Detection of Ionospheric Spatial Gradients

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This thesis titled
Detection of Ionospheric Spatial Gradients

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ABSTRACT

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Spatial ionospheric gradients cause differences in satellite ranging measurements at different locations. Three algorithms are analyzed in this thesis to estimate the variations in the TEC gradient between two locations. These variations are then used to characterize the size of spatial gradients. Error sources are analyzed for each algorithm and detection thresholds are calculated. Data collected from Ohio University airport and Jordan sites with a 5-km baseline are used to test the feasibility and performance of the three algorithms.

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## DEFINITIONS OF SYMBOLS AND ABBREVIATIONS

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<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>AD</td>
<td>Accumulated Doppler frequency shift measurement in meters</td>
</tr>
<tr>
<td>c</td>
<td>Speed of light in m/s</td>
</tr>
<tr>
<td>C/A</td>
<td>Coarse/Acquisition code of GPS</td>
</tr>
<tr>
<td>$c\Delta t_{PR}$</td>
<td>Receiver clock offset in meters</td>
</tr>
<tr>
<td>$c\Delta t_{sv}$</td>
<td>Satellite clock offset in meters</td>
</tr>
<tr>
<td>CORS</td>
<td>Continuously Operating Reference Stations</td>
</tr>
<tr>
<td>DD</td>
<td>Double difference</td>
</tr>
<tr>
<td>DFS</td>
<td>Divergence-Free Smoothing</td>
</tr>
<tr>
<td>DGPS</td>
<td>Differential Global Positioning System</td>
</tr>
<tr>
<td>D/U</td>
<td>Desired to Undesired Ratio</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth-Centered-Earth-Fixed</td>
</tr>
<tr>
<td>GIM</td>
<td>Global Ionosphere Map</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>H</td>
<td>Antenna Height</td>
</tr>
<tr>
<td>I</td>
<td>Delay introduced by the ionosphere</td>
</tr>
<tr>
<td>i</td>
<td>i th satellite</td>
</tr>
<tr>
<td>ICA</td>
<td>Ionospheric Correction Algorithm</td>
</tr>
<tr>
<td>j</td>
<td>j th satellite</td>
</tr>
<tr>
<td>JPL</td>
<td>Jet Propulsion Laboratory</td>
</tr>
<tr>
<td>L1</td>
<td>GPS link one</td>
</tr>
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</table>
L2    GPS link two
LAAS  Local Area Augmentation System
MLA   Multipath-Limiting Antenna
MP    Multipath error
N     Integer ambiguity of the carrier phase measurement
OF    Obliquity Factor
POS   Receiver position vector in ECEF coordinate
PR    Pseudorange measurement in meters
R     True range from user to satellite
SBAS  Space-Based Augmentation System
SD    Single difference
SPS   Standard Positioning Service
STEC  Slant TEC
T     Delay introduced by the troposphere
TEC   Total electron content
VTEC  Vertical TEC
ε     Line-of-sight orbit error of the satellite
η     Noise
θ     Satellite elevation angle measured upward from the local horizon
ϑ     Satellite along-track or cross-track orbit error in meters
λ     Wavelength
σ     Standard deviation of measurement
$\tau_G$  Antenna group delay

$\tau_{ph}$  Antenna phase delay

$\psi$  Satellite azimuth angle measured clockwise with respect to north
1. INTRODUCTION

1.1 Global Navigation Satellite Systems

The Global Positioning System (GPS) is a satellite-based radionavigation system consisting of at least 24 satellites configured in 6 orbital planes with a minimum of 4 satellites in each plane. The satellite orbits are approximately 11,000 miles above the earth’s surface and the satellites complete one orbit around the earth approximately every 12 hours. The Russian Space Forces operate the Global Navigation Satellite System (GLONASS), while China and Europe started development of COMPASS and GALILEO, respectively. In general, satellite-based radionavigation systems like GPS, GLONASS, COMPASS and GALILEO are referred to as Global Navigation Satellite Systems (GNSS) [1] [2].

Signals from these four systems propagate through the ionosphere before they reach the earth’s surface. Errors introduced into the GPS range measurements due to the ionosphere are the focus of this thesis. More specifically, this thesis investigates differences in ionospheric errors as observed by two receivers separated by approximately 5 km, where both receivers rely on measurements using either one or two satellite broadcast frequencies. Characterization of these ionospheric spatial gradients are important for high-performance, single-frequency GPS applications where errors observed at one receiver location are used to correct the other receiver’s errors – an operation referred to as differential GPS (DGPS). DGPS is used for aircraft precision approach and landing, precision agriculture, harbor entrance and approach, and several other applications that require high-levels of performance to
satisfy safety and/or economic requirements. At the time of this writing, all GPS satellites broadcast Standard Positioning Service (SPS) Coarse/Acquisition (CA) code signals on the L1 frequency centered at 1575.42 MHz. Some satellites also broadcast a civilian signal at the L2 frequency centered at 1227.6 MHz. In addition, several civilian GPS receivers manage to obtain measurements from both the L1 and L2 frequencies using signals intended for military use.

1.2 GPS Error Model

As the signal travels from the GPS satellites to the receiver, several errors are introduced in the measurements. There are two basic measurements made by a GPS receiver: one is the code phase measurement and the second is the carrier phase measurement. Since the code phase measurement is biased due to the GPS receiver clock offset relative to GPS time, this measurement is also referred to as the pseudorange measurement. The pseudorange measurement error model is given below [3].

\[
PR_{i,L}(t) = R_i(t) - c\Delta t_{i,SV,PR,L}(t) + I_{i,PR,L}(t) + \varepsilon_i(t) + c\Delta t_{PR,L}(t) + \tau_{G,L}\{\theta_i(t),\psi_i(t)\} + MP_{i,PR,L}(t) + \eta_{i,PR,L}(t)
\]

(1.1)

Where:

\(i\): a subscript to indicate a specific satellite;

\(L\): a subscript to indicate either the L1 or the L2 GPS frequency;

PR: a subscript to indicate a pseudorange measurement;

\(R\): true range from user to satellite;

\(c\Delta t_{SV}\): satellite clock offset;
I: delay introduced by the ionosphere;

T: delay introduced by the troposphere;

ε: line-of-sight orbit error of the satellite;

cΔt_{PR}: receiver clock offset;

τ_a: antenna group delay;

θ: satellite elevation angle measured upward from the local horizon;

ψ: satellite azimuth angle measured clockwise with respect to north;

MP: multipath;

η: noise;

The carrier phase measurement provides the accumulated phase of Doppler frequency shift. The error source model for accumulated Doppler (AD) is given below [3].

\[ AD_{i,L}(t) = R_i(t) - c\Delta t_{i,SV,AD,L}(t) - I_{i,AD,L}(t) + T_i(t) + \varepsilon_i(t) + c\Delta t_{AD,L}(t) + N_i\lambda + \tau_{ph,L} \{ \theta(t), \psi(t) \} + MP_{i,AD,L}(t) + \eta_{i,AD,L}(t) + \frac{\lambda}{2\pi} \psi(t) \]

(1.2)

Where:

N: integer ambiguity of the carrier phase measurement;

λ: wavelength of L1/L2 frequency;

τ_{ph}: antenna phase delay.

In this thesis, the emphasis is on ionosphere error characterization, such that Equation (1.1) can be simplified as follows.

\[ PR_{i,L}(t) = R_{i,L}(t) + I_{i,L}(t) + E_{i,L}(t) \]

(1.3)

Where:

E(t): a combination of all errors except ionospheric error.
The ionosphere structure and characteristics will be discussed in the next section.

1.3 Ionospheric Effects on GPS Signals

The ionosphere is a dispersive medium which is typically located at heights from 50 km to more than 1000 km above the earth’s surface. In a typical ionosphere, three layers can be observed [4]:

D region from 50 km to 90 km

E region from 90 km to 150 km

F region consisting of the F1 and F2 layers from 150 km to more than 1000 km

Due to sun radiation, atoms are ionized in the ionosphere, which results in free electrons. These electrons affect the propagation velocity of signals passing through the ionosphere. In addition, the propagation velocity is frequency dependent and the effects on the code and carrier phase measurements are opposite: the carrier phase measurement is advanced while the code phase measurement is delayed. This phenomenon is known as ionospheric divergence [5].

Electron densities in the ionosphere follow a diurnal cycle, while the overall peak level of solar activity follows an eleven year cycle [4]. In GPS, ionosphere delay error during elevated solar activity is the largest error source. Due to the dispersive nature of the ionosphere, most of the error can be mitigated using measurements at two frequencies. At the time of this writing, GPS provides only one frequency for safety-of-life applications, which necessitates a detailed understanding of single-frequency ionosphere errors. Even when a second civilian frequency becomes available, it will still be important to understand single-frequency performance in case
of the loss of one frequency due to interference or high levels of ionosphere signal attenuation.

### 1.4 Total Electron Content (TEC) Calculation

The GPS signal velocity through the ionosphere is directly related to the Total Electron Content (TEC), which is the number of electrons in a cylinder with a cross section of \(1 \text{ m}^2\) connecting the satellite to the GPS receiver [1][5].

\[
TEC = \int_{\text{Satellite}}^{\text{User}} n_e(l)dl
\]  

(1.4)

Where:

- \(n_e(l)\): the electron density along the signal path from the satellite to the receiver.
- \(l\): signal path distance in the direction from the satellite to the receiver.

Neglecting higher order terms, the ionosphere delay for the code phase and advance for the carrier phase is given by:

\[
I_{PR} = -I_{AD} = \frac{40.3 \cdot TEC}{f^2}
\]  

(1.5)

TEC is normally measured in TEC Units (TECU), where 1 TECU=10\(^{16}\) electrons/m\(^2\) and it induces a delay of 0.162 m on L1 signals and 0.267 m on L2 signals. TEC is a function of time, user location, satellite elevation angle, season, ionizing flux, magnetic activity, sunspot cycle and scintillation. It usually lies in the range of 10\(^{16}\) to 10\(^{19}\) electrons/m\(^2\).

The TEC value here is usually referred to as slant TEC (STEC or just TEC). To map the STEC to the zenith delay or vertical TEC (VTEC), the ionosphere is modeled as a thin shell [1]. A mapping function or obliquity factor (OF) is introduced in this model.
to scale STEC to VTEC

The ionospheric OF as a function of elevation angle is defined as follows [1]:

$$OF(\theta) = \frac{1}{\sqrt{1 - \left(\frac{R_e}{R_e + h_i} \cos \theta\right)^2}}$$ (1.6)

Where:

$\theta$: elevation angle at the user location.

$R_e$: average earth radius of the earth.

$h_i$: mean height of the ionospheric shell, which is usually taken in the range of 300-400 km.

Then TEC and VTEC can be related by OF as below:

$$TEC = VTEC \cdot OF(\theta)$$ (1.7)

The range of OF is from one (zenith direction) to around three for a satellite at an elevation angle of five degrees.

Equation (1.7) can be used to relate ionospheric delay to the ionospheric zenith delay as follow [1]:

$$I(\phi) = I_z \cdot OF(\theta)$$ (1.8)

Where:

$I$: ionospheric delay;

$I_z$: ionospheric zenith delay;

For mid-latitude locations, $I_z$ is about 1-3 m at night and 5-15 m in the mid-afternoon (around 2 pm at local time) when $I_z$ typically reaches its highest value during the day [1].
1.5 Thesis Objectives

Due to its dynamic characteristics, both over time and over distance, the ionospheric delay must be fully characterized to assure high-accuracy GPS positioning performance. Of particular interest are the differences in ionospheric errors as observed by two receivers separated by approximately 5 km. Existing methods to calculate absolute TEC are reviewed. Based on these methods, three algorithms are analyzed that can be used to measure time-varying differences in TECs observed at two locations. For each method, the errors that are not due to the ionosphere will be characterized statistically and bounded to ensure that only spatial TEC gradients are detected by the algorithms.

1.6 Thesis Organization

Chapter 2 provides the background and literature review of existing techniques of absolute TEC value estimation and data analysis based on some of the existing methods is provided. Chapter 3 introduces three algorithms for TEC gradient calculations. Chapter 4 provides TEC gradient error sources analysis and thresholds for each algorithm. Chapter 5 contains the data analysis results for three algorithms using GPS data collected from a 5-km baseline in Ohio. Chapter 6 contains the summary, conclusions and recommendations for future research.
2. DIFFERENT APPROACHES TO OBTAIN TEC GRADIENT VALUES

2.1 Absolute TEC Estimation

2.1.1 TEC Estimation with a Single Receiver

Due to the fact that the ionosphere is a dispersive (frequency dependent) medium, the first-order ionospheric delay can be mitigated by using dual frequency measurements. Several techniques have been introduced in [9] [10] [11] [12]. In principle, the ionospheric delay on the L1 frequency can be adjusted using a correction term as shown in Equation (2.1) [8].

\[ I_{PR,L1} = \frac{PR_{L1} - PR_{L2}}{1 - \left( \frac{f_{L1}}{f_{L2}} \right)^2} \] (2.1)

where \( PR_{L1} \) and \( PR_{L2} \) are pseudorange measurements for L1 and L2, respectively.

After the ionospheric delay is calculated, then the TEC can be estimated by Equation (1.5). However, in reality, this estimation is corrupted by several error sources, such as the receiver inter-frequency bias, the satellite inter-frequency bias, multipath and noise. The biases can be corrected either by hardware calibration or by using a method introduced in [8], which estimates the total bias at night when the ionosphere is quiet.

There are other methods using single frequency measurements to mitigate ionospheric delay, among which the simplest way is using the broadcast Ionospheric Correction Algorithm (ICA). This algorithm uses eight coefficients broadcast in the GPS satellite ephemeris data, and it can correct around 50% of the total ionospheric error. There are more complicated and accurate models such as the International Reference Ionosphere...
(IRI) and the Bent model in which hundreds of coefficients are used to calculate TEC, and delay correction can be improved to about 70%. Further improved delay corrections are available from the Space-Based Augmentation System (SBAS), which corrects up to 90% of the ionospheric delay in near-real time. To use SBAS ionosphere corrections, the GPS receiver must be SBAS capable and within the SBAS service volume [9].

A new single frequency algorithm is introduced in [13]. The basic idea of this algorithm is to keep the vertical TEC floating as an unknown in the range equation.

2.1.2 TEC Estimation with a Network of Receivers

A lot of research has been done by different research organization to calculate TEC and produce global TEC maps since the 1990s. One of these organizations is the Jet Propulsion Laboratory (JPL), which provides Global Ionosphere Maps (GIM). Data are collected from over 100 continuously operating GPS receivers and used to produce global TEC maps. The GIM is updated every 5-15 minutes.

2.2 Relative TEC Estimation

There are two ways of estimating the TEC difference between two locations. Using the first method, the absolute TEC value measured at receiver B $TEC_B(t)$ is subtracted from the absolute TEC value $TEC_A(t)$ measured at receiver A. The outcome can be expressed as $TEC_A(t) - TEC_B(t)$. The second method makes use of direct gradient measurements, which is discussed for the different levels of ionosphere spatial gradient and ionosphere treat models in [6]. The algorithms introduced in this thesis are based on the both the absolute and relative TEC gradient methods. Due to
unknown biases encountered by each of the methods, the focus will be on the
detection of changes in the spatial gradients. A detailed analysis on error distribution,
ionosphere spatial gradient detection and expected performance will be provided for
each method in Section 3.

2.3 Carrier Smoothing Process

Equation (2.1) relies on pseudorange (code phase) measurements. Accumulated
Doppler (carrier phase) measurements are less noisy but biased with an unknown
ambiguity. Raw code phase measurements can be smoothed by carrier phase before
being used in the TEC calculation. The main goal of carrier smoothing is to attenuate
the high frequency disturbance in code phase, which could be due to either multipath
or noise.

Traditional Hatch filter smoothing and divergence-free smoothing (DFS) are
discussed in [6] [8], two methods that are widely used.

A 2.5-hour long data set collected at Arecibo Observatory is used to test the
performance of these two smoothing techniques. The results are shown in Figure 2.1.
In Figure 2.1, the blue line is the TEC value calculated from raw code phase measurements using Equation (2.1); the red line is the TEC value smoothed by a Hatch filter; the black line is the TEC value smoothed by DFS. First, it is noticed that the TEC estimate is partially negative due to interfrequency hardware biases in both the satellite and the GPS receivers. As can also be observed from Figure 2.1, the red line is much smoother than the blue line which contains noise and multipath fluctuations. The DFS technique does not accumulate ionospheric divergence [6], however, the ionospheric delay still remains in the divergence-free smoothed pseudorange. It also demonstrates the fact that employing a smoothing filter will
unavoidably impose a time delay on the output, which is approximately \( \frac{1}{3} T_{\text{smooth}} \)

where \( T_{\text{smooth}} \) is the duration time of the smoothing filter [8].

2.4 Discussion of TEC Methods

Unless the interfrequency receiver and satellite hardware delays are both calibrated, the absolute TEC value will be biased even after all other sources are removed. Therefore, this thesis focuses on the changes of TEC values over time and distance, instead of absolute TEC values. The objective is to identify spatial gradients, which are not present continuously and can therefore be identified and characterized by analyzing the changes in TEC between two locations. Specifically, relative TECs can be calibrated (“set to zero”) when a period of time with no ionospheric gradient activity has been observed. This method is appropriate for characterization of the size of ionospheric spatial gradients, but it is not sufficient for real time detection when new satellites appear above the horizon. Since characterization of ionospheric spatial gradients is the focus of this thesis, relative methods are sufficient, and are also helpful for the development of real time algorithms.
3. ALGORITHMS FOR TEC GRADIENT CALCULATIONS

3.1 Overview

As described in Chapter 2, the precision of absolute TEC estimation is limited by several error sources. Hence, instead of trying to obtain the absolute TEC values, TEC variation between two locations is the primary goal in this thesis. The TEC difference between two locations over a baseline (in km) is referred to as the TEC gradient, as is shown in Figure 3.1.

Figure 3.1 TEC gradient calculation

Equation (3.1) can be used to calculate the TEC gradient between two locations A and B. Only ionospheric delay at the L1 frequency is addressed in this equation.
\[ G = \frac{(TEC_{A,L1} - TEC_{B,L1})}{\text{baseline}} = \frac{f_{L1}^2 (I_{A,L1} - I_{B,L1})}{40.3} \]

(3.1)

where:

- \( G \) : TEC gradient between two locations;
- \( TEC \) : TEC value at one site;

Baseline: geometric range between locations A and B;

- \( I_{A,L1} \) : L1 ionospheric delay at location A;
- \( I_{B,L1} \) : L1 ionospheric delay at location B;
- \( I_{AB,L1} \) : L1 ionospheric delay difference between locations A and B.

By investigating the TEC gradient changes over time, errors due to receiver biases are avoided and the resulting gradient can be initialized to zero when no gradient variations are present.

In this chapter, three algorithms are introduced to calculate the TEC gradient using GPS measurement data from two locations. The first algorithm uses the combination of single frequency carrier phase and code phase measurements. The second algorithm is also based on single frequency measurements, but only carrier phase measurements. The third method is a dual frequency method, with only carrier phase measurements. These algorithms are discussed in detail in the following sections.

3.2 Algorithm 1: Single Frequency Carrier and Code Phase Measurements

In this algorithm, only L1 code and carrier phase measurements are used, as shown in Equation (3.2).

\[ CMC_{SD_{i,AB}} = (PR_{i,A,L1} - AD_{i,A,L1}) - (PR_{i,B,L1} - AD_{i,B,L1}) \]  

(3.2)
where $SD_{i,AB}$ is the single difference of code minus carrier between two receivers with respect to the $i$th satellite.

Equation (3.2) can be expanded by substituting Equations (1.1) and (1.2) into it (refer to Appendix A for details). The gradient of the ionospheric delay is given by:

$$I_{i,L1,AB,1} = 0.5 \times (CMC - SD_{i,AB} + c \Delta t_{i,SV,PR,AD,L1,AB} - c \Delta t_{i,PR,AD,L1,AB} - MP_{i,PR,AD,L1,AB} - \tau_{i,G,Ph,L1,AB} - \eta_{i,PR,AD,L1,AB} + N_{i,AB,L1} \Delta_{L1} + \frac{\lambda_{L1}}{2\pi} \Psi_{i,AB}) / \text{baseline}$$

(3.3)

where $I_{i,L1,AB,1}$ is the ionospheric delay gradient between two sites A and B at L1 frequency with respect to the $i$th satellite. The subscript 1 indicates that this ionospheric gradient is derived from algorithm 1. $c \Delta t_{i,SV,PR,AD,L1,AB}$ is the $i$th satellite clock offset difference between code and carrier phase measurements on L1 at antenna A subtracted from the $i$th satellite clock offset difference between code and carrier phase measurements on L1 at antenna B. This error is canceled out because it’s common for receivers with respect to the same satellite. $c \Delta t_{i,PR,AD,L1,AB}$ is the receiver clock offset difference between code and carrier phase measurements on L1 at antenna A subtracted from the clock offset difference between code and carrier phase measurements on L1 at antenna B. This error is assumed to be constant as well. $MP_{i,PR,AD,L1,AB}$ is the multipath error difference between code and carrier phase measurements on L1 at antenna A subtracted from the multipath error difference between code and carrier phase measurements on L1 at antenna B. $\tau_{i,G,Ph,L1,AB}$ is the difference between antenna group and phase delay on L1 at antenna A subtracted from the difference between antenna group and phase delay on L1 at antenna B. $\eta_{i,PR,AD,L1,AB}$ is the thermal noise error difference between code and carrier phase...
measurements on L1 at antenna A subtracted from the thermal noise error difference between code and carrier phase measurements on L1 at antenna B. \( N_{i,AB,L1,L1} \) is the ambiguity difference of carrier phase measurements on L1 between antennas A and B. \( \frac{\lambda_{L1}}{2\pi} \Psi_{i,AB} \) is the phase wrap up difference at L1 between antennas A and B.

As discussed in Chapter 1, the first order ionospheric delay in code and carrier phase measurements has the same magnitude, but opposite signs, which is also known as ionospheric divergence. Due to the divergence, all of the error differences between the two receivers are halved in Equation (3.3).

### 3.3 Algorithm 2: Single Frequency Carrier Phase Measurements

In this algorithm, a double difference method using L1 frequency carrier phase measurements from two satellites is applied to eliminate the time-varying receiver clock bias.

\[
DD_{g,AB} = SD_{i,AB} - SD_{j,AB} = (AD_{i,A,L1} - AD_{i,B,L1}) - (AD_{j,A,L1} - AD_{j,B,L1})
\]  

(3.4)

where:

\( i, j \) : the subscripts indicate measurements from \( i \) th or \( j \) th satellite

Refer to the Appendix B for the expansion of Equation (3.4). The gradient of ionospheric delay between two receivers is calculated as follows.

\[
I_{g,L1,AB,2} = (R_{g,AB} + T_{g,AB} + \varepsilon_{g,p,AB} + MP_{g,AD,L1,AB} + \tau_{g,ph,L1,AB} + \frac{\lambda_{L1}}{2\pi} \Psi_{g,AB} + N_{g,AB} \lambda_{L1} + \eta_{g,AD,L1,AB} - DD_{g,AB}) / \text{baseline}
\]  

(3.5)

where:

\( I_{L1,AB,2} \) is the ionospheric delay gradient difference between the \( i \) th and \( j \) th satellites for sites A and B at the L1 frequency. The subscript 2 indicates that this ionospheric
gradient is derived from algorithm 2;

A double difference technique applied in this method contains error components from four carrier phase measurements: two sets of carrier phase measurements from two different satellites.

3.4 Algorithm 3: Dual Frequency Carrier Phase Measurements

In this algorithm, both L1 and L2 frequency carrier phase measurements are used, as shown in Equation (3.6).

\[
AD_{L1L2}^{SD} = I_{i,L1,A} - I_{i,L1,B} = \frac{1}{f_1^2} (AD_{i,L1,A} - AD_{i,L2,A}) - \frac{1}{f_2^2} (AD_{i,L1,B} - AD_{i,L2,B})
\]

(3.6)

Refer to Appendix C for the expansion resulting in Equation (3.7)

\[
I_{i,L1,AB,3} = [AD_{L1L2}^{SD} + \frac{1}{\alpha} (c\Delta t_{i,AD,L1,L2,AB} + MP_{i,AD,L1,L2,AB} + \tau_{i,Ph,L1,L2,AB})
\]
\[
+ N_{i,L1,AB} (\lambda_{\ell_1} - \lambda_{\ell_2}) + \frac{\lambda_{\ell_1} - \lambda_{\ell_2}}{2\pi} \psi_{i,AB} + n_{i,AD,L1,L2,AB})] / \text{baseline}
\]

where \( \alpha = \frac{f_1^2}{f_2^2} - 1 \)

The subscript 3 in \( I_{L1,AB,3} \) indicates that this ionospheric delay gradient is derived from algorithm 3.

Since the ionospheric delay introduced by the L1 and L2 signals has the relation given by \( I_{L2} = \left( \frac{f_1}{f_2} \right)^2 I_{L1} \), a factor \( \frac{1}{\alpha} \) is a multiplier in Equation (3.7).

Equation (3.7) contains the relative clock offset between L1 and L2 for receivers A and B which can not be neglected. In order to investigate the effect of the clock drift between L1/L2 over two antennas, 24 hours of data are processed to obtain filtered
dual frequency carrier phase measurement residuals. Assuming that no ionospheric gradients are present, then the clock drift between L1 and L2 over antennas A and B, $\Delta t_{i,AD,L1,L2,AB}$ can be estimated. Figure 3.2 shows the clock drift estimate between L1 and L2 for the two antennas over 24 hours.

![Smoothed & Unsmoothed SD residual](image)

**Figure 3.2 Clock drift estimate over 24 hours**

From Figure 3.2, the maximum clock drift is approximately 3 cm/hour and the peak to peak value over 24 hours is approximately 6 cm. This drift needs to be taken into consideration, since it is up to 30% of the detection threshold goal of 10 cm over 5-km baseline. It is difficult to guarantee a bound on this clock drift as the long-term behavior is unknown. Therefore, the clock offset is removed by differencing between two satellites $i$ and $j$: 
\[ AD_{L1,L2,DD} = I_{y,L1,A} - I_{y,L1,B} = \frac{1}{(f_1/f_2)^2} (AD_{y,L1,A} - AD_{y,L2,A}) - \frac{1}{(f_1/f_2)^2} (AD_{y,L1,B} - AD_{y,L2,B}) \]

(3.8)

Such that Equation (3.7) is rewritten as:

\[
I_{y,L1,AB} = [AD_{L1,L2,DD} + \frac{1}{\alpha} (MP_{y,AD,L1,L2,AB} + \tau_{y,PK,L1,L2,AB} + N_{y,L1,AB}(\lambda_{l1} - \lambda_{l2})) + \frac{\lambda_{l1} - \lambda_{l2}}{2\pi} \psi_{y,AB} + \eta_{y,AD,L1,L2,AB})]/\text{baseline} \]

(3.9)

3.5 Summary

From the description of the three gradient algorithms above, the advantages and disadvantages of the three methods are summarized as follows:

Method 1: Uses single frequency GPS code and carrier phase measurements. Although the error sources are cut in half due to the code and carrier differencing, the code phase measurements are noisy.

Method 2: Uses single frequency GPS carrier phase measurements. A double difference method is applied in this method to eliminate the time-varying receiver clock offset which results in increased error differences and the resulting gradient is also sensitive to tropospheric gradients and antenna positions.

Method 3: Uses dual frequency GPS carrier phase measurements. This method is not sensitive to tropospheric gradients or antenna positions. However, the effect of the clock drift \( c\Delta t_{AD,L1,L2,AB} \) may not be neglected and therefore a single difference between two satellites must be applied, which increases noise on the gradient estimate.
4. TEC Gradient Error Sources Analysis

4.1 Overview

Three algorithms are discussed in Chapter 3 to obtain the TEC gradient with single or dual frequency measurements between two locations. In this chapter, each error source is evaluated to determine the accuracy of the gradient estimates and to identify options to improve the estimates.

4.2 Error Sources Analysis

Table 4.1 shows how different error sources affect the three algorithms. An “X” in a cell indicates that the specific error source affects the algorithm, while a “-” means the error source doesn’t affect the algorithm. Each error source is discussed in detail in the following sections from 4.2.1 to 4.2.10.
Table 4.1: Errors in Three TEC Gradient Calculations

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Method 1 (single frequency code minus carrier)</th>
<th>Method 2 (single frequency double difference)</th>
<th>Method 3 (dual frequency double difference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troposphere Decorrelation</td>
<td>-</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Orbit Error</td>
<td>-</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Antenna Position Error</td>
<td>-</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>Multipath</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Thermal Noise</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Phase Wrap Up</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Satellite Clock Offset</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Antenna Phase/Group Delay</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Receiver Clock Offset</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ambiguities</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 4.2 shows the expression for each error source in each algorithm. (refer to Appendices A to C for details)

**Table 4.2: Error Source Expression**

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Method 1 (single frequency code minus carrier)</th>
<th>Method 2 (single frequency double difference)</th>
<th>Method 3 (dual frequency double difference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troposphere Decorrelation</td>
<td>-</td>
<td>$T_{ij,AB}$</td>
<td>-</td>
</tr>
<tr>
<td>Multipath</td>
<td>$-0.5MP_{PR,AD,I1,AB}$</td>
<td>$MP_{ij,AD,I1,AB}$</td>
<td>$\frac{1}{\alpha}MP_{ij,AD,I1,I2,AB}$</td>
</tr>
<tr>
<td>Antenna Position Error</td>
<td>-</td>
<td>$A_{ij,AB}$</td>
<td>-</td>
</tr>
<tr>
<td>Orbit Error</td>
<td>-</td>
<td>$\epsilon_{g,p,AB}$</td>
<td>-</td>
</tr>
<tr>
<td>RF Phase/Group Delay</td>
<td>$-0.5\tau_{G,Ph,I1,AB}$</td>
<td>$\tau_{ij,Ph,I1,AB}$</td>
<td>$\frac{1}{\alpha}\tau_{ij,Ph,I1,I2,AB}$</td>
</tr>
<tr>
<td>Satellite Clock Offset</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Receiver Clock Offset</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ambiguities</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Phase Wrap up</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Thermal Noise</td>
<td>$-0.5\eta_{PR,AD,I1,AB}$</td>
<td>$\eta_{ij,AD,I1,AB}$</td>
<td>$\frac{1}{\alpha}\eta_{ij,AD,I1,I2,AB}$</td>
</tr>
</tbody>
</table>

4.2.1 Tropospheric Spatial Decorrelation Error

The troposphere is the lowest part of earth’s atmosphere which extends up to about 16 km in the tropical regions and about 9 km in the polar areas [1]. It mainly contains dry gases and water vapor, thus there are a variety of well established methods that model the tropospheric air with a dry component and a wet component. The dry component, which is responsible for around 90% of the total tropospheric delay, can be predicted with relatively high accuracy. The wet component, which accounts for approximately
10% of the total troposphere delay, is difficult to predict and model. Unlike the ionosphere, the troposphere is not a dispersive medium for frequencies up to 15 GHz [15]. As a result, the troposphere delay cannot be mitigated by using dual frequency measurements. Since the code and carrier velocities are the same within this layer, the code and carrier measurements on L1 and L2 experience the same delay, which typically ranges from 2.5 m to 25 m [1] [5]. Tropospheric delay is a function of satellite elevation angle, temperature, pressure and relative humidity [5] [14].

Most of the tropospheric delay is the same for two locations within a few km. The remaining tropospheric delay difference is referred to as the tropospheric spatial decorrelation error. In relatively quiet weather conditions, this error can be mitigated by implementing tropospheric propagation delay models. But in severe weather conditions, the entire troposphere profile lacks understanding and it is more difficult to model the path from satellites to GPS receivers [14]. Consequently, the propagation delay models become less accurate.

For ionospheric gradient methods 1 and 3, the tropospheric gradient does not affect the gradient estimation, but for method 2,  is the difference between tropospheric delay at antenna A and tropospheric delay at antenna B with respect to th and jth satellites. For a 5-km baseline, this error is within 0.1 m (95% confidence) and 0.2 m (1-10^-7 confidence) [14].

4.2.2 Satellite Orbit Error

Similar to the troposphere error, the orbit does not affect methods 1 and 3. For method 2, orbit error  is the difference between orbit error projections of th and jth
satellites on the line-of-sight directions to two receivers A and B. An orbit error vector can be decomposed in three directions: along track, cross track and range directions. Typical GPS satellite orbit error (using broadcast ephemeris data) for the block IIA satellites in the along track direction is approximately 4.2 m (95%) and, in the cross track direction the error is approximately 2 m (95%), in radial direction the error is approximately 0.72 m (95%), and the three dimensional radial position error is approximately 4.6 m (95%) [16]. Orbit errors for the newer satellites (Block IIR and Block IIR-M) are smaller than those of the Block IIA satellites. Also, the bounding probabilities are found from [16] by multiplying the overbound sigmas by a factor of 5.33 to obtain (1-10^{-7}) confidence: along track 30.9 m, cross track 18.6 m, radial 21.4 m, and the three-dimensional radial position 33.2 m. The magnitude of the satellite differential orbit error can be approximated as follows [14].

\[ e_{i,p,AB}(t) \approx \frac{|\overline{POS}_i - \overline{POS}_b|}{Range_i(t)} |\delta_i(t)| \]  \hspace{1cm} (4.8)

where:

- \overline{POS} : receiver position vector in ECEF coordinates;
- \delta_i(t) : i the satellite along-track or cross-track orbit error in meters;
- Range_i(t) : distance from receivers to i th satellite.

Since the distance between the two receivers is 5 km, the worst case in along-track or cross-track directions is 33.2 m (1-10^{-7} confidence) and the range to the satellite is approximately 20 million meters, then the differential orbit error is approximately \[
\frac{5\times10^3}{20\times10^6} \times 33.2 = 8.3 \text{ mm.}
\] This number is doubled to 16.6 mm as the double difference
method is applied in method 2. Note that for bounding purposes, the errors are assumed to add in the worst possible way. For 95% performance, the horizontal orbit error will be less than the three-dimensional radial error of 4.6 m and the error is assumed independent between satellites, such that the differential orbit error is approximately \( \sqrt{2 \times \frac{5 \times 10^3}{20 \times 10^6} \times 4.6} = 1.6 \text{ mm} \).

It is noted that the above statistics are only valid in the absence of major satellite orbit anomalies that could occur up to 3 times per years [16]. If an orbit error is larger than 33 m, it is considered an anomaly that needs to be addressed in a different way. One solution for ionospheric gradient monitoring would be to use a combination of methods 2 and 3 or 2 and 1 to detect an anomalous orbit error condition.

4.2.3 Code Phase and Carrier Phase Multipath Errors

When a GPS signal arrives at an antenna, it might be reflected or diffracted from nearby structures such as buildings, trees or ground, etc., and this phenomenon is referred to as multipath. An antenna can receive the direct signal together with one or more reflected replica signals. Because multipath signals travel longer than the direct signal does, multipath signals are usually delayed and attenuated relative to direct signals. Multipath affects the code and carrier phase measurements in different ways. Typical multipath error magnitude on code phase measurements is on the order of 1 m to more than 5 m. In comparison, the multipath error magnitude on carrier phase measurements is relatively small and can’t be worse than one quarter wavelength. [5][14] In this thesis, the formulas used to calculate the code and carrier multipath error bounds are given in Equation (4.9) and (4.11) respectively:
\[ MP_{i,pr}(H, \frac{D}{U}, \theta) = 2\gamma H \sin(\theta) \] (4.9)

with \[ \gamma = \frac{1}{\frac{D}{(U \cdot \text{Loss})}} \] (4.10)

where:

\( i \): \( i \)th satellite;

D/U: signal desired to undesired ratio;

Loss: reflection coefficient;

H: antenna height;

\( \theta \): satellite elevation angle.

\[ MP_{i,AD,L} = \frac{\lambda_L}{2\pi} \tan^{-1}(\gamma) \] (4.11)

From Equations (4.9) and (4.11), we can observe that the multipath error bound for the code is dependent on signal D/U ratio, elevation angle, reflection coefficient and antenna height while the multipath error bound for the carrier depends on signal D/U ratio and reflection coefficient [25].

This thesis presents data collected from a NovAtel Pinwheel antenna. Expected performance of an integrated multipath-limiting antenna (MLA) is also discussed and it will be used in future work. The multipath error on code phase measurements collected from an MLA can be reduced to approximately 0.1 m (95% confidence) and 0.2 m (1-10^{-7} confidence) [17][18][19].

Since the multipath environment of receivers at two locations is different, multipath errors of both locations are considered independent. Although the multipath errors on L1 and L2 are related, their phase relationship is not known. Therefore, the multipath errors on L1 and L2 are assumed to be unrelated as well.
The Pinwheel antenna is discussed first. For method 1,
\[ 2(M_{\text{PR}_i,\text{AD}_j,\text{L}_1,\text{AB}} + M_{\text{PR}_i,\text{AD}_j,\text{L}_1,\text{AB}}) \] stands for the difference between multipath errors on code phase measurement at L1 and carrier phase measurements at L1 of antenna A subtracted from the difference between multipath on code phase measurements at L1 and carrier phase measurement at L1 of antenna B with respect to \( i \)th satellite. For method 2, \( M_{\text{PR}_i,\text{AD}_j,\text{L}_1,\text{AB}} \) is the difference between multipath error on carrier phase measurements at L1 of antenna A and multipath error on carrier phase measurements at L1 of antenna B with respect to \( i \)th and \( j \)th satellites. For method 3, \( M_{\text{PR}_i,\text{AD}_j,\text{L}_1,\text{L}_2,\text{AB}} \) is the difference between multipath on L1 carrier phase measurements and multipath on L2 carrier phase measurements of antenna A subtracted with the difference between multipath on L1 carrier phase measurements and multipath on L2 carrier phase measurements of antenna B with respect to \( i \)th and \( j \)th satellites.

For 95% confidence, approximately by 2 standard deviations, the total combined multipath error is
\[ 0.5 \times 2 \times \sqrt{2 \times (M_{\text{PR}_i,\text{L}_1}^2 + M_{\text{AD}_j,\text{L}_1}^2)} = \sqrt{2(M_{\text{PR}_i,\text{L}_1}^2 + M_{\text{AD}_j,\text{L}_1}^2)} \] for method 1, \[ 2 \sqrt{2M_{\text{PR}_i,\text{AD}_j,\text{L}_1}^2 + 2M_{\text{AD}_j,\text{L}_1}^2} \] for method 2 and \[ \frac{2}{\alpha} \sqrt{2M_{\text{PR}_i,\text{AD}_j,\text{L}_1}^2 + 2M_{\text{PR}_i,\text{AD}_j,\text{L}_2}^2 + 2M_{\text{AD}_j,\text{L}_1}^2 + 2M_{\text{AD}_j,\text{L}_2}^2} \] for method 3 with \( \alpha \) provided by Equation (3.7). For 1-10\(^{-7}\) confidence, the total combined multipath error is
\[ 0.5 \times 2(M_{\text{PR}_i,\text{L}_1} + M_{\text{AD}_j,\text{L}_1}) \] for method 1, \( 2(M_{\text{PR}_i,\text{L}_1} + M_{\text{AD}_j,\text{L}_1}) \) for method 2 and \[ \frac{2}{\alpha} (M_{\text{PR}_i,\text{AD}_j,\text{L}_1} + M_{\text{PR}_i,\text{AD}_j,\text{L}_2} + M_{\text{AD}_j,\text{L}_1} + M_{\text{AD}_j,\text{L}_2}) \] for method 3. Notice that the code and carrier multipath errors in the 1-10\(^{-7}\) confidence calculation use a prime notation to indicate that these represent multipath error bounds. Also, for the 1-10\(^{-7}\) confidence, the multipath errors are assumed to add in the worst possible way to reflect that
multipath error distributions are sinusoidal, which increases the likelihood that more than one multipath error is at a maximum value at the same time.

Figure 4.1 shows an overbound for Pinwheel antenna ground multipath D/U ratio at L1 and L2 as a function of elevation angle [18].

For example, the following parameters could be used for a 95% confidence calculation:

50 degrees elevation angle, 22.5 dB D/U ratio at L1 and 20 dB D/U ratio at L2, H = 5 m, Loss = 0.5 [18].

D/U = 22.5 dB \approx 13 \text{ at L1}, \text{ D/U = 20 dB = 10 at L2}. 
\[ MP_{i,PR} = 2\alpha H \sin(\theta)=2 \times 0.5 \times (1/13) \times 5 \times \sin(50) \approx 0.29m \]

\[ MP_{i,AD,L1} = \frac{\lambda_{L1}}{2\pi} \tan^{-1}(\gamma) = \frac{0.19}{2\pi} \tan^{-1}(0.5/13) \approx 0.001m \]

\[ MP_{i,AD,L2} = \frac{\lambda_{L2}}{2\pi} \tan^{-1}(\gamma) = \frac{0.24}{2\pi} \tan^{-1}(0.5/10) \approx 0.002m \]

Then the total multipath error for each method is calculated as follows:

**Method 1:** \[ 0.5 \times 2 \sqrt{2 \times (0.29^2 + 0.001^2)} \approx 0.4m \]

**Method 2:** \[ 2\sqrt{4 \times 0.001^2} = 0.004m \]

**Method 3:** \[ 1.5457 \times 2 \sqrt{2 \times (2 \times 0.001^2 + 2 \times 0.002^2)} \approx 0.01m \]

The following parameters are an example for 1-10^{-7} confidence calculation:

5 degrees elevation, 2.2 dB D/U ratio at L1 and 2 dB at L2, H = 5 m, Loss =1 [18] [25].

D/U = 2.2 dB \approx 1.29 at L1, D/U = 2dB \approx 1.26 at L2.

\[ MP_{i,PR} = 2\alpha H \sin(\theta)=2 \times (1/1.29) \times 5 \times \sin(5) \approx 0.68m \]

\[ MP_{i,AD,L1} = \frac{\lambda_{L1}}{2\pi} \tan^{-1}(\gamma) = \frac{0.19}{2\pi} \tan^{-1}(1/1.29) \approx 0.02m \]

\[ MP_{i,AD,L2} = \frac{\lambda_{L2}}{2\pi} \tan^{-1}(\gamma) = \frac{0.24}{2\pi} \tan^{-1}(1/1.26) \approx 0.03m \]

Then the multipath error bounds for 1-10^{-7} confidences for each method are calculated as follows:

**Method 1:** \[ 0.5 \times 2 \times (0.68+0.02) = 0.7m \]

**Method 2:** \[ 4 \times 0.02 = 0.08m \]

**Method 3:** \[ 1.5457 \times 2 \times (0.02+0.03) \approx 0.3m \]

Next, the MLA performance is discussed. According to [21], the desired and undesired (D/U) ratio of an MLA is above 35 dB between elevation angles of 5 to 13 degrees as shown in Figure 4.2 [17]. Between 13 and 34-degree elevation angles, the
D/U ratio is above 30 dB.

![Ground multipath D/U for MLA](image)

**Figure 4.2 Ground multipath D/U for MLA, [17]**

For example, if the MLA D/U parameters are used, then the 95% confidence could be based on the following parameters:

- 20 degrees elevation angle, 30 dB D/U ratio, \( H = 5 \) m, Loss = 0.5.

\[
MP_{i,PR,MLA} = 2\alpha H \sin(20) \approx 0.05 \text{m}
\]

\[
MP_{i,AD,L1,MLA} = \frac{\lambda_{11}}{2\pi} \tan^{-1}(\gamma) \approx 0.0005 \text{m}
\]

Then the total multipath error for method 1 is

\[
0.5 \times 2 \sqrt{2 \times (0.05^2 + 0.0015^2)} \approx 0.07 \text{m}
\]

For an example 1-10⁻⁷ confidence calculation:

- 35 degrees elevation angle, 25 dB D/U ratio, \( H = 5 \) m, Loss = 1.

\[
MP_{i,PR,MLA} = 2\alpha H \sin(35) \approx 0.3 \text{m}
\]
\[
MP_{i,AD,L1,MLA} = \frac{\lambda_{L1}}{2\pi} \tan^{-1}(\gamma) = \frac{0.19}{2\pi} \tan^{-1}(1/18) \approx 0.002m
\]

Then the total multipath error for method 1 is \(0.5 \times 2 \times (0.3 + 0.002) \approx 0.3m\)

4.2.4 Thermal Noise

Thermal noise affects both code and carrier phase measurements and it is induced by hardware components such as antenna, cables, amplifiers, etc. Noise is independent between receivers and L1/L2 frequencies. Thermal noise is considered to exhibit a Gaussian distribution.

Thermal noise observed on carrier phase measurements is given as follows [25]:

\[
\sigma_{\eta,AD,L} = \frac{\lambda_{L}}{2\pi} \sqrt{\frac{B_{AD}}{0.1C_{\eta}N_{o}}} = \frac{\lambda_{L1}}{2\pi} \sqrt{\frac{15}{10}} \approx 0.0012m.
\]

Where:

\(\sigma_{\eta,AD} \): standard deviation of carrier phase tracking error in meters;

\(\lambda_{L} \): GPS signal wavelength on L1/L2 in meters, \(\lambda_{L1} = 0.19 \text{ m}, \lambda_{L2} = 0.24 \text{ m}\);

\(B_{AD} \): carrier loop tracking bandwidth in Hz, which is 15 Hz in this case;

\(C_{\eta}N_{o} \): signal carrier to noise ratio in dB-Hz.

For instance, for a signal with \(C_{\eta}N_{o}\) of 40 dB-Hz and carrier loop tracking bandwidth of 15 Hz,

\[
\sigma_{\eta,AD,L1} = \frac{\lambda_{L1}}{2\pi} \sqrt{\frac{15}{10}} = 0.19 \sqrt{\frac{15}{10}} \approx 0.0012m.
\]

\[
\sigma_{\eta,AD,L2} = \frac{\lambda_{L2}}{2\pi} \sqrt{\frac{15}{10}} = 0.24 \sqrt{\frac{15}{10}} \approx 0.0015m.
\]

Thermal noise observed on code phase measurement is given as follows [26]:

\[
\sigma_{\eta,chip} = \sqrt{\frac{d \times B_{chip}}{2 \cdot 10}} \sqrt{\frac{0.1C_{\eta}N_{o}}{N_{o}}}
\]

where:
\( \sigma_{\eta,PR} \): standard deviation of pseudorange noise in meters;

chip: the C/A code bit length in meters, which is approximately 293 m;

d: the correlator spacing in chips, which is 0.1.

\( B_{SS} \): the single-sided loop bandwidth in Hz.

For example, for a signal with \( \frac{C}{N_0} \) of 40 dB-Hz and single-sided bandwidth of 0.125 Hz, \( \sigma_{\eta,PR} = 293 \sqrt{\frac{0.1 \times 0.125}{2 \times 10^4}} = 0.2316m \). After 100-s smoothing filter, the equivalent single-sided bandwidth is 0.0025 Hz and the pseudorange noise is reduced to 0.033 m.

For method 1, \( \eta_{i,PR,AD,L1,AB} \) is the difference between thermal noise on code phase measurement at L1 and thermal noise on carrier phase measurement at L1 of antenna A subtracted from the difference between thermal noise on code phase measurement at L1 and thermal noise on carrier phase measurement at L1 of antenna B with respect to \( i \) th satellite. Thus the total combined noise error for 95% confidence is \( 0.5 \times 2 \sigma_1 = 0.5 \times 2 \sqrt{2 \sigma_{PR}^2 + \sigma_{AD,L1}^2} = \sqrt{2} \sqrt{0.033^2 + 0.0012^2} \approx 0.047m \); for 1-10\(^{-7}\) confidence is \( 0.5 \times 5.33 \sigma_1 = 0.5 \times 5.33 \times \sqrt{2 \sigma_{PR}^2 + \sigma_{AD,L1}^2} = 2.665 \times \sqrt{2 \sqrt{0.033^2 + 0.0012^2}} \approx 0.1m \).

For method 2, \( \eta_{ij,AD,L1,AB} \) is the difference between thermal noise on carrier phase measurement at L1 of antenna A and B with respect to \( i \) th and \( j \) th satellites. Then the total combined error for 95% confidence is \( 2 \sigma_2 = 2 \sqrt{4 \sigma_{AD,L1}^2} = 2 \times 2 \sqrt{0.0012^2} \approx 0.005m \); for 1-10\(^{-7}\) confidence is \( 5.33 \sigma_2 = 5.33 \times \sqrt{4 \sigma_{AD,L1}^2} = 5.33 \times 2 \sqrt{0.0012^2} \approx 0.01m \). For method 3, \( \eta_{ij,AD,L1,L2,AB} \) is the difference between thermal noise on carrier phase measurement at L1 and L2 of Antenna A subtracted from the difference between thermal noise on carrier phase measurement at L1 and L2 of Antenna B.
measurement at L1 and L2 of Antenna B with respect to \(i\) th and \(j\) th satellites. Thus the total combined error for 95\% confidence is

\[
\frac{1}{\alpha} \sigma_3 = \frac{1}{\alpha} 2 \sqrt{2(\sigma_{i,AD,L1}^2 + \sigma_{j,AD,L2}^2 + \sigma_{j,AD,L1}^2 + \sigma_{j,AD,L2}^2)} \approx 1.5457 \cdot 4\sqrt{0.0012^2 + 0.0015^2} \approx 0.01 m;
\]

for 1-10^{-7} confidence, the total combined error is

\[
\frac{1}{\alpha} 5.33 \sigma_3 = \frac{5.33}{\alpha} 2 \sqrt{2(\sigma_{i,AD,L1}^2 + \sigma_{j,AD,L2}^2 + \sigma_{j,AD,L1}^2 + \sigma_{j,AD,L2}^2)} \approx 1.5457 \cdot 5.33 \cdot 2\sqrt{0.0012^2 + 0.0015^2} \approx 0.03 m.
\]

4.2.5 Phase Wrap up

\[
\frac{\lambda}{2\pi} \Psi_{i,AB}
\]

is the difference in antenna wrap up error on both frequencies between two receivers with respect to \(i\) th satellite. This error depends on the difference in azimuth angle to the same satellite between two receivers, which can be calculated based on the position of the satellites and both receivers. Remaining errors in this calculation are considered negligible.
4.2.6 Satellite Clock Offset

For all three methods, the satellite clock offset error between two receivers cancels because for different receivers with respect to the same satellite, the satellite clock offset is common.

4.2.7 Antenna Group and Phase Delay

Different antennas have different group and phase delay characteristics. According to [20], antenna phase and group delay corrections are functions of elevation/azimuth angle and frequency. In this thesis, we consider both antenna phase and group delay as fixed numbers. The reason we can implement the simplicity is that compared to the errors induced by other error sources, these two errors are relatively small. The antenna used for collecting data is a NovAtel Pinwheel 600. The nominal phase delay is 0.2 mm at L1 and 0.1 mm at L2 and nominal group delay is 0.01 m. [20] That is,

\[ \sigma_{G, pinwheel} = 0.01 m, \quad \sigma_{Ph, pinwheel, L1} = 0.0002 m, \quad \sigma_{Ph, pinwheel, L2} = 0.0001 m. \]

For method 1, \( \tau_{i, G, Ph, L1, AB} \) is the difference between group delay of antenna A at L1 and phase delay of antenna A at L1 subtracted from the difference between group delay of antenna B at L1 and phase delay of antenna B at L1 with respect to \( i \) th satellite. Therefore the total error bound for method 1 95% confidence is

\[ 0.5 \times 2\sqrt{2 \times (0.01^2 + 0.0002^2)} \approx 0.01 m; \text{ for } 1-10^{-7} \text{ confidence is } 0.5 \times 5.33\sqrt{0.01^2 \times 2 + 0.0002^2 \times 2} \approx 0.038 m. \]

For method 2, \( \tau_{j, Ph, L1, AB} \) represents the phase delay difference between antenna A and B at L1, with respect to \( i \) th and \( j \) th satellites. The total error bound for 95% confidence is \( 2 \sqrt{4 \times 0.0002^2} = 0.0008 m; \) for 1-10^{-7} confidence is \( 5.33\sqrt{4 \times 0.0002^2} \approx 0.002 m. \) For method 3, \( \tau_{j, Ph, L1, L2, AB} \) stands
for the difference between phase delay at L1 and L2 of antenna A subtracted with
difference between phase delay at L1 and L2 of antenna B with respect to $i$th and
$j$th satellite. The total error bound for 95% confidence
is $1.5457 \times 2 \times 2 \sqrt{0.0002^2 + 0.001^2} \approx 0.001m$; for $1 \times 10^{-7}$ confidence
is $1.5457 \times 5.33 \times 2 \sqrt{0.0001^2 + 0.0002^2} \approx 0.004m$.

For a multipath-limiting antenna the performance characteristics are $\sigma_{g,MLA} = 0.02m$, $\sigma_{Ph,MLA} = 0.01m$ [21]. For method 1, the total error bound for 95% confidence is
$0.5 \times 2 \sqrt{0.02^2 \times 2 + 0.01^2 \times 2} \approx 0.03m$; for $1 \times 10^{-7}$ confidence the error bound is
$0.5 \times 5.33 \sqrt{0.02^2 \times 2 + 0.01^2 \times 2} \approx 0.085m$.

Group and phase delay are elevation/azimuth angle dependent and they are similar for
both antennas at two locations if same type of antennas are used and both antennas are
orientated in the same way. In this case, group and phase delay can be mitigated.

4.2.8 Receiver Clock Offset

For method 1, the clock offset between code and carrier phase measurements are
assumed as constant. The clock drift between carrier phase measurements at L1 and
L2 over two antennas can not be neglected which is discussed in detail in section 3.4.
Thus, a double difference method between two different satellites is applied in method
2 and 3 to remove the clock offset between L1 and L2, which, however, increases the
noise on both methods 2 and 3.

4.2.9 Ambiguities

For all three algorithms, this error term is part of the calibration when no ionospheric
gradients are present.
4.2.10 Antenna Position Error

This error affects method 2 only. According to Continuously Operating Reference Stations (CORS) accuracy, this error item is approximately 1 cm [26].

4.3 Thresholds for Different Antennas

Table 4.1 illustrates the error items affecting each algorithm. Table 4.2 lists the expression of each error source for the three methods and tables 4.3 through 4.6 describe the average values that each error source can reach for each of the three methods, with respect to different antennas (Pinwheel and MLA) and different confidence levels (95% and $10^{-7}$).
Table 4.3 95% Value of Each Error Source in Meters for Three Methods

**(NovAtel Pinwheel 600)**

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Method 1 (code minus carrier)</th>
<th>Method 2 (double difference)</th>
<th>Method 3 (dual frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troposphere Decorrelation</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>Multipath^1</td>
<td>0.4</td>
<td>0.004</td>
<td>0.01</td>
</tr>
<tr>
<td>Antenna Position Error</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>Orbit Error</td>
<td>0</td>
<td>0.0025</td>
<td>0</td>
</tr>
<tr>
<td>Antenna Phase/Group Delay</td>
<td>0.01</td>
<td>0.0008</td>
<td>0.001</td>
</tr>
<tr>
<td>Receiver Clock Offset</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Thermal Noise</td>
<td>0.047</td>
<td>0.005</td>
<td>0.01</td>
</tr>
<tr>
<td>RSS</td>
<td>0.4 m</td>
<td>0.016 m</td>
<td>0.014 m</td>
</tr>
<tr>
<td>Gradient</td>
<td>80 mm/km</td>
<td>3.2 mm/km</td>
<td>2.8 mm/km</td>
</tr>
</tbody>
</table>

Note 1: Multipath error is elevation angle dependent; the numbers in the table are based on the calculation using an average elevation angle of 50 degrees for each method of Pinwheel and MLA. This is the same for tables 4.4 and 4.5.
Table 4.4 \(1 \times 10^{-7}\) Error Value of Each Error Source in Meters for Three Methods

(\textit{NovAtel Pinwheel 600})

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Method 1 (code minus carrier)</th>
<th>Method 2 (double difference)</th>
<th>Method 3 (dual frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troposphere Decorrelation</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>Multipath</td>
<td>0.7</td>
<td>0.08</td>
<td>0.3</td>
</tr>
<tr>
<td>Orbit Error</td>
<td>0</td>
<td>0.005</td>
<td>0</td>
</tr>
<tr>
<td>Antenna Phase/Group Delay</td>
<td>0.038</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>Satellite Clock Offset</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Receiver Clock Offset</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ambiguities</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wrap up</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Thermal Noise</td>
<td>0.1</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>Total Error Bound</td>
<td>0.838 m</td>
<td>0.297 m</td>
<td>0.334 m</td>
</tr>
<tr>
<td>Gradient</td>
<td>167.6 mm/km</td>
<td>59.4 mm/km</td>
<td>66.8 mm/km</td>
</tr>
</tbody>
</table>
Table 4.5 95% and $10^{-7}$ Value of Each Error Source in Meters for Method 1 (MLA)

<table>
<thead>
<tr>
<th>Error Source</th>
<th>95% value</th>
<th>$10^{-7}$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troposphere Decorrelation</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Multipath</td>
<td>0.07</td>
<td>0.3</td>
</tr>
<tr>
<td>Orbit Error</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Antenna Phase/Group Delay</td>
<td>0.03</td>
<td>0.085</td>
</tr>
<tr>
<td>Satellite Clock Offset</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Receiver Clock Offset</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ambiguities</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wrap up</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Thermal Noise</td>
<td>0.047</td>
<td>0.1</td>
</tr>
<tr>
<td>RSS</td>
<td>0.09 m</td>
<td>-</td>
</tr>
<tr>
<td>Total Error Bound</td>
<td>-</td>
<td>0.485 m</td>
</tr>
<tr>
<td>Gradient over 5 km</td>
<td>18 mm/km</td>
<td>97 mm/km</td>
</tr>
</tbody>
</table>

4.4 Summary

Different error bounds are analyzed with different confidence levels (95% and $10^{-7}$) for both Pinwheel and multipath-limiting antennas. It is convenient to check tables 4.3, 4.4, 4.5 for error thresholds of different algorithms for gradient detection to see if a particular method is feasible or not to automatically detect gradients with the desired probabilities. Comparing thresholds in method 1 using Pinwheel and MLA from tables 4.3 and 4.5, we can see that using the MLA for method 1 reduces the threshold significantly. With these multiple choices, different algorithms are chosen depending on which type of antennas are used and in which environment the antennas are set up.
5. Data Analysis Results

5.1 Overview

The data used to implement the three ionospheric gradient detection methods described in Chapter 4 were collected from Ohio University Airport (KUNI) and the Jordan site with a baseline of approx. 5 km, on December 16th, 2006, from 10am to 11am. There was no strong ionospheric activity on that day, which is important for establishing the nominal performance of the algorithms. Ionospheric delay variations at each receiver are calculated. The ionospheric spatial gradients obtained from the three methods will be presented with and without the effect of simulated ionospheric activity.

5.2 Antenna Environment Description

The Ohio University Airport site is shown in the left of Figure 5.1. It is an experimental Local Area Augmentation System (LAAS) Ground Facility (LGF) consisting of an MLA and a NovAtel Pinwheel GPS-600 antenna [17].

The Jordan site is located on a low hill 5 km south-west of the LGF and this site has a low-obstruction horizon. It also uses both MLA and PinWheel antennas, which is shown in the right of Figure 5.1. In this thesis, data used in the gradient simulation was collected from both Pinwheel antennas at the two locations.
5.3 Satellite Geometry

A skyplot for visible satellites passing over the KUNI site area on December 16th, 2006, from 10am to 11am, is shown in Figure 5.2. Satellite PRN 20 and PRN 25 are selected for detailed analysis as well as the gradient simulation.
According to Figure 5.2, the elevation angle of satellite PRN 20 ranged from approximately 44 degrees to 31 degrees. The elevation angle of satellite PRN 25 ranged from approximately 60 degrees to 43 degrees.

5.4 Detection Thresholds of Three algorithms (Pinwheel and MLA)

Based on the threshold calculations described in Chapter 4, figures 5.3-5.8 provide the thresholds for method 1 as a function of elevation. Thresholds for method 2 and 3 are not provided because they depend on two sets of elevation angles which can not be plotted in the same plot.
Figure 5.3 Pinwheel 95% thresholds for method 1

Figure 5.3 shows the 95% and $1\times10^{-7}$ confidence thresholds of Pinwheel antenna for method 1. In this plot, the two elevation angle dependent thresholds (95% in blue line and $1\times10^{-7}$ in red line) have the same trend. They increase at first until reaches the maximum value around 18 degrees of elevation angle and then start decreasing till 60 degrees of elevation angle and after that they start to increase again.

Figure 5.4 shows the 95% and $1\times10^{-7}$ thresholds of MLA which are elevation angle dependent as well. In operation, the MLA is used between 2 and 35 degrees. Figure 5.5 shows the thresholds for the MLA with a smaller elevation angle range.
Figure 5.4 MLA 95% and $1 \times 10^{-7}$ thresholds for method 1

Figure 5.5 MLA thresholds for method 1 with smaller elevation angle range
5.5 Ionospheric Spatial Gradient over a 5 km Baseline

The ionospheric delay on L1 is calculated as follows:

$$I_{i,L1} = \frac{-AD_{i,L1} + AD_{i,L2}}{1 - \left( \frac{f_{L1}}{f_{L2}} \right)^2}$$  \hspace{1cm} (5.1)

Figure 5.6 shows the ionospheric delay variation on L1 (subtracted by the mean value of ionospheric delay on L1) at the KUNI site on December 16th, 2006, from 10 am to 11 am. The blue line is ionospheric delay variation in meters for satellite PRN 20 and the red line is for satellite PRN 25.
Figure 5.6 Ionospheric delay variations for KUNI site,
December 16\textsuperscript{th}, 2006, 10am to 11am

It shows that for PRN 20 and PRN 25, there was no significant ionosphere activity along the path from the satellites to KUNI site during this hour.
Figure 5.7 shows ionospheric delay variations at the Jordan site at the same time. Similar to figure 5.6, the blue line is ionospheric delay variation (subtracted by mean value) in meters for satellite PRN 20 and the red line is satellite PRN 25.

Figure 5.7 Ionospheric delay variations for Jordan site, December 16th, 2006, 10am to 11am

It shows that for the same two satellites, there was no significant ionosphere activity along the path from the satellite to the Jordan site.

Figure 5.8 shows the nominal ionospheric gradient in mm/km between KUNI and Jordan site for the three methods. The top plot is the ionospheric gradient between two sites in mm/km for method 1. The middle plot is for method 2 and the lower plot is for method 3. The two red lines in each plot represent the 95% thresholds.
As we can see from Figure 5.8, the ionospheric spatial gradient derived from method 1 is noisy. For methods 2 and 3, a double difference method is applied between satellites PRN 20 and PRN 25 to remove the receiver clock offsets between two receivers which explains why only one line is shown in the middle and lower plots. Methods 2 and 3 have similar performance.

### 5.6 Documented Ionospheric Storms

On November 20th, 2003, there were well-documented, severe ionospheric storms in northern Ohio [22] [23] [24]. Data were obtained from two northern Ohio Continuously Operating Reference Stations (CORS), FREO and LSBN (with a baseline around 73.5 km) for that day to exercise the three algorithms. The results are
shown in Figure 5.9.

Figure 5.9 Three detection algorithms, northern Ohio, November 20th, 2003, from 8 pm to 10 pm

In Figure 5.9, a severe ionospheric gradient was observed on satellite PRN 28 on November 20th, 2003, from 8 pm to 10 pm. According to [23], the gradient variation observed was up to 400 mm/km. Take method 3 for example, it shows a peak to peak value of approximately 400 mm/km. The typical $1\times10^{-7}$ threshold value for method 3 is 66.8 mm/km (refer to table 4.4). Thus, for this ionospheric gradient, it could be in the range 333.2 mm/km to 466.8 mm/km.

5.7 Model Establishment and Simulation

Severe ionospheric anomalies have a plume-shaped characteristic and move fast in general [23]. For simplicity, the anomalies are modeled as a linear wedge as
illustrated in Figure 5.10 [23] [24]. Although this linear model is a simplification of the actual ionosphere behavior, it provides a helpful starting point to study the impact of ionospheric fronts on LAAS users. There are three parameters to characterize the ionospheric model: front speed, gradient width, and maximum vertical delay. These parameters are determined based on the analysis of past storms [23] [24]. In this thesis, the gradient width is 100 km and the maximum vertical delay is 10 m, which results in a zenith-delay gradient of 100 mm/km. As shown in Figure 5.10, the simulated front is moving from right to left (from Jordan site to KUNI site) at a front speed of 360 km/hr.

Figure 5.10 Ionosphere anomaly model from [23] [24]
The ionospheric anomaly is added in a way that it only affects one satellite during the simulation time. Figure 5.11 shows how the gradient impacts the data set with the 5 km baseline. The red line in each plot represents the $10^{-7}$ thresholds for each algorithm. It shows that method 1 performs the worst and it may not detect 100 mm/km ionospheric gradient at all times. Methods 2 and 3 perform very well can detect 100 mm/km ionospheric gradient with $10^{-7}$ confidence.

![Graph of Method 1](image1)

![Graph of Method 2](image2)

![Graph of Method 3](image3)

Figure 5.11 Three detecting algorithms with the effect of modeled 100mm/km ionospheric gradient plus $10^{-7}$ thresholds on 12/16/2006, 10 am to 11am
5.8 Discussion and Summary

In this chapter, an ionospheric anomaly is modeled as a 100 mm/km ionospheric gradient. GPS data collected from two sites in Ohio with a 5-km baseline are analyzed with and without the gradient. The three ionospheric gradient detection methods are evaluated and detection thresholds are provided for each algorithm based on the error analysis in Chapter 4. Method 1 performs noisy which is mostly caused by the multipath error and thermal noise on the pseudorange measurements. The multipath error can be reduced significantly by using an MLA. Method 2, using L1 carrier phase measurements, perform very well and can detect the 100 mm/km ionospheric gradient. But this method is sensitive to tropospheric gradients and antenna position error. Method 3, using dual frequency carrier phase measurements, can also detect the 100 mm/km ionospheric gradient. Compared to method 2, method 3 is not sensitive to tropospheric gradients and antenna position errors.
6. Summary, Conclusions and Future Work

6.1 Summary and Conclusions

Several methods for obtaining absolute TEC are reviewed in this thesis. Due to interfrequency GPS receiver biases, the precision of TEC values calculated with these methods is usually limited. What we are interested in this thesis is the relative TEC characterization between two locations, which can be used for applications such as aircraft precision approach and landing, and precision agriculture. Hence, this thesis concentrates on three algorithms that calculate variations in the TEC gradient between two locations, instead of the absolute TEC values at each location.

Data analysis and simulation results are used to demonstrate the feasibility of the three methods for a 5-km baseline data set. Merits and shortcomings of the methods are also discussed. In general, method 1 has more noise because of the code phase measurements used in this method. In method 2, in order to eliminate the effect of receiver clock error residuals, a double difference method is applied which results in increased errors. Method 3 is based on dual frequency measurements. Only single frequency GPS receivers are needed for methods 1 and 2. For method 3, dual frequency GPS receivers are required.

Different confidence levels (95% and $10^{-7}$) for the detection thresholds for Pinwheel and MLA (method 1 only) are provided through the analysis of error sources, which provides detection options for different receiver and antenna configurations.

6.2 Future Work

Based on the error source analysis in Chapter 4, several improvements can be
investigated, such as better multipath limiting on both code and phase measurements by using an MLA. Another possible improvement can be made by trying to solve for the receiver clock errors in method 2, instead of implementing the double difference method. We need to investigate more on the L1/L2 clock drift for method 3 as well. We can also attempt to process more data sets collected from geographically diverse locations, to test the reliability and robustness of these three methods.
7. References


Appendix A

This section provides the detailed expansion for Equation (3.2). Since the single difference is with respect to the same satellite, the subscript $i$ indicating $i$th satellite is omitted for simplicity in the expansion.

\[
\begin{align*}
CMC_{SD_{AB}} &= (PR_{L1,A} - AD_{L1,A}) - (PR_{L1,B} - AD_{L1,B}) = [(R_A - c\Delta t_{SV,PR,L1,A} + I_{L1,A}) + T_A + \varepsilon_{P,A} + c\Delta t_{PR,PR,L1,A} + MP_{PR,PR,L1,A} + \tau_{G,PR,L1,A} + \eta_{PR,PR,L1,A}) \\
-&(R_A - c\Delta t_{SV,AD,L1,A} - I_{L1,A} + T_A + \varepsilon_{P,A} + c\Delta t_{AD,AD,L1,A} + MP_{AD,AD,L1,A} + \tau_{PR,PR,L1,A}) \\
+N_{L1,A}\lambda_{L1} + \frac{\lambda_{L1}}{2\pi}\Psi_{A} + \eta_{AD,AD,L1,A} - [(R_B - c\Delta t_{SV,PR,L1,B} + I_{L1,A}) + T_B + \varepsilon_{P,B} + c\Delta t_{PR,PR,L1,B} + MP_{PR,PR,L1,B} + \tau_{G,PR,L1,B} + \eta_{PR,PR,L1,B}) \\
-&(R_B - c\Delta t_{SV,AD,L1,B} - I_{L1,B} + T_B + \varepsilon_{P,B} + c\Delta t_{AD,AD,L1,B} + MP_{AD,AD,L1,B} + \tau_{PR,PR,L1,B}) \\
+N_{L1,B}\lambda_{L1} + \frac{\lambda_{L1}}{2\pi}\Psi_{B} + \eta_{AD,AD,L1,B}]
\end{align*}
\]

\[
= [(R_A - R_A) - (c\Delta t_{SV,PR,L1,A} - c\Delta t_{SV,AD,L1,A}) + (I_{L1,A} + I_{L1,A}) + (T_A - T_A) \\
+(\varepsilon_{P,A} - \varepsilon_{P,A}) + (c\Delta t_{PR,PR,L1,A} - c\Delta t_{AD,AD,L1,A}) + (MP_{PR,PR,L1,A} - MP_{AD,AD,L1,A}) + (\tau_{G,PR,L1,A} - \tau_{PR,PR,L1,A}) \\
+((\eta_{PR,PR,L1,A} - \eta_{AD,AD,L1,A}) - (N_{L1,A}\lambda_{L1} + \frac{\lambda_{L1}}{2\pi}\Psi_{A})] \\
+(R_B - R_B) - (c\Delta t_{SV,PR,L1,B} - c\Delta t_{SV,AD,L1,B}) + (I_{L1,B} + I_{L1,B}) + (T_B - T_B) \\
+(\varepsilon_{P,B} - \varepsilon_{P,B}) + (c\Delta t_{PR,PR,L1,B} - c\Delta t_{AD,AD,L1,B}) + (MP_{PR,PR,L1,B} - MP_{AD,AD,L1,B}) + (\tau_{G,PR,L1,B} - \tau_{PR,PR,L1,B}) \\
+((\eta_{PR,PR,L1,B} - \eta_{AD,AD,L1,B}) - (N_{L1,B}\lambda_{L1} + \frac{\lambda_{L1}}{2\pi}\Psi_{B})] \\
= [2I_{L1,A} - c\Delta t_{SV,PR,AD,L1,A} + MP_{PR,AD,L1,A} + \tau_{G,PR,PR,L1,A} + \eta_{PR,PR,L1,A} \\
-(N_{L1,A}\lambda_{L1} + \frac{\lambda_{L1}}{2\pi}\Psi_{A})] - [2I_{L1,B} - c\Delta t_{SV,PR,AD,L1,B} + c\Delta t_{PR,PR,L1,B} \\
+MP_{PR,AD,L1,B} + \tau_{G,PR,PR,L1,B} + \eta_{PR,PR,L1,B} - (N_{L1,B}\lambda_{L1} + \frac{\lambda_{L1}}{2\pi}\Psi_{B})] \\
= 2I_{L1,AB} - c\Delta t_{SV,PR,AD,L1,AB} + c\Delta t_{PR,PR,L1,AB} + MP_{PR,AD,L1,AB} + \tau_{G,PR,PR,L1,AB} + \eta_{PR,PR,L1,AB} \\
-(N_{L1,AB}\lambda_{L1} + \frac{\lambda_{L1}}{2\pi}\Psi_{AB})
\]
Appendix B

This section contains the detailed expansion of Equation (3.4)

\[ DD_{i,AB} = SD_{i,AB} - SD_{j,AB} = (AD_{i,L1,A} - AD_{i,L1,B}) - (AD_{j,L1,A} - AD_{j,L1,B}) \]

\[ = (R_{i,A} - c\Delta t_{i,SV,AD,L1,A} - I_{i,L1,A} + T_{i,A} + \varepsilon_{i,P,A}) \]

\[ + c\Delta t_{i,AD,L1,1} + MP_{i,AD,L1,1} + \tau_{i,Ph,L1,1,A} + N_{i,L1,1,1} + \frac{l_{i,L1}}{2\pi} \Psi_{i,A} + \eta_{i,AD,L1,1,A} \]

\[ - (R_{i,B} - c\Delta t_{i,SV,AD,L1,B} - I_{i,L1,B} + T_{i,B} + \varepsilon_{i,P,B}) \]

\[ + c\Delta t_{i,AD,L1,1,B} + MP_{i,AD,L1,1,B} + \tau_{i,Pph,L1,1,B} + N_{i,L1,1,1} + \frac{l_{i,L1}}{2\pi} \Psi_{i,B} + \eta_{i,AD,L1,1,B} \]

\[ - (R_{j,A} - c\Delta t_{j,SV,AD,L1,A} - I_{j,L1,A} + T_{j,A} + \varepsilon_{j,P,A}) \]

\[ + c\Delta t_{j,AD,L1,1,A} + MP_{j,AD,L1,1,A} + \tau_{j,Pph,L1,1,A} + N_{j,L1,1,1} + \frac{l_{j,L1}}{2\pi} \Psi_{j,A} + \eta_{j,AD,L1,1,A} \]

\[ + (R_{j,B} - c\Delta t_{j,SV,AD,L1,B} - I_{j,L1,B} + T_{j,B} + \varepsilon_{j,P,B}) \]

\[ + c\Delta t_{j,AD,L1,1,B} + MP_{j,AD,L1,1,B} + \tau_{j,Pph,L1,1,B} + N_{j,L1,1,1} + \frac{l_{j,L1}}{2\pi} \Psi_{j,B} + \eta_{j,AD,L1,1,B} \]

\[ = R_{g,AB} - I_{g,L1,AB} + T_{g,AB} + \varepsilon_{g,p,AB} + MP_{g,AD,L1,1,AB} \]

\[ + \tau_{g,Pg,L1,1,AB} + \frac{l_{i,L1}}{2\pi} \Psi_{g,AB} + N_{g,AB} + \eta_{g,AD,L1,1,AB} \]
Appendix C

This section is the detailed expansion of Equation (3.6) for simplicity, the subscript $i$ indicating the satellite is omitted.

\[ AD_{L1L2}SD = I_{L1A} - I_{L1B} = \frac{1}{(f_1/f_2)^2 - 1} (AD_{L1A} - AD_{L2A}) - \frac{1}{(f_1/f_2)^2 - 1} (AD_{L1B} - AD_{L2B}) \]

\[ = \frac{1}{(f_1/f_2)^2 - 1} [ (R_A - c\Delta t_{SV', AD, L1A} - I_{L1A} + T_A + \epsilon_{P,A} + c\Delta t_{AD, L1A, A} + MP_{AD, L1A}, \]

\[ + \tau_{\phi_{L1A}} + \frac{\lambda_{L1A}}{2\pi} \Psi_A + \eta_{AD, L1A,A} - (R_A - c\Delta t_{SV', AD, L2A} - (f_1/f_2)^2) I_{L1A} \]

\[ + T_A + \epsilon_{P,A} + c\Delta t_{AD, L2A, A} + MP_{AD, L2A, A} + \tau_{\phi_{L2A}} + N_{L2A, A} \lambda_{L2A} + \frac{\lambda_{L2A}}{2\pi} \Psi_A + \eta_{AD, L2A,A} ) \]

\[ - (R_B - c\Delta t_{SV', AD, L1B} - I_{L1B} + T_B + \epsilon_{P,B} + c\Delta t_{AD, L1B, B} + \tau_{\phi_{L1B}} + N_{L1B, B} \lambda_{L1B} + \frac{\lambda_{L1B}}{2\pi} \Psi_B + \eta_{AD, L1B,B} ) \]

\[ + (R_B - c\Delta t_{SV', AD, L2B} - (f_1/f_2)^2 I_{L1B} + T_B + \epsilon_{P,B} + c\Delta t_{AD, L2B, B} + \tau_{\phi_{L2B}} + N_{L2B, B} \lambda_{L2B} + \frac{\lambda_{L2B}}{2\pi} \Psi_B + \eta_{AD, L2B,B} ) ] \]

\[ = \frac{1}{(f_1/f_2)^2 - 1} [ -c\Delta t_{SV', AD, L1L2, A} + \frac{f_1^2}{f_2^2} I_{L1B} + c\Delta t_{AD, L1L2, A} + MP_{AD, L1L2, A} + \tau_{\phi_{L1L2}} \]

\[ + (N_{L1L2, A} \lambda_{L1L2} - N_{L1L2, B} \lambda_{L1L2}) + \frac{\lambda_{L1L2} - \lambda_{L2L2}}{2\pi} \Psi_A + \eta_{AD, L1L2,B} + c\Delta t_{SV', AD, L1L2, B} \]

\[ - \frac{f_1^2}{f_2^2} I_{L1L2} - c\Delta t_{AD, L1L2, B} - MP_{AD, L1L2, B} + \tau_{\phi_{L1L2,B}} - (N_{L1L2, A} \lambda_{L1L2} - N_{L1L2, B} \lambda_{L1L2}) - \frac{\lambda_{L1L2} - \lambda_{L2L2}}{2\pi} \Psi_B - \eta_{AD, L1L2,B} ] \]

\[ = \frac{1}{\alpha} [ -c\Delta t_{SV', AD, L1L2, A} + cI_{L1L2} + c\Delta t_{AD, L1L2, A} + MP_{AD, L1L2, A} + \tau_{\phi_{L1L2, A}} \]

\[ + (N_{L1L2, A} \lambda_{L1L2} - N_{L1L2, B} \lambda_{L1L2}) + \frac{\lambda_{L1L2} - \lambda_{L2L2}}{2\pi} \Psi_A + \eta_{AD, L1L2,B} ] \]