THE TRANSIENT RESPONSE OF A CAM-AND-FOLOWER SYSTEM
WITH UNFORMLY VARYING FREQUENCY OF EXCITATION/

A Thesis Presented to
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Degree of Master of Science

by
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ABSTRACT

The purpose of this paper is to find the transient response of a cam-and-follower system with uniformly varying frequency of excitation for four different classical cam profiles as input functions. Using computer programming, the results of the follower displacements and accelerations are listed in nondimensional form as $S_{WD}$ and $S_{WA}$ respectively. The Figures of $S_{WD}$ and $S_{WA}$ are presented as a way to compare cam performance and to serve as an aid in design.
ACKNOWLEDGMENTS

The author wishes to express sincere thanks to Dr. Chen for his guidance and Dr. Adams for his advise and help in carrying out this paper.
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Chapter I

NOMENCLATURE

The following symbols are used in this paper:

\( c \)  viscous damping coefficient
\( k \)  spring's stiffness
\( m \)  follower's mass
\( \omega_n \)  natural angular frequency
\( \zeta \)  damping ratio equals \( c/2m/\omega_n \)
\( T_1 \)  input function duration
\( T_n \)  natural period
\( H \)  total cam lift
\( \beta \)  angular displacement of cam shaft to achieve cam lift \( H \)
\( y \)  cam input function
\( \text{ABS} \)  absolute value

[i]  number in bracket refers to the reference cited.

ARPA  angular acceleration of the cam shaft
SNA  non-dimensional acceleration at the follower of a cam-and-follower system
SND  non-dimensional displacement of the follower of a cam-and-follower system
NAAF  normalized acceleration amplification factor
Chapter II
INTRODUCTION

Researchers in recent years have examined a number of cam profiles of the dwell-rise-dwell type with regard to the important vibrational response characteristics. An excitation consisting of a constant camshaft angular acceleration is assumed for a single degree of freedom system and a second order differential equation results for the cam-and-follower system. The one basic assumption made is that the residual vibration is damped out during the dwell period. This paper examines and compares four different cam profiles as input functions to the system under a uniformly varying frequency of excitation. The comparison of the dynamic response of the cam-and-follower system with the uniformly varying frequency of excitation of a cam will be assessed by a nondimensional quantity called normalized acceleration amplification factor (NAAF).

The dynamic characteristic of the cam-and-follower system with dwell-rise-dwell type cam (see Appendix pp. 45) operating at high speed has been investigated for years through the application of advanced numerical methods and large storage computers. The cam-and-follower system cannot run at high speed until it passes through a period of
acceleration. By starting the engine or by suddenly applying the accelerator of a car one finds that the engine speed increase from zero to more than 2000 RPM. When accelerating, from idle speed to around 3000 RPM in less than a half second, the acceleration is high and cannot be ignored. In this paper we will investigate the dynamic response of the follower driven by a dwell-rise-dwell cam under high constant camshaft acceleration using the nondimensional term of NAAF to compare the results with regard to excessive vibration and acceleration performance.

W.J. Strange in his paper [4] 'Vibration due to an Uniformly Varying Frequency of Excitation', uses a physical model slightly different from ours for which the dwell period of the excitation function did not exist and with the important assumption that the damping force was large enough to damp out the residual vibration during the dwell period.

We can obtain a closed form solution for the problem of a car (the follower system) running on a SINE wave road (the input function) using Strong's model and a convolution integral and Fresnel's integral. The author pointed out that the effect of an accelerating forcing function is to increase the resonant frequency and reduce the amplitude of the response envelope in comparison with
constant speed input function. His results were verified by computer calculations and are listed in Table 1. However, the acceleration of the follower in the up and down direction is more important than the consideration of follower displacement. Due to the severity of the dynamic loads expressed in terms of acceleration, excessive vibration and impact results and subsequent wear of cam profiles would occur. The dynamic response characteristic of the follower will be emphasized in the form of the normalized acceleration amplification factor (NAAF). The computer program for calculating the value of NAAF can be seen in the Appendix. Any cam profile can be converted into FORTRAN language and put into the SUBROUTINE program. The nondimensional dynamic response of the follower can be plotted as a function of the ratio of input period to natural period.

A card computer program deck was offered by Dr. Chen as a test program to solve and test the cam-and-follower system under uniformly varying frequency of excitation. The following method was used to test the program and is listed as follows:

1. Solve the previously solved problem for the Dynamic Response of a cam-and-follower system due to the excitation of cam shaft operation at cam-and-follower system due to a constant speed camshaft, and compare the results with my analysis.
II. After the first part was proved, make a comparison between the results of Stronge's and mine.

III. Check the results to see whether at $T_1/T_N >> 1$ the relative displacement between the follower and cam approaches zero. This can be explained in one of the following two ways. The first explanation is that for a given spring-mass-and-damper system $T_N$ is fixed and $T_1/T_N >> 1$ means that $T_1$ is large and the cam shaft takes a longer time to complete a cycle. This implies that the acceleration of the cam shaft is comparatively slow and under very slow excitations we can not expect high relative displacements between cam and follower when $T_1/T_N >> 1$. The other explanation is that for a given input function under the fixed acceleration of the cam shaft $T_1=constant$. Under this situation, $T_1/T_N >> 1$ means that $T_N$ is short. For a system with a short natural period one of the following situations should result, either the mass of the system is less or the spring constant is high, so as to make a hard spring. There will be no relative displacement if we put a rigid body (very hard spring) between the cam and the follower. This phenomenon can help us in checking the computer outputs and if the relative displacements approach zero when $T_1/T_N$ is much greater than 1, then we can say that the physical model and equation of motion probably are correct.
Chapter III
DYNAMIC ANALYSIS OF THE CAM-AND-FOLLOWER SYSTEM

3.1 Physical Model and Formulation

The model of the cam-and-follower system is shown as a linear-single-degree of freedom system with spring, damper and mass. The mass of the spring and damper and point A of Fig. 2 are assumed negligible in comparison with the mass M of Fig. 2. Damping in the follower system is represented by the damping coefficient c. It is also assumed that point A of Fig. 2 is in contact with the cam during the entire event and is free of backlash. We also assumed cam profile is not changed by the force acting on it and the cam support is rigid and free of wind-up and flexibility. Furthermore, it is assumed that the damping force of the system is large enough to damp out the residual vibration during the dwell period. If this were not true the resonant phenomena would occur and the system would be more complicated and difficult to analyze.

In Fig. 2, y(t) represents the input function due to the different cam profiles then the system equation can be derived and based on relative coordinates as following:
In Figure 3, assume \( y > x \)

Based on relative coordinate

\[ Z = y - x; \; \ddot{Z} = \dot{y} - \dot{x}; \; \dddot{Z} = \ddot{y} - \ddot{x} \]

From Fig. 3 we can see

\[ m \dddot{Z} = c(\dot{y} - \dot{x}) + k(y - x) \quad \cdots \quad (1) \]

where \( \dddot{Z} = \ddot{y} - \ddot{x} \)

Then Equation (1) becomes

\[ m \dddot{Z} = -cZ - kZ = 0 \quad \cdots \quad (2) \]

Equation (2) divided by \( m \) on both sides,

\[ \dddot{Z} + \frac{c}{m} Z + \frac{k}{m} Z = \dot{y} \quad \cdots \quad (3) \]

These can be expressed in state-variable form as

\[
\begin{align*}
\dot{Z} &= -\frac{c}{m} Z - \frac{k}{m} Z = v \\
\dot{v} &= -2 \zeta Wn Z - 2 Wn Z + \dddot{y}(t) \quad \cdots \quad (4)
\end{align*}
\]

where \( Wn \) is the natural frequency of the system of Fig. 2

\( \zeta \) is the damping ratio. We assume \( \zeta = 0.0, 0.05, 0.1, 0.2 \) and 0.3.

Equation (4) was solved by computer programming using the
Runge-Kutta method to obtain the transient response for each time period ranging from 0 to 2T1.

A flow chart of the computer program can be seen in the appendix pp. 43.
Input function of four different cam profiles in the computer program were normalized by dividing the input function by $T_1$. This means that when $t$ went from 0 to $T_1$, the cam shaft traveled only one stroke and during this stroke the normalized follower displacement was a unit.

We already have assumed that during the dwell period the damping force is large enough to damp out the residual vibration. Therefore, the initial condition for the next cycle will be the same as for the first cycle. Moreover, the angular acceleration is constant throughout the entire event. These conditions provide a common basis for comparison of the four cam profiles. Any different responses among the four cam profiles will result from the characteristics of the cam profiles themselves.

3.2 Definition of SND and SNA

In order to provide a common basis for comparison, the dynamic responses of the follower are normalized. The normalized follower displacement relative to cam profiles is defined as

$$\text{SND} = \frac{\text{ABS}(Z_{\text{MAX}})}{H}$$

where $\text{ABS}(Z_{\text{MAX}})$: is the absolute maximum value of $Z$ occurring during primary and residual vibration. $Z$: is the relative displacements of
follower (see Fig. 2)

The normalized response in acceleration is defined as:

\[ SNA = \left( \frac{Wn^2 Z_{MAX}}{\ddot{Y}_{MAX}} \right) \]

where \( SNA \): may be called NAAF.

\( \ddot{Y}_{MAX} \): maximum value of input function \( \ddot{Y}(t) \).

From the flow chart we can see that absolute maximum value of \( Z \) is obtained by setting \( SND = \text{ABS}(Z_{MAX}) / H \) its normalized value. \( SNA \) is obtained by setting \( SNA = \left( \frac{Wn^2 Z_{MAX}}{\ddot{Y}_{MAX}} \right) \). \( Z \) and \( Z \) of the follower response occur during the period of \( 0 < t < t_1 \) of the excitation function. The resulting follower response is referred to as a forced vibration or primary vibration. The response during \( T_1 < t < 2T_1 \), when the excitational function is not acting, is called the follower response of residual vibration or free vibration. (input function not acting means that equation (3) equals zero)

3.3 Input Functions
The following four different dwell-rise-dwell type cam profiles have been selected as input functions.

Classical cam profiles or trigonometric function types:

1. Simple Harmonic.

2. Cycloidal

3. Modified Trapezoidal

4. 4-5-6-7 degree polynomial.

The reason for these four cam profiles is that, the characteristics of the above four cam profiles for constant cam shaft angular velocity are well known and have been investigated for years. The technique in inventing modern cam profiles is in synthesizing the basic cam profiles. This approach is simplified, so the results of this paper can be compared with the constant angular velocity case and serve as a reference paper for the researcher in this field.

The simple harmonic function expressed in terms of the instantaneous angular displacement of the cam shaft is:
\[ y(\theta) = \frac{H}{2} \left( 1 - \cos \frac{\pi \theta}{\beta} \right) \ldots (1) \]

\( y(\theta) \): is cam lift when cam shaft has
instantaneous angular displacement.

\( H \): is the total cam lift.

\( \beta \): is the angular displacement to
achieve the cam lift \( H \).

We assume the initial condition to be when
\[ \theta = 0; \quad y(\theta) = 0; \]

when
\[ \dot{\theta} = 0; \quad \dot{y}(\theta) = 0; \]

where dot (\( . \)) means the derivative with respect to
time \( t \). Under constant angular acceleration or so called
uniformly varying frequency of excitation, angular
displacement can be expressed in terms of angular
acceleration and time \( t \) as \( \theta = \frac{1}{2} \alpha t^2 \) when the initial
conditions are assumed. Equation (1) can be expressed as,

\[ y(t) = \frac{H}{2} \left( 1 - \cos \frac{\frac{1}{2} \alpha t^2}{\beta} \right) \ldots (2) \]

By assuming \( \beta = \frac{1}{2} \alpha T^2 \) equation (2) can be
normalized, where \( T \) is the period of the excitational
function.

\[ y(t) = \frac{H}{2} \left( 1 - \cos \frac{\frac{\alpha t^2}{T^2}}{\frac{1}{2} \alpha T^2} \right) \ldots (3) \]
Taking the derivation with respect to \( t \) twice,

\[
\ddot{y}(t) = \frac{2Hn^2}{T^4} \cos\left(\frac{\pi t^2}{T^2}\right) \quad \cdots \quad (4)
\]

Equation (4) is then substituted into the SUBROUTINE of the computer program. A set of input data for this computer program is \( m = 0.00329 \text{ lb.-sec/inch} \), \( k = 16,050 \text{ lb./in.} \), \( H = 0.5 \text{ inch} \), which was ranging from 0.0 to 0.3 and \( \ddot{y}_{\text{MAX}} \) were from Table 1 of Dr. Chen's paper 'Assessment of the Dynamic Quality of a Class of Dwell-Rise-Dwell Cams' [1].
Chapter IV

RESULTS OF DYNAMIC RESPONSE

SND and SNA are plotted versus T1/Tn in Fig. 4-7 and Fig. 8-11. SND is the normalized amplification of displacement factor. SNA is called normalized amplification of acceleration factor (NAAF). Fig. 4-7 and Fig. 8-11 shows the SND and SNA for four different cams. Solid lines represent primary dynamic response of the follower system and dashed line stand for the residual dynamic response. A primary response is defined, for the case of acceleration, as the maximum absolute acceleration response of the system when the input excitation is applied and the residual response is defined as the highest absolute acceleration response of the system after the applied input function had been terminated. The primary and residual response for SND is defined in the same way as SNA. Each value (or point) of SNA in Fig. 8-11 is actually the absolute maximum absolute acceleration response during the period of primary vibration (0<t<T1) and the maximum residual response during the period of residual vibration. T1 range from 0.05 times the natural period TN of the system to 15.05 times TN with the increment 0.05*TN. In the computer program we set DO LOOPS to take care of the increment of T1 and for each
value of $T_1$ the computer used Runge-Kutta method to solve the differential equation from $t=0$ to $t=T_1$. The maximum value of the response during this period was determined and then the response was converted into one value of SND and SNA. For each increment of $T_1$ we will have one value of SND and SNA until $T_1=15.05*TN$ at the time three hundred values of SND and SNA are obtained. The computer outputs can be seen in the Appendix.

The purpose of having Fig. 8-11 and Fig. 12-17 is to compare the characteristic of each cam profile under accelerations. For simplicity zero damping was assigned in the computer program. We shall see the effect of damping on the dynamic response in the last section of this chapter. We are going to discuss each of the cam profiles separately in the following.

4.1 Simple Harmonic

In Fig. 8, the simple harmonic curves have a peak acceleration response of magnitude $SNA=4.880$ at $T_1/TN=1.550$ and has maximum displacement response of magnitude $SND=0.451$ at $T_1/TN=0.150$. In Table 1 the maximum residual value of SNA for a simple harmonic motion cam profile is the lowest among the different four cam profiles. However, from Fig. 8 we can see that simple
harmonic cam profile has persistant residual vibration of $SNA=4.02$ up to $T1/TN=15.05$. This means that we are going to expect excessive vibration on the follower during dwell period when the cam-and-follower system is under uniformly varying frequency of excitation. The result makes the simple harmonic cam profile undesirable for a cam-and-follower system with constant angular acceleration. $SND$ is the normalized relative displacement between the cam profile and follower system. The higher the $SND$ the higher the inaccuracy of the cam-and-follower system. As far as precision is concerned, the simple harmonic cam profile is not recommended for a cam-and-follower system under constant angular acceleration because the simple harmonic cam has the highest persistant $SND$ which occurred at $T1/TN=15.05$.

4.2 Modified Trapezoidal

Modified Trapezoidal was developed by Nuklutin based upon an intuitive observation and result from actual machine tests [1]. Before researchers could apply advanced numerical analysis using high storage computers to analyze cam-and-follower systems, Nuklutin's method of analysis was quite practical. However, Fig. 10 shows that the response of the modified trapezoidal cam is neither a smooth curve nor does the $SNA$ curve approach zero at
$T_1/T_N > 0$. It has the highest peak $S_{WA}$ which is equal to 8.61 at $T_1/T_N=2.800$ for primary vibration and $S_{WA}=6.80$ at $T_1/T_N=2.81$ for residual vibration. We do have some low values of $S_{WA}$ in Fig. 10, but any attempt to pin down the values for element of cam and follower system at local lowest values is not easy. Since the friction force between machine components which results from, for example, lubricant, changes the machine speed and the friction force is difficult to predict. Due to the unpredictable factors of application we might achieve the local highest $S_{WA}$ just next to the local lowest $S_{WA}$. Therefore, the modified trapezoidal cam profile is not an ideal cam profile for a uniformly varying frequency of excitation.

4.3 Cycloidal

The cycloidal cam profile has peak values of $S_{WA}=6.26$ for primary case at $T_1/T_N=1.75$ and $S_{WA}=6.24$ for residual vibration at $T_1/T_N=1.70$. Each value of $S_{WA}$ is higher than that of simple harmonic but the rate of decline $S_{WA}$ for residual vibration is so high that at $T_1/T_N = 15.05$ $S_{WA}$ equals only 0.54. This result makes cycloidal cam profiles a possible ideal one for constant angular acceleration. Fig. 5 shows the peak value of $S_{HD}$ for different cam profiles. Dashed lines represent residual vibration and
solid lines stand for primary vibration.

4.4 4-5-6-7 Polynomial

As far as these four cam profiles are concerned, the 4-5-6-7 cam profiles is highly recommended for the constant angular acceleration input function. Although the peak value of SNA for primary case is 6.21 and 6.24 for residual vibration which are higher than that of a simple harmonic cam, it's residual vibration declines so rapidly that at \( \frac{T1}{Tn} = 15.05 \), SNA is equal to only 0.198 (Fig. 11). That means under constant angular acceleration will the follower will not suffer high values of SNA when the cam-and-follower system reaches high angular velocities. SND for residual vibration equal to 0.000167 at \( \frac{T1}{TN} = 15.05 \) this also tells us that the 4-5-6-7 polynomial cam profile is an acceptable cam profile under uniformly varying frequencies of excitation.

4.5 Dynamic Response Due to Different Degrees of Damping Factors:

Damping is the most difficult parameter to pin down in dynamic analysis. Surface condition of contact surfaces, lubrication condition between constituent components, and operating speed all affect the value of
damping. In practical design, damping is established by experiment. In this paper damping factors =0.0,0.05,1.0,2.0 and 3.0 were used to see the effect of the damping factor on primary response and residual response. Cycloidal and 4-5-6-7 polynomial cam profiles were chosen for different degree of damping factors due to their better performance when operating at a constant angular acceleration.

The response for different degrees of damping can be seen in Fig. 14. The peak value for the primary case due to increasing damping shows a shift to the right of T1/TN when the damping factor is increasing. (Fig. 14 primary case).

We do not see the peak value shifting to the right for the residual vibration when the damping factor is increasing but the peak value decreases with increasing damping factors. 'Ripple' lines can be detected in Fig. 14 and Fig. 16 with increasing damping factor.
Chapter V
CONCLUSION

This paper provides a quantitative comparisons of four classical cam profiles under uniformly varying frequency of excitation. The analysis is based on a single-degree-of-freedom physical model with a second order differential equation and using computer programming to solve the equation. We can either analyze all the of existing cam profiles the way this paper did and pin down a few cam profiles which have the best performance in uniformly varying frequency of excitation or we can analyze only those basic cam profiles and find new profiles by synthesizing the basic cam profiles. Before this latter step can be done the following should be mentioned:

I. Of the four cam profiles analyzed, 4-5-6-7 polynomial cam with $S_{NA} = 0.198$ at $T_1/T_N = 115.05$ of residual vibration is the lowest response among the four cam profiles.

II. With the trigonometric term in cam profiles such as in the case of the simple harmonic and modified trapezoidal cam, the residual vibration does not disappear but maintains a high value of $S_{NA}$ during the dwell period. Therefore, simple harmonic and modified trapezoidal cams
are not recommended for service at constant angular acceleration.

III. We can not see the touch downs (zero SNA) of residual vibration due to the high constant speed excitation. (as we saw in Dr. Chen's paper) [1]. This means that in accelerating the follower will have a reduced value of residual vibration in the dwell period with increasing $T_1/T_2$. However, zero SNA will not happen at all the input frequencies.

IV. From Table 1 we can see that the maximum value of SNA for constant angular acceleration is higher than that of constant angular velocity. Moreover, ideal cam profiles for constant speed might not be as perfect in constant angular acceleration. Using the value of SNA of constant velocity in designing constant acceleration problem will result in out-of-safty-bound design.

V. A test program (which can be seen in appendix), must mention the CPU time. It took around 100 seconds to obtain a cam profile with the damping ranging from 0.0 to 0.3. This can be improved by using a modified Runge-Kutta method which will adjust time interval automatically by checking an error bound set.
FIGURE 4

4-5-6-7 POLYNOMIAL

FIGURE 5

CYCLOIDAL

FIGURE 6

SIMPLE HARMONIC

FIGURE 7

MODIFIED TRAPEZOIDAL
FIGURE 8

SIMPLE HARMONIC

FIGURE 9

CYCLOIDAL
RESIDUAL RESPONSE
4-5-6-7 POLYNOMIAL

PRIMARY RESPONSE
4-5-6-7 POLYNOMIAL
FIGURE 16
PRIMARY RESPONSE
4-5-6-7 POLYNOMIAL

FIGURE 17
RESIDUAL RESPONSE
4-5-6-7 POLYNOMIAL
### PRIMARY RESPONSE

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<th>ARFA = Cont.</th>
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<td>T1/TN</td>
<td>SND</td>
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<td>SIMPLE HARMONIC</td>
<td>0.100</td>
<td>0.921</td>
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<td>CYCLOIDAL PROFILE</td>
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<td>0.947</td>
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<tr>
<td>MODIFIED TRAPEZOIDAL</td>
<td>0.115</td>
<td>0.933</td>
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<tr>
<td>4-5-6-7 POLYNOMIAL</td>
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<td>4-5-6-7 POLYNOMIAL</td>
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**TABLE I**
RESIDUAL RESPONSE

<table>
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<th>Cam Profile</th>
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<td>T1/TN</td>
<td>SND</td>
<td>T1/TN</td>
<td>SND</td>
</tr>
<tr>
<td>Simple Harmonic</td>
<td>0.300</td>
<td>0.975</td>
<td>0.200</td>
<td>1.590</td>
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<td>Cycloidal Profile</td>
<td>0.350</td>
<td>0.924</td>
<td>0.100</td>
<td>1.560</td>
</tr>
<tr>
<td>Modified Trapezoidal</td>
<td>0.350</td>
<td>0.890</td>
<td>0.200</td>
<td>1.670</td>
</tr>
<tr>
<td>4-5-6-7 Polynomial</td>
<td>0.350</td>
<td>0.934</td>
<td>0.100</td>
<td>1.750</td>
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</table>

<table>
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<tr>
<th>Cam Profile</th>
<th>W = Cont.</th>
<th>ARFA = Cont.</th>
<th></th>
<th></th>
</tr>
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<tr>
<td></td>
<td>T1/TN</td>
<td>SNA</td>
<td>T1/TN</td>
<td>SNA</td>
</tr>
<tr>
<td>Simple Harmonic</td>
<td>0.900</td>
<td>2.760</td>
<td>1.600</td>
<td>5.230</td>
</tr>
<tr>
<td>Cycloidal Profile</td>
<td>1.150</td>
<td>3.240</td>
<td>1.700</td>
<td>6.270</td>
</tr>
<tr>
<td>Modified Trapezoidal</td>
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<td>3.570</td>
<td>2.250</td>
<td>8.010</td>
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<tr>
<td>4-5-6-7 Polynomial</td>
<td>1.250</td>
<td>3.200</td>
<td>1.850</td>
<td>6.240</td>
</tr>
</tbody>
</table>

**Table 2**
REFERENCE
REPERBICE


5. A Midha, 'Periodic Response of High Speed Cam Mechanism with Flexible Follower and Camshaft Using a Closed-Form Numerical Algorithm," ASME.

APPENDICES
APPENDIX

Input functions for Cycloidal, Modified Trapezoidal, and 4-5-6-7 Polynomial cam profiles.

Cycloidal:

\[ \ddot{y}(t) = \frac{8H\pi t^2}{T_1^4} \sin\left(\frac{2\pi t^2}{T_1^2}\right) \]

Modified Trapezoidal:

1. For \( 0 < t < \frac{T_1}{8} \)

\[ \ddot{y}(t) = 19.552496 \frac{Ht^2}{T_1^4} \sin\left(\frac{4\pi t^2}{T_1^2}\right) \]

2. For \( \frac{T_1}{8} < t < \frac{3T_1}{8} \)

\[ \ddot{y}(t) = 19.552496 \frac{Ht^2}{T_1^4} \]

3. For \( \frac{3T_1}{8} < t < \frac{T_1}{2} \)

\[ \ddot{y}(t) = 19.552496Ht^2 \frac{\sin\left(\frac{4\pi t^2}{T_1^2} - \pi\right)}{T_1^4} \]

4. For \( \frac{T_1}{2} < t < \frac{6T_1}{8} \)

\[ \ddot{y}(t) = -\frac{19.552496Ht^2}{T_1^4} \sin\left(\frac{4\pi t^2}{T_1^2} - 2\pi\right) \]
\[ \ddot{y}(t) = -\frac{19.552496Ht^2}{T_1^4} \sin\left(\frac{4\pi t^2}{T_1^2} - 2\pi \right) \]

5. For \( \frac{6T_1}{8} < t < \frac{7T_1}{8} \)

\[ \ddot{y}(t) = -\frac{19.552496Ht^2}{T_1^4} \]

6. For \( \frac{7T_1}{8} < t < T_1 \)

\[ \ddot{y}(t) = -\frac{19.552496Ht^2}{T_1^4} \sin\left(\frac{4\pi t^2}{T_1^2} - 3\pi \right) \]

4-5-6-7 Polynomial:

\[ \ddot{y}(t) = 1680 \frac{Ht^2}{T_1^4} \left( R^4 - 4R^6 + 5R^8 - 2R^{10} \right) \]

where \( R = \frac{t}{T_1} \)
FLOW CHART

START

READ $H, E$ (DAMPING FACTOR)
MASS, $K, \dot{Y}_{\text{MAX}}, \ddot{Y}_{\text{MAX}}$

$B1 \rightarrow N = 0 \rightarrow B$

$N = N + 1$
$T1 = 0.1 + (N-1) \times 0.05$

INITIAL CONDITION
$T = 0.0, \dot{Z} = 0.0$
$\ddot{T} = 0.0, \ddot{Z} = 0.0$

USING RUNGE-KUTTA METHOD TO SOLVE

$\ddot{Z} + 2E \omega_n \dot{Z} + \omega_n^2 Z = \ddot{Y}(t)$

2 SETS OF DATA ARE OBTAINED DURING THE PERIOD OF
(1) $0 < t < T1$ (PRIMARY)
(2) $T1 < t < 2T1$ (RESIDUAL)

FIND THE MAXIMUM OF $Z$
IN LAST STEP

$A$
LET SND = ABS(ZMAX)/H 
DURING PERIOD (1) AND (2)

BY CONVOLUTION INTEGRAL 
FIND SNV AND SNA 

IF N > 300 
NO → B1 
YES → PRINT T, SND, SNV, SNA 

IF E > 0.3 
NO → C 
YES → STOP 
END
Dwell-Rise-Dwell Cam Profile
**MAIN PROGRAM**

```plaintext
REAL K0, K1, K2, K3, M0, M1, M2, M3
DIMENSION TR(300)
DIMENSION PHI(300), RES(300), Y(2), DATA(2, 2000)
DIMENSION PDISP(300), PVEL(300), PACC(300)
DIMENSION RDISP(300), RVEL(300), RACC(300)
COMMON DAMP, WN, PI, TRK, T1, T, DLTAT

READ (5, 900) DAMP, H, XM, VMAX, AMAX
WRITE (6, 880)
WRITE (6, 890) DAMP
NUMB = 300
WN = SQRT(XM/XM)
PI = 3.1415926
DLTAT = 0.0001
TN = 2.*PI/WM
DO 140 M = 1, NUMB
NT = 1
TR(N) = 0.1 + (N-1)*0.05
T1 = TR(N)*TN
XST = 1.0
TLAST = 2.*T1
COUNT = 2.*T1/DLTAT
ICT = COUNT
MCT = ICT/2 + 1
NCT = MCT + 1
DO 10 I = 1, 2
Y(I) = 0.0
DO 10 J = 1, 2000
10 DATA(I, J) = 0.0
T = 0.0
```

**RUNGE-KUTTA METHOD WITH TWO VARIABLES**

```plaintext
M0 = Y(2)
TRK = T
CALL FCT(Y(1), Y(2), K0)
M1 = Y(2) + K0*DLTAT/2.
X1 = Y(1) + M0*DLTAT/2.
X2 = Y(2) + K0*DLTAT/2.
TRK = T + DLTAT/2.
CALL FCT(X1, X2, K1)
M2 = Y(2) + K1*DLTAT/2.
X1 = Y(1) + M1*DLTAT/2.
X2 = Y(2) + K1*DLTAT/2.
TRK = T + DLTAT/2.
CALL FCT(X1, X2, K2)
```
M3 = Y(2) + K2*DLTAT
X1 = Y(1) + M2*DLTAT
X2 = Y(2) + K2*DLTAT
TBK = T + DLTAT
CALL FCT(X1, X2, K3)
Y(1) = Y(1) + (M0+2.*M1+2.*M2+M3)/6.*DLTAT
Y(2) = Y(2) + (K0+2.*K1+2.*K2+K3)/6.*DLTAT
IF(INT-MCT) 80, 80, 90
80 DATA(1, INT) = ABS(Y(1)/XST)
GO TO 100
90 DATA(2, INT) = ABS(Y(1)/XST)
100 CONTINUE
INT = INT + 1
T = T + DLTAT
IF(T-TLAST) 20, 20, 70
70 CONTINUE
PBI(N) = DATA(1, I)
DO 110 I = 1, MCT
IF(PBI(N)-DATA(1, I)) 120, 110, 110
120 PBI(N) = DATA(1, I)
110 CONTINUE
RES(N) = DATA(2, NCT)
DO 140 I = NCT, ICT
IF(RES(N)-DATA(2, I)) 130, 140, 140
130 RES(N) = DATA(2, I)
140 CONTINUE
DO 800 N = 1, 300
C******************************************************************************
C                        *                        *
C NEXT SIX STATEMENTS DEFINE SND, SNV,      *                        *
C AND SNA.                  *                        *
C******************************************************************************
PDISP(N) = PRI(N)/H
RDISP(N) = RES(N)/H
PVEL(N) = PRI(N)*WN*TN*TR(N)/H/VMAX
RVEL(N) = RES(N)*WN*TN*TR(N)/H/VMAX
PACC(N) = PRI(N)*WN*WN*(TN*TR(N))*2/H/AMAX
RACC(N) = RES(N)*WN*WN*(TN*TR(N))*2/H/AMAX
800 CONTINUE
WRITE(6, 920)
WRITE(6, 910)
DO 850 N = 1, 300
850 WRITE(6, 950) TR(N), PDISP(N), PVEL(N), PACC(N)
CALL FLOT(TR, PDISP, PVEL, PACC, NUMB)
WRITE(6, 880)
WRITE(6, 890) DAMP
WRITE(6, 940)
WRITE(6, 910)
DO 855 N = 1, 300
855 WRITE(6, 950) TR(N), RDISP(N), RVEL(N), RACC(N)
CALL FLOT(TR, RDISP, RVEL, RACC, NUMB)
IF (DAMP - 0.3) 555, 556, 556
555 GO TO 333
556 STOP
880 FORMAT(1H1,'5X, 'NORMALIZED DATA FOR HARMONIC CASE')
890 FORMAT(5X,'DAMPING FACTOR = ',F6.3)
900 FORMAT(6F12.6)
910 FORMAT(12X,'T/T1, TN, W, PI, T, T1, TIME, DLTAT')
A = -2.*DAMP*WN*X2-X1*WN**2
R = T/T1
IF(T-T1) 15, 15, 16
15 ZZ = (H*2.+R*R*PI*PI/(T1*T1)) * COS(PI*R*B)
S = A + ZZ
GO TO 10
16 S = A
10 RETURN
END
C*****************************************************************************
C SUBROUTINE FOR INPUT FUNCTIONS, THE FOLLOWING IS THE EXAMPLE FOR SIMPLE
C HARMONIC FUNCTION
C*****************************************************************************
SUBROUTINE PLOT(X1,X2,S)
COMMON DAMP,WN,H,PI,T,T1,TIME,DLTAT
A = -2.*DAMP*WN*X2-X1*WN**2
R = T/T1
IF(T-T1) 15, 15, 16
15 ZZ = (H*2.+R*R*PI*PI/(T1*T1)) * COS(PI*R*B)
S = A + ZZ
GO TO 10
16 S = A
10 RETURN
END
C*****************************************************************************
C SUBROUTINE FOR PLOTTING '.' REPRESENTS
C SND, '+' REPRESENTS SNV AND '*' PEPRE-
C SENTS SNA.
C*****************************************************************************
SUBROUTINE PLOT(T,A,B,C,N)
COMMON S(300)
DATA ELANK, DOT, PL, ST/ ',', ',', ',', '+' /
A_MAX = A(1)
A_MIN = A(1)
DO 111 I = 2, N
1 IF(A(I) - A_MAX) 2, 2, 3
3 A_MAX = A(I)
2 IF(A(I) - A_MIN) 4, 5, 5
4 A_MIN = A(I)
5 IF(B(I) - B_MAX) 6, 6, 7
6 B_MAX = B(I)
7 B_MIN = B(I)
6 IF(B(I) - BMIN) 8, 9, 9
8 BMIN=B(I)
9 IF(C(I) - CMAX) 10, 10, 11
11 CMAX=C(I)
10 IF(C(I) - CMIN) 12, 111, 111
12 CMIN=C(I)
111 CONTINUE
   DO 13 I=1,N
    NA=(A(I) - AMIN) * 110. / (AMAX - AMIN) + 1.5
    NB=(B(I) - BMIN) * 110. / (BMAX - BMIN) + 1.5
    NC=(C(I) - CMIN) * 110. / (CMAX - CMIN) + 1.5
   DO 14 L=1,112
14 S(L)=BLANK
   S(NA)=DOT
   S(NB)=PL
   S(NC)=ST
13 WRITE(6, 15) T(I), (S(III), III=1, 112)
15 FORMAT(1X, E10.5, 5X, 112A1)
   RETURN
END