APPLICATION OF H∞ OPTIMAL CONTROL TO LARGE SPACE STRUCTURES

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Chapter 1

Introduction

Since the early 1970's the use of large structures in space has become increasingly important. Programs such as Skylab, the Hubble Space Telescope, and the proposed orbiting space station involve the launch, deployment, and subsequent control of large, flexible structures. As a result, the subject of controlling the behavior of these structures has become very important to engineers in the aerospace field.

1.1 LSS Control Issues

There are usually two major aspects of controlling a large space structure (LSS). One involves some sort of performance criterion such as pointing, rapid slewing, or tracking. The other, structural vibration suppression, is considered to be vital in controlling lightly damped space structures, even though it may not be directly related to performance. An example of this case might be the need to suppress vibrations induced by movement of the crew within a spacecraft.

There are several factors contributing to the challenging nature of LSS control. Because of their size and complexity, LSS usually have many sensors and actuators, thus requiring a multi-input multi-output (MIMO) control design. They are also quite flexible with very little structural damping (typically
less than 1%). In addition, LSS exhibit a high number of closely spaced, low frequency modes.

The physical characteristics listed above translate into a difficult task when the engineer attempts to derive a model as a design tool. Typical methods in use today include finite-element modeling techniques and various system identification techniques which use experimental data taken from the actual structure to create a mathematical model. These methods are typically successful in identifying low frequency behavior but often do not perform well at higher frequencies. As a result, the control system designer must design a controller which is "robust" to modeling errors as well as nonlinearities, noise, and time-varying properties the system may have, while at the same time achieving the desired performance and structural damping goals.

1.2 Previous Approaches

Although literature abounds on the subject of LSS control, very few designs have been actually implemented with hardware. The majority of these designs have been applications of some variety of optimal control with at least some relation to the Linear Quadratic Gaussian (LQG) approach. Classical techniques have also been employed, utilizing sequential SISO designs and closing sensor/actuator loops one at a time.
In the Air Force sponsored Active Control Technique Evaluation for Spacecraft (ACES) program [1], three techniques were applied to a test article at the NASA Large Space Structure Ground Test Facility. The goal of the program was to evaluate different control system design techniques for applicability to the problem of LSS control. Two of the techniques were:

(a) High Authority Control / Low Authority Control (HAC/LAC)

(b) Filter Accommodated Model Error Sensitivity Suppression (FAMESS)

HAC/LAC is a method developed by Lockheed Missiles and Space Corporation. The most prominent aspect of this approach is the separation of the control system design problem into two parts. The first part employs a High Authority Control (HAC) design, which is a high gain, low-bandwidth controller used to achieve performance goals. The second is a Low Authority Control (LAC) design, which is a low-gain, broad-bandwidth controller used to stabilize modes destabilized by the HAC controller. The HAC controller design has the following requirements:

(a) The HAC model must include all modes which have significant effect on performance.

(b) The HAC model must be quite accurate.

(c) Actuators used in HAC design must have high authority over modes to be controlled.
(d) Performance goals must be met by the HAC controller. The HAC design is accomplished using LQG techniques. Since an LQG design typically yields a controller with the same order as that of the model, most applications will necessitate some reduced order model for the HAC design. Model reduction schemes typically make no robustness guarantees when the controller is used with a system other than the one used for design; therefore, some means of ensuring robustness is needed. This is where the LAC controller is used.

The LAC controller is designed under the assumption that the HAC controller has destabilized some modes not included in the HAC model. The LAC controller is comprised of simple gains in the feedback path. Because of this, the LAC controller is best implemented using collocated and consistent sensor/actuator pairs. The term "collocated" means that the sensor and actuator have the same physical location on the structure, while a "consistent" pair indicates that the sensor and actuator are of the same type (e.g. rate sensor and force actuator). In general terms, the LAC controller is designed to shift poles to the left in the s-plane while at the same time using small gains. The necessary calculations involve the Jacobi Root Perturbation formula. By using small gains, it is hoped that unmodeled modes will not be destabilized.

The HAC/LAC method seems at first to be a good approach. Further thought reveals, however, that there are limitations involved. Since the HAC design will usually be done first, it
should be included in the model used for LAC design. The HAC design is used for performance so it would be advantageous to design the HAC controller in the digital domain so as to include the sampled-data effects (computational delay etc.) inherent to a system using a digital controller. The LAC controller, however, must be designed using a continuous plant. Thus, the designer must either complete the HAC design in the continuous domain and attempt to approximate sampled-data effects with filters (thus adding to the order of the design plant), or make some continuous approximation of the digital HAC controller. Another limitation is the required use of collocated sensor/actuator pairs. If the system to be controlled has few or no such sensor/actuator pairs, or if the collocated pairs have no control authority over behavior which the LAC controller is intended to improve, the LAC controller will not be able to achieve the design goals.

Filter Accommodated Model Error Sensitivity Suppression (FAMESS) is based upon Model Error Sensitivity Suppression (MESS) which is a modified LQG approach to control system design. The MESS technique attempts to decouple the controller from system states corresponding to "suppressed" modes which are assumed to have little effect on system performance, thus reducing the possibility of these modes being destabilized. This is accomplished by modifying the usual LQG performance index by only weighting the states to be controlled, as well as adding an explicit weighting term
penalizing the feedback of states corresponding to the suppressed modes. As a result, the full-state feedback MESS design will have as many states as the unsuppressed portion of the model.

Filter Accommodation (FA) attempts to ensure that the controller has low-pass characteristics by including low-pass filters in the design plant, thus decreasing the influence of less well known high frequency modes on the performance index. The low pass filters are cascaded at each of the system outputs, but the FA technique constrains the design equations such that the filter states are not fed back.

FAMESS appears to be an improvement over standard LQG techniques, but it still suffers from many of the same shortcomings. First, the separation of modes for suppression requires that these dynamics be well known, and no guarantees can be made about spillover elimination if there is significant model error. The disadvantage of FA is that the incorporation of low-pass filters at each input or output adds significantly to the order of the design model.

Another approach to LSS controller design has been the use of "one loop at a time" classical techniques. This method requires the designer to do sequential SISO designs, closing each loop after each design. Typically, loops with collocated, complimentary sensors and actuators are closed first, since damping can be added without any fear of stability problems. Next, loops with non-collocated but
complimentary sensors and actuators are closed. Finally, if position control is desired, position loops are closed.

One advantage to this approach is that when the design is completed, the designer will have intimate knowledge of the characteristics of each loop. In addition, most frequency domain oriented design goals can be easily evaluated when such classical techniques are employed. Some questions do arise, however, concerning the question of robustness. Though the designer can design for stability margins for each loop, he has no clear way to ensure that he does not affect the stability margins for the loops that have already been closed.

In light of the fact that no methods existed which directly dealt with the problem of MIMO robustness issues, Francis, Helton, and Zames solved the MIMO $H^\infty$ minimum sensitivity optimization problem [2] as a means of optimizing robustness. Further work by Kwakernaak [3] and others showed that feedback system design objectives such as disturbance rejection, bandwidth limitation, and robustness could be expressed in terms of required frequency domain bounds on the sensitivity function

$$S(s) = (I + G(s)H(s))^{-1} \quad (1-1)$$

where $G(s)$ is the plant transfer function and $H(s)$ is the controller transfer function in the cascade configuration. The parametrization of all stabilizing controllers satisfying
an $H^\infty$ bound on a certain transfer function, as well as the complete solution to the MIMO $H^\infty$ problem may be found in Francis [4] and the references therein.

Though the complete solution exists, issues concerning compensator order, numerical difficulty in algorithm implementation, and practicality of algorithm assumptions when applied to real problems are still under investigation. The work presented here is aimed at the practical application of $H^\infty$ theory to controller design for LSS. In particular, an $H^\infty$ controller is designed and implemented on the NASA Large Space Structure Ground Test Facility at Marshall Space Flight Center in Huntsville, Alabama. Both the factorization approach of Francis et. al. and the more recent state-space approach of Doyle and Glover [5] are investigated as to their applicability to the solution of LSS control problems.

This report is organized into seven chapters. The first chapter is this introduction. The second chapter describes in detail both the factorization approach and the state-space approach to $H^\infty$ design. The third chapter describes the particular control problem being used to evaluate the technique, with a complete facility description. Chapter 4 describes the frequency dependent weighting scheme used to enforce performance and robustness constraints. The fifth chapter shows the design evolution, while Chapter 6 illustrates the experimental test results of the controller implementation. Chapter 7 presents conclusions and
recommendations for future work. Finally, the appendix describes the original software package developed to perform $H^\infty$ control synthesis and analysis.
CHAPTER 2
H∞ Theory

The main thrust of $H^\infty$ optimization is to minimize the $H^\infty$ norm of a certain partition of the closed-loop transfer function matrix of the system, where the $H^\infty$ norm is given by

$$\|G(j\omega)\|_\infty = \sup \{\|G(j\omega)\|_2 : \omega > 0\}$$

$$= \sup_{\omega} \sigma(G(j\omega)).$$

where $\sigma$ denotes the maximum singular value. In general terms, this amounts to minimizing the output due to the worst case input. To accomplish this optimization, the problem is typically cast in the standard form shown below in Figure 2.1.

Figure 2.1 Block Diagram for Standard Problem

$W(s)$ is a vector of command and/or disturbance inputs, $U(s)$ is the vector of controller outputs, $Z(s)$ is a performance
oriented vector of controlled outputs, and $Y(s)$ is the measurement vector. The accompanying equations are

$$Z = G_{11}W + G_{12}U, \quad (2-2)$$

$$Y = G_{21}W + G_{22}U, \quad (2-3)$$

and

$$U = KY. \quad (2-4)$$

Once cast in this form, the problem is treated as a disturbance attenuation problem, where the norm of the transfer function from $W$ to $Z$ is minimized. This transfer function is given by

$$Z = [G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}]W. \quad (2-5)$$

Other problems such as tracking and robust stabilization problems can be cast in this form by making appropriate definitions for $W$, $U$, $Y$, and $Z$.

As an example of casting a tracking problem in standard form, consider the system of Figure 2.2.
In this tracking problem, the performance goal is to minimize the error signal $E$. This can also be stated in the frequency domain as minimizing the transfer function from $R$ to $E$. Making the substitutions

$$Z = E = R - C,$$
$$Y = C,$$
$$U = KC,$$

and

$$W = R$$

gives

$$Z = W - G(W - U)$$
$$= W(I - G) + GU$$

and
\[ Y = G(W - U). \]

These equations are now in standard form such that

\[ G_{11} = I - G, \]
\[ G_{12} = G, \]
\[ G_{21} = G, \]

and

\[ G_{22} = -G. \]

2.1 Factorization Approach

The first solutions to the H^\infty optimization problem were accomplished using a factorization approach which relied heavily on the theory of operators and interpolation. A thorough derivation of the factorization approach is given in Francis [4]. The basic steps to calculating the optimal H^\infty controller using this approach are:

(a) Cast the problem in standard form
(b) Parameterize all stabilizing controllers
(c) Parameterize the closed loop transfer function
(d) Solve the model matching problem indicated by (c)
(e) Calculate the controller
Before proceeding with the explanation of these steps, some definitions are needed.

**DEFINITION**

The space \( \mathbb{R}^n \) consists of all matrices whose elements are rational functions which have real coefficients, no poles in the right half of the complex plane (stable), and are proper. A proper function satisfies the inequality \( |G(\infty)| < \infty \).

**DEFINITION**

Two matrices \( F \) and \( G \) in \( \mathbb{R}^n \) are **right-coprime** if they have equal column dimension and there exist matrices \( X \) and \( Y \) in \( \mathbb{R}^n \) such that

\[
\begin{bmatrix} X & Y \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix} = XF + YG = I. \tag{2-6}
\]

Similarly, \( F \) and \( G \) are **left-coprime** if they have the same row dimension and satisfy

\[
\begin{bmatrix} F \\ G \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = FX + GY = I. \tag{2-7}
\]

A **right-coprime factorization** of an \( \mathbb{R}^n \) matrix \( G \) is a factorization \( G = NM^{-1} \) where \( N \) and \( M \) are right-coprime \( \mathbb{R}^n \) matrices. A **left-coprime factorization** of \( G \) is a factorization \( G = \tilde{M}^{-1}\tilde{N} \) where \( \tilde{N} \) and \( \tilde{M} \) are left-coprime.
DEFINITION

For every matrix $G$ in $\mathbb{RH}^s$ there exist eight matrices in $\mathbb{RH}^s$ which satisfy

$$G = NM^{-1} = \tilde{M}^{-1}\tilde{N}$$  \hspace{1cm} (2-8)

and

$$\begin{bmatrix} \dot{X} & -\dot{Y} \\ -\dot{N} & M \end{bmatrix} \begin{bmatrix} M & Y \\ N & X \end{bmatrix} = I.$$  \hspace{1cm} (2-9)

Equations (2-8) and (2-9) define a doubly coprime factorization of $G$.

The problem solution begins with the parameterization of all stabilizing controllers. Though it will not be proven here, it is true that if $G$ is stabilizable, a controller $K$ stabilizes $G$ iff $K$ stabilizes $G_{22}$. The set of all proper real-rational $K$'s which stabilize $G_{22}$ can be parameterized in terms of the doubly-coprime factorization of $G_{22}$ and the $\mathbb{RH}^s$ parameter $Q$. To derive the parameterization for all such $K$'s, first the doubly coprime factorization of $G_{22}$ is introduced:

$$G_{22} = NM^{-1} = \tilde{M}^{-1}\tilde{N}$$  \hspace{1cm} (2-10)

and
The equation for the $(1,2)$ block is

\[
\begin{bmatrix}
\dot{X} & -\dot{Y} \\
-N & M
\end{bmatrix}
\begin{bmatrix}
M & Y \\
N & X
\end{bmatrix}
= I.
\] (2-11)

From (2-11)

\[
\begin{bmatrix}
I & Q \\
0 & I
\end{bmatrix}
\begin{bmatrix}
\dot{X} & -\dot{Y} \\
-N & M
\end{bmatrix}
\begin{bmatrix}
M & Y \\
N & X
\end{bmatrix}
\begin{bmatrix}
I & -Q \\
0 & I
\end{bmatrix}
= I,
\] (2-12)

so

\[
\begin{bmatrix}
\dot{X} - Q\dot{N} & -(\dot{Y} - Q\dot{M}) \\
-N & M
\end{bmatrix}
\begin{bmatrix}
M & Y - MQ \\
N & X - NQ
\end{bmatrix}
= I.
\] (2-13)

The equation for the $(1,2)$ block is

\[
(\dot{X} - Q\dot{N})(Y - MQ) - (\dot{Y} - Q\dot{M})(X - NQ) = 0,
\] (2-14)

so that

\[
(Y - MQ)(X - NQ)^{-1} = (\dot{X} - Q\dot{N})^{-1}(\dot{Y} - Q\dot{M}).
\] (2-15)

Now let $K$ be defined by

\[
K = (Y - MQ)(X - NQ)^{-1}
= (\dot{X} - Q\dot{N})^{-1}(\dot{Y} - Q\dot{M}),
\] (2-16) (2-17)
and define

\[ U = Y - MQ, \quad (2-18) \]
\[ V = X - NQ, \quad (2-19) \]
\[ \dot{U} = \dot{Y} - Q\dot{M}, \quad (2-20) \]

and

\[ \dot{V} = \dot{X} - Q\dot{N} \quad (2-21) \]

so that

\[ K = UV^{-1} = \dot{V}^{-1}\dot{U}. \quad (2-22) \]

From equation (2-13)

\[
\begin{bmatrix}
\dot{Y} & -\dot{U} \\
-N & \hat{M}
\end{bmatrix}
\begin{bmatrix}
M & U \\
N & V
\end{bmatrix} = I. \quad (2-23)
\]

In [4] Francis shows that for K given by (2-22) and \( G_{22} \) with the factorization of (2-10), equation (2-23) is true iff K stabilizes \( G_{22} \). Thus for every \( Q \) in \( RH^\infty \), there is a corresponding K which stabilizes \( G_{22} \) and therefore stabilizes G.

Now that all stabilizing controllers have been parameterized, the next step in the problem solution is to
parameterize the closed loop transfer function from \( W \) to \( Z \).

First, define

\[
T_1 = G_{11} + G_{12} \tilde{M} G_{21} \quad (2-24)
\]

\[
T_2 = G_{12} \tilde{M} \quad (2-25)
\]

\[
T_3 = \tilde{M} G_{21}. \quad (2-26)
\]

It turns out that these matrices belong to \( RH^\infty \). Substituting \( G_{22} = NM^{-1} \) and (2-17) into \( (I - KG_{22}) \) yields

\[
(I - KG_{22}) = (\dot{X} - Q\tilde{N})^{-1} \left[ \dot{X} - Q\tilde{N} - (\dot{Y} - Q\tilde{M})NM^{-1} \right]. \quad (2-27)
\]

Using (2-10) gives

\[
= (\dot{X} - Q\tilde{N})^{-1} \left[ \dot{X} - Q\tilde{N} - (\dot{\breve{Y}} - Q\breve{M})\tilde{M}^{-1}\tilde{N} \right] \quad (2-28)
\]

\[
= (\dot{X} - Q\tilde{N})^{-1} (\dot{X} - \dot{\breve{Y}}\tilde{M}^{-1}\tilde{N}) \quad (2-29)
\]

\[
= (\dot{X} - Q\tilde{N})^{-1} (\breve{X}M - \breve{Y}N)M^{-1}. \quad (2-30)
\]

Applying (2-11) gives

\[
(I - KG_{22}) = (\dot{X} - Q\tilde{N})^{-1} \tilde{M}^{-1} \quad (2-31)
\]

and

\[
(I - KG_{22})^{-1} = M(\breve{X} - Q\tilde{N}). \quad (2-32)
\]
Applying (2-17) again,

\[(I - KG_{22})^{-1}K = M(\bar{Y} - \bar{M}).\]  

(2-33)

Substituting (2-33) into (2-5) and applying the definitions for \(T_1\), \(T_2\), and \(T_3\) yields

\[Z = (T_1 - T_2QT_3)W.\]  

(2-34)

The model matching problem of equation (2-34) can be represented by the block diagram of Figure 2.3.

![Figure 2.3 Block Diagram For Model Matching Problem](image)

\[T_1\] represents a model which is to be matched by the cascade combination \(T_2QT_3\). The quantity \(Q\), the "controller", is to be designed such that \(Q\) is in \(RH^*\) and

\[\|T_1 - T_2QT_3\|_\infty = \text{minimum}.\]  

(2-35)

For each allowable \(Q\), there is a model matching error
\[ \| T_1 - T_2 QT_3 \|_\ast. \]

The smallest possible error shall be denoted

\[ \alpha = \inf \{ \| T_1 - T_2 QT_3 \|_\ast : Q \text{ in } RH^\circ \}. \quad (2-36) \]

The corresponding \( Q \) is said to be optimal.

Obviously, this problem is trivial if a \( Q \) in \( RH^\circ \) exists such that

\[
T_1 = T_2 QT_3 \\
Q = T_2^{-1} T_1 T_3^{-1}.
\]

However, since \( T_1, T_2, T_3, \) and \( Q \) are in \( RH^\circ \), this solution only exists when \( T_2 \) and \( T_3 \) have inverses and the inverses are in \( RH^\circ \). This is equivalent to requiring that \( T_2 \) and \( T_3 \) are non-strictly proper and have no zeros in the right half-plane. This seldom occurs in practice, since it means that the optimal model matching error is zero and that \( W \) can be completely decoupled from \( Z \).

For the case when the optimal model matching error is nonzero, the \( H^\infty \) optimization step becomes necessary. The following explanation summarizes the process for the MIMO case with \( T_2 \) and \( T_3 \) square (called the 1-block case), as given in Francis [4]. Francis also gives the solution to the more complicated MIMO case with nonsquare \( T_2 \) and \( T_3 \). A simpler
treatment relating the SISO case to the 1-block MIMO case is given in [6]. Before continuing, more definitions are needed.

DEFINITION

Every matrix $G$ in $\mathbb{RH}^n$ has an **inner-outer factorization**

$$G = G_i G_o$$

where

$$G_i G_o = I$$

with $G_i$ *inner* and $G_o$ *outer*. In the scalar case, this means that $G_o$ is minimum phase and $G_i$ is an all-pass function such that $G_i G_i^{-1} = 1$.

DEFINITION

A matrix $G$ has a **co-inner-outer factorization** of the form

$$G = G_{co} G_{ci}$$

where

$$(G_{ci}^\dagger) (G_{co}^\dagger) = I$$

and $G_{co}^\dagger$ has a right inverse in $\mathbb{RH}^n$. 
Now, to begin the $H^\infty$ optimization, first introduce an inner-outer factorization of $T_2$ and a co-inner-outer factorization of $T_3$ given as

$$T_2 = T_{2i} T_{2o} \quad (2-41)$$
$$T_3 = T_{3co} T_{3ci} \quad (2-42)$$

Now

$$T_1 - T_2 Q T_3 = T_1 - T_{2i} T_{2o} Q T_{3co} T_{3ci} \quad (2-43)$$

By definition, inner and co-inner factors have unity infinity norm so

$$\| T_1 - T_{2i} T_{2o} Q T_{3co} T_{3ci} \|_\infty = \| T_{2i}^{-1} T_1 T_{3ci}^{-1} - T_{2o} Q T_{3co} \|_\infty \quad (2-44)$$

Making the definitions

$$R = T_{2i}^{-1} T_1 T_{3ci}^{-1} \quad (2-45)$$
$$X = T_{2o} Q T_{3co} \quad (2-46)$$

yields

$$\| T_1 - T_2 Q T_3 \|_\infty = \| R - X \|_\infty \quad (2-47)$$
Thus, the co-inner-outer factorization of $T_3$ is used to apply a norm preserving transformation to the form of (2-47). Now an $X$ must be found that minimizes the norm, after which (2-46) can be used to find $Q$. Once $Q$ is found, (2-16) can be used to calculate the controller $K$.

To find the optimal $X$, a minimal stable/antistable decomposition of $R$ is needed such that

$$R = R_1 + R_2$$

(2-48)

where $R_1$ is minimal and completely unstable, and $R_2$ is minimal and completely stable. The stable part of $R$ is now included in $X$ by letting

$$X = X_1 + R_2.$$  

(2-49)

Now

$$\|T_1 - T_2 Q T_3\|_\infty = \|R_1 - X_1\|_\infty$$  

(2-50)

and $R_1$ is completely unstable and $X_1$ is completely stable. Thus, the model matching problem has been reduced to finding the closest stable approximation of an antistable matrix. This problem is sometimes called the Nehari problem.

The next step is to find $\alpha$, the optimal model-matching error:
\[ \alpha = \min(\|T_1 - T_2 QT_3\|) = \min(\|R_1 - X_1\|). \] (2-51)

In the case when \( T_2 \) and \( T_3 \) are square and of equal dimension, \( \alpha \) can be calculated via

\[ \alpha^2 = \max \{ \text{eigenvalues of } W_c W_o \} \] (2-52)

where \( W_c \) and \( W_o \) are the controllability and observability grammians of a state-space representation of \( R_1 (A,B,C) \). The grammians can be calculated by solving the Lyapunov equations

\[ AW_c + W_c A^T = BB^T \] (2-53)
\[ A^T W_o + W_o A = C^T C. \] (2-54)

Note that these equations differ from the usual Lyapunov equations for computing the grammians of stable systems. In the more general case, an upper bound for \( \alpha \) is found by iteration and spectral factorization. This more general case is illustrated by Francis in [4].

Francis casts the problem such that \( \alpha \) need only be less than unity. This is not restrictive, as in [6] the authors give the following explanation which shows that it is possible to get arbitrarily close to the optimal \( \alpha \).

From (2-51)

\[ \alpha/(\alpha + \beta) = \min(\|R_1/(\alpha + \beta) - X_1/(\alpha + \beta)\|) \] (2-55)
where \( \beta > 0 \). Now define

\[ R' = R_1/ (\alpha + \beta) \]  \hspace{1cm} (2-56)

and

\[ X' = X_1/ (\alpha + \beta). \] \hspace{1cm} (2-57)

Equation (2-55) becomes

\[ \alpha/(\alpha + \beta) = \min(\|R' - X'\|_\infty) < 1. \] \hspace{1cm} (2-58)

Now if \( X' \) can be found such that

\[ \|R' - X'\|_\infty \leq 1, \] \hspace{1cm} (2-59)

the inequality

\[ \alpha/(\alpha + \beta) \leq \min(\|R' - X'\|_\infty) \leq \|R' - X'\|_\infty \] \hspace{1cm} (2-60)

can be formed. Since \( \beta \) is an arbitrary parameter, \( X' \) can be pushed as close to optimal as desired.

Finally, \( X' \) must actually be calculated such that (2-59) holds. Let \((A,B,C)\) be a state-space realization of \( R' \) and define

...
\[ N = (I - W_0 W_e)^{-1} \quad (2-61) \]
\[ L_1 = (A, N^T, C, 0) \quad (2-62) \]
\[ L_2 = (-A^T, NW_0 B, B^T, I). \quad (2-63) \]

A nonunique \( X' \) which satisfies (2-59) is

\[ X' = R' - L_1 L_2^{-1}. \quad (2-64) \]

Now, by stepping back through the various substitutions and parameterizations, the \( H^\infty \) controller can be calculated. First, \( X_1 \) is calculated from (2-57). Next, \( X \) is found via (2-49). Now \( Q \) can be calculated by solving (2-46), and \( K \) can then be found by applying (2-16) or (2-17).

Though the solution to the \( H^\infty \) optimization problem has been described by a relatively simple algorithm here, it is important to note that some important difficulties become apparent when the algorithm is implemented. First, most of the steps in the solution require several calculations themselves. Because of this, as many as twenty-nine Riccati equation solutions may be required before a controller is calculated. In addition, as the algorithm progresses, the order of the realizations increases geometrically. This is because of the repeated algebraic operations on transfer function matrices. As a result, it is not uncommon for the controller calculated using this approach to have five times the number of states as that of the design plant.
2.2 State-space Approach

The recent work of Glover and Doyle [5] is a significant advance in the $H^\infty$ field. They give state-space design equations that result in a controller of the same order as the design model. Furthermore, only two Riccati solutions are required. Most of the proofs relating to the equations in [5] may be found in [7].

For the problem cast in standard form, the system equations are

$$\dot{x} = Ax + B_1w + B_2u \quad (2-65)$$

$$z = C_1x + D_{11}w + D_{12}u \quad (2-66)$$

$$y = C_2x + D_{21}w + D_{22}u \quad (2-67)$$

where $w$ is in $\mathbb{R}^m$, $u$ is in $\mathbb{R}^n$, $z$ is in $\mathbb{R}^p$, and $y$ is in $\mathbb{R}^q$. The vector $w$ is the disturbance vector, $z$ is the performance related vector, and $y$ is the vector of measurements. The problem being solved here is the suboptimal $H^\infty$ problem of finding a stabilizing controller $K$ satisfying a norm constraint on the closed loop transfer function from $w$ to $z$ such that

$$\|G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}\|_\infty < \Gamma \quad (2-68)$$
for some prespecified real \( \Gamma \). The optimal solution may be found by iterating to find the smallest \( \Gamma \) for which a stabilizing \( K \) exists.

There are several assumptions relating to problem setup that must be satisfied before the design equations may be applied. The first of these is that \((A, B_2, C_2)\) be stabilizable and detectable. This is required so that a stabilizing controller exists. The next assumption is related to the requirement of including the control signal in the performance index for an LQG problem:

\[
\text{rank } D_{12} = m_2. \tag{2-69}
\]

Also, there is a requirement that the disturbance appear in the measurements via

\[
\text{rank } D_{21} = p_2. \tag{2-70}
\]

An assumption closely related to the above rank requirements is that \( D_{12} \) and \( D_{21} \) be of the form

\[
D_{12} = [0 \ I]^T \tag{2-71}
\]

and

\[
D_{21} = [0 \ I]. \tag{2-72}
\]
The norm preserving transformation which takes $D_{12}$ and $D_{21}$ to this form is derived in [6]. Finally, the last assumption requires that the realizations for $G_{12} (A,B_2,C_1,D_{12})$ and $G_{21} (A,B_1,C_2,D_{21})$ be minimal and that

\begin{align*}
\text{rank } G_{12}(j\omega) &= m_2 \text{ for all } \omega, \quad (2-73) \\
\text{rank } G_{21}(j\omega) &= p_2 \text{ for all } \omega. \quad (2-74)
\end{align*}

If any of the above assumptions is violated, a change in problem setup is required. As will be shown later, frequency dependent weighting functions are typically needed when applying these equations to most physical systems. This is because most physical systems are low-pass and the rank conditions on $G_{12}$ and $G_{21}$ will not be satisfied without adding weighting functions.

For ease of notation, the solution to the algebraic Riccati equation

$$Q + XA + A^TX - XPX = 0 \quad (2-75)$$

will be denoted by its Hamiltonian matrix

$$X = \text{Ric} \begin{bmatrix} A & -P \\ -Q & -A^T \end{bmatrix}. \quad (2-76)$$
First, $D_{11}$ is partitioned

\[
D_{11} = \begin{bmatrix}
D_{1111} & D_{1112} \\
D_{1121} & D_{1122}
\end{bmatrix}
\begin{bmatrix}
p_1 - m_2 \\
m_2
\end{bmatrix}
\]

and two intermediate variables are defined

\[
D_{1x} = [D_{11} \ D_{12}] \tag{2-78}
\]

\[
D_{x1} = [D_{11}^T \ D_{21}^T]^T. \tag{2-79}
\]

The following definitions are made:

\[
R = D_{1x}^T D_{1x} - \begin{bmatrix}
\Gamma^2 I_{m1} & 0 \\
0 & 0
\end{bmatrix}
\]

and

\[
\tilde{R} = D_{x1} D_{x1}^T - \begin{bmatrix}
\Gamma^2 I_{p1} & 0 \\
0 & 0
\end{bmatrix}. \tag{2-81}
\]

Next, two Riccati equations are solved:
\[ X_0 = \text{Ric} \begin{bmatrix} U & -P \\ -Q & -U^T \end{bmatrix} \]

where

\[ P = BR^{-1}B^T, \quad U = A - BR^{-1}D_{1x}C_1, \quad \text{and} \quad Q = C_1^T C_1 - C_1^T D_{1x} R^{-1} D_{1x}^T C_1 \]

and

\[ Y_0 = \text{Ric} \begin{bmatrix} U & -P \\ -Q & -U^T \end{bmatrix} \]

where now

\[ P = C^T \tilde{R}^{-1} C, \quad U = A^T - C^T \tilde{R}^{-1} D_{x1} B_{1x}^T, \quad \text{and} \quad Q = B_{1x} B_{1x}^T - B_{1x} D_{x1} \tilde{R}^{-1} D_{x1} B_{1x}^T. \]

The state feedback and "output injection" matrices are defined as

\[ F = -R^{-1} \left[ D_{1x}^T C_1 + B^T X_0 \right] = \begin{bmatrix} F_{11} & F'_{12} \\ F_{12} & F_{2} \end{bmatrix} \]

\[ H = -[B_{1x} D_{x1}^T + Y_0 C^T] \tilde{R}^{-1} = [H_{11}, H_{12}, H_{2}]. \]

Once the above calculations have been made, it is necessary to perform some tests to ensure that a stabilizing controller exists which satisfies the norm constraint of (2-68). The first test is again related to proper problem setup and is related to the direct feedthrough of \( w \) to \( z \):
Next, the Riccati equation solutions must both be positive semi-definite as well as satisfy

\[ \mu_{\text{max}}(X_n Y_n) < \Gamma, \]  

(2-87)

where \( \mu \) is an eigenvalue. If any of these tests is failed, \( \Gamma \) may be increased and the process may be repeated. If all the tests are passed, an \( n \)th order controller satisfying the norm constraint of (2-68) is given by

\[ K = (\hat{A}, \hat{B}_1, \hat{C}_1, \hat{D}_{11}) \]  

(2-88)

where

\[ \hat{B}_2 = (B_2 + H_{12}) \hat{D}_{12}, \]  

(2-89)

\[ \hat{C}_2 = -\hat{D}_{21} (C_2 + F_{12}) Z, \]  

(2-90)

\[ \hat{B}_1 = -H_2 + \hat{B}_2 \hat{D}_{12}^{-1} \hat{D}_{11}, \]  

(2-91)

\[ \hat{C}_1 = F_2 Z + \hat{D}_{11} \hat{D}_{21}^{-1} \hat{C}_2, \]  

(2-92)

\[ \hat{A} = A + H C + \hat{B}_2 \hat{D}_{12}^{-1} \hat{C}_1 \]  

(2-93)

and

\[ Z = (I - \Gamma^{-2} X_n Y_n)^{-1}. \]  

(2-94)
Also, \( \hat{D}_{12} \) and \( \hat{D}_{21} \) are any matrices (e.g. Cholesky factors) which satisfy

\[
\hat{D}_{12} \hat{D}_{12}^T = I - D_{1121} (\Gamma^2 I - D_{1111}^T D_{1111}^T)^{-1} D_{1121}^T \tag{2-95}
\]

and

\[
\hat{D}_{21}^T \hat{D}_{21} = I - D_{1112}^T (\Gamma^2 I - D_{1111}^T D_{1111}^T)^{-1} D_{1112}. \tag{2-96}
\]

Note that this controller is in different coordinates than the original plant, since a norm preserving transformation was applied to satisfy equations (2-71) and (2-72). When the transformation is reversed, the resulting controller is in the original coordinates and satisfies the norm constraint of (2-68).
Chapter 3
Facility Description

The facility used for this work is the NASA Large Space Structure Ground Test Facility (LSS GTF) at Marshall Space Flight Center in Huntsville, Alabama. The facility was created as a test bed for the comparison of system identification and control system design techniques, as well as different sensor, actuator, and control computer hardware. The basic configuration is shown in Figure 3.1.

Figure 3.1 MSFC LSS Ground Test Facility
3.1 Structure

The ACES configuration consists of a flexible beam which is attached to a 3-axis gimbal via a base mounting plate, which is in turn attached to a 2-axis motion table. The beam is the deployable/retractable flight spare magnetometer boom for the Voyager spacecraft. It has three longerons which are continuous through its 45 foot length, and is triangular in cross-section with each side measuring eight inches. There are 91 flexible batons in compression along the length of the boom connected by diagonal cross-members in tension. In the present configuration, the beam exhibits a 260 degree longitudinal twist of the tip in relation to the base.

A three-legged, non-symmetrical cruciform is attached at the beam tip. On one leg of the cruciform rests the antenna with optical detector and mirror. The other two legs are used as counterweights. Two appendages are attached near the base of the mast, one of which supports the Image Motion Compensation (IMC) mirror pointing gimbal assembly. The other is used as a counterweight.

In the present configuration, the structure shows 43 vibrational modes below 9 Hz, with the lowest frequency mode being a torsional mode at roughly .08 Hz.
3.2 Actuators

The current set of actuators includes:

(a) the two-axis Base Excitation Table (BET),
(b) three torque motors on the augmented Advanced Gimbal System (AGS),
(c) two Linear Momentum Exchange Devices (LMED's), each with two actuators, and
(d) two Image Motion Compensation system (IMC) torque motors.

Though the BET can be commanded by the control computer as well as a programmable signal generator, it is not used as a control actuator, but rather as a means of disturbance input. Various disturbance profiles are stored on disk in the control computer and can be used to program the signal generator.

The AGS is a high precision 2-axis gimbal system which was originally designed for high-accuracy pointing applications. It has been augmented with a third gimbal and a torque motor for control in the roll axis. The AGS torquers have +/- 30 degrees of range and are capable of 50 N-m of torque, while the roll axis has a +/- 5 degree range and a torque limit of 18.7 N-m.

The two LMED's are mounted on the mast at positions 6.43 and 11.43 meters, respectively, from the base mounting plate. Each device consists of two orthogonal proof-mass actuators.
aligned with the AGS X and Y axes, along with collocated accelerometers and linear variable differential transformers (LVDT's) in both axes. The LVDT position measurements are used in analog control loops which command the proof mass actuators. The optional digital loop which includes the control computer will not be used in this application.

The IMC system consists of a laboratory-fixed 5 mW laser source, two circular mirrors 30.5 cm in diameter, an optical position detector with associated microprocessor and electronics, two pointing gimbals, a servo-controller, and control electronics. One of the mirrors is fixed on the antenna near the optical detector. The other is mounted on the IMC pointing gimbals. The IMC torquers have a bandwidth of 50 Hz and a range of approximately +/- 5 degrees. In this application, the torquers will receive torque commands from the control computer so the analog servo loops will not be used.

3.3 Sensors

Though the LSS GTF facility has several sensor packages located at various locations on the structure, in this application only one will be used as inputs to the control law implemented by the control computer.

The IMC detector, located inside the antenna mounted on the cruciform structure at the mast tip, is a 144 cell
photodetector which measures the location of the laser beam
to an accuracy of .025 cm. A local Intel Z80 microprocessor
is used to resolve the beam position and then provide X and
Y displacement information to the control computer. A
separate presence signal is also sent to the control computer
to indicate whether or not the beam is hitting the detector.
If not, the IMC servo-controller initiates a scan sequence to
place the beam back on the detector.

3.4 Computer System

The current computer system in use at the GTF consists
of:

(a) an MSFC built COSMEC computer which performs all I/O
functions,

(b) an HP-9000 computer running a version of the
HP-BASIC operating system with compiling capability,
and

(c) an Analogic Array Processor which utilizes
the HP-BASIC operating system.

The COSMEC is an in-house built microcomputer used for
I/O processing. The tasks it performs include sensor input
processing, force and torque command output to the actuators,
and sensors and actuators interfacing with the HP-9000.
Currently, the COSMEC processes 25 sensors and 9 actuators at
a sample rate of 50 Hz. Of the 20 millisecond sample period,
the COSMEC requires roughly 5 milliseconds.

The HP-9000 is a 32-bit machine running at 18 MHz which is responsible for performing the strapdown algorithm, control algorithm, data storage, real-time plotting, and review of safety information. Previous testing shows that the strapdown algorithm and other overhead requires 6 milliseconds which leaves roughly 10 milliseconds to implement the control law.

The Analogic array processor was added to increase the size of the controller which can be implemented in the allotted 10 milliseconds. Currently, controllers with 100 states can be implemented while still maintaining the 50 Hz sample rate.
In order to apply $H^\infty$ techniques to most problems, it is necessary to incorporate frequency dependent weighting functions to achieve performance and robustness goals. Not only are these weighting functions used as design "knobs", but they are often necessary so that the rank conditions on the open loop D matrix are satisfied. The goal of the plant weighting process is to form an $H^\infty$ problem such that satisfying an infinity norm constraint guarantees certain levels of performance and robustness.

4.1 Weighting Theory

The weighting scheme illustrated here is that of Safonov and Chiang [8]. The closed loop configuration is shown in Figure 4.1.

![Figure 4.1](image-url)
The transfer functions from $R$ to the signals $E$ and $Y$ are given by

$$S(s) = E(s)/R(s) = [I + G(s)K(s)]^{-1} \quad (4-1)$$

and

$$T(s) = Y(s)/R(s) = G(s)K(s)[I + G(s)K(s)]^{-1}. \quad (4-2)$$

$S(s)$ and $T(s)$ are sometimes called the "sensitivity function" and "complementary function", respectively. For a disturbance attenuation problem, a performance constraint might be defined as

$$\|S(j\omega)\|_\infty \leq |W_1^{-1}(j\omega)| \quad (4-3)$$

where $|W_1^{-1}(j\omega)|$ is a frequency dependent bound on disturbance attenuation. This constraint will be satisfied if

$$\|S(j\omega)W_1(j\omega)\|_\infty \leq 1. \quad (4-4)$$

As the sensitivity function is used to analyze performance in multivariable systems, the complementary sensitivity function is used to study the robustness of multivariable systems when they are subjected to multiplicative plant perturbations. Suppose the plant in
Figure 2.1 is perturbed by a stable $M(s)$ so that the open loop transfer function matrix is given by

$$G'(s) = G(s) + G(s)P(s).$$

(4-5)

As stated in [8], the smallest (in infinity norm) $P(s)$ which will cause $G'(s)$ to be unstable is given by

$$\|P(j\omega)\|_\infty = 1/\|T(j\omega)\|_\infty.$$  

(4-6)

Thus, as the norm of $T(s)$ is decreased, the norm of the smallest destabilizing perturbation increases. A robustness constraint can be expressed as

$$1/\|T(j\omega)\|_\infty \geq \|W_3(j\omega)\|_\infty,$$  

(4-7)

which is equivalent to requiring

$$\|T(j\omega)W_3(j\omega)\|_\infty \leq 1.$$  

(4-8)

Note that since the complimentary sensitivity function is just the closed loop transfer function of the system in Figure 4.1, this norm constraint also provides a means for requiring high frequency roll-off of the closed loop system.

If the performance variable $Z$ is formed such that the transfer function from $W$ to $Z$ in Figure 2.1 is given by
and the controller is designed such that

\[ \|G_{cl}(j\omega)\|_\infty \leq 1, \]

the performance and robustness constraints will be satisfied. In [8], the authors show that a necessary condition for the existence of a solution to the "small gain" problem is that

\[ \sigma(W_1^{-1}(j\omega)) + \sigma(W_3^{-1}(j\omega)) > 1 \text{ for all } \omega \]

4.2 Plant Augmentation

To form the performance variable \( Z \) as described, the block diagram of Figure 4.2 is used.

![Figure 4.2 Block Diagram for Plant Augmentation](image-url)
The plant augmentation appears to be straightforward. However, to enforce high frequency closed loop plant roll-off as well as the rank requirements on the open loop plant $D$ matrix, it may be necessary that $W_3$ be represented by an improper transfer function (numerator order greater than denominator order). Since such a transfer function cannot be represented by state equations, this presents difficulties when the designer attempts to use state space techniques to form the augmented plant. The strategy taken here is to incorporate the improper weight $W_3$ into the proper plant.

The improper weighting algorithm described here relies on two limiting assumptions. The first is that the open loop plant is in modal form, and the second is that $W_3$ has the form

$$W_3 = K(s^2 + 2\zeta \omega_n s + \omega_n^2)$$ \hspace{1cm} (4-11)

For each mode, the original plant can be represented by the block diagram of Figure 4.3.

![Block diagram of Figure 4.3](image-url)
The associated state matrices for each mode are shown below.

\[
A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ k_1 & k_2 & \ldots & k_p \end{bmatrix}
\]

\[
C = \begin{bmatrix} g_{1n} & 0 \\ g_{2n} & 0 \\ \vdots & \vdots \\ g_{mn} & 0 \end{bmatrix} \quad D = 0
\]

where \( p \) is the number of inputs and \( m \) is the number of outputs. The gains \( k_1 - k_p \) and \( g_{1n} - g_{mn} \) denote the actuator and sensor gains for the \( n \)th mode.

Inserting the improper weight \( W_3 \) into the second order block results in the block diagram of Figure 4.4, which can be simplified to that of Figure 4.5.
The state equations describing the relationship between v and p can be written as

\[ r = \ddot{v} + 2\zeta_n \dot{v} + \omega_n^2 v. \]  

(4-12)

Letting

\[ x_1 = v \]  

(4-13)

\[ x_2 = \dot{v} \]  

(4-14)

gives

\[ \dot{x}_1 = x_2 \]  

(4-15)

\[ \dot{x}_2 = -\omega_n^2 x_1 - 2\zeta_n \omega_n x_2 + r \]  

(4-16)

and
\[ p = (2\zeta_3 \omega_3 - 2\zeta \omega_n) x_2 + (\omega_3^2 - \omega_n^2) x_1 + r \quad (4-17) \]

so the relationship between \( r \) and \( p \) can be represented by the state matrices:

\[
A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta \omega_n \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ 1 & 1 & \ldots & 1 \end{bmatrix} \\
C = \begin{bmatrix} K(\omega_3^2 - \omega_n^2) & 2K(\zeta_3 \omega_3 - \zeta \omega_n) \end{bmatrix} \quad D = K
\]

Now, to include the actuator and sensor gains, we need only to incorporate them into the \( B, C \) and \( D \) matrices above. If we note that the \( B \) matrix for the original plant contains the actuator gains, and that the \( C \) matrix for the original plant contains the sensor gains, we can see that the addition of the improper weight only requires that we change the \( C \) matrix and add a \( D \) matrix.

The new \( C \) matrix is the product of the \( C \) matrix for the unweighted plant and the \( C \) matrix shown above. The \( D \) matrix is simply the product of \( K \) and the actuator and sensor gain matrices. This case is easily generalized to the multi-mode case.
Chapter 5

H∞ Controller Design

This chapter describes the evolution of the H∞ controller design as well as the models used for design and evaluation of each design. In Chapter 5 and Chapter 6, most of the figures show two curves. In the figures that do not explicitly state an individual LOS axis, both LOS axes are included. The dominant behavior in the BET to LOS data is that of the "on axis" loops (e.g. BETx to LOSx). The dominant behavior in the IMC to LOS data is that of the "cross axis" loops (e.g. IMCx to LOSy).

5.1 Model Description

Two models were used for the design and evaluation of the H∞ controller. Both were taken from a continuous finite element model (FEM) generated by Control Dynamics Company. The Control Dynamics FEM has 50 modes ranging in frequency from zero to approximately 18 Hz.

Since the control problem being addressed here is that of minimizing the LOS error at the detector by controlling the IMC pointing gimbals, the design model has the IMC gimbal torques as inputs and the LOS error positions as outputs. The singular value based model reduction scheme proposed in [9] was used to reduce the number of states to 12 (6 modes).
Next, the reduced order continuous model was discretized and a sample period delay (.02 seconds) was added at each output. Finally, the resultant 14 state discrete model was transformed to the w-plane via the bilinear transformation

\[ w = \frac{2(z - 1)}{T(z + 1)} \]  \hspace{1cm} (5-1)

where \( T \) is the sample period. The magnitude frequency responses from the IMC gimbles to the LOS error positions are shown in Figure 5.1 and Figure 5.2.

![Design Model Frequency Response From IMCx to LOS](image)

**Figure 5.1** Design Model Frequency Response From IMCx to LOS
The evaluation model is different from the design model in that it is a discrete model and contains all 50 modes from the original FEM. In addition, the X and Y BET forces are added as inputs and the IMC pointing gimbal torques are added as outputs. The open loop frequency responses from the BET and the IMC gimbals to the LOS are shown in figures 5.3-5.6.
Figure 5.3 Evaluation Model Frequency Response From BETx to LOS

Figure 5.4 Evaluation Model Frequency Response From BETy to LOS
Figure 5.5 Evaluation Model Frequency Response From IMCx to LOS

Figure 5.6 Evaluation Model Frequency Response From IMCy to LOS
5.2 Design Approach

The approach used here was to design a "tight" IMC loop so that disturbances at the BET could not hurt performance within the design bandwidth. A software package developed by the author and Dr. R.D. Irwin, including a module employing the state-space method of Glover and Doyle, was used to augment the plant and calculate the controller.

Given that both the design model and evaluation model were derived from an FEM, a certain amount of conservatism was indicated in terms of the level of performance to be achieved by the design. Previous experience by other investigators using this FEM showed that modeling errors existed at frequencies at least as low as 8 Hz. Because of this and the fact that the sample rate was only 50 Hz, a closed loop bandwidth just beyond the frequency of the IMC pendulum behavior (approximately 10 rad/s) could be termed a success.

Each design iteration consisted of the following steps:

(a) Design the weighting functions, ensuring that the inequality described in [8] holds:

\[ \hat{\sigma} (W_1^{-1}(j\omega)) + \hat{\sigma} (W_3^{-1}(j\omega)) > 1 \text{ for all } \omega. \]

(b) Incorporate the improper weight \( W_3 \) into the design plant.

(c) Augment the plant as shown in Figure 4.2.
(d) Calculate the $H^\infty$ w-plane controller.

(e) Form the closed loop system using the unaugmented, w-plane design model and w-plane controller.

(f) Check that the sensitivity function and complementary sensitivity functions satisfy constraints.

(g) Analyze the singular value response (or frequency response) of the controller.

(h) Transform the controller to the z-plane using the bilinear transformation.

(i) Form the closed loop system using the discrete evaluation model.

(j) Simulate BET disturbances and evaluate performance.

The weighting functions used in the initial design are shown below in Figure 5.7. The 0 dB crossover of $W_1^{-1}$ requires disturbance attenuation up to approximately 20 rad/s, while the 0 dB crossover of $W_3^{-1}$ requires a 20 dB per decade closed loop roll-off above 100 rad/s. In addition, these weights prescribe a tolerance to multiplicative plant variations less than 40 dB, and 10:1 disturbance attenuation below approximately 10 rad/s.
Figure 5.7 Inverse Weighting Functions For First Design

Figure 5.8 shows the singular value response of the resultant $w$-plane controller, while Figure 5.9 shows the resultant closed loop $w$-plane system. Clearly, the bandwidth constraint has been met. However, both figures show an undesired peaking at 100 rad/s.
Figure 5.8 Controller Singular Value Response For First Design

Figure 5.9 Closed Loop Singular Value Response For First Design
Successive attempts at lowering the 0 dB crossover frequency of $W_3^{-1}$ while maintaining a bandwidth just over 10 rad/s resulted in the final design. The weighting functions for this design are shown in Figure 5.10.

Any attempt to further lower the 0 dB crossover of $W_3^{-1}$ resulted in failure of one or more of the tests posed by the controller calculation algorithm. As the crossover was lowered, the two Riccati equations became more ill-conditioned, causing the solutions not to be positive semidefinite, or for the eigensystem of the solutions to become so ill-conditioned that the eigenvalues could not even
be accurately calculated to determine positive semi-
definiteness.

The singular value responses of the resulting controller and closed loop system are shown below in Figure 5.11 and 5.12. With this design, the controller still shows the peaking behavior near 100 rad/s but the closed loop system no longer peaks above 0 dB.

![Controller Singular Value Response](image)

**Figure 5.11 Controller Singular Value Response For Final Design**
Figure 5.12 Closed Loop Singular Value Response For Final Design

Figure 5.13 shows the singular value responses of the sensitivity function and $W_3^{-1}$. That the two curves are so close at lower frequencies shows that the design is at the limit of performance with the given robustness constraint. Figure 5.14 shows the singular value responses of the complimentary sensitivity function (closed loop transfer function) and $W_1^{-1}$. The bound is tight here also, so the design is indeed at the limit of what is possible with this particular control problem.
Figure 5.13 Sensitivity Function and Performance Weighting For Final Design

Figure 5.14 Complimentary Sensitivity Function and Robustness Weighting For Final Design
It is important to note that all controllers calculated during the design iterations showed strong notching at 10 rad/s. This shows that the controller is "trying" to notch out the IMC pendulum modes. In fact, the loop singular value response shown in Figure 5.15 shows that the controller has completely notched these modes. This raises questions as to the actual robustness achieved. Though stability is guaranteed via $W_3$ if multiplicative plant disturbances are less than 40 dB, failure to exactly identify the frequency of the lightly damped IMC pendulum behavior in the model may cause an error in excess of 40 dB, with possibly destabilizing results.

Figure 5.15 Loop Singular Value Response With Final Design
The open loop evaluation model LOS response to an X axis BET position pulse is shown in Figure 5.16. Figure 5.17 shows the closed loop LOS response to the same disturbance. Figures 5.18 and 5.19 show the open and closed loop LOS response to Y axis BET position pulses. Figures 5.20 and 5.21 show the open and closed loop LOS response due to the X axis "demonstration" disturbance. The LOS responses to the Y axis demonstration disturbance are shown in Figure 5.22 and Figure 5.23.

The demonstration disturbance is comprised of three main segments. For the first .08 seconds, the position profile ramps up. For the next .3 seconds the profile is flat, and for the final .08 seconds the profile ramps back down to zero.

![Open Loop Response to BETx Pulse](image)

Figure 5.16 Open Loop Response to BETx Pulse
Figure 5.17 Closed Loop Response to BETx Pulse

Figure 5.18 Open Loop Response to BETy Pulse
Figure 5.19  Closed Loop Response to BETy Pulse

Figure 5.20  Open Loop Response to Demo X Disturbance
Figure 5.21  Closed Loop Response to Demo X Disturbance

Figure 5.22  Open Loop Response to Demo Y Disturbance
The closed loop results show that the controller is achieving moderate performance improvement to both the pulse and demonstration disturbances in both axes. The dominant behavior in both the open and closed loop responses at approximately 1.6 Hz is the IMC pendulum behavior noted in the discussion of the design model frequency responses. The damping of these modes has not been significantly increased since the controller depends on notching in the IMC to LOS loop. Since the BET was not included in the design model, the notching is not completely effective when the pendulum modes are excited by the BET. This fact is illustrated in the block diagram of Figure 4.1 which shows that the forward path from
the disturbance to the measurement does not include the cascade combination of the plant and the controller. As a result, modes in the transfer function from the disturbance to the measurements may not be affected.

It is also interesting to note that an $H^\infty$ controller was also designed using the PRO-MATLAB Robust Control Toolbox function "hinf" and the weighting functions of the final design. This controller did not roll off at high frequency. Obviously, it is desirable for the controller to roll off at frequencies above the design bandwidth so that higher frequency modes will not be destabilized. The low frequency characteristics of the MATLAB controller were identical to that of the controller calculated by the author's software up to the design bandwidth.
Chapter 6
Experimental Results

The experiment was conducted in three segments. The first involved open loop transfer function identification for the BET to LOS and IMC to LOS loops. The next involved a closed loop time response analysis of three controllers. The third step was closed loop transfer function identification using the best performing controller.

Transfer function identification for both open and closed loop tests was completed as follows. For each of the four inputs \((\text{BET}x, \text{BET}y, \text{IMC}x, \text{IMC}y)\), a 328 second random noise excitation was given. The response at both LOS axes was recorded, and the resulting time response data was collected resulting in 16,400 data points (due to 50 Hz sample rate) for each test. The first 16,384 points were used to calculate and average eight 2048 point FFT's. The "spectrum" function in the MATLAB Signal Processing Toolbox was used to calculate the FFT's as well as the coherence function for each test. Time and data storage constraints prevented longer tests which may have yielded better results. The open loop magnitude frequency responses from the BET to the LOS are shown in figures 6.1-6.4
Figure 6.1 Open Loop Frequency Response From BETx to LOSx

Figure 6.2 Open Loop Frequency Response From BETx To LOSy
Figure 6.3 Open Loop Frequency Response From BETy To LOSx

Figure 6.4 Open Loop Frequency Response From BETy To LOSy
Figure 6.5 shows the coherence function for the BETx to LOSx transfer function, which is similar to the coherences seen for the other three BET to LOS transfer function tests. The low coherence values, especially at higher frequencies, show that the transfer functions are not well characterized. Repeated attempts to characterize the BET to LOS transfer functions using different excitation and different FFT lengths failed to improve the coherence values. This is probably due to nonlinear effects present at the BET, and that the BET position command signal, rather than the actual BET position, was used to calculate the transfer functions.

Another limitation is due to the fact that FFT's produce frequency domain data with linearly spaced points. As a result, to obtain a desirable number of points in the low frequency range, a very large number of time domain data points must be used. With a fixed length of time domain data, it was decided not to sacrifice the number of averages to take higher resolution FFT's. Clearly, the low frequency behavior (below 1 rad/s) is not characterized well here. However, since the problem at hand is to compare open and closed loop results rather than to perform system identification, the results obtained are still useful.
Figure 6.5  Coherence Function For Open Loop BETx To LOSx Transfer Function

Figure 6.6 and Figure 6.7 show the open loop transfer function magnitudes from the IMC to the LOS. Figures 6.8 and 6.9 show the phase for the IMCx to LOSy and the IMCy to LOSx transfer functions. Figure 6.10 shows the coherence function for the IMCx to LOSy test.
Figure 6.6 Open Loop Frequency Response From IMCx To LOS

Figure 6.7 Open Loop Frequency Response From IMCy To LOS
Figure 6.8 Open Loop Phase Response From IMCx to LOSy

Figure 6.9 Open Loop Phase Response From IMCy to LOSx
From the coherence function shown in Figure 6.10, which is similar to the coherence functions for the other three IMC to LOS transfer functions, it is evident that at least the behavior within the design bandwidth has been characterized fairly well. However, it is clear that the low frequency gains of the IMC to LOS frequency responses do not completely agree with those of the design model. In addition, the IMC pendulum modes (near 10 rad/s) appear to have more damping than in the design model.

The low frequency gain discrepancy was probably due to a slight misalignment between the cruciform arm supporting the antenna and the arm supporting the IMC gimbals. This
problem was accounted for by cascading an appropriate two-input two-output constant matrix after each controller to form "scaled" controllers.

Testing of the "initial design" controller quickly showed that it destabilized a mode at approximately 5 Hz. Successive attempts at scaling the controller outputs to maintain stability revealed that the closed loop system remained unstable until the scales were lowered to the point of achieving open loop behavior.

Next, the scaled version of the final design was tested. Figure 6.11 and Figure 6.12 show the open and closed loop LOS responses to a BET X axis pulse disturbance, while Figure 6.13 and Figure 6.14 show the responses to Y axis pulses.

![Open Loop LOS Response to BETx Pulse](image)

**Figure 6.11 Open Loop Response To BETx Pulse**
Figure 6.12 Closed Loop Response To BETx Pulse

Figure 6.13 Open Loop Response To BETy Pulse
Slight performance gains can be seen in the response to the BET X axis pulse, mostly in the removal of the initial offsets in both axes. The closed loop response to the BET Y axis pulse also shows removal of the initial offsets, but otherwise the behavior has not been significantly improved. In fact, the initial large amplitude response has been worsened. Also interesting to note is the 5 Hz resonance in the LOS X axis in both closed loop responses. This is the same behavior which was destabilized by the first controller tested. The open and closed loop responses to the demonstration disturbance in both BET axes are shown in figures 6.15-6.18.
Figure 6.15 Open Loop Response to Demo X Disturbance

Figure 6.16 Closed Loop Response to Demo X Disturbance
Figure 6.17 Open Loop LOS Response to Demo Y Disturbance

Figure 6.18 Closed Loop LOS Response to Demo Y Disturbance
The closed loop response to the X axis demonstration disturbance shows a moderate performance improvement, but again the improvement is less in the Y axis. By comparing the open loop responses, it is evident that the Y axis disturbance excites more higher frequency modes than the X disturbance which seems to only excite two modes (one near .8 Hz and one at .1 Hz). This more complex Y axis behavior may explain the difference in performance improvement.

The final disturbance used in closed loop testing was a 1 Hz sine wave at the BET. This disturbance differed from the others in that it was persistent throughout each test. Figures 6.19-6.22 show the results.

![Figure 6.19 Open Loop Response to BETx 1 Hz Sine Disturbance](image-url)
Figure 6.20  Closed Loop Response to BETx 1 Hz Sine Disturbance

Figure 6.21  Open Loop Response to BETy 1 Hz Sine Disturbance
Again, the better performance gains are seen in response to the X axis disturbance. One should notice, however, that even though the amplitude of the response has been decreased, no significant damping of the dominant behavior has been achieved. This, as mentioned previously, is most likely due to notching characteristics of the controller. The closed loop response to the Y axis disturbance, as seen in the other tests, has been slightly worsened.

To fully analyze the closed loop system, it is also necessary to study the frequency domain characteristics of the closed loop system. Figures 6.23-6.26 show the closed loop BET to LOS frequency responses, which were calculated in the same way as the open loop frequency responses.
Figure 6.23  Closed Loop Frequency Response From BETx to LOSx

Figure 6.24  Closed Loop Frequency Response From BETx to LOSy
Figure 6.25 Closed Loop Frequency Response From BETy to LOSx

Figure 6.26 Closed Loop Frequency Response From BETy to LOSy
The closed loop transfer functions show that low frequency attenuation has been improved for all but the BET Y axis to LOS Y axis loop which shows little or no change from the open loop transfer function. It is also important to note that for the axes that do show attenuation improvement, the improvement ends just below 10 rad/s, which is the frequency of the dominant behavior seen in the LOS time responses. Figure 6.27 and Figure 6.28 show the closed loop frequency responses for the IMC to LOS loops.

Figure 6.27 Closed Loop Frequency Response From IMCx to LOS
The closed loop IMC to LOS frequency responses show very clearly why no performance improvement was seen in the LOSy axis. No detectable change is evident in the IMCx to LOSy or IMCy to LOSy frequency response, and therefore no performance improvement from open loop can be expected in the LOS Y axis. The reason for the complete lack of effectiveness in this axis is unclear. Figure 6.28 shows that though the low frequency behavior in the IMCy to LOSx has been improved, the dominant modes near 10 rad/s have not been significantly affected. Since in the design stage, these modes were completely notched, it is clear that unmodeled behavior is responsible for the lack of performance improvement for these modes. In addition, Figure 6.27 shows that the controller has added a
mode at approximately 40 rad/s in the IMCx to LOSx transfer function. This explains the 5 Hz resonance seen in the closed loop time responses.

The lack of performance improvement points out that a controller which uses notching for performance depends heavily on modeling accuracy, regardless of what robustness guarantees are made. Thus, it would seem that though the $H^\infty$ design method can guarantee robustness to modeling uncertainty, no guarantees can be made concerning performance when modeling errors are present. It is also evident that notching controllers are ineffective when disturbances enter the system at locations which are not included as inputs in the design model. Certainly these questions warrant further study, since it is not clear what performance gains would be possible when employing an $H^\infty$ controller on an experimentally derived model of the ACES facility.
Chapter 7
Conclusions and Recommendations

The work presented here has addressed several issues concerning the application of $H^\infty$ control theory to the design and implementation of controllers for LSS. First, the theoretical and practical aspects of the factorization approach were investigated. It was found that the computational requirements using this approach are quite heavy, requiring the solution of almost thirty matrix Riccati equations. In addition, the order of controllers calculated was often as many as five times that of the design model.

Next, the state space approach of Glover and Doyle was investigated. This method requires the solution of only two matrix Riccati equations and yields a controller with the same order as the design model, including frequency dependent weights. Though this method is much less difficult in the computational sense, there are several assumptions relating to problem setup that can make the design problem difficult. In particular, the rank conditions on the open loop $D$ matrix make it necessary to add frequency dependent weighting functions when working with most physical systems. Further study is indicated as to the use of acceleration measurements since such measurements affect the characteristics of the open loop $D$ matrix.

A weighting scheme which allows frequency dependent performance and robustness constraints to be simultaneously
included was also studied. Though this scheme is analytically elegant, it requires the use of an improper transfer function to satisfy the rank requirements of the open loop $D$ matrix. This presents difficulties when using normal state space techniques to form the augmented plant. For the work presented here, the method used for including the improper weight was limited to use with a plant in modal form and a weight with no higher than a second derivative term. Clearly this restricts the applicability of this weighting scheme. A more general method for including the improper weight is needed for this method to be applicable to problems where the plant is not given in modal form.

A software system was developed for a VAXstation 2000 running the VMS operating system. Both the factorization approach and state space method were implemented, though the factorization approach was abandoned when it was found to be so much more computationally burdensome than the state space method.

An $H^\infty$ controller was designed using a 12 state FEM as a design model. Simulation results showed moderate performance gains, with the controller exhibiting notching behavior to achieve the desired performance goals. The order of the controller had the same order as the augmented design plant (36).

Since no attempts were made to reduce the order of the augmented design model past the original model reduction used
to form the unaugmented design plant, it remains to be seen if a reduced order $H^\infty$ controller could have been calculated. Even though the state space formulae yield a controller with the same order as the augmented plant, reduced order controllers will be necessary for the method to be truly applicable to the control of LSS, which typically have a large number of modes.

Finally, the $H^\infty$ controller was tested at the NASA Large Space Structure Ground Test Facility at Marshall Space Flight Center. Modest performance levels were achieved, though differences between the design model and the real system proved significant enough to greatly reduced the performance seen in the simulation results. This points out that controllers which depend on notching for performance depend heavily on modeling accuracy, regardless of what robustness margins are mandated. In addition, such notching behavior raises questions as to the robustness margins which may be required when dealing with lightly damped modes. Small errors in the modal frequencies of the design model may result in plant perturbations that are quite large.

It remains to be seen what performance levels may be possible using an $H^\infty$ controller in this particular application since the work here represents only one design iteration. The next iteration requires the use of an experimentally derived model and investigation into reducing the order of the controller.
References


Appendix
Software System

Due to the complexity of the algorithms for accomplishing $H^\infty$ control designs, a software system was developed. The purpose of the software is to lend flexibility and reliability to the software development process. The philosophy on which the software system is based is to develop a relatively small number of independent commands accessible via the particular operating system in use. Data is shared between these different low-level commands via a "state space system" representation which is sufficiently flexible to allow for all types of numerical data storage and which relies on the sequential file capabilities of the particular computer. An added benefit of this approach is that internal memory requirements are those of each particular module, rather than the memory required for the total design process. This approach is currently implemented on a VAXstation 2000 running the VMS operating system. However, the approach is fully consistent with any operating system and set of high level language compilers capable of providing an interface between the operating system command line interpreter and program modules.

A.1 Data Representation

The first representation for shared data between modules
is that of state space system representations stored in sequential files on system disk space. This representation allows for the storage of general linear continuous and discrete-time systems, two-dimensional array data, one-dimensional array data, and scalar data. For example, the system

\[ \dot{x} = Ax + Bu \]

\[ y = Cx + Du \]

would be stored in a file named "system1" in the following form:

<table>
<thead>
<tr>
<th>1st Record</th>
<th># outputs(p), # inputs(m), # states(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Next n Records</td>
<td>rows of A</td>
</tr>
<tr>
<td>Next n Records</td>
<td>rows of B</td>
</tr>
<tr>
<td>Next p Records</td>
<td>rows of C</td>
</tr>
<tr>
<td>Next p Records</td>
<td>rows of D.</td>
</tr>
</tbody>
</table>

An \( i \times j \) matrix \( Z \) would be stored in file "Z" as:

<table>
<thead>
<tr>
<th>1st Record</th>
<th>i, j, 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Next i Records</td>
<td>rows of Z</td>
</tr>
</tbody>
</table>

so that a matrix is stored in the form of a system with \( j \)
inputs, i outputs, and 0 states. A scalar quantity is stored as a 0 state system with 1 input and 1 output.

The second representation for shared data is that of frequency responses stored in direct access unformatted files (with record length 16) on system disk space. This format was chosen so that typically large frequency response files will require a more reasonable amount of disk space and I/O time is reduced. Frequency responses can be scalar, one-dimensional, or two-dimensional, depending on the number of inputs and outputs of the transfer function corresponding to the response.

Frequency responses are generated using the module FREQ (see section A.2 for more detail) which requires a file containing the desired frequency points as one of its inputs. This frequency file must also be a direct access, unformatted file with record length 16. It can be generated by using the interactive module PREFREQ.

For a frequency response of a system with p outputs, m inputs, and n states, the frequency response would be stored in the following form:

| Record 1 | m |
| Record 2 | p |
| Record 3 | n |
| Record 4 | minimum frequency |
| Record 5 | maximum frequency |
Record 6 # of frequency points
Record 7 w1 (minimum frequency)
Next p records 1st column of frequency response matrix evaluated at w1.
Next p records 2nd column of frequency response matrix evaluated at w1.
Next p records m'th column of frequency response matrix evaluated at w1.
Next record w2
Next p records 1st column of frequency response matrix evaluated at w2.
A.2 Command Summary

ACEMODST FEMDAT G FREQS

Interactive program which allows user to pick inputs, outputs and desired modes from modal data in FEMDAT to form state-space continuous system G. The chosen modal frequencies are stored in FREQS.
AGTBTEST A B FLAG

Compares two scalars A and B. If A is greater than B, FLAG is set to 1.

AUGMENT GWT3 W1 GW3 GAUG

Forms the augmented plant GAUG for $H^\infty$ controller synthesis. GWT3 is the original plant with improper weight W3 already "absorbed". W1 is the proper performance weighting function.

BALANCE SYS1 SYS2

Calculates the balanced realization of the minimal, continuous realization SYS1 and places result in SYS2.

CASCADE ASYS BSYS ABSYS

Cascades the two systems ASYS and BSYS to form ABSYS.

CASDCMP SYS STABLE UNSTABLE D

Decomposes the continuous system SYS into its stable and unstable components. D is the d matrix of the original system.

CCLOSE G11 G12 G21 G22 K TCL

Forms the closed loop transfer function from W to Z for the standard $H^\infty$ problem.
CCONGRAM SYS LC

Calculates the controllability Grammian of the continuous system SYS and places result in LC.

CDCOFAC G N M X Y NT MT XT YT

Calculates the doubly co-prime factorization of G.

CFINDK Q N2 M2 X2 Y2 K

Calculates the $H^\infty$ controller K from its parameterization.

CHNORM SYS HNORM

Calculates the Hankel norm of an unstable continuous system.

CHOLESKY A R

Calculates the Cholesky factor R of the symmetric matrix A such that $A = R^T R$.

CIOFAC G UI UO

Calculates the inner-outer factorization of G.

CLIASOL A Q X

Solves the continuous Lyapunov equation $A^T X + XA + Q = 0$
COLCAT A B C
Concatenates the columns of two matrices A, B of equal row dimension to form C.

COLSPLIT D N1 D1 D2
Splits an MxN matrix D into two matrices with column dimension N. D1 has N1 rows and D2 has M-N1 rows.

COBSGRAM SYS LO
Calculates the observability Grammian of the continuous system SYS.

CQBLOCK1 X UO UCO SCALE Q
Forms Q for the 1-block H∞ problem.

CRBLOCK1 T1 T2 T3 UO UCO R SCALE RSTAB
Forms R for the 1-block H∞ problem.

CRICSOL A Q G K
Solves the continuous Riccati equation $KA + A^T K + Q - KGK = 0$

CSCHURS A U U11 U21
Calculates the similarity U which takes A to real Schur form. U11 and U21 are partitions of U useful in Laub's Schur method of Riccati equation solution.
CSPECFAC G G-

Calculates the spectral factorization of G.

CXBLOCk1 R X RSTAB

Find X within 1 unit of R for the 1-block problem when R is minimal, antistable, strictly proper, and has Hankel norm less than unity.

DIAGSqrt A SQRTA

Find the square root of a diagonal matrix A.

DRICSOL A B Q R K

Solves the discrete Riccati equation and places result in K.

\[ A^TA - X - A^TXB(B^TXB + R)^{-1}B^TXA + Q = 0 \]

EIGMAX A MAX

Calculates the eigenvalues of the matrix A and stores the maximum eigenvalue in MAX.

DUALSYS SYS1 SYS1PRIME

Forms the dual of SYS1 and places result in SYS1PRIME.
FDBKSYS G H CL

Calculates the closed-loop state-space representation resulting from putting G in the forward path and H in the negative feedback path.

FOURCONCAT A11 A12 A21 A22 A

Combines the four matrices A11, A12, A21, A22 as partitions of the matrix A.

FMAXMAG FRESP MAXMAG

Searches a scalar frequency response contained in FRESP to find the maximum magnitude of the response over its range of frequency. This value is placed in MAXMAG.

FRADD RESP1 RESP2 RESP3

Adds the frequency responses RESP1 and RESP2 to form RESP3.

FRCTRAN RESP1 RESP2

Computes the conjugate transpose of the frequency response in RESP1 and places the result in RESP2.

FREQ SYS FREQS FRESP FLAG OUTFILE

Calculates the frequency response of the system contained in SYS at the frequencies contained in FREQS and stores the result in FRESP. Both FREQS and FRESP are direct
access, unformatted files with record length 16. FLAG should be S if a continuous response is desired, Z for a discrete response. If the fifth argument, OUTFILE, is present on the command line, a formatted version of FRESP will be placed in OUTFILE.

FRINV RESP RESPINV

Computes the inverse of the frequency response RESP by inverting the frequency response matrix at each frequency.

FRMAXSVD SVDRESP MAXSV

Retrieves the max singular value of a singular value frequency response at each frequency to form the scalar max singular value response.

FRMINSVD SVDRESP MINSV

Same as FRMAXSVD except that the minimum singular value is retrieved.

FRMULT A B AB

Multiplies the two frequency responses A, B to form AB.

FRNEGATE A -A

Negates the frequency response A.
FRQRANK A B C D SCALAR FREQS FLAG
Tests the rank of the matrix
\[
\begin{bmatrix}
A - j\omega I & B \\
C & D
\end{bmatrix}
\]
over the frequency range defined in FREQS. If at any frequency the rank of the matrix is less than the scalar contained in SCALAR, the output FLAG is zero. Otherwise, FLAG is set to one.

FRSMULT RESP1 SCALAR RESP2
Multiplies the frequency response in RESP1 by the scalar in SCALAR to form the frequency response RESP2.

FRSUB RESP1 RESP2 RESP3
Calculates the frequency response resulting from subtracting the response in RESP2 from that in RESP1. The result is placed in RESP3.

FRSVD RESP1 SVRESP U V
Computes the singular value decomposition of the frequency response RESP1 at each frequency point. The singular values are stored as a frequency response (SVRESP) with one output and \( \min(\#\text{inputs}, \#\text{outputs}) \) inputs. The left singular vectors are stored in U as a square frequency response (\( \#\text{inputs} \times \#\text{inputs} \)) and the
right singular vectors are stored in $V$ as a frequency
response with dimension (#outputs x #outputs).

**FRTRAN RESP1 RESP2**
Calculates the transpose of the frequency response RESP1 and stores the result in RESP2.

**GAMCNVRG SCALAR1 SCALAR2 TOL FLAG**
Checks the absolute value of the difference between the two scalars contained in SCALAR1 and SCALAR2 to see if it is less than the scalar contained in TOL. If so, FLAG is set to 1.

**GAMMA GAUG M1 M2 P1 P2 W GAMMA**
Calculates parameters needed to form an $H^\infty$ controller using the Doyle-Glover state-space formulae. GAUG should contain the augmented plant, M1 is the # of disturbance inputs, M2 is the # of control inputs, P1 is the # of controlled outputs, and P2 is the number of measured outputs. These scalars should all be stored in files. W is a file containing the frequencies for infinity-norm calculations, and GAMMA contains the scalar defining the infinity-norm constraint the closed loop system must satisfy. HINFINK should be run immediately after GAMMA to calculate the controller.
GAMMAIT G M1 M2 P1 P2 W MINGAMMA

Same as GAMMA except that an iteration (binary search) is performed to calculate the minimum feasible gamma. MINGAMMA should contain the lower bound for the binary search (i.e. 0).

GWEIGHT3 G GWT3 W3INV

"Absorbs" a second order improper weight into the system in G. The user is queried for the frequency, damping, gain, and dimension of W3. The weighted system is stored in GWT3 and the proper inverse of W3 is stored in W3INV.

HINFINK M1 M2 P1 P2 K

Calculates the $H^\infty$ controller K from parameters generated by GAMMA or GAMMAIT. M1, M2, P1, P2 should be the same filenames used with GAMMA or GAMMAIT.

IDFORM A B I

Creates an identity matrix with the same row dimension as the matrix in A and the column dimension of the matrix in B.

MADD A B C

Adds the matrix in A to the matrix in B and stores the result in C.
MATEXP CSYS DT DSYS
Discretizes the system in CSYS with sample period contained in DT using a Pade approximation technique. The resulting discretized system is stored in DSYS.

MINBAL SYS MINSYS
Calculates the minimal realization of SYS by first forming the balanced realization and then keeping only those states corresponding to nonzero singular values of the Grammian of the balanced realization.

MINIMAL SYS MINSYS
Calculates the minimal realization of SYS by first extracting the observable part, and then extracting the controllable part of the remaining system.

MINV A AINV
Calculates the inverse of the matrix in A.

MMUL A B AB
Matrix multiplication AB = A*B.

MNEGATE A -A
Negates the matrix in A.
MSQRT A SQRTA

Calculates the square root of A which is known to have singular value decomposition of the form $A = U S U^T$ (A symmetric).

MSUB A B C

Matrix subtraction $C = A - B$.

MTRANS A ATRANS

Matrix transpose.

NULLFORM A B NULL

Forms a zero matrix with row dimension equal to that of A and column dimension equal to that of B.

PARA SYS1 SYS2 SYS3

Forms system resulting from combining SYS1 and SYS2 in parallel. All inputs and outputs are preserved.

PREFREQ

Interactive program used to generate frequency files for frequency response calculation.

RANKTEST SIGMA RANK FLAG

Compares the rank of a diagonal matrix SIGMA (usually the singular value matrix of the matrix being tested) to the
scalar in RANK and sets FLAG=1 if the rank is \( \geq \) the scalar.

**RMAXSVD** \( A \) **MAXSV**

Performs a singular value decomposition on the matrix \( A \) and returns the max singular value in **MAXSV**.

**ROWCAT** \( A1 \) **A2** **A**

Concatenates two matrices \((A1, A2)\) with equal column dimension to form **A**.

**ROW_SPLIT** \( A \) **N1** **A1** **A2**

Splits an \( M \times N \) matrix \( A \) into two matrices with row dimension \( M \). \( A1 \) has \( N1 \) columns and \( A2 \) has \( N-N1 \) columns.

**RSVD** \( A \) **U** **S** **V**

Performs a singular value decomposition on the matrix \( A \) such that \( A = U S V^T \).

**SCHUR_C** \( A \) **U** **U11** **U21** **NSTABLE** **FLAG**

Calculates the similarity \( U \) which will take \( A \) to real Schur form. \( NSTABLE \) contains information about the number of stable and unstable eigenvalues of \( A \). \( FLAG \) is 1 if \( A \) is positive semi-definite. Same as the module **CSCHURS** except for the additional outputs **NSTABLE** and **FLAG**.
SBALANCE SYS1 SYSBAL

Calculates the balanced realization of the system in SYS1 and places result in SYSBAL.

SCLPARA G11 G12 G21 G22 M2 YT2 T1 T2 T3

Forms T1, T2, and T3 to parameterize the closed loop transfer function from the disturbance inputs to the controlled outputs.

SCOM A B C D SYS

Takes individual matrix elements of a system representation A, B, C, D in separate files and places them in a single system file format in SYS.

SCONMUL SYS1 K SYS2

Performs state-space operations equivalent to multiplying the transfer function matrix of SYS1 by K.

SINV G GINV

Performs state-space operations equivalent to taking the inverse of the transfer function matrix of the system in G.

SMATMUL K A KA

Multiplies the matrix in A by the scalar in K.
SPLIT FOUR D M1 M2 N1 N2 D11 D12 D21 D22
Separates the matrix D into four partitions to be placed in files D11, D12, D21, and D22. The dimensions of these partitions are determined by the scalars contained in M1, M2, N1, N2. D11 is M1 x N1, D12 is M1 x N2, D21 is M2 x N1, and D22 is M2 x N2.

SSEP G A B C D
Takes the system representation in G and places the matrices A, B, C, D in separate files.

SSUB G1 G2 G3
Performs state variable operations equivalent to negating G2 and placing it in parallel with G1 to form G3.

SSTRAN G T GPRIME
Applies the similarity transform T to the system G to form GPRIME.

WEIGHT1 W1SYS
Creates a 2nd order state-space system with user specified gain, damping, and natural frequency. The system is written out in modal form with position output.
ZCRIC A B Q R Z

Forms the "Hamiltonian" or "Z-matrix" required for the Riccati equation solution. A, B, Q, R, are the equation coefficient matrices and Z is the resulting Hamiltonian matrix (continuous form).

ZCLIA A Q Z

Forms the "Z-matrix" required for solution of the continuous Lyapunov equation.

A.3 Example Command Sequence For $H^\infty$ Controller Design

The following sequence of commands is an example of how an $H^\infty$ controller would be designed using the state-space formulae. The unaugmented plant is contained in "G.SYS", the error signal weighting is in "W1.SYS". "M1.", "M2.", "P1.", and "P2." contain scalars defining the dimensions of the input and output vectors as described in section 6.2 in the description for GAMMA and GAMMAIT.

Step 1: Incorporate improper weight W3 into plant

GWEIGHT3 G.SYS GWT3.SYS W3INV.SYS

Step 2: Form augmented plant

AUGMENT GWT3.SYS W1.SYS W3INV.SYS GAUG.SYS

Step 3: Generate frequency file for frequency response calculations in module GAMMA.
PREFREQ (During interactive session select direct access unformatted output file option and name the file "FREQS.DAT")

Step 4: Calculate parameters needed to generate controller satisfying infinity-norm constraint represented as a scalar in file GAMMA (use gamma=1 for small gain problem formulation).

GAMMA GAUG.SYS M1. M2. P1. P2. FREQS.DAT GAMMA.
The present working directory now contains approximately 100 files with suffix ".GAM". Do not delete any of these files yet, as they are needed to calculate the controller.

Step 5: Calculate the H∞ controller

"K.SYS" now contains the controller. At this point, the files ending in ".GAM" may be deleted if desired.