INVESTIGATION THROUGH SIMULATION TECHNIQUES OF THE APPLICATION OF
DIFFERENTIAL GPS TO CIVIL AVIATION

A Thesis Presented to
The Faculty of the College of Engineering and Technology
Ohio University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
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August, 1985
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II.</td>
<td>GLOBAL POSITIONING SYSTEM</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>A. Background</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>B. Signal Structure</td>
<td>7</td>
</tr>
<tr>
<td>III.</td>
<td>DIFFERENTIAL GPS</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>A. The Differential GPS Uplink</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>B. The Differential GPS Uplink Using a Pseudolite</td>
<td>16</td>
</tr>
<tr>
<td>IV.</td>
<td>SIMULATION OF GPS/DGPS</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>A. 18 Satellite Configuration</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>B. DGPS User Flight Path</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>C. The Navigation Algorithms</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>1. Position-Fixing Algorithm</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>2. Alpha-Beta Tracking Filter</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>D. Simulation of GPS Range Errors</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>1. Multipath</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>2. Ionospheric Range Errors</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>3. Tropospheric Range Error</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>4. Receiver Clock Error</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>5. Selective Availability Errors</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>E. GPS Simulation Operation</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>1. Sequential/Single-Channel Architecture</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>2. Four-Channel Architecture</td>
<td>56</td>
</tr>
<tr>
<td>V.</td>
<td>SIMULATION RESULTS</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>A. Receiver Performance, Conventional-Mode, No SA Errors</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>B. Receiver Performance, Conventional-Mode, With SA Errors</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>C. Receiver Performance, Fixed-Mode, Without SA Errors</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>1. Diffuse Multipath, Range-Correction Effectiveness</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>2. Atmospheric Delays, Range-Correction Effectiveness</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>3. Receiver Performance, Fixed-Mode, General</td>
<td>73</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>D. Receiver Performance, Fixed-Mode, With SA Errors</td>
<td>76</td>
</tr>
<tr>
<td>E. Receiver Performance, Differential-Mode, Without SA Errors</td>
<td>82</td>
</tr>
<tr>
<td>F. Receiver Performance, Differential-Mode, With SA Errors</td>
<td>87</td>
</tr>
<tr>
<td>G. Receiver Performance, Differential-Mode, With SA Errors, Using Corrections From Different Receivers of Different Update Periods</td>
<td>92</td>
</tr>
<tr>
<td>VI. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>97</td>
</tr>
<tr>
<td>VII. ACKNOWLEDGEMENTS</td>
<td>101</td>
</tr>
<tr>
<td>VIII. BIBLIOGRAPHY</td>
<td>103</td>
</tr>
<tr>
<td>Figure</td>
<td>Illustration Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>2-1</td>
<td>The Uniform 6-Plane, 18 Satellite Configuration (From Kruh [2]).</td>
</tr>
<tr>
<td>2-2</td>
<td>NAVSTAR Block II Production and Launch Schedule as of November 1982. The Numbers Shown are NAVSTAR (From Kruh [3]).</td>
</tr>
<tr>
<td>2-3</td>
<td>Typical and Composite Outage Areas for the 6-Plane Constellation (From Kruh [7]).</td>
</tr>
<tr>
<td>2-4</td>
<td>Construction of the Digital GPS PPS Signal and Modulation of the Carrier Using PSK Techniques.</td>
</tr>
<tr>
<td>2-5</td>
<td>The Power Spectra of the GPS L1 and L2 Signals (After Spilker [8]).</td>
</tr>
<tr>
<td>3-1</td>
<td>The Functional Components of a Differential GPS Uplink.</td>
</tr>
<tr>
<td>3-2</td>
<td>The Functional Components of a Differential GPS Pseudolite Uplink.</td>
</tr>
<tr>
<td>4-1</td>
<td>Phasing of the Uniform 6-Plane, 18 Satellite Configuration (After Kruh [13]).</td>
</tr>
<tr>
<td>4-2</td>
<td>The Teardrop Flight Path Ground Track.</td>
</tr>
<tr>
<td>4-3</td>
<td>The Teardrop Flight Path Altitude Profile.</td>
</tr>
<tr>
<td>4-4</td>
<td>Graphic Representation of the Pythagorean Solution for Satellite Range.</td>
</tr>
<tr>
<td>4-5</td>
<td>The Alpha-Beta Filter Estimation Loop (From Shively [18]).</td>
</tr>
<tr>
<td>4-6</td>
<td>An Example of the Computer-Generated Diffuse Multipath Range-Error.</td>
</tr>
<tr>
<td>4-7</td>
<td>Average Monthly Value of the Ionospheric Propagation Delay, Measured at Zenith, and the Raised Cosine Model (From Klobuchar [20]).</td>
</tr>
<tr>
<td>4-8</td>
<td>Ionospheric Range-Error Model Parameters (After Klobuchar [20]).</td>
</tr>
<tr>
<td>4-9</td>
<td>An Example of the Computer-Generated Ionospheric Range-Error.</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4-10</td>
<td>An Example of the Computer-Generated Tropospheric Range-Error.</td>
</tr>
<tr>
<td>4-11</td>
<td>An Example of the Computer-Generated Values for Errors Induced by an Imperfect Receiver Oscillator.</td>
</tr>
<tr>
<td>4-12</td>
<td>Selective Availability Rate Distribution (From Kalafus [24]).</td>
</tr>
<tr>
<td>4-13</td>
<td>Selective Availability Second-Derivative Distribution (From Kalafus [24]).</td>
</tr>
<tr>
<td>4-14</td>
<td>Selective Availability Model Rate Distribution.</td>
</tr>
<tr>
<td>4-15</td>
<td>Selective Availability Model Second-Derivative Distribution.</td>
</tr>
<tr>
<td>4-16</td>
<td>Example of the Computer-Generated Selective Availability Range-Errors for Four Satellites.</td>
</tr>
<tr>
<td>4-17</td>
<td>Sequential/Single-Channel Receiver Simulation Program Flow.</td>
</tr>
<tr>
<td>4-18</td>
<td>Four-Channel Receiver Simulation Program Flow.</td>
</tr>
<tr>
<td>5-1</td>
<td>Altitude Errors, Run C-1, Without SA, Sequential/Single-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-2</td>
<td>3D Position Error, Run C-1, Without SA, Sequential/Single-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-3</td>
<td>2D Position Error, Run C-1, Without SA, Sequential/Single-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-4</td>
<td>Altitude Error, Run C-4, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-5</td>
<td>3D Position Error, Run C-4, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-6</td>
<td>2D Position Error, Run C-4, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-7</td>
<td>Altitude Error, Run C-1, With SA, Sequential/Single-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-8</td>
<td>3D Position Error, Run C-1, With SA, Sequential/Single-Channel, 0.3 second.</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>5-9</td>
<td>2D Position Error, Run C-1, With SA, Sequential/Single-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-10</td>
<td>Altitude Error, Run C-4, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-11</td>
<td>3D Position Error, Run C-4, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-12</td>
<td>2D Position Error, Run C-4, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-13</td>
<td>Altitude Error, Run F-4, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-14</td>
<td>3D Position Error, Run F-4, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-15</td>
<td>2D Position Error, Run F-4, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-16</td>
<td>Cross Correlation of the Fixed and Airborne Receivers' Multipath Range Errors.</td>
</tr>
<tr>
<td>5-17</td>
<td>Cross Correlation of the Fixed and Airborne Receivers' Ionospheric Range Errors.</td>
</tr>
<tr>
<td>5-18</td>
<td>Cross Correlation of the Fixed and Airborne Receivers' Tropospheric Errors.</td>
</tr>
<tr>
<td>5-19</td>
<td>Cross Correlation of the Fixed and Airborne Receivers' Range Errors, Consisting of Multipath, Ionospheric, and Tropospheric Combined.</td>
</tr>
<tr>
<td>5-20</td>
<td>Altitude Error, Run F-4, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-21</td>
<td>3D Position Error, Run F-4, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-22</td>
<td>2D Position Error, Run F-4, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-23</td>
<td>Cross Correlation of the Fixed and Airborne Receivers' Selective Availability Range Errors.</td>
</tr>
<tr>
<td>5-24</td>
<td>Cross Correlation of the Fixed and Airborne Receivers' Range Errors, Consisting of Multipath, Ionospheric, Tropospheric, and Selective Availability Combined.</td>
</tr>
<tr>
<td>Figure</td>
<td>Illustration Description</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>5-25</td>
<td>Altitude Error, Run D-8, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-26</td>
<td>3D Position Error, Run D-8, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-27</td>
<td>2D Position Error, Run D-8, Without SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-28</td>
<td>Sequential/Single-Channel Performance, Without SA.</td>
</tr>
<tr>
<td>5-29</td>
<td>Four-Channel Performance, Without SA.</td>
</tr>
<tr>
<td>5-30</td>
<td>Altitude Error, Run D-8, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-31</td>
<td>3D Position Error, Run D-8, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-32</td>
<td>2D Position Error, Run D-8, With SA, Four-Channel, 0.3 second.</td>
</tr>
<tr>
<td>5-33</td>
<td>Sequential/Single-Channel Performance, With SA.</td>
</tr>
<tr>
<td>5-34</td>
<td>Four-Channel Performance, With SA.</td>
</tr>
<tr>
<td>5-35</td>
<td>Altitude Error, Run D-6, With SA, Sequential/Single-Channel, 2.4 seconds.</td>
</tr>
<tr>
<td>5-36</td>
<td>3D Position Error, Run D-6, With SA, Sequential/Single-Channel, 2.4 seconds.</td>
</tr>
<tr>
<td>5-37</td>
<td>2D Position Error, Run D-6, With SA, Sequential/Single-Channel, 2.4 seconds.</td>
</tr>
<tr>
<td>5-38</td>
<td>3D Position Errors of a 0.3 second/Differential-Mode Receiver, Using Corrections from either a 1.2 seconds or 2.4 seconds, Fixed-Mode Receiver, With SA Errors.</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table | Description                                                                 | Page |
------|------------------------------------------------------------------------------|------|
2-1   | GPS Received Signal Power Levels, as Would be Measured at the Output of a 0 dBIC Antenna with Right-Hand Circular Polarization, With the Satellite at an Elevation of 5° or Greater (After Spilker [9]). | 13   |
4-1   | NAVSTAR GPS Orbit Description as Implemented in the Computer Simulation, Longitude Relative to Earth and Astronomical Coordinates (After Kruh [15]). | 21   |
5-1   | The 32 Basic Receiver Configurations Used in This Study. | 59   |
I. INTRODUCTION

Civil aviation will soon have access to a new radio navigation system called NAVSTAR GPS, which stands for Navigational Satellite Time and Range Global Positioning System. GPS, conceived as a strategic navigation system by and under the Department of Defense (DOD), will provide an all-weather, world-wide satellite-based navigation system for both military and civilian applications. The entire 18 satellite constellation is scheduled to be in orbit and the system fully operational by 1988 [1]. It is significant that GPS system concepts be introduced now into civil aviation. It is noteworthy that without exception, today's civilian aviation radio navigation systems are spinoffs of a technology developed during the period of 1940-1955.

Several current navigation systems exist for use by the civil aviator, but no one system can match the characteristics and capabilities of GPS. GPS will be used passively and therefore will be non-saturable. Also, due to the signal structure of GPS, the system will be available under virtually any weather condition with an accuracy and continuous coverage area greater than any conventional radial-range or point-position system available today (VORTAC, OMEGA, LORAN-C). However, an important capability of GPS lies in the available accuracies obtainable by the application of differential techniques (i.e. differential GPS or DGPS).

DGPS is a method whereby the bias error components induced by the signal channel and use of imperfect satellite ephemeris data can
be substantially reduced. These errors are measured at a fixed station and are transmitted to local DGPS users via a passive telemetry uplink in the form of range corrections. These corrections, once applied to the user's range measurements, enable the user to compute a more precise position fix. The application of this technique may render an accuracy sufficient to support the requirements of a Category I precision approach to landing for all of general aviation.

The focus of this paper is to determine whether DGPS has the potential to provide a precision approach and landing capability. The results and conclusions of this study were derived from the output of a computer simulation. The use of simulation techniques offered three important advantages:

1) The computer simulation offers a controlled environment with no unknown distortions.

2) DGPS simulation is the most time-effective at this point in the development of the GPS constellation. Currently, there are only enough satellites to offer a geometry sufficient for 3-D navigation a few hours each day.

3) Simulation is the most cost-effective means of performing a feasibility study. The technology exists whereby these tests could have been accomplished in real-time and with prototype hardware, but it would have come about at the cost of millions of dollars. This point is particularly impor-
tant in light of the fact that this study demanded considera-

tion of the civilian aviator and funding is not available from the civilian agencies to support the research and development of a prototype DGPS.

Furthermore, computer studies such as these will provide sufficient foresight to conceive, test, and establish GPS applications long before the system becomes fully operable.

This paper is structured as follows:

Chapter 2 discusses the basic concepts of GPS and its signal structure to provide the reader with a basic background of the conventional Global Positioning System.

Chapter 3 describes differential GPS (DGPS) concepts and two techniques that appear to be the most practical methods for implementing DGPS.

The computer simulation, its models, and execution are the topic of Chapter 4. Detail is given to the models used, sequence of execution to emulate either a single-channel or multiple-channel receiver, the GPS position-fixing algorithm, and the use of an alpha-beta filter for smoothing the navigation data.

The simulation results and the discussion of these results are presented within Chapter 5.

Finally, the conclusions and recommendations are presented in Chapter 6.
II. GLOBAL POSITIONING SYSTEM

A. Background. The Global Positioning System (GPS) is a worldwide, all-weather navigation system currently under development by the Department of Defense. The baseline space segment configuration will consist of 18 satellites equally spaced in six orbits (see Figure 2-1 [2]). The orbits will be uniformly distributed about the earth, each inclined 55° with respect to the spin axis of the earth. This baseline system is scheduled to begin deployment in January, 1986 and will tentatively continue as per the schedule shown in Figure 2-2 [3].

GPS will offer two systems, the Precise Positioning System (PPS) which implements the P-code and the Standard Positioning System (SPS) which will use the C/A code. The 1984 Federal Radionavigation Plan (FRP) states that the PPS will provide a relative and best predictable positioning accuracy of 17.8 meters (2 drms) horizontally and 27.7 meters (2 sigma) vertically [4]. In the interest of national security, the PPS will most likely be available for DOD applications and only to those users authorized by the DOD. Flight tests performed by Campbell and LaFrey [5] show that the C/A code offered an accuracy of 49.1 meters (1 sigma) horizontally and 28.4 meters (1 sigma) vertically. This accuracy will, for reasons of security, be purposely degraded such that the available accuracy will be 100 meters (2 drms) horizontally [6]. The DOD annually will review the practicality of increasing the accuracy of the SPS.

The user of either system can determine his three-dimensional position and receiver clock error by measuring the time of propagation
Figure 2-1. The Uniform 6-Plane, 18 Satellite Configuration (From Kruh [2]).
Figure 2-2. NAVSTAR Block II Production and Launch Schedule as of November 1982. The Numbers Shown are NAVSTAR (From Kruh [3]).
of four GPS signals, each received from four different satellites which provide good geometry. The ideal geometry would consist of one satellite at zenith and three satellites close to the horizon, each of the latter uniformly spaced by 120° in azimuth. Because errors due to the effects of the atmosphere are significantly amplified when a satellite is positioned near the horizon, a mask angle of 5-10° elevation is usually implemented.

The user may compute his position and clock error by tracking fewer than four satellites if he has good redundant position information available, such as knowledge of altitude or differential LORAN-C position fixes. Alternately, the user may track only three satellites to compute three dimensions of position if the user's receiver implements an extremely stable frequency standard in its tracking loop hardware.

The general aviation (GA) user, with degraded C/A code compounded by areas of poor geometry and satellite outages, will be limited to the enroute and, perhaps, the non-precision approach environment. A projected global outage map is shown in Figure 2-3 [7]. This outage map is calculated on visible satellites and the geometries offered by the visible constellation.

B. Signal Structure. The P-code of the PPS will be transmitted on two frequencies, L1 = 1575.42 MHz and L2 = 1227.6 MHz. The C/A code will be transmitted on the L1 frequency only. Receivers that track both carriers will have the capability to determine the
Figure 2-3. Typical and Composite Outage Areas for the 6-Plane Constellation (From Kruh [7]).
phase/delay and carrier/advance effects of the atmosphere. Other receivers will have a significantly less accurate determination of the atmospheric effects by means of a model provided in the GPS navigation message data. Either the two frequency measurements or the model calculations will be used to reduce the effects of the propagation delays of the GPS signals.

The PPS signal will be modulated on both GPS carrier frequencies. The PPS signal is a digital pulse sequence generated by the modulo-2 addition of the P-code, which is a 10.23 MHz PseudoRandom Noise (PRN) code, and a 50 bps data block message. This digital sum is then modulated with the carrier frequencies using Phase Shift Keying (PSK) techniques (see Figure 2-4). More specifically, GPS implements Bi-Phase Shift Keying (BPSK) where the in-phase carrier component is modulated by the P signal and the quadrature phase component is modulated by the C/A signal.

The C/A signal consists of the modulo-2 addition of the C/A code and the 50 bps data message. The C/A code is a 1.023 MHz digital Gold code formed by the product of two 1023 bit PRN codes. The C/A signal is used to modulate the quadrature component of only the L1 frequency.

The frequency power spectra of the L1 and L2 signals are shown in Figure 2-5 [8]. The phased antenna array of the transmitting satellite distributes the power of the two signals such that more power is distributed towards the outer circumference of the main lobe and less is directed below or at the center of the main lobe. In this
Figure 2-4. Construction of the Digital GPS PPS Signal and Modulation of the Carrier Using PSK Techniques.
Figure 2-5. The Power Spectra of the GPS L1 and L2 Signals (After Spilker [8]).
way, the power is distributed so that a user tracking a satellite near the horizon will receive a power level equal to the power received from a satellite tracked at zenith.

Under normal operating conditions and for satellites at least 5° above the horizon, Table 2-1 [9] depicts the minimum received power levels for the Phase I satellites [10].
<table>
<thead>
<tr>
<th>Frequency</th>
<th>GPS Signal Component (Minimum Strength)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P-Code</td>
</tr>
<tr>
<td>L1 = 1575.42 MHz</td>
<td>-163 dBw</td>
</tr>
<tr>
<td>L2 = 1227.6 MHz</td>
<td>-166 dBw</td>
</tr>
</tbody>
</table>

Table 2-1. GPS Received Signal Power Levels, as Would be Measured at the Output of a 0 dBIC Antenna with Right-Hand Circular Polarization, With the Satellite at an Elevation of 5° or Greater (After Spilker [9]).
Differential GPS (DGPS) is a technique whereby the accuracy of a GPS calculated position is improved by providing, to the GPS user, knowledge as to the bias distortions contained in GPS range measurements. Several methods have been discussed by others [10, 11] as how these corrections may be implemented. However, only two methods have drawn significant attention [12]; the uplink method and the uplink method by pseudolite (i.e. a ground-based NAVSTAR transmitter).

A. The Differential GPS Uplink. The basic components of a DGPS uplink ground unit consist of a GPS receiver, a central processor, and a telemetry unit with the antenna of the GPS receiver placed at a precisely known location (see figure 3-1). The DGPS ground system's processor is programmed with the coordinates of the receiver's antenna. The central processor is interfaced to the GPS receiver in such a manner that it can obtain the satellite data block information stored in the receiver as well as time/Sat ID-tagged range/range-rate information.

With the knowledge of GPS system time, derived either from the satellite range data or by an external standard, the central processor can determine the true range from the receiver's antenna to any particular GPS satellite. The difference between the processor's calculated range and the receiver's measured range is the range correction provided to the DGPS user(s) in the local area via a one-way data telemetry uplink. Similarly, calculations and measurements can be made to provide the user with a range-rate or Doppler correction.
Airborne Differential GPS Receiver Processor

Radio Uplink

Fixed GPS Receiver Processor

Figure 3-1. The Functional Components of a Differential GPS Uplink.
The DGPS user would have a similar breakdown of basic components consisting of a GPS receiver, a central processor, and a telemetry receiver. Once the correction has been received, the central processor would then apply the correction to the user's measured ranges and then proceed to compute a position fix which is significantly more accurate than those using conventional GPS. It should be noted that the user's processor would be less sophisticated than the DGPS ground system's processor. If commercially produced, the user's central processor would probably be shared with the GPS receiver.

The corrections could also take the form of position coordinate corrections. The application of position-coordinate corrections to a conventional GPS position fix would only provide consistently improved results if the correction and the conventional position fix were derived from the identical selection of satellites. The application of corrections to a position fix, both of which were calculated from separate sets of satellites, would produce meaningless results.

B. The Differential GPS Uplink Using a Pseudolite. This method is very similar to the DGPS uplink, however the distinct advantage to this method comes from the use of a pseudolite (see figure 3-2). A pseudolite is a static, ground-based NAVSTAR transmitter. Its signal format would be identical to that of the space-based satellites, however, the data content of the pseudolite would be somewhat different. Instead of several complex ephemeris constants, its position data would consist of three earth centered coordinates. The data
Figure 3-2. The Functional Components of a Differential GPS Pseudolite Uplink.
message would also contain the DGPS corrections calculated at the pseudolite. These corrections would be derived using the techniques described in the uplink method.

A major advantage to using a pseudolite uplink is the improved geometry offered by obtaining a range measurement from a transmitter beneath the user. In conventional GPS all of the satellites offer a positive elevation angle, thus diluting one's ability to accurately resolve his position with respect to altitude. The pseudolite decreases the effects of the residual range errors in the DGPS range measurements and tends to provide an overall improvement to the altitude calculation when using either GPS or DGPS.

One problem with the pseudolite uplink is in coverage. The uplink method without pseudolites could employ the use of transmission frequencies with enough ground wave to service 100 miles or more over the horizon. Because the pseudolite will transmit at the NAVSTAR frequencies, there will be no ground wave, therefore all receptions of the differential information will be done within line-of-sight of the pseudolite. The coverage of the pseudolite can not be expanded by merely increasing its transmission power; to do so would risk saturating the RF front-ends of the all of the GPS receivers in the immediate area and either degrade or jam the user's receiving capabilities.
IV. SIMULATION OF GPS/DGPS

The following is a discussion of the operation of the computer simulation used to derive the results in this paper.

A. 18 Satellite Configuration. The simulation uses 18 satellites distributed equally into six circular orbits about the earth. Each orbit is inclined 55° with respect to the spin axis of the earth. Each satellite has an earth centered radius of 14,568 nautical miles and thus the orbital period of each satellite is approximately 12 hours.

The orbits are sequentially phased 40° in such a way that if a satellite is directly above the equator, the plane adjacent to the east will have a satellite 40° ahead and the plane adjacent to the west will have a satellite 40° lagging (see Figure 4-1 [13]). Table 4-1 [14] shows the longitudes of the ascending nodes and the angles of right ascension for each satellite used. The angles of right ascension, between each orbital plane, are evenly spaced 60° apart.

B. DGPS User Flight Path. The user dynamics used in this simulation were modeled after those generally encountered in the GA helicopter terminal/approach environment. The mission contains six legs, each of which can contain the following characteristics:

* straight, level, constant velocity paths

* constant heading, constant velocity, constant ascent/descent paths (descent/ascent speeds varying from -300 fpm to 750 fpm).
Figure 4-1. Phasing of the Uniform 6-Plane, 18 Satellite Configuration (After Kruh [13]).
Table 4-1. NAVSTAR GPS Orbit Description as Implemented in the Computer Simulation. Longitude Relative to Earth and Astronomical Coordinates (After Kruh [15]).

<table>
<thead>
<tr>
<th>SATELLITE NUMBER</th>
<th>ORBIT PLANE</th>
<th>LONGITUDE OF THE ASCENDING NODE, DEG.</th>
<th>RIGHT ASCENSION OF THE ASCENDING NODE, DEG.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0. 180.</td>
<td>0.</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>240. 60.</td>
<td>0.</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>300. 120.</td>
<td>0.</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>260. 80.</td>
<td>60.</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>320. 140.</td>
<td>60.</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>20. 200.</td>
<td>60.</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>340. 160.</td>
<td>120.</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>40. 220.</td>
<td>120.</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>100. 280.</td>
<td>120.</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>60. 240.</td>
<td>180.</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>120. 300.</td>
<td>180.</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>180. 0.</td>
<td>180.</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>140. 320.</td>
<td>240.</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>200. 20.</td>
<td>240.</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
<td>80. 260.</td>
<td>240.</td>
</tr>
<tr>
<td>16</td>
<td>6</td>
<td>220. 40.</td>
<td>300.</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>280. 100.</td>
<td>300.</td>
</tr>
<tr>
<td>18</td>
<td>6</td>
<td>160. 340.</td>
<td>300.</td>
</tr>
</tbody>
</table>

* Referenced to astronomical coordinates of 1950.0 as of 0 hr 0 min GMT, the simulation does not allow the right ascending node to regress.
various turns, including a teardrop maneuver

several different speeds (89-120 knots) and accelerations
(-2 knots/second to 5 knots/second)

The final approach to the runway is made using the teardrop maneuver.

Figures 4-2 and 4-3 depict the ground track and altitude profile of the user. The entire scenario was similar to that used in a study of degraded GPS for non-precision approaches [15].

C. The Navigation Algorithms. The following text describes the position-fixing algorithm used to derive user navigational information given time and four pseudorange measurements. A brief discussion is devoted to the alpha-beta tracking filter employed to smooth the position calculations.

1. Position-Fixing Algorithm. The following discussion is based upon the information presented by Noe and Myers [16].

The algorithm used to calculate the user's position requires the inputs of time, an estimate of the user's position and four user pseudorange measurements. To assist in the discussion of this algorithm, assume that the pseudorange is equal to the true range between the user and a satellite, plus user clock error.

The following expression relates the user's position \( (u_1, u_2, u_3) \) and receiver clock bias \( b \), \( i \)th satellite position \( (x_{i1}, x_{i2}, x_{i3}) \) and the measured pseudorange \( r_i \):
Figure 4-2. The Teardrop Flight Path Ground Track.
Figure 4-3. The Teardrop Flight Path Altitude Profile.
The equation and solving for \( r_i \), the following expression is obtained:

\[
\sqrt{\sum_{j=1}^{3} (x_{ij} - u_j)^2 + (x_{i2} - u_2)^2 + (x_{i3} - u_3)^2 + b}
\]  \hspace{1cm} (4-2)

To obtain the desired navigation algorithm, it is first necessary to expand the pseudorange measurement \( r_i \) with a Taylor series about the user position estimate \( \bar{U} \). This results in the following equation:

\[
\frac{\partial r_i}{\partial U} = r_i + \frac{1}{2!} \frac{\partial^2 r_i}{\partial U^2} + \frac{1}{3!} \frac{\partial^3 r_i}{\partial U^3} + \ldots \hspace{1cm} (4-3)
\]

This equation can now be linearized by deleting all of the partial derivatives of an order greater than one, giving the expression:

\[
r_i = r_i + \frac{\partial r_i}{\partial U} |_{\bar{U}} \delta U \hspace{1cm} \text{or}
\]

\[
\delta r_i = \left[ \begin{array}{c} \frac{\partial r_i}{\partial U_1} \\ \frac{\partial r_i}{\partial U_2} \\ \frac{\partial r_i}{\partial U_3} \\ \frac{\partial r_i}{\partial b} \end{array} \right] |_{\bar{U}} \delta U \hspace{1cm} (4-5)
\]

Where \( \delta r = r_i - \bar{r}_i \) and \( \delta U = U - \bar{U} \).

Define \( \delta r \) as follows:
Figure 4-4. Graphic Representation of the Pythagorean Solution for Satellite Range.
\[ \delta r_i = h_i \delta U \] 

(4-6)

where \( h_i \) is the row vector of first order partial derivatives;

\[
h_i = \left[ \begin{array}{c} \frac{\partial r_i}{\partial u_1} \\ \frac{\partial r_i}{\partial u_2} \\ \frac{\partial r_i}{\partial u_3} \end{array} \right] \left[ \begin{array}{c} \frac{\partial r_i}{\partial b} \end{array} \right] \]

(4-7)

Now, replace \( r_i \) in the \( h_i \) row vector with Eq. 4-2, expand the partial derivatives, substitute \( r_i - b \) from Eq. 4-2, and obtain;

\[
h_i = \left[ \begin{array}{c} \frac{u_1-x_{i1}}{r_i-b} \\ \frac{u_2-x_{i2}}{r_i-b} \\ \frac{u_3-x_{i3}}{r_i-b} \end{array} \right] 1 \]

(4-8)

This may now be expanded to four pseudorange measurements and solved for the user position vector \( U \);

\[
\delta R = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} \delta U = H \delta U
\]

(4-9)

\[
\delta U = H^{-1} \delta R
\]

(4-10)

Now, given time, four pseudorange measurements, and an user position estimate, this algorithm will provide a user position and clock bias update.

2. Alpha-Beta Tracking Filter. The data output from the position fix algorithm is filtered using a simple, fixed gain alpha-beta tracking filter. This is the same filter used in the GPS non-precision approach study simulations developed for the Department of Transportation. The following discussion is paraphrased from the documentation of that study [17].
Figure 4-5 [18] depicts the alpha-beta filter estimation loop and its corresponding system equations. Rearranging these filter equations yields:

\[
\frac{U_p(t + T) - U_p(t)}{T} = \frac{\alpha \Delta U}{T} + V_p(t + T) \quad (4-11)
\]

\[
\frac{V_p(t + T) - V_p(t)}{T} = \frac{\beta \Delta U}{T^2} \quad (4-12)
\]

where \( U_p \) is the predicted user position component, \( V_p \) is the predicted user velocity component, \( T \) is the update period, \( \Delta U \) is the position update from the position fix algorithm, and alpha and beta are the filter gain constants.

These equations can be approximated by the following differential equations, assuming \( U_p \) and \( V_p \) are constant over the sampling period;

\[
\frac{dU_p}{dt} = \frac{\alpha \Delta U}{T} + V_p \quad (4-13)
\]

\[
\frac{dV_p}{dt} = \frac{\beta \Delta U}{T^2} \quad (4-14)
\]

and then converted to these integral relations;

\[
U_p = \int \frac{\alpha \Delta U}{T} + V_p \ dt \quad (4-15)
\]

\[
V_p = \int \frac{\beta \Delta U}{T^2} \ dt \quad (4-16)
\]

By means of the Laplace Transform and deriving the transfer function of this system, it is possible to obtain the characteristic
Figure 4-5. The Alpha-Beta Filter Estimation Loop (From Shively [18]).
equation to determine the relationship of the filter gain constants to the damping ratio of the system. A damping ratio of unity was chosen for the non-precision approach study to compromise between filter rise time, overshoot, and ringing of the transient response. This gives the relationship between alpha and beta:

\[ \alpha^2 = 4\beta \]  

(4-17)

Again, using the transfer function of the system, the unit step response, \( u(t) \), of the filter is found to be:

\[ u(t) = 1 + \frac{at}{2T} e^{-\frac{at}{2T}} - \frac{at}{2T}, \quad \text{for } t > 0 \]  

(4-18)

Now define a time constant \( \tau_f \) such that at \( t = 2\tau_f \), the step response is within 13.5% \( (e^{-2}) \) of the final value of the unit step response, this yields:

\[ e^{-2} = e^{-\frac{\alpha \tau_f}{T}} - \frac{\alpha \tau_f}{T}, \quad \text{for } t > 0 \]  

(4-19)

This transcendental equation can be solved and found to have at least one real solution:

\[ \tau_f = \frac{.72T}{a} \]  

(4-20)

The values used for the DOT simulations were found by experimentation. Reasonable performance was achieved for alpha = .2 and beta = .01 for a dwell time of 1.2 seconds. This gave an overall time constant of \( \tau_f = 4.3 \) seconds. These filter gain constants were used
throughout this simulation, regardless of receiver update rate. Therefore, \( T_f \) was allowed to vary proportional to the receiver update rate, a receiver with a shorter update period filtered its position data using a filter with a faster reaction time. A receiver with an update rate of 0.3 second had a time constant of approximately 1.075 seconds and a receiver with an update rate of 2.4 seconds had a time constant of 8.6 seconds.

D. Simulation of GPS Range Errors. The following is a discussion of the error models used in this study and their derivations.

1. Multipath Range-Error. Multipath can take on two basic forms of interference to GPS signals, specular and diffuse multipath. Specular multipath produces a delayed replica of the direct path GPS signal and, depending upon the media from which it is reflected, can be received at various signal levels. This degradation can be substantial if the receiver does not lock on the earliest received signal; usually this can only occur if the signal delay is greater than one PseudoRandom Noise (PRN) code chip width. Specular multipath was not included in the simulation. The diffuse multipath was modeled as follows.

Diffuse multipath produces a wide-band interference rather than inducing a large range offset and can be approximated by a random process. The diffuse multipath is caused by the addition of the many indirect signals, reflected mainly from the user's vehicle surface.
These errors have an rms value of 1.0 to 3.0 meters for the P-code and 10.0 to 30.0 meters for the C/A code [19].

The error values are generated by the simulation by calling a zero-mean Gaussian distribution with a standard deviation of one and then multiplying the elements of the distribution by ten (see Eq. 4-21). This produces a sequence of random numbers with a standard deviation of ten, which is then scaled to the appropriate units of measure used by the simulation. A simulated diffuse-multipath error sequence is shown in Figure 4-6.

\[ \text{Multipath Error} = 10.0 \ [N(0,1)] \quad (4-21) \]

Where \( N(0,1) \) denotes a sequentially accessed, zero-mean Gaussian number sequence, with a unity standard deviation.

2. Ionospheric Range-Error. The delay caused by the ionosphere is due to the free electrons encountered by the signal along the ray path between the satellite and the user. The delay encountered when a satellite is at zenith varies from about 5 nanoseconds during the evening, to a peak of 35 nanoseconds during the middle of the day, for the L-band frequencies. A plot of the average value of zenith propagation delay versus time, over a 24 hour period, is shown in Figure 4-7 [20].

Superimposed with the ionospheric data is a plot of the raised cosine model approximation of the ionosphere. This model was deve-
Figure 4-6. An Example of the Computer-Generated Diffuse Multipath Range-Error.
Figure 4-7. Average Monthly Value of the Ionospheric Propagation Delay, Measured at Zenith, and the Raised Cosine Model (From Klobuchar [20]).
loped by Klobuchar [20] and has been incorporated into the GPS simulation to calculate the ionospheric propagation delay, at zenith, given the time and location at the point where the ray path, between the user and the satellite, pierces the bottom of the ionospheric layer. This location is called the local ionospheric point (see Figure 4-8 [20]).

The local ionospheric point is calculated from the satellite's earth angles, elevation and azimuth angle as viewed from the user, and the user's location using a flat earth approximation. By applying a few simple trigonometric rules and identities to the triangle depicted in Figure 4-8, the earth angle $A$ can be found to be described by the following equation:

$$A = 90^\circ - el - \arcsin \left[ \frac{r_0 + \text{alt}}{r_0 + h} \left( \cos el \right) \right]$$

(4-22)

Where $el$ is the satellite elevation angle, $\text{alt}$ is the user altitude, $r_0$ is the radius of the earth, and $h$ is the height of the ionosphere.

The location of the local ionospheric point may now be calculated using:

$$\phi_I = \phi_0 + A \cos AZ$$

(4-23)

$$\lambda_I = \lambda_0 + \frac{A \sin AZ}{\cos \phi_I}$$

(4-24)

Where $\phi_0$ is the user latitude, $\lambda_0$ is the user longitude, $AZ$ is the satellite azimuth angle, $\phi_I$ is the latitude of
Figure 4-8. Ionospheric Range-Error Model Parameters (After Klobuchar [20]).
the local ionospheric point, and $\lambda_I$ is the longitude of the local ionospheric point.

Given time, in GMT hours at the user location, and the calculated longitude of the local ionospheric point, the time at the local ionospheric point is calculated from:

$$t = \frac{\lambda_I}{15} + \text{GMT}$$  \hspace{1cm} (4-25)

Where $t: 0 \leq t \leq 24$ in hours

Now that time is known at the local ionospheric point, the zenith propagation delay can now be computed using the raised cosine model. For values of local time at the ionospheric point between 0600 and 2200 hours, the zenith propagation delay can be represented by:

$$\text{Ion Zenith Delay} = DC + A \cos \left( \frac{(t - \phi)2\pi}{P} \right)$$  \hspace{1cm} (4-26)

Where $DC =$ nighttime component of the zenith propagation delay, approximately 5 nanoseconds; $A =$ the peak zenith propagation delay, 30 nanoseconds; $P =$ period of the cosine model, 32 hours; $t =$ the local time at the local ionospheric point; and $\phi =$ local time of the peak value of ionospheric delay.

For all other times, the delay will equal the nighttime component only.
Thus far, the computation of the ionospheric delay has only been concerned with a signal ray path at zenith, however rarely are there any satellites available exactly at zenith. For those ray paths not coinciding with zenith, a slant factor or obliquity factor must be calculated. The slant factor is defined as the secant of the zenith angle, $Z$, at the local ionospheric point. The angle $Z$ is equal to:

$$Z = \arcsin \left( \frac{r_0 + \text{alt}}{r_0 + h} \cos \theta \right)$$ \hspace{1cm} (4-27)

which, if user altitude (alt) is considered negligible, is approximately:

$$Z = \arcsin[0.94792 \cos(\theta)]$$ \hspace{1cm} (4-28)

The slant factor, $SF$, is then equal to:

$$SF = \sec[Z]$$ \hspace{1cm} (4-29)

The propagation delay, due to the true signal ray path, is then calculated to be a product of the zenith propagation delay and the slant factor (see equation 4-30).

$$\text{Total Time Delay} = SF \times \text{Ion Zenith Delay}$$ \hspace{1cm} (4-30)

Figure 4-9 shows typical ionospheric range error produced by this model.

3. Tropospheric Range-Error. This particular error model was developed by Altshuler and Kalaghan [21]. The inputs required for
Figure 4-9. An Example of the Computer-Generated Ionospheric Range-Error.
this model included user altitude, satellite elevation angle, and the season or month of the year. The basic equation used is:

\[
\Delta R(\theta, h, N_s) = G(\theta) \cdot H(h) \cdot F(h, N_s) \quad (4-31)
\]

Where \( G(\theta) \) is a function of satellite elevation angle, \( H(h) \) is a function of user altitude, and \( F(h, N_s) \) is a function of user height and surface refractivity \( N_s \).

The functions of the tropospheric range error are found from the following equations:

\[
G(\theta) = (g_0 + g_1\theta^{-1} + g_2\theta^{-2} + g_3\theta^{-3}) \left[ g_4 + g_6(\theta-g_5)^2 \right] \quad (4-32)
\]

Where satellite elevation angle \( \theta \), is valid only for \( \theta > 5^\circ \).

\[
H(h) = [(b_0+b_1(h+8.6286)^{-1}+b_2(h+8.6286)^{-2}+b_3(h+8.6286)^{-3})] \quad (4-33)
\]

\[
F(h, N_s) = c_0 \left. \left. \frac{c_1}{h+c_0} + c_2(h+c_0) + c_3N_s-c_4 \right) \left[ 1-c_5(N_s-c_6)^2 \right] \right) \quad (4-34)
\]

\[
N_s(h, L, M) = a_0 + a_1h + a_2L + a_3hs^2 + a_4Ls^2 + a_5hc + a_6Lc \quad (4-35)
\]

Where \( L \) is the user latitude in degrees and:

\[
s = \sin\left( \frac{\pi}{12M} \right) \quad (4-36)
\]

\[
c = \cos\left( \frac{\pi}{12M} \right) \quad (4-37)
\]

Good results can be obtained by using an average global surface refractivity value of 324.8 N units and deleting the calculation of \( N_s \).
The constants required for these equations are as follows:

<table>
<thead>
<tr>
<th>$g_0$</th>
<th>$b_0$</th>
<th>$c_0$</th>
<th>$a_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1556</td>
<td>0.00970</td>
<td>3.28084</td>
<td>369.0300</td>
</tr>
<tr>
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<td>$b_1$</td>
<td>$c_1$</td>
<td>$a_1$</td>
</tr>
<tr>
<td>138.8926</td>
<td>-2.08809</td>
<td>6.81758</td>
<td>-.01553</td>
</tr>
<tr>
<td>$g_2$</td>
<td>$b_2$</td>
<td>$c_2$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>-105.0574</td>
<td>122.73592</td>
<td>0.30480</td>
<td>0.92442</td>
</tr>
<tr>
<td>$g_3$</td>
<td>$b_3$</td>
<td>$c_3$</td>
<td>$a_3$</td>
</tr>
<tr>
<td>31.5070</td>
<td>703.82166</td>
<td>0.00423</td>
<td>0.00160</td>
</tr>
<tr>
<td>$g_4$</td>
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<td>$c_4$</td>
<td>$a_4$</td>
</tr>
<tr>
<td>1.000</td>
<td></td>
<td>1.33333</td>
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</tr>
<tr>
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<td>$c_5$</td>
<td>$a_5$</td>
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<td>1.41723X10^-6</td>
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<td>$c_6$</td>
<td>$a_6$</td>
</tr>
<tr>
<td>1.0X10^-4</td>
<td></td>
<td>315.00000</td>
<td>-0.05958</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Season</th>
<th>M Value (Month Number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>1.5</td>
</tr>
<tr>
<td>Spring</td>
<td>4.5</td>
</tr>
<tr>
<td>Summer</td>
<td>7.5</td>
</tr>
<tr>
<td>Fall</td>
<td>10.5</td>
</tr>
</tbody>
</table>

The error values are calculated for each satellite by supplying to these equations the necessary data and scaling the results to the required units for the simulation to process the range measurements. Figure 4-10 depicts the error profile generated by this model for the teardrop flight path. Notice the deviation due to the change in altitude. The tropospheric error behaves in much the same manner as the ionospheric error with respect to elevation angles. The tropospheric error induced by a satellite close to the horizon may be as much as an order of magnitude greater than that exhibited by a satellite located near zenith.
Figure 4-10. An Example of the Computer-Generated Tropospheric Range-Error.
4. Receiver Clock Error (Model used by Shively [22]). The receiver clock error model used in this simulation calculates an instantaneous clock offset biased upon a starting clock offset (SO), frequency error based upon clock temperature stability (TS), and frequency drift based upon aging rate (AR) characteristics. Given SO, TS, and AR, the clock parameters are derived using the following equations:

\[
\text{Freq. Error} = FE = (TS) (DELT) (C) \tag{4-38}
\]

\[
\text{Freq. Drift} = FD = (AR/86400.) (.5 DELT^2) (C) \tag{4-39}
\]

Where 86400. scales the aging rate from days to seconds, DELT is the update period of the receiver, and C is the speed of light.

The clock bias is then calculated to be;

\[
\text{Clock Bias} = SO + (FE) (TIM) + (FD) (TIM^2) + SS \tag{4-40}
\]

Where TIM is the number of elapsed update periods and SS is a short term stability term derived from a Gaussian random sequence of \(N(0,SS)\). SS is assigned a value of 50 nanoseconds thus giving the Gaussian sequence a standard deviation of 50.

Figure 4-11 shows the output of the clock error model for the following values;
Figure 4-11. An Example of the Computer-Generated Values for Errors Induced by an Imperfect Receiver Oscillator.
5. Selective Availability Errors. A statistical analysis has been performed by Kalafus [23], to determine the characteristics of the selective availability errors to be induced by the Department of Defense. These studies were performed upon unclassified samples of the SA error, given the original premise that the error was to induce a 500 meter (2d rms) horizontal position error. This has been lowered by a factor of five so now the expected horizontal-position error will now be 100 meters (2d rms).

The SA model used in this study was scaled to the 100 meter criteria and maintained the dynamic characteristics found by Kalafus. These dynamics are characterized by the probability density functions, as derived by Kalafus, and are shown in Figures 4-12 and 4-13 [24].

These statistical characteristics are closely modeled by the following equation:

$$\text{SA Error}(i) = \text{SA Error}(i-1) + \text{VEL}(\text{DELT}) + 0.5(\text{ACC})(\text{DELT}^2) \quad (4-41)$$

Where VEL is a constant and ACC is a random walk generated from the summation of a gaussian random-number sequence of $N(0,\text{SIGA})$. SIGA was assigned a value of 0.07 feet. This value was achieved through experiment.
Figure 4-12. Selective Availability Rate Distribution (From Kalafus [24]).

Figure 4-13. Selective Availability Second-Derivative Distribution (From Kalafus [24]).
The calculation of ACC was as follows:

\[ ACC = ACC + (\text{SIGA}) [N(0,1)] \]  

Figures 4-14 and 4-15 show the typical statistical characteristics of the SA error rate and accelerations, as computed by the model, for one satellite.

Comparing the statistics of the model and those found by Kalafus, some discrepancies can be seen. The velocity distribution does not appear entirely Gaussian. However, if the model is allowed to run for longer periods of time, it is found that the distribution takes on a more zero-mean, Gaussian appearance. Another discrepancy can be seen by comparing the endpoints of the distributions between those found by Kalafus and those generated by the model.

The accelerative values of the error generated by the model can take on values double those found by Kalafus, during a significant portion of the mission. Also, the velocity values generated by the model are somewhat lower than those calculated from the SA samples. The standard deviation of the random-number sequence used to generate the accelerative random walk, was modeled to best reflect those results produced by Kalafus. Although the accelerative values are excessive, it was felt that the velocity values were representative of those to be encountered in the field. Figure 4-16 displays the data output of this model for four satellites.
Figure 4-14. Selective Availability Model Rate Distribution.
Figure 4-15. Selective Availability Model Second-Derivative Distribution.
Figure 4-16. Example of the Computer-Generated Selective Availability Range-Errors for Four Satellites.
E. GPS Simulation Operation. Thus far, programming tools have been defined which can compute;

1) User/Receiver Position, given local time

2) Satellite Position and Range to User, given local time and the user's position

3) Satellite Range Error Values, given local time, satellite's observed azimuth and elevation angles, and the user's position

4) Position fix and user clock error estimate based upon the best estimate of four satellite ranges

These basic tools can be used together to emulate either a sequential/single-channel receiver or a four channel receiver. Each architecture is able to operate in a conventional, fixed, or differential GPS mode. These architectures were constructed to perform at 0.3, 1.2, and 2.4 second range update periods.

When operating in the conventional-mode, the GPS receiver uses the range measurements to compute the receiver's position, without any corrections except for those intrinsic to operating with four satellites. The conventional-mode also uses the flight path simulator to predetermine the position and velocity of the receiver.

When operating in the fixed-mode, the GPS receiver collects range measurements as in the conventional-mode. However, the fixed
receiver and its antenna are positioned at a precisely known location, allowing the fixed receiver to compute its true range from a particular satellite at a given time. The difference between the true range and measured range is computed and provided to the differential receiver at specified uplink rate. The rate used in these studies is one correction every 12 seconds.

The differential-mode incorporates many of the characteristics of the conventional-mode and the application of the corrections generated by the fixed receiver. The differential receiver applies a range correction to each one of its measurements to obtain a better position fix. This correction is updated at the rate specified by the fixed receiver's uplink rate. For these experiments, the flight path and dynamics incurred during the conventional-mode and the differential-mode are identical.

1. Sequential/Single-Channel Architecture. The flowchart for the sequential/single-channel receiver is shown in Figure 4-17. The following text elaborates upon the program flow:

1) First the program is initialized, by defining; 1) local mission-start and mission-end time, 2) receiver update period, 3) initial user position/velocity, 4) four satellites to be tracked and assigned names KTH one through four, 5) initial range-error and user clock offset values. The initialization defines satellite KTH=1 to be the first satellite tracked.
Figure 4-17. Sequential/Single-Channel Receiver Simulation Program Flow.
2) Next the true position of the receiver is determined. This position is determined by the flight path generator, given time.

3) With the knowledge of the receiver's position and time, the position for the KTH satellite can be determined and the resulting range between the satellite and the receiver.

4) The programmer selected range errors are now computed for the current KTH satellite given time, receiver position, and the observed satellite azimuth and elevation angles. These errors can include diffuse multi-path, ionospheric delay, tropospheric delay, receiver clock error, and a selective availability error.

5) Differential-Mode -- If the receiver is operating in the differential-mode, the receiver obtains the most recent range correction from the fixed receiver and applies that correction to the most recent estimate of the corresponding range measurement.

Fixed-Mode -- If the receiver is operating in the fixed mode and one uplink period has elapsed, the receiver will compute a correction for the most recent satellite measurement, and then provide the most recent corrections of all of the selected satellites to the differen-
tial receiver. The minimum number of satellites to be serviced by the fixed receiver is four.

Conventional-Mode -- If operating in the conventional mode, once the necessary range and range error calculations have been made and properly combined, the program will then proceed to calculate a GPS position fix based upon the degraded range measurements.

6) A position fix is not calculated by the sequential receiver until at least four separate range measurements are available. Once four satellite ranges have been computed, a new position fix is made with every new range measurement. Therefore, the position fix calculation is made with ranges that are either instantaneous, or one, two, or three update periods old.

7) Next, time is incremented by one receiver update period.

8) If time is equal to or greater than the mission-end time, a subroutine is called to generate the statistical data and desired plot files for post program execution evaluation. Program execution is terminated after this subroutine call.

If the mission-end time has not been reached, the value of KTH is sequenced and the process recycles beginning with step two.
2. **Four-Channel Architecture.** The operation of the four-channel set is very similar to that of the sequential/single-channel receiver. The subroutines used in both programs are almost identical. The difference between the two architectures is the sequential/single-channel receiver measures one satellite range each update period and the four-channel receiver can measure four separate satellite ranges during the same period.

The program steps executed for the four-channel receiver is shown in Figure 4-18. The explanation of this flowchart is the same as the sequential/single-channel receiver with the exception of the computation of satellite range and range errors. Each time the program is sequenced through the satellite position/range and satellite range error subroutines, new values for each of the four satellites are calculated. Likewise in the differential- or fixed-mode, either four of the latest corrections are applied or calculated at one time.

The calculation of the true receiver position, GPS position fix, time, and program statistics for both receiver types are identical.
Figure 4-18. Four-Channel Receiver Simulation Program Flow.
Shown in Table 5-1 are the basic receiver configurations used for the three different modes of operation. Two sets of data were collected for each run. One set of data was collected with the Selective Availability error evoked and the second set was run without the SA error.

The entire file of statistics generated in these experiments is not presented. This is in an attempt to lower the amount of data to be assimilated by the reader and to avoid the presentation of redundant information. Only selections of representative data and data of particular interest are shown and discussed.

A. Receiver Performance, Conventional-Mode, No SA Errors.

Figures 5-1, 5-2, and 5-3 show the altitude errors, three dimensional errors, and two dimensional errors encountered during the entire teardrop scenario using a sequential/single-channel receiver, with an update period of 0.3 second, and operating in the conventional mode (run ID # C-1). Figures 5-4, 5-5, and 5-6 depict the same scenario and mode; however, this receiver has four channels with an update period of 0.3 second (run ID # C-4).

Because the four-channel receiver's measurements are more accurate, the errors induced by the alpha-beta tracking filter are amplified, particularly in Figure 5-6. At times equal to approximately 0.039-0.053, 0.162-0.188, and 0.284-0.293 hours, one can see the
### CONVENTIONAL-MODE RECEIVER CONFIGURATIONS

<table>
<thead>
<tr>
<th>RUN ID</th>
<th>RECEIVER TYPE</th>
<th>PERIOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-1</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>0.3 second</td>
</tr>
<tr>
<td>C-2</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>1.2 seconds</td>
</tr>
<tr>
<td>C-3</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>2.4 seconds</td>
</tr>
<tr>
<td>C-4</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
</tr>
<tr>
<td>C-5</td>
<td>FOUR-CHANNEL</td>
<td>1.2 seconds</td>
</tr>
<tr>
<td>C-6</td>
<td>FOUR-CHANNEL</td>
<td>2.4 seconds</td>
</tr>
</tbody>
</table>

### FIXED-MODE RECEIVER CONFIGURATION

<table>
<thead>
<tr>
<th>RUN ID</th>
<th>RECEIVER TYPE</th>
<th>PERIOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-1</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>0.3 second</td>
</tr>
<tr>
<td>F-2</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>1.2 seconds</td>
</tr>
<tr>
<td>F-3</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>2.4 seconds</td>
</tr>
<tr>
<td>F-4</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
</tr>
<tr>
<td>F-5</td>
<td>FOUR-CHANNEL</td>
<td>1.2 seconds</td>
</tr>
<tr>
<td>F-6</td>
<td>FOUR-CHANNEL</td>
<td>2.4 seconds</td>
</tr>
</tbody>
</table>

### DIFFERENTIAL-MODE RECEIVER CONFIGURATIONS

<table>
<thead>
<tr>
<th>RUN ID</th>
<th>RECEIVER TYPE</th>
<th>PERIOD</th>
<th>FIXED RECEIVER MODE</th>
<th>RUN ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-1</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>0.3 second</td>
<td>F-1</td>
<td></td>
</tr>
<tr>
<td>D-2</td>
<td>SEQ/SINGLE-CHAN.</td>
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<td>F-4</td>
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</tr>
<tr>
<td>D-3</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>1.2 seconds</td>
<td>F-2</td>
<td></td>
</tr>
<tr>
<td>D-4</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>1.2 seconds</td>
<td>F-5</td>
<td></td>
</tr>
<tr>
<td>D-5</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>2.4 seconds</td>
<td>F-3</td>
<td></td>
</tr>
<tr>
<td>D-6</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>2.4 seconds</td>
<td>F-6</td>
<td></td>
</tr>
<tr>
<td>D-7</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
<td>F-1</td>
<td></td>
</tr>
<tr>
<td>D-8</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
<td>F-4</td>
<td></td>
</tr>
<tr>
<td>D-9</td>
<td>FOUR-CHANNEL</td>
<td>1.2 seconds</td>
<td>F-2</td>
<td></td>
</tr>
<tr>
<td>D-10</td>
<td>FOUR-CHANNEL</td>
<td>1.2 seconds</td>
<td>F-5</td>
<td></td>
</tr>
<tr>
<td>D-11</td>
<td>FOUR-CHANNEL</td>
<td>2.4 seconds</td>
<td>F-3</td>
<td></td>
</tr>
<tr>
<td>D-12</td>
<td>FOUR-CHANNEL</td>
<td>2.4 seconds</td>
<td>F-6</td>
<td></td>
</tr>
<tr>
<td>D-13</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>0.3 second</td>
<td>F-2</td>
<td></td>
</tr>
<tr>
<td>D-14</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
<td>F-2</td>
<td></td>
</tr>
<tr>
<td>D-15</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>0.3 second</td>
<td>F-5</td>
<td></td>
</tr>
<tr>
<td>D-16</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
<td>F-5</td>
<td></td>
</tr>
<tr>
<td>D-17</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>0.3 second</td>
<td>F-3</td>
<td></td>
</tr>
<tr>
<td>D-18</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
<td>F-3</td>
<td></td>
</tr>
<tr>
<td>D-19</td>
<td>SEQ/SINGLE-CHAN.</td>
<td>0.3 second</td>
<td>F-6</td>
<td></td>
</tr>
<tr>
<td>D-20</td>
<td>FOUR-CHANNEL</td>
<td>0.3 second</td>
<td>F-6</td>
<td></td>
</tr>
</tbody>
</table>

Table 5-1. The 32 Basic Receiver Configurations Used in This Study.
Figure 5-1. Altitude Errors, Run C-1, Without SA, Sequential/Single-Channel, 0.3 second.
Figure 5-2. 3D Position Error, Run C-1, Without SA, Sequential/Single-Channel, 0.3 second.

Figure 5-3. 2D Position Error, Run C-1, Without SA, Sequential/Single-Channel, 0.3 second.
Figure 5-4. Altitude Error, Run C-4, Without SA, Four-Channel, 0.3 second.
Figure 5-5. 3D Position Error, Run C-4, Without SA, Four-Channel, 0.3 second.

Figure 5-6. 2D Position Error, Run C-4, Without SA, Four-Channel, 0.3 second.
peaks recorded in the two-dimension error plots. These peaks occur
during the three turns incurred during the teardrop flight path. The
alpha-beta tracking filter does perform well in constant velocity
dynamics. However, when submitted to the rates of velocities induced
by these turns, the performance of this fixed gain filter is degraded
considerably.

B. Receiver Performance, Conventional-Mode, With SA Errors.

Figures 5-7, 5-8, and 5-9 depict the positional errors encountered
during the teardrop flight path while operating in the conventional
mode, with a sequential/single-channel, and an update period of 0.3
second with the SA error evoked (run ID # C-1). Figures 5-10, 5-11,
and 5-12 depict the same type of errors for a similar receiver type
with four tracking channels (run ID # C-4).

The cause for the increasing magnitudes of all of the positional
errors during the second half of the mission is evident when these
results are compared with the individual satellite SA errors shown in
Figure 3.4.5.5. in Section 3.4.5. This increase corresponds to a
significant increase in range error for three of the four satellites
during this time frame. These errors may also be amplified by a
slight increase in PDOP/GDOP. These values increase respectively from
2.44/2.67 at the beginning of the mission to the final values of
2.78/3.01 at the mission-end time.

The effects of the alpha-beta tracking errors are still notice-
ceable in the 3D and 2D error plots. The net effect of these errors
Figure 5-7. Altitude Error, Run C-1, With SA, Sequential/Single-Channel, 0.3 second.
Figure 5-8. 3D Position Error, Run C-1, With SA, Sequential/Single-Channel, 0.3 second.

Figure 5-9. 2D Position Error; Run C-1, With SA, Sequential/Single-Channel, 0.3 second.
Figure 5-10. Altitude Error, Run C-4, With SA, Four-Channel, 0.3 second.
Figure 5-11. 3D Position Error, Run C-4, With SA, Four-Channel, 0.3 second.

Figure 5-12. 2D Position Error, Run C-4, With SA, Four-Channel, 0.3 second.
are diluted somewhat by the large positional errors induced by the SA errors.

C. Receiver Performance, Fixed-Mode, Without SA Errors. Figures 5-13, 5-14, and 5-15 depict the positional errors calculated at the fixed receiver without the SA error model evoked. It is interesting to see that the errors induced by the alpha-beta tracking filter, seen in the dynamic cases, are no longer present.

The performance of the fixed receiver is best measured by the quality of its calculated range corrections. This comparison is rendered by the cross-correlation of the range errors experienced at the fixed receiver and those experienced at the airborne receiver for a particular satellite. The following cross-correlation functions are presented in the form of plots with the computed $R_{xy}(t)$ versus the uplink period. The value of the cross-correlation coefficient is inversely proportional to the value of the uplink period. The shorter uplink periods will provide better correlation than the longer, undersampled, uplink periods.

1. Diffuse Multipath, Range Correction Effectiveness. The cross-correlation of multipath errors experienced at the fixed receiver and at the airborne receiver is shown in Figure 5-16. This plot clearly shows, that in the case of diffuse multipath, there is no correlation between the errors experienced at the two receivers. This should be expected as the paths taken by the many indirect signals cannot possibly be the same for two spatially separated antennas and
Figure 5-13. Altitude Error, Run F-4, Without SA, Four-Channel, 0.3 second.
Figure 5-14. 3D Position Error, Run F-4, Without SA, Four-Channel, 0.3 second.

Figure 5-15. 2D Position Error, Run F-4, Without SA, Four-Channel, 0.3 second.
Figure 5-16. Cross Correlation of the Fixed and Airborne Receivers' Multipath Range Errors.
therefore the addition of those indirect signals should not be identi-

cal.

2. **Atmospheric Delays, Range Correction Effectiveness.** The

delays induced by the ionosphere and the troposphere are very similar
in that both are very correlative over relatively long periods of

time. The differences lie within the diurnal dynamic properties of
the ionosphere and the sensitivity of tropospheric error to receiver
altitude.

Figure 5-17 shows the correlation between the ionospheric errors
measured at the fixed receiver and those measured at the airborne
receiver. The correlation between the two measurements is nearly
constant over an uplink period of instantaneous corrections (uplink
period = receiver update period) to a value of almost 5 minutes.

The cross-correlation coefficients for the tropospheric errors,
as shown in Figure 5-18, are comparably well behaved. The correlation
between the two receiver measurements is fairly constant over the
range from zero to five-minute uplink periods.

It is clear from these correlation plots that the corrections
provided by the fixed receiver can produce significant improvements
when applied to the range measurements at the airborne receiver.

3. **Receiver Performance, Fixed-Mode, General.** Although a
period of 12 seconds is recommended by the RTCM Special Committee
[25], these results suggest that comparable improvements could be
Figure 5-17. Cross Correlation of the Fixed and Airborne Receivers' Ionospheric Range Errors.
Figure 5-18. Cross Correlation of the Fixed and Airborne Receivers' Tropospheric Errors.
expected with longer uplink periods, given the above conditions.

Figure 5-19 displays the correlative properties of the combined multi-path and atmospheric range errors between the two receivers. This also shows that uplink periods greater than 12 seconds could provide an equally good correction. The 12 second uplink period does provide a better capability when the system is subjected to SA errors.

D. Receiver Performance, Fixed-Mode, With SA Errors. Figures 5-20, 5-21, and 5-22 show the positional errors of the fixed receiver with the SA errors. The cross-correlation plot of the Selective Availability error is shown in Figure 5-23. This plot shows that the SA error is highly correlative, but shows significant decreases in correlation for higher uplink periods. The degradation in correlation appears to be linear for first 60-90 seconds and then begins to worsen with an some type of exponential decay-like function, however, continuing with the decreasing tendency displayed by the 60-90 second linear function. This phenomenon is most likely due to the fixed receiver providing under-sampled corrections at the high uplink periods (i.e., aliasing).

The effect of injecting the SA errors with the diffuse multipath, ionospheric delay, and tropospheric delay is shown in the cross correlation plot shown in Figure 5-24. As seen in the previous section, the resulting correlation plot reflects the culmination of the more negative characteristics of the error models. In this case the correlative properties are degraded by the white noise induced by the diffuse multipath model and the correlation function.
Figure 5-19. Cross Correlation of the Fixed and Airborne Receivers' Range Errors, Consisting of Multipath, Ionospheric, and Tropospheric Combined.
Figure 5-20. Altitude Error, Run F-4, With SA, Four-Channel, 0.3 second.
Figure 5-21. 3D Position Error, Run F-4, With SA, Four-Channel, 0.3 second.

Figure 5-22. 2D Position Error, Run F-4, With SA, Four-Channel, 0.3 second.
Figure 5-23. Cross Correlation of the Fixed and Airborne Receivers' Selective Availability Range Errors.
Figure 5-24. Cross Correlation of the Fixed and Airborne Receivers' Range Errors, Consisting of Multipath, Ionospheric, Tropospheric, and Selective Availability Combined.

The positional errors shown in Figures 5-25 through 5-27 are for a four-channel, differential-mode, 0.3-second-update-period receiver using corrections provided by a four-channel, fixed-mode, 0.3-second-update-period-receiver (run ID D-8). New corrections are provided to the differential set once every 12 seconds. These results are indicative of the statistical characteristics of the other receiver types. The differences lie mainly in alpha-beta tracking filter's inability to track turns especially for the 2.4-second update period receivers.

The overall positive effect of differential GPS can be seen in Figures 5-28 and 5-29. The plots contained in these figures are broken up into four columns of three bars each; column one contains the 3D approach results while operating in the conventional mode, column two contains the 3D positional information generated by the fixed-mode receiver during the approach, column three displays the 3D positional approach information while operating in the differential mode with corrections provided by a sequential/single-channel fixed receiver with a common update rate, and column four displays the 3D data from a receiver operating in the differential-mode using corrections from a four-channel fixed receiver with a common update rate. Each of the columns are labeled with the appropriate receiver configuration run ID number shown in table 5-1 of Section 5 and the differential data are labeled with their respective abscissa values in meters.
Figure 5-25. Altitude Error, Run D-8, Without SA, Four-Channel, 0.3 second.
Figure 5-26. 3D Position Error, Run D-8, Without SA, Four-Channel, 0.3 second.

Figure 5-27. 2D Position Error, Run D-8, Without SA, Four-Channel, 0.3 second.
Figure 5-28. Sequential/Single-Channel Performance, Without SA.
Figure 5-29. Four-Channel Performance, Without SA.
The data displayed by these bar charts, and those that follow, contain the 3D positional information relevant to the approach segment of the teardrop flight path. This was done to eliminate the negative effects of the alpha-beta tracking filter. The dynamics of the approach involve constant velocities and therefore the tracking filter can perform reasonably well and provide an indication of the accuracies obtainable by using differential GPS.

The most impressive results were obtained by the 0.3 second update period, four channel, differential-mode receiver. The values of one-standard-deviation for along-track error, cross-track error, and altitude error were equal to or less than 2.8 meters and were generally about 2 meters or less. The values for the sequential/single-channel receiver were good but not nearly impressive with a one-standard-deviation value for each dimension of error not exceeding 10 meters.

F. Receiver Performance, Differential-Mode, With SA Er
The data output for the four-channel, 0.3-update-period, differential-mode receiver using a four-channel, 0.3-update-period, fixed-mode receiver, both subjected to SA errors, is shown in Figures 5-30, 5-31, and 5-32. Again, the results are good except when the alpha-beta tracking filter is expected to track the receiver's position during turns.

Figures 5-33 and 5-34 show the same type of bar graphs as described in the previous section, except that these results reflect
Figure 5-30. Altitude Error, Run D-8, With SA, Four-Channel, 0.3 second.
Figure 5-31. 3D Position Error, Run D-8, With SA Four-Channel, 0.3 second.

Figure 5-32. 2D Position Error, Run D-8, With SA Four-Channel, 0.3 second.
Figure 5-33. Sequential/Single-Channel Performance, With SA.
Figure 5-34. Four-Channel Performance, With SA.
the effects of the SA range errors. When compared with the bar graphs in Section 4.0.5, it can be seen that for an uplink period of 12 seconds, the positional accuracies obtainable with DGPS are comparable whether or not the SA errors are present. The value of standard deviation never exceeded 2.3 meters for along track (ATE), cross track (CTE), or altitude error for the four-channel, 0.3-update-period, differential mode receiver. The standard deviation for the ATE and altitude errors incurred by the longer update period receivers never exceeded 10.5 meters and the CTE never exceeded 15.7 meters.

The sequential/single-channel, 2.4-update-period, differential-mode receiver was the most susceptible to the position errors due to the alpha-beta filter. These prevalent deviations can be seen in Figures 5-35, 5-36, and 5-37. The large deviations, which appear in the 3D and 2D errors, directly correspond with the execution of turns in the teardrop flight path.

G. Receiver Performance, Differential-Mode, With SA Errors, Using Corrections From Different Receivers of Different Update Periods. The problems of using a receiver with an update period larger than 0.3 second have been presented for the dynamic case. This set of experiments was conducted to see the effects of using fixed receivers of longer update periods than that used by the airborne unit. In each case, the differential-mode airborne unit was either a sequential/single-channel or four-channel receiver with an update period of 0.3 second. The fixed-mode receiver varied in both channel
Figure 5-35. Altitude Error, Run D-6, With SA, Sequential/Single-Channel, 2.4 seconds.
Figure 5-36. 3D Position Error, Run D-6, With SA, Sequential/Single-Channel, 2.4 seconds.

Figure 5-37. 2D Position Error, Run D-6, With SA, Sequential/Single-Channel, 2.4 seconds.
number and update period, the period being either 1.2 seconds or 2.4 seconds.

Figure 5-38 displays the 3D position errors incurred by the 0.3-second-update-period, differential-mode receiver when using correction data provided by a fixed receiver of a slower update period. The performance of the differential-mode, sequential/single-channel receivers (run ID numbers D-13, D-15, D-17, and D-19) are comparable with mean error values of 11.3-13.3 meters, rms error values of 12.5-14.7 meters, and standard deviation of 5.4-6.4 meters. These values correlate well with the previously discussed 0.3-second-update-period, sequential/single-channel, differential-mode receivers. Likewise, the four-channel, differential-mode receiver errors (run ID numbers D-14, D-16, D-18, and D-20) correlated well and performed comparably to the previously mentioned 0.3-update-period, four-channel, differential-mode receivers.
Figure 5-38. 3D Position Errors of a 0.3 second/Differential-Mode Receiver, Using Corrections from either a 1.2 seconds or 2.4 seconds, Fixed-Mode Receiver, With SA Errors.
VI. CONCLUSIONS AND RECOMMENDATIONS

The simulation results show that DGPS has the potential to meet ILS/CAT I accuracy criteria. The 1984 Federal Radionavigation Plan states that the accuracy of a Category I ILS, at a 100-foot decision height, is ± 9.1 meters (2 sigma) horizontally and 3.0 meters (2 sigma) vertically for a precision approach and landing. Most all of the four-channel receivers, operating in the differential-mode, met or exceeded this figure. The two of the four-channel receivers did potentially exceed these values, however these receivers had update periods of 2.4 seconds, a value most likely unsuitable for airborne/NAV applications.

Although none of the sequential/single-channel receivers met the CAT I ILS criteria, they should not be discounted as having a precision approach capability. The poorer performance of the sequential channel receivers can be attributed to non-optimal filtering techniques and lack of other readily available information from the GPS signal. One characteristic not introduced in this study included Doppler measurements from the GPS signal. Measuring the carrier shift in the GPS signal can provide very accurate user velocity information which can be very useful when incorporated into the estimation of the user's position. This information would significantly improve the performance of a sequential receiver and would be necessary if sequential receivers were to be used for civil aviation approach and landing applications.
The ramifications of DGPS offering a local area Category I precision approach capability will have a significant positive effect on civil aviation. As an example, consider the impact to the state of Ohio. Ohio currently has 194 commercial airports, 597 private airports, and 253 heliports or a total of 1044 facilities. Of these facilities, 46 have a NDB approach capability, 35 have a VOR approaches, 6 have NDB and VOR approaches, and 12 have an ILS or LOC and NDB or VOR approach. A total of 74 airports or only 7 percent of the airport facilities have an established approach. If it is proven that a single DGPS ground unit can service an circular area of a radius equal to 50-60 miles, then all of the airport facilities in the State of Ohio and all the real estate between these facilities could have a Category I precision approach capability. Using the 50-60 mile service radius, the State of Ohio would have to establish 5-6 DGPS ground units to provide 100 percent Category I coverage.

The reduction of the DGPS user's position error, due to white noise, will be a high-priority task in the implementation of a DGPS. Considering the high correlative characteristics of the atmospheric delays and the selective availability errors, the highest contributor of uncertainty to the differential (range) corrections is white noise. And, as a result of this uncertainty, can be considered the highest contributor of position error while operating in the differential-mode. This phenomenon presents itself via the independent random processes contained in the system, sources of which include multipath, fixed receiver tracking and thermal noise, airborne receiver tracking
and thermal noise, spurious channel interference, and others and will have to be reduced to assure the safe implementation and use of DGPS.

For future work and improvements to the DGPS system, the following items should be given consideration:

1) The results generated in this study do not use all of the data available from GPS to produce the best estimate of the user's position. Doppler or Delta-Range values should be used to update the ranges measurements, particularly for the slow sequencing, single channel receivers.

2) Doppler or Delta-Range corrections, provided with the DGPS range corrections, can provide a better estimate of bias component of the range error. This should improve overall DGPS system performance, especially if longer DGPS correction uplink periods are to be considered.

3) More sophisticated filtering techniques should be investigated. The airborne receiver will require some type of adaptive filter to overcome the acceleration and dynamics which may be encountered during a precision approach or missed approach. The ground receiver filter should be designed to minimize the content of white noise in the DGPS corrections. Future investigations should be designed to prove whether linear filter or simple integration techniques are sufficient or if the complexity of a Kalman or adaptive filter is required.
4) Once the launch schedule for the Phase II satellites has begun, investigations and tests should be initiated to determine the integrity of the control and space segments of GPS. Considerable concern has surrounded GPS's capability for quick notification to the user of a system problem. Before this system can be incorporated into the National Airspace System, this system failure notification response time must be identified and quantified.

5) A feasibility study should be performed to determine the cost, impact, and logistics of integrating DGPS with the National Airspace System.

6) If DGPS is considered to be a feasible landing system, studies will have to performed to determine if the DGPS ground unit should be an area-service (50-100 mile service range) or a terminal service (0-25) miles.

7) Finally, work should be performed to determine to what extent local meteorological phenomenon will have on the DGPS corrections.
VII. ACKNOWLEDGEMENTS

The beginning of this work stemmed from an internship, which I served at NASA Langley Research Center, Hampton, Virginia, as an augmentation of NASA Grant NGR 36-009-017. The bulk of this work was performed under NASA Grant NAG 2-231 with continued support from NASA Contract NAS2-11969, both sponsored by NASA Ames Research Center, Moffett Field, California.

The author wishes to acknowledge the following individuals for their assistance and contributions to this work:

Mr. William Howell, Branch Head of ATRB, NASA Langley Research Center, for making it possible for me to receive this internship and providing a position for me on the GPS effort, and his helpful suggestions throughout this work. Mr. Thomas Bundick, ATRB Research Engineer and my assigned mentor throughout my internship, NASA Langley Research Center, for his guidance, support, and friendship which has continued beyond the internship.

Mr. Fred G. Edwards, Branch Head of the Aircraft Guidance and Navigation Branch at NASA Ames Research Center for enabling this work to continue and for his helpful suggestions during this work.

Dr. Richard H. McFarland, Director of the Avionics Engineering Center at Ohio University and Advisor/Chairman of my thesis committee, for providing invaluable guidance and advice during the creation of this paper.
Messrs. Dennis Scanlon, Graduate Research Assistant, Howard B. Maidlow, Undergraduate Intern, and Andrew Nelson, Undergraduate Intern have assisted me in the development of the DGPS programs and performed a significant amount of the data collection tasks required to present this work.

Dr. Butch Hill and Dr. Roger Radcliff, Associate Professors of Electrical and Computer Engineering at Ohio University and Dr. Roy Lawrence, Professor of Mechanical Engineering at Ohio University, for reviewing this document and serving as members of my Thesis Defense Committee.

Additional thanks go to Dr. Robert W. Lilley, Associate Director, Mr. James D. Nickum and Mr. Richard N. Turner, my co-engineers, and Mr. Frank Van Graas, Graduate Research Assistant, for reviewing this document. And special thanks go for the efforts of Ms. Alicya Shade and Ms. Michele Nutter during the production of this paper.


[9] Ibid.


[14] Ibid.


[18] Ibid.


[22] Shively, Curtis A., op. cit., Appendix C.


[24] Ibid.