MULTIAXIAL FATIGUE TESTING MACHINE

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Mu-Hsin Liu
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CHAPTER 1

INTRODUCTION

For most engineers, machine design is one of the most important jobs. Engineers and scientists have developed hundreds of theories and techniques for engineering design during the last several centuries. One important topic in machine design is safety. Ensuring the machines work safely is necessary and important. Many mechanical engineers however are not sufficiently familiar with material properties, although most machine design is involved with all kinds of materials.

The fundamental material properties such as yield strength, tensile strength, Young’s modulus, etc. have been well established for many engineering materials for a long time. With the progress of industrial technology, the traditional material properties can not fully explain some phenomena of material failures. Fatigue failure is one of the most common types of failure and is still not well understood. This thesis is devoted to design a new multiaxial fatigue testing machine that can provide the fatigue properties of materials in multiaxial loading conditions.

Chapter 2 provides the basic fatigue theory. In this chapter, existing fatigue theories are reviewed in both mechanical engineering and material science point. Chapter 3 is focused on the testing methods of tension and torsion tests for both static and dynamic cases. Some existing testing machines are also introduced in this chapter. Chapter 4 is the design of the new multiaxial fatigue testing machine. This idea comes from Dr. H. Pasic of Ohio University, who in 1994 designed a torsion fatigue testing
machine. This is a new concept about fatigue testing machines. This machine is small, convenient, and low in cost. It can be used to conduct tests in both lab and industrial environments. Chapter 5 contains some basic test results obtained with the new multiaxial fatigue testing machine and their comparison with existing results.
CHAPTER 2
FATIGUE OF MATERIALS

Fatigue failure is a multi-stage process. It begins with the initiation of cracks and the cracks propagate with continued cyclic loading, finally leading to a rupture. The demarcation between the above stages is not well-defined yet. Interest in the study of fatigue failure began with the machine components subjected to repeated loading. Today, this subject is subdivided into several fields: fatigue crack propagation, low cycle fatigue (LCF), high cycle fatigue (HCF), fatigue in notched member, etc. In this chapter, the fatigue behavior is presented from a macroscopic viewpoint [4].

2.1 S-N Curve And Fatigue Limit

Many engineering components must withstand numerous load reversals during their service life. Examples of these types of loading include alternating stresses associated with a rotating shaft and the load fluctuations affecting the wings during flight of airplanes. Depending on a number of factors, these load excursions may be introduced either between fixed stress or strain limits. Hence, the fatigue process in a given situation may be governed by the stress-controlled or strain-controlled condition. Engineering fatigue data are represented in the S-N curves, where the stresses (S) are plotted against the number of cycles to failure (N). However, some S-N curves use the strain instead of the stress.
There are three typical loading types: bending, torsion, and tension (compression). Each type of loading can cause fatigue failure. The service life is closely related to the magnitude of applying load. As the load decreases, the service life will increase. It is obvious that lower forces will give components longer service lives.

There are two basic shapes of S-N diagrams. First, when the load is decreased below a certain value, the material will not break regardless of how many cycles are experienced. There is an evident "knee" in the S-N curve. Usually, steel alloys and some ferrous metals have this tendency; i.e., they have the so-called fatigue limit. However, there exist some other materials, like aluminum alloys, which do not have the minimum limit of permanent service life. There is not a "knee" in this type of S-N curve. The fatigue limit (or fatigue strength) is the minimum stress of the S-N curve. For those materials which do not have the minimum fatigue strength, the stress value at loading cycle 10^7 (or 10^8) is taken as their fatigue limit. Figure 2-1 represents the constant load amplitude S-N curve for aluminum alloy 7075-T6 [5].

![Figure 2.1 S-N curve of aluminum alloy 7075-T6 [5].](image)
2.2 High-Cycle Fatigue (HCF) And Low-Cycle Fatigue (LCF)

The fatigue failure takes place at a different number of cycles (N) due to different stress (S) levels. The basic difference between high-cycle and low-cycle fatigue is the service life of the component. If N is less than $10^4$, it is called low-cycle fatigue. If N is larger than $10^5$, it is considered high-cycle fatigue.

High-cycle fatigue happens at a low stress level, where the stress level is far below the yielding strength and the strains are essentially elastic. Low-cycle fatigue is caused by high fluctuating stresses and strains. Due to these high stress levels, plastic deformations take place during the service life. Typical examples structures operating at of low-cycle fatigue conditions include nuclear reactor vessels, steam turbines, and some power machinery. Generally, low-cycle fatigue research is based on strain analysis, while high cycle fatigue is best described by analyzing the stress field [4].

2.3 Cyclic Stress-Controlled Fatigue

The very first systematic study of metal fatigue using the cyclic stress-controlled method was done by Wholer on rotating axles [13]. Since Wholer’s 1871 work, the fatigue data have been presented in the S-N curves. Over the years, laboratory tests have been conducted in bending (rotating or reversed flexure), torsion, pulsating tension, or tension-compression axial loading. Figure 2.2 shows one of the typical cyclic stress patterns and standard definitions.
where

\[\Delta\sigma = \text{stress range}\]
\[\sigma_a = \text{stress amplitude}\]
\[\sigma_m = \text{mean stress}\]
\[\sigma_{\text{max}} = \text{maximum stress}\]
\[\sigma_{\text{min}} = \text{minimum stress}\]
\[R = \frac{\text{minimum stress ratio}}{\text{max stress ratio}}\]

The stress data plotted in the S-N curve can be presented by the amplitude \((\sigma_a)\), maximum\((\sigma_{\text{max}})\) or minimum stress\((\sigma_{\text{min}})\), along with the mean stress\((\sigma_m)\) or stress ratio \((R)\). Most S-N curves are experimentally obtained at \(R = -1\).

The advantage of stress-controlled analysis is it is easier to compute the stress. For high cycle fatigue with a relatively large number of cycles, the material is subjected to small stress and the strain is elastic. In this region, stress-controlled fatigue provides a precise and simple approach to analyze the fatigue failure.
2.4 Cyclic Strain-Controlled Fatigue

Strain-controlled fatigue analysis predicts the life expectancy of the component based on the magnitude of fluctuating strain. The analysis has its beginning in the early 1950s with the observations of metals subjected to low-cycle fatigue. For most materials, large stress will not only cause elastic deformation but also plastic deformation. Furthermore, some inelastic deformation may also occur with high temperatures and relatively high stress density accompanied by creep. In such conditions, strain-controlled analysis can provide a better approach to analyze the fatigue behavior. Figure 2.3 is the comparison of ideally elastic material and material undergoing both elastic and plastic deformation during cyclic loading.

Figure 2.3 Hysteresis loops for cyclic loading in (a) ideally elastic material and (b) material undergoing elastic and plastic deformation [5].
2.4.1 Cycle-Dependent Response

Cycle-dependent material responses under stress and strain control are shown in Figure 2.4, which reflects changes in the shape of the hysteresis loop. After cycling the material for a certain duration (often less than 100 cycles), the hysteresis loops generally stabilize and the material achieves an equilibrium condition. This cyclically stabilized stress-strain response may be quite different from the initial monotonic response in Figure 2.5. It is important to understand the difference between the monotonic response and the cyclically stabilized response, because the fatigue failure is caused by the cyclic loading.

![Figure 2.4 Cycle-dependent response](image)

*Figure 2.4 Cycle-dependent response [5].*
Figure 2.5 Monotonic and cyclic stress-strain curves for SAE 4340 steel [5].

The varieties of cyclically stabilized responses are also quite different for different materials. Some materials will become stronger while others will become softer. The cyclic-hardening or cyclic-softening behavior depends on the ratio of ultimate strength to yield strength. Manson observed that when the ratio \( \sigma_{\text{ult}} / \sigma_{\text{ys}} \) is > 1.4, the material will harden and if the ratio is < 1.2, the material will soften. Therefore, the initially hard and strong materials will gradually soften under cyclic strain and initially soft materials will gradually harden. The reason for this property appears to be related to dislocation distribution. For initially soft materials, the dislocation density is relatively low. As a result of cyclic plastic strain, the dislocation density increases rapidly and
makes the material stronger. For initially strong and hard materials with relatively high dislocation density, cyclic plastic strain will rearrange the dislocation distribution and offer less resistance to further deformation.

2.4.2 Strain Life Curves

To consider the service life of a component subjected to strain-controlled loading, it is convenient to begin the analysis by considering the elastic and plastic strain components separately. From the Holloman relation[5] given by \( \sigma = K\varepsilon^n \), it is possible to mathematically describe the material stress-strain response in either a monotonic or cyclically stabilized state. The total strain range \( (\Delta \varepsilon) \) consists of both elastic strain \( (\Delta \varepsilon_e) \) and plastic strain \( (\Delta \varepsilon_p) \) components [5]:

\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} = \frac{\Delta \sigma}{2} + \left( \frac{\Delta \sigma}{2K'} \right)^{\frac{1}{n}}
\]

where \( \Delta \sigma \) is the stress range.

The plastic component of strain is well described by the Coffin-Mason relation [5]:

\[
\frac{\Delta \varepsilon_p}{2} = \varepsilon_f \left( 2N_f \right)^c
\]

where

\( \Delta \varepsilon_p = \) plastic strain amplitude

\( \varepsilon_f = \) fatigue ductility coefficient, defined by the strain intercept at one load (2Nf=1)

2Nf=total strain reversals to failure
c = fatigue ductility exponent (-0.5 to -0.7)

The elastic component is often described by Basquin's equation [5]:

\[ \frac{\Delta \varepsilon_e E}{2} = \sigma_a = \sigma_f (2N_f)^b \]  \hspace{1cm} (2.3)

where

\( \Delta \varepsilon_e \) = elastic strain amplitude
\( E \) = modulus of elasticity
\( \sigma_a \) = stress amplitude
\( b \) = fatigue strength exponent

The fatigue-life curve tends toward the plastic curve at large total strain amplitude (low-cycle fatigue), and toward the elastic curve at small total strain amplitude (high-cycle fatigue).

Figure 2.6 Superposition of elastic and plastic strain life curves [5].
2.5 Fatigue Failure Mechanism

The fatigue process begins with the accumulation of damage at a localized region or regions due to cyclic loads, which eventually leads to formation of cracks and their subsequent propagation. When one of the cracks has grown to such an extent that the remaining net-section is insufficient to support the applied load, a sudden fracture (rapture) takes place. Therefore, when viewing a fatigue fractured surface, three features may be distinguished:

1. an initiation site (or sites);
2. a crack growth surface area with distinct features;
3. a final fractured surface.

A macroscopic examination of fatigue failures reveals distinct fracture surface markings. For one thing, the fracture surface is generally flat, indicating the absence of an appreciable amount of gross plastic deformation during service life. In many cases, the fracture surface contains lines referred to as “clam shell markings” or “striations”. From these markings, one can make conclusions about the fatigue failure mechanism. As an illustration, Figure 2.7 shows the fractured surface of a rotating steel shaft. It is to be emphasized that these markings reflect “the period of growth” and are not representative of “individual loading history”.
2.5.1 Initiation Of Fatigue Cracks

There is general agreement that persistent slip bands (PSB) are major nucleation sites for cracks of many smooth metal materials. The cracks tend to form at the interface of PSB and matrix, and generally initiate after a localized saturation of the dislocation effects. Many investigations have shown that the cyclic strains produce sharp peaks (extrusions) and troughs (intrusions) resulting from irreversible slips at the initiation sites (see Figure 2.8). It has also been noticed that fatigue cracks are usually initiated on the free surface.

Figure 2.7 Fatigue fracture markings [5].
2.5.2 Crack Propagation

Once cracks have initiated, they may grow as a result of further cyclic deformation. There are two major stages called “Stage I” and “Stage II” for fatigue crack propagation. In uniaxial loading (tension and compression), the maximum shear stress is on the plane which is about 45° to the applied stress direction. Thus, the cracks of Stage I may propagate along the active slip band of approximate maximum shear stress in the
suitably oriented crystal (about a 45° angle). In this stage, the crack growth is in sliding or in-plane shear mode (Mode II, Figure 2.9).

With further cyclic deformation, the dominant crack gradually emerges and changes direction to become perpendicular to the loading axis. The transition from the active slip direction (Stage I) to the non-crystallographic plane perpendicular to the loading axis (Stage II), occurs over a few grains. The crack length of the Stage I depends on the material and loading amplitude, and is usually of the order of three to four grain sizes. It increases with an increase of grain size of the material and is inversely proportional to the loading amplitude. The crack growth in the Stage II is often called Mode I—opening, or tensile mode.

![Figure 2.9 Crack growth in stage I and stage II [4].](image)

2.5.3 Final Failure (Rupture)

After the crack has propagated to a certain length, even if the applied cyclic stress is much below the ultimate stress, due to stress concentration the real stress at the crack
tip is high enough to damage the component. This stage is called the final failure stage and rupture will occur and destroy the component rapidly. This stage takes very little time and usually happens suddenly. Thus, the whole fatigue failure procedure is completed.

2.6 Factors Affecting Fatigue Life

2.6.1 The Effect Of Mean Stress

Mean stress can represent an important test variable for fatigue. There is a general trend in fatigue: tension will increase fatigue failure, while compression will decrease it. From experimental data, one can find that at the same stress amplitude (σₐ) level, the component life (N) will increase if the mean stress (σₘ) is decreased. Some empirical relations have been developed to account for the mean stress effect. For example, Goodman proposed the following relationship[13]

\[
\sigma_0 = \sigma_{fat} \left(1 - \frac{\sigma_m}{\sigma_{is}}\right)
\]

(2.4)

where

- \(\sigma_s\) = fatigue strength when \(\sigma_m \neq 0\)
- \(\sigma_m\) = mean stress
- \(\sigma_{fat}\) = fatigue strength when \(\sigma_m = 0\)
2.6.2 Notch Effect

The shape of the component is an important factor in fatigue. It has long been known that stress concentration plays a decisive role in fracture. Due to different shapes, the same materials have different stress concentration factors. Neuber’s formula to account for the notch effect reads [3]

\[
K_f = 1 + q(K_r - 1)
\]

\[
q = \frac{1}{1 + \frac{a}{r}}
\]

where

- \(K_f\) = fatigue limit of smooth specimen / fatigue limit of notched specimen
- \(K_r\) = stress concentration factor
- \(r\) = notch radius
- \(a\) = material property constant with dimension in length
- \(q\) = notch sensitivity

2.6.3 Surface Roughness

Processing techniques, such as forging and rolling, may change materials’ microstructure and, generally, will cause some defects (flaws) inside the materials and on
the surface. The surface roughness can cause high stress concentration, and therefore provides a good location for crack site formation. Figure 2.10 presents the surface roughness factor as a function of the surface finishing process, such as machining, grinding, rolling, and forging. As is seen, the surface roughness factor may vary from 0.3 to 0.95.

![Graph showing surface roughness factor as a function of tensile strength for different machining processes.]

**Figure 2.10 Surface roughness under different machining conditions [13].**

### 2.6.4 Environment Factors

A corrosive environment causes degradation of material—the most visible effect being pitting or surface roughness. The notch-like region increases stress and is generally the site of nucleation. The most important locations are those where fatigue slip activity is taking place. It has been noted that cyclic loading enhances the corrosion effect by continually breaking the protective oxide layer and exposing a fresh surface.
Another important environmental factor affecting the fatigue is temperature. Some investigations have indicated that an increase in temperature reduces fatigue resistance of most metals. The creep of most metals can be negligible at room temperature. However, at high temperatures, the time-dependent inelastic deformation becomes very important in fatigue. Increased temperature generally increases the fatigue crack propagation rate [4].

2.6.5 Loading Mode Factor

There are three typical loading modes: bending, torsion, and tension. Each mode has its own fatigue characteristic. Generally, for the same stress level, bending has a higher limit than tension. This is because the stress distribution in tension is uniform for all cross-section areas, while in bending the stresses vary from zero in the center to the maximum value on the surface. The loading factor can be estimated by (K) [13]:

- Bending \( K = 1.0 \)
- Tension \( K = 0.9 \)
- Torsion \( K = 0.577 \)

2.7 Multiaxial Fatigue

Multiaxial stress states are encountered in a majority of mechanical components, e.g., shafts, beams, pressure vessels. Indeed, the early investigations into fatigue were on
the shafts of rotating machines where the bending and torsion couples were applied to simulate service loads. The motivation for multiaxial fatigue research has been to produce design guidelines for developing the new multiaxial fatigue testing machine.

The analysis of multiaxial fatigue can be done in two stages: first, the "equivalent" stresses and strains are found and, second, based on the equivalent stresses and strains, the fatigue life is computed utilizing the one-dimensional theory [13].

There are several known theories for computing the equivalent stress. One of the best known theories is the von Mises. Von Mises theory assumes that hydrostatic pressure or tension does not affect the critical behavior of a stressed body. The equivalent stress tensor \( \sigma_e \) is

\[
\sigma_e = \begin{bmatrix}
\sigma_e & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  

(2.7)

In accordance with the assumption that the hydrostatic stress cannot cause plastic deformation, the stress deviator tensor should be considered instead. It follows that

\[
\begin{bmatrix}
\frac{\sigma_e}{3} & 0 & 0 \\
0 & -\frac{\sigma_e}{3} & 0 \\
0 & 0 & -\frac{\sigma_e}{3}
\end{bmatrix}
= \begin{bmatrix}
\frac{2\sigma_1 - \sigma_2 - \sigma_3}{3} & 0 & 0 \\
0 & \frac{2\sigma_2 - \sigma_1 - \sigma_3}{3} & 0 \\
0 & 0 & \frac{2\sigma_3 - \sigma_1 - \sigma_2}{3}
\end{bmatrix}
\]  

(2.8)
According to von Mises' criterion, the equivalent stress is

\[
\sigma_e = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}
\]

\[
= \frac{1}{\sqrt{2}}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]
\]

(2.9)

where \(\sigma_1, \sigma_2, \sigma_3\) are the principal stresses and \(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}\) are normal and shear stresses at a point, as defined in the Cartesian coordinate system.

The equivalent strain is found by a similar procedure. One can easily find the equivalent strain from the equivalent stress

\[
\varepsilon_e = \frac{1}{(1+\nu)\sqrt{2}}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}
\]

(2.10)

where \(\nu\) is Poisson’s ratio.
CHAPTER 3
FATIGUE TEST

Fatigue failure has been generally subdivided into high-cycle and low-cycle fatigue, depending on the applied stress level. As for the state of damage and its nature, two distinguished stages are fatigue initiation and propagation. Other common divisions in fatigue are fatigue damage in smooth and notched components. Each of these fields requires a specific experimental approach and corresponding use of the testing equipment [4].

There are three typical loading situations for the mechanical parts: bending, tension (compression), and torsion. Most materials are tested in one of these modes. However, most machine parts experience combined loadings during their service life. The multiaxial fatigue testing, therefore, needs to be applied and the corresponding testing equipment must be developed.

3.1 Bending Test

When the specimen or component is subject to bending, it will be subjected to internal bending moment and the shear force along the cross-sectional area. The longitudinal stress distribution varies linearly from zero at the neutral axis to the maximum value at the farthest edge (see Figure 3.1). The magnitude of the normal stress is
\[ \sigma = -\frac{My}{I} \]  \hspace{1cm} (3.1)

where

\( y \) = the perpendicular distance from the neutral axis

\( I \) = moment of inertia of the cross-sectional area computed w/r to the neutral axis

\( M \) = the resultant bending moment

Figure 3.1 Stress distribution in bending [4].

Generally, bending testing machines are subdivided into rotary-bending and plate-bending testing machines. Figure 3.2 shows the rotary-bending testing machine. The weight produces the constant bending moment and the specimen experiences either tension or compression in each half revolution. The rotary-bending testing machine may be used only for the specimens with the circular cross-sectional area. For specimens having a non-circular cross-section, one needs to use the plate-bending test. The plate-bending test generally uses either the cantilever beam (one end is fixed and the other is free and subjected to the perpendicular force), the three-point loaded beam (the beam
rests on two supports and is centrally loaded), or the four-point loaded beam (the beam rests on two supports and is acted upon by two loadings equally spaced from each end).

![Figure 3.2 Rotary-bending fatigue testing machine [13].](image)

3.2 Axial Loading Test

3.2.1 Tensile Test

The engineering tension test is the most fundamental and frequently used testing method in both static and fatigue testing conditions. It provides the basic design information on the strength of materials. The static testing results are usually represented by a stress-strain curve. The engineering stress-strain curve is constructed from load-elongation measurements (see Figure 3.3). The engineering stress used in this curve is the average longitudinal stress in the specimen. It is obtained by dividing the load by the original cross-sectional area

\[ \sigma = \frac{P}{A_o} \] (3.2)
The strain used in this curve is the average linear engineering strain, which is obtained by dividing the elongation of the gage length of the specimen by its original length

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

(3.3)

There are two significant points in the engineering stress-strain curve: yield strength and tensile strength (ultimate strength). The yield point is the onset of the plastic deformation. Sometimes it is also called elastic limit (or proportional limit). Before this point, the stress-strain curve is a straight line and the slope represents the Young’s modules. Because of the practical difficulties in measuring the real yield strength, it is widely accepted to use 0.2% offset of the strain to find the yield strength [3].

Figure 3.3 Engineering stress-strain curve. A: yielding point. C: ultimate point [3].
However, engineering stress-strain curve does not give a true indication of plastic deformation. The true stress-strain curve, which is based on the real cross-section area and length, provides an accurate indication of material deformation during the whole test (see Figure 3.4).

![Figure 3.4 Comparison of engineering and true stress-strain curve [3].](image)

Because of the strain-hardening of the metal all the way up to fracture, it is found that the stress continues to increase in the true stress-strain curve. The true stress is the load divided by the true, current, cross-sectional area

\[
\sigma_T = \frac{P}{A_i} \tag{3.4}
\]

The true strain is defined as
\[ \varepsilon_r = \ln \frac{L_t}{L_0} \]  

(3.5)

In the plastic region, the relationship between the stress and the strain can be expressed by

\[ \sigma = K\varepsilon^n \]  

(3.6)

where \( K \) is the stress at \( \varepsilon = 1.0 \) and \( n \) is the strain-hardening coefficient. If one plots the true stress-strain curve in log-log scale, the strain-hardening coefficient \( (n) \) is found to be identical with the slope of this curve.

In a tensile test, the maximum load defines the tensile strength. Generally, necking of ductile materials begins at the maximum load. An ideal plastic material with no strain hardening will begin to neck as soon as yielding takes place. However, real metal undergoes strain hardening which tends to increase the capacity of the material as deformation increases. The plastic deformation is a constant volume process, which means there is no change in total volume during the plastic deformation. By using the constancy-of-volume relationship and that at the maximum load \( dP = 0 \), one can find the maximum load point in true stress-true strain \( (\varepsilon_t) \) and true stress-engineering strain \( (\varepsilon_E) \) plot.
\[ P = A \sigma \]
\[ dP = Ad \sigma + \sigma dA = 0 \]
\[ \frac{dA}{A} = -\frac{d\sigma}{\sigma} \]
\[ V = A L \]
\[ dV = AdL + LdA = 0 \]
\[ \frac{dA}{A} = -\frac{dL}{L} \]
\[ \frac{dL}{L} = \frac{d\sigma}{\sigma} = d\varepsilon_t = \frac{d\varepsilon_E}{1 + \varepsilon_E} \]

Therefore,

\[ \frac{d\sigma}{d\varepsilon_T} = \sigma \] \hspace{1cm} (3.7)
\[ \frac{d\sigma}{d\varepsilon_E} = \frac{\sigma}{1 + \varepsilon_E} \] \hspace{1cm} (3.8)

Equation 3.7 states that the necking occurs at the stress equals the slope of the true stress-strain curve. If the flow curve is given by Equation 3.6, it is possible to determine the strain at which the necking occurs readily.

\[ \sigma = K\varepsilon^n \]
\[ \frac{d\sigma}{d\varepsilon} = \sigma = nK\varepsilon^{n-1} \] \hspace{1cm} (3.9)

Equation 3.8 is based on the true stress-engineering strain curve. At this point, it is where the tensile strength \( \sigma_u \) is defined. The relation between the true stress \( \sigma_T \) and engineering stress \( \sigma_E \) is
At the beginning of this section, it was pointed out that a tension test is used for basic testing of materials. However, there are some disadvantages to the axial-loading test. First, a large force is needed to achieve the necessary stress, which increases the size of the testing machine. In addition, it is difficult to line up the specimen in a perfectly straight line, which can cause the buckling instability associated with compression when necking occurs[3].

3.2.2 Axial Fatigue Test

Most fatigue data are obtained at constant amplitude loading conditions. Therefore the results are suitable for direct application to design only when the service conditions exactly parallel the test conditions.

The axial fatigue testing system should be able to provide the repeated cyclic loading to the specimen. The loading can be performed by mechanical (eccentric crank, power screws, or rotating mass), electromechanical, magnetically driven, or hydraulic equipment. Figure 3.5 shows the schematics of a testing machine operated by hydraulic force. The specimen (a) is loaded by the hydraulic actuator (b). The actuator is controlled by the servo-valve (c). The range of testing frequency can be up to 10,000 cpm (170 Hz)
However, the testing frequency should be chosen carefully according to the dimensions of the specimen and the applied loading. To avoid resonance, the natural frequency of the specimen should be at least two times the testing frequency.

As discussed above, for axial-loading tests, the eccentrically applied loading can cause the specimen buckling. Because in fatigue testing the specimen is subjected to dynamic loading, it seems impossible to avoid the bending that occurs throughout the test. The bending is defined as

\[
\text{Percent bending} = \left( \frac{\varepsilon_{\text{max}} - \varepsilon_{\text{avg}}}{\varepsilon_{\text{avg}}} \right) \times 100
\]  

(3.11)

and its maximum value should be less than 5% [1].

Figure 3.5 Hydraulic axial-loading fatigue testing system [4].
3.3 Torsion Fatigue Test

Thus far the torsion test has not been widely used nor has it been standardized to the same extent as the tension test. However, it is useful in many practical engineering applications, such as shafts, axles, and twist drills. Torsion test is usually used to determine such properties as the shear modulus, the torsion yield strength, and the modulus of rupture in shear.

Figure 3.6 shows the torsion fatigue test system first developed in 1994 at Ohio University. It contains a rotary servomotor providing the required torque, twist angle, and speed and a servo controller with high precision. One end of the specimen is mounted to the motor and the other end is fixed to the clamping head.

Figure 3.6 Torsion fatigue testing system.
When a cylindrical specimen is subjected to a torsion moment at one end, this moment is resisted by shear stresses over the cross-section of the specimen. In the elastic range, the shear stress varies linearly from zero at the center to the maximum value at the surface (see Figure 3.7). Its magnitude is

\[ \tau = \frac{Mr}{J} \]  

(3.10)

where

\( \tau \) = shear stress

\( M \) = moment of the applied force

\( R \) = radial distance measured from center

\( J \) = polar moment of inertia

Figure 3.7 Shear stress distribution in torsion member [6].
In the torsion test, the data are generally plotted in a moment (M) versus twist angle (θ) diagram instead of stress-strain curve. If the specimen’s gage length is L, the shear strain is

$$\gamma = \frac{r}{L} \theta$$  \hspace{1cm} (3.11)

Within the elastic region, the shear stress can be considered proportional to the shear strain, i.e., where G is the modulus of elasticity in shear (modulus of rigidity)

$$\tau = G\gamma$$  \hspace{1cm} (3.12)

Substituting Equation 3.10 and 3.11 into Equation 3.12 gives an expression for the shear modulus in terms of the geometry of the specimen.

$$G = \frac{ML}{J\theta}$$  \hspace{1cm} (3.13)

### 3.4 Axial Loading Test Versus Torsion Test

The objective of the new multi-axial fatigue testing machine is to combine both tension and torsion tests. The tension test provides the fundamental information of material strength. However, the torsion test can give a more precise measurement of the material’s plastic deformation than the tension test. A large value of strain can be obtained in torsion without complications, such as necking in tension and barreling or buckling in compression. Moreover, in the torsion test, tests can be made easier at high
strain rates and large torque and twist angles can be produced without particular problems.

The comparison of tension and torsion tests in terms of the state of stress and strains are [3]

<table>
<thead>
<tr>
<th>Tension</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_1 = \sigma_{\text{max}}, \sigma_2 = \sigma_3 = 0 )</td>
<td>( \sigma_1 = -\sigma_3, \sigma_2 = 0 )</td>
</tr>
<tr>
<td>( \tau_{\text{max}} = \frac{\sigma_1}{2} = \frac{\sigma_{\text{max}}}{2} )</td>
<td>( \tau_{\text{max}} = \frac{2\sigma_1}{2} = \sigma_{\text{max}} )</td>
</tr>
<tr>
<td>( \varepsilon_{\text{max}} = \varepsilon_1, \varepsilon_2 = \varepsilon_3 = -\frac{\varepsilon_1}{2} )</td>
<td>( \varepsilon_{\text{max}} = \varepsilon_1 = -\varepsilon_3, \varepsilon_2 = 0 )</td>
</tr>
<tr>
<td>( \gamma_{\text{max}} = \frac{3\varepsilon_1}{2} )</td>
<td>( \gamma_{\text{max}} = \varepsilon_1 - \varepsilon_3 = 2\varepsilon_1 )</td>
</tr>
<tr>
<td>( \sigma_e = \sigma_1 )</td>
<td>( \sigma_e = \sqrt{3}\sigma_1 )</td>
</tr>
</tbody>
</table>

\[
\sigma_e = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}} \quad (3.14)
\]

By using von Miese’s equivalent stress criteria, one can calculate the equivalent stresses in both tension and torsion and then compare them with the above values. It is seen that the torsion test produces \( \sqrt{3} \) larger equivalent stress than the tension test. It is, therefore, easier to produce large stresses and deformation in torsion than in tension. Also, the comparison shows in Figure 3.8 that \( \tau_{\text{max}} \) will be twice as great in torsion as in tension for a given value of \( \sigma_{\text{max}} \). Since the plastic deformation occurs upon reaching a critical value of shear stress, the opportunity of ductile behavior is greater in torsion than
in tension. Therefore the torsion test is more conveniently used in the low cycle fatigue regime which is characterized by large stresses and deformations.

Figure 3.8 also shows that in the torsion test the critical shear stress for plastic flow is reached before the critical normal stress for fracture, while in the tension test the critical normal stress is reached before the shear stress for the plastic flow. This phenomenon exists in most brittle materials—even in ductile metal in the tension test. Figure 3.8 also shows that the amount of plastic deformation is greater in torsion than in tension. This explains why more and more researchers in plasticity are using the torsion test instead of the tension test.

The tensile stress-strain curve can be derived from the curve for torsion when the stress-strain curve is plotted in terms of significant stress and strain or the octahedral shear stress and strain. Figure 3.9 shows a true stress-strain curve for low-carbon steel in both tension and torsion. When both curves are plotted in terms of significant stress and
strain, the two curves superimpose within fairly close limits. However, because the data derived from the torsion test in the large plastic deformation region were obtained without presence of necking, it should provide better characterization of the material than the tension test.

Figure 3.9 Tension and torsion true stress-strain curves [3].

The failure of the brittle material in axial loading is caused by large normal stress, and due to the lack of plastic deformation, there is no evident necking around the failure area. The crack surface is perpendicular to the longitude direction. Conversely, for ductile materials, the failure is caused by shear stress and the crack surface is along the maximum shear direction, which is 45° to the longitude and transverse directions.

Torsion failures are different from tensile failures in that there is little localized reduction in area or elongation. A ductile material fails by shear along one of the planes of the maximum shear stress. Generally, at rupture the plane is normal to the longitudinal axis. Therefore, a brittle material raptures along a plane perpendicular to the direction of
the maximum normal stress, which in torsion makes an angle of $45^\circ$ to the longitudinal and transverse direction (see Figure 3.10).

![Figure 3.10 Failure of brittle material in tension and torsion [6].](image)

When the specimen is subjected to a simultaneous axial force and torsion, as in Figure 3.11, the stress state is

$$
\sigma_x = \frac{P}{A}
$$

$$
\sigma_y = \sigma_z = 0
$$

$$
\tau_{xy} = -\frac{Tc}{J}
$$

$$
\tau_{yz} = \tau_{zx} = 0
$$

From Mohr’s cycle, the principle stresses and orientation are

$$
\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}
$$

(3.15)

$$
2\theta_p = \tan^{-1} \left( \frac{\tau_{xy}}{\sigma_x/2} \right)
$$

(3.16)
The maximum shear stress is:

\[ \tau_{\text{max}} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \]  

(3.17)

Its angle is 45° to the principle stress.

Figure 3.11 Stresses in tension and torsion.

3.5 Multiaxial Fatigue Test

In the early stages, the multiaxial fatigue analysis combined the results of several uniaxial tests run independently. The major disadvantage of these methods is the difficulty of interpretation of the results. Furthermore, the geometry of the specimen is an important factor affecting the fatigue life. With the development of servo-hydraulic test systems, it is now possible to perform fatigue tests under complex stress-strain conditions [4]. Although scientists and engineers have been researching multiaxial fatigue for several decades, it is difficult to translate and apply the research results to the engineering
design reality. It is generally agreed that the test results can be applied to design only in identical loading situations. With the lack of reliable data, the multiaxial fatigue test systems are, indeed, required for both laboratory research and engineering design.

Multiaxial fatigue test systems are not popular in the market, and only a few systems are available currently. The existing systems are exclusively servo-hydraulic and combine both tension and torsion loads. Figures 3.12 to 3.17 show some of these existing devices together with their basic properties.

1. **MTS 858 tabletop series (see Figure 3.12):**
   
   Application: tension and torsion fatigue test.
   
   Capable of dynamic loads: 10-25 kN / 100 Nm.
   
   Displacement: ± 50 mm / ±135°.

2. **MTS 809 Axial-Torsion series (see Figure 3.13):**
   
   Application: tension and torsion fatigue test.
   
   Capable of dynamic loads from 25 kN / 200 Nm up to 1000 kN / 10000 Nm.

3. **INSTRON 8874 series (see Figure 3.14):**
   
   Application: tension and torsion fatigue test.
   
   Capable of dynamic loads: 25 kN / 100 Nm.
Figure 3.12 MTS 858 series.
Figure 3.13 MTS 809 series.
Figure 3.14 INSTRON 8874 series.
Because of the lack of a commercial testing machine, some laboratories have built multiaxial fatigue test systems for research. The first such example is the fatigue test system for a notched shaft in combined bending and torsion (see Figure 3.15). This machine is used for testing shafts of ground vehicles. Two hydraulic actuators are controlled by two individual controllers, and each actuator has 44 kN maximum load and 150 mm stroke. The bending and torsion moment arms are 155 and 203 mm, respectively.

![Block-diagram of bending-torsion testing machine](image)

**Figure 3.15 Block-diagram of bending-torsion testing machine [10].**

The next example is a multiaxial fatigue testing machine for polymers (see Figure 3.16). The idea of this machine is to use axial loading and the inner and outer pressure difference of thin-walled tubes of plastic materials to get biaxial stress states. It also can
provide torsion moment. The basic loading capacity is ±30 kN (tension) and 1400 Nm (torsion), and the two ranges of pressure are 0 to 25 MN/m² and 0 to 125 MN/m².

![Figure 3.16 Loading systems of polymer fatigue testing machine [10].](image)

Yet another example of a specialized multiaxial fatigue testing machine is a machine designed for testing thin-walled tubular specimens. This machine was designed by the Department of Mechanical Engineering of the University of Sheffield [10]. This machine may be used in biaxial and triaxial stress testing. It is capable of straining a thin-walled tube in three modes: axial loading, torsion, and internal and external pressure (see Figure 3.17). The loading capacity is ±400 kN (tension) and 1000 Nm (torsion). The pressure range is 0 to 1700 bar.
Figure 3.17 Thin-walled tube testing machine [10].
CHAPTER 4

NEW MULTIAXIAL FATIGUE TESTING MACHINE

In Chapter 3, several existing multiaxial fatigue testing machines were reviewed. Most of the testing systems are hydraulic, and thus, capable of producing high stress. In fatigue tests (especially high cycle fatigue), the stresses and strains are usually not as high as in static tests. The fatigue ratio \( \frac{\sigma_{\text{Fat}}}{\sigma_{\text{Ult}}} \) for most steel alloys is between 0.35 and 0.6, and even lower for aluminum alloys. Fatigue tests are very expensive, because they involve testing many specimens at different stress levels to construct the S-N curve. In addition, it takes far more time to break a specimen than the static test does. Generally, the test frequency can be from negligibly small to as high as 170 Hz (10,000 cpm) [1].

The objective of this thesis work is to develop a new multiaxial fatigue testing machine by using the electric servomotor. The machine has to be small in size, fast, and precise. The advantages of using a servomotor are:

1. **Unlimited range of twist angle:**

   Hydraulic systems used in torsion have a limited twist angle. Generally the twist angle of hydraulic systems is less than \( \pm 180^\circ \). For most materials it is more than sufficient, but not for all materials. In 1985 Lawrence proposed using step motors to improve the twist angle problem [6].
2. Improved precision:

The testing machine should be able to provide precise testing results. The servomotor systems are controlled and already equipped by very precise and inexpensive encoders. The precision is typically $10^{-2}$ to $10^{-3}$ of a degree.

3. Low cost:

Being small in size, the machine is suitable for testing small size components and specimens only. It is, therefore, ideal for multiaxial fatigue testing of plane (unnotched) specimens in both low- and high-cycle fatigue regions.

4.1 Identification Of Need And Goal

The idea of this new testing machine is to provide a tension and torsion fatigue testing equipment that is capable of dynamic loading up to 1kN (in tension) and 25Nm (in torsion) with potential infinite rotational displacement and a maximum frequency to 50Hz. The design should prevent collision between linear and rotary motion to protect the motors. The loading capacity is good for most metal materials with a diameter of 3 to 5 mm and non-metal materials, see Table 4.1, of even larger diameter.

In 1994, Dr. H. Pasic of Ohio University proposed a new torsion fatigue testing machine described earlier, see Figure 3.6. It consisted of a servomotor, mounting plate, and grips. Based on this idea, another motor was added to provide linear motion to achieve multiaxial stress and strain states, figure 4.1. The main components of the new
testing machine are: (1) the structural components, (2) the servomotor systems, and (3) computer data control system (see Figure 4.2).

Table 4.1. Fatigue limit of selected engineering materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Strengh</th>
<th>Tensile Strength</th>
<th>Fatigue Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL 2024-T6</td>
<td>463</td>
<td>511</td>
<td>140</td>
</tr>
<tr>
<td>AL 6061-T6</td>
<td>276</td>
<td>300</td>
<td>97</td>
</tr>
<tr>
<td>AL 7075-T6</td>
<td>470</td>
<td>580</td>
<td>165</td>
</tr>
<tr>
<td>AISI 1015</td>
<td>275</td>
<td>455</td>
<td>240</td>
</tr>
<tr>
<td>AISI 4340</td>
<td>855</td>
<td>965</td>
<td>380</td>
</tr>
<tr>
<td>70Cu-30Zn Brass</td>
<td>435</td>
<td>542</td>
<td>145</td>
</tr>
<tr>
<td>HK 31A-T6</td>
<td>215</td>
<td>110</td>
<td>83</td>
</tr>
<tr>
<td>AZ 91A</td>
<td>235</td>
<td>160</td>
<td>96</td>
</tr>
<tr>
<td>ABS</td>
<td></td>
<td></td>
<td>46</td>
</tr>
<tr>
<td>Nylon 66</td>
<td></td>
<td></td>
<td>83</td>
</tr>
<tr>
<td>PVC</td>
<td></td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

Figure 4.1 Block diagram of “multiaxial fatigue testing machine”.
4.2 Motor Description

The major reasons for choosing the electric servomotor are the reliability, low cost, and its precise control system. Its operating frequency is also very high--up to 50Hz.

Another comparison is the linear motor and rotary motor. A rotary motor can be used to provide linear motion as well as the linear motor. The cost is a major consideration in choosing the rotary motor. A linear motor, which provides 300 N force, is over $8,000 and a rotary motor--under $3,000--can achieve 25Nm torque.
consistently. It means that if one uses a 25 mm moment arm, 1,000N (1kN) force will be produced. Since the goal of the new multiaxial fatigue testing machine is to provide 1kN in axial loading fatigue test and 25Nm in torsion fatigue test, the motor used here is APEX 640 made by Parker Hannifin Corporation with APEX 40 drive and 6250 2-axis controller. Table 4.1 shows the characteristics of the motor.

**Table 4.2 APEX 640 motor specification.**

<table>
<thead>
<tr>
<th>Continuous Torque</th>
<th>Peak Torque</th>
<th>Rated Power</th>
<th>Rated Speed</th>
<th>Rated Current</th>
<th>Peak Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Nm)</td>
<td>(Nm)</td>
<td>(k Watts)</td>
<td>(rpm)</td>
<td>(A)rms</td>
<td>(A)rms</td>
</tr>
<tr>
<td>APEX 640</td>
<td>25</td>
<td>55</td>
<td>4.7</td>
<td>14</td>
<td>45</td>
</tr>
</tbody>
</table>

**4.3 Structural Components**

Figure 4.3 represents the basic design of the new multiaxial fatigue testing machine. The main parts of this machine are linear motion assembly and rotation assembly. The control system has been described earlier in Figures 4.1 and 4.2.
Figure 4.3 Drawing of the “multiaxial fatigue testing machine”.
4.3.1 Linear Motion Assembly

The major issues for linear motion are (1) how to transmit motor torque into a proper linear force and (2) how to avoid collision between rotation and translation. Therefore, the mechanism of the new machine must be able to convert the torque into linear force and avoid the interference between rotation and translation.

**Gear and rack:**

There are several possible solutions to convert servomotor rotation into translation. Rack and gear (pinion) was chosen here because: (1) it is standardized, (2) it has a low cost but high quality, and (3) it has a flexible displacement control. Gear and rack had been well developed for a long history and are highly standardized. They are used to transmit torque and angular velocity in a wide variety of applications. Most testing machines have a potential problem of limited displacement. Generally, the fatigue testing machines operate at relatively small displacements. However, it is hard to predict in advance how much displacement is needed for testing a new material. In the rack and gear machine, the length of the rack can be increased to prolong the displacement as much as necessary.

Because the maximum force needed for this machine was estimated to be about 1KN, the gear pitch diameter is about 50 mm. From AGMA standard, the Browning YSS 1018 spur gear (pitch diameter = 1.8 in) and 6YRS 10x1.25 rack were chosen. The maximum dynamic linear force is (see Figure 4.4 and Table 4.2)
The maximum static force will be (see Figure 4.4 and Table 4.2)

\[ F = \frac{M}{r} = \frac{25}{0.02286} = 1093.6 N = 250 lb \]  \hspace{1cm} (4.1)

\[ F_t = \frac{M}{r} = \frac{55}{0.02286} = 2406 N = 550 lb \]  \hspace{1cm} (4.2)

\[ F_r = F_t \tan \theta = 2406 \times \tan 20^\circ = 875 N = 200 lb \]  \hspace{1cm} (4.3)

Figure 4.4 Forces on gear and rack.

Angle plate:

The purpose of the angle plate, see Figure 4.3, is to hold the grip without slip by a counter bore. The angle surface is ground to ensure the rack is on the horizontal
line. The angle plate is attached to the linear slider with nine 1/4 screws and two 3/8 taper pins.

**Linear slider:**

The purpose of the linear slider, Figure 4.3 and 4.5, is to prevent rotation interference between torsion and tension tests and to provide smooth linear motion. The force in the slider for linear motion is identical with gear radial force, which is 875N (200lb) maximum.

During the torsion test, the slider is subjected to the maximum torque of 55Nm which is transmitted to the two bearings, Figure 4.5. The force in each bearing and rod is, therefore,

\[
F = \frac{M}{d} = \frac{55}{0.1016} = 541N = 124lb
\]

\[(4.4)\]

[Diagram of linear slider]
Alignment plate:

To ensure the rack and slider run in a straight line, the alignment plate, Figure 4.3, has a key in the bottom which makes the linear motion assembly parallel to the line of action. The alignment plate is adjustable with 10mm long slots. The linear slider is attached to the alignment plate with four 5/16 screws and two 3/8 taper pins providing the assembly precision.

The motion of linear motion assembly, Figure 4.2, is accomplished by the gear and rack, which transmit the motor torque into linear force. The rack is attached to the linear slider and moves back and forth.

4.3.2 Rotation Assembly

The rotation assembly consists of: (1) the motor (APEX 640), (2) two gears (YSS 1018), (3) two bearings (pillow block), and (4) the shaft, see Figure 4.3. When the torsion test is conducted, the motor transmits torque or angular displacement by the gear set. The second gear is attached to the shaft with the grip in the other end.

Bearings:

The purpose of the bearing is to resist the axial force during the tension test and protect the motor. There are several types of bearings that meet the need--the pillow block, the flange-mount bearing, etc. In order to avoid further damage to the motor due
to bearing failure, two pillow blocks are used. The part number of the pillow blocks is Browning SPB 1000E 1&3/16”.

Because there is no force along the axial direction (Z-axes) and the moment is rF,
y, there are four unknowns (R1x, R1y, R2x, and R2y) and four equilibrium equations, Figure 4.4:

\[ \Sigma F_x = F_x + R1_x + R2_x = 0 \]
\[ \Sigma F_y = F_y + R1_y + R2_y = 0 \]
\[ \Sigma M_{ox} = R1_y d_1 + R2_y (d_1 + d_2) = 0 \]
\[ \Sigma M_{oy} = R1_x d_1 + R2_x (d_1 + d_2) = 0 \] (4.5)

Figure 4.6 Forces distribution in bearings.
Using matrix rotation

\[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
d_1 & d_1 + d_2 & 0 & 0 \\
0 & 0 & d_1 & d_1 + d_2
\end{bmatrix}
\begin{bmatrix}
R1_x \\
R2_x \\
R1_y \\
R2_y
\end{bmatrix}
= \begin{bmatrix}
-F_x \\
-F_y \\
0 \\
0
\end{bmatrix}
\]  

(4.6)

For \(d_1 = 2.5\) in (63.5 mm), \(d_2 = 5\) in (127 mm), \(F_x = 875\) N (radial force), and \(F_y = 2406\) N (tangent force), from above equation one gets

\[R1_x = -1750\] N,

\[R1_y = -4092\] N,

\[R2_x = 875\] N,

\[R2_y = 2046\] N.

The resultant forces in each bearing are 4231 (950 lb) N and 2225 N (500 lb).

Shaft:

The shaft design is perhaps the most important part of the whole mechanism. There are six potential degrees of freedom (D.O.F) for one shaft with respect to a second shaft and four possible misalignment types [11]. One end of the shaft is attached to the gear and the other end to the grip, Figure 4.3. The gear used here is YSS 1018 with bore diameter 0.875 inch and 1.875 inch in length. The grip and the shaft are connected by the mounting plate, Figure 4.7.
The maximum torque is 55Nm (static) and 25Nm (dynamic loading) for torsion test, and 2406 N (static) and 1093 N (dynamic) axial force for tension test, Table 4.2.

During the tension test, the maximum axial stress of the shaft at the gear section is:

$$\sigma = \frac{F}{A} = \frac{2406}{\pi (0.022225/2)^2} = 4.7\text{MPa}$$  \hspace{1cm} (4.7)

For most steel materials, this value is far below the yield strength and it can be considered safely under repeated loading. The detailed mechanical drawings are presented in the Appendix.

![Figure 4.7 Shaft.](image-url)
4.4 Control System

The motors are individually controlled by APEX40 drives along with a 6250 controller. The software for control programs is Motion Architect which allows for automatic generation of commented setup code, creation, edition and execution of motion control programs, and creation of a custom test panel.

The heart of Motion Architect is the shell, which provides an integrated environment to access four main modules. These consist of:

1. **System Configurator:** This module prompts to fill in all pertinent set-up information (via pull-down menus) in order to get motion. Configurable to the 6000 Series products, the information is then used to generate actual 6000 language code at the beginning of the program to configure the product.

2. **Program Editor:** This module allows one to edit the code. It also has commands available through Help menus where the entire contents of the software reference guide is available.

3. **Terminal Emulator:** One can interact directly with the 6000 product to upload and download programs or request product status. Help is again available with all commands and their definitions are available for reference.

4. **Test Panel:** One can simulate programs, debug programs, and check for program flow using this module.
The requirements of Motion Architect are: 386 or higher IBM personal computer or compatible, Windows 3.1, Windows 95, Windows NT or higher, and 2 MB of RAM and 6 MB of hard disk space.

Figure 4.8 View of multiaxial fatigue testing machine (I).
Figure 4.9 View of multiaxial Fatigue testing machine (II).
Figure 4.10 Linear motion assembly.
Figure 4.11 Rotation assembly.
CHAPTER 5

TEST RESULTS

Since the new multiaxial fatigue testing machine is unique, the verification of machine performances is important and becomes the first priority before any other tests will be conducted. As described in previous chapters, this new machine is based on the idea of a torsion fatigue testing machine, which was first built in 1994 at Ohio University. Therefore, the verification of the new machine is focused on the linear motion mechanism. The machine performance verification is important in this stage not only because of the concern for the testing accuracy, but for safety reasons as well.

5.1 System Performance

The servomotor adapted here is capable of providing torque up to 25Nm at high speed consistently. During the tension test, this torque is converted into linear force and its magnitude is about 1.1KN (250lb). Therefore, before starting further experiments, some simple tests to verify this machine are conducted. The linear motion is accomplished by the gear and rack which transmit torque into linear force and translation. The gear is attached to the motor and the rack is mounted on the slider with the angle plate and jaw (see Figure 4.3). Since the fatigue testing will be operated in stress-controlled or strain-controlled mode, the displacement accuracy is highly demanded, as well as force and torque. The objective of displacement verification is to
get the accuracy of the displacement while operating the strain-controlled fatigue testing. By measuring the initial and final position of the slider, one can get the error range between the desired and actual displacement. The measuring instrument used is the dial caliper with accuracy of ±0.001”.

From Table 5.1, the error between the desired and actual displacement is under ±0.001” (0.025mm).

**Table 5.1 Displacement verification results.**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired displacement (in)</td>
<td>0.1</td>
<td>0.125</td>
<td>0.25</td>
<td>0.5</td>
<td>0.625</td>
<td>0.75</td>
</tr>
<tr>
<td>Test 1</td>
<td>0.0997</td>
<td>0.1248</td>
<td>0.2493</td>
<td>0.4999</td>
<td>0.6248</td>
<td>0.7496</td>
</tr>
<tr>
<td>Test 2</td>
<td>0.1002</td>
<td>0.1253</td>
<td>0.2499</td>
<td>0.4997</td>
<td>0.6253</td>
<td>0.7502</td>
</tr>
</tbody>
</table>

Another conducted test measures the difference between the desired force and actual force by using the force gage. The range of the force gage is 12kg(118N) and the accuracy is ±0.5%. The hook of force gage is attached to the slider and the other end of the force gage is fixed. This test meets the ASTM standards and could be used for further calibration of this machine for additional tests [8].

**5.2 Experimental Result**

The experimental results in this section were performed by torsion fatigue tests and compared with the existing data from “Databook of Fatigue Strength of Metallic Materials” [7]. In order to produce the S-N curves, the torsion tests were conducted on 13 specimens. The specimens were made from aluminum alloy 6061-T6 with gage
diameter 4 mm, Figure 5.1. The diameter of the each specimen is measured by using the
dial caliper and the maximum stress is calculated by $\tau = 2T/\pi r^3$. The results are given in
table 5.2.

Figure 5.1 Specimen.

The ASTM standards do not include the torsion fatigue test and test results for
aluminum alloy 6061-T6 are not available. One simple method to estimate the fatigue
limits for the torsion is to use the results of the tension test multiplied by 0.577 (see
discussion on equivalent stresses on page 34). The comparison of the test results and
existing data is shown in Figure 5.2. The comparison shows that the test results are a
little higher than the data multiplied by 0.577 from tension tests, but the overall
agreement is satisfactory.
Table 5.2 Numerical data of torsion tests

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<th>NO.</th>
<th>Diameter at Rupture (mm)</th>
<th>Torque (Nm)</th>
<th>R</th>
<th>Cycles to failure</th>
<th>( \tau_{\text{max}} ) (Mpa)</th>
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<td>68.07627265</td>
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</table>

Figure 5.2 S-N curves of aluminum alloy 6061-T6. Curve I: our results. Curve II: data from [7].
5.3 Conclusion

As a result of this thesis, a multiaxial fatigue testing machine has been built at Ohio University, Department of Mechanical Engineering. The machine is capable of applying tension, torsion, and the combination of both tension and torsion loadings. The electric servomotors provide the torque, and the gear and rack mechanism is used to translate the torque from one of the two motors into linear force for the tension test. The linear slider then provides the linear motion for tension and compression tests. The pillow blocks prevent the force that could cause the motor damage.

The computer hardware and software provide a user-friendly environment for setup and execution of tests with simple programs and data acquisition. This machine has many noteworthy features including:

1. The size and cost of this machine are much less than in corresponding hydraulic systems.

2. Its setup is simple.

3. High precision servomotor systems provide accuracy for testing small specimens and components.

4. High frequency saves testing time. The machine can be used at as high a frequency as 10 Hz in multiaxial testing, and as high as 50Hz in torsion testing only.
References


12. Thomas A. Stellman, G.V. Krishnan and Robert A. Rhea, Harnessing AutoCAD, Delmar Publisher, 1996.

Appendix: Mechanical Drawings

Drawing 001  Base Plate.................................................................71
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BAES PLATE
MATERIAL: 3/4 A36 HR PLATE
001 01-15-99 MU-HSIN LIU
PIN ANGLE PLATE TO SLIDER
WITH #0.375 PIN, 2 PLACES.
FIT TO BE DETERMINED
BY MACHINIST.

SECTION A-A

#0.250 6 PLACES

#0.250 9 PLACES

M.H. LIU

ANGLE PLATE
3/8" x 1" DEEP TAPPED HOLE

BEARING MOUNTING BLOCK

MATERIAL: CF STEEL

003 03-08-99 M.H LIU
PIN SLIDER TO MOUNTING PLATE WITH 3/8" PIN, 2 PLACES.
FIT TO BE DETERMINED BY MACHINIST

SLIDER MOUNTING PLATE
MATERIAL: CF STEEL 6.5x12x.875
REMOVE 3 TEETH BOTH END (1/4 DEEP)

11 TEETH

SECTION A-A

(BOTTOM)

RACK

BROWNING PART # 6YRS 10X1.25 PRESSURE ANGLE=20

005 02-27-99 M.H LIU
ADDITIONAL 1/4-20 TAPPED HOLES TO BE DRILLED. 8 PLACES.

GRAY INDICATES EXISTING TAPPED HOLES.

PIN SLIDER TO MOUNTING PLATE WITH 3/8" PIN, 2 PLACES. FIT TO BE DETERMINED BY MACHINIST.

PIN ANGLE PLATE TO SLIDER WITH 3/8" PIN, 2 PLACES. FIT TO BE DETERMINED BY MACHINIST.

CONCEPT DRAWING OF SLIDER MODIFICATIONS
DIMENSIONS ON ANGLE PLATE DRAWING(002-03)
MOUNTING PLATE & WASHER
CF STEEL D=2' X L=11.5'
009  08-15-99  MH LIU