Nonlinear Normal Force Indicial Responses for a
2-D NACA 0015 Airfoil /

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NOMENCLATURE

AR = aspect ratio
C = chord length of airfoil
$C_1, C_2, C_3$ = calibration constants
$F_n$ = normal force
$F_t$ = axial force
$\alpha$ = angle of attack
$\rho$ = density of water
k = non-dimensional pitching rate
$\dot{\alpha}$ = pitching rate
$C_N, C_A$ = normal and axial force coefficients
$\frac{\partial C_N(t)}{\partial \alpha}, C_N\alpha$ = normal force indicial response
$C_{L_{\text{max}}}$ = maximum steady state section lift
q = reference pressure
S = planform area
$m_o$ = section lift curve slope
$A_L$ = dynamic augmentation parameter
U = free stream velocity
Introduction

Chapter 1

1.1 Background

A number of approaches have been taken with regard to the prediction of airfoils experiencing time dependent motions. These include numerical solutions of the Navier Stokes equation[1,2], models based on simulating the large scale vortical structure of the flow[3], and the mathematical models involving convolution integrals of the indicial responses to characteristic motions[4]. The term "nonlinear aerodynamics modeling" has been used in connection with the latter of these approaches. In particular, the loading on airfoils operating at high angles of attack is a nonlinear flight problem. In this context, nonlinear refers to the fact that the loading response to a step change in one or more of the motion variables is influenced by the flow conditions which exist at the onset of the step. The flow conditions at the step onset will be determined by the airfoil motion history prior to the step. Mathematically, the loading response to a step change in a motion variable as the step amplitude approaches zero and wherein an accounting is made of past motion effects is called the nonlinear indicial response[4,5]. The loading on an airfoil in arbitrary motion can, in principle, be computed through a convolution of indicial responses.

This thesis investigates experimentally the nonlinear indicial normal force response to step changes in angle of attack. The normal force acting on a 2-D airfoil undergoing a large-amplitude ramp motion is computed from the experimentally measured nonlinear indicial responses. A description of the experimental apparatus is made in the next chapter followed by a discussion of nonlinear aerodynamic modeling using simplified indicial responses.
Experimental Apparatus
Chapter 2

2.1 Tow tank facility

Tow tank studies were done on a six inch chord NACA 0015 airfoil at a Reynolds number of $9.5 \times 10^4$. The pitch axis was at the airfoil quarter chord. The tank has a length of 30 feet, a width of 12 feet and a depth of 4 feet as shown in the Figure 2.1a. The airfoil was supported vertically and the clearance between the free end and the tank bottom was one inch. The pitching motion is imparted by a high-power (3 hp) stepper motor linked to the airfoil via a 10 to 1 speed reducer. The stepper motor is interfaced to an IBM p.c. equipped with a stepper motor translator card. The computer/translator may be programmed to give prescribed changes in angle of attack and pitching rate. The motor has the rapid starting and stopping response necessary to impart rapid small changes in angle of attack. The translational motion is imparted by a 3 hp 220V three-phase electric motor and cable drive assembly as shown in the Figure 2.1b. The translational speed may be varied in the range of 0.5 to 4 feet per second by varying the diameter of the pulley as shown in the Figure 2.1b. In the present study one translational velocity of $2.03 \text{ ft/s}$ is considered. The airfoil normal force, $C_N$, and axial force, $C_A$, are measured using two strain gauge bridges placed on the shaft which connects the airfoil to the speed reducer. The normal and axial forces are defined as shown in the Figure 2.1c. The angle of attack is measured using a precision rotational potentiometer. The uncertainty in the absolute angle of attack is believed to be less than $\pm 0.5$ degrees while the uncertainty in relative changes in angle of attack is thought to be...
Figure 2.1a. Tow Tank Facility.
Figure 2.1b. Tow Tank Translational Drive Mechanism.
Figure 2.1c. Definition of Airfoil Loads.
less than ±0.1 degrees. The uncertainty in the normal force measurements is less than ±3%, while for the axial load the uncertainty is ±10%.

2.2 Strain Gauge Measurement

Measurements of normal and axial forces as functions of angle of attack were performed on NACA 0015 airfoil at the Reynolds number $10^5$. As indicated in Figure 2.2 the forces were measured using the strain gauges located on a load cell mounted to the quarter chord of the airfoil. Each strain gauge bridge was arranged so that it was sensitive only to the desired force. Each bridge consisted of four OMEGA(Model No. EA-13-250MQ-350) 350 ohm strain gauges connected in a Wheatstone bridge configuration, a 15V dc OMEGA MODEL PST-430 Bridge excitation power supply, and a OMEGA series OM3 amplifier. The output voltage was amplified 200 times to increase the signal level to the ±5V range. Each bridge has a trim pot for balancing the bridge before each test. The amplified strain gauge signals and the signal from the angle of attack sensor were introduced into the computer memory using a MetraByte model DASH8 high speed A/D converter which has a conversion rate of 4000 data points per second.

The voltage output from the strain gauge bridges $E_1$ and $E_2$ are related to the normal force $(F_n)$ and the tangential force $(F_t)$ by:

\[ F_n = C_1 E_1 \]  \hspace{1cm} (2.1a)

\[ F_t = C_2 E_2 \]  \hspace{1cm} (2.1b)

while the angle of attack of the airfoil is related to the voltage drop across the
Figure 2.2. Load Cell Configuration.
variable potentiometer $\Delta E_3$ by:

$$\alpha = C_3 \Delta E_3$$  \hspace{1cm} \text{(2.2)}$$

In equation 2.1 and 2.2, $C_1$, $C_2$ and $C_3$ are calibration constants. The calibration constants for the strain gage bridges $C_1$ and $C_2$ were found by applying known loads to the airfoil at the center of mass of the submerged portion and noting the bridge outputs.
3.1 The Nonlinear Indicial Response

The indicial response is the transient airfoil loading due to an instantaneous step change in a motion variable. This could be, for example, the normal force response due to a step change in angle of attack. This is shown schematically in Figure 3.1. The normal force indicial response, \( \frac{\partial C_N}{\partial \alpha} \), is defined by Tobak and Schiff [4] as:

\[
\frac{\partial C_N}{\partial \alpha} (t) = \lim_{\Delta \alpha \to 0} \frac{C_L(t) - C_L(0)}{\Delta \alpha}, \quad t \geq \tau
\]

(3.1)

If the indicial response is known, the loading during an arbitrary motion can, in principle, be computed using Duhamel's superposition integral.

In linear aerodynamics, the indicial response is independent of the flow conditions which exist at the onset of the step. The linear indicial response is thus totally insensitive to prior motion effects, and \( \alpha (\xi) \) in Figure 3.1 is immaterial insofar as the linear indicial response is concerned. This immediately excludes inherent and strong wake effects. These effects may be important for motions in which the structure of the flow near the airfoil is dependent on things such as initial conditions and/or pitch rate [6]. Consequently, the range of validity of the linear response model warrants investigation. Furthermore, it would be desirable to experimentally measure the approximate form of the indicial response in a nonlinear flight regime. We use the term "approximate form" because it is, of course, impossible to experimentally duplicate a step function.
where:

\( \alpha(\xi) \) is the "reference motion," defined for \(-\infty < t \leq \tau\);
\( \alpha_1 \) consists of \( \alpha(\xi) \), for \( t \leq \tau \), and is held constant at \( \alpha(\tau) \) for \( t > \tau \);
\( \alpha_2 \) consists of \( \alpha(\xi) \), for \( t \leq \tau \), but jumps instantaneously to \( \alpha(\tau) + \Delta \alpha \), for \( t > \tau \).

Figure 3.1. Motions Defining the Indicial Response[4].
Tobak and Schiff[4] have shown that the nonlinear indicial response to a step change in angle of attack should be expressed as a functional given by:

\[ \frac{\partial C_N}{\partial \alpha} = C_{N\alpha}[\alpha(\xi) ; t, \tau] \] (3.2)

where the terms in brackets have been defined in Figure 3.1. The normal force at some time, \( C_N(t) \), may then be found using the superposition integral given by:

\[ C_N(t) = C_N(0) + \int_0^1 C_{N\alpha}[\alpha(\xi) ; t, \tau] \frac{d\alpha}{dt} d\tau \] (3.3)

Tobak and Chapman[5] show that the convolution integral in this form contains all the "direct" effects from the pulses describing the motion plus the effects of "interference" between pairs and/or groups of pulses. This formulation allows the loading due to the current state of the motion to be influenced by the prior events which give rise to interference effects.

The form of the indicial response in the integral of Equation 3.2 is not tractable and simplification of the response is needed. Tobak and Schiff[4] suggest that for relatively slow motions the response may be expressed in terms of elapsed time, \( \tau_e = t - \tau \), and a finite number of motion variables at step onset. This may be written:

\[ C_{N\alpha}[\alpha(\xi) ; t, \tau] = C_{N\alpha}[\tau_e ; \alpha(\tau), \dot{\alpha}(\tau), \ddot{\alpha}(\tau), \ldots] \] (3.4)

where the retention of successively higher order derivatives effectively includes motion
events further into the past. The indicial response has thus been simplified, however, the number of motion variables which must be included in the representation of Equation 3.4 is still unknown. The present work focuses on the dependence of the normal force indicial response on the first order term, $\alpha(\tau)$, in Equation 3.4. First, we give some background on existing representations of the indicial response given by the Wagner function and the Kussner function. The Kussner function has been used in the work of Beddoes[7] and Leishman[8].

In an early contribution to the theory of unsteady airfoils, Wagner derived a relationship for the variation in the airfoil lift due to an instantaneous change in angle of attack. This is precisely the condition for which the indicial response is defined. On the other hand, Kussner derived a function for the lift on an airfoil which is penetrating a sharp edged gust. Both the Wagner and Kussner analyses assume inviscid incompressible flow, low angle of attack, and negligible wake distortions. Sears[9] has used Heaviside's operators to show that the relationship giving the change in the total circulation about an airfoil (this relationship excludes the circulation in the wake) undergoing a step change in angle of attack, is identical to the Kussner function. It appears that Beddoes and Leishman have used this analogy to base the indicial response on the Kussner function.

In References[7,8] the normal force indicial response, $C_{N\alpha}$, is represented as the sum of a circulatory component, $C_{N\alpha C}$, and an initial component which is noncirculatory and decays rapidly, $C_{N\alpha I}$. The response is a function of only elapsed time, $\tau_e$, and may be written:

$$C_{N\alpha}(\tau_e) = C_{N\alpha C}(\tau_e) + C_{N\alpha I}(\tau_e)$$ (3.5)
Note that Equation 3.5 is a special case of Equation 3.4 wherein the response is independent of alpha and all its derivatives. Leishman has used a modified form of the Kussner function to represent the circulatory component, and the results of piston theory for the noncirculatory component. The resulting normal force indicial response to a step change in angle of attack is:

\[ C_{N\alpha}(\tau_e) = \left[ 2\pi / \beta \right] \phi_C(\tau_e) + \left[ 4 / M \right] \phi_I(\tau_e) \]  

(3.6)

where

\[ M = \text{Mach number} \]
\[ \beta = \text{compressibility factor} = (1-M^2)^{1/2} \]
\[ \phi_C(\tau_e) = \text{circulatory component of the normal force response} \]
\[ = 1 - A_1 \exp(-b_1 \beta^2 S) - A_2 \exp(-b_2 \beta^2 S) \]
\[ \phi_I(\tau_e) = \text{noncirculatory component of the indicial normal force response} = \exp(-S / T_1) \]
\[ T_1 = 4M/[2(1-M) + 2\pi \beta M^2(A_1 b_1 + A_2 b_2)] \]
\[ S = \text{nondimensional elapsed time in semi-chords} = \tau_e U/b, \ b = \text{semi-chord length} \]

The constants appearing in the above relations are given by: \( A_1=0.3, A_2=0.7, b_1=0.14, \) and \( b_2=0.53. \) Note that Equation 3.6 includes Mach number effects, which was is not indicated explicitly in Equation 3.5. Equation 3.6 may also be recast in the form of the Wagner function by changing the constants to: \( A_1=0.165, A_2=0.335, b_1=0.0455, \) and \( b_2=0.3. \) Later we will compare Equation 3.6 with the experimentally measured indicial responses of the present study.

Leishman has used the Kussner function form of Equation 3.6 to compute the normal force coefficient for harmonic forcing using the Laplace transform method.
This method expresses in operational form the relationship between the load and the forcing motion. The function describing the airfoil motion must be Laplace transformable. In a similar way, Beddoes has derived transfer functions for harmonic, ramp, and plunge motions. This operational approach is permissible because the coefficients multiplying the functions \( \phi_C(t_e) \) and \( \phi_I(t_e) \) are independent of the conditions which exist at the step onset. This response is thus linear in the sense described above. Beddoes has also used Equation 3.6 in Equation 3.3 to develop useful recursion-type formulae to evaluate the integral in Equation 3.3. These formulae may be applied to arbitrary motions.

3.2 Incompressible Case

The present study has focused on experimental measurement of the indicial normal force response at very low Mach number. The tests were conducted in water at a translational speed of 2 ft/s. Under these conditions the Mach number is nearly zero and, henceforth, this discussion relates only to incompressible flow. For zero Mach number, Jenkins[6] has shown that Equation 3.5 may be written:

\[
C_{N\alpha}(\tau_e, \alpha(\tau)) = C_{N\alpha_C}(\tau_e, \alpha(\tau)) + \delta_I(\tau_e) \tag{3.7}
\]

where we now extend Equation 3.5 to include the dependence of the circulatory component on the first order motion parameter \( \alpha(\tau) \). In Equation 3.7, \( \delta_I(\tau_e) \) is the Dirac delta function acting at the step onset (\( \tau_e=0 \)) due to the initial incompressible apparent mass reaction. Thus, the noncirculatory component is instantaneous and acts only at the step origin. This is also consistent with the exponential representation of the
noncirculatory component used by Leishman in Equation 3.6. The normal force at time, \(t\), may be computed by substituting Equation 3.7 into Equation 3.3. Jenkins shows the result to be:

\[
C_N(t) = C_N(0) + C_{N\alpha m} \tilde{\alpha}(t) + \int_{0}^{t} C_{N\alpha C}[\tau_e, \alpha(t)] \frac{d\alpha(\tau)}{d\tau} d\tau \tag{3.8}
\]

where the second term on the RHS is an apparent mass term and is the result of integrating across the delta function. Evaluation of Equation 3.8 requires knowledge of the apparent mass reaction and the nonlinear normal force circulatory indicial response, \(C_{N\alpha C}[\tau_e, \alpha(t)]\). We now discuss briefly the apparent mass effect and then will turn our attention to the circulatory term.

In general, the nonlinear apparent mass reaction, like the circulatory reaction, may be a functional of the function describing the airfoil motion. There is little information in the literature as to the form of the nonlinear incompressible apparent mass reaction. For low angles of attack and in the absence of strong wake distortions, reference[10] provides the following linear relationship for the noncirculatory component of the normal force coefficient, \(C_{NI}\):

\[
C_{NI}(t) = \pi \left[ \ddot{h}(t) \frac{b}{U^2} + (\ddot{\alpha}(t) \frac{b}{U}) - (\dddot{\alpha}(t) \frac{ab^2}{U}) \right] \tag{3.9}
\]

where \(b\) is the semichord length, and \(a=-1/2\). The first term in Equation 3.9 is the reaction due to plunging where \(\ddot{h}\) is the rate of plunge of the aerodynamic center and is positive downward. The second term is the reaction due to rotation about the
aerodynamic center, and the third term is the reaction due to the upwash at the 3/4 chord due to rotation about the aerodynamic center. Measurement of the apparent mass reactions for nonlinear flows may require novel experimentation and remains a topic for future study. The present thesis investigates only the circulatory component of the nonlinear indicial response.

At first, it may be tempting to introduce the effect of \( \alpha(t) \) into the circulatory response by replacing the \( 2\pi \) in Equation 3.6 with the local value of the steady state normal force curve slope, yielding:

\[
C_{N\alpha C}[\tau_e, \alpha(t)] = C_{N\alpha S} \phi_C(\tau_e)
\]  

(3.10)

where \( C_{N\alpha S} \) is the steady state normal force curve slope at \( \alpha(t) \). For large values of elapsed time the function \( \phi_C(\tau_e) \) as defined in Equation 3.6 approaches unity. From Equation 3.10, the indicial response will approach the steady state normal force curve slope, as it should regardless of the relationship used to represent the response. Equation 3.10 can be seen as a special case of Equation 3.4 wherein the dependence of the response on \( \alpha(t) \) is introduced through the steady state lift curve slope, \( C_{N\alpha S} \), and the time dependence is contained within the function \( \phi_C(\tau_e) \). While this is nonlinear, Equation 3.10 has the disadvantage that the "functional form" of the response given by \( \phi_C(\tau_e) \) does not itself depend on \( \alpha(t) \). As such, the response given by Equation 3.10 does not encompass first order dependence in the general sense implied by Equation 3.4. Furthermore, it will show that for large variations in onset angle of attack, the first order indicial response cannot be adequately represented by a single "type" of function as in Equations 3.6 or 3.10.
Discussion of Results

Chapter 4

4.1 Steady State Test Results

Static tests were conducted to determine the steady state normal force curve slope. The results are shown in Figure 4.1. These data show significant nonlinearities in the steady state normal force curve slope with angle of attack. The nonlinearity is present even at lower angles of attack where linear behavior is usually assumed. For comparison, the theoretical thin-airfoil normal force curve slope given by $2\pi$ is also shown. Notice that for angles where the actual slope is greater than $2\pi$, Equation 3.6 will underpredict the steady state response and for angles where the slope is less than $2\pi$, Equation 3.6 overpredicts.

4.2 Experimental Measurement of the Nonlinear Indicial Response

The normal force indicial response due to angle of attack has been measured by considering the three motions shown in Figure 4.2. In each motion, the airfoil moves for twelve chord lengths at constant angle of attack so that in the notation of Figure 3.1 we have $\alpha(\xi) = \alpha(\tau) = \alpha_o$. In motion 0, the angle is held constant to provide a measure of the static normal force at $\alpha_o$. In motion 1, the airfoil undergoes a rapid change in angle of attack to $\alpha_o + \Delta\alpha_u$, and in motion 2 to $\alpha_o - \Delta\alpha_d$. The angle of attack parameters $\alpha_o$, $\Delta\alpha_u$, and $\Delta\alpha_d$ are measured quantities. The distinction between $\Delta\alpha_u$ and $\Delta\alpha_d$ was necessary because the experimental apparatus is not capable of imparting perfectly symmetric steps about $\alpha_o$. The time dependent normal
Figure 4.1. Static Normal Force Curve Slope.
Figure 4.2. Test Motion for first Order Indicial Response.
force indicial response corresponding to the initial angle $\alpha_o$ was computed using a three point formula given by:

\[
C_{N\alpha}[\tau_e, \alpha_o] = C_{N0}[\Delta\alpha_u - \Delta\alpha_d (\Delta\alpha_u \Delta\alpha_d)^{-1}] + C_{N+}[\Delta\alpha_d (\Delta\alpha_u + \Delta\alpha_d)^{-1}] + C_{N-}[\Delta\alpha_u (\Delta\alpha_d + \Delta\alpha_d)^{-1}]
\]

(4.1)

where

- $C_{N0}=C_{N0}(\alpha_o)$ = steady state normal force coefficient during motion 0
- $C_{N+}=C_{N+}[\tau_e, \alpha_o]$ = transient normal force coefficient during motion 1
- $C_{N-}=C_{N-}[\tau_e, \alpha_o]$ = transient normal force coefficient during motion 2

For symmetric steps Equation 4.1 reduces to a two point central difference. Table 4.1 is a list of onset angles of attack ($\alpha_o$) for which the normal force response was measured.

The uncertainty in the normal force response given by Equation 4.1 has been estimated to be ±25%. The large uncertainty is primarily due to the uncertainty in $\Delta\alpha_u$ and $\Delta\alpha_d$, and the fact that the magnitudes of these terms are small. Also, the magnitudes of $\Delta\alpha_u$ and $\Delta\alpha_d$ are not completely repeatable and variations on the order of ±15% have been observed from run to run. The variations in the step amplitudes can lead to variations in the transient part of the indicial response. To account for this each response was measured five times and the results were ensemble averaged. The uncertainty analysis is given in Appendix A.

Due to the rapid starting and stopping of the airfoil during a step, structural vibrations led to significant levels of noise occurring at the natural frequencies of the
Table 1. First Order Onset Angles

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>2.09°</th>
<th>25.72°</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8°</td>
<td>31.0°</td>
<td></td>
</tr>
<tr>
<td>8.33°</td>
<td>35.77°</td>
<td></td>
</tr>
<tr>
<td>11.32°</td>
<td>41.0°</td>
<td></td>
</tr>
<tr>
<td>14.52°</td>
<td>46.0°</td>
<td></td>
</tr>
<tr>
<td>15.47°</td>
<td>51.0°</td>
<td></td>
</tr>
<tr>
<td>17.55°</td>
<td>56.0°</td>
<td></td>
</tr>
<tr>
<td>20.77°</td>
<td>60.0°</td>
<td></td>
</tr>
</tbody>
</table>
towing structure. To remove the noise the strain gauge output signals were low-pass filtered to remove all frequencies above 170 Hz. The data acquisition rate was then set at 340 Hz (twice the Nyquist frequency) to ensure detection of any remaining frequencies. The natural vibrational frequencies in the normal and axial force directions were determined by draining the tank and running the airfoil across the tank at various angles of attack. These "empty tank" data were transformed into the frequency domain using FFT where the natural frequencies of vibration became apparent. The vibrational frequencies were removed from the actual aerodynamic test data using digital notch filters tuned to the natural frequencies. The filters introduce a time lag which has been corrected for in the data reduction. The time lag is approximately 1.0 in semichords. The time lag was estimated by assuming that shortly after the step occurs the rise in the normal force signal would occur near the same frequency as the rise in the signal from the angle of attack sensor. The angle of attack data were passed through the normal force notch filter and the time lag was estimated by comparing the filtered output with the unfiltered input. The natural frequencies and filtering technique are discussed in Appendix C.

The noncirculatory component of Equation 3.5 was not detected in the filtered test results. This is not surprising since, as previously discussed, the initial noncirculatory loading is nearly an impulse which would either not be detected in discrete time sampling or, in any case, would be eliminated by low pass filtering. As a result, the present tests yielded information on only the circulatory component of the response.
4.3 Nonlinear Indicial Response Results

Typical test results for the case of $\alpha_o = 2^\circ$ are shown in Figures 4.3a and 4.3b. Figure 4.3a shows the angle of attack data as a function of chords of travel for motion 1 described in general terms in Figure 4.2. As may be seen, the step height is approximately one degree and the step occurs quite rapidly. The high frequency electrical noise was removed by filtering. These data were averaged before and after the step and $\Delta\alpha_{ui}$ in Equation 4.1 was taken as the difference between the average values. Figure 4.3b shows corresponding normal force coefficient data. To illustrate the effect of the filtering, the unfiltered force data are also shown in Figure 4.3b. The step occurs at a time of 9.7 chords. The filter eliminates the large inertial component of the signal which occurs due to the rapid step. The filter also removes the structural vibration component and this results in a substantial decrease in the noise level in the signal.

Five separate sets of measurements were made and the indicial normal force response for each set has been computed using Equation 4.1. The results for five runs at $\alpha_o = 2^\circ$ are shown in Figure 4.4a which shows some scatter in the data from run to run. The scatter is believed to be due to uncertainties in the step amplitudes and, perhaps more importantly, the effect of the step amplitude on the transient load response. In any event, Figure 4.4a illustrates a worst case and the responses for other onset angles showed repeatability better than that of Figure 4.4a. To emphasize this point Figure 4.4b shows the indicial response results of five runs for $\alpha_o = 11^\circ$. These data are repeatable. The response data have been ensemble averaged and the results for $\alpha_o = 2^\circ$ are shown in Figure 4.5. The response approaches the correct steady state value which has been taken from Figure 4.2. Also shown in Figure 4.5 are the responses given by Equations 3.6 and 3.10 using the Kussner function to evaluate $\Phi_C(\tau_e)$. Equation 3.10 was evaluated using the present experimental result.
Figure 4.3a. Angle of Attack Data for Step-Up for $\alpha_0 = 2.09^\circ$. 
Figure 4.3b. Normal Force Data for Step-Up for $\alpha_o = 2.09^\circ$. 
Figure 4.4a. Normal Force Indicial Responses for $\alpha_o = 2^\circ$. 
Figure 4.4b. Normal Force Indicial Responses for $\alpha_0 = 11^\circ$. 
Figure 4.5. Averaged Normal Force Indicial Response for $\alpha_0 = 2.09^\circ$. 
for the static normal force curve slope shown in Figure 4.1 \( C_{N_{\alpha_s}} = 8.07/\text{rad} \). There is relatively good agreement between the experimental data and Equation 3.10 over most of the test interval. Equation 3.6 is seen to give results substantially below the experimental results.

The normal force response at \( \alpha = 4.8^\circ \) is shown in Figure 4.6. The response computed from Equation 3.10 is in reasonably good agreement with the experimental results. Equation 3.10 has again been evaluated using the present results for the steady state slope given in Figure 4.1. Equation 3.6 is now slightly larger than the steady state experimental result. Increasing \( \alpha_0 \) to 8.33° results in the normal force indicial response shown in Figure 4.7. The experimental response differs from Equation 3.10 at elapsed times below 3.0. Equation 3.6 gives a vast overestimate of the steady state normal force response.

Figure 4.8 shows the normal force indicial response at \( \alpha_0 = 11.32^\circ \). Neither Equation 3.6 or 3.10 adequately represents the experimental response. Note, however, that the experimental result still approaches the steady state lift curve slope as elapsed time from step onset increases. More importantly, if Equation 3.10 rather than the actual response is used in Equation 3.8 to calculate the normal force, the contribution to the convolution of the response at 11° will be too small for elapsed times less than that for steady state.

Figure 4.9 is an overlay of the indicial responses for onset angles less than 16°. These responses are similar in that they each appear to approach a single-valued steady state. This is due to the fact that the steady state flow is attached. Notice also that the response decreases substantially as the static stall angle is approached, even for small values of elapsed time.

The experimental results for onset angles in the range 15° < \( \alpha_0 < 26^\circ \) are shown in Figure 4.10a. The responses for \( \alpha_0 > 17^\circ \) are obviously fundamentally
Figure 4.6. Normal Force Indicial Response for $\alpha_0 = 4.8^\circ$. 
Equation 3.6

$c_{Na}$

Equation 3.10

Figure 4.7. Normal Force Indicial Response for $\alpha_0 = 8.33^\circ$. 
Figure 4.8. Normal Force Indicial Response for $\alpha_o = 11.32^\circ$. 
Figure 4.9. Overlay of Normal Force Indicial Responses for $0^\circ < \alpha < 15^\circ$.
Figure 4.10a. Overlay of Normal Force Indicial Responses for $15^\circ < \alpha_o < 30^\circ$. 
different than either the Kussner or Wagner function. Flow visualization studies have shown that for angles beyond $\alpha_o = 16^\circ$ massive leading edge separation due to static stall has occurred before the step onset. Notice in Figure 4.1 that the steady state lift curve slope is nearly zero at $\alpha = 16^\circ$ and becomes negative for $\alpha > 16^\circ$, which is another indication of static stall. Referring again to Figure 4.10a, the indicial response is seen to change substantially in form between $\alpha_o = 15.47^\circ$ and $\alpha_o = 17.55^\circ$. This may be indicative of a bifurcation in the aerodynamic response located at the static stall angle. This conclusion may be further justified by two observations. First, notice that for onset angles less than $16^\circ$ the indicial response approaches steady state, more-or-less, without oscillations(cf. Figure 4.9). Whereas at onset angles larger than $16^\circ$ the responses have large oscillations which may arise due to vortex shedding, even for large values of elapsed time(i.e. the steady state may be oscillatory). Oscillations in the steady state response is a characteristic of a Hopf bifurcation[5]. Secondly, the responses for $\alpha_o = 17.55^\circ$ and $\alpha_o = 20.77^\circ$ have a different time-scale than those for $\alpha_o = 15.47^\circ$ and below. This indicates that the flow has fundamentally changed and the response has undergone a bifurcation which reflects the new flow regime.

The responses at $\alpha_o = 17.55^\circ$ and $\alpha_o = 20.77^\circ$ display some interesting behavior. First, for elapsed times below 2.0 the response magnitudes are small and the response at $20.77^\circ$ actual becomes slightly negative. For small elapsed times these responses will have only small contributions to the convolution integral of Equation 3.8 which, as will be shown, leads to a significant deficit in the integrated normal force results. As elapsed time increases the responses become extremely large in relation to their respective steady state values given in Figure 4.1. These large responses will translate into large normal forces for airfoils operating slightly beyond the static stall
angle. This is significant from the standpoint of using first order indicial responses to model dynamic stall where it is well known that large forces may ensue. Preliminary results from flow visualizations suggest that the large forces may be due to rapid flow reattachment shortly after the step onset. This flow reattachment may be similar to the phenomenon of flow separation delay in unsteady airfoils, where the delay gives rise to large suction pressures (and lift) over the upper surface of the airfoil. Assuming these phenomena are related then this observation would provide some physical grounds for computing dynamic effects using first order indicial responses. Notice, however, that the occurrence of separation delay in dynamic situations will also enter into the nonlinear formulation through attached flow onset conditions. There is then an important distinction between the rapid flow reattachment which has been observed for first order responses at onset angles where the flow is initially separated and separation delay in dynamic situations. Flow reattachment was observed in the range of $17^\circ < \alpha_0 < 30^\circ$. At larger angles flow reattachment was not observed and leading edge separation exists throughout the motion.

The responses for $30^\circ < \alpha_0 < 45^\circ$ are shown in 4.10b and those for $50^\circ < \alpha_0 < 60^\circ$ are shown in Figure 4.10c. There is undoubtedly work to be done in interpreting the physical mechanisms involved in the experimental indicial responses (notice in Figure 4.10c, for instance, the unusual behavior of the response at $51^\circ$). However, the data presented above clearly show that the indicial response is (at least) first order dependent even for moderate angles of attack. Furthermore, for motions with large changes in angle of attack there will be a wide variation in "functional form" of the indicial responses. As has been discussed in connection with Equation 3.10, this discounts the possibility of simply scaling the linear response according to
Figure 4.10b. Overlay of Normal Force Indicial Responses for $30^\circ < \alpha_o < 45^\circ$. 

\[ c_{Na} \]

$0.00 \rightarrow 10.00$

$t - r$ (chords)

-5.00

35.77°

41.0°

31.0°

46.0°
Figure 4.10c. Overlay of Normal Force Indicial Responses for $45^\circ < \alpha_o < 60^\circ$. 
nonlinearities in the static load slope only.

The normal force coefficient for a ramp motion is computed below by integrating the experimental indicial responses. The indicial responses presented above were not corrected for time lag due to filtering. This correction is done during the integration process.

4.4 Integrated Experimental Indicial Response

The integral in Equation 3.8 has been evaluated numerically for the ramp-up motion given by \( K = \dot{\alpha}C/2U = \text{constant} \), using the experimental indicial responses. The airfoil begins at \( \alpha = 0^\circ \) and ramps up to \( \alpha > 30^\circ \). As shown in Figure 4.11, the motion was approximated by steps of \( \Delta \alpha = +1^\circ \). The experimental indicial responses were measured in intervals of at most five degrees alpha. The response at intermediate angles which were not measured directly were computed by interpolation between the test data. For example, the indicial response at, say, \( \alpha_0 = 22^\circ \) was not measured directly but was computed by interpolating between the responses measured at \( \alpha_0 = 20^\circ \) and \( \alpha_0 = 25^\circ \). The circulatory component of the normal force at some time \( t_N \) and corresponding angle of attack \( \alpha(t_N) = K t_N \) is given by:

\[
C_N(t) = \sum_{i=1}^{N} c_N \alpha \left[ \tau_{e_i} , \alpha_0(\tau_i) \right] \Delta \alpha_i
\]  

(4.2)

where \( \tau_{e_i} = t_N - \tau_i + \tau_{\phi} \), and \( \Delta \alpha_i = \pi/180 \) radians. The quantity, \( \tau_{\phi} \), is a correction for the time lag introduced by the filtering process and, in the present study, was found to be nearly equal to 1.0 in semichords. The value of the elapsed time \( \tau_{e_i} \) will depend on the pitching rate, and for constant pitch rate (and zero initial angle) may
Figure 4.11. Ramp Motion.
also be expressed as $\tau_{\text{ei}} = (\alpha(t_{N}) - \alpha_{o}(t_{\text{i}})) / K + \tau_{\text{f}}$. The calculations are relatively insensitive to the time lag for values in the range $1.0 < \tau_{\text{f}} < 2.0$ chords.

The uncertainty in Equation 4.1 has been estimated on the basis of the uncertainties in $C_{N\alpha}[\tau_{\text{ei}}, \alpha_{o}(t_{\text{i}})]$ and $\Delta \alpha_{\text{i}}$ and will be indicated on the subsequent figures by error bars. The uncertainty analysis indicates that even for relatively accurate angle of attack and force measurements needed to construct the indicial response, the error in the force calculation of Equation 4.1 can become large (~20%). This is primarily due to the fact that the step amplitudes are small.

4.5 Normal Force Results for $K \leq 0.01$

The computed normal force for $K = 0.01$ is shown in Figure 4.12 along with actual ramp motion data taken concurrently with the same airfoil used to measure the indicial responses. Forces computed using the linear response of Equation 3.6 are also shown. The agreement between the analysis and the ramp data is generally within the experimental uncertainty. At an angle near $16^\circ$ the integrated indicial response results flatten out while the actual ramp data continue to increase. We believe this to be a first order effect embedded within the responses at onset angles above $16^\circ$. Returning to Figure 4.10a it can be observed that for elapsed times less than about 2 chords, the responses at onset angles of $17.55^\circ$ and $20.77^\circ$ are small compared to the peak values which occur in the range of 4 to 6 chords. Furthermore, the magnitude of these responses for $(t - t_{\text{e}}) < 2$ are significantly less than those for lower onset angles of attack shown in Figure 4.10b. This deficiency in the response leads to the flattening of the integrated results near $16^\circ$ in Figures 4.12 through 4.16. In the actual ramp data, the effects of dynamic stall are evident near $25^\circ$ where the force data flattens out. The
Figure 4.12. Normal Force Results for Ramp Motion, $K = 0.01$. 
Figure 4.13. Normal Force Results for Ramp Motion, $K = 0.0075$. 
Figure 4.14. Normal Force Results for Ramp Motion, K = 0.015.
Figure 4.15. Normal Force Results for Ramp Motion, $K = 0.025$. 
Figure 4.16. Normal Force Results for Ramp Motion, $K = 0.05$. 
nonlinear indicial response model predicts the stall with reasonable accuracy. The linear model of Equation 3.6 is in general agreement with the ramp data up to approximately 15°, but does not predict the dynamic stall.

The static normal force curve for the present airfoil is also shown in Figures 4.12 through 4.16. In each case the dynamic ramp data is substantially larger than the static data which is a well known result. For the pitch rates considered here the dynamic loads are approximately twice as large as their static counterparts. The point to be made is that the first order responses predict with some accuracy the dynamic augmentation in the loading and this can be traced directly to the large magnitude indicial responses that occur in the range 16° < α₀ < 30°, two of which have been shown in Figure 4.10a.

The normal force results for K = 0.0075 are shown in Figure 4.13. In the case of K = 0.0075, the test was started with the airfoil at an initial angle of 8° to provide enough track so that dynamic stall would be reached. The pitching motion was initiated shortly after the motion began so that the steady state had not yet been reached at the inception of the pitch. This explains the discrepancy in the ramp data and the indicial response calculations at the lower angles of attack. On the other hand, by the time the airfoil had reached the dynamic stall angle it had moved approximately 9 chord lengths so that initial angle of attack effects should have diminished. Furthermore, the first order response model predicts the initial loading quite well for K = 0.01 as seen in Figure 4.12. In Figure 4.13 the dynamic stall is again predicted relatively well.
4.6 Normal Force Results for $K \geq 0.015$

Increasing the pitch rate to $K = 0.015$ results in data of Figure 4.14. The initial loading is predicted reasonably well. At angles beyond $20^\circ$ rate effects are becoming increasingly important and flow separation is significantly delayed by the unsteady motion. Flow separation for $K = 0.015$ occurs near $23^\circ$ [11]. The first order response model introduces flow separation effects too early in the motion and, as a result, underpredicts the loading during dynamic stall.

The results for higher values of pitch rate, $K$, are shown in Figure 4.15 for $K = 0.025$, and Figure 4.16 for $K = 0.05$. In both Figures, the effect of static stall at onset, which is naturally embedded within the first order indicial responses near $\alpha = 16^\circ$, is apparent. Furthermore, a comparison of Figures 4.13 and 4.17 shows that as the pitching rate increases from $K = 0.01$ to 0.025, the first order indicial response model does not accurately describe the loading for higher angles of attack. This is not too surprising when considering that at a pitch rate of $K = 0.025$ the flow does not separate until near $\alpha = 27^\circ$ [11], prior to which the flow is more or less attached. This means that for the duration of the ramp motion between the dynamic stall angle of attack and the static stall angle of attack, $16^\circ < \alpha < 27^\circ$, Equation 4.1 has been computed using indicial responses which do not contain the physically correct step onset conditions. Furthermore, as a consequence of the convolution process these errors will propagate throughout the calculation for all subsequent times. Rate effects on the indicial response become important for motions in which the angle of attack at which separation actually occurs differs significantly from the static stall angle. This situation occurs at high pitch rates.

The normal force results for $K = 0.05$ given in Figure 4.16 also show the
consequences of neglecting rate effects on the flow separation. For $K = 0.05$, leading edge separation occurs near $33^\circ$. Improvement can only be achieved by including higher order terms in the indicial response given by Equation 3.4. The present results are sufficiently promising to warrant such an effort.

Finally, it should be restated that the integrated normal force results given above do not fully contain the apparent mass effects. This deficiency may explain some of the discrepancies in the integrated forces and the ramp data. Without knowledge of the apparent mass contribution to the nonlinear indicial response a full comparison between the present results and the actual force data is not possible.

The first order results have also been integrated for the case of a sinusoidal motion and these results are given in Appendix A.
The present study has investigated the possibility of using experimentally measured first order nonlinear indicial responses to model a ramp motion. The conclusion can be made as the following:

* For large amplitude motions, a large amount of indicial response data may be required. The functional form of the indicial response changes radically over a range of angle of attack of $0^\circ \leq \alpha \leq 60^\circ$. As such, it is difficult to make use of regression techniques and/or curve fitting to simplify the evaluation of the convolution integral. The alternative is to numerically integrate the indicial response data, the accuracy of which depends on the amount of data available.

* Integrated first order indicial responses appear to model dynamic stall effects (for ramp up motion) reasonably well for $K \leq 0.01$. For pitch rates higher than 0.01, the use of only first order responses begins to give increasingly incorrect results. This is primarily due to static stall effects which are naturally embedded within the first order responses. However, the results do show promise and the investigation of second order nonlinear indicial responses needed to model higher pitch rates is warranted.

* The contribution of the apparent mass reaction to the nonlinear indicial response needs to be studied. Without knowledge of the nonlinear noncirculatory effects, full prediction of nonlinear aerodynamic loading using the present method is not possible.

* Direct measurements of nonlinear indicial responses in a tow tank facility, along with associated flow visualization, is a promising technique for gaining a fundamental understanding of nonlinear and unsteady flow mechanisms of practical interest.
6.1 Introduction

Reference[14] describes the results of an analytical study of the performance of a hypothetical dynamically "augmented lift vehicle"(ALV). To produce the dynamic augmentation in lift the ALV wings were uncoupled from the fuselage so that the wings could be mechanically pitched while, at the same time, the vehicle body maintained at a minimum drag trim. The ALV airfoil motion history was essentially a ramp up to a high angle of attack followed almost immediately by a ramp down to the original angle of attack. The study assumed that the wings of the ALV could be reduced in size(due to the dynamic increase in lift) and the design criteria was that the ALV wings should produce the same total lift as the wings of a conventional vehicle(CV) of similar design. The ALV and CV wings were taken to be NACA 0015 airfoils and the following relationship for wings of finite span was used to size the ALV wings:

\[
\frac{A_L C_{L_{\text{max}}} q S_{\text{ALV}}}{(1 + m_o /\pi \text{AR}_{\text{ALV}})} = \frac{0.8C_{L_{\text{max}}} q S_{\text{CV}}}{(1 + m_o /\pi \text{AR}_{\text{CV}})}
\]

(6.1)

where \( C_{L_{\text{max}}} \) is the maximum steady state section lift, \( q \) is the reference pressure, \( S \) is the planform area, \( m_o \) is section lift-curve slope and \( \text{AR} \) is the aspect ratio. The constant of 0.8 on the RHS is due to the fact that the CV used only 80% of the
maximum static lift. On the LHS the term $A_L$ is a parameter which quantifies the
dynamic augmentation in lift and is defined as the ratio of the time-average section lift
over the combined ramp up and down motions to the maximum steady
state section lift. Reference[14] presents NACA 0015 dynamic section lift data
for which the maximum time-average lift yielded $A_L = 1.114$. However, the
existence of higher values of $A_L$ was presupposed and the ALV mission analysis was
carried out using a value of $A_L = 1.5$. The results showed a near 20% increase in
straight-and-level range over the CV and improved range and maneuverability for a
terrain-following mission. This chapter presents NACA 0015 dynamic section lift data
at conditions for which time-average lift values of $A_L = 1.5$ and above were measured.

6.2 Experiment

The strain gauge signals were low-pass filtered at a cut off frequency of 170 hz and
the data sampling rate was 340 hz per signal. The axial load measurements were also
filtered using a digital notch filter tuned to the natural frequency of the towing structure
which was determined to be near 4 hz. Large vibrations did not occur in the normal
direction and, as such, the normal force signal was not digitally filtered. A more
detailed description of the data acquisition and reduction is given in chapter 2.

Steady state tests were conducted initially to determine the static section lift curve
for the present airfoil. These data indicated that the static stall angle of attack was 15°
and that the maximum lift coefficient prior to stall was 1.22. The unsteady motion
considered in this study is shown in Figure 6.1. The airfoil moves initially at a low
angle of attack, $\alpha_o$, and then undergoes a ramp up motion to a large angle of attack ,
$\alpha_{\text{max}}$, followed by a ramp down motion at the same pitch rate. Three pitch rates of
Figure 6.1. Airfoil Test Motion.
K = 0.05, 0.1, and 0.2 were considered where \( K = \dot{\alpha}C/2U_\infty \). Figure 6.1 shows present angle of attack data versus chords of travel for the cases where \( \alpha_0 = 0^\circ \) and \( \alpha_{\text{max}} = 40^\circ \). Other values of \( \alpha_0 \) and \( \alpha_{\text{max}} \) were considered as shown in Table 6.1. The three pitch rates were selected to investigate slightly higher pitching rates than those reported in Reference [14]. The idea being that increasing the pitch rate may increase the time-average lift.

6.3 Results and Conclusions

Shown in Figure 6.2a are representative lift coefficient results plotted as a function of angle of attack for a pitch rate of \( K = 0.1 \) and an initial angle of \( 0^\circ \). Arrows on the load history curves indicate the direction of pitch. Figure 6.2a gives some indication of the effect of \( \alpha_{\text{max}} \) on the relative contributions of the lift during ramp up and that during ramp down to the total (ramp up and down) time-average lift. For the ramp up to \( \alpha_{\text{max}} = 60^\circ \) the lift begins to decrease at an angle of attack near \( 45^\circ \). Dynamic stall inception data[11] indicate that at a pitch rate of \( K = 0.1 \) large scale flow separation occurs during ramp up at an angle of attack near \( 40^\circ \). The data of Figure 6.2a indicate that most of the dynamic augmentation in lift occurs before flow separation and increases only slightly beyond this point. Consequently, the time-average lift during pull up displays the same behavior. The lift force data during the ramp down motion indicate that for large values of \( \alpha_{\text{max}} \) the lift forces are significantly reduced below their ramp up counterparts. A significant reduction in the lift during ramp down as \( \alpha_{\text{max}} \) increased beyond the dynamic stall threshold was also observed for \( K = 0.05 \) and 0.2. The reduction of the lift during ramp down appears then to be due to the flow separation and the accompanying loss of suction pressure. The flow separation also appears to be related to the occurrence of negative lift coefficients during the ramp.
Figure 6.2. Representative Lift and Drag Coefficient Data for $K = 0.1$ and $\alpha_0 = 0^\circ$. 
down from large $\alpha_{\text{max}}$ as in the data for $\alpha_{\text{max}} = 60^\circ$ for angles of attack near 10° and below. Negative lift coefficients have also been reported in Reference [14]. As $\alpha_{\text{max}}$ is reduced into the range $30^\circ < \alpha_{\text{max}} < 40^\circ$, the lift forces during ramp down approach the ramp up lift values and as pointed out in Reference [14] this suggests that the flow remains, to some extent, "contour" following.

Unsteady airfoils may also experience a dynamic augmentation in drag due primarily to augmented normal force loading. Figure 6.2b indicates that for values of $\alpha_{\text{max}}$ near the dynamic stall onset angle the drag loading during ramp up and ramp down is nearly the same. This further suggests that the flow remains contour following. For $\alpha_{\text{max}}$ well beyond the stall limit the drag during ramp down is significantly less than that for ramp up.

The time-average lift has been defined in this study as:

$$C_{L_{\text{avg}}} = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} C_L(\xi) d\xi \quad (6.2)$$

where $\xi$ is time. The dynamic augmentation parameter, $A_L$, in Equation 6.1 is given by:

$$A_L = C_{L_{\text{avg}}} / C_{L_{\text{max}}} \quad (6.3)$$

where for the present airfoil $C_{L_{\text{max}}} = 1.22$. In Figure 6.3a, the time-average lift over the ramp up interval of the motion is plotted as a function of $\alpha_{\text{max}}$. Data at $K = 0.05$ and 0.1 for which the onset angle was $10^\circ$ are also shown. In Equation 6.2, $\tau_1$ is the time at which the ramp up begins and $\tau_2$ is the time when the airfoil reaches $\alpha_{\text{max}}$. At each pitch rate the average lift initially increases rapidly with $\alpha_{\text{max}}$ and then levels out.
Figure 6.3. Time-Average Lift Parameter Results.
at high $\alpha_{\text{max}}$. For the data at $K = 0.1$ and 0.2 there appears little to be gained by increasing $\alpha_{\text{max}}$ above 45°. Increasing the onset angle to 10° increases the time-average lift significantly, however, examination of the load histories such as those in Figure 6.2 indicates that this is mainly a numerical effect and not some further dynamic lift augmentation.

The time-average lift for the ramp down motion is shown in Figure 6.3b. Here, $\tau_1$ in Equation 6.2 is the time when the ramp down begins and $\tau_2$ is the time when the airfoil returns to $\alpha_o$. The lift initially increases with $\alpha_{\text{max}}$, achieves a maximum at an intermediate $\alpha_{\text{max}}$ and then decreases rapidly. Dynamic stall inception data[11] indicate that for ramp up motion at low Mach numbers a NACA 0015 airfoil will experience large-scale flow separation during ramp up motion at the approximate angles of attack indicated by the arrows in Figure 6.3b for each pitch rate. For values of $\alpha_{\text{max}}$ beyond the dynamic stall threshold the average lift during ramp down decreases rapidly. As shown below, this decline in sustained lift during ramp down limits the total time-average lift for maneuvers to large $\alpha_{\text{max}}$.

The effect of $\alpha_{\text{max}}$ on the total time-average lift is shown in Figure 6.3c. Insofar as the total average lift is concerned, there is little or no advantage in increasing $\alpha_{\text{max}}$ beyond the dynamic stall angle of attack. A comparison of Figures 6.3a, b, and c shows that the average lift during the ramp down motion is the dominant factor in the decline of the total average lift at high $\alpha_{\text{max}}$. In the analysis in Reference[14] a dynamic lift augmentation of $A_L = 1.5$ was assumed and this resulted in an ALV with a near 20% improvement in straight-and-level range over the conventional baseline vehicle. In Figure 6.3c, an augmentation of $A_L = 1.5$ was achieved at the highest pitch rate of $K = 0.2$ and $\alpha_o = 0^\circ$ and for both the cases of $K = 0.05$ and 0.1 for starting
Figure 6.4. Time-Average Drag Coefficient Results.
angles of $\alpha_o = 10^\circ$. The assumption of Reference[14] concerning the existence of augmentation parameters near 1.5 and above would appear then to have some justification.

Any benefits which might be gained from augmented lift must be weighed against the penalty levied by the increase in dynamic drag loading. Figure 6.4 shows the total time-average drag coefficients for each test motion. Clearly from Figure 6.4 decreasing $\alpha_{\text{max}}$ is advantageous from the standpoint of average drag reduction. From Figure 6.3c at the pitch rates of $K = 0.05$ and $0.1$ and starting angles of $10^\circ$, a value of $\alpha_{\text{max}} = 35^\circ$ results in nearly optimum lift augmentation with respective $A_L$ values of 1.5 and 1.63. The corresponding average drag coefficient is seen in Figure 6.4 to be near 0.7 which by some standards is large. Notice however that the average drag decreases rapidly with decreasing $\alpha_{\text{max}}$ and for $\alpha_{\text{max}} = 25^\circ$ and pitch rates of $K = 0.05$ and $0.1$, the average drag coefficient has dropped to 0.3 and 0.24 respectively. At the same time, the lift augmentation remains well above unity and from Figure 6.3c has values of 1.38 and 1.34 respectively. Thus for the rate of $K = 0.1$, decreasing $\alpha_{\text{max}}$ from $35^\circ$ to $25^\circ$ results in a drop in the average lift of 18%(though still maintaining significant lift augmentation) while the average drag decreases over 60%.

In the present study the pitch rate for the ramp up and ramp down motions was the same. For a stopping angle of $\alpha_{\text{max}} = 25^\circ$ the data of of Figure 6.3a indicate that the average lift during ramp up generally increases with pitch rate while in Figure 6.3b the average lift during ramp down decreases with pitch rate. There may then be some advantage in ramping up at high rate followed by ramp down at a lower rate. In the motions studied in Reference[14] the rate during pull up was lower the that for pitch down. Maintaining acceptable drag loading may be a limiting condition for defining
airfoil motions for the purpose of utilizing augmented lift.

Table 6.1. Test Conditions

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<th>$\alpha^*_{0}$</th>
<th>$\alpha^*_{\text{max}}$</th>
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<td>0</td>
<td>15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65</td>
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References

Chapter 7


Appendix A

A.1 Oscillatory results

This appendix contains the integrated normal force of NACA 0015 airfoil loading undergoing oscillatory sinusoidal motions for non-dimensional pitch rates $K$ of 0.02, 0.05, 0.1 and 0.2 computed from first order indicial responses and compared with the actual normal force of NACA 0012 airfoil loading.

The angle of attack for the oscillatory sinusoidal motion is defined by:

$$\alpha = \alpha_0 + A \sin(\omega t)$$

where

- $\alpha_0$ is the initial angle of attack.
- $A$ is the amplitude.

But the pitch rate in this case is

$$K = \omega c / 2U$$

where

- $c$ is the chord length.
- $U$ is the free stream velocity.
- $\omega$ is the frequency.

Then the angle of attack turns out to be

$$\alpha = \alpha_0 + A \sin((2UK/c)t)$$
$$= \alpha_0 + A \sin(2K t^*)$$

where
\( t^* = \frac{Ut}{c} \)

Note that for \( t^* = 0 \), \( \alpha = \alpha_0 + 0 \)

Shift so that at \( t^* = 0 \), \( \alpha = \alpha_0 - A \)

Then the angle of attack becomes

\[
\alpha = \alpha_0 + A \sin(2Kt^* - \pi/2)
\]

On the basis of above information, the first order indicial responses were integrated numerically for the pitch rates of 0.02, 0.05, 0.1 and 0.2. The amplitude and the initial angle of attack are 10° and 15° respectively. The integrated results are compared with the actual oscillatory sinusoidal motion data in the reference [13], which is carried out with a different airfoil and Reynold number of 2.5 x 10^6. Overlays of integrated results and the actual results are shown in Figures A-1 to A-4. Result from linear theory for the pitch rate of 0.1 is also plotted in the Figure A-3. However the results show promise, but full prediction is not possible without the actual oscillatory results from the same airfoil from which the indicial responses are obtained.
Figure A-1. Normal Force Results for Sinusoidal Motion, $K = 0.02$. 

[Graph showing normal force $C_N$ against angle of attack $\alpha$ for sinusoidal data and first order result with NACA 0012 and Reynolds number $2.5 \times 10^6$.]
Figure A-2. Normal Force Results for Sinusoidal Motion, K = 0.05.
Figure A-3. Normal Force Results for Sinusoidal Motion, $K = 0.1$. 

[NACA 0012, Reynold number $2.5 \times 10^6$]
Figure A-4. Normal Force Results for Sinusoidal Motion, $K = 0.2$. 
Appendix B

B.1 Error analysis

There are basically two ways to estimate the uncertainty in an experimental result; Worse-Case Combination and Constant Odds Combination [15,16]. They are illustrated by the following equations:

Worse-Case Combination:

\[ \partial R \approx \left| \frac{\partial R}{\partial x_1} \partial x_1 \right| + \left| \frac{\partial R}{\partial x_2} \partial x_2 \right| + \cdots + \left| \frac{\partial R}{\partial x_n} \partial x_n \right| \]

Constant Odds Combination:

\[ \partial R = \pm \left[ \left( \frac{\partial R}{\partial x_1} \right)^2 + \left( \frac{\partial R}{\partial x_2} \right)^2 + \cdots + \left( \frac{\partial R}{\partial x_n} \right)^2 \right]^{0.5} \]

where: \( \partial R \) is the uncertainty in the final result \( R \)

\( x_i \) is the uncertainty in each of the variable \( x_i \)

We use the latter method for our present analysis. The uncertainties of each input variable is at the odd of 20 to 1.

Calibration Constant:

There are many possible sources of error which give rise to the uncertainties in the calibration constants of strain gauges \( C_1 \) and \( C_2 \). Some of them are alignment of
applied load normal and tangential to the chord of airfoil, the location of centroid of the section of airfoil submerged under the center of strain gauges, friction, error in weights and accidental errors of instruments used. Accidental errors are those varying errors which cause repeated readings to differ without apparent reasons. It arises from instrument friction, amplification, time lag, personal errors etc.

Calibration constant in the normal direction:

\[ C_1 = \frac{f}{FL} \text{ (lbf/volt)} \]

where

Estimated distance the location of centroid from the center of strain gauges \((L)\) is 34 ± 0.5 in.

Estimated distance of applied load from the center of strain gauges \((l)\) is 13.5 ± 0.25 in.

Load applied \((f)\) is 10 ± 0.25 lbf.

The amplified output signal \((E)\) is 0.911 ± 0.005 V

Thus:

\[ \frac{\partial C_1}{C_1} = \pm \left[ \left( \frac{\partial E}{E} \right)^2 + \left( \frac{\partial L}{L} \right)^2 + \left( \frac{\partial f}{f} \right)^2 + \left( \frac{\partial l}{l} \right)^2 \right]^{0.5} = \pm 0.035 \]

Similarly for \(C_2\), we have

\[ \frac{\partial C_2}{C_2} = \pm \left[ \left( \frac{\partial E}{E} \right)^2 + \left( \frac{\partial L}{L} \right)^2 + \left( \frac{\partial f}{f} \right)^2 + \left( \frac{\partial l}{l} \right)^2 \right]^{0.5} = \pm 0.035 \]

As for calibration constant of potentiometer, besides the number of errors
mentioned above, additional errors include the mechanical play in speed reducer and bevel gear drive and alignment of airfoil. It is estimated that $E = 2.383 \pm 0.005$ V and $\alpha = 45^\circ \pm 0.05^\circ$.

$C_3 = \alpha / E$

$\partial C_3 / C_3 = \pm \left( \left( \partial E / E \right)^2 + \left( \partial \alpha / \alpha \right)^2 \right)^{0.5}$

Normal and Tangential forces:

The normal and tangential forces can first be non-dimensionalized as:

$F_n^* = F_n / \left( \left( \rho U^2 C \right) / 2 \right) = \left( C_1 E_1 \right) / \left( \left( \rho U^2 C \right) / 2 \right)$

$F_t^* = F_t / \left( \left( \rho U^2 C \right) / 2 \right) = \left( C_2 E_2 \right) / \left( \left( \rho U^2 C \right) / 2 \right)$

where it is estimated that $C = 6.00 \pm 0.001$ in., $U = 2.033 \pm 0.025$ ft/sec., and $\rho = 62.25 \pm 0.005$ lb/in$^3$, $E_1 = 0.512 \pm 0.005$ V, $E_2 = 0.251 \pm 0.025$ V.

$\partial F_n^* / F_n^* = \pm \left[ \left( \partial E_1 / E_1 \right)^2 \left( \partial C_1 / C_1 \right)^2 \left( \partial \rho / \rho \right)^2 \left( 2 \partial U / U \right)^2 \right]^{0.5}$

$= \pm 0.026$

$\partial F_t^* / F_t^* = \pm \left[ \left( \partial E_2 / E_2 \right)^2 \left( \partial C_2 / C_2 \right)^2 \left( \partial \rho / \rho \right)^2 \left( 2 \partial U / U \right)^2 \right]^{0.5}$

$= \pm 0.102$

Normal force Indicial response:

$\omega_{cn} \alpha / C_{n\alpha} = \pm \left[ \left( \omega_{cn} / \Delta C_n \right)^2 + \left( \omega_\alpha / \Delta \alpha \right)^2 \right]^{0.5}$

$= \pm 0.25$
where $\Delta C_n = 2\pi (\alpha_+ - \alpha_- )$

\[ = 0.40 \]

$\omega_{cn} = 0.033$, $\omega_{a} = 0.035$, $\Delta \alpha = 0.35$
Appendix C

Filtering Technique

The filtering technique used to eliminate the structural frequency from the force measurement is described below:

C.1 Filtering technique using Z-transform

Definition of notch filter[17] in S-plane is:

\[
\frac{(S^2+2*Drn*\omega_0+\omega_0^2)}{(S^2+2*Drd*\omega_0+\omega_0^2)}
\]

where \( \omega_0 \) the notch frequency (radian/sec.) and Drn and Drd are the numerator and denominator damping ratios respectively. Conversion to the Z-plane is:

\[
S = \frac{\omega_0}{(\tan(\omega_o*Dx/2))}\left[\frac{(z-1)}{(z+1)}\right]
\]

Note to get the valid results, the sample "time" must be close to the assumed 0.01 chords; i.e., 3000 points over 30 chords of travel.

The form of the Z-domain notch filter function turns out to be:

\[
\frac{[A_1(z^0-A_2z^{-1}+A_3z^{-2})]}{[(z^0-A_4z^{-1}+A_5z^{-2})]} \quad (C-1)
\]

As with the transfer function in the Laplace transform domain, this is output over input. Thus denoting \( Y(z) \) as the output, \( X(z) \) the input, \( G(z) \) the numerator, and \( H(z) \) the denominator:

\[
Y(z) =[G(z)/H(z)]*X(z)
\]
or

\[ H(z)Y(z) = G(z)X(z) \]

It turns out that:

\[ A_1z^{-n}Y(z) \text{ corresponds to } A_1Y_{-n}(t) \]

where \( Y_{-n}(t) = Y(t-n\Delta t) \)

So the equation C-1 leads to the time domain finite difference equation:

\[ Y(t) - A_4Y(t-\Delta t) + A_5Y(t-2\Delta t) = A_1(X(t) - A_2X(t-\Delta t) + A_3X(t-2\Delta t)) \]

Thus under the assumption that the last two values of both the filter input and output are available, along with the current value of input, the above equation can be solved for the output at current time.

From the above technique, a computer program is developed to eliminate the following structural frequencies in table C-1:

| Table C-1: |
|-----------------|-----------------|
| 2.620 radian/chord | 13.93 radian/chord |
| 5.455 radian/chord | 6.809 radian/chord |
| 7.350 radian/chord | 77.40 radian/chord |
| 151.7 radian/chord | 134.0 radian/chord |
| 10.45 radian/chord | 46.40 radian/chord |
C.2 Technique to design a nonrecursive filter using fourier series

A causal nonrecursive system designed by fourier series described by the difference equation[12]

\[ Y(n) = \sum_{k=0}^{k=L} b_k x(n-k) \]

where the \( b_k \) the filter coefficient and \( x \) is the data points.

A summary of the steps that need to be taken in the design of an approximation to an ideal bandstop(notch) filter:

a. Obtain the desired cutoff frequency \( \theta_c \).

b. The filter coefficients for a lowpass filter are given by

\[ h_{LP}(n) = (K/\pi n)\sin(n\theta_c) \quad n=0, \pm 1, \pm 2, \pm 3, \ldots \]

Substituting the numerical value of \( \theta_c \) to obtain the \( h_{LP}(n) \) values.

c. Truncate the coefficients with \( \pm I \) terms which yeilds the \((2I+1)\) filter coefficients

\[ h_{LP}(n) = (K/\pi n)\sin(n\theta_c) \quad n=0, \pm 1, \pm 2, \pm 3, \ldots, \pm I \]

where \( h(0) = K\theta_c/\pi \).

d. Shift \( h_{LP}(n) \) to right by \( I \) terms to make the filter causal giving

\[ h_{LP}(n) = [K/(\pi(n-I))]\sin([n-I]\theta_c) \quad n=0, 1, 2, 3, \ldots, 2I \]

e. With the above coefficients and the following frequency transformations, the coefficients of bandstop filter can be obtained.

For Highpass filter, the coefficients are

\[ h_{HP}(n) = (-1)^n h_{LP}(n) \quad n=0, \pm 1, \pm 2, \pm 3, \ldots, \pm I \]
For Bandpass filter, the coefficients are

\[ h_{BP}(n) = [2 \cos(n\theta_O)] h_{LP}(n) \quad n=0, \pm 1, \pm 2, \pm 3, \ldots, \pm I \]

\[ \theta_u - \theta_l = 2\theta_c, \quad \theta_O = (\theta_u + \theta_l)/2 \]

where \( \theta_O \) is the center of the passband.

\( \theta_u \) & \( \theta_l \) are the upper and lower band.

For Bandstop filter, the coefficients are

\[ h_{BS}(0) = K - h_{BP}(0) \]

\[ h_{BS}(n) = - h_{BP}(n), \quad n=0, \pm 1, \pm 2, \pm 3, \ldots, \pm I \]

Both the techniques were used to eliminate the structural frequencies. Listing of the

Computer Programs are in Appendix D.
Appendix D

Computer Programs

A listing of the data acquisition source code is given in the next page.
Using Mode 12 **
Base Address of DASCON-1 Board
Set PB as Output And PLOWER as Input
Base Address of DASH8 Board
Define The Current Segment of Memory For Dash8

Initial Angle in Degrees = ALPO
Amplication Factor == AMP
step Height , + For Step Up, - For Step Down

10 BASE1% = &H3000
20 OUT BASE1% + 11, &H89
30 BASE8% = &H340
40 SG8 = &H300
50 CLS : LOCATE 4, 5 : INPUT " Initial Angle in Degrees = ", ALPO

INPUT " step Height , + For Step Up, - For Step Down > ", DALP%
60 CLS : LOCATE 4, 5 : PRINT " Input 1 for k = 0.1 "
70 LOCATE 5, 5 : PRINT " Input 2 for k = 0.075 "
80 LOCATE 7, 5 : PRINT " Input 4 for k = 0.035 "
90 LOCATE 8, 5 : PRINT " Input 5 for k = 0.025 "
100 LOCATE 9, 5 : PRINT " Input 6 for k = 0.02 "
110 LOCATE 10, 5 : INPUT "Desired Pitch Rate , 1, 2, 3, 4, 5, 6 ", K%
120 CLS : LOCATE 4, 5 : INPUT " Step Onset Angle in Degrees ", ALPS
130 RATIO = 1 / 10
140 NT% = -( ALPS ) / (.9 * RATIO)
150 NTO% = -ALPO / (.9 * RATIO)
160 NTS%=-DALP%
170 ND% = -( NT% + NTO% +NTS% )
180 U = 2.033

Initialize The Stepper Motor

200 CALL MSTEP (MD%, 0%(0), STP#, FLAG%)
390 CALL MSTEP (MD%, 0%(0), STP#, FLAG%)
400 IF FLAG% < 0 THEN LOCATE 10, 5 : PRINT " ! ! ! ERROR # = " ; FLAG% ; " In Initialization of MSTEP ! ! ! " : STOP
410 IF FLAG% = 0 THEN LOCATE 12, 5 : PRINT " **** Initialization Completed ****
420 --- Using Mode 12 **

-- 初始化 DASH8 板 --

430 MOS% = 0
440 CALL DASH8 (MOS%, BASES%, FLAGS%)
450 CALL DASH8 (MD8%, BASE8%, FLAGS8%)
460 IF FLAGS8% < 0 THEN LOCATE 10, 5 : PRINT " ! ! ! ERROR # = " ; FLAGS8% ; " In Initialization of DASH8 ! ! ! " : STOP
470 IF FLAGS8% = 0 THEN LOCATE 12, 5 : PRINT " **** Initialization Completed ****
480 PTOT = 0
490 MD8% = 1 : LT%(0) = 1 : LT%(1) = 1
500 CALL DASH8 (MD8%, LT%(0), FLAGS8%)
510 MD8% = 4
520 FOR I% = 1 TO 100
530 CALL DASH8 (MD8%, POS8%, FLAGS8%)
540 PTOT = PTOT + POS8%
550 NEXT I%
560 PZERO = PTOT / 100
570 MD% = 10
580 Dk(0) = 0
590 Dk(1) = 15
600 CALL MSTEP (MD%, Dk(0), STP#, FLAG%)
610 MD% = 4
620 Dk(1) = 20
630 Dk(0) = 0
640 STP# = NTO%
650 CALL MSTEP (MD%, Dk(0), STP#, FLAG%)
660 IF (K% = 1 ) THEN X% = 1
670 IF (K% = 2 ) THEN X% = 1
680 IF (K% = 3 ) THEN X% = 1
690 IF (K% = 4 ) THEN X% = 10
700 IF (K% = 5 ) THEN X% = 10
710 IF (K% = 6 ) THEN X% = 10
720 IF (K% = 1 ) THEN RA = 96
730 IF (K% = 2 ) THEN RA = 128
740 IF (K% = 3 ) THEN RA = 193
750 IF (K% = 4 ) THEN RA = 50
760 IF (K% = 5 ) THEN RA = 70
770 IF (K% = 6 ) THEN RA = 88
780 CLS : LOCATE 4,5 : INPUT " *** Has the water calmed down ? (Y/N) *** ", R$
790 IF (R$ = "Y") GOTO 810
800 GOTO 780
810 NTOT = 0 : TTOT = 0
820 MD8% = 1 : LT%(0) = 2 : LT%(1) = 3
830 CALL DASH8 (MD8%, LT%(0), FLAG8%)
840 MD8% = 4
850 FOR 1% = 1 TO 100
860 CALL DASH8 (MD8%, NORM%, FLAG8%)
870 CALL DASH8 (MD8%, TANG%, FLAG8%)
880 NTOT = NTOT + NORM%
890 TTOT = TTOT + TANG%
900 NEXT 1%
910 NZERO = NTOT / 100
920 TZERO = TTOT / 100
930 MD8% = 1 : LT%(0) = 1 : LT%(1) = 3
940 CALL DASH8 (MD8%, LT%(0), FLAG8%)
950 MD8%=6
960 DIO%(0)=2
970 DIO%(1)=0
980 CALL DASH8(MD8%,DIO%(0),FLAG8%)
990 MD8% = 10
1000 DIO%(0) = 2
1010 DIO%(1) = 3
1020 CALL DASH8 (MD8%, DIO%(0), FLAG8%)
1030 MD8% = 8
1040 DIO%(0) = 9000
1050 DIO%(1) = 15000
1060 CALL DASH8 (MD8%, DIO%(0), FLAG8%)
1070 MD8% = 9
1080 TRAN%(1) = 1
1090 TRAN%(2) = 17998
1100 TRAN%(0) = VARPTR(CHECK%(0))
1110 CALL DASH8 (MD8%, TRAN%(0), FLAG8%)
1120 AC% = CHECK%(0)
1130 MD8% = 11
1140 DIO%(0) = 2
1150 DIO%(1) = 1962
1160 NP = NZERO * .00244
1170 TP = TZERO * .00244
1180 PP = PZERO * .00244
1190 'CLS : LOCATE 4,5 : PRINT " Initial Offsets : NOR, TAN,POS "
1200 PRINT : PRINT , NP; TP; PP
1210 LOCATE 10,5 : INPUT" *** Would you like to start the test? (Y/N) **", R$
1220 IF (R$ = "Y") THEN GOTO 1240
1230 GOTO 1210
1240 OUT BASE1% + 9, &H2
1250 DEF SEG = 0
1260 A = PEEK(&H46C) : B = PEEK(&H46D) : C = PEEK(&H46E)
1270 CALL DASH8(MD8%, D10%(0), FLAG8%)
1280 DEF SEG=0
1290 A77 = PEEK(&H46C) : B77= PEEK(&H46D) : C77= PEEK(&H46E)
1300 T55 = A + B * 256 + C * 65535!
1310 T66= A77+ B77* 256 + C77* 65535!
1320 F77=(T66-T55)/18.2
1330 IF(F77<1.48) GOTO 1280
1340 IF (NTP%=0) THEN 1510
1350 MD%=10: D8%(0)=0 : D8%(1)=X%
1360 CALL MSTEP(MD%, D8%(0), STP#, FLAG%)
1370 MD%=4: D8%(0)=0 : D8%(1)=RA : STP#=NTP%
1380 CALL MSTEP (MD%, D8%(0), STP#, FLAG%)
1390 IF (NTS%=0) THEN 1510
1400 MD%=8: D8%(0)=0
1410 CALL MSTEP(MD%, D8%(0), STP#, FLAG%)
1420 IF (FLAG% = 1) THEN GOTO 1410
1430 MD%=10: D8%(0)=0 : D8%(1)=1
1440 CALL MSTEP (MD%, D8%(0), STP#, FLAG%)
1450 IF (FLAG% = 0) THEN GOTO 1470
1460 GOTO 1440
1470 MD%=4: D8%(0)=0 : D8%(1)=20 : STP#=NTS%
1480 CALL MSTEP (MD%, D8%(0), STP#, FLAG%)
1490 IF (FLAG% = 0) THEN GOTO 1520
1500 GOTO 1480
1510 DEF SEG=0
1520 A2 = PEEK(&H46C) : B2 = PEEK(&H46D) : C2 = PEEK(&H46E)
1530 T1 = A + B * 256 + C * 65535!
1540 T2 = A2 + B2 * 256 + C2 * 65535!
1550 F1=(T2-T1)/18.2
1560 IF(F1<7.41) GOTO 1510
1570 DEF SEG = 0
1580 A1 = PEEK(&H46C) : B1 = PEEK(&H46D) : C1 = PEEK(&H46E)
1590 OUT BASE1% + 9, 0
1600 MD8% = 9
1610 LEV%=2
1620 CALL DASH8(MD8%, LEV%, FLAGS%)
1630 T1 = A + B * 256 + C * 65535!
1640 T2 = A1 + B1 * 256 + C1 * 65535!
1650 DIM ARRAY%(18000)
1660 MD8% = 9
1670 TRAN%(1) =18000
1680 TRAN%(2) = 0
1690 TRAN%(0)=VARPTR(ARRAY%(0))
1700 CALL DASH8 (MD8%, TRAN%(0), FLAG8%)
1710 DT= ( (T2-T1) / (16.2*2999) ) * U / .5
1720 F=(T2-T1)/18.2
1730 CN = 3.295 * 32.174 / AMP
1740 CT = 1.25 * 32.174 / AMP
1750 CP = 48.0128
1760 DIM WT(10), DENSITY(10), KEVIS(10)
WT(2)=15: DENSITY(2)=999.1: KEVIS(2)=1.14E-06 'DENSITY, RG/CU M
1780 WT(3)=20: DENSITY(3)=998.2: KEVIS(3)=1.004E-06 'KINEMATIC VISCOSITY,
1790 WT(4)=25: DENSITY(4)=997.1: KEVIS(4)=8.93E-07 'M SQ/SEC
1800 WT(6)=30: DENSITY(5)=9957: KEVIS(5)=8.009E-07
1810 WT(6)=35: DENSITY(6)=994.1: KEVIS(6)=7.237E-07
1820 WT(7)=40: DENSITY(7)=992.2: KEVIS(7)=6.568E-07
1830 WT(8)=45: DENSITY(8)=990.3: KEVIS(8)=6.018E-07
1840 WT(9)=50: DENSITY(9)=988.1: KEVIS(9)=5.534E-07
1850 WT(10)=55: DENSITY(10)=985.7: KEVIS(10)=5.113E-07
1860 CLS: LOCATE 5,5: INPUT "ENTER THE WATER TEMPERATURE (DEG. C) = ", TEMP
1870 IF (TEMP<WT(1)) OR (TEMP>WT(10)) GOTO 1960
1880 FOR M% = 1 TO 10
1890 IF (TEMP = WT(M%)) THEN RO = DENSITY(M%)*.06243: VIS = KEVIS(M%)*10.765: GO
1900 TO 1880
1910 RO = DENSITY(M%-1) + (TEMP-WT(M%-1))*(DENSITY(M%)-DENSITY(M%-1)) / (WT(M%)-
1920 WT(M%-1))
1930 VIS = KEVIS(M%-1) + (TEMP-WT(M%-1))*(KEVIS(M%)-KEVIS(M%-1)) / (WT(M%)-WT(M%
1940 -1))
1950 VIS = VIS*10.765
1960 GOTO 1880
1970 CLS: LOCATE 5,5: INPUT "ENTER THE WATER TEMPERATURE (DEG. C) = ", TEMP
1980 IF (TEMP<WT(1) OR (TEMP>WT(10)) GOTO 1960
1990 FOR M% = 1 TO 10
2000 IF (TEMP = WT(M%)) THEN RO = DENSITY(M%)*.06243: VIS = KEVIS(M%)*10.765: GO
2010 TO 1990
2020 RO=RO*.06243
2030 VIS = VIS*10.765
2040 OPEN FILES FOR OUTPUT AS #2
2050 FOR J% = 1 TO 3000
2060 JP% = (J%-1) * 3
2070 JN% = JP% + 1
2080 JT% = JP% + 2
2090 AOA = CP*(ARRAY%(JP%)- PZERO)
2100 NORMAL = CN / NOND * .00244
2110 TANGENT = CT / NOND * .00244
2120 TSTAR = DT * (J%-1)
2130 'PRINT USING" ### ### ### *** *** *** *** *** " ; J% ; TSTAR ; AOA ; NORMAL ; TANGENT
2140 PRINT #2, J%; TSTAR; AOA; NORMAL; TANGENT
2150 NEXT J%
2160 CLOSE #2
2170 CLS: PRINT " **** PRINT THE GENERAL RESULTS ON SCREEN ****
2180 PRINT " REYNOLD NUMBER, RE = " , RE
2190 PRINT " VELOCITY, ACTU = " , U
2200 PRINT " WATER TEMPERATURE, TEMP = " , TEMP
2210 INPUT "SEE NEXT PAGE (Y/N) ? ", RR$ 2220 IF (RR$ = "Y") GOTO 2240
2230 GOTO 2340
2240 CLS : PRINT "Return Airfoil To Zero Degrees (Y/N) ? " , R$ 2250 IF (R$ = "N") GOTO 2340
2260 MD% = 10
2270 DS%(0) = 0
2280 DS%(1) = 15
2290 CALL MSTEP (MD%, DS%(0), STP#, FLAG%)
2300 MD% = 4
2310 DS%(1)=20
2320 STP# = NTOT%
2330 CALL MSTEP (MD%, D%(O), STP%, FLAG%)  
2340 END
A listing of the Filtering source code using z-transform is given in the next page.
10 INPUT "name of the data file? ", FILES
20 OPEN "i", #1, FILES
30 INPUT "name of the filtered file? ", FILE1$
40 OPEN FILE1$ FOR OUTPUT AS #4
50 '------ Filter coeff.-------------
60 F1 = .9973897
70 F2 = 1.999308
80 F3 = .9999948
90 F4 = 1.99409
100 F5 = .9947743
110 A1 = .9632738
120 A2 = 1.996949
130 A3 = .9999236
140 A4 = 1.923608
150 A5 = .9264741
160 B1 = .9511583
170 B2 = 1.994498
180 B3 = .9998972
190 B4 = 1.897083
200 B5 = .9022189
210 C1 = .52715
220 C2 = 1.074442
230 C3 = .9982042
240 C4 = .0566392
250 C5 = .0533534
260 AA1 = .9320198
270 AA2 = 1.988944
280 AA3 = .999854
290 AA4 = 1.853736
300 AA5 = .8693035
310 BB1 = .9123009
320 BB2 = 1.978704
330 BB3 = .998058
340 BB4 = 1.805173
350 BB5 = .8228301
360 CC1 = .9545844
370 CC2 = 1.99527
380 CC3 = .9999048
390 CC4 = 1.904654
400 CC5 = .9097799
410 D1 = .6142216
420 D2 = 1.429342
430 D3 = .9987426
440 D4 = .8779324
450 D5 = .2276708
460 DD1 = .5334775
470 DD2 = .4571051
480 DD3 = .9982492
490 DD4 = .2438552
500 DD5 = .0660208
510 EE1 = .7131594
520 EE2 = 1.787819
530 EE3 = .9991948
540 EE4 = .275
550 EE5 = .4257445
560 PASS1 = .0009431
570 PASS2 = 1.911271
580 PASS3 = .9150438
590 JJ = 0
600 FOR I% = 1 TO 3000
ClO INPUT#1,X,ALP,CL,CD,CN,CA
620 IF (I1>1) GOTO 820
630 XI1=CA
640 XII1=CA
650 Y003=CA
660 Y03=CA
670 Y3=CA
680 CAA3=CA
690 CA3=CA
700 CNI3=CN
710 CNII3=CN
720 CNO03=CN
730 CNO03=CN
740 CN03=CN
750 CN3=CN
760 CDN3=CN
770 CDND3=CN
780 CNEE3=CN
790 ALPI3=ALP
800 ALPO3=ALP
810 GOTO 1770
820 IF (I1>2) GOTO 1200
830 XI2=XI1
840 XI3=CA
850 XII2=XII1
860 XII3=CA
870 Y002=Y003
880 Y003=CA
890 Y02=Y03
900 Y03=CA
910 Y2=Y3
920 Y3=CA
930 CAA2=CAA3
940 CAA3=CA
950 CA2=CA3
960 CA3=CA
970 CNII2=CNI3
980 CNI3=CN
990 CNII2=CNII3
1000 CNII3=CN
1010 CNO002=CNO003
1020 CNO003=CN
1030 CNO02=CNO03
1040 CNO03=CN
1050 CN02=CNO3
1060 CN3=CN
1070 CN2=CN3
1080 CN3=CN
1090 CDN2=CDN3
1100 CDN3=CN
1110 CDN2=CDN3
1120 CDN3=CN
1130 CNEE2=CNEE3
1140 CNEE3=CN
1150 ALPI2=ALPI3
1160 ALPI3=ALP
1170 ALPO2=ALPO3
1180 ALPO3=ALP
1190 GOTO 1770
1200 XI1=XI2
XI2 = XI3
XI3 = CA
XI1 = XI2
XI2 = XI3
XI3 = F1*(XI3-F2*XI2+F3*XI1)+F4*XI12-F5*XI1
Y001 = Y002
Y002 = Y003
Y003 = BB1*(XI3-BB2*XI2+BB3*XI1)+BB4*Y002-BB5*Y001
Y01 = Y02
Y02 = Y03
Y03 = A1*(Y003-A2*Y002+A3*Y001)+A4*Y02-A5*Y01
Y1 = Y2
Y2 = Y3
Y3 = B1*(Y03-B2*Y02+B3*Y01)+B4*Y2-B5*Y1
CAA1 = CAA2
CAA2 = CAA3
CAA3 = AA1*(Y3-AA2*Y2+AA3*Y1)+AA4*CAA2-AA5*CAA1
CA1 = CA2
CA2 = CA3
CA3 = D1*(CAA3-D2*CAA2+D3*CAA1)+D4*CA2-D5*CA1
CN11 = CN12
CN12 = CN13
CN13 = CN
CN14 = CN15
CN15 = CN16
CN16 = CN17
CN17 = CN18
CN18 = CN19
CN19 = CN20
CN20 = CN21
CN21 = CN22
CN22 = CN23
CN23 = B1*(CN03-B2*CN02+B3*CN01) + B4*CN2 - B5*CN1
CND1 = CND2
CND2 = CND3
CND3 = C1*(CN3-C2*CN2+C3*CN1) + C4*CND2 - C5*CND1
CND1 = CND2
CND2 = CND3
CND3 = DD1*(CND3-DD2*CND2+DD3*CND1) + DD4*CND2-DD5*CND1
CNEE1 = CNEE2
CNEE2 = CNEE3
CNEE3 = EE1*(CND3-EE2*CND2+EE3*CND1)+ EE4*CNEE2 -EE5*CNEE1
ALP11 = ALP12
ALP12 = ALP13
ALP13 = ALP
ALP1 = ALP02
ALP2 = ALP03
ALP3 = PASS1*(ALP13+2*ALP12+ALP11) + PASS2*ALP02 - PASS3*ALP01
JJ = JT + 1
IF(JJ = 1) THEN PRINT#4,X;ALP03;CNEE3;CA3;CN;CA
IF(JJ=3) THEN JJ=0
NEXT IT
CLOSE #1
CLOSE #4
END
A listing of the Filtering source code using fourier series technique is given in the next page.
10 INPUT "NAME OF THE DATA FILE?? ", FILE$
20 INPUT "ENTER THE NUMBER OF DATA POINTS?? ", N10%
30 DIM HL(101)
40 DIM HBP(101)
50 DIM HBS(101)
60 DIM AA(3000)
70 DIM YY(3000)
80 OPEN "I",#1,FILES
90 INPUT "LOWER FREQ. OF BAND ?? ", Y1
100 INPUT "UPPER FREQ. OF BAND ?? ", Y2
110 'INPUT "SAMPLING FREQUENCY ?? ", Y3
120 LET Y3=203
130 PI=3.1415927#
140 THL=(21*PI*Y1)/Y3
150 THU=(21*PI*Y2)/Y3
160 THC=(THU-THL)/2!
170 THO=(THU+THL)/2!
180 'CALCULATION OF FILTER COEFF.
190 FOR N%=O TO 100
200 IF (N%=50) THEN ijL(N%)=THC/PI
210 IF (N%=50) THEN HBP(N%)=(21*COS«N%-50)*THO»*HL(N%)
220 IF (N%=50) THEN HBS(N%)=1-HBP(N%)
230 IF (N%=50) THEN GOTO 270
240 HL(N%)=(1/(PI*(N%-50)))*SIN((N%-50)*THC)
250 HBP(N%)=(21*COS((N%-50)))*SIN((N%-50)*THC)
260 HBS(N%)=-HBP(N%)
270 NEXT N%
280 FOR J%=1 TO N10%
290 INPUT#1,B,C,X1,A
300 'XX(J%)=X1
310 AA(J%)=A
320 NEXT J%
330 FOR K%=1 TO N10%
340 Y=0:X=0!
350 FOR L%=O TO 50
360 N5=50+L%
370 N4=50-L%
380 P%=K%+L%
390 Q%=K%-L%
400 IF (P%>N10%) GOTO 430
410 IF (L%=0) THEN Y=Y+AA(P%)*HBS(N5)
420 IF (Q%<1) GOTO 440
430 X=X+AA(Q%)*HBS(N4)
440 NEXT L%
450 Z=Y+X
460 YY(K%)=Z
470 NEXT K%
480 CLOSE#1
490 OPEN "I",#2,FILES
500 CLS
510 INPUT "NAME OF THE FILTERED FILE?? ", FILE1$
520 OPEN FILE1$ FOR OUTPUT AS #4
530 FOR N3%=1 TO N10%
540 INPUT#2,B,C,X1,A
550 PRINT#4,B;C;X1;YY(N3%)
560 NEXT N3%
570 CLOSE#2
580 CLOSE #4
590 END