INDUCTION MOTOR OPERATION WITH SERIES CAPACITANCE.

A Thesis Presented to
The Faculty of the College of Engineering and Technology
Ohio University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
Deib, Deib Ali
November, 1986
In the name of God, Most Gracious, Most Merciful.

"Verily, this is My Way, Leading straight: follow it"

(Al-Qur'an VI:153)

"O my Lord! advance me In knowledge"

(Al-Qur'an XX:114)
Acknowledgments

My greatest gratitude is to Al Mighty Allah s.a.w. for helping me in every second of my life especially in preparing this thesis.

I would like to thank Dr. H. W. Hill for his helpful guidance and advice throughout my study.

I would also like to thank my brother Shamsul for his help in editing.

The utmost sincere thanks are given to my parents and my wife for their assistance and moral support.
Table of Contents

<table>
<thead>
<tr>
<th>Content</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgments</td>
<td>iv</td>
</tr>
<tr>
<td>Table of Figures</td>
<td>vii</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>viii</td>
</tr>
<tr>
<td>Abstract</td>
<td>1</td>
</tr>
</tbody>
</table>

Chapter 1

1 - 1 Introduction                      | 2    |
1 - 2 Objectives of this Thesis         | 4    |

Chapter 2 Induction Motor Performance

2 - 1 Simplified Model of Induction Motor | 5    |
2 - 2 Resonance in the Induction Motor Circuit | 8    |
2 - 3 Current and Torque Equation        | 10   |
2 - 4 Extra High Starting Torque         | 18   |
2 - 5 Output Power                       | 23   |
2 - 6 Motor Efficiency                   | 28   |
2 - 7 Input Power Factor                 | 31   |
2 - 8 Starting Time of the Induction Motor | 32   |
2 - 9 Experimental Work                  | 34   |
2 - 10 Discussion                       | 37   |
Table of Figures

(2 - 1) A per-phase circuit model of a three-phase induction motor...... 5
(2 - 2) Approximate equivalent circuit..... 7
(2 - 3) Torque-slip curves for different capacitive reactances.............15
(2 - 4) Current-slip curves for different capacitive reactances.............17
(2 - 5) Current-slip curves for normal starting and extra-high torque.....20
(2 - 6) Mechanical power-slip curves for different capacitive reactances....27
(2 - 7) Efficiency-slip curve.................30
(3 - 1) Torque and slip limits..............44
(3 - 2) Speed-torque characteristics for the suggested method and the rotor capacitor method..................45
(4 - 1) Induction motor motion after sudden change in load torque..........56
List of Symbols

$R_1$ stator resistance ($\Omega$)
$R_2$ rotor resistance referred to stator-side ($\Omega$)
$X_1$ stator leakage reactance ($\Omega$)
$X_2$ rotor leakage reactance referred to stator-side ($\Omega$)
$X_L$ total reactance ($X_1 + X_2$)
$X_m$ magnetizing reactances ($\Omega$)
$X_c$ capacitive reactance ($\Omega$)
$f$ supply frequency (Hz)
$w_s$ $2 \cdot f$ (rad/sec)
$w_m$ rotor speed (rad/sec)
$s$ rotor slip in per unit
$p$ number of poles
$V_s$ supply voltage (V)
$I_s$ stator current (A)
$I_r$ rotor current (A)
$P_g$ air-gap power (W)
$P_m$ mechanical power (W)
ABSTRACT

An improvement in the performance of the induction motor can be achieved by the insertion of a series capacitor in the stator circuit. A reduction in the rotor slip at a given load torque is achieved, which yields to an increase in the motor efficiency. The machine stability is highly increased by increasing the machine stability limit (pull-out torque). A high starting torque is achieved by the insertion of a stepped resistor in the rotor circuit of the slip-ring induction motor. In addition, the capacitor-resistor combination can be used for speed control for given torque and speed settings.
The induction machine is the most commonly used of all rotating electrical machines. Many types of motors are used in a wide variety of applications - from fractional horsepower two-phase motors to 45,000 horsepower polyphase motors for wind tunnels, and tremendous numbers of induction motors are used in many home appliances. The features that justify this popularity are largely economic; the induction motor is mechanically simple and therefore inexpensive. It is rugged and requires very little maintenance, but in addition to these features, its performance characteristics can be adjusted to fit a number of different operating conditions by relatively simple design changes.\(^{(1)}\)

During the starting period of the induction motor, the input current must not be excessive, typically, no more than six or seven times the full-load current. The starting torque should be at least 1.5 times the full-load torque or higher. The starting torque is dependent on rotor resistance. A high rotor resistance results in a high starting torque. In wound-rotor machines, external resistance in the rotor circuit may be conveniently used. Thus a high starting torque can be maintained throughout the
accelerating period, and the time required to reach full speed is thereby reduced. (2)

The induction motor exhibits the characteristics of the shunt direct-current motors between zero slip and the slip at which the maximum torque is developed (3). Secondary impedance control with wound-rotor motors is a method of discrete-speed operation of induction motors. Use of a series resistor-capacitor network in the rotor circuit has been suggested by Shepherd (4) for any desired speed setting below the synchronous speed. Also, M. Ayyadurai (3) suggested the use of a resistor-capacitor-inductor combination in the rotor circuit to reduce the capacitor size, and to improve the speed regulation. However, the required capacitor size is still very large due to the low frequency in the rotor circuit.
1.2. Objectives of this Thesis

This thesis report suggests a method of getting a very high starting torque in the wound-rotor motors by the insertion of a series capacitor in the stator circuit and a stepped resistor in the rotor circuit. This resistor-capacitor combination increases the starting torque up to 5 times the normal starting one, while keeping the starting current the same as normal. Also, the series terminal capacitor increases the pull-out torque of the motor up to 8 times the rated torque, which yields to a very stable machine under sudden changes in load torque.

For speed control, the use of the capacitor in the stator side reduces its size by a factor of \((\text{slip})^2\) from that one which is needed by Shepherd or Ayyadurai in their suggestions for a given torque and slip settings. The reduction factor in the capacitor size is about \(10^{-3} - 10^{-4}\) at the rated slip. This reduction is due to the relatively high frequency in the stator circuit compared with the rotor circuit. In addition, the series capacitor reduces the speed regulation of the motor, especially at heavy loads.

Also, the equation of motion of the induction motor has been derived.
CHAPTER 2.
Induction Motor Performance

2.1. Simplified Model of the Induction Motor

The circuit of figure (2.1) is a per-phase equivalent circuit, referred to the primary side, of a three-phase induction machine with a series terminal capacitor.

\[ V_{th} = V_s \frac{jX_m}{R_1 + j(X_1 + X_m - X_C)} \]  \hspace{1cm} (2-1)
For approximate calculation, very little error is involved in using the following approximation:

\[ jX_m = R_1 + j(X_1 - X_C + X_m) \quad (2-2) \]

since

\[ R_1 \ll (X_1 - X_C + X_m) \]

and

\[ (X_1 - X_C) \ll X_m \]

Then

\[ V_{th} \approx V_s \quad (2-3) \]

Also, the Thevenin equivalent impedance will be the impedance appearing at terminals 1-2 with \( V_s \) replaced by a short:

\[ Z_{th} = \frac{jX_m [R_1 + j(X_1 - X_C)]}{R_1 + j(X_1 - X_C + X_m)} \quad (2-4) \]

Using the above approximation of equation (2-2), \( Z_{th} \) can be approximated by :-
From equations (2-3) and (2-5), the circuit of fig. (2-1) can be approximated to that of fig. (2-2) with an error of only about 5 percent at normal operation ($X_C = 0$ or $X_C = 2X_L$) (5), and with a less error if $0 < X_C < 2X_L$ due to the decrease in the voltage drop across $(X_1 - X_C)$.

\[ Z_{th} = R_1 + j(X_1 - X_C) \]  \hspace{1cm} (2-5)

Fig (2-2) Approximate equivalent circuit

Let $X_L$ be the overall reactance of the machine referred to the stator side

\[ X_L = X_1 + X_2 \]  \hspace{1cm} (2-6)
2.2. Resonance in the Induction Motor Circuit

By looking to the right into terminals 3 and 4 of fig. (2-1), the equivalent impedance will be:

\[ Z_{\text{eq}} = \frac{jX_m (R_2/S + jX_2)}{R_2/S + j(X_m + X_2)} \] (2-7)
\[ = R + jX \] (2-8)

where
\[ R = \frac{(R_2/S)X_m^2}{(R_2/S)^2 + (X_2 + X_m)^2} \] (2-9)

and
\[ X = \frac{X_m [(R_2/S)^2 + X_2 (X_2 + X_m)]}{(R_2/S)^2 + (X_2 + X_m)^2} \] (2-10)

The input impedance into terminals 5 and 6 becomes

\[ Z_{\text{in}} = R_1 + j (X_I - X_C) + Z_{\text{eq}} \] (2-11)
\[ = (R_1 + R) + j(X_I + X - X_C) \] (2-12)

Note that:

1) \( R_2/S \) is function of the speed. Its minimum value equals \( R_2 \), and its maximum value equals \( R_2/S_r \) (\( S_r \) is the rated slip).

2) \( X_m \) is not constant. Its value depends on the saturation (or the excitation current). If the saturation increases, \( X_m \) will decrease.

Therefore, the circuit of fig (2-1) is a non-linear circuit, and does not have a single resonance. Its resonance depends on the speed \((R_2/S)\), and the saturation \((X_m)\). However, at standstill \( X_m \gg R_2 + jX_2 \).
Then

\[ R = R_2, \quad X = X_2. \]

and the required capacitive reactance for resonance at standstill equals to \((X_1 + X_2)\).

At any speed the required capacitive reactance to resonate with the machine inductive reactance is given by \((X_1 + X)\). This term \((X_1 + X)\) is not easy to predict because \(X\) depends on \(X_m\) which depends on the degree of saturation of the machine.
2.3. Current and Torque Equations

Referring to fig.(2-2), the magnitude of the stator current is

\[ I_s = \frac{V_s}{\sqrt{\left(\frac{R_1 + R_2}{S}\right)^2 + \left(X_L - X_C\right)^2}} \]  \hspace{1cm} (2-13)

Per-phase air gap power can be expressed in terms of the applied voltage and the impedance of fig.(2-2), in particular the rotor impedance and current as

\[ P_g = (R_2/S) I_s^2 \quad \text{W} \]  \hspace{1cm} (2-14)

The per-phase power converted to mechanical form is

\[ P_m = (1 - S) P_g \quad \text{W} \]  \hspace{1cm} (2-15)

and the motor speed is

\[ w_m = \frac{2}{P} (1 - S) w_s \quad \text{rad/s} \]  \hspace{1cm} (2-16)

Then, the internal per-phase torque exerted on the rotor is

\[ T = \frac{P_m}{w_m} \quad \text{N.m} \]  \hspace{1cm} (2-17)
From equations (2-13) to (2-17)

\[ T = K \left( \frac{R_2}{S} \right) \frac{1}{\left( R_1 + \frac{R_2}{S} \right)^2 + (X_L - X_C)^2} \]  

(2-18)

where

\[ K = \frac{P V_s^2}{2 w_s} \]  

(2-19)

From equation (2-18) it is clear that the addition of the capacitive reactance decreases the machine impedance, yielding an increase in the motor torque at a specified rotor slip (speed).

At small slips, the rotor resistance is very high and much larger than machine inductive reactance. Therefore, the capacitor effect on increasing the motor developed torque is small. But as the slip increases, the rotor resistance decreases and the capacitor effect becomes more and more dominant.

In the torque equation (2-18), the only variable on the right side for a given capacitor setting is the slip S. The maximum magnitude of the developed torque, and the slip at which it occurs, can therefore be determined by setting \( dT/dS = 0 \).
The slip at which the maximum torque occurs is

\[ S_m = \pm \frac{R_2}{\left[R_1^2 + (X_L - X_C)^2\right]^{1/2}}. \quad (2-20) \]

The negative sign corresponds to generator operation, and the positive sign corresponds to motor operation.

Note that the slip at which maximum torque occurs \((S_m)\) is directly proportional to rotor resistance \((R_2)\), and inversely proportional to the total reactance of the machine. The capacitor effect clearly decreases the total reactance, or increases the slip corresponding to the maximum torque \((S_m)\).

As \(S_m\) increases, the stable operation zone of the motor widens. Thus, the terminal series capacitor increases the motor stability by increasing the stable operation zone.

The maximum torque may be found by inserting the expression for \(S_m\) (equation 2-20) into the equation for the developed torque (2-18). Note that in equation (2-20) the term \(R_2/S_m\) is independent of \(R_2\). Therefore \(T_{max}\) is also independent of \(R_2\) (equation 2-21).
From equations (2-18) and (2-20)

\[ T_{\text{max}} = \frac{K}{2} \frac{1}{R_1 + \left[R_1^2 + (X_L - X_C)^2\right]^{1/2}}. \] (2-21)

This torque is limited by the stator resistance, machine reactance, and the capacitor reactance. The stator resistance cannot be reduced to increase \( T_{\text{max}} \). But, the total reactance can be reduced because \( X_L - X_C \) is less than \( X_L \) for \( 0 < X_C < 2X_L \). For this reason, if \( X_C \) increases, the total reactance \( (X_L - X_C) \) will decrease, and thus the maximum torque increases. The highest possible torque for a certain machine can be achieved at resonance \( (X_L = X_C) \). This torque is equal to

\[ T = \frac{K}{4R_1}. \] (2-22)

This torque is as high as 10 times the rated torque for a typical machine, and it is independent of rotor resistance, fig (2-3).

Inspection of equation (2-18) indicates that the starting torque at resonance can be found by setting \( S = 1 \), and \( X_L = X_C \).
Then,

\[ T_s = K \frac{R_2}{(R_1 + R_2)^2} \]  \hspace{1cm} (2-23)

This torque is the highest possible starting torque. For a typical set of machine parameters, it equals about 8 times the rated torque, fig.(2-3), and it is almost constant from stand still up to about 0.3 slip. This very high torque decreases the starting time of the motor. But the corresponding starting current is also very high (about 20 times the rated current, fig.(2-4)).
Fig (2-3) Torque slip curves for different capacitive reactances

- $X_C = X_L$
- $X_C = 1.5X_L$
- $X_C = 2X_L$ or $X_C = 0$
This very high starting current may cause two kinds of stresses:

1. Mechanical stresses, because the force between any two wires carrying the same current ($I$) is proportional to $I^2$. As the current increases the forces between the coils and between the wires inside the coil also increase.

2. Thermal stresses, since the dissipated power in a resistor is proportional to $I^2$ ($P = RI^2$). Therefore, the dissipated power will be very high if the starting current exceeds its limits (about 5 times the rated current) and it may overheat the machine if it continues for a long time.

In the following section a method is developed to decrease the starting current while keeping a high starting torque.
Fig (2-4) Current slip curves for different capacitive reactances.
2.4. Extra High Starting Torque With Normal Starting Current

From equation (2-13) the magnitude of the starting current is

\[ I_s = \frac{V_s}{\sqrt{[(R_1 + R_2)^2 + (X_L - X_C)^2]}} \quad (2-24) \]

This current is inversely proportional to the machine impedance. The capacitor decreases the total impedance, thus it increases the starting current. At resonance \((X_L = X_C)\) the starting current is much higher than the normal starting current. If an external resistor \((R_{ex})\) is inserted in the rotor circuit, the total impedance will increase countering the decrease due to the capacitor, and the starting current will decrease. The rotor external resistance \((R_{ex})\) can be adjusted to give a normal starting current.

Therefore,

\[ \frac{V_s}{\sqrt{[(R_1 + R_2)^2 + X_L^2]}} = \frac{V_s}{R_1 + R_2 + R_{ex}} \]

\[ (2-25) \]

From which

\[ R_{ex} = [(R_1 + R_2)^2 + X_L^2]^{1/2} - (R_1 + R_2) \quad (2-26) \]
Now the rotor resistance is higher than the stator resistance. Thus, from equation (2-20), the slip at which the motor will break-down ($S_m$) is higher than unity. This means that the machine can run at any speed from standstill up to its rated speed (depending on the load torque).

The starting torque becomes

$$T_s = k \frac{(R_2 + R_{ex})}{(R_1 + R_2 + R_{ex})^2}. \quad (2-27)$$

This torque is still very high (about 7 times the rated torque), and it is equal to the pull-out torque of the machine, as in fig.(2-5). The torque and current drop continuously as the speed decreases.

It should be noted that the maximum developed torque may be maintained throughout the accelerating period, with a reasonable starting current, if a continuously decreasing resistance is used in the rotor external circuit. When the slip approaches its rated value, then the external resistor can be shorted while the terminal series capacitor stays in the circuit. At this instant the breakdown torque is given by equation (2-22). The machine now becomes highly stable under sudden changes in the load torque.
Fig (2-5) Current-slip curves for starting torque

a) Normal operation
b) Extra-high operation
A useful relation in calculating starting performance of high inertia loads is:

The total heat generated in the rotor windings of an induction motor in bringing the load from rest to full speed is equal to the kinetic energy supplied to the rotating parts, if friction and load torques are neglected. That is, whatever the shape of the speed-torque curve, every kilowatt-second of kinetic energy stored in the rotating parts requires an equal number of kilowatt-seconds to be dissipated in secondary copper loss, when an induction motor accelerates a load from rest to its rated speed.\(^{(6)}\)

Consider now the starting currents of fig.(2-5). Comparing the starting currents of the machine with:— (1) the suggested starting method (graph 'b'), and the normal starting, no capacitor nor rotor external resistance, (graph 'a'). From the above relations between rotating-parts kinetic energy and dissipated heat in the rotor circuit, it is clear that the dissipated heat in the rotor circuit (including the external rotor resistance) is the same for both the suggested method and the normal method. However, for the suggested method most of the heat is dissipated in the external resistance (outside the machine, which is about 3 times the rotor resistance. This means that in the
suggested method the heat generated in the rotor circuit (inside the machine) has been reduced by 75 percent of that of the normal starting method. Therefore, the motor has the capability to start more frequently, and to start higher inertia loads.

Also, from graphs a & b, the starting current of the suggested method is lower than that of the normal starting method at slips lower than unity. In addition, the starting period is reduced due to the high starting torque. This means that the dissipated energy in the stator circuit is also reduced by the suggested method.

From the above discussion, it is clear that the starting torque is much higher than normal, the initial acceleration of the motor-load combination is increased, the starting time required to reach steady state is reduced, the starting current throughout the starting period is decreased, and the generated heat inside the rotor and stator circuits is reduced by the suggested method, which decrease the initial thermal stresses in the machine-winding insulation.
2.5. Output Power

From equations (2-16), (2-17), and (2-18), the per-phase power converted to mechanical form is

\[ P_m = R_2 \frac{(1 - S)}{S} \frac{V_s^2}{(R_1 + R_2/S)^2 + (X_L - X_C)^2} \]  

(2-28)

Inspection of the mechanical power equation, (2-28), indicates that for a specified speed (or slip), the mechanical power can be increased if the machine impedance is decreased. This can be achieved by a series terminal capacitor which reduces the circuit impedance, fig.(2-2).

For a given power setting, the motor slip is less than normal if the series capacitor is inserted in the stator circuit. But the rotor efficiency is given by \((1 - S)\). Therefore, the terminal capacitor increases the machine efficiency at a given output mechanical power. This enables the machine to drive higher loads with the same heat loss inside the windings.
The per-phase maximum power that can be developed by the motor can be found by setting $dP/dS = 0$ in equation (2-28). This can be done easily by breaking $R_2/S$ into its components: 1) $R_2$ which represents the rotor copper loss, and 2) $R_2(1-S)/S$ which represents the mechanical load, and set $dP/dr = 0$

\[
\text{where} \quad r = R_2 \frac{(1-S)}{S} \quad (2-29)
\]

then

\[
(R_1 + R_2 + r)^2 + (X_L - X_C)^2 - 2r(R_1 + R_2 + r) = 0
\]

\[ (2-30) \]

or

\[
r = \left( (R_1 + R_2)^2 + (X_L - X_C)^2 \right)^{1/2}
\]

\[ (2-31) \]

From equations (2-29) and (2-31)

\[
S_{mp} = \frac{R_2}{\left[ (R_1 + R_2)^2 + (X_L - X_C)^2 \right]^{1/2} + R_2}
\]

\[ (2-32) \]
and from equations (2-28) and (2-31)

\[ P_{\text{max}} = \frac{V_s^2}{2\left[(R_1 + R_2)^2 + (X_L - X_C)^2\right]^{1/2} + (R_1 + R_2)} \]  

(2-33)

The efficiency at this condition is slightly higher than 50 percent for a typical machine. Therefore, the machine should not reach this point of operation. If it runs at this condition for a long time it will overheat, and the winding insulation will be damaged.

Comparing equations (2-20) and (2-32) it is clear that the slip at which maximum delivered power occurs is less than that at which maximum torque occurs.

In equation (2-32) if the capacitor reactance resonates with the machine reactance \((X_L = X_C)\), then the slip \(S_{mp}\) becomes higher than that of the normal operation (without series capacitor). \(S_{mp}\) increases almost by a factor of 2.

In addition, equation (2-33) indicates that at resonance the maximum power is
\[ P_{\text{max}} = \frac{V_s^2}{4(R_1 + R_2)} \quad (2-34) \]

and from equation (2-13) the corresponding current is
\[ I_s = \frac{V_s}{2(R_1 + R_2)} \quad (2-35) \]

which is about 7 times the rated current for a typical machine parameters.

Also, the corresponding efficiency is 50 \%. Therefore, the machine should never be allowed to reach this point of operation at steady-state, because the dissipated power inside the machine in the form of heat is about 50 times the dissipated power at rated load (note that the dissipated power in the machine windings resistance is proportional to \( I^2 \)).
Fig. (2-6) Mechanical power as a function of the rotor slip.
2.6. Motor Efficiency

The power loss of an induction motor consists of five elements:

1 - The primary (stator) copper loss.
2 - The slip loss, or rotor copper loss.
3 - The core losses ($P_{h+e}$).
4 - The stray-load losses ($P_{SL}$).
5 - The friction and windage losses.

The core losses depend on the applied voltage and frequency. They are independent of the load torque. In practice the core losses are constant for a specified source voltage and frequency, and they may be about 2 percent of the rated power (6). Also the stray-load losses may be about 1 percent of the load power (6).

The friction and windage losses are usually empirically determined. They are function of the speed. For a good approximation they are about 1 percent of the rated load (6).

The major losses are the copper losses ($P_{Cu}$). They depend on the load ($P_{Cu} = (R_1 + R_2) I_s^2$). As the load torque increases, the current increases, and the copper losses increase.
Then, the per-phase output power of the motor is

\[ P_{\text{out}} = P_m - P_{fw} - P_{h+e} - P_{SL} \]  
\[ = P_m - 0.01P_r - 0.02P_r - 0.01P_m \]  
\[ = 0.99P_m - 0.03P_r \]  

(2-36)

(2-37)

where

- \( P_m \) is given by equation (2-28)
- \( P_r \) is the per-phase rated power
- \( P_{\text{out}} \) is the per-phase mechanical power delivered to the load.

The per-phase input electrical power to the motor is

\[ P_{\text{in}} = P_m + I_s^2(R_1 + R_2) \]  

(2-38)

where

- \( I_s^2(R_1 + R_2) \) represents the stator and rotor copper losses.

Then the efficiency becomes

\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \]  

(2-39)
Fig. (2-7) Motor efficiency as a function of the slip.
2.7. Input Power Factor

The input power factor for the circuit of fig. (2-2), under steady state operation, is given by

\[ p.f = \cos \left( \frac{1}{Z_{\text{in}}} \right) \]

\[ = \frac{R_1 + R_2/S}{\left[ (R_1 + R_2/S)^2 + (X_L - X_c)^2 \right]^{1/2}} \]  \hspace{1cm} (2-40)

Actually the input power factor is lower than that of equation (2-40) due to the lagging magnetizing current \( I_m \) (fig.(2-1)) which was neglected in the approximate circuit of fig.(2-2) in section (2-1).

From equation (2-40) it is clear that the capacitor improves the source power factor. If the circuit is at resonance, the power factor will be unity for all load settings. If the capacitive reactance is higher than the machine inductive reactance, then the input power factor will be leading. If \( X_c > 2X_L \), then from equations (2-18), (2-21), (2-33), and (2-40), the starting torque, pull-out torque, maximum developed mechanical power, and input power factor will be less than those of the normal machine. Therefore, \( X_c \), should be less than twice \( X_L \).
2.8. Starting Time of the Induction Motor

Neglecting the friction, windage, and load torques throughout the starting period, the equation of motion of an induction motor becomes

\[ T = J \frac{dw_m}{dt} \]  \hspace{1cm} (2-41)

and

\[ w_m = \frac{(2/p)(1 - S) w_s}{\text{rad/sec}} \]  \hspace{1cm} (2-42)

where

\[ w_m = \text{the motor speed in rad/sec.} \]
\[ T = \text{the induced torque given by equation (2-18)} \]
\[ P = \text{number of poles}. \]

Then from equations (2-18), (2-41), and (2-42)

\[
K \left( \frac{R_2}{S} \right) \left( \frac{1}{(R_1 + R_2/S)^2 + (X_L - X_C)^2} \right) = \frac{2}{p} \left( \frac{Jw_s}{dt} \right)
\]  \hspace{1cm} (2-43)

By separation of variables

\[
- \left\{ \left( \frac{R_1^2 + X_2}{K_1} \right) s + \frac{2R_1R_2}{K_1} + \frac{R_2^2}{K_1S} \right\} ds = dt
\]  \hspace{1cm} (2-44)
where
\[ K_1 = \frac{KR_2 P}{2Jw_s} , \quad X = X_L - X_c . \]  \hspace{1cm} (2-45)

At standstill \( t = 0 \), and \( S = 1 \), and at the rated speed \( S = S_0 \).

Then by integrating both sides of equations (2-44)
\[
\left(\frac{R_1^2 + X_2}{2K_1}\right)(1 - S_0^2) + \frac{2R_1 R_2}{K_1}(1 - S_0) - \frac{R_2^2}{K_1} \ln S_0 = t_0
\]
\hspace{1cm} (2-45)

where
\[ t_0 = \text{starting time of the machine from standstill up to its rated speed.} \]

However, the starting time is higher than \( t_0 \) because we have neglected all mechanical load torques. A more accurate starting time can be computed from equation (4-16) in section (4-2), where the mechanical load torque has been taken into consideration.
2.9. Experimental Work

The test machine is a three-phase squirrel-cage induction motor. The machine data and parameters are:

- power = 1/4 hp
- speed = 1670 rpm
- line voltage = 208 V
- line current = 1.2 A

From the machine tests (no load and locked-rotor tests) the following parameters have been calculated:

\[ R_1 + R_2 = 19.5 \ \Omega \]
\[ X_1 + X_2 = 25.4 \ \Omega \ (X_1 \approx X_2) \]

\[ R_1 \] can be measured by a direct current supply. It is found to be

\[ R_1 = 12 \ \Omega \]

therefore,

\[ R_2 = 7.5 \ \Omega \]
a) Table (2-1) shows the starting torque of the test machine at rated voltage for different capacitor settings.

<table>
<thead>
<tr>
<th>Capacitance (F)</th>
<th>Measured Torque (lb.in)</th>
<th>Calculated Torque (lb.in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>no capacitor</td>
<td>18</td>
<td>12.86</td>
</tr>
<tr>
<td>40</td>
<td>3.6</td>
<td>6.42</td>
</tr>
<tr>
<td>80</td>
<td>15</td>
<td>29.93</td>
</tr>
<tr>
<td>120</td>
<td>23</td>
<td>33.7</td>
</tr>
<tr>
<td>240</td>
<td>27</td>
<td>24</td>
</tr>
</tbody>
</table>

Table (2-1)

b) Table (2-2) shows the relations between the magnetizing reactance and the applied voltage.

<table>
<thead>
<tr>
<th>$V_{ph}$ (volt)</th>
<th>$I_m$ (A)</th>
<th>$X_m$ (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>.52</td>
<td>192</td>
</tr>
<tr>
<td>110</td>
<td>.59</td>
<td>186</td>
</tr>
<tr>
<td>120</td>
<td>.65</td>
<td>184</td>
</tr>
<tr>
<td>130</td>
<td>.73</td>
<td>178</td>
</tr>
<tr>
<td>140</td>
<td>.82</td>
<td>170</td>
</tr>
<tr>
<td>150</td>
<td>.92</td>
<td>163</td>
</tr>
<tr>
<td>160</td>
<td>1.08</td>
<td>148</td>
</tr>
<tr>
<td>180</td>
<td>1.44</td>
<td>124</td>
</tr>
<tr>
<td>200</td>
<td>1.93</td>
<td>103</td>
</tr>
<tr>
<td>210</td>
<td>2.25</td>
<td>93</td>
</tr>
</tbody>
</table>

Table (2-2)
c) The load test with the series capacitor failed because the machine began to resonate with the series capacitor yielding to a very high current equal to or exceeding the short-circuit current of the machine. As an example:

1) with a 40 F series capacitor, the machine started-up to 1250 rpm. At this speed a high in-rush current (higher than the short-circuit current) flowed into the machine and its speed dropped down to 1000 rpm.

2) the experiment was repeated with higher series capacitances, but the resonant problem remained. As the capacitance was increased the machine began to vibrate at low speed although it was unloaded.

3) while the motor was connected to the 208 V three-phase supply, it was driven by another dc motor at the synchronous speed and the following tests were done.

i) a 40 F capacitor was introduced in series with the motor leading to a higher stator current (higher than short-circuit) and a very high terminal voltage (484 V) appeared across the machine terminals.
ii) with an 80 F capacitor a high voltage and current also appeared, the speed dropped down to 1650 rpm, and the induction motor exhibited some vibration.

iii) with 120 F, the current and voltage were lower than before, but the machine speed dropped down to 1400 rpm and the induction motor exhibited a very high vibration.

2.10. Discussion

1. From table (2-2), it is clear that the magnetizing reactance drops as the voltage increases.

2. From equation (2-10), at high speeds, $X_2 < X < X_m$, which means that the machine will resonate at some speed below its synchronous speed for any value of $X_c$ less than $X_m$.

3. The resonance between the machine and the capacitor causes a high current to flow into the machine yielding to a higher saturation in the magnetic circuit which results into a drop in $X_m$ and $X$.

4. When $X_m$ drops down at resonance, the resonance slip increases (resonance speed decreases). This may be the reason of the speed drop at resonance.
CHAPTER 3.
Speed Control of Induction Motor
by an External Impedance

3.1. Previous Work

The induction motor suffers from the drawback that, in contrast to dc motors, its speed cannot be easily and efficiently varied continuously over a wide range of operating conditions. However, it exhibits the characteristics of the shunt dc motor between its synchronous speed and that of break-down torque. Many attempts have been made to obtain shunt characteristics at more than one speed at fixed voltage and frequency, by changing the number of poles. The number of poles can be changed in the ratio of 2 to 1 by reconnecting the stator winding of the squirrel-cage motor. This method is not practical for wound-rotor motors, because the rotor windings would also have to be reconnected to have the same number of poles as the stator. The number of poles can be changed also by the pole amplitude modulation method. It has the advantage that the two speed, need not have a two-to-one ratio. But these methods need a special design(2).
Rotor-impedance control with wound-rotor motors is another method of discrete speed control. This was done by:
(a) conventional external rotor resistance control (2),(5),(6). (b) saturable reactors(7), (c) by passive impedance(3),(4).

The external rotor-resistance method suffers from the poor efficiency of the system at reduced speed (high slip), since the rotor circuit copper loss increases with the slip increase.

The use of a series resistor-capacitor in the rotor circuit has been suggested by Shepherd(4) for any desired speed setting below the synchronous speed. Also, Ayyadurai(3) suggested different schemes in the rotor circuit of the induction motor. These schemes are: (a) capacitor, (b) series inductor-capacitor combination, (c) series resistor-inductor-capacitor combination, and (d) series resistor-capacitor network. These schemes suffer from the very large required capacitor size due to the low frequency in the rotor circuit. Also, these schemes suffer from bad speed regulation, and poor efficiency due to the power loss in the resistive element in schemes (b),(c), & (d). Scheme (a) suffers from the very sharp drop in the torque as the slip decreases. At low slip scheme (a) produces very low output torque.
If $X_C$ of fig. (2-2) is moved from the stator side to the rotor circuit, then its value referred to the stator side becomes $X_C/s^2$ (3), (4).

Then the developed torque of equation (2-12)

$$T = K \frac{R_2}{s} \frac{1}{(R_1 + R_2/s)^2 + (X_L - X_C/s^2)^2} . \quad (3-1)$$

For a given speed (slip) and torque settings, the required capacitive reactance is

$$X_C = S^2 \left( X_L \pm \frac{KR_2}{TS} - \left[ R_1 + R_2/s \right]^2 \right)^{1/2} . \quad (3-2)$$

$X_C$ has to be as large as possible to give lower value of capacitance (reduce capacitor size), and also to make the term $(X_L - X_C/s^2)$ negative so as to give net capacitive effect (leading power factor) at the set speed. Hence equation (3-2) is rewritten taking the plus sign before the square root as (3)

$$X_C = S^2 \left( X_L + \left[ \frac{KR_2}{TS} - (R_1 + R_2/s)^2 \right]^{1/2} \right) . \quad (3-3)$$

From equation (3-3), for a given speed and torque setting, $X_C$ can, therefore be evaluated.
3.2. Speed Control by Series Capacitor

From equation (2-18), for a given torque and speed settings, the required capacitive reactance, becomes

\[ X_C = X_L \pm \left[ \frac{K R_2}{T S} - \left( R_1 + R_2/s \right)^2 \right]^{1/2} \]  

(3-4)

Also, for lower capacitor size and leading power factor, the positive sign should be taken into account as

\[ X_C = X_L + \left[ \frac{K R_2}{T S} - \left( R_1 + R_2/s \right)^2 \right]^{1/2} \]  

(3-5)

In equations (3-3), and (3-5) the following condition must be satisfied

\[ \frac{K R_2}{T S} - \left( R_1 + R_2/s \right)^2 \geq 0 \]  

(3-6)

from which

\[ T \leq \frac{K R_2 S}{(S R_1 + R_2)^2} \]  

(3-7)

for a specified speed setting.

If T equals the right side of equation (3-7), then the required capacitor size is \( X_C = X_L \), or the circuit becomes
resonant. The torque (T) limits of equation (3-7) are those of fig. (3-1), which show the highest possible torque at any slip.

Also, equation (3-6) may be rewritten as

\[ S \geq \frac{R_2}{2R_1^2} - \frac{K}{T} - 2R_1 \left( \frac{K}{T} - 4R_1 \right) \]  

which is another form of equation (3-7), but it gives the slip limits for any given torque setting.

If the slip (S) equals the right side of equation (3-8), then the speed is the highest possible speed at that load torque (T) and, from equation (3-5), the circuit is at resonance \((X_C = X_L)\). The graph of fig. (3-1) shows the minimum slip at any load torque.

Now comparing equations (3-3) and (3-5), it is clear that at any torque and slip settings, the required rotor capacitive reactance equals \(S^2\) times that of the terminal capacitive reactance. This means that by using the terminal capacitor instead of the rotor capacitor, its size have been reduced by a factor of \(S^2\). This factor equals about \(10^{-3}-10^{-4}\) at rated speed.

If the machine is required to run at slips higher than the pull-out slip, then an external resistance should be
added in the rotor circuit (and this can be done only in slip-ring motors) to increase the pull-out slip. But this will reduce the machine efficiency, due to the power loss in this resistor.

In this case the network parameters can be synthesized as follows:

1 - From equation (3-5) calculate the required capacitive reactance for the specified load conditions (torque and slip).

2 - Calculate the pull-out slip from equation (2-14). It should be higher than the operating slip.

3 - If the pull-out slip is less than the operating slip, then add an external resistor in the rotor circuit to achieve the above requirements. The required external resistor is given by

\[ R_{ex} = S_m \left[ R_1^2 + (X_L - X_C)^2 \right]^{1/2} \quad (3-9) \]

4 - Then repeat step 1 with the new rotor resistance \( R_{ex} + R_2 \) instead of \( R_2 \).

5 - Repeat steps 2, 3, and 4 for several iterations. The number of iterations depend on the required accuracy of the spare pull-out slip value.

Fig. (3-2) shows the speed-torque characteristics for both the suggested method and rotor capacitor method for a given torque and speed setting (\( 1.5T_r, 1.56S_r \)).
Fig (3-1) Torque and slip limits
Fig (3-2) Speed-torque Characteristics:
(a) suggested method
(b) rotor capacitor method
In comparing both methods it's clear that:

(a) The required rotor capacitance is about 3000 times that of the series capacitance.

(b) The suggested method has better speed regulation than the other method.

(c) The suggested method pull-out torque is higher than that of the other method.

(d) The suggested method starting torque is higher than that of the other method (about 2 times).
4.1. Stability Problem

Stability and stability limit are defined in "American Standard Definitions of Electrical Terms," as follows:

[Stability, when used with reference to a power system, is that attribute of the system, or part of the system, which enables it to develop restoring forces between the elements thereof, equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements.

A stability limit is the maximum power flow possible through some particular point in the system when the entire system or the part of the system to which the stability limit refers is operating with stability].

The terms stability and stability limit are applied to both steady-state and transient conditions. Steady-state stability limit refers to the maximum flow of power possible through a particular point without the loss of stability when the power is increased very gradually. Transient stability limit refers to the maximum flow of power, possible through a point without the loss of stability when a sudden disturbance occurs.

For an induction motor the above definitions are slightly different. The steady-state stability limit refers to the maximum electromagnetic induced torque (which almost equals the load torque if the friction and windage losses) are neglected through a particular point without the loss of stability when the load torque is increased very gradually. The transient stability limit refers to the maximum induced torque through a point without the loss of stability when a sudden disturbance occurs.

Consider an induction motor connected to a large power source (constant voltage and frequency). The induced torque is determined by the voltage of the source, its frequency, the rotor slip, and the machine parameters (as shown in equation (2-18)). The induced torque of the motor when it is running at steady-state is, of course, equal to the mechanical output torque plus friction and windage losses.
If the mechanical load torque on the motor is increased, the motor cannot supply the entire load until its induced torque increases. Therefore, the motor slows down. The rotor slip of the motor increases until the motor induced torque is equal to the mechanical load torque plus losses. While the slip is increasing (rotor is slowing down), the excess of the torque required by the motor over the electric torque (induced torque) is supplied by the stored energy in the rotating system.

In the following section, the equation of the motion of the motor is derived.
4.2. Induction Machine Equation of Motion

Any difference between the overall mechanical torque (load torque plus the torque caused by the friction, windage, and core losses) and the electromagnetic torque developed must cause acceleration or deceleration of the machine.

The basic equation of motion is

\[ T_e - T_m = J \frac{dW_m}{dt} + BW_m \]  \hspace{1cm} (4-1)

where

- \( T_m \) = Mechanical load torque in N.m.
- \( T_e \) = Electromagnetic torque in N.m (given by equation (2-18)).
- \( J \) = Moment of inertia of all rotating parts (motor's rotor and load) referred to the motor shaft, in kg.m\(^2\).
- \( B \) = Damping torque coefficient of the motor and the load referred to the motor shaft, in Nm.sec.
- \( W_m \) = Rotor shaft velocity in rad/sec.

The electrical transient that occurs in the machine is usually neglected, because its duration time is very small compared to the mechanical transient, and the machine current will reach its steady-state value before the mechanical transient begins.
The damping torque, $BW_m$, is caused by rotor-bearing friction, windage, magnetic losses (core losses), and other drag torques that oppose rotation. The damping torque is small compared to the other torques; for this reason the damping torque can be disregarded to simplify the development.

Therefore:

$$T_e - T_m = J \frac{dW_m}{dt} \quad (4-2)$$

but

$$W_m = \frac{2}{P} W_s (1 - S) \quad (4-3)$$

By differentiating both sides of equation (4-3) with respect to time (t).

$$\frac{dW_m}{dt} = - \frac{2}{P} W_s \frac{dS}{dt} \quad (4-4)$$

but

$$W_m = \frac{2}{P} W_s(1 - S) \quad (4-3)$$

By differentiating both sides of equation (4-3) with respect to time (t).

$$\frac{dW_m}{dt} = - \frac{2}{P} W_s \frac{dS}{dt} \quad (4-4)$$
Substituting equation (4-4) into equation (4-2)

\[ T_m - T_e = \frac{2}{P} JW_s \frac{dS}{dt} \]  \hspace{1cm} (4-5)

Substituting the value of \( T_e \) from (2-12) in (4-5)

\[ T_m - k \frac{R_2}{S} \frac{1}{(R_1 + R_2/S)^2 + (X_L - X_C)^2} = \frac{2JW_s}{P} \frac{dS}{dt} \]  \hspace{1cm} (4-6)

let

\[
\begin{aligned}
X_L - X_C &= X \\
\frac{T_m P}{2JW_s} &= K_1 \\
\frac{KR_2 P}{2JW_s} &= K_2
\end{aligned}
\]  \hspace{1cm} (4-7)

Then equation (4-6) becomes

\[
\frac{(R_1^2 + X^2)^2 K_1 S^2 + (2R_1 R_2 K_1 - K_2) S + R_2^2 K_1}{(R_1^2 + X^2)^2 S^2 + 2R_1 R_2 S + R_2^2} = \frac{dS}{dt} \]  \hspace{1cm} (4-8)

This is a simple first order differential equation. At steady-state \( dS/dt = 0 \).

Thus

\[
(R_1^2 + X_2) S^2 + (2R_1 R_2 - (K_2/K_1)) S + R_2^2 = 0 \]  \hspace{1cm} (4-9)
The roots of equation (4-9) are

\[ S_{1,2} = \frac{(KR_2) - 2R_1R_2 \pm \sqrt{(KR_2)^2 - 4R_2(KR_1R_2 + R_2X^2)}}{2(R_1^2 + X^2)} \]  

where

\[ S_2 > S_1 > 0 \] for motor operation.

\[ S_1 \] represents the operating slip corresponds to a load torque \((T_m)\).

Now by separation of variables in equation (4-8)

\[ \frac{(R_1^2 + X^2)S^2 + 2R_1R_2S + R_2^2}{(R_1^2 + X^2)K_1S^2 + 2R_1R_2K_1 - K_2)S + R_2^2K_1} \]  

This equation can be rewritten as

\[ \frac{1}{K_1} + \frac{(K_2 / K_1^2)S}{(R_1^2 + X^2)S^2 + (2R_1R_2 - K_2/K_1)S + R_2^2} \]  

From equations (4-9), (4-10), and (4-12)

\[ \frac{1}{K_1} + \frac{(K_2 / K_1^2)S}{(S - S_1)(S - S_2)} \]  

where \( S_1 \) and \( S_2 \) are the same as those of equation (4-10).

By partial fraction expansion equation (4-13) becomes

\[ \frac{1}{K_1} + \frac{K_3}{S - S_1} + \frac{K_4}{S - S_2} \]  

(4-14)
where

\[ K_3 = \frac{K_2 S_1}{K_1^2 (S_2 - S_1)}, \quad K_4 = \frac{K_2 S_2}{K_1^2 (S_2 - S_1)} \]  \hspace{1cm} (4-15)

The initial conditions of equation (4-14) are at \( t=0, S=S_0 \)

By integrating both sides of equation (4-14)

\[ \frac{1}{K_1} (S - S_0) - K_3 \ln \left[ \frac{S - S_1}{S_0 - S_1} \right] + K_4 \ln \left[ \frac{S - S_2}{S_0 - S_2} \right] = t \]  \hspace{1cm} (4-16)

Equation (4-16) can be written as

\[ \frac{(S - S_0)}{K_1} + \ln \left[ \frac{S - S_2}{S_0 - S_2} \right]^{K_4} \left[ \frac{S_0 - S_1}{S - S_1} \right]^{K_3} = t \]  \hspace{1cm} (4-17)

This is the equation of motion of the induction motor during its motion from an initial slip \( S_0 \) to any other slip \( S \) due to a change in the load torque.

From equation (4-17) it is clear that the slip is an exponential function of time. Therefore, the induction motor does not have any overshoot or oscillations during its transient period.

Also, equation (4-17) can be used to calculate the transition time from any slip to another slip. To calculate the starting time, just let \( S_0 = 1 \), and \( S = S_1 + \varepsilon \) (which
is the steady state operating point), where $\epsilon$ is a very small positive number.

Figure (4-1) shows the slip as a function of the time when the load torque is increased suddenly from its rated value to 1.5 times the rated value.
Fig (4-1) Induction motor motion after sudden change in load torque
CHAPTER 5.
Conclusions

This thesis opens the way for further research to be done in the operation of induction motors with a series capacitor. The theoretical analysis showed that it is possible to get an extra-high starting torque with limited starting current, and a highly stable machine due to the very high pull-out torque. However, the experimental work had many problems which should be resolved as further research.

The experimental work should be done on a slip-ring induction motor in order to be able to measure the rotor current which will help in explaining the dynamics of the resonance problem. By measuring rotor and stator currents, the magnetization current \( I_m \) can be calculated as \( I_m = I_s - I_r \). Then the magnetization reactance \( X_m \) will be known from table (2-2). From equations (2-10) and (2-12), the value of the capacitive reactance can be calculated as \( X_1 + X \).
REFERENCES


