STRESS ANALYSIS OF SINGLE LAP ADHESIVE BONDED JOINTS

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Ohio University

In partial fulfillment
of the Requirements for the Degree of
Master of Science

by
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NOMENCLATURE

\[
\begin{align*}
\sigma & = \text{normal 'peel' stress, psi} \\
\tau & = \text{shear stress, psi} \\
L & = \text{length of overlap, inches} \\
h & = \text{adhesive thickness, inches} \\
t & = \text{adherend thickness, inches} \\
E_{\text{as}} & = \text{adhesive time independent elastic modulus, psi} \\
E_{\text{ad}} & = \text{adherend elastic modulus, psi} \\
\nu & = \text{Poisson's ratio} \\
T & = \text{time of loading, hours} \\
E_{\text{v}} & = \text{adhesive viscoelastic modulus, psi} \\
E_{\text{t}} & = \text{adhesive time dependent elastic modulus, psi}
\end{align*}
\]
Chapter I
INTRODUCTION

1.1 ADHESIVE BONDED JOINTS:
The design of adhesive bonded joints involves selecting the proper geometry, choosing the type of adhesive and substrates to be employed, surface preparation methods and considering the ease by which it can be tooled, fabricated and mass produced. The main criteria for a particular application are the type of structure, mechanical strength factors, service requirements and fabrication costs. The average technician should be able to identify the various types of joining and fastening techniques and have a basic knowledge of their function and the primary reason for a particular joint design selection.

When joining thin-gauge substrates or plastics, adhesives show exceptional performances and have the following advantages:

1. They allow fabrication of smooth parts and do not break through or deform the surface of the assembly - which is important aerodynamically for exterior airframe, missile and space structures.

2. Adhesive bonding minimizes stress concentrations that commonly occur in mechanical fasteners such as
screws, rivets, bolts and spot welds due to the whole joint being bonded rather than a small area. This also provides a more uniform stress distribution resulting in better fatigue resistance under vibrational loads.

3. They can join dissimilar metals and prevent corrosion problems caused by their electromotive series relationship as the adhesive isolates the two materials.

4. Adhesives permit easier fabrication of complex and unique contoured surfaces and can also act as a seal against many liquids and gases.

There is a need for a complete selection process to be developed to select an adhesive bonded joint for a particular application. At the present time, there is a design problem as engineers used to designing with conventional fasteners are now faced with adhesives and are not fully aware of their potential and the general design concepts of adhesive-joint stresses and behaviour under service conditions. All stress analysis work done till now involved extensive experimental methods with limited analytical treatment. Analytical stress analysis of bonded joints has been restricted due to the complex adherend-adhesive interface and has neglected the effect of various joint parameters.
1.2 LAP JOINT ANALYSIS INVESTIGATIONS

The selection of the type of joint configuration is largely dependent on the end use. Lap joints and their variations are the most commonly used and provide good load resistance in metal-to-metal joints. The type of lap joints used include simple offset, tapered edge, joggle, double butt and double scarf joints as shown in Figure 1A. The single lap joint has one bondline and the double lap joint has two bondlines. The double lap joint under tension is relatively simpler to analyse as there is no bending incurred during deformation. An exhaustive study of the double lap joint under a tensile loading has been done by Jayanto Sen(1) (1) in 1980.

The first analytical model of the more common and practical single lap joint was developed in 1944 by Goland and Reissner (2), who considered two modes of behaviour to be possible in single lap analysis. One was based on the assumption that most of the deformation occurred in the adhesive layer, as in metal to metal bonds; and the other case treated the adhesive as relatively inflexible such as in wood to wood bonds. The equations derived assumed plane strain by considering the bonded joint to be very 'wide' and under cylindrical bending and used the theory of linear elastic plates.

(1) Numbers in parenthesis correspond to the list of references in the Bibliography.
Butt - unsatisfactory

Beveled lap - good, but impractical, difficult mate

Joggle lap - good, practical

Double strap - good, desirable

Good - requires machining

Double lap - good, sometimes desirable

Lap - good, practical

Scarf - good, requires machining

Strap - fair, but seldom used

Recessed double strap - good, expensive to machine

Beveled strap - good, difficult to produce

Very good - difficult to mate, requires machining

FIG. 1A  TYPES OF JOINTS
To make the analytical model possible, Goland and Reissner neglected the effect of many parameters such as the thickness of the adhesive and the stresses due to different loading conditions.

A few years later, in 1953, Cornell (3) analysed the joint by simplifying Goland and Reissner's model and considered the adhesive as a system of tension and shear springs. This neglected the stresses parallel to the joint and the effects of Poisson's ratio in the adhesive. The complexity of the model and the assumptions necessary to provide analytical solutions, prompted investigators to turn to experimental techniques (4-6) to study the stress distribution in single lap joints and to predict their behaviour under service conditions. Bikerman (7), in 1961, was the first to publish data comparing and contrasting all experimental and theoretical work done till then and his book has a reliable treatment of the science of single lap adhesive joints.

Renton and Vinson (8), have presented a detailed analysis of single lap joints but assumed the adhesive to be perfectly elastic and with no variation of the shear stress through the adhesive thickness. It was in 1978, 34 years after the classical model proposed by Goland and Reissner, that another pair of investigators, Ojalvo and Eidnoff (9) finally take into account the thickness of the adhesive in their mathematical model. They used a similar flat plate theory and they predicted a more practical stress distribution in the joint.
1.3 **AIM OF PRESENT STUDY**

Previous analyses were restricted to one or more of the following assumptions:

1. The adhesive is isotropic and linearly elastic.
2. Negligible Poisson's ratio.
3. No variations of stress along the adhesive thickness.

Limited work has been done using numerical techniques to analyse single lap joints and application of the finite element method has been restricted to two-dimensional tensile loading or plane stress models (10 and 11). To the author's knowledge, no work has been done to attack the problem using three-dimensional models and to consider the effect of various loadings. The plane stress and plane strain models did not take into account loading along the width of the joint, bending loads, and uniformly varying loads. The adhesives commonly used are either epoxies, cyanoacrylates, polyimides, or are either epoxy based amides, nitriles and phenols. These, in all previous work were assumed to be perfectly elastic, while they are actually viscoelastic. The aim of this study is to investigate the behaviour of single lap joints under different loadings and service conditions. A three-dimensional finite element model was developed to be analysed using the general purpose finite element program, MSC/NASTRAN(2) so that the loading and its effects could be

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(2) **MSC/NASTRAN** is a proprietary version of NASTRAN, the NASA developed program for structural analysis. NASTRAN is a registered trademark belonging to NASA, MSC/NASTRAN is marketed by McNeal Schwendler Corp..
analysed in three dimensions. The assumptions of plane stress analysis used by previous investigators was also verified by using a two-dimensional model. The viscoelastic nature of the adhesive and the dependence of the strength of the joint to the time and temperature of loading was also considered.

The adhesive joint was subjected to the following conditions:

1. Simple tensile lap shear loading (2-d and 3-d models).
2. Bending loads (3-d model).
3. Uniformly varying tensile load (3-d model).
4. Uniform load along the width of the joint (3-d model).
5. Effect of time of the loading and the temperature by considering the adhesive as a viscoelastic solid (2-d model).

An experimental study was also conducted to examine the location and nature of the failure of the joint and correlate it to the stress distribution obtained by the finite element model.
Chapter II
EXPERIMENTAL INVESTIGATION

2.1 INTRODUCTION
An experimental study of the single-lap joint under a tensile loading was conducted to study the mode of failure of the joint. This was done to investigate the effect of the region of high stress concentration and to correlate it to the results obtained using the finite element method.

To test the joint as a lap shear specimen, dimensions similar to the American Society for Testing and Materials standards were used - ASTM D1002, 1979 (12). Figure 1B shows the joint geometry and dimensions. Samples were made from 6061-T6 Aluminium strips 0.1875 inches thick and required lengths were cut and milled to the right dimensions. Care was taken not to overheat or mechanically damage the joint specimens while machining. The test was conducted for overlaps of different lengths of 0.5 inches, 0.75 inches, 1.0 inches, and 1.5 inches.
2.2 **PREPARATION OF THE SPECIMEN**

Each joint strip was pre-treated for adequate metal-metal bonding. It was necessary to clean the joint specimen surfaces of dirt and impurities which would otherwise lower the bond strength. The machined samples were first washed in tapwater and dried to remove metal chips and dirt particles sticking to the specimens. Light grade 600-A sandpaper was used to clean the bond area and remove slight burrs and eliminate unevenness in the sample. The bond area was made as flat as possible for good adhesion and this was followed by solvent degreasing to remove grease and oily particles from the surface. The reagent used for degreasing was Acetone and this process was repeated to ensure removal of all surface impurities. A general purpose commercial brand of a two-part epoxy system was used as the adhesive which included the application of the resin and hardener in equal quantities. The well-mixed adhesive was applied to the bond area and light pressure applied to form the joint. The joint samples were cured at room temperature for eight hours.

2.3 **TESTING**

After the test joints were bonded in the required geometry, they were tested to failure using a tensile load. The testing machine used was a TINIUS OLSEN machine and a tensile load was applied to the joint so that the joint behaved as a lap-shear specimen. The highest load at the point of fai-
lure was recorded to give the failure load. All loading is not in shear in the plane of the bond as the joint is anti-symmetric, and application of the tensile load causes a bending moment that tends to deform the test part. As a result, stresses are concentrated at the edges of the bond area. This was verified using a finite element model, as discussed in the following chapters. The experimental investigation was not meant to be rigorous and extensive to obtain the stress distribution for various lap geometries and material properties. The specimens and testing procedures did not consider laps of various adhesive and adherend thickness, geometry, and various types of adhesives. Instead, the aim of this experimental investigation was to study the origin and nature of failure in lap shear specimens and relate them to the results of the stress distribution obtained using the finite element model. The result of this experimental investigation is included in Section 4.5.
3.1 **Introduction**

Analytical solutions by earlier investigators were limited due to the non-linear relationship between the lap joint properties and the stress distribution. The closed form stress field solutions did not completely conform to practical solutions due to the complex adherend-adhesive interface and led to simplified and idealized models of the real joint.

The non-linear behaviour of the joint under loading makes a complete analytical solution very complex and a numerical technique, the finite element method, was adopted for analysing the joint.

The finite element model allows the structural continuum to be replaced by an equivalent system of discrete elements of finite dimensions. Two and Three-dimensional bodies such as plates, shells, dams, etc. may be thought of as internally statically indeterminate structures for which the degree of indeterminacy is infinite.

The finite element model of the single lap joint consists of a number of elements obtained by means of fictitious cuts through the original structure with each adjoining element
connected at common points called 'nodes'. The boundary and element constraints are then set and the model subjected to the various loading conditions. The finite element program used is MSC/NASTRAN.

3.2 **OVERVIEW OF MSC/NASTRAN**

MSC/NASTRAN is a system that will create and manipulate a database to solve problems using structural analysis (13). The system, shown in Figure 2 is composed of a database, an executive system, and modules that perform modelling, database manipulation, and program input and output.
FIG. 2

MODELING MODULES

FUNCTIONAL MODULES

I/O MODULES

EXECUTIVE SYSTEM

DATA BASE
Stiffness, Mass Matrices, etc.

MSG/NASTRAN SCHEME
The data base can be created by the input-output modules, or it can be created by the modelling modules (11). The data base is then manipulated by the functional modules (addition, subtraction, equation solving, etc.) to obtain a solution data set that is then selectively displayed by the user. The whole process is controlled by the NASTRAN executive, which is, in turn under user control by means of the MSC/NASTRAN language that is called DMAP (Direct Matrix Abstraction Programming). The MSC/NASTRAN executive is always controlled by a sequence of DMAP statements, but sets of pre-coded DMAP sequences, called 'rigid formats', have been included in the program. The user can then manipulate the entire set of DMAP instructions associated with a particular rigid format by using a single directive in the MSC/NASTRAN data deck.

3.3 **MSC/NASTRAN DATA DECK**

1. Defining the physical problem or the system of equations to be solved. This is called the Bulk Data Deck.

2. Providing user control over input-output, called the Case Control Deck.

3. Providing user control over the executive functions, called the Executive Control Deck.

The input data for the finite element analysis program consists of the geometric idealization, the material proper-
ties, and the loading and boundary conditions. The major problem in using NASTRAN, and a large portion of the input, is in the geometric representation of the joint by a suitable mesh. This data preparation stage is extremely tedious and time consuming. There are several types and families of elements offered by NASTRAN and for this application, the isoparametric family of elements was used.

The study included the use of two and three-dimensional models. The two dimensional model was used to verify existing solutions assuming a plane stress distribution in the joint and to study the viscoelastic behaviour of the adhesive at various temperatures and durations of loading. The three-dimensional model was used to verify the accuracy of the assumption used for two dimensional analysis and also to consider loads applied to the joint along its width.
3.4 THE TWO DIMENSIONAL MODEL

Figure 3 shows the single lap joint under consideration. The joint is antisymmetric and consists of a single adhesive bondline. For analysis in two dimensions, a plane stress model is assumed with no movement of the joint in the X-direction along the width.
3.4.1 Choice of element axes and dimensions

MSC/NASTRAN offers a variety of two-dimensional elements depending on the type of application and element geometry, such as triangles and quadrilaterals, with and without mid-side nodes. The major element chosen is the 'isoparametric' element CQUAD8 which has four nodes to define the corners of the quadrilateral and four midside nodes. The isoparametric elements are elements which have their shape functions describing the element coordinates identical to those used for prescribing the variation of the function in the element.

The finite element model of the joint is designed to yield a good approximation to the stress distribution in the joint. From the lap geometry and the results of previous investigators, the critical section of the joint is known to be at or near the ends of the overlap where the magnitudes of the stresses and their gradients are high. In contrast, over the middle two thirds of the overlap, the stress distribution is approximately uniform. The critical regions of the joint are at the adherend-adhesive interfaces; and the joint is therefore divided into a mesh of elements of different sizes with smaller elements in the regions of high stress gradients. To change from a coarse mesh to a finer mesh towards the overlap region, triangular CTRIA6 elements were used which have three nodes to define the corners and three midside nodes. This transition region was necessary
to include more elements of smaller size in the region of high stress concentration which are at the ends of the overlap and the adhesive interface due to material discontinuity.

The model has four elements across the thickness of each adherend and two across the thickness of the adhesive, making a total of ten elements across the thickness of the whole joint. The axes are chosen conforming to the element geometries and the origin chosen such that all grid points have positive coordinates as shown in Figure 4.
3.4.2 **Number of elements and grid points**

The single lap joint in two dimensions was analysed under a tensile load. The adhesive was considered as perfectly elastic and also as a viscoelastic solid and the viscoelastic model was subjected to various temperatures and durations of loading. To determine the effect of varying the lap geometry and material properties, three basic models in two dimensions were generated, keeping element dimensions and mesh geometry constant so as to be able to compare the results. Overlap lengths 'L' of 0.5 inches, 1.0 inch and 1.5 inches were modelled as shown in the following table:

<table>
<thead>
<tr>
<th>Lap Length</th>
<th>No. of Elements</th>
<th>No. of Gridpoints</th>
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<tbody>
<tr>
<td>L=0.5</td>
<td>172</td>
<td>218</td>
</tr>
<tr>
<td>L=1.0</td>
<td>212</td>
<td>273</td>
</tr>
<tr>
<td>L=1.5</td>
<td>262</td>
<td>328</td>
</tr>
</tbody>
</table>

Table 1.
The smallest dimension was along the Y-axis, the thickness of the adhesive being 0.002 inches resulting in a maximum aspect ratio of 50. It was not possible to reduce the length in the Y-direction or to change element thickness as at least two elements were required to model the thickness of the adhesive to obtain the variation of stress across the thickness. To maintain low aspect ratios of 1, 5, 10 and 50, element lengths of nearly equal magnitude were chosen along the X-direction. The model was analysed using various aspect ratios and it was noticed that the results did not vary appreciably and nearly twice the amount of computer time was necessary. An optimum trade-off between the accuracy obtained and the computer time taken to process the data was considered and an element thickness of 0.0016 inches along the adhesive was maintained as the smallest dimension for a total adhesive thickness of 0.0032 inches.

3.4.3 Preparation of input and analysis of the results
The data preparation stage of element connectivities and location of grid points in space is time consuming, often repetitive and subject to considerable errors since hundreds of program statements describing the joint geometry are required. The joint is geometrically regular and can be considered to be made of quadrilaterals. It was desirable to write a program to generate a mesh for the individual quadrilaterals. A simple grid connectivity processor program
was written to construct a mesh for the joint and produce an output which would conform to the format of MSC/NASTRAN input cards. Appendix A.1 gives a listing of the program.

Similarly, a processor was necessary to analyse the results generated by the finite element program. The output requested by the user can be selected for a particular region of the model and could then be analysed. Even selective printing of the results prohibits easy interpretation as it becomes necessary to study stresses or deformations only in certain known directions. In two dimensional models, four adjacent elements share a common node and in the output, each element contributes to the stress at a grid point. A program was written to read-off the values of the stresses at a grid point contributed by every element and to calculate the average stress at that grid point. To accomplish this, the output from the computer was redirected to the Reader(3) instead of the line printer so that the output could then be manipulated using the post processing program. Appendix A.2 gives a listing of the program. The method adopted was to use the output file residing in the reader as a data file and the processing program would read-off the values of the required stresses at the required grid points and calculate the average and the stress concentration due to the loading and lap geometry. The output file generated

---------------

(3) A storage device existing for every virtual machine on the IBM mainframe system which can be used to store generated output and data files.
by this program could then, in turn, be used directly as a
data file for plotting the results obtained.

3.4.4 **Material properties, loadings and boundary conditions**

MSC/NASTRAN has the facility to apply forces at grid points
in any direction, uniform pressure loads on surfaces, mo-
ments at grid points and temperature dependent loadings for
static analysis. The two dimensional model was subjected to
tensile loadings and time dependent material properties when
cosidering the adhesive as viscoelastic. A tensile load of
1000 lb. was applied to the end grid point on roller support
and the ends of the joint were modelled to facilitate single
point loadings and boundary conditions as shown in Figure 4.

The time dependent properties of the adhesive at various
time intervals after loading were approximated from linear
viscoelastic theory using Findley's (14) equation. There is
no experimental or empirical data available to predict the
properties of adhesives at various loadings and tempera-
tures. This part of the analysis was hence restricted to us-
ing approximate values for the constants in Findley's equa-
tion and the aim was to obtain the trend of the results for
such a case. The viscoelastic model was analysed for time
periods of 0, 1000, and 100,000 hours of loading with temp-
erature loads 77, 150 and 250 degrees Kelvin for each time
period of loading.
The boundary conditions are an important part of the program and for the tensile load model, one end of the joint was pin-connected and the other on roller support as shown in Figure 4. Single point constraints were introduced to simulate these end conditions and the whole joint was modelled to be under plane-stress.

The material properties were specified on separate cards with the value of the Modulus of Elasticity, Poisson's ratio and the coefficient of thermal expansion.

A listing of the complete input program deck is given in Appendix A.3. The various two-dimensional models for the different overlap lengths under tensile loading were run for different values of material and geometric parameters as given in Table 2. The output in each case was processed by the post-processing program to obtain the stress distribution.
ADHEREND THICKNESS = 0.064 INCHES  ADHEREND MODULUS = 10.0 E+6
TENSILE LOADING

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TABLE 2  MATERIAL AND GEOMETRIC PARAMETERS FOR 2-D MODE OF JOINT
3.4.4.1 Findley's Theory:

Findley's equation (14) for linear viscoelastic behaviour is based on experimental values and the constants for the equation are obtained from experimental data.

The equation is:

\[
\frac{1}{E_v} = \frac{1}{E_o} + \frac{t^n}{E_t}
\]

\(E_v\) is the viscoelastic modulus and is composed of two elements: first is the elastic time-independent component, \(E_o\), and the second is the time-dependent component, \(E_t\).

The time under loading in hours is denoted as 't' and 'n' is a material constant; substantially independent of stress magnitude and is a dimensionless quantity. For the epoxy-based adhesive under consideration, the value of \(E_t\) can be taken as,

\[E_t = 20.0(E_o) = 20.0 \times 0.5E6 = 10.0E6 \text{ psi.}\]

The material and physical properties of epoxy-based materials and plastics were compared to those of the adhesive. From the trend of the values for those epoxy based materials having similar properties, the values of the material constant 'n' for the epoxy adhesive can be assumed to be equal to 0.15. Substituting in Findley's equation we get,

\[
E_v = \frac{10.0E6 \times 0.5E6}{0.5E6 t^{0.15} + 10.0E6}
\]
The above equation gives the values of the viscoelastic modulus at various time intervals under loading. The calculated values of the modulus are used as the material constants and temperature are also applied to obtain the behaviour of the joint under combined tensile and temperature loadings.
3.5 **THE THREE-DIMENSIONAL MODEL**

3.5.1 **Choice of Element**

A three dimensional model was developed to obtain the stress distribution when the joint was loaded along the width of the joint and also to verify the assumption of plane stress analysis in two dimensional models. The application of the finite element method in three dimensional theory of elasticity is a simple extension of the techniques of two dimensions. The isoparametric 'CHEXA' hexahedral elements are used, each element having eight corner nodes to define the solid element with a facility to introduce mid-side nodes on the edges, making a total of twenty nodes for the element. Any or all of the mid-side nodes may be omitted for analysis, depending on the geometry and the mesh requirements. The transition region from a coarse mesh to a fine mesh in three dimensions involved a grid similar to two dimensions and the five-faced 'CPENTA' elements were used, with care taken to maintain the same number of nodes at the common boundaries. Figure 5 gives the three dimensional mesh geometry.
As the joint is composed of the adherend and adhesive of different materials and elastic properties, the element boundaries are chosen such that they coincide with the material boundaries. The strains are continuous parallel to the material boundary, whereas the continuity in the direction perpendicular to the boundary concerns stresses and rotations.

As a trial run, the three dimensional model was initially analysed by a multi-step approach. The critical region was the portion of the adherend-adhesive interface with two adherends and the adhesive. The joint was first analysed as a whole, including the overhang portion of the adherend, using a coarse grid. The region where a more detailed analysis was required was then considered as a new system and the nodal displacements obtained in the first step was introduced as external displacements on the new system and the stress distribution and concentrations obtained. A comparison of the results obtained showed that there was no computer time saved by this method and the accuracy of the results were not affected by considering the joint as a whole and using more elements. The step-wise method, moreover, involved the preparation and input of more models and with no significant gain in accuracy. Hence, the approach of using the whole joint as a model was used. The computational errors due to roundoff were not excessive when the complete model was used instead of adopting the stepwise method due to refined MSC/NASTRAN software.
The hexahedron curved elements are superior to the tetrahedron and plane face elements. The total computer time was reduced by defining the grid and elements with nodes at the corners only wherever possible as the model was geometrically regular. Tetrahedron shaped elements generally are preferable for regions of stress concentration but the great amount of computer time necessary for processing such a model prohibited the use of tetrahedral elements to define model structure. The average computer time necessary for one run of the three dimensional model was sixteen CPU minutes, and as the model had to be analysed for various parameters, the total computer time required would be very large, and prohibited the use of smaller element dimensions for analysis.

3.5.2 Choice of axes and element dimensions:

A correct choice of axes was essential for the model so that the input of data and analysis of results was error-free, and no co-ordinate transformations were needed at the time of interpretation of the results.

NASTRAN has the facility to define co-ordinate systems based on structure geometry. The co-ordinate system chosen was a right-handed system with the Y-axis along the length of the joint and X-axis along the width as shown in Figure 6. The stresses are output in the 'element co-ordinate system' and to maintain the same orientation and axes direc-
tions, the size of each element was adjusted to conform to the output format of NASTRAN. Figure 6 describes the element co-ordinate system of the CHEXA element and the X-direction is the direction of the longest length. This was maintained along the width of the joint so that user-defined input axes coincided with the output axes configuration.
X : longest line joining centroids of opposite faces
XY: plane containing longest and next longest lines lines
joining centroids of opposite faces.

θ : material property orientation angle

Figure 6. Coordinate system for elements
3.5.3 Number of elements and grid points

The three-dimensional models developed differed in their shape at the ends of the joint so that suitable boundary conditions could be applied. Two models were used to analyse the behaviour of the joint under the following loadings:

1. Uniform tensile loading across the width.
2. Uniformly varying tensile load across the width.
3. Bending load
4. Uniform shear load across the width.

The model for the tensile loadings had 710 elements with 1044 grid points and the model for the bending and uniform shear load across the width had 740 elements with 1050 grid points.

A simple processor was written to generate the 3-D element connectivities and location of grid points. A listing is included in Appendix A.1. The output of this processor confirms with the data input format of MSC/NASTRAN and this overcomes the tedious task of individually inputting the adhesive joint geometry.

The resequencing of grid points internally by MSC/NASTRAN to obtain an optimum band-width for the equation matrix was activated using the necessary control cards in the Executive Control Deck. The sequence processor renumbers the grid points internally to obtain a reduced band-width and saves processing time; and the output is automatically transferred
back into user-defined grid point numbers for easy interpretation of the results.

A method to automatically analyse the output for the stresses and deformations was necessary for the three-dimensional model and one similar to that used for the two-dimensional model was written. A listing is given Appendix A.2 and is similar to the program described in article 2.3.3. The output file from NASTRAN is processed and the average stresses and stress concentrations at the required grid points calculated. The output from this program is then used as a data file for plotting the results.
Chapter IV
RESULTS AND DISCUSSION

4.1 INTRODUCTION
The single lap joint construction is antisymmetric and two of the major factors influencing the design of single lap joints are the magnitude and the direction of the load the joint will have to bear. The lap joint was subjected to the following conditions:

1. Simple tensile lap shear loading. (2-D & 3-D models)
2. Bending load. (3-D model)
3. Uniformly varying tensile load. (3-D model)
4. Uniform load along the width of the joint. (3-D model)
5. Effect of time of the loading and the temperature by considering the adhesive as a viscoelastic solid. (2-D model)

4.2 TENSILE LOADING
Figure 7 shows the stress distribution in the joint under an uniform tensile load. The influences of the parameters are shown relating to the stress concentration factors caused by the variation of the joint parameters. The major factors which influence the stress distribution are i) the mecha-
cal properties of the materials of the adherend and adhesive and ii) the lap geometry. The adherend selected is Aluminum which is the most commonly used in metal-metal bonding applications. The variables for analysis were the adhesive elasticity modulus, $E_a$, the adhesive thickness, $h$, and the length of the overlap, $L$. The adherend thickness and elasticity modulus was kept constant for the entire two-dimensional analysis. The values obtained were plotted for non-dimensionalized values of length and thickness.

The influence of the Poisson's ratio of the adhesive on the stress distribution has not been studied because only a large change in Poisson's ratio affect the distributions and magnitudes of the stresses. The range of Poisson's ratio for commonly used adhesives does not vary much and also there is a scarcity of information available regarding the Poisson's ratio for the various adhesives.

The three-dimensional model used was analysed by varying the adherend thickness, $t$, and the adhesive thickness, $h$. The values of the stress concentrations obtained were compared with the stress distributions for the corresponding $h/t$ ratios in the two-dimensional model. Figure 7 shows the stress distribution in a single lap joint under a tensile load.

It is seen from Figure 7 that the shear stresses are much greater than the average applied stresses and are concentrated at the edges of the joint. The normal 'Peel' stresses are also greater than the average stress and is also con-
centrated at the edges. The transfer of load from one adherend to another through the adhesive bond is not uniform across the joint. The shear stress distribution shows a region of nearly zero stress in the lap joint which is approximately centered between the two narrow zones of highly stressed adhesive. This zero stress region is seen to be always within a distance of two to three times the adherend thickness. The normal peel stress (tearing stress) is the stress perpendicular to the bondline and tends to peel the joint. This stress is high at the edge, then changes sign and becomes compressive, with nearly zero stress for the middle portion of the bond length.
Tables 3, 4, 5 give the values of the stresses obtained for the various lap and material properties. Most adhesives used for bonding metals are relatively rigid, and strong in shear and not so strong in peel. Care should be taken to minimize the effect of loading along the peel direction and loads should be applied such that the adhesive is in shear.
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ADHEREND ELASTIC MODULUS=10.0E+6 PSI
LAP LENGTH= 0.5 INCHES

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TABLE 3. STRESS CONCENTRATION FACTORS FOR 2-D TENSILE LOADING
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LAP LENGTH= 1.0 INCHES

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**Table 4. Stress Concentration Factors for 2-D Tensile Loading**
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LAP LENGTH= 1.5 INCHES

PEEL STRESS SHEAR STRESS

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<td>11.98</td>
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</table>

Table 5. Stress Concentration Factors for 2-D Tensile Loading
The adhesive was modelled by using two elements along its thickness so that the stress distribution across the thickness could be analysed. The stress distribution obtained along the thickness of the adhesive is not constant, and is maximum at the centerline of the adhesive with the variation following a parabolic trend. Figures 8A and 8B show the stress variation along the thickness of the adhesive.
Shear Stress Concentration

Distance along adhesive thickness, × 0.001 inches

E-adhesive = 5.0E+6 psi
Adhesive Thickness = 0.004 inches

LEGEND

△ L-0.5 INCHES
× L-1.0 INCHES
○ L-1.5 INCHES

SHEAR STRESS DISTRIBUTION ALONG ADHESIVE THICKNESS

FIGURE 8A
Single lap joints when loaded in tension result in out-of-plane normal deflections due to the eccentricity in the load path. A high bending moment at the ends of the overlap caused by the eccentricity results in the peel of the adhesive. The effect of bending can be reduced by increasing the adhesive thickness and the peel stress decreases with increasing h/t ratios as shown in Figures 9, 10 and 11.
VARIATION OF MAXIMUM STRESS WITH ADHESIVE THICKNESS

FIGURE 9
VARIATION OF MAXIMUM STRESS WITH ADHESIVE THICKNESS

FIGURE 11
For short overlap lengths, the peel stress decreases by a small quantity for increasing h/t ratios when the modulus of elasticity of the adhesive is high. At a low elasticity modulus, there is an appreciable change in the peel stress for long overlaps, with the peel stress decreasing for increasing adhesive thickness.

The shear stress is concentrated at the edges of the joint and tapers down to nearly zero stress for the middle two-thirds of the lap length. The finite element analysis does not result in precisely zero magnitudes of the shear stress in the elements adjacent to the free boundaries, but their magnitudes are very small and can be considered nearly zero.

At low values of adhesive elastic modulus and for long overlaps, there is a steep change in the shear stress as the adhesive thickness is increased.
4.3 **VISCOELASTIC ANALYSIS**

Adhesives may be considered as actually viscoelastic in nature and the effect of the time and temperature of loading the joint was investigated. Figures 12, 13 and 14 give the variation of stress with respect to time. The shape of the stress distribution curves along the length of the joint remains the same with the value of the stress gradually decreasing with time. This is attributed to the relaxation effect of the stress caused due to the viscoelasticity of the adhesive.
The model was analysed for time periods of 0, 1000 and 100,000 hours of loading with temperature loads of 77, 150, and 250 degrees Kelvin applied for each time period of loading. The lap length was maintained constant at 1.0 inches and the thickness of the adhesive layer being varied to study the effect of varying lap geometry. Table 6 gives the values obtained and the stress distribution was found to follow a similar pattern at all levels of time. The model was restricted as the adhesives generally lose their strength above 250 degree K. Lack of data relating to adhesive time and temperature dependent material properties prevented a more rigorous viscoelastic analysis.
ADHEREND THICKNESS = 0.064 INCHES  ADHEREND ELASTIC MODULUS = 10.0E+6 PSI
LAP LENGTH = 1.0 INCHES

<table>
<thead>
<tr>
<th>$x/L$</th>
<th>$E_{as}$</th>
<th>PEEL STRESS</th>
<th>SHEAR STRESS</th>
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</thead>
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<td>1.0E+3</td>
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<tr>
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<td>5.49</td>
<td>5.39</td>
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<tr>
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<td>-3.00</td>
<td>-2.89</td>
<td>-2.79</td>
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<tr>
<td>0.20</td>
<td>0.22</td>
<td>0.15</td>
<td>0.09</td>
</tr>
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<td>0.13</td>
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<td>-0.01</td>
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<td>-0.01</td>
<td>-0.01</td>
<td>0.0</td>
</tr>
<tr>
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<td>0.10</td>
<td>0.12</td>
<td>0.13</td>
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<td>0.90</td>
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<td>-2.79</td>
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<tr>
<td>1.00</td>
<td>5.49</td>
<td>5.39</td>
<td>5.29</td>
</tr>
</tbody>
</table>

**Table 6. Stress Concentration Factor Variation with Time**
SHEAR STRESS DISTRIBUTION WITH RESPECT TO TIME.

FIGURE 12
Figure 13: Peel Stress Distribution with Respect to Time

Legend:
- △ 0 hours
- × 1000 hours
- ✗ 10000 hours

Thicknes: Adherend: 0.064 Adhesive: 0.004

Distance along overlap, - - - - - - -
MAXIMUM STRESS VARIATION WITH TIME

FIGURE 14
4.4 THREE-DIMENSIONAL ANALYSIS

4.4.1 TENSILE LOADING
The results of the two-dimensional model under tension loading and assuming plane stress, was compared to the results obtained using a complete three dimensional model. Figure 15 compares the results. The models differed in the stress gradients obtained. The two-dimensional model has a steeper stress gradient from the point of maximum stress at the end of the overlap to a point inside the joint. This difference, however, can be attributed to the difference in element sizes between the two models.

There is no significant difference between them and the three-dimensional model is used mainly to investigate the effects of loading conditions across the width of the joint. The two-dimensional plane stress model is accurate as the uniform tension loading causes only a small stress in the direction along the width and does not contribute to the strength of the joint. Table 7 gives the values obtained for this model. A plane stress model is accurate for analysis when the joint is subjected to an uniform tensile loading.
Table 7. Stress Concentration Factors for J-0 Tensile Loading

<table>
<thead>
<tr>
<th>x/L</th>
<th>Full Stress</th>
<th>Shear Stress</th>
</tr>
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<tr>
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<td>0.0084</td>
</tr>
<tr>
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<td>5.75</td>
<td>5.30</td>
</tr>
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<td>0.125</td>
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<td>0.25</td>
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<td>0.00</td>
</tr>
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<td>0.375</td>
<td>0.00</td>
<td>0.00</td>
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</tr>
<tr>
<td>0.625</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.75</td>
<td>1.10</td>
<td>0.00</td>
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<tr>
<td>0.875</td>
<td>-3.25</td>
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</tr>
<tr>
<td>1.00</td>
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<td>5.30</td>
</tr>
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</table>
Stress Distribution for Tensile Loading

Figure 15
4.4.2 **UNIFORMLY VARYING LOAD ACROSS WIDTH**

An uniformly varying load was applied across the width of the joint and the results obtained are shown in Figures 16 and 17 and Tables 8 and 9. The stresses along the width of the joint are distributed as expected by the moment caused by the non-uniform loading. The stress is maximum at the ends of the adhesive where the maximum load is applied. The stress decreases across the width as the loading decreases.
ADHESIVE ELASTIC MODULUS=5.0E+6 PSI  ADHESIVE ELASTIC MODULUS=10.0E+6 PSI
LAP LENGTH= 1.0 INCHES

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<td>-0.17</td>
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**TABLE 8. STRESS CONCENTRATION FACTORS ALONG THE LENGTH OF THE JOINT FOR 3-D UNIFORMLY VARYING LOAD**
Stresses along the length for 3-D triangular loading

Figure 16
ADHESIVE ELASTIC MODULUS = 5.0E+6 PSI
ADHESIVE THICKNESS = 0.006 INCHES
ADHESIVE THICKNESS = 0.064 INCHES
LAP LENGTH = 1.0 INCHES

<table>
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TABLE 9. STRESS CONCENTRATION FACTORS ALONG THE WIDTH OF THE
JOINT FOR 3-D TRANSVERSE LOADING
Stresses along the width for 3-D triangular loading

Figure 17
4.4.3 **SHEAR LOAD ACROSS THE WIDTH**

Figures 18 and 19 show the stress distribution along the length and width of the joint when subjected to a shear load across its width. Tables 10 and 11 give the values obtained. Along the length of the joint, the peel stress is a maximum at the ends of the overlap, but opposite in sign, due to the shear loading. From the ends of the overlap to the centre, the stresses change sign from each end as seen in Figure 18. The shear loading causes the shear stresses to be a maximum at the edges of the joint and the stresses are symmetrical about the centre of the joint. Along the width of the joint, the shear loading causes equal and opposite stresses at each end of the joint, as shown in Figure 19.
**ADHESIVE ELASTIC MODULUS = 5.0E+6 PSI**  **ADHEREND ELASTIC MODULUS = 10.0E+6 PSI**  **LAP LENGTH = 1.0 INCHES**

**Table 10: Stress Concentration Factors Along the Length of the Joint for 3-D Transverse Loading**

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</table>
FIGURE 18

Stresses along the length for 3-D transverse loading.

Legend:
- Δ Peel stress
- × Shear stress
- □ Shear stress

Stress Concentration: $\tau_x$, $\tau_y$

Distance along length of overlap, $x$.
ADHESIVE ELASTIC MODULUS = 5.0E+6 PSI  ADHEREND ELASTIC MODULUS = 10.0E+6 PSI
LAP LENGTH = 1.0 INCHES

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<th>σᵧ</th>
<th>τₓ</th>
<th>τᵧ</th>
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<td>-1.53</td>
<td>-1.53</td>
<td>4.92</td>
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</table>

**TABLE 11** STRESS CONCENTRATION FACTORS ALONG THE WIDTH OF THE JOINT FOR 3-D TRANSVERSE LOADING
Stresses along the width for 3-D transverse loading

Figure 19
A bending load was applied to the joint and the results obtained are shown in Figures 20 and 21 and the values tabulated in Table 12. The stress distribution obtained is similar to that for a tensile loading with the peel stress changing sign near the ends of the joint. The bending moment applied to the end of the joint causes a high peel stress and tends to peel the adherends. Along the width of the joint, the stresses are approximately uniform and the small change in the values of stress shown in Figure 21 can be attributed to the finite element model.
ADHESIVE ELASTIC MODULUS=5.06 x 10^6 PSI  ADHEREND ELASTIC MODULUS=10.0 x 10^6 PSI
ADHESIVE THICKNESS= 0.0040 INCHES  ADHEREND THICKNESS= 0.064 INCHES
LAP LENGTH= 1.0 INCHES

<table>
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</tr>
<tr>
<td>0.250</td>
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<td>0.20</td>
</tr>
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<tr>
<td>0.500</td>
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<td>0.00</td>
</tr>
</tbody>
</table>

**Table 12. Stress Concentration Factors for 3-D Bending Load**
4.5 **EXPERIMENTAL INVESTIGATION**

An experimental investigation was conducted to obtain the mode of failure of the joint under a tensile load and to relate it to the stress distribution obtained by the finite element method.

A number of joint samples of various lap lengths of 0.5, 0.75, 1.0 and 1.5 inches were loaded to failure. Adhesive failure is the common mode and bond failures occur mostly along the bond adherend interfaces. This can be seen from the pattern of the adhesive material which remains on the adherend strips after failure. Figure 22A shows a test specimen after it has been loaded to failure.

The results obtained are consistent with a possible theory of failure suggested by Ojalvo and Eidinoff (9) in 1978. As the shear stress is highest at points A and D, Figure 22B, they represent logical points for cracks to originate. This point of maximum stress is also obtained from the finite element models developed. As the cracks formed propagate, they create a shorter joint with high regions of stress at A' and D'. Thus, the crack propagates to points A'' and D'' in the bond as shown in Figure 22C. At A'' and D'', the crack jumps across the adhesive, resulting in failure.
Figure 22. COMPARISON OF ACTUAL JOINT FAILURE WITH POINTS OF MAXIMUM STRESS
4.6 DISCUSSION

There is no complete analytical solution to the problem and the comparison to work done by earlier investigations is restricted. There are also no analytical solutions for loadings across the width, involving three-dimensional models; and all two-dimensional models evolved so far consider only uniform tensile loadings so that the joint could then be analysed in plane stress or plane strain.

The solution proposed in 1978 by Ojalvo and Eidinoff (9), is the closest to that obtained using the two and three-dimensional finite element models. However, the results obtained by Ojavlo and Eidinoff (9) and by Goland and Reissner (2) assumed plane strain whereas the two-dimensional model used assumes plane stress conditions.

The stress distribution obtained using the three-dimensional models and various loadings satisfied boundary conditions and followed an expected pattern. The finite element model of the joint is designed to yield a good approximation to the real situation and the results of the finite element compare favorably with the analytical solutions proposed. The aspect ratios were kept to a minimum by choosing proper element sizes and the maximum aspect ratio among all the models used was 1.5625 in the adherend and 50 in the adhesive as the adhesive thickness is very small compared to the length of the overlap. As a check, the stresses in the two-
dimensional model for an overlap length of 1.0 inches was recomputed with an aspect ratio of 25 as a maximum in the adhesive. The maximum stress concentration factor obtained by such a model was lower by 3% using the refined model, but the stress gradient from the end of the overlap was steeper. However, this model required an additional 4 minutes of CPU time which prohibited its use and the model with an aspect ratio of 50 gave satisfactory results.
BIBLIOGRAPHY


APPENDIX A.1

PROGRAM FOR GENERATING MSC/NASTRAN INPUT CARDS

A simple program was written to generate the input cards for MSC/NASTRAN. Geometrically regular CQUAD8 quadrilateral elements and CHEXA cubic elements were generated with the format being the free-field format of MSC/NASTRAN.
PROGRAM FOR PREPARING MSC/NASTRAN INPUT CARDS

THIS PROGRAM GENERATES ELEMENT CONNECTIVITIES FOR 3-D
HEXAHEDRA ELEMENTS. A SIMILAR PROGRAM IS WRITTEN FOR 2-D
ELEMENTS.

INTEGER ELEM,A,B,C,D,E,F,G

INITIALIZATION :

ELEM=101
A=1
B=31
C=32
D=2
E=7
F=37
G=1101

WRITE (6,1) ELEM,A,B,C,D,E,F,G
1 FORMAT ('HEX,'I3,','200,'G(I3,','),I4)

A=A+1
B=B+1
C=C+1
D=D+1
E=E+1
F=F+1
IF ((A.EQ.6).OR.(A.EQ.12).OR.(A.EQ.18)) GOTO 10
IF (A.EQ.24) GOTO 20
IF ((A.EQ.346).OR.(A.EQ.552).OR.(A.EQ.558)) GOTO 10
IF (A.EQ.564) GOTO 50
ELEM=ELEM+1
J=J+1
GOTO 5

20 A=A+517
B=B+517
C=C+517
D=D+517
E=E+505
F=F+505
ELEM=ELEM+361
G=G+361
GOTO 5

TO GENERATE GRID COORDINATES

50 GR1=0.0
GR2=0.5
GR3=0.064
IG=1

WRITE(6,2) IG,GR1,GR2,GR3
2 FORMAT('GRID,'I3,','2(FS,3,','),FS,3)
IG=IG+1
QR1=QR1+.2
IF(IG.EQ.7).OR.(IG.EQ.13).OR.(IG.EQ.19)) GOTO 60
IF(IG.EQ.25) GOTO 60
IF(IG.EQ.31) STOP
GOTO 5
60
QR1=0.0
QR3=QR3-0.016
GOTO 5
END
APPENDIX A.2

PROGRAM FOR ANALYSING MSC/NASTRAN OUTPUT

The values computed by MSC/NASTRAN were analysed by reading them off the output file using this program and the program calculates the average values which can be used directly for plotting the results.
This program reads the values of the Nastran output files residing in the reader of the virtual machine. The format statements correspond to Nastran output format and each time a value is read, it is stored as a temporary variable and is added to the existing value for that particular grid point. A counter increments each time the same grid number is read so that the average can be calculated and used for plotting.

For each variation in geometry in the 2-D models, a separate but similar program was written to analyze the different lap geometries.

```fortran
DIMENSION SSTR(800),PSTR(800),SCSSTR(800),SCPSTR(800)
DIMENSION COUNT(600),SSTR(600),PSTR(600),SCSSTR(600),SCPSTR(600)
DIMENSION AVSSTR(600),AVPSTR(600)

INITIALIZATION OF VARIABLES:
SSTR(I),PSTR(I): SHEAR AND PEEL STRESS VARIABLES.
COUNT(I): COUNTER
N1=77
N2=193
DO 10 I = N1,N2
PSTR(I)=0.0
SSTR(I)=0.0
COUNT(I)=0.0
CONTINUE

END OF INITIALIZATION

READ(5,99) N
99 FORMAT(2X,I3)

IF((N.GE.N1).AND.(N.LE.N2)) GO TO 20
IF(N.EQ.900) GO TO 200
GO TO 100

20 READ(5,98) PSTRN, SSTRN

98 FORMAT(5X,E13.6,1X,E13.6)
PSTR(N)=PSTRN+PSTR(N)
SSTR(N)=SSTRN+SSTR(N)
COUNT(N)=COUNT(N)+1.0
```
CONTINUE

PRINTING OUT THE RESULTS

WRITE(6,96)
FORMAT('/','10X,'THE AVERAGE STRESSES AND THE STRESS ',
1 'CONCENTRATIONS',/,'10X,'*** ******** ******** **** *** *** *** **
3** ***)
WRITE(6,94)
FORMAT(5X,'NODE',5X,'PEEL STRESS',5X,'SHEAR STRESS',
2 5X,'SCF(PEEL)',3X,'SCF(SHEAR)'),/)
C
WRITE(6,697)
FORMAT(15X,'RESULTS FOR THE INTERFACE PLANE IN ADHRED + ADHS',//)
DO 30 I=127,127,5
WRITE(6,697) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I)
30 CONTINUE
WRITE(6,697)
FORMAT('/','15X,'RESULTS FOR STRESS IN THE ADHESIVE',/)
C
PRINTING THE RESULTS FOR THE ADHESIVE
DO 50 I = 132,142,1
WRITE(6,97) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I)
50 CONTINUE
STOP
END
THIS PROGRAM READS THE VALUES OF THE NASTRAN OUTPUT FILES RESIDING IN THE READER OF THE VIRTUAL MACHINE. THE FORMAT STATEMENTS CORRESPOND TO NASTRAN OUTPUT FORMAT AND EACH TIME A VALUE IS READ IT IS STORED AS A TEMPORARY VARIABLE AND IS ADDED TO THE EXISTING VALUE FOR THAT PARTICULAR GRID POINT. A COUNTER INCREMENTS EACH TIME THE SAME GRID NUMBER IS READ SO THAT THE AVERAGE CAN BE CALCULATED AND USED FOR PLOTTING.

FOR EACH VARIATION IN GEOMETRY IN THE 3-D MODELS, A SEPERATE BUT SIMILAR PROGRAM WAS WRITTEN TO ANALYSE THE DIFFERENT LAP GEOMETRIES.

```
DIMENSION SSSTR(800),PSTR(800),SCSSTR(800),SCPSTR(800)
DIMENSION AVSSTR(800),AVPSTR(800),COUNT(800)
DIMENSION SSSTRZX(800), ASTRZX(800), SCSTRZX(800)
```

N1,N2 ARE THE GRID POINTS IN THE FINITE ELEMENT MODEL.
SSSTR(I),PSTR(I) ARE THE SHEAR AND PEEL STRESSES.
INITIALIZING THE VARIABLES.

```
N1=161
N2=760
DO 10 I = N1,N2
   PSTR(I)=0.0
   SSSTR(I)=0.0
   SSSTRZX(I)=0.0
   COUNT(I)=0.0
10   CONTINUE
```

END OF INITIALIZATION

```
100 READ(5,99) N
99 FORMAT(1X,I3)
   IF((N.EQ.N1).AND.(N.LE.N2)) GO TO 20
   IF(N.EQ.900) GO TO 200
   GO TO 100
20 READ(5,98) SSTRN , PSTRN , STRZXN
98 FORMAT(1X,2F13.6,1X,2F13.6,1X)
```

```
PSTR(N)=PSTRN+PSTR(N)
SSSTR(N)=SSSTRN+SSSTR(N)
SSSTRZX(N)=SSSTRZX+SSSTRZX(N)
COUNT(N)=COUNT(N)+1.0
```
DO 40 J = N1,N2
IF(COUNT(J).LT.1) GOTO 40
AVPSTR(J)=PSTR(J)/COUNT(J)
AVSSTR(J)=SSTR(J)/COUNT(J)
ASTRZX(J)=SSTRZX(J)/COUNT(J)

LSCPSTR(J)=AVPSTR(J)/1000.0
SCSSTR(J)=AVSSTR(J)/1000.0
SCSTZX(J)=ASTRZX(J)/1000.0

CONTINUE

PRINTING THE RESULTS:
WRITE(6,292) N,PSTRN,SSTRN,STRZRN,COUNT(N)
FORMAT(I3,2X,3(I3,E13.6),F4.1)
GO TO 100

WRITE(6,96)
FORMAT(IOX,'THE AVERAGE STRESSES ARE':/,IOX, '************
************',//)
WRITE(6,94)
FORMAT(3X,'NODE',3X,'PEEL STRESS',3X,'SHEAR STRESS',3X,
2 'SCF(PEEL)',3X,'SCF(SHEAR ZY)',//)

WRITE(6,197)
FORMAT(/,IOX,'RESULTS ALONG THE LENGTH OF INTERFACE 1')
DO 50 I = 211,451,30
WRITE(6,97) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I),SCSTZX(I)
CONTINUE
WRITE(6,297)
FORMAT(/,IOX,'RESULTS ALONG LENGTH IN ADHESIVE')
DO 51 I = 481,529,6
WRITE(6,97) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I),SCSTZX(I)
CONTINUE
WRITE(6,397)
FORMAT(/,IOX,'RESULTS ALONG THE LENGTH OF INTERFACE 2')
DO 52 I = 535,775,30
WRITE(6,97) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I),SCSTZX(I)
CONTINUE
WRITE(6,497)
FORMAT(/,IOX,'RESULTS ALONG THE WIDTH IN INTERFACE 1')
DO 53 I = 211,216
WRITE(6,97) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I),SCSTZX(I)
CONTINUE
WRITE(6,597)
FORMAT(/,IOX,'RESULTS ALONG THE WIDTH FOR ADHESIVE')
DO 54 I = 481,486
WRITE(6,97) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I),SCSTZX(I)
CONTINUE
WRITE(6,697)
FORMAT(/,IOX,'RESULTS ALONG THE WIDTH FOR INTERFACE 2')
DO 55 I = 535,540
WRITE(6,97) I,AVPSTR(I),AVSSTR(I),SCPSTR(I),SCSSTR(I),SCSTZ(I)
CONTINUE
END
APPENDIX A.3

LISTINGS OF MSC/NASTRAN INPUT PROGRAMS

The source listings of the two-dimensional and three-dimensional MSC/NASTRAN models used are given in this section. The program uses Rigid Format SOLN 24 for Linear Static Analysis. The important control cards used are explained.

The material properties and joint geometry is inputted by the MAT1, CHEXA, CQUAD8, CTRIA6, CPENTA and GRID cards. The default time limit for NASTRAN is 5 minutes and for jobs requiring more than 5 minutes, the format of the Job Control Card is //EXEC NASTRAN, T=nn, where nn is the number of minutes required.

Parametric options used include:

i) PARAM, AUTOSPC, YES : This automatically identifies singularities with a stiffness ratio smaller than 1.E-8 and constrains them with single point constraints.

ii) PARAM, PRGPST, NO : This suppresses the print out of the singularities in the output. For a three-dimensional model, this table becomes very lengthy which is suppressed by this statement.

iii) PARAM, SEQOUT, 0 : This controls the printing of the resequenced grid points after the resequencing processor has been activated. To avoid a large output, the printing of this resequencing processor is suppressed.
The analysis of the single lap joint involved the generation of various models for the different geometry and lap lengths. Models were developed for lap lengths of 0.5 inches, 1.0 inches and 1.5 inches. The geometry and material parameters could be altered by changing or inserting cards in the NASTRAN data deck.

The two-dimensional models differed only in the number of elements and the finite element mesh pattern was kept similar for all models so that there is no loss in accuracy in comparison of the results. The three-dimensional models differed only in the loadings and the boundary conditions. Only a representative listing of Two-dimensional and Three-dimensional models is given as all the NASTRAN listings for the various models are similar and differ only in the number of grid points and elements.
TWO-DIMENSIONAL MODEL

LISTING OF THE TWO-DIMENSIONAL MODELS USED TO ANALYZE THE LAP JOINT.

PROGRAM USED : MSC/NASTRAN (VERSION 62A)

LAP LENGTH = 1.0 INCHES. THICKNESS OF ADHESIVE = 0.004 INCHES.
NO. OF ELEMENTS = 212 NO. OF GRID POINTS = 273

//N9206GNC JOB (,
// G1267SUR,10M),CHOKSI.GURANG,
// MSGLEVEL=2,0
///* DEST=RSCS41/MEGRAD1
// EXEC NASTRAN,T=10

CASE CONTROL DECK
ID THESIS, ADHRD.THCK=0.064, OVERLAP=1.0, WIDTH OF JOINT=0.02
SOL 24
TIME 10
CEND

EXECUTIVE CONTROL DECK
TITLE=2-D MODEL. ADHRD.THCK=0.064; E-ADHR=10.0E6. FILE TH910
SUBTITLE=E-ADHR=10.0E6, ADHS THCK=0.004, OVERLAP=1.0.
LOAD=ALL

SELECTIVE PRINTING OF THE STRESSES AT REQUIRED GRID POINTS ONLY
SET 99 = 33,57,61,69,73,77,81,85,89,93,37 THRU 116
STRESS=99
SUPRESS SORTED BULKDATA LISTING
ECHO=NONE
LABEL=LOAD USING FORCE1 CARD (TENSILE LOAD).
LOAD=400
SPC=450

BULK DATA DECK

BEGIN BULK

CONNECTIVITY OF ELEMENTS — CTRIA6 AND CQUAD8 :

CTRIA6,1,200,1,2,3
CQUAD8,2,200,2,4,5,3
CQUAD8,3,200,4,6,7,5
=C,*(1),=,*(2),*(2),*(2),*(2)
=(7)
CTRIA6,12,200,22,24,25
CTRIA6,13,200,22,25,23
CTRIA6,14,200,23,25,26
CQUAD8,15,200,24,27,28,25
=*,(2),=,*,(3),*(3),*(3),*(3)
>(8)
QUAD8, 16, 200, 25, 28, 29, 26
=*(2), =*(3), =*(3), =*(3), =*(3)
=(8)
TRI36, 35, 200, 54, 57, 58
TRI36, 36, 200, 55, 54, 58
TRI36, 37, 200, 55, 58, 59
TRI36, 38, 200, 59, 59, 50
TRI36, 39, 200, 56, 55, 60
TRI36, 40, 200, 56, 60, 61
QUAD8, 41, 200, 57, 62, 63, 58
=(12)
QUAD8, 42, 200, 58, 63, 64, 59
=(12)
QUAD8, 43, 200, 59, 64, 65, 60
=(12)
QUAD8, 44, 200, 60, 65, 66, 61
=(12)
QUAD8, 96, 300, 132, 133, 82, 77
=*(2), =*(1), =*(1), =*(5), =*(5)
=(8)
QUAD8, 97, 300, 143, 148, 133, 132
=*(2), =*(5), =*(5), =*(1), =*(1)
=(8)
QUAD8, 117, 200, 144, 149, 148, 143
=(12)
QUAD8, 118, 200, 145, 150, 149, 144
=(12)
QUAD8, 119, 200, 146, 151, 150, 145
=(12)
QUAD8, 120, 200, 147, 152, 151, 146
=(12)
TRI36, 173, 200, 213, 214, 218
TRI36, 174, 200, 214, 219, 218
TRI36, 175, 200, 214, 219, 219
TRI36, 176, 200, 216, 219, 215
TRI36, 177, 200, 216, 220, 219
TRI36, 178, 200, 217, 220, 216
QUAD8, 179, 200, 219, 222, 221, 218
=*(2), =*(3), =*(3), =*(3), =*(3)
=(8)
QUAD8, 180, 200, 220, 223, 222, 219
=*(2), =*(3), =*(3), =*(3), =*(3)
=(8)
TRI36, 179, 200, 248, 249, 249
TRI36, 200, 248, 252, 251
TRI36, 201, 200, 249, 250, 252
CONSTRAINTS FOR PLANE STRESS ANALYSIS

GRIDSET,3456

GRID POINT COORDINATES

GRID.1.,0.,0.100
GRID.2.,0.1,0.084
GRID.3.,0.1,0.116
GRID.4.,0.2,0.068
GRID.5.,0.2,0.132
GRID.6.,1.2,0.068
GRID.7.,2.3,0.068
GRID.8.,2.3,0.084
GRID.9.,2.3,0.100
GRID.10.,2.3,0.116
GRID.11.,2.3,0.132
GRID.12.,2.7,0.066
GRID.13.,2.7,0.064
GRID.14.,2.7,0.048
GRID.15.,2.7,0.032

CQUAD8,202,200,252,254,253,251
  =*(1),=*(2),*(2),*(2),*(2)
  *(8)
CSTRIA6,212,200,271,272,273
  *(8)

$ $ CONSTRAINTS FOR PLANE STRESS ANALYSIS

GRIDSET,3456
$ $ GRID POINT COORDINATES

GRID.1.,0.,0.100
GRID.2.,0.1,0.084
GRID.3.,0.1,0.116
GRID.4.,0.2,0.068
GRID.5.,0.2,0.132
GRID.6.,1.2,0.068
GRID.7.,2.3,0.068
GRID.8.,2.3,0.084
GRID.9.,2.3,0.100
GRID.10.,2.3,0.116
GRID.11.,2.3,0.132
GRID.12.,2.7,0.066
GRID.13.,2.7,0.064
GRID.14.,2.7,0.048
GRID.15.,2.7,0.032

  =*(2),*(0.1),=
  *(8)
  =*(2),*(0.1),=
  *(8)
  =*(3),*(0.1),=
  *(9)
  =*(3),*(0.1),=
  *(9)
  =*(5),*(0.1),=
  *(13)
  =*(5),*(0.1),=
  *(15)
  =*(5),*(0.1),=
  *(13)
  =*(5),*(0.1),=
  *(13)
  =*(5),*(0.1),=
  *(13)
= (13)
GRID, 146, 2.7, 0.016
=,*(3),*(0.1),=
= (13)
GRID, 147, 2.7, 0.0
=,*(5),*(0.1),=
= (13)
GRID, 218, 4.2, 0.064
=,*(3),*(0.1),=
= (9)
GRID, 219, 4.2, 0.032
=,*(3),*(0.1),=
= (9)
GRID, 220, 4.2, 0.0
=,*(3),*(0.1),=
= (9)
GRID, 251, 5.3, 0.064
=,*(2),*(0.1),=
= (8)
GRID, 252, 5.3, 0.0
=,*(2),*(0.1),=
= (8)
GRID, 271, 6.3, 0.048
GRID, 272, 6.3, 0.016
GRID, 273, 6.4, 0.032
$ MATERIAL PROPERTIES FOR ADHEREND AND ADHESIVE
$ PSHELL, 200, 201, 0.02, 201
PSHELL, 300, 301, 0.02, 301
MAT, 201, 10.0E6, 0.33
MAT, 301, 0.1E6, 0.35
$ $ LOADING OF 1000.00 LBS ALONG X AXIS.
FORCE, 400, 273, 1000.0, 267, 269
$ $ SINGLE POINT CONSTRAINTS:
SPCADD, 450, 350.351
SPC1, 350, 123.1
SPC1, 351, 23, 273
$ $ ACTIVATE GRID POINT SINGULARITY PROCESSOR
PARAM, AUTOSPC, YES
$ $ SUPPRESS PRINTING OF GRID POINT SINGULARITY TABLE
PARAM, PRGPST, NO
ENDDATA
/*
//