EXPERIMENTAL AND NUMERICAL INVESTIGATION OF A DEEP-CORRUGATED STEEL, BOX-TYPE CULVERT,

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ABSTRACT

A comprehensive investigation of the behavior of a corrugated steel, box-type culvert is described. The culvert design is unique in its use of a deep-corrugated cross section (5.5 inch depth) and a welded corner connection to form a culvert span of 15 feet with a rise of less than 5 feet. A full scale field test was undertaken to measure the response of the culvert under construction and service loads at a highway installation. Extensive instrumentation that measured deformations and forces was installed on the culvert before it was backfilled. Data was acquired at frequent stages during construction of the sand fill and under the influence of a stationary truck with three loads at five locations on the completed roadway. Construction procedures that nearly overstressed the culvert are discussed. A numerical simulation of the construction and service loads on the soil-structure system was undertaken using the finite element code CANDE. To reduce numerical error associated with soil modelling, the sand used to backfill the culvert was tested extensively utilizing a cubical multiaxial testing device. Parameters for a hyperbolic soil model were derived with special provisions to model the soil along non-triaxial compression stress paths. The numerical simulations are compared with the measured field response.
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A relatively new concept in corrugated metal drainage products is the box-type culvert. In cross section, these culverts appear as low, flattened arches and are most useful in locations requiring a large drainage capacity under low vertical clearance. This report is concerned with the performance of a corrugated metal box-type culvert manufactured by the Syro Steel Company and pictured in Figure 1.1 before placement of the sand backfill. Instead of employing transverse stiffening ribs to increase the moment capacity of the corrugated section, as is commonly used in long-span culverts, Syro uses the deep-corrugated section pictured in Figure 1.2.

Box culverts, with their long and rather flat spans under relatively shallow soil cover, represent a complex soil-structure interaction problem. As a result, box culvert designs are frequently analyzed with finite element techniques. Many designers use the finite element code Culvert ANalysis and DEsign (CANDE) which has been specifically tailored to handle the culvert problem.

This research effort is primarily concerned with field measurements of the performance of a Syro deep-corrugated, box-type culvert. Measured field performance is compared with finite element predictions from CANDE. Since the backfill soil is very important to the structural integrity of the culvert system, the sand used to backfill the culvert was brought to the lab and tested to allow proper numerical modelling. Hence, this report discusses the:

- field instrumentation of a Syro deep-corrugated steel, box-type culvert,
- measured performance of the culvert under construction and service loads,
• laboratory testing of the backfill sand with the development of a constitutive model,
• finite element analysis of the Syro box culvert using CANDE, and
• comparison of finite element predictions with the measured culvert performance.

1.2. DEVELOPMENT OF THE CORRUGATED METAL, BOX-TYPE CULVERT

Corrugated metal culverts have long provided an economical alternative to short bridges for stream crossings and grade separations. Culverts are relatively quick and easy to install and require minimal maintenance. Improved understanding of the soil-structure interaction problem and the development of finite element methods have permitted the safe design of very large span (> 50 ft.) culverts. In the late 1970's, the box-type culvert emerged to fill the need for large drainage structures in locations with low vertical clearance [Duncan et al., 1985]. This culvert shape (Figure 1.1) is particularly well suited for replacement of short highway bridges over small streams.

When culverts are round and placed in relatively deep fills, loads are primarily supported through ring compression and arching within the backfill soil. In box-type culverts, shallow soil cover hinders the development of soil arching and the larger culvert spans require greater bending capacity in the culvert walls. Further, as the soil cover over the culvert becomes more shallow, surface traffic loads constitute a greater portion of the loads which must be supported by the culvert itself. In fact, failures frequently occur due to plastic hinge formation under heavy construction traffic loads [Seed and Raines, 1988].

These factors thus combine to challenge the designer of the box-type culvert. Under the conditions of shallow cover over a long, relatively flat culvert span, soil support and bending moment capacity are crucial for safe culvert installations.
Syro Steel's deep-corrugated box-type culvert. Several manufacturers have developed culvert designs which use some type of transverse stiffener ribs, bolted to the corrugated plate, to increase the bending capacity of the corrugated section. The Syro Steel Company has developed a deep-corrugated section that can be used without any additional stiffening ribs. This corrugated section (Syro DEEP COR™) has a pitch of 15 inches with a depth of 5½ inches (Figure 1.2). Other manufacturers generally use corrugated sections which are on the order of 6 inches by 2 inches.

Additionally, the Syro culvert is formed by welding the curved top section to vertical side plates at a sharp corner as can be seen in Figures 1.1 and 1.3. Other manufacturers often incorporate a curved plate section in this corner, or haunch, of the culvert.

1.3. CULVERT DESIGN

In general, culvert design entails consideration of many factors including hydraulic requirements, corrosion resistance, constructability, etc. This report deals only with structural considerations; i.e., the ability of the culvert system to support service loads. It is important to note that most culvert failures are not the result of design deficiencies but are due to poor backfill or construction procedures [Selig et al., 1977].

In the 1920's, the American Railway Engineering Association inspected numerous large culverts and concluded that failure usually occurs when deflections exceed 20% of the culvert diameter. To ensure a safety factor of 4, deflections were recommended to be limited to 5% of the culvert's diameter. These ideas were incorporated by Marston and Spangler in the 1940's to develop the so-called "Iowa Formula". This design technique, in popular use for many years, unfortunately relies on a soil modulus that can not be evaluated in the laboratory [Katona et al., 1976].
In later years, various elasticity solutions were proposed for use in the design of buried culverts [Moore, 1987; Katona et al., 1976]. These methods are often limited to circular pipes and linear soil behavior and are thus not well suited for the design of box-type culverts. Others have proposed analysis techniques which model soil-structure interactions with normal and tangential springs attached to nodes around the culvert, but this method was intended only for the study of culvert buckling [Ghobrial and Abdel-Sayed, 1985].

By far, the most useful tool for the design and analysis of culverts is the finite element method. This numerical technique allows consideration of any culvert shape, nonlinear soil behavior, and a host of other important factors. Based on extensive finite element analyses of several culverts, other researchers have developed simplified design methods using empirical relationships and design charts [Duncan and Drawsky, 1983; Flint and Kay, 1982; Allgood and Takahashi, 1972]. Finite element analyses coupled with field testing lead to simplified design techniques for box-type culverts [Boulanger et al., 1989; Duncan et al., 1985; Duncan and Drawsky, 1983].

Due to unique installation conditions and other variables, many culvert designs are still checked with a finite element analysis. Since the finite element code CANDE (available from the Federal Highway Administration [Katona et al., 1981; Katona et al., 1976]) is specifically written for the design and analysis of culverts, it is widely used by culvert designers. In fact, CANDE was used by Syro Steel to verify the design of the culvert which is the subject of this investigation [Powell, 1987].

Since CANDE is routinely used to evaluate culvert designs, it is imperative to check the code's predictions against actual field performance. This is especially true with new and unique culvert products such as Syro's deep-corrugated box-type culvert.
1.4. CULVERT TESTING -- LITERATURE REVIEW

Most laboratory testing of culverts is done on samples of the corrugated plate to determine the bending capacity of a section (onset of plastic hinge formation or buckling) [Cary, 1986; Duncan and Drawsky, 1983; Gorman, 1981]. However, no published information is available on the measured capacity of Syro’s deep-corrugated section.

Laboratory testing of culvert scale models has been used to investigate the effects of various design parameters. Most model tests have been conducted in controlled test pits [Dessouki and Monforton, 1986; Nielson, 1972] but some work has been done using photoelasticity techniques [Abel et al., 1973]. More promising model studies are being attempted using geotechnical centrifuges [Craig and Mokrani, 1988], although more development is needed to accurately model culvert behavior.

The behavior of full size culverts has been studied in a number of field tests over the past 20 years. Most of these investigations were concerned with elliptical or arched long-span culverts with only a few published test results from box-type culverts. Most researchers attempt to gauge culvert performance by measuring:

- deformations of the culvert,
- stresses in the culvert plate, due to bending moments and axial thrusts, and
- stresses and displacements in the backfill soil.

Most investigators record culvert responses during both the construction phase (backfilling) and under surface traffic loads (live load).

**Measurement of culvert deflections.** The simplest method of measuring culvert deformation utilizes surveying techniques to monitor movements of control points on the culvert plate. Vertical control is maintained with a surveyor's level while a steel tape is used to measure chord lengths between the control points. With triangulation, the changing coordinates of the control points can
be deduced [Bacher and Kirkland, 1986; Seed and Ou, 1986; Selig and Musser, 1985; Kay and Flint, 1982; Selig et al., 1979]. Unfortunately, this technique is usually not very precise. Others have used dial gauges mounted on reference frames to track culvert deflections [Gorman, 1981]. Generally, this data can not be decomposed into deflections normal and tangential to the culvert plate.

More precise measurements of culvert deformation can be accomplished using electrical transducers. This scheme uses cables that are stretched across various culvert diameters [Bakht, 1985; Beal, 1982; Duncan and Jeyapalan, 1982; McVay and Selig, 1982] or attached to a reference frame inside the culvert [Selig, 1975; Selig and Calabrese, 1975]. Electrical transducers mounted on the cables measure the relative movement between the anchor points. This system yields perhaps the most precision, but accurate chord measurements are still required to establish the true initial shape of the culvert before loading.

**Measurement of plate forces.** Bending moments and axial thrusts in the culvert plate can be determined from strain measurements. Some researchers use weldable type strain gauges [Gorman, 1981; Selig et al., 1979] while most use bonded foil, electrical resistance strain gauges.

Unfortunately, several investigators [Bakht, 1985, 1981; Selig and Musser, 1985] overlook the influence of the biaxial strain field in the culvert plate. Because of the bellows action of the corrugated section, small longitudinal bending moments can cause significant, localized strains transverse to the circumferential direction. Due to the Poisson effect, longitudinal strains must be considered when trying to deduce the circumferential stress due to bending and thrust. In other words, biaxial strain measurements are necessary to correctly measure culvert response. Using only uniaxial strain gauges can introduce errors in the plate forces derived from the strain measurements. Beal [1982] properly recognized the biaxial strain field and used biaxial strain gauges. Further, Beal conducted full scale laboratory calibration tests to ensure the reliability of his field measurements.

**Measurement of stresses and displacements in the backfill soil.** Selig and his various co-workers have done extensive work to measure the response of backfill soil around a culvert [McVay
and Selig, 1982; Selig et al., 1979; Selig, 1975; Selig and Calabrese, 1975]. This is usually done using buried vertical and horizontal extensometers which measure the change in distance between two buried control points. Another instrument, developed by Selig [1975], determines the soil strains by monitoring changes in inductance between two buried electrical coils. Observations of backfill behavior aid in the understanding of how the soil and culvert interact to support service loads.

**Field tests of corrugated metal, box-type culverts.** A full size box culvert was field tested by Duncan and Drawsky [1983] in support of their work to develop a simplified design method for aluminum box culverts. Lane Metals has also published a report [Gorman, 1981] on a full scale test of their steel box culvert design. Perhaps the most comprehensive study of box-type culvert behavior was performed by Beal [1986] using an aluminum culvert with stiffener ribs.

In addition to Syro's deep-corrugated box culvert, which is the subject of this report, three other box culvert designs were field tested as part of a larger research effort at Ohio University. Each of these other three culverts had slightly different design features but all were approximately the same size and all used stiffener ribs. Instrumentation and data analysis techniques [Abdel-Karim; 1987, Byrne, 1987; Tan, 1987] were modified for this investigation.
Figure 1.1. Syro Steel's box-type culvert during construction.

Figure 1.2. Syro's deep-corrugated steel plate.
Figure 1.3. Detail of welded connection between crown and side plates.
CHAPTER 2

FIELD TESTING OF DEEP-CORRUGATED CULVERT

2.1. INTRODUCTION

To meet the objectives of this research effort, a deep-corrugated steel, box-type culvert was monitored under construction and service conditions. After the culvert was assembled on its foundation, a full range of instrumentation was installed. Data was collected at frequent stages of construction as the culvert was backfilled with sand and the roadway was paved. With the culvert structure complete, data was obtained with a loaded truck positioned at different points over the top of the culvert. The completed culvert is a permanent installation which remained in service at the completion of the research project.

The culvert was manufactured by the Syro Steel Company of Centerville, Utah, and Girard, Ohio. The structure is owned by the Ohio Department of Transportation (ODOT) and was installed by a local contractor under ODOT supervision. The culvert was assembled and backfilled in April and May of 1988 and was opened to traffic that June. The structure provides a stream crossing for State Route 554 in rural Gallia County, Ohio. The culvert is designated as ODOT Structure No. GAL-554-0841 and is located approximately one mile northeast of Porter, Ohio.
2.2. DESCRIPTION OF THE CULVERT AND THE DEEP-CORRUGATED PLATE

Syro's box-type culvert has a low, flattened arch appearance as seen in Figure 1.1. Dimensions of the cross sectional area are shown in Figure 2.1. The rise, or vertical clearance from the bottom of the side to the inside surface at the crown, is 59 inches. The top plate is curved with an inside radius of 294 inches. The top is welded at 110° angles to the side plates which are straight and 48 inches long. The resulting span is 15 feet (see Figure 2.1).

The 44 foot long culvert is set perpendicular to the roadway. The bottoms of the side plates are seated into concrete foundations that are 3 ft. by 3 ft. in cross section. The side plates are bolted to a steel bracket that is tied into the foundation along the entire 44 ft. length. The foundations are located such that the normal stream flow line is about 2 inches above the top of the concrete. Dimensions of the corrugated cross section are reproduced in Figure 2.2. The corrugated section, cold formed from 10 gauge steel and galvanized, has a 15 inch pitch and a 5.5 inch depth. No additional stiffeners are attached to the culvert plate.

Separate corrugated plates are bolted together on 30 inch centers to form the culvert. The additional steel plate in the lap joint significantly influences the cross sectional properties of the assembled plate. Since the lap joint is always toward the outside, the neutral bending axis is not at mid-depth but is 3.0189 inches from the innermost surface. The resulting moment of inertia is 8.5756 in.⁴/lineal ft. with a cross sectional area of 2.2596 in.²/lineal ft. These quantities, which were available from the manufacturer, are based on the uncoated plate thickness of 0.1345 inches and were verified by hand calculations.
2.3. STRENGTH AND ELASTIC PROPERTIES OF STEEL PLATE

The corrugated section is formed from AASHTO M-167 Structural steel plate. This specification requires a minimum yield point of the flat plate of 28,000 psi and a minimum tensile strength of 42,000 psi [AASHTO, 1982]. For design purposes, Syro uses a 33,000 psi yield strength and a tensile strength of 45,000 psi [Powell, 1987]. These higher strengths are probably based on actual test data on steel samples and are certainly above the required minimums to meet the AASHTO specification.

Because the actual strength of the structural steel is crucial to determining failure conditions in the culvert, a laboratory tensile test was performed using a coupon cut from the deep-corrugated plate. The specimen was cut from the radiused part of the plate and, since this location is furthest from the neutral axis, material in this region would be expected to experience the highest stresses. Results from the tensile test are shown in Figure 2.3 and, as indicated in that figure, a yield stress of 54,600 psi was observed. This higher measured yield stress is due to strain hardening imparted in the steel as it was cold worked into the corrugated shape. In fact, the test coupon was cut from the highly curved region where the cold work would be greatest. The increase in tensile strength due to cold working has been well understood for many years [Winter, 1970].

For design, the strength of the flat plate before cold working must be used to assure a conservatively safe structural design. When no laboratory data is available, the AASHTO minimum of 28,000 psi for M-167 plate must be assumed. Syro's design value of 33,000 psi yield stress is most likely based on actual data on samples taken from the flat plate and would therefore be safe for design. For observations of the instrumented culvert in this study, the measured yield point of 54,600 psi can be used to determine actual yield stresses.
The tensile test data also gives a direct measure of Young's Modulus as shown in Figure 2.3. For analysis purposes throughout this work, Young's Modulus (E) is taken as 29,300,000 psi and Poisson's ratio (ν) is assumed to be 0.3.

2.4. PLACEMENT OF FILL AND PAVEMENT

After the culvert was completely assembled on the foundations, reinforced concrete headwalls were constructed at both ends of the culvert. The dimensions of the headwalls, as well as the general limits of excavation around the culvert, are displayed on the left side of Figure 2.4. With the headwalls completed, the culvert was backfilled in layers with a select granular material (sand). The sand was brought to a height of 80 inches above the top of the foundations. A five inch base of crushed limestone was placed directly on the sand followed by three layers of asphalt. Details of the backfill zones are shown in Figure 2.4.

Backfill placement. Proper construction of the soil fill is critical to the safe functioning of the culvert system. To prevent overstressing of the culvert during construction, it is imperative to keep heavy construction equipment away from the culvert until it is sufficiently covered. Further, it is important to keep fill levels on both sides of the culvert approximately equal. To a much lesser degree, the culvert may lean to one side if fill is always placed first on the same side. Poor construction techniques can lock in culvert deformations that can potentially lead to collapse under service loads.

At this site, sand was trucked from a local quarry to provide a clean, granular, backfill material. Samples of this sand were collected for testing in the laboratory. Various characteristics and properties of the sand are detailed in Chapter 8. Backfilling of the culvert proceeded by dumping sand along alternate sides of the culvert and spreading by hand. Each layer, which did not
exceed five inches in thickness, was heavily watered to attain an optimal density. This was followed by compaction with many passes of a vibratory plate, as shown in Figure 2.5.

At various stages of fill, the backfill compaction was checked using a Troxler nuclear density gauge (Figure 2.6) operated by ODOT personnel. These measurements indicated an average, in place, dry density of 106.4 pcf.

Sound construction practices were followed up to the completion of backfill level with the culvert crown (Figure 2.7). After covering the crown, however, the contractor completed backfilling the culvert by spreading sand from only the south side. This was done by building up a mound of sand in layers and spreading the material toward the north. This created an uneven fill over the culvert as depicted in Figure 2.8. Furthermore, the front wheels of the loader were driven out over the culvert to spread the sand, as in Figure 2.9. However, the back wheels of the loader were never on the culvert until the sand fill was complete. This procedure produced unsymmetric load distribution in the culvert, as will be shown in Chapter 5, and demonstrates the importance of careful backfill practices for large span culverts.

All of the sand backfill was placed during four days from May 16 to May 19, 1988. Approximately 5 inches of crushed limestone was placed, graded, and compacted with a vibratory plate on May 20, 1988. This layer served as a stabilized subbase for the asphalt pavement.

**Placement of asphalt.** Eight inches of asphaltic concrete was placed in two layers on May 23 and May 25, 1988. During placement, loaded asphalt trucks were driven across the culvert and a small bulldozer was used to spread these two asphalt layers. This equipment, working on the hot asphalt or on the limestone subbase, produced very high loads on the culvert. The cured pavement, being much stiffer, acts to spread surface loads and significantly reduce the forces imparted to the culvert. Naturally, heavy, steel drum rollers and other equipment operating on the hot asphalt will produce large loads on the culvert. Thus, placement of the asphalt roadway can generate the most significant loads on the culvert even when accepted construction practices are followed.
2.5. MEASUREMENT OF CULVERT BEHAVIOR DURING CONSTRUCTION

Data on culvert behavior was taken regularly during the backfill and paving phases of construction. Instrumentation, described in the next two chapters, was installed prior to backfilling. Measurements of culvert plate forces were acquired at the completion of each layer of backfill. Deformation measurements, being acquired by hand (see Chapter 3), were very time consuming. To minimize interference with construction work, deformation measurements were sometimes taken only after every second layer. In all, data was acquired under 25 different construction load conditions. While any data was being acquired, no additional fill was placed but the vibratory plate compactor was allowed to operate as its effect on the data was negligible.

To determine the approximate height of fill for each load condition, depth markers were painted on the inside of each headwall. These markers, visible in Figure 2.10, were referenced to the top of the foundations and provided a measure of the backfill height at each of the four corners of the culvert. Naturally, the fill surface was only roughly level and these measures could only be used to give an average height of backfill for each load condition.

Full sets of data were also obtained upon completion of the crushed limestone subbase and each of the two layers of asphalt base course. No data was acquired on the top 1.75 inch layer of asphalt that was placed before the application of live loads.

2.6. MEASUREMENT OF CULVERT BEHAVIOR UNDER SERVICE LOADS

After completion of 9.75 inches of asphalt pavement, loaded trucks (Figure 2.11) were used to simulate surface loads expected on the in-service culvert. The trucks were positioned at various locations to produce static surface loads, as approximately one half hour was required to obtain a
complete set of data on the culvert's response. In all, three different truck loads were placed at five
different locations over the culvert. This work was performed on June 1, 1988.

The truck loads, or live loads, were planned to simulate 50%, 100%, and 130% of the
standard AASHTO H20-44 truck load. This corresponded to weights of 16,000 lbs, 32,000 lbs, and
42,000 lbs (16, 32, and 42 kips) on the rear axle of each truck. The trucks were loaded with crushed
stone at a local quarry and driven to the site. There, the tires on the center axle were deflated as
can be seen in Figures 2.11 and 2.12. This ensured that the desired weight would be transferred
mostly through the rear axle. The total weight of the truck was obtained to deduce the front wheel
loads. Front axle loads and the wheelbase of the loaded trucks are given in Table 2.1.

Each loaded truck was placed at 5 positions to produce 15 sets of data under live loading
(Figure 2.14). These positions, detailed in Figure 2.13, were chosen to produce the greatest loads
in the instrumented cross section of the culvert. Position #1 (see Figure 2.13) was located so that
the centroid of the rear axle was placed directly over the instruments at the crown. The truck was
backed up for Positions #2 and #3 to produce unsymmetric loading on the culvert. Positions #4
and #5 placed one set of wheels directly over the crown at the instrumented section. Since this
section of highway is straight with no superelevation, the depth of soil cover at each live load position
was equal. The 16 kip load was placed first in Positions #1 through #5 followed by the 32 kip load.
Lastly, the 42 kip truck load was placed on all five positions in order.
Table 2.1. Configuration of loaded trucks used for live loads.

<table>
<thead>
<tr>
<th>Live Load*</th>
<th>Rear Axle Load, (lbs)</th>
<th>Front Axle Load, (lbs)</th>
<th>Wheelbase; front axle to rearmost axle, (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 kip</td>
<td>16,000</td>
<td>4,000</td>
<td>16</td>
</tr>
<tr>
<td>32 kip</td>
<td>32,000</td>
<td>8,300</td>
<td>14</td>
</tr>
<tr>
<td>42 kip</td>
<td>42,000</td>
<td>10,700</td>
<td>16</td>
</tr>
</tbody>
</table>

* 1 kip = 1000 lbs
Welded corner connection

Concrete foundation

Crown

Rise = 59°

Radius = 294°

Span = 15 ft = 180°

Crown

Figure 2.1. Cross sectional dimensions of the deep-corrugated culvert.

Moment of inertia = 8.5756 in^4/lineal ft
Cross sectional area = 2.2596 in^2/lineal ft
AASHTO M-167 Structural Steel Plate

Bolt

0.1345" thick (10 Ga.)

Outside surface

Depth = 5.5"

Inside surface

N.A.

\[ \bar{y} = 3.0189" \]

Pitch = 15"

7.5"

30"

Figure 2.2. Dimensions of deep-corrugated steel plate.
Figure 2.3. Laboratory tensile test on coupon from the deep-corrugated plate.
Figure 2.4. Limits of fill constructed around the culvert.
Figure 2.5. Construction of sand backfill.

Figure 2.6. Nuclear density test equipment.
Figure 2.7. Backfill approaching level of culvert crown.

Figure 2.8. Sloping fill profile during construction process.
Figure 2.9. Using front-end loader to spread sand fill.

Figure 2.10. Depth markers used to determine height of fill.
Figure 2.11. Dump truck used for live load application.

Figure 2.12. Deflated tires on center axle of loaded truck.
Figure 2.13. Live load Positions #1 through #5.
Figure 2.14. Stationary truck during live load data acquisition.
3.1. OVERVIEW OF DEFORMATION MEASUREMENTS

Culvert deformations were monitored under both construction and live loads. The most important measurements were made to determine the vertical and horizontal movement of thirteen reference points around the culvert periphery. Foundation settlements, as well as vertical movements of two reference points in the stream bed, were monitored with a hose level. A split light beam was used to measure bending in the welded haunch corner. Also, a dial gauge and bracket was mounted near the crown to measure flattening of the corrugations under load. Finally, four extensometers were buried in the backfill to determine deformations in the soil around the culvert. This chapter will discuss each of these instrumentation systems, data reduction methods, and some preliminary conclusions. Most of the deformation data is presented in Chapter 5.

3.2. FOUNDATION SETTLEMENTS

The corrugated box culvert is seated upon two concrete foundations as shown in Figure 2.1. Settlement of these foundations were expected to occur under the loads carried by the culvert. Small, equal, vertical movements of the north and south foundation would result in a simple
translation of the culvert that would not impose any additional bending in the culvert. On the other hand, unequal settlements could generate significant bending forces in the culvert plate.

Foundation settlements were measured using a very simple device known as a "hose level" that has been used since ancient times [Dunnicliff, 1988]. A hose level consists of a long, water-filled tube with both ends attached to vertical scales. Since the water level in both ends of the tube will be level, elevation differences between two points can be readily deduced. The hose level used on this culvert study is pictured in Figures 3.1 and 3.2. A translucent 3/8" plastic tube was filled with colored water. One end of this tube was attached to a reference pole, located about 20 feet upstream from the culvert, that would not be affected by settlements of the culvert. The reference pole consisted of a steel pipe that was driven to refusal and firmly concreted in place. The other end of the tube was fixed to a steel rod that could be carried to each point of interest.

Accuracy of these measurements is limited mostly by how precisely the water level can be read in either end of the tube. In this field effort, all data is assumed to be accurate to ±1/16 inch which is the smallest division on the scales on each pole. Because the vertical clearance inside the culvert is low, the movable pole could be no longer than three feet. Consequently, the reference pole had to be set at approximately the same elevation as the foundations. In general, the system worked well and was relatively easy to use. Naturally, much better precision would be possible with more sophisticated instruments.

Results from the hose level measurements are plotted in Figure 3.3. Data from both the north and south foundations show nearly the same trend with measurable settlements occurring as a function of time. Because of the time dependent nature of the settlements, and due to the widely varying backfill loads during this period, a true time (or fill height) could not be used for the abscissa in this plot.

As an independent check, the hose level data is compared with culvert deflection data in Figures 3.4 and 3.5. Deflection measurements, described fully later in this chapter, were made using
control points mounted on the culvert and reference points located in the stream bed. Plotted in Figures 3.4 and 3.5 are the vertical deflections of points located 24 inches above the foundation. Since these points are located on the nearly vertical sections of the culvert close to the foundation, any vertical movements that are detected can be attributed to foundation settlements. Two reference points located just 8 inches above the foundations indicated very erratic vertical movements and could not be used in this comparison. However, as will be discussed later, this is due to significantly higher measurement errors at these two points.

The agreement between the hose level data and the culvert deflection data is quite good. Unquestionably, the culvert did settle by a measurable amount. Furthermore, both foundations settled at approximately the same rate as can be seen in Figure 3.3. This would indicate a simple vertical translation of the entire culvert structure with no significant differential settlement. Thus, no detrimental structural loads in the culvert would be expected due to foundation settlements.

The assumed settlement time history is given in Table 3.1 and plotted in Figures 3.3, 3.4, and 3.5. (These values are subtracted from the measured vertical deflections of the culvert as described in Section 3.4). Settlements are observed to occur over a period of time after foundation loads are significantly increased. From May 16 through May 18, backfill was placed along the sides of the culvert. Most settlement occurred on May 19 when the backfill was being placed over the culvert crown up to the elevation of the stone subbase. Placement of fill over the culvert crown would generate the greatest foundation contact pressures and therefore the greatest settlements. Further settlements are seen in the week before June 1 which further illustrates the time dependent nature of these settlements.
3.3. TAPE EXTENSOMETER MEASUREMENTS

Deflections in the culvert cross section were determined using a triangularization technique (see Section 3.5). To accomplish this, precise measurements of distance between various control points were obtained using a tape extensometer manufactured by the Slope Indicator Company.

The tape extensometer is pictured in use in Figures 3.6 and 3.7. The instrument consists of a steel tape with holes punched precisely every two inches. The tape is mounted on a reel attached to the extensometer body. To determine the length between two control points, hooks located on the instrument and on one end of the tape are clipped into two eyebolts. The extensometer is then clipped into one of the holes, spaced every two inches, in the steel tape. A screw and spring mechanism is then tightened to a repeatable tensile force. This tension in the tape is consistently reproduced by alignment of two scribed lines on a spring loaded indicator. Having attained the desired tension, the measurement is read from the length to the punched hole and an integral vernier. The vernier scale, with a dial gauge to make readings to .001 inch, is visible in Figure 3.7.

The manufacturer claims an accuracy of ±0.005 inches for measurements made with this instrument. Based on ten trial measurements made between two fixed points in the lab, the instrument readings are probably repeatable to only ±0.015 inch. Furthermore, temperature changes will affect the length of the steel tape. Considering the worst temperature extremes in the field (temperature change of 40°F) and the longest length measured (134 inches), the maximum probable error due to temperature effects is 0.035 inches. Taken together, the maximum probable error in any one measurement is ±0.050 inches.

Finally, the tape extensometer does not read out in a measure of the true length between two points. Instead, a calibration constant must be added to the instrument reading to yield the true length. For the instrument used in this study, a calibration constant of 17.458 inches was determined in the lab. Adding this constant to each reading yields the total length between the centers of the
eyebolt control points. Since culvert deformations are indicated by changes in these measured lengths, errors in the determination of the true calibration constant will have negligible effect on the computed culvert deflections.

3.4. MOVEMENT OF DEFLECTION REFERENCE POINTS

Deformations of the culvert cross section were determined from measurements to two reference points located in the stream bed. These reference points, labeled A and B in Figure 3.8, were monitored for any potential movement during the study. Both reference points consisted of eyebolts that were screwed into a 6 inch length of rebar. This bar was set in concrete that topped a 2 inch pipe that was driven to refusal at each location (7.5 ft. deep at A and 4.5 ft. deep at B).

Vertical movements of reference points A and B were monitored using the same hose level instrument, described in Section 3.2, that was used to measure the foundation settlements. This data is plotted in Figure 3.9. Considering the accuracy of the system, no measurable vertical movement of reference points A and B is observed. This data was also used to determine the elevation of point B to be -1.06 inches relative to point A. This effectively defines the horizontal and vertical planes to be used later in determining the deflections of the culvert shape.

Changes in the horizontal distance between reference points A and B were monitored using the tape extensometer discussed in Section 3.3. This data is plotted in Figure 3.10. Recalling the maximum probable error in this data is .050 inches, Figure 3.10 shows that the reference points have indeed moved horizontally apart. These movements appear to roughly follow the same trends observed in the settlement of the foundations. Noting this trend, an average distance between the reference points was calculated over three sets of readings. These average distances are given in Table 3.2 and are plotted with the data in Figure 3.10.
Figure 3.11 illustrates the probable mechanism that would produce the observed movements of the reference points. As the foundation settled (Section 3.2), the underlying soil would have moved down and away. Because the reference points are anchored on long steel pipes, this soil movement would have resulted in the rotation of the pipes as depicted in Figure 3.11. This movement would result in negligible vertical movements even while the points moved measurably apart. Furthermore, this mechanism would be time dependent and related to the foundation settlements.

With no further data available, the measured movements were assumed to occur equally in reference points A and B. That is, half of the total change in horizontal distance is attributed to each reference point. This is consistent with the equal settlements observed in both foundations. Movements of the reference points, as well as the foundation settlements, are included in the calculation of deflections as discussed in the next section.

3.5. MEASUREMENT OF CULVERT DEFLECTIONS

The tape extensometer, described in Section 3.3, was used to monitor the movement of the control points in the plane of the instrumented cross section. Movements of the 13 control points shown in Figure 3.8 were tracked independently. Each control point is a stainless steel eyebolt that was bolted to the culvert plate prior to backfilling. Measurements to each eyebolt on the culvert were made relative to the two reference points set in the stream bed as depicted in Figure 3.12. The tape extensometer is used to measure the length from each reference point (A,B) to each control point (1-13). Thus, for each load condition, 26 readings are taken to record the deformed shape of the culvert cross section.
Data reduction. As discussed in Section 3.4, the elevation of reference point B with respect to A can be taken as a constant -1.06 inches. This effectively defines a constant horizontal plane from which the x-y positions of the control points are computed. The field data is reduced by computing the x-y coordinates of each control point based on the initial position of reference A as (0,0). Deflections are computed by simply subtracting the control point coordinates as determined for various load conditions. A computer program was written to permit the rapid reduction of raw data into culvert deflections.

A typical set of measurements to an arbitrary point P is shown in Figure 3.12. The lengths r, s, and t are determined using the tape extensometer. This data is then fed into the computer along with the elevation of B with respect to A. Referring to Figure 3.12, the angle $\beta$ is calculated as:

\[
\beta = \sin^{-1}\left(\frac{1.06}{r}\right)
\]

Eq. 3.1.

The angle $\theta$ is computed from the cosine law:

\[
\theta = \cos^{-1}\left(\frac{r^2 + s^2 - t^2}{2rs}\right)
\]

Eq. 3.2.

This gives $\alpha$, the angle measured from the horizontal plane, as:

\[
\alpha = \theta - \beta
\]

Eq. 3.3.

Now, the x-y coordinates of point P are computed simply as:

\[
P_x = s \cos \alpha
\]

Eq. 3.4.
As was discussed in Section 3.4, the reference points A and B were observed to move horizontally apart. To compensate for this movement in the calculation of point coordinates, one half of the observed change in horizontal distance is attributed to the x coordinate of each reference point. This procedure has the effect of correcting all of the coordinates to be based on the initial position of reference point A.

To compute culvert deflections, coordinates from an initial data set are subtracted from the data set of interest. Vertical deflections were further corrected to eliminate the vertical translation of the culvert due to foundation settlements (see Section 3.2). Table 3.1 lists the vertical settlement values that were subtracted to produce vertical deflections of the culvert relative to the foundations. This yields the desired horizontal/vertical deformations of the culvert at the 13 control points around the cross section.

Errors in deflection data. As discussed in Section 3.3, the maximum probable error in any length measured with the tape extensometer is .05 inches. This includes the maximum error due to the greatest temperature difference experienced during field activities. The tension in the tape during each measurement, consistently a force of 20 lbs, is strong enough to eliminate nearly all sag in the tape. Because the primary interest is in the movements of the control points, only the changes in the measured lengths are really important to the accuracy of the data. Since the tension in the tape is the same, the sag in the tape is unchanged from one set of readings to the next and the sag does not introduce any errors in determining the movements of the control points.

The data reduction algorithm was tested in an attempt to assess the maximum errors in the deflection data due to the maximum probable errors in any single length measurement. Errors tend to be magnified the most in the vertical movements computed for points closest to the foundations. For these points, an error in the measurement of lengths s or t (Figure 3.12) produces errors in the
angle $\alpha$ which is either close to 0° or 180°. From Equation 3.5, a small error in $\alpha$ will generate a relatively large error in the computed vertical position when $\sin \alpha$ is close to zero.

For this reason, the error in the reported deflections will depend on the position of the control point. Trial runs were made with the data reduction code to assess these errors given the maximum probable error in the measured lengths. Based on these runs, the accuracy of the deflection data for this culvert are taken as:

- points in the crown region: accuracy = $\pm 0.05$ inch
- points located on vertical sides of culvert: accuracy = $\pm 0.10$ inch
- points #1 and #13 (see Figure 3.8) located 8" above foundations accuracy = $\pm 0.40$ inch

**Determination of actual culvert shape.** Generally, only the deflections of the control points are desired from this data. However, the same data can be used to determine the actual shape of the culvert cross section. This was done to check for conformance of the constructed culvert shape to the design shape. Furthermore, this information is invaluable in deriving the correct coordinates for the finite element mesh described in Chapter 9.

Point coordinates, computed as described previously, will correspond to the center of the eyebolt. To determine the true position of the culvert plate, this coordinate must be adjusted to compensate for the length of the eyebolt. The design shape was used to determine the normal angle to the plate at each control point location. The average eyebolt length was taken to be 1.25 inches and each control point was adjusted accordingly in the normal direction (the eyebolts are perpendicular to the culvert plate). Because the actual length of each eyebolt is unknown, determination of the culvert shape in this manner is probably no more accurate than $\pm 0.5$ inches.

This procedure was employed to plot the culvert shape as measured just before backfilling began. The measured culvert shape very nearly overlaid the design shape. Hence, the design shape
was used to determine the nodal coordinates for the finite element mesh in Chapter 9. More importantly, this proves that no significant distortions were induced into the culvert during assembly.

3.6. BENDING OF THE HAUNCH CORNER

Syro's box-type culvert design uses a distinctive 110° welded connection in the haunch, as shown in Figure 2.1. Under load, high moments would be expected in this region if the culvert behaves like a frame. To monitor the integrity of this connection, a split light beam scheme was devised to economically measure any changes in the haunch angle.

A schematic of the instrumentation setup for one corner is shown in Figure 3.13. Identical setups were used to monitor the haunch angles on either side of the culvert. A high intensity, focused light projector is used to produce a long, narrow band of light. This beam is reflected off of a target mirror mounted to the culvert's side. The reflected beam then hits two smaller mirrors at the top which splits the light beam. Two narrow lines of light are thus projected onto a scale which is mounted on a vertical pole. The change in the separation of the light beam at this point can be related to the change in the haunch angle. The light projector and the target mirrors, mounted in the culvert haunch corner, are pictured in Figure 3.14. A more detailed rendering of the important mirror targets is shown in Figure 3.15. The large, lower mirror (A) is mounted on a bracket that is glued to the culvert side with epoxy. This mirror will thus rotate with the culvert sides. The metal bracket also extends to support Mirror B which is mounted at approximately 110° to the plane of Mirror A. Mirror B is not attached to the crown plate and will thus remain at a constant angle to the plane of Mirror A. Mirror C is glued directly to the culvert's crown plate. As the haunch corner angle changes, the angle between the planes of Mirrors B and C will also change causing the light beams to converge or diverge. All of the target mirrors are located within 2 inches
of the innermost corner of the welded connection so that only changes in the haunch connection itself will affect the readings.

**Data reduction.** Only the separation of the light beams at the scaled target is required to determine changes in the haunch corner angle. The geometry used to reduce this data is represented in Figure 3.16. In this figure, greek letters refer to angles whereas other symbols denote lengths. The initial haunch angle \( \alpha \) is taken from the design shape as 110°. Note also that the horizontal scale is greatly shortened in Figure 3.16. The length \( L \) from the target to the scale is measured in the field after the instruments are set up. The separation distance \( d \) is read, in millimeters, for each data set. The change in this separation distance, \( \Delta d \), is used to compute the change in the haunch angle, \( \Delta \alpha \).

Recall that the angle of incidence for a light ray equals the angle of reflection and also, for any triangle, the sum of the interior angles is 180°. Using a triangle formed by the planes of the mirrors in Figure 3.16, it can be shown that:

\[
\text{incident angle at Mirror } B = 180^\circ - \theta - \beta \quad \text{Eq. 3.6.}
\]

\[
\text{incident angle at Mirror } C = 180^\circ - \theta - \alpha \quad \text{Eq. 3.7.}
\]

By inspection, it can be deduced that the angle between the planes of Mirrors B and C is equal to \( \alpha - \beta \). Furthermore, the angle from the plane of Mirror A to the light ray "L" can be expressed in two ways which incorporate Equations 3.6 and 3.7:

\[
\text{angle from plane of } A \text{ to } L = \alpha - (180^\circ - \theta - \alpha) - \beta -(180^\circ - \theta - \beta) + \gamma \quad \text{Eq. 3.8.}
\]

Simplified, Equation 3.8 yields:
Knowing that $\gamma$ in Figure 3.16 is much less than 5°:

\[ d' = (L+M)\gamma \quad \text{Eq. 3.10.} \]

From the small triangle at the lower right of Figure 3.16 and the law of sines:

\[ d \sin \delta = d' \sin \left(90^\circ + \frac{\gamma}{2}\right) \quad \text{Eq. 3.11.} \]

Substituting 3.9 and 3.10 into 3.11 gives:

\[ d \sin \delta = 2(L+M)(\alpha - \beta) \sin(90^\circ + \alpha - \beta) \quad \text{Eq. 3.12.} \]

Using the isosceles triangle containing $M$, the angle $\tau$ is:

\[ \tau = 90^\circ - \frac{\gamma}{2} = 90^\circ - \alpha + \beta \quad \text{Eq. 3.13.} \]

The law of sines and the small triangle containing $s$ and $s'$ gives:

\[ s' \sin \tau = s \sin 2(180^\circ - \theta - \beta) \quad \text{Eq. 3.14.} \]

Then, again recalling $\gamma$ is less than 5°:

\[ s' = M\gamma = 2M(\alpha - \beta) \quad \text{Eq. 3.15.} \]

Bringing Equations 3.13 through 3.15 together will give:

\[ M = \frac{s'}{2(\alpha - \beta)} = \frac{s \sin 2(180^\circ - \theta - \beta)}{2(\alpha - \beta) \sin(90^\circ - \alpha + \beta)} \quad \text{Eq. 3.16.} \]

Finally, substitution of Equation 3.16 into 3.12 gives:

\[ d = \frac{\sin(90^\circ + \alpha - \beta)}{\sin \delta} \left[ 2L(\alpha - \beta) + s \frac{\sin 2(180^\circ - \theta - \beta)}{\sin(90^\circ - \alpha + \beta)} \right] \quad \text{Eq. 3.17.} \]
Equation 3.17 is used to convert the field data (d) to changes in the haunch angle (a). The bracket angle $\beta$ remains constant throughout the field tests. The parameters L, s, $\theta$, and $\delta$ are assumed constant such that changes in d are solely a function of changes in a. The length of L is measured in the field for each set up. The small length s and the angles $\delta$ and $\theta$ could not be measured in the field and had to be estimated. As the change in a is desired, the true initial values of these various parameters is of secondary importance. Only changes in the assumed constants will yield erroneous results for changes in a.

**Estimates of errors.** Errors in the change in haunch angle determined from the split light beam arise mostly from slight changes in the lengths and angles that are assumed constant. This is expected to happen as the culvert shifts under load or as the projector or scale positions change as the foundations settle.

To ascertain the degree to which each of these factors may induce errors in the data, Equation 3.17 was used to compute errors in d due to changes in the constants L, s, $\theta$, and $\delta$. This process pointed out the relative insignificance of some of these errors when compared to how accurately the distance d could be read in the field. Based on these calculations, it is assumed that the true change in the haunch angle could be measured to within $\pm 0.1^\circ$.

Greater care and attention to various details could probably improve the accuracy attainable with this system. Particular attention should be paid to isolating the projector and the vertical scale from culvert settlements and measuring the initial lengths and angles discussed above. Furthermore, a low power laser beam, which could be focused more sharply, would yield better measurements of the separation d. Also, laboratory calibrations would yield greater confidence in the data and the data reduction technique. Regardless, this particular setup produced reasonable results that are presented in Chapter 5.
3.7. CHANGES IN CORRUGATION DEPTH

This culvert was constructed of Syro Steel’s deep-corrugated steel plate. Field measurements were made to check the behavior of this unique geometry under load. Specifically, a 0.001 inch gradation dial gauge was mounted on a steel bracket to measure changes in the depth of the corrugation. The bracket and dial gauge are pictured in Figure 3.17. The bracket spanned one full corrugation with the dial gauge mounted perpendicularly. One side of the bracket was bolted to the culvert plate using a slotted hole to prevent any localized stiffening due to the bracket itself. This instrument was installed about one foot from the culvert’s crown where maximum bending moments were expected.

Data from this instrument is plotted in Figure 3.18 versus backfill height. This data indicates that the corrugation depth decreased by about 0.014 inches under the construction loads. No measurable changes occurred during the live loading sequence.

The maximum observed change corresponds to about 0.25% of the initial depth. Viewed another way, this would change the theoretical moment of inertia by about 1% (based on the moment of inertia being related to the depth\(^4\)). This small change is judged to be insignificant in determining the stiffness and behavior of the culvert. Moreover, these small changes would be expected as normal behavior as the culvert flexes under load and do not indicate any instabilities of the corrugated section.

3.8. DISPLACEMENTS IN SAND BACKFILL

Four extensometers were buried during construction to measure displacements in the sand fill. The four 10 ft. extensometers, supplied by the Slope Indicator Company, are pictured in Figure
The extensometers consist of a long steel rod isolated inside a PVC pipe. Each end of the extensometer is anchored in the soil using a 24 inch steel flange visible in Figure 3.20. The protective PVC pipe has special joints to permit movement of the anchor flanges relative to one another. The steel rod inside the pipe is attached to the soil anchor on one end and to an electronic displacement transducer on the other end. As the anchors move within the soil mass, the transducer, visible in Figure 3.20, measures the apparent movement of the steel rod to indicate the change in length between the steel flanges.

Each extensometer was placed in the fill during construction. The initial set length, ranging from 103 inches to 113 inches, was recorded before burying the instrument. Electrical cables were fed inside the culvert through holes drilled in the plate. Inside the culvert, remote readings were taken using a special electronic indicator pictured in Figure 3.21.

The four soil extensometers were placed in the positions shown in Figure 3.22. The first two instruments, R1 and R2, were placed midway up the culvert side with one end bolted directly to the culvert plate. This connection is visible in Figure 3.23. The remaining extensometers, R3 and R4, were placed over the culvert crown about midway between the top of the culvert and the bottom of the asphalt pavement. One of these rods is pictured during installation in Figure 3.24.

The manufacturer supplied a calibration factor for each individual displacement transducer. Displacements are computed simply as:

\[
\text{displacement} = \frac{\Delta \text{reading}}{\text{calibration factor}}
\]

Eq. 3.18.

The extensometer is reportedly accurate to ±0.0005 inches with a full range of 6 inches.

To validate the deformation data, the horizontal displacement of the culvert side as measured with the soil extensometers was compared with deflections computed from the tape extensometer data. Soil extensometers R1 and R2 were bolted to the culvert 30 inches above the foundation. To get deflection data at the same point, horizontal deflections at the first control points located above
and below this point were interpolated. This data is compared in Figures 3.25 and 3.26 for both sides of the culvert. In some places, the comparison of data is reasonable. However, there is too much scatter in the deflection data to permit a good validation. This results from the better precision of the soil extensometers when compared to the deflection data. Recall from Section 3.5 that the deflections computed in the side region can be in error by as much as 0.10 inches.
Table 3.1. Foundation settlement time history.

<table>
<thead>
<tr>
<th>Dates</th>
<th>Loading Conditions</th>
<th>Settlement (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 16-18</td>
<td>0&quot; to 60&quot; of backfill</td>
<td>0.00</td>
</tr>
<tr>
<td>May 19</td>
<td>60&quot; to 70&quot; of backfill</td>
<td>-0.04</td>
</tr>
<tr>
<td>May 19</td>
<td>70&quot; to 78&quot; of backfill</td>
<td>-0.16</td>
</tr>
<tr>
<td>May 19</td>
<td>78&quot; to 80&quot; of backfill</td>
<td>-0.29</td>
</tr>
<tr>
<td>May 20-25</td>
<td>Limestone subbase, asphalt</td>
<td>-0.42</td>
</tr>
<tr>
<td>June 1</td>
<td>Live loads</td>
<td>-0.56</td>
</tr>
</tbody>
</table>

Table 3.2. Measured distance between deflection reference points A and B.

<table>
<thead>
<tr>
<th>Dates:</th>
<th>Loading conditions:</th>
<th>Average distance (in):</th>
<th>Change in distance (in):</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 16-20</td>
<td>All backfill including crushed limestone subbase</td>
<td>95.123</td>
<td>0.0</td>
</tr>
<tr>
<td>May 23-25</td>
<td>Asphalt placement</td>
<td>95.187</td>
<td>+0.064</td>
</tr>
<tr>
<td>June 1</td>
<td>Live loads</td>
<td>95.224</td>
<td>+0.101</td>
</tr>
</tbody>
</table>
Figure 3.1. Stationary end of hose level system.

Figure 3.2. Close-up of hose level showing water tube and vertical scale.
Foundation Settlements
Hose Level Measurements

Figure 3.3. Foundation settlements as measured with hose level.

Settlement of North Foundation
Hose Level vs. Deflection Data

Figure 3.4. Settlement of north foundation measured two ways.
Settlement of South Foundation
Hose Level vs. Deflection Data

Figure 3.5. Settlement of south foundation measured two ways.

Figure 3.6. Tape extensometer device.
Figure 3.7. Close-up of tape extensometer.

Figure 3.8. Locations of deflection control points and strain gauges.
Movement of Deflection Reference Points
Hose Level Measurements

Figure 3.9. Vertical movement of deflection reference points A and B.

Movement of Deflection Reference Points
Horizontal Length from Ref. A to Ref. B

Figure 3.10. Horizontal movement of deflection reference points A and B.
Figure 3.11. Proposed mechanism that would cause observed reference point movement.

Figure 3.12. Determination of control point coordinates using triangularization.
Bracket and mirrors
Focused light projector
Scale

Figure 3.13. Split light beam for measuring bending in the haunch corners.

Figure 3.14. Light projector and mirrors in north haunch corner.
Mirrors A and B are mounted to the bracket which is attached to the vertical culvert side. Mirror C is mounted directly to the culvert crown plate.

Figure 3.15. Bracket and mirrors which split light beam corner angle measurement.

Figure 3.16. Geometry of split light beam. (Note that horizontal scale is greatly shortened).
Figure 3.17. Dial gauge and bracket for measuring changes in corrugation depth.

Change in Corrugation Depth
Dial Gauge Measurement Near Crown

Figure 3.18. Measured change in corrugation depth.
Figure 3.19. Soil extensometers before installation.

Figure 3.20. Displacement transducer and steel anchor on soil extensometer.
Figure 3.21. Indicator for reading soil extensometers.

Figure 3.22. Locations of soil extensometers.
Figure 3.23. Installation of soil extensometer bolted to culvert side.

Figure 3.24. Installation of soil extensometer in backfill over top of culvert.
Comparison of Soil Extensometer and Deflection Data on North Side

![Graph showing comparison of soil extensometer and deflection data on north side.]

Positive displacement is outward
- Extensometer data
- Deflection data

Live Load and Position

Figure 3.25. Comparison of deflection and soil extensometer data on north side.

Comparison of Soil Extensometer and Deflection Data on South Side

![Graph showing comparison of soil extensometer and deflection data on south side.]

Positive displacement is outward
- Extensometer data
- Deflection data

Live Load and Position

Figure 3.26. Comparison of deflection and soil extensometer data on south side.
CHAPTER 4

MEASUREMENT OF PLATE FORCES

4.1. OVERVIEW OF STRAIN GAUGE MEASUREMENTS

To determine the axial and bending forces carried in the corrugated plate, a large number of strain gauges were installed on two cross sections near mid-length of the culvert. These gauges were organized and distributed into the same thirteen instrument locations used for the deflection control points. Three types of strain gauges were used: biaxial bonded foil gauges, rosette bonded foil gauges, and uniaxial vibrating wire strain gauges. The bonded foil, electrical resistance strain gauges, 53 biaxials and 8 rosettes, were read using a computerized data acquisition system. The 10 vibrating wire strain gauges were read one at a time using a hand-held pick up sensor and indicator. Biaxial, plane stress assumptions were used to compute circumferential stresses in the culvert plate. The validity of using simple beam theory to analyze the plate forces was checked. Finally, all of the strain gauge data was used together to compute bending moments and thrusts as presented in Chapter 5 and Appendix A.

4.2. BONDED FOIL, ELECTRICAL RESISTANCE STRAIN GAUGES

Bonded foil, electrical resistance strain gauges were selected to provide the primary measurements of strain in the culvert plate. Fifty-three biaxial gauges, consisting of two gauge elements laminated directly over one another in perpendicular directions, were used to measure
circumferential and longitudinal strains in the culvert plate. Uniaxial gauges, with only one gauge element, were not used because a biaxial strain field was expected in the corrugated plate. Biaxial gauges were thus needed to compute the correct stresses in the plate. Ten rosette resistance gauges, consisting of three gauge elements laminated together, measured strains in perpendicular directions as well as 45° to the perpendicular. This third strain measurement allows the determination of shear stresses in the plate.

The biaxial foil gauges selected were Micro-Measurements No. CEA-06-125WT-350 while the rosettes used were No. CEA-06-250UR-350. All of the bonded foil gauges had a nominal resistance of 350 ohms in each element. Because of the relatively long lead wires used on each gauge, the higher gauge resistances was needed to minimize desensitization due to the resistance of the lead wires. The biaxial gauges have a nominal gauge factor of 2.09 with a nominal transverse sensitivity of \( K_t = 0.8\% \). The rosette gauges have a nominal gauge factor of 2.07 with a nominal \( K_t = 0.0\% \). All of the gauges were self temperature compensating; i.e., the gauges had the same thermal expansion properties as the steel plate to eliminate erroneous strain readings due to changes in temperature.

**Installation.** Two biaxial strain gauges are pictured in Figure 4.1 before installation. All of the foil electrical resistance gauges were mounted to the inside of the culvert plate using Micro-Measurement M-BOND 200 adhesive. At each gauge location, the galvanized plate was sanded smooth prior to installation of the gauge. A mounted biaxial gauge, with a temporary cellophane tape covering, is shown in Figure 4.2.

Each element of the biaxial and rosette gauges was wired independently. To speed the installation process, cables were made up in the lab, labeled, and carried to the field. These cables were then hung inside the culvert and lead wires were routed to the individual gauges and secured with liberal quantities of duct tape. One of the biaxial gauges is shown in Figure 4.3 after the lead wires have been attached. Approximately one inch from the gauge, the lead wires are firmly glued to the culvert plate.
A three-wire, quarter bridge circuit was used to wire each strain gauge element. The three wire system was employed to mitigate erroneous strain readings due to temperature changes in the lead wires. Since similar gauge elements were employed in both the biaxial and rosette gauges, each element of any strain gauge could be wired and used in the same way. This simplified the task of running wire to each gauge. Yet, running all of the necessary wires and keeping track of gauge numbers and locations proved to be the most time consuming aspect of the installation work. Conditions inside the culvert were obviously very wet. In the event of heavy rain, some gauges located near the foundation would be expected to flood. Consequently, it was very important to protect and seal each strain gauge to keep the gauge dry. Protective measures included an acrylic sealant brushed directly onto the gauge, a tar sealant pad, a rubber pad, and a foil covering sealed with nitrile rubber. An installed strain gauge, with all of the protective coverings, is shown in Figure 4.4. Figure 4.5 is a photograph showing two biaxial strain gauges (left and right) with a rosette gauge (middle) after installation on the culvert plate.

**Data acquisition system.** The fifty-three biaxial gauges and eight rosette gauges produced 130 strain readings. To handle this volume of data quickly and accurately, a computerized data acquisition system was employed. The system was mounted in the back of the four wheel drive field vehicle in Figure 4.6.

All 130 lead wire sets were strung along the culvert crown to the western end. Here, the cables were organized into groups of ten and terminated with connectors. A matching group of connectors and cables lead to the data acquisition electronics in the truck. These connectors, shown in Figure 4.7, made it possible to disconnect and remove the electronic system at the end of each day. Unfortunately, each time the connectors were disconnected and then reconnected, the resistance across the connection changed. Tests showed this could have the effect of producing an erroneous strain reading of 8 micro-strain. Because of this effect, careful consideration of the initial conditions had to be employed during data analysis. This will be discussed in the next section.
Figure 4.8 is a photograph of the electronics and computer used to acquire strain readings from the bonded foil strain gauges. All of the lead wires were connected to a Hewlett-Packard Model 3497A data acquisition unit. This unit contained the necessary electronics to complete a quarter-bridge circuit for each of the 130 strain gauge elements. The unit also contained a digital voltmeter which could be switched to read each strain channel. The data acquisition unit was controlled using a desktop Hewlett-Packard computer.

Software was written to sweep each of the strain gauge channels five times and record the data. Later, these five readings were averaged together to reduce the influence of electrical noise in the readings. The data was stored on floppy disk and printed out on paper in the field. It is not known how much electrical noise was present in the strain data recorded with this system. More reliable strain data might have been achieved if greater care had been taken to isolate electrical noise in the system by using shielded cable and proper grounding techniques.

4.3. REDUCTION OF ELECTRICAL RESISTANCE STRAIN GAUGE DATA TO STRAINS

A schematic of the quarter bridge circuit, with a three wire system, used for the bonded foil gauges is shown in Figure 4.9. The data acquisition system reads the two voltages, $V_{IN}$ and $V_{OUT}$, and returns the ratio of $V_{OUT}/V_{IN}$. This ratio is stored on disk and analyzed later by computer to produce the strains experienced by each individual gauge element.

Reducing the measured voltage ratios to strains employs an analysis of the quarter bridge circuit [Hewlett Packard, 1981]. Referring to Figure 4.9, the current flowing from b to c must be equal to current flowing from a to c. Thus:
Similarly, from the current flow through the strain gauge ($R_g$):

\[ \frac{V_{dc}}{R_1 + R_3} = \frac{V_{IN}}{2R_1 + R_3 + R_g} \]  

Eq. 4.2.

The potential difference $V_{OUT}$ is given by:

\[ V_{OUT} = V_{dc} - V_{bc} \]  

Eq. 4.3.

Substituting Equations 4.1 and 4.2 gives:

\[ V_R = \frac{V_{OUT}}{V_{IN}} - \frac{R_1 + R_3}{2R_1 + R_3 + R_g} - \frac{R_2}{R_1 + R_2} \]  

Eq. 4.4.

where $V_R$ is the voltage ratio recorded by the data acquisition system.

When strained, the gauge resistance will change by $\Delta R$. The change in the voltage ratio is then:

\[ \Delta V_R = \left( \frac{R_1 + R_3}{2R_1 + R_3 + R_g + \Delta R} \right) - \left( \frac{R_1 + R_3}{2R_1 + R_3 + R_g} \right) \]  

Eq. 4.5.

The unstrained resistance of the gauge is matched on the electronics board such that $R_3 = R_g = 350$ ohms. Equation 4.5 can then be simplified to:
This can be rearranged to give:

\[
\frac{\Delta R}{R_L} = \frac{-4\Delta V_R}{1+2\Delta V_R} \left(1+\frac{R_L}{\Delta R}\right)
\]  

Eq. 4.7.

The term in parentheses on the right of Equation 4.7 accounts for lead wire desensitization [Hewlett Packard, 1981]. This factor can become significant if the lead wires are very long. For this installation, about 75 feet of wire (22 and 24 AWG) yields an estimated lead wire resistance of \( R_L = 1.48 \) ohm. The desensitization factor will thus yield a correction of only 0.42% in the strain data. Also note that the data acquisition system utilizes the ratio of \( V_{OUT} \) over \( V_{IN} \). This eliminates potential errors due to fluctuations in the power supply for the strain gauges.

Initial conditions. Equation 4.5 implies that a strain is measured as a change from some assumed initial condition. For the strain data taken during live loading, this initial condition is taken as the gauge readings on the morning the live loads were applied. Similarly, strains due to construction loads are based on an initial condition from when the culvert was free-standing before any backfill was placed. As was pointed out in Section 4.2, it was necessary to unhook the computer from the strain gauges on the culvert each night during construction. Unfortunately, the resistance across the connectors would change each time this was done and would have the effect of producing erroneous strain data. To avoid this, strains were computed for all of the bonded foil gauges using the following two step process:

1. Strains were computed using as an initial the first set of readings taken on the same day before any loads were applied.

2. These strains were then added to the strains computed for the last data set from the previous day.
Since forces on the culvert are theoretically the same for the last and first data sets on consecutive days, the strains could be accumulated in this manner to produce strains that were consistently based on the same initial condition. For the live load data, strains were not accumulated to the total strains due to backfill loads.

Unfortunately, this procedure cannot account for relaxation or redistribution of stresses in the culvert that might occur overnight. For example, assume a very large increase in load is measured in the last data set acquired on a given day. This large load will then be reflected in the stresses computed for all subsequent data sets. It is possible that the load will redistribute with time, but this effect cannot be measured with this instrumentation. The necessity of unhooking the strain gauge data acquisition system every night, and thereby changing the resistance across the connection, inflicts a significant deficiency in the measurement system. Using this system, it is not possible to assess the changes in stress distribution within the culvert between two consecutive days.

**Correction for transverse sensitivity.** The biaxial strain gauges were also corrected for transverse sensitivity. This is necessary because the gauges will experience a small change in resistance due to strains perpendicular to the axial direction of the gauge elements. The following development is taken from Dally and Riley [1978].

The transverse sensitivity, \( K_t \), of a strain gauge element is defined as:

\[
K_t = \frac{S_t}{S_a}
\]

where \( S_t \) and \( S_a \) are the gauge's sensitivity in the transverse and axial directions, respectively. The sensitivity \( S_g \), or gauge factor, of a strain gauge is determined in a uniaxial strain field such that:

\[
\frac{\Delta R}{R_s} = S_g e_a - S_a e + S_t e_t = S_t (e_a - K_t v e_o)
\]
where \( v \) is Poisson’s ratio. Rearranging will give the true axial sensitivity as a function of the measured gauge factor:

\[
S_a = \frac{S_g}{1-vK_t}
\]

Eq. 4.10.

Now, the response of a gauge in an axial strain field can be expressed as:

\[
\frac{\Delta R}{R_g} = S_a(e_a + K_r e_i) - \frac{S_g}{1-vK_t}(e_a + K_r e_i)
\]

Eq. 4.11.

Writing this equation for each element of a biaxial gauge:

\[
\left( \frac{\Delta R}{R_g} \right)_{\text{axial}} = \frac{S_g}{1-vK_t}(e_a + K_r e_i)
\]

Eq. 4.12.

\[
\left( \frac{\Delta R}{R_g} \right)_{\text{trans.}} = \frac{S_g}{1-vK_t}(e_r + K_r e_a)
\]

Eq. 4.13.

The quantities on the left side of these last two equations are determined from the field data using Equation 4.7. Solving Equations 4.12 and 4.13 simultaneously will yield the desired strain values:

\[
e_{\text{axial}} = \frac{1-vK_t}{S_g(1-K_t^2)} \left[ \left( \frac{\Delta R}{R_g} \right)_{\text{axial}} - K_t \left( \frac{\Delta R}{R_g} \right)_{\text{trans.}} \right]
\]

Eq. 4.14.

\[
e_{\text{trans.}} = \frac{1-vK_t}{S_g(1-K_t^2)} \left[ \left( \frac{\Delta R}{R_g} \right)_{\text{trans.}} - K_t \left( \frac{\Delta R}{R_g} \right)_{\text{axial}} \right]
\]

Eq. 4.15.

For the rosette gauges, the transverse sensitivity \( (K_r) \) has a negligible value. Thus, for each element on a rosette gauge, Equation 4.14 will reduce to:

\[
e_{\text{axial}} = \frac{1}{S_g} \left( \frac{\Delta R}{R_g} \right)_{\text{axial}}
\]

Eq. 4.16.
4.4. VIBRATING WIRE STRAIN GAUGES

Bonded foil strain gauges provided the primary means of measuring forces in the culvert plate. However, even with the efforts to provide adequate protection, these gauges remained susceptible to water damage. Thus, to provide for long-term strain measurements and to back up the electrical resistance strain gauges, ten weldable vibrating wire strain gauges were also installed on the culvert.

The vibrating wire gauges were manufactured by the Slope Indicator Company (Model 52621) and had a length of two inches. The gauge consists of a hollow tube mounted on a flat flange which is spot welded to the culvert plate. Inside the sealed tube is a wire, restrained only at the ends, that is stretched over the two inch gauge length. As the gauge is strained, either in compression or expansion, the tension in the wire changes. The natural frequency of the wire will vary with the square root of the tension in the wire, which can then be related to the strain.

Installation. The vibrating wire gauges were spot welded to the culvert plate using the battery powered, capacitor type welder shown in Figure 4.10. To ensure strong welds to the base metal, the zinc galvanizing had to be ground off at each gauge location as shown in Figure 4.11. The gauges were then permanently attached using a multitude of small spot welds as in Figure 4.12. It was impossible to weld the flat gauges on the curved sections of the corrugated plate; therefore, all of the vibrating wire gauges are mounted on the flat portions of the corrugated section. An installed vibrating wire gauge is shown in Figure 4.13. After the welding was complete, the entire gauge was coated with a rubber sealant to inhibit rust.

Data acquisition. Strains were read from the vibrating wire gauges using a pick-up sensor and indicator box as demonstrated in Figure 4.14. A small sensor was held, by hand, over each vibrating wire gauge. This sensor induces a constant oscillation, at the natural frequency of the wire, using a magnetic field. When the natural frequency is reached, the indicator box reads the frequency.
Internally, this frequency is converted to the strain in the wire. The electronic box reads out directly in micro-strain.

When initially installed, the wire inside the gauge has a nominal pretension of about .2% strain. As the culvert plate expands or compresses, the tensile strain in the wire increases or decreases respectively. Strains in each vibrating wire gauge are computed by subtracting the current strain reading from the initial reading for each individual gauge. The vibrating wire gauges are constructed of steel and thus have similar thermal expansion properties as the base metal of the culvert. Thus, no corrections for temperature changes are necessary.

Due to the two inch length of the gauges, the vibrating wires could not be mounted across the corrugated section. That is, the gauges could only be welded in the circumferential direction of the culvert cross section where there was a sufficiently flat area to mount the gauges. Consequently, only circumferential strains could be recorded with the vibrating wire gauges. This precludes measuring the biaxial strain field completely and this problem will be discussed later in this chapter.

4.5. LOCATION OF STRAIN GAUGES

All of the strain gauges were distributed in the same thirteen circumferential locations as the deflection control points (refer to Figure 3.8). Locations of all of the gauges, and the deflection eyebolts, within each section are shown in Figure 4.15. Twenty-six biaxial gauges and ten vibrating wire gauges are organized into pairs in the primary instrumented circumferential section. An additional twenty-six biaxial gauges are paired in a secondary cross section as shown in Figure 4.15. One additional biaxial gauge is placed in the primary section at the crown. Eight rosette gauges are evenly distributed around the circumference in the thirteen gauge locations. A typical installation showing two biaxial gauges, two vibrating wire gauges, and one rosette is pictured in Figure 4.16.
As can be seen in Figure 4.15, all of the biaxial gauges were placed in either the very top or very bottom of the corrugation. However, the vibrating wire and rosette gauges were placed on the flat portion of the corrugation. The depth of each gauge (the distance to the neutral axis of the corrugated section) must be known accurately to minimize errors in the bending moments calculated from the strain gauge data. For each rosette and each vibrating wire gauge, this depth was measured approximately by placing a straight edge across two peaks in the corrugation and measuring the depth to a particular gauge. More accurate measurements were made by using a flexible ruler to measure the surface length along the corrugated steel from the gauge to the very peak of the corrugation. Knowing this length and the geometry of the corrugations, the depth of the gauge could be determined more accurately.

**Damaged strain gauges.** Even with the extensive efforts to protect the bonded foil gauges, some gauge elements produced highly erratic data or no data at all. This probably resulted from water intrusion or poor electrical connections on isolated gauges. In all, six gauge elements (out of 130 total) produced bad data at some point during the study. All of the data from these six elements was discarded even if the gauge appeared to be giving reasonable results during part of the field tests. This resulted in the loss of data from two rosette gauges and four biaxial gauges. Even so, 95% of the 130 bonded foil gauge elements produced good data throughout the field study.

### 4.6. CALCULATION OF SHEAR STRESSES FROM ROSETTE DATA

Data from the rosette gauges were used to compute shear stresses and subsequently the beam shear. A rosette gauge provides strain readings in two perpendicular directions and along a 45° arm as shown in Figure 4.17. Using index notation, the strain in the direction of a general vector \( \vec{n} \) can be written as:
\[ \varepsilon_{mn} = \varepsilon_y n_i n_j \]  
Eq. 4.17.

Expanding this equation gives:

\[ \varepsilon_{mn} = \varepsilon_x n_i n_x + 2\varepsilon_y n_i n_y + \varepsilon_y n_y n_y \]  
Eq. 4.18.

Referring to Figure 4.17, it is observed for the rosette gauges \( \varepsilon_{XX} = \varepsilon_A, \varepsilon_{YY} = \varepsilon_C, \varepsilon_{mn} = \varepsilon_B \).

Substitution yields:

\[ \varepsilon_{xy} = \varepsilon_y - \frac{\varepsilon_A}{2} - \frac{\varepsilon_C}{2} \]  
Eq. 4.19.

Noting that the shear strain, \( \gamma_{xy} = 2\varepsilon_{xy} \), the following equation gives the shear strain from the three rosette strain measurements [Dally and Riley, 1978]:

\[ \gamma_{xy} = 2\varepsilon_y - \varepsilon_A - \varepsilon_B \]  
Eq. 4.20.

Having the shear strain, \( \gamma_{xy} \), the shear stress (\( \tau \)) is given simply by:

\[ \tau = G \gamma_{xy} \]  
Eq. 4.21.

where \( G \) is the shear modulus. For the base steel in the culvert with a Young's modulus of 29,300 ksi and a Poisson's ratio of 0.30, the shear modulus is 11,270 ksi. The next section demonstrates the validity of treating the corrugated plate using beam theory. The shear stresses, derived from the rosette data, can thus be used to compute beam shear (\( V \)) as [Popov, 1976]:

\[ V = \frac{\tau t l}{Q} \]  
Eq. 4.22.

where \( I \) is the moment of inertia, \( t \) is the beam thickness (at the location of \( \tau \)), and \( Q \) is the statical moment around the neutral axis of the area outside of the gauge location. Hence, it is necessary to determine the values of \( t \) and \( Q \) from the geometry of the corrugated cross section for each rosette gauge location.
4.7. VALIDITY OF SIMPLE BEAM THEORY ASSUMPTIONS

Corrugated steel plate is typically treated as a simple beam with the relevant beam forces of axial thrust and bending moments. The development of simple beam theory assumes that plane sections in a beam remain plane under bending loads [Popov, 1976]. For the deep-corrugated section employed in this study, it might not have been valid to apply simple beam theory; i.e., plane sections might not have remained plane due to gross distortions in the corrugated section.

Strains measured in the circumferential direction, using the biaxial and rosette gauge elements as well as the vibrating wire gauges, are plotted in Figures 4.18 and 4.19. These figures show the circumferential strains in seven instrumented sections (refer to Figure 3.8 for locations) around the culvert cross section under the 42 kip live load. During construction, more erratic data is observed due to localized effects that arise out of the construction loads. Hence, only the strains due to the live loads are plotted in these figures to demonstrate culvert behavior under a uniform loading.

As can be seen in Figures 4.18 and 4.19, all of the circumferential strain data varies linearly with depth from the neutral bending axis of the cross section. This clearly demonstrates the validity of assuming that plane sections do remain plane under load and, therefore, that the assumption of simple beam theory can be used in further analysis. Figures 4.18 and 4.19 also demonstrate that the strains recorded using the bonded foil gauges and the vibrating wire gauges are in good agreement. This provides an important check on the validity of the data from each instrumentation system.

4.8. REDUCTION OF STRAINS TO AXIAL THRUSTS AND BENDING MOMENTS

As was discussed in Chapter 1, the culvert plate should be analyzed assuming a biaxial strain field. Since the corrugations are circumferential around the culvert, and loads can be expected to
be carried in that direction, the principal strain directions in the corrugated plate can be assumed to lie in the circumferential and longitudinal directions of the culvert. Figure 4.20 is a scatter plot of the circumferential and longitudinal strains measured with each element of all of the biaxial strain gauges under all loading conditions. This figure indicates the relative predominance of the measured circumferential strains over the longitudinal strains. Again, this would be expected as loads are carried circumferentially to the foundations. However, Figure 4.20 also shows that longitudinal strains can be very significant and even greater than the circumferential strains in some of the data. Consequently, the strain data should be reduced to the desired stresses using the appropriate biaxial equations. This is done assuming plane stress equations and using the familiar relation:

\[
\sigma_{\text{circum}} = \frac{E}{1-\nu^2} (\varepsilon_{\text{circum}} + \nu \varepsilon_{\text{long}})
\]

Eq. 4.23.

Equation 4.23 is easily applied to the longitudinal and circumferential strain data from the appropriate element in the biaxial gauges as well as the rosette gauges. However, it was not possible to mount vibrating wire strain gauges across the corrugations to measure longitudinal strains. To permit the consistent use of the plane stress concept, the average longitudinal strains measured by the bonded foil gauges in the same circumferential location was employed as the longitudinal strain for computing stress from the vibrating wire data.

Circumferential plate stresses are plotted in Figures 4.21 and 4.22 for the 42 kip live load at Positions #1 and #5. These plots correspond to the plots of strain in Figures 4.18 and 4.19. Again, note that the observed stresses are generally linear. Figures 4.21 and 4.22 also show a linear regression fit through the data in each instrumentation section. Note also that, where a sufficient number of gauges exist, separate linear fits are made for circumferential stresses in the primary and secondary cross sections (refer to Figure 4.15). This was done to detect potentially different stresses in the cross sections due to the relative position of the truck's wheels over the culvert. This effect
will be discussed in Chapter 5, but notice that the linear regression lines in Figures 4.21 and 4.22 are very similar.

Chapter 5 presents the plate forces derived from analysis of the strain gauge data. These plate forces, consisting of axial thrust and bending moment diagrams, were derived from the linear regression fit through the appropriate circumferential stresses. The axial thrust is simply the circumferential stress at the neutral bending axis (as found from the regression fit) multiplied by the area of the corrugated plate (per lineal foot). The bending moment is similarly the product of the moment of inertia and the slope of the regression line.
Figure 4.1. Bonded foil, electrical resistance strain gauges before installation.

Figure 4.2. Bonded foil strain gauge after gluing to culvert plate.
Figure 4.3. Bonded foil strain gauge with lead wires attached.

Figure 4.4. Bonded foil strain gauge installation with protective coatings.
Figure 4.5. Strain gauge location showing two biaxial and one rosette gauge.

Figure 4.6. Data acquisition system mounted in field vehicle.
Figure 4.7 Lead wire cables and connectors for bonded foil gauges.

Figure 4.8. Electronic data acquisition system.
Figure 4.9. Quarter-bridge circuit used for bonded foil strain gauges.

Figure 4.10. Portable spot welder used to attach vibrating wire strain gauges.
Figure 4.11. Grinding off galvanized coating before welding vibrating wire strain gauges.

Figure 4.12. Spot welding a vibrating wire strain gauge to the culvert plate.
Figure 4.13. Vibrating wire strain gauge after installation.

Figure 4.14. Reading strain from a vibrating wire gauge.
Figure 4.15. Locations of strain gauges within the corrugated section.

Figure 4.16. Location with two biaxial, one rosette, and two vibrating wire strain gauges.
Figure 4.17. Layout of rosette strain gauge.
Circumferential Strains under 42 kip Live Load at Position 1

Figure 4.18. Measured strains under 42 kip live load, Position #1.
Circumferential Strains under 42 kip Live Load at Position 5

Legend:
- Biaxial gages
- Rosette gages
- Vibrating wire gages
- Linear regression fit

Scale for strains: 1 inch = 0.05%

Figure 4.19. Measured strains under 42 kip live load, Position #5.
Biaxial Strain Field at Biaxial Strain Gages
SYRO Culvert — All Load Conditions

Figure 4.20. Composite of biaxial strains measured at all locations under all loads.
Circumferential Stresses under 42 kip Live Load at Position 1

Legend:

○ Biaxial gages
□ Rosette gages
△ Vibrating wire gages

——— Linear fit in primary cross section
—- — Linear fit in secondary cross section

Scale for stress:
1 inch = 10 ksi

Figure 4.21. Measured stresses under 42 kip live load, Position #1.
Circumferential Stresses under 42 kip Live Load at Position 5

Figure 4.22. Measured stresses under 42 kip live load, Position #5.
5.1. OVERVIEW

This chapter presents most of the results and conclusions from the filed test data. Graphs showing the measured culvert deflections, bending in the haunch corners, and displacements within the backfill are contained in this chapter. Some plots of bending moments and axial thrusts in the plate are also discussed in this chapter; however, most of that data is presented as Appendix A. Data is presented for two categories of culvert loads: construction (backfill) loads and service (live) loads. These load conditions are discussed in Chapter 2. This chapter also contains a discussion of a potential yield condition in the corrugated plate.

5.2. CONSTRUCTION LOADS

The behavior of the culvert was monitored throughout the backfilling and paving phases of construction. For the purposes of this discussion, asphalt is treated as fill because it produces a dead load on the culvert. This section describes the culvert responses measured during construction.

Deflections. Culvert deflections were determined from the tape extensometer data. Figure 5.1 shows the deformed shape of the culvert, with the deflections magnified 50 times, at various levels of fill. Note that the primary deformation in the culvert shape is for a downward movement of the culvert's top.
Figure 5.2 depicts the progressive vertical deflection of the culvert top during construction. Most of the deflection occurs after 60 inches of fill have been placed. This is also apparent in Figure 5.3 which is a plot of the vertical deflection of the crown point as a function of the average backfill height. Note that downward deflections are plotted as negative values in Figure 5.3. Very small vertical deflections are observed until the backfill begins to cover the top of the culvert, but the deflections increase significantly as fill is placed over the top of the culvert. The largest vertical deflections occur during placement of the asphalt (after the top of the subgrade is reached). Also, rather surprisingly, the crown point does not deflect upward in the early stages of construction as fill is brought up the sides of the culvert. This behavior, known as "peaking", would be expected as the culvert sides are pushed inward. Apparently, the stiffness of the deep corrugated section, as well as the stiffness of the welded corner connection, are sufficient to resist this type of deformation.

Section 2.4 describes the backfilling procedures used to construct this culvert with Figure 2.8 depicting an uneven backfill profile that was formed in the later stages of construction. Figure 5.4 depicts the deformed shape of the culvert as measured under the uneven backfill loads. To a seemingly minor degree, the culvert appears to be shifted to the left in Figure 5.4 under the effect of the unsymmetric backfill loading. This behavior is also observed in the bending moments as discussed later.

**Bending in the haunch corners.** Data from the split light beam was used to determine the changes in the angle of the haunch corner plotted in Figure 5.5. This angle does not change significantly until the fill is placed over the top of the culvert when the maximum measured change is about -0.7 degrees. Note that negative changes correspond to a closing or decreasing angle. Figure 5.5 correlates well with the observed deflection data in that virtually no changes, due to peaking, are seen as the backfill is built up the sides of the culvert. When the fill is placed over the top of the culvert, the crown moves downward and the haunch corner angles are observed to close.
This data does not, however, indicate any adverse response due to the uneven backfill profile. It is probable that the split light beam measurements are not precise enough to detect this behavior.

**Moments and thrusts in the culvert plate.** Bending moments and axial thrusts were determined from the strain gauge data as described in the previous chapter. Recall that the strain gauges are organized into two circumferential cross sections of the culvert (see Figure 4.15) and are distributed to the same thirteen locations as the deflection control points (see Figure 3.8).

Bending moment and thrust diagrams are presented in Figure 5.6 and 5.7 at two stages of backfill construction. In these figures, as in others in this chapter and Appendix A, bending moments are presented in the upper plot with the corresponding thrust diagram in the lower plot of the same figure. Moments and thrusts are computed per lineal inch; i.e., per unit length of the culvert. This convention is consistent with the plane strain assumption for the culvert-soil system as used in the finite element analysis in Chapter 9. Furthermore, a consistent sign convention is employed throughout this study. Positive bending moments generate tensile strains on the inside of the culvert; or, viewed another way, positive moments imply a concave-up bending in the crown region. For thrusts, tensile forces are positive.

Figure 5.6 shows the loads carried in the culvert under 65 inches of backfill. This corresponds to the time when the sand fill is high enough to just cover the top of the culvert. Note that the bending moments in both the primary and secondary cross sections are nearly the same and also that the bending moment is approximately equal around the circumference of the culvert. The measured thrusts are somewhat more erratic but also show similar trends in both the primary and secondary sections. It appears that at this stage, loads are being carried by the culvert mainly through ring compression.

Figure 5.7 shows the moments and thrusts after completion of the sand backfill. This stage corresponds to a net increase of just 15 inches of fill over the fill height in Figure 5.6. With sand being placed over the top of the culvert, the culvert carries more load through beam action in the
crown. Note that the thrusts in Figure 5.7 have increased over the values in Figure 5.6, but the bending moments have increased by a much larger amount. Also note that the positive bending moments in Figure 5.7 correspond correctly to the concave-up deflections measured for the same load condition as plotted in Figure 5.2.

Recall from the discussion of culvert deflections that the culvert might have shifted slightly to the north under the uneven backfill profile. Figures 5.8 and 5.9 are plots of the measured moments and thrusts, as measured in the primary cross section, at important stages of the backfilling process. Note that maximum bending moments and axial thrusts occur just to the north of the crown point. This evidently results from the unsymmetric backfill profile. A greater fill height over the south half of the culvert induced greater compression in the southern half of the top plate. Further, greater tensile thrusts are seen to the north. These trends tend to suggest that the culvert has shifted slightly due to the sloping fill profile. This shift appears to be locked in as the moment and thrust distributions do not change when the fill profile is leveled.

Appendix A contains plots of the moments and thrusts, plotted versus the average backfill height, for each of the thirteen instrument locations in the primary cross section. Most of these plots show a symmetric load distribution around the culvert, particularly those points on the culvert sides. However, in the nearest locations to the north and south of the crown point (Sections #6 and #8), the measured thrusts are opposite in sign. Again, this is likely the result of the sloping backfill profile.

Also notice the similarities in trends between the bending moments in Sections #3 and #11 in Appendix A and the changes in the haunch corner angle in Figure 5.3. Sections #3 and #11 are the closest gauge locations to the corner itself. Comparison of these figures helps to validate the results from the split light beam instrumentation. The negative bending moments (compression on inside) measured in these locations correspond nicely to the observed closing of the haunch angle in Figure 5.3.
A beam shear diagram, determined from the rosette gauge data, is shown for the total backfill load in the top of Figure 5.10. Unfortunately, damaged gauge elements permitted the calculation of beam shear at only five locations around the culvert. For this figure, a positive shear force corresponds to an upward force on the right end of a beam element. A beam shear diagram under the 42 kip live load is plotted in the lower half of Figure 5.10.

**Culvert loads during paving operations.** The observed responses due to the unsymmetric backfill profile have been discussed above. However, the most significant loads on the culvert appear to have resulted from paving operations.

The top of the crushed limestone subbase corresponds to 85 inches of fill. Approximately ten inches of asphalt were then placed in three lifts with a full set of data acquired at the completion of each lift. This data showed the maximum deflections and loads experienced by the culvert during construction as evidenced by the crown deflection in Figure 5.3, and the bending moments in Appendix A.

Near the end of the backfill operations, some construction equipment crossed over the culvert. This was mainly a small backhoe that was used to spread the crushed limestone. During paving operations, trucks loaded with asphalt were driven across the culvert several times, sometimes on just the stone subbase. For the eight inches of asphalt base course, the hot asphalt was spread with a small bulldozer (weighing approximately six tons) and then compacted with the small vibratory compactor. These practices appear to have caused the greatest loads on the culvert. The final two inches of pavement were placed after the base courses had cured and stiffened. Heavy rollers on the top course did not appear to have a detrimental effect on the culvert because of the load spreading action of the base courses.
5.3. SERVICE LOADS

To simulate traffic loads (live loads) that could be expected during the life of the culvert, loaded trucks were positioned on the roadway surface as described in Section 2.6. This section will describe the observed culvert responses due to these live loads. It is important to note that all of the deflections and loads presented in this section are computed using as an initial condition a full data set acquired just before application of the live loads. Hence, the culvert responses shown here are those responses due only to the truck loads themselves and are not accumulated to the responses measured during construction.

Deflections. Vertical deflections of the crown for each of the live loads are plotted in Figure 5.11. These deflections, as expected, increase significantly with the greater loads. Crown deflections under the 16 kip live load are roughly the same regardless of the load position. Maximum deflections appear to occur at Positions #1, #3, and #5 under the 32 kip and 42 kip loads. These load positions are described in Figure 2.13.

Vertical deflection of the culvert top under each load at Position #1 is shown in Figure 5.12. Again, as expected, the deflections increase under greater load. Much more significantly, note that the culvert does not rebound completely after removal of the loaded trucks. That is, the culvert has been left with a small, permanent deformation. As the 42 kip truck is probably the greatest load placed on the culvert up to this time, this behavior probably represents an initial seating of the culvert-soil system. It is also important to note that some rebound has occurred which would suggest that similar loads in the near future will probably not cause further, detrimental deformations of the culvert.

A plot depicting the deformed shape of the culvert under the 42 kip live load is shown in Figure 5.13. Recall from Figure 2.13 that Position #1 is over the crown, Position #2 is to the south of the crown, and Position #3 is over the south haunch. Positions #2 and #3 would therefore be
expected to cause the culvert to shift slightly to the north. This behavior is seen, upon close inspection of Figure 5.13, where north is to the left. However, this effect is very slight in the deflection data.

**Bending in the haunch corners.** Data from the split light beam for the live loads is plotted in Figure 5.14. As with deflections, greater changes in the haunch angle occur under greater loads. All of this data indicates that the haunch angle closed under the influence of the live loads. However, this data is probably not precise enough to distinguish the relative effects of the specific load positions. Yet, Figure 5.14 also shows that most, but not all, of the culvert deformations rebound after removal of the loaded trucks.

**Displacements in the soil backfill.** Figure 5.15 is a plot of the data from the four soil extensometers during live load application. The upper plot in Figure 5.15 shows displacements measured above the culvert while the lower plot shows the data from the extensometers placed along the sides. Data from extensometers R1 and R2 indicate compression which would correspond to an outward movement of the culvert sides. This movement would result from the arching action of the culvert itself. In the upper plot under the 42 kip live load, R3 indicates compression for load Positions #1, #4, and #5 while R4 indicates extension for all positions. This result is not well understood but may arise from the relative location and weight of the front and rear axles on the loaded truck. Recall that the front axle of the truck is always positioned to the north. It is difficult to assess from this data the degree to which the soil backfill might be carrying the traffic loads away from the culvert crown.

**Moments and thrusts in the culvert plate.** Bending moments and axial thrusts measured during live load application are plotted in Figures 5.16 through 5.18 and in Appendix A. These plots depict the change in the plate forces due to the truck loads and are not accumulated with the loads induced by construction. Moreover, recall that the sign convention dictates that a positive bending
moment induces tensile strains on the innermost fibers of the plate and tensile axial thrusts are positive.

Moments and thrusts in the crown region, measured in both the primary and secondary instrumented cross section are plotted for each live load condition in Figure 5.16. Load Positions #1, #4, and #5 are all positioned over the culvert crown as depicted in Figure 2.13. In Figure 5.16 for the 32 kip and 42 kip loads, slightly greater moments are measured in the primary section for Positions #4 and #5 while the moments are slightly greater in the secondary section for loads at Position #1. This is probably the result of the wheel positions relative to each cross section. Bending moment and thrust diagrams for the 42 kip truck load at all five live load positions are plotted in Appendix A. These plots also show a slight effect in the different cross sections due to the relative positions of the truck wheels. These relationships can be understood by referring to the wheel positions (Figure 2.13) relative to the primary and secondary cross section (Figure 4.15). In Position #1, the truck wheels straddle the primary cross section while being vertically over the secondary cross section. Thus, greater moments are measured in the secondary cross section. When the truck is positioned at #4, the wheels are over the primary section and straddle the secondary. At Position #5, the wheels are again over the primary section, but the other wheel is about 8.5 feet from the secondary section. This would explain why the moments in the secondary section are much less for Position #5 than for Position #4 (refer to Appendix A). This data suggests that the contact stresses under each wheel of the truck are being spread by the asphalt and soil along the longitudinal direction of the culvert.

Axial thrusts in all of these plots are more erratic than the bending moments which might indicate a greater sensitivity of thrusts to small errors in the data. Residual crown moments, measured after the removal of all of the live loads, are plotted on the far right of Figure 5.16. Once again, an incomplete rebound of the forces in the culvert suggest an initial seating of the culvert under these loads.
Bending moment and thrust diagrams for all of the truck loads at each of the live load positions are shown in the plots at the end of Appendix A. In general, all of the bending moments show very nearly the same moment distribution with greater magnitude under greater loads. These figures, which show an excellent correlation between the various live load data, demonstrate that loads are carried by a similar mechanism for all of the truck loads. The culvert behaves something like a frame; when loaded on the top, large positive moments are seen in the crown region while smaller negative moments are seen in the haunch area. Again, measured thrusts are more erratic but show consistent trends.

Figures 5.17 and 5.18 depict the bending moments and thrusts under the 42 kip live load at all positions. For Positions #2 and #3, located to the south of the crown point, the maximum moments occur to the south of the crown. Again, this would be expected if the culvert was behaving like a frame under an unsymmetric point load.

More significantly, Figure 5.18 shows that when the wheels are placed directly over the crown (true for Positions #1, #4, and #5), an unsymmetric moment distribution occurs. Specifically, greater moments are seen to the north of the crown than are found at a similar location to the south of the crown. This phenomena can also be seen in the data from the 32 kip truck load in Appendix A. It is possible that this may result from a permanent shift in the culvert due to the uneven backfill profile described in Section 5.2. However, this phenomena more likely results from the method of applying the live load. Recall from Section 2.6 that the air was let out of the tires on the center axle of the truck to transfer the load to the rear axle. This method is probably imperfect and some load might still be transferred through the center axle. Since the truck faced north at all positions, this would explain the greater moments measured in the culvert to the north of the rear axle positions than to the south.
5.4. POTENTIAL FOR YIELDING IN CULVERT PLATE

As other researchers have pointed out [Duncan and Drawsky, 1983; Duncan, 1979], culverts with relatively flat tops under shallow fill are most susceptible to collapse from excessive bending moments. Rapid changes in the deflections, corner angles, and bending moments, which were observed near the end of the construction process, suggested that the loads in the culvert plate might be reaching a maximum safe value.

An effort was made to relate the bending in the haunch corner to the bending moment across the haunch. However, no simple analogy could be formulated that would reliably relate this deformation to the bending moment. The complicated geometry of the corrugated plates meeting at a welded corner was too complicated to allow anything but rough estimates of the moment in the plate from the split light beam data.

Figure 5.19 shows the maximum outer fiber stresses in the culvert plate as determined from the computed moments and thrusts. Stresses shown in the lower plot of Figure 5.19 are for the live load positions which produced the greatest moments in the culvert. Also, for this figure only, stresses due to the live loads are accumulated to the backfill stresses in the plot at the top from Figure 5.19. In reality, some of the stresses imposed during construction may have been redistributed due to relaxation of the soil-culvert system before the application of live load. It is impossible to assess how much, if any, of the stress induced during construction had been redistributed before live load application. Hence, the absolute value of the stresses shown in the lower plot of Figure 5.19 may be somewhat lower.

Nevertheless, the stresses in the top of Figure 5.19 at one circumferential position do clearly exceed the design yield strength ($F_y$) values of 28 ksi or 33 ksi. Recall from Section 2.3 that the culvert plate has a minimum yield strength of at least 28 ksi while Syro Steel used $F_y=33$ ksi for design. Laboratory tensile tests, discussed in Section 2.3, showed that the actual yield strength in the
outermost fibers is 54.6 ksi. This higher yield strength, the result of cold working the flat steel into the corrugated shape, is not exceeded at any point in the plots of Figure 5.19.

Yielding in the outer fibers of the corrugated plate would imply the formation of a plastic hinge in the culvert. However, formation of one plastic hinge would not entail the imminent collapse of a culvert as other hinges would have to form to create a collapse mechanism. [Duncan, 1979].

Conversely, the actual plastic moment capacity of the deep corrugated section is unknown. Work by Cary [1986] suggests that buckling of a corrugated section can occur prior to reaching the theoretical plastic yield moment. Cary's experimental results are not applicable to this culvert because of the smaller dimensions of the corrugated sections used in his study. Laboratory tests are needed to determine the maximum bending stresses that can be carried by the deep-corrugated plate.

Factor of safety against formation of plastic hinges. Other researchers have proposed a formulation for assessing the factor of safety against plastic hinge formation in corrugated culvert sections. [Leonards, et al., 1985; Duncan and Drawsky, 1983; and Duncan, 1979]. The factor of safety \( F_p \) is computed as:

\[
F_p = 0.5 \left( \frac{P_p}{P} \right) \left[ \sqrt{ \left( \frac{M}{M_p} \right)^2 + 4 \left( \frac{M}{M_p} \right)^2} - \left( \frac{M}{M_p} \right) \left( \frac{P_p}{P} \right) \right]
\]

Eq. 5.1.

where \( P \) is the thrust, \( P_p \) is the theoretical squash thrust load, \( M \) is the moment, and \( M_p \) is the theoretical plastic moment capacity of the section with no thrust. This equation includes a consideration of the interaction of moment and thrust to create plastic yielding. These researchers recommend that a minimum value of \( F_p = 1.65 \) be accepted for culverts. Maintaining the factor of safety provides an allowance for buckling failure prior to reaching the theoretical plastic moment capacity.

Assessment of the safety factor \( F_p \) requires the calculation of the theoretical plastic moment capacity \( (M_p) \) and squash load \( (P_p) \) which are a function of the yield strength of the steel. Using
the measured plate forces and the three values of yield strength, as discussed earlier, Equation 5.1 was used to compute the safety factors plotted in Figure 5.20. Plotted at the top of this figure is $F_p$ at the end of construction. Notice that $F_p$ falls below the safe minimum of 1.65 even for a yield strength of 54.6 ksi. The plot in the lower part of Figure 5.20 is for the live load which produced the greatest stresses in the culvert. Under the live load, the required $F_p = 1.65$ is easily met; but, the moment and thrusts used for this calculation were not accumulated to the forces due to construction loads.

While the safety factor does fall below the recommended minimum value, it is reasonable to conclude the culvert has not yielded. The yield strength value of 54.6, as measured in the lab, probably better represents the true yield strength of the corrugated plate. Assuming this, Figure 5.19 demonstrates that the culvert has not yielded. Laboratory studies are needed to establish the bending capacity of the deep-corrugated section. Moreover, the high bending moments incurred during construction should, with time, redistribute around the culvert as a new equilibrium state is formed between the culvert and the backfill soil.

5.5. ADVERSE CONSTRUCTION PRACTICES

Figure 5.20 demonstrates that prudent design criteria for the culvert has been exceeded. While it is felt that the culvert is not in danger of collapse, this data does show the vulnerability of these culvert-soil structures to accepted construction procedures. Specifically, permitting the uneven backfilling, even though the culvert was completely covered at the time, generated unsymmetric loading in the culvert. More importantly, construction equipment and loaded trucks on top of the culvert during early paving operations posed the greatest threat to the structural integrity of the culvert.
During construction at this site, the density of the constructed backfill was checked regularly with a nuclear density gauge. It is not clear that density measurements are always made at such installations. Informal discussions with the construction crew suggested that the enforcement of backfill compaction specifications is often lax. (Sometimes this may result from the lack of access to nuclear density gauges.) This would seem to agree with an apparently prevalent attitude on many construction sites that backfill compaction specifications are not crucial to the quality of the finished project.

These facts illuminate a major concern for the safe installation of metal box-type culverts. Contractors and construction inspectors must be made to understand that a culvert is really a steel and soil structure. Without a sound backfill and prudent construction procedures, safety is compromised. The long, flat span of a box-type culvert makes this culvert shape particularly vulnerable to collapse due to a weak backfill.

Specifically, data from this field test strongly suggests the following guidelines for the future construction of box-type culverts:

1. The density of the placed backfill should be monitored frequently throughout construction and the construction specifications must be enforced.

2. Uneven backfill profiles should not be allowed at any time during construction. This requirement is crucial for shallow cover culvert installations.

3. Heavy equipment, particularly loaded asphalt trucks, must be kept off of the culvert until most of the pavement has been placed and cured. Construction equipment on the fill during initial paving operations can unquestionably produce the greatest stresses in the culvert and could very well lead to a collapse.
Deformed Shape under Construction Loads
Deflections Magnified 50 times

○ Eyebolt control points
— Undeformed shape
--- 43" fill on south, 48" fill on north
---- 80" of fill, top of sand backfill
----- 93" of fill, top of asphalt

Figure 5.1. Deformed shape of culvert under construction loads.

Vertical Deflection of Culvert Top
Under Construction Loads

Avg. Backfill Heights:
— 22.5 inch
--- 42.0 inch
---- 60.0 inch
----- 80.0 inch
------ 90.0 inch
-------- 93.0 inch

Figure 5.2. Vertical deflection of culvert top under construction loads.
Vertical Deflection of Crown Point
Under Construction Loads

![Graph](image)

Figure 5.3. Vertical deflection of crown under construction loads.

Deformed Shape under Sloping Backfill
Deflections Magnified 100 times

![Graph](image)

Figure 5.4. Deformed shape of culvert under sloping backfill profile.
Bending in Haunch Corner
Data from Split Light Beam

Figure 5.5. Bending in haunch corner under construction loads.
Figure 5.6. Measured plate forces with backfill up to the height of the crown.
Figure 5.7. Measured plate forces after completion of sand backfill.
Figure 5.8. Measured plate forces at various stages of backfill construction.
Measured Plate Forces — SYRO Culvert
Backfill Loads, Primary Cross Section

Figure 5.9. Measured plate forces during paving operations.
Figure 5.10. Measured beam shear under construction and service loads.
Vertical Deflection of Crown Point Under Service Loads

Figure 5.11. Vertical deflection of crown under service loads.

Vertical Deflection of Culvert Top Under Service Loads

Figure 5.12. Vertical deflection of culvert top under service loads.
Deformed Shape under Service Loads
Deflections Magnified 100 times

Figure 5.13. Deformed shape of culvert under service loads.

Bending in Haunch Corner
Data from Split Light Beam

Figure 5.14. Bending of haunch corner under service loads.
Deformations in Sand Backfill
Data from Soil Extensometers

Figure 5.15. Deformations in sand backfill under service loads.
Figure 5.16. Measured plate forces under service loads.
Figure 5.17. Measured plate forces under 42 kip live load, Positions #1, #2, and #3.
Figure 5.18. Measured plate forces under 42 kip live load, Positions #1, #4, and #5.
Figure 5.19. Maximum plate stresses computed from bending moment and thrust.
Measured Plate Forces — SYRO Culvert

Factor of Safety Against Plastic Hinge Formation

93 in. of fill, top of asphalt

42 kip live load at Position #4

Figure 5.20. Calculated factor of safety against plastic hinge formation.
CHAPTER 6

LABORATORY TESTING OF BACKFILL MATERIAL

6.1. INTRODUCTION

To analyze the culvert problem using numerical techniques, it is very important to properly characterize the behavior of the backfill material. Toward this end, samples of the sand used to construct the culvert were taken to the laboratory for extensive testing. The results were then used to derive constitutive parameters to fit Duncan's hyperbolic soil model. This model and the corresponding parameters are discussed in the next two chapters. Subsequently, the parameters were used in the finite element analysis of the culvert as discussed in Chapter 9.

The soil was tested in the geotechnical laboratory at Ohio University using two different devices: a triaxial device and a cubical multiaxial device (also known as a truly triaxial device). Multiaxial tests were performed along each of the standard stress paths described in the next section. Appendix B contains the data from these tests which could be used to derive parameters to fit other soil models.

6.2. DESCRIPTION OF STRESS PATHS

The observed response of a soil depends on the stress history as well as its physical properties. The concept of stress history, or stress path, entails how the soil element has proceeded to the
current stress state—the magnitude, direction, and sequence of three dimensional loads previously applied to the soil. To simulate in-situ loading histories in the laboratory, several stress paths, described in this section, were followed. A stress path is defined by the particular relationships maintained between orthogonal loads during the soil test.

A comprehensive set of tests were performed in this study using each of the important stress paths [Desai and Siriwardane, 1984]. Experiments were planned to model loading conditions experienced by the backfill material. All soil specimens were first confined under hydrostatic pressure before applying loads incrementally along the stress path as illustrated in Figure 6.1. The tests which were run are summarized in Table 8.2 and the stress paths are individually described in the following paragraphs.

**Hydrostatic Compression - HC:** \( \Delta \sigma_x = \Delta \sigma_y = \Delta \sigma_z \).

During this test, stresses on all sides of the test specimen are kept equal as the pressure is increased in increments. This stress path imparts no shear stress to the specimen and provides data for evaluating the bulk modulus.

**Conventional Triaxial Compression - CTC:** \( \Delta \sigma_x = \Delta \sigma_y = 0, \ \Delta \sigma_z \) increasing).

This is the most common stress path performed on a triaxial device. The test begins with equal pressure on all sides of the specimen and proceeds by increasing the load on one axis in increments. During this loading, the stresses on the two axes orthogonal to the load, termed "confining pressure", remain constant and equal.

**Conventional Triaxial Extension - CTE:** \( \Delta \sigma_x = \Delta \sigma_y \) increasing, \( \Delta \sigma_z = 0 \).

This test measures the extension along one axis due to compressing the other two axes. In other words, the test observes the axial behavior under a constant load as the confining pressure is increased.
Reduced Triaxial Compression - RTC: \( \Delta\sigma_x = \Delta\sigma_y \text{ decreasing, } \Delta\sigma_z = 0 \).

The RTC test induces axial compression by decreasing the confining pressure while holding the axial stress constant.

Reduced Triaxial Extension - RTE: \( \Delta\sigma_x = \Delta\sigma_y = 0, \Delta\sigma_z \text{ decreasing} \).

This stress path invokes axial extension by holding the confining pressure constant as the axial load is reduced.

Simple Shear - SS: \( \Delta\sigma_z = 0, \Delta\sigma_y = -\Delta\sigma_x, \Delta\sigma_y \text{ increasing, } \Delta\sigma_x \text{ decreasing} \).

While one axial load is unchanged, the other two stresses are increased and decreased by equal amounts. Thus, the mean pressure (average of three normal stresses) remains constant and the observed response is due to shearing stresses.

Triaxial Compression - TC: \( \Delta\sigma_z = -2\Delta\sigma_x = -2\Delta\sigma_y, \Delta\sigma_z \text{ increasing, } \Delta\sigma_x = \Delta\sigma_y \text{ decreasing} \).

In a TC test, one axis is subjected to increased loads while the confining stresses are decreased. The stress on each confining axis is decreased in increments equal to one half the increment applied to the other axis. Thus, this load path also maintains a constant mean pressure and isolates the effects of shear or deviator stress.

Triaxial Extension - TE: \(-\Delta\sigma_z = 2\Delta\sigma_x = 2\Delta\sigma_y, \Delta\sigma_z \text{ decreasing, } \Delta\sigma_x = \Delta\sigma_y \text{ increasing}\).

This stress path is similar to a TC test except that it is run to produce axial extension. That is, the axial stress is decreased in increments double the magnitude of concurrent increases in confining pressure.

Proportional Loading - PL.

These tests are designed to measure the influence of the intermediate principal stress, \(\sigma_2\). PL tests are generally run in increments which keep the ratio ‘b’, defined in Equation 6.1, constant.
Stress-strain curves from each of the 36 laboratory tests are shown in Appendix B. These graphs use values of stress and strain as defined below:

\[
b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}
\]

Eq. 6.1.

\[
\text{mean stress} = \sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}
\]

Eq. 6.2.

\[
\text{deviator stress} = (\sigma_1 - \sigma_3)
\]

Eq. 6.3.

\[
\text{volumetric strain} = e_v = \frac{\Delta V}{V} = e_1 + e_2 + e_3
\]

Eq. 6.4.

\[
\text{octahedral shear stress} = \tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}
\]

Eq. 6.5.

In these relations, the numerical subscripts refer to the principal values, i.e., \(\sigma_1\) is the major principal stress and \(\sigma_3\) is the minor principal stress. Since neither the triaxial nor multiaxial equipment impart shear stresses to any face of the specimen, the principal stress directions correspond to X, Y, and Z axial directions of the testing apparatus.

6.3. TRIAXIAL TESTING DEVICE

Four soil tests were performed using a Soiltest T-1500 triaxial testing device. Triaxial equipment has been widely used for many years to measure the strength and constitutive response
of soils. Consequently, triaxial test results provide an important confirmation of the performance of the more recently developed multiaxial apparatus.

While triaxial equipment is familiar and reliable, it is not capable of executing all the stress paths listed in Section 6.2. The triaxial device applies a confining pressure to the specimen inside the test cell using water pressure (Figure 6.2). As a result, two of the stresses are always equal; that is, $\sigma_x = \sigma_y$. The SS and PL stress paths cannot be run on a triaxial device making it difficult to investigate the influence of the intermediate principal stress in constitutive studies. This limitation lead to the development of multiaxial equipment in the 1970's.

The backfill material tested was a dry, cohesionless sand which was formed inside a latex rubber membrane that had been stretched inside a cylindrical mold. Once formed, the rubber jacket was sealed and the sample was evacuated to a 3-5 psi vacuum to hold the specimen shape while the triaxial testing chamber was assembled. Then, the vacuum pressure was reduced in small increments as the confining pressure within the triaxial cell was increased equally to compensate, thereby maintaining a fairly constant confinement on the sand. Once the vacuum was completely removed, the tube leading into the specimen was left open to ensure drained testing conditions (no pore pressure). Additional confining pressure, as required by the particular test, was exerted inside the triaxial cell.

To measure volumetric strains during the test, changes in the volume of water inside the triaxial cell were monitored with a graduated tube. Strain controlled axial loading was exerted with a ram and the load was measured using a proving ring. To compute axial stresses during the test, the cross sectional area of the specimen was corrected to account for Poisson effects. The instantaneous area was computed using volumetric strain data and assuming the cross section was equal along the length of the specimen.

The cylindrical soil sample tested in the triaxial was 2.8 inches in diameter and approximately 5.5 inches high. This shape made forming relatively more difficult than forming a cubical specimen.
for the multiaxial. A particularly troublesome problem was consistently reproducing the desired soil density, an extremely important specimen property. Moreover, the rubber jacket containing the sand had to be completely sealed to prevent water from entering the specimen. Even a small leak, which often occurred where the membrane was wrapped over the end caps, would invalidate the volumetric strain data and require a new test run.

6.4. PREPARATION OF CUBICAL SPECIMEN FOR MULTIAXIAL

The multiaxial equipment, described in Section 6.5, requires a cubical testing specimen measuring 4.00 inches on each side. The technique used to make a cube of dry sand [Mould and Sture, 1979] is similar to that for making a cylindrical triaxial specimen. Since the cubical multiaxial apparatus is less familiar, cubic specimen preparation will be described in detail.

The specimen is formed inside a cubical, aluminum mold which measures 4.00 inches on the inside. Thin latex rubber sheets, cut from membranes manufactured for large triaxial specimen, are glued to form an open cylinder of about 15 inches in circumference. Inside the assembled sides of the mold, this latex membrane is neatly stretched and held by lapping over the bolt heads on the outside of the mold. This must be done carefully to smooth out all wrinkles and stretch the membrane well into the corners. Next, sheets of latex are smoothed over the mold's top and bottom plates and held in place using a thin film of grease. The bottom is then glued to the membrane in the mold to form an open box. Liberal amounts of rubber cement produced the best seal of this joint.

Into this prepared mold, the dry sand is placed in layers. It is very important to simulate field compaction with this process and to produce the same density. The culvert backfill was compacted
using a vibrating plate as pictured in Figure 6.3. Hence, each layer of the sand specimen was compacted with a firm tapping from a metal block as in Figure 6.4. It was important not to pound the sand as this could shatter the sand particles. The idea is to induce compaction by vibration with some vertical force. To produce the right density, the correct amount of material was weighed out before tamping into the cubic mold. As it turned out in this case, the desired dry density of 112.5 pcf was obtained by fully compacting, but not heavily pounding, the sand into the mold.

After the sample was formed, it was weighed to check the exact density. A small plastic tube (1/16 inch OD), with a small piece of filter paper attached over the end, was placed in a groove cut in the top edge of the mold. The top membrane, clinging to the top plate, was then carefully glued to close the specimen. Again, liberal amounts of rubber cement were used to seal this joint. A vacuum pressure of 3-5 psi, maintained with the simple regulator pictured in Figure 6.5, was applied through the small tube. When no leakage occurred, the specimen would become firm inside the rubber jacket. The mold could be disassembled, the excess membrane cut away, and the specimen was ready for testing (Figure 6.6).

6.5. CUBICAL MULTIAXIAL TESTING DEVICE

The cubical multiaxial apparatus used in this investigation was built from a design developed at Virginia Polytechnic Institute [Mould and Sture, 1979] and is shown assembled in Figure 6.7. Although several variations of truly triaxial testing equipment have been proposed over the last twenty years, this particular design stands out in its simplicity. Moreover, soil loading is entirely stress controlled (the specimen "floats" within the test chamber) with no significant problems encountered from specimen-apparatus interaction.
The multiaxial equipment is assembled around a reaction frame, pictured in Figure 6.8, milled from a solid block of aluminum. Six walls, one for each face of the specimen, are bolted onto the reaction frame. Attached to each wall is a silicone rubber membrane filled with the pressurizing fluid. To measure the specimen strains, three probes are located inside each of the six membranes producing 18 deformation readings. When all six walls are bolted to the frame, a central cubical cavity, 4.00 inches to a side, is formed in which the test specimen is placed. This multiaxial cell can safely apply hydrostatic pressures of up to 250 psi.

Wall assemblies. Each of the six wall assemblies, pictured in Figures 6.9 and 6.10, are bolted to the reaction frame using eight ¼ inch steel bolts. A rubber membrane, filled with hydraulic oil, is glued to the front of each wall. When the multiaxial equipment is assembled, the face of each membrane is in full contact with one face of the test specimen. Loads are transmitted to the soil by applying pressure to the hydraulic oil inside the membranes. Different stress paths are simulated by application of the appropriate pressures in each of the six walls.

An exploded view of the multiaxial walls is drawn in Figure 6.11. Inside each membrane, engulfed in hydraulic oil, are three Linear Variable Differential Transformers (LVDT's) which are in contact with the back side of the membrane face. These probes measure specimen deformations and are discussed in a subsequent section.

Membranes. Perhaps the most crucial component of the multiaxial system is the rubber membrane which transmits stress to the soil. These membranes must produce an even normal stress distribution, prevent any shear loads, easily deform to match (without impeding) any changes in the specimen surface, and not rupture during the test. To meet all of these criteria, a flexible silicone rubber was used with a nominal thickness of 0.09 inches at the membrane face.

The membranes were made in the lab using General Electric's RTV664 silicone rubber. This material comes in two components which must be accurately and thoroughly mixed. To remove air bubbles, the mix must be held under vacuum (20 to 30 psi) for several minutes before pouring into
the membrane mold. The cured membranes are then glued to the clean surface of the multiaxial walls using GE's RTV silicone sealant.

Thin teflon sheets and a light grease are spread over the face of the membranes to allow the specimen to slide easily between them. In effect, this prevents any shear stresses from building up between the silicone membrane and the specimen during the test. Furthermore, grease and teflon sheets are used between the sides of each membrane and the aluminum reaction frame. This reduces any friction that might diminish the total stress being transmitted by the membrane as it stretches to follow the deforming specimen. Overall, the silicone membranes permit the assumption that the normal stresses being applied to the test specimen are equal to the hydraulic oil pressure inside each wall assembly.

The corners of each membrane are beveled at 45 degrees as can be seen by close examination of Figure 6.10. The flat face of each membrane is a four inch square that exactly matches each face of the specimen. The corners of adjacent membranes contact each other along the beveled edge as shown in Figure 6.12. Since the corner of the reaction frame is separated from the corner of the specimen by about 3/8 inch, no interference along the edges will occur. Thus, the specimen can be said to float with no edge constraints within the stress field.

Linear Variable Differential Transformers (LVDT's). To measure specimen deformation under load, a total of 18 LVDT probes are located inside the walls as in Figure 6.13. These probes are spring loaded to ensure constant contact of the rounded tip with the inside surface of the membrane face. The three LVDT's on each wall are bolted into position along 120° radial lines and oriented so that probes on opposing walls are matched up along axial lines.

LVDT's used in this multiaxial device were manufactured by Schaevitz Engineering (model GPA-121-250-0608). An LVDT contains a movable metal core, acting as a mechanical probe, inside three coils. As the core slides inside the coils, displacement is determined by applying an excitation current to the primary coil and observing changes in the mutual inductance of the two secondary
coils. Electronic equipment attached to each LVDT supplies the required excitation current and conditions the output signal to permit data acquisition using a computer. The primary advantage of using an LVDT is the linearity of the analog output. The relationship of output voltage (-10 to +10 VDC) to probe displacement remains very linear over the ½ inch range of the LVDT. This feature greatly simplifies reduction of raw data to specimen strains. Moreover, LVDT’s are reliable and durable which ideally suits them for use in the multiaxial apparatus.

**Pressure system and pressure transducers.** Hydraulic oil, under controlled pressure, must be delivered to each of the six wall assemblies during multiaxial testing. To do this, the pressure panel of Figures 6.14 and 6.15 is used. Hydraulic pressure is supplied by three hand driven Enerpac P-39 pumps. On the front of the panel (Figure 6.14), three pressure gauges allow the operator to apply stresses to the desired level on each of the three axes before taking data. The pressure on each axis is accurately read using three Heise model 623 pressure transducers visible in Figure 6.15. These instruments have a range of 0-1000 psi with an accuracy of ± 1 psi. Output from the pressure transducers range from 0 to 10 VDC and is fed directly into the computer, along with LVDT data, for analysis.

The pressure panel contains a series of valves and cross-over connections, as can be seen in Figure 6.15, which permit the control of axial stresses when following various stress paths. Each pressure axis can be independently controlled, or two axes can be connected to give equal pressures, or all three axes can be connected to produce hydrostatic pressures.

**Data acquisition.** All output from the 18 LVDT’s and three pressure transducers are fed into an analog/digital converter for data acquisition using a Micro PDP-11 computer from Digital Equipment. The A/D converter is a DEC model ADV11-C. All data acquisitions are controlled from a FORTRAN program described later.

**Assembly of equipment for testing.** Once the cubic specimen is readied, it is slipped into the testing cell as in Figure 6.16. The vacuum pressure holding the specimen shape has to be maintained
while the multiaxial device is assembled. Thus, the vacuum tube leading into the specimen is fed through a small hole drilled into the corner of the reaction frame.

After carefully placing the specimen in the center and affixing all the required teflon sheets, each of the six walls are bolted to the frame. For safety, the eight bolts on each wall must be tightened carefully. As each wall is bolted down, the pressure port on the back of each wall is left open to drain any pressure that might develop from having too much oil in the membranes. It is best to overfill the membranes each time and drain the excess oil during assembly as this forces out any air that might be trapped inside.

Once all six walls are secured and the LVDT's and pressure hoses have been hooked up, the vacuum that is confining the specimen is incrementally reduced and replaced by hydraulic pressure from the multiaxial. Generally, the vacuum pressure is only 5 psi but is never more than the confining pressure to be used during that particular test. After complete removal of the vacuum pressure and subsequently during the test, the vacuum tube is left open to ensure drained conditions; i.e., no pore pressures are allowed to develop. Finally, hydrostatic pressure is adjusted to the desired starting point for the planned test. This is taken as the starting point of zero strain where the membranes and all of the LVDT tips are in firm contact with the specimen.

Processing of raw data. The stresses are applied in increments during the multiaxial test. At the end of each increment, the system is allowed two minutes before the data acquisition begins. This gives sufficient time, as proved in trial runs, for complete straining of the sand to occur under the new stress state. When the data is swept with the computer, all 18 LVDT channels and the three pressure channels are read at the same time.

A FORTRAN code named MULAX was written to control data acquisitions on the Micro PDP-11. When the data sweep is initiated by the operator, this program sweeps all the channels ten times and then uses the average to compute pressures and displacements. This raw data is then stored on line for further analysis at the end of the test.
To compute the measured strains, MULAX begins by adding the averaged displacements of opposing LVDT's (two corresponding probes on opposite faces). This has the effect of subtracting measured displacements due to specimen sliding. This yields the total deformation as measured along three lines in each axial direction. These three lineal displacements are then averaged to give the deformation of each axis of the specimen. Finally, dividing by the initial length of 4.0 inches gives the axial strain desired.

Each LVDT is calibrated when installed in the multiaxial and a calibration constant for each individual gauge is used by the MULAX program. Also, calibration runs were performed on the complete multiaxial system by testing a solid block of aluminum. Since the elastic properties of aluminum are well known, the deformations of the test chamber could be evaluated. These "device strains" can result from expansion of the reaction frame under load or, more significantly under low stresses, the compression of the silicone rubber between the LVDT tip and the test specimen. These deformations were modelled with a polynomial function and are subtracted by MULAX from the measured strains to give the correct axial strains of the soil specimen.
Figure 6.1. Stresses applied to test specimen.

Figure 6.2. Triaxial test chamber.
Figure 6.3. Culvert backfill being compacted with vibratory plate.

Figure 6.4. Compaction of multiaxial specimen.
Figure 6.5. Multiaxial specimen under vacuum.

Figure 6.6. Cubical sand specimen ready for multiaxial testing.
Figure 6.7. Cubical multiaxial testing device.

Figure 6.8. Multiaxial reaction frame.
Figure 6.9. Wall assembly with silicone rubber membrane and teflon sheets.

Figure 6.10. Multiaxial wall and reaction frame.
Figure 6.11. Cubical multiaxial testing equipment.
Figure 6.12. Corner boundary inside multiaxial test cell.

Figure 6.13. LVDT probes.
Figure 6.14. Front of multiaxial pressure panel.

Figure 6.15. Rear of pressure panel with pressure transducers.
Figure 6.16. Inserting specimen during assembly of multiaxial cell.
CHAPTER 7

DUNCAN'S HYPERBOLIC SOIL MODEL

7.1. DEVELOPMENT AND BASIC PRINCIPLES

The finite element program, CANDE, used to analyze the culvert in Chapter 9, was updated in 1980 to include a hyperbolic soil model developed by J.M. Duncan and his associates at the University of California, Berkeley [Duncan et al., 1980; Wong and Duncan, 1974; Duncan and Chang, 1970]. There are problems associated with using Duncan's hyperbolic model to characterize the culvert backfill sand; namely, problems with unloading/reloading cycles, shear dilatancy, and the influence of stress path (of particular concern with the culvert backfill). Yet, this soil model remains the best available within CANDE for the analysis of culvert soil-structure interactions [Leonards et al., 1985; McVay and Selig, 1982]. Working within these limits, Duncan's model has been used to obtain the best possible numerical model of the culvert problem.

Duncan's hyperbolic soil model, or Duncan's model, has its basis on a 1963 study by Kondner which showed that constitutive response curves for soils could be approximated reasonably well using hyperbolas [Kondner, 1963]. Accordingly, Duncan's soil model consists of fitting hyperbolic curves to observed data--it is not founded in the principles of continuum mechanics. While this can result in some theoretical objections, the model succeeds in being easy to understand and apply. The model parameters can be readily derived from conventional triaxial compression tests; familiar tests often performed for constitutive modelling. Likewise, the model is relatively simple to program in finite element codes. These factors have lead to extensive use of the Duncan soil model and specifically to its inclusion in CANDE.
Duncan's model relies on good experimental data to obtain model constants that can be subsequently used to predict the response of a soil mass under analysis. It is imperative that laboratory tests be performed in a manner closely simulating the in situ conditions since the model predictions depend on empirical constants. These parameters must inherently account for factors such as soil density, moisture content, stress history, etc. which influence the behavior of the soil. These soil conditions must be reproduced in the laboratory to obtain reliable predictions using the Duncan model. However, by using strictly conventional triaxial compression data to obtain the parameters, the model will not account for behavior under different load paths. The culvert backfill being analyzed in this report is subjected to mixed loading paths and parameters have been derived in Chapter 8 which include this influence in the model. This required some extensions to Duncan's model as described in Section 7.8.

A basic premise of this model, as with many constitutive laws, is that the axial response of the soil is a function of the greatest stress difference, or \((\sigma_1 - \sigma_3)\). This is consistent with our understanding of geologic materials whose behavior is dependent not so much on absolute stress levels, but on the magnitude of the stress difference. Moreover, Duncan's model incorporates observations that some soil properties, such as the initial stiffness and the failure stress, are functions of the minor principal stress.

The primary feature of Duncan's soil model is a tangent modulus which allows a piece-wise linear approximation of the hyperbolic response curve. However, since a hyperbolic approximation is only applicable below failure, the model is not valid for stresses above the peak value. This can limit the applications of the model, especially in solving problems with soil instability. Duncan uses the familiar Mohr-Coulomb criterion to define the failure surface.

Originally, Duncan modelled the lateral response of the soil using a tangent Poisson ratio formulation [Wong and Duncan, 1974]. Later, studies showed this feature gave erratic results in some cases. In 1980, Duncan and his co-workers improved the model with a bulk modulus
formulation, though still based entirely on CTC tests [Duncan et al., 1980]. Recently, Selig has proposed a more logical technique for obtaining the same bulk modulus parameters from hydrostatic compression data [Selig, 1988].

7.2. DUNCAN SOIL MODEL IN CANDE

The finite element program CANDE [Katona et al., 1981] computes soil element stresses and strains using an isotropic, incremental form of Hooke’s law:

\[
\begin{bmatrix}
\Delta \sigma_x \\
\Delta \sigma_y \\
\Delta \tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & 0 \\
c_{12} & c_{22} & 0 \\
0 & 0 & c_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta \epsilon_x \\
\Delta \epsilon_y \\
\Delta \gamma_{xy}
\end{bmatrix}
\]

Eq. 7.1.

where the constants can be written in terms of Young’s modulus (E) and Poisson’s ratio (\(v\)) or bulk modulus (B):

\[
c_{11} - c_{22} = \frac{E(1-v)}{(1+v)(1-2v)} - \frac{3B(3B+E)}{(9B-E)}
\]

Eq. 7.2.

\[
c_{12} = \frac{Ev}{(1+v)(1-2v)} - \frac{3B(3B-E)}{9B-E}
\]

Eq. 7.3.

\[
c_{33} = \frac{E}{2(1+v)} - \frac{3BE}{9B-E}
\]

Eq. 7.4.

Duncan’s soil model is incorporated to compute the tangent value of E and B as functions of the minor principal stress (\(\sigma_3\)) and the deviator stress (\(\sigma_1 - \sigma_3\)) at each increment along the modelled response of the soil. These expressions are Equations 7.5 and 7.6 and are derived in Section 7.3. Atmospheric pressure, \(P_a = 14.7\) psi, is included to permit the use of any system of units.
Experience shows that the friction angle is not constant (i.e., the failure envelope is really curved); rather, $\phi$ decreases with increasing pressure. To model this, the angle of internal friction is determined as a function of the minor principal stress:

$$\phi = \phi_0 - \Delta \phi \log_{10} \left( \frac{\sigma_3}{P_a} \right)$$  \hspace{1cm} \text{Eq. 7.7.}$$

These last three equations and the eight constants they contain define the Duncan hyperbolic soil model in CANDE. The model parameters are described in Table 7.1. It should be noted that Duncan uses a different modulus number, $K_u$, to model unloading and reloading cycles. This feature is not used by CANDE and is discussed further in Section 7.3.

CANDE computes stresses by applying the load in increments and computing the tangent modulus for each increment. To improve the modelled soil response, CANDE calculates the value of $E_t$ at the beginning and end of the load increment and uses the average given by:

$$E_{\text{avg}} = (1-r)E_1 + rE_2$$  \hspace{1cm} \text{Eq. 7.8.}$$

where $E_1$ and $E_2$ are the tangent moduli at the beginning and end of the load increment, respectively. The variable $r$ is an averaging ratio; in most cases, $r$ is set to $\frac{1}{2}$ to produce a true average. For in situ elements, $r$ is given as 1.0 so that $E_t = E_2$. 

\[
E_t = K P_a \left( \frac{\sigma_3}{P_a} \right)^n \left[ 1 - \frac{R_f (1-\sin\phi) (\sigma_1-\sigma_2)}{2c \cos\phi + 2\sigma_3 \sin\phi} \right]^2 \hspace{1cm} \text{Eq. 7.5.}
\]

\[
B = K_b P_a \left( \frac{\sigma_3}{P_a} \right)^m \hspace{1cm} \text{Eq. 7.6.}
\]
7.3. THEORETICAL DEVELOPMENT

As proposed by Kondner [1963], the compressive response curve of a soil can be approximated by a hyperbola of the form:

\[
(\sigma_1 - \sigma_3) = \frac{e}{a + be}
\]

Eq. 7.9.

where \(\sigma_1\) is the major principal stress, \(\sigma_3\) is the minor principal stress, and \(e\) is the compressive axial strain. The constant "a" is equal to the inverse of the initial slope and "b" is the inverse of the horizontal asymptote as shown in Figure 7.1.

The horizontal asymptote is denoted as the ultimate stress difference, \((\sigma_1 - \sigma_3)_{ult}\). This stress is reached only at infinite strains; consequently, the failure stress will be reached before the ultimate stress. This suggests defining a failure ratio, \(R_f\), whose value will always be less than 1.0.

\[
R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}}
\]

Eq. 7.10.

With this definition, the hyperbolic curve can be expressed as:

\[
(\sigma_1 - \sigma_3) = e \left[ \frac{1}{E_i} + \frac{eR_f}{(\sigma_1 - \sigma_3)_f} \right]^{-1}
\]

Eq. 7.11.

However, for the nonlinear model, an expression for the tangent modulus as a function of the stress difference is needed. Since the tangent modulus is the instantaneous slope of the curve in Figure 7.1, Equation 7.11 is differentiated with respect to axial strain [Duncan and Chang, 1970] to give:
\[ E_t = \frac{\delta(\sigma_1 - \sigma_3)}{\delta e} \]  
Eq. 7.12.

which yields:

\[ E_t = \frac{1}{E_i \left(1 + \frac{eR_f}{(\sigma_1 - \sigma_3)_f}\right)^2} \]  
Eq. 7.13.

Now, Equation 7.11 is rearranged to give an expression for \( \epsilon \):

\[ \epsilon = \frac{\sigma_1 - \sigma_3}{E_i \left(1 - \frac{(\sigma_1 - \sigma_3)R_f}{(\sigma_1 - \sigma_3)_f}\right)} \]  
Eq. 7.14.

Substituting this into Equation 7.13 will, after lengthy manipulations, give \( E_t \) as a function of the stress difference:

\[ E_t = E_i \left(1 - \frac{R_f (\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f}\right)^2 \]  
Eq. 7.15.

It now becomes necessary to define the failure stress state. This is done using the familiar Mohr-Coulomb failure envelope defined as \( \tau = c + \sigma \tan \phi \). An expression for the peak stress is developed based on this failure criterion. First, from Figure 7.2:

\[ \sin \phi = \frac{R}{D} = \frac{(\sigma_1 - \sigma_3)/2}{\left(\frac{\sigma_1 + \sigma_3}{2} + \frac{c}{\tan \phi}\right)} \]  
Eq. 7.16.

This equation is algebraically manipulated to give the desired expression in Equation 7.17.

\[ (\sigma_1 - \sigma_3)_P = \frac{2c \cos \phi + 2\sigma_3 \sin \phi}{1 - \sin \phi} \]  
Eq. 7.17.
Also, it would be desirable to express the initial Young's modulus of the soil as a function of the stress state. This is done empirically with Equation 7.18 [Janbu, 1963] where the atmospheric pressure, \( P_a \), is included to accommodate different systems of units.

\[
E_i = K \frac{\sigma_3}{P_a}^{n}
\]

Eq. 7.18.

Finally, substituting Equations 7.17 and 7.18 back into Equation 7.15 will give Equation 7.5. This expression permits a computation of the instantaneous tangent modulus solely as a function of the major and minor principal stresses.

Duncan's model includes a separate formulation for the behavior of overly consolidated soils. The observed response of an unloading and reloading cycle will contain some hysteresis as in Figure 7.3. Duncan models both unloading and reloading using a straight line approximation [Duncan, 1980]. Further, the constant modulus \( E_{ur} \) is determined as a function of the minor principal stress:

\[
E_{ur} = K_{ur} \frac{\sigma_2}{P_a}^{n}
\]

Eq. 7.19.

This is of the same form as \( E_i \) and the same exponent, \( n \), is used for both moduli. However, this characterization is not valid for stress paths where \( \sigma_3 \) is not constant [Katona et al., 1981] during a decrease in the stress difference (unloading). This is not a problem in Duncan's work, which generally considers only CTC stress paths, but is of concern when solving a problem with general loading paths. For this reason, CANDE does not incorporate the separate unloading/reloading formulation of Equation 7.19. In fact, values of \( K_{ur} \) as determined from the laboratory tests shown in Appendix B were highly erratic and therefore discarded.

Lately, a new soil model, which can be used in conjunction with the hyperbolic model, has been introduced to handle this loading/unloading behavior [Seed and Duncan, 1986]. Specifically, this hysteretic model can be used in a finite element analysis to compute the residual stresses...
resulting from the cyclic applications of compaction forces. Unfortunately, this new development is not currently included in CANDE and could not be used here.

This leaves only the formulation for the bulk modulus which will be necessary to characterize the soil behavior in directions orthogonal to the compressed axis. The bulk modulus is defined as the mean stress divided by the volumetric strain which, upon expansion, gives:

\[ B = \frac{\Delta \sigma_m}{\varepsilon_v} = \frac{\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3}{3 \varepsilon_v} \]

Eq. 7.20.

In this soil model, the basic assumption is that the bulk modulus varies with the confining pressure [Duncan et al., 1980]. Specifically, this relation is modelled with Equation 7.6. For a given value of \( E \), the bulk modulus is uniquely related to Poisson's ratio and, as such, \( B \) is given limits within the finite element code to ensure Poisson's ratio is between the theoretical limits of 0 and 0.5.

Using a bulk modulus formulation does not permit predictions of volumetric strains due to shear dilatancy. That is, Duncan's model predicts volume changes only when the mean stress changes. For a simple shear (SS) stress path, the mean stress remains constant but a volume increase is observed due to the movement of sand particles up and over one another. Yet, Duncan's model will predict zero volumetric strains for this case of shear dilatancy. This particular shortcoming causes significant accuracy problems with dilatant materials such as dense sand, as will be seen in Chapter 8.

7.4. DETERMINATION OF PARAMETERS -- \( K, n, \) AND \( R_f \)

Having the complete model in hand, the task remains to get the model parameters from experimental data. To facilitate the graphical determination of parameters, Equation 7.9 can be rewritten as:
Now, by plotting values of $\epsilon/(\sigma_1 - \sigma_3)$ versus $\epsilon$, the hyperbola will become a straight line [Duncan and Chang, 1970]. Graphing experimental data on these transformed axes, it is simple to determine the values of $E_i$ and $(\sigma_1 - \sigma_3)_{ult}$ as the inverse of the intercept and slope, respectively. This is demonstrated in Figure 7.4.

When actual experimental data is plotted on the transformed axes, a slight curve is generally produced instead of a perfectly linear fit. Based on extensive experience, Duncan recommends fitting the hyperbola through points corresponding to 70% and 95% of the strength [Wong and Duncan, 1974]. This is done by first finding the peak deviator stress for each test, taking 70% and 95% of this peak value, then reading off the corresponding strains from the experimental results. For each laboratory test, plotting just these two points on the transformed axes effectively fits a hyperbola to the data.

Subsequently, the inverse of the intercept and slope for each test is used to determine the initial modulus and ultimate stress difference. The failure ratio ($R_f$) is determined as the ratio of failure stress over ultimate stress for each test. $R_f$ from all of the tests can then be averaged.

It is now possible to determine the modulus parameters $K$ and $n$ as required by Equation 7.18. A plot of Equation 7.18 on log-log paper yields a straight line of the form:

$$\log_{10}\left(\frac{E_i}{P_a}\right) = \log_{10}K + n \log_{10}\left(\frac{\sigma_3}{P_a}\right)$$

Hence, the constants are determined by first plotting $(E_i/P_a)$ versus $(\sigma_3/P_a)$ from each test on log-log paper. A linear regression is performed to determine the best straight line fit through data points from all of the laboratory tests. Then, $K$ is seen as equal to $(E_i/P_a)$ where $\log_{10}(\sigma_3/P_a)$ is equal to zero; that is, at $(\sigma_3/P_a) = 1.0$. Also, the parameter $n$ is equal to the slope of this line. Numerically:
7.5. DETERMINATION OF PARAMETERS -- c, $\phi_0$, AND $\Delta \phi$

Using the peak stress, the Mohr-Coulomb failure criterion is applied to determine the angle of internal friction ($\phi$) and the cohesion intercept (c) for each soil test. For a cohesionless material such as a clean sand, c can be taken as zero. Then, referring to Figure 7.2:

$$\sin \phi = \frac{R}{D} - \frac{2}{\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}}$$

Eq. 7.24.

$$\phi = \sin^{-1} \left( \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \right)$$

Eq. 7.25.

For a cohesive soil such as clay, $c \neq 0$ and it is best to use a p-q diagram to evaluate c and $\phi$ [Lambe and Whitman, 1969].

It remains to determine the hyperbolic parameters $\phi_0$ and $\Delta \phi$ which will indicate the decrease in the friction angle at higher pressures. The friction angle and minor principal stress are plotted on a semi-logarithmic plot (Figure 7.5) through which a straight line is passed using linear regression. Duncan has modelled this phenomenon with Equation 7.7 whose constants $\Delta \phi$ and $\phi_0$ can be readily seen as the slope and intercept on Figure 7.5.
7.6. DETERMINATION OF PARAMETERS -- \( K_b \) AND \( m \) -- DUNCAN'S TECHNIQUE

In the original formulation for the bulk modulus [Duncan et al., 1980], only CTC stress paths were considered. For this specific case, Equation 7.20 simplifies to:

\[
B = \frac{\sigma_1 - \sigma_3}{3 \varepsilon_v}
\]

Eq. 7.26.

Therefore, to determine the bulk modulus for a CTC test, it is necessary to use only one point from a volumetric strain curve. In choosing this point, Duncan and his co-workers provide these guidelines:

a. use the point corresponding to 70% of the peak stress; but only if no horizontal tangent develops in the volumetric strain prior to reaching this point; or

b. use the point where the volumetric strain curve reaches a horizontal tangent; but only if this occurs prior to reaching 70% of the failure stress.

Having chosen a point on the volumetric strain curve using the above criteria, the bulk modulus for each test is calculated. The variation of \( B \) with the minor principal stress is characterized with Equation 7.6. To determine \( K_b \) and \( m \), a procedure is used similar to that for finding \( K \) and \( n \). First, Equation 7.6. is written in the form:

\[
\log_{10} \left( \frac{B}{P_a} \right) = \log_{10} K_b + m \log_{10} \left( \frac{\sigma_3}{P_a} \right)
\]

Eq. 7.27.
Hence, values of \( \frac{B}{P_a} \) for each test are plotted versus \( \frac{\sigma_3}{P_a} \) on a log-log scale and a straight line is fit with linear regression. The values of \( K_b \) and \( m \) are then read as the intercept and slope, respectively, as in Figure 7.6. Since this is a logarithmic plot, the intercept value is really \( \log_{10} K_b \) and occurs where the logarithm is zero; i.e., where \( \frac{\sigma_3}{P_a} \) equals 1.0.

### 7.7. DETERMINATION OF PARAMETERS -- \( K_b \) AND \( m \) -- SELIG'S TECHNIQUE

Alternately, the bulk modulus parameters can be derived from hydrostatic compression (HC) tests [Selig, 1988]. This method represents a more logical approach because HC tests are a direct measure of bulk modulus behavior. Selig's suggested technique, while not changing the basic hyperbolic soil model, does provide for more reasonable model constants. This was found to be especially true when dealing with a dilatant soil (dense sand) in Chapter 8.

Selig begins by representing HC test data with yet another hyperbola:

\[
\sigma_m = \frac{B_i}{\frac{1}{\epsilon_v} - \frac{1}{\epsilon_{ult}}} 
\]

Eq. 7.28.

where \( \sigma_m \) is the mean stress and \( \epsilon_v \) is the volumetric strain (sum of the three axial strains). \( B_i \) is the initial bulk modulus and \( \epsilon_{ult} \) represents the asymptotic volumetric strain at large stress, as in Figure 7.7. This expression can then be rewritten to produce Equation 7.29.

\[
\frac{\sigma_m}{\epsilon_v} - B_i + \frac{\sigma_m}{\epsilon_{ult}} 
\]

Eq. 7.29.
As seen before, this produces a linearized graph of the hyperbola when plotted on the transformed axes of Figure 7.8.

By plotting several data points from each HC test on the transformed axes and then using a linear regression analysis, values of \( B_i \) and \( \varepsilon_{ult} \) can be determined as in Figure 7.8. The values of \( B_i \) and \( \varepsilon_{ult} \) from each test are then averaged. It remains to convert these two parameters to \( K_b \) and \( m \) as required for Duncan's formulation.

The tangent bulk modulus is found by differentiating Equation 7.28:

\[
B = \frac{\partial \sigma_m}{\partial \varepsilon_Y} - \frac{\partial}{\partial \varepsilon_Y} \left[ \frac{B_i}{\varepsilon_Y - \varepsilon_{ult}} \right] = \frac{B_i}{\left( 1 - \frac{\varepsilon_Y}{\varepsilon_{ult}} \right)^2} \\
\text{Eq. 7.30.}
\]

Rearranging Equation 7.28 will give an expression for \( \varepsilon_Y \):

\[
\varepsilon_Y = \frac{\sigma_m}{B_i + \frac{\sigma_m}{\varepsilon_{ult}}} \\
\text{Eq. 7.31.}
\]

which will, upon substitution back into Equation 7.30, yield:

\[
B = B_i \left( 1 + \frac{\sigma_m}{B_i \varepsilon_{ult}} \right)^2 \\
\text{Eq. 7.32.}
\]

Having determined \( B_i \) and \( \varepsilon_{ult} \) from the HC data on transformed axes, Equation 7.32 can be used to compute the bulk modulus at various levels of the mean stress. Recalling that the minor principal stress is equal to the mean stress during an HC test, "data points" of \( (B/P_a) \) versus \( (\sigma_3/P_a) \) can be computed. When these points are plotted on log-log paper, as in Figure 7.6, they should be linear to fit Equation 7.27. When this is done, however, some small curvature exists in the computed data points. By passing a linear regression fit through the points, however, Selig effectively fits Duncan's model to the hydrostatic compression data.
7.8. MODEL EXTENSIONS TO PERMIT INCLUSION OF NON-CTC TESTS

Previously, parameters for Duncan’s soil model have been derived only from CTC stress paths where the minor principal stress is constant. To incorporate the effects of other stress paths into the model predictions, the basic assumptions of the hyperbolic model must be clarified. This does not require any new formulations, however, some extension of the definitions used in deriving the model parameters are necessary.

Duncan’s hyperbolic model operates on the major and minor principal stresses, \( \sigma_1 \) and \( \sigma_3 \), both of which may be changing under general loading. Moreover, as soil cannot resist tensile loading, only axial compression can be modelled with Equation 7.5. Axial extension resulting from Poisson effects are handled with the bulk modulus portion of the model. Therefore, to determine model parameters from non-CTC data, the following clarifications should be followed:

1. Fit the hyperbola to axes under compressive strains. This may mean fitting the hyperbola to the axis which is under constant stress but is nevertheless compressed by a decreasing load on another axis. If two axes are being compressed under the same load conditions, then fit the hyperbola using an average of the two response curves.

2. Use the minor principal stress at the start of the test when deriving the parameters \( K \) and \( n \). These two parameters are associated with the initial modulus of the soil.

3. Use the minor principal stress at failure to derive the failure parameters \( \phi_0 \) and \( \Delta \phi \).
4. Use the method proposed by Selig to determine the bulk modulus parameters $K_b$ and $m$ from HC data. This is a more direct method of measuring the bulk modulus and avoids the inconsistencies encountered when trying to apply Duncan's technique to non-CTC stress paths.

Using these guidelines, the model parameters can be derived from any stress path using the same techniques described in previous sections of this chapter. The resulting parameters will produce model predictions which reflect more general loading conditions, depending on the stress paths included in deriving the parameters.
Table 7.1. Duncan's hyperbolic soil model parameters for CANDE.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>Modulus number</td>
<td>from relating initial modulus to confining pressure.</td>
</tr>
<tr>
<td>$n$</td>
<td>Modulus exponent</td>
<td>from relating initial modulus to confining pressure.</td>
</tr>
<tr>
<td>$R_f$</td>
<td>Failure ratio</td>
<td>failure shear stress / ultimate shear stress.</td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesion intercept</td>
<td>from Mohr-Coulomb failure criterion.</td>
</tr>
<tr>
<td>$\phi_o$</td>
<td>Friction angle</td>
<td>from Mohr-Coulomb failure criterion and relating $\phi$ to $\sigma_3$.</td>
</tr>
<tr>
<td>$\Delta\phi$</td>
<td>Reduction in $\phi$ for 10-fold increase $\sigma_3$</td>
<td>from Mohr-Coulomb failure criterion and relating $\phi$ to $\sigma_3$.</td>
</tr>
<tr>
<td>$K_b$</td>
<td>Bulk modulus number</td>
<td>relating $B$ to $\sigma_3$.</td>
</tr>
<tr>
<td>$m$</td>
<td>Bulk modulus exponent</td>
<td>relating $B$ to $\sigma_3$.</td>
</tr>
</tbody>
</table>
\( \sigma = a + b (\sigma_1 - \sigma_3)_{ult} \)

Figure 7.1. Hyperbolic stress-strain curve.

\[ \tau = c + \sigma \tan \phi \]

Figure 7.2. Mohr-Coulomb failure criterion.
Figure 7.3. Unloading / reloading modulus.

Figure 7.4. Hyperbolic curve on transformed axes.
Figure 7.5. Variation of $\phi$ with $\sigma_3$. 

$\phi = \phi_o - \Delta \phi \log(\sigma_3/P_a)$

Figure 7.6. Variation of bulk modulus with $\sigma_3$. 

$\log(\sigma_3/P_a)$
Figure 7.7. Hyperbolic representation of hydrostatic compression test.

Figure 7.8. Hydrostatic compression test on transformed axes.
CHAPTER 8

HYPERBOLIC SOIL MODEL PARAMETERS FOR BACKFILL SAND

8.1. RECOMMENDED HYPERBOLIC PARAMETERS

Table 8.1 presents parameters recommended for simulating behavior of the culvert backfill using Duncan's hyperbolic constitutive model. CTC Path Parameters were derived using only data from conventional triaxial compression stress paths performed on both the triaxial and multiaxial devices. The General Path Parameters in Table 8.1 were derived by extending the basic Duncan model to include all stress paths performed on the multiaxial apparatus. This set of values is intended to reflect the behavior of the material under a wide range of loading paths. Hence, the General Path Parameters effectively sacrifice the accuracy of the prediction along specific stress paths while permitting the prediction of responses under unknown or mixed loading conditions. The CTC Path Parameters correspond to the original features and assumptions of Duncan's model.

These parameters should be used with an understanding of the specific laboratory tests and techniques used to determine their values. Later sections of this chapter will describe the material, the laboratory tests, and the conclusions reached when deriving the parameters. Duncan's hyperbolic model is discussed in the previous chapter together with the extensions for non-CTC tests.
8.2. DESCRIPTION OF BACKFILL MATERIAL TESTED

As part of the larger culvert research project at Ohio University, four box culverts were investigated which were constructed using sand backfill acquired from two separate quarries. The first three culverts, installed in 1986, were backfilled with sand quarried along the Muskingum River in southeastern Ohio. The fourth culvert, which is the subject of this report, was installed in the spring of 1988 with sand backfill from the Ohio River valley. Backfill material was collected from each construction site and brought to the laboratory for extensive testing.

Samples from the two quarry sites were examined and were judged to be physically similar, as would be expected from their similar geologic histories. Some small difference did exist in the gradation of the sand from the two sites, as indicated by sieve analyses, but the difference was judged to be insignificant. Therefore, the backfill sand was considered to be the same and any backfill data would be applicable to any of the four culverts.

The backfill material is a medium, brown, quartz sand quarried from river valley deposits and commercially graded for 100% passing the U.S. No. 4 sieve (no particles larger than 4.75 mm). The sieve analysis indicated apparent diameters of $D_{10} = 0.47$ mm, $D_{30} = 0.85$ mm, and $D_{60} = 1.73$ mm; i.e., 60% of the grains are smaller than 1.73 mm, etc. These values yield a coefficient of uniformity $C_u = D_{60} / D_{10} = 3.7$ and a coefficient of concavity of $C_c = D_{30}^2 / (D_{10}D_{60}) = 0.89$. The material classifies as A-1-b under the AASHTO system while under the Unified Soil Classification, it is grouped as SP - poorly graded sand.

To further describe the sand being tested, the maximum and minimum dry density of the sand was determined [Bowles, 1986] to be 120.0 pcf and 97.2 pcf, respectively. The sand was tested at a dry density of 112.5 pcf; consequently, the relative density is found to be $D_r = 72\%$. This classifies the material as a dense sand [Lambe and Whitman, 1969].
8.3. LABORATORY TESTING CONDITIONS

It is important when running laboratory tests on soil to closely simulate the field conditions. The sand was backfilled around the culverts in a wet condition and then further flooded with water. Due to very good drainage, however, most of this water quickly dissipated. Hence, it is impossible to determine the true wet density of the sand as placed. In addition, the effect of water on the constitutive response of an unsaturated, porous sand under drained conditions is negligible. For these reasons, the sand was tested in an oven dry state. The density of the backfill as placed around the culverts was measured by personnel from the Ohio Department of Transportation using a nuclear density gauge. Field conditions were simulated in the laboratory specimen using a dry density of 112.5 pcf.

Moreover, it is important to simulate the same loading path encountered by the material under the conditions being analyzed. The actual general loading conditions of the culvert backfill, however, are not easily simulated. The stress path of a given soil element depends on its location and the overall loading conditions of the culvert system. For example, an element of soil near the side of the culvert will undergo a conventional triaxial compression type loading during placement of backfill layers directly above it. Later, as backfill is placed over the crown of the culvert and the sides of the culvert are forced outward, the same soil element may follow a conventional triaxial extension type loading. For these reasons, it was felt that an improvement in the finite element predictions using Duncan's hyperbolic model could be obtained by including data from tests along a variety of stress paths.

Another important factor in planning these tests is the confining pressure from which the test begins. In all of the experiments, the sand specimen was first consolidated under hydrostatic stress which defined the starting point of zero strain. Subsequently, loading along the stress path was applied. The choice of confining pressure should also reflect the anticipated field conditions. Given
the wet density of the backfill in the field of approximately 125 pcf and a backfill height of 8 feet, this would result in a confining pressure of approximately 7 psi due to the overburden alone. Additional confining pressure could result from the action of the culvert itself. With these factors in mind, confining pressures were chosen for each test in the range of 3 psi to 25 psi.

8.4. RESULTS OF TESTS ON BACKFILL SAND

The laboratory tests performed on the backfill sand are summarized in Table 8.2, which indicates the stress path used and the confining pressure at which the tests were initiated. Only four tests were performed on the triaxial device, each along a CTC path. All other tests shown in Table 8.2 were run in the cubical multiaxial device.

The results of each test are presented in Appendix B. In these graphs, the X, Y, and Z axial response curves correspond to arbitrary directions in the multiaxial device for which the vertical direction is taken as the Z axis. During several calibration runs, it was proven that the multiaxial apparatus behaves isotropically; that is, each axis of the device responds equally. Therefore, it will not matter how the test specimen is oriented in the testing apparatus.

From the response of individual specimen axes during the HC tests and considering the overall accuracy of the testing apparatus, one can sufficiently consider the sand as isotropic. This is also demonstrated in the very similar response of the X and Y axes during CTE, RTE, and RTC loading (see Appendix B). After demonstrating the isotropy of the sand in HC tests, the sand specimen were consistently tested in the multiaxial device with the direction of the compaction force placed vertically.
8.5. DERIVATION OF HYPERBOLIC PARAMETERS - GENERAL COMMENTS

The guidelines suggested by Duncan [Duncan et al., 1980] were followed in deriving the hyperbolic model parameters. Any data obviously inconsistent with other results were discarded. Additionally, each curve displayed in Appendix B was smoothed by hand before picking off data points. For the tests where two axes followed equal stress paths, some small differences result from experimental error. Thus, the two response curves were averaged by using one smooth fit through both curves.

Duncan recommends fitting the hyperbola through only two points on the stress-strain curve: 70% and 95% of the failure strength of each test. This is accomplished on the plot of transformed axes where a hyperbola plots as a straight line (see section 7.4). Initially, it was felt that perhaps an improvement could be made by using a linear regression line to fit the hyperbola. However, this procedure yields a model that is too stiff by inherently weighting the hyperbolic fit closer to points at the beginning of the curve. Better overall representations of the experimental data were obtained by fitting to the 70% and 95% points as advised by Duncan.

Finally, the results from multiaxial CTC Test #7 were not used to derive the hyperbolic parameters. This particular test was withheld to permit an independent verification of the model predictions by direct comparison with laboratory data. The comparisons are shown in the next two sections.
8.6. PARAMETERS FROM CTC TEST DATA

Hyperbolic model parameters were derived using only CTC data from the multiaxial and triaxial CTC tests. This was done to verify the legitimacy of the multiaxial tests by showing a direct comparison with the results from the more standardized and familiar triaxial device. These parameters are given in Table 8.3. The graphs used to determine these values are shown in Figures 8.3, 8.4, and 8.5. It should be noted that Triaxial Test #3 was judged to be inconsistent and was only used to determine the failure parameters, \( \phi_0 \) and \( \Delta \phi \).

These parameters were then tested against the multiaxial CTC Test #7 as shown in Figure 8.1. The predicted curves were computed by inserting the parameters of Table 8.3 into Duncan's hyperbolic model, and then plotting the results directly against the experimental data. Obviously from Figure 8.1, the predictions based solely on the triaxial data are not satisfactory. One problem with the triaxial tests is the difficulty in reproducing the desired density of the cylindrical specimen. It was easier to achieve a consistent density of 112.5 pcf in the cubical specimen for the multiaxial whereas densities of the triaxial specimen ranged from 110 to 114 pcf.

Additionally, the range of confining pressures used in only four triaxial tests made it difficult to correctly ascertain the correct relationship between the minor principal stress and the friction angle. To fit a straight line through the four triaxial data points in Figure 8.5, the multiaxial CTC test results were used as a guide. That is, the linear fit through the triaxial CTC points was made using the same slope as indicated by linear regression through the multiaxial CTC data. This slope agrees satisfactorily with data from several direct shear tests, although these tests cannot be used to directly determine the friction angle parameters. Other experimenters have found the friction angle for sand will be somewhat too high when obtained from direct shear tests [Bowles, 1986].

Due to the poor predictions illustrated in Figure 8.1 and the inconsistent specimen density in the triaxial, the triaxial CTC parameters were modified by using the same value of \( \phi_0 \) as determined
from the multiaxial CTC tests. Making this one important modification yields the results shown in Figure 8.2. Whereas the triaxial prediction in Figure 8.1 uses a value of $\phi_0 = 42^\circ$, the modified triaxial prediction of Figure 8.2 differs only due to using $\phi_0 = 39^\circ$. This clearly demonstrates the relative importance of the friction angle parameters in the predictions of the hyperbolic model. More importantly, Figure 8.2 tends to validate the use of the multiaxial device to obtain the hyperbolic model parameters.

However, Figure 8.2 illustrates a problem in the bulk modulus parameters derived from the multiaxial tests. The data used to derive the bulk modulus parameters are shown in Figure 8.3. Specifically, a negative value of $m$ would indicate a stiffer material under lower confining pressures, a seemingly anomalous behavior. This might result from the repositioning of sand particles under higher confining pressures [Duncan, 1980]. However, this observed behavior does not appear in the triaxial tests or other multiaxial tests. Furthermore, Selig reported unsatisfactory results when applying the bulk modulus formulation to CTC tests [Selig, 1988].

More importantly, this hyperbolic soil model does not include volume changes due to shear stresses (shear dilatancy). For a dense sand under low confining pressures, this portion of the total volume change is very significant and thus limits the accuracy of volume change predictions. These factors suggest using methods proposed by Selig for obtaining the bulk modulus parameters from hydrostatic compression tests as is done in Section 8.7.

With the above considerations in mind, the recommended CTC path parameters were chosen as shown in Table 8.1. Since the material is sand, the cohesion intercept ($c$) was simply taken as zero. The value of $\phi_0$ from the triaxial tests is ignored as are the bulk modulus parameters $m$ and $K_b$ from the multiaxial tests. These CTC parameters thus represent the classical application of Duncan's hyperbolic model to the culvert backfill soil and agree favorably with parameters reported by other investigators [Duncan et al., 1980; Beal, 1986]. Predictions based on these parameters are satisfactorily demonstrated in Figure 8.7 against the measured response of multiaxial CTC Test #7.
8.7. GENERAL PATH PARAMETERS USING NON-CTC STRESS PATHS

Multiaxial data from load paths other than conventional triaxial compression were included in the derivation of parameters to produce a more general soil model. This required some extensions to Duncan's hyperbolic model as described in Section 7.8. Basically, this consisted of defining the loaded axis as the axis or axes under compressive loading and determining what stress should be used as the confining pressure for a given point in a test. The parameters derived using all of the test data are displayed in Table 8.1 under General Path Parameters.

Results from non-CTC tests are shown in the back of Appendix B. Graphically, the points of 70% and 95% of the failure stress were read from the response curves after being smoothed by hand. These points represent 70% and 95% of the mobilized strength of the material and were subsequently used to fit the hyperbola. As described in Section 7.4, this is done by plotting the points on the transformed axes which give a linear plot of the hyperbola. From these transformed plots for each test, the ultimate stress difference is equal to the inverse of the slope and the initial modulus is equal to the inverse of the intercept.

The failure ratio, \( R_f \), of each test was determined by taking the ratio of failure shear stress over ultimate shear stress. The values from the appropriate tests were averaged to give the parameter \( R_f \) as shown in Table 8.1.

The parameters \( K \) and \( n \) are used to model the relationship between the initial modulus, \( E_i \), and the initial minor principal stress, \( \sigma_3 \). The values of \( E_i \) from the transformed plots of each test were plotted versus the initial \( \sigma_3 \) on a log-log scale as shown in Figure 8.4 where \( P_a \), atmospheric pressure, is 14.7 psi. For the CTC path parameters, a linear regression line was fit only through the appropriate CTC test points. The linear fit through these points is fairly good in accordance with Duncan's original model. For the general path parameters, a linear regression was fit through all of the test data. Obviously, a straight line does not fit closely to all of this data and thus indicates
Duncan's model is poorly suited to predicting non-CTC stress paths. However, there is enough of a fit to permit squeezing a more general prediction from the Duncan model by including these other stress paths.

Figure 8.5 is the plot used to determine the parameters $\phi_o$ and $\Delta \phi$ relating the friction angle to the minor principal stress at failure. Once again, this graph shows an adequate fit of Duncan's model to CTC stress paths and a poor fit through non-CTC tests. A more significant problem is the narrow range of confining pressures in the experimental data which tends to exaggerate the poor fit of the model through this data. However, the culvert problem to be analyzed is also under a narrow range of confining pressures. This tends to make the value of $\Delta \phi$, the slope of the lines in Figure 8.5, relatively insignificant in the finite element analysis; again permitting the Duncan model to be stretched to include non-CTC load paths. Additionally, the cohesion intercept is taken as zero.

As discussed in the previous chapter, Duncan's bulk modulus formulation is incapable of characterizing volume changes due to shear stresses alone. Additionally, problems can arise when trying to determine the parameters as in Figure 8.3. For these reasons, and especially to avoid theoretical objections along non-CTC paths, the bulk modulus parameters were derived from HC test data as suggested by Selig and described in Section 7.7. These are the values of $K_0$ and $m$ recommended under General Path Parameters in Table 8.1.

Selig first fits a hyperbola to the HC response curves. The initial bulk modulus and the ultimate volumetric strain are determined from a transformed plot of each test and then averaged. These two values are then used to determine the bulk modulus as a function of the mean stress. Plotting these computed values on Figure 8.6 permits the determination of the bulk modulus parameters for Duncan's hyperbolic model. Note that the linear fit in Figure 8.6 is quite good.
8.8. MODEL PREDICTIONS VERSUS EXPERIMENTAL RESULTS

The recommended hyperbolic parameters of Table 8.1 were tested along three loading paths in Figures 8.7, 8.8, and 8.9. This was done by programming Duncan's hyperbolic model to compute the response along the various testing paths using the two sets of parameters. The experimental data from CTC Test #7 was not used in deriving any of the parameters; thus, Figure 8.7 serves as an independent check on the predictions of the model. As expected, the prediction based on CTC path parameters more closely fits the test data than the general path parameters. For the CTE test of Figure 8.8, the CTC path parameters again give a better prediction of the compressive response. However, the general path parameters give a better prediction along the RTC stress path as shown in Figure 8.9.

Overall, the general path parameters reflect a stiffer material. This results because reverse loading gives a stiffer constitutive response as seen in the lab data in Appendix B. Inclusion of the RTC, RTE, TC, TE, and PL tests produces a stiffer soil model than that derived from the softer CTC response curves alone. Consequently, the general parameters will be expected to give less accurate modelling along the CTC stress path as demonstrated in Figure 8.7. The general path parameters sacrifice accuracy along specific load paths to permit a model which encompasses more general loading paths.

Ultimately, the engineer must judge the anticipated occurrence or importance of the non-CTC stress paths in the problem to be analyzed. If the soil is being loaded along predominately CTC stress paths, then the CTC path parameters should be used. The general path parameters should only be applied where the loading path is mixed or unknown. When using this soil model in conjunction with a finite element solution, it would be advisable to make this choice on an element by element basis. For instance, for a culvert analyzed with CANDE, the general stress path parameters can be used for elements along the sides of the culvert. This will be discussed in more
detail in Chapter 9. Additionally, it would be possible to derive hyperbolic parameters from specific stress paths. This would be advisable if using the hyperbolic model to predict the soil response along a known stress path.

Overall, Duncan's hyperbolic model is poorly suited for modelling the stress-strain response of soils along non-CTC stress paths. Presented with such a problem, it may be best to employ a different soil model. However, if the hyperbolic model is the best available, as when using CANDE, parameters can be derived for the more general loading paths.

8.9. EVALUATION OF MULTIAXIAL DEVICE

Clearly from Figure 8.2, the multiaxial device produces acceptable results and can be used to obtain parameters for any soil model. It would be advisable, however, to determine the bulk modulus parameters using HC tests as suggested by Selig. This may be advisable even with triaxial tests as HC tests are easy to run and represent a direct method of experimentally measuring the bulk modulus.

An advantage of using the multiaxial device lies in the preparation of the specimen itself. As described in Chapter 6, cubical cohesionless specimens are no more difficult to form than cylindrical cohesionless specimens. Also, it is much easier to produce the correct specimen density with the cubical shape. The cube has a larger open surface during forming which permits more even and controlled compaction of the specimen. Obviously, controlled specimen density is very important for producing good, consistent laboratory results.

In addition, the multiaxial device at Ohio University has computerized data acquisitions which makes collecting and processing the test data more accurate and much easier. Of course, triaxial
devices can be equipped with similar data acquisition equipment, but the triaxial device used in this study required hand readings. Overall, the multiaxial device is easier to use although it does require more time to assemble before conducting a test.

One problem with the multiaxial device used in this study was the system used to read and apply pressure. This pressure system is capable of applying and reading pressures of 1000 psi. All pressures applied during this testing program were well below 100 psi and thus in the lower working range of the gauges and transducers -- where precision becomes a problem. Consequently, it was difficult to apply and maintain a certain pressure at any time during a test and no stresses are known more accurately than 1.0 psi. This lead to some of the erratic behavior observed but reasonable results were still obtainable. Since this work was performed, a new multiaxial pressure system that will work accurately in this pressure range has been constructed for use on projects at Ohio University.

Finally, the primary advantage of using the multiaxial device is to permit the testing of soils along a wide variety of stress paths. This study was restricted by the finite element codes to using Duncan's model which is not generally used to model the non-CTC stress paths. Consideration of general load paths in the constitutive relations is important for producing a reliable and accurate analysis of the culvert problem.
Table 8.1. Recommended parameters for Duncan's Hyperbolic Soil Model -- culvert backfill sand.

<table>
<thead>
<tr>
<th>Soil Model Parameter</th>
<th>CTC Path Parameters</th>
<th>General Path Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>39°</td>
<td>39°</td>
</tr>
<tr>
<td>$\Delta\phi$</td>
<td>5.5°</td>
<td>3.0°</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>K</td>
<td>450</td>
<td>1200</td>
</tr>
<tr>
<td>n</td>
<td>0.35</td>
<td>1.1</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.60</td>
<td>0.70</td>
</tr>
<tr>
<td>$K_b$</td>
<td>300</td>
<td>350</td>
</tr>
<tr>
<td>m</td>
<td>0.30</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 8.2. Tests performed on culvert backfill sand.

<table>
<thead>
<tr>
<th>Stress Path:</th>
<th>Number of Tests</th>
<th>Confining Pressure (psi):</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC Hydrostatic Compression</td>
<td>3</td>
<td>--</td>
</tr>
<tr>
<td>CTC Conventional Triaxial Compression -- triaxial device</td>
<td>4</td>
<td>5,7,10,15</td>
</tr>
<tr>
<td>CTC Conventional Triaxial Compression -- multiaxial device</td>
<td>8</td>
<td>3,5,7,10,12</td>
</tr>
<tr>
<td>CTE Conventional Triaxial Extension</td>
<td>3</td>
<td>5,7,10</td>
</tr>
<tr>
<td>RTC Reduced Triaxial Compression</td>
<td>3</td>
<td>15,20,25</td>
</tr>
<tr>
<td>RTE Reduced Triaxial Extension</td>
<td>4</td>
<td>10,15,20</td>
</tr>
<tr>
<td>SS Simple Shear</td>
<td>3</td>
<td>15,17,20</td>
</tr>
<tr>
<td>TC Triaxial Compression</td>
<td>3</td>
<td>10,15,20</td>
</tr>
<tr>
<td>TE Triaxial Extension</td>
<td>3</td>
<td>10,15,20</td>
</tr>
<tr>
<td>PL Proportional Loading</td>
<td>2</td>
<td>10 (b = .2,.8)</td>
</tr>
</tbody>
</table>
Table 8.3. Parameters for Duncan's Hyperbolic Soil Model -- based on CTC stress paths.

<table>
<thead>
<tr>
<th>Soil Model Parameter:</th>
<th>Triaxial CTC Tests:</th>
<th>Multiaxial CTC Tests:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_0 )</td>
<td>42°</td>
<td>39°</td>
</tr>
<tr>
<td>( \Delta \phi )</td>
<td>5.7°</td>
<td>5.7°</td>
</tr>
<tr>
<td>C</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>K</td>
<td>430</td>
<td>470</td>
</tr>
<tr>
<td>n</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>( R_f )</td>
<td>0.73</td>
<td>0.52</td>
</tr>
<tr>
<td>( K_b )</td>
<td>280</td>
<td>130</td>
</tr>
<tr>
<td>m</td>
<td>0.34</td>
<td>-0.22</td>
</tr>
</tbody>
</table>
Figure 8.1. Triaxial and multiaxial CTC parameters vs. test data.
CTC TEST # 7
10 psi confining pressure

Figure 8.2. Modified triaxial and multiaxial CTC parameters vs. test data.
\[
\left( \frac{B}{P_a} \right) = K_b \left( \frac{\sigma_3}{P_a} \right)^m
\]

Figure 8.3. Variation of bulk modulus parameters with confining pressure.
\[( E_i/P_a ) = K ( \sigma_3/P_a )^n \]

- CTC -- triaxial
- CTC -- multiaxial
- non CTC -- multiaxial

General Parameters
\( K = 1200 \)
\( n = 1.1 \)

Multiaxial
CTC
\( K = 470 \)
\( n = 0.35 \)

Triaxial
CTC
\( K = 430 \)
\( n = 0.30 \)

Figure 8.4. Variation of initial modulus with initial confining pressure.
\[
\phi = \phi_o - \Delta \phi \log( \sigma_3 / P_a )
\]

Figure 8.5. Variation of the friction angle with minor principal stress at failure.
Figure 8.6. Determination of bulk modulus parameters using Selig's technique with HC data.

\[
\frac{B}{P_a} = K_b \left( \frac{\sigma_3}{P_a} \right)^m
\]

\[K_b = 350\]
\[m = 0.25\]
CTC TEST # 7
10 psi confining pressure

Figure 8.7. Model predictions vs. CTC test data.
Figure 8.8. Model predictions vs. CTE test data.
Figure 8.9. Model predictions vs. RTC test data.
CHAPTER 9

FINITE ELEMENT SIMULATION OF CULVERT

9.1. FINITE ELEMENT MODELLING OF CULVERTS: LITERATURE REVIEW

Numerical simulations of the culvert-soil structure was carried out using the finite element code Culvert ANalysis and Design (CANDE). CANDE was originally released in 1976 with an update in 1980 [Katona, et al., 1976; 1980]. The code is written specifically for simulating culvert behavior and is available from the Federal Highway Administration.

Numerous finite element simulations of culverts have been described in the literature and a large number of these studies used the CANDE code. Work by Leonards, et al. [1985], and concludes that CANDE was the best available code at that time for modelling culverts and that CANDE contained most of the important features for culvert analysis. Other culvert simulations that successfully used CANDE are reported by Beal [1986], Leonards and Juang [1985; 1984], Selig and Musser [1985], Chang, Espinoza, and Selig [1980], and Katona [1978]. It appears that CANDE is widely used in the United States for the analysis of culverts.

Incremental construction. To properly analyze culvert-soil interactions, it is important to model the incremental construction of the soil backfill. The importance of simulating the correct construction sequence has been pointed out in the work of Leonards, et al. [1985; 1982], McVay and Selig [1982], and Katona [1978]. Modelling of the incremental backfill was also carried out by Seed and Ou [1986], Duncan and Drawsky [1983], Hafez and Abdel-Sayed [1983], Duncan and Jeyapalan [1982], and Duncan [1979].
Modelling of soil backfill. As mentioned in Chapter 7, Duncan's hyperbolic soil model is often judged to be the best soil model available in CANDE [Leonards, et al., 1985; 1984; 1982; McVay and Selig, 1982]. Other researchers who have successfully used Duncan's model for culvert-soil characterization include: Boulanger et al. [1989]; Seed and Raines [1988]; Beal [1986]; Dessouki and Monforton [1986]; Seed and Duncan [1986]; Seed and Ou [1986]; Selig and Musser [1985]; Hafez and Abdel-Sayed [1983]; Duncan and Jeyapalan [1982]; and Duncan [1979].

Modelling of shear failure in some soil elements has been addressed by Dessouki and Monforton [1986] and Hafez and Abdel-Sayed [1983]. Since the Duncan hyperbolic model is not well suited for modelling soil beyond failure (see Chapter 7), these two studies used finite element codes that would redistribute element stresses if soil failure occurred. This feature is not available in CANDE.

Modelling of soil compaction. Various construction equipment operating on the backfill soil can be expected to impart additional stresses on the culvert. Because soil generally does not act elastically, some of these compaction stresses can be expected to remain upon completion of the culvert. However, it is difficult to model this loading in a finite element analysis. Usually, compaction loading is modelled using a uniform pressure acting on each layer of fill [Beal, 1986; Selig and Musser, 1985; Hafez and Abdel-Sayed, 1983; Duncan and Jeyapalan, 1982; Chang, et al., 1980; Katona, et al., 1976]. This pressure is usually assumed to be around 5 psi and can be removed at the completion of each lift by applying an equal load in the opposite direction. Improved results were obtained by Seed and Ou [1986] and Seed and Duncan [1986] using a soil model specifically formulated to model hysteretic, cyclic loadings. However, using a uniform surface pressure to simulate construction equipment is not a satisfactory technique to model this phenomena [Leonards, et.al., 1985; 1982; McVay and Selig, 1982]. This technique suffers mainly from the inability to choose a realistic pressure to simulate the construction equipment. It is possible that using some assumed compaction pressure may just offset errors due to other factors in the finite element model.
Interface elements. The behavior of the soil-structure interface is relatively more difficult to model. Several researchers have pointed out the necessity of characterizing this interface with special elements in a finite element analysis [Leonards, et al., 1985; 1982; Hafez and Abdel-Sayed, 1983; Duncan and Jeyapalan, 1982; Katona, 1982; McVay and Selig, 1982]. CANDE incorporates a constraint-type interface element that models the soil-steel surface as fully bonded, freely slipping, or completely debonded. Some authors have used this element in their CANDE analyses [Beal, 1986; Selig and Musser, 1985; and Katona, 1983]. Others have reported problems with numerical convergence when using the CANDE interface element [Abdel-Karim, 1987; Katona, 1983; Leonards, et al., 1982; Chang, et al., 1980]. It appears that convergence problems are often encountered with nonlinear soil models or when specific elements experience rapidly changing bonding to debonding conditions.

9.2. FINITE ELEMENT MODEL OF THE DEEP-CORRUGATED CULVERT

Analysis of the deep-corrugated culvert was carried out using the CANDE code with considerations given to the various factors discussed in the previous section. CANDE will perform elasticity solutions for round pipes or finite element analyses using a "canned" mesh in the code. Neither of these features were used for this work as a detailed finite element mesh was carefully prepared to simulate the field test. Furthermore, CANDE can be used to back-calculate the required culvert wall sectional properties for design purposes; only the analysis routines were used in this study.

CANDE contains features for modelling different culvert materials including reinforced concrete, plastic, and corrugated aluminum. A corrugated steel model and a user-defined pipe material is also available. The corrugated steel feature allows for a bilinear stress-strain relation for
the culvert plate to model yielding behavior. When using this option, the code uses an iterative procedure to compute bending and thrust in the culvert wall when yielding occurs. To do this, the code approximates the arc and tangent corrugation with an equivalent "sawtooth" cross section [Katona, et al., 1976]. Unfortunately, this sawtooth cross section (composed of only straight segments) is a poor approximation of Syro's deep-corrugated cross section. In fact, the sawtooth approximation will underestimate the moment of inertia of the deep-corrugated section by about 30%. This results because the sawtooth approximation cannot account for the greater amount of steel found in the arc section of the corrugation furthest from the neutral bending axis. Therefore, the CANDE analysis of this culvert was performed with the user-defined pipe model. The area, moment of inertia, and elastic properties given in Chapter 2 were used for the beam elements representing the culvert in the mesh.

All CANDE solutions assume plane strain conditions, small displacement theory, and quasi-static responses. Hence, CANDE cannot model load distribution along the longitudinal axis of the culvert or dynamic loadings. Time-dependent material responses cannot be considered directly. The small displacement assumption impairs the code's ability to model culvert buckling or failure. Various soil models are available; however, Duncan's hyperbolic soil model was chosen for simulation of the soils around the culvert. Duncan's model has been judged as the best constitutive model available in CANDE for soils [Leonards, et al., 1985; McVay and Selig, 1982].

Meshes. Figure 9.1 shows the half-mesh used to simulate symmetric loads on the culvert. A second full-mesh, shown in Figure 9.2, was developed for the simulation of unsymmetric loads. The half-mesh, consisting of 241 nodes and 226 elements, has a finer grid spacing than the full-mesh and can thus be expected to yield a more accurate simulation. Construction loads and those live loads positioned over the crown were analyzed using the half-mesh of Figure 9.1. The full-mesh of Figure 9.2, containing 365 nodes and 344 elements, was developed primarily to model unsymmetric
live loads at Positions #2 and #3. The full-mesh, while having a coarser gridding, required significantly more computer time to solve and nearly exceeded the available memory capacity.

Most of the elements in both meshes are two-dimensional, isoparametric, quadrilateral elements. The culvert plate is simulated using a series of straight, beam-column elements which model both bending and thrust loads in the culvert wall. The half-mesh models half of the culvert with 25 beam-columns elements while the full-mesh uses 30 elements for the culvert.

Reports by Amla [1990], Boulanger, et al., [1989], Hurd and Sargand [1988], and Katona [1978] suggest that the initial shape of the culvert can affect the finite element solution. The true initial shape of the deep-corrugated culvert was determined from field measurements made just before backfilling began, as discussed in Section 3.5. When the measured shape was compared to the design shape of the culvert, no significant differences could be observed. Hence, the coordinates for the culvert nodes where computed directly from the design shape. These nodes are located at the theoretical neutral bending axis of the deep-corrugated section. Other coordinates for the mesh were taken from design values or field observation. For example, the asphalt thickness and foundation dimensions were taken from design drawings while layers in the backfill were located to approximate the observed construction layering.

Both meshes utilize roller-type boundary conditions along the vertical sides of the mesh and hinge-type boundary conditions along the bottom of the mesh. The connection of the culvert plate to the concrete foundation is modelled as a hinge—a close approximation of the actual connection which consists of a steel angle section running the length of the culvert.

Construction increments. As pointed out earlier, modelling of the backfill construction in increments is an important aspect of culvert simulation. Figure 9.3 depicts how this was carried out for the CANDE analysis using the half-mesh. CANDE solves the culvert mesh fifteen times, once after adding each construction increment. That is, the mesh is first solved with all of the elements and loads in construction increment #1. The elements and loads comprising the second construction
increment are then added and the mesh is solved again. This continues until the simulated live load is applied as construction increment #15. The backfill lift layers, as depicted in Figure 9.3, were chosen to closely match the backfill heights present when the field data was acquired. Backfill layers were similarly chosen for the full-mesh; however, the full-mesh of Figure 9.2 utilizes only 13 construction increments.

CANDE solves the entire mesh with each additional construction increment and outputs the total culvert forces and displacements. The culvert elements are present in construction increment #1 and forces and displacements are thus computed for the in situ conditions. In the field test, the free-standing culvert (before backfilling begins) is taken as having no loads or displacements. To remain consistent in the finite element analysis, the culvert loads and deflections computed for the first construction increment are subtracted from the results of subsequent construction increments. Furthermore, to maintain consistency with analysis of the field data, live load responses in the culvert are computed by subtracting the results from the last two construction increments. These manipulations are carried out in a simple post-processor written for this study.

Interface elements. As discussed in Section 9.1, special elements are needed to accurately simulate the behavior of the culvert-soil interface. CANDE has an interface element that uses a constraint-type approach; i.e., the element models the interface as bonded, slipping, or debonded. Behavior of the interface element is controlled by parameters of interface tensile strength and a coefficient of friction. A mesh was developed for this analysis that included interface elements and numerous runs using several different tensile strengths and friction coefficients were made. Each time, CANDE encountered problems with numerical convergence. Any results from the mesh containing the interface elements were thus deemed unreliable. As discussed in Section 9.1, other CANDE users have reported this difficulty. Hence, interface elements were not used for any of the CANDE simulations reported in this study. This means that the culvert elements and the adjoining soil elements are treated as fully bonded in the finite element meshes.
9.3. BACKFILL AND CONSTRUCTION SIMULATION-PARAMETRIC STUDIES

Various material zones in the finite element meshes are depicted in Figures 9.4 and 9.5. Selection of the various material models was made using parametric studies, as described in this section.

**In situ soils.** In situ soil around the sand backfill was modelled using Duncan’s hyperbolic soil model. This soil, however, was not tested in the laboratory and the appropriate constitutive parameters could not be directly determined. Instead, hyperbolic parameters contained in a library within the CANDE code were used to model these soils.

The half-mesh was run using four different soil types from the CANDE library to model the in situ soils. These candidate soil types were:

- CL100 - clay, 100% compaction
- CL90 - clay, 90% compaction
- SC100 - silty-clay, 100% compaction
- SC90 - silty-clay, 90% compaction

Comparisons of the predicted foundation settlement from each of these four soil types are compared with the measured foundation settlements in Figure 9.6. From this plot, it is concluded that the in situ soil beneath the foundations are best modelled using the CL90 soil type from the CANDE library.

However, this soil model would probably be too soft to model the in situ soils beneath the roadway but above the water table. Hence, the CL100 soil type from the CANDE library was chosen to characterize, using the hyperbolic model, in situ soils above the level of the stream. The locations of these soil types are depicted for the half- and full meshes in Figures 9.4 and 9.5.
Backfill soils. Laboratory tests were performed to develop hyperbolic model parameters for the backfill sand as given in Table 8.1. A parametric study was undertaken to ascertain the influence of the backfill soil model on the CANDE solution. Three sets of parameters were tested:

- parameters for SM100 (silty-sand, 100% compaction) from the CANDE soil library;
- CTC parameters, derived from all of the laboratory tests along conventional triaxial compression stress paths (parameters given in Table 8.1);
- General path parameters, derived from laboratory tests along a variety of stress paths (parameters given in Table 8.1).

Three CANDE runs were made with each run using one set of these parameters to model the backfill soil. A fourth run was made using the laboratory derived CTC parameters in elements expected to experience mostly CTC loading and general path parameters for the other backfill elements. The general loading paths would be mostly expected in the soil elements along the sides of the culvert and would result from load distribution by the culvert. Vertical crown deflections, as predicted by each of these CANDE runs, is compared with the field data for construction loadings in Figure 9.7. Crown deflections were chosen as the simplest representation of culvert behavior.

Figure 9.7 implies that CANDE’s prediction of culvert behavior may not be overly sensitive to the parameters used in Duncan’s hyperbolic model. While the CANDE prediction of crown deflection is significantly less than the measured data, it appears that this is not due to the soil model parameters used for the backfill. Hence, the laboratory-derived CTC and general path parameters were used in specific zones as the most logical way to model this culvert. The locations of these backfill soil zones, as used in subsequent CANDE analyses, are depicted in Figures 9.4 and 9.5 for both meshes.
Construction equipment. The comparatively low predictions of crown deflections in Figure 9.7 might result from neglecting loads from construction equipment operating on the fill. The sand backfill was compacted using the small vibratory plate pictured in Figures 2.5 and 6.3. The machine weighed approximately 100 lbs. and would exert perhaps only 0.25 psi static pressure. Even considering the dynamic loading and multiple passes, this equipment would probably not exert a significant load on the culvert. However, in the later stages of construction, a front loader was driven onto the fill as discussed in Section 2.4. Even more significantly, a small bulldozer and loaded asphalt trucks were driven across the culvert during paving operations. As discussed by Duncan and Jeyapalan [1982], heavy construction loads such as these can cause noticeable deflections in a culvert.

To investigate the possible effect of these construction loads, a uniform surface pressure was applied to the top of the half-mesh. The small dozer was chosen as a representative load of 12,000 lbs. supported on tracks with a contact surface area of approximately 18 ft\(^2\). This yields an "equivalent" surface pressure of about 5 psi. This pressure is only a rough estimate but agrees well with magnitudes used by others for similar analyses [Beal, 1986; Duncan and Jeyapalan, 1982; and Katona, et al., 1976].

The effect of adding the 5 psi surface pressure is shown in Figure 9.8. This loading more than doubles the crown deflections and results in better agreement with the field data. Subsequent unloading, modelled with CANDE by an upward 5 psi pressure in the next construction increment, produces a rebound in culvert deflections back to a level very near those predicted without the added surface pressure. Near total rebound of the culvert would not be expected due to the plastic behavior of the soils. This probably results from CANDE's inability to accurately model soil unloading using the hyperbolic model.

Hence, these CANDE predictions of culvert behavior probably suffer from inadequate modelling of construction equipment loads. Besides not being able to simulate soil unloading correctly, the code does not permit the use of "uncompacted" material properties. That is, a soil
element should theoretically be very soft when initially placed and then stiffen after being compacted by construction equipment. This can not be simulated using CANDE.

Finally, the correct magnitude to use for the construction equipment pressure is unknown. Different pieces of equipment operated on the culvert at various times at various places. It is really not feasible to attempt to model each individual loading. This parametric study has shown however, that the construction equipment may explain the greater crown deflections observed than would be predicted by CANDE.

**Modelling of asphalt pavement.** The crushed limestone subgrade layer was also simulated using the hyperbolic soil model. Parameters were obtained from the CANDE soil library by assuming the soil could be represented by "CA105" described as coarse aggregate at 105% compaction.

Asphalt pavement was placed in three layers on top of the crushed limestone. This is modelled in the finite element meshes with one row of elements along the top as in Figures 9.4 and 9.5. The asphalt is characterized simply as linear elastic but with properties that reflect the asphalt temperature. As the asphalt cools and cures, it will stiffen and act to spread any surface loads. The question becomes how to simulate the asphalt behavior during construction (uncured) and then during live load (cured).

Two CANDE runs were made to study the effect of the asphalt cure. The first run assumed the asphalt was cured and was characterized by a Young’s modulus (E) of 400 ksi and a Poisson’s ratio (ν) of 0.41. The second run assumed E = 7 ksi and ν = 0.20 to simulate an uncured, hot asphalt layer. The results showed a difference of 7% in crown deflections and 9% in crown moments. These crown deflections are plotted in Figure 9.8. As would be expected, the elastic properties of the asphalt do have a noticeable effect. However, increasing E by a factor of over 50 reduced the crown moments by only 9%. Consequently, the assumed values of E = 7 ksi and ν = 0.2 should be adequate for simulating culvert response due to the added weight of the uncured
For simulations of the live loads, the asphalt was naturally considered to be cured. However, the spreading characteristics of the pavement is more crucial to accurate simulation of surface loads. Hence, the true elastic properties of the asphalt, a function of temperature, can be expected to have a greater influence on the predicted culvert responses under traffic loads. This hypothesis was tested with three CANDE runs that assumed three different asphalt temperatures. Asphalt properties at 70°F, 75°F, and 80°F, given in Table 9.1, were obtained from Kelly [1986]. Note that asphalt gets significantly stiffer with just a 5°F decrease in temperature.

For the same surface load conditions, the CANDE predictions of culvert moments and thrust were affected by up to 10% by a 10°F change in the asphalt temperature. Consequently, for simulation of culvert live loads, the elastic properties of the asphalt pavement were carefully chosen to reflect the temperature at the time field data was acquired. Recall that live loads were applied in three sets on June 1, 1988. That particular day started out cool and became hotter as the day progressed. Since only approximate air temperatures were known during live loading, the asphalt temperature was taken as:

- 70°F for 16 kip live loads
- 75°F for 32 kip live loads
- 80°F for 42 kip live loads.

The appropriate elastic parameters from Table 9.1 were used for the simulation of each live load.
9.4. EQUIVALENT LINE LOADS -- LITERATURE REVIEW

When using a plane strain finite element analysis of the culvert cross section, point loads on the surface (representing wheel loads under a truck) can only be modeled as line loads running parallel to the longitudinal axis of the culvert. Difficulties arise from trying to compute a line load that is equivalent to the wheel loads. Usually, the criteria is to select a line load that will produce the same effects on the culvert as the vehicular load. This is done by estimating a line load that would produce the same vertical stresses at the level of the culvert crown as produced by the wheel loads. The most thorough discussion of the various estimation techniques is given by Bakht [1981].

The simplest method for estimating an equivalent line load uses an assumed spreading factor. To compute vertical stresses at a given depth, the surface load is assumed to spread to be effective over a length equal to the depth times the spreading factor. Discussion often centers around the correct value for this spreading factor [Abdel-Sayed and Bakht, 1982; Bakht, 1981]. According to these papers, the American Association of State Highway and Transportation Officials (AASHTO) recommends a spreading factor of 1.75 such that a live load is spread to a width of 1.75H at a depth of H. Conversely, the Ontario Bridge Design Code specifies a spreading factor of 2.0. Bakht [1981] also argues that different spreading factors should be used for the parallel or perpendicular directions of the culvert. This is generally not considered when computing equivalent line loads for a plane strain finite element analysis. For the design analysis of this culvert, Syro used a spreading factor of 1.75 and included a consideration of the width of the tire [Powell, 1987]. This technique will be described completely later in the next section.

Equivalent line loads can also be estimated using elasticity solutions for point loads and line loads on a on a semi-infinite half space as described in the CANDE manual [Katona, et al., 1976]. This method is also described by Abdel-Sayed and Bakht [1982] and Bakht [1981]. Beal [1986] also employs this technique but points out correctly that no closed form elasticity solution exists for a
layered media. Hence, besides considering the soil as linearly elastic, this technique suffers from not being able to account for the stiff pavement layer found under normal service loads.

Duncan [1979; Duncan and Drawsky, 1983] has devised a simplified equation for computing a line load for design studies using finite element analyses. This simplified formula is a function of the axle loads and the depth of cover over the culvert. The equation is based on elasticity solutions for an HS-20 vehicle. The equivalent line load is intended to produce the same maximum vertical soil stresses at the height of the crown as the wheel loads. However, it is not clear how Duncan has derived this formulation. It appears that some allowance has been made for the stiff pavement layer, perhaps by using empirical relationships. Nonetheless, Duncan's simplified formula is frequently used for the finite element analysis of culverts [Boulanger, et al., 1989; Seed and Raines, 1988; Bakht, 1981].

9.5. SIMULATION OF LIVE LOADS

Dimensions, weights, and positions of the truck loads applied in the field study are given in Section 2.6, Table 2.1, and Figure 2.13. The wheel loads of the truck are really distributed over a small area under each tire. It is assumed here that the contact area under each tire measures 8 inches by 10 inches (8 inches by 20 inches for dual tires). To simulate surface loads in CANDE, the load must be expressed as an equivalent line load, applied to a given node, that extends parallel to the culvert.

The equivalent line load is necessary because of the plane strain assumption used by CANDE. This means that CANDE is able to compute stress and strain distributions within the circumferential cross section. However, CANDE is not able to compute load distribution along the length of the culvert (perpendicular to the plane strain mesh). In reality, the wheel loads of the truck will be
distributed along the length of the culvert with maximum stresses expected directly beneath the wheels.

With these factors in mind, the equivalent line load should be chosen to reflect the distance along the length of the culvert between the instrumented cross section and the wheel loads. The magnitude of the line load should not reflect the circumferential distance between the crown and the wheels as the finite element code will compute the stress distribution in this direction. Now, refer to Figure 2.13 for the wheel positions at the five live load positions. The same line load can be used to model Positions #1 through #3. A different line load should be used for Positions #4 and #5 because of the different longitudinal offset from the instrumented cross section. However, Positions #4 and #5 are equivalent with one set of wheels directly over the primary instrumented cross section.

Line loads are generally considered "equivalent" if they produce the same vertical stresses at some depth (usually the depth to the culvert crown) as the actual wheel loads. Again, since the finite element code will compute the stress distributions around the culvert, the presence of the culvert can be ignored when computing the equivalent line load. That is to say the line load is computed to produce equivalent effects on a pavement underlain by just soil. This equivalent line load is then placed on the culvert mesh to study the effects of the traffic load.

The half-mesh can only be used to simulate symmetric loads. Hence, this mesh can only be used to simulate the rear wheels at Positions #1, #4, and #5. The nodal location of the equivalent line load is shown in Figure 9.1. Because the half-mesh models only half of the culvert, the applied line load is only one-half the magnitude of the computed equivalent line load; i.e., half the load is carried in the other side of the culvert not included in the mesh. The full-mesh can be used to simulate any of the live loads and can include the front axle load. Figure 9.9 shows the nodal locations of the line loads for both axles under the 16 kip and 42 kip live loads. Recall that a shorter truck was used for the 32 kip load necessitating the different front axle positions depicted in Figure
Line loads for Positions #4 and #5 are applied at the same nodes as Position #1 in Figures 9.9 and 9.10.

The remainder of this section will discuss four techniques for estimating the equivalent line load. Not all of these techniques are capable of accurately simulating all of the live load positions. Others can not account for the stiff asphalt layer. A new technique, proposed in this paper, determines an equivalent line load using a finite element model of a two-way slab on a spring foundation.

**Equivalent line load using a spreading factor.** An equivalent line load for the deep-corrugated culvert was determined using a spreading factor of 1.75 as recommended by AASHTO. In equation form [Powell, 1987]:

\[
q = \frac{\text{wheel load}}{1.75H + W}
\]

where \(q\) is the equivalent line load, \(H\) is the height of soil cover over the crown (32.5 inches), and \(W\) is the width of the wheel. This representation of load spreading is depicted schematically in Figure 9.11. For the front wheels, \(W\) is taken as 10 inches while for the rear dual wheels, \(W=20\) inches. Note also that the wheel load is equal to one half of the axle load.

This simple technique does not permit consideration of the relative position of the wheels with respect to the instrumented cross section. Consequently, the same line load would be used for all of the live load positions. Also, this technique does not consider the load spreading effect of the pavement.

**Equivalent line load from elasticity solutions.** This development follows the formulation given by Katona, et al. [1976]. Elasticity solutions for a point load and a line load acting on a homogeneous, semi-infinite half space are used to compute an equivalent line load that will yield the same vertical stresses at the level of the crown.

Using the elasticity solution, the vertical stress \(\sigma_v\) below a point load of \(Q\) is given by:
Eq. 6.2.

where \( H \) is the depth and \( L \) is the length from the point of application to the point of interest. For a line load \( q \), the stress is:

\[
\sigma_v = \frac{2}{\pi} \frac{q H^3}{L^4}
\]

Eq. 9.3.

Equating these two equations yields the equivalent line load as:

\[
q = \frac{3Q}{4L}
\]

Eq. 9.4.

Note that this method treats the wheel loads as a point load and does not consider the affect of the pavement. In fact, no closed form elasticity solution exists for a line load on a layered half-space.

This technique does permit the calculation of an equivalent line load for the offset live load positions (#4 and #5). Figures 9.12 and 9.13 demonstrate how this is done. Each wheel load is treated independently with the correct distance \( L \) used for the distance between each wheel and the crown at the instrumented cross section. The equivalent line load for each wheel is then summed to produce the line load for the finite element simulation. Thus, for Positions #4 and #5, the equivalent line load is computed as (refer to Figure 9.13):

\[
q = \frac{3Q}{4} \left( \frac{1}{L_1} + \frac{1}{L_2} \right)
\]

Eq. 9.5.

where \( Q \) is the load of one wheel.

Equivalent line load from Duncan’s simplified equation. As discussed in Section 9.4, Duncan and his co-workers have developed a simplified equation for computing an equivalent line load [Duncan 1979; Duncan and Drawsky, 1983]. This equation for the equivalent line load \( q \) is:
where $K_4$ is a tabulated coefficient that is a function of the wheel configuration and depth of cover over the culvert. For the front axle (two wheels) and a depth of 32.5”, $K_4$ is given as 7.1 ft. For the rear axle with four wheels, $K_4$ is 8.0 ft. Equivalent line loads computed with Equation 9.6 are tabulated in Table 9.2.

Duncan's equation is based on elasticity solutions although the specific assumptions used in the derivation are not clear. Wheel loads are considered to be distributed over a small area and some consideration might be given for the presence of the asphalt. It appears that the formulation might have been modified based on field data or finite element solutions. Again, this is not clear. However, Equation 9.6 is derived from consideration of the peak vertical stress beneath the wheel loads. Hence, Duncan's equation is appropriate for computing an equivalent line load for Positions #4 and #5 where the wheels are directly over the instrumented cross section. Duncan's technique yields the same line load for all positions.

**Equivalent line load from finite element slab model.** All of the techniques discussed thus far have some deficiency. A different approach is needed to permit accurate modelling of each live load position while considering the load spreading effects of the pavement. Towards this end, the truck loads were modelled using the finite element code ILLISLAB which was developed at the University of Illinois in the mid-1970's [Ioannides, 1984].

The pavement over the culvert is modelled as a two-way slab on an elastic foundation using ILLISLAB. Wheel loads are input as a uniform surface pressure and the resulting deflection at the point of interest is recorded. Then, by trial and error, a line load is found that yields the same deflection at the point of interest.

The slab mesh used for finding the equivalent load is shown in Figure 9.14. Note that full advantage is taken of symmetry and only one fourth of the slab is included in the mesh. Wheel loads
are modelled as uniform pressures acting over an area 8 inches long by 10 inches wide for each tire (20 inches wide for dual tires). Line loads are modelled as strip loads one inch wide running the full width of the slab.

This code allows the consideration of various layers. The asphalt pavement (9.75 inches thick) is characterized as an elastic material with $E = 400$ ksi and $v = 0.41$. The 5 inch limestone subbase is modelled using $E = 20$ ksi and $v = 0.35$. The underlying sand is modelled as a spring foundation with a modulus of subgrade reaction of $k_s = 150$ pci for a medium dense sand. The correct values of these material parameters should not influence the line load determined because all of the runs use the same parameters. Changing one of these parameters should still yield approximately the same equivalent line load. Complete bonding between the various layers is assumed.

To account for the different load positions, a different control point is chosen for equating the vertical deflections due to the wheel and strip loads. The control point is selected to represent a point directly over the instrumented cross section. As shown in Figure 9.14, the control point is midway between the wheels for Positions #1 through #3. For Positions #4 and #5, the control point is located directly under the wheel. The use of ILLISLAB allows for the correct consideration of the pavement layers and permits modelling of all of the live load positions.

Comparison of results from live load simulations. Line loads representing the 42 kip truck load, computed using each of the four techniques described earlier, were input to CANDE. Positions #1 and #5 were simulated using the half-mesh with the line loads given in Table 9.2. The resulting deflections and bending moments in the crown were compared with the field data to assess how well each technique simulated the truck loads.

The results of these CANDE runs are given in Table 9.3. In this table, field data and CANDE predictions represent the effect due to the live load; i.e., deflections and moments resulting from construction loads are subtracted out. The field data points in this table do not seem to indicate the expected influence of the wheel positions. The measured crown deflections are equal
for both positions while the measured crown moment is actually less with the truck at Position #5. It is not clear if this is due to the actual culvert response or if the field experiment is just not sensitive enough to reflect the influence of truck position.

Nevertheless, the results shown in Table 9.3 can be used to reach some general conclusions regarding live load characterization. The equivalent line load from the elasticity solution appears to be much too high resulting in moments and deflections at least twice the magnitude measured in the field. Overall, the line loads found from the finite element slab model seem to produce the best agreement with the field data. Line loads from the spreading factor and Duncan's equation both yield reasonable results. More importantly, Duncan's equation appears to be conservative by yielding results consistently greater than the field data. Based on this limited analysis, the elasticity solution should not be used for determining an equivalent line load. For routine design analysis, Duncan's equation should be employed. Equivalent line loads from the finite element slab model, which produce the most consistent results, will be used in this study but requires too much work to be warranted for routine design studies.

9.6. RESULTS FROM CONSTRUCTION SIMULATION

Using the information from the parametric studies described earlier, three CANDE simulations were selected for comparisons with field data from the construction phase of this project. Since nearly all of the construction loads were nominally symmetric, the half-mesh (with finer grid spacing) was used for most of these comparisons. One run using the half-mesh did not model construction equipment on the fill while a second half-mesh run used a 5 psi surface pressure to simulate equipment loads. A third run was made with the full-mesh to simulate the observed uneven backfill profile. All of these CANDE simulations used the zoned backfill soil parameters (see
Figures 9.4 and 9.5) and assumed elastic properties for hot asphalt \((E = 7\) ksi, \(v = 0.2)\). Because of the convergence problems discussed earlier, interface elements could not be used. Results from these simulations are compared with the deflection data and the plate forces measured in the primary instrumented cross section.

Vertical deflection of the crown point as a function of backfill height, as predicted by CANDE with the half-mesh, are compared to the field data in Figure 9.8. Similarly, predicted bending moments and axial thrusts are compared with the field data in Figure 9.15. With backfill up to the level of the asphalt, the predicted deflections and moments compare reasonably well with the field data. However, deflections and moments are generally less than those measured in the field. When no compaction pressure is used, deflections and moments are significantly under-predicted by CANDE. Addition of the 5 psi compaction pressure produces more reasonable predictions. Comparisons with the measured thrusts are generally poor. However, thrust in the crown region is small and rather insignificant.

Vertical deflection of the culvert top at two important stages of construction, upon completion of the sand backfill and the asphalt pavement, are shown in Figures 9.16 and 9.17. The corresponding bending moments and thrusts are given in Figures 9.18 and 9.19. Upon completion of the sand backfill, the CANDE predictions of deflection (Figure 9.16) are generally low when compared with the field data. At the same stage, the predicted bending moments (Figure 9.18) agree very well with the field data. The measured axial thrusts are rather erratic and do not agree well with the CANDE predictions.

Vertical deflections predicted at the end of paving with no compaction pressure are a factor of three less than the measured values (Figure 9.17). Inclusion of the 5 psi compaction pressure on top of the asphalt yields better agreements between the CANDE predictions and the field data. Bending moments and thrusts for these conditions are compared in Figure 9.19. Moments in the crown region compare reasonably well only when the 5 psi compaction pressure is included.
However, in the haunch and side regions, excellent agreement with the field data is achieved only when no compaction pressure is included in the CANDE simulation.

The post-processor used to analyze the CANDE output also computed the factor of safety against plastic hinge formation (Equation 5.1). The maximum outer fiber stresses were also computed from the predicted moments and thrusts then compared to a yield stress value. As discussed in Section 2.3, Syro used a yield stress of 33 ksi for this culvert while the measured value was 54.6 ksi. Throughout the CANDE predictions, the computed factor of safety against plastic hinge formation was never less than 1.0 (which would indicate a plastic hinge had formed) for a yield stress of 33 ksi. The predicted maximum outer fiber stress only exceeded 33 ksi when the 5 psi compaction pressure was applied. This occurred in the haunch region and is due to the high moments predicted under these loadings. At no time did CANDE predict culvert stresses greater than the more realistic 54.6 ksi yield stress.

Figure 9.20 compares the culvert deflections predicted using the half- and full-meshes with no compaction pressure. The results are nearly the same and proves the virtual equivalence of results from either mesh. When loads are known to be symmetric, the half-mesh can be expected to yield better results due to the finer mesh (more elements) used and a more accurate representation of the backfill layers.

The full-mesh, however, must be used to model unsymmetric loads such as the uneven backfill profile represented in Figure 2.8. This backfill loading was modelled by altering the construction sequence of some elements in the full-mesh. Results from this prediction are also plotted on Figure 9.20. This uneven backfill has a very small effect on the predicted deflections. Which corresponds to the observed field data. It is likely that an uneven backfill profile occurring earlier in the construction sequence would have a more adverse effect on the CANDE predictions.

Assessment of CANDE predictions. In general, CANDE predictions of culvert bending moments during construction are reasonably good. Comparisons of predicted and measured thrusts
are more difficult due to the apparently erratic thrusts from the field data. Overall, the predicted
deflections are significantly less than the measured values. This is particularly true when no attempt
is made to include construction equipment loads in the CANDE simulations.

The CANDE finite element meshes may be too stiff because slippage along the culvert-soil
interface is not modelled. The interface element in CANDE needs to be altered or replaced to
permit better simulations. Furthermore, the CANDE predictions of culvert response may be too low
due to the manner in which new elements are added to simulate construction. It appears that
CANDE includes the stiffness of the new element in the mesh before adding the element's body
weight. In reality, the weight of an added soil layer will generate loads before any additional stiffness
results from compaction of the new layer. The magnitude of this effect on the CANDE results is
unknown.

Finally, the 5 psi compaction pressure, acting on top of the asphalt, demonstrates the largest
probable error in the CANDE simulation of construction loads. In the field, heavy equipment,
including a small dozer and loaded trucks, crossed the culvert numerous times during paving
operations. The 5 psi surface pressure in only an estimate of an equivalent pressure that could be
used to simulate these loads. While a better technique is needed to simulate these loads, it is clear
that some accounting must be taken of the construction equipment to achieve accurate predictions
of culvert behavior from CANDE.

9.7. RESULTS FROM SERVICE LOAD SIMULATION

For modelling of the truck loads, equivalent line loads from the finite element slab model
(Table 9.2) were used. When these line loads were applied at the axis of symmetry on the half-
mesh, only half the magnitude of the line load was applied (the other half is carried in the symmetric
part that is not modelled). Each line load was selected to simulate the culvert responses in the primary instrumented cross section. See Figure 2.13 for the specific live load positions. The elastic properties of the asphalt were selected for each simulation to reflect the temperatures during the field study (see Table 9.1). Finally, to maintain consistency with techniques used to analyze the field data, predicted culvert responses are reported as the incremental change due to the applied live load. That is, culvert responses predicted for the end of construction are subtracted from the total response predicted when the live load is applied. This also has the effect of removing errors that accumulate during simulation of culvert construction and allow direct comparisons with field data.

Figure 9.21 shows the deflection of the culvert top under the 32 kip load at the centered position as predicted by the full- and half-mesh. Both meshes, when only simulating the rear axle, predict nearly the same deflection profile. The apparent jaggedness of the deflection profile from the half-mesh results from the limited number of significant digits available from the CANDE output. CANDE yields only three digits for each output quantity; when these numbers are subtracted by the post-processor as described above, the jagged profile in Figure 9.21 can result. Hence, a more smooth curve would certainly be expected if CANDE output greater precision.

Parametric studies performed by Abdel-Karim [1987] using CANDE showed that effect of the front axle was negligible in simulations of live loads placed directly over the crown. Figure 9.21 includes a plot of predicted deflections when both axles of the 32 kip truck are simulated using the full-mesh. The front axle is to the north and greater deflections are seen in the northern half of the culvert. In the southern half, the predicted deflections are nearly unchanged due to the presence of the front axle. Since the 32 kip truck had the shortest wheel base of the trucks used in this study, Figure 9.21 represents the most severe error that might result from neglecting the front axle. Hence, for modelling of the live loads, the half-mesh without the front axle is used to simulate the symmetric live load positions (#1, #4, #5). For Positions #2 and #3, the full-mesh is used and the front axle is simulated.
Predicted deflections of the culvert top under live loads at Position #1 are compared with the field data in Figure 9.22. The results for the 32 kip load agree quite well while the predicted deflections under the 42 kip load are somewhat low. Greater deflections measured in the field to the north of the crown may have resulted from load transfer through the center axle as discussed in Chapter 5.

Vertical deflections, bending moments, and axial thrusts in the crown as measured and predicted are plotted in Figures 9.23 and 9.24 for all of the live loadings. As was observed for construction loads, CANDE predictions of bending moment agree favorably with the field data. Note that CANDE consistently predicts the smallest deflections and bending moments in the crown for Position #3. Also, CANDE significantly under-predicts thrusts at Position #4.

Figures 9.25 through 9.28 present distributions of bending moments and thrusts as measured and predicted for the 42 kip at all five positions. In general, the CANDE bending moments agree fairly well with the measured values. However, the measured thrusts appear too erratic for comparisons. Even so, the CANDE predictions of thrust generally agree in magnitude with the measured values.

Bending moments for the unsymmetric live loads (Positions #2 and #3) are plotted in Figures 9.26 and 9.27. Notice that, as would be expected, CANDE computes the maximum bending moment in a location near the rear wheels of the truck (in this case, to the south of the crown). This effect is also seen in the field data, but is less pronounced.

**Soil stresses.** The total stresses in the backfill sand, as predicted by CANDE, are shown in Figure 9.29 for the 42 kip live load at Position #1. Plotted in this figure is the maximum principal stress and direction with the magnitude represented by the length of the arrow. Over the top of the culvert, the soil is redistributing the load away from the culvert crown. Along the sides, however, the soil is loading toward the culvert. In other words, positive arching is seen above the culvert with negative arching observed along the culvert side. As was also discussed by Leonards, et al. [1982],
these results illustrate that the concept of positive or negative arching can be misleading as a gauge of culvert performance. Finally, Figure 9.29 shows the maximum soil stresses occur directly over the haunch region and, when combined with the downward stresses along the culvert side, result in axial thrusts in the culvert.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>Young’s Modulus E (ksi)</th>
<th>Poisson’s Ratio (v)</th>
<th>Live Load Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>500</td>
<td>0.40</td>
<td>16 kip</td>
</tr>
<tr>
<td>75</td>
<td>400</td>
<td>0.41</td>
<td>32 kip</td>
</tr>
<tr>
<td>80</td>
<td>300</td>
<td>0.42</td>
<td>42 kip</td>
</tr>
</tbody>
</table>

Table 9.1. Average elastic properties for asphalt concrete as a function of temperature [from Kelly, 1986].
Table 9.2. Equivalent line loads for plane strain finite element simulation of live loads.

For **REAR** Axle

<table>
<thead>
<tr>
<th>Load and Position</th>
<th>Rear Axle Load (kips)</th>
<th>Spreading Factor</th>
<th>Elasticity Solution</th>
<th>Duncan's Equation</th>
<th>Finite Element Slab Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 kip, #1,2,3</td>
<td>16.0</td>
<td>104.1</td>
<td>247</td>
<td>167</td>
<td>101.8</td>
</tr>
<tr>
<td>16 kip, #4,5</td>
<td>16.0</td>
<td>104.1</td>
<td>261</td>
<td>167</td>
<td>126.7</td>
</tr>
<tr>
<td>32 kip, #1,2,3</td>
<td>32.0</td>
<td>208.1</td>
<td>495</td>
<td>333</td>
<td>203.6</td>
</tr>
<tr>
<td>32 kip, #4,5</td>
<td>32.0</td>
<td>208.1</td>
<td>521</td>
<td>333</td>
<td>253.5</td>
</tr>
<tr>
<td>42 kip, #1,2,3</td>
<td>42.0</td>
<td>273.2</td>
<td>649</td>
<td>438</td>
<td>266.7</td>
</tr>
<tr>
<td>42 kip, #4,5</td>
<td>42.0</td>
<td>273.2</td>
<td>684</td>
<td>438</td>
<td>332.1</td>
</tr>
</tbody>
</table>

For **FRONT** Axle

<table>
<thead>
<tr>
<th>Load and Position</th>
<th>Front Axle Load (kips)</th>
<th>Spreading Factor</th>
<th>Elasticity Solution</th>
<th>Duncan's Equation</th>
<th>Finite Element Slab Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 kip, #1,2,3</td>
<td>4.0</td>
<td>29.9</td>
<td>58.3</td>
<td>46.9</td>
<td>21.5</td>
</tr>
<tr>
<td>16 kip, #4,5</td>
<td>4.0</td>
<td>29.9</td>
<td>63.5</td>
<td>46.9</td>
<td>31.5</td>
</tr>
<tr>
<td>32 kip, #1,2,3</td>
<td>8.3</td>
<td>62.0</td>
<td>121</td>
<td>97.4</td>
<td>44.6</td>
</tr>
<tr>
<td>32 kip, #4,5</td>
<td>8.3</td>
<td>62.0</td>
<td>132</td>
<td>97.4</td>
<td>65.4</td>
</tr>
<tr>
<td>42 kip, #1,2,3</td>
<td>10.7</td>
<td>80.0</td>
<td>156</td>
<td>126</td>
<td>57.4</td>
</tr>
<tr>
<td>42 kip, #4,5</td>
<td>10.7</td>
<td>80.0</td>
<td>170</td>
<td>126</td>
<td>84.3</td>
</tr>
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</table>
Table 9.3. CANDE predictions of culvert response under various line loads compared to field data.

<table>
<thead>
<tr>
<th>Method</th>
<th>Equivalent Line Load (lb/in)</th>
<th>42 kip, Position #1</th>
<th>42 kip, Position #5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Field Data</td>
<td>0.211</td>
<td>1.63</td>
<td>0.211</td>
</tr>
<tr>
<td>Spreading Factor</td>
<td>273.2, Pos#1</td>
<td>0.164</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>273.2, Pos#5</td>
<td>-22.3%</td>
<td>+4.3%</td>
</tr>
<tr>
<td>% Diff. from Measured</td>
<td>649, Pos#1</td>
<td>0.416</td>
<td>4.30</td>
</tr>
<tr>
<td></td>
<td>684, Pos#5</td>
<td>+97.2%</td>
<td>+164%</td>
</tr>
<tr>
<td>Elasticity Solution</td>
<td>438, Pos#1</td>
<td>0.279</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>438, Pos#5</td>
<td>+32.3%</td>
<td>+74.2%</td>
</tr>
<tr>
<td>Duncan's Equation</td>
<td>266.7, Pos#1</td>
<td>0.166</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>332.1, Pos#5</td>
<td>-21.3%</td>
<td>+1.8%</td>
</tr>
<tr>
<td>% Diff. from Measured</td>
<td>266.7, Pos#1</td>
<td>0.166</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>332.1, Pos#5</td>
<td>-21.3%</td>
<td>+1.8%</td>
</tr>
</tbody>
</table>
Live loads

No rotation on end of this beam element

Hinge

Figure 9.1. Half-mesh of culvert used for CANDE analysis.
Figure 9.2. Full-mesh of culvert used for CANDE analysis.
Figure 9.3. Construction increments used for finite element simulation.
Asphalt

Limestone subgrade

Backfill sand, CTC path parameters

Corrugated steel plate

Backfill sand, general path parameters

Concrete

In situ clay, 90% compaction

In situ clay, 100% compaction

Figure 9.4. Material zones used in half-mesh.
Asphalt Limestone subgrade

In situ clay, 100% comp

Backfill sand, CTC path parameters

Concrete

In situ clay, 90% compaction

Corrugated steel plate

Backfill sand,

In situ clay, 100% comp

General path

General path

Figure 9.5. Material zones used in full-mesh.
Influence of In Situ Soil Model

CANDE Predictions of Foundation Settlements

Field Measurements:
- North foundation
- South foundation

CANDE prediction, in situ soil model:
- Duncan's model, "CL90" parameters
- Duncan's model, "CL100" parameters
- Duncan's model, "SC90" parameters
- Duncan's model, "SC100" parameters

Figure 9.6. Parametric study of in situ soil.
Influence of Backfill Soil Model
CANDE Predictions of Crown Deflections

![Graph showing CANDE predictions of crown deflections with various backfill models.]

Figure 9.7. Parametric study of backfill soil model parameters.

Influence of Asphalt Placement
CANDE Predictions of Crown Deflections

![Graph showing CANDE predictions of crown deflections with different asphalt placement conditions.]

Figure 9.8. Parametric study of asphalt paving.
Figure 9.9. Placement of equivalent line loads for 16 and 42 kip live loads.

Figure 9.10. Placement of equivalent line loads for 32 kip live loads.
Figure 9.11. Assumed load distribution when using spreading factor.
Figure 9.12. Lengths from wheel loads at centered positions for elasticity solutions.

Figure 9.13. Lengths from wheel loads at offset positions for elasticity solutions.
Figure 9.14. Mesh for finding equivalent line loads using finite element slab model.
Figure 9.15. Plate forces as a function of backfill height.
Vertical Deflection of Culvert Top
Upon Completion of Sand Backfill

Figure 9.16. Vertical deflection of culvert top upon completion of sand backfill.

Vertical Deflection of Culvert Top
Upon Completion of Asphalt Pavement

Figure 9.17. Vertical deflection of culvert top upon completion of asphalt pavement.
Figure 9.18. Distribution of culvert loads upon completion of sand backfill.
Plate Forces under Backfill Loads
Upon Completion of Asphalt Pavement

Figure 9.19. Distribution of culvert loads upon completion of asphalt pavement.
Predicted Vertical Deflection of Culvert Top upon Completion of Asphalt Pavement

Figure 9.20. Predicted deflections using half- and full-meshes after construction.

Predicted Vertical Deflection of Culvert Top under 32 kip Live Load at Position #1

Figure 9.21. Predicted deflections using half- and full-meshes under live load.
Vertical Deflection of Culvert Top
Under Service Loads

Figure 9.22. Vertical deflection of culvert top under service loads.

Vertical Deflection of Crown Point
Under Service Loads

Figure 9.23. Vertical deflection of crown under service loads.
Plate Forces in Crown Section
Under Service Loads

- Measured, primary cross section
- Predicted, Pos. #1,4,5 - half-mesh
- Pos. #2,3 - full-mesh

Figure 9.24. Bending moment and thrusts in crown under service loads.
Plate Forces

42 kip Live Load, Position #1

Figure 9.25. Bending moments and thrusts for 42 kip load at Position #1.
Figure 9.26. Bending moments and thrusts for 42 kip load at Position #2.
Plate Forces
42 kip Live Load, Position #3

Figure 9.27. Bending moments and thrusts for 42 kip load at Position #3.
Plate Forces

42 kip Live Load, Position #4 & #5

Figure 9.28. Bending moments and thrusts for 42 kip load at Positions #4 and #5.
Figure 9.29. Predicted maximum principal stresses in backfill soil under 42 kip load.
CHAPTER 10

PROJECT SUMMARY AND RECOMMENDATIONS

10.1. FIELD STUDY

Full-scale field testing provided valuable information on the behavior of the deep-corrugated culvert. Field data from various instrumentation was consistent although some instruments were more accurate or precise than others. Specifically:

- The hose level worked well to measure actual vertical settlements of the foundations. However, a more precise version of this instrument would have been desirable.
- The split light beam did indicate bending changes in the haunch corners. Better accuracy is needed from this instrument to determine the true change in the corner angle. It was not possible to relate this data to the bending forces acting in this region.
- The tape extensometer worked very well to accurately measure deformation in the culvert cross-section. Obtaining these measurements, however, was very time consuming.
- Extensometers buried in the backfill measured displacements in the soil very accurately. Yet, this data does not provide much information on the culvert-soil interactions. The extensometers were too long to indicate the mechanism of load transfer from the culvert to the backfill soil. Shorter extensometers in a different configuration, or different instruments such as soil pressure gauges, would provide more valuable information on load transfer in the backfill soil.
Vibrating wire strain gauges were used successfully to measure culvert strains and will be most useful, together with deflection measurements, to monitor long-term culvert behavior. The vibrating wire strain gauges could not, however, be mounted across the curved sections of the corrugations and thus could not measure the longitudinal strains.

Bonded foil, electrical resistance strain gauges worked satisfactorily to provide the bulk of the data on stresses in the culvert plate. A major deficiency in this system was the requirement that the leadwires had to be disconnected at the end of each day. This prevented the detection of probable stress redistribution that might have occurred overnight.

It has been shown that it is valid to analyze the deep-corrugated section using simple beam theory. The moment and thrust capacity of the section is unknown and needs to be determined in the laboratory. Laboratory tensile tests on samples cut from the corrugated plate showed that the actual yield strength in the outer fibers is over 50 ksi.

Overall, the culvert performed satisfactorily under construction and service loads. As would be expected, the culvert did settle slightly and this effect was accounted for when computing deflections of the culvert under load.

A sloping backfill profile over the crown did generate a small unsymmetric load distribution in the culvert. Construction equipment operating over the culvert during paving operations nearly overstressed the culvert. This information clearly demonstrates the importance of diligently following conservative procedures during construction of box-type culverts. The flattened arch cross section and shallow cover appears to make these structures vulnerable to failure when careless construction procedures are employed. This is particularly true during paving operations when heavy equipment and loaded trucks are driven across the culvert backfill.
10.2. MODELLING OF THE BACKFILL SAND

The sand used to backfill the culvert was characterized with laboratory tests along a wide variety of stress paths. Most laboratory testing was performed using a cubical multiaxial testing device while four tests were run using conventional triaxial equipment. The laboratory data was then fit to Duncan's hyperbolic soil model and subsequently used for the finite element analysis of the soil-culvert system.

The multiaxial device proved to be very versatile and yielded results that agreed favorably with results from the triaxial equipment. The multiaxial test specimens were easier to produce, at a more consistent density, than the triaxial specimens. Computer data acquisition on the multiaxial made analysis of the data quick and more accurate than the triaxial test data. The multiaxial data did suffer from relatively imprecise control of the applied stresses. This resulted mainly because the pressure control system was designed for testing at much higher pressures than was used for this study.

Data from all of the lab soil tests were used to develop parameters for Duncan's hyperbolic soil model. The fit through the conventional triaxial compression (CTC) data was satisfactory, although there was insufficient test data from the triaxial device to define the failure parameters for those particular tests. Parameters from the multiaxial CTC tests agree favorably with the parameters from the triaxial CTC tests.

The definitions used in the model were extended to permit modelling of the soil along non-CTC stress paths. The fit of the model through the assorted stress path data was not very good and statistical techniques had to be employed. It was found that a more reasonable determination of the bulk modulus parameters could be obtained using hydrostatic compression data. Finite element calculations of the culvert behavior proved to be rather insensitive to the soil model parameters used.

Recommendations arising out of the laboratory testing and modelling of the sand backfill are:
• The multiaxial device can be used to determine the constitutive behavior of sand.

• Duncan's hyperbolic soil model can be extended to characterize non-CTC stress paths; however, the fit along general loading paths is relatively poor.

• Selig's technique for deriving the bulk modulus parameters for Duncan's model from hydrostatic compression data is highly recommended.

10.3. FINITE ELEMENT SIMULATION

Numerical predictions of culvert behavior were made using the finite element code Culvert ANalysis and Design (CANDE). The culvert was modelled using beam-column elements in a symmetric half-mesh as well as a full-mesh. The construction sequence and backfill layering was carefully simulated. Furthermore, the different truck loads at all positions were simulated with CANDE. The backfill soil was characterized using laboratory-derived parameters for Duncan's hyperbolic soil model.

Results from the finite element simulations were compared directly with the field data to assess CANDE's predictions. Overall, CANDE predictions of bending moments in the culvert agree very well with the measured values. Predicted deflections are generally less than measured. Predicted axial thrusts in the culvert plate appear to be of the correct magnitude; however, the measured thrusts are too erratic for comparisons. Relatively low thrusts in this culvert may account for the erratic thrust data.

Simulation of the culvert with CANDE lead to the following conclusions:

• The CANDE predictions are not overly sensitive to the parameters used to characterize the backfill sand with Duncan's hyperbolic soil model. Good results would be expected
if the soil parameters were derived only from conventional triaxial compression tests. A satisfactory soil model could have been derived without the sophisticated multiaxial soil tests.

- Interface elements, for the boundary between the soil and the culvert plate, would not converge in this simulation. Other users of CANDE have also reported this problem. A good interface model would probably yield improved CANDE predictions of culvert behavior, particularly during the construction phase when most interface slippage would be expected.

- Improved CANDE predictions were obtained by approximating construction equipment loads with a uniform surface pressure. However, the magnitude of this pressure is only a rough estimate and better techniques are needed to model multiple passes of heavy construction equipment.

- When simulating loads on the road surface, it is important to consider the temperature dependent properties of the asphalt. A 10°F temperature change will cause significant changes in the elastic properties of the asphalt and noticeably affect the predicted culvert response.

- Four techniques were employed to estimate equivalent line loads for simulating truck loads on the plane strain meshes. Of these, equivalent line loads from a finite element slab model yielded the best results. Line loads from Duncan's simplified formula appear adequate for routine design work with CANDE. Using simple elasticity solutions appears to greatly overestimate the magnitude of the line load and should not be used.

- CANDE predicts soil stresses acting away from the culvert crown but toward the culvert sides under the 42 kip truck load. This illustrates how the concept of positive or negative soil arching, in reference to box-type culverts under shallow fill, can be misleading.
10.4. RECOMMENDATIONS

**Construction.** Deep-corrugated steel, box-type culverts can be constructed to produce a stable, safe structure. But it cannot be said enough: failure of these structures can result from seemingly harmless construction practice. Level, symmetric backfill profiles must be maintained during construction. Most importantly, all heavy equipment must be kept well off the culvert until the paved roadway has had ample opportunity to cure and stiffen. Soil compaction should be checked frequently, with the appropriate equipment, during backfilling operations.

During this particular project, no deficiencies were observed in the assembly of the culvert plates or in the construction of the concrete foundations. Yet, construction of the soil fill and the base courses of asphalt could have collapsed the culvert.

Contractors, and construction inspectors, must be aware that following prudent procedures is imperative to building safe culvert installations. These personnel must understand that a culvert really acts to transfer loads to the backfill soil. Without the soil, the culvert shell would easily collapse. Put another way, a culvert is really a soil bridge with backfill integrity controlling structural stability. These ideas must be understood by the construction inspectors in the field.

**Design.** CANDE appears to be a powerful tool for the analysis of culvert designs. From the experience of this project, designers should consider the possible overloading of the culvert during pavement operations. Under shallow cover, attention should be paid to how a contractor plans to spread and compact the base courses of pavement. During this stage, the culvert is most vulnerable to overstressing. The concept of soil arching should be dropped in reference to box-type culverts under shallow fill. Under these conditions, the true nature of "positive" or "negative" soil arching is unclear and confusing.

**Further Research.** Foremost, laboratory testing of the deep-corrugated section is needed to establish the bending and thrust capacities. It is possible that the section will buckle prior to
attaining the theoretical plastic yield moment as computed from the yield strength of the steel. Work
is also needed to study the bending behavior and capacity of the welded haunch connection.

Long-term monitoring of the culvert would be useful to establish the behavior of the culvert
after numerous load cycles while in service. It is likely that a truly elastic response of the culvert
would eventually be observed.

CANDE remains a suitable tool for analyzing culverts. A working interface element would
probably yield better results. Also, a better, simpler technique is needed to model wheel loads for
plane strain analysis. An empirical model might be developed based on numerous finite element
runs.

Finally, pre- and post-processors would be most useful for facilitating these numerical
simulations. Since computer drafting systems are being used increasingly to prepare design drawings,
the most useful pre-processor would be able to generate the CANDE mesh from these drawings.
This would allow for the rapid analysis and evaluation of various design configurations.
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APPENDIX A

MEASURED BENDING MOMENTS AND THRUSTS
Measured Plate Forces — SYRO Culvert
Crown Location on Primary Cross Section

Bending Moment (kip-in / lineal inch)

Section #7

Height of Backfill (inch)

Axial Thrust (kip / lineal inch)

Top of crown
Top of sand
Top of asphalt

Height of Backfill (inch)
Measured Plate Forces -- SYRO Culvert
Symmetric Locations on Primary Cross Section

Bending Moment (kip-in / lineal inch)

Height of Backfill (inch)

Axial Thrust (kip / lineal inch)

Height of Backfill (inch)
Measured Plate Forces — SYRO Culvert

Symmetric Locations on Primary Cross Section

---

**Bending Moment (kip-in / lineal inch)**

- **Section #5**
- **Section #9**

**Height of Backfill (inch)**

---

**Axial Thrust (kip / lineal inch)**

- Top of crown
- Top of sand
- Top of asphalt

**Height of Backfill (inch)**
Measured Plate Forces — SYRO Culvert
Symmetric Locations on Primary Cross Section

- Bending Moment (kip-in / lineal inch)
- Axial Thrust (kip / lineal inch)

Sections #4 and #10 vs Height of Backfill (inch)
Measured Plate Forces -- SYRO Culvert

Symmetric Locations on Primary Cross Section

- Bending Moment (kip-in / lineal inch)
- Axial Thrust (kip / lineal inch)

Height of Backfill (inch)
Measured Plate Forces — SYRO Culvert
Symmetric Locations on Primary Cross Section

Graph 1: Pinned Moment (kip-in / lineal inch)
- Section #2
- Section #12

Graph 2: Axial Thrust (kip / lineal inch)
- Top of crown
- Top of sand
- Top of asphalt

Height of Backfill (inch)
Measured Plate Forces — SYRO Culvert
Symmetric Locations on Primary Cross Section

Bending Moment (kip-in. / lineal inch)

Section #1
Section #13

Height of Backfill (inch)

Axial Thrust (kip / lineal inch)

Top of crown
Top of sand
Top of asphalt

Height of Backfill (inch)
Measured Plate Forces — SYRO Culvert

42 kip Live Load at Position #1

- Bending Moment (kip-in / lineal inch)
- Axial Thrust (kip / lineal inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces — SYRO Culvert

42 kip Live Load at Position #2

Bending Moment (kip-in / linear inch)

Axial Thrust (kip / linear inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces — SYRO Culvert

42 kip Live Load at Position #3

Bending Moment (kip-in / lineal inch)

Axial Thrust (kip / lineal inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces — SYRO Culvert

42 kip Live Load at Position #4

Bending Moment (kip-in / lineal inch)

Axial Thrust (kip / lineal inch)
Measured Plate Forces -- SYRO Culvert

42 kip Live Load at Position #5

Bending Moment (kip-in / lineal inch)

Axial Thrust (kip / lineal inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces — SYRO Culvert

Live Loads at Position #1, Primary Cross Section

Bending Moment (kip-in / linear inch)

Circumferential Distance from North Foundation (inches)

Axial Thrust (kip / linear inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces -- SYRO Culvert

Live Loads at Position #2, Primary Cross Section

- Bending Moment (kip-in / lineal inch)
- Axial Thrust (kip / lineal inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces — SYRO Culvert

Live Loads at Position #3, Primary Cross Section

Bending Moment (kip-in / lineal inch)

Axial Thrust (kip / lineal inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces — SYRO Culvert

Live Loads at Position #4, Primary Cross Section

Bending Moment (kip-in / lineal inch)

Circumferential Distance from North Foundation (inches)

Axial Thrust (kip / lineal inch)

Circumferential Distance from North Foundation (inches)
Measured Plate Forces — SYRO Culvert

Live Loads at Position #5, Primary Cross Section

Circumferential Distance from North Foundation (inches)

Bending Moment (kip-in / lineal inch)

Axial Thrust (kip / lineal inch)

North haunch
Crown
South haunch
South found.
APPENDIX B

LABORATORY DATA FROM
TRIAXIAL AND MULTIAXIAL TESTS
ON BACKFILL SAND
CULVERT BACKFILL SAND
H.C. TEST # 3

(mean pressure, psi)

VOLUMETRIC STRAIN, %

AXIAL STRAIN, %

X AXIS RESPONSE
Y AXIS RESPONSE
Z AXIS RESPONSE

MEAN PRESSURE, psi

0.0 0.1 0.2 0.3 0.4

0 10 20 30 40 50 60 70 80 90

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

0 10 20 30 40 50 60 70 80 90

MEAN PRESSURE, psi

0 10 20 30 40 50 60 70 80 90

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

VOLUMETRIC STRAIN, %

MEAN PRESSURE, psi

0 10 20 30 40 50 60 70 80 90

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

VOLUMETRIC STRAIN, %

MEAN PRESSURE, psi

0 10 20 30 40 50 60 70 80 90

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

VOLUMETRIC STRAIN, %
CULVERT BACKFILL SAND
TRIAXIAL TEST # 2
7 psi confining pressure
5 psi confining pressure

DEVORR
STRESS (0.1 - 0.3) psi

AXIAL STRAIN %
VOLUMETRIC STRAIN %

compression

CULVERT BACKFILL SAND
TRIAXIAL TEST # 1
5 psi confining pressure

DEVORR
STRESS (0.1 - 0.3) psi

AXIAL STRAIN %
VOLUMETRIC STRAIN %

compression
CULVERT BACKFILL SAND
TRIAXIAL TEST # 3
10 psi confining pressure

CULVERT BACKFILL SAND
TRIAXIAL TEST # 4
15 psi confining pressure
CULVERT BACKFILL SAND
CTE TEST # 1
5 psi initial confining

CULVERT BACKFILL SAND
CTE TEST # 2
7 psi initial confining
CULVERT BACKFILL SAND

CTE TEST #3
10 psi initial confining

AXIAL STRAIN, %
EXPANSION
COMPRESSION

CULVERT BACKFILL SAND

RTC TEST #1
15 psi initial confining

AXIAL STRAIN, %
CULVERT BACKFILL SAND
RTC TEST # 2
20 psi initial confining

CULVERT BACKFILL SAND
RTC TEST # 3
25 psi initial confining
CULVERT BACKFILL SAND
RTE TEST # 1
10 psi confining pressure

CULVERT BACKFILL SAND
RTE TEST # 2
15 psi confining pressure
CULVERT BACKFILL SAND
RTE TEST # 3
15 psi confining pressure

CULVERT BACKFILL SAND
RTE TEST # 4
20 psi confining pressure
CULVERT BACKFILL SAND
SS TEST # 1
15 psi initial confining

CULVERT BACKFILL SAND
SS TEST # 2
17 psi initial confining
CULVERT BACKFILL SAND
TC TEST # 2
15 psi initial confining

CULVERT BACKFILL SAND
TC TEST # 3
20 psi initial confining
CULVERT BACKFILL SAND
TE TEST # 1
10 psi initial confining

CULVERT BACKFILL SAND
TE TEST # 2
15 psi initial confining
CULVERT BACKFILL SAND
TE TEST # 3
20 psi initial confining

CULVERT BACKFILL SAND
PL TEST # 1
b=0.2, inter. stress =10 psi
CULVERT BACKFILL SAND
PL TEST # 2
b=0.8, inter. stress=10 psi

OCTAHEDRAL SHEAR STRESS, psi

AXIAL STRAIN, %

EXPANSION

COMPRESSION

○ X AXIS RESPONSE
△ Y AXIS RESPONSE
□ Z AXIS RESPONSE