Development of a Real-Time Monitor for Satellite Anomalous Clock and Orbit Errors

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ABSTRACT

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Development of a Real-Time Monitor for Satellite Anomalous Clock and Orbit Errors
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The thesis focuses on the development of a real-time software monitor to detect anomalous Global Positioning System (GPS) satellite clock and orbit errors for any one of the satellites in view. The monitor not only detects the anomaly, but also identifies the satellite and quantifies the magnitude of the anomaly to alert the users of the system.

Different methods based on the pseudorange residual are used to detect the anomaly and to identify the satellite. Both differenced pseudorange residual and subset formation method are used. In addition, accumulated Doppler range residuals were used to characterize the error growth over time. All methods used were verified using precise orbits and clock products. Detection threshold values are calculated based on error statistics for pseudorange and ADR measurements.

Approved: _____________________________________________________________

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1 INTRODUCTION

The Global Positioning System has enabled a wide spectrum of applications ranging from precise timing applications to precision farming, from aircraft attitude and heading determination using multiple antennas to spacecraft radio occultation methods, from location based services (LBS) such as Emergency – 911 to Geo – fencing [1]. Owned by the US Government (USG) and operated by the US Air Force (USAF), GPS can be used by an unlimited number of users for Position Navigation and Timing applications, with high accuracy, in any weather, anywhere in the world [2]. A GPS receiver calculates its three-dimensional position and clock offset from GPS time, using range measurements to at least four satellites in combination with satellite orbit and clock data. The range measurements are often referred to as pseudo range measurements as each measurement contains the receiver clock offset with respect to GPS time in addition to the geometric range between the satellite and the user reception antenna. The GPS system is divided into the Space Segment (SS), the Control Segment (CS) and the User Segment.

1.1 Space Segment

At the time of this writing, the SS consists of 31 satellites in 6 orbital planes at an altitude of 20200 km above the surface of the Earth. The original concept validation satellites, designed by Rockwell International, with satellite vehicle numbers (SVN) from 1-11 are categorized as Block I satellites. The last of the Block I satellites was decommissioned in 1995 [2]. Block II satellites are the first operational satellites, designed to provide service for 14 days without contact from the CS. Satellites from Block IIA through IIF provide 60 days of positioning services without contact from the CS. A history of satellites and launch dates are given in [3].

As defined in [2], operational satellites from Block II through IIF can operate in one of the 3 modes; Normal operation, Short Term Extended Operation and Long Term Extended Operation, depending upon the level of contact from the CS. In Normal Operation, satellites are uploaded with fresh navigation data uploads once per day, in
general. When navigation data uploads are not possible, the satellites continue to provide positioning services with degraded accuracy for up to 14 days in the Short Term Extended Operation mode, and the fit interval flag in the broadcast navigation data will be set to 1. If a SV cannot be loaded with a fresh navigation upload for 14 days, the SV is considered to be in Long Term Extended Operation and accuracy will be degraded.

### 1.2 Control Segment (CS) or Operational Control System (OCS)

The CS consists of

- A Master Control Station (MCS) at Schriever Air Force Base, Colorado
- A Backup Master Control Station (BMCS)
- Monitor Stations (MS) at 6 locations spread out on the earth
- Four Ground Antennas

One of the most important features of GPS is its ability to provide some level of integrity. Integrity is defined “to be the trust that can be placed in the correctness of the information provided by the SPS SIS”, which includes the Time-To-Alert (TTA) required to detect a Not-To-Exceed (NTE) error [4]. GPS addresses this objective by having unmanned remote monitor stations, controlled from the MCS, spread throughout the world. There used to be 6 Air Force monitor stations whose coverage was limited [5]; however, as a result of the Accuracy Improvement Initiative (AII), a total of 11 National Geospatial-Intelligence Agency (NGA) Monitor Stations were added to the Air Force Monitor Stations [6] to improve the accuracy and monitoring of the satellite constellation. Figure 1.1 shows the locations of the GPS monitor stations used by the MCS [6].
Monitor stations of the CS along with NGA monitor stations provide 100% global coverage of the satellite constellation. The 4 Ground Antennas provide near real time Telemetry Tracking & Command (TT&C) interfaces in S – band between the MCS and the satellites [4].

The Master Control Station is at the heart of the CS and operations are maintained 24 hours a day, 7 days a week. Responsibilities of the MCS include [7] [1],

- Monitoring satellite health
- Monitoring orbit & clock errors
- Maintaining GPS time
- Predicting and uploading ephemeris & clock parameters
Commanding small maneuvers in order to maintain satellite orbits

Even though the CS tries to maintain the accuracy of the GPS system at the highest level, errors at various levels are still present. These errors may occur in various stages, from erroneous prediction of the ephemeris and clock parameters at the MCS Kalman filter, to on board satellite subsystem errors, propagation errors, user environment and receiver errors [1, 7, 8]. This thesis is primarily concerned with the errors from the Space Segment and the Control Segment.

1.2.1 Accuracy & Integrity of SPS Signal in Space

The positioning service provided by GPS for civil users is referred to as the Standard Positioning Service (SPS), which supports the coarse acquisition code, navigation data and the L1 carrier [4]. The Signal-in-Space (SIS) User Range Error (URE) is the error in the measured range contributed by the Space and Control Segments (this does not include propagation and receiver errors).

There are several potential causes for accuracy degradation of the satellite orbit and clock errors, including:

- The MCS cannot upload a fresh navigation message at regular intervals due to limitations on GA (a particular satellite may not be visible)
- The satellite itself cannot process the upload
- The navigation data upload is bad due to the an erroneous prediction of the satellite ephemeris/clock parameters by the MCS Kalman filter
- Unpredictable acceleration of a satellite
- Satellite clock malfunction

Some of the errors are mitigated by the MCS by performing a contingency upload, for example if the MCS detects that the satellite clock has errors, it provides a fresh upload of the navigation data. However, if the accuracy of the SIS exceeds the Not To Exceed
(NTE) limit, then the user needs to be alerted/ warned within 10 s. This limit is defined as 
\[ \pm 4.42 \times URA \], where URA is an upper bound obtained from the URA index of the
navigation message and is defined as the 1 sigma error estimate in the navigation
message (due to CS and SS) [4]. If the accuracy of a satellite does not meet this
requirement and if the user is not alerted/ warned within 10 s, then it is said to be both a
major service failure and an integrity failure. As provided by Appendix A in [4], Table
1.1 summarizes some of the potential integrity failure modes for the CS and SS.

Table 1.1. GPS Failure Modes (from [Appendix A -22 in [4]])

<table>
<thead>
<tr>
<th>Segment/ System</th>
<th>Failure Mode</th>
</tr>
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<tbody>
<tr>
<td>Satellite</td>
<td>Thruster Firing</td>
</tr>
<tr>
<td></td>
<td>Shift in clock frequency or instability</td>
</tr>
<tr>
<td></td>
<td>Corrupted Navigation data</td>
</tr>
<tr>
<td>Control</td>
<td>Delayed/ Missed Upload</td>
</tr>
<tr>
<td></td>
<td>Bad Upload: wrong/ irrelevant data</td>
</tr>
<tr>
<td></td>
<td>Bad upload: bad ephemeris/ clock</td>
</tr>
<tr>
<td></td>
<td>Operational Error: Health Settings</td>
</tr>
<tr>
<td></td>
<td>Errors induced by GA</td>
</tr>
<tr>
<td></td>
<td>Errors induced by MS</td>
</tr>
<tr>
<td>Input data to CS</td>
<td>Bad Earth orientation parameters</td>
</tr>
<tr>
<td></td>
<td>Bad UTC offset data</td>
</tr>
</tbody>
</table>

Most of the failures indicated in Table 1.1 are detected by the MCS, and the user is
alerted. The kind of alert depends on the condition detected, and whether it was detected
by the satellite on board monitoring unit or by the MCS. For example, if there is a failure
detected by the satellite on board monitoring unit, then the satellite could stop
transmitting, or it could broadcast a Non Standard Code (NSC & NSP) instead of the
regular C/A and regular P pseudorandom noise (PRN) code. For failures detected by the MCS, several methods are available to alert the user [4]

- Cease the signal transmission
- Eliminate the C/A code on the L1 frequency
- Switch to a Non Standard Code
- Switch the satellite to transmit PRN 37

Although integrity is defined in the GPS Performance Standard [4], this same document also states “The DoD does not currently monitor and assess SPS performance in real time.” Also, the number of major service failures should be at most three per year. Unfortunately, thirteen major satellite orbit and clock errors occurred during the period June 2005 through July 2008, as analyzed and documented in [9]. These malfunctions were analyzed using recorded data broadcast by the satellites in combination with truth data from multiple sources that are available at 15-minute time intervals with a delay of several days. The goal of this thesis is to detect satellite orbit and clock malfunctions in real time and to examine malfunctions in detail with high update rates on the order of at least once per second.

1.3 Problem Introduction

GPS anomalous orbit and clock errors are of concern and the user must be warned/alerted of such anomalies in time. Even though the OCS continuously monitors the constellation with the help of Monitoring Stations and responds to the failure by automatically or manually removing the satellite from service in a prompt manner, this may be limited by Monitor Station visibility (e.g. stations that are out-of-service), Ground Antenna visibility, OCS equipment, and reliability of communications. Time to Alert (TTA), defined as the maximum time allowed from the time an unsafe condition exists to the time the user is alerted [5], is also very critical for aviation applications. For example, Category I (CAT I) precision approach operations require a TTA of 6 seconds [5]. Even if the MCS observes a failure, it often takes 5 – 15 minutes to alert the user (section 4.3.3 in [5]). To verify that the actual GPS performance is in agreement with the SPS
Performance Standard, a real-time monitor for satellite orbit and clock errors is desirable.
This monitor can also be used for high-integrity applications.
2 BACKGROUND

This chapter provides a brief review of the work documented in the area of GPS satellite orbit and clock error monitoring, and also provides the problem statement for this thesis. Much effort has been expended to measure the performance of the GPS satellite orbits and clocks. Systems like the Federal Aviation Administration’s (FAA) Wide Area Augmentation System (WAAS) and the Jet Propulsion Laboratory’s (JPL) Global Differential GPS (GDGP) were developed to provide corrections for satellite orbit and clock parameters in real time. Although these systems provide an excellent solution for dealing with malfunctioning satellites, their corrections are not available to all GPS users at all times, and may not meet integrity requirements for all applications. Also, high-accuracy, high-integrity correction systems, such as FAA’s Local Area Augmentation System (LAAS), rely on the assumption that unannounced satellite maneuvers do not occur when the satellite is set healthy [10]. In addition, monitoring of the GPS SPSPS [4] requires a real time assessment of the actual SIS range error compared to its integrity bound.

Previous monitoring efforts have employed a post processing methodology in which the broadcast orbit and clock data are compared against a truth source, also known as precise orbit & clock products. These products are available from various organizations; for example, both NGA and the International Global Navigation Satellite Systems (GNSS) Service (IGS) provide precise orbits and clock products at 15-minute intervals. These orbit and clock products are discussed in more detail in Chapter 4.

Using precise orbit and clock data from IGS, Langley, et al. [11] calculated the GPS broadcast orbit errors from January 1999 through June 2000. Several anomalies were documented and the orbit errors were given for each PRN for the time period investigated. A web-based system [12] for calculating day-to-day errors was developed. Unfortunately, IGS final orbit products are only available after 12-18 days, while less-precise rapid orbits are available after 17-41 hours. Signal-In-Space Range Error (SISRE) was used as the performance metric. SISRE is the RMS range error averaged over the
satellite’s earth coverage, and represents the fidelity of the orbit and clock parameters as they are observed by a GPS receiver [11]:

\[ SISRE = \sqrt{\left( R - B \right)^2 + \frac{1}{49} \left( A^2 + C^2 \right)} \]  

(2.1)

Where,
R – Radial component of the orbit error in m
B – Clock error expressed in m (equals the clock error in seconds multiplied by the speed of light)
A – Along track orbit error in m
C – Cross track orbit error in m

From Eq. 2.1, the user primarily observes the combination of the satellite clock and the radial component of the orbit error. The projection of the along track and cross track errors on the line-of-sight between the user and the satellite is such that only a small portion of these errors affects the user (represented by the factor 1/49 in Eq. 2.1).

Dieter, Hatten, and Taylor [13] calculated the Zero Age of Data (ZAOD) error by comparing it to NGA precise orbit and clock data. ZAOD represents the highest quality of the ephemeris and clock corrections; this is the navigation message that will be transmitted by the satellite right after being uploaded by the CS. Several ZAOD outliers are reported, due to thruster firings or because of unstable clocks and aging of the SVs. Some of the examples include SVN 36 with periodic clock errors with a period close to the 12-hour orbital period and SVN 32 with ephemeris errors due to occasional thruster firings.

Warren and Raquet [14] compared broadcast orbits with IGS final orbits for the time period from 14 November 1993 through 31 December 2002 to evaluate the impact of the Ephemeris Enhancement Endeavor (EEE). Statistics of these errors were presented by providing the probability density functions of the errors before and after the EEE. Clock errors were not considered in the evaluation and data discrepancies larger than 50 m were removed from the processing.
Taylor and Barnes [15], document the improvements in SISRE with CS improvements. NGA data was used as truth and the performance of SIS is given in terms of Estimated Range Deviation (ERD), as monitored by MCS, ZAOD URE and as broadcast URE. All of these parameters are compared for a single day for SVN 38. ERDs are obtained by assuming the MCS Kalman filter estimates as truth, to compare with the broadcast navigation message’s orbit and clock. The differenced orbit and clock are then projected onto line-of-sight of fictitious sites on the ground in the coverage area of the SV. ERDs are indicators of the MCS Kalman filter estimation accuracy.

Most recently, in [9], a detailed orbit and clock error analysis is provided for the 3-year time period from June 2005 to 2008. Approximately 25,000 precise and broadcast orbit and clock data files were analyzed. Various data sources, NGA, IGS and the Center for Orbit Determination in Europe (CODE) were used as truth for comparison of broadcast ephemerides. Anomalies were identified among the outliers by verifying against the IGS truth and actual measurement data from IGS receiver sites. Results are provided for 7 metrics including, along track, cross track, radial, clock errors and for 3 operational scenarios for ephemeris selection. Cumulative distribution functions for the errors were also presented.

The next step in clock and orbit error monitoring would be a real time system that monitors the accuracy of the broadcast ephemerides against its guaranteed performance of $4.42 \times$ URA. In addition, techniques are needed that enable update rates higher than once every 15 minutes to characterize anomalous error growth in detail, and to evaluate the efficacy of correction methods. The latter might be accomplished using the highly accurate accumulated Doppler frequency shift observable that enables the determination of changes in the pseudorange measurements at the cm-level.

Therefore, the research in this thesis is focused on the following topics:

- Development of a real time monitor to detect anomalous satellite orbit and clock errors.

- Development of a technique to obtain error estimates for update rates of at least once per second using the accumulated Doppler frequency shift observables from
a GPS receiver to enable the characterization of satellite clock and orbit error
growth in between 15-minute updates based on precise clock and orbit data.

The remainder of this document is organized as follows. Chapter 3 discusses the
development of a real time monitor in the range domain, by forming the residuals from
measured pseudo ranges. The corresponding equations are explained in detail with a
block diagram of the procedure. Chapter 4 discusses the analysis of an anomaly discussed
in chapter 3 by using precise orbits and Accumulated Doppler Range residuals (ADR).
Chapter 5 provides the conclusions and recommendations for future research.
3 DEVELOPMENT OF REAL TIME URE MONITOR

The orbit and clock error monitors developed in this chapter are designed for real time applications. The line-of-sight combined components of satellite clock and orbit errors are monitored using the known location of the GPS reception antenna. Section 3.1 of this chapter is focused on pseudorange residual methods, while Section 3.2 addresses a carrier phase method for accurate characterization of clock and orbit error variations over time.

3.1 Pseudo Range Residuals

The measured range from a GPS satellite to the user can be written as the summation of all terms that affect the GPS signal as it propagates from the satellite to the user. Depending on the required accuracy of the user position, the number of error terms to be considered will be different. For example, if the accuracy needed for the user position is within a decimeter (1 sigma horizontal) as in precise point positioning, several errors are corrected that are usually neglected in less precise calculations. Examples of these errors include earth tides, ocean tides, and satellite antenna errors [21].

The most common error sources that need to be addressed for accuracy at the meter level (1 sigma) are given in Eq. 3.1

\[
P_{R_{i,c}}(t) = r_i(t) + c[ \delta_{r_{i,c,PR}}(t) - \delta_{s_{i,c,PR}}(t - \tau) + \delta_{s_{i,c,PR}}(t - \tau)] + T_i(t) + I_{i,c,PR}(t) + \\
+ \varepsilon(t) + \tau_{PR,L} \{ \theta_i(t) \psi_i(t) \} + MP_{i,c,PR}(t) + \eta_{i,c,PR}(t)
\]  

(3.1)

Where:

- L - subscript for frequency of the carrier signal 1 or 2
- i - subscript referring to a specific satellite
- c - subscript for code type C/A or P
- r - true range between satellite and the receiver (m)
- \(\delta_{r_i}\) - receiver clock offset from GPS time (seconds)
- \(\delta_{s_i}\) - clock offset of satellite ‘i’ from GPS time (seconds)
In order to monitor the satellite orbit and clock errors, these error terms need to be separated from the equation. Rewriting Equation (3.1) for observing satellite orbit and clock errors results in

\[
\delta t_i(t) = PR_{i,L,c}(t) - r_i(t) + c[\delta t_{s_{i,L,PR,c}}(t) - \delta t_{r_{i,L,PR,c}}(t)] - T_i(t) - I_{i,L,PR}(t) + \tau_{PR,L} \{\theta_i(t)\psi_i(t)\} - MP_{i,PR,L,c}(t) - \eta_{i,PR,L,c}(t)
\]

(3.2)

As the ionosphere is a dispersive medium, refraction is different for different frequencies; which enables a direct measurement of the first-order ionospheric delay using two different frequencies. The first-order ionospheric delay is inversely proportional to the frequency squared, such that the delay on the L1 frequency is related to the delay on the L2 frequency as follows

\[
I_{L1} = \frac{f_{L2}^2}{f_{L1}^2} I_{L2}
\]

(3.3)

It then follows that the difference between the L1 and L2 pseudorange delays can be written as:
\[ \begin{align*}
PR_{i, L_1, c} - PR_{i, L_2, c} &= I_{i, L_1, PR} - I_{i, L_2, PR} + \tau_{PR, L_1} \{ \theta_i(t) \psi_i(t) \} - \tau_{PR, L_2} \{ \theta_i(t) \psi_i(t) \} + \\
&+ \eta_{i, PR, L_1, c} - \eta_{i, PR, L_2, c} + MP_{i, PR, L_1, c} - MP_{i, PR, L_2, c} \\
&= I_{i, L_1, PR} - I_{i, L_1, PR} \frac{f_{L_2}^2}{f_{L_1}^2} + \tau_{PR, L_1} \{ \theta_i(t) \psi_i(t) \} - \tau_{PR, L_2} \{ \theta_i(t) \psi_i(t) \} + \\
&+ \eta_{i, PR, L_1, c} - \eta_{i, PR, L_2, c} + MP_{i, PR, L_1, c} - MP_{i, PR, L_2, c} \\
&\Rightarrow \frac{f_{L_2}^2 - f_{L_1}^2}{f_{L_2}^2} \ast I_{i, L_1, PR} = PR_{i, L_1, c} - PR_{i, L_2, c} - \tau_{PR, L_1} \{ \theta_i(t) \psi_i(t) \} + \tau_{PR, L_2} \{ \theta_i(t) \psi_i(t) \} + \\
&- \eta_{i, PR, L_1, c} + \eta_{i, PR, L_2, c} - MP_{i, PR, L_1, c} + MP_{i, PR, L_2, c}
\end{align*} \]

(3.4)

Substituting (3.4) in (3.2),
\[ \begin{align*}
\varepsilon_i(t) + c \times \delta_i(t) &= \left( 1 + \alpha \right) PR_{i, L_1, c}(t) - \alpha PR_{i, L_2, c}(t) - \tau_i(t) + \\
&+ \left[ \delta_{s, L_1, PR, c}(t - \tau) - \delta_{s, L_1, PR, c}(t) \right] - T_{s, PR}(t) + \\
&- \left( 1 + \alpha \right) \tau_{PR, L_1} \{ \theta_i(t) \psi_i(t) \} + \alpha \tau_{PR, L_2} \{ \theta_i(t) \psi_i(t) \} + \\
&- \left( 1 + \alpha \right) MP_{i, PR, L_1, c}(t) + \alpha MP_{i, PR, L_2, c}(t) + \\
&- \left( 1 + \alpha \right) \eta_{i, PR, L_1, c}(t) + \alpha \eta_{i, PR, L_2, c}(t)
\end{align*} \]

(3.5)

Equation (3.5) provides the desired satellite orbit and clock error term for monitoring.

Where: \( \alpha = \frac{f_{L_2}^2}{f_{L_1}^2} = 1.5457 \)

### 3.1.1 Error Source Mitigation & Obtaining PR Residual

The following error sources can be mitigated:

- SV clock correction & Relativistic correction
- Earth rotation correction for SV position
- Ionosphere delay correction, by dual frequency measurement combination
  - P1C1 bias
  - Group delay differential
- Troposphere delay correction, by using the modified Hopfield model
The SV clock correction is applied through a second order polynomial, whose coefficients are given in the broadcast navigation message [2]. As mentioned in IS GPS 200D, utilizing this polynomial will allow the user to find the SV PRN clock offset at the time of transmission, with reference to the satellite antenna phase center with respect to GPS time. The relativistic correction is a result of the application of the fact, from the special theory of relativity, that the clock on board a satellite, traveling at a constant speed, appears to lose time due to the difference in the gravitational potential. As the satellite orbits are not circular, the speed of the SV is not constant and a correction needs to be applied [7]. The earth rotation correction must be applied in order to obtain the position of the satellite at the time of signal transmission expressed in the coordinate frame at time of signal reception. The Earth-Center-Earth-Fixed (ECEF) coordinate frame is used for the satellite position. The ECEF frame is different at the times of signal transmission and reception due to earth rotation during the signal transmission in space. The ionosphere delay error is mitigated using the dual frequency combination of the pseudo ranges on L1 and L2. In practice, the L1 pseudorange measurement is based on the C/A code, while the L2 pseudorange measurement is based on the P code. The two codes are not exactly aligned with each other due to different hardware circuitry [17]. As a result, there will be a bias that needs to be corrected before using pseudoranges from different codes for the ionosphere delay calculation. In this particular case, the Inter Code Bias between P code and C/A code on L1 needs to be corrected. These correction values are available online [22]. The Inter Frequency Bias between L1 and L2, also known as the group delay differential, needs to be addressed as well. It turns out that if an ionospheric correction is made, the group delay differential does not need to be corrected, as discussed in 20.3.3.3.3 of IS GPS 200D. The troposphere delay is calculated using the Modified Hopfield model. Multipath and noise on the code pseudo range measurements can be reduced using time-differenced carrier phase measurements for smoothing, known as the Hatch filter [16]. Although a topic for further research, multipath and noise error are not further reduced in this research. As shown in Figure 3.1 (see also Equation (3.5)), satellite orbit and clock error can be obtained by compensating for the propagation errors due to Troposphere and Ionosphere,
receiver clock offset, multipath and noise in the measured range and by subtracting the true range from the measured range.

Figure 3.1. Block Diagram for obtaining pseudorange residuals for satellite clock & orbit error determination

The user clock bias is calculated from a least squares user state vector as follows. After the pseudoranges have been corrected for known errors, each visible satellite position is calculated at the time of transmissions in the ECEF coordinate frame at the time of reception. The nonlinear relationship between the pseudoranges and the user state vector is given by:

\[ PR_i(t) = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} + B \]

(3.6)
Where \((x,y,z)\) are the user coordinates in ECEF in m
\((x_i,y_i,z_i)\) are the coordinates for the \(i^{th}\) satellite in ECEF in m

\(B\) is the receiver clock offset from GPS time in m

For \(n\) satellites, the equation is linearized and written in matrix format:

\[
\begin{pmatrix}
PR_1 \\
\vdots \\
PR_n
\end{pmatrix} =
\begin{pmatrix}
\frac{x - x_1}{R} & \frac{y - y_1}{R} & \frac{z - z_1}{R} & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\frac{x - x_n}{R} & \frac{y - y_n}{R} & \frac{z - z_n}{R} & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
B
\end{pmatrix}
\]

or

\[y = Hx\] (3.7)

Orbit and clock error for each satellite obtained from the measured pseudo ranges at this point contain jumps in the values caused by satellites entering or leaving the solution. For example, when a satellite enters the solution, it introduces a bias that changes the receiver position and clock solution.

After compensation of the pseudorange for known errors sources, consider that a bias given by \(\Delta b_i\) remains in each \(i^{th}\) pseudorange measurement. The receiver clock offset calculated from Equation 3.7 will have a bias given by

\[
\Delta b = \frac{\sum_{i=1}^{n} \Delta b_i}{n}
\]

Where: \(\Delta b\) is the bias in the receiver clock offset solution, \(n\) is the number of satellites available for solution. This bias changes whenever a satellite enters or leaves the solution. Keeping track of the changes in the bias can compensate sudden jumps due to this bias,

\[
\hat{B}(t) = B(t) + \frac{\sum_{i=1}^{n} \Delta b_i}{n}
\]

This bias is modified whenever there is a satellite transition.
Once the residuals are calibrated for the bias introduced by the satellite transitions, errors in the orbit and clock are monitored using one of the following 3 methods.

1. Differencing between the range residuals from two satellites

2. Formation of Subsets for Receiver Clock calculation

3. Using a highly-stable receiver clock (e.g. Rubidium or Cesium standard) to maintain the Receiver Clock offset solution

The third option is not used in this research as a stable clock was not available, but is recommended for future research. Once the receiver clock offset is known, the third option enables a direct observation of the URE.

3.1.2 Differencing between Range Residuals from Two Satellites

Satellite orbit and clock error from the range residuals can be written as,

$$\Delta R_{i,\text{res}} = \Delta R_{i,\text{true}} + \Delta B_{\text{res}} + \eta_{i,\text{res}}$$

Where,

$\Delta R_{i,\text{res}}$ - Range Residual for satellite ‘$i$’, which is a combination of true satellite clock and orbit error and a clock offset that is common to all satellites. In addition, $\eta_{i,\text{res}}$, representing noise, multipath and modeling errors also are part of this residual.

The receiver clock offset $\Delta B_{\text{res}}$ can be removed by differencing the residuals from two satellites. Differencing between the residuals is obtained by taking one of the satellites as the reference satellite and by subtracting residuals of all the other SVs from the reference SV.

$$\begin{bmatrix}
\Delta R_{\text{ref}} - \Delta R_1 \\
\Delta R_{\text{ref}} - \Delta R_2 \\
\vdots \\
\Delta R_{\text{ref}} - \Delta R_{n-1}\text{ or } (n-1)X_1
\end{bmatrix}$$

Differenced residual range vector from n satellites

If there is a large error (above the threshold value) in one of the differenced range values, clearly the satellite may have the resulting error in the orbit and clock, and can be
detected by the monitor. Once the error is detected, it is analyzed in more detail as discussed in Section 3.2.

If there is a bias in all the differenced residuals, the error may be resulting from the reference SV itself. The latter can be verified by changing the reference SV chosen for obtaining the differenced residuals.

Differencing combines the noise from two satellites, which will be taken into account for the setting of the detection threshold.

3.1.3 Formation of Subsets for Receiver Clock Calculation

Another way of identifying SV orbit and clock errors, is to form sub-sets leaving out one satellite at the time, thereby forming \( n \) different receiver clock solutions at each epoch, where \( n \) is the number of satellites available (above the elevation mask).

A large error for any of the satellites is directly observed in the residual range of that particular satellite. For example, if the first satellite has a large error, then the clock offset calculated without the first satellite will not be affected by the large error, and the residual for the first satellite will be directly observable using this clock offset. In general, all possible combinations are calculated as shown below.

\[
\begin{bmatrix}
\Delta R_1 - \Delta t_{r_1} & \Delta R_1 - \Delta t_{r_2} & \cdots & \Delta R_1 - \Delta t_{r_2} \\
\Delta R_2 - \Delta t_{r_1} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
\Delta R_n - \Delta t_{r_1} & \cdots & \cdots & \Delta R_n - \Delta t_{r_{(n-1)}}
\end{bmatrix}
\]

In the above matrix, \( n \) is the number of visible satellites above the elevation mask, \( \Delta R_1, \Delta R_2 \ldots \Delta R_n \) are the residual pseudoranges without accounting for the receiver clock bias. \( \Delta t_{r_1}, \Delta t_{r_2}, \ldots \Delta t_{r_n} \) are the receiver clock bias solutions with \( n-1 \) satellites in each of the solutions, formed by eliminating one of the satellites for each solution.

In effect, each satellite is tested for a large error as represented by each of the columns in the above matrix. If only element in a column is larger than the threshold, the satellite with the large error will have been found as well as an estimate of its error. If more than one element in a column exceed the threshold, the column is not used as its clock offset was calculated using the large error.
3.1.4 Threshold Calculation

As the anomalous orbit and clock error monitor triggers whenever a residual crosses the threshold value, the threshold setting should be based on the desired probabilities of false alarms and missed detections. The threshold is calculated based on the potential error resulting from the residual errors remaining in the pseudorange. As discussed in the section 3.1.1, the troposphere delay is corrected by using the Modified Hopfield model and the first order ionosphere delay is corrected by using the dual frequency combination of the pseudoranges. However, these delays are not removed completely and there is still some residual error left that could result from non-nominal weather conditions, represented by temperature, pressure and relative humidity. In the case of the ionosphere delay, only the first order bias is eliminated but the bias resulting from the higher order Ionosphere delay is still present [25, 26]. In addition to the residual errors resulting from the corrected errors, Multipath, noise and group delay terms that are not mitigated must be considered in the calculation of threshold.

3.1.4.1 Residual Troposphere delay

The error bounds on the troposphere delay can be obtained by varying the standard temperature, pressure, and relative humidity values, used in the Modified Hopfield model, to their corresponding extremes and by subtracting the delay computed with standard conditions from these values. This can be written as,

\[
TC_{\text{bound}} = \max \left[ \left| TC(T_{E,H}, RH_{E,H}, P_{E,H}) - TC(T_{S}, RH_{S}, P_{S}) \right| \right]
\]

Where:

\( TC_{\text{bound}} \) – Error bound on the residual troposphere delay due to extreme weather conditions (m)

\( T \) – Temperature in Kelvin

\( RH \) – Relative Humidity in percentage
P – Pressure in milli bars

S – Subscript for standard weather conditions

E – Subscript for extreme weather conditions

Extreme weather conditions: \( T_{E,L} = 260.93 \) Kelvin, \( R_{H,E,L} = 20\% \) and \( T_{E,H} = 310.93 \) Kelvin, \( R_{H,E,H} = 100\% \). Since the effect of pressure on the troposphere delay is negligible; atmospheric pressure of 1013 milli bars at mean sea level is considered for all the calculations.

Standard weather conditions: \( T_{S} = 288.15 \) Kelvin, \( R_{H,S} = 50\% \).

The standard deviation is calculated by using the LAAS troposphere model presented in [23]. Equation (1) in [23] for the Tropospheric Correction (TC) is:

\[
TC = N_R h_0 \frac{10^{-6}}{\sqrt{0.002 + \sin^2(\theta)}} \left( 1 - e^{-\frac{\Delta h}{h_0}} \right)
\]

(3.9)

Where:

TC – Tropospheric Delay

\( N_R \) – Refractivity Index

\( h_0 \) – Scale Height

\( \theta \) - Elevation Angle of the SV

\( \Delta h \) – Differential Altitude between Reference Station and the user in (DGPS)

From Equation 3.9, for a standalone user, the troposphere delay can be written as:

\[
TC = N_R h_0 \frac{10^{-6}}{\sqrt{0.002 + \sin^2(\theta)}}
\]

(3.10)

From Eq. 3.10, the standard deviation can be calculated as

\[
\sigma_{TC}(\theta) = \sigma_N h_0 \frac{10^{-6}}{\sqrt{0.002 + \sin^2(\theta)}}
\]

(3.11)
Where:
\(\sigma_{TC}\) - Uncertainty in the Troposphere delay
\(\sigma_N\) – Uncertainty in the Refractivity

As presented in the [23], the yearly average values substituted in the Eq. 3.11 are:

\[N_R = 332 \text{ (unit less)}\]
\[h_0 = 15730 \text{ meters}\]
\[\sigma_N = 33 \text{ (unit less)}\]

As can be seen from Equation 3.11, the standard deviation of the troposphere delay is a function of satellite elevation angle.

3.1.4.2 Multipath, Noise, Group Delay and Residual Ionosphere delay

The maximum multipath error that is possible on the L1 pseudorange measurement is 15 meters, as seen from the multipath error envelope (p. 290 in [1]). The uncertainty can be calculated from the Code Minus Carrier (CMC) corrected for ionosphere divergence. The effect of receiver noise is different for different receivers and the noise on the pseudorange can be obtained from [24]:

\[
\sigma_{PR} = \text{chip} \sqrt{\frac{d(BW_{SS})}{2 \cdot 10^{(C/N_0/10)}}}
\]

(3.12)

Where:
\(\sigma_{PR}\) – Standard deviation of the noise on pseudorange in meters.
chip – The C/A code length in meters (293).
d – The correlator spacing in code chips (0.1)
\(BW_{SS}\) – Single sided loop bandwidth in Hz
\(C/N_0\) – Carrier to Noise ratio in dB-Hz

Antenna group delay and phase delay values are dependent on the elevation and azimuth angles of the satellite and are repeatable approximately every 24 hours. Just observing the
previous day’s pseudorange residuals, one can identify any bias resulting from these delays. The residual delay due to the second and higher order Ionosphere delay is discussed in detail in [25, 26]. A Summary of all the residuals and maximum possible errors are provided in Table 3.1.

Table 3.1. Summary of the residual errors in the pseudo range for threshold calculation

<table>
<thead>
<tr>
<th></th>
<th>σ (Standard deviation)</th>
<th>Bound</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropo</td>
<td>Eq. 3.10</td>
<td>Eq. 3.8</td>
<td>Elevation Dependent [23]</td>
</tr>
<tr>
<td>Iono</td>
<td>1cm</td>
<td>10cm</td>
<td>Higher order iono delays [25, 26]</td>
</tr>
<tr>
<td>Group/Phase</td>
<td>~5cm/~0.5mm</td>
<td>~20cm/~2mm</td>
<td>Satellite Elevation and Azimuth dependent</td>
</tr>
<tr>
<td>Multipath</td>
<td>σ(CMC-2ΔI)</td>
<td>15m</td>
<td>From [1], can be limited to 5m, see Note 1.</td>
</tr>
<tr>
<td>Noise</td>
<td>Eq. 3.11</td>
<td>6σ</td>
<td>Gaussian distribution [24]</td>
</tr>
</tbody>
</table>

Note 1: Multipath bound given in this table is the worst case for an unknown antenna environment. The delay can be limited to 5m for the case of an antenna located at a height of less than 2 m above the ground without large nearby obstructions.

3.1.4.3 Threshold for Differencing
From Eq. 3.5, it can be observed that the uncorrected Multipath and noise from both L₁ and L₂ will be scaled by a factor. As a result, the threshold will be higher than the actual un-corrected errors. For differencing, the threshold will be even higher as the errors such
as noise add up in terms of variance. The threshold can be calculated based on the following equation.

\[
res_{ij, \text{diff}}(t) = \alpha MP_{i,PR,L2,e}(t) - (1 + \alpha) MP_{i,PR,L1,e}(t) + \alpha \eta_{i,PR,L2,e}(t) - (1 + \alpha) \eta_{i,PR,L1,e}(t) + \alpha \eta_{i,PR,L2,c}(t) + (1 + \alpha) \eta_{i,PR,L1,c}(t) + \alpha MP_{j,PR,L2,e}(t) + (1 + \alpha) MP_{j,PR,L1,e}(t) - \alpha \eta_{j,PR,L2,e}(t) - (1 + \alpha) \eta_{j,PR,L1,e}(t)
\]

(3.13)

Where:

- \( res_{\text{diff}} \) – Residual for the differencing method
- \( i,j \) – Subscripts for the satellites \( i \) and \( j \) which are differenced.

As can be seen from the Eq. 3.13, multipath adds up with a scaled factor, which is the dominant factor for the threshold.

The threshold is calculated by adding the six-sigma noise value to the worst possible combination of the bias contributions to the residual. Using Eq. 3.13, threshold can be written as,

\[
Th_{ij, \text{diff}} = \alpha \mu_{i,MP,PR,L2} + (1 + \alpha) \mu_{i,MP,PR,L1} + \alpha \mu_{j,MP,PR,L2} + (1 + \alpha) \mu_{j,MP,PR,L1} + 6 \sqrt{(1 + \alpha) \sigma^2_{i,PR,L2,e} + \alpha ^2 \sigma^2_{i,PR,L1,e} + (1 + \alpha) \sigma^2_{j,PR,L2,e} + \alpha ^2 \sigma^2_{j,PR,L1,e}}
\]

(3.14)

Where:

- \( Th_{ij, \text{diff}} \) – Threshold for the differenced pseudorange residuals from \( i^{\text{th}} \) and \( j^{\text{th}} \) satellites
- \( \mu_{MP} \) – Maximum possible bias from the multipath
- \( \sigma \) - Standard deviation of the noise

### 3.1.4.4 Threshold for Subset Method

In the computation of the threshold in section 3.1.4.3, receiver clock offset is eliminated after differencing. But in the case of subsets, receiver clock offset is still present. As a result, the residual consists of two components: one is the multipath and noise residual and the other resulting from the receiver clock offset. The combined residual can be written as,

\[
res_{i, \text{subset}}(t) = T_{i, \text{res}}(t) + I_{i,PR,L1,\text{res}}(t) + \tau_{i,PR,L1} \left\{ \theta(t) \psi_i(t) \right\}(t) + \alpha MP_{i,PR,L2,e}(t) + (1 + \alpha) MP_{i,PR,L1,e}(t) + B_{\text{bias,subset}}(t) + (\alpha \eta_{i,PR,L2,c}(t) - (1 + \alpha) \eta_{i,PR,L1,c}(t)) + \eta_{B, \text{subset}}(t)
\]
Where:

\( r_{es i, \text{subset}} \) – Residual for the pseudorange from \( i^{th} \) satellite of the subset

\( B_{bias} \) – Bias due to the receiver clock offset calculated for the subset

\( \eta_B \) – Noise due to the receiver clock offset calculated for the subset

The threshold is calculated by adding the six-sigma value of the resultant noise to the worst possible combination of the bias contribution to the residual. Using Eq. 3.15, this can be written as,

\[
T_{h_i, \text{subset}} = T_{C_i, \text{bound}} + \mu_{i, \text{L1,res}} + \mu_{i, \text{PR,L},1} + \alpha \mu_{i, \text{MP,PR,L1,c}} + (1 + \alpha) \mu_{i, \text{MP,PR,L1,c}} + B_{\text{max}} + 6 \sqrt{\left( \sigma^2_{i, \text{MP,PR,L1,c}} + \sigma^2_{i, \text{MP,PR,L1,c}} + \sigma^2_{B, \text{subset}} \right)}
\]

(3.16)

Where:

\( B_{\text{max}} \) – Maximum bias resulting from the receiver clock offset calculation

\( \sigma_B \) – Standard deviation of the receiver clock offset noise calculated from the subset

To calculate the bias \( B_{\text{max}} \) and noise \( \sigma_B \), consider the solution for the receiver clock offset by inverting Equation Eq. 3.7,

\[
\begin{pmatrix}
x \\
y \\
z \\
B
\end{pmatrix} = \left( (H^T H)^{-1} H^T \right) \begin{pmatrix} \mu_{\text{MP,PR,L1,c}} \\
PR_1 \\
PR_n \end{pmatrix}
\]

(3.17)

If we write the vector \((x \ y \ z \ B)^T\) as \(X\) and \((H^T H)^{-1} H^T\) as \(G\), then the matrix \(G\) will have size of \((n-1)x4\), where \(n\) is the number of available satellites. The bias component \(B_{\text{max}}\) can be obtained as the sum of products of the 4th column with the corresponding pseudo ranges, and can be written as:
\[ B_{\text{max}} = \sum_{i=1}^{n-1} |G_{i,\mu_{i,\text{max}}} | \]  

(3.18)

The noise component can be obtained by computing the covariance for the position vector in Eq. 3.17, and can be written as:

\[
\text{cov}(X) = \text{cov}\left( H^T H^{-1} H^T PR \right) \\
= E\left( \left( H^T H^{-1} H^T PR \right) \left( H^T H^{-1} H^T PR \right)^T \right)
\]

(3.19)

Where \( \text{cov()} \) is the covariance matrix of the vector \( X \).

Eq. 3.19 can be written as:

\[
\text{cov}(X) = \left( H^T H^{-1} H^T \right) E\left( PR PR^T \right) \left( H^T H^{-1} H^T \right)^T
\]

(3.20)

The pseudorange variances are known for the pseudorange residuals. In general pseudorange variance for each satellite will be different as the receiver tracks each satellite at a different carrier to noise ratio. But in setting up the threshold, a satellite is considered not trackable if the carrier to noise ratio is below 32 dB-Hz, and the same value is used for every satellite. As a result, variances of all the satellites are the same and Eq. 3.20 is further simplified. In practice carrier to noise ratio values given by the receiver can be used to feed into the calculation of the variances, which are used to calculate variance of the clock offset using Eq. 3.20. The variance is the square of the clock offset noise component \( \sigma_B \).

### 3.2 Accumulated Doppler Residual based

The carrier phase measurement (\( \Phi \)) at time \( t \) is the Accumulated Doppler (AD) from the time when the receiver is phase locked onto the signal. Unlike the pseudorange measurement, the AD is a relative measurement and has an unknown number of cycles,
known as the ambiguity, associated with it. Accumulated Doppler, in meters, can be written as,

\[ AD_{i,L}(t) = \Phi_{i,L}(t) = \lambda_L \phi_{i,L}(t) = r_i(t) + c[\delta t_{r,AD,L}(t) - \delta t_{s,AD,L}(t - \tau) + \delta t_i(t)] + \epsilon_i(t) - I_{i,L,AD}(t) \]

\[ + T_{i,AD}(t) + \lambda_L N_L + \tau_{AD,L} \{ \theta_i(t), \psi_i(t) \} + \eta_{i,AD,L}(t) \]

(3.21)

Where:

- \( \Phi \) - Carrier phase (CP) measurement (m)
- \( \phi \) - Carrier phase measurement (cycles)
- \( \lambda \) - Wavelength of the carrier signal (m)
- \( \tau \) - Receiver hardware delay for carrier (m)
- \( N \) - number of cycles present in the carrier signal from receiver to satellite, when phase is locked

As can be seen in the above equation, the ionosphere advances the carrier phase as opposed to delaying, as is the case for the pseudorange. Rewriting Equation (3.21) for satellite orbit and clock errors,

\[ AD_{i,L}(t) = \Phi_{i,L}(t) = \lambda_L \phi_{i,L}(t) = r_i(t) + c[\delta t_{r,AD,L}(t) - \delta t_{s,AD,L}(t - \tau) + \delta t_i(t)] + \epsilon_i(t) - I_{i,L,AD}(t) \]

\[ + T_{i,AD}(t) + \lambda_L N_L + \tau_{AD,L} \{ \theta_i(t), \psi_i(t) \} + \eta_{i,AD,L}(t) \]

(3.22)

Similar to the Code Ionosphere delay, we can calculate the Phase Ionosphere delay as

\[ \frac{f_{L,1}}{f_{L,2}} \cdot \frac{f_{L,1}}{f_{L,2}} \cdot I_{i,L,AD} = \Phi_{i,L,2} - \Phi_{i,L,1} + \lambda_{L,1} N_{L,1} - \lambda_{L,2} N_{L,2} + \]

\[ + \tau_{AD,L,1} \{ \theta_i(t), \psi_i(t) \} - \tau_{AD,L,2} \{ \theta_i(t), \psi_i(t) \} + \eta_{i,AD,L,1} - \eta_{i,AD,L,2} \]

(3.23)

Note that the signs are reversed when compared to the Code Ionosphere delay term.

Substituting (3.23) in (3.22),
\[ e_i(t) + c \times \delta \tau_i(t) = -\alpha \Phi_{i,L2} + (1 + \alpha )\Phi_{i,L1} - r_i(t) + \\
\] 
\[ c \left[ \delta \chi_{AD,i,c}(t - \tau) - \delta \chi_{r,AD,i,c}(t) - T_i(t) + \\
- (1 + \alpha )\lambda_{L1} N_{L1} + \alpha \lambda_{L2} N_{L2} + \\
- (l + \alpha )\tau_{AD,i,c}(t) + \alpha \tau_{AD,L1} \{ \theta_i(t), \psi_i(t) \} + \\
- (l + \alpha )\eta_{i,AD,L1}(t) + \alpha \eta_{i,AD,L2}(t) \right] \\
\] 
(3.24)

### 3.2.1 Differencing between Successive Accumulated Doppler Residuals

As can be seen from Equation (3.22), after mitigating the effects of troposphere and ionosphere, the receiver clock offset cannot be determined as each observable has its own ambiguity in addition to the common clock offset. One way the AD can be used is by differencing over time such that the ambiguity cancels. The differenced ADs represent the change in range over the differencing time interval. Next, these differenced ADs can be used in the same way as the pseudoranges, except that the errors on the ADs are much smaller than the errors on the pseudoranges. This allows for a precise characterization of the changes in the URE over time.
4 MEASUREMENT RESIDUALS COMPARED WITH TRUTH CLOCK AND ORBIT DATA

This chapter presents the comparison calculations of the pseudorange and carrier phase residual methods with post-processed precise orbit and clock data from the NGA. The purpose of this comparison is to verify the pseudorange residual calculations against a truth source.

4.1 Orbit & Clock Error Using Precise Orbit and Clock Data

Precise orbit and clock data are obtained from monitor stations by processing the satellites’ broadcast data from a worldwide GPS receiver network. Precise orbits and clock products are used for surveying and Precise Point Positioning (PPP) [19, 9]).

These data are available online in various latency vs. accuracy formats from several organizations, including IGS and NGA. IGS provides four products with different latencies and accuracies. The following table lists the IGS products, as well as their accuracies and latencies.
Table 4.1. IGS precise clock & orbit products summary [21]

<table>
<thead>
<tr>
<th>GPS Satellite Ephemerides/ Satellite &amp; Station Clocks</th>
<th>Accuracy</th>
<th>Latency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broadcast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbits</td>
<td>100cm</td>
<td>Real time</td>
</tr>
<tr>
<td>Sat. Clocks</td>
<td>5ns RMS, 2.5ns Sdev</td>
<td></td>
</tr>
</tbody>
</table>

| Ultra - Rapid (Predicted half)                      |          |               |
| Orbits                                              | 5cm      | Real time     |
| Sat. Clocks                                         | 3ns RMS, 1.5ns Sdev |       |

| Ultra - Rapid (Observed half)                       |          |               |
| Orbits                                              | 3cm      | 3 - 9 hours   |
| Sat. Clocks                                         | 150ps RMS, 50ps Sdev |       |

| Rapid                                               |          |               |
| Orbits                                              | 2.5cm    | 17 - 41 hours |
| Sat. Clocks                                         | 75ps RMS, 25ps Sdev |       |

| Final                                               |          |               |
| Orbits                                              | 2.5cm    | 12 - 18 days  |
| Sat. Clocks                                         | 75ps RMS, 20ps Sdev |       |

IGS provides the XYZ positions of the satellites with respect to the center of mass of the satellite. NGA provides the data both with respect to the Antenna Phase Center (APC) as well as the center of mass [9]. If the precise orbits are used with respect to the center of
mass, they need to be translated to the antenna phase center for comparison with the pseudorange residuals.

The precise orbit and clock products are provided in Standard Product 3 (SP3) format, which contain XYZ positions of the satellites in ECEF coordinates in units of kilometers and satellite clock offset in microseconds. The time interval between orbit and clock updates is 15 minutes. At each of the 15-minute updates, the broadcast ephemerides can be used to calculate the broadcast satellite clock and orbit parameters. The difference between the broadcast and precise data is considered to be the error in the broadcast data. The block diagram in Figure 4.1 depicts the procedure for calculating the error in the broadcast orbits compared to the NGA precise orbit and clock truth data.

Figure 4.1. Block diagram for computing satellite clock and orbit error using NGA precise orbit and clock products
The orbit error calculated will be in ECEF coordinates. This error is translated into the line-of-sight direction, from the satellite to the user, to determine the URE. The URE can then be compared with the pseudorange residuals.

4.2 Carrier Phase Residuals

The precise orbit and clock data are only available every 15 minutes. To accurately determine the actual error signature between the 15-minute updates, the carrier phase residuals are used as described in Section 3.2.1. The single differenced carrier phase residual can be written as follows,

$$SD_{ij,L} = AD_{i,L} - AD_{j,L}$$

$$⇒ SD_{ij,L} = \delta t_i(t) - \delta t_j(t) + \lambda_L N_i,L - \lambda_L N_j,L + \eta_{i,AD,\lambda}(t) - \eta_{j,AD,\lambda}$$

(4.1)

Assuming that there are no undetected cycle slips, the initial offset between the truth and the differenced residuals can be subtracted. The resulting ADR residual provides the details of the URE behavior at the rate the carrier phase measurements are collected. The block diagram of Figure 4.2 depicts the procedure to obtain the differenced carrier phase residuals.
Figure 4.2. Block diagram representing the computation procedure differenced accumulated Doppler range residuals.
5 DATA EXAMPLES & DISCUSSION

This chapter discusses the implementation of the real time monitor, both using pseudorange residuals and accumulated Doppler range residuals. Various methods discussed in Chapter 3 are also implemented in MATLAB® using measurement data from different sites. Some of the datasets used in this chapter are downloaded from CORS STKR, corresponding to data collected from an Ashtech receiver at Ohio University’s Stocker Center. Other data were collected at Ohio University’s Airport using NovAtel receivers. As there is a slight difference in the way receiver clock steering is implemented, care should be taken while using the measurements. For example, the Ashtech receiver clock is allowed to grow an error of 1 millisecond and then jump back. This is depicted in Figure 5.1.

![Receiver Clock offset with 1 milli-second jumps](image)

Figure 5.1. Receiver clock offset with 1 ms jumps
In order to deal with the jumps, the clock offset is calculated first, and then subtracted from the observation time epochs, whenever this kind of receiver is used. A Stocker Center user solution where there are no anomalies in the satellite clock and orbit data is presented in Figure 5.2.

**Figure 5.2. East vs. North Error plot for a scenario without anomalous clock & orbit error**

The east, north and up error are generated by comparing the user solution with the CSRS PPP [19] position in the ECEF coordinate frame followed by a conversion to the locally-level coordinate frame with the Stocker Center GPS antenna as the origin. Figures 5.3
and 5.4 shows an example when an anomalous orbit error is present in PRN18. This example is from April 10, 2007, when the Control Segment performed an orbit adjustment without setting the satellite to the status of unhealthy, which resulted in a growing URE that reached several hundreds of meters [20]. The datasets containing this problem are used to demonstrate the real-time clock and orbit error monitor performance using different methods.

Figure 5.3. East vs. North Error plot for a scenario with anomalous clock & orbit error
At the Stocker Center location, PRN 18 was only visible for a few minutes around 5:00 pm as shown in the satellite visibility plot of Figure 5.5. PRN 18 was rising and came into view with a large error. Shortly after that, the satellite was set unhealthy and was no longer tracked by the receiver. From Figure 5.4 and 5.5, it is clear that PRN 18 was the bad satellite as the vertical position error occurs when PRN 18 is available to the solution. In general, it will not always be this easy to identify the bad satellite, and the detectors developed in Chapter 4 need to be used.
5.1 Pseudo Range Residual based

The pseudo range measurements obtained from the RINEX observation file are used to form the pseudorange residual for each satellite above the elevation mask, as depicted in the pseudo range residual block diagram and equations presented in chapter 3. After correcting for the known errors and the range to the satellite, the pseudorange residual can be displayed as illustrated in Figure 5.6 for PRN 20 on November 2, 2009.
Figure 5.6. Pseudorange residual plot for a scenario without anomalous clock & orbit error

In Figure 5.6, both the true URE and the pseudorange residuals are shown. The noise in the residuals is due to uncompensated noise and multipath errors. The true URE is derived from the NGA precise clock and orbit data and is available in 15-minute time increments illustrated by the green dots. Connecting two successive green dots draws the blue lines in Figure 5.6.

Since the pseudo range residuals contain clock and orbit errors along with multipath, noise and other un-calibrated biases of the pseudorange from an SV, the pseudorange residual plots for all SVs, except for the SV with the anomalous clock and orbit error, should have similar statistics. However, this is not the case when a satellite with a large bias is in the solution such that the clock offset will be in error. Figure 5.7 illustrates this effect for PRN 26, which is biased when PRN 18 is present. To avoid this problem, the sub-set solution is used as shown in Section 5.1.2. The pseudorange residuals for PRN 18
are shown in Figure 5.8. Clearly, a large error is present during the few minutes that the satellite was visible.

Figure 5.7. Pseudorange residual plot for a scenario with anomalous clock & orbit error
5.1.1 Differencing Method

The pseudorange residuals for different satellites are differenced with respect to a reference satellite, which can be either the one with highest elevation angle or the one that is in view for longest duration, as described in Section 3.1.2. If the reference satellite is free of anomalies, then all differenced residuals will be within the threshold, except for the one with the anomalous satellite. If the reference satellite is the anomalous satellite, then more than one of the differenced residual will be out of the bounds, which is easily identified by using a different reference satellite. Figures 5.9 and 5.10 depict the differenced PR residuals of PRN 6 and PRN 18 with PRN 10 as the reference satellite.

Figure 5.8. Pseudorange residual plot for the PRN 18 with anomalous clock & orbit error
Figure 5.9. Differencing method applied to pseudorange residuals in a scenario with anomalous clock and orbit errors
Figure 5.10. Differencing method applied to pseudorange residuals for PRN 18 with anomalous clock and orbit errors

As can be seen from figures 5.9 and 5.10, by differencing the pseudorange residuals, the PRN 18 with an anomalous clock and orbit error can be identified, and the size of the anomaly can be estimated. It is noted that the differenced truth in Figure 5.10 is offset by approx. -65 m with respect to the differenced pseudorange residuals.

5.1.2 Subset Method
The subset method isolates one satellite at a time and computes the receiver clock offset for each subset, with the one satellite eliminated. Clock and orbit errors for each satellite using different subsets are plotted over time. Assuming that there is only one satellite with the anomaly, each satellite will contain more than one subset that is outside the
detection thresholds, except for the subset in which the SV with the anomaly is eliminated.

Figure 5.11. Subsets method applied for obtaining pseudorange residuals for PRN 10 in a scenario with anomalous clock and orbit errors
Figures 5.11 and 5.12 show two satellite PR residuals with different subsets in different
subplots. When PRN 18 is used in the solution for the clock offset, all the residuals are
biased as shown in Figure 5.11. The satellite visibility plot (Figure 5.5) can be used along
with Figure 5.11 to identify the satellite eliminated in each subset; to detect the satellite
with anomalous clock and orbit errors. As can be seen from Figure 5.11 and Figure 5.5,
when PRN 18 is removed from the solution, the residual is well within the bounds.

5.2 Accumulated Doppler Range Residual based

The ADR residual is used to quantify the change in the error after detection and
identification of the anomalous satellite. Specifically, the differenced ADR residuals are
set equal to that obtained from a truth source (e.g. NGA) and the variations can then be
examined at the cm-level between two truth updates separated by 15 minutes in time. Figure 5.13 shows an example of differenced ADR residuals for PRN 5 and PRN 10. Both satellites are without significant anomalies and the differenced ADRs provide detailed information on the actual URE signature between the truth points. If the ADR is not reset to the truth points every 15 minutes, a small ramp error, causing an error of up to 1 m after a few hours, due to elevation-dependent satellite biases [17].

Figure 5.13. Differenced method applied on accumulated Doppler range residuals for a scenario without anomalous clock & orbit errors

Figure 5.14 shows the differenced ADRs for the anomalous PRN 18.
Figure 5.14. Differenced method applied on accumulated Doppler range residuals for the PRN 18 with anomalous clock and orbit errors

As can be seen from Figure 5.14, the differenced ADRs provide detailed information on the URE growth during the time period of the anomaly.
6 CONCLUSIONS AND FUTURE WORK

6.1 Summary and Conclusions
A real-time monitor to detect anomalous satellite clock and orbit errors is developed. Different methods based on pseudo range residuals and accumulated Doppler range residuals are implemented successfully. The pseudo range residual based methods detected and identified the PRN with the anomaly; the Subsets method also computed an approximate value of the anomaly. The Accumulated Doppler range residual based differenced method, used for the analysis of the anomaly, presented a clear picture of the error growth over time. NGA precise clock and orbit products are used for the verification of all the methods that are implemented in Matlab®. The monitor was implemented on a previous anomaly on PRN 18 that occurred on 4/10/2007. All the datasets used were RINEX observation and navigation files obtained from different CORS sites.

6.2 Future Work
All the methods implemented in the research can detect the anomalies associated with the satellite clock and orbit errors with any one of the visible satellites, but it is difficult to detect multiple satellite failures. In order to address this problem, a highly-stable clock (e.g. Cesium standard) is recommended to be used as the frequency reference for the user receiver. This clock, after synchronizing with GPS time, can be used to detect multiple satellite failures as it avoids the necessity of computing the receiver satellite clock offset at the range measurement data rate, and pseudorange residuals from each satellite can be monitored independently. Using a highly-stable clock also helps in setting up tighter detection thresholds, as the receiver clock bias does not add to the threshold.
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