading of the textbooks as a way to encourage mathematical connection. For them, students can learn about how mathematics concepts are connected with each other by looking at their textbooks where they can read about how any topics have real-life applications. The observation of the participants demonstrated that all of them give students the opportunity to take a couple of minutes at the beginning of the lesson to read the opening page of the chapter in their textbook as a way to understand how the new lesson has real-life applications. Discussion with the participants shows that they do not have to worry about how to help students understand the mathematical connections embedded in the lesson because their textbooks are doing so in the opening of the lesson. In a teacher’s own words: “In the new curriculum, there is a connection to the lessons with our real life. Every lesson gives us a real-life application. We show them this connection and ask them if they know anything other than this connection” (HI.5, December 10, 2011).

In addition, TA.6 demonstrates in this lengthy quote how she uses both venues, using her brief lecturing or by allowing students to read the textbook, to demonstrate connection in mathematics. TA.6 indicates that she can support students’ understanding of connection by her declarative way of teaching. TA.6 shows that giving students examples of real-life application to mathematics would help them to understand the

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\(^2\)The middle school textbooks contain opening paragraphs in some lessons that are presented as an option for the readers. This reading discusses information on how the lesson is significant to the students’ lives.
meaning of these applications by relying on the textbook or on her declarative way of teaching about connection:

Our book provides the examples of real life, and possibly I provide other examples to the students. For example, in a lesson about scale drawing and enlarging and shrinking an object, and also the similarity of polygons, I brought to them, for example, a small miniature of [famous building] model. I told them that this is a scaling down of the [building]; I also can bring to them an enlarged image of something specific. I bring these models to them from time to time and these relate to their lives. Possibly, an artificial sample of human organs, why not? ...the student should consider the previous lessons as a continuum especially if we are studying the same field, for example, in the field of algebra or geometry I tell them … that we will look at something more in the next lesson and we will mention it more as well…. For example, we studied similar polygons and moved onto the congruent polygons; there is connection between congruent and similar polygons. We might ask whether the congruent polygons are similar at the same time? The concepts of similarity and congruence are related to each other, but each one has its own conditions…. On this basis, we can apply this idea to the rest of the mathematics concepts. Since these concepts have a great relationship between them, I try to link the lessons and concepts together. For example, equations and inequalities … What are the similarities and differences among them? (TA.6. December 12, 2011)
**Theme 3. detailed reasoning.** The third theme that emerged from the observation and initial interviews describes the teachers’ knowledge regarding how to support students’ processes of reasoning, proof, and communication. When discussing with participants about these mathematics processes through the interviews, it seems that they talk about mathematical reasoning and communication interchangeably as one concept. For them mathematical reasoning and communication at the same time means to be able to express thinking in a verbal and non-verbal manner. Discussion with the participants regarding their approach to support students’ reasoning and communication reveal that they promote students’ expressing of thinking in each step of the lesson similar to the teacher’s own way of reasoning about mathematical problems. The participants discuss that students should demonstrate their communication and reasoning processes in written and verbal manner.

*Written modeled reasoning.* This category describes how Saudi teachers support their students to express their thinking and mathematical reasoning using memorized formulas and detailed solutions similar to what their teachers have previously modeled for them. Teachers stressed the importance of having detailed steps that demonstrate how a student moves from one step of solving the problem to the other. When they learn about a formula, they should write it down using accurate mathematical terminologies when they have to apply it in a problem.

During the observation, teachers were insisting that the students should write any formula they will use for solving a particular problem. On one of the observations, the teacher asked the students who were working on a problem that involved percentage that
they should indicate the formula they used to arrive at their solution. AL.3 explained in the interview that her students should follow her own way of reasoning when solving a particular problem in the same detailed manner that she used to demonstrate to them: “First students should solve the problem in a modeled and detailed way [in the classroom]. Second, I ask the students to mention their mathematical justifications and reasoning in the test” (AL.3, December 13, 2011).

Participants highlighted the role of writing detailed solutions to problems. Students should show their understanding of the learned materials by writing their mathematical reasoning in a detailed manner similar to what their teachers have been modeling for them. The use of mathematical reasoning in the manner of writing up formulas and detailed transcripts is in fact a requirement by teachers to be included when they are testing. Such detailed solutions that students are presenting in the test would demonstrate the level of students’ understanding of the lesson. The following quotes exhibit that students must illustrate their understanding of the concept to their teachers by writing up the details needed for the solutions as was modeled by their teachers:

- AR.7: If there is a solution to the question, they should include the formula they use and this is compulsory to the students in the test. (December 5, 2011)
- NO.4: I encourage students to use mathematical proofs. For example, in solving inequalities, today we had been doing practice exercises. I asked the student about writing the reasons for moving to the next step, and it is considered as an algebraic proof. For example, this step is simplified by the associative property
and the next step by the distributive property …at each step they must indicate the reason. (November 28, 2011)

- DA.2: My excellent students do not usually focus on the detailed solution. They give me their results without the method they follow…. However, I encourage them to write their strategy. Now they get used to writing all the details of their solution. (December 17, 2011)

*Verbal reasoning.* This category of the theme modeled detailed reasoning describes how teachers in Saudi Arabia are encouraging their students’ expression of thinking as a way to demonstrate mathematical reasoning and communication. In this regard, all the students should participate in classroom discussions where they should engage in discourse regarding particular mathematical topics. This category portrays how middle school teachers in Saudi Arabia stress the substantial role of having active students during mathematics instruction.

During the interview session, the participants convey that having an active classroom is the nature of mathematics instruction. Most of the teachers have indicated that every individual in the middle school classrooms, which are occupied by between 27 to 40 students, must participate in the lesson no matter what the level of their abilities or their mathematical interests. Expressing students’ thinking verbally should be presented at each part of the lesson and at each part of working on whole class activity. When a student is demonstrating a problem solution to the whole class she should explain the steps she has followed and the reasoning she has employed. For example: “I encourage mathematical reasoning. If one student comes to the board, I critique what she is writing.
I ask the students to solve the problem part by part and I discuss each part of the solution with them” (RA.6, December 14, 2011).

The classroom observations convey similar points; the teachers put higher emphasis on the students’ expression of thinking in the classroom despite the nature of such an action to their learning.

This category is presented under the theme *detailed mathematical reasoning* because students are required to verbally express their thinking at each step of the lesson. The observation confirmed that students’ discussion takes many forms. One of these forms is answering the teacher’s questions in the warm-up activities. Four teachers reveal that their use of classroom discussion is concentrated in the introductory portion of the lesson where they encourage students to answer their questions as a form of warming up to the lesson. MO.1, as one of the four teachers, indicated that “Discussion is included in the introduction of the lesson” (December 11, 2011). Another discussion form in the classroom is about justifying their answer verbally to the teachers in each step of the solution. Even when students are in the middle of working on individual tasks, they are required to illustrate their steps and verbally express what kind of definitions or formulas they will apply to solve a particular activity. In addition, if the students are at the board demonstrating their solution, they have to verbally explicate their work to the whole class.

The following quotes from the interview transcription present the substantial role of students’ verbal expression during the instruction:
• DA.2: I encourage discussion in the beginning of presenting the activities. I concentrate on students’ expression of thinking during their work on problems. This would help the students to speak about what is in their minds. (December 17, 2011)

• MA.4: If she comes up to the board, she should explain what she is doing and she should not go back to her seat without understanding. Speaking helps the student to discover their own error. (November 30, 2011)

**Theme 4. teachers’ views of students’ mathematics capabilities.** The interview transcription and classroom observation enfold discussion of teachers’ views of how much students in Saudi Arabia are capable of achieving in learning of mathematics. Early coding stages explicate various categories such as: confidence in students’ thinking, understanding of students’ anxiety, and knowledge about students’ abilities. In the stage of axial coding, these open codes were grouped together under one theme: teachers’ views of students’ mathematical capabilities. This theme describes how teachers in Saudi Arabia view the level of middle school students’ abilities in learning mathematics. Two categories have emerged from the data which describe this theme.

*The negative view.* This category describes the Saudi teachers’ view of their students that they have less enthusiasm towards learning mathematics, especially through working on individual problem solving. The teachers express many causes to their view that students are less capable of learning mathematics. Some of these reasons are because those students were taught by unqualified elementary teachers who did not support them to accomplish basic mathematical knowledge such as computation skills. Another cause
indicates that students do not have the support system at home to show them that they are capable at learning mathematics.

This negative view of students’ capability in learning mathematics can be seen through various classroom observations. The classroom observations show that participants tend to allow less time for students to think individually during problem solving because of their underestimation of how much students can accomplish through their individual classroom activity. This category was mainly discussed by the two participants in School 1. TH.1 expressed her belief about her students in the following quote:

But the problem is that makes us stop here is the basics from elementary education. Many of the primary teachers they do not know how to teach and those students have more knowledge than them. Primary teachers have really limited knowledge and in middle school we are teaching elementary and middle school topics at the same time…I have a lot of creative ways of teaching but it is greater than their age and we are establishing concepts in middle school not completing (as it is supposed to be doing in middle school). How do I provide new ways of teaching for students who do not know the multiplication table? This is a disaster for us that the students did not learn the basics in elementary school. So far, we do not know the value of mathematics, we think it depends on the four operations …we do not have any mathematical reasoning because students are copying what the teacher is saying and apply it directly. I did not notice that one student came and said to me I solve the problem in a different way, or that this way is a wrong
way of solving a problem (this is practically impossible) (TH.1, December 11, 2011).

MO.1, the other teacher in School 1, reported:

Students’ levels become worse than before because when they were in elementary school their teachers did not give them accurate assessment and they gave them higher grades that they deserve; they go to middle school while they are not eligible. (MO.1, December 11, 2011)

The final comment that explains the teacher’s negative view of students’ capabilities when learning mathematics came from a teacher at another school. It shows that students are not able to engage in high-level thinking during problem solving:

Students are not thinking, and they do not try, and they depend either on the teacher or on their group leader [during group activity]. There is no solution for that... maybe the socioeconomic status of the area around us. Students only want to move to the next grade and do not want to think. (MA.5, December 10, 2011)

The Saudi teachers’ view of students’ capabilities especially emerged from the Initial interviews during the participants’ discussion of how they approach reasoning and proof mathematical processes. A teachers’ knowledge about how to engage students in working on formal proofs indicated their dislike to the use of formal abstract proofs. Saudi teachers view the formal proof as using symbolic mathematical language to verify the truthfulness of a mathematics statement. For those teachers, middle school is not the time for writing formal mathematical proofs. Instead, students should have the ability to justify their answers verbally or in a written manner by discussion with the teachers about
why they solve a problem in a certain way. Analyzing observation field notes did not identify any negative or positive knowledge of the role of formal proofs in students’ learning. The absence of this idea in the observation field notes more than likely takes place because none of the observed lessons could possibly have included formal proofs as indicated by the middle school textbooks. The following quotes describe the undesirable use of mathematics abstract proofs for middle grades:

- **HE.4**: Mathematical argument is always done orally but sometimes we use mathematical proof in a limited manner because we are in the middle grades. (November 30, 2011)

- **MA.4**: I direct my students’ attention to the idea of the lesson and the smart students should understand what I mean. I do not encourage the students to use abstract proofs because they are too young for these things. (November 30, 2011)

*The positive view.* The positive category describes the positive view of students’ ability in learning mathematics. This view was discussed by various participants who recognize their students to have intellectually unlimited ability when learning mathematics. They can study various mathematical concepts in an active manner. Very limited evidence from the interview and from the classroom observation can support this positive view of how much students are able to accomplish during mathematics instruction. The following quotes discuss the teachers’ positive view of their student abilities’ in learning mathematics: “My students’ thinking is always active in learning... they always come to me and show me they solve high-level thinking problems by their own without my direction” (AR.7, December 5, 2011).
In the words of another teacher: “My students are becoming more educated intellectuals. They had become more conscious and logical. A large proportion of my students have the ability of logical thinking” (MA.4, November 30, 2011).

**Emergent Grounded Theory**

The analysis of the classroom observations and initial interviews concludes four main themes in the grounded theory. Table 2 summarizes the themes, categories and properties that emerged in this study. The explication of the second part of the data analysis will help to either confirm or contradict these themes presented from the observations and initial interviews. These themes presented from the first part of data analysis will provide explanation to the Saudi teachers’ knowledge regarding how they promote their students’ mathematical processes.
Table 2

The List of Themes, Categories, and Properties

<table>
<thead>
<tr>
<th>Themes</th>
<th>Categories</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guided Problem Solving</td>
<td>Semi-discovery problem solving approach</td>
<td>Specific students and specific methods</td>
</tr>
<tr>
<td></td>
<td>Partial opportunities of alternative solutions</td>
<td>Selectivity and inconsistent implementation</td>
</tr>
<tr>
<td></td>
<td>Limited implementation of cognitively demanding tasks</td>
<td></td>
</tr>
<tr>
<td>Declarative Connections</td>
<td>Declarative by the teacher</td>
<td></td>
</tr>
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<td></td>
<td>Declarative by the textbook</td>
<td></td>
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<tr>
<td>Detailed Reasoning</td>
<td>Written modeled reasoning</td>
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<td></td>
<td>Verbal reasoning</td>
<td></td>
</tr>
<tr>
<td>Views of Students’ Mathematical Capabilities</td>
<td>The negative view</td>
<td></td>
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<tr>
<td></td>
<td>The positive view</td>
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</table>

II. Findings From the Scenario-Based Interviews

The coding procedures. This section is composed of the analysis of the scenario-based interview discussed in Chapter 3. For this portion of data, the interview transcriptions were coded using open and axial coding, similar to the analysis of the observation field notes and the initial interview. Each teaching scenario was coded separately using the same procedure. Because the transcription was brief for each teaching scenario (as compared to the initial interviews transcription), I was able to identify only one level of categories for most teaching scenarios. These categories described the overreaching findings that were presented across all participants. The next
chapter will discuss in detail how the two parts of the findings provide understanding for the Saudi middle school teachers’ knowledge on students’ mathematical learning and how their knowledge promotes students’ mathematical processes.

Findings from the Rental-Car scenario. As with all teaching scenarios in the study, the intention for presenting the first scenario to the participants is to get an understanding of the teachers’ instructional strategies that are embedded in this artificial teaching scenario (see Figure 3). I had to explain to the participants that the intention is not to see whether they are aware of how to answer the question or not but to get to learn about their possible teaching strategy and how they have knowledge about supporting students’ learning. Nonetheless, all the teachers in the study were able to arrive at the correct solution or the correct possible strategy for arriving at the solution of the task.

With regard to the teachers’ own solution approach to the problem, all of those in the study have followed algebraic representation to arrive at their own solution. The differences within the participants stemmed from their knowledge about students’ learning in mathematics and how they promote the students’ engagement in the mathematical processes. Three categories have emerged that indicated the teachers’ knowledge on how to support students to experience various mathematical processes.
Imagine that the textbook presents the following problem to help students explore the concept of function:

*Your family will need to rent a car to go on a family vacation. Two rental companies provide different prices:*

Company A: 39.95 Riyal, but they charge an additional 0.19 Saudi cents per kilometer  
Company B: 19.95 Riyal, but they charge an additional 0.49 Saudi cents per kilometer

*Determine for how many kilometers the two companies will cost the same. After how many kilometers, will Company B have a lower price?*

Figure 3. The Rental-Car scenario.

**Numerical representation or computing the table is more guided than other approaches.** Teachers attempt to find a way to present the problem to the students by making it easier for them. Because those middle school students may see this problem as challenging for them, participant teachers have decided that they could direct students to use numerical solutions to the problem. Nine out of twelve teachers think that they could do that by giving students guidance in computing tables to discover how many kilometers it takes for the two companies’ services to cost the same price.

TH.1 was one of those teachers, who assumed that her students would feel excited about working on the problem using the numerical approach because there is a sequentially organized way of achieving the solution to the problem. TA. 6 discussed in the interview that using this sequential approach of filing out numerical information could help students to be organized in their thinking and understand the problem better. She brought in a connection between this arrangement of numbers and organizing the students’ study subjects in their school schedule:
Students would like to use the table to solve the problem because it is organizing their solution similar to how their class schedule is organizing their time, even when I teach students about the inputs and the outputs for functions. If the table organizes the inputs and outputs of the function it is easier for them. However, using graphs or using an algebraic equation for the solution would be very hard for them. (TA. 6, December 12, 2011)

Some of those teachers who consider that their students would like to use the numerical way for solving the problem indicated that they would direct their student to the solution method even though the students might have the freedom to choose their own way of working through the problem. MA. 4 indicated her preference about using tables because it is her approach to any demanding problem in mathematics. “I love to tabulate my answer for any problem. The students will estimate their method but I will require them to use a table to solve the problem…tabulation can organize the answer. In the beginning students do not know how to work with equations” (MA.4, November 30, 2011).

**Graphical representation is not suitable for middle school students.** Exposing participants to this problem reveals less of a preference toward the implementation of graphical representation to teach the concept of function. The teachers argue that it would not help their students to understand the concept of function by representing the function on the Cartesian coordinate plane. Fifty percent of the participants believe that using graphical representation to teach the concepts of function can be confusing for students because of the complicated steps of implementation.
One of the participants would not suggest this method for her students because her students do not like using the coordinate system. For most of these participants who disagree on using graphical representation, using numerical representation will be enough for students to help them understand the pattern of the function. Another teacher mentioned that she would avoid demanding that her students approach the problem in this scenario graphically to arrive at the solution because such a solution approach would require a specific kind of intelligence that she is not seeing through her teaching of middle school mathematics:

Students can use many methods to arrive at the solution, such as table or algebraic representation. However, a very few number of students use graphing representation because those students who lean toward a graphing strategy have a special kind of intelligence. I missed this kind of thinking and most of our students are abstract in their thinking. (TA.6, December 12, 2011)

**Semi-discovery problem solving approach for teaching students the concept of function.** A portion of the participants in the study plan their instruction for the artificial teaching scenario by using the *semi-discovery problem solving approach*. In this regard, the teacher could give the students the problem and ask them to solve it individually and then give students a series of reminders so that they could be able to achieve the solution to the problem.

HI.5 indicated that she would achieve her semi-discovery problem solving approach by requiring students to divide the problem into two parts, which could be done within two days. The first part would answer the question: *Determine for how many
kilometers the two companies will cost the same? The second part is answering the question: After how many kilometers, company B will cost a lower price? Doing that would help her to give a reminder or review about linear equations and then they could work out the problem. The second half would require her to give a review on inequalities and then require her students to employ her review to solve the second portion of the problem.

AL.3 would devote herself to reading the problem a couple of times to her students and break it down into small parts so that her students would know how to work through the problem.

**Findings from the Adding and Subtracting Integers scenario.** I presented the teaching situation to the participants to examine how they would discuss their teaching strategies that unfold to help students to develop an understanding of the adding and subtracting of integers (see Figure 4). The data analysis reveals one overreaching category that was presented across most of the participants.

**Teaching middle school integers requires visual representation and making real life connections.** Most of the participants have agreed on the use of visual representation to teach the concept of integers for seventh grade students. “It is not because the concept is difficult, but because there is confusion from the term negative” (MA.4, November 30, 2011).

This is why teachers have indicated that supporting students to understand integers and addition and subtraction of integers requires them to implement visual representation
such as using counters and the integer number line to show how to add and subtract integers.

You are going to teach a lesson about adding and subtracting integers. You find that the textbook presents three models to help students understand the concepts of adding and subtracting negative numbers: by the number line of integers, by using counters, and by using algebraic formulas of adding and subtracting.

Figure 4. The Adding and Subtracting Integers scenario.

Furthermore, all of the teachers have indicated that visual representations are not enough on their own. The teacher should plan on some real-life examples that demonstrate integers conceptually. The suggested examples from the participants are such concepts as temperature, gaining and losing money or building’s floors.

For those teachers, real-life situations and tangible manipulatives are important representations for students to help them reinforce the meaning of negative and positive integers. This would be essential in the introduction to a lesson that involves the addition and subtraction of integers. The following statements illustrate how teachers approach this:

- NO.4: These representations are important because the students imagine the problem before they think about it logically or algebraically. She will think about it as a tangible situation. (November 28, 2011)

- TA. 6: The counters would be great for my students because the student would move the counter back and forth and this is nice for them. Also, if we connect the
positive integers with money gain and the negative integers with money loss and we could give them other real-life examples…also by using this connection for teaching addition. The students would understand addition very well. (December 12, 2011)

**Findings from the Which Is More scenario.** The participants in the study were presented with the Which Is More scenario (see Figure 5). Conferencing with the participants around this teaching scenario helps the researcher to learn about the participants’ instructional approach of teaching proportional reasoning to middle school students. This teaching scenario helps the researcher to learn about how the participants would engage students in these mathematical processes if they would present this problem to their students. The data of the study reveal three major categories.

*Algorithmic approach to the problem.* In this problem the teachers have agreed on its benefit to teach proportional reasoning to middle school students. They also agreed that solving this problem by students can be achieved algorithmically. The phrase *which class has more as compared to the whole class* in the problem suggested to them that their students should compare each quantity using fractions. “Students will know that comparing here means ratio. The students would know that by middle school” (MA.5, December 10, 2011).

NO.4 also indicated the same thing. She suggested that the problem can be done in the introductory part of the lesson. She agreed that students would want to compare two quantities by following algorithmic procedure to grasp proportional reasoning:
The first and second case is easy for students and does not require a lot of thinking from students. I guess I would give my students these two cases in the lesson introduction if we reach the chapter on ratio and rate. I will use it to help students understand the ratio. I will clarify to them that comparing two quantities can be done using fractions. (NO.4, November 28, 2011)

Facilitating the cognitive demanding and overcoming students struggle. Two ideas of the data show that teachers are intending to overcome students’ struggle when this task was given to them. They are doing this by allowing the problem to be presented
during the applying phase of the instruction and by suggesting an easier similar example that can be substituted for the problem.

The first suggestion shows that participants in the study tend to teach the lesson by their own way on how to compare two quantities multiplicatively and then give them this example as an individual exercise to the lesson. This teaching strategy would support students in their understanding of how to work through the problem and would support the teacher in explaining this problem to the student. MA.4, HE.4 and HI.5 indicated that to support their students to approach this problem fluently, they would give students this problem after the main lesson has been discussed regarding how to compare two ratios:

If I ask my students about how to compare two groups, they will say that comparison can be done using a greater and less than kind of analogy. They will not compare the two groups proportionally. Hence, I will use this example as an application exercise after teaching the main concept of the lesson.

Another suggestion from the participants spans around giving students easier problems than the problem in the scenarios to help students experience the proportional reasoning with more relevant problems. These examples are supposed to be more relevant to the students than the ones presented in the third scenario. TH.1 suggested the following problem: “Sara read six books in two days but Nora read 7 books in 3 days”. In her opinion, this problem can make it easier for students to see the clear comparison between Sara and Nora in the example.

Five other teachers suggested substituting examples for the problem, rather than the one presented in the scenario, to help students see the connection between comparing
groups and proportional reasoning. MA.5 suggested that she might ask her students to compare the number of girls who have an interest in mathematics in their classroom with the number of girls who are interested in mathematics in another classroom and see which class has the most students who like mathematics. For those teachers, such modifications would help students more to think about proportional situations:

I try to simplify the idea for my students because this example is not easy. I will compare the number of students who took an A and A- in our classroom with the students who took an A and A- in another classroom within the school. (HE.4, November 30, 2011)

The difficulty of proportional reasoning is beyond teachers’ control. Teachers in the sample indicated that their teaching strategies and their instruction are not the main reasons for students’ misunderstanding proportional reasoning when comparing quantities. They argue that the difficulties came from things that are not controlled by the teachers such as students’ mismemorization of the times table or misplacing the numbers of the denominator or numerator (if we consider that the students are using cross product method to compare two ratios). The following quotes illustrate the teachers’ idea of the students’ reasons for struggle in understanding proportional reasoning:

- TA.6: When students are comparing two quantities she might mess up. For example instead of writing a/b, she might write b/a. Writing up ratios is problematic but proportional reasoning is an easy concept for my students. (December 12, 2011)
• HE.4: The multiplication tables are the problem in this situation. We suffer extensively from that and we do not want them to use calculators. (November 30, 2011)

• DA.7: My students have many problems with memorizing the multiplication and division tables. In addition, imagining the concepts half and quarter are challenging for some students and most of these problems have to do with their elementary education. (December 17, 2011)

• HI.5: The arithmetic is the problem for my students. The proportional concept is an easy one for my students. (December 10, 2011)

**Findings from the Perimeter scenario.** The final teaching scenario was a fencing task that involved perimeter (Stein, Smith, Henningsen, & Silver, 2009, p. xvii; see Figure 6). The task helped the participants to explore how they could teach the connections between area and perimeter within a real-life situation. The tasks helped the participants to explore how they could support students’ connections between the concepts of perimeter and area. One category emerged from the interview transcription of this scenario.
Ms. Brown’s class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep rabbits.

a. If Ms. Brown’s students want their rabbits to have as much room as possible how long would each of the sides of the pen be?

b. How long would each of the sides of the pen be if they had only 16 feet of fencing?

c. How would you go about determining the pen with the most room for any amount of fencing?


Figure 6. The Perimeter scenario

*Whole class activity to support students’ connecting and discussing concepts.* In this scenario, most teachers have agreed in giving this problem as a whole class problem solving activity. As stated by one teacher:

I would use this problem in the beginning of the lesson. I would ask students gradual questions about the definition of polygon…or…rectangle… And ask them about the definition of perimeter that they already studied in elementary school. Maybe I will do that in the introduction. I will ask my students to give me all the possible numbers that give me the largest area…drawing would be essential here…I will draw the rectangles and next to them I would record all the lengths that my students gave me…we would also need to approach the problem algebraically at the end of the lesson. (MO.1, December 11, 2012)

Participants have suggested whole class activity because they assume that their students would not see the connection between the concepts of area and perimeter in the
problem that requires finding a maximum length of the sides of rectangular rabbit pen. This is because most of those participants have struggled themselves to work through the problem when I introduced it to them. The teachers indicated that they would use questioning techniques with the whole classroom to help students pay attention to the two concepts together and how they can be used to solve the problem:

I would present to them area as coloring the whole figure where as coloring the edges of the figures means perimeter and this will make the connection for them.

(MA.5, December 10, 2011)

AL.3 demonstrated that her students should see the connection between the concepts of area and perimeter in the problem and to be able to apply their understanding in real-life situations as she said: “This is an interesting question for my students. It would help my students to see the connection between the area and perimeter of our own classroom this year and the year before it” (TH.1, December 11, 2011).

Furthermore, some participants explain their choice of having this problem as a whole class group activity because it would generate classroom discussion and support students to express their solutions. As one teacher explained:

As in any mathematics lesson, discussion would be in each step of the solution to the problem. We would not continue to the next step unless everybody understands it. Mathematics is about discussing students’ thinking. (TA.6, December 12, 2011)

TH.1 pointed out the importance of classroom discussion for helping students understand this problem: “Discussion would be from my students’ thinking and I would not want to
only ask and she gave me the answer. I would want her to ask me and I will answer her questions” (TH.1, December 11, 2011).
Chapter 5: Discussion of Findings

Generalizability and Applicability of the Qualitative Findings

This study explored the three research questions using three data sources: classroom observations, initial interviews, and scenario-based interviews. The first research question addressed studying how Saudi teachers practiced their knowledge of student mathematical learning as they support students’ engagement in the mathematical processes. Chapter 4 presented the findings that generated a grounded theory of the teachers’ knowledge of students’ mathematical learning. The grounded theory emerged from the classroom observations and initial interviews. The data presented in the grounded theory was used to answer the first research question. The second and third questions addressed studying Saudi teachers’ knowledge that they reveal across the three data sources. Chapter 4 also presented findings that came from the scenario-based interviews. The first three sections of this chapter aim to respond to the three research questions.

I include phrases such as “Saudi teachers do” or “Saudi teachers do not do” as a part of my discussion about teachers’ knowledge and perspectives on how they would support students’ learning. Throughout the elaboration of the first three research questions, the reader may view these phrases as an attempt to generalize the findings to generate results of the whole population of Saudi middle school teachers. Glaser and Strauss (1967) commented on the level of applicability of grounded theory based on including “vast number of diverse qualitative facts on many different situations in the area” (p. 243). These facts allow the reader to judge how applicable the findings are to
the general population. Answering the research questions by including diverse number of facts and variables can help the reader get insight into the teachers’ practices and ideas about teaching in Saudi Arabia. The researcher endeavors to supplement the reader with the thick descriptions of the “facts” surrounding each interpretation that I am claiming in this chapter. As it is the nature of qualitative inquiry, through this chapter I invite the reader to judge the applicability and generalizations of the discovered results to his or her own research focus (Glaser & Strauss, 1969).

Being aware of the issues above, the chapter is organized in the following manner. The first three sections offer discussions, interpretations, comparisons, and triangulations of the findings in light of literature review. The chapter offers general interpretations of the whole picture in the study. Implications and recommendations are presented at the conclusion of the chapter.

Knowledge for Engaging Students as Practiced

In keeping with the first research question of this study, one of its substantial objectives was to understand how female middle school teachers practice their knowledge of student mathematical learning in their instruction with regard to how to promote students’ experiences of various mathematical processes. To be able to highlight the teachers’ knowledge within the classroom, the discussion derived from the classroom observation findings only used to underline the Saudi teachers’ knowledge in practice.

This section provides interpretation of the themes in the grounded theory: guided problem solving, declarative connection, detailed reasoning, and teachers’ views of students’ mathematical capabilities. The four themes will be guiding my discussion of the Saudi
teachers’ position on how they support the students’ mathematical processes. In addition, the discussion of the teacher’s position will exclude the categories presented in the second part of findings.

To offer a clear picture of the Saudi teachers’ knowledge on how they support students’ mathematical processes in the instruction, this section will explicate each process separately. The discussion will be presented in relation to the literature presented in Chapter 2.

**Problem solving.** A major finding that emerged from the grounded theory presented in Table 2 is related to the teachers’ knowledge of students’ learning that promotes students’ engagement through the process of problem-solving. The grounded theory in this regard specifies three emerging categories that can be implemented to explain the teachers’ understanding on how to support their students’ problem solving approaches. These categories include *semi-discovery problem solving approach*, *limited opportunities of alternative solution* and *limited implementation of cognitively demanding tasks*. The interpretation of these categories and their relation to the teacher’s exhibited knowledge is examined in the following paragraphs.

The first category that emerged from the observations and the initial interviews is the teachers’ approach to implement what I called *semi-discovery problem-solving approach*. Through some of the observed lessons, teachers have accomplished their understanding of students’ learning by allowing them to experience mathematical tasks on their own. Through learning by discovery problem solving, students should have the opportunities to experience control over their learning by engaging in problem solving.
that supports their learning processes (Bruner, 1960). Through Chapter 1, discussion about the importance of learning by discovery problem solving and how it impacts students’ transfer of learning was presented. Chapter 1 also discusses literature that advocates for the empirical role of full engagement in the mathematical process, which can be achieved through discovery problem solving (Bloom & Broder, 1950; Bruner, 1960; NCTM, 1980). The first category was titled in this manner to highlight the Saudi teachers’ intention to engage their students in discovery problem solving, which they expressed through their teaching.

The Saudi teachers exhibited limited knowledge on how to achieve discovery problem solving as they intended. The first category of problem solving discussed in Chapter 4 shows that teachers focus on giving students sequential questioning, reminders and hints to support them to arrive at the solution that the teacher intends for. Instead of having discovery problem solving, where students can experience open opportunities to invent their solution strategy and are able to synthesize and apply their learning in new situations, they would embed aspects to their instruction, causing the problem-solving process to be less beneficial to students’ learning. Such roles alter students’ learning in a way that makes them focus on achieving a particular solution to the problem rather than aiming for acquiring thinking skills, as should be the case with discovery problem solving (Bruner, 1960).

The Saudi teachers’ problem-solving approach in the instruction can be interpreted as a limitation of the Saudi teachers’ knowledge of students’ learning because of many aspects. One of these aspects is that students’ learning from this problem solving
process will be questioned because their thinking was guided through problem solving. Classroom discussion based on solving problems using the *semi-discovery problem solving approach* might not have the advantage for students to think deeply of the problems, which could possibly result in less conceptual understanding. This is because students’ thinking in mathematics instruction is significant for them to be able to acquire deep understanding of the problem (Rasmussen, Zandieh, King, & Teppo, 2005; Carpenter & Leher, 1999). Another aspect that drives limitation of the teachers’ problem-solving approach is that students’ learning would become based on students’ recalling of information and answering questions that are presented by their teachers, which contradict with the teachers’ intention in this case to allow students to work on the problem on their own and devise their own solutions.

The second category presented the teachers’ knowledge on how to direct students to use solution strategies during problem solving. Through Chapter 2, literature studies were presented on the characteristics of mathematical instructional practices that foster students’ learning in mathematics. A major finding was drawn from Franke, Fennema and Carpenter (1997) who indicate that classroom environment should have the essential features to provide opportunities for students to use multiple and nonalgorithmic solution strategies to solve problems. Literature that explores such a factor reveals that it is significant to provide students the power to their thinking, creativity and unlimited abilities (Hiebert & Grouws, 2007). The category *partial opportunities of alternative solution during problem solving* discusses the Saudi teachers’ knowledge on giving students’ opportunities to think and investigate their thinking with two directions of
limitations. The first limitation stems from the property *specific students*, which shows that not all students are required to experience their opportunity of solving problems on their own. If students are interested and motivated enough to try multiple approaches to solving problems in their own ways, then the teacher is welcoming their intent. On the other hand, if the students are not interested in being creative in their problem solution strategies, then they would find a direction to work on their activities using algorithmic pre-taught strategies. The second limitation stems from the property *specific method*, which indicates that Saudi teachers’ exhibit the knowledge for providing students freedom to work on problems, but such opportunity is limited to the teachers’ own solution suggestions. This limitation reveals that students’ solutions are pre-structured by the teachers’ own approaches instead of allowing students to experience ways of solving the problem beyond what their teachers have taught them to do.

The third category that is related to Saudi teachers’ approaches to support students’ problems solving is *limited implementation of high level tasks*. The category suggests that teachers reveal knowledge on how they choose tasks that are cognitively demanding for students. Most of the observed tasks were scored as Level 3 or Level 4 according to the IQA observation protocol for scoring the potential of the academic tasks. However, all of the observed tasks were scored as level 1 or 2 in the stage of the implementation of the tasks in the instruction. Literature studies in Chapter 2 indicate that the complexity and the high level of thinking tasks can create; produce greater opportunity for students’ learning. When implementing tasks to be presented at level 1 or 2 means that students are learning mathematics through memorization and mimicking
previously taught procedures which may not be effective for students’ learning (Stein, Remillard, & Smith, 2007)

The property selectivity suggests that Saudi teachers do not hold consistent knowledge for all students to expose to high-level thinking during instruction. Only the students are elected whether to solve high-level thinking tasks and non routine problem solving or not. The properties inconsistent implementation and selectivity suggest that teachers in Saudi Arabia generate limited knowledge on how to provide opportunities for students to experience high-level problem solving.

**Reasoning, proof and communication.** This study investigated the teachers’ knowledge of student mathematical learning with regards to promoting students’ mathematical processes of reasoning, proof, and communication during instruction. Combining the discussion of reasoning and proof with that of communication at the same time is done because of the nature of my findings. The participants were elected to discuss these two processes as one idea. In Chapter 2, the discussion of reasoning and proof processes is defined as the students’ ability for “analyzing problem situation…explaining strategies and checking the reasonableness of the results” (Kim & Kasmer, 2006, p. 90). On the other hand, engaging students in the communication process can allow them to become part of the classroom social interaction by encouraging them to express their thinking using appropriate verbal and written mathematical language (NCTM, 2000). The findings of the study indicate that teachers tend to practice detailed reasoning as a means to support their students’ processes of communication,
reasoning and proof. Two main aspects of the teachers’ knowledge emerged that
underline how teachers in Saudi Arabia practice their support of these processes.

A key finding of the study was the teachers’ intention to require their students to
provide full explanation and responses to the students’ solution strategies as a part of
their reasoning processes. This portion of my findings was labeled as modeled written
reasoning. The study shows that teachers in Saudi Arabia practice their knowledge on
supporting students’ reasoning by requiring them to provide full mathematical
justification to their solutions. “If there is a solution to the question, they should include
the formula they use and this is compulsory to the students in the test” (AR.7, December
5, 2011). The Saudi teachers’ requirements in this regard allow them to judge the
students’ understanding of the solutions they provide. The nature of students’
explanations, as indicated by the teachers, should be similar to the teacher’s previous way
of reasoning.

Teachers’ approach in supporting students’ reasoning using modeled and detailed
written explanations clearly can help students to express their thinking and support them
in verifying their own problem solving. However, because students’ reasoning is mostly
just a copy of what the teachers have told them to do, the nature of students’ learning that
results from such a way of reasoning can be under questioning. No literature has been
found to explain the advantage of supporting pretaught reasoning on students’ learning.

The category verbal reasoning refers the Saudi teachers’ desire for their students
to express their thinking verbally using shared understandable mathematical language to
the whole class. The teachers in Saudi Arabia understand the importance of students’
expressing to their thinking using appropriate mathematical language and being confident in speaking mathematically (NCTM, 2000). Teachers in Saudi Arabia emphasize students’ expression of thinking within each step of the lesson. In addition, the teachers in Saudi Arabia show that they support students’ verbal reasoning during their problem solving. “I encourage discussion in the beginning of presenting the activities...I concentrate on students’ expression of thinking during their work on problems. This would help the students to speak about what is in their minds” (December 17, 2011). The Saudi teachers’ emphasis can have the potential to help increase students’ beliefs and autonomy and give them confidence to be part of the classroom environment (Yackel & Cobb, 1996).

**Connections.** A major finding that emerged from the classroom observing is the Saudi female teachers’ declarative approach to promoting students’ mathematical connections. In Chapter 2, the process of connections was defined by Leikin and Levav-Waynberg as “linking new ideas to related ones and solving challenging mathematical tasks by seeking familiar concepts and procedures that may help in new situations“ (2007, p. 350).

The theme *declarative connections* indicated a shortcoming in the teachers’ knowledge regarding their implementation of mathematical connections among their students. The study determined that participants support mathematical connections mostly in two traditions: by relying on the textbooks or the teachers’ declarative way of teaching students about how to understand mathematical connections presented in the lesson. Saudi female teachers did not express any knowledge of how to support students’
understanding of connections through the active role of problem solving. The Saudi teachers’ approach in teaching students about mathematical connections contradicts with most mathematics education literature. These studies advocate for the role of engaging students to work on challenging mathematics problem situations that allow them to apply their understanding of concepts in new situations (NCTM, 2000; Leikin and Levav-Waynberg, 2007).

**Representation.** In the regard of mathematical representation, no themes emerged from the observation field notes that explain the teachers’ knowledge of how to promote representations. The discussion of the Saudi teachers’ approach to promote mathematical representation will be presented in the next section.

**Overall Knowledge for Engaging Students**

This section gives broader elaboration into the Saudi teachers’ knowledge of student mathematical learning, as it is the main investigation of the second research question of the study. To give the broader picture of the investigated teachers’ knowledge, discussion will be based on the findings of all three data sources of the study. More specifically, the discussion of the teachers’ knowledge of students’ learning that supports students’ mathematical processes is resulted from two parts of findings in Chapter 4.

Each mathematical process is discussed individually using supportive findings. The study applied three data sources to achieve triangulation to help increase the credibility of the findings and avoid unintended biases that result from a single data source (Patton, 2002). It should be mentioned that the findings of the study are presented
through two sections in Chapter 4. The first section presents four themes that appear to emerge from the classroom observations and initial interviews. The second section presents themes that emerge from the scenario-based interviews. The discussion derived from the findings of both sections could appear contradictive or coincidental with each other. To better understand the Saudi teachers’ knowledge of students’ learning, at the end of each section I give a summary of what I see as the Saudi teachers’ knowledge on promoting a specific process as informed by multiple sources of findings. The summary will present only the points that were common across at least two sources of findings. For example, if the finding emerged only from the teaching scenario and is contradicted with findings from the observation and initial interview, a finding with more supportive evidence will be included in the summary. In addition, if a finding is presented from only one source yet is not contradicted with data in other source, then I will include it in the summary as a main theme with weak evidence.

Problem solving. The main findings of the study were related to the Saudi teachers’ knowledge of student mathematical learning with regard to promoting students’ problem solving approaches. The grounded theory in Chapter 4 indicated three major findings that describe teachers’ approaches of guiding students through problems solving through their processes. Common findings resulted from the teaching scenarios as well.

One of the main findings presented in the theme guided problem solving indicate that teachers in Saudi Arabia are trying to make individual activities more accessible to their students by using semi-discovery problem solving approach. This category was discussed through answering the first research question. In addition, the data in the
scenarios supported this claim. For example, two emerging categories that support the Saudi teachers’ *semi-discovery problem solving approach* developed from the Rental-Car scenario. These categories from the scenario include: *numerical representation or filling out table is more guided than other approaches* and *semi-discovery problem solving approach for teaching students the concept of function*.

Through the Rental-Car scenario, the teachers in Saudi Arabia tried to limit students’ efforts resulting from their mathematical representation solution choice that would make their solution appear complex to their learning. Because the first scenario is complex for students as indicated by the participants, they suggested that they could facilitate these difficulties by guiding the students’ discovery of the solution through requiring them to use a certain kind of mathematical representation. One participant responded during the discussion of the first scenario as follows:

> I love to tabulate my answer for any problem. The students will guess out their method but I will require them to use the table to solve the problem…tabulation can organize the answer. In the beginning students do not know how to work with equations. (MA.4, November 30, 2011)

In addition, the participants suggested other methods to help make the first scenario more accessible for the students and give them the opportunity to achieve its solution. One of these methods, as suggested by HI.5, is to give a review on equations and inequalities previous to the problem and then require the students to implement the teacher’s overview to solve the task in the first scenario. The finding derived from the
Rental-Car scenario resembled the Saudi teachers’ approach similar to what is presented in the category *semi-discovery problem solving approach*.

The Saudi teachers’ problem solving teaching approach is also presented through the category *partial opportunities of alternative solutions*. From the property *specific methods*, the main finding in this regard pointed out that teachers in Saudi Arabia have limited knowledge on how to support students’ alternative solutions because of the limitations that come with the approach. It was discussed through answering the first research question that teachers perceived multiple solutions by giving students a number of strategies to work with. The findings also suggested that multiple solutions have limitations because not all students are required to try different solutions with what has been presented in the class. In contrast, evidence from the teaching scenarios did not support the level of teacher’s knowledge with regard to supporting alternative solutions during problem solving. Through the teaching scenarios, especially the Rental-Car scenario, the teachers tend to advocate their students to use one solution strategy and disregard other solutions that might cause difficulties to the students’ learning.

In addition, the findings emerging from the property *specific students* indicate that teachers are giving opportunities for specific students to integrate multiple solution strategies rather than focusing on one specific method. There are no contradictory or supportive findings from the teaching scenarios that are related to teachers permitting specific students to integrate multiple solutions during problem solving.

The third aspect that reveals the Saudi teachers’ knowledge on supporting students’ problems solving process demonstrates their tendency and intention to lower
the cognitive demands of a presented task on their students. This aspect emerged from two parts of data in Chapter 4. The participants’ responses to the Rental-Car, Which Is More and Perimeter scenarios reveal the Saudi teachers’ intention to lower the cognitive level of the tasks. For example, through the Which Is More scenario, one suggestion offered by one of the participants is to present the task in the application phase of the lesson after teaching the main concept to help avoid students’ misconceptions that result from their thinking about the task:

If I ask my students about how to compare two groups, they will say that comparison can be done using greater and less than kind of analogy. They will not compare the two groups proportionally. Hence, I will use this example as an application exercise after teaching the main concept of the lesson. (HE.4, November 30, 2011)

In addition, the category *limited implementation of cognitively demanding tasks* suggested the teachers’ knowledge is geared toward supporting students to avoid the difficulties and struggle that result from the process of problem solving. Classroom observation shows that participants are trying to lower the cognitive level of implemented cognitively demanding problems. Also, the property *selectivity* suggests that teachers in Saudi Arabia allow specific students in the classroom to engage in high level thinking. This finding was not presented through the teaching scenario interview. However, the teachers’ view to allow specific students to solve high level thinking tasks is supported by evidence from both observations and the initial interview.
The summarizing points of Saudi teachers’ knowledge of student mathematical learning with regard to supporting students’ problem solving processes are as follow

- Teachers are approaching discovery problem solving using the semi-discovery problem solving approach. This finding emerged mainly from the category semi-discovery problem solving approach for teaching students the concept of function, and numerical representation or filling out table is more guided than other approaches.

- Specific students are approaching alternative solutions. This finding emerged mainly from the category limitation of alternative solution during problem solving.

- Saudi teachers intentionally try to lower the cognitive demand when implementing high level tasks. This finding emerged mainly from the property inconsistent implementation and from the category facilitating the cognitive demand and overcoming students’ struggle.

- Approaching high level tasks is optional for students. This finding emerged from the property selectivity.

**Reasoning, proof, and communication.** The data of the study aimed to investigate the teachers’ knowledge role to promote students; reasoning, proof and, communication processes. The previous section offers discussion of the Saudi teachers’ approaches to support students’ processes of reasoning, proof and communication as they practice in the classroom. This section will elaborate on the same discussion using supportive findings from the observation, Initial interviews and scenario-based interview. Common themes are presented across the three data sources describing the Saudi
teachers’ knowledge that support students’ processes of reasoning, proof and communication.

The first emerging category from the first part of findings indicates that teachers in Saudi Arabia support *written modeled reasoning* as a means to help students express their reasoning and thinking in solutions. In this regard:

First students should solve the problem in a modeled and detailed way. Second, I ask the students to mention their mathematical justifications and reasoning in the test. (AL.3, December 13, 2011)

Discussion of the reasoning, proof, and communication processes through answering the first research question reveals that there is no interpretation of the teachers’ requirement of *written modeled reasoning* from the literature. In addition, the findings derived from the teaching scenarios did not directly highlight this category as an emerging theme. However, the data did not specify otherwise. The data presented from the Rental- Car scenario and Perimeter scenario show two emergent categories that can be related to the category: *written modeled reasoning*. These categories are: *numerical representation or filling out table is more guided than other approach* and the category in the forth scenario: *whole class activity to support students’ connections and discussion*. The interpretation of these categories can exhibit the teachers’ tendency to require their students to achieve a level of detailed mathematical reasoning in their solution as modeled by the teachers. Also, the same interpretation can be found through the Perimeter scenario. One participant reported:
…I will ask my students to give me all the possible numbers that give me the largest area…drawing would be essential here…I will draw the rectangles and next to them. I would record all the lengths that my students gave me...we would also need to approach the problem algebraically at the end of the lesson. (MO.1, December 11, 2012)

The category about *verbal reasoning* demonstrates Saudi teachers’ approach regarding supporting students’ processes of reasoning, proof, and communication. Through answering the first research question, it was mentioned that Saudi teachers’ support of students’ verbal expression can help increase their confidence and understanding of learning mathematics. One participant reported:

If she comes out to the board, she should explain what she is doing and she should not go back to her seat without understanding. Speaking helps the student to discover their own error. (MA.4, November 30, 2011)

I presented evidence on the teachers’ support of students’ verbal reasoning through the Perimeter scenario. During the Perimeter scenario, participants agree to use whole class activity to help students understand the concept of area and perimeter. The classroom discussion would play a crucial role as the teachers would require students to respond to a set of questions that could break down the difficulty of the task and help them to understand the solution strategy of this problem. The following quote was presented from the Perimeter scenario interview transcription:

As in any mathematics lesson, discussion would be in each step of the solution to the problem. We would not continue to the next step unless everybody
Mathematics is about discussing students’ thinking. (TA. 6, December 12, 2011)

The last portion of findings that demonstrate the teachers’ view on supporting students’ reasoning, proof, and communication is presented through the fourth theme, teachers’ views of students’ mathematical capabilities. The teachers express that their ideas about implementation of formal proof to middle school should be not necessary because formal proof is more suitable for high school students where they learn formal mathematical knowledge. The Saudi teachers’ view towards formal proofs is in fact coincided with a previously reviewed study. In Chapter 2, a major finding that appears from Francisco and Maher (2005) indicates that when students are not mature enough to provide formal proof, then informal proof will be significant in supporting the students’ understanding and building of their reasoning processes. They suggest that it is imperative for students to provide clear “convincing justification” to their problem solving processes rather than focusing on writing their justification formally (p. 368). The finding of the study corresponds with the Saudi teachers’ standpoint on their disagreement of encouraging formal proof for middle school students.

Discussion of the teachers’ ideas about the use of formal proof in Chapter 4 indicates that this category has emerged only from the initial interview. The findings also show the teachers’ trajectory toward formal proof was not presented from the Scenario-based interview. This shows that the teachers’ negative view of the use of formal proof does not have enough findings to support it. However, the data of the observation and
scenario-based interview did not reveal any contradiction with the teachers’ negative view on the implementation of formal proof.

The summarizing points that demonstrate the Saudi teachers’ knowledge regarding how they support students’ reasoning, proof and communication are as follow.

- Teachers are supporting detailed mathematical justification as modeled by the teachers. The finding mainly emerged from the category: *written modeled reasoning*.

- Saudi teachers appear to support students to verbally express their thinking. This finding emerged mainly from the categories: *verbal reasoning* and *whole class activity to support students’ connections and discussion of concepts*.

- Saudi teachers have a negative view regarding the use of formal proof with middle school students. This finding emerged from the theme *teachers’ views of students’ mathematical capabilities*.

**Connections.** The study highlighted the teachers’ knowledge on students’ learning with regard to their knowledge of supporting students’ connections process. The results of the study demonstrate two properties under the connections category. In addition, the findings drawn from the teaching scenarios generate two main results that are related to the teachers’ knowledge on how to support connection, specifically drawn from the second and third scenarios.

Throughout the discussion of the first research question, it was mentioned that Saudi middle school teachers practice providing connections for their students in a way that is distinct from how the literature has advocated. Through answering the first
research question it was indicated that teachers tend to highlight the connections between topics using brief discussion either at the beginning or end of the lesson. Problem solving was not a main source for students to understand mathematics connections. However, two main results emerge from the second part of findings that contradicted with the teachers’ knowledge of mathematical connection as practiced in instruction.

During the Adding and Subtracting Integers scenario, participants specify to use visual representations, such as counters in this case, to convey the meaning of the negative numbers and their connection to the students’ real lives. In addition, the Saudi teachers elected themselves to allow students to work with real-life examples that would help build their conceptual understanding of integers. One participant reported:

The counters would be great for my students because that the student would move the counter back and forth and this is nice for them…also, if we connect the positive integers with money gain and the negative integers with money loss and we could give them other real-life examples…also using this connection for teaching addition… The students would understand addition very well. (TA.6, December 12, 2011)

The Saudi teachers’ reaction to the Adding and Subtracting Integers scenario shows their willingness to allow their students to participate in problem solving that involves adding and subtracting integers using counters and using the number line at the same time. The teachers’ approach is advocated by mathematics education reform recommendations that promote the students’ engagement in problem solving to be able to connect mathematics in other contexts (NCTM, 2000; Leikin and Levav-Waynberg, 2007).
The previous finding from the Adding and Subtracting Integers Scenario does not accommodate with the theme *declarative connections* that was presented through the first part of my findings. Discussion of question one demonstrates that Saudi teachers are not supporting students’ activity to understand mathematical connections as shown through their knowledge as practiced in instruction. The same finding is also presented from the initial interview, as one participant stated:

[I encourage] the development of scientific thinking for the students where she can apply mathematics in all life situations and development of problem solving. At the beginning of every lesson I tell them that they have previously taken a concept, and now we will investigate it in a broader way. For example, students who start middle school [are reminded] that they studied the lesson in elementary school. Now we will build upon that but in a broader sense in ninth grade. (NO.4, November 28, 2011)

The second observed point here emerged from the finding in the Which Is More scenario, which indicated a similar contradiction with the theme *declarative connections*. Exploring the teachers’ knowledge on how to enact the task in the third scenario in the lesson shows how teachers exhibited understanding and appreciation of real-life examples to convey the meaning of the lesson and better understand proportional reasoning.

Through the third scenario, and in the category *facilitating the cognitive demand and overcoming students struggle* some participants pointed out giving real-life examples that are easier for students so they could connect these examples with the task in the
Which Is More scenario. One participant offered this suggestion: “Sara read six book in two days but Nora read 7 books in 3 days” (TH.1, December 11, 2011). The teachers’ use of connection here is not only to motivate students to learning nor to simply show them that there are real-life applications by brief reporting, but this would encourage them to achieve a level of their instrumental understanding where they could apply it to the example presented in the Which Is More scenario (Skemp, 1987). To summarize the knowledge of how to support students’ connection, evidence in this regard is not consistent. Some findings that are presented from the grounded theory developed from classroom observations and the initial interviews while others are presented from the teaching scenarios.

- The Saudi teachers are reporting on connections aspects of the concepts briefly. This finding emerged from the category *declarative by the teacher.*

- The teachers are asking students to read their textbook to learn about the connections aspects of the lesson. This finding emerged from the category *declarative by the textbook.*

**Representation.** The findings presented from the observation field notes and the Initial interviews did not show any evidence of the Saudi teachers’ knowledge on how to support students’ mathematical representation. Hence, the discussion of this teachers’ knowledge domain will be drawn from the scenario-based interview.

The key idea advocated by the current mathematics education reform requirement indicates that students do not only need to develop single mathematics representation. Rather, students are demanded to develop flexibility with mathematics representations by
being able to translate among them (NCTM, 2000; Lesser & Tchoshanov, 2005). The interpretation and analyzing of the teachers’ use of representation will be based on how representation is advocated by the literature.

Through the Rental-Car scenario, an emerging idea explains the teachers’ use of numerical representation to help students understand the concept of function. The teachers’ decision on applying this representation stems from the definition of function itself. Because the participants view the function as correspondence that associates each input with exactly one output, they see that using numerical representation would help students to relate to the meaning of function. The numerical representation would maintain students’ understanding of inputs and outputs through organizing their numerical data on a table. The Rental-Car scenario shows that participants are not interested in using graphical representation for the same problem because it would be a source of confusion.

The teachers’ advocacy for the use of numerical representation to support student learning can encourage students to connect the definition of function using numerical representation. One of the main limitations is that teachers were trying to escape from embracing more than one representation in the same problem. The teachers were not satisfied with supporting the students’ working and translation among graphical and numerical or numerical and algebraic representation. Instead, the teachers’ understanding of representation is to carry through allowing students to investigate the best representation that conveys meaning to the lesson without trying to work through more than one representation as advocated by mathematics education reform.
Although little evidence from the findings illustrate the Saudi approach to support students’ representation, the summarizing points of the Saudi teachers’ knowledge on how they support mathematical representation are as follows:

- Developing only a single mathematical representation when needed for solving problems. This finding emerged from the category *numerical representation or computing table is more guided than other approaches*.
- Developing only mathematical representation that is easier for the students. This finding emerged from *graphical representation is not suitable for middle grade students*.

**Teachers’ Knowledge of Student Mathematical Learning**

This section elaborates on the Saudi teachers’ knowledge of student mathematical learning as the primary focus of the third research question of the study. Throughout the study, the researcher implements the NCTM’s teaching standards in order to define the teachers’ knowledge of student mathematical learning. The NCTM’s (2007) framework calls for teachers to know and recognize the importance of

- what is known about the ways students learn mathematics;
- methods of supporting students as they struggle to make sense of mathematical concepts and procedures;
- ways to help students build on informal mathematical understandings;
- a variety of tools for use in mathematical investigation and the benefits and limitations of those tools, and
• ways to stimulate engagement and guide the exploration of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representations. (p. 25)

The answer to the third research question will identify the middle school Saudi teachers’ knowledge of student mathematical learning derived from the three data sources in the study. Part of the teachers’ knowledge of students’ learning is the teachers’ ways of guiding students’ involvement in mathematical processes. The discussion in this regard was presented separately through answering the second research question. Overall elaboration on the teachers’ knowledge of student mathematical learning as discovered through the data is discussed through the following paragraphs.

One of the main findings consistent through two parts of findings in Chapter 4 is the Saudi teachers’ level or extent of understanding of the teaching role in mathematics and how this role is transferred to students’ learning. The study investigated the Saudi middle school teachers’ knowledge of student mathematical learning. One of the central emergent themes from the study is the teachers’ understanding of “ways students learn mathematics” (NCTM, 2007, p. 25).

Schoenfeld and Kilpatrick (2008) advocate for such teacher understanding of learning theories and perspectives on how students learn mathematics. They couch this in terms of “knowing students as learners,” which they define as “being aware of one’s theory of learning and how that plays out in terms of classroom activities and interactions with individual students” (p. 333). The findings of this research highlight many aspects of the Saudi teachers’ knowledge of student mathematical learning.
One of the main aspects of findings regarding the teachers’ understanding of students’ learning in mathematics was presented through the theme *teachers’ views of students’ capabilities*. Most of the Saudi teachers in the study assert that students need extensive assistance with regard to the learning of mathematics. They are not well prepared to enter middle school. Others do not have the family support to see mathematics as worth learning. One of the participants speaks to that point:

The problem [with teaching mathematics] is not the curriculum but our students and their negative way of thinking about mathematics...Students do not like mathematics. The parents also raise the difficulty of mathematics to their children. (HI.5, December 10, 2011)

Because Saudi teachers carry negative views regarding their students’ capabilities, they think that teaching for conceptual understanding is not a problem of their own instruction. Considering the Which Is More scenario, teachers express the difficulties that students are facing because of their poor studying of multiplication facts. They believe that their instructional approach is not the main reason that students experience proportional reasoning misconceptions. Rather, reasons such as students’ arithmetic problems and students negative’ beliefs towards mathematics are the main causes for students’ under achievement in mathematics. The category *the difficulties of proportional reasoning is beyond teachers’ control* shows that participants do not view proportional reasoning as a challenging concept for middle school students and that any difficulty the students are facing is based on their problems with arithmetic and involves very little with their conceptual understanding as a result of the teachers’ instructional approaches. The
teachers in the scenario indicate that their emphasis was on students’ procedural fluency rather than thinking about their conceptual understanding.

Because the teachers in Saudi Arabia view their students as having limited capabilities for achievement in mathematics, they tend to provide all the support they can to overcome the difficulties involved in complex learning structures. The middle school teachers in Saudi Arabia would not want their students to experience a struggle with learning complex problems. Through the Which Is More scenario, the teachers planned the artificial instruction in a manner that allowed their students to experience less difficulty with problem solving. This can be seen through the category facilitating the cognitive demand and overcoming students’ struggles. The finding from this scenario illustrates that participants tend to include the task in the application phase after giving direct instruction on how a similar problem can be solved. They try to allow less complexity to become involved in students’ thinking. The students’ thinking will likely be minimal because it will be easier for them to solve the problem. In addition, the grounded theory that emerged from the observations and initial interviews generated similar findings that elaborate on the teachers’ tendency to overcome students’ struggles in thinking through the categories semi-discovery problem solving approach and limited implementation of cognitively demanding tasks.

Providing sustainable help for students in the process of learning mathematics is not an issue that questions the Saudi teachers’ knowledge about their students’ learning. In fact, helping students overcome their struggles in learning is an indication of the teachers’ knowledge of student mathematical learning as specified by the NCTM.
framework of knowledge on student mathematical learning. However, middle school Saudi teachers, by trying to facilitate the complexity involved in learning, would cause students to experience low cognitive thinking in mathematics instruction. Boston and Smith (2009) and Doyle (1988) indicate that using complex non algorithmic thinking can produce a high level of conceptual understanding. The Saudi teachers do not seem confident about their students’ thinking as they could invent their solutions or deal with complex problem solving. As such, the teachers’ mistrust of students’ capabilities gives them the power to provide too much help for their students during their work and that affects the benefit the students get from engaging in complex tasks.

On the contrary, the data of the study generates less and inconsistent evidence on the teachers’ view of students as being capable in learning mathematics. This was discussed under the positive dimension of the category teachers’ views of students’ mathematical capabilities. The positive side of the Saudi teachers’ knowledge was implied from a few quotes in the initial interviews such as “My students’ thinking is always active in learning. They always come to me and show me they solve high-level thinking problems by their own without my direction” (AR.7, December 5, 2011). Teachers such as AR.7 believe that the classroom constrains many levels of students’ capabilities. If the students are interested in learning and experiencing complex mathematics concepts then they will find their teacher is welcoming their efforts. A similar view came from the properties selectivity and specific students, which demonstrate the Saudi teachers’ support to students to engage in high level thinking and invent solutions when they show their interests to do so.
Overarching Interpretation of the Data

**Saudi middle school teachers in transition of changing features.** Looking at the whole picture of the findings, there are some interpretations that can explain the Saudi teachers’ knowledge of students’ learning and how their knowledge supports students’ mathematical processes. This section will discuss the conclusion that is implied from the findings of this research.

One aspect to point out is that Saudi teachers are coping with new reform practices by the Ministry of Education of Saudi Arabia that require them to implement reform that is aligned with the mathematical process standards. The process of overcoming the new educational system was complex on them. The study focuses on understanding the Saudi teachers’ knowledge of how to support the mathematical processes, which allows me to see how these teachers are in the middle of shifting their teaching in contrast with my own previous educational experiences.

The focal point of this study was not to evaluate the teacher’s changes with regard to addressing reform requirements in Saudi Arabia, although no known information in this regard has been published. Rather, the study aims at understanding the Saudi teachers’ knowledge of student mathematical learning in general and also with regard to supporting students’ mathematical processes in specific. A holistic interpretation to the findings of the teachers’ actions is that those teachers are addressing mathematics processes in a way that is suitable to their system and their own role as teachers. Interpretation of the findings of the study shows that Saudi teachers are striving to provide a variety of experiences for students within mathematical processes. While
addressing some of these processes does not appear successful for students’ learning, other approaches are considered as valuable strategies to reforming the education. In the long run, the Saudi teachers’ implementation of specific mathematical processes does not seem to alternate their system of teaching as superior figures in the classroom who should control students’ thinking and direction of solutions during problem solving.

Stigler and Hiebert (1999) discuss in the following quote how implementing reform requirements in mathematics education within the United States system had been limitedly sufficient because while it appears that the U.S. teachers change some features of their teaching, the most essential elements of the instructions are still the same:

It has now been documented in several studies that teachers asked to change features of their teaching often modify the features to fit within their preexisting system instead of changing the system itself. The system assimilates individual changes and swallows them up. Thus, although surface features appear to change, the fundamental nature of the instruction does not. When this happens anticipated improvements in student learning fail to materialize, and everyone wonders why.

(p. 98)

According to my experience as the researcher, I experienced how the teachers in Saudi Arabia appear to have complete power of the class instruction even though they want to improve their instruction and allow students to experience their own problem solving approaches. Saudi teachers, who were taught through their career professional development that discovery problems are significant for students’ learning, were documented to apply them in an insufficient way. For them, solving problems using
complete discovery approach would be opposed to their role as powerful respected figures in the classroom. Saudi teachers appear to understand the power of addressing various features of problem solving such as discovery approach, high level tasks, and providing opportunities for multiple solutions. However, as discussed in answering the first research question, these aspects lacked the necessary features to support students’ understanding of mathematics.

Teachers in Saudi Arabia appear to understand the power of promoting the connections process because their textbooks are asking for such and because they have to respond to the reform requirements. However, literature studies suggested that teachers in Saudi Arabia seem to practice their knowledge on how to support students’ connections differently than engaging students in activities that allow them to experience and think of mathematics connections. Thus, as with the problem solving process, teachers in Saudi Arabia translate their understanding of how to promote mathematics connections in a way that is suitable to their traditional teaching role in the classroom.

As for the representation process, little evidence was developed from the findings of the study. This evidence indicates that teachers in Saudi Arabia tend to encourage single mathematics representation that is the most helpful for students to grasp particular mathematical concepts. However, Saudi teachers do not challenge their teaching manner to develop inconvenient mathematical representations or to support flexibilities of working through multiple representations.

The study reveals that Saudi teachers account for supporting students’ verbal and written expression of thinking through instruction. The teachers in Saudi Arabia also
support students to express their pre-taught reasoning to the teachers. The study also shows that participants tend to encourage students to express their solution verbally in the classroom. The Saudi teachers’ approaches with how to promote students’ reasoning, proof and communication indicate that they see the power of students’ verbal and non-verbal justifications to their thinking. The teacher’s role in this regard is preserved as the main source of knowledge in the classroom. Thus, this part of Saudi teachers’ knowledge demonstrates a source of willingness to promote students’ activities that is not contradicted with their main role as the teachers.

The Saudi teachers’ changing features of how to promote mathematics processes implies that they are in the middle of pursuing a path of improvement, as opposed to my own experiences as the main instrument in the study. Although Stigler and Hiebert (1999) argued in the previous quote that changing features of teaching while preserving the teaching system cannot develop effective teaching change, Ma (1999) argued otherwise:

The change of a classroom mathematics tradition may not be a "revolution" that simply throws out the old and adopts the new. Rather, it may be a process in which some new features develop out of the old tradition. In other words, the two traditions may not be absolutely antagonistic to each other. Rather, the new tradition embraces the old—just as a new paradigm in scientific research does not completely exclude an old one but includes it as a special case. (p. 153)

Avoiding struggle. One of the main findings of the study shows that participants in Saudi Arabia are likely to avoid any sources of confusion that may appear from students’ learning. Based on my own experience with the participants and my own
education in Saudi Arabia, I can conclude that Saudi teachers are highly protective to their students through learning of mathematics. The teachers hold many sources of knowledge and understanding of processes that the students should experience in a mathematics lesson. However, the sense of caring they carry out causes their knowledge about students to be inconsistent with the recent mathematics education reforms.

An important interpretation of the findings signified that teachers in Saudi Arabia are not accepting that their students should struggle to some degree when they are working in complex problem solving. Many signs in the findings show that the teachers in Saudi Arabia avoid engaging their students in complex thinking. For example, through the discussion of the teacher’s problem solving approaches with their students, teachers demonstrated how they intentionally tried to lower the level of thinking the students may go through when solving unfamiliar problems. Further, individual problem solving is achieved through gradual questioning and hints that the teachers convey to the students. The researcher interpreted this evidence as that teachers in Saudi Arabia affirm limited knowledge regarding the demanding need to involve students in high level thinking and unfamiliar problem solving situations in order to achieve effective teaching of mathematics (e.g. Bruner, 1960; Doyle, 1988; Boston & Smith, 2009).

**Recommendation for Saudi Educators and Policy Makers**

This study provide in-depth analysis of the Saudi teaching approaches when teaching mathematics, specifically their approaches that support students learning of mathematics. The study highlighted the Saudi teaching approaches, philosophy of teaching and ideas about how they can support students’ engagement in mathematics
processes. The recommendations presented in the following paragraphs can be incorporated through continued professional developments, teaching support groups, or through preservice teaching curricula.

One of the most important recommendations for educators and policy makers is to supplement teachers in Saudi Arabia with the necessary knowledge on learning philosophies, especially in mathematics education. The study implemented the construct knowledge of student mathematical learning to capture the Saudi teachers’ understanding into how to support students’ thinking and what is the effective learning philosophies is suitable for teaching mathematics. The study shows that challenging students’ thinking and engaging them into complex processes of problem solving where the nature of the solution is not expected for the learner is not part of the Saudi teachers’ knowledge in mathematics. Educators and policy makers in Saudi Arabia should provide professional developments that support teachers to enrich their knowledge about students’ mathematics learning. Giving students opportunities to struggle or take sufficient time to make efforts to understand mathematics problems does not always resemble that there is misconceptions in learning. The higher cognitive demanding the problem the more that it is helping students to develop conceptual understanding. As such, Saudi teachers should consider allowing students to experience complex thinking process where they can help them to develop their own solution approaches and acquire greater conceptual understanding.

Educators in Saudi Arabia should support teachers to develop understanding into how better they can improve students’ experiences of the investigated mathematics
processes. Teachers in Saudi Arabia should have access into how to develop students’ experiences with mathematical processes of reasoning and proof, communication, connections and problem solving. The related information will help them to make improvement into how they implement them in the classroom.

Educators and policy makers should help teachers to increase their knowledge into research in mathematics education, especially, research that corresponds to new reform requirements that the teachers are expected to implement in their instruction. Teachers in Saudi Arabia do not seem understanding the ideas behind reform requirements includes in their implemented curricula. When their textbooks includes connections ideas and high-level tasks to be used by the teachers, Saudi teachers seem to have less understanding into their benefit to their students’ learning. As such, Saudi teachers should have access to information and practitioner research studies that can help them increase their understanding into how and why newly implemented reform requirements can improve students’ learning.

The study examined middle school Saudi teachers’ knowledge of student mathematical learning. The study indicated that the teachers need to refine their ideas about how students’ should be learning mathematics. An important recommendation is to offer professional development that one of its aims to help teachers improve their knowledge about students’ learning for specific grade level. This recommendation is originated from the CGI research group and professional developments. A major findings of the CGI research group is that “learning to understand the development of children’s mathematical thinking could lead to fundamental changes in teachers’ beliefs and
practices and that these changes were reflected in students’ learning” (Carpenter et al., 2000, p. 1). The study advocates for professional development programs that help teachers’ to understand students’ thinking at specific grade levels and for specific mathematical concepts.

**Recommendations for Future Research**

This study provides general overview into how the teachers in Saudi Arabia support students to experience various mathematical processes. This research did not aim to focus on the study of single mathematical process in isolation with other processes. Thus, future research should be geared towards replicating the same study design with focusing in on a single process of problems solving, communication, connections, reasoning and proof and communication.

As the qualitative approach is not intended for generalization and more intended for understanding the current knowledge of student mathematical learning, a future recommended research is to use the qualitative data to build instruments that evaluate the found four themes in the grounded theory. In addition, the study highlights the role of using the construct knowledge of students’ mathematical learning to capture the teachers’ philosophy of teaching that promote students’ learning. Conducting quantitative research to test the current knowledge of Saudi teachers is one recommendation for future research.

Little findings emerged in the study regarding how teachers in Saudi Arabia make use of mathematics representations and how they promote them in the classroom. Data that explores the teachers’ practices to promote the process of representation resulted only from the initial interviews. Thus, this research provided little evidence that supports
the findings in this regard. The study implicated for more research in this area. Future research can examine the kind of representations that Saudi teachers tend to focus on when teaching specific mathematical concepts.

This study did not explore students’ learning or their perspectives toward what they experience in the classroom. Therefore, other recommended research study should investigate the Saudi teachers’ approach to support specific mathematics process while evaluating students’ learning or perspectives. Collecting data from the students’ side can bring to light into the level of effectiveness of the Saudi teachers’ instructional approaches or their approaches in promoting a single mathematical process.

Saudi teachers face some challenges because of implicating of new reform requirements. While this research did not aim to study these challenges, nor to focus on the changing aspects that teachers are going through. As such, a future research recommendation can be designed towards this lane. Further research should explore the Saudi teachers changing aspects and perspectives resulted from addressing specific reform requirements.

**Conclusion**

I offer in this study a view to Saudi teachers’ instructional approaches, perspectives, and understanding into students’ learning. The information presented in this research can help the reader to gain insights into the unique aspects of middle school female mathematics teachers in Saudi Arabia. The researcher examined these instructional practices and compared them with current reform document and literature studies.
It should be noted that the current Saudi middle school teachers’ approaches could be the most suitable ways to teach mathematics in the Saudi system. However, it may be that significant changes must be addressed. Consequently, the study opens the door for other researchers to carefully listen to teachers and understand their instructional approaches.
References


Appendix A: The Ministry of Education Approval Letter

This appendix contains the translated version of the approval letter from the Ministry of Education in Saudi Arabia, followed by the actual official version written in the Arabic language:

To whom it may concern:

This document is written to support the request from researcher Maha Alsaeeed to obtain approval from the Research and Project Administration Department at the Ministry of Education in Alahsa, Saudi Arabia to conduct her study. The researcher will ask questions and administrate observation protocol that seek to understand middle school teachers’ knowledge about students’ learning. The Ministry of Education for Girls in Al-Ahsa, Saudi Arabia has approved the researcher’s request to conduct the study as long as this research is not conflict with the Ministry’s rules in Saudi Arabia.

Wafa Mubark Alsaief

The Director of Research and Project Administration
بسم الله الرحمن الرحيم

الملكة العربية السعودية
وزارة التربية والتعليم
(200)
الإدارة العامة للتربية وتعليم البنات بالأحساء
وحدة التخطيط والتطوير
إدارة البحث والمشاريع

من يهمه الأمر

السلام عليكم ورحمة الله وبركاته

بناء على خطاب الباحثة / مها سعد السعد / تاريخ 3/4/1432 هـ بشأن طلب موافقة إدارة البحوث والمشاريع لتسهيل مهمتها البحثية والتي ستُطبق على عينة من معلومات مادة الرياضيات المرحلة المتوسطة من خلال استمارة تقييم للتعرف على وجهة نظرهم في تعزيز النشاط الطلابي عند حل المشكلات والمسائل الرياضيات خلال الفصل الدراسي الثاني من العام الحالي 1432 هـ والذي بيدى فيه تحكيم استعدادنا للتعاون وتسهيل مهمتها البحثية بما لا يتعارض مع الإجراءات التنظيمية المعول بها إدارة تربية وتعليم البنات بالأحساء.

والفاء مبارك فهد السيف

مديرة إدارة البحوث والمشاريع التربوية

سترال 777 - 5801933 - 5802137 - البريد الالكتروني 31982
www.age.gov.sa
Appendix B: Institutional Review Board Approval

A determination has been made that the following research study is exempt from IRB review because it involves:

Category 2. research involving the use of educational tests, survey procedures, interview procedures or observation of public behavior

Project Title: Saudi Arabian Middle School Teachers' Knowledge That Supports Female Students' Processes in Learning Mathematics

Primary Investigator: Maha Saad Aalaood

Co-Investigator(s): 

Advisor: Gregory D. Foley

Department: Education

Robin Stack, CIP, Human Subjects Research Coordinator
Office of Research Compliance

07/20/2011

The above exemption is effective provided the study is conducted exactly as described in your application for review. Any additions or modifications to the project must be approved (as an amendment) prior to implementation.
Appendix C: Instructional Quality Assessment Toolkit.

Academic Rigor in Mathematics

Adapted from Boston and Wolf (2006). Used with permission of the author. See the following two pages.
**RUBRIC 1: Potential of the Task**

Did the task have potential to engage students in rigorous thinking about challenging content?

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
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| 4     | The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:  
  - Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR  
  - Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.  
  The task must explicitly prompt for evidence of students’ reasoning and understanding.  
  For example, the task MAY require students to:  
    - solve a genuine, challenging problem for which students’ reasoning is evident in their work on the task;  
    - develop an explanation for why formulas or procedures work;  
    - identify patterns and form and justify generalizations based on these patterns;  
    - make conjectures and support conclusions with mathematical evidence;  
    - make explicit connections between representations, strategies, or mathematical concepts and procedures.  
    - follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. |
| 3     | The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a “4” because:  
  - the task does not explicitly prompt for evidence of students’ reasoning and understanding.  
  - students may be asked to engage in doing mathematics or procedures with connections, but the underlying mathematics in the task is not appropriate for the specific group of students (i.e., too easy or too hard to promote engagement with high-level cognitive demands);  
  - students may need to identify patterns but are not pressed for generalizations or justification;  
  - students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them;  
  - students may be asked to make conjectures but are not asked to provide mathematical evidence or explanations to support conclusions. |
| 2     | The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. There is little ambiguity about what needs to be done and how to do it. The task does not require students to make connections to the concepts or meaning underlying the procedure being used. Focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).  
  OR The task does not require student to engage in cognitively challenging work; the task is easy to solve. |
<p>| 1     | The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions. The task does not require students to make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or... |</p>
<table>
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<tr>
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<tbody>
<tr>
<td>0</td>
<td>Students did not engage in a mathematical activity.</td>
</tr>
<tr>
<td>N/A</td>
<td>Reason:</td>
</tr>
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</table>
**RUBRIC 2: Implementation of the Task**

At what level did the teacher guide students to engage with the task in implementation?

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
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| 4     | Students engaged in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:  
- Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR  
- Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.  
There is explicit evidence of students’ reasoning and understanding. For example, students may have:  
- solved a genuine, challenging problem for which students’ reasoning is evident in their work on the task;  
- developed an explanation for why formulas or procedures work;  
- identified patterns and formed generalizations based on these patterns;  
- made conjectures and supported conclusions with mathematical evidence;  
- made explicit connections between representations, strategies, or mathematical concepts and procedures.  
- followed a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. |
| 3     | Students engaged in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the implementation does not warrant a “4” because:  
- there is no explicit evidence of students’ reasoning and understanding.  
- students engaged in doing mathematics or procedures with connections, but the underlying mathematics in the task was not appropriate for the specific group of students (i.e., too easy or too hard to sustain engagement with high-level cognitive demands);  
- students identified patterns but did not make generalizations;  
- students used multiple strategies or representations but connections between different strategies/representations were not explicitly evident;  
- students made conjectures but did not provide mathematical evidence or explanations to support conclusions. |
<p>| 2     | Students engaged in using a procedure that was either specifically called for or its use was evident based on prior instruction, experience, or placement of the task. There was little ambiguity about what needed to be done and how to do it. Students did not make connections to the concepts or meaning underlying the procedure being used. Focus of the implementation appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm). OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class. |
| 1     | Students engage in memorizing or reproducing facts, rules, formulae, or definitions. Students do not make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced. |</p>
<table>
<thead>
<tr>
<th>0</th>
<th>The students did not engage in mathematical activity.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td><strong>Reason:</strong></td>
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</table>
Appendix D: Initial Interview Protocol

1. Tell me about your experience teaching mathematics. For example how long have you been teaching; what is you educational background; what grades have you taught?

2. Do you view your teaching experience as a successful one? For example, do you enjoy teaching? Why or why not?

3. What is your opinion about the mathematics education reform that has taken place recently in Saudi Arabia? What effect has this had on your instructional practices?

4. To what extent have you changed your instructional practice as a result of the reform requirements (with regards to your teaching style, students’ roles in the classroom, the nature of mathematics taught)?

5. Talk to me about what teaching methods do you think will help students to learn mathematics. How do you think these methods can provide support for students to learn mathematics?

6. What teaching methods do you use to help students become good problem solvers? Do you think your teaching methods in this regard will help them to succeed outside the classroom (i.e., international tests and real-life situations?)

7. How do you help your students to be confident in discussing their thinking? How do you encourage them to invent their own problem solving strategies? How do you encourage them to talk about their thinking and reasoning process?

8. When students are solving mathematical activity, how do you encourage them to explain the reasons behind solving the problem in a particular way? what kind of
mathematical arguments do you push your students to use in working through a problem?

9. In what ways do you use discussion as a teaching strategy in your classroom? In what part of the lesson you would like to promote classroom discussion?

10. How do you support your students to make learning of mathematics relevant to them? In what ways do you guide students to explore connections between mathematical (topic, procedures, or representations)
Arabic Translation of the Initial Interview Protocol

1. أخبرني عن خبراتك السابقة في تدريس الرياضيات. مثلاً: منذ كم سنة وانت تدرسين، من أي تخرجت. ما هي المراحل الدراسية التي قمت بتدريسها.

2. هل تتظاهر خبراتك في تدريس الرياضيات على أنها من التجارب الناجحة، مثال: فقد أجبني، تدرس وانت سن عمر بالغ من تدريس.

3. ما وجهة نظرك في ما يحدث حالياً من ناحية تطوير تعلم الرياضيات في المملكة العربية السعودية؟ كيف اثر هذا التطور على طريقة تدرسيك؟

4. إلى أي درجة غيرت طريقة تدرسيك كنتيجة لمتبوعات التطور الذي تشهده المملكة؟ فيما يتعلق بنمط التدريس. دور الطلاب في الفصل، طبيعة المواضيع التي تدرس في الرياضيات.

5. أخبرني عن طرق التدريس التي تعتقد أنها تساعد الطلاب في تعلم الرياضيات، كيف تظن أن هذه الطرق سوف تساعد الطلاب في تعلم الرياضيات؟

6. ما الطرق التي تستخدمين لتساعد الطلاب على أن يطوروا طريقة حل المشاكل والتفكير العلمي، هل تظن أن هذه الطرق تساعد الطلاب للنجاح خارج نطاق المدرسة (مثلاً أن يبتزوا الاختبارات الدولية أو الظروف البدنية). كيف تساعد طالب على أن يكون واقع وقادرين على مناقشة أو التحدث عن تفكيرهم؟ كيف تساعدهم على إبداع طرقهم الاستراتيجية لحل المشاكل الحسابية؟ كيف تساعدهم على التحدث عن أسباب اختياراتهم لطريقة بذاتها عند المسألة.

7. عندما يحل الطلاب نشاط في مادة الرياضيات، كيف تشجعهم لذكر الأسباب التي دعتهم إلى حل المسألة بطريقة معينة، مانع الجدل الرياضي الذي تحت طابع لاستخدامه أثناء حل مسألة ما؟ كيف تمتى تستخدم المناقشة كوسيلة للتدريس في الفصل الدراسي، في أي جزء من الدروس تكون المناقشة فعالة بالنسبة لك؟
كيف تجعل درس الرياضيات متصل بواقعهم في الحياة، ماهي الطرق التي ترشد الطلاب لاستكشاف الاتصال بين
(مواضيع الرياضيات المختلفة، الصيغ الرياضية، القوانين الرياضية)
Appendix E: Protocol for Teaching Scenarios

The questions in this section will function to help me understand teachers’ knowledge of students when learning various mathematical topics. The questions will further provide an understanding of the participant teachers’ capacities to support their students to explore mathematical processes during classroom instruction.

Instruction

In this section, I will ask you to respond to four teaching scenarios that unfold discussion of mathematical tasks that you might face in your teaching. You will have to talk about how you may incorporate these tasks in your instruction. Note that some of these tasks can be trivial for you and some of them are not. However, this is neither a test to your knowledge nor to your ability as a teacher. I only would like to understand your thoughts regarding how you could support your students in learning these tasks during instruction.

The Rental- Car Scenario

Imagine that the textbook presents the following problem to help students explore the concept of function.

*Your family will need to rent a car to go over for a family vacation. Two rental companies provide different prices:*

*Company A: 39.95 Riyal, but they charge in addition, 0.19 Saudi cents per kilometer*

*Company B: 19.95 Riyal, but they charge in addition, 0.49 Saudi cents per Kilometer*

*Determine for how many kilometers, the two companies will cost the same. After how many kilometers, company B will cost lower price?*
• How could you use this task in instruction? Talk to me to about possible lesson plan that would incorporate this task?

• What kind of mathematical representation would think that your student might use when working in this task?

• If you are going to provide some suggestions to the students for solving the problem, what kind of representation would you prefer the students start with, and why would you choose this particular one (graph, table, or algebraic equation); which one do you feel most comfortable teaching first? Which do you think the students find easiest to use first to solve the problem?

• What concepts would you stress in this problem? What relationship or connections between mathematical concepts would you ask students to pay attention to?

• To what extent can you use this problem or similar one to help students understand the definition of function?

• How can this problem be used to help students improve their problem solving strategies?

• How can you use this task to initiate classroom discussion?

• How do you see this example helping students connect mathematics with real-life situations?

• How can you know whether your students understand newly taught concepts in mathematics? What do you do to measure (test) their understanding? explain what to do with such instructional information?
The Adding and Subtracting Integers Scenario

You are going to teach a lesson about adding and subtracting integers. You find that the textbook presents three models to help students understand the concepts of adding and subtracting negative numbers: by the number line of integers, by using counters, and by using algebraic formulas of adding and subtracting.

- How would you incorporate these three models in your teaching about adding and subtracting integers?
- Which model would you use first to help students best understand the concepts of adding and subtracting integers?
- How can these models help students connect adding and subtracting integers with everyday life?
- Do you have better representations for adding and subtracting integers other than the ones I have proposed? What are they?
- What kind of difficulties do you think your students might face in using the models? How would you resolve their difficulties?

The Which Is More Scenario

You are teaching the proportional reasoning concept to students. You have the following activity to use in teaching the introductory lesson. In the activity, the students are asked to think about the following situations to explore proportional reasoning. The task is

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- Discuss how you could use this task to develop students’ understanding of proportional quantities
- How could you modify the task and implement it with your students to engage them in proportional reasoning.
• What other possible topics or concepts would you discuss with the students to enhance their mathematical connections between this topic and others?

• How can these problems be used to help students practice problem solving?

• How do you see this example helping students connect mathematics with real-life situations?

• How can you use this example to promote classroom discussion?

• What kind of difficulties do you think that students may experience in understanding the concepts of proportion and ratio? How can you deal with such problems?

**The Perimeter Scenario**

Please read the following problem and the scenario that follows. The problem is taken from Stein, Smith, Henningsen, and Silver (2009, p. xvii).

*Ms. Brown’s class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep rabbits.*

*a. If Ms. Brown’s students want their rabbits to have as much room as possible how long would each of the sides of the pen be?*

*b. How long would each of the sides of the pen be if they had only 16 feet of fencing?*

*c. How would you go about determining the pen with the most room for any amount of fencing?*

A similar problem is presented in the chapter about measurement for middle school students. The problem is supposed to help students understand the concept of area. Students can investigate this problem using multiple methods such as: by using grid
paper and drawing different configurations that have the perimeter of 24 ft; or by filling out a table that shows all the possible configurations that have a perimeter of 24 ft but different areas.

- How would you implement this problem in your instruction?
- How would you incorporate the two proposed solution approaches within the instruction?
- Do you have any other solution approaches for this problem?
- How can this problem be used to help students improve their problem solving strategies?
- How can this example help students understand and connect measurement concepts with everyday life?
- What type of discussion would like to generate if you implemented this problem in your instruction? When and how would you discuss this problem with the students?
- What kind of difficulties do you think your students might face in solving this problem? How would you resolve their difficulties?
١. كيف يمكن استخدام هذه المسألة في التعليم؟ ناقش خطة الدروس المحتملة التي قد تستخدمها لمساعدة الطلاب على فهم وحل هذا السؤال؟

**Arabic Translation of the Protocol for Teaching Scenarios**

سياقات (مواقف) تعليمية

سوف تعليمي السؤال في هذا الجزء على قياس مدى فهم المعلمين لل الطلاب عند دراسة مفاهيم الرياضية المختلفة. سيتم طرح السؤال كشف قدرات المعلمين المشاركون في الدراسة لدعم الطلاب لاستكشاف العملية الرياضية أثناء التدريس الفصول الدراسية.

**تعليمات**

في هذا الدرس، سوف أطلب منك الرد على سيناريوهات (مواقف) تدريسية أربعة تناقش مسائل رياضية قد تواجهها في التدريس. سوف أوضح لك المجال للحديث حول كيفية استخدام هذه المسائل الرياضية في التدريس. لاحظ أن بعض هذه المسائل الرياضية يمكن أن تكون صعبة بالنسبة لك وبعض منها صعبة. ومع ذلك، هذا ليس اختياراً لمعالمتك ولا ليكراتك كمدرس. أود فقط أن أفهم أفكارك حول كيفية دعمك للطلاب عند تعلم هذه المسائل والنظرية المطلوبة للمناقشة.

الموقف التعليمي الأول

تخيل أن يكون المنهج المسألة التالية لمساعدة الطلاب على استكشاف مفهوم الدالة.

عائِلتك لديها الحاجة لاستئجار سيارة للسفر خلال إجازة الربيع. اثنين من شركات تأجير السيارات تقدم أسعار مختلفة للزوار:

١. شريك: بـ ١٩٩.٥ ريال سعودي لكنها تتطلب بالإضافة إلى ذلك، ٠.١٩ هللة لكل كيلومتر.

٢. شريك: بـ ٢٠٩.٥ ريال سعودي لكنها تتطلب بالإضافة إلى ذلك، ٠.٤٩ هللة لكل كيلومتر.

احسب بعد كم كيلومتر سوف تكون تسعة الشريكين متساويه. احسب بعد كم كيلومتر سوف تكون تسعةيرة

الشركة ب اختر:

كيف يمكن استخدام هذه المسألة في التعليم؟ ناقش خطة الدروس المحتملة التي قد تستخدمها لمساعدة الطلاب على فهم وحل هذا السؤال؟
ماهي الصيغ (التعبيرات) الرياضية التي تعتقد أن الطالب قد يستخدمها عند حل هذا السؤال؟ المقصود

بالصيغ الرياضية مثل: رسم بياني - تعبينه جداول أو معادلة جبرية.

إذا كنت على وشك تقديم بعض الاقتراحات للطلاب من أجل حل المسألة، فإن أي نوع من الصيغ الرياضية قد تفضل الطلاب البداية بها، ولماذا اخترت هذه الصيغة عن البقيّة؟ أي من الصيغ الرياضية قد لا تتغير بالراحة لتدريسها أولاً؟ أي من الصيغ الرياضية تعتقد أن يجدها الطلاب أسهل للاستخدام أولاً عند حل هذه المشكلة?

ما هي المفاهيم الرياضية التي قد تطرّحها بجانب حل هذه المسألة؟ ماهي العلاقات بين المفاهيم الرياضية التي قد تحاول تتبع الطالب إلى أهميتها؟

إلى أي مدى يمكن استخدام هذه المشكلة أو أخرى مماثلة لمساعدة الطلاب على فهم المفاهيم؟

كيف يمكن استخدام هذه المسألة لمساعدة الطلاب على تحسين استراتيجيات التفكير العلمي و حل المشكلات؟

كيف يمكن استخدام هذه المسألة لتعزيز أسلوب المناقشة خلال(dr) الدروس؟

كيف يمكن استخدام هذه المسألة لتعزيز أسلوب المناقشة خلال(dr) الدروس؟

أ manual رو عن الصيغ الجبرية

إذا كنت قد احتفظت ببعض النماذج مثل

إذا كنت قد احتفظت ببعض النماذج لتمثيل مجموعة من الأعداد، ماذا تعبّر عن ذلك؟ كيف يمكن استخدام هذه النماذج لتمثيل مجموعة من الأعداد في الكلية؟

كيف يمكن استخدام النماذج لتمثيل مجموعة من الأعداد في الكلية؟

كيف يمكن استخدام النماذج لتمثيل مجموعة من الأعداد في الكلية؟

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أفضل
3. كيف يمكن لهذه النماذج أن تساعد الطلاب على ربط الرياضيات بالحياة العملية؟
4. هل لديك تفسير أفضل يساعد الطلاب على فهم جمع وطرح الأعداد الصحيحة أفضل مما كنت بطرحه سابقاً؟
5. ما هي المصاعب التي قد تواجه الطلاب عند استخدام هذه النماذج؟ كيف يمكن لك حل هذه الصعوبات؟

الموقف التعليمي الثالث

سوف تقوم بتقديم مفهوم النسبة والتناسب للطلاب. لديك النشاط التالي لاستخدامه في التمهد للدرس. في هذا النشاط يطلب من الطلاب التفكير في الحالات التالية لاستكشاف المنطقة النسبية والعلاقات المناسبة والغير مناسبة.

الحالة الأولى: أي من المجموعتين التاليتين تحتوي على عدد أكبر من الفتيات؟

الحالة الثانية: أي من الحوضتين التاليين تحتوي على عدد أكبر من السمك الصغير؟

الحالة الثالثة: أي من المجموعتين يحتوي على عدد أكبر من الدوائر؟

الحالة الرابعة: ريم لديها قطتين: أولي وساندي. عندما اخترعتهم للمنزل أولي كانت تزن أربعة باوند وساندي كانت تزن سبعة باوند. الآن أولي تزن ثمانية باوند وساندي تزن اثني عشر باوند. أي من القطتين تزن أكثر؟
1. ناقش كيف يمكن استخدام هذه المسألة لمساعد الطلاب على استيعاب مفهوم النسبة والتناسب؟

2. كيف يمكن لك تطوير أو تعديل هذه المسألة واستخدامها لمساعد الطلاب على تعزيز التعلم النشط (النشاط الطلابي) عند تعلم مفهوم النسبة والتناسب؟

3. ما هي المفاهيم الرياضية أو المواضيع التي قد تطرحها بجانب حل هذه المسألة لتعزيز الربط بين المفاهيم والورود المختلفة؟

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4. كيف يمكن استخدام هذه المسألة لمساعدة الطلاب على تحسين استراتيجيات التفكير العلمي و حل المشاكل؟

5. كيف يمكن استخدام هذه المسألة لتعزيز أسلوب المناقشة خلال الدرس؟

6. كيف يمكن استخدام هذه المسألة لتعزيز أسلوب المناقشة خلال الدرس؟

7. كيف يمكن استخدام هذه المسألة لمساعدان الطالب عند مواجهة أصعب مفهوم نفسه والمعدل؟ كيف تواجه هذه الصعاب؟

الموقف التعليمي الرابع

ارجعوا ملاحظة هذه المسألة والموقف التعليمي التابع لها

1. كيف يمكن استخدام هذه المسألة في خطط الدرس 1
2. كيف يمكن استخدام الحلول المطروحة للكيف زرع هذه المسألة
3. هل لديك حلول أخرى لهذه المسألة غير متأت طرحه
4. كيف يمكن استخدام هذه المسألة لمساعدة الطلاب على تحسين استراتيجيات التفكير العلمي و حل المشاكل
5. كيف يمكن استخدام هذه المسألة لمساعدة الطلاب على ربط مفهوم النسبه ب حياتهم العمليه
6. ما نوع المناقشة الفصلية التي قد ترغب في تفعيلها في الفصل عند حل هذه المسألة؟ متى وفي أي مرحلة من الشرح سوف تقوم بتعييم اسلوب المناقشة مع الطلاب؟

7. ما هي المصاعب التي قد تواجه الطلاب عند محاولة استيعاب مفهوم النسبه والمعدل؟ كيف تواجه هذه الصعاب؟