PREDICTIVE METHODS

FOR

SUBSIDENCE DUE TO LONGWALL MINING

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by

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To Mom, Dad, and Li-chu
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CHAPTER 1

INTRODUCTION

1.1 PURPOSE OF STUDY

The purpose of this investigation is to determine the most reliable method for predicting subsidence due to longwall mining. The method can then be used to assess the damages to surface structures if mining occurs underneath. If damages to structures are expected to be substantial, so that mining underground is not feasible, then the predicted subsidence can be used to estimate the safe stopping distance of the longwall face.

1.2 BACKGROUND

Longwall mining is the preferred method for removing underground coal since a larger amount of coal can be removed faster and with less waste when compared to the room-and-pillar type mining. Thus, the coal tonnage removed each year by longwall mining methods has increased dramatically (31)*. However, a high extraction rate

* Numbers in parentheses indicate references.
results in surface subsidence which has adverse effects on the environment and on surface structures. To prevent damages to surface structures, one must be able to predict the magnitude of subsidence and the area which is affected. Also for the case when damages are allowed to occur, predictions of structural damage are also dependent on the magnitude of the expected horizontal strains.

In general, there are three methods for predicting subsidence characteristics: the graphical method, the profile method, and the finite element method. All three methods can be used to predict ground movements and horizontal strains. However, the finite element method, which takes into account the structural properties of the stratigraphy and equilibrium of the overburden, is theoretically the more logical choice for use in subsidence prediction.

1.3 SCOPE OF STUDY

In this study an investigation was conducted to predict the subsidence caused by longwall mining in Southeastern Ohio. Measurements were taken along S.R. 556 in Monroe County and S.R. 689 in Meigs County, Ohio. The
methods examined for predicting subsidence characteristics were the graphical method, the profile method, and the finite element method.

The graphical method, was developed by the National Coal Board (NCB) of Great Britain. This method was selected for comparison since it is perhaps the best known and most widely used method in the world.

The profile function used was developed by Peng and Chiang (24). This function resulting from numerous subsidence investigations conducted in the Northern Appalachian coal fields. Peng's profile function, with empirical constants, was used in this study since the stratigraphy was expected to be similar in this region.

The finite element analysis was accomplished through the use of the finite element program SEQCON developed at the Virginia Polytechnical Institute and State University. SEQCON was chosen for this investigation because of its ability to model excavation sequences. In addition, this program also offers constitutive models that are highly suitable for modeling geologic materials.
Two characteristics of the predicted subsidence profile were investigated. These were the subsidence profile and the resulting horizontal strains. The prediction accuracies, of all methods, were determined by comparing with field measurements.

1.4 LITERATURE REVIEW OF FINITE ELEMENT MODELS AND LABORATORY SIMULATION

1.4.1 Sequential Construction Finite Element Approach

Sequential Construction (SEQCON) finite element program used in this investigation was the result of a research done by Lightner and Desai (18). The derivation of the equations for SEQCON is based on a variational formulation of the potential energy equation. This method has been outlined in detail by Desai and Abel (9). SEQCON was developed primarily to analyze geotechnical problems by simulating sequences of construction. This program was expected to work well in subsidence analyses since mining sequences can be numerically simulated. The excavation simulation in a finite element method was first pioneered by Goodman and Brown (13). Later, this method was
improved further by Clough and Duncan (6,7).

Four constitutive models are offered in SEQCON. They are the linear elastic, variable moduli, Drucker-Prager, and Sandler-Cap models. The complete description of each method is available in several books: two of the best references are by Christian and Desai (5), and Desai and Siriwardane (11).

Together with the four different constitutive models, SEQCON has the capability to simulate slippage between two layers of elements where material constants greatly differ from one another. This is accomplished through the use of interface or joint elements. One of the earliest use of an interface element was developed by Goodman, Taylor and Brekke (14). Zienkiewicz and others (33), then, incorporated non-linear material behavior of an element for use in modeling movements of rock joint. Later, Desai et. al. (10,12) adopted this element for use in simulating slippage between soil and structure. The formulation of the interface element based on the work by Desai et. al. (10,12), is the one available in SEQCON.
1.4.2 Rubble Formulation in the Finite Element Model and Laboratory Simulation to Rock Failure

There have been many recent theoretical and experimental investigations into numerical and physical simulations of mine overburden. Probably the most extensive research lately has been done at Sandia National Laboratories. This numerical simulation, using the finite element approach, was presented by Benzley and Krieg (3). The physical simulation was done by Sutherland et. al. (27,29). The theoretical part of the finite element simulation incorporates strata failure, bulking of rock rubble, and recompaction of the rubble. Benzley and Krieg (3) assumed that the overburden rock fractured into small pieces and then subsided into the excavated area forming rubbles. When reloading took place, rock rubble was allowed to recompact before carrying any load. Comparisons are drawn to a laboratory study of subsidence conducted by Sutherland et. al. (27,29).

In this study, a core hole, OU-3, was drilled into a subsided longwall panel along S.R. 689. This was done to investigate the hydraulic balance of mine overburden. Another benefit was the examination of rubble formation.
Corings removed from OU-3 were visually inspected for fractures. Based on this information, it was concluded that the overburden rock failed in large continuous segments and not in small broken pieces as assumed by Benzley and Krieg (3). It would be logical, therefore, to incorporate the crack analysis pattern into the finite element program.

An investigation of experimental aspects of subsidence, conducted by Sutherland and others (27,29), used scale subsidence models tested in a laboratory centrifuge. Since subsidence over a longwall panel is the result of the weight of overburden, the centrifuge is an ideal laboratory device to simulate gravity force. Results of tests performed on a layered media show that the strata breaks progressively upward as the weight is increased. Strata fractures form an inverted "V" as they progress upwards towards the surface. Because of this fracture behavior the subsidence area at the top of the surface is smaller than the extracted area. However, this does not agree with field observations since the area subsidence influence is always larger than the mined area. The differences may be attributed to the process in which load is applied. In the laboratory experiments,
load is gradually applied to the already excavated mined area. Whereas, in the actual field condition insitu stresses are present before excavation takes place. Nonetheless, this beam-like failure of strata emphasize the importance of interface elements for use when modeling slippage between layers in any numerical simulation. Future research needs to be done on selecting appropriate materials and experimental procedures since the experimental results do not model field conditions.

1.4.3 Considerations on Rigid Finite Element Approach

Another interesting finite element model has been applied to subsidence predictions in Japan. This finite element model presented by Kawai (17) includes a slip line formulation. Accordingly, it is assumed that the elements move as rigid segments and slip will occur with relative motions between segments. Stiffness is lumped on the contact surfaces of neighboring rigid elements. The method is new but shows promise of providing a subsidence pattern that correlates well with field measurements.
1.5 LITERATURE REVIEW OF EMPIRICAL METHODS OF LONGWALL SUBSIDENCE PREDICTIONS

One of the best descriptions of the empirical methods employed in the European coal fields to predict subsidence in the United States coal fields is given by Munson and Eichfeld (19). The European empirical methods can be divided into either graphical or mathematical methods with the mathematical methods utilizing either a profile or an influence function. The National Coal Board (NCB) of Great Britain has compiled a large number of field measurements into graphical form. These results are published in the Subsidence Engineer's Handbook (21) and are widely quoted in the United States.

When matched against measurements taken in the Illinois coal basin, Munson and Eichfeld (19) and Munson and Sutherland (20) show that maximum subsidence predicted by the graphical method is consistent with the field observation, but the subsidence profile does not compare well. The NCB method predicts a wider trough, and consequently, a lower strain level.

Profile functions are mathematical functions based
on the two-dimensional cross section of a subsidence profile. These functions contain variables such as the critical width, the maximum possible subsidence and the distance between the point of half maximum subsidence and the longwall face. Most profile functions are in the form of a trigonometric or exponential function. For applications in the Northern Appalachian coal fields the following equations are suggested by Peng and Chyan (25):

\[ S(x) = S_0 e^{-6.67 \left( \frac{x}{L} \right)^{1.8}} \]

\[ S(x) = \frac{1}{2} S_0 \left( 1 - \tanh \frac{8.3x}{h} \right) \]

where 
- \( x \) = horizontal distance from the point of maximum subsidence.
- \( L \) = length of the half subsidence profile
- \( S_0 \) = maximum subsidence
- \( h \) = seam depth

These equations have been used by such authors as Peng and Chen (22), Adamek and Jeran (1), and Hood, Ewy, and Riddle (16). Since constants in these equations can be adjusted to local mining conditions and stratigraphy, the predicted subsidence profiles correspond to measured values. However, as this report illustrates, strains predicted by the above exponential function are larger
than the measured value.

Like the profile functions, influence functions are based upon mathematical equations of subsidence profile. The difference between the two methods lies in the treatment of a chosen mathematical function. Unlike the profile function, where the whole subsidence profile is described by one equation, the influence function predicts subsidence of a small extracted element and the total subsidence trough is obtained by summing up subsidence values for all of the elements. As pointed out by Sutherland and Munson (28), this method predicts significantly more subsidence directly over the rib side. This results from the fact that the influence function method calculates the point of one half maximum subsidence (point of inflection) to be directly over the panel edge. Sutherland and Munson (28) solved this problem by proposing two separate influence functions for mined and unmined areas.

Adamek and Jaren (1), while applying Bals' theory (2) to the Shoemaker Mine in Northern West Virginia, took a different approach by curve fitting a known subsidence profile by using angle of draw and effective
mining width as dependent variables. They also suggested that migration of the inflection point towards the center of a longwall panel is caused by the presence of stronger rock defined to be limestone and sandstone located in the upper zone of the overburden rock. However, based on laboratory tests by Hazen and Sargand (15) as well as these specimens reported in this study (see Appendix A), shale often exhibits yield strength higher than sandstone. The migration of the inflection point is, therefore, more likely caused by the tendency of each rock strata to behave as an independent beam (or plate in three dimension). Sutherland and others (27,29) while conducting experiments on scale models in a centrifuge have shown beam-like failure of strata.

The obvious advantage of this method is the ability to work well with a complex mine layout such as an entry area to a longwall panel. However, since longwall panels have a simple geometric shape and the effect of subsidence over the entry was measured to be small, the influence function method was not used in this investigation.

1.6 LITERATURE REVIEW OF SUBSIDENCE PREDICTIVE METHODS THAT INCLUDE STRATIGRAPHY OF THE OBERBURDEN
Empirical methods of Europe have been compared to measured values of subsidence in the United States by Munson and Eichfeld (19). The lack of agreement in the case of the NCB graphical method is probably due to the presence of mostly shales in the overburden of the Great Britain coal fields whereas the United States coal fields occur beneath both shale and sandstone. One approach would be to compile graphical data on subsidence for a geographical region. This has been done for seven longwall panels in four mines by Peng and Cheng (23) which are located in the Northern Appalachian coalfield. Another approach is to adjust the profile shape to local conditions. Along this approach a profile function has been modified by Peng and Chen (22) to describe the subsidence trough for six longwalls in the Northern Appalachian coal fields. The maximum subsidence and empirical constants are calculated according to the strength of mine overburden. Tandanand and Powell (30) of the United States Bureau of Mines have also considered strength of the overburden in predicting the maximum subsidence factor. They assumed that the maximum subsidence is controlled by the amount of "strong rock" in the overburden. They have indicated that the subsidence factor is linearly dependent upon the
percentage of "strong rock" which was defined as limestone and sandstone. However, based on the measured subsidence and stratigraphy of the investigated area along S.R. 556 and 689, the predicted subsidence factor based on Tandanand and Powell (30) can have as much as 50 percent error. Based upon the measured subsidence profile and the stratigraphy of longwall panel along S.R. 689 (see section 2.2 and 3.2), Tandanad and Powell's equation predicts a maximum subsidence of 2.02 ft while the actual value was 4.10 ft. However, the equation is consistent with the stratigraphy along S.R. 556 where over 70 percent of the mine overburden is shale (see Table 3.1). Also, the assumption that "strong rock" consists of only limestone and sandstone may be invalid since shale exhibits yield strength comparable to that of sandstone.
2.1 INTRODUCTION

There are three methods for predicting subsidence characteristics over a longwall. These are the graphical method, the profile method, and a finite element method. All of these methods can be used to obtain a subsidence profile once the width and depth of the extraction is known. In addition, local conditions such as the rock types and composition of the overburden enter into refinements to the predicted profile.

The graphical method was developed by the National Coal Board of Great Britain (NCB) and published in the Subsidence Engineers' Handbook. It is based on subsidence readings taken over many longwall panels, and while local conditions will affect the predictions, results have been found to be conservative when applied to mines in the United States. As an example, Figure 2.1a illustrates a typical subsidence profile encountered during mining of a panel having a width less or equal to a critical value. The latter is the value in which the
FIGURE 2.1a TROUGH SUBSIDENCE FOR PANEL WIDTH LESS OR EQUAL TO CRITICAL VALUE

FIGURE 2.1b TROUGH SUBSIDENCE FOR PANEL WIDTH LARGER THAN THE CRITICAL VALUE
maximum possible subsidence is reached. When the panel is larger than the critical value the trough subsidence profile will appear flattened as shown in Figure 2.1b.

On the other hand, the profile method is based on an algebraic expression which fits the transitional zone from unsubsided surface to a fully subsided surface. The expression which is felt to be the most appropriate is:

\[ S(x) = S_0 e^{-ax^b} \]

where \( S_0 \) is the maximum subsidence on the cross section, \( a \), and \( b \) are site specific constants which can be derived from curve fitting known subsidence profiles. The site specific constants \( a \), \( b \) do not have any significant meaning but rather are equation constants that must be known before accurate predictions can be made for subsidence, strain, and curvature. The expression was applied to six longwall panels in the Appalachian region by Peng and Chiang (24) to obtain average values for \( a \) and \( b \) of 8.97 and 2.03, respectively. The dimensionless distance \( Z \) equals \( X/R \) where \( X \) is the horizontal distance from the point of maximum subsidence and \( R \) is the distance across subsidence transition region. For a fully
developed subsidence trough, figure 2.1b, $S_o$ equals 0.74 $\times$ \( H \) with values for subcritical areas given by Peng.

2.2 SUBSIDENCE MEASUREMENT AND PREDICTION

2.2.1 Measurements Taken Along S.R. 689

Three sections of roadway were investigated. The most complete investigation was conducted along S.R. 689 in Meigs County from mileage posts 0.11 to 0.26. Along this section of highway survey pins were installed every 25 feet. Reinforcing rods, 5 ft long, were driven into the ground. A 10 in. by 6 in. diameter concrete top was added to the steel rods spaced every 100 ft. The untopped rods (pins) were found to be as stable as the topped rods (monuments), and thus no differentiation was observed when analyzing readings. Leveling was accomplished with a Kern tilting level with a standard rod graduated to 0.01 ft. All leveling measurements closed to maintain an accuracy in ft to within $0.1 \sqrt{\text{miles}}$ of the circuit. Changes in horizontal distances between survey pins were measured with a steel survey tape to within an accuracy of 0.03 ft. Standard third order surveying equipment was used for all measurements. A bench mark was established
outside mined area as shown on Figure 2.4.

The original ground profile is shown in Figure 2.2. The local terrain is hilly where the longwall advanced up the hollow from right to left. Figure 2.3 shows the location where the longwall crossed S.R. 689. The longwall panel, with its advance labeled with respect to subsidence and deformation dates is shown in Figure 2.4. Survey pins are labeled along the roadway. The longwall panel is 700 feet across, and advances at the rate of approximately 50 ft. per day, perpendicular with respect to S.R. 689. Five feet of coal was removed.

Figure 2.5 shows the results of several subsidence measurements. There was very little subsidence as the longwall passed beneath the monument line on January 18, 1985. Then, the rate of settlement increased as the longwall progressed for about 150 ft, and then the rate of subsidence decreased until settlement was almost complete at the date when the longwall had progressed to 380 ft past the monument line. The last occasion, when settlement was measured, was on April 4. The average depth of coal at this location is 380 ft.

The center region of the subsidence profile is
FIGURE 2.3: LOCATION OF S.R. 689 WHERE LONGWALL MINING TOOK PLACE.
FIGURE 2.4. LONGWALL PANEL WITH THE POSITION OF THE FACE LABELED
discontinuous and exhibits compressive features. The smooth shape and change in curvature along the sides indicates a tensile region. It is interesting to note just how localized the subsidence pattern was with respect to the longwall since very little settlement occurred outside the mined area.

2.2.2 Comparison of Measurement with Prediction

Two empirical methods are commonly used to predict maximum subsidence and shape of the subsidence trough when coal is removed with a longwall mining procedure. The National Coal Board of Great Britain has developed monographs based on years of experience and numerous measurements (21) in overburden consisting predominantly of shale and siltstone. The accuracy level, when applied to a new location, is expected to be within ten percent. But experience in the United States (19) has shown that NCB predictions are conservative and the subsidence profile are not representative of the United States mining experience. However, since NCB data represents the largest group of data on longwall mining, it has been compared for the locations under investigation. Examination of Figure 2.6 shows that NCB predicts a
subsidence trough that is deeper and wider than the one measured. The data are not symmetric because of a previously mined panel located to the right. The effect of the previously mined panel is the crushing of pillars between panels resulting in additional subsidence.

The profile method shows that the prediction of maximum subsidence is excellent, but the trough is again wider than the field data indicate. Since the structural response of overburden (which determines the profile shape) and maximum subsidence could be further refined to apply to a particular mine, agreement can be improved with the calculation of profile equation coefficients \( S_0 \), a and b.

2.2.3 Measurements Taken Along S.R. 556

Two longwall panels were monitored along S.R. 556 from 11.86 to 12.29 mileage posts in Monroe County. The terrain in this area is rugged with deep valleys and steep slopes. State Route 556 rises from the Ohio River along a creek valley at an approximate 20 percent slope. The slope is uniform enough so that subsidence produced a noticeable depression to the road. Figure 2.7 shows S.R.
FIGURE 2.7. UPPER LOCATION WHERE S.R. 556 INTERSECTS LONGWALL MINING OPERATIONS; A LEVELING MONUMENT IS SHOWN.
where the upper longwall mining occurred. Survey
monuments were installed every 100 ft. Monuments on the
upper longwall panel (between mileage posts 11.86 and
11.98) were concrete cylinders 6 in. in diameter and 2.5
to 3 ft. deep with stove bolts set as the leveling
targets. Monuments over the lower longwall panel (between
mileage posts 12.03 and 12.29) were 3/4 in. reinforcing
rods driven to 5 ft. of rejection. A 6 in. diameter by 10
in. long cylinder of concrete was added to the top of each
rod for stability. Figure 2.8 shows the original road
profile.

The progress of mining with the location of survey
monuments is shown in Figure 2.9 and 2.10. Consistent
with observations made on S.R. 689, subsidence lags the
removal of coal. The first measurement of subsidence took
place on July 21 when the face was 50 feet beyond the road
as shown in Figure 2.11. When the face was 380 ft. beyond
the road, subsidence was up to 70 percent completed. It
is interesting to note that an additional 10 percent of
subsidence took place when the second, adjacent longwall
panel was extracted which is shown as the readings taken
on January 5 versus the reading taken on August 4. The
relative location of the two longwall panels can be seen
in Figure 2.10. Monument No. 1 was used as the initial
FIGURE 2.8. ORIGINAL ROAD PROFILE IN SUBSIDED REGION FROM 11.86 TO 12.29 MILEAGE POSTS ON S.R. 556
FIGURE 2.9  UPPER LONGWALL MINING PROGRESS AND LOCATION OF SUBSIDENCE MONUMENTS FOR MONITORING SUBSIDENCE ON STATE ROUTE 556 BETWEEN MILEAGE POSTS 11.86 AND 11.98
FIGURE 2.10 LOWER LONGWALL MINING PROGRESS AND LOCATION OF SUBSIDENCE MONUMENTS FOR MONITORING SUBSIDENCE ON STATE ROUTE 556 BETWEEN MILEAGE POSTS 12.03 AND 12.29
benchmark for the upper longwall. The bottom of the coal is an average of 630 ft below S.R. 556 at this panel.

The lower panel was extracted parallel to S.R. 556 for a considerable distance in contrast to the other panels investigated which were approximately perpendicular to the roadway. Thus, an indication of rate of subsidence in the line of mining progress can be found in Figure 2.11. The curvature of subsidence profile is greater along monuments 9 through 5 during extraction than it was when measured across the panel. However, the profile curve levels off as extraction passes beyond a critical distance. The bottom of the coal is at an average of 533 ft. below the surface.

2.2.4 Comparison of Measurement with Prediction

The experimental data from the upper panel on S.R. 556 shows excellent agreement to both profile and the NCB prediction as shown in Figure 2.12. Since both NCB predictions were conservative on the other longwall panels, this agreement may be the result of additional subsidence due to failure of entry pillars as the adjacent longwall panel was mined.
Comparison of a perpendicular panel profile to measurements taken along the lower longwall panel, which crosses but parallels S.R. 556 for a short distance, is compared using an average depth value. The agreement with the profile method in Figure 2.13 is not good. Data on the greater depth side (right side of figure) of the longwall panel indicates a subsidence which is less than expected. The NCB profile, again, exhibits more vertical movement and a wider trough than measured data indicates.

2.3 STRAIN MEASUREMENT AND PREDICTION

Strain is a direct measure of loading on surface structures and, consequently, a measure of damage which may result to highways and highway structures. In the cases investigated, highways were asphalt laid over a gravel base. Two concrete culverts with concrete entries were situated above the longwall panels mined on S.R. 556. Both of these culverts were located in the compression zone near their respective panel centers.

Examination of the strains along S.R. 689 shows the discontinuous nature of subsidence with strain values in the range of +7.2 micro in./in. to -14 micro in./in. in
FIGURE 2.13, SUBSIDENCE COMPARISON WITH PREDICTION OF LOWER PANEL ON S.R 556 BETWEEN 12.03 AND 12.29 MILEAGE POSTS
Figure 2.14. Tensile cracks perpendicular to the road were recorded at panel edges. Figure 2.15, upper photo, illustrates some alligator and block cracking that were caused by longwall mining along S.R. 556 and aggravated by the winter weather. Approximately twenty percent of the cracks existed before mining took place. Most cracks along S.R. 556 and 689 were minor as shown in Figure 2.15, upper photo. However, one crack was measured 3.5 in. across and displaced 2 in. in the direction parallel to the longwall (Figure 2.15), lower photo. This crack opened at core hole OU-1 on January 17 before measurable subsidence took place. When the survey crew leveled monuments on January 22, the crack had closed, and it remained closed during the observation period. Compression movements were recorded in the center of the panel, but no damage attributable to mining was noted.

Along S.R. 556 some ground movements occurred as shown in Figure 2.15, upper photo. The berm at this location was regraded to road level. Yet along S.R. 556 road damage was observed to be slight which may be the result of the greater depth of mining.

Based on the erratic behavior of measured strains, with respect to time as well as location (Figure 2.16), it
FIGURE 2.14. STRAIN LEVELS MEASURED DURING LONGWALL MINING BENEATH S.R. 689
FROM 0.11 TO 0.26 MILEAGE POSTS
FIGURE 2.15. UPPER VIEW IS OF BERM REPAIR ALONG S.R. 556 WITH A SOFTNESS IN PAVEMENT NOTED; LOWER VIEW IS OF TEMPORARY CRACK WHICH OPENED ALONG S.R. 689
FIGURE 2.16. STRAIN LEVELS MEASURED DURING LONGWALL MINING OF LOWER PANEL ALONG S.R. 556 FROM 11.86 TO 12.29 MILEAGE POSTS
was felt that the 100 ft. interval between monuments were too long a distance to pick up peak strain readings. Thus, there was no comparison made with predicted strains along S.R. 556. The erratic nature of the measured strain is probably due to the influence of the terrain which is dictated by the layout of the stratigraphy. It is still interesting to note that strain magnitudes are much less than those measured along S.R. 689.

Comparison of strains predicted by the NCB graphical method and the profile method to measured values shown in Figure in 2.17, illustrates that the trend is the same in both cases with tension present at the panel edges and compression at the panel center. However, the magnitudes are substantially different. Compressive strains calculated using the NCB method are much less than the measured magnitudes.

Examination of the strain measurements along S.R. 689 shows the discontinuous nature of subsidence with strain values in the range of +7.2 micro in./in. to -14 micro in./in. in Figure 2.14. Tensile cracks perpendicular to the road were recorded at panel edges. Most cracks were minor as shown in Figure 2.15. However, one crack was measured 6 in. across and displaced 2 in. in the
FIGURE 2.17 COMPARISON OF MEASURED STRAIN TO PREDICTED VALUES FOR S.R. 689
FROM 0.11 TO 0.26 MILEAGE POSTS
direction parallel to the longwall (Figure 2.15). This crack has now completely closed. Compression movements were recorded in the center of the panel, but no damage attributable to mining was noted. The concrete culverts, located in the compression region, experienced no visual damage.

Comparison of strains predicted by the NCB graphical method and profile method to measured values in Figure 2.17 shows that the trend is the same in both cases with tension present at the panel edges and compression at the panel center. However, the magnitudes are substantially different compressive strains calculated using the NCB method are much less than measured values. Based on the erratic behavior of measured strains, with respect to time as well as location (Figure 2.14), it was felt that the 100 ft interval between monuments was too long to pick up peak strain readings. Thus, there was no comparison with predictions along S.R. 556. It is still interesting to note that strain magnitudes are much less than those measured on S.R. 689.

2.4 CONCLUSIONS

Three longwall panels were examined for subsidence
effects on highways and highway structures. Based on comparison of measured values to predicted values by the NCB graphical method and the profile method, the profile method can be relied on to give reasonable values for displacements on all investigated longwall panels.

The values measured for strain was very erratic with respect to time and position. Among the empirical methods it is felt that neither the NCB method, which predicts a smooth transition and low strains from experience acquired in England nor the profile method, which is based on the equation of a line, accurately calculates strain. A numerical method which accounts for the structural properties of rock is the more logical model to predict damage to surface structures.
CHAPTER 3

LABORATORY TESTING AND THE CONSTITUTIVE MODEL

3.1 INTRODUCTION

When a numerical procedure, like the finite element method, is used to solve a geotechnical problem, the most important factor that governs accuracy will probably depend upon how well the existing soil or rock is described with a mathematical model. To select the best model for load-respond behavior, extensive testing of these materials must be performed.

Rock specimens, for laboratory testing in this investigation, were obtained from two NX** corings along S.R.689 from 0.11 to 0.26 mileage posts. The first coring, OU-1, was located at approximately 100 ft from the edge of the 700 ft panel with the second, OU-2, located at the center of this panel.

Unconfined and confined compression tests were

** NX (NWG,NWM) designates a 3 in. nominal diameter coring bit and a rock core diameter of 2 in.
conducted on the specimens collected using various strain and displacement measuring devices.

This chapter will discuss the stratigraphy obtained from the NX coring and the laboratory experiments. The basic relations of the linear elastic and the Drucker-Prager models will be included in this section. Likewise, a brief overview of the linear interface model used for simulating fractured claystone layers will also be explained.

3.2 STRATIGRAPHY

The stratigraphic descriptions, obtained from OU-1 and OU-2 corings, are shown in Figures 3.1 and 3.2, respectively.

An examination of these Figures show numerous layers of sandstone, shale and claystone. A four foot layer of limestone is located at the 371 ft depth. The overlying soil is 25 ft thick and is composed of clay-soil and sandstone fragments. These Figures also show four thin layers of coal which are laminated in the shale or claystone layers. Observation of rock corings and the
FIGURE 3.1. THE STRATIGRAPHY COLUMN FOR OU-1 LOCATED AT 0.12 MILEAGE ON S.R. 689 (continued on next page)
FIGURE 3.1. THE STRATIGRAPHY COLUMN FOR OU-1 LOCATED AT 0.12 MILEAGE ON S.R 689.
Figure 3.2. The stratigraphy column for OU-2 located at 0.17 mileage on S.R. 689 (continued on next page)
FIGURE 3.2. THE STRATIGRAPHY COLUMN FOR QU-2 LOCATED AT 0.17 MILEAGE ON S.R. 689
visual inspection of the core holes with a geoprobe showed that these layers were highly fractured before mining.

The sandstone corings recovered were mostly light grey and appear to be homogeneous. Coring through sandstone was easily accomplished with a 15 ft specimen often removed intact. On the other hand, with its highly laminated structure, shale was very weak and was mostly recovered in a highly fractured state.

At all times during coring operations a graduate geologist was present to log the stratigraphy and to collect laboratory samples. As samples were recovered they were identified according to rock type and depth and placed in air tight bags.

A similar stratigraphy is expected throughout this region, but the thickness of each layer may vary. As summarized in Table 3.1, the strata of OU-1 and OU-2 are very similar. Stratigraphic composition of S.R.556, for analysis done between mileage posts 11.86 and 12.29, were obtained from an unpublished report on subsidence (8) in the area.
## TABLE 3.1

**STRATIGRAPHY SUMMARY**

<table>
<thead>
<tr>
<th>Lithologic Composition</th>
<th>OU-1</th>
<th>OU-2</th>
<th>SR 556</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Sandstone</td>
<td>34.5</td>
<td>34.8</td>
<td>14.3</td>
</tr>
<tr>
<td>% Shale</td>
<td>34.3</td>
<td>29.6</td>
<td>70.9</td>
</tr>
<tr>
<td>% Coal</td>
<td>3.4</td>
<td>3.2</td>
<td>3.0</td>
</tr>
<tr>
<td>% Limestone</td>
<td>1.8</td>
<td>1.9</td>
<td>8.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Major Layers</th>
<th>OU-1</th>
<th>OU-2</th>
<th>SR 556</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone</td>
<td>8</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Shale</td>
<td>14</td>
<td>9</td>
<td>55</td>
</tr>
<tr>
<td>Coal</td>
<td>5</td>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

### 3.3 UNCONFINED COMPRESSION TESTS

The simplest ways to find the basic material parameters and the overall stress-strain relationship for
a geologic material is to conduct an unconfined compression test. One of the reasons that this test is particularly suitable is the ease with which specimens can be prepared. Preparation of specimens required cutting of the 2 in. diameter rock cores into cylinders of 4 in. length. This was accomplished with a diamond blade masonry saw. The specimens were then capped with a sulfur capping compound to prevent stress concentration along the loading surfaces.

Three deformation measuring devices were used to measure material responses; they are the electric extensometer, uniaxial and biaxial electrical strain gages. The biaxial electrical strain gages were used primarily for determining Poisson's ratio. As the specimens were easy to prepare, several different tests were conducted for materials obtained from a single layer. This provided the means for establishing limits on the variability of strength and elastic moduli.

The results obtained from these tests are found in "The Effect of Longwall Mining on Surface Subsidence of Highways and Bridges," published by the Ohio University Civil Engineering Department for the Ohio Department of Transportation and Federal Highway Administration (15).
3.4 CONFINED COMPRESSION TESTS

Since rocks are a frictional type material which exhibit load-respond behavior that is dependent upon the hydrostatic pressure, confined compression tests are needed to examined the failure mechanism. The triaxial cell was used in this investigation to determine the effect of confining pressure on rock strength and response.

The preparation of specimens for the triaxial cell required recoring a 2 in. diameter NX corings to a 1 and 1/2 in. diameter. For sandstone this was accomplished easily but for shale and claystone the samples often break during the recoring process. Consequently, only a limited amount of specimens were prepared. The successfully dimensioned specimens were cut to a 3 in. length and fitted into a rubber sleeve in preparation for placing into the triaxial cell.

The confining pressure of the triaxial cell was determined by using the theory of elasticity. In this theory, if a material is assumed to behave linear elastically, then Hooke's Law may be used. If a plane strain condition is applied, the coefficient of lateral
earth pressure, \( k \), will be equal to \( \frac{1}{1-\nu} \). However, when this method was used in this experiment, the confining pressure obtained was very small (less than 50 psi for rock at 200 ft depth). Confining pressure of 500 psi and 300 psi were used for sandstone and shale respectively since mining will add an additional compressive stress to the overburden rock before failure.

The conventional triaxial test procedure, where loading and unloading was done in the axial direction only, was used in most of the tests. However, when more than one specimen per layer was successfully prepared, a reduced triaxial compression test procedure was also used. In this procedure, the specimens were kept under a confining pressure and then loaded axially until yield. Then, the confining pressure was reduced in order to induce failure. The reduced triaxial compression test simulates field failure condition where the confining stresses are lowered as rocks in the roof formed a rubble in the void space created by mining.

The axial deformations of the rock specimens were obtained by using a mechanical extensometer with an accuracy of 0.001 in. Because a high pressure triaxial cell was used in this experiment, the measurements for the
change in volume necessary for determining materials parameters for the Sandler-Cap model were difficult to determine. Additional tests are necessary to utilize the full capability of the SEQCON computer program.

Specimens subjected to confined and reduced compression tests failed with high angle fractures similar to fractures observed in the field corings and viewed on the geoprobe after mining. The results of the triaxial compression tests can be found in Appendix A.

3.5 MOISTURE AND DENSITY CONTENT

Density and moisture were measured in order to determine in situ loading as coal was extracted since equilibrium must be restored to support the overburden load. Density and moisture content were determined for specimens taken from all major rock seam. In several instances density and moisture content were determined from specimens used for strength test.

3.6 CONSTITUTIVE MODELS
Upon examining the stress-strain relationship, it is clear that most material behave linearly almost to failure. However, they exhibit some plastic strains as well as showing some strength dependency on the confining pressure.

The constitutive equations chosen to be the most suitable for modeling these materials were the linear elastic and the Drucker-Prager formulations. Although Sandler-Cap equation is also suitable, it was not possible to use in this investigation since additional measurements of specimen volume changes are required.

Along with the triaxial compression testing device, a newly developed multiaxial testing device was also available at the Ohio University. This device had been used to determine the orthotropic behavior of shale in the investigation for the Ohio Department of Transportation (ODOT) (26). The operating principle of the multiaxial testing device is basically the same as the traxial testing device except that loading and unloading can be done in three perpendicular directions, independently. Displacements in all six directions can be measured using the linear voltage differential transformer (LVDT). This device is highly suitable for obtaining material
parameters in the Sandler-Cap model. Due to high costs associated with recovering six inch diameter cores, and the time required preparing the specimens, this device was not used for this investigation.

3.6.1 Linear Elastic Model

Linear elastic modeling is based on Hooke's law which states that the deformation is directly proportional to the force. For the plane strain condition, where the longitudinal direction is large, the relationship between \( \sigma \) and \( \varepsilon \) reduces to:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}
\begin{bmatrix}
1-\nu & \nu & 0 \\
\nu & 1-\nu & 0 \\
0 & 0 & \frac{1-2\nu}{2}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z
\end{bmatrix}
\]

The plane strain condition applied to this investigation since the length of the longwall panel is much larger than its width.

3.6.2 Drucker-Prager Model
Since fracturing was observed to extend to the surface the overburden materials were obviously in a failed condition. The failure criterion introduced into the finite element model was developed by Drucker-Prager who modified the well known Mohr-Coulomb yield criterion by including the effects of all the minor principal stresses. The Drucker-Prager yield criteria can therefore be written as:

\[ f = \sqrt{J_{2D}} - \alpha J_1 - k \]

where

- \( \alpha \) - positive material parameter
- \( k \) - positive material parameter
- \( J_1 \) - the first invariant of the stress tensor
- \( J_{2D} \) - the second invariant of the deviatoric stress tensor

This is shown graphically in Figure 3.3. The Drucker-Prager failure criterion will reduce to the Von Mises criterion if \( \alpha = 0 \). The values of \( \alpha \) and \( k \) can be expressed in terms of angles of internal friction \( \phi \) and cohesion \( c \). When the triaxial compression test is used their relationship can be written as:
Figure 3.3  DRUCKER-PRAGER FAILURE CRITERION USED TO MODEL FAILURE IN ROCK STRATA OVER LONGWALL PANELS
The derivation of the incremental stress-strain relationship which is based on the associated plasticity theory will not be given here. However, the final form of this relationship can be written as:

\[
d\sigma_{ij} = 2Gd\epsilon_{ij} - 2G \left[ A (\sigma_{mn}\delta_{ij} + \sigma_{ij}\delta_{mn}) + B\delta_{mn}\delta_{ij} + C\sigma_{mn}\sigma_{ij} \right] d\epsilon_{mn}
\]

\[
a = \frac{2 \sin \phi}{\sqrt{3} (3 - \sin \phi)}
\]

\[
k = \frac{6c \cos \phi}{\sqrt{3} (3 - \sin \phi)}
\]

\[
A = \frac{h}{p'k}
\]

\[
p' = \frac{\sqrt{J_{2D}}}{k} (1 + 9\alpha^2 k / G)
\]

\[
h = \frac{-k(p' - 1)}{6a\sqrt{J_{2D}}} = - \left( \frac{3aK}{2G} + \frac{J_1}{6\sqrt{J_{2D}}} \right)
\]

\[
B = \frac{2h^2\sqrt{J_{2D}}}{p'k} - \frac{3vK}{E} = \frac{2h^2}{1 + 9\alpha^2 K / G} - \frac{3vK}{E}
\]

\[
C = \frac{1}{2kp'\sqrt{J_{2D}}}
\]
The matrix form of this equation is shown in Table 3.2.

3.6.3 Linear Interface Model

Modeling of cracks in rocks were done with a linear interface model. Linear interface elements were designed for simulating interactions between two very different types of materials. This proved very successful for simulating the interactions of soil and structure but is used here to model the highly fractured claystone layers. Figure 3.4 shows this claystone coring that was modeled as interface elements. In the Desai's interface model the shear stiffness $G$ is totally independent of Young's modulus $E$ and Poission's ratio $\nu$. Thus a frictional interface can be numerically simulated.

For a two-dimensional plane strain conditions, the strain- stress relations can be written as:

\[
\begin{pmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_{xy}
\end{pmatrix} =
\begin{bmatrix}
\frac{1-\nu^2}{E} & -\frac{\nu(1+\nu)}{E} & 0 \\
-\frac{\nu(1+\nu)}{E} & \frac{1-\nu^2}{E} & 0 \\
0 & 0 & \frac{1}{G_i}
\end{bmatrix}
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix}
\]
| TABLE 3.2 Stress–Strain Matrix for Ducker–Prager Material Model |

\[
\begin{pmatrix}
    d\sigma_{11} \\
    d\sigma_{22} \\
    d\sigma_{23} \\
    d\sigma_{13}
\end{pmatrix} = 2G \begin{pmatrix}
    1 - T_1\sigma_{11} - R_1 & - (T_1\sigma_{22} + R_1) & - (T_1\sigma_{31} + R_1) & - (T_1\sigma_{12}) & - (T_1\sigma_{13}) & - (T_1\sigma_{11}) \\
    - (T_2\sigma_{11} + R_2) & 1 - T_2\sigma_{22} - R_2 & - (T_2\sigma_{32} + R_2) & - (T_2\sigma_{12}) & - (T_2\sigma_{23}) & - (T_2\sigma_{11}) \\
    - (T_3\sigma_{11} + R_3) & - (T_3\sigma_{23} + R_3) & 1 - T_3\sigma_{33} - R_3 & - (T_3\sigma_{12}) & - (T_3\sigma_{23}) & - (T_3\sigma_{11}) \\
    - T_1\sigma_{12} & - T_2\sigma_{12} & - T_3\sigma_{12} & \frac{1}{2} - C_{12} & - C_{12} & - C_{12} \\
    - T_1\sigma_{23} & - T_2\sigma_{23} & - T_3\sigma_{23} & - C_{12} & \frac{1}{2} - C_{23} & - C_{12} \\
    - T_1\sigma_{13} & - T_2\sigma_{13} & - T_3\sigma_{13} & - C_{12} & - C_{12} & \frac{1}{2} - C_{13}
\end{pmatrix} \begin{pmatrix}
    d\epsilon_{11} \\
    d\epsilon_{22} \\
    d\epsilon_{23} \\
    d\gamma_{12} \\
    d\gamma_{13} \\
    d\gamma_{11}
\end{pmatrix}
\]

where \( T_a = A + Ca_a \) and \( R_a = Aa_a + B; a = 1, 2, 3 \). Here a repeated subscript does not mean summation.

(after Desai and Siriwardane (11))
4.1 INTRODUCTION

The finite element program (SEQCON) developed at Virginia Polytechnical Institute (18), designed to calculate displacements and stress during stages of construction was utilized in this investigation. This program has been used in studies of structural stability during embankment and excavation.

The need for using 'SEQCON' in the excavation of the coal layer arises from the fact that the computed results based on single stage construction, differs greatly from one based on sequential construction. This fact can be easily understood by realizing that stresses in rock mass after an excavation is dependent upon the pre-existing stress (i.e., insitu stress).

When solving excavation problem, SEQCON first calculates the insitu stresses in the medium. Next, to simulate the open space due to excavation, the weight of the excavated elements computed and are applied in the opposite direction. Finally, displacements, strains and other unknowns are calculated. This procedure can be
described in the following equation:

\[ (\sigma_i) = (\sigma_o) + \sum_{i} (\Delta\sigma_i) \]

where \( (\sigma_i) \) = the final stress after the \( i \)th cycle.
\( (\sigma_o) \) = the stress due to insitu loads.
\( (\Delta\sigma_i) \) = the change in stress due to the \( i \)th cycle.

In SEQCON the formulation of the stiffness matrices, body forces and surface tractions are based upon 8-nodes isoparametric elements. This chapter will, therefore, give an overview of the derivation of equation using the said element.

4.2 GENERAL DERIVATION OF THE GOVERNING EQUATION

For any body acted upon by external forces, the potential energy equation of this system can be written as:

\[
\Pi_p = \frac{h}{2} q^T \int_A [B]^T[C][B] dx dy \{ q \} - h \{ q \} \int_A [N]^T \{ \bar{u} \} dx dy \\
- h \{ q \} \int_{S_1} [N]^T \{ \bar{T} \} ds
\]
The solution to the governing equation can be found by incorporating the variational principle to the above equation.

Hence, as a general solution we can write:

\[ [K](q) = (Q_1) + (Q_2) \]

where \([K]\) = the stiffness matrix, and is defined as:

\[
[K] = h \int_A [B]^T [C] [B] dx dy \]

\( h \) = the transverse thickness of the element

\( \{q\} \) = the displacement vector

\( \{Q_1\} \) = the body force, and is defined as:

\[
\{Q_1\} = h \int_A [N]^T \{\bar{X}\} dx dy
\]

\( \{Q_2\} \) = the surface traction, defined as:

\[
\{Q_2\} = h \int_{S_1} [N]^T \{\overline{T}\} ds
\]

\([B]\) = the 'B' matrix describing the strain to
The displacement model can be approximated using many equations in mathematics but because of the ease in manipulation, hence, polynomial equation is the most popular. Therefore, for an 8-nodes isoparametric element the displacement function can be written as:

\[ u = a_1 + a_2 x + a_3 y + a_4 xy + a_5 x^2 + a_6 y^2 + a_7 x^2 y + a_8 xy^2 \]

\[ v = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 xy + \beta_5 x + \beta_6 y^2 + \beta_7 x^2 y + \beta_8 xy^2 \]

and the interpolation function 'N' written in terms of natural coordinates \( s, t \) is in the form:

\[ N = c_1 + c_2 s + c_3 t + c_4 st + c_5 s^2 + c_6 t^2 + c_7 s^2 t + c_8 st^2 \]

by using the layout in Figure 4.1 and substituting the following condition:
\[ N_1 = 1 \quad \text{for} \quad s = s_i, \quad t = t_i \]
\[ N_i = 0 \quad \text{for} \quad s \neq s_i, \quad t \neq t_i \]

where \[ i = 1, 2, 3 \ldots 8 \]

the interpolation function is then reduced to:

\[ N_1 = \frac{1}{4} (1-s)(1-t)(-1-s-t) \]
\[ N_2 = \frac{1}{2} (1-s^2)(1-t) \]
\[ N_3 = \frac{1}{4} (1+s)(1-t)(-1+s-t) \]
\[ N_4 = \frac{1}{2} (1+s)(1-t^2) \]
\[ N_5 = \frac{1}{4} (1+s)(1+t)(-1+s+t) \]
\[ N_6 = \frac{1}{2} (1-s^2)(1+t) \]
\[ N_7 = \frac{1}{4} (1-s)(1+t)(-1-s+t) \]
\[ N_8 = \frac{1}{2} (1-s)(1-t^2) \]

with these interpolation functions defined in the above manner; the position and the displacement vectors can now be described by the following equation:

\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} = \begin{bmatrix} [N] & 0 \\ 0 & [N] \end{bmatrix} \begin{bmatrix} \{x_i\} \\ \{y_i\} \end{bmatrix}
\]
4.4 THE STIFFNESS MATRIX AND ITS COMPONENTS

The stiffness matrix which was defined earlier in Section 4.2, equation 4.2a, has three unknowns, namely: \( h \), \([C]\) and \([B]\). Out of these three unknowns the thickness \( h \) must be given. Therefore, only \([C]\), and \([B]\) are left to be defined.

\[
\begin{pmatrix}
  u \\
  v
\end{pmatrix} =
\begin{bmatrix}
  [N] & 0 \\
  0 & [N]
\end{bmatrix}
\begin{Bmatrix}
  \{u_i\} \\
  \{v_i\}
\end{Bmatrix}
\]

where: \([N] = [N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8]\)

![Node connectivity diagram](image.png)

Figure 4.1
The discussions on the constitutive matrix, \([C]\) were given earlier in chapter 3. Before discussing matrix \([B]\) one should bear in mind that this matrix is the one which describes the relationship between strains and displacements in differential forms. Hence, the expressions relating the differentials in global and local coordinates systems are needed. This expression is termed as the Jacobian matrix.

4.4.1 The Jacobian Matrix

Through the use of the chain rule, the following equation can be written:

\[
\frac{\partial N_i}{\partial s} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial s}
\]

\[
\frac{\partial N_i}{\partial t} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial t}
\]

or written in a matrix form:

\[
\begin{pmatrix}
\frac{\partial}{\partial s} \\
\frac{\partial}{\partial t}
\end{pmatrix} =
\begin{bmatrix}
\frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\
\frac{\partial x}{\partial t} & \frac{\partial y}{\partial t}
\end{bmatrix}
\begin{pmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y}
\end{pmatrix}
\]
Hence, the Jacobian matrix can be written as:

\[
[J] = \begin{bmatrix}
\frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\
\frac{\partial x}{\partial t} & \frac{\partial y}{\partial t}
\end{bmatrix}
\] (4.4a)

and where the inverse relationship of local to global coordinates is required; the inverse can be written as:

\[
[J]^{-1} = \frac{1}{|J|} \begin{bmatrix}
\frac{\partial y}{\partial t} & \frac{\partial y}{\partial s} \\
\frac{\partial x}{\partial t} & \frac{\partial x}{\partial s}
\end{bmatrix}
\]

where: \(|J|\) is the determinant of the Jacobian matrix. The derivation of the Jacobian matrix and its determinants can be found in Appendix B.

With the Jacobian matrix defined; \([B]\) can now be obtained through a direct substitution of the kinematic equations in solid mechanics.

The kinematic equations used in finding \([B]\) can be written in a rectangular coordinated system for plane stress/strain case and a cylindrical coordinate system for...
4.4.2 Plane Strain and Plane Stress

The kinematic equations from solid mechanics are defined as:

\[ \epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]

Applying the chain rule to these equations, we can write:

\[ \epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial u}{\partial t} \frac{\partial t}{\partial x} \]

\[ \epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial v}{\partial t} \frac{\partial t}{\partial y} \]

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left( \frac{\partial u}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial u}{\partial t} \frac{\partial t}{\partial y} \right) + \left( \frac{\partial v}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial v}{\partial t} \frac{\partial t}{\partial x} \right) \]

After appropriate substitutions, strain can be written in matrix form as:

\[
\begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
[B]' & 0 \\
0 & [B]''
\end{bmatrix}
\begin{bmatrix}
\{u_i\} \\
\{v_i\}
\end{bmatrix}
\]

(4.4.2a)
where \([B]' = [B_{11}, B_{12}, B_{13}, B_{14}, B_{15}, B_{16}, B_{17}, B_{18}]\)

\([B]'' = [B_{21}, B_{22}, B_{23}, B_{24}, B_{25}, B_{26}, B_{27}, B_{28}]\)

and

\[
B_{1k} = \frac{1}{|J|} \sum_{j=1}^{8} \left( \frac{\partial N_k}{\partial s} \frac{\partial N_j}{\partial t} - \frac{\partial N_k}{\partial t} \frac{\partial N_j}{\partial s} \right) y_j \tag{4.4.2b}
\]

\[
B_{2k} = \frac{1}{|J|} \sum_{j=1}^{8} \left( \frac{\partial N_k}{\partial s} \frac{\partial N_j}{\partial t} - \frac{\partial N_k}{\partial t} \frac{\partial N_j}{\partial s} \right) x_j \tag{4.4.2c}
\]

The reduced forms of \([B]', [B]''\) are presented in Appendix C.

Finally, through substitutions of \(h, [B]\) and \([C]\) the \([K]\) matrix can be obtained. Before \([K]\) can be evaluated, however, \([K]\) can be simplified further to accommodate the procedure for numerical integration. This required \([K]\) to be written in the form:

\[
[K] = h \int_{-1}^{1} \int_{-1}^{1} [B]^T[C][B] |J| dsdt
\]

The stiffness matrix for individual element can, then, be summed up to form the complete stiffness matrix for the whole problem.

4.5 THE BODY FORCE
The general form of the body force as given in section 4.2 can be written as follows:

\[ \{Q_1\} = h \int \int_A [N]^T(\bar{X})dxdy \]

This general form can be re-written in the natural coordinates system for the purpose of applying the numerical integration procedure as:

\[ \{Q_1\} = h \int \int_A [N]^T(\bar{X})|J|dsdt \]

where \(|J|\) is the determinant of the Jacobian matrix (see Appendix B).

For the case in which we consider only the gravity loading, as in the case of longwall mining problems, \(\{Q_1\}\) can be written as:

\[ \{Q_1\} = h \int_{-1}^{1} \int_{-1}^{1} [N]^T \left[ \begin{array}{c} 0 \\ y \end{array} \right] |J|dsdt \]

The reduced form of this equation can be found in Appendix D.

4.6 THE SURFACE TRACTION
As stated earlier in section 4.2, the general equation for surface traction can be written in this form:

\[
\{Q_2\} = h \int_{S_1} [N]^T \{\mathbf{T}\} dS
\]

and again for the purpose of numerical integration, this equation can be written as:

\[
\{Q_2\} = h \int_{-1}^{1} [N]^T \begin{bmatrix} T_{x1} \\
4T_{x1} \\
T_{x1} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \end{bmatrix} |J| ds
\]

Since tractions are applied only on the side of an element, the interpolation function \([N]\) is reduced to one for a quadratic line element. After substituting this into the above equation \(\{Q_2\}\) is then simplified to:

\[
\{Q_2\} = \frac{hL}{6} \begin{bmatrix} 4T_{x1} \\
T_{x1} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \end{bmatrix}
\]
4.7 TECHNIQUES FOR SOLVING PROBLEMS WITH MATERIAL NONLINEARITY

From the laboratory tests it is quite obvious that most materials collected behaved as non-linear materials.

To model these materials' nonlinearity, $[C]$ matrix can be expressed as a stress dependent variable and, therefore, the stiffness matrix $[K]$ can be re-written as:

$$[K] = \iint_{V} [B]^T [C(\sigma)] [B] \, dv$$

The technique employed in SEQCON for computing displacements and other unknown falls under the procedure called the initial strain method.

The basic procedure for this method is shown graphically in Figure 4.2. When a load $\{Q\}$ is applied to a system, a resulting displacement $A$ can be computed as in the linear elastic case. However, the correct displacement according to the nonlinear elastic curve is $B$. The difference between the correct displacement $\{Q_B\}$ and the displacement computed $\{q_a\}$ is called $\{q_0\}$. The corresponding initial strain $\{\epsilon_0\}$ can be computed using the $[B]$ matrix as:
Figure 4.2 Basis of the initial strain method

Figure 4.3 Basis of the iterative initial strain method
Therefore, the correction load \( Q_0 \) can be found by using the following equation:

\[
\{ \epsilon_0 \} = [B] \{ q_0 \}
\]

This correction load \( Q_0 \) is, therefore, the additional load required to push the system to achieve the correct displacement.

When this method is applied to the analysis of plastic materials the above procedure can be expanded by including the plastic strains. However, to include the plastic strains incremental procedure is required. The schemes used in SEQCON are the mixed incremental and iterative initial strain method.

The basic equations for this approach can be written as:

\[
[K] \{ q_i \} = \{ Q \} + \{ Q_0, i-1 \}
\]

for this procedure the incremental correction load which is now in terms of plastic strain can be written as:
\( (Q_{0,i-1}) = \iiint_{V} [B]^T [C^e] (\epsilon^P_i) \, dv \)

where \( (\epsilon^P_i) \) can be found from subtracting the computed elastic strain \( (\epsilon^e_i) \) from the known total strain \( (\epsilon_i) \).

The equation for this step is written as:

\[
(\epsilon^P_i) = (\epsilon_i) - (\epsilon^e_i)
\]

The whole procedure of this mixed incremental and iterative method can best be described graphically in Figure 4.3.
5.1 INTRODUCTION

As previously described in detail in Chapter 4, the finite element program, SEQCON, was used to predict subsidence in this investigation. When using this program to analyze the subsidence resulting from longwall mining, understanding the failure behavior of the complete structure for application to the finite element method is essential, otherwise, the results obtained are unrealistic. Thus, it is only through the understanding of material and structural response that the finite element analysis can be applied to obtain subsidence values consistent with observation.

The SEQCON program was installed into the IBM main frame facility at the Ohio University with minor modifications made to SEQCON prior to usage. The installation and the modification of this program will be treated separately in the next section. This chapter will also discuss the design of the finite element mesh, adapting experimental data, and the analysis of the results.
5.2 INSTALLATION AND MODIFICATIONS OF SEQCON

The installation of SEQCON into the IBM main frame was relatively simple. It was simply read from a magnetic tape and stored on disk. SEQCON was then compiled with the FORTRAN compiler.

Prior to compiling the program a few changes were made. The first was modifying how SEQCON uses external storage devices. Since the program gives provision to either a direct access or a sequential file operation, a choice must be made to select the one which calculates better. Sequential file was selected in our case because of its efficiency. This required commenting out all of the direct access commands and reactivating the sequential commands. Commenting out a statement in FORTRAN is to turn this statement into a non-executable statement. This can be done by putting a letter 'c' in the first column of a line to be commented out. The second and more important change made was the modification of equations for calculating positive material parameters in the Drucker-Prager constitutive model. The equations that were originally programmed are for laboratory tests which we ran under a plane strain condition. These equations were changed to the ones appropriate to triaxial
During the finite element design process, several fine meshes with elements numbering in excess of 200 were used. Using these fine meshes required increasing of the variables in the DIMENSION statement and the maximum front width. These were accomplished through the equations given within the program which is written as:

\[ A = 22 \times NNP + 190 \times NEL + 5 \times MFRON + (MFRON \times 2 - MFRON)/2 \]

where:

- \(A\) - variable in the DIMENSION statement
- \(NNP\) - number of node points
- \(NEL\) - number of elements
- \(MFRON\) - maximum number of equations on the front in the frontal routine

SEQCON was run under the BATCH MONITOR SYSTEM (or commonly known as BATCHMON). An example of the job stream can be found in Appendix E.

5.3 DESIGNING OF THE FINITE ELEMENT MESH
When designing a finite element mesh layout for geological media four important factors must be considered: first, the geometry of the boundary; second, the natural subdivision which occurs at discontinuities; third, the regions of high stress concentration; and fourth, the number of elements used in a mesh. The explanations are as follows:

First: For a longwall mining, boundary geometry is not an obstacle since mining of the longwall panel leads to a rectangular opening, and so isoparametric elements can be designed to fit the opening easily. In addition, 8 node elements, used in the present investigation, can be arranged to fit curve boundaries exactly.

Second: Because there are many different layers of rock above and below the excavated zone, it is desirable to have a corresponding layer of elements for each layer of rock. This cannot be fully achieved since an extended computational effort would result. Therefore, in this study, eight layers of elements were chosen for the finite element mesh. This is a problem with all finite element solutions. However, regions of primarily shale or sandstone can be modeled and some special features such as the interface and limestone strata effects on subsidence
can be examined.

Third: The region of high stress concentration and material failure occurs at the entry support adjacent to the opening area. Material failure is expected to take place there as well as in the panel roof. For greater accuracy, the mesh layout has smaller size elements in this region.

Fourth: In general, the finer the mesh divisions the more accurate the results will be. However, this has a drawback because with too fine of a mesh division excessive computational effort is required. Since the results must be physically realistic and computationally possible, the final mesh division is a compromise of accuracy and computational time needed.

Taking into account the factors discussed, two final mesh designs illustrated in Figures 5.1 and 5.2 were chosen. Both meshes are almost identical except mesh B (Figure 5.2) includes the layer of limestone above the coal seam. Also incorporated into both meshes were two layers of claystone at 45 ft and 250 ft from the bottom of the mesh. As the result of geoprobing bore holes OU-1 and OU-2 before and after coal was mined and as the result of
FIGURE 5.1. FINITE ELEMENT MESH USED TO MODEL LONGWALL MINING WITHOUT SEPARATE LIMESTONE
Figure 5.2  FINITE ELEMENT MESH USED TO MODEL LONGWALL MINING WITH LIMESTONE ROOF INCLUDED
examining recovered cores, it was noticed that these two layers were weak and highly fractured. Therefore, by including layers at this location, the behavior of fractured rock was modeled in the finite element analysis with either a very weak element or an interface element.

5.4 ADAPTING EXPERIMENTAL DATA TO SEQCON

By dividing the finite element meshes into 8 and 10 layers as shown in Figures 5.1 and 5.2, the material constants obtained in the laboratory were averaged before inputting into SEQCON. The overburden being modeled as an upper sandstone and a lower shale for purposes of keeping the CPU time of calculation within reasonable limits. Material constants were selected according to the following procedure:

First: For coal (10 ft to 15 ft), claystone (45 ft to 55 ft and 250 ft to 260 ft), and in mesh B limestone (19 ft to 27 ft), material constants determined from laboratory testing were input directly into SEQCON.

Second: The elements located to the depth of 265 ft were averaged according to the thickness of each stratum.
Third: For the elements below node 116 in Mesh A and node 162 in Mesh B, which represents a depth of 265 ft, the material constants of rocks were again averaged over their corresponding strata.

Fourth: For linear interface elements, which are designed to simulate the effect of fracturing in the claystone layers, the material constants of claystone were used.

All constants obtained in accordance with the above procedure are shown in Figure 5.3. The coefficient of the lateral earth pressure used is dependent upon the Poisson's ratio as shown in the following equation:

\[ k = \frac{1}{(1-\nu)} \]

This equation is based on Hooke's Law and can be obtained by assuming transverse stress and strain to be zero.

5.5 FINITE ELEMENT RESULTS AND INTERPRETATIONS

When SEQCON was used to examine the subsidence caused by longwall mining, one major problem was the inability to limit deflections so that overlapping of elements above
E = 3.66 \times 10^{6} \text{ psi}

u = 0.18

c = 372 \text{ psi}

G = 35.3"

Figure 5.3

MATERIAL CONSTANTS AS DETERMINED FOR
SUBSTITUTION INTO SEQCON
and below the extracted coal seam could not occur. One of the methods employed in this respect was to manually input sufficient surface loadings on these nodes to prevent overlapping. The amount of surface loadings required were determined by trial and error. This method was adequate for analyses made with the linear elastic and the linear interface models. However, when used with Drucker-Prager constitutive model, this method resulted in an ill-conditioning of solutions. The ill-conditioning occurred because of the reloading of materials after material yield. To prevent overlapping of elements when Drucker-Prager model was used, the number of increments and iterations in the nonlinear solution process were limited to 4 and 2 respectively. It must be pointed out that by doing this the numerically simulated behavior of materials no longer represent the true stress-strain responses obtained from laboratory testing. The resulted subsidence profile using the Drucker-Prager model is shown here for the purpose of illustration only.

The finite element analyses based on the above procedure were conducted using Meshes A and B. The results were then compared with the measured values for subsidence and horizontal strain.
As seen in Figure 5.4, Drucker-Prager constitutive model gives more subsidence over the entry than both the linear elastic prediction and the measured field value. The difference is the result of the region of high stress which occurs in the region of entry support. Since the Drucker-Prager constitutive failure criterion permits large strain values, a compressive arch effect is believed to have resulted with more subsidence over the edges of the panel and less panel center subsidence than would be expected. Field measurements show that the compression arch effect fails to develop.

The effect of limestone on subsidence in the finite element analysis can be considered negligible (see Figure 5.5). Subsequently, only linear elastic modelling was used on Mesh B.

The subsidence analysis using the linear elastic constitutive equation is shown in Figures 5.4 and 5.5. It is seen that the linear elastic model predicts a wider subsidence trough and less maximum subsidence than measured values. The predicted curve results from two considerations: (1) the inability of the linear elastic model to reflect rock fracturing under tension and conditions where slippage between seams occur. (2) since
Figure 5.4  SUBSIDENCE COMPARISON WITH PREDICTION ON S.R. 689 BETWEEN 0.11 AND 0.26 MILEAGE POSTS
Figure 5.5
SUBSIDENCE COMPARISON WITH PREDICTION ON S.R. 689 BETWEEN 0.11 AND 0.26 MILEAGE

0 = MEASURED
△ = LINEAR ELASTIC WITH LIMESTONE
▼ = LINEAR ELASTIC WITHOUT LIMESTONE

[Diagram showing subsidence values]
the rock samples used for testing in the laboratory were intact, the material constants obtained may be higher than actual field values. The linear elastic model in this analysis is, therefore, further refined by incorporating interface elements and by reducing Young's modulus for the elements in the tension zone.

For the case in which moduli of the elements in the tension zone are reduced, analysis of stress output from the linear elastic solution is first performed in order to locate the elements which are in tension. These elements are then given lower moduli to represent failed regions. Figure 5.6 shows failed elements in Mesh A that were modeled in SEQCON; also a visual interpretation of this failure mechanism is also shown in Figure 5.7. The profile curve for this procedure, illustrated in Figure 5.8, matches very closely to that of the measured values.

An additional core hole, OU-3, was drilled into the center of the adjacent longwall panel along S.R. 689 after subsidence took place. This was done to investigate the hydraulic balance after mining. However, additional information was obtained regarding fracturing of the overburden which suggested the use of the reduced modulus method. Corings removed from OU-3 were less fractured
Figure 5.6  FINITE ELEMENT MESH SHOWING FAILED ELEMENTS USED IN SEQCON
Figure 5.7  FAILURE MECHANISM OF ROCK ABOVE A LONGWALL MINE
Figure 5.8  SUBSIDENCE COMPARISON WITH PREDICTION BETWEEN MILEAGE POST 0.11 AND 0.26 ON S.R. 689
than expected. Only the black shale strata, immediately above the excavated coal seam, were found in a rubble state. Therefore, it is concluded that the overburden rock failed with high angles cracks extending, in an irregular pattern, from the entry towards the surface. The failed materials in the overburden settled into the excavated zone in continuous segments and not in small broken fragments first believed. This failure mechanism is illustrated graphically in Figure 5.7. Based on these assumptions, it is important to include a crack analysis procedure into future finite element analysis program. Since it is not possible to implement this in SEQCON, the reduced modulus was used as an alternate procedure.

Results of a linear elastic, finite element solution with interface elements in Figure 5.8 predicts a maximum subsidence which is unreasonably large, but the profile above the panel corresponds to measured values.

Figures 5.9 and 5.10 show strain comparison to all elastic models. The three models predict reasonable strain values. The solution using interface elements predicts the best agreement to the average field values.

Since the numerical solution is based on equilibrium,
Figure 5.9

COMPARISON OF MEASURED STRAIN TO PREDICTED VALUES FOR S.R. 689
FROM 0.11 TO 0.26 MILEAGE POSTS
Figure 5.10 COMPARISON OF MEASURED STRAIN TO PREDICTED VALUES FOR S.R. 689 FROM 0.11 TO 0.26 MILEAGE POSTS
strains are expected to be more accurate for all numerical models than strains computed from a predicted subsidence profile.

In the case in which one of these models is to be used for finding the safe stopping distance of the longwall face or the expected damages if mining under is permitted, the criteria limiting the maximum permissible tensile and compression strains can be selected and a cost analysis carried out.

5.6 CONCLUSION

1. Because the Drucker-Prager constitutive model allows higher strains in the region of high stress found near the entry ways, the linear elastic constitutive model gives better subsidence estimates.

2. Because the material constants obtained from intact rock specimens in the laboratory may be higher than the actual values, the subsidence from the linear elastic model may predict smaller values than measured.
3. Interface elements used in the linear elastic solution provided a consistent but larger subsidence than measured. Moreover, this method predicted strain values closer to the average measured value.

4. By reducing moduli for the elements in which tensile cracks occur, the resulting subsidence profile matches more closely to the measured values.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSION

The examination of subsidence measurements from this investigation, gives subsidence profiles over three longwall panels which were mined beneath state highways in southeastern Ohio. Maximum subsidence were 4.0 ft, 3.3 ft, and 3.1 ft for S.R. 556 upper panel, and S.R. 556 lower panel. The rate of subsidence decreased to zero when the longwall mining face had proceeded to 600 ft, 650 ft, and 610 ft respectively, beyond the roadways. The average depths of mining were 380 ft, 630 ft, and 533 ft, respectively. A crack appeared on S.R. 689 before the longwall face passed beneath it. But surface subsidence was not considerable until the face had passed beyond the roadway. Ten percent additional subsidence was recorded on S.R. 556 on the upper panel when the lower panel was mined.

Comparison of the results from this investigation with an exponential profile method which was modified by Peng and Chiang (24) to the Appalachian coal region shows reasonable agreement between predicted and measured values
of subsidence. Such a prediction would be important when examining drainage effects, induced bending strains in structures, and differential settlement of structures. The largest surface curvature was measured in the direction of mining on the lower panel on S.R. 556. The curvature due to the moving longwall, however, is only temporary.

Maximum compressive strains measured tangent to the roadway were -14.0 and -5.7 micro in./in. for S.R. 689 and S.R. 556, respectively. Base lengths on which deformations were measured 25 ft on S.R. 589 and 100 ft on S.R. 556, respectively. Because of the rapid variation in surface movement, measurements taken on a 25 ft gage length are felt to provide a more reasonable estimate of horizontal strains experienced by any surface structure.

The most appropriate method for predicting strain levels before mining takes place is with a finite element program of the type which was adapted. Such a program must include an elastic response with a failure criterion. In this case the Drucker-Prager criterion proved unsatisfactory. Also an interface element was found to be necessary to model differential movement of rock strata. Examination of core specimens was found to
be adequate for determination of interface layer locations. Material constants were determined from unconfined compression tests. However, recent investigation by Sargand and Hazen (26) on grey shales have shown the shales to be elastic and transversely isotropic almost to failure. Thus a more accurate formulation will result from larger (6 in. diameter) cores tested in a multiaxial apparatus.

6.2 DAMAGE PREDICTION AND PREVENTION

As an outline procedure for estimating damages, the responsible party may determine the distance from the longwall face in which a 0.002 in./in. or more tensile strain and a 0.003 in./in. or more compressive strain occurs. This can best be done by applying the finite element method described in this investigation since the finite element method gives the most reliable strain results. The cost of resurfacing pavement and shoulders will then dictate the cost of such bond.

For the case in which a structure is located in the subsidence zone, the type of structure must be considered. If the structure has supports which allow
vertical and horizontal movements, then the damages should be minor. However, if the structure consists of rigid supports, strain and deformation produced by mining would determine the extent of damage and a cost estimate must be done on case to case basis.

If it is determined that coal must remain to protect against displacements or strains, then the distance at which mining should cease can be deduced from the measurements taken along S.R. 689. For example, a large surface crack (see Figure 2.15) was recorded along S.R. 689 on January 17. This would indicate the highway to be located at about 50 ft ahead of the longwall face. At this position, the maximum strain recorded along the highway indicates a strain level of 0.004 in./in. If 0.002 in./in. is taken as a maximum permissible tensile strain of the pavement, then, based upon this strain reading at this location, the longwall face should stop advancing 70 ft prior to reaching the spot directly under the pavement if damage has to be prevented. In such a case the responsible party would be required to negotiate for the value of the coal. However, for the usual case, it will be more economical to maintain the structure.

It must be noted that the distance estimate is
dependent upon the condition at each location and should take into account such factors as: (1) the stratigraphy of the area which includes depth of the overburden, height of the coal seam, and the inclination of the overburden materials as well as the coal seam, (2) the width (length) of the longwall panel which will be mined under the highway, and (3) the dimensions of structures to be protected.

6.3 RECOMMENDATIONS

Although the finite element program, SEQCON, has proved to be the most reliable in predicting subsidence and strain profile, it is still complicated to apply and the full capability of the program cannot be used. Some of the difficulties involved are:

1. The difficulty of the program to limit displacements of the elements above and below the coal seam so that overlapping will not occur.

2. The inability of the program to reflect the actual behavior of the materials in the mine overburden when failure strength is reached. When failure
occurs in the material above the mine, cracks will begin to form. At this point the material exhibits discontinuity and, therefore, the condition of compatibility assumed by the method is no longer valid.

Based on the inability of the program to limit displacements, one method for improvement could be the addition of a subroutine in the program to check overlapping of elements. When overlapping occurs the nodal points associated with the overlapping will be fixed, thus, limiting further movements. It is important that material loading, simulated in the program, must be done incrementally to insure that only a very small overlapping will occur.

Because the finite element method, when deriving from the potential energy equation, assumes that the compatibility of elements is maintained it is impossible to modify the program to account for the material's behavior after cracks have formed. However, the more advanced hybrid finite element method which does not assume the condition of compatibility can be adapted for use. The method is now at an early stage of development.
and further studies are needed before implementation can be made.
REFERENCES


APPENDICES
APPENDIX A

Triaxial Compression Tests
FIGURE A,3 TRIAXIAL COMPRESSION TEST FOR SANDSTONE DRILL HOLE OU-1 (DEPTH = 159'0")

FAILURE STRESS = 6885 PSI
CONFINING PRESSURE REDUCED TO FAILURE

FIGURE A.11 TRIAXIAL COMPRESSION TEST FOR SANDSTONE
DRILL HOLE OU-2 (DEPTH = 133’2")

FAILURE STRESS = 6490 PSI
Figure A.16 Triaxial Compression Test for Sandstone Drill Hole OU-2 (Depth = 302'8")

Failure Stress = 6855 PSI
FIGURE A.17 TRIAXIAL COMPRESSION TEST FOR SANDSTONE
DRILL HOLE OU-2 (DEPTH = 320.3°)
FIGURE A.18 TRIAXIAL COMPRESSION TEST FOR SHALE
DRILL HOLE OU-2 (DEPTH = 343'2")
FIGURE A.19 TRIAXIAL COMPRESSION TEST FOR LIMESTONE
DRILL HOLE OU-2 (DEPTH = 350' 5")

FAILURE STRESS = 21000 PSI
APPENDIX B

The Determinant of The Jacobian Matrix
Derivations for the Determinant of Jacobian Matrix

The Jacobian matrix was defined earlier in section 4.4.1, equation 4.4a as:

\[
[J] = \begin{bmatrix}
\frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\
\frac{\partial x}{\partial t} & \frac{\partial y}{\partial t}
\end{bmatrix}
\]

The determinant of this matrix can be written as:

\[
|J| = \frac{\partial x}{\partial s} \frac{\partial y}{\partial t} - \frac{\partial y}{\partial s} \frac{\partial x}{\partial t}
\]

For the case in which 8-nodes isoparametric elements are used, the determinant of the Jacobian matrix can be written as follow:

\[
|J| = \sum_{i=1}^{8} \sum_{j=1}^{8} \left( \frac{\partial N_i}{\partial s} \frac{x_i}{\partial t} \frac{\partial N_j}{\partial y} - \frac{\partial N_i}{\partial t} \frac{x_i}{\partial s} \frac{\partial N_j}{\partial y} \right)
\]

\[
|J| = \sum_{i=1}^{8} \sum_{j=1}^{8} \left( x_i \left( \frac{\partial N_i}{\partial s} \frac{\partial N_j}{\partial t} - \frac{\partial N_i}{\partial t} \frac{\partial N_j}{\partial s} \right) \right)
\]

After performing necessary substitutions, the final form of the determinant of the Jacobian matrix can be written as:
\[ |J| = \frac{1}{8} \left[ Y_1 + Y_2s + Y_3t + Y_4st + Y_5s^2 + Y_6t^2 + Y_7s^2t^2 + Y_8st^2 + Y_9s^2t^2 + Y_{10}s^3 + Y_{11}t^3 \right] \]

where \( Y_1 - Y_{11} \) is defined as:

\[ Y_1 = 2 \left[ x_{26}y_{48} - x_{48}y_{26} \right] \]
\[ Y_2 = \left[ x_{48}(y_{13} + y_{57}) - y_{48}(x_{13} + x_{57}) + 2\left( x_{26}(y_{12} - y_{23} - y_{25} - y_{27}) \right) \right] \]
\[ Y_3 = \left[ x_{26}(y_{13} + y_{57}) - y_{26}(x_{13} + x_{57}) + 2\left( x_{48}(y_{14} + y_{34} - y_{45} - y_{47}) \right) \right] \]
\[ Y_4 = 2 \left[ (x_{14}y_{24} - x_{24}y_{14}) + (x_{16}y_{18} - x_{18}y_{16}) + (x_{25}y_{45} - x_{45}y_{25}) + (y_{78}y_{27} - x_{27}y_{78}) + (x_{36}y_{34} - x_{34}y_{36}) + (x_{38}y_{28} + x_{28}y_{38}) + (x_{47}y_{46} + x_{46}y_{47}) + (x_{58}y_{68} - x_{68}y_{58}) + 2\left( x_{12}y_{18} - x_{18}y_{12} + x_{34}y_{23} - x_{23}y_{34} + x_{45}y_{46} - x_{46}y_{45} + y_{78}y_{67} - x_{67}y_{78} \right) \right] \]
\[ Y_5 = \left[ y_{48}(x_{17} + x_{35}) - x_{48}(y_{17} + y_{35}) + 2\left( x_{12}y_{17} - x_{17}y_{12} \right) + x_{36}y_{13} + x_{13}y_{36} + x_{23y_{25} - x_{25}y_{23}} + x_{48}y_{26} - x_{26}y_{48} + (x_{56}y_{57} - x_{57}y_{56}) \right] \]
\[ Y_6 = \left[ x_{26}(y_{57} - y_{13}) - y_{26}(x_{57} - x_{13}) + 2\left( x_{13}y_{18} - x_{18}y_{13} + x_{47}y_{14} - x_{14}y_{47} + x_{48}y_{26} - x_{26}y_{48} + x_{34}y_{35} - x_{35}y_{34} + x_{57}y_{58} - x_{58}y_{57} \right) \right] \]
\[ Y_7 = \left[ 2(x_{23}y_{12} - x_{12}y_{23}) + 3(x_{15}y_{12} - x_{12}y_{15} + x_{16}y_{36} - x_{36}y_{16} + x_{27}y_{23} - x_{23}y_{27}) + 4(y_{48}(y_{16} + y_{36} - y_{25} - y_{27}) - y_{48}(x_{16} + x_{36} - x_{25} - x_{27})) + 5(x_{56}y_{57} - x_{57}y_{56}) \right] \]
\[ \gamma_8 = [2(x_{78}y_{17} - x_{17}y_{78}) + 3(x_{14}y_{17} - x_{17}y_{14}) + x_{58}y_{15} \\
- x_{15}y_{58} + x_{78}y_{37} - x_{37}y_{78}) + 4(x_{26}(y_{18} + y_{38} - y_{45} - y_{47}) \\
- y_{26}(x_{18} + x_{38} - x_{45} - x_{47})) + 5(x_{34}y_{35} - x_{35}y_{34})] \]

\[ \gamma_9 = 3 [x_{37}y_{15} - x_{15}y_{37} + y_{48}(x_{17} + x_{35}) - x_{48}(y_{17} + y_{35}) \\
+ x_{26}(y_{34} + y_{57} - y_{18} - y_{48}) - y_{26}(x_{34} + x_{57} - x_{18} - x_{48})] \]

\[ \gamma_{10} = 2 [(x_{12} - x_{23})(y_{56} - y_{67}) - (x_{56} - x_{67})(y_{12} - y_{23})] \]

\[ \gamma_{11} = 2 [(x_{34} - x_{45})(y_{18} + y_{78}) - (x_{18} + x_{78})(y_{34} - y_{45})] \]

Note: It should be pointed out that the subscript in the above equations denote subtractions. For example \( x_{78} = x_{7} - x_{8} \) and \( y_{26} = y_{2} - y_{6} \), etc. Where \( x_{7}, x_{8}, y_{2}, y_{6} \) are the distances from origin for node 7, 8, 2, 6 in the x and y directions.
APPENDIX C

The [B] Matrix
The [B] Matrix and its contents

As defined in section 4.4.2, equation (4.4.2a), [B] can be written as:

\[
[B] = \begin{bmatrix}
[B]' & 0 \\
0 & [B]''
\end{bmatrix}
\]

where \([B]' = [B_{11}, B_{12}, B_{13}, B_{14}, B_{15}, B_{16}, B_{17}, B_{18}]\]
\([B]'' = [B_{21}, B_{22}, B_{23}, B_{24}, B_{25}, B_{26}, B_{27}, B_{28}]\]

also \(B_{1k}\) and \(B_{2k}\) which are defined by equations (4.4.2b) and (4.4.2c) as:

\[
B_{1k} = \frac{1}{|J|} \Sigma_{j=1}^{8} \left( \frac{\partial N_k}{\partial s} \frac{\partial N_j}{\partial t} - \frac{\partial N_k}{\partial t} \frac{\partial N_j}{\partial s} \right) y_j
\]

\[
B_{2k} = \frac{1}{|J|} \Sigma_{j=1}^{8} \left( \frac{\partial N_k}{\partial s} \frac{\partial N_j}{\partial t} - \frac{\partial N_k}{\partial t} \frac{\partial N_j}{\partial s} \right) x_j
\]

where \(k = 1, 2, 3 \ldots 8\)

By substitutions of appropriate terms \(B_{1k}\) and \(B_{2k}\) can be written as:

\[
B_{11} = \frac{1}{8|J|} \left[ s(-2y_{26}-y_{48})+t(-y_{26}-2y_{48})+2st(3y_{28}-y_{46})+ \\
s^2(2y_{23}+2y_{67}+y_{48})+t^2(2y_{34}+2y_{78}+y_{26})+s^2t(3y_{56}-4y_{48} \\
-5y_{23})+st^2(3y_{45}-4y_{26}-5y_{78})+3s^2t^2(y_{23}+y_{46}+y_{78})+ \right]
\]
\[+2s^3(y_{56} - y_{67}) + 2t^3(y_{43} + y_{45})\]

\[B_{12} = \frac{1}{8|J|} \left[ 2y_{48} + 2s(y_{16} + y_{36} + y_{56} + y_{76}) + t(y_{13} + y_{57}) + 2st(y_{43} + y_{85} + y_{87} + 2y_{83} + 3y_{41}) + 2s^2(y_{31} + y_{75} + y_{84}) + t^2(y_{31} - y_{75} + 2y_{84}) + s^2t(5y_{13} + 3y_{75} + 8y_{48}) + 4st^2(y_{14} + y_{34} + y_{58} + y_{78}) + 3s^2t^2(y_{31} + y_{57} + 2y_{84}) + 4s^3(y_{65} + y_{67}) \right] \]

\[B_{13} = \frac{1}{8|J|} \left[ s(y_{48} - 2y_{26}) + t(y_{26} - 2y_{48}) + 2st(y_{68} + 3y_{24}) + s^2(y_{48} + 2y_{12} + 2y_{56}) - t^2(y_{26} + 2y_{14} + 2y_{58}) + s^2t(3y_{67} - 4y_{48} - 5y_{12}) - st^2(3y_{78} + 4y_{26} - 5y_{48}) + 3s^2t^2(y_{12} + y_{45} + y_{68}) + 2s^3(y_{56} - y_{67}) + 2t^3(y_{18} + y_{78}) \right] \]

\[B_{14} = \frac{1}{8|J|} \left[ 2y_{62} + s(y_{13} + y_{57}) + 2t(y_{18} + y_{38} + y_{58} + y_{78}) + 2st(y_{12} - y_{67} - 3y_{23} + 3y_{56}) - s^2(y_{12} + y_{35} + 2y_{56}) + 2t^2(y_{13} + y_{26} + y_{57}) + 4s^2t(y_{12} - y_{67} - y_{23} + y_{56}) - st^2(3y_{17} + 5y_{35} - 8y_{26}) - 3s^2t^2(y_{17} + y_{35} - 2y_{26}) - 4t^3(y_{18} + y_{78}) \right] \]

\[B_{15} = \frac{1}{8|J|} \left[ -s(y_{48} + 2y_{26}) - t(y_{26} + 2y_{48}) + 2st(y_{28} + 3y_{46}) + s^2(2y_{23} - y_{48} + 2y_{67}) - t^2(y_{26} - 2y_{34} + 2y_{78}) - s^2t(3y_{12} + 4y_{48} - 5y_{67}) + st^2(3y_{18} - 4y_{26} + 5y_{34}) - 3s^2t^2(y_{26} - y_{37} + y_{48}) - 2s^3(y_{12} - y_{23}) + 2t^3(y_{18} + y_{78}) \right] \]

\[B_{16} = \frac{1}{8|J|} \left[ -2y_{48} - 2s(y_{12} - y_{23} - y_{25} - y_{27}) - t(y_{13} + y_{57}) \right] \]
\[
-2st(y_{14}+y_{38}+3y_{45}+3y_{78})-2s^2(y_{13}+y_{57}-y_{48})+t^2(y_{13}-y_{57}+2y_{48})
+4s^3(y_{12}-y_{23})
\]

\[
B_{17} = \frac{1}{8|J|} \left[ -s(y_{48}+2y_{26})+t(y_{26}-2y_{48})+2st(y_{24}+3y_{68}) 
+s^2(2y_{12}-y_{48}+2y_{56})+t^2(y_{26}-2y_{14}-2y_{58})-s^2t(3y_{23}+4y_{48} 
-5y_{56})+st^2(3y_{34}-4y_{26}+5y_{18})-3s^2t^2(y_{12}+y_{45}+y_{68}) 
-2s^3(y_{12}-y_{23})-2t^3(y_{34}-y_{45}) \right]
\]

\[
B_{18} = \frac{1}{8|J|} \left[ 2y_{26} - s(y_{13}+y_{57}) - 2t(y_{14}+y_{34}-y_{45}-y_{47}) + 2st(y_{56} 
-y_{23}+3y_{12}-3y_{67})+s^2(y_{15}+y_{37}-2y_{26})+2t^2(y_{12}-y_{35}+y_{67}) 
-4s^2t(y_{12}-y_{23}+y_{56}-y_{67})-st^2(3y_{35}+5y_{17}-8y_{26}) 
+3s^2t^2(y_{12}-y_{23}-y_{56}+y_{67})+4t^3(y_{34}-y_{45}) \right]
\]

By replacing \( y_{ij} \) with \( x_{ij} \), \( B_{2k} = -B_{1k} \) for \( k = 1, 2, 3 \ldots 8 \)

for example:

\[
B_{26} = \frac{1}{8|J|} \left[ 2x_{48} + 2s(x_{12}-x_{23}-x_{25}-x_{27})+t(x_{13}+x_{57})+2st(x_{14} 
+x_{38}-3x_{45}+3x_{78})+2s^2(x_{13}+x_{57}-x_{48})-t^2(x_{13}-x_{57}+2x_{48}) 
-s^2t(3x_{13}-5x_{57}+8x_{48})-3s^2t^2(x_{13}-x_{57}+2x_{48})-4s^3(x_{12} 
-x_{23}) \right]
\]
APPENDIX D

Body Force Vector
The Body Force

As mentioned previously in Section 4.5, the body force can be written as:

\[
\{Q_1\} = h \int_1^1 \int [N]^T_1 \begin{bmatrix} 0 \\ \gamma \end{bmatrix} |J| \, dsdt
\]

By performing necessary substitutions the reduced form of this equation can be shown below:
\[ \{ Q_1 \} = \frac{h}{2} y \begin{pmatrix}
\frac{1}{4} \left( -\frac{y_1}{3} - \frac{y_2}{9} - \frac{y_3}{9} + \frac{y_4}{9} - \frac{y_5}{45} - \frac{y_6}{45} - \frac{y_7}{15} - \frac{y_8}{45} - \frac{y_9}{15} - \frac{y_{10}}{15} - \frac{y_{11}}{15} \right) \\
\frac{1}{4} \left( \frac{y_1}{3} - \frac{y_2}{9} + \frac{y_3}{9} - \frac{y_4}{45} - \frac{y_5}{45} - \frac{y_6}{45} - \frac{y_7}{15} - \frac{y_8}{45} - \frac{y_9}{15} - \frac{y_{10}}{15} - \frac{y_{11}}{15} \right) \\
\frac{1}{4} \left( -\frac{y_1}{3} + \frac{y_2}{9} - \frac{y_3}{9} - \frac{y_4}{45} - \frac{y_5}{45} - \frac{y_6}{45} + \frac{y_7}{15} + \frac{y_8}{45} + \frac{y_9}{15} + \frac{y_{10}}{15} + \frac{y_{11}}{15} \right) \\
\frac{1}{4} \left( \frac{y_1}{3} + \frac{y_2}{9} + \frac{y_3}{9} + \frac{y_4}{45} - \frac{y_5}{45} - \frac{y_6}{45} + \frac{y_7}{15} + \frac{y_8}{45} + \frac{y_9}{15} + \frac{y_{10}}{15} + \frac{y_{11}}{15} \right) \\
\frac{1}{4} \left( -\frac{y_1}{3} + \frac{y_2}{9} + \frac{y_3}{9} - \frac{y_4}{45} - \frac{y_5}{45} + \frac{y_6}{45} + \frac{y_7}{15} + \frac{y_8}{45} + \frac{y_9}{15} + \frac{y_{10}}{15} + \frac{y_{11}}{15} \right) \\
\frac{1}{4} \left( \frac{y_1}{3} + \frac{y_2}{9} + \frac{y_3}{9} - \frac{y_4}{45} + \frac{y_5}{45} + \frac{y_6}{45} + \frac{y_7}{15} + \frac{y_8}{45} + \frac{y_9}{15} + \frac{y_{10}}{15} + \frac{y_{11}}{15} \right)
\end{pmatrix} \]

Note: \( y_1 - y_{11} \) are constants from the determinant from the Jacobian matrix.
APPENDIX E

Computer Input Cards and Examples
SEQCON COMPUTER PROGRAM GUIDE

The purpose of this appendix is to provide the basic information for running SEQCON program. The user should already have some background in Fortran language and finite element analysis before attempting to use SEQCON.

E.1 USER'S GUIDE

Problem Information

<table>
<thead>
<tr>
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<th>Column</th>
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</thead>
<tbody>
<tr>
<td>Problem Title</td>
<td>1-80</td>
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(8I5, 4E10.3) Problem Parameters

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<tr>
<td>Number of nodes</td>
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<td>Number of elements</td>
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<td>Number of materials</td>
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<td>Constitutive model</td>
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<td>1 - linear elastic</td>
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<tr>
<td>2 - variable moduli</td>
<td></td>
</tr>
<tr>
<td>3 - Drucker-Prager</td>
<td></td>
</tr>
<tr>
<td>4 - Sandler-Cap</td>
<td></td>
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<tr>
<td>Number of load increments</td>
<td>21-25</td>
</tr>
<tr>
<td>(for 2 sequences of construction input)</td>
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</tr>
<tr>
<td>2 - load increments</td>
<td></td>
</tr>
<tr>
<td>Maximum number of iterations</td>
<td>26-30</td>
</tr>
<tr>
<td>Problem type</td>
<td>31-35</td>
</tr>
<tr>
<td>0 - Axisymmetric</td>
<td></td>
</tr>
<tr>
<td>1 - Plane strain</td>
<td></td>
</tr>
<tr>
<td>2 - Plane stress</td>
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Tape output flag  
0 - No write  
1 - Write

Graph option (To obtain graph  
computer must have the facility,  
if not input 0)  
0 - No graph  
1 - Graph

Unit weight of water

Atmospheric pressure (Only  
needed for variable moduli)

Thickness of elements (Default - 1.00)

Size of graph (Y-axis height in  
inches default - 9, maximum - 30)

Material Properties (3 cards/material)

CARD 1:  (2I5, 4E10.3)

<table>
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<td>1 - variable moduli</td>
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<td>2 - Drucker-Prager</td>
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<td>3 - Sandler-Cap</td>
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</tr>
<tr>
<td>4 - linear interface</td>
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<tr>
<td>5 - hyperbolic interface</td>
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</tr>
<tr>
<td>Element type</td>
<td>6-10</td>
</tr>
<tr>
<td>1 - soil</td>
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</tr>
<tr>
<td>2 - structure</td>
<td></td>
</tr>
<tr>
<td>3 - interface</td>
<td></td>
</tr>
<tr>
<td>4 - 3 node bar</td>
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</tr>
<tr>
<td>5 - 2 node bar</td>
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Young's modulus for solid and bar elements; normal stiffness for interface elements

Poisson's ratio for solid elements; area for bar elements; shear stiffness for interface elements

Unit weight

Coefficient of lateral earth pressure

Unit weight of water

CARD 2 (5E10.3)

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<td>V3</td>
<td>21-30</td>
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<tr>
<td>V4</td>
<td>31-40</td>
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<td>V5</td>
<td>41-50</td>
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CARD 3 (5E10.3)

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<td>V7</td>
<td>11-20</td>
</tr>
<tr>
<td>V8</td>
<td>21-30</td>
</tr>
<tr>
<td>V9</td>
<td>31-40</td>
</tr>
<tr>
<td>V10</td>
<td>41-50</td>
</tr>
</tbody>
</table>
NOTE: Variables V1 - V10 on cards 2 and 3 are defined below:

1. Linear elastic
   V1 - V10 are not used (Input 2 blank cards)

2. Variable moduli
   a) Solid element
      V1 - angle of internal friction (PHI)
      V2 - cohesion (C)
      V3 - tension cutoff (T)
      V4 - failure ratio (RF)
      V5 - exponent (N)
      V6 - loading factor (K)
      V7 - unloading factor (XU)
      V8 - Poisson parameter (G)
      V9 - Poisson parameter (D)
      V10 - Poisson parameter (F)
   b) Interface element
      V1 - angle of interface friction (PHI)
      V2 - cohesion of interface (C)
      V3 - tension cutoff normal to interface (T)
      V4 - failure ratio (RF)
      V5 - exponent (N)
      V6 - loading factor (K)
      V7 - V10 not used (Input blank card for card 3)

Nodal Point Data

(I5, 3X, 2I1, 4E10.3, T80, I1)

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<td>Fixity node for x-direction</td>
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<tr>
<td>1 - Fixed</td>
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<tr>
<td>X coordinate</td>
<td>11-20</td>
</tr>
<tr>
<td>Y coordinate</td>
<td>21-30</td>
</tr>
<tr>
<td>Water Head</td>
<td>31-40</td>
</tr>
<tr>
<td>Change in head</td>
<td>41-50</td>
</tr>
</tbody>
</table>
Data generation code

1 - generated fixity codes same as last one
0 - generated fixity code is 0
(Note: Automatic generation is possible. Nodes must be equidistant and on a straight line.
Input first and last node. Fixity code is set to zero unless the last node card data generation code is set to 1. Then the fixity code is set the same as the last node.)

Element Data

(12I5, 12X, 8I1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element number</td>
<td>1-5</td>
</tr>
<tr>
<td>Node I</td>
<td>6-10</td>
</tr>
<tr>
<td>Node J</td>
<td>11-15</td>
</tr>
<tr>
<td>Node K</td>
<td>16-20</td>
</tr>
<tr>
<td>Node L</td>
<td>21-25</td>
</tr>
<tr>
<td>Node M</td>
<td>26-30</td>
</tr>
<tr>
<td>Node N</td>
<td>31-35</td>
</tr>
<tr>
<td>Node O</td>
<td>36-40</td>
</tr>
<tr>
<td>Node P</td>
<td>41-45</td>
</tr>
<tr>
<td>Material type number</td>
<td>46-50</td>
</tr>
<tr>
<td>Integration type</td>
<td>51-55</td>
</tr>
<tr>
<td>0 - 2x2 point</td>
<td></td>
</tr>
<tr>
<td>1 - 3x3 point</td>
<td></td>
</tr>
<tr>
<td>Flag for midpoint node generation</td>
<td>56-60</td>
</tr>
<tr>
<td>0 - 2x2 generation</td>
<td></td>
</tr>
<tr>
<td>1 - no generation</td>
<td></td>
</tr>
<tr>
<td>Element generation data</td>
<td>72-80</td>
</tr>
</tbody>
</table>
(Note: Automatic generation is possible. Input first element of pattern and first element to break the pattern. In 73-80 place number to be added to previous's element's nodal values to obtain net element's values. Place this data on card which breaks the pattern, not the first card. Referring to Fig. 1: to generate an element besides the one shown, look at node I and the corresponding node in the next element. Take the difference between their global node numbers. Place this number in column 73 as the pattern code. Continue for each node J, K, L, M, N, P and columns 74, 75, 76, 77, 78, 79, 80 respectively.

Fig. 1. 2D element

Fig. 2. 3-node bar element

Fig. 3. 2-node bar element

Fig. 4. Interface element
## Output Control Information

\[
\text{(A4/16I5 as many cards as needed)}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of output control</strong> (Note</td>
<td>1-1</td>
</tr>
<tr>
<td>if input blank card default is ALL)</td>
<td></td>
</tr>
<tr>
<td>ALL - Print all nodes and elements, (skip 1 and 2 below)</td>
<td></td>
</tr>
<tr>
<td>DISP - nodes limited (Do 1 and skip 2)</td>
<td></td>
</tr>
<tr>
<td>STRS - elements limited (Skip 1 and do 2)</td>
<td></td>
</tr>
<tr>
<td>BOTH - nodes and elements limited (Do both 1 and 2)</td>
<td></td>
</tr>
</tbody>
</table>

1. **Nodes specified** (Number of nodes for which output is wanted)
   - Nodes for which output is wanted | 1-5 |
   - 1-5 | |
   - 6-10, etc. | |

2. **Elements specified** (Number of elements for which output is wanted)
   - Element for which output is wanted | 1-5 |
   - 1-5 | |
   - 6-10, etc. | |

## Increment Control Parameters

\[
\text{(4I5, E10.3, 2I5, 1X, 4I1)}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sequence type</strong></td>
<td>1-5</td>
</tr>
<tr>
<td>0 - in situ</td>
<td></td>
</tr>
<tr>
<td>1 - dewatering</td>
<td></td>
</tr>
<tr>
<td>2 - excavating</td>
<td></td>
</tr>
<tr>
<td>3 - embankment</td>
<td></td>
</tr>
<tr>
<td>4 - surface or point load only</td>
<td></td>
</tr>
<tr>
<td>5 - initial bar stresses</td>
<td></td>
</tr>
</tbody>
</table>
Number surface load cards  
(1 card per element)  

Number concentrated loads  
(at nodes only)  

Mesh modification loads  
0 - mesh is good  
1 - delete mesh  
2 - add mesh  

Number of load increments for this  
sequence to be divided into (Default  
is 1.00)  

Stress transfer flag  
0 - no transfer  
1 - transfer  

Number of specified nodal  
displacements  

Flag for output for this  
sequence (FLAG1)  
0 - output  
1 - no output  

Flag for output of iterations (FLAG2)  
0 - no output  
1 - output  

Limited for sequence output (FLAG3)  
1 - all increments output  
2 - every second increment output  
3 - every third increment output  
4 - etc. (Last increment is always output)  

Type of solution  
0 - frontal  
1 - band  

Note: Repeat this card for each load increment.
Mesh Modification Data
(Not needed if mesh is not to be changed)

CARD 1  (3I5)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLAG1 for elements added or deleted</td>
<td>1-5</td>
</tr>
<tr>
<td>FLAG2 for nodes added or deleted</td>
<td>6-10</td>
</tr>
<tr>
<td>FLAG3 for elements which have changes in material properties</td>
<td>11-15</td>
</tr>
<tr>
<td>Note: Input either 1 or 0</td>
<td></td>
</tr>
</tbody>
</table>

CARD 2  (I5, 1X, A4)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element number added or deleted</td>
<td>1-5</td>
</tr>
<tr>
<td>Key</td>
<td>6-10</td>
</tr>
<tr>
<td>Blank - no generate</td>
<td></td>
</tr>
<tr>
<td>GEN     - generate</td>
<td></td>
</tr>
<tr>
<td>LAST    - last element</td>
<td></td>
</tr>
<tr>
<td>Note: Input as many cards as needed; if not needed FLAG1</td>
<td>0.</td>
</tr>
<tr>
<td>Note: Input as many cards as needed; if not needed FLAG1</td>
<td>0.</td>
</tr>
<tr>
<td>Note: Input as many cards as needed; if not needed FLAG1</td>
<td>0.</td>
</tr>
</tbody>
</table>

CARD 3  (E5, 3X, 2I1, A4)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node number to be added or deleted</td>
<td>1-5</td>
</tr>
</tbody>
</table>
Fixity code for X degree of freedom 9
0 - added
1 - deleted variable

Fixity code for Y degree of freedom 10
0 - added
1 - deleted

Key 12-15
Blank - no generate
GEN - generate
LAST - last node

Note: Use as many cards as needed; if not needed FLAG2 = 0.

CARD 4 (2I5, 1X, A4)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element number to have material changed 1-5</td>
<td></td>
</tr>
<tr>
<td>New material number</td>
<td>6-10</td>
</tr>
</tbody>
</table>

Key 12-15
Blank - no generate
GEN - generate
LAST - last element

Notes:
1. Use as many cards as needed; if not needed FLAG3 = 0.
2. Elements or nodes in numerical order may be generated by specifying first element or node in a string with key being blank and the last element or node of string with key being GEN or LAST.
### Initial Bar Stresses

(2I5, E10.3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number elements for which stresses is specified</td>
<td>1-5</td>
</tr>
<tr>
<td>Element number</td>
<td>6-10</td>
</tr>
<tr>
<td>Initial stress</td>
<td>11-20</td>
</tr>
</tbody>
</table>

Note: Not needed if there are no initial stresses.

### Surface Load

(3I5, 4E10.3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISC</td>
<td>1-5</td>
</tr>
<tr>
<td>JSC</td>
<td>6-10</td>
</tr>
<tr>
<td>KSC</td>
<td>11-15</td>
</tr>
<tr>
<td>STXI</td>
<td>16-25</td>
</tr>
<tr>
<td>STXK</td>
<td>26-35</td>
</tr>
<tr>
<td>STYI</td>
<td>36-45</td>
</tr>
<tr>
<td>STYK</td>
<td>46-55</td>
</tr>
</tbody>
</table>

Note:
1. Card for surface load is not needed if there is no surface load.
2. For definition of the above variables see figure shown.

### Sequencing Construction

For two or more load increments, input increment control parameter card here. If needed, the mesh
modification card can be input after the increment control parameter card.

E.2 INPUT EXAMPLE

Following is an example of inputting data for an excavation problem. The finite element mesh is divided into 416 elements and 1333 nodes. There are two load increments used: (1) In situ stress and (2) for excavation stress. Nine elements are excavated in the second load increment.
** Data for Longwall Mining Project (linear elastic) 56-Elements **

```
<table>
<thead>
<tr>
<th>Node</th>
<th>100</th>
<th>56</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>3.088E+05</td>
<td>0.320</td>
<td>0.09744</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1.333E+05</td>
<td>0.422</td>
<td>0.05580</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3.651E+05</td>
<td>0.187</td>
<td>0.09545</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1.690E+05</td>
<td>0.250</td>
<td>0.08863</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1.550E+05</td>
<td>0.300</td>
<td>0.72210</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

11 0.0 0.0
22 600.0 0.0
33 1200.0 0.0

**** cards for node 4 to 98 is here ****

```
<table>
<thead>
<tr>
<th>Node</th>
<th>100</th>
<th>56</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>00</th>
</tr>
</thead>
<tbody>
<tr>
<td>199</td>
<td>10</td>
<td>6000.0</td>
<td>4680.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>35</td>
<td>53</td>
<td>52</td>
<td>51</td>
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<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>36</td>
<td>55</td>
<td>54</td>
<td>53</td>
</tr>
</tbody>
</table>
```

**** cards for element #1 to #55 input here ****
<table>
<thead>
<tr>
<th>FORMAT STATEMENT</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>56</td>
<td>174</td>
<td>175</td>
<td>176</td>
<td>184</td>
<td>199</td>
<td>198</td>
<td>197</td>
</tr>
<tr>
<td></td>
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<td>16</td>
<td>16</td>
<td>16</td>
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<tr>
<td></td>
<td>11</td>
<td>LAST</td>
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<tr>
<td></td>
<td>41</td>
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<td>5</td>
<td>LAST</td>
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</tr>
</tbody>
</table>
Job Stream Example

ID BATCHMON
/M
/JOB HAZEN N1986ZZZ ZLONGWLL HAZEN
/TELL
CP LINK HAZEN 191 400 OVER
ACCESS 400 C
GLOBAL TXTLIB FORTLIB TEKLIB CALCOMP
LOAD SEQCONX
FI 2 DISK OUT2 DATA
FI 3 DISK OUT3 DATA
FI 5 DISK LONWAL9 DATA C
FI 6 DISK LONGOUT DATA
START
/*
/*