Single Platform Relative Positioning for Sensor Stabilization

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by

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Abstract

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Intelligence, Surveillance, and Reconnaissance (ISR) sensors such as radio detecting and ranging, laser detecting and ranging, electro-optic / infra-red, and other remote sensing activities are becoming increasingly dependent on their position and orientation in time and space. The higher dynamics of flight and the increased sensor requirements have led to the need for sensor stabilization by direct motion measurement. A stabilization system based on the Global Positioning System (GPS) can provide good performance, but high-frequency ISR sensor pointing applications have led to the need for additional measurement bandwidth, accuracy, and robustness. Two significant issues will be addressed in this dissertation to improve the stabilization system robustness and accuracy at the mm level: GPS Carrier Phase measurement noise using inertial measurements and carrier phase multipath. Higher bandwidth requirements (i.e., hundreds of Hz) will be addressed with the incorporation of a high-rate inertial measurement unit. All concepts developed in this dissertation will be illustrated using real sensor data from either static aircraft tests on the tarmac or dynamic flight tests.

The work to be described in this document will expand the state-of-the-art in the area of navigation sensor noise reduction while preserving high measurement bandwidth. Coupling GPS and inertial measurements has demonstrated the required improvements in similar applications, but an approach was sought which was tailored for the stabilization
application. This dissertation will examine both optimal and non-optimal coupling of
navigation sensors to form optimized high-accuracy single-platform relative position
measurements with the intent to coherently stabilize ISR sensors.

This dissertation also examines GPS carrier phase multipath as an error contributor in the
ground calibration of the ISR sensor antenna baseline. Several indicators will be
examined as a means to exclude satellites from the calibration data. Also, the narrow-
lane measurement combination technique will be used as a means to mitigate the
remaining carrier phase multipath in the baseline solution.

The contributions of this work include a framework for the single-platform stabilization
problem, sensor integration and alignment techniques for single-platform baseline
stabilization, an inertial synthesized baseline technique for smoothing noise in the GPS
measurements, multipath considerations pertaining to the ground-calibration of sensors,
and a demonstration of the utility of the narrow-lane linear combination for dual
frequency GPS noise and multipath reduction.

Approved: _________________________________________________________________

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<th>Definition</th>
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<tr>
<td>AEC</td>
<td>Ohio University Avionics Engineering Center</td>
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<tr>
<td>AGC</td>
<td>Automatic Gain Control</td>
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<td>ALS</td>
<td>Airborne Laser Scanner</td>
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<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<td>CORS</td>
<td>Continuously Operating Reference Station</td>
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<td>CNR</td>
<td>Carrier to Noise Ratio</td>
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<tr>
<td>C/A</td>
<td>Coarse Acquisition (GPS Ranging Code)</td>
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<tr>
<td>CMC</td>
<td>Code Minus Carrier (i.e., pseudorange – carrier phase)</td>
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<tr>
<td>CP</td>
<td>Carrier Phase</td>
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<tr>
<td>DCM</td>
<td>Direction Cosines Matrix</td>
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<tr>
<td>DD</td>
<td>Double Difference</td>
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<tr>
<td>DDCP</td>
<td>Double Difference Carrier Phase</td>
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<tr>
<td>DGPS</td>
<td>Differential Global Positioning System</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution Of Precision</td>
</tr>
<tr>
<td>DTED</td>
<td>Digital Terrain Elevation Database</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth Centered Earth Fixed (geodetic coordinate frame)</td>
</tr>
<tr>
<td>ENU</td>
<td>East, North, Up (local-level coordinate frame)</td>
</tr>
<tr>
<td>EO</td>
<td>Electro Optic</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<td>IMU</td>
<td>Inertial Measurement Unit</td>
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<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>IR</td>
<td>Infra-Red</td>
</tr>
<tr>
<td>IRU</td>
<td>Inertial Reference Unit</td>
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<tr>
<td>ISR</td>
<td>Sensors Used for Intelligence, Surveillance, and Reconnaissance</td>
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<tr>
<td>KUNI (UNI)</td>
<td>Ohio University Airport</td>
</tr>
<tr>
<td>KVM</td>
<td>Keyboard, Video, Mouse</td>
</tr>
<tr>
<td>LADAR</td>
<td>Laser Detection and Ranging</td>
</tr>
<tr>
<td>NED</td>
<td>North, East, Down (local-level coordinate frame)</td>
</tr>
<tr>
<td>NL</td>
<td>Narrow-Lane</td>
</tr>
<tr>
<td>OPUS</td>
<td>Online Positioning User Service</td>
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<tr>
<td>PPM</td>
<td>Parts Per Million</td>
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<tr>
<td>PPS</td>
<td>GPS Pulse Per Second</td>
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<tr>
<td>PVT</td>
<td>Position, Velocity, Time</td>
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<tr>
<td>QNX</td>
<td>Real Time Operating System (vendor)</td>
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<tr>
<td>RADAR</td>
<td>Radio Detection and Ranging</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RM</td>
<td>Resource Manager (i.e., QNX Device Driver)</td>
</tr>
<tr>
<td>SAR</td>
<td>Synthetic Aperture Radar</td>
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<tr>
<td>SDCP</td>
<td>Sequentially Differenced Carrier Phase</td>
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<tr>
<td>SF</td>
<td>Single Frequency</td>
</tr>
<tr>
<td>SMR</td>
<td>Signal to Multipath Ratio</td>
</tr>
<tr>
<td>SV</td>
<td>Space Vehicle (GPS Satellite)</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Arial Vehicle</td>
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<tr>
<td>ZAB</td>
<td>Zero-Attitude Baseline</td>
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1 Introduction

1.1 Overview

Intelligence, Surveillance, and Reconnaissance (ISR) sensors such as RAdio Detecting And Ranging (radar), LAser Detecting And Ranging (ladar), Electro-Optic (EO) / Infra-Red (IR), and other remote sensing activities are becoming increasingly dependent on their position and orientation (i.e., the sensor pose) in time and space. The pose label will be used throughout this document to jointly refer to position and orientation. Certain types of uncompensated ISR sensor motion can lead directly to ISR sensor measurement and geolocation errors, so motion must be accounted for. Many of these sensors have simplistic positioning requirements because the sensors are used on the ground or in similar low-dynamic conditions or the sensor may operate at a low frequency (e.g., radar in the C-band); Lower frequencies tend to decrease the stabilization complexity because the required accuracy is generally specified as a fraction of the wavelength of the ISR sensor. Under these conditions, the motion of the platform could be approximated and removed with a motion model or mechanically stabilized using motion suppression techniques. Motion suppression is commonly achieved using gimbals to remove low-rate motion and shock-mounts to remove high-rate motion. With the increase in computational power, analytical techniques such as low-pass filtering can be employed to blindly suppress the detrimental high rate motion and vibration when knowledge of this motion is inconsequential to the overall system performance. With increasing stabilization requirements, it is desirable to be able to measure and analytically remove the high rate motion and vibration. A major topic to be described throughout this
document is an analytical technique whereby motion is compensated using measurements made directly at the ISR sensor or at a rigidly connected location nearby.

Many sensors are able to measure motion, but navigation systems such as the Global Positioning System (GPS) and Inertial Navigation Systems (INS) provide the best potential to directly measure platform motion with high precision and at high update rates while georeferencing the ISR measurements in a global temporal and spatial reference frame. The inertial sensors referred to in this dissertation will always refer to those in a strapdown INS configuration unless stated otherwise. A combination of GPS and INS sensors will be at the core of the stabilization techniques described in this dissertation.

As a general convention, sensor stabilization will generically describe a technique to remove motion from ISR sensor measurements including mechanical and analytical techniques as subsets. Mechanical techniques will refer to the active motion suppression of gimbals and shock mounts while analytical techniques will refer to the direct measurement and compensation of motion as well as the passive use of filters.

1.1.1 Problem to Be Solved
ISR sensors are now being used on airborne platforms with higher dynamics and at higher frequencies with increased accuracy requirements. Some airborne applications are calling for ISR sensor placement in locations with challenging stabilization requirements such as aircraft wingtips. In such cases, blind filtering is no longer appropriate and a model is not adequate to ensure performance under varying platform dynamics, so
motion measurements must account for a range of conditions from low-rate position changes to platform flexure and vibrations of several mm at hundreds of Hz. Stabilization is particularly challenging when making angular measurements (e.g., ISR sensor pointing) since a small angular pointing error becomes a large position error over a long distance.

A GPS-only system was shown to provide a stabilization capability over short baselines (tens of meters) at update rates of 100 Hz (Bartone, van Graas, & Arthur, 2005). A system of this type will have performance limitations caused by the inherent noise threshold of a given GPS receiver and its tracking loop bandwidth (Braasch & van Graas, 1991), as well as errors due to signal reflection or diffraction known as multipath (Braasch, 1996). Secondly, GPS is susceptible to interference and signal blockages that must be overcome in a robust stabilization system (Volpe, 2001). Thirdly, even state-of-the-art GPS receivers are bandwidth limited and can only provide measurements at 100 Hz, which can be a limitation for some motion dynamics. Due to the accuracy demands of a high-frequency ISR sensor pointing application, accuracy and robustness improvements were sought.

A new system was conceived as part of this research to be more robust and lower noise through the use of INS measurements. INS sensors provide rotation and translation measurements at update rates greater than GPS (commonly 400 Hz and higher) with much lower noise levels and are not susceptible to external factors like interference and
signal blockage. INS position, velocity, and orientation measurements drift over time, but they are stable for short-term (sub-second) integration applications. When properly aligned and synchronized, the 100 Hz GPS updates can be used to remove the drift from every fourth 400 Hz INS measurement while the INS can simultaneously smooth the GPS noise to yield a low-noise and drift-free composite navigation measurement. Measurement integration can be achieved in many ways, so an approach must be identified which is suitable for relative stabilization applications. If GPS measurements are not available for an extended period of time, the INS drift can be too severe to provide useful measurements. An additional sensor such as a ladar could provide measurements to constrain the INS drift during such outages.

To summarize the problem, the higher dynamics of flight and the increased ISR requirements have led to the need for direct-measurement sensor stabilization. A GPS-only system can provide good performance, but high-frequency ISR sensor pointing applications have led to the need for additional accuracy and robustness. Two significant problems need to be addressed to improve the system accuracy and robustness: GPS Carrier Phase (CP) noise, and CP multipath. Coupling GPS and INS measurements has demonstrated the required improvements in similar applications, but an approach is sought which is tailored to the stabilization application.
1.1.2 Implementation Challenges
The coupling of INS measurements with relative GPS baseline measurements can be a powerful solution to many stabilization problems, but in practice there are several implementation challenges to be overcome.

Navigation measurements used for stabilization will ultimately be coupled together and then blended with ISR sensor data, so the time synchronization is critical to performing accurate motion compensation especially in high dynamics. Similarly, the coordinate frame alignment is critical to removing the correct amount of motion from each of the three Euclidian axes while not introducing additional error. A static calibration might provide sufficient alignment in some cases, but a more complex continuous alignment process might be required in other situations. The navigation sensors themselves also pose a significant challenge due to their own error sources. In many cases, simply coupling two sensors does not eliminate the navigation system errors and in some cases it introduces new ones. One such example is ground multipath during a multi-sensor alignment and calibration prior to takeoff.

1.1.3 Implementation Methodology
The research presented in this dissertation will examine the coupling of navigation sensors to form optimized high-accuracy platform pose measurements with the intent to coherently stabilize ISR sensors. A basic description of the methodology will be provided in this section as an overview while saving the more technical discussion for later chapters. In general, GPS baseline determination from CP Double Differences
(DDs) will be the basis for the integrated sensor stabilization system, so strategies that can use and improve these measurements will be addressed.

The noise reduction aspect of this dissertation will be addressed in several ways. GPS measurement differencing increases its noise, so taking fewer differences is one way to inherently reduce the system noise as described in (Dickman & Bartone, 2007) (Van Graas & Soloviev, 2004) for measurement alignment. This approach may help to solve part of the alignment and integration problem, but not the positioning and absolute pointing part. INS data can be used for simultaneous alignment and GPS noise reduction. Several of the approaches to be presented integrate these measurements in an unconventional way: a three-state orientation filter, a three-state baseline filter, a DD range domain filter, and a non-optimal baseline-smoothing filter. Raw INS angular rate measurements are only integrated once to form orientation so they contain less drift from Brownian motion and thus have longer-term stability than the double integrated accelerometer measurements used to derive the position, so the angular rate measurements will be used primarily. The first integration approach is a loose three-state orientation Kalman filter whereby INS orientation is loosely integrated with GPS-derived orientation estimates for alignment and GPS orientation smoothing. Similarly, a loose three-state baseline filter can be formed using relative GPS position measurements and INS-synthesized position estimates by making a rigid-body assumption. The range-domain integration provides a technique to integrate the measurements prior to forming a GPS navigation solution and has several consequential advantages. Another approach
will also be presented to reduce the GPS noise by smoothing the baseline estimates recursively in the relative position domain without a Kalman filter when the frame alignment has already been performed.

The system robustness aspect of this dissertation will be considered by addressing multipath, a dominant GPS interferometric CP error source. Multipath can affect ground alignment and system calibration in addition to airborne GPS measurements, so it is an important error source to deal with.

All of these concepts will be illustrated using real sensor data collected while field testing the aircraft-mounted sensors on the tarmac or in flight tests.

1.1.4 Research Contributions
Simple motion compensation using navigation sensors has become a generally accepted technique, but innovation is still rapidly advancing. The work to be described in this document will expand the state-of-the-art in GPS sensor stabilization by considering noise reduction while preserving high measurement bandwidth. This dissertation will provide navigation sensor position measurements to stabilize single-platform ISR sensors at the mm level or better. Many of the issues required for this level of system performance will be considered and flight test data will be analyzed to demonstrate the efficacy of the techniques described. The contributions of this work include a framework to use when considering the stabilization problem, sensor alignment and high-accuracy Kalman integration techniques for single-platform baseline stabilization, an INS
synthesized DD technique for smoothing noise in the GPS measurements, some multipath considerations pertaining to the ground-calibration of sensors, and a demonstration of the utility of the narrow-lane (NL) linear combination for dual frequency GPS noise and noise-like multipath reduction.

1.1.5 Organization
This overview section was intended to paint the big picture and to provide the scope of the problem to be solved. The next section provides some system architecture descriptions. After that, chapter two will provide some of the necessary background information for the remainder of the dissertation. The third chapter deals with the coordinate frame alignment and coupling of the GPS and INS sensors and provides several different measurement integration approaches. These approaches will be demonstrated using static and in-flight data. The fourth chapter deals with CP multipath as it relates to the short baseline (tens of meters) interferometry case considered in this dissertation. Field test data will be used to illustrate the effect of multipath and the dual-frequency NL processing technique will be applied for multipath mitigation. The fifth chapter will describe a GPS DD measurement smoothing technique using INS synthesized DDs. The sixth chapter will provide some conclusions to be drawn from the research presented in this document. Finally, chapter seven will suggest future work that should be considered as follow-on research to build on the work presented in this dissertation. The appendices contain additional descriptive material such as more details on the data collection system, flight-test results of the NL baseline solution, and in-flight spectral content of the aircraft motion at two nodes of the aircraft.
1.2 Sensor Stabilization System Architecture

The ISR sensor architecture can be configured in several different ways depending on the desired application. Two of the major configurations will be briefly discussed: single-node and multiple-node stabilization, but the latter will be considered in greater detail throughout this document. In general, the architecture to be discussed in this chapter will refer to measurements combined in a single epoch “snapshot” as opposed to ones integrated over time (e.g., Synthetic Aperture Radar (SAR)).

1.2.1 Single-Node ISR Sensor Aperture

The first configuration involves a single-node ISR sensor aperture whereby measurements are made from a single point (e.g., ladar, EO camera, or basic single-aperture radar). Only an instantaneous snapshot of the sensor pose must be determined at each measurement epoch to geo-reference the sensor measurements in this configuration. The main distinguishing characteristic for this type of system architecture is the size and lack of physical separation in the sensor aperture since there is only a single node. Navigation measurements can often be made directly at the ISR sensor and applied directly to its measurements since the single-point aperture does not flex. In some cases, such as a terrain scanning ladar, motion must be removed from high-rate (e.g., tens of kHz) sequentially pulsed measurements so they can be combined over a time window to form a terrain snapshot. Significant vibration that is not observable can seriously degrade the accuracy of this type of measurement by distorting its angular pointing accuracy and thus causing a blurry or distorted picture (Dickman & Uijt de Haag, 2007). This type of
system architecture can be considered a special case of the more general one to be described next and will not be directly considered beyond this description.

### 1.2.2 Multiple-Node ISR Sensor Aperture

The second configuration is an extension of the first whereby the ISR sensor aperture includes multiple nodes and is thus physically separated to form an extended ISR sensor aperture, which is often important for ranging multi-lateration. In this case, the same single aperture navigation problem of motion compensation persists, but this time the motion must be captured at multiple locations and be accurately combined to form a sensor array (e.g., interferometric radar). The main reason to extend the sensor aperture is to provide better ranging measurement geometry for a given range to the target (e.g., time difference of arrival or frequency difference of arrival measurements).

A simple 2D graphical example is shown in Figure 1-1 to illustrate one benefit of extending the ISR sensor aperture. In subfigure A, a single node ranging ISR sensor is shown at the center of the concentric circles which representing ranging lines. The measurement uncertainty is shown with a darker color band with the target somewhere in the middle of this band. When two ranging sensors are combined, the target location can be defined by the intersection of two circles as shown in subfigure B. Technically, there is an ambiguity when only two measurements are made because there are two intersection points, but one is assumed to be the obvious choice for the sake of simplicity. A sub-optimal case is shown on the left side of subfigure B where the baseline between the sensors is too short for the given range and the measurement uncertainty is amplified.
The optimal case is shown on the right side of subfigure B where the range lines are orthogonal to each other at the point of intersection and the measurement uncertainty is minimized. Subfigure C shows the optimal range for a given aperture baseline length and subfigure D shows that the optimal range is extended by increasing the aperture baseline length.

Figure 1-1: Extended Aperture Baseline Illustration

Figure 1-1 demonstrates the advantage of extending the aperture baseline for improved geometry. In the case of poor geometry, i.e., non-orthogonal intersection lines, the composite measurement uncertainty from the two nodes of the ISR sensor is amplified as shown in the left side of subfigure B. The composite geometry is an optimal square when the ranging lines intersect orthogonally.

The total ISR measurement uncertainty is comprised of the ISR sensor measurement accuracy and the uncertainty in the position of the ISR sensor. The major limitation to the total system accuracy is the composite ISR position uncertainty, which is derived
from the ISR sensor aperture baseline length, and the constituent measurement uncertainties in the position of each node. The subsequent chapters of this document will describe techniques to minimize the positioning errors and thus the constituent bands of uncertainty using high-accuracy navigation measurements.

The baseline length can be extended in two ways: by placing the ISR sensors at the furthest extents possible on a single platform (e.g., aircraft nose and tail or the wingtips) or by placing the ISR sensors on multiple platforms.

1.2.2.1 Single Platform

One multi-nodal sensor configuration involving multiple ISR sensors on a single platform is shown in Figure 1-2. Navigation sensors (e.g., GPS or integrated GPS/INS) are installed nearby to the ISR sensor to measure its pose information.

![Figure 1-2: Single Platform Multiple Sensor Aperture](image)

For this illustration, ISR sensor antennas (represented by a solid circle) are shown on the aircraft wingtips and fuselage with navigation sensors mounted nearby (represented by a cross hatched circle Figure 1-2) for motion compensation. The ISR sensor data and the
precise baseline position measurements can be optimally combined to enable many of the remote sensing applications described previously. Historically, the ability to measure high rate motion was often ignored because it was considered insignificant for a given ISR sensor accuracy at a given frequency. Today, however, the ability to accurately measure vibration, wing flexure, and fuselage bending at a given instant of time is critical for high-performance system operation. These types of navigation measurements tend to be spatial snapshots for the single aperture case, but coherent integration of measurements from multiple navigation and ISR sensors is required for the multi-nodal extended aperture case.

A single platform relative positioning system has been previously developed and demonstrated by Ohio University to provide stabilization accuracy using the GPS L1 and L2 CP at the mm-level (Bartone, van Graas, & Arthur, 2005). This system, similar to the one shown conceptually in Figure 1-2, uses GPS double differenced carrier phase (DDCP) measurements to provide precise in-flight relative baseline positioning at a 100 Hz update rate for short baselines, e.g., tens of meters, with a relative horizontal and vertical position accuracy on the order of 4 and 10 mm, respectively.

1.2.2.2 Multiple Platform

Another configuration might include ISR sensor nodes on multiple platforms, which would offer a theoretically unlimited aperture size. Multiple platform measurement integration is considerably more complicated than the single platform case because of measurement synchronization, datalink issues, and spatial decorrelation of the GPS errors
between nodes and so the performance is not as good. Figure 1-3, shows a conceptual measurement system involving multiple nodes installed on separate platforms where stabilization occurs on a platform-to-platform basis.

In the platform-to-platform case, the navigation sensor must account for the motion of one platform with respect to the other platforms with minimal timing error. Platform-to-platform measurements provide capabilities for long baseline (kilometers) sensor integration whereby the aperture size (e.g., aircraft, UAV, flight path, or platform separation) is no longer limited. As the baseline length increases, ISR sensor measurements would provide additional performance, but the GPS measurements would contain additional error. For this integration, the platform/array baseline position and timing accuracy are critical to accomplishing coherent sensor integration. Other applications include ground feature imaging and geolocation as well as air-to-air refueling.
Due to the additional system complexity and GPS error decorrelation over longer distances, the accuracy is often an order of magnitude worse than the single platform case (Misra & Enge, 2001). A system has been demonstrated by the Air Force Institute of Technology with automatic airborne refueling applications (Spinelli, Raquet, & Kish, 2006). The multi-platform configuration is beyond the scope of this research, so only the high-accuracy single-platform coherent sensor integration will be considered after this.

2 Background

2.1 GPS Measurements

GPS measurements have been used extensively for positioning and navigation for many years. This system provides different levels of accuracy for civil users depending on the technique used to calculate the user Position, Velocity, and Time (PVT). In its most basic form, GPS coarse/acquisition (C/A) code phase measurements result in position accuracies on the order of 10-100 meters (Misra & Enge, 2001). Users with more restrictive position accuracy requirements have made use of code phase Differential GPS (DGPS) in order to cancel out common error sources and provide additional position accuracy on the order of 1-10 meters (Misra & Enge, 2001). Both types of position solutions have limited position accuracy primarily due to their use of C/A code phase measurements, which have a significant noise component (up to meter level). Users who require even more accuracy make GPS CP measurements that have less noise (less than one cm) and can result in sub meter-level position error (Misra & Enge, 2001). Differential CP measurements can then be used to provide relative positioning capable of
position accuracy of under 10 cm with respect to a surveyed ground reference (Cosentino & Diggle, 1996). If absolute positioning is not as important as locating a point with respect to another point, then interferometric relative positioning (i.e., baseline determination) can provide mm-level position accuracy for short baselines, e.g., tens of meters, (Bartone, van Graas, & Arthur, 2005).

For the high-accuracy applications to be described in this dissertation, interferometric baseline measurements will primarily be used. The details of CP baseline processing will be explained in the next section.

### 2.1.1 Carrier Phase Baseline Processing

The CP measurement model can be used to illustrate the error sources and the effect of double differencing (Misra & Enge, 2001). A single frequency CP model is described by equation (2-1) for the space vehicle (SV) under consideration equal to $j$ (ARINC, 2000):

$$
\Phi^j = \left[ r^j - I^j + T^j + c(\Delta t^j + \delta t) \right] + N^j + \varepsilon_{\phi}^j + \varepsilon_{\phi\text{mp}}^j
$$  \hspace{1cm} (2-1)

where:
- $r$ = true range from user antenna to SV antenna \([\text{m}]\)
- $I$ = ionospheric phase advance \([\text{m}]\)
- $T$ = troposphere delay \([\text{m}]\)
- $\Delta t^j$ = SV transmission clock bias \([\text{sec}]\)
- $\delta t$ = user receiver clock bias \([\text{sec}]\)
- $N$ = CP ambiguity \([\text{m}]\)
- $\varepsilon_{\phi\text{mp}}$ = CP multipath error \([\text{m}]\)
- $\varepsilon_{\phi}$ = CP receiver noise \([\text{m}]\)

For a short baseline (tens of meters in this case), the error terms within the brackets cancel in the DD leaving only the true DD range, the DD ambiguity, CP multipath, and receiver noise. The ambiguity can be solved using various techniques, but in this case
will be performed using fixed-baseline constraint as described in (Bartone, van Graas, & Arthur, 2005). The DD operation introduces new notation for the key satellite, $k$, the main receiver, $m$, and a secondary receiver denoted by $s$. After DD formulation and removing the ambiguity, the range vector difference between two receivers and two SVs becomes:

$$
\Phi_{sm}^{jk} = r_{sm}^{jk} + \epsilon_{\phi_{mp},sm}^{jk} + \epsilon_{\phi,sm}^{jk}
$$

where:

- $r$ = true DD range [m]
- $j$ = satellite identifier [unitless]
- $k$ = key satellite identifier [unitless]
- $m$ = main receiver identifier [unitless]
- $s$ = secondary receiver identifier [unitless]

Range measurements are nonlinear quantities, so the DDs must be linearized using vectors that relate ranges to positions in a linear equation. For DDs, the geometry matrix must include a line of sight vector to each of the two SVs involved in a measurement. The coordinate frame of this vector determines the coordinate frame of the baseline vector, so it must be chosen with consideration for measurement integration with other navigation or ISR sensors. A convenient frame is a local level navigation from such as East, North, Up (ENU) or North, East, Down (NED). The following equation describes the DD line of sight vector:

$$
\tilde{e}_{jk} = e_{sj}^{j} - e_{sk}^{k}
$$

where:

- $e_{sj}^{j}$ = SV $j$ line of sight unit vector from the secondary receiver [m]
- $e_{sk}^{k}$ = SV $k$ line of sight unit vector from the secondary receiver [m]

It should be noted that the linearization only takes place around the secondary receiver, $s$. This is an acceptable approximation since the baseline is so short. A simple geometric
correction could be applied to correct the single difference for dissimilar unit vectors with longer baselines, as would be the case in a multi-platform integrated system.

Each row of the geometry matrix is populated with the three components (i.e., \(x,y,z\)) of the vector, \(\tilde{e}_{jk}\), for each SV (\(j\)) paired with the key SV (\(k\)). The line of sight vector is not a unit vector, as might be inferred from the choice of variables, because it is the difference of two unit vectors as shown in equation (2-3) and motivates the tilde notation.

The following equation shows the geometry matrix, \(H\), for a minimum satellite set to determine a position solution:

\[
H = \begin{bmatrix}
\tilde{e}_{x}^{1k} & \tilde{e}_{y}^{1k} & \tilde{e}_{z}^{1k} \\
\tilde{e}_{x}^{2k} & \tilde{e}_{y}^{2k} & \tilde{e}_{z}^{2k} \\
\tilde{e}_{x}^{3k} & \tilde{e}_{y}^{3k} & \tilde{e}_{z}^{3k}
\end{bmatrix} \quad \text{[unitless]} \quad (2-4)
\]

Three or more unambiguous DDs from equation (2-2) can be grouped in a matrix and can then be related to the relative baseline vector with the following linear equation:

\[
\Phi = H \cdot \mathbf{b} \quad \text{[m]} \quad (2-5)
\]

where:
- \(\Phi\) = double difference vector \quad \text{[m]}
- \(H\) = satellite geometry matrix \quad \text{[unitless]}
- \(\mathbf{b}\) = relative baseline vector from receiver \(s\) to \(r\) \quad \text{[m]}

An ordinary least squares solution can be performed using the GPS DDCP measurements, \(\Phi\), to solve for the baseline solution, \(\mathbf{b}\), between the main and the secondary GPS antenna phase centers as shown:

\[
\mathbf{b} = (H^T \cdot H)^{-1} \cdot H^T \cdot \Phi \quad \text{[m]} \quad (2-6)
\]
As a general labeling convention in this dissertation, the secondary node will be listed first and then the main node will be second. So, baseline12 would indicate measurements from node 1 relative to node 2.

Since this is a high performance application with mm-level performance in the baseline solution, there is no absolute truth reference system available. Hence, the least squares residual values can provide an indication of the error in the range domain and can thus be mapped back to the position domain.

\[
R = \left( I - H \cdot (H^T \cdot H)^{-1} \cdot H^T \right) \Phi \quad [m] \quad (2-7)
\]

where:
- \( R \) = least squares residual vector \([m]\)
- \( I \) = identity matrix \([\text{unitless}]\)

Similarly, parity space (Brown, 1996) can also be used to assess the baseline solution accuracy (Van Graas, 1996) as previously demonstrated in (Bartone, van Graas, & Arthur, 2005). Equation (2-8) illustrates the same baseline solution as in equation (2-6) in the upper half partition and the parity vector, \( p \), calculated in the lower partition:

\[
\begin{bmatrix}
  b \\
  \vdots \\
  p
\end{bmatrix}
= \begin{bmatrix}
  (H^T H)^{-1} H^T \\
  \vdots \\
  P
\end{bmatrix} \Phi \quad [m] \quad (2-8)
\]

where:
- \( H \) = double differenced satellite geometry matrix
- \( p \) = parity vector \([m]\)
- \( P \) = parity matrix \([\text{unitless}]\)

Assuming the parity vector is zero-mean over a given time interval, the magnitude of its variance can be related to position space using the DD dilution of precision (DOP). The
parity vector can thus be used as a performance metric to assess the overall baseline solution accuracy (Van Graas, 1996).

2.1.2 GPS Noise and Multipath
The dominant remaining errors in short-baseline relative CP positioning include CP noise and multipath (Cosentino & Diggle, 1996). CP multipath is an environmentally sensitive error in that small changes to environmental parameters such as the reflection/diffraction surface or antenna height dramatically change the error. Consequently, it is difficult to predict, validate, and remove in an airborne environment. Multipath is usually characterized by changes in measurement amplitude and fading frequency; it can be caused by reflections or diffractions from airframe surfaces and will change with varying platform dynamics, configurations, and body flexure. Because of these variations and the short wavelength of the GPS carrier frequency, the airborne GPS CP multipath is difficult to isolate and remove, but can be considered noise-like (Braasch & van Graas, 1991). CP noise is also affected by receiver parameters such as tracking loop bandwidth and user-controlled parameters such as preamplifier quality. Due to the noise-like qualities of both of these errors (CP multipath and carrier tracking noise), they can be reduced in two ways. The first involves a smoothing algorithm; however, the challenge is to smooth the measurements in a way that preserves the dynamic bandwidth and does not introduce a time lag. The second is using the NL measurement combination for error mitigation.

2.2 Inertial Measurements
Inertial sensors have also been used extensively for many years. They are typically used when there is a requirement for high rate, low noise measurements and/or if a self-
contained measurement system is required. Gyroscopic sensors are used to provide information about angular motion while accelerometers are used to provide velocity rate (acceleration) information from specific force measurements. There are many different levels of accuracy provided by inertial sensors ranging from navigation/space grade (best performance, highest cost, and largest in size) to automotive/consumer grade (lowest performance, lowest cost, and smallest size) (Titterton & Weston, 1997). Throughout this dissertation specific reference will be made to different sensor grades such as a navigation-grade inertial reference unit (IRU), a tactical-grade inertial measurement unit (IMU) and generic references to either sensor will be simply referred to as an inertial navigation system (INS).

Gyro operation can be thought of as a measurement of the amount of torque required to null the motion induced angular momentum. Gyroscopes commonly range in performance from 0.001 deg/h to 50 deg/s of induced error. Several errors are present in the output of such devices; the largest ones will be briefly described in this section. A fixed bias occurs when a sensor output exists even when no input rotation is present. This error is usually expressed in deg/h for accurate sensors and deg/s for the lower performance units. An acceleration dependent bias (expressed in deg/h/g) changes in proportion to the input acceleration. A scale factor error is a magnification error that increases as a function of the sensed input rate and is usually expressed as a ratio of output error to input rate in parts per million (ppm) or a percentage. Cross coupling errors are erroneous outputs from rotations about orthogonal axes (normal to input axis)
and are usually expressed in ppm or a percentage. As with most sensors, gyroscopes also exhibit random errors often referred to as angular random walk and are typically expressed in deg/root-hour (Titterton & Weston, 1997).

Accelerometer operation can be thought of as the force required to set a pendulum or a proof mass to zero displacement. Accelerometers commonly range in bias performance from a few micro-g to a few milli-g. Several sources of error are also present for accelerometers; only the most significant ones will be described in this section. A fixed bias occurs when there is a residual measurement output when there is no specific force being applied to the sensor and is usually measured in fractions of a g. A scale factor, usually expressed in ppm, may be present due to errors in the ratio of the measured output to the input acceleration and can be linear or non-linear. They may be caused by temperature variation or imperfect component behavior. Cross-axis coupling is a type of bias induced by a sensed force that is exerted orthogonal to the sensing axis and is usually expressed as a percentage of the applied acceleration. A random bias is one that was induced by instabilities within the sensor and is expressed in fractions of a g (Titterton & Weston, 1997).

Absolute angular and velocity measurements are obtained by integrating the raw measurements over the measurement interval. Any errors in the measurements will accumulate over time and result in drift, which is the primary limitation of inertial measurements.
2.2.1 Inertial Orientation Processing
The aircraft platform attitude and heading are the primary quantities of interest for this research since they can be used to relate changes in position from one frame to another. The inertial orientation measurement convention used throughout this document provides the relationship between the body frame (with respect to the INS center of gravity) and the north-pointing dynamic navigation frame expressed in a North-East-Down (NED) convention. Measurements in one frame will be transformed to the other via a Direction Cosines Matrix (DCM). A single DCM to transform body coordinates to navigation coordinates can be formed as a series of three Euler angle rotations which have been multiplied together in the conventional order as shown (Titterton & Weston, 1997):

\[
C_v^b = \begin{bmatrix}
\cos \phi \cos \psi & -\cos \phi \sin \psi + \sin \phi \sin \theta \cos \psi & \sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi \\
\cos \theta \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \theta \sin \psi & -\sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi \\
-\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \\
\end{bmatrix}
\]  
\tag{2-9}

where:
\[\phi = \text{Platform Roll} \quad \text{[rad]}\]
\[\theta = \text{Platform Pitch} \quad \text{[rad]}\]
\[\psi = \text{True Platform Heading} \quad \text{[rad]}\]

Using this DCM, a vector expressed in the body frame will be converted to the NED local-level navigation frame conventionally used for inertial navigation.

A static attitude and heading survey must be conducted to relate the GPS measurements in the navigation frame to the body frame via the transpose of equation (2-9). Further discussion of the static survey and sensitivity to error can be found in (Dickman & Bartone, 2007).
2.3 Measurement Integration Requirement Guidelines

There are many important requirements to consider when performing sensor stabilization. One goal of this research was to determine what reasonable accuracy could be attained without constraining the solution to a particular set of requirements. Since absolute requirements cannot be clearly defined without loss of generality, several different types of requirements will be initially described and the details provided later as needed.

When using any navigation sensor, there is a need to characterize its measurement accuracy. Navigation measurements might include PVT, and/or orientation. The performance requirements for each measurement greatly differ depending on the ISR sensor to be stabilized and the architecture in which it is being used. For example, in the GPS stabilized multi-node/single-platform case, the position and time accuracy dominate the others, while in the INS stabilized single-node/single-platform case, the orientation accuracy dominates.

Another important consideration is the update rate of the navigation sensor. Higher rate motion is observable with a higher rate navigation sensor as constrained by the Nyquist Rate. The required update rate must be larger than double the maximum rate of motion to be observed. If the rate of motion is greater than the navigation sensor update rate, aliasing will occur and it will be difficult to tell the true motion from the alias. When the frequency spectrum of the motion is not known, it is a good idea to initially sample the motion at a much higher rate than required so that a requirement can be created. In order to determine the spectral content of typical aircraft motion, INS data was collected from
several locations on Ohio University’s Douglas DC-3 aircraft as shown in Figure 2-5. The frequency content of this motion was calculated in blocks using a windowed FFT. An example of this is shown in Figure 2-1, where the spectral content of the platform motion is plotted as the engines throttle up for takeoff.

A more detailed description of the procedure and the spectral content at multiple nodes on the DC-3 are shown in Chapter 10 (Appendix C). Factors that affect the update rate requirement include baseline motion type (i.e., first order or harmonic), expected platform dynamics, and vibration. It is important to know how much and what type of motion is expected on the platform because this information will influence the navigation sensor requirements. The expected motion also influences the type of navigation output required, i.e., position, velocity, orientation, etc… Since wing flexure frequency is inversely proportional to its displacement, a rough estimate of wing motion can be
determined from physical platform measurements such as baseline length and material rigidity.

Another important requirement to be identified is the timing accuracy for measurement integration. This integration can either be between two navigation sensors, between the navigation sensor and the ISR sensor, or both. If the motion dynamics are low, then a slight timing error will have less impact on the integration error because the sensor pose would not have changed significantly during the timing offset. For high dynamics, careful attention must be paid to the timing accuracy, so that the motion measurement is temporally aligned and properly removed from the ISR sensor measurement.

The final category of performance requirements for sensor stabilization involves the measurement of the vector offset between the sensors. The offset can be translational or rotational in nature and is often referred to as a lever-arm vector. This lever-arm offset can be between navigation sensors in a coherent multi-nodal integration or between the ISR sensor and the navigation sensor in single-aperture integration. As with the other performance requirement categories, the lever-arm accuracy requirements depend on the sensor integration architecture and stabilization measurements. More information about lever-arm accuracy requirements and flight test examples can be found in the reference (Dickman & Bartone, 2007).
2.4 Measurement Alignment Considerations

This section will describe both temporal and spatial alignment considerations that are important for GPS and INS integration. The material presented in this section is based on the work published in (Dickman & Bartone, 2007). A characterization of the basic INS self-alignment approach will be given as well as an external alignment based on a precise integrated GPS velocity vector.

2.4.1 Temporal Alignment Error Analysis

Timing accuracy requirements are largely driven by the platform dynamics and will differ depending on the type of measurement to be integrated between sensors (e.g., position, velocity, orientation, etc.). Platform motion limits need to be established so that a timing requirement can be determined; otherwise the timing performance will restrict the tolerable dynamics of the system. Figure 2-2 illustrates the integration error that can occur if two measurements are not temporally aligned.

![Figure 2-2: Timing Error Illustration](image)

A kinematic equation for position change can be used to relate position error to timing uncertainty as shown in equation (2-10) for acceleration measurements and equation (2-11) for integrated acceleration (delta velocity) (Halliday, Resnick, & Walker, 1993).
Integrated acceleration needs to be considered separately since it is often provided as a sensor output where the integration has been performed internally and is not subject to timing uncertainty. This is typically the case for inertial sensors. In both equations, noise has been neglected.

\[
\Delta R_{\text{error}} = V_0 t_e + A_0 \frac{t_e}{2} t_e^2
\]

(2-10)

where:
- \(\Delta R\) = Position Change during Update Interval [m]
- \(V_0\) = Average Velocity over Update Interval [m/sec]
- \(t_e\) = Timing Uncertainty [sec]
- \(A_0\) = Average Acceleration over Update Interval [m/sec^2]

\[
\Delta R_{\text{error}} = (V_0 + \Delta V) t_e
\]

(2-11)

where:
- \(\Delta R\) = Position Change during Update Interval [m]
- \(V_0\) = Average Velocity over Update Interval [m/sec]
- \(\Delta V\) = Integrated Acceleration [m/sec]
- \(t_e\) = Timing Uncertainty [sec]

The position error (\(\Delta R\)) increases proportionally to timing error (\(t_e\)), average velocity (\(V_0\)), integrated acceleration (\(\Delta V\)), and average acceleration (\(A_0\)). Similarly, these equations can be reversed to find the maximum acceptable timing tolerance (\(t_e\)) for a given velocity and position error. For example, if the platform is moving at \(V_0 + \Delta V = 100\) m/s (i.e., 195 kn), equation (2-11) says that the timing accuracy must be within 40 \(\mu\)s to keep the error within 4 mm. The accuracy requirement decreases with decreasing maximum velocity. The update rate is not involved in this equation unless acceleration is considered.
The timing requirements for matching velocity measurements are less demanding because the change in velocity over time is less than that of position. A slight time offset will create less of a measurement combination error.

2.4.2 Spatial Alignment Error Analysis
Spatial alignment of an integrated sensor system involves a progression of alignments beginning with the individual sensors. A pure strapdown inertial sensor, for example, must be told its initial position and how it is pointing. Next, the individual sensors must be related to each other (and to the aircraft body-frame) in terms of lever-arm offset and pointing alignment (i.e., boresighting). The total alignment process can be complex and involves minimizing many different error sources. This section will primarily deal with the alignment of an IMU with the navigation frame. Chapter 3 will discuss the alignment between navigation systems (e.g., GPS and INS). The final alignment between the navigation subsystem and the ISR sensors will not be considered in this document.

Literature generally refers to two types of inertial alignment: a self-aligning method and an external (transfer) alignment from one sensor to another (Titterton & Weston, 1997). With good enough measurements, the self-aligning technique would be preferable because there is the least chance of propagating erroneous measurements from one sensor to another. Self-alignment will be considered first and then external alignment will be considered.
To illustrate the accuracy of self-alignment from a tactical-grade IMU, consider the orientation error equations shown in equation (2-12) and equation (2-13) (Titterton & Weston, 1997):

\[
\delta_{\text{tilt}} = \pm \frac{B_{y,\text{accel}}}{g}, \quad (2-12)
\]

\[
\delta_{\text{azimuth}} = \frac{D_{y,\text{gyro}}}{\Omega \cos(Lat)} + \frac{B_{y,\text{accel}} \tan(Lat)}{g}, \quad (2-13)
\]

where:
- \( \delta_{\text{tilt}} \) = Pitch or roll tilt error [rad]
- \( \delta_{\text{azimuth}} \) = Azimuth (i.e., heading) error [rad]
- \( B_a = \) Accelerometer Bias a = x or y axis [g]
- \( g = \) Acceleration due to gravity = 9.81 [m / s^2]
- \( D_y = \) Gyroscope Drift, y axis [rad / hr]
- \( \Omega = \) Earth Rotation Rate 262.51614 E -3 [rad / hr]
- \( Lat = \) Geographic Latitude [rad]

If typical tactical-grade sensor accuracy numbers are used for accelerometer bias and gyroscope drift, the alignment accuracy can be determined as summarized in Table 2-1.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Sensor Accuracy</th>
<th>Alignment Accuracy, Lat = 45 deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tilt (pitch or roll)</td>
<td>Accel Bias = 300 ( \mu )g</td>
<td>0.3 mrad = 0.017 deg</td>
</tr>
<tr>
<td></td>
<td>Accel Bias = 1.5 mg</td>
<td>1.3 mrad = 0.086 deg</td>
</tr>
<tr>
<td>Azimuth (heading)</td>
<td>Gyro Drift = 1 deg / hr</td>
<td>94.3 mrad = 5.4 deg</td>
</tr>
<tr>
<td></td>
<td>Gyro Drift = 3 deg / hr</td>
<td>284.0 mrad = 16.3 deg</td>
</tr>
</tbody>
</table>

As can be seen from the table, a typical tactical-grade IMU can resolve tilt error with reasonable accuracy from its own accelerometer measurements. The same typical tactical-grade IMU, however, is not able to accurately resolve heading for most applications since the gyro drift rate leads to an azimuth alignment error > 1 deg shown in Table 2-1 (both using a 1.5 mg accelerometer bias). For better azimuth (i.e., heading)
alignment some external sensor would be required to measure the heading and then transfer the body to navigation frame alignment information to the IMU.

Assuming a static alignment, averaging can also be utilized to lessen the measurement noise according to $\sigma/\sqrt{N}$ where $N$ is the number of averaged samples. The limitation on averaging is the sensor bias. As the averaging window becomes longer the inertial bias grows in significance, the inertial measurements become non-stationary, and alignment biases will result.

Any sensor that can observe aircraft orientation relative to the navigation frame can aid in transfer alignment. Typically a navigation-grade IRU is used to measure heading because its gyro drift is on the order of 0.01 deg / hr (and thus a theoretical azimuth alignment accuracy of ~1.24 mrad). A more typical number might be a closer to a few mrad. One could also make use of GPS velocity, an ALS, flash ladar, or high-resolution camera to make a similar measurement. The GPS integrated velocity vector alignment will be considered subsequently.

2.4.3 Error Analysis of a GPS Velocity Vector Azimuth Alignment

GPS integrated velocity can be used as an external alignment for the IMU azimuth as long as the aircraft is in motion (Dickman & Bartone, 2007). Since the GPS measurements from a single node configuration cannot sense pure yaw rotation about the GPS antenna phase center, the heading measurement will not reflect sideslip and simply provides the aircraft ground track displacement. A multi-antenna GPS orientation
measurement would not have this limitation, but would contain more noise and would be more complex as will be considered in Chapter 3. The most reliable time for a velocity vector alignment would likely be on the ground during surface movement. It should be noted, however, that specific airborne maneuvers are often used to observe and calibrate some error sources and a ground calibration would be incomplete. The GPS velocity vector alignment could be used as an initial prediction in a Kalman filter to improve convergence time. This section will consider the potential azimuth accuracy from a GPS velocity vector azimuth alignment.

The total position displacement is calculated according to equation (2-14) where \( r \) is a three element East, North, and Up (ENU) position vector as a function of time, \( v_0 \) is the three element average ENU velocity vector, and \( \zeta \) is a three element ENU displacement noise vector. This integrated velocity (displacement) vector is used to calculate the change in orientation from one epoch to the next:

\[
\ddot{r}_i = \dot{r}_i + v_0 \cdot dt + \zeta = \dot{r}_i + \delta \dot{r} + \zeta
\]  \[m\]  \hspace{1cm} (2-14)

Figure 2-3 shows the simplified contribution of the displacement noise, \( \zeta \), to pointing error, \( \varepsilon \). For this analysis, a zero acceleration and constant \( v_0 = 51 \text{ m/s} \) (i.e., 99 kn) east velocity profile is assumed over one second. For the sake of theoretical comparison, rough numbers will be presented for velocity and velocity noise. Real numbers for velocity error will be presented at the end of this section from the flight test data.
Figure 2-3 considers motion in the East / North plane and shows the parameters for calculation of azimuth error (i.e., heading). The geometric relationship between integrated velocity error and pointing error is shown in Table 2-2. The GPS integrated velocity vector provides information about tilt and azimuth errors, so the error analysis will be presented, but only azimuth will be used in the alignment process since the IMU vertical self alignment is sufficient. Consider first, a theoretical integrated velocity error vector of [4,4,10] mm and a time interval of 10 ms for example; the azimuth and tilt errors work out to be 0.446 and 1.114 deg respectively as shown in Table 2-2 for the given assumptions.

<table>
<thead>
<tr>
<th>Error (10 ms Interval)</th>
<th>Defining Equation</th>
<th>Integrated ENU Velocity Error [mm]</th>
<th>Pointing Error [deg (mrad)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth</td>
<td>$\varepsilon_a = \tan^{-1}\left(\frac{\varsigma_e}{\delta r_e + \varsigma_r}\right)$</td>
<td>$\varsigma = [4, 4, 10]$</td>
<td>0.446 (7.78)</td>
</tr>
<tr>
<td>Tilt</td>
<td>$\varepsilon_t = \tan^{-1}\left(\frac{\varsigma_t}{\delta r_t + \varsigma_r}\right)$</td>
<td>$\varsigma = [4, 4, 10]$</td>
<td>1.114 (19.45)</td>
</tr>
</tbody>
</table>

Notice that the denominator of both azimuth and tilt error equations in Table 2-2 contain the displacement term $\delta r$. As a result, both errors are inversely proportional to craft velocity and proportional to measurement update rate.
The true displacement noise determined from flight test data can then be used to determine a more representative angular pointing accuracy, which can be attained from stand-alone GPS measurements. Based on the integrated velocity accuracy presented in (Dickman & Bartone, 2007), the $1\sigma$ integrated velocity error vector used for this analysis will be $\zeta = [2.25, 2.85, 4.03]$ mm. By applying the equations from Table 2-3 to the data-derived integrated velocity error, the $1\sigma$ azimuth error is found to be 0.319 deg (5.56 mrad) and the $1\sigma$ tilt error is found to be 0.453 deg (7.90 mrad) as shown in Table 2-3.

<table>
<thead>
<tr>
<th>Error (10 ms Interval)</th>
<th>Defining Equation</th>
<th>Integrated ENU Velocity Error [mm]</th>
<th>Pointing Error [deg (mrad)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth</td>
<td>$\varepsilon_a = \tan^{-1}\left(\frac{\zeta_e}{\zeta_n + \zeta_e}\right)$</td>
<td>$\zeta = [2.25, 2.85, 4.03]$</td>
<td>0.319 (5.56)</td>
</tr>
<tr>
<td>Tilt</td>
<td>$\varepsilon_t = \tan^{-1}\left(\frac{\zeta_u}{\zeta_n + \zeta_e}\right)$</td>
<td>$\zeta = [2.25, 2.85, 4.03]$</td>
<td>0.453 (7.90)</td>
</tr>
</tbody>
</table>

The azimuth accuracy number is the important one for IMU alignment using GPS. The 5.56 mrad azimuth pointing error shown in these results is approximately 2 times worse than the typical navigation-grade inertial azimuth accuracy, but this is still good enough to allow a Kalman filter to converge. A longer time interval would provide a longer time-baseline in the GPS velocity measurement and thus lead to an alignment accuracy improvement, but the motion dynamics in the interval must then be considered.

### 2.5 Flight Test and Data Collection Description

Flight test data will be used for illustration throughout this dissertation, so it will be introduced in this section for later use. Two flight tests were conducted as part of this research, but only data from the second will be used for illustration. Two field tests were also conducted using a stationary aircraft sitting parked on the tarmac using the same
sensor configuration aboard the aircraft. The static test will be described in more detail in section 4.3.1. The data collection hardware will be generally described and additional detail will be provided later as required.

2.5.1 Flight Test Description
A flight test was conducted on April 12, 2006 using the Ohio University DC-3 Aircraft. Flight dynamics include takeoff, landing, level flight, and some extensive banking. The GPS ground track is shown in Figure 2-4 to illustrate the flight profile from the Ohio University Airport (UNI), in Albany, OH to the Parkersburg, WV Airport (PKB), approximately 30 nmi away.

![Figure 2-4: GPS Ground Track During Flight Test](image-url)

To support the research described in this dissertation, a data collection system was assembled with extensive in-flight measurement and remote sensing capabilities onboard the Ohio University Douglas DC3 flying laboratory. The hardware consisted of two data
collection computers, five NovAtel OEM4 GPS receivers, three tactical-grade Northrop Grumman LN200 fiber-optic gyro equipped IMUs (two sample-synchronized units, and one 3600 Hz gyro measurement update unit), three low-cost Microstrain 3DM-GX1 IMUs, a navigation-grade Honeywell HG1150 IRU, a Riegl LMSQ140 Airborne Laser Scanner (ALS), battery backup power, and a rack-mounted keyboard, video, and mouse unit. The sensors used during the flight test were arranged as shown in Figure 2-5.

![Figure 2-5: DC3 Sensor Arrangement](image)

GPS L1 and L2 CP data were collected from five nodes using five NovAtel OEM4 receivers connected to five dual-frequency patch antennas installed as shown in Figure 2-5. GPS L1 CP measurements were logged at a 100 Hz rate from nodes 1-4 and at 10 Hz from node 5. A set of five GPS SVs was chosen to be free of cycle slips for processing as shown in the skyplot of Figure 2-6 for the duration of the flight test. The SV identification number is shown at the beginning of its corresponding track. A 20-
degree mask angle was also applied to provide the best measurement data. SV 2 was chosen as the key for differencing, so only four satellite pairs were used to calculate the baseline.

A Honeywell HG1150 navigation grade IRU was used to measure platform attitude orientation and was located at the INS location in Figure 2-5. This unit was made for commercial aviation and was not intended for data processing and analysis. As a result the measurements are heavily filtered to provide a smooth display and have a latency of a few milliseconds according to the ARINC 429 specification. Additionally, the orientation resolution is limited by quantization at 1/10 mrad (Honeywell). The IRU was installed near the aircraft center of gravity and physically aligned approximately with the aircraft body frame.
Two types of IMUs are listed in Figure 2-5. The low-cost model has an LC subscript after its label while the tactical-grade model has a TG subscript. Each GPS antenna location will be referred to as a node and two nodes constitute a baseline. The main node will always be listed second in the baseline label. For example, baseline 12 indicates that node 1 is secondary and node 2 is the main. A more detailed description of the hardware installation at each node will be provided in chapter 8 (Appendix A).

The data was collected using the QNX 6.3 real-time operating system. The real-time data collection software will be described in further detail in chapter 8 (Appendix A). The stabilization solution was determined in post-processing software for the purpose of easier algorithm development and analysis. It should be noted that the ambiguity resolution and 100 Hz GPS baseline solution for an hour of data could be processed in approximately ten minutes on a 3 GHz Pentium IV with 2 gigabytes of RAM, so a real-time stabilization solution is feasible.

**2.5.2 Data Collection Hardware**

The conceptual block diagram for the main GPS/INS data collection system is shown in Figure 2-7.
A single remote node is enclosed in a dashed line to indicate the equipment to be installed at each stabilization node. GPS RF, inertial data, timing, and synchronization were all connected to the data collection rack in the center of the aircraft cabin. The RF from all five nodes (fifth not shown in Figure 2-7) was brought back to the GPS receiver module and the associated data was then sent to the data collection computer for processing and recording. Similarly, the HG1150 navigation-grade IRU was connected to the data collection computer via an ARINC 429 bus for processing and recording. Another item not shown in this block diagram is the LMS-Q140 ALS. It was connected to a second data collection computer along with the 3600 Hz LN200 IMU and the fifth OEM4 GPS receiver.

The data collection equipment rack is shown in Figure 2-8.
The switching control to sequence the IMU power is shown on line 1. The GPS receiver module is shown in line 2. The primary data collection computer is shown in line 3. The KVM switch and display are shown by line 4. The secondary data collection computer is shown by line 5. The battery backup to prevent IMU power loss is shown in line 6. Finally the air data computer and IRU control-head are shown in line 7.
3 Frame Alignment and Measurement Integration

3.1 Frame Alignment and Measurement Integration Introduction

3.1.1 Frame Alignment and Measurement Integration Overview

The overall goal of the work presented in this dissertation is to provide high accuracy relative positions for the purpose of sensor stabilization. To that end, measurements from several navigation sensors must be integrated with the ISR sensors and measurement alignment is needed to ensure that the measurements are integrated in a common coordinate frame. Two stages of alignment are required: one to align the navigation subsystem sensors with each other and another to align the ISR subsystem with the navigation subsystem. To remain within the scope of this dissertation, it will be assumed that the ISR subsystem alignment is simply a static calibration that has already been determined and consequently, only the navigation sensor alignment will be emphasized. To that end, this chapter will describe several ways to estimate the frame alignment between the navigation sensors and to optimally combine their measurements. Although the intent of an alignment and integration estimator is primarily to keep the navigation frames of the GPS and INS sensors aligned using dynamic measurement data, a secondary benefit of this integration (in the Kalman filter cases) is the provision of a lower noise output. Navigation sensor integration can be complex when the sensors are not co-located. The coordinate frame alignment strategy can often differ depending on the installed sensor locations and requires some analysis to determine the best approach as was described in previous work (Dickman & Bartone, 2007). For this chapter, it will be assumed that the sensors are either co-located or mounted on a rigid body so a GPS
alignment tradeoff versus a higher quality INS transfer alignment will not be described further; only GPS data will be used for the alignment. There are several ways to implement an alignment estimator using GPS measurements and four of them will be described in this chapter. It should also be noted that the GPS velocity vector alignment approach described in (Dickman & Bartone, 2007) will not be described in detail, but some of the performance results will be used for comparison.

The first technique to be described in section 3.2 is a loose coupling of the orientation measurements from the GPS and INS sensors. In this case, the INS orientation comes directly from the INS sensor rotation measurements and the GPS orientation is derived from the pointing angle of the baseline position vector. This approach involves minimal INS computation; the attitude and heading are either extracted directly from an IRU output or they are mechanized from the rotation rate measurements of an IMU. The GPS computation is moderate; the CP baseline must be determined along with the CP ambiguities, and then the baseline-pointing angle must be determined relative to the navigation frame to form attitude and heading estimates. The GPS and INS orientation quantities are then combined in a three-state complementary Kalman filter to estimate the INS orientation error states. This filter will be able to maintain a fine orientation alignment as well as smooth the GPS orientation measurement noise.

The second technique, described in section 3.3, loosely couples the measurements from the GPS and INS sensors in the position domain. In this case, the INS baseline position
vector is synthesized from a static baseline survey and dynamic INS orientation measurements while the GPS baseline solution is derived from the ambiguity-resolved DDCP. Moderate INS computation is required to synthesize the baseline measurement and minimal computation is required to form the GPS baseline solution. This three-state complementary Kalman filter maintains a fine alignment by unbiasing the baseline differences and minimizes the baseline noise.

Section 3.4 will discuss the third technique, which does not involve a continuous estimator, but rather a one-time mean coordinate frame offset calibration. The orientation estimates from the GPS and INS sensors are averaged over some interval to determine their mean frame offset. This is the same approach as used in (Dickman & Bartone, 2007) as a coarse estimate. Determination of the static frame bias in a single data set does not preclude the later use of a GPS/INS Kalman filter because the GPS & INS measurements remain independent and would allow more rapid filter convergence. A static calibration would be sufficient for use in a system that does not experience significant sensor drift (i.e., to an extent that causes the stabilization accuracy to exceed the system accuracy requirements). This approach might also be preferable to running a Kalman filter for continuous frame alignment if the INS measurements need to stay independent of the GPS measurements for GPS error detection and correction. If the frame alignment is already known, a Kalman filter can also be designed to refine the antenna position survey used for baseline synthesis.
The final technique to be described in section 3.5 tightly integrates measurements in the range domain to maintain a fine orientation alignment in the presence of sensor drift. The INS computation is the most complex because DD measurements must be synthesized from a static baseline survey and dynamic INS orientation measurements. The GPS measurement processing is minimal because the ambiguity-resolved DDs are used directly in the measurement vector. The structure of the filter takes care of the domain conversion between the raw DD measurements and the output orientation error states. The filter makes use of the low-noise INS measurements to minimize the noise in the raw GPS DD measurements. The concept and algorithm will be presented in subsequent sections, but the results and analysis will be left for future work.

Each of these approaches will be described in subsequent sections; all will yield the desired alignment, but with different limitations and varying levels of performance. The inertial sensor used in each of the integration approaches can either be a navigation-grade IRU whereby the filter will provide the dynamic frame alignment and limited IRU drift compensation or the inertial sensor can be a tactical-grade IMU, which would require the filter to estimate the dynamic frame alignment as well as compensate for more severe IMU drift. The remainder of section 3.1 will provide the background information for the rest of the chapter.

### 3.1.2 Sensor Coordinate Frames and Alignment Error

A critical step in measurement coupling is bringing the measurements from different sensors and different coordinate frames into agreement as illustrated by Figure 3-1.
Several parameters must be determined for the GPS and INS attitude alignment: the physical GPS antenna locations must be related to the body frame, a coarse estimate of the between-sensor frame alignment should be identified as an initial prediction into the filter (optional), the variations in the dynamic local-level navigation frame (for fixed reference points), and the INS attitude drift. For a simple filter, there may not be enough measurement observability to independently determine each of the alignment parameters due to error masking and since there are likely more unknowns than measurements, so the filter would attempt to correct the composite errors. It should be noted that the combination of a more complex filter and certain flight maneuvers might cause some errors to become observable that were previously masked. A single-axis composite alignment error term, intentionally shown without units to be applicable to both position and orientation domains, would consist of several error terms as shown:

\[ \varepsilon_{\text{align}} = \varepsilon_{\text{bias}} + \varepsilon_{\text{synth}} + \varepsilon_{\text{noise, GPS}} + \varepsilon_{\text{drift, INS}} + \varepsilon_{\text{flex}} + \varepsilon_{g} + \varepsilon_{\text{cross}} \]  

where:
- \( \varepsilon_{\text{bias}} \) = primary frame offset bias
- \( \varepsilon_{\text{synth}} \) = measurement synthesis errors
- \( \varepsilon_{\text{noise, GPS}} \) = GPS measurement noise
- \( \varepsilon_{\text{drift, INS}} \) = INS drift
- \( \varepsilon_{\text{flex}} \) = baseline flexure
- \( \varepsilon_{g} \) = gravity anomaly error
- \( \varepsilon_{\text{cross}} \) = cross-axis misalignment error
Many of these error terms can be minimized and neglected under some conditions such as gravity anomaly (with a typical vertical deflection of 25 μrad) and baseline flexure for rigid baselines. Many of the errors will only manifest themselves when measurements are used for baseline synthesis and will be described more in section 3.1.4. The bias error can be thought of as a calibration and will be constant with time. The variance of the GPS noise term will remain approximately the same although it will vary slightly with GPS constellation geometry. The INS drift error will grow as random walk because of the integration of noise. Finally, the baseline flexure term will be a function of platform dynamics and thus causing the relative baseline position to change without observation by the INS. Many cross-coupled errors will be neglected by focusing on applications during flat and level flight.

This chapter aims to resolve the frame misalignments by observing and then minimizing \( \varepsilon_{\text{align}} \). The total navigation frame error definition can be represented as truth plus error:

\[
\theta_{\text{nav}} = \theta_{\text{true}} + \varepsilon_{\text{align}}
\]  

(3-2)

While there is only a single true navigation frame, \( n \), each sensor will have its own modeled navigation frame alignment errors due to position and north-pointing errors so the sensor name will be used when referring to its misaligned navigation frame (e.g., \( n_{,\text{gps}} \) refers to the misaligned GPS navigation frame). The body frame, \( b \), is usually the most convenient for calibration because the sensor positions can be physically surveyed with respect to the body-frame axes. The body-frame follows the forward, right wing,
down convention. An example of a body-frame calibration is the measurement of the lever-arm between the GPS antennas and the INS sensor. A total-station can typically provide cm-level accuracy in the body frame and then a rotation DCM can be used to dynamically relate the body frame to the navigation frame. If the desired level of alignment accuracy cannot be attained in the body frame, then navigation sensor measurements can be used to calibrate the coordinate frames using their respective erroneous navigation frames, \( n \), as will be described in this chapter. The navigation frame used for measurement integration follows the INS north-pointing NED local-level convention. It should be noted that a measurement-derived alignment option might not be available for the ISR subsystem alignment, so the attainable alignment accuracy might limit the overall system accuracy and must be considered in a feasibility study.

For this dissertation research, the sensor positions were initially measured using an optical total measurement station with about 1 cm of accuracy, but this was insufficient for orientation calibration so the raw sensor measurements in the GPS and INS navigation frames (\( n,_{\text{gps}} \) and \( n,_{\text{ins}} \) respectively) were used for the initial orientation calibration. GPS baselines are typically expressed in a local-level coordinate frame (e.g., ENU) with respect to a reference position: either static or dynamic. A frame will be labeled with a subscript 0 to indicate that it uses a static reference point (e.g., \( b_0 \) or \( n_0 \)). When using INS data, a dynamic North-pointing inertial navigation frame (e.g., NED) is a convenient frame to dynamically relate the GPS local-level measurements to those from the INS because most inertial mechanizations are written in this frame.
3.1.3 Kalman Filter Overview

This section will provide a generalized framework for the Kalman filter, which follows that of (Brown & Hwang, 1997). The filters described in this chapter will be complementary in nature, which means that the measurement vector is the difference between two sensor measurements of some quantity, e.g., position, attitude, range, etc, and the output states are the combined sensor error as described by equation (3-1). A block diagram is shown in Figure 3-2 for a generic complementary Kalman filter (Brown & Hwang, 1997).

![Figure 3-2: Complementary Kalman Filter](image)

This figure is generic because the sensor measurements, \( y \), are not specifically described and the domain transformation, \( H \), is not specific to a certain type of measurement. This is intentional, so that the figure applies to each different type of measurement coupling to be described in subsequent sections. In general, the INS measurement will be referred to as the reference data and the GPS measurements will be referred to as the calibration data. Bias errors in the reference data will be corrected in the filter using the calibration data and noise in the calibration data will be smoothed with the reference data. Each filter must be tailored to the specific application and the type of measurement integration;
however, the framework presented in this section will provide a common point of reference for the discussions to follow.

### 3.1.3.1 Step 0 – Filter Initialization

This step is conducted to initialize the filter computations. An initial prediction of each error state, $\hat{x}_{k^-}$, must be specified along with an estimate of the error prediction covariance, $P_{k^-}$. Since the filter is estimating error states, it is usually reasonable to predict the errors in all states to be zero. The associated error covariance matrix for this prediction would then equal the uncertainty in the measurements. Initial prediction errors will mainly affect the convergence time and will not greatly impact the overall performance as long as they are reasonable. In some cases where the error estimates are too conservative in one state and too liberal in another, the filter may diverge; but this is less likely in a simple filter.

Next, the parameters that define the filter structure must be assigned. The *system noise covariance matrix*, $Q$, is typically a diagonal matrix calculated from the variance of the system noise and it tells the filter what kind of dynamic uncertainty to expect in the system. The *measurement error covariance matrix*, $R$, is typically a diagonal matrix calculated from the variance of the measurement noise and it tells the filter how much noise is expected in the measurements. The *measurement transformation matrix*, $H$, tells the filter how to relate the *filter measurement matrix*, $z$, to the *estimation states*, $x$. Finally, the *state transition matrix*, $\Phi$, tells the filter how to predict the estimation states and prediction error covariance at the next time epoch based on the current information.
3.1.3.2 Step 1 – Form the Kalman Gain

The Kalman gain is a unitless quantity that tells the filter how much emphasis to place on the input measurement vector versus the state predictions:

\[
K_k = \frac{P_k - H_k^T (H_k P_k^{-1} H_k^T + R_k)^{-1} H_k}{P_k} = \text{Kalman Gain}
\]  

(3-3)

where:

- \(H_k\) = Measurement Transformation Matrix
- \(P_k\) = Predicted Estimation Error Covariance
- \(R_k\) = Measurement Error Covariance

A large gain value says that the filter has high confidence in the measurement vector and it is weighted heavier while a small gain value indicates a high confidence in the prediction and deemphasizes the measurement vector.

3.1.3.3 Step 2 – Form the Measurement Matrix

The measurement matrix in a complementary filter contains the difference between the two sensor measurements. It is conventional for the INS measurement, \(y_{INS}\), to be the reference quantity and the calibration quantity to be the GPS measurement, \(y_{GPS}\), such that the reference is subtracted from the calibration:

\[
Z_k = y_{GPS} - y_{INS} = \text{Filter Measurement Matrix}
\]  

(3-4)

The measurement matrix will contain the combined measurement noise and bias errors from both sensors. The measurement quantities used to form the filter measurement matrix do not necessarily have to be in their raw form, but they must both be measurements of the same quantity (e.g., position or velocity).

3.1.3.4 Step 3 – Determine the State Estimates

The state error vector provides an estimate of the errors in the measurement vector. It uses the Kalman gain, the measurement, and state prediction vectors to come up with an
optimal state error estimate; in this case optimality is defined by a minimum variance unbiased estimate:

\[ \hat{x}_k = \hat{x}_k^- + K_k(z_k - H_k \cdot \hat{x}_k^-) = \text{State Estimate Vector} \]  \hspace{1cm} (3-5)

where:

- \( \hat{x}_k^- \) = State Prediction Vector
- \( z_k \) = Filter Measurement Matrix

The quantity in parenthesis is commonly referred to as the innovation since it contains the new information provided by the measurement with respect to the prediction. In this equation, the measurement transformation matrix converts the state prediction into the measurement domain so that it can be differenced with the filter measurement matrix. The Kalman gain scales the innovation appropriately with respect to the state predictions and when combined, they produce optimal error state estimates.

### 3.1.3.5 Step 4 – Estimation Error Covariance

The Kalman gain also determines how to scale the estimation error covariance for the current estimate:

\[ P_k = (I - K_k \cdot H_k)P^-_k = \text{Estimation Error Covariance} \]  \hspace{1cm} (3-6)

where:

- \( P^-_k \) = Predicted Estimation Error Variance
- \( I \) = Identity Matrix

A large gain places more weight on the measurement covariance because the measurement vector is statistically deemed more important that the state vector.
3.1.3.6 Step 5 – Forward Prediction
Based on the state transition matrix, the error states and the error covariance matrix can be predicted for the next time epoch. The state transition matrix is formed using a linear relationship between the states:

\[
\hat{x}_{k+1} = \phi_k \cdot \hat{x}_k \quad (3-7)
\]

\[
P_{k+1} = \phi_k \cdot P_k \cdot \phi_k^T + Q_k \quad (3-8)
\]

where:
\[\phi_k = \text{State Transition Matrix}\]
\[Q_k = \text{System Noise Covariance}\]

If higher order states are available, they are used in conjunction with the update interval to predict ahead to the next time. If they are not available, the current state is used as the best prediction for the next epoch.

3.1.3.7 Step 6 – Coast or Update Decision
At this point, a decision must be made whether to coast on the predictions or whether to incorporate a new measurement. In most cases, the measurement should be included and allow the filter to weigh it appropriately, but if there is cause to doubt the validity of a measurement, it may be better to discard it in favor of the prediction. This decision can be based on the magnitude of the filter innovation or on some other quantity. If the next measurement is to be incorporated then the process should return to step 1, but if the filter is to coast, it should repeat step 5 until valid new information is available.

3.1.4 Dynamic Baseline Synthesis
The purpose of this section is to describe how to synthesize baseline estimates in a dynamic navigation frame using a body-frame GPS antenna survey and dynamic INS orientation measurements. This approach does not use body flexure information, and
thus makes an implicit assumption of a rigid body. The first step is to perform a static frame calibration to determine the relationship between the static GPS navigation frame, $n_{0,GPS}$, and the static INS navigation frame, $n_{0,INS}$, and will be described as finding the orientation offset.

A static GPS antenna survey will be used to form a static GPS antenna baseline, $b^{n_{0,GPS}}$, which will then be used to determine the static baseline orientation with respect to the GPS navigation frame as will be described in section 3.1.5. A typical GPS baseline survey would be conducted using dual-frequency CP data for high accuracy measurements. The static data could be processed using the Online Positioning User Service (OPUS) (NOAA, 2006), for example, to provide an estimate of the precise point positions of each antenna.

An IRU should be used to find the time-concurrent orientation of the body with respect to the INS navigation frame. The static INS orientation provides a means to relate the static INS navigation frame to the INS body frame, $C^{b,INS}_{n,INS}$. A coarse static frame offset can be formed by subtracting the INS orientation measurement from the GPS orientation estimate, and then used to form a calibration DCM, $C^{n,INS}_{n_{0,INS}}$.

Once the static GPS baseline survey and frame calibration are known, the GPS baselines can be expressed in the INS body frame (referred to as the zero attitude baseline (ZAB)) according to the procedure described here. The ZAB vector is formed by converting the
GPS baseline survey into the inertial body frame, \( b_{\text{ins}} \), which is coincident with the \( b_{\text{zab}} \) frame. Formation of the ZAB can be achieved with a series of coordinate transformations as shown:

\[
\hat{b}_{\text{zab}} = \hat{b}_{\text{ins}} = C_{n_0,\text{gps}}^{n_0,\text{ins}} \left( b_{n_0,\text{gps}}^{n_0,\text{gps}} + \varepsilon_{\text{survey}}^{n_0,\text{gps}} + \varepsilon_{\text{align}}^{n_0,\text{gps}} \right) \quad [\text{m}] (3-9)
\]

where:

- \( b_{n_0,\text{gps}} \) = initial static survey baseline vector in the GPS navigation frame [m]
- \( C_{n_0,\text{gps}}^{n_0,\text{ins}} \) = static GPS to INS orientation calibration DCM [unitless]
- \( C_{n_0,\text{ins}}^{n_0,\text{ins}} \) = static INS navigation frame to zero attitude body frame DCM [unitless]
- \( \hat{b}_{\text{zab}} \) = ZAB vector in the body frame [m]
- \( \varepsilon_{\text{survey}}^{n_0,\text{gps}} \) = error in the initial static survey baseline error [m]
- \( \varepsilon_{\text{align}}^{n_0,\text{gps}} \) = error due to GPS/INS frame misalignment in the initial survey [m]

The error terms in equation (3-9) are important because they will affect the quality of the baseline synthesis. The \( \varepsilon_{\text{survey}} \) error is a result of either the static GPS survey or the total station measurements. The \( \varepsilon_{\text{align}} \) error is the measurement frame bias error described in equation (3-1). Both errors will be addressed in the upcoming sections. \( C_{n_0,\text{gps}}^{n_0,\text{ins}} \) is a static survey frame alignment DCM and aligns the initial survey baseline with the INS. The ZAB vector can then be propagated dynamically using inertial attitude estimates to form a synthesized baseline in the navigation frame as shown:

\[
\hat{b}_{\text{ins}}^{n_0,\text{ins}} = C_{b_{\text{ins}}}^{n_0,\text{ins}} \hat{b}_{\text{ins}} + \varepsilon_{\text{flex}}^{n_0,\text{ins}} \quad [\text{m}] (3-10)
\]

where:

- \( \hat{b}_{\text{ins}}^{n_0,\text{ins}} \) = zero attitude baseline vector in the body frame, rigid-body [m]
- \( C_{b_{\text{ins}}}^{n_0,\text{ins}} \) = dynamic INS body to navigation frame DCM [unitless]
- \( \hat{b}_{\text{ins}}^{n_0,\text{ins}} \) = dynamic synthesized baseline vector in the INS navigation frame [m]
- \( \varepsilon_{\text{flex}}^{n_0,\text{ins}} \) = error due to baseline flexure [m]

Flex error is caused by baseline flexure after the static survey has been conducted and is not observed by a remote IMU. This error will primarily affect the dynamic synthesized...
baseline. The synthesized dynamic baseline, $\hat{b}^n_{ins}$, can be aligned with the GPS navigation frame using a dynamic frame calibration DCM, $C^n_{n,ins}$. This DCM is treated separately from the static frame calibration DCM used in equation (3-9) because different processing and often different data sets are used to form them. The dynamic frame calibration DCM can also account for the residual errors in the static survey alignment. The details of how to calculate this matrix will be described in future sections, but at this point it will simply be stated that it is able to estimate the INS orientation drift and in most cases it can estimate the static frame alignment bias. The entire baseline synthesis procedure, from static survey to dynamic baseline estimate, can be described in a single equation as:

$$\hat{b}^n_{n,gps} = C^n_{n,ins} C^{b,ins}_{n,ins} C^{n,ins}_{n_0,gps} b^{n_0,gps}$$

It is often convenient to have an equation that already includes the static frame calibration and simply propagates an existing ZAB using dynamic orientation measurements. The first two terms in equation (3-11) can be combined to provide the propagation while the last three can be combined to provide the static frame calibration. In this case, the ZAB will be transformed into the dynamic GPS navigation frame in a way similar to equation (3-10) except that the body to navigation frame DCM, $C^b_{n,gps}$, includes an alignment with the GPS navigation frame as will be described in sections 3.2 through 3.5:

$$\hat{b}^n_{n,gps} = C^n_{b,ins} \hat{b}^{b,zab}$$
3.1.5 Multi-Antenna GPS Orientation Determination

3.1.5.1 Multi-Antenna GPS Orientation Overview
This section will describe the procedure to determine the GPS attitude and heading from multiple GPS antennas. GPS velocity provides one alignment technique (Dickman & Bartone, 2007), but since it only uses a single antenna, it is not able to determine the body-frame pointing angle separately from the angle of motion (e.g., sideslip). While orientation is not the final use of the GPS CP measurements in this chapter, it is necessary for alignment with the inertial navigation frame, which is needed for high accuracy applications at the mm level. The navigation frame was chosen to adhere to the NED navigation frame standard so that measurement integration and DCM usage would be more conventional. The body frame and platform orientation sign conventions are shown in Figure 3-3 where X, Y, and Z coincide with the forward, right wing, down convention, respectively. Roll, pitch, and yaw are right handed rotations about these axes.

![Figure 3-3: Body Frame and Orientation Sign Convention](image)

In order to estimate the absolute platform attitude and heading from GPS CP data, the baseline must be determined between the two or more antennas. This baseline is then
converted into Euler angles with respect to the local level navigation frame. A more complete least squares approach to orientation determination is described in (Parikh, Soloviev, & van Graas, 2007). A simpler geometric approach is favored in this chapter since the measurement coupling will only be performed using a single baseline. The generic procedure for multi-node GPS orientation determination will be described in this section, but a special case simplification will be used for the results to be shown in the next section.

Platform orientation is determined in the body frame with respect to the navigation frame, so GPS measurements may need to be transformed into the proper coordinate frame. If the GPS antenna baseline was not installed in alignment with the body frame, the navigation-frame baseline vector measurement must be rotated into alignment with the body frame axes using a DCM such as the one in equation (2-9), and then re-expressed in the navigation frame in order to determine the platform orientation.

The magnitude of the baseline projected in each axis of the body frame and expressed in the navigation frame will give an indication of the relative observability of each orientation component. A projection as described by the unit vector representation shown in equation (3-13) provides a means of approximating the aligned vector components:

\[
\vec{p}_{x,y,z}^n = \vec{b}_{x,y,z}^{n,GPS} \cdot \vec{b}_{x,y,z}^{n,GPS} = \vec{b}_{x,y,z}^{n,GPS} \cdot \hat{\vec{b}}_{x,y,z}^{n,GPS}
\]  

(3-13)

where:
\( \bar{p}^n_{x,y,z} = \) Scalar projection of the baseline onto the navigation-frame body axes \([\text{m}]\)  
\( \vec{b}^n_{\text{GPS}} = \) Measured GPS baseline vector \([\text{m}]\)  
\( B^n_{\text{GPS}} = \) Body axes expressed in the navigation frame \([\text{m}]\)

This equation makes the assumption that a relationship is already known between the body frame and the navigation frame. Another way to express the scalar projection is using the cosine of the angle between the vectors as shown in equation (3-14):

\[
\bar{p}^n_{x,y,z} = |\vec{b}^n_{\text{GPS}}| \cdot \cos \Theta_{\text{cal}} \quad (3-14)
\]

where:
\( \Theta_{\text{cal}} = \) Calibration angle between the measured baseline and the body-frame axis \([\text{rad}]\)

The projection approach is not optimal for several reasons. First, a scalar projection does not make complete use of the information in the vector unless it is already aligned with the desired body-frame axis. Some orientation information might be strongly or weakly observable from any given vector. Second, the calibration angles require a rigid body assumption unless some measurement of baseline flexure can be made using a remote IMU. Third, the body axes are difficult to observe and could potentially make a projection inaccurate.

In the special case where the baseline vector is already aligned with the body frame, the projection drops out and will not change the vector.

The platform orientation can then be determined using components of the rotated baseline vector, the baseline projection, or the pre-aligned baseline vector as appropriate:
\[ \phi = \arctan \left( \frac{p_x^n}{p_y^n} \right) = Roll \quad (3-15) \]

\[ \theta = \arctan \left( \frac{-p_x^n}{p_y^n} \right) = Pitch \quad (3-16) \]

\[ \psi = \arctan2 \left( \frac{p_y^n}{p_x^n} \right) = Heading \quad (3-17) \]

where:
\[ \phi = \text{platform pitch angle} \quad [\text{rad}] \]
\[ \theta = \text{platform roll angle} \quad [\text{rad}] \]
\[ \psi = \text{platform heading angle} \quad [\text{rad}] \]

The \( p \) variable represents the components of the scalar projection described in equation (3-13). The same expressions can be written without the need for a projection if the GPS antenna baseline had been already installed in the body frame \( x \)-axis by replacing the \( \vec{p}_s^n \) vector with the navigation frame baseline vector \( \vec{b}^n \). This special case will be considered more in the next section.

### 3.1.5.2 Multi-Antenna GPS Orientation Illustration

For the results shown in this section, a special case of equation (3-16) and equation (3-17) is used to simplify the processing. Baseline12 (node 1 relative to node 2) lies in the body frame \( x \)-axis, so no projection is required to extract pitch and heading information. Static GPS CP data will be processed according to section 2.1.1 to determine the baseline, which will then be used to find the baseline orientation according to equation (3-16) and equation (3-17).
The GPS baseline relative position solution in the navigation frame, $\vec{b}^n$, is shown in Figure 3-4. The first three subplots show the respective East, North, and Up components of the baseline vector. The last subplot contains the baseline magnitude.

![Figure 3-4: GPS Carrier Phase Baseline Results for Static Tarmac Data](image)

Baseline is aligned with the body $x$-axis so the components of $\vec{b}^n$ can be used directly in equation (3-16) and equation (3-17) to determine the pitch and heading respectively, which have been plotted in Figure 3-5. With no baseline data in the body frame $y$-axis, there is no roll observability in this data set. Since the measurements are static, the mean baseline measurements shown in the subplot legends can be used to predict the mean orientation.
The mean orientation numbers will now be compared using the mean baseline components from static data as an illustration of the orientation determination process.

For example, the pitch is derived from the Down-component of the baseline vector (third subplot of Figure 3-4), -1.44 m, and the vector magnitude (fourth subplot of Figure 3-4) according to equation (3-16) to yield ~10.66 deg. This prediction is close to the mean shown in the second subplot of Figure 3-5. Similarly, according to equation (3-17), the inverse tangent of the mean East and North components from the first and second subplots of Figure 3-4 yields a heading angle of ~151.76 deg. These predictions match the mean values in Figure 3-5 to the extent possible given the multipath errors present in the data. A comparison and calibration of these measurements will be made with concurrent INS measurements in the following sections.
3.2 **Loose Orientation Integration Filter**

3.2.1 **Loose Orientation Filter Overview**

This section will describe a technique to determine and constantly maintain a fine alignment between the GPS and INS frames so that an accurate alignment DCM, $C_{n,\text{gps}}^{n,\text{ins}}$, can ultimately be used to synthesize INS baseline estimates according to equation (3-12) as baseline positions are needed by the system. The most obvious way to keep the GPS and INS coordinate frames aligned is to loosely couple their orientation measurements in a complementary Kalman filter. A loose integration of this type requires knowledge of both the INS and GPS orientation (i.e., roll, pitch, and true heading). The INS orientation can either be a direct output from a navigation-grade IRU or it can be from the attitude and heading mechanization of a tactical-grade IMU. To determine the GPS attitude and heading, the relative antenna baselines are computed as described in section 2.1.1 and the baseline orientation is then used to find the platform attitude and heading according to section 3.1.5. The INS orientation values are subtracted from the corresponding GPS values and the difference becomes the measurement matrix input to a complementary Kalman Filter. The three output error states correspond to the attitude and heading measurement errors. The complementary filter acts to smooth the noise in the GPS measurements while removing the alignment and drift errors from the INS measurements. The aligned and smoothed orientation measurements can then be used to synthesize a smoother baseline by propagating the ZAB vector in the current navigation frame according to equation (3-12).
3.2.2 Loose Orientation Filter Implementation

This section will describe a simple three-state complementary Kalman filter that loosely integrates orientation measurements from a GPS baseline with INS orientation. The three states correspond to the orientation error between the GPS and INS estimates of roll, pitch, and heading. The first step to implementing a loosely integrated orientation filter is to define the filter parameters. The system noise covariance matrix, \( Q \), is defined based on the uncertainty in the system (i.e., the true orientation residual error). In this case, the use of the INS as a reference reduces the residual system dynamics to the point where the measurement vector can be considered stationary and the values in the covariance matrix simply represent the system uncertainty. The system noise covariance should represent the expected dynamic uncertainty in the low noise sensor. In this section, the residual system dynamics were considered minimal (i.e., approximating a stationary process) and the system uncertainty covariance matrix was chosen based on an orientation noise standard deviation of 0.1 mrad (1\( \sigma \)). The measurement error covariance matrix, \( R \), was chosen based on both quantities in the measurement vector, \( z \). In this case, the GPS orientation measurement noise was much more dominant than the INS noise, so the measurement covariance was chosen based on the GPS orientation variance. The relationship between the GPS baseline noise and the orientation noise is dependent on the baseline length in a manner similar to Figure 2-3. For the baseline length of 7.8 meters, and the respective horizontal and vertical baseline noise level of 4 and 10 mm, the measurement error covariance was chosen to be 1.3, 1.3, and 0.5 mrad for roll, pitch, and heading respectively. For this loosely integrated filter, the measurement domain is
the same as the quantity being estimated (i.e., orientation), so the measurement transformation matrix, $H$, is a diagonal matrix of ones. No higher order orientation states are included in the filter, so the state transition matrix, $\phi$, is also a diagonal matrix of ones; the current estimate is the best prediction of the next epoch based on the available states. These initial configuration parameters are summarized in Table 3-1.

<table>
<thead>
<tr>
<th>Table 3-1: Orientation Filter Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{x}_k$, [rad]</td>
</tr>
<tr>
<td>$P_k$, [rad$^2$]</td>
</tr>
<tr>
<td>$Q$, [rad$^2$]</td>
</tr>
<tr>
<td>$R$, [rad$^2$]</td>
</tr>
<tr>
<td>$H$, [unitless]</td>
</tr>
<tr>
<td>$\phi$, [unitless]</td>
</tr>
<tr>
<td>X (Roll)</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>$(4 \times 10^{-3})^2$</td>
</tr>
<tr>
<td>$(1 \times 10^{-4})^2$</td>
</tr>
<tr>
<td>$(1.3 \times 10^{-3})^2$</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Y (Pitch)</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>$(2 \times 10^{-3})^2$</td>
</tr>
<tr>
<td>$(1 \times 10^{-4})^2$</td>
</tr>
<tr>
<td>$(1.3 \times 10^{-3})^2$</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Z (Heading)</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>$(2 \times 10^{-3})^2$</td>
</tr>
<tr>
<td>$(1 \times 10^{-4})^2$</td>
</tr>
<tr>
<td>$(5 \times 10^{-4})^2$</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

It should be noted that the longer the baseline, the smaller will be the GPS orientation error because the magnitude of the fixed baseline errors used for synthesis will cause smaller orientation errors according their arc-tangent relationship. GPS azimuth variance will be generally smaller because the noisier vertical measurements are not used in its formulation.

Now that the filter has been defined, the orientation measurements will be formed from both sensors so they can be combined. Dynamic orientation measurements will be considered so that the GPS and INS orientation measurements can be coupled while in motion. The dynamic INS orientation measurement, $y_{INS}$, can either be found using an IMU and a standard strapdown mechanization (Titterton & Weston, 1997) or directly from an IRU. Orientation is not a raw measurement from GPS, but orientation
determination was briefly described in section 3.1.5 and has been well documented in the literature (Van Graas & Braasch, 1991) (Cohen, Parkinson, & McNally, 1994). The GPS orientation measurement, \( y_{GPS} \), will be determined using a standard multi-antenna technique (Misra & Enge, 2001) as described by equation (3-15) for roll, equation (3-16) for pitch, and equation (3-17) for heading. The measurement matrix of the filter, \( z_k \), is the difference between the two sensor orientation measurements as described by equation (3-4) and shown specifically here:

\[
z_{k,\theta} = y_{GPS,\theta} - y_{INS,\theta} \tag{3-18}
\]

where:
- \( z_{k,\theta} \) = Filter measurement matrix for the loose orientation filter [rad]
- \( y_{GPS,\theta} \) = Calculated GPS orientation [rad]
- \( y_{INS,\theta} \) = Mechanized INS orientation [rad]

Since the complementary filter provides error estimates, they must be removed from the reference measurement in order to determine the total corrected orientation states as shown:

\[
\hat{y}_{\theta} = y_{INS,\theta} + x_{\theta} \tag{3-19}
\]

where:
- \( \hat{y}_{\theta} \) = Filter corrected orientation estimate [rad]
- \( y_{INS,\theta} \) = Mechanized INS orientation [rad]
- \( x_{\theta} \) = Filter states, orientation corrections [rad]

The improved orientation measurements can then be used to synthesize an enhanced baseline measurement as described by equation (3-12). The primary limitation of this filter is in the application of equation (3-12). The filter has not minimized some of the error terms present in this equation (via its derivation in equation (3-9) and equation (3-10)); in this case, the rigid-body assumption is a limitation.
3.2.3 Loose Orientation Filter Results

In this section, the results of the loose orientation coupling will be shown using static GPS/IRU data, flight GPS/IRU data, and flight GPS/IMU data. Unless otherwise stated, the unit labels provided in the figure legends of this section will apply to both the mean and standard deviation when separated by a comma. The loosely coupled orientation filter performance for the complete static data set is shown in Figure 3-6. An initial 20-second segment of the data is shown in Figure 3-7 to illustrate the typical filter performance and to show the filter convergence time. In both cases, the smooth red line represents the INS data from an IRU, the noisy black line represents the GPS data, and the smooth blue line represents the attitude and heading data corrected by the loosely coupled orientation filter.

Figure 3-6: Loosely Coupled Attitude Comparison for the Static Tarmac Test, Baseline12, Roll Data not Estimated
As mentioned previously in section 3.1.5.2, baseline12 is installed approximately in line with the body frame x-axis and has limited roll observability so the roll data in the first subplot of Figure 3-6 and Figure 3-7 is taken directly from the INS data and is not estimated in the filter.

The filter was able to bring the INS orientation measurements into alignment with the GPS navigation frame at the same time as it reduced the GPS CP attitude noise. The most dramatic noise reduction can be seen in the pitch axis because the GPS pitch estimate contains more noise due to its use of the noisier vertical baseline data (due to a higher vertical dilution of precision) with a 10 mm typical $1\sigma$ standard deviation compared to a 4 mm typical horizontal $1\sigma$ standard deviation.
As can be seen in Figure 3-7, this filter does not exclude the CP multipath (sinusoidal variations) because the filter is too simple and does not correctly model the fading behavior. This additional functionality could be added, but will be left for future work.

Using a representative 20-second static data set when the filter has reached steady state (not shown here), the performance can be quantified with actual data. For the pitch axis, mean offset between the INS and GPS frames was about 0.5 degrees; the GPS 1σ standard deviation was 1.2 mrad, the IRU 1σ standard deviation was 0.02 mrad, and the filtered pitch 1σ standard deviation was 0.1 mrad. Similarly for the heading axis, the mean offset was 0.3 degrees; the GPS 1σ standard deviation was 0.4 mrad, the IRU 1σ standard deviation was 0.04 mrad, and the filtered pitch 1σ standard deviation was 0.1 mrad. Figure 3-7 shows a segment of data at the start of the data set in order to illustrate the convergence time of the filter. In this case, no coarse correction was applied to get the initial prediction closer to the actual orientation offset and the convergence time was only 2-3 seconds. Table 3-2 provides a summary of the static orientation filter performance where the mean offset between sensors is indicated by \( \mu \) and the standard deviations are indicated by \( \sigma \).

<table>
<thead>
<tr>
<th>Table 3-2: Static Orientation Filter Performance</th>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Roll</td>
</tr>
<tr>
<td>Pitch</td>
</tr>
<tr>
<td>Heading</td>
</tr>
</tbody>
</table>

It is difficult to quantify the filter performance in the midst of flight dynamics, but the results of the flight test will still be presented here to demonstrate that the filter is able to
track the in-flight dynamics. Figure 3-8 shows the attitude comparison for the entire flight test duration while Figure 3-9 shows the filter agreement with GPS.

Figure 3-8: Loosely Coupled Orientation Comparison for the Dynamic Flight Test, Baseline12, Roll Data not Estimated

Figure 3-9: Loosely Coupled Orientation Agreement with GPS for the Dynamic Flight Test Case, Baseline12, Roll Data not Estimated
Both of these figures show that the loosely coupled attitude filter is able to track the flight dynamics and keep the INS orientation measurements aligned with the GPS navigation frame.

Now, since the system was designed to stabilize the ISR sensors in the position domain, the aligned orientation must be used to synthesize baseline measurements according to section 3.1.4. An error vector time series will be considered which represents the difference between the measured GPS baseline and the filter-synthesized baseline as shown in Figure 3-10.

![Figure 3-10: Difference Between the GPS Baseline Measurements and the Baseline Synthesized Using Aligned IRU Orientation Measurements](image)

Notice that the mean errors in all three axes are approximately 1 mm or better. This indicates good agreement with the GPS baseline measurements (both temporal and spatial) as well as minimum baseline flexure. The standard deviation in each axis is
primarily due to the residual GPS measurement noise and is not indicative of the filter performance, which would be much smaller for the filtered baseline. Also, since the filter erroneously follows the multipath error, the residuals shown in Figure 3-10 do not contain the multipath-fading pattern.

The same filter architecture was used to estimate and correct the IMU orientation drift. No significant modifications were made to the filter except to use IMU data as the INS sensor instead of the IRU. The errors are too small to observe in an overview of the in-flight orientation; so only the uncorrected GPS-INS orientation difference residuals have been plotted in Figure 3-11.

![Figure 3-11: GPS Minus IMU Orientation Difference for the Uncorrected Dynamic Flight Test Case, Baseline12, Roll not Calculated](image)

Figure 3-11 illustrates the uncorrected orientation drift contributed by the IMU measurements with respect to the GPS orientation, especially in the heading axis. This
residual quantity is also the measurement matrix into the filter, which will attempt to correct the orientation drift and misalignment. After the filter has corrected the INS orientation, a similar comparison can be made using the filtered orientation as shown in Figure 3-12.

![Figure 3-12: Loosely Coupled Filter Orientation Minus GPS Orientation for the Corrected Dynamic Flight Test Case, Baseline 12, Roll not Calculated](image)

Figure 3-12 illustrates that the GPS and INS frames have been brought into agreement because the mean error in each axis is zero and the only remaining error is a noise-like quantity from the GPS orientation measurements. As stated previously, this noise-like residual is not indicative of the filter performance and simply shows short-term agreement with the GPS measurements. The typical vertical and horizontal GPS noise performance of 10 mm and 4 mm predicts the corresponding residual orientation noise to be 1 mrad and 0.5 mrad, so the results are as expected. This is flight data, so it also demonstrates that the filter is able to track the platform dynamics with the most notable
exception being the touchdown point at 4200 seconds where a spike can be seen in the pitch axis. This spike is most likely due to fuselage flexure.

As mentioned previously, the intent of the system is to stabilize the antenna baseline and thus ISR sensors in the position domain, so a baseline error vector will be formed similar to the one shown in Figure 3-10 except that this error vector will be the difference between the GPS baseline measurements and the IMU synthesized baseline as shown in Figure 3-13.

![Figure 3-13: Difference Between the GPS Baseline Measurements and the Baseline Synthesized Using Aligned IMU Orientation Measurements](image)

This error quantity is quite accurate given the amount of inertial drift evident in Figure 3-11. The most notable exception is that the vertical component of the baseline has a slight negative bias of approximately 21 mm (between 800 and 4100 sec) that emerges around the time of takeoff and vanishes around the time of touchdown. This error is
thought to be caused by the baseline synthesis because the magnitude subplot does not reflect an error. This also indicates that baseline flexure is unlikely. A plausible explanation is that the IMU was allowed to tilt on its installation platform due to motion not reflected by the GPS orientation. The installation platform is quite rigid, but for a 7.78 m baseline only a 2.75 mrad tilt would be required to cause the -21.39 mm vertical baseline error shown in the third subplot of Figure 3-13.

### 3.3 Loose Baseline Integration Filter

#### 3.3.1 Loose Baseline Filter Overview

The previously described orientation filter requires an additional conversion of its output from orientation into baseline measurements before it is useful for position-domain ISR stabilization. To avoid this extra conversion, it makes sense to directly couple the GPS and INS baseline measurements rather than the orientation data as described previously. The challenge for this approach is to maintain the frame alignment using only the baseline measurements. The INS baseline can be synthesized using a highly accurate baseline survey in the body frame, which can then be expressed in the navigation frame using inertial orientation measurements as described by equation (3-10). There are several ways to blend the two sets of measurements, but the simplest is to loosely integrate them in the position domain. This section will describe a three-state complementary Kalman filter that loosely integrates GPS baseline measurements with those synthesized using INS orientation. The three states correspond to the baseline error between the GPS and INS estimates in the ENU navigation frame. In this case, the
measurements provided to the filter and the states estimated by the filter are both local-level baseline positions. By unbiasing the GPS/INS baseline differences in the filter, the two coordinate frames will be brought into agreement by minimizing the frame bias term in equation (3-1). If the baseline is rigid, the filter will track the frame alignment bias and inertial drift projected into the position domain. Otherwise, the filter will also treat the baseline flexure and other lower order error terms as inertial biases.

### 3.3.2 Loose Baseline Filter Implementation

The same filter parameters must be defined for the position domain integration as with the orientation filter described in section 3.2. The baseline filter will also be complementary in nature, so the INS will again be used as the reference to reduce the system dynamics and thus reduce the values in the system noise covariance matrix, $Q$. The composite GPS/INS orientation noise standard deviation was found to be on the order of 0.1 mrad (1σ), so the system noise covariance matrix was chosen based on a projection of this performance into the position domain. By making a small angle approximation, it can be seen that 0.1 mrad leads to a baseline error of .78 mm for the 7.8 m baseline12 length. The measurement vector, $z_k$, is formed as described by equation (3-4) and is shown here for the loose baseline filter:

$$z_{k,bl} = y_{GPS,bl} - y_{INS,bl}$$  \hspace{1cm} (3-20)

where:
- $z_{k,bl}$ = Filter measurement matrix for the loose baseline filter [m]
- $y_{GPS,bl}$ = Calculated GPS baseline [m]
- $y_{INS,bl}$ = Synthesized INS baseline [m]
The measurement error covariance matrix, $R$, must be chosen based on both quantities comprising the measurement vector. In this case, it was chosen based on the GPS baseline noise since it is much more dominant than the INS noise. A 16 and 100 mm$^2$ variance was chosen for the measurement error variance in the horizontal and vertical directions respectively. The measurement domain is the same as the estimation domain (i.e., position), so the measurement transformation matrix, $H$, is a diagonal matrix of ones. No velocity states are included in the filter, so the state transition matrix, $\phi$, is also a diagonal matrix of ones; the current estimate is the best prediction of the next epoch based on the available states. These initial filter configuration parameters have been summarized in Table 3-3.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{x}_k^-$ [m]</th>
<th>$P_{k-}^-$ [m$^2$]</th>
<th>$Q_k$ [m$^2$]</th>
<th>$R$ [m$^2$]</th>
<th>$H$ [unitless]</th>
<th>$\phi$ [unitless]</th>
</tr>
</thead>
<tbody>
<tr>
<td>X (East)</td>
<td>0</td>
<td>$(4 \times 10^{-3})^2$</td>
<td>$(8 \times 10^{-4})^2$</td>
<td>$(4 \times 10^{-3})^2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Y (North)</td>
<td>0</td>
<td>$(4 \times 10^{-3})^2$</td>
<td>$(8 \times 10^{-4})^2$</td>
<td>$(4 \times 10^{-3})^2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Z (Up)</td>
<td>0</td>
<td>$(10 \times 10^{-3})^2$</td>
<td>$(8 \times 10^{-4})^2$</td>
<td>$(10 \times 10^{-3})^2$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

At this point, the filter has been configured, the GPS baselines have been determined from equation (2-6), and the synthesized INS baselines are available from equation (3-10). All that is left is to loosely couple the GPS and INS baseline measurements in the complementary baseline estimation filter to determine the enhanced baseline positions. The GPS and INS coordinate frames are implicitly aligned in the filter and the GPS baseline measurements are smoothed using the ultra-low noise synthesized INS baseline. Since the filter estimates position error states, they must be removed from the reference
measurements (synthesized INS baselines) in order to form a total baseline position estimate in the GPS navigation frame:

\[
\hat{y}_{bl} = y_{INS,bl} + x_{bl}
\]

(3-21)

where:

- \(\hat{y}_{bl}\) = Filter corrected baseline estimate [m]
- \(y_{INS,bl}\) = Synthesized INS baseline [m]
- \(x_{bl}\) = Filter states, baseline corrections [m]

The primary advantage of this filter over the orientation integration filter is that all errors are included in the measurement and no assumptions (e.g., rigid-body) are made after the filter has been applied. If required, however, the filtered baselines can also be used to form an estimate of platform orientation in the GPS navigation frame whose accuracy will then be limited by the validity of the rigid-body assumption.

### 3.3.3 Loose Baseline Filter Results

In this section, the results of the loose baseline coupling will be shown using static GPS/IRU data, flight GPS/IRU data, and flight GPS/IMU data. Unless otherwise stated, the unit labels provided in the figure legends of this section will apply to both the mean and standard deviation when separated by a comma. The loosely coupled baseline filter performance for the complete static data set is shown in Figure 3-14 and a 20-second zoomed segment from the beginning of the test is shown in Figure 3-15 to illustrate the filter performance and also the convergence time. In both cases, the smooth red line represents the INS data, the noisy black line represents the GPS data, and the smooth blue line represents the baseline position data corrected by the loosely coupled baseline filter.
The filter was able to bring the INS baseline estimates into alignment with the GPS navigation frame at the same time as it reduced the GPS CP baseline noise. Using a
representative 20-second static data set when the baseline filter has reached steady state (not shown here, but similar to Figure 3-15), the performance can be quantified with actual data. For the East-axis, the GPS 1σ standard deviation was 2.25 mm, the INS 1σ standard deviation was 0.96 mm, and the filtered East position 1σ standard deviation was 0.87 mm. Similarly, for the North-axis, the GPS 1σ standard deviation was 2.64 mm, the INS 1σ standard deviation was 0.42 mm, and the filtered North position 1σ standard deviation was 1.07 mm. Finally, for the up-axis, the GPS 1σ standard deviation was 8.79 mm, the INS 1σ standard deviation was 0.48 mm, and the filtered up position 1σ standard deviation was 1.61 mm. Table 3-4 provides a summary of the static filter performance where the mean sensor offset is indicated by \( \mu \) and the standard deviations are indicated by \( \sigma \).

<table>
<thead>
<tr>
<th></th>
<th>( \mu ) [mm]</th>
<th>( \sigma_{GPS} ) [mm]</th>
<th>( \sigma_{IRU} ) [mm]</th>
<th>( \sigma_{Filt.} ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>-3.87</td>
<td>2.25</td>
<td>0.96</td>
<td>0.87</td>
</tr>
<tr>
<td>North</td>
<td>2.40</td>
<td>2.64</td>
<td>0.42</td>
<td>1.07</td>
</tr>
<tr>
<td>Up</td>
<td>-1.80</td>
<td>8.79</td>
<td>0.48</td>
<td>1.61</td>
</tr>
</tbody>
</table>

The mean offsets between the GPS, INS, and filtered axes do not provide any insightful information because they are a composite of many different errors such as frame alignment and baseline survey errors. The convergence time of this filter was less than two seconds, so it can be concluded that no extra effort is required to estimate the initial offset for more rapid convergence.
The baseline comparison for the in-flight case has been shown in Figure 3-16 and a representative 20-second segment from a section of level flight has been shown in Figure 3-17. The zoomed segment is approximately indicated with a vertical line in Figure 3-16. As before, the smooth red line represents the INS data, the noisy black line represents the GPS data, and the smooth blue line represents the filter-corrected baseline position.

Figure 3-16: Loosely Coupled Baseline Comparison for the Dynamic Flight Test, Vertical Line Indicates Zoomed Segment
The in-flight baseline dynamics tend to mask the filter performance, but the results of the flight test still demonstrate that the filter is able to track these dynamics. Now consider the same flight test example using the IMU as the reference data source. The difference between the GPS and synthesized INS baselines is shown in Figure 3-18.
The residuals shown in each subplot are largely due to errors in the baseline synthesis caused by the drifting IMU. In the East and North and Up axes, the errors are due to coordinate frame misalignment. The East and North errors are more pronounced because more flight dynamics are present in these axes. Also, the heading error is more pronounced than the pitch and roll. The misalignment errors grow as the attitude and heading drift, which then produces disagreement with the GPS measurements. By using the loosely coupled complementary baseline filter, the drift and alignment bias errors can be removed and the measurement variance can be minimized as shown in Figure 3-19.
The residual baseline noise is a result of the GPS baseline measurement and reflects its expected noise performance. The most significant baseline residual error was found in the vertical axis at around 4200 seconds when the aircraft lands on the runway and experiences the largest vertical dynamics. Adding higher order states to the filter may be able to react more quickly to these dynamics and reduce the residual errors. It is important to note that the loosely coupled baseline filter, unlike the attitude filter, has smoothed all three axes. Since the goal of the system is to form the best baseline measurements possible, this is a favorable result since roll bias errors might affect the synthesized baseline from section 3.2.

### 3.4 One-Time Coordinate Frame Calibration

This section will describe an approach to determine a one-time calibration alignment between the inertial navigation frame and the GPS navigation frame. The calibration
terminology is meant to imply a one-time procedure that can be used multiple times as long as the hardware installation is not changed. While this calibration would not be appropriate for use with orientation measurements from a drifting IMU, it may be sufficient for use with an IRU because it drifts much less and a coarse calibration would be valid for a longer period of time (e.g., a few hours). In many cases a coarse calibration might also be a first step to a finer alignment from a Kalman filter.

One approach might be considered that uses components of the baseline error vector to determine relative orientation corrections (Dickman & Bartone, 2007). This approach has the advantage of not requiring GPS orientation information, but it relies on a priori knowledge of the antenna baseline vector. The accuracy of this information will then limit the calibration accuracy.

A more obvious approach has been selected for this section that calibrates the frame offset by simply differencing the orientation measurements from two sensors. This adds some complexity to the GPS computation, but provides a direct solution of the orientation offset. Several techniques can be utilized to determine the absolute GPS attitude and heading vector. For example, GPS velocity can provide accurate orientation vectors via sequentially differenced CP (Dickman & Bartone, 2007) when aerodynamic slip motion is not present. Also, GPS can provide an absolute attitude and heading estimate using the relative positions from multiple antennas as described in section 3.1.5 (Van Graas & Braasch, 1991) (Cohen, Parkinson, & McNally, 1994). The multi-antenna GPS
orientation approach was chosen for the INS navigation frame calibration and the theory and results will be presented in the next section.

3.4.1 Frame Calibration Using Absolute Orientation Measurements
In this section, the orientation calibration will be determined between the GPS and INS measurements using an absolute GPS orientation technique that provides the antenna baseline pointing angles as described in section 3.1.5. The orientation difference vector is calculated using the mean of the time series differences as shown and both orientation vectors are expressed in the body frame with respect to the navigation frame as indicated by the subscript \(nb\):

\[
\bar{E}_{align}^{nb} = \frac{\sum_{i=1}^{N} (\tilde{\theta}_{nb}^{GPS} - \tilde{\theta}_{nb}^{INS})}{N} \quad \text{[rad]} \tag{3-22}
\]

where:
- \(\tilde{\theta}_{nb}\) = vector time series containing elements of roll, pitch, and heading \([\text{rad}]\)
- \(N\) = total number of elements in the time series to be averaged \([\text{unitless}]\)

The INS orientation is subtracted from GPS orientation and the error vector time series, \(E_{align}^{nb}\), is shown in Figure 3-20. The components of the mean error vector that were calculated according to equation (3-22) are shown in the legend of each subplot. Unless otherwise stated, the unit labels provided in the figure legends of this section will apply to both the mean and standard deviation when separated by a comma.
As mentioned in section 3.1.5.2, there is limited GPS roll observability since baseline12 is predominantly along the roll axis (body frame x-axis), so it must be calibrated with another GPS baseline or using a stand-alone INS self-alignment as described in 2.4.2 using gravity information from static accelerometer data. Referring to Figure 3-20 again, the pitch axis contains a mean bias of -11.6 mrad while the heading axis contains a 6.25 mrad bias. The frame calibration DCM that relates the GPS to the INS orientation, $C_{\text{gps}, \text{ins}}^{\text{no}, \text{gps}}$, can be determined directly from the mean bias components using equation (2-9). The sinusoidal pattern seen in most of the static data is due to CP multipath and a detailed discussion of this error will be provided in chapter 4.

If the uncalibrated INS orientation vector were used to synthesize a baseline, according to section 3.1.4, the misalignment would produce baseline biases. To see this effect, a
baseline error quantity will be considered, which is carefully constructed to show errors due to misalignment and not due to baseline survey errors. This is accomplished by synthesizing two baseline quantities according to equation (3-10) as shown in equation (3-23), one that is aligned with GPS and one that is not:

\[
\hat{b}_\text{aligned}^n - \hat{b}_\text{uncal}^n = \epsilon_\text{align}^n \quad \text{[m]}
\]  

(3-23)

where:

\( \hat{b}_\text{aligned}^n \) = synthesized baseline which is aligned with GPS  [m]
\( \hat{b}_\text{uncal}^n \) = uncalibrated synthesized baseline  [m]
\( \epsilon_\text{align}^n \) = baseline error vector due to alignment  [m]

The first baseline, \( \hat{b}_\text{aligned}^n \), was aligned with GPS using the orientation filter described in section 3.2. The second baseline, \( \hat{b}_\text{uncal}^n \), was synthesized using the raw INS orientation measurements. Both synthesized baselines contain survey errors as shown in equation (3-9), and other errors, which cancel in the difference, shown in equation (3-23), leaving the alignment error. The same alignment error shown in Figure 3-20 now manifests as a baseline error in Figure 3-21.
The magnitude error is exactly zero since both the raw synthesized baseline and the filtered baseline are rotated versions of the same vector. The baseline error magnitudes can be crosschecked geometrically using the knowledge that the fixed baseline length between nodes 1 and 2 of 7.78 m. A pitch alignment error of -11.60 mrad would lead to a 90.3 mm vertical baseline error. Similarly, a 6.25 mrad heading error would lead to a 48.6 mm horizontal error (normalization of East and North mean errors). Both of these predicted errors approximately match those found in the mean values of each axis of Figure 3-21; the vertical error magnitude was shown to be 88.76 mm and the normalized horizontal error was found to be 50.28 mm.

Performing the frame alignment calibration and thus eliminating the errors visible in Figure 3-20, effectively reduced the mean baseline synthesis errors to zero as shown in Figure 3-22.
3.4.2 Relative Antenna Baseline Survey Refinement

Absolute GPS position survey accuracies are subject to the quality of the measurements provided. A typical 2-hour survey will provide cm-level survey accuracy at each node. For the static data used in this chapter, there is substantial CP multipath in the measurements, so the survey will also contain errors due to the multipath. As will be described in chapter 4, the error due to multipath may not completely average out over time because the reflection/diffraction point is not constant and so a bias error might remain. The filtered INS baseline uses the erroneous GPS baseline survey (with cm accuracy and multipath errors) as part of the INS baseline synthesis and the relative error in the survey between the two nodes will be transferred to the synthesized baselines. Since the GPS baseline is short, the raw baseline solution without multipath will be presumably more accurate (within a few mm) due to the cancellation of correlated errors.
in the DD measurements. Consequently, there is potential to improve the baseline survey by looking at the difference between the raw baselines and the synthesized ones. The refined baseline survey will then inherit the same relative accuracy of a few mm (perhaps better after averaging the residual over 4500 seconds).

At this point, it is productive to revisit the baseline synthesis error equation as repeated from equation (3-9):

\[
\hat{b}_{\text{int, raw}} = \hat{b}_{\text{int}} = C_{\text{int, int}} C_{\text{int, gps}} \left( b_{\text{int, gps}} + \varepsilon_{\text{survey}} \right) + \varepsilon_{\text{align}} \quad \text{[m]} \quad (3-24)
\]

The intent of the previous alignment calibration was to optimize \( C_{\text{int, int}} \) to minimize \( \varepsilon_{\text{align}} \), so the alignment error residual can now be assumed negligible because it has already been filtered. Thus, a Kalman filter can provide a reduced-noise baseline estimate that is aligned with GPS:

\[
\hat{b}_\text{filt} = b_\text{true} + \varepsilon_{\text{survey}} + \varepsilon_{\text{residual \ multipath}} + \varepsilon_{\text{residual \ filternoise}} \quad \text{[m]} \quad (3-25)
\]

where:

- \( b_\text{true} \) = true GPS baseline [m]
- \( \varepsilon_{\text{survey}} \) = baseline survey error [m]
- \( \varepsilon_{\text{residual \ multipath}} \) = residual multipath after filter [m]
- \( \varepsilon_{\text{residual \ filternoise}} \) = residual noise after filter [m]

Now consider a raw GPS DD baseline measurement with presumably less measurement error due to the short baseline length:

\[
\hat{b}_\text{DD} = b_\text{true} + \varepsilon_{\text{noise}} + \varepsilon_{\text{multipath}} \quad \text{[m]} \quad (3-26)
\]

where:

- \( \varepsilon_{\text{noise}} \) = GPS CP baseline measurement noise [m]
- \( \varepsilon_{\text{multipath}} \) = GPS CP multipath error [m]
If we subtract equation (3-25) from equation (3-26), then the survey error, $\varepsilon_{\text{survey}}^n$, can be detected to the extent that it is larger than the combined noise and multipath error, $\varepsilon^{\text{noise}}_{\text{multipath}}${:

$$b_{\text{DD}}^n - \hat{b}_{\text{fit}}^n = \varepsilon_{\text{survey}}^n + \varepsilon^{\text{noise}}_{\text{multipath}} [\text{m}] \quad (3-27)$$

The measured GPS baseline will contain some errors due to noise and multipath, but the difference shown in equation (3-27) will primarily expose the errors due to the static GPS baseline survey that are greater than the GPS measurement errors as shown in Figure 3-23.

![Figure 3-23: Baseline12 Survey Errors in Static Data](image)

The residual multipath errors that were partially cancelled in equation (3-27) are readily identifiable via the sinusoidal fading pattern. The expected GPS measurement noise level is found in the standard deviation of the baseline residual. If the combined noise and multipath error could be assumed to be zero-mean, then the mean values shown in the legend of Figure 3-23 would represent the relative survey error. The relative survey
correction can only be as good as the raw GPS DD baseline solution, but is typically at the mm level and likely better than the original survey. The relative survey error can then be removed from the static baseline survey to form a refined survey.

After correcting the GPS baseline survey errors, the updated form of Figure 3-23 will contain exactly zero bias, but this does not demonstrate that the survey has been improved. A more convincing way to demonstrate an improvement is to show that the synthesized INS baseline minus measured GPS residuals are of the expected accuracy using the same survey refinement on another data set. In this case, baselines measurements from flight data will be used and the refined filter baseline minus the measured baseline error is shown in Figure 3-24.

Figure 3-24 shows that the calibrated baseline biases in all three directions are minimal and are as expected from the typical CP measurement noise. This figure also provides a
reasonable performance characterization of the overall GPS baseline standard deviation since the filtered baseline is not expected to contribute much noise to the residual.

At this point, a thorough coordinate frame alignment calibration has been performed and the baseline survey has been refined using static data. The static data does contain errors due to multipath, but averaging has reduced this effect, so the synthesized baselines that are derived from these calibrations will be as accurate as possible given the available data.

### 3.5 Tight Double Difference Integration Filter

#### 3.5.1 Tight Double Difference Filter Overview

Tight integration makes use of the raw GPS DDCP to align and stabilize the INS attitude estimate. As in the loose integration case, the inertial sensor in the INS can either be an IRU whereby the filter will merely provide the dynamic frame alignment along with minimal drift correction or it can be an IMU, which would require the filter to estimate the dynamic frame alignment as well as compensate for more substantial IMU drift. The measurements will be coupled in the range domain, so the synthesized INS DD quantity must be generated in the filter using raw INS orientation measurements. This integration technique is the most complex of the ones presented in this chapter because orientation error correction states must be estimated using DD range measurements and the filter must keep track of the domain conversions. The tight range-domain filter provides the ability to operate when the available GPS constellation is underdetermined. It also
operates on a per-DD basis and could allow the multipath to be addressed before it becomes part of the GPS or filtered solution.

The tightly coupled range-domain filter will provide orientation correction estimates to align the INS measurements with the GPS coordinate frame without bias using DD measurements and with a minimum variance using INS data. The heart of the tightly coupled range domain filter is the measurement domain transformation matrix, $H$. This matrix will convert the raw INS measurements into the DD range domain for use in the filter. The output states will be estimates of the INS orientation errors, which can then be used to correct the reference INS attitude and heading. This stabilized orientation measurement can then be used to synthesize a baseline measurement, which is aligned with the GPS navigation frame and with noise qualities similar to the INS measurements. The procedure to synthesize DD measurements is similar to the one used to synthesize baseline measurements described in section 3.1.4.

### 3.5.2 Tight Double Difference Filter Implementation

The GPS DDCP measurements can be described as the true DD plus noise as described by:

$$DD_{GPS} = DD_{truth} + \varepsilon_{DD, noise}$$  \hspace{1cm} (3-28)

Similarly, the synthesized inertial DD quantity can be described using a synthesized baseline that is converted into the range domain using the satellite geometry matrix, $G$. This quantity can also be described as the true DD plus a bias from baseline survey errors.
and orientation alignment errors. It will also contain integrated noise, but is considered negligible compared to the GPS DD noise:

$$DD_{INS} = GC_n \hat{C}_{b,ins} b_{b,ins} = DD_{true} + \epsilon_{bias}$$  (3-29)

The $C_n$ DCM is an orientation perturbation term, which represents the orientation difference between the INS and GPS navigation frames. This term will be estimated in the filter, but it must first be isolated from the other terms in the expression. To that end, it will first be replaced with an identity matrix plus the small angle cross-matrix representing the matrix exponential, a customary approximation in orientation estimation (Titterton & Weston, 1997):

$$C_n = \expm(\delta \Gamma \times) = I + \delta \Gamma \times$$  (3-30)

By substituting equation (3-30) into equation (3-29), an INS DD expression is developed which parallels the GPS DD of equation (3-28):

$$DD_{INS} = G(I + \delta \Gamma \times)C_{b,ins} b_{b,ins} = GC_{b,ins} b_{b,ins} + G\delta \Gamma \times C_{b,ins} b_{b,ins}$$  

$$DD_{INS} = DD_{true} + G\delta \Gamma \times C_{b,ins} b_{b,ins}$$  (3-31)

When the INS DD is subtracted from the GPS DD, an expression can be written for the measurement residual as in the filter measurement matrix:

$$DD_{GPS} - DD_{INS} = -G\delta \Gamma \times C_{b,ins} b_{b,ins} + \epsilon_{DD,noise}$$  (3-32)

The linear equation that governs the filter operation is of the form:

$$z = DD_{GPS} - DD_{INS} = H \cdot x + \epsilon_{noise} - \epsilon_{bias}$$  (3-33)

Equation (3-32) must be re-arranged so that the known quantities can be combined into $H$ and the remaining terms can be estimated as $x$. This can be accomplished by reversing the order of the cross product in equation (3-32), which causes a sign change to maintain
the right-hand orthogonal nature of the expression. This results in the expression shown in equation (3-34):

\[
z = GC_{b,ins}^{n,ins} \times \delta \Gamma + \varepsilon_{DD,noise}
\]

(3-34)

By equating (3-34) with equation (3-33), an expression for the measurement transformation matrix can be written as:

\[
H = GC_{b,ins}^{n,ins} \times
\]

(3-35)

The error state vector estimates can then be written as:

\[
x = \delta \Gamma = \begin{bmatrix}
\delta \gamma_{roll} \\
\delta \gamma_{pitch} \\
\delta \gamma_{heading}
\end{bmatrix}
\]

(3-36)

By running the filter, the INS orientation errors are estimated and the total orientation states are estimated by using the error estimates given in equation (3-36) to align the INS and GPS navigation frames according to equation (3-29). The baseline estimates used for ISR sensor stabilization are formed by performing a least squares solution according to equation (2-6) except the DD measurements are provided by equation (3-29) with the inclusion of the frame alignment error correction DCM, \( C_n^{\tilde{c}} \). This approach is convenient because the raw GPS and INS measurements are integrated without pre-processing steps. It has additional advantages because it is able to operate on individual DD measurements and can potentially remove per-SV errors such as multipath before corrupting the position or orientation solution. The measurement noise covariance matrix was chosen to be a diagonal matrix of 2 mm². The remaining initial configuration parameters have been summarized in Table 3-5.
Table 3-5: Tight Range-Domain Filter Parameters

<table>
<thead>
<tr>
<th></th>
<th>$\hat{x}_{k^-}$ [rad]</th>
<th>$P_{k^-}$ [rad²]</th>
<th>$Q$ [rad²]</th>
<th>$\phi$ [unitless]</th>
</tr>
</thead>
<tbody>
<tr>
<td>X (Roll)</td>
<td>0</td>
<td>$(4 \times 10^{-3})^2$</td>
<td>$(1 \times 10^{-4})^2$</td>
<td>1</td>
</tr>
<tr>
<td>Y (Pitch)</td>
<td>0</td>
<td>$(2 \times 10^{-3})^2$</td>
<td>$(1 \times 10^{-4})^2$</td>
<td>1</td>
</tr>
<tr>
<td>Z (Heading)</td>
<td>0</td>
<td>$(2 \times 10^{-3})^2$</td>
<td>$(1 \times 10^{-4})^2$</td>
<td>1</td>
</tr>
</tbody>
</table>

3.5.3 Tight Double Difference Filter Results

In this section, the results of the tight DD range-domain filter will be shown using static GPS/IMU data and flight GPS/IMU data. Unless otherwise stated, the unit labels provided in the figure legends of this section will apply to both the mean and standard deviation when separated by a comma. The performance for the static data collection using the IMU data is shown in Figure 3-25. As before, the smooth red line represents the INS data, the noisy black line represents the GPS data, and the smooth blue line represents the baseline position data corrected by the tightly coupled range-domain filter.
The effect of the IMU drift is apparent in Figure 3-25, but the filter is still able to bring the measurements into agreement. The filter operation is shown in Figure 3-26 for a segment of the static tarmac data from baseline. Only this portion of the data is shown due to a loss of GPS data for the beginning of the data set.
The filter agreement with GPS is not as good for baseline32 in Figure 3-26 as for previous baselines. This could be due to GPS measurement errors or measurement alignment errors. More investigation is required to improve this performance and will be left to future work.

The performance of the range-domain filter for dynamic flight IMU data is shown in Figure 3-27 for baseline12. Only the filter agreement with GPS is shown to demonstrate that the filter is able to compensate for the inertial drift and that the filter is in agreement with the GPS baseline measurements. The magnitude subplot illustrates that the baseline flexure or minimal.
Similarly, the in-flight tight range-domain filter agreement with GPS is shown for dynamic flight IMU data from baseline32 in Figure 3-28.
It can be seen here that the baseline32 filter agreement with GPS is not as good as in other baselines. Initial coordinate frame calibration for this baseline is challenging, so a more accurate coarse alignment might improve the performance. As with the static measurements from baseline12, a more complex filter is also expected to yield better results.

An advantage of range-domain measurement integration is the ability to operate on a per-DD basis as pointed out previously. To illustrate this benefit, the components of the Kalman measurement vector for baseline12 are shown in Figure 3-29.

![Figure 3-29: Kalman Measurement Vector for Baseline12 Using IRU Data](image)

The Kalman measurement vector is able to isolate error sources such as multipath before being introduced to the position solution. The multipath error is evident from the sinusoidal variation in the first subplot of Figure 3-29, which will be discussed further in
chapter 4. The other filters used in this chapter are based on the GPS position domain measurements and would be unable to isolate the contributor of the error since it would be distributed throughout as will be shown in Figure 4-22. With the proper filter tuning, the impact of this measurement error could be reduced, but this work will be left for future research.

3.6 Frame Alignment and Measurement Integration Conclusions
This chapter has provided a fairly comprehensive description of the sensor frame alignment for the single platform sensor stabilization application. Techniques were described to align the measurements in the position, orientation, and range domains. A one-time approach was described to calibrate the coordinate frames so that the measurements remain independent for future integration and verification options. A technique was also described to refine the relative baseline survey by minimizing the baseline error after frame alignment. Results show promising performance for both static and in-flight data using both IRU and IMU measurements. While longer baselines provide smaller orientation errors, they will lead to larger baseline synthesis errors. Each technique provided performance that was very low noise and largely unbiased with respect to GPS, a more complex filter would need to be designed to isolate error sources such as multipath from the filtered values. The baseline integration filter handles baseline flex fairly well and treats it as an error source to be minimized, but the orientation filter makes a rigid body assumption in the baseline synthesis after the filter has been run, so the error would be less observable. The one-time calibration of position and alignment was shown to improve the integrated sensor performance for the static and
in-flight data sets. The tight DD range-domain filter demonstrated promising results and the potential to individually improve the GPS measurements and also to operate with an underdetermined solution. From this work, it is recommended to perform a static calibration to improve the initial alignment before using any filter and then use either a baseline or tight DD integration filter for enhanced measurements for position-domain ISR stabilization.
4 Carrier Phase Multipath

4.1 Carrier Phase Multipath Introduction
CP processing is important for millimeter-level accuracy applications such as sensor stabilization. Differential techniques are traditionally used to remove CP measurement errors that are spatially correlated, but errors that are uncorrelated are not removed. CP multipath is one of the dominant error sources remaining in differential CP analysis (Cosentino & Diggle, 1996) and can be as large as a quarter wavelength, ~5cm for L1 (Braasch, 1996), which is significant for high accuracy applications. Multipath occurs when a direct GPS signal deflects and/or reflects off of any surface and enters the reception antenna with a slight delay. A GPS receiver that is unable to differentiate the direct and multipath signals would attempt to track the composite sum of both, and thus cause deviations in the phase measurement. Since a GPS receiver extracts its range measurements from phase aligned signals, multipath results in range errors. Multipath affects both types of GPS measurements: pseudorange (code phase) and CP. The measurements that are most important for high accuracy relative positioning applications such as the ones described in this document are the CP measurements, so code-multipath will not be extensively considered. Code-multipath cannot be completely ignored since it is interrelated with the carrier measurements, but it has already been extensively documented elsewhere (Kalyanaraman, Braasch, & Kelly, 2006) and the references provide sufficient background.
The direct and multipath signals add constructively and destructively in phase, which induces a beat frequency in the composite signal referred to as the fading frequency since it causes varying amplitude and phase of the composite signal. The geometric point of reflection/diffraction is sensitive to changes in airborne dynamics and is difficult to observe and thus hard to analyze, so airborne-multipath will not be directly considered although the techniques described herein are still applicable.

Ground-multipath is an important concern for airborne users because it can affect the accuracy of the sensor calibration and alignment on the ground. Ground calibration using GPS CP measurements is often used to align multi-sensor position, velocity, and orientation, as described in section 3.4, but ground-multipath can dramatically reduce its accuracy unless multipath is minimized to a negligible level. The analysis conducted in this chapter was largely motivated by evidence of ground-multipath in the ground calibration and alignment data shown earlier. SV elevation angle is often used as a ground-multipath predictor and as the basis for SV exclusion, but a high elevation SV used for the ground calibration in this research contained substantial multipath, so other indicators were sought.

This chapter will describe CP multipath errors, the models to quantify them, and discuss possible range-domain error minimization. Field test data will be used to validate existing multipath models and the modeled predictions can then be used to verify the presence of CP multipath in the field test data. Multipath errors are dependent on many
environmental factors such as antenna height and reflection/diffraction surface that make it difficult to detect and directly quantify using raw range measurement data. Several other quantities can be combined to detect and predict multipath errors under a few assumptions. Multipath detection can be conducted in the time or frequency domain. The time domain provides a coarse detection method that will work in real-time, but lacks robustness. The frequency domain offers a more robust detection technique, but suffers from time resolution limitations since it depends on the number of time blocks within a dataset. The time resolution problem can be avoided by directly tracking the multipath using raw data from a block processing software receiver (i.e., correlation) (Zhu & van Graas, 2005), but this technique is rather complicated to implement and requires RF data collection and/or real time RF data processing. Consequently, the time and frequency domain detection techniques will be used for excluding measurements with the most potential multipath and dual-frequency measurement combinations will be used to mitigate the remaining multipath error. DD techniques are used to solve for the relative CP antenna baseline (Cosentino & Diggle, 1996) and the NL measurement combination technique will be applied to reduce the noise and multipath error; the accuracy improvement will be quantified.

Smoothing techniques must be applied cautiously in the presence of multipath. Even for a static user, there can be enough change in the reflection/diffraction surface within a fading cycle to alter the multipath strength and fading frequency, or even to cause discontinuity in the fading pattern; this might introduce a bias when smoothing or
filtering techniques are applied. Also, the multipath-fading rate must be several times larger than the smoothing window; slow fading frequencies and short smoothing windows will also introduce a bias.

4.2 Carrier Phase Multipath Theory

4.2.1 Multipath Parameter Estimation Models

Multipath characteristics are largely driven by environmental variables such as path delay (i.e., antenna distance to the point of reflection or diffraction) and multipath strength. With knowledge of these parameters, the composite multipath corrupted signal can be predicted using equations and the corresponding receiver measurements can be modeled. Nevertheless, CP multipath is sensitive to small changes in these parameters and would be difficult to perfectly model, even for a single specular reflection. This section presents the basic models sufficient to describe CP multipath and for comparison with field test data.

The first parameter to be estimated is the strength of the multipath with respect to the direct signal. Since multipath interferes with the GPS signal constructively and destructively, the power level varies correspondingly. The received signal strength is generally estimated using a post-correlation signal amplitude ratio such as the carrier to noise ratio (CNR). CNR is affected by multipath as a function of the phase variation between the multipath and direct signals. If the noise level is assumed to stay constant as the multipath phase varies, it can be neglected and the relative amplitude variation is all
that needs to be considered. The composite signal amplitude can be modeled as shown in equation (4-1) for a single specular reflection (Van Nee, 1995):

\[ A_c = |A_d e^{j\theta_d} + \alpha A_d e^{j(\theta_d + \theta_m)}| \]  \hspace{1cm} (4-1)

where:
- \( A_c \) = composite signal amplitude
- \( A_d \) = direct signal amplitude
- \( \alpha \) = amplitude ratio of the multipath to direct signal
- \( \theta_d \) = direct signal phase
- \( \theta_m \) = multipath signal phase offset

A similar expression often used as an approximation is shown in equation (4-2) (Axelrad, Comp, & Macdoran, 1996):

\[ A_c = A_d + \alpha A_d \cos(\theta_m) \]  \hspace{1cm} (4-2)

The composite amplitude variation can then be used to predict the direct signal to multipath ratio (SMR). In a traditional receiver the SMR must be estimated rather than measured because the receiver only tracks the composite signal (superposition of multipath and direct signals). The SMR can be estimated by looking at maximum and minimum deviations from the nominal CNR. The nominal value, \( CNR_{nom} \), can be estimated by taking the mean of the peak and trough of the one cycle of the CNR. The relative increase and decrease of the CNR from the nominal provides a first order estimate of the peak-to-peak amplitude of the multipath signal. The SMR is based on the peak amplitude variation from the nominal as shown in equation (4-3):

\[ SMR = \frac{1}{\alpha} = \frac{2 \cdot CNR_{nom}}{(CNR_{Max} - CNR_{Min})} = \frac{2 \cdot A_{c,Nom}}{(A_{c,Max} - A_{c,Min})} \]  \hspace{1cm} (4-3)
Now, with an estimate of the multipath strength, the CP multipath error can also be estimated. An equation describing the CP error solely due to multipath can be formulated as shown (Braasch, 1996):

\[
\theta_c = \arctan \left( \frac{\alpha R(\tau_c - \delta) \sin(\theta_m)}{R(\tau_c) + \alpha R(\tau_c - \delta) \cos(\theta_m)} \right)
\]

where:
- \( R \) = pseudorandom code correlation function [unitless]
- \( \tau \) = delay lock loop code tracking error [sec]
- \( \delta \) = multipath delay with respect to the direct signal [sec]
- \( \theta_m \) = phase relation between the direct and multipath signal [rad]

By neglecting path delay and other CP tracking errors, the worst-case error due to CP multipath can be approximated by assuming equal direct and multipath strength. Because of the tangent function, it can be seen that the CP multipath tracking error, \( \theta_c \), is a maximum of \( \pi/2 \) radians for a given multipath strength when the multipath phase difference, \( \theta_m \), is \( \pi \) radians (Braasch, 1996). The signal wavelength determines the number of meters per cycle and there are \( 2\pi \) radians per cycle, so the maximum range error works out to be \( \lambda/4 \) meters. Figure 4-1 shows the phasor diagram representing the geometry of equation (4-4) with the assumption of zero path delay and unity SMR.
Figure 4-1: Carrier Phase Multipath Phasor Diagram

Since the SMR = 1, both direct and multipath signals, $D$ and $M$ respectively, can be shown rotating on a unit circle. The vector sum of these two signals is the composite signal, $C$. The angle between the multipath and direct signals is the multipath phase, $\theta_m$. Two cases are shown in Figure 4-1, each with a different multipath phase offset. Case 2 is shown for a large $\theta_m$ with a “dash dot” line and Case 1 is shown with a dashed line.

The parameter of interest here is the composite multipath phase error, $\theta_c$. As the multipath phase progresses from case two to case one, the multipath phase error approaches its maximum of $\pi/2$ radians.

Next, the multipath fading frequency can be estimated using the equivalent distance to the point of reflection or diffraction, the GPS signal wavelength, the satellite elevation angle, and its rate of change (Axelrad, Comp, & Macdoran, 1996):

$$f = \frac{2 \cdot h \cdot \cos(\zeta)}{\lambda} \cdot \frac{d\zeta}{dt}$$  \hspace{1cm} \text{(4-5)}$$

where:

$\lambda$ = multipath fading frequency [Hz]
$h$ = reflection/diffraction distance [m]
4.2.2 Multipath Ground Reflection Point Prediction

This section discusses a coarse technique to predict the multipath ground-reflection point in order to estimate the surface characteristics. The potential for diffraction is not considered in this section to keep the illustration at a higher level. To perfectly model the distance to the reflector, more information would be required about the shape of the reflecting surface and the exact point of reflection. For a perfectly flat ground, the reflection point can be geometrically approximated as shown in Figure 4-2 and can be a helpful tool for assessing the characteristics of the reflecting surface.

The North and East coordinates of the ground reflection point can be determined using the antenna height and the SV azimuth angle (from North = 0) and elevation angle (Local Level = 0):

\[
N = h \cdot \cos(\zeta) \cdot \cos(\psi) \\
E = h \cdot \cos(\zeta) \cdot \sin(\psi)
\]

(4-6)

where:
- \(N\) = North coordinate of reflection point [m]
- \(E\) = East coordinate of reflection point [m]
\( \zeta = \text{satellite elevation angle} \quad [\text{rad}] \)
\( \psi = \text{satellite azimuth angle} \quad [\text{rad}] \)

The reflection geometry changes when the reflection point is inclined as shown in Figure 4-3. The local level surface can represent the flat ground and the inclined surface might represent a sloping aircraft wing surface. This geometry is further complicated if the inclination is three-dimensional as would likely be the case in reality.

Notice that the reflected signal has a different elevation angle than the direct. Snell’s law says that the angle of reflection equals the angle of incidence, so when the reflecting surface is inclined, the angle of incidence is the difference between the satellite elevation angle, \( \zeta \), and the inclination angle, \( \gamma \). The reflection surface has changed the angle at which the multipath would enter an antenna as shown:

\[ \tilde{\zeta} = \zeta - 2 \cdot \gamma \quad (4-7) \]

If multipath is present in the measurements, it will be combined from two antennas and two SVs and then contribute to the overall baseline error, so double difference processing quadruples the potential CP multipath exposure.
4.2.3 Narrow-Lane Measurement Combination Theory

It has already been shown in equation (4-4) that the maximum CP error due to multipath is $\pi/2$ radians. One way to reduce the impact of CP multipath is to reduce the effective wavelength of the ranging signal. This is accomplished using the NL combination of dual-frequency measurements to derive a measurement vector to replace equation (2-2). It should be noted that the true NL carrier is not tracked, but the multipath error can still be proportionally scaled as a function of the NL wavelength. In addition, the NL measurement combination exhibits a statistical improvement due to a reduction in the range-domain noise variance.

The DD noise variance can be reduced using the synthetic NL formed by adding the separately tracked L1 and L2 signals. The total NL variance comes from the additive contribution of both the L1 and L2 signals as shown in equation (4-8):

$$
\sigma_{NL,\text{rad}}^2 = \sigma_{L1,\text{rad}}^2 + \sigma_{L2,\text{rad}}^2 \quad \text{[rad]}
$$

(4-8)

If the standard deviations of the CP measurements are the same on both frequencies, the CP standard deviation can be expressed as in equation (4-9):

$$
\sigma_{CP,\text{rad}} = \sigma_{L1,\text{rad}} = \sigma_{L2,\text{rad}} \quad \text{[rad]}
$$

(4-9)

With this assumption, the two variance terms can be combined as in equation (4-10):

$$
\sigma_{NL,\text{rad}}^2 = 2\sigma_{CP,\text{rad}}^2 \quad \text{[rad]}
$$

(4-10)

The standard deviation is then the square root of the variance as in equation (4-11):

$$
\sigma_{NL,\text{rad}} = \sigma_{CP,\text{rad}} \sqrt{2} \quad \text{[rad]}
$$

(4-11)
The NL standard deviation, in length units, is then formed by evaluating equation (4-12) at the NL frequency:

\[
\sigma_{NL,m} = \sigma_{NL,rad} \frac{\lambda_{NL}}{2\pi} \quad [\text{m}]
\]  

(4-12)

Finally, the standard deviation of the NL can be determined relative to the L1 standard deviation by substituting the L1 portion of equation (4-9) into equation (4-11) and then substituting the result into equation (4-12). The result is shown in equation (4-13):

\[
\sigma_{NL,m} = \left( \frac{\lambda_{NL}}{\lambda_{L1}} \sqrt{2} \right) \sigma_{L1,m} \quad [\text{m}]
\]  

(4-13)

The factor in parenthesis works out to be approximately 0.79, so it can be concluded that the L1 standard deviation can be reduced by 21\% using the NL.

The combined NL measurement provides a real-time approach to statistically reduce the measurement noise and scale the multipath error. It can be used whenever dual-frequency measurements are available and when the update rate of both measurements (GPS L1 and L2) is sufficient to completely observe the baseline motion. It should be noted that the NL ambiguity could be challenging to resolve if typical GPS error source (e.g., ionosphere, troposphere, orbit, and clock errors) are not removed in the DD. For this application, the baseline was short (tens of meters) and most of the errors cancelled due to their correlation over this distance. The baseline survey was used as an ambiguity constraint and allowed the NL ambiguity to be rapidly resolved on the fly.
The NL measurement can be defined using DDCP measurements from both the L1 and
L2 frequencies. The multi-satellite and multi-receiver notation used in equation (2-2)
will be replaced with a multi-frequency notation:

\[ \Phi_{NL} = \Phi_{L1} + \Phi_{L2} \quad [\text{m}] \]  

(4-14)

The combined NL wavelength, \( \lambda_{NL} \), is defined similarly as a sum of the constituent
measurement frequencies:

\[ \lambda_{NL} = \frac{c}{f_{L1} + f_{L2}} \quad [\text{m}] \]  

(4-15)

where:
- \( c \) = speed of light = 299792458 [m/s]
- \( f_{L1} \) = GPS L1 frequency = 1575.42 [MHz]
- \( f_{L2} \) = GPS L2 frequency = 1227.60 [MHz]
- \( \lambda_{NL} \) = GPS NL wavelength \( \approx \) 10.7 [cm]

For GPS L1, the maximum CP multipath error (\( \lambda/4 \)) is approximately 4.8 cm. The
maximum multipath error would be scaled to approximately 2.7 cm using the NL.

The NL baseline can be determined using the NL measurement vector in the traditional
least squares solution shown in equation (2-6). The same least squares residuals from
equation (2-7) can also be used as a performance metric as with the Single Frequency
(SF) analysis. Further improvement will be possible using different combinations of
measurements when additional signals become available (Richert & El-Sheimy, 2007).
4.3 Multipath Reduction Results and Analysis

4.3.1 Field Test Description

A static data collection was conducted on April 12th, 2006 on the tarmac of the Ohio University airport (UNI) with all of the same sensors and log configurations used in that day’s flight test (described in section 2.5). Static data provides an opportunity to calibrate the coordinate frames and assess some measurement errors like ground-multipath. The data provides a real-world illustration of the errors that can occur and their effect on the overall system.

The entire system was powered by a generator on the ground rather than aircraft power for the 82-minute duration of the test. GPS L1 CP data was collected at a 100 Hz update rate while the L1 and L2 pseudorange and L2 CP were collected at 2 Hz from all nodes. The receivers’ WAAS corrected position estimate as well as the raw ephemeris data were logged at a 1 Hz rate. The navigation-grade inertial collected platform orientation information for body frame calibration. The NovAtel OEM4 log of the best position for the static test is shown in Figure 4-4. The GPS constellation sky plot during the time of the static data collection is shown in Figure 4-5. The mask angle for data processed from this data collection was 20 deg and the key satellite for DD processing was SV10.
Figure 4-4: Static Data Collection Profile

Figure 4-5: GPS Constellation Sky Plot for Static Test
4.3.2 Multipath Parameter Estimation

Data from the tarmac field test will be used to provide an example of multipath parameter estimation for a given set of conditions measured in the field. The intent of this section is primarily to use the models to identify the CP errors observed in the field as those caused by CP multipath and secondarily to demonstrate limitations in the models.

Typical GPS receivers provide some estimate of the received signal strength, which can be used to predict the multipath strength as described in equation (4-3) because multipath causes the CNR to fluctuate as the multipath signal interferes with the direct signal. A short segment of the CNR data collected from a static GPS receiver is shown in Figure 4-6 along with a model of the CNR generated using amplitude variation estimates derived from equation (4-1).

![Figure 4-6: CNR Data with Model Agreement](image)
This figure also shows that the data likely contains multipath with an approximate SMR of 13 dB. It also demonstrates how the multipath strength and phase offset can be estimated using the GPS receiver’s estimate of CNR even though the data is quantized at the unit dB-Hz level. It is apparent that it would be difficult to estimate the strength of the multipath over a short time span (less than 1 cycle) because at least a half cycle of the multipath fading is required to determine the peak variation. The parameters used in this model were generated manually, but a more automated procedure is possible.

Two amplitude estimation models were presented in equation (4-1) for an exponential amplitude variation and equation (4-2) for the sinusoidal approximation. The effect of strong multipath on the CNR is shown in Figure 4-7 while Figure 4-8 shows the same for weaker multipath. The simulated nominal CNR (no multipath) is 48.23 dB-Hz and the mean of each CNR estimator is shown for each curve as a comparison metric.
The sinusoidal model can be a good approximation when only weak multipath is present as is visually apparent by comparing Figure 4-7 and Figure 4-8. Also, note that the CNR becomes less sinusoidal as the SMR decreases.

Based on the antenna height, the satellite elevation angle and elevation change rate, the fading frequency can be predicted using equation (4-5) for a single ground reflection. Figure 4-9 shows the prediction based on the static node-2 antenna height for SV2.
Figure 4-9: Predicted Fading Frequency Increases with Increasing Time

The variation in the predicted fading frequency is only due to the changes in the satellite elevation angle. Any changes in environmental parameters such as surface flatness or surface curvature will dramatically change the multipath fading frequency prediction.

Fading frequency can also be directly observed in the CNR and Code Minus Carrier (CMC) values extracted from the GPS measurements. Determining the spectral content of each parameter over a block of 500 seconds allows the fading frequency variation to be plotted over time. Figure 4-10 shows the time/frequency variation of the CNR data while Figure 4-11 shows the time/frequency variation of the CMC data.
As with the multipath strength estimation, the fading frequency estimation improves with more cycles of fading in each time block. Time resolution with this approach is fairly
limited, but both figures illustrate that the fading frequency is generally decreasing over
time, which does not agree with the model shown in Figure 4-9. It is difficult to model
the fading frequency completely because many of the previously mentioned
environmental factors cannot be perfectly modeled. One reason for the disagreement
could be the curvature of the presumed reflecting surface. If the distance from that
surface to the antenna (height) were decreasing at a faster rate than the other parameters
in equation (4-5) are increasing, then the actual fading frequency would decrease; at the
same time, the predicted fading frequency of the constant antenna height model would
increase.

The ground reflection point can be predicted according to equation (4-6) with respect to
the antenna location. For the static data collection, the reflection points are shown in
Figure 4-12 for the forward antenna (node 1) and in Figure 4-13 for the aft antenna (node
2). Solid lines are shown between the nodes to represent the span between the wingtip
antennas and the fuselage antennas. The satellite elevation angle is encoded in each trace
with colors according to the color map shown on the right of each figure. All SVs in
view were plotted for this example even though some were not used in the navigation
solution.
Figure 4-12: Node 1 Predicted Horizontal Ground Reflection Position Over Time, With Color Coded SV Elevation Angle

Figure 4-13: Node 2 Predicted Horizontal Ground Reflection Position Over Time, With Color Coded SV Elevation Angle
This is not a complete model because the width of the fuselage and wings are not considered as reflection/diffraction sources; however, it does provide a first order approximation and can be used to estimate the location and characteristics of the surface.

### 4.3.3 Corroboration of Single Antenna Multipath Indicators

Multipath affects each SV differently, so it is prudent to analyze individual SVs when trying to identify multipath rather than the navigation solution. Several measurements can be used as multipath indicators and each will be presented together in order to substantiate the presence of multipath in the data. The first measurement is the CP data itself. Although the raw measurement contains a trend due to satellite and user induced Doppler changes, the sequentially differenced carrier phase (SDCP) can provide a direct indication of variation in the data as long as the multipath is strong enough. Next, the CNR will be presented since multipath is evident in signal amplitude variations as previously described. Third, the CMC data provides an indication of the presence and magnitude of multipath. If multipath is present, it will be found with varying levels in both the code and carrier measurements. The CP multipath is very small compared to the code phase and will be negligible in the CMC quantity. A polynomial-fit must be subtracted from the single SV CMC in order to remove the ionospheric effects and the integer ambiguity (Dickman, Bartone, Zhang, & Thornburg, 2003). Finally, the SV elevation angle is a good predictor for ground multipath as SVs with low elevation angles are more likely to contain multipath because it is difficult to build an antenna with a multipath rejecting gain pattern near the horizon. Two examples will be used to demonstrate the value of these multipath indicators using data collected from two antenna
locations at the same time for the same satellite. The first example, shown in Figure 4-14, was collected from the forward fuselage antenna (node 1) and the second, shown in Figure 4-15, was collected from the aft fuselage antenna (node 2). Figure 4-15 contains vertical bars to highlight the sinusoidal oscillation pattern that is characteristic of multipath fading.

Figure 4-14: Node 1 Multipath Indicators
These figures illustrate the sensitivity of multipath to environmental factors (mainly antenna height in this case) since they contain greatly different multipath characteristics even though they are located on the aircraft fuselage with an approximately 8-meter separation. Figure 4-14 shows only a hint of weak multipath during the latter half of the data set while Figure 4-15 contains some obvious multipath characteristics. A closer examination of Figure 4-15 is warranted to provide some insight into the data. Some variation can be seen in the SDCP in the first subplot; when the CNR is at a minimum, the SDCP error is at a maximum. It is possible, however, that the variations in the SDCP values might be caused by variations in the CNR other than multipath, i.e., the receiver noise level has increased, but other factors will refute that possibility in this case. The CNR variation in the second subplot shows the higher rate variation expected from multipath, but is also contains a low rate trend that is also evident in Figure 4-14. This trend is likely due to variations in the antenna gain pattern and is observed in both figures.
because the signal can be assumed to arrive at approximately the same point in the gain pattern. This assumption can be made since both antennas are the same model and the satellites are tens of thousands of kilometers away and thus enter both antennas at nearly the same azimuth and elevation angles. The third subplot shows the single antenna CMC, which largely reflects the effect of multipath on the pseudorange. Since the variation in the code corresponds to that of the CNR, it reinforces the assumption that the variation is due to multipath. The final subplot shows that the satellite elevation angle decreases from 47 to 20 degrees. This is an unusually high elevation angle to see ground multipath because the antenna gain and shading would normally attenuate the signal, but by examining the ground reflection map in Figure 4-13 it can be surmised that the reflection point for SV2 is actually on the top of the port wing. In this case, the elevation angle would have to be reduced by the inclination of the reflecting surface as approximately described by equation (4-7) (since the wing is inclined in the pitch and roll axes). The DC-3 aircraft is inclined by approximately 12 degrees in pitch when stationary and the wing might be further pitched by a few degrees at the same time as a roll inclination of a few degrees as can be seen in Figure 4-16.

Figure 4-16: Location and Orientation of the DC-3 During Static Testing
If the total inclination were approximately 15 degrees, for example, the multipath would initially enter the antenna at 17 degrees and decrease until insignificant by the time the SV reaches its final elevation angle of 20 degrees. This example illustrates the value of the elevation angle as well as reflection point estimation when trying to assert multipath as the error source in the data. The consistent reinforcement of each of the presented multipath indicators build confidence that multipath is the likely source of the CP ranging error in the data for SV2.

4.3.4 Double Difference Carrier Phase Multipath Indicators

In this section, the CP measurements from two antennas and two satellites will be differenced to form a DD measurement as described by equation (2-2) with the intent to solve for the baseline separation between the antennas as described by equation (2-6). Multipath can affect any one of the four raw measurements. It is important to consider the single-antenna multipath indicators from the previous section to isolate the errors, but assuming a multipath-free key SV and negligible multipath at the second antenna location, the DD measurements provide the ability to directly analyze the ranging error on a particular satellite. The example given in Figure 4-14 and Figure 4-15 provides a good candidate for DD analysis because it was shown that node 1 (Figure 4-14) was relatively free of multipath and node 2 (Figure 4-15) had about 3000 seconds of multipath fading. The key was chosen to be SV 4 as shown in Figure 4-5 with a red line. This satellite was chosen because of its high elevation angle and its persistence throughout the data collection. While not shown here, the indicators for this SV seem to be relatively free of multipath. A new set of multipath indicators can be formed using DD
measurements as shown in Figure 4-17 with SV10 as the key satellite. There are two notable differences from Figure 4-14 and Figure 4-15; the SF L1 DDCP data in the first subplot and the DD CMC in the fourth subplot, which will be discussed subsequently.

**Figure 4-17: Double Difference Multipath Indicators for SV2**

The second and third subplots are simply the CNR values from the reference and roving nodes respectively. The last subplot contains the SV elevation angle for reference. The SF (L1) DDCP error shown in the first subplot is calculated by subtracting a measurement prediction from the DDCP measurement. The prediction can be formed using accurate knowledge of the true baseline, baseline orientation, and satellite positions as described in the reference (Dickman & Bartone, 2008). The DD residual in the first subplot contains unmodeled errors such as multipath as is apparent from the variation corresponding to the same fading pattern seen in the CNR data. The DD error residual is more compelling than the sequentially differenced CP shown in Figure 4-15 since the ranging error can be directly quantified. The DD CMC quantity is formed by subtracting
the DDCP and its associated ambiguity from the DD pseudorange. No polynomial removal is required because the DD is formed over a short baseline (tens of meters) and the ambiguity has been resolved. The DD CMC is a more convincing indicator of multipath because the true error would not be mistakenly removed by a polynomial in this case.

### 4.3.5 Carrier Phase Multipath Error Analysis and Mitigation

Now that the multipath parameters have been modeled and used to verify the presence of multipath in the field-test data via the single antenna and DD multipath indicators, the results of the multipath error analysis and mitigation will be shown.

Equation (4-4) shows that the CP multipath error is a function of the multipath strength and the relative phase difference caused by multipath. This relationship is shown graphically in Figure 4-18 assuming no path delay for a range of multipath strength values and a range of multipath phase offsets. Admittedly, this is not a realistic scenario, but serves as the worst-case error estimate. The ranging error has been converted to units of centimeters for the GPS L1 carrier frequency. Figure 4-19 represents the pathological case as a single slice of Figure 4-18 where the CP multipath error is largest, i.e., when multipath signal is as strong as the direct signal and the magnitude of the multipath error is a maximum.
These figures agree with literature that the maximum L1 CP multipath error is roughly 4.8 cm (Braasch, 1996). Having an estimate of the multipath strength allows a decision
to be made as to the significance of the potential ranging error. In some cases, the error might be within the error budget, but in others the satellite may need to be excluded from the navigation solution.

By applying the same theory to the NL quantity, equivalent maximums can be found for the shorter wavelength described in equation (4-15). Since the receiver is locked to both L1 and L2, it is as if the receiver were tracking the true NL signal (i.e., at a frequency of 2803.02 MHz). Figure 4-20 shows the theoretical NL ranging error (even though the NL signal is not directly tracked in practice) when the multipath strength is equal to the direct signal.

![Figure 4-20: Worst Case NL Ranging Error Due to Carrier Phase Multipath with SMR = 1](image)

The scale in this plot has been maintained with that of Figure 4-18 and Figure 4-19 so the reduction in ranging error magnitude is visually apparent. As mentioned in the previous
theoretical discussion, the $\lambda/4$ worst-case error would be reduced to approximately 2.7 cm for the NL measurement. The next section will illustrate the statistical performance improvement gained by using the NL measurement combination.

4.3.6 Narrow-Lane Noise and Multipath Mitigation Illustration
Consideration will now be given to field-test data to verify this error reduction in a real-world scenario. First consider the measured DD minus prediction residual as shown in Figure 4-21 for SV2.

![SV 2 DD Measurement Comparison](image)

![Narrow Lane Error](image)

Figure 4-21: Measured DDCP Minus Predicted DDCP Residual for Static Data

The L1 SF residual mean (calculated over the entire 4500 second interval) is approximately 7.8 mm. This value gets reduced to 6.3 mm when the NL is used (19% improvement). Similarly, the L1 SF standard deviation is 7.3 mm as calculated over the entire interval while the NL standard deviation is only 4.93 mm (32% improvement). It is important to note that the predicted DD might contain some error due to imperfect
frame calibration and baseline survey as described in the reference (Dickman & Bartone, 2008), but these errors are unlikely to be sinusoidal and in agreement with the CNR variation. Prediction error is more likely the cause of the gradually increasing mean in Figure 4-21.

Up till now, only the raw measurements have been considered because it is easier to isolate the errors and take corrective steps in the measurement domain, but the end user is more concerned with the navigation solution in the position domain. The position-domain navigation solution is found by grouping all of the measurements as described by equation (2-5) and then solving for the baseline as described by equation (2-6). The set of CP measurements used in the navigation solution is shown in Figure 4-5 where the key satellite, SV10, is marked in red and node 2 was chosen as the reference receiver. Since the static baseline is known, it can be subtracted from the baseline solution to yield baseline error. The resulting baseline solution error in the ENU frame is shown in the first three subplots of Figure 4-22 and the baseline magnitude is shown in the last subplot.
The important thing to focus on in the baseline error plot is the position variation, which is most likely due to multipath since it corresponds to the variation seen in the previous sets of multipath indicator figures. It should be noted that the same multipath-laden CP data was used in static post-processing to determine the baseline survey (without excluding SVs), so the survey may contain error due to CP multipath. The mean errors in Figure 4-22 are most likely due to baseline survey errors since the aircraft is static. Several other SVs in the data set contain multipath errors, but were excluded from the baseline solution, so multipath error from SV2 is strongly visible.

Another benefit of the least squares solution is that the residuals can be used as another performance metric for noise and multipath reduction. The least squares residuals are formed according to equation (2-7) and are shown in Figure 4-23 for the SF data.
Unlike the position error, the residuals do not rely on an external truth reference and can stand on their own as an assessment of the noise and multipath left over from the least squares estimation. The mean and standard deviation are shown in the legend for each SV pair used in the least squares solution. While the labels imply a single satellite data set, they are each differenced with respect to the key SV and reference receiver. The statistics presented in Figure 4-23 will be used as a benchmark for comparisons with the NL processing.

A similar performance metric is the parity space solution (Brown, 1996), (Van Graas, 1996) and (Bartone, van Graas, & Arthur, 2005), which can also be used to assess the noise and multipath performance. The total parity vector calculated using the lower half partition of equation (2-8) was split in two pieces using a 0.5 Hz low pass filter as shown in Figure 4-24. The low frequency component is considered to be a parity space trend or
bias term while the higher frequency component is considered to be parity space noise term as described in (Dickman & Bartone, 2008).

The noise component is at an expected level for the type of equipment used in the data collection and for the SF processing technique used in the baseline solution. The parity space trend component, however, has a multipath-like oscillation pattern with a magnitude that is expected from the multipath evident in the SV2 data.

Combining the measurements according to equation (4-14) and then solving equation (2-6) using the NL measurements instead of the SF data leads to less deviation in the baseline as shown in Figure 4-25 compared with Figure 4-22.
In each axis the mean error increases when the NL data is used, but the standard deviation is reduced. As mentioned previously, the baseline survey likely contains error due to the multipath environment, and would lead to a constant bias in the error. The standard deviation improves by approximately 32%, 19%, 17%, and 19% in the East, North, Up, and magnitude subplots respectively.

The least squares residuals can also be formed according to equation (2-7) for NL data as shown in Figure 4-26 for each satellite in the baseline solution.
As mentioned previously, the least squares residuals do not have the truth-reference accuracy limitations and show an unambiguous improvement. The mean error has been almost completely removed (less than a tenth of a mm remains) while the standard deviation has been improved by almost 30% for each SV. The parity space residuals are shown Figure 4-27 for the NL baseline solution.
Figure 4-27: Parity Space Residuals from Static Narrow-Lane Baseline12 Solution

The parity space residuals show a noise standard deviation reduction of about 20% and a trend bias and standard deviation reduction of 86% and 35% respectively. All of the performance statistics as well as the improvement percentages have been summarized in Table 4-1 with the mean indicated by $\mu$ and the standard deviation indicated by $\sigma$.

| Performance Metric | Single Frequency [mm] | Narrow-Lane [mm] | Improvement [%] |
|--------------------|-----------------------|------------------|-----------------
|                    | $\mu$  $\sigma$ $\mu + \sigma$ | $\mu$  $\sigma$ $\mu + \sigma$ | $\mu$  $\sigma$ $\mu + \sigma$ |
| Range Error        |                        |                  |                |
| SV2                | 7.8  7.3  15.1         | 6.3  4.9  11.2   | 19.2  32.5  25.6 |
| Position Error     | $\mu$  $\sigma$ $\mu + \sigma$ | $\mu$  $\sigma$ $\mu + \sigma$ | $\mu$  $\sigma$ $\mu + \sigma$ |
| East               | 11.4  5.2  16.6        | 14.2  3.5  17.8  | -24.3  31.5  -6.9 |
| North              | 4.1   5.6  9.7         | 4.6   4.5  9.2   | -13.5  19.3  5.5  |
| Up                 | 28.8  14.7  43.5       | 29.1  12.3  41.4 | -1.0   16.6  4.9  |
| Magnitude          | 6.8   7.2  14.0        | 7.6   5.8  13.4  | -12.0  18.7  3.7  |
| LS Residuals       | $\mu$  $\sigma$ $\mu + \sigma$ | $\mu$  $\sigma$ $\mu + \sigma$ | $\mu$  $\sigma$ $\mu + \sigma$ |
| SV 2               | 0.2   0.6  0.8         | 0.01  0.4  0.4   | 95.2   29.8  47.4 |
| SV 6               | 0.2   0.8  1.0         | 0.05  0.6  0.6   | 79.2   29.5  41.2 |
| SV 26              | -0.6  2.5  1.9         | -0.06 1.8  1.8   | 90.0   26.5  6.3  |
The same NL solution can be applied to the in-flight data to provide similar error mitigation. The reflection and diffraction surfaces are more dynamic when in-flight, so the same statistical performance improvement is not as evident. However, the NL is still valid and would offer added performance in the situation where multipath was a problem. The complete results of the in-flight NL baseline solution in comparison with the SF baseline solution are shown in Appendix B.

### 4.4 Carrier Phase Multipath Conclusions

This chapter described the relevant CP multipath error theory, presented models to quantify the error characteristics, and discussed the NL measurement combination technique as a means of range-domain noise and multipath error reduction. Field test data was provided to validate the multipath models and some geometric reflection point theory was presented to demonstrate the sensitivity of multipath errors and as a means to explain the data disagreement with the simple models. Several different quantities were described as multipath indicators for time-domain detection and were compared with the frequency-domain characterization. Finally, the NL baseline processing technique was presented as a real-time approach to mitigate the noise and multipath errors. The NL solution was practical for this application because the baseline uncertainty could be controlled through error cancellation due to the short baseline length. Least squares residual errors were reduced by almost 30% and the baseline position magnitude errors
were reduced by 19%. It is concluded that the NL is a good way to minimize noise and multipath in real-time applications and should be used whenever possible. If this reduction is not significant enough for the requirements, additionally processing such as complementary smoothing might offer performance improvements, but care must be taken to not introduce biases due to non-even fading patterns in the multipath fading frequency. In general, it is too difficult to perfectly model the reflection and diffraction surfaces and other variables affecting multipath errors, so an approach that does not rely on this information will be more robust.
5 Recursive Smoothing

5.1 Recursive Smoothing Introduction

With CP noise and multipath limiting the ISR stabilization accuracy, smoothing techniques were sought to enhance the GPS DDCP measurements on a single platform by smoothing them while preserving measurement bandwidth and without introducing a time lag. This enhancement will reduce the impact of CP noise and high frequency CP multipath to increase the performance for ISR stabilization applications. The technique described in this chapter is generic in nature such that inertial data can be provided by a host IRU or a remotely mounted IMU in close proximity to the GPS antenna(s). References to an INS are intended to generically refer to either source of inertial data.

The approach taken to smooth the GPS DDCP measurements without losing dynamic bandwidth was to synthesize a complementary DD quantity with ultra-low-noise characteristics that follows the airborne dynamics. A highly accurate static measurement of the baseline can be rotated using INS attitude and heading measurements to form a dynamic synthesized DD quantity as described in section 3.5. The synthesized DD can then be used to smooth the GPS DDCP measurements without causing a lag and enhance performance of the overall relative position solution. The smoothing operation is performed to reduce the noise on the GPS DDCP measurements using the synthesized DD quantity, which is a combination of a precise antenna baseline survey and the INS attitude and heading measurements. While this operation is performed primarily to
benefit the GPS DDCP measurements, the DDs could also be used in an optimal filter to also improve the inertial measurements as described in section 3.5. Flight test data will be presented to demonstrate the performance improvement in the midst of aircraft dynamics. It should be noted that a recursive smoothing approach using complementary measurements is unique compared to a complementary Kalman filter integration because smoothing can be practically implemented with no a priori knowledge of the system and its dynamics while attaining a substantial performance improvement over raw measurements. Additionally, a recursive smoothing filter is non-optimal in comparison with the typical Kalman filter which constrains its estimates to a minimum variance unbiased estimate. Consequently, a recursive smoothing filter can smooth over longer windows to attain a desired variance at the cost of some tolerable bias.

The smoothing terminology used throughout this chapter refers to the averaging of residual time-invariant noise (assumed to be white after the removal of platform dynamics). It could be described as a causal filter in that it does not make use of future information and is useful for real-time applications. This terminology is also consistent with that employed when using the GPS carrier to smooth the code measurements (Hatch, 1982). Another common smoothing definition (Gelb, 1974) would not be appropriate for real-time applications because it utilizes a forward and backward filter to form an optimally smoothed estimate.
Several “operational assumptions” are made for the single platform case that will affect the overall performance gained by this smoothing technique and will be referred to throughout this chapter. These operational assumptions for this single-platform case are: 1) a rigid body assumption from the INS sensor to the GPS antenna, 2) a rigid body along the baseline between GPS antennas, 3) an accurate antenna position survey, 4) accurate temporal and spatial alignment between GPS and INS sensors, and 5) high accuracy and low latency inertial orientation measurements. Each of these is important to consider and will affect the level of performance enhancement derived from smoothing to varying degrees. Violation of operational assumptions 1-5 will only introduce errors when in high turn rates and their effect will be negligible in flat and level flight. Operational assumption 3 and 5 will affect the magnitude of the error in dynamics and as long as their error can be reduced below the performance requirement (i.e., 4 mm), it can be considered negligible. With dynamics removed, the smoothing window can be increased until the stated assumptions become less valid. When this happens, smoothing divergence will occur; however, the smoothing window will be shown to be on the order of thousands of samples. Longer smoothing times can be obtained with more careful considerations of the stated assumptions.

The scenario shown in Figure 1-2 is used to illustrate one potential application. While it is true that the wing flexure would violate, the first two assumptions to some degree, this can be addressed by using an inertial sensor at the wingtip or by accepting the wing motion as an accuracy limitation. Two antennas on the aircraft fuselage are used here to
present the concept of recursive GPS DDCP Smoothing, which can be expanded upon later for wingtip applications. For the remainder of this chapter, data from two points on the centerline of the fuselage (baseline12) will be presented which is consistent with the operational assumptions.

5.2 Recursive Double Difference Smoothing Theory

5.2.1 Recursive Smoothing Overview
It is important to preserve measurement bandwidth without introducing observation lag when smoothing for high performance applications. As mentioned in the previous section, this can be accomplished by blending an ultra-low-noise secondary measurement that tracks the platform dynamics with an unbiased, but noisy primary measurement. The secondary measurement can be biased as long as the bias is stable in the short-term. Conceptually, this type of smoothing can be thought of as removing the dynamics to leave a stationary noise process and then averaging the residuals as if they were static measurements. For this chapter, the DDCP measurement was selected as the primary measurement rather than the raw CP because several hard to model error sources such as clock offsets and atmospheric biases have been substantially reduced as shown in the progression from equation (2-1) to equation (2-2). The presence of these error sources would cause the measured and synthesized DDs to diverge and the smoothed DD would accumulate biases. The typical CP measurement contains 1-2 mm of noise (1σ), which is magnified by a factor of 2 in the DD operation (Misra & Enge, 2001). The secondary quantity was chosen to be a synthesized DD, which was created using a static baseline survey and dynamic orientation measurements from the inertial sensors. The synthesized
quantities are very low noise since the survey coordinates are constant in the body frame and the platform orientation is derived from very low noise inertial measurements. Thus, while the noise on the DDCP measurements is twice that of the raw CP measurements, it can be smoothed with the synthesized DD to produce less overall noise. As will be shown later, the increase in noise due to the differencing process can be effectively removed after smoothing over a window of 4 samples.

5.2.2 Baseline and Double Difference Synthesis
The synthesized dynamic DD can be created using an accurate measurement of the baseline (e.g., from a static survey) in the body frame, $b^h$, along with a dynamic estimate of the baseline orientation, which will be expressed in terms of a coordinate rotation DCM from the body frame to the navigation frame. The first part of this section describes the baseline synthesis and follows the one described in 3.1.4, but the second part of the section uses the synthesized baseline to form synthetic DDs. When a rigid body assumption is made between the inertial measurement device and the GPS antenna(s), the baseline orientation can be approximated with the platform orientation. If this assumption is not valid, the baseline orientation must be directly measured at the flexing node. The first step in synthesizing a dynamic baseline is to accurately survey the baseline vector under static conditions. This can be done using an optical laser survey or by using a precise survey from GPS data. An optical survey can be very accurate, but is difficult to rotationally align with the body frame and it is difficult to physically locate the phase center of the antenna. The GPS survey has an advantage because the position can be averaged over longer data sets to improve accuracy and the phase center location
is included in the measurement data. The disadvantage is that there might be a slight misalignment of the GPS navigation frame with respect to the inertial navigation frame. Despite this disadvantage, the GPS survey method is used here and a calibration is applied to account for the frame misalignment as was discussed in chapter 3. The GPS survey provides antenna positions in an Earth-Centered-Earth-Fixed (ECEF) frame and they must be converted to a local-level NED frame with respect to a static reference point so the inertial DCM from equation (2-9) can be used. The next step is to determine the relationship between the navigation frame used for the static survey, \( b_{n,sta} \), and the inertial-referenced body frame, \( b^b \). The static baseline survey and the static navigation to body frame DCM are then used to form the Zero Attitude Baseline (ZAB) in the body frame as described in equation (3-9) and simplified here (in meters):

\[
b^{b,zab} = C_{n,sta}^{b,zab} \left( b_{n,sta} + \varepsilon_{bl}^{n,sta} \right) \tag{5-1}
\]

where:
- \( b^{b,zab} \) = body frame ZAB vector \([m]\)
- \( C_{n,sta}^{b,zab} \) = static orientation navigation to ZAB body frame DCM \([\text{unitless}]\)
- \( b_{n,sta} \) = initial static survey baseline vector \([m]\)
- \( \varepsilon_{bl}^{n,sta} \) = initial static survey baseline vector error \([m]\)

\( b^{b,zab} \) will contain small errors due to the static survey and will consequently limit the accuracy of the synthesized DDs and thus the final baseline solution. As stated in the operational assumptions, this error will be considered negligible. Also, it is assumed that the frames are aligned, so the alignment error from equation (3-9) is not shown in equation (5-1).
A series of two operations must be applied to the ZAB in order to form a dynamic synthesized DD. The first is a rotation into the dynamic navigation frame using measured rotational information. This frame is dynamic in that it is north pointing and rotates with the platform transport rate. The synthesized dynamic baseline shown here is a repeat of (3-10) and is shown again for convenience:

\[ b_{n,dyn}^* = C_{b,zab}^{n,dyn} \cdot b_{b,zab}^{n,dyn} + \varepsilon_{bl}^{n,dyn} \]  (5-2)

where:
- \( b_{n,dyn}^* \) = inertial-derived baseline vector wrt dynamic main antenna [m]
- \( C_{b,zab}^{n,dyn} \) = body to navigation frame DCM [unitless]
- \( b_{b,zab}^{n,dyn} \) = ZAB vector [m]
- \( \varepsilon_{bl}^{n,dyn} \) = error in the inertial-derived baseline from body flexure [m]

It should be noted that \( b_{n,dyn}^* \) will be biased by any baseline flexure that is not accounted for in the rotational measurements. As long as this flexure is constant or slowly changing, it will not affect the smoothing algorithm as will be described later.

The second operation in the synthesis process is to transform the inertial-derived baseline vector into the ECEF frame, so that it matches the GPS range measurements. This transformation is achieved with a rotation from the navigation to ECEF frame and a translation using the dynamic main antenna location expressed in the ECEF frame as follows (in meters):

\[ b_{ecef} = C_{n,dyn}^{ecef} \cdot b_{n,dyn}^{ecef} + X_m^{ecef} \]  (5-3)

where:
- \( b_{ecef} \) = ECEF frame baseline vector with respect to the master antenna [m]
- \( C_{n,dyn}^{ecef} \) = navigation to ECEF frame DCM [unitless]
- \( X_m^{ecef} \) = Absolute position of the master antenna location for translation [m]
The synthesized DD accuracy is not very sensitive to the absolute accuracy of the main antenna location, because the error cancels in the single difference between receivers. Similarly, errors in the DCM from an inaccurate main antenna position may cause slight frame misalignments, but can be neglected because the angular error resulting from a 100 m position error at the Earth’s semi-major axis would only be approximately 15 µrad. It is important to note that the inertial-derived baseline vector quantity in equation (5-3) could be the basis for a position domain smoothing approach similar to the approach described in section 3.3. The synthesized range to each SV \((\hat{r}^j & \hat{r}^k)\) is calculated for both ends of the baseline by normalizing the position difference between the SV antenna and the user antenna location. These ranges are then differenced as is done with the GPS measurements to form a synthesized DD quantity as described below (in meters):

\[
\hat{\phi}_{sm}^{jk} = \left(\hat{r}_s^j - \hat{r}_s^k\right) - \left(\hat{r}_m^j - \hat{r}_m^k\right) = \hat{r}_{sm}^{jk} + \varepsilon_{sm,align}^{jk} + \varepsilon_{sm,flexure}^{jk} \tag{5-4}
\]

where:

- \(\hat{\phi}_{sm}^{jk}\) = synthesized DD range (with error) [m]
- \(\hat{r}\) = synthesized range between SV antenna and GPS user antenna [m]
- \(\hat{r}_{sm}^{jk}\) = synthesized DD true range (estimated) [m]
- \(\varepsilon_{sm,align}^{jk}\) = DD GPS/INS static and dynamic frame alignment errors [m]
- \(\varepsilon_{sm,flexure}^{jk}\) = DD error due to body flexure [m]

As an aside, the synthesized DD could be formed by simply multiplying the geometry matrix in equation (2-8) by the baseline vector from equation (5-3). This approach is not used because it gives up error observability and the ability to crosscheck individual range and SD measurements.
5.2.3 Recursive Double Difference Smoothing

The synthesized DD quantities shown in equation (5-4) are very low noise measurements, but may contain some bias due to violations of the operational assumptions made earlier (i.e., rigid body, alignment, survey accuracy, etc.) and can be used as a basis to smooth the noisier GPS DDCP measurements shown in equation (2-2). By differencing equation (2-2) and equation (5-4), it can be seen that the baseline dynamics have been removed and the remaining quantity represents CP measurement error, CP multipath, and errors due to violations of the operational assumptions as (in meters):

\[ \epsilon_{sm, resid} = \Phi_{sm} - \hat{\Phi}_{sm} = \epsilon_{sm, \phi} + \epsilon_{sm, \phi MP} + \epsilon_{sm, align} + \epsilon_{sm, flexure} \]  

(5-5)

A recursive smoothing algorithm can be used to reduce the CP noise and CP multipath if the smoothing window duration is longer than the inverse of the fading frequency. As mentioned previously, airborne dynamics are expected to cause multipath to be noise-like (Braasch & van Graas, 1991). This smoothing filter is similar to a Hatch filter used for carrier smoothing of the pseudorange measurements (Hatch, 1982). In this case the synthesized DD can be used to smooth the measured GPS DDCP measurements. The same notation is used for the smoothed DDs as was used previously for the raw GPS DDCP measurements, but the receiver and satellite labels have been omitted as shown (in meters):

\[ \Phi_t = \frac{1}{M} \Phi_t + \frac{M - 1}{M} \left[ \Phi_{t-1} + (\hat{\Phi}_t - \hat{\Phi}_{t-1}) \right] \]  

(5-6)

where:
- \( \Phi \) = smoothed DDCP (using synthesized and measured DDCP) [m]
- \( \Phi \) = raw GPS DDCP measurement [m]
- \( \hat{\Phi} \) = synthesized DD (using inertial attitude) [m]
- \( M \) = smoothing window duration [samples]
- \( t \) = time index [unitless]
Notice that the synthesized DD in equation (5-6) is sequentially differenced. As a result, the synthesized DD can contain a constant bias between epochs and it would be removed in the differencing operation.

The theoretical relation between the respective measurement and the smoothed standard deviation, $\sigma_m$ and $\sigma_{sm}$, for a time-invariant random noise process smoothed by a moving average filter is shown in equation (5-7) (Hwang, McGraw, & Bader, 1999) (in meters):

$$\sigma_{sm} = \frac{\sigma_m}{\sqrt{2M}} \quad (5-7)$$

Even though the DD CP measurements are correlated, a time-invariant random noise process can be used to approximate the remaining noise when the dynamics have been effectively removed. As the smoothing window size linearly increases, the noise standard deviation decreases in an approximately quadratic manner. The noise in the smoothed DDCP can be reduced by half after only two samples and to 1/10 of its original value after 50 samples.

**5.3 Recursive Smoothing Results and Analysis**

**5.3.1 Raw Double Difference Baseline**

To demonstrate the performance of smoothing the measured GPS DDCP with the synthesized DD under dynamic conditions, flight data were collected as described in section 2.5 for analysis.
GPS DDCPs were formed according to equation (2-2) and then used to calculate a platform relative baseline as described in equation (2-8). The GPS baseline solution is shown in Figure 5-1 for baseline12. All of the results presented in this chapter will be from baseline12 unless otherwise stated. The first three subplots of Figure 5-1 represent the relative position of node 1 from the main antenna (node 2) in a local level coordinate frame. The fourth subplot of Figure 5-1 shows the baseline12 magnitude. A deviation from the “true baseline magnitude”, under the rigid body assumption, represents measurement error and body flexure. The aircraft taxied until takeoff at about 500 sec. The aircraft then maneuvered and traversed to PKB until about 1600 seconds and then entered a flat and level segment continuing until about 2900 seconds. After returning to UNI a series of banking maneuvers were executed over the UNI airport until touchdown at about 4200 seconds.

![Figure 5-1: GPS SF L1 Baseline Solution](image-url)
Parity space was used as a performance metric as described by equation (2-8). The parity space residual was broken down into two parts using a low pass filter with a cutoff frequency of 0.5 Hz to illustrate the noise and the bias/trend performance separately as shown in Figure 5-2. Only a single parity vector was computer because there were four DD measurements available to solve the three baseline position unknowns, leaving only one redundant measurement to use for parity space analysis. The trend quantity was derived by subtracting the parity space noise from the total parity vector so that there were no issues with overlap in the 0.5 Hz filter roll off. The raw DD noise standard deviation for baseline12 was 2.49 mm as shown in the upper subplot of Figure 5-2. The lower subplot of Figure 5-2 represents the baseline12 bias/trend inconsistency over time. Both subplots of Figure 5-2 will be used as a basis to assess the smoothing technique.

Figure 5-2: GPS SF L1 Parity Space Residuals
5.3.2 Smoothed Double Differences

Complementary smoothing can be used to reduce the CP noise and multipath by using synthesized DDs as described by equation (5-6) to produce smoothed DDCP. Figure 5-3 shows both the measured and smoothed DD for SV10 (with SV2 as the key) with a smoothing window of \( M = 200 \) samples. The top subplot contains both the measured DD and smoothed DD for SV10, however they overlay so closely that only one curve is visible. The bottom subplot of Figure 5-3 represents the noise residual that remains after the smoothed DDCP is subtracted from the measured DDCP. Unless stated otherwise, the statistical values shown in each figure legend were calculated using the constituent data over the same interval shown in the plot.

![Figure 5-3: DD Comparison for a 200 Sample Smoothing Window](image)

Figure 5-4 shows a representative 8-second segment of the same quantities to more closely examine the smoothing effect and to show that no significant lag or bias was
introduced in the smoothed DDCP with respect to the measured DDCP. There are two lines in the first subplot with the noisier one being the measured DD and the smoother one being the smoothed DD.

![Figure 5-4: Zoomed-In DD Comparison for a 200 Sample Smoothing Window](image)

It is apparent in both Figure 5-3 and Figure 5-4 that the noise in the smoothed DDCP has been reduced and the smoothed DDCP does not contain a significant bias or lag. By solving equation (2-8) using the smoothed DDCPs, a smoothed baseline solution can be determined. The smoothed baseline parity space residual is shown in Figure 5-5 with the \( M=200 \) sample smoothing window (i.e., 2 seconds).
The parity space noise quantity has reduced from 2.49 mm (in Figure 5-2) to 0.10 mm in Figure 5-5, approximately the theoretical noise reduction predicted by equation (5-7). The standard deviation is better than theoretical due to the noise separation filter cutoff frequency of 0.5 Hz. Much of the noise has also been removed from the trend quantity and its shape has remained unchanged with no additional biases. The smoothing window can be increased to $M=1000$ samples (i.e., 10 seconds) to achieve more performance improvement in the parity space trend. A smoothing comparison is made for this smoothing window size as shown in Figure 5-6 for the same representative data segment used previously to illustrate the smoothing performance. The corresponding parity vectors are shown next in Figure 5-7.
As the smoothing window increases, the smoothed DDCP follows the synthesized DDCP more closely, so any errors in the synthesis process, equation (5-4), will be transferred to the smoothed DDCPs. A 0.86 mm smoothing bias is visible in the range residuals of
Figure 5-6 with $M=1000$, where the smoothing bias was previously 0.39 mm with $M=200$, so measurement agreement errors have become slightly more apparent. This bias is believed to be from a slight error in the inertial-synthesized baseline and from a violation of the operational assumptions made earlier. The parity space noise has been almost eliminated in Figure 5-7 and the standard deviation in the trend has been reduced further below 3 millimeters. The performance enhancement contributed by complementary smoothing is summarized in Table 5-1 as observed in the parity space solution.

<table>
<thead>
<tr>
<th></th>
<th>Parity Space Mean, [mm]</th>
<th>Parity Space Standard Deviation, [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 0$, Noise</td>
<td>0.00</td>
<td>2.49</td>
</tr>
<tr>
<td>$M = 0$, Trend</td>
<td>0.16</td>
<td>3.28</td>
</tr>
<tr>
<td>$M = 200$, Noise</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>$M = 200$, Trend</td>
<td>-0.15</td>
<td>2.96</td>
</tr>
<tr>
<td>$M = 1000$, Noise</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>$M = 1000$, Trend</td>
<td>-0.11</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Substantial performance improvements were noticed from smoothing for a very short period of time. In addition to the noise performance improvement, examination of the synthesized DDCP provides an independent measurement quantity for comparison and can make the system more robust. As expected, the smoothing performance is particularly noticeable in the noise standard deviation as highlighted in Table 5-1.

### 5.4 Recursive Smoothing Conclusions

This chapter presented a technique to smooth GPS DDCP measurements using inertial synthesized DDCP quantities & lower noise levels were demonstrated that would then
enhance the single-platform relative baseline measurement performance for many high performance ISR stabilization applications. Additionally, this type of smoothing was shown theoretically to remove the additional noise from CP measurement double differencing with a smoothing window of only 4 samples. The theoretical basis and equations were presented for this generic technique and flight data was used to demonstrate its effectiveness and that it follows the theoretical performance.

While raw GPS DDCP measurements from a flight test were shown to provide a very precise 100 Hz estimate of the baseline between two antennas in previous work, the DDCP measurement noise and CP multipath were seen as the biggest limitations to further performance improvement. Due to the noise-like nature of these error sources when in flight, they can be reduced using a recursive complementary smoothing technique. A recursive complementary smoothing algorithm was described to synthesize ultra low noise DDs using inertial attitude. This algorithm was then shown to reduce the noise on DDCP measurements without losing measurement bandwidth by removing the platform dynamics from the DD measurements. The parity space noise value above 0.5 Hz was reduced from 2.5 mm without smoothing to 0.1 mm with a 2 sec smoothing window. Noise reductions were even more evident in the parity space trend below 0.5 Hz when the smoothing window was increased to 10 sec. A very slight bias (< 1mm) was observed in the smoothed DDs, but it was determined that this bias likely resulted from slight violations of the operational assumptions. Smoothing did not add
measurement lag or bias unless the smoothing window was too large for the given level of operational assumptions.
6 Summary and Conclusions
ISR sensors such as radar, ladar, electro-optic / infra-red, and other remote sensing activities are becoming increasingly dependent on their position and orientation in time and space. The higher dynamics of flight and the increased sensor requirements have led to the need for direct-measurement sensor stabilization. A GPS-based stabilization system can provide good performance, but high-frequency sensor pointing applications have led to the need for additional accuracy and robustness.

The integration of INS measurements was shown to provide a substantial performance increase over the GPS-only approach to single-platform ISR stabilization application. A sensor ground-calibration step is key to the system operation, but is susceptible to GPS ground-multipath, so strategies were presented to identify, characterize, and mitigate this error for the single-platform ISR stabilization application.

Sensor integration using navigation sensors requires accurate time tagging and synchronization as well as spatial alignment to provide high performance stabilization. The platform dynamics were shown to dictate the synchronization and alignment requirements. Spectral analysis was shown as a way to assess the bandwidth requirements to capture the desired motion dynamics.

Chapter 3 provided information on sensor alignment and measurement integration using a variety of techniques. It was shown that longer baselines provide smaller orientation
errors, but could also lead to larger baseline synthesis errors. Three different Kalman Filtering alternatives were presented to optimally reduce the errors in a minimum variance and unbiased sense. A position-based filter provided a more robust integration than the orientation-based filter because errors due to baseline flexure were minimized inside the filter. The tight range-domain filter provides the ability to operate when the available GPS constellation is underdetermined. It also operates on a per-DD basis and would allow the multipath to be addressed before it becomes part of the navigation solution. An offline frame calibration and survey refinement technique was shown as a way to provide a coarse alignment while maintaining measurement independence. Static and flight results were used to demonstrate a combined solution that contains less noise than the mm-level GPS measurements and one that is unbiased from the GPS measurements. For static data, the orientation filter provided $1\sigma$ standard deviation performance of approximately 0.1 mrad in each observable axis. With the same static data, the baseline filter provided $1\sigma$ standard deviations of approximately 0.87, 1.07, and 1.61 mm in the respective East, North, and vertical directions. A tightly integrated range-domain filter was also demonstrated for the static tarmac test data as well as the flight test data using measurements from the wing and fuselage baselines.

Chapter 4 was largely motivated by multipath errors in the ground-calibration used for baseline synthesis and ambiguity resolution. These errors were inconsistent with the simple models due to the complex reflection/diffraction surfaces found on and around aircraft. Several quantities were identified as multipath indicators to detect the presence
of multipath, which can then be used for satellite exclusion. The NL measurement combination technique was analyzed as a means to mitigate noise and multipath in the baseline solution. The least squares residual error was reduced by approximately 30% while the position errors were reduced by approximately 19%.

Chapter 5 presented a technique to reduce the measurement noise and noise-like errors in a non-optimal sense using complementary smoothing techniques. This technique provided a means to reduce the GPS measurement noise using INS measurements while maintaining measurement bandwidth and without introducing a lag. A smoothing window of only two samples was sufficient to eliminate the measurement noise added by double differencing. The one sigma standard deviation in the parity space solution above 0.5 Hz was reduced from 2.5 mm to 0.1 mm with a two-second smoothing window without a noticeable bias. This error was further reduced to 0.02 mm for a ten-second smoothing window, but with evidence of a sub-mm bias.

It can be concluded from this work that a high-rate mm-level ISR stabilization system is realizable. The hardware and software were developed and demonstrated on two flight tests and in two ground-calibration field tests. The real-time data collection software processed the raw 100 Hz GPS L1 carrier phase measurements in a manor that makes real-time GPS-based stabilization possible. Since the post-processing software was able to process an hour of high-rate GPS data in under ten minutes, real-time ISR stabilization would be possible.
The addition of high-rate IMU data brings more system complexity through temporal and spatial alignment, but offers a substantial increase in system performance through noise reduction. A range-domain Kalman filter has many convenient features for measurement integration and would be preferred over ones that require a complete GPS position solution. All of the integration techniques demonstrated an improvement over the GPS-only solution for a rigid baseline; however, more work would be required to realize the same improvement for a flexible baseline. Multipath is still one of the dominant error sources for single-platform relative ISR stabilization and ground calibration, but it can be mitigated via satellite exclusion and NL processing. If an optimally unbiased solution were required, a Kalman filter integration technique would be favored. However, if a non-optimal solution is acceptable, the complementary smoothing technique offers a more dramatic noise reduction.
7 Recommendations and Future Work
A significant amount of work for this dissertation went into the development and expansion of data collection and processing capabilities to the point where real-time GPS-based ISR stabilization is possible. The next logical step would be a real-time in-flight demonstration of the high-rate relative GPS-only stabilization system followed by a demonstration of the integrated GPS/INS stabilization system.

Two flight tests and two ground-calibration field tests provided a wealth of navigation sensor data. Only a small portion of the data was shown in this document. There is more work to be done to integrate measurements from the other sensors and multiple baselines. The basic three-state filters presented in this dissertation were not able to properly handle the wing dynamics, so more work is required to add higher order states to these filters. Also, a more elaborate Kalman filter could be incorporated to model the navigation sensor measurement errors.

In every location on the DC-3 where a tactical-grade IMU was installed, a low cost IMU was also installed. Some research should be conducted to determine the benefits that a low cost sensor might provide in a short-term integration. With high-rate (100 Hz) GPS carrier phase measurements, a low cost IMU would not drift very much between updates. As long as the IMU measurements are low noise and can be accurately aligned and calibrated, some benefit might be gained.
Also, more spectral analysis of the inertial data from each node would provide additional insight into the requirements for motion stabilization. More focused attention could be given to key phases of flight to help understand the platform dynamics at multiple locations. This understanding might be useful in the development of a platform-specific structural motion model.

More research is needed to reduce the impact of carrier phase multipath in the ground calibration. Ultra-short baseline data (several centimeters) was collected at node 1 during each test and would provide a means to remove the spatially correlated multipath from this node. Additionally, some work could be done to tune the Kalman filter to exclude multipath so that it does not impact the integrated solution. More research is needed to investigate the severity of in-flight multipath. The first step in that direction might be the development of airframe multipath models that would help predict the multipath reflection or diffraction point given the SV location and the aircraft position and orientation.
References


8 Appendix A – Data Collection Configuration

The information presented in this chapter is complementary to section 2.5 because it provides more detail about the sensor installation at each node of the DC-3 aircraft used for the static field test and flight test on April 12, 2006. It was not considered essential to the understanding of the material being presented, so the detail was saved for this appendix. This appendix also provides information about the real-time data collection software used on the aircraft.

8.1 Hardware Installation

8.1.1 Remote Node 3

The wing-mounted IMUs (IMU#0) were mounted below the port wing of the DC3 and co-located with the GPS antenna as shown in Figure 8-1 and Figure 8-2. Figure 8-1 also illustrates where the Microstrain IMU and the remote power supply were installed. The power supply provided a switchable DC power source close to the IMUs to avoid a large voltage drop over the length of the wing.
The LN200 was installed at an odd angle to be roughly aligned with the aircraft body frame. The device was installed as close as physically possible to the GPS antenna.

8.1.2 Remote Node 1
A second IMU was mounted inside the fuselage (IMU#1) which was mounted directly to a table-top platform that was attached to the HG1150 Navigation-Grade Inertial Reference Unit (IRU) as shown in Figure 8-3. Figure 8-4 shows a closer look at the hardware at this node.
This picture shows most of the hardware that was required for the data collection at a remote node. It also illustrates the sensor proximity to the Honeywell HG1150 IRU. Figure 8-4 shows a closer view with each component labeled.

The remote power supply provided three DC voltages power to the LN200 IMU and allowed each of them to be independently switched. It also provided the DC power to the low-cost IMU as well as the two circuit boards. These boards, labeled in Figure 8-4 as
RS485 transceivers, were designed and built in-house to take RS232 data and convert it to differential signaling, which is more immune to common mode noise and interference and allow it to traverse longer distances. These boards were used as transmitters at node 2 and node 3 while the ones shown in Figure 8-4 were acting as receivers. The differential signaling was converted back to RS232 for use by the data collection PC serial ports.

8.1.3 Remote Node 2
A third IMU (IMU #3) was also installed on a platform connected directly to the belly-mounted ladar as shown in Figure 8-5 and Figure 8-6. This platform was shock mounted to protect the ladar from vibration damage while in flight, but the IMU would experience the same dynamics as the ladar. This IMU provided gyro measurements at a 3600 Hz output and accelerometer data at 360 Hz.
The hardware installation at this node was the same as those seen at the other two nodes except that it is mounted below the floor boards (and the aircraft control surface cabling).

### 8.2 Data Collection Software

The data collection system software was run under the QNX 6.3 real-time operating system. This allowed some of the data to be processed in real-time and recorded in a more intelligent way. It also allowed some of the sensor data to be visually monitored for real-time troubleshooting. Under QNX, a flexible architecture exists, referred to as a resource manager (RM), for hardware interfacing. This architecture can be thought of as a device driver because of their similarities. Several RMs were developed for this research, several were reused from previous projects, and several were modified to suit new requirements.

An overview of the data collection system is given in the software block diagram shown in Figure 8-7.
There are four categories of blocks shown in Figure 8-7. The first, numbered from 1 to 4, represent the physical sensors providing data to the system. The second category, numbered 5 to 9, represent the resource managers in the system. The blocks numbered 10 and 12 represent the graphical interfaces to the resource managers and block 11 is a client program that continuously reads data from each available RM and writes it to the hard drive. The last category consists of single-use programs for sensor configuration. Block 13 was modified for this effort to operate natively under QNX 6 to align the HG1150 IRU and block 14 was used to configure the LMSQ140 laser scanner. After these programs have run, the sensors are ready to collect data. All of the other resource managers are spawned by a third program, not listed, called SpawnRMs that starts each resource manager in the correct order. The low-cost Microstrain software is also not
listed because it was considered as an LN200 backup device, so the data collection functionality (RM and graphical interface) would mirror that of the LN200.

The OEM4, shown in the first block, provides the GPS code and carrier measurements as well as a 1 PPS signal synchronous system timing. The Honeywell HG1150, shown in the second block, provides data over an ARINC 429 interface to an interface card in the data collection computer. LMSQ140 ALS, shown in block three, is configured via the serial port and then provides a data output via the PC parallel port. The fourth block represents the Northrop Grumman LN200 IMU, which outputs its data and a timing pulse via a synchronous data link control (SDLC) differential interface. The data and timing pulse are connected to an SDLC interface card inside the data collection computer. The time-tag in each data frame is applied based on the time of receipt of the timing pulse. Block five represents the timing resource manager. This RM functions to provide a current estimate of the GPS time based on the PC clock that was calibrated to GPS time and aligned using interrupts generated by the PPS signal. The NovAtel RM in block 6 decompresses the pseudorange and CP measurements. A new processing module was added for this research to decode the high-rate, 100 Hz, CP log and decompress its measurements using the pseudorange measurements. Both the ARINC and ALS RMs in blocks seven and eight were reused from previous research to make the IRU and ALS measurements available to the system. Two versions of the LN200 RM were developed for this research: a 400 Hz version and a 3600 Hz version for each of the two types of IMUs that were installed. The RM managed the configuration of the SDLC interface
card, clock synchronization, and time tagging of the data frames. More detail is provided in Figure 8-8.

**Figure 8-8: LN200 Resource Manager Flow Diagram**

After both ports of the SDLC interface card have been configured, the interrupt thread of the LN200 RM operates in a loop waiting for data to arrive. When an interrupt is
generated, it is processed in the interrupt service routine and then processed by the interrupt processor. The data is put into a circular buffer to be available for clients in the system and the interrupt waiting thread continues.

Block ten represents the graphical interface block to display the data provided by the OEM4. This software was written as a client that connects to the NovAtel RM and displays much of the data contained in its circular buffer. The interface, as is shown in Figure 8-9, was modeled after one used for previous research at Ohio University.

![Figure 8-9: Ohio University NovAtel GPS Data Collection Display Software](image)

Three windows are shown in Figure 8-9 as a sample. The first one is the back layer; it consists of a title panel on top and a messages panel in the bottom, which is common to all windows. The primary window is shown in the bottom right corner, which provides channel information about the receiver. The top line provides a receiver identifier, clock status, automatic gain control (AGC) indicator, a jamming indicator, and the GPS time. All of this information is extracted directly from the receiver logs. The L1 data is shown
in the top half of this panel and the L2 data is on the bottom. Each channel lists the satellite identification, pseudorange measurement, Doppler measurement, carrier to noise ratio estimate, lock time, and a channel tracking state indicator. A third window, shown in the upper right hand corner of Figure 8-9, provides more receiver-specific details in two panels. Each panels contains the calculated receiver position in latitude, longitude, and height. It also provides hex codes for the receiver and clock status. The solution status and position type are displayed in text while the number of satellites tracked is indicated as a decimal. An indication of the serial port buffer overflow status and receiver CPU status were added to the original panel because the high-rate CP log was more demanding of these resources. The fine time indicator is a repeat from the channel summary window.

The next block to be considered represents the graphical interface to the LN200 RM. All of the available data from the sensor is displayed in real-time as shown in Figure 8-10.
Figure 8-10: Ohio University LN200 IMU Data Collection Display Software

Figure 8-10 is arranged similar to the GPS receiver. The back layer consists of a title panel and a common messages panel. The primary window for the 3600 Hz IMU is shown in the bottom right corner. On the top line, an indication of the SDLC port number is displayed on the left and the GPS time from the PPS RM is shown in the right. The measurements of delta velocity and delta angular rate are shown for each of the six axes along with a primary failure indicator. The next line provides an indication of the time offset from the frame-timing signal, results of the RM data frame CRC check, the IMU frame check and the IMU mode indicator. The next block of indicators provide several IMU temperature and voltage measurements. In the event of some IMU failure, the detailed failure indicators are provided in the last three lines. The window shown in the upper right corner of Figure 8-10 is the primary window for the 400 Hz model with the sample-synchronizing clock. All of the same indicators are provided except that an indicator is added for the sample synchronization status. In this figure, the IMU was not locked to the clock because it was a test device and was not able to synchronize itself.
with an external clock, but the devices that were flown maintained continuous synchronization lock during the April 16th flight test.

Block eleven in Figure 8-7 represents the data recorder program. This program is simply an RM client that connects to each RM running on the system, reads all of the current data from their circular buffers, and writes it to the hard drive. This program interleaves data from every different sensor into one large binary file. If the time tags are corrupted from a given sensor for any reason, an estimate of the time can be made from the surrounding measurements. The filename is automatically named with the current date and time so as to enable rapid program execution and prevent accidental data loss by overwriting files.
9 Appendix B – Narrow-Lane Results for In-Flight Data
Baseline measurements can be calculated using the NL measurement combination technique described in equation (4-14). This chapter will provide additional results for the dynamic baseline measurements made in flight. For comparison, the low-rate L1 SF data must be presented since the NL is only available at 2 Hz due to the 2 Hz L2 CP measurements. These results are similar to the full 100 Hz L1 measurement data shown in Chapter 5.

9.1 In-Flight Single Frequency L1 Results
The GPS-only SF L1 baseline was solved according to equation (2-6) for the set of SVs described in Chapter 5 using SV2 as the key. The predicted baseline error is found by removing the synthesized baseline from the measured and the remaining portion is shown in Figure 9-1.
It should be noted again that the mean errors and the baseline error variation shown in Figure 9-1 cannot be solely attributed to the GPS baseline solution and are likely due to several different error sources included in the baseline synthesis. Next, the least squares residuals can be formed according to equation (2-7) for the SF L1 baseline solution as shown in Figure 9-2.
The total parity vector is again calculated using the lower half partition of equation (2-8) and split using a 0.5 Hz low pass filter as shown in Figure 9-3. It should be noted that this parity space solution differs slightly from the one shown in Figure 5-2 for the same flight conditions due to the difference in measurement update rates.

Figure 9-2: Least Squares Residuals for the In-Flight SF L1 Baseline Solution
9.2 In-Flight Narrow-Lane Results

Now that the low-rate SF baseline performance has been established, the NL equivalents will now be compared. The predicted NL baseline error is shown in Figure 9-4 for comparison with Figure 9-1.
The NL baseline error in Figure 9-4 does not contain significant differences in the East and North directions, but the vertical direction improves slightly in both bias (12%) and standard deviation (15%). Next, The least squares residuals are shown in Figure 9-5 for comparison with Figure 9-2.
In Figure 9-5, there is not a noticeable statistical improvement in comparison with Figure 9-2, but the NL residuals are visually more stationary than those of the SF solution. This is particularly true in the first 500 seconds of the data. It is not known for sure what caused the variation during this time, but the aircraft was conducting more extreme maneuvers at the beginning and end of the data set, so airframe-multipath could be a contributor. Finally, the NL parity vector is shown in Figure 9-6.
Figure 9-6: Parity Space Residuals for the In-Flight NL Baseline Solution

Similar to the least-squares residuals, the NL parity space analysis does not reflect a statistical performance improvement, but the improvement in low frequency variation is evident in the bottom subplot of Figure 9-6. Consequently, the standard deviation of the trend component shows a reduction from 2.31 to 1.43 as a result of the NL solution.

The results of the in-flight NL solution are not as dramatic as the static tarmac data shown in Chapter 4, but the solution is felt to be more robust due to the reduction in mean variation over time. Whether due to airframe-multipath or some other error source, the NL solution appears to mitigate its impact on the baseline solution.
10 Appendix C – DC3 Platform Spectral Content

10.1 Spectral Estimation Introduction

The following section will assess the platform dynamics of the Ohio University Douglas DC3 flying laboratory during flat and level condition of flight at a given cruising speed. A major reason for performing this analysis is to assess the bandwidth requirements for motion stabilization on this platform. The aircraft was instrumented with three Northrop Grumman LN200 IMUs in different locations on the aircraft. This device was selected because it has no internal shock mounts, so it was able to directly observe platform motion and also because of its relatively high measurement bandwidth of 200 Hz. The default IMU configuration provides coincident gyroscope and accelerometer measurements at 400 Hz. One of the three units provided high-rate 3600 Hz gyro outputs and 360 Hz accelerometer outputs. This unit was placed atop a shock-mounted ALS to provide high-rate pointing measurements since scanning ladar measurements are sensitive to pointing accuracy. The frequency content at this location would be uncharacteristically less than the aircraft itself due to the shock mounting, so measurements from this unit will not be considered further. The devices in the fuselage and port wingtip, which will be considered next, had an internal phase lock loop to synchronize their internal sampling to a GPS-derived 100 Hz clock. The measurements were then time-tagged using GPS time. Three IMU locations were selected as shown in Figure 2-5 as part of the April 12, 2006 sensor stabilization flight test. A more detailed flight test description and explanation is provided in section 2.5 and (Dickman & Bartone, 2008). The spectral content at two of the installed locations will be described in
this section, node 3 (port wing tip), and the node 1 (fuselage near center-of-gravity).

Rotational measurements processing can be described from the gyroscopes, but since the
displacement stabilization is more critical and for brevity, their results will not be
presented. The spectral analysis results from the displacement measurements of the
accelerometers, however, will be presented.

10.2 Measurement Pre-Processing
There are two ways to properly process the INS data in a meaningful way. The
measurements should be processed in a body coordinate frame so the spectral content is
not blurred with aircraft turns and other dynamics. Additionally, gravity should be
removed from the accelerometer measurements and the transport rate and Earth rotation
rate should be removed from the gyroscope measurements. There are two ways to
accomplish this. One is to perform a standard mechanization in a local-level NED frame,
estimate gravity, transport rate, and Earth rate, remove them, and return the
measurements in the body frame. This approach requires less processing and complexity,
but suffers from inertial drift. The second way is to integrate the measurements in a
Kalman filter to eliminate drift and align the measurements in the navigation frame. The
measurements can be rotated into the navigation frame for removal of unwanted motion
content and then rotated back into the body frame for spectral processing. Both methods
will be described and results will be presented from the second approach.
10.2.1 Stand-Alone Mechanization

The raw IMU data from each sensor was mechanized in a NED navigation frame and the raw delta-velocity measurements were aligned with this coordinate frame using estimates of the IMU orientation as shown in equation (10-1):

\[ \delta V^n = C_b^n \cdot \delta V^b \] (10-1)

where:
\[ \delta V^b = \text{INS delta-velocity measurement in the body-frame} \quad [\text{m/s}] \]
\[ C_b^n = \text{Body to navigation-frame DCM} \quad [\text{unitless}] \]
\[ \delta V^n = \text{INS delta-velocity expressed in the navigation-frame} \quad [\text{m/s}] \]

While in the navigation frame, gravity was estimated and removed and the same DCM was used to rotate the corrected delta-velocity measurements back to the body frame as shown in equation (10-2):

\[ \delta \hat{V}^b = C_n^b (\delta V^n - G) \] (10-2)

where:
\[ \delta \hat{V}^b = \text{Gravity-compensate delta-velocity re-expressed in the body-frame} \quad [\text{m/s}] \]
\[ G = \text{Change in gravity estimated over integration interval} \quad [\text{m/s}] \]

Similarly, the Earth rate and transport rate were removed from the gyro measurements to obtain rotations only due to the platform dynamics as shown in equation (10-3) (Titterton & Weston, 1997):

\[ \hat{\omega}_{nb}^b = \delta \hat{\theta}^b_{ib} - C_n^b \cdot (\omega^e_{ie} + \omega^n_{en}) \] (10-3)

where:
\[ \omega^n_{en} = \text{Transport rate} \quad [\text{rad/sec}] \]
\[ \omega^e_{ie} = \text{Earth rate} \quad [\text{rad/sec}] \]
\[ \delta \hat{\theta}^b_{ib} = \text{Measured angular rate} \quad [\text{rad/sec}] \]
\[ \hat{\omega}^b_{nb} = \text{Compensated angular rate} \quad [\text{rad/sec}] \]

The corrected measurements denoted with using the hat notation in equation (10-2) and equation (10-3), can then be used for spectral estimation.
10.2.2 Measurement Reconstruction from a Kalman Filter

This technique makes use of a Kalman filter to stabilize the inertial drift and maintain navigation frame alignment. The raw GPS CP and inertial measurements were integrated in a high-rate Kalman filter to estimate orientation and velocity at each measurement epoch. The resulting states were then sequentially differenced and the delta-velocity was expressed in the body frame to approximately reconstruct the measurements without gravity, Earth rate, and transport rate. The main reason not to use this approach is the complexity and processing time. Since neither were an issue, this approach was selected and the results to be described were derived using this method.

10.3 Spectral Estimation Theory

Since the delta-velocity and delta-orientation estimates are sequentially differenced in time, the low frequency components are not present and the higher frequency content will be exposed (effectively high-pass filtered). The data is shown in a time versus frequency plot of spectral amplitude. Spectral estimates will be derived using the Fast Fourier Transform (FFT) of segmented time blocks of data. The amplitude of the delta-velocity measurements will be presented, so a constant velocity must be assumed over the measurement interval to extract displacement. It should also be noted that some amount of amplitude averaging would take place over the time block, so amplitude estimation from an FFT is not perfectly deterministic. There is an inherent tradeoff between time and frequency resolution when performing this type of analysis, so care must be taken to optimize both the time and frequency resolution. The time-series data will be segmented into time blocks to calculate the spectrum of each block; larger time blocks provide more frequency resolution and less time resolution while a smaller time block provides more
time resolution and less frequency resolution. This is because a larger time block allows more points in the FFT. The flight data contains approximately 4314 seconds of data, which were broken into 842 segments (giving 2048 samples per segment), which yield a time resolution of approximately 5.12 seconds and a frequency resolution of 0.195 Hz (assuming a 2048 length FFT).

The FFT amplitude spectrum was shifted and doubled to only consider positive frequency and then normalized by the FFT length to maintain the amplitude units:

\[
S_a = \frac{2 \cdot |FFT(X|_{\text{block}})|}{N_{\text{FFT}}}
\]  

(10-4)

where:

- \( S_a \) = amplitude spectrum [data units]
- \( X \) = input data of length \( \text{block} \) samples [data units]
- \( N_{\text{FFT}} \) = number of points in the FFT [unitless]

Estimates of the amplitude as a function of both time and frequency were determined for the accelerometers separately for IMU#0\text{TG} and then for IMU#1\text{TG}. The rotational spectra were also found from the gyroscopes, but did not impact the displacement bandwidth requirements and will be left for future analysis.

10.4 Spectral Estimation Results and Analysis

10.4.1 Wingtip IMU#0, Node 3

The node 3 IMU data will be considered first. The data will be presented individually for each body frame axis with \( X \) pointing forward, \( Y \) pointing toward the right wing, \( Z \) pointing downward. Both the time block and FFT length will be 2048 samples as discussed above for the full-flight spectral estimation. The measurement update rate was
400 Hz, and thus the update period was .0025 sec. To estimate the displacement amplitude, the delta-velocity measurements amplitude estimates must be scaled by the update period. The X-axis accelerometer spectral estimates are shown in Figure 10-1.

During the first 800 seconds of the data, there are several peaks and the spectrum shows few noticeable patterns. This is during the static alignment, engine run-up, and taxing phases. Once airborne in the flat and level phase, a noticeable pattern emerges in the data whereby spectral lines are evident at 20, 35, 52, 60, 70, and 90 Hz. The most dominant line appears to be at 60 Hz. Another transition occurs around 3100 seconds into the data when the engines throttle up for a climb prior to some turning maneuvers. At this time all of the spectral lines are scaled to higher frequencies. Similar events happen at 3445 for a frequency reduction to the flat and level conditions, another increase at 3548 to match the climbing conditions, and finally at 4168 for landing. The flat and level conditions are
considered more important because that is when the ISR sensors would need to be stabilized. During this time, peak delta-velocity amplitudes of 30-35 mm/sec can be seen.

The next set of data to be considered comes from the Y-axis accelerometer as shown in Figure 10-2.

![Figure 10-2: Node 3, Body-Frame Y Accelerometer Time/Frequency Plot, Full Flight](image)

A similar behavior to the X-axis data can be seen for the first 800 seconds of flight during ground operations. During flight, the same harmonics are observed at 20, 35, 60, 70, and 90 Hz and the 52 Hz component is still present, but with less amplitude. The 60 Hz spectral line is still the strongest component. The peak amplitudes during the flat and level segments were 20-25 mm/sec.
The Z-axis accelerometer data is the final set to be considered at this node as shown in Figure 10-3.

![Figure 10-3: Node 3, Body-Frame Z Accelerometer Time/Frequency Plot, Full Flight](image)

In this figure, the same spectral behavior is observed in the first 800 seconds and many of the same spectral lines can be observed during the flight. Weak 40 and 52 Hz components are also visible. The most notable difference in the Z-axis is the presence lower frequencies (15 Hz and below), which show more amplitude than in the X and Y axes. The peak displacement during the flat and level phase of flight was between 15 and 20 mm/sec.
10.4.2 Center Fuselage IMU#1, Node 1
The node 1 IMU data will be considered next. The X-axis accelerometer is shown in Figure 10-4.

![Figure 10-4: Node 1, Body-Frame X Accelerometer Time/Frequency Plot, Full Flight](image)

In the fuselage, the spectral content is similar to that of node 3. During the flat and level phase of flight, the spectral lines reside at 20, 35, 60, 70, 88, 118, and 123 Hz. For this node, the 35 Hz frequency is dominant. The two high frequency lines at 118 and 123 are very weak contributors. It is believed that these higher frequencies are not as strongly observed because the physical structure of the wing acts, to some extent, as a low-pass filter. The peak amplitudes for the node 1 X-axis were found to be between 6 and 10 mm/sec.

The Y-axis data is shown next in Figure 10-5.
No significant structural differences in spectrum are observed in the Y-axis other than slight variations in amplitude. The same spectral components are observed during the flat and level phase of flight: 20, 35, 60, 70, 88, 118, and 123. The peak amplitude displacements were found to be between 8 and 10 mm/sec during the flat and level phase of flight.

The final full-flight data set to be considered is the Z-axis of node 1. The spectral content for this axis is shown in Figure 10-6.
The spectral content of Figure 10-6 is more unique than the others at this node because of the presence of the low frequency content (near DC) and the relative strength of the 118 and 123 Hz spectral lines. The peak amplitudes were found to range from 6 to 8 mm/sec during the flat and level phase of flight.
10.4.3 Taxi to Takeoff Transition

Now, two examples will be considered to demonstrate the spectral changes from low frequency dynamic motion of taxiing to the higher frequency dynamics of takeoff. Due to the smaller data length, the block size and FFT length have been reduced to 512 for more time resolution in this section. Only the $X$ accelerometer data will be considered as is shown in Figure 10-7 for node 3.

![Figure 10-7: Node 3 Spectrum During Transition from Taxi to Takeoff](image)

It is apparent that the amplitude during the initial time period of this figure is small compared to when the engines are throttled up for takeoff toward the end of the segment. The main peak is at 45 Hz while the secondary one is at 75 Hz. Also at this time, several additional weak terms become more observable around 90 and 112 Hz.

Similarly, the $X$ accelerometer data is shown in Figure 10-8 for node 1.
Figure 10-8: Node 1 Spectrum During Transition from Taxi to Takeoff

During the time when the engines are throttled up for takeoff, several high frequency terms are observable above 100 Hz (primarily 120 and 150 Hz), much higher than for node 3, but the peak amplitude is much less than that of node 3. This is most likely due to the beam-like properties of the wing wherein there is a structural tradeoff between frequency and displacement. In this case, it is easier to move the IMU with more amplitude at the wing than in the fuselage, but the fuselage is more likely to experience higher frequencies.

Based on the relationship between the motion at the two nodes shown in Figure 10-7 and Figure 10-8, it is surmised that the frequency content is some multiple of the engine throttle and structural dampening of the airframe. Similarly, the amplitude is some other multiple of throttle and structural amplification or dampening by the airframe. This chapter does not seek to explain all of the mechanical reasons for the motion and related
frequency beyond the elementary relationships described in this section. It is interesting to note that node 3 generally experiences more displacement than node 1.

10.5 Spectral Content Conclusions

The purpose of this appendix was to demonstrate that there is significant high frequency motion in the DC-3. The level of significance of the displacement amplitude must be assessed based on the sensor bandwidth and the stabilization requirements. For this high-accuracy ISR stabilization application, it can be seen that motion occurs at tens of mm/s at frequencies above the 50 Hz bandwidth of the GPS receiver, but the displacement may not be significant enough at the GPS update rate to cause aliasing above the accuracy threshold. Another important observation to note is that node 3 experiences substantially more displacement motion due to the wing structure than the fuselage, but at lower frequencies. The likely fundamental frequency for the throttle setting during flat and level flight was approximately 35 Hz, but a 60 Hz harmonic was more dominant at the wingtip.